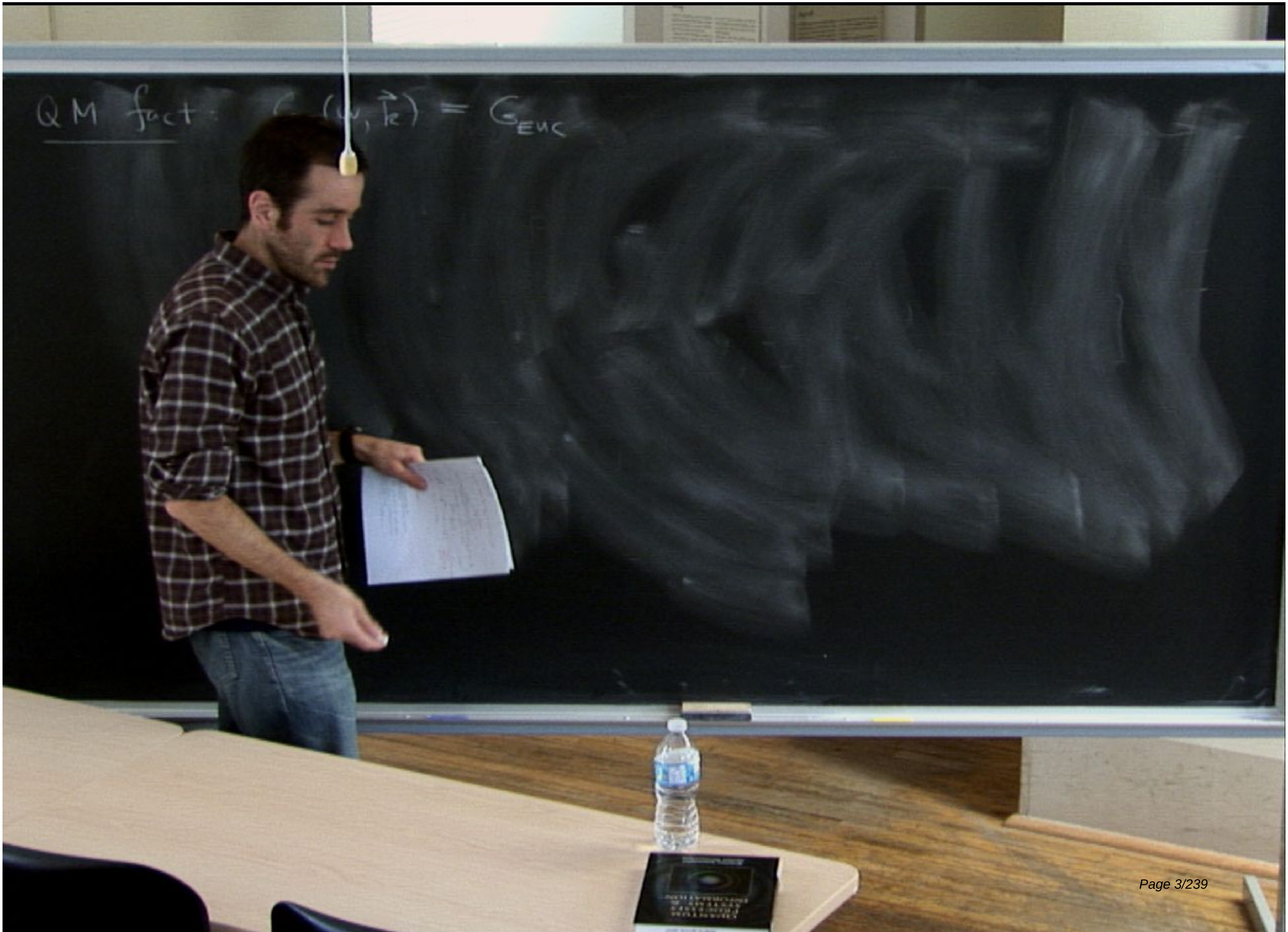
Title: Explorations in String Theory - Lecture 8

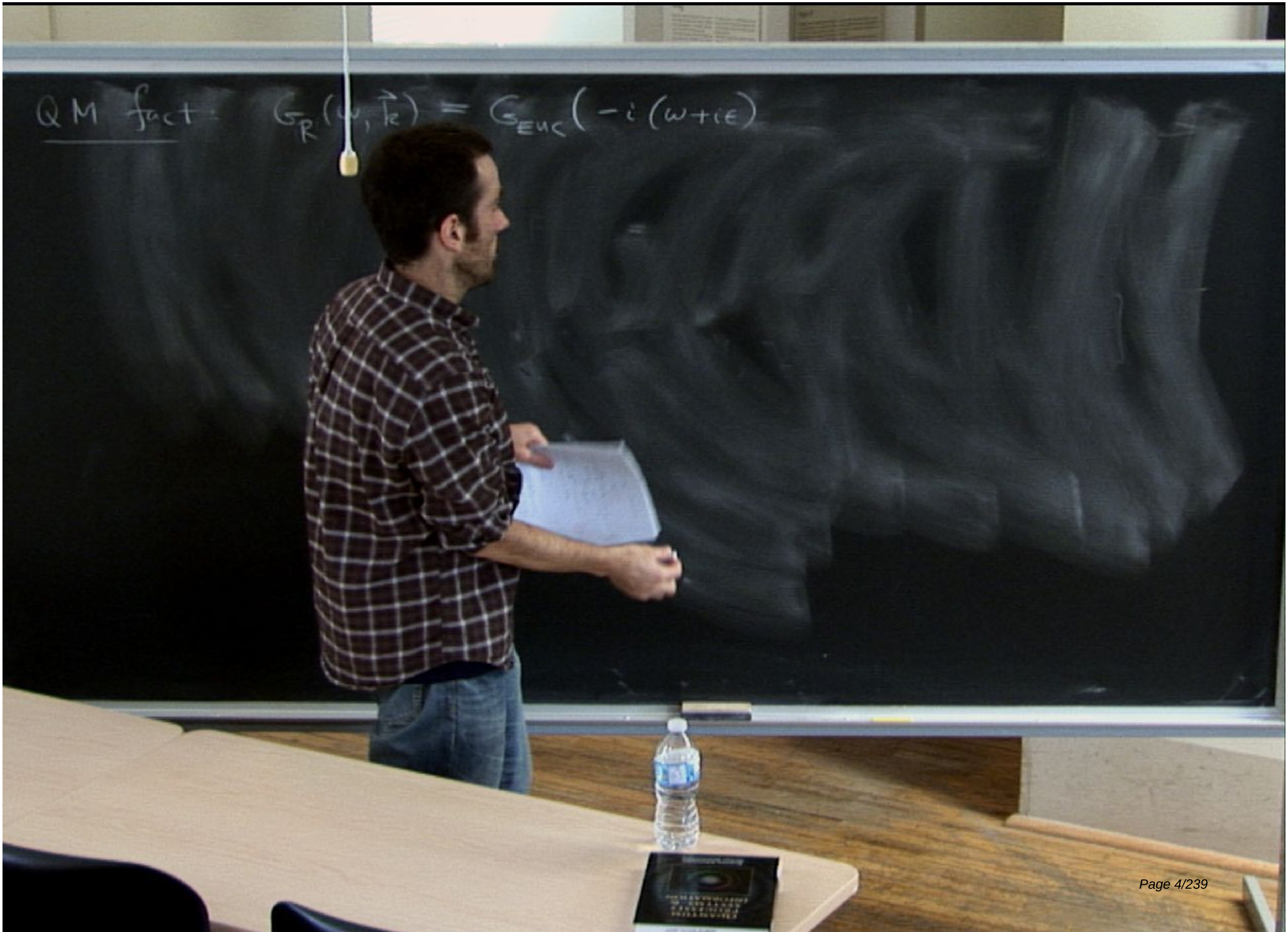
Date: Mar 23, 2011 11:30 AM

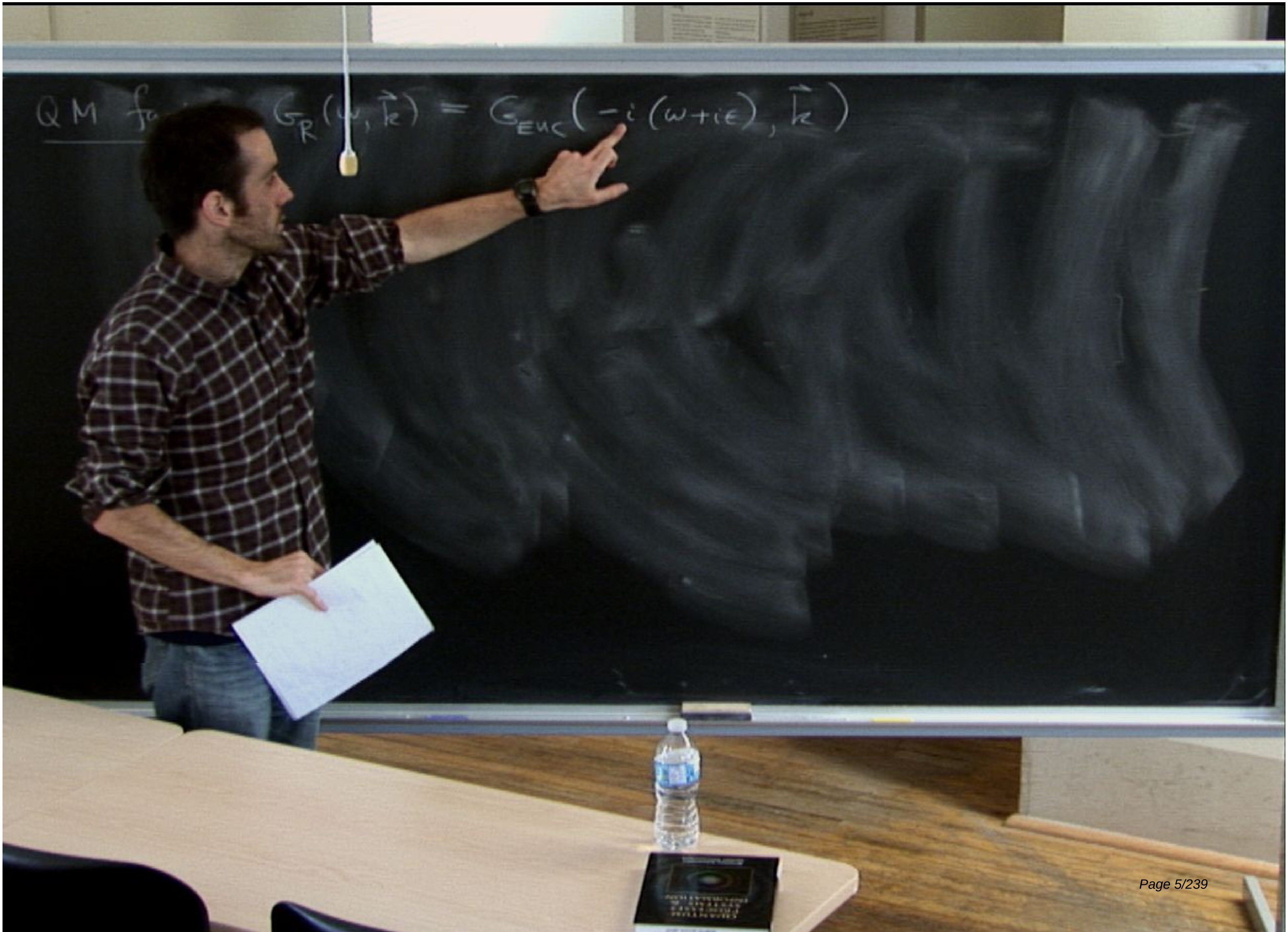
URL: http://pirsa.org/11030057

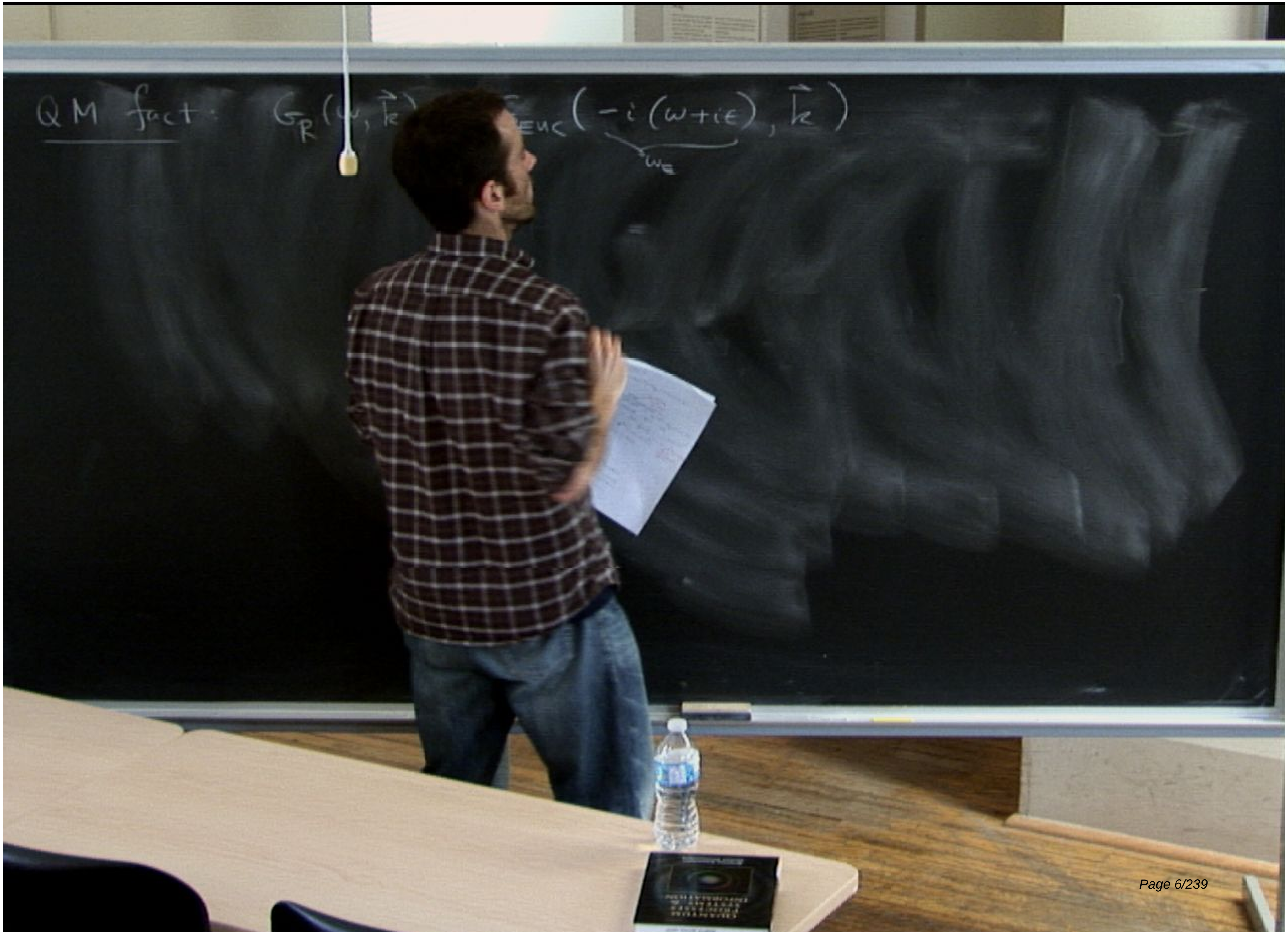
Abstract:

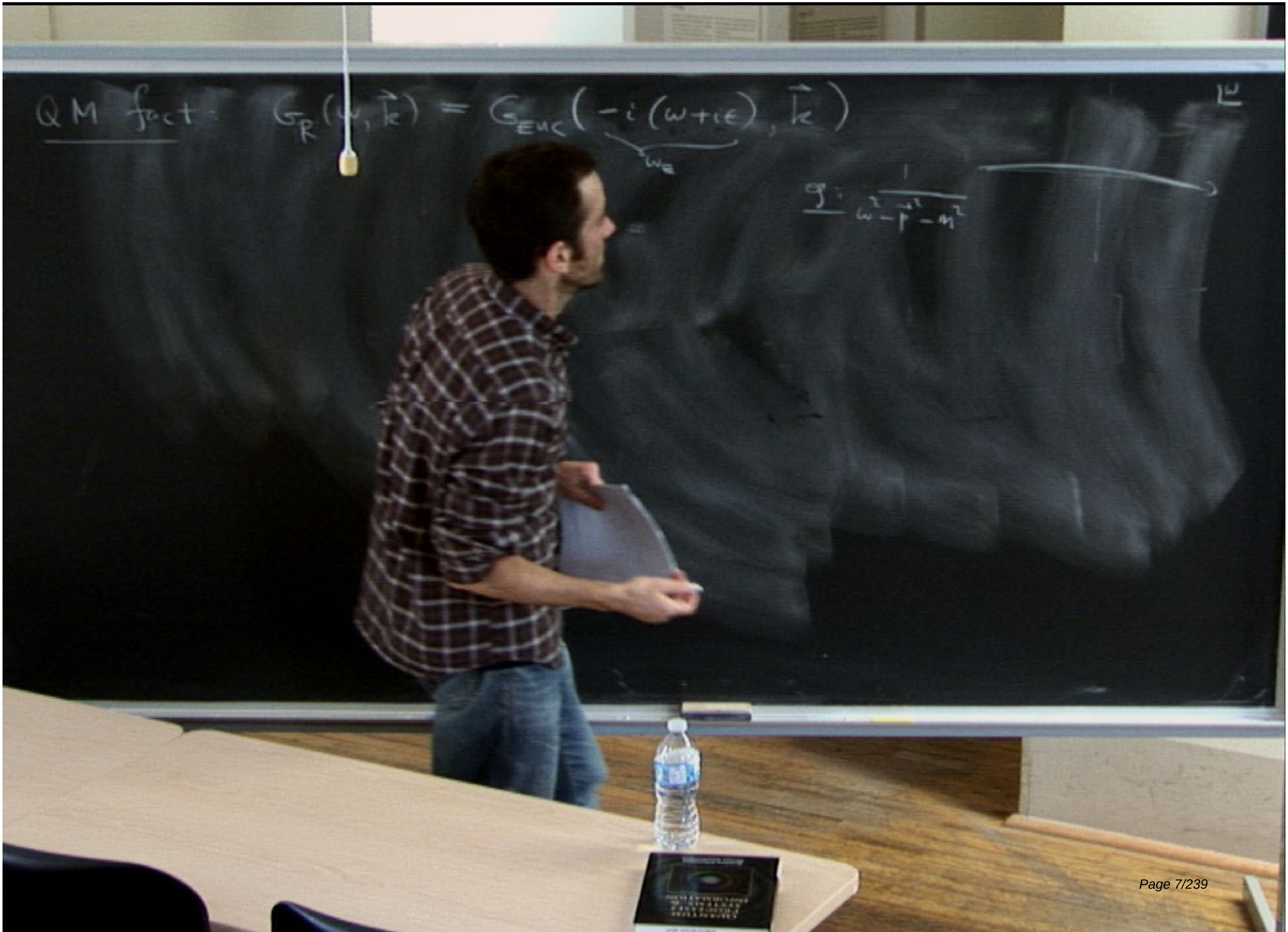

M fact: G_R(ω, k) = G_Euc

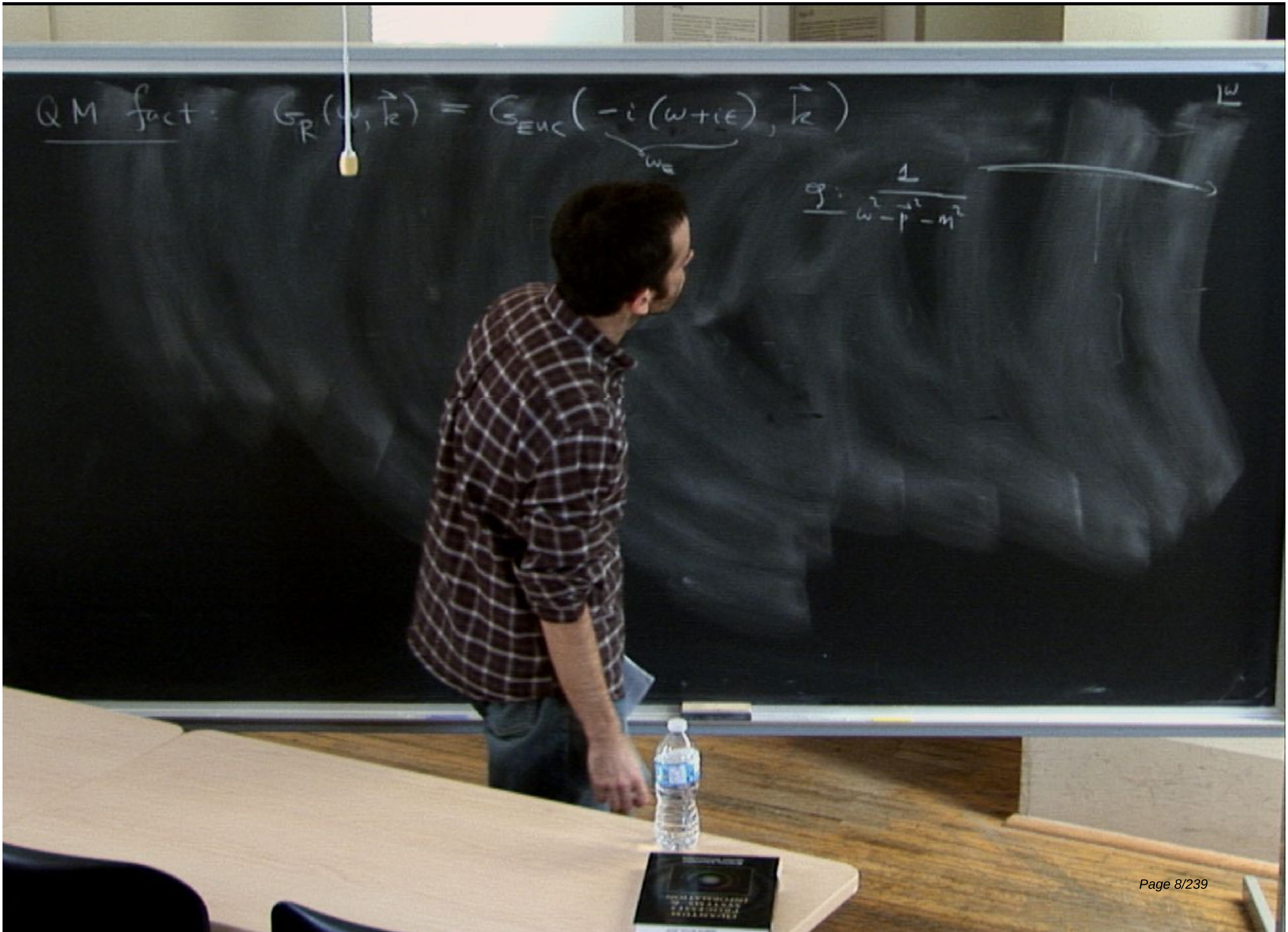

QM fact :

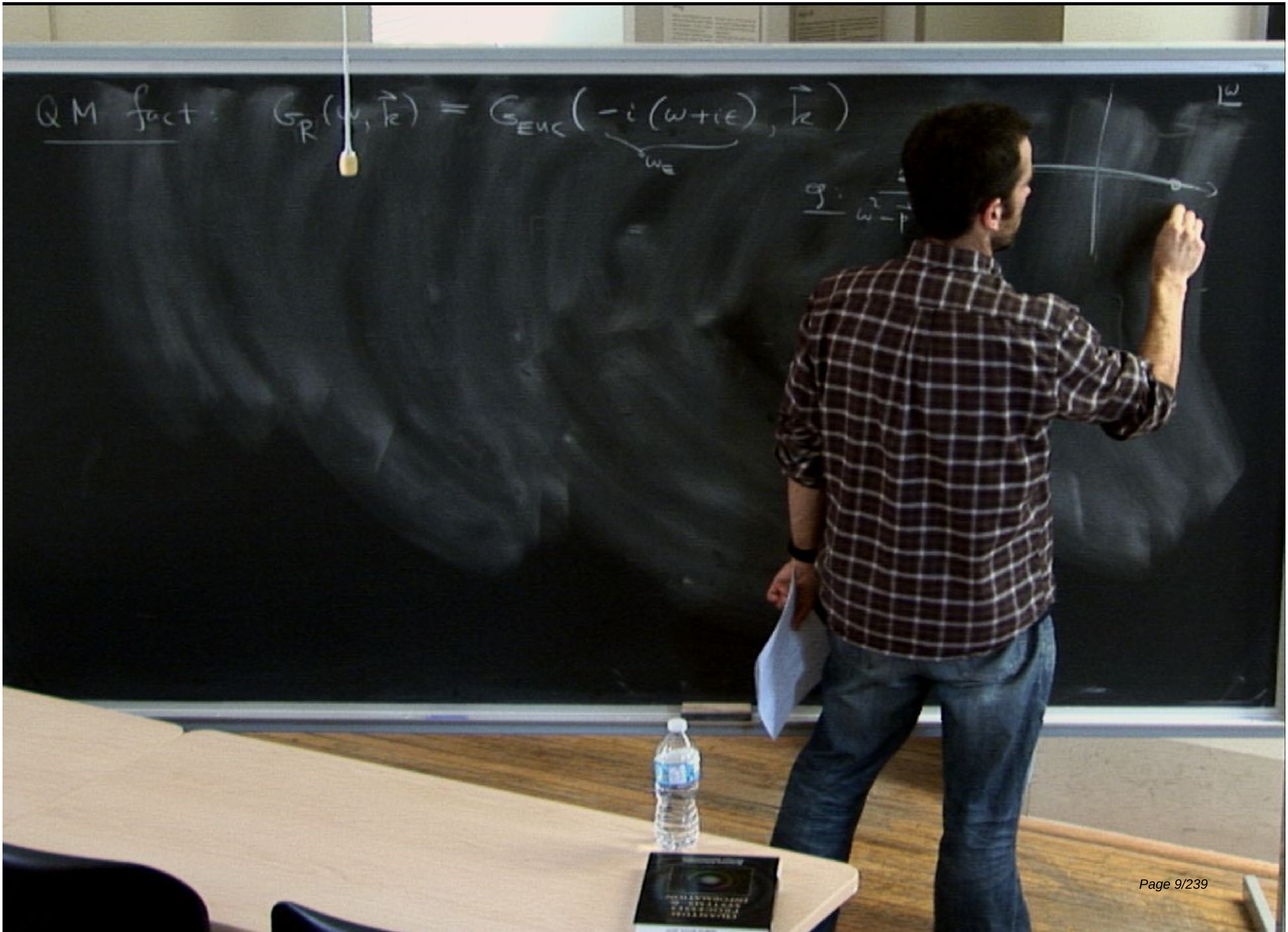

QM fact: $G_R(\omega, \vec{k}) = G_{Euc}(-i(\omega+i\epsilon)$

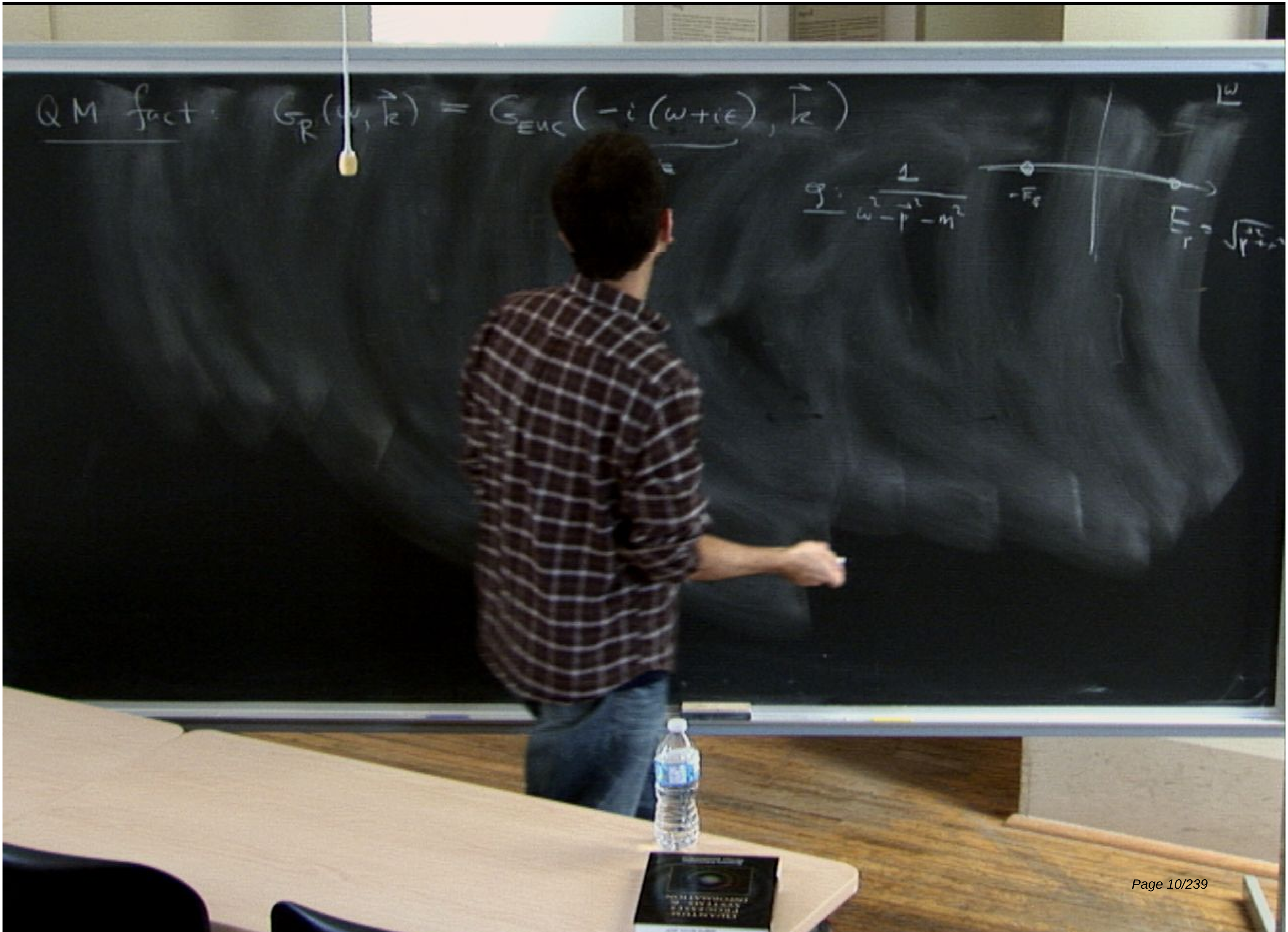

QM fo
$G_R(\omega, \vec{k}) = G_{Euc}(-i(\omega + i\epsilon), \vec{k})$

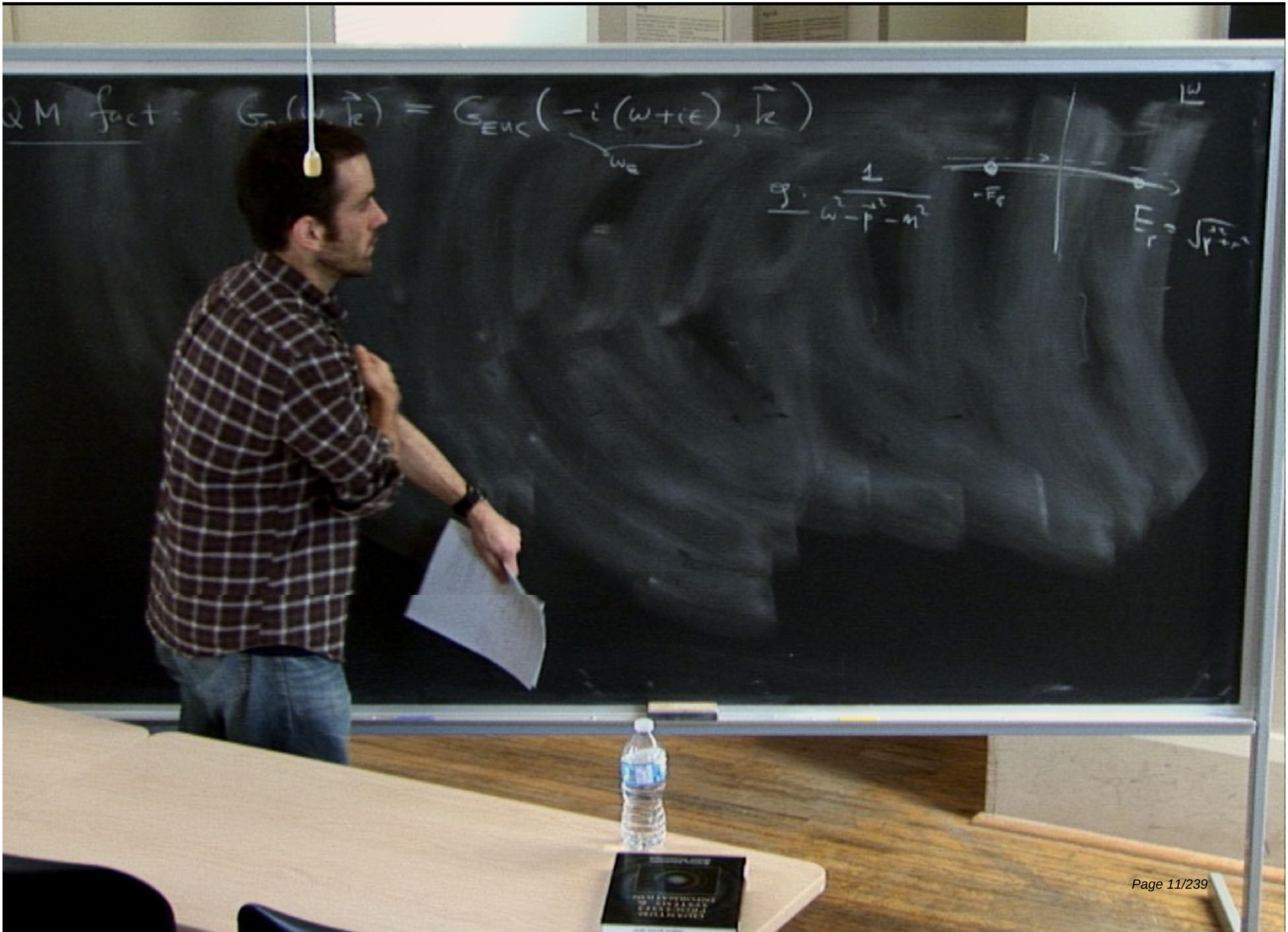

QM fact:
$G_R(\omega, \vec{k})$
$(-i(\omega + i\epsilon), \vec{k})$
ω_E

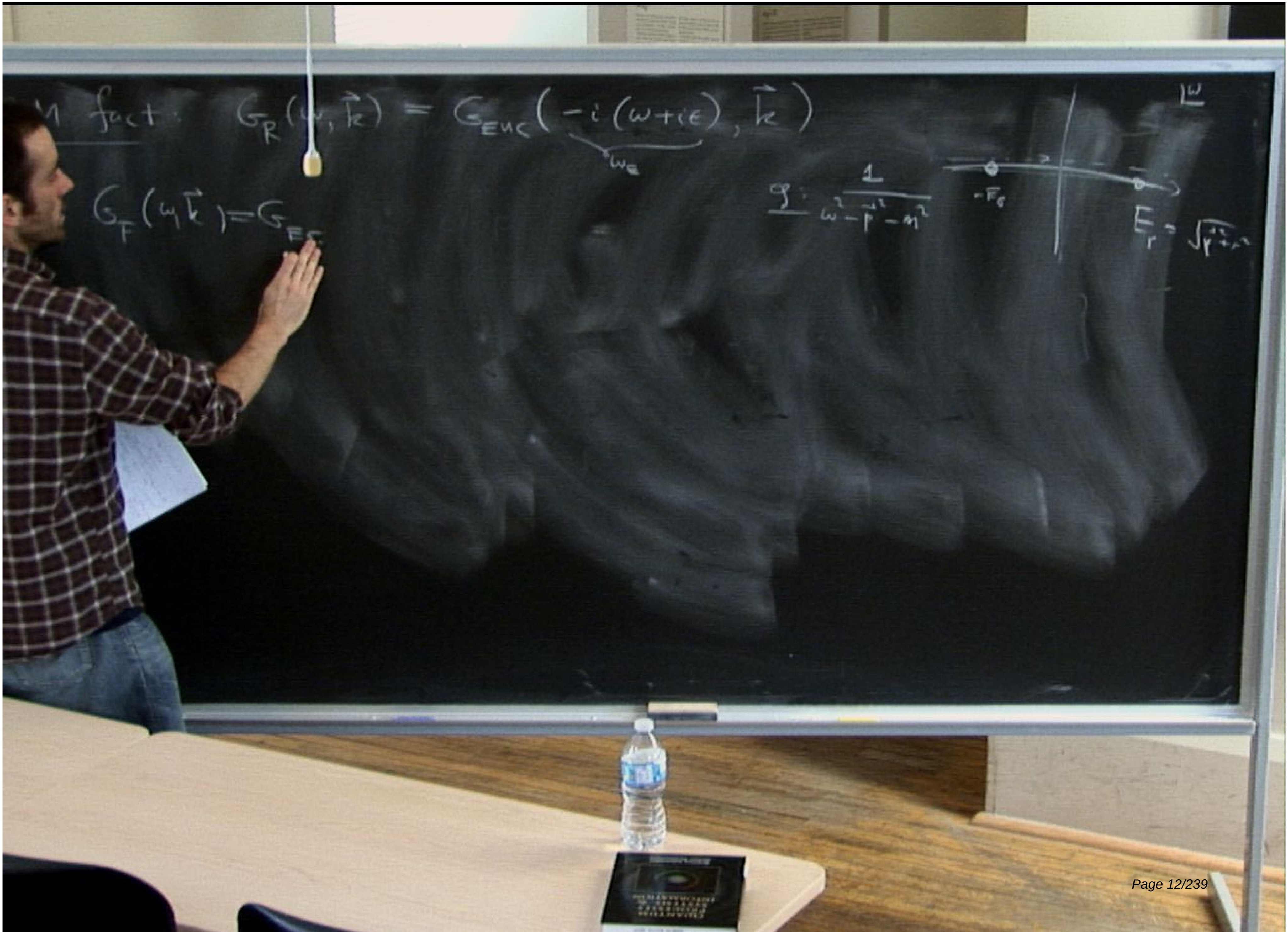

QM fact: G_R(ω, k) = G_EUC(-i(ω+iε), k)
ω_E
g: 1 / (ω² - p² - m²)

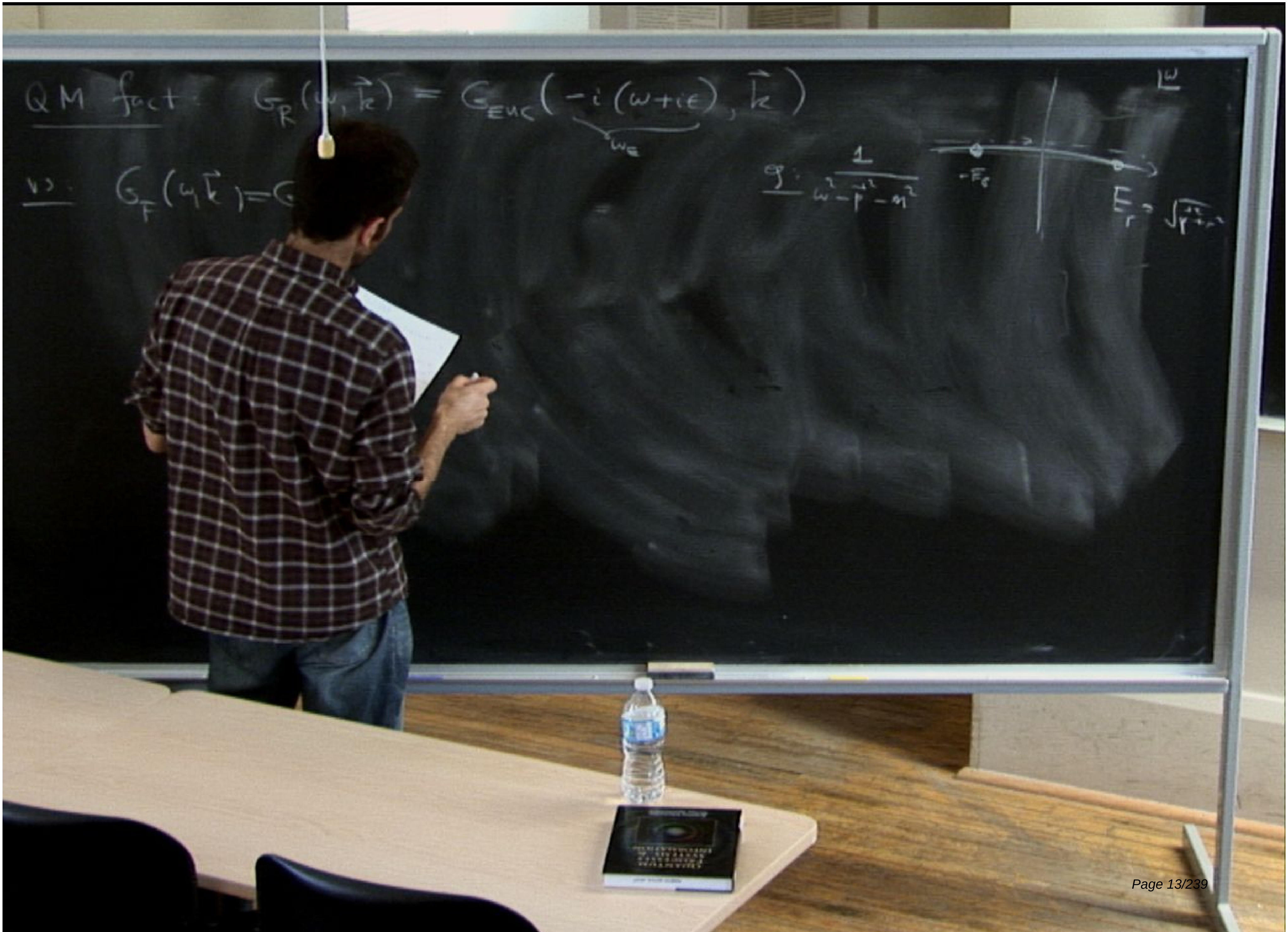

QM fact: G_R(ω, k) = G_EUC(−i(ω+iε), k)
ω_E
eg: 1 / (ω² − p² − m²)
ω

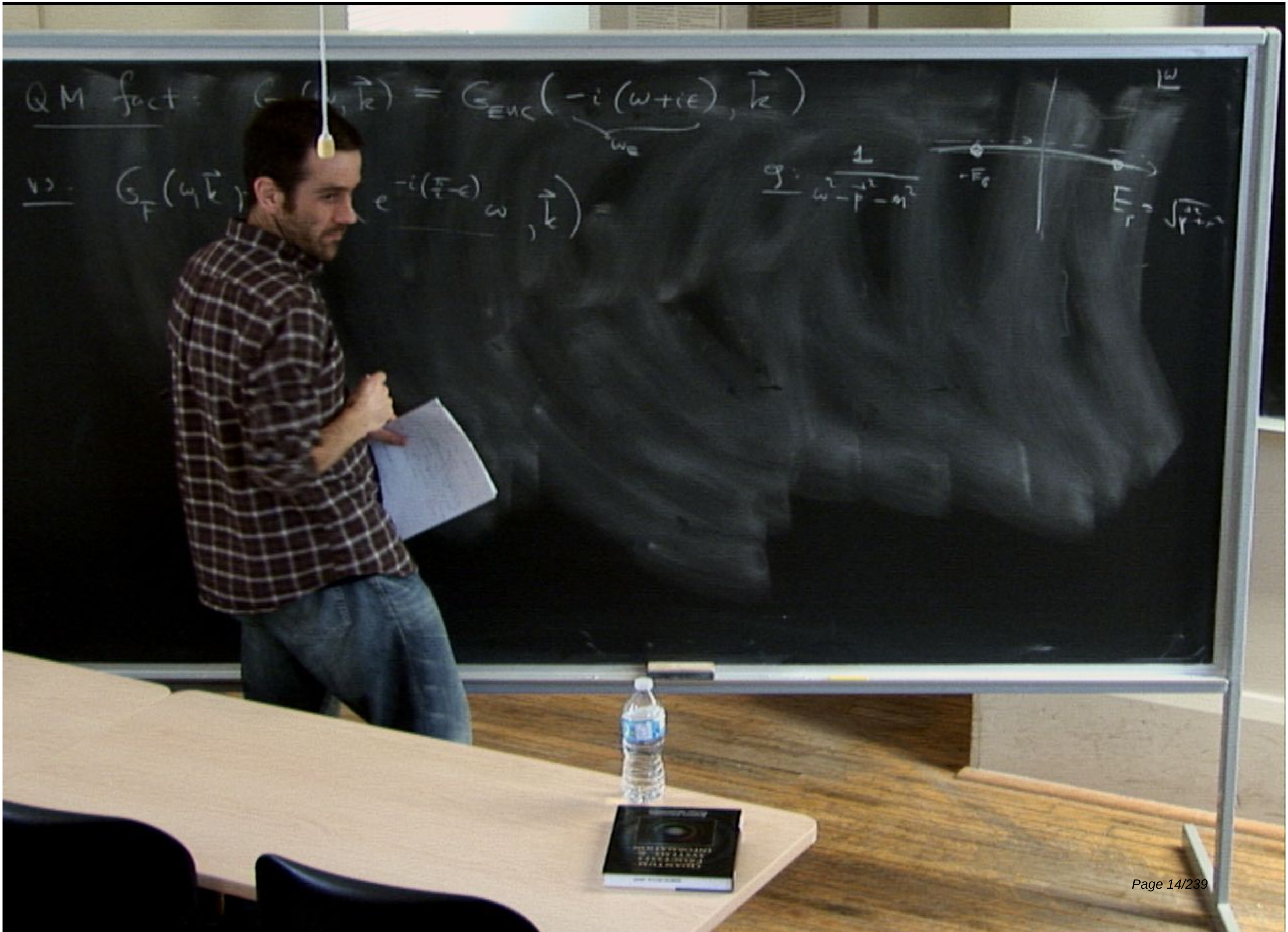

QM fact: $G_R(\omega, \vec{k}) = G_{EUC}(-i(\omega + i\epsilon), \vec{k})$
ω_E

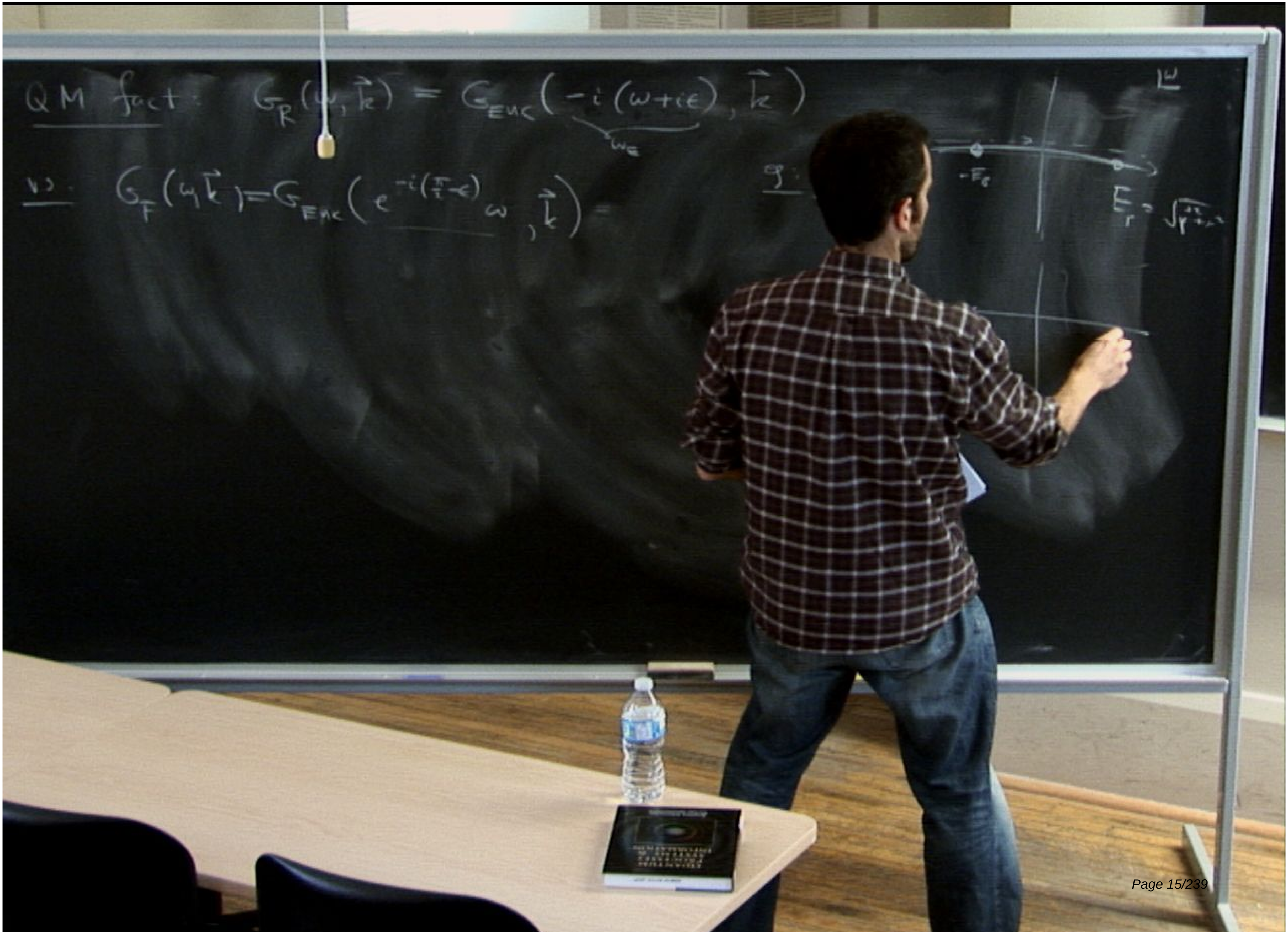

QM fact: G_R(ω, k) = G_Euc(-i(ω+iε), k)
g: 1 / (ω² - p² - m²)
-E_p
ω

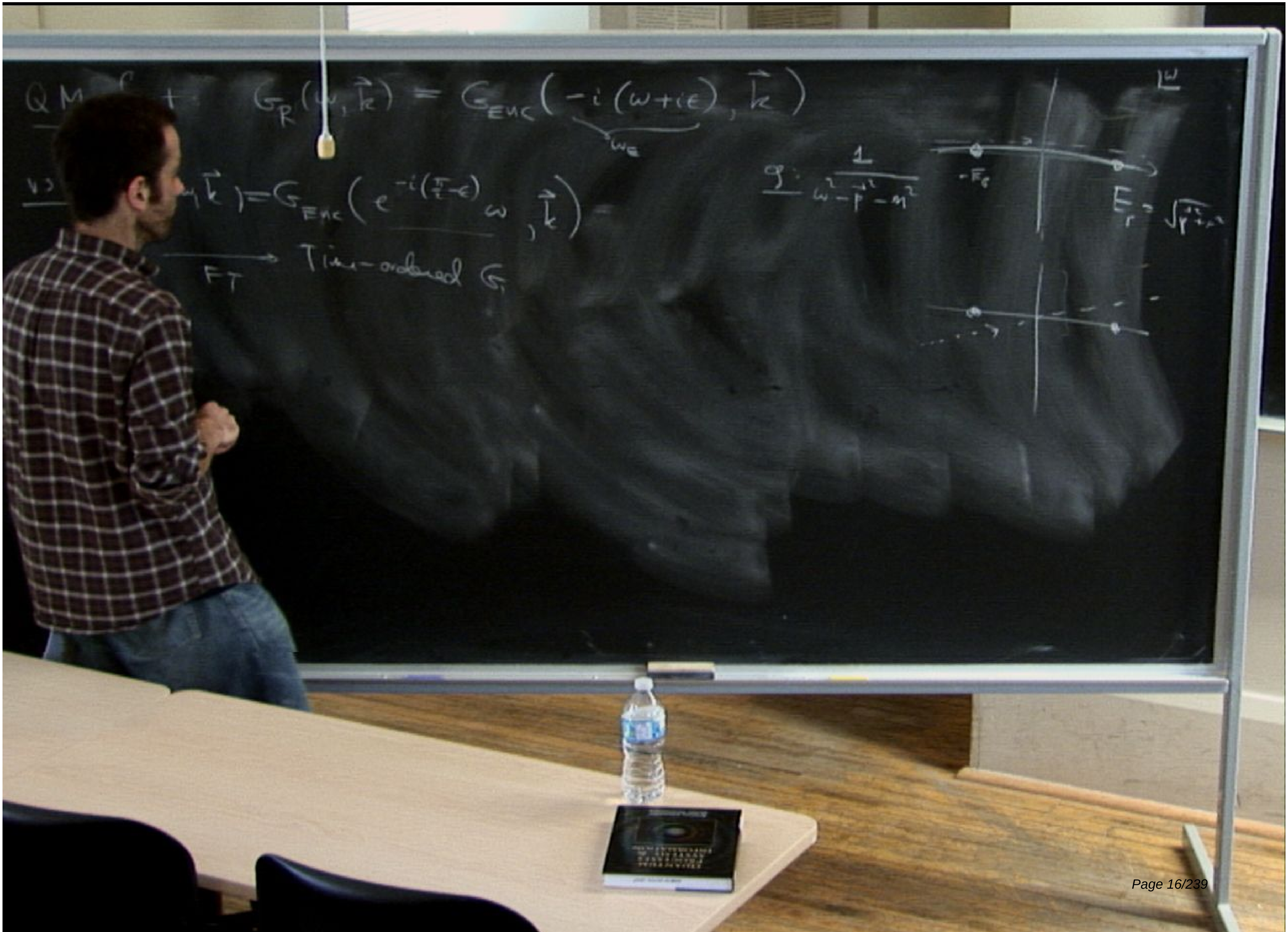

QM fact: $G_R(\omega, \vec{k}) = G_{EUC}(-i(\omega + i\epsilon), \vec{k})$
ω_E
$g: \frac{1}{\omega^2 - \vec{p}^2 - m^2}$
$-E_p$
$E_p = \sqrt{\vec{p}^2 + m^2}$
ω

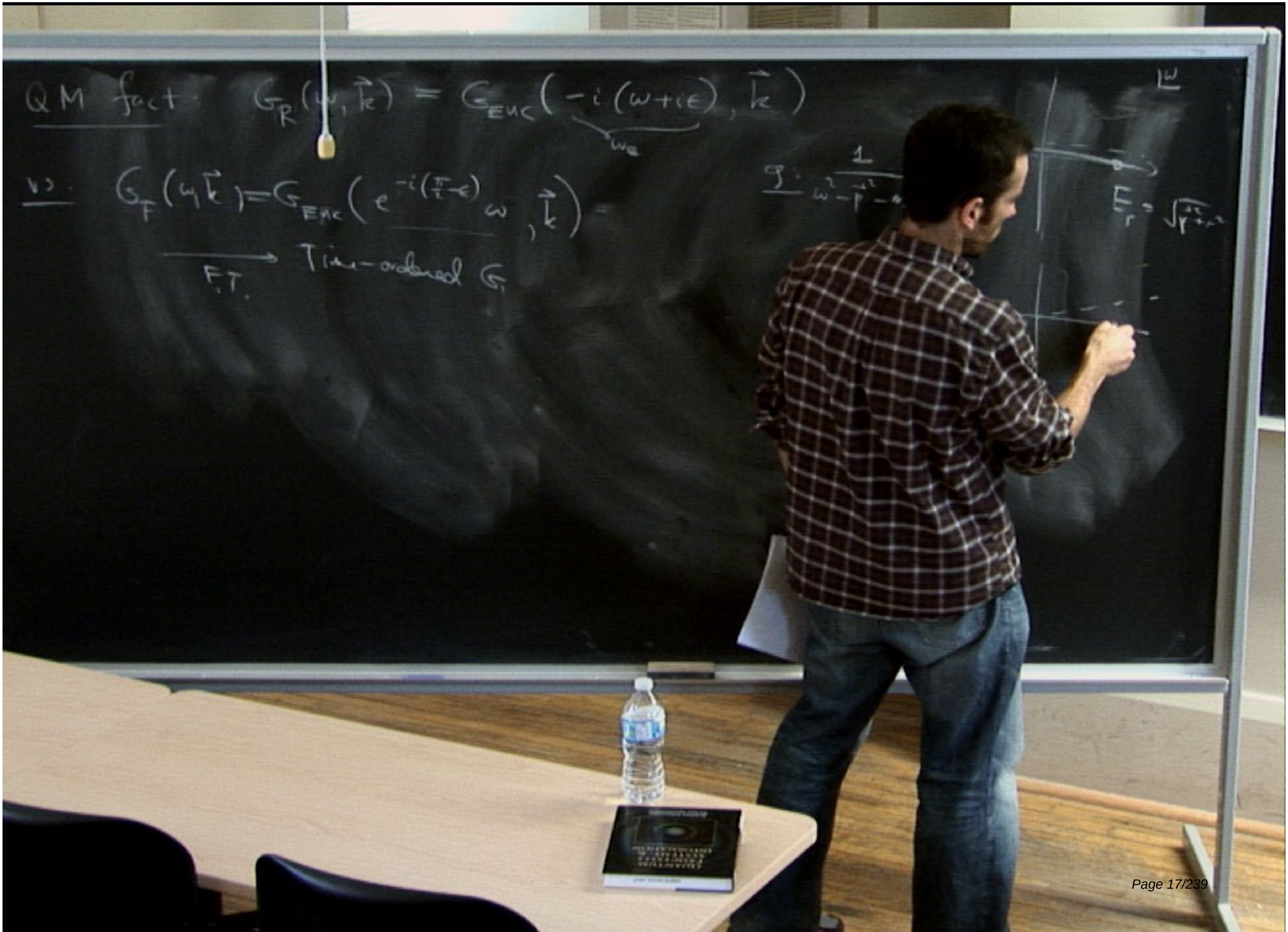

In fact: G_R(ω, k) = G_EUC(-i(ω+iε), k)
ω_E
G_F(ω, k) = G_E
g: 1 / (ω² - p² - m²)
-E_p
E_p = √(p² + m²)

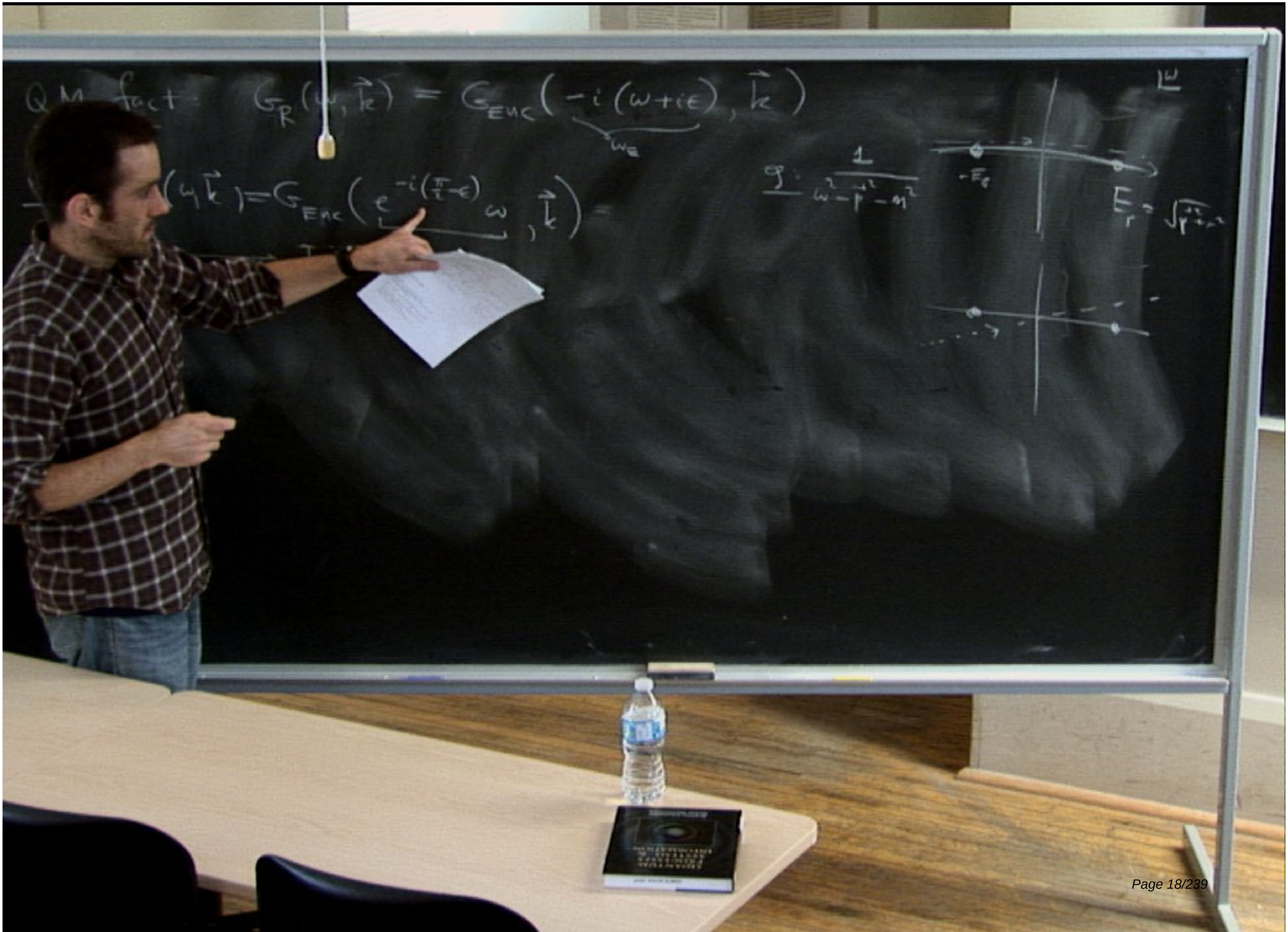

QM fact:
vs.

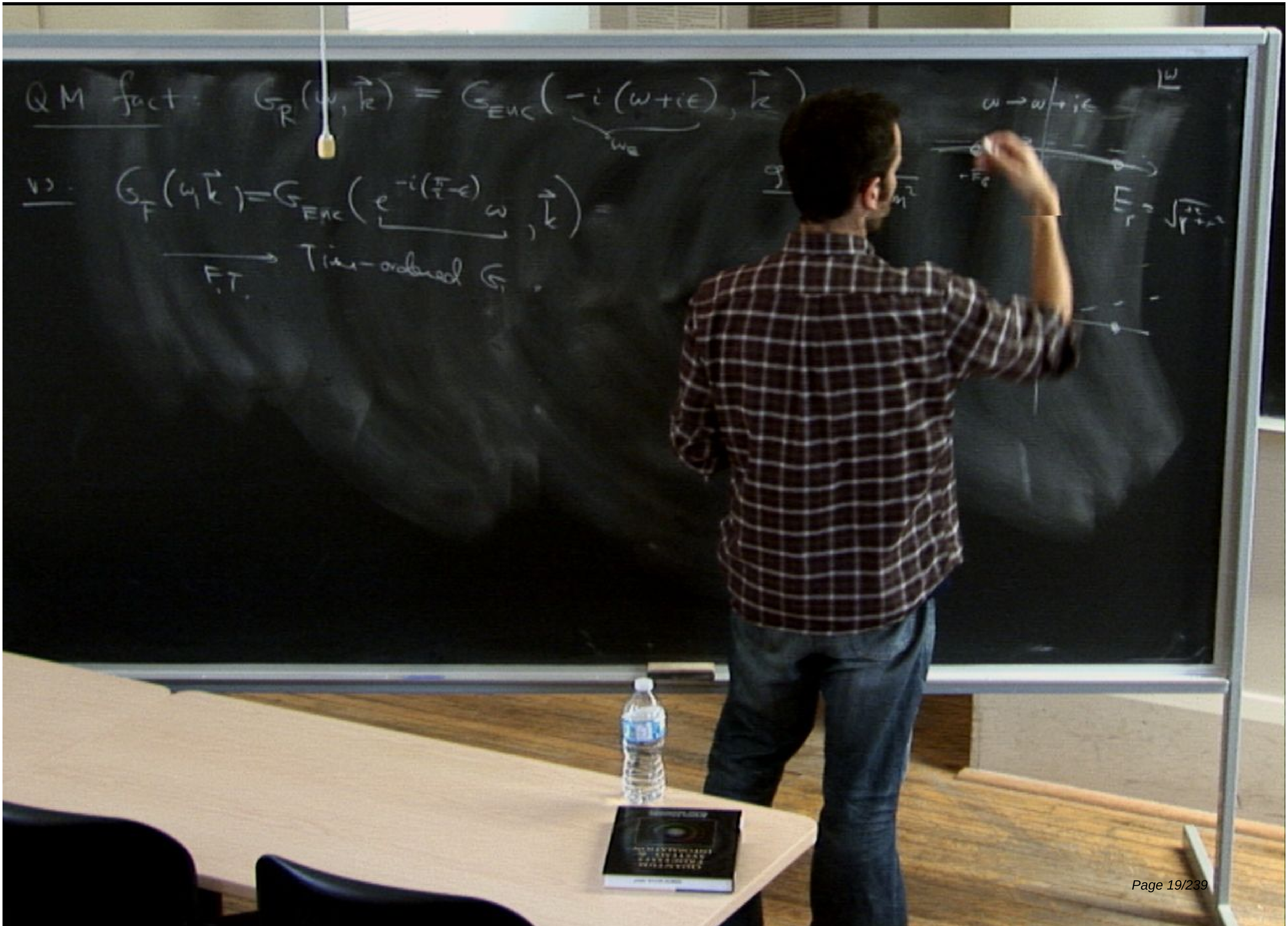

QM fact

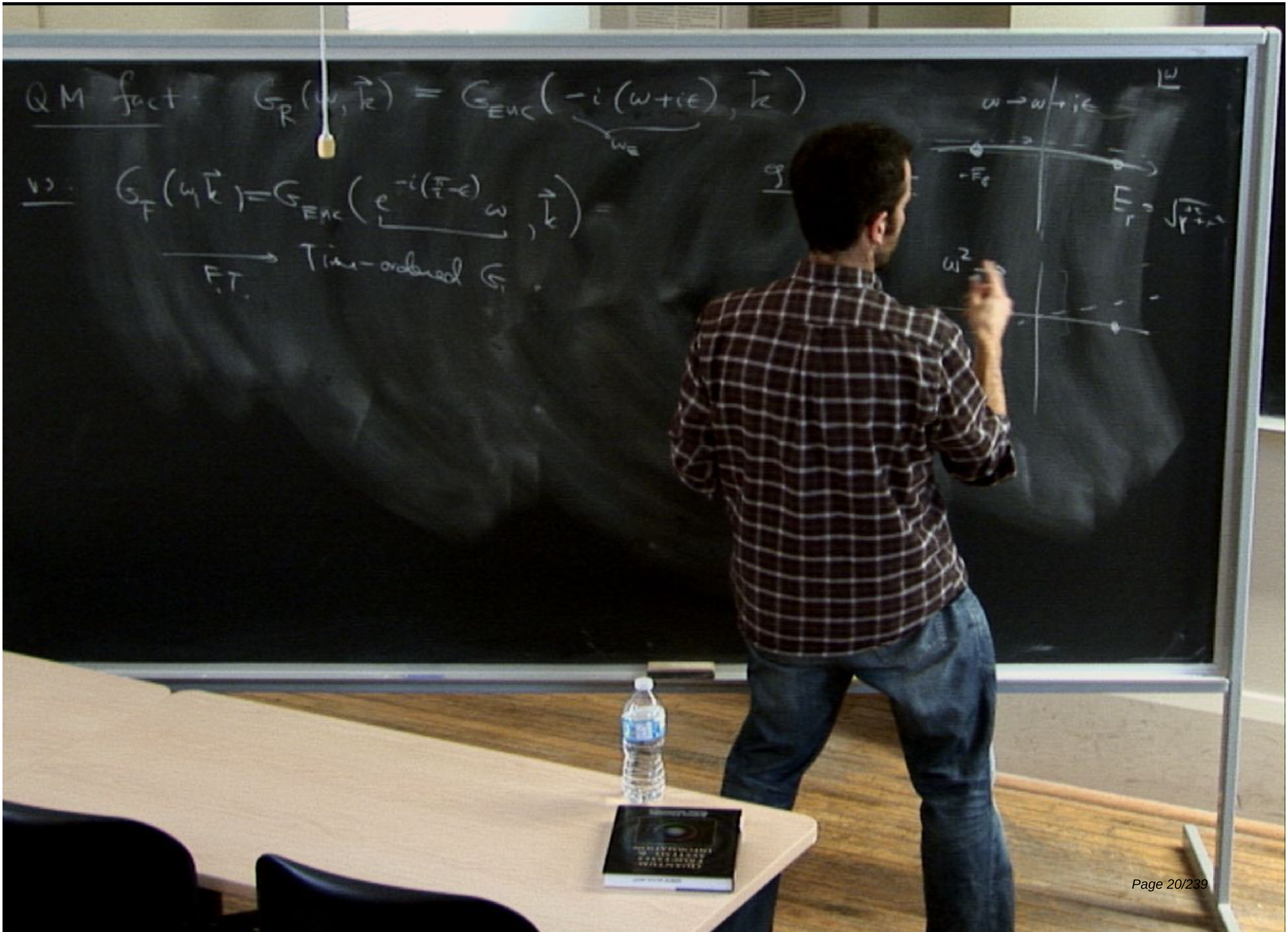

QM fact: $G_R(\omega, \vec{k}) = G_{Euc}(-i(\omega + i\epsilon), \vec{k})$
vs. $G_F(\omega, \vec{k}) = G_{Euc}(e^{-i(\frac{\pi}{2}-\epsilon)}\omega, \vec{k})$

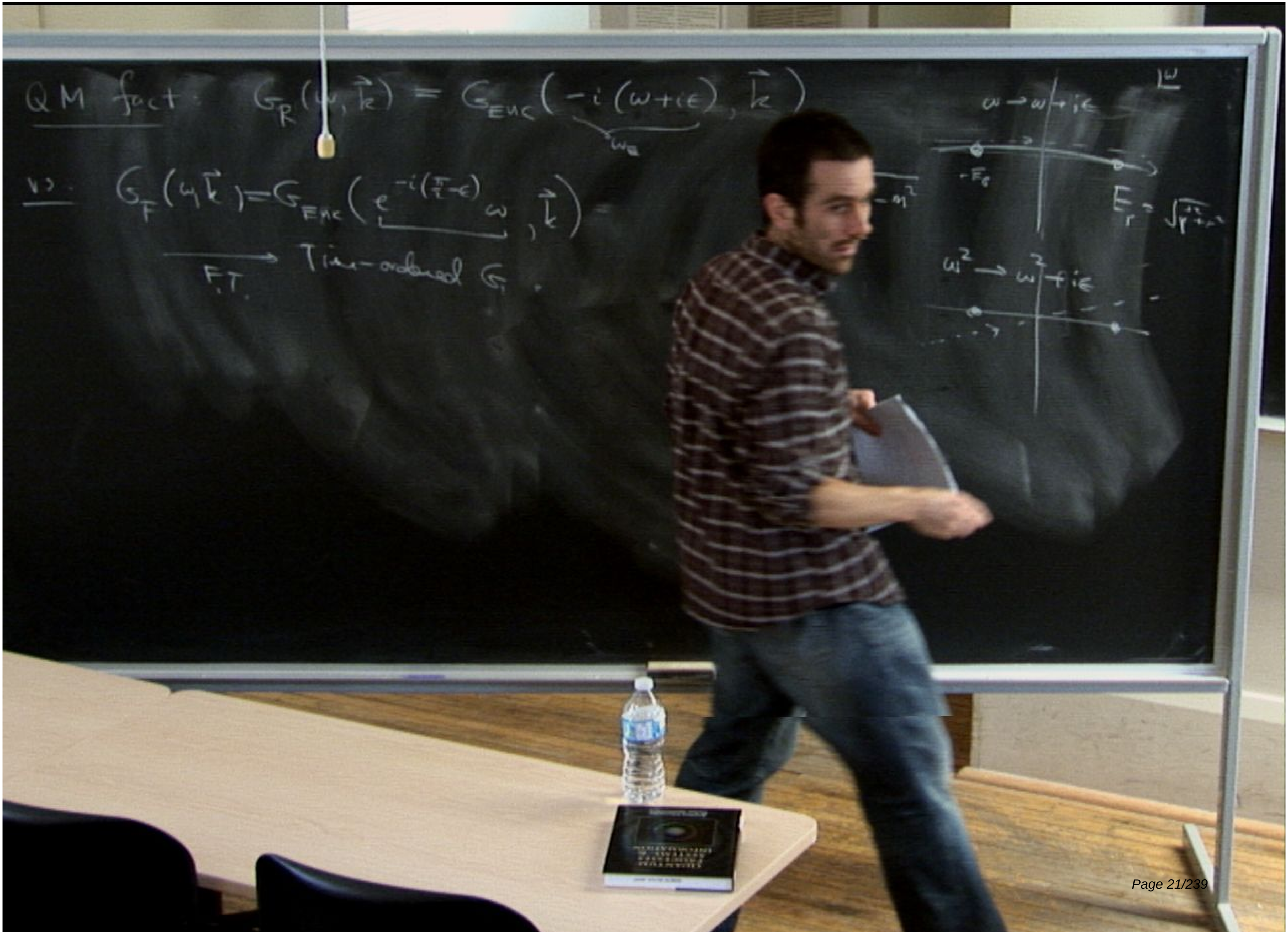

QM
G_R(ω, k) = G_Euc(−i(ω+iε), k)
FT
Time-ordered G

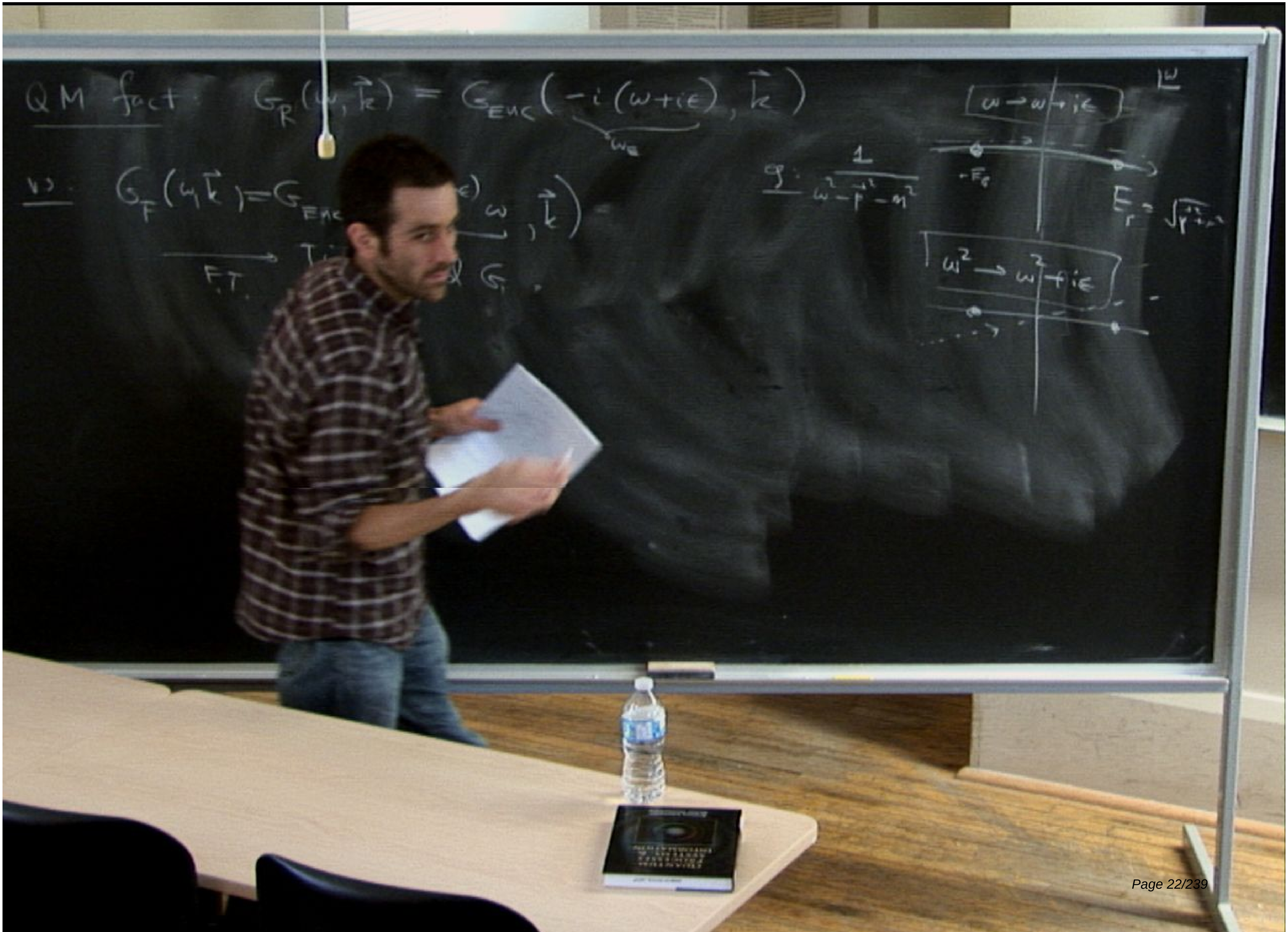

QM fact: G_R(ω, k) = G_Euc(−i(ω+iϵ), k)
ω_E
vs. G_F(ω, k) = G_Euc(e^{−i(π/2−ϵ)} ω, k)
F.T.
Time-ordered G
ω

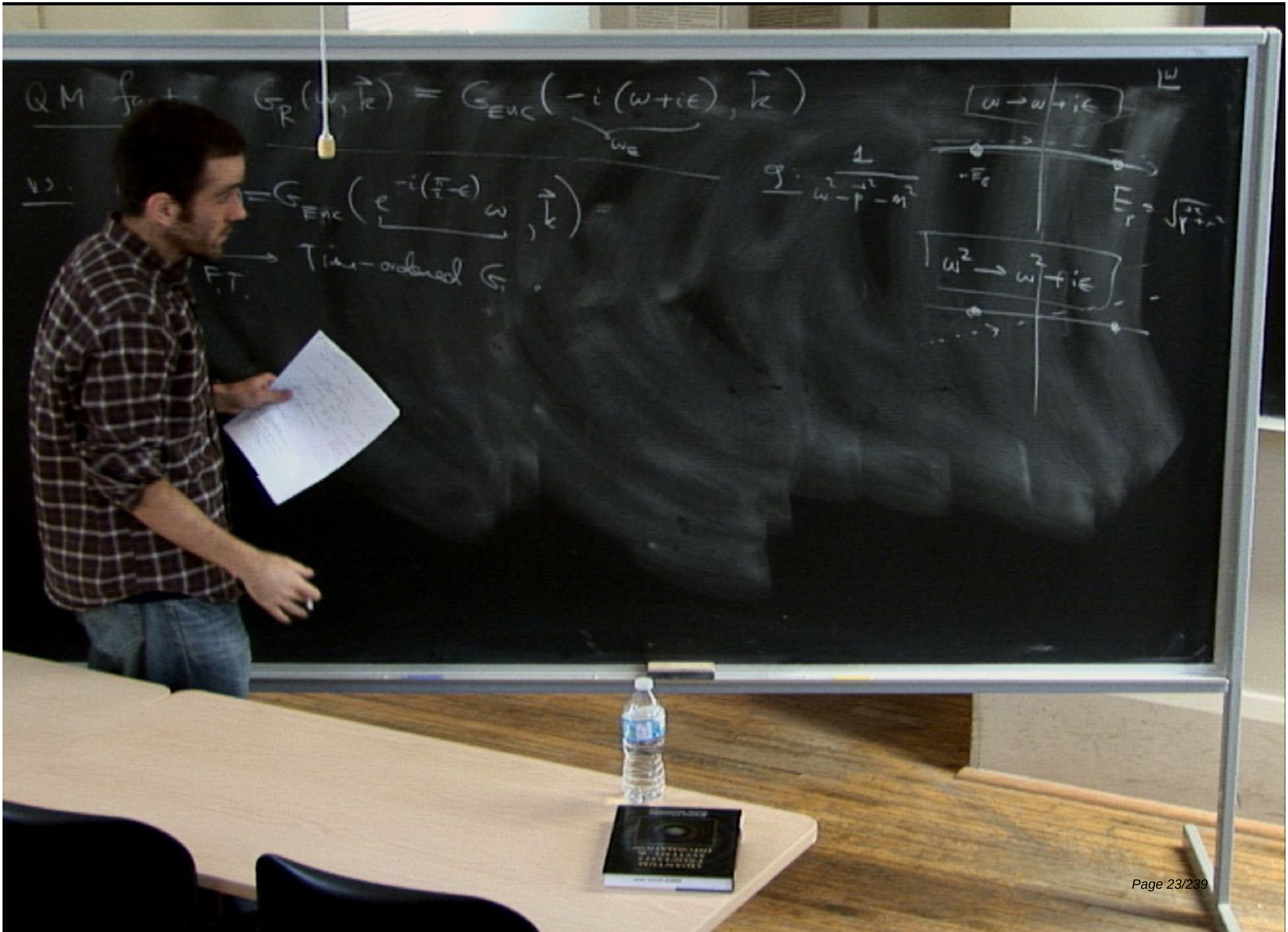

QM fact: G_R(ω, k) = G_Euc(−i(ω+iε), k)

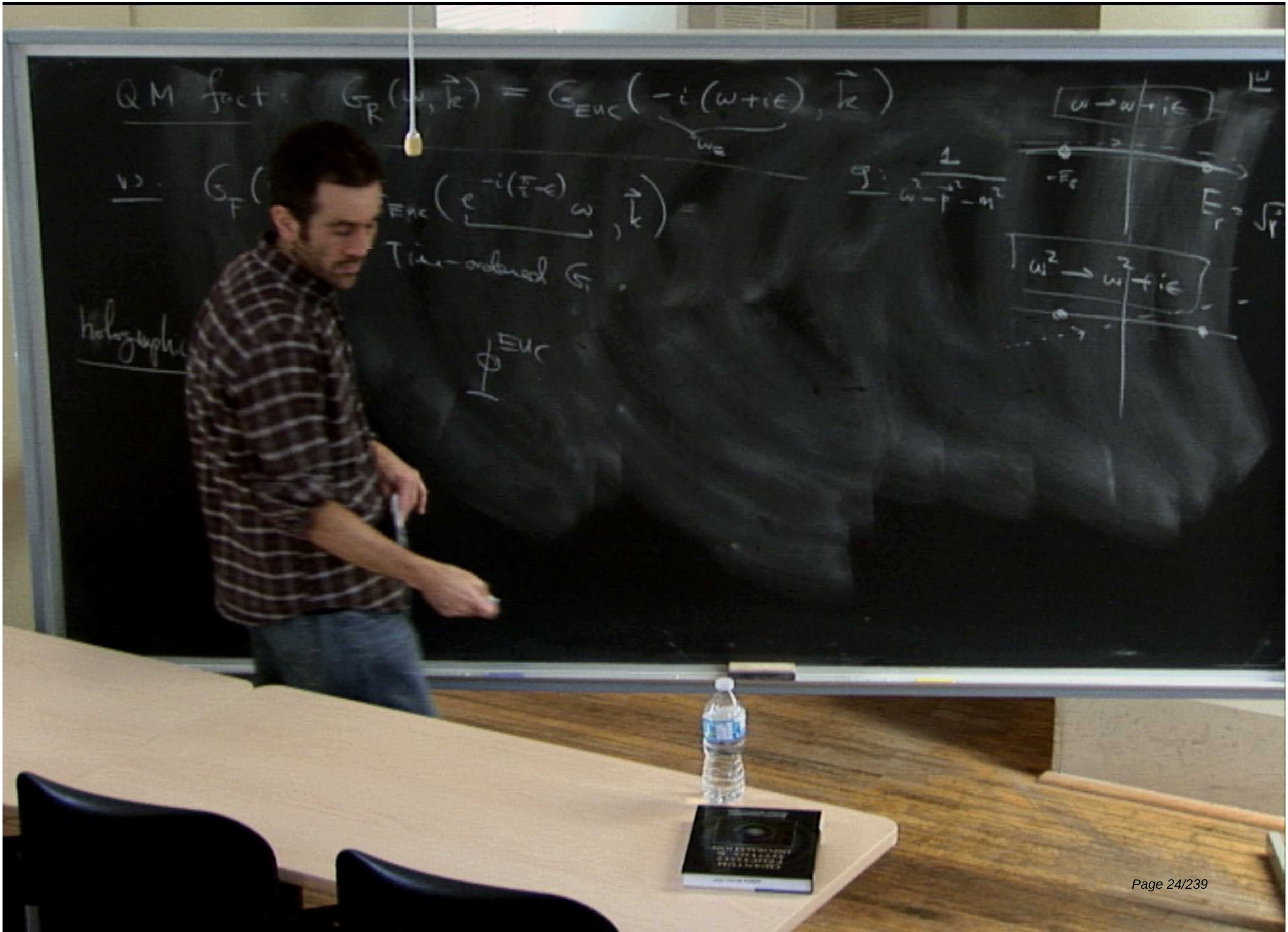

QM fact: $G_R(\omega, \vec{k}) = G_{EUC}(-i(\omega+i\epsilon), \vec{k})$
ω_E
vs. $G_F(\omega, \vec{k}) = G_{EUC}(e^{-i(\frac{\pi}{2}-\epsilon)}\omega, \vec{k})$
F.T.
Time-ordered G
$\omega \to \omega + i\epsilon$

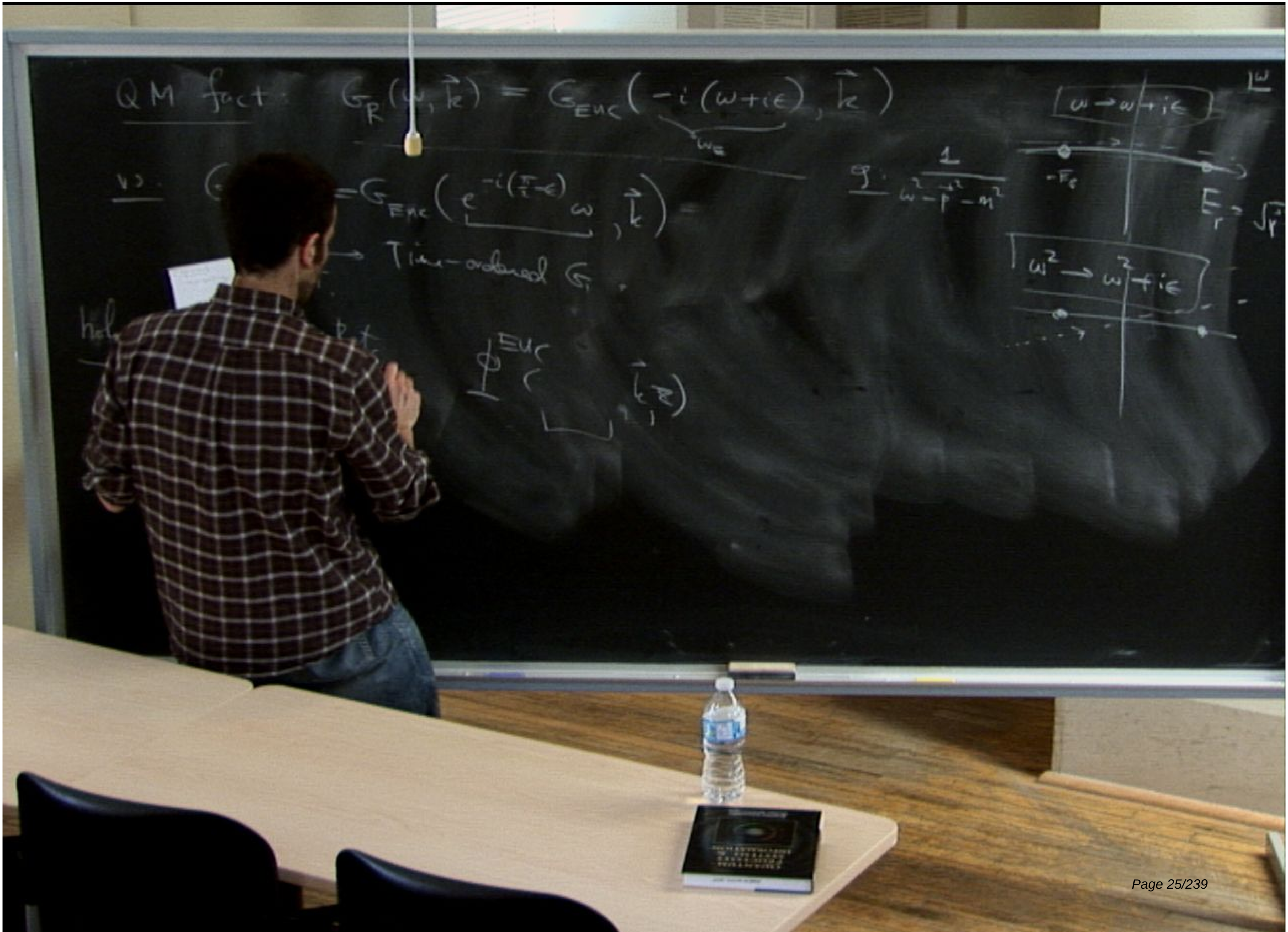

QM fact
G_R(ω, k) = G_Euc(−i(ω+iε), k)
ω_E
vs.
G_F(ω, k) = G_Euc(e^{−i(π/2−ε)} ω, k)
F.T.
Time-ordered G
ω → ω + iε

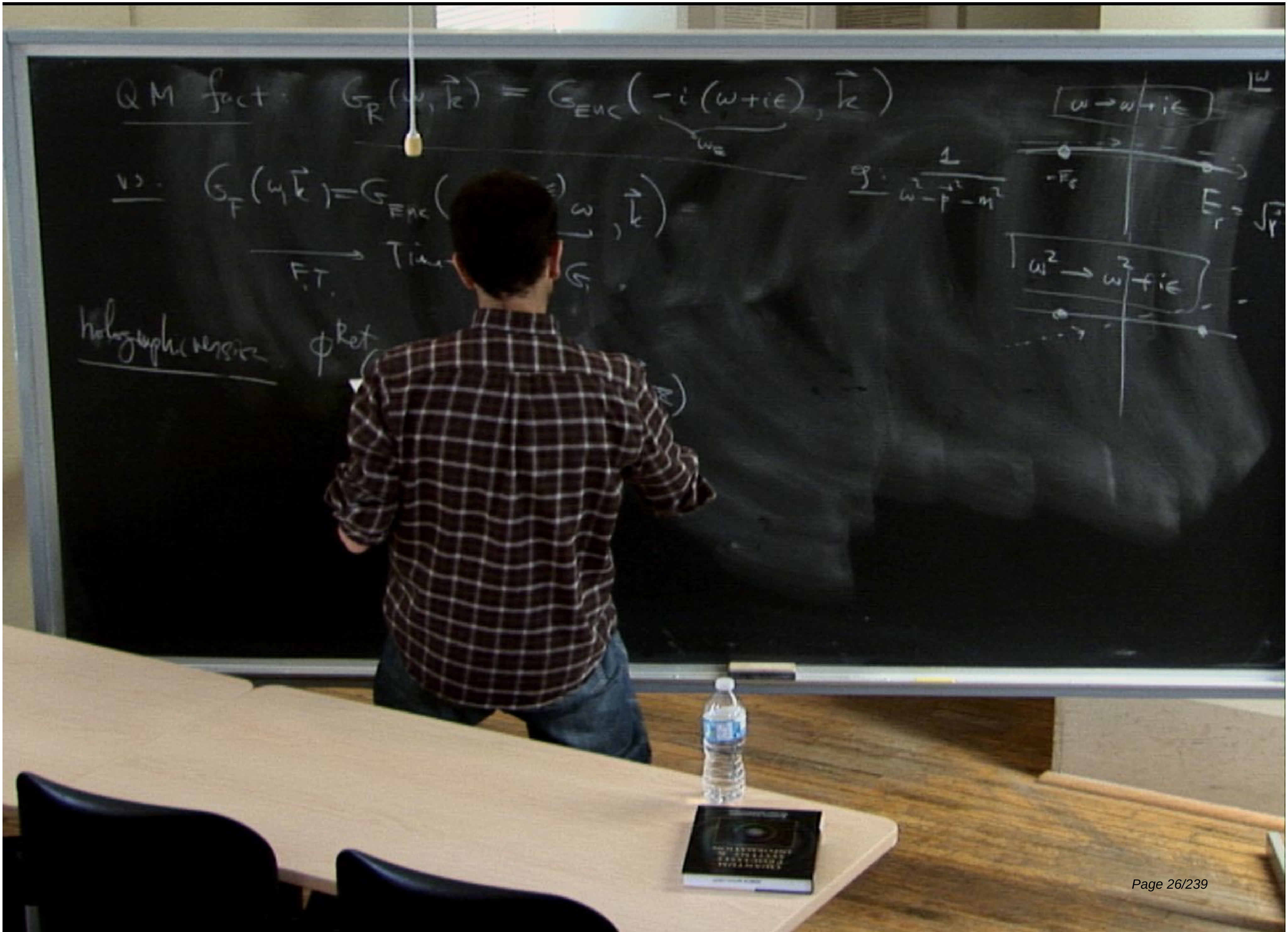

QM fact:
vs.
F.T.
Time-ordered G

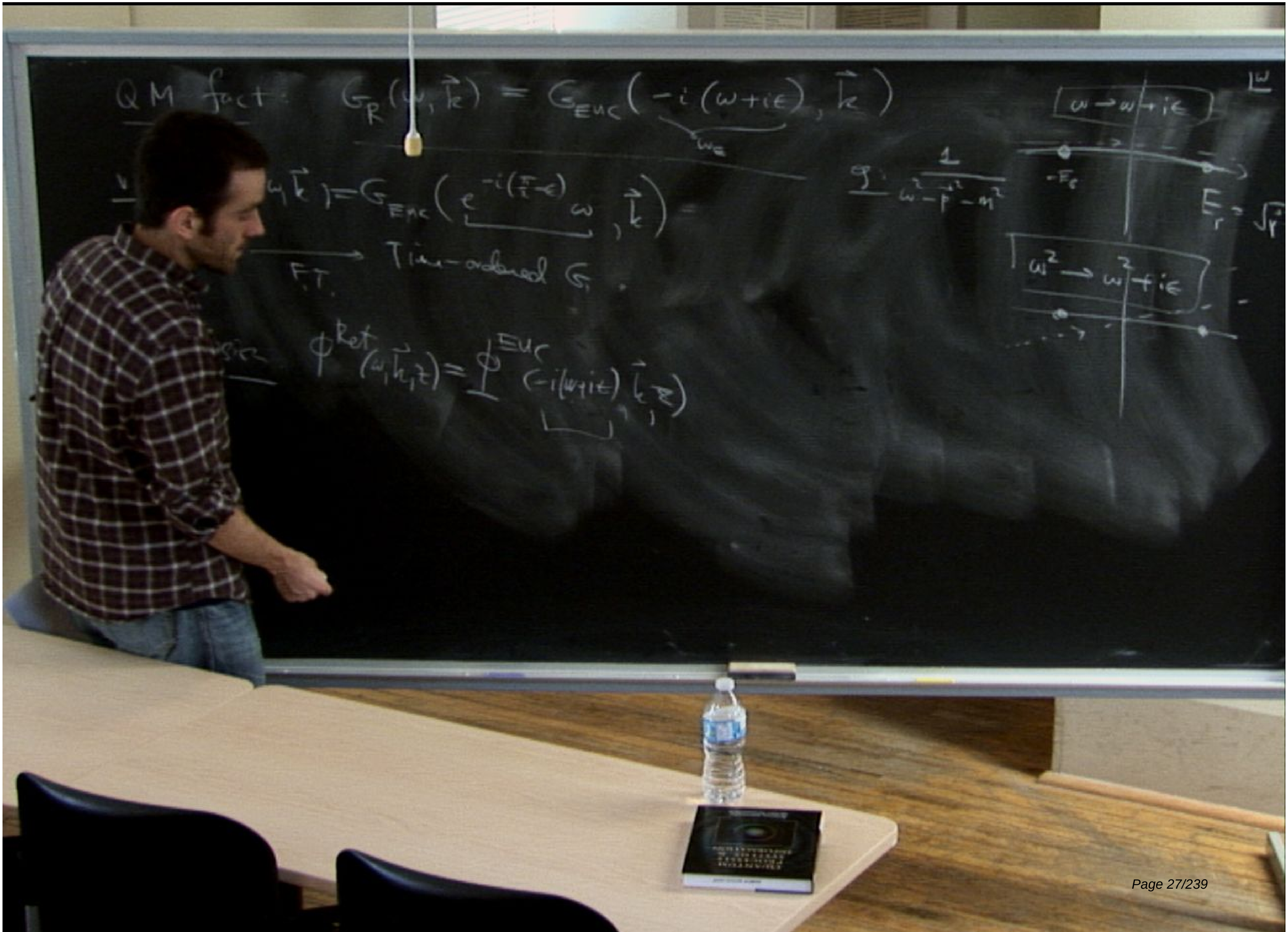

QM fact
vs.
F.T.

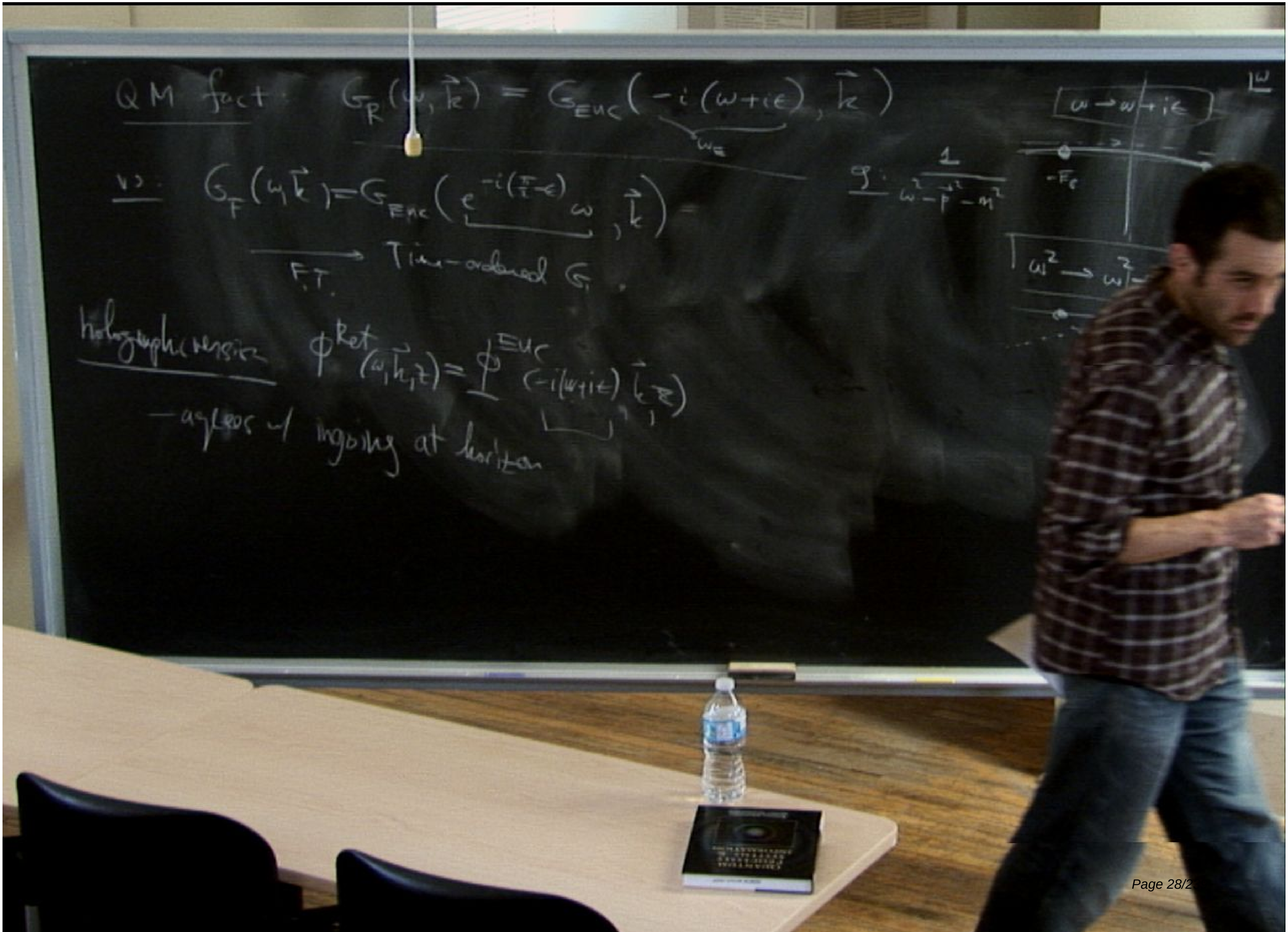

QM
G_R(ω, k) = G_Euc(-i(ω+iε), k)
ω_E
vs.
= G_Enc(e^{-i(π/2-ε)} ω, k)
F.T.
Time-ordered G
ω → ω + iε
-E_p
E_p = √(p² + m²)
g: 1/(ω² - p² - m²)
ω² → ω² + iε


QM fact:
vs.
Time-ordered G
holographic

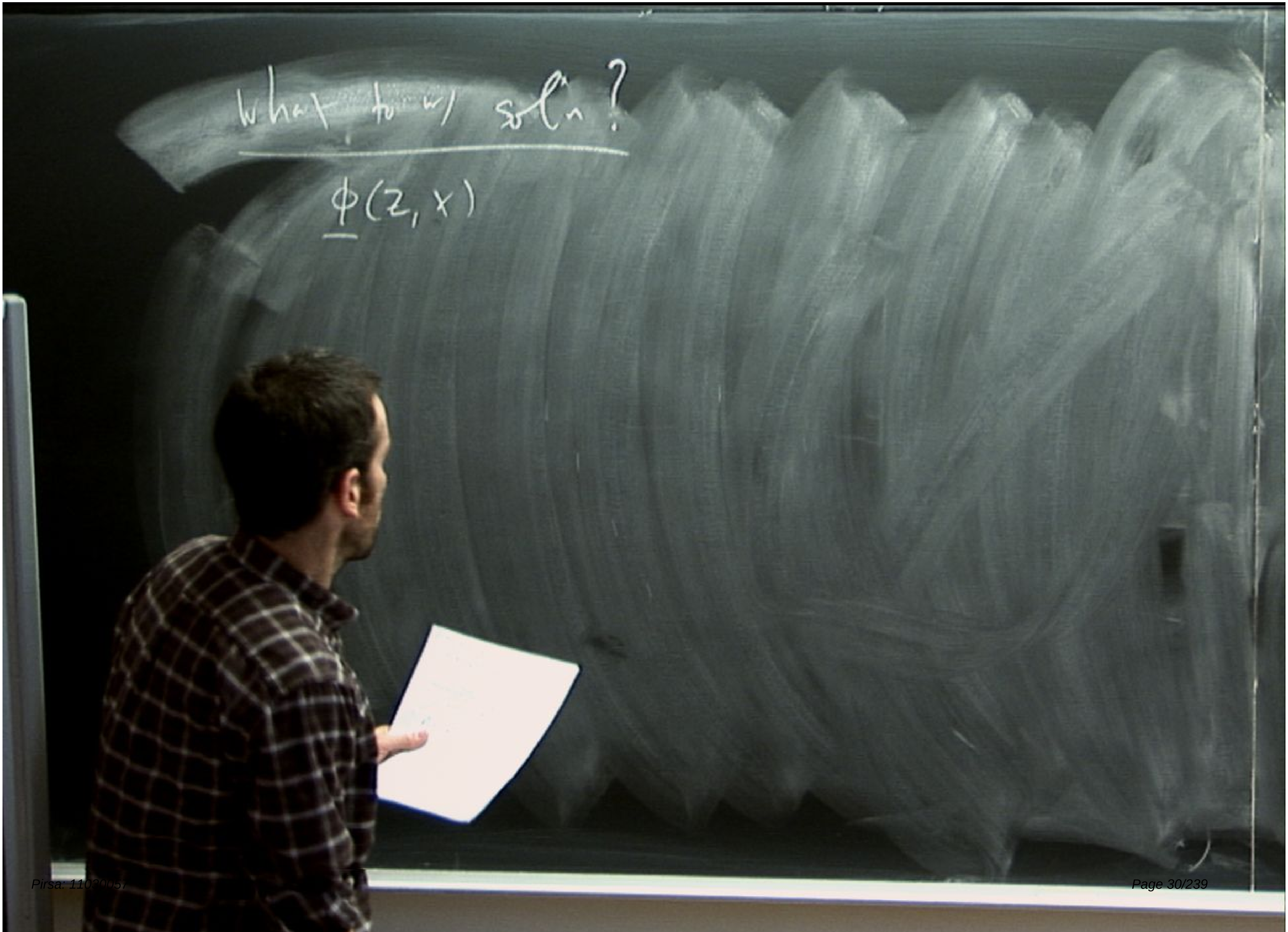

QM fact:
Time-ordered G


QM fact
vs.
F.T.


QM fact:
F.T.
Time-ordered G

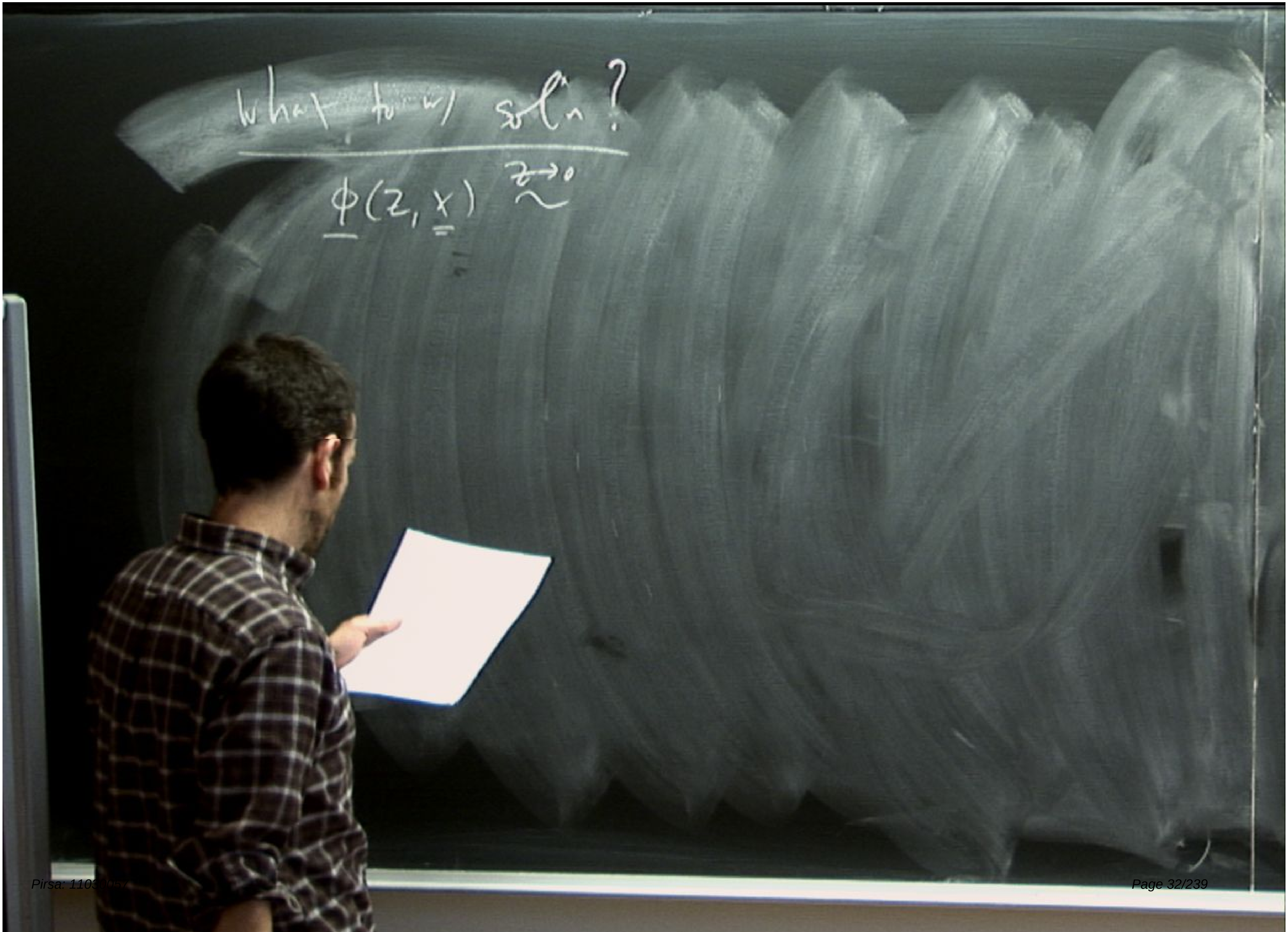

QM fact
holographic version
ingoing at horizon
Time-ordered G
F.T.

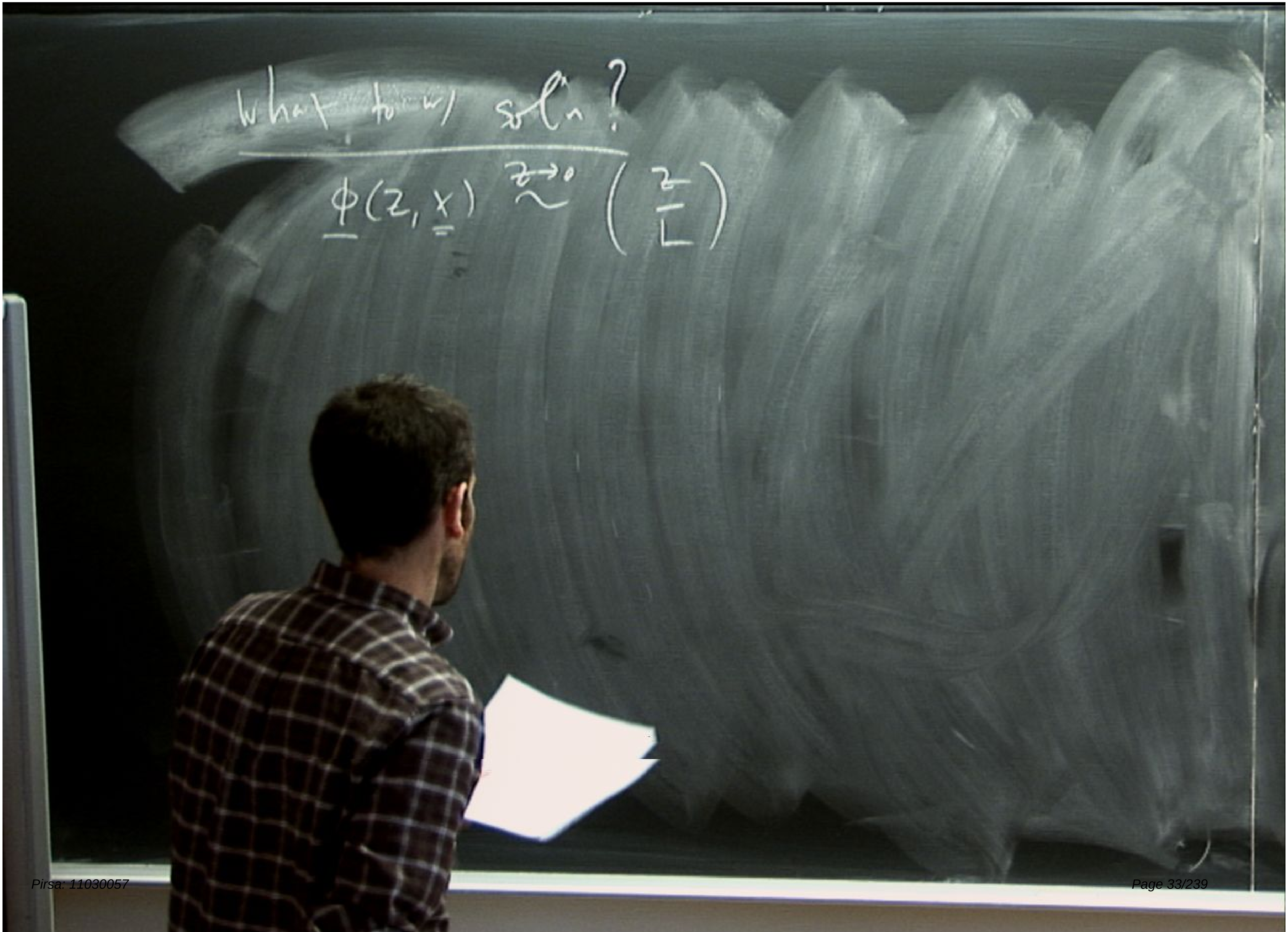

soln ?

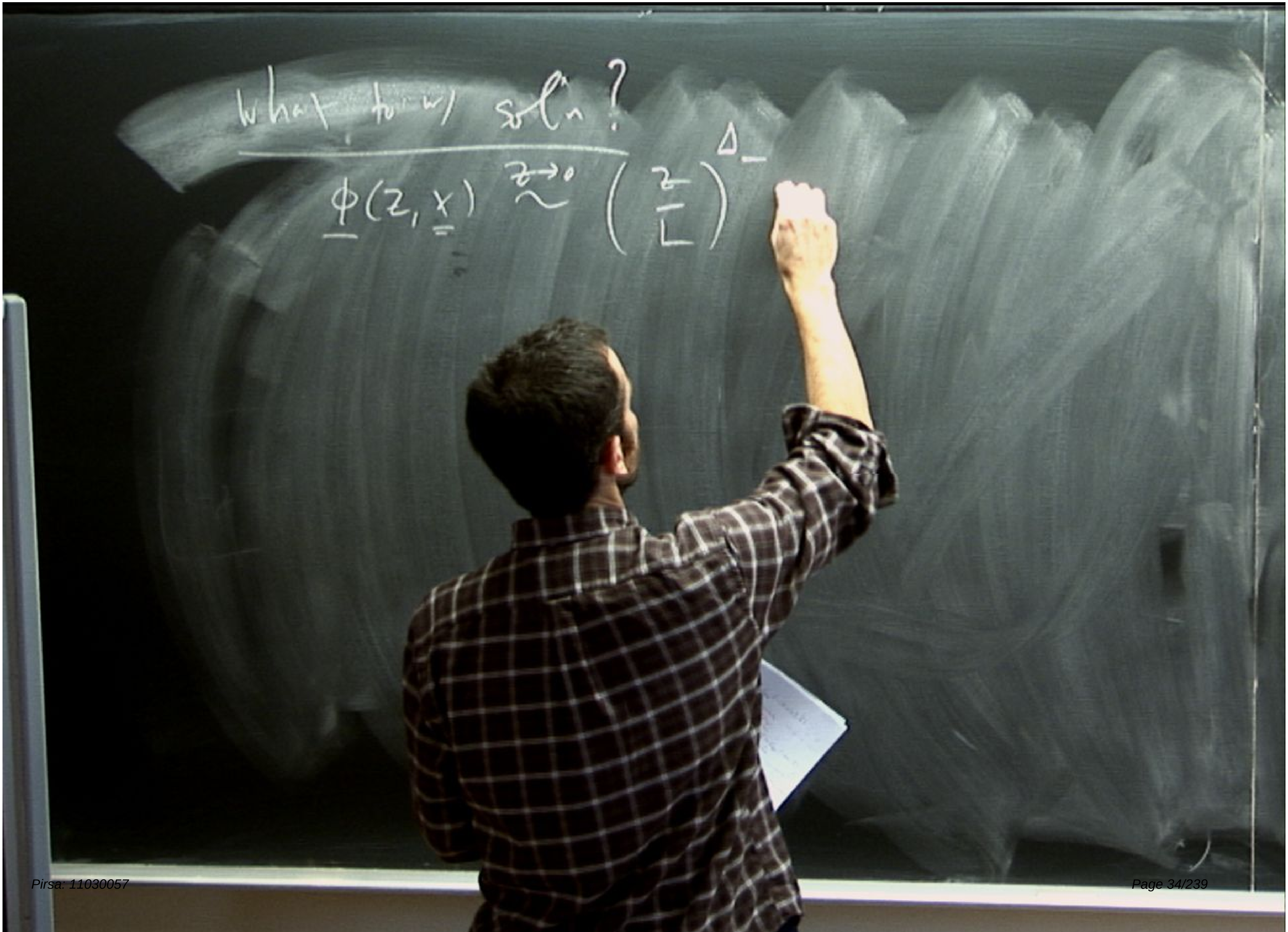

what to w/ sol^n ?
φ(z, x)

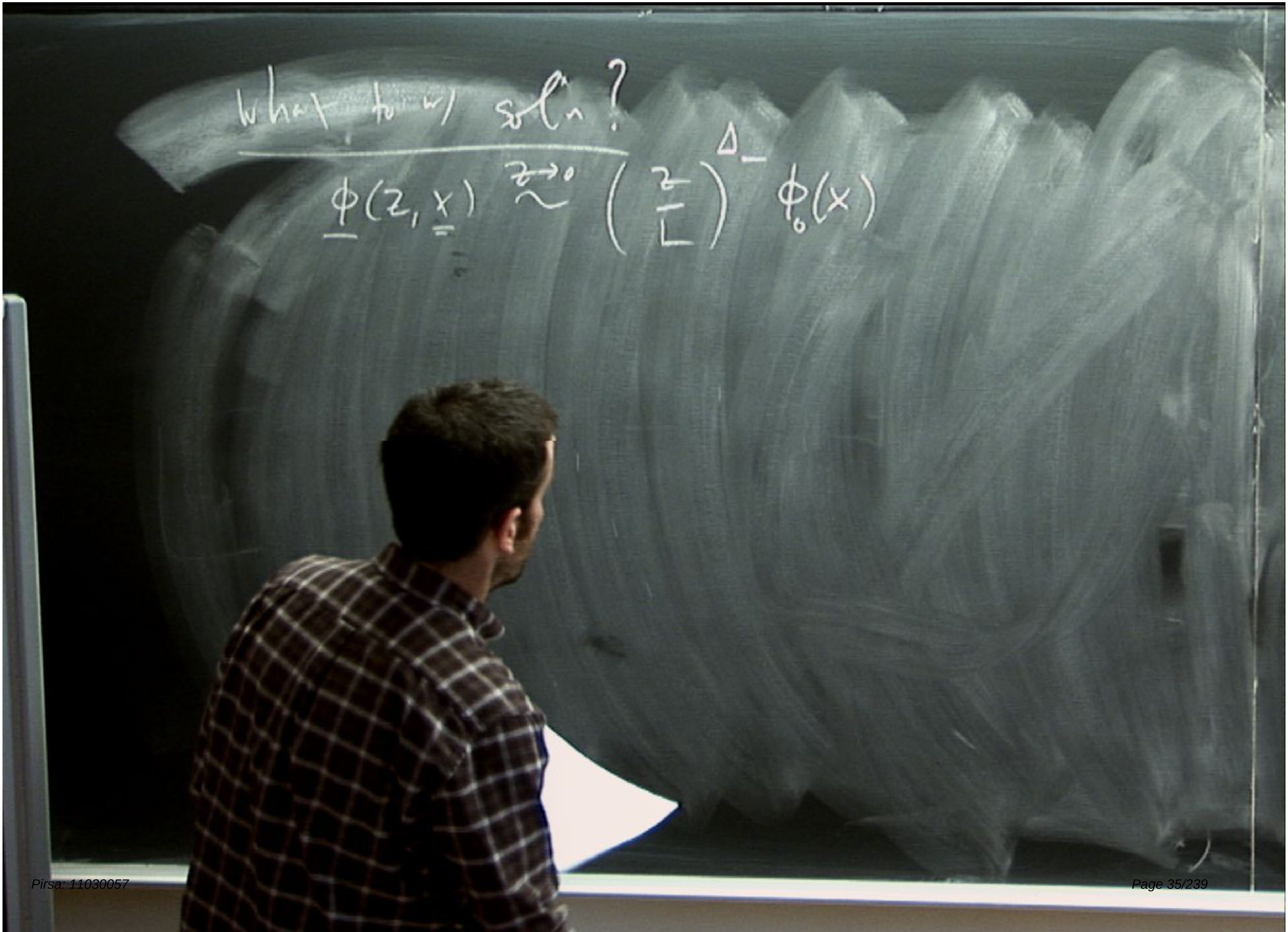
what to w/ sol^n?

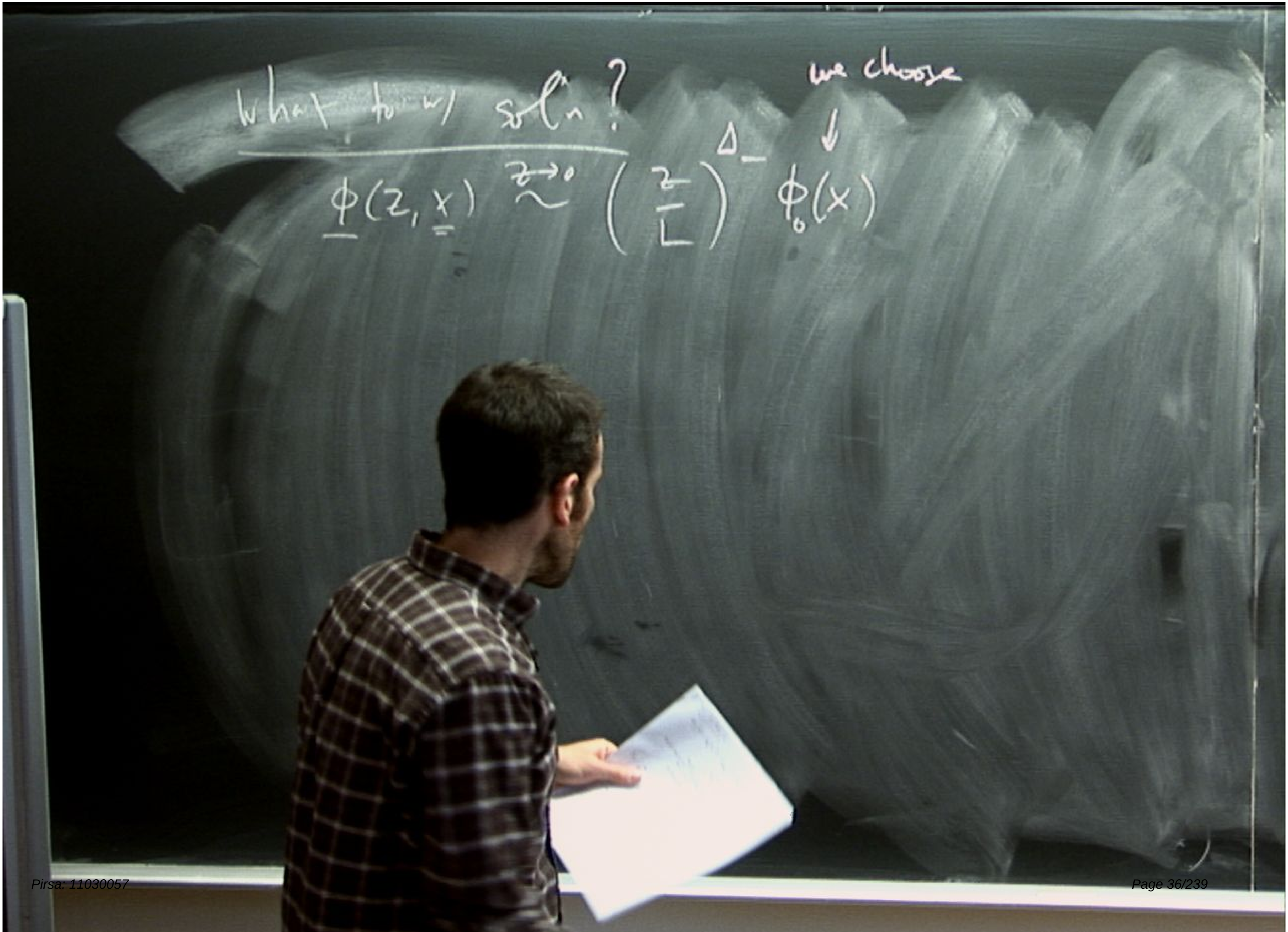
$\underline{\Phi}(z, \underline{\underline{x}})$

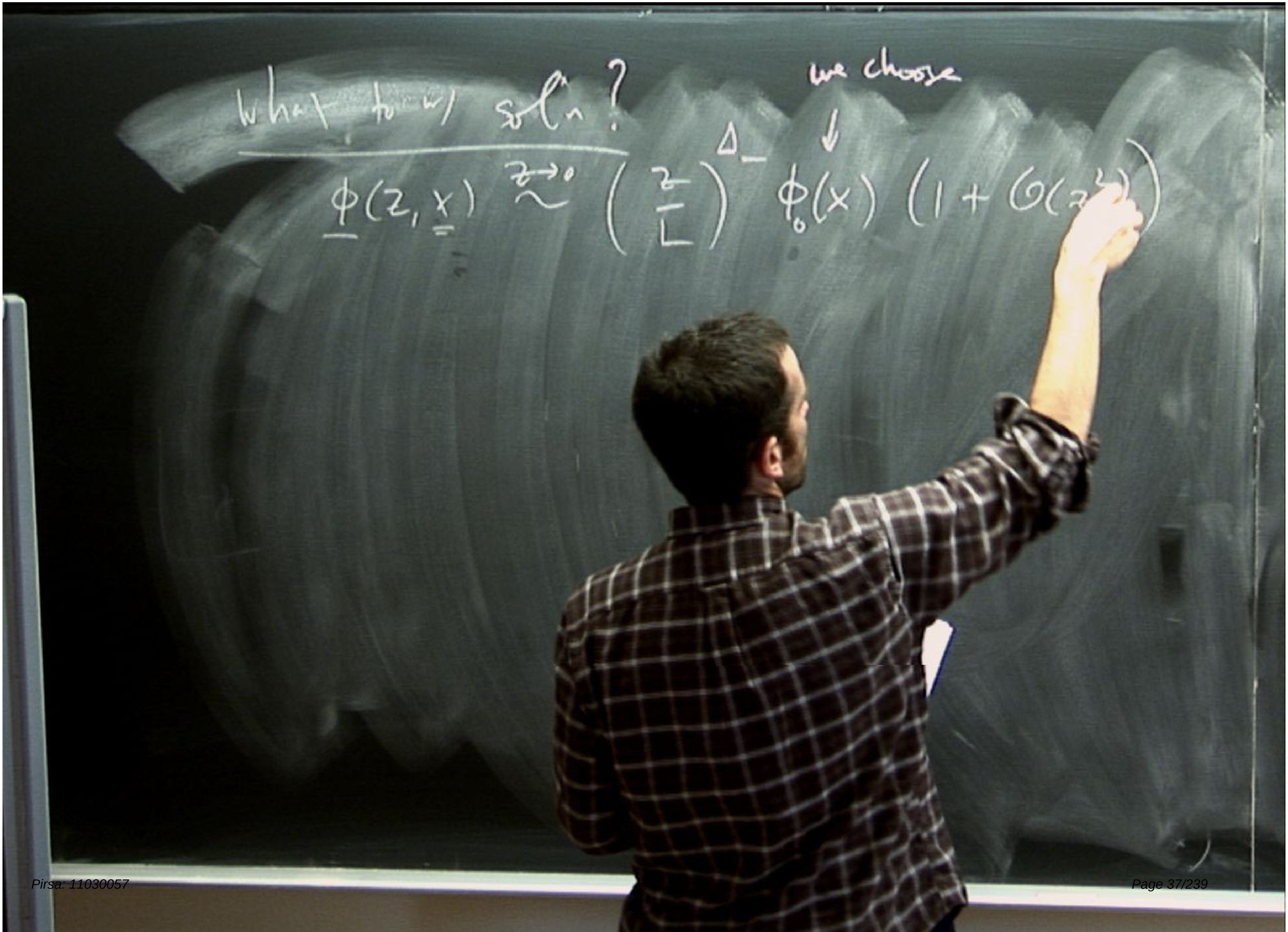

what to w/ sol'n ?
φ(z, x) z→0 ~

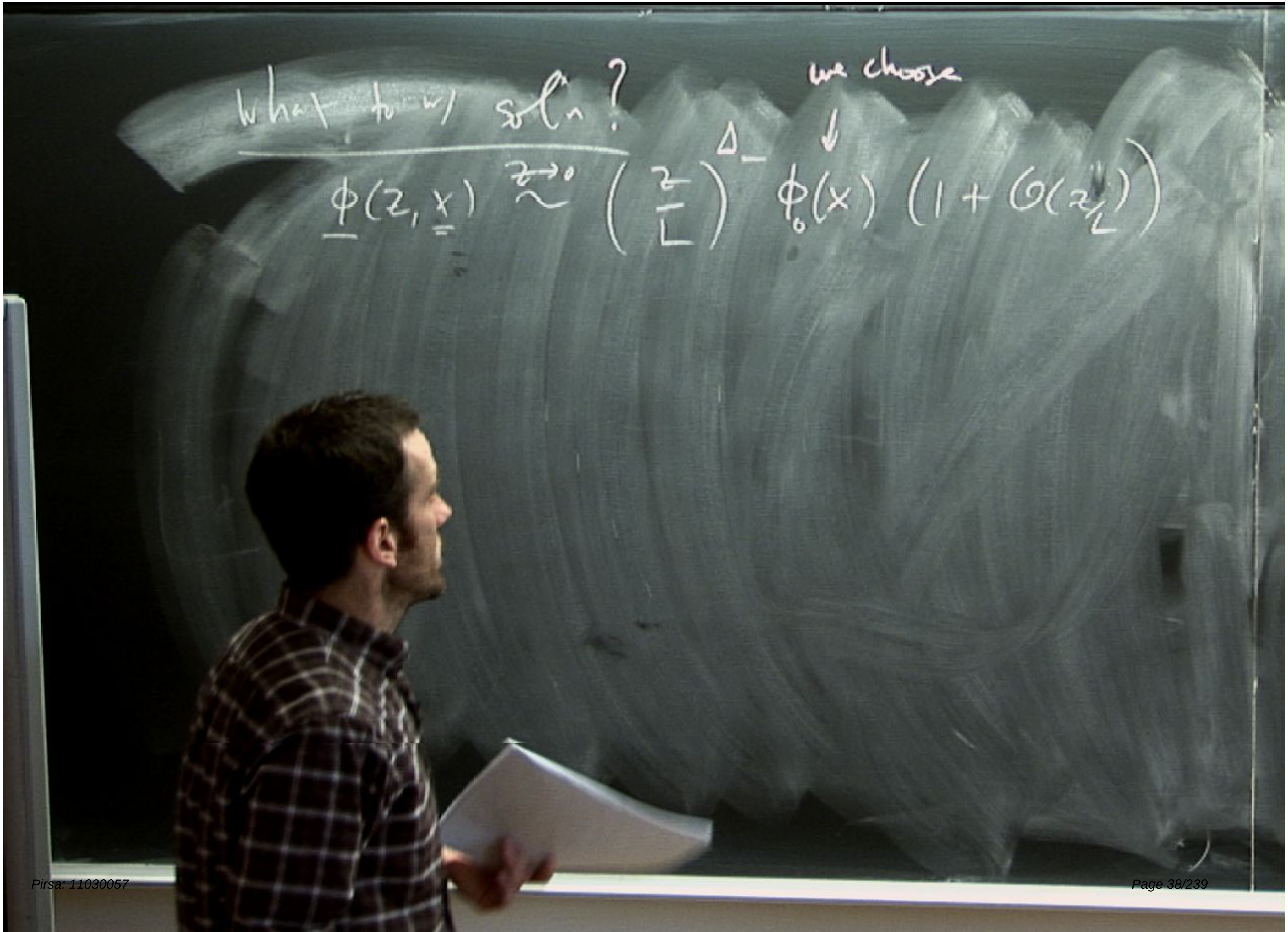

what to w/ sol'n ?
φ(z, x) z→0 ~ (z / L)

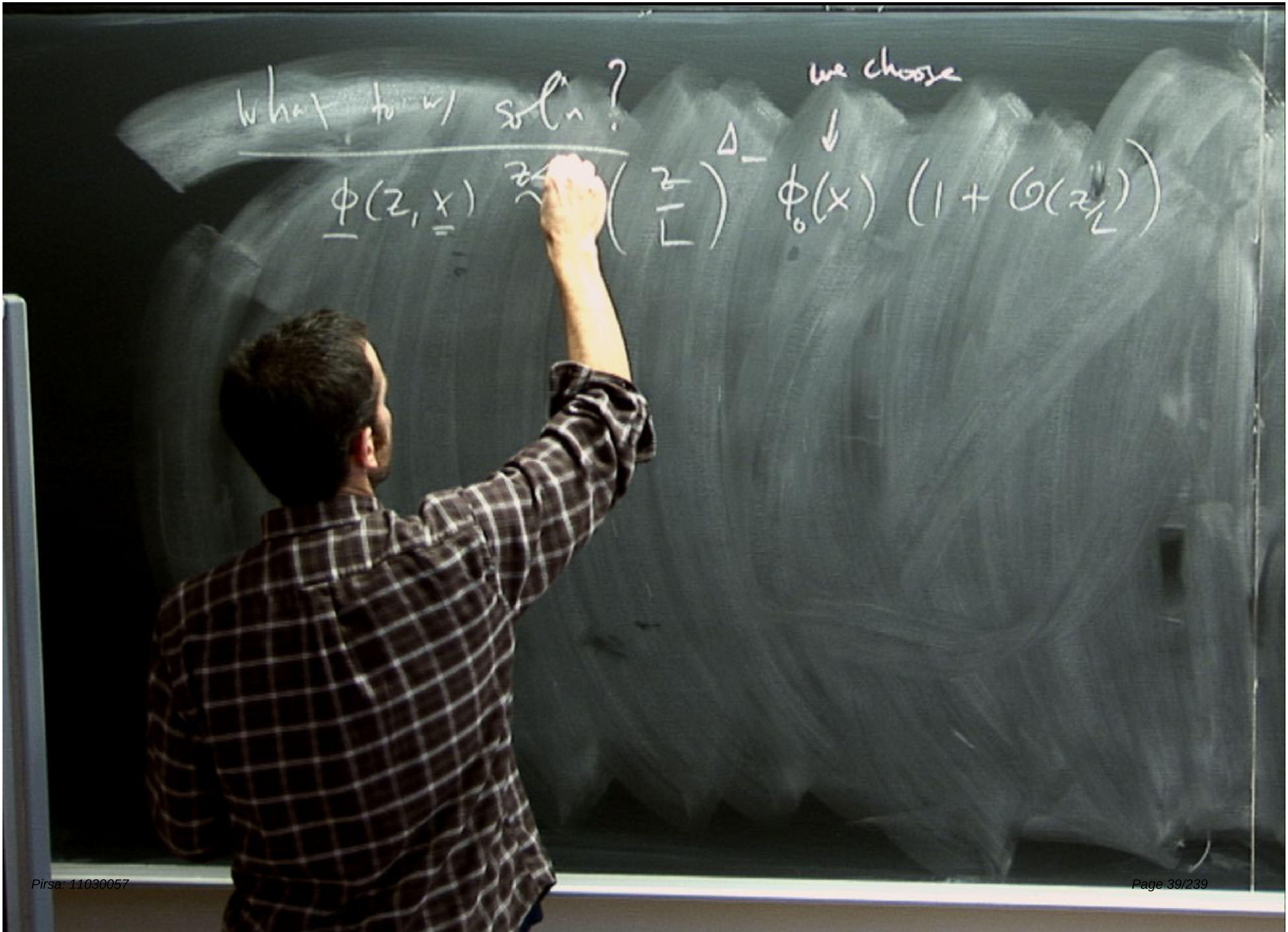

$\underline{\phi}(z,\underline{x}) \overset{z\to 0}{\sim} \left(\frac{z}{L}\right)^{\Delta_-}$

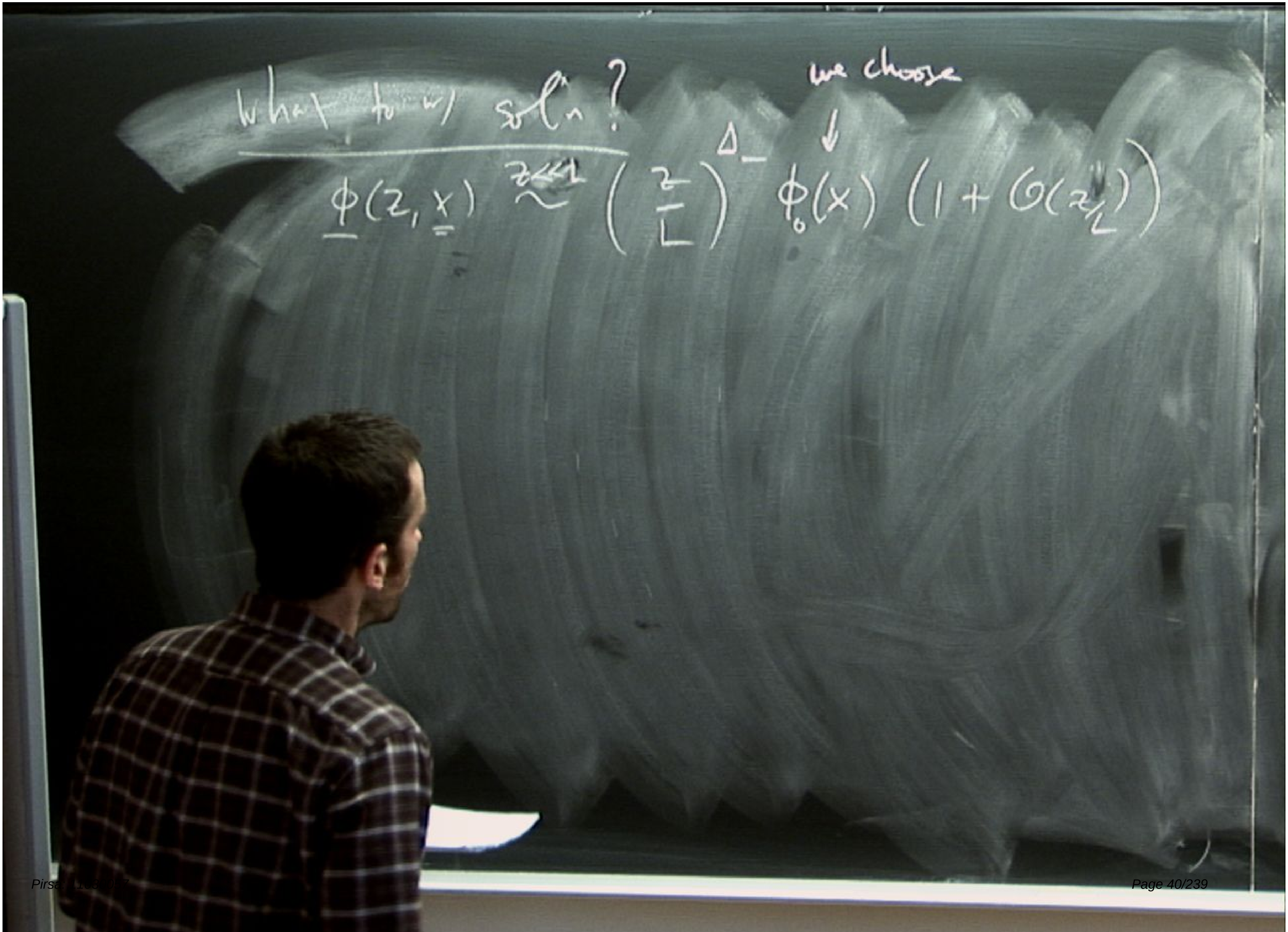

what to w/ sol'n ?
$\underline{\phi}(z, \underline{x}) \overset{z\to 0}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x)$

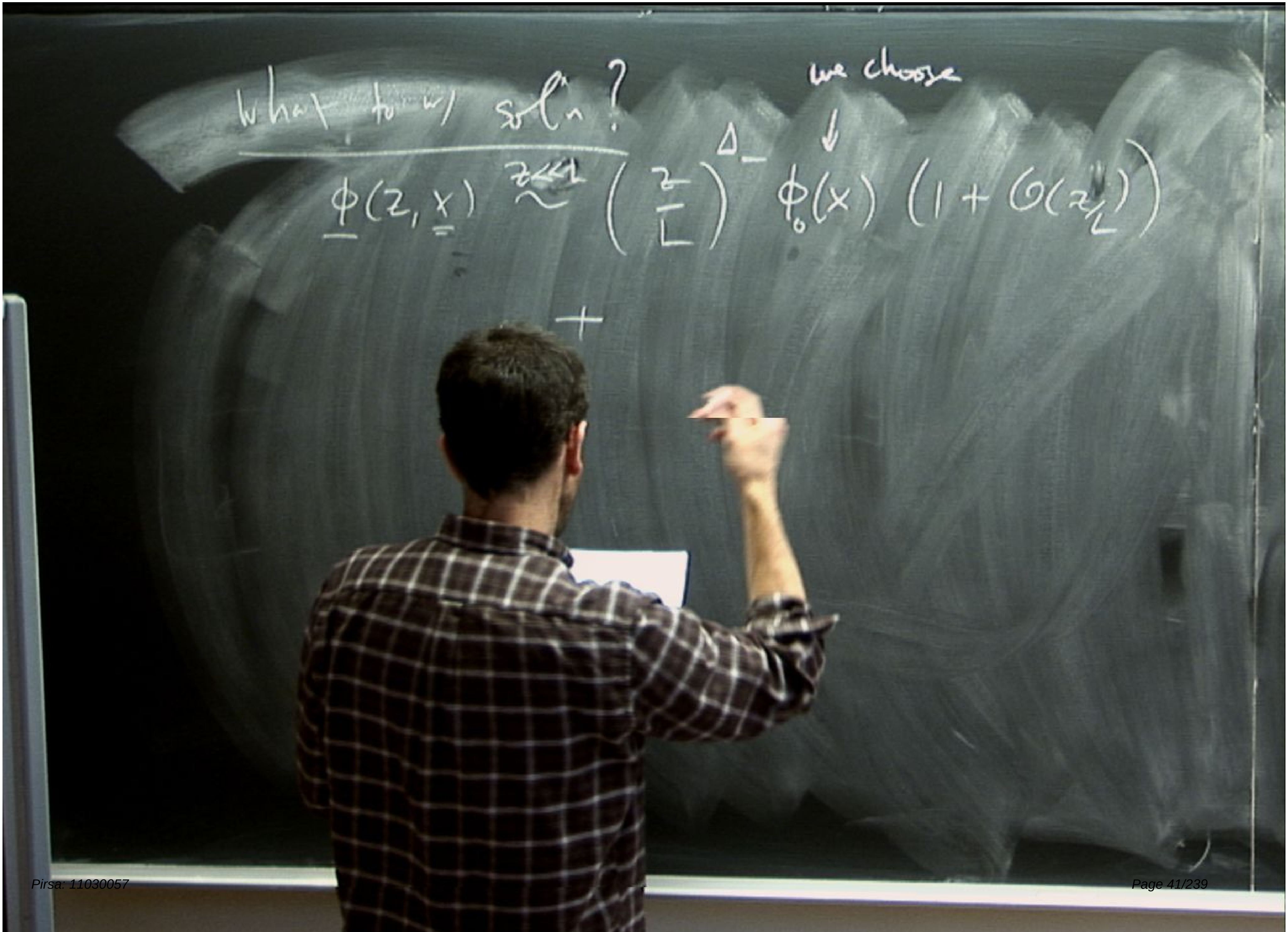

what to w/ sol'n ?
we choose
$\underline{\phi}(z, \underline{x}) \overset{z\to 0}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x)$

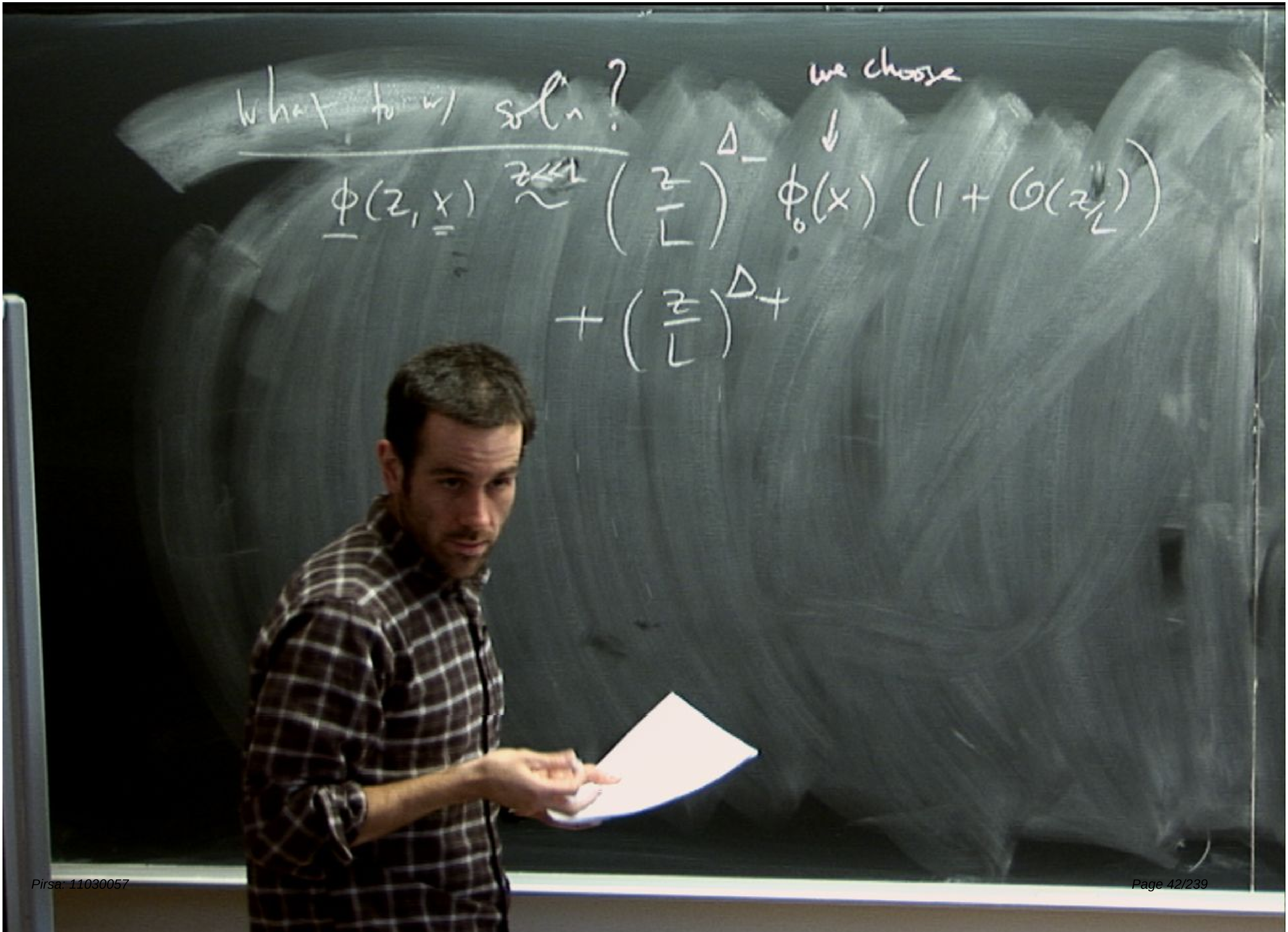

what to w/ sol'n ?
we choose
$\underline{\phi}(z,\underline{x}) \overset{z\to 0}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x)\,(1 + O(z^2))$

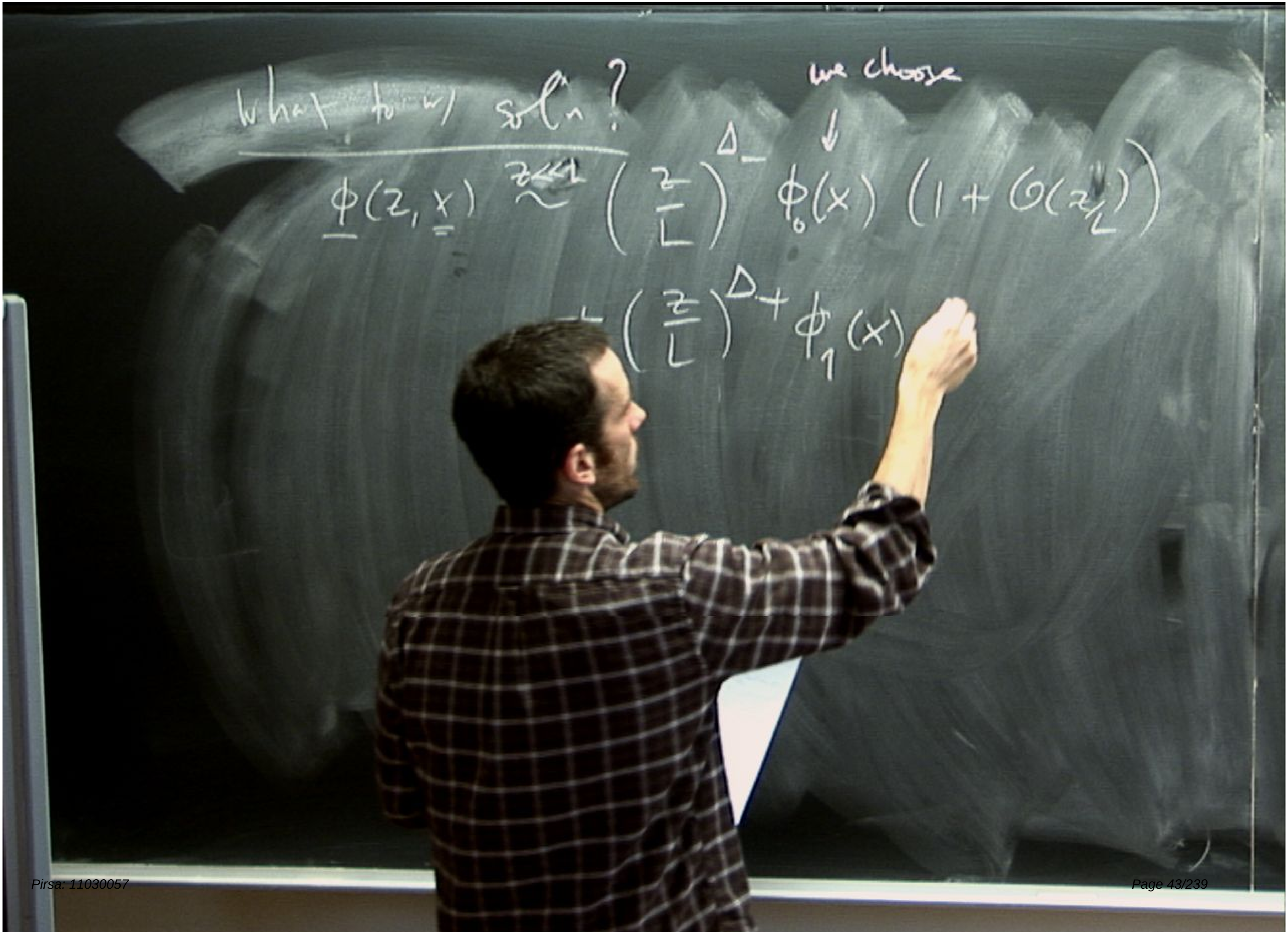

what to w/ soln?
we choose
$\underline{\phi}(z, \underline{\underline{x}}) \overset{z \to 0}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) \left(1 + O(z/L)\right)$

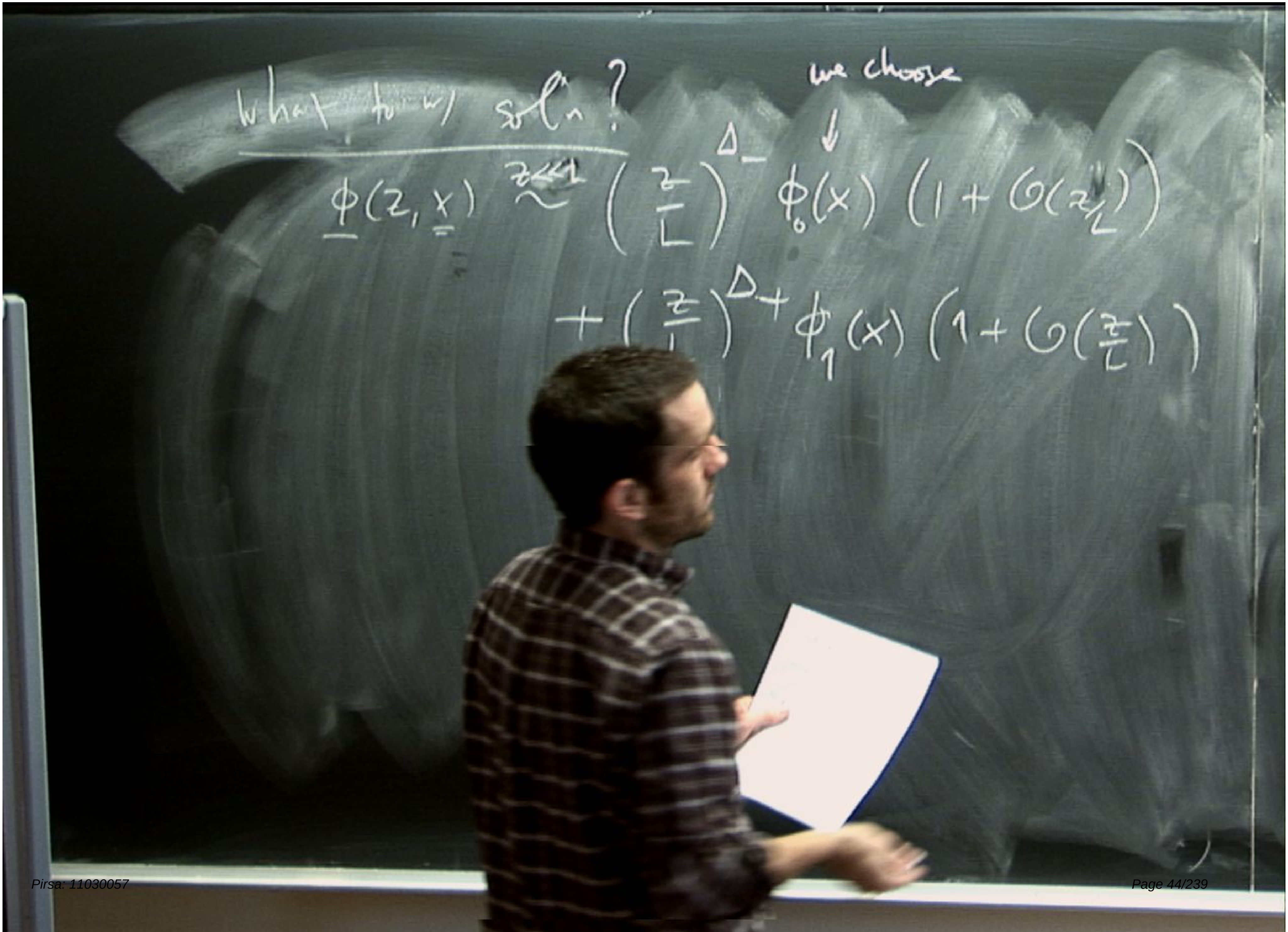

what to w/ soln ?
we choose
φ(z, x) ~ (z/L)^Δ₋ φ₀(x) (1 + O(z/L))

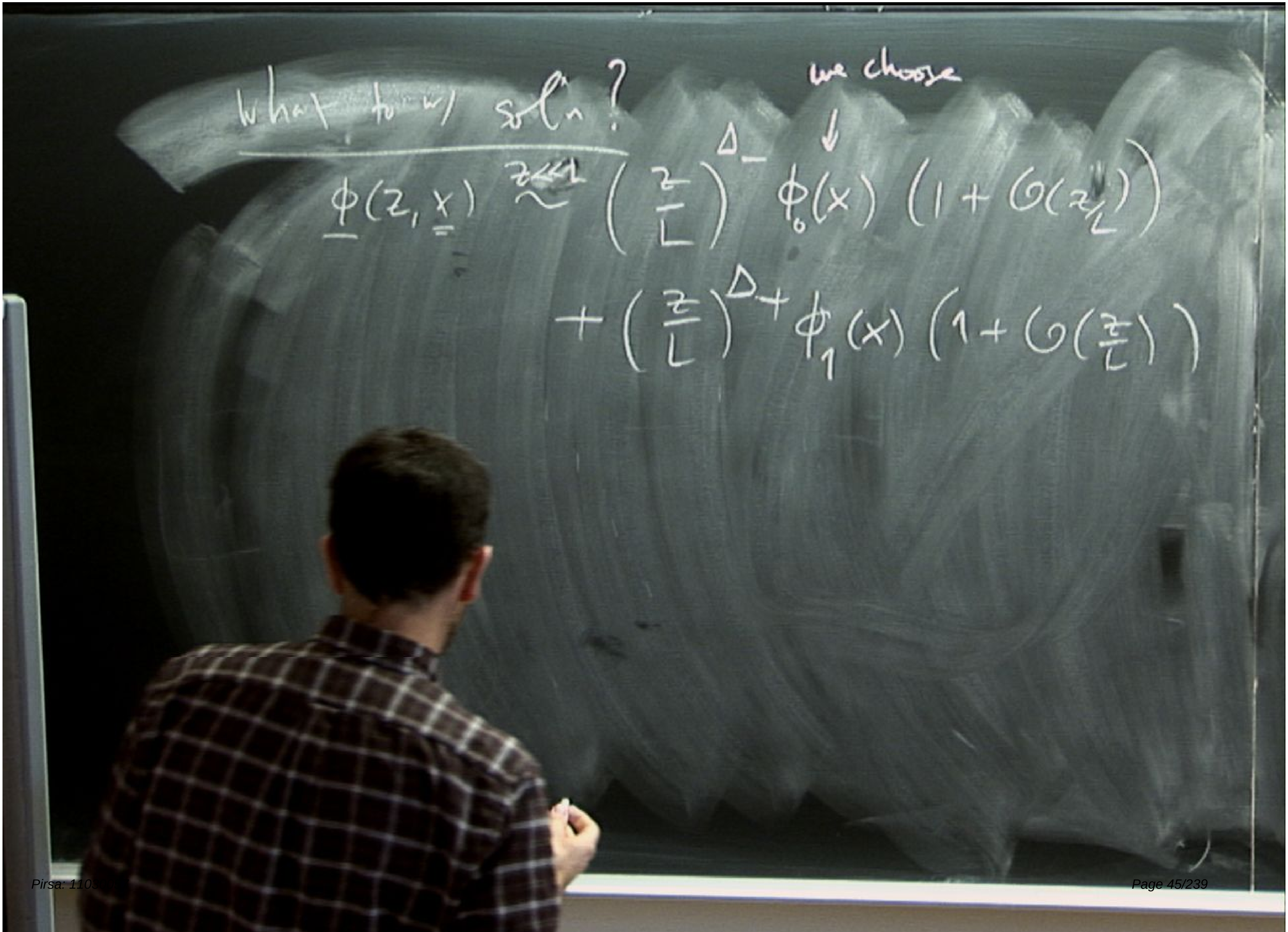

what to w/ soln?
we choose
$\phi(z, x) \overset{z \ll 1}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) \left(1 + O(z/L)\right)$


what to w/ soln ?
we choose
φ(z, x) ~ (z/L)^Δ₋ φ₀(x) (1 + O(z/L))
z<<1
+

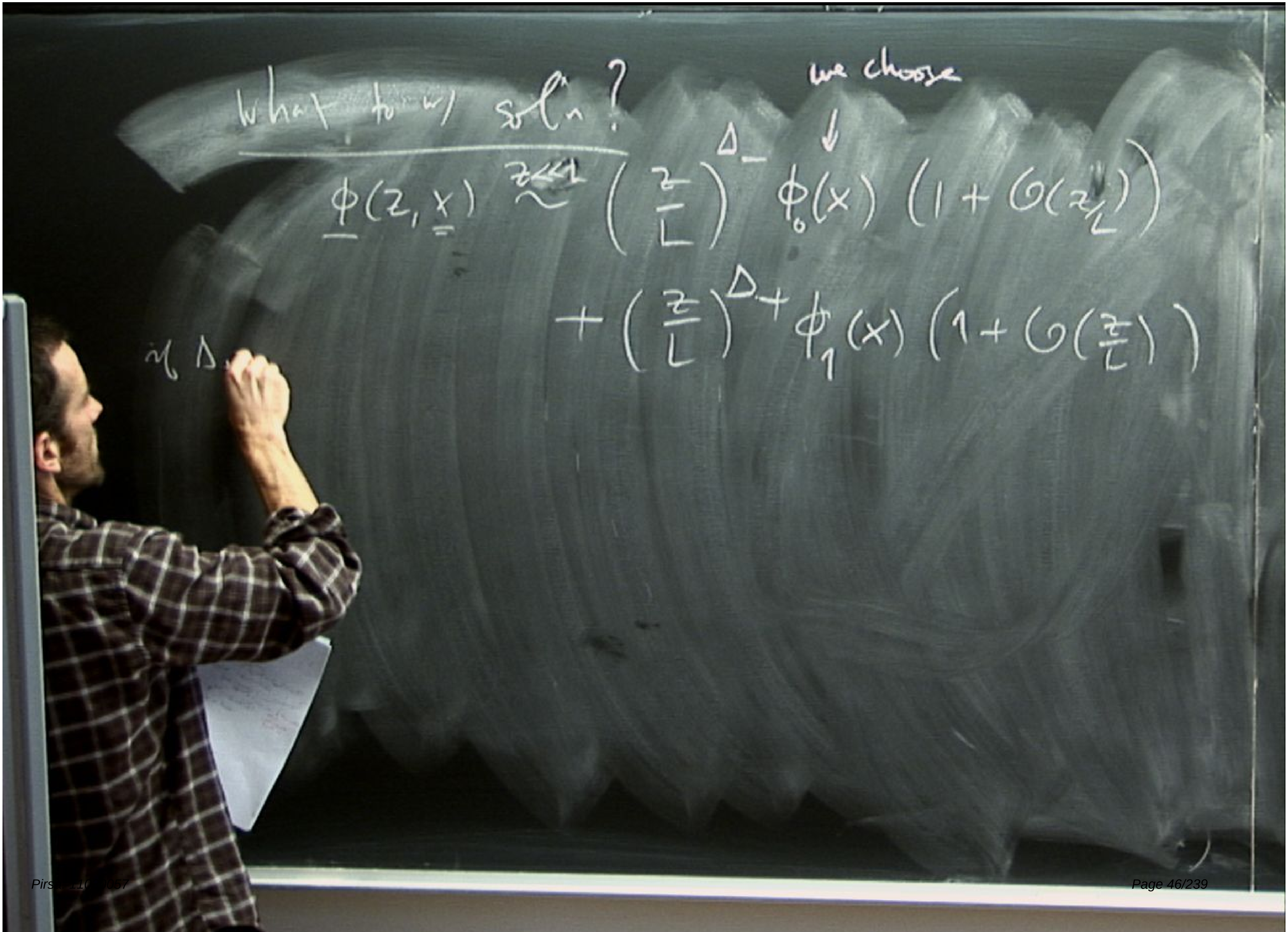

what to w/ sol'n ?
we choose
$\underline{\phi}(z, \underline{x}) \overset{z \ll 1}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) \left(1 + O(z/L)\right)$
$+ \left(\frac{z}{L}\right)^{\Delta_+}$

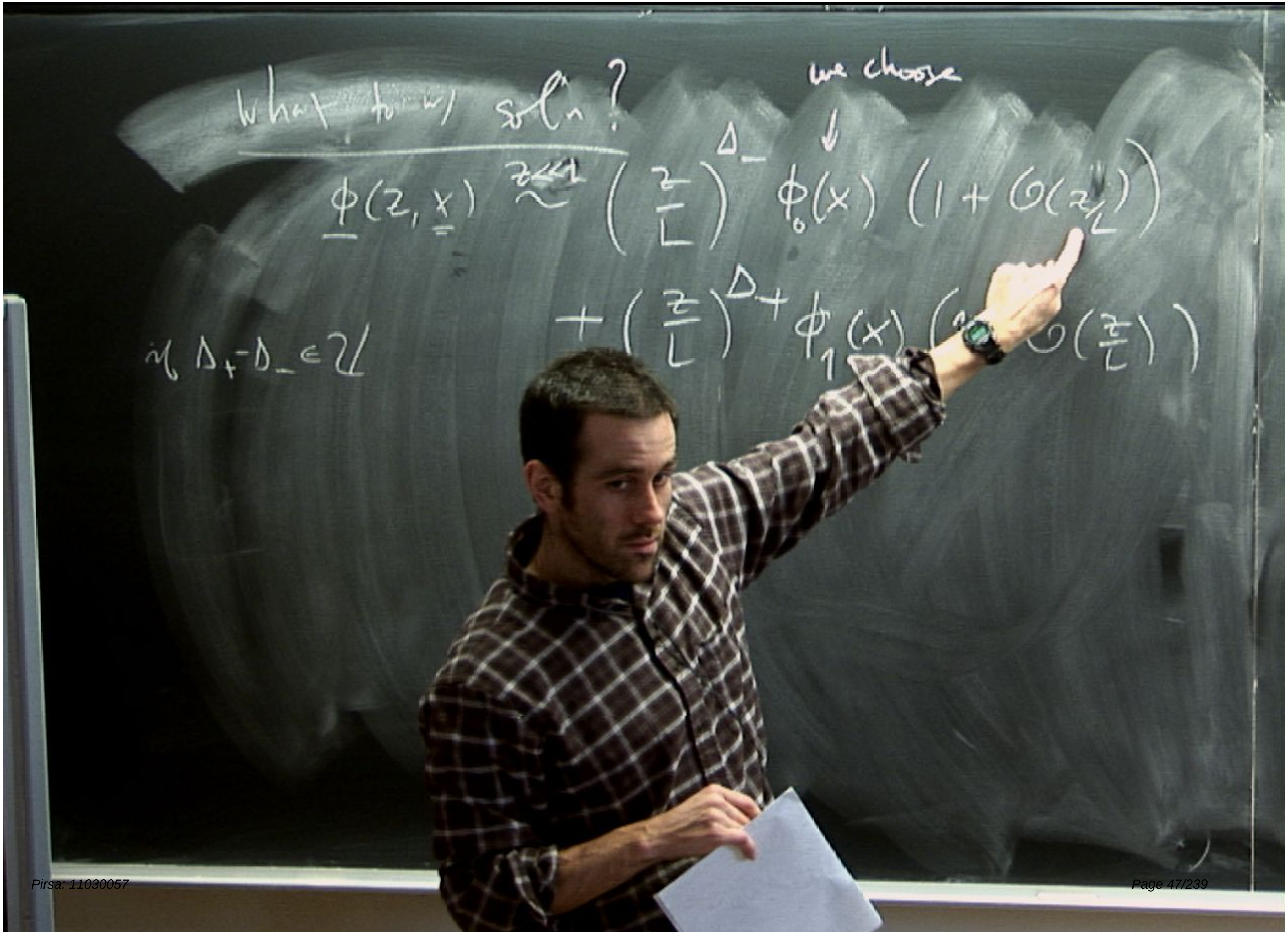

what to w/ soln ?
we choose
φ(z, x) ~ (z/L)^Δ- φ0(x) (1 + O(z/L))
+ (z/L)^Δ+ φ1(x)


we choose
φ(z, x) z<<1 ~ (z/L)^Δ- φ0(x) (1 + O(z/L))
+ (z/L)^Δ+ φ1(x) (1 + O(z/L))



what to w/ soln ?
we choose
$\underline{\phi}(z, \underline{x}) \overset{z \ll 1}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) \left(1 + \mathcal{O}(z/L)\right)$
$+ \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) \left(1 + \mathcal{O}\left(\frac{z}{L}\right)\right)$


what to w/ soln ?
we choose
$\underline{\phi}(z, \underline{x}) \overset{z \ll 1}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) \left(1 + O(z/L)\right)$
$+ \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) \left(1 + O(\frac{z}{L})\right)$
if Δ


what to w/ sol'n ?
we choose
φ(z, x) ~ (z/L)^Δ- φ0(x) (1 + O(z/L))
z<<1
+ (z/L)^Δ+ φ1(x) (1 + O(z/L))
if Δ+ - Δ- ∈ Z

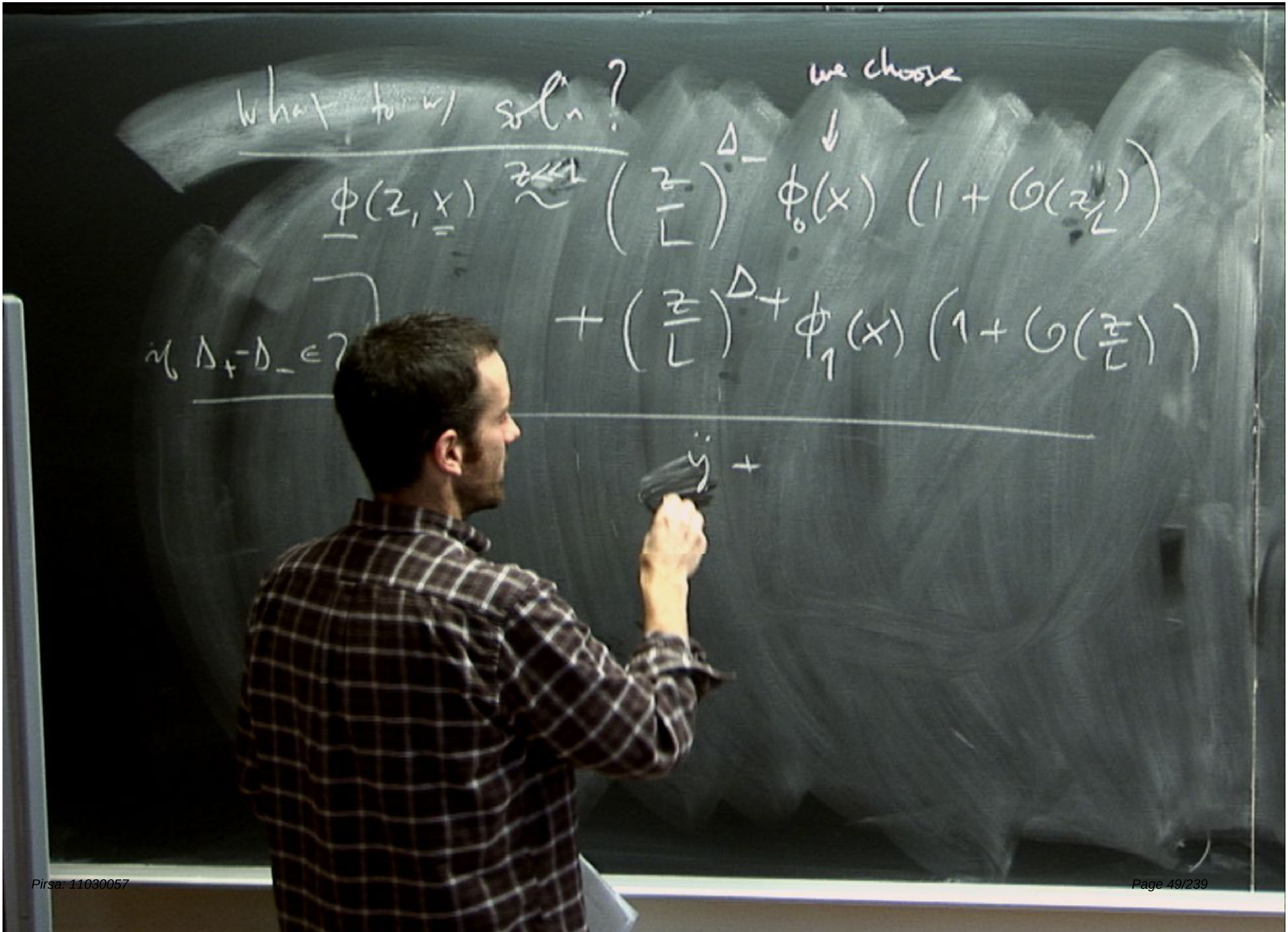
What to w/ sol'n ?

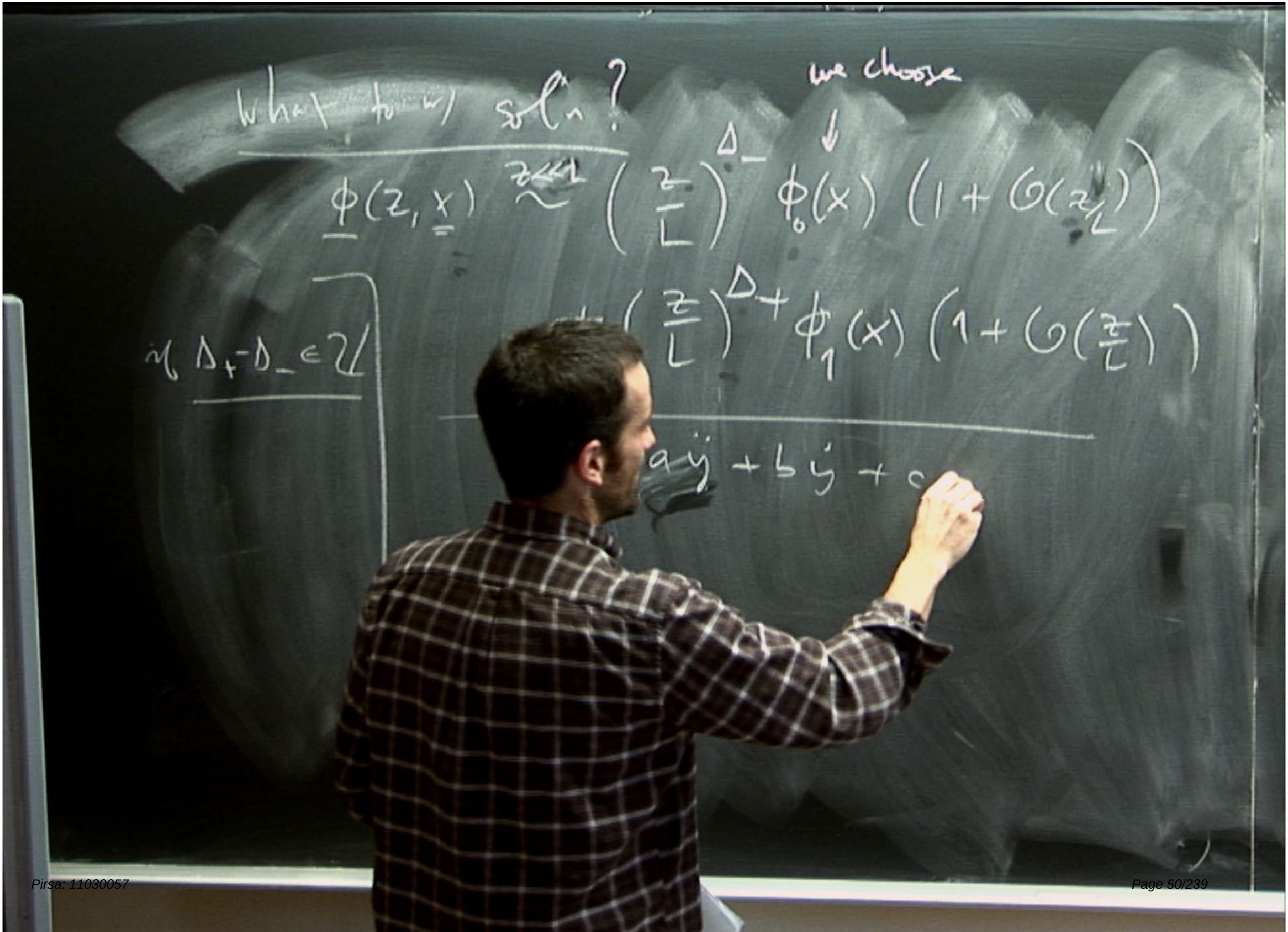
we choose ↓

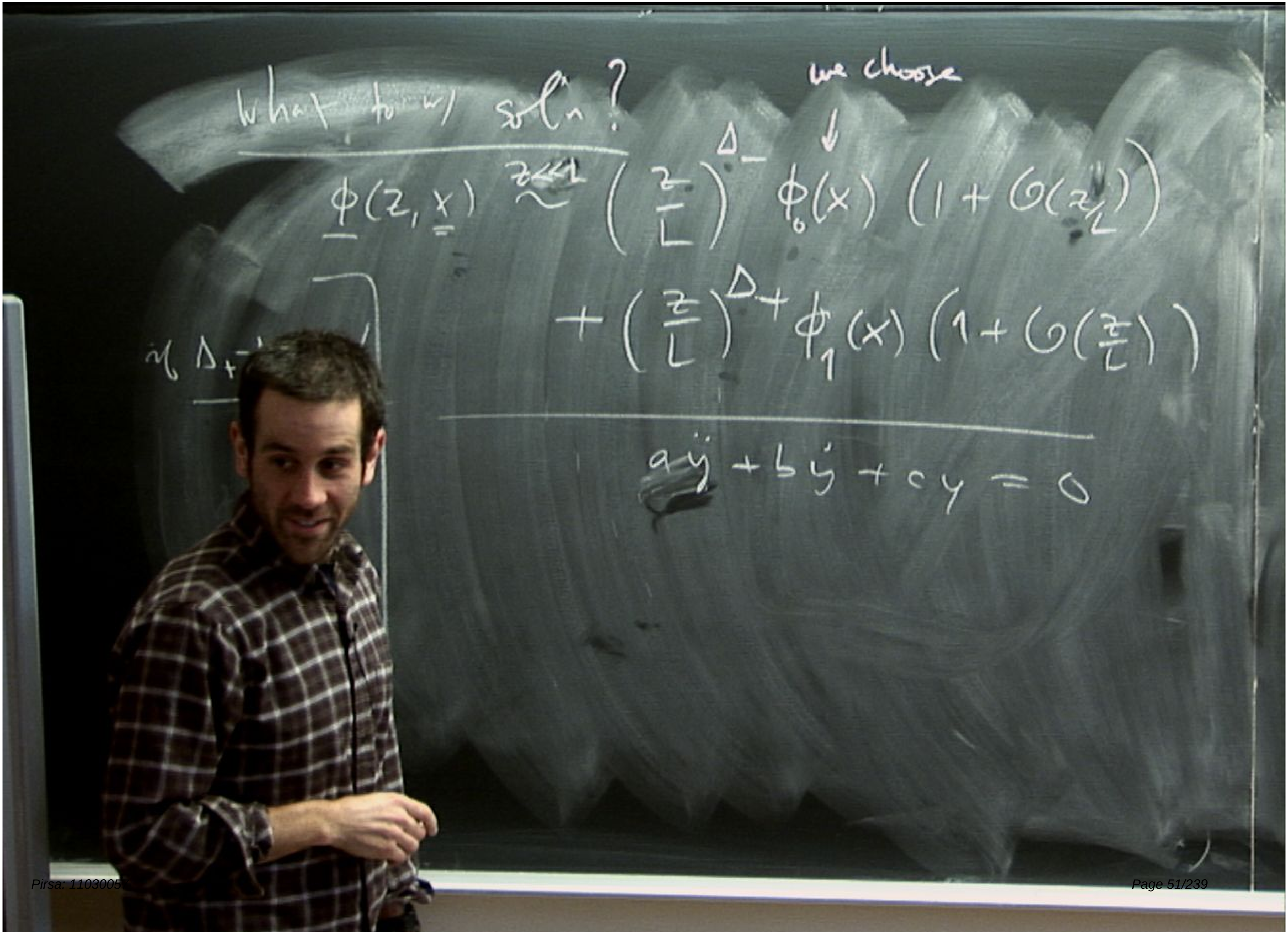
$$\underline{\phi}(z,\underline{x}) \overset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x)\left(1+\mathcal{O}(z/L)\right)$$

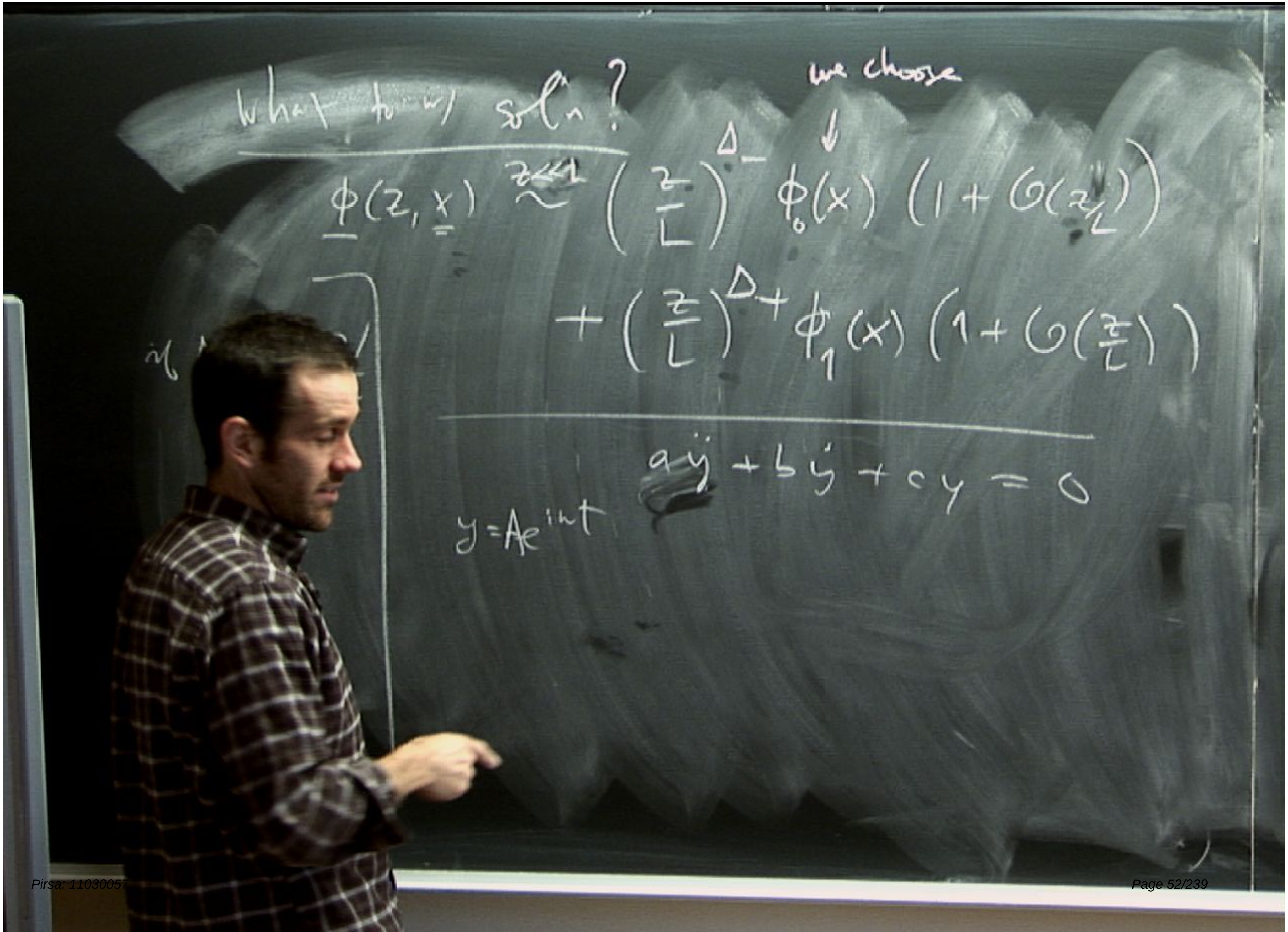
$$+ \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x)\left(1+\mathcal{O}\left(\frac{z}{L}\right)\right)$$

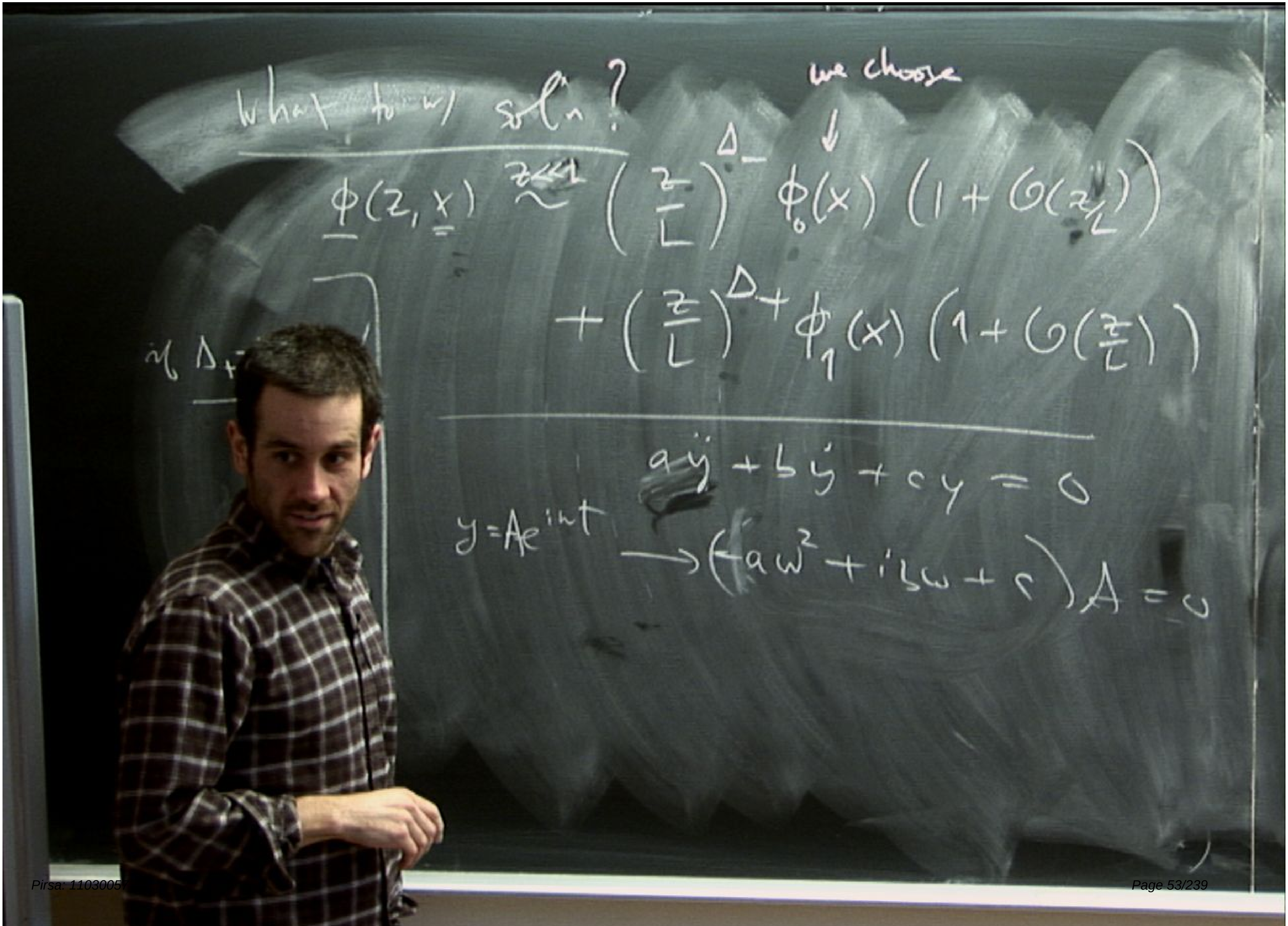
if $\Delta_+ - \Delta_- \in \mathbb{Z}$

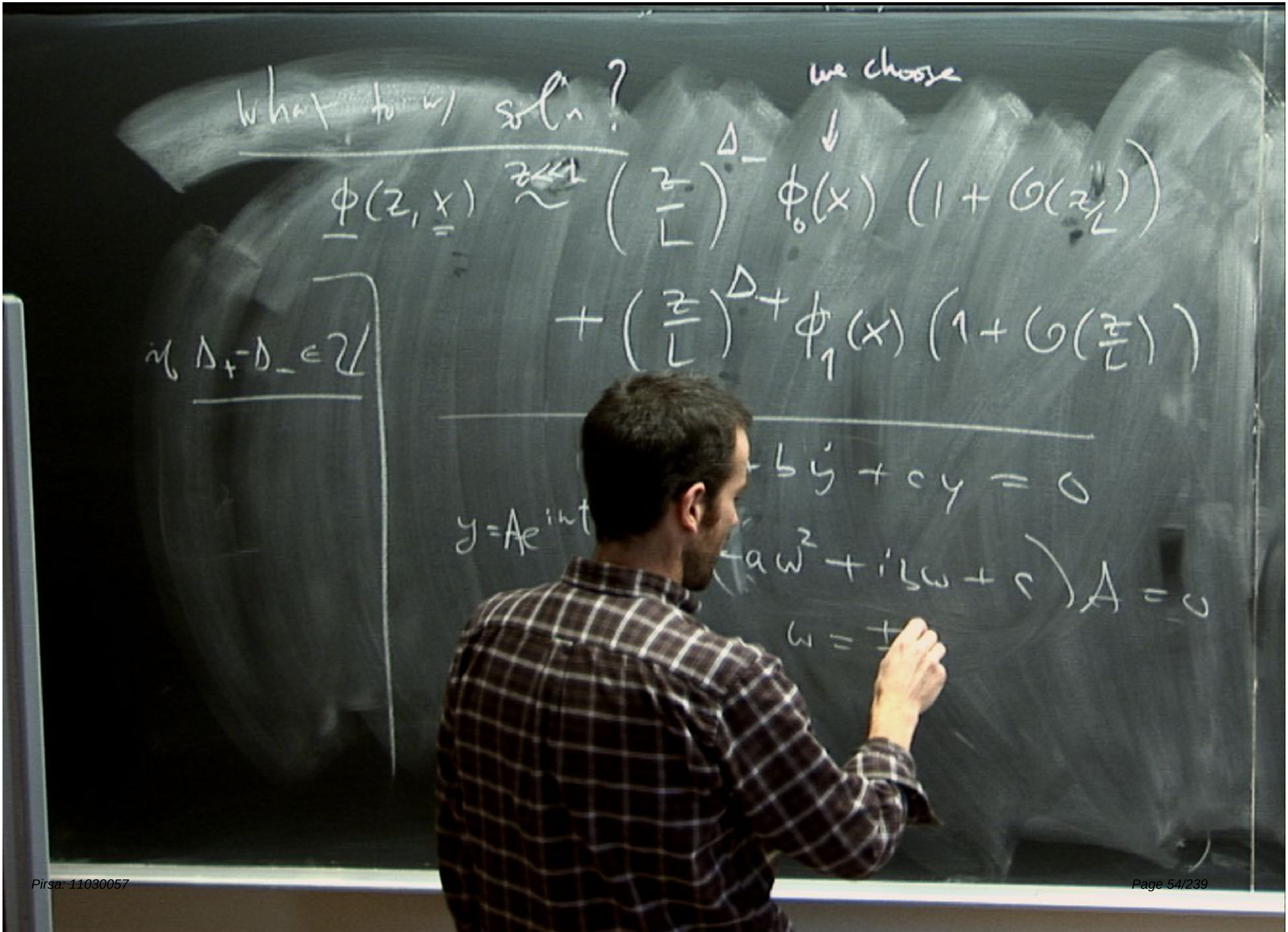

what to w/ soln ?
we choose
φ(z, x) ~ (z/L)^Δ- φ0(x) (1 + O(z/L))
z<<1
if Δ+ - Δ- ∈ Z
+ (z/L)^Δ+ φ1(x) (1 + O(z/L))

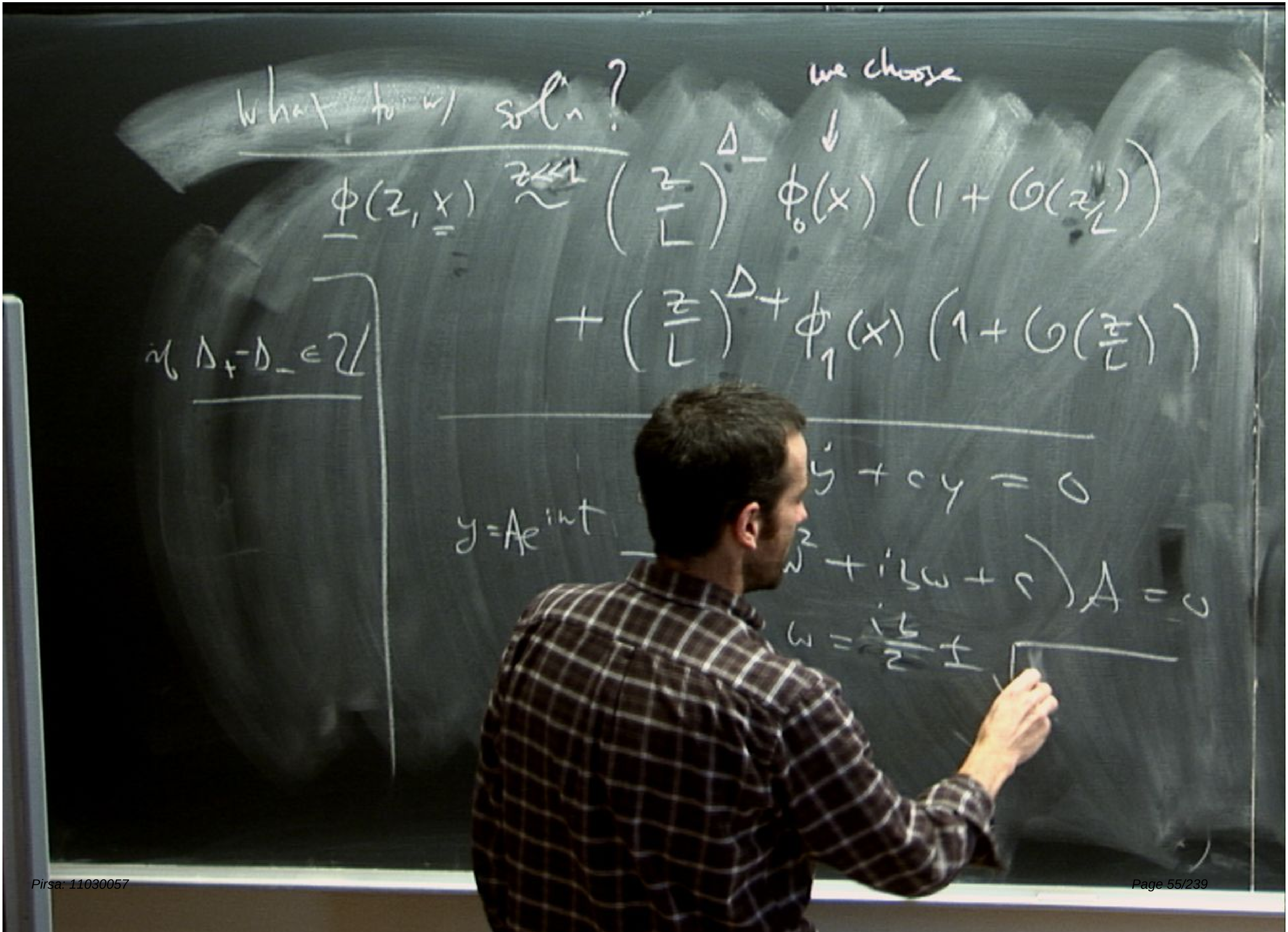

what to w/ sol'n ?
we choose
$\phi(z, x) \overset{z \ll 1}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) \left(1 + O(z/L)\right)$
$+ \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) \left(1 + O(\frac{z}{L})\right)$
if $\Delta_+ - \Delta_- \in \mathbb{Z}$
$a\ddot{y} + b\dot{y} + c$

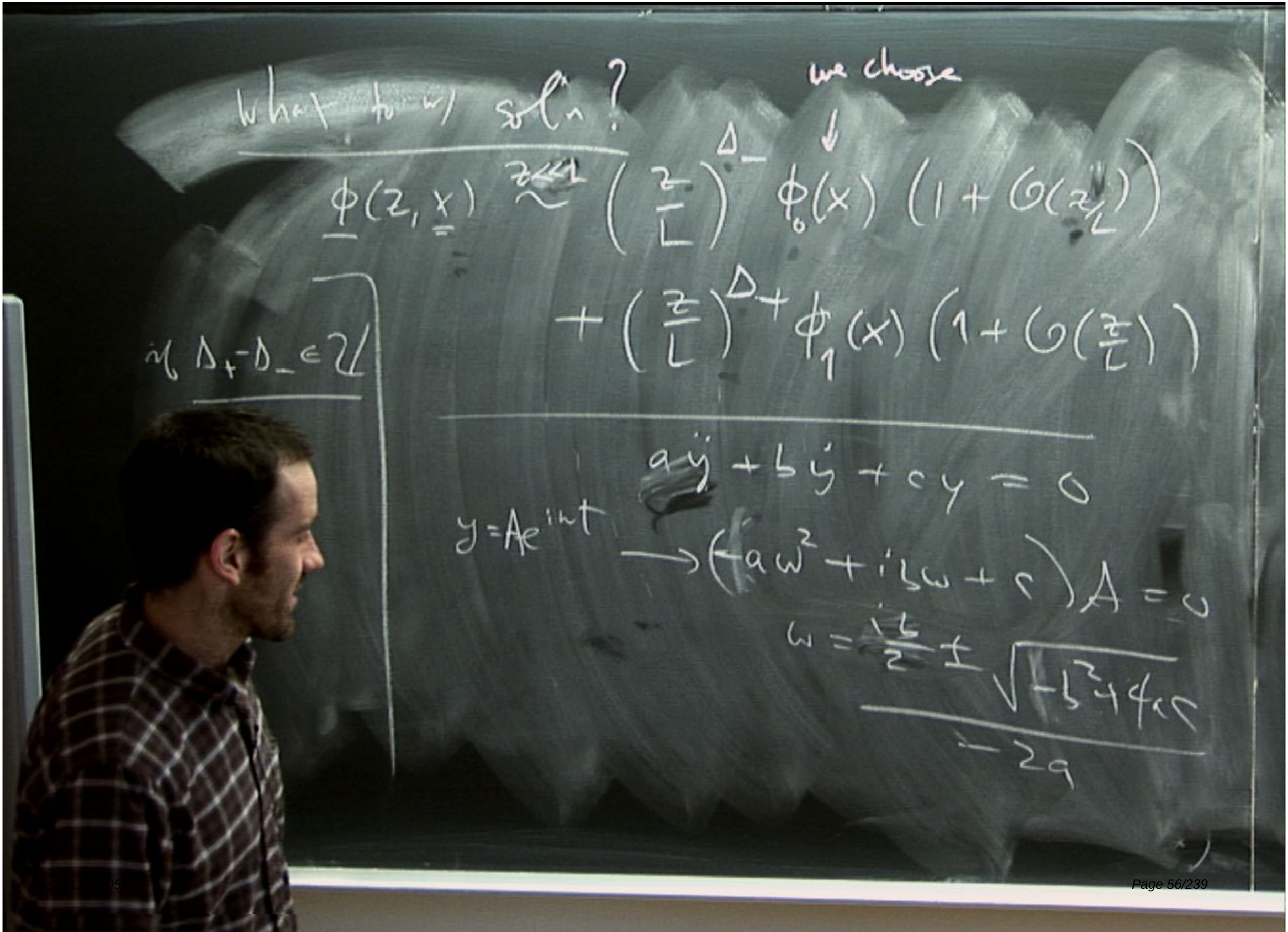

what to w/ soln ?
we choose
ϕ(z, x) z<<1 ~ (z/L)^Δ- ϕ0(x) (1 + O(z/L))
+ (z/L)^Δ+ ϕ1(x) (1 + O(z/L))
aÿ + bẏ + cy = 0

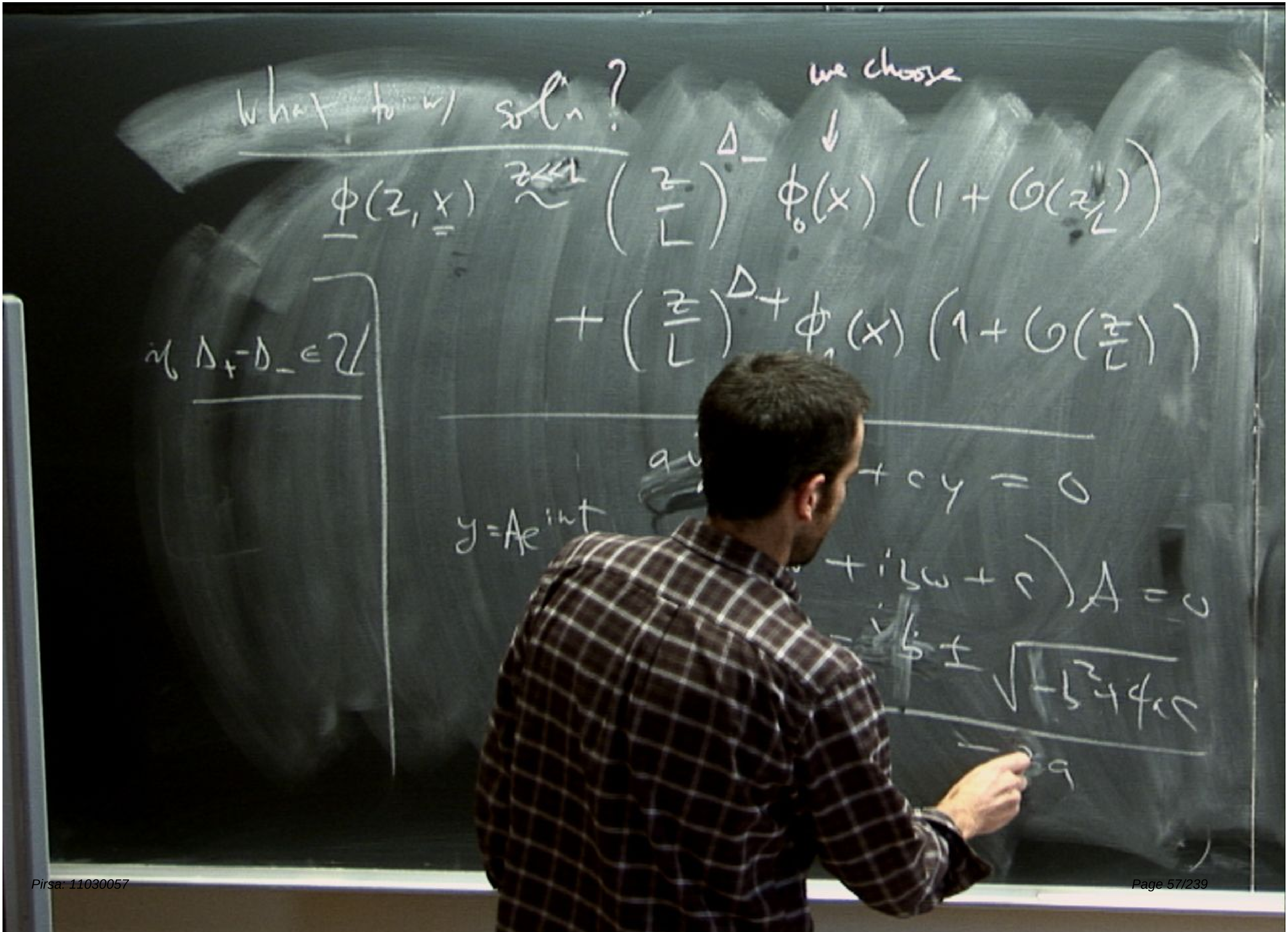

we choose
φ(z, x) z<<1 ~ (z/L)^Δ- φ0(x) (1 + O(z/L))
+ (z/L)^Δ+ φ1(x) (1 + O(z/L))
aÿ + bẏ + cy = 0
y = Ae^iωt

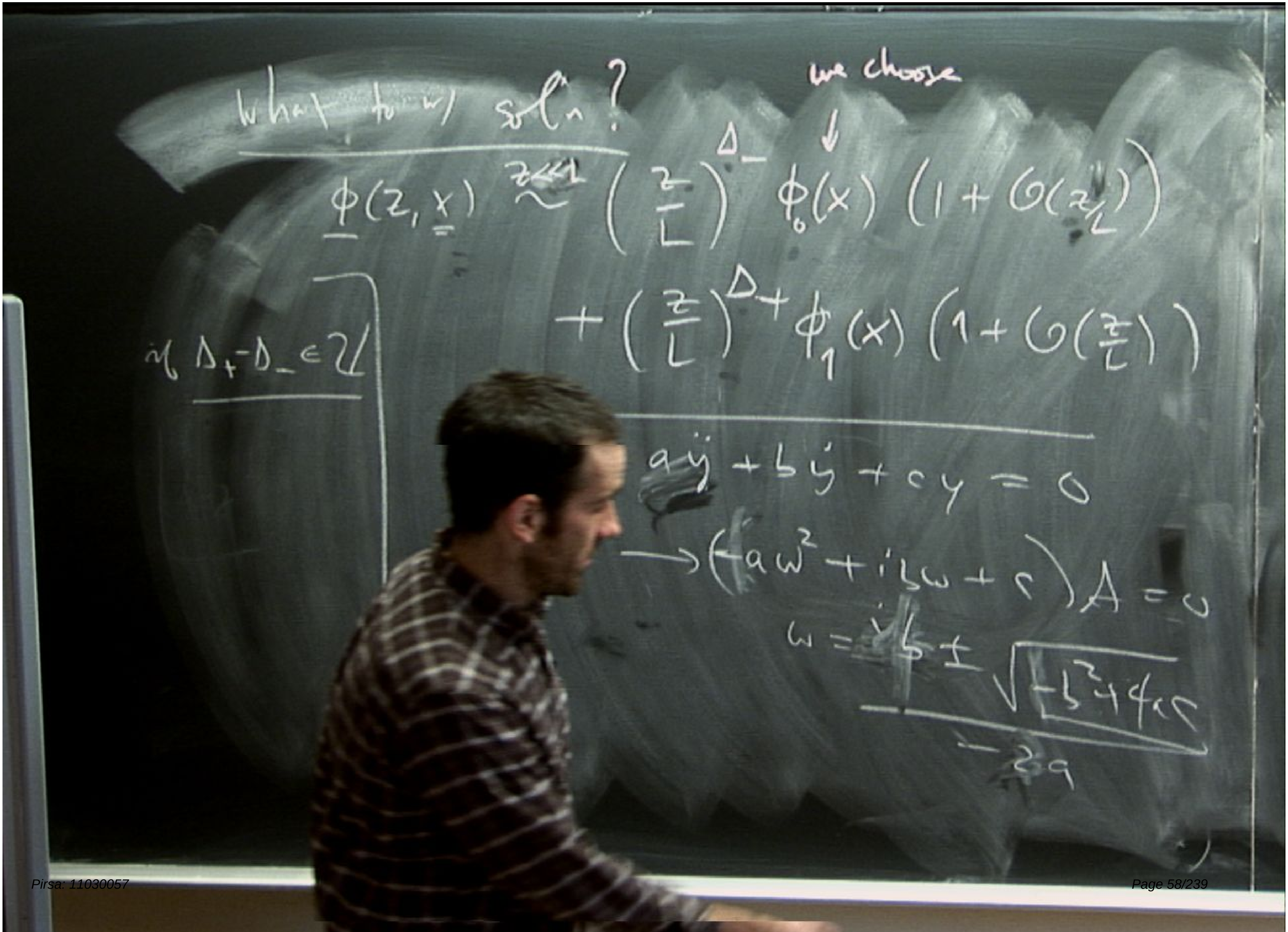

what to w/ soln?
we choose
$\phi(z,x) \overset{z \ll 1}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x)\left(1+O(z/L)\right)$
$+ \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x)\left(1+O(\frac{z}{L})\right)$
$a\ddot{y} + b\dot{y} + cy = 0$
$y = Ae^{i\omega t}$
$\rightarrow (-a\omega^2 + ib\omega + c)A = 0$

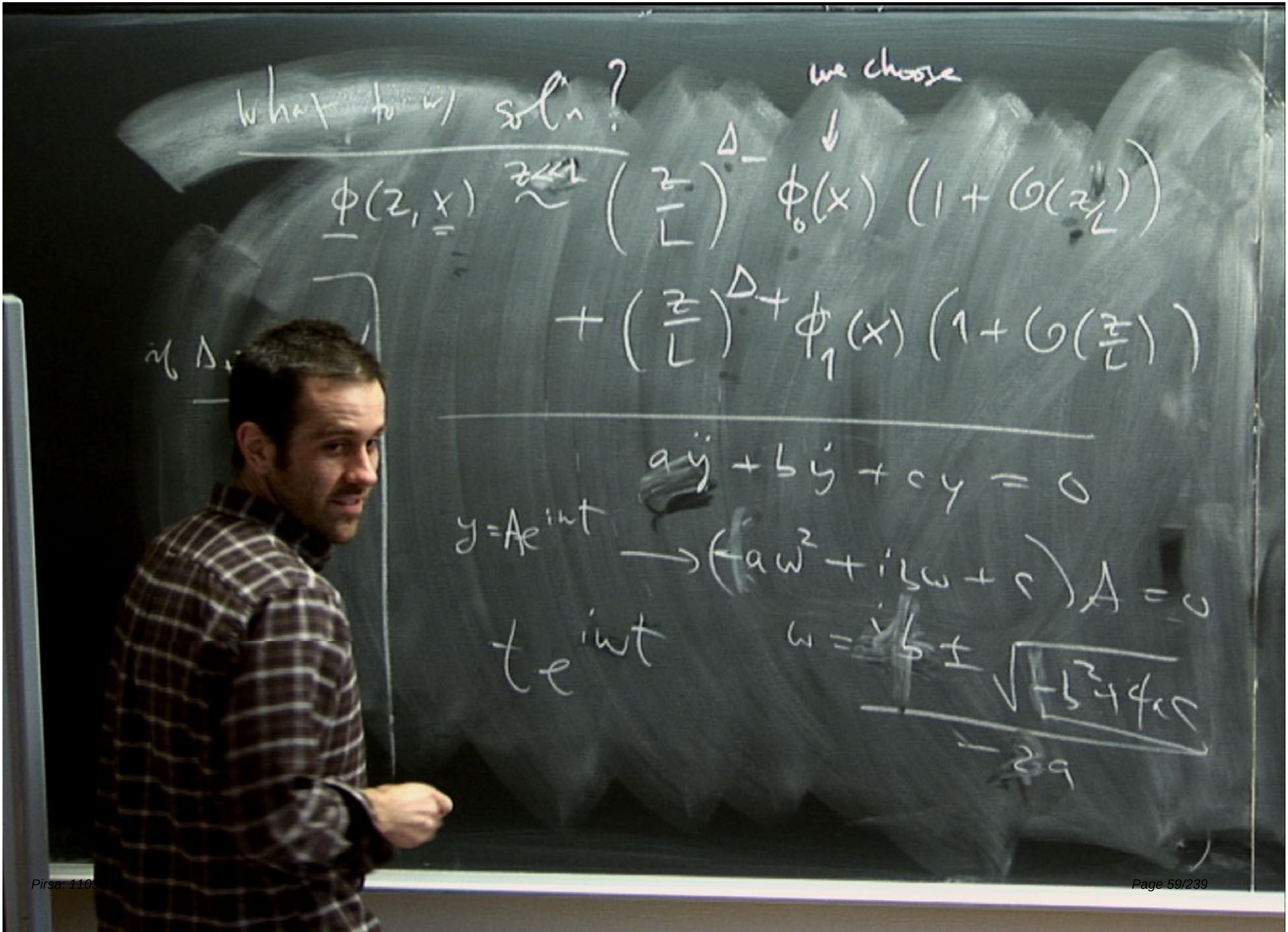

what to w/ soln ?
we choose
φ(z, x) ~ (z/L)^Δ− φ0(x) (1 + O(z/L))
+ (z/L)^Δ+ φ1(x) (1 + O(z/L))
if Δ+ − Δ− ∈ Z
+ bẏ + cy = 0
y = Ae^iωt
(−aω² + ibω + c) A = 0
ω = ±

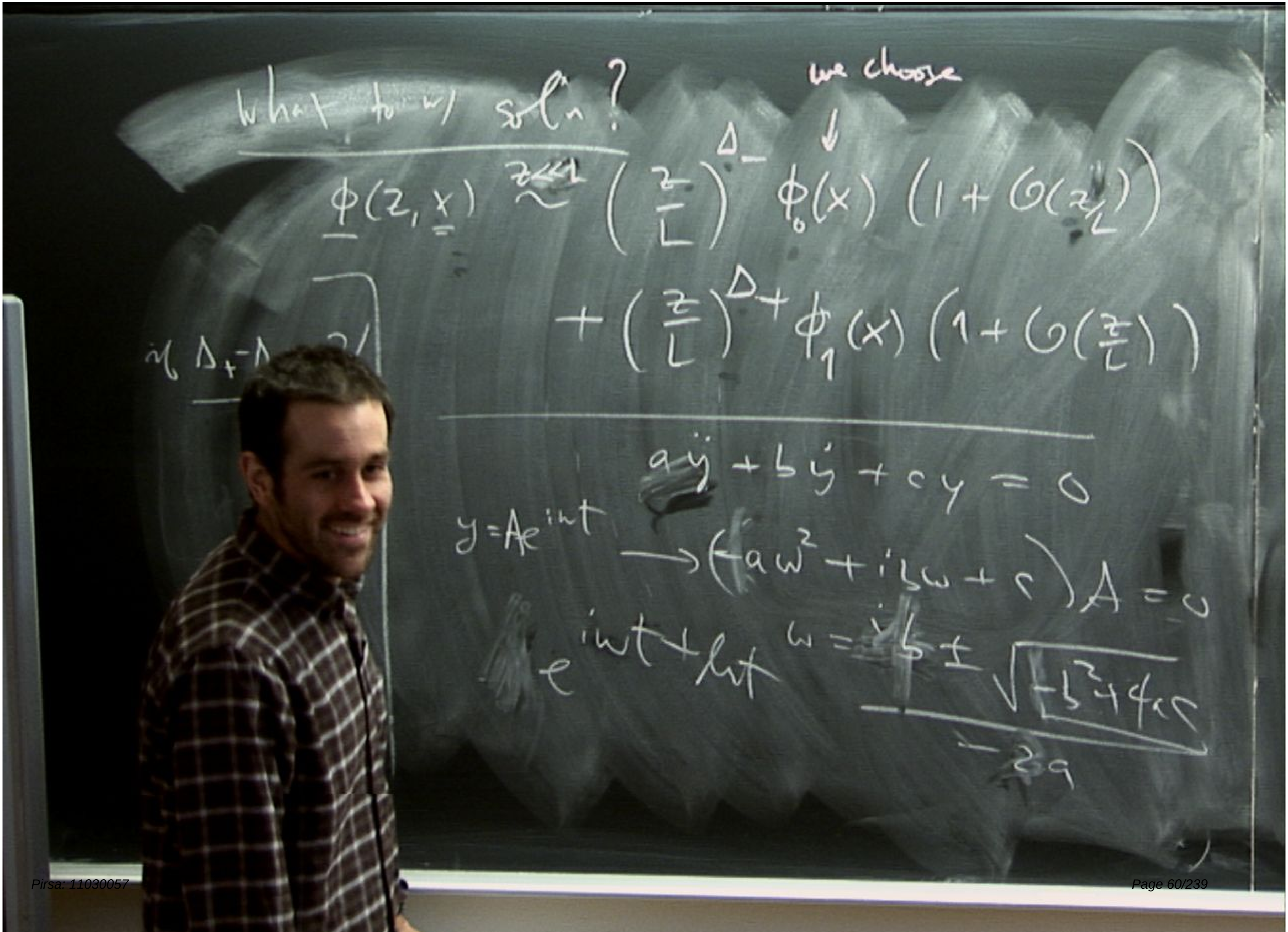

what to w/ sol'n ?
we choose
φ(z, x) ~ (z/L)^Δ− φ0(x) (1 + O(z/L))
+ (z/L)^Δ+ φ1(x) (1 + O(z/L))
if Δ+ − Δ− ∈ Z
+ c y = 0
y = Ae^iωt
+ ibω + c) A = 0
ω = ib/2 ±

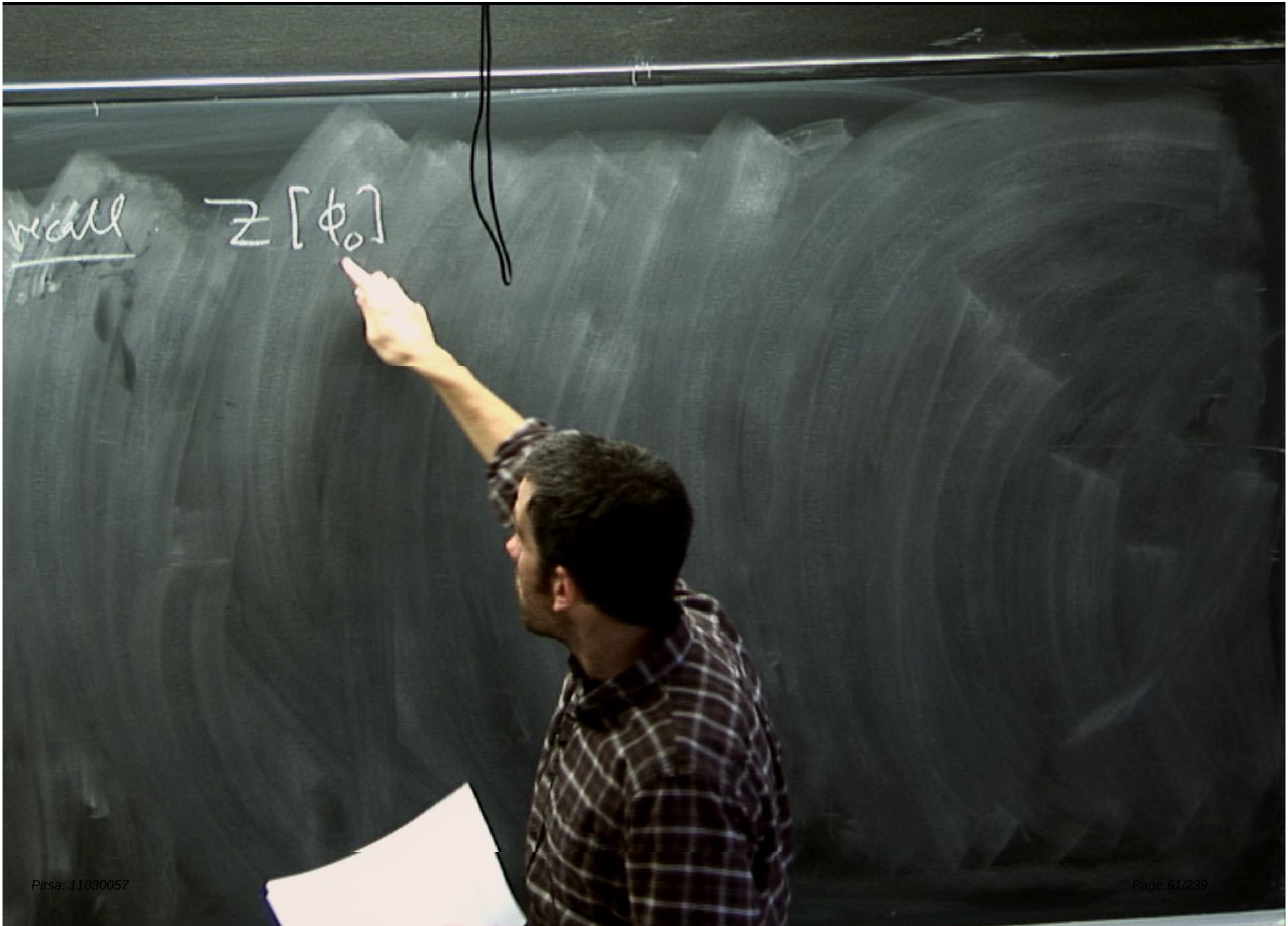

what to w/ sol'n ?
we choose
φ(z, x) ~ (z/L)^Δ_- φ_0(x) (1 + O(z/L))
z<<1
+ (z/L)^Δ_+ φ_1(x) (1 + O(z/L))
if Δ_+ - Δ_- ∈ Z
a ÿ + b ẏ + c y = 0
y = Ae^iωt
→ (-aω^2 + ibω + c) A = 0
ω = ib/2 ± √(-b^2 + 4ac) / -2a

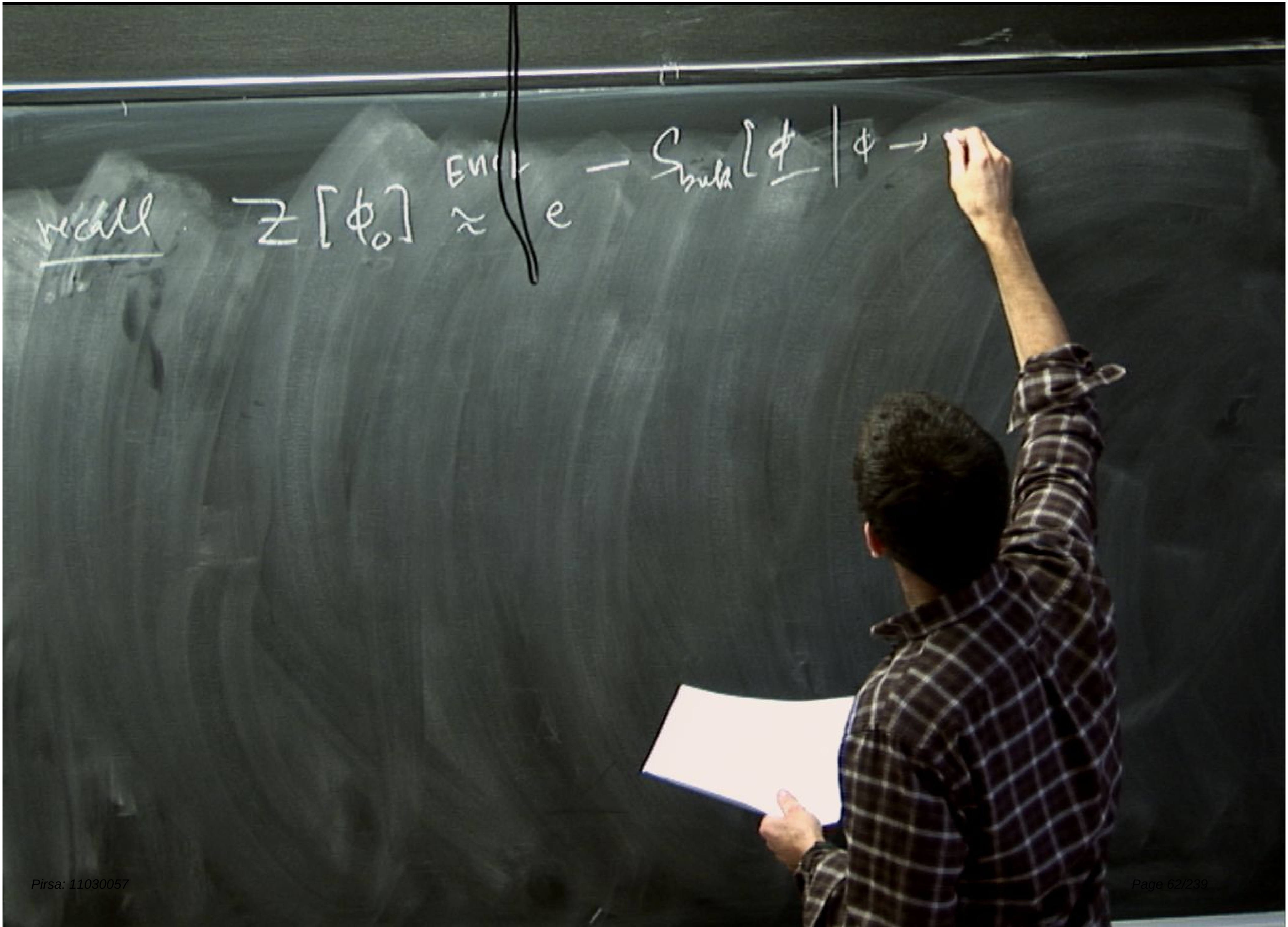

what to w/ soln ?
we choose
φ(z, x) ~ (z/L)^Δ− φ0(x) (1 + O(z/L))
+ (z/L)^Δ+ φ1(x) (1 + O(z/L))
if Δ+ − Δ− ∈ Z
+ cy = 0
y = Ae^iωt
+ ibω + c) A = 0


what to w/ soln ?
we choose
φ(z, x) z<<1 ~ (z/L)^Δ- φ0(x) (1 + O(z/L))
+ (z/L)^Δ+ φ1(x) (1 + O(z/L))
if Δ+ - Δ- ∈ Z
aÿ + bẏ + cy = 0
→ (-aω² + ibω + c) A = 0


what to w/ soln ?
we choose
φ(z, x) ~ (z/L)^Δ− φ₀(x) (1 + O(z/L))
+ (z/L)^Δ+ φ₁(x) (1 + O(z/L))
a ÿ + b ẏ + c y = 0
y = Ae^iwt
→ (−aw² + ibw + c) A = 0
te^iwt

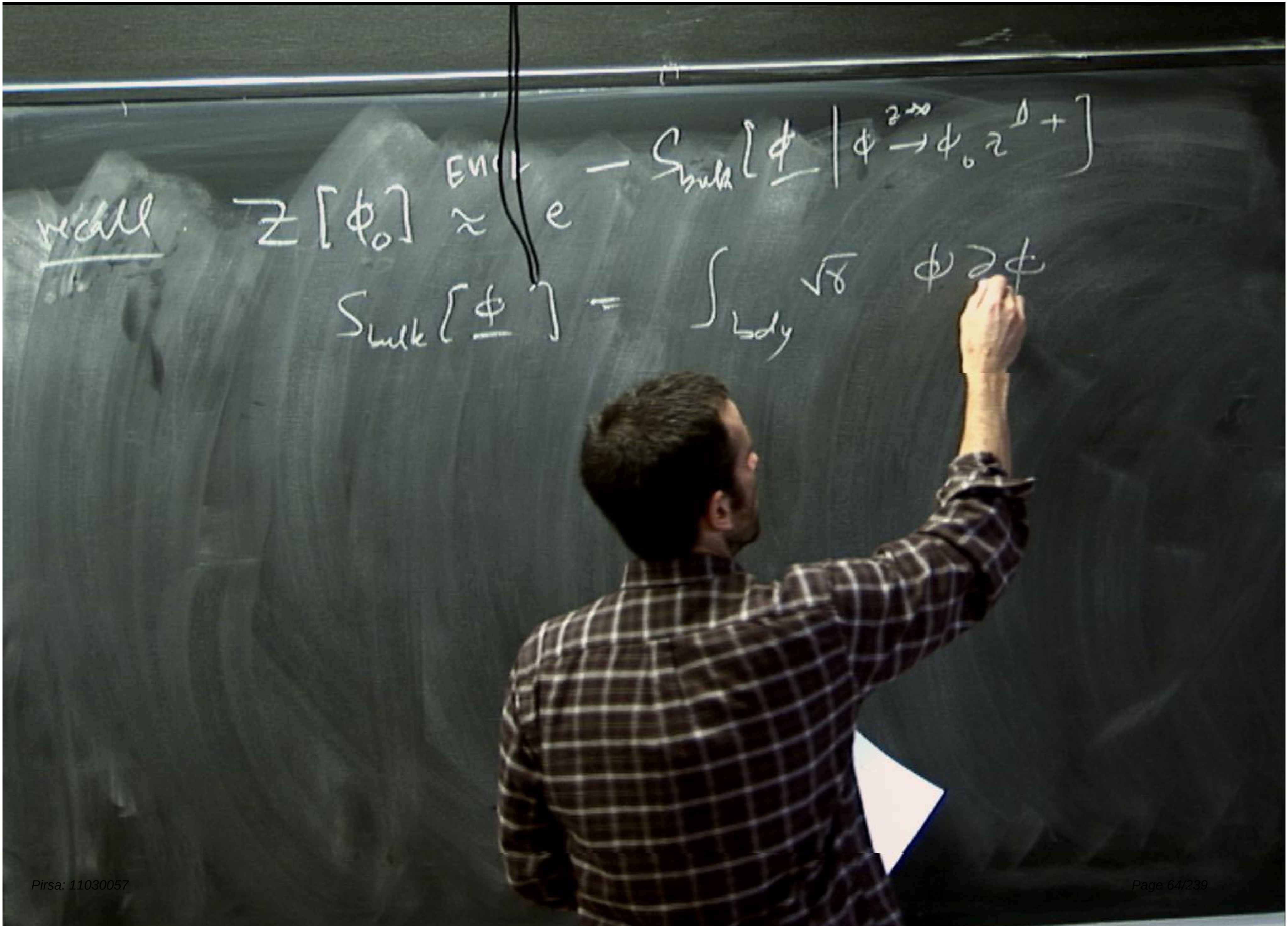

what to w/ soln ?
we choose
z<<L
φ(z, x) ≃ (z/L)^Δ− φ0(x) (1 + O(z/L))
+ (z/L)^Δ+ φ1(x) (1 + O(z/L))
aÿ + bẏ + cy = 0
y = Ae^iωt
→ (−aω² + ibω + c) A = 0


recall
$Z[\phi_0]$


recall
$Z[\phi_0] \approx e^{\text{Eucl} - S_{\text{bulk}}[\phi \,|\, \phi \to}$

recall: $Z[\phi_0] \approx e^{-S^{\text{Eucl}}_{\text{bulk}}[\underline{\phi}\,|\,\phi \xrightarrow{z\to 0} \phi_0 z^{\Delta} + \,]}$

$S_{\text{bulk}}[\underline{\phi}$

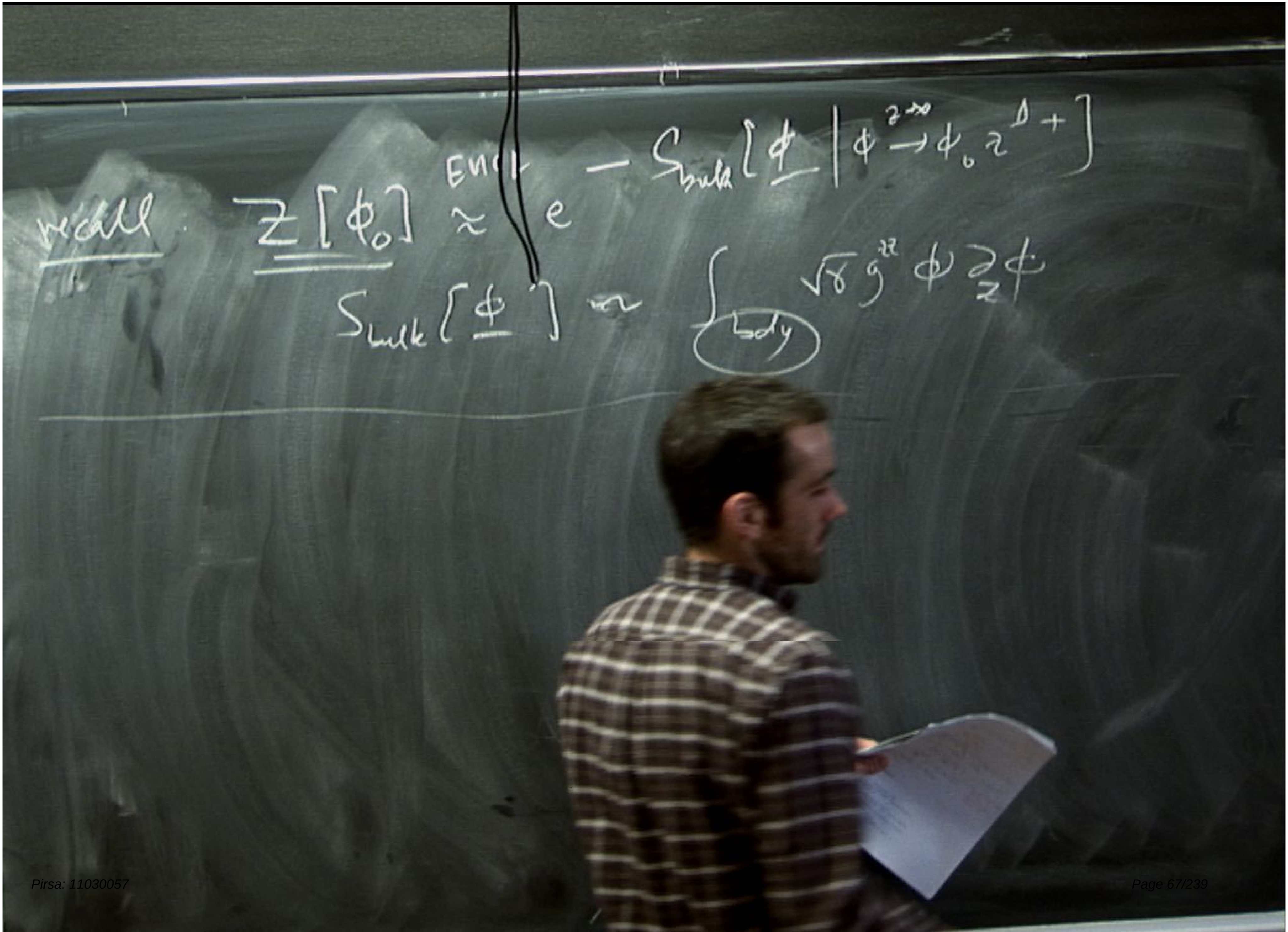

recall:
$Z[\phi_0] \approx e^{-S^{Eucl}_{bulk}[\phi \,|\, \phi \xrightarrow{z\to 0} \phi_0 z^{\Delta} +]}$
$S_{bulk}[\phi] = \int_{bdy} \sqrt{\gamma} \; \phi \partial \phi$

recall: $Z[\phi_0] \approx e^{\text{Eucl}\; -S_{bulk}[\underline{\phi} \,|\, \phi \xrightarrow{z\to 0} \phi_0 z^{\Delta} + \dots]}$

$$S_{bulk}[\underline{\phi}] = \int_{bdy} \sqrt{\gamma}\, g^{zz}\, \phi\, \partial_z \phi$$

recall: $Z[\phi_0] \approx e^{-S_{bulk}^{Eucl}[\underline{\phi} \,|\, \phi \xrightarrow{z\to 0} \phi_0 z^{\Delta} + \ldots]}$

$$S_{bulk}[\underline{\phi}] \approx \int_{bdy} \sqrt{\gamma}\, g^{zz} \phi\, \partial_z \phi$$


recall
Z[φ₀] ≈ e^{-S_bulk[φ | φ → φ₀ z^Δ +]}
S_bulk[φ] ≈ ∫_bdy √γ g^{zz} φ ∂_z φ

recall: $Z[\phi_0] \overset{\text{Eucl}}{\approx} e^{-S_{bulk}[\underline{\phi} \,|\, \phi \xrightarrow{z\to 0} \phi_0 z^{\Delta} + \dots]}$

$S_{bulk}[\underline{\phi}] \approx \int_{bdy} \sqrt{\gamma}\, g^{zz}\, \phi\, \partial_z \phi$

recall: $Z[\phi_0] \approx e^{-S_{bulk}[\underline{\phi} \,|\, \phi \xrightarrow{z\to 0} \phi_0 z^{\Delta} + \ldots]}$ Eucl.

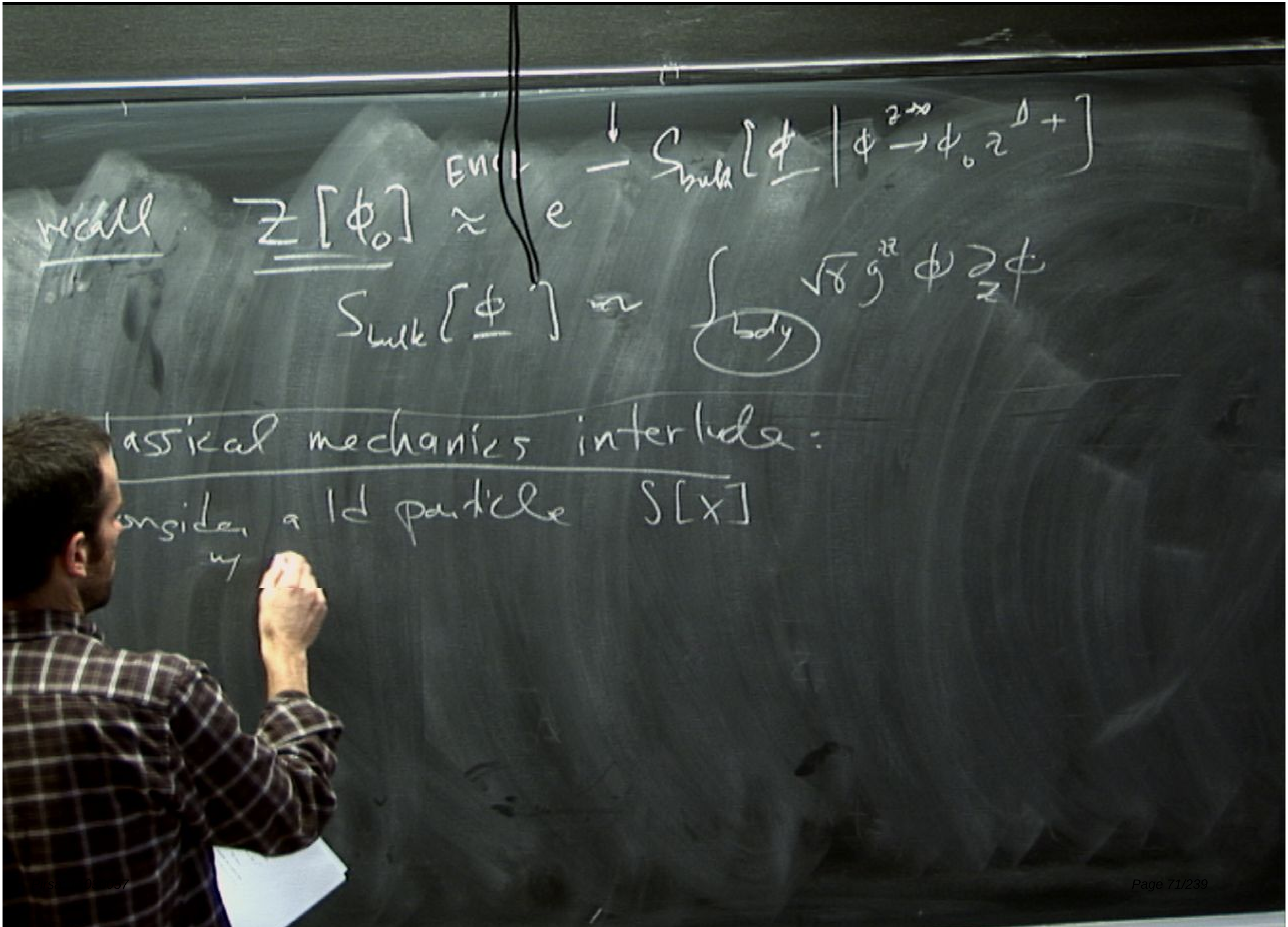
$S_{bulk}[\underline{\phi}] \approx \int_{bdy} \sqrt{\gamma}\, g^{zz} \phi \partial_z \phi$

cla

recall: $\underline{\underline{Z}}[\phi_0] \approx e^{-S_{bulk}[\underline{\phi} \,|\, \phi \xrightarrow{z\to 0} \phi_0 z^{\Delta} +]}$ (Eucl)

$$S_{bulk}[\underline{\phi}] \sim \int_{\text{bdy}} \sqrt{\gamma}\, g^{zz} \phi\, \partial_z \phi$$

classical mechanics interlude:

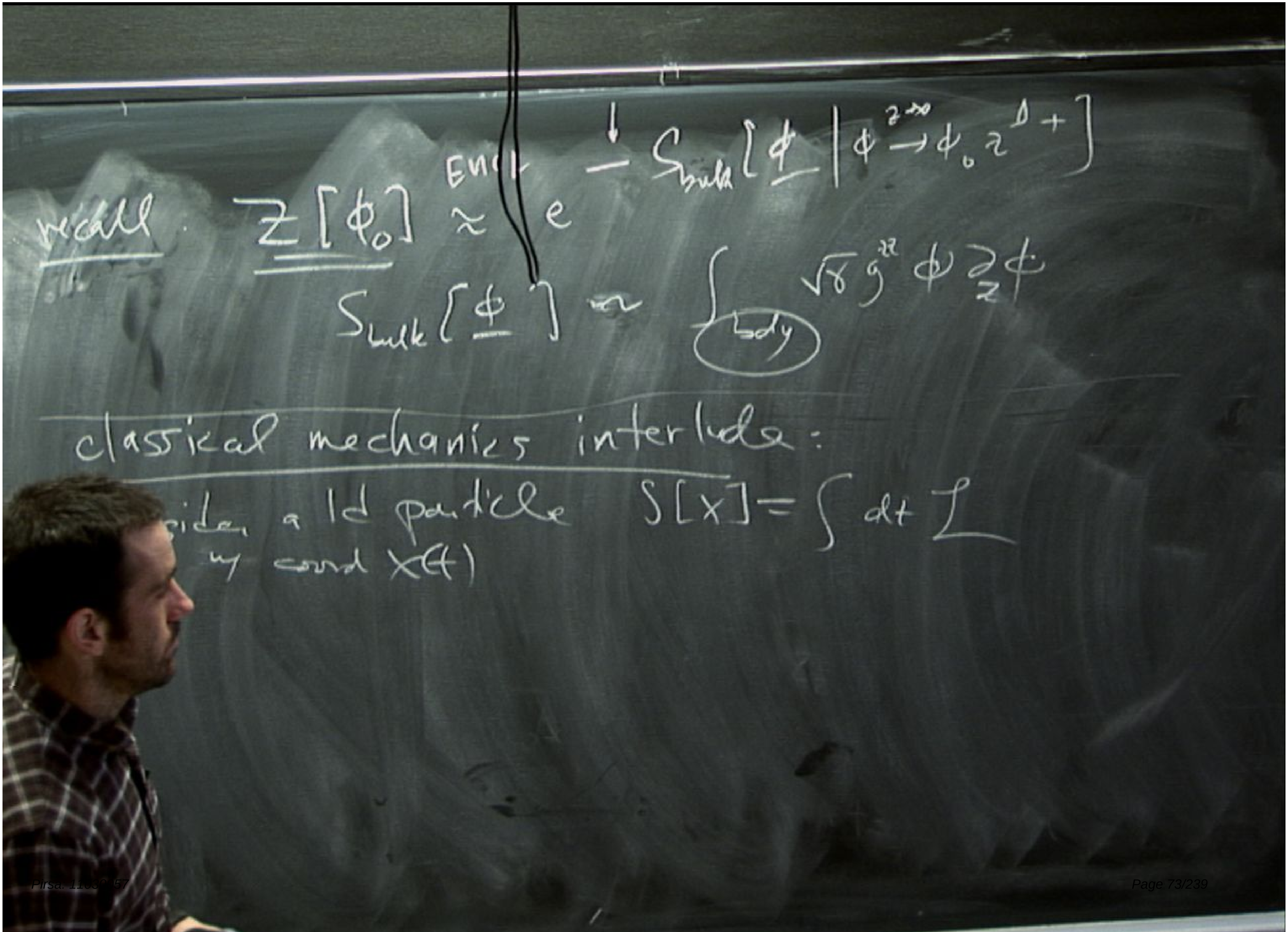

recall
Eucl
Z[φ0] ≈ e − Sbulk[φ | φ → φ0 z^Δ +]
Sbulk[φ] ≈ ∫bdy √γ g^zz φ ∂z φ
lassical mechanics interlude:
onsider a 1d particle S[x]
w/

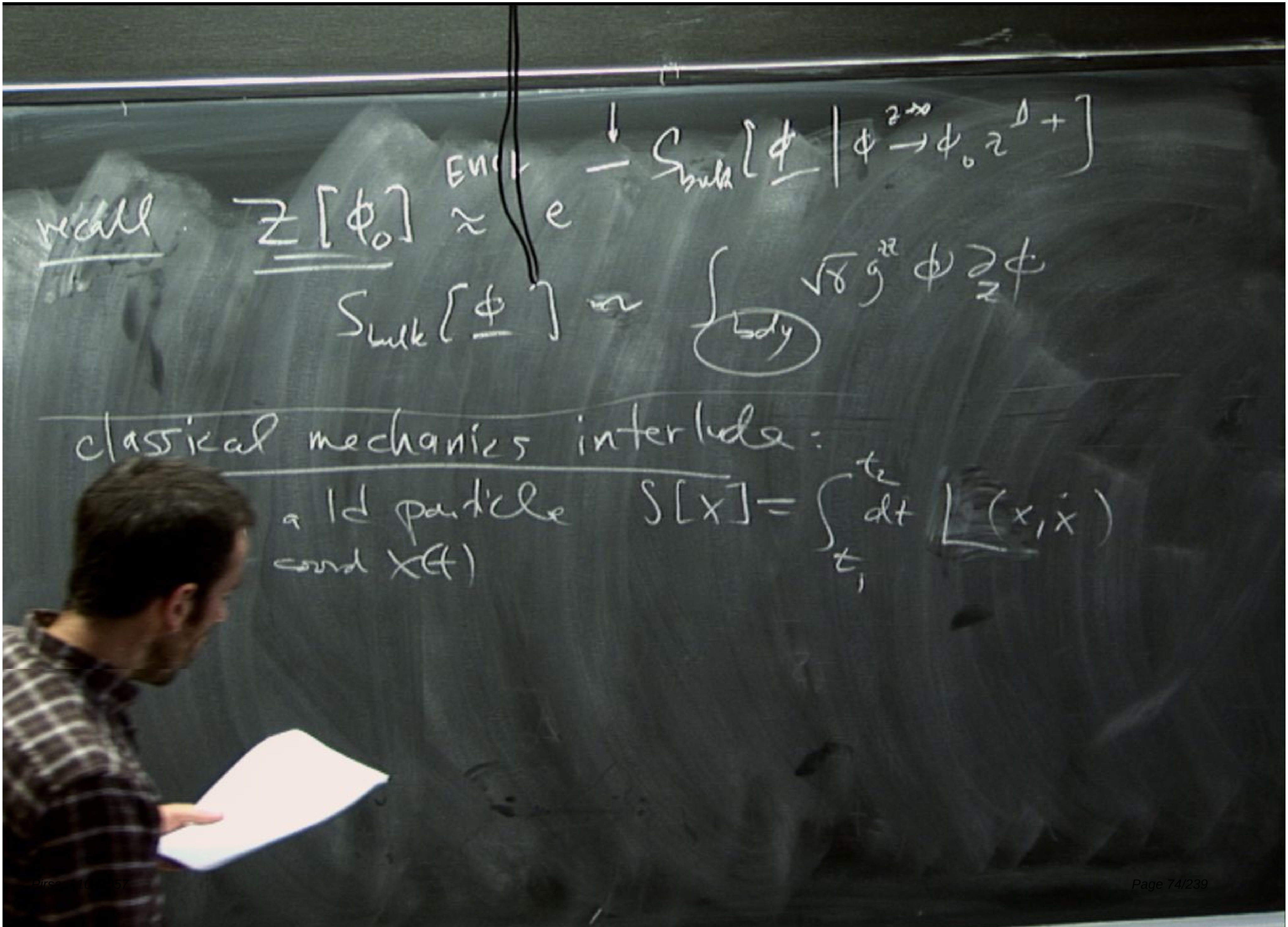
recall: $Z[\phi_0] \approx e^{-S_{bulk}[\phi \,|\, \phi \xrightarrow{z\to 0} \phi_0 z^{\Delta} + \ldots]}$ (Eucl)

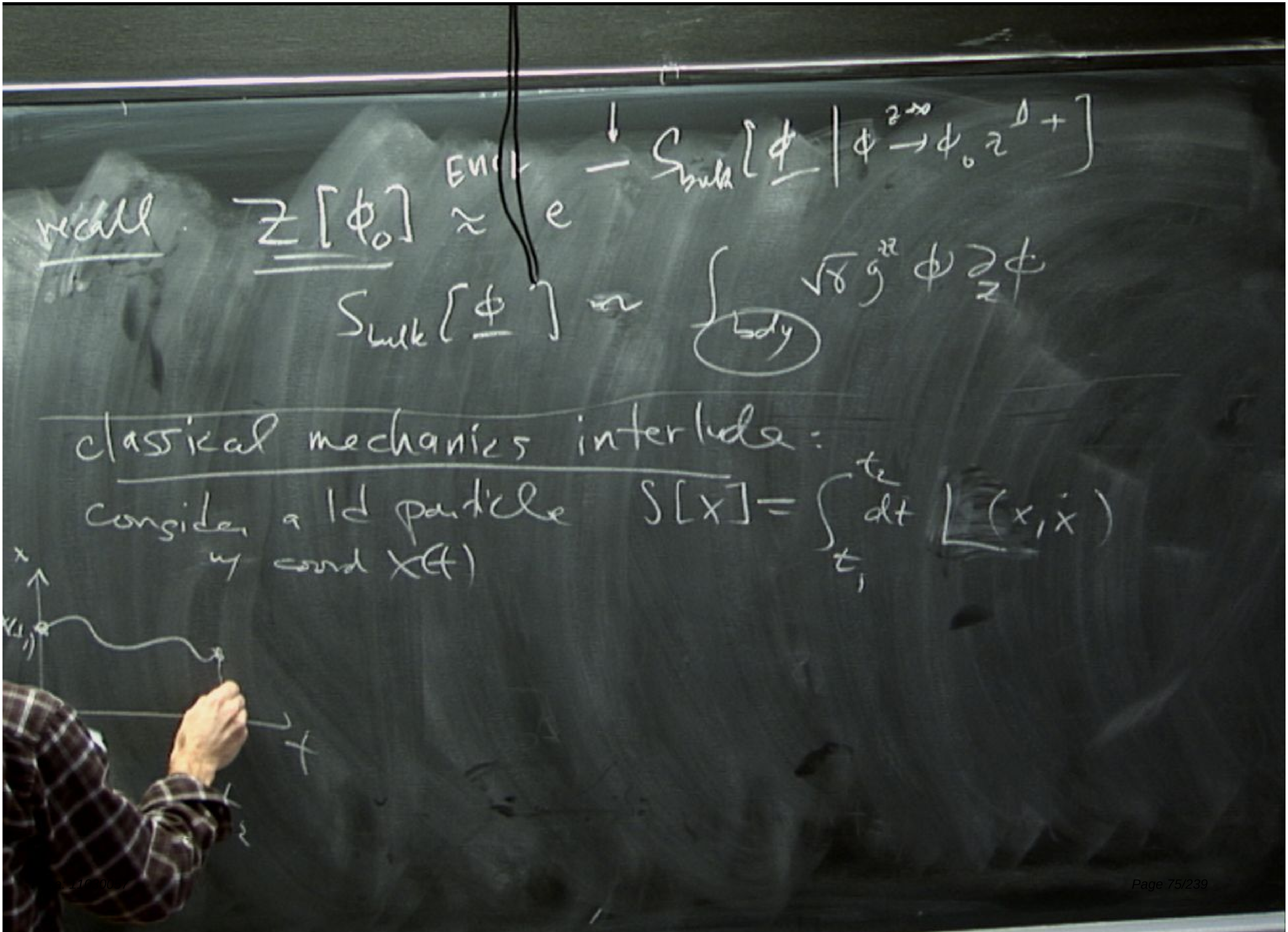
$S_{bulk}[\phi] \sim \int_{bdy} \sqrt{\gamma}\, g^{zz} \phi \partial_z \phi$

classical mechanics interlude:

consider a 1d particle $S[x]$
w/ coord $x(t)$


recall:
classical mechanics interlude:
a 1d particle S[x] = ∫ dt L
coord x(t)


recall
Z[φ0] ≈ e^(-1/ħ S_bulk[φ | φ → φ0 z^Δ +])
S_bulk[φ] ~ ∫bdy √γ g^zz φ ∂z φ
classical mechanics interlude:
a 1d particle
coord x(t)
S[x] = ∫ from t1 to t2 dt L(x, ẋ)

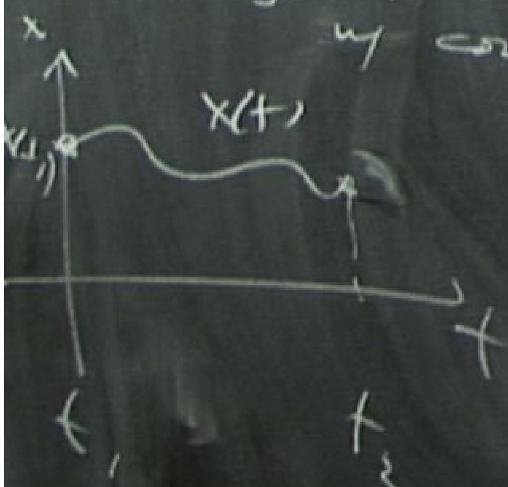

recall
$Z[\phi_0] \approx e^{-S_{bulk}[\phi | \phi \to \phi_0 z^{\Delta} +]}$
$S_{bulk}[\phi] \sim \int_{bdy} \sqrt{\gamma} g^{zz} \phi \partial_z \phi$
classical mechanics interlude:
consider a 1d particle
w/ coord x(t)
$S[x] = \int_{t_1}^{t_2} dt\, L(x, \dot{x})$

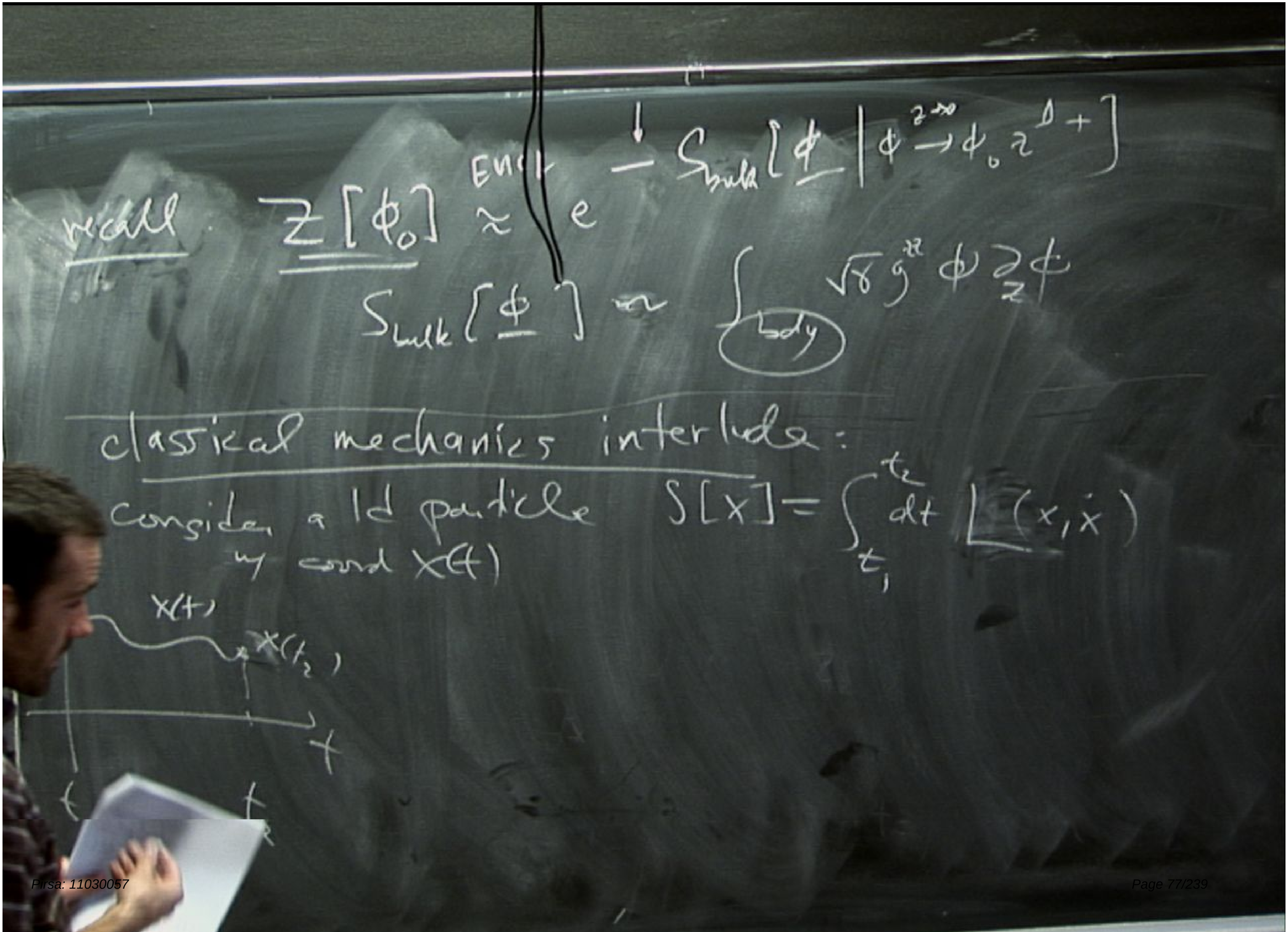
recall: $Z[\phi_0] \approx e^{-S_{bulk}[\underline{\phi} \,|\, \phi \xrightarrow{z\to 0} \phi_0 z^{\Delta} + \ldots]}$ (Eucl)

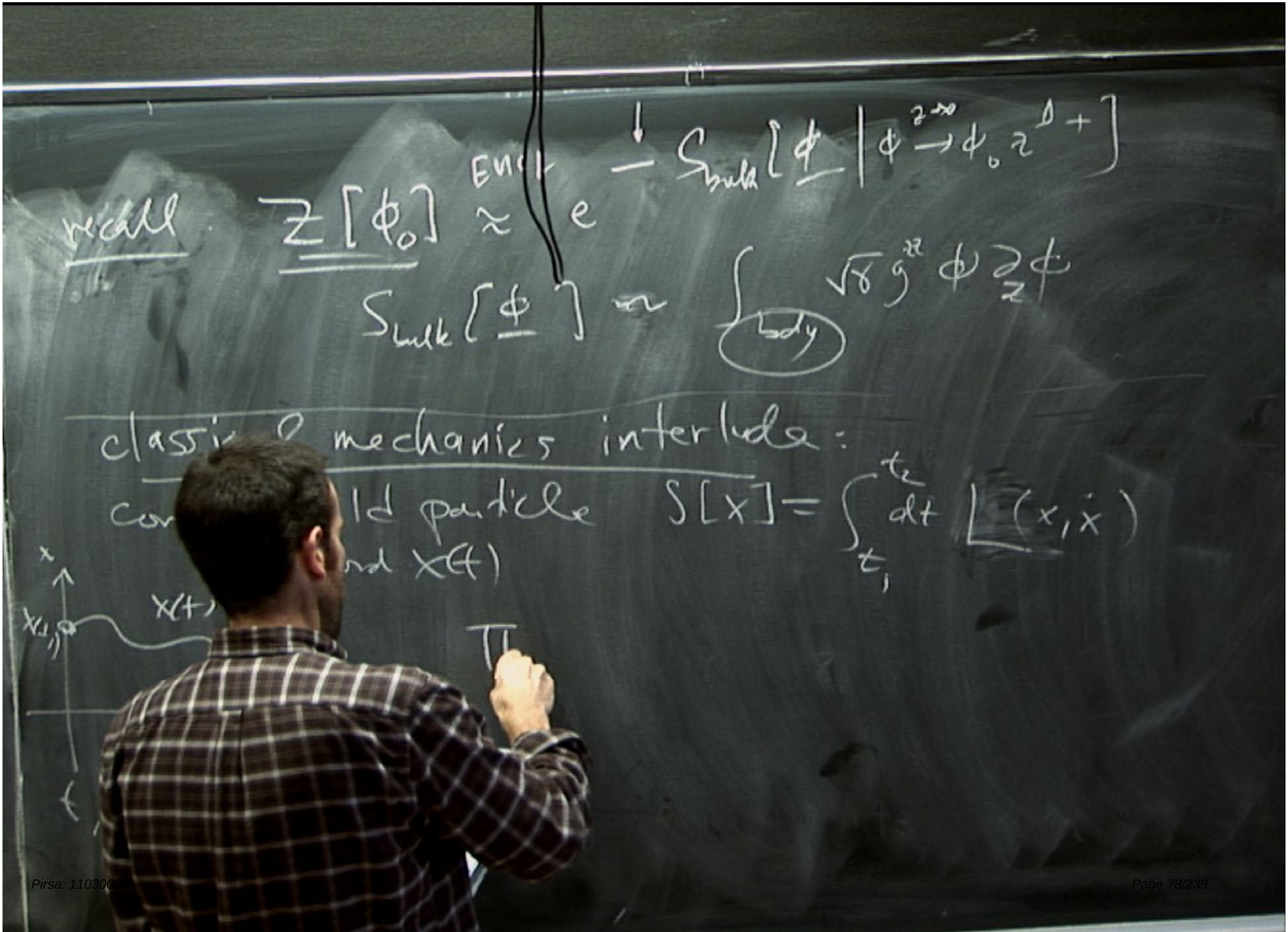
$S_{bulk}[\underline{\phi}] \sim \int_{bdy} \sqrt{\gamma}\, g^{zz} \phi\, \partial_z \phi$

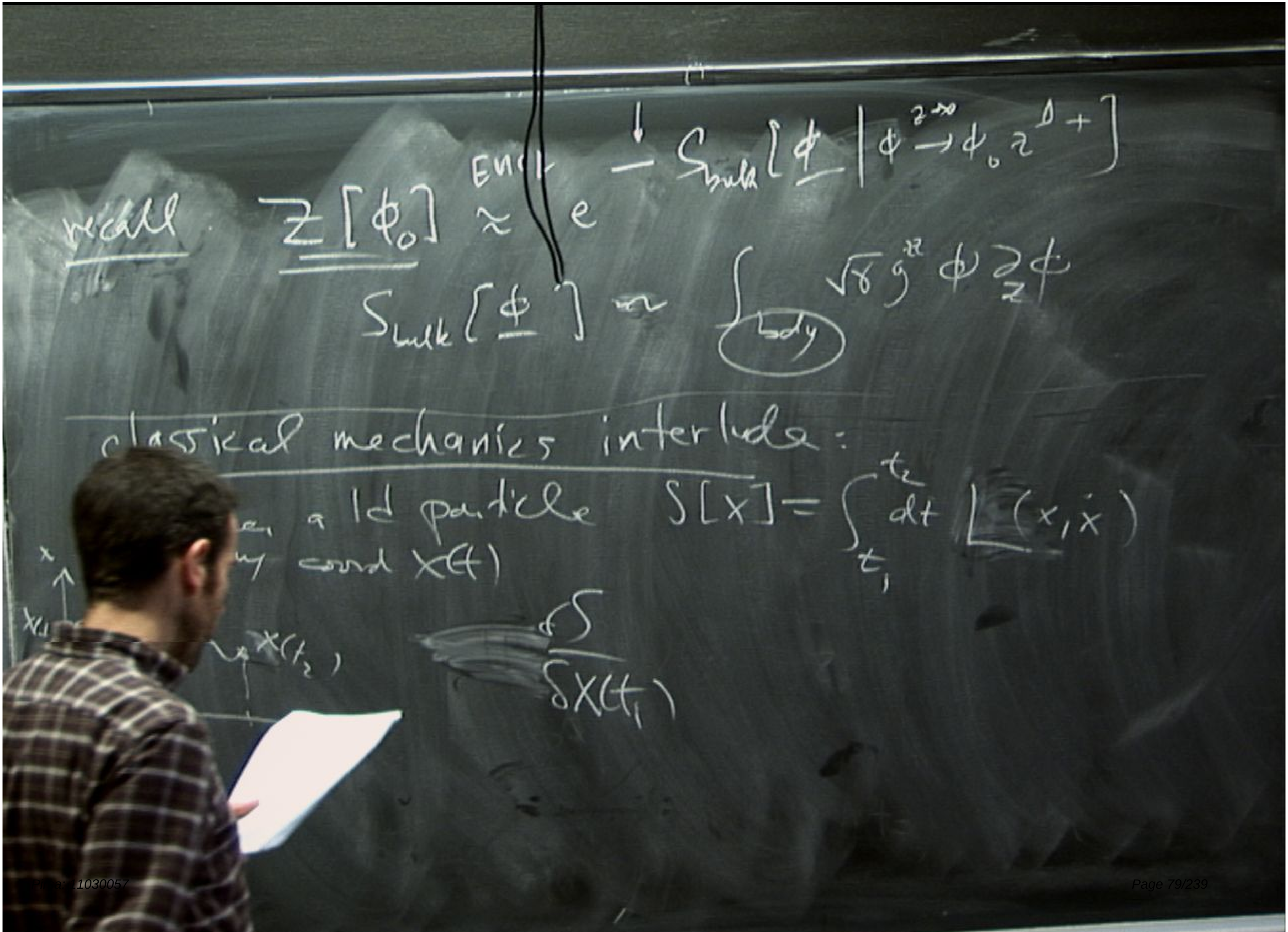
classical mechanics interlude:

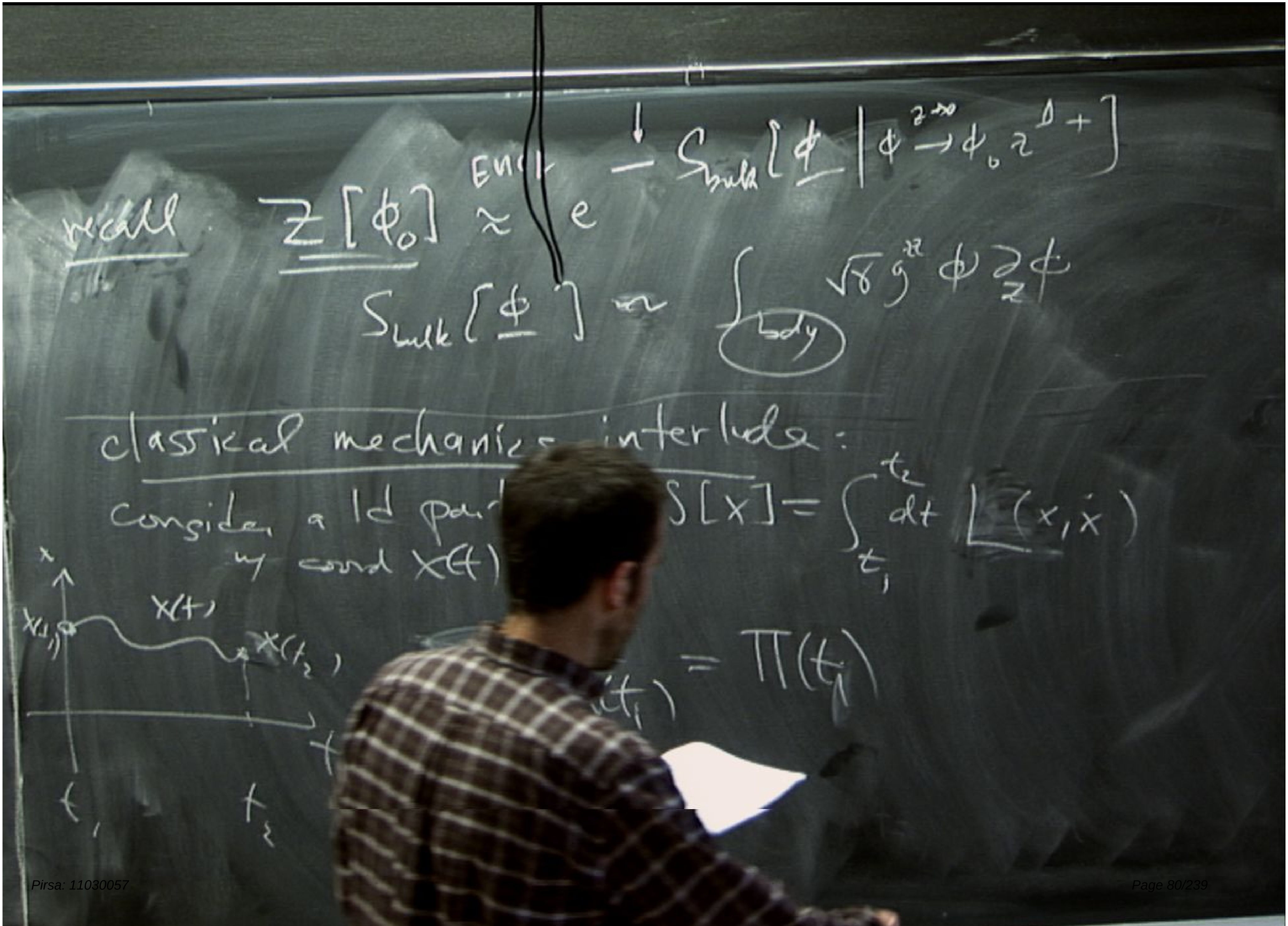
consider a 1d particle w/ coord $x(t)$ $\quad S[x] = \int_{t_1}^{t_2} dt\, L(x, \dot{x})$

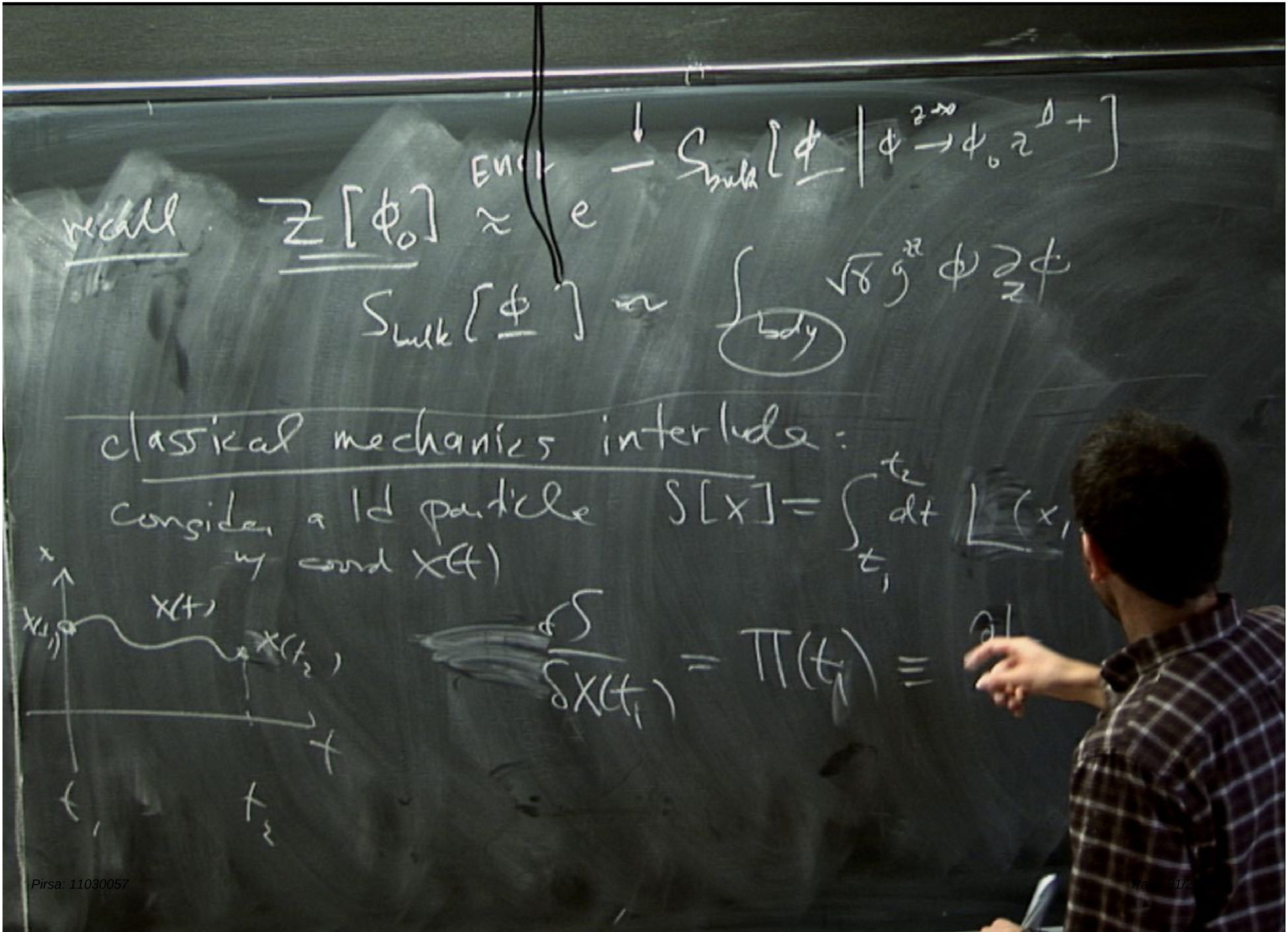


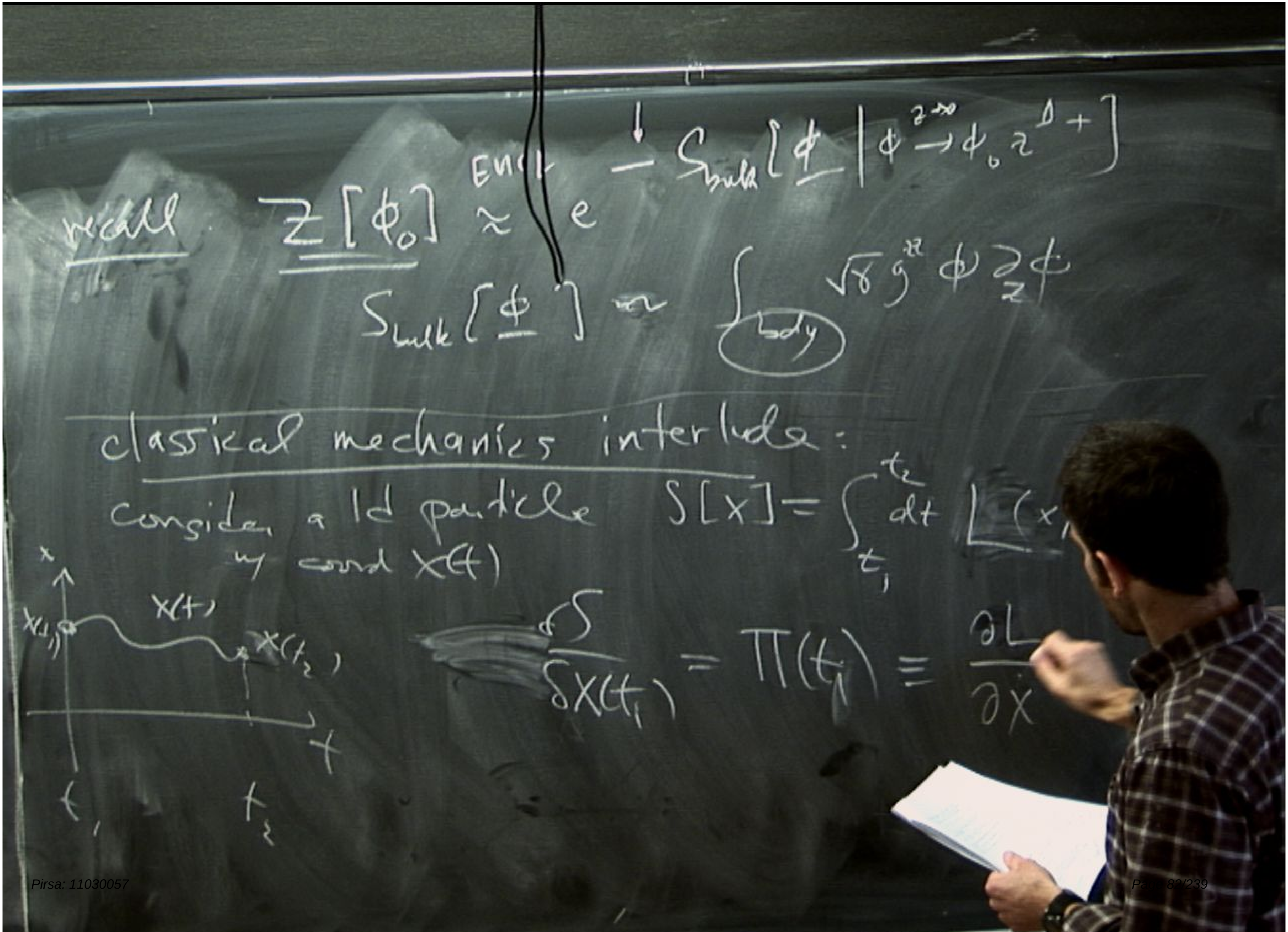

recall
Eucl
$Z[\phi_0] \approx e^{-S_{bulk}[\phi \,|\, \phi \xrightarrow{z\to 0} \phi_0 z^{\Delta} +]}$
$S_{bulk}[\phi] \sim \int_{bdy} \sqrt{\gamma}\, g^{zz} \phi \partial_z \phi$
classical mechanics interlude:
consider a 1d particle w/ coord x(t)
$S[x] = \int_{t_1}^{t_2} dt\, L(x, \dot{x})$
x(t)
$x(t_2)$
t
t_1
t_2


recall
Z[φ₀] ≈ e^{-S_bulk[Φ | Φ → φ₀ z^Δ +]}
Encl
S_bulk[Φ] ~ ∫_bdy √γ g^zz Φ ∂_z Φ
classical mechanics interlude:
ld particle S[x] = ∫_{t_i}^{t_2} dt L(x, ẋ)
nd x(t)
x(t)


recall
Z[φ₀] ≈ e^{−S_bulk[φ | φ → φ₀ z^Δ +]}
Eucl
S_bulk[φ] ~ ∫_bdy √γ g^{zz} φ ∂_z φ
classical mechanics interlude:
a 1d particle
coord x(t)
S[x] = ∫_{t₁}^{t₂} dt L(x, ẋ)
x(t₂)
δS / δx(t₁)


recall
classical mechanics interlude:
consider a 1d particle
w/ coord x(t)
x(t)


recall
Z[φ₀] ≈ e^{- S_bulk[φ | φ → φ₀ z^Δ +]}
Eucl
z→0
S_bulk[φ] ~ ∫_bdy √γ g^{zz} φ ∂_z φ
classical mechanics interlude:
consider a 1d particle w/ coord x(t)
S[x] = ∫_{t₁}^{t₂} dt L(x,
x
x(t)
x(t₁)
x(t₂)
t
t₁
t₂
δS/δx(t₁) = Π(t₁) ≡ ∂L

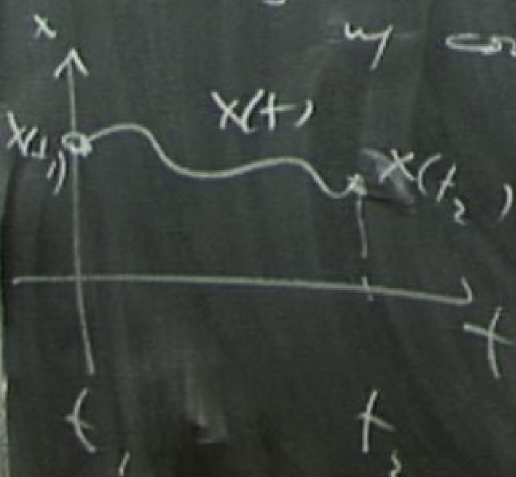

recall Z[φ₀] ≈ e^{−S_bulk[φ | φ → φ₀ z^Δ + …]}
Eucl
S_bulk[φ] ~ ∫_bdy √γ g^zz φ ∂_z φ
classical mechanics interlude:
consider a 1d particle w/ coord x(t)
S[x] = ∫_{t₁}^{t₂} dt L(x,
δS/δx(t₁) = π(t₁) ≡ ∂L/∂ẋ
x(t)
x(t₁)
x(t₂)
t₁
t₂
t
x

recall $Z[\phi_0] \approx e^{\text{Eucl} \; -\frac{1}{\hbar} S_{bulk}[\underline{\phi} \,|\, \phi \xrightarrow{z\to 0} \phi_0 z^{\Delta} + \ldots]}$

$S_{bulk}[\underline{\phi}] \sim \int_{bdy} \sqrt{\gamma}\, g^{zz} \phi\, \partial_z \phi$

classical mechanics interlude:

consider a 1d particle w/ coord $x(t)$ $\quad S[x] = \int_{t_1}^{t_2} dt\, L(x, \dot{x})$



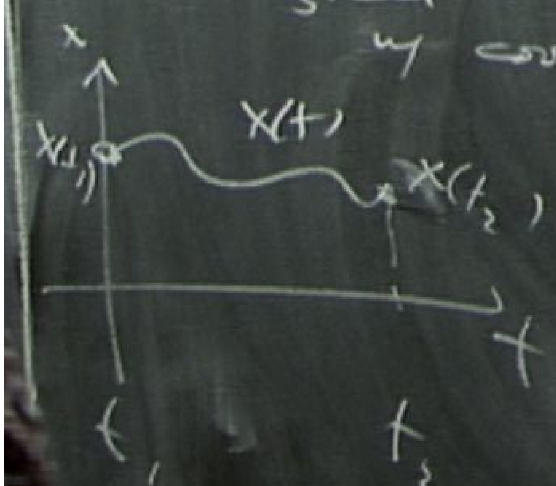
$$\frac{\delta S}{\delta x(t_1)} = \pi(t_1) \equiv \frac{\partial L}{\partial \dot{x}}$$

recall: $Z[\phi_0] \approx e^{-S_{bulk}[\underline{\phi} \,|\, \phi \xrightarrow{z\to 0} \phi_0 z^{\Delta} + \dots]}$ (Eucl)

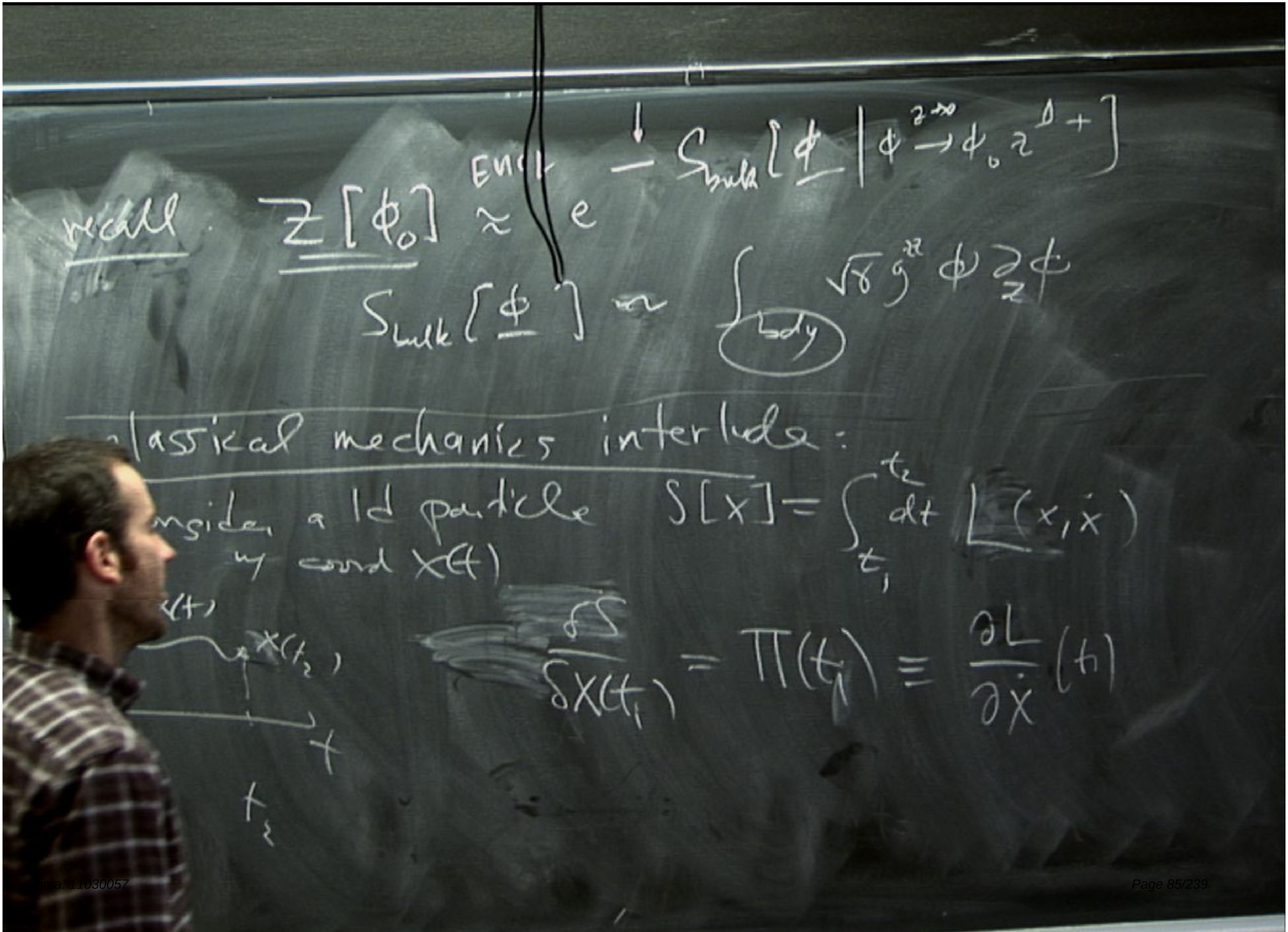
$$S_{bulk}[\underline{\phi}] \approx \int_{bdy} \sqrt{\gamma}\, g^{zz} \phi\, \partial_z \phi$$

classical mechanics interlude:

consider a 1d particle w/ coord $x(t)$ $\quad S[x] = \int_{t_1}^{t_2} dt\, L(x, \dot{x})$



$$\frac{\delta S}{\delta x(t_1)} = \pi(t_1) \equiv \frac{\partial L}{\partial \dot{x}}(t_1)$$

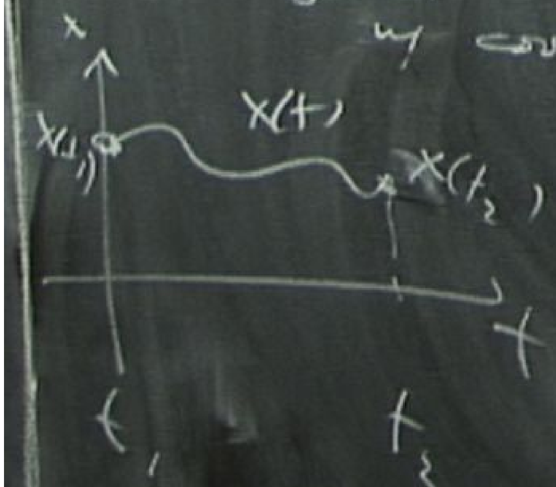

recall $Z[\phi_0] \approx e^{-S_{bulk}[\Phi \mid \Phi \to \phi_0 z^{\Delta} +]}$
Eucl
$S_{bulk}[\Phi] \sim \int_{bdy} \sqrt{\gamma} g^{zz} \Phi \partial_z \Phi$
classical mechanics interlude:
consider a 1d particle
w/ coord $x(t)$
$S[x] = \int_{t_1}^{t_2} dt \, L(x, \dot{x})$
$\frac{\delta S}{\delta x(t_1)} = \Pi(t_1) \equiv \frac{\partial L}{\partial \dot{x}}(t_1)$

recall: $Z[\phi_0] \approx e^{-S_{bulk}[\phi \,|\, \phi \xrightarrow{z\to 0} \phi_0 z^{\Delta} + \dots]}$ (Eucl)

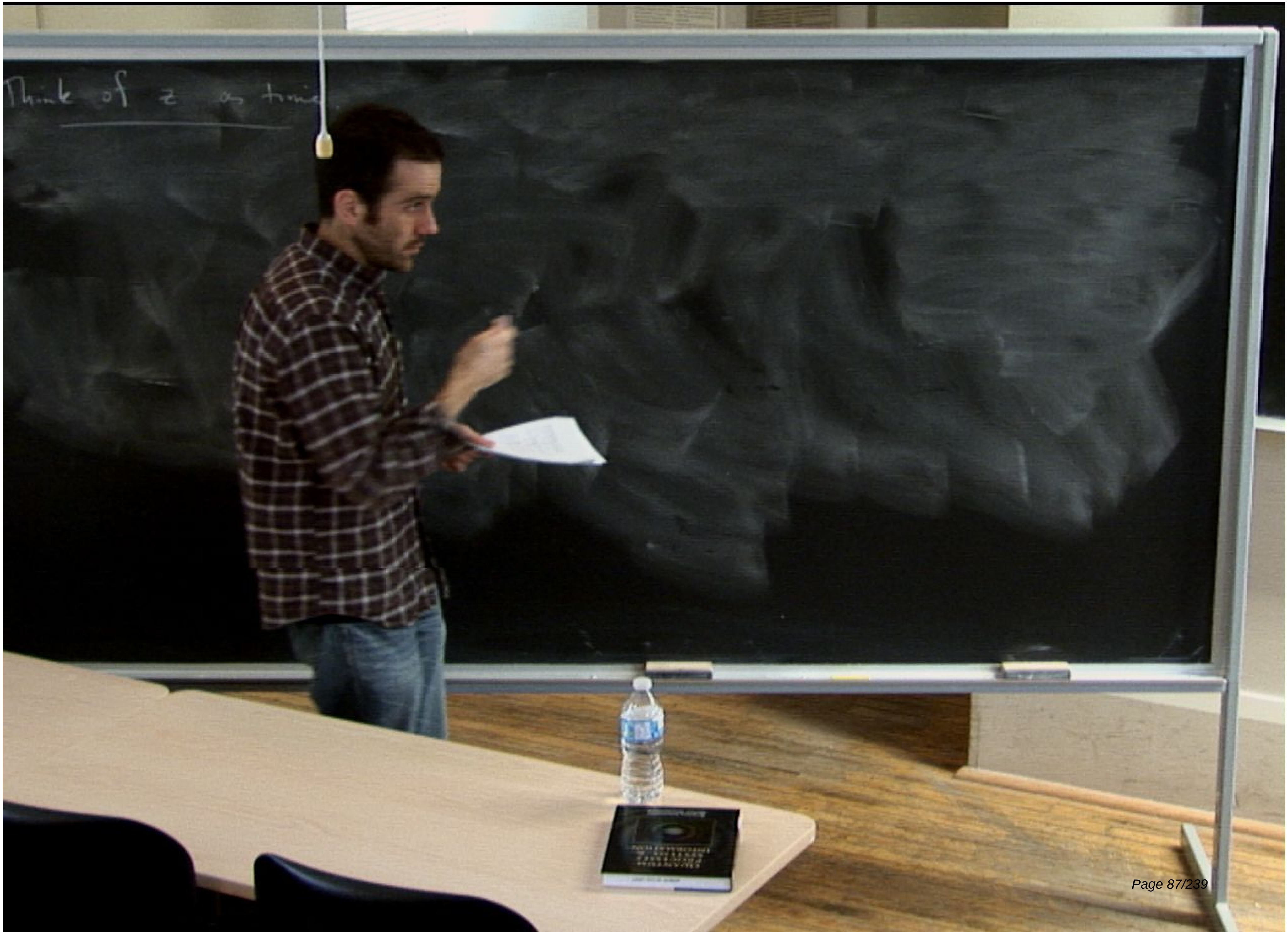
$$S_{bulk}[\phi] \sim \int_{bdy} \sqrt{\gamma}\, g^{zz} \phi \, \partial_z \phi$$

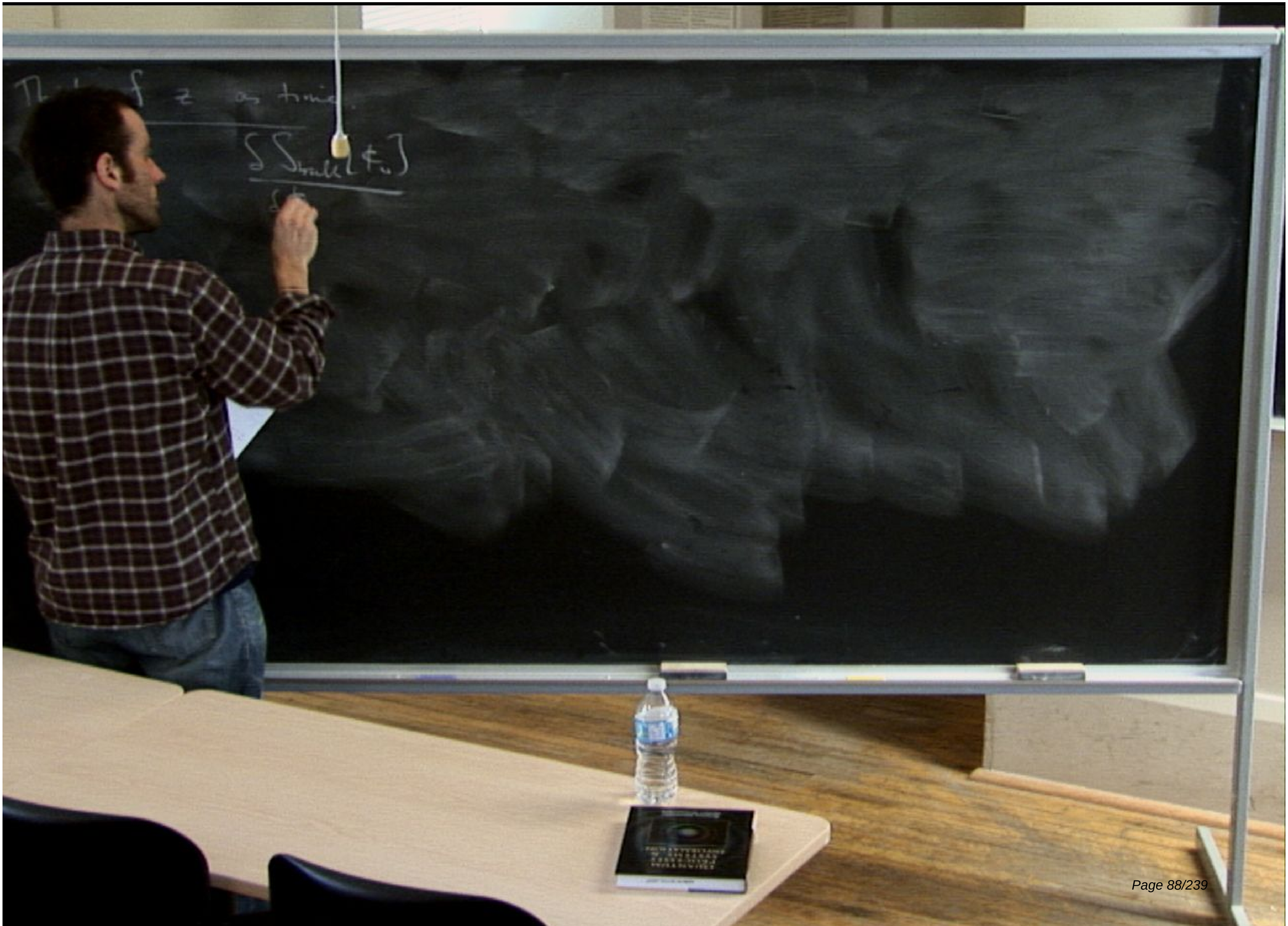
classical mechanics interlude:

consider a 1d particle w/ coord $x(t)$ $S[x] = \int_{t_1}^{t_2} dt\, L(x, \dot{x})$



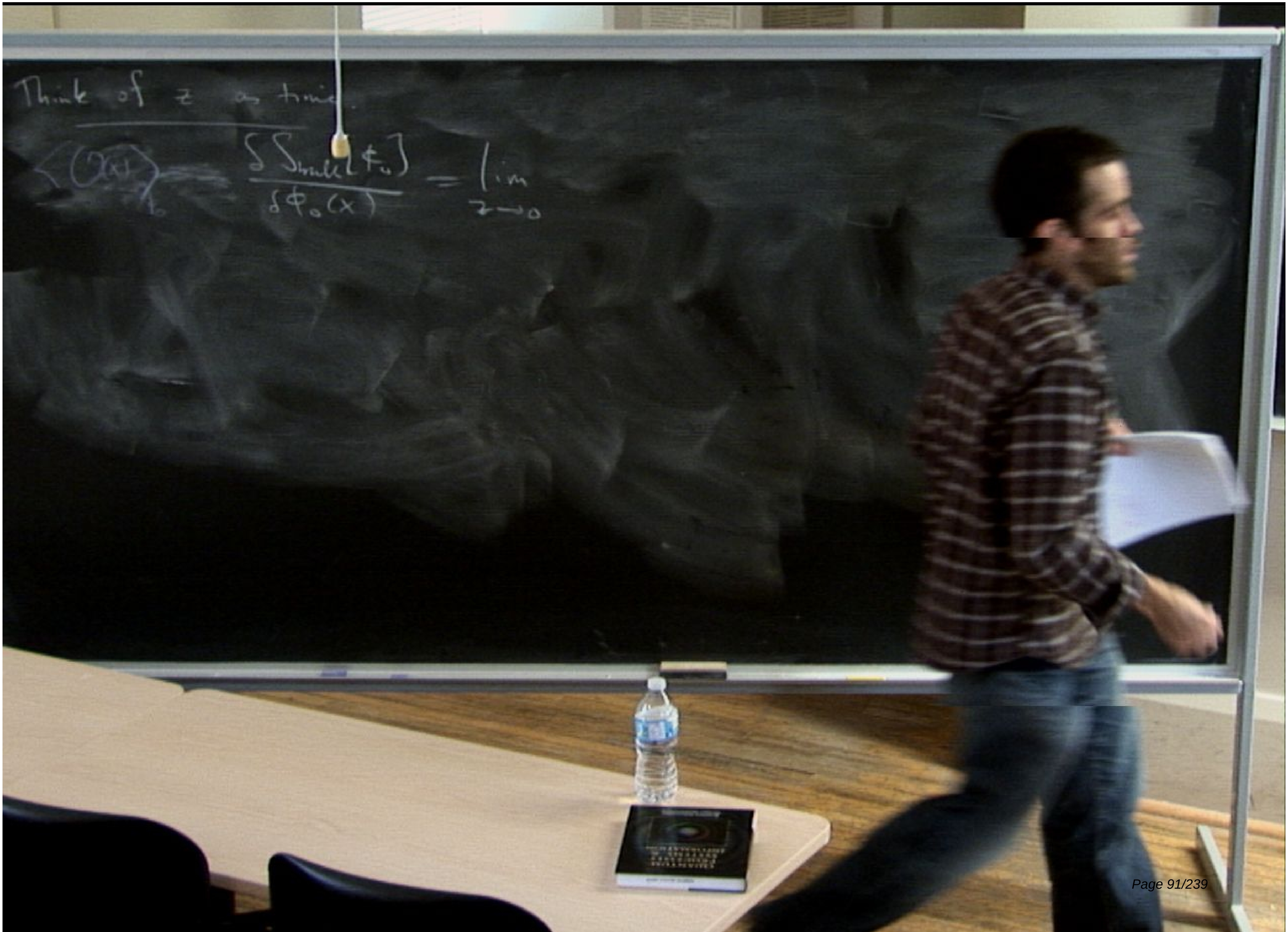
$$\frac{\delta S}{\delta x(t_1)} = \Pi(t_1) \equiv \frac{\partial L}{\partial \dot{x}}(t_1)$$

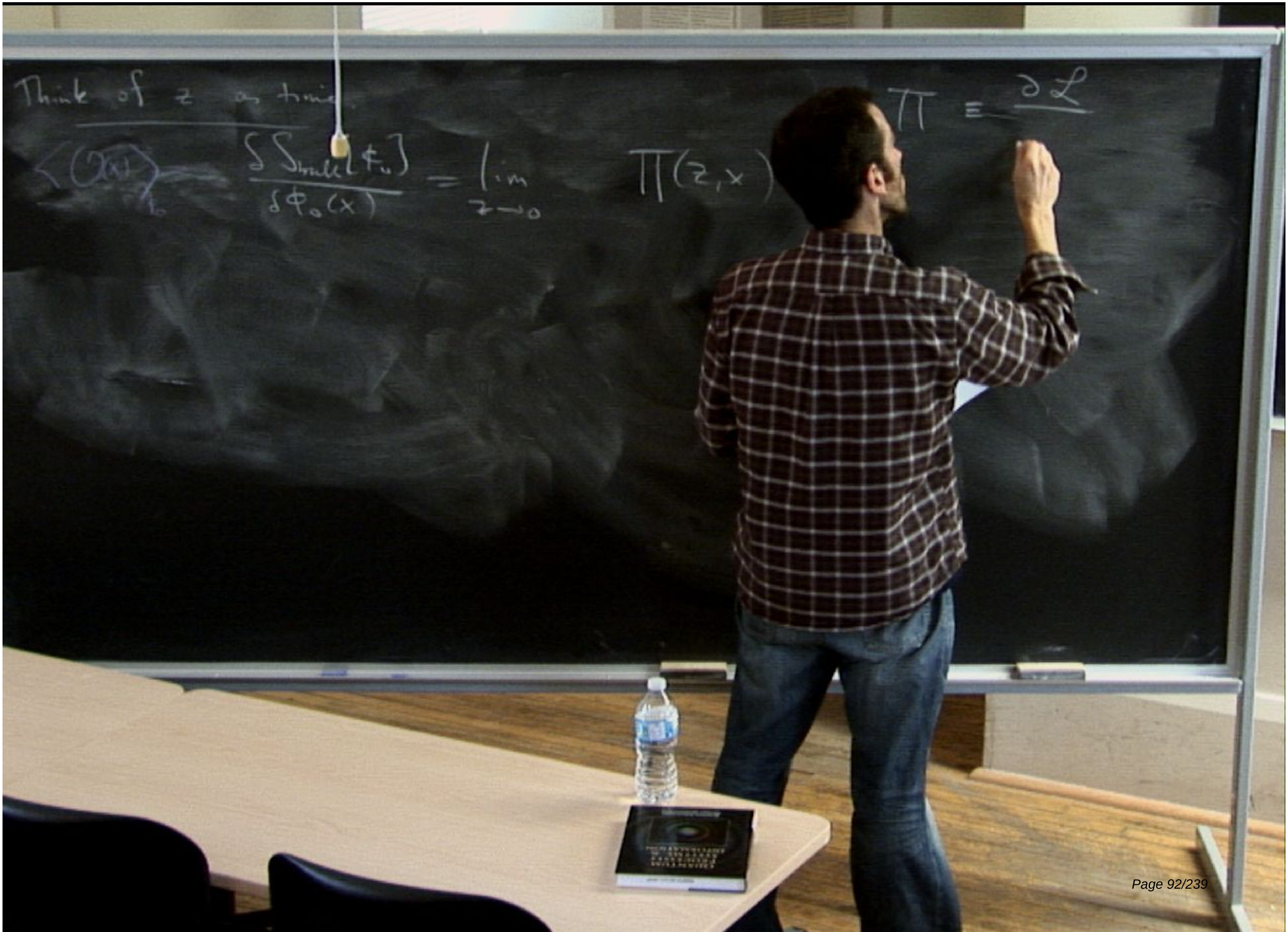

Think of z as time

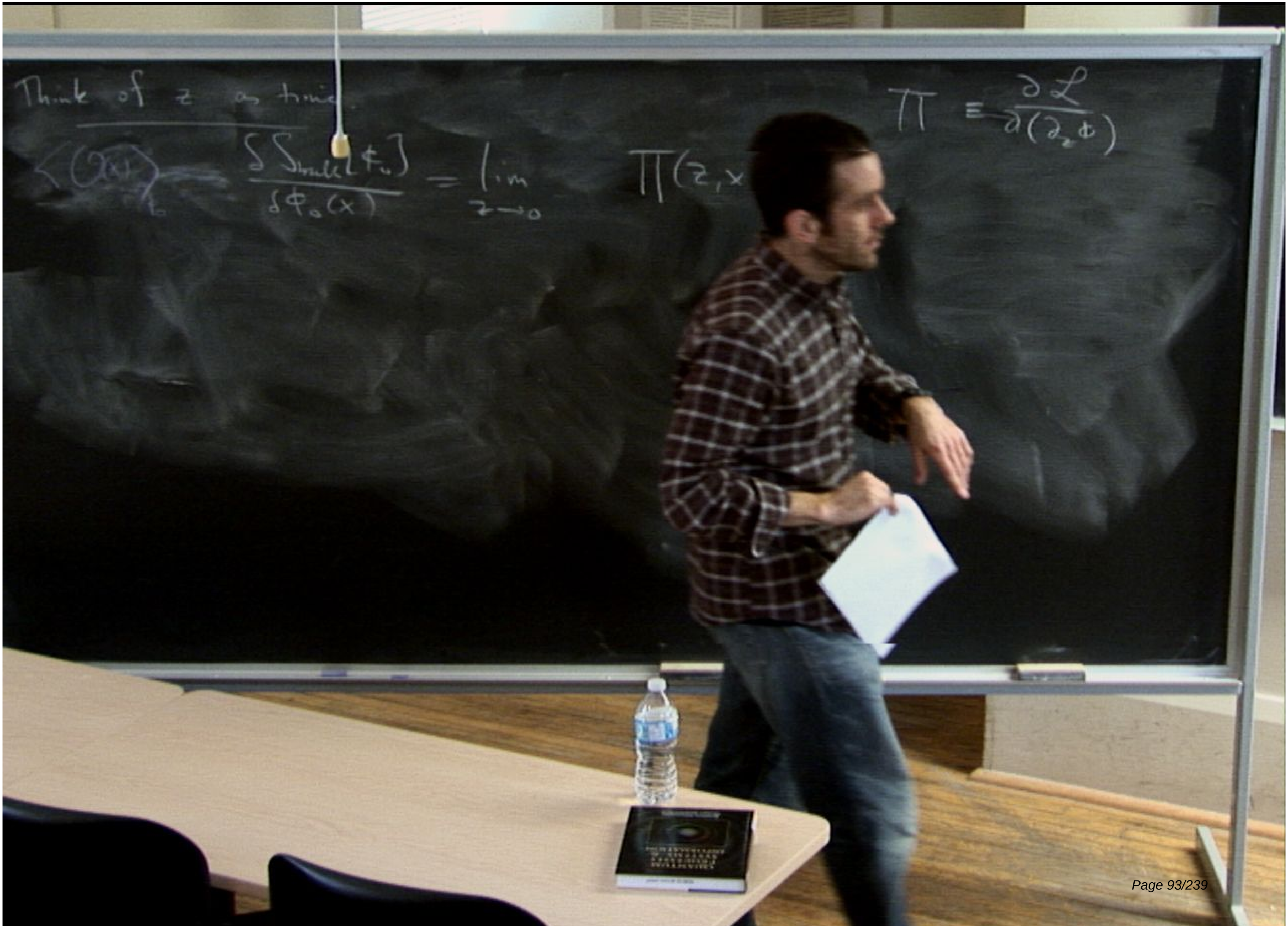


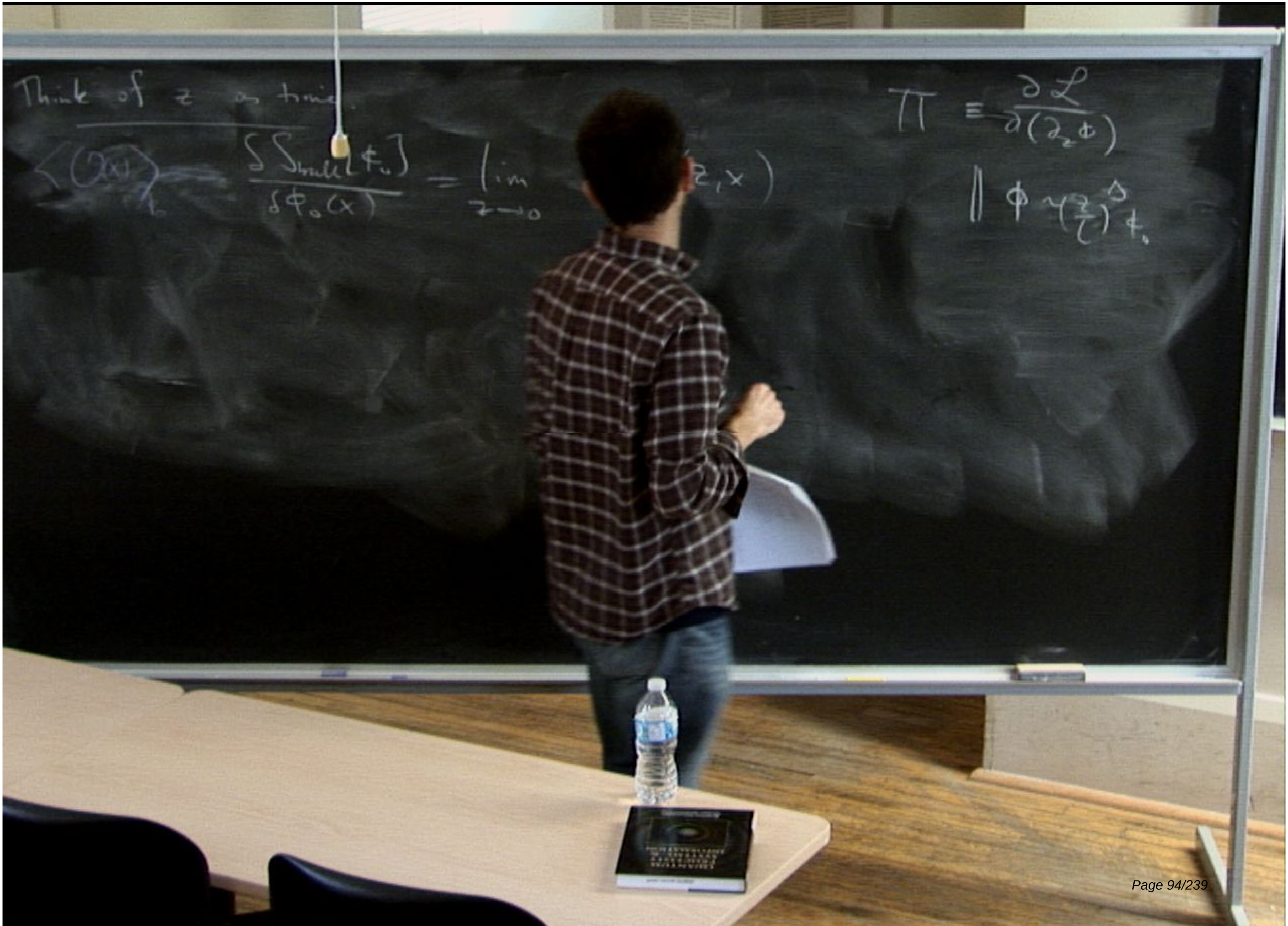

Think of z as time.
δS_bulk[φ_0] / δφ_0(x)

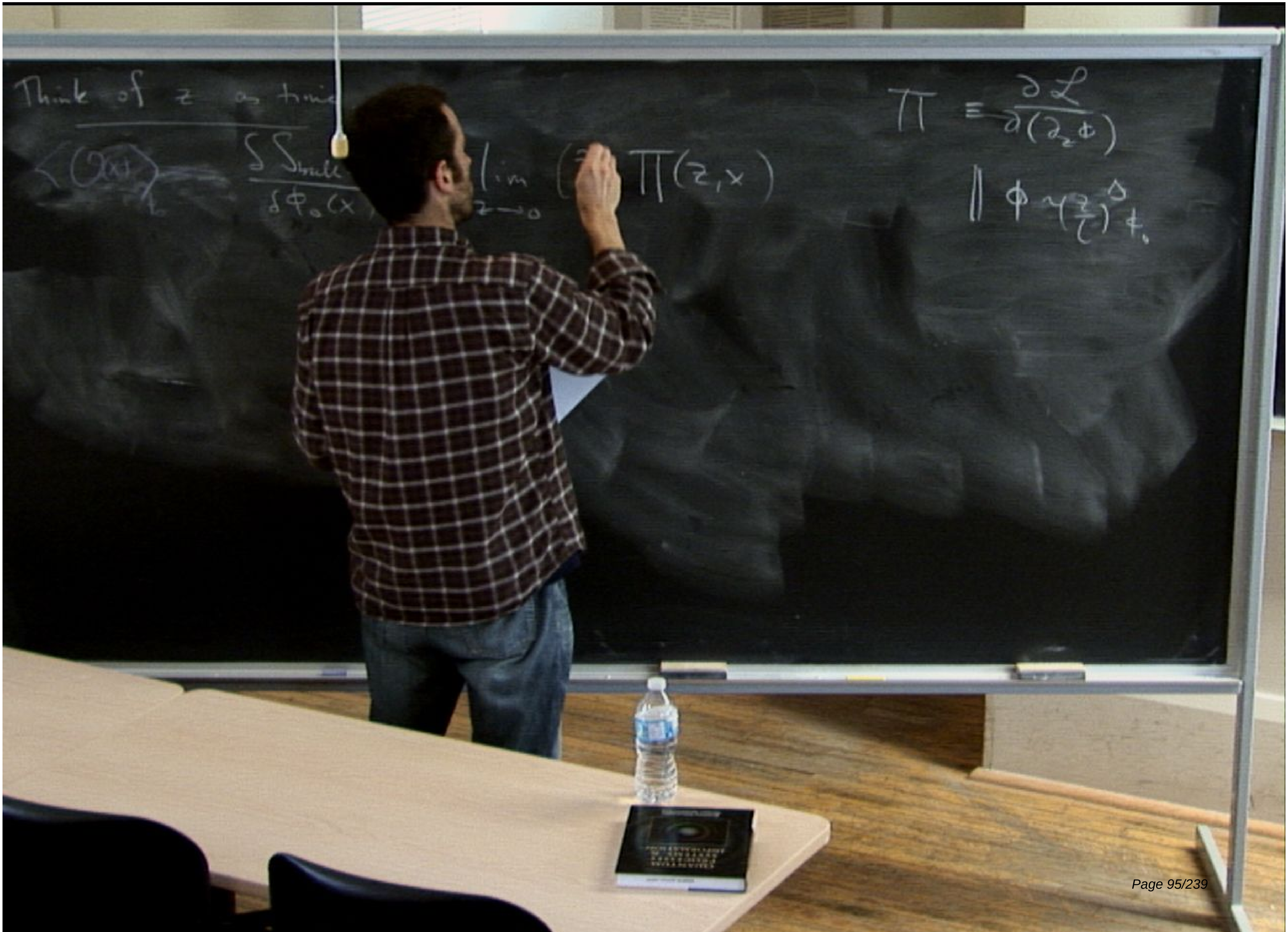

Think of z as time.

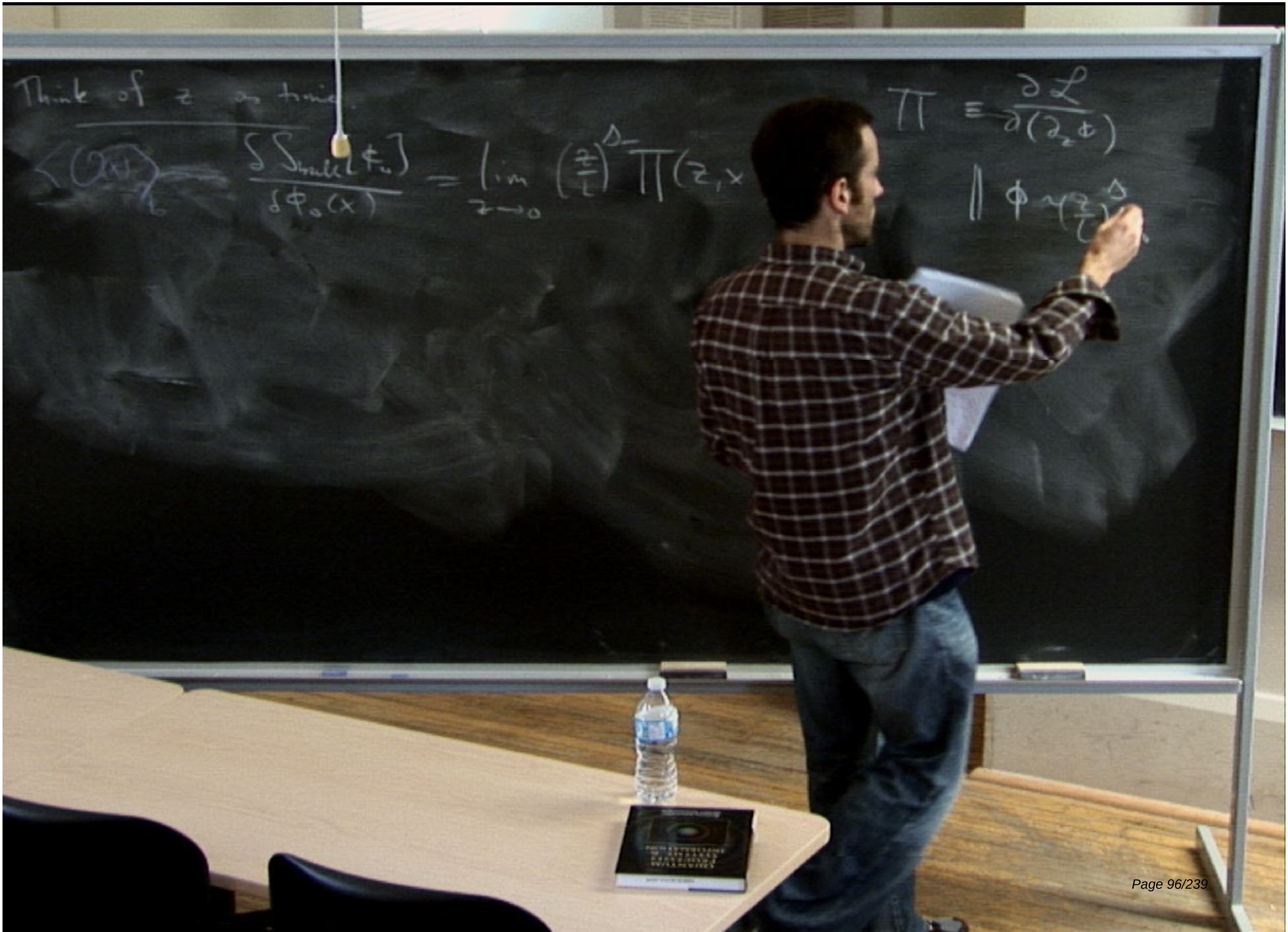

Think of z as time.
⟨O(x)⟩
δS_bulk[φ0] / δφ0(x) = lim z→0

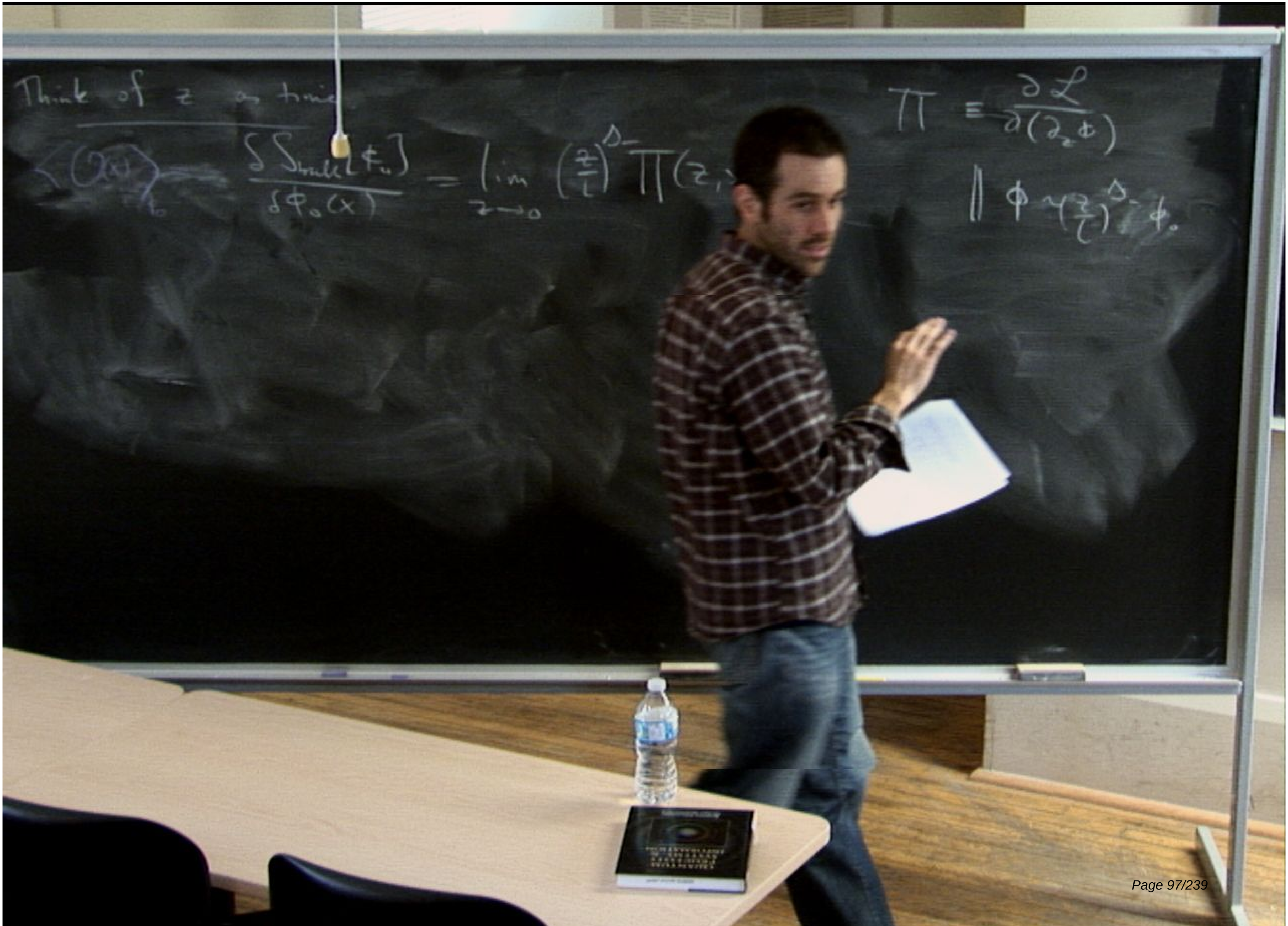

Think of z as time.
δS_bulk[φ_0] / δφ_0(x) = lim z→0
Π(z, x)
Π ≡ ∂L

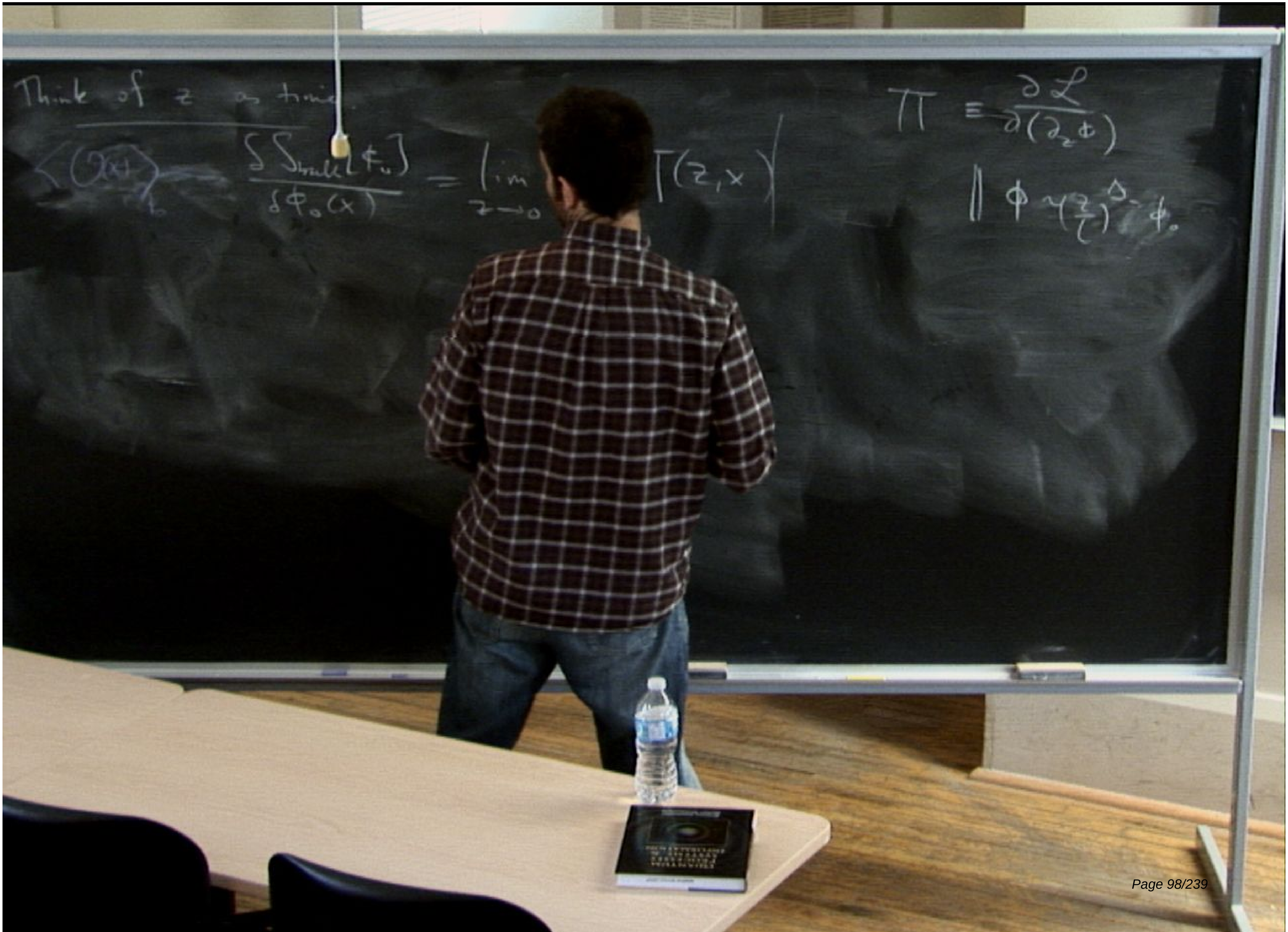

Think of z as
Π ≡ ∂ℒ / ∂(∂_z φ)
= lim z→0
Π(z, x

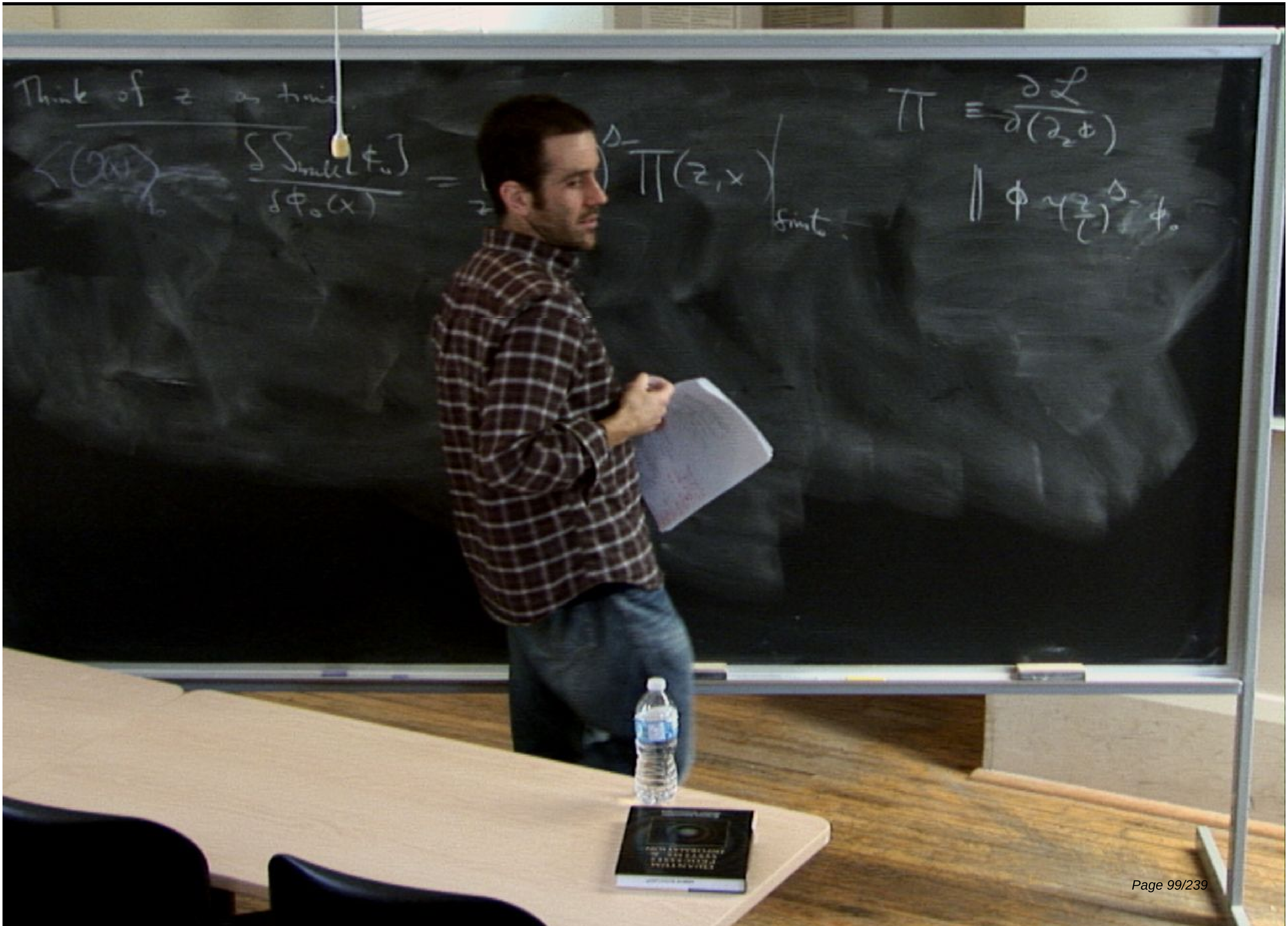

Think of z as time.

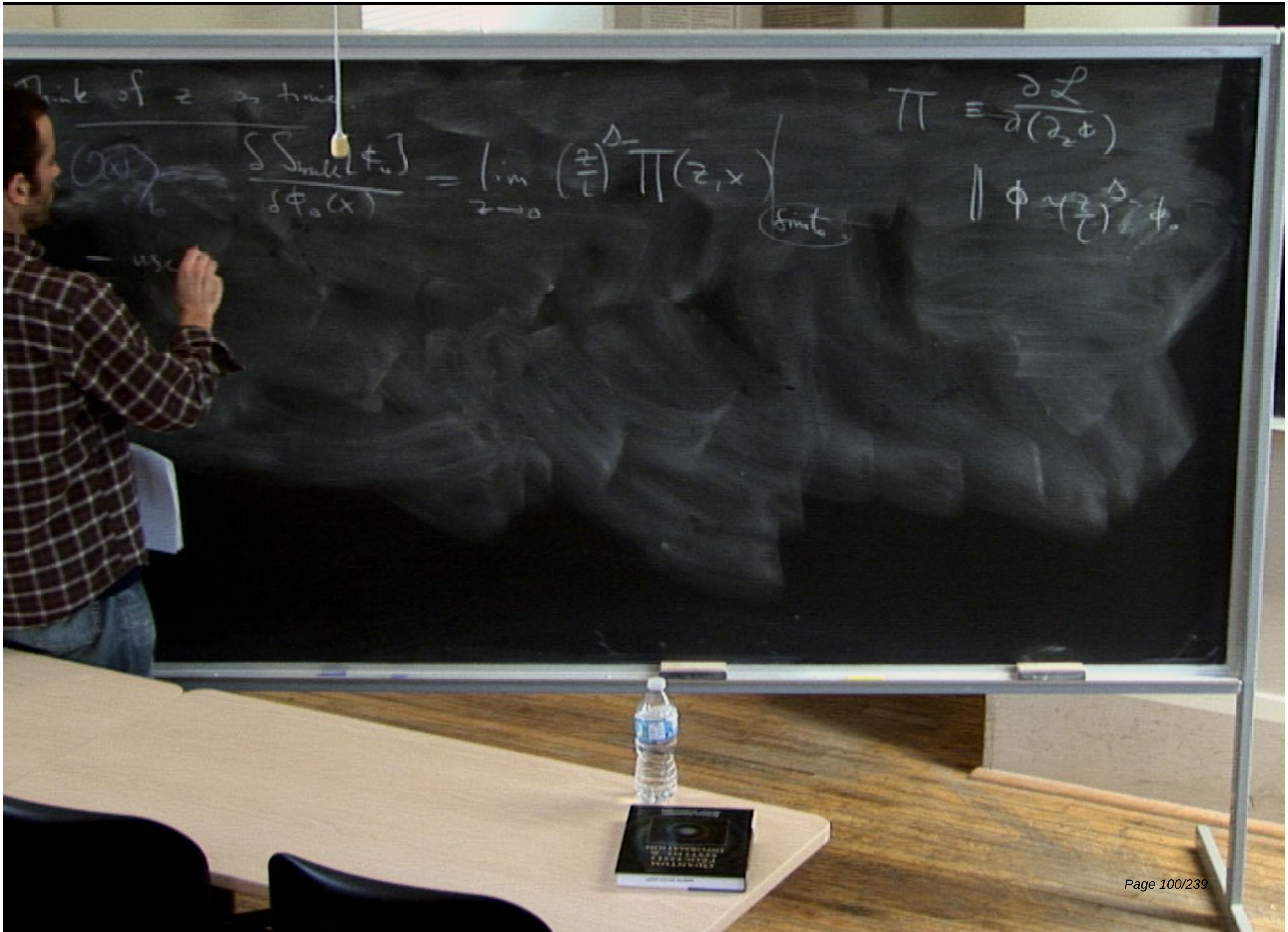

Think of z as tim
Π ≡ ∂L / ∂(∂z φ)
lim
Π(z, x)

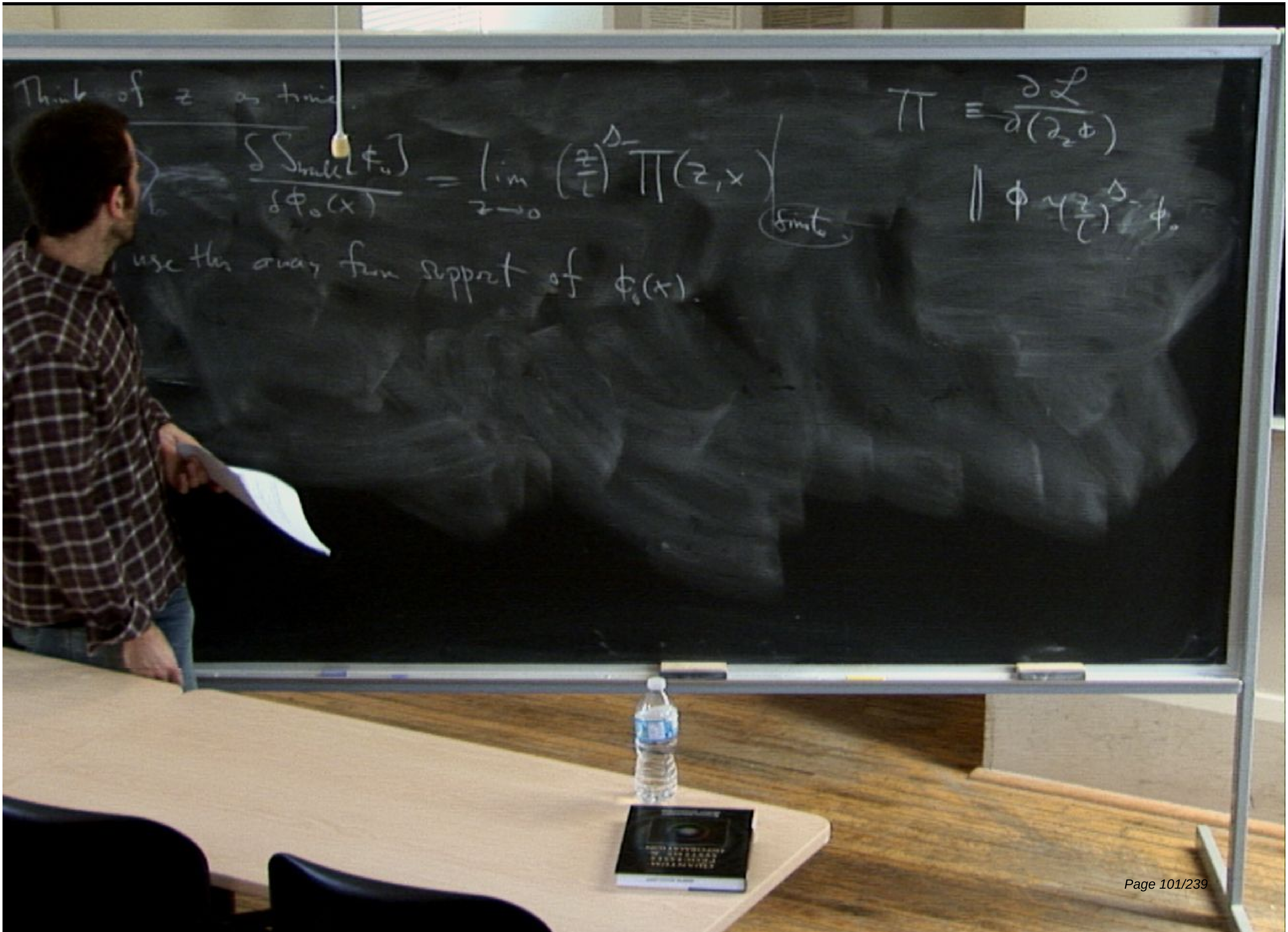

Think of z as time.

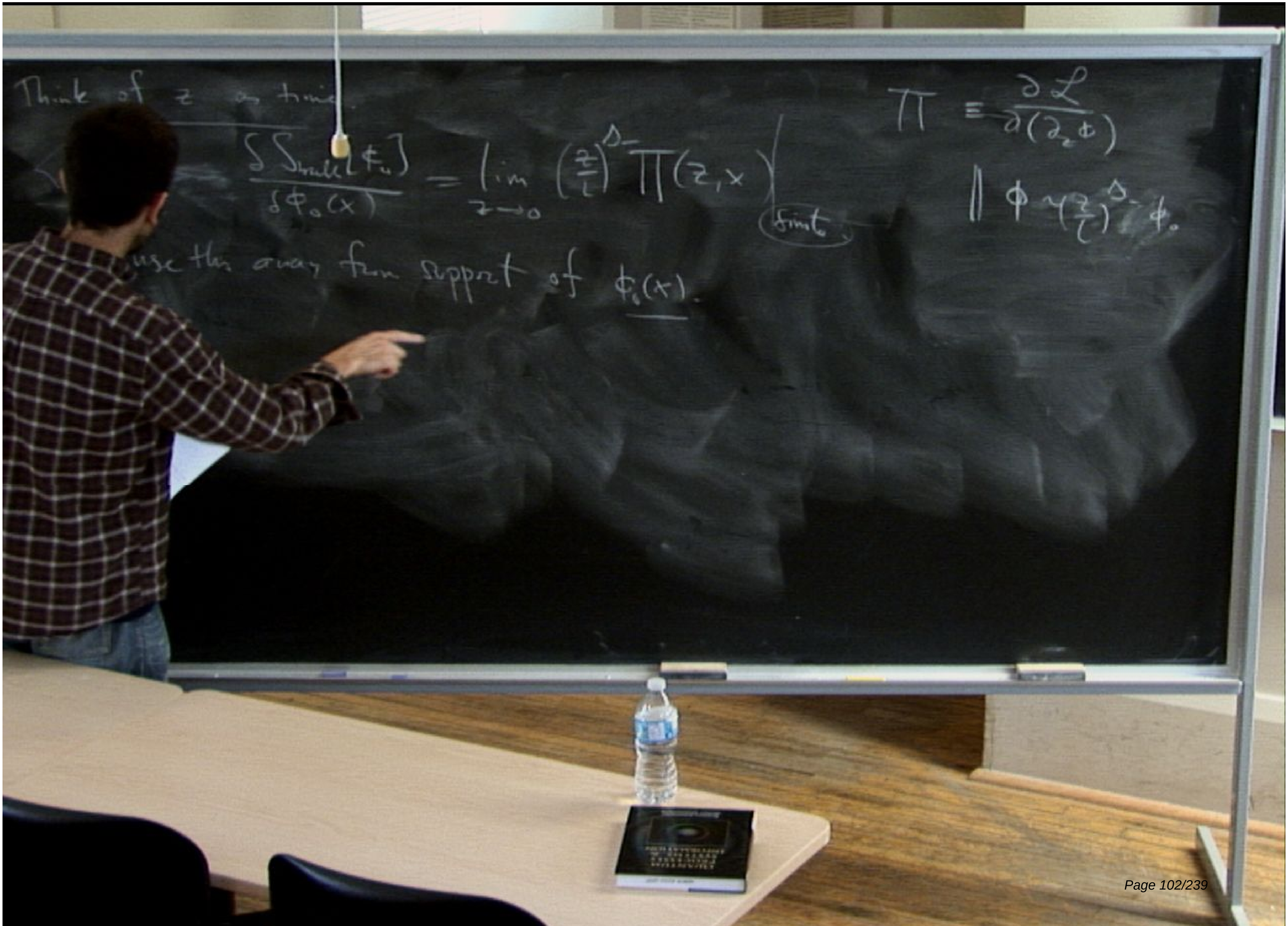

Think of z as time

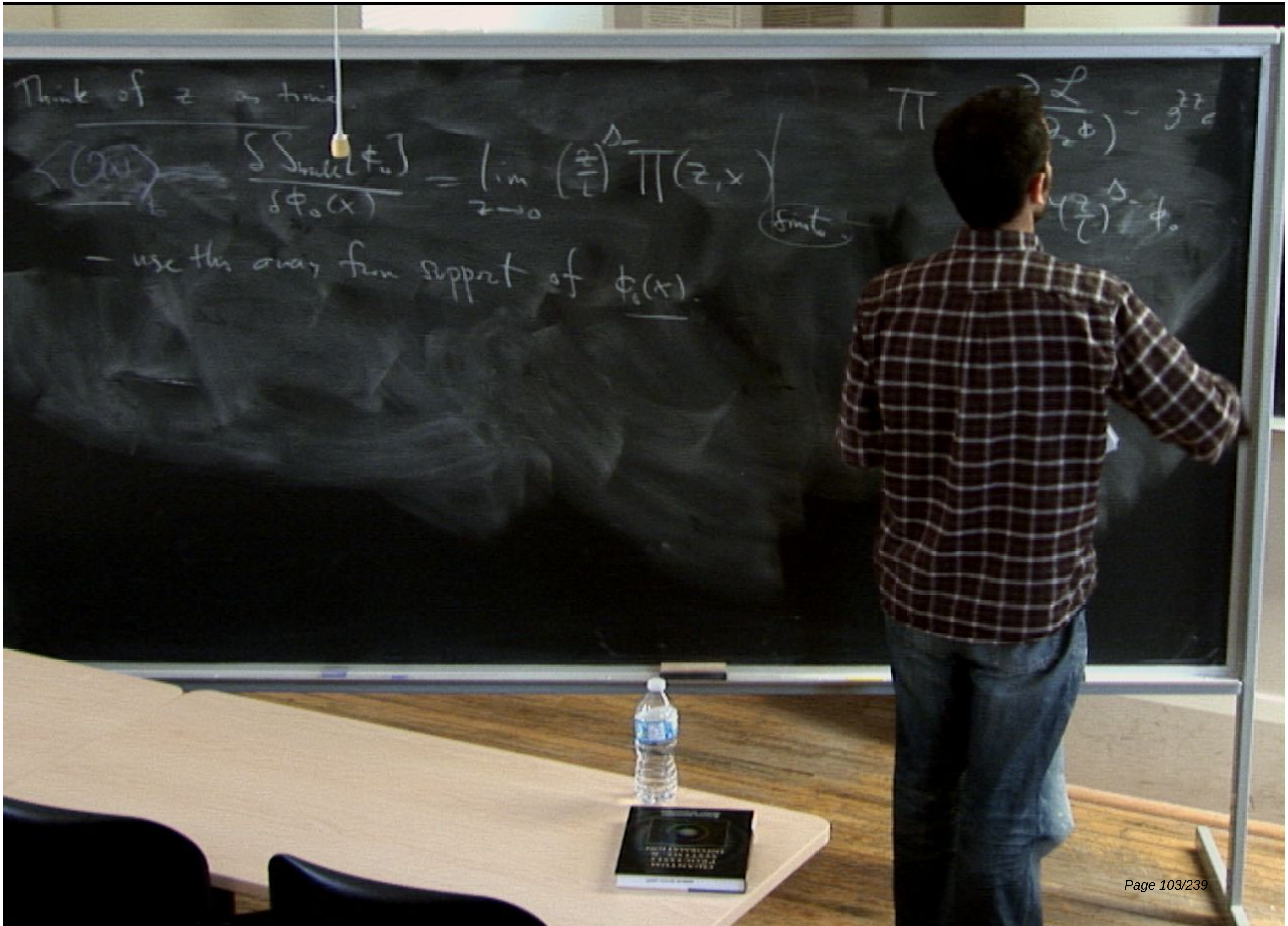

Think of z as time

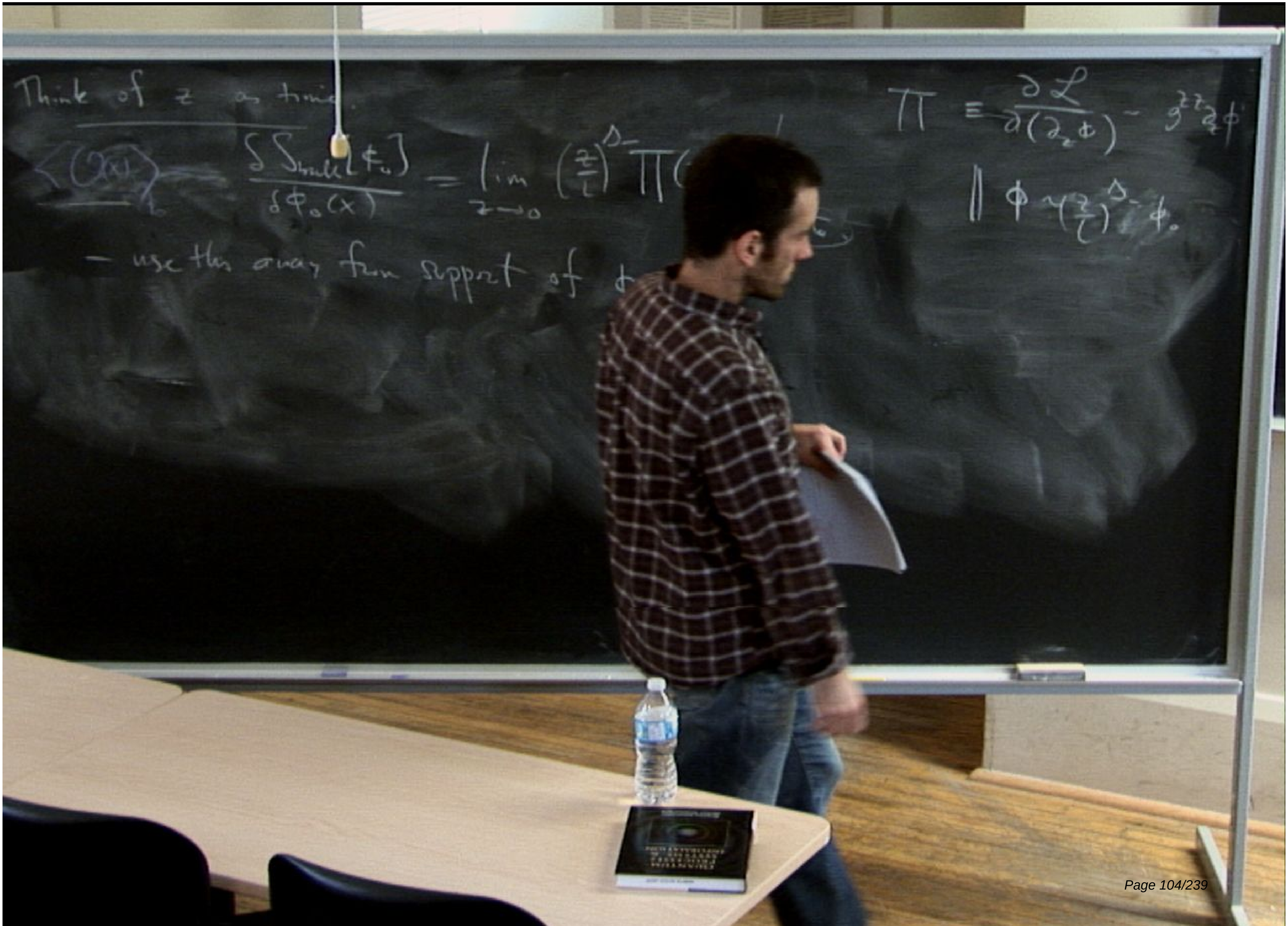

Think of z as time

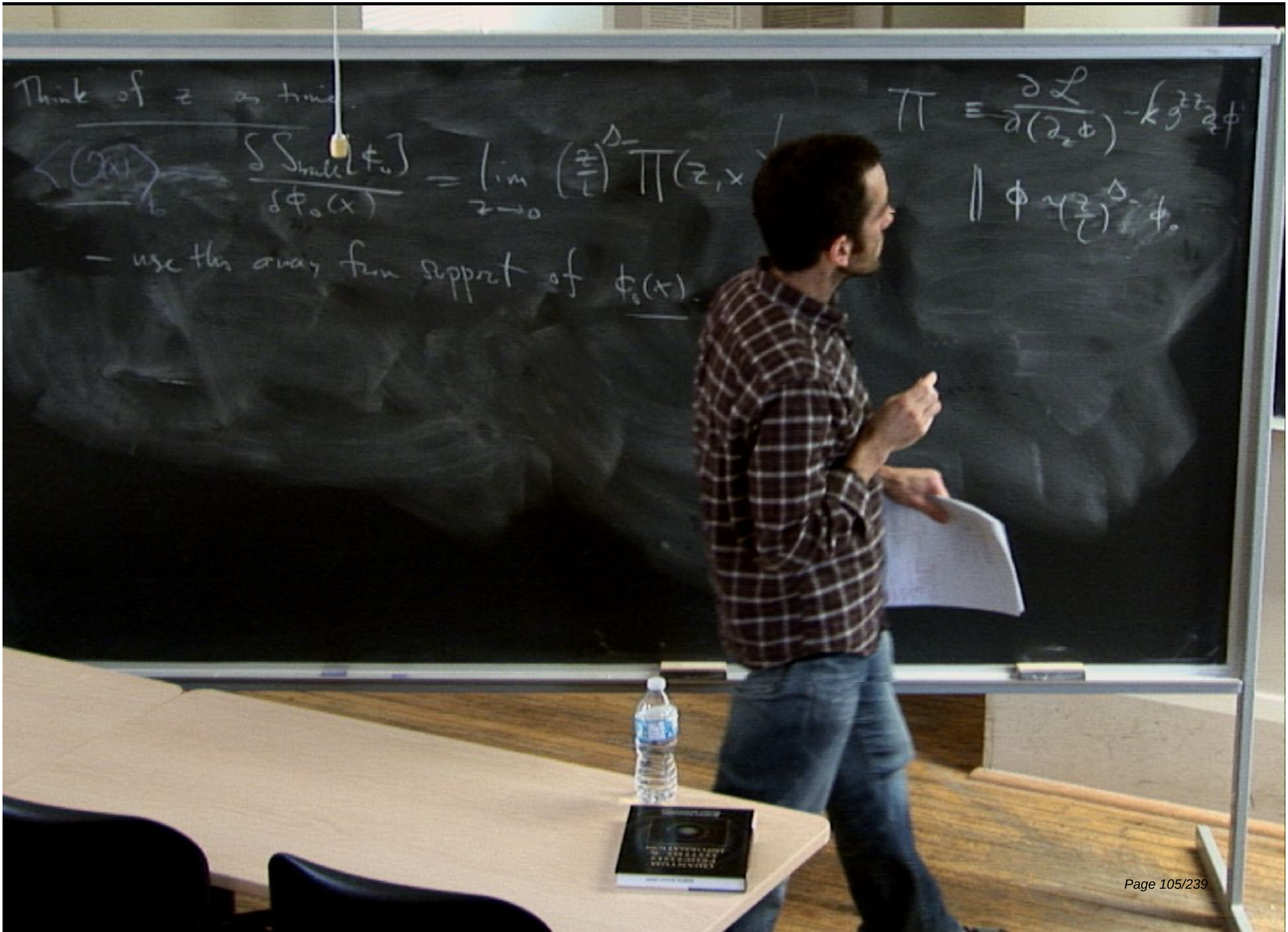


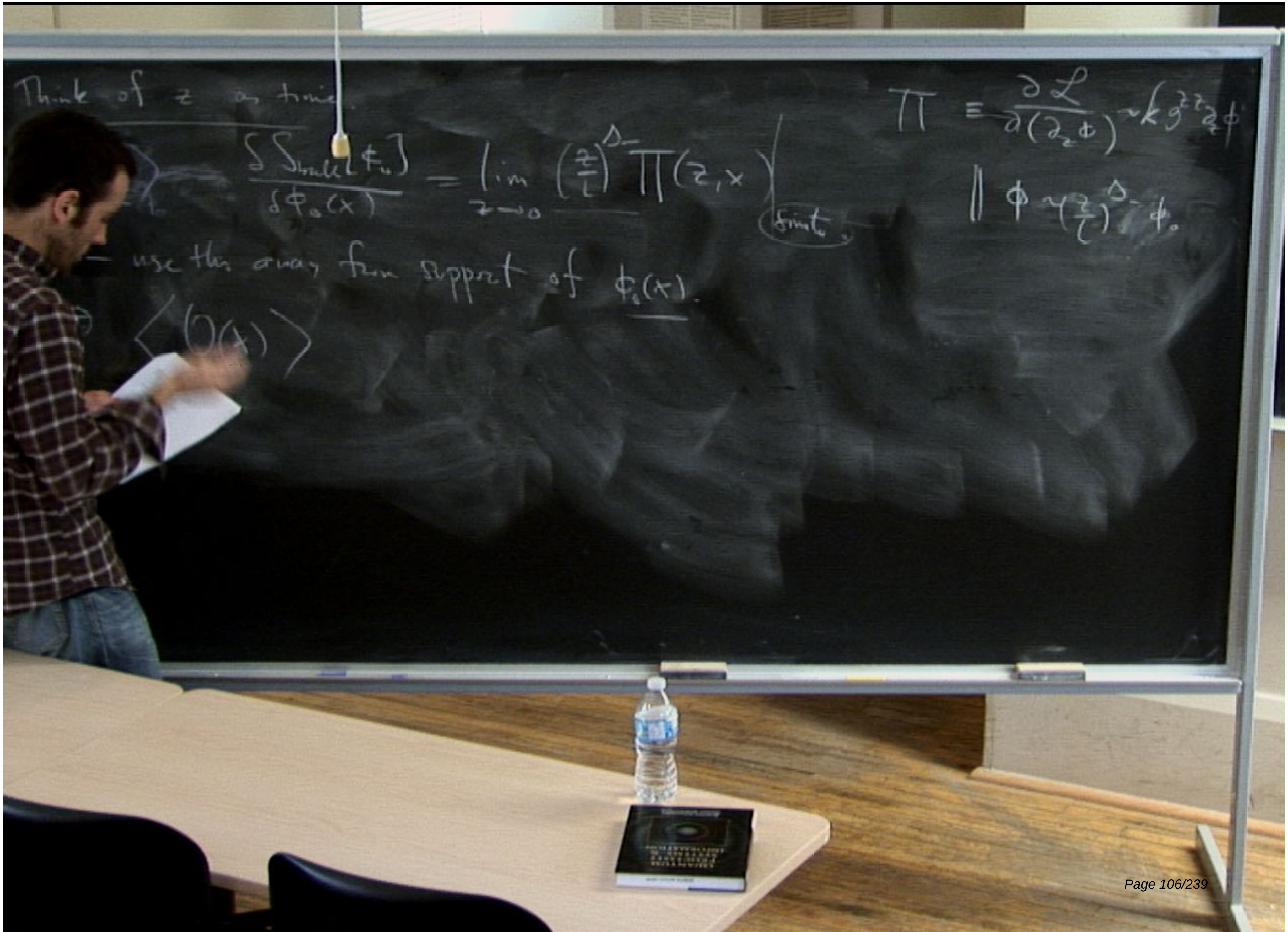

Think of z as time.
$\frac{\delta S_{bulk}[\phi_0]}{\delta \phi_0(x)} = \lim_{z \to 0} \left(\frac{z}{l}\right)^{\Delta_-} \Pi(z, x) \Big|_{finite}$
$\Pi \equiv \frac{\partial \mathcal{L}}{\partial(\partial_z \phi)}$
$\phi \sim \left(\frac{z}{l}\right)^{\Delta_-} \phi_0$
use this away from support of $\phi_0(x)$.

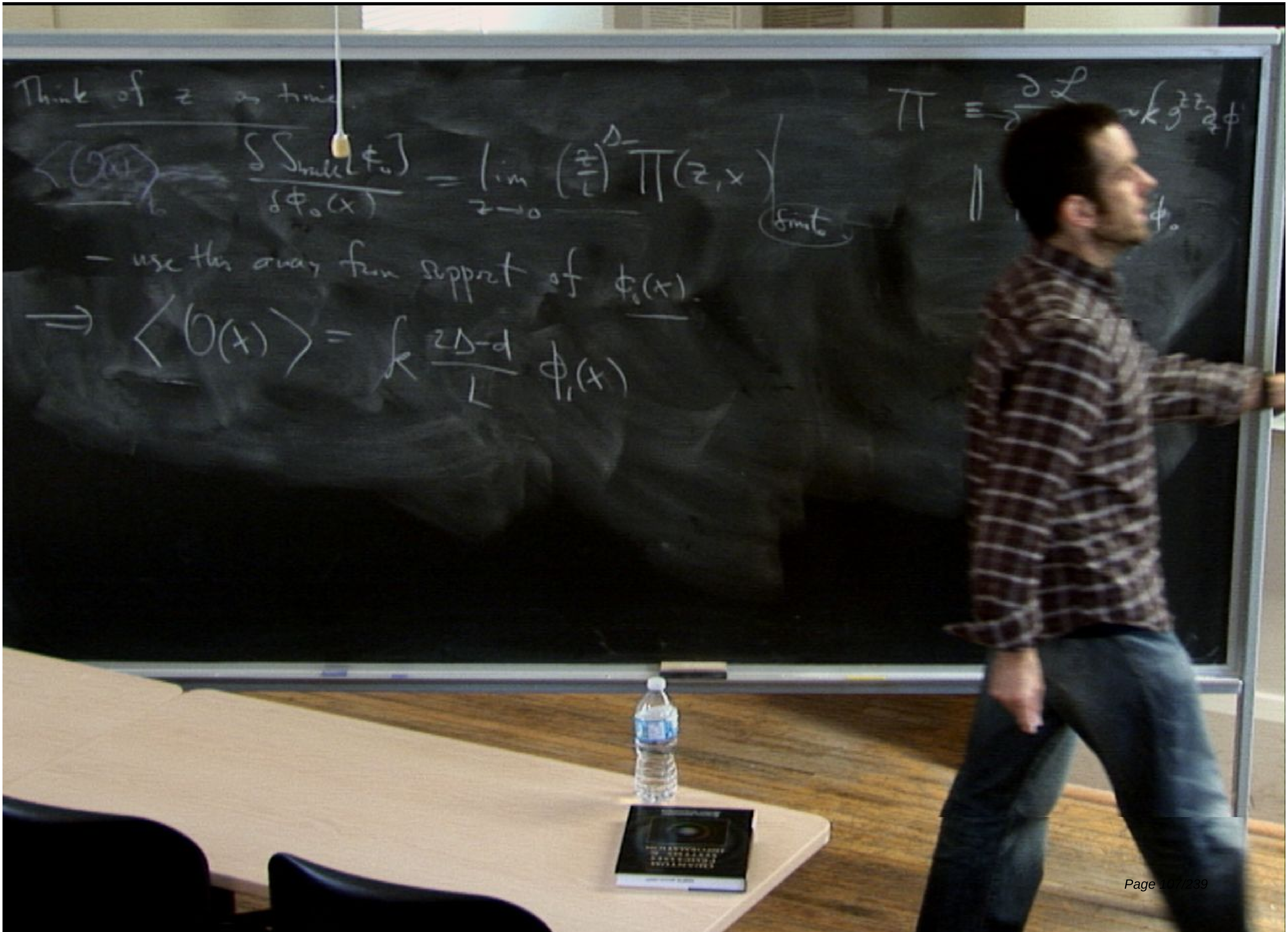

Think of z as time.
δS_bulk[φ₀] / δφ₀(x) = lim z→0 (z/L)^Δ₋ Π(z,x) | finite
Π ≡ ∂L / ∂(∂_z φ)
|| φ ~ (z/L)^Δ₋ φ₀
use this away from support of φ₀(x)

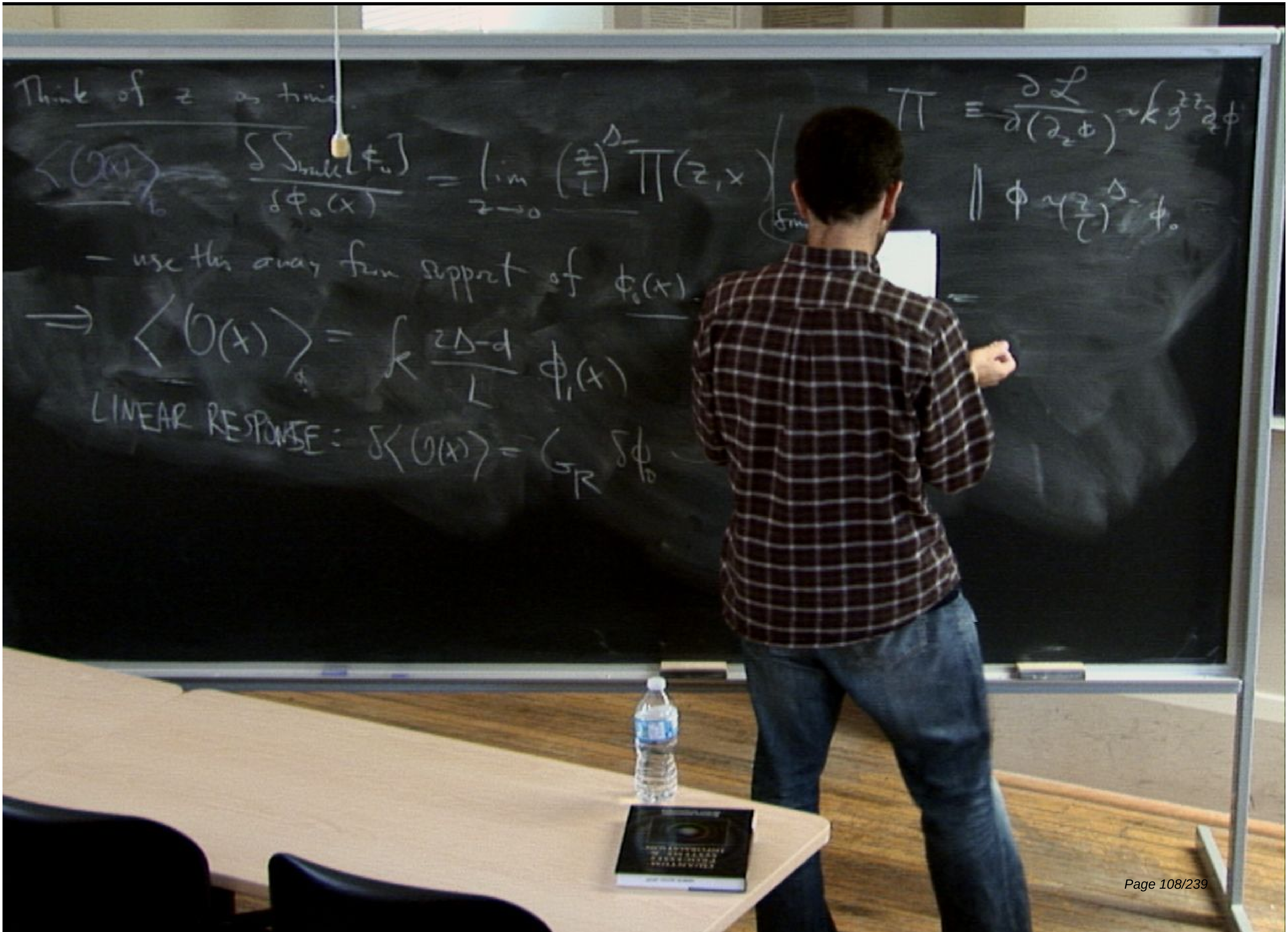

Think of z as time.
finite
– use this away from support of φ₀(x)

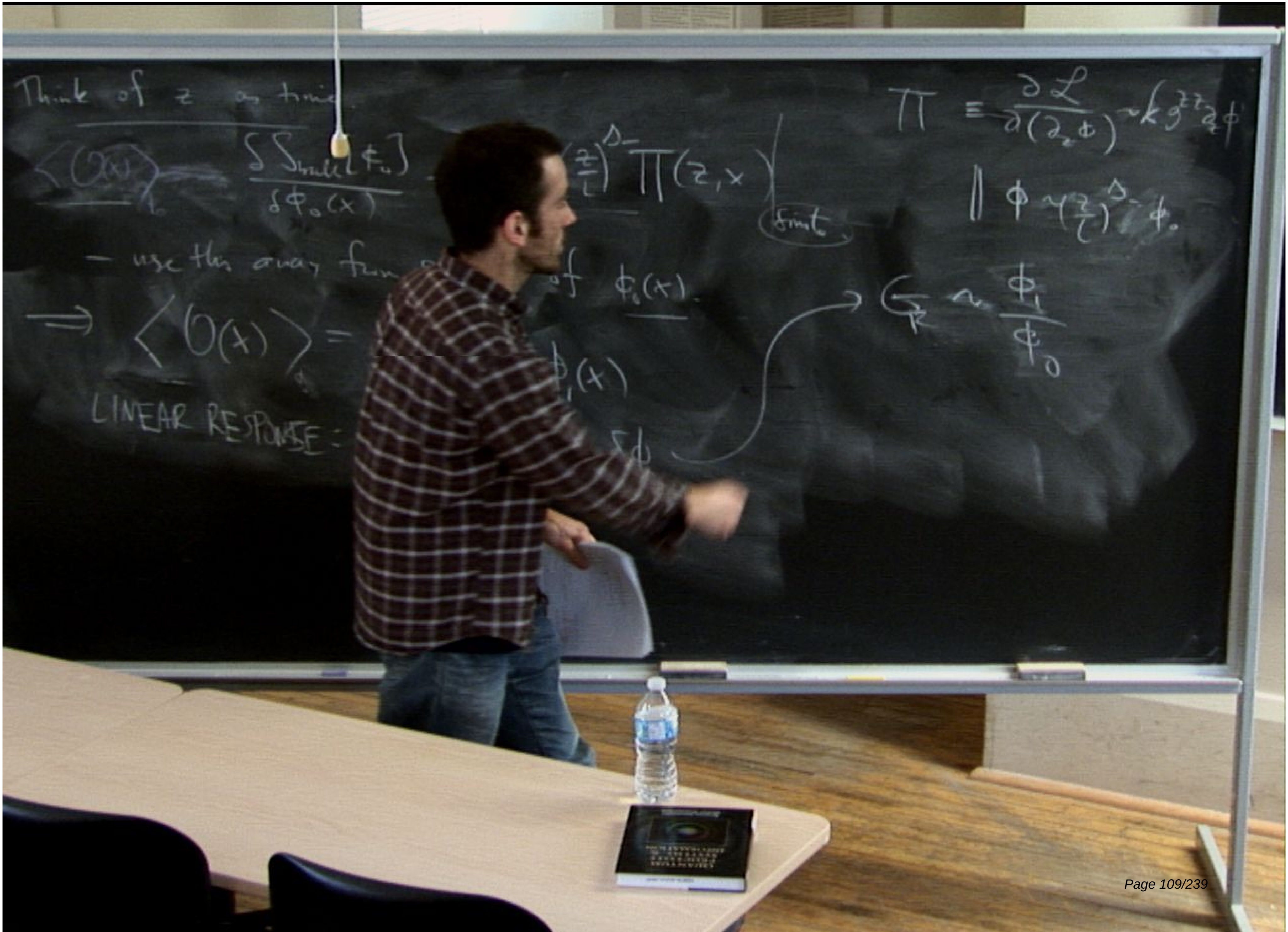

Think of z as time.
– use this away from support of φ

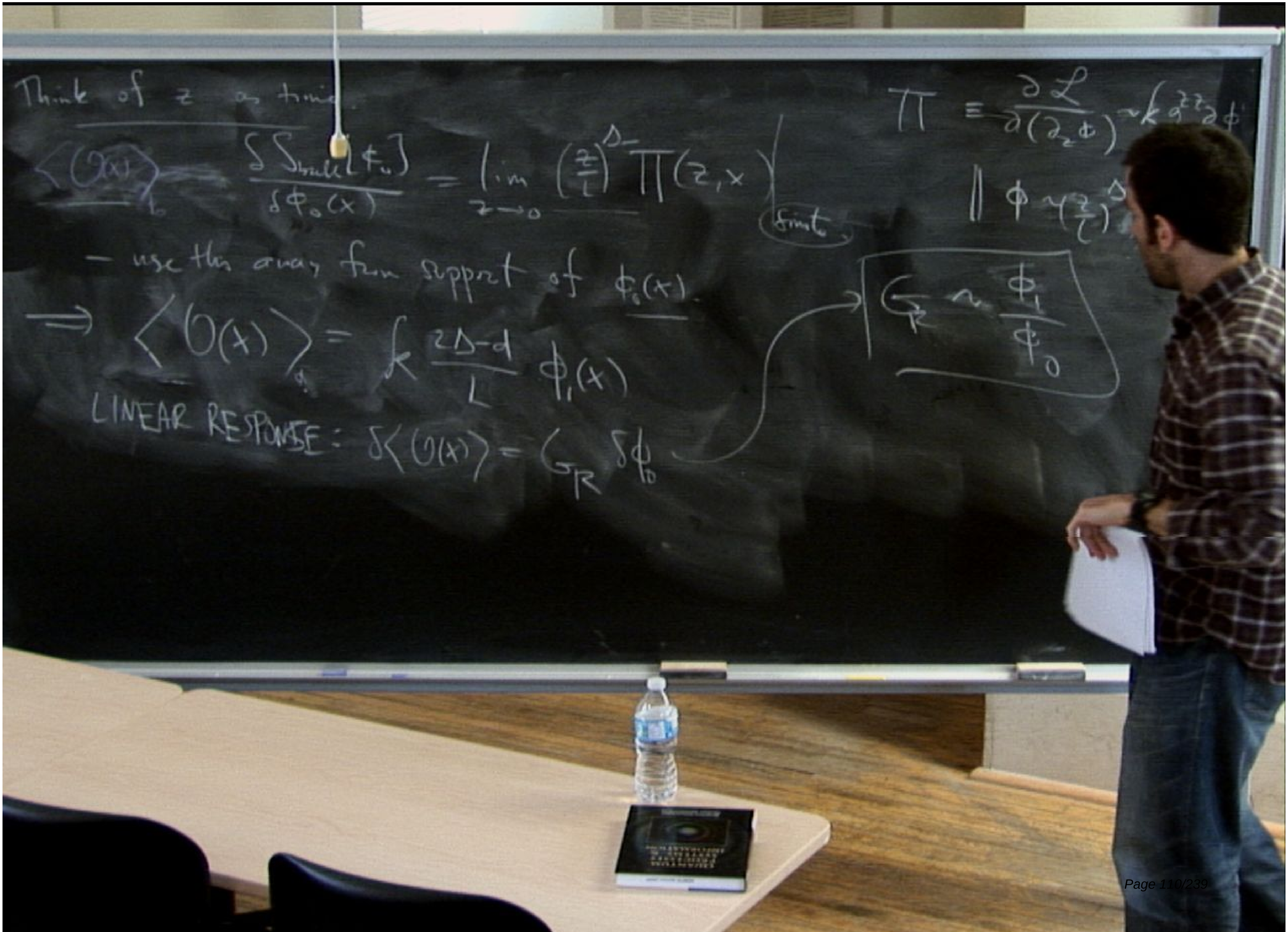

Think of z as time.
– use this away from support of φ₀(x)

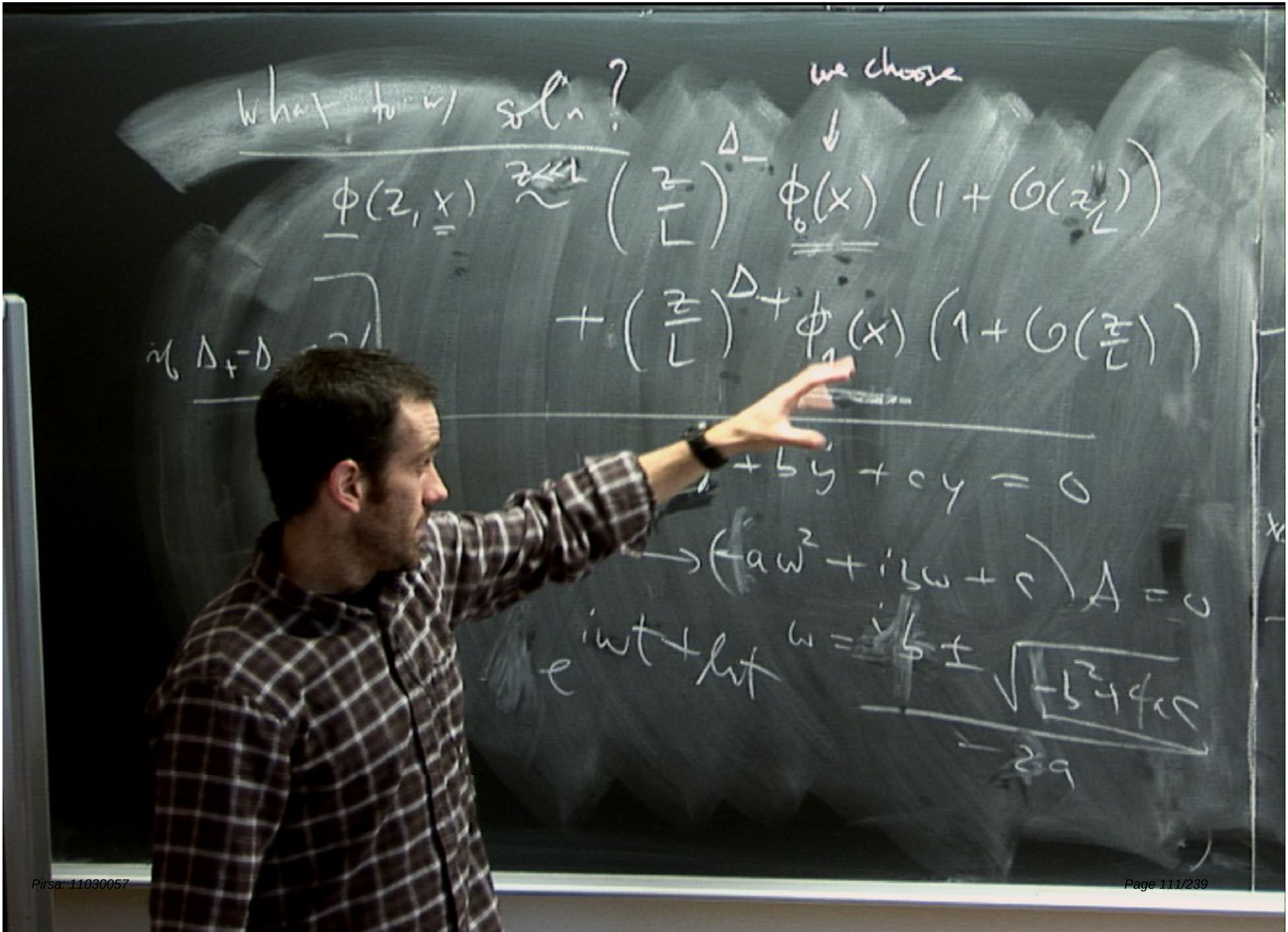

Think of z as time
finite
use this away from support of

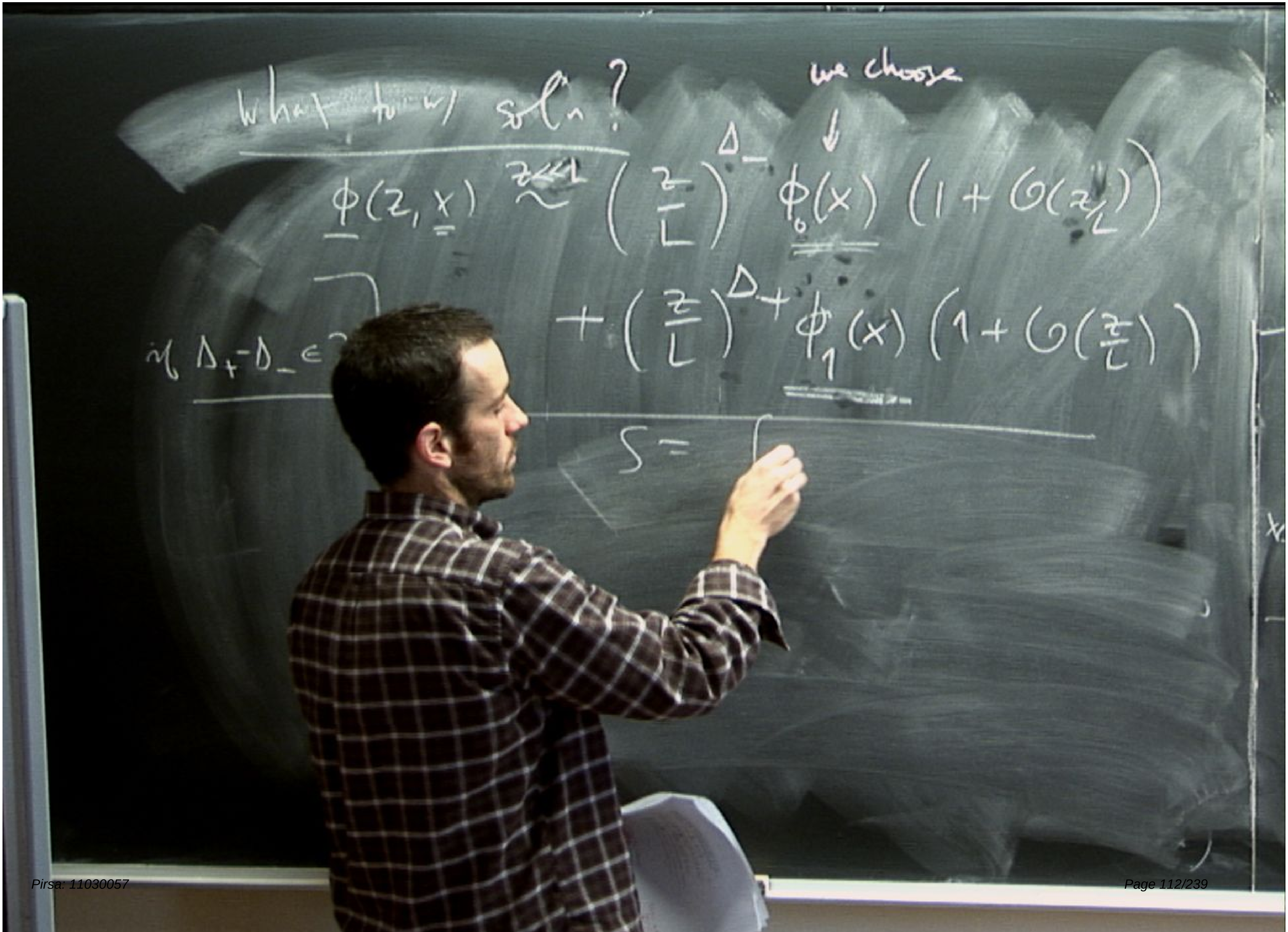

Think of z as time.
– use this away from support of

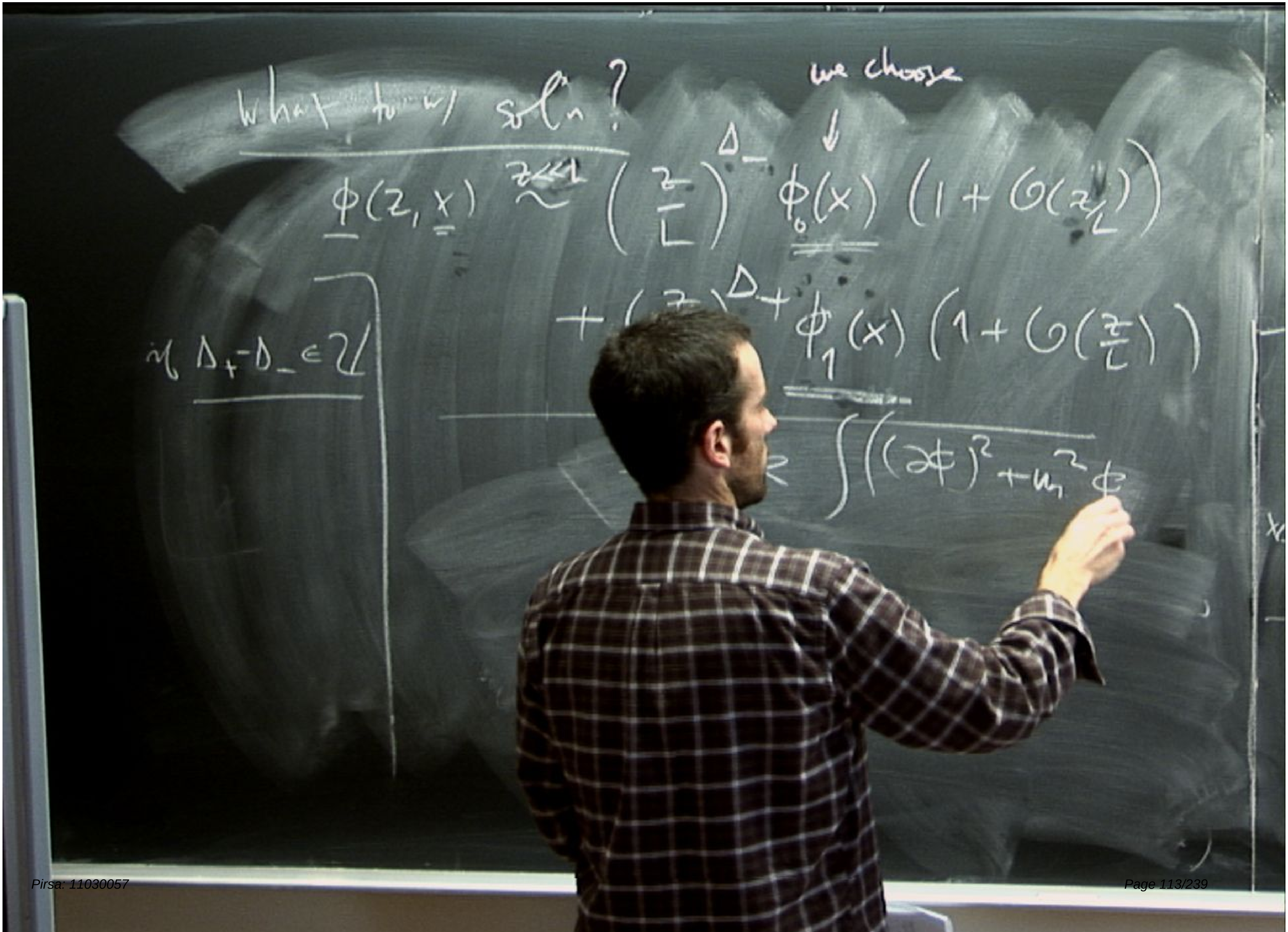

Think of z as time
– use this away from support of φ₁(x)
LINEAR RESPONSE:

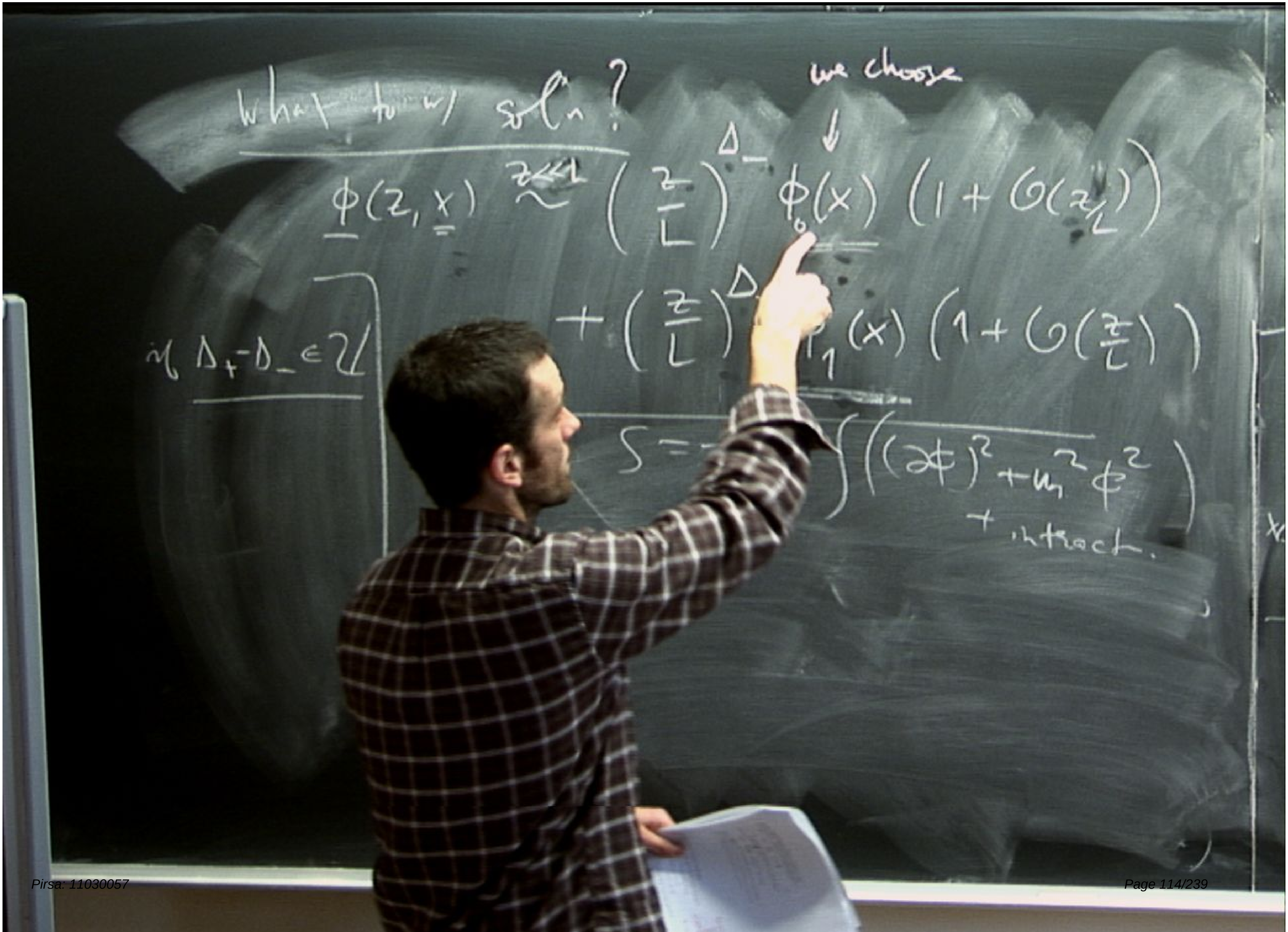

Think of z as time
– use this away from
LINEAR RESPONSE:


Think of z as time.
– use this away from support of
LINEAR RESPONSE:
finite

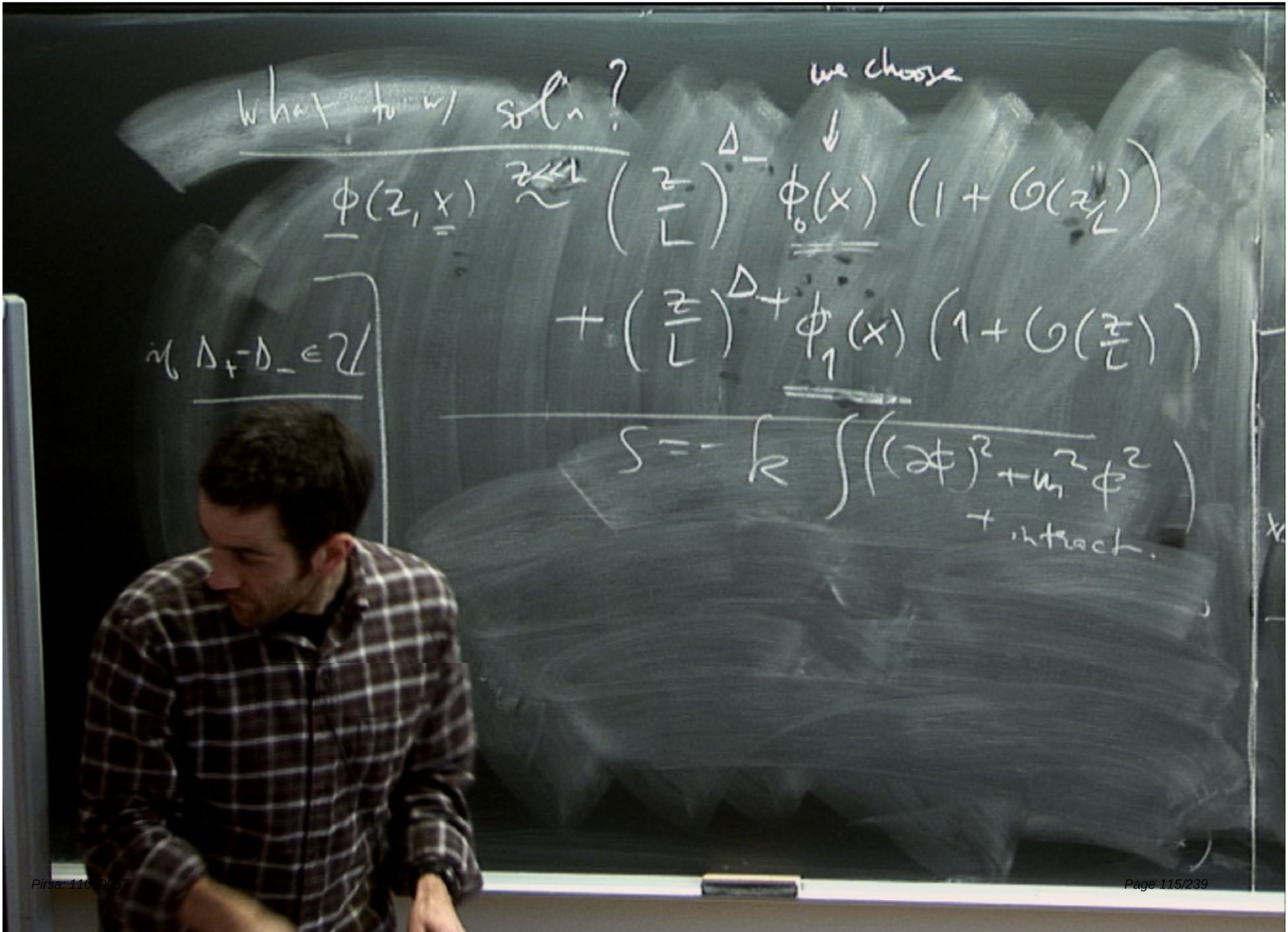

what to w/ soln ?
we choose

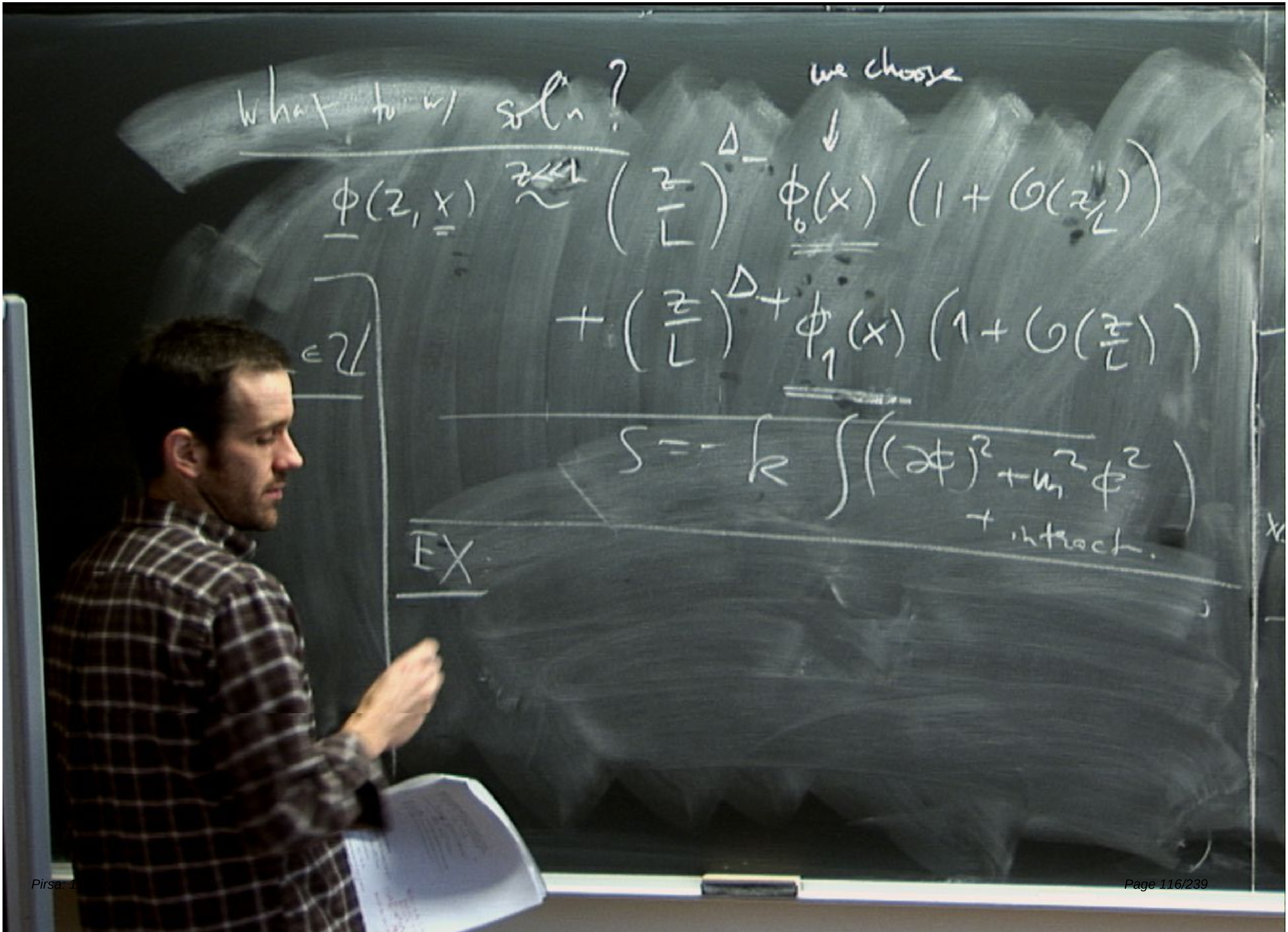

we choose
z<<L
φ(z, x) ~ (z/L)^Δ- φ0(x) (1 + O(z/L))
+ (z/L)^Δ+ φ1(x) (1 + O(z/L))
S =

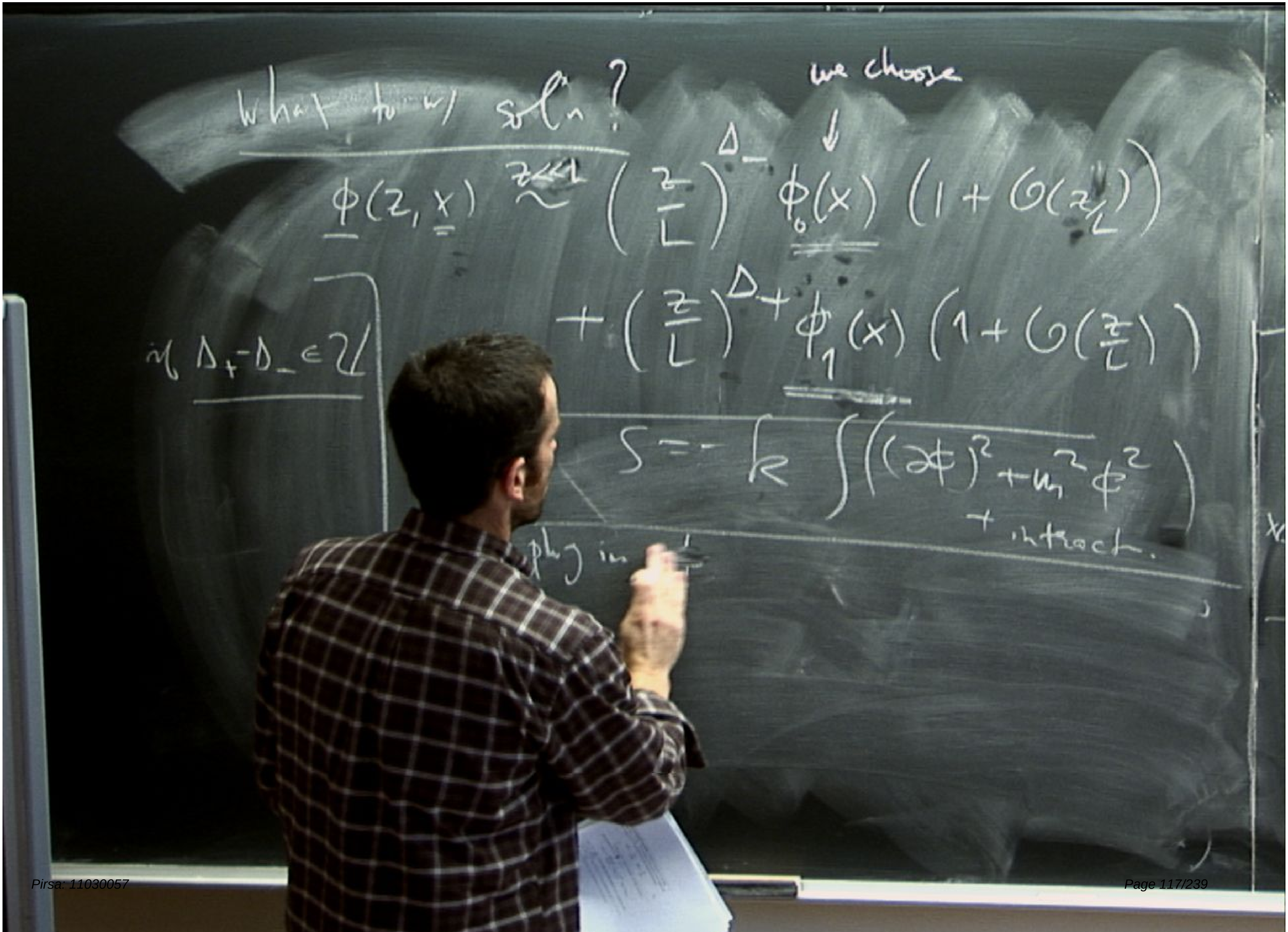

what to w/ soln ?
we choose

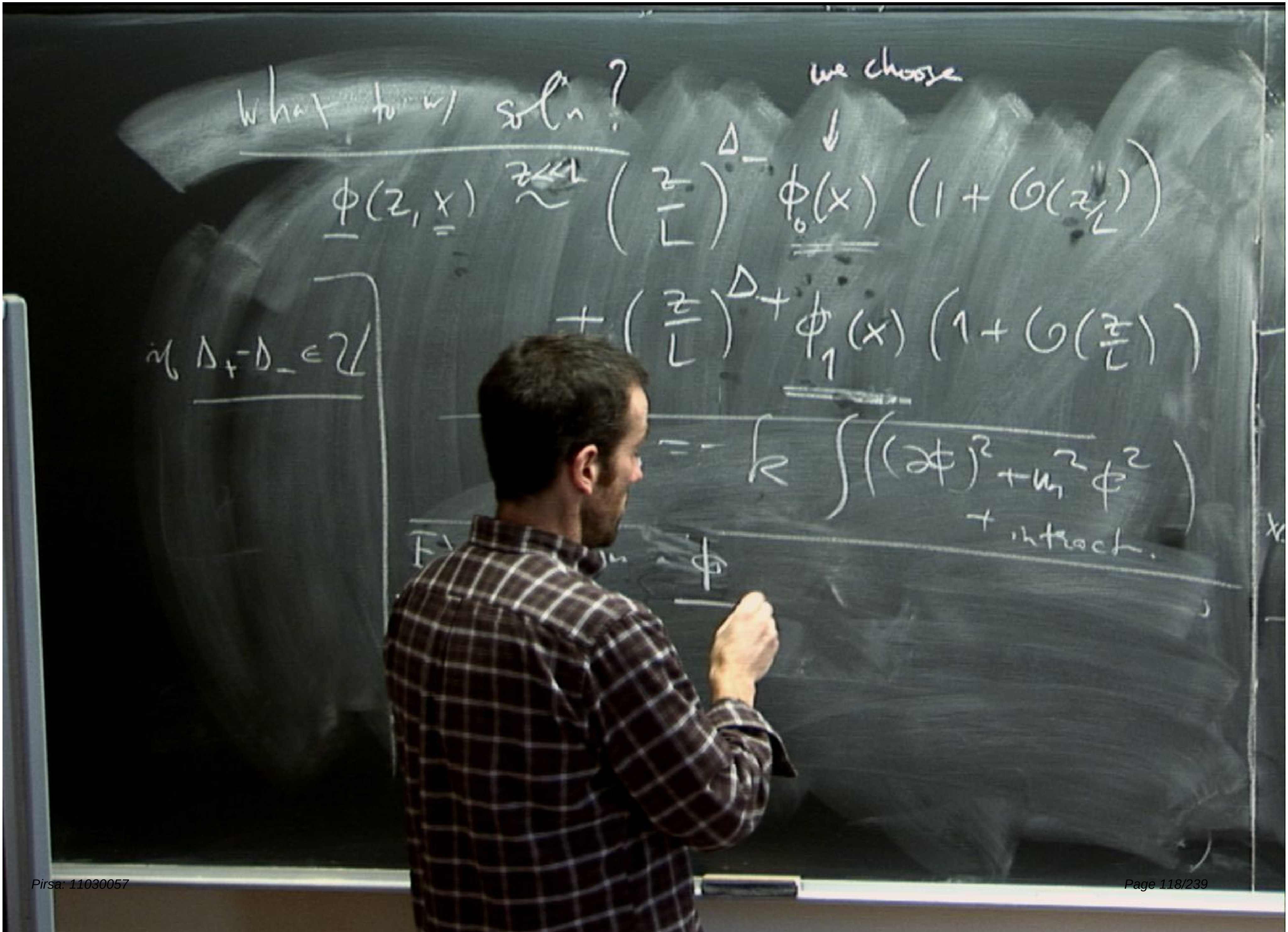


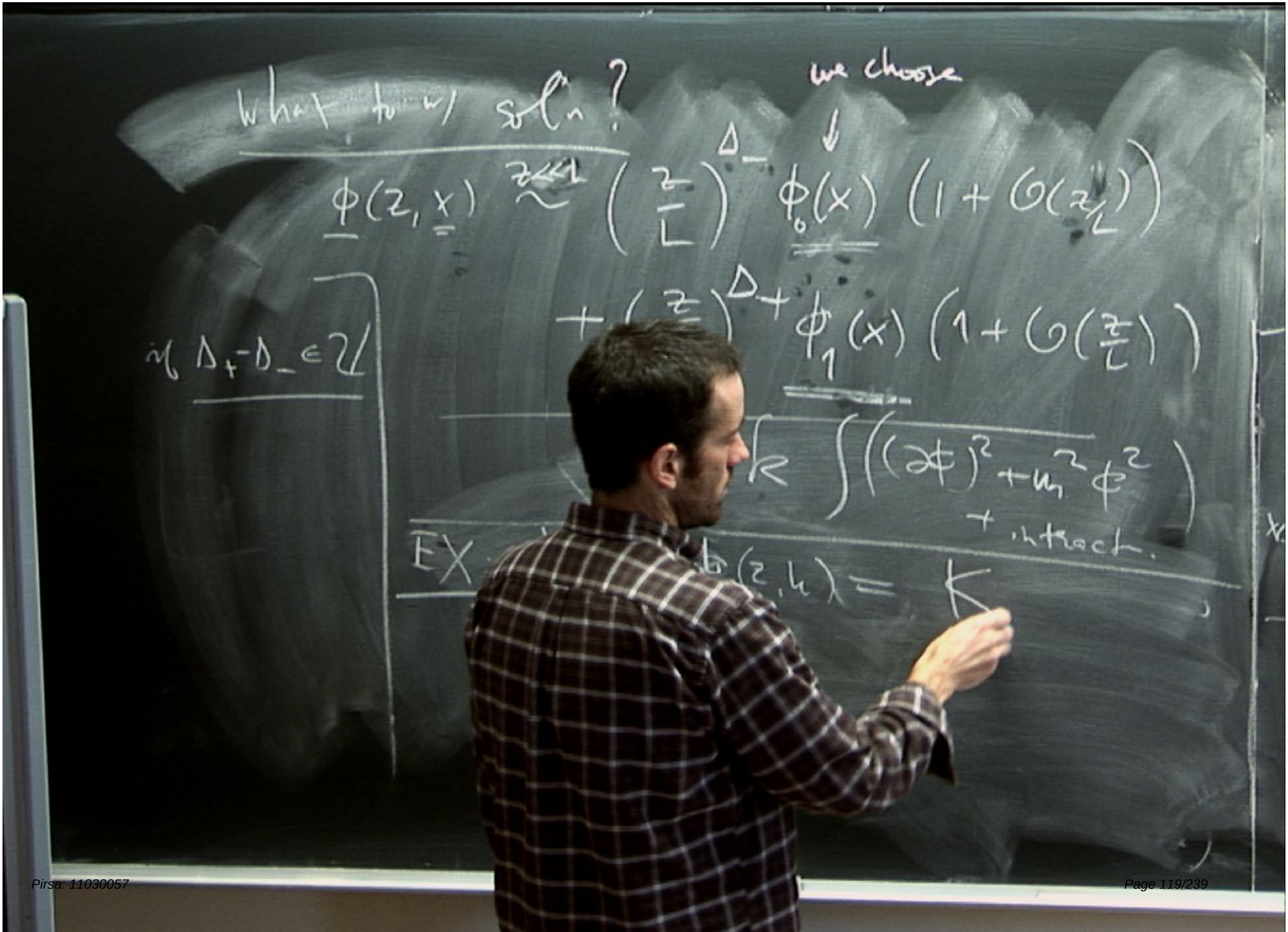
what to w/ soln ?
we choose
φ(z, x) z≪1 ~ (z/L)^Δ₋ φ₀(x) (1 + O(z/L))
+ (z/L)^Δ₊ φ₁(x) (1 + O(z/L))
if Δ₊ − Δ₋ ∈ ℤ
S = ∫ ((∂φ)² + m² φ²
+ interact.

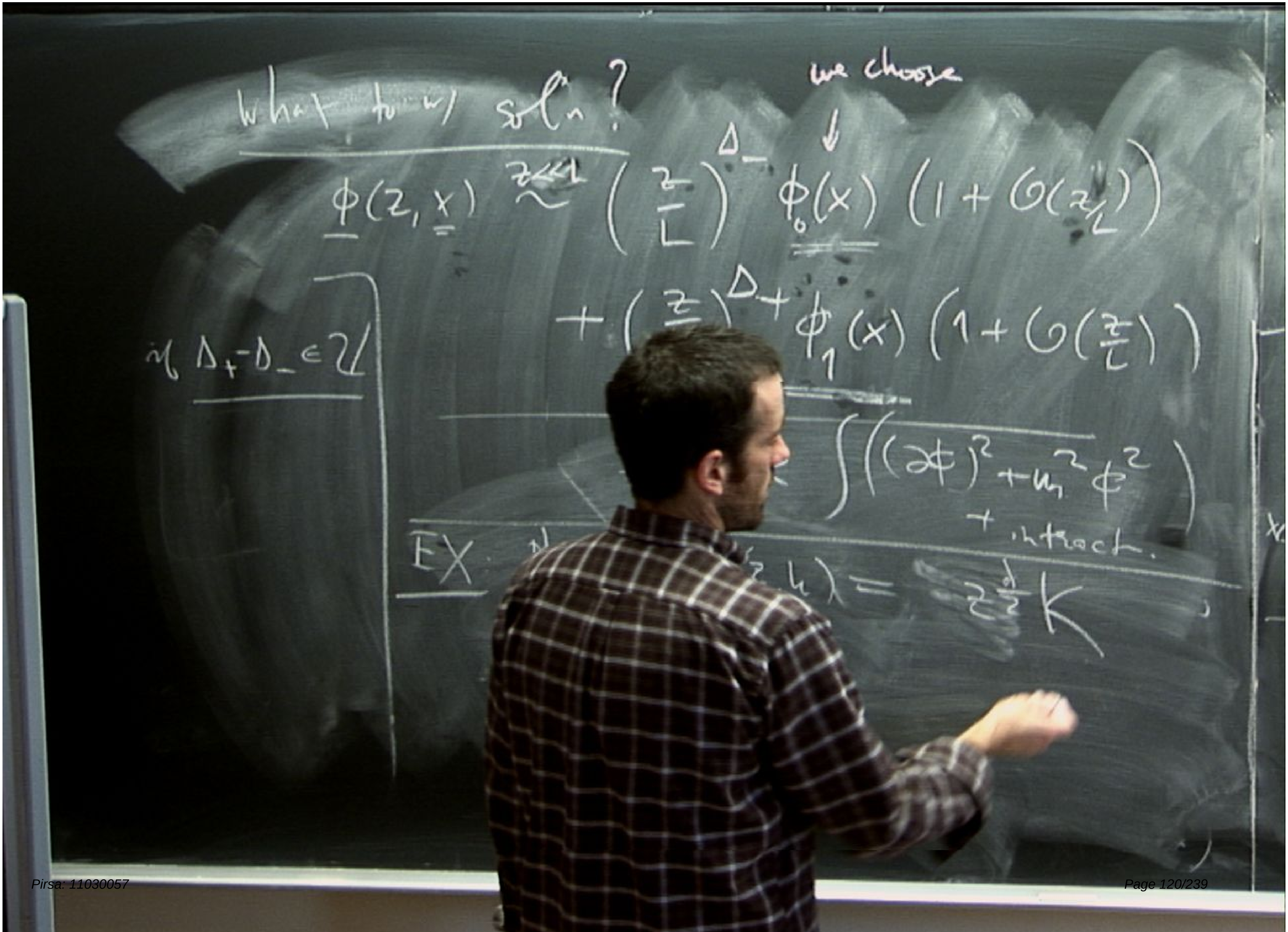

what to w/ soln ?
we choose
φ(z, x) ~ (z/L)^Δ- φ0(x) (1 + O(z/L))
+ (z/L)^Δ+ φ1(x) (1 + O(z/L))
if Δ+ − Δ− ∈ Z
S = −k ∫ ((∂φ)² + m² φ²
+ interact.)

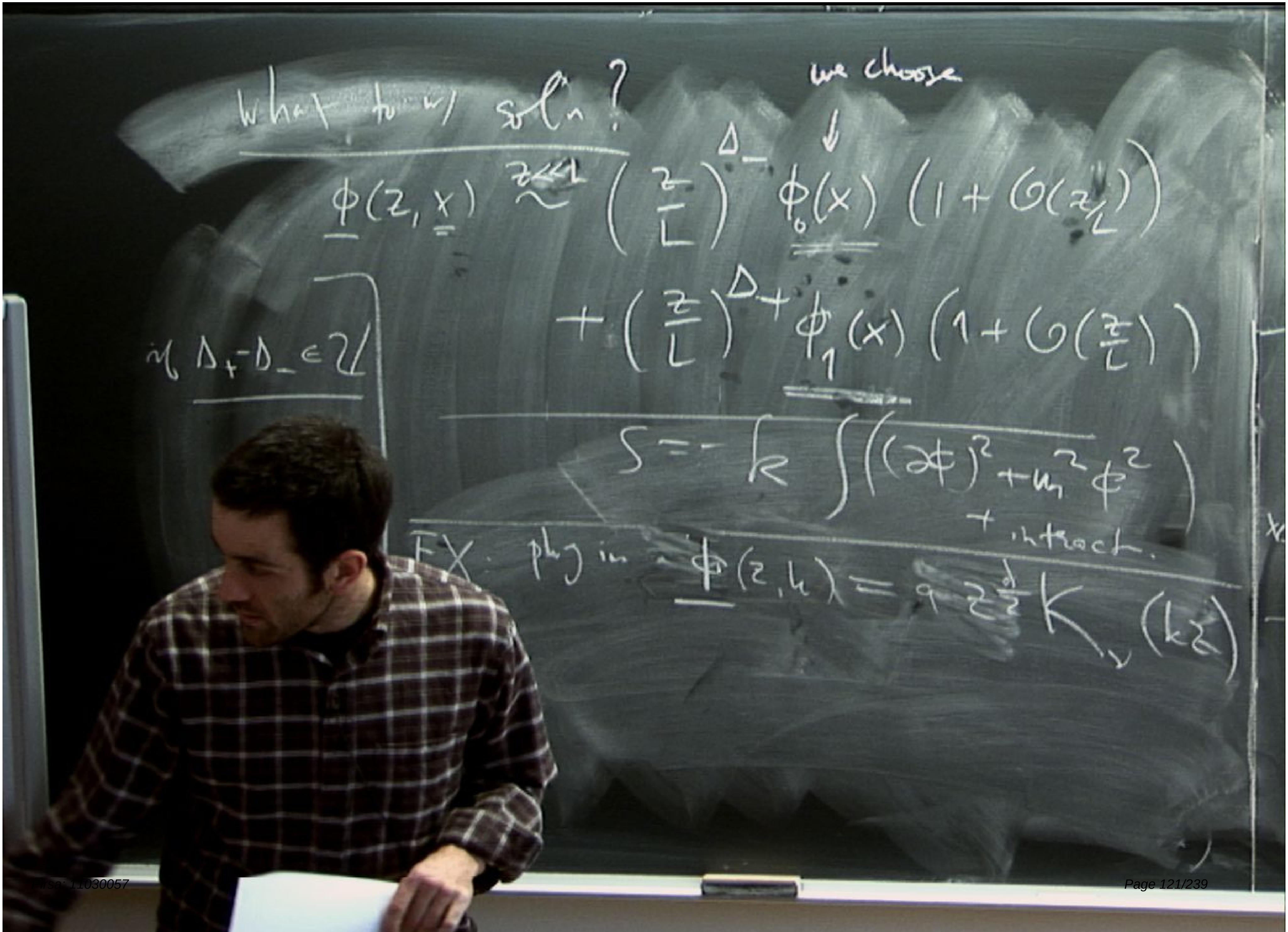

what to w/ soln ?
we choose
EX
+ interact.

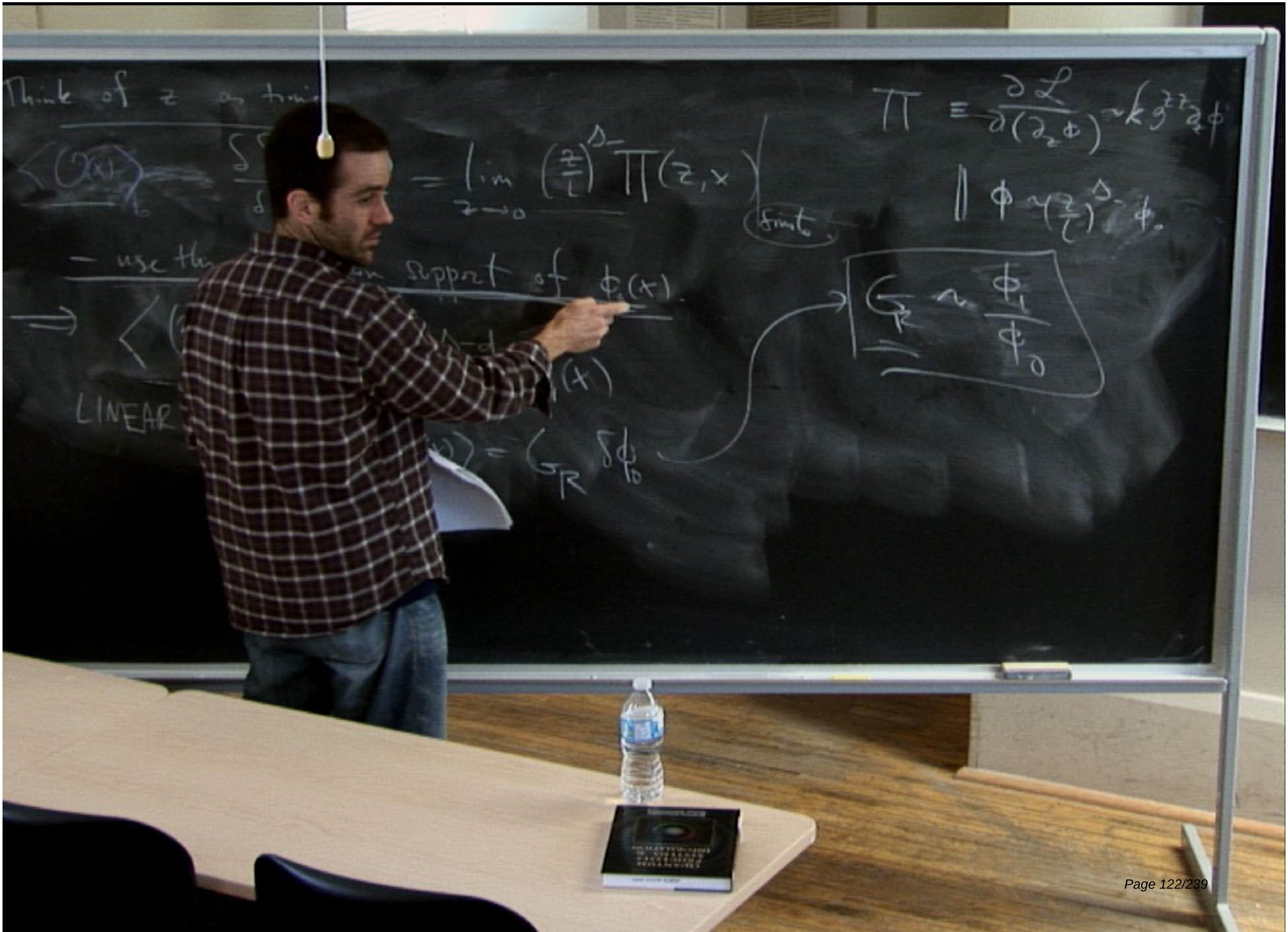

what to w/ soln ?
we choose
φ(z, x) ~ (z/L)^Δ− φ0(x) (1 + O(z/L))
+ (z/L)^Δ+ φ1(x) (1 + O(z/L))
if Δ+ − Δ− ∈ Z
S = − k ∫((∂φ)² + m²φ²)
+ interact.
plug in

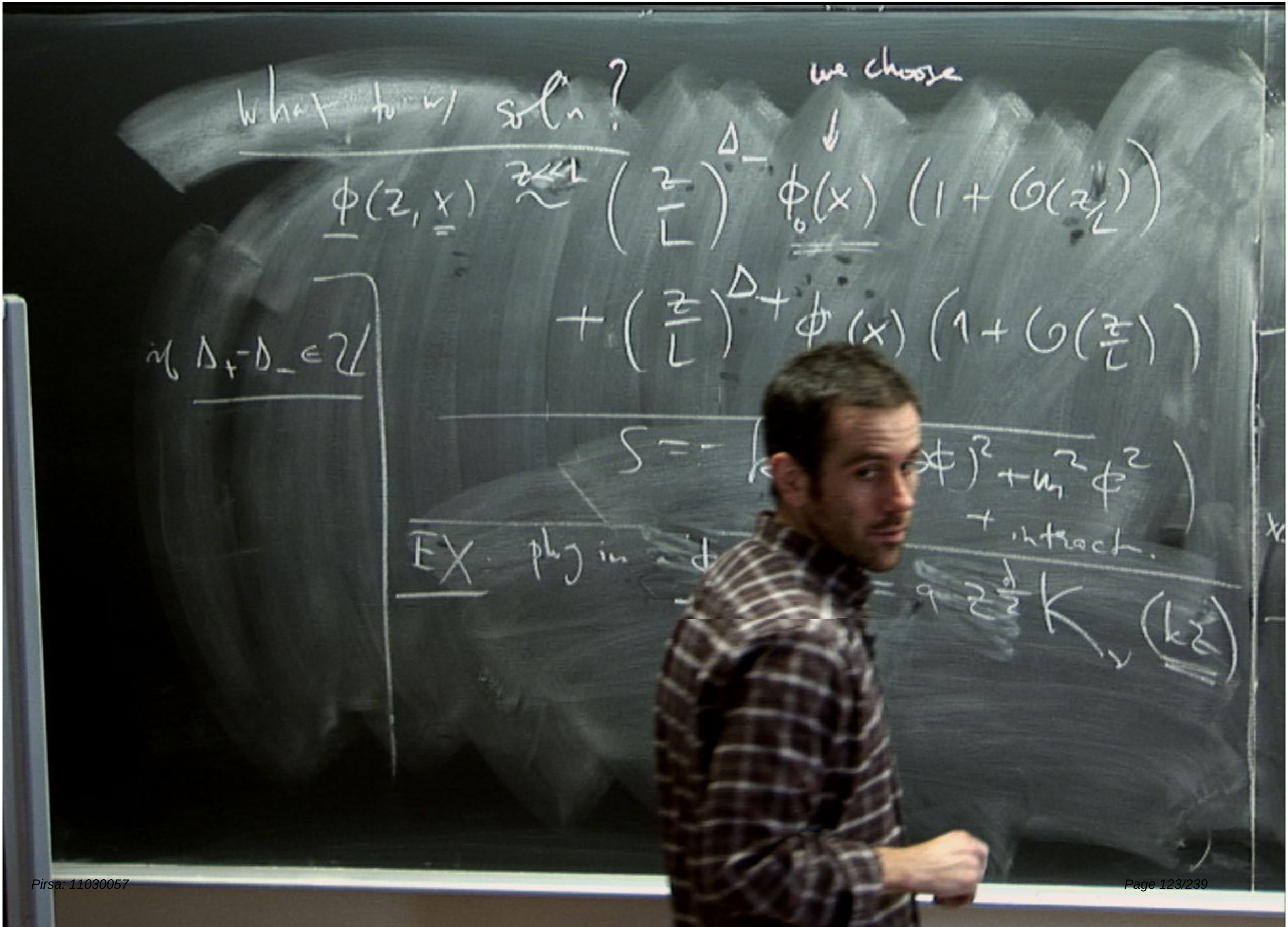

what to w/ soln ?
we choose
z<<L
φ(z, x) ~ (z/L)^Δ- φ0(x) (1 + O(z/L))
+ (z/L)^Δ+ φ1(x) (1 + O(z/L))
if Δ+ - Δ- ∈ Z
= - k ∫((∂φ)^2 + m^2 φ^2
+ interact.
φ

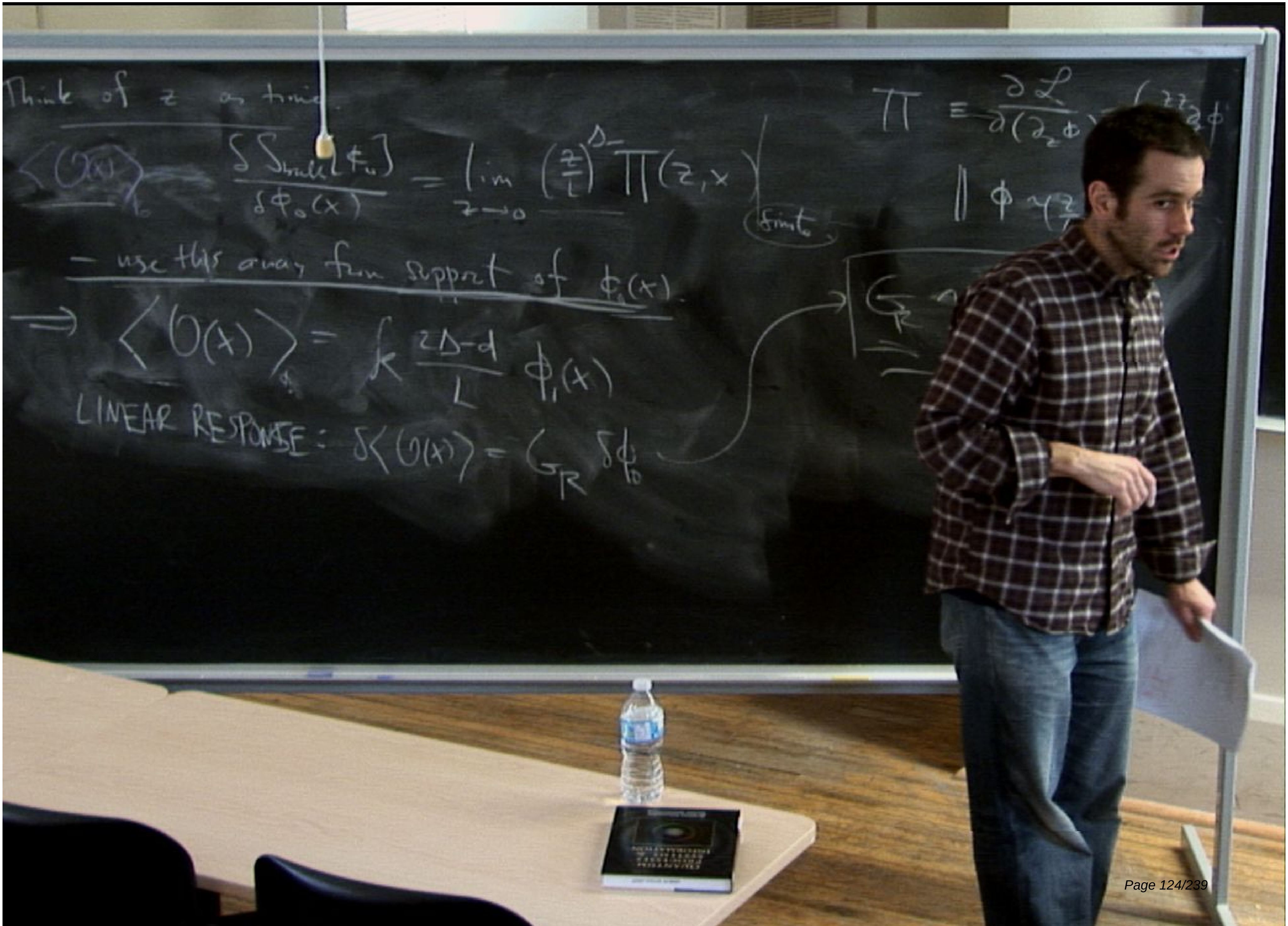

what to w/ soln ?
we choose
if Δ+ − Δ− ∈ Z
+ intrxn.
EX

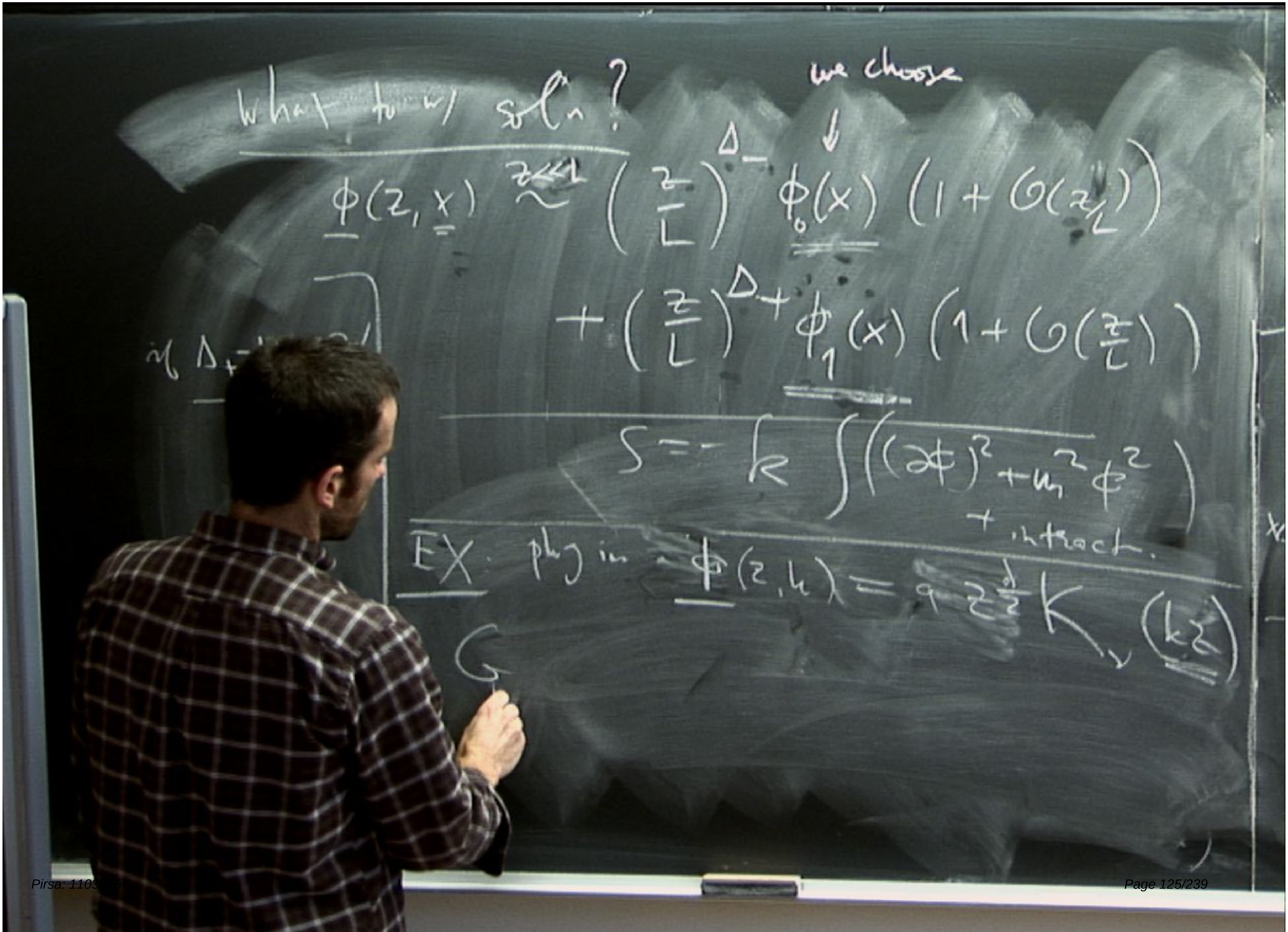

what to w/ soln ?
we choose
φ(z, x) ~ (z/L)^Δ− φ0(x) (1 + O(z/L))
+ (z/L)^Δ+ φ1(x) (1 + O(z/L))
if Δ+ − Δ− ∈ Z
∫((∂φ)² + m²φ²
+ interact.
EX
z^(d/2) K

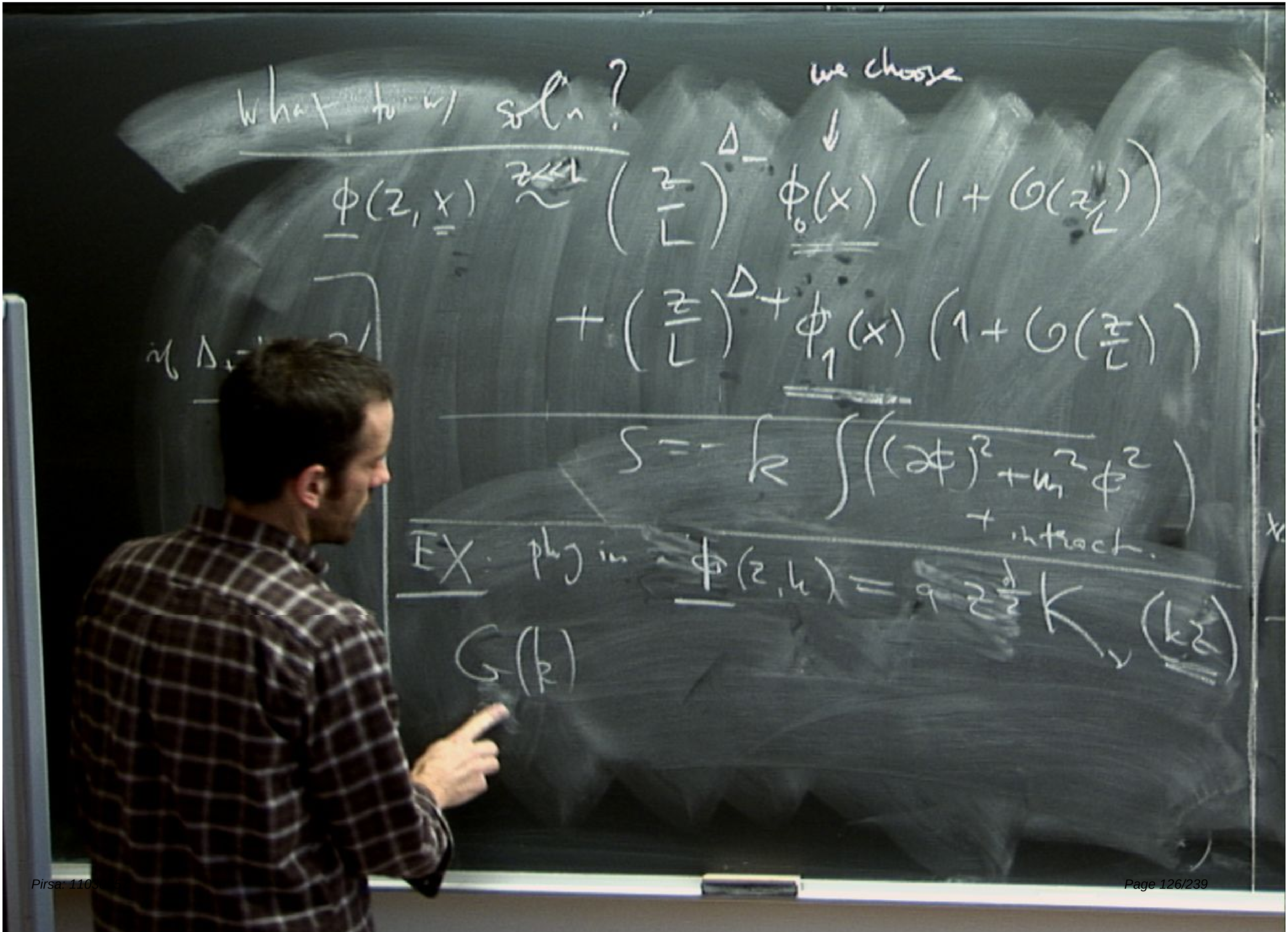

what to w/ soln ?
we choose
φ(z, x) ≈ (z/L)^Δ- φ0(x) (1 + O(z/L))
+ (z/L)^Δ+ φ1(x) (1 + O(z/L))
if Δ+ - Δ- ∈ Z
S = -k ∫((∂φ)^2 + m^2 φ^2 + interact.)
EX: plug in φ(z, k) = a z^(d/2) K_ν(kz)

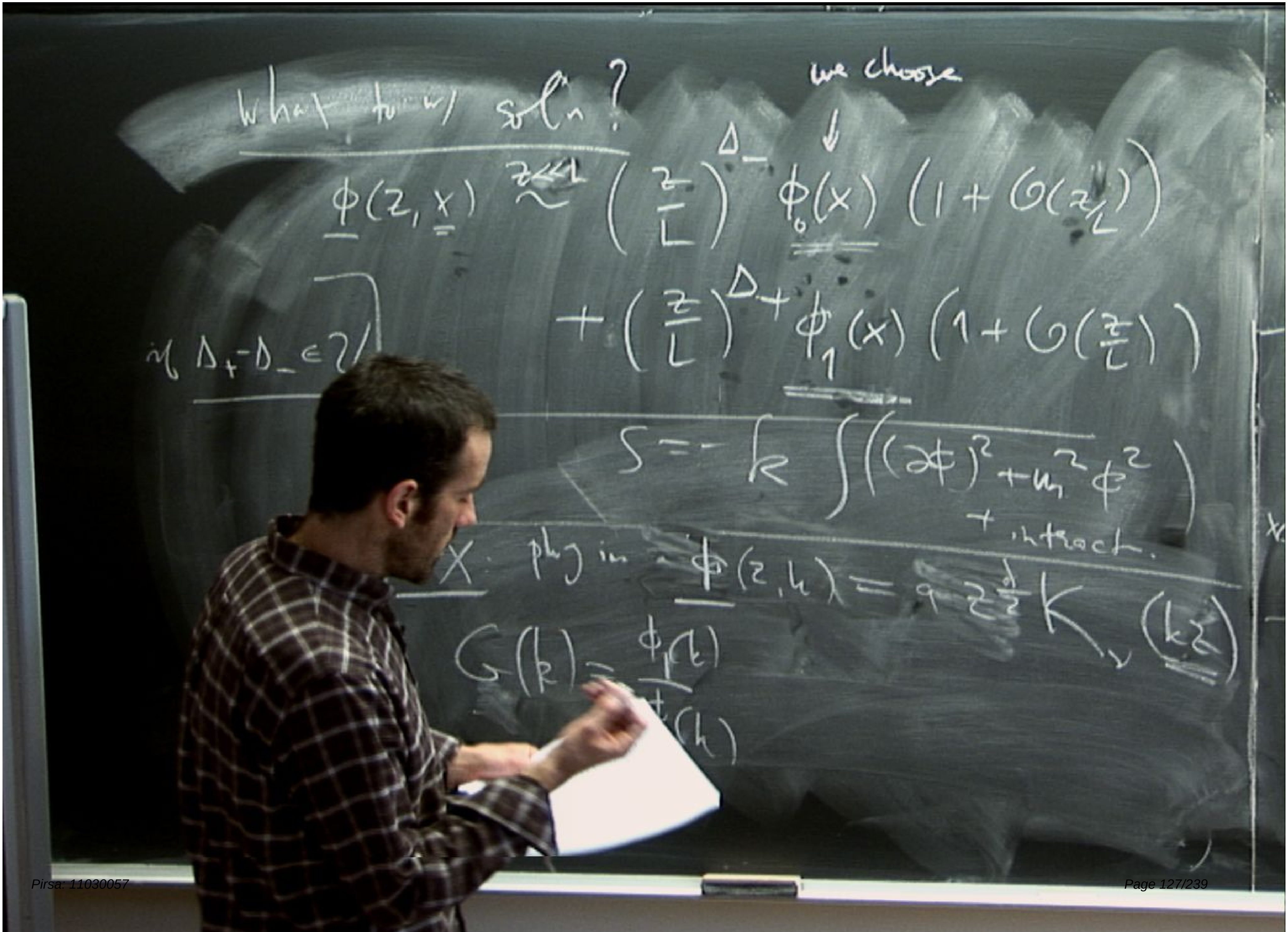

Think of z as time
finite
LINEAR

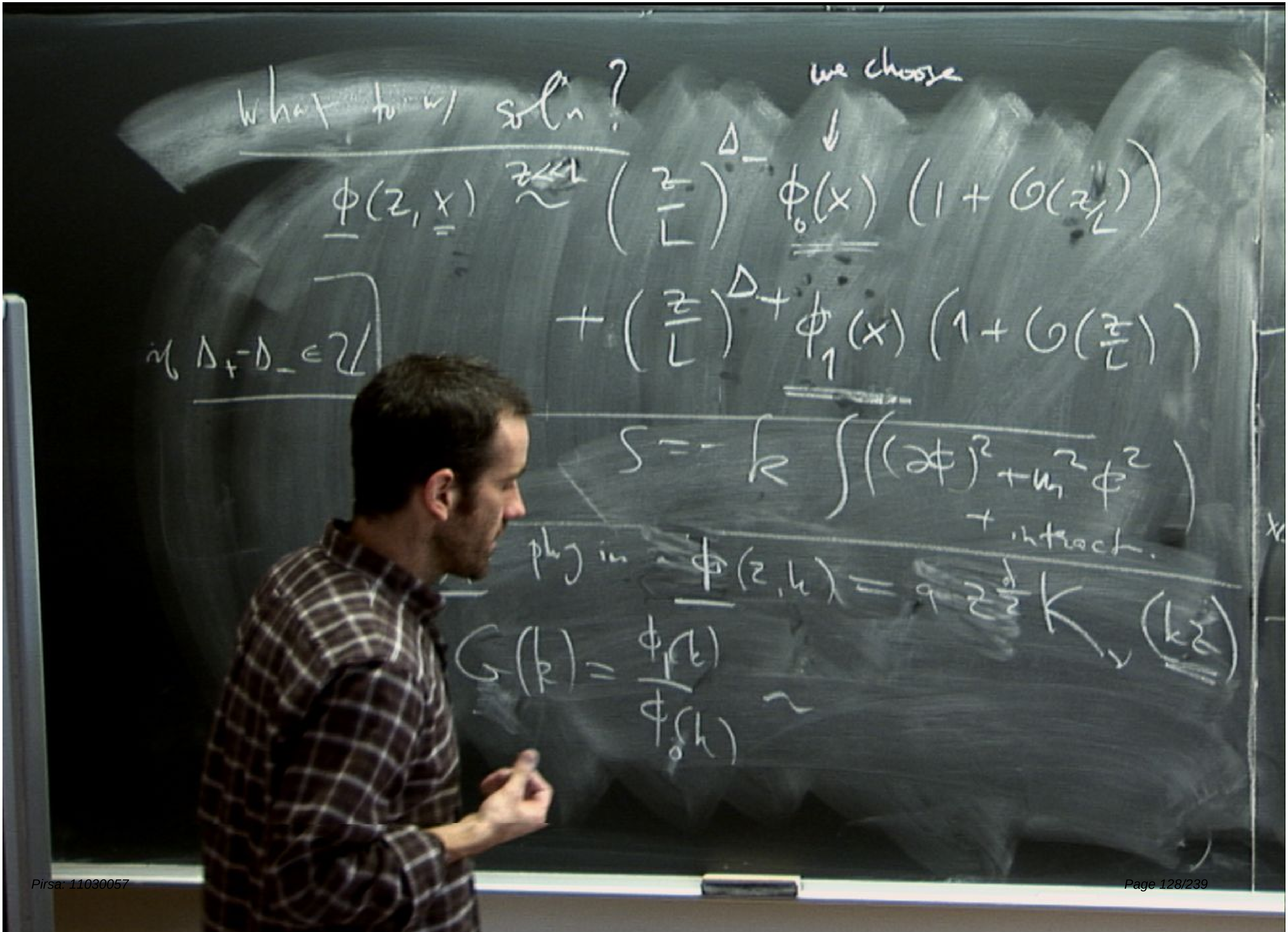

what to w/ soln ?
we choose
φ(z, x) ≈ (z/L)^Δ− φ0(x) (1 + O(z/L))
+ (z/L)^Δ+ φ(x) (1 + O(z/L))
if Δ+ − Δ− ∈ Z
+ interact.
a z^(d/2) K_ν (kz)


Think of z as time.
finite
− use this away from support of φ1(x)
LINEAR RESPONSE:

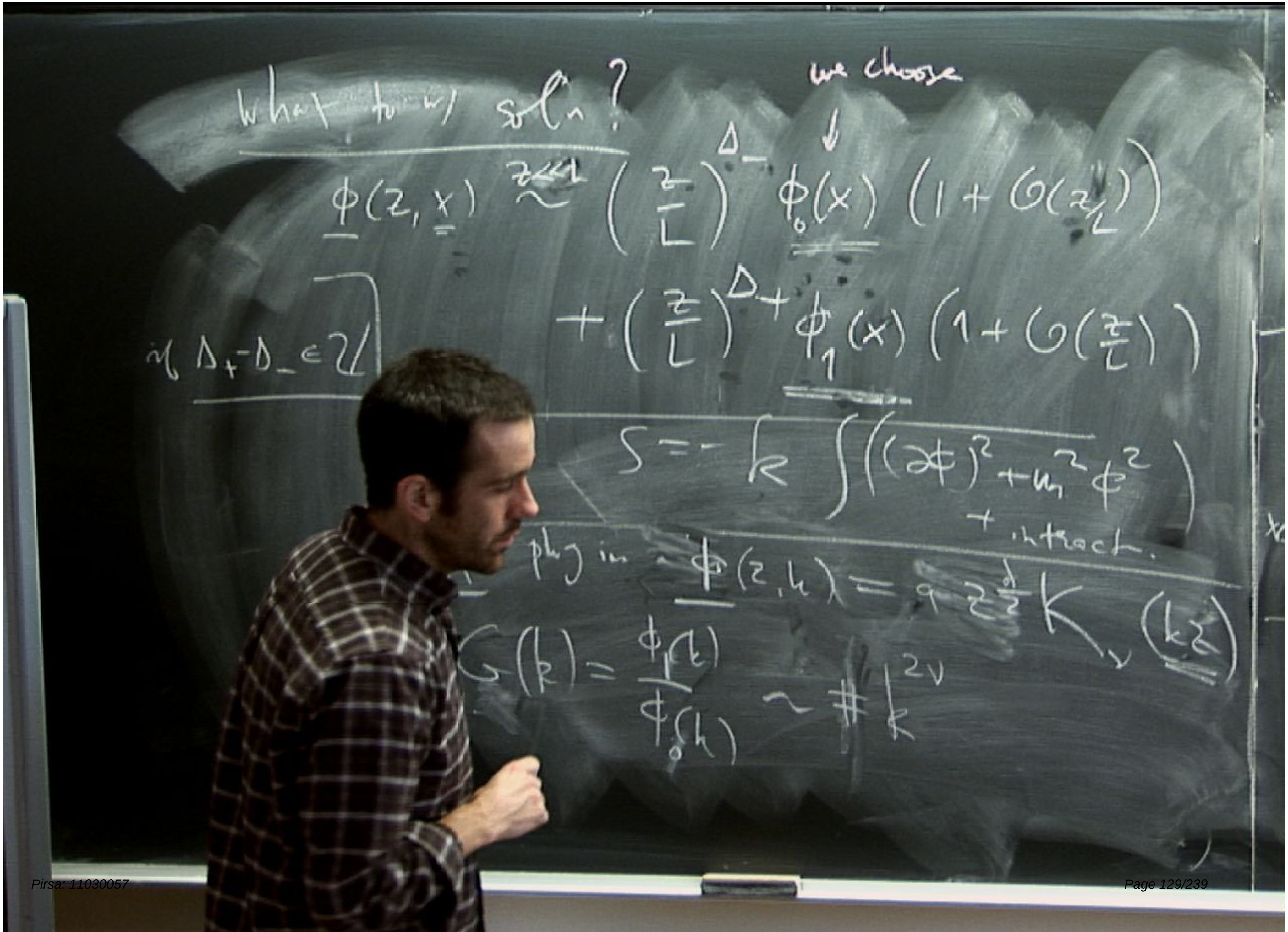

what to w/ soln ?
we choose
$\phi(z,x) \overset{z \ll 1}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x)\left(1+O(z/L)\right)$
$+ \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x)\left(1+O(\frac{z}{L})\right)$
$S = -k \int \left((\partial\phi)^2 + m^2\phi^2 + \text{interact.}\right)$
EX. plug in $\phi(z,k) = a\, z^{\frac{d}{2}} K_\nu(kz)$

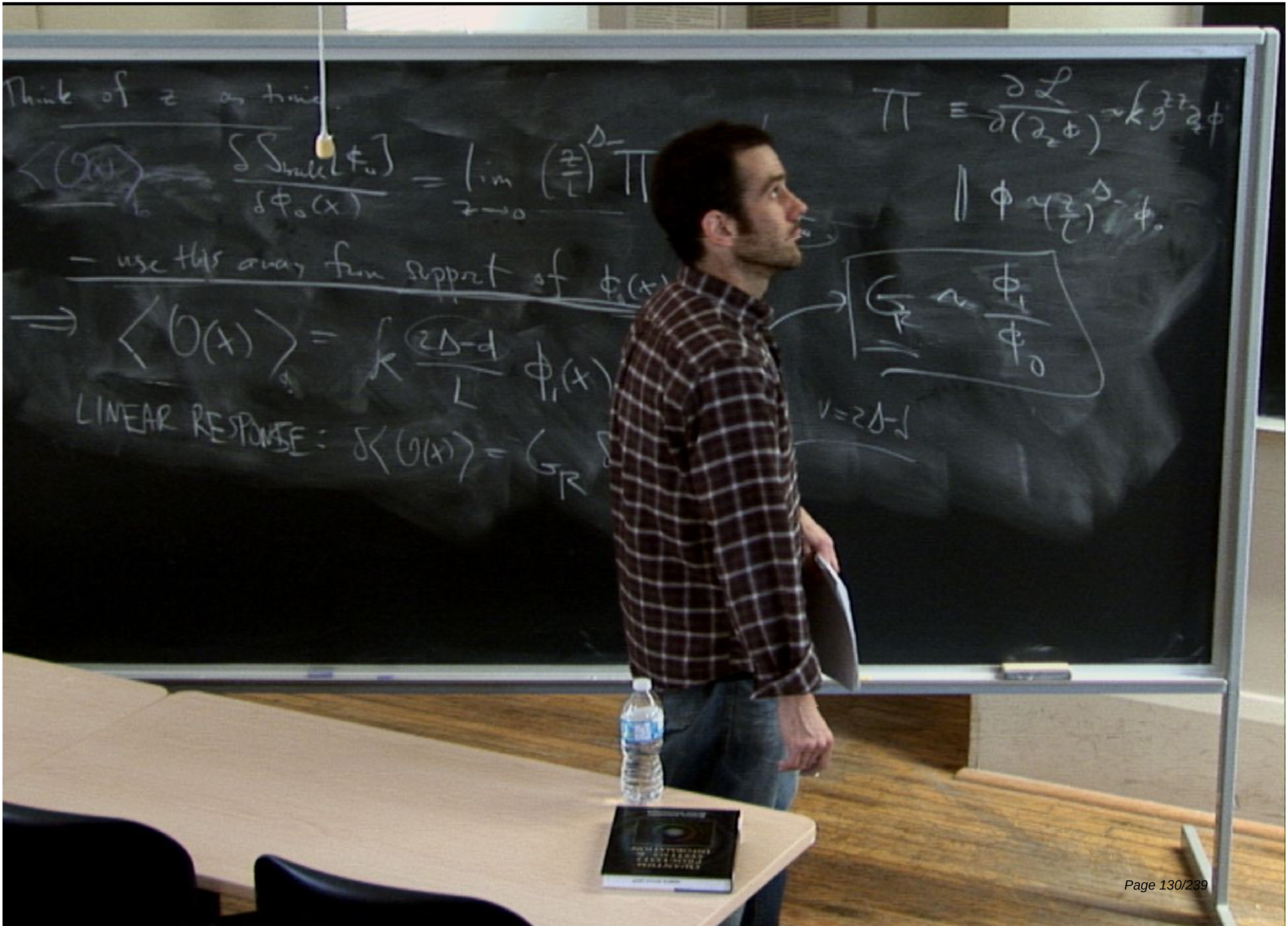

what to w/ sol^n ?
we choose
ϕ(z,x) ~ (z/L)^Δ- ϕ0(x) (1 + O(z/L))
+ (z/L)^Δ+ ϕ1(x) (1 + O(z/L))
S = -k ∫((∂ϕ)^2 + m^2 ϕ^2 + interact.)
EX. plug in ϕ(z,k) = a z^(d/2) K_ν(kz)
G(k)

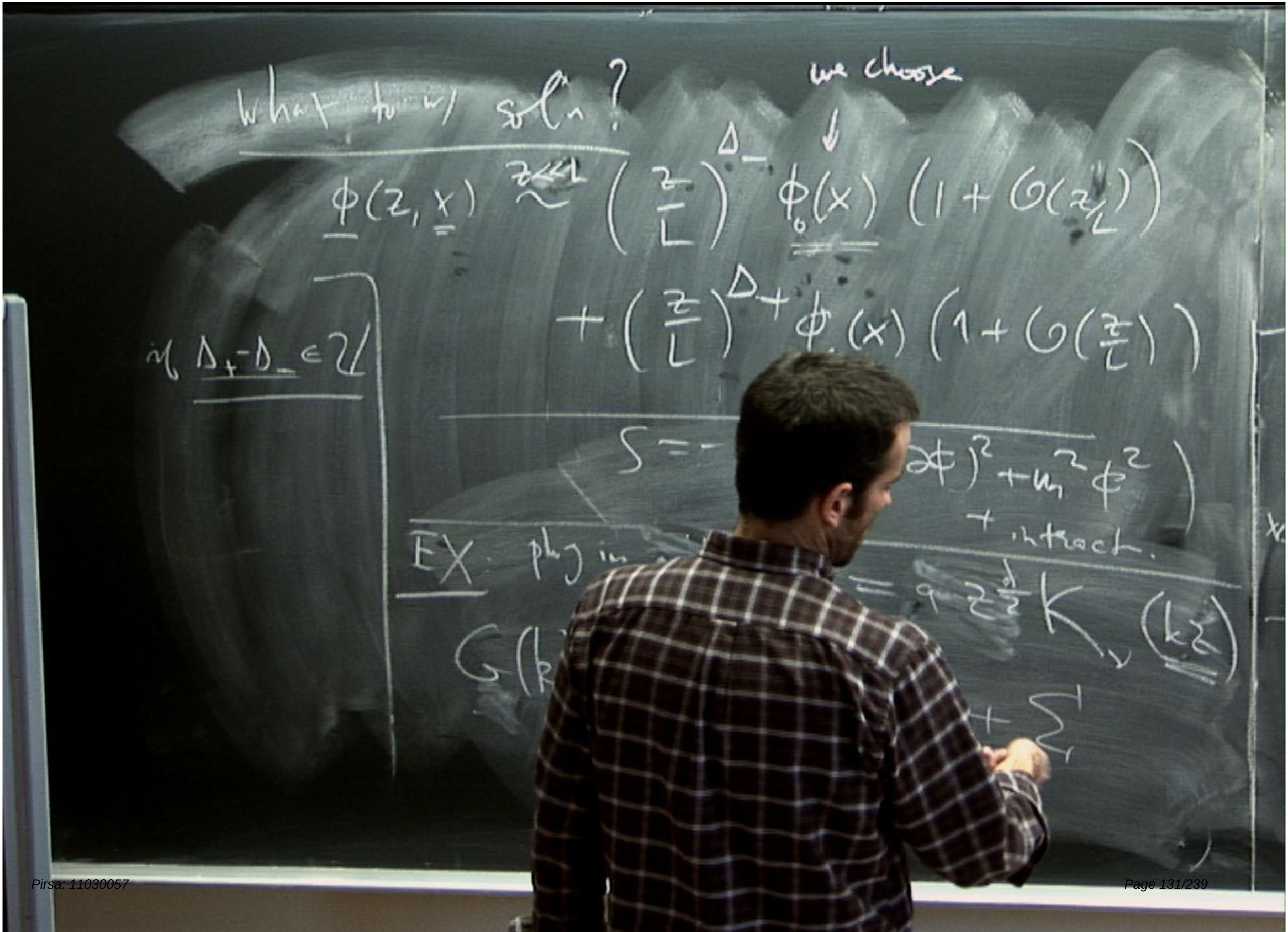

what to w/ soln ?
we choose
+ interact.

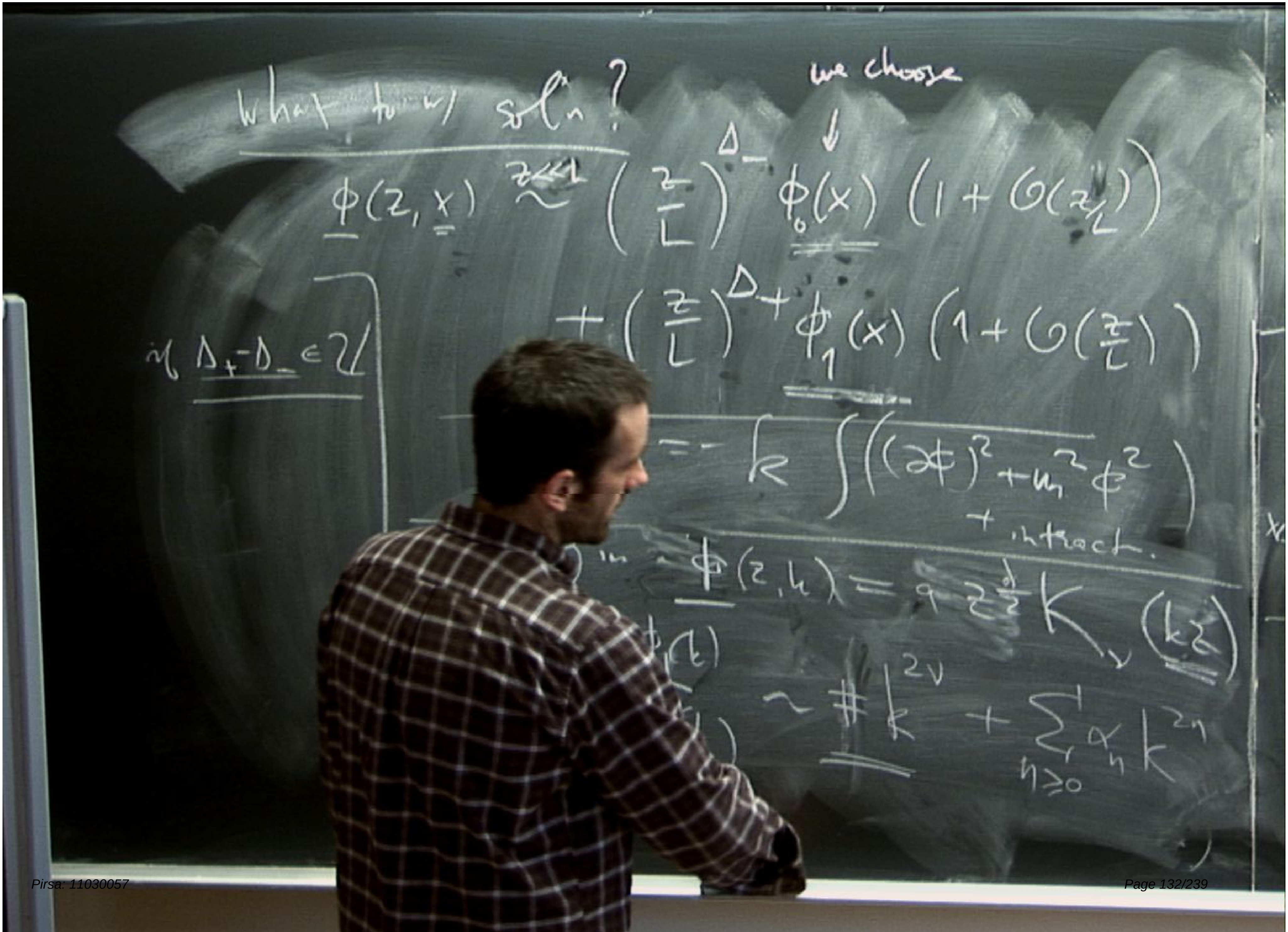


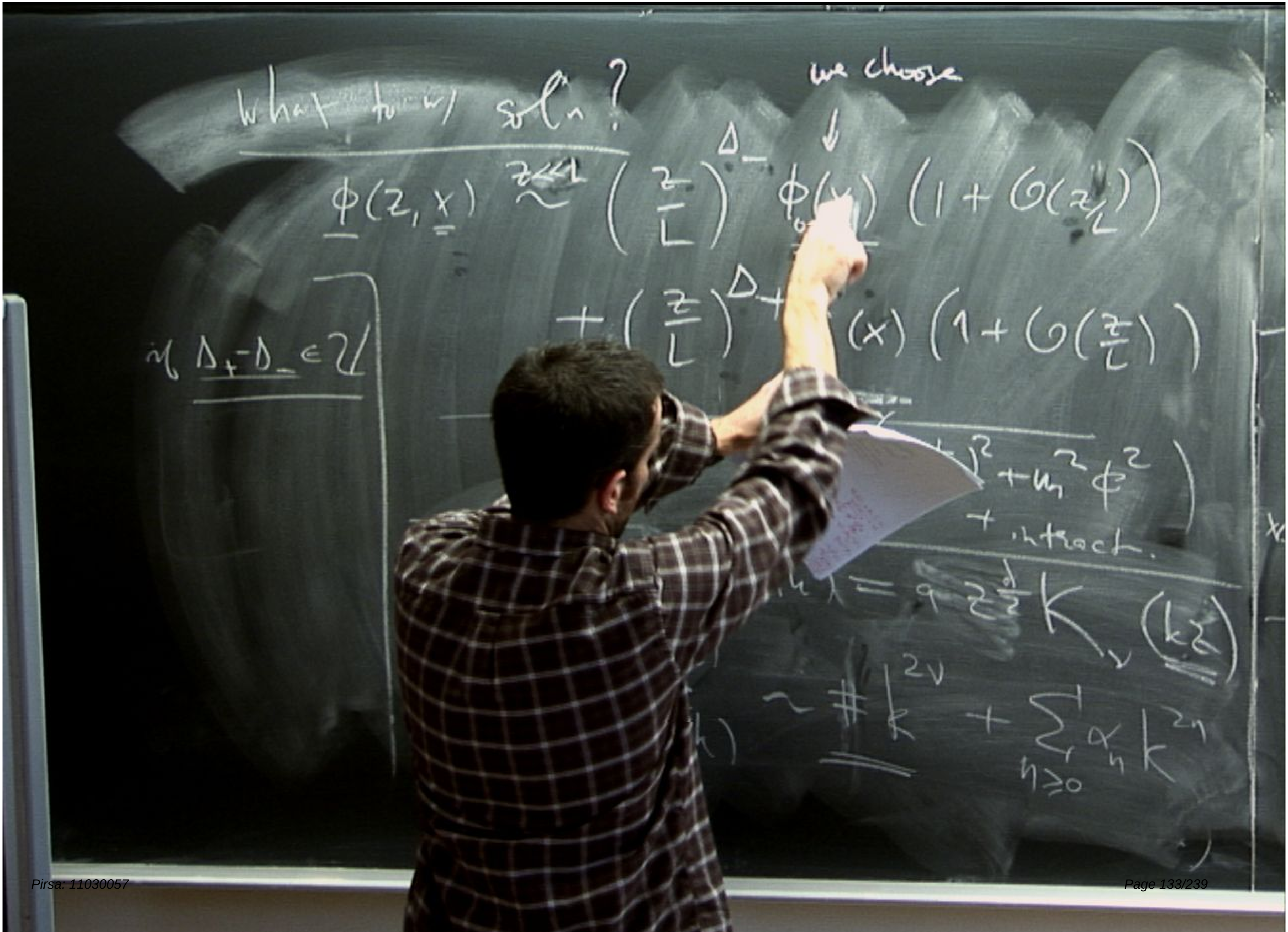
what to w/ soln ?
we choose
$\phi(z,x) \overset{z \ll 1}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x)\left(1 + O(z/L)\right)$
$+ \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x)\left(1 + O(\frac{z}{L})\right)$
if $\Delta_+ - \Delta_- \in \mathbb{Z}$
$S = -k \int \left((\partial\phi)^2 + m^2\phi^2 + \text{interact.}\right)$
plug in $\phi(z,k) = a\, z^{\frac{d}{2}} K_\nu(kz)$
$G(k) = \frac{\phi_1(k)}{\phi_0(k)} \sim$

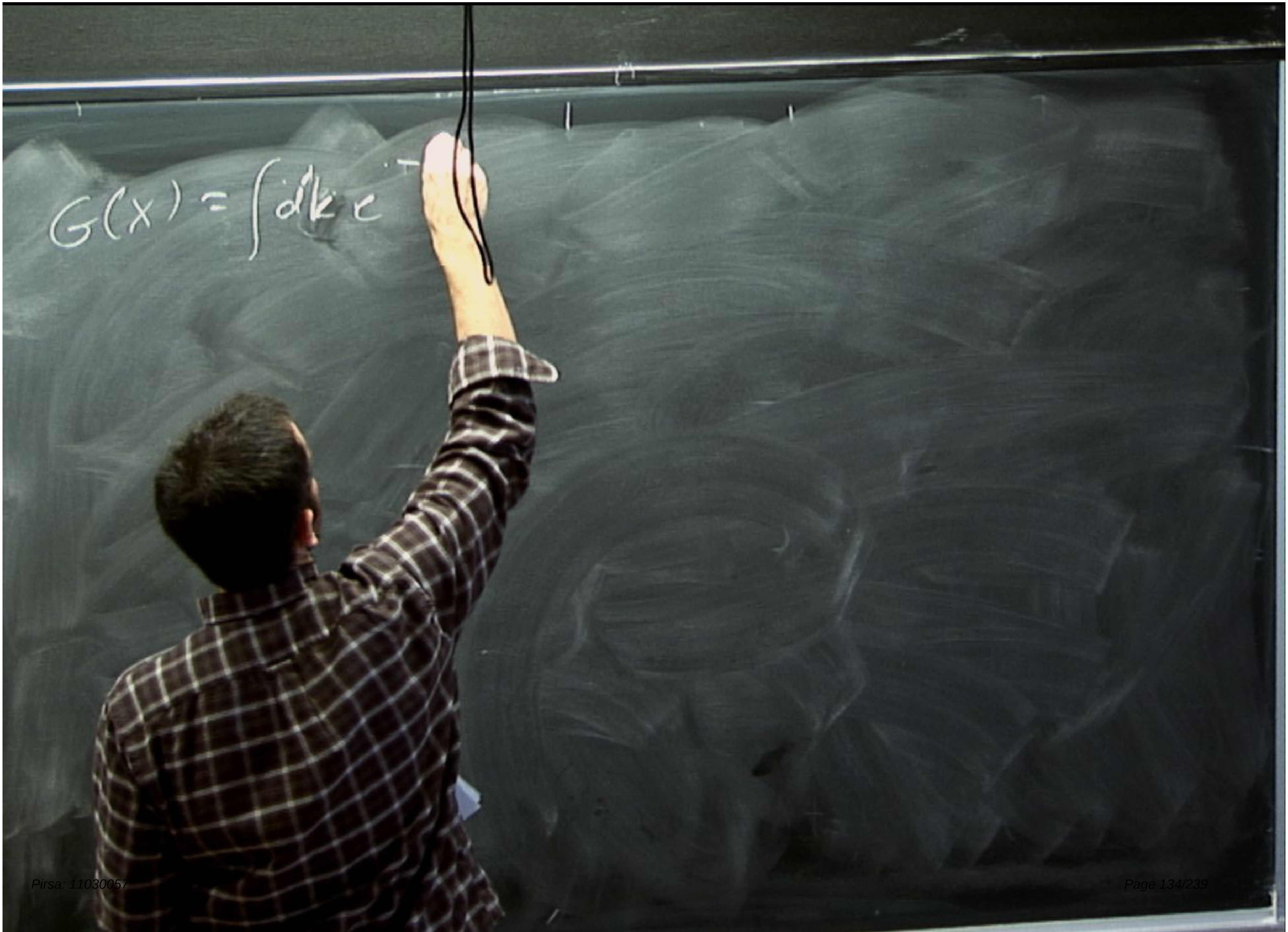

what to w/ soln?
we choose
+ intrach.
phg in

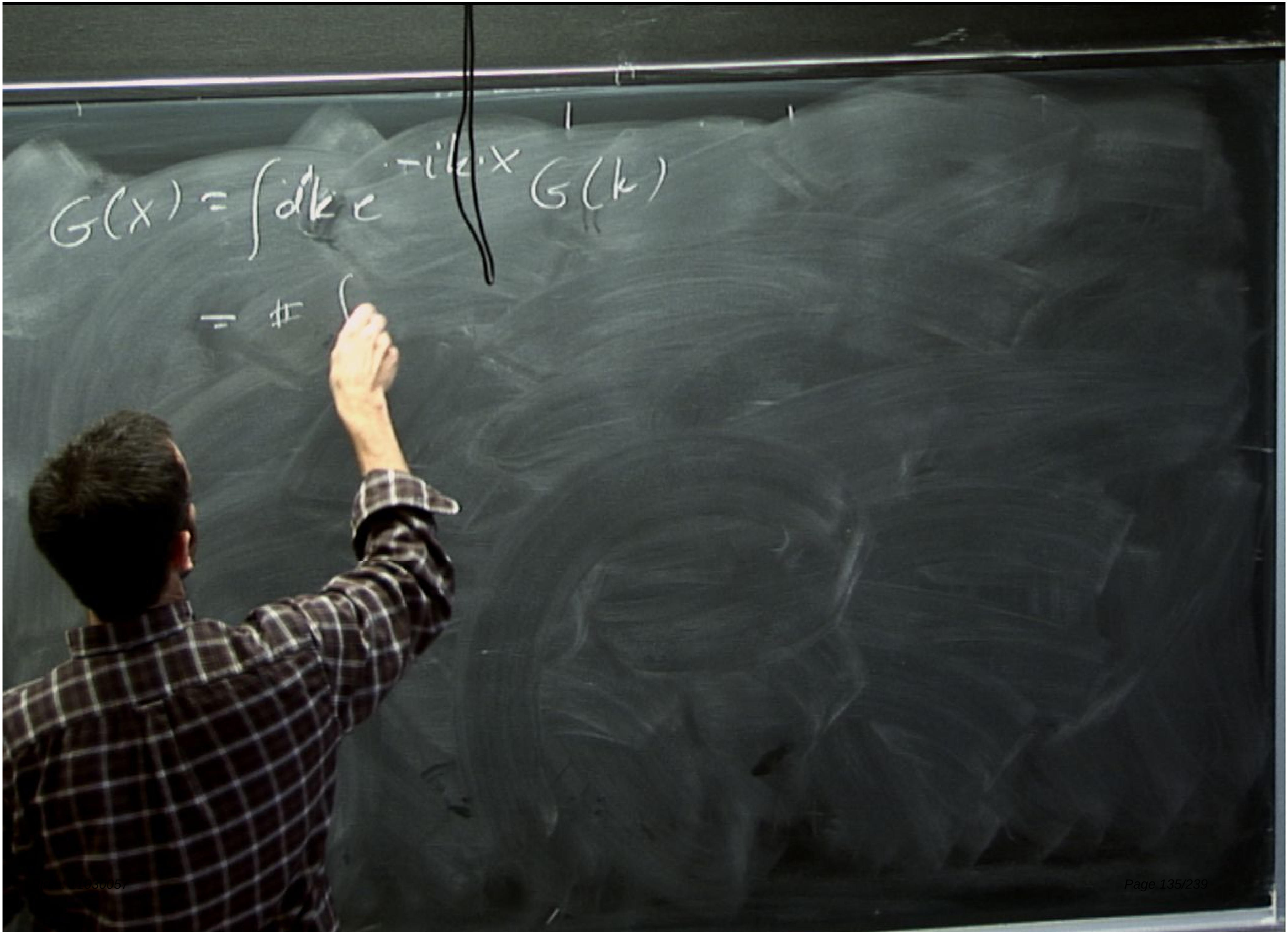

Think of z as time
use this away from support of
LINEAR RESPONSE:

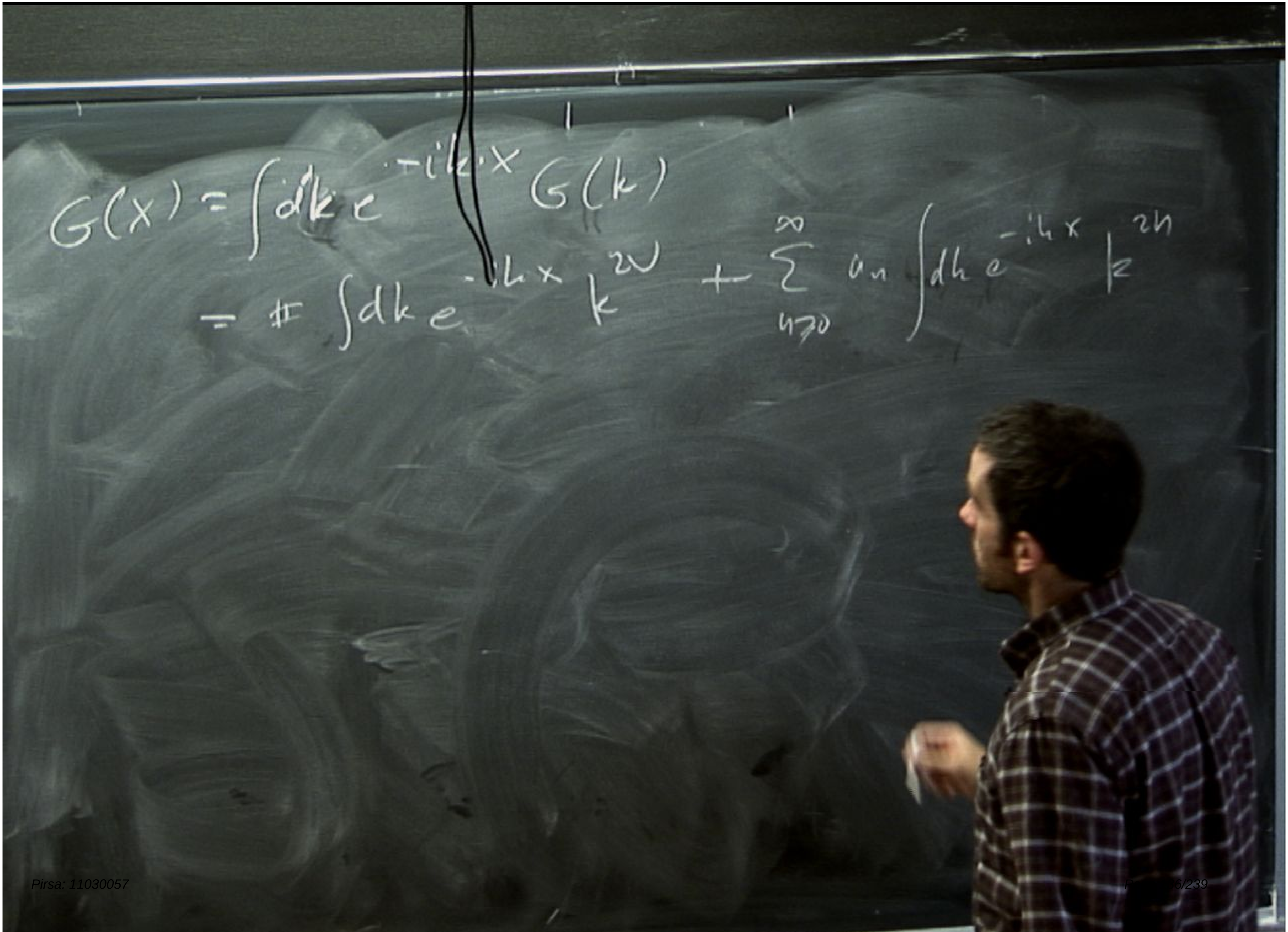

what to w/ sol'n ?
we choose
φ(z, x) ≈ (z/L)^Δ₋ φ₀(x) (1 + O(z/L))
+ (z/L)^Δ₊ φ₁(x) (1 + O(z/L))
if Δ₊ - Δ₋ ∈ ℤ
+ interact.
EX


what to w/ soln ?
we choose
if Δ+ - Δ- ∈ Z
+ intract.


what to w/ soln ?
we choose
if Δ+ − Δ− ∈ Z
+ intrxn.


G(x) = ∫ dk e


$G(x) = \int dk\, e^{-ik\cdot x}\, G(k)$
$= \# \int$


$G(x) = \int dk\, e^{-ik\cdot x} G(k)$
$= \# \int dk\, e^{-ikx} k^{2\nu} + \sum_{n\geq 0}^{\infty} a_n \int dk\, e^{-ikx} k^{2n}$

$$G(x) = \int d^d k \, e^{-ik\cdot x} \, G(k)$$

$$= \# \int d^d k \, e^{-ik\cdot x} \, k^{2\nu} + \sum_{n \geq 0}^{\infty} a_n \int dk \, e^{-ikx} \, k^{2n}$$

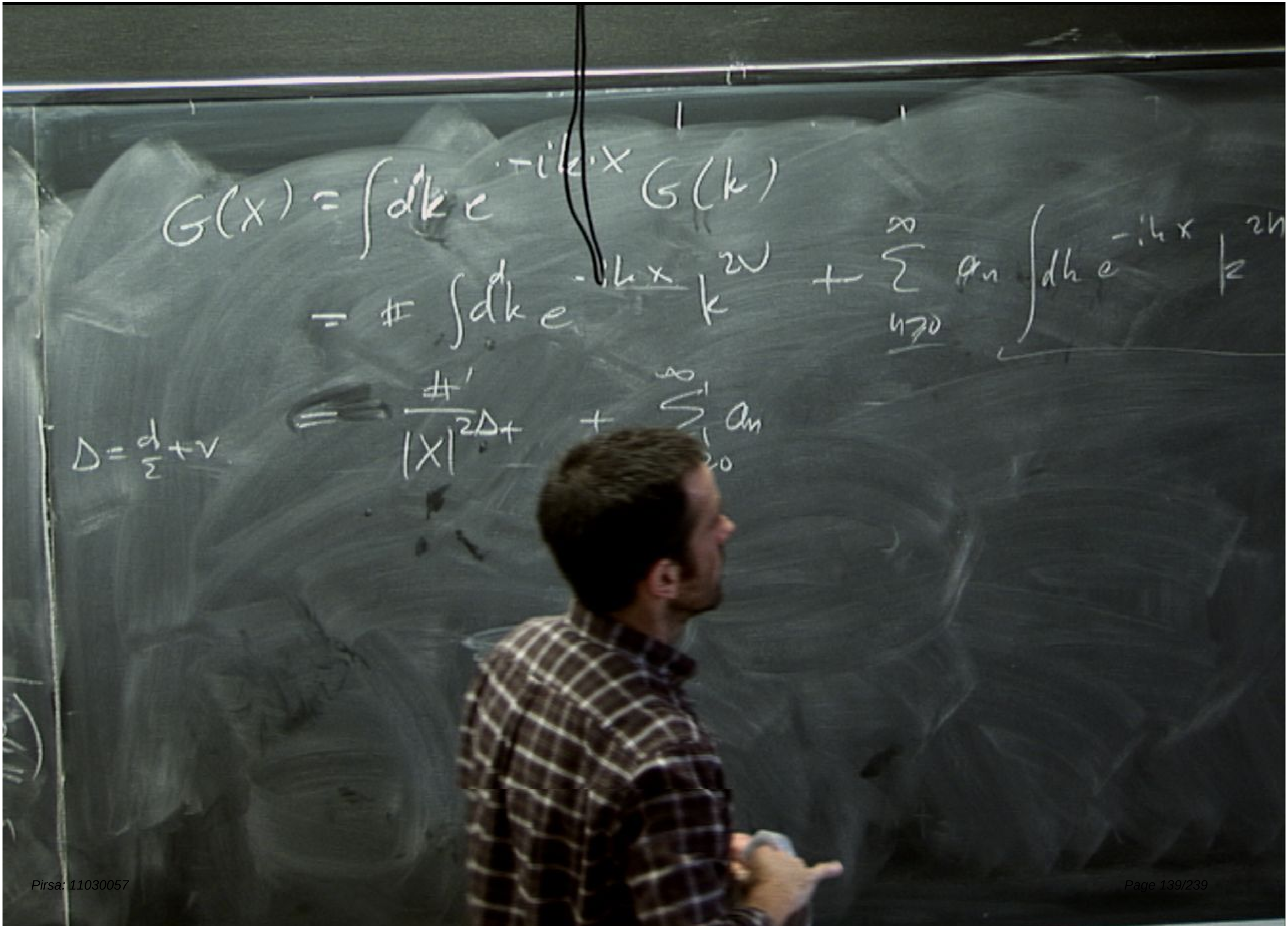
$$= \frac{1}{|x|^{2\Delta_+}}$$

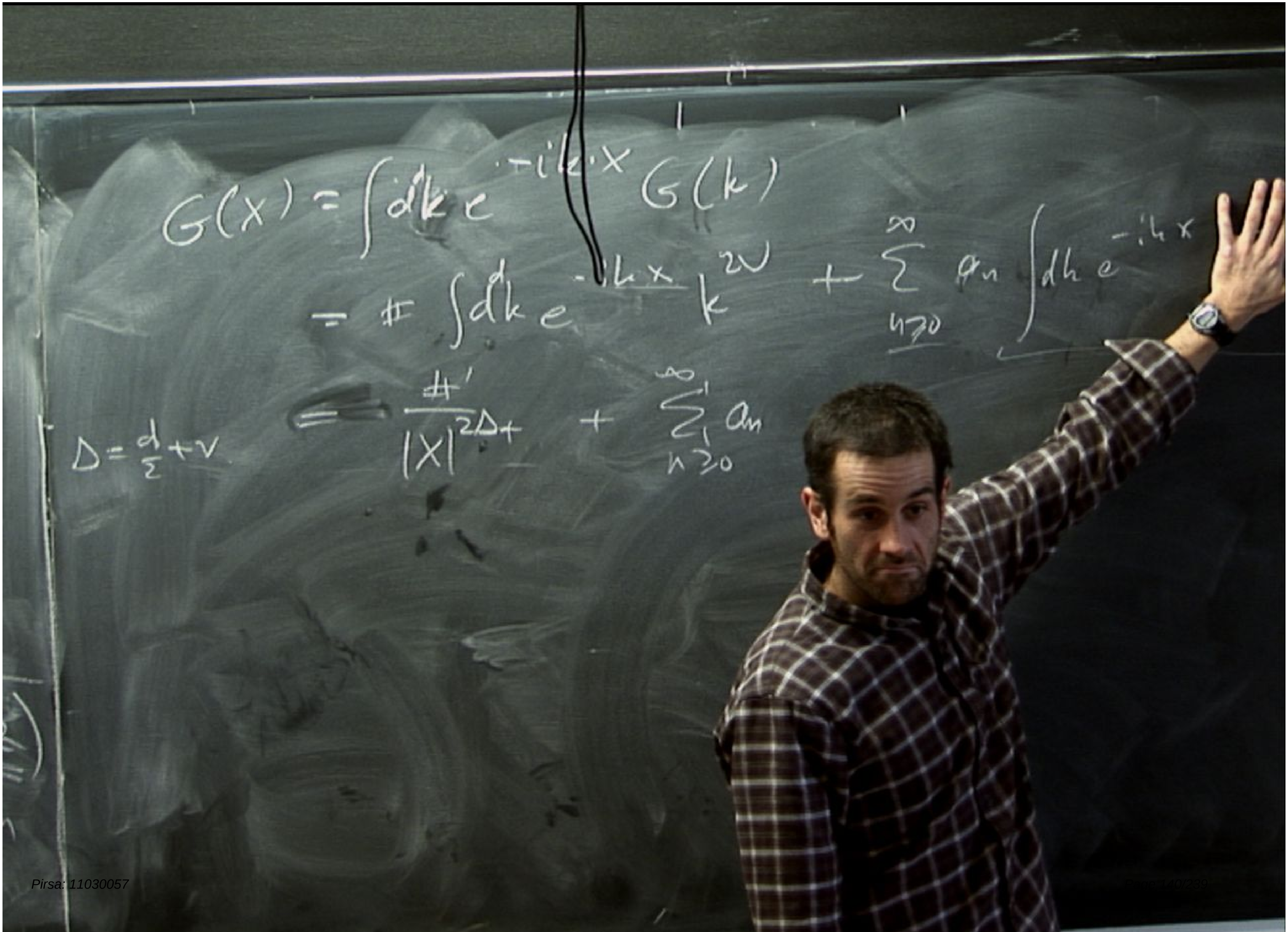
$$G(x) = \int d^dk \, e^{-ik\cdot x} \, G(k)$$

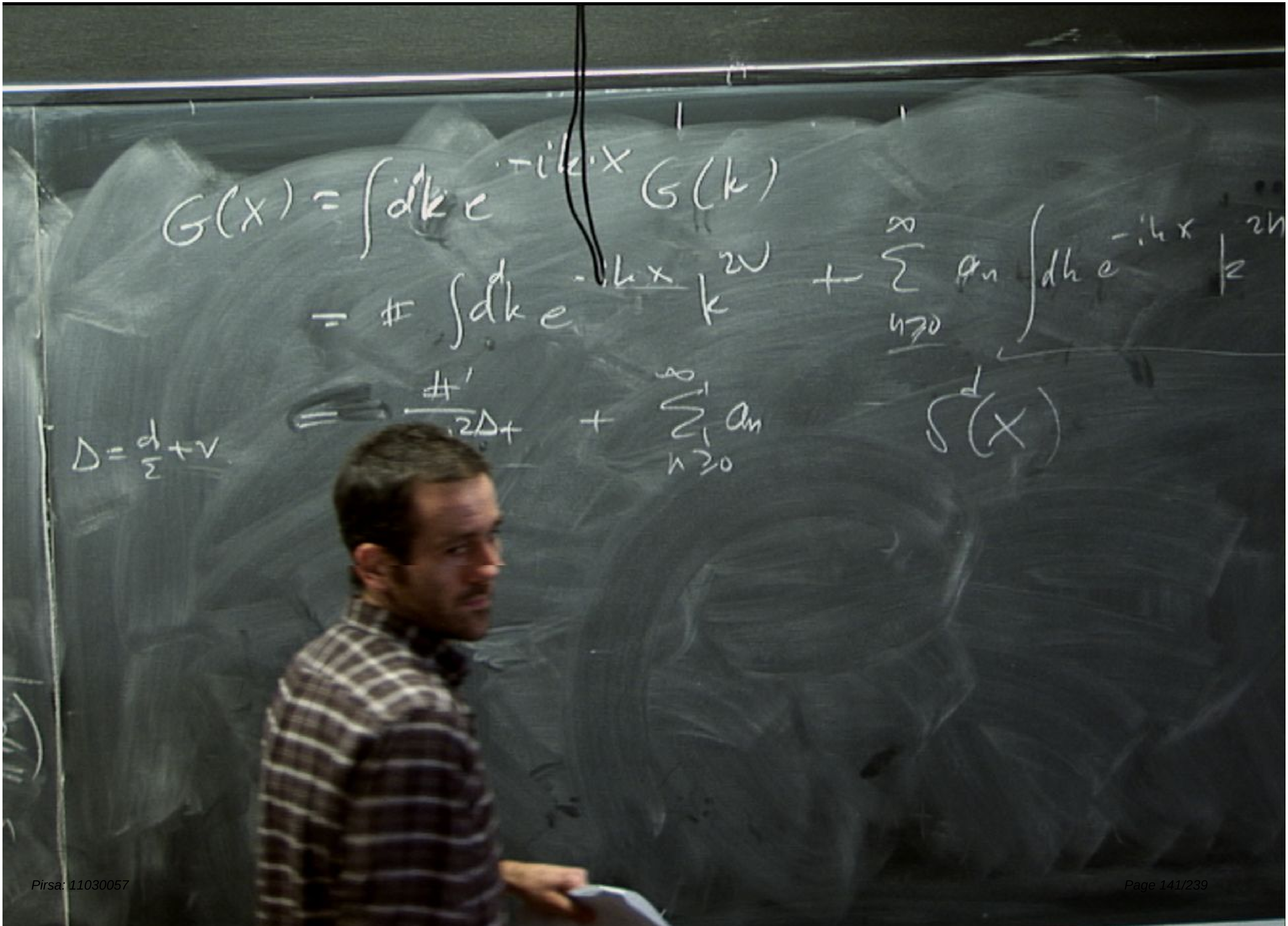
$$= \# \int d^dk \, e^{-ik\cdot x} \, k^{2\nu} + \sum_{n\geq 0}^{\infty} a_n \int d^dk \, e^{-ik\cdot x} \, k^{2n}$$

$$= \frac{\#'}{|x|^{2\Delta_+}}$$

$$\Delta = \frac{d}{2} + \nu$$


G(x) = ∫d^dk e^{-ik·x} G(k)
= # ∫d^dk e^{-ikx} k^{2ν} + Σ_{n≥0}^∞ a_n ∫dk e^{-ikx} k^{2n}
= #' / |x|^{2Δ+} + Σ a_n
Δ = d/2 + ν


G(x) = ∫d^d k e^{-ik·x} G(k)
= # ∫d^d k e^{-ik·x} k^{2ν} + Σ_{n≥0}^∞ a_n ∫dk e^{-ikx}
= #' / |x|^{2Δ+} + Σ_{n≥0}^∞ a_n
Δ = d/2 + ν



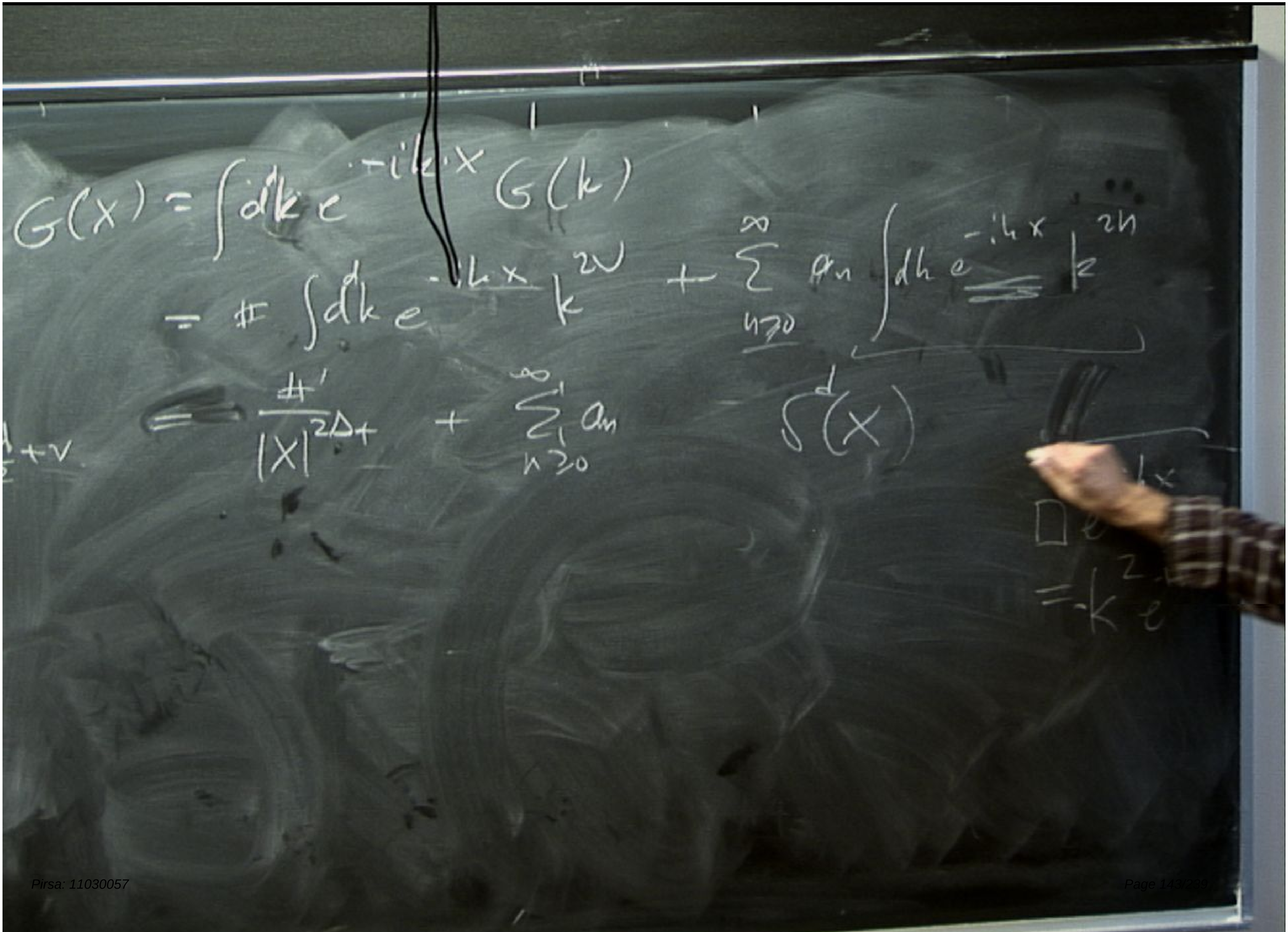
G(x) = ∫ d^dk e^{-ik·x} G(k)
= # ∫ d^dk e^{-ik·x} k^{2ν} + Σ_{n≥0}^∞ a_n ∫ d^dk e^{-ik·x} k^{2n}
= #' / x^{2Δ+} + Σ_{n≥0}^∞ a_n δ^d(x)
Δ = d/2 + ν

$$G(x) = \int d^d k \, e^{-ik\cdot x} \, G(k)$$

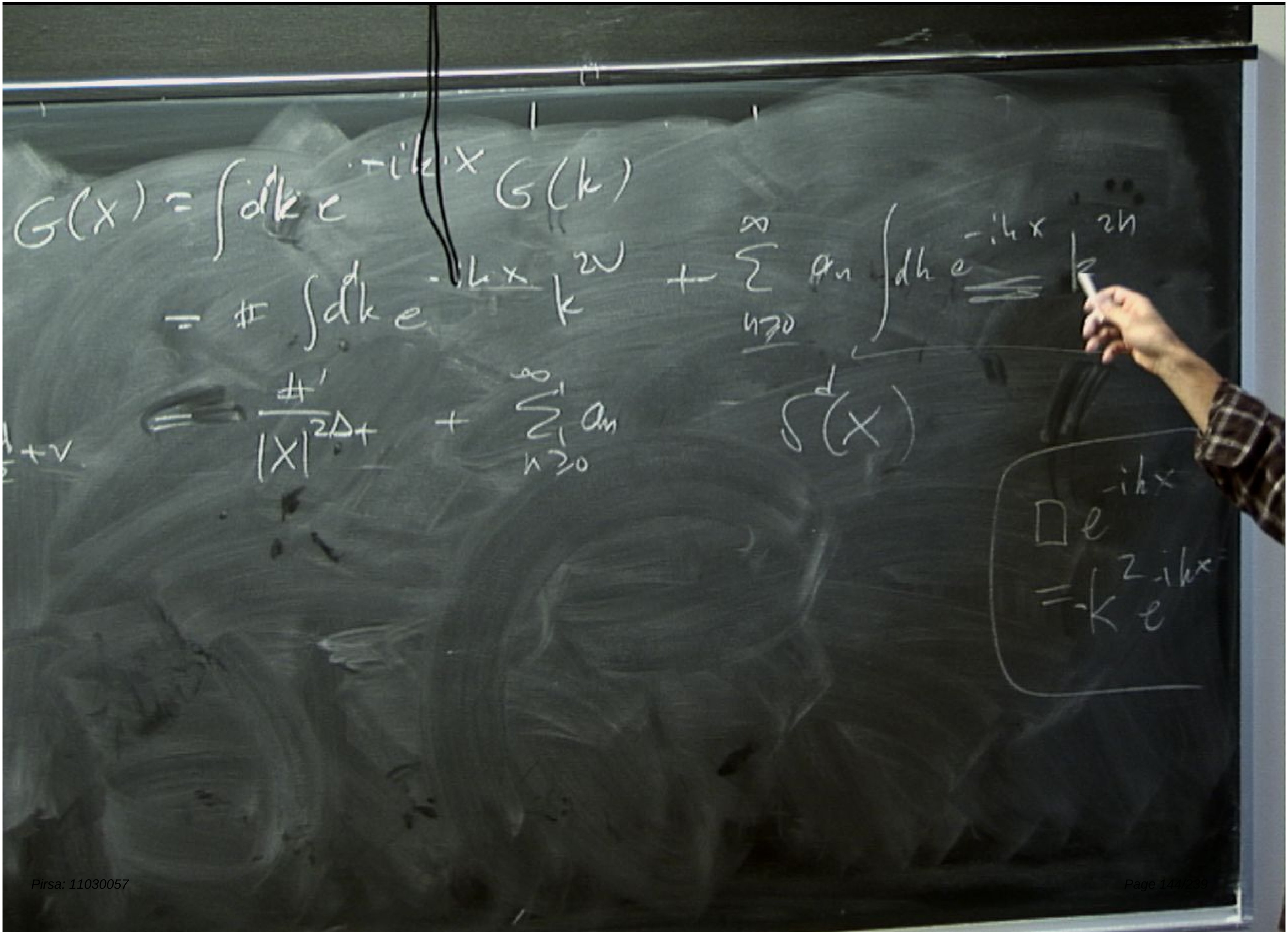
$$= \# \int d^d k \, e^{-ik\cdot x} \, k^{2\nu} + \sum_{n\geq 0}^{\infty} a_n \int d^d k \, e^{-ik\cdot x} \, k^{2n}$$

$$= \frac{\#'}{|x|^{2\Delta +}} + \sum_{n\geq 0}^{\infty} a_n \, \delta^d(x)$$

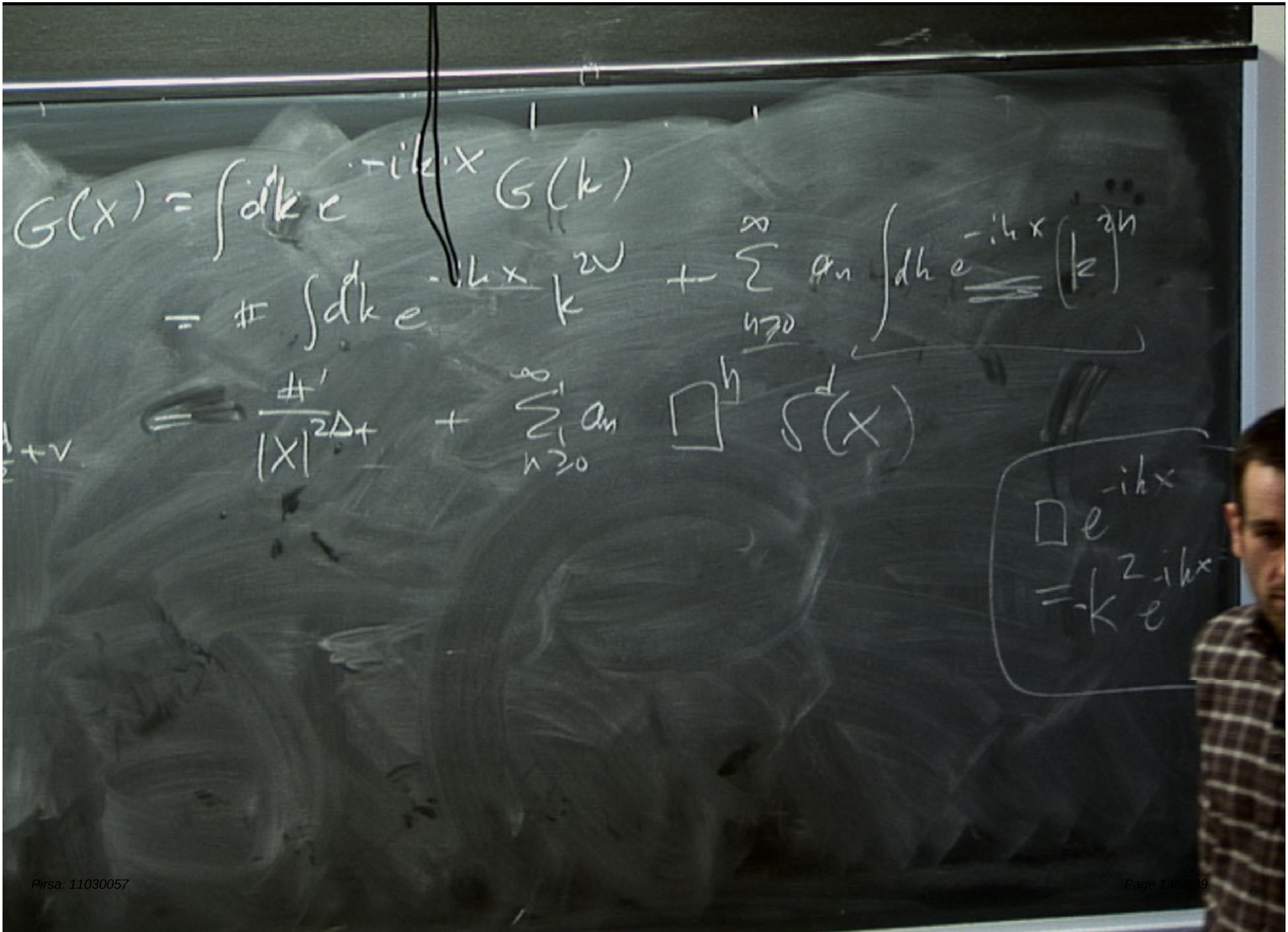
$$\Delta = \frac{d}{2} + \nu$$



G(x) = ∫ d^d k e^{-ik·x} G(k)
= ∫ d^d k e^{-ik·x} k^{2ν} + Σ_{n≥0}^{∞} a_n ∫ d^d k e^{-ik·x} k^{2n}
= 1/|x|^{2Δ+} + Σ_{n≥0}^{∞} a_n δ^d(x)
□ e
= -k^2 e



G(x) = ∫d^dk e^{-ik·x} G(k)
= # ∫d^dk e^{-ik·x} k^{2ν} + Σ_{n≥0}^∞ a_n ∫d^dk e^{-ikx} k^{2n}
= #' / |x|^{2Δ+} + Σ_{n≥0}^∞ a_n δ^d(x)
□e^{-ikx} = -k^2 e^{-ikx}


G(x) = ∫d^dk e^{-ik·x} G(k)
= # ∫d^dk e^{-ik·x} k^{2ν} + Σ_{n≥0}^{∞} a_n ∫d^dk e^{-ik·x} (k)^{2n}
= #' / |X|^{2Δ+} + Σ_{n≥0}^{∞} a_n □^n δ^d(x)
□ e^{-ikx} = -k^2 e^{-ikx}

$$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle$$

$$= G(x) = \int d^dk\, e^{-ik\cdot x}\, G(k)$$

$$= \# \int d^dk\, e^{-ik\cdot x}\, k^{2\nu} + \sum_{n\geq 0}^{\infty} a_n \int d^dk\, e^{-ikx}\, (k^2)^n$$

$$= \frac{\#'}{|x|^{2\Delta}} + \sum_{n\geq 0}^{\infty} a_n\, \Box^n\, \delta^d(x)$$

$$\Delta = \frac{d}{2} + \nu$$

$$\Box e^{-ikx} = -k^2 e^{-i\ldots}$$

$$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle = G(x) = \int d^dk\, e^{-ik\cdot x}\, G(k)$$

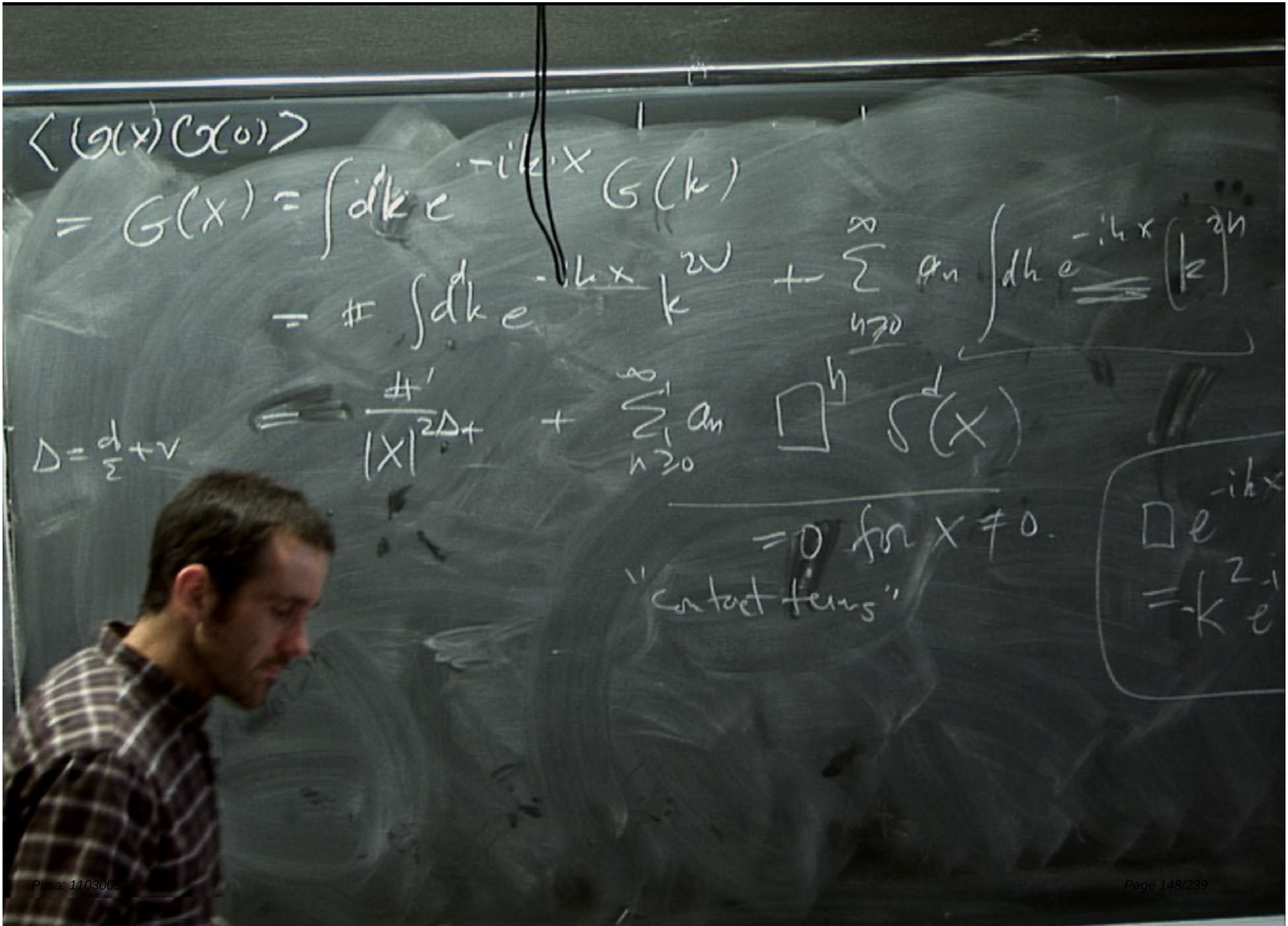
$$= \# \int d^dk\, e^{-ik\cdot x}\, k^{2\nu} + \sum_{n\geq 0}^{\infty} a_n \int d^dk\, e^{-ikx} \left(k^2\right)^n$$

$$\Delta = \frac{d}{2} + \nu$$

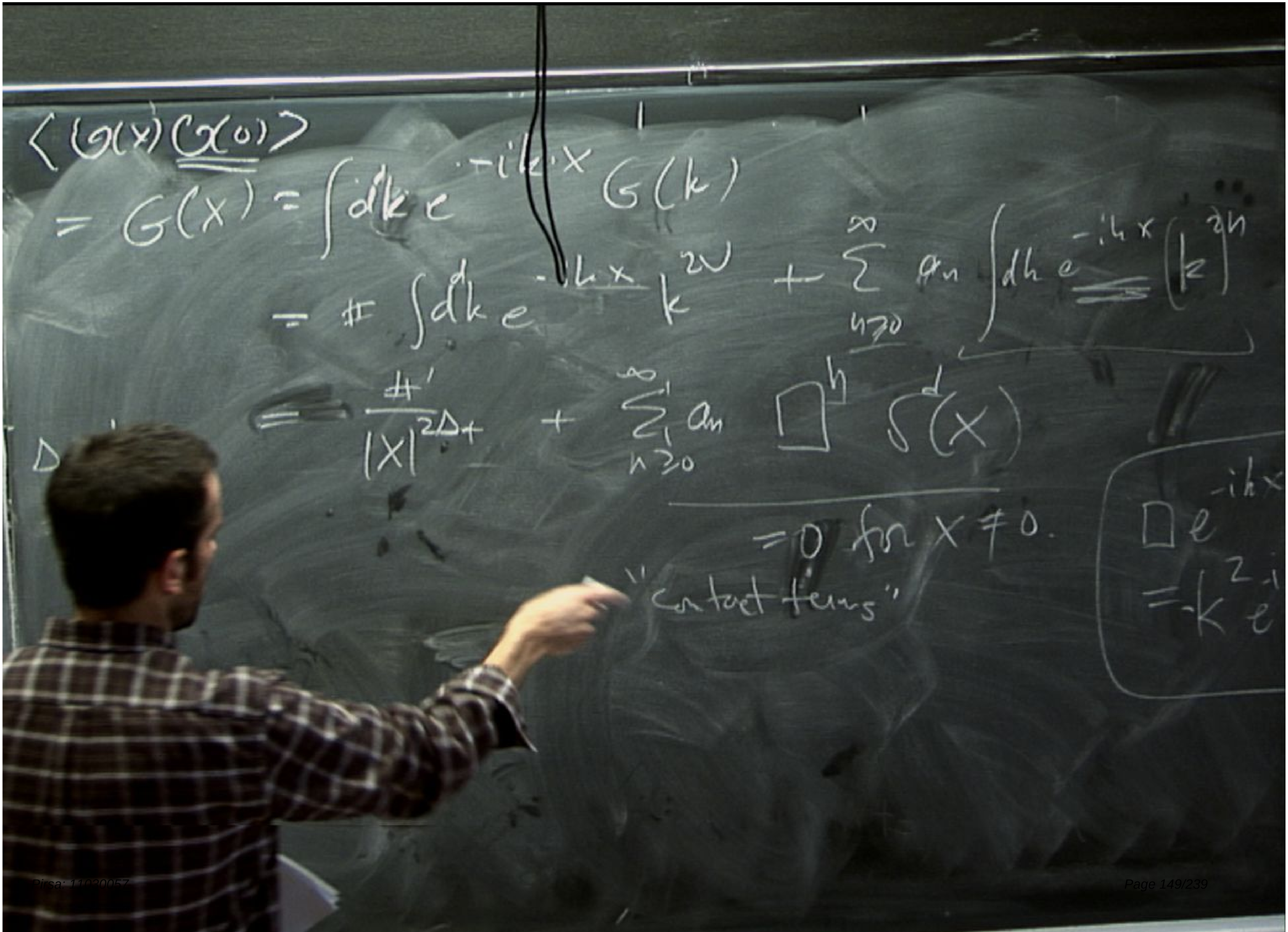
$$= \frac{\#'}{|x|^{2\Delta_+}} + \sum_{n\geq 0}^{\infty} a_n\, \Box^n\, \delta^d(x)$$

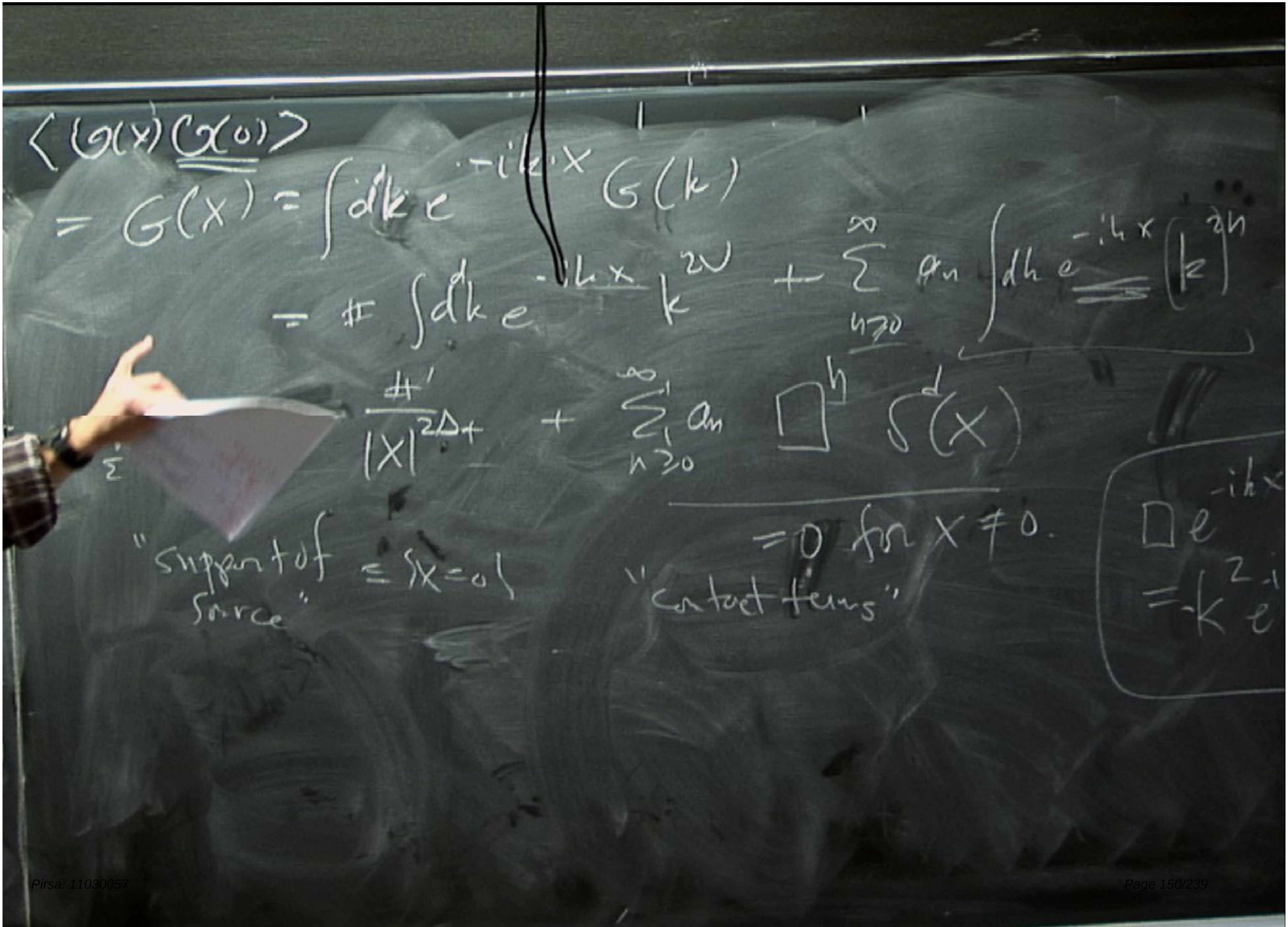
$$= 0 \text{ for } x \neq 0.$$

$$\Box\, e^{-ikx} = -k^2 e^{-i\ldots}$$



⟨O(x)O(0)⟩
= G(x) = ∫d^dk e^(-ik·x) G(k)
= # ∫d^dk e^(-ik·x) k^(2ν) + Σ_(n≥0)^∞ a_n ∫d^dk e^(-ik·x) k^(2n)
= #'/|x|^(2Δ) + Σ_(n≥0)^∞ a_n □^n δ^d(x)
Δ = d/2 + ν
= 0 for x ≠ 0.
"contact terms"
□ e^(-ikx)
= -k^2 e


=0 for x ≠ 0.
"contact terms"


⟨O(x) O(0)⟩
= G(x) = ∫ d^d k e^{-ik·x} G(k)
= # ∫ d^d k e^{-ik·x} k^{2ν} + Σ_{n≥0}^∞ a_n ∫ d^d k e^{-ikx} (k)^{2n}
#' / |x|^{2Δ+} + Σ_{n≥0}^∞ a_n □^n δ^d(x)
= 0 for x ≠ 0.
"support of source" ⊆ {x = 0}
"contact terms"
□ e^{-ikx} = -k^2 e

$$\langle \mathcal{O}(x)\,\mathcal{O}(0)\rangle$$

$$= G(x) = \int d^d k\, e^{-ik\cdot x}\, G(k)$$

$$= \# \int d^d k\, e^{-ik\cdot x}\, k^{2\nu} + \sum_{n\geq 0}^{\infty} a_n \int d^d k\, e^{-ikx} (k^2)^n$$

$\Delta = \frac{d}{2} + \nu$

$$= \frac{\#'}{|x|^{2\Delta_+}} + \sum_{n\geq 0}^{\infty} a_n\, \Box^n\, \delta^d(x)$$

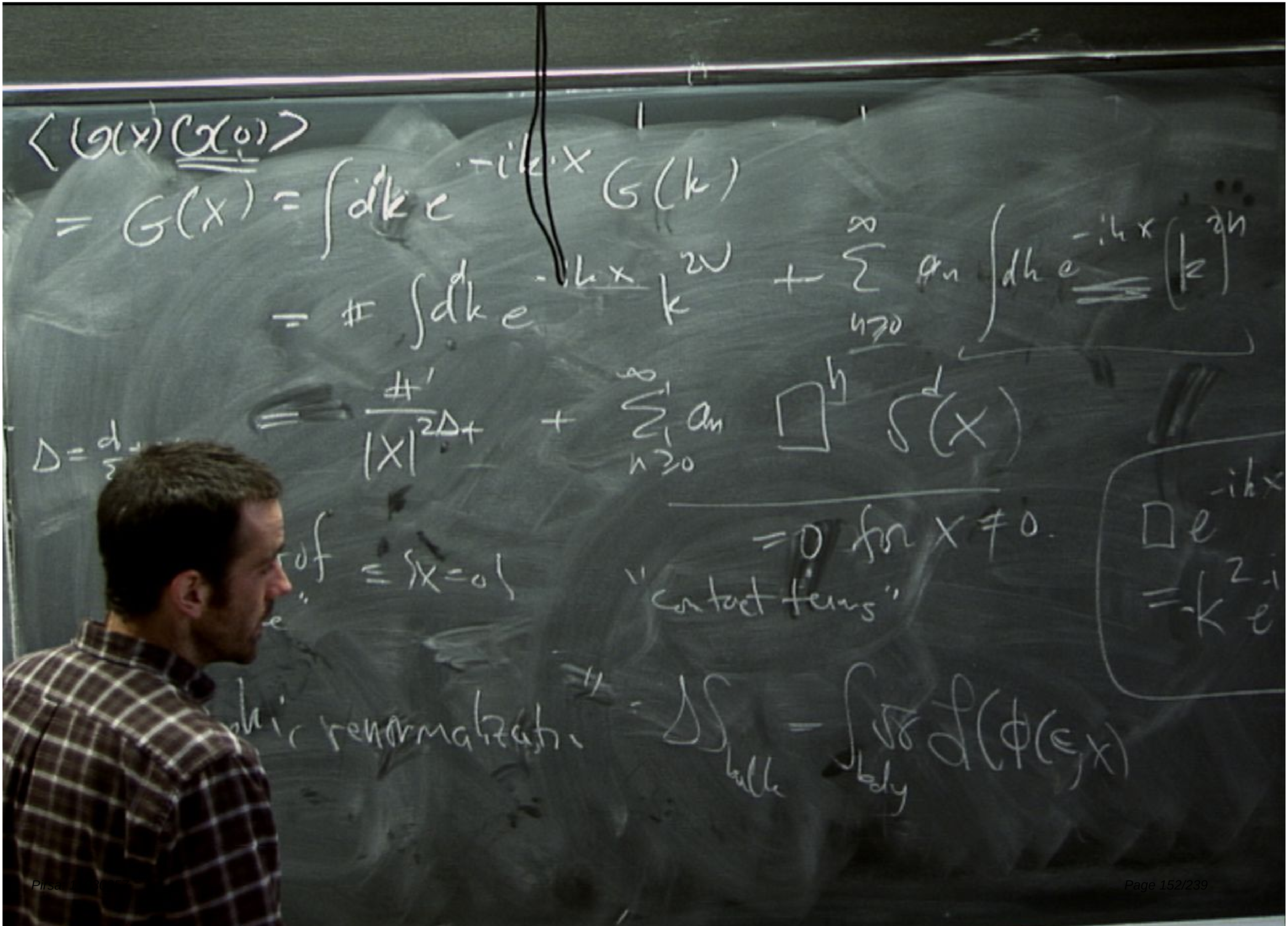
$= 0$ for $x \neq 0$.

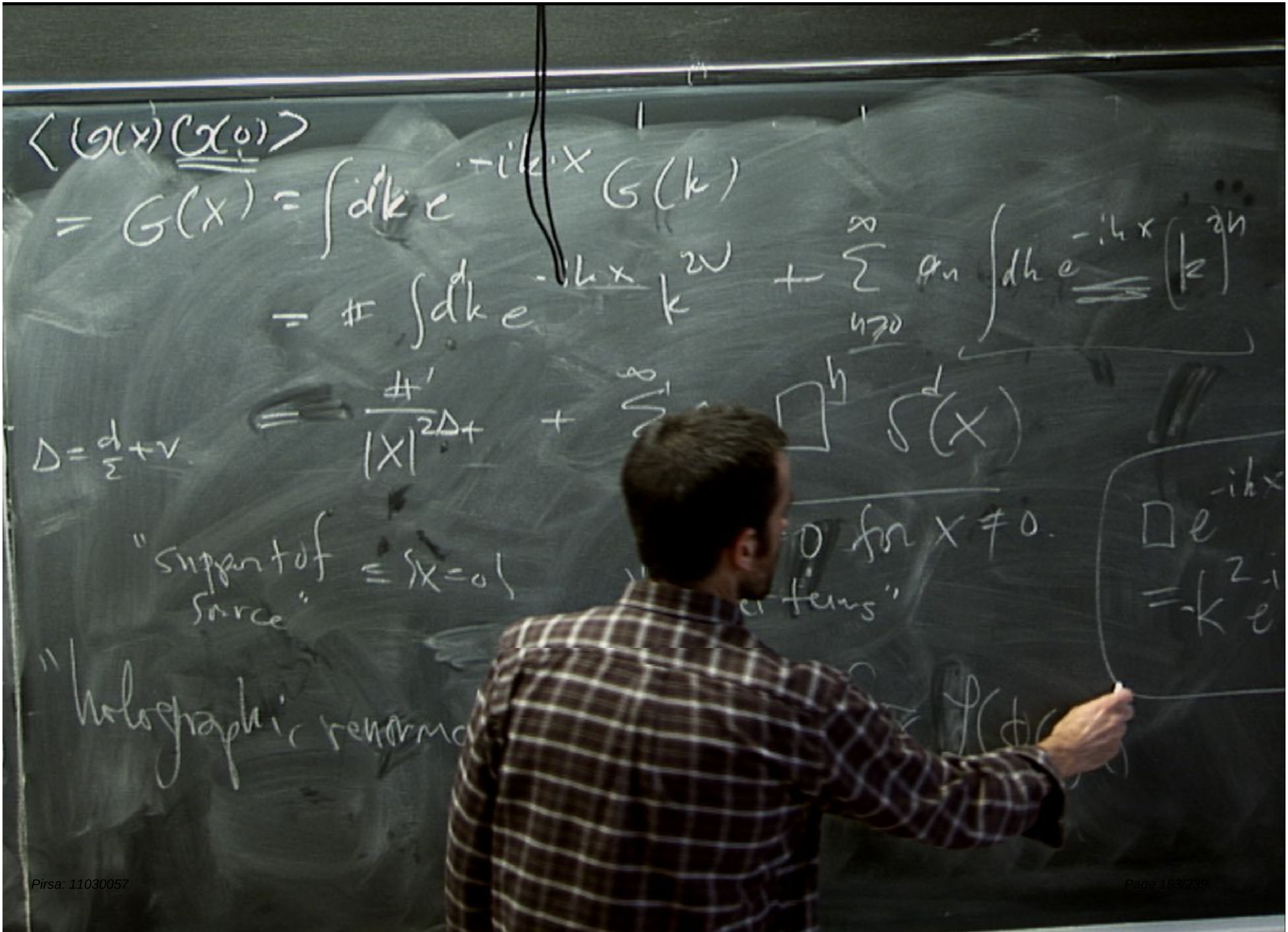
"support of source" $\equiv \{x = 0\}$

"contact terms"

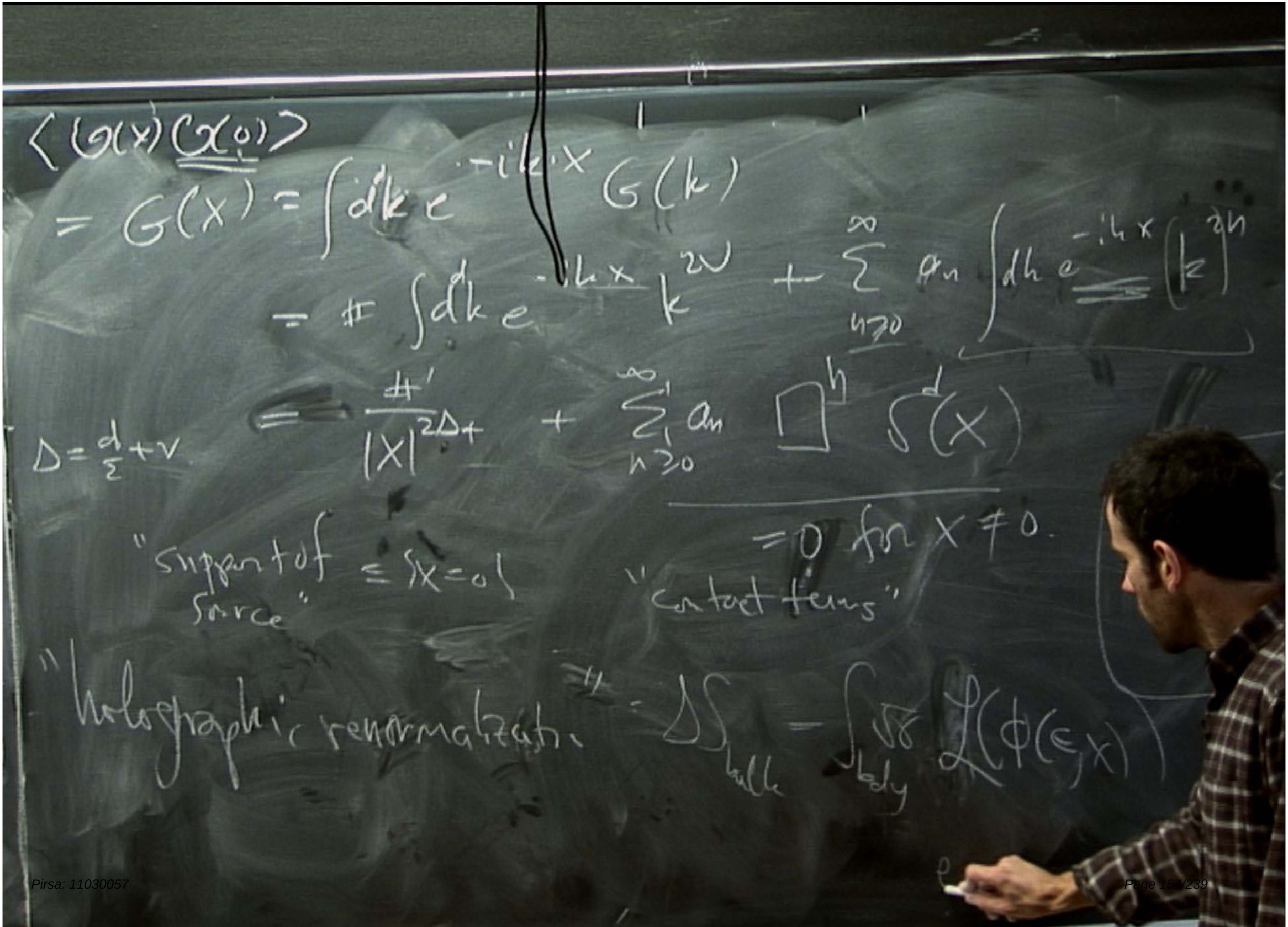
$$\Box e^{-ikx} = -k^2 e^{-i\ldots}$$

"holographic renormalization"


= 0 for x ≠ 0.
"contact terms"



⟨O(x) O(0)⟩
= G(x) = ∫d^d k e^{-ik·x} G(k)
= # ∫d^d k e^{-ik·x} k^{2ν} + Σ a_n ∫d^d k e^{-ik·x} (k)^{2n}
= #' / |x|^{2Δ+} + Σ □^n δ^d(x)
Δ = d/2 + ν
"support of source" ⊆ {x=0}
0 for x ≠ 0.
terms"
□ e^{-ikx}
= -k^2 e
"holographic renorma



⟨O(x) O(0)⟩
= G(x) = ∫d^d k e^{-ik·x} G(k)
= # ∫d^d k e^{-ik·x} k^{2ν} + Σ_{n≥0}^∞ a_n ∫d^d k e^{-ik·x} (k^2)^n
Δ = d/2 + ν
= #'/|x|^{2Δ} + Σ_{n≥0}^∞ a_n □^n δ^d(x)
= 0 for x ≠ 0.
"support of source" ⊆ {x=0}
"contact terms"
"holographic renormalization"
= S_bulk − ∫_bdy √γ L(φ(ε,x))

$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle$

$= G(x) = \int d^dk\, e^{-ik\cdot x} G(k)$

$= \# \int d^dk\, e^{-ik\cdot x} k^{2\nu} + \sum_{n\geq 0}^{\infty} a_n \int d^dk\, e^{-ik\cdot x} |k|^{2n}$

$\Delta = \frac{d}{2}+\nu$

$= \frac{\#'}{|x|^{2\Delta_+}} + \sum_{n\geq 0}^{\infty} a_n \Box^n \delta^d(x)$

$= 0$ for $x \neq 0$.

"support of source" $= \{x=0\}$

"contact terms"

$\Box e^{-ikx} = -k^2 e^{-i\dots}$

"holographic renormalization" $= \Delta S_{bulk} - \int_{bdy} \sqrt{\gamma}\, \mathcal{L}(\phi(\epsilon, x))$

e.g. $\mathcal{L} =$

$$\langle \mathcal{O}(x)\,\mathcal{O}(0)\rangle$$

$$= G(x) = \int d^dk\, e^{-ik\cdot x}\, G(k)$$

$$= \# \int d^dk\, e^{-ik\cdot x}\, k^{2\nu} + \sum_{n\geq 0}^{\infty} a_n \int d^dk\, e^{-ik\cdot x}\, (k^2)^n$$

$$= \frac{\#'}{|x|^{2\Delta_+}} + \sum_{n\geq 0}^{\infty} a_n\, \Box^n\, \delta^d(x)$$

$\Delta = \frac{d}{2} + \nu$

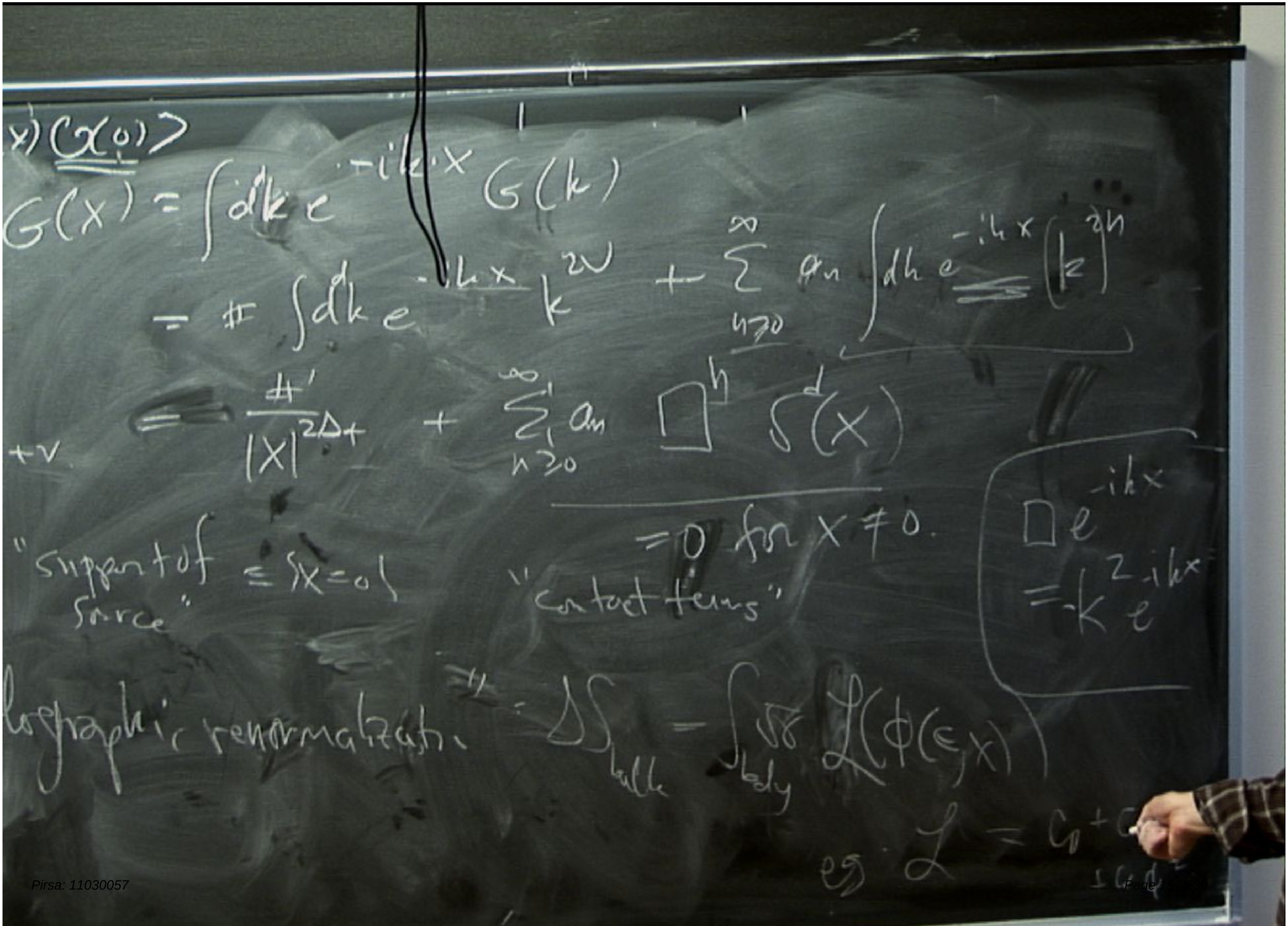
"support of source" $= \{x=0\}$

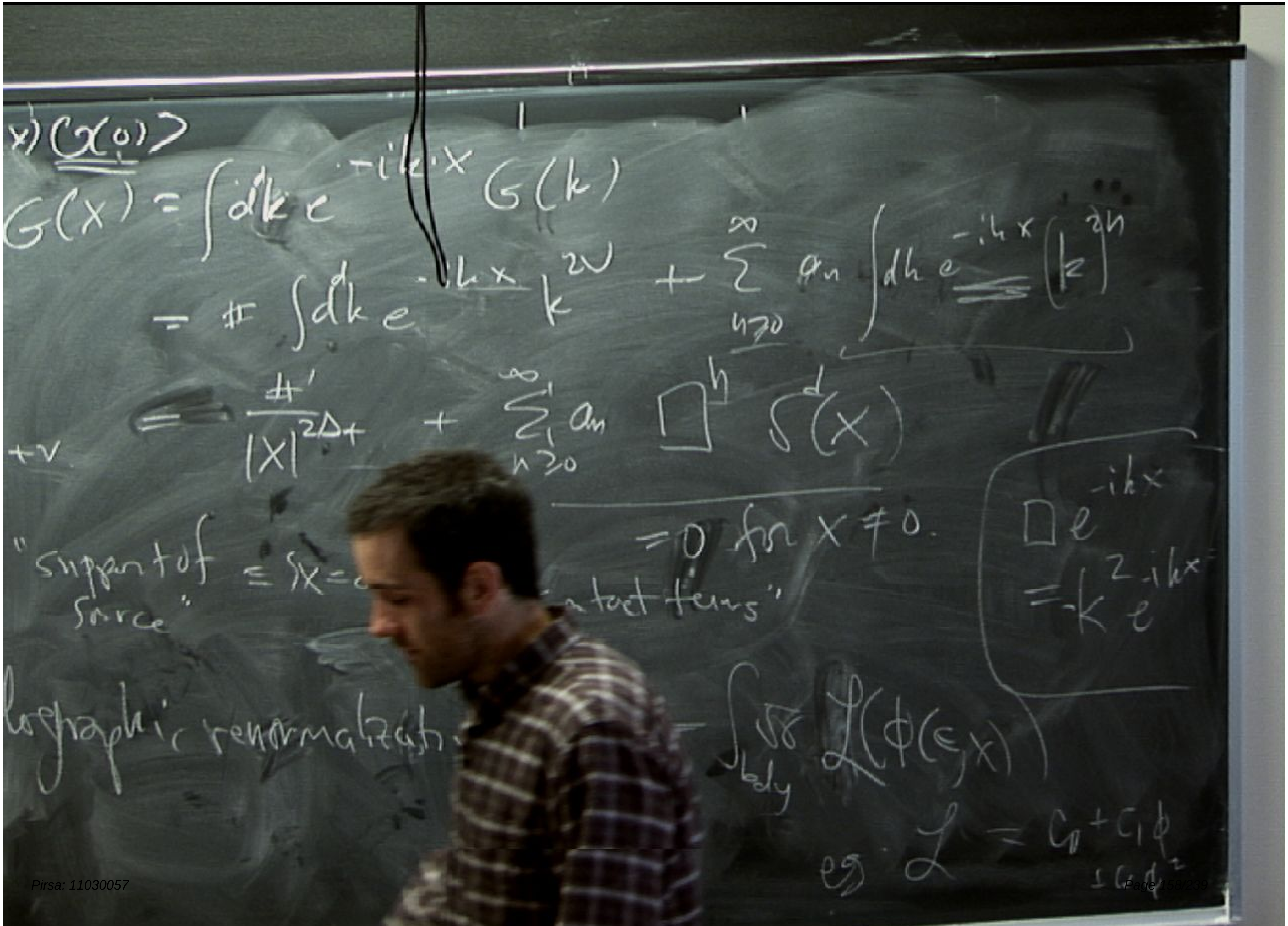
$= 0$ for $x \neq 0$. "contact terms"

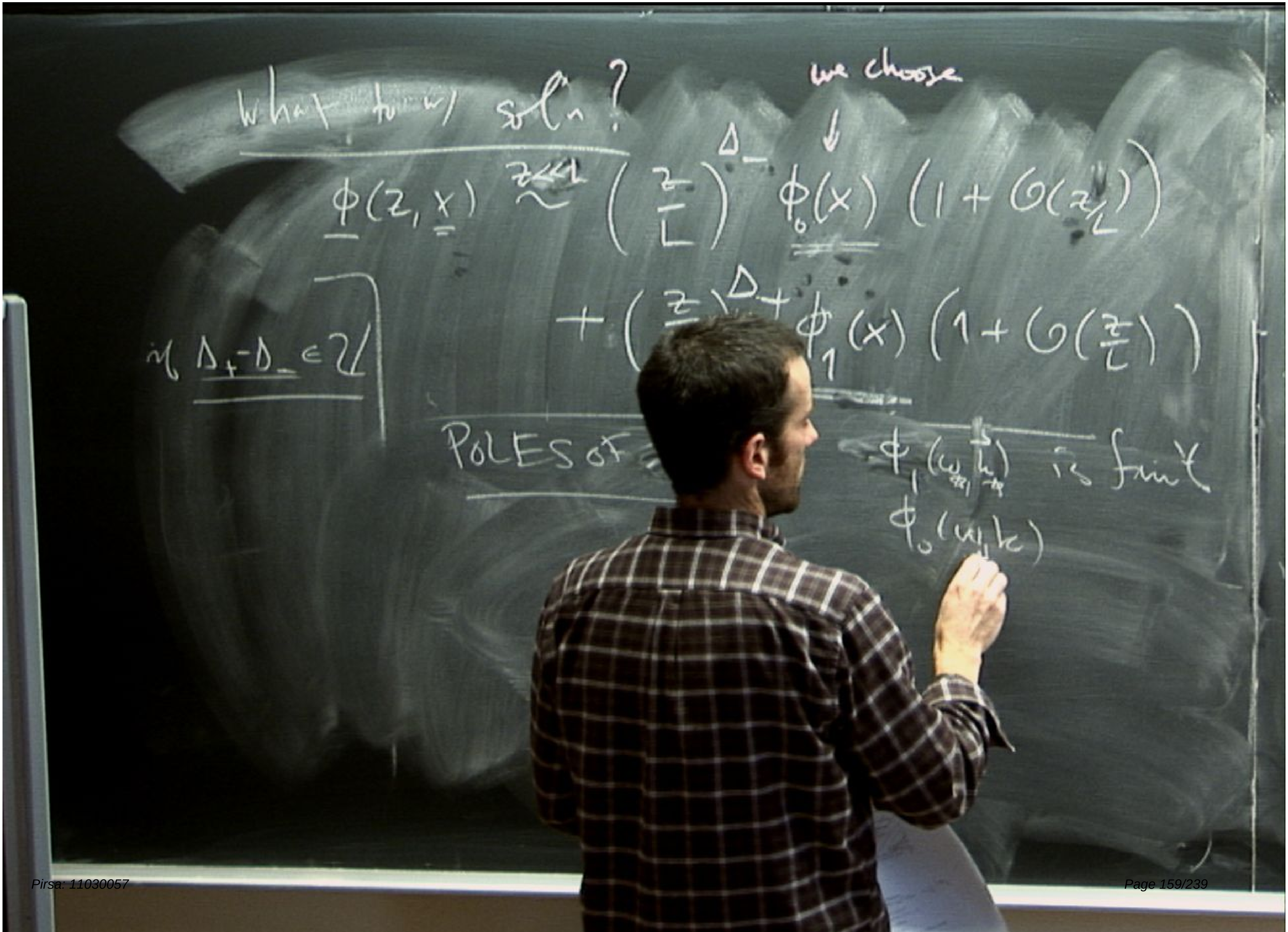
$$\Box e^{-ikx} = -k^2 e^{-i\dots}$$

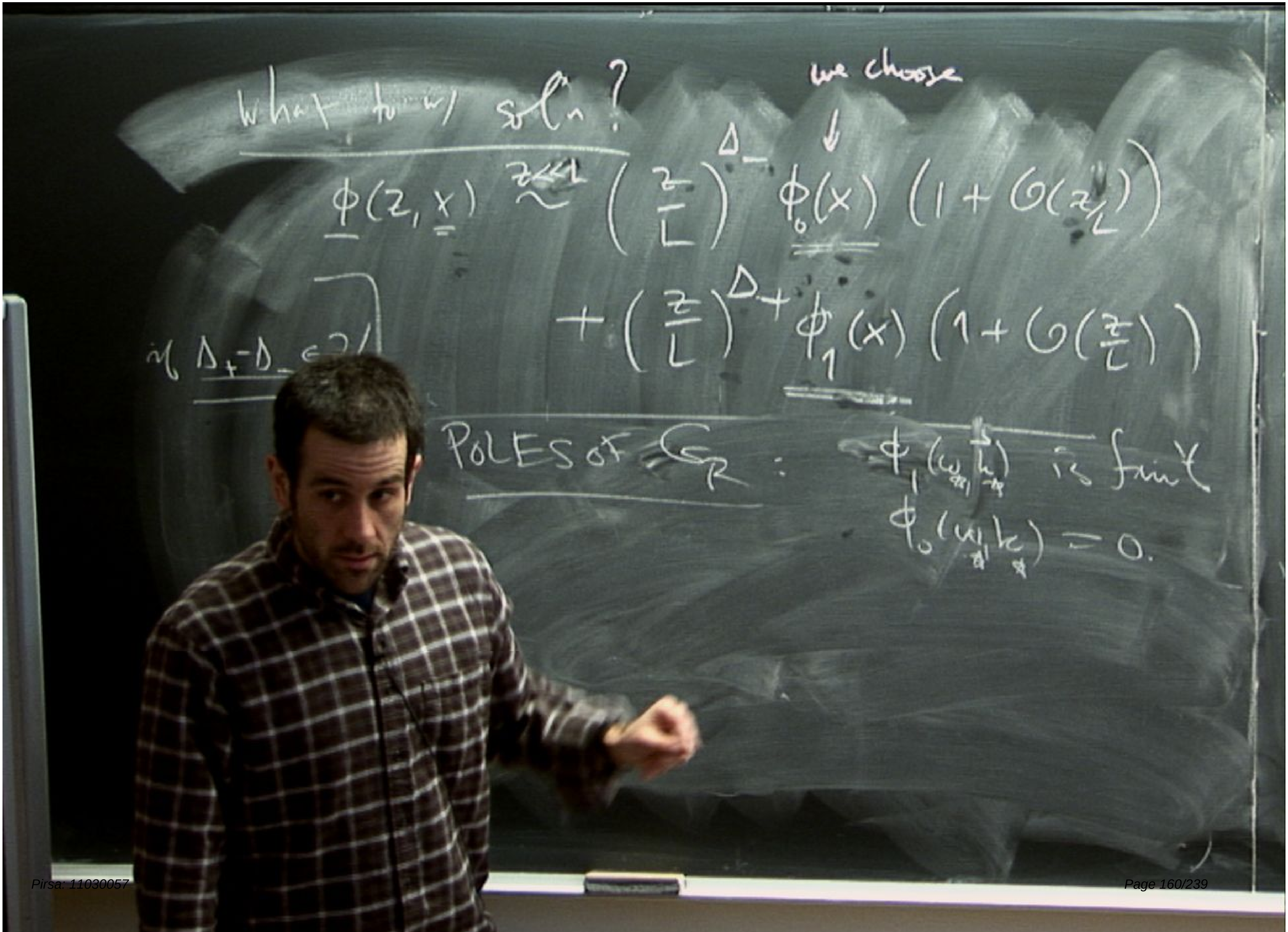
"holographic renormalization" $= \Delta S_{bulk} = \int_{bdy} \sqrt{\gamma}\, \mathcal{L}(\phi(\epsilon, x))$

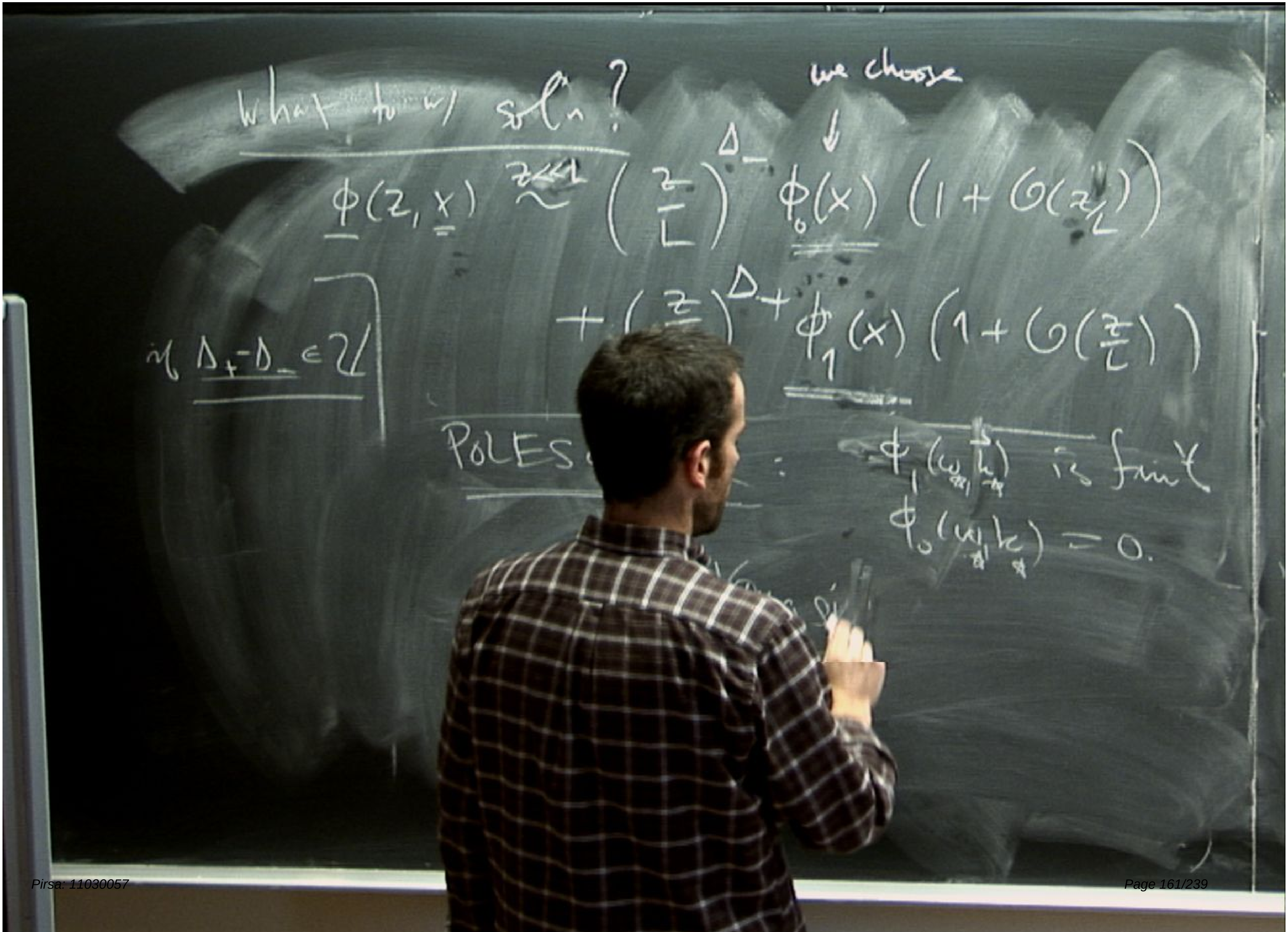
eg $\mathcal{L} = c_0$

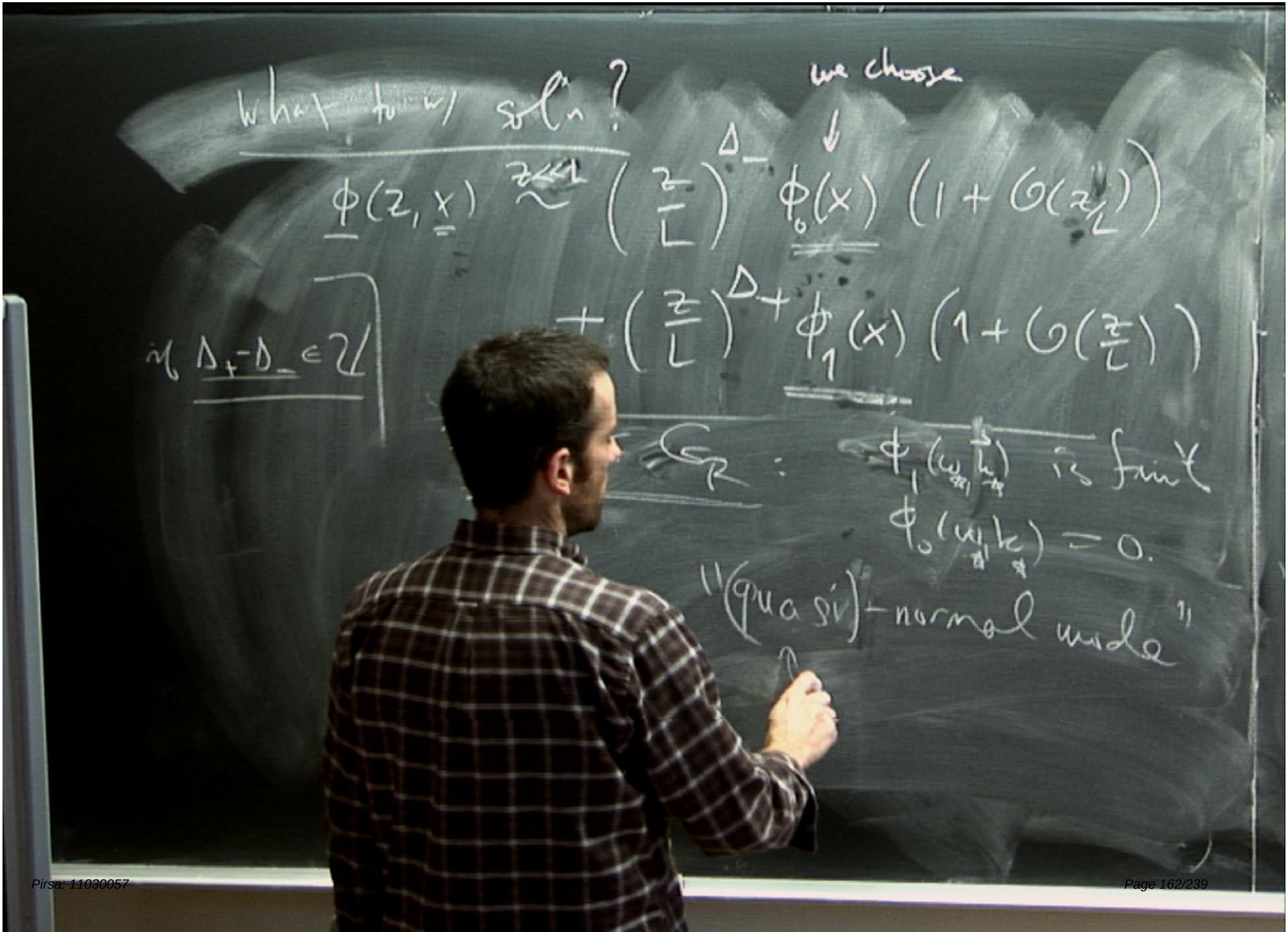

G(x) = ∫ dk e^{-ik·x} G(k)
= # ∫ dk e^{-ik x} k^{2ν} + Σ a_n ∫ dk e^{-ik x} |k|^{2n}
= #' / |x|^{2Δ+} + Σ a_n □^n δ^d(x)
= 0 for x ≠ 0.
"support of source" ⊆ {x = 0}
"contact terms"
□ e^{-ikx} = -k^2 e^{-ikx}
holographic renormalization
ΔS_bulk = ∫_bdy √γ ℒ(φ(ε, x))
eg. ℒ = c_0 + c

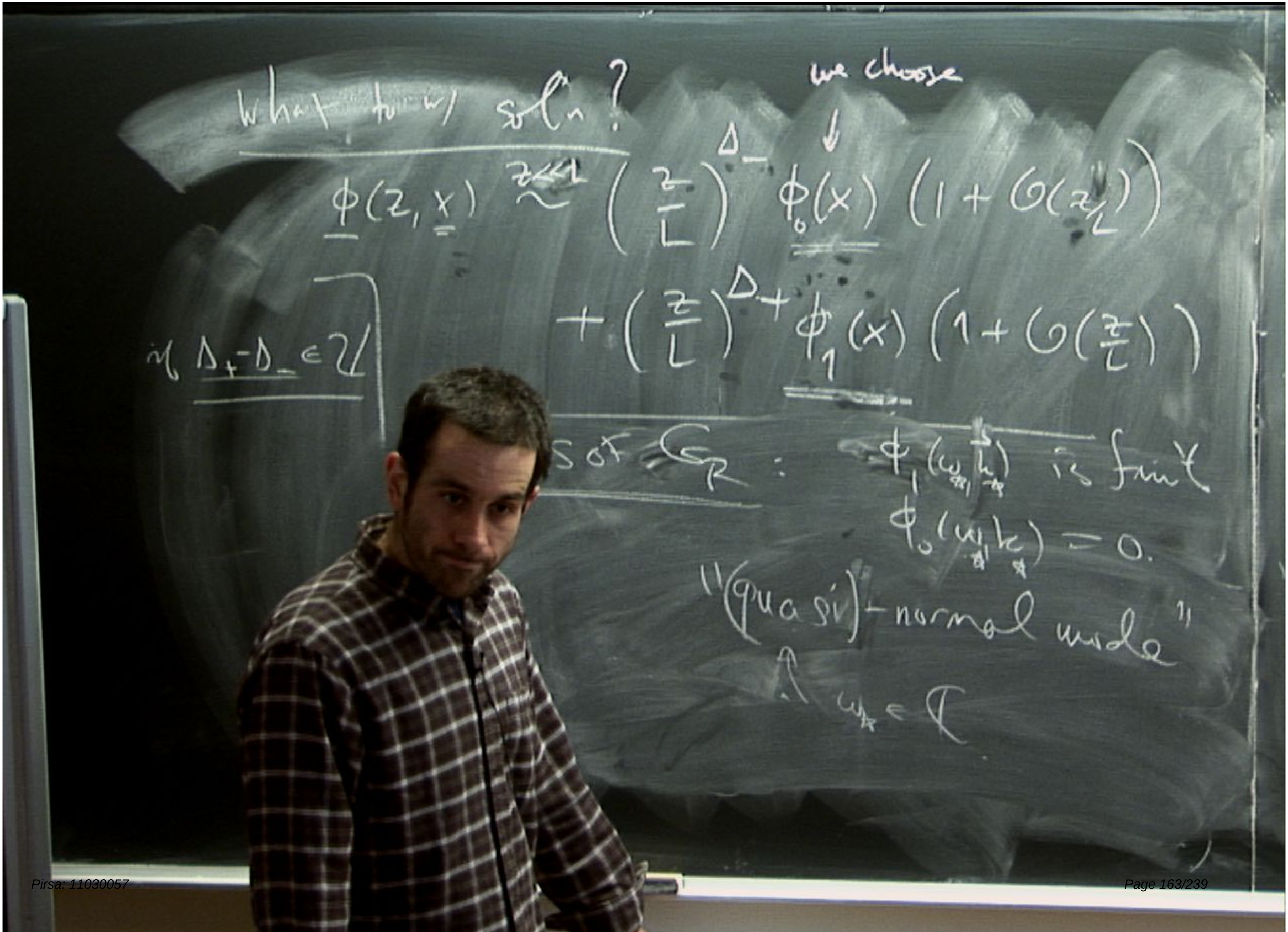

G(x) = ∫d^dk e^{-ik·x} G(k)
= # ∫d^dk e^{-ik·x} k^{2ν} + Σ_{n≥0}^{∞} a_n ∫d^dk e^{-ik·x} (k^2)^n
= #' / |x|^{2Δ+} + Σ_{n≥0}^{∞} a_n □^n δ^d(x)
= 0 for x ≠ 0.
"support of source"
"contact terms"
□ e^{-ikx} = -k^2 e^{-ikx}
holographic renormalization
∫_{bdy} √γ L(φ(ε,x))
eg L = c_0 + c_1 φ + c_2 φ^2


what to w/ soln ?
we choose
φ(z, x) ~ (z/L)^Δ- φ0(x) (1 + O(z/L))
+ (z/L)^Δ+ φ1(x) (1 + O(z/L))
if Δ+ - Δ- ∈ ℤ
POLES OF
φ1(ω, k) is finite
φ0(ω, k)

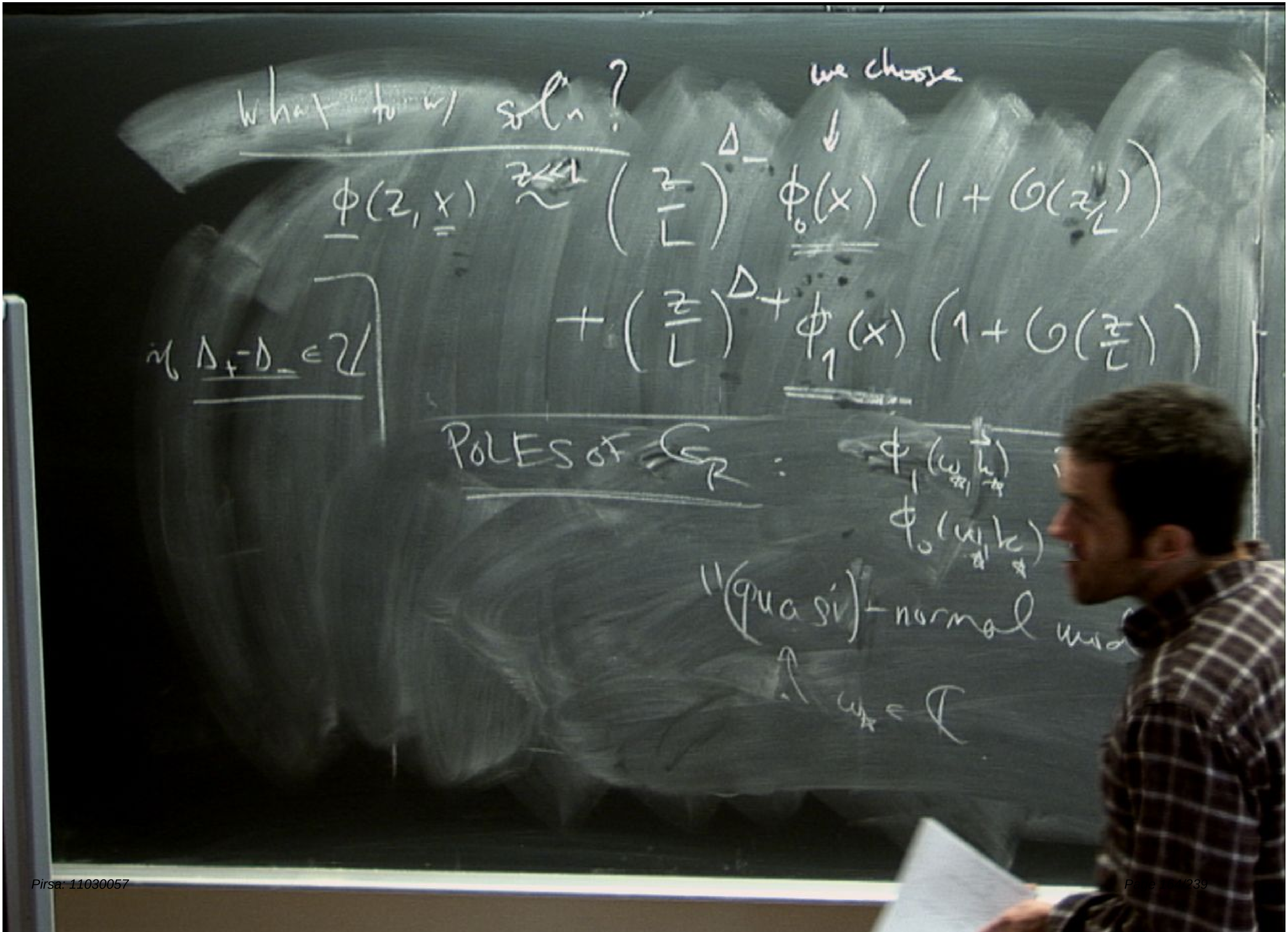

what to w/ soln ?
we choose
POLES OF
is finite
= 0.

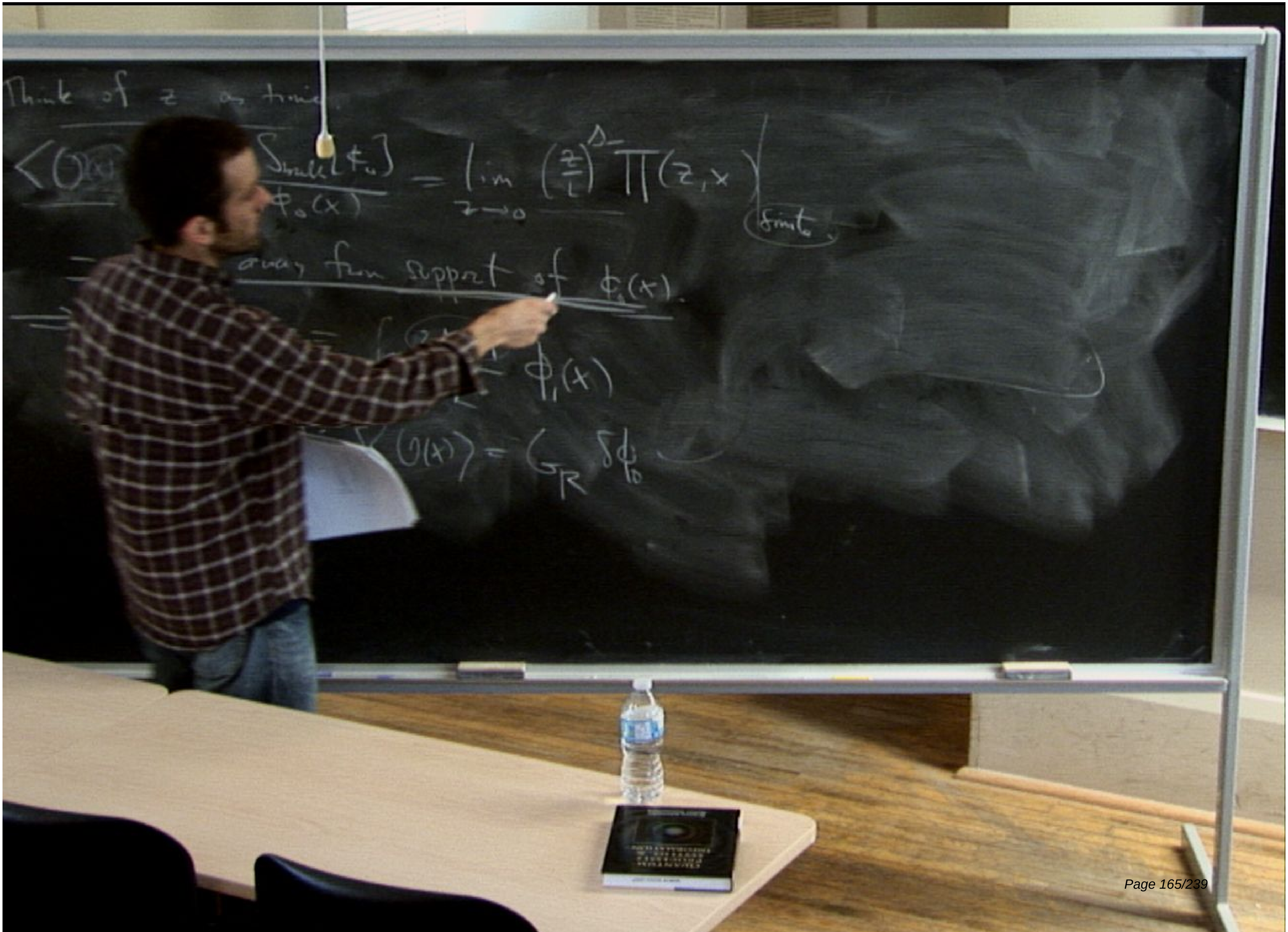

what to w/ soln ?
we choose
φ(z, x) ~ (z/L)^Δ- φ0(x) (1 + O(z/L))
+ (z/L)^Δ+ φ1(x) (1 + O(z/L))
if Δ+ - Δ- ∈ Z
POLES
φ1(ω*, k*) is finite
φ0(ω*, k*) = 0.

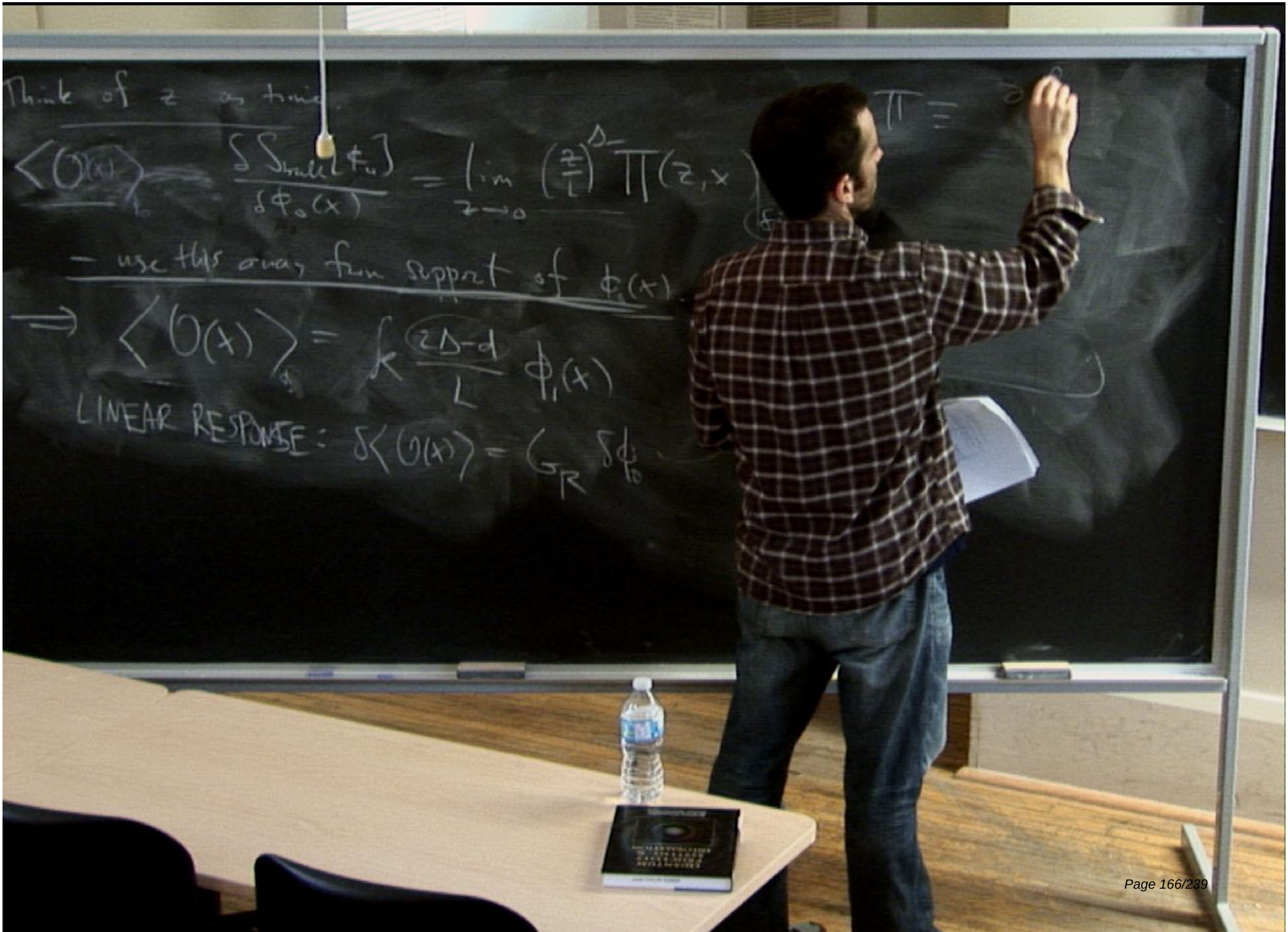

what to w/ soln ?
we choose
φ(z, x) ~ (z/L)^Δ− φ0(x) (1 + O(z/L))
+ (z/L)^Δ+ φ1(x) (1 + O(z/L))
if Δ+ − Δ− ∈ Z
φ1(ω, k) is finite
φ0(ω, k) = 0.
"(quasi)-normal mode"

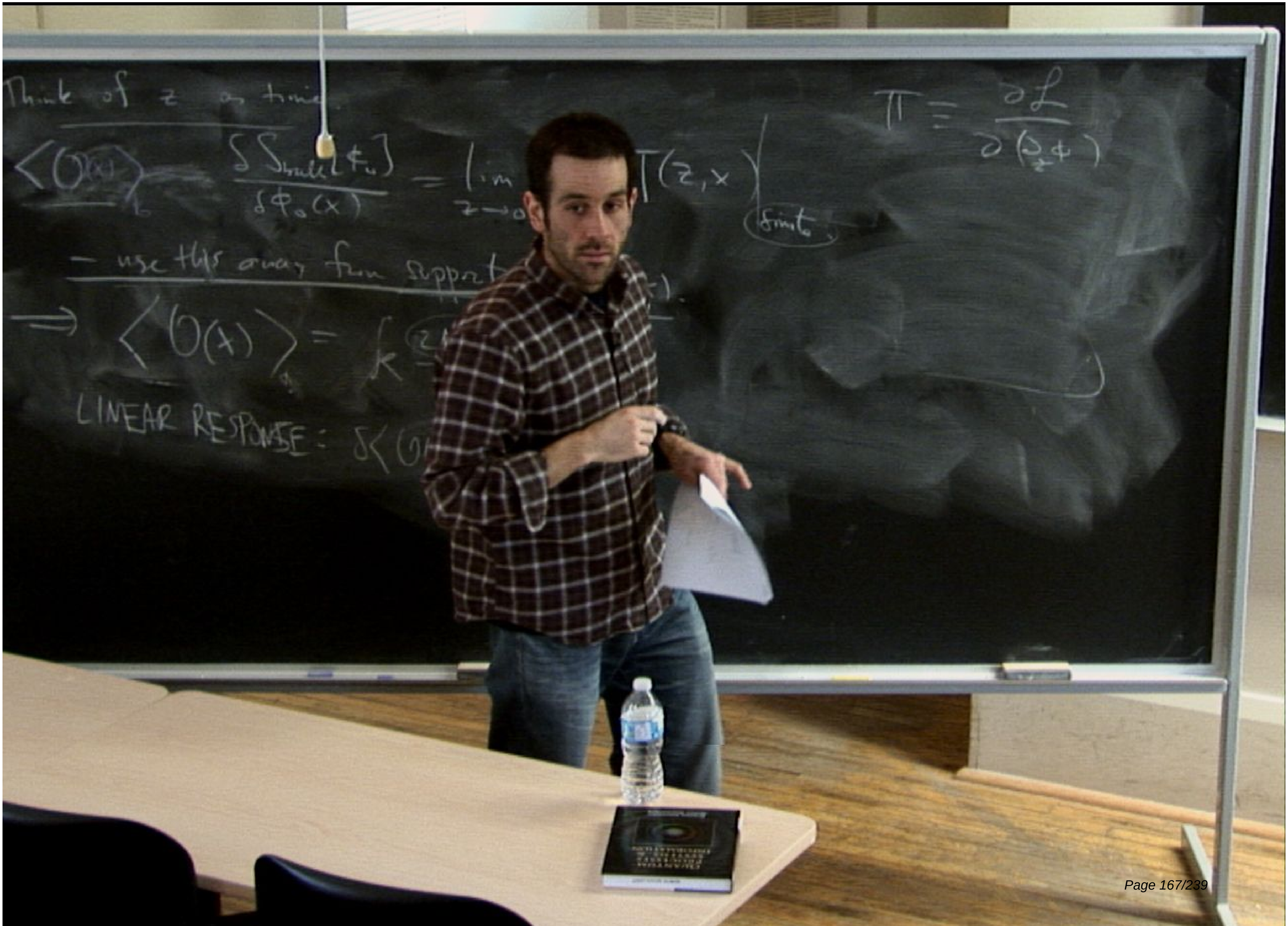


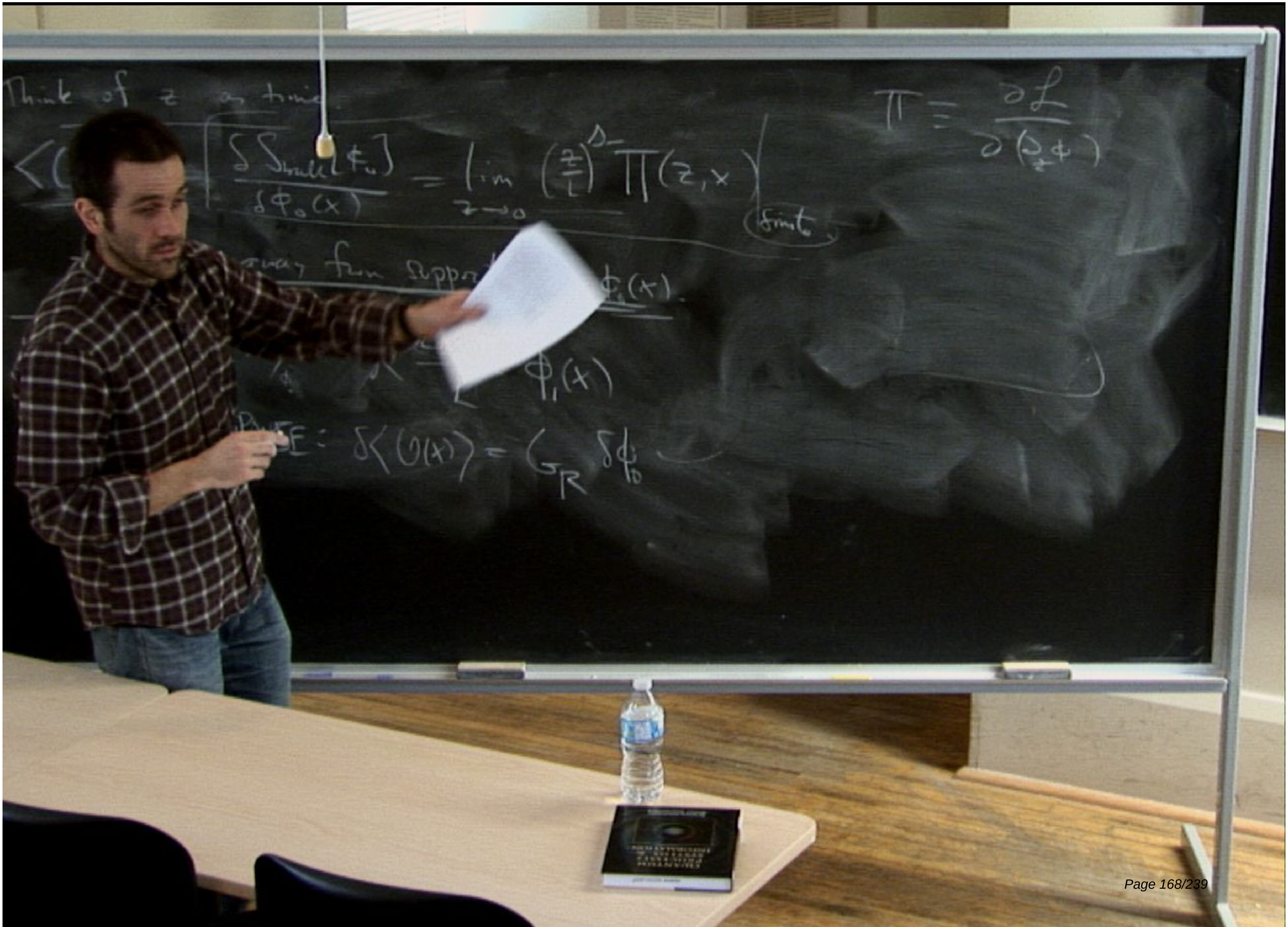
what to w/ soln ?
we choose
is finite
"(quasi)-normal mode"

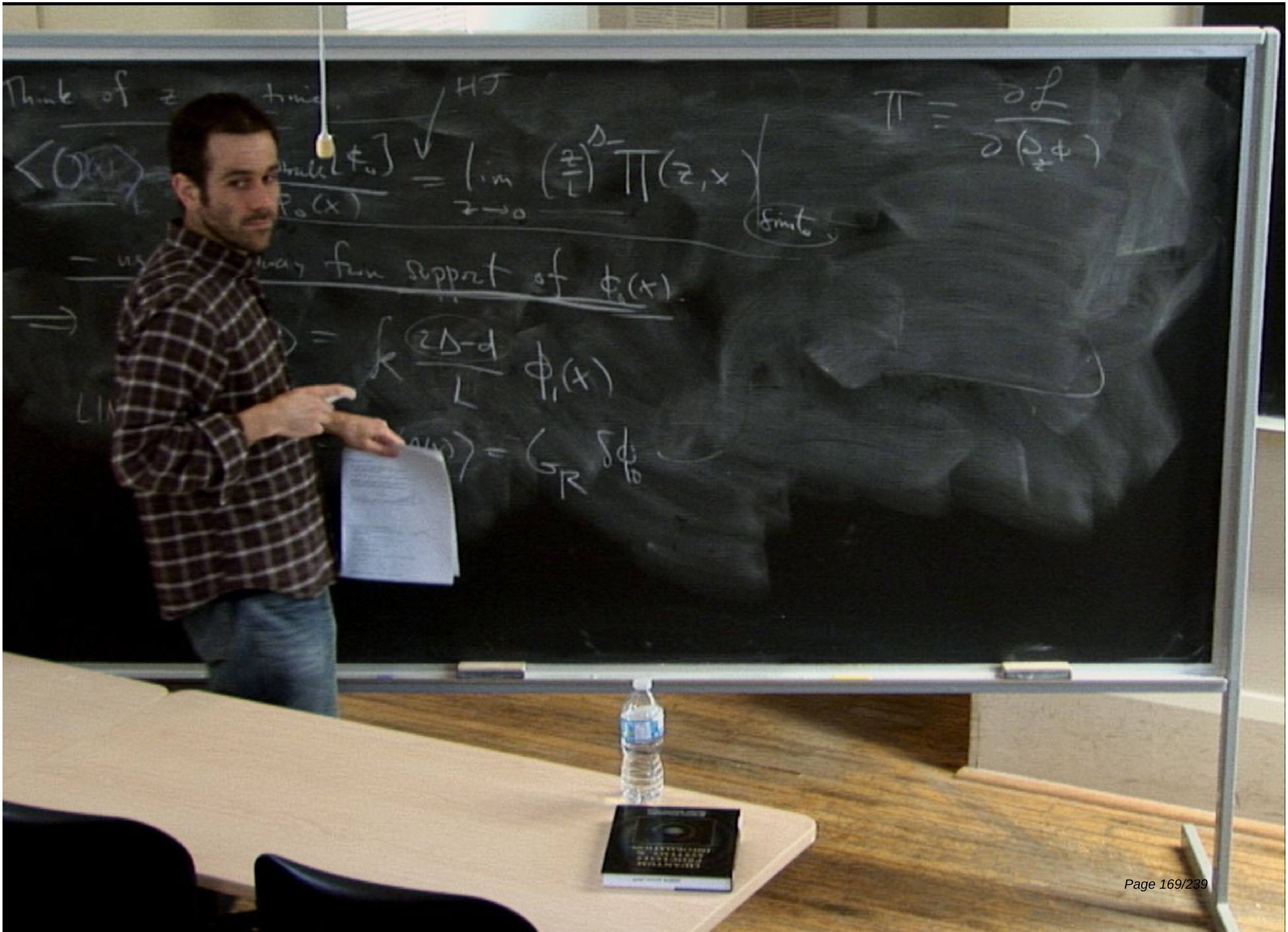

what to w/ sol'n ?
we choose
$\phi(z, x) \overset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) \left(1 + O(z/L)\right)$
$+ \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) \left(1 + O(\frac{z}{L})\right)$
if $\Delta_+ - \Delta_- \in \mathbb{Z}$
POLES OF G_R :
"(quasi)-normal
$\omega_* \in \mathbb{C}$

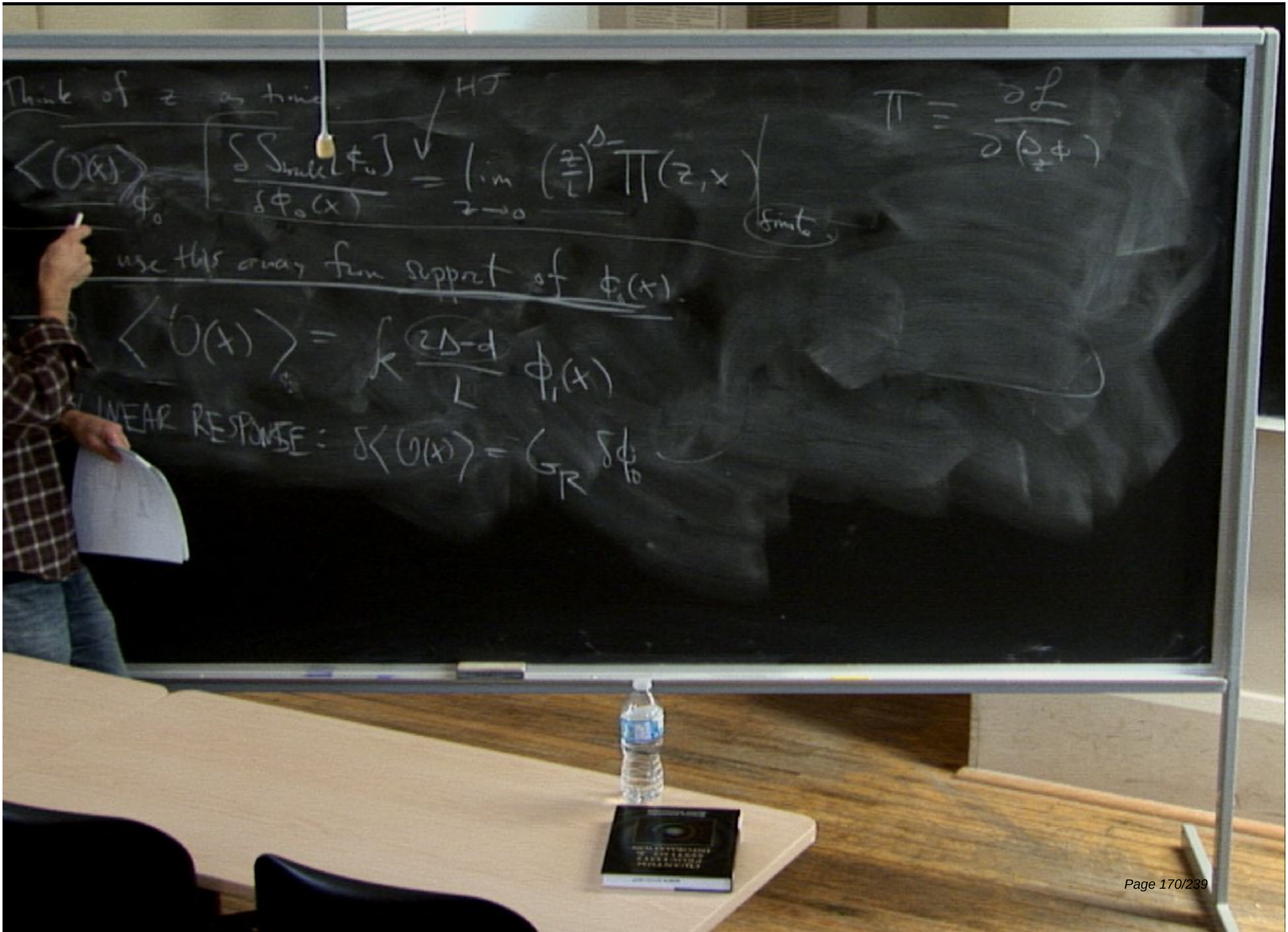

Think of z as time
finite
away from support of ϕ1(x)

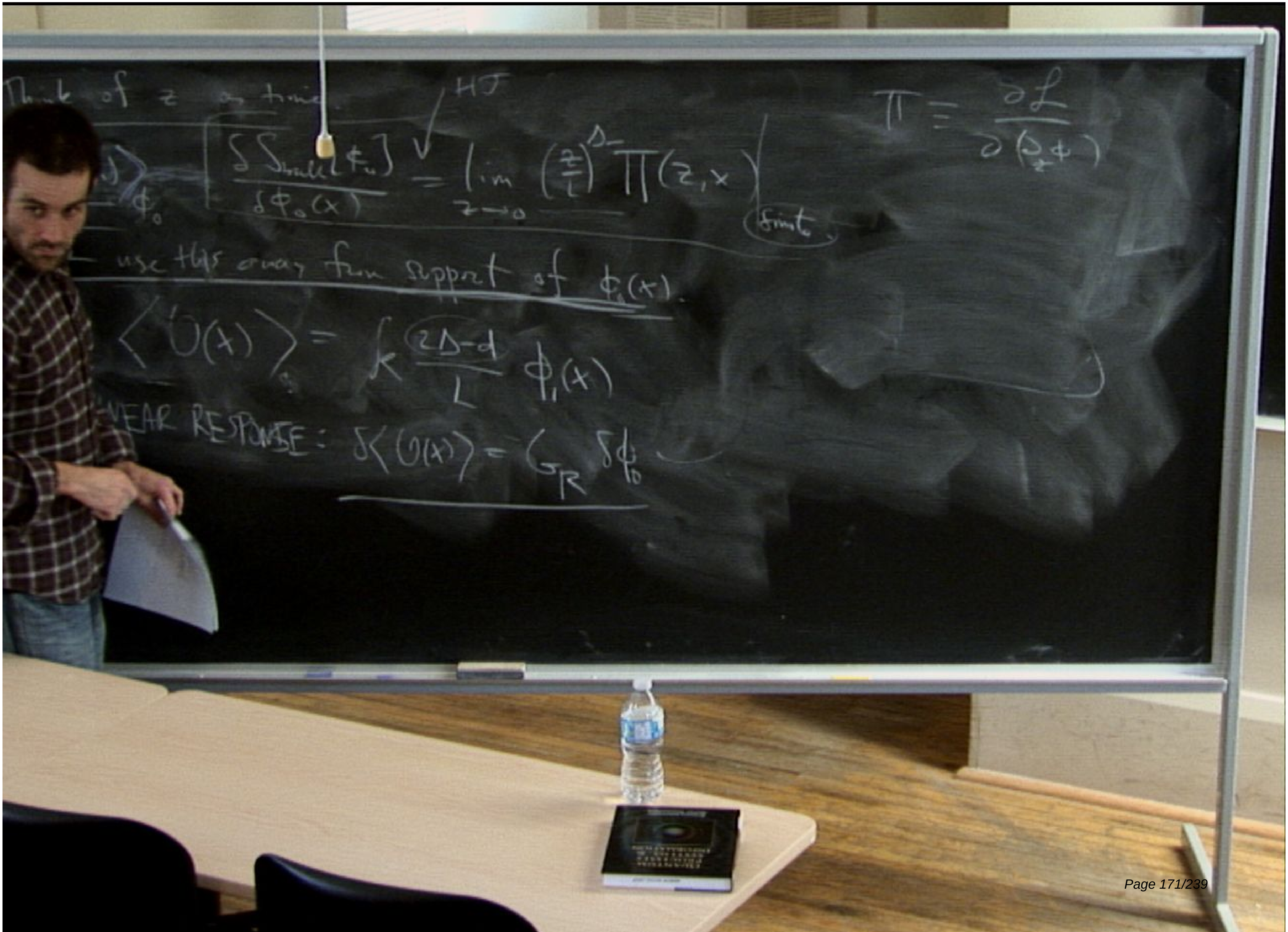

Think of z as time
– use this away from support of ϕ₁(x)
LINEAR RESPONSE:

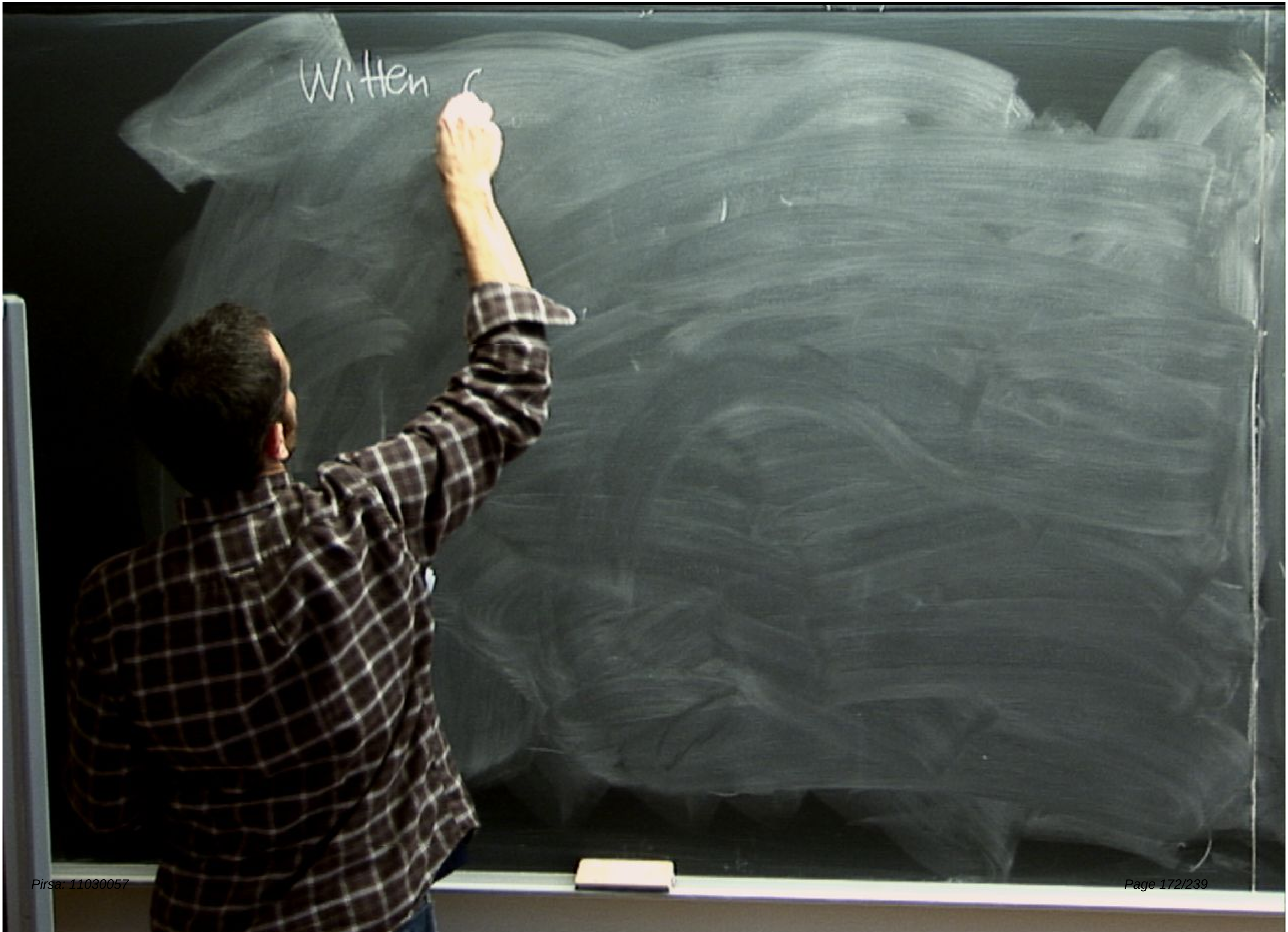

Think of z as time.
finite
LINEAR RESPONSE:

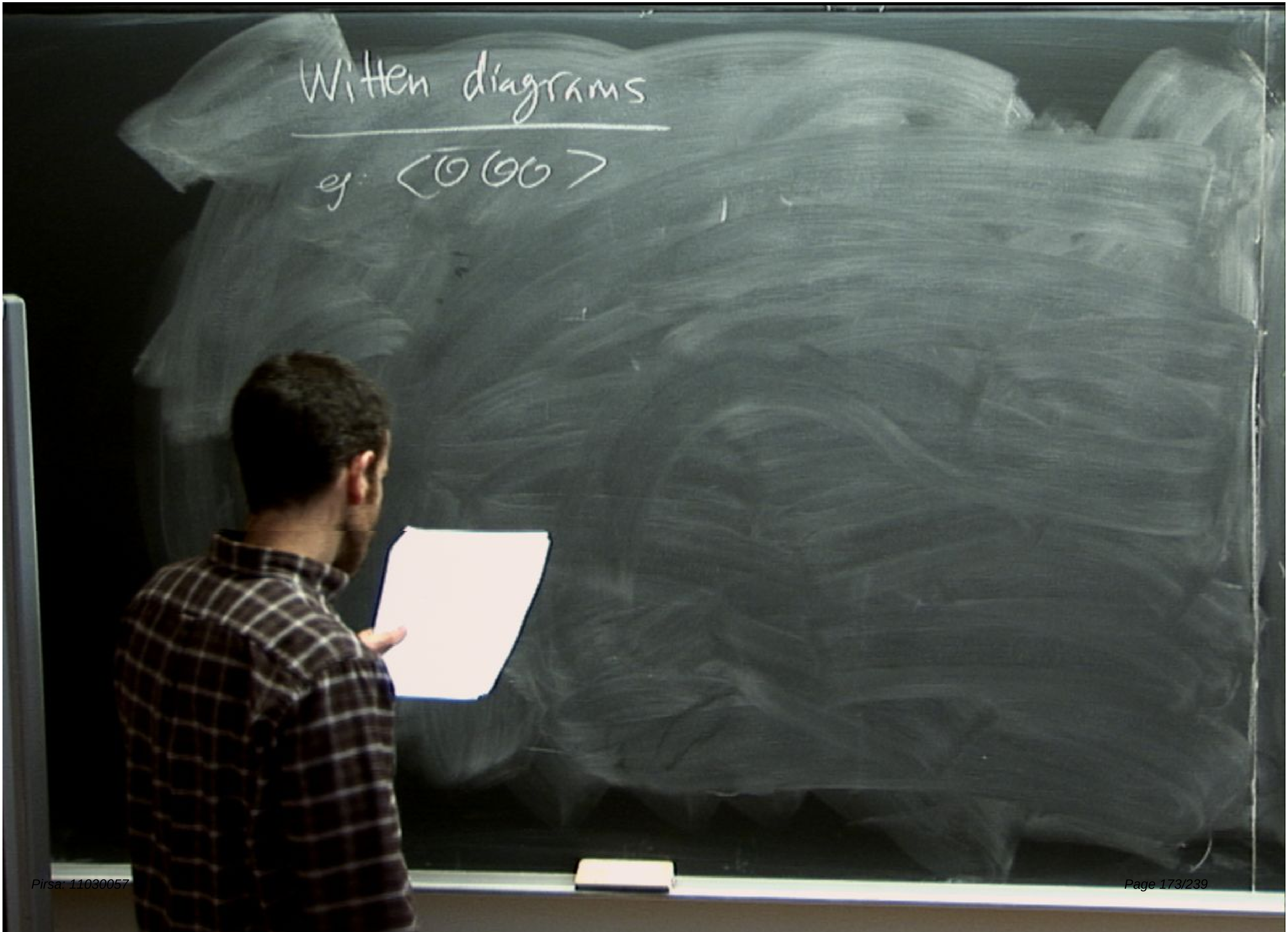

Think of z as time
finite

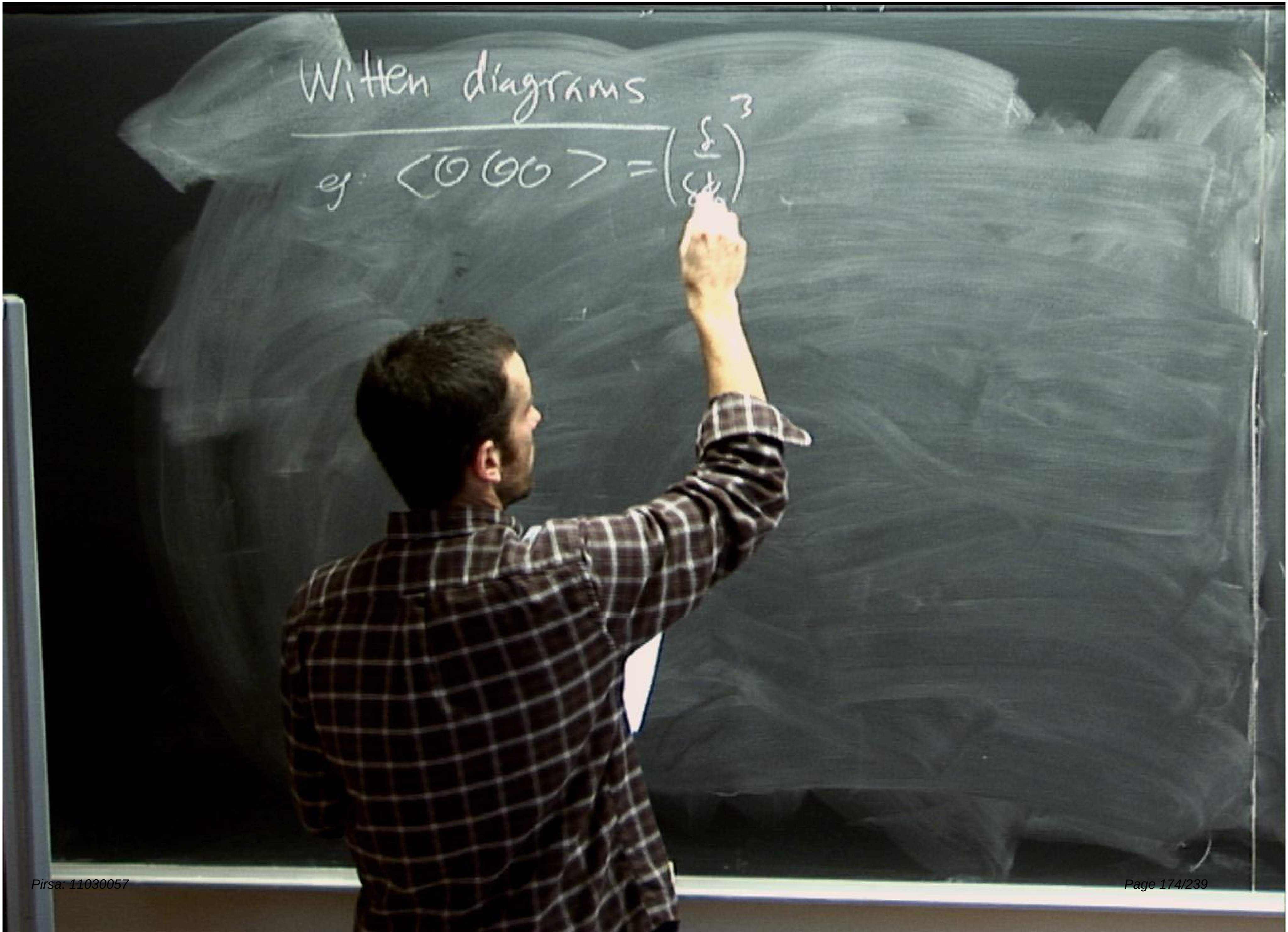

Think of z
time
HJ
lim
finite
support of
Π = ∂L / ∂(∂_z φ)

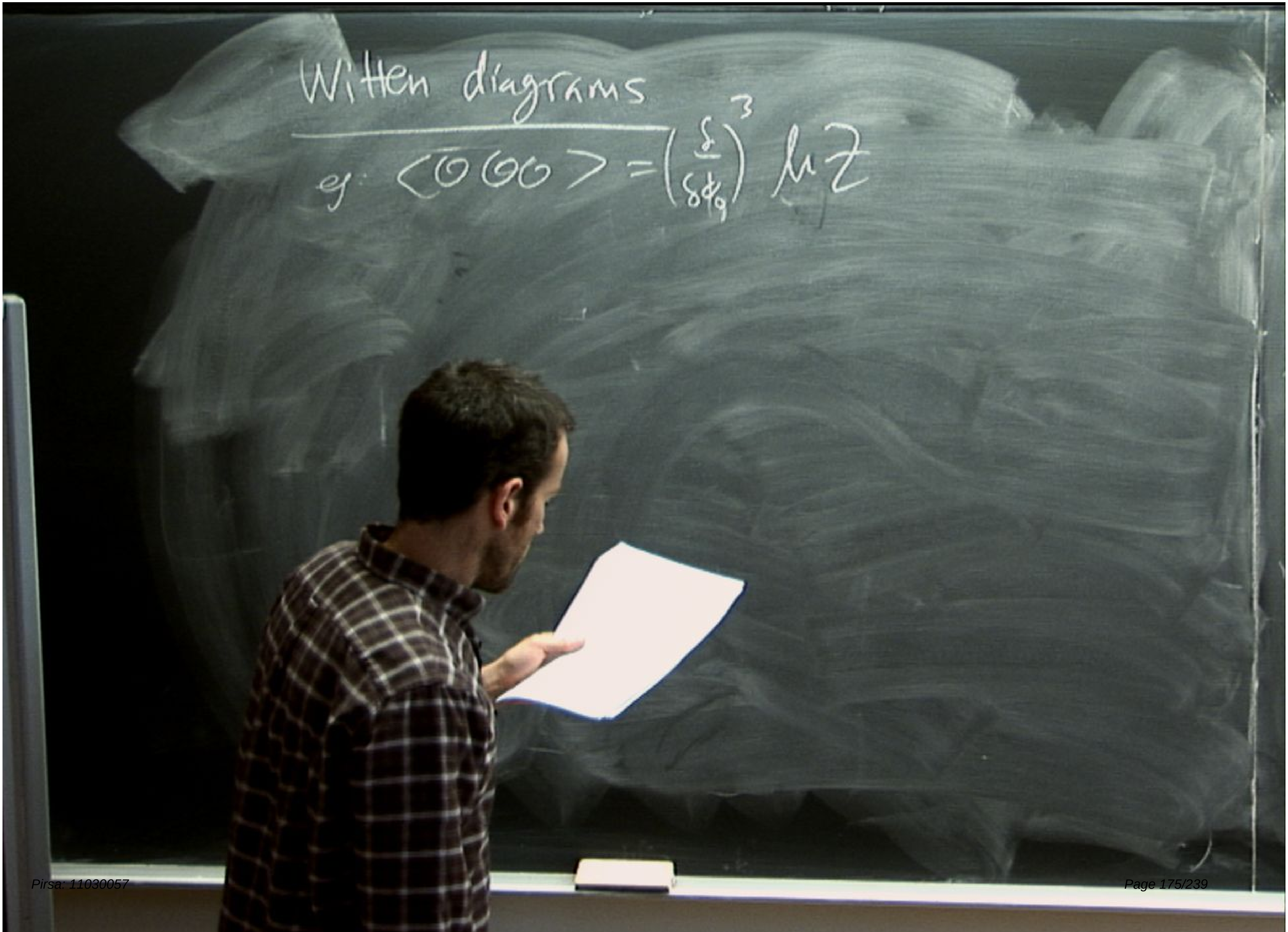

Think of z as time
HJ
finite
use this away from support of φ1(x)
LINEAR RESPONSE:

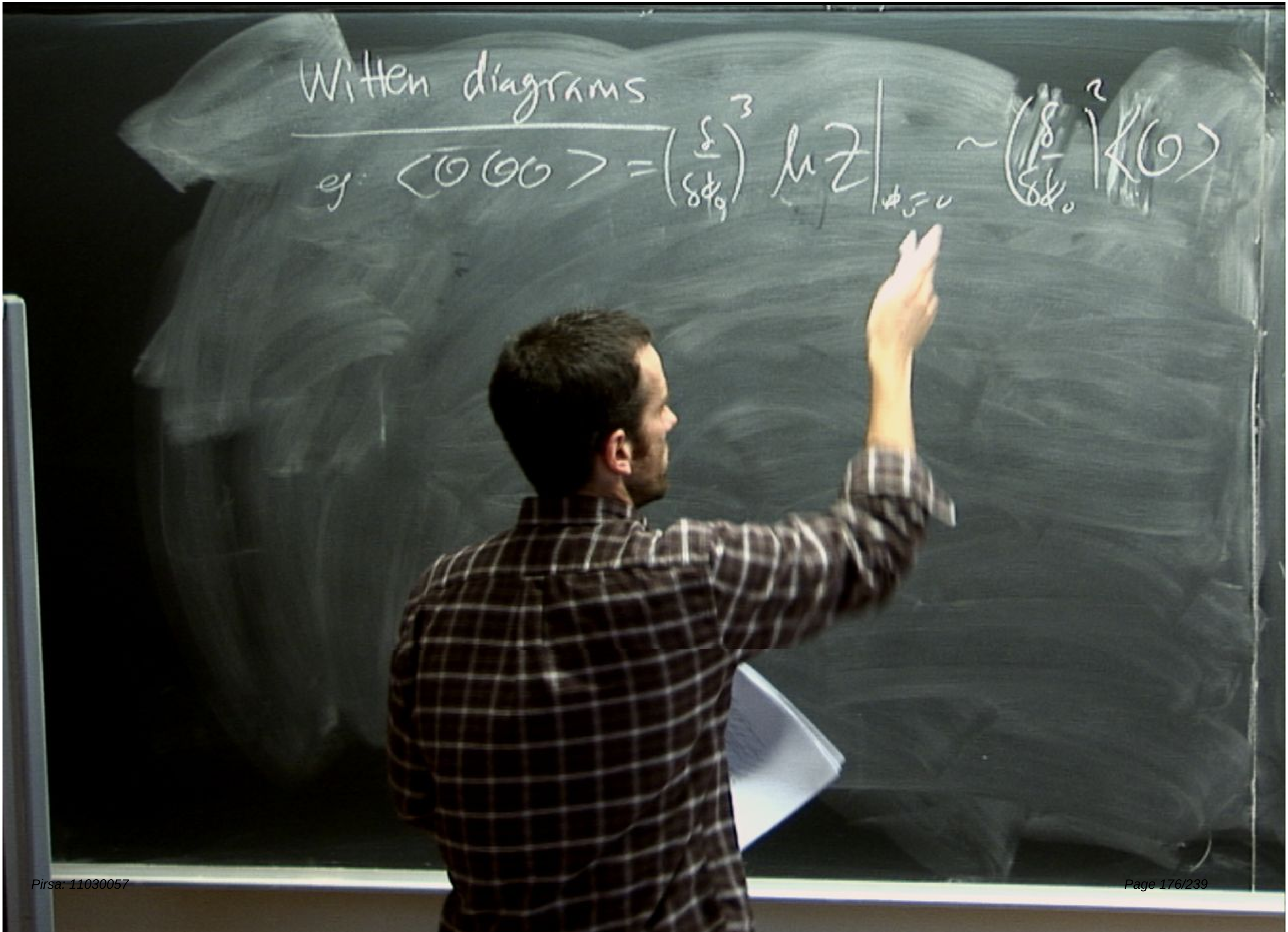

of z as time
HJ
π = ∂L / ∂(∂zφ)
lim z→0
finite
use this away from support of φ1(x)
NEAR RESPONSE:

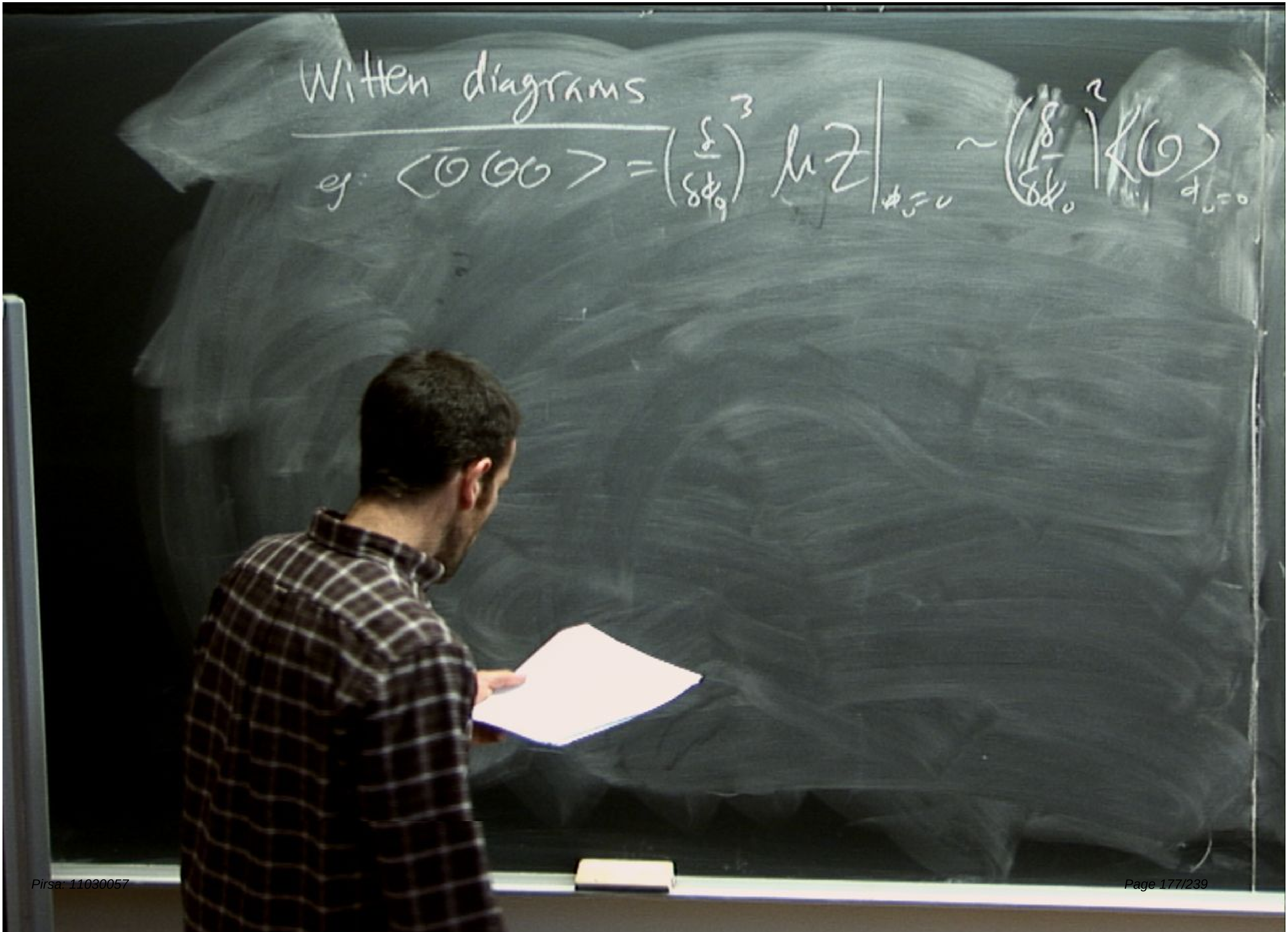

Witten

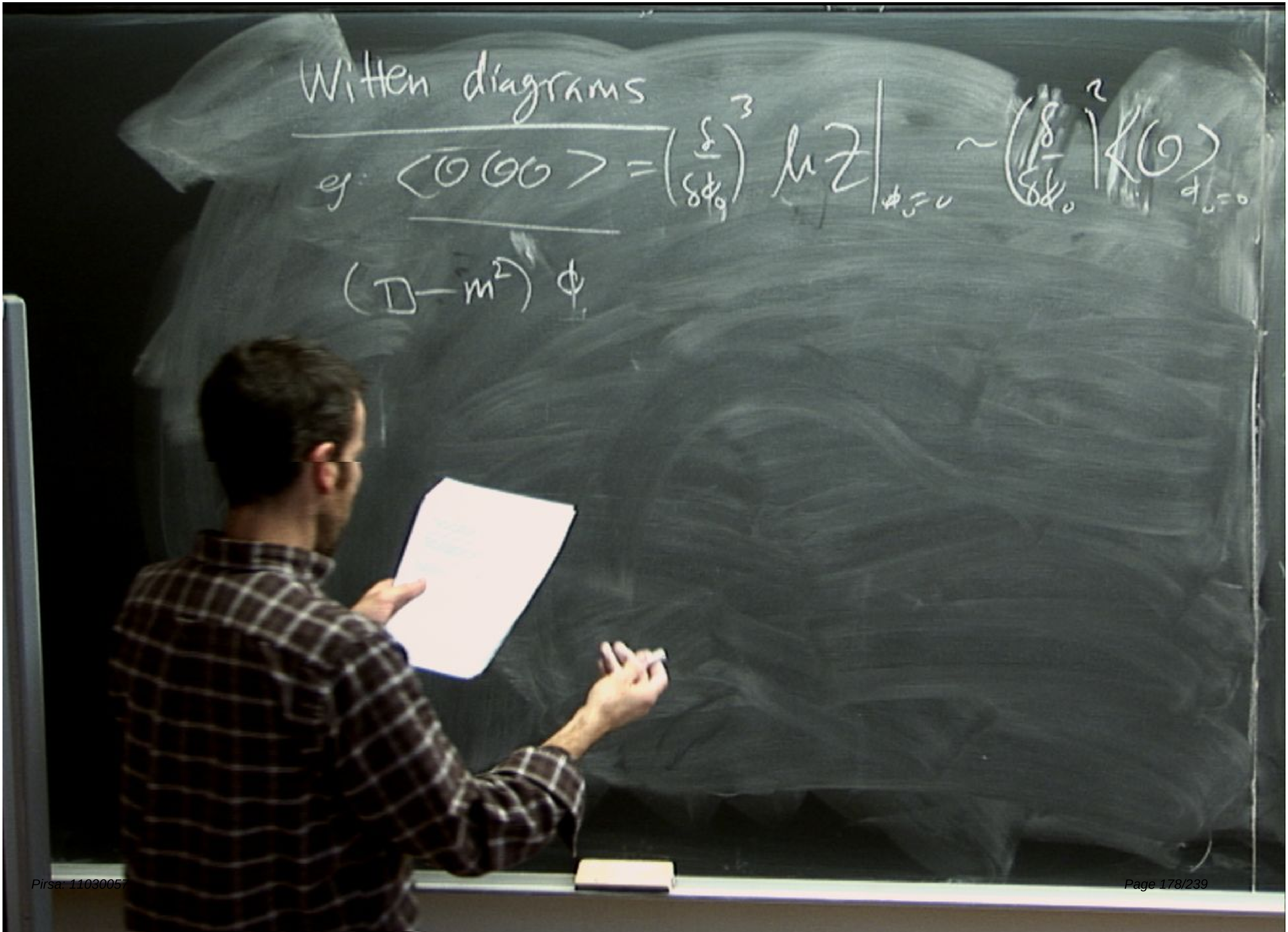

Witten diagrams
eg. <OOO>

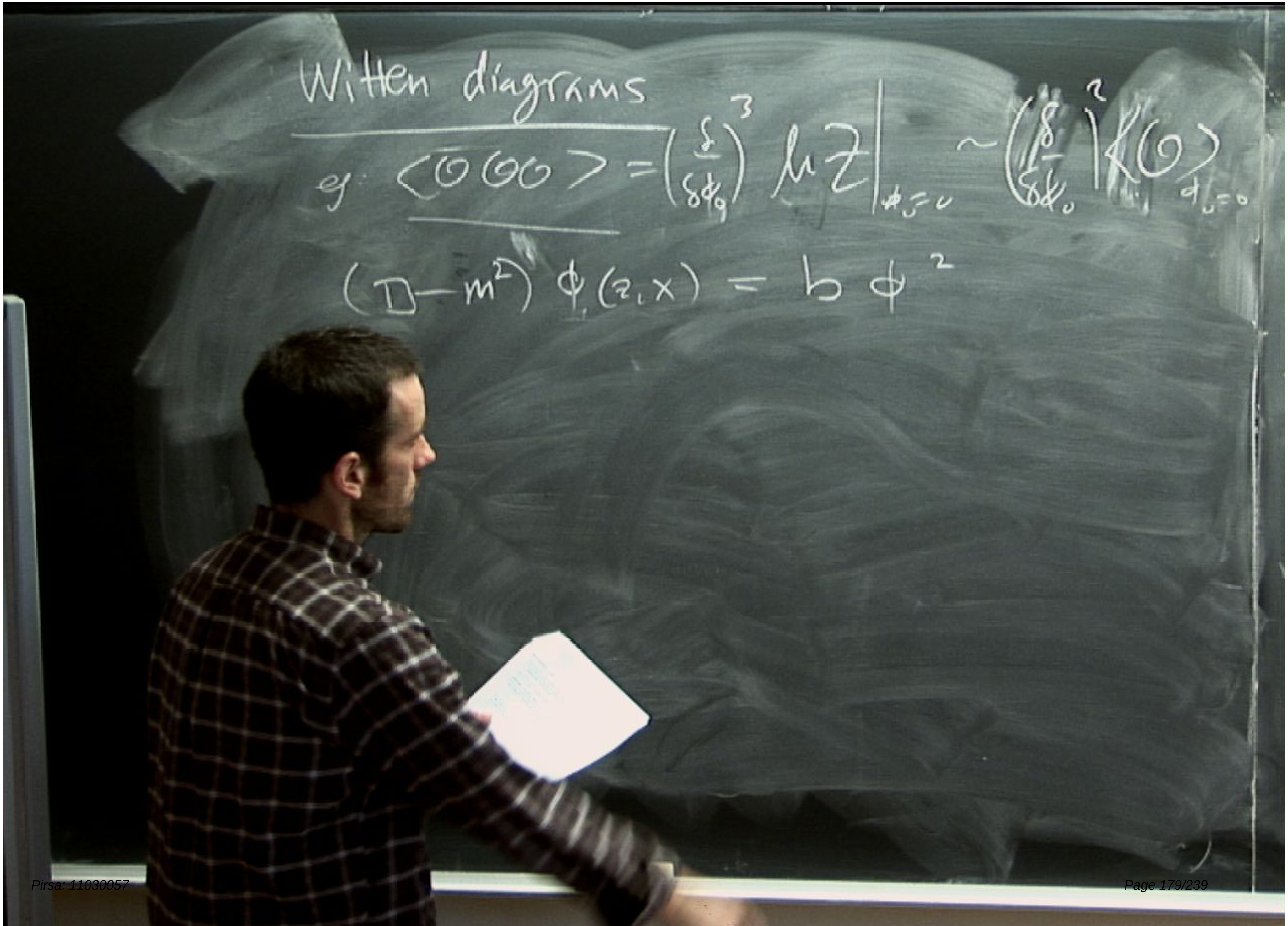

Witten diagrams
eg. <OOO> = (δ/δφ)³

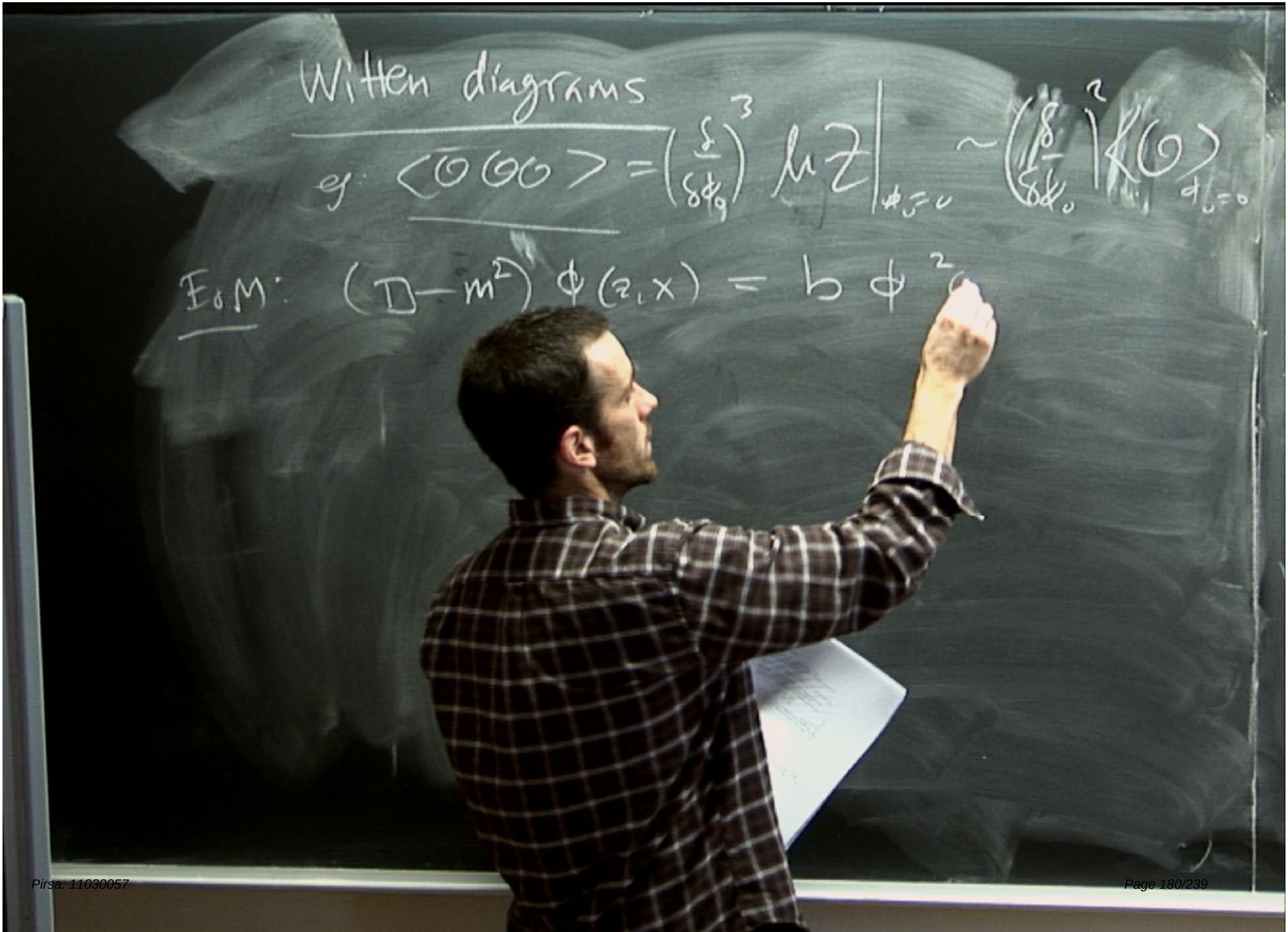

Witten diagrams
e.g. $\langle \mathcal{O}\mathcal{O}\mathcal{O} \rangle = \left(\frac{\delta}{\delta\phi_0}\right)^3 \ln Z$

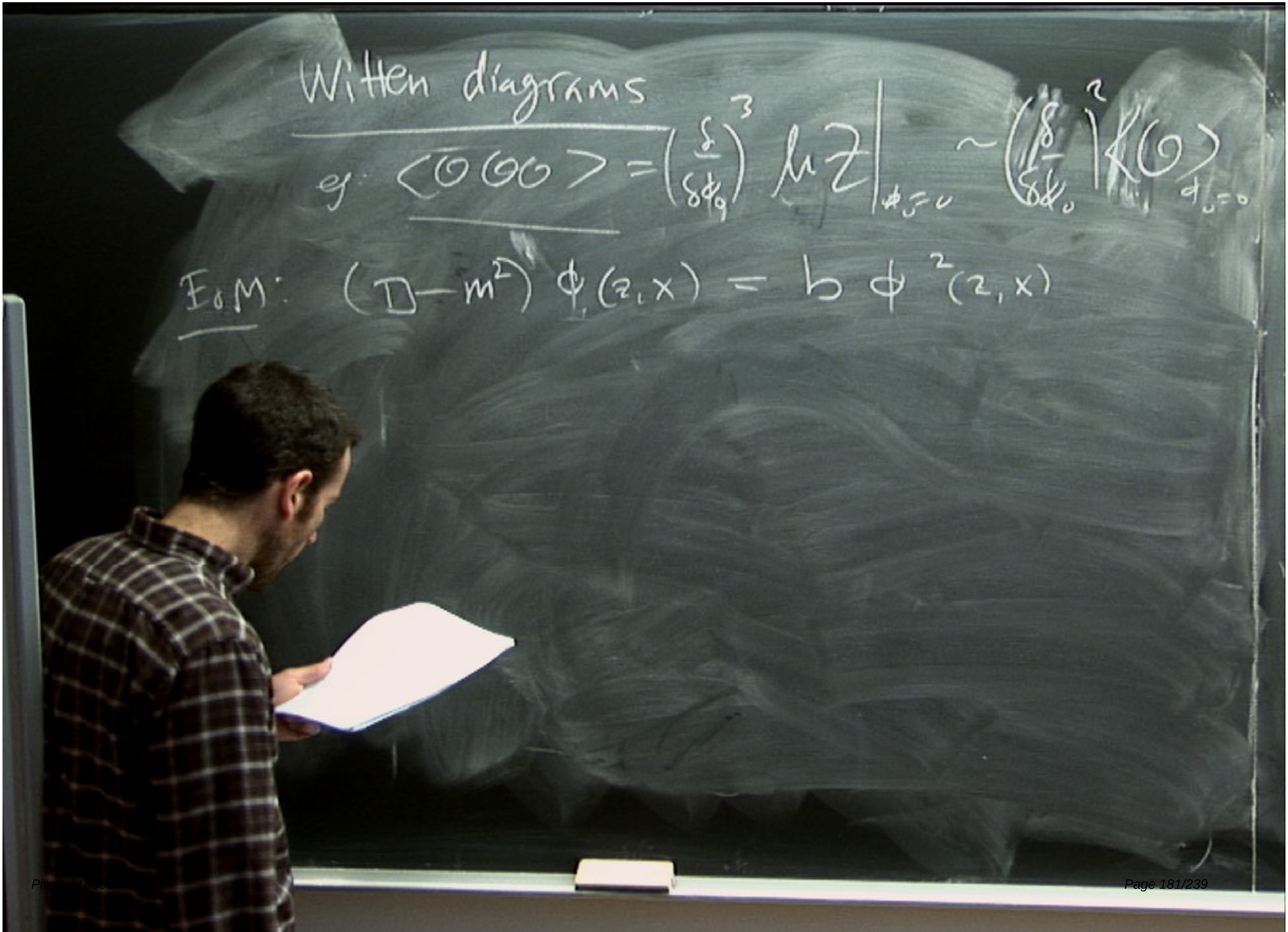

Witten diagrams

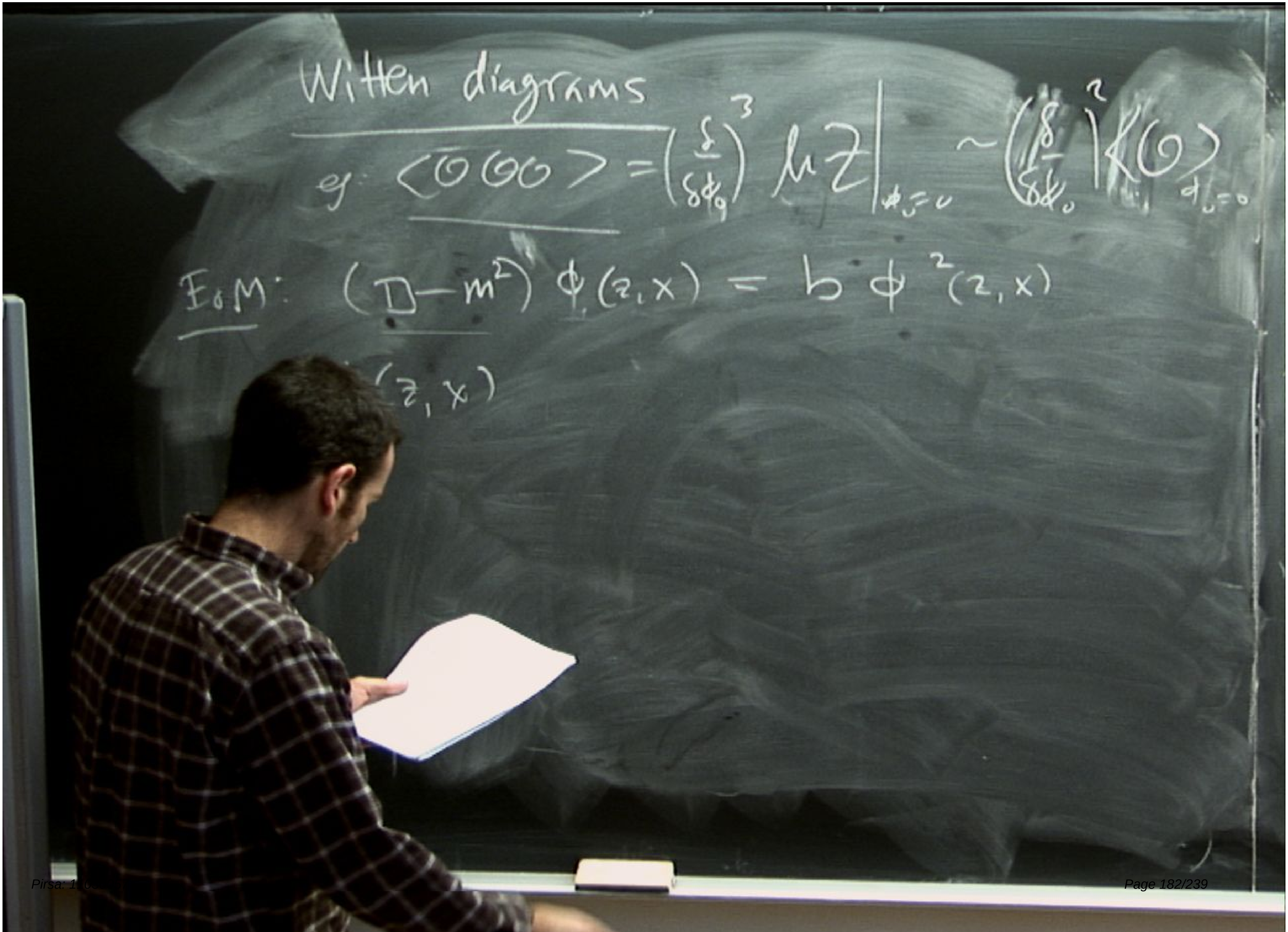

Witten diagrams
eg. ⟨OOO⟩ = (δ/δφ₀)³ ln Z|φ₀=0 ~ (δ/δφ₀)² ⟨O⟩φ₀=0

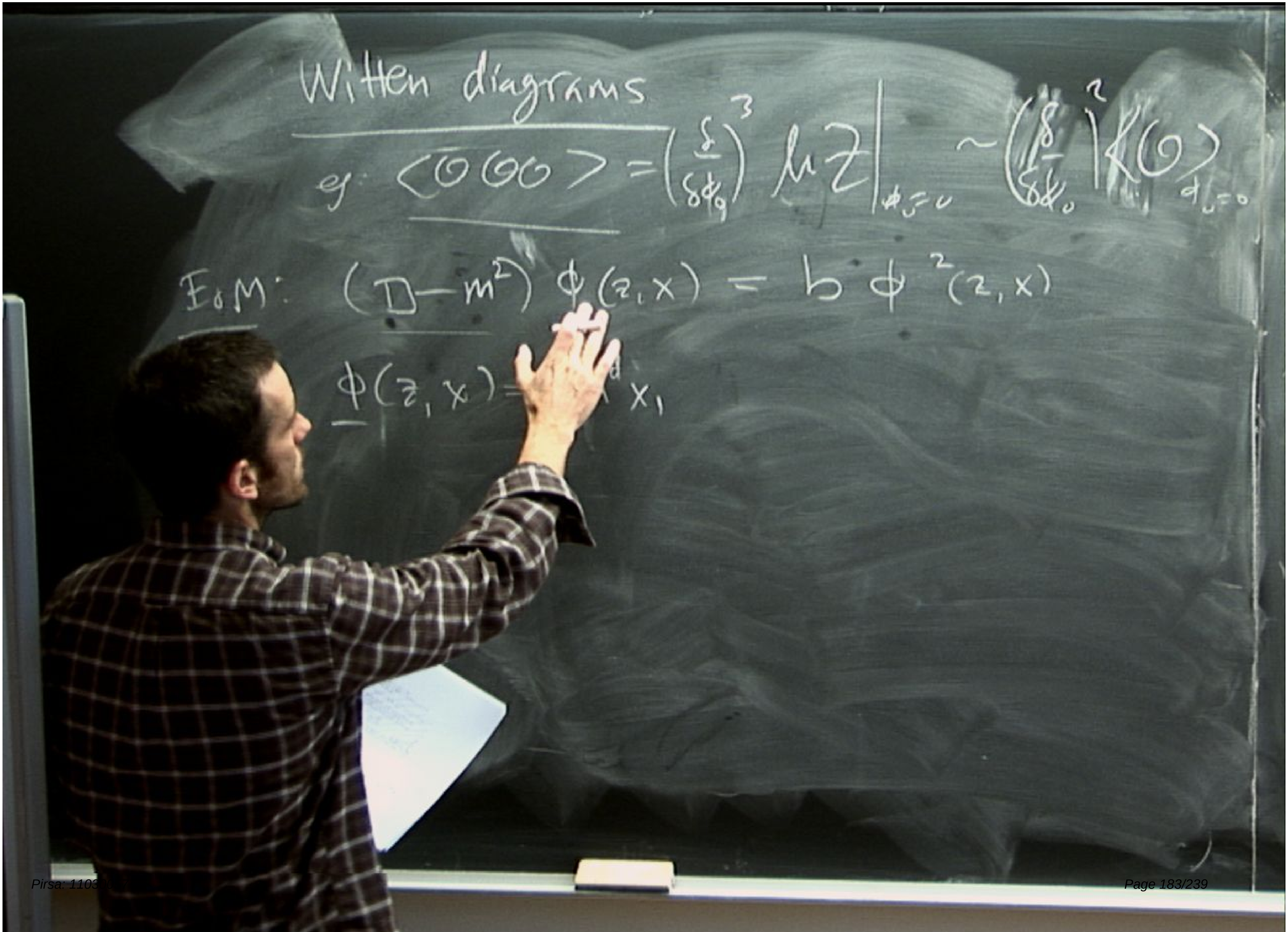

Witten diagrams
e.g. $\langle \mathcal{O}\mathcal{O}\mathcal{O} \rangle = \left(\frac{\delta}{\delta\phi_0}\right)^3 \ln Z \Big|_{\phi_0=0} \sim \left(\frac{\delta}{\delta\phi_0}\right)^2 \langle \mathcal{O} \rangle_{\phi_0=0}$
$(\Box - m^2)\,\phi$

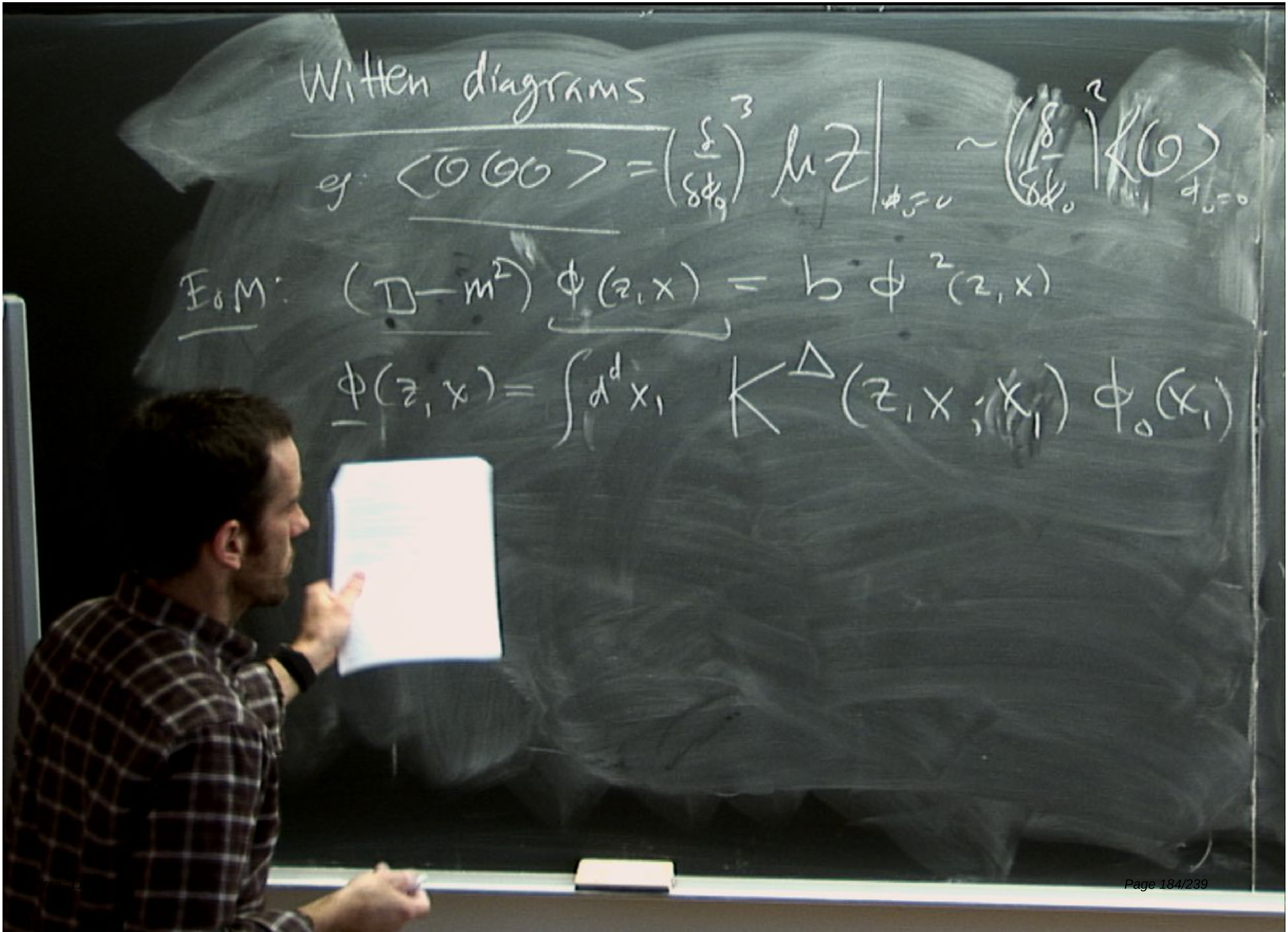

Witten diagrams
eg. ⟨OOO⟩ = (δ/δφ_0)^3 ln Z |_{φ_0=0} ~ (δ/δφ_0)^2 ⟨O⟩_{φ_0=0}
(□ − m^2) φ_1(z,x) = b φ^2


Witten diagrams
eg. $\langle OOO \rangle = \left(\frac{\delta}{\delta\phi_0}\right)^3 \ln Z \Big|_{\phi_0=0} \sim \left(\frac{\delta}{\delta\phi_0}\right)^2 \langle O \rangle_{\phi_0=0}$
EoM: $(\Box - m^2)\,\phi(z,x) = b\,\phi^2$

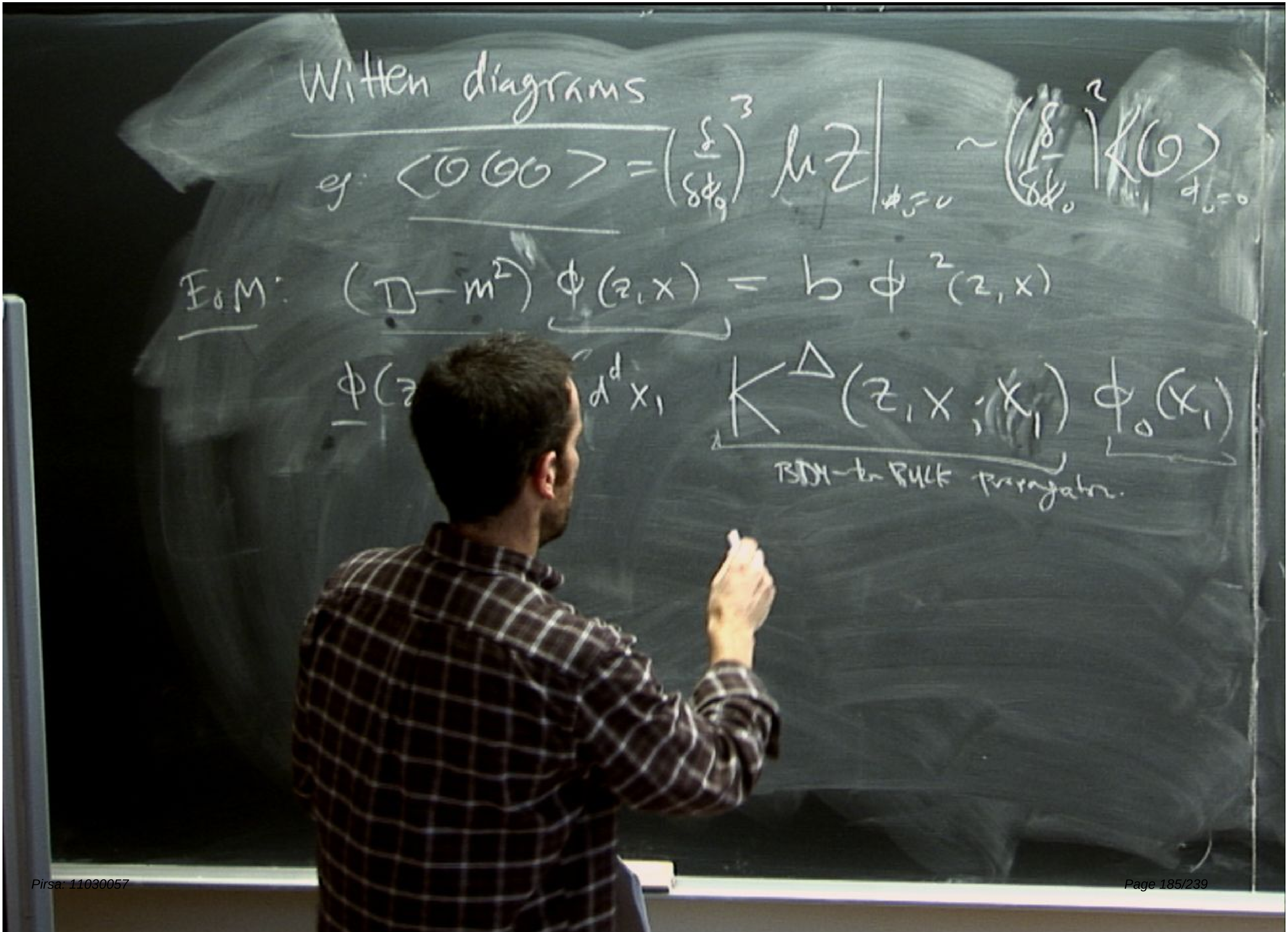

Witten diagrams
eg. ⟨OOO⟩ = (δ/δφ0)^3 ln Z |φ0=0 ~ (δ/δφ0)^2 ⟨O⟩φ0=0
EoM: (□ − m^2) φ(z,x) = b φ^2(z,x)

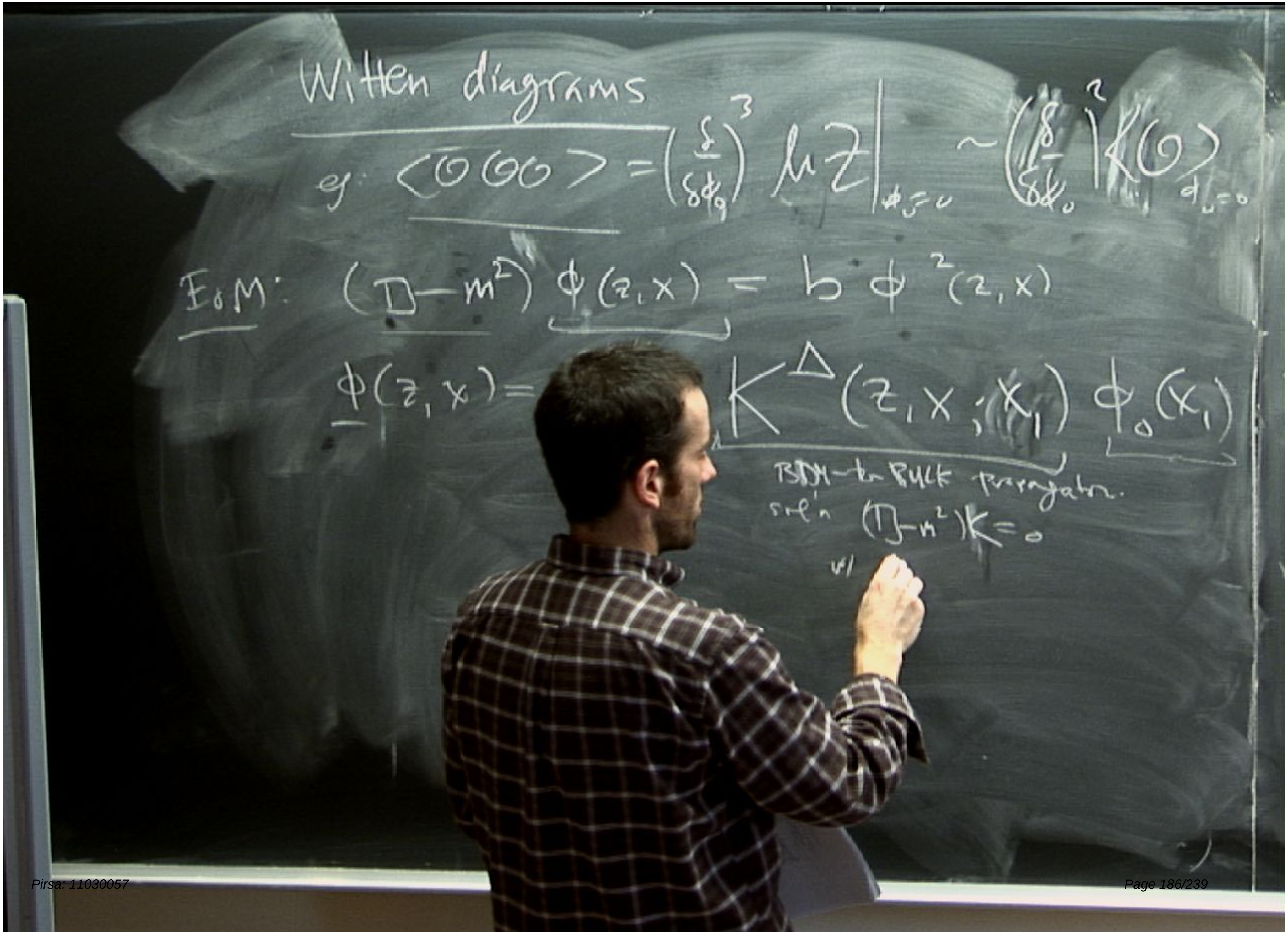

Witten diagrams
eg. ⟨OOO⟩ = (δ/δφ0)^3 ln Z |φ0=0 ~ (δ/δφ0)^2 ⟨O⟩ φ0=0
EoM: (□ − m^2) φ(z, x) = b φ^2(z, x)
(z, x)

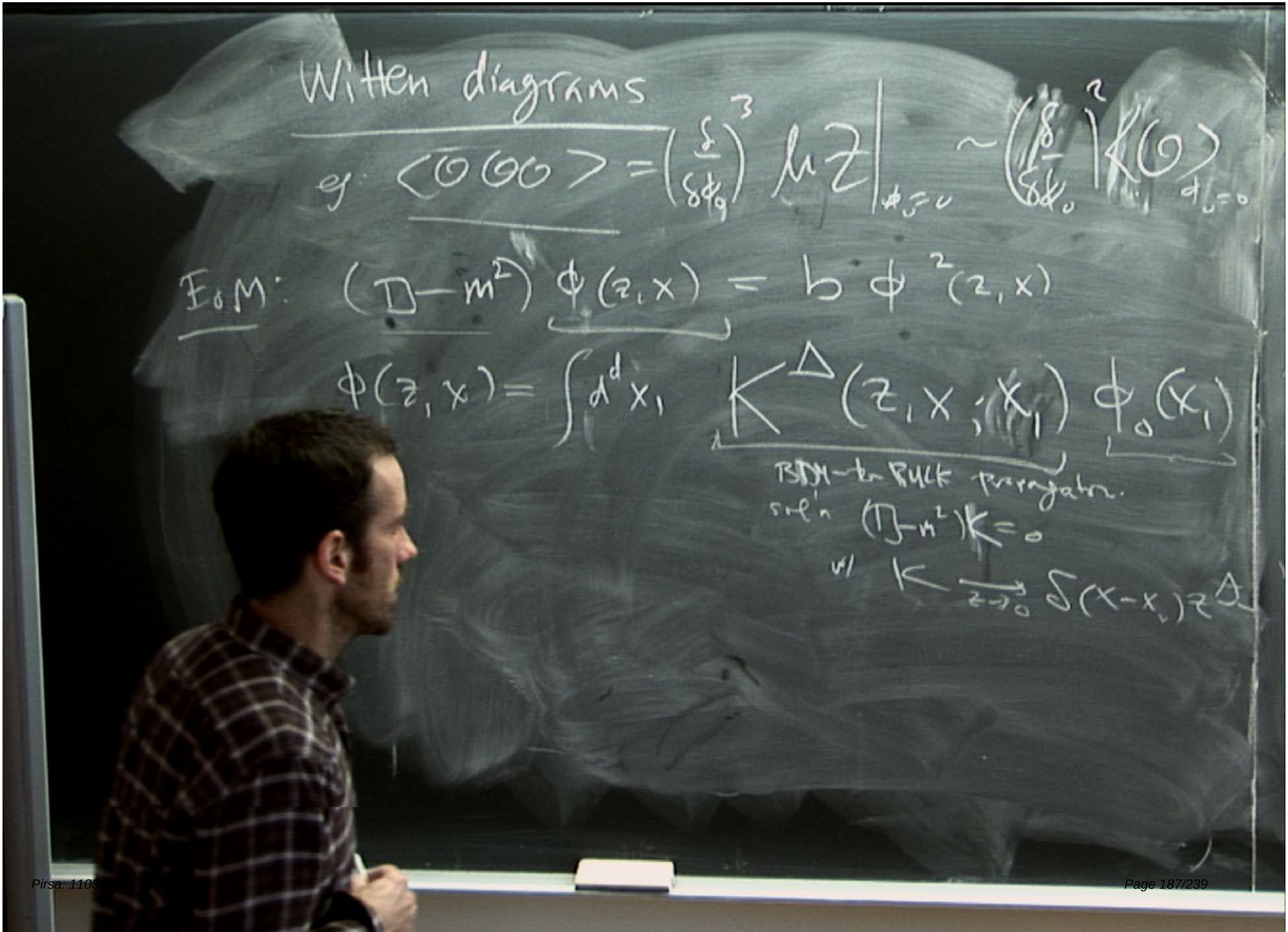

Witten diagrams
eg. ⟨OOO⟩ = (δ/δφ0)^3 ln Z|φ0=0 ~ (δ/δφ0)^2 ⟨O⟩φ0=0
EoM: (□ − m²) φ(z, x) = b φ²(z, x)
φ(z, x) =
x1

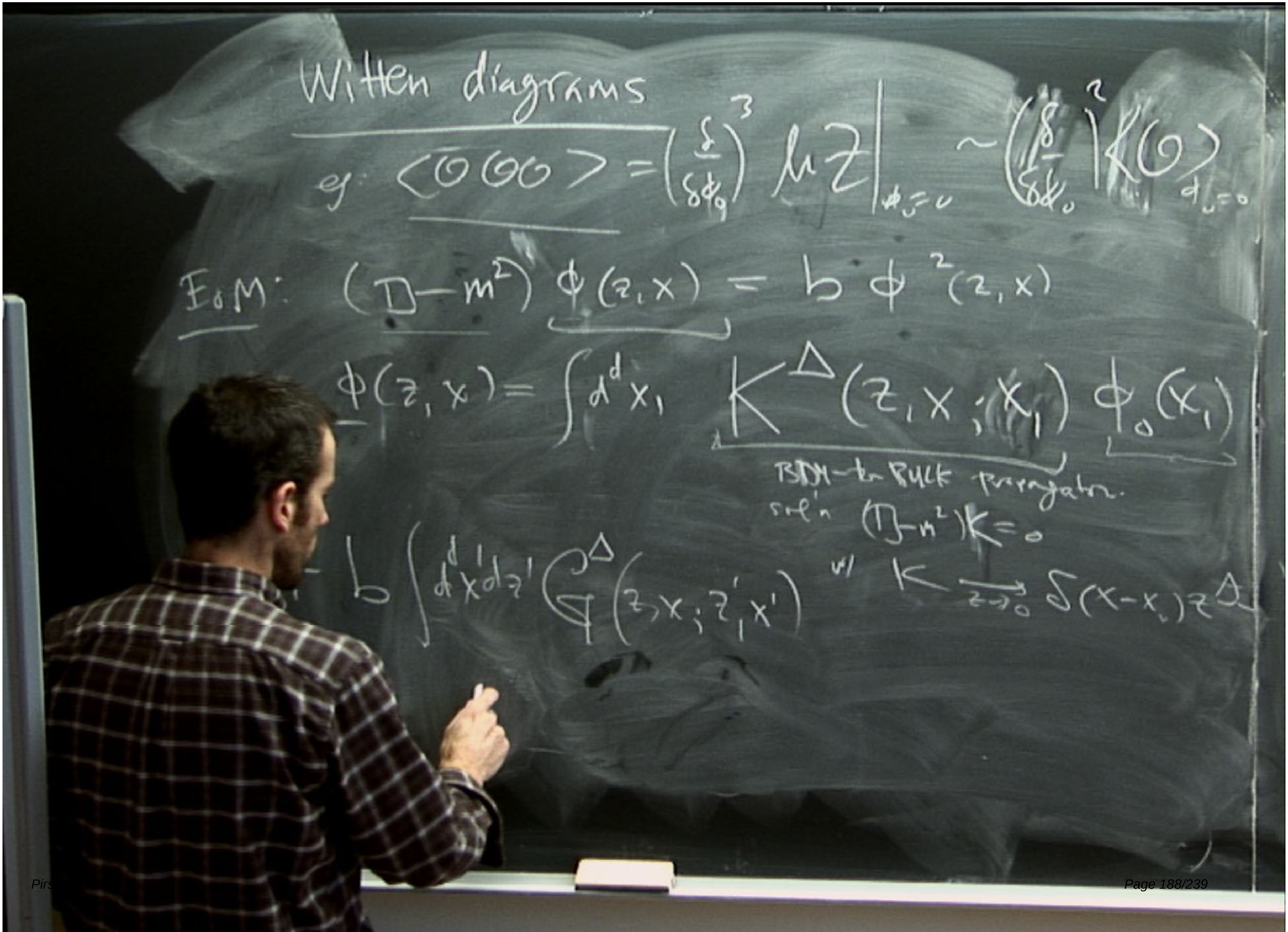


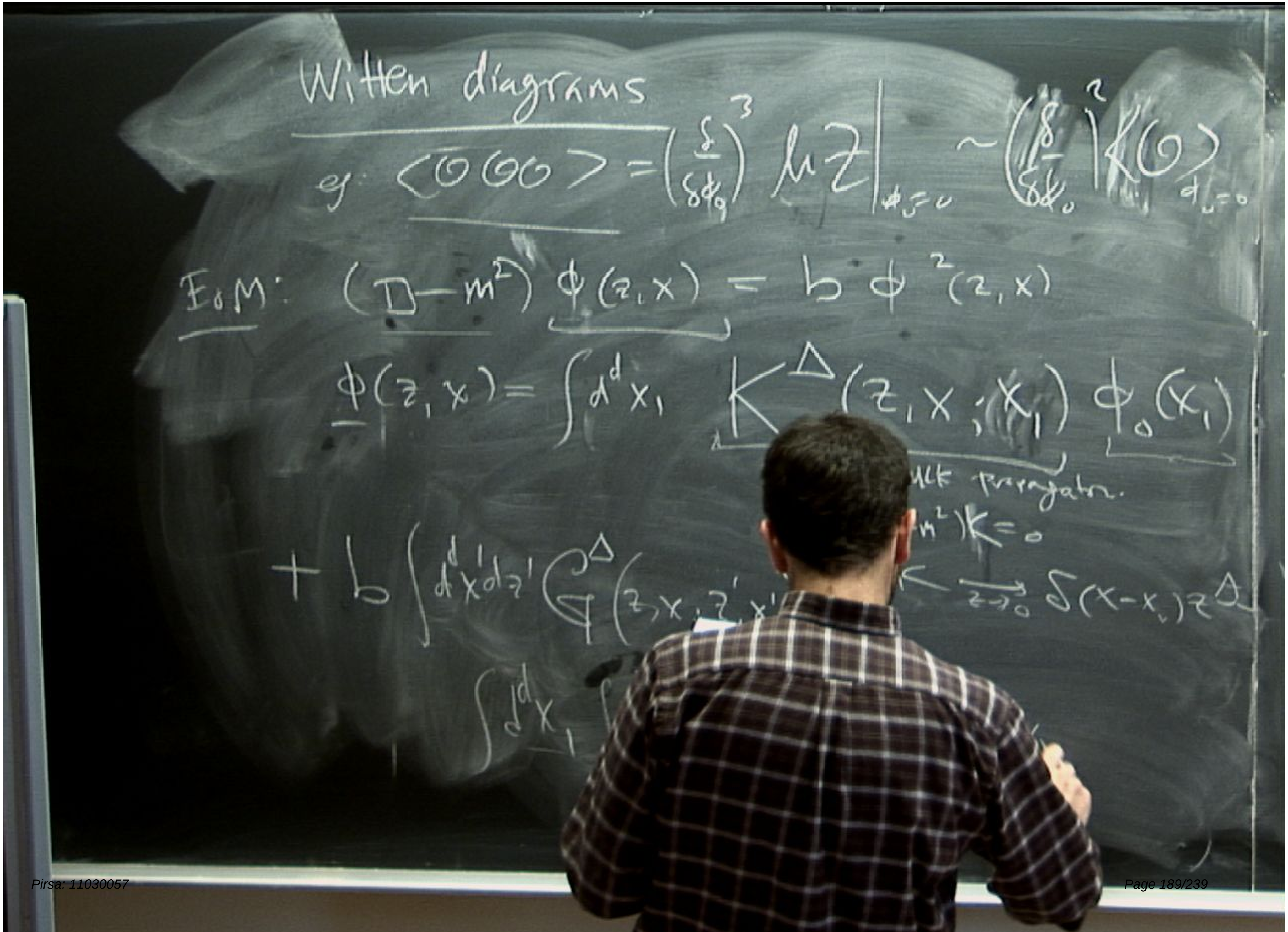
Witten diagrams
e.g. $\langle \mathcal{O}\mathcal{O}\mathcal{O} \rangle = \left(\frac{\delta}{\delta\phi_0}\right)^3 \ln Z \Big|_{\phi_0=0} \sim \left(\frac{\delta}{\delta\phi_0}\right)^2 \langle \mathcal{O} \rangle_{\phi_0=0}$
EoM: $(\Box - m^2)\,\phi(z,x) = b\,\phi^2(z,x)$
$\phi(z,x) = \int d^d x_1 \, K^\Delta(z,x;x_1)\,\phi_0(x_1)$


Witten diagrams
e.g. ⟨OOO⟩ = (δ/δφ0)^3 ln Z |φ0=0 ~ (δ/δφ0)^2 ⟨O⟩ φ0=0
EoM: (□ − m^2) φ(z,x) = b φ^2(z,x)
d^d x1 K^Δ(z,x;x1) φ0(x1)
BDY-to-BULK propagator


Witten diagrams
eg. ⟨OOO⟩ = (δ/δφ₀)³ ln Z |φ₀=0 ~ (δ/δφ₀)² ⟨O⟩ φ₀=0
EoM: (□ − m²) φ(z, x) = b φ²(z, x)
φ(z, x) = K^Δ(z, x; x₁) φ₀(x₁)
BDY-to-BULK propagator.
sol'n (□ − m²) K = 0
w/


Witten diagrams
eg. <OOO> = (δ/δφ0)^3 ln Z | φ0=0 ~ (δ/δφ0)^2 <O> φ0=0
EoM: (□ − m^2) φ(z,x) = b φ^2(z,x)
φ(z,x) = ∫ d^d x1 K^Δ(z,x;x1) φ0(x1)
BDY-to-BULK propagator.
sol'n (□ − m^2) K = 0
w/ K → (z→0) δ(x − x1) z^Δ


Witten diagrams
eg: $\langle \mathcal{O}\mathcal{O}\mathcal{O} \rangle = \left(\frac{\delta}{\delta\phi_0}\right)^3 \ln Z \Big|_{\phi_0=0} \sim \left(\frac{\delta}{\delta\phi_0}\right)^2 \langle \mathcal{O} \rangle_{\phi_0=0}$
EoM: $(\Box - m^2)\,\phi(z,x) = b\,\phi^2(z,x)$
$\phi(z,x) = \int d^dx_1\, K^\Delta(z,x;x_1)\, \phi_0(x_1)$
BDY-to-BULK propagator.
sol'n $(\Box - m^2)K = 0$
w/ $K \xrightarrow[z\to 0]{} \delta(x-x_1) z^\Delta$
$+ b \int d^dx' dz'\, G^\Delta(z,x;z',x')$


Witten diagrams
e.g. $\langle \mathcal{O}\mathcal{O}\mathcal{O} \rangle = \left(\frac{\delta}{\delta\phi_0}\right)^3 \ln Z \Big|_{\phi_0=0} \sim \left(\frac{\delta}{\delta\phi_0}\right)^2 \langle \mathcal{O} \rangle_{\phi_0=0}$
EoM: $(\Box - m^2)\,\phi(z,x) = b\,\phi^2(z,x)$
$\phi(z,x) = \int d^dx_1\, K^\Delta(z,x;x_1)\,\phi_0(x_1)$
ulk propagator
$m^2)K = 0$
$K \xrightarrow{z\to 0} \delta(x-x_1)\,z^\Delta$
$+\, b \int d^dx'\,dz'\, G^\Delta(z,x,z',x'$
$\int d^dx_1$

Witten diagrams

e.g. $\langle \mathcal{O}\mathcal{O}\mathcal{O} \rangle = \left(\frac{\delta}{\delta\phi_0}\right)^3 \ln Z \Big|_{\phi_0=0} \sim \left(\frac{\delta}{\delta\phi_0}\right)^2 \langle \mathcal{O} \rangle_{\phi_0=0}$

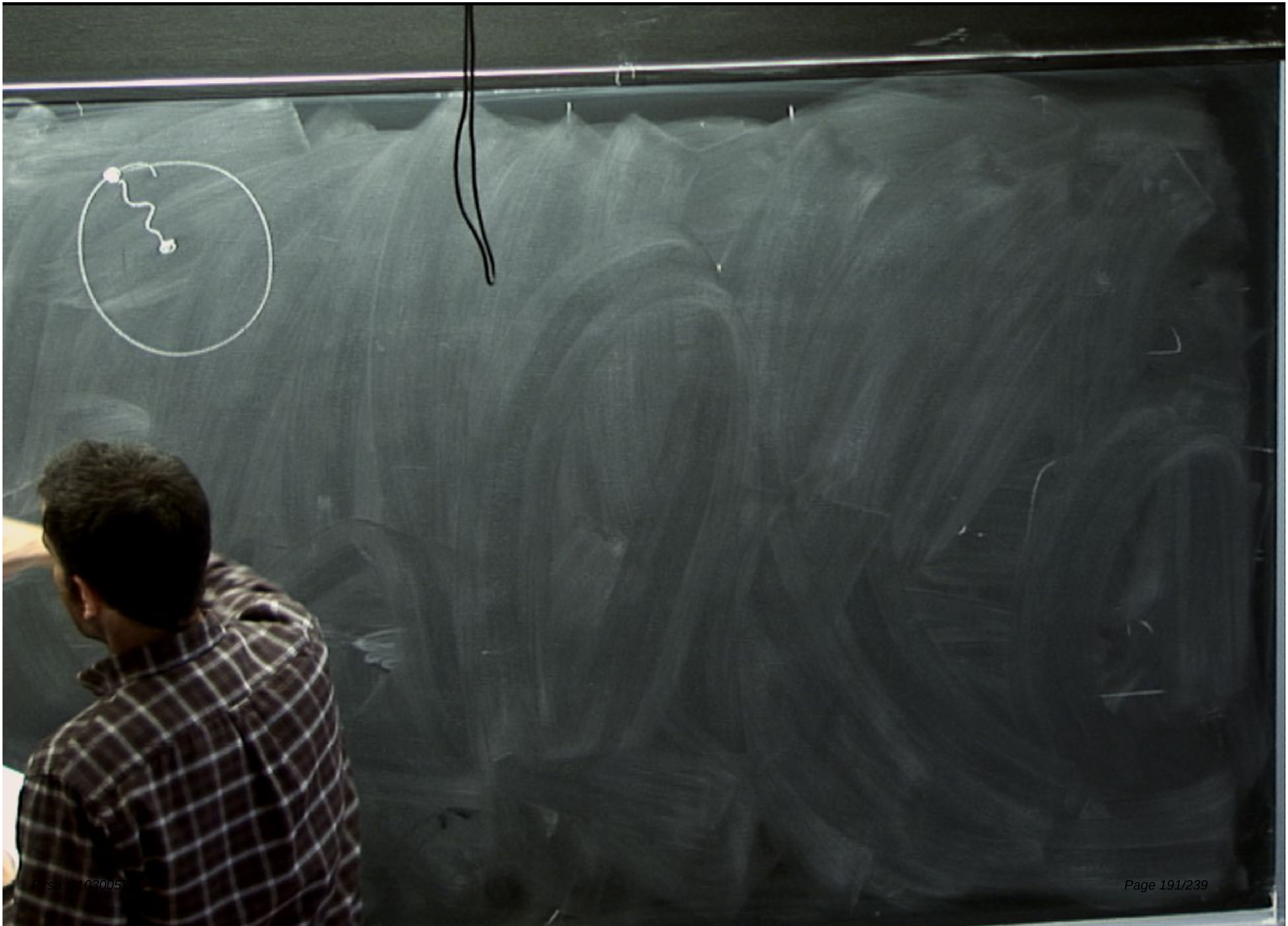
EoM: $(\Box - m^2)\, \phi(z,x) = b\, \phi^2(z,x)$

$$\phi(z,x) = \int d^d x_1\, K^\Delta(z,x;x_1)\, \phi_0(x_1)$$

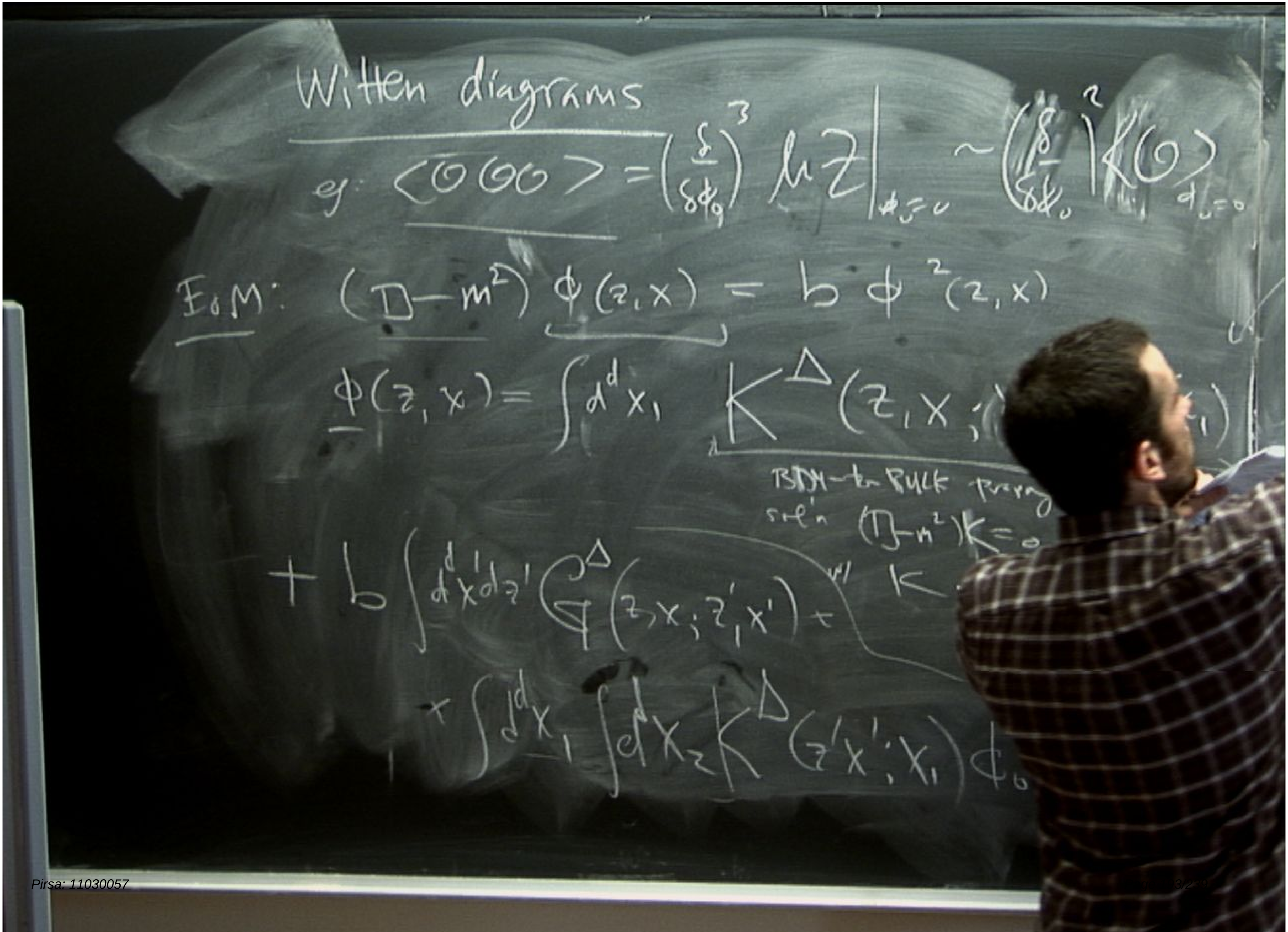
Boundary-to-bulk propagator.
sol'n $(\Box - m^2) K = 0$ w/ $K \xrightarrow[z\to 0]{} \delta(x-x_1)\, z^{\Delta \ldots}$ [illegible]

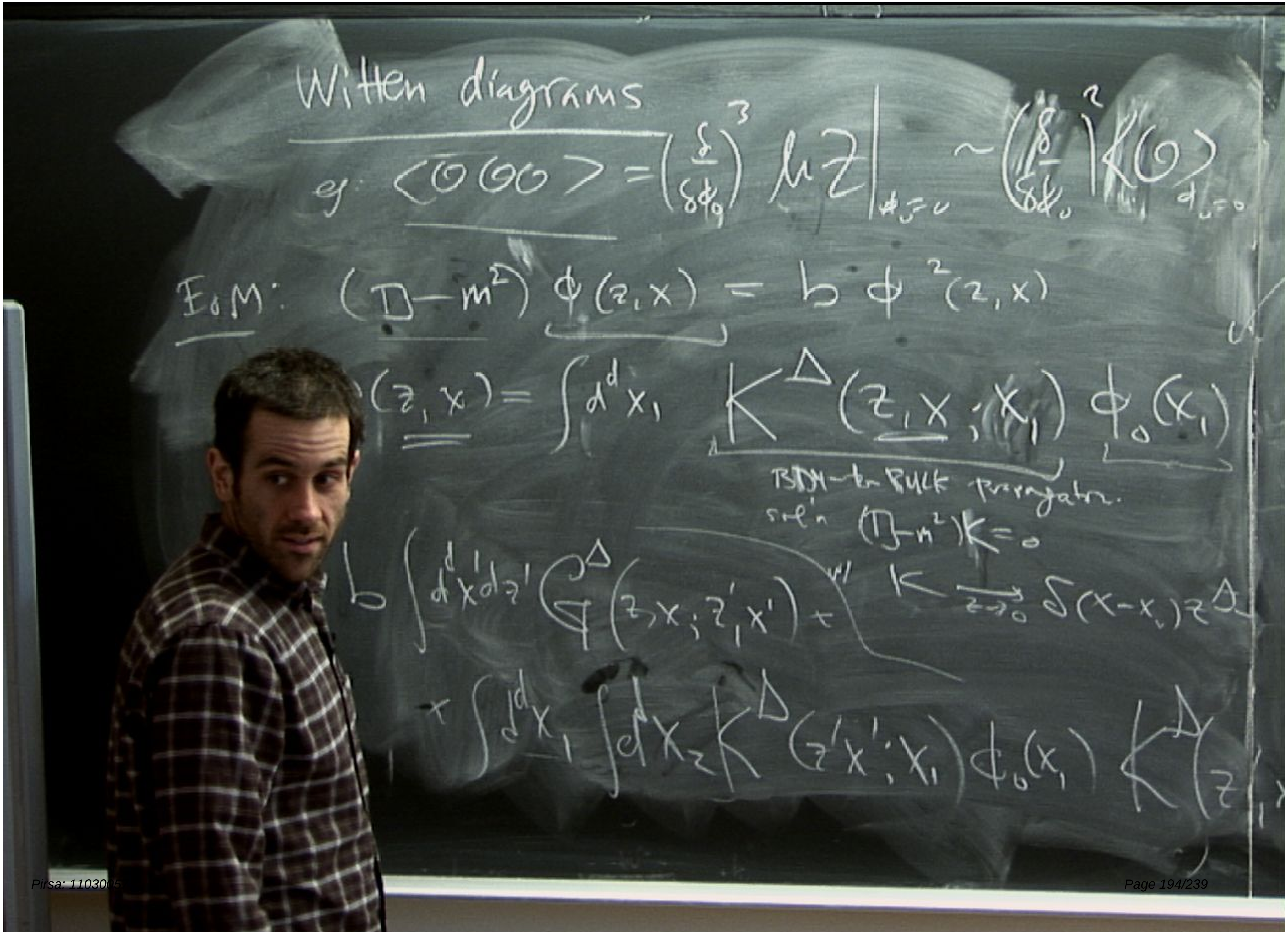
$$+\, b \int d^d x'\, dz'\, G^\Delta(z,x;z',x')$$

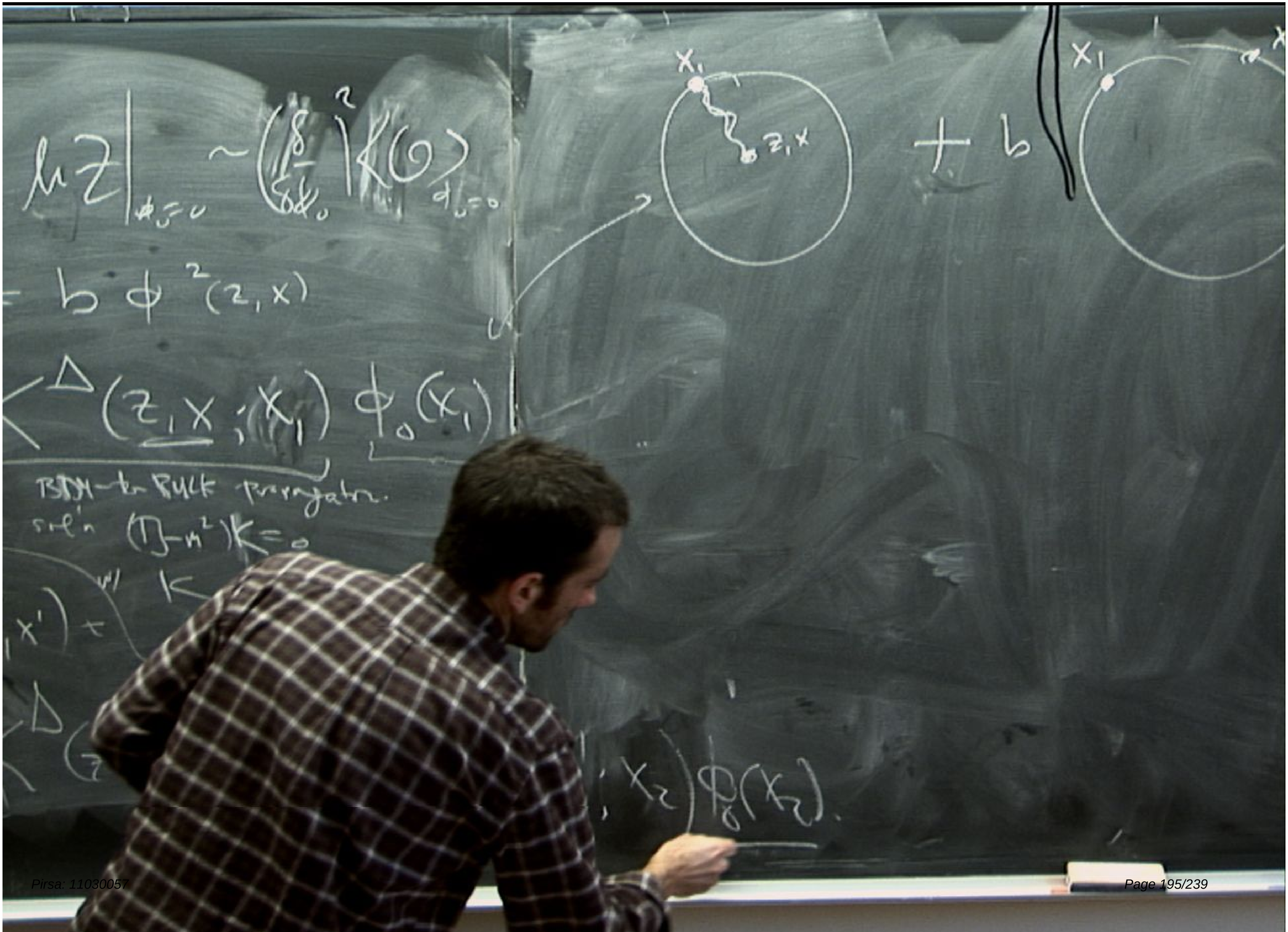
$$\int d^d x_1 \int d^d x_2\, K^\Delta(z',x';x_1)\, \phi_0(x_1)\, K^\Delta(z', \ldots$$ [illegible]



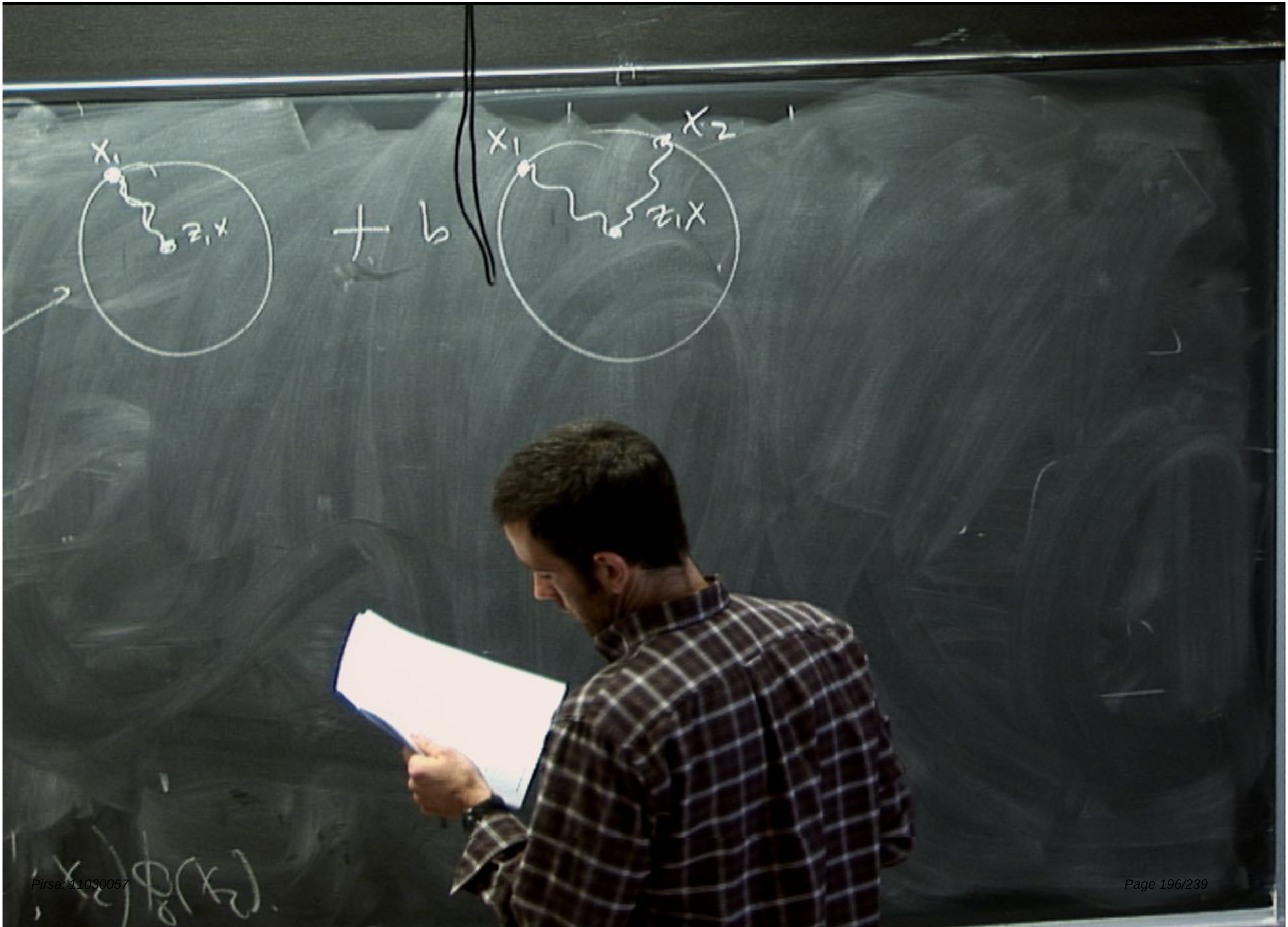

X_1
z, x

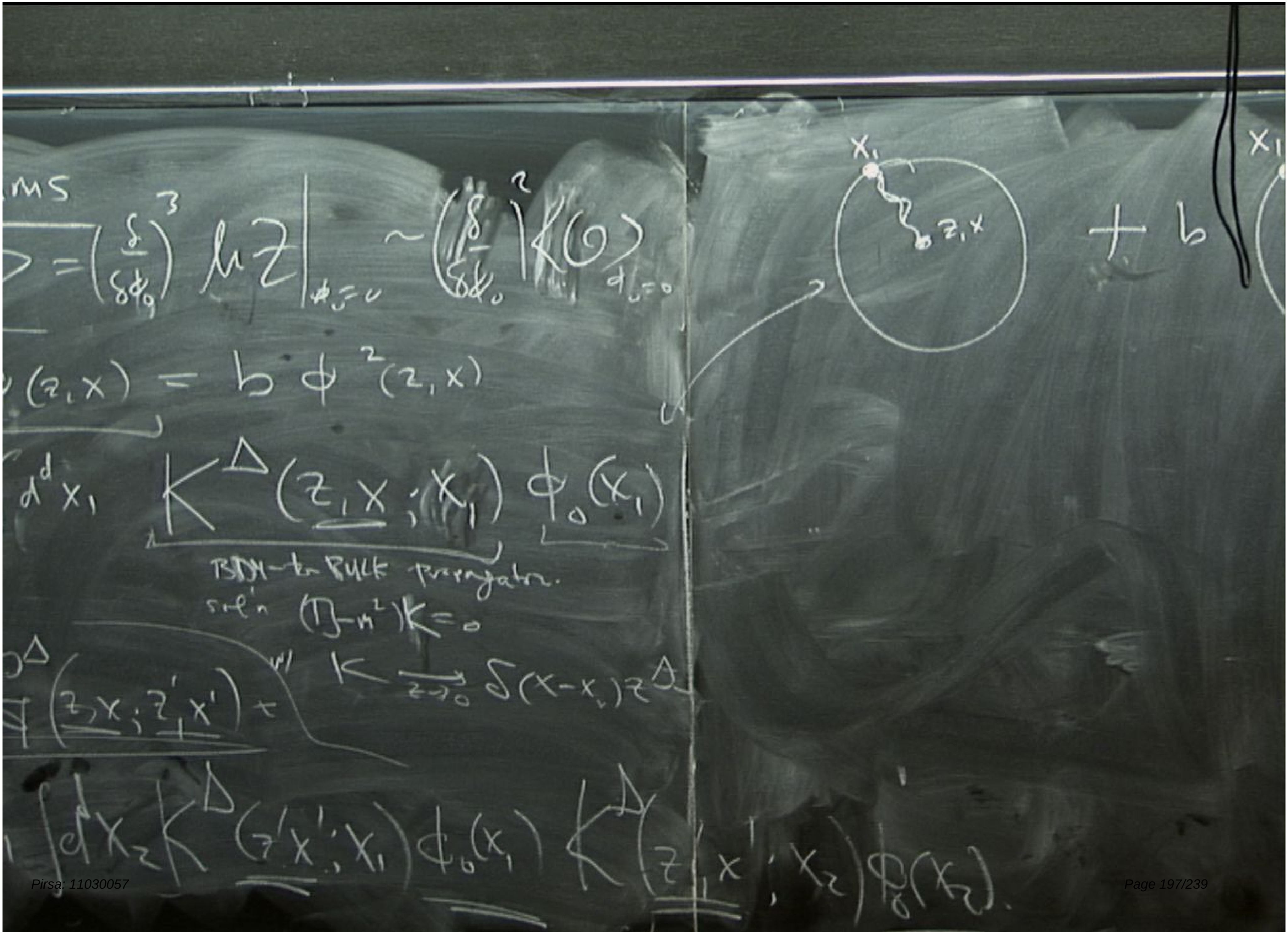

Witten diagrams
eg: ⟨O O O⟩ = (δ/δφ0)^3 ln Z|φ0=0 ~ (δ/δφ0)^2 ⟨O⟩φ0=0
EoM: (□ − m^2) φ(z, x) = b φ^2(z, x)
φ(z, x) = ∫d^d x1 K^Δ(z, x;
BDY-to-BULK propag
sol'n (□ − m^2)K = 0
+ b ∫d^d x' dz' G^Δ(z, x; z', x') ×
K
+ ∫d^d x1 ∫d^d x2 K^Δ(z' x'; x1) φ0

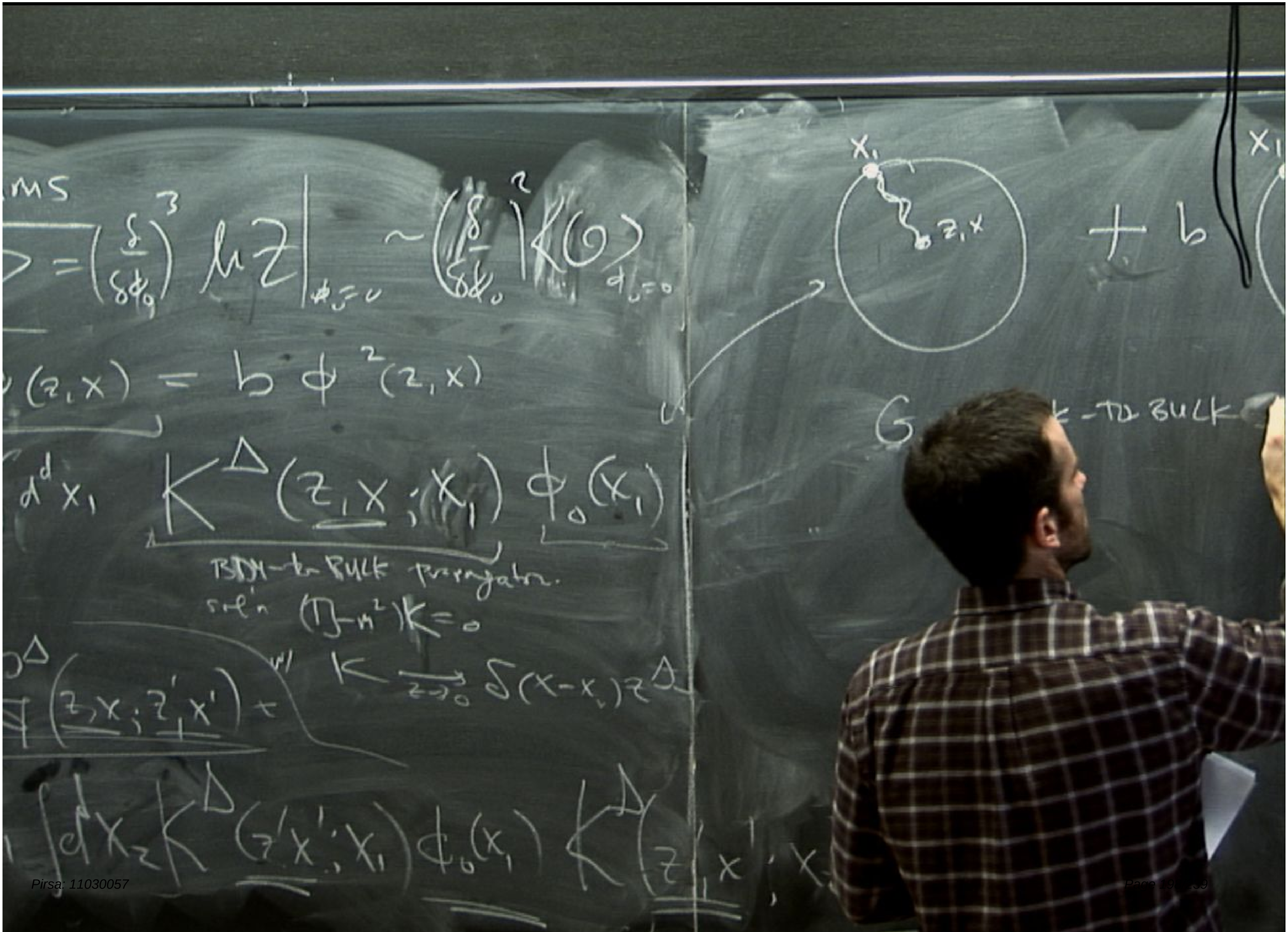

Witten diagrams
eg: ⟨OOO⟩ = (δ/δφ0)^3 ln Z |φ0=0 ~ (δ/δφ0)^2 ⟨O⟩φ0=0
EoM: (□ − m^2) φ(z, x) = b φ^2(z, x)
φ(z, x) = ∫d^d x1 K^Δ(z, x; x1) φ0(x1)
BDY-to-BULK propagator.
sol'n (□ − m^2) K = 0
K → δ(x − x1) z^Δ (z → 0)
b ∫d^d x' dz' G^Δ(z, x; z', x') ×
× ∫d^d x1 ∫d^d x2 K^Δ(z' x'; x1) φ0(x1) K^Δ(z,

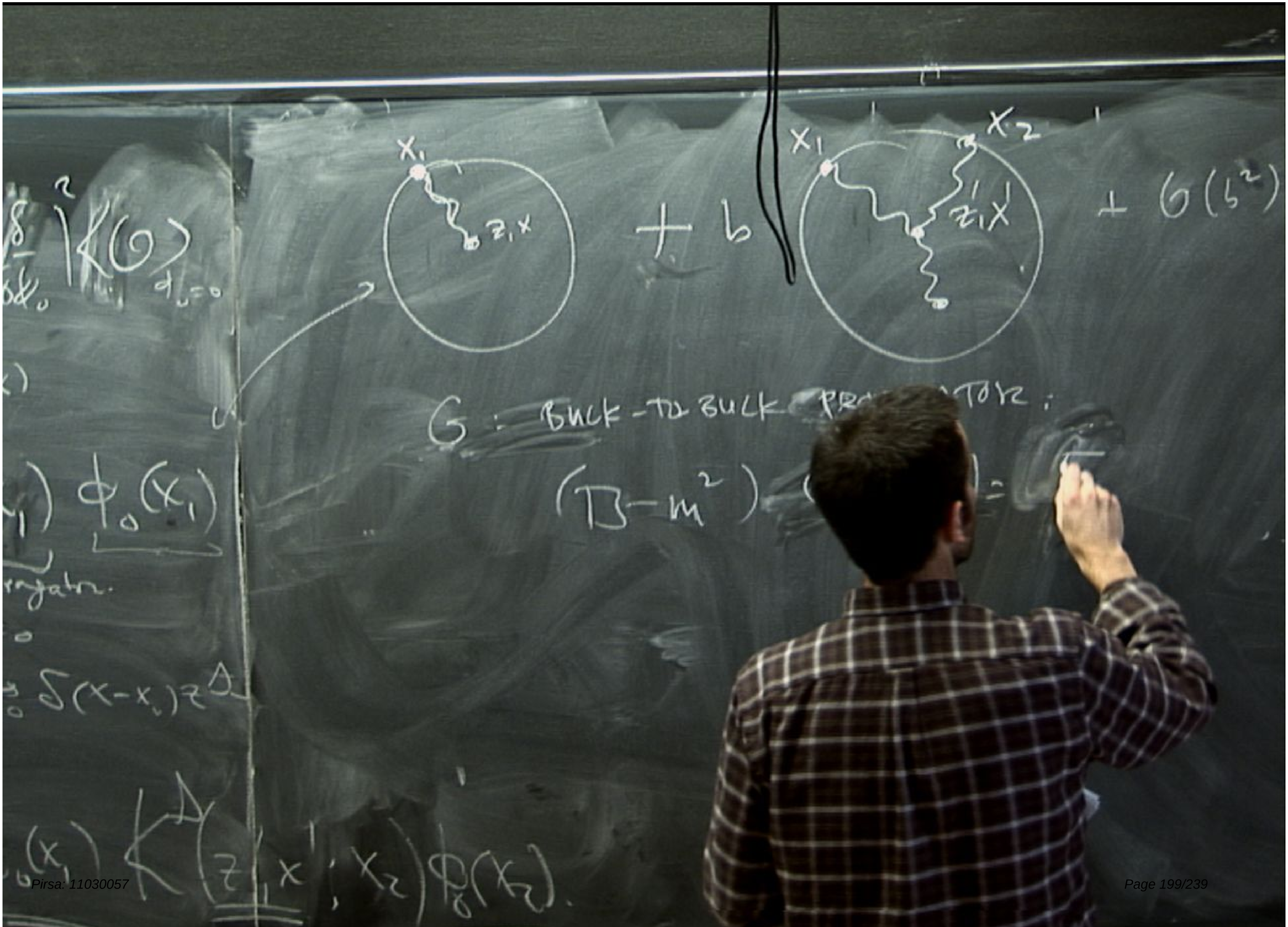


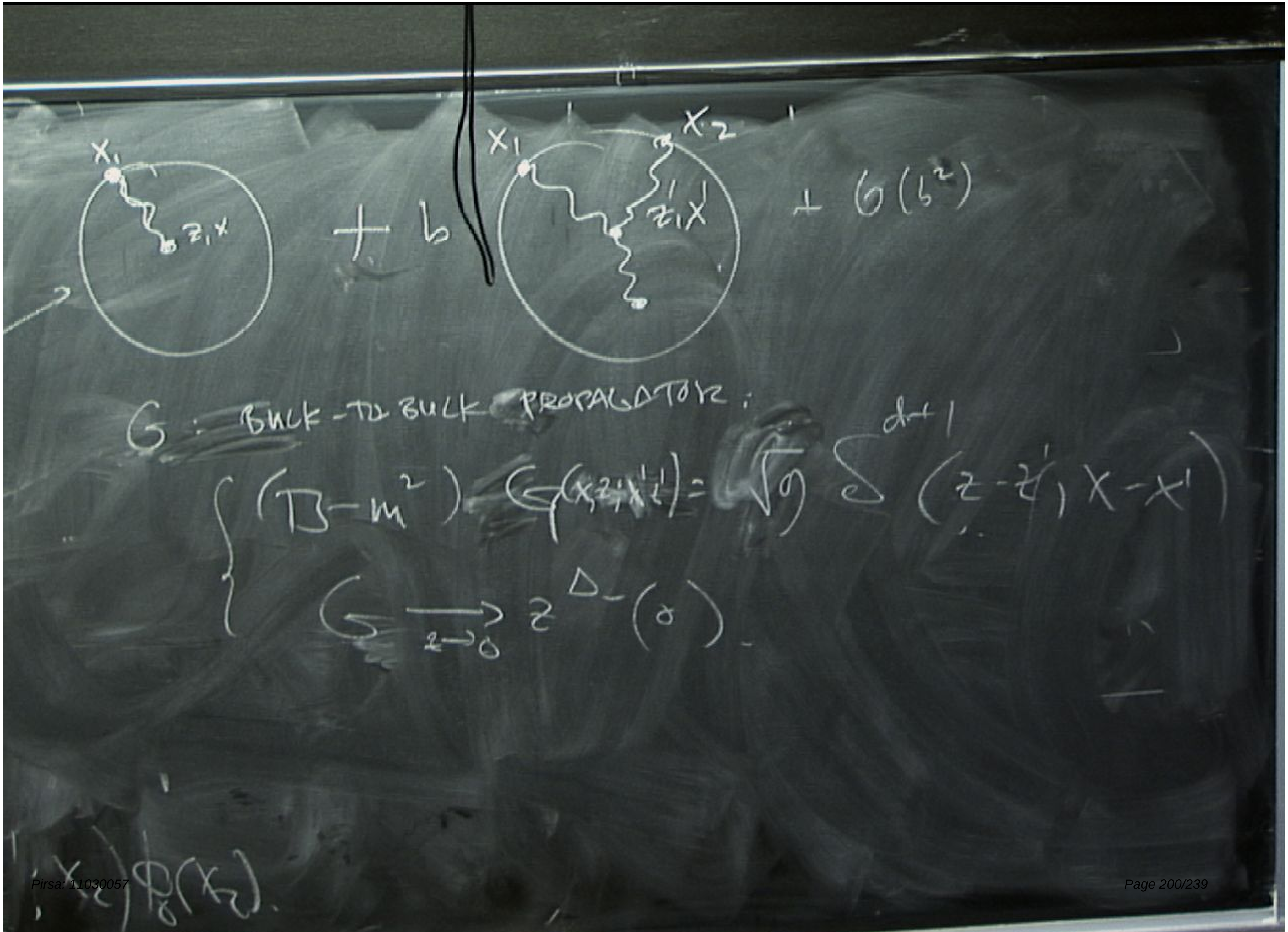
BDY-to-BULK propagator.
+ b

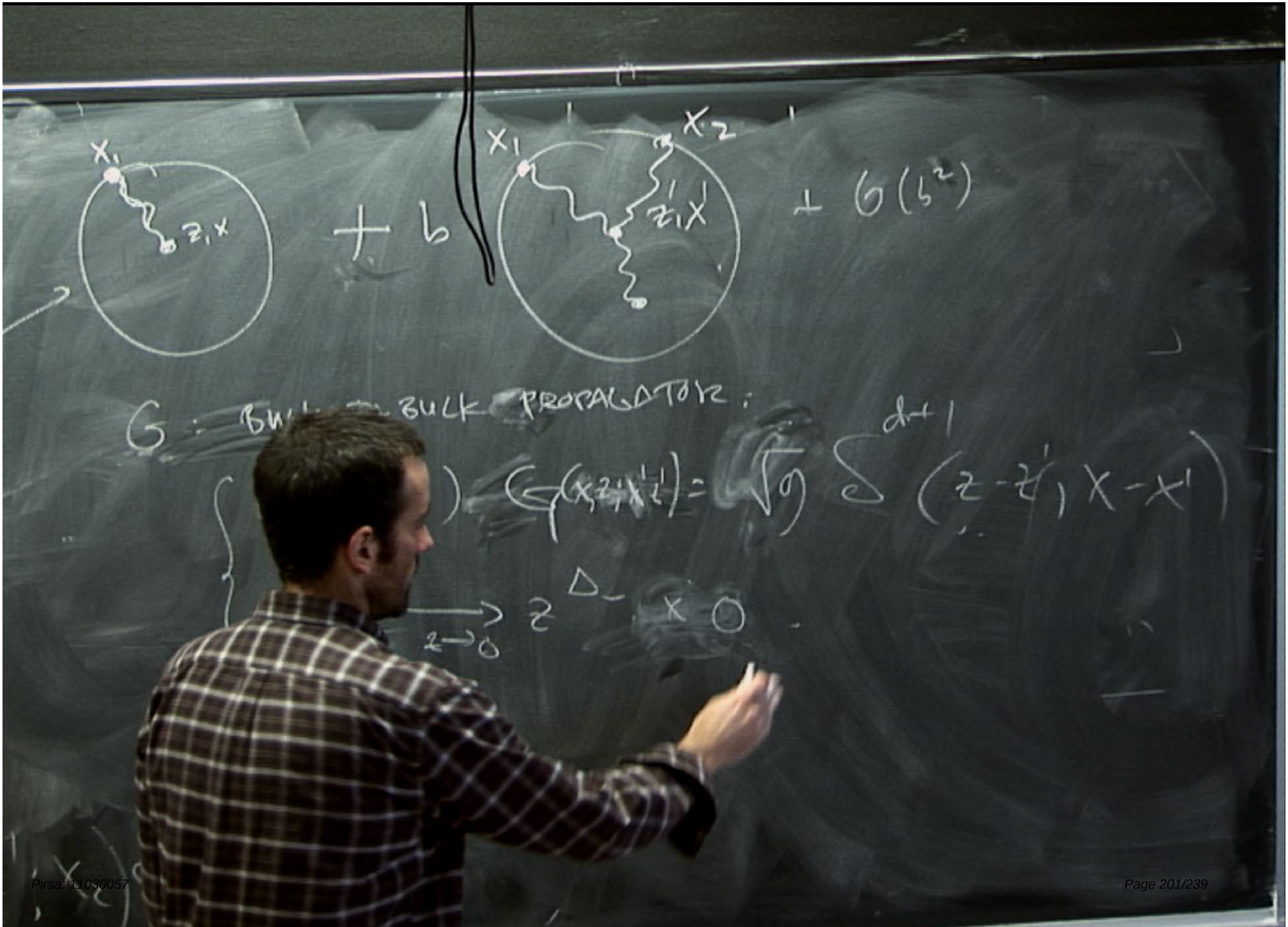

X1
z, x
+ b
X1
X2
z, X

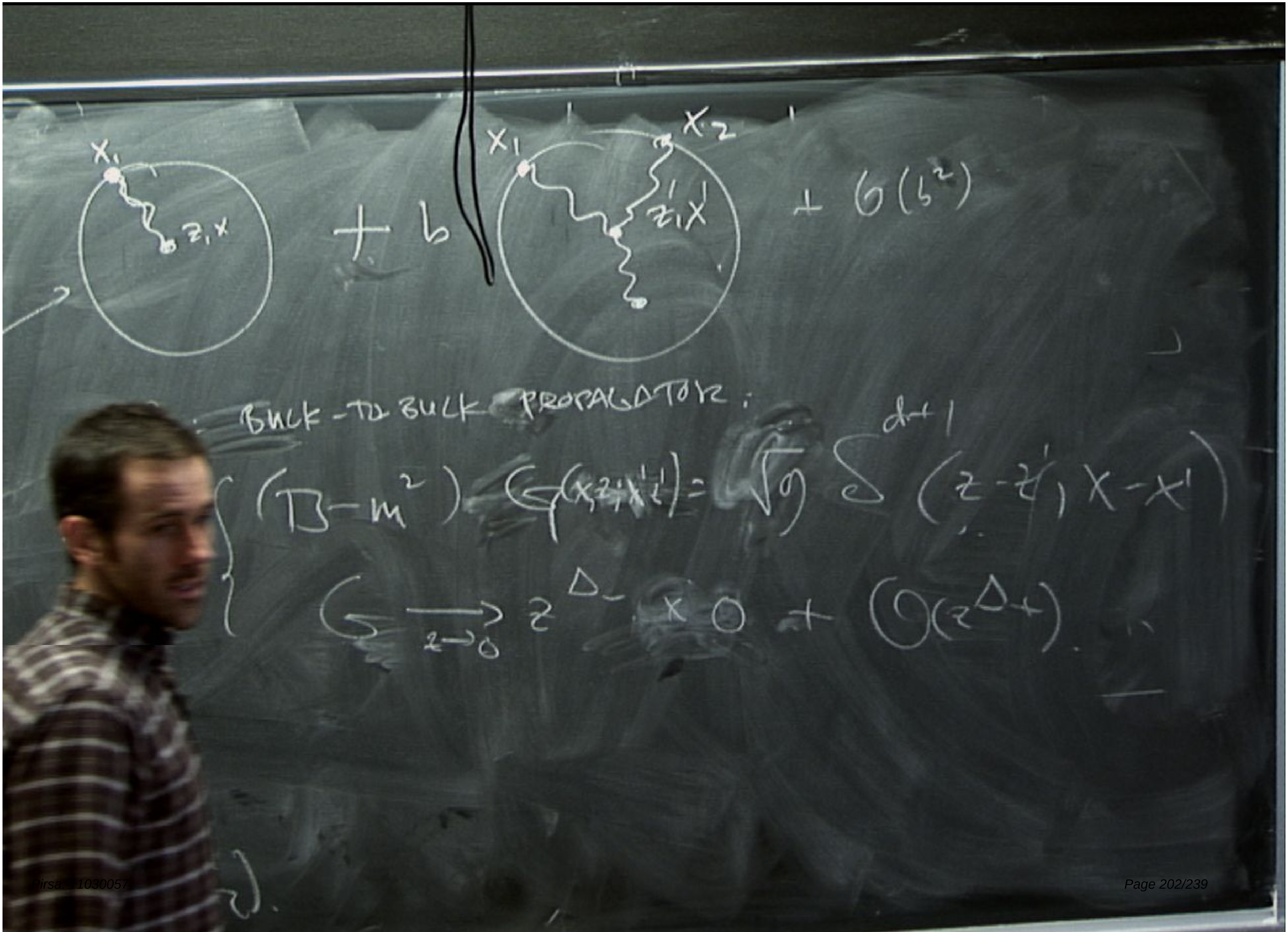

BDY-to-BULK propagator.
sol'n
+ b

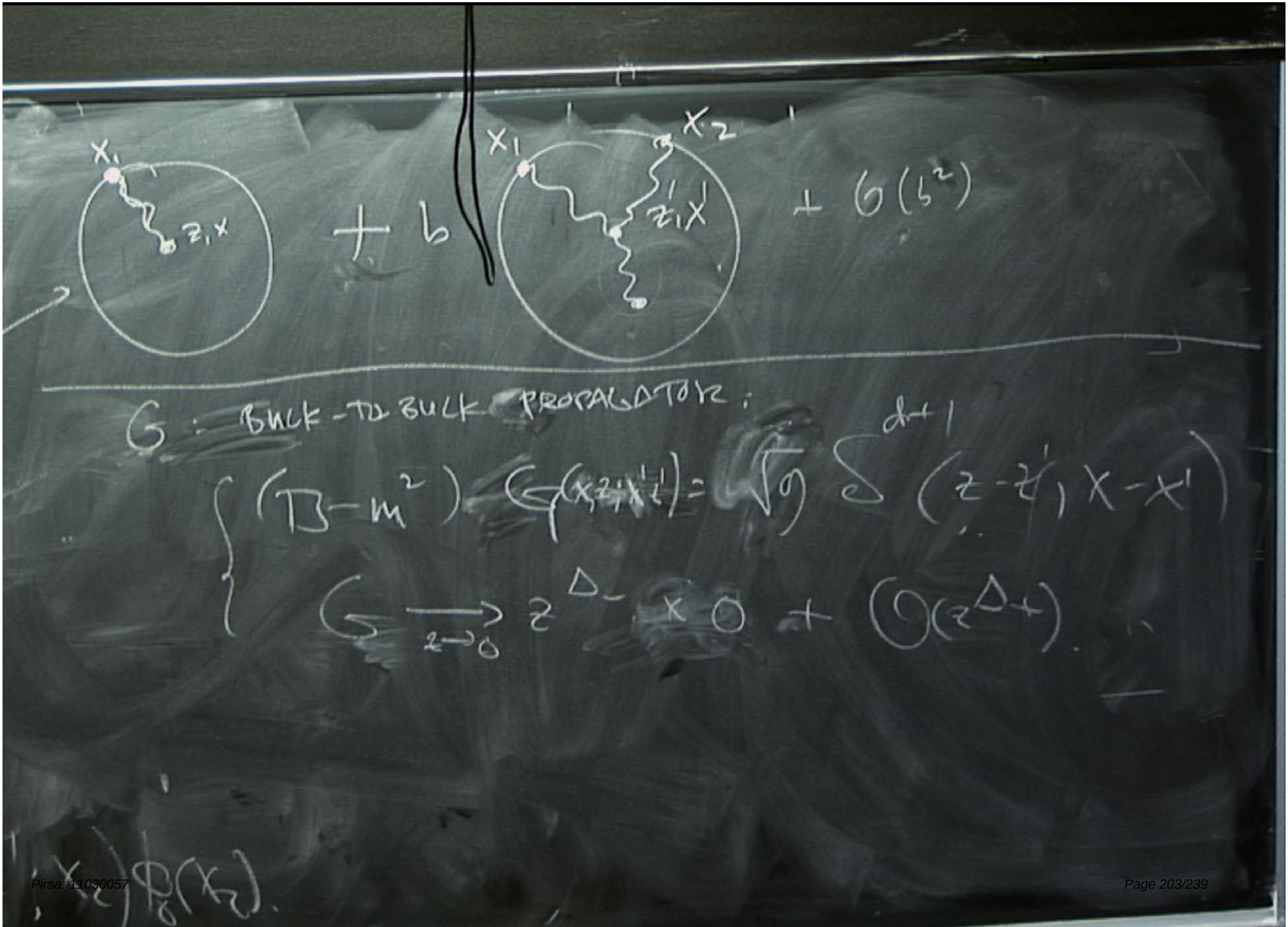

BDY-to-BULK propagator.
-TO BULK
+ b

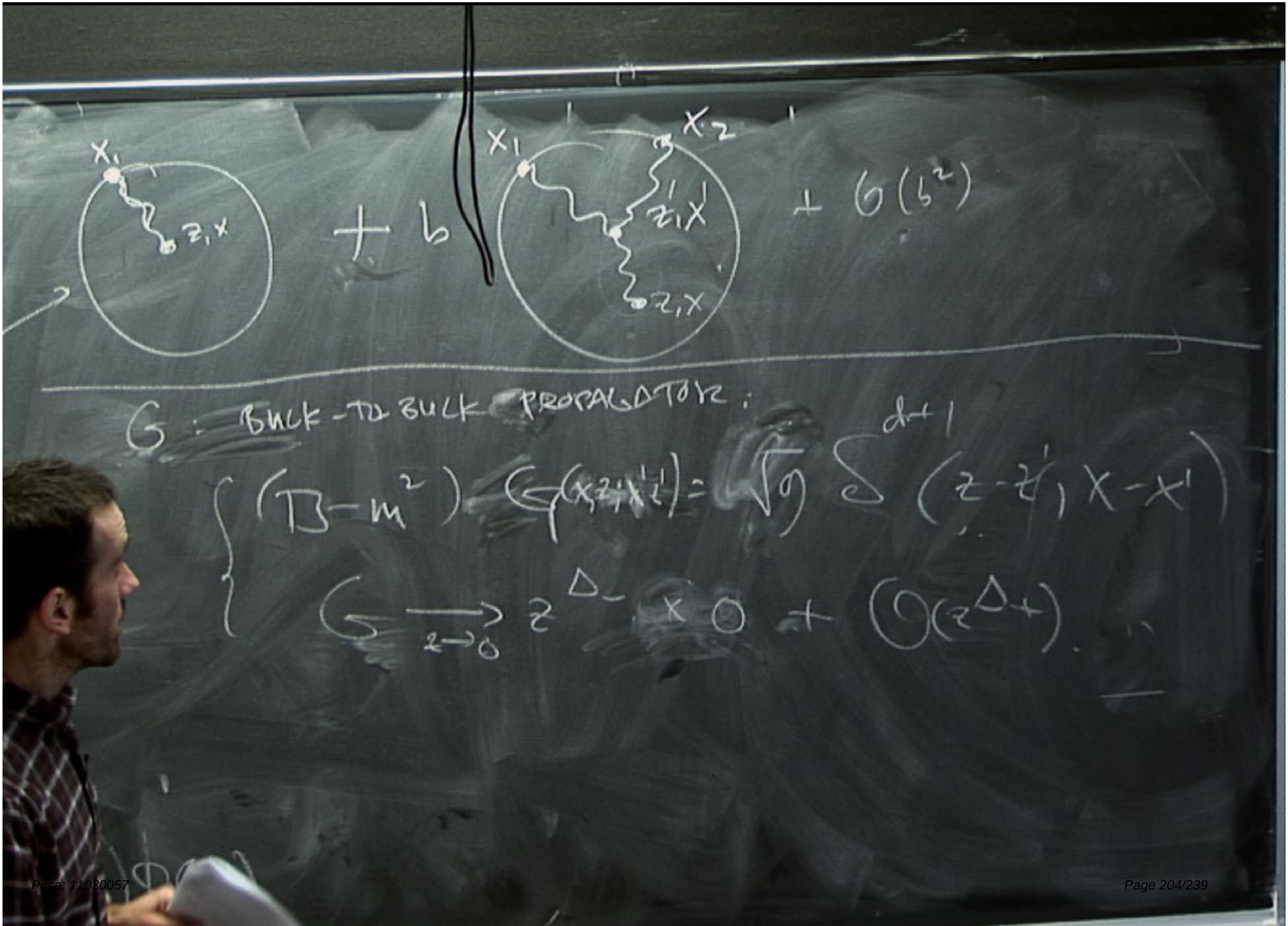

G : BULK-TO BULK PR ATOR :
+ b
+ G(b²)

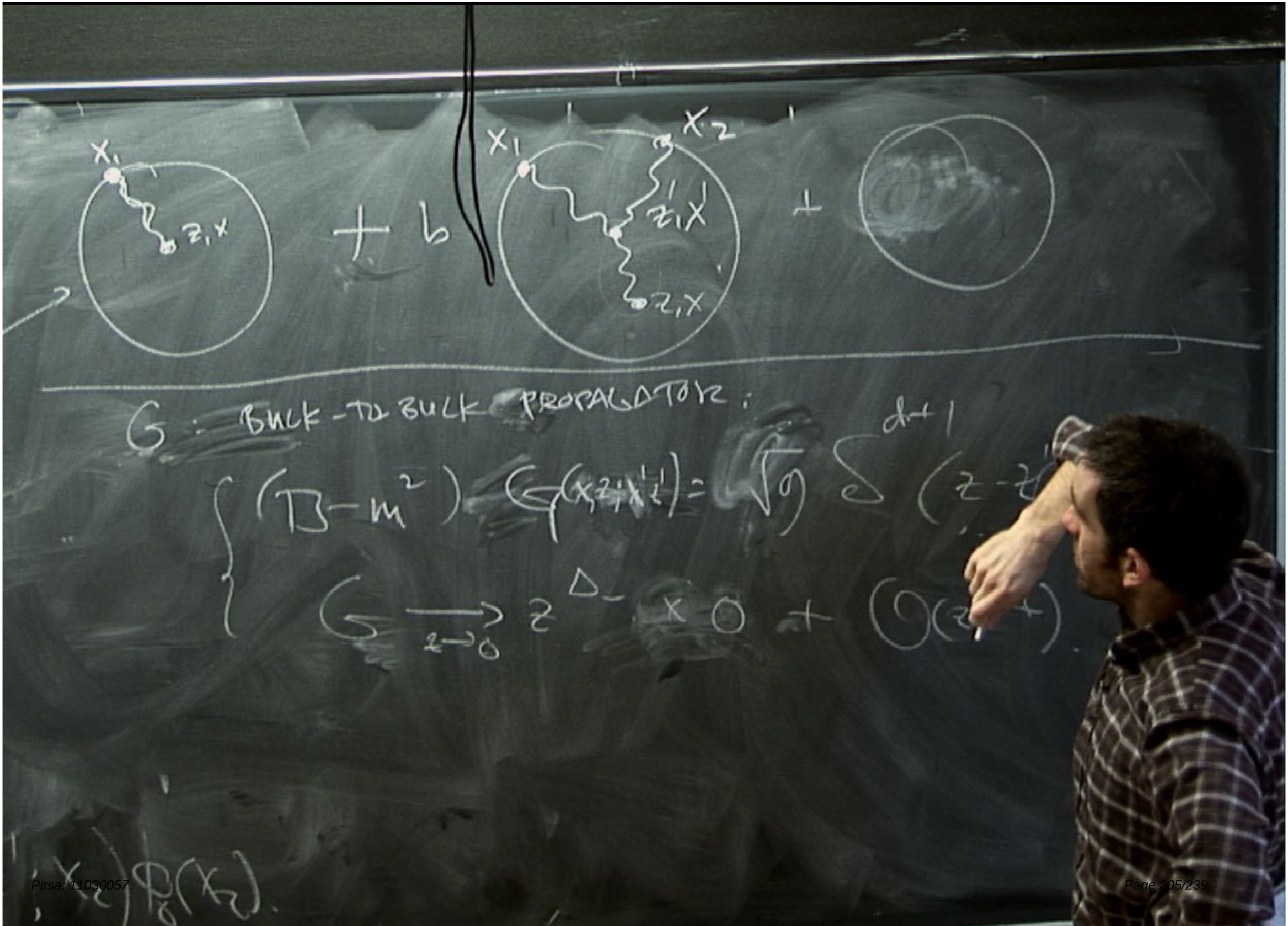

x_1
z, x
$+ b$
x_1
x_2
z', x'
$+ O(b^2)$
G : BULK-TO-BULK PROPAGATOR :
$(\Box - m^2)\, G(x,z;x',z') = \sqrt{g}\, \delta^{d+1}(z-z', x-x')$
$G \xrightarrow[z \to 0]{} z^{\Delta_-}(\sigma)$
$\phi_0(x_2)$

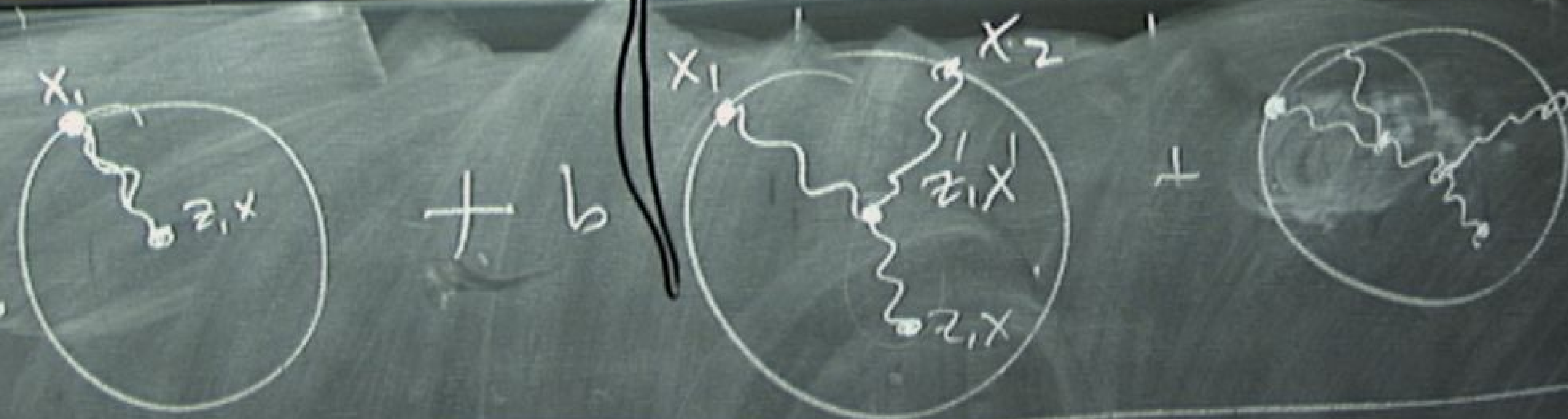

X1
z, x
+ b
X1
X2
z', x'
+ O(b²)
G :
BULK PROPAGATOR :
d+1
z → 0


x1
z, x
+ b
x1
x2
z', x'
+ O(b^2)
BULK-TO-BULK PROPAGATOR:


x1
z, x
+ b
x1
x2
z', x'
+ O(b²)
G : BULK-TO BULK PROPAGATOR :
(□ - m²) G(x,z;x',z') = √g δ^(d+1) (z - z', x - x')
G → z^(Δ-) × 0 + O(z^(Δ+))
z→0


X1
z, x
+ b
X1
X2
z', x'
z, x
+ O(b^2)
G : BULK-TO-BULK PROPAGATOR :
(□ − m^2) G(x,z;x',z') = √g δ^{d+1} (z − z', x − x')
G → z^{Δ−} × O + O(z^{Δ+})
z→0

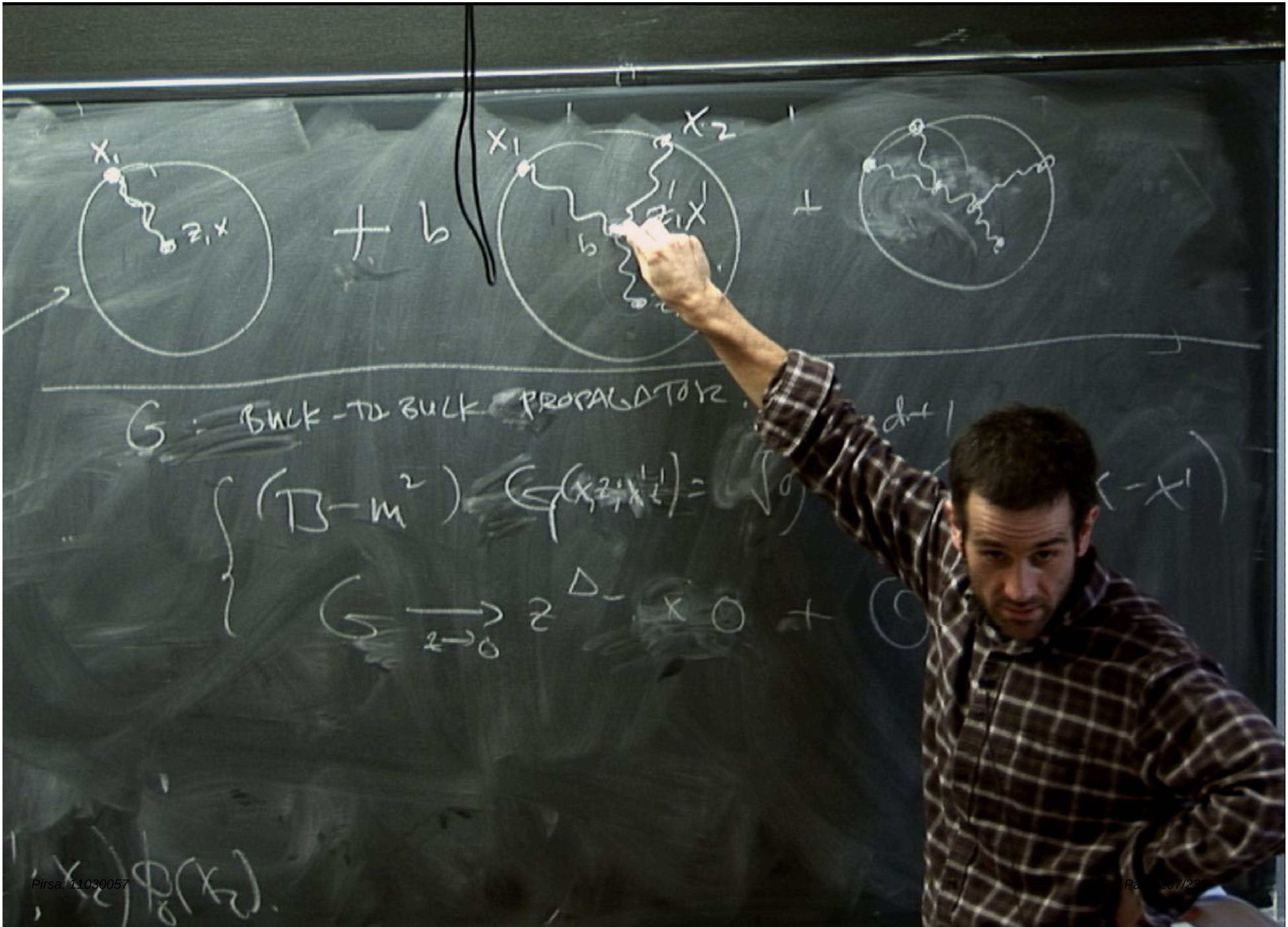

x1
z, x
+ b
x1
x2
z, x
z, x
+
G : BULK-TO BULK PROPAGATOR :

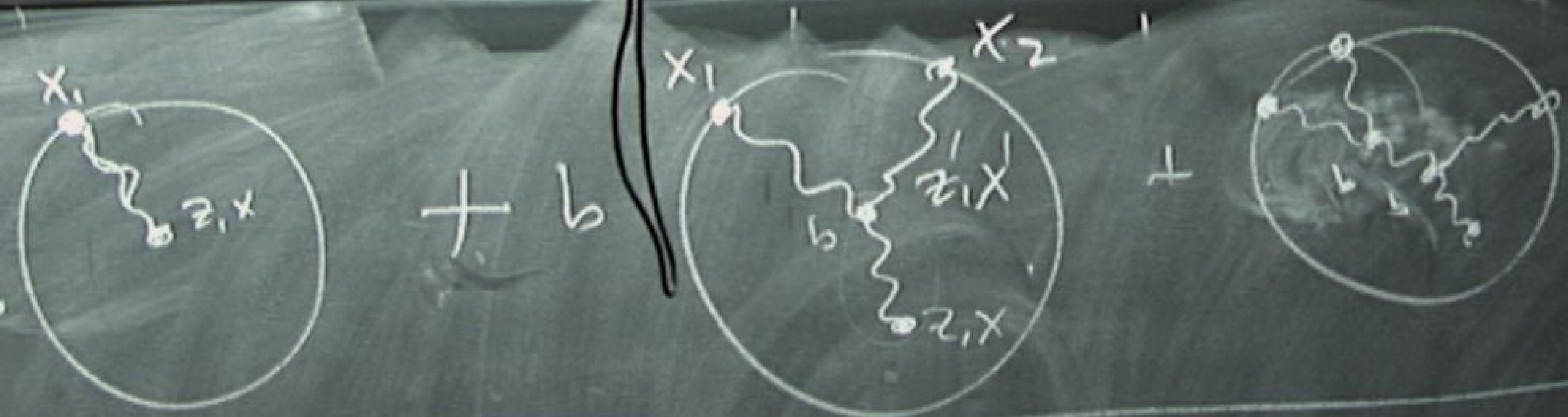


G : BULK-TO-BULK PROPAGATOR :

$$\begin{cases} (\Box - m^2)\, G(x,z;x',z') = \sqrt{g}\, \delta^{d+1}(z-z', x-x') \\ G \xrightarrow[z\to 0]{} z^{\Delta_-} \times \bigcirc + O(z^{\Delta_+}). \end{cases}$$

$; x_2)\, \phi_0(x_2).$

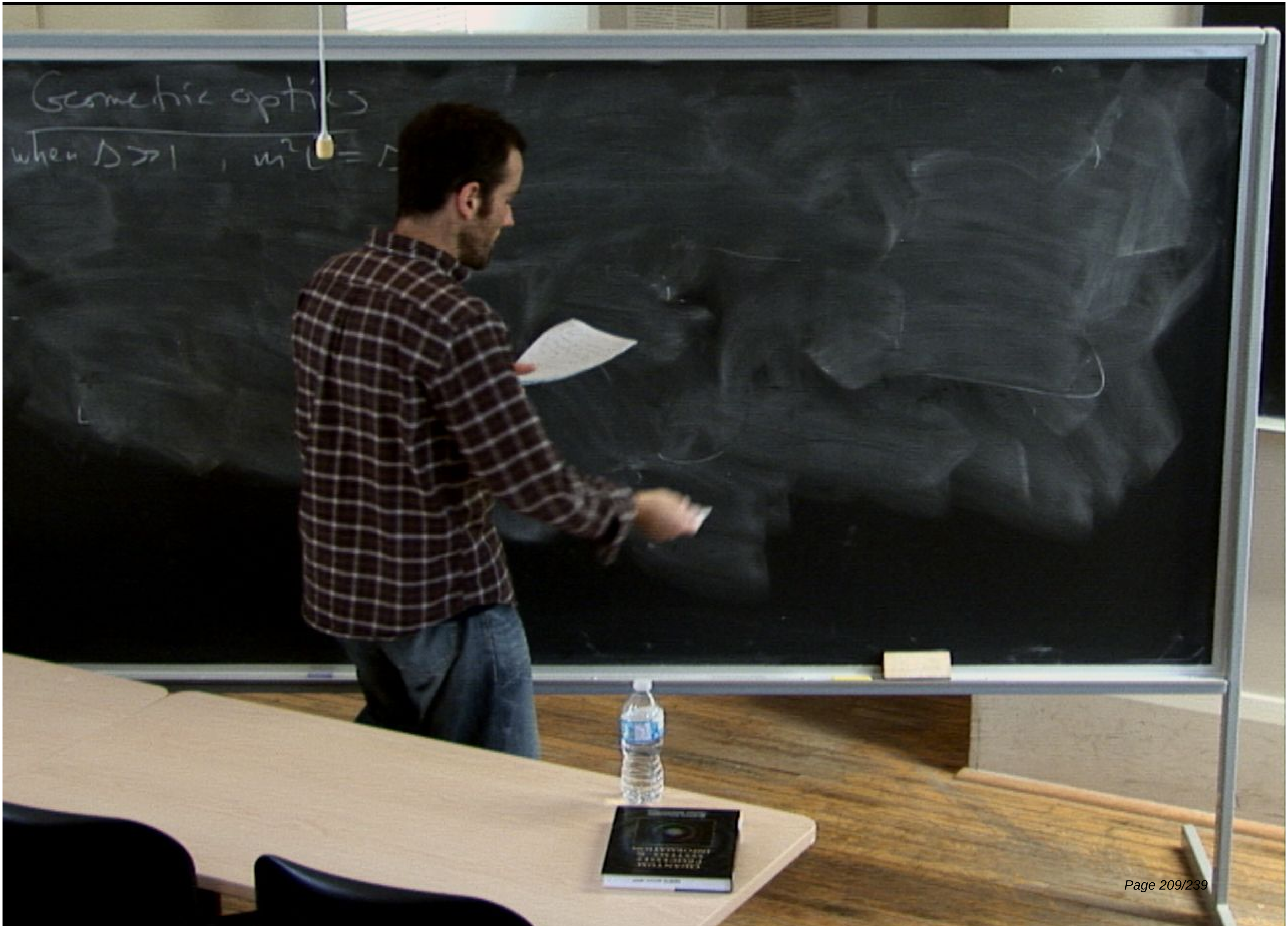

X1
z, x
+ b
X1
X2
G : BULK-TO-BULK PROPAGATOR

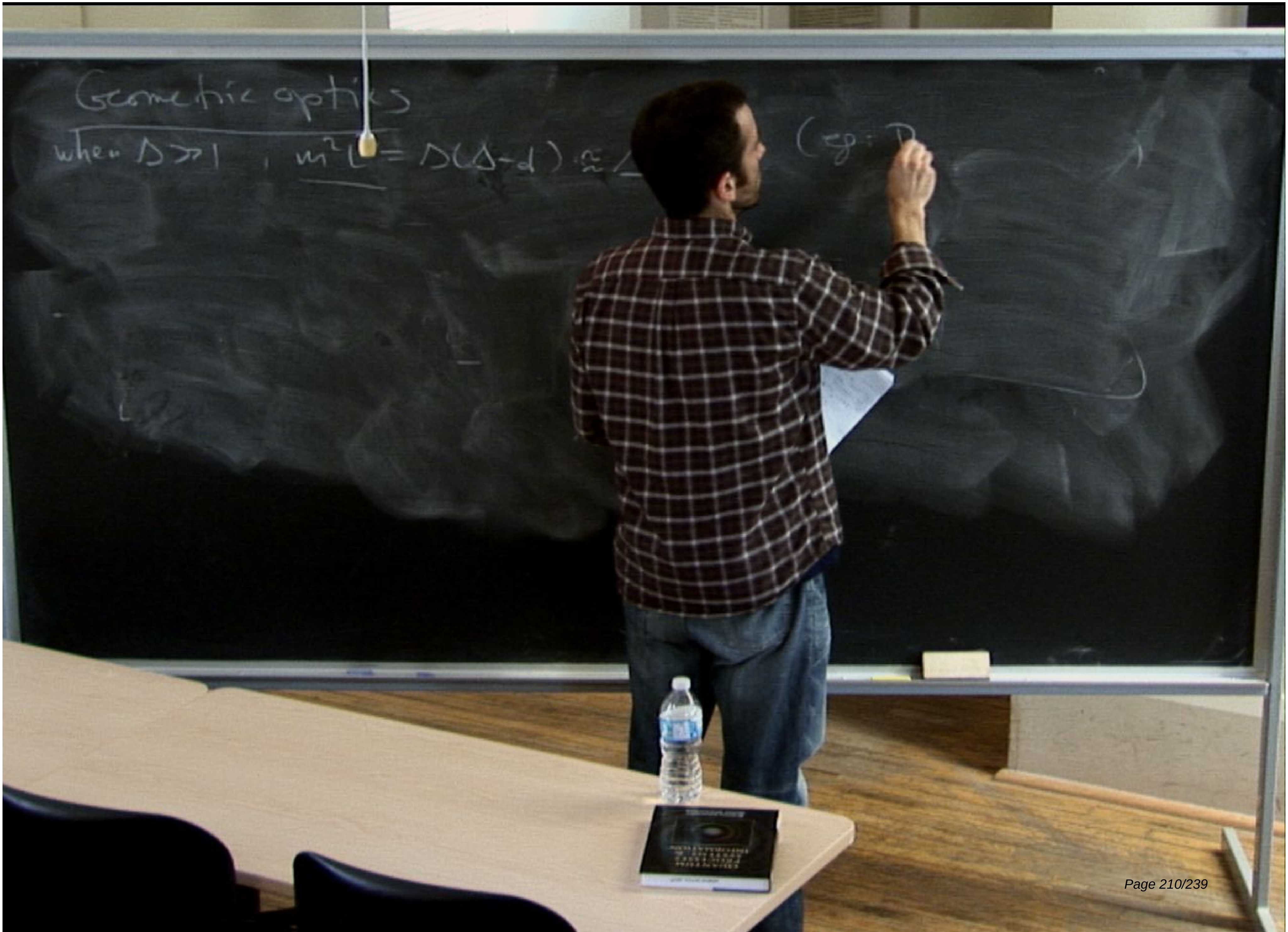


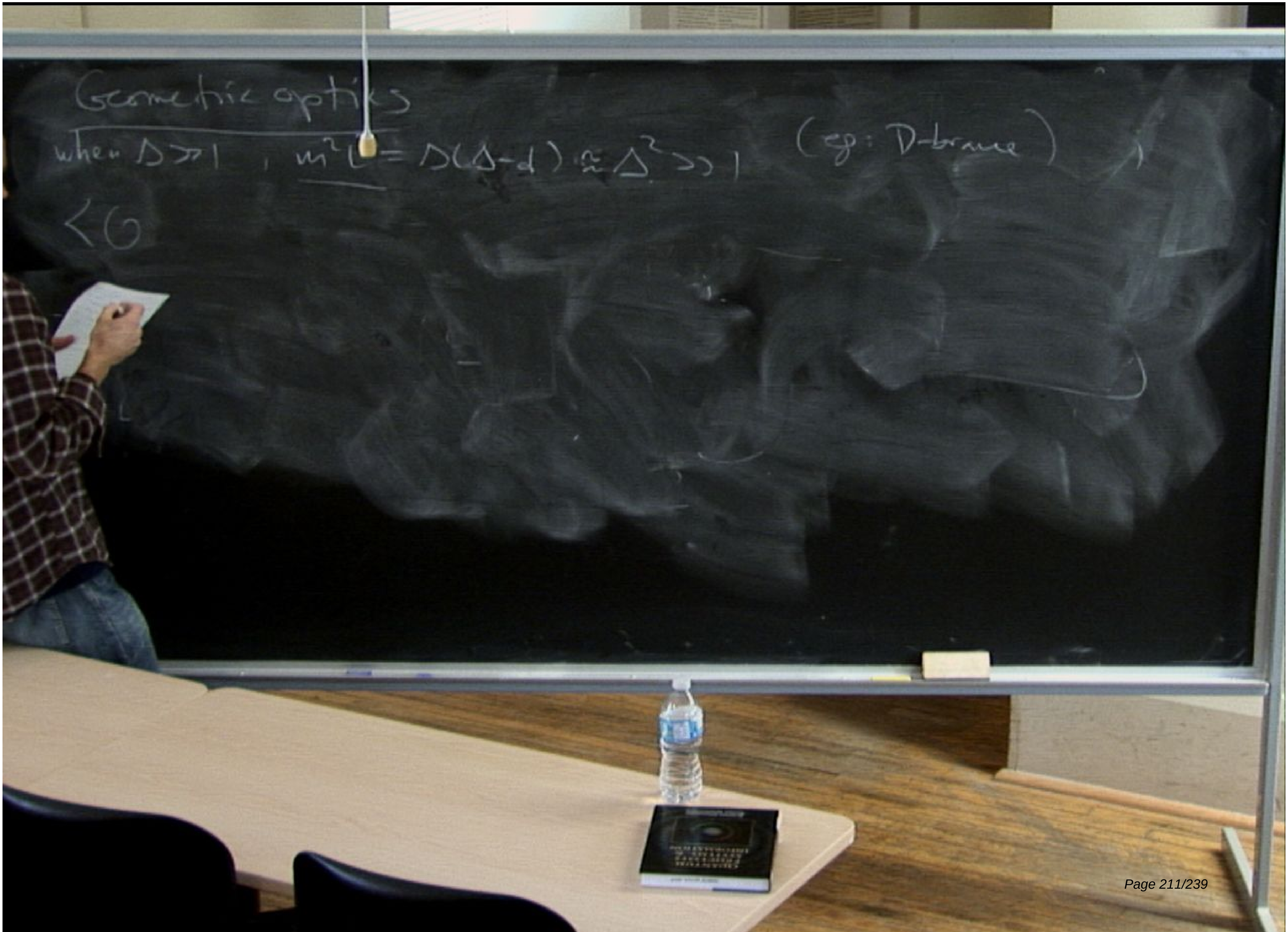
G : BULK-TO-BULK PROPAGATOR:

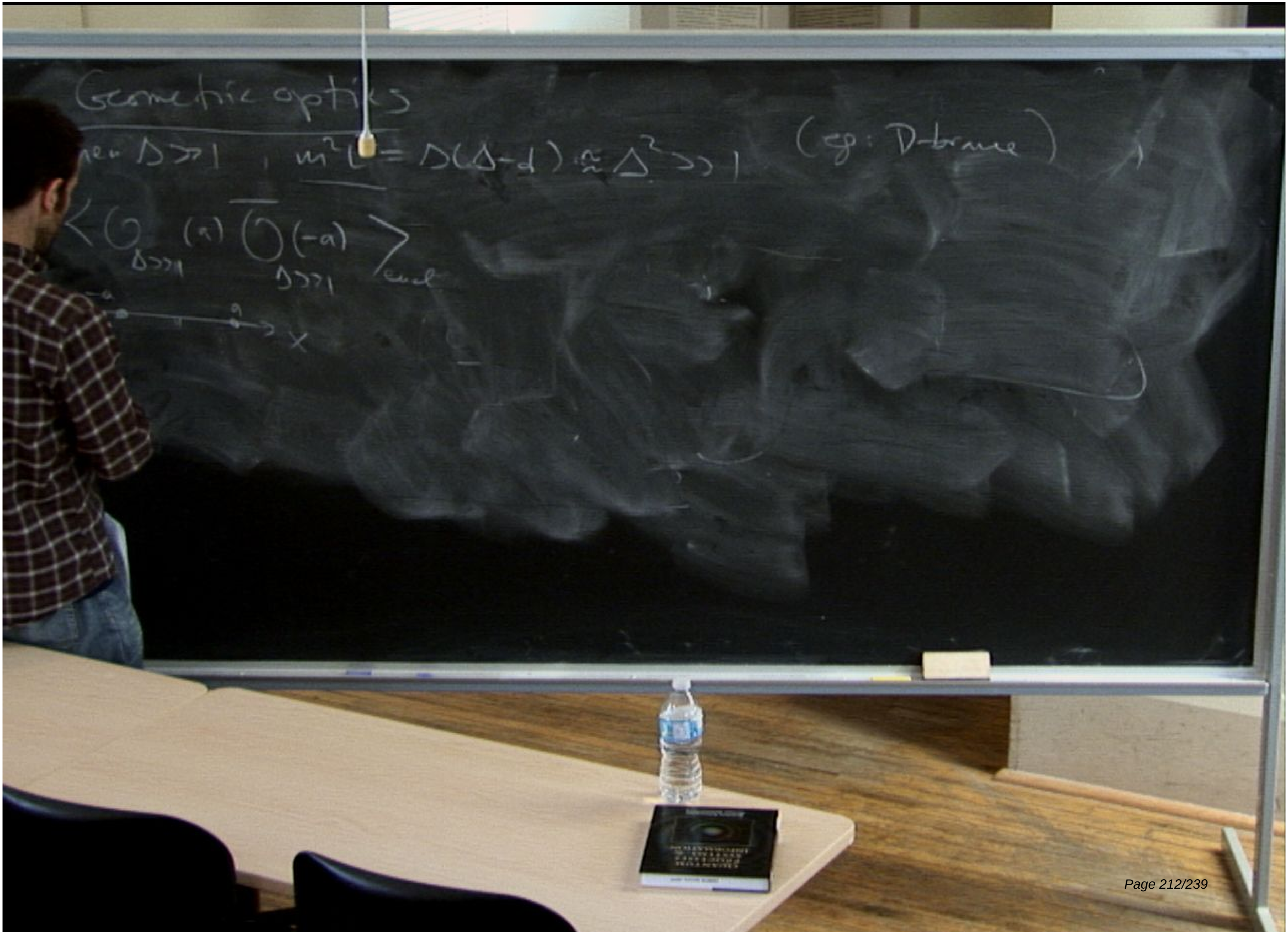
$$\begin{cases} (\Box - m^2)\, G(x,z;x',z') = \sqrt{g}\, \delta^{d+1}(z-z', x-x') \\ G \xrightarrow[z\to 0]{} z^{\Delta_-} \times \mathcal{O} + O(z^{\Delta_+}) \end{cases}$$

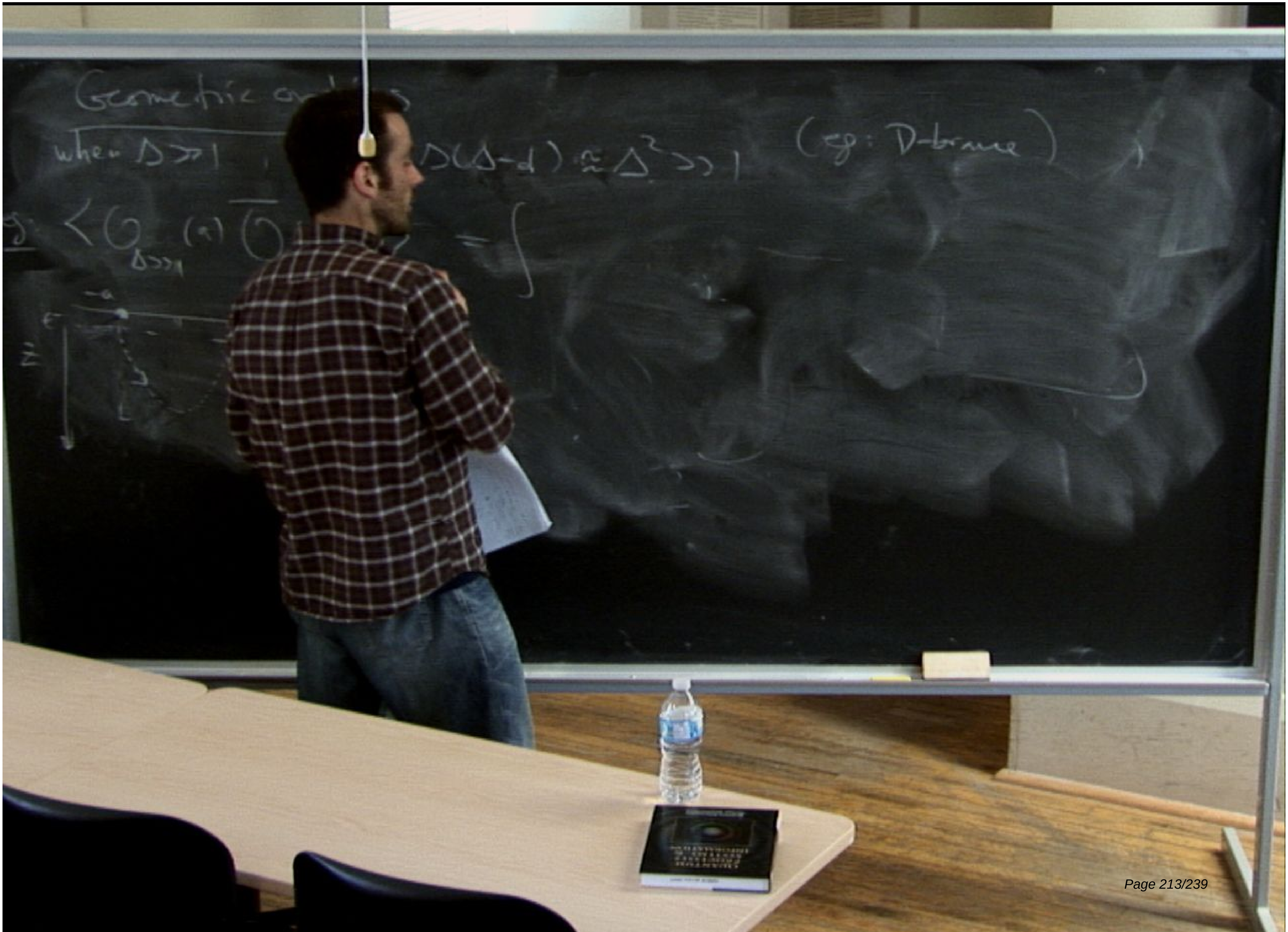
$; x_2)\, \phi_0(x_2).$

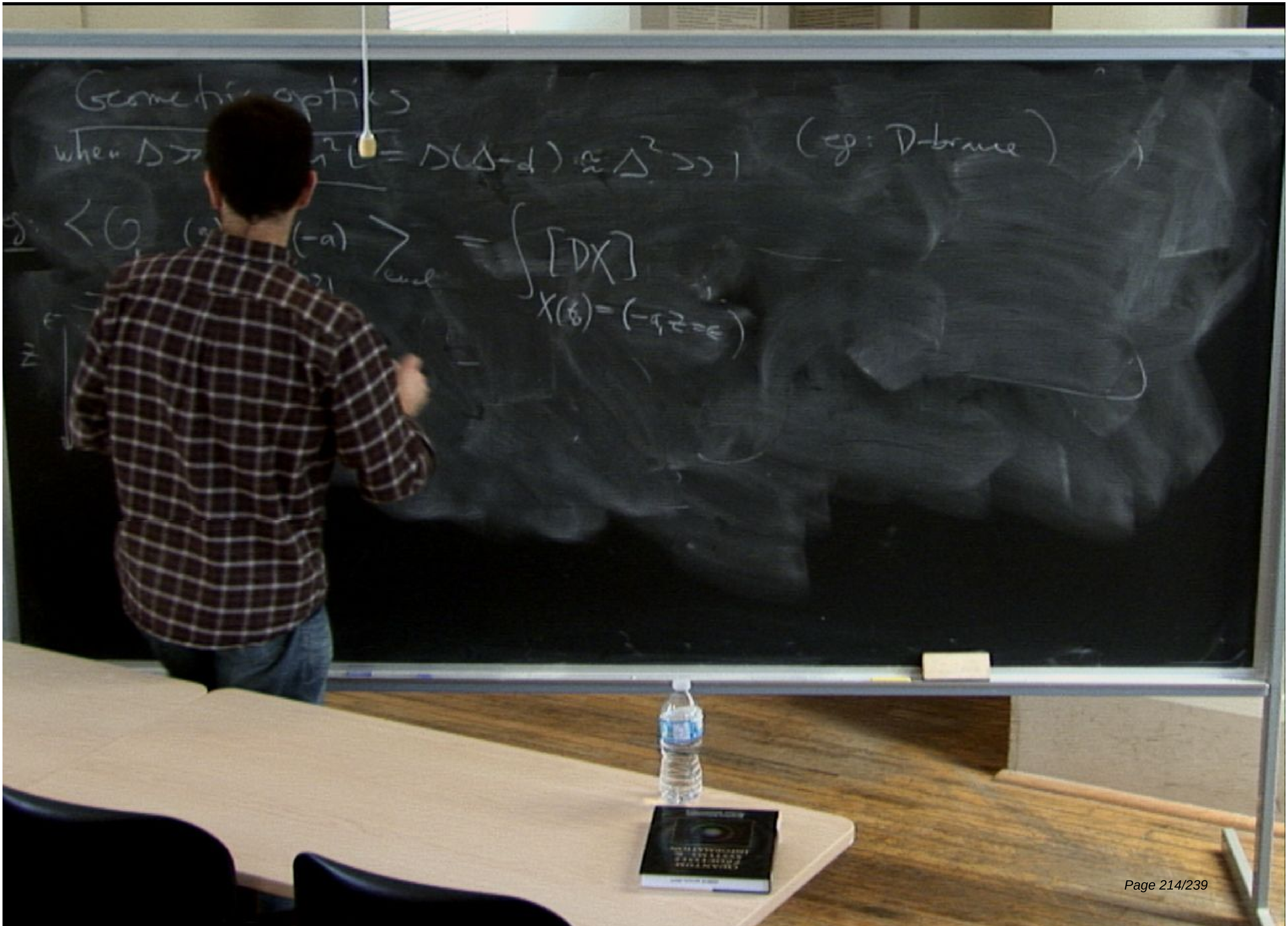

Geometric optics
when Δ ≫ 1 , m²L =

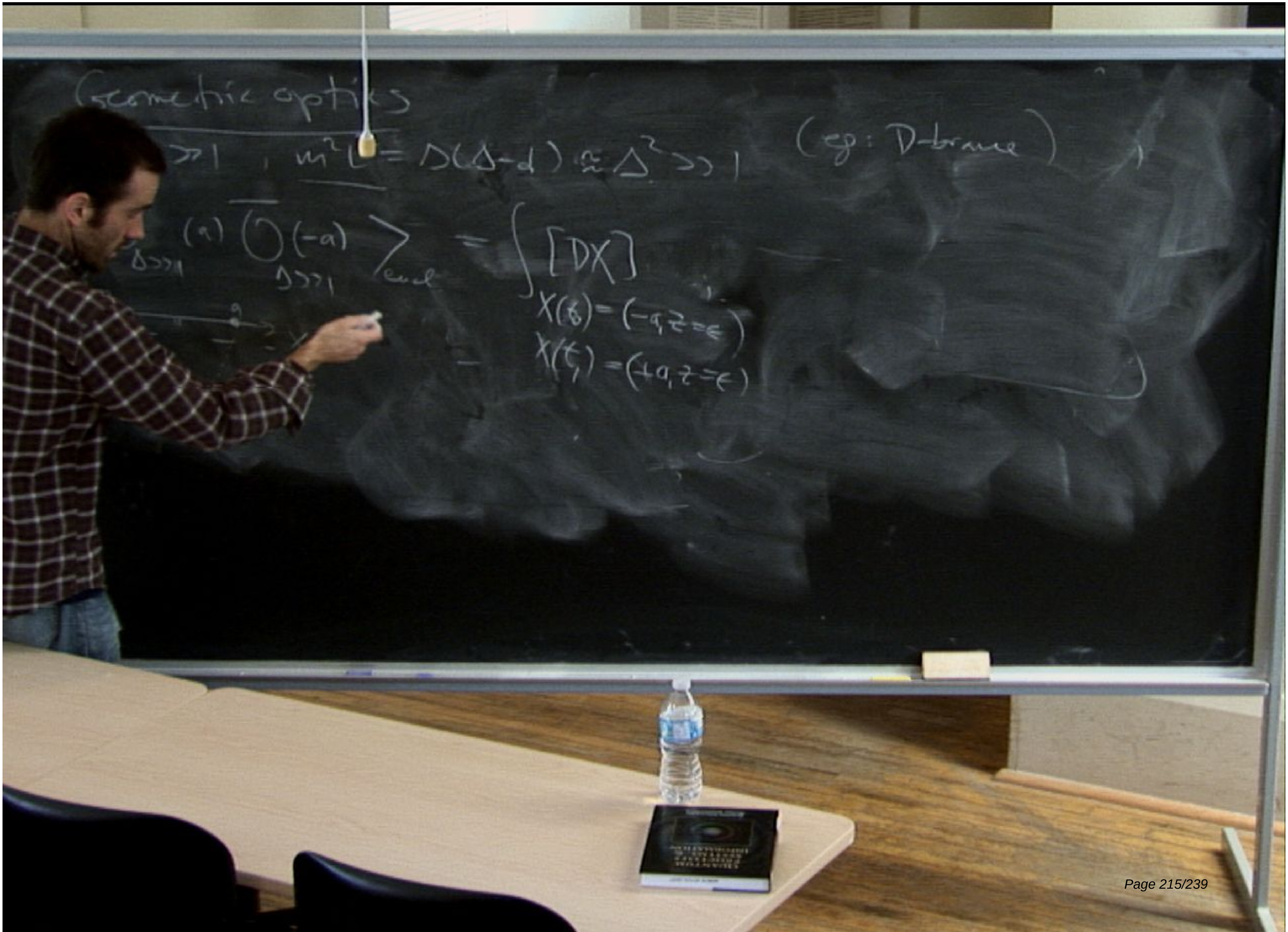

Geometric optics

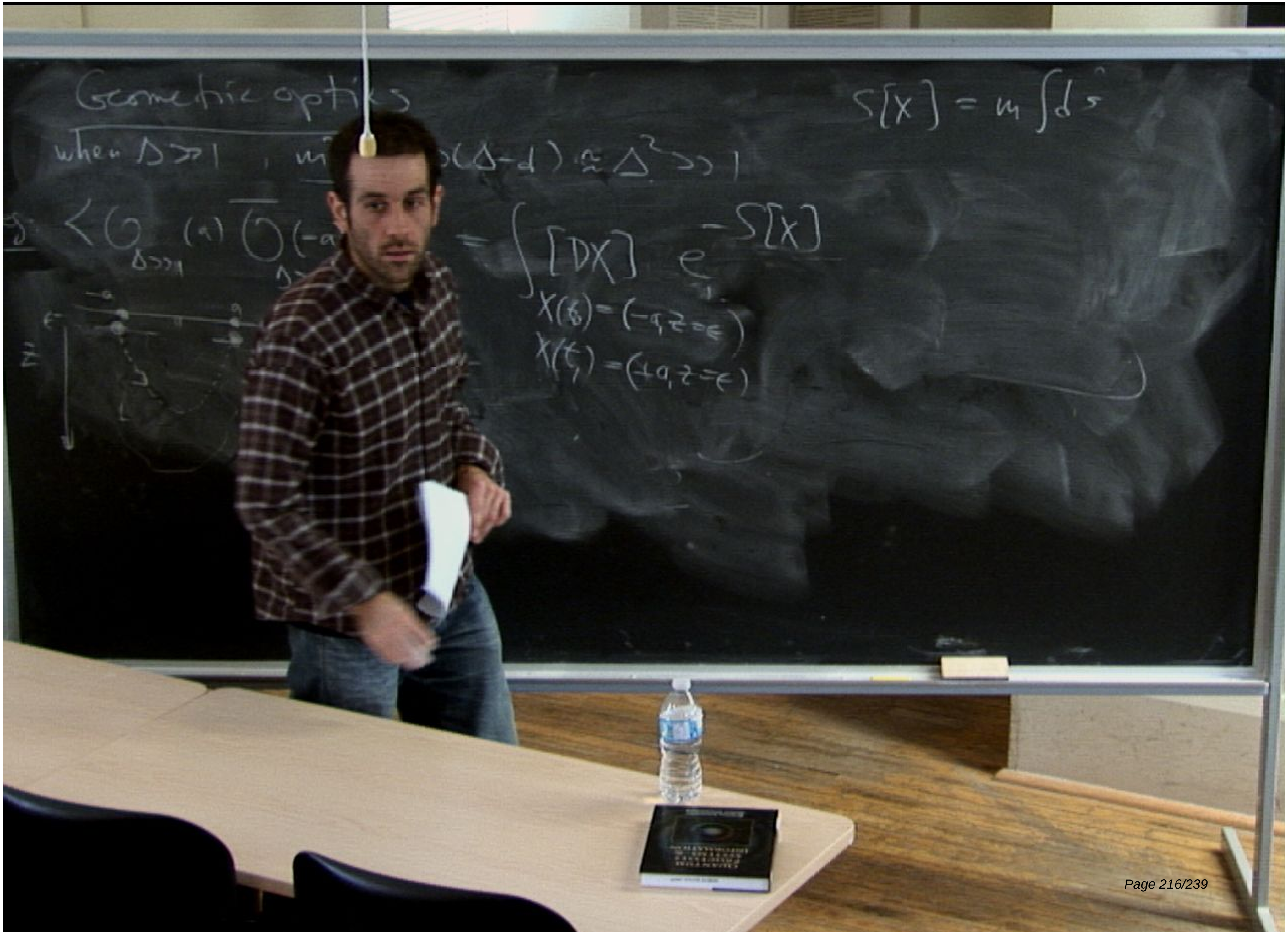

Geometric optics
when Δ >> 1 , m²L² = Δ(Δ-d) ≈ Δ² >> 1
(eg: D-brane)

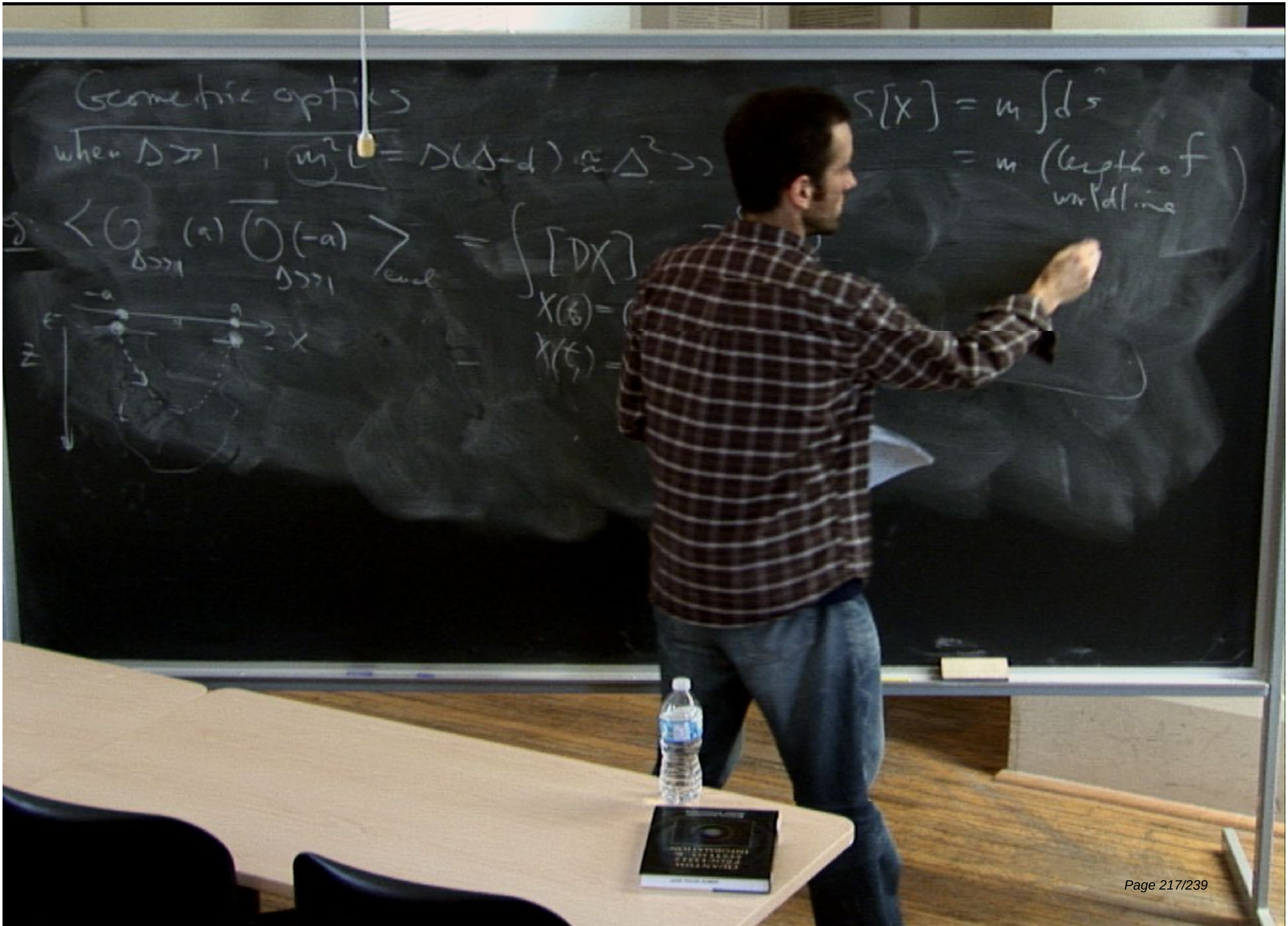

Geometric optics
(eg: D-brane)

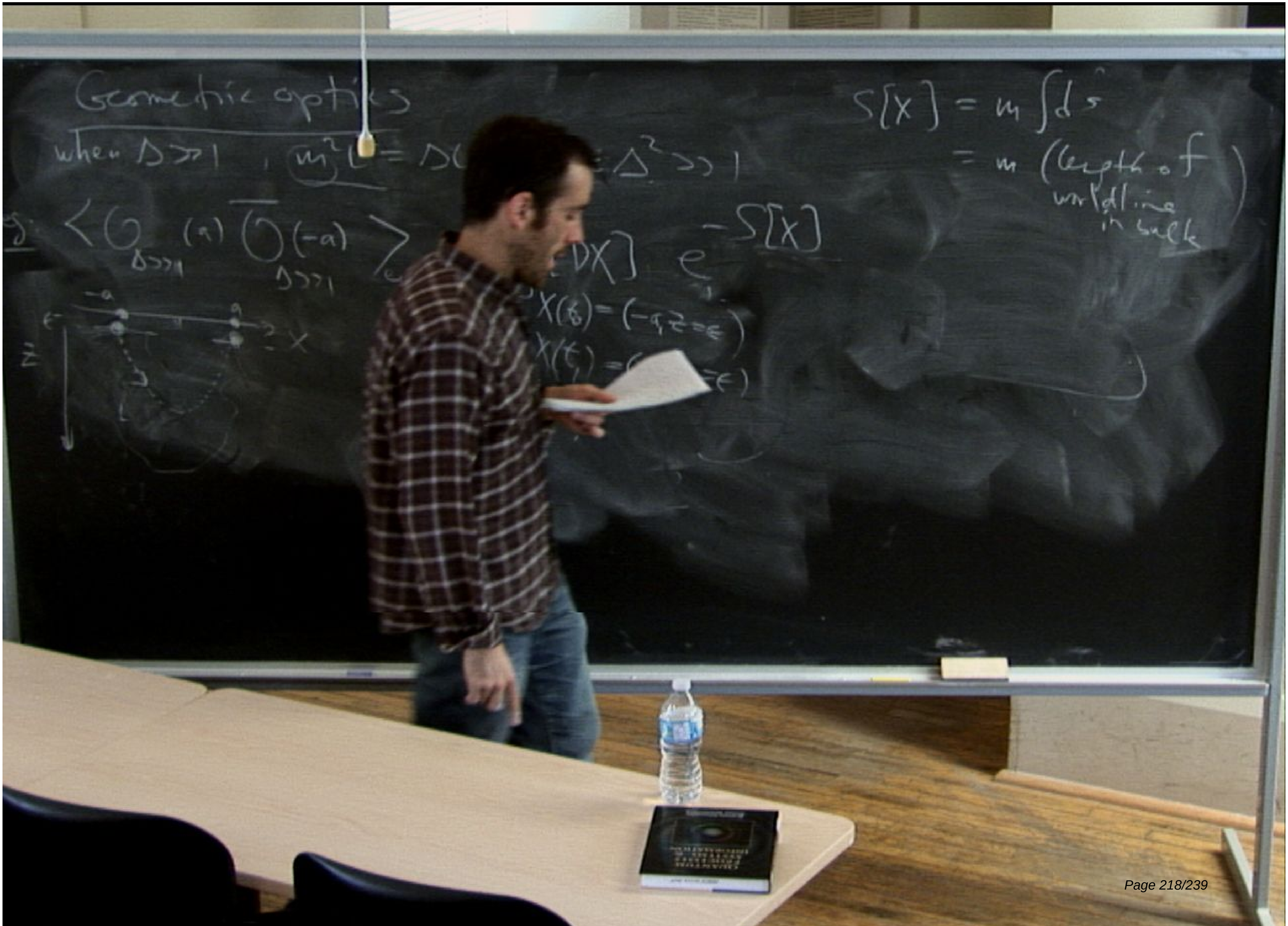

when Δ≫1
Δ(Δ-d) ≃ Δ² ≫ 1
(eg: D-brane)

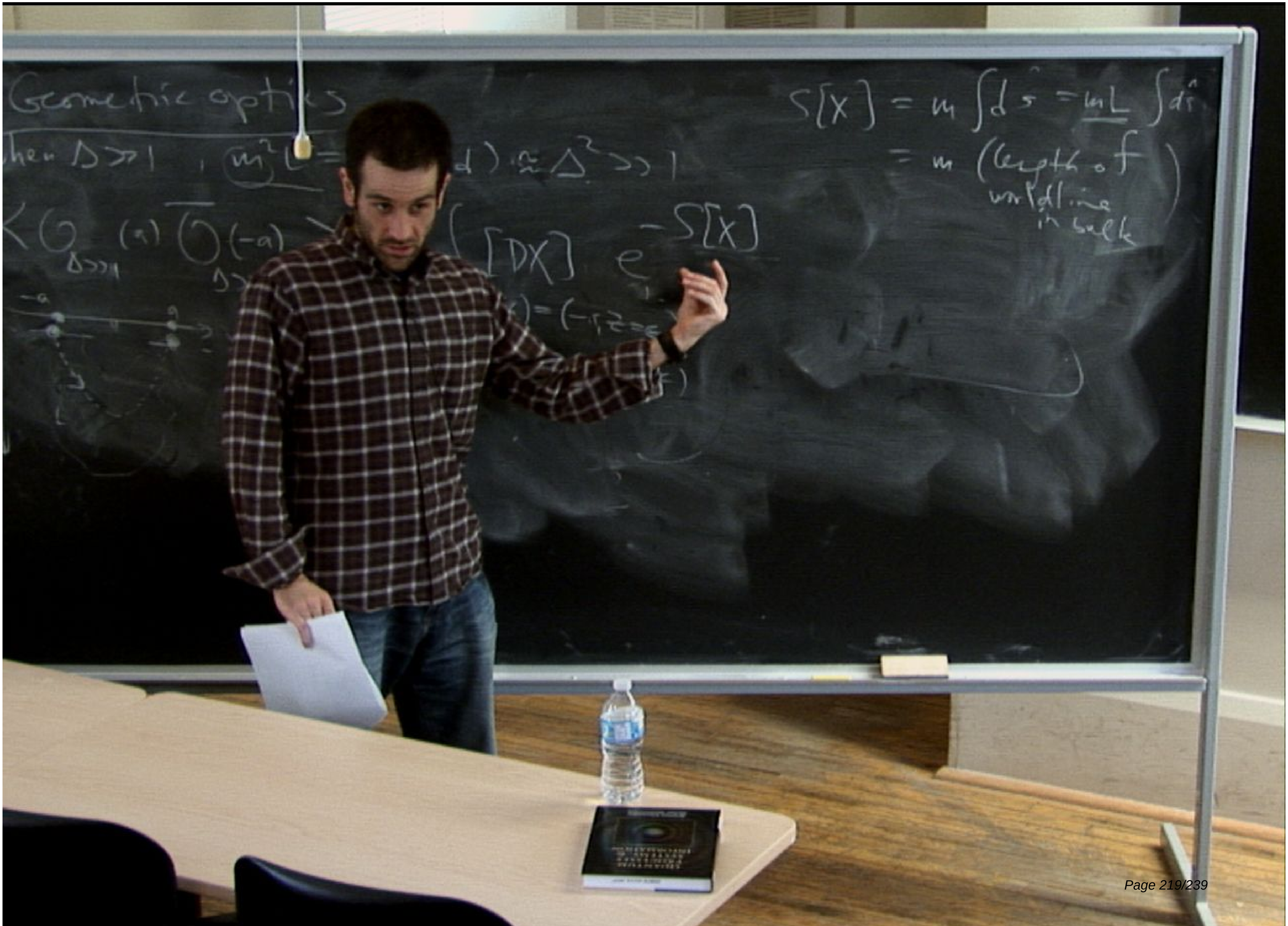

(eg: D-brane)
[DX]

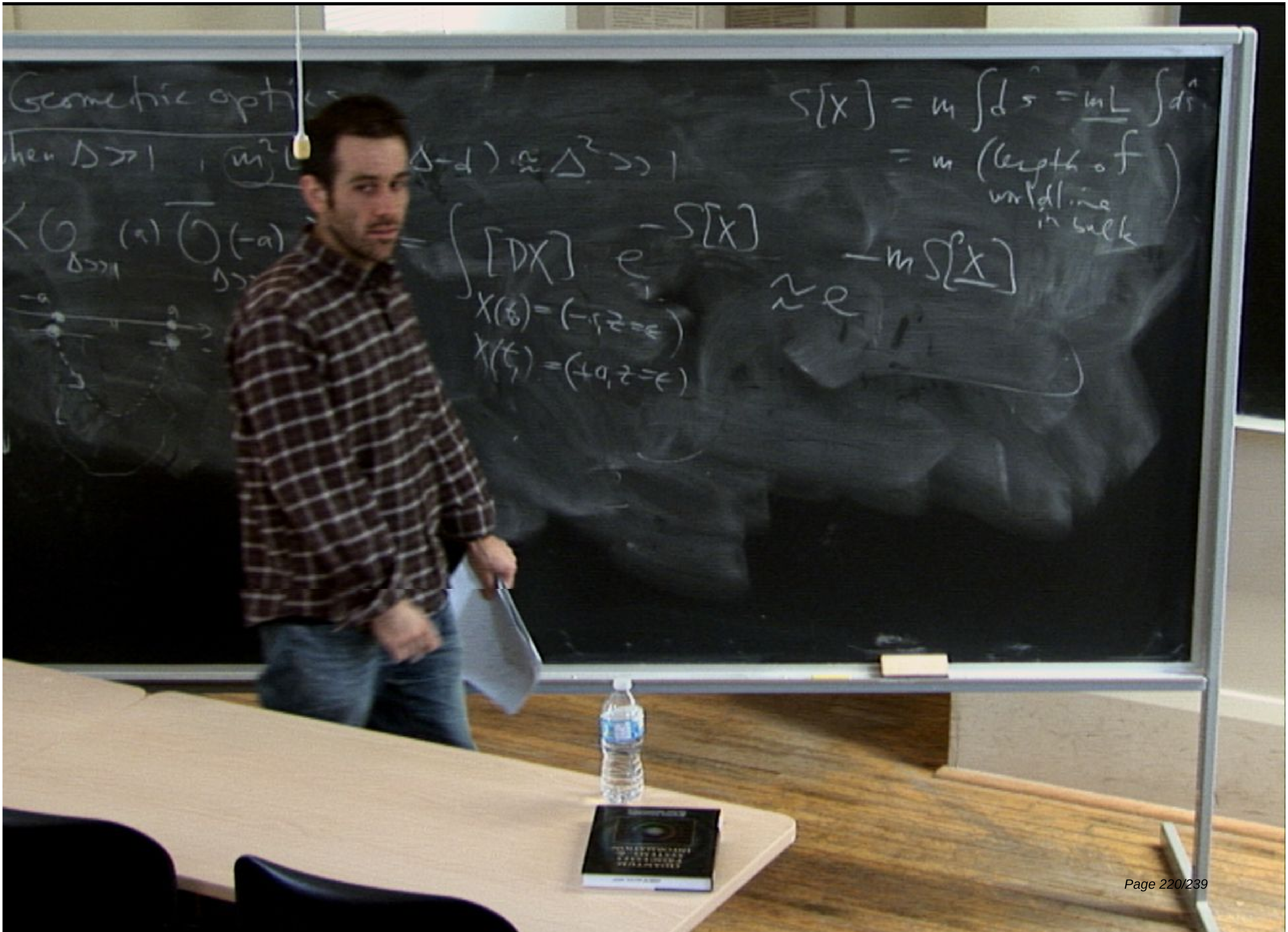

Geometric optics
(eg: D-brane)
[DX]
X(t₀) = (-a, z = ε)
X(t₁) = (+a, z = ε)

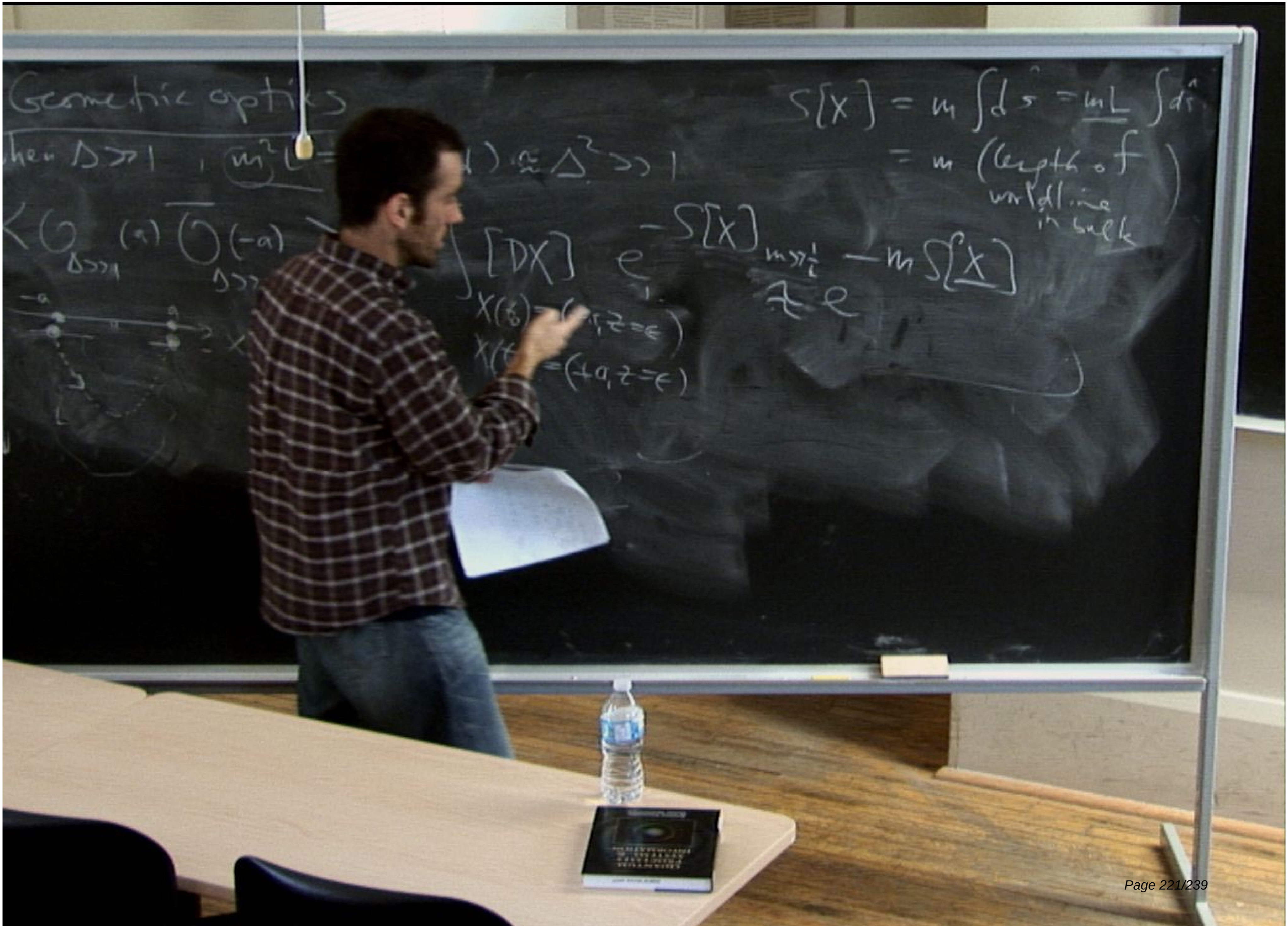

Geometric optiks
S[X] = m ∫ ds
when Δ >> 1
[DX] e^{-S[X]}
X(t_0) = (-a, z = ε)
X(t_1) = (+a, z = ε)

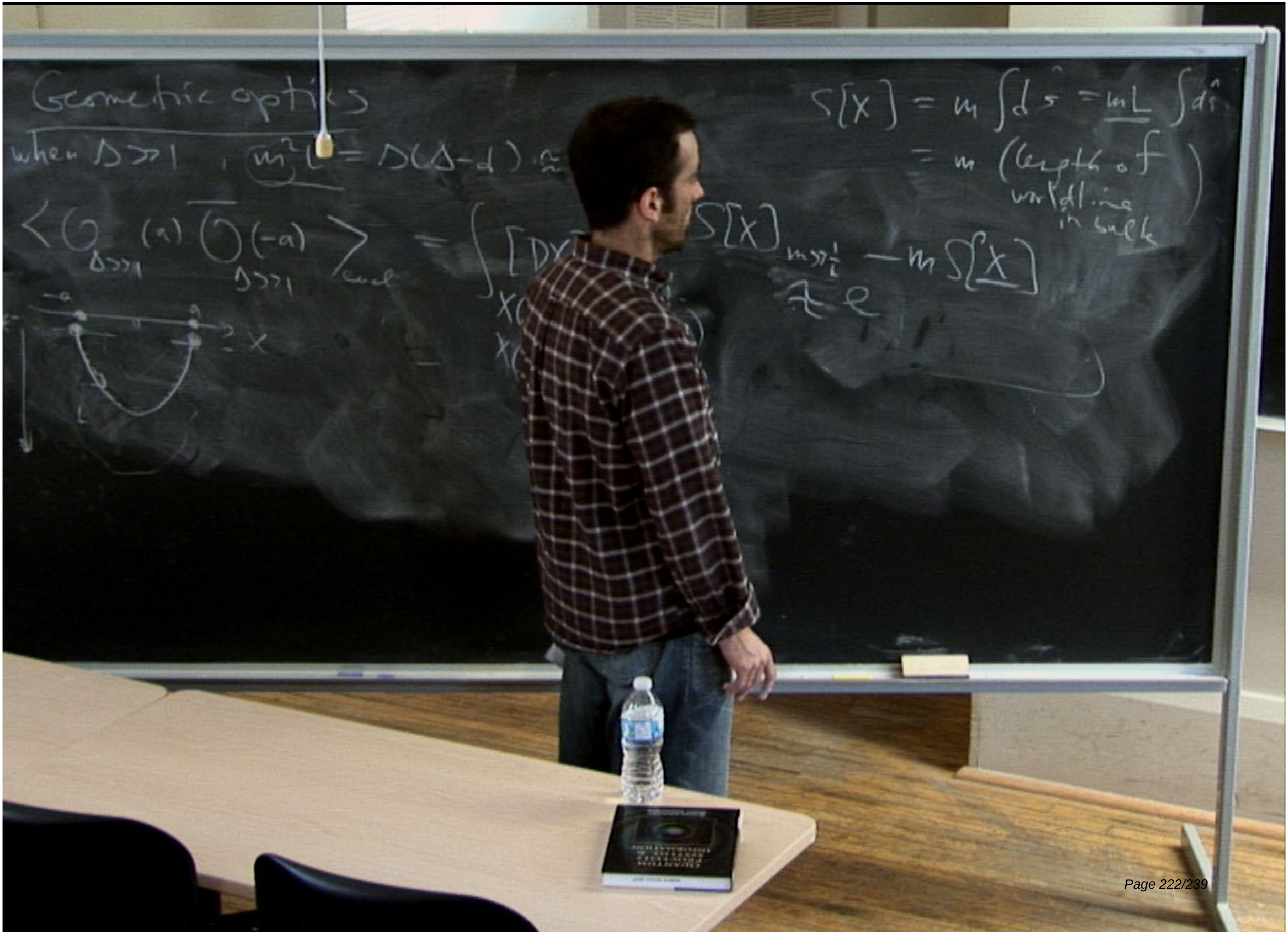

Geometric optics
when Δ ≫ 1
S[X] = m ∫ ds
= m (length of worldline)
[DX]

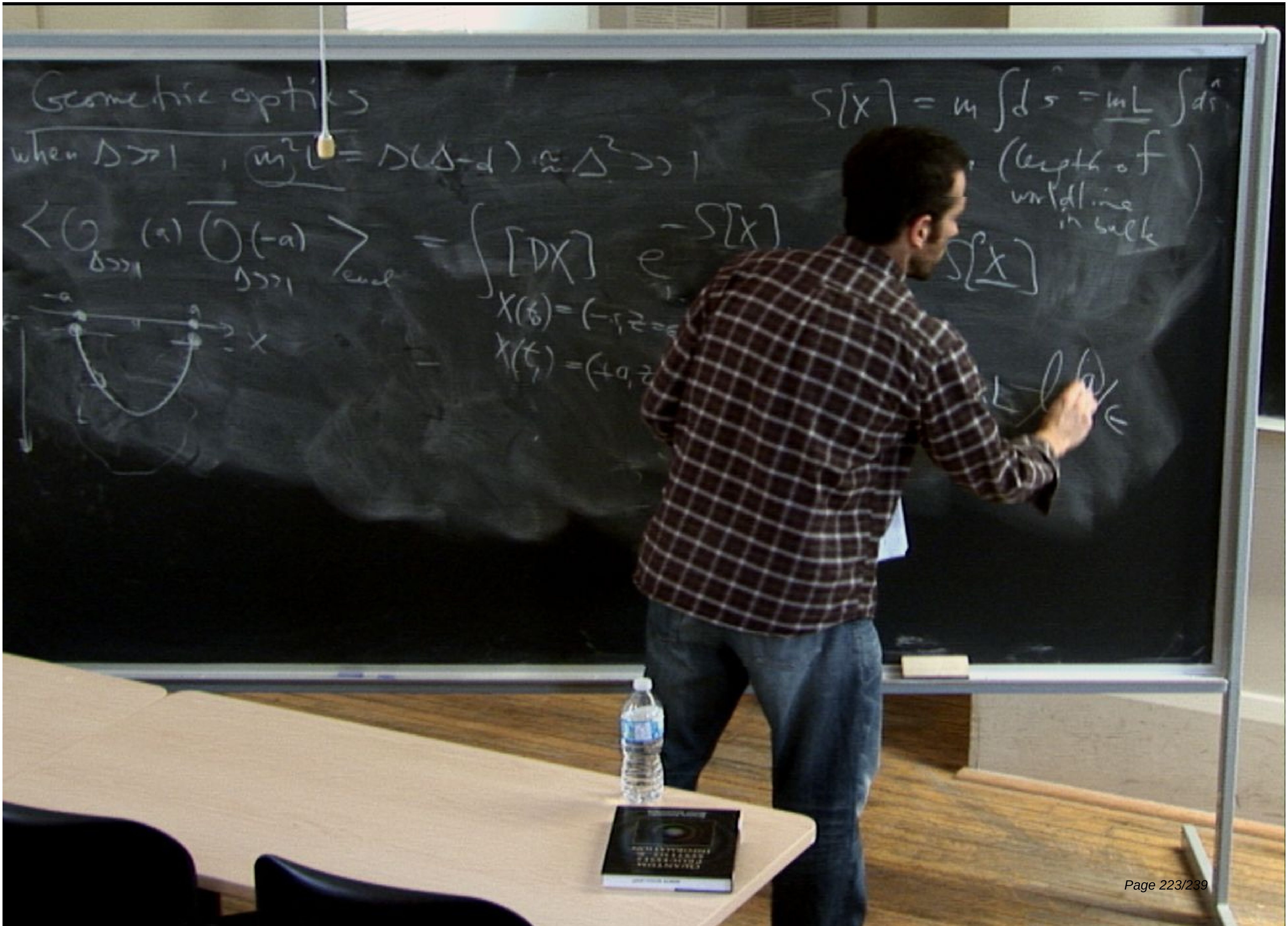

Geometric optics
S[X] = m ∫ ds

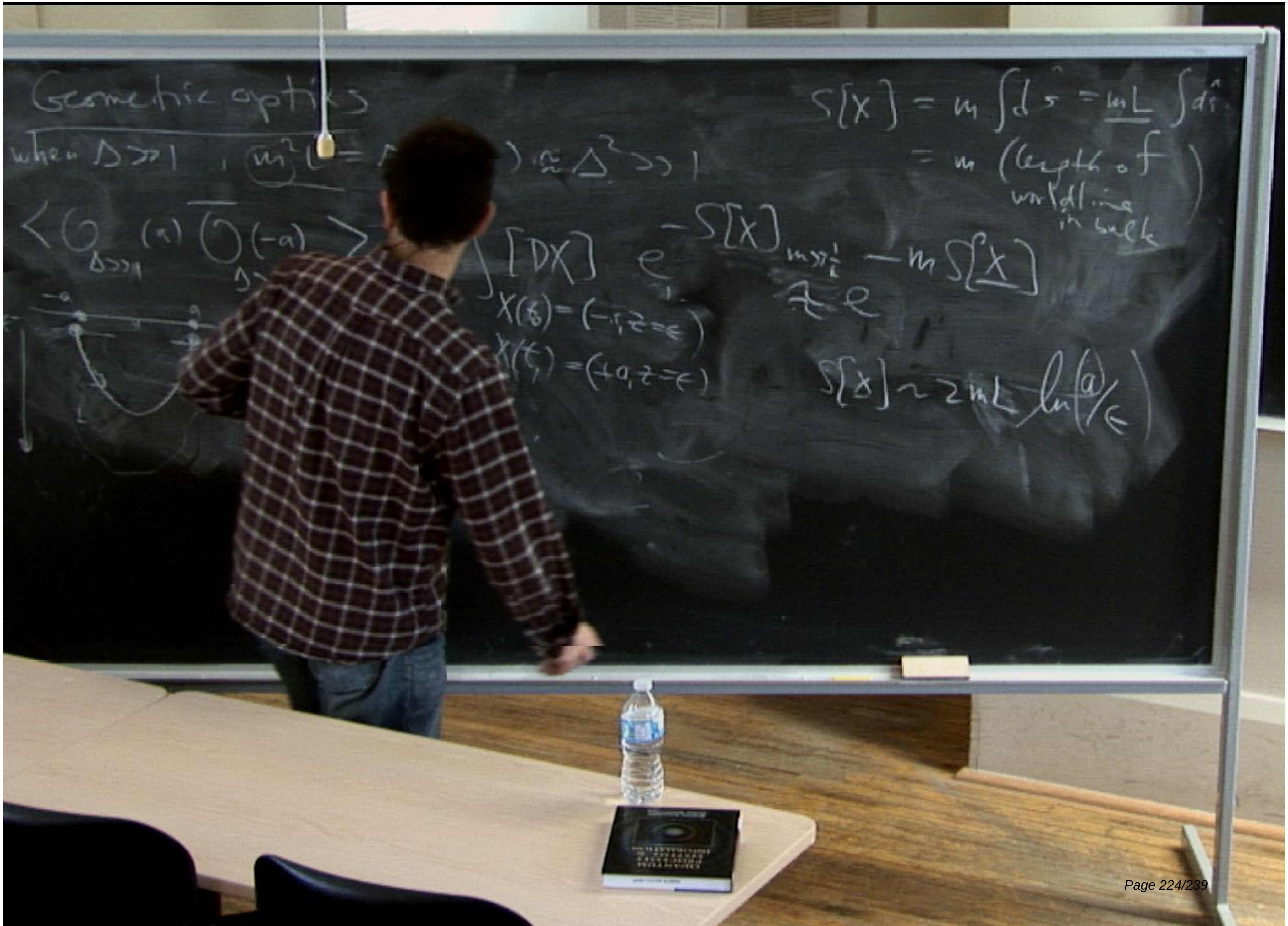

Geometric optics
S[X] = m ∫ds = mL ∫ds
= m (length of worldline in bulk)

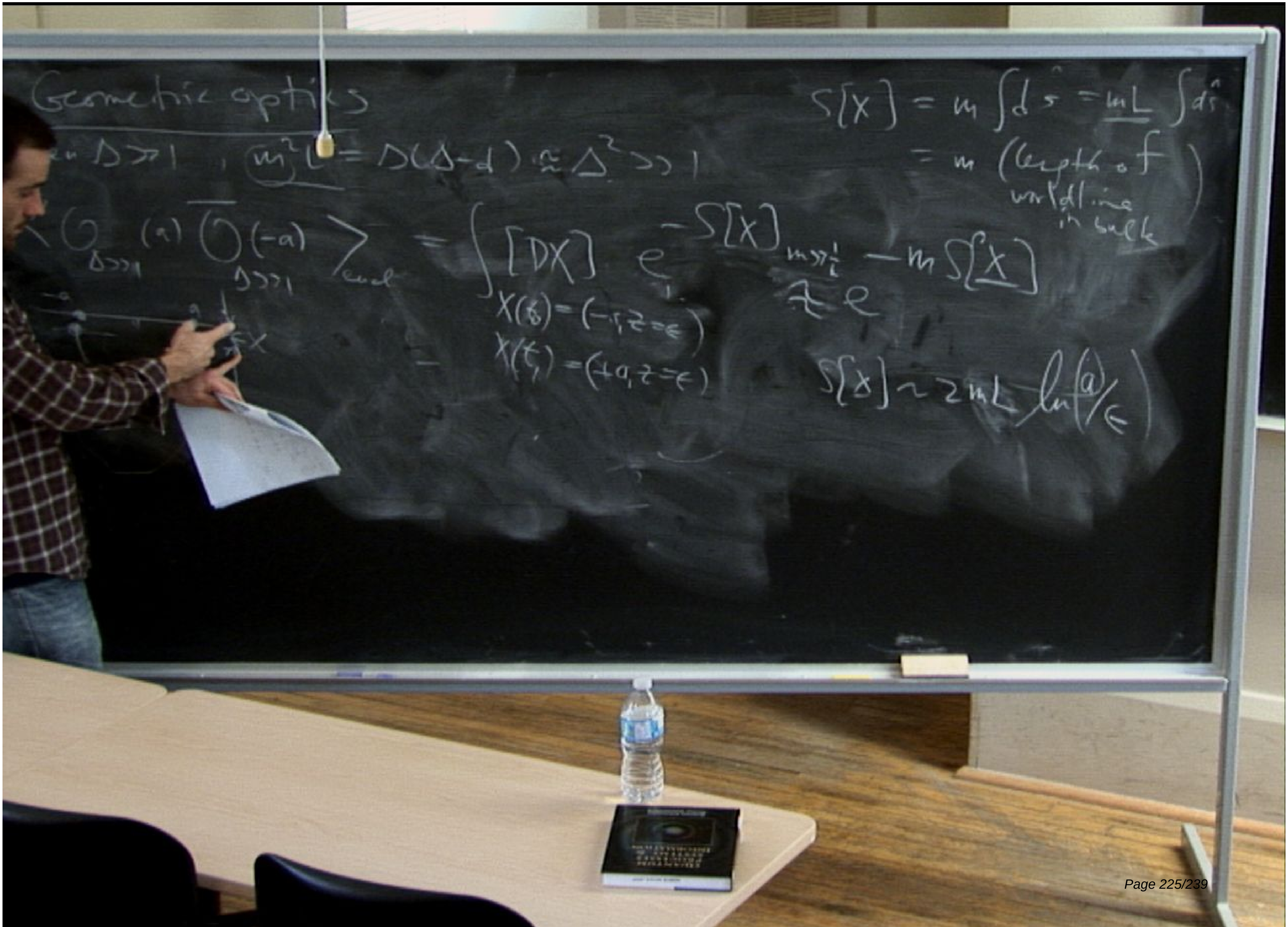

Geometric optic
S[X] = m ∫ds = mL ∫ds
= m (length of worldline in bulk)
[DX] e^{-S[X]}
≈ e^{-m S[X]}
X(τ0) = (-a, z=ε)
X(τ1) = (+a, z=ε)

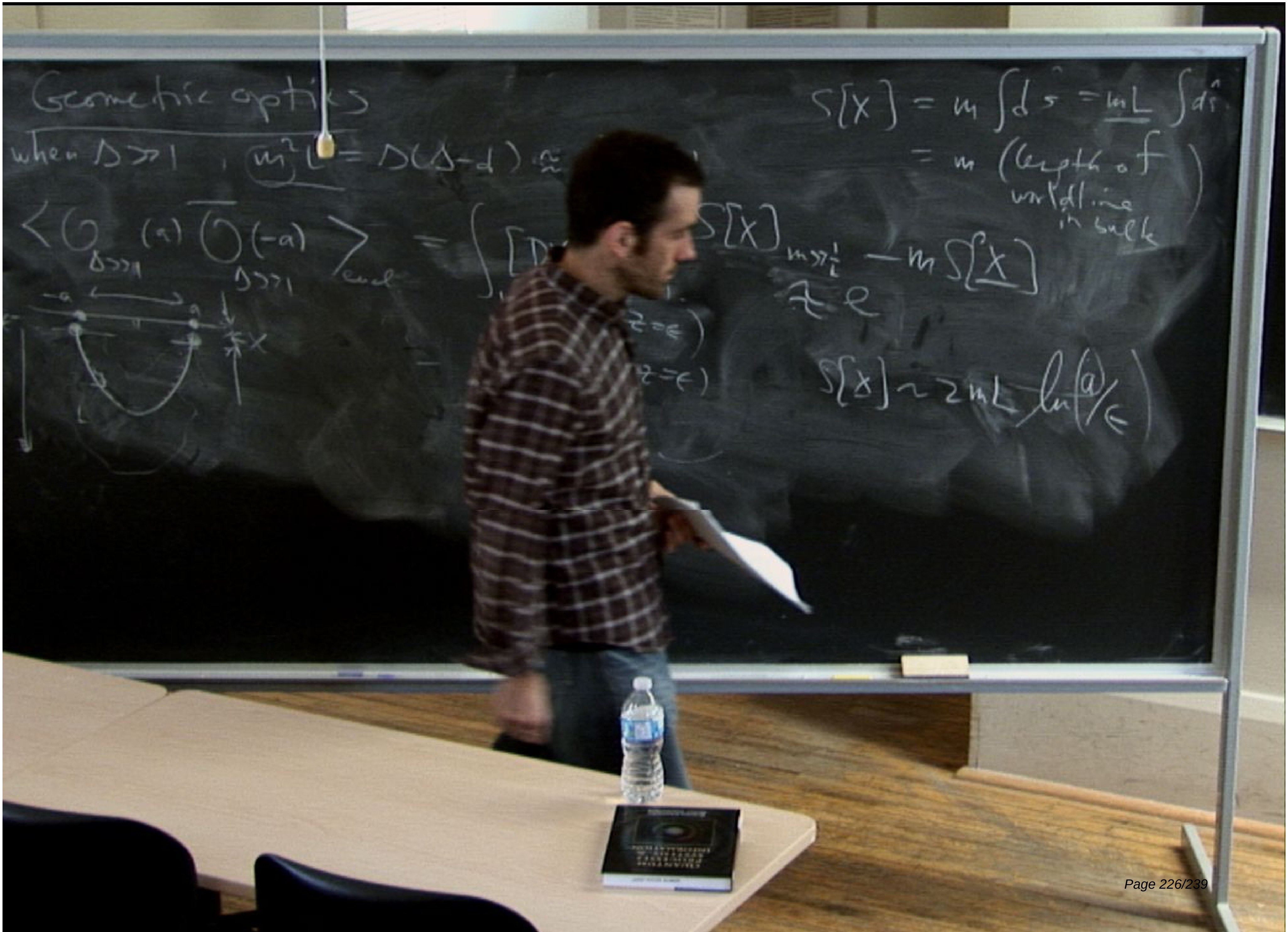

Geometric optics
= m (length of worldline in bulk)

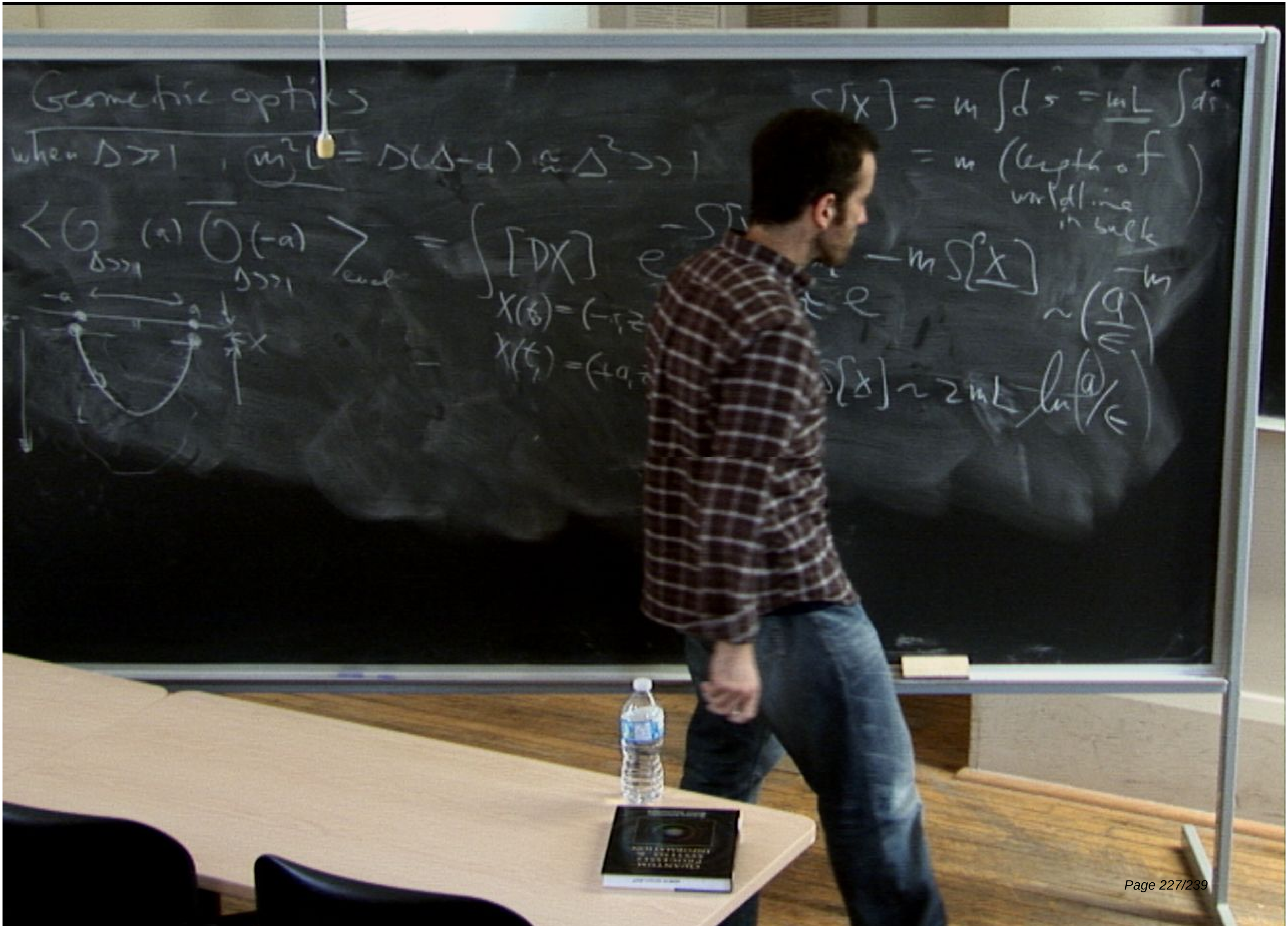


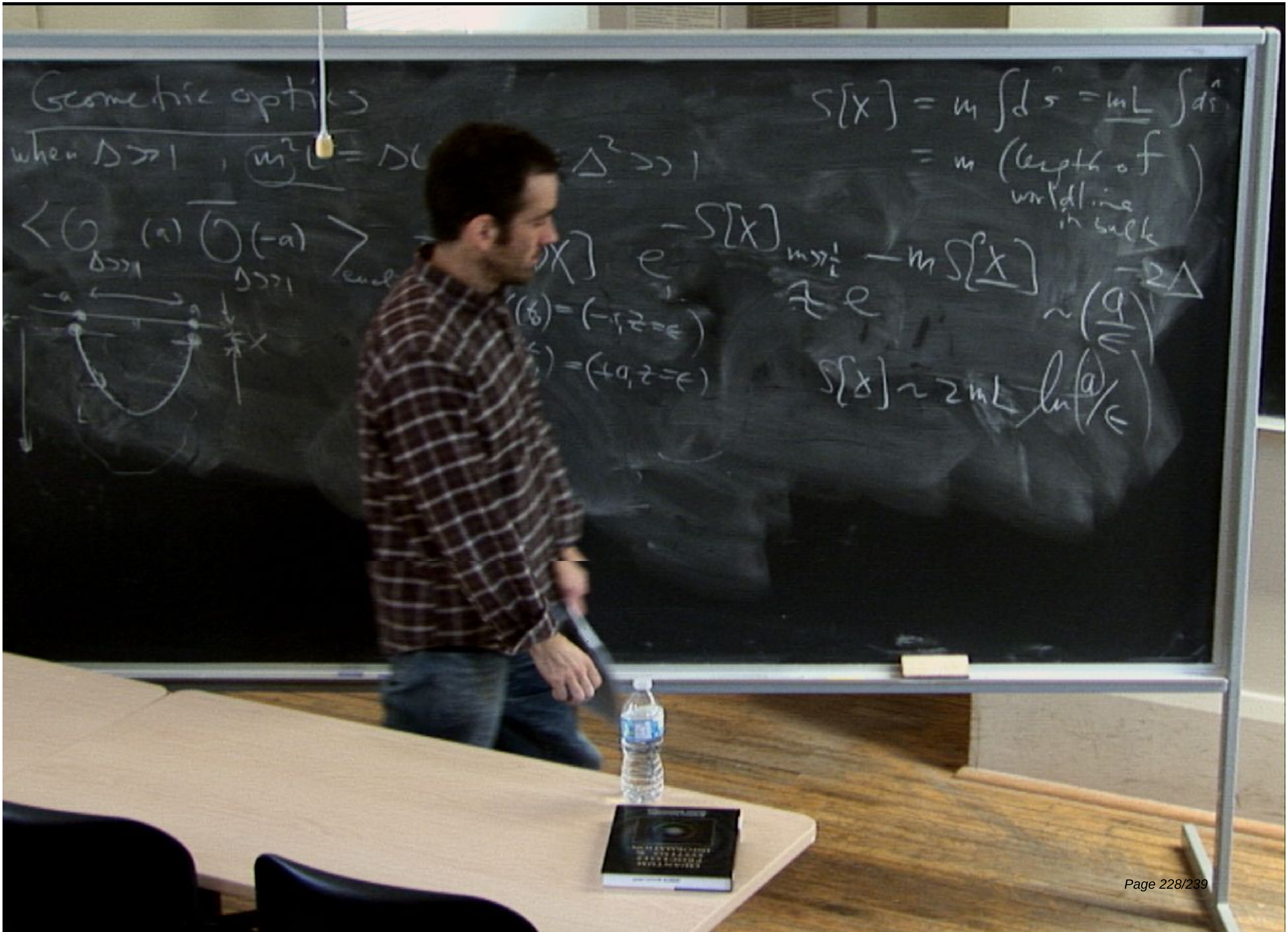

Geometric optics
when Δ>>1 , m²L² = Δ(Δ-d) ≈ Δ² >> 1
S[X] = m ∫ds = mL ∫ds̃
(length of worldline in bulk)
[DX] e^{-S[X]}

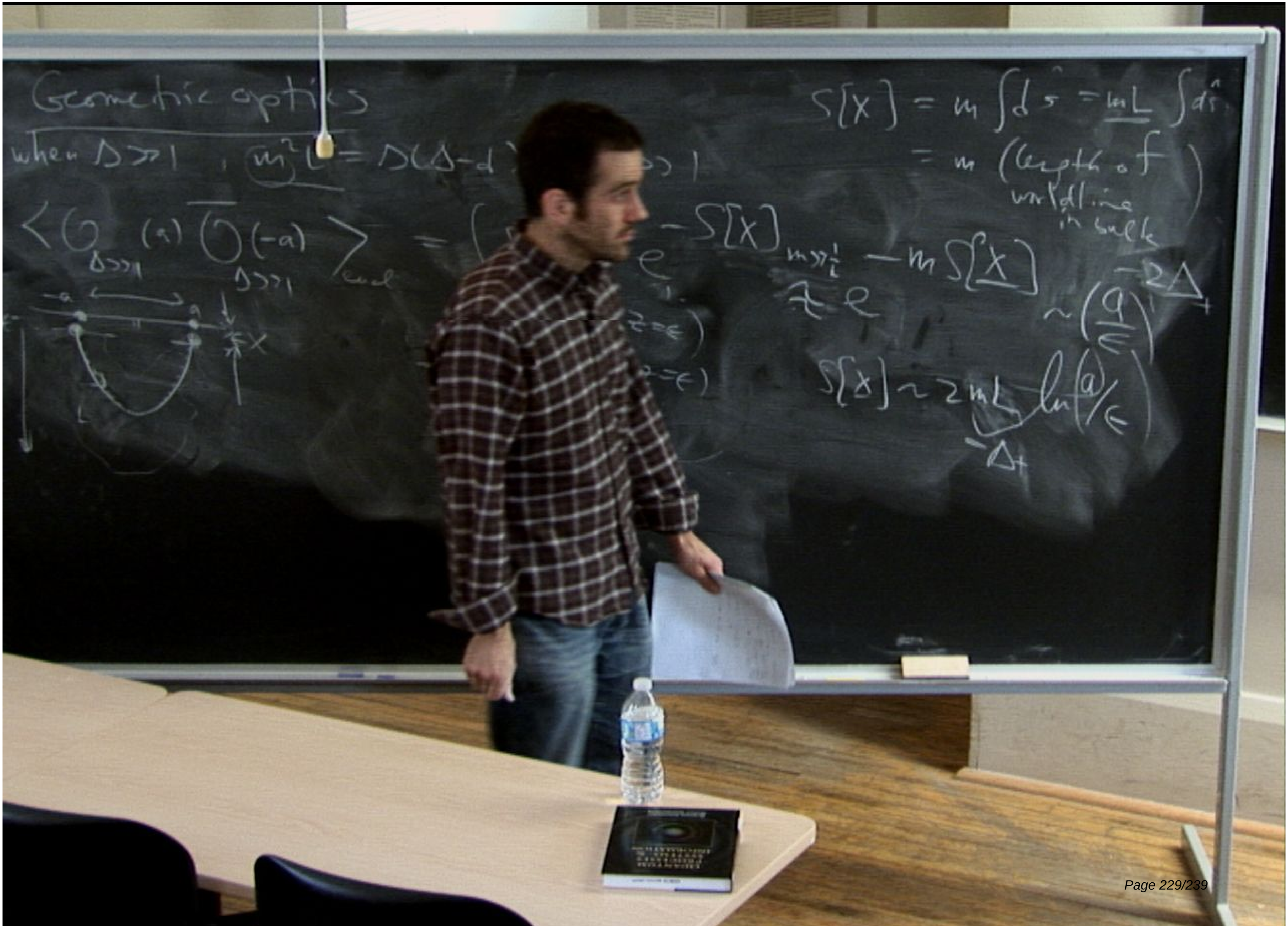

Geometric optics
S[X] = m ∫ds = mL ∫ds
= m (length of worldline in bulk)

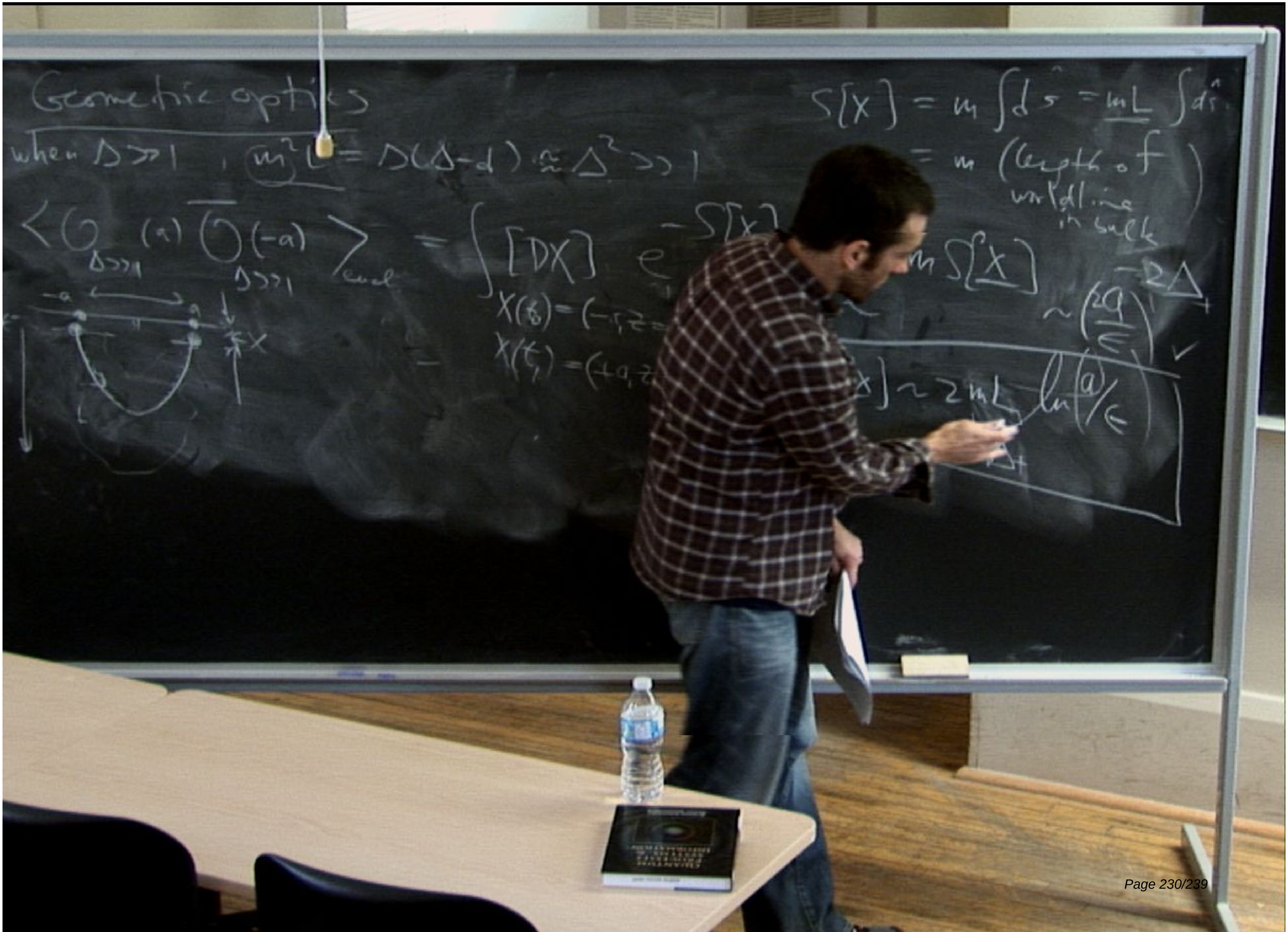

Geometric optics
Δ >> 1 , m²L² = Δ(Δ−d) ≈ Δ² >> 1
S[X] = m ∫ds = mL ∫ds
= m (length of worldline in bulk)
[DX] e^{−S[X]}
X(t0) = (−a, z = ε)
X(t1) = (+a, z = ε)
S[X] ≈ 2mL ln(a/ε)

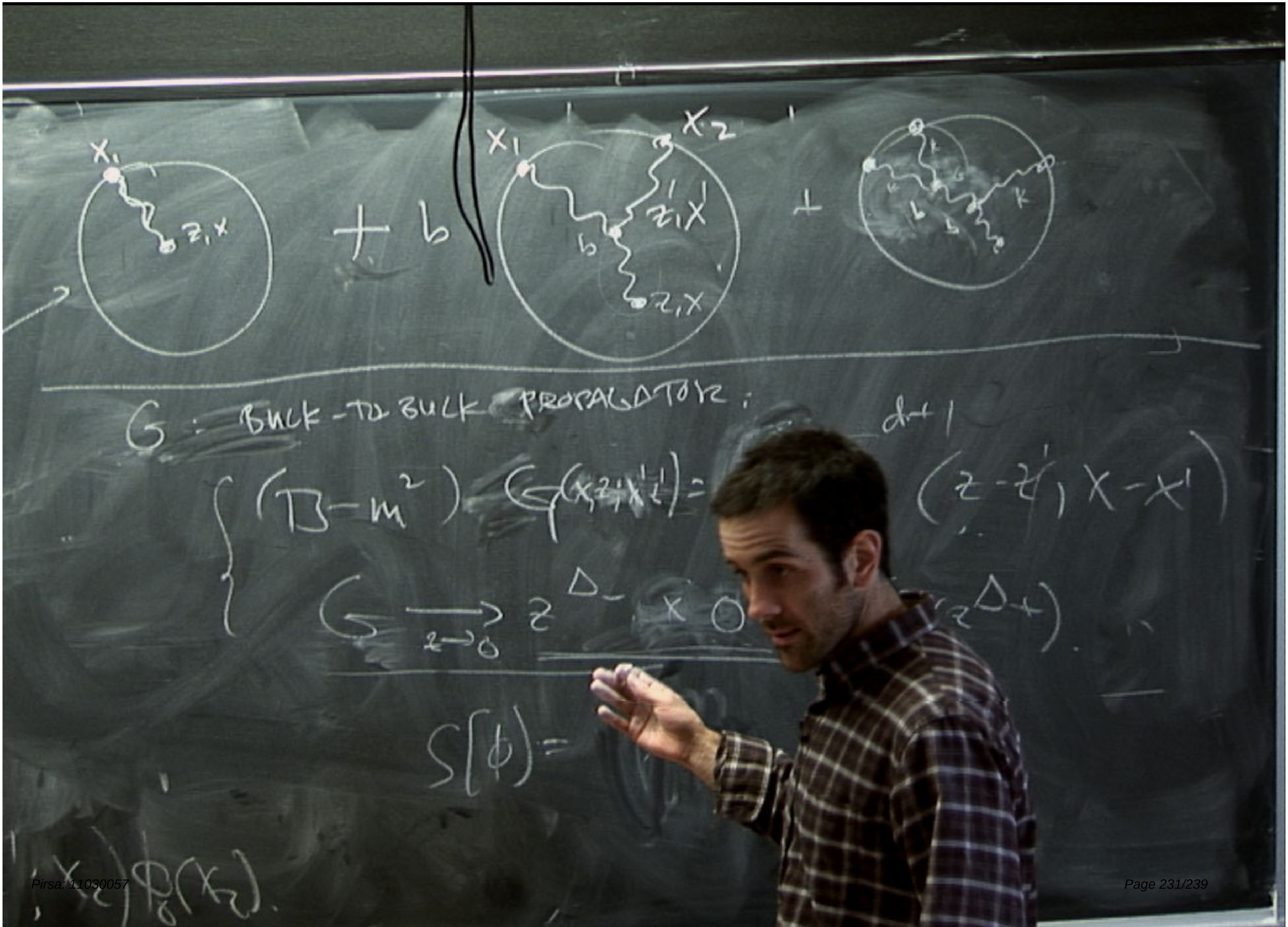

Geometric optics

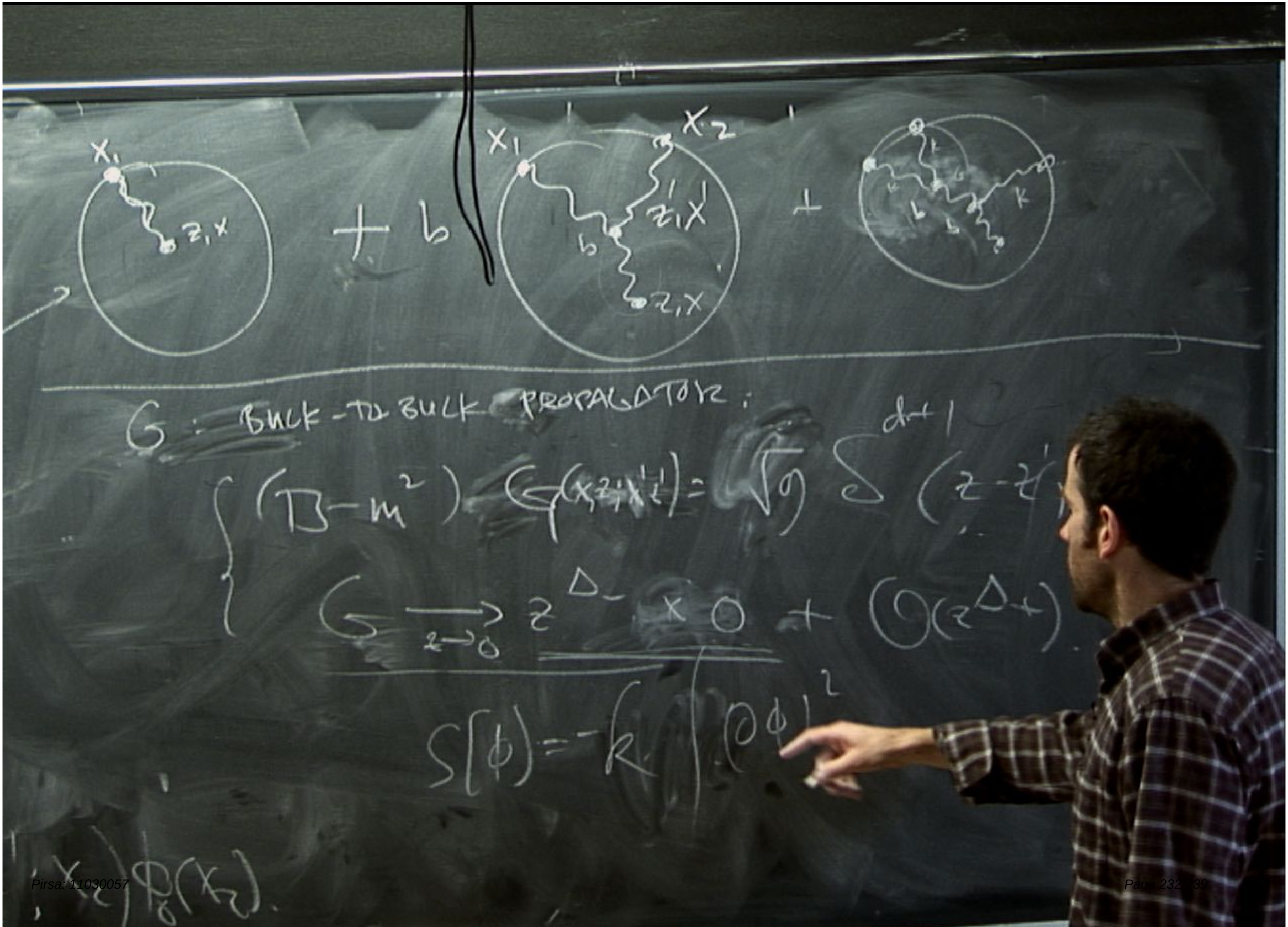

Geometric optics
when Δ >> 1
= m (length of worldline in bulk)

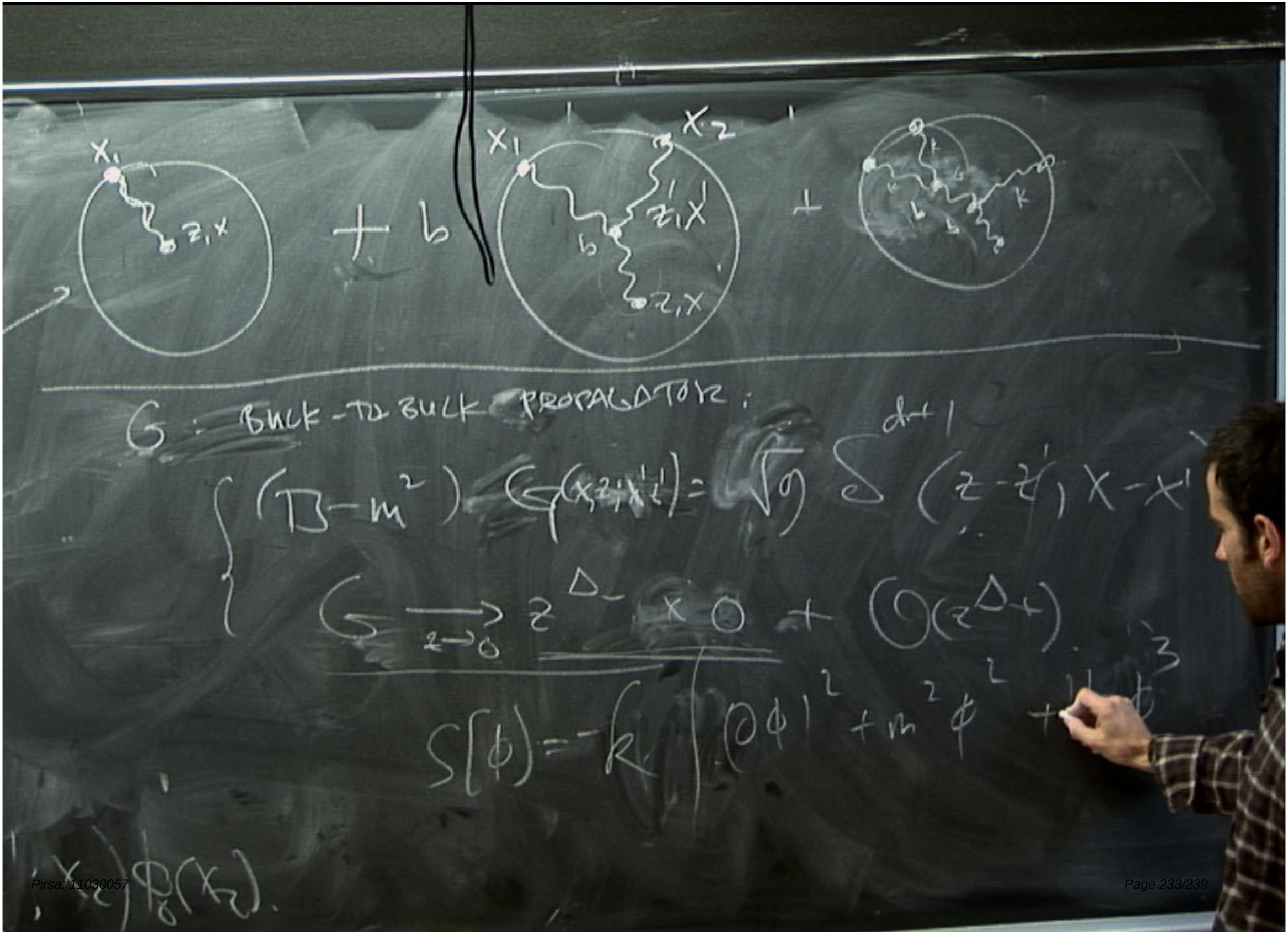

Geometric optics
when Δ >> 1
= m (length of worldline in bulk)

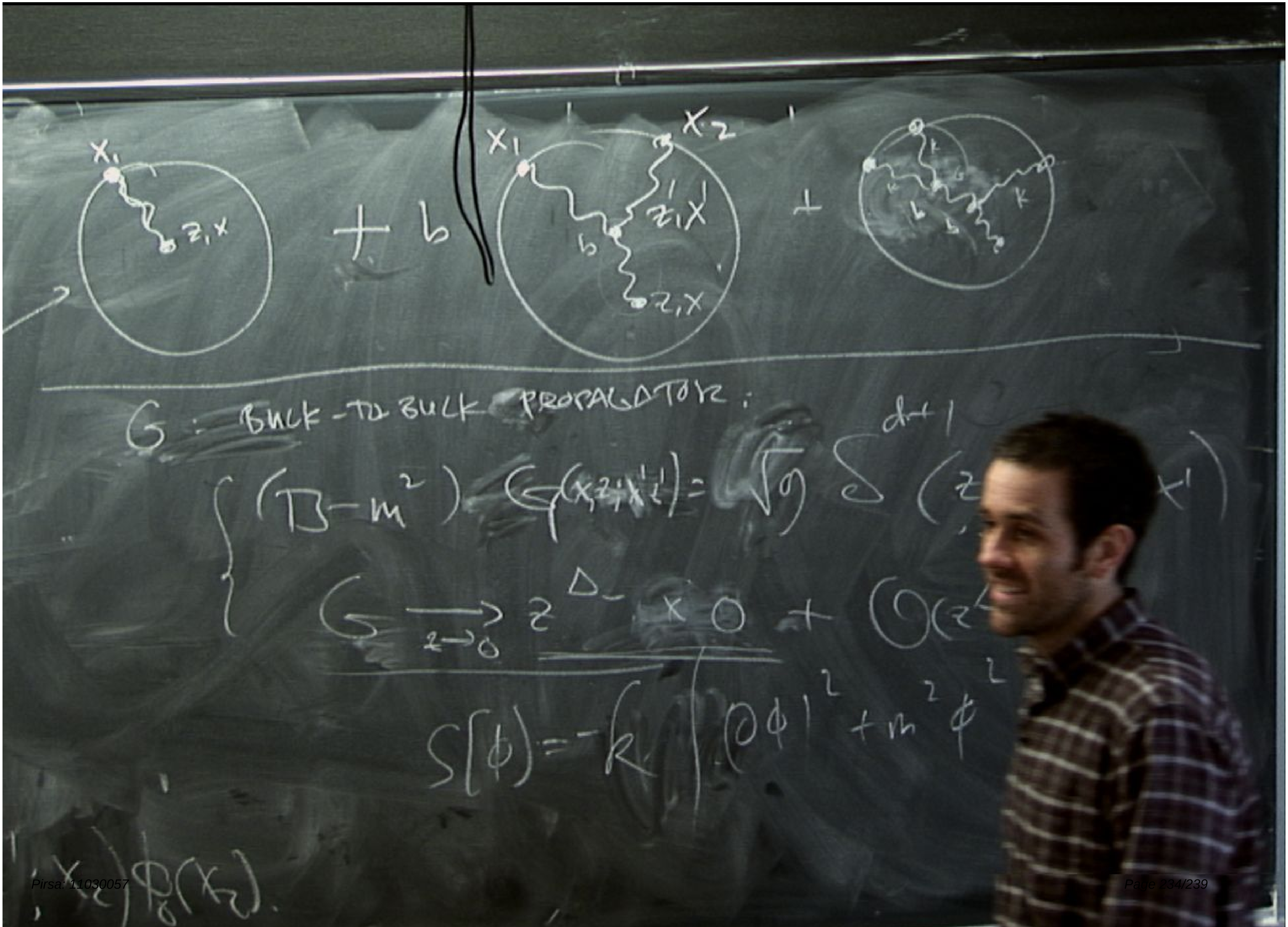

Geometric optics
S[X] = m ∫ ds


Geometric optics
when Δ >> 1
= m (length of worldline in bulk)


X1
z, x
+ b
X1
X2
z', X'
b
z, X
G : BULK-TO BULK PROPAGATOR :
d+1
z - z', X - x'
S[φ] =


G : BULK-TO BULK PROPAGATOR :
S[φ] = -k ∫ (∂φ)²


x1
z, x
+ b
x1
x2
z', x'
b
z, x
+
k
G : BULK-TO BULK PROPAGATOR :
(□ − m²) G(x,z; x',z') = √g δ^(d+1) (z − z', x − x')
G → z^Δ− × 0 + O(z^Δ+)
z→0
S[φ] = −k ∫ (∂φ)² + m² φ² + λ φ³

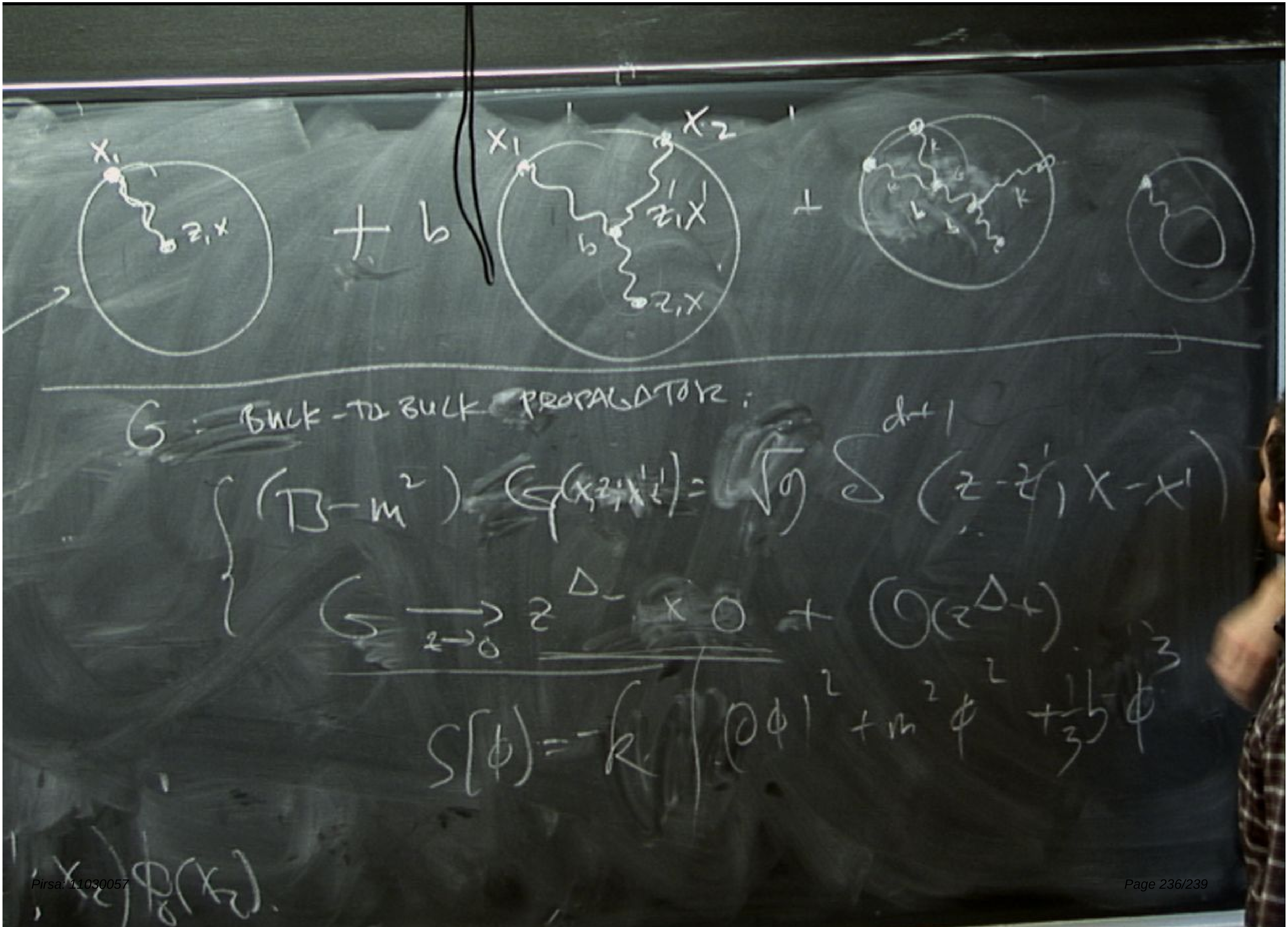

X1
z, x
+ b
X1
X2
z, x
b
G : BULK-TO BULK PROPAGATOR :
d+1
G z→0 z Δ- x O + O(z
S[φ] = -k ∫ (∂φ)² + m² φ²

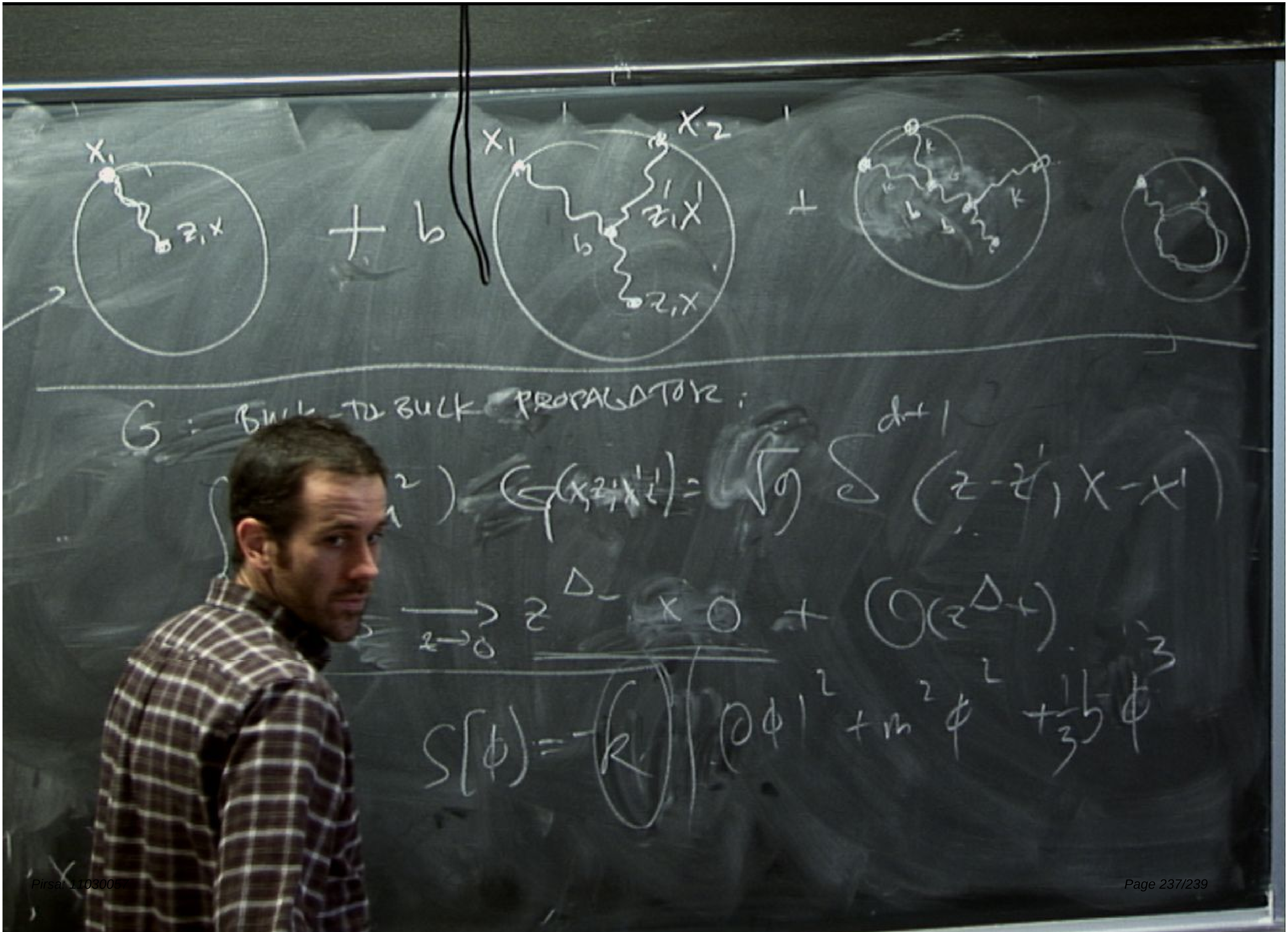
G : BULK-TO-BULK PROPAGATOR :

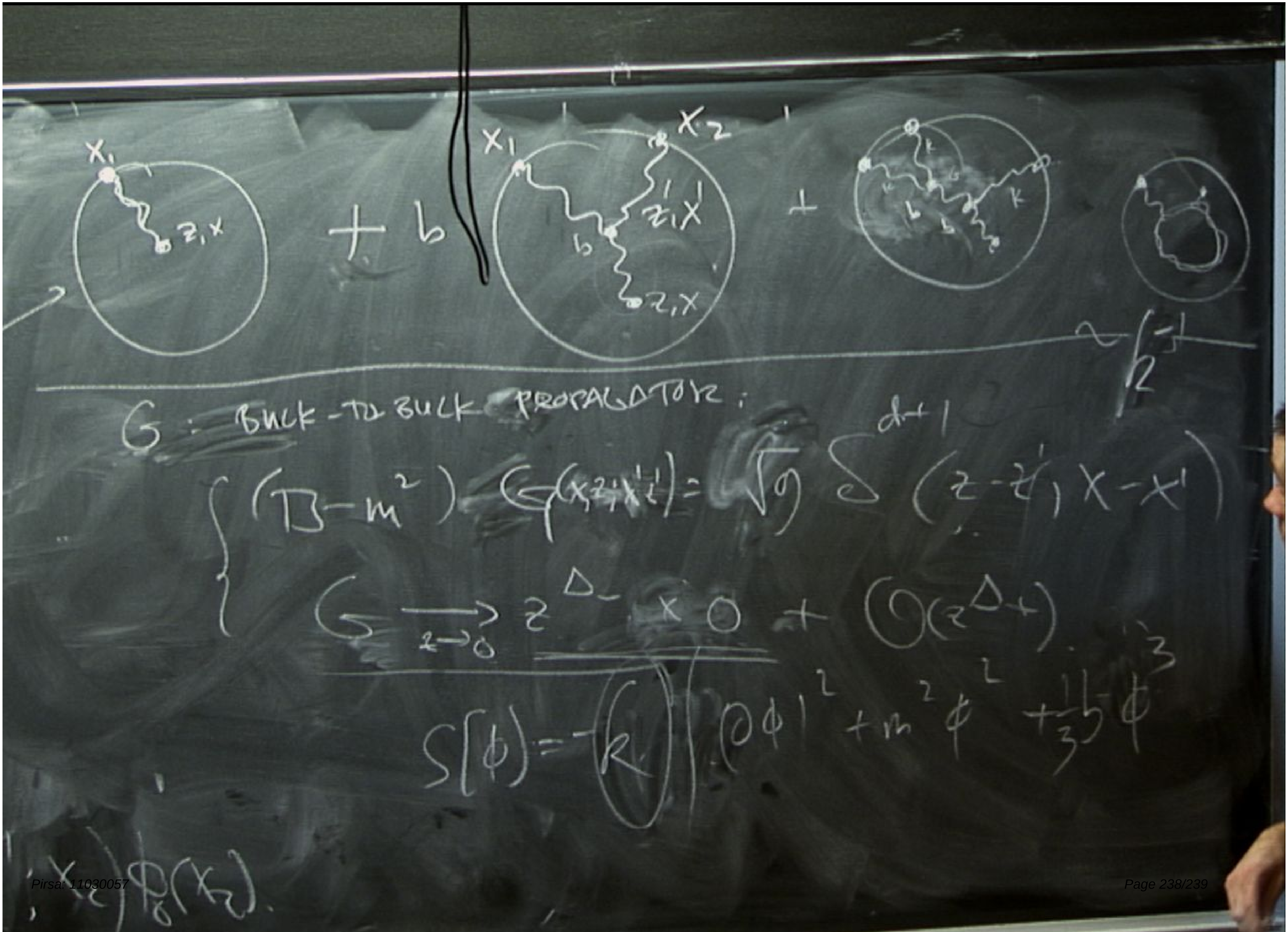
$$\begin{cases} (\Box - m^2)\, G(x,z;x',z') = \sqrt{g}\, \delta^{d+1}(z-z', x-x') \\ G \xrightarrow[z \to 0]{} z^{\Delta_-} \times \mathcal{O} + \mathcal{O}(z^{\Delta_+}) \end{cases}$$

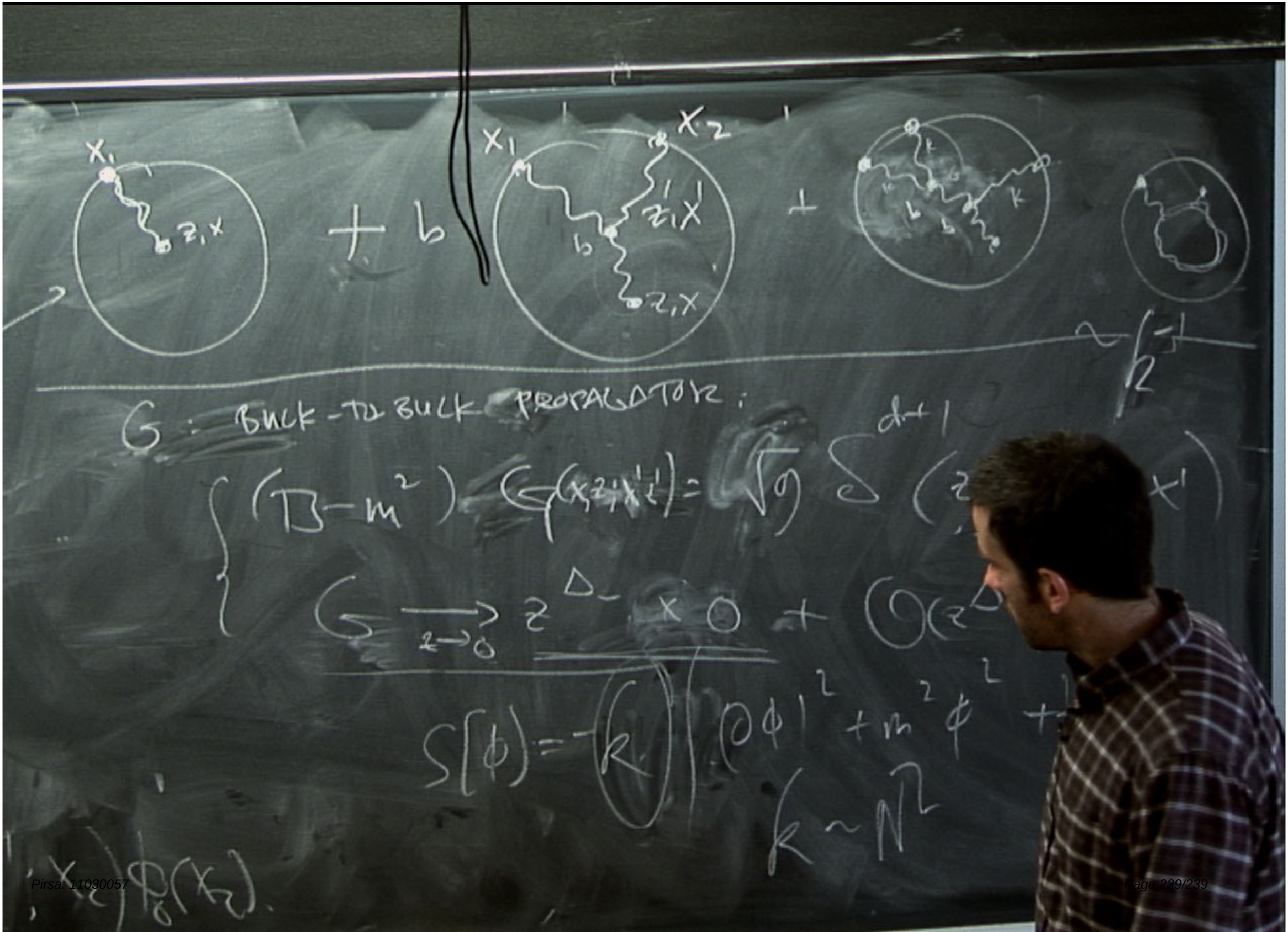
$$S[\phi] = -k \int (\partial\phi)^2 + m^2\phi^2 + \tfrac{1}{3} b\, \phi^3$$

$;x_2)\, \phi_0(x_2).$


x1
z, x
+ b
x1
x2
z', x'
b
z, x
+
G : BULK-TO-BULK PROPAGATOR :
(□ − m²) G(x,z;x',z') = √g δ^(d+1) (z − z', x − x')
G → z^Δ− × 0 + O(z^Δ+)
z→0
S[φ] = −k ∫ (∂φ)² + m² φ² + 1/3 b φ³


X1
z,x
+ b
X2
z',x'
G : BULK-TO-BULK PROPAGATOR :
z→0


X1
z, x
+ b
X1
X2
z', x'
b
z, x
+
k
b
k
G : BULK-TO BULK PROPAGATOR :
(□ − m²) G(x, z; x', z') = √g δ^{d+1}(z − z', x − x')
G → z^{Δ−} × 0 + O(z^{Δ+})
z→0
S[φ] = −k ∫ (∂φ)² + m²φ² + 1/3 b φ³
x2) φ0(x2).



G : BULK-TO BULK PROPAGATOR :
k ~ N²