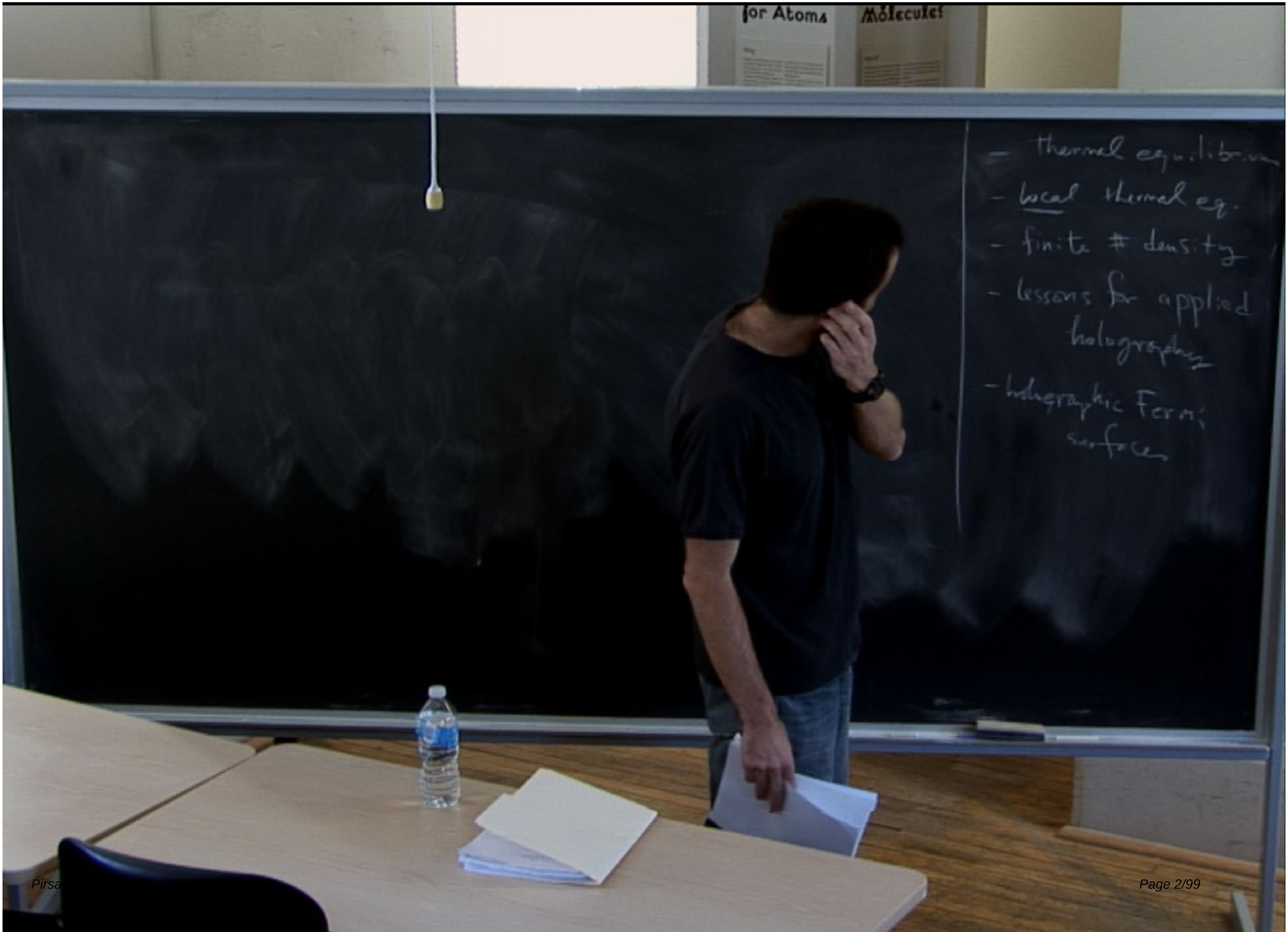
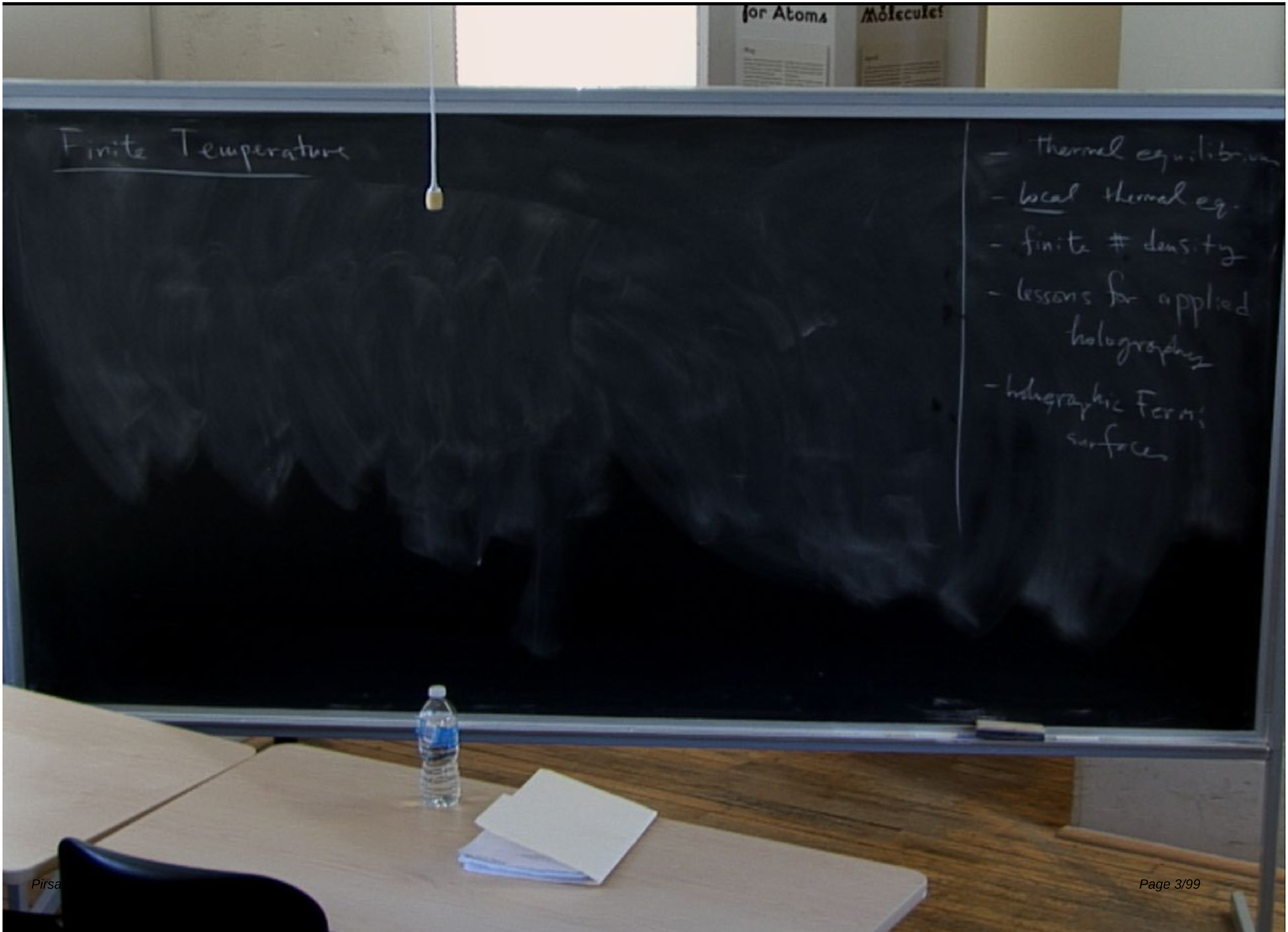
Title: Explorations in String Theory - Lecture 9

Date: Mar 25, 2011 10:15 AM

URL: http://pirsa.org/11030058

Abstract:


for Atoms
Molecules
- thermal equilibrium
- local thermal eq.
- finite # density
- lessons for applied holography
- holographic Fermi surface


for Atoms
Molecules
Finite Temperature
– thermal equilibrium
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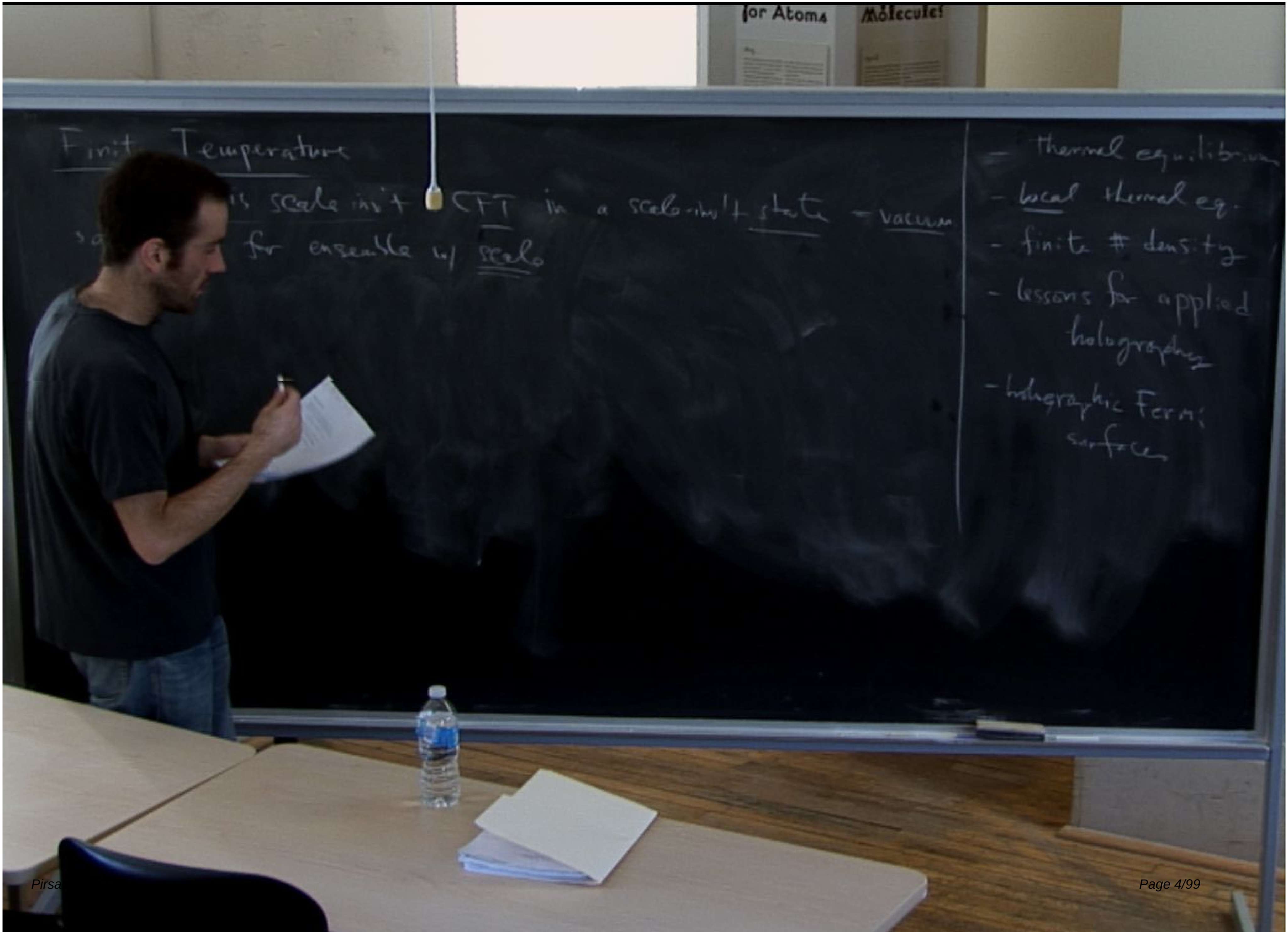
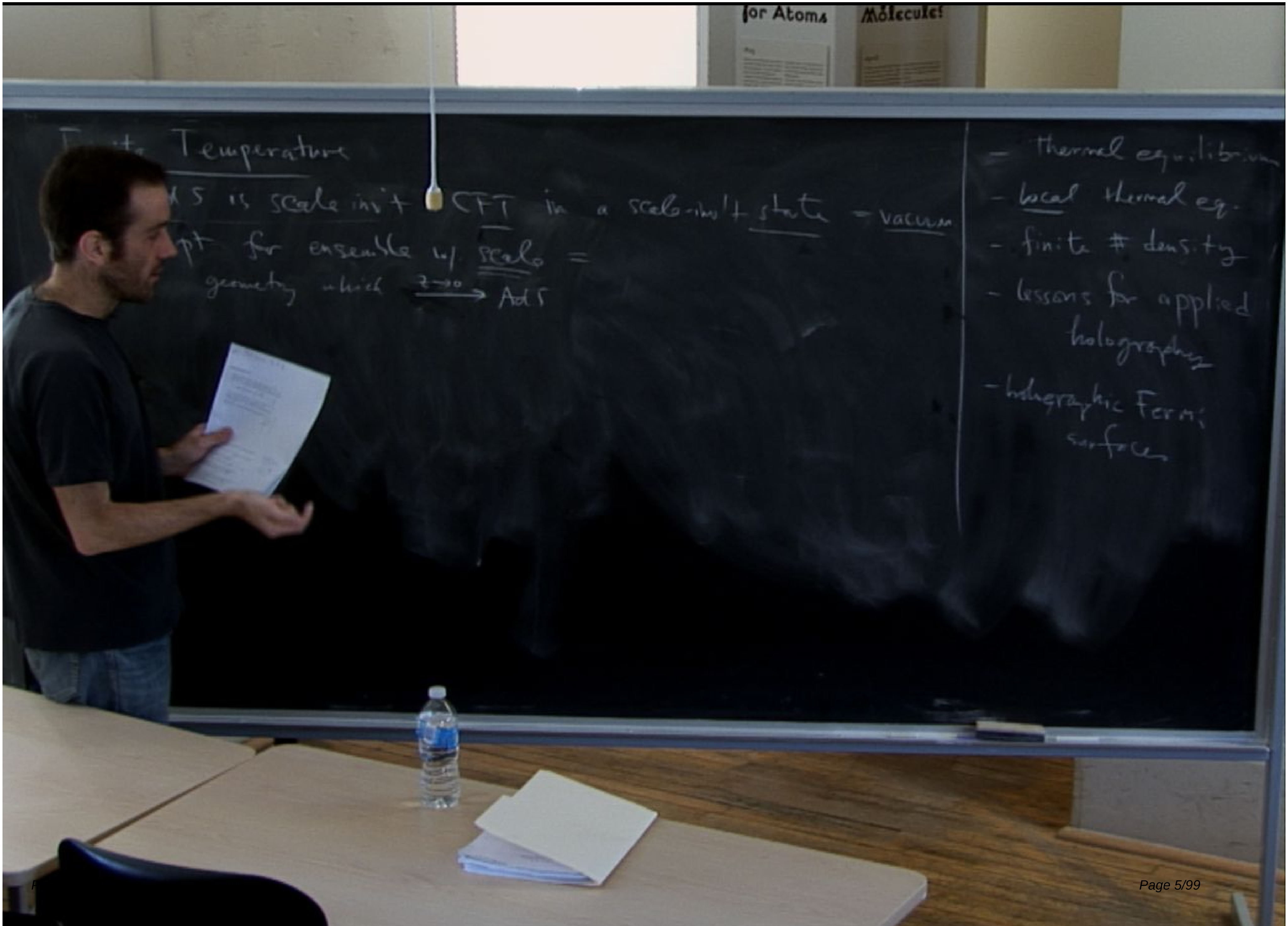
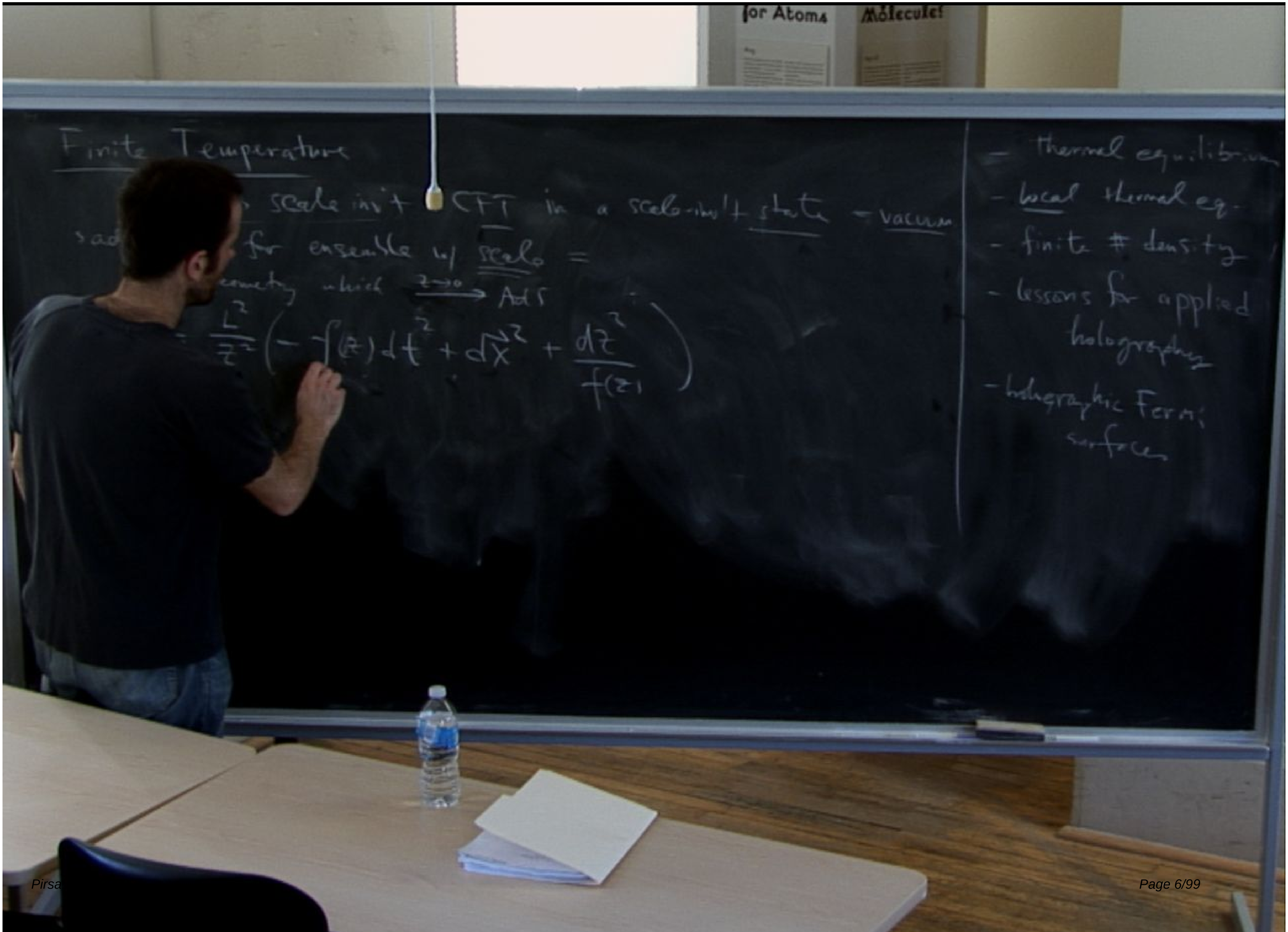
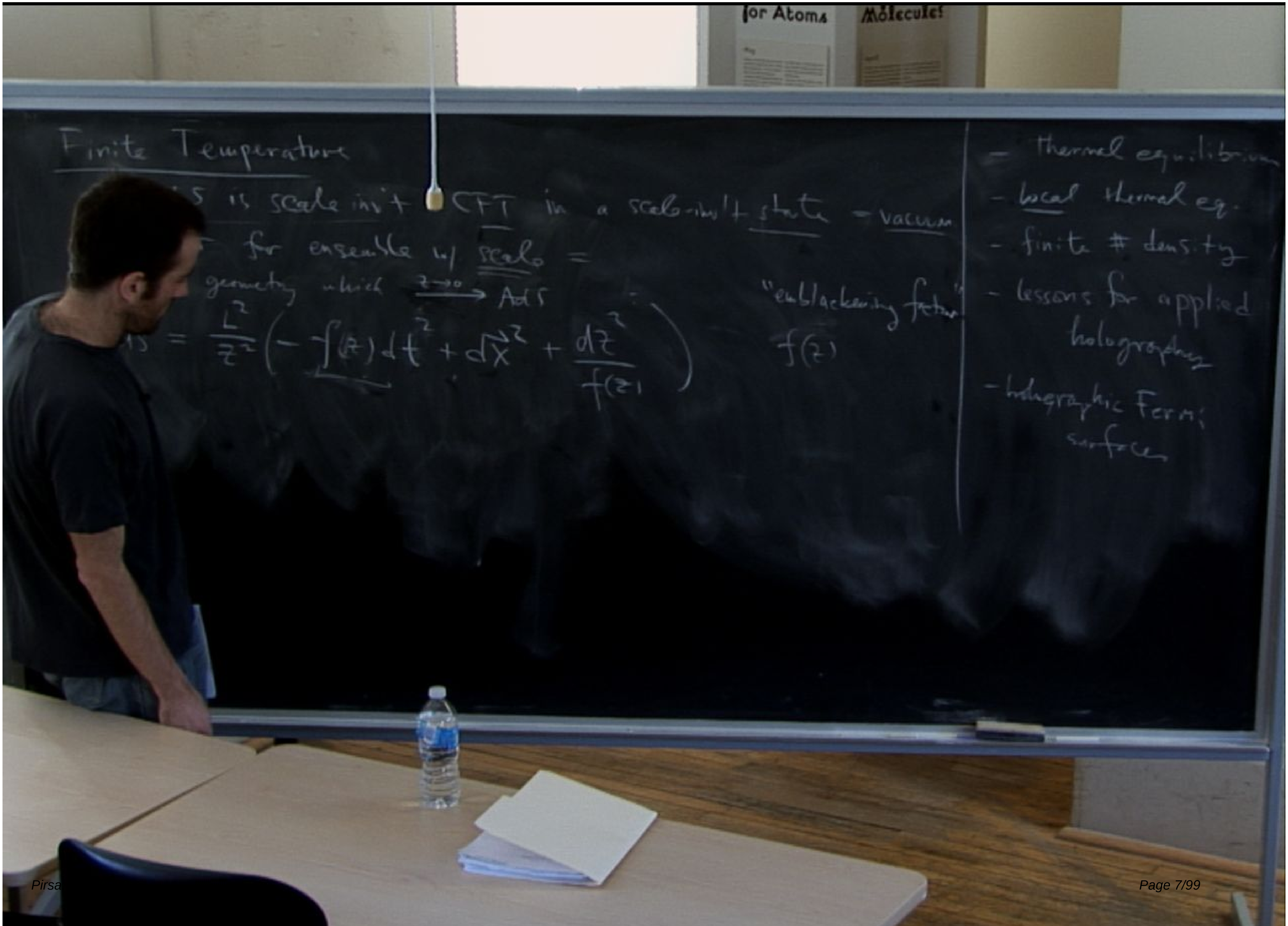
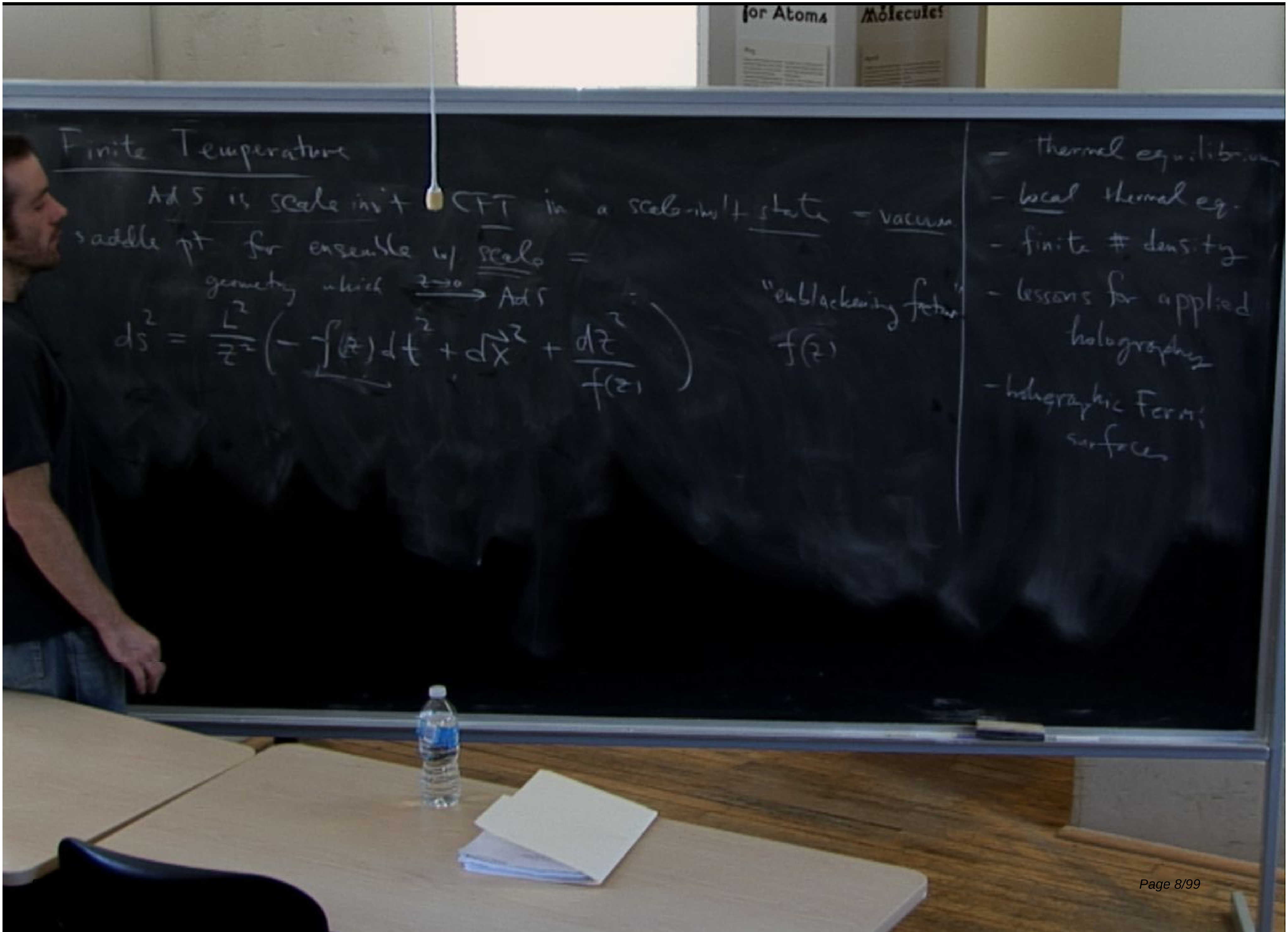
– holographic Fermi surface

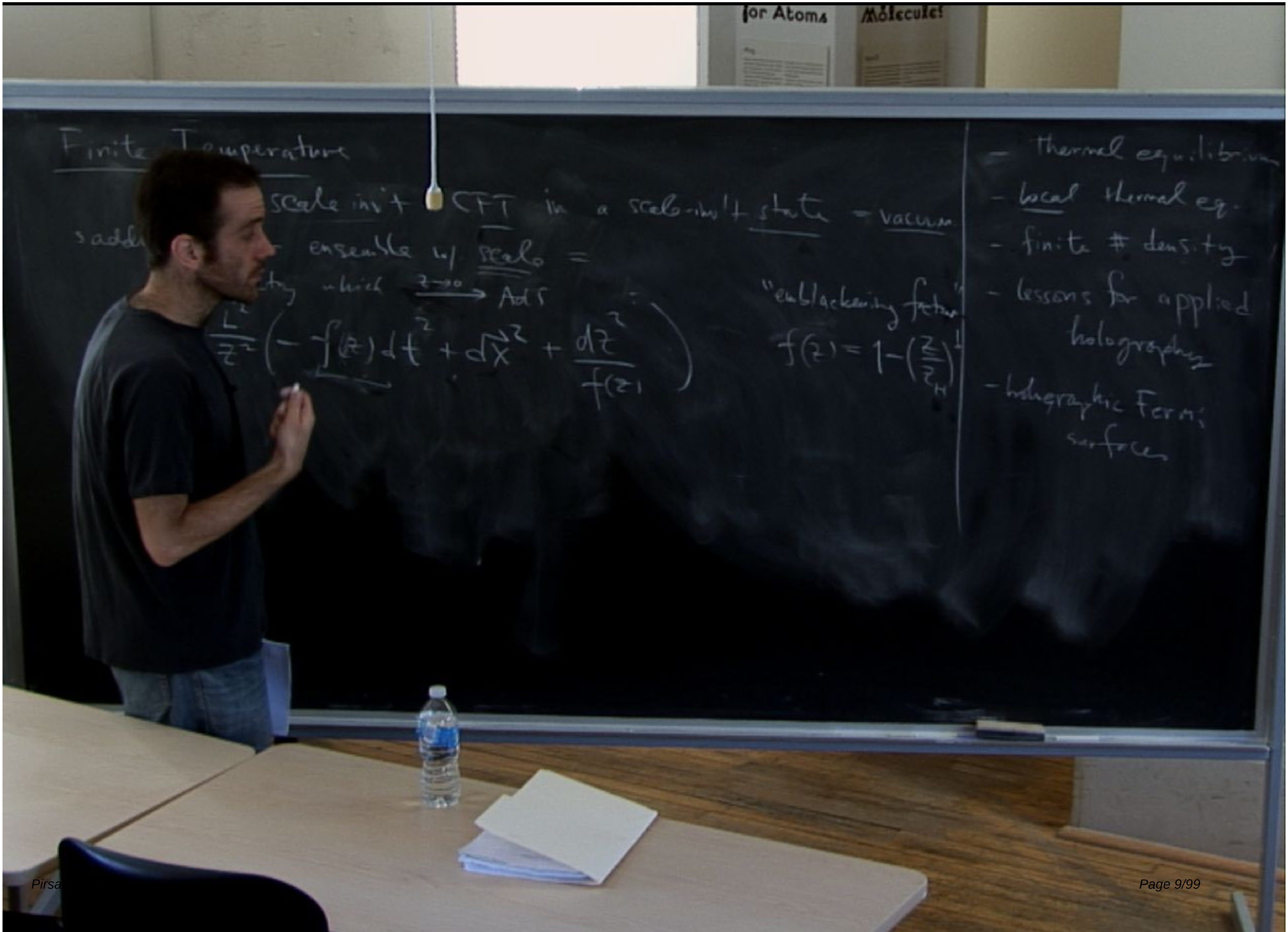

for Atoms
Molecules
Finite Temperature
scale inv't CFT in a scale-inv't state - vacuum
for ensemble w/ scale
- thermal equilibrium
- local thermal eq.
- finite # density
- lessons for applied holography
- holographic Fermi surface

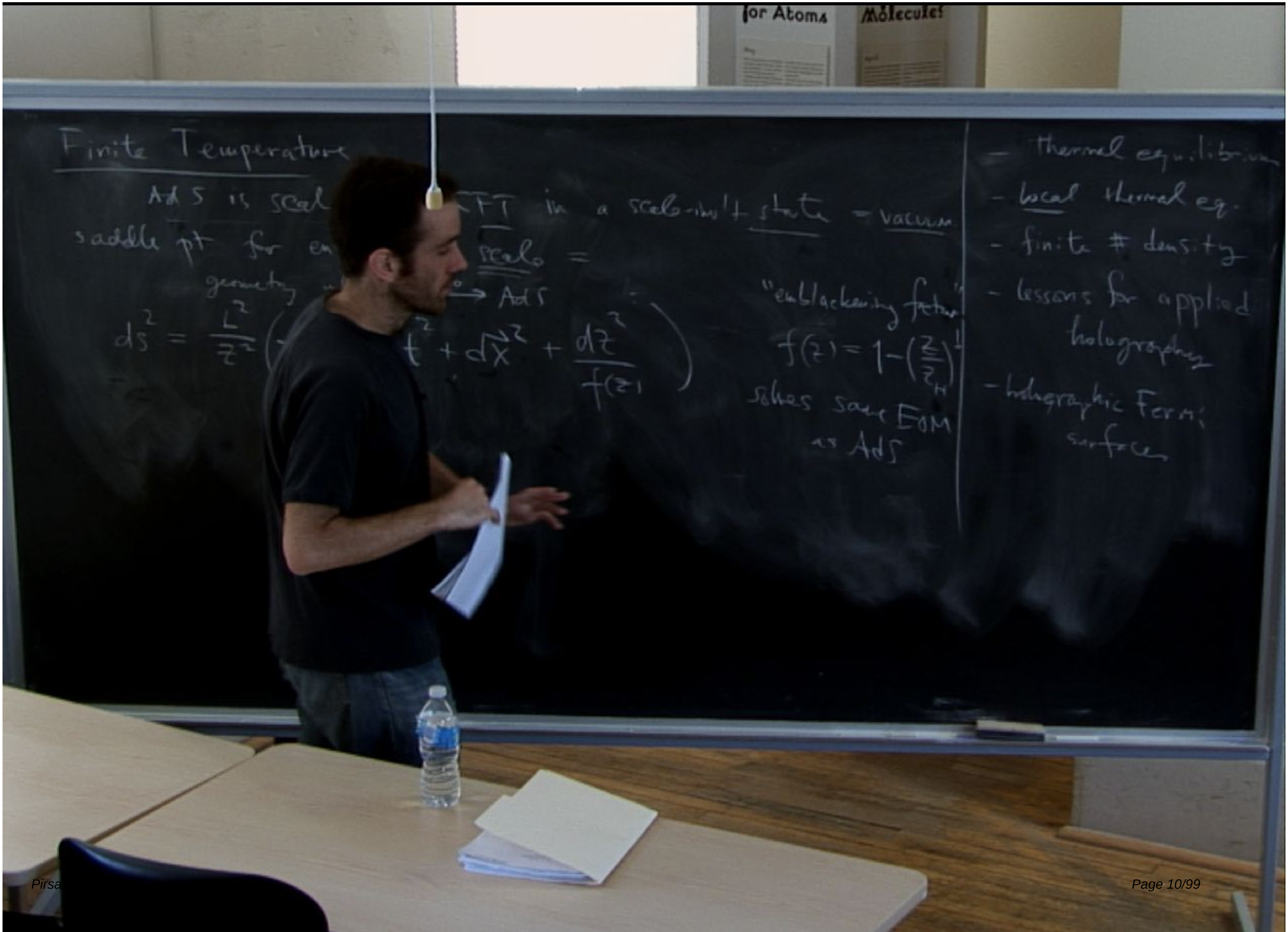

for Atoms
Molecules
Temperature
is scale inv't CFT in a scale-inv't state = vacuum
for ensemble w/ scale =
geometry which → AdS
– thermal equilibrium
– local thermal eq.
– finite # density
– lessons for applied holography
– holographic Fermi surface

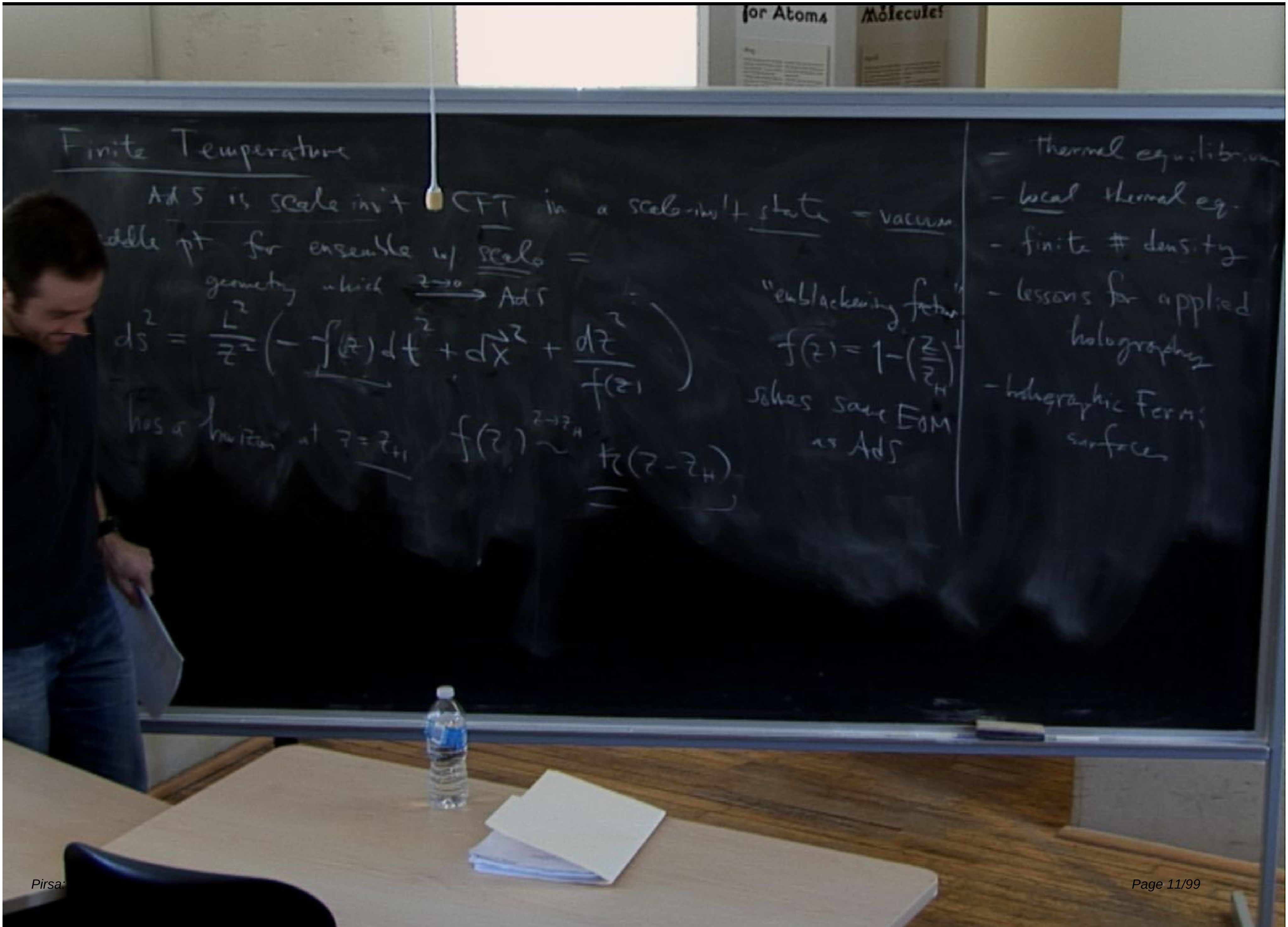

for Atoms
Molecules
Finite Temperature
scale inv't CFT in a scale-inv't state - vacuum
for ensemble w/ scale =
which $\xrightarrow{z\to 0}$ AdS
$\frac{L^2}{z^2}\left(-f(z)\,dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)}\right)$
- thermal equilibrium
- local thermal eq.
- finite # density
- lessons for applied holography
- holographic Fermi surface

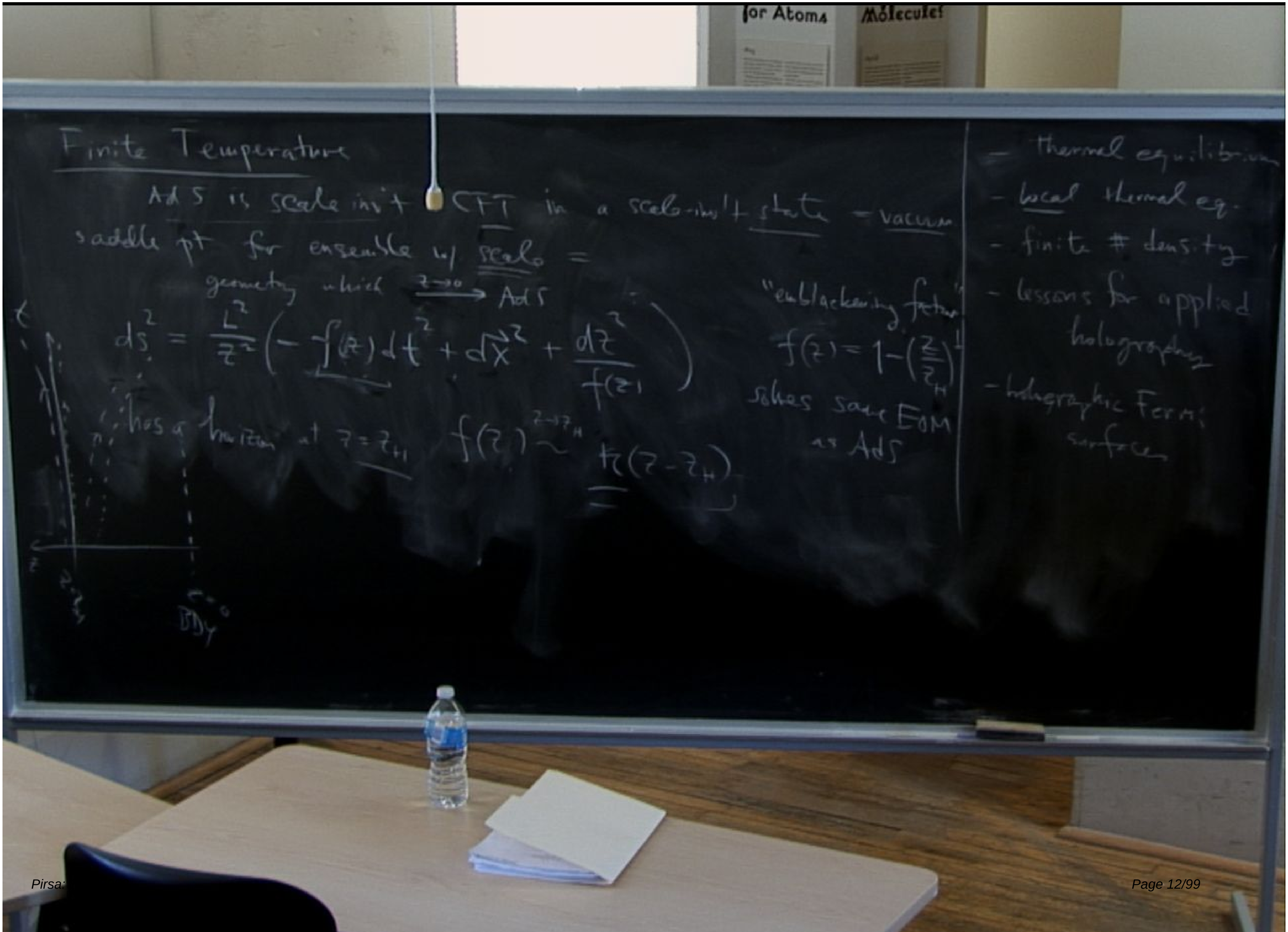

for Atoms
Molecules
Finite Temperature
is scale inv't CFT in a scale-inv't state - vacuum
for ensemble w/ scale =
geometry which z→0 → AdS
= L^2/z^2 (- f(z) dt^2 + dx^2 + dz^2/f(z))
"emblackening factor"
f(z)
- thermal equilibrium
- local thermal eq.
- finite # density
- lessons for applied holography
- holographic Fermi surface


for Atoms
Molecules
Finite Temperature
AdS is scale inv't CFT in a scale-inv't state - vacuum
saddle pt for ensemble w/ scale =
geometry which z→0 → AdS
ds^2 = L^2/z^2 (- f(z) dt^2 + dx^2 + dz^2/f(z))
"emblackening factor"
f(z)
- thermal equilibrium
- local thermal eq.
- finite # density
- lessons for applied holography
- holographic Fermi surface


for Atoms
Molecules!
Finite Temperature
scale inv't CFT in a scale-inv't state = vacuum
ensemble w/ scale =
which AdS
"emblackening factor"
f(z) = 1 − (z/z_H)
thermal equilibrium
local thermal eq.
finite # density
lessons for applied holography
holographic Fermi surface


for Atoms
Molecules
Finite Temperature
"emblackening factor"
solves same EOM as AdS
- thermal equilibrium
- local thermal eq.
- finite # density
- lessons for applied holography
- holographic Fermi surface


for Atoms
Molecules
Finite Temperature
AdS is scale inv't CFT in a scale-inv't state - vacuum
saddle pt for ensemble w/ scale =
geometry which z→0 → AdS
ds² = L²/z² (- f(z) dt² + dx² + dz²/f(z))
has a horizon at z = z_H   f(z) ~ (z→z_H) k(z - z_H)
"emblackening factor"
f(z) = 1 - (z/z_H)^d
solves same EOM as AdS
- thermal equilibrium
- local thermal eq.
- finite # density
- lessons for applied holography
- holographic Fermi surface


for Atoms
Molecules
Finite Temperature
AdS is scale inv't CFT in a scale-inv't state - vacuum
saddle pt for ensemble w/ scale =
geometry which z→0 → AdS
$ds^2 = \frac{L^2}{z^2}\left(-f(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)}\right)$
"emblackening factor"
$f(z) = 1-\left(\frac{z}{z_H}\right)^d$
solves same EOM as AdS
has a horizon at $z = z_H$
$f(z) \overset{z\to z_H}{\sim} (z - z_H)$
Bdy
- thermal equilibrium
- local thermal eq.
- finite # density
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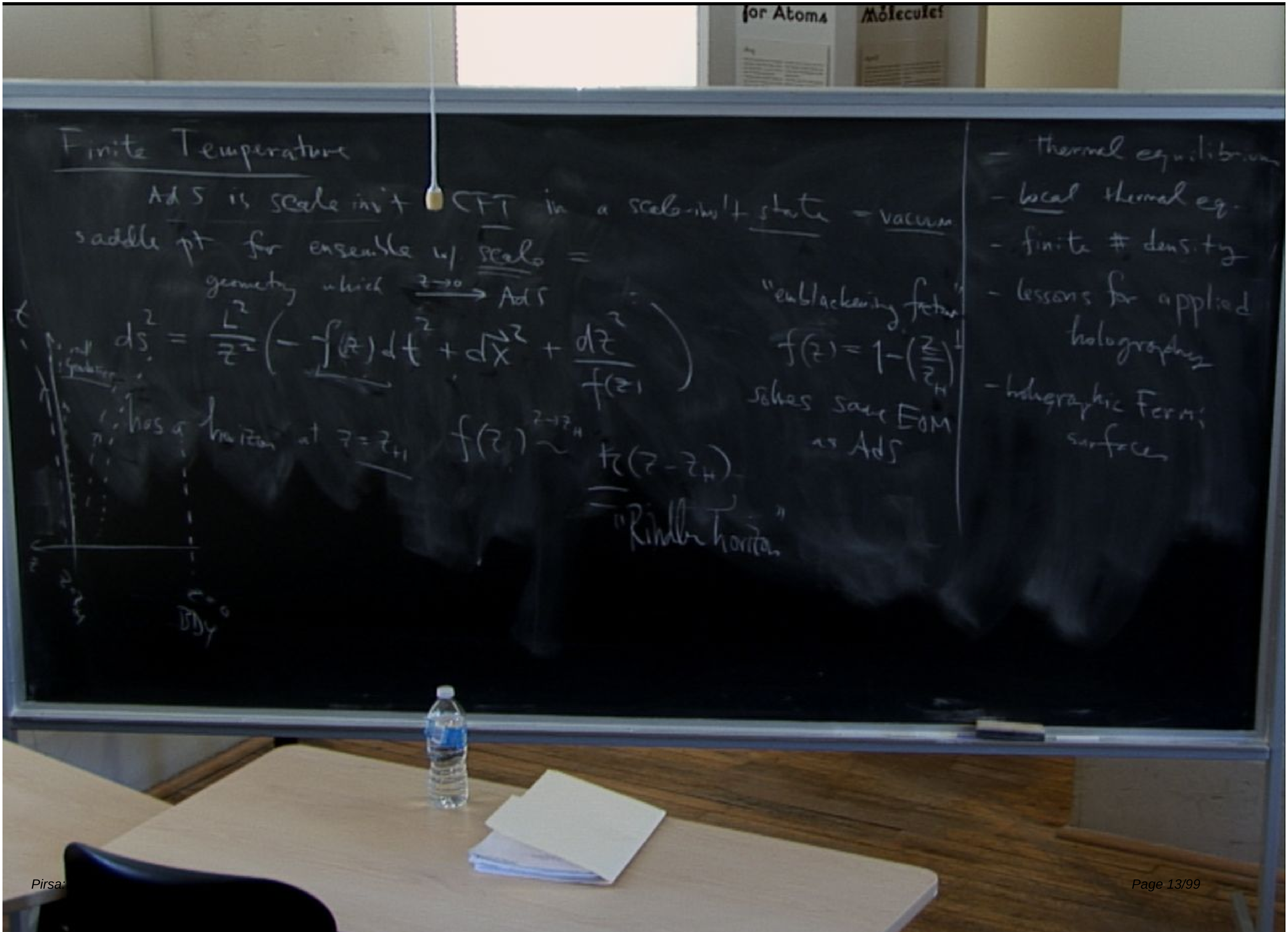
for Atoms
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AdS is scale inv't CFT in a scale-inv't state = vacuum
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$ds^2 = \frac{L^2}{z^2}\left(-f(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)}\right)$
"emblackening factor"
$f(z) = 1 - \left(\frac{z}{z_H}\right)^d$
solves same EOM as AdS
has a horizon at $z = z_H$
$f(z) \overset{z \to z_H}{\sim} \kappa(z - z_H)$
"Rindler horizon"
BDY
- thermal equilibrium
- local thermal eq.
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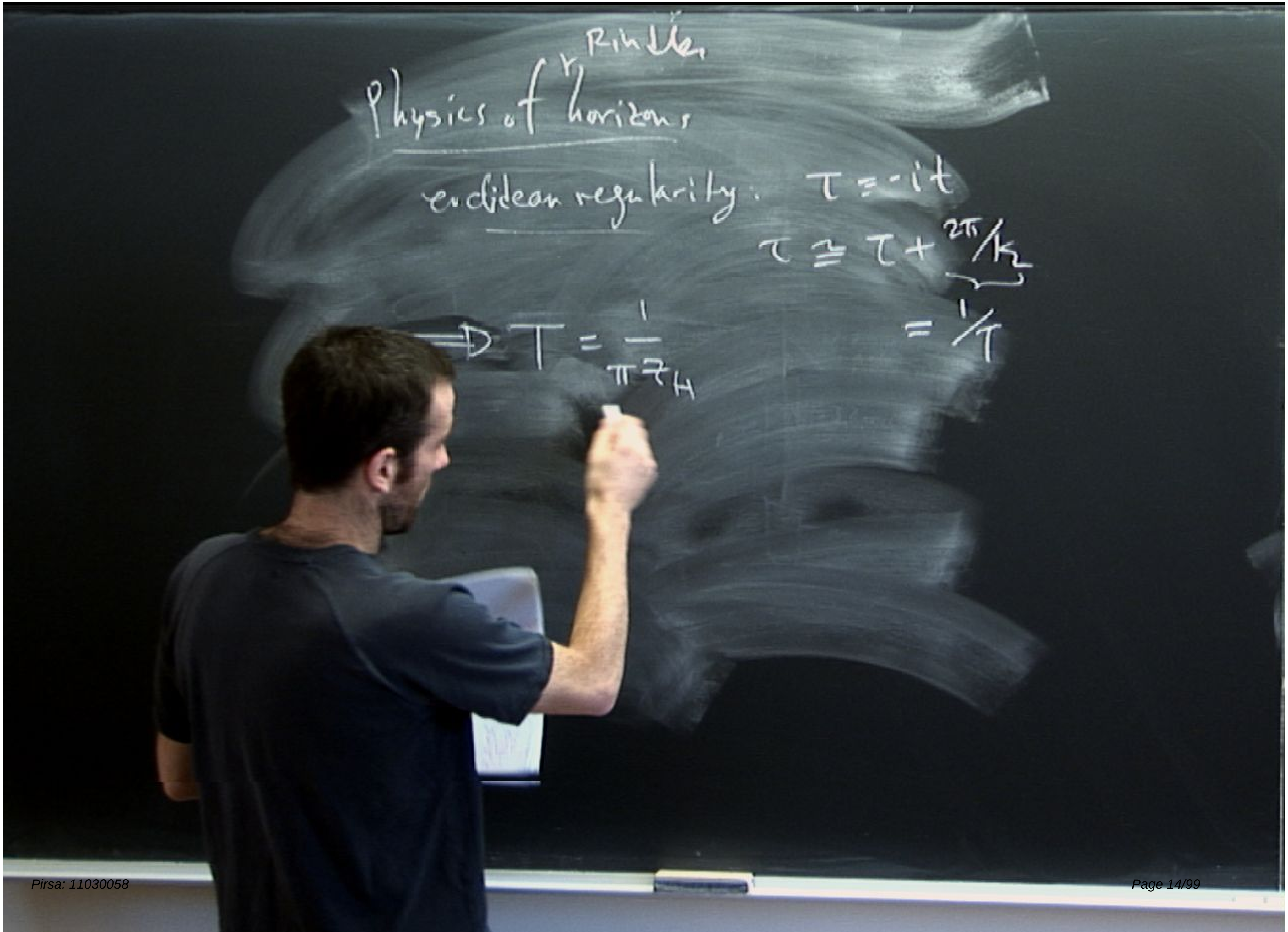
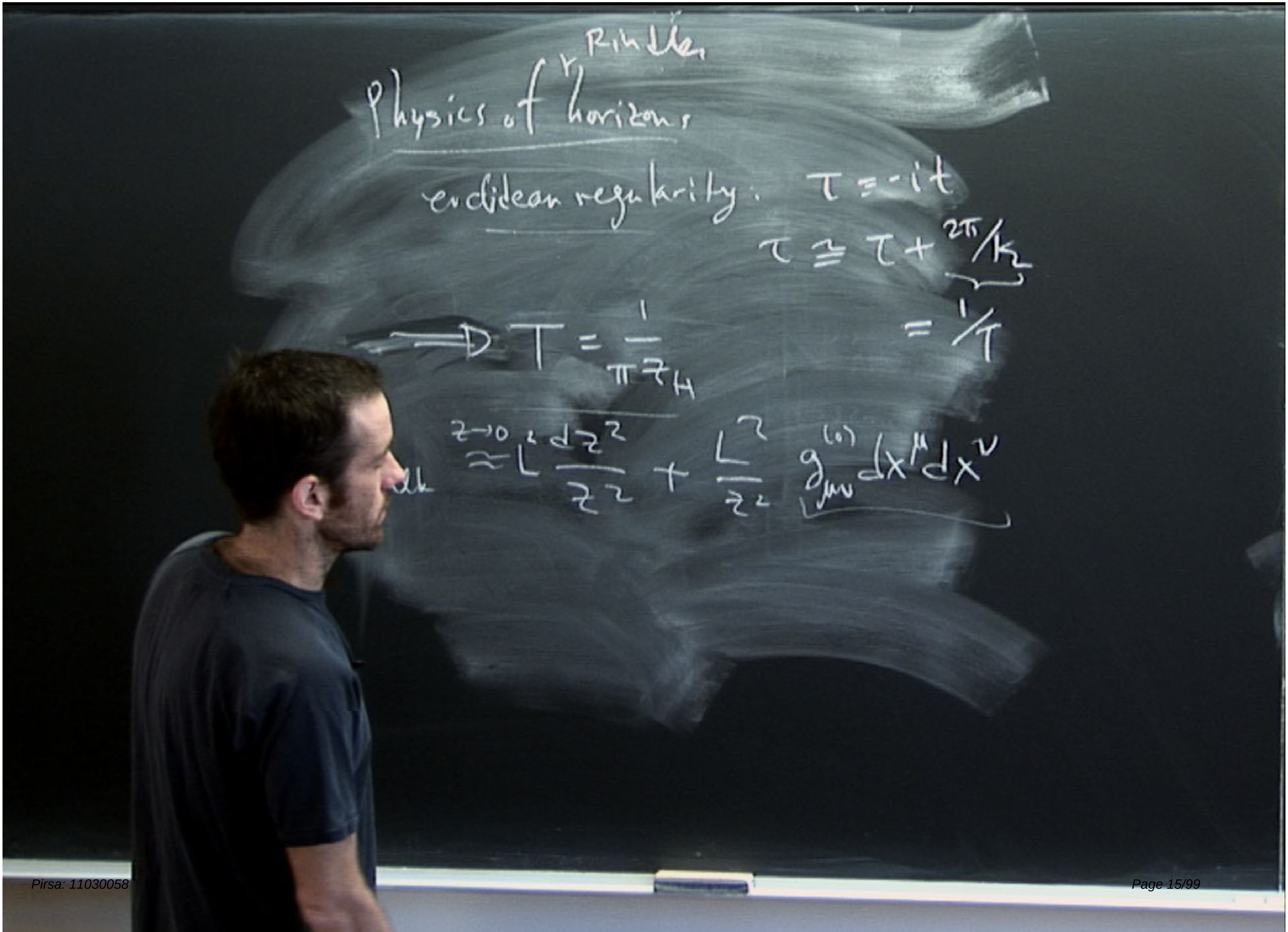
- holographic Fermi surface


Physics of Rindler horizons
euclidean regularity: τ = -it
τ ≅ τ + 2π/κ
= 1/T
⇒ T = 1/πr_H


Rindler
Physics of horizons
euclidean regularity: $\tau = -it$
$\tau \cong \tau + 2\pi/\kappa$
$= 1/T$
$\Rightarrow T = \frac{1}{\pi z_H}$
$\overset{z \to 0}{\simeq} L^2 \frac{dz^2}{z^2} + \frac{L^2}{z^2} g^{(0)}_{\mu\nu} dx^\mu dx^\nu$

Physics of Rindler horizons

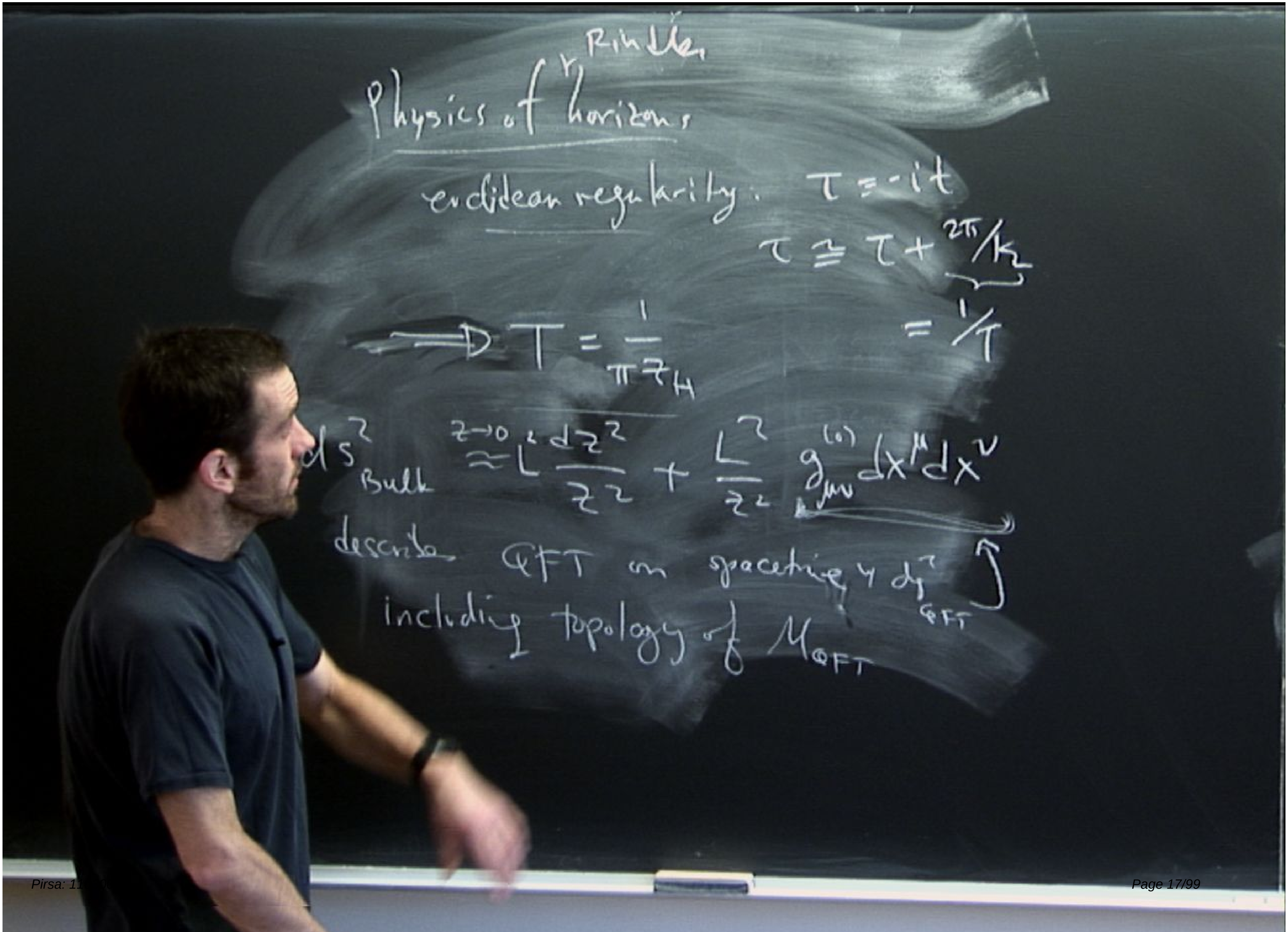
euclidean regularity: $\tau = -it$

$$\tau \cong \tau + \underbrace{2\pi/\kappa_2}_{=\,1/T}$$

$$\Longrightarrow T = \frac{1}{\pi z_H}$$

if: $ds^2_{\text{Bulk}} \overset{z\to 0}{\simeq} L^2 \frac{dz^2}{z^2} + \frac{L^2}{z^2}\, g^{(0)}_{\mu\nu}\, dx^\mu dx^\nu$

describes QFT on spacetime w/ g_{QFT}


Rindler
Physics of horizons
euclidean regularity: τ = -it
τ ≅ τ + 2π/κ
⟹ T = 1/(π z_H) = 1/T
z→0
ds²_Bulk ≈ L² dz²/z² + L²/z² g^(0)_μν dx^μ dx^ν
describe QFT on spacetime w/ g_QFT
including topology of M_QFT

Physics of $^{\text{Rindler}}_{\text{r}}$ horizons

euclidean regularity: $\tau \equiv -it$

$\tau \cong \tau + \underbrace{2\pi/\kappa_H}$

$$\Longrightarrow \boxed{T = \frac{1}{\pi z_H}} \qquad = 1/T$$

if: $ds^2_{\text{Bulk}} \overset{z\to 0}{\approx} L^2 \frac{dz^2}{z^2} + \frac{L^2}{z^2}\, g^{(0)}_{\mu\nu} dx^\mu dx^\nu$

describes QFT on spacetime w ds^2_{QFT}

including topology of M_{QFT}

$\Longrightarrow$ eucl. time of QFT: $\tau \cong \tau + \frac{1}{T}$

Physics of Rindler horizons

euclidean regularity: $\tau = -it$

$$\tau \cong \tau + \underbrace{2\pi/\kappa_H}_{= 1/T}$$

$$\Longrightarrow \boxed{T = \frac{1}{\pi z_H}}$$

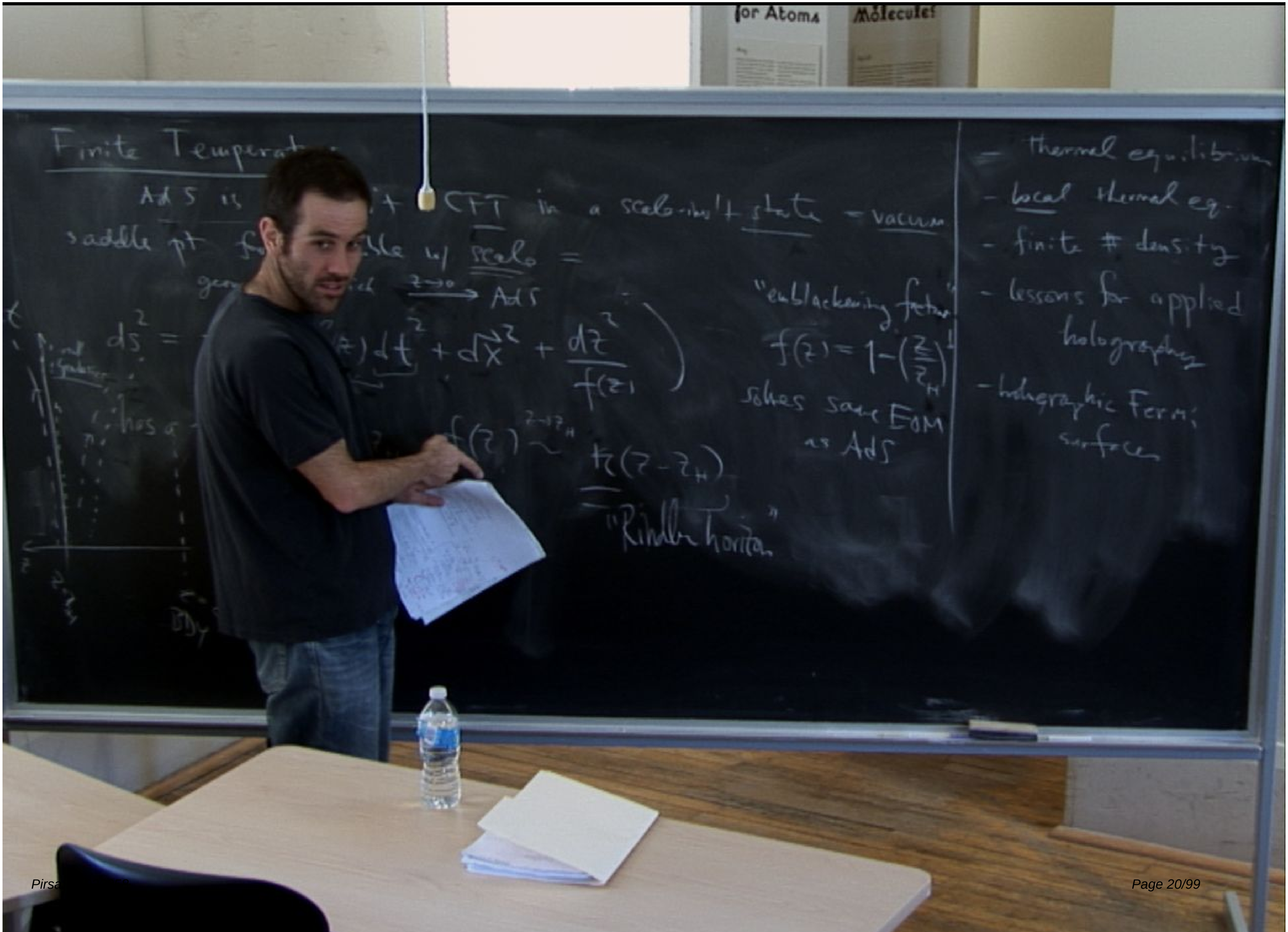
if: $ds^2_{Bulk} \overset{z\to 0}{\simeq} L^2 \frac{dz^2}{z^2} + \frac{L^2}{z^2} g^{(0)}_{\mu\nu} dx^\mu dx^\nu$

describe QFT on spacetime w/ g_{QFT}

including topology of M_{QFT}

$\Longrightarrow$ eucl time of QFT: $\tau \cong \tau + \frac{1}{T}$

$\Longrightarrow$ static AdS BH = thermal eq. of dual CFT.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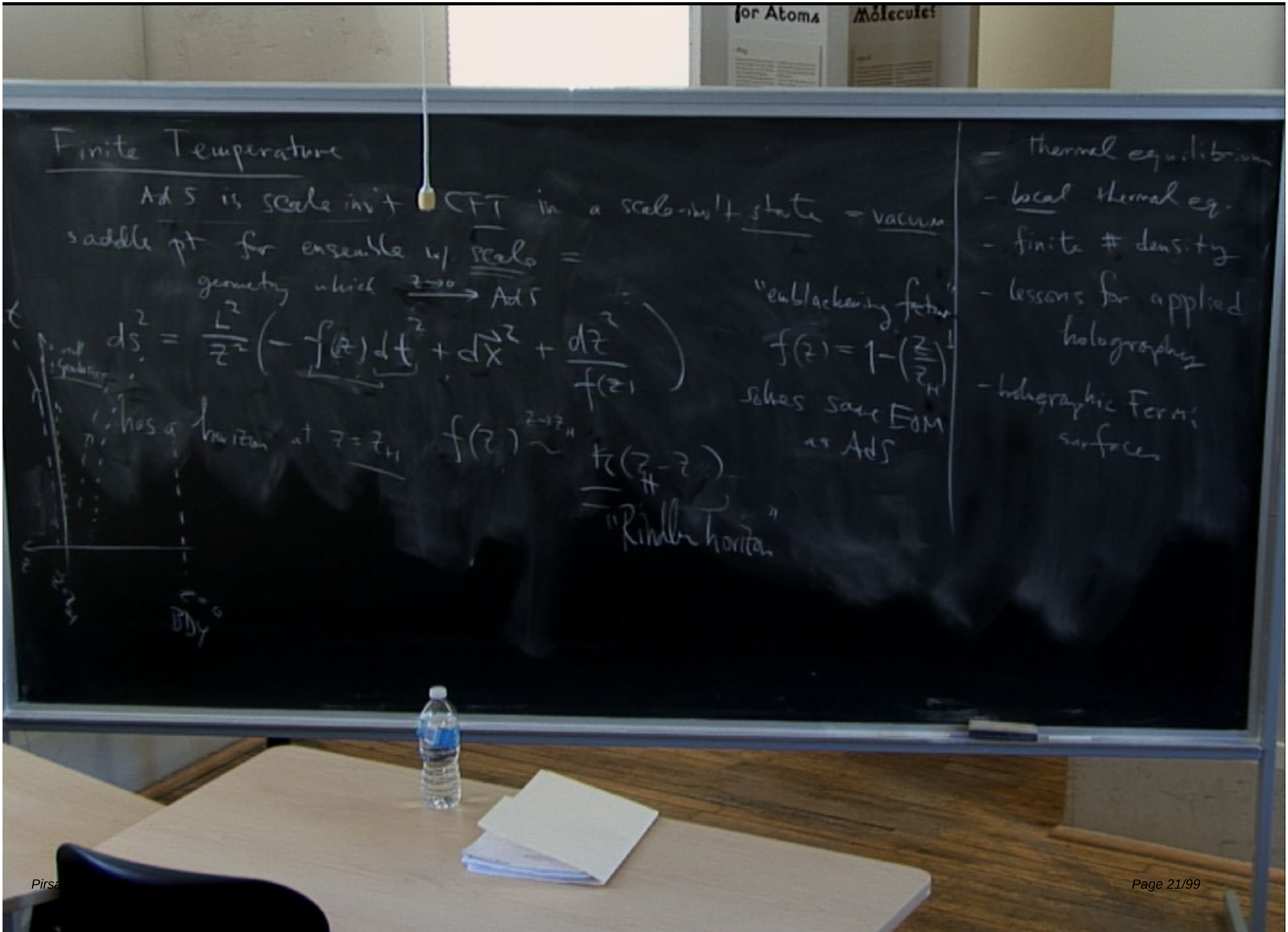
CFT in a scale-inv't state - vacuum
saddle pt
scale =
AdS
"emblackening factor"
$ds^2 =$
$d\vec{x}^2 + \frac{dz^2}{f(z)}$
$f(z) = 1 - \left(\frac{z}{z_H}\right)^d$
solves same EOM
as AdS
$f(z) \sim$
"Rindler horizon"
- thermal equilibrium
- local thermal eq.
- finite # density
- lessons for applied
holography
- holographic Fermi
surface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saddle pt for ensemble w/ scale =
geometry which z→0 → AdS
$ds^2 = \frac{L^2}{z^2}\left(-f(z)\,dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)}\right)$
"emblackening factor"
$f(z) = 1 - \left(\frac{z}{z_H}\right)^4$
solves same EOM as AdS
has a horizon at $z = z_H$
$f(z) \overset{z \to z_H}{\sim} (z_H - z)$
"Rindler horizon"
BDY
- thermal equilibrium
- local thermal eq.
- finite # density
- lessons for applied holography
- holographic Fermi surface
for Atoms
Molecules

Physics of horizons (Rindler)

euclidean regularity: $\tau = -it$

$\tau \cong \tau + 2\pi/\kappa_2$

$\Longrightarrow \boxed{T = \frac{1}{\pi z_H}} = \beta = 1/T$

$Z(T) = \mathrm{tr}\, e^{-\beta H}$

$\beta = \frac{1}{T}$

if: $ds^2_{\text{Bulk}} \overset{z\to 0}{\simeq} L^2 \frac{dz^2}{z^2} + \frac{L^2}{z^2} g^{(0)}_{\mu\nu} dx^\mu dx^\nu$

describes QFT on spacetime w g^{τ}_{QFT}

including topology of M_{QFT}

$\Longrightarrow$ eucl. time of QFT: $\tau \simeq \tau + \frac{1}{T}$

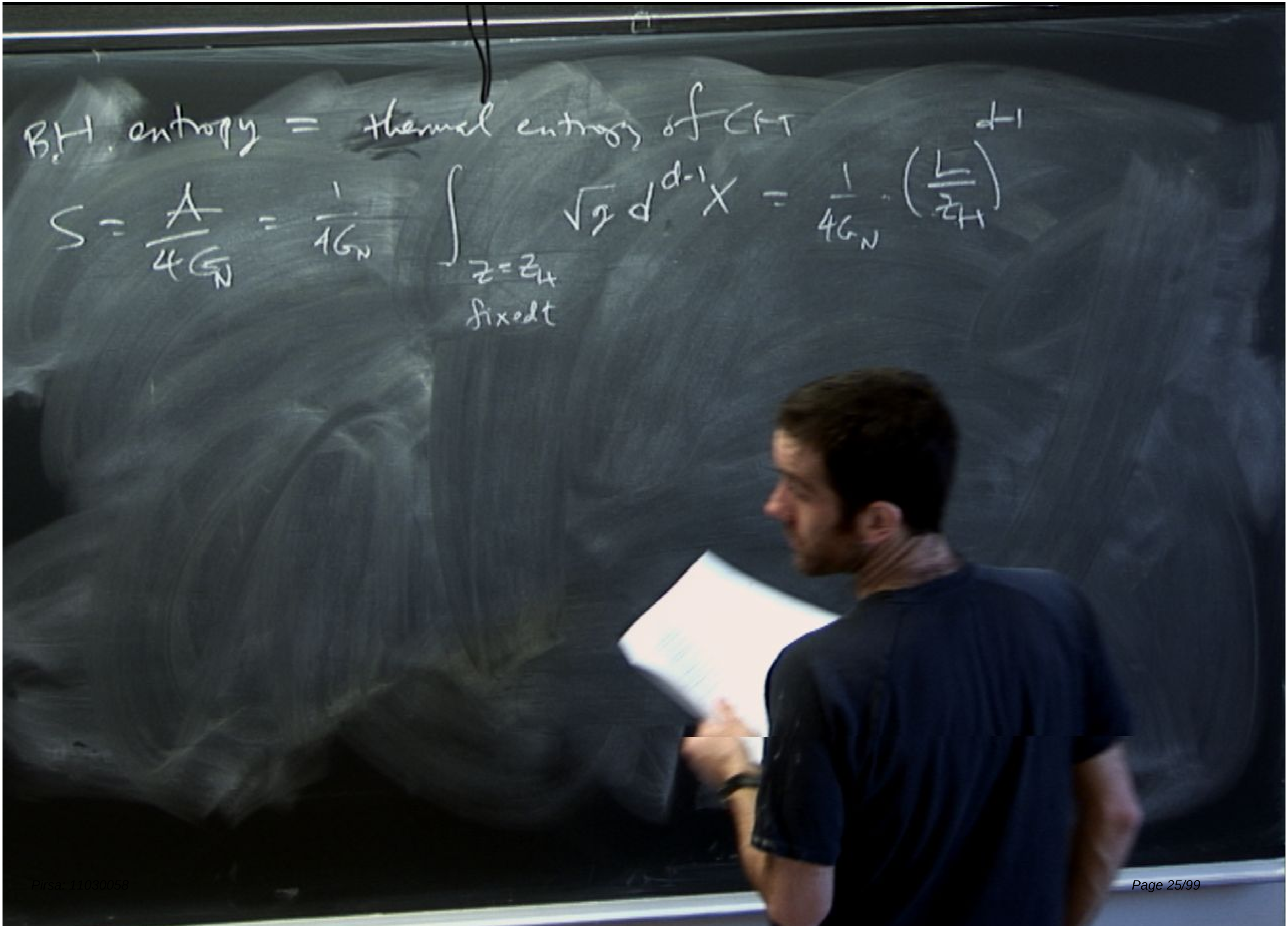
$\Longrightarrow$ static AdS BH = thermal eq. of dual CFT.

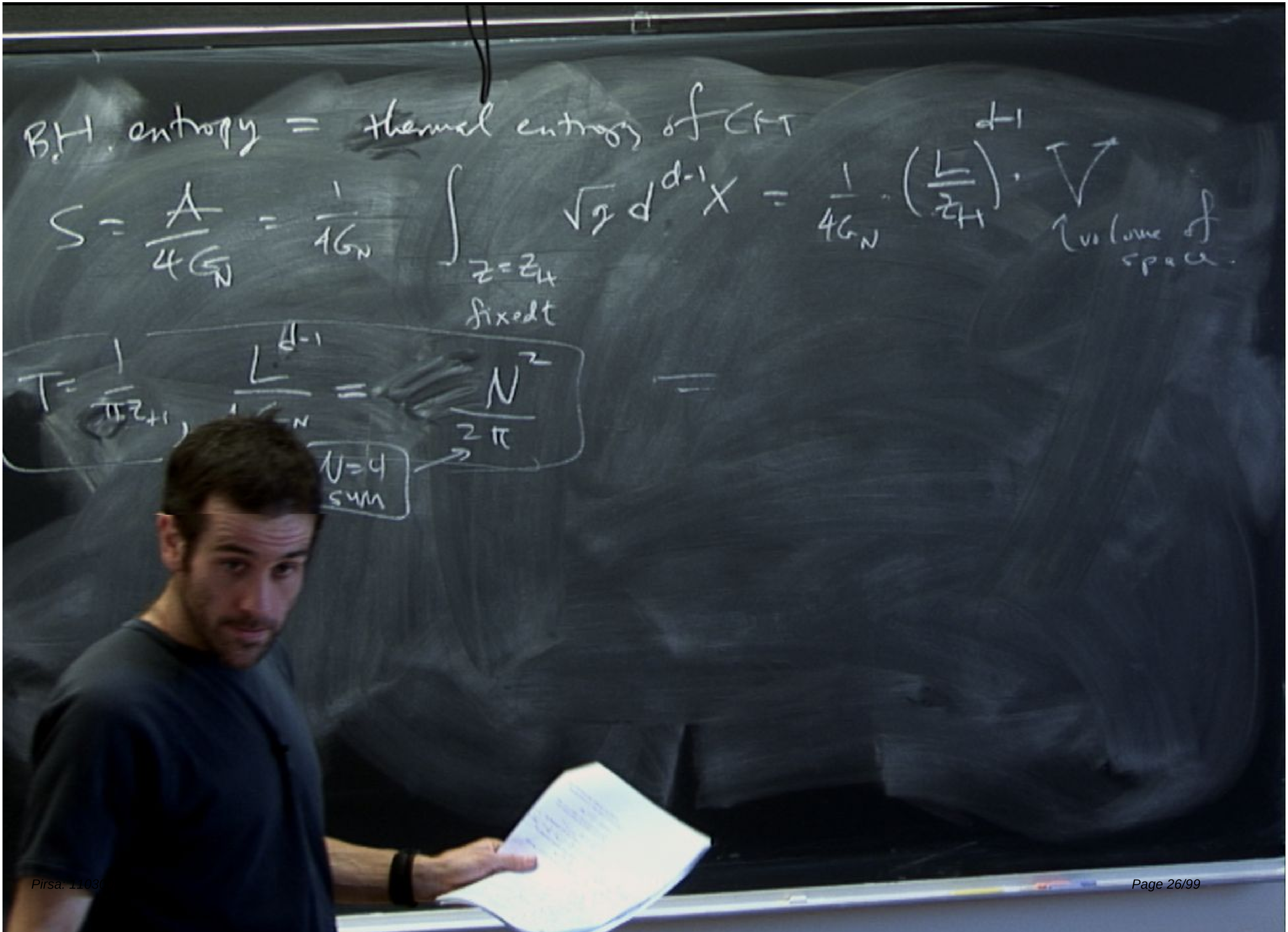
B.H. entropy = thermal entropy of CFT

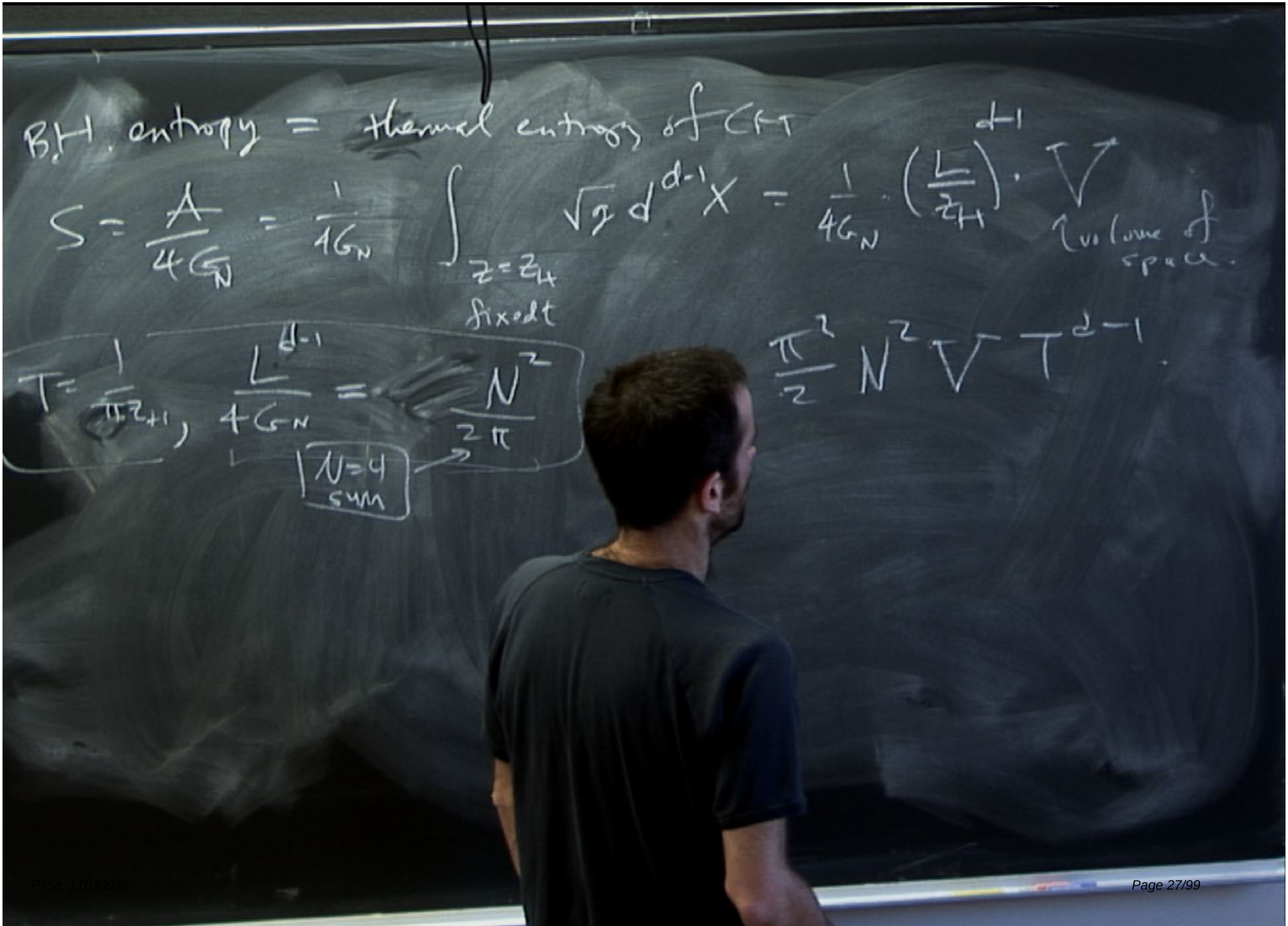
$$S = \frac{A}{4G_N}$$

B.H. entropy = thermal entropy of CFT

$$S = \frac{A}{4G_N} = \frac{1}{4G_N} \int_{\substack{z=z_H \\ \text{fixed } t}}$$


B.H. entropy = thermal entropy of CFT
$S = \frac{A}{4G_N} = \frac{1}{4G_N} \int_{\substack{z=z_H \\ \text{fixed } t}} \sqrt{g}\, d^{d-1}x = \frac{1}{4G_N} \left(\frac{L}{z_H}\right)^{d-1}$


B.H. entropy = thermal entropy of CFT
S = A/4G_N = 1/4G_N ∫ √g d^{d-1}x = 1/4G_N (L/z_H)^{d-1} · V
z = z_H
fixed t
volume of space
T = 1/πz_H
L^{d-1}/4G_N = N^2/2π
N=4
SYM


B.H. entropy = thermal entropy of CFT
$S = \frac{A}{4G_N} = \frac{1}{4G_N} \int_{z=z_H, \text{ fixed } t} \sqrt{g}\, d^{d-1}x = \frac{1}{4G_N} \left(\frac{L}{z_H}\right)^{d-1} \cdot V$
volume of space
$T = \frac{1}{\pi z_H}, \quad \frac{L^{d-1}}{4G_N} = \frac{N^2}{2\pi}$
$\mathcal{N}=4$ SYM
$\frac{\pi^2}{2} N^2 V T^{d-1}$

B.H. entropy = thermal entropy of CFT

$$S = \frac{A}{4G_N} = \frac{1}{4G_N}\int_{\substack{z=z_H \\ \text{fixed } t}} \sqrt{g}\, d^{d-1}x = \frac{1}{4G_N}\left(\frac{L}{z_H}\right)^{d-1} \cdot V$$

volume of space

$$\boxed{T = \frac{1}{\pi z_H}, \quad \frac{L^{d-1}}{4G_N} = \frac{N^2}{2\pi}}$$

$\boxed{\mathcal{N}=4 \text{ SYM}}$

$$= \frac{\pi^2}{2} N^2 V T^{d-1}.$$

B.H. entropy = thermal entropy of CFT

$$S = \frac{A}{4G_N} = \frac{1}{4G_N}\int_{\substack{z=z_H \\ \text{fixed } t}} \sqrt{g}\, d^{d-1}x = \frac{1}{4G_N}\left(\frac{L}{z_H}\right)^{d-1} \cdot V$$

volume of space

$$\boxed{T = \frac{1}{\pi z_H}, \quad \frac{L^{d-1}}{4G_N} = \frac{N^2}{2\pi}} \qquad = \frac{\pi^2}{2} N^2 V T^{d-1}$$

$\boxed{\mathcal{N}=4 \text{ SYM}}$

$$s = \frac{S}{V} = \frac{a_{BH}}{4G_N}$$

B.H. entropy = thermal entropy of CFT

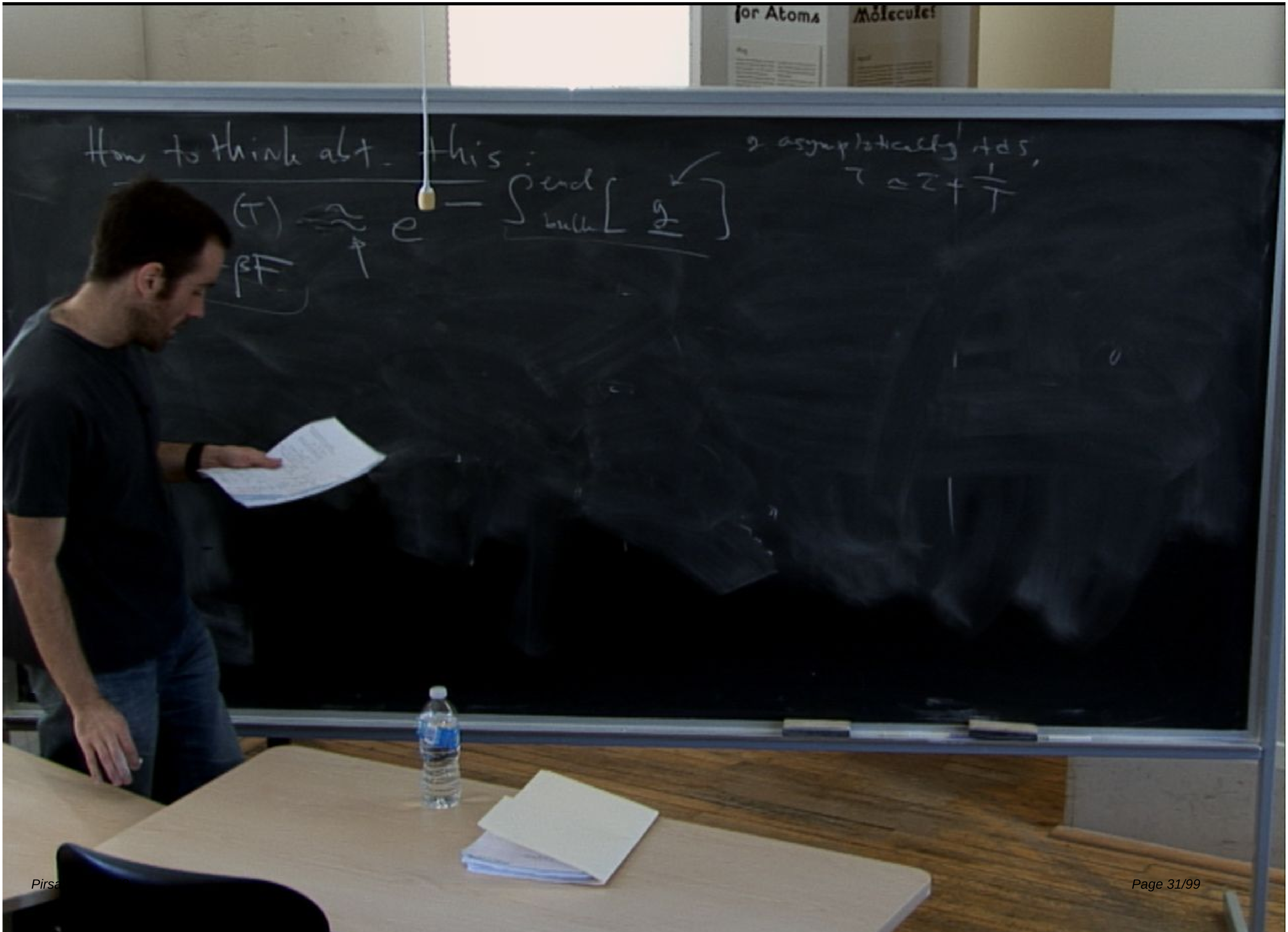
$$S = \frac{A}{4G_N} = \frac{1}{4G_N}\int_{\substack{z=z_H \\ \text{fixed } t}} \sqrt{g}\, d^{d-1}x = \frac{1}{4G_N}\left(\frac{L}{z_H}\right)^{d-1} \cdot V$$

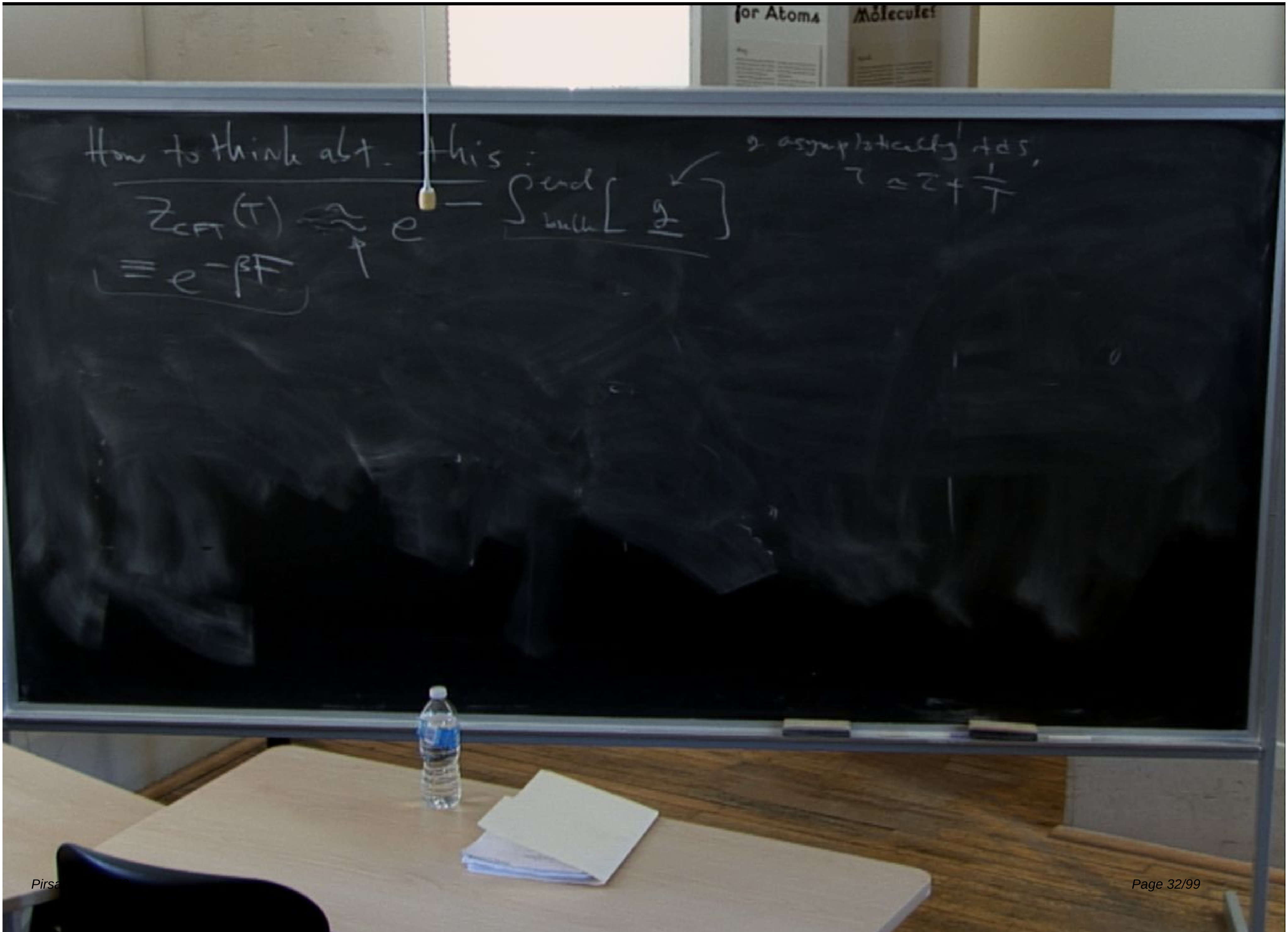
volume of space

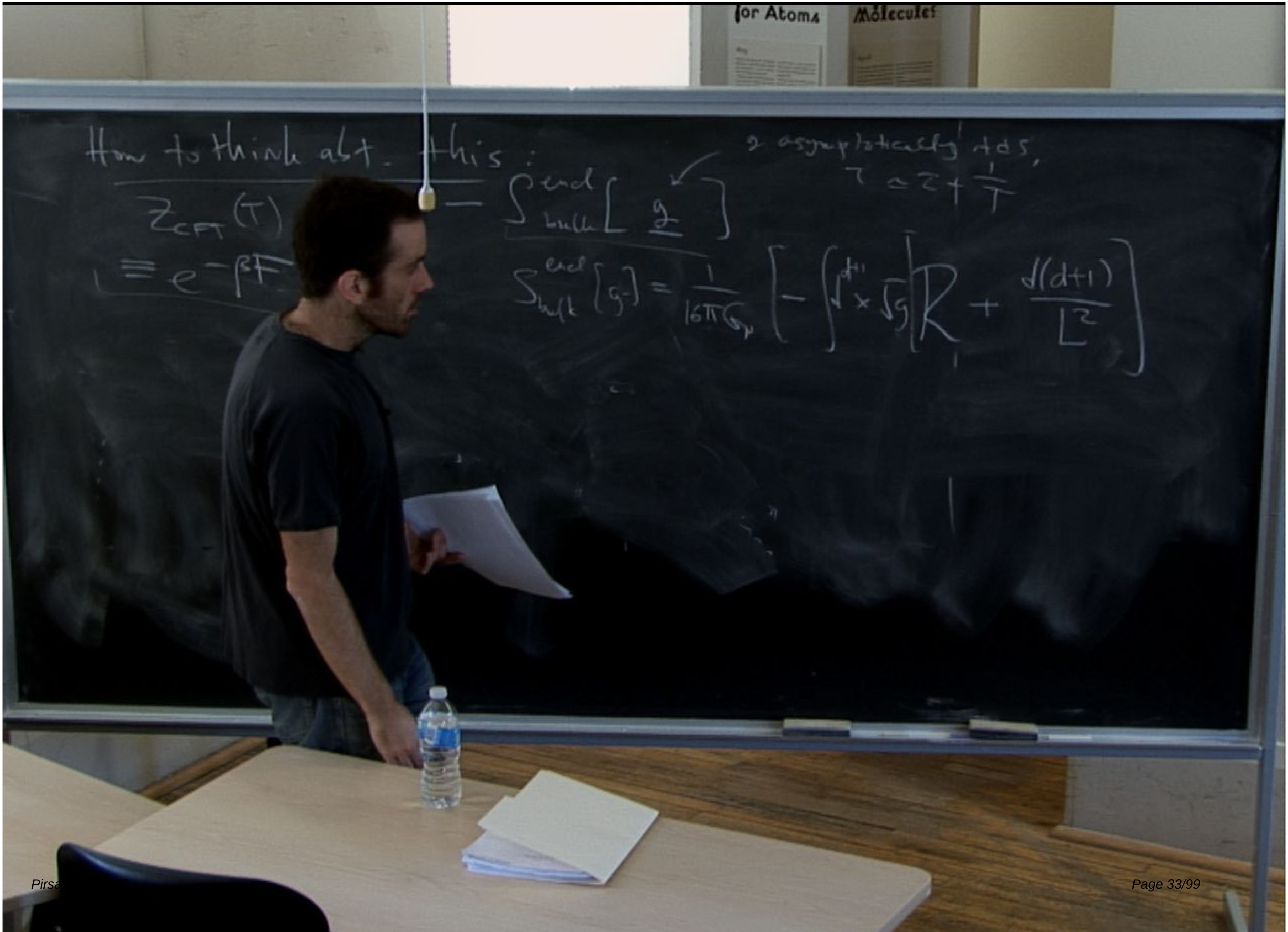
$$\boxed{T = \frac{1}{\pi z_H}, \quad \frac{L^{d-1}}{4G_N} = \frac{N^2}{2\pi}} = \underbrace{\frac{\pi^2}{2}}\,\underbrace{N^2}\,\underbrace{V}\;\underbrace{T^{d-1}}.$$

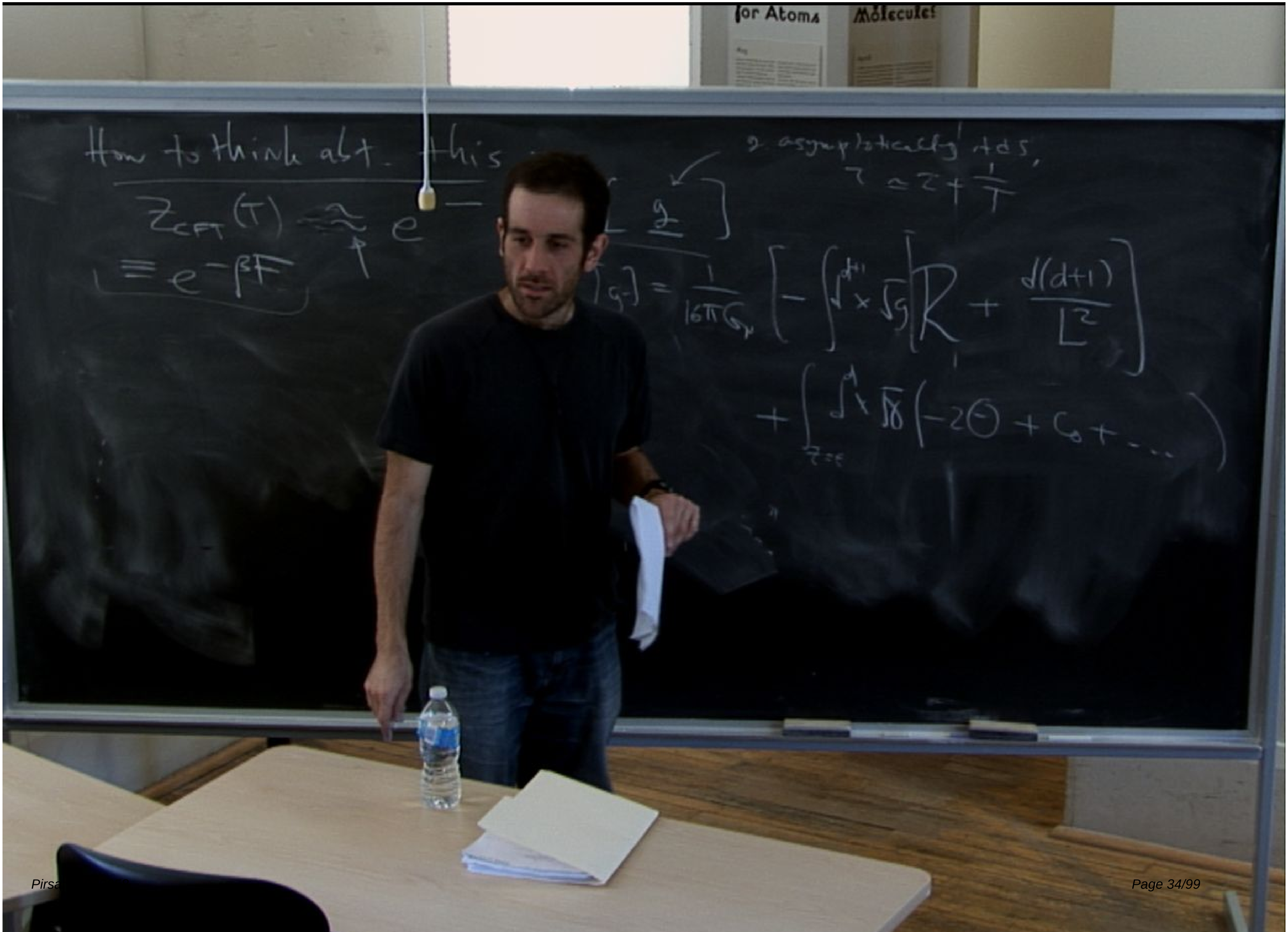
$\boxed{\mathcal{N}=4 \text{ SYM}}$

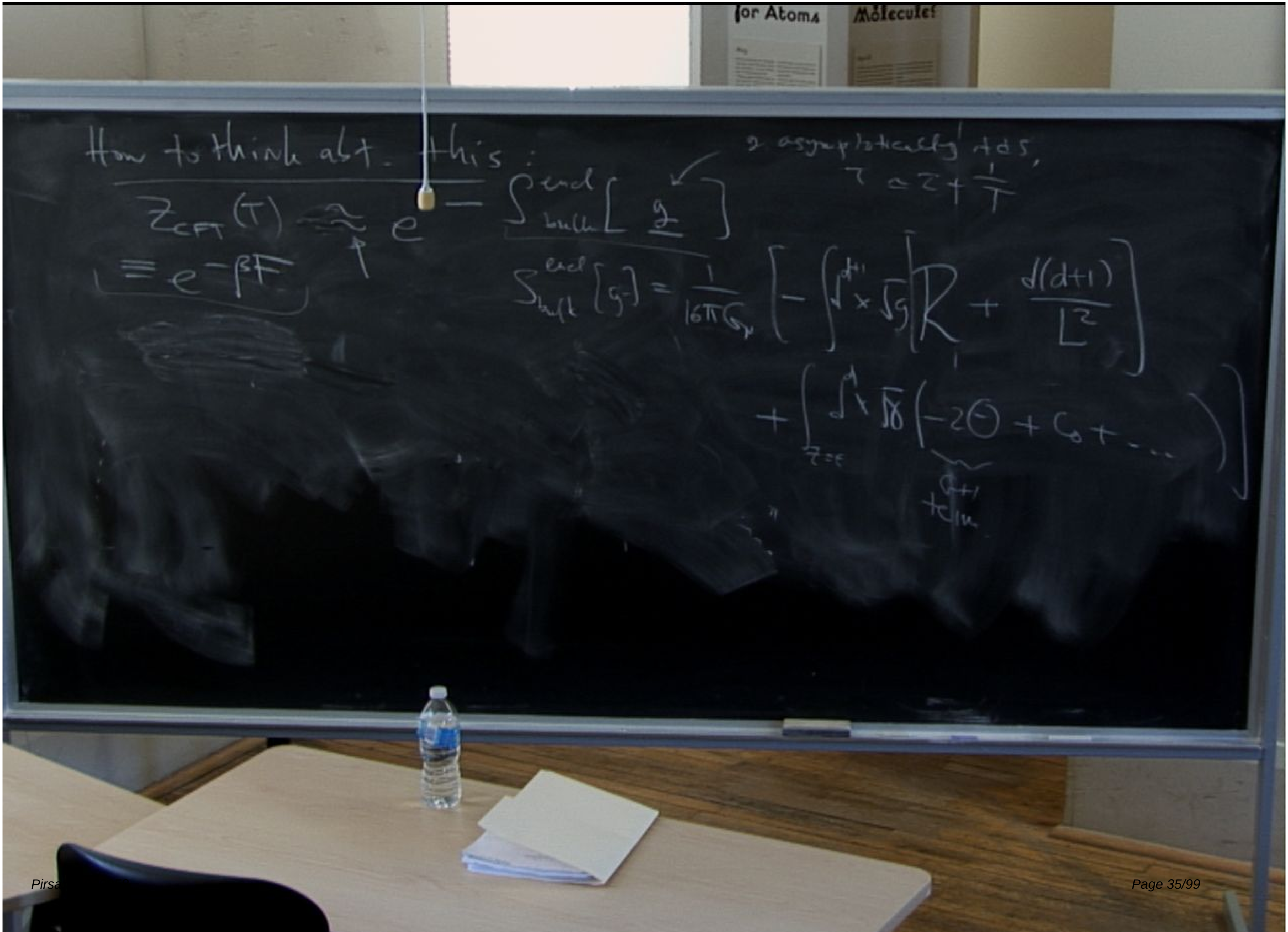
$$s = \frac{S}{V} = \frac{a_{BH}}{4G_N} = \frac{\pi^2}{2} N^2 T^{d-1}.$$


How to think abt. this :
for Atoms
Molecules


How to think abt. this:
Z_CFT(T) ≈ e^(−∫_bulk^end [g])
≡ e^(−βF)
g asymptotically AdS,
τ ≃ τ + 1/T
for Atoms
Molecules


for Atoms
Molecules
How to think abt. this :
asymptotically AdS,
$Z_{CFT}(T)$
$\int^{end}_{bulk}[g]$
$\equiv e^{-\beta F}$
$S^{end}_{bulk}[g] = \frac{1}{16\pi G_N}\left[-\int d^{d+1}x\sqrt{g}\left(R+\frac{d(d+1)}{L^2}\right)\right]$


for Atoms
Molecules
How to think abt. this
asymptotically AdS,


for Atoms
Molecules
How to think abt. this:
$Z_{CFT}(T) \approx e^{-S^{encl}_{bulk}[g]}$
$\equiv e^{-\beta F}$
g asymptotically AdS,
$\tau \simeq \tau + \frac{1}{T}$
$S^{encl}_{bulk}[g] = \frac{1}{16\pi G_N}\left[-\int d^{d+1}x\sqrt{g}\left(R + \frac{d(d+1)}{L^2}\right) + \int_{z=\epsilon} d^dx\sqrt{\gamma}\left(\underbrace{-2\Theta}_{\text{G-H term}} + c_0 + \dots\right)\right]$

Physics of Rindler horizons

euclidean regularity: $\tau = -it$

$\tau \cong \tau + 2\pi/\kappa$

$\Longrightarrow \boxed{T = \frac{1}{\pi z_H}} = \beta = 1/T$

$Z(T) = \mathrm{Tr}\, e^{-\beta H}$

$\beta = \frac{1}{T}$

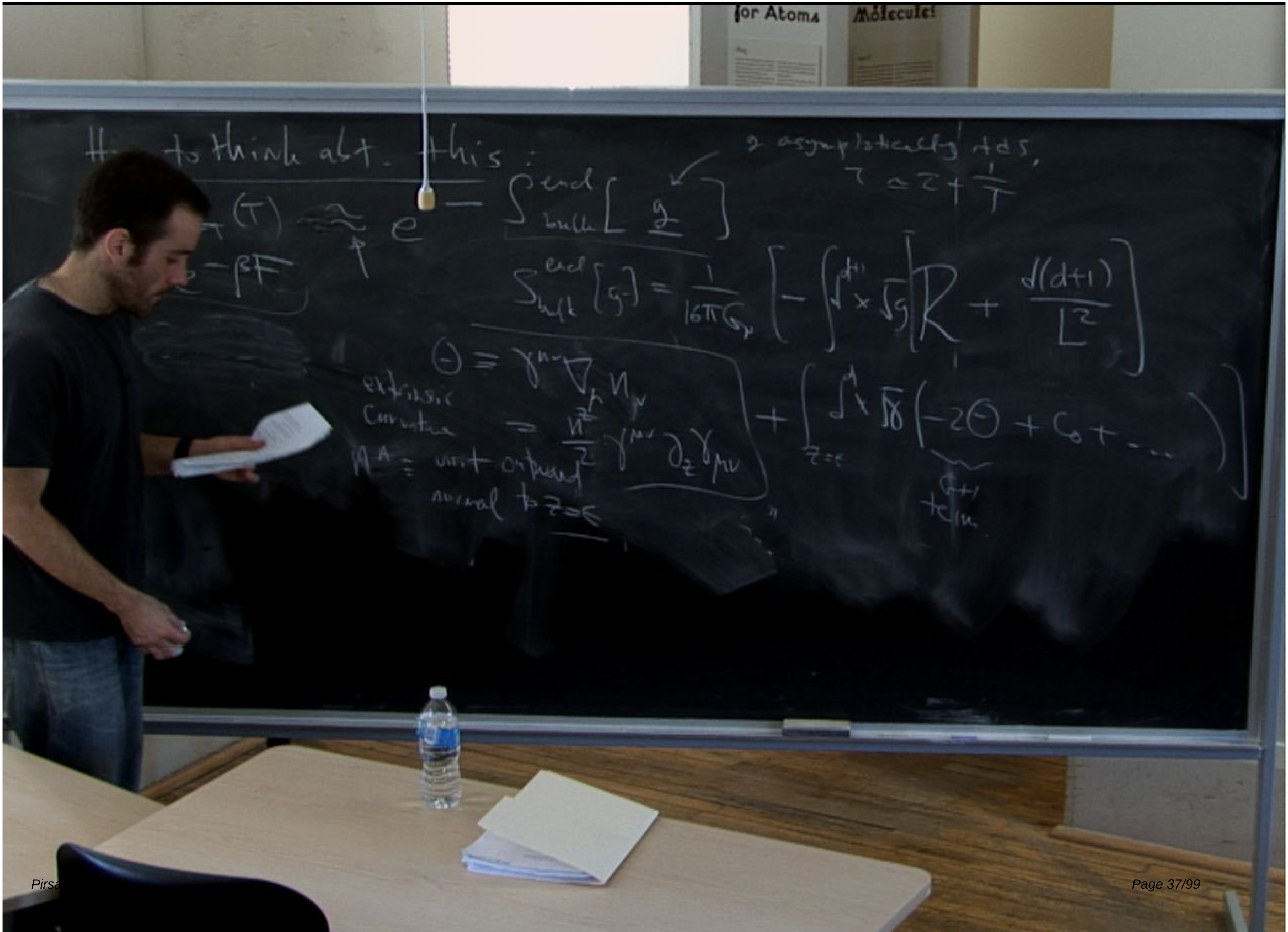
if: $ds^2_{Bulk} \overset{z\to 0}{\simeq} L^2 \frac{dz^2}{z^2} + \gamma_{\mu\nu} dx^\mu dx^\nu$

describe QFT on spacetime w ds^2_{QFT}

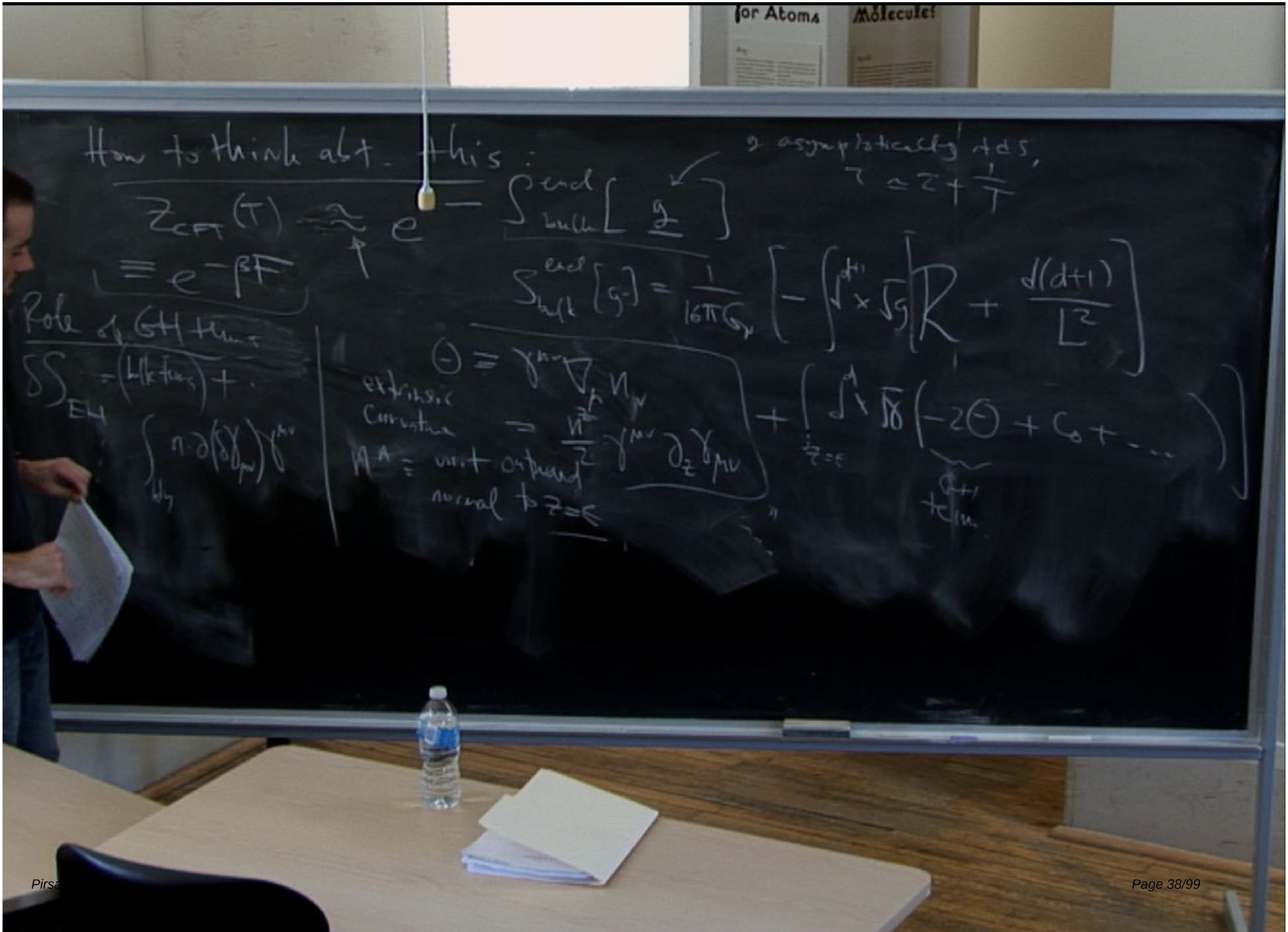
including topology of M_{QFT}

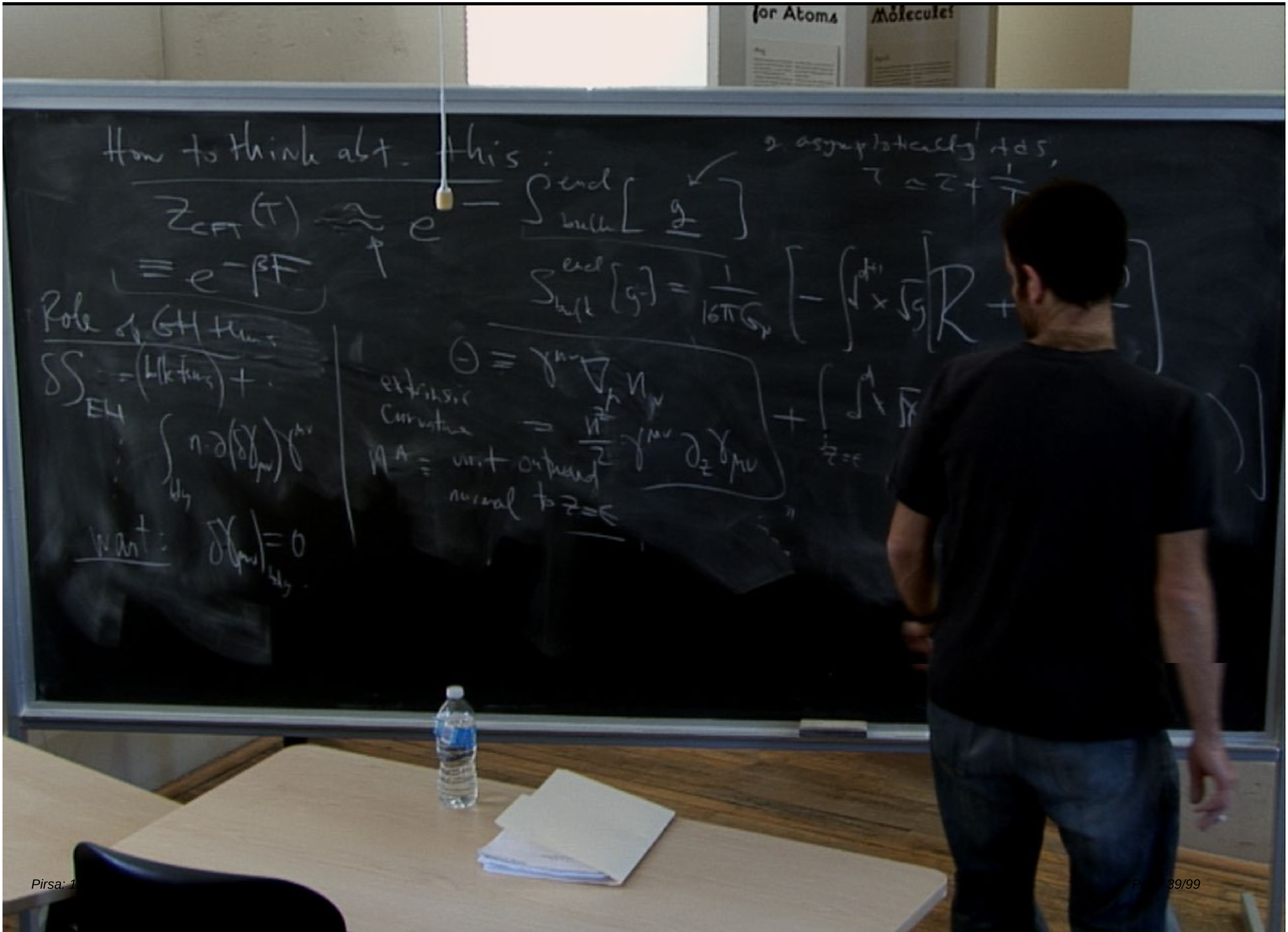
$\Longrightarrow$ eucl. time of QFT: $\tau \simeq \tau + \frac{1}{T}$

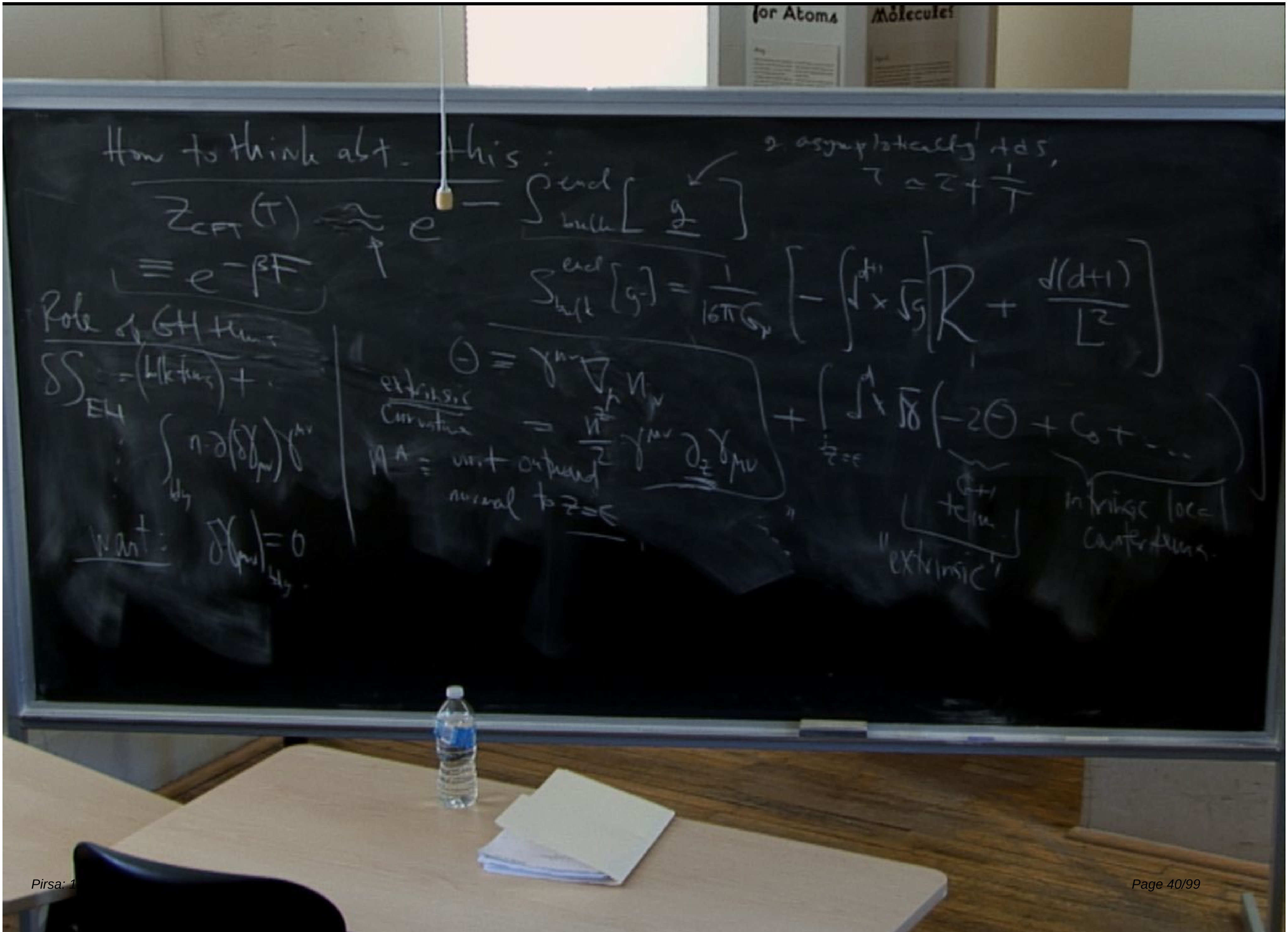
$\Longrightarrow$ static AdS BH = thermal eq. of dual CFT.



for Atoms
Molecules
to think abt. this :
extrinsic
Curvature
unit orthonormal
normal to z=ε


How to think abt. this :
asymptotically AdS,
$Z_{CFT}(T) \approx e^{-S^{eucl}_{bulk}[g]}$
$\equiv e^{-\beta F}$
$S^{eucl}_{bulk}[g] = \frac{1}{16\pi G_N}\left[-\int d^{d+1}x\sqrt{g}\left(R + \frac{d(d+1)}{L^2}\right) + \int_{z=\epsilon} d^dx\sqrt{\gamma}\left(-2\Theta + C_0 + \dots\right)\right]$
Role of GH term
extrinsic curvature
$\Theta = \gamma^{\mu\nu}\nabla_\mu n_\nu = \frac{n^z}{2}\gamma^{\mu\nu}\partial_z\gamma_{\mu\nu}$
n^A = unit outward normal to $z=\epsilon$


for Atoms
Molecules
How to think abt. this:
Role of GH term
extrinsic curvature
unit outward normal to z=ε
want:


for Atoms
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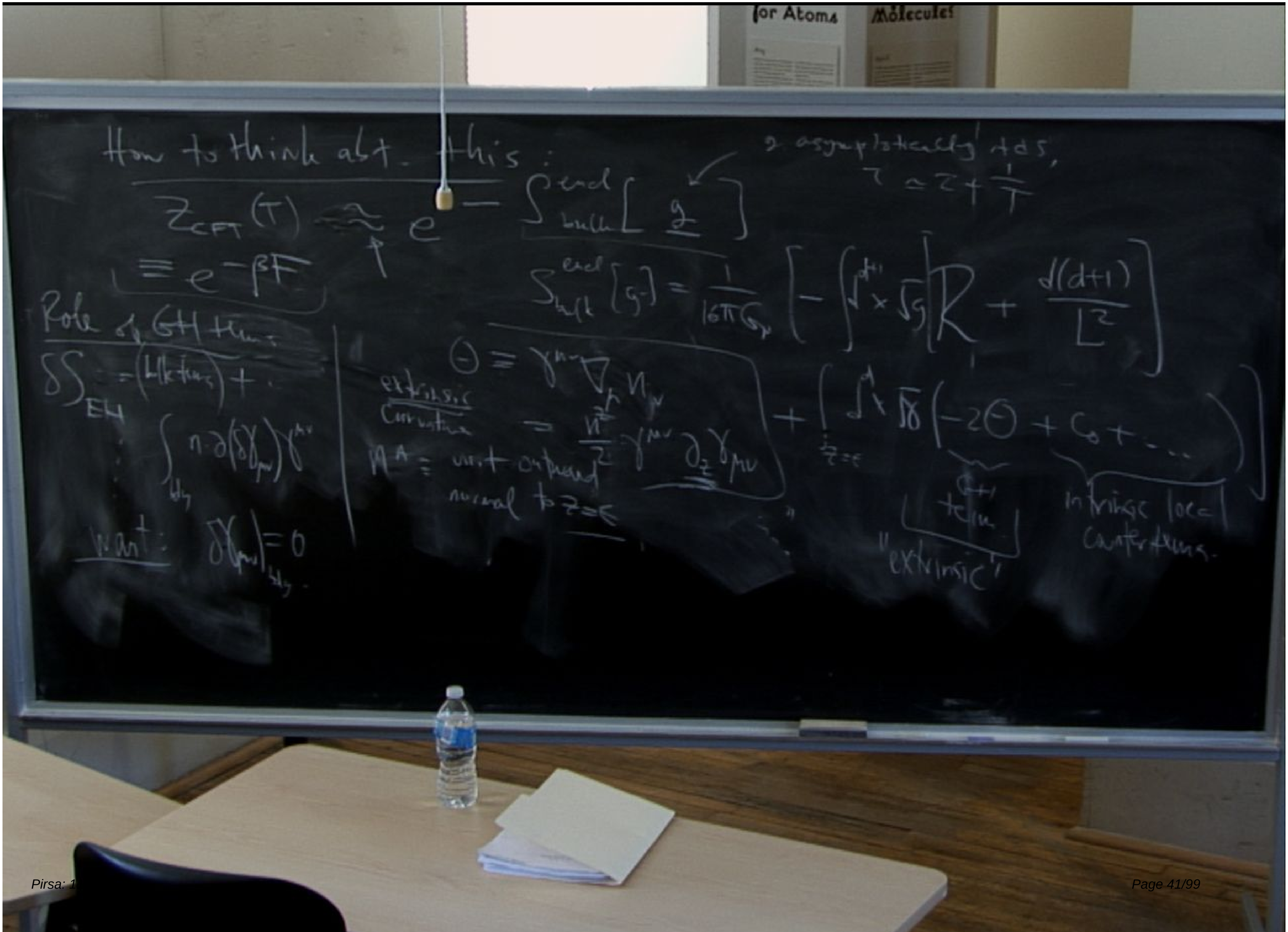
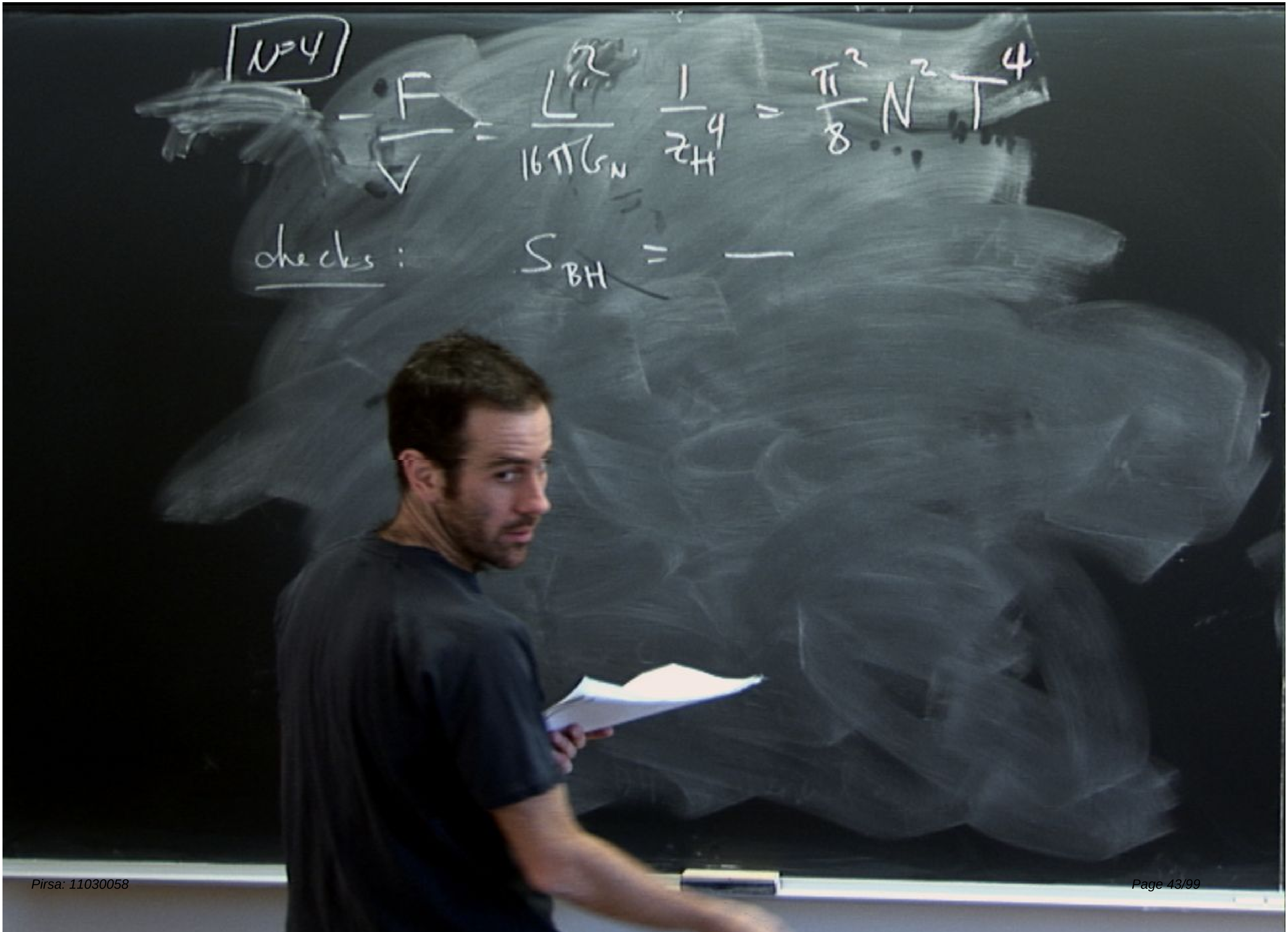
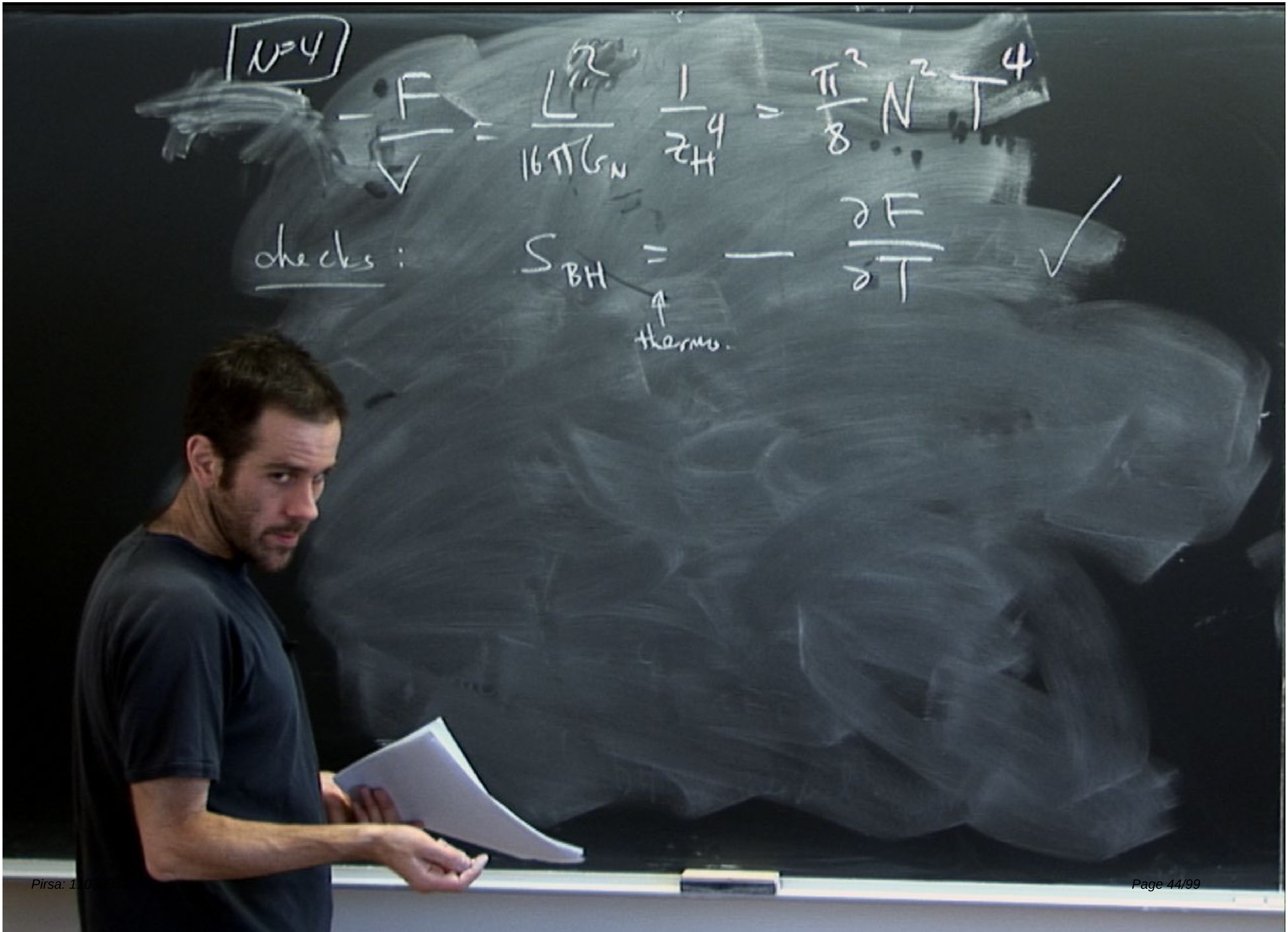
How to think abt. this :
Z_CFT(T) ≃ e^{- S^eucl_bulk [g]}
≡ e^{-βF}
g asymptotically AdS,
S^eucl_bulk [g] = 1/(16πG_N) [- ∫ d^{d+1}x √g (R + d(d+1)/L^2) + ∫_{z=ε} d^d x √γ (-2Θ + c_0 + ...)]
Role of GH term
δS_EH = (blk terms) + ∫_bdy n·∂(δγ_μν) γ^μν
want: δγ_μν|_bdy = 0
Θ = γ^μν ∇_μ n_ν
extrinsic curvature = (z/2) γ^μν ∂_z γ_μν
n^A = unit outward normal to z=ε
GH term
"extrinsic"
intrinsic local counterterms



for Atoms
Molecules
How to think abt. this:
Role of GH term
extrinsic curvature
unit outward normal to z = ε
want:
"extrinsic"
intrinsic local counterterms.


N=4
d=3+1,


N=4
-F/V = L^3/16πG_N 1/z_H^4 = π²/8 N² T⁴
checks: S_BH = —


N=4
$-\frac{F}{V} = \frac{L^3}{16\pi G_N}\frac{1}{z_H^4} = \frac{\pi^2}{8}N^2T^4$
checks:
$S_{BH} = -\frac{\partial F}{\partial T}$
thermo.

$\boxed{\mathcal{N}=4}$

$$-\frac{F}{V} = \frac{L^3}{16\pi G_N}\frac{1}{z_H^4} = \frac{\pi^2}{8}N^2T^4$$

checks: $S_{BH} = -\frac{\partial F}{\partial T}$ ✓

thermo.

B.H. entropy = thermal entropy of CFT

$$S_{BH} = \frac{A}{4G_N} = \frac{1}{4G_N}\int_{\substack{z=z_H \\ \text{fixed } t}} \sqrt{g}\, d^{d-1}x = \frac{1}{4G_N}\left(\frac{L}{z_H}\right)^{d-1} \cdot \underbrace{V}_{\text{volume of space}}$$

$$T = \frac{1}{\pi z_H}, \qquad \frac{L^{d-1}}{4G_N} = \frac{N^2}{2\pi} \qquad = \frac{\pi^2}{2} N^2 V T^{d-1}.$$

$\boxed{\mathcal{N}=4 \text{ SYM}}$

$$\frac{S}{V} = \frac{a_{BH}}{4G_N} = \frac{\pi^2}{2} N^2 T^{d-1}.$$

$\boxed{\mathcal{N}=4}$

$$-\frac{F}{V} = \frac{L^3}{16\pi G_N}\,\frac{1}{z_H^4} = \frac{\pi^2}{8} N^2 T^4$$

checks: • $S_{BH} = -\frac{\partial F}{\partial T}$ ✓

(1st law)

horizon

thermo.

$\int_{\text{spacetime}}(\;)$

• $C_V = \left.\frac{\partial E}{\partial T}\right|_V$

$\boxed{N=4}$

$$-\frac{F}{V} = \frac{L^3}{16\pi G_N}\frac{1}{z_H^4} = \frac{\pi^2}{8}N^2T^4$$

checks: • $S_{BH} = -\frac{\partial F}{\partial T}$ ✓

(1st law)

horizon

thermo.

$\int_{\text{spacetime}}(\ )$

• $c_V = \frac{\partial E}{\partial T}\Big|_V > 0$

(AdS BH are different!)

$\boxed{\mathcal{N}=4}$

$$-\frac{F}{V} = \frac{L^{3}}{16\pi G_N}\frac{1}{z_H^4} = \frac{\pi^2}{8}N^2 T^4$$

checks: • $S_{BH} = -\frac{\partial F}{\partial T}$ ✓

(1st law)

horizon

thermo.

$\int_{\text{spectrum}}(\ )$

• $C_V = \left.\frac{\partial E}{\partial T}\right|_V > 0$

(AdS BH are different!)

• uniqueness of stationary BH ("no hair")

$\boxed{\begin{matrix}N=4\\ SYM\end{matrix}}$

$$-\frac{F}{V} = \frac{L^2}{16\pi G_N}\frac{1}{z_H^4} = \frac{\pi^2}{8} N^2 T^4$$

checks: • $S_{BH} = -\frac{\partial F}{\partial T}$ ✓

(1st law) horizon thermo. $\int_{\text{spacetime}} (\ )$

• $C_V = \left.\frac{\partial E}{\partial T}\right|_V > 0$ (AdS BH are different!)

• uniqueness of stationary BH $\longleftrightarrow$ few state vars
("no hair") in eq. thermo.

$\boxed{\mathcal{N}=4 \text{ SYM}}$

$\{\lambda \ll 1 \,\&\, N\}$

$$-\frac{F}{V} = \frac{L^2}{16\pi G_N}\frac{1}{z_H^4} = \frac{\pi^2}{8} N^2 T^4$$

checks: • $S_{BH} = -\frac{\partial F}{\partial T}$ ✓

(1st law)

horizon

thermo.

$\int_{\text{spacetime}}(\ )$

• $C_V = \frac{\partial E}{\partial T}\Big|_V > 0$

(AdS BH are different!)

• uniqueness of stationary BH ("no hair!") $\longleftrightarrow$ few state vars in eq. thermo.

$\mathcal{N}=4$ SYM

$\{\backslash \mathrm{cal} \, \mathcal{N}\}$

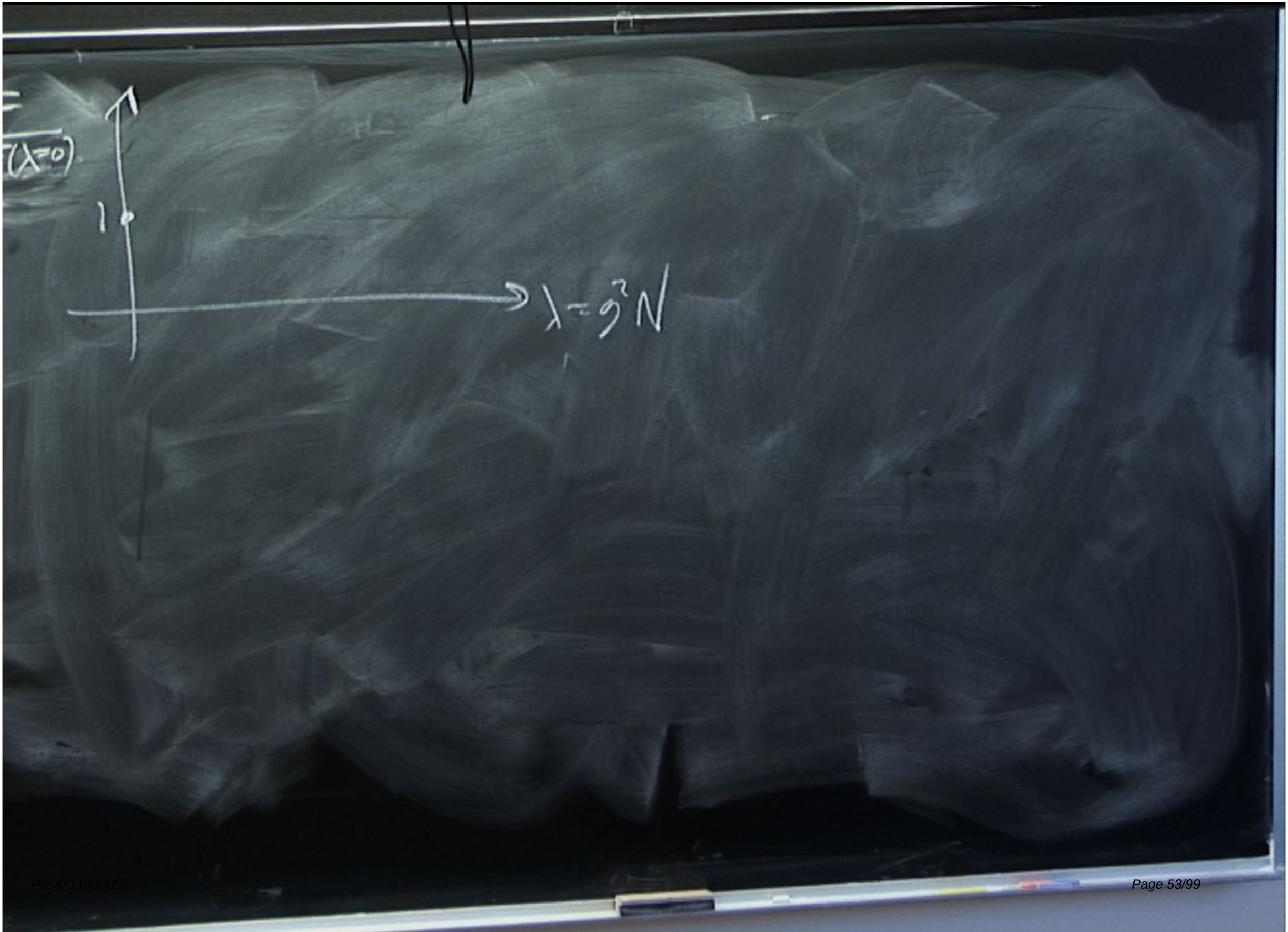
$$\frac{F}{V} = \frac{L^3}{16\pi G_N} \frac{1}{z_H^4} = \frac{\pi^2}{8} N^2 T^4$$

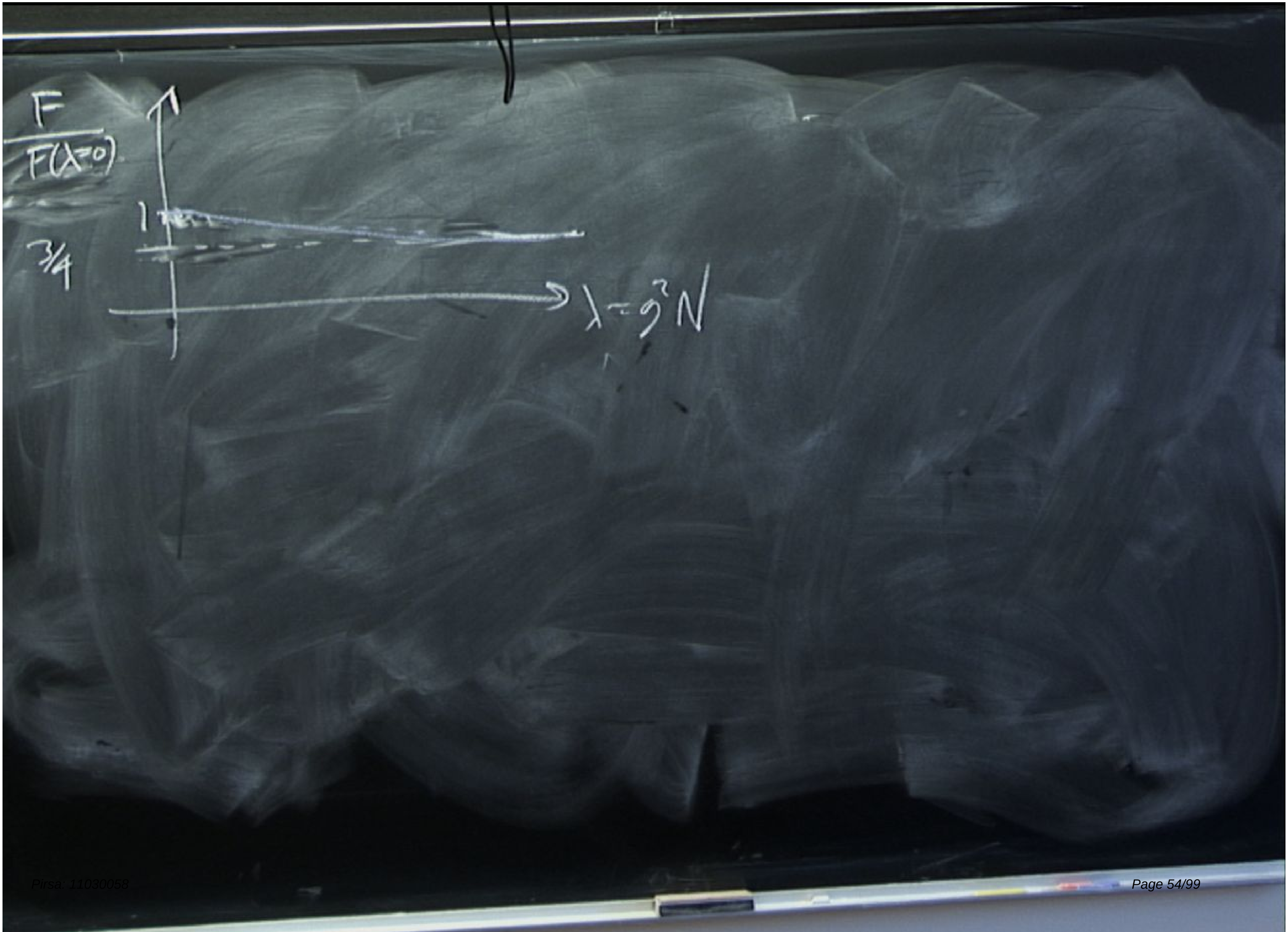
checks: • $S_{BH} = -\frac{\partial F}{\partial T}$ ✓

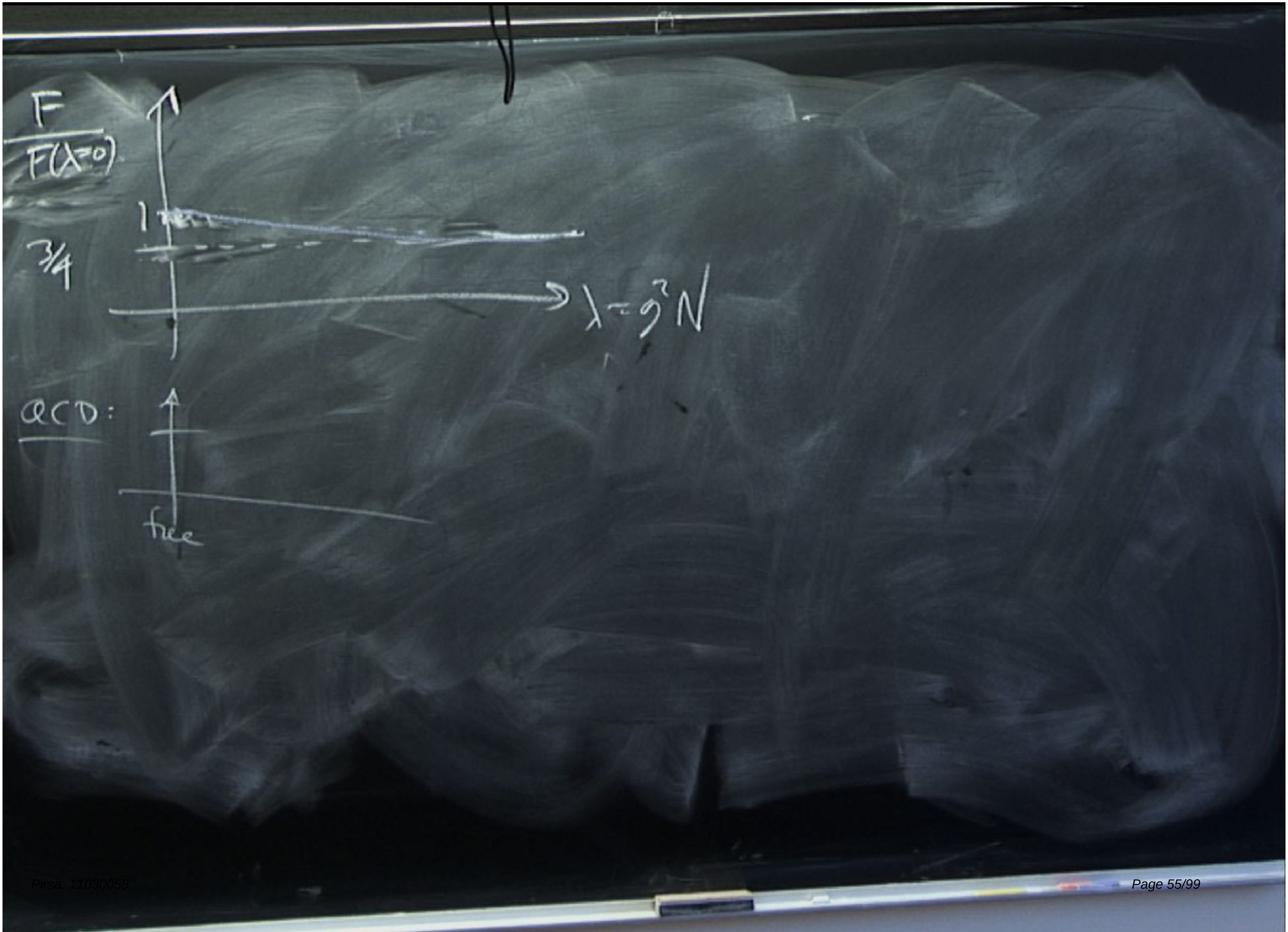
(1st law) horizon thermo. $\int_{\text{spacetime}} (\ )$

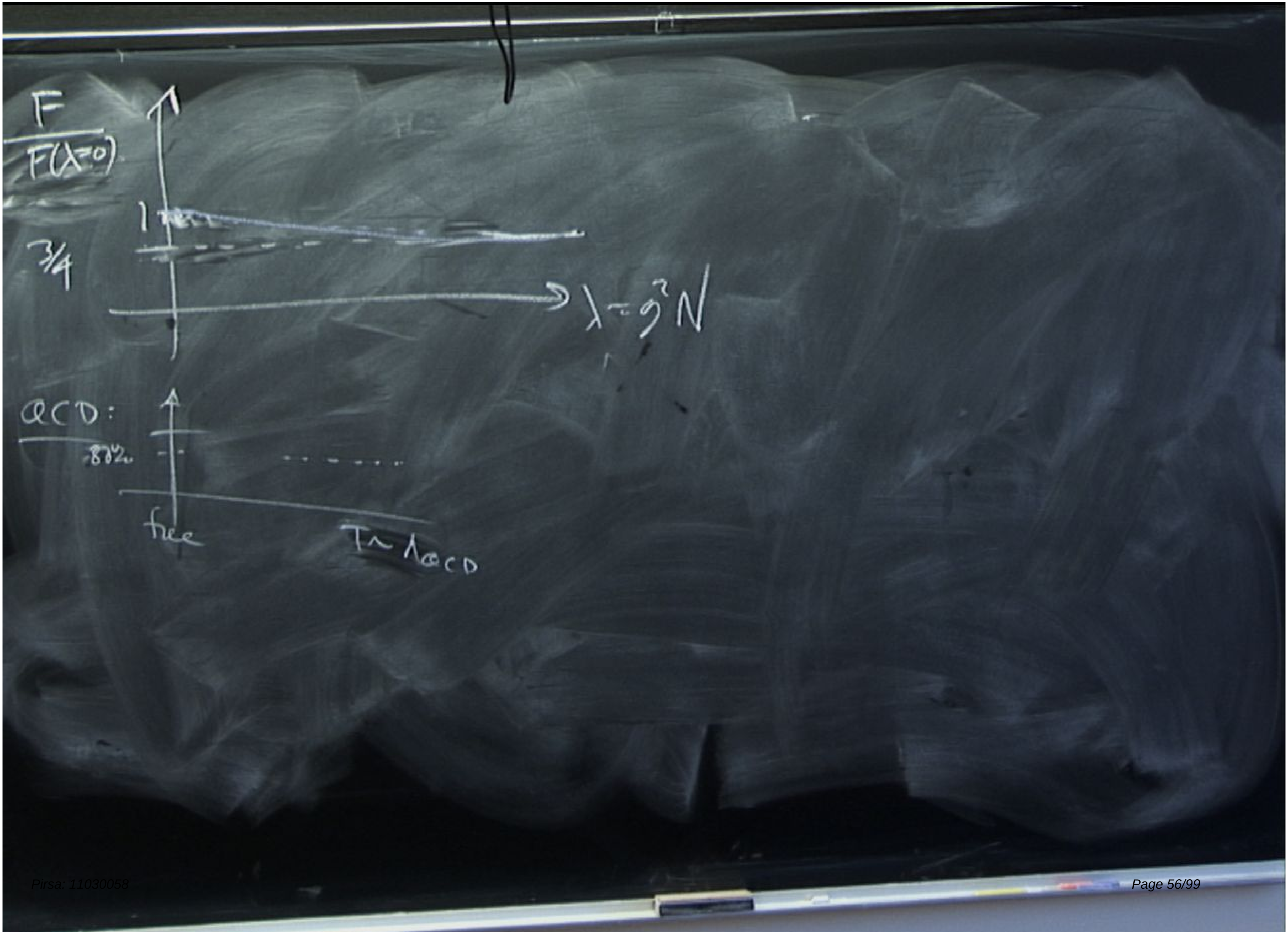
• $C_V = \left.\frac{\partial E}{\partial T}\right|_V > 0$ (AdS BH are different!)

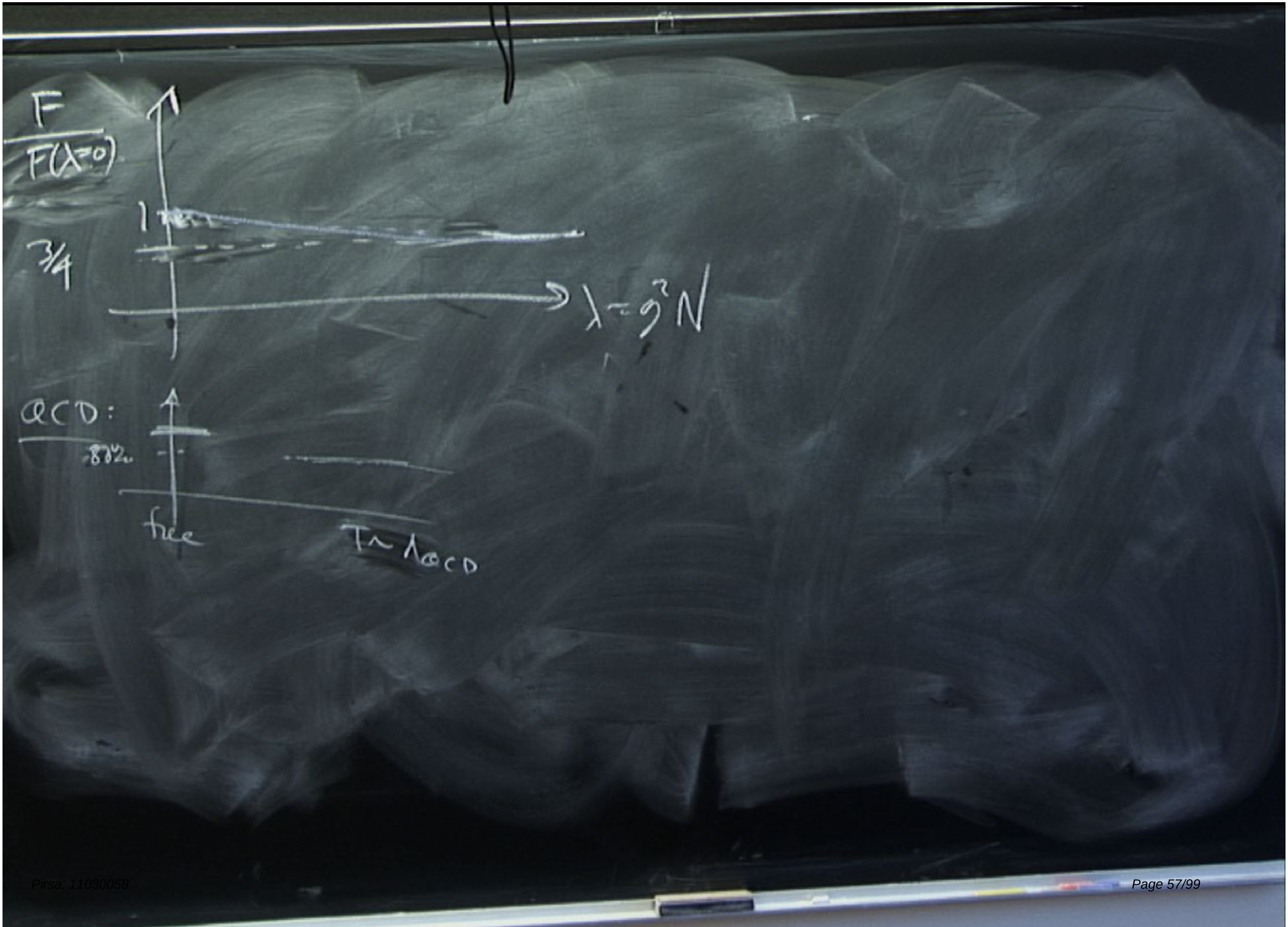
• uniqueness of stationary BH ("no hair") $\longleftrightarrow$ few state vars in eq. thermo.


(λ=0)
1
λ = g²N


$\frac{F}{F(\lambda=0)}$
1
3/4
$\lambda = g^2 N$


F/F(λ=0)
1
3/4
λ = g²N
QCD:
free


F / F(λ=0)
1
3/4
λ=g²N
QCD:
80%
free
T~Λ QCD


$\frac{F}{F(\lambda=0)}$
1
3/4
$\lambda = g^2 N$
QCD:
80%
free
$T \sim \Lambda_{QCD}$

$\frac{F}{F(\lambda=0)}$

1

3/4

$\lambda = g^2 N$

QCD:

80%

free

$T \sim \Lambda_{QCD}$

Compute

$\boxed{\begin{array}{c}\mathcal{N}=4\\ SYM\\ \{\lambda, N\}\end{array}}$

$$\frac{F}{V} = \frac{L^3}{16\pi G_N}\frac{1}{z_H^4} = \frac{\pi^2}{8}N^2T^4$$

checks: • $S_{BH} = -\frac{\partial F}{\partial T}$ ✓

(1st law) horizon thermo. $\int_{\text{spacetime}}(\ )$

• $C_V = \frac{\partial E}{\partial T}\Big|_V > 0$ (AdS BH are different!)

• uniqueness of stationary BH ("no hair") $\longleftrightarrow$ few state v... in eq. th...

$(\lambda=0)$

1

$3/4$

$\lambda = g^2 N$

QCD:

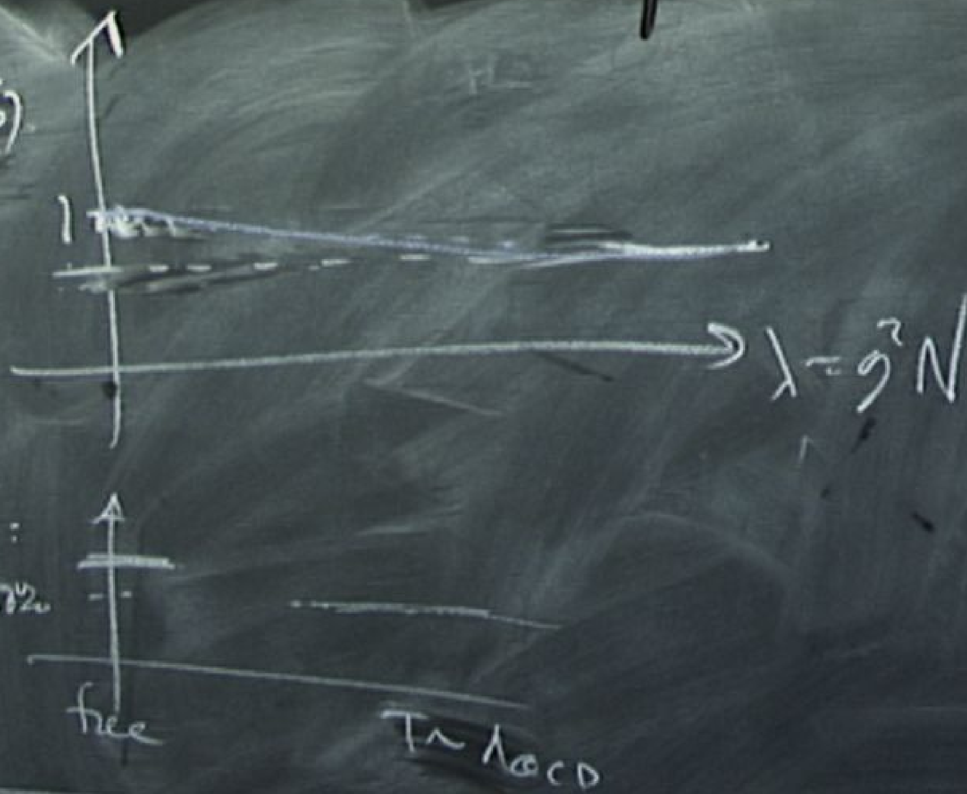
$.8?$

free

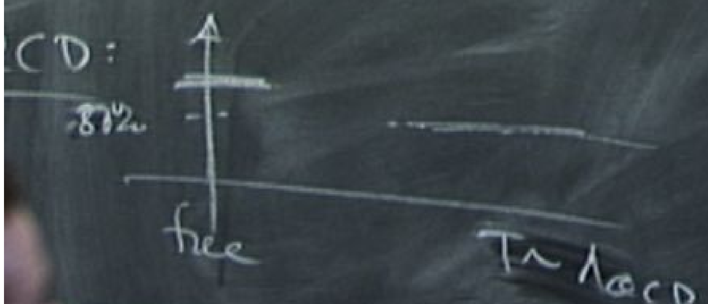
$T \sim \Lambda_{QCD}$

compute QFT stress tensor: $\langle T^{\mu}{}_{\nu} \rangle = \frac{2}{\sqrt{\gamma}} \frac{\delta S[g]}{\delta \gamma_{\mu}{}^{\nu}}$

$(\lambda=0)$

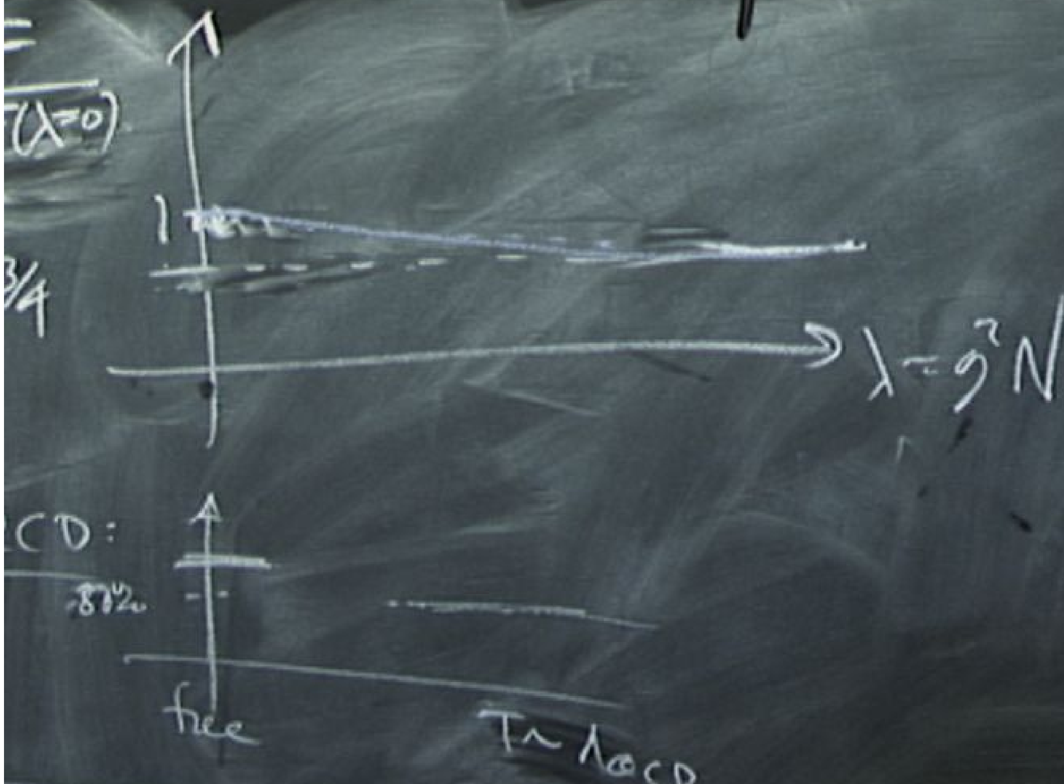


Compute QFT stress tensor: $\langle T^{\mu}{}_{\nu} \rangle = \frac{2}{\sqrt{\gamma}} \frac{\delta S_r[g]}{\delta \gamma_{\mu}{}^{\nu}}$ ← perturb bc of metric



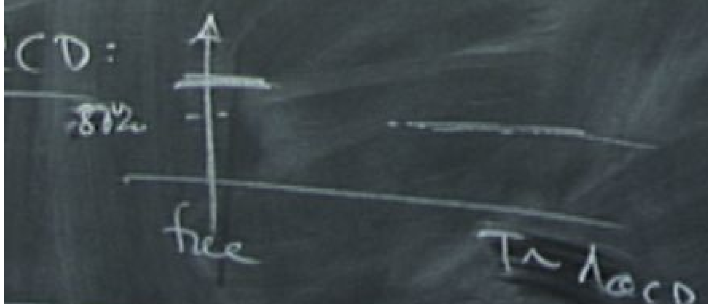
compute QFT stress tensor : $\langle T^{\mu}{}_{\nu} \rangle = \frac{2}{\sqrt{\gamma}} \frac{\delta S_{\epsilon}[g]}{\delta \gamma_{\mu}{}^{\nu}}$ ⟵ perturb by δ metric

$T^{t}{}_{t} = \epsilon, \; T^{i}{}_{i} = P$



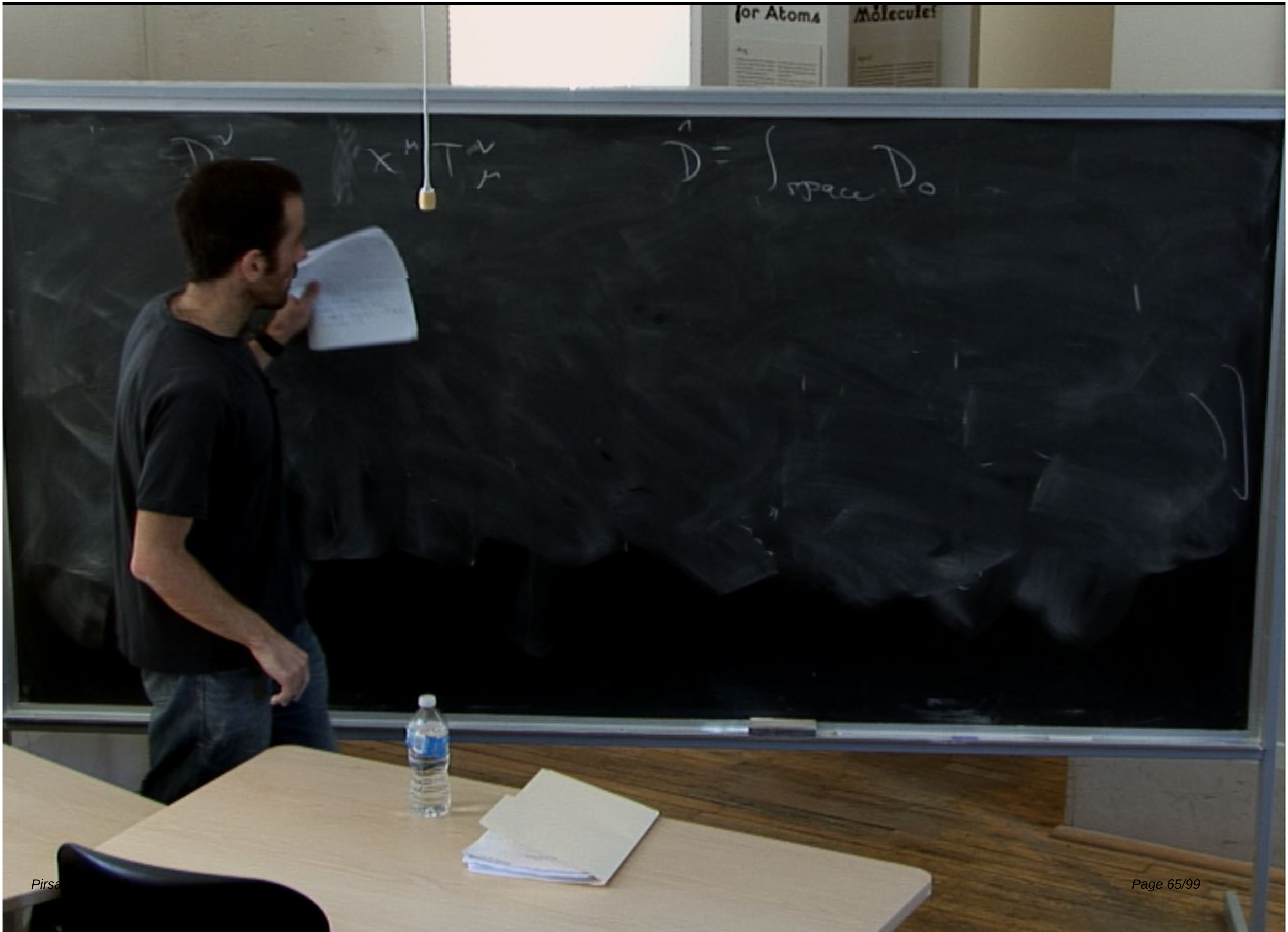
compute QFT stress tensor: $\langle T^{\mu}{}_{\nu} \rangle = \frac{2}{\sqrt{\gamma}} \frac{\delta S_r[g]}{\delta \gamma_{\mu\nu}}$ ← perturb bc of metric

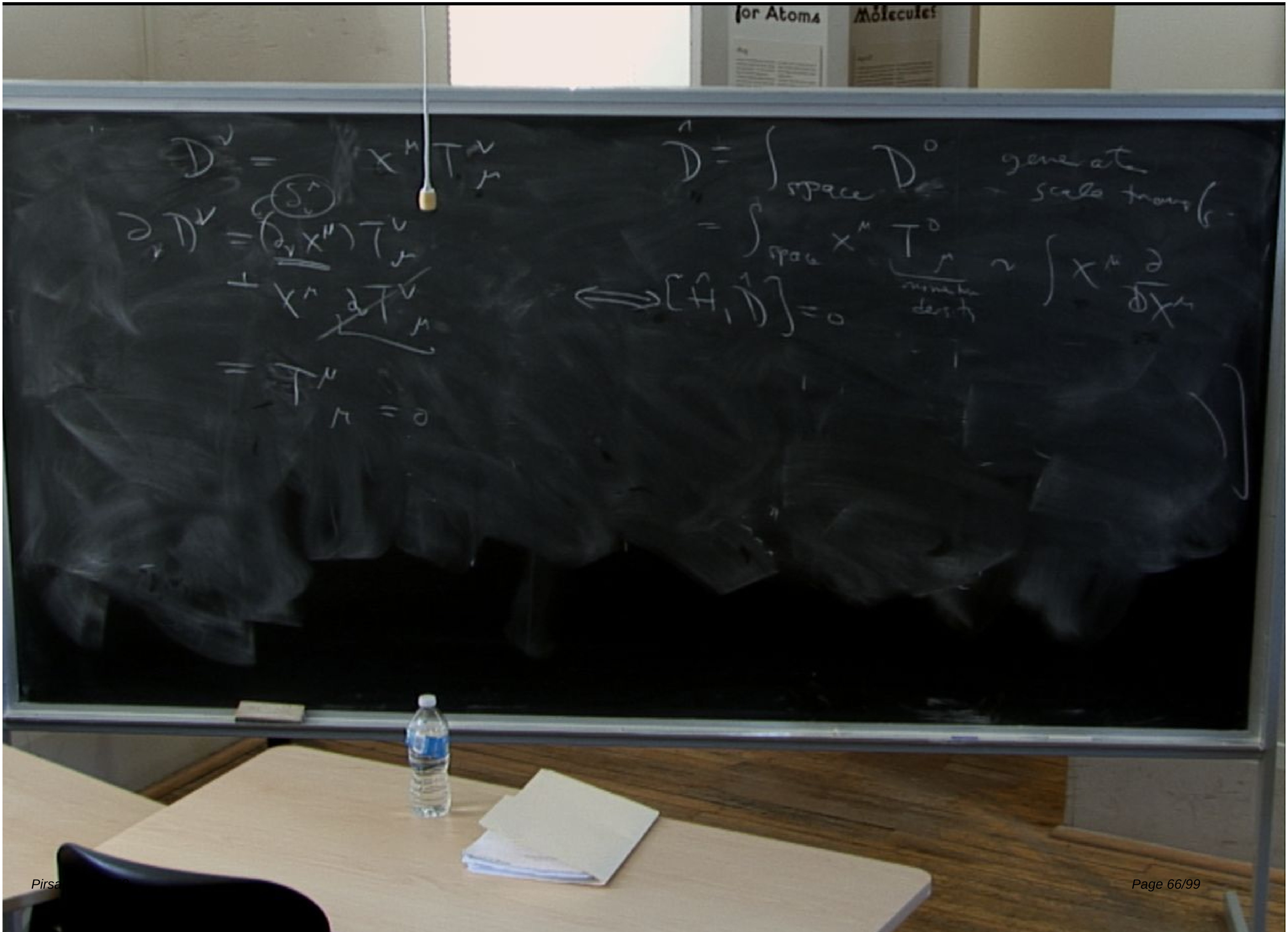
$T^{t}{}_{t} = \epsilon, \quad T^{i}{}_{i} = P \implies \epsilon = (d-1)P$ ie $\langle T^{\mu}{}_{\mu} \rangle = 0$



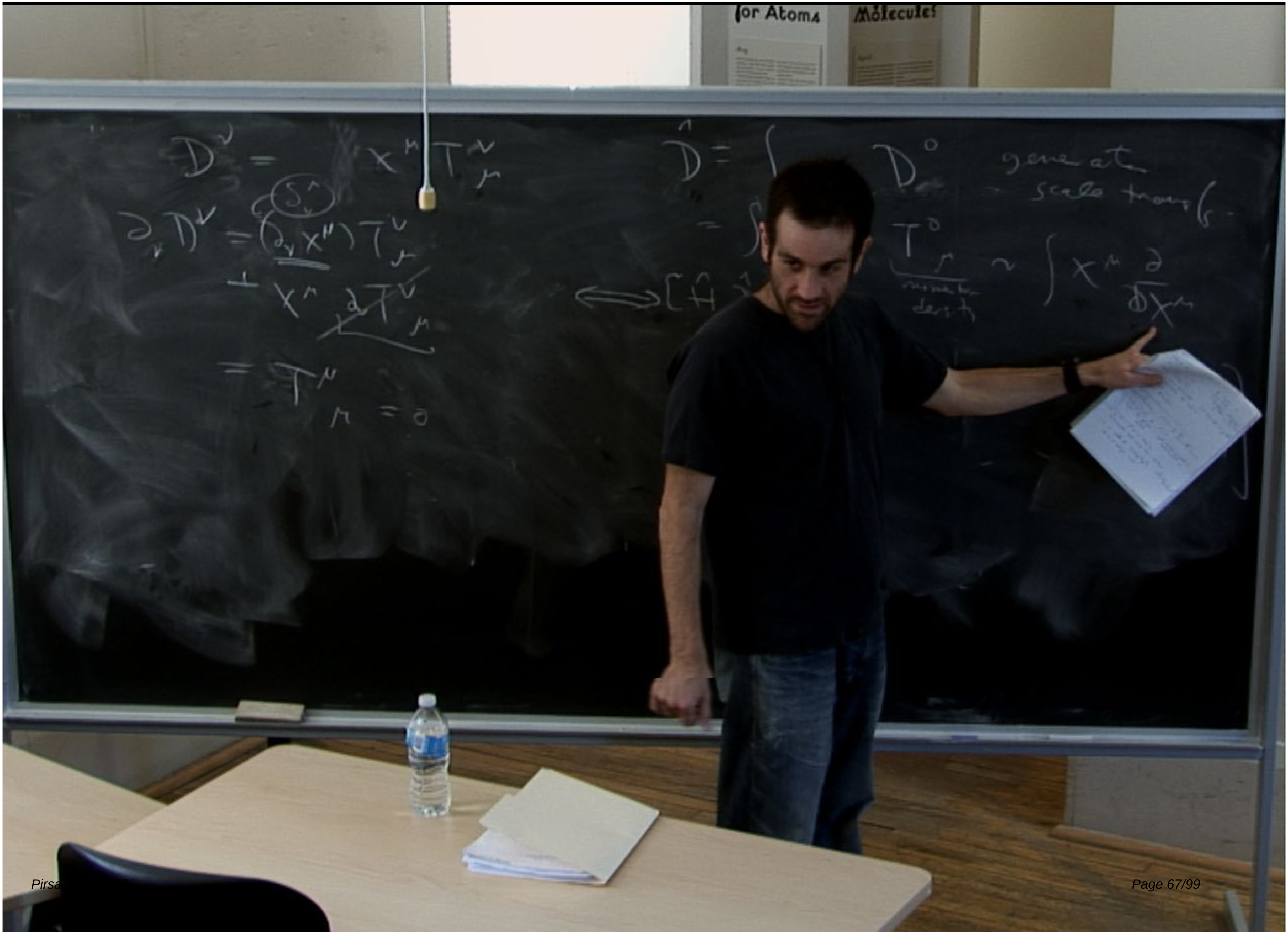
compute QFT stress tensor: $\langle T^{\mu}{}_{\nu} \rangle = \frac{2}{\sqrt{\gamma}} \frac{\delta S_r[g]}{\delta \gamma^{\nu}{}_{\mu}} \leftarrow$ perturb bc of metric

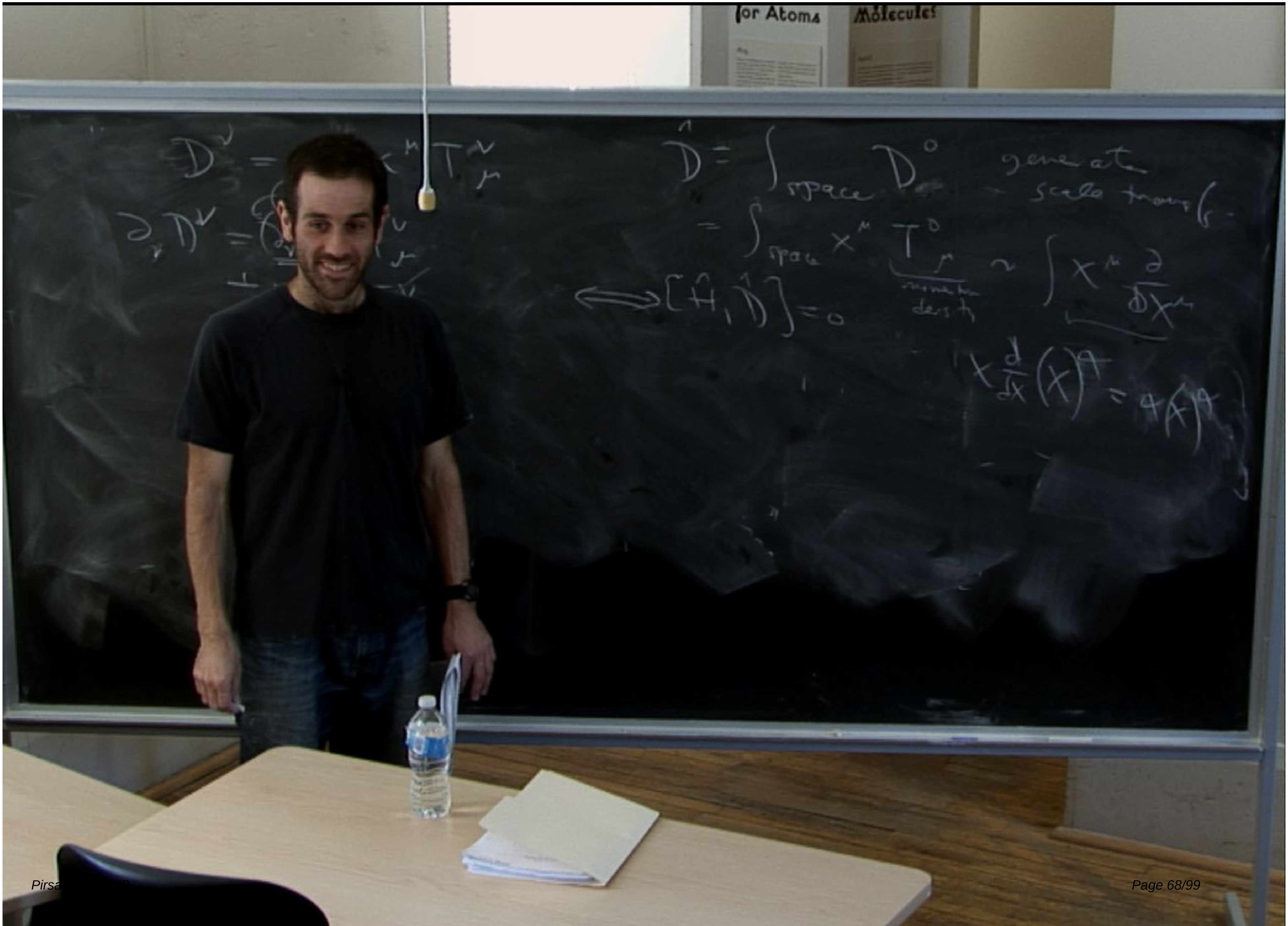
$T^{t}{}_{t} = \epsilon, \; T^{i}{}_{i} = P \implies \epsilon = (d-1)P$ ie $\langle T^{\mu}{}_{\mu} \rangle = 0$ ✓

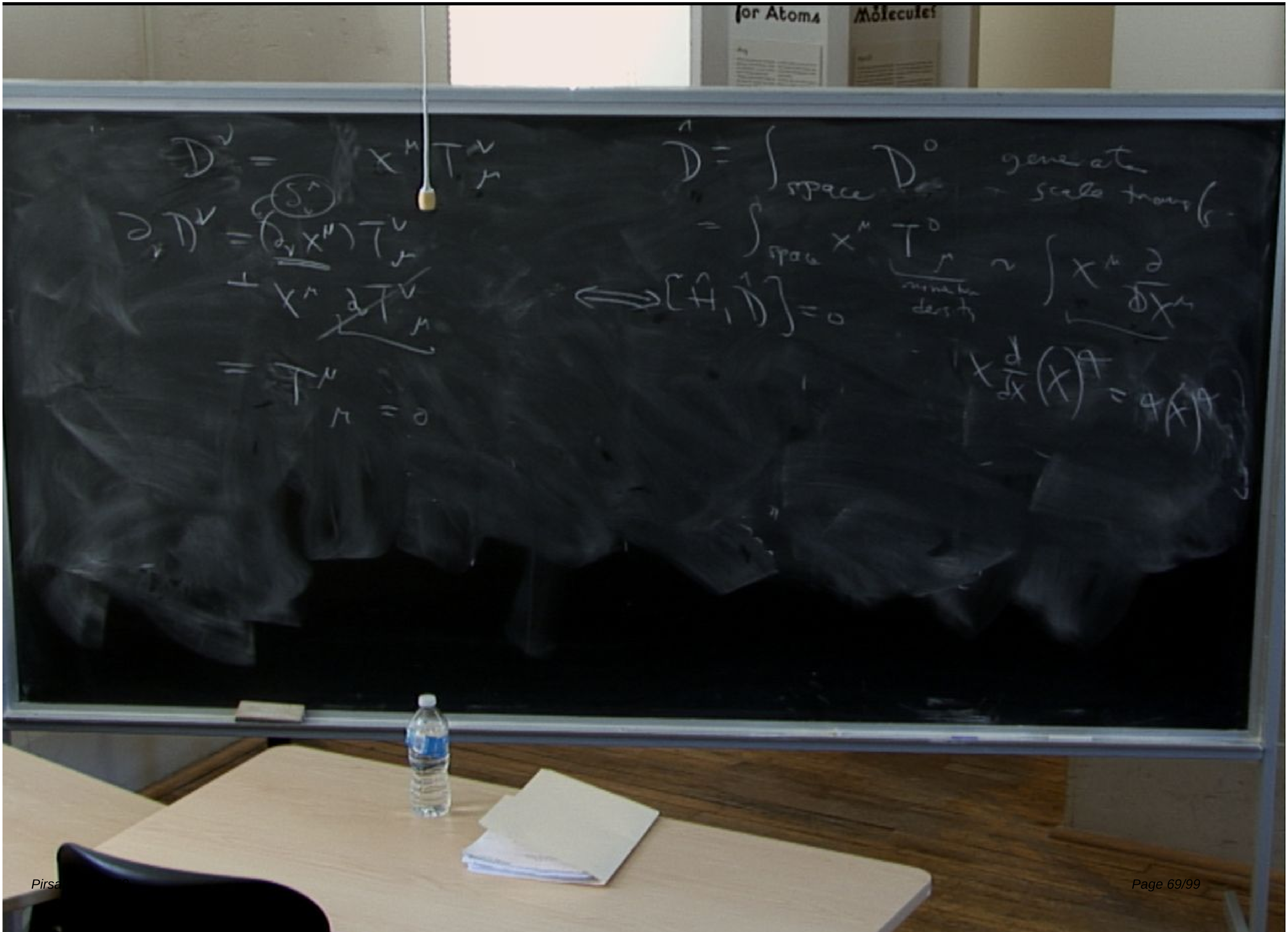

for Atoms
Molecules
$x^\mu T^\nu_\mu$
$\hat{D} = \int_{space} D_0$



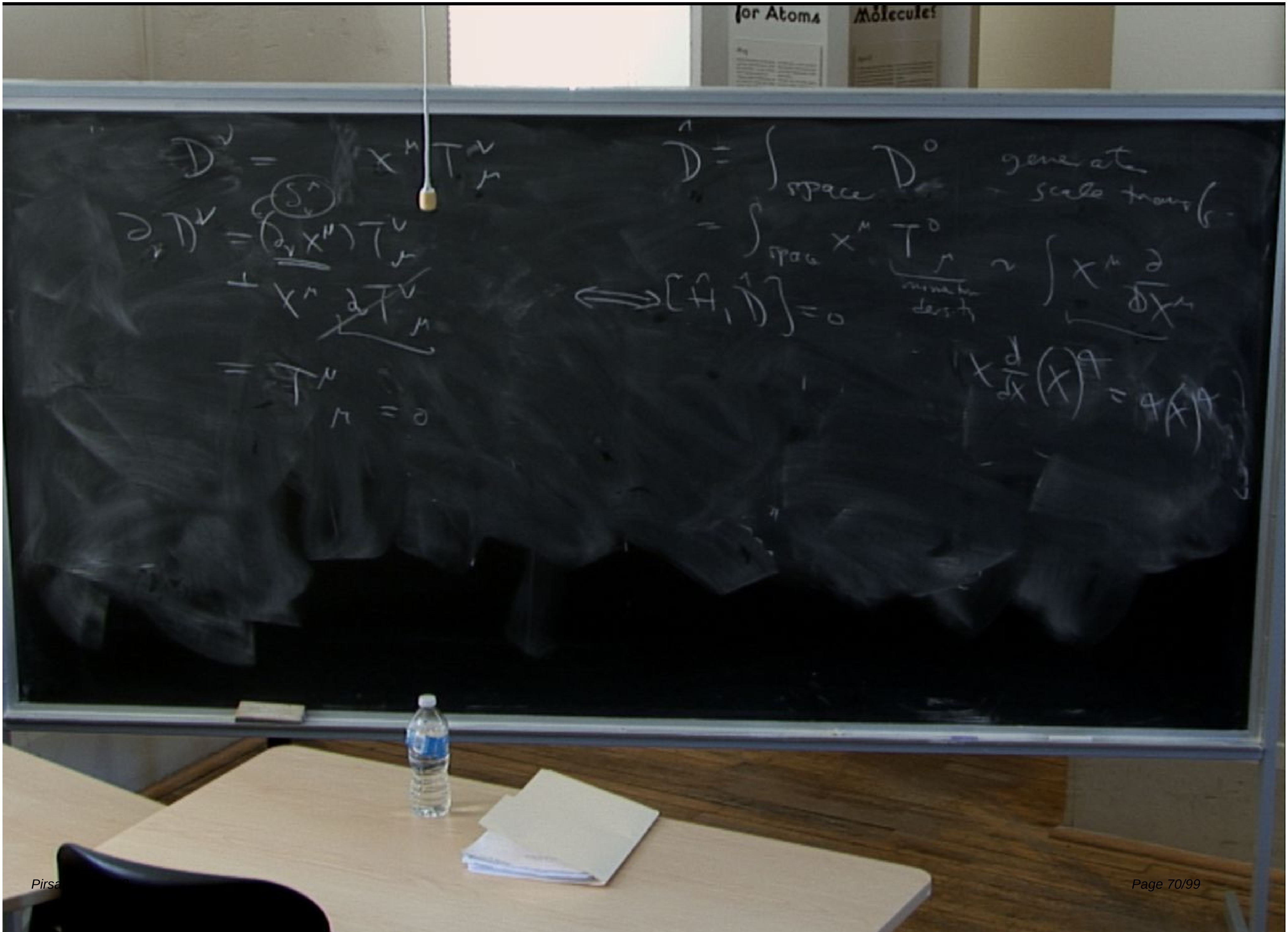
for Atoms
Molecules
$D^\nu = x^\mu T^\nu_\mu$
$\partial_\nu D^\nu = (\partial_\nu x^\mu) T^\nu_\mu + x^\mu \partial_\nu T^\nu_\mu$
$= T^\mu_\mu = 0$
$\hat{D} = \int_{space} D^0$ generate scale transf
$= \int_{space} x^\mu \underbrace{T^0_\mu}_{\text{momentum density}} \sim \int x^\mu \frac{\partial}{\partial x^\mu}$
$\Longleftrightarrow [\hat{H}, \hat{D}] = 0$


for Atoms
Molecules
generate
scale transfs


for Atoms
Molecules
$\hat{D} = \int_{space} D^0$
generate
$= \int_{space} x^\mu T^0_\mu$
$[\hat{H}, \hat{D}] = 0$

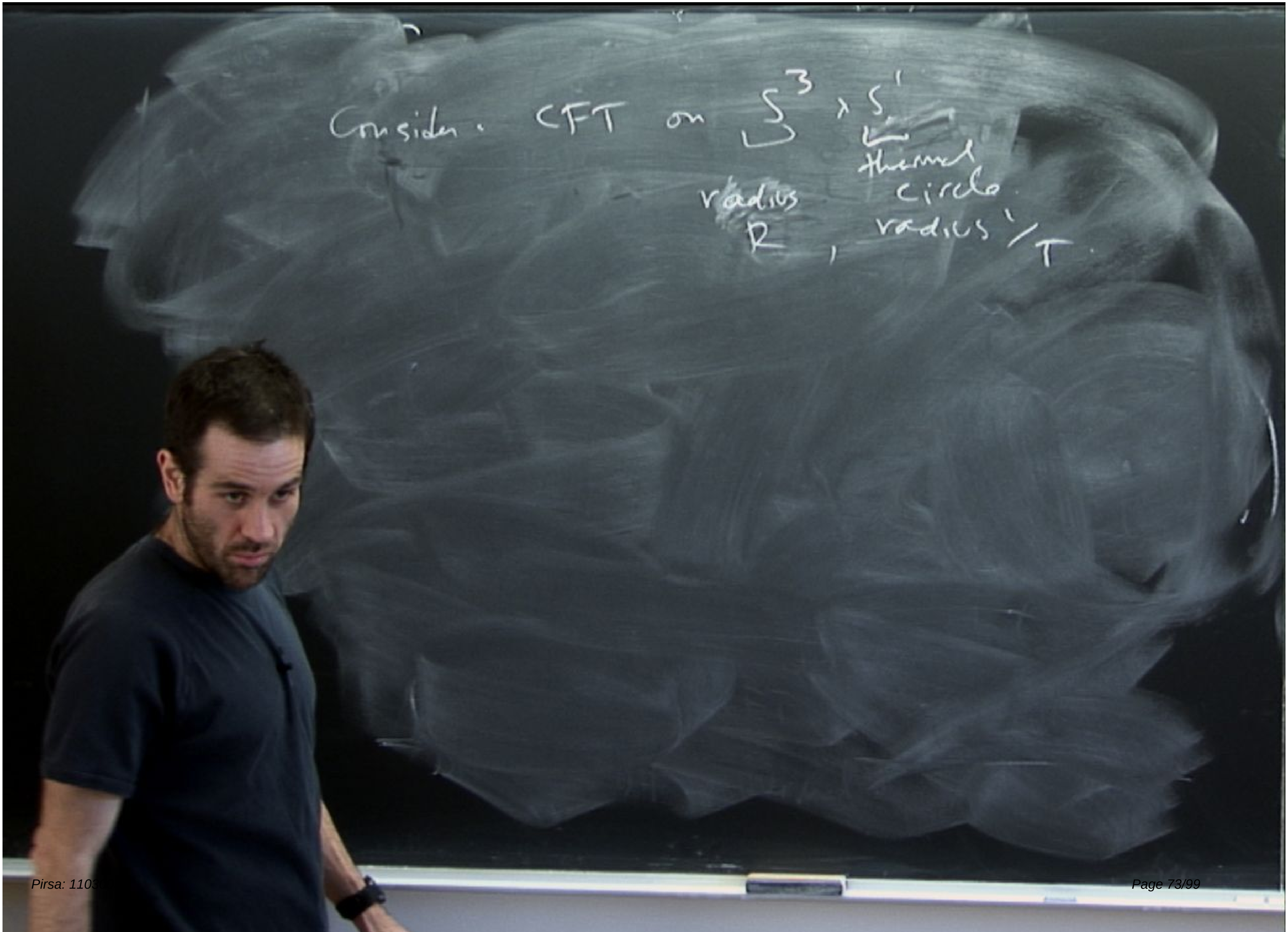


for Atoms
Molecules
$D^\nu = x^\mu T^\nu_\mu$
$\partial_\nu D^\nu = (\partial_\nu x^\mu) T^\nu_\mu + x^\mu \partial_\nu T^\nu_\mu$
$= T^\mu_\mu = 0$
$\hat{D} = \int_{space} D^0$
generate scale transf
$= \int_{space} x^\mu T^0_\mu \sim \int x^\mu \frac{\partial}{\partial x^\mu}$
momentum density
$\Longleftrightarrow [\hat{H}, \hat{D}] = 0$
$x \frac{d}{dx} (x)^q = q (x)^q$


for Atoms
Molecules
D^ν = x^μ T^ν_μ
∂_ν D^ν = (∂_ν x^μ) T^ν_μ + x^μ ∂_ν T^ν_μ
= T^μ_μ = 0
D̂ = ∫_space D^0
generate scale transfs
= ∫_space x^μ T^0_μ
momentum density
⟺ [Ĥ, D̂] = 0

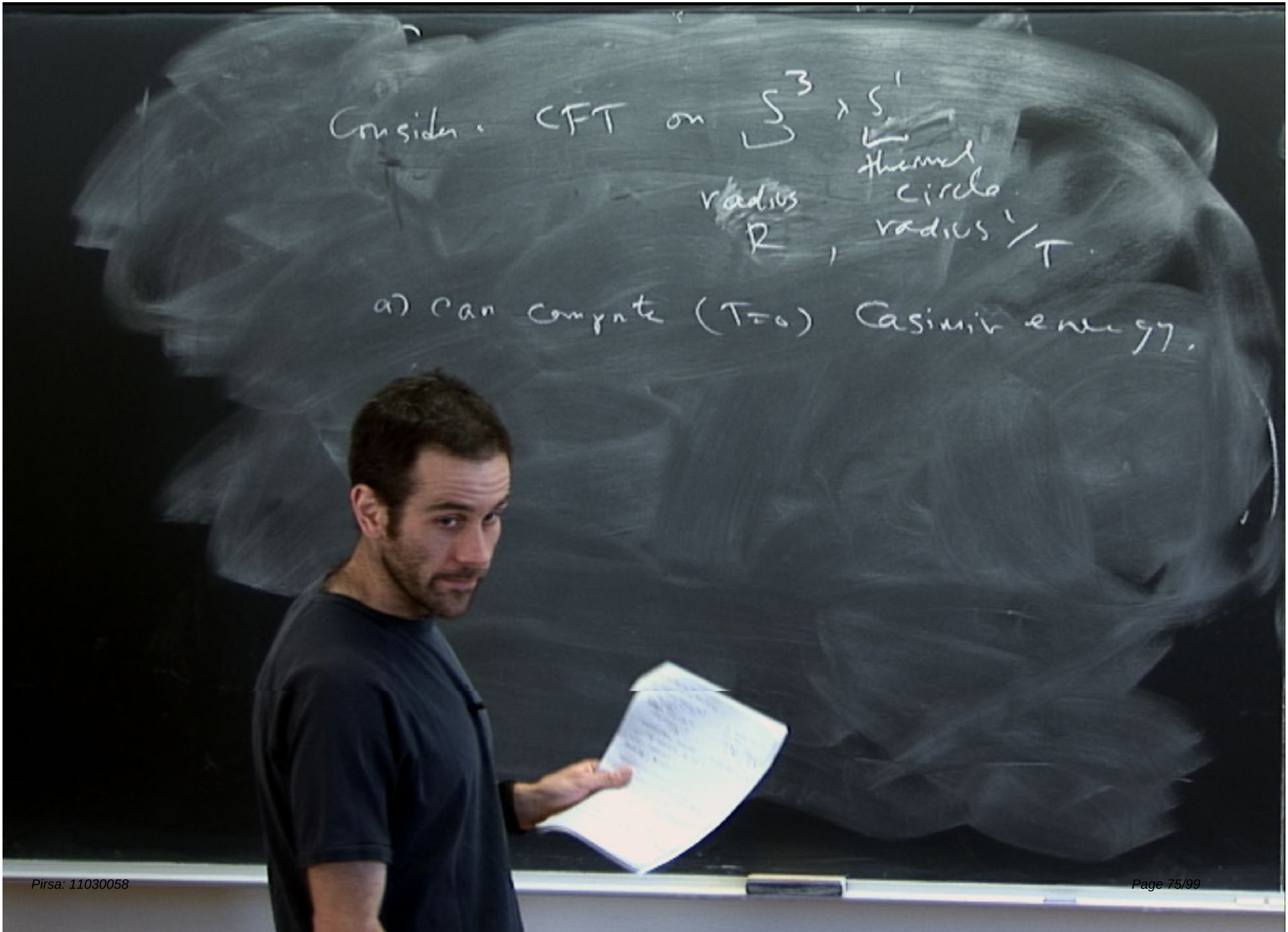

Consider: CFT

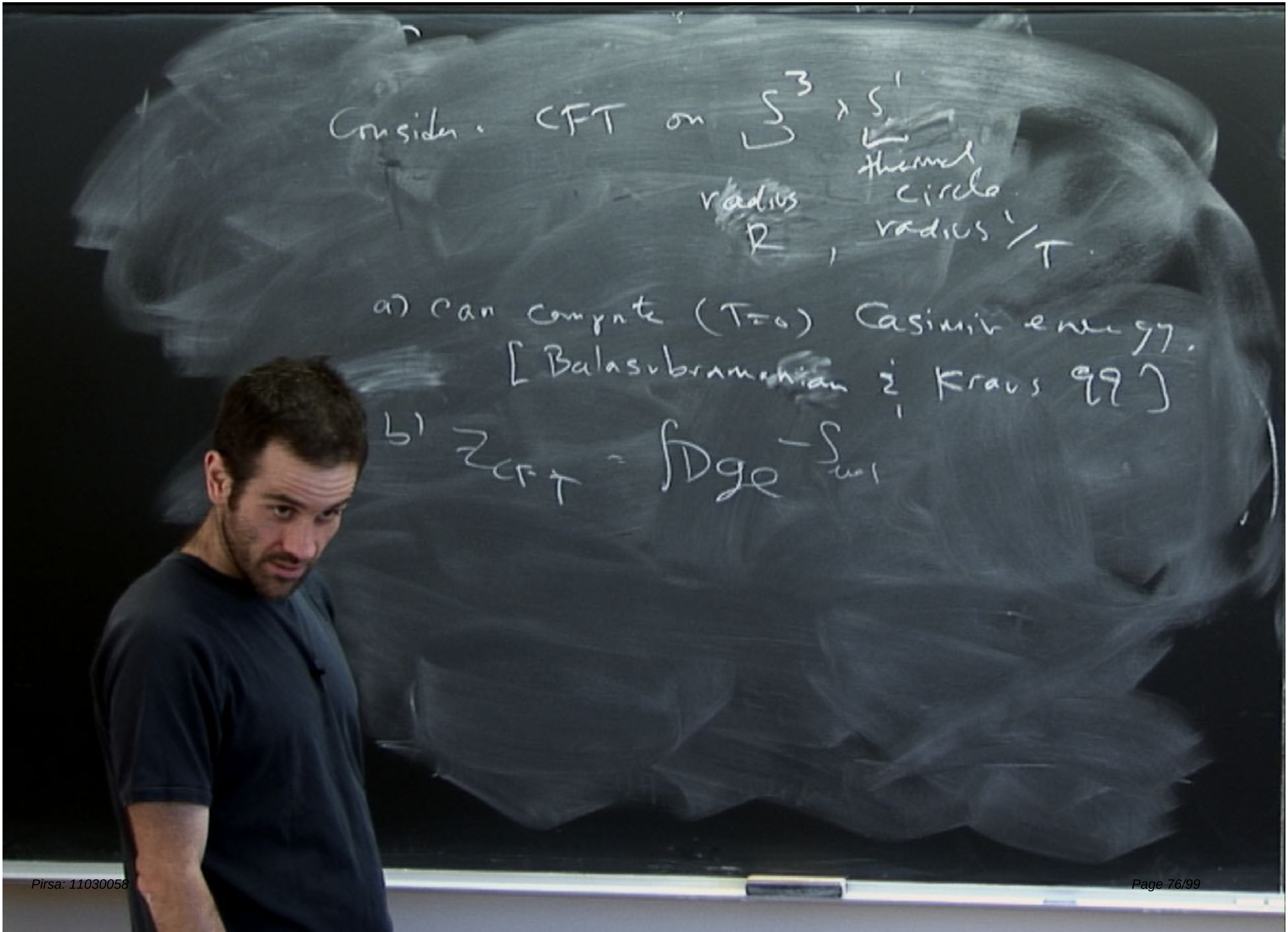

Consider: CFT on $S^3 \times S^1$
thermal circle

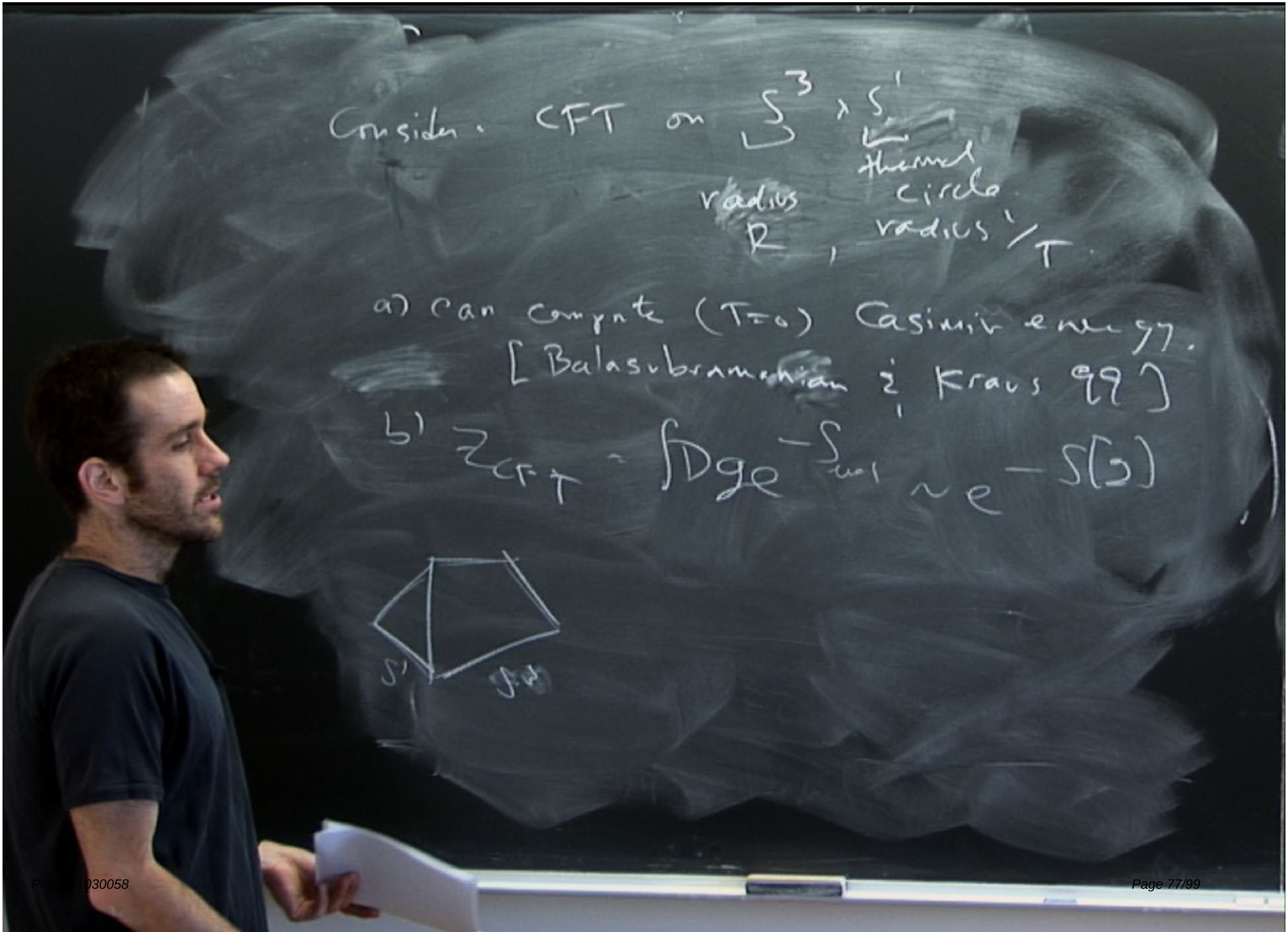

Consider: CFT on $S^3 \times S^1$
radius R
thermal circle, radius $1/T$.

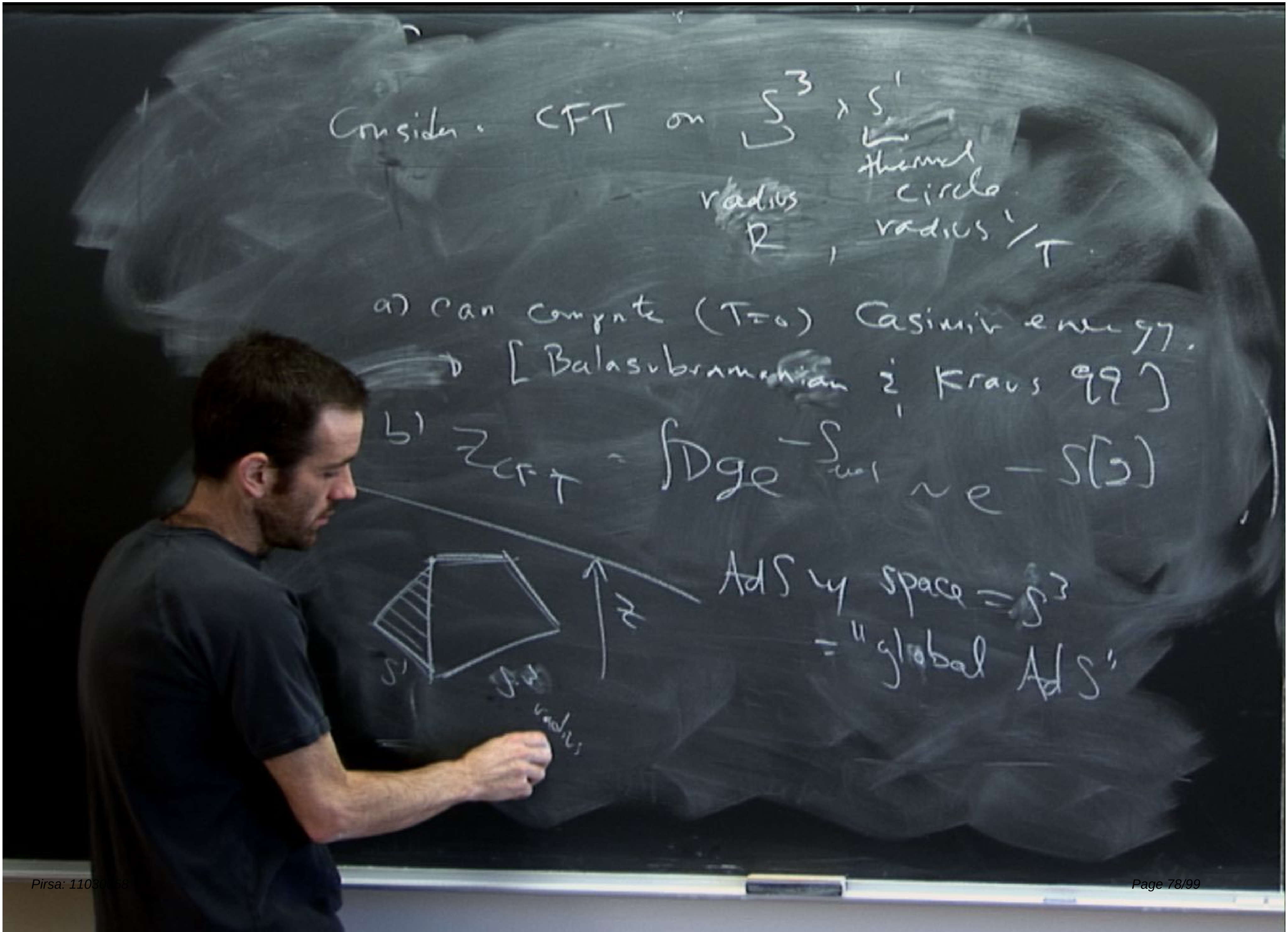
Consider: CFT on $\underbrace{S^3}_{\text{radius } R} \times \underbrace{S^1}_{\text{thermal circle, radius } 1/T}$.

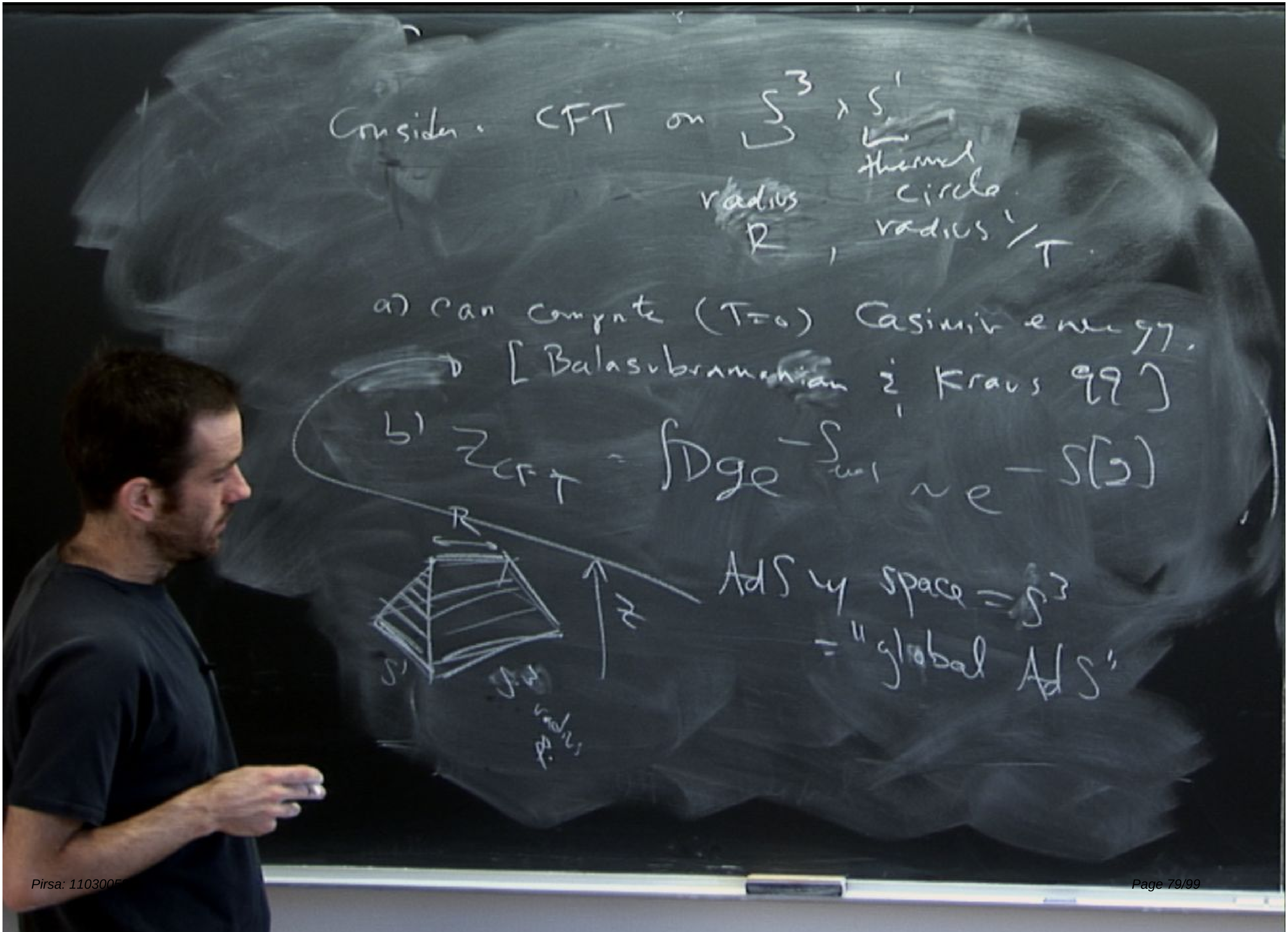
a) can compute ($T=0$) Casimir energy.

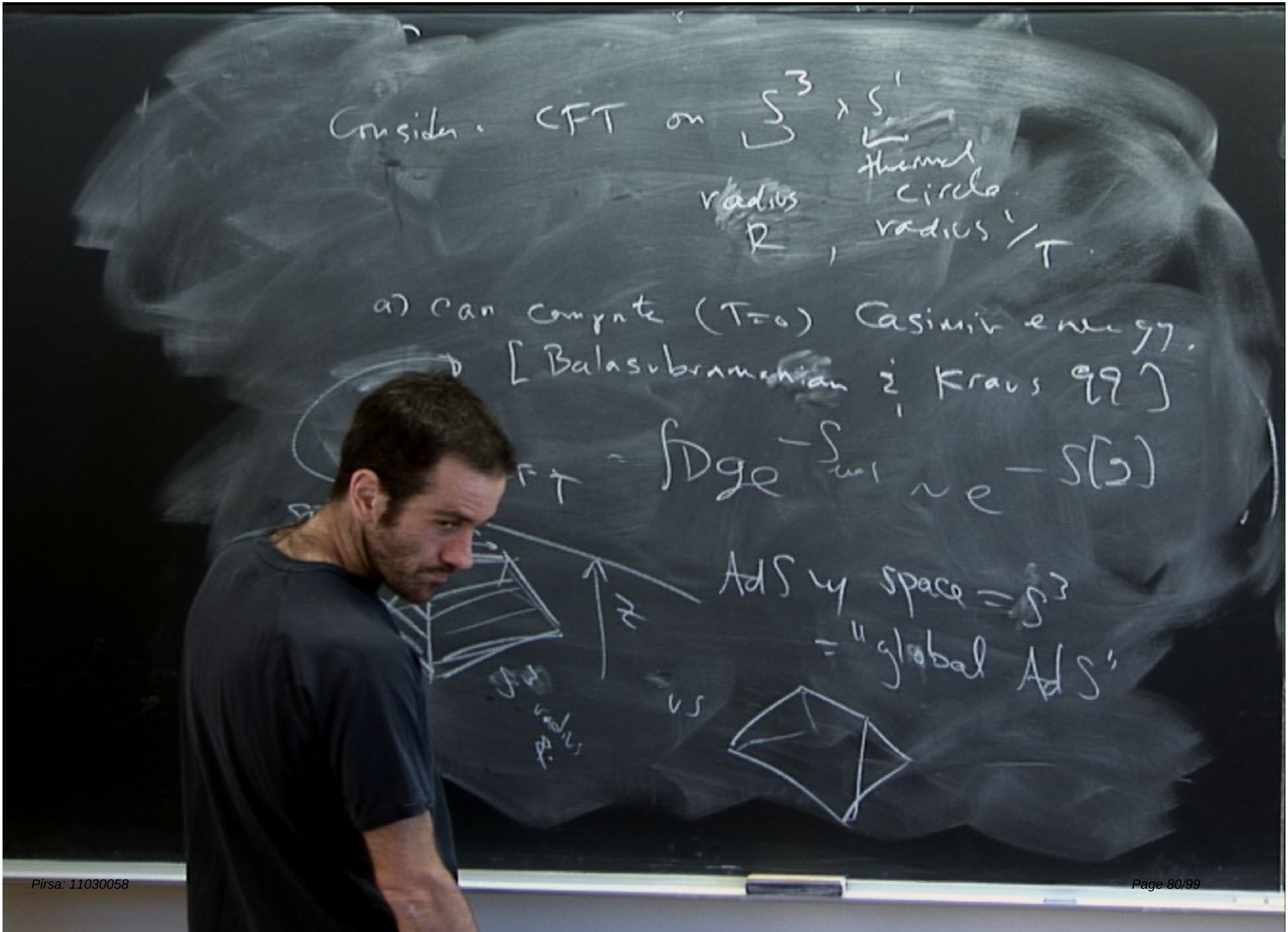

Consider: CFT on $S^3 \times S^1$
thermal circle
radius R, radius 1/T.
a) can compute (T=0) Casimir energy.

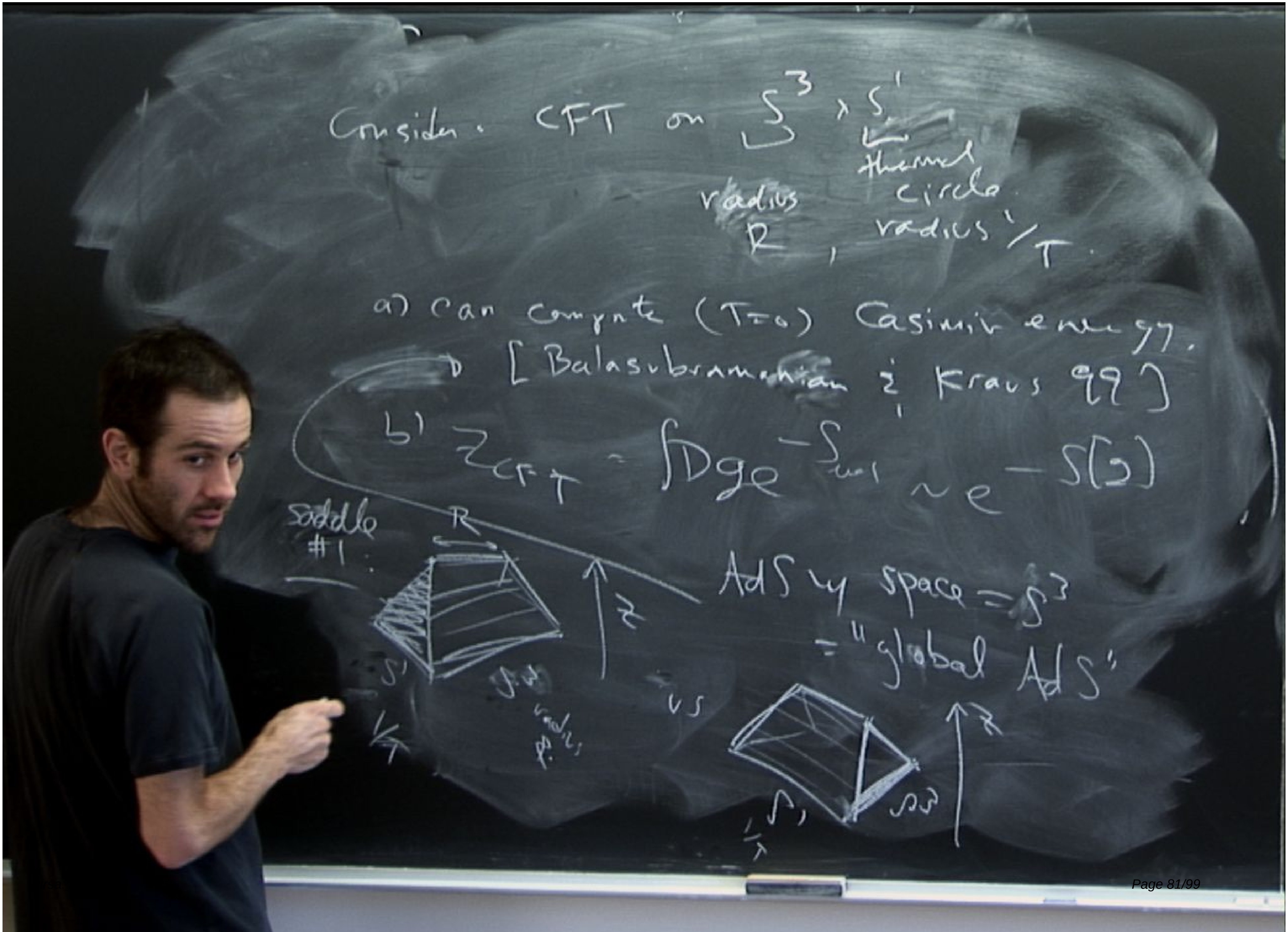

Consider: CFT on $S^3 \times S^1$
thermal circle
radius R
radius $1/T$.
a) can compute (T=0) Casimir energy.
[Balasubramanian & Kraus 99]
b) $Z_{CFT} = \int Dg\, e^{-S_{grav}}$

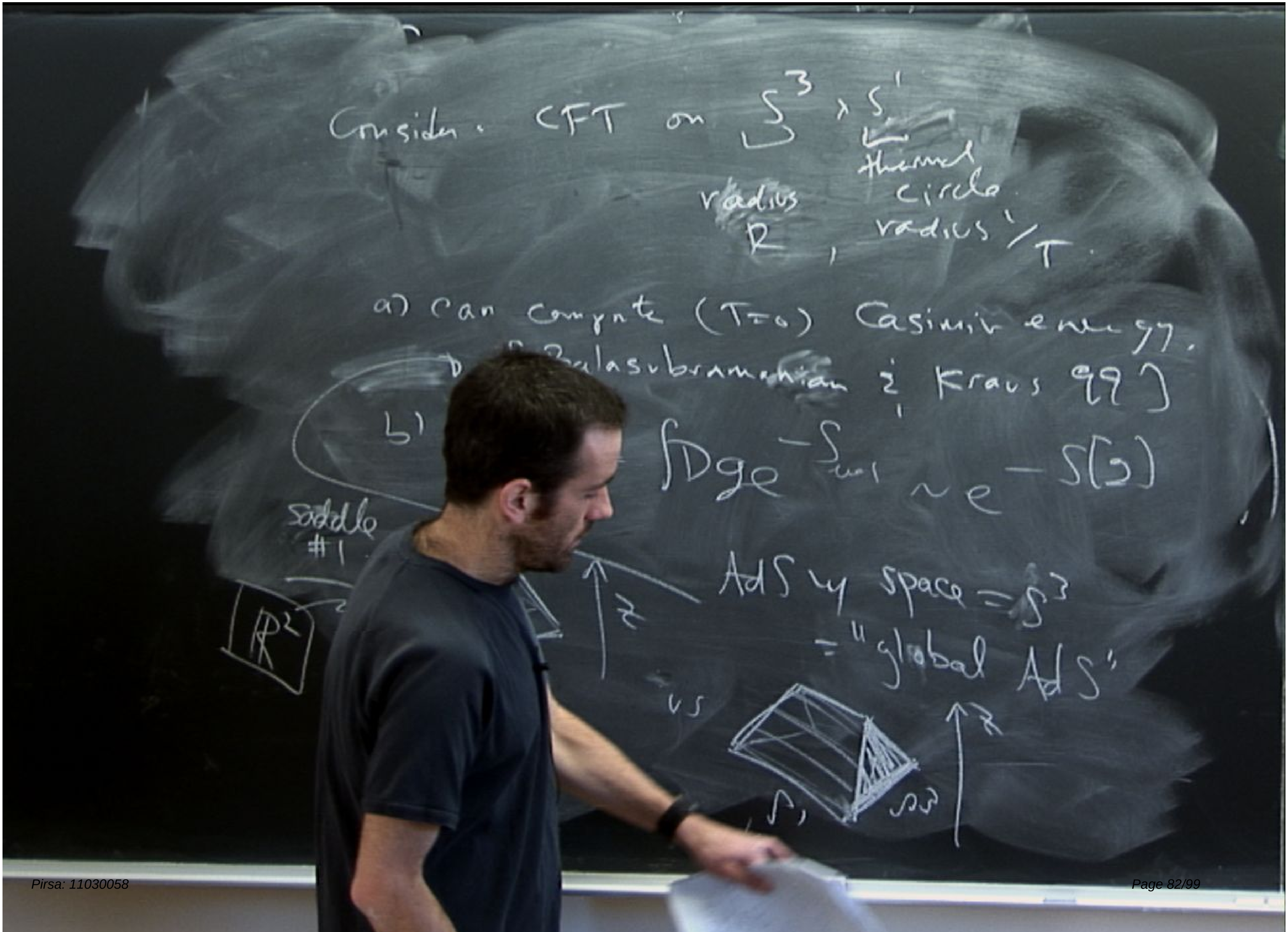

Consider: CFT on $S^3 \times S^1$
radius R , thermal circle radius 1/T
a) can compute (T=0) Casimir energy.
[Balasubramanian & Kraus 99]
b) $Z_{CFT} = \int Dg\, e^{-S_{wel}} \sim e^{-S(g)}$

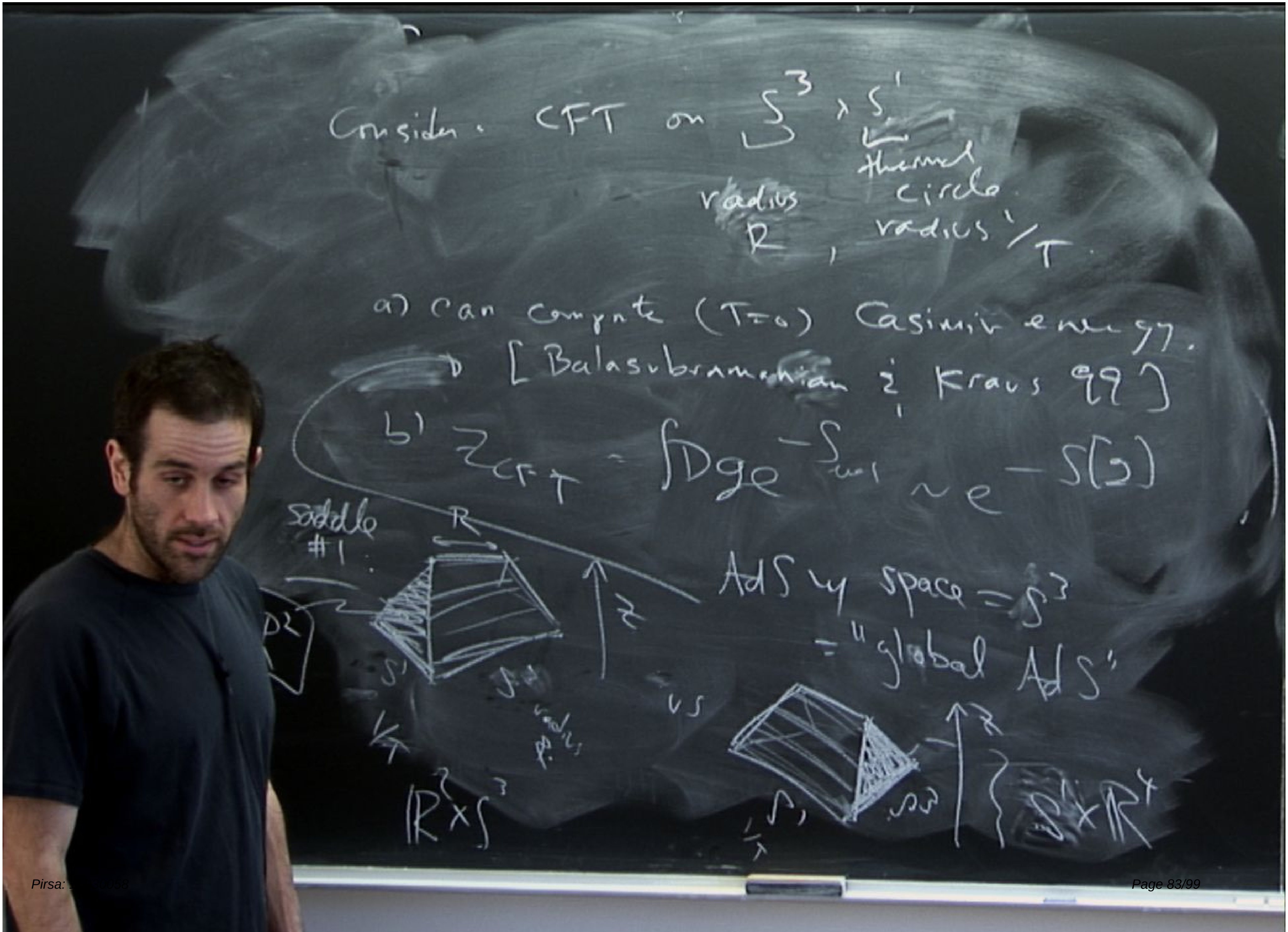

Consider: CFT on $S^3 \times S^1$
radius R
thermal circle
radius $1/T$
a) can compute (T=0) Casimir energy.
[Balasubramanian & Kraus 99]
b) $Z_{CFT_T} = \int Dg\, e^{-S_{grav}} \sim e^{-S(g)}$
AdS_4 space = S^3
= "global AdS"
S^1
S^3
radius
z

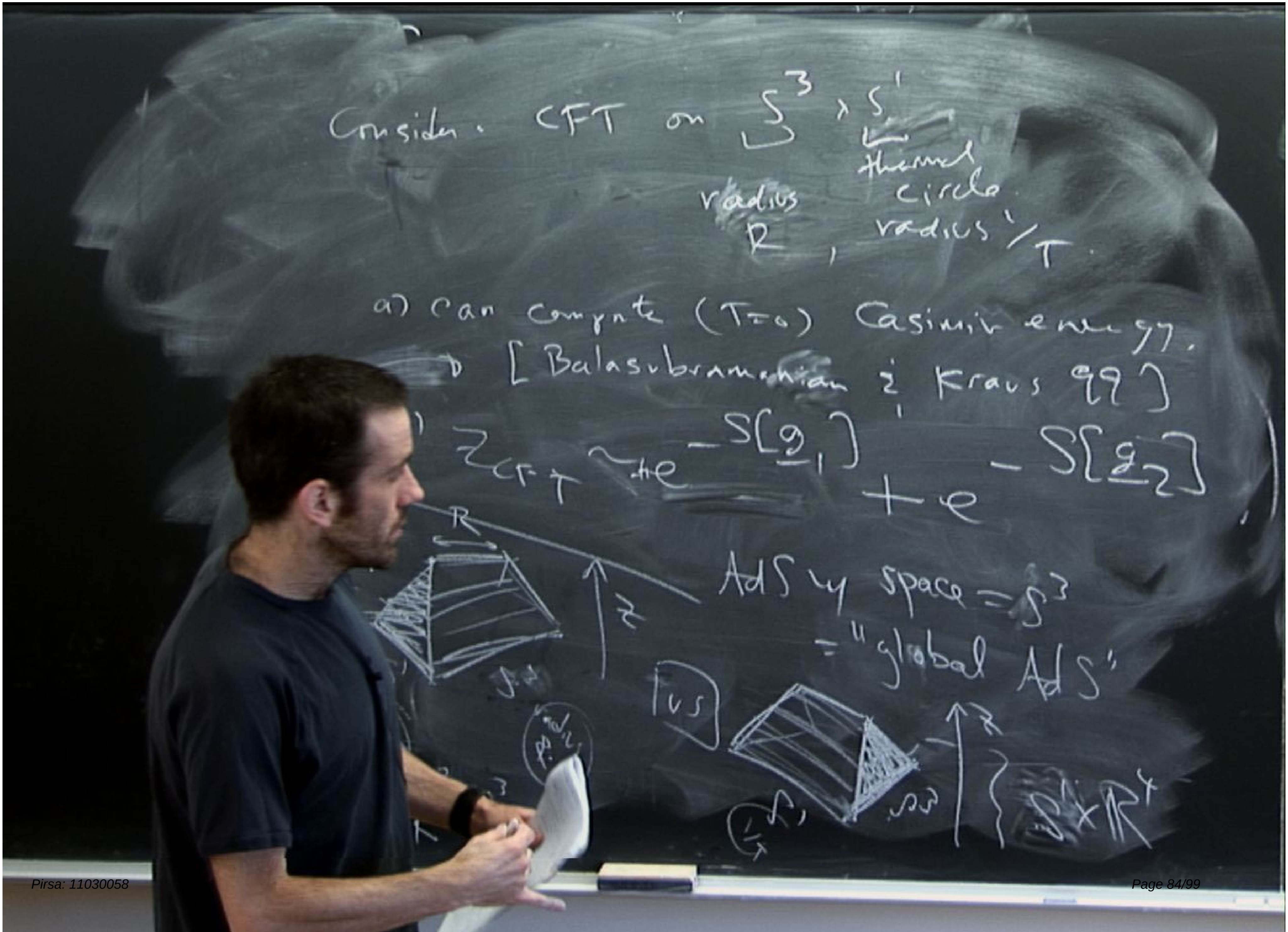

Consider: CFT on S^3 x S^1
radius R
thermal circle, radius 1/T.
a) can compute (T=0) Casimir energy.
[Balasubramanian & Kraus 99]
b) Z_CFT = ∫Dg e^{-S_grav} ~ e^{-S(g)}
R
z
S'
AdS w/ space = S^3
= "global AdS"

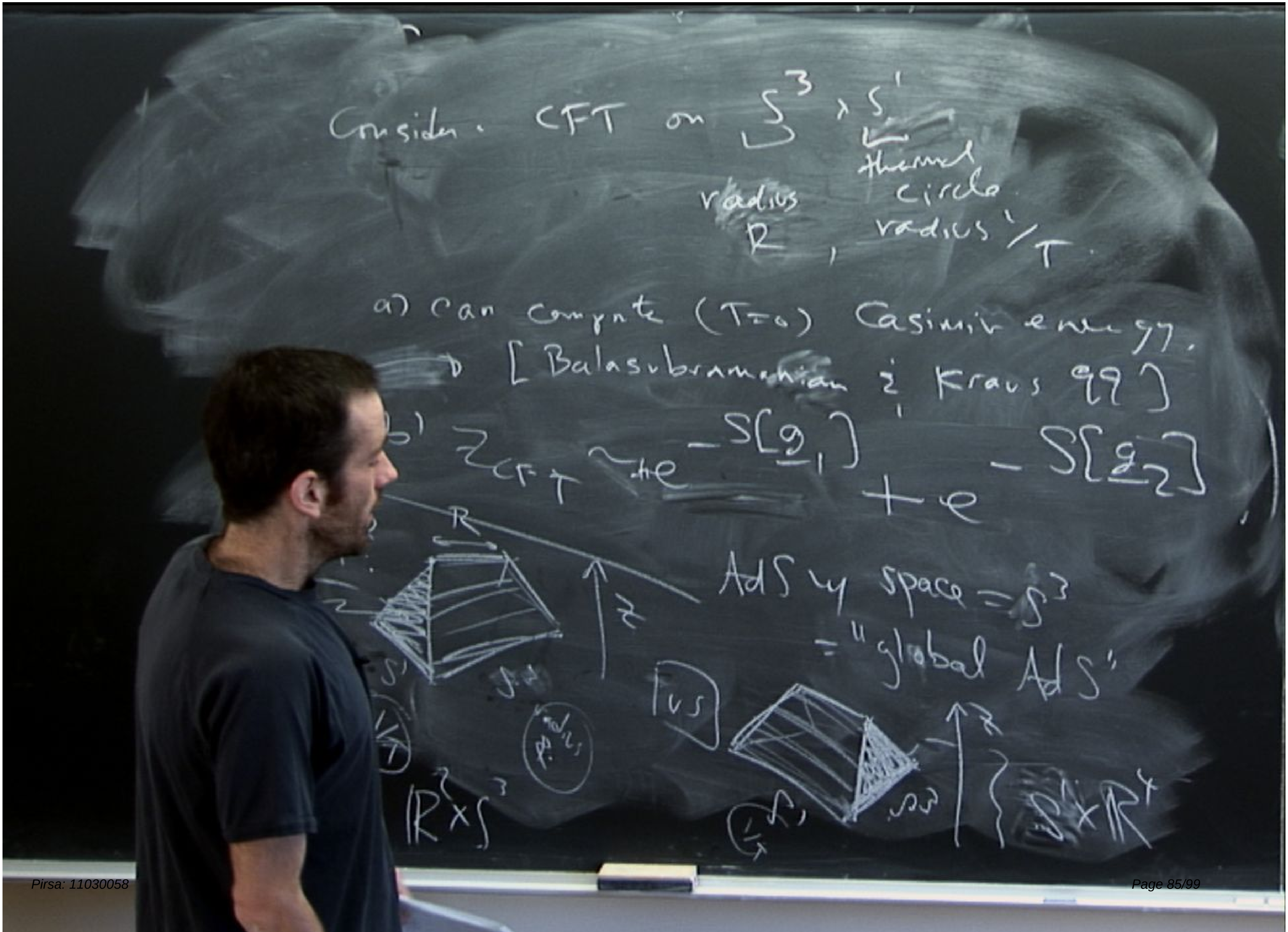

Consider: CFT on S3 x S1
thermal circle
radius R
radius 1/T
a) can compute (T=0) Casimir energy.
[Balasubramanian & Kraus 99]
AdS w/ space = S3
= "global AdS"


Consider: CFT on S^3 x S^1
radius R
thermal circle, radius 1/T
a) can compute (T=0) Casimir energy.
[Balasubramanian & Kraus 99]
b) Z_CFT = ∫Dg e^{-S_grav} ~ e^{-S(g)}
saddle #1:
R
z
AdS_4 space = S^3
= "global AdS"
S^1
vs
z
S^3

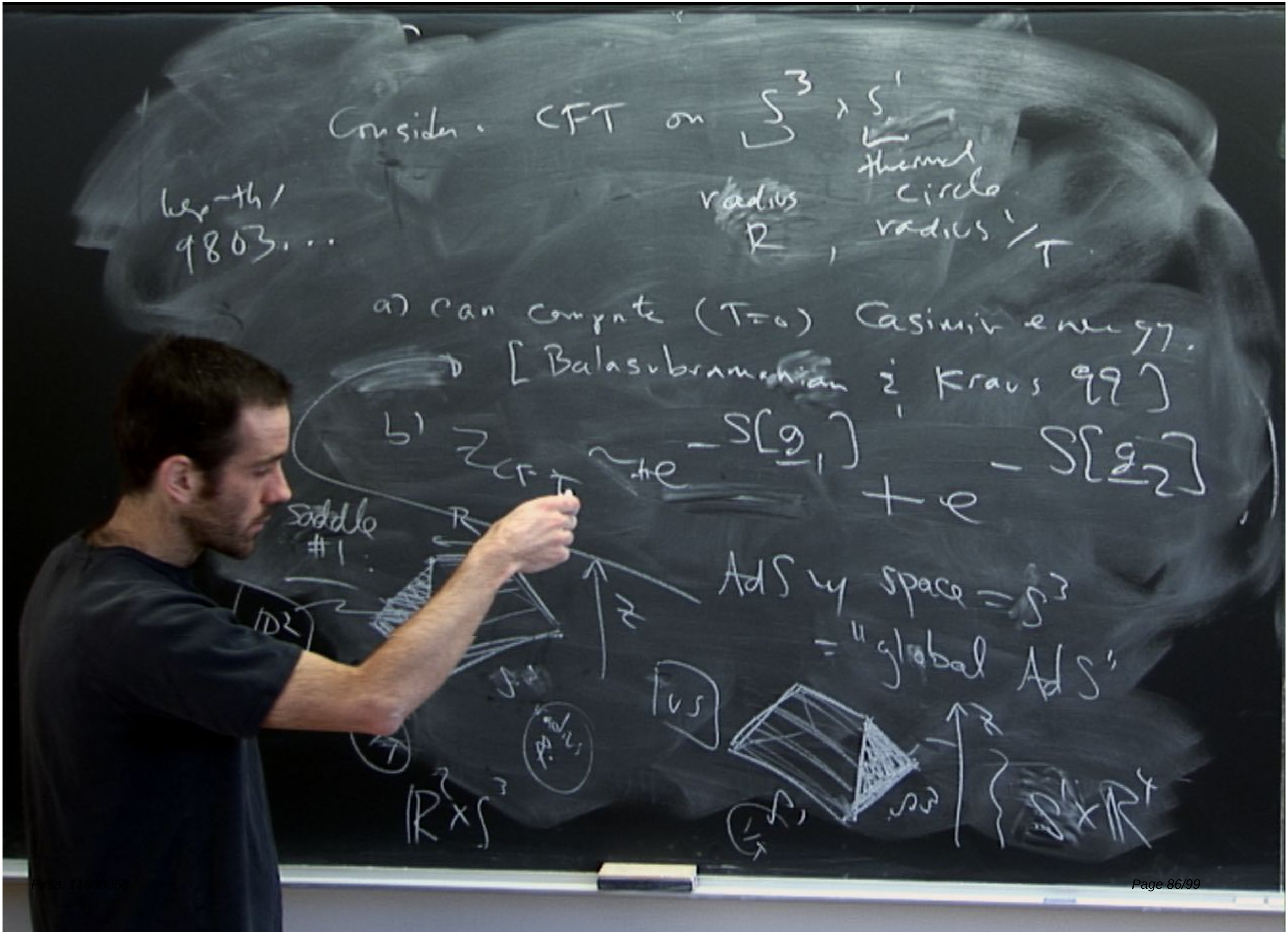

Consider: CFT on S^3 x S^1
thermal circle
radius R, radius 1/T
a) can compute (T=0) Casimir energy.
[Balasubramanian & Kraus 99]
b)
∫Dg e^{-S_grav} ~ e^{-S(g)}
saddle #1
R^2
AdS_4 w/ space = S^3
= "global AdS"
z

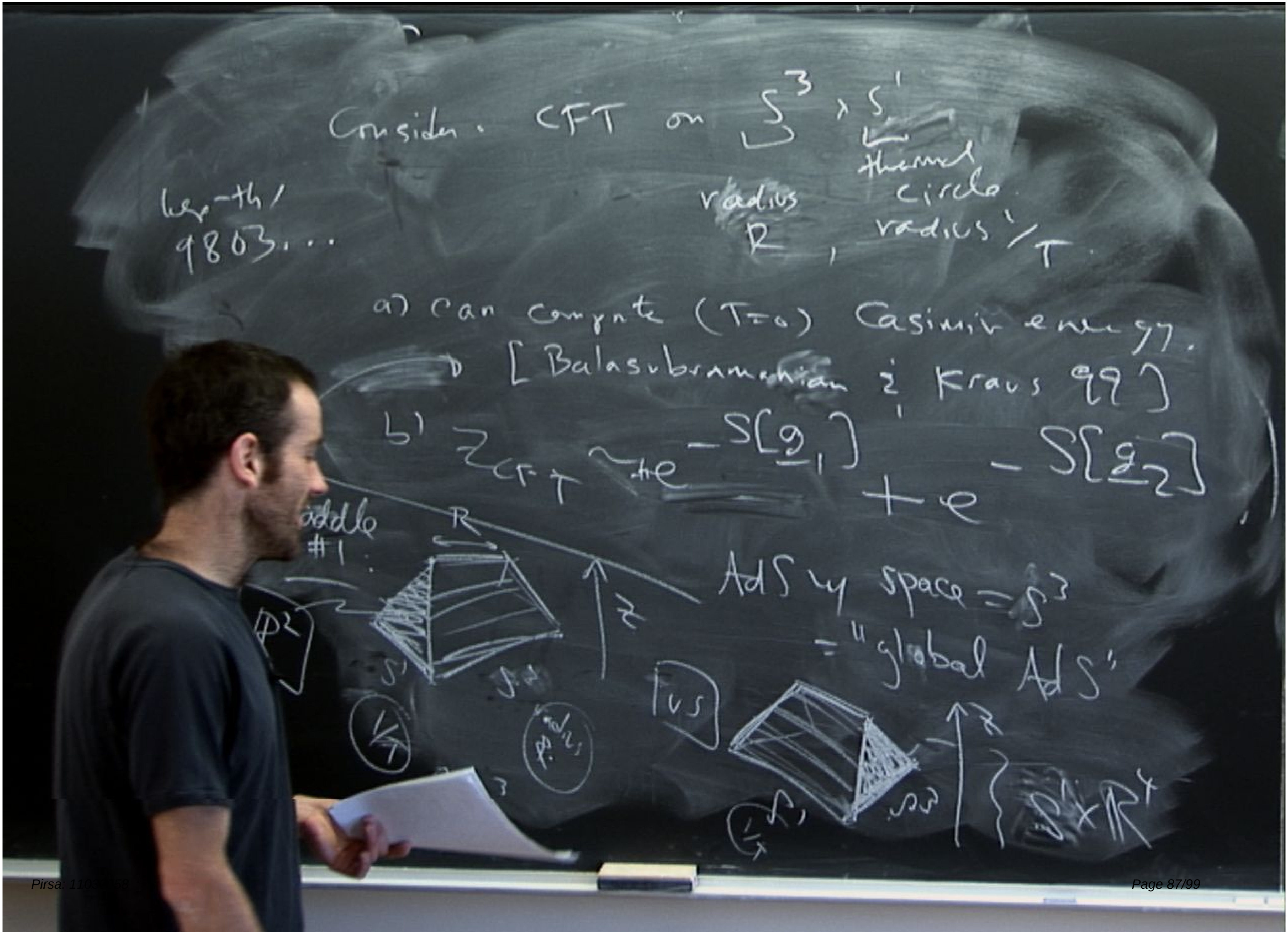

Consider: CFT on S^3 x S^1
radius R
thermal circle
radius 1/T.
a) Can compute (T=0) Casimir energy.
[Balasubramanian & Kraus 99]
b) Z_CFT = ∫Dg e^{-S_grav} ~ e^{-S(g)}
saddle #1
R
z
AdS_4 space = S^3
= "global AdS"
S^1
1/T
vs
R^2 x S^3

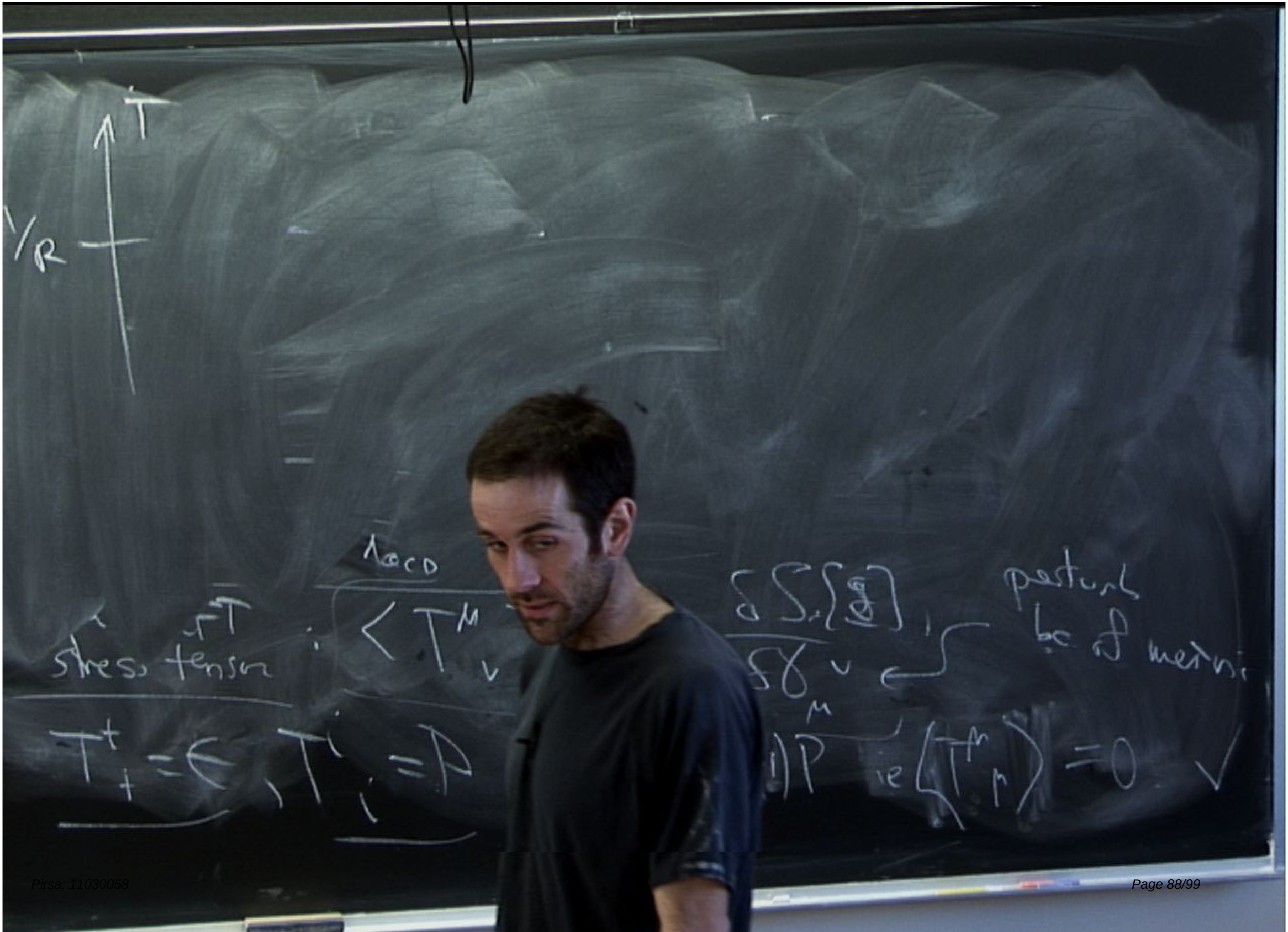

Consider: CFT on S^3 × S^1
radius R
thermal circle
radius 1/T
a) can compute (T=0) Casimir energy.
[Balasubramanian & Kraus 99]
Z_CFT_T ~ e^{-S[g_1]} + e^{-S[g_2]}
R
z
AdS_4 space = S^3
= "global AdS"
z

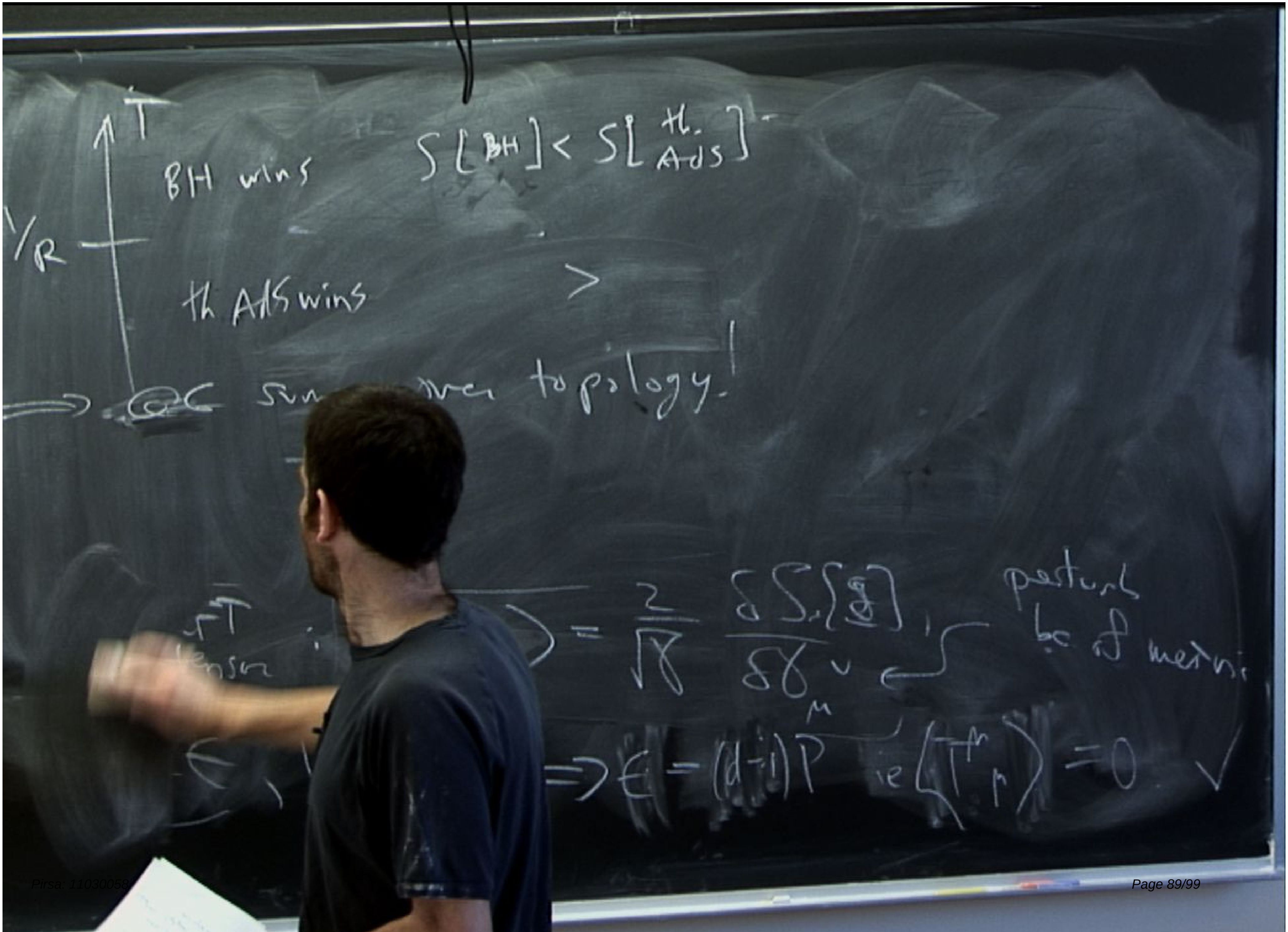


Consider. CFT on S³ × S¹
thermal circle
radius R, radius 1/T
a) Can compute (T=0) Casimir energy.
[Balasubramanian & Kraus 99]
Z_CFT ~ e^{-S[g_1]} + e^{-S[g_2]}
AdS w/ space = S³
= "global AdS"
R × S³
S¹ × R⁴


Consider: CFT on S^3 x S^1
radius R
thermal circle
radius 1/T
length/ 9803...
a) can compute (T=0) Casimir energy.
[Balasubramanian & Kraus 99]
b) Z_CFT,T ~ e^-S[g_1] + e^-S[g_2]
saddle #1
AdS_4 space = S^3
= "global AdS"
R^2 x S^3
S^1 x R^4


Consider: CFT on $S^3 \times S^1$
radius R
thermal circle, radius 1/T
hep-th/9803...
a) can compute (T=0) Casimir energy.
[Balasubramanian & Kraus 99]
b) $Z_{CFT} \sim e^{-S[g_1]} + e^{-S[g_2]}$
saddle #1
R
z
AdS$_4$ space = S^3
= "global AdS"
$S^1 \times R^4$


stress tensor
perturb
bc of metric


T
BH wins
S[BH] < S[th AdS]
1/R
th AdS wins
>
sum over topology!
perturb
bc of metric
ie ⟨T^μ_μ⟩ = 0

T

BH wins $\quad S[\mathrm{BH}] < S[\text{th. AdS}]$

$1/R$

th AdS wins $\quad >$

QG sums over topology!

T

BH wins $\quad S[\mathrm{BH}] < S[\text{th. AdS}]$

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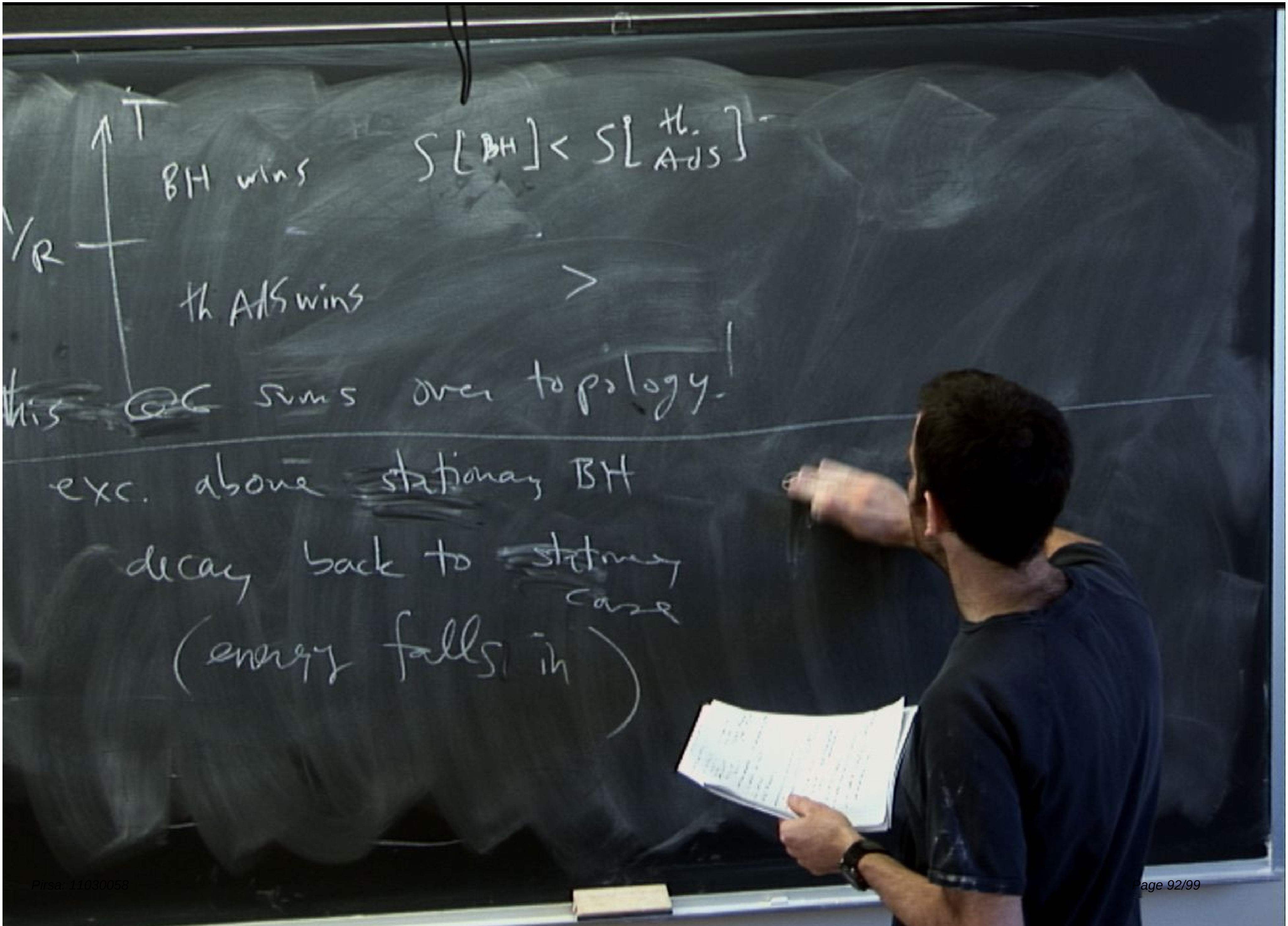
this QG sums over topology!

exc. above stationary BH


T
BH wins
S[BH] < S[th. AdS]
1/R
th AdS wins
>
this QG sums over topology!
exc. above stationary BH
decay back to stationary case
(energy falls in)

T

BH wins $\quad S[BH] < S[\text{th. AdS}]$

$1/R$

th AdS wins $\quad >$

this QG sums over topology!

exc. above ~~stationary~~ BH

decay back to ~~stationary~~ case

(energy falls in)

by "ring-down"

via quasinormal modes of BH

$\mathrm{Im}\,\omega_A < 0$

$\phi(t) \sim e^{-i\omega_A t}$

T

BH wins $\quad S[\text{BH}] < S[\text{th. AdS}]$

$1/R$

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this QG sums over topology!

exc. above stationary BH

decay back to stationary case

(energy falls in)

by "ring-down"

via quasinormal modes of BH

$\mathrm{Im}\,\omega_* < 0$

$\phi(t) \sim e^{-i\omega_* t} \sim e^{-\gamma t}\, e^{-i\mathrm{Re}\,\omega_* t}$

T

BH wins $\quad S[\mathrm{BH}] < S[\text{th. AdS}]$

$1/R$

th AdS wins $\quad >$

this QG sums over topology!

exc. above ~~stationary~~ BH

decay back to ~~stationary~~ case by "ring-down"

via quasinormal modes of BH

(energy falls in)

$\mathrm{Im}\,\omega_* < 0$

$\boxed{\gamma \sim \frac{T}{T}}$

$\phi(t) \sim e^{-i\omega_* t} \sim \underbrace{e^{-\gamma t}}\, e^{-i\mathrm{Re}\,\omega_* t}$

T

BH wins $\quad S[BH] < S[\text{th. AdS}]$

$1/R$

th AdS wins $\quad >$

this ~~QC~~ sums over topology!

exc. above ~~stationary~~ BH

decay back to ~~stationary~~ case by "ring-down"

of (energy falls in) via quasinormal modes of BH

$\boxed{\gamma \sim \frac{1}{T}}$

$\mathrm{Im}\,\omega_* < 0$

$\phi(t) \sim e^{-i\omega_* t} \sim e^{-\gamma t} e^{-i\mathrm{Re}\,\omega_* t}$

T

BH wins $\qquad S[\text{BH}] < S[\text{th. AdS}]$

$1/R$

th AdS wins $\qquad >$

$\Rightarrow$ this QG sums over topology!

exc. above stationary BH

decay back to stationary case by "ring-down"

(energy falls in) via quasinormal m

presence of horizon $\Rightarrow$ dissipation. $\boxed{\gamma \sim \frac{1}{T}}$

$\mathrm{Im}\,\omega_* < 0$

$\phi(t) \sim e^{-i\omega_* t} \sim e^{-\gamma t}$

T

BH wins $\quad S[BH] < S[\text{th. AdS}]$

$1/R$

th AdS wins $\quad >$

$\Rightarrow$ this ~~QG~~ sums over topology!

exc. above stationary BH $\quad$ by "ring-down"

decay back to stationary case $\quad$ via quasinormal m

(energy falls in) $\quad \mathrm{Im}\,\omega_* < 0$

presence of horizon

$\Rightarrow$ dissipation. $\quad \boxed{\gamma \sim \frac{1}{T}}$ $\quad \phi(t) \sim e^{-i\omega_* t} \sim e^{-\gamma t}$

T

BH wins $\quad S[BH] < S[\text{th. AdS}]$

$1/R$

th AdS wins $\quad >$

this QG sums over topology!

[Chesler - Yaffe]

numerical relativity in AdS.

exc. above stationary BH

decay back to stationary case

(energy falls in)

presence of horizon

$\Rightarrow$ dissipation.

$\boxed{\gamma \sim \frac{E}{T}}$

by "ring-down"

via quasinormal modes of BH

$\mathrm{Im}\,\omega_* < 0$

$\phi(t) \sim e^{-i\omega_* t} \sim e^{-\gamma t} e^{-i \mathrm{Re}\,\omega_* t}$