

Title: Explorations in String Theory - Lecture 8

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URL: <http://pirsa.org/11030057>

Abstract:

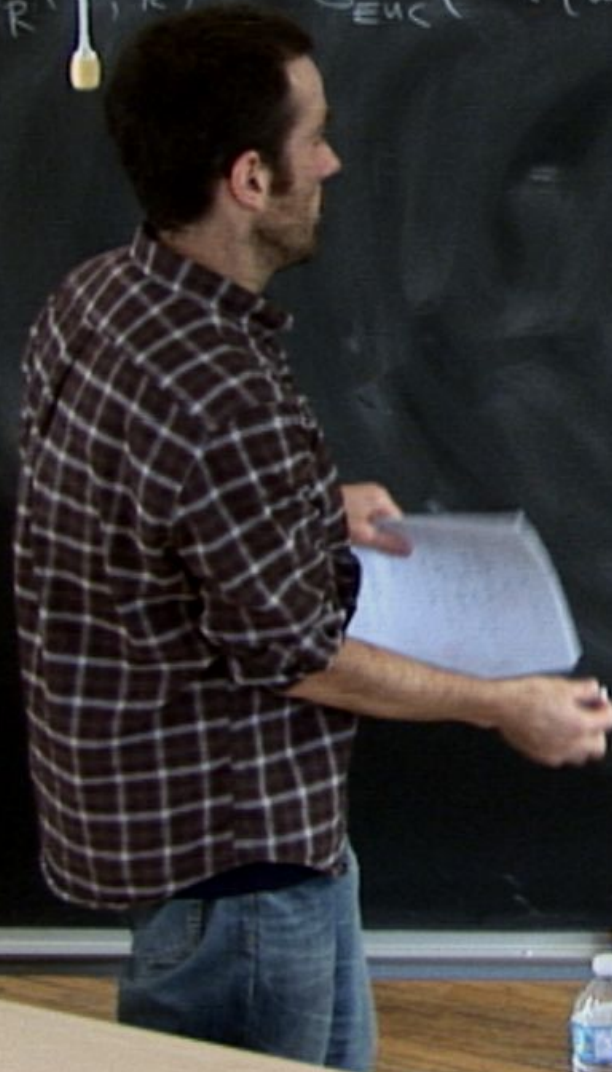


M fact: $G_R(\omega, \vec{k}) = G_{ENC}$

QM fact: $\langle \psi, \vec{k} \rangle = G_{EWC}$



QM fact: $G_R(\omega, \vec{k}) = G_{EUC}(-i(\omega + i\epsilon))$



QM for $G_R(\omega, \vec{k}) = G_{ENC}(-i(\omega + i\epsilon), \vec{k})$

QM fact: $G_R(\omega, \vec{k}) = \frac{1}{E - E_{\vec{k}} - i\epsilon}$

QM fact: $G_R(\omega, \vec{k}) = G_{ENC}(\underbrace{-i(\omega + i\epsilon)}_{\omega_E}, \vec{k})$

eg: $\frac{1}{\omega^2 - \vec{p}^2 - m^2}$

QM fact: $G_P(\omega, \vec{k}) = G_{EUC}(\underbrace{-i(\omega + i\epsilon)}_{\omega_E}, \vec{k})$

eg: $\frac{1}{\omega^2 - \vec{p}^2 - m^2}$

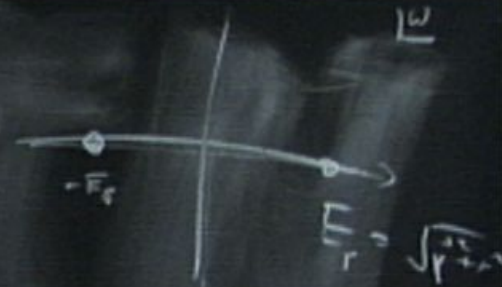
QM fact: $G_R(\omega, \vec{k}) = G_{EUC}(\underbrace{-i(\omega + i\epsilon)}_{\omega_E}, \vec{k})$

g: $\omega^2 - \vec{k}^2$



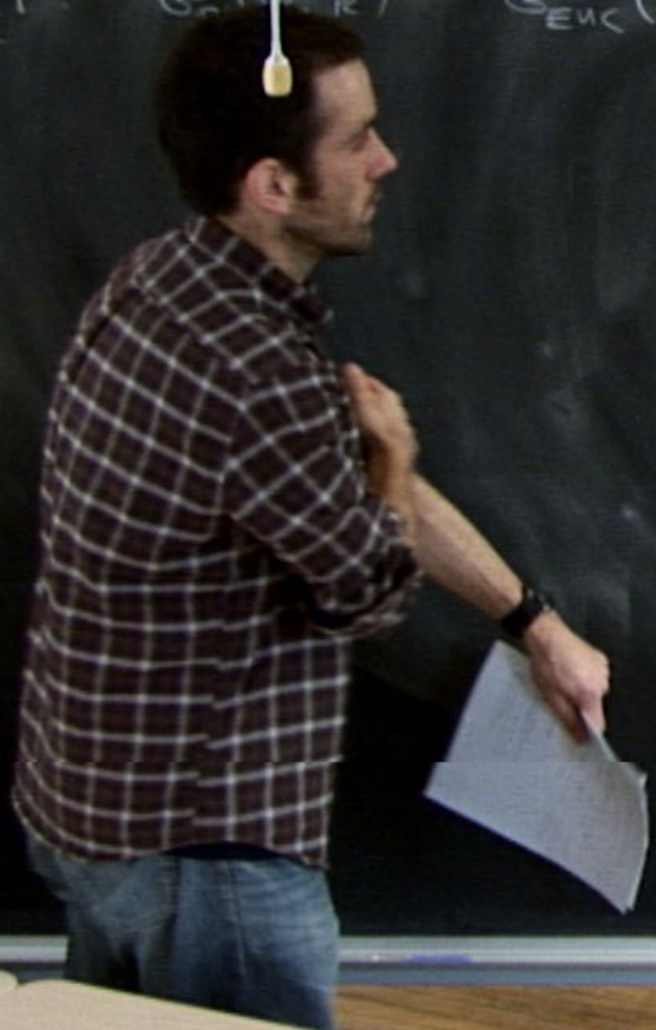
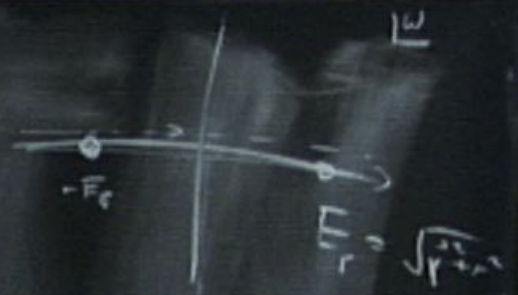
QM fact: $G_P(\omega, \vec{k}) = G_{EUC}(-i(\omega + i\epsilon), \vec{k})$

$$g: \frac{1}{\omega^2 - \vec{p}^2 - m^2}$$



QM fact: $G_n(\omega, \vec{k}) = G_{\text{EMC}}(\underbrace{-i(\omega + i\epsilon)}_{\omega_E}, \vec{k})$

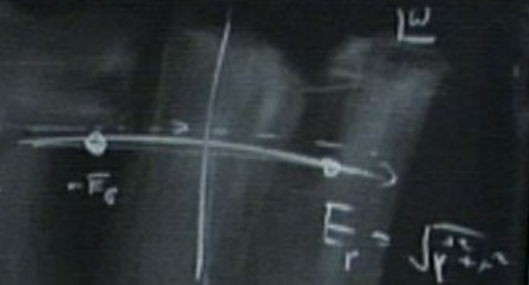
eg. $\frac{1}{\omega^2 - \vec{p}^2 - m^2}$



1 fact: $G_R(\omega, \vec{k}) = G_{ENC}(\underbrace{-i(\omega + i\epsilon)}_{\omega_+}, \vec{k})$

$G_F(\omega, \vec{k}) = G_{FF}$

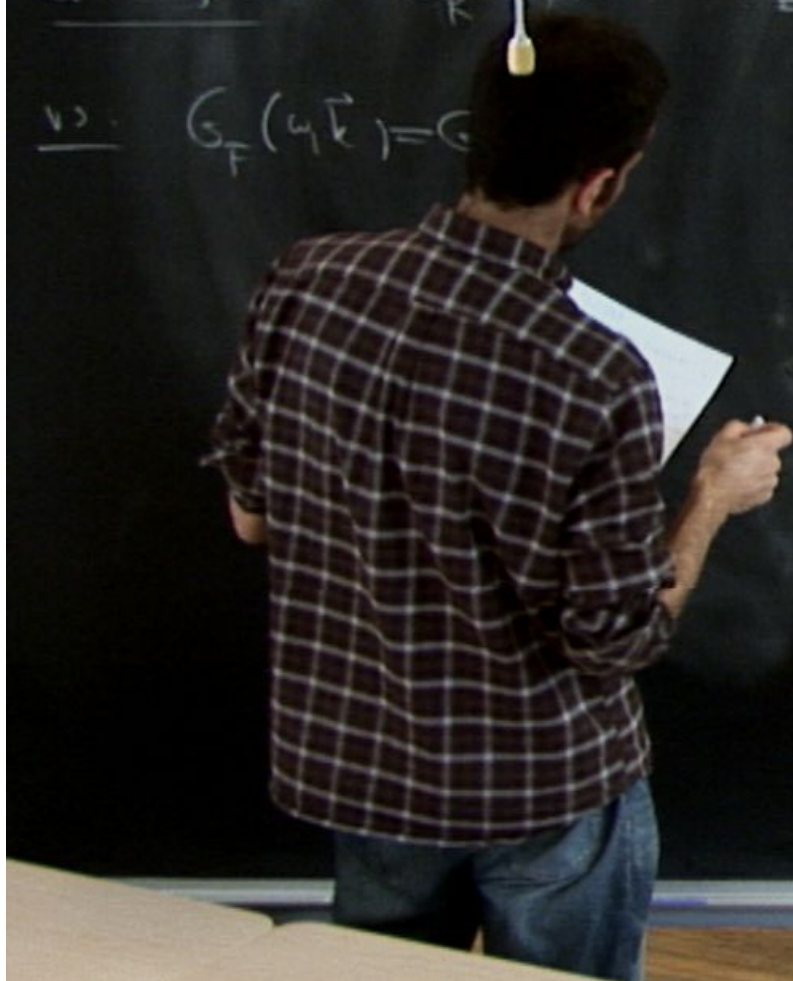
$g = \frac{1}{\omega^2 - p^2 - m^2}$



QM fact: $G_R(\omega, \vec{k}) = G_{EUC}(\underbrace{-i(\omega + i\epsilon)}_{\omega_+}, \vec{k})$

$\Rightarrow G_F(\omega, \vec{k}) = G$

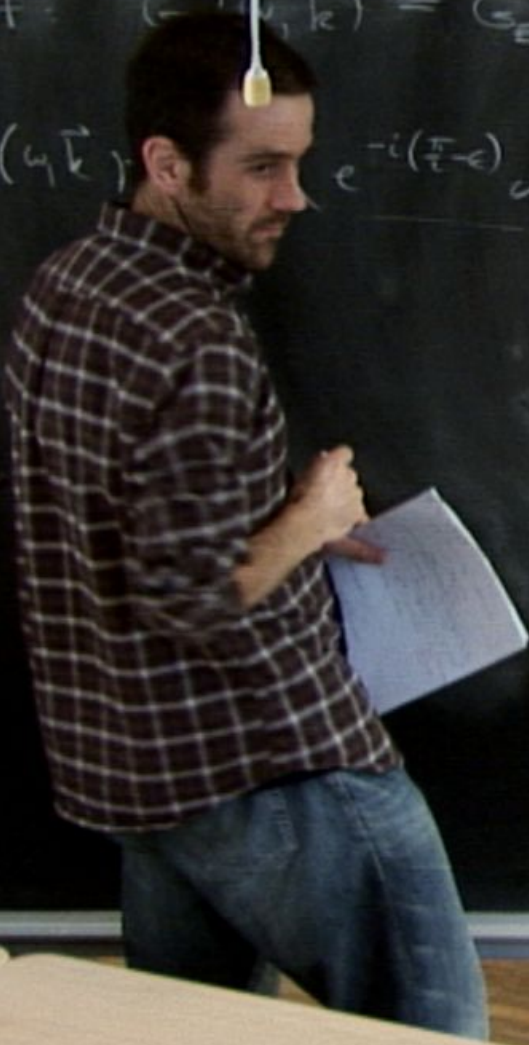
eg: $\frac{1}{\omega^2 - p^2 - m^2}$



QM fact: $\psi(\omega, \vec{k}) = \mathcal{G}_{\text{EUC}}(-i(\omega + i\epsilon), \vec{k})$

$\Rightarrow \mathcal{G}_{\text{F}}(\omega, \vec{k}) = e^{-i(\frac{\pi}{2} - \epsilon)\omega} \psi(\omega, \vec{k})$

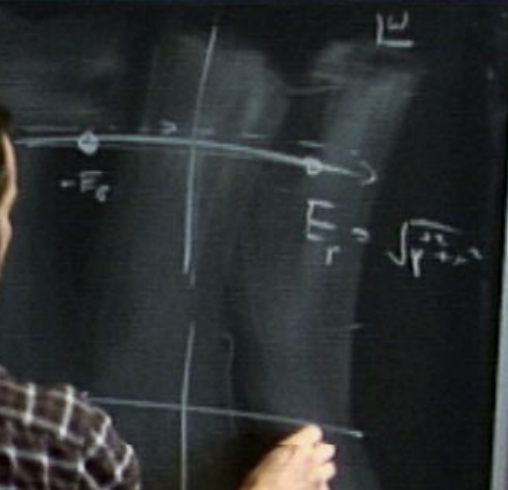
$g = \frac{1}{\omega^2 - p^2 - m^2}$



QM fact: $G_R(\omega, \vec{k}) = G_{EUC}(\underbrace{-i(\omega + i\epsilon)}_{\omega_E}, \vec{k})$

v.s. $G_F(\omega, \vec{k}) = G_{EUC}(e^{-i(\frac{\pi}{2} - \epsilon)} \omega, \vec{k}) =$

g.



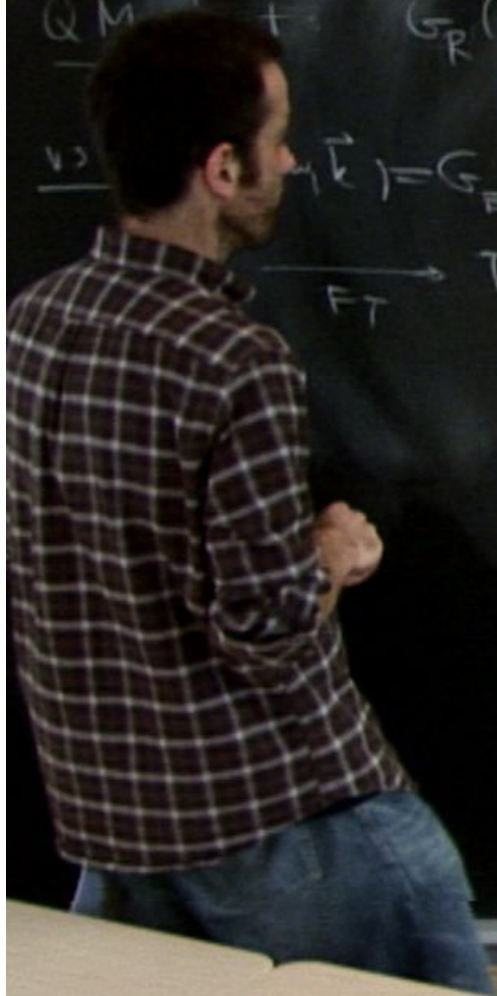
QM $\epsilon \rightarrow +$ $G_R(\omega, \vec{k}) = G_{ENC}(-i(\omega + i\epsilon), \vec{k})$

$\omega \rightarrow -i\epsilon$ $G_R(\omega, \vec{k}) = G_{ENC}(e^{-i(\frac{\pi}{2} - \epsilon)\omega}, \vec{k})$

\xrightarrow{FT} Time-ordered G

$g = \frac{1}{\omega^2 - p^2 - m^2}$

$E_0 = \sqrt{p^2 + m^2}$

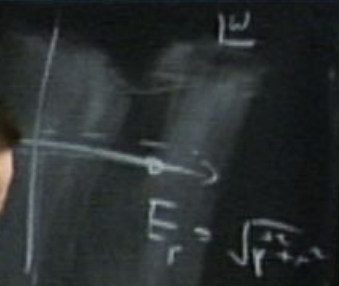


QM fact: $G_R(\omega, \vec{k}) = G_{EUC}(\underbrace{-i(\omega + i\epsilon)}_{\omega_0}, \vec{k})$

v3. $G_F(\omega, \vec{k}) = G_{EUC}(e^{-i(\frac{\pi}{2} - \epsilon)} \omega, \vec{k})$

FT. \rightarrow Time-ordered G

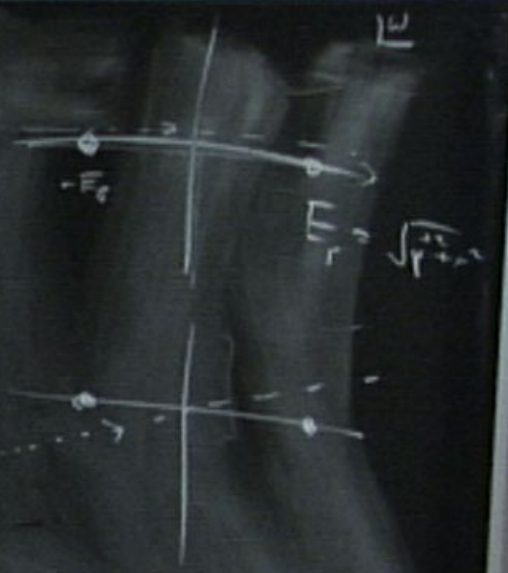
$$g = \frac{1}{\omega^2 - \vec{k}^2 - m^2}$$



Q.M. fact: $G_R(\omega, \vec{k}) = G_{ENC}(\underbrace{-i(\omega + i\epsilon)}_{\omega_0}, \vec{k})$

$G(\omega, \vec{k}) = G_{ENC}(e^{-i(\frac{\pi}{2} - \epsilon)}, \omega, \vec{k})$

eg: $\frac{1}{\omega^2 - p^2 - m^2}$



QM fact: $G_R(\omega, \vec{k}) = G_{ENC}(\underbrace{-i(\omega + i\epsilon)}_{\omega_0}, \vec{k})$

v) $G_F(\omega, \vec{k}) = G_{ENC}(e^{-i(\frac{\pi}{2} - \epsilon)\omega}, \vec{k})$
 FT. \rightarrow Time-ordered G



QM fact: $G_R(\omega, \vec{k}) = G_{ENC}(\underbrace{-i(\omega + i\epsilon)}_{\omega_+}, \vec{k})$

$\Rightarrow G_F(\omega, \vec{k}) = G_{ENC}(e^{-i(\frac{\pi}{T} - \epsilon)} \omega, \vec{k}) =$
 $\xrightarrow{F.T.} \text{Time-ordered } G$



QM fact: $G_R(\omega, \vec{k}) = G_{ENC}(\underbrace{-i(\omega + i\epsilon)}_{\omega_0}, \vec{k})$

$\Rightarrow G_F(\omega, \vec{k}) = G_{ENC}(e^{-i(\frac{\pi}{2} - \epsilon)\omega}, \vec{k})$

\xrightarrow{FT} Time-ordered G

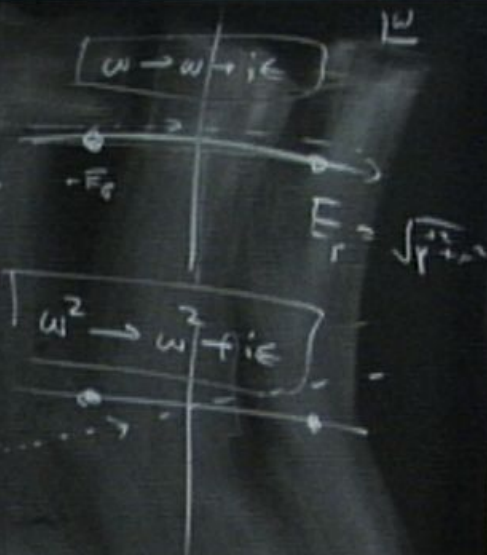


QM fact: $G_R(\omega, \vec{k}) = G_{EUC}(\underbrace{-i(\omega + i\epsilon)}_{\omega_0}, \vec{k})$

\hookrightarrow $G_F(\omega, \vec{k}) = G_{EUC}(\omega, \vec{k})$

FT \rightarrow TI \leftarrow G

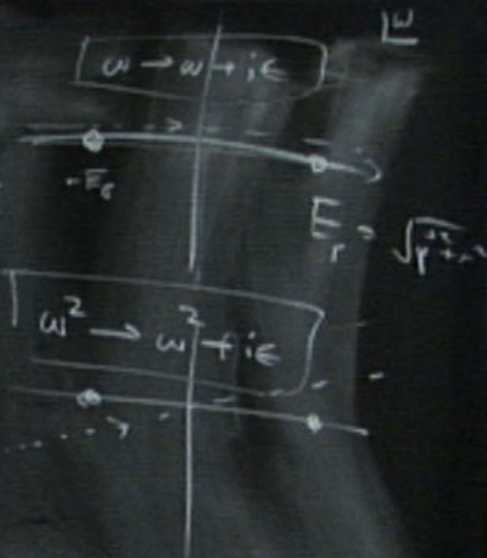
$g: \frac{1}{\omega^2 - p^2 - m^2}$



QM fact: $G_R(\omega, \vec{k}) = G_{ENC}(-i(\omega + i\epsilon), \vec{k})$

vs. $G_{ENC}(e^{-i(\frac{\pi}{T} - \epsilon)\omega}, \vec{k}) =$
 \xrightarrow{FT} Time-ordered G .

eg: $\frac{1}{\omega^2 - p^2 - m^2}$



QM fact: $G_R(\omega, \vec{k}) = \langle \phi_{EUC}(-i(\omega+i\epsilon), \vec{k}) \rangle$

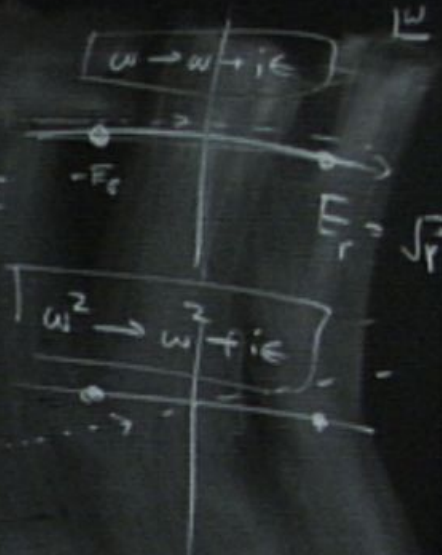
v) $G_F(\omega, \vec{k}) = \langle \phi_{EUC}(e^{-i(\frac{\omega}{T}-\epsilon)\omega}, \vec{k}) \rangle$

Time-ordered G .

Holographic

ϕ_{EUC}

$g: \frac{1}{\omega^2 - \vec{p}^2 - m^2}$

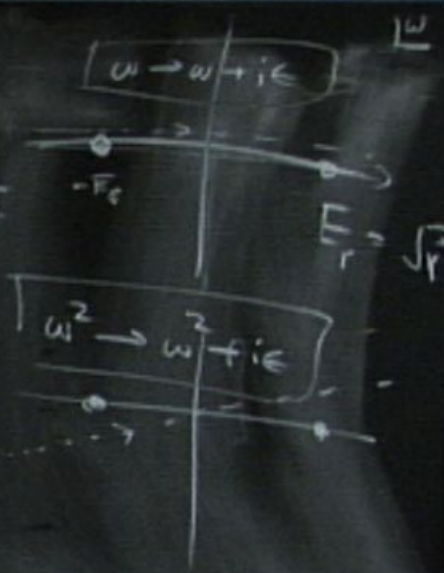


QM fact: $G_R(\omega, \vec{k}) = G_{EUC}(-i(\omega+i\epsilon), \vec{k})$

vs. $G_{EUC}(e^{-i(\frac{\pi}{2}-\epsilon)\omega}, \vec{k})$

Time-ordered G

$g = \frac{1}{\omega^2 - p^2 - m^2}$



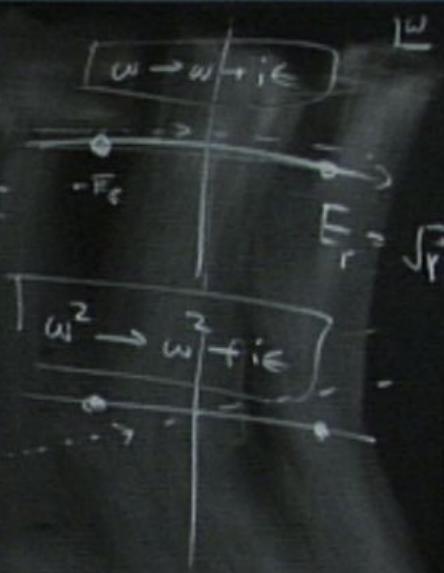
$\phi^{EUC}(\vec{k})$

QM fact: $G_R(\omega, \vec{k}) = G_{ENC}(-i(\omega + i\epsilon), \vec{k})$

vs: $G_F(\omega, \vec{k}) = G_{ENC}(\omega, \vec{k})$

FT. Time

eg: $\frac{1}{\omega^2 - p^2 - m^2}$



holographic version

ϕ_{Ret}

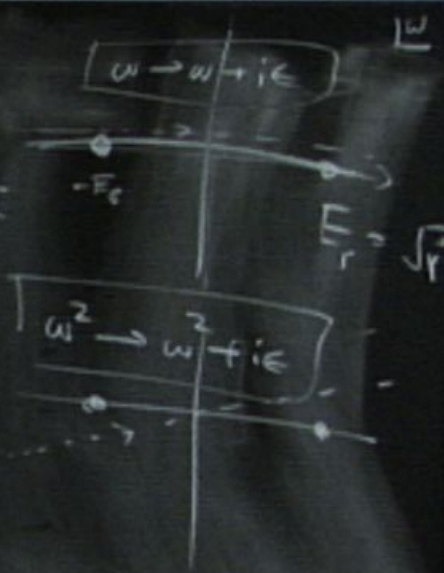
QM fact: $G_R(\omega, \vec{k}) = G_{\text{EUC}}(-i(\omega+i\epsilon), \vec{k})$

$G_R(\omega, \vec{k}) = G_{\text{EUC}}\left(\underbrace{e^{-i(\frac{\omega}{T}-\epsilon)} \omega}_{\omega_0}, \vec{k}\right)$

FT → Time-ordered G_i

$\phi^{\text{Ret}}(\omega, \vec{k}, z) = \phi^{\text{EUC}}(-i(\omega+i\epsilon), \vec{k}, z)$

$g = \frac{1}{\omega^2 - p^2 - m^2}$



QM fact: $G_R(\omega, \vec{k}) = G_{\text{EUC}}(\underbrace{-i(\omega+i\epsilon)}_{\omega_E}, \vec{k})$

\Rightarrow $G_F(\omega, \vec{k}) = G_{\text{EUC}}(e^{-i(\frac{\pi}{T}-\epsilon)} \omega, \vec{k})$

$\xrightarrow{\text{FT}}$ Time-ordered G_i

$g = \frac{1}{\omega^2 - \vec{p}^2 - m^2}$



Holographic version $\phi^{\text{Ret}}(\omega, \vec{k}, z) = \phi^{\text{EUC}}(-i(\omega+i\epsilon), \vec{k}, z)$

- agrees w/ ingoing at horizon

What is the soln?

What is the soln?

$\phi(z, x)$

What is the soln?

$$\underline{\phi(z, \underline{x})}$$

What is the sol'n?

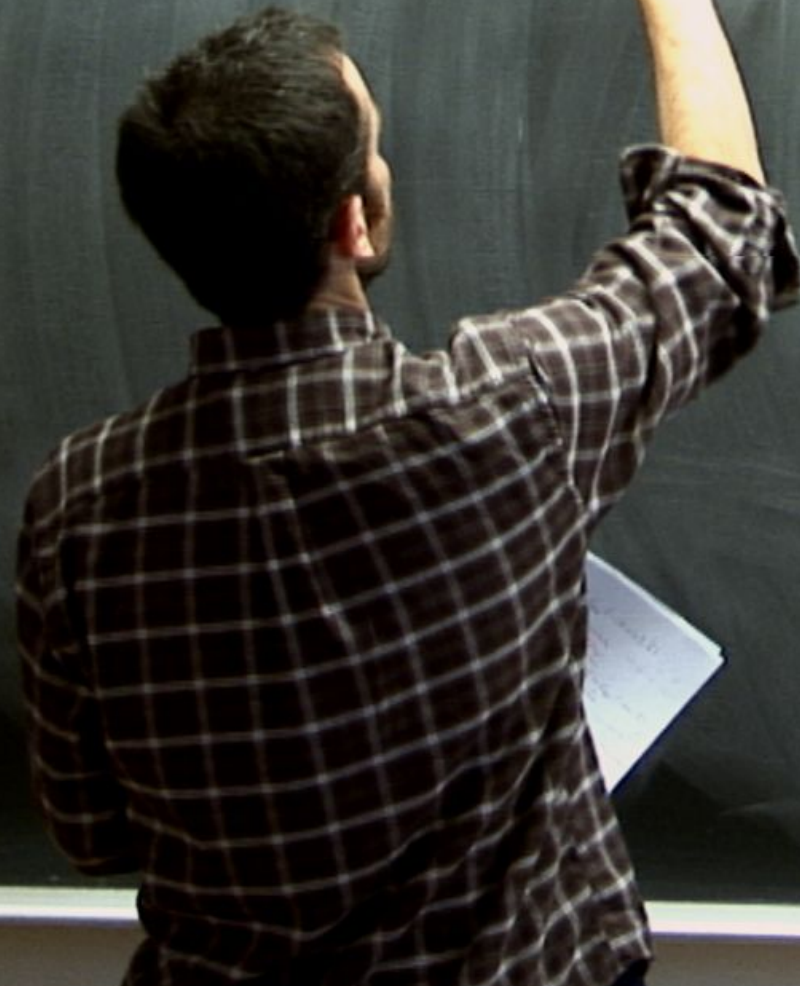
$$\underline{\phi(z, x)} \underset{z \rightarrow 0}{\sim}$$

What to w/ soln?

$$\underline{\phi(z, \underline{x})} \underset{z \rightarrow 0}{\sim} \begin{pmatrix} z \\ \underline{L} \end{pmatrix}$$

What is the soln?

$$\underline{\phi}(z, \underline{x}) \underset{z \rightarrow 0}{\sim} \left(\frac{z}{L} \right)^{\Delta_-}$$



What is the sol'n?

$$\underline{\phi(z, \underline{x})} \underset{z \rightarrow 0}{\sim} \left(\frac{z}{L} \right)^{\Delta_-} \phi_0(x)$$

What is the sol'n?

we choose

$$\underline{\phi(z, \underline{x})} \underset{z \rightarrow 0}{\sim} \begin{pmatrix} z \\ \underline{L} \end{pmatrix}^{\Delta_-} \phi_0(x)$$

What is the solⁿ?

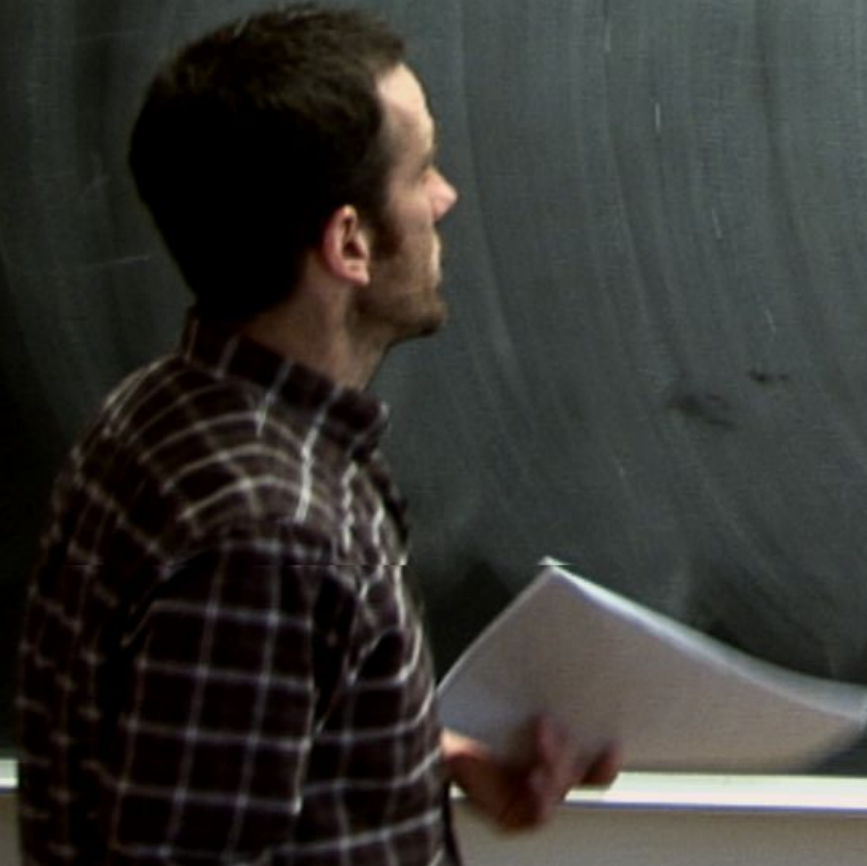
we choose

$$\underline{\phi(z, \underline{x})} \underset{z \rightarrow 0}{\sim} \left(\frac{z}{L} \right)^{\Delta_-} \phi_0(x) \left(1 + \mathcal{O}(z^2) \right)$$

What is the soln?

$$\underline{\phi(z, x)} \underset{z \rightarrow 0}{\sim} \left(\frac{z}{L} \right)^{\Delta_-} \phi_0(x) (1 + \mathcal{O}(z/L))$$

we choose



What is the sol'n?

we choose

$$\phi(z, x) \approx \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) (1 + \mathcal{O}(z/L))$$

What is the sol'n?

we choose

$$\underline{\phi(z, \underline{x})} \underset{z \ll L}{\sim} \left(\frac{z}{L} \right)^{\Delta_-} \phi_0(x) \left(1 + \mathcal{O}\left(\frac{z}{L}\right) \right)$$

What is the solⁿ?

we choose

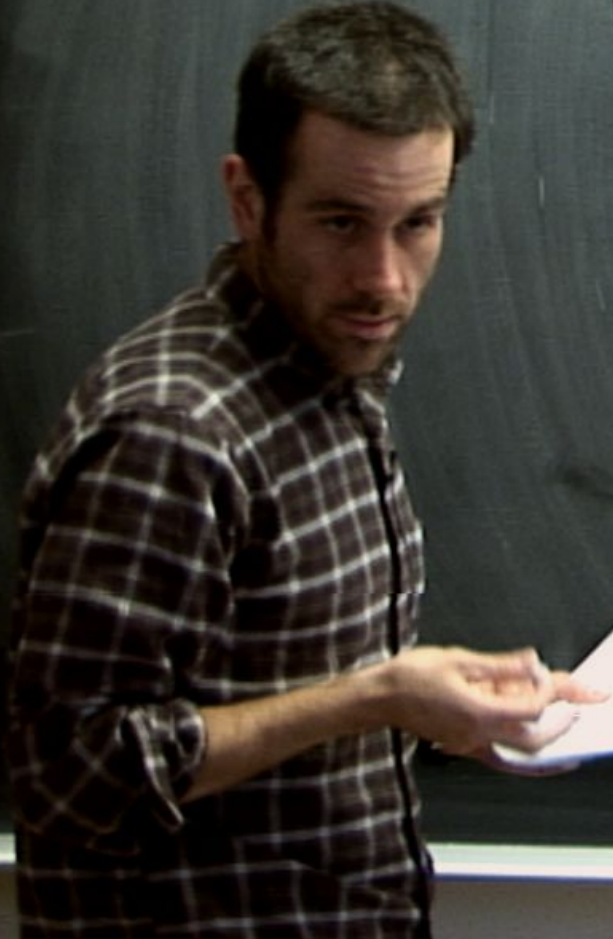
$$\underline{\phi}(z, \underline{x}) \underset{z \ll L}{\sim} \left(\frac{z}{L} \right)^{\Delta_-} \phi_0(x) \left(1 + \mathcal{O}\left(\frac{z}{L}\right) \right)$$

+

What is the solⁿ?

we choose

$$\underline{\phi(z, x)} \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) (1 + \mathcal{O}(z/L)) + \left(\frac{z}{L}\right)^{\Delta_+}$$



What is the solⁿ?

we choose

$$\underline{\phi(z, x)} \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) \left(1 + \mathcal{O}\left(\frac{z}{L}\right)\right)$$

$$+ \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x)$$

What form sol_n?

we choose

$$\underline{\phi(z, x)} \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) \left(1 + \mathcal{O}\left(\frac{z}{L}\right)\right) + \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) \left(1 + \mathcal{O}\left(\frac{z}{L}\right)\right)$$

What is the soln?

we choose

$$\underline{\phi(z, x)} \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) \left(1 + \mathcal{O}\left(\frac{z}{L}\right)\right) \\ + \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) \left(1 + \mathcal{O}\left(\frac{z}{L}\right)\right)$$

What to w/ sol'n?

we choose

$$\underline{\phi(z, x)} \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) \left(1 + \mathcal{O}\left(\frac{z}{L}\right)\right) + \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) \left(1 + \mathcal{O}\left(\frac{z}{L}\right)\right)$$

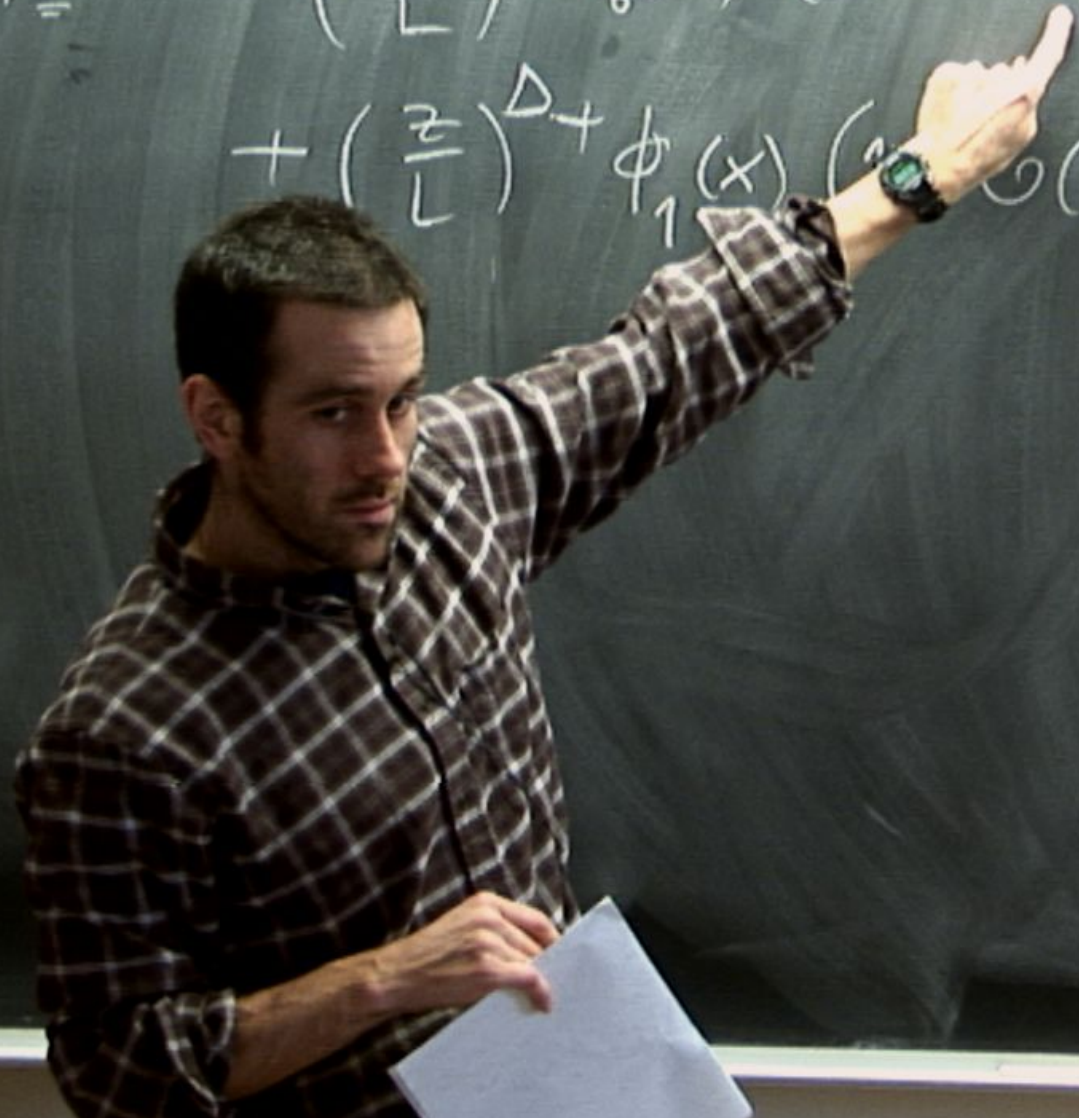
Δ_0

What is the soln?

we choose

$$\underline{\phi(z, x)} \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) \left(1 + \mathcal{O}\left(\frac{z}{L}\right)\right) + \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) \left(1 + \mathcal{O}\left(\frac{z}{L}\right)\right)$$

$$\text{if } \Delta_+ - \Delta_- \in \mathbb{Z}$$



What is the solⁿ?

we choose

$$\underline{\phi(z, x)} \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) \left(1 + \mathcal{O}\left(\frac{z}{L}\right)\right) \\ + \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) \left(1 + \mathcal{O}\left(\frac{z}{L}\right)\right)$$

$\Delta_+ - \Delta_- \in \mathbb{Z}$

What form soln?

we choose

$$\underline{\phi(z, x)} \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) (1 + \mathcal{O}(z/L))$$
$$+ \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) (1 + \mathcal{O}(z/L))$$

$\Delta_+ - \Delta_- \in \mathbb{Z}$

$y_i +$

What form solⁿ?

we choose

$$\phi(z, x) \sim \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) \left(1 + \mathcal{O}\left(\frac{z}{L}\right)\right) + \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) \left(1 + \mathcal{O}\left(\frac{z}{L}\right)\right)$$

$$\Delta_+ - \Delta_- \in \mathbb{Z}$$

$$ay + by + c$$

What is the sol'n?

we choose

$$\underline{\phi(z, x)} \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) (1 + \mathcal{O}(z/L))$$
$$+ \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) (1 + \mathcal{O}(z/L))$$

Δ_+

$$ay'' + by' + cy = 0$$

What is the solⁿ?

we choose

$$\underline{\phi(z, x)} \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) (1 + \mathcal{O}(z/L))$$
$$+ \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) (1 + \mathcal{O}(z/L))$$

$$y = Ae^{i\omega t}$$
$$ay'' + by' + cy = 0$$

What is the soln?

we choose

$$\underline{\phi(z, x)} \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) (1 + \mathcal{O}(z/L))$$
$$+ \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) (1 + \mathcal{O}(z/L))$$

Δ_+

$$y = Ae^{i\omega t}$$
$$a\ddot{y} + b\dot{y} + cy = 0$$
$$\rightarrow (a\omega^2 + i b\omega + c)A = 0$$

What is the solⁿ?

we choose

$$\underline{\phi(z, x)} \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) (1 + \mathcal{O}(z/L))$$
$$+ \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) (1 + \mathcal{O}(z/L))$$

$$\Delta_+ - \Delta_- \in \mathbb{Z}$$

$$y'' + by' + cy = 0$$
$$y = Ae^{i\omega t}$$
$$(-a\omega^2 + i b\omega + c)A = 0$$
$$\omega = \pm$$

What is the solⁿ?

we choose

$$\underline{\phi(z, x)} \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) (1 + \mathcal{O}(z/L)) + \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) (1 + \mathcal{O}(z/L))$$

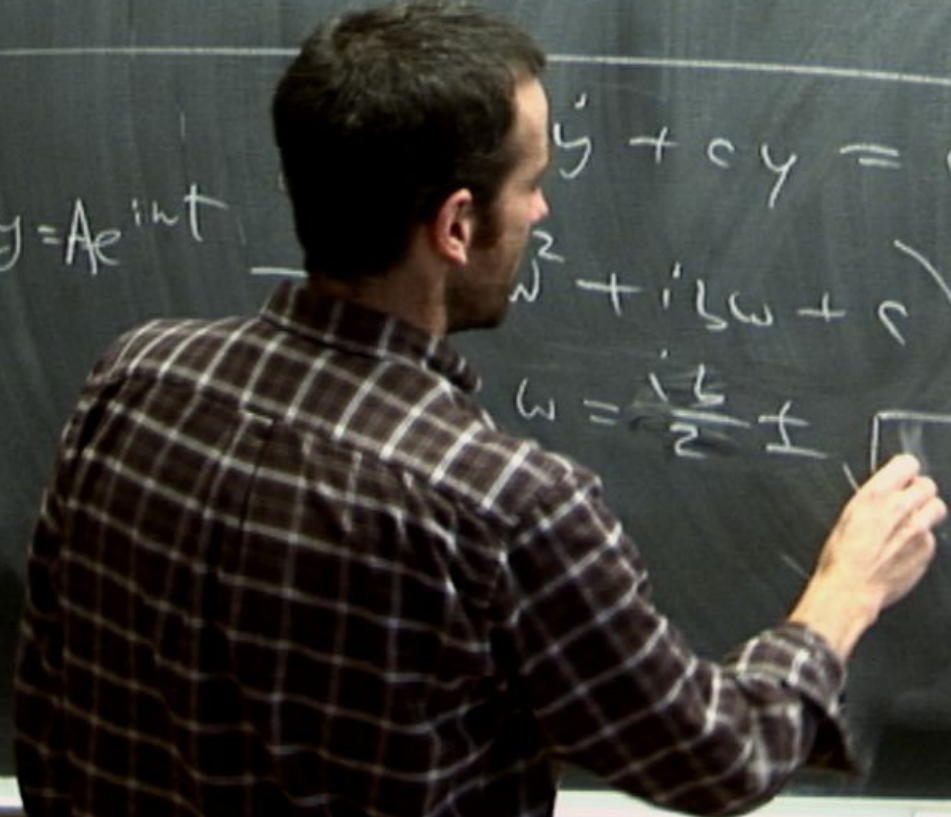
$$\Delta_+ - \Delta_- \in \mathbb{Z}$$

$$y = Ae^{i\omega t}$$

$$\ddot{y} + cy = 0$$

$$(-\omega^2 + i\zeta\omega + c)A = 0$$

$$\omega = \frac{i\zeta}{2} \pm \sqrt{\dots}$$



What is the soln?

$$\phi(z, x) \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) (1 + \mathcal{O}(z/L)) + \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) (1 + \mathcal{O}(z/L))$$

we choose



$$\Delta_+ - \Delta_- \in \mathbb{Z}$$

$$ay'' + by' + cy = 0$$
$$y = Ae^{i\omega t} \rightarrow (-a\omega^2 + ib\omega + c)A = 0$$
$$\omega = \frac{ib}{2} \pm \frac{\sqrt{-b^2 + 4ac}}{-2a}$$

What is the soln?

we choose

$$\phi(z, x) \sim \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) (1 + O(\frac{z}{L})) + \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) (1 + O(\frac{z}{L}))$$

$$\Delta_+ - \Delta_- \in \mathbb{Z}$$

$$y'' + ay' + cy = 0$$

$$y = Ae^{i\omega t}$$

$$(-b \pm \sqrt{b^2 - 4ac}) A = 0$$

What is the solⁿ?

we choose

$$\underline{\phi(z, x)} \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) (1 + \mathcal{O}(z/L))$$
$$+ \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) (1 + \mathcal{O}(z/L))$$

$$\Delta_+ - \Delta_- \in \mathbb{Z}$$

$$ay'' + by' + cy = 0$$

$$\rightarrow (a\omega^2 + i b\omega + c)A = 0$$

$$\omega = \frac{-ib \pm \sqrt{-b^2 + 4ac}}{2a}$$

What to w/ soln?

we choose

$$\underline{\phi(z, x)} \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) (1 + \mathcal{O}(z/L))$$
$$+ \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) (1 + \mathcal{O}(z/L))$$

Δ_+

$$ay'' + by' + cy = 0$$
$$y = Ae^{i\omega t} \rightarrow (a\omega^2 + i b\omega + c)A = 0$$
$$t e^{i\omega t} \quad \omega = \frac{-b \pm \sqrt{b^2 - 4ac}}{-2a}$$

What is the sol'n?

we choose

$$\underline{\phi(z, x)} \sim \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) (1 + \mathcal{O}(z/L))$$
$$+ \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) (1 + \mathcal{O}(z/L))$$

$\Delta_+ - \Delta_-$

$$ay'' + by' + cy = 0$$
$$y = Ae^{i\omega t} \rightarrow (a\omega^2 + i b\omega + c)A = 0$$
$$e^{i\omega t} + h e^{i\omega t} \quad \omega = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

recall

$$\mathbb{Z}[\phi_0]$$

recall

$$Z[\phi_0] \approx e^{-S_{\text{min}}[\phi]|\phi \rightarrow}$$

Encl

recall

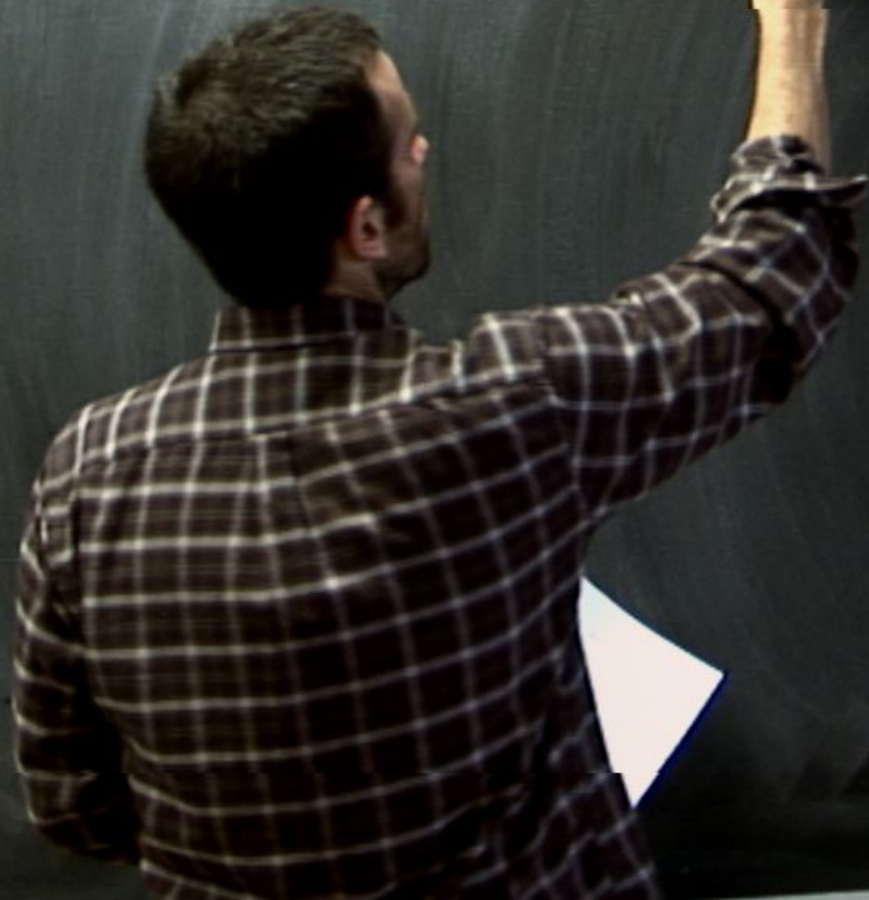
$$Z[\phi_0] \stackrel{\text{Euler}}{\approx} e^{-S_{\text{min}}[\phi | \phi \rightarrow \phi_0 z^{\Delta+}]}$$

$$S_{\text{min}}[\phi]$$

recall

$$Z[\phi_0] \stackrel{\text{Eucl}}{\approx} e^{-S_{\text{bulk}}[\phi | \phi \rightarrow \phi_0 z^{\Delta+}]}$$

$$S_{\text{bulk}}[\phi] = \int_{\text{body}} \sqrt{g} \phi \partial \phi$$



recall

$$Z[\phi_0] \stackrel{\text{Eucl}}{\approx} e^{-S_{\text{bulk}}[\phi | \phi \rightarrow \phi_0 z^{\Delta+}]}$$

$$S_{\text{bulk}}[\phi] = \int_{\text{body}} \sqrt{g} \mathcal{L}(\phi, \partial \phi)$$

recall

$$Z[\phi_0] \stackrel{\text{Eucl}}{\approx} e^{-S_{\text{bulk}}[\phi | \phi \rightarrow \phi_0 z^{\Delta+}]}$$

$$S_{\text{bulk}}[\phi] \approx \int_{\text{body}} \sqrt{g} \phi \square \phi$$

recall

$$Z[\phi_0] \approx e^{-S_{\text{bulk}}[\phi | \phi \rightarrow \phi_0 z^{\Delta+}]}$$

$$S_{\text{bulk}}[\phi] \approx \int_{\text{body}} \sqrt{g} \phi \partial \phi$$

recall

$$Z[\phi_0] \approx e^{-S_{\text{bulk}}[\phi_0 | \phi \rightarrow \phi_0 z^{\Delta+}]}$$

$$S_{\text{bulk}}[\phi] \approx \int_{\text{body}} \sqrt{g} \phi^2$$

recall

$$\underline{Z}[\phi_0] \stackrel{\text{Eucl}}{\approx} e^{-S_{\text{bulk}}[\phi | \phi \rightarrow \phi_0 z^{\Delta+}]}$$

$$S_{\text{bulk}}[\phi] \approx \int_{\text{body}} \sqrt{g} \phi \square \phi$$

cla

recall

$$\underline{Z}[\phi_0] \approx e^{-\frac{E_{\text{min}}}{\hbar} S_{\text{min}}[\phi_0 | \phi \rightarrow \phi_0 z^{\Delta+}]} \quad \downarrow$$

$$S_{\text{min}}[\phi] \approx \int_{\text{body}} \sqrt{g} g^{\mu\nu} \phi \partial_\mu \phi \partial_\nu \phi$$

classical mechanics interlude:

recall

$$Z[\phi_0] \stackrel{\text{Eucl.}}{\approx} e^{-S_{\text{bulk}}[\phi | \phi \rightarrow \phi_0, z \rightarrow z^+]}$$

$$S_{\text{bulk}}[\phi] \approx \int_{\text{body}} \sqrt{g} \phi \partial_z \phi$$

classical mechanics interlude:

consider a 1d particle $S[x]$

recall

$$\underline{Z}[\phi_0] \stackrel{\text{Eucl}}{\approx} e^{-S_{\text{bulk}}[\phi | \phi \rightarrow \phi_0 z^{\Delta+}]}$$

$$S_{\text{bulk}}[\phi] \approx \int_{\text{body}} \sqrt{g} \phi \partial^2 \phi$$

classical mechanics interlude:

consider a 1d particle $S[x]$
 w/ coord $x(t)$

recall

$$\underline{Z}[\phi_0] \stackrel{\text{Eucl}}{\approx} e^{-S_{\text{bulk}}[\phi | \phi \rightarrow \phi_0, z^D \rightarrow 1]}$$

$$S_{\text{bulk}}[\phi] \approx \int_{\text{body}} \sqrt{g} \phi \partial_z \phi$$

classical mechanics interlude:

consider a 1d particle
by coord $x(t)$

$$S[x] = \int dt \mathcal{L}$$

recall

$$Z[\phi_0] \stackrel{\text{Eucl}}{\approx} e^{-S_{\text{bulk}}[\phi | \phi \rightarrow \phi_0, z \rightarrow z^+]}$$

$$S_{\text{bulk}}[\phi] \approx \int_{\text{body}} \sqrt{g} g^{ij} \phi \partial_i \phi \partial_j \phi$$

classical mechanics interlude:

a 1d particle
coord $x(t)$

$$S[x] = \int_{t_1}^{t_2} dt L(x, \dot{x})$$

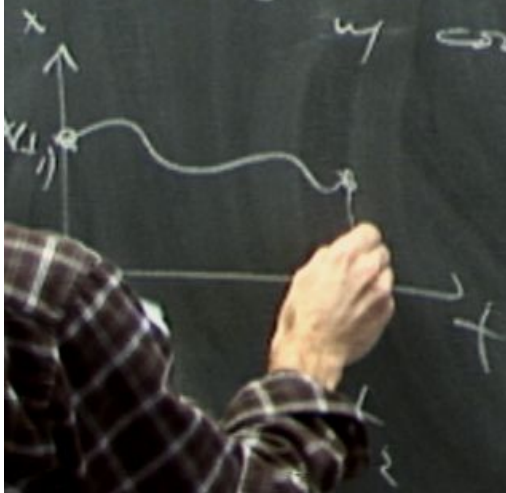
recall

$$Z[\phi_0] \stackrel{\text{Eucl}}{\approx} e^{-S_{\text{bulk}}[\phi | \phi \rightarrow \phi_0 z^{\Delta+}]}$$

$$S_{\text{bulk}}[\phi] \approx \int_{\text{body}} \sqrt{g} \phi \partial_z \phi$$

classical mechanics interlude:

consider a 1d particle by coord $x(t)$ $S[x] = \int_{t_1}^{t_2} dt L(x, \dot{x})$



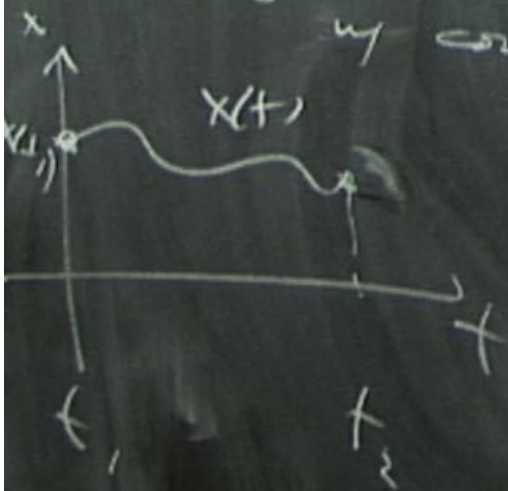
recall

$$Z[\phi_0] \stackrel{\text{Eucl}}{\approx} e^{-S_{\text{bulk}}[\phi | \phi \rightarrow \phi_0 z^{\Delta+}]}$$

$$S_{\text{bulk}}[\phi] \approx \int_{\text{body}} \sqrt{g} \phi \partial_z \phi$$

classical mechanics interlude:

consider a 1d particle by coord $x(t)$ $S[x] = \int_{t_1}^{t_2} dt L(x, \dot{x})$



recall

$$Z[\phi_0] \approx e^{-S_{\text{bulk}}[\phi | \phi \rightarrow \phi_0, z^D+]}$$

$$S_{\text{bulk}}[\phi] \approx \int_{\text{body}} \sqrt{g} g^{\mu\nu} \phi \partial_\mu \phi \partial_\nu \phi$$

classical mechanics interlude:

consider a 1d particle
w/ coord $x(t)$

$$S[x] = \int_{t_1}^{t_2} dt L(x, \dot{x})$$

$x(t)$
 $x(t_2)$

x

recall

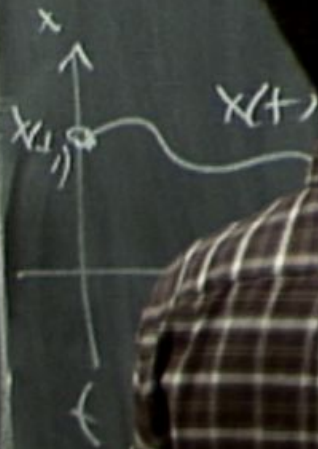
$$Z[\phi_0] \approx e^{-S_{\text{bulk}}[\phi | \phi \rightarrow \phi_0, z^D \rightarrow +]}$$

$$S_{\text{bulk}}[\phi] \approx \int_{\text{body}} \sqrt{g} g^{\mu\nu} \phi \partial_\mu \phi \partial_\nu \phi$$

classical mechanics interlude:

consider 1d particle
and $x(t)$

$$S[x] = \int_{t_1}^{t_2} dt L(x, \dot{x})$$



Π

recall

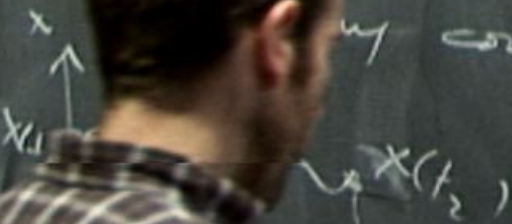
$$Z[\phi_0] \approx e^{-S_{\text{bulk}}[\phi | \phi \rightarrow \phi_0, z^D+]}$$

$$S_{\text{bulk}}[\phi] \approx \int_{\text{body}} \sqrt{g} g^{\mu\nu} \phi \partial_\mu \phi \partial_\nu \phi$$

classical mechanics interlude:

consider a 1d particle
with coord $x(t)$

$$S[x] = \int_{t_1}^{t_2} dt L(x, \dot{x})$$



$$\int_{x(t_1)}^{x(t_2)} dx$$

recall

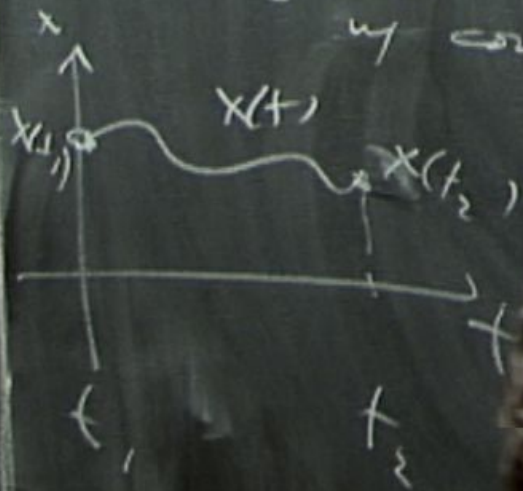
$$Z[\phi_0] \approx e^{-S_{\text{bulk}}[\phi | \phi \rightarrow \phi_0, z^D \rightarrow 1]}$$

$$S_{\text{bulk}}[\phi] \approx \int_{\text{body}} \sqrt{g} g^{\mu\nu} \phi \partial_\mu \phi \partial_\nu \phi$$

classical mechanics - interlude:

consider a 1d particle
with coord $x(t)$

$$S[x] = \int_{t_1}^{t_2} dt L(x, \dot{x})$$



$$= \Pi(t_1)$$

recall

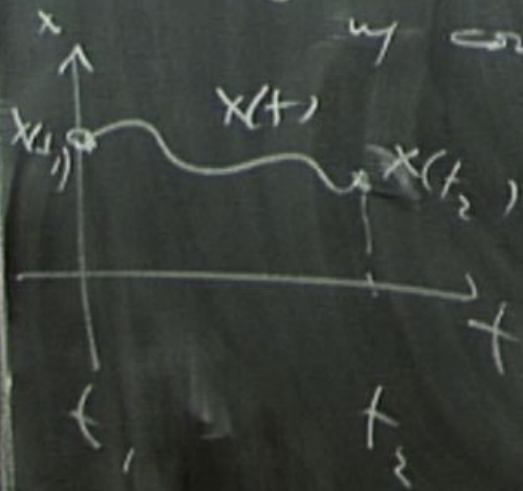
$$Z[\phi_0] \stackrel{\text{Eucl}}{\approx} e^{-S_{\text{bulk}}[\phi | \phi \rightarrow \phi_0, z^D \rightarrow +]}$$

$$S_{\text{bulk}}[\phi] \approx \int_{\text{body}} \sqrt{g} g^{\mu\nu} \phi \partial_\mu \phi \partial_\nu \phi$$

classical mechanics interlude:

consider a 1d particle
w/ coord $x(t)$

$$S[x] = \int_{t_1}^{t_2} dt L(x, \dot{x}, t)$$



$$\delta S_{X(t_1)} = \Pi(t_1) \equiv \frac{\partial S}{\partial x(t_1)}$$

recall

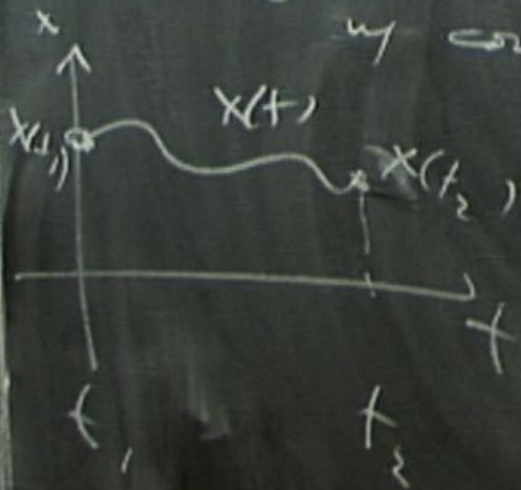
$$Z[\phi_0] \approx e^{-S_{\text{bulk}}[\phi | \phi \rightarrow \phi_0, z^D \rightarrow z^D +]}$$

$$S_{\text{bulk}}[\phi] \approx \int_{\text{body}} \sqrt{g} g^{\mu\nu} \phi \partial_\mu \phi \partial_\nu \phi$$

classical mechanics interlude:

consider a 1d particle
w/ coord $x(t)$

$$S[x] = \int_{t_1}^{t_2} dt L(x, \dot{x}, t)$$



$$\frac{\delta S}{\delta x(t_1)} = \Pi(t_1) \equiv \frac{\partial L}{\partial \dot{x}}$$

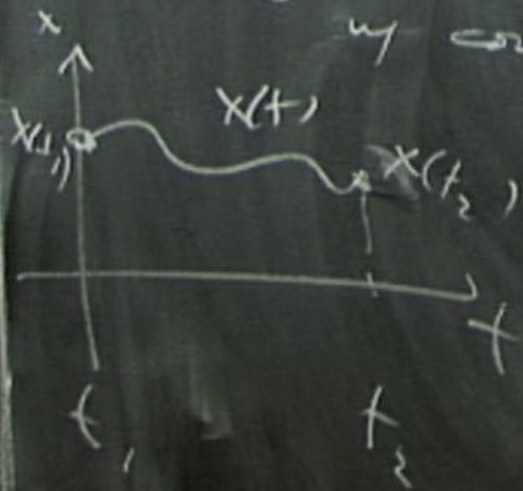
recall

$$Z[\phi_0] \stackrel{\text{Eucl}}{\approx} e^{-S_{\text{bulk}}[\phi | \phi \rightarrow \phi_0, z^D \rightarrow 1]}$$

$$S_{\text{bulk}}[\phi] \approx \int_{\text{body}} \sqrt{g} g^{\mu\nu} \phi \partial_\mu \phi \partial_\nu \phi$$

classical mechanics interlude:

consider a 1d particle $S[x] = \int_{t_1}^{t_2} dt L(x, \dot{x}, t)$
by cond $x(t)$



$$\delta S[x(t_1)] = \Pi(t_1) \equiv \frac{\partial L}{\partial \dot{x}}$$

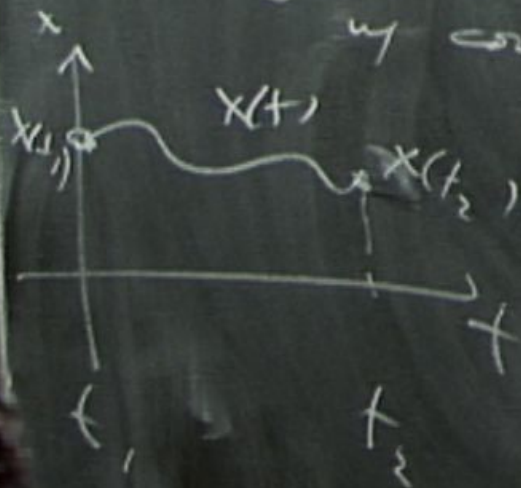
recall

$$Z[\phi_0] \approx e^{-S_{\text{bulk}}[\phi | \phi \rightarrow \phi_0, z^D +]}$$

$$S_{\text{bulk}}[\phi] \approx \int_{\text{body}} \sqrt{g} g^{\mu\nu} \phi \partial_\mu \phi \partial_\nu \phi$$

classical mechanics interlude:

consider a 1d particle $S[x] = \int_{t_1}^{t_2} dt L(x, \dot{x})$
w/ coord $x(t)$



$$\frac{\delta S}{\delta x(t_1)} = \Pi(t_1) \equiv \frac{\partial L}{\partial \dot{x}}(t_1)$$

recall

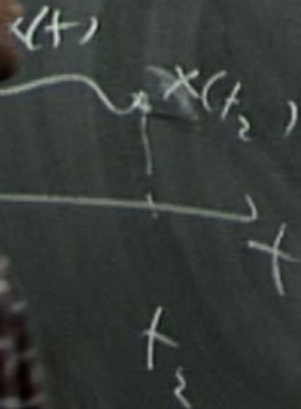
$$Z[\phi_0] \approx e^{-S_{\text{bulk}}[\phi | \phi \rightarrow \phi_0, z^D+]}$$

$$S_{\text{bulk}}[\phi] \approx \int_{\text{body}} \sqrt{g} g^{\mu\nu} \phi \partial_\mu \phi \partial_\nu \phi$$

classical mechanics interlude:

consider a 1d particle
by coord $x(t)$

$$S[x] = \int_{t_1}^{t_2} dt L(x, \dot{x})$$



$$\frac{\delta S}{\delta x(t_1)} = \Pi(t_1) \equiv \frac{\partial L}{\partial \dot{x}}(t_1)$$

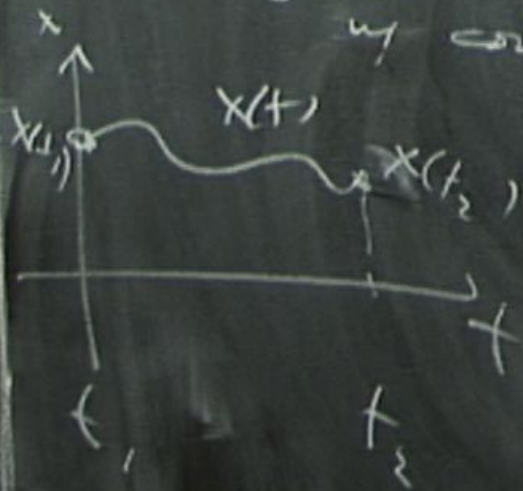
recall

$$Z[\phi_0] \stackrel{\text{Eucl}}{\approx} e^{-S_{\text{bulk}}[\phi | \phi \rightarrow \phi_0, z^D \rightarrow 1]}$$

$$S_{\text{bulk}}[\phi] \approx \int_{\text{body}} \sqrt{g} g^{\mu\nu} \phi \partial_\mu \phi \partial_\nu \phi$$

classical mechanics interlude:

consider a 1d particle $S[x] = \int_{t_1}^{t_2} dt L(x, \dot{x})$
w/ coord $x(t)$



$$\frac{\delta S}{\delta x(t_1)} = \Pi(t_1) \equiv \frac{\partial L}{\partial \dot{x}}(t)$$

Think of z as time



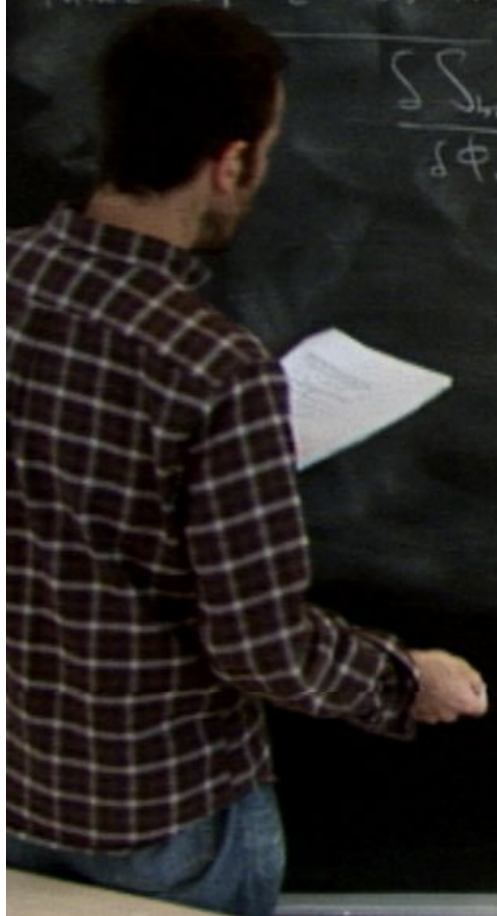


The f z as time
 $\{ \text{small } f_u \}$



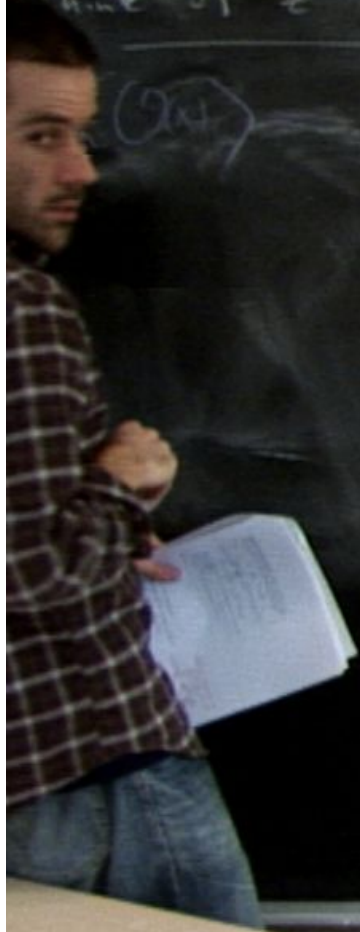
Think of z as time.

$$\frac{\delta S_{\text{small}}(\phi_0)}{\delta \phi_0(x)}$$



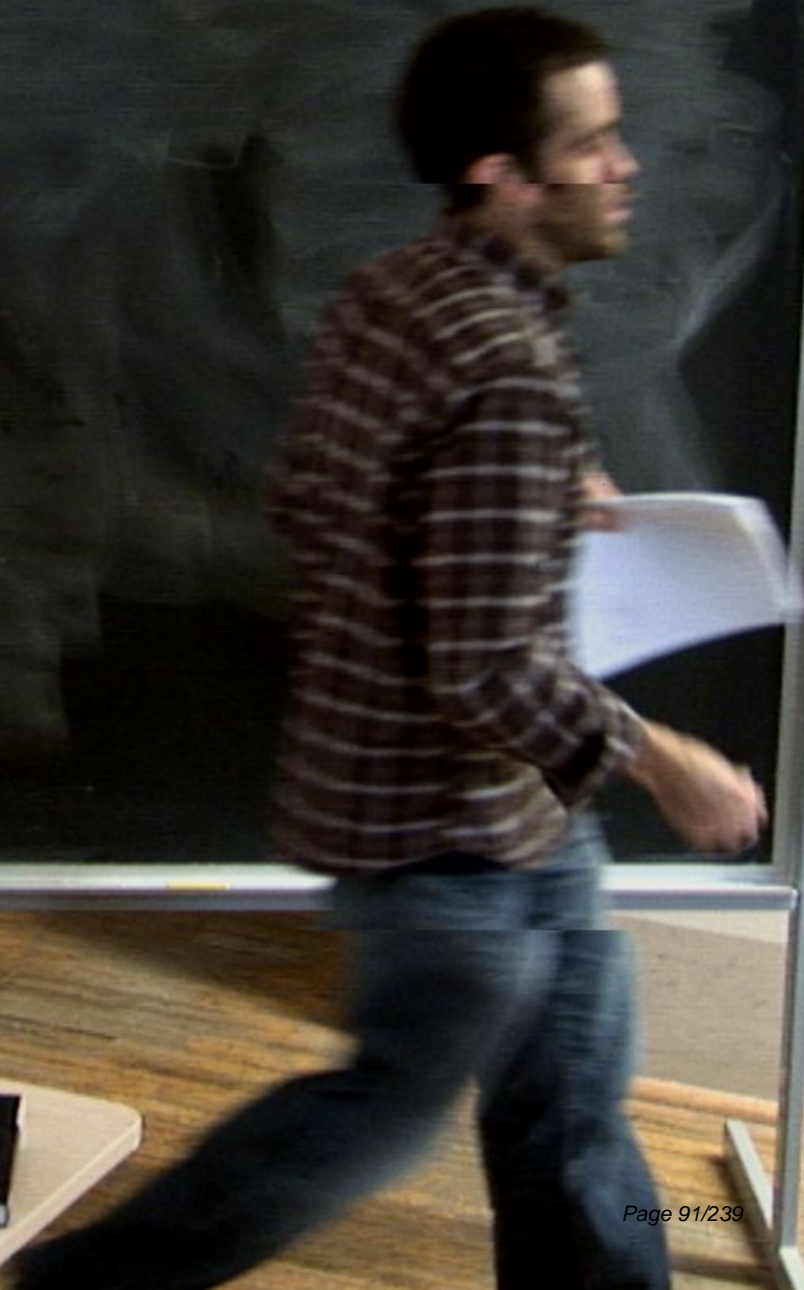
Think of z as time.

$$(24) \rightarrow \frac{\delta S_{\text{small}}(\phi_u)}{\delta \phi_0(x)}$$



Think of z as time.

$$\langle \dot{\phi}_0 \rangle = \frac{\delta S_{\text{small}}(\phi_0)}{\delta \phi_0(x)} = \lim_{z \rightarrow 0}$$



Think of z as time.

$\langle \phi_0 \rangle$

$$\frac{\delta S_{\text{small}}(\phi_0)}{\delta \phi_0(x)} = \lim_{z \rightarrow 0} \Pi(z, x)$$

$$\Pi(z, x)$$

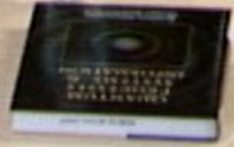
$$\Pi \equiv \frac{\partial \mathcal{L}}{\partial \dot{\phi}}$$

Think of z as time

$\langle \phi_0 \rangle$ $\frac{\delta S_{\text{small}}(\phi_0)}{\delta \phi_0(x)} = \lim_{z \rightarrow 0}$

$\Pi(z, x)$

$\Pi \equiv \frac{\partial \mathcal{L}}{\partial(\partial_z \phi)}$



Think of z as time.

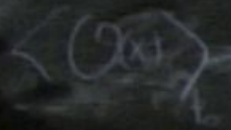
$$\left\langle \frac{\delta S_{\text{small}}(\phi_0)}{\delta \phi_0(x)} \right\rangle = \lim_{z \rightarrow 0} \langle \dots(z, x) \rangle$$

$$\Pi \equiv \frac{\partial \mathcal{L}}{\partial (\partial_z \phi)}$$

$$\| \phi \sim \left(\frac{z}{l}\right)^\Delta \phi_0$$



Think of z as time



$$\frac{\delta S_{\text{small}}}{\delta \phi_0(x)}$$

$$\lim_{z \rightarrow 0} \Pi(z, x)$$

$$\Pi \equiv \frac{\partial \mathcal{L}}{\partial (\partial_z \phi)}$$

$$\parallel \phi \sim \left(\frac{z}{\ell}\right)^\Delta$$

Think of z as time

$$\langle \psi | \psi \rangle = \frac{\delta S_{\text{small}}[\phi_0]}{\delta \phi_0(x)} = \lim_{z \rightarrow 0} \left(\frac{z}{L}\right)^{\Delta} \Pi(z, x)$$

$$\Pi \equiv \frac{\delta \mathcal{L}}{\delta (\partial_z \phi)}$$

$$\| \phi \sim \left(\frac{z}{L}\right)^{\Delta}$$



Think of z as time

$$\left\langle \frac{\delta S_{\text{total}}(\phi_0)}{\delta \phi_0(x)} \right\rangle = \lim_{z \rightarrow 0} \left(\frac{z}{L} \right)^{\Delta} \Pi(z, x)$$

$$\Pi \equiv \frac{\delta \mathcal{L}}{\delta (\partial_z \phi)}$$

$$\| \phi - \left(\frac{z}{L} \right)^{\Delta} \phi_0 \|$$



Think of z as time

(x)

$$\frac{\delta S_{\text{small}}(\phi_0)}{\delta \phi_0(x)}$$

$$= \lim_{z \rightarrow 0}$$

$$T(z, x)$$

$$\Pi \equiv \frac{\partial \mathcal{L}}{\partial (\partial_z \phi)}$$

$$\| \phi \approx (\frac{z}{l})^\Delta \phi_0$$

Think of z as time.

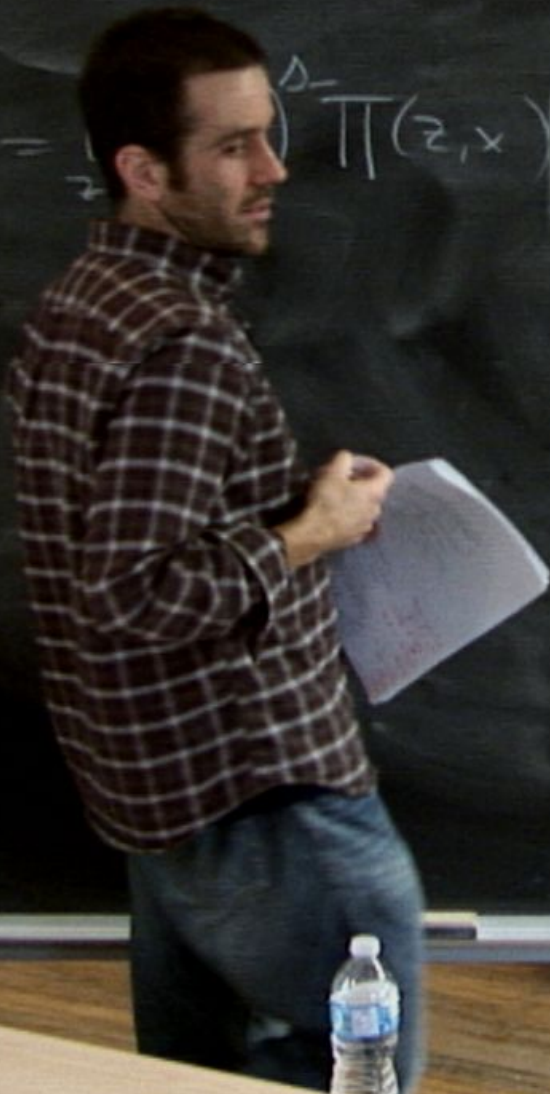
$\langle \psi |$

$$\frac{\delta S_{\text{small}}(\phi_0)}{\delta \phi_0(x)}$$

$$= \int_{z'} \Pi(z, x)$$

$$\Pi \equiv \frac{\delta \mathcal{L}}{\delta (\partial_z \phi)}$$

$$\| \phi \sim \left(\frac{z}{l}\right)^{\Delta} \phi_0$$



Think of z as time

(20)

$$\frac{\delta S_{\text{small}}[\phi_0]}{\delta \phi_0(x)} = \lim_{z \rightarrow 0} \left(\frac{z}{L} \right)^{\Delta_\phi} \Pi(z, x) \quad \text{limit}$$

$$\Pi \equiv \frac{\delta \mathcal{L}}{\delta (\partial_z \phi)}$$

$$\|\phi \sim \left(\frac{z}{L} \right)^{\Delta_\phi} \phi_0$$

use



Think of z as time.

$$\frac{\delta S_{\text{small}}(\phi_0)}{\delta \phi_0(x)} = \lim_{z \rightarrow 0} \left(\frac{z}{L} \right)^{\Delta_\phi} \Pi(z, x) \quad \text{finite}$$

use this away from support of $\phi_0(x)$.

$$\Pi \equiv \frac{\delta \mathcal{L}}{\delta (\partial_z \phi)}$$

$$\| \phi - \left(\frac{z}{L} \right)^{\Delta_\phi} \phi \|$$

Think of z as time.

$$\frac{\delta S_{\text{small}}(\phi_0)}{\delta \phi_0(x)} = \lim_{z \rightarrow 0} \left(\frac{z}{L} \right)^{\Delta_\phi} \Pi(z, x) \quad \text{finite}$$

$$\Pi \equiv \frac{\delta \mathcal{L}}{\delta (\partial_z \phi)}$$

$$\| \phi - \left(\frac{z}{L} \right)^{\Delta_\phi} \phi_0$$

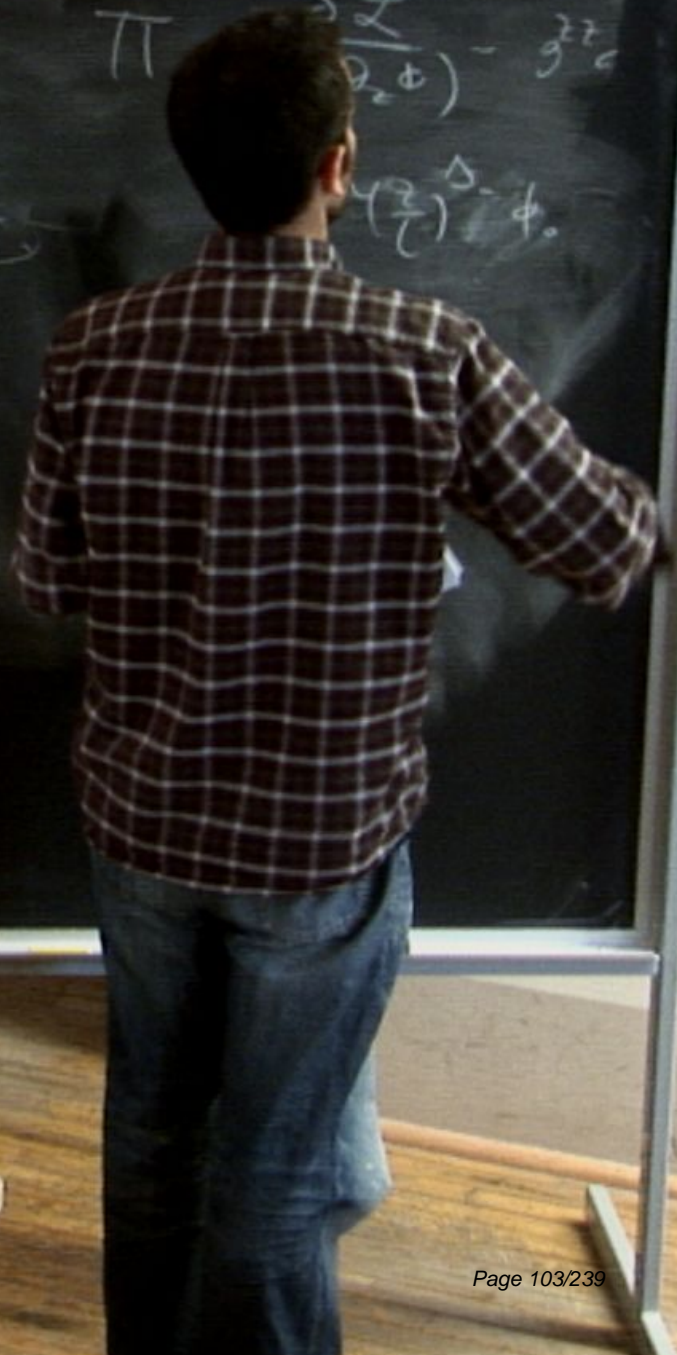
use this away from support of $\phi_0(x)$.

Think of z as time.

$$\langle \text{Ans} \rangle \frac{\delta S_{\text{small}}[\phi_0]}{\delta \phi_0(x)} = \lim_{z \rightarrow 0} \left(\frac{z}{L} \right)^{\Delta_\phi} \Pi(z, x) \quad \text{limit}$$

- use this away from support of $\phi_0(x)$.

$$\Pi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} - g^{\mu\nu} \partial_\mu \phi$$
$$\left(\frac{z}{L} \right)^{\Delta_\phi} \phi_0$$



Think of z as time.

$$\langle \psi | \frac{\delta S_{\text{small}}(\phi_0)}{\delta \phi_0(x)} \rangle = \lim_{z \rightarrow 0} \left(\frac{z}{L} \right)^{\Delta} \Pi(x)$$

- use this away from support of ϕ

$$\Pi \equiv \frac{\delta \mathcal{L}}{\delta (\partial_z \phi)} - z^{\Delta} \partial_z \phi$$

$$\| \phi - \left(\frac{z}{L} \right)^{\Delta} \phi \|$$

Think of z as time.

$$\langle \psi | \frac{\delta S_{\text{small}}[\phi_0]}{\delta \phi_0(x)} \rangle = \lim_{z \rightarrow 0} \left(\frac{z}{L} \right)^{\Delta_\phi} \Pi(z, x)$$

- use this away from support of $\phi_0(x)$.

$$\Pi \equiv \frac{\delta \mathcal{L}}{\delta (\partial_z \phi)} - k g^{zz} \partial_z \phi$$

$$\| \phi - \left(\frac{z}{L} \right)^{\Delta_\phi} \phi_0$$



Think of z as time.

$$\frac{\delta S_{\text{small}}[\phi_0]}{\delta \phi_0(x)} = \lim_{z \rightarrow 0} \left(\frac{z}{L} \right)^{\Delta_\phi} \Pi(z, x) \quad \text{limit}$$

use this away from support of $\phi_0(x)$.

$$\langle \mathcal{O}(x) \rangle$$

$$\Pi \equiv \frac{\delta \mathcal{L}}{\delta (\partial_z \phi)} \sim k g^{zz} \partial_z \phi$$

$$\| \phi \sim \left(\frac{z}{L} \right)^{\Delta_\phi} \phi_0$$

Think of z as time

$$\langle O(x) \rangle = \lim_{z \rightarrow 0} \left(\frac{z}{L} \right)^{\Delta} \Pi(z, x) \quad \text{finite}$$

- use this away from support of $\phi_0(x)$.

$$\Rightarrow \langle O(x) \rangle = k \frac{z^{\Delta-d}}{L} \phi_1(x)$$

$$\Pi \equiv \frac{\partial \mathcal{L}}{\partial \phi} \sim k g^{zz} \partial_z \phi$$

Think of z as time.

$$\langle \psi(x) \rangle = \frac{\delta S_{\text{small}}(\phi_0)}{\delta \phi_0(x)} = \lim_{z \rightarrow 0} \left(\frac{z}{L} \right)^{\Delta} \Pi(z, x)$$

- use this away from support of $\phi_0(x)$.

$$\Rightarrow \langle \psi(x) \rangle = \int \frac{z^{\Delta-d}}{L} \phi_1(x)$$

$$\text{LINEAR RESPONSE: } \delta \langle \psi(x) \rangle = \int_{\mathcal{R}} \delta \phi_0$$

$$\Pi \equiv \frac{\delta \mathcal{L}}{\delta (\partial_z \phi)} \sim k g^{zz} \partial_z \phi$$

$$\| \phi \sim \left(\frac{z}{L} \right)^{\Delta} \phi_0$$

Think of z as time

$$\langle O(x) \rangle$$

$$\frac{\delta \langle \text{small } \phi_0 \rangle}{\delta \phi_0(x)}$$

$$\left(\frac{z}{L}\right)^{\Delta} \Pi(z, x)$$

$$\Pi \equiv \frac{\delta \mathcal{L}}{\delta (\partial_z \phi)} \sim k g^{zz} \partial_z \phi$$

$$\| \phi \sim \left(\frac{z}{L}\right)^{\Delta} \phi_0$$

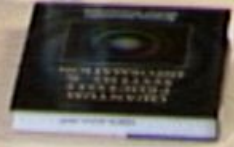
- use this away from $\phi_0(x)$

$$\Rightarrow \langle O(x) \rangle =$$

$$\phi_1(x)$$

$$\sim \frac{\phi_1}{\phi_0}$$

LINEAR RESPONSE =



Think of z as time

$$\langle \psi(x) \rangle = \frac{\delta \langle \psi(x) \rangle}{\delta \phi_0(x)} = \lim_{z \rightarrow 0} \left(\frac{z}{L} \right)^{\Delta} \Pi(z, x)$$

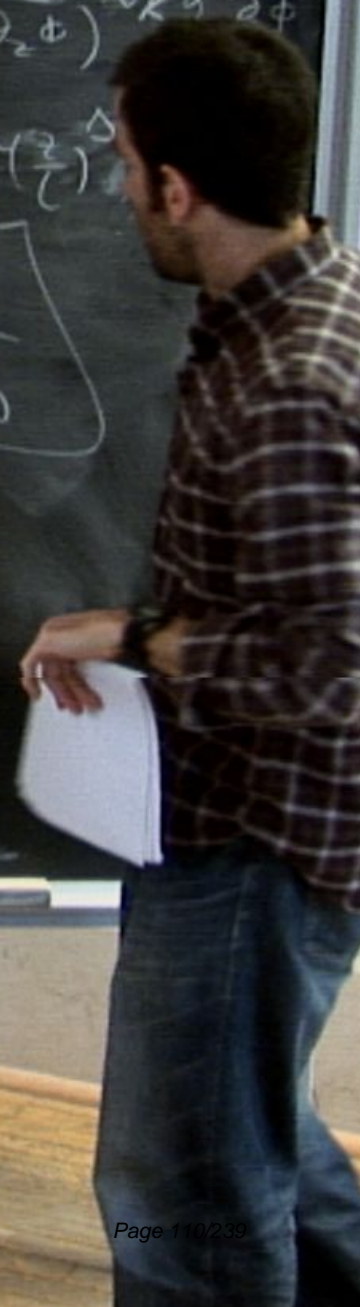
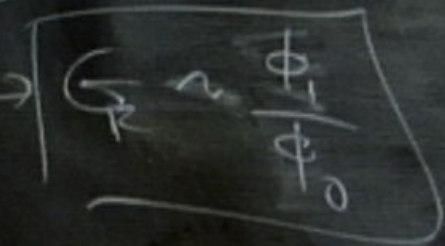
- use this away from support of $\phi_0(x)$

$$\Rightarrow \langle \psi(x) \rangle = \int \frac{z^{\Delta-d}}{L} \phi_1(x)$$

LINEAR RESPONSE: $\delta \langle \psi(x) \rangle = \int_{\mathbb{R}} \delta \phi_0$

$$\Pi \equiv \frac{\delta \mathcal{L}}{\delta (\partial_z \phi)} \sim k \partial_z^2 \phi$$

$$\|\phi \sim \left(\frac{z}{L} \right)^{\Delta}$$



What is the soln?

we choose

$$\underline{\phi(z, x)} \sim \left(\frac{z}{L}\right)^{\Delta_-} \underline{\phi_0(x)} (1 + \mathcal{O}(z/L))$$

$$+ \left(\frac{z}{L}\right)^{\Delta_+} \underline{\phi_1(x)} (1 + \mathcal{O}(z/L))$$

$\Delta_+ - \Delta_-$

$$+ by + cy = 0$$

$$\rightarrow (-aw^2 + ibw + c)A = 0$$

$$e^{i\omega t + \lambda x} \quad \omega = \frac{b \pm \sqrt{-b^2 + 4ac}}{-2a}$$

What is the soln?

we choose

$$\phi(z, x) \sim \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) (1 + \mathcal{O}\left(\frac{z}{L}\right)) + \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) (1 + \mathcal{O}\left(\frac{z}{L}\right))$$

if $\Delta_+ - \Delta_- \in \mathbb{Z}$

$$S =$$

What is the solⁿ?

we choose

$$\phi(z, x) \sim \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) \left(1 + \mathcal{O}\left(\frac{z}{L}\right)\right) + \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) \left(1 + \mathcal{O}\left(\frac{z}{L}\right)\right)$$

$$\Delta_+ - \Delta_- \in \mathbb{Z}$$

$$\int \left((\partial\phi)^2 + m^2 \phi \right)$$

What to w/ soln?

we choose

$$\phi(z, x) \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) (1 + \mathcal{O}(z/L))$$

$$+ \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) (1 + \mathcal{O}(z/L))$$

$$\Delta_+ - \Delta_- \in \mathbb{Z}$$

$$S = - \int \left((\partial\phi)^2 + m^2 \phi^2 \right) + \text{interact.}$$

What to w/ soln?

we choose

$$\underline{\phi(z, x)} \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \underline{\phi_0(x)} (1 + \mathcal{O}(z/L))$$

$$+ \left(\frac{z}{L}\right)^{\Delta_+} \underline{\phi_1(x)} (1 + \mathcal{O}(z/L))$$

$$\Delta_+ - \Delta_- \in \mathbb{Z}$$

$$S = -k \int \left((\partial\phi)^2 + m^2 \phi^2 \right) + \text{interact.}$$

What to w/ soln?

we choose

$$\phi(z, x) \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) (1 + \mathcal{O}(z/L))$$
$$+ \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) (1 + \mathcal{O}(z/L))$$

$\epsilon \ll$

$$S = -k \int \left((\partial\phi)^2 + m^2 \phi^2 \right) + \text{interact.}$$

EX

What to w/ soln?

we choose

$$\underline{\phi(z, x)} \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \underline{\phi_0(x)} (1 + \mathcal{O}(z/L))$$

$$+ \left(\frac{z}{L}\right)^{\Delta_+} \underline{\phi_1(x)} (1 + \mathcal{O}(z/L))$$

$\Delta_+ - \Delta_- \in \mathbb{Z}$

$$S = -k \int \left((\partial\phi)^2 + m^2 \phi^2 \right) + \text{interact.}$$

plug in

What is the soln?

we choose

$$\underline{\phi(z, x)} \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \underline{\phi_0(x)} (1 + \mathcal{O}(z/L))$$

$$+ \left(\frac{z}{L}\right)^{\Delta_+} \underline{\phi_1(x)} (1 + \mathcal{O}(z/L))$$

$$\Delta_+ - \Delta_- \in \mathbb{Z}$$

$$= -k \int \left((\partial\phi)^2 + m^2 \phi^2 \right) + \text{interact.}$$



What to w/ soln?

we choose

$$\underline{\phi}(z, x) \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \underline{\phi}_0(x) (1 + \mathcal{O}(z/L))$$

$$+ \left(\frac{z}{L}\right)^{\Delta_+} \underline{\phi}_1(x) (1 + \mathcal{O}(z/L))$$

$$\Delta_+ - \Delta_- \in \mathbb{Z}$$

$$k \int ((\partial\phi)^2 + m^2 \phi^2) + \text{interact.}$$

EX: $\underline{\phi}(z, u) = \leftarrow$

What to w/ soln?

we choose

$$\phi(z, x) \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) (1 + \mathcal{O}(z/L))$$

$$+ \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) (1 + \mathcal{O}(z/L))$$

$$\Delta_+ - \Delta_- \in \mathbb{Z}$$

$$\int \left((\partial\phi)^2 + m^2 \phi^2 \right) + \text{interact.}$$

EX. $\phi(z, x) = z^{\frac{d}{2}} K$

What to w/ soln?

we choose

$$\underline{\phi(z, x)} \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \underline{\phi_0(x)} (1 + \mathcal{O}(z/L))$$

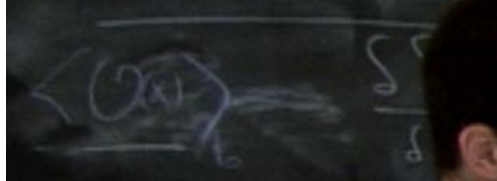
$$+ \left(\frac{z}{L}\right)^{\Delta_+} \underline{\phi_1(x)} (1 + \mathcal{O}(z/L))$$

$$\Delta_+ - \Delta_- \in \mathbb{Z}$$

$$S = -k \int \left((\partial\phi)^2 + m^2 \phi^2 \right) + \text{interact.}$$

FX. plug in $\underline{\phi(z, u)} = g z^{\frac{d}{2}} K_{\nu}(kz)$

Think of z as time



- use the support of $\phi(x)$

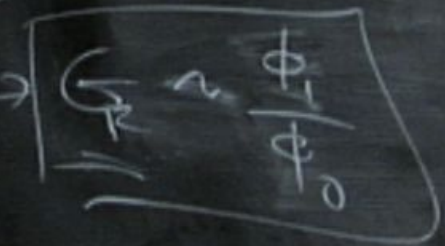
LINEAR

$$= \lim_{z \rightarrow 0} \left(\frac{z}{L} \right)^{\Delta} \Pi(z, x)$$

limit

$$\Pi \equiv \frac{\partial \mathcal{L}}{\partial (z_2 \phi)} \sim k g^{zz} z_2 \phi$$

$$\| \phi \sim \left(\frac{z}{L} \right)^{\Delta} \phi_0$$



support of $\phi(x)$

$$\int_{\mathbb{R}} \delta \phi_0$$



What is the soln?

we choose

$$\phi(z, x) \sim \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) (1 + \mathcal{O}(z/L)) + \left(\frac{z}{L}\right)^{\Delta_+} \phi(x) (1 + \mathcal{O}(z/L))$$

$\Delta_+ - \Delta_- \in \mathbb{Z}$

$$S = - \int (\dot{\phi})^2 + m^2 \phi^2 + \text{interact.}$$

EX. plug in $\phi = \int \frac{d^d k}{(2\pi)^d} K(k, z)$

$$K(k, z)$$

Think of z as time.

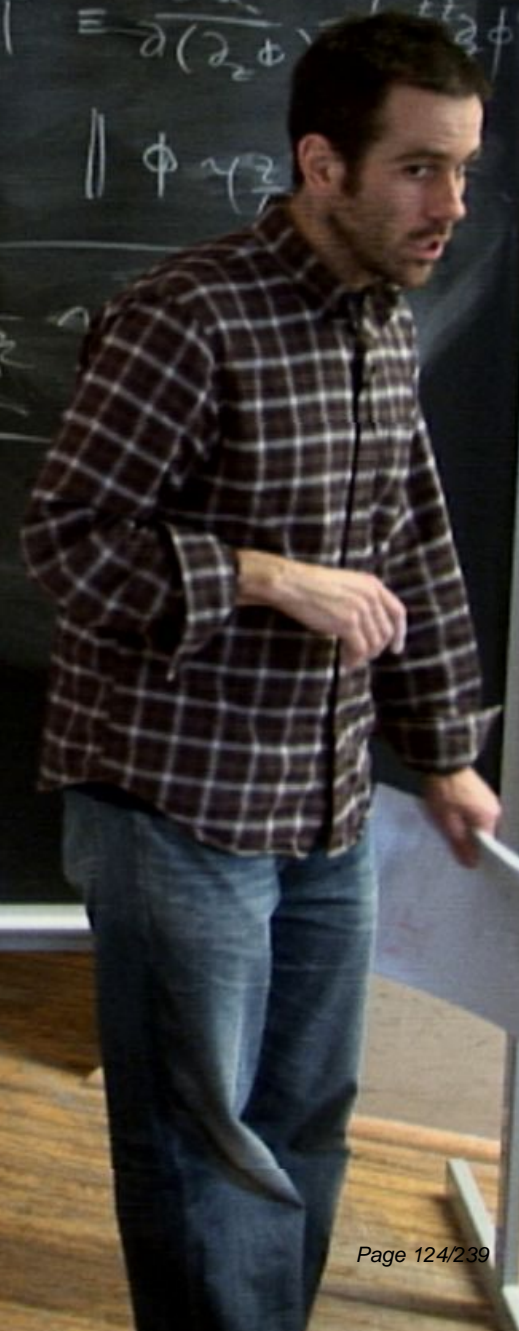
$$\langle O(x) \rangle = \frac{\delta S_{\text{bulk}}[\phi_0]}{\delta \phi_0(x)} = \lim_{z \rightarrow 0} \left(\frac{z}{L} \right)^{\Delta_O} \Pi(z, x)$$

$$\Pi \equiv \frac{\delta \mathcal{L}}{\delta(\partial_z \phi)} = \left(\frac{z}{L} \right)^{\Delta_O} \phi$$

- use this away from support of $\phi_0(x)$

$$\Rightarrow \langle O(x) \rangle = k \frac{z^{\Delta_O - d}}{L} \phi_0(x)$$

LINEAR RESPONSE: $\delta \langle O(x) \rangle = \int_{\mathcal{R}} \delta \phi_0$



What to w/ soln?

we choose

$$\phi(z, x) \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) (1 + \mathcal{O}(z/L))$$

$$+ \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) (1 + \mathcal{O}(z/L))$$

Δ_+

$$S = -k \int \left((\partial\phi)^2 + m^2 \phi^2 \right) + \text{interact.}$$

EX. plug in $\phi(z, u) = g z^{\frac{d}{2}} K_{\nu}(kz)$

What to w/ soln?

we choose

$$\underline{\phi(z, x)} \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \underline{\phi_0(x)} (1 + \mathcal{O}(z/L))$$

$$+ \left(\frac{z}{L}\right)^{\Delta_+} \underline{\phi_1(x)} (1 + \mathcal{O}(z/L))$$

$$S = -k \int \left((\partial\phi)^2 + m^2 \phi^2 \right) + \text{interact.}$$

EX. plug in $\underline{\phi(z, u)} = g z^{\frac{d}{2}} K_{\nu}(kz)$

$$G(k)$$

What to w/ soln?

we choose

$$\underline{\phi(z, x)} \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \underline{\phi_0(x)} (1 + \mathcal{O}(z/L))$$

$$+ \left(\frac{z}{L}\right)^{\Delta_+} \underline{\phi_1(x)} (1 + \mathcal{O}(z/L))$$

$\Delta_+ - \Delta_- \in \mathbb{Z}$

$$S = -k \int ((\partial\phi)^2 + m^2 \phi^2) + \text{interact.}$$

plug in $\underline{\phi(z, h)} = \dots$

$$G(k) = \frac{\phi_1(k)}{\dots} \rightarrow (kz)$$

What to w/ soln?

we choose

$$\underline{\phi(z, x)} \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \underline{\phi_0(x)} (1 + \mathcal{O}(z/L))$$

$$+ \left(\frac{z}{L}\right)^{\Delta_+} \underline{\phi_1(x)} (1 + \mathcal{O}(z/L))$$

$\Delta_+ - \Delta_- \in \mathbb{Z}$

$$S = -k \int \left((\partial\phi)^2 + m^2 \phi^2 \right) + \text{interact.}$$

plug in $\underline{\phi(z, h)} = \int \frac{d^d k}{(2\pi)^d} \underline{G(k)}$

$$G(k) = \frac{\phi_1(k)}{\phi_0(k)} \sim$$

What to w/ soln?

we choose

$$\underline{\phi(z, x)} \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \underline{\phi_0(x)} (1 + \mathcal{O}(z/L))$$

$$+ \left(\frac{z}{L}\right)^{\Delta_+} \underline{\phi_1(x)} (1 + \mathcal{O}(z/L))$$

$\Delta_+ - \Delta_- \in \mathbb{Z}$

$$S = -k \int \left((\partial\phi)^2 + m^2 \phi^2 \right) + \text{interact.}$$

plug in $\underline{\phi(z, h)} = \underline{g} z^{\frac{d}{2}} \underline{K} \underline{(kz)}$

$$G(k) = \frac{\phi_1(k)}{\phi_0(k)} \sim \# |k|^{2\nu}$$

Think of z as time.

$$\langle \mathcal{O}(x) \rangle = \frac{\delta S_{\text{bulk}}[\phi_0]}{\delta \phi_0(x)} = \lim_{z \rightarrow 0} \left(\frac{z}{L} \right)^{\Delta} \Pi$$

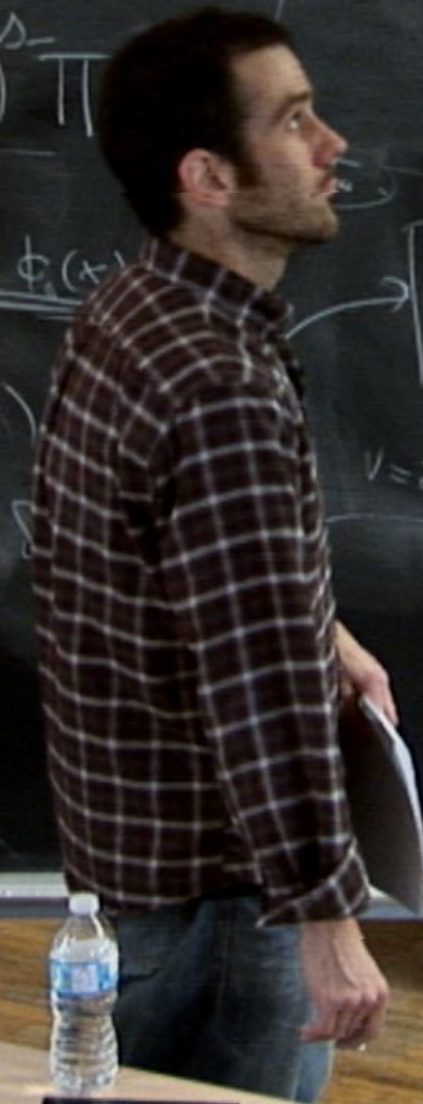
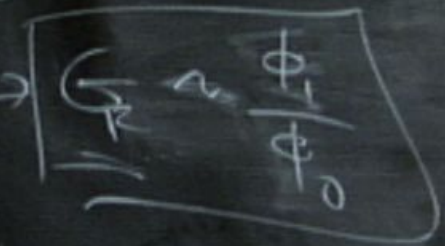
- use this away from support of $\phi_0(x)$

$$\Rightarrow \langle \mathcal{O}(x) \rangle = k \frac{(2\Delta-d)}{L} \phi_1(x)$$

LINEAR RESPONSE: $\delta \langle \mathcal{O}(x) \rangle = \int_{\mathcal{R}} \delta \phi_1(x)$

$$\Pi \equiv \frac{\delta \mathcal{L}}{\delta (z \phi)} \sim k g^{zz} z \phi$$

$$\| \phi \sim \left(\frac{z}{L} \right)^{\Delta} \phi_0$$



What to w/ soln?

we choose

$$\phi(z, x) \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) (1 + \mathcal{O}(z/L))$$

$$+ \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) (1 + \mathcal{O}(z/L))$$

$\Delta_+ - \Delta_- \in \mathbb{Z}$

$$S = - \int (\partial\phi)^2 + m^2 \phi^2 + \text{interact.}$$

EX. plug in

$$G(k) = \int z^{\frac{d}{2}} K_{\nu}(kz) (kz)$$

$$+ \sum_{\nu}$$

What is the soln?

we choose

$$\phi(z, x) \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) (1 + \mathcal{O}(z/L))$$

$$+ \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) (1 + \mathcal{O}(z/L))$$

$\Delta_+ - \Delta_- \in \mathbb{Z}$

$$= -k \int ((\partial\phi)^2 + m^2 \phi^2) + \text{interact.}$$

$$\phi(z, k) = \int z^{\frac{d}{2}} K(kz)$$

$$\sim \# k^{2\nu} + \sum_{n \geq 0} \alpha_n k^{2n}$$

What is the soln?

we choose

$$\phi(z, x) \sim \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) \left(1 + \mathcal{O}\left(\frac{z}{L}\right)\right) + \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) \left(1 + \mathcal{O}\left(\frac{z}{L}\right)\right)$$

$\Delta_+ - \Delta_- \in \mathbb{Z}$

$+ \mu^2 \phi^2 + \text{interact.}$

$\mu = g z^{\frac{d}{2}} K(kz)$
 $\sim \sum_{k \neq 0} k^{2\nu} + \sum_{n \geq 0} \alpha_n k^{2n}$

$$G(x) = \int dk e$$

$$G(x) = \int dk e^{-ik \cdot x} G(k)$$

$$= \# \int$$

$$G(x) = \int dk e^{-ikx} G(k)$$

$$= \# \int dk e^{-ikx} k^{2\nu} + \sum_{n=0}^{\infty} a_n \int dk e^{-ikx} k^{2n}$$

$$G(x) = \int dk e^{-ikx} G(k)$$

$$= \# \int dk e^{-ikx} k^{2\omega} + \sum_{n=0}^{\infty} a_n \int dk e^{-ikx} k^{2n}$$

$$= \frac{1}{|x|^{2\Delta+}}$$

$$G(x) = \int dk e^{-ikx} G(k)$$

$$= \# \int dk e^{-ikx} k^{2\nu} + \sum_{n=0}^{\infty} a_n \int dk e^{-ikx} k^{2n}$$

$$\Delta = \frac{d}{2} + \nu$$

$$= \frac{\#'}{|x|^{2\Delta}}$$

$$G(x) = \int dk e^{-ik \cdot x} G(k)$$

$$= \# \int dk e^{-ik \cdot x} k^{2\nu} + \sum_{n=0}^{\infty} a_n \int dk e^{-ik \cdot x} k^{2n}$$

$$\Delta = \frac{d}{2} + \nu$$

$$= \frac{\#'}{|x|^{2\Delta}} + \sum_{n=0}^{\infty} a_n$$

$$G(x) = \int dk e^{-ik \cdot x} G(k)$$

$$= \# \int dk e^{-ik \cdot x} k^{2\nu} + \sum_{n \geq 0} a_n \int dk e^{-ik \cdot x}$$

$$\Delta = \frac{d}{2} + \nu$$

$$= \frac{\#'}{|x|^{2\Delta}} + \sum_{n \geq 0} a_n$$

$$G(x) = \int dk e^{-ikx} G(k)$$

$$= \# \int dk e^{-ikx} k^{2\nu} + \sum_{n \geq 0} a_n \int dk e^{-ikx} k^{2n}$$

$$\Rightarrow \frac{\#'}{2\Delta_+} + \sum_{n \geq 0} a_n \int dk e^{-ikx} k^{2n}$$

$$\Delta = \frac{d}{2} + \nu$$

$$G(x) = \int d^d k e^{-ik \cdot x} G(k)$$

$$= \# \int d^d k e^{-ik \cdot x} k^{2\nu} + \sum_{n \geq 0} a_n \int d^d k e^{-ik \cdot x} k^{2n}$$

$$\Delta = \frac{d}{2} + \nu$$

$$= \frac{\#'}{|x|^{2\Delta}} + \sum_{n \geq 0} a_n$$

$$\int d^d(x)$$

$$G(x) = \int dk e^{-ikx} G(k)$$

$$= \# \int dk e^{-ikx} k^{2\nu}$$

$$+ \sum_{n \geq 0} a_n \int dk e^{-ikx} k^{2n}$$

$$= \frac{\#'}{|x|^{2\nu+1}} + \sum_{n \geq 0} a_n$$

$$\int dx$$

$$\square e^{-ikx} = -k^2 e^{-ikx}$$

$$G(x) = \int d^d k e^{-ik \cdot x} G(k)$$

$$= \# \int d^d k e^{-ik \cdot x} \frac{1}{k^{2\omega}}$$

$$+ \sum_{n \geq 0} a_n \int d^d h e^{-ih \cdot x} \frac{1}{k^{2n}}$$

$$= \frac{\#'}{|x|^{2\Delta+}} + \sum_{n \geq 0} a_n$$

$$\int d^d(x)$$

$$\square e^{-ik \cdot x} = \frac{1}{k^2} e^{-ik \cdot x}$$

$$G(x) = \int dk e^{-ikx} G(k)$$

$$= \# \int dk e^{-ikx} k^{2\nu} + \sum_{h \geq 0} a_n \int dk e^{-ikx} \left(\frac{2h}{k} \right)$$

$$= \frac{\#'}{|x|^{2\Delta+}} + \sum_{h \geq 0} a_n \square^h \int dk e^{-ikx}$$

$$\square e^{-ikx} = \frac{2}{k} e^{-ikx}$$

$$\langle G(x) G(0) \rangle$$

$$= G(x) = \int d^d k e^{-ik \cdot x} G(k)$$

$$= \# \int d^d k e^{-ik \cdot x} k^{2\omega} + \sum_{n \geq 0} a_n \int d^d k e^{-ik \cdot x} k^{2n}$$

$$\Delta = \frac{d}{2} + \nu$$

$$= \frac{\#'}{|x|^{2\Delta}} + \sum_{n \geq 0} a_n \square^h \int d^d k$$

$$\square e^{-ik \cdot x} = \frac{1}{k^{2n}} e^{-ik \cdot x}$$

$$\langle G(x) G(0) \rangle$$

$$= G(x) = \int d^d k e^{-ik \cdot x} G(k)$$

$$= \# \int d^d k e^{-ik \cdot x} k^{2\nu} + \sum_{n \geq 0} a_n \int d^d k e^{-ik \cdot x} k^{2n}$$

$$\Delta = \frac{d}{2} + \nu$$

$$= \frac{\#'}{|x|^{2\Delta}} + \sum_{n \geq 0} a_n \square^{\nu} \int d^d k$$

= 0 for $x=0$.

$$\square^{\nu} e^{-ik \cdot x} = \frac{1}{k^{2\nu}} e^{-ik \cdot x}$$

$$\langle G(x) G(0) \rangle$$

$$= G(x) = \int d^d k e^{-ik \cdot x} G(k)$$

$$= \# \int d^d k e^{-ik \cdot x} k^{2\nu} + \sum_{n \geq 0} a_n \int d^d k e^{-ik \cdot x} k^{2n}$$

$$\Delta = \frac{d}{2} + \nu$$

$$= \frac{\#'}{|x|^{2\Delta}} + \sum_{n \geq 0} a_n \square^h \int d^d k$$

= 0 for $x \neq 0$.

"contact terms"

$$\square^h e^{-ik \cdot x} = \frac{1}{k^{2n}} e^{-ik \cdot x}$$

$$\langle G(x) G(0) \rangle$$

$$= G(x) = \int d^d k e^{-ik \cdot x} G(k)$$

$$= \# \int d^d k e^{-ik \cdot x} k^{2\omega} + \sum_{n \geq 0} a_n \int d^d k e^{-ik \cdot x} k^{2n}$$

$$= \frac{\#'}{|x|^{2\Delta+}} + \sum_{n \geq 0} a_n \square^h \int d^d k$$

= 0 for $x \neq 0$.

"contact terms"

$$\square^h e^{-ik \cdot x} = \frac{1}{k^2} e^{-ik \cdot x}$$

$$\langle G(x) \delta(x) \rangle$$

$$= G(x) = \int d^d k e^{-ik \cdot x} G(k)$$

$$= \# \int d^d k e^{-ik \cdot x} k^{2\omega} + \sum_{n \geq 0} a_n \int d^d k e^{-ik \cdot x} k^{2n}$$

$$\frac{\#'}{|x|^{2\Delta+}} + \sum_{n \geq 0} a_n \square^h \int d^d k$$

"support of source" $\equiv \{x=0\}$

"contact terms" $= 0$ for $x \neq 0$.

$$\square^h e^{-ik \cdot x} = \frac{1}{k^{2\omega}} e^{-ik \cdot x}$$

$$\langle G(x) \underline{G(0)} \rangle$$

$$= G(x) = \int d^d k e^{-ik \cdot x} G(k)$$

$$= \# \int d^d k e^{-ik \cdot x} k^{2\Delta} + \sum_{n \geq 0} a_n \int d^d k e^{-ik \cdot x} k^{2n}$$

$$\Delta = \frac{d}{2} + \nu$$

$$= \frac{\#'}{|x|^{2\Delta}} + \sum_{n \geq 0} a_n \square^h \int d^d x$$

"support of source" $\equiv |x|=0$

"contact terms" $= 0$ for $x \neq 0$

$$\square^h e^{-ik \cdot x} = k^{2n} e^{-ik \cdot x}$$

"holographic renormalization"

$$\langle G(x) G(0) \rangle$$

$$= G(x) = \int d^d k e^{-ik \cdot x} G(k)$$

$$= \# \int d^d k e^{-ik \cdot x} k^{2\omega} + \sum_{n \geq 0} a_n \int d^d k e^{-ik \cdot x} k^{2n}$$

$$= \frac{\#'}{|x|^{2\Delta+}} + \sum_{n \geq 0} a_n \square^h \int d^d k$$

$$\Delta = \frac{d}{2}$$

def $\equiv |x|=0$

$= 0$ for $x \neq 0$.

"contact terms"

$$\square^h e^{-ik \cdot x} = k^{2n} e^{-ik \cdot x}$$

renormalization

$$\Delta S_{\text{bulk}} = \int_{\text{body}} \sqrt{g} d(\phi(\epsilon, x))$$

$$\langle G(x) \delta(x_0) \rangle$$

$$= G(x) = \int d^d k e^{-ik \cdot x} G(k)$$

$$= \# \int d^d k e^{-ik \cdot x} k^{2\omega} + \sum_{\omega > 0} a_n \int d^d k e^{-ik \cdot x} k^{2n}$$

$$\Delta = \frac{d}{2} + \nu$$

$$= \frac{\#'}{|x|^{2\Delta}} + \sum_{\omega > 0} \int d^d k e^{-ik \cdot x} k^{2\omega}$$

"support of source" $\equiv \{x=0\}$

0 for $x \neq 0$.
"criteria"

"holographic renormalization"

$$\square e^{-ik \cdot x} = -k^2 e^{-ik \cdot x}$$

$$\langle G(x) G(x_0) \rangle$$

$$= G(x) = \int d^d k e^{-i k \cdot x} G(k)$$

$$= \# \int d^d k e^{-i k \cdot x} k^{2\omega} + \sum_{h \geq 0} a_h \int d^d k e^{-i k \cdot x} k^{2h}$$

$$\Delta = \frac{d}{2} + \nu$$

$$= \frac{\#'}{|x|^{2\Delta}} + \sum_{h \geq 0} a_h \int d^d x \dots$$

"support of source" $\equiv |x|=0$

"contact terms" $= 0$ for $x \neq 0$

"holographic renormalization"

$$\Delta S_{\text{bulk}} = \int_{\text{body}} \mathcal{L}(\phi(\epsilon, x))$$

$$\langle G(x) G(0) \rangle$$

$$= G(x) = \int d^d k e^{-ik \cdot x} G(k)$$

$$= \# \int d^d k e^{-ik \cdot x} k^{2\omega} + \sum_{h \geq 0} a_h \int d^d k e^{-ik \cdot x} k^{2h}$$

$$\Delta = \frac{d}{2} + \nu$$

$$= \frac{\#'}{|x|^{2\Delta}} + \sum_{h \geq 0} a_h \square^h \int d^d x$$

"support of source" $\equiv \{x=0\}$

"contact terms" $= 0$ for $x \neq 0$

$$\square^h e^{-ik \cdot x} = \frac{1}{k^{2h}} e^{-ik \cdot x}$$

"holographic renormalization"

$$\Delta S_{\text{bulk}} = \int_{\text{body}} \mathcal{L}(\phi(\epsilon), x)$$

eg $\mathcal{L} =$

$$\langle G(x) G(0) \rangle$$

$$= G(x) = \int d^d k e^{-ik \cdot x} G(k)$$

$$= \# \int d^d k e^{-ik \cdot x} k^{2\Delta} + \sum_{n \geq 0} a_n \int d^d k e^{-ik \cdot x} k^{2n}$$

$$\Delta = \frac{d}{2} + \nu$$

$$= \frac{\#'}{|x|^{2\Delta}} + \sum_{n \geq 0} a_n \square^h \int d^d x$$

"support of source" $\equiv |x|=0$

"contact terms" $= 0$ for $x \neq 0$.

$$\square^h e^{-ik \cdot x} = k^{2n} e^{-ik \cdot x}$$

"holographic renormalization"

$$\Delta S_{\text{bulk}} = \int_{\text{body}} \mathcal{L}(\phi(\epsilon), x)$$

eg. $\mathcal{L} = G$

v) $\langle \alpha_0 \rangle$

$$G(x) = \int dk e^{-ikx} G(k)$$

$$= \# \int dk e^{-ikx} k^{2\nu} + \sum_{n \geq 0} a_n \int dk e^{-ikx} k^{2n}$$

$$= \frac{\#'}{|x|^{2\nu+1}} + \sum_{n \geq 0} a_n \square^h \int dx$$

"support of source" $\equiv \delta(x=0)$

$= 0$ for $x \neq 0$.

"contact terms"

$$\square^h e^{-ikx} = k^{2h} e^{-ikx}$$

logarithmic renormalization

$$\Delta S_{\text{bulk}} = \int_{\text{body}} \mathcal{L}(\phi(\epsilon, x))$$

eg. $\mathcal{L} = c_0 + c_1 \dots$

x) $\langle \alpha(0) \rangle$

$$G(x) = \int d^d k e^{-ik \cdot x} G(k)$$

$$= \# \int d^d k e^{-ik \cdot x} k^{2\omega} + \sum_{h \geq 0} a_h \int d^d k e^{-ik \cdot x} k^{2h}$$

$$\stackrel{+v}{=} \frac{\#'}{|x|^{2\Delta+}} + \sum_{h \geq 0} a_h \square^h \int d^d k e^{-ik \cdot x} k^{2h}$$

"support of source" $\Rightarrow \delta(x=0)$

$= 0$ for $x \neq 0$.

"contact terms"

$$\square e^{-ik \cdot x} = k^{2h} e^{-ik \cdot x}$$

logarithm; renormalization

$$\int_{\text{body}} \mathcal{L}(\phi(\epsilon, x))$$

eg. $\mathcal{L} = C_0 + C_1 \phi + \dots$

What is the solⁿ?

we choose

$$\underline{\phi(z, x)} \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \underline{\phi_0(x)} (1 + \mathcal{O}\left(\frac{z}{L}\right))$$

$$+ \left(\frac{z}{L}\right)^{\Delta_+} \underline{\phi_1(x)} (1 + \mathcal{O}\left(\frac{z}{L}\right))$$

$\Delta_+ - \Delta_- \in \mathbb{Z}$

POLES of

$\phi_1(\omega, k)$ is finite
 $\phi_0(\omega, k)$

What is the soln?

we choose

$$\phi(z, x) \sim \left(\frac{z}{L}\right)^{\Delta_-} \phi_0(x) (1 + \mathcal{O}(z/L)) + \left(\frac{z}{L}\right)^{\Delta_+} \phi_1(x) (1 + \mathcal{O}(z/L))$$

$\Delta_+ - \Delta_- \in \mathbb{Z}$

POLES of ζ_R : $\phi_1(\omega_{\pm} \frac{s}{d})$ is finite
 $\phi_0(\omega_{\pm} \frac{s}{d}) = 0$.

What to w/ soln?

we choose

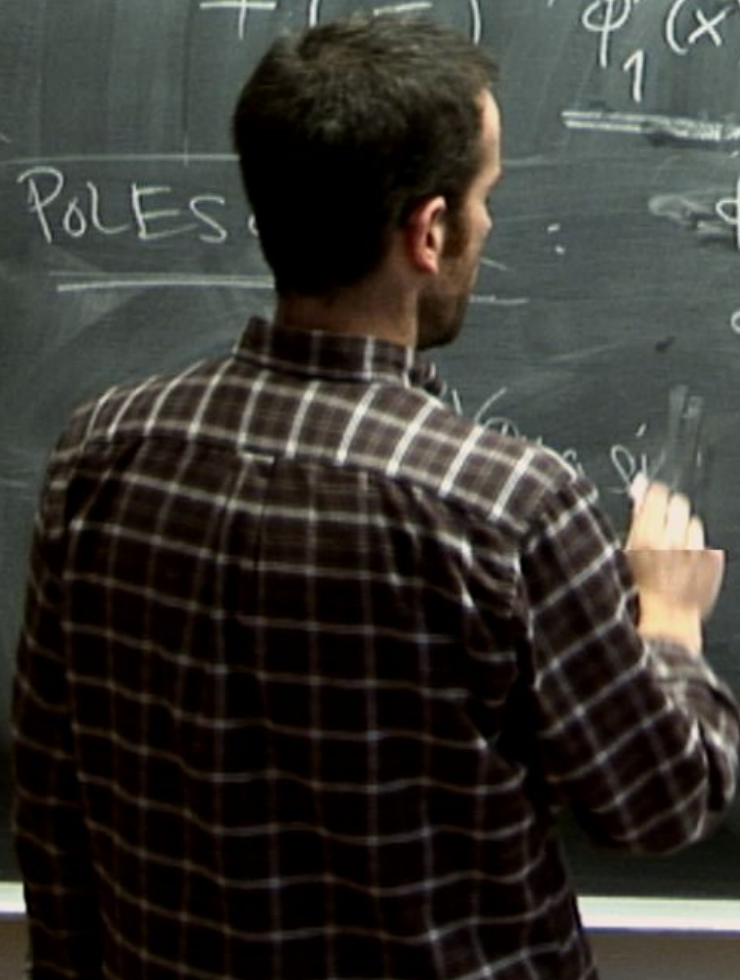
$$\underline{\phi(z, x)} \underset{z \ll 1}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \underline{\phi_0(x)} (1 + \mathcal{O}(z/L))$$

$$+ \left(\frac{z}{L}\right)^{\Delta_+} \underline{\phi_1(x)} (1 + \mathcal{O}(z/L))$$

$\Delta_+ - \Delta_- \in \mathbb{Z}$

POLES

$\phi_1(\omega_{\pm} \frac{s}{\alpha})$ is finite
 $\phi_0(\omega_{\pm} \frac{s}{\alpha}) = 0$.



What to w/ soln?

we choose

$$\underline{\phi(z, x)} \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \underline{\phi_0(x)} (1 + \mathcal{O}(z/L))$$

$$+ \left(\frac{z}{L}\right)^{\Delta_+} \underline{\phi_1(x)} (1 + \mathcal{O}(z/L))$$

$\Delta_+ - \Delta_- \in \mathbb{Z}$

\mathcal{S}_R : $\phi_1(\omega_{\pm}^{\pm})$ is finite
 $\phi_0(\omega_{\pm}^{\pm}) = 0$.

"(quasi)-normal mode"

What to w/ soln?

we choose

$$\underline{\phi(z, x)} \sim \left(\frac{z}{L}\right)^{\Delta_-} \underline{\phi_0(x)} (1 + \mathcal{O}(z/L))$$

$$+ \left(\frac{z}{L}\right)^{\Delta_+} \underline{\phi_1(x)} (1 + \mathcal{O}(z/L))$$

$$\Delta_+ - \Delta_- \in \mathbb{Z}$$

Set \mathcal{C}_R : $\phi_1(\omega_{\pm}^s)$ is finite
 $\phi_0(\omega_{\pm}^s) = 0$.

"(quasi)-normal mode"

$$\omega_{\pm} \in \mathcal{I}$$

What to w/ soln?

we choose

$$\underline{\phi(z, x)} \underset{z \ll L}{\sim} \left(\frac{z}{L}\right)^{\Delta_-} \underline{\phi_0(x)} (1 + \mathcal{O}(z/L))$$

$$+ \left(\frac{z}{L}\right)^{\Delta_+} \underline{\phi_1(x)} (1 + \mathcal{O}(z/L))$$

$\Delta_+ - \Delta_- \in \mathbb{Z}$

POLES of ζ_R :

$\phi_1(\omega_{\pm} \frac{s}{d})$

$\phi_0(\omega_{\pm} \frac{s}{d})$

"(quasi)-normal mod"

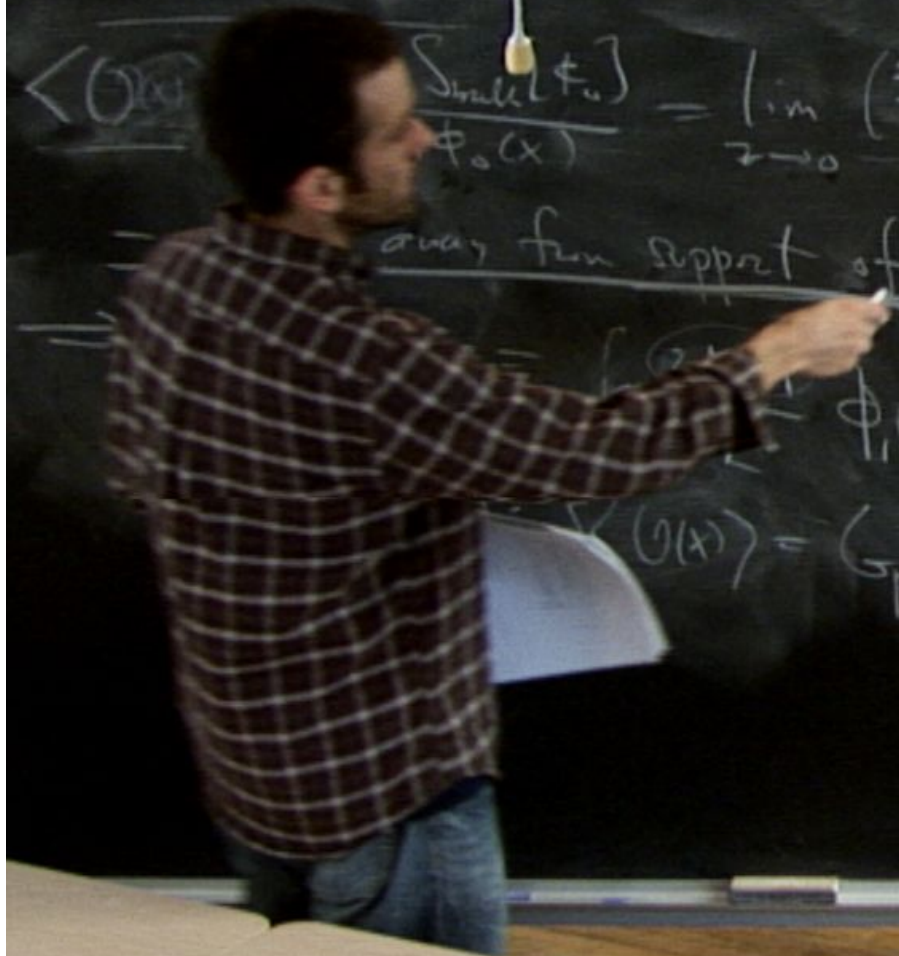
$\omega_{\pm} \in \mathbb{C}$

Think of z as time.

$$\langle \phi_0 | \phi_0 \rangle = \lim_{z \rightarrow 0} \left(\frac{z}{L} \right)^{\Delta_0} \Pi(z, x) \quad \text{finite}$$

away from support of $\phi_0(x)$.

$$\langle \phi_1 | \phi_1 \rangle = \int_{\mathbb{R}} \phi_1^2(x) dx$$



Think of z as time

$$\left\langle \frac{\partial \phi_0(x)}{\partial t} \right\rangle = \lim_{z \rightarrow 0} \left(\frac{z}{L} \right)^{\Delta - d} \Pi(z, x)$$

- use this away from support of $\phi_0(x)$

$$\Rightarrow \left\langle \frac{\partial \phi_0(x)}{\partial t} \right\rangle = k \frac{z \Delta - d}{L} \phi_0(x)$$

LINEAR RESPONSE: $\delta \left\langle \frac{\partial \phi_0(x)}{\partial t} \right\rangle = \chi_{\text{LR}} \delta \phi_0$

$$\Pi = \dots$$

Think of z as time.

$$\langle \phi(x) \rangle = \frac{\delta S_{\text{shell}}(\phi_0)}{\delta \phi_0(x)} = \lim_{z \rightarrow 0} T(z, x)$$

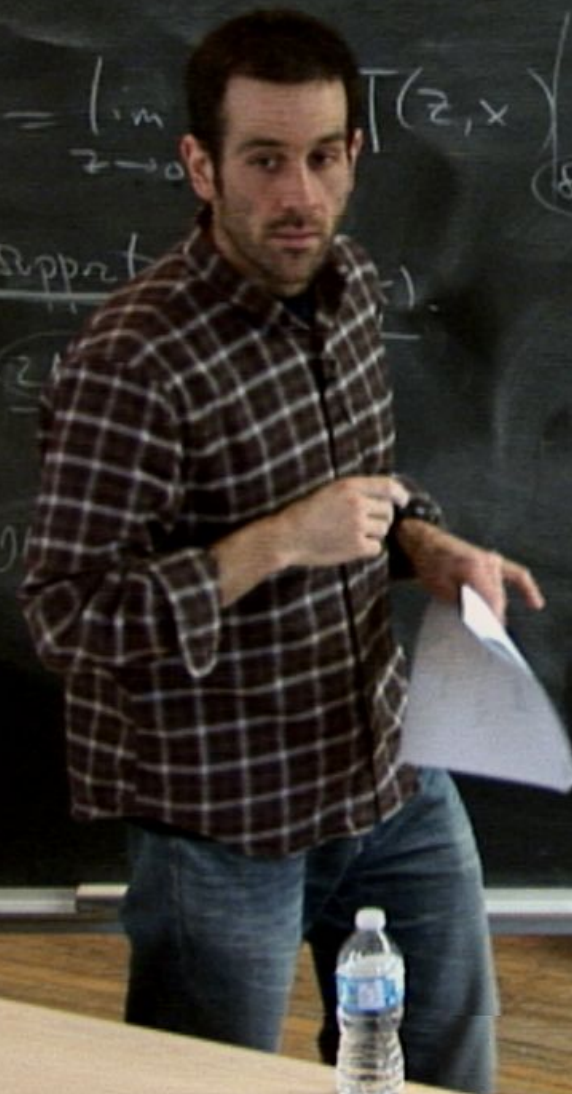
$$\Pi = \frac{\partial \mathcal{L}}{\partial (\partial_z \phi)}$$

finite

- use this away from support

$$\Rightarrow \langle \phi(x) \rangle = k \phi(x)$$

LINEAR RESPONSE = $\delta \langle \phi \rangle$



Think of z as time

$$\left\langle \frac{\delta S_{\text{shell}}(\phi_0)}{\delta \phi_0(x)} \right\rangle = \lim_{z \rightarrow 0} \left(\frac{z}{L} \right)^{\Delta_\phi} \Pi(z, x)$$

$$\Pi = \frac{\delta \mathcal{L}}{\delta (\delta \phi)}$$

away from support $\phi_0(x)$

$$\phi_1(x)$$

$$P_{\text{WF}} = \delta \langle \psi(x) \rangle = \int_{\mathcal{R}} \delta \phi_0$$

Think of z time

$\langle O(x) \rangle = \lim_{z \rightarrow 0} \left(\frac{z}{L} \right)^{\Delta} \Pi(z, x)$ (finite)

$\Pi = \frac{\partial \mathcal{L}}{\partial (\Delta \phi)}$

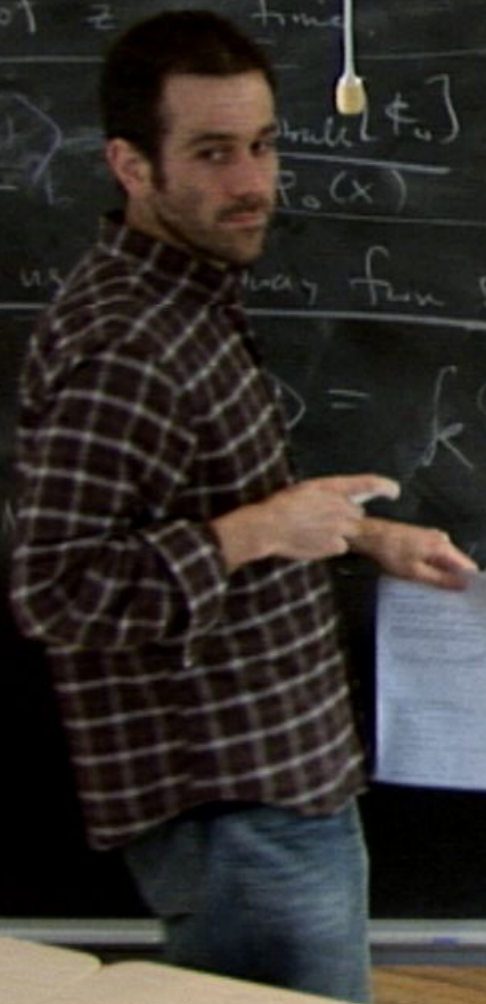
\checkmark HJ

$\frac{\partial \mathcal{L}}{\partial \phi(x)}$

way from support of $\phi_1(x)$

$\Rightarrow \langle O(x) \rangle = \int \frac{z \Delta - d}{L} \phi_1(x)$

$\langle O(x) \rangle = \int_{\mathbb{R}} \delta \phi_0$



Think of z as time.

$$\langle \psi(x) \rangle_{\phi_0} = \frac{\int \mathcal{D}\psi \psi(x) e^{-\int \mathcal{L}(\psi, \dot{\psi})}}{\int \mathcal{D}\psi e^{-\int \mathcal{L}(\psi, \dot{\psi})}} = \lim_{z \rightarrow 0} \left(\frac{z}{L} \right)^{\Delta_\psi} \Pi(z, x)$$

$\Pi \equiv \frac{\partial \mathcal{L}}{\partial (\partial_z \psi)}$

use this away from support of $\phi_0(x)$.

$$\langle \psi(x) \rangle = k \frac{z^{\Delta_\psi - d}}{L} \phi_1(x)$$

LINEAR RESPONSE: $\delta \langle \psi(x) \rangle = \int_{\mathcal{R}} \delta \phi_0$

Think of z as time.

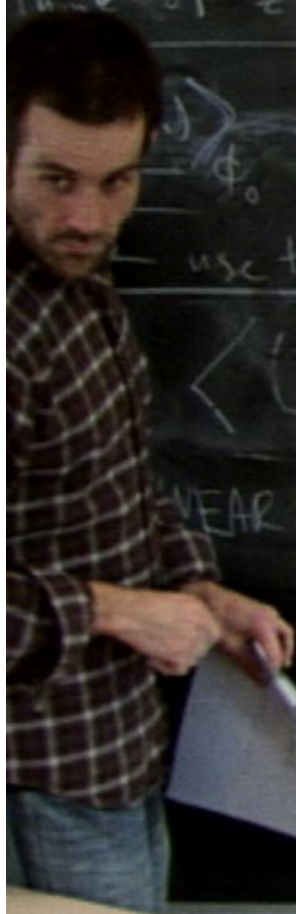
$$\left[\frac{\delta S_{\text{shell}}(\phi_0)}{\delta \phi_0(x)} \right] \stackrel{HJ}{=} \lim_{z \rightarrow 0} \left(\frac{z}{L} \right)^{\Delta_-} \Pi(z, x) \quad \text{finite}$$

$$\Pi \equiv \frac{\delta \mathcal{L}}{\delta \phi(z)}$$

use this away from support of $\phi_0(x)$.

$$\langle \psi(x) \rangle = k \frac{(2\Delta - d)}{L} \phi_1(x)$$

$$\text{NEAR RESPONSE: } \delta \langle \psi(x) \rangle = \left(\frac{\delta \mathcal{L}}{\delta \phi_0} \right)$$



Written

Witten diagrams

eg. $\langle \text{O} \text{O} \text{O} \rangle$

Witten diagrams

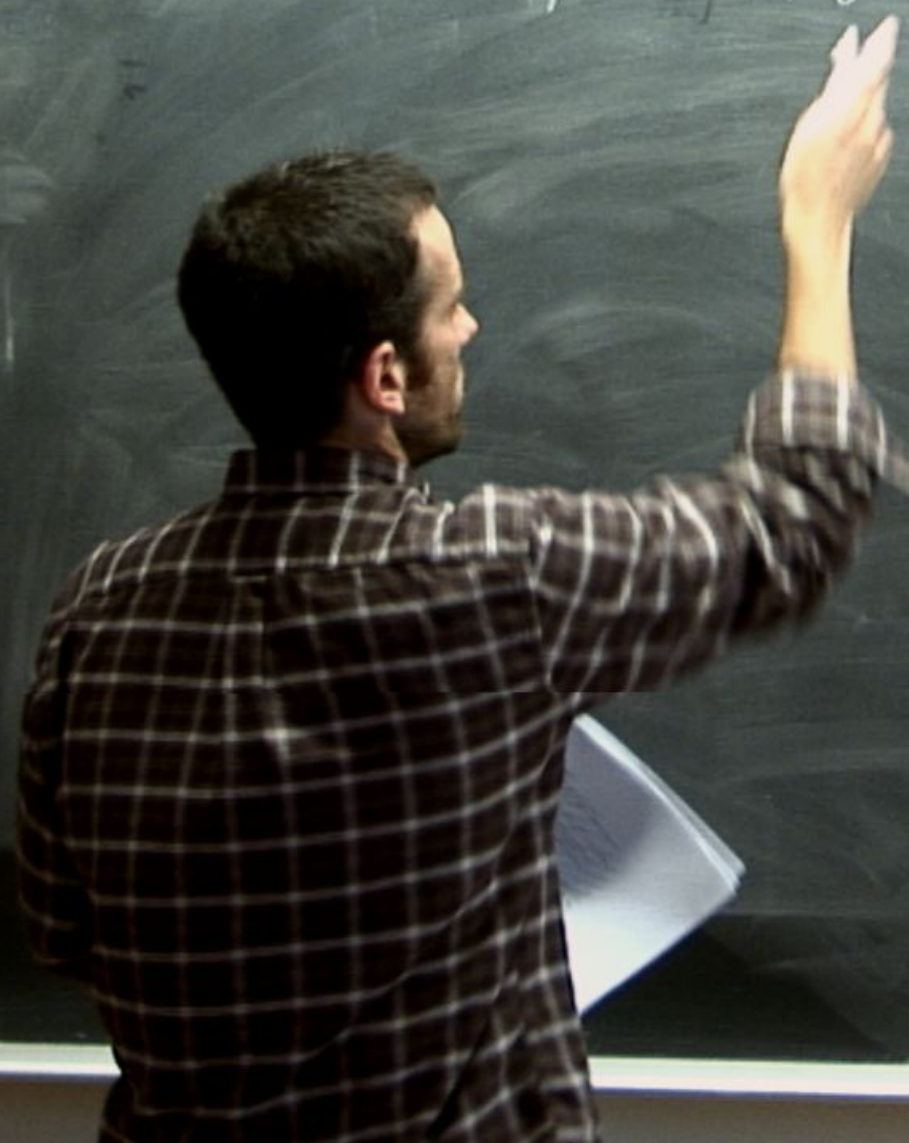
$$\text{eg. } \langle 000 \rangle = \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix}^3$$

Witten diagrams

$$\text{eg. } \langle \bar{0} 0 0 \rangle = \left(\frac{\delta}{\delta \phi_0} \right)^3 \mu Z$$

Witten diagrams

$$\text{eg. } \langle 000 \rangle = \begin{pmatrix} \delta \\ \delta \end{pmatrix}^3 \mu z \Big|_{\phi=0} \sim \begin{pmatrix} \delta \\ \delta \end{pmatrix}^2 \langle 0 \rangle$$



Witten diagrams

$$\text{eg. } \langle \bar{0} 00 \rangle = \begin{pmatrix} \delta \\ \delta \phi_9 \end{pmatrix}^3 \mu z \Big|_{\phi_3=0} \sim \begin{pmatrix} \delta \\ \delta \phi_0 \end{pmatrix}^2 \langle \bar{0} \rangle \Big|_{\phi_0=0}$$

Witten diagrams

$$\text{eg. } \langle \text{O O O} \rangle = \left(\frac{\delta}{\delta \phi_0} \right)^3 \mu Z \Big|_{\phi_0=0} \sim \left(\frac{\delta}{\delta \phi_0} \right)^2 \langle \text{O} \rangle \Big|_{\phi_0=0}$$

$$(\mathbb{D} - m^2) \phi$$

Witten diagrams

$$e.g. \langle \text{O} \text{O} \text{O} \rangle = \left(\frac{\delta}{\delta \phi_0} \right)^3 \mu z \Big|_{\phi_0=0} \sim \left(\frac{\delta}{\delta \phi_0} \right)^2 \langle \text{O} \rangle_{\phi_0=0}$$

$$(\mathbb{D} - m^2) \phi(z, x) = b \phi^2$$

Witten diagrams

$$e.g. \langle \mathcal{O} \mathcal{O} \mathcal{O} \rangle = \left(\frac{\delta}{\delta \phi_0} \right)^3 \mu z \Big|_{\phi_0=0} \sim \left(\frac{\delta}{\delta \phi_0} \right)^2 \langle \mathcal{O} \rangle_{\phi_0=0}$$

EoM: $(\mathbb{D} - m^2) \phi(z, x) = b \phi^2$

Witten diagrams

$$e.g. \langle \phi \phi \phi \rangle = \left(\frac{\delta}{\delta \phi_0} \right)^3 \mu z \Big|_{\phi=0} \sim \left(\frac{\delta}{\delta \phi_0} \right)^2 \langle \phi \rangle_{\phi=0}$$

EoM: $(\square - m^2) \phi(z, x) = b \phi^2(z, x)$

Witten diagrams

$$e.g. \langle \text{O} \text{O} \text{O} \rangle = \left(\frac{\delta}{\delta \phi_0} \right)^3 \mu z \Big|_{\phi_0=0} \sim \left(\frac{\delta}{\delta \phi_0} \right)^2 \langle \text{O} \rangle_{\phi_0=0}$$

EoM: $(\square - m^2) \phi(z, x) = b \phi^2(z, x)$

(z, x)

Witten diagrams

$$e.g. \langle \bar{0} 00 \rangle = \left(\frac{\delta}{\delta \phi_0} \right)^3 \mu z \Big|_{\phi_0=0} \sim \left(\frac{\delta}{\delta \phi_0} \right)^2 \langle \bar{0} \rangle \Big|_{\phi_0=0}$$

$$EOM: (\square - m^2) \phi(z, x) = b \phi^2(z, x)$$

$$\phi(z, x) = \dots x,$$

Witten diagrams

$$e.g. \langle \phi \phi \phi \rangle = \left(\frac{\delta}{\delta \phi_0} \right)^3 \mu z \Big|_{\phi=0} \sim \left(\frac{\delta}{\delta \phi_0} \right)^2 \langle \phi \rangle_{\phi_0=0}$$

EoM: $(\square - m^2) \phi(z, x) = b \phi^2(z, x)$

$$\phi(z, x) = \int d^d x_1 K^\Delta(z, x; x_1) \phi_0(x_1)$$

Witten diagrams

$$e.g. \langle \phi \phi \phi \rangle = \left(\frac{\delta}{\delta \phi_0} \right)^3 \mu z \Big|_{\phi=0} \sim \left(\frac{\delta}{\delta \phi_0} \right)^2 \langle \phi \rangle_{\phi_0=0}$$

EoM: $(\square - m^2) \phi(z, x) = b \phi^2(z, x)$

$$\phi(z, x) = \int d^d x_1 \underbrace{K^\Delta(z, x; x_1)}_{\text{BDH-to-BULK propagator}} \underbrace{\phi_0(x_1)}$$

Witten diagrams

$$e.g. \langle \mathcal{O} \mathcal{O} \mathcal{O} \rangle = \left(\frac{\delta}{\delta \phi_0} \right)^3 \mu z \Big|_{\phi_0=0} \sim \left(\frac{\delta}{\delta \phi_0} \right)^2 \langle \mathcal{O} \rangle_{\phi_0=0}$$

EoM: $(\square - m^2) \phi(z, x) = b \phi^2(z, x)$

$$\phi(z, x) = \int K^\Delta(z, x; x_1) \phi_0(x_1)$$

BDM-to-Bulk propagator.
 solve $(\square - m^2) K = 0$
 w/

Witten diagrams

$$g \langle \phi \phi \phi \rangle = \left(\frac{\delta}{\delta \phi_0} \right)^3 \mu z \Big|_{\phi=0} \sim \left(\frac{\delta}{\delta \phi_0} \right)^2 \langle \phi \rangle_{\phi_0=0}$$

EoM: $(\square - m^2) \phi(z, x) = b \phi^2(z, x)$

$$\phi(z, x) = \int d^d x_1 \underbrace{K^\Delta(z, x; x_1)}_{\text{BDM-to-Bulk propagator}} \underbrace{\phi_0(x_1)}_{\text{boundary value}}$$

BDM-to-Bulk propagator.
solves $(\square - m^2) K = 0$

$$\text{w/ } K \xrightarrow{z \rightarrow 0} \delta(x - x_1) z^\Delta$$

Witten diagrams

$$e.g. \langle \bar{\phi} \phi \phi \rangle = \left(\frac{\delta}{\delta \phi_0} \right)^3 \mu z \Big|_{\phi_0=0} \sim \left(\frac{\delta}{\delta \phi_0} \right)^2 \langle \phi \rangle_{\phi_0=0}$$

EoM: $(\square - m^2) \phi(z, x) = b \phi^2(z, x)$

$$\phi(z, x) = \int d^d x_1 \underbrace{K^\Delta(z, x; x_1)}_{\text{BDM-to-Bulk propagator}} \underbrace{\phi_0(x_1)}_{\text{source}}$$

BDM-to-Bulk propagator.
solves $(\square - m^2)K = 0$

$$= b \int d^d x' d^d z' \underbrace{\square^\Delta(z, x; z', x')}_{\text{bulk-to-bulk propagator}} \text{ w/ } K \xrightarrow{z \rightarrow 0} \delta(x-x_0) z^\Delta$$

Witten diagrams

$$e.g. \langle \phi \phi \phi \rangle = \left(\frac{\delta}{\delta \phi_0} \right)^3 \mu z \Big|_{\phi_0=0} \sim \left(\frac{\delta}{\delta \phi_0} \right)^2 \langle \phi \rangle_{\phi_0=0}$$

EoM: $(\square - m^2) \phi(z, x) = b \phi^2(z, x)$

$$\phi(z, x) = \int d^d x_1 \underbrace{K^\Delta(z, x; x_1)}_{\text{bulk propagator.}} \underbrace{\phi_0(x_1)}_{m^2 K=0}$$

$$+ b \int d^d x_1 d^d x_2 \underbrace{\Delta(z, x; x_1, x_2)}_{\xrightarrow{z \rightarrow 0} \delta(x-x_1) z^\Delta}$$

$$\int d^d x_1$$

Witten diagrams

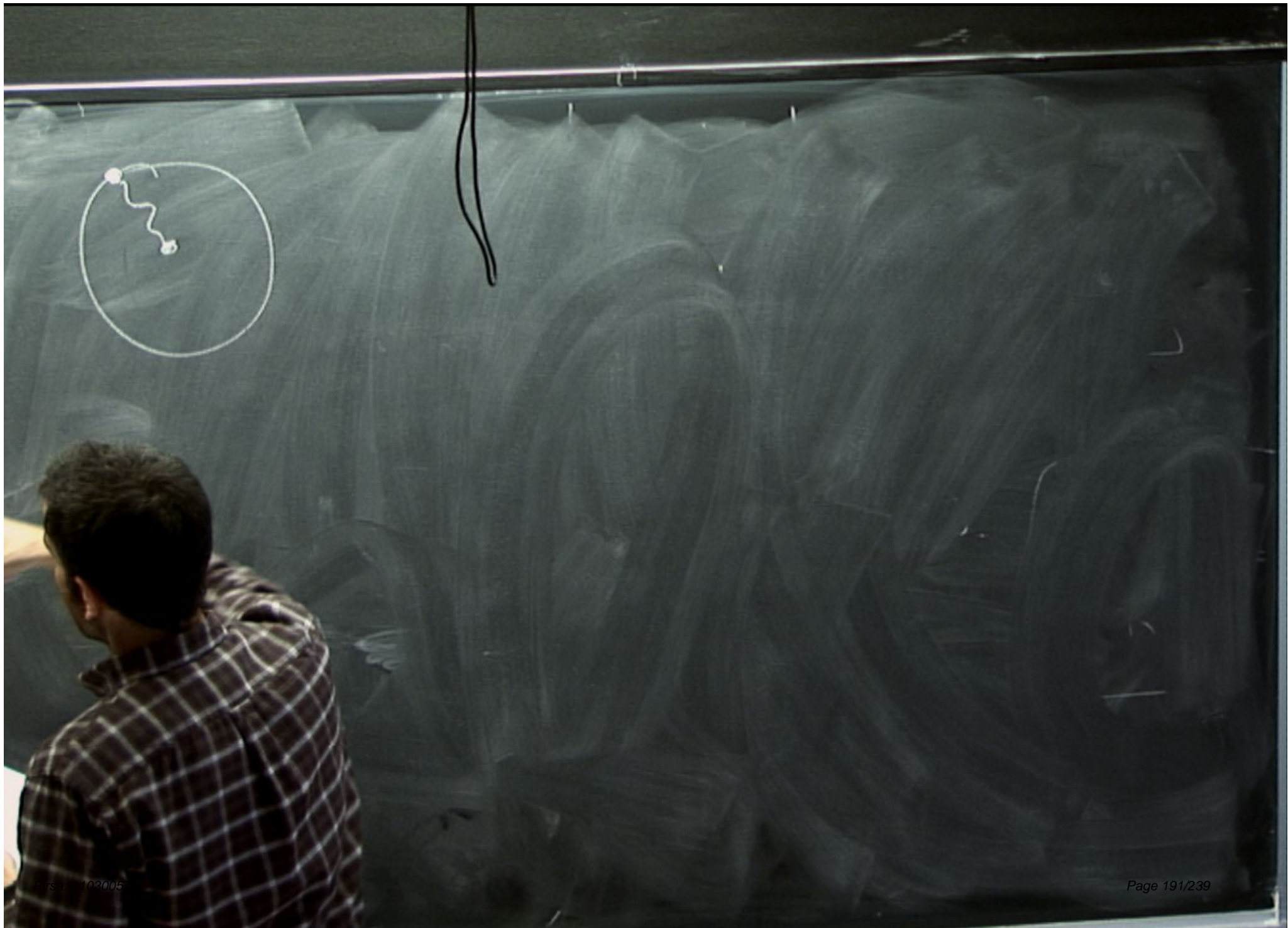
$$e.g. \langle \phi \phi \phi \rangle = \left(\frac{\delta}{\delta \phi_0} \right)^3 \mu z \Big|_{\phi_0=0} \sim \left(\frac{\delta}{\delta \phi_0} \right)^2 \langle \phi \rangle_{\phi_0=0}$$

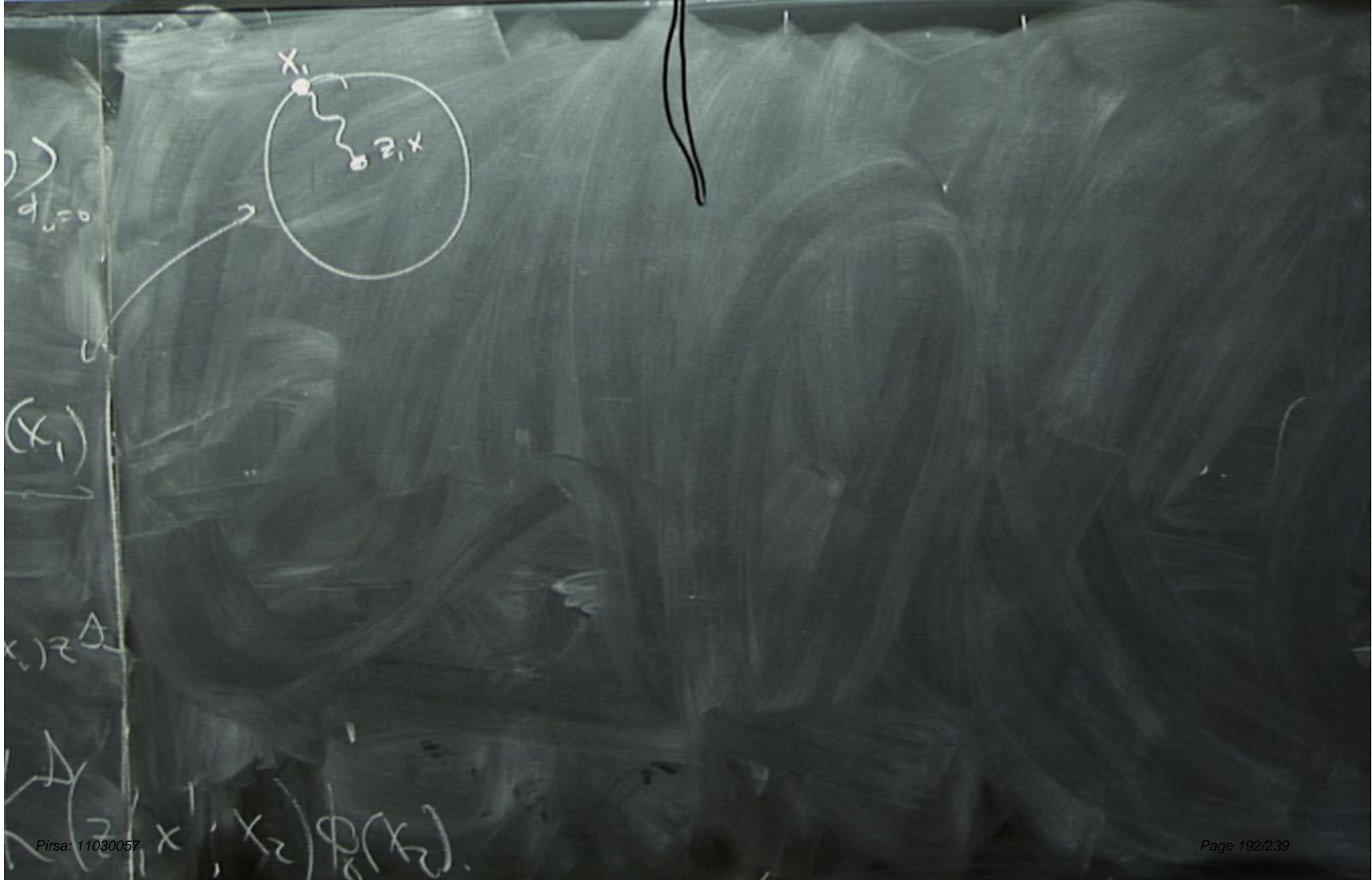
EoM: $(\square - m^2) \phi(z, x) = b \phi^2(z, x)$

$$\phi(z, x) = \int d^d x_1 \underbrace{K^\Delta(z, x; x_1)}_{\text{BDM-to-Bulk propagator.}} \underbrace{\phi_0(x_1)}_{\text{in } (\square - m^2)K=0}$$

$$+ b \int d^d x_1 d^d x_2 \underbrace{G^\Delta(z, x; z_1, x_1)}_{\text{w/ } K \xrightarrow{z \rightarrow 0} \delta(x-x_1) z^\Delta}$$

$$\int d^d x_1 \int d^d x_2 K^\Delta(z_1, x_1; x_2) \phi_0(x_2) K^\Delta(z, x; z_1, x_1)$$



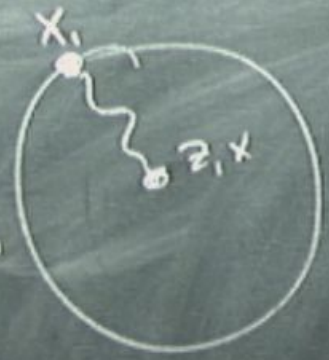


$d_u=0$

(x_1)

(x_2)

$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}$



Witten diagrams

$$e.g. \langle \phi \phi \phi \rangle = \left(\frac{\delta}{\delta \phi_0} \right)^3 \mu z \Big|_{\phi=0} \sim \left(\frac{\delta}{\delta \phi_0} \right)^2 \langle \phi \rangle_{\phi=0}$$

EoM: $(\square - m^2) \phi(z, x) = b \phi^2(z, x)$

$$\phi(z, x) = \int d^d x_1 K^\Delta(z, x; x_1)$$

BDH to BULK theory
 with $(\square - m^2) K = 0$

$$+ b \int d^d x_1 d^d x_2 \phi(z, x; x_1, x_2) + \int d^d x_1 \int d^d x_2 K^\Delta(z, x; x_1, x_2) \phi_0$$

Witten diagrams

$$g. \langle \phi \phi \phi \rangle = \left(\frac{\delta}{\delta \phi_0} \right)^3 \mu z \Big|_{\phi=0} \sim \left(\frac{\delta}{\delta \phi_0} \right)^2 \langle \phi \rangle_{\phi_0=0}$$

EoM: $(\square - m^2) \phi(z, x) = b \phi^2(z, x)$

$$\phi(z, x) = \int d^d x_1 \underbrace{K^\Delta(z, x; x_1)}_{\text{BDM-to-Bulk propagator}} \underbrace{\phi_0(x_1)}$$

BDM-to-Bulk propagator
solves $(\square - m^2)K = 0$

$$b \int d^d x_1 \int d^d x_2 \underbrace{G^\Delta(z, x; z', x')}_{\text{bulk-to-bulk propagator}} + \underbrace{K^\Delta(z, x; x_1)}_{\text{BDM-to-Bulk propagator}} \xrightarrow{z \rightarrow 0} \delta(x - x_1) z^\Delta$$

$$+ \int d^d x_1 \int d^d x_2 K^\Delta(z', x'; x_1) \phi_0(x_1) K^\Delta(z, x; x_2)$$

$$\mu z \Big|_{d_0=0} \sim \left(\frac{\delta}{\delta d_0} \right)^2 \langle \phi \rangle_{d_0=0}$$

$$- b \phi^2(z, x)$$

$$\langle \Delta(z, x; x_1) \phi_0(x_1) \rangle$$

TDN-to-Bulk propagator.
 solve $(\mathcal{D} - m^2) \phi = 0$

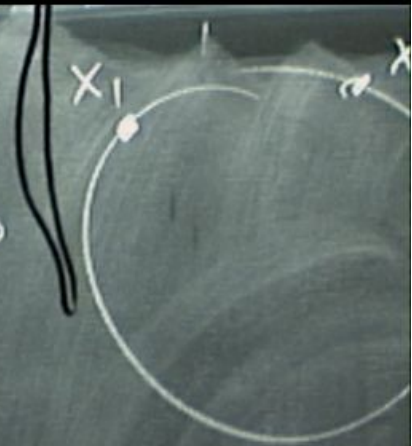
$$\phi(x_1) + \dots$$

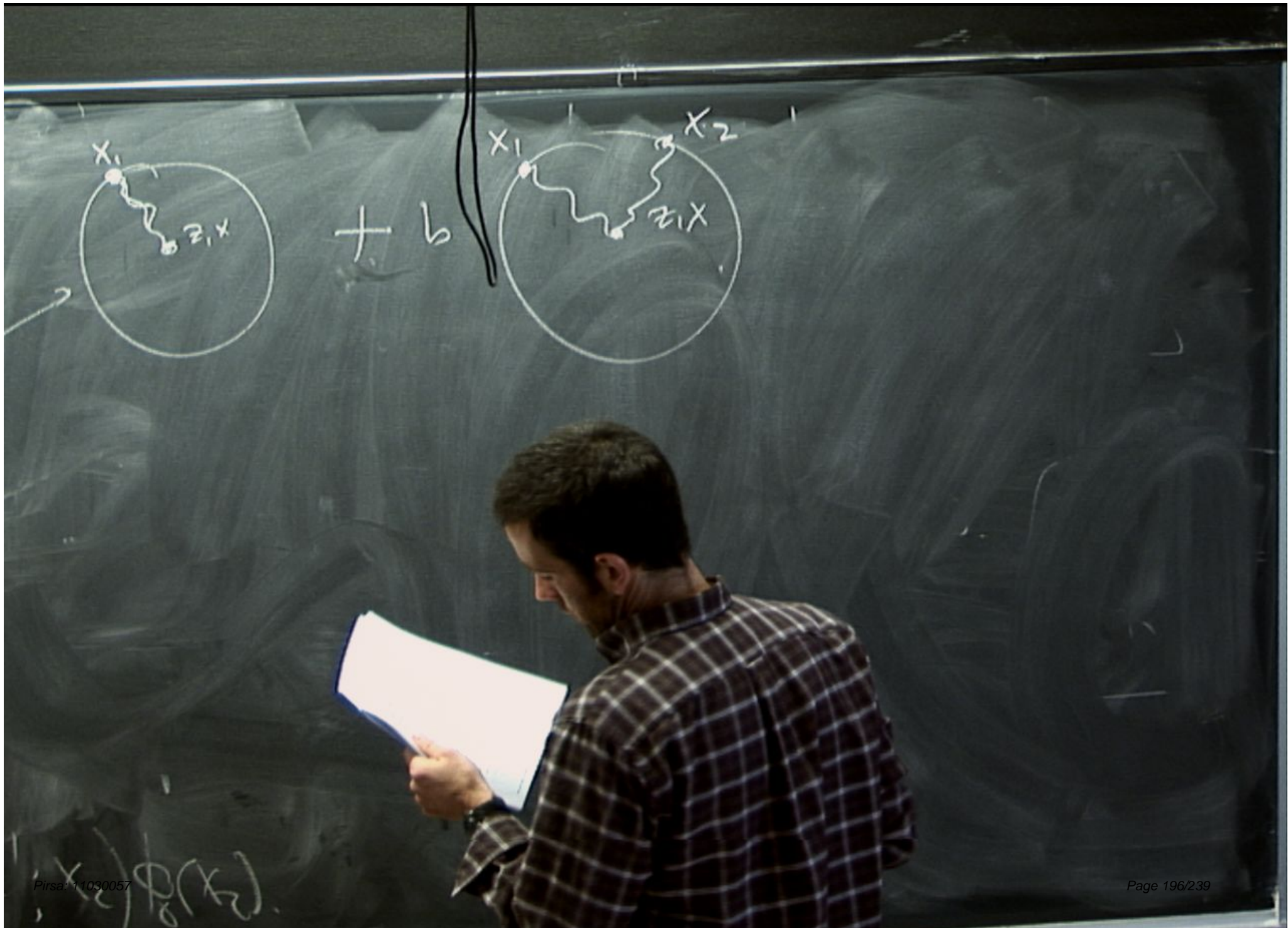
$$\Delta(z, x)$$

$$\phi_0(x_2)$$



+ b





$$\frac{\delta S}{\delta \phi_0} \ln z \Big|_{\phi_0=0} \sim \left(\frac{\delta}{\delta \phi_0} \right)^2 \langle \phi \rangle \Big|_{\phi_0=0}$$

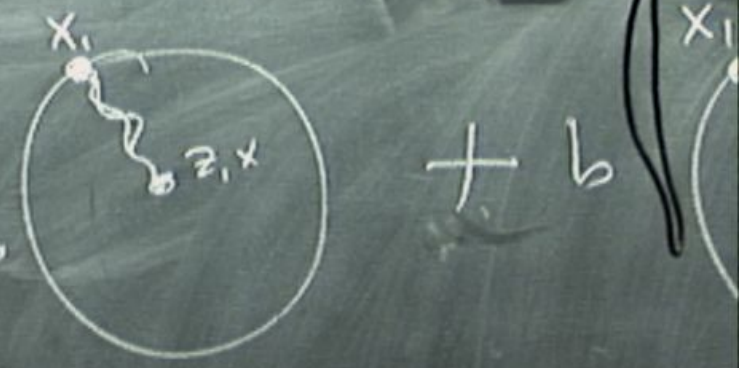
$$\phi(z, x) = b \phi^2(z, x)$$

$$\int d^d x_1 K^\Delta(z, x; x_1) \phi_0(x_1)$$

BRN-to-BULK propagator.
 sein $(\mathbb{D} - m^2)K = 0$

$$\int d^d x_1 K^\Delta(z, x; z_1, x_1) \xrightarrow{z \rightarrow 0} \delta(x - x_1) z^\Delta$$

$$\int d^d x_2 K^\Delta(z, x; x_1) \phi_0(x_1) K^\Delta(z_1, x_1; x_2) \phi_0(x_2)$$



$$\frac{\text{MS}}{\delta\phi_0} = \left(\frac{\delta}{\delta\phi_0}\right)^3 \ln Z \Big|_{\phi_0=0} \sim \left(\frac{\delta}{\delta\phi_0}\right)^2 \langle \phi \rangle \Big|_{\phi_0=0}$$

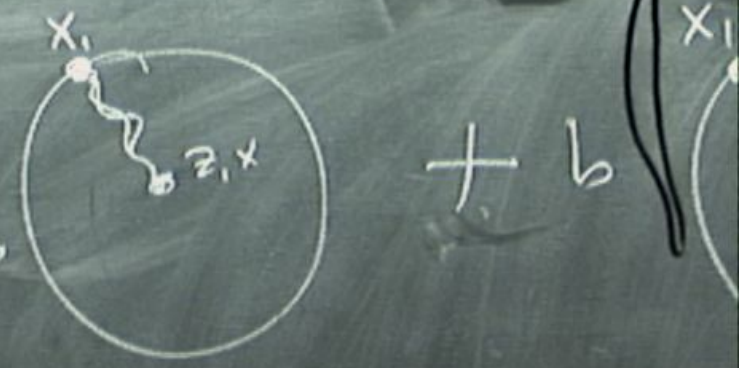
$$\langle \phi(z, x) \rangle = b \phi^2(z, x)$$

$$\int d^d x_1 K^\Delta(z, x; x_1) \phi_0(x_1)$$

IRDM-to-BULK propagator.
 sein $(\mathbb{D}-m^2)K=0$

$$\int d^d x_1 K^\Delta(z, x; z_1, x_1) \xrightarrow{z \rightarrow 0} \delta(x-x_1) z^\Delta$$

$$\int d^d x_2 K^\Delta(z, x; x_1) \phi_0(x_1) K^\Delta(z_1, x_1; x_2)$$



G \xrightarrow{K} TO BULK

$$\delta \langle K(\phi) \rangle_{\phi_0=0}$$

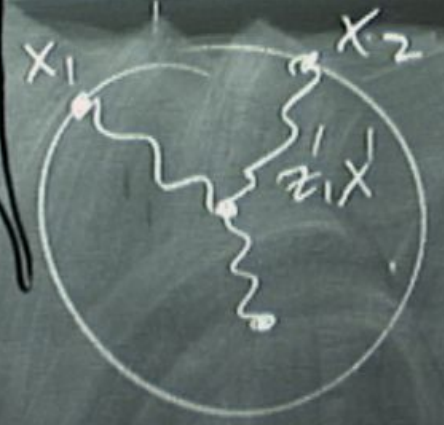
$$\phi_0(x_1)$$

$$\delta(x-x_0) \approx \Delta$$

$$\langle K(z_1, x; x_2) \phi_0(x_2) \rangle$$



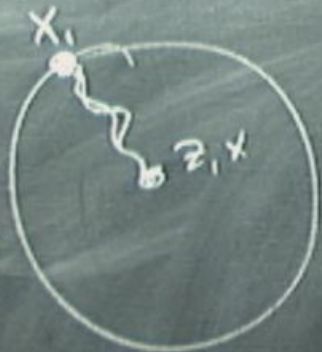
$$+ b$$



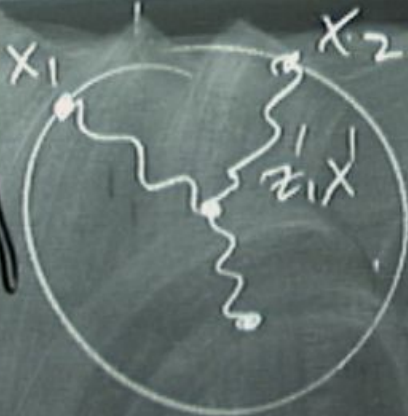
$$+ O(b^2)$$

G = BULK-TO-BULK PROPAGATOR:

$$(\mathbb{D} - m^2) G = \delta$$



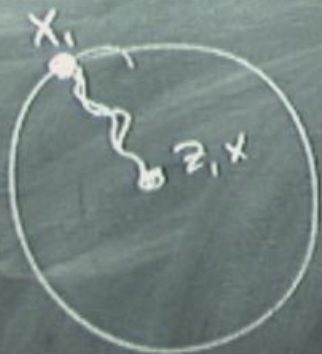
$+ b$



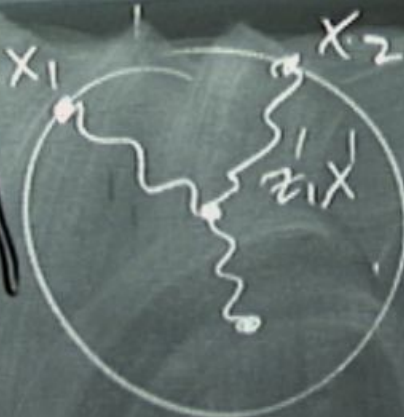
$+ O(b^2)$

G : BULK-TO-BULK PROPAGATOR :

$$\left\{ \begin{array}{l} (\mathbb{D} - m^2) \langle \phi(x_2; X_2) | \phi(x_1; X_1) \rangle = \sqrt{g} \int_{\Sigma}^{d+1} \delta(z - z', X - X') \\ \xrightarrow{z \rightarrow 0} z^{\Delta}(\sigma) \end{array} \right.$$



$+ b$



$+ O(b^2)$

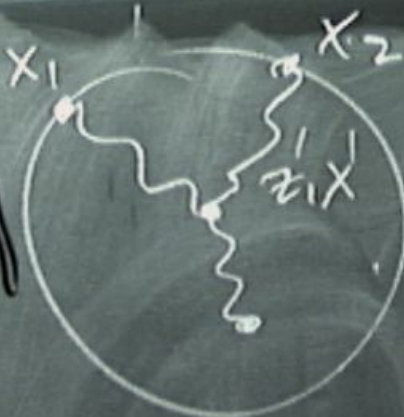
$G =$ BULK-BULK PROPAGATOR:

$$G(x_2, x_1 | z) = \sqrt{g} \int \Delta^{d+1}(z-z', X-X')$$

$$\Delta^{d+1}(z, X) = \frac{1}{(z^2 - X^2)^{\frac{d+1}{2}}}$$



$+ b$



$+ O(b^2)$

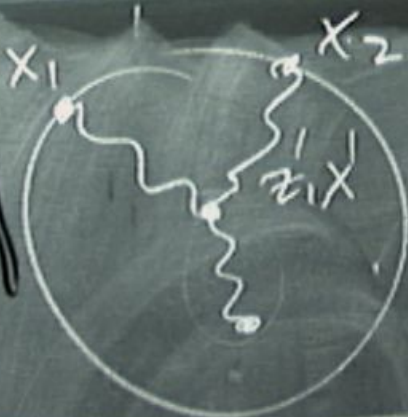
BULK-TO-BULK PROPAGATOR:

$$\left(\mathbb{D} - m^2 \right) \left(\frac{1}{|x_2 - x_1|} \right) = \sqrt{g} \int_{\mathcal{D}}^{d+1} \Delta(z - z', X - X')$$

$$\xrightarrow{z \rightarrow 0} z^{\Delta_-} X^0 + O(z^{\Delta_+})$$



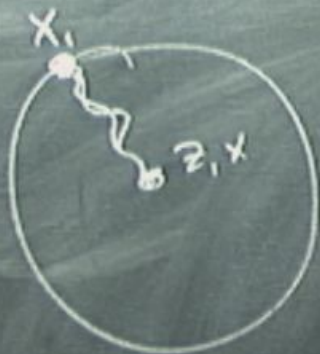
$+ b$



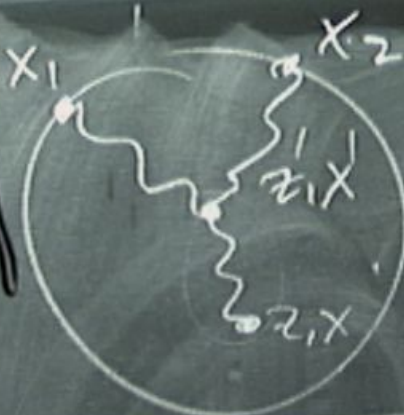
$+ O(b^2)$

G : BULK-TO-BULK PROPAGATOR :

$$\left\{ \begin{aligned} (\mathbb{D} - m^2) \langle \phi(x_2; x_1') \rangle &= \sqrt{g} \int_{\mathcal{D}} \Delta^{d+1}(z-z', X-X') \\ \lim_{z \rightarrow 0} \langle \phi(x_2; x_1') \rangle &\rightarrow z^{\Delta_-} \langle \phi(x_0) \rangle + \mathcal{O}(z^{\Delta_+}) \end{aligned} \right.$$



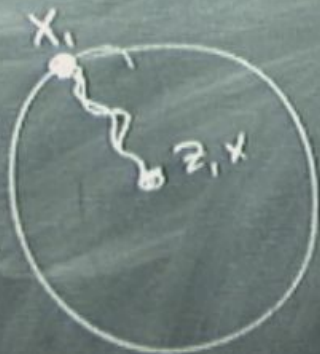
$+ b$



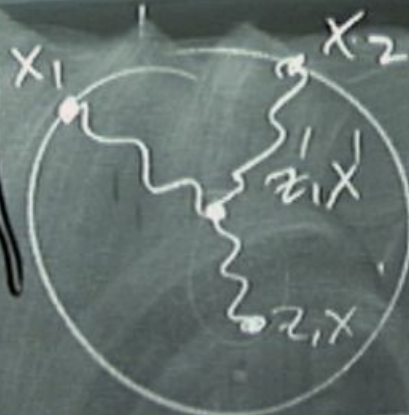
$+ O(b^2)$

$G =$ BULK-TO-BULK PROPAGATOR:

$$\left\{ \begin{array}{l} (\mathbb{D} - m^2) \langle \phi(x_2, X_2) | \phi(x_1, X_1) \rangle = \sqrt{g} \int_{\mathcal{D}} \Delta^{d+1}(z, X; z_1, X_1) \\ \xrightarrow{z \rightarrow 0} z^{\Delta_-} \phi(x_0) + O(z^{\Delta_+}) \end{array} \right.$$



+ b

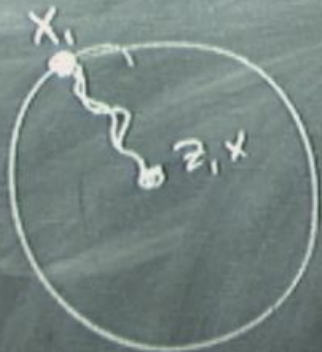


+

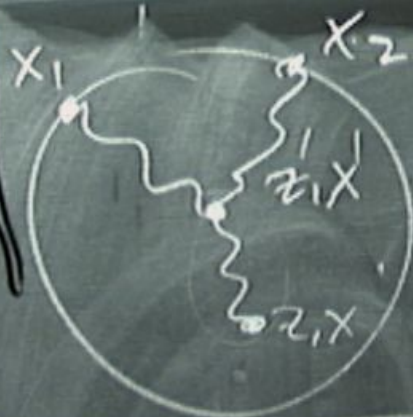


G = BULK-TO-BULK PROPAGATOR:

$$\left\{ \begin{aligned} (\mathbb{D} - m^2) \langle \phi(x_2, x_2') | \phi(x_1, x_1') \rangle &= \sqrt{g} \int_{\mathcal{D}} \delta(z - z') \\ \lim_{z \rightarrow 0} \langle \phi(x_2, x_2') | \phi(x_1, x_1') \rangle &\sim z^{\Delta} \langle \phi(x_1, x_1') | \phi(x_2, x_2') \rangle + \mathcal{O}(z^{\Delta+1}) \end{aligned} \right.$$



+ b

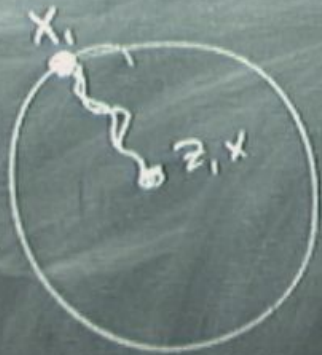


+



G = BULK-TO-BULK PROPAGATOR:

$$\left\{ \begin{aligned}
 (\mathbb{D} - m^2) \langle \phi(x_2, X_2) | \phi(x_1, X_1) \rangle &= \sqrt{g} \int_{\mathcal{D}} \Delta^{d+1}(z - z', X - X') \\
 \lim_{z \rightarrow 0} \langle \phi(z, X) | \phi(x_0, X_0) \rangle &+ \mathcal{O}(z^{\Delta_+})
 \end{aligned} \right.$$



+ b

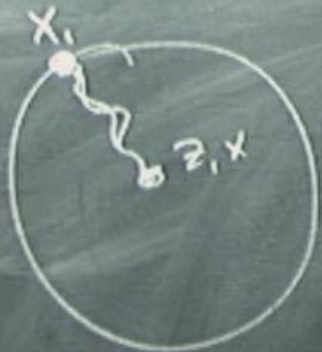


+ b

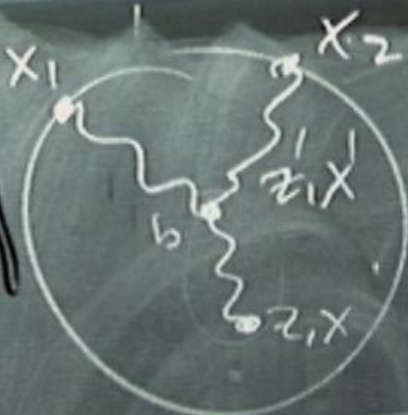


G = BULK-TO-BULK PROPAGATOR

$$\left\{ \begin{array}{l} (\mathbb{D} - m^2) \langle \phi(x_2, x_2') | \phi(x_1, x_1') \rangle = \sqrt{g} \delta(x_2 - x_2') \delta(x_1 - x_1') \\ \langle \phi(x_2, x_2') | \phi(x_1, x_1') \rangle \xrightarrow{z \rightarrow 0} z^{\Delta} \langle \phi(x_2, x_2') | \phi(x_1, x_1') \rangle \end{array} \right.$$



+ b



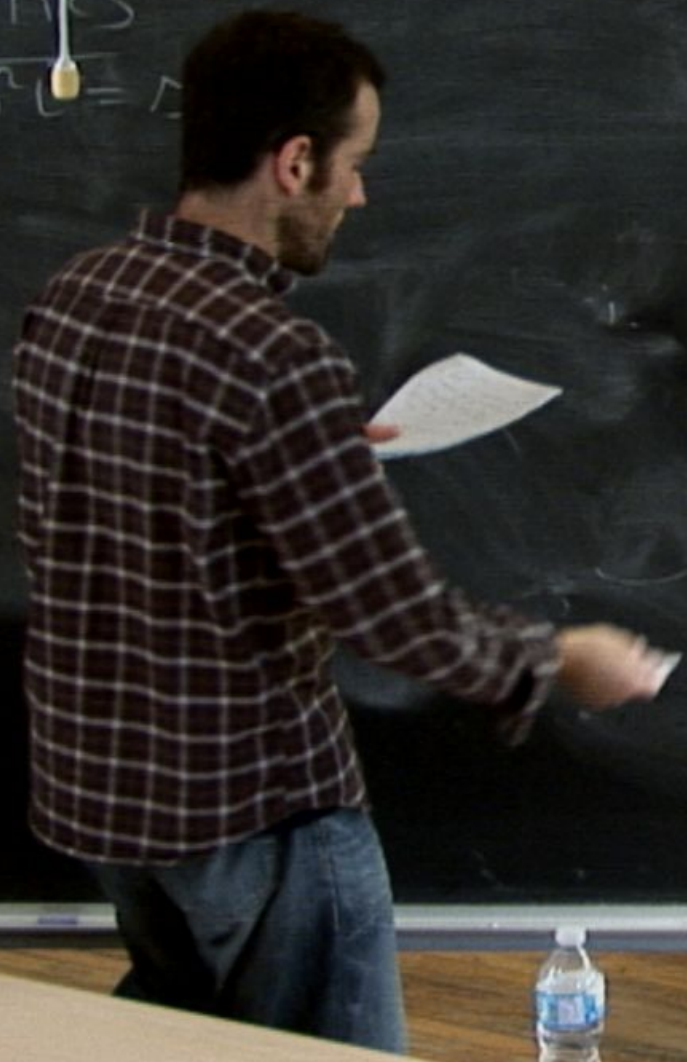
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G : BULK-TO-BULK PROPAGATOR :

$$\left\{ \begin{aligned} (\mathbb{D} - m^2) \langle \phi(x_2; X_2) | \phi(x_1; X_1) \rangle &= \sqrt{g} \int_{\mathcal{D}} \delta^{d+1}(z - z', X - X') \\ \lim_{z \rightarrow 0} \langle \phi(x_2; X_2) | \phi(x_1; X_1) \rangle &\sim z^{\Delta_-} X^0 + \mathcal{O}(z^{\Delta_+}) \end{aligned} \right.$$

Geometric optics
when $\Delta \gg 1$, $m^2 L = \Delta$



Geometric optics

when $\Delta \gg 1$, $m^2 L = \Delta(\Delta - d) \approx \Delta^2$

$(g = D)$

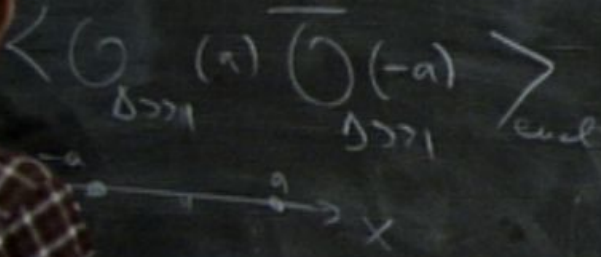
Geometric optics

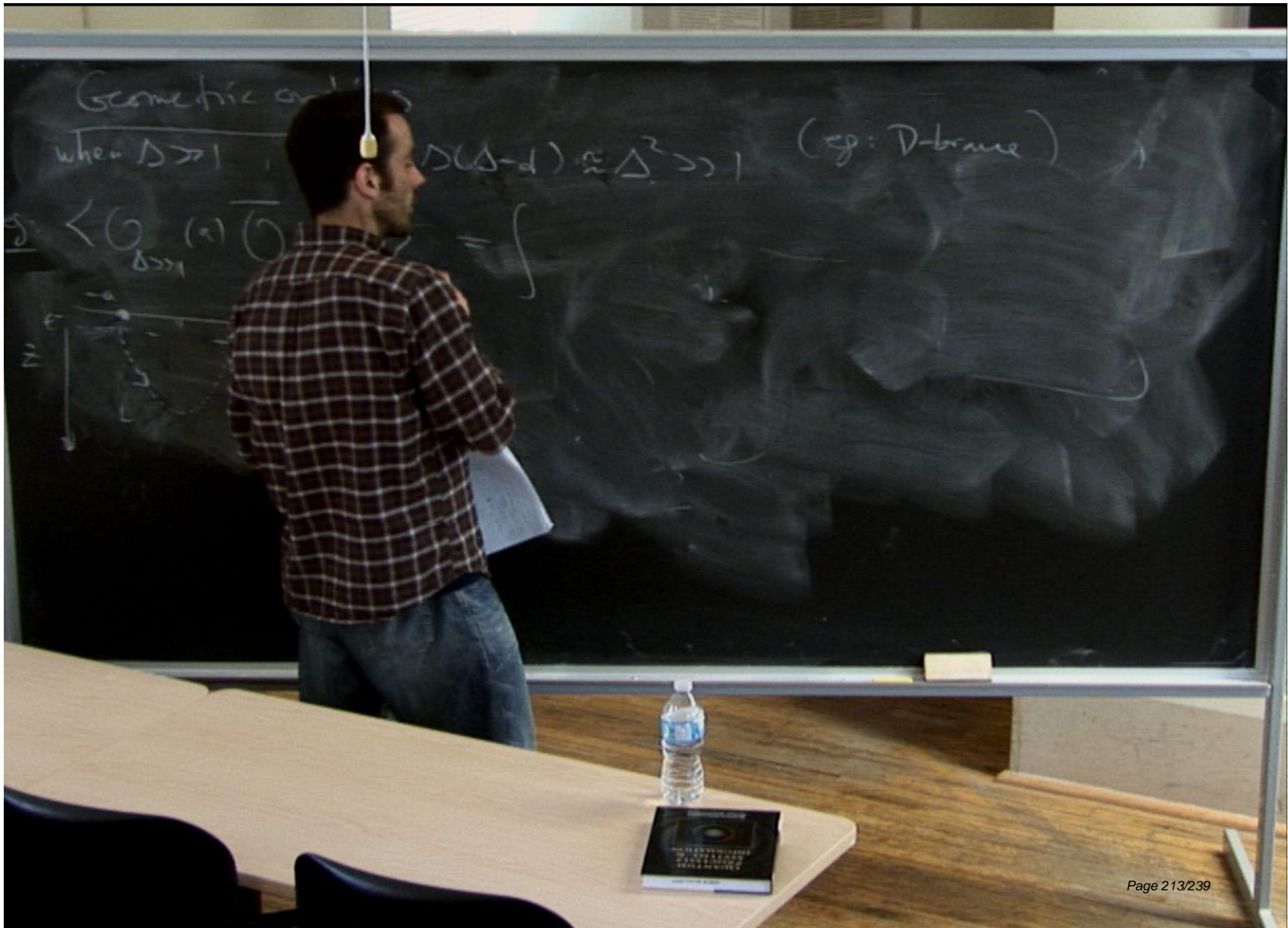
when $\Delta \gg 1$, $m^2 L = \Delta(\Delta - d) \approx \Delta^2 \gg 1$ (eg: D-brane)

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Geometric optics

for $\Delta \gg 1$, $m^2 L = \Delta(\Delta - d) \approx \Delta^2 \gg 1$ (eg: D-brane)

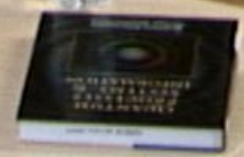
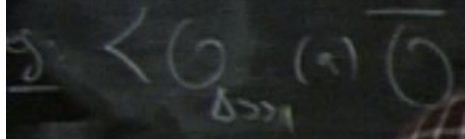




Geometric analysis

when $\Delta \gg 1$

$$\Delta(\Delta-d) \simeq \Delta^2 \gg 1 \quad (\text{eg: D-brane})$$



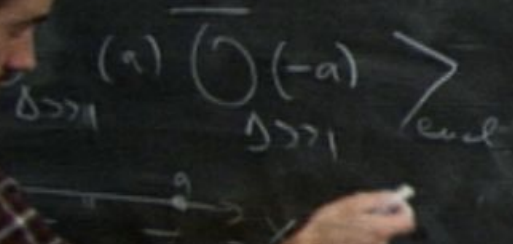
Geometric optics

when $\Delta \gg \lambda^2 L = \Delta(\Delta - d) \approx \Delta^2 \gg \lambda$ (eg: D-brane)

$\langle \psi | \hat{G} | \psi \rangle = \int [DX] X(\beta) = (-1, z = \epsilon)$

Geometric optics

$$\Rightarrow 1, m^2 L = \Delta(\Delta-d) \approx \Delta^2 \gg 1 \quad (\text{eg: D-brane})$$



$$= \int [DX]$$

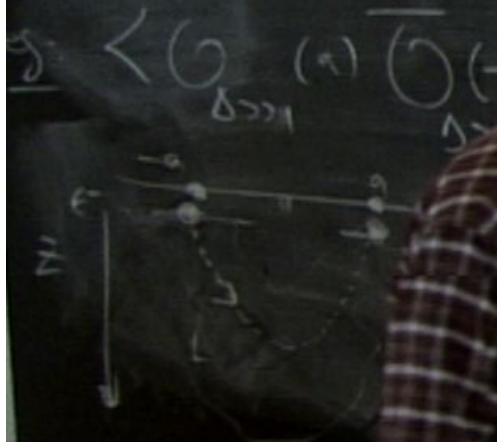
$$X(\beta) = (-a, z = \epsilon)$$

$$X(\xi) = (+a, z = \epsilon)$$

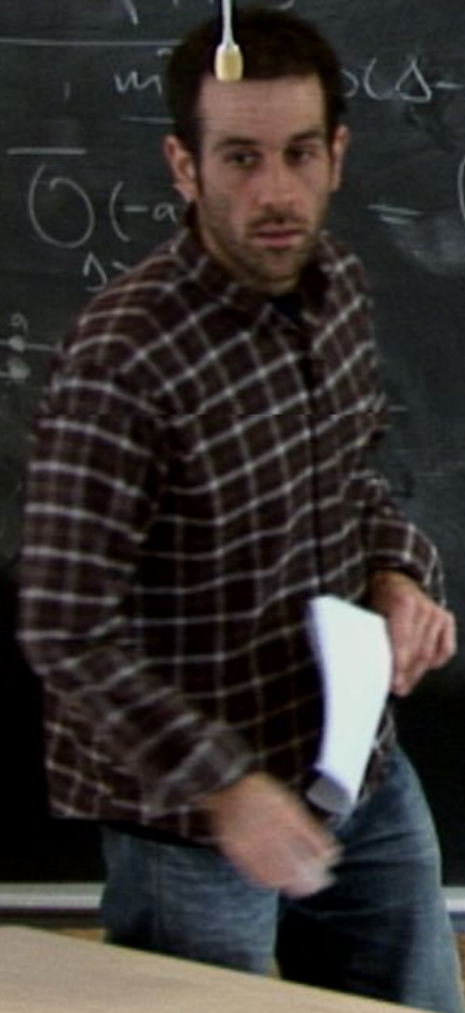
Geometric optics

when $\Delta \gg 1$, $m \approx (\Delta - d) \approx \Delta^2 \gg 1$

$$S[x] = m \int ds$$

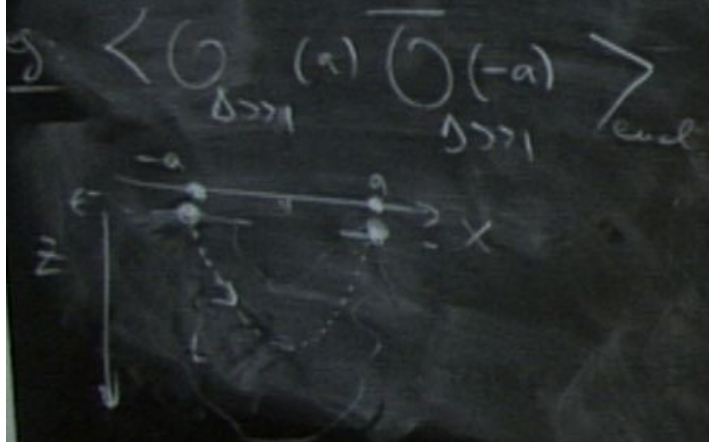


$$\int [DX] e^{-S[X]}$$
$$X(\beta) = (-q, z = \epsilon)$$
$$X(\epsilon) = (-q, z = \epsilon)$$



Geometric optics

when $\Delta \gg 1$, $(m^2 L = \Delta(\Delta - d) \approx \Delta^2 \gg 1)$



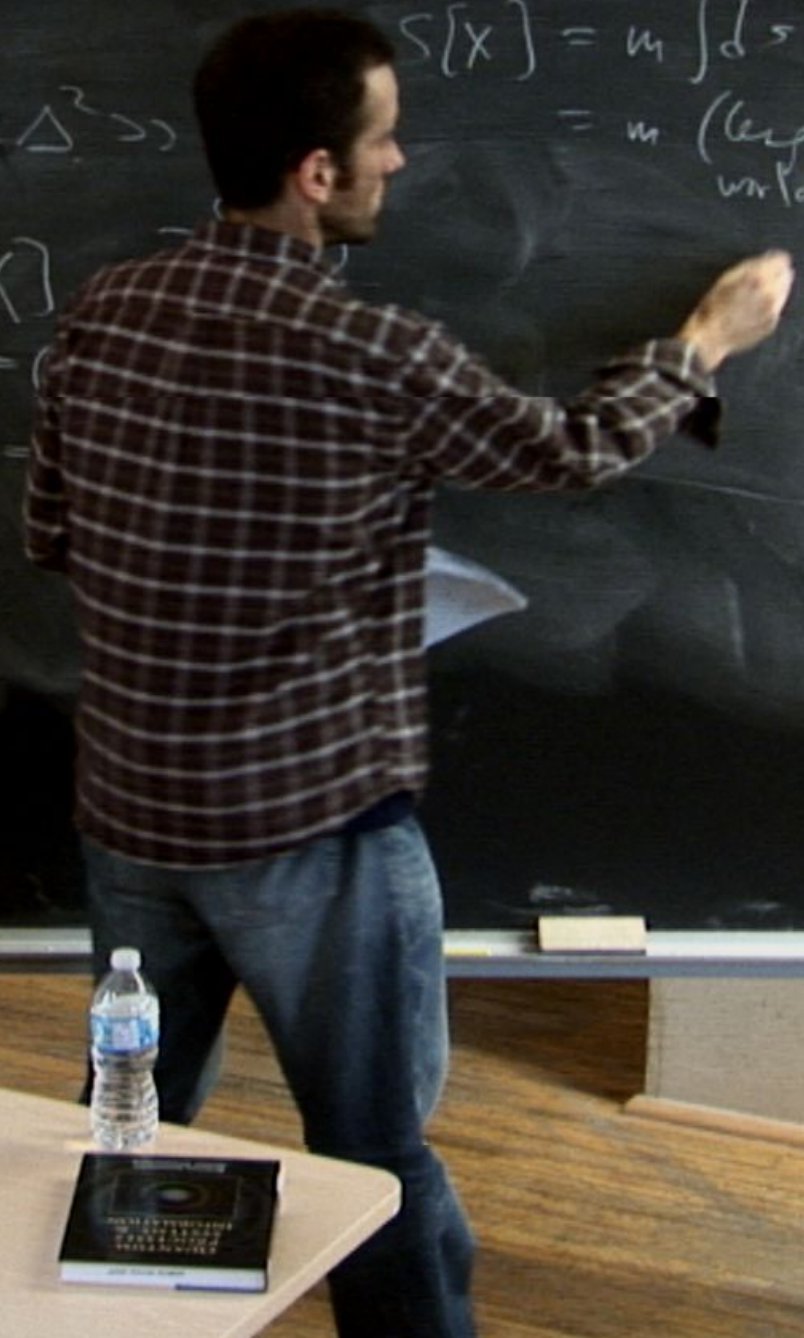
$$\int [DX]$$

$$X(\xi) = \dots$$

$$X(\eta) = \dots$$

$$S[X] = m \int ds$$

$$= m (\text{length of worldline})$$

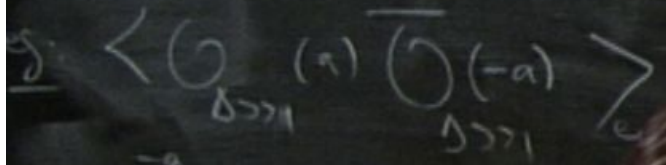


Geometric optics

when $\Delta \gg 1$, $(m^2 L = \Delta C = \Delta^2 \gg 1)$

$$S[x] = m \int ds$$

$$= m (\text{length of worldline in bulk})$$



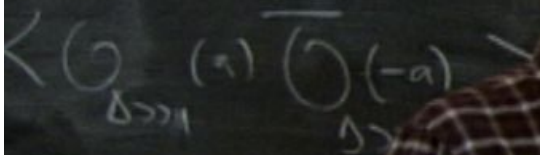
$$X(\tau) = (-\tau z = \epsilon)$$

$$X(\tau) = (-\tau z = \epsilon)$$



Geometric optics

when $\Delta \gg 1$, $(m^2 L = d) \approx \Delta^2 \gg 1$



$$[DX] e^{-S[X]}$$
$$(-r_1^2 = r_2^2)$$

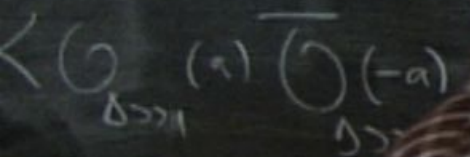
$$S[X] = m \int ds = \underline{mL} \int ds$$

= m (length of worldline in bulk)



Geometric optics

when $\Delta \gg 1$, $(m^2 L^2 (\Delta - d) \approx \Delta^2 \gg 1)$



$$\int [DX] e^{-S[X]}$$

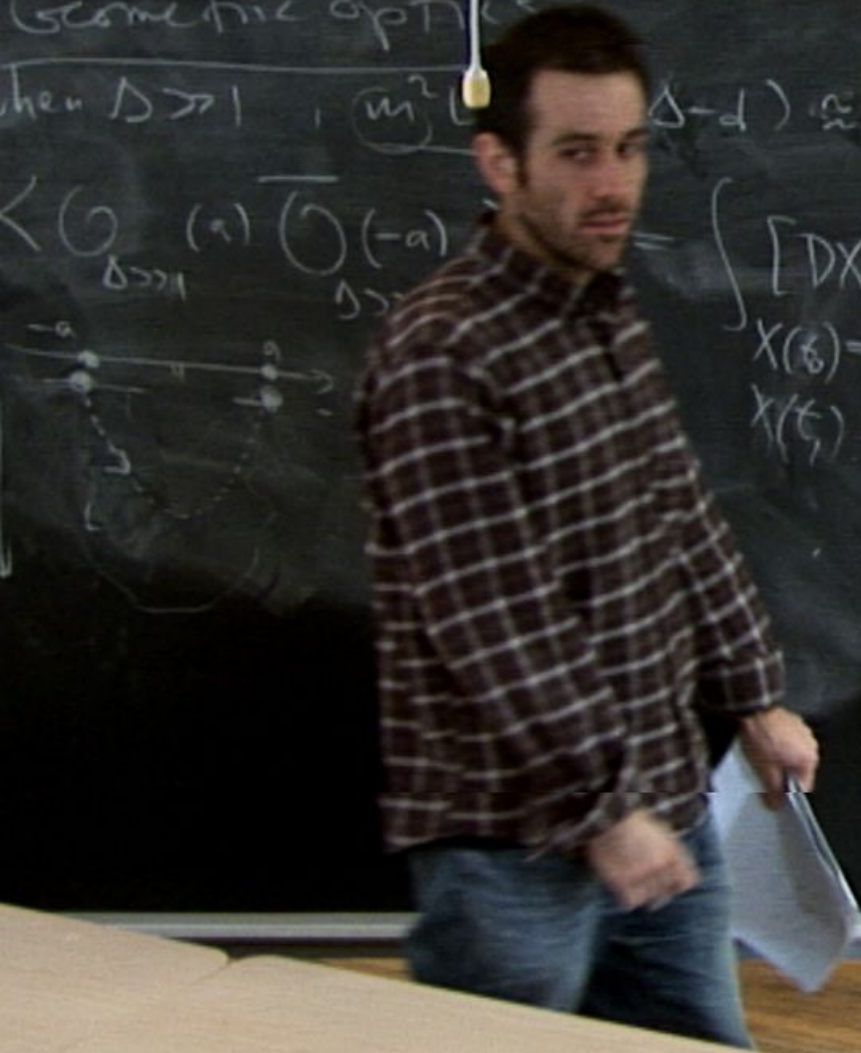
$$X(\beta) = (-r, z = \epsilon)$$

$$X(\alpha) = (-l, z = \epsilon)$$

$$S[X] = m \int ds = \underline{mL} \int ds$$

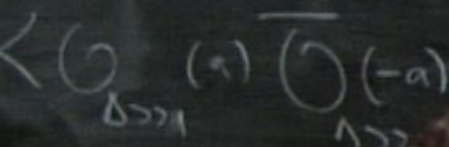
$$= m \text{ (length of worldline in bulk)}$$

$$\approx e^{-m S[X]}$$



Geometric optics

when $\Delta \gg 1$, $(m^2 L)^2 \approx \Delta^2 \gg 1$



$$\int [DX] e^{-S[X]}$$

$$X(\delta) = (-a, z=\epsilon)$$

$$X(\epsilon) = (-a+\delta, z=\epsilon)$$

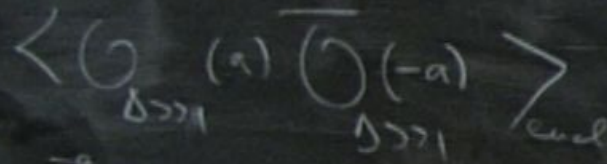
$$S[X] = m \int ds = \underline{mL} \int ds$$

$$= m (\text{length of worldline in bulk})$$

$$\frac{m \gg \frac{1}{L}}{z} e^{-m S[X]}$$

Geometric optics

when $\Delta \gg 1$, $m^2 L = \Delta(\Delta - d) \approx$



$$= \int_{x_0}^{x_1} [Dy]_{\text{eval}}$$

$$S[x] = m \int ds = \underline{mL} \int ds$$

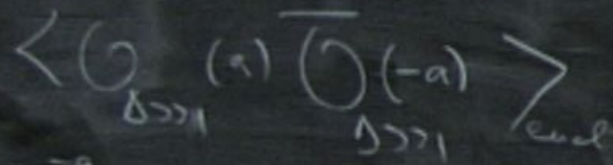
= m (length of worldline in bulk)

$$m \gg \frac{1}{L} \rightarrow m S[x]$$



Geometric optics

when $\Delta \gg 1$, $(m^2 L = \Delta(\Delta - d) \approx \Delta^2 \gg 1)$



$$\langle \dots \rangle_{\text{encl}} = \int [DX] e^{-S[X]}$$

$$X(\xi) = (-r, z = \xi)$$

$$X(\xi) = (+a, z = \xi)$$

$$S[X] = m \int ds = mL \int ds$$

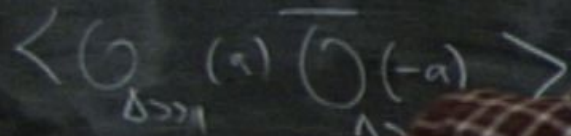
(length of worldline in bulk)

$$S[X]$$



Geometric optics

when $\Delta \gg 1$, $(m^2 L = \Delta^2 \gg 1)$



$$\int [DX] e^{-S[X]}$$

$$X(\beta) = (-r, z = \epsilon)$$

$$X(\xi) = (+a, z = \epsilon)$$

$$S[X] = m \int ds = mL \int ds$$

$$= m (\text{length of worldline in bulk})$$

$$\sim e^{-m S[X]}$$

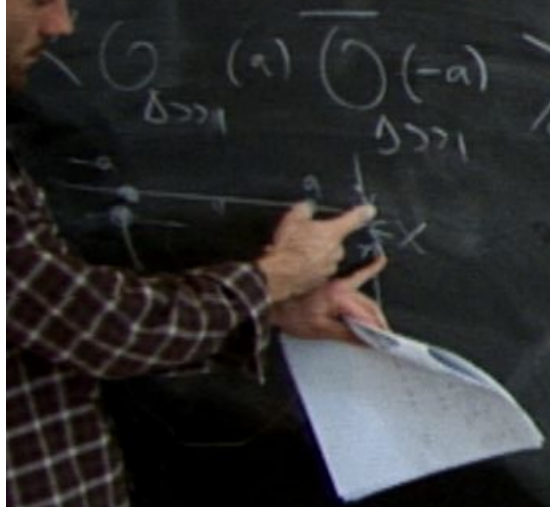
$$S[X] \sim 2mL \ln(a/\epsilon)$$



Geometric optics

$n \gg 1, \quad m^2 L = n(n-1) \approx n^2 \gg 1$

$S[x] = m \int ds = \underline{mL} \int ds$
 $= m$ (length of worldline in bulk)



$\int [DX] e^{-S[x]}$
 $X(\tau) = (-\tau, z = \epsilon)$
 $X(\xi) = (+a, z = \epsilon)$

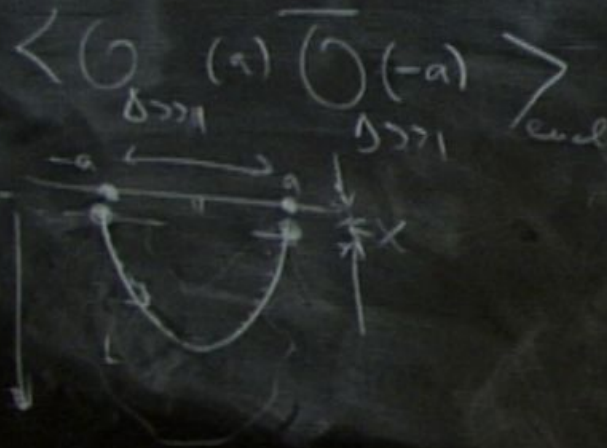
$m \gg \frac{1}{L} \Rightarrow -m S[x]$
 $\tau \ll \epsilon$

$S[x] \sim 2mL \ln(a/\epsilon)$



Geometric optics

when $\Delta \gg 1$, $(m^2 L = \Delta(\Delta - d) \approx$



$$S[x] = \int_{-a}^a \sqrt{\Delta^2 - z^2} dz$$

$$S[x] = m \int ds = \underline{mL} \int ds$$

= m (length of worldline in bulk)

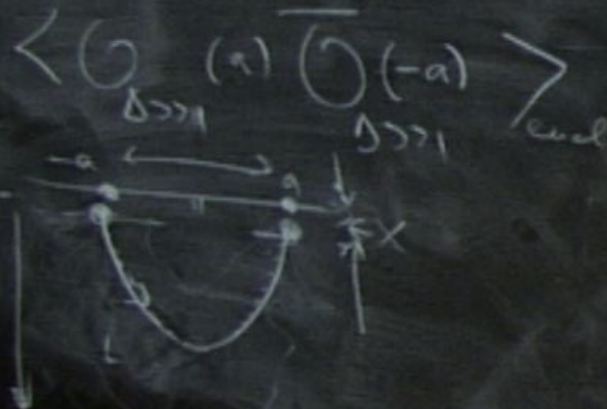
$$m \gg \frac{1}{L} \rightarrow m S[x]$$

$$S[x] \sim 2mL \ln(a/\epsilon)$$



Geometric optics

when $\Delta \gg 1$, $(m^2 L = \Delta(\Delta - d) \approx \Delta^2 \gg 1)$



$$\langle \dots \rangle_{\text{encl}} = \int [DX] e^{-S[X]}$$

$$X(\beta) = (-\tau, z)$$

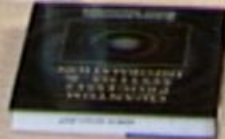
$$X(\tau) = (+a, \bar{z})$$

$$\langle S[X] \rangle = m \int ds = \frac{mL}{\epsilon} \int ds$$

= m (length of worldline in bulk)

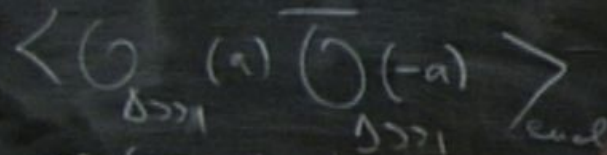
$$-m S[X] \sim \left(\frac{g}{\epsilon} \right)^{-m}$$

$$S[X] \sim 2mL \ln(a/\epsilon)$$



Geometric optics

when $\Delta \gg 1$, $(m^2 L = \Delta L) \Delta^2 \gg 1$



$$S[x] = m \int ds = mL \int ds$$

$$= m \text{ (length of worldline in bulk)}$$

$$e^{-S[x]} \xrightarrow{m \gg \frac{1}{L}} -m S[x] \sim \left(\frac{a}{\epsilon}\right)^{-2\Delta}$$

$$z \sim \epsilon$$

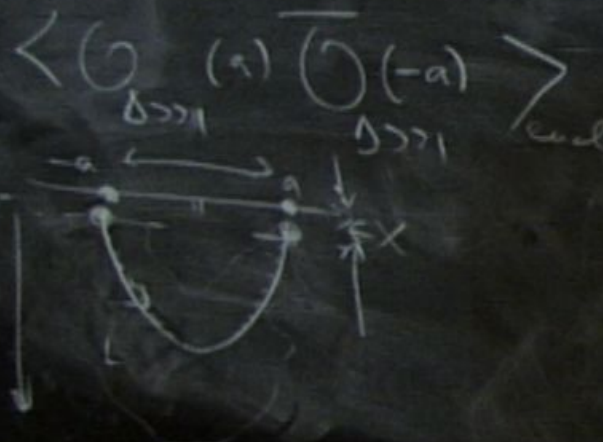
$$S[x] \sim 2mL \ln(a/\epsilon)$$

$$(x) = (-r, z = \epsilon)$$

$$(x) = (-a, z = \epsilon)$$

Geometric optics

when $\Delta \gg 1$, $m^2 L = \Delta(\Delta - d)$

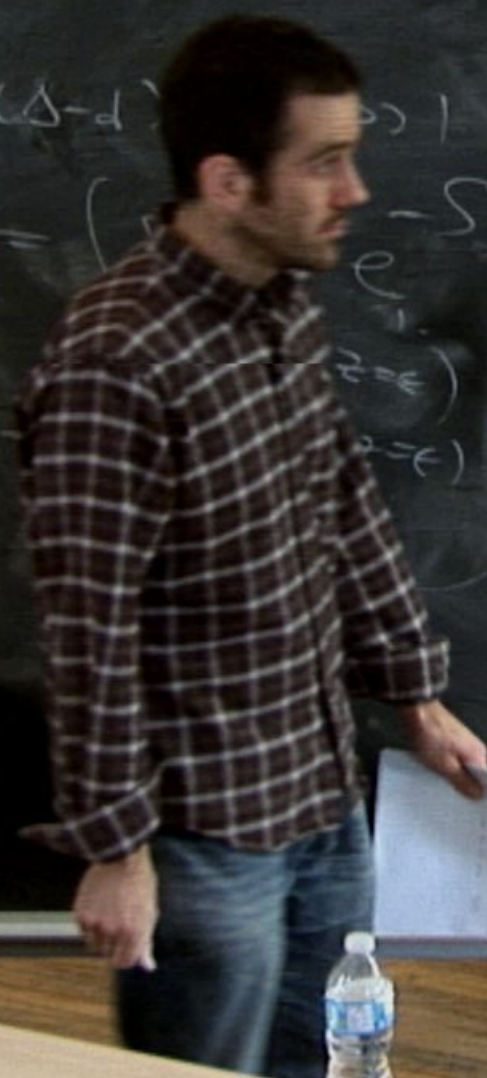


$$S[x] = m \int ds = \underline{mL} \int ds$$

= m (length of worldline in bulk)

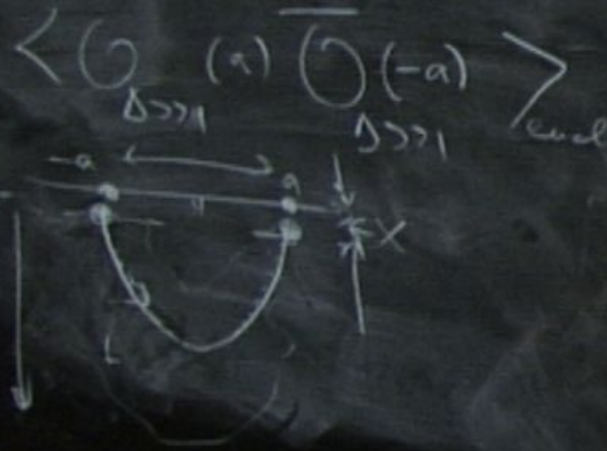
$$e^{-S[x]} \sim e^{-mS[x]} \sim \left(\frac{a}{c}\right)^{-2\Delta}$$

$$S[x] \sim 2mL \ln\left(\frac{a}{c}\right)$$



Geometric optics

when $\Delta \gg 1$, $(m^2 L = \Delta(\Delta - d) \approx \Delta^2 \gg 1)$



$$= \int [DX] e^{-S[X]}$$

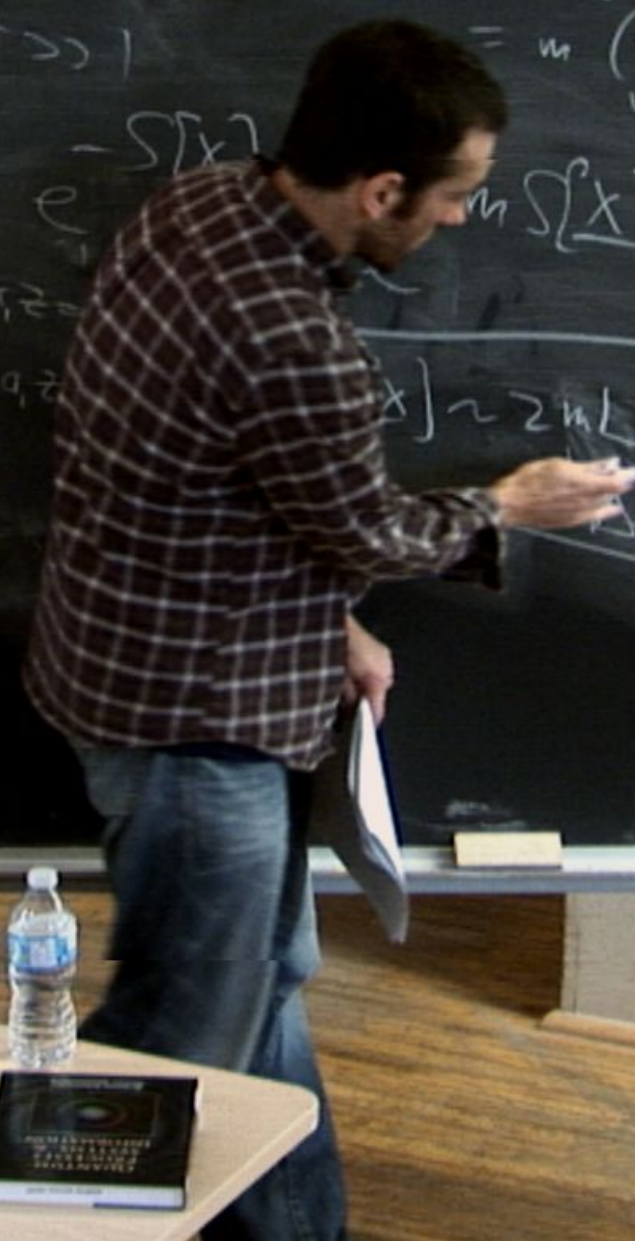
$$X(\xi) = (-\tau, z)$$

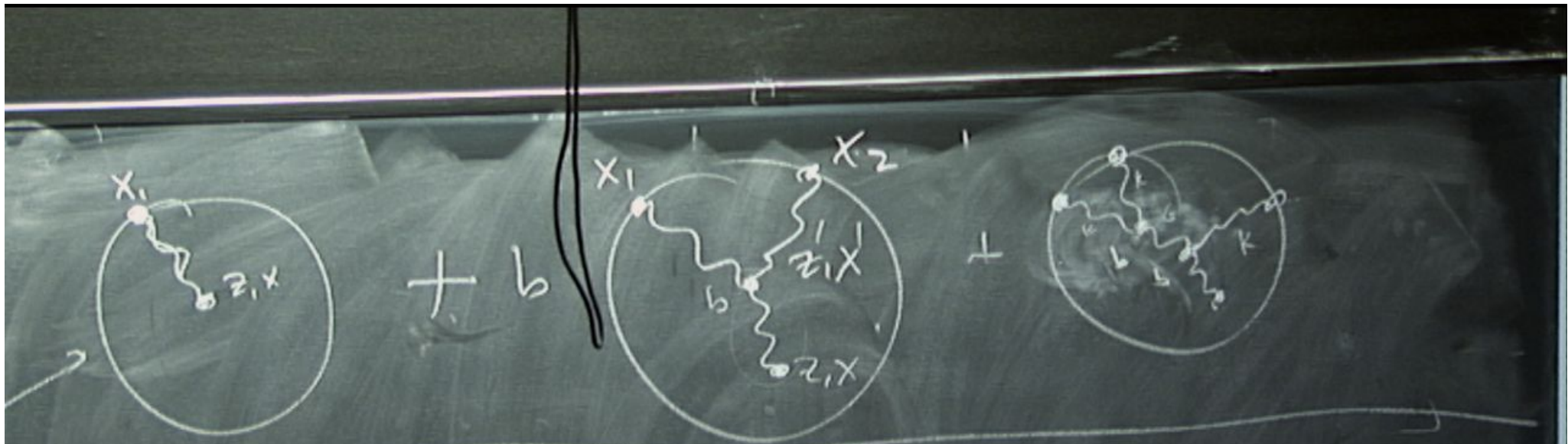
$$X(\xi) = (-t, z)$$

$$S[X] = m \int ds = mL \int ds$$

$$= m (\text{length of worldline in bulk})$$

$$m S[X] \sim \left(\frac{2a}{\epsilon}\right)^2 \sim 2mL \ln(a/\epsilon)$$

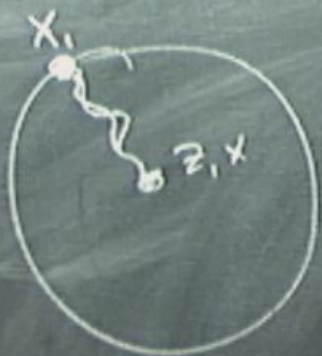




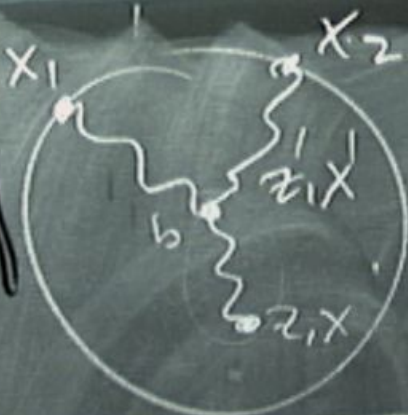
G : BULK-TO-BULK PROPAGATOR :

$$\left\{ \begin{array}{l} (\mathbb{D} - m^2) \langle \phi(x_2, X_2) | \phi(x_1, X_1) \rangle = \delta(x_2 - x_1, X_2 - X_1) \\ \langle \phi \rangle \xrightarrow{z \rightarrow 0} z^{\Delta_-} \quad \langle \phi \rangle \xrightarrow{z \rightarrow \infty} z^{\Delta_+} \end{array} \right.$$

$$S[\phi] =$$



+ b



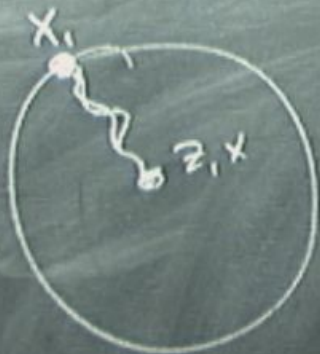
+ b



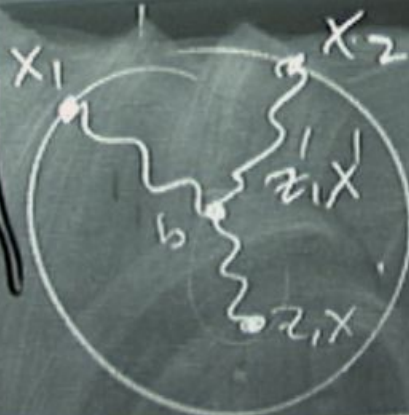
G = BULK-TO-BULK PROPAGATOR:

$$\left\{ \begin{aligned} (\mathbb{D} - m^2) \langle \phi(x_2) \phi(x_1) \rangle &= \sqrt{g} \Delta^{d+1} (z - z') \\ \langle \phi \rangle &\xrightarrow{z \rightarrow 0} z^{\Delta} \times 0 + \mathcal{O}(z^{\Delta+1}) \end{aligned} \right.$$

$$S[\phi] = -k \int (\partial\phi)^2$$



+ b



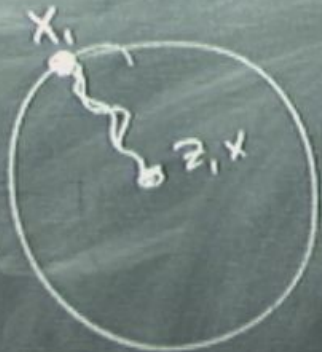
+ b



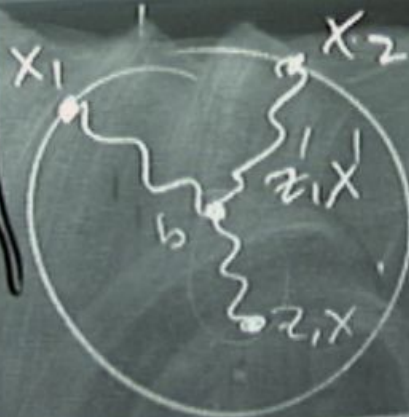
G = BULK-TO-BULK PROPAGATOR:

$$\left\{ \begin{aligned} (\mathbb{D} - m^2) \langle \phi(x_2, x_2') \rangle &= \sqrt{g} \delta(z - z', X - X') \\ \langle \phi \rangle &\xrightarrow{z \rightarrow 0} z^{\Delta_-} \times 0 + \mathcal{O}(z^{\Delta_+}) \end{aligned} \right.$$

$$S[\phi] = -k \int (\partial\phi)^2 + m^2 \phi^2 + \dots$$



+ b



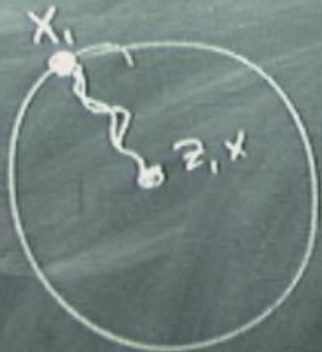
+



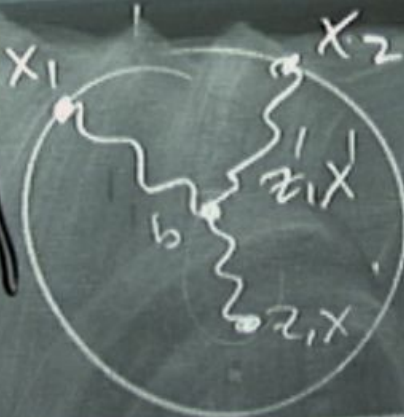
G = BULK-TO-BULK PROPAGATOR:

$$\left\{ \begin{aligned} (\mathbb{D} - m^2) \langle \phi(x_2; x_1) \rangle &= \sqrt{g} \delta^{d+1}(z, x) \\ \lim_{z \rightarrow 0} z^{\Delta} \langle \phi(x_2; x_1) \rangle &= \mathcal{O}(z^{-\Delta}) \end{aligned} \right.$$

$$S[\phi] = -k \int (\partial\phi)^2 + m^2 \phi^2$$



+ b



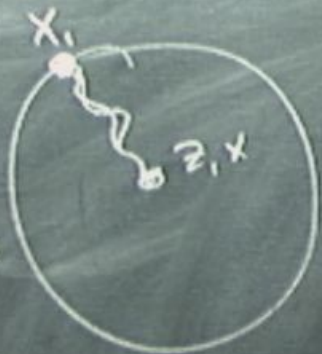
+ b



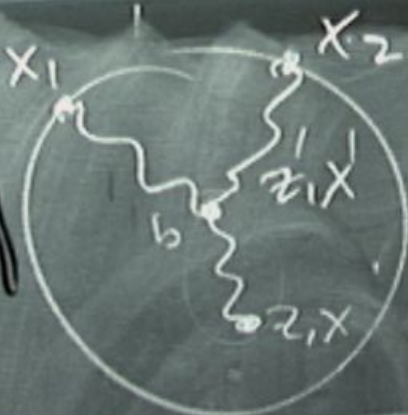
G : BULK-TO-BULK PROPAGATOR :

$$\left\{ \begin{aligned} (\mathbb{D} - m^2) \langle \phi(x_2, X_2) | \phi(x_1, X_1) \rangle &= \sqrt{g} \Delta^{d+1}(z-z', X-X') \\ \langle \phi \rangle &\xrightarrow{z \rightarrow 0} z^{\Delta_-} \times 0 + \mathcal{O}(z^{\Delta_+}) \end{aligned} \right.$$

$$S[\phi] = -R \int (\partial\phi)^2 + m^2 \phi^2 + \frac{1}{3} b \phi^3$$



+ b



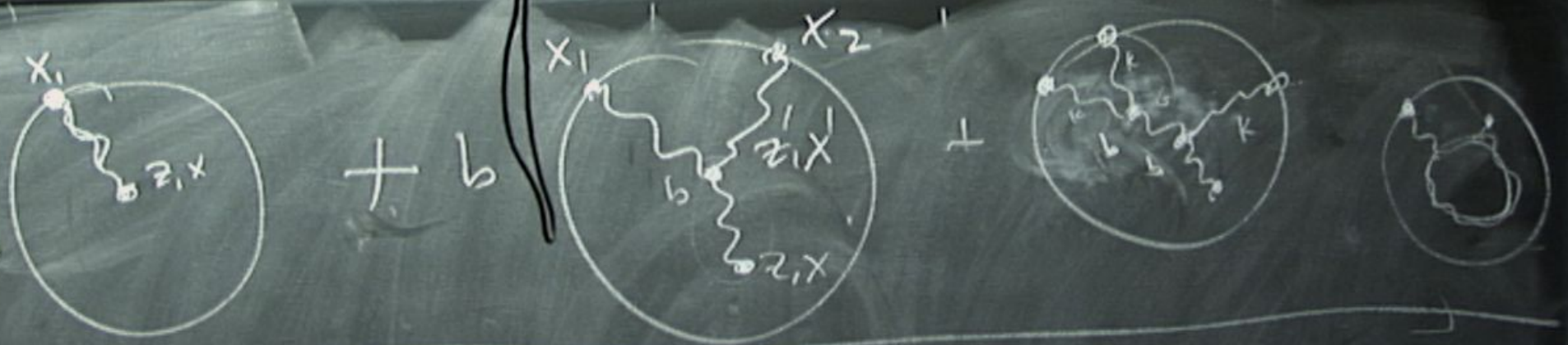
+ b



G : BULK-TO-BULK PROPAGATOR :

$$\left\{ \begin{array}{l} (\mathbb{D} - m^2) \langle \phi(x_2, X_2 | x_1, X_1) \rangle = \sqrt{g} \Delta^{d+1} (z - z', X - X') \\ \lim_{z \rightarrow 0} \langle \phi \rangle \rightarrow z^{\Delta_-} \times 0 + \mathcal{O}(z^{\Delta_+}) \end{array} \right.$$

$$S[\phi] = -R \int (\partial\phi)^2 + m^2 \phi^2 + \frac{1}{3} b \phi^3$$

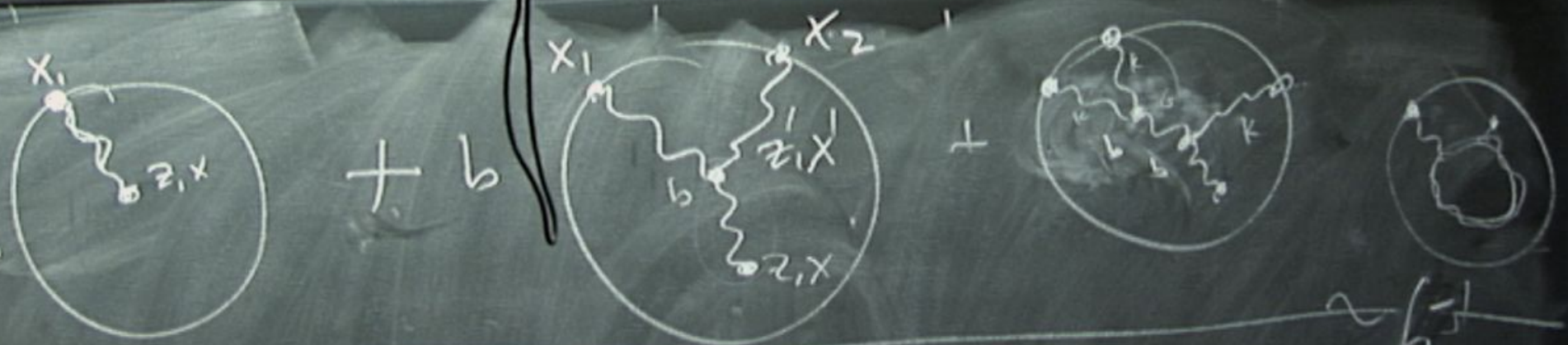


G : Bulk-to-bulk propagator:

$$G(x_2, x_1; z) = \int_{x_1}^{x_2} \sqrt{g} \delta(z - z', x - x')$$

$$\xrightarrow{z \rightarrow 0} z^{\Delta} x^0 + \mathcal{O}(z^{\Delta+1})$$

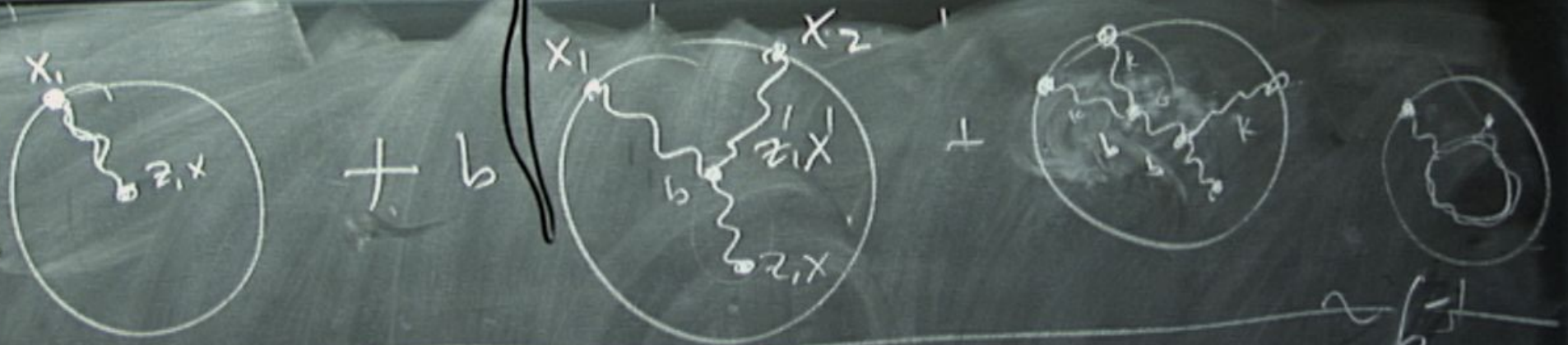
$$S[\phi] = -R \int (\partial\phi)^2 + m^2 \phi^2 + \frac{1}{3} \phi^3$$



G : BULK-TO-BULK PROPAGATOR:

$$\left\{ \begin{aligned} (\mathbb{D} - m^2) \langle \phi(x_2, x_2') \rangle &= \sqrt{g} \delta(z - z', x - x') \\ \langle \phi \rangle &\xrightarrow{z \rightarrow 0} z^{\Delta_-} x^0 + \mathcal{O}(z^{\Delta_+}) \end{aligned} \right.$$

$$S[\phi] = -R \int (\partial\phi)^2 + m^2 \phi^2 + \frac{1}{3} \phi^3$$



G : BULK-TO-BULK PROPAGATOR :

$$\left\{ \begin{aligned} (\mathbb{D} - m^2) \langle \phi(x_2; x_1) \rangle &= \sqrt{g} \delta(z_2, x_1) \\ \langle \phi \rangle &\xrightarrow{z \rightarrow 0} z^{\Delta} \phi(x=0) + \mathcal{O}(z^{\Delta}) \end{aligned} \right.$$

$$S[\phi] = -R \int (\partial\phi)^2 + m^2 \phi^2 + \dots$$

$R \sim N^2$