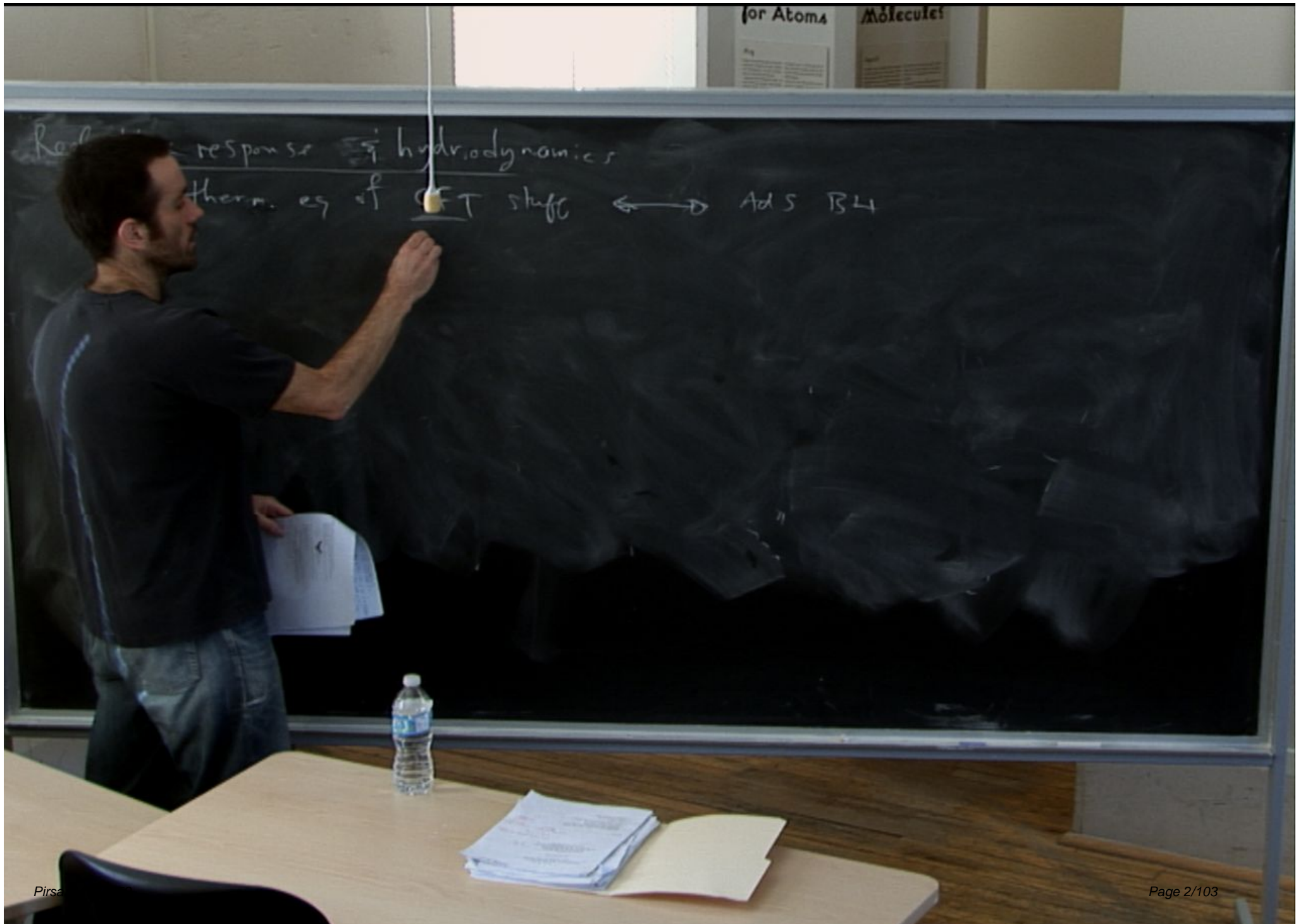


Title: Explorations in String Theory - Lecture 10

Date: Mar 25, 2011 11:30 AM

URL: <http://pirsa.org/11030059>

Abstract:



Real-time response  $\approx$  hydrodynamics

therm. eq of FT stuff

T



AdS BH



location of horizon

$z = z_H$

Real-time response  $\equiv$  hydrodynamics

therm. eq of CFT stuff

$T$



AdS BH



location of horizon

$$z = z_H$$

therm. eq of CFT  $\dots$

$\equiv$  hydrodynamics



slowly-varying def.

of AdS BH

$$z = z_H(t, \vec{x})$$

Real-time response  $\approx$  hydrodynamics

therm. eq of CFT stuff  $\longleftrightarrow$  AdS BH  
T  $\longleftrightarrow$  location

local therm. eq of CFT  $\dots$   
 $\equiv$  hydrodynamics  $\longleftrightarrow$  slowly-var of AdS

(Janik-Pesl  
Bhattacharya)

$$z = z_H$$

$$= z_H(t, \vec{x})$$



Real-time response  $\equiv$  hydrodynamics

therm. eq of  $\rho, T$  stuff

$\longleftrightarrow$  AdS BH

location of horizon  $z = z_H$

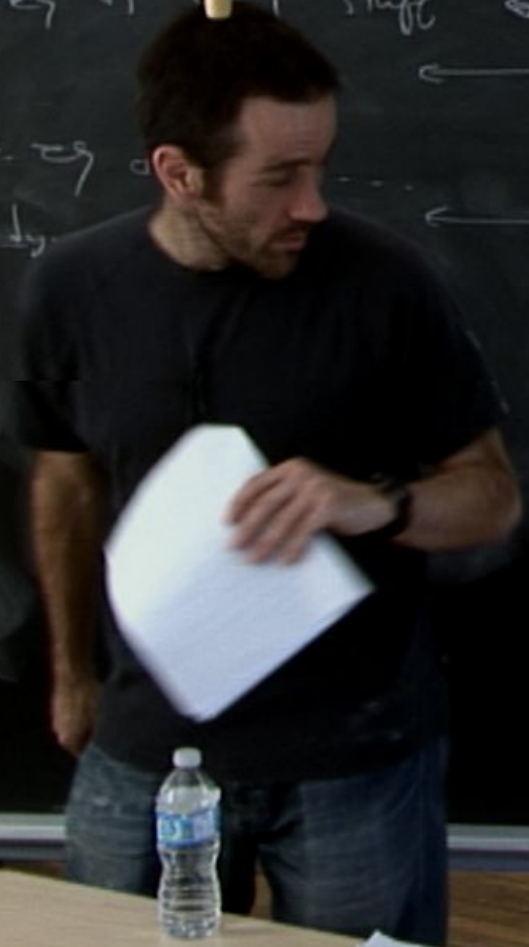
local therm. eq  $\equiv$  hydrody

$\longleftrightarrow$

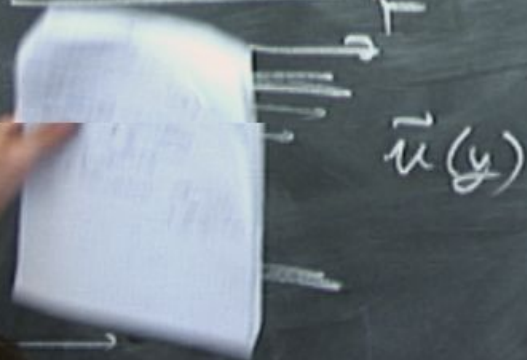
slowly-varying def. of AdS BH

$$z = z_H(t, \vec{x})$$

(Janik-Peschke-Rhattacharyya et al)



viscosity

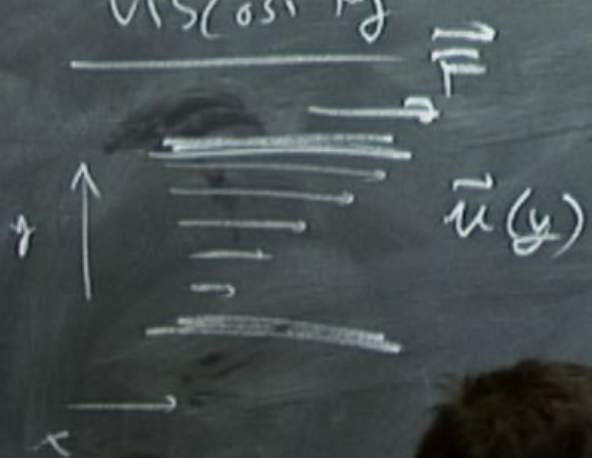


apply force  
measure vel. gradient

$$\vec{F} = \left( \begin{array}{c} \text{area} \\ \text{of plates} \end{array} \right)$$

viscosity

apply force  
measure vel. gradient



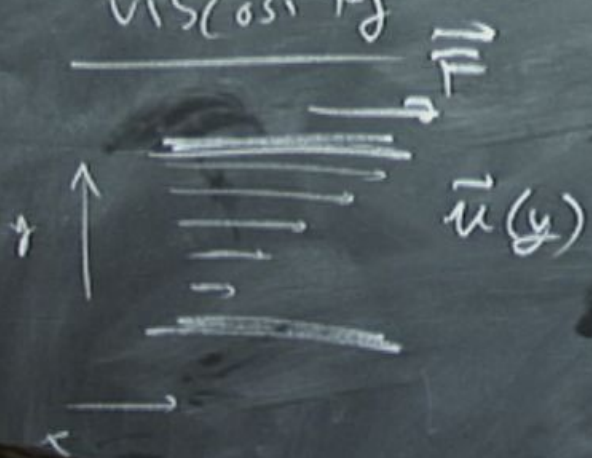
$$\vec{F} = \left( \begin{array}{l} \text{area} \\ \text{of plates} \end{array} \right)$$



viscosity

apply force

measure vel. gradient



$$\vec{F} = (\eta) (\text{area of plates}) \frac{d\vec{u}}{dy}$$

↑  
≡ viscosity.

viscosity



apply force  
measure vel. gradient

$$\vec{F} = (\eta) (\text{area of plates}) \frac{d\vec{u}}{dy}$$

↑  
≡ viscosity.

Stress:

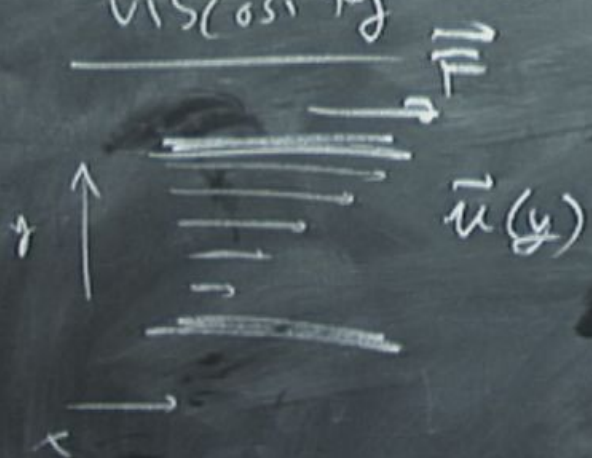
$$\tau_{xy}$$

response:  $\tau_{yx}$

viscosity

apply force

measure vel. gradient



$$\vec{F} = (\eta) (\text{area of plates}) \frac{d\vec{u}}{dy}$$

$\eta \equiv \text{viscosity}$

source:  $T^x$

response

i.e.  $\Delta H = \int_{\text{sp}}$

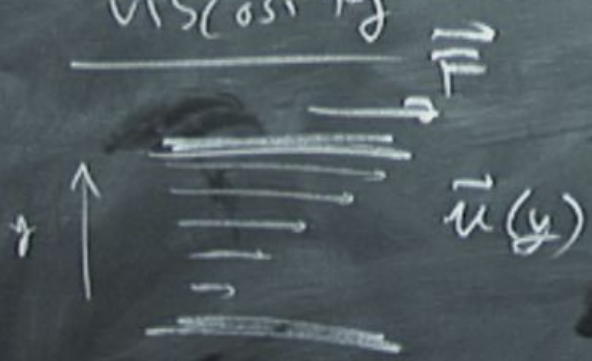
$$\langle T^x_y \rangle = i\omega \gamma$$

$$= \lim_{\omega \rightarrow 0} \frac{\langle T^x_y \rangle}{i\omega}$$

viscosity

apply force

measure vel. gradient



$$\vec{F} = (\eta) (\text{area of plates}) \frac{d\vec{u}}{dy}$$

↑  
≡ viscosity.

force:  $T_x$

$y$

response:  $T_y$

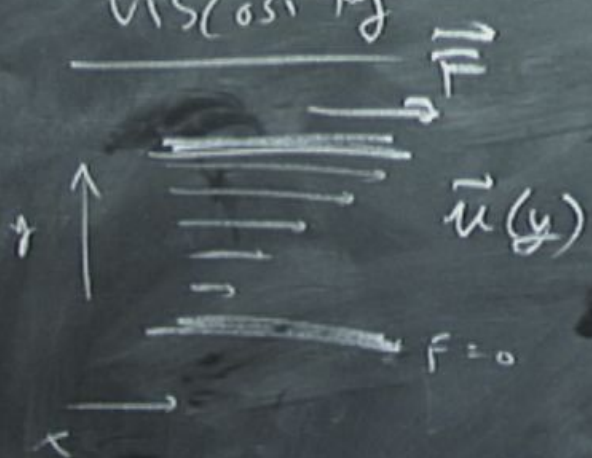
ie:  $\delta H = \int \delta y T_x$

↑  
spec.

cube:

$$T_y = i\omega \eta \delta y \Rightarrow \eta = \lim_{\omega \rightarrow 0} \frac{\frac{R}{T_y T_x} (T_x)}{i\omega}$$

viscosity



apply force  
measure vel. gradient

$$\vec{F} = (\eta) (\text{area of plates}) \frac{d\vec{u}}{dy}$$

↑  
≡ viscosity.

source:  $T_x^x$

response:  $T_y^x$

i.e.  $\delta H = \int \delta y T_x^x$

$$\langle T_y^x \rangle = i\omega \eta \delta y^x \Rightarrow \eta = \lim_{\omega \rightarrow 0} \frac{\langle T_y^x \rangle}{i\omega \delta y^x}$$

Kubo:

$$\eta = \lim_{\omega \rightarrow 0} \frac{\langle T_y^x \rangle}{i\omega \delta y^x}$$

very general bulk metric:

$$ds^2 = g_{tt} dt^2 + g_{zz} dz^2 + g_{ij} dx^i dx^j$$

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s.t.:  $g_{AB} = g_{AB}(z)$

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s.t. 1)  $g_{AB} = g_{AB}(z)$

2)  $\rightarrow$  AdS near  $z \rightarrow 0$ .



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[holographic  
interpretation]

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3)  $g_{tt} \xrightarrow{z \rightarrow z_H} -2K(z_H - z)$

$$g_{zz} \xrightarrow{z \rightarrow z_H} \frac{1}{2K(z_H - z)}$$

[ holographic interpretation ]

very general bulk metric:

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[holographic interpretation]

Rindler horizon

very general bulk metric:

$$ds^2 = g_{tt} dt^2 + g_{zz} dz^2 + g_{ij} dx^i dx^j$$

s.t. : 1)  $g_{AB} = g_{AB}(z)$

2)  $\rightarrow$  AdS near  $z \rightarrow 0$ .

3)  $\xrightarrow{z \rightarrow z_H} -2K(z_H - z)$   
 $\xrightarrow{\quad} \frac{1}{2K(z - z_H)}$

[holographic interpretation]

Rindler horizon

Consider

$$\frac{1}{z} \int dz \sqrt{g} (\partial\phi)^2 \frac{1}{g(z)}$$

very general bulk metric:

$$ds^2 = g_{tt} dt^2 + g_{zz} dz^2 + g_{ij} dx^i dx^j$$

s.t. : 1)  $g_{AB} = g_{AB}(z)$

2)  $\rightarrow$  AdS near  $z \rightarrow 0$ .

3)  $g_{tt} \xrightarrow{z \rightarrow z_H} -2K(z_H - z)$   
 $g_{zz} \xrightarrow{z \rightarrow z_H} \frac{1}{2K(z_H - z)}$

[holographic interpretation]

Rindler horizon

Consider:

$$S = -\frac{1}{2} \int dt \int d^3x \sqrt{g} (\partial\phi)^2$$

$\phi \leftrightarrow \langle O \rangle$   
 eg  $\phi = \frac{1}{\sqrt{2}} \psi$

very general bulk metric:

$$ds^2 = g_{tt} dt^2 + g_{zz} dz^2 + g_{ij} dx^i dx^j$$

s.t. : 1)  $g_{AB} = g_{AB}(z)$

2)  $\rightarrow$  AdS near  $z \rightarrow 0$ .

3)  $g_{tt} \xrightarrow{z \rightarrow z_H} -2K(z_H - z)$   
 $g_{zz} \xrightarrow{z \rightarrow z_H} \frac{1}{2K(z_H - z)}$

[holographic interpretation]

Rindler horizon

Consider:

$$S = -\frac{1}{2} \int d^4x \sqrt{g} \left[ (\partial\phi)^2 + \frac{1}{g(z)} \right]$$

$\phi \leftrightarrow 0$   
 eg.  $\phi = \frac{1}{2} \ln \frac{x}{y}$   
 $0 = \frac{1}{2} \ln \frac{x}{y}$

very general bulk metric:

$$ds^2 = g_{tt} dt^2 + g_{zz} dz^2 + g_{ij} dx^i dx^j$$

s.t. : 1)  $g_{AB} = g_{AB}(z)$

2)  $\rightarrow$  AdS near  $z \rightarrow 0$ .

3)  $\left. \begin{aligned} g_{tt} &\xrightarrow{z \rightarrow z_H} -2K(z_H - z) \\ g_{zz} &\xrightarrow{\quad} \frac{1}{2K(z - z_H)} \end{aligned} \right\}$

Consider:

$$S = -\frac{1}{2} \int dt \int d^3x \sqrt{g} \left[ (\partial\phi)^2 + \frac{1}{g(z)} \right]$$

eg.  $\phi = \frac{1}{\sqrt{g}}$   
 $\partial\phi = -\frac{1}{2} \frac{g'}{g^2}$   
 $\partial\phi^2 = \frac{1}{4} \frac{g'^2}{g^4}$

$$g = 16\pi G_N$$

very general bulk metric:

$$ds^2 = g_{tt} dt^2 + g_{zz} dz^2 + g_{ij} dx^i dx^j$$

s.t. : 1)  $g_{AB} = g_{AB}(z)$

2)  $\rightarrow$  AdS near  $z \rightarrow 0$ .

3)  $\left. \begin{aligned} g_{tt} &\xrightarrow{z \rightarrow z_H} -2K(z_H - z) \\ g_{zz} &\xrightarrow{\quad} \frac{1}{2K(z_H - z)} \end{aligned} \right\}$

$$q = 16\pi G_N$$

Consider:

$$S = -\frac{1}{2} \int dt \int d^4x \sqrt{g} \left[ (\partial\phi)^2 + \frac{1}{g(z)} \right]$$

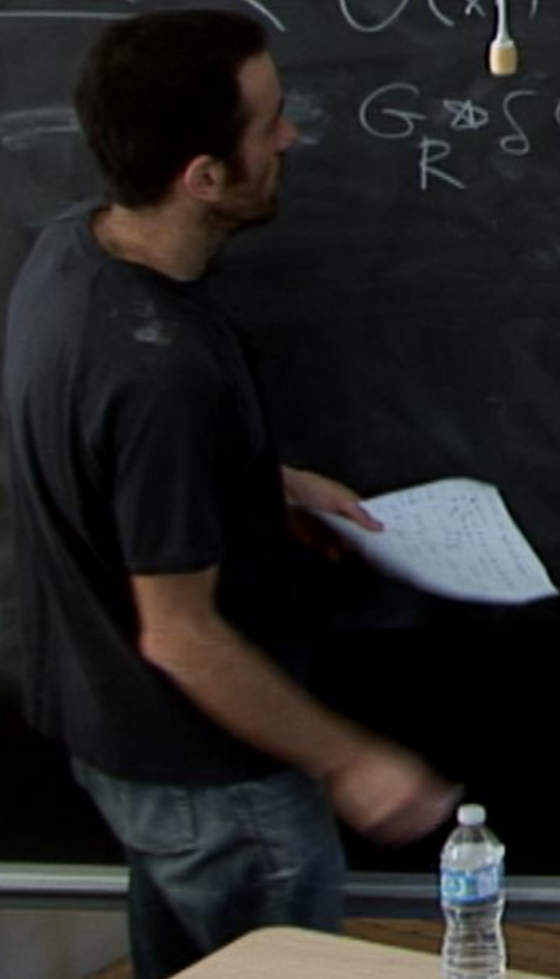
$$m = 0$$

eg.  $\phi = \frac{1}{\sqrt{g}}$   
 $0 =$



Recall:  $\langle \mathcal{O}(x) \rangle_{\text{CFT}} = \lim_{z \rightarrow 0} \Pi_{\phi}(x, z)$

$G \otimes_{\mathbb{R}} \mathcal{S}\phi$

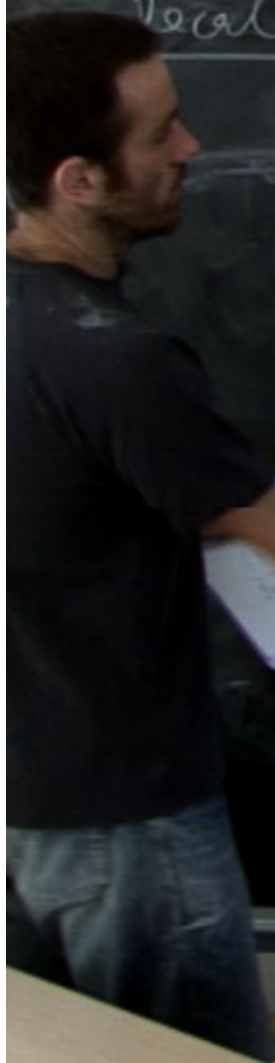


for Atoms

Molecules

Recall:  $\langle \psi(x) \rangle_{\text{CFT}} = \lim_{z \rightarrow 0} \Pi_{\phi}(x, z)$

$= G_{\mathbb{R}}^{\Delta} \psi$



Recall:  $\langle \mathcal{O}(x) \rangle_{\text{CFT}} = \lim_{z \rightarrow 0} \Pi_{\phi}(x, z)$

$= G \star \phi$   
 $R$

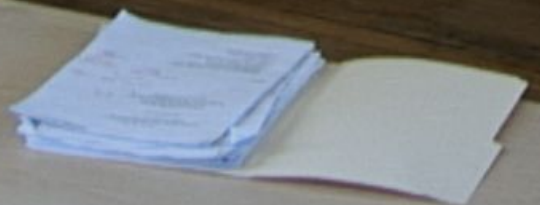
$$\eta = \lim_{z \rightarrow 0} \left[ \frac{\Pi(z, k_{\mu})}{\omega \phi(z, k_{\mu})} \right]$$



Recall:  $\langle \mathcal{O}(x) \rangle_{\text{CFT}} = \lim_{z \rightarrow 0} \Pi_{\phi}(x, z)$

$= G \otimes_{\mathbb{R}} \phi$

$\eta = \lim_{\substack{z \rightarrow 0 \\ k \rightarrow 0 \\ \text{THEN} \\ w \rightarrow 0}} \left[ \frac{\Pi(z, k, w)}{w \phi(z, k, w)} \right]$



Recall:  $\langle \mathcal{O}(x) \rangle_{\text{CFT}} = \lim_{z \rightarrow 0} \Pi_{\phi}(x, z)$

$\equiv G \otimes_{\mathbb{R}} \phi$

$\eta = \lim_{\substack{z \rightarrow 0 \\ k \rightarrow 0 \\ \text{THEN} \\ w \rightarrow 0}} \left[ \frac{\Pi(z, k, w)}{w \phi(z, k, w)} \right]$

$\Pi_{\phi} = \frac{\partial^2 \mathcal{L}}{\partial (\partial \phi)^2}$   
 $= \frac{1}{g} \sqrt{g} g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi$



Recall:  $\langle \mathcal{O}(x) \rangle_{\text{CFT}} = \lim_{z \rightarrow 0} \Pi_{\phi}(x, z)$

$= G \otimes_{\mathbb{R}} \phi$

$\eta = \lim_{\substack{z \rightarrow 0 \\ k \rightarrow 0 \\ \text{THEN} \\ w \rightarrow 0}} \left[ \frac{\Pi(z, k, w)}{w \phi(z, k, w)} \right]$

$\Pi_{\phi} = \frac{\delta \mathcal{L}}{\delta \phi}$   
 $= \frac{1}{g} \sqrt{g} g^{\mu\nu} \partial_{\mu} \phi$



two steps

two steps

1) near horizon:  $\phi(z) \approx (z - z_+)^{\pm i\omega/4\pi T}$



two steps

1) near horizon:  $\phi(z) \sim (z - z_H)$

$$\oplus i\omega / 4\pi T$$

$$\phi(t, z) = e^{-i\omega t} \phi(z)$$

$$i\omega / 4\pi T$$

at horizon:  $\Gamma_{\phi}(z_H, k) = \frac{1}{g(z_H)} \sqrt{\frac{|g|}{g_{zz} |g_H|}}$

two steps

1) near horizon:  $\phi(z) \approx (z - z_H)$

$$\oplus i\omega / 4\pi T$$

$$i\omega / \alpha r$$

$$\psi(t, z) = e^{-i\omega t} \phi(z)$$

at horizon

$$\pi / \left( g(z_H) \sqrt{\frac{|g|}{g_{zz} |g_H|}} \right)$$

two steps

1) near horizon:  $\phi(z) \approx (z - z_H)$

$\oplus i\omega/4\pi T$

$i\omega/4\pi T$

$\phi(t, z) = e^{-i\omega t} \phi(z)$

horizon

$\Pi_\phi(z_H, k) = \frac{1}{g(z_H)} \sqrt{\frac{|g|}{g_{zz}|g_{tt}|}}$

$i\omega \phi$

2) p

eqn:  $\partial_z \Pi_\phi = \left( \dots + k_\mu k_\nu g^{\mu\nu} \right) \phi$   
 $\omega, k \rightarrow 0$

two steps

1) near horizon:  $\phi(z) \approx (z - z_H)$

$$\oplus i\omega / 4\pi T$$

$$\phi(t, z) = e^{-i\omega t} \phi(z)$$

$i\omega / 4\pi T$

at horizon:  $\Pi_\phi(z_H, k) = \frac{1}{g(z_H)} \sqrt{\frac{|g|}{g_{zz} |g_H|}} i\omega \phi$

2) propagate to bdy

eqn:  $\partial_z \Pi_\phi = \left( \dots, + \frac{h_{\mu\nu} k_\nu}{g_{zz}} \right) \phi$

$$\partial_z(\phi \omega) = \frac{g}{\sqrt{g_{zz}}} \omega \Pi_\phi$$

$\omega, k \rightarrow 0$   $\bigcirc$

two steps

1) near horizon:  $\phi(z) \sim (z - z_H)$

$\oplus i\omega / 4\pi T$

$i\omega / 4\pi T$

$\psi(t, z) = e^{-i\omega t} \phi(z)$

at horizon:  $\Pi_\phi(z_H, k) = \frac{1}{g(z_H)} \sqrt{\frac{|g|}{g_{zz} |g_H|}} i\omega \phi$

2) propagate to bdy

eqn:  $\partial_z \Pi_\phi = \left( \dots + \frac{h_{\mu\nu} k_\mu k_\nu}{g_{zz}} \right) \phi$

$\partial_z(\phi\omega) = \frac{g}{\sqrt{g_{zz}}} \omega \Pi_\phi$

$\omega \rightarrow 0$   
 $\omega \phi$  fixed

$$\frac{\prod \phi}{\omega \phi} \Big|_{z=0} = \frac{\prod \phi}{\omega \phi} \Big|_{z=z_H}$$

$$z=z_H$$

$$\frac{\prod \phi}{\omega \phi} \Big|_{z=0} = \frac{\prod \phi}{\omega \phi} \Big|_{z=z_H}$$

$z=z_H$

$$\frac{\prod \phi}{\omega \phi} \Big|_{z=0} = \frac{\prod \phi}{\omega \phi} \Big|_{z=z_H}$$

"membrane  
paradigm"

$z=z_H$



$$\frac{\Pi_\phi}{w\phi} \Big|_{z=0} = \frac{\Pi_\phi}{w\phi} \Big|_{z=z_H}$$

"membrane  
paradigm"

$$\Rightarrow \eta = \frac{1}{g(z_H)} \sqrt{\frac{|g|}{g_{zz}|g_H|}}$$

entropy density :  $s = \frac{a}{4G_N} = \frac{1}{4G_N} \sqrt{\frac{|g|}{g_{zz}|g_H|}}$

$$\left. \frac{\Pi_\phi}{w\phi} \right|_{z=0} = \left. \frac{\Pi_\phi}{w\phi} \right|_{z=z_H}$$

"membrane paradigm"

$$\Rightarrow \eta = \frac{1}{16\pi G_N} \sqrt{\frac{|g|}{g_{zz}|g_{tt}|}}$$

entropy density:  $s = \frac{a}{4G_N} = \frac{1}{4G_N} \sqrt{\frac{|g|}{g_{zz}|g_{tt}|}}$

$$\eta =$$

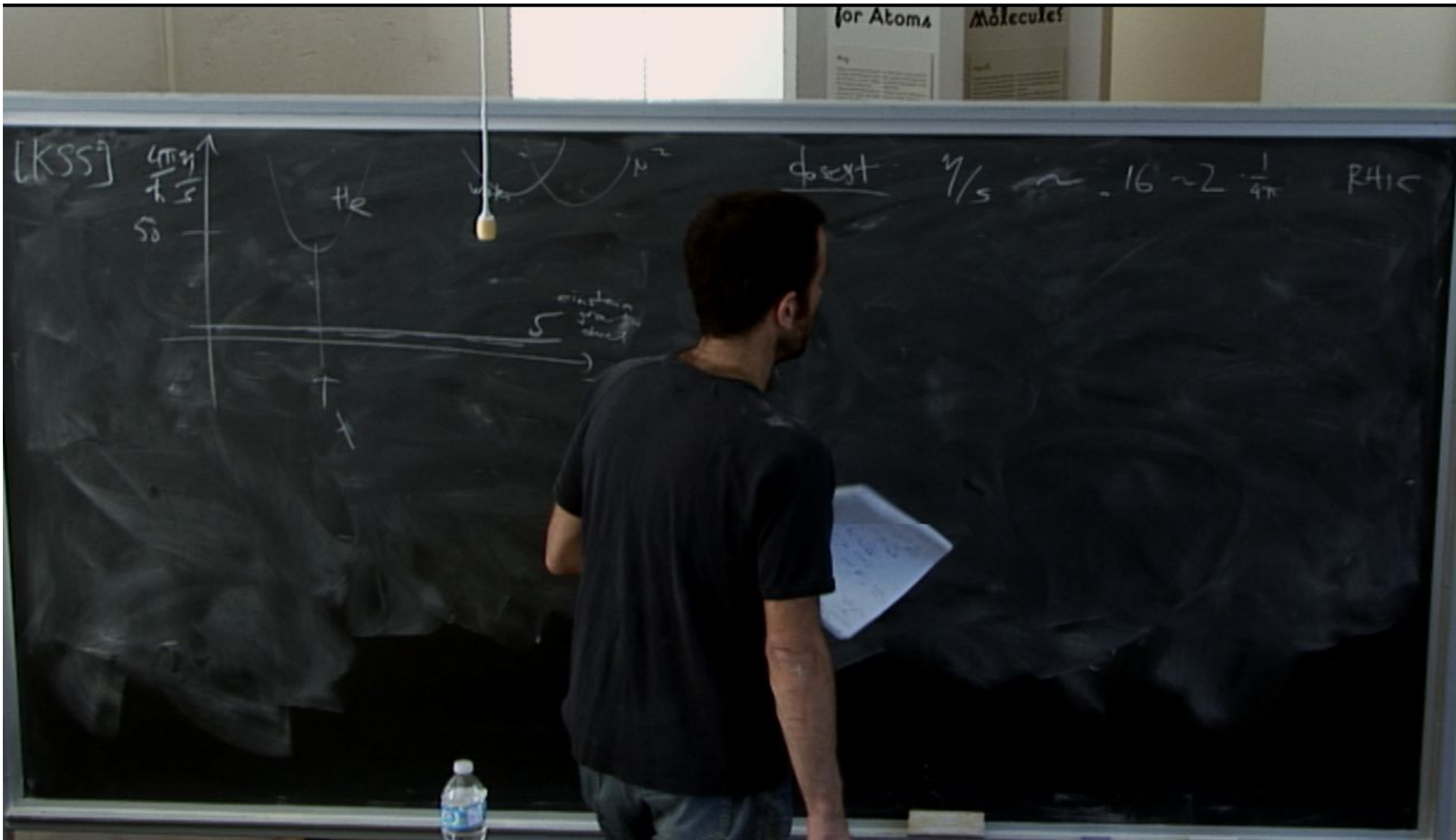
$$\left. \frac{\pi_\phi}{\omega \phi} \right|_{z=0} = \left. \frac{\pi_\phi}{\omega \phi} \right|_{z=z_H}$$

"membrane"  
paradigm

$$\Rightarrow \eta = \frac{1}{16\pi G_N} \sqrt{\frac{|g|}{g_{zz}|g_{tt}|}}$$

entropy density:  $S = \frac{a}{4G_N} = \frac{1}{4G_N} \sqrt{\frac{|g|}{g_{zz}|g_{tt}|}}$

$$\eta = \frac{1}{4\pi} S \sim 0.08 S$$



[KSS]

$\frac{4\pi\epsilon_0}{h^2}$   
50

He

$N^2$

$\phi_{ext}$

$\frac{7}{5}$

$16 \sim 2 \cdot \frac{1}{4\pi}$

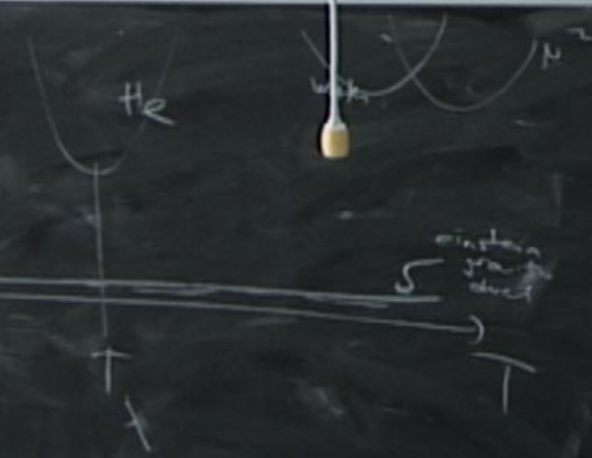
R41C

for Atoms

Molecules

[KSS]

$\frac{4\pi m \omega}{\hbar^2}$   
50



$\phi_{\text{ext}} \quad \eta/s \sim 16 \sim 2 \cdot \frac{1}{4\pi} \quad R_{410}$   
 $\sim \frac{1}{2}$   
 Unitary fermions



[KSS]  $\frac{4\pi\hbar^2}{h^2} \uparrow$   
 $50$  He  $\psi^2$   $N^2$

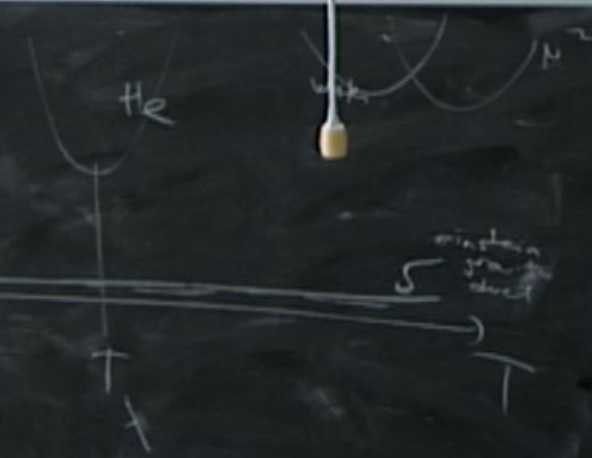
$\psi_{\text{ext}} \quad \eta/5 \sim 16 \sim 2 \cdot \frac{1}{4\pi} \quad \text{R41C}$   
 $\sim \frac{1}{2} \quad \text{unitary fermions}$

$\psi_5$ : weakly coupled fields  $\eta/5 \sim \frac{1}{2} \log 1 \rightarrow 1$   
 elliptic flow.



[KSS]

$$\frac{4\pi\hbar^2 n}{h^2} \approx \frac{50}{\dots}$$



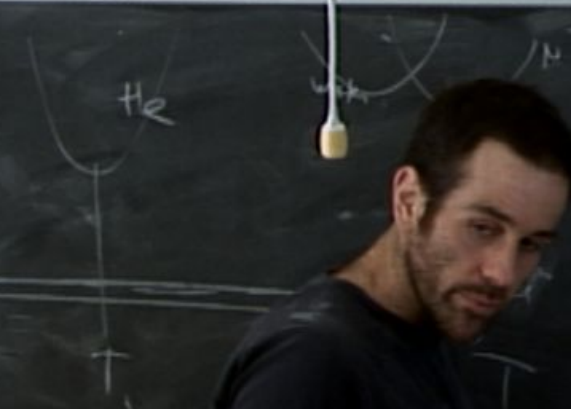
$\phi_{\text{ext}} \quad \eta/5 \sim 16 \sim 2 \cdot \frac{1}{4\pi} \quad \text{RHIC}$   
 $\sim \frac{1}{2}$   
 unitary fermions

$\nu/5 =$  weakly coupled fields :  $\eta/5 \sim \frac{1}{2} \log \dots$   
 elliptic flow

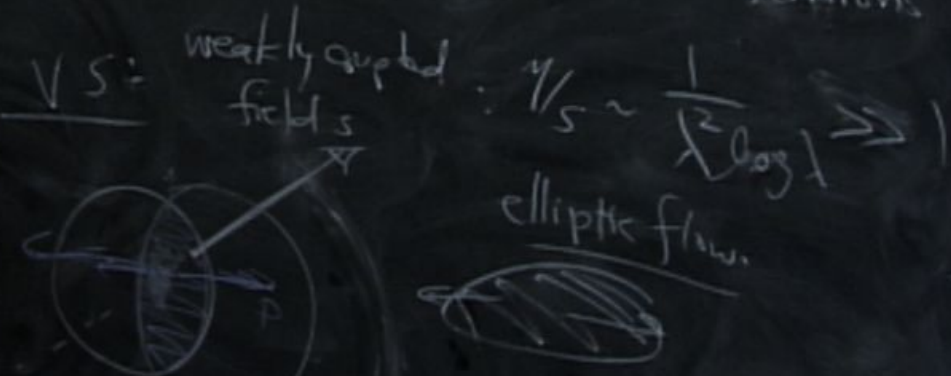


[KSS]

$\frac{4\pi\hbar^2}{h^2} \frac{1}{5}$

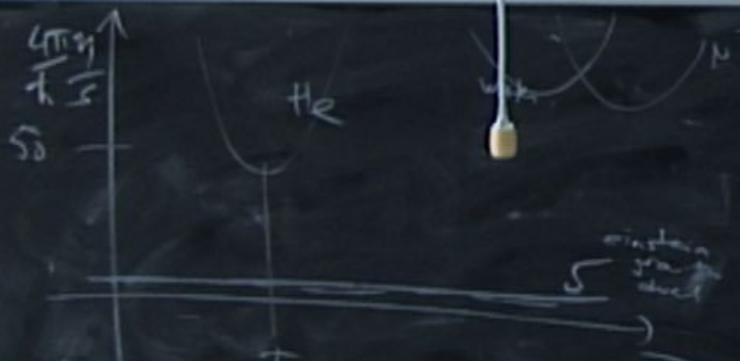


$\chi_{\text{ext}} \quad \chi/5 \sim 16 \sim 2 \cdot \frac{1}{4\pi} \quad R_{410}$   
 $\sim \frac{1}{2}$   
unitary fermions





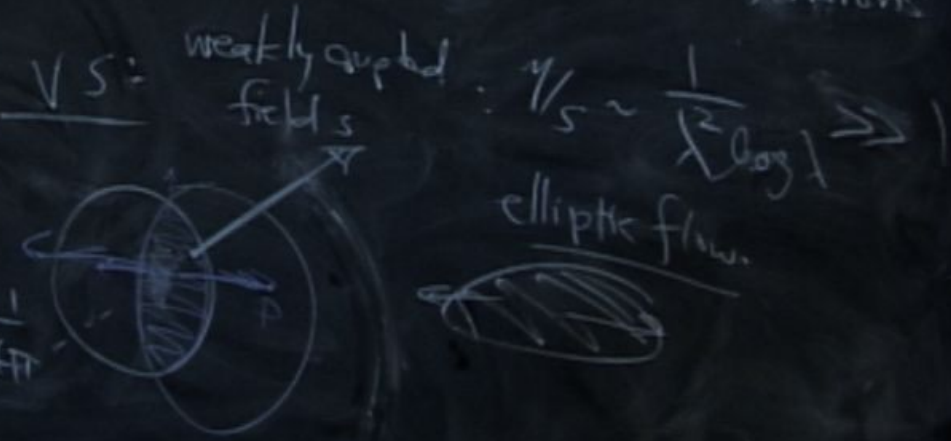
[KSS]



$\chi_{\text{ext}} \quad \chi/s \sim 16 \sim 2 \cdot \frac{1}{4\pi} \quad R_{410}$   
 $\sim \frac{1}{2}$   
 unitary fermions

NET A BOARD

$$\frac{1}{5} = \frac{1}{4\pi g(z_H)} = \frac{1-4\lambda}{4\pi} \approx \frac{16}{25} \approx \frac{1}{4}$$



[KSS]

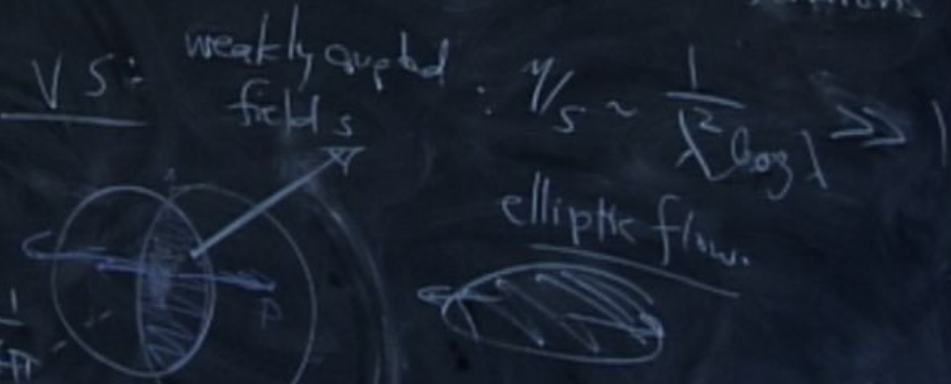


$\phi_{\text{ext}} \quad \gamma/5 \sim 16 \sim 2 \cdot \frac{1}{4\pi} \quad \text{RHIC}$   
 $\sim \frac{1}{2}$   
 unitary fermions

NET A BOARD

$$\frac{\gamma}{5} = \frac{1}{4\pi g(z_H)} = \frac{1-4\lambda}{4\pi} \approx \frac{16}{25} \cdot \frac{1}{4\pi}$$

[Myers et al]



# Finite chemical potential

# Finite chemical potential

---

suppose  $\exists$  U(1) current  $J_\mu$

$\longrightarrow$  massless gauge field  $A_\mu$   
in bulk

$$\Delta S_{\text{bulk}} = -\frac{1}{4g^2} \int d^{d+1}x \sqrt{|g|} F_{AB} F^{AB}$$

# Finite chemical potential

$$T_{AB} = \partial_A A_B - \partial_B A_A$$

suppose  $\exists$  U(1) current  $J_M$

$\rightarrow$  massless gauge field  $A_M$   
in bulk

Wilson

$$S_{\text{eff}} = -\frac{1}{4g^2} \int d^{d+1}x \sqrt{g} F_{AB} F^{AB}$$

max eq  
 $\delta = \frac{\delta}{\delta A}$

$$A \sim_{z \rightarrow 0} A^{(0)}(x) + \left(\frac{z}{L}\right)^{d-2} A^{(1)}(x)$$

# Finite chemical potential

$$T_{AB} = \partial_A A_B - \partial_B A_A$$

suppose  $\exists$  U(1) current  $J_M$

$\rightarrow$  massless gauge field  $A_M$   
in bulk

Wilson

$$\Delta S_{\text{bulk}} = -\frac{1}{4g^2} \int d^{d+1}x \sqrt{g} F_{AB} F^{AB}$$

$\Rightarrow$   $A \underset{z \rightarrow 0}{\sim} A^{(0)}(x) + \left(\frac{z}{L}\right)^{d-2} A^{(1)}(x)$

# Finite chemical potential

$$T_{AB} = \partial_A A_B - \partial_B A_A$$

suppose  $\exists$  U(1) current  $J_M$

$\rightarrow$  massless gauge field  $A_M$   
in bulk

$$\Delta S_{\text{bulk}} = -\frac{1}{4g^2} \int d^{d+1}x \sqrt{g} F_{AB} F^{AB}$$

$$\Rightarrow A \underset{z \rightarrow 0}{\sim} A^{(0)}(x) + \left(\frac{z}{L}\right)^{d-2} A^{(1)}(x)$$

$$\delta S = \int J + M$$

# Finite chemical potential

$$T_{AB} = \partial_A A_B - \partial_B A_A$$

suppose  $\exists$  U(1) current  $J_M$

$\longrightarrow$  massless gauge field  $A_M$   
in bulk

$$\Delta S_{\text{bulk}} = -\frac{1}{4g^2} \int d^{d+1}x \sqrt{g} F_{AB} F^{AB}$$

$q^M \Rightarrow$

$$A \underset{z \rightarrow 0}{\sim} \underline{\underline{A^{(0)}(x)}} + \left(\frac{z}{L}\right)^{d-2} \underline{\underline{A^{(1)}(x)}}$$

$$\delta S = \int J + M$$



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$\delta S_A$  eqn  $\Rightarrow$

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Wilson

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max eqn  $\Rightarrow$   $A \sim \underset{z \rightarrow 0}{A^{(0)}(x)} + \left(\frac{z}{L}\right)^{d-2} A^{(1)}(x)$

$\partial =$

$$\frac{\delta S}{\delta A_A}$$

$$\delta S = \int J + M$$

$$M + \left(\frac{z}{L}\right)^{d-2} (P)$$

$$\int \frac{\delta S}{\delta A} = P$$

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 $\delta = \frac{\delta S}{\delta A_A}$

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Saddle pt w/ these BC :

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$f(z)$

Saddle pt w/ these BC: charged BH (AdS-RN)

$$f(z) = 1 - Mz^d + Qz^{2d-2}$$

$$A = dt A_t = dt \mu \left( 1 - \left( \frac{z}{z_0} \right)^{d-2} \right)$$

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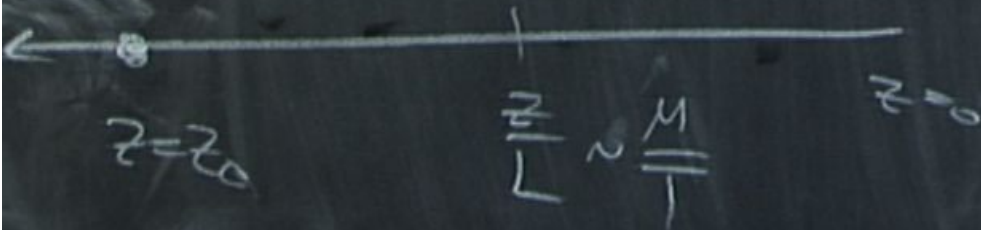
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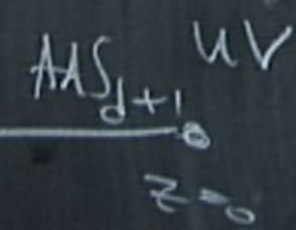


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$$T \ll \mu$$

$$\mu \approx \frac{M}{L}$$

saddle pt w/ these BC: charged BH (AdS-RN)

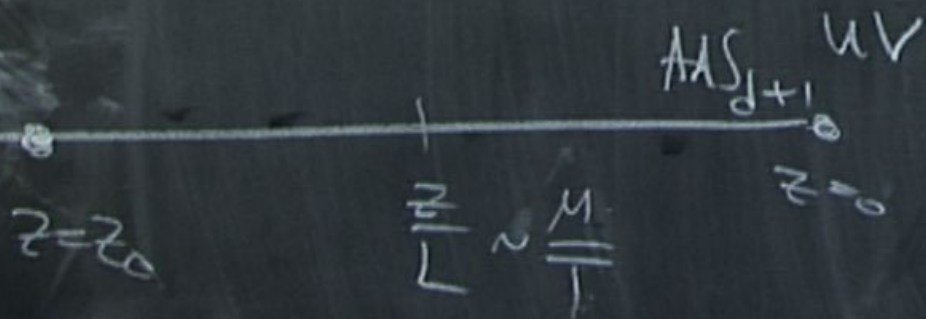
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IR



$T \ll \mu$ :

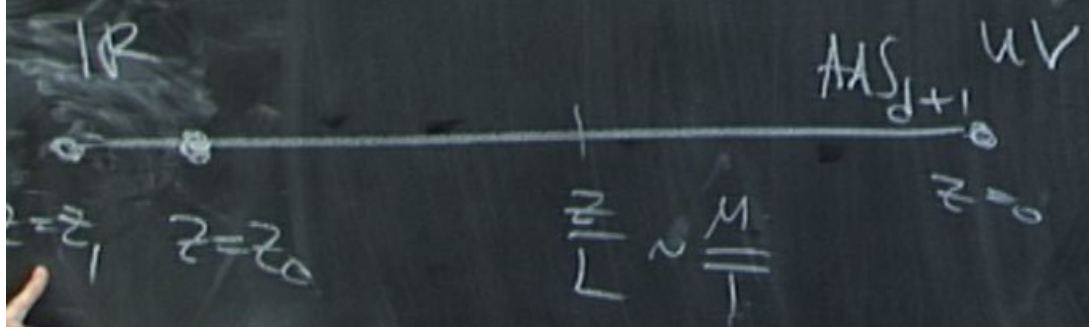
saddle pt w/ these BC: charged BH (AAS-RN)

$$f(z) = 1 - Mz^d + Qz^{2d-2}$$

$$A = dt A_t = dt \mu \left( 1 - \left( \frac{z}{z_0} \right)^{d-2} \right)$$

$$f(z_0) = 0$$

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$T \ll \mu$

saddle pt w/ these BC: charged BH (AdS-RN)

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$$f(z_0) = 0$$

$$Q = \frac{\mu L^2}{z_0 g_F}$$

IR

AdS<sub>d+1</sub> UV  
z=0

$$T \ll \mu : z_1 \rightarrow z_0$$

$$f \sim a(z-z_0)^2 + b(z-z_0)$$

$$\frac{a(z-z_0) dt^2 + dz^2}{(z-z_0)^2}$$

saddle pt w/ these BC: charged BH (AdS-RN)

$$f(z) = 1 - Mz^d + Qz^{2d-2}$$

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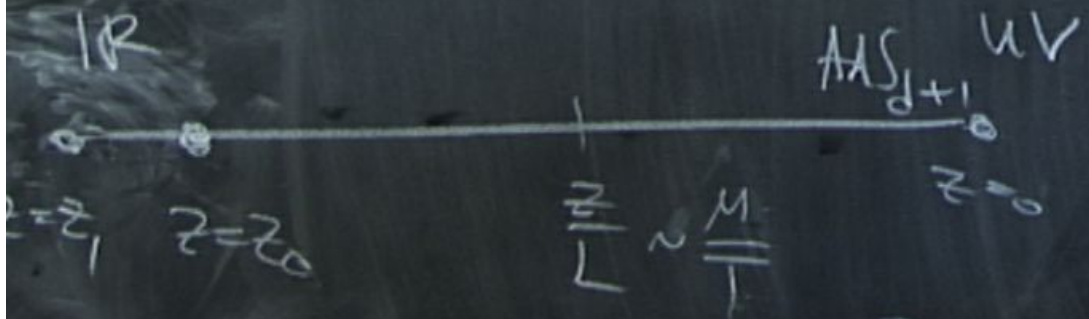


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$T \ll \mu : z_1 \sim \dots$

$$f \sim a/(z - z_0)^2$$

$$ds^2 = -a(z - z_0) dt^2 + \frac{dz^2}{(z - z_0)^2} + dx^2$$

saddle pt w/ these BC: charged BH (AdS-RN)

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IR  $z=0$   $z=z_0$   $z=z_1$   $z=z_2$   $z=z_3$   $z=z_4$   $z=z_5$   $z=z_6$   $z=z_7$   $z=z_8$   $z=z_9$   $z=z_{10}$   $z=z_{11}$   $z=z_{12}$   $z=z_{13}$   $z=z_{14}$   $z=z_{15}$   $z=z_{16}$   $z=z_{17}$   $z=z_{18}$   $z=z_{19}$   $z=z_{20}$   $z=z_{21}$   $z=z_{22}$   $z=z_{23}$   $z=z_{24}$   $z=z_{25}$   $z=z_{26}$   $z=z_{27}$   $z=z_{28}$   $z=z_{29}$   $z=z_{30}$   $z=z_{31}$   $z=z_{32}$   $z=z_{33}$   $z=z_{34}$   $z=z_{35}$   $z=z_{36}$   $z=z_{37}$   $z=z_{38}$   $z=z_{39}$   $z=z_{40}$   $z=z_{41}$   $z=z_{42}$   $z=z_{43}$   $z=z_{44}$   $z=z_{45}$   $z=z_{46}$   $z=z_{47}$   $z=z_{48}$   $z=z_{49}$   $z=z_{50}$   $z=z_{51}$   $z=z_{52}$   $z=z_{53}$   $z=z_{54}$   $z=z_{55}$   $z=z_{56}$   $z=z_{57}$   $z=z_{58}$   $z=z_{59}$   $z=z_{60}$   $z=z_{61}$   $z=z_{62}$   $z=z_{63}$   $z=z_{64}$   $z=z_{65}$   $z=z_{66}$   $z=z_{67}$   $z=z_{68}$   $z=z_{69}$   $z=z_{70}$   $z=z_{71}$   $z=z_{72}$   $z=z_{73}$   $z=z_{74}$   $z=z_{75}$   $z=z_{76}$   $z=z_{77}$   $z=z_{78}$   $z=z_{79}$   $z=z_{80}$   $z=z_{81}$   $z=z_{82}$   $z=z_{83}$   $z=z_{84}$   $z=z_{85}$   $z=z_{86}$   $z=z_{87}$   $z=z_{88}$   $z=z_{89}$   $z=z_{90}$   $z=z_{91}$   $z=z_{92}$   $z=z_{93}$   $z=z_{94}$   $z=z_{95}$   $z=z_{96}$   $z=z_{97}$   $z=z_{98}$   $z=z_{99}$

$$T \ll \mu : z_1 \rightarrow z_0$$

$$f \sim a(z-z_0)^2 + b(z-z_0)$$

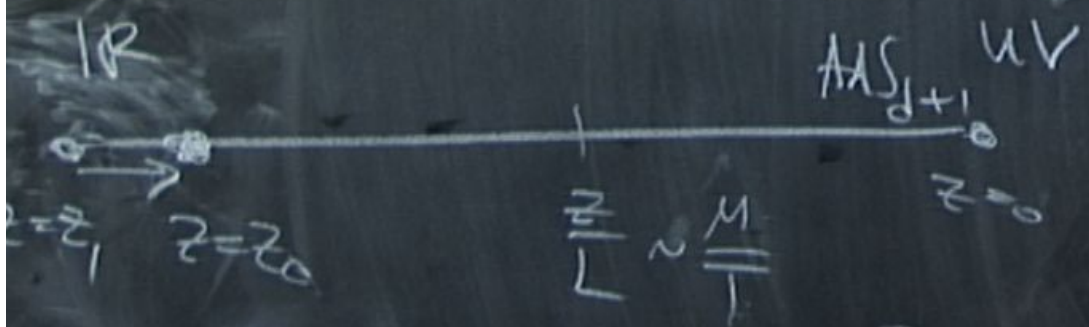
$$ds^2 \sim -a(z-z_0)dt^2 + \frac{dz^2}{(z-z_0)^2} + \frac{d\vec{x}^2}{z_0^2}$$

saddle pt w/ these BC: charged BH (AdS-RN)

$$f(z) = 1 - Mz^d + Qz^{2d-2}$$

$$A = dt A_t = dt \mu \left( 1 - \left( \frac{z}{z_0} \right)^{d-2} \right)$$

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IR  
 $z_1$   
 $z = z_0$

AdS<sub>d+1</sub> UV  
 $z = z_0$   
 $\frac{z}{L} \sim \frac{\mu}{1}$

$$T \ll \mu : z_1 \rightarrow z_0$$

$$f \sim a(z - z_0)^2 + b(z - z_0)^3$$

$$ds^2 = -a(z - z_0) dt^2 + \frac{dz^2}{(z - z_0)^2} + \frac{d\vec{x}^2}{z_0^2} = -b dt^2 + \frac{dz^2}{z^2} + d\vec{x}^2$$

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IR AdS<sub>2</sub> x IR<sup>d-1</sup>

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IR

saddle pt w/ these BC: charged BH (AdS-RN)

$$f(z) = 1 - \frac{Mz^d}{L^{d-2}} + Qz^{2d-2}$$

$$A = dt A_t = dt \mu \left( 1 - \left( \frac{z}{z_0} \right)^{d-2} \right)$$

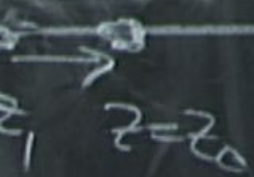
~~$f(z_0) = 0$~~

$$Q = \frac{\mu L^2}{z_0^{d-2} g_F}$$

IR AdS<sub>2</sub> x IR<sup>d-1</sup>

AdS<sub>d+1</sub> UV  
z=0

$T \ll \mu : z_1 \rightarrow z_0$



$$\frac{z}{L} \sim \frac{\mu}{T}$$

$$f \sim a(z-z_0)^2 + b(z-z_0)^3$$

$$ds^2 \xrightarrow{z \rightarrow z_0} -a(z-z_0) dt^2 + \frac{dz^2}{(z-z_0)^2} + \frac{d\vec{x}^2}{z_0^2} = -\frac{dt^2}{y^2} + \frac{dx^2}{y^2}$$

IR<sup>d-1</sup>

- New emergent scale inv. in IR

AdS<sub>2</sub> →

- "IR CFT" is non-relativistic.:

scale transfs:  $\begin{cases} t \rightarrow \lambda t \\ \vec{x} \rightarrow \lambda^{\frac{1}{2}} \vec{x} \end{cases}$

$z \equiv \text{dynamical exp.} \rightarrow \infty$



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PRACTICE: HOLOGRAPHY 'RESPONSIBILITY'

1) critical exponents are determined

by parameters of bulk actions  
→ affected by "landscape issues"

2) the analysis is sensitive to string





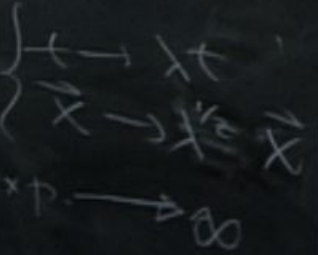
- New emergent scale inv. in IR

$AdS_2 \rightarrow$

- "IR CFT"

scale

- relativist.:



PRACTICE HOLOGRAPHY RESPONSIBILITY

- 1) critical experiments are determined by parameters in bulk actions
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- 2) thermodynamics is not sensitive to string coupling
- 3) red-fine dynamics, transport



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$$\text{scale transfs: } \begin{cases} t \rightarrow \lambda t \\ \vec{X} \rightarrow \lambda^{\nu} \vec{X} \end{cases}$$

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(eg. RPP)
- 4)

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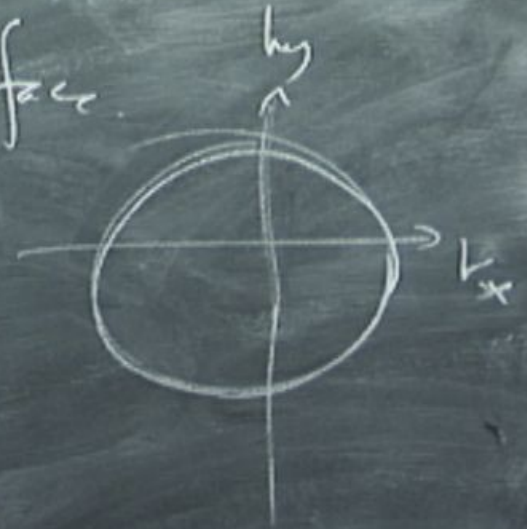
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## PRACTICE: HOLOGRAPHY 'RESPONSIBILITIES'

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- 4)

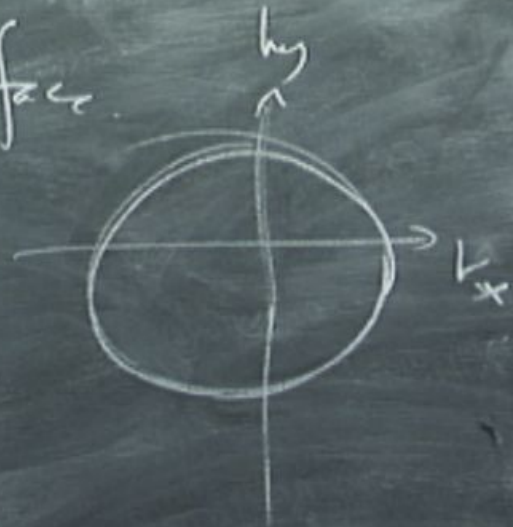
holography for metals

$\equiv$  systems w/ Fermi surface



holography for metals

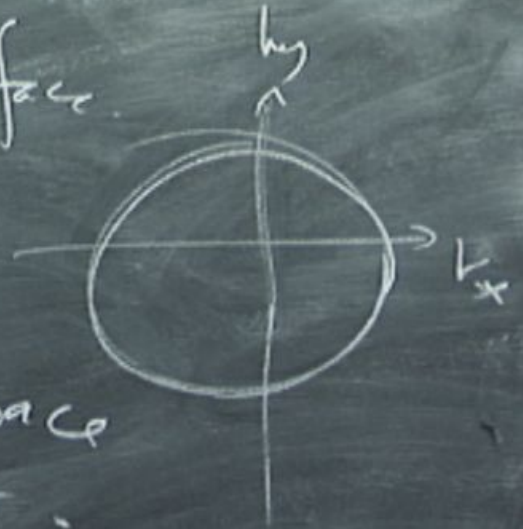
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# holography for metals

= systems w/ Fermi surface

gaps exc. on  
some surface in  
momentum space



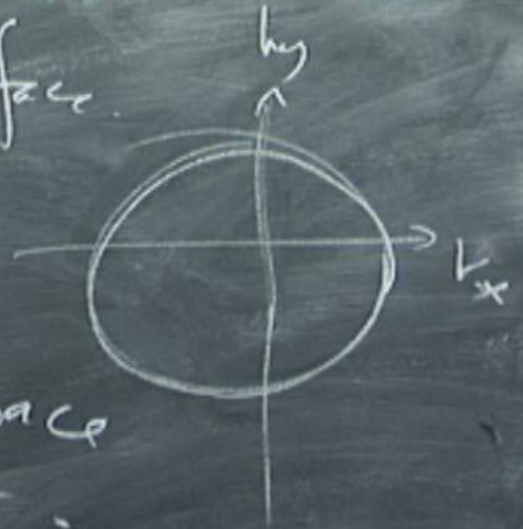
relativistic QFT:

$d$

# holography for metals

= systems w/ Fermi surface

gaps exc. on  
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vs  
statistic @ FT:  
gaps stuff at  $|\mathbf{k}| \rightarrow \infty$

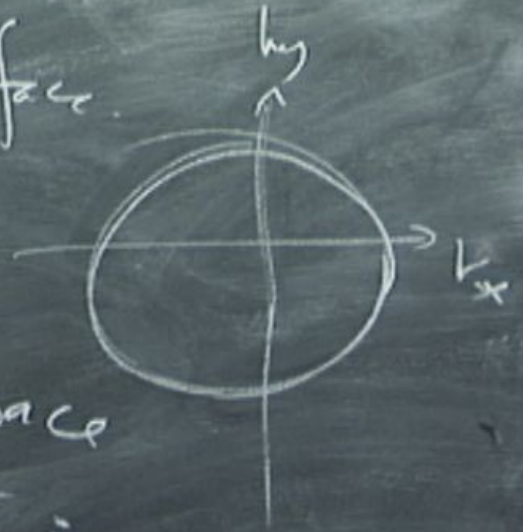
$$\omega^2 = \mathbf{k}^2$$



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relativistic @ FT:

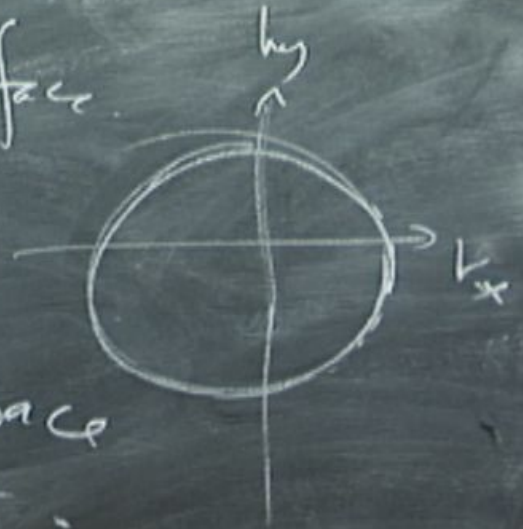
gaps stuff at  $|k| = \infty$

$$\omega^2 = \vec{k}^2$$

# holography for metals

= systems w/ Fermi surface

gaps exc. on  
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momentum space



vs: relativistic QFT:

gaps still at  $|k| = \infty$

$$\omega^2 = \vec{k}^2$$

Suppose:  $T_{\mu\nu}$ ,  $J^\mu$ ,  $\Psi$

Suppose:  $T_{\mu\nu}$ ,  $J^\mu$ ,  $\Psi_\Delta$

→  $g_{\mu\nu}$   $A_\mu$ ,  $\Psi$  charge  $q$   
mass  $m$

Suppose:  $T_{\mu\nu}$ ,  $J^\mu$ ,  $\Psi_\Delta$

$\rightarrow$   $g_{\mu\nu}$   $A_\mu$ ,  $\Psi$  charge  $q$   
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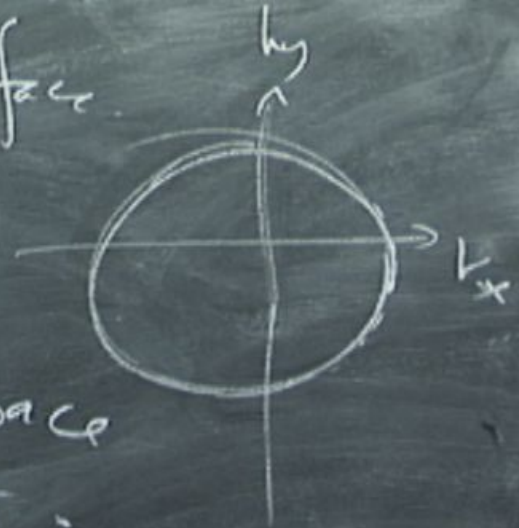
Suppose:  $T_{\mu\nu}$ ,  $J^\mu$ ,  $\Psi_\Delta$

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# holography for metals

= systems w/ Fermi surface

gaps exc. on  
some surface in  
momentum space



vs: relativistic QFT:

gaps stuff at  $|\mathbf{k}| = 0$

$$\omega^2 = \mathbf{k}^2$$

FS:

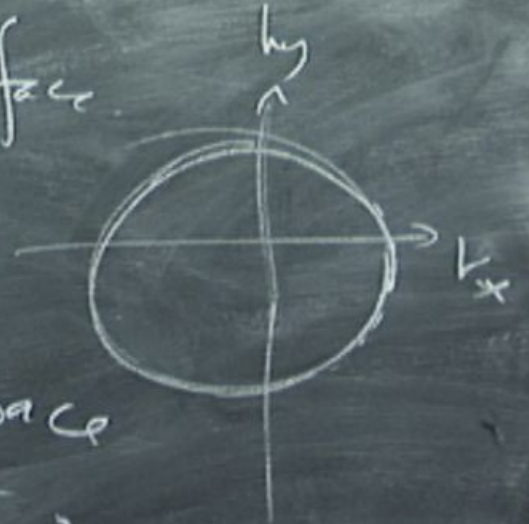




# holography for metals

= systems w/ Fermi surface

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vs: relativistic QFT:

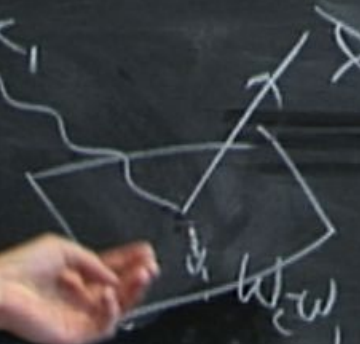
gaps stuff at  $|\mathbf{k}| = \infty$

FS

$\omega_{\mathbf{k}}$

$$\omega^2 = \mathbf{k}^2$$

FS:



# holography for metals

= systems w/ Fermi surface

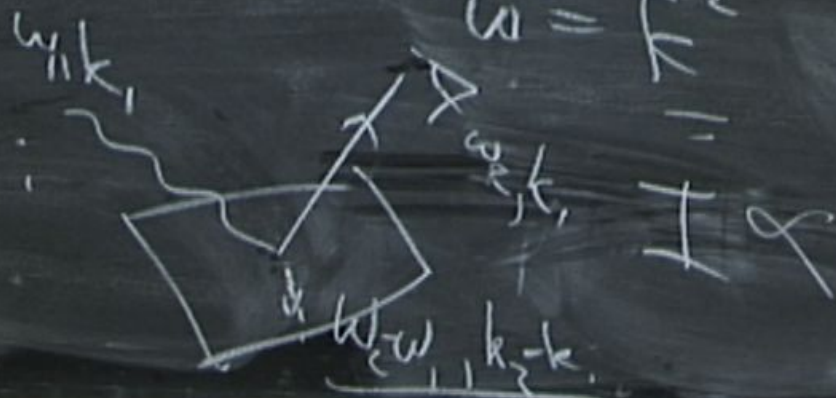
gaps exc. on  
some surface in  
momentum space



vs: relativistic QFT:

gaps stuff at  $|\mathbf{k}|=0$

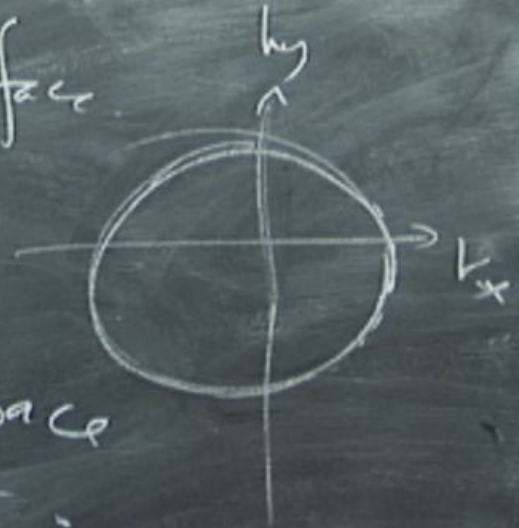
ARPES  
observable  
to see a FS:



# holography for metals

= systems w/ Fermi surface

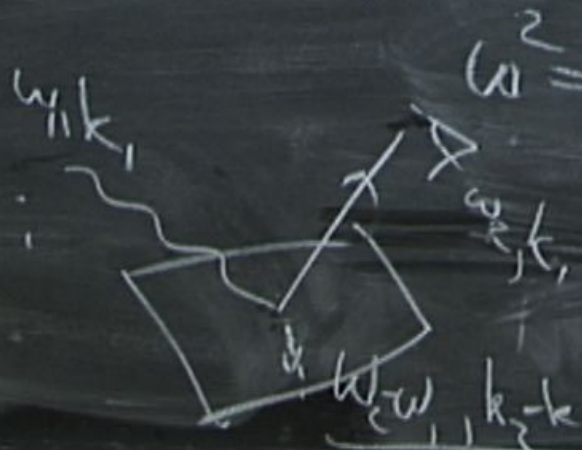
gaps exc. on some surface in momentum space



vs: relativistic QFT:

gaps stuff at  $|k| = \infty$

ARPES  
observable  
to see a FS:



$I \propto \text{Im} G_R(\omega_2, \mathbf{k}_2)$

- Suppose:  $T_{\mu\nu}$ ,  $J^\mu$ ,  $\Psi_\Delta$

$\rightarrow$   $g_{\mu\nu}$ ,  $A_\mu$ ,  $\Psi$  charge  $q$   
mass  $m$

$$G(p) = \int dm^2 \frac{\rho(p)}{p^2 + m^2}$$

$$G^R = G^R \Psi \Psi$$

$$\rho = \text{Im} G$$

$(k_0, \vec{k})$

Suppose:  $T_{\mu\nu}$ ,  $J^\mu$ ,  $\Psi_\Delta$

$\rightarrow$   $g_{\mu\nu}$ ,  $A_\mu$ ,  $\Psi$  charge  $q$   
mass  $m$

$$G(p) = \int dm^2 \frac{\rho(p)}{p^2 + m^2}$$

$$\rho = \text{Im} \zeta$$

$$G^R = G \Psi \bar{\Psi}$$

$$S_{\text{bulk}} = \dots + \int d^4x \sqrt{g} \bar{\Psi} (\not{D} - m) \Psi$$

$$(\not{D} - m) \Psi = 0$$

$(\not{k}_2 - m) \psi_2 = 0$   
 $(\not{k}_1 - m) \psi_1 = 0$

Suppose:  $T_{\mu\nu}$ ,  $J^\mu$ ,  $\Psi_\Delta$

$\rightarrow$   $g_{\mu\nu}$ ,  $A_\mu$ ,  $\Psi$  charge  $q$   
mass  $m$

$$G(p) = \int dm^2 \frac{\rho(p)}{p^2 + m^2}$$

$$\rho = \text{Im} \zeta$$

$$G^R = G \Phi \Psi$$

$$S_{\text{bulk}} = \dots + \int d^4x \sqrt{-g} \bar{\Psi} (\not{D} - m) \Psi$$

$$(\not{D} - m) \Psi = 0$$

Suppose:  $T_{\mu\nu}$ ,  $J^\mu$ ,  $\Psi_\Delta$

→  $g_{\mu\nu}$ ,  $A_\mu$ ,  $\Psi$  charge  $q$   
mass  $m$

$$G(p) = \int d^4m^2 \frac{\rho(p)}{p^2 + m^2}$$

$$\rho = \text{Im} G$$

$$G^R = G \Psi \Psi^R$$

$$S_{\text{bulk}} = \dots + \int d^4x \sqrt{-g} \Psi (\not{D} - m) \Psi$$

$$(\not{D} - m) \Psi = 0$$