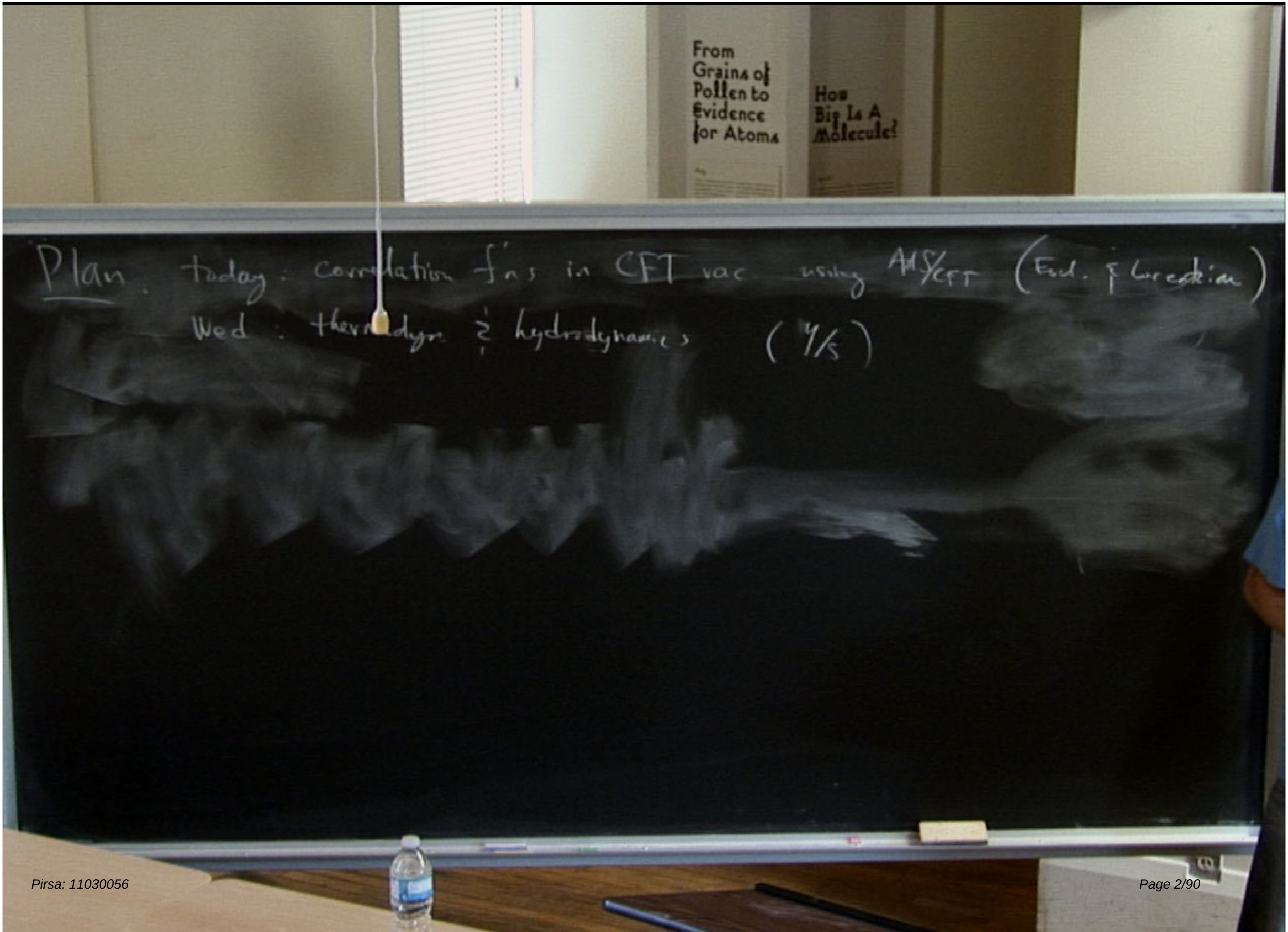
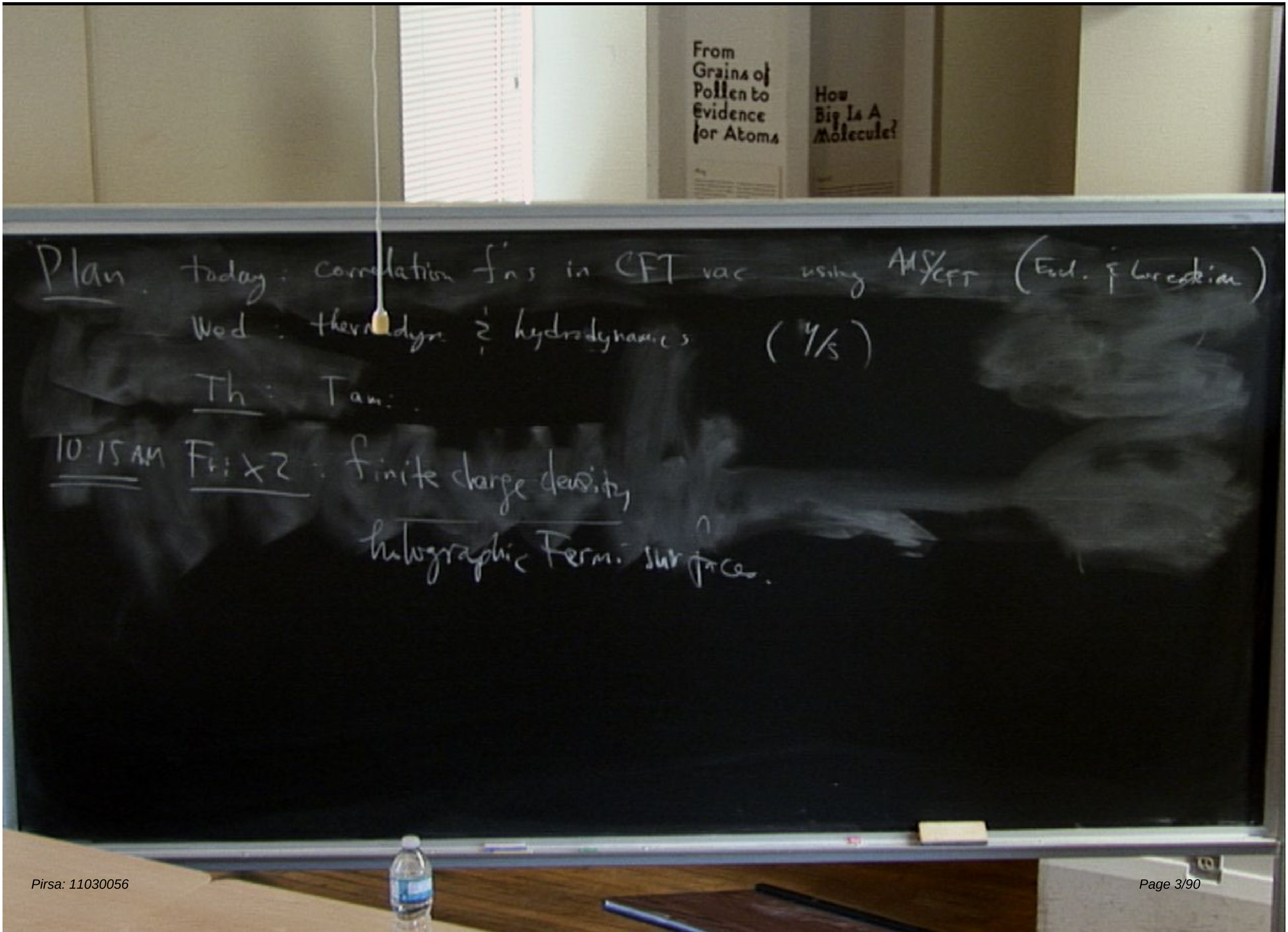
Title: Explorations in String Theory - Lecture 7

Date: Mar 22, 2011 11:30 AM

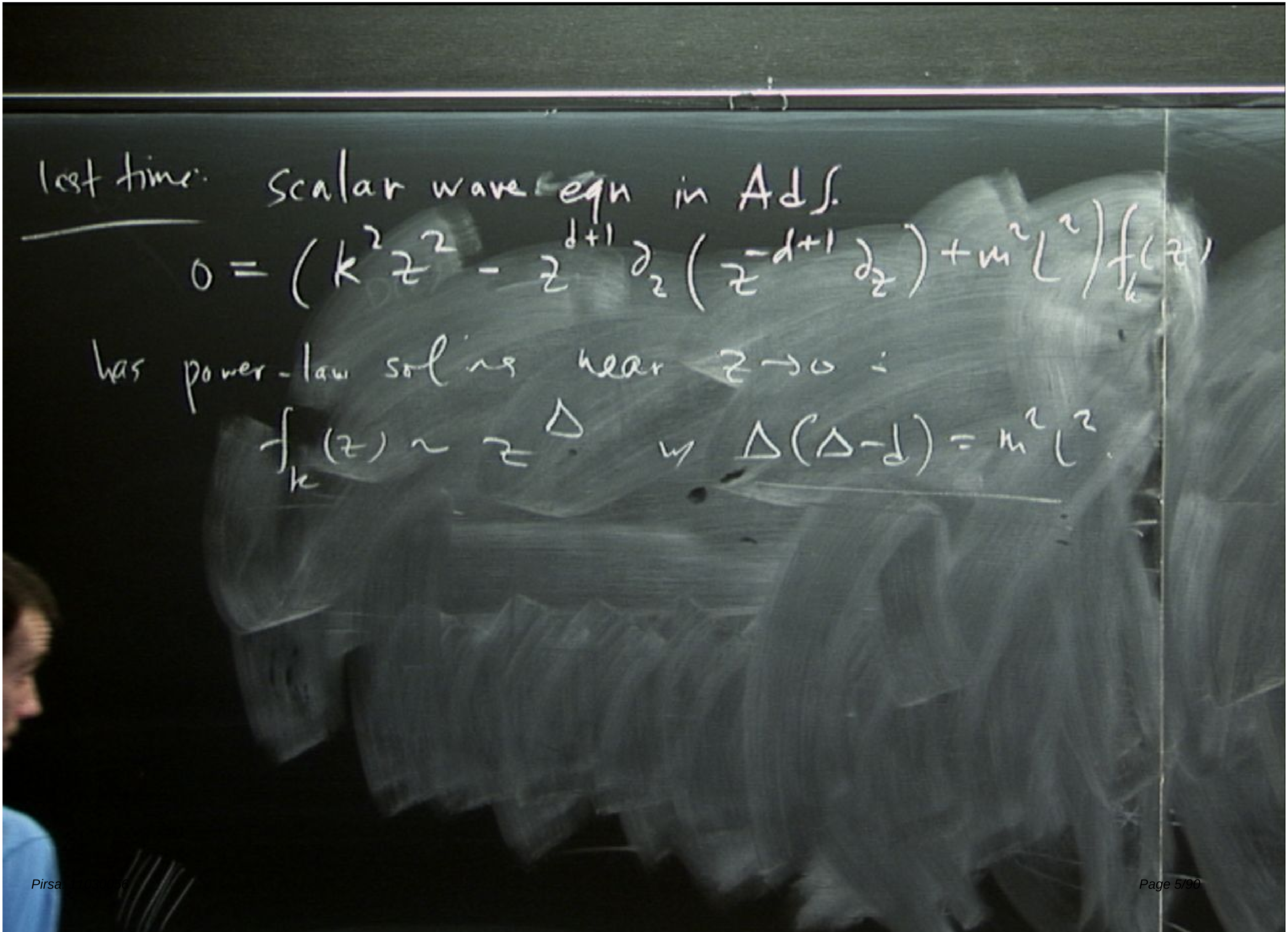
URL: http://pirsa.org/11030056

Abstract:


From Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
Plan. today: correlation f'ns in CFT vac using AdS/CFT
Wed: thermodyn & hydrodynamics (η/s)


From Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
Plan. today: correlation fns in CFT vac using AdS/CFT (Eucl. & Lorentzian)
Wed: thermodyn. & hydrodynamics (η/s)
Th: Tam:
10:15 AM Fri x2: finite charge density
holographic Fermi surface.

last time: scalar wave eqn in AdS.


last time: scalar wave eqn in AdS.
$0 = \left(k^2 z^2 - z^{d+1}\partial_z\left(z^{-d+1}\partial_z\right) + m^2 L^2\right) f_k(z)$
has power-law sol'ns near $z \to 0$:
$f_k(z) \sim z^{\Delta}$ w/ $\Delta(\Delta - d) = m^2 L^2$.

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Frobenius:

$$f_k(z) = a\, z^{\Delta_+} \left(1 + \#_1 z + \mathcal{O}(z^2) \right) + b\, z^{\Delta_-} \left(1 + \#_2 z + \mathcal{O}(z^2) \right)$$

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initial data

determined by ODE

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initial data; determined by ODE

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$$\Delta_\pm = \frac{d}{2} \pm \sqrt{\left(\frac{d}{2}\right)^2 + m^2 L^2}$$

• the $\left(\frac{z}{L}\right)^{\Delta_-}$ sol'n is <u>bigger</u> near $z \to 0$

• $\Delta_+ > 0$

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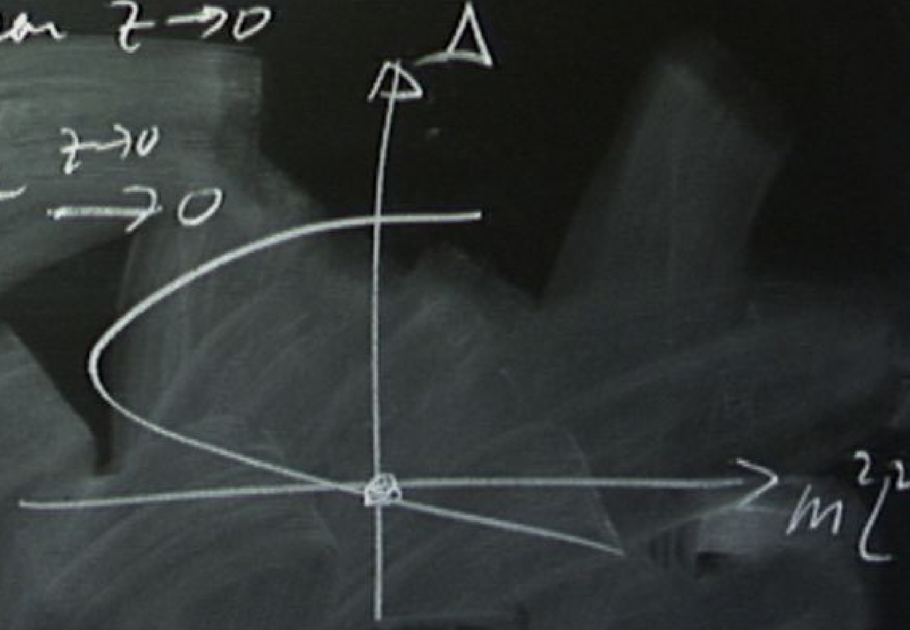
- $\Delta_+ + \Delta_- = d$



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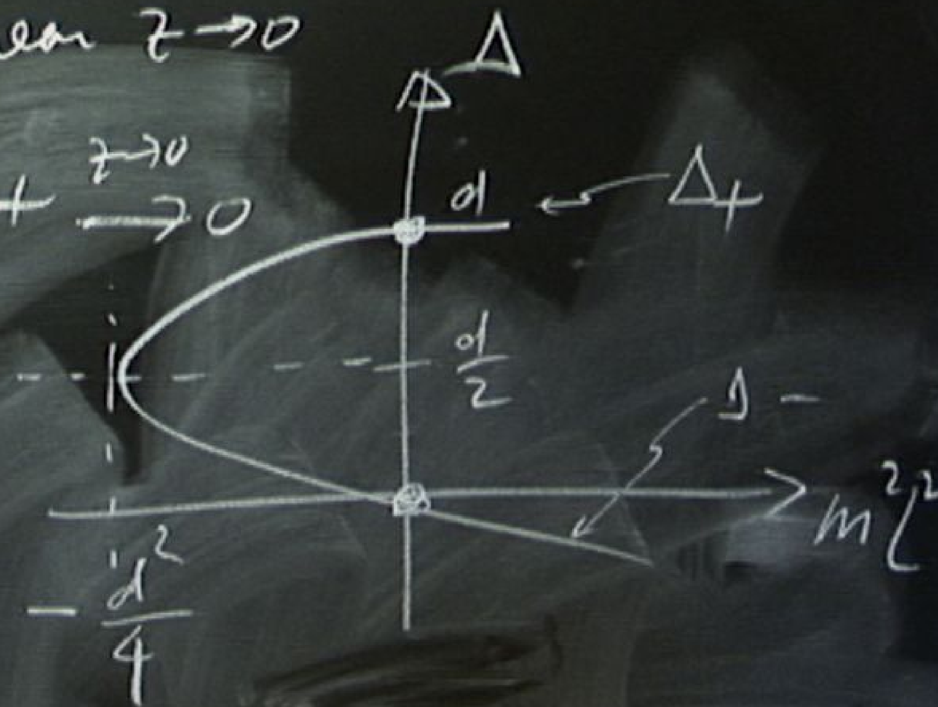

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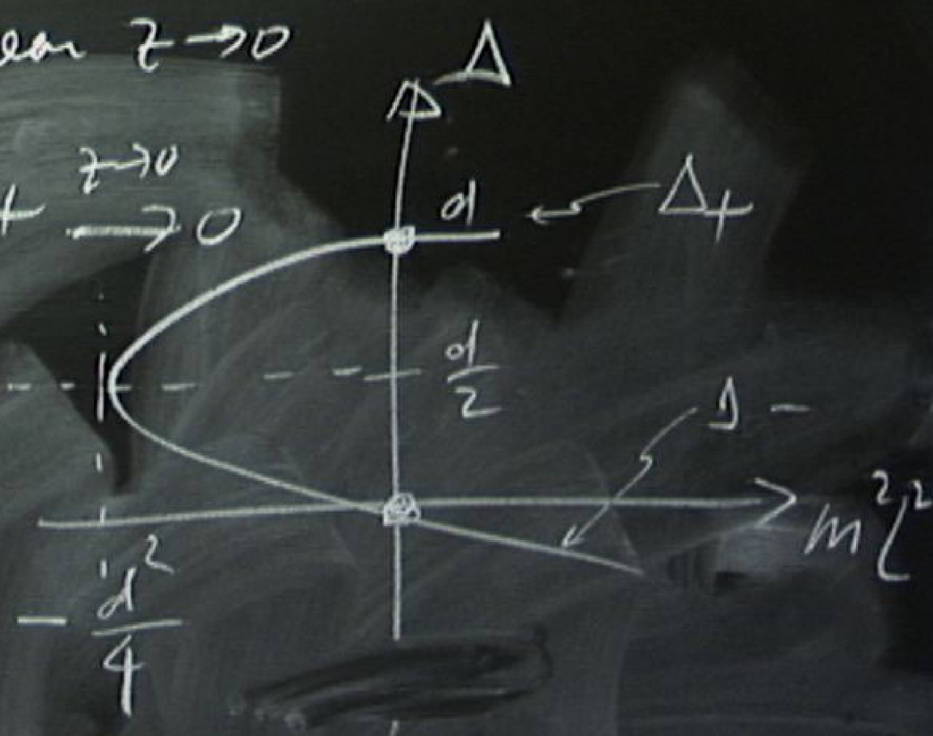
mpose b.c.'s that allow
solutions



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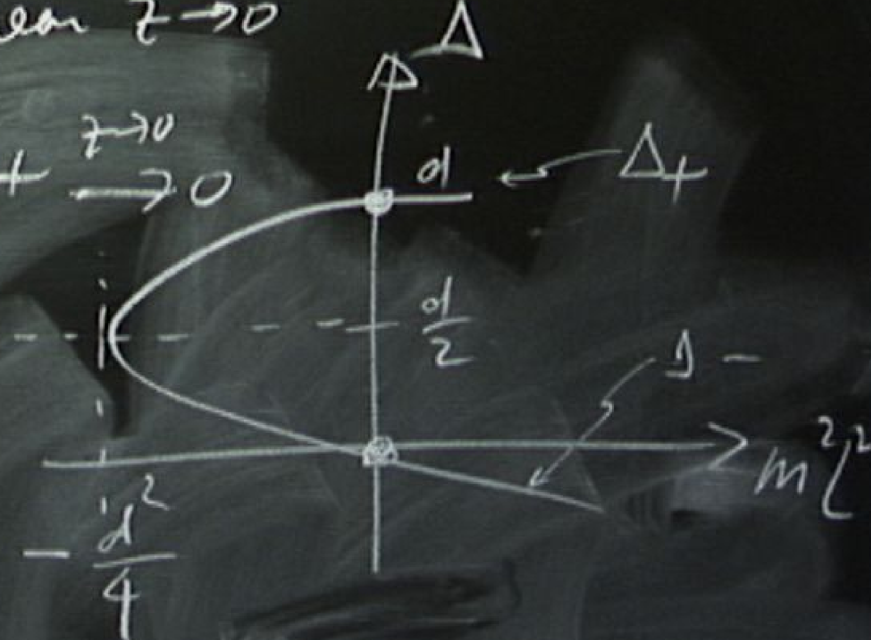

Impose b.c's that allow solutions

"$\phi(z,x) \longrightarrow \phi_0(x)$"

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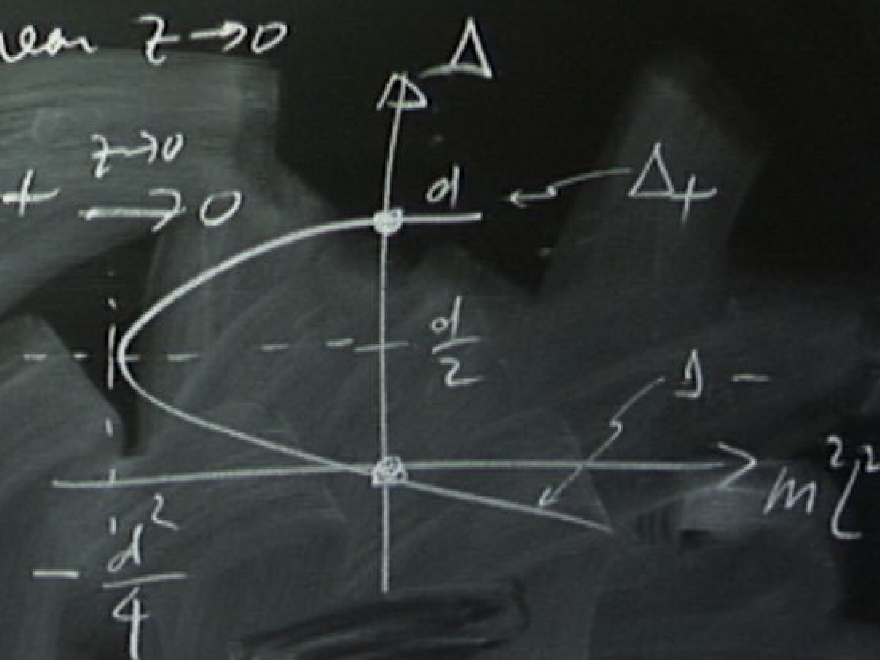

Impose b.c's that allow solutions

$$\text{"}\phi(z,x) \xrightarrow{z\to 0} \phi_0(x)\text{"}$$

Better: $\phi(z,x)\big|_{z=\epsilon} \equiv \phi_0(\epsilon, x)$

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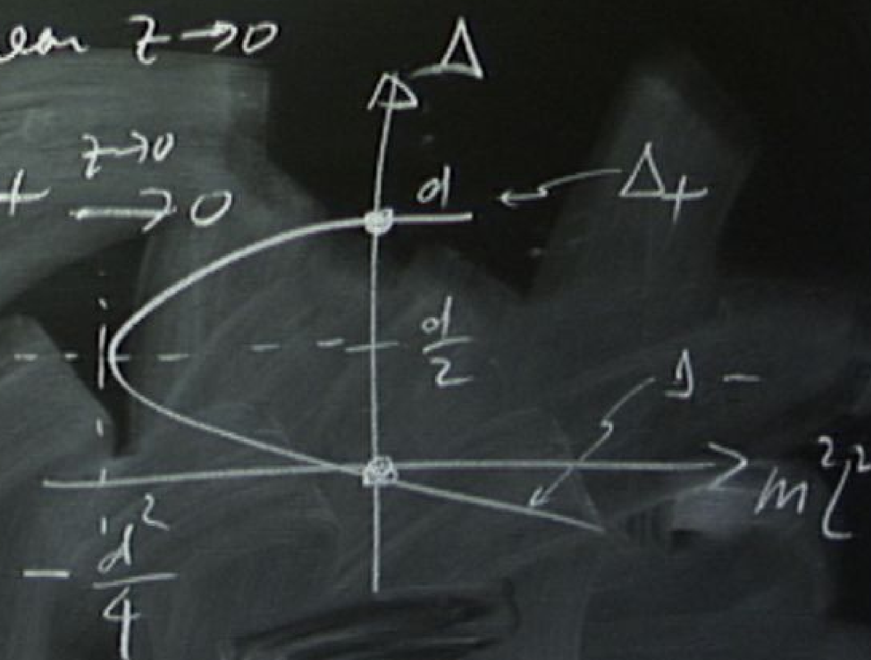

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Better: $\phi(z,x)\big|_{z=\epsilon} \equiv \phi(\epsilon,x) \equiv \epsilon^{\Delta_-} \phi_0^{Ren}(x)$

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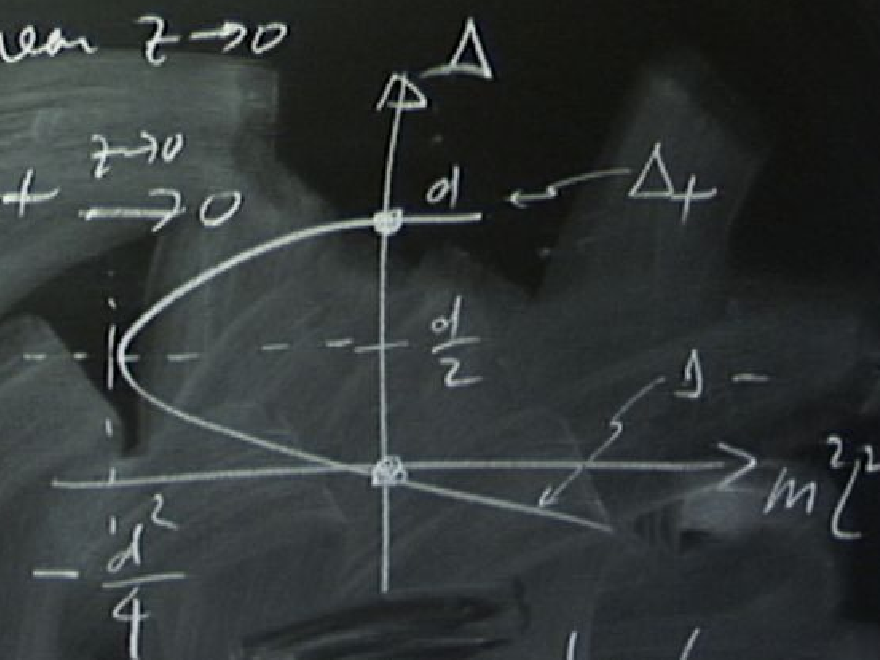

Impose b.c's that allow solutions

"$\phi(z,x) \xrightarrow{z \to 0} \phi_0(x)$"

renormalized source

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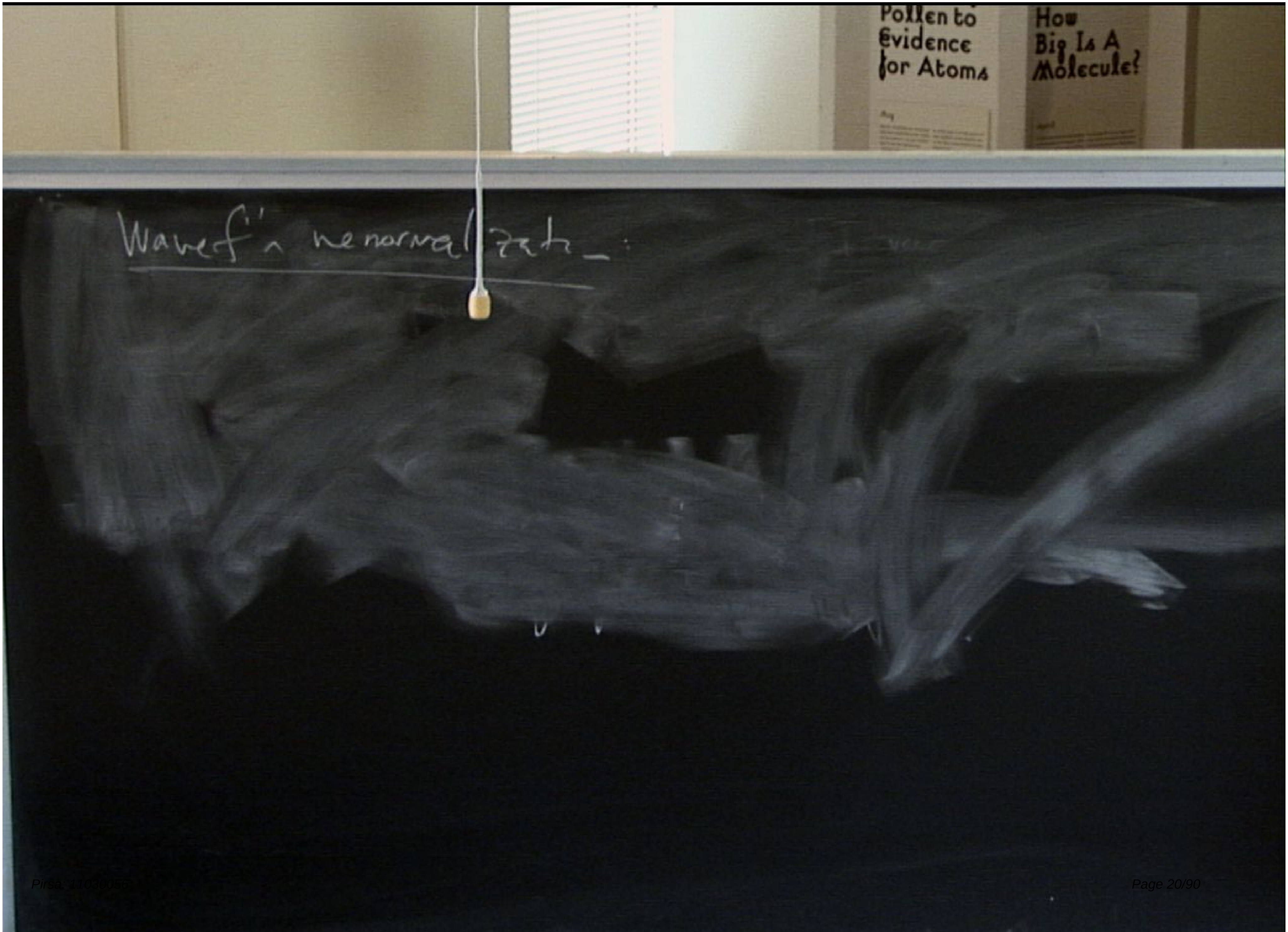
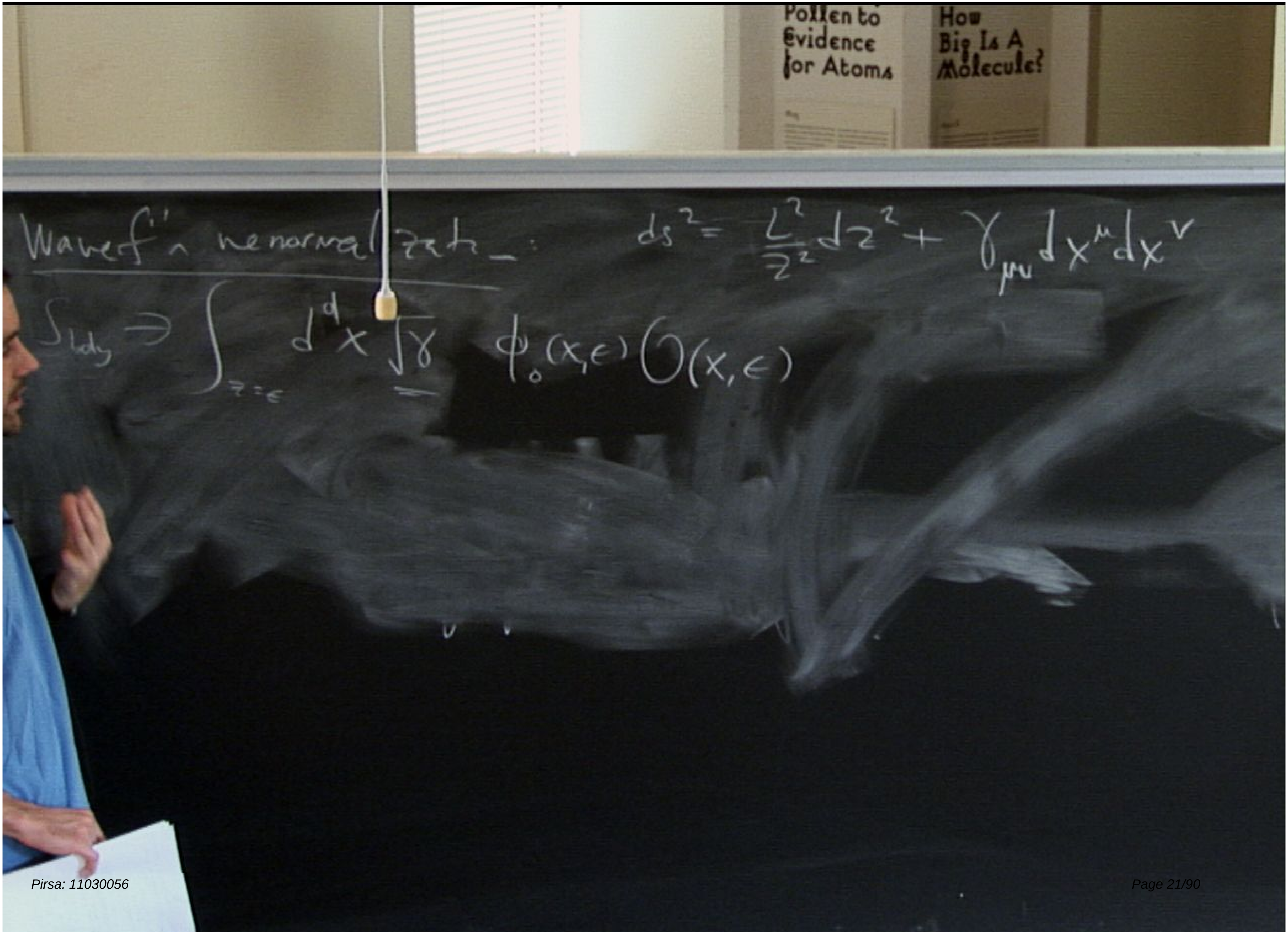

Pollen to Evidence for Atoms
How Big Is A Molecule?
Wavef'n renormalizat_ :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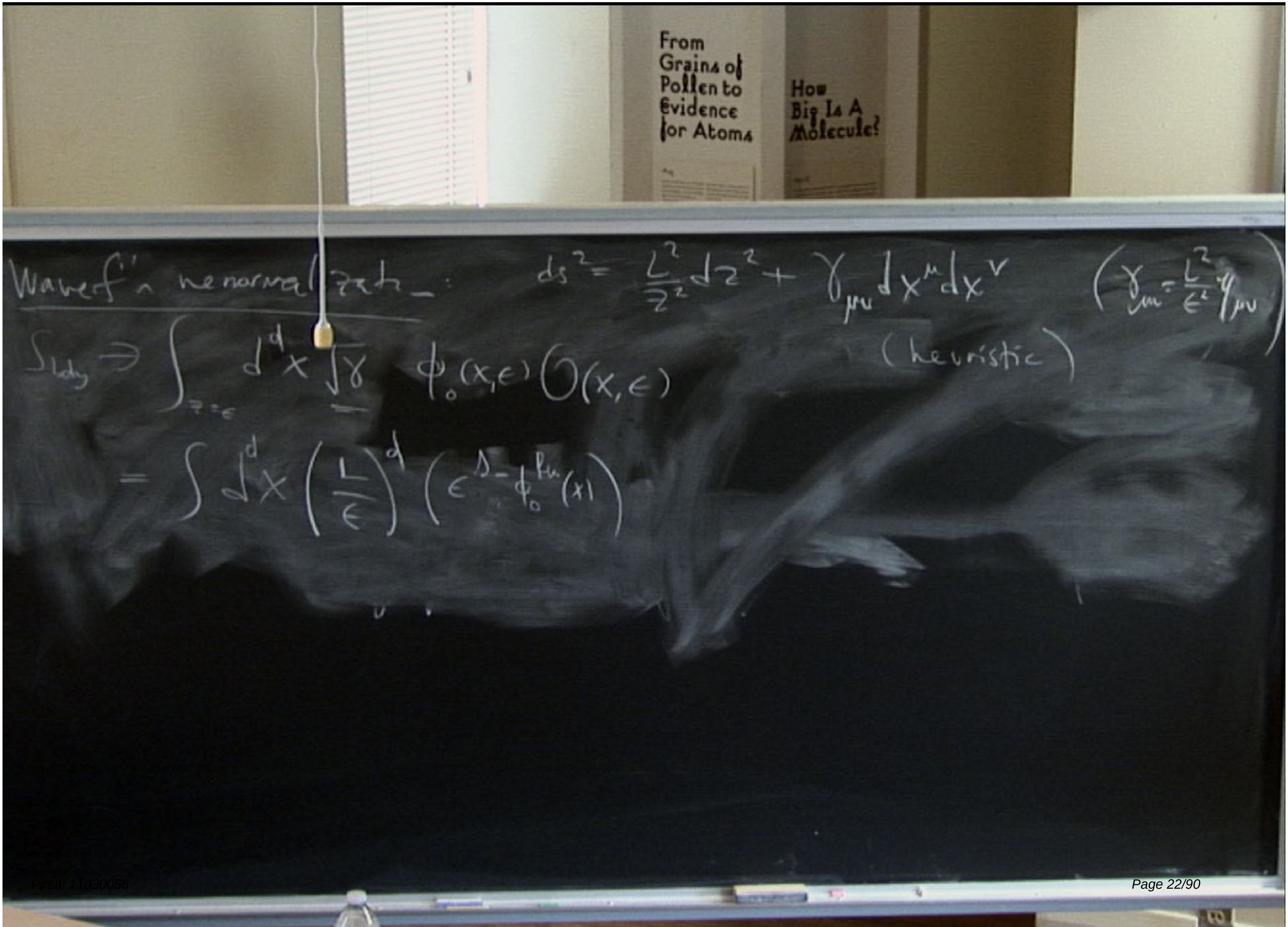
Wavef'n renormalization:
$ds^2 = \frac{L^2}{z^2} dz^2 + \gamma_{\mu\nu} dx^\mu dx^\nu$
$S_{bdy} \ni \int_{z=\epsilon} d^d x \sqrt{\gamma}\, \phi_0(x,\epsilon)\, \mathcal{O}(x,\epsilon)$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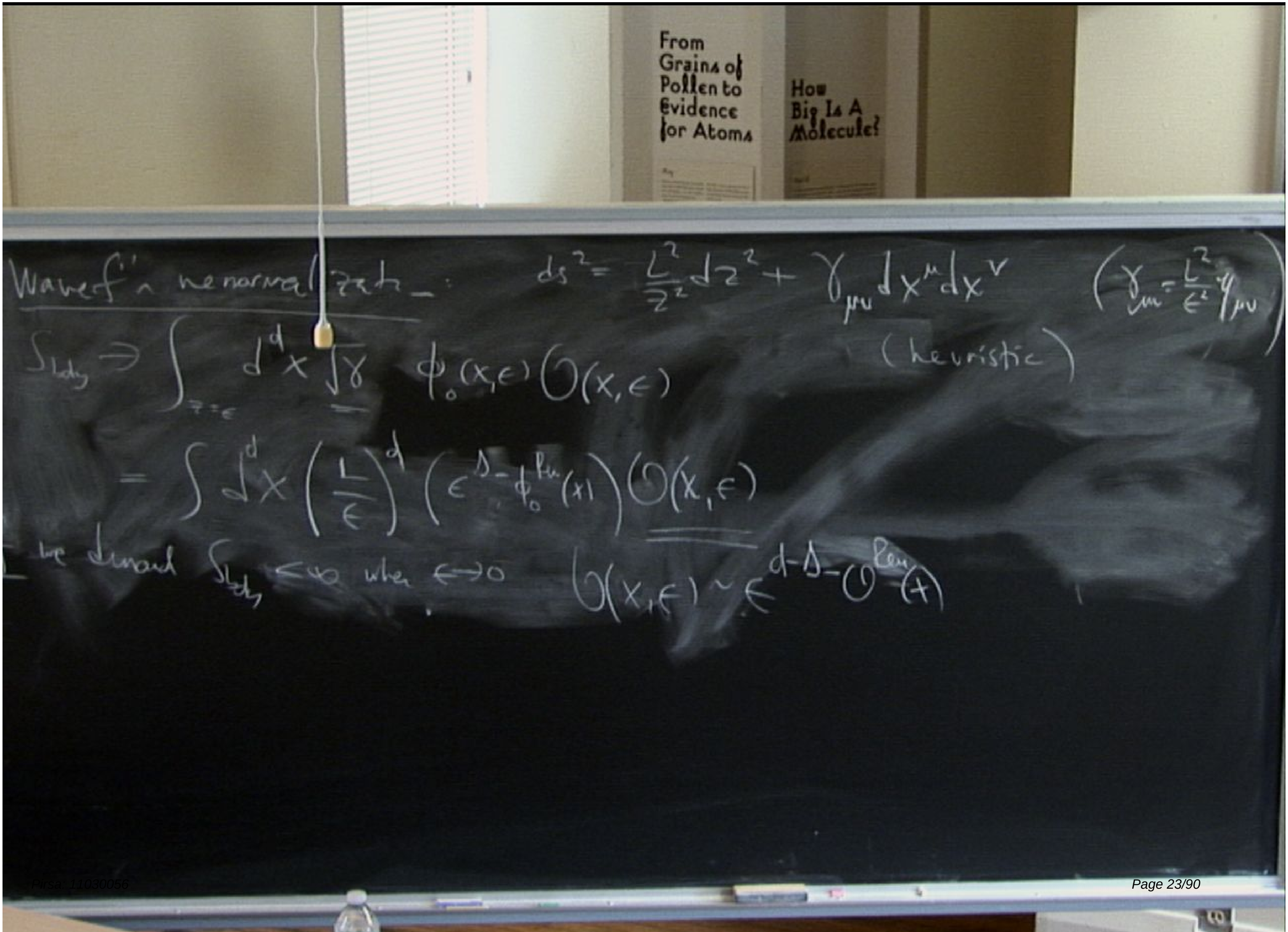
(heuristic)
$S_{bdy} \ni \int_{z=\epsilon} d^d x \sqrt{\gamma}\, \phi_0(x,\epsilon)\, \mathcal{O}(x,\epsilon)$
$= \int d^d x \left(\frac{L}{\epsilon}\right)^d \left(\epsilon^{\Delta_-} \phi_0^{ren}(x)\right)$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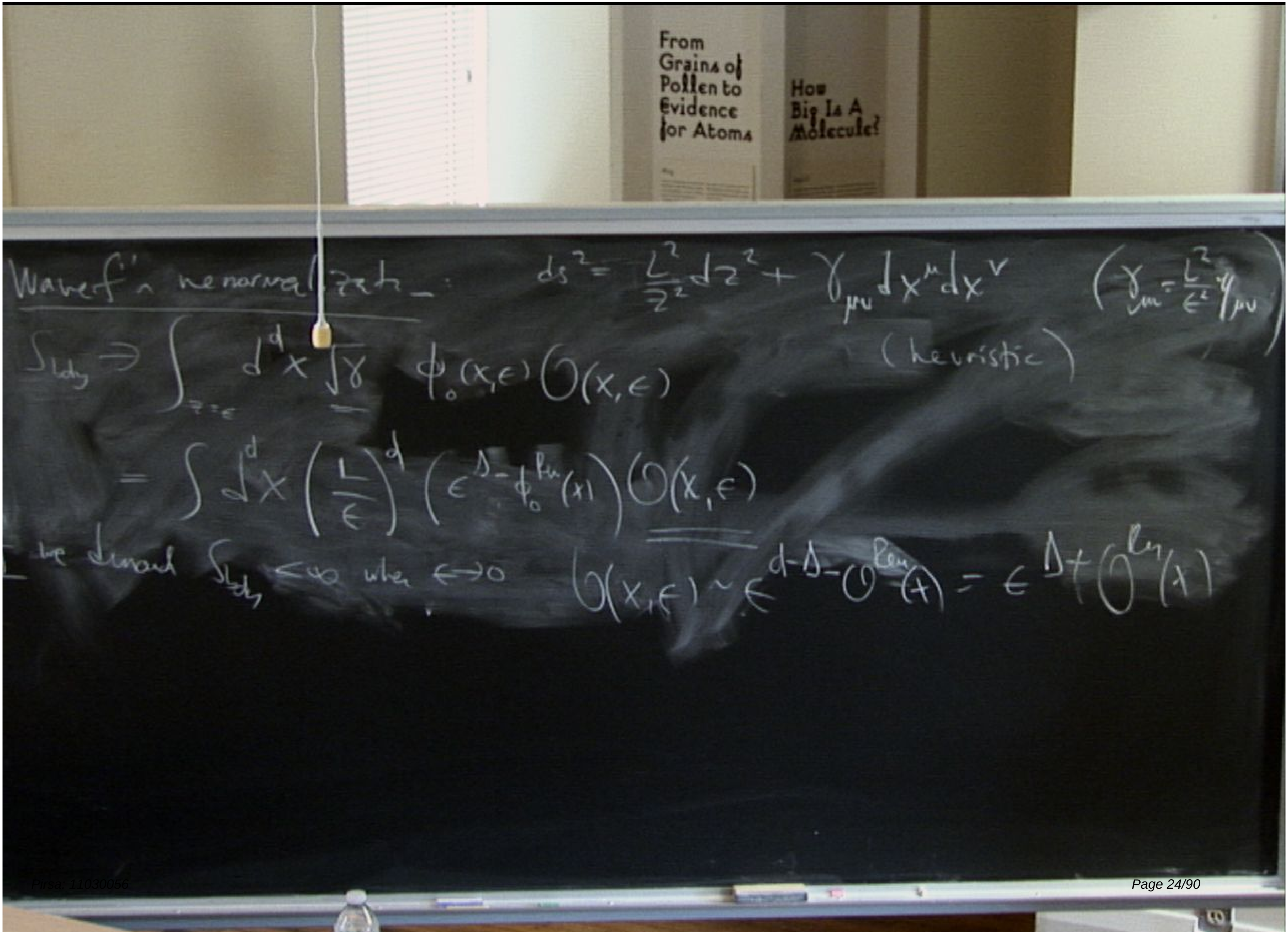
(heuristic)
$= \int d^d x \left(\frac{L}{\epsilon}\right)^d \left(\epsilon^{\Delta-d} \phi_0^{Ren}(x)\right) \mathcal{O}(x,\epsilon)$
we demand $S_{bdy} < \infty$ when $\epsilon \to 0$
$\mathcal{O}(x,\epsilon) \sim \epsilon^{d-\Delta} \mathcal{O}^{Ren}(x)$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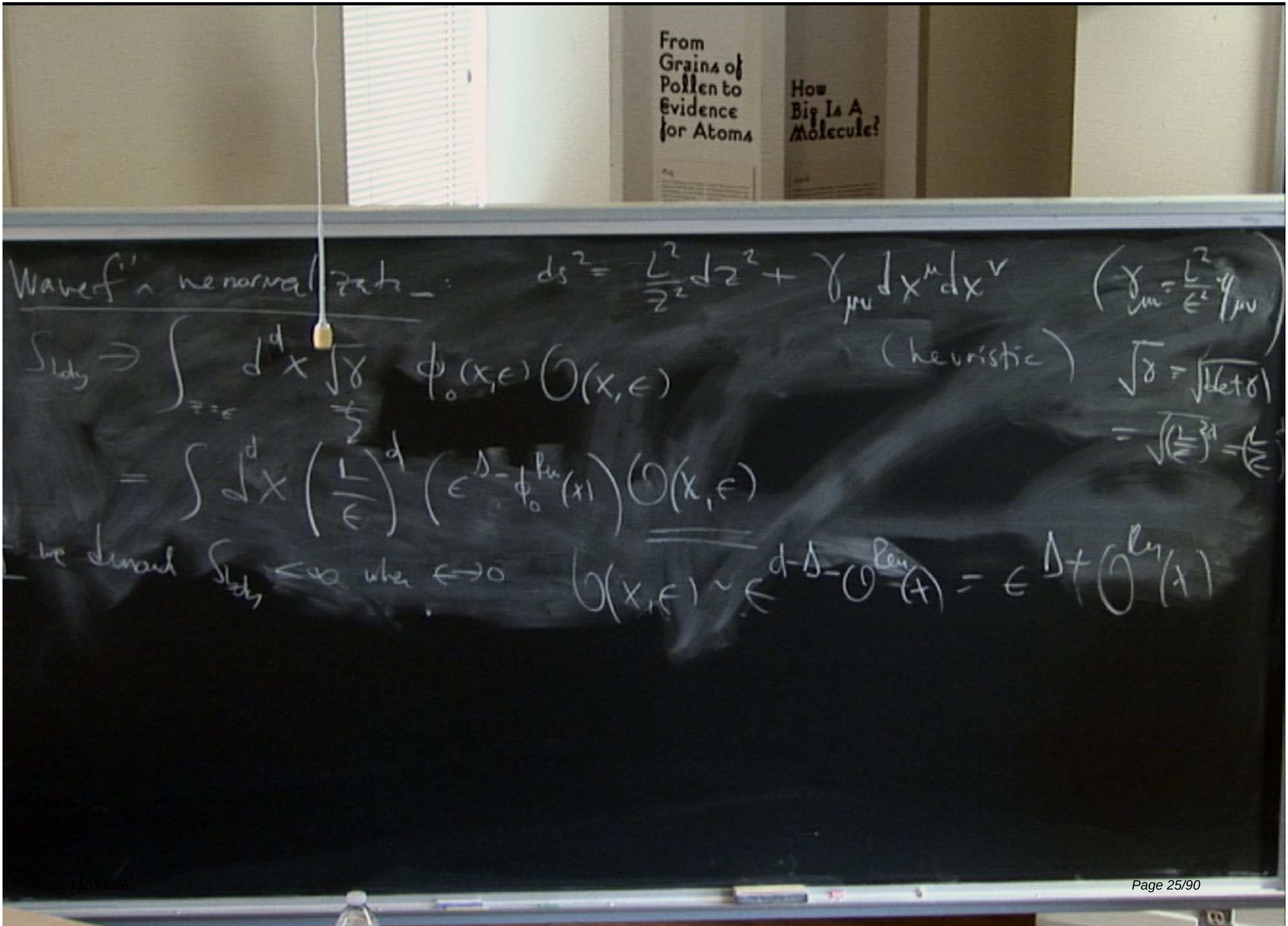
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$\left(\gamma_{\mu\nu} = \frac{L^2}{\epsilon^2} \eta_{\mu\nu} \right)$
$S_{bdy} \Rightarrow \int_{z=\epsilon} d^d x \sqrt{\gamma}\, \phi_0(x,\epsilon)\, \mathcal{O}(x,\epsilon)$
(heuristic)
$\sqrt{\gamma} = \sqrt{|\det \gamma|} = \sqrt{\left(\frac{L}{\epsilon}\right)^{2d}} = \left(\frac{L}{\epsilon}\right)^d$
$= \int d^d x \left(\frac{L}{\epsilon}\right)^d \left(\epsilon^{\Delta_-} \phi_0^{ren}(x)\right) \mathcal{O}(x,\epsilon)$
we demand $S_{bdy} < \infty$ when $\epsilon \to 0$
$\mathcal{O}(x,\epsilon) \sim \epsilon^{d-\Delta_-} \mathcal{O}^{ren}(x) = \epsilon^{\Delta_+} \mathcal{O}^{ren}(x)$

Wavef'n renormalization: $ds^2 = \frac{L^2}{z^2} dz^2 + \gamma_{\mu\nu}$

$$S_{bdy} \ni \int_{z=\epsilon} d^d x \sqrt{\gamma}\, \phi_0(x,\epsilon)\, \mathcal{O}(x,\epsilon)$$

$$= \int d^d x \left(\frac{L}{\epsilon}\right)^d \left(\epsilon^{\Delta_-} \phi_0^{\text{ren}}(x)\right) \mathcal{O}(x,\epsilon)$$

If we demand $S_{bdy} < \infty$ when $\epsilon \to 0$ $\quad \mathcal{O}(x,\epsilon) \sim \epsilon^{d-\Delta_-} \mathcal{O}$

$$ds^2 = \frac{L^2}{z^2}dz^2 + \gamma_{\mu\nu}dx^\mu dx^\nu \qquad \left(\gamma_{\mu\nu} = \frac{L^2}{\epsilon^2}\eta_{\mu\nu}\right)$$

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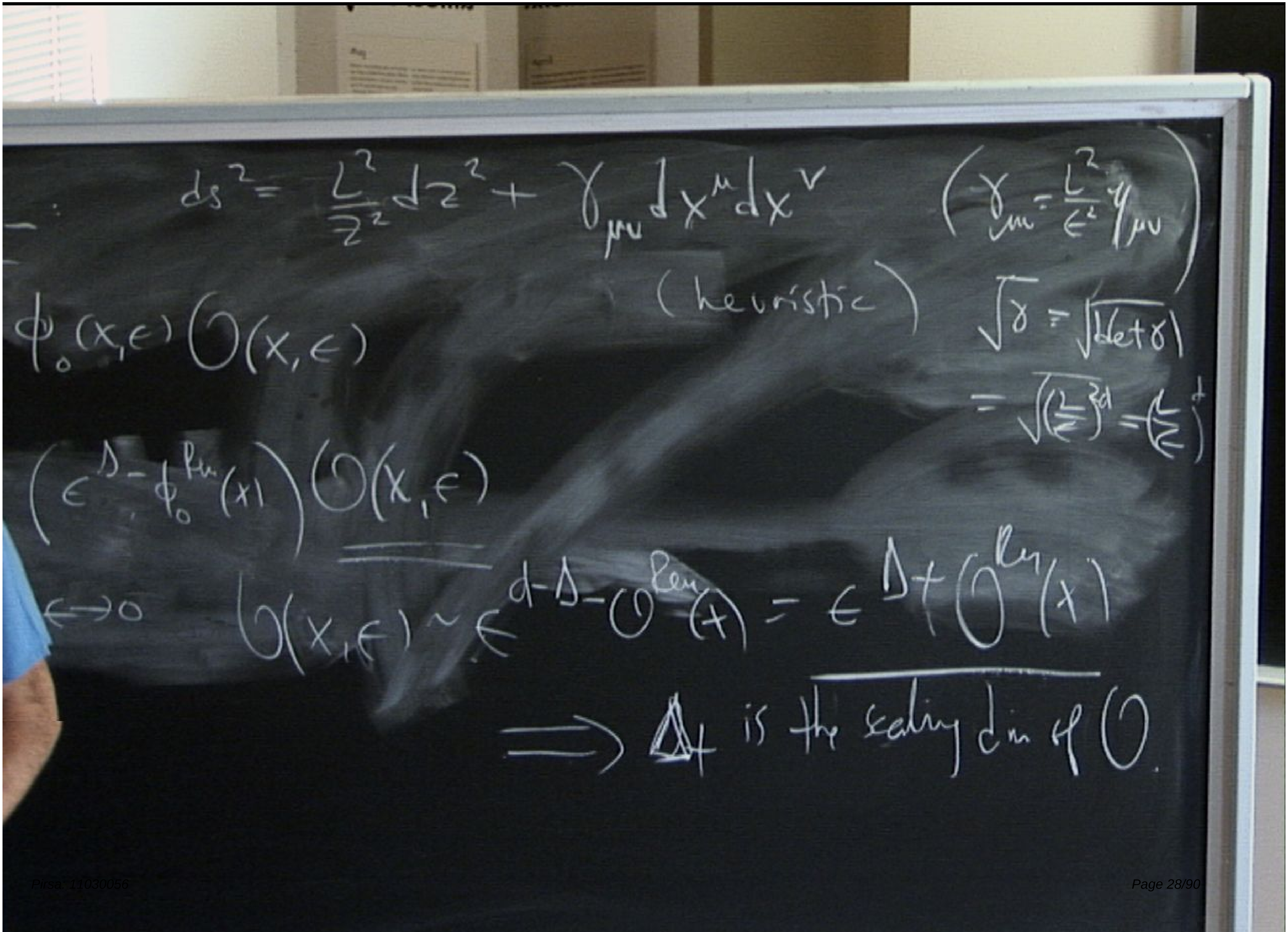
$$\phi_0(x,\epsilon)\,\mathcal{O}(x,\epsilon)$$

$$\left(\epsilon^{\Delta_-}\phi_0^{\mathrm{Ren}}(x)\right)\underline{\underline{\mathcal{O}(x,\epsilon)}}$$

$\epsilon \to 0$

$$\mathcal{O}(x,\epsilon) \sim \epsilon^{d-\Delta_-}\mathcal{O}^{\mathrm{Ren}}(x) = \underline{\epsilon^{\Delta_+}\mathcal{O}^{\mathrm{Ren}}(x)}$$



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$\sqrt{\gamma} = \sqrt{|\det \gamma|} = \sqrt{\left(\frac{L}{\epsilon}\right)^{2d}} = \left(\frac{L}{\epsilon}\right)^{d}$
$\phi_0(x,\epsilon)\, \mathcal{O}(x,\epsilon)$
$\left(\epsilon^{\Delta_-} \phi_0^{ren}(x)\right) \mathcal{O}(x,\epsilon)$
$\epsilon \to 0 \quad \mathcal{O}(x,\epsilon) \sim \epsilon^{d-\Delta_-} \mathcal{O}^{ren}(x) = \epsilon^{\Delta_+} \mathcal{O}^{ren}(x)$
$\Longrightarrow \Delta_+$ is the scaling dim of $\mathcal{O}$.

Wavef'n renormalization:

$$ds^2 = \frac{L^2}{z^2}dz^2 + \gamma_{\mu\nu}dx^\mu \ldots$$

$$S_{bdy} \ni \int_{z=\epsilon} d^dx \sqrt{\gamma}\, \phi_0(x,\epsilon)\, \mathcal{O}(x,\epsilon) \qquad (\text{he}\ldots$$

$$= \int d^dx \left(\frac{L}{\epsilon}\right)^d \left(\epsilon^{\Delta_-} \phi_0^{ren}(x)\right) \mathcal{O}(x,\epsilon)$$

If we demand $S_{bdy} < \infty$ when $\epsilon \to 0$ $\qquad \mathcal{O}(x,\epsilon) \sim \epsilon^{d-\Delta}\ldots$

Simpler: $\langle \mathcal{O}(x)\mathcal{O}(0)\rangle \sim \frac{1}{|x|^{2\Delta}} + \ldots \qquad \Longrightarrow \Delta\ldots$

Wavef'n renormalization:

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(he

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If we demand $S_{bdy} < \infty$ when $\epsilon \to 0$ $\quad \mathcal{O}(x,\epsilon) \sim \epsilon^{d-\Delta_-}\mathcal{O}^{Ren}(x)$

Simpler: $\langle \mathcal{O}(x)\mathcal{O}(0)\rangle \sim \frac{1}{|x|^{2\Delta_+}}$ $\Longrightarrow \Delta_+$ i

the $\left(\frac{z}{L}\right)^{\Delta_-}$ sol'n is bigger near $z \to 0$

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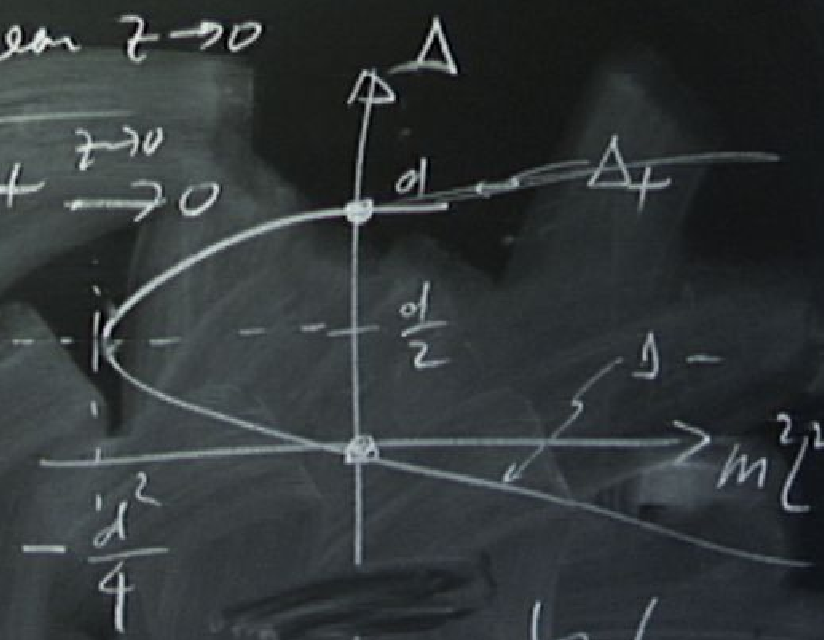

mpose b.c's that allow solutions

"$\phi(z,x) \xrightarrow{z \to 0} \phi_0(x)$"

renormalized source

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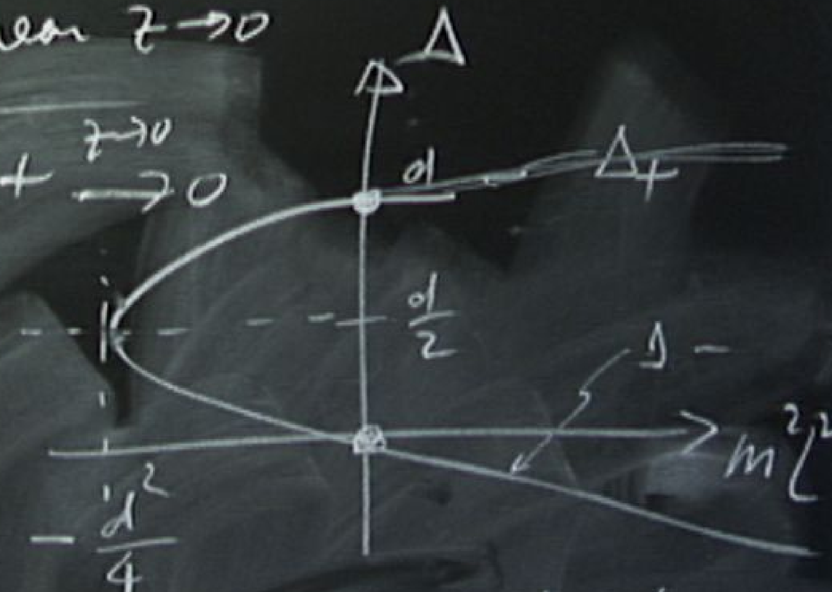

impose b.c.'s that allow solutions

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renormalized source

"alternative quantization"

Klebanov-Witten

hepth/9908

Relevantness

$$\Delta_\pm = \frac{d}{2} \pm \sqrt{\left(\frac{d}{2}\right)^2 + m^2 \ell^2}$$

- $m^2 > 0$ $\quad \Delta_+ > d$

Relevantness

$$\Delta_\pm = \frac{d}{2} \pm \sqrt{\left(\frac{d}{2}\right)^2 + m^2 L^2}$$

- $m^2 > 0$ $\quad \underbrace{\Delta_+ > d}_{} \Rightarrow \mathcal{O}$ is an irrelevant operator.

$$\Delta S = \int d^d x \, \underbrace{(\text{mass})^{d-\Delta}}_{} \, \mathcal{O}$$

goes away when $E <$ mass

$\underline{\underline{\phi(z)}}$ grows in the UV

$\sim z^{\Delta < 0} \longrightarrow \infty$

Relevantness

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ODE

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A $\underline{\text{finite}}$ ϕ_b destroys AdS asymptotics

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$$\Delta S = \int d^d x \, \underbrace{(\text{mass})^{d-\Delta}}_{\phi(z)} \mathcal{O}$$

goes away when $E <$ mass

$\phi(z)$ grows in the UV

$\sim z^{\Delta < 0} \longrightarrow \infty$

A finite ϕ_0 destroys AdS asymptotics

$\Rightarrow$ more data

ADE

Relevantness

$\Delta_\pm = \frac{d}{2} \pm \sqrt{\left(\frac{d}{2}\right)^2 + m^2 L^2}$

- $m^2 > 0$ $\quad \underbrace{\Delta_+ > d} \Rightarrow \mathcal{O}$ is an irrelevant operator

$\Delta S = \int d^d x \, \underbrace{(\text{mass})^{d-\Delta}}_{\phi(z)} \mathcal{O}$ goes away when $E <$ mass

$\phi(z)$ grows in the UV

$\sim z^{\Delta < 0} \longrightarrow \infty$

A finite ϕ_b destroys AdS asymptotics

$\Longrightarrow$ more data required.

ADE

• $m^2 = 0 \Rightarrow \Delta = d$ marginal.

• $m^2 < 0$

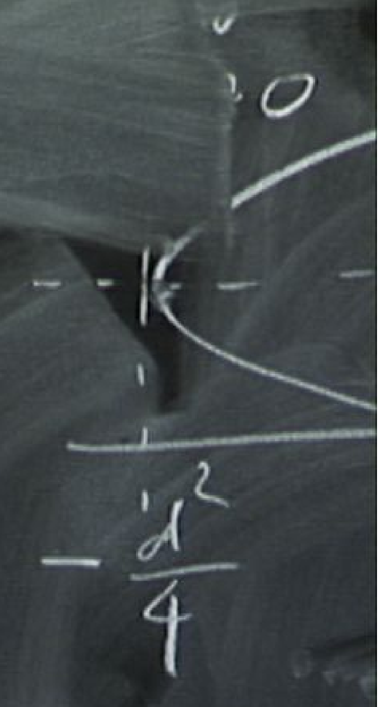
ODE

$-\frac{d^2}{4}$

Δ_-

- $m^2 = 0 \Rightarrow \Delta = d$ marginal.
- $m^2 < 0 \Rightarrow \Delta < d$.

ODE

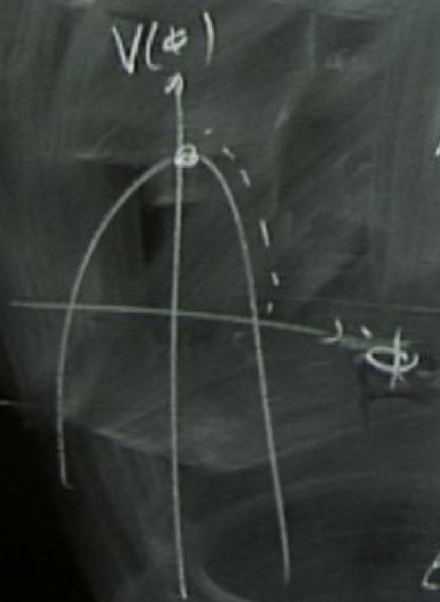


Relevantness

$$\Delta_\pm = \frac{d}{2} \pm \sqrt{\left(\frac{d}{2}\right)^2 + m^2 L^2}$$

- $m^2 > 0$

$\underbrace{\Delta_+ > d}$ $\Rightarrow$ $\mathcal{O}$ is an irrelevant operator.



$$\Delta S = \int d^d x \,\underbrace{(\text{mass})^{d-\Delta}}\, \mathcal{O}$$

goes away when $E <$ mass

$\phi(z)$ grows in the UV

$\sim z^{\Delta<0} \longrightarrow \infty$

A finite ϕ destroys AdS asymptotics,

$\Longrightarrow$ more data required.

$m^2 L^2$

operator

goes away

$E <$ mass

ODE

the UV

∞

asymptotics

$\cdot\ m^2 = 0 \Rightarrow \Delta = d$ marginal

$\circ\ m^2 < 0 \Rightarrow \Delta < d$

$$\phi(z) \overset{z\to 0}{\sim} z^{\Delta_-} \to 0$$

• $m^2 = 0 \Rightarrow \Delta = d$ marginal.

• $m^2 < 0 \Rightarrow \Delta < d$.

$$\phi(z) \overset{z \to 0}{\sim} z^{\Delta_-} \to 0$$

$\Rightarrow$ no instability for

$$m^2 > m^2_{BF} = -\frac{d^2}{4}.$$

ator

way

mass ODE

$-\frac{d^2}{4}$

• $m^2 = 0 \Rightarrow \Delta = d$ marginal.

• $m^2 < 0 \Rightarrow \Delta < d$.

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$\Rightarrow$ no instability for

$$m^2 > m^2_{BF} = -\frac{d^2}{4}.$$

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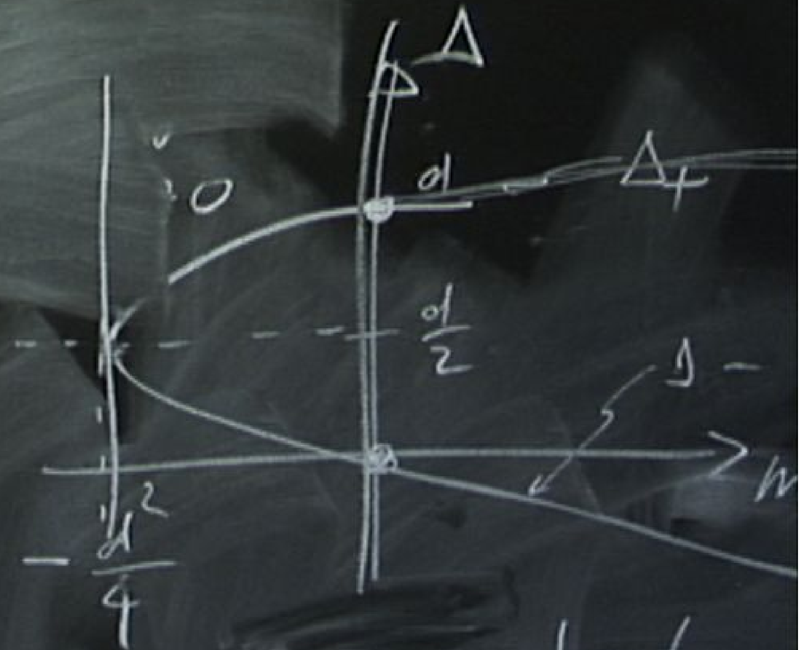
Δ is a relevant op.



Renormalized source

Δ_- $\boxed{\phi^{Ren}_0(x)}$

"alternat quantiz

Klebanov - Witten

hep-th/9908

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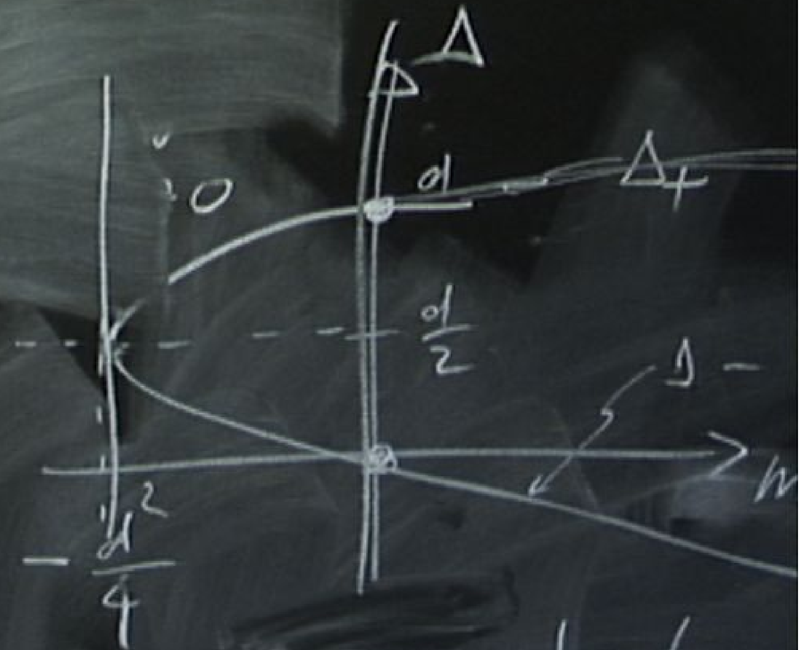
$z \to \infty$.



renormalized source

$\Delta_- \quad \boxed{\Phi_0^{Ren}(x)}$

"alternat[ive] quantiz[ation]"

Klebanov–Witten hep-th/9908

eg vector field : A_μ

$$\Delta_J(\Delta_J - d + \#) = m^2 \ell^2$$

$$S_{bdy} \supset \int A^{(0)}_\mu J^\mu$$

$$\underline{\underline{m_A = 0}} \Longrightarrow \Delta_J = d - 1$$

$\Longleftrightarrow J$ is conserved

Vacuum of CFT, euclidean case

$$0 = \left(z^{d+1}\partial_z\left(z^{-d+1}\partial_z\right) - m^2 l^2 - z^2 k^2\right) f_k(z)$$

euclidean: $k^2 \equiv k^\mu k^\nu \eta_{\mu\nu} \geq 0$

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Sol'n: $f_k(z) = a_k\, z^{d/2} K_\nu(kz) + a_I\, z^{d/2} I_\nu(kz)$

$$\nu = \Delta_+ - \frac{d}{2} = \sqrt{\left(\frac{d}{2}\right)^2 + m^2 l^2}\,.$$

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IR behavior: $z \to \infty$

$$K_\nu(kz) \overset{z\to\infty}{\sim} e^{-kz}, \quad I_\nu(kz) \overset{z\to\infty}{\sim} e^{+kz}$$

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$$Z \sim e^{-S[f]}$$

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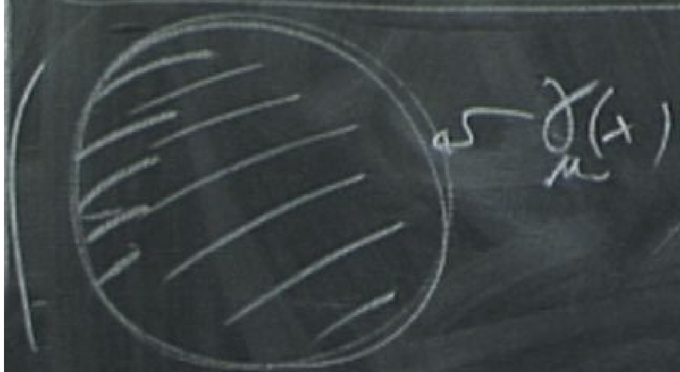

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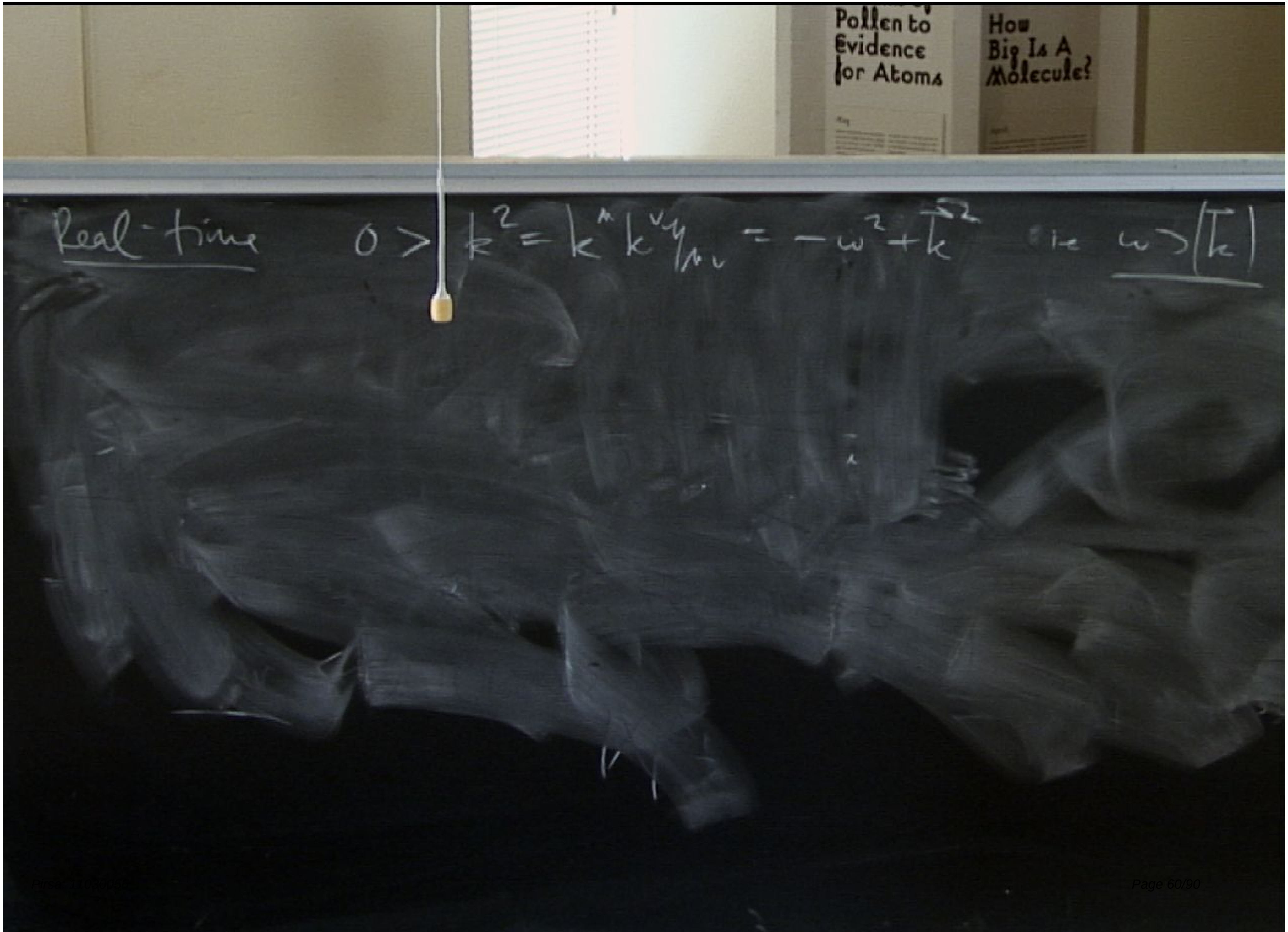
$\gamma_\mu(x)$

$\Longrightarrow \exists !\, g_{\mu\nu}$


Pollen to Evidence for Atoms
How Big Is A Molecule?
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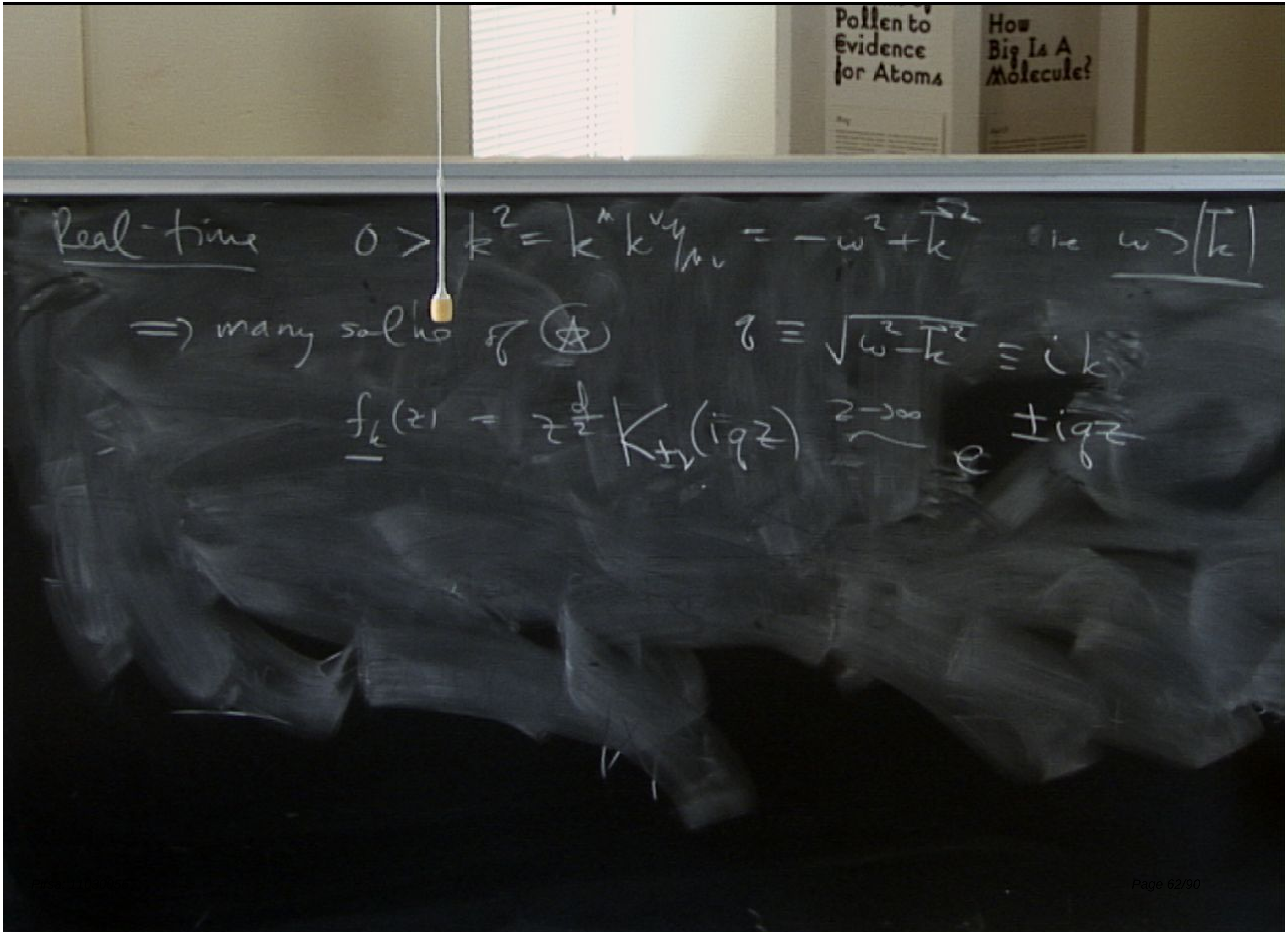
$$f_k(z) = z^{\frac{d}{2}} K_{\pm\nu}(iqz)$$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$$= -\omega^2 + \vec{k}^2 \qquad \text{i.e. } \omega > |\vec{k}|$$

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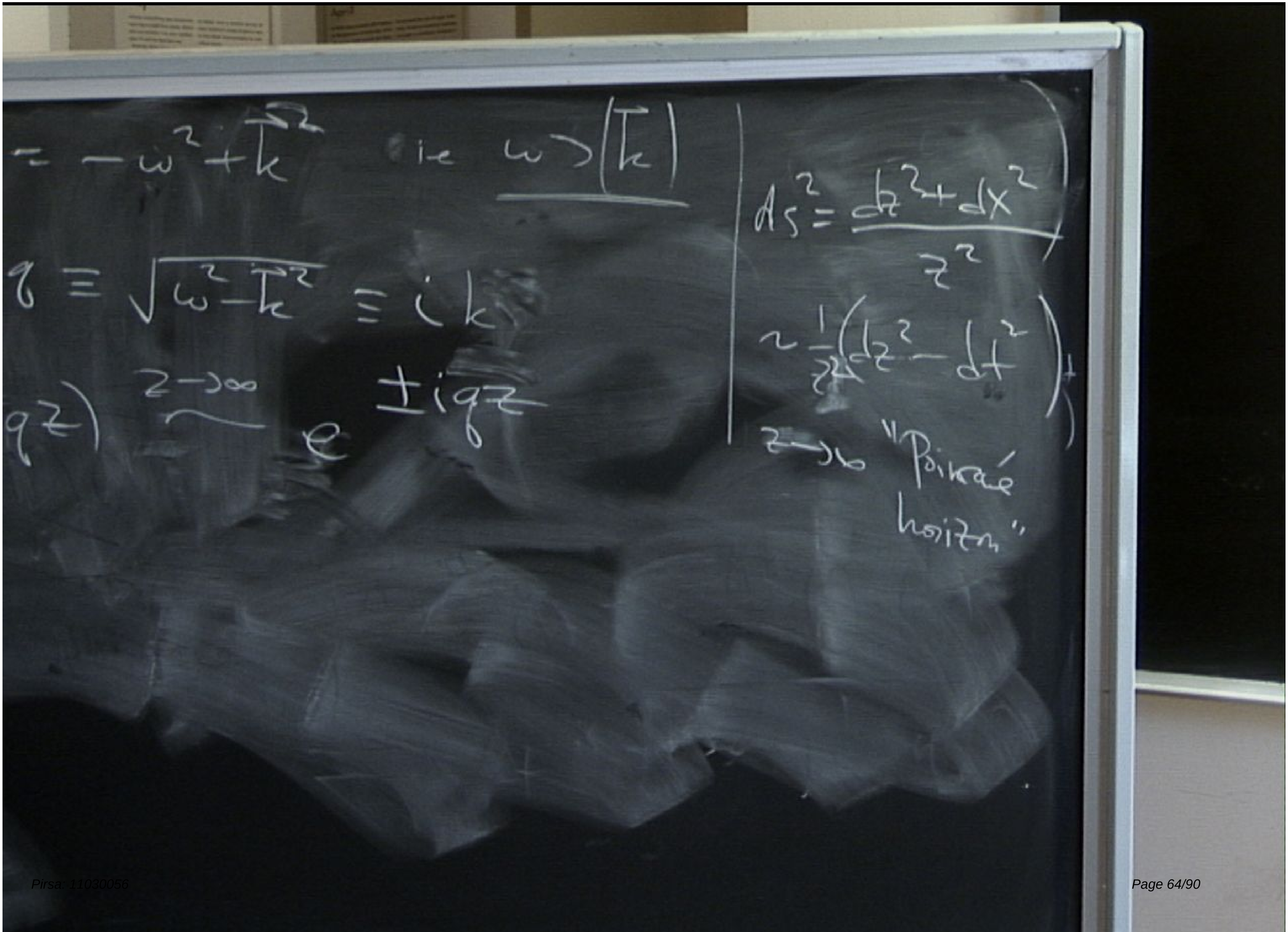
$$q z) \quad \underset{z \to \infty}{\sim} \quad e^{\pm i q z}$$

$$ds^2 = \frac{dz^2 + dx^2}{z^2}$$

$$\sim \frac{1}{z_0^2}\left(dz^2 - dt^2\right)$$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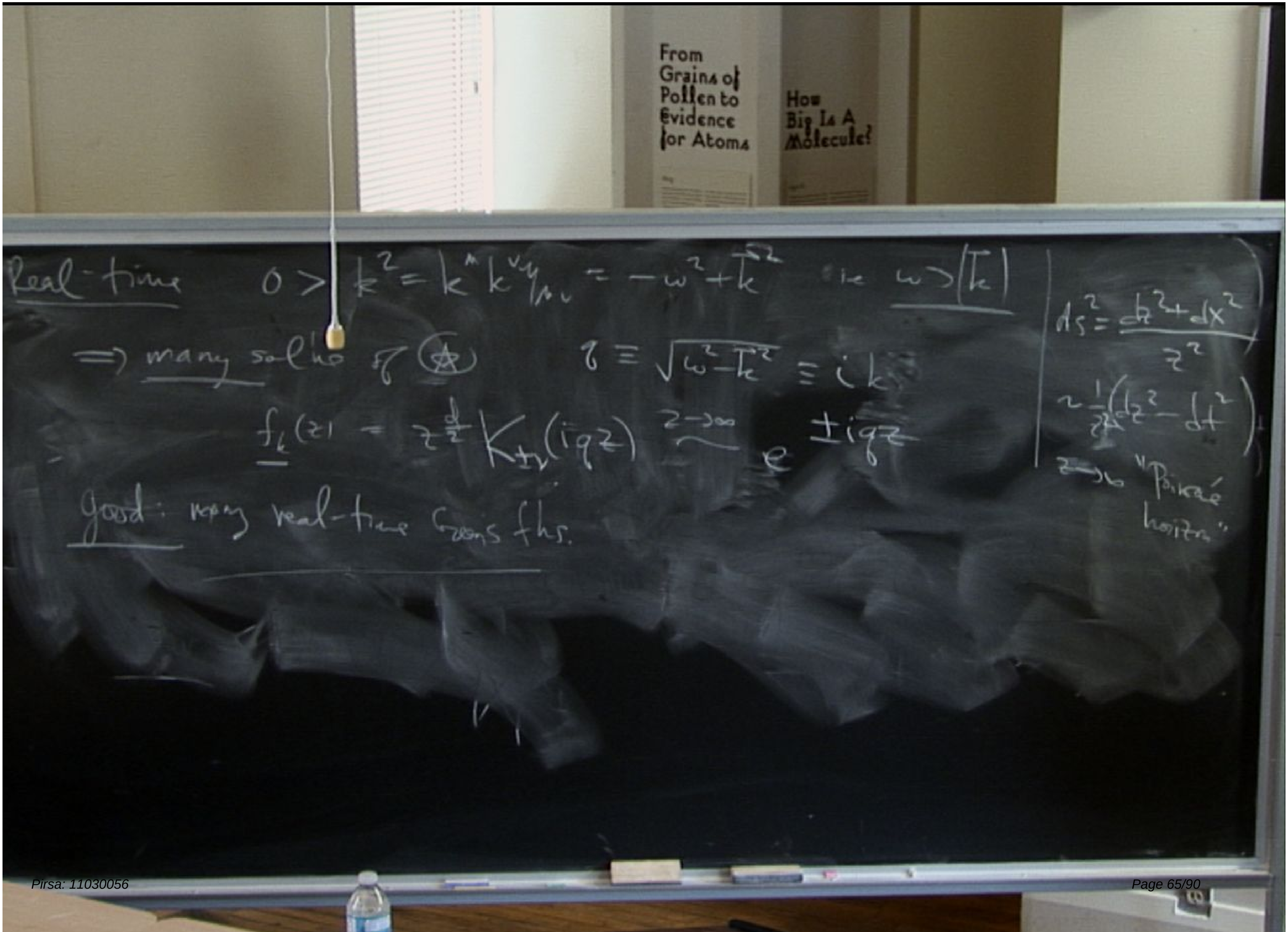
$z \to \infty$ "Poincaré horizon"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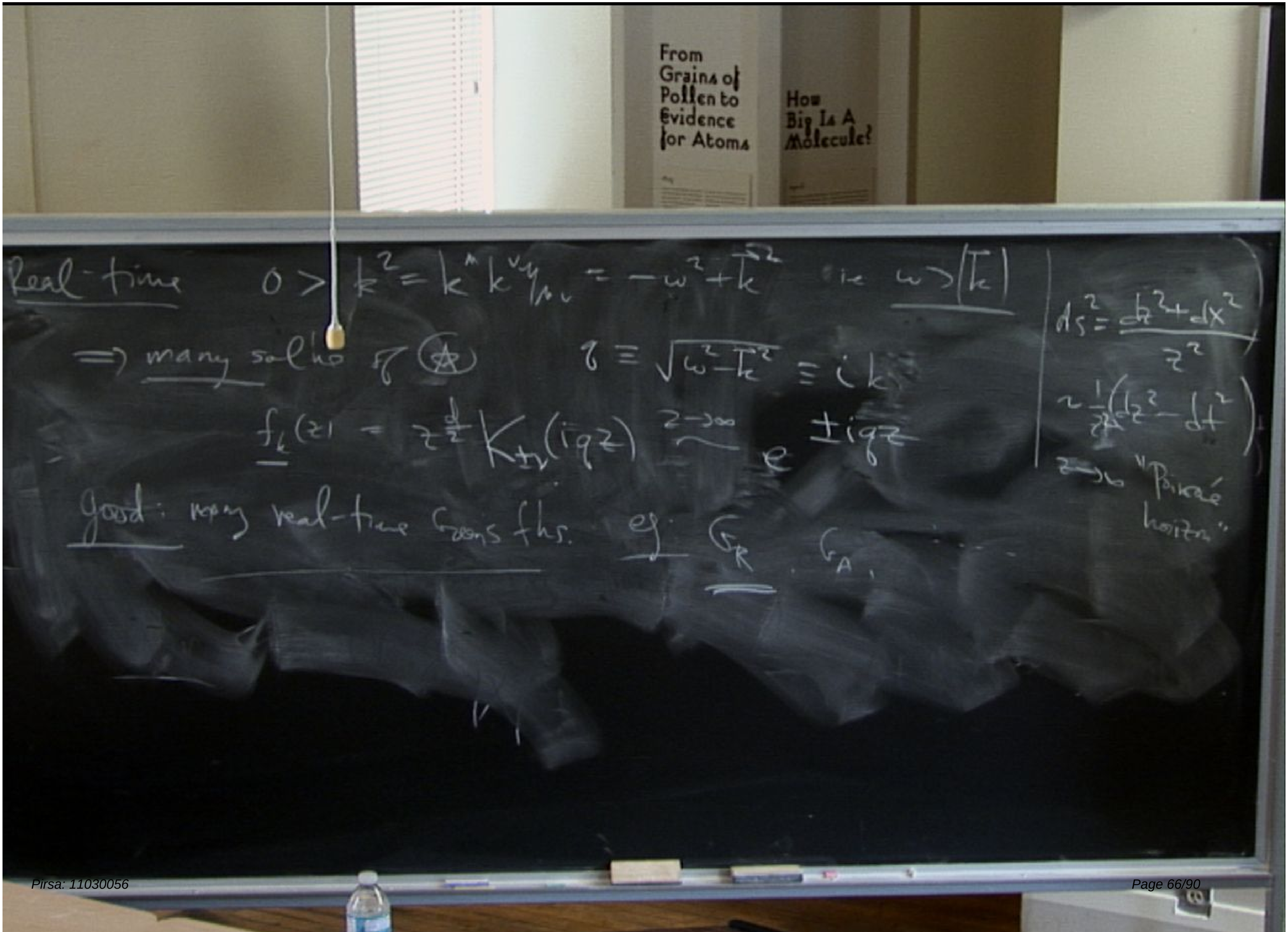
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Linear Response

$$G^R_{\mathcal{O}_A \mathcal{O}_B}(\omega, k) \equiv -i \int d^{d-1}x\, dt\, e^{i\omega t - i\vec{k}\cdot\vec{x}}\, \theta(t) \left\langle \left[\mathcal{O}_A(t,\vec{x}), \mathcal{O}_B(0,0) \right] \right\rangle$$

$\mathcal{O}_A$ ↑ Response

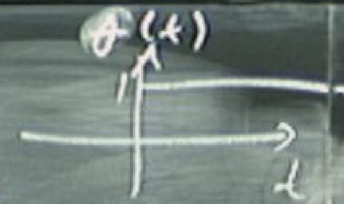
$\mathcal{O}_B$ ↑ source

$\theta(t)$

Why: $\delta H(t) = \int d^{d-1}x\; \phi_{B(0)}(t,\vec{x})\, \mathcal{O}_B(\vec{x})$

e.g. $\delta(t)\,\delta(\vec{x}$

Linear Response (QM)



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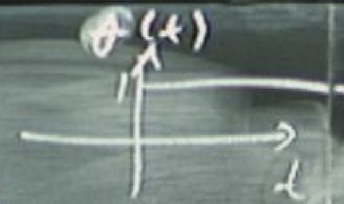

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$$\langle \mathcal{O}_A(t,\vec{x}) \rangle_{\delta H} = \mathrm{tr}\left[\rho(t)\, \mathcal{O}_A(t,x) \right]$$

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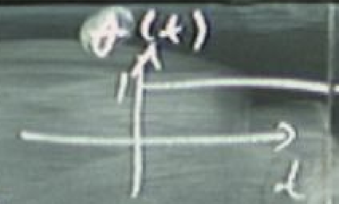

$$G^R_{\mathcal{O}_A\mathcal{O}_B}(\omega,k) \equiv -i\int d^{d-1}x\,dt\; e^{i\omega t-\vec{k}\cdot\vec{x}}\;\theta(t)\;\langle[\mathcal{O}_A(t,\vec{x}),\mathcal{O}_B(0,0)]\rangle$$

Response ↑ source ↑

Why: $\delta H(t) = \int d^{d-1}x\;\phi_{B(0)}(t,\vec{x})\,\mathcal{O}_B(x)$ (hz)

$$\langle\mathcal{O}_A(t,\vec{x})\rangle_{\delta H} = \mathrm{tr}\left[\rho(t)\,\mathcal{O}_A(t,x)\right]$$

e.g. $\delta(t)\,\delta(\vec{x})$

$$\langle O_A \rangle = \mathrm{tr}\, \rho_0\, U^{-1}(t)\, O_A(t,x)\, U(t)$$

$$\rho_0 = e^{-\beta H_0}$$

$$\langle \mathcal{O}_A \rangle = \mathrm{tr}\, \rho_0\, U^{-1}(t)\, \mathcal{O}_A(t,x)\, U(t)$$

$$\rho_0 = e^{-\beta H_0} \qquad \left(\underline{gs:} \;\; \beta \to \infty \right)$$

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$$U(t) = T e^{-i\int^t \delta H(t')dt'}$$

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$\Big\} \Leftarrow$ Schwind

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$$= -i \int^t dx'\, dt' \left\langle \left[\mathcal{O}_A(t,x'),\, \mathcal{O}_B(t',x') \right] \right\rangle \phi_B(t',\vec{x}')$$

$$= \int dx'\, G_R(x,x')\, \phi_{B(0)}(x')$$

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$$\delta\left\langle \mathcal{O}_A(\omega,\vec{k})\right\rangle = G^R_{\mathcal{O}_A\mathcal{O}_B}(\omega,\vec{k})\,\delta\phi_{B(0)}(\omega,\vec{k})$$

G_F

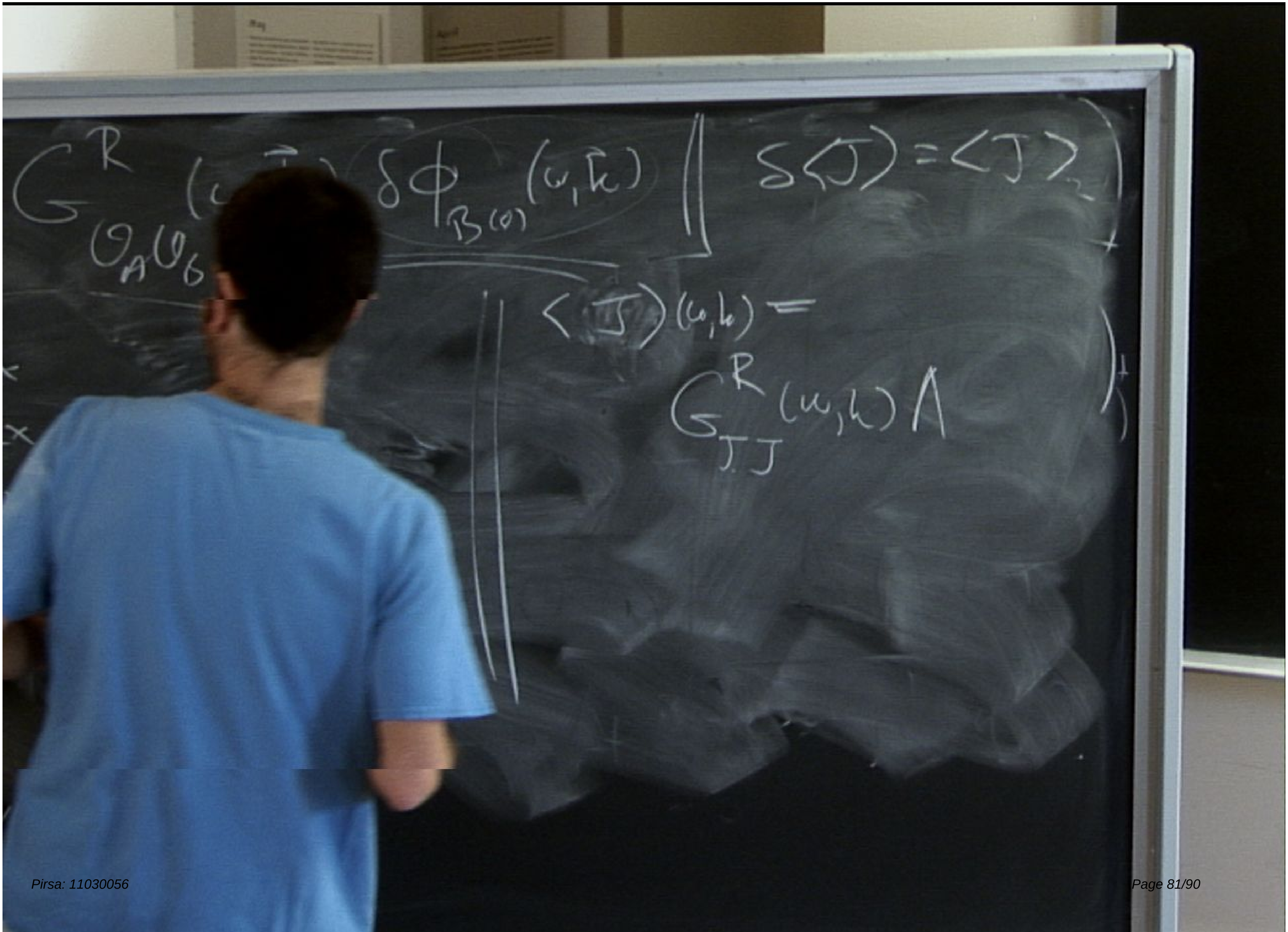
$$\delta\langle \mathcal{O}_A(\omega,\vec{k})\rangle = G^R_{\mathcal{O}_A\mathcal{O}_B}(\omega,\vec{k})\,\delta\phi_{B(0)}(\omega,\vec{k})$$

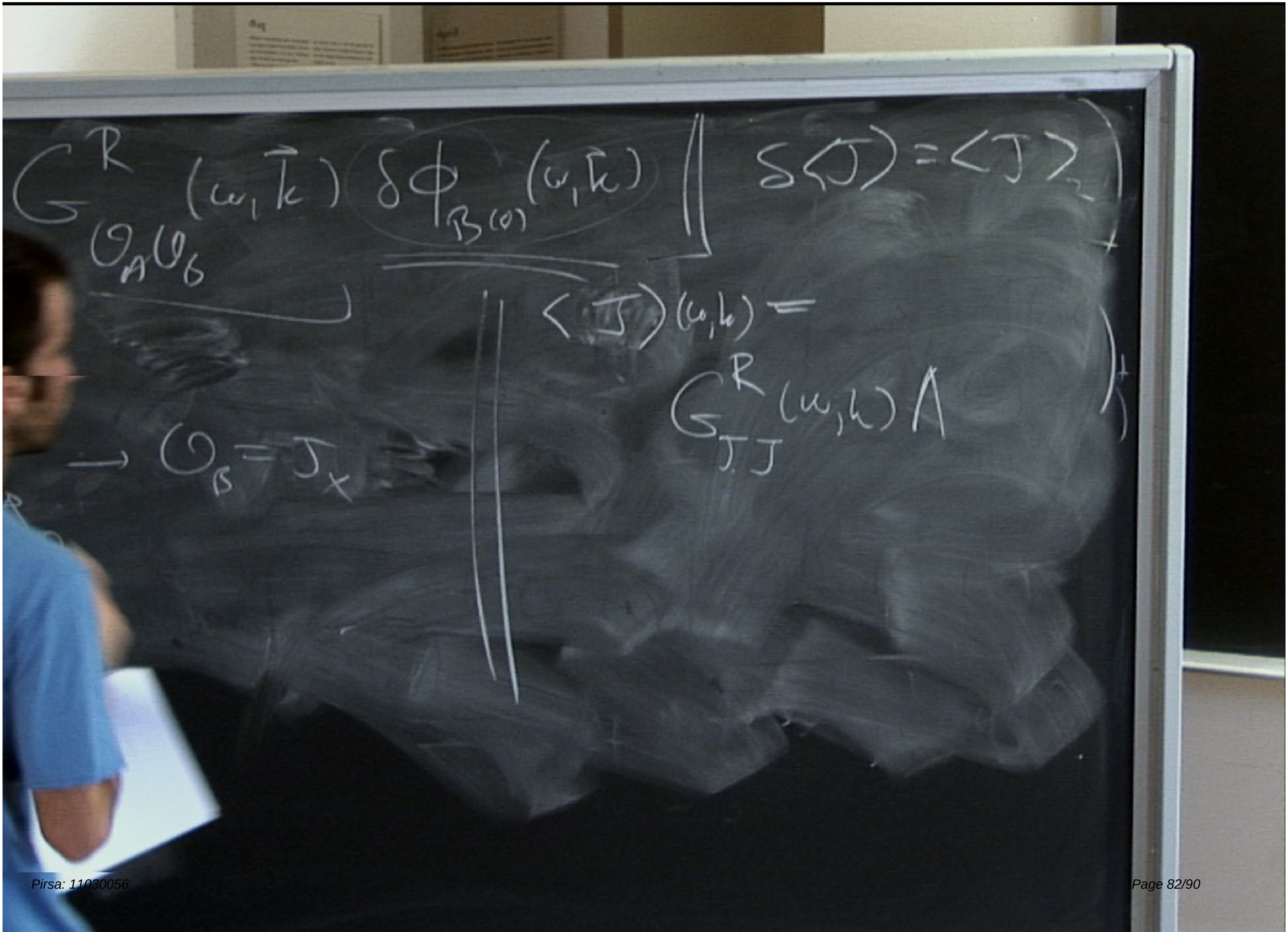
eg: perturbation: $E_x = i\omega A_x$

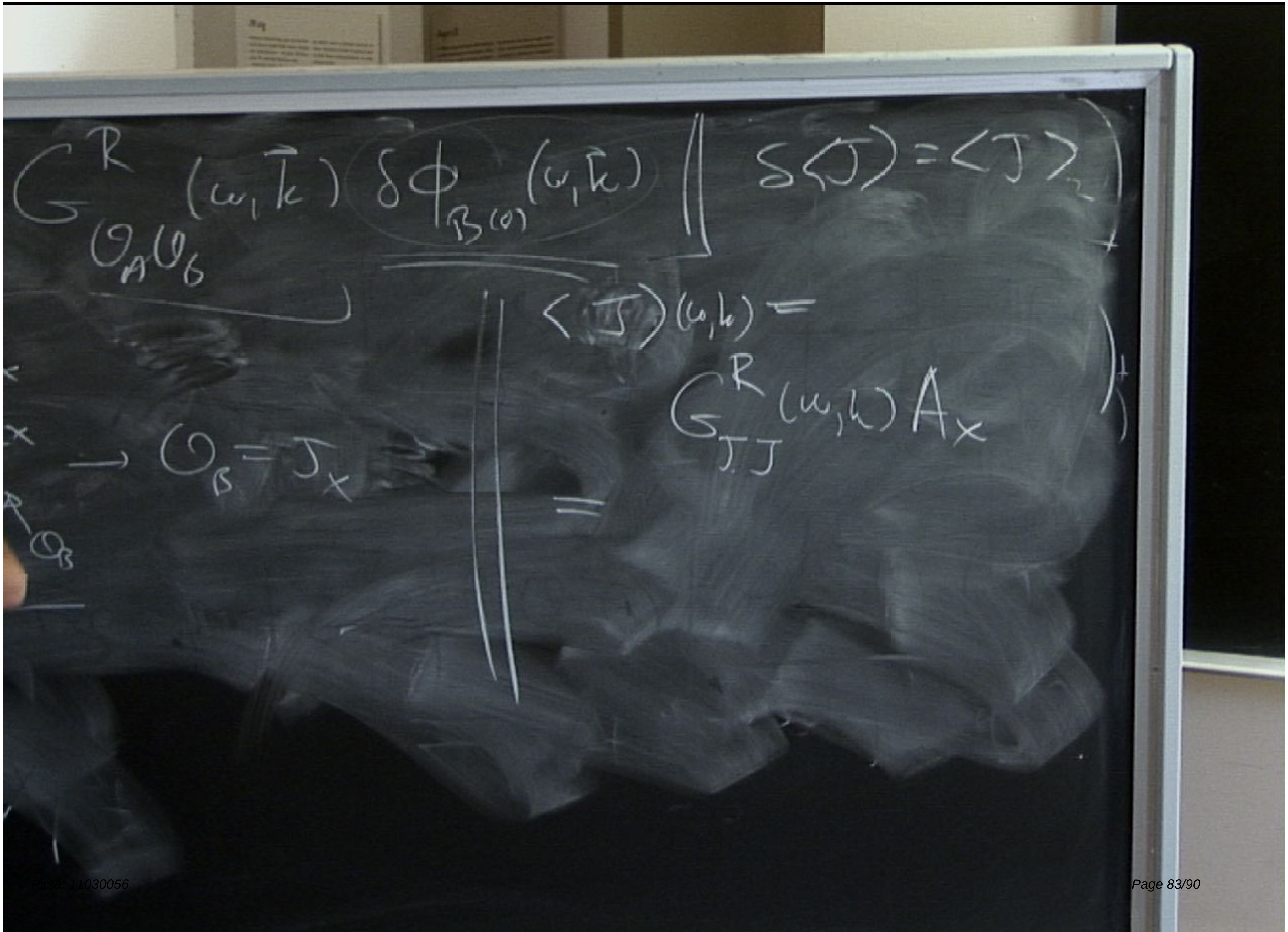
couples via $\delta H = \int \underbrace{A_x}_{\phi_B^{(0)}} \underbrace{J^x}_{\mathcal{O}_B} \longrightarrow \mathcal{O}_B = J_x$

response: $\mathcal{O}_A = J_x$

$\longrightarrow J = \sigma E$


δ⟨J⟩ = ⟨J⟩₂
⟨J⟩(ω,k) =
G^R_{JJ}(ω,k) Λ


$G^R_{O_A O_B}(\omega, \vec{k})\, \delta\phi_{B(0)}(\omega, \vec{k})$
$\delta\langle J\rangle = \langle J\rangle_2$
$\langle J\rangle(\omega, k) =$
$G^R_{J J}(\omega, k)\, A$
$\rightarrow O_B = J_x$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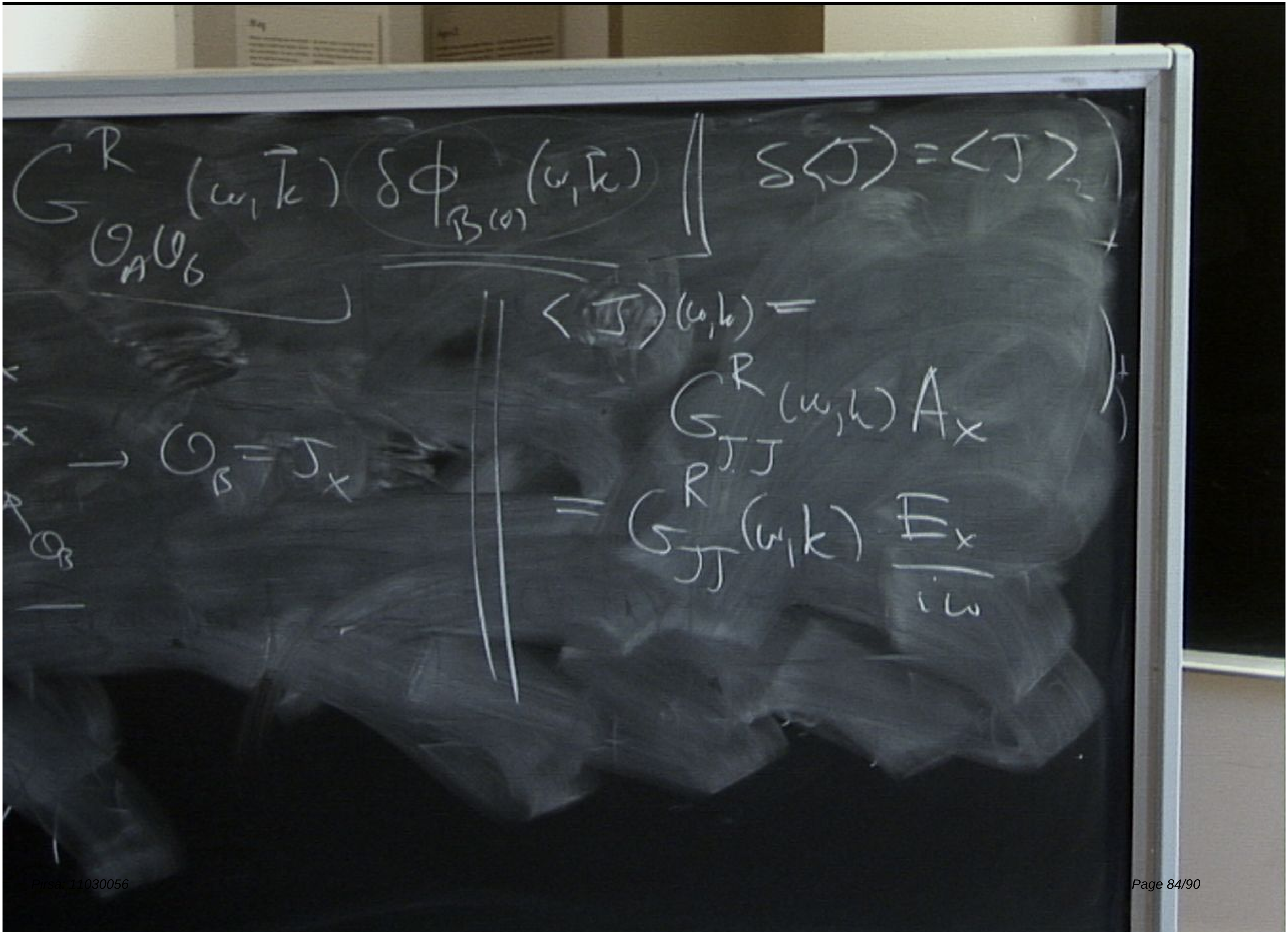
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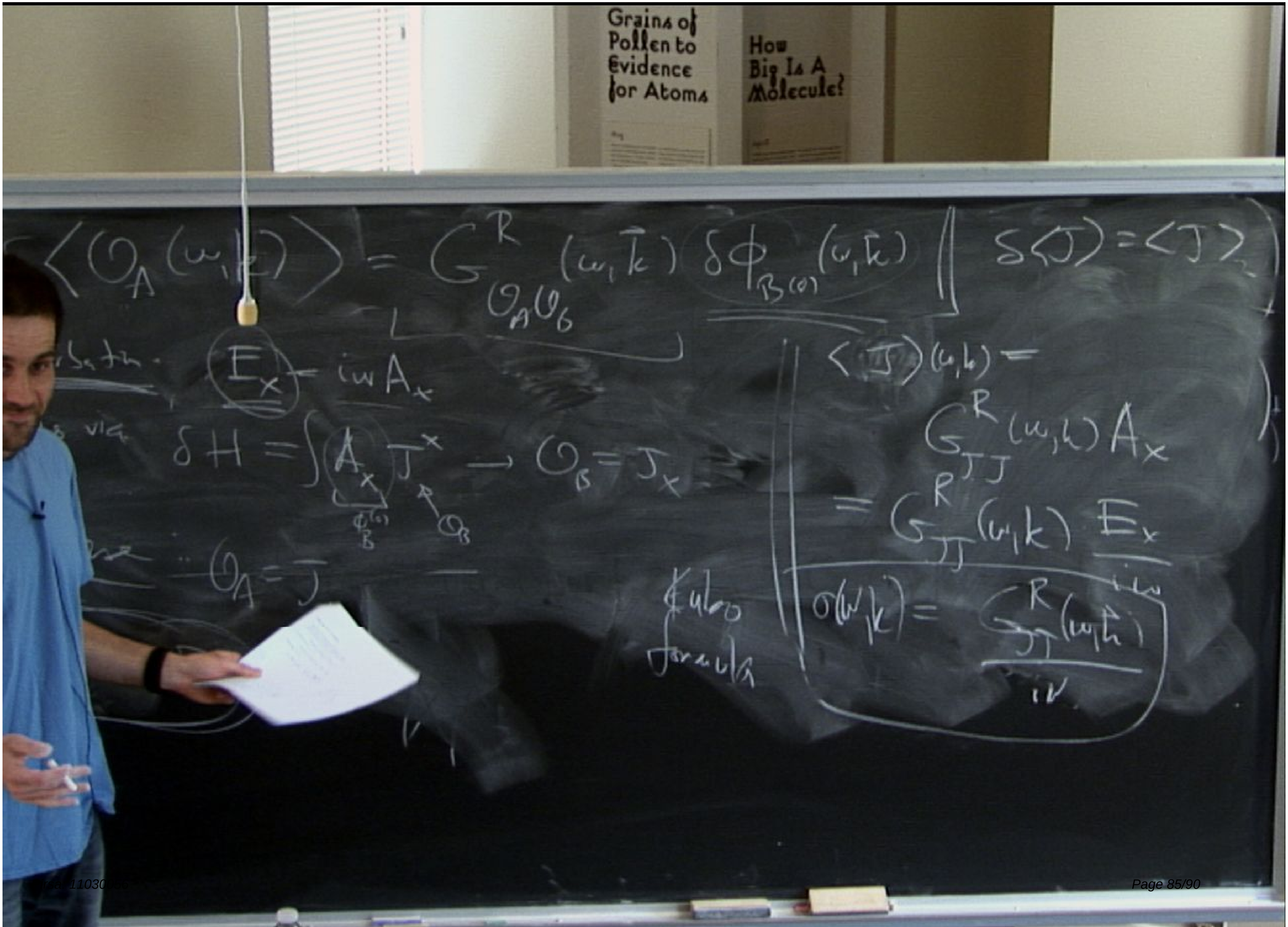
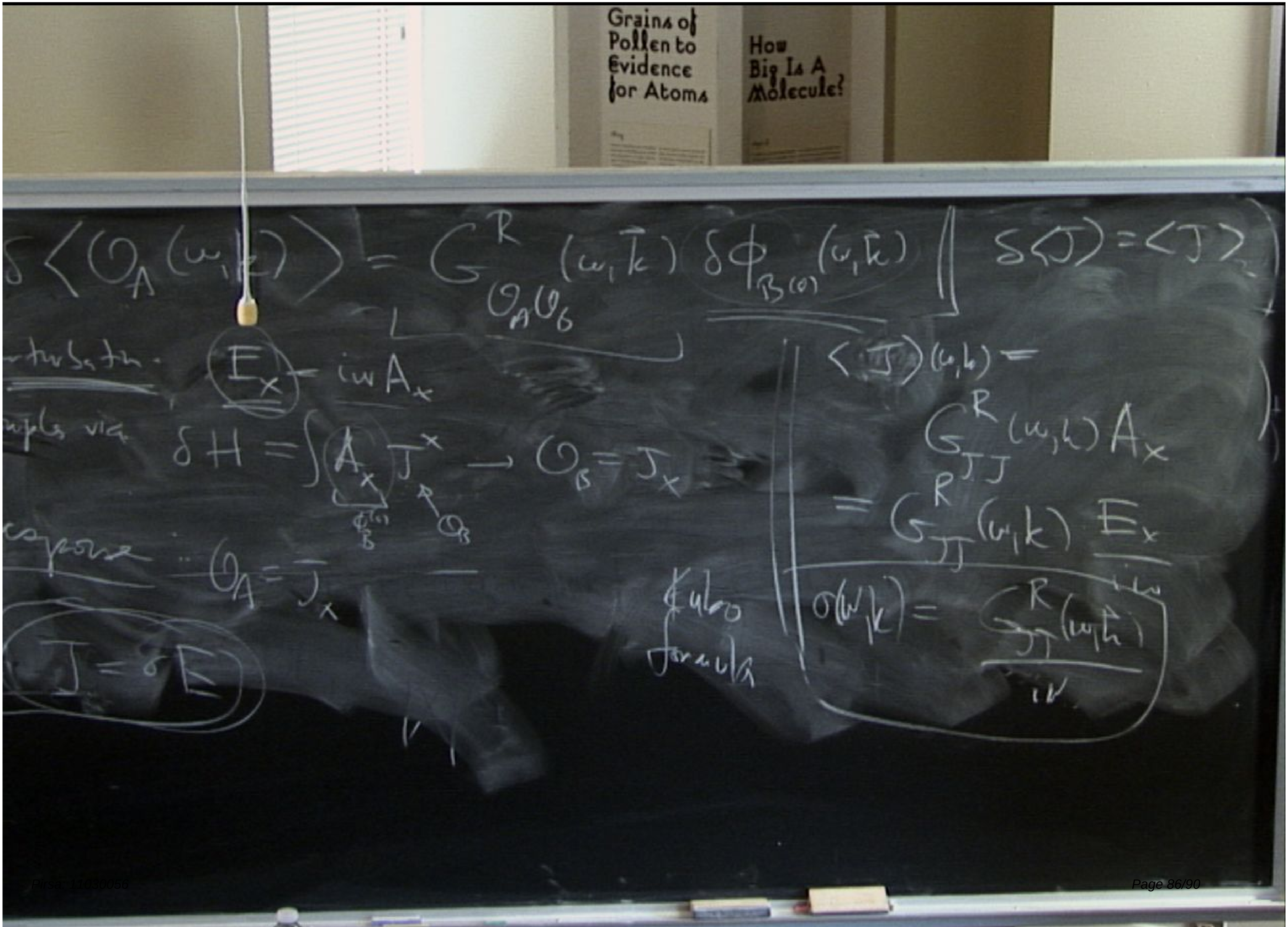
$G^R_{JJ}(\omega,k)\,A_x$
$= G^R_{JJ}(\omega,k)\,\frac{E_x}{i\omega}$


Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
Kubo formula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$\delta\langle \mathcal{O}_A(\omega,k)\rangle = G^R_{\mathcal{O}_A\mathcal{O}_B}(\omega,\vec{k})\,\delta\phi_{B(0)}(\omega,\vec{k})$
$\delta\langle J\rangle = \langle J\rangle_1$
perturbation: $E_x = i\omega A_x$
couples via $\delta H = \int A_x J^x \longrightarrow \mathcal{O}_B = J_x$
response: $\mathcal{O}_A = J_x$
$J = \sigma E$
$\langle J\rangle(\omega,k) = G^R_{JJ}(\omega,k)\,A_x = G^R_{JJ}(\omega,k)\,E_x$
$\sigma(\omega,k) = \frac{G^R_{JJ}(\omega,k)}{i\omega}$
Kubo formula

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$$A = J_x$$

$$\langle J\rangle(\omega, k) = G^R_{JJ}(\omega, k)\, A_x = G^R_{JJ}(\omega, k)\, \frac{E_x}{i\omega}$$

Kubo formula

$$\sigma(\omega, k) = \frac{G^R_{JJ}(\omega, k)}{i\omega}$$

Holographic real-time prescription

which sol'n

claim: the sol'n corresponding to retarded response is the one describing stuff falling IN to the horizon

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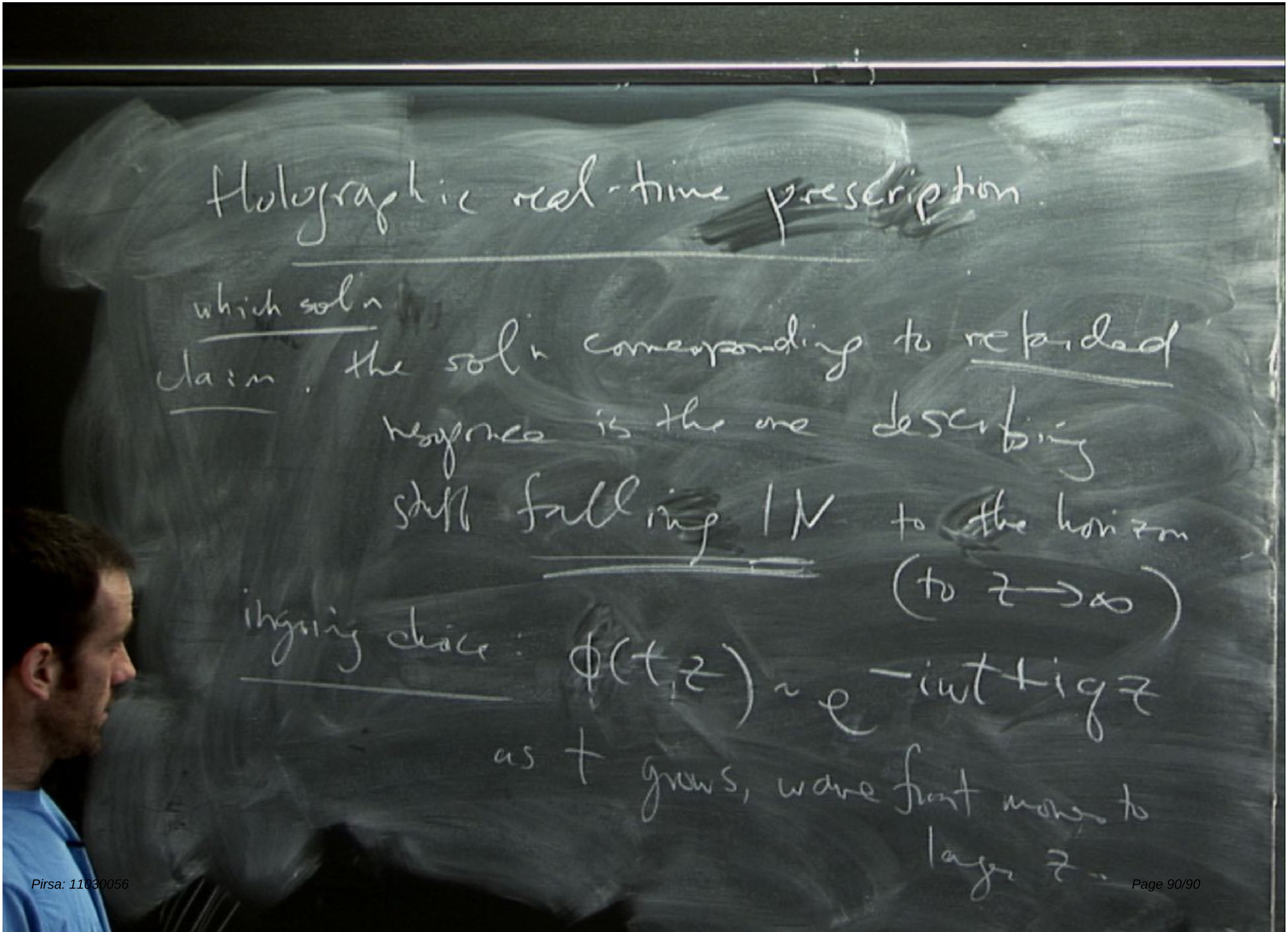
which sol'n

claim: the sol'n corresponding to retarded response is the one describing stuff falling IN to the horizon (to $z \to \infty$)


Holographic real-time prescription
which sol'n
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stuff falling IN to the horizon
(to $z \to \infty$)
ingoing choice: $\phi(t,z) \sim e^{-i\omega t + iqz}$
as t grows, wave front moves to
larger z.