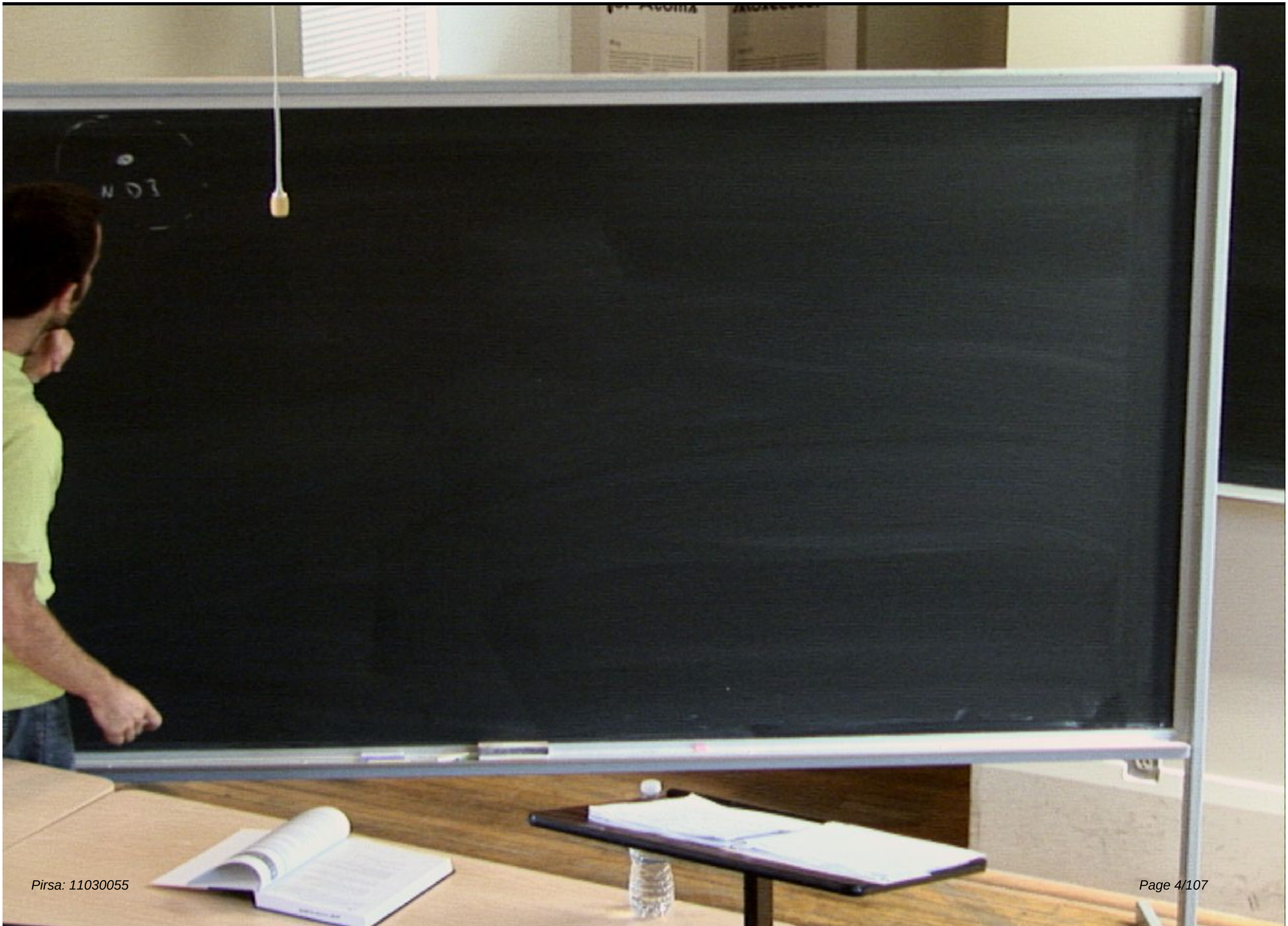
Title: Explorations in String Theory - Lecture 6

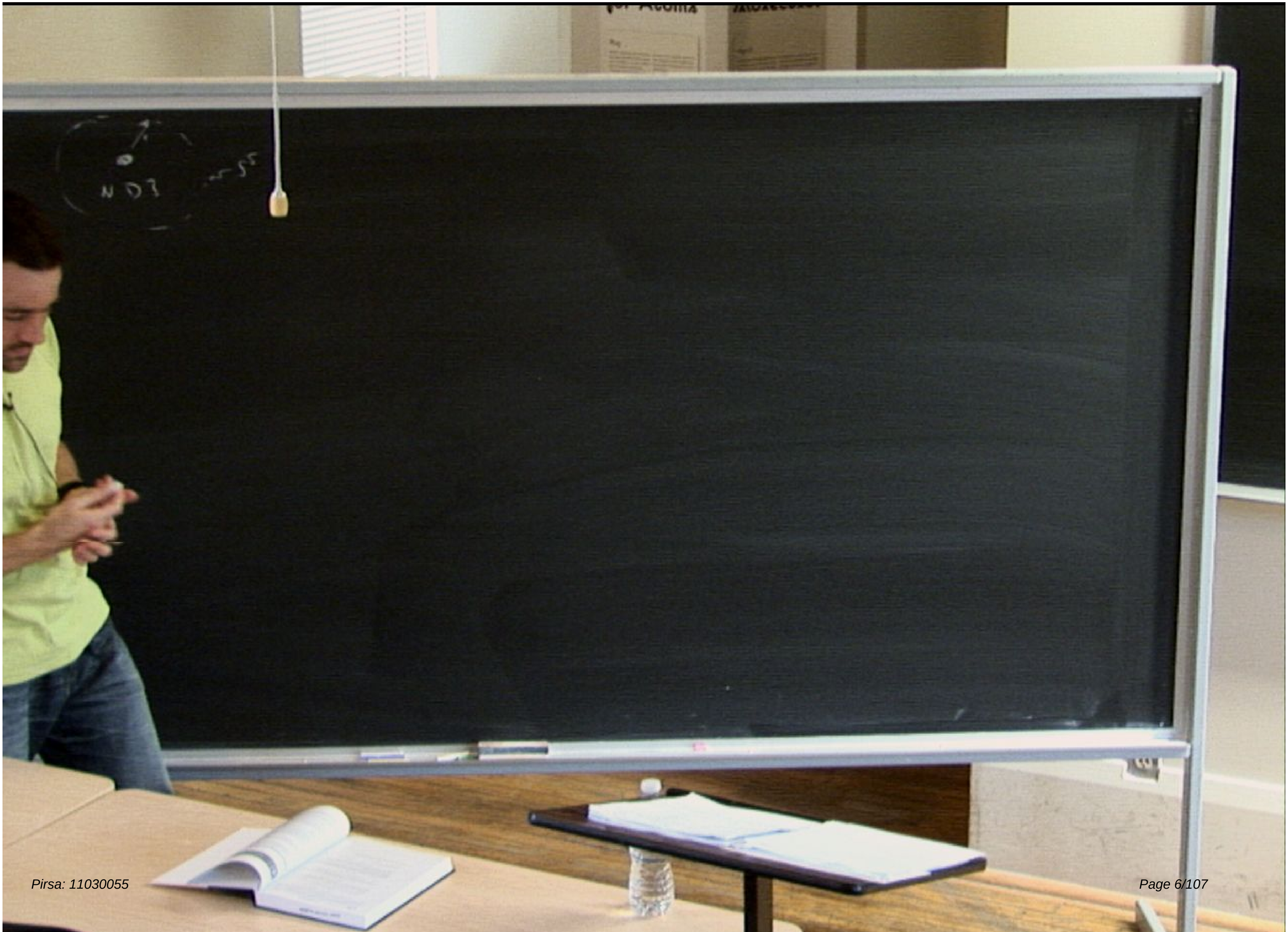
Date: Mar 21, 2011 11:30 AM

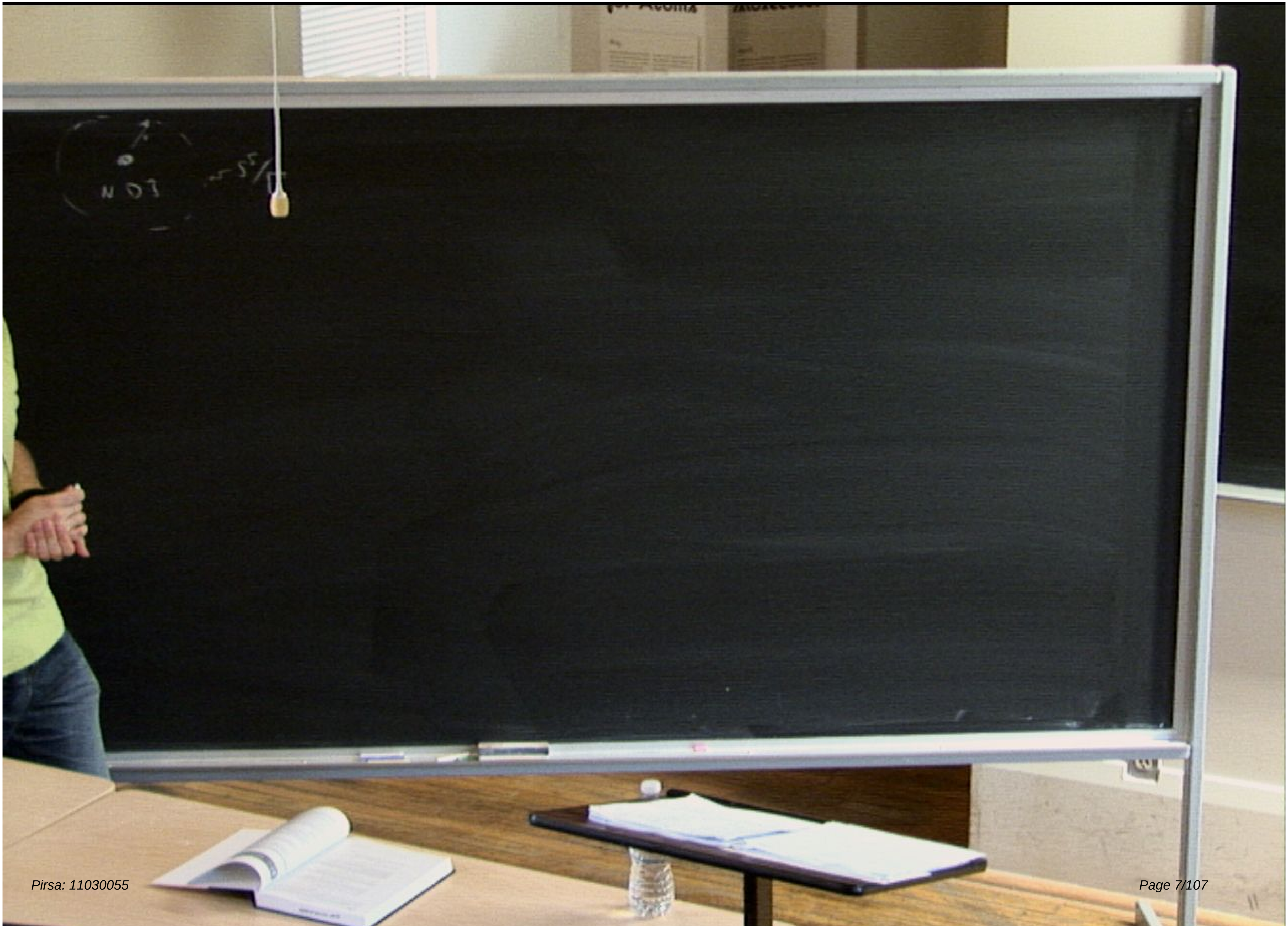
URL: http://pirsa.org/11030055

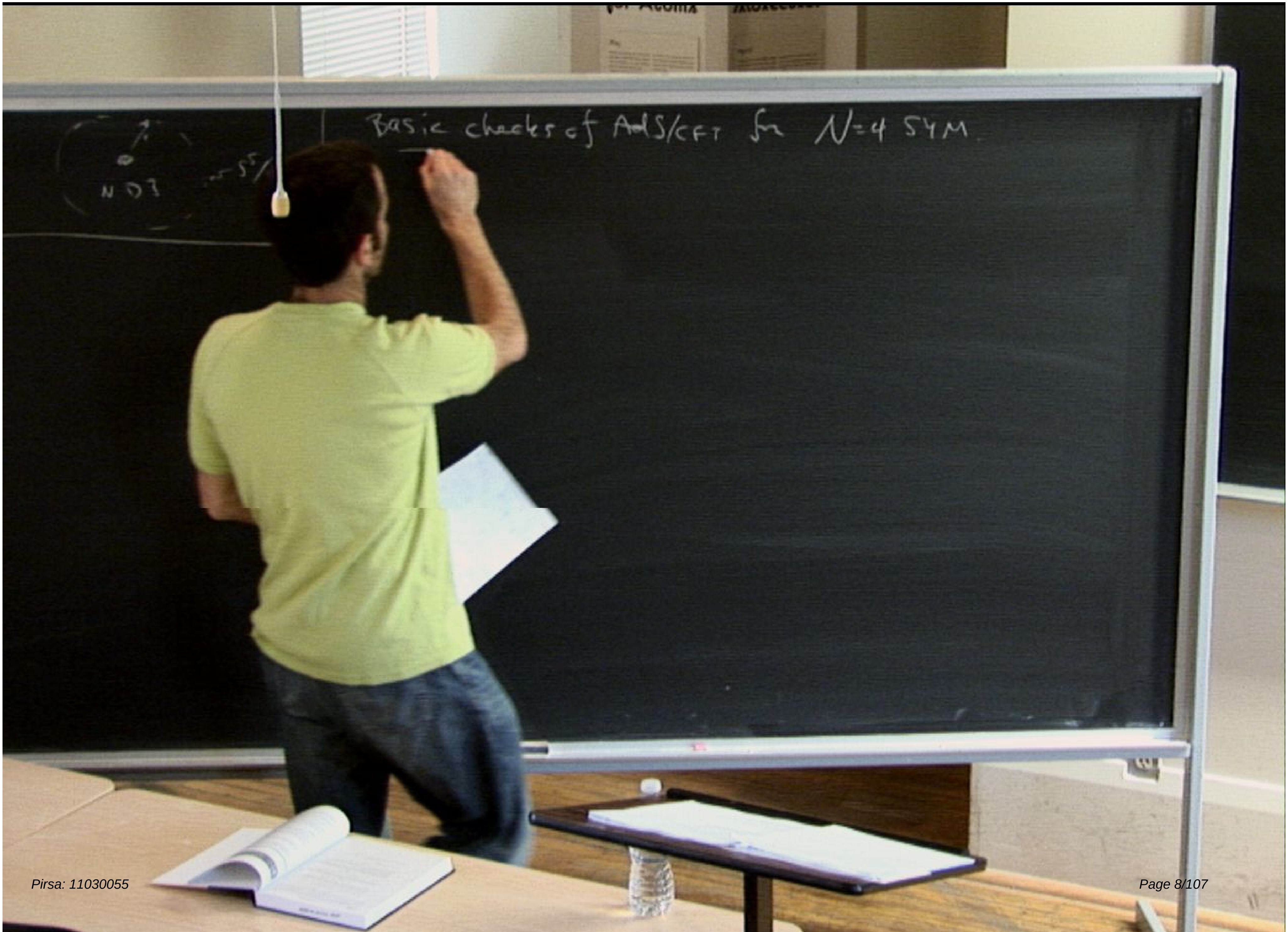
Abstract:

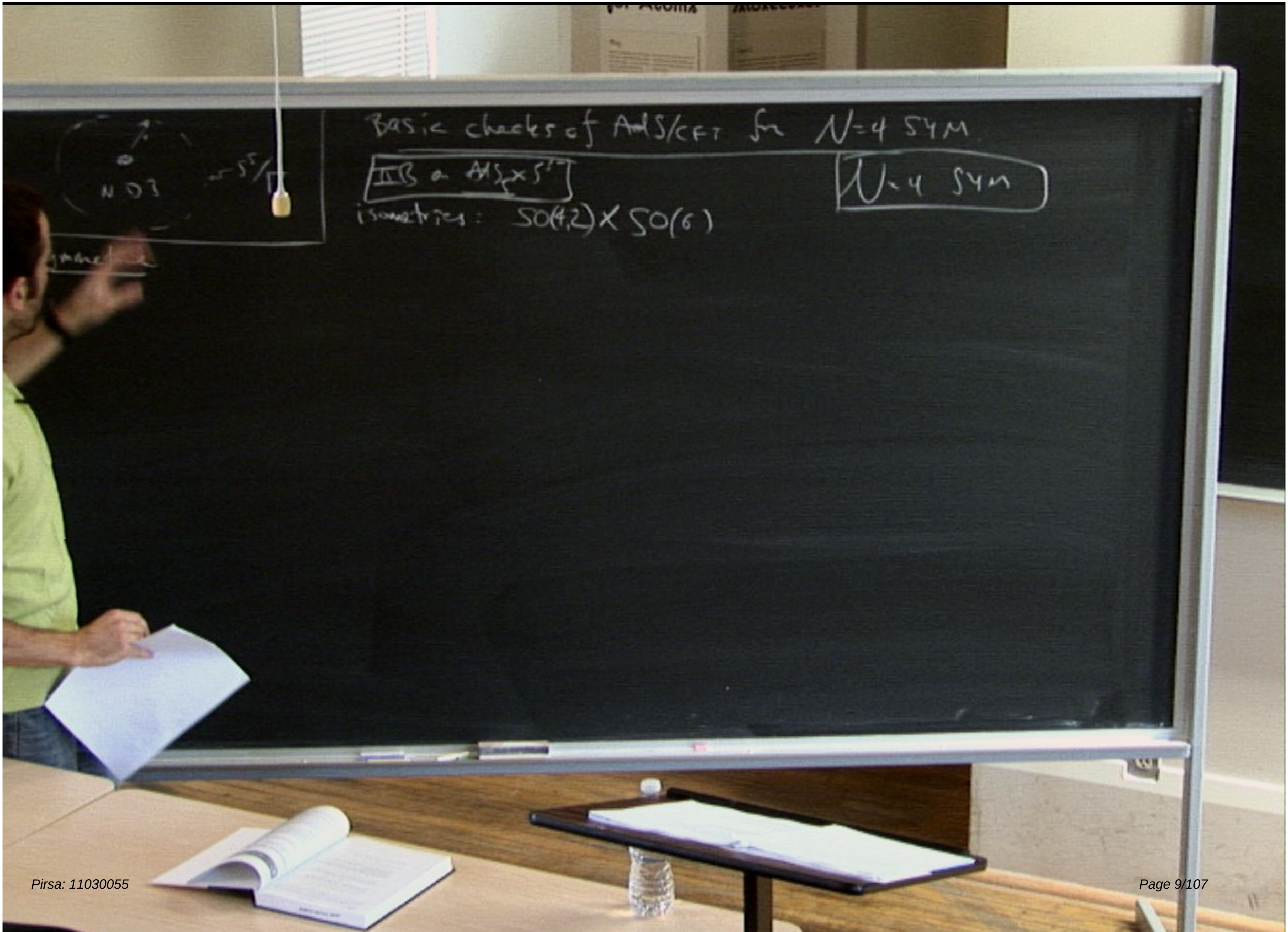
perimeter scholars
international

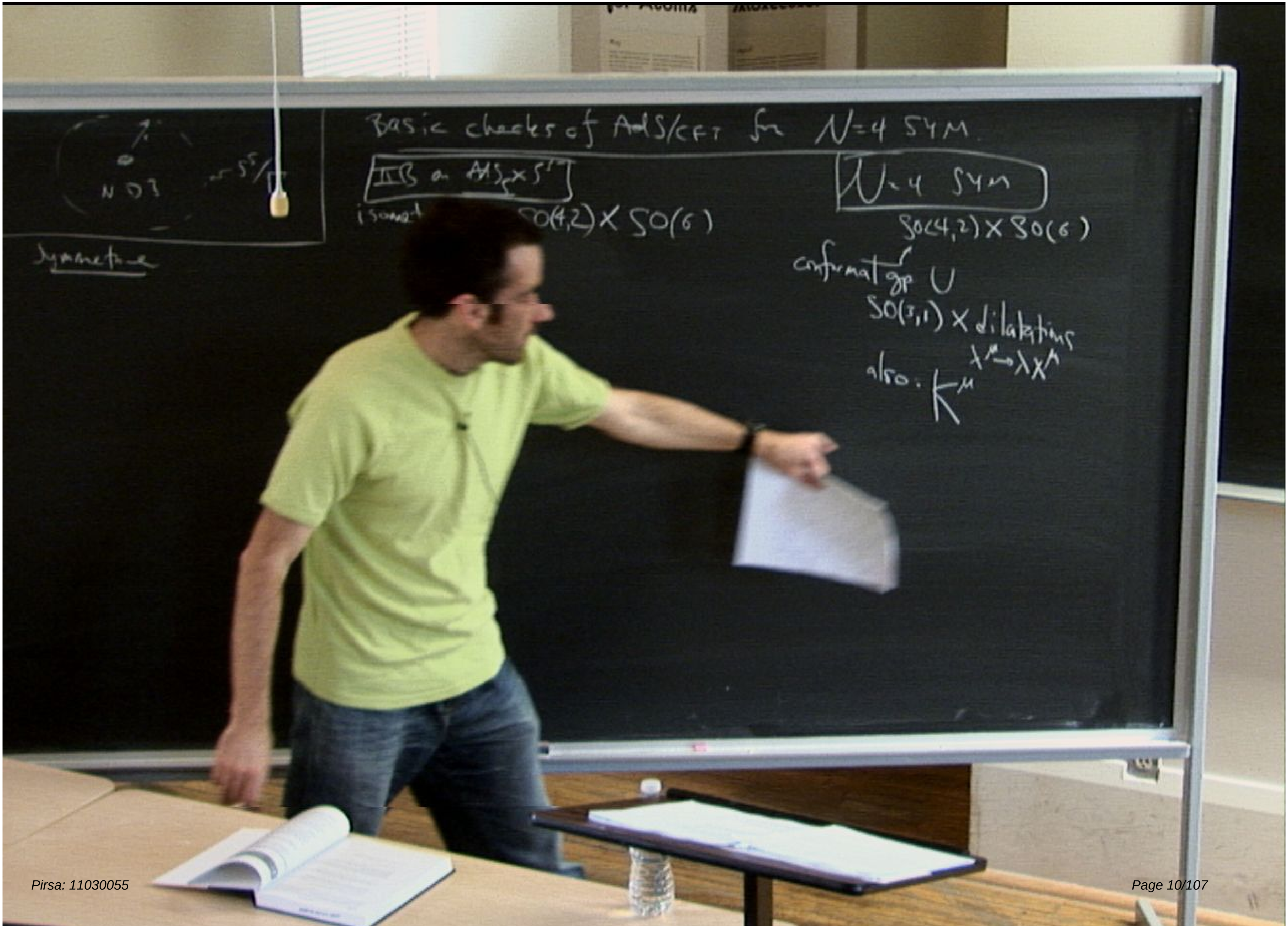




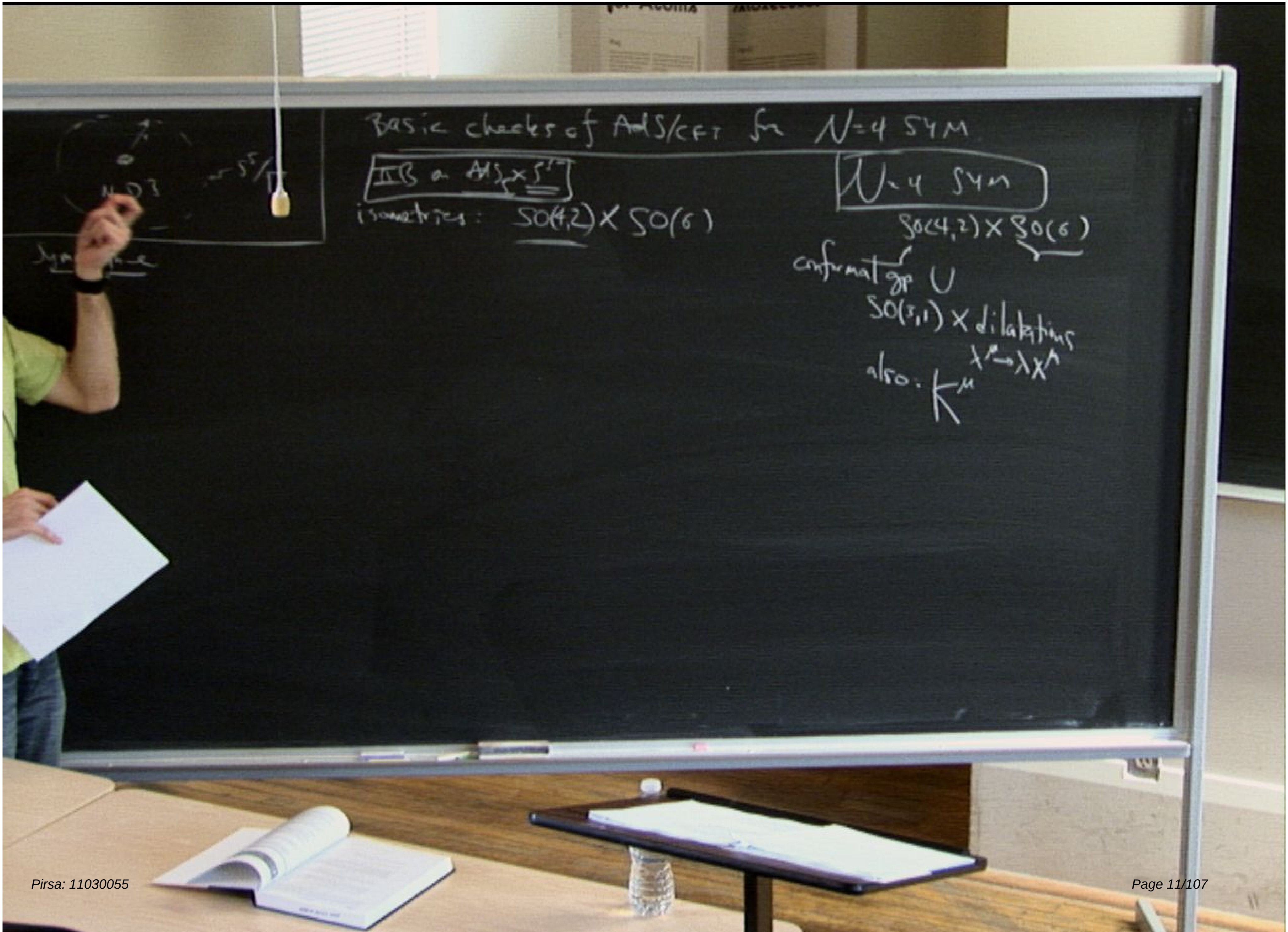



Basic checks of AdS/CFT for N=4 SYM
N D3


Basic checks of AdS/CFT for N=4 SYM
IIB on AdS5×S5
N=4 SYM
isometries: SO(4,2)×SO(6)
N D3



Basic checks of AdS/CFT for N=4 SYM
IIB on AdS5 x S5
N=4 SYM
SO(4,2) x SO(6)
SO(4,2) x SO(6)
conformal gp
SO(3,1) x dilatations
also: K^μ
symmetries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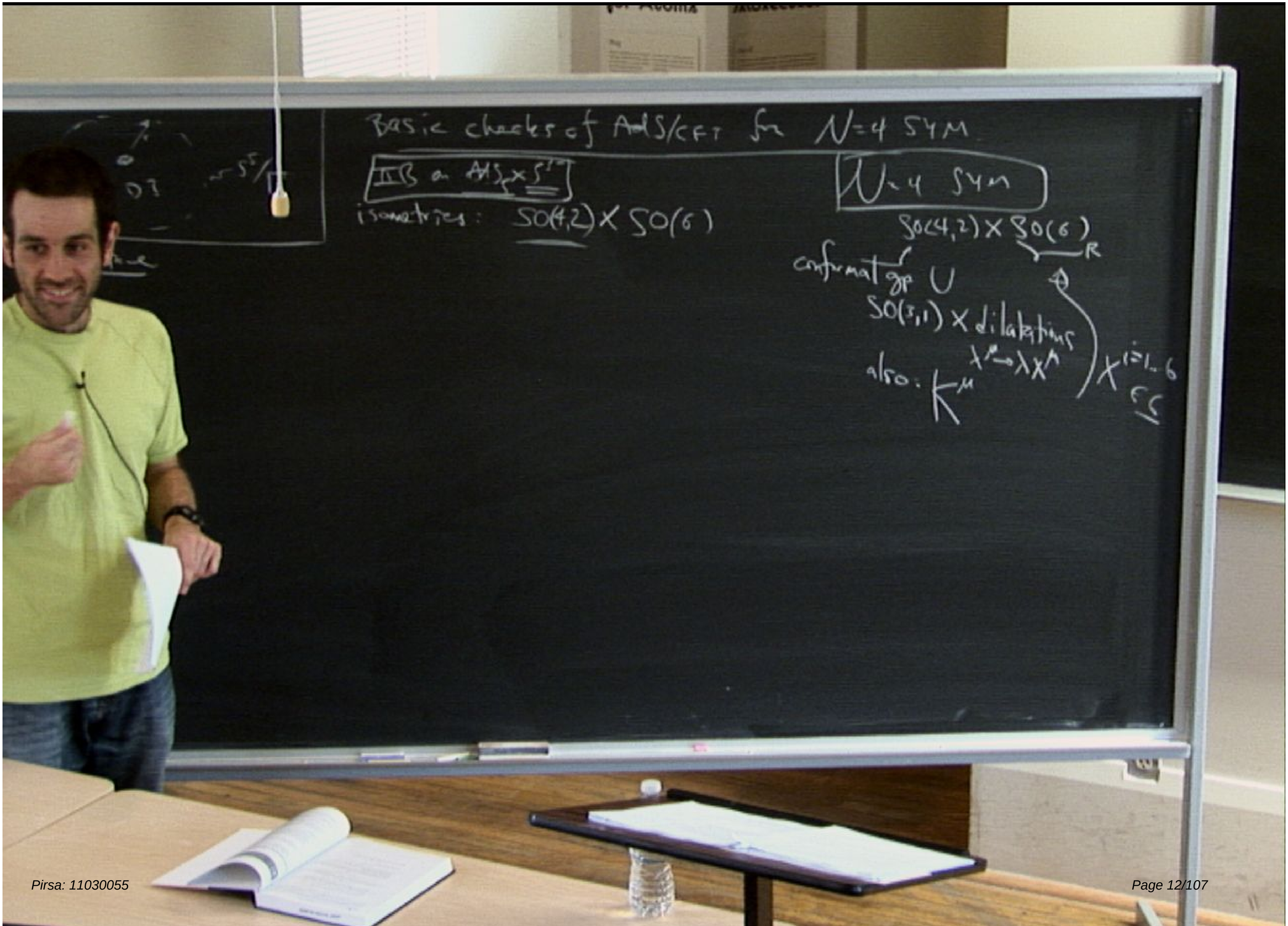
x^μ → λx^μ
also: K^μ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SO(4,2) x SO(6)
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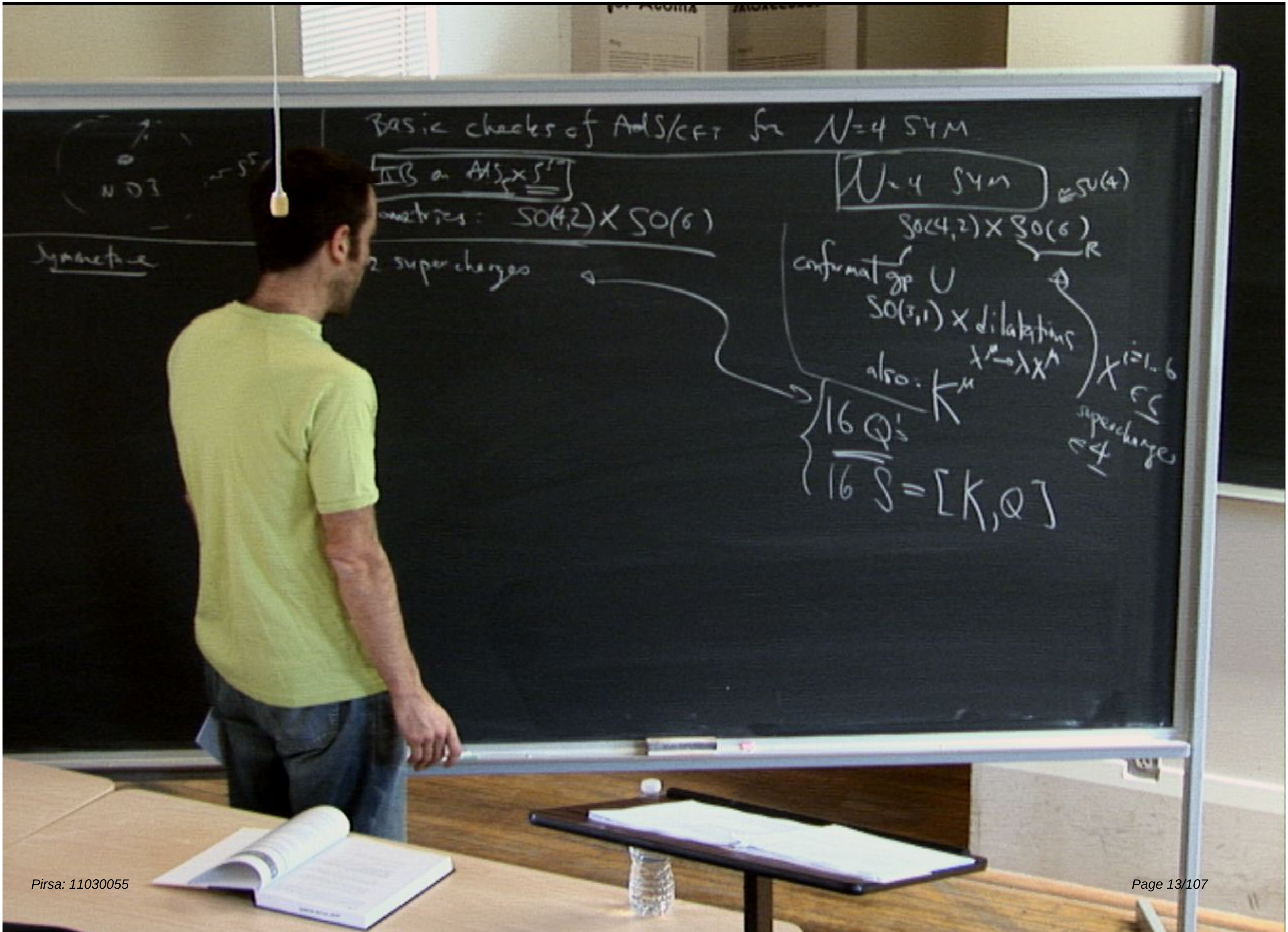

Basic checks of AdS/CFT for N=4 SYM
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N=4 SYM
SU(4)
symmetries: SO(4,2) X SO(6)
SO(4,2) X SO(6)
R
32 supercharges
conformal gp U
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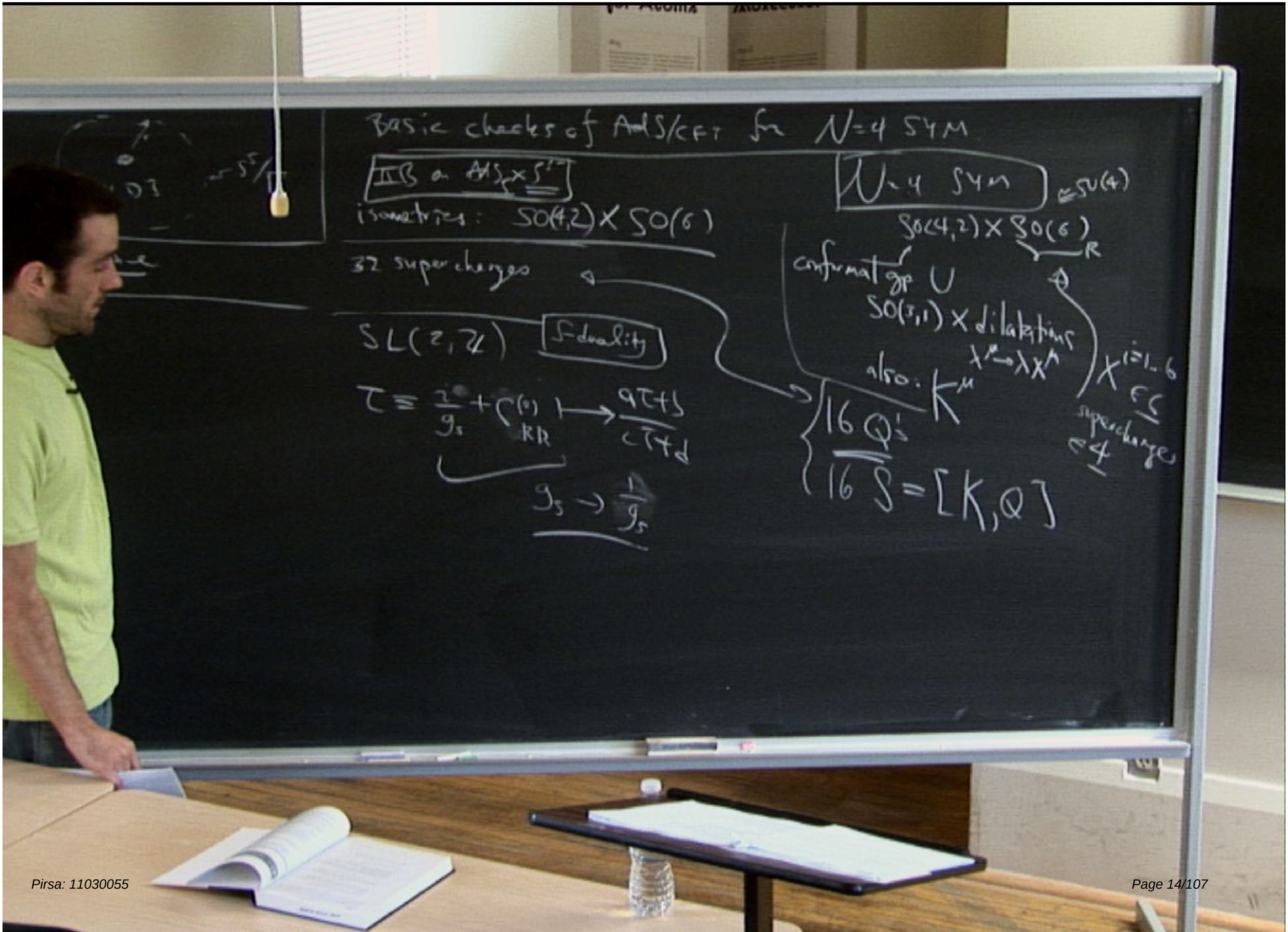
also: K
16 Q's
16 S = [K,Q]
supercharges
Symmetries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also: K
SL(2,Z)
S-duality
16 Q's
16 S=[K,Q]
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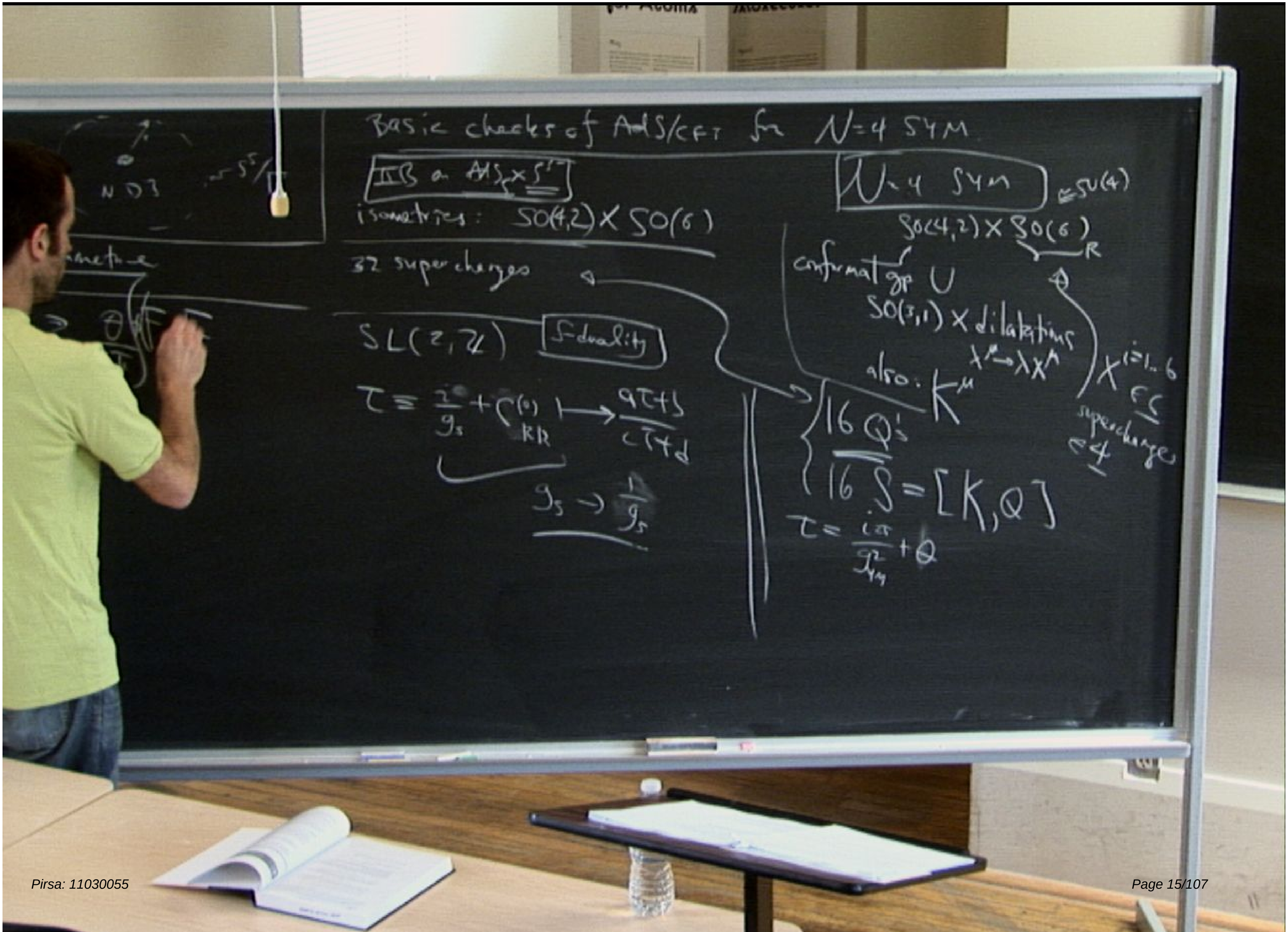
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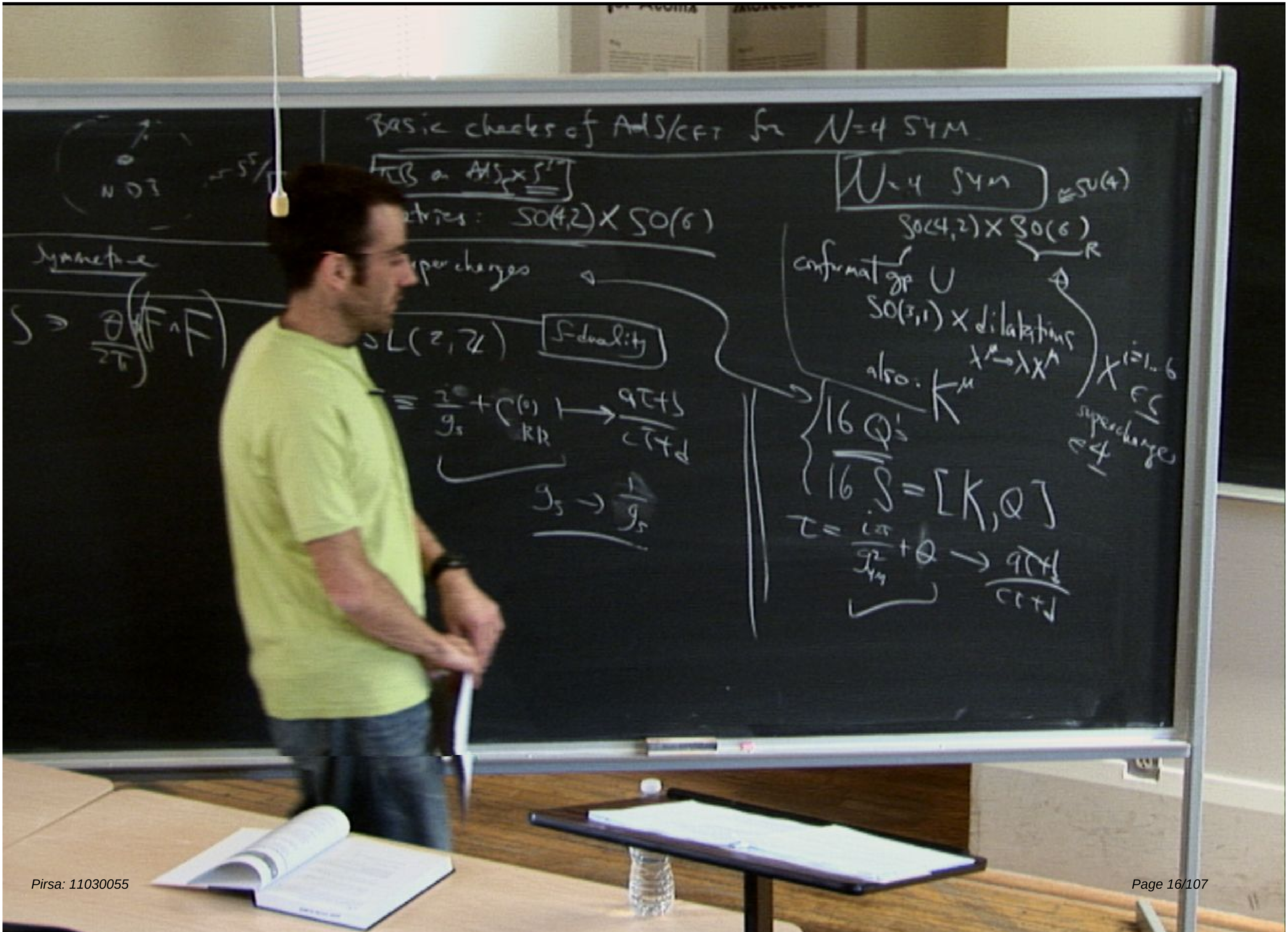
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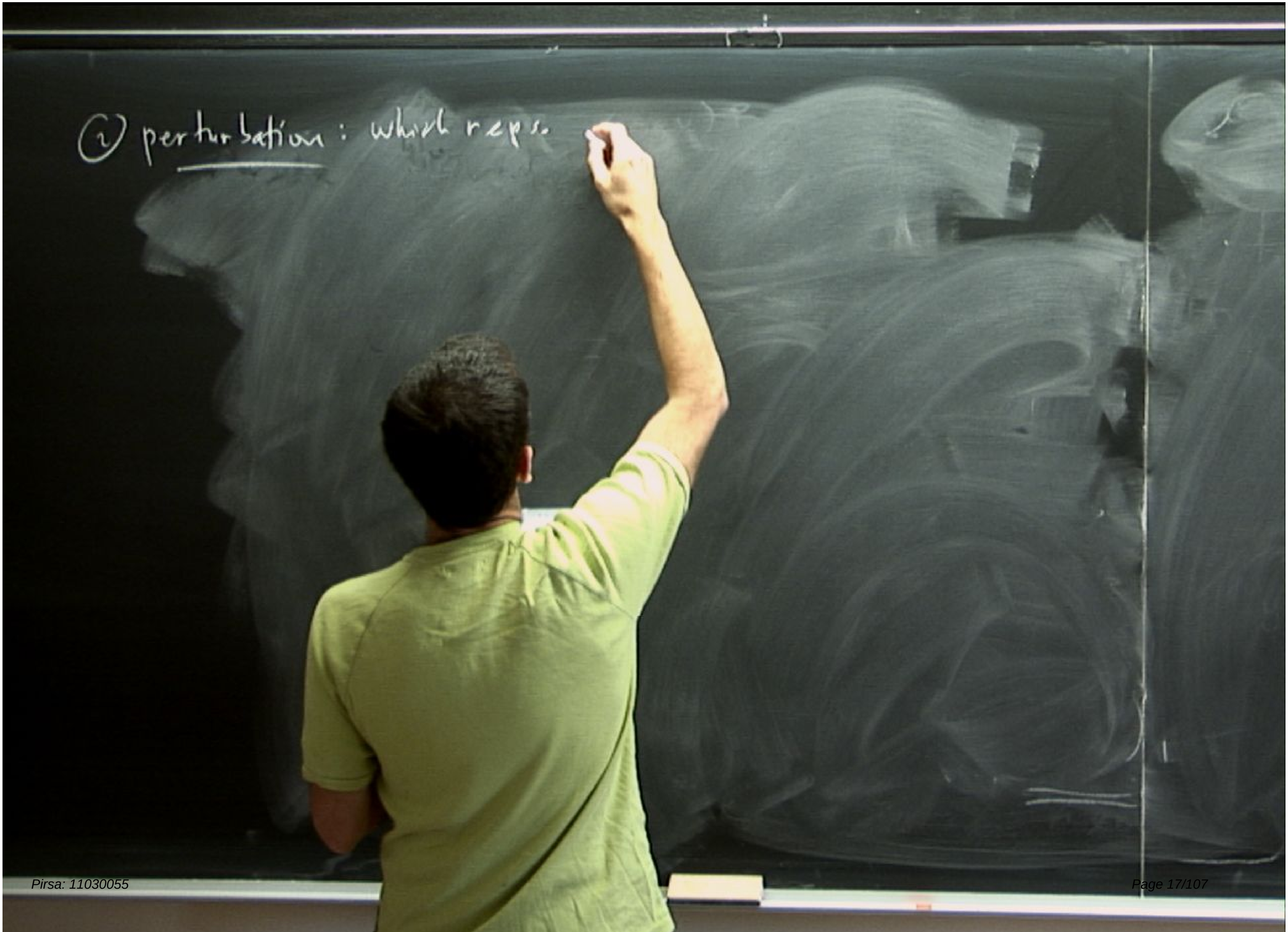
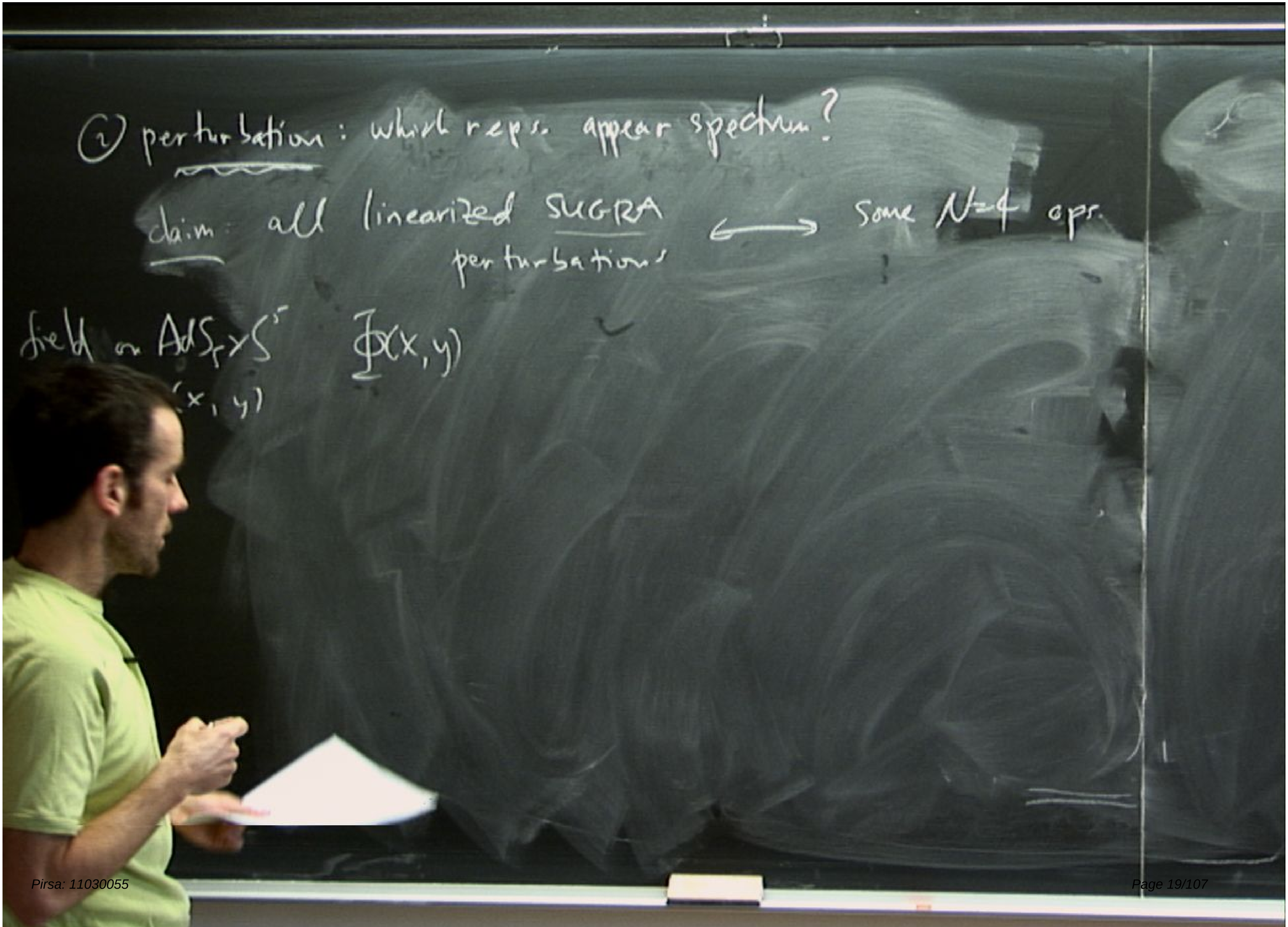
Symmetries


② perturbation : which reps.

(2) perturbation: which reps. appear spectrum?

claim: all linearized SUGRA perturbations $\longleftrightarrow$ some $\mathcal{N}=4$ ops.



② perturbation: which reps. appear spectrum?
claim: all linearized SUGRA perturbations ⟷ some N=4 ops.
field on AdS5×S5
Φ(x,y)
(x,y)

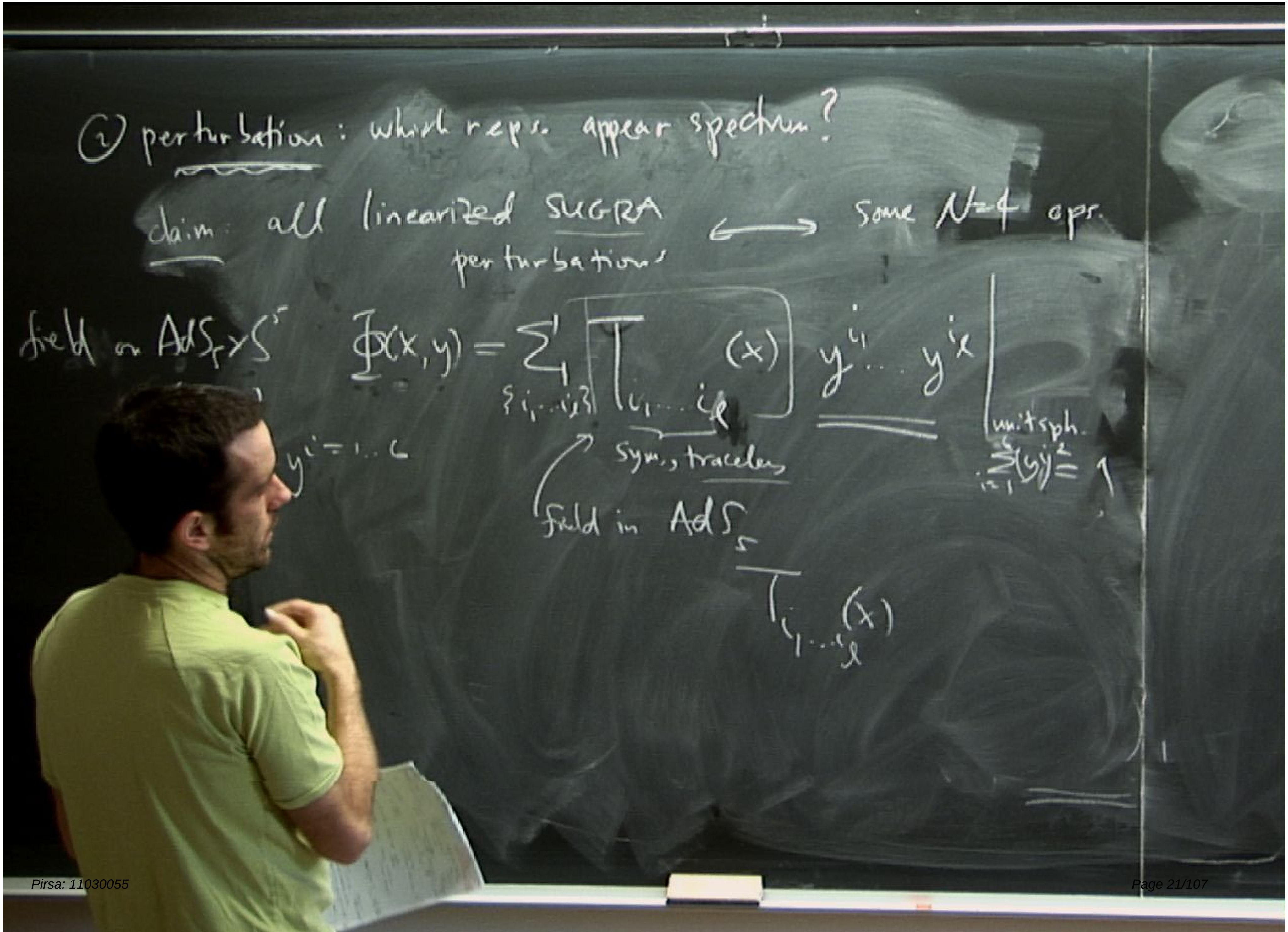
(2) perturbation: which reps. appear spectrum?

claim: all linearized SUGRA perturbations $\longleftrightarrow$ some $\mathcal{N}=4$ ops.

field on $AdS_5 \times S^5$ (x, y)

$$\Phi(x,y) = \sum_{\{i_1 \dots i_\ell\}}' \underbrace{T_{i_1 \dots i_\ell}}_{\text{sym., traceless}}(x)\, y^{i_1} \dots y^{i_\ell} \Big|_{\text{unit sph.}\ \sum_{i=1}^{6} (y^i)^2 = 1}$$

$y^{i=1 \dots 6}$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field on $AdS_5 \times S^5$
$\Phi(x,y) = \sum'_{\{i_1 \dots i_\ell\}} T_{i_1 \dots i_\ell}(x)\, y^{i_1} \dots y^{i_\ell}$
unit sph.
$\sum_{i=1}^{6} (y^i)^2 = 1$
$y^i = 1 \dots 6$
sym., traceless
field in AdS_5
$T_{i_1 \dots i_\ell}(x)$

(2) perturbation: which reps. appear spectrum?

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field on $AdS_5 \times S^5$

$(x, \ldots$

$$\Phi(x,y) = \sum_{\{i_1 \ldots i_l\}}' \boxed{\overline{T}_{i_1 \ldots i_l}(x)}\, y^{i_1} \ldots y^{i_l} \Big|_{\text{unit sph. } \sum_{i=1}^{6} (y^i)^2 = 1}$$

sym., traceless

field in AdS_5

$$\overline{T}_{i_1 \ldots i_l}(x)$$

$$m_{5d}(T) \sim \frac{l}{L}$$

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$y^i = 1 \ldots 6$

sym., traceless

field in AdS_5

$\sum_{i=1}^{6} (y^i)^2 = 1$

$$T_{i_1 \ldots i_\ell}(x)$$

$$\boxed{m^2_{5d}(T) \sim \left(\frac{\ell}{L}\right)^2}$$

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field in AdS_5

unit sph. $\sum_{i=1}^{6} (y^i)^2 =$

$T_{i_1 \ldots i_l}(x)$

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...s. appear spectrum?

SUGRA $\longleftrightarrow$ some $\mathcal{N}=4$ opr.

...urbations

$$\sum_{\{i_1 \dots i_\ell\}}' \boxed{T_{i_1 \dots i_\ell}(x)}\; y^{i_1} \cdots y^{i_\ell}$$

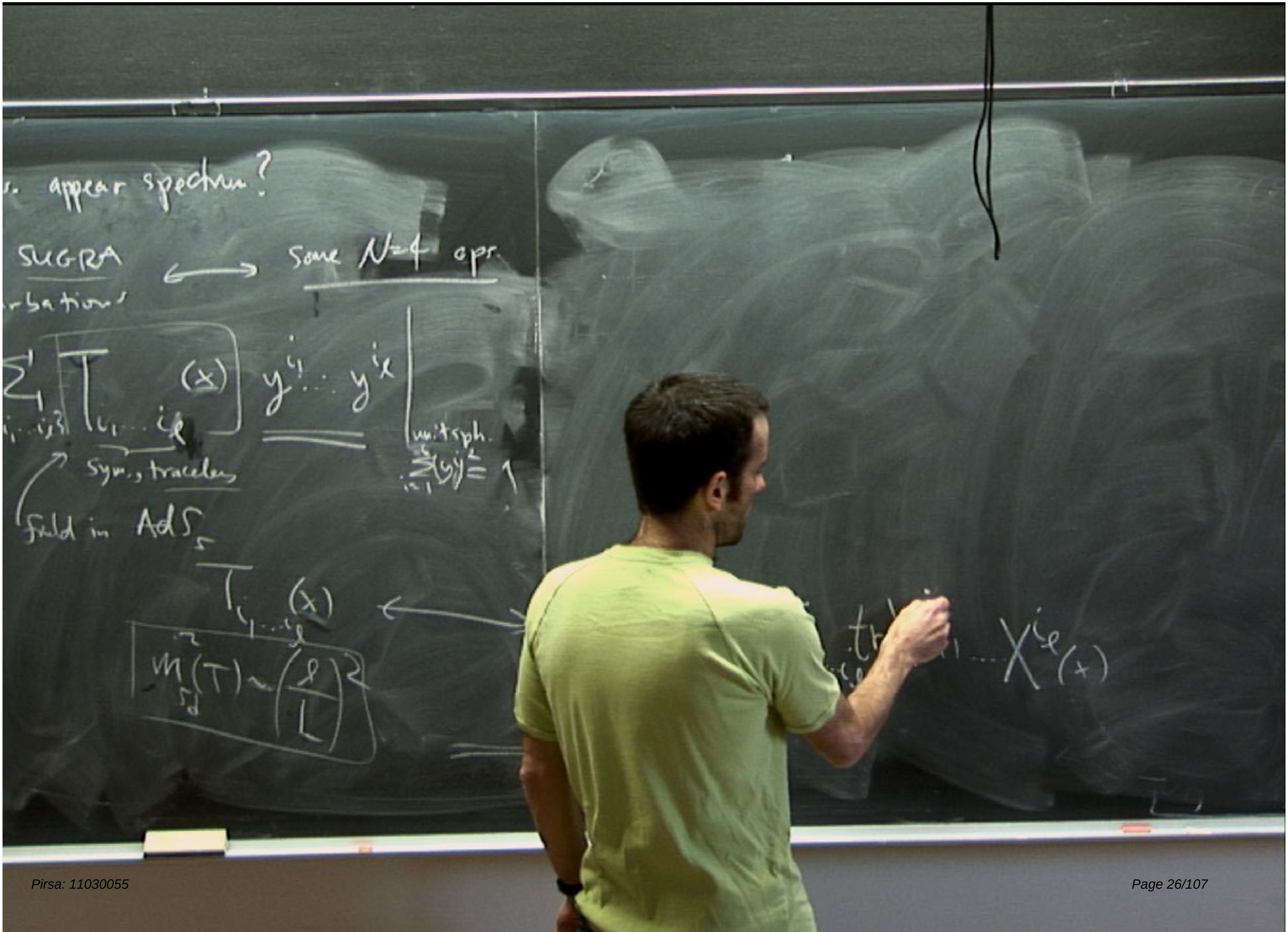
sym., traceless

field in AdS_5

$T_{i_1 \dots}$

$m^2_{5d}(T) \sim$

$$\mathcal{O}_T(x^{\mu=0\text{-}3}) = T_{i_1 \dots i_\ell}\, \mathrm{tr}\, X^{i_1}(x) \cdots X^{i_\ell}(x)$$


appear spectrum?
SUGRA
Some N=4 opr.
rbations
sym., traceless
unit sph.
field in AdS5

.. appear spectrum?

SUGRA $\longleftrightarrow$ Some $N=4$ opr.

-rbations

$$\sum_{\{i_1 \ldots i_\ell\}}' \boxed{T_{i_1 \ldots i_\ell}(x)}\; y^{i_1} \cdots y^{i_\ell} \Big|_{\text{unit sph.}}$$

sym., traceless

$$\sum_{i=1}^{6} (y^i)^2 = 1$$

.d in AdS_5

$$T_{i_1 \ldots i_\ell}(x) \longleftrightarrow \mathcal{O}_T(x^{\mu=0..3}) = T_{i_1 \ldots i_\ell}\, \mathrm{tr}\, X^{i_1}(x) \cdots X^{i_\ell}(x)$$

$$\boxed{m^2_{S_5}(T) \sim \left(\frac{\ell}{L}\right)^2}$$

.. appear spectrum?

SUGRA $\longleftrightarrow$ some $N=4$ opr.

..rbations

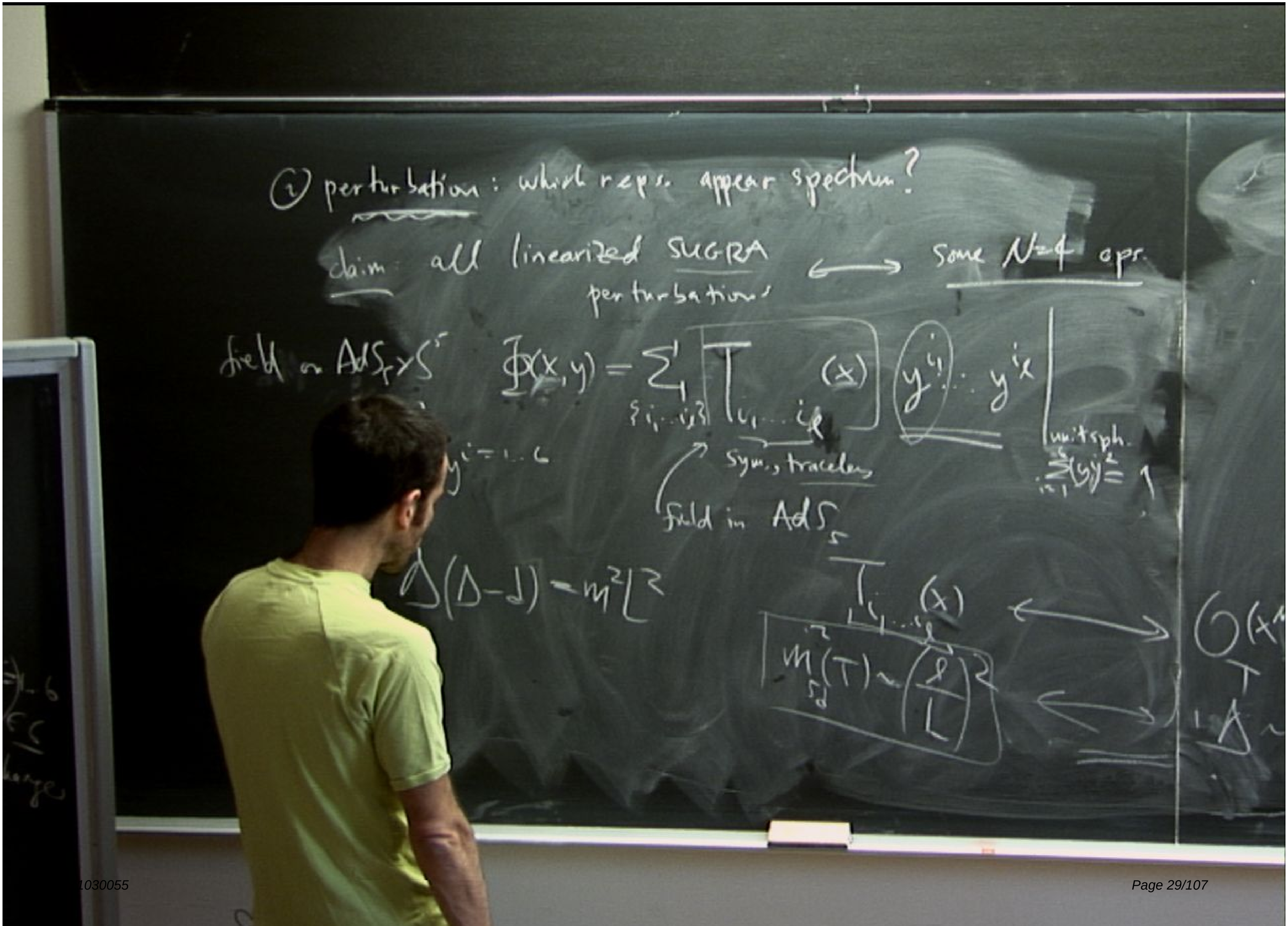
$\sum_{\{i_1 \dots i_\ell\}}' T_{i_1 \dots i_\ell}(x)\; y^{i_1} \cdots y^{i_\ell}$ unit sph. $\sum_{i=1}^{6} (y^i)^2 = 1$

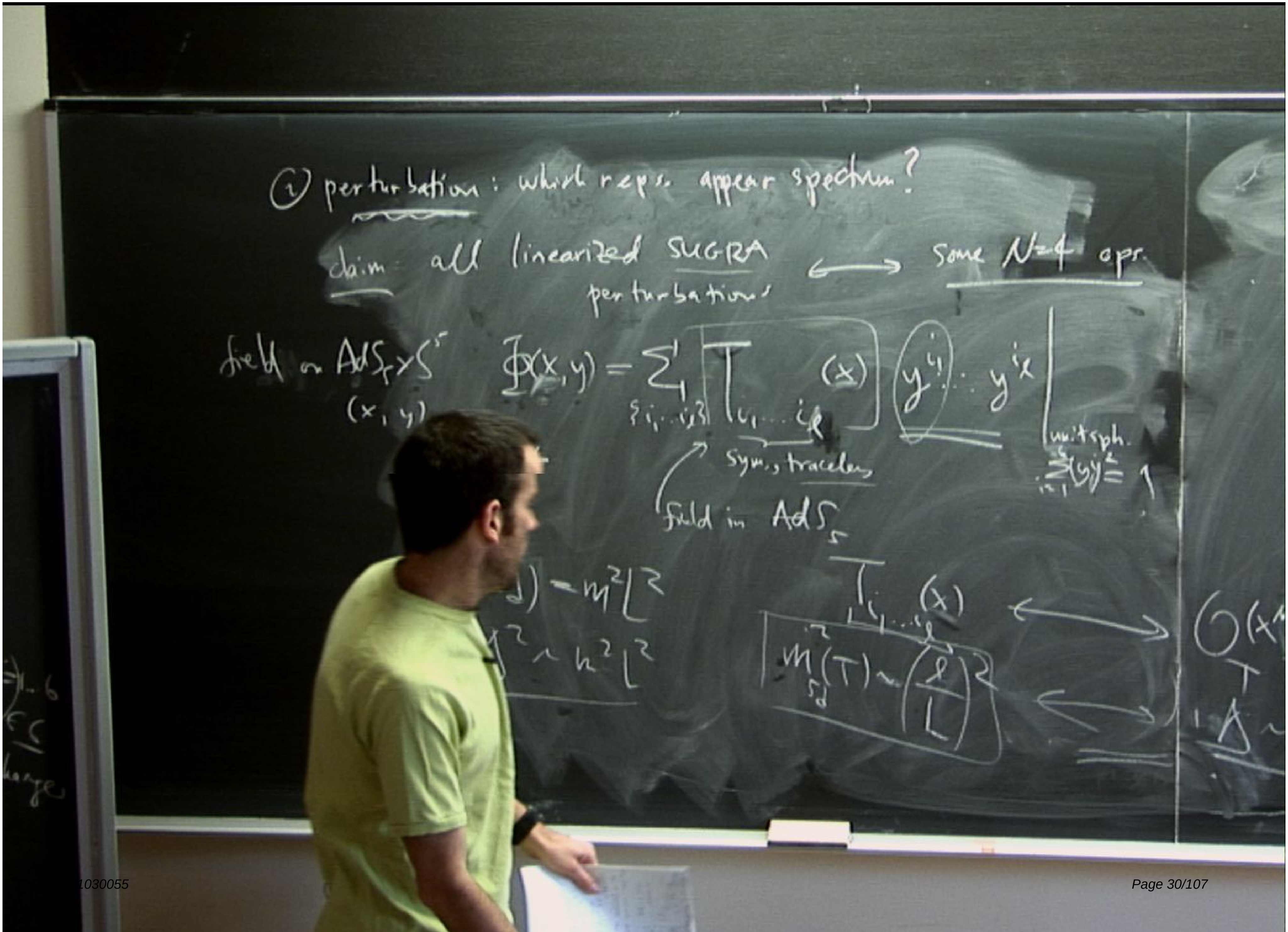
sym., traceless

field in AdS_5

$T_{i_1 \dots i_\ell}(x) \longleftrightarrow \mathcal{O}_T(x^{\mu=0..3}) = T_{i_1 \dots i_\ell}\, \mathrm{tr}\, X^{i_1}(x) \dots X^{i_\ell}(x)$

$m^2_{5d}(T) \sim \left(\frac{\ell}{L}\right)^2 \longleftrightarrow \Delta \sim \ell$


① perturbation: which reps. appear spectrum?
claim: all linearized SUGRA perturbations ⟷ some N=4 opr.
field on AdS5×S5
Φ(x,y) = Σ' T_{i1...il}(x) y^{i1}...y^{il} | unit sph.
{i1...il}
sym., traceless
field in AdS5
Σ(y^i)^2 = 1
Δ(Δ−d) = m^2 L^2
T_{i1...il}(x)
m^2_{5d}(T) ~ (l/L)^2



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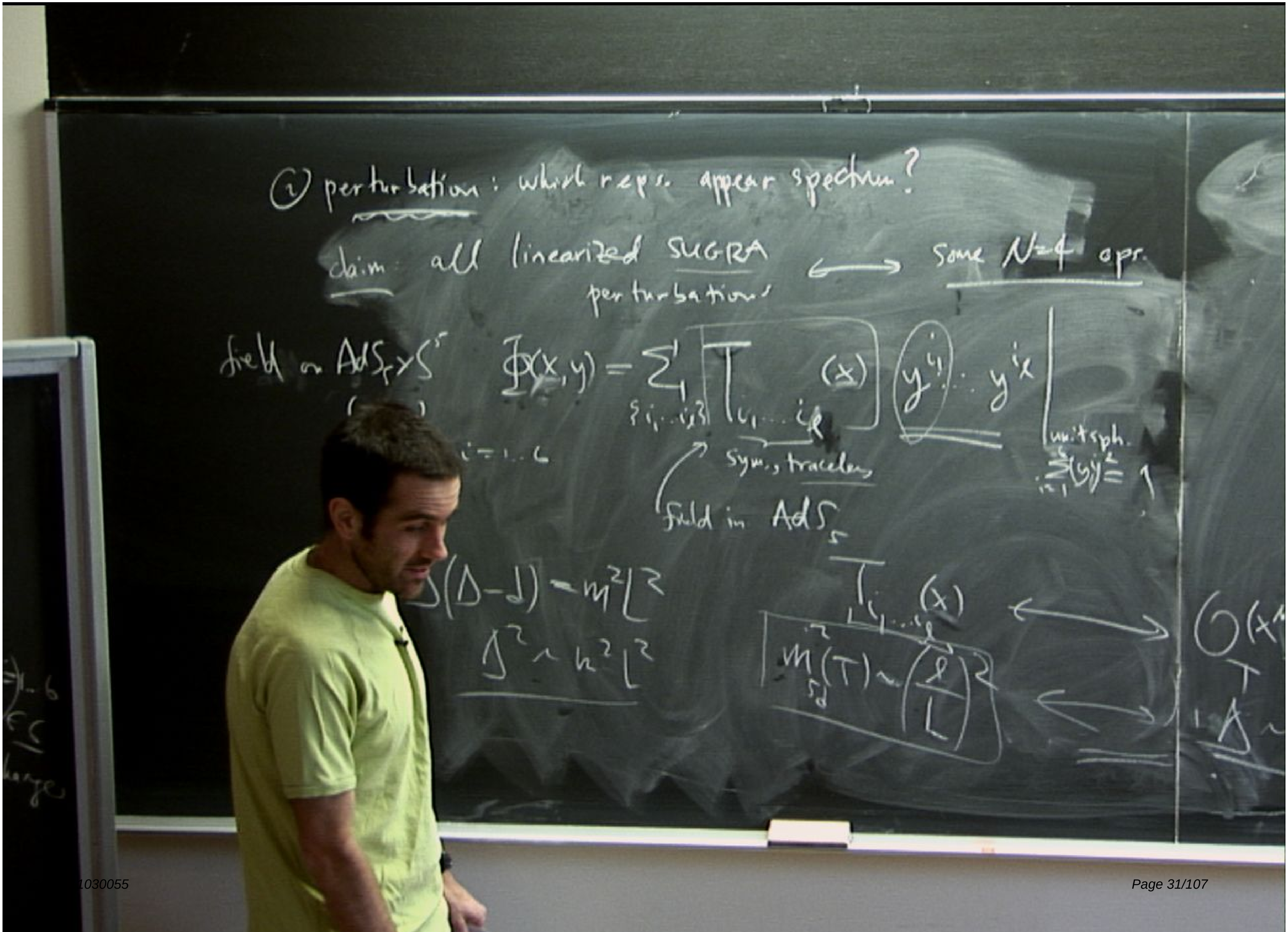
{i1...il}
sym., traceless
Σ(y^i)^2 = 1
field in AdS5
) = m^2 L^2
^2 ~ h^2 L^2
T_{i1...il}(x) ⟷ O(x
m^2_{5d}(T) ~ (l/L)^2 ⟷ Δ ~



① perturbation: which reps. appear spectrum?
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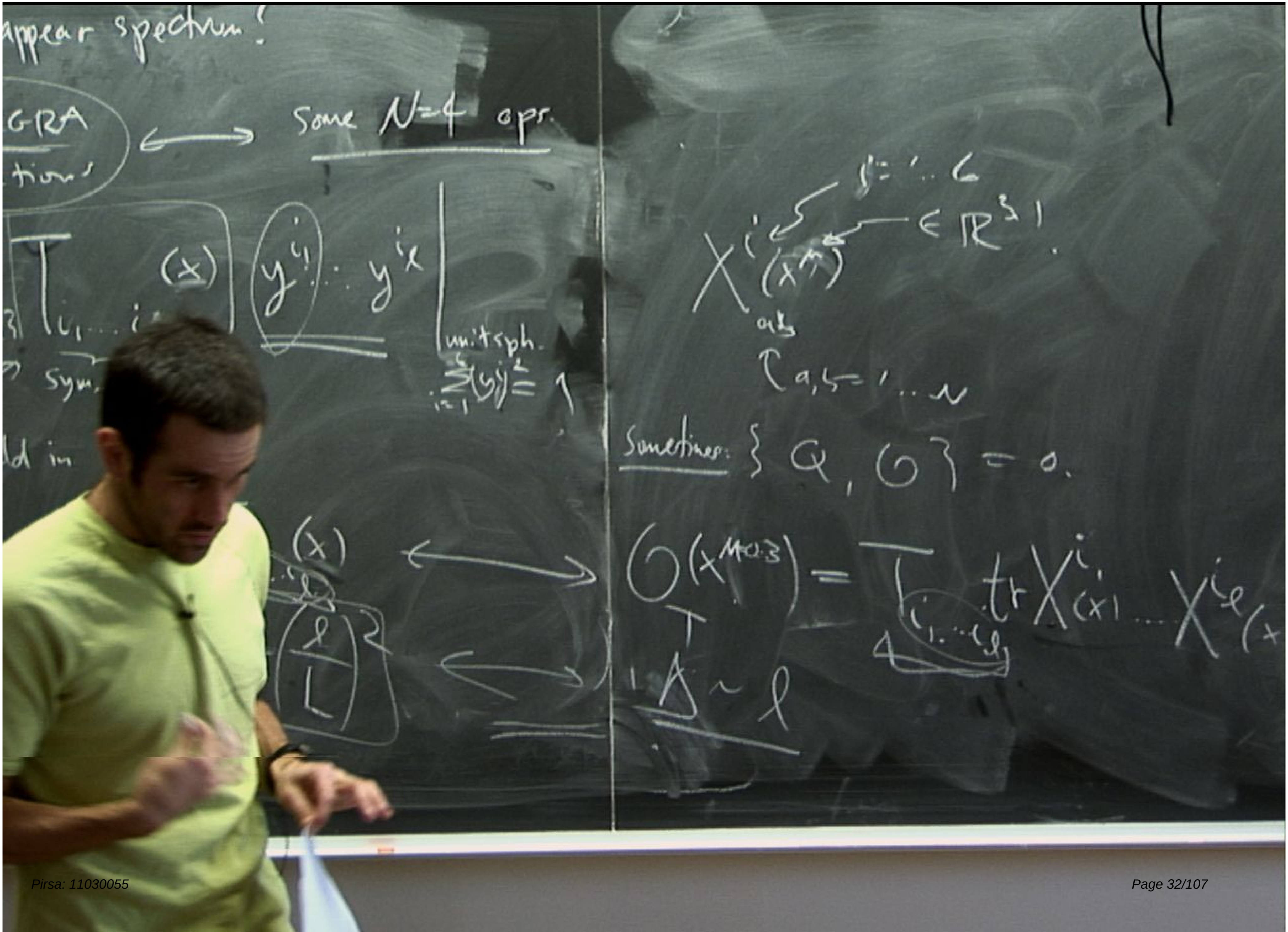
field on AdS5×S5
Φ(x,y) = Σ' T_{i1...il}(x) y^{i1}...y^{il} | unit sph.
sym., traceless
field in AdS5
Σ(y^i)² = 1
Δ(Δ-d) = m²L²
Δ² ~ m²L²
T_{i1...il}(x)
m²_{5d}(T) ~ (l/L)²


appear spectrum?
GRA
Some N=4 opr.
unit sph.
Sometimes: { Q , O } = 0.

...rum!

$\longrightarrow$ some $\mathcal{N}=4$ ops.

(x) $y^{i_1} \dots y^{i_\ell}$ unit sph. $\sum_{i=1}^{6} (y^i)^2 = 1$

$X^i_{(x^\mu)}$ $i = 1 \dots 6$ $\in \mathbb{R}^3$!

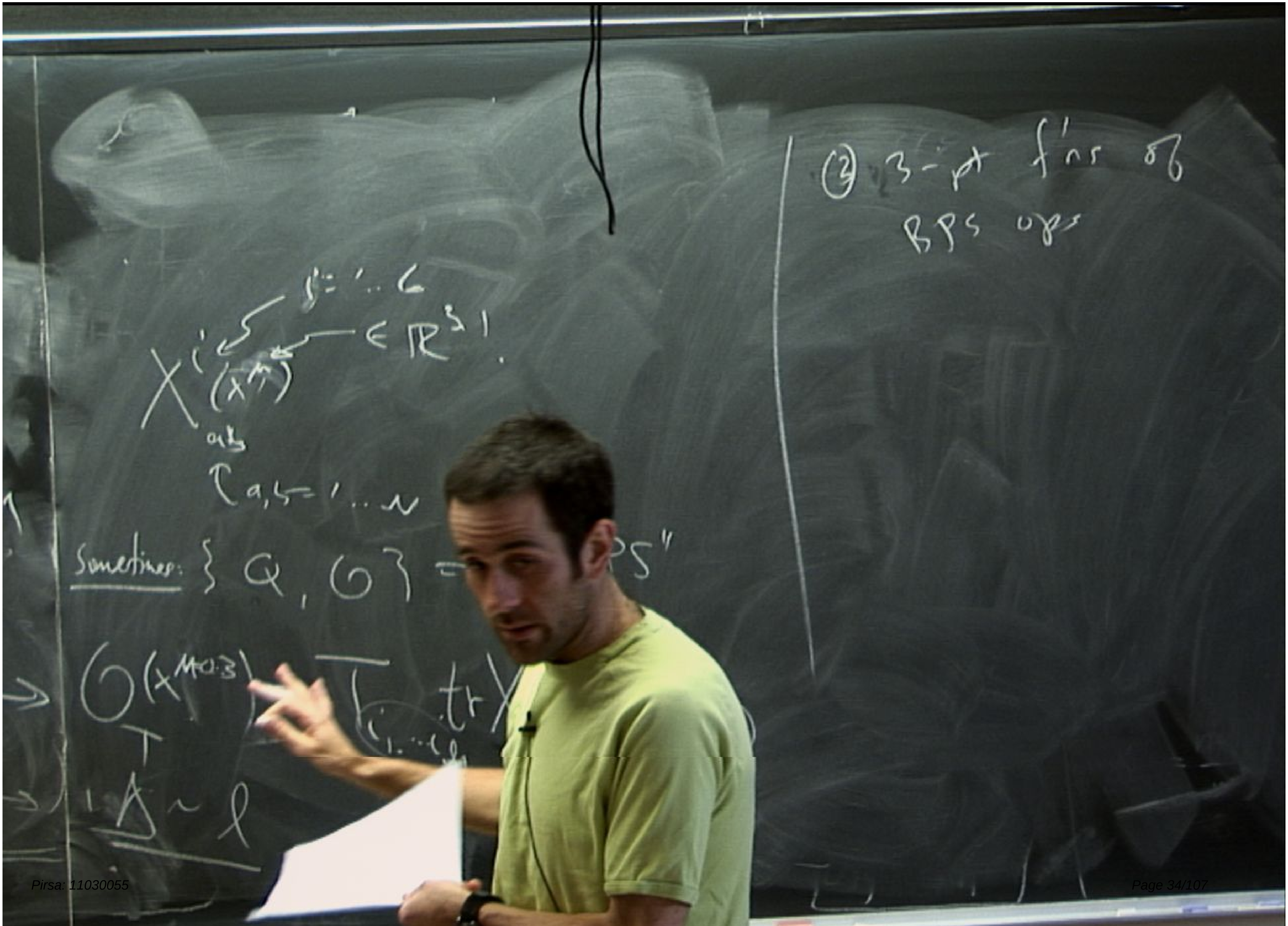
ab

$a, b = 1 \dots N$

Sometimes: $\{ Q, \mathcal{O} \} = 0$. "BPS"

$T_{i_1 \dots i_\ell}(x) \longleftrightarrow \mathcal{O}(x^{\mu=0..3}) = T_{i_1 \dots i_\ell} \operatorname{tr} X^{i_1}_{(x)} \dots X^{i_\ell}_{(x)}$

$(T) \sim \left(\frac{\ell}{L}\right)^2 \longleftrightarrow \Delta \sim \ell$



(2) 3-pt f'ns of
BPS ops
Sometimes:

$i = 1 \dots 6$

$X^{i}_{ab}(x^{\mu}) \in \mathbb{R}^{3,1}$.

$a,b = 1 \dots N$

Sometimes: $\{ Q , \mathcal{O} \} = 0$. "BPS"

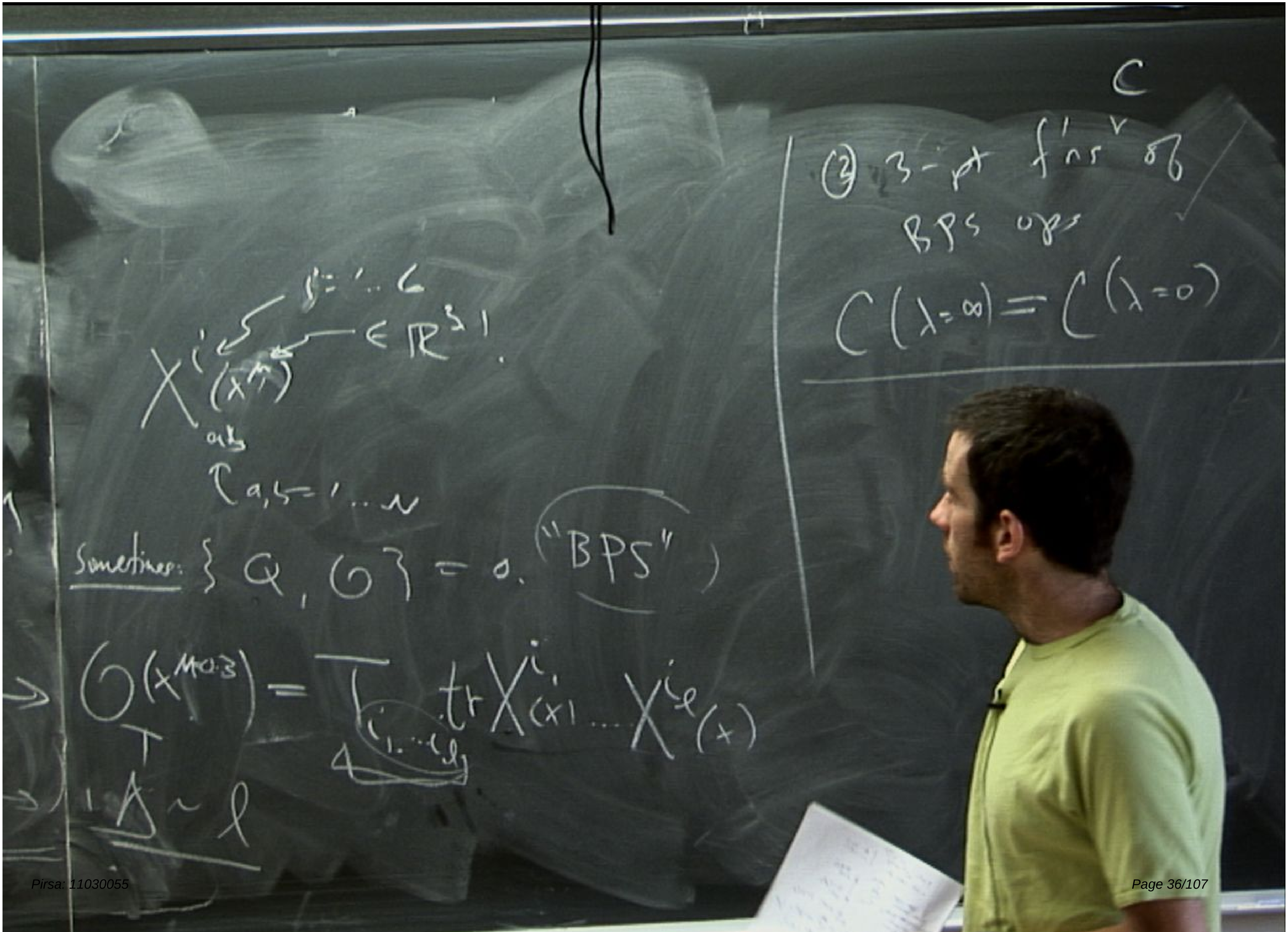
$\Rightarrow \mathcal{O}(x^{\mu=0..3}) = T_{i_1 \dots i_\ell}\, \mathrm{tr}\, X^{i_1}(x) \dots X^{i_\ell}(x)$

$\Delta \sim \ell$

(2) 3-pt f'ns

BPS ops

($\lambda = 0$


C
(2) 3-pt fn's of BPS ops
C(λ=∞) = C(λ=0)
i=1..6
X^i_ab (x^μ) ∈ R^3,1
a,b=1...N
Sometimes: {Q, O} = 0. "BPS"
O(x^μ=0..3) = T_i1...iℓ tr X^i1(x) ... X^iℓ(x)
Δ ~ ℓ

C

$i = 1 \ldots 6$

$X^i_{(x^\mu)} \in \mathbb{R}^{3,1}$

$a b$

$a, b = 1 \ldots N$

Sometimes: $\{ Q, \mathcal{O} \} = 0$. "BPS"

$\mathcal{O}(x^{\mu=0..3}) = T_{i_1 \ldots i_\ell} \, \mathrm{tr} X^{i_1}_{(x)} \ldots X^{i_\ell}_{(x)}$

$\Delta \sim \ell$

(2) 3-pt f'ns of BPS ops

$C(\lambda = \infty) = C(\lambda = 0)$

A more gen'l operator of N

$\mathcal{O}(x) = \mathrm{tr}[XXX \ldots$

C

(2) 3-pt f'ns of BPS ops

$$C(\lambda=\infty) = C(\lambda=0)$$

A more gen'l operator of N=

$$\mathcal{O}(x) = \mathrm{tr}[XXX \ldots D_\mu X Y$$

$$X^i_{ab}(x^\mu) \in \mathbb{R}^{3,1}$$

$i = 1 \ldots 6$

$a,b = 1 \ldots N$

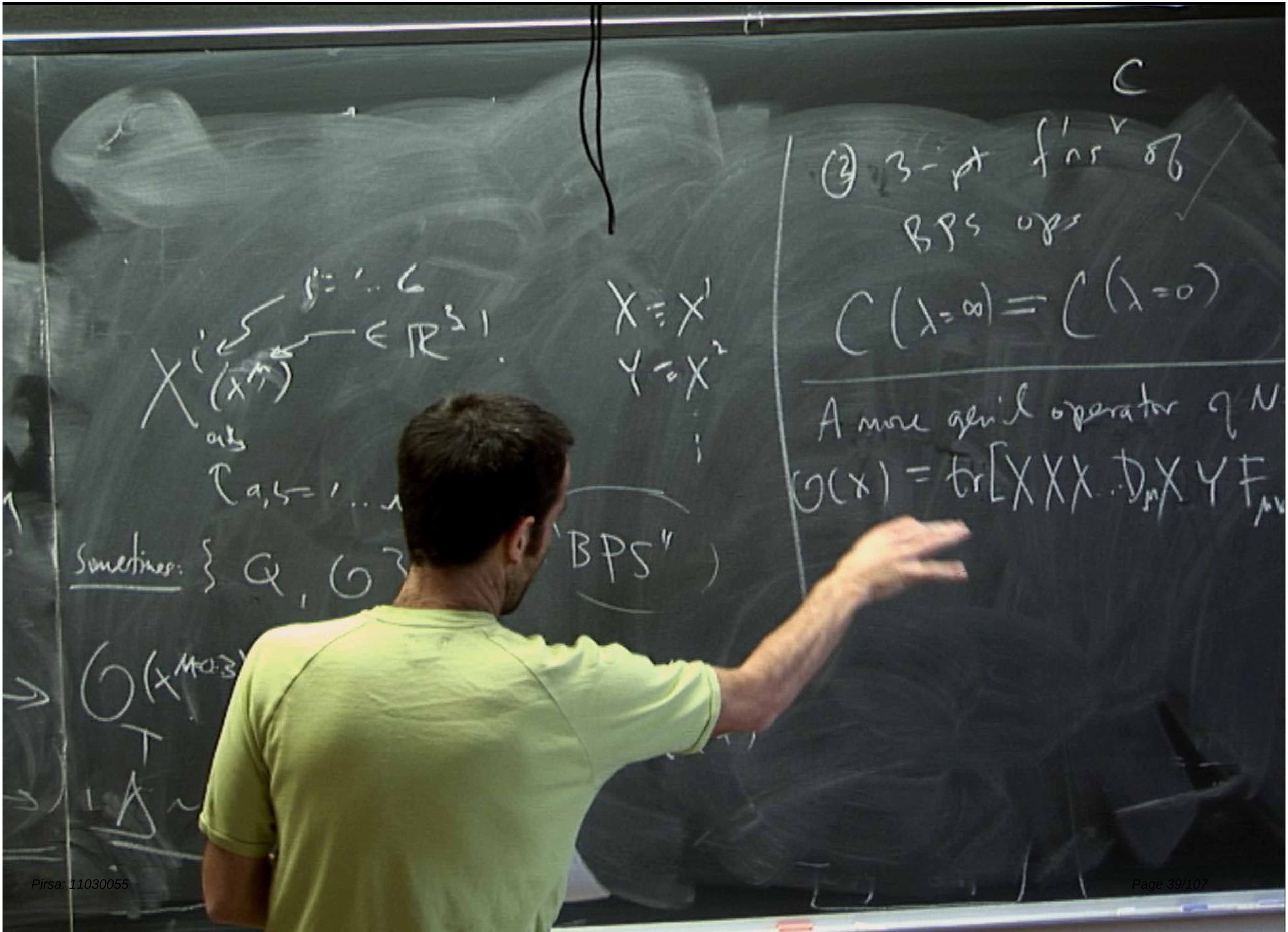
$X = X^1$

$Y = X^2$

Sometimes: $\{Q, \mathcal{O}\} = 0$. "BPS"

$$\mathcal{O}(x^{\mu=0..3}) = T_{i_1 \ldots i_\ell} \mathrm{tr} X^{i_1}(x) \ldots X^{i_\ell}(x)$$

$$\Delta \sim \ell$$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C(λ=∞) = C(λ=0)
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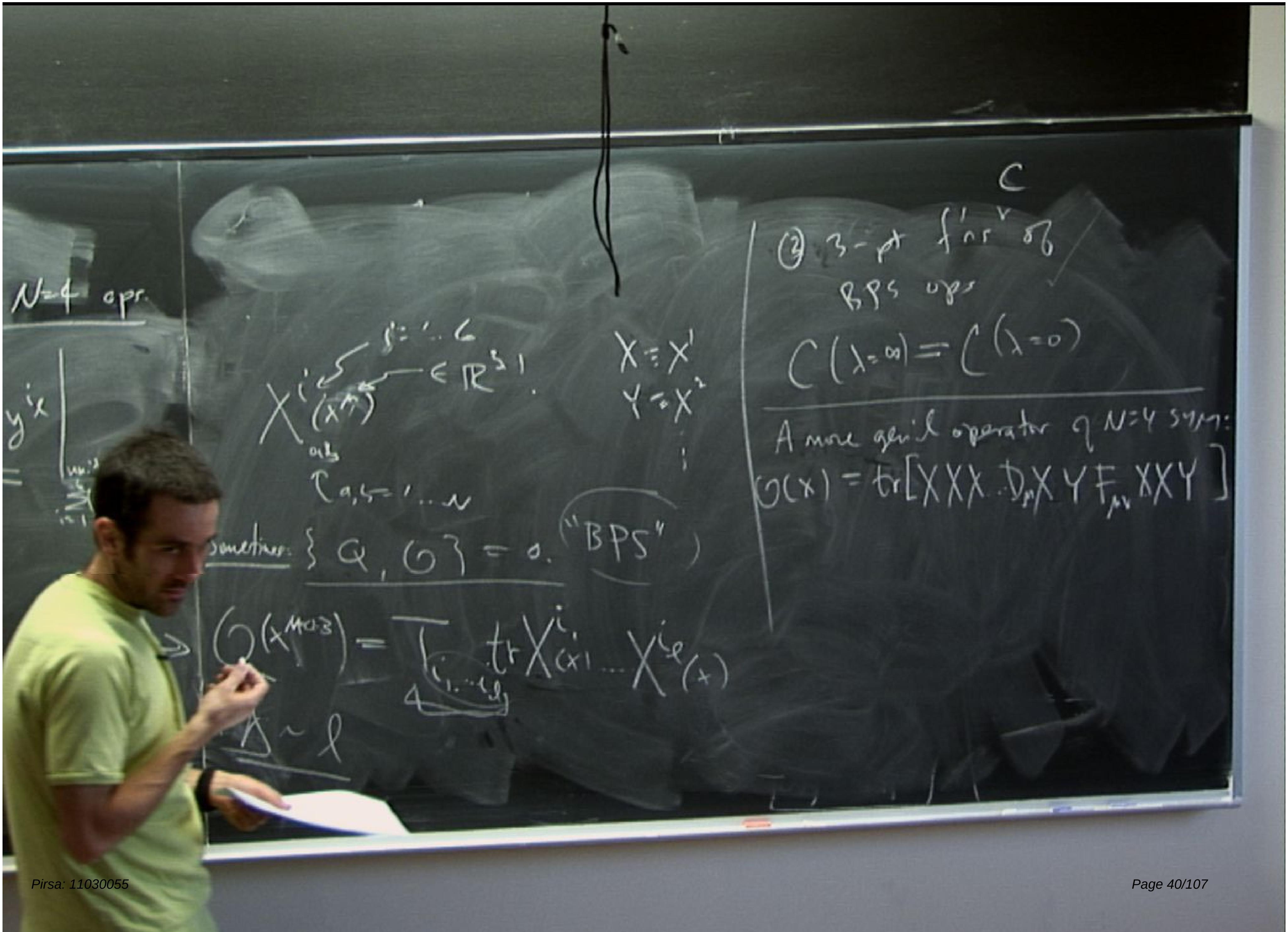
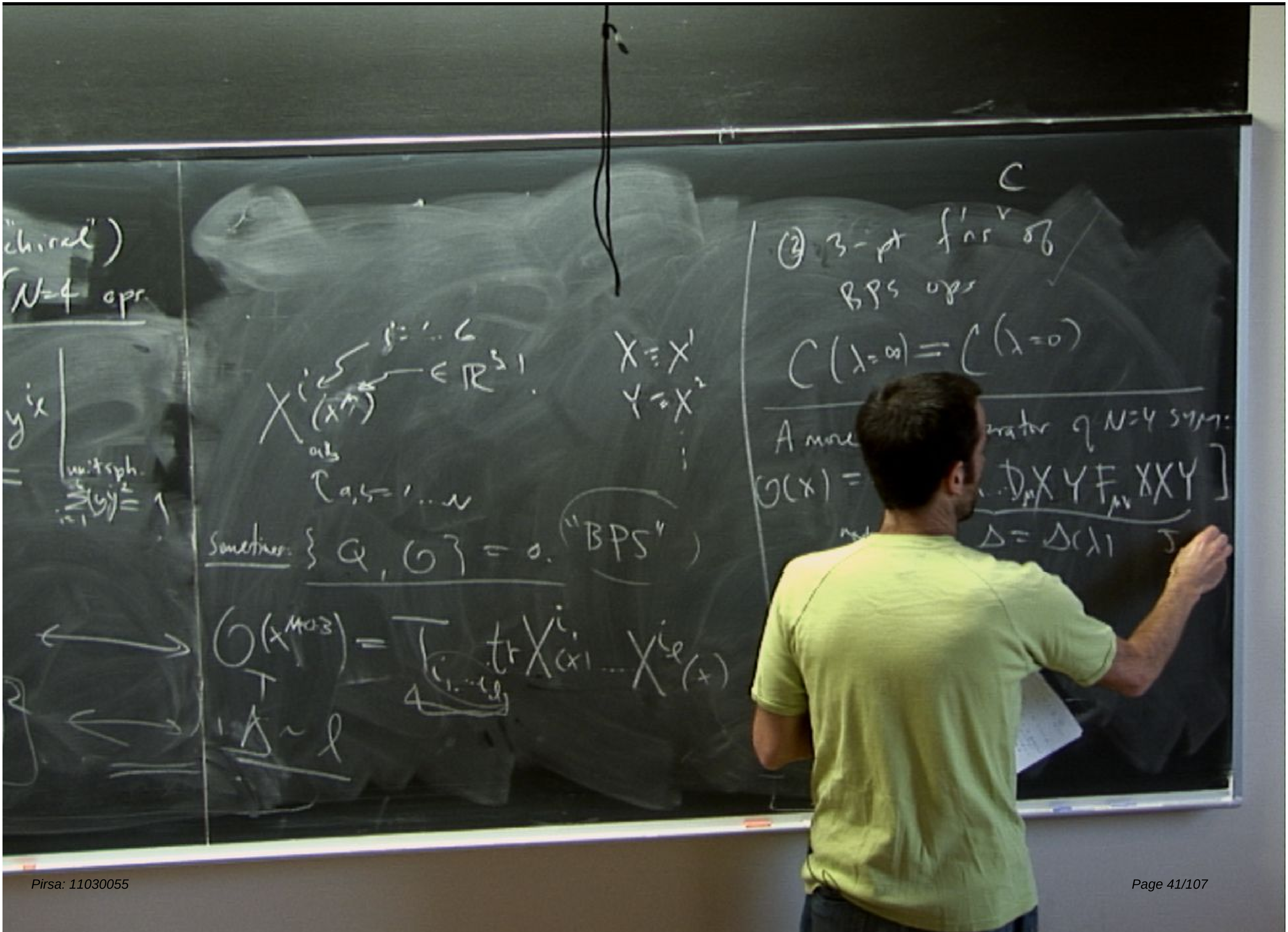
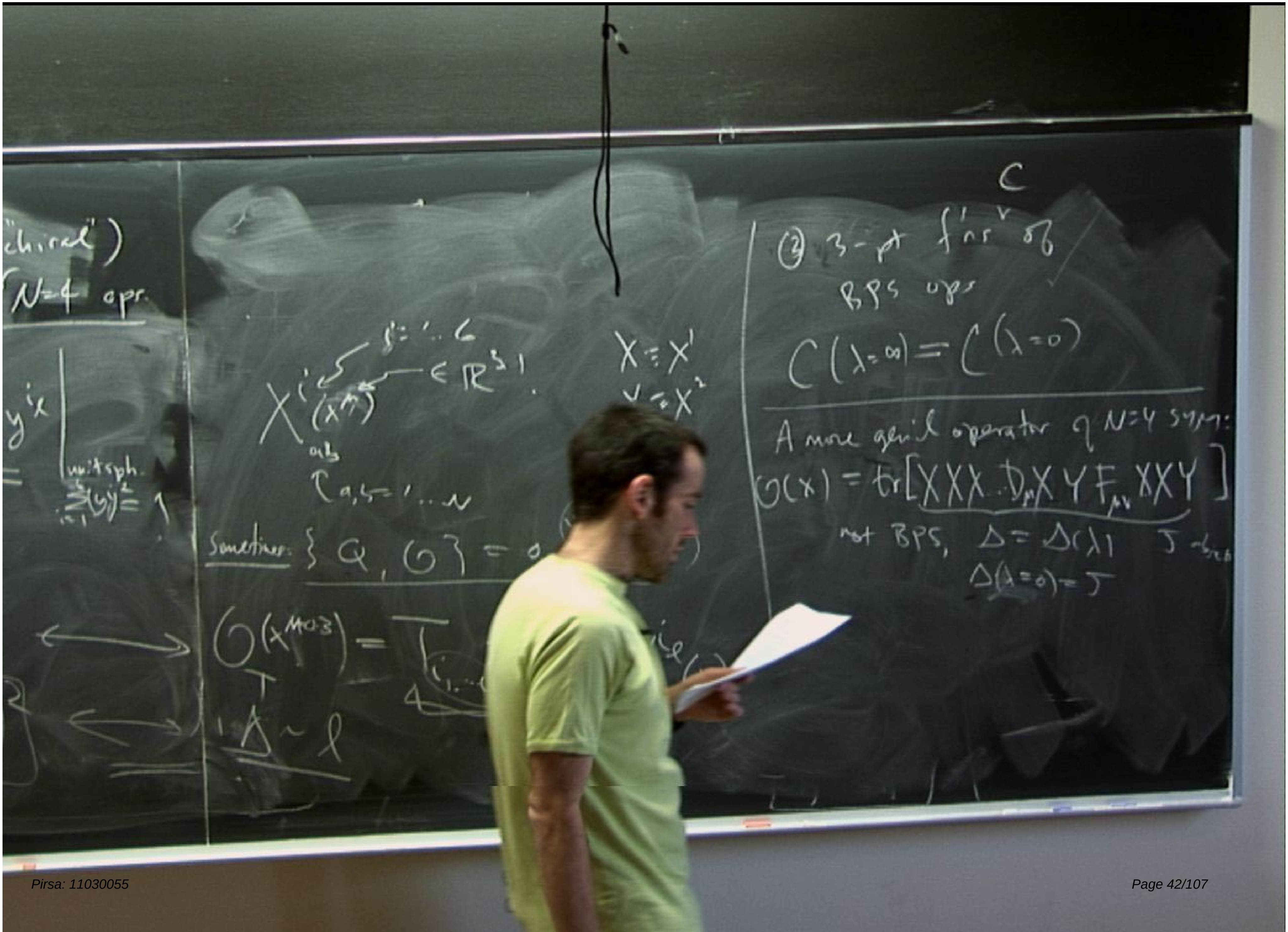
X = X'
Y = X²
"BPS"
Sometimes: { Q , O


N=4 opr.
C
③ 3-pt f'nr's of BPS ops
C(λ=∞) = C(λ=0)
A more gen'l operator of N=4 SYM:
O(x) = tr[XXX DμX Y Fμν XXY]
X = X¹
Y = X²
Sometimes: {Q, O} = 0. "BPS"
Δ ~ l


"chiral")
N=4 opr.
unit sph.
Sometimes {Q, O} = 0. "BPS"
C
(2) 3-pt f'ns of
BPS ops
C(λ=∞) = C(λ=0)
A more ... erator of N=4 sym:
O(x) = ... D_μ X Y F_μν XXY]
Δ = Δ(λ)
Δ ~ l


("chiral")
N=4 opr.
C
(2) 3-pt f'n's of
BPS ops
C(λ=∞) = C(λ=0)
A more gen'l operator of N=4 sym:
O(x) = tr[XXX D_μ X Y F_μν XXY]
not BPS, Δ = Δ(λ)
Δ(λ=0) = J
Sometimes {Q, O} = 0
X = X'
Y = X²
unit sph.
Δ ~ l

("chiral")

$\mathcal{N}=4$ opr.

$y^i x$

unit sph. $\sum_{i=1}^{6} (y^i)^2 = 1$

$i = 1 \dots 6$

$X^i_{ab}(x^\mu) \in \mathbb{R}^{3,1}$

$a,b = 1 \dots N$

$X = X^1$

$Y = X^2$

Sometimes $\{ Q, \mathcal{O} \} = 0$. ("BPS")

$\mathcal{O}(x^\mu) = T_{i_1 \dots i_\ell} \operatorname{tr} X^{i_1}(x) \dots X^{i_\ell}(x)$

$\Delta \sim \ell$

C

(2) 3-pt f'ns of BPS ops

$C(\lambda=\infty) = C(\lambda=0)$

A more gen'l operator of $\mathcal{N}=4$ SYM:

$\mathcal{O}(x) = \operatorname{tr}[\underline{X X X D_\mu X Y F_{\mu\nu} X X Y}]$

not BPS, $\Delta = \Delta(\lambda)$ J

$\Delta(\lambda=0) = J$

C

$X^i_{(x^\mu)}$ $i = 1 \ldots 6$ $\in \mathbb{R}^{3,1}$

a,b

$a,b = 1 \ldots N$

$X = X^1$

$Y = X^2$

⋮

Sometimes: $\{ Q, \mathcal{O} \} = 0.$ "BPS"

$\mathcal{O}(x^{M=0..3}) = T_{i_1 \ldots i_\ell}\, \mathrm{tr}\, X^{i_1}_{(x)} \ldots X^{i_\ell}_{(x)}$

Δ

$\Delta \sim \ell$

(2) 3-pt f'ns of BPS ops

$C(\lambda = \infty) = C(\lambda = 0)$

A more gen'l operator of $N=4$

$\mathcal{O}(x) = \mathrm{tr}[XXX \ldots D_\mu X\, Y\, F_{\mu\nu} X$

not BPS, $\Delta = \Delta(\lambda)$

$\Delta(\lambda = 0) = 5$

$= \mathrm{tr}[YXXX \ldots D_\mu X\, Y\, F_{\mu\nu} XX$

$\to$ closed string

$)$

opr.

:tsph.

$(y\cdot y)^2 = 1$

$X^i_{(x^\mu)}$ $i = 1 \ldots 6$, $\in \mathbb{R}^{3,1}$.

adj

$\mathbb{1}_{a,b=1 \ldots N}$

$X = X^1$

$Y = X^2$

$\vdots$

Sometimes: $\{ Q, \mathcal{O} \} = 0$. "BPS"

$\mathcal{O}(x^{\mu}) = T_{i_1 \ldots i_\ell} \, \mathrm{tr} X^{i_1}_{(x)} \ldots X^{i_\ell}_{(x)}$

$\Delta \sim \ell$

C

(3) 3-pt f'n's of BPS ops

$C(\lambda=\infty) = C(\lambda=0)$

A more gen'l operator of N=4 sym

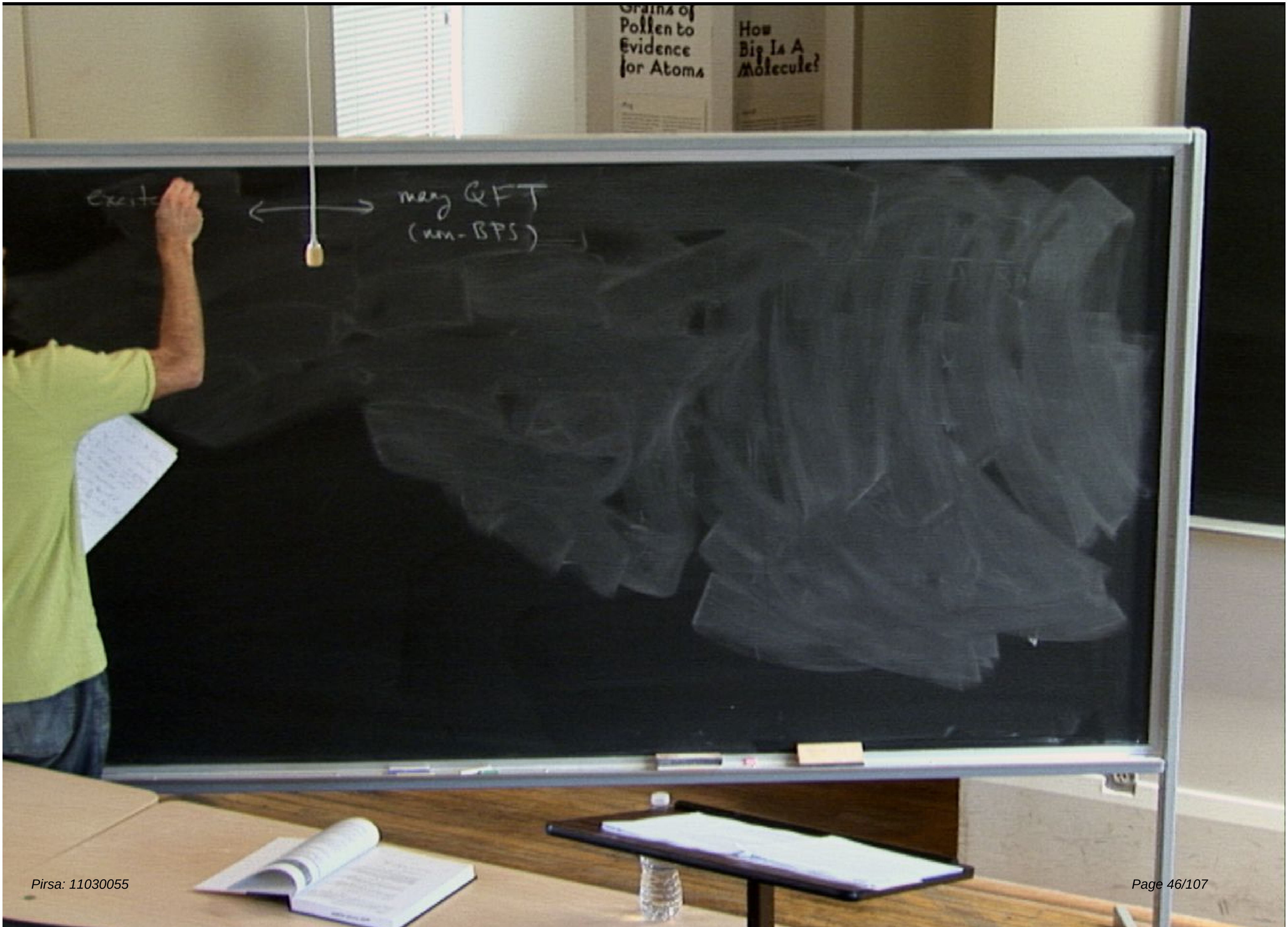
$\mathcal{O}(x) = \mathrm{tr}[XXX \ldots D_\mu X Y F_{\mu\nu} XXY$

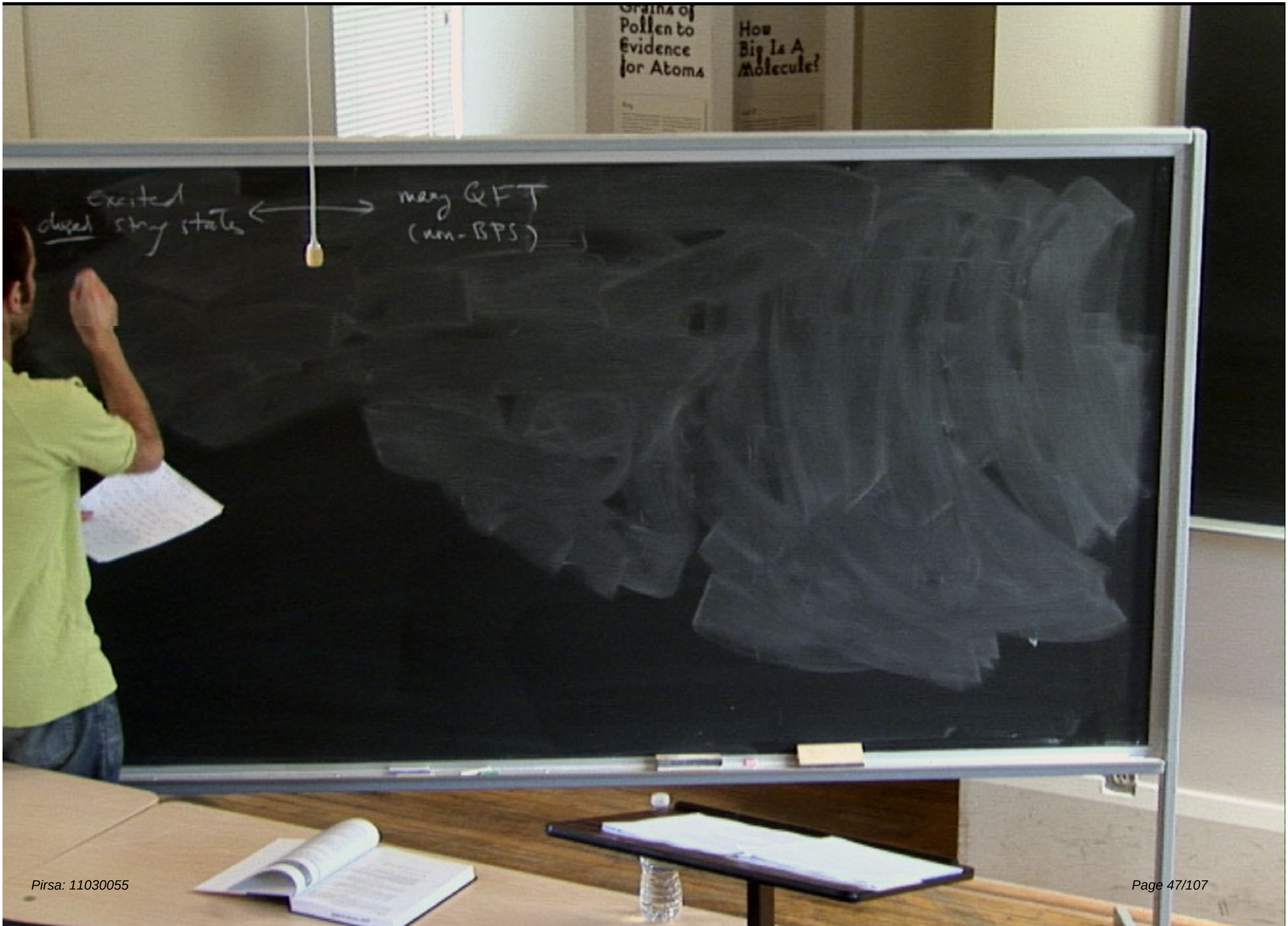
not BPS, $\Delta = \Delta(\lambda)$ J

$\Delta(\lambda=0) = J$

$= \mathrm{tr}[YXXX \ldots D_\mu X Y F_{\mu\nu} XX)$

$\rightarrow$ closed string


Pollen to
Evidence
for Atoms
How
Big Is A
Molecule?
many QFT
(non-BPS)


Pollen to
Evidence
for Atoms
How
Big Is A
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Excited
closed string states
many QFT
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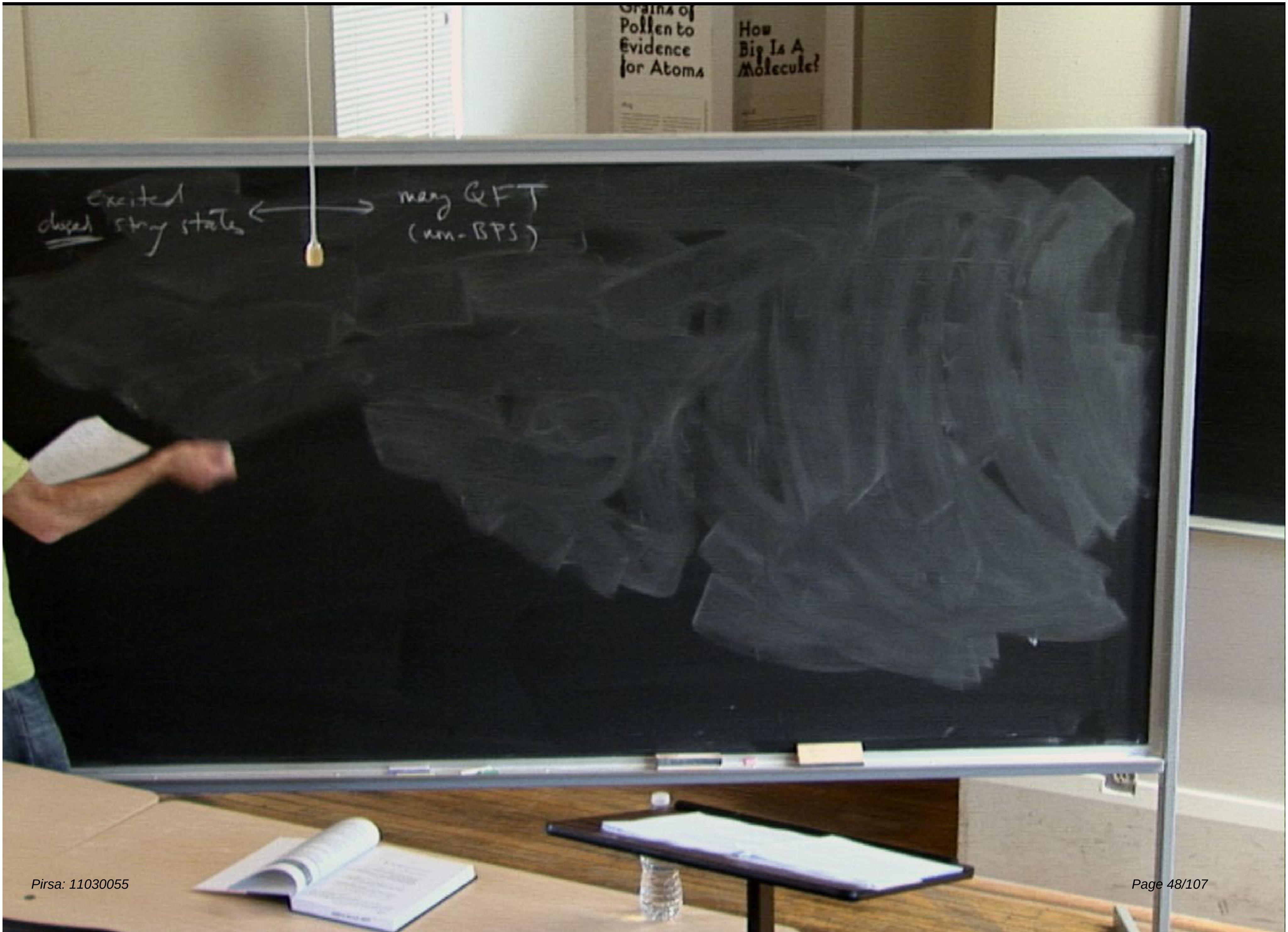
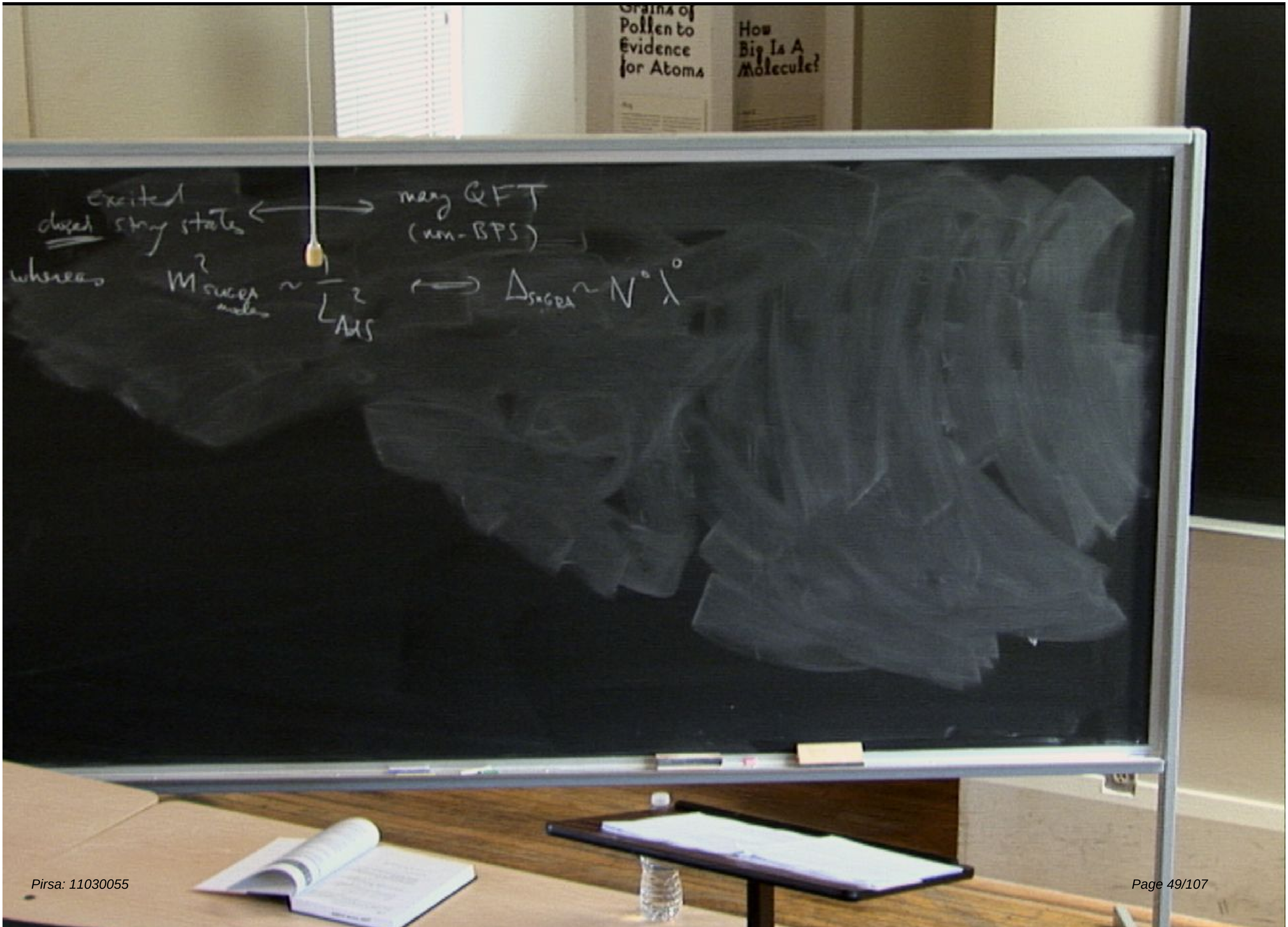
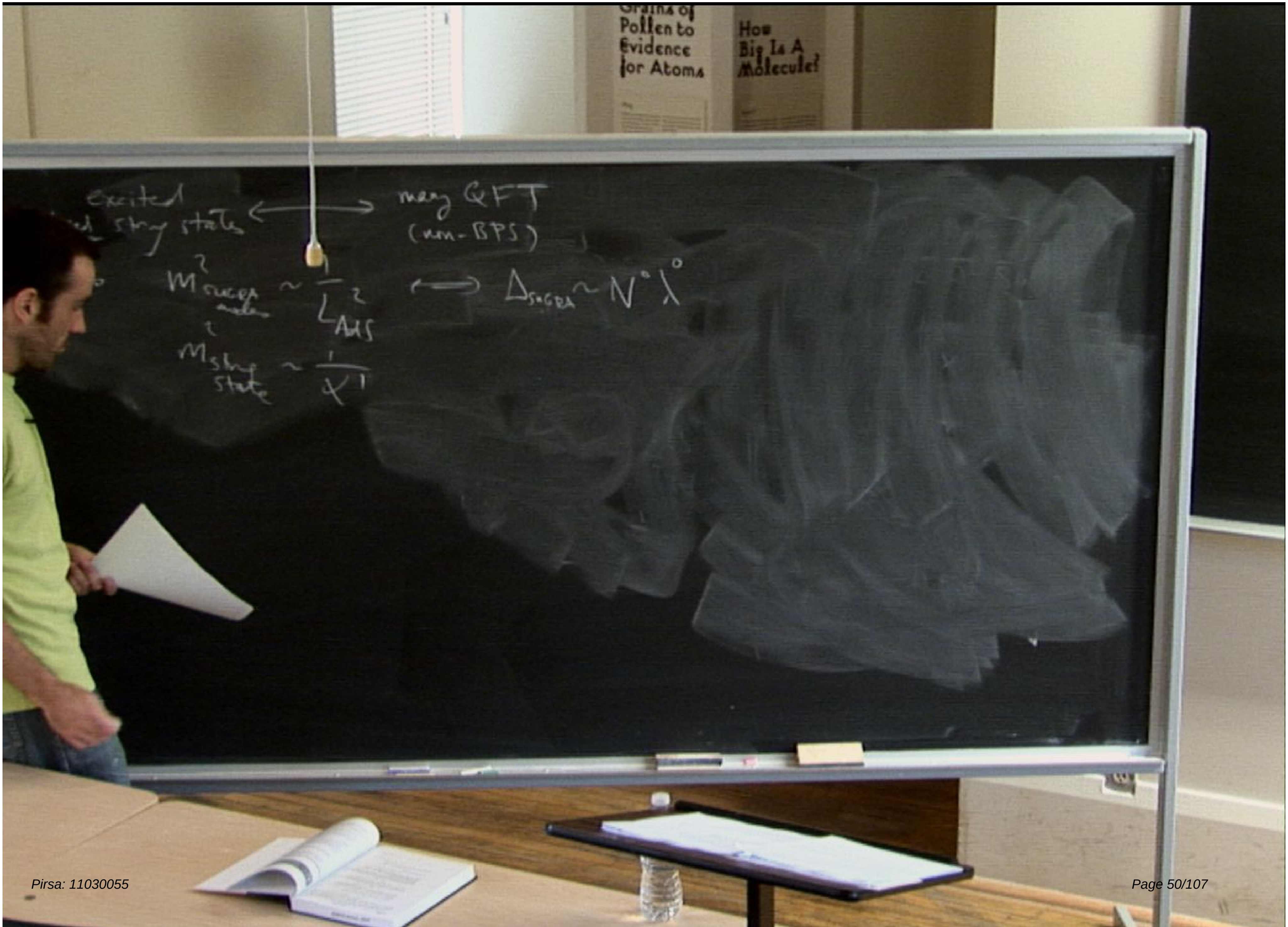
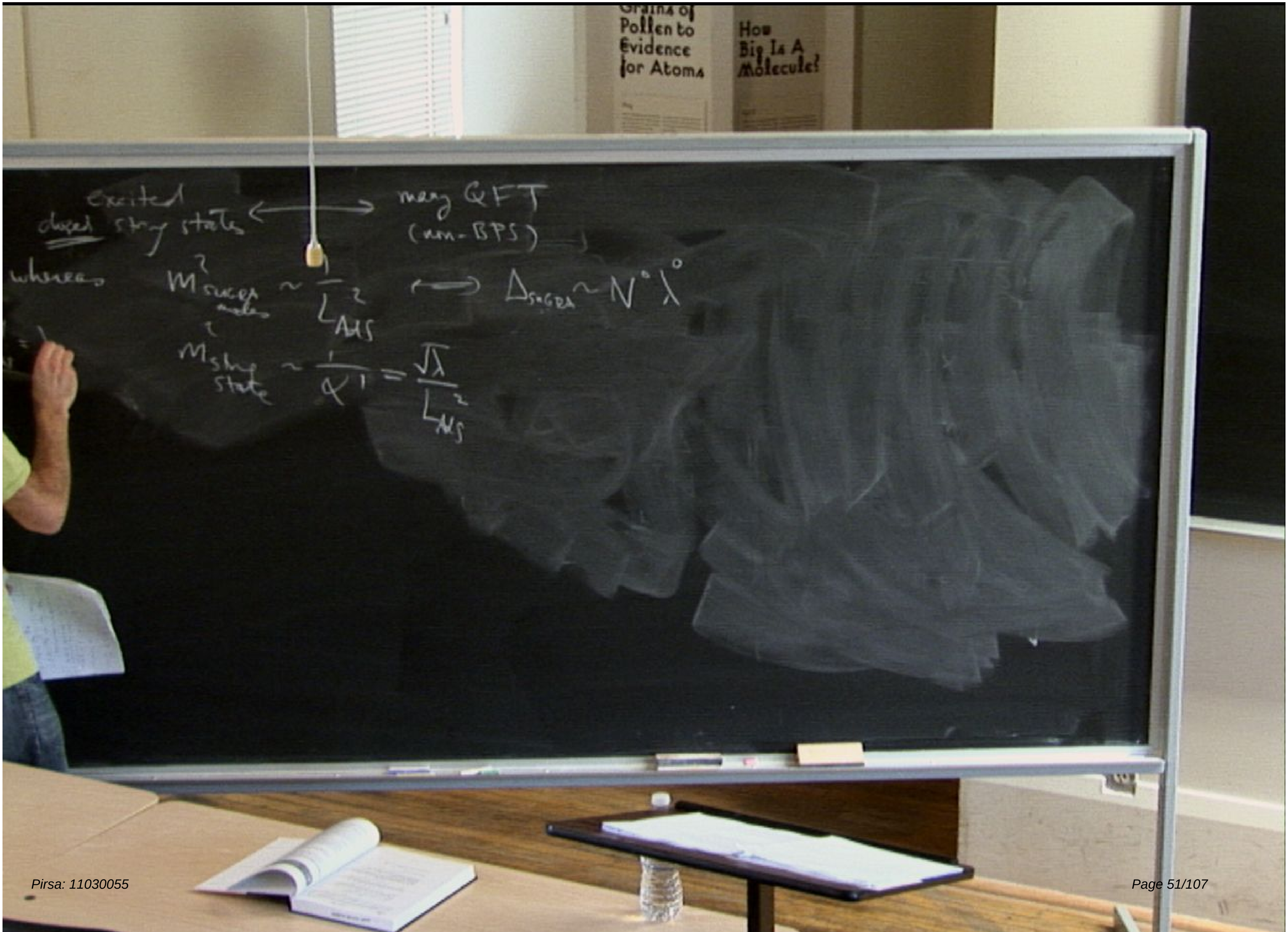
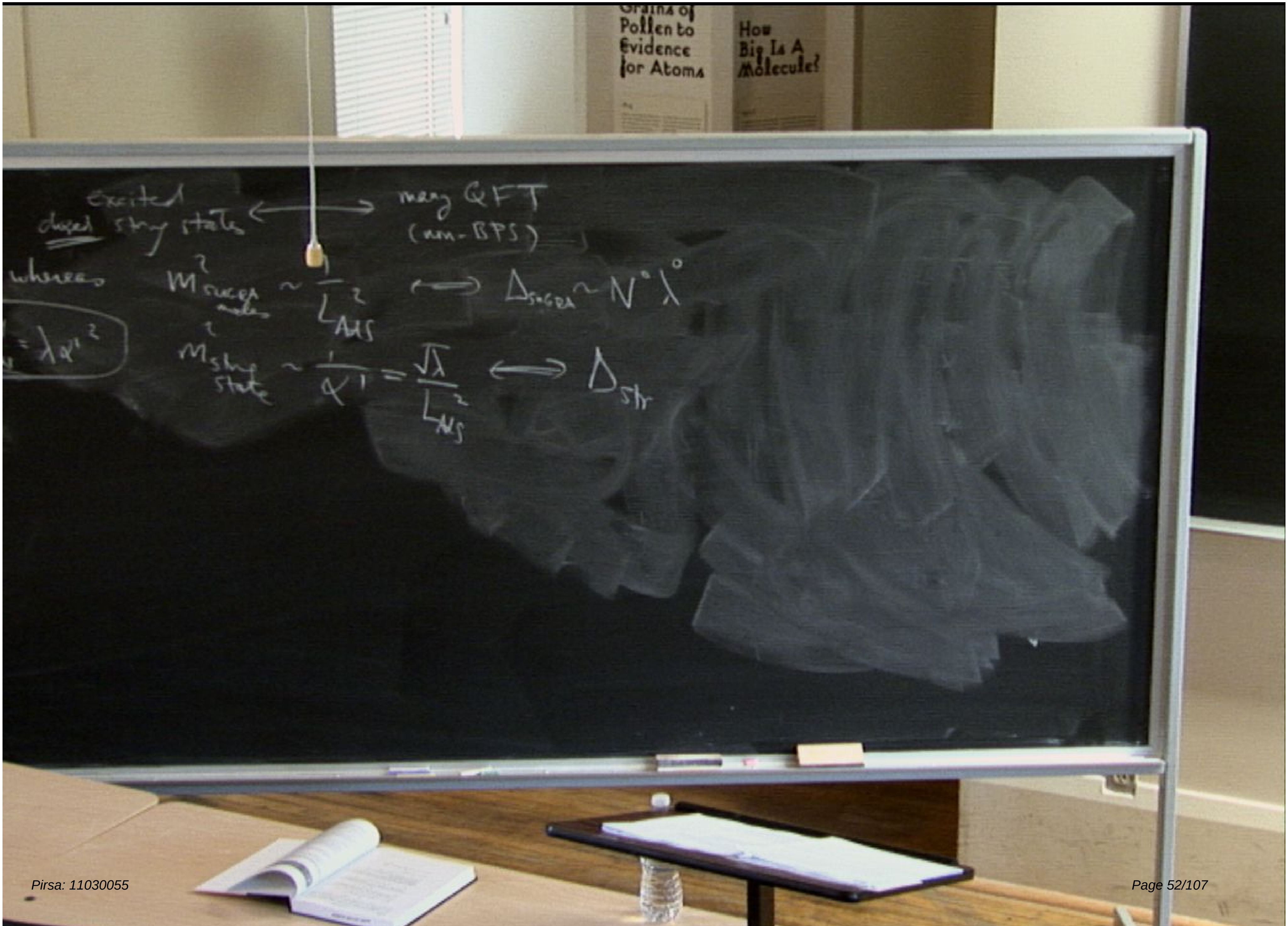

Grains of
Pollen to
Evidence
for Atoms
How
Big Is A
Molecule?
excited
closed string states ⟷ many QFT
(non-BPS)

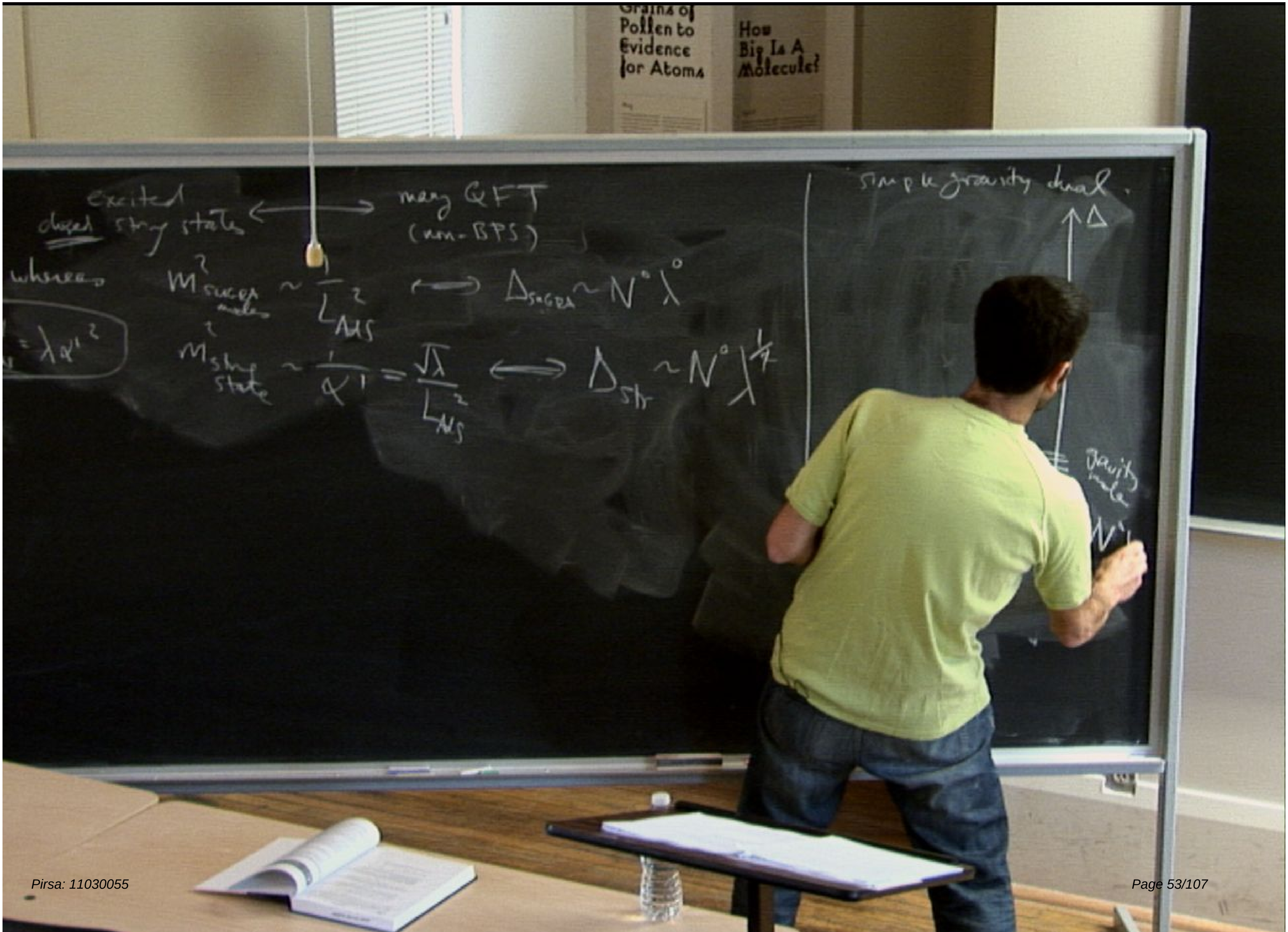

Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
excited closed string states ⟷ many QFT (non-BPS)
whereas $M^2_{\text{sugra modes}} \sim \frac{1}{L^2_{AdS}}$ ⟷ $\Delta_{SUGRA} \sim N^0 \lambda^0$

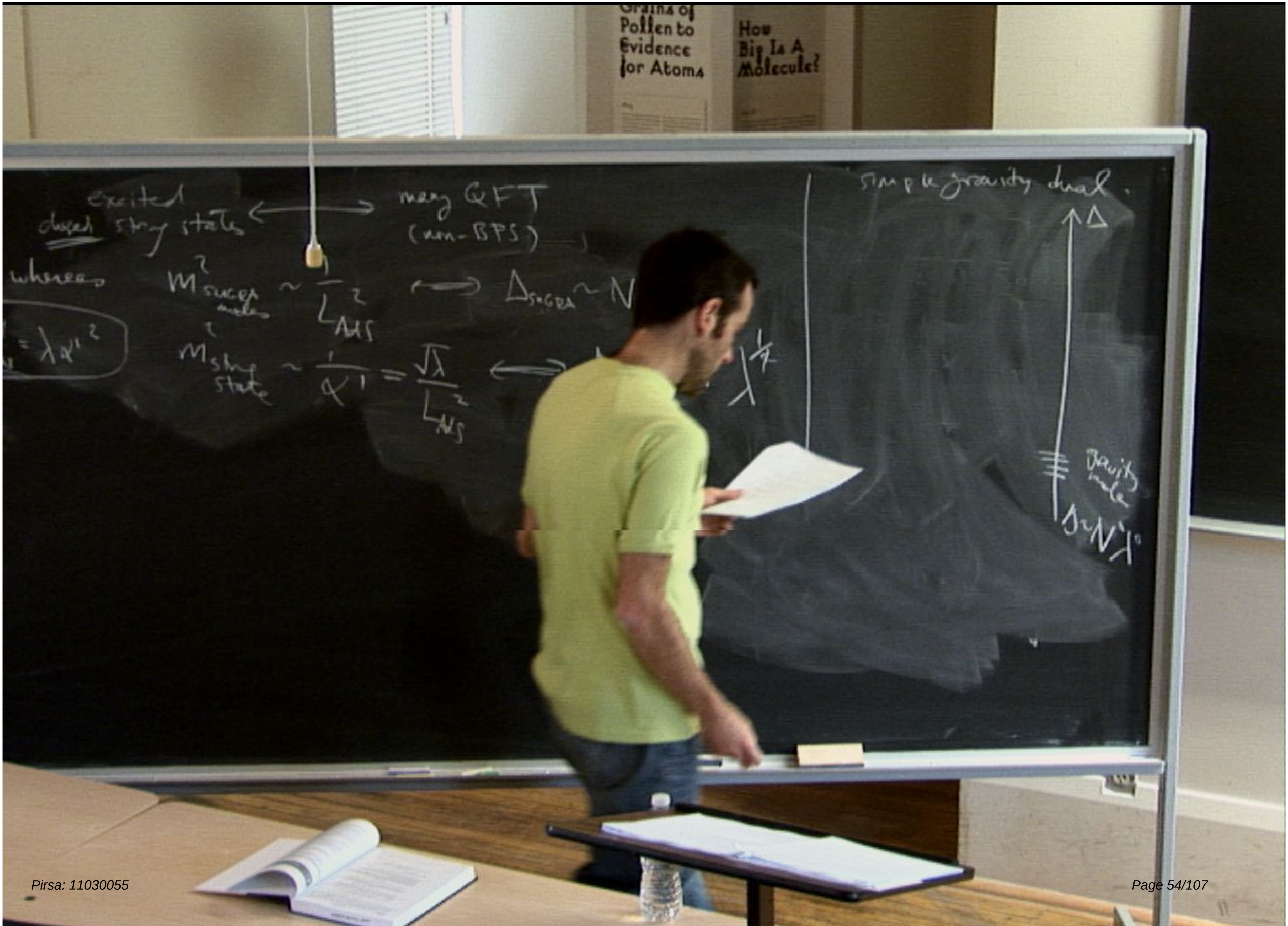

Pollen to
Evidence
for Atoms
How
Big Is A
Molecule?
Excited
string states
many QFT
(non-BPS)
$M^2_{\text{sugra modes}} \sim \frac{1}{L^2_{AdS}}$
$\Delta_{\text{sugra}} \sim N^0 \lambda^0$
$M^2_{\text{string state}} \sim \frac{1}{\alpha'}$


Pollen to
Evidence
for Atoms
How
Big Is A
Molecule?
excited
closed string states
many QFT
(non-BPS)
whereas
$M^2_{\text{sugra modes}} \sim \frac{1}{L^2_{AdS}} \longleftrightarrow \Delta_{\text{sugra}} \sim N^0 \lambda^0$
$M^2_{\text{string state}} \sim \frac{1}{\alpha'} = \frac{\sqrt{\lambda}}{L^2_{AdS}}$



Pollen to Evidence for Atoms
How Big Is A Molecule?
Excited closed string states ⟷ many QFT (non-BPS)
whereas M²sugra modes ~ 1/L²AdS ⟷ ΔsuGRA ~ N⁰λ⁰
= λα'²
M²string state ~ 1/α' = √λ/L²AdS ⟷ Δstr


Pollen to
Evidence
for Atoms
How
Big Is A
Molecule?
Excited
closed string states
many QFT
(non-BPS)
simple gravity dual.


Pollen to
Evidence
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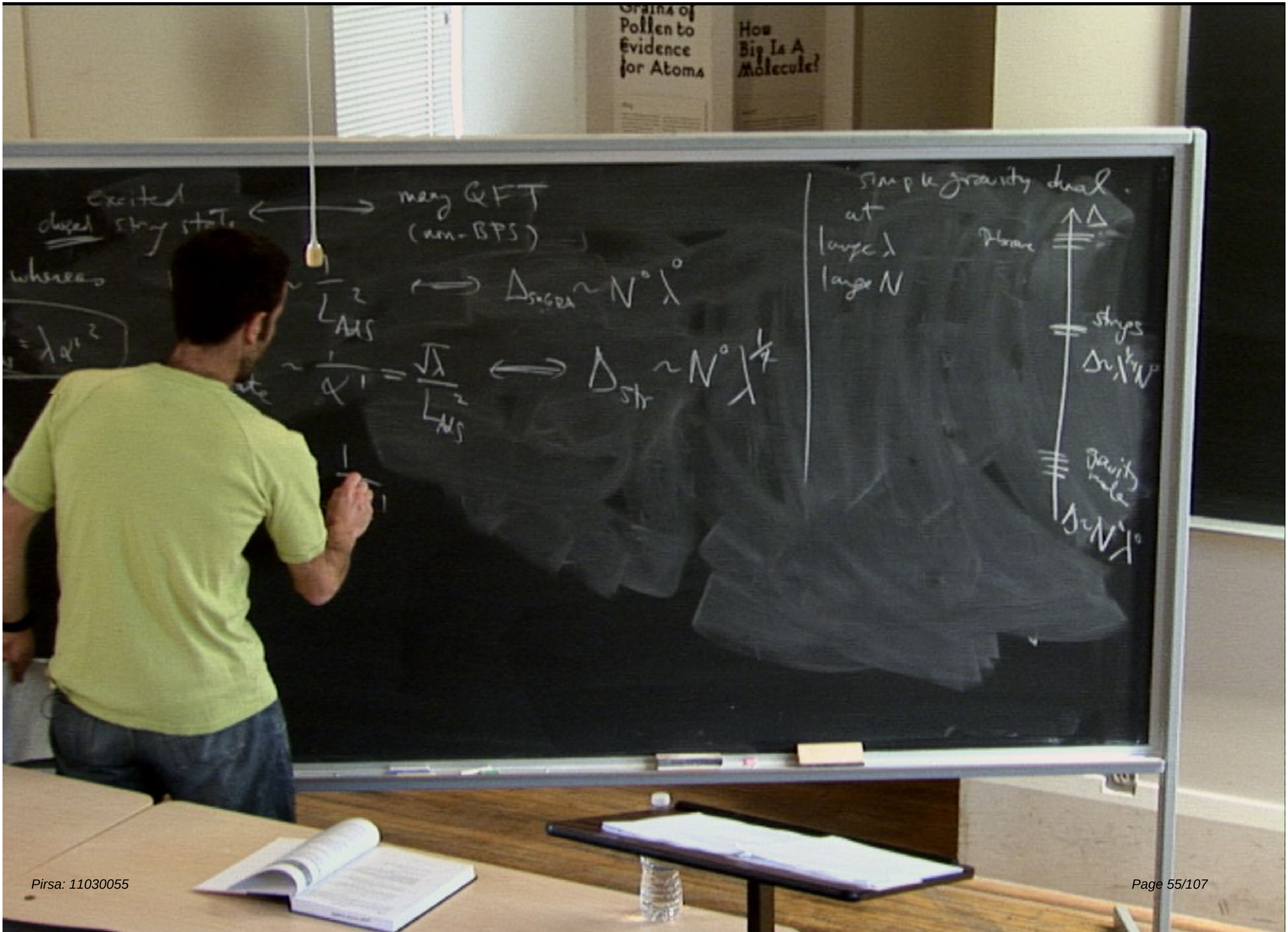
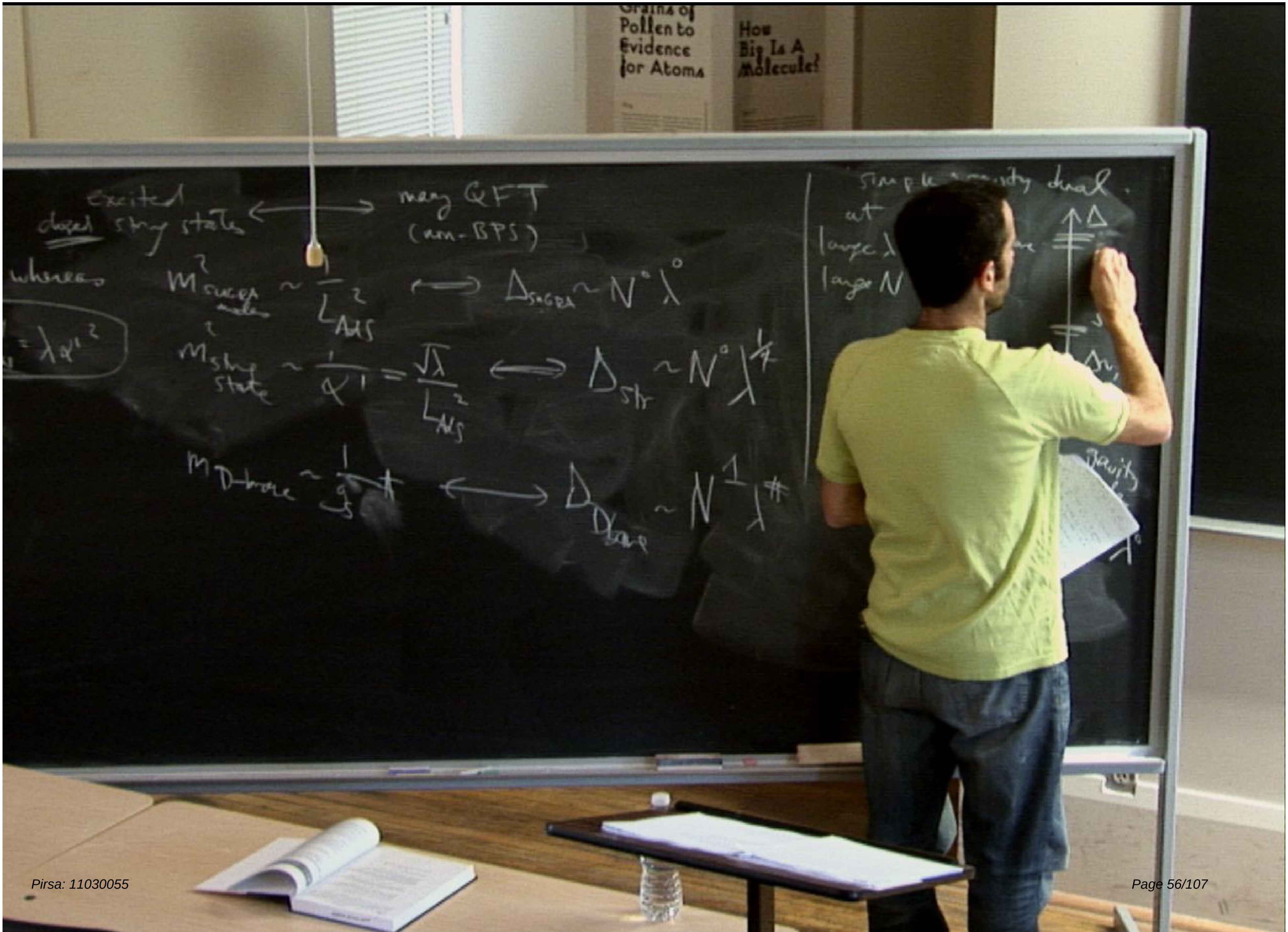
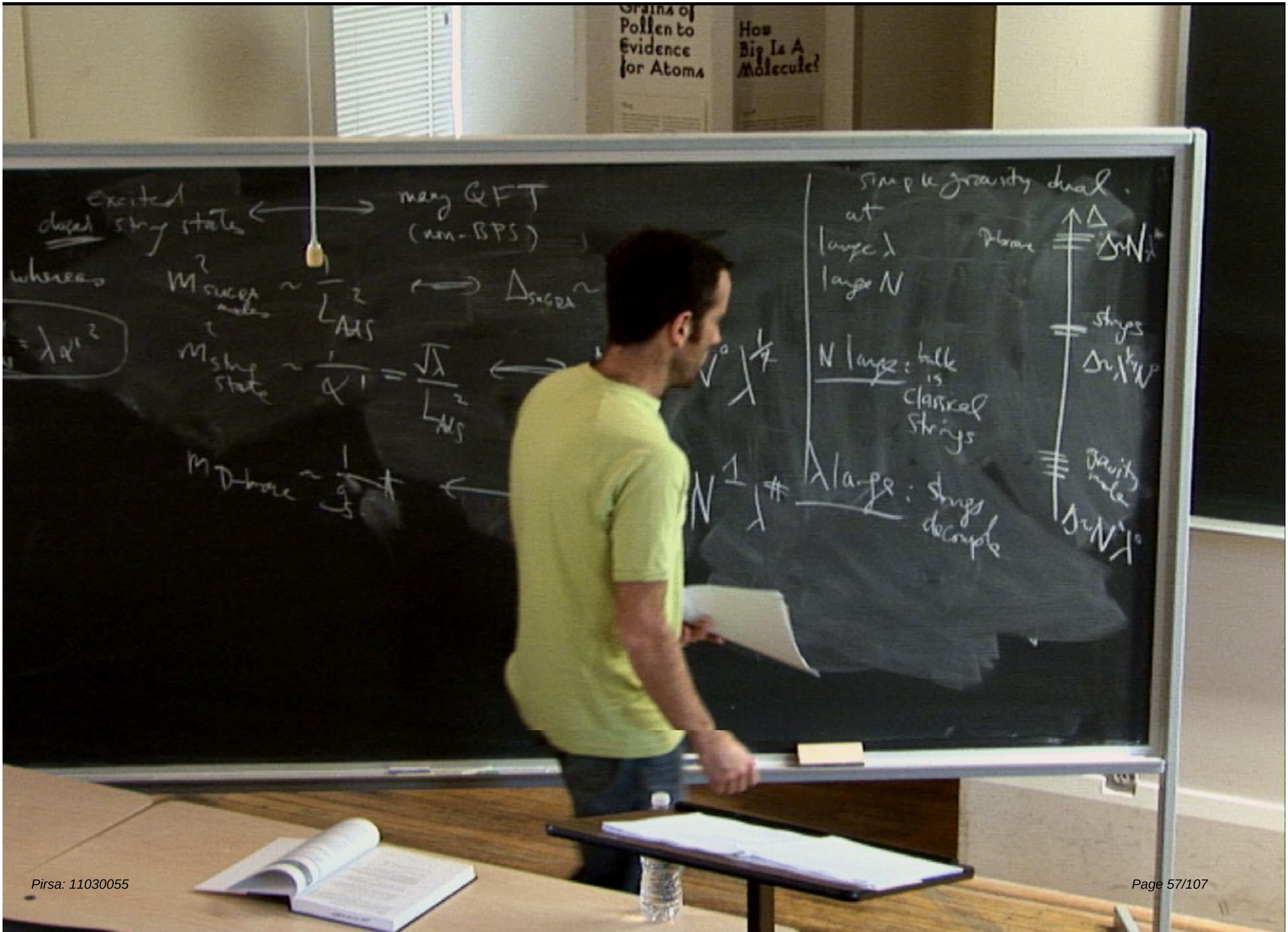
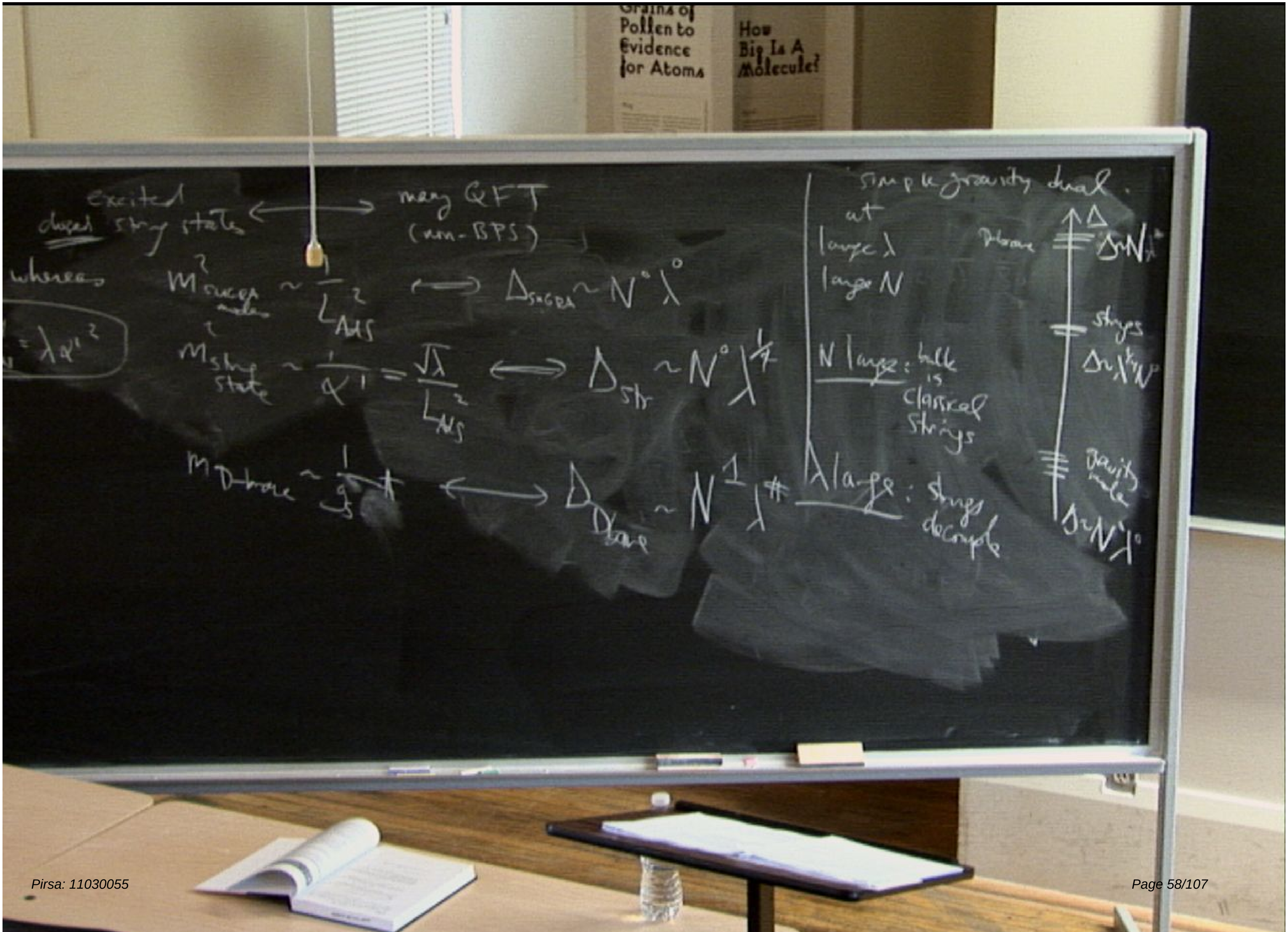
How
Big Is A
Molecule?
excited
closed string states
many QFT
(non-BPS)
whereas
simple gravity dual.
gravity mode

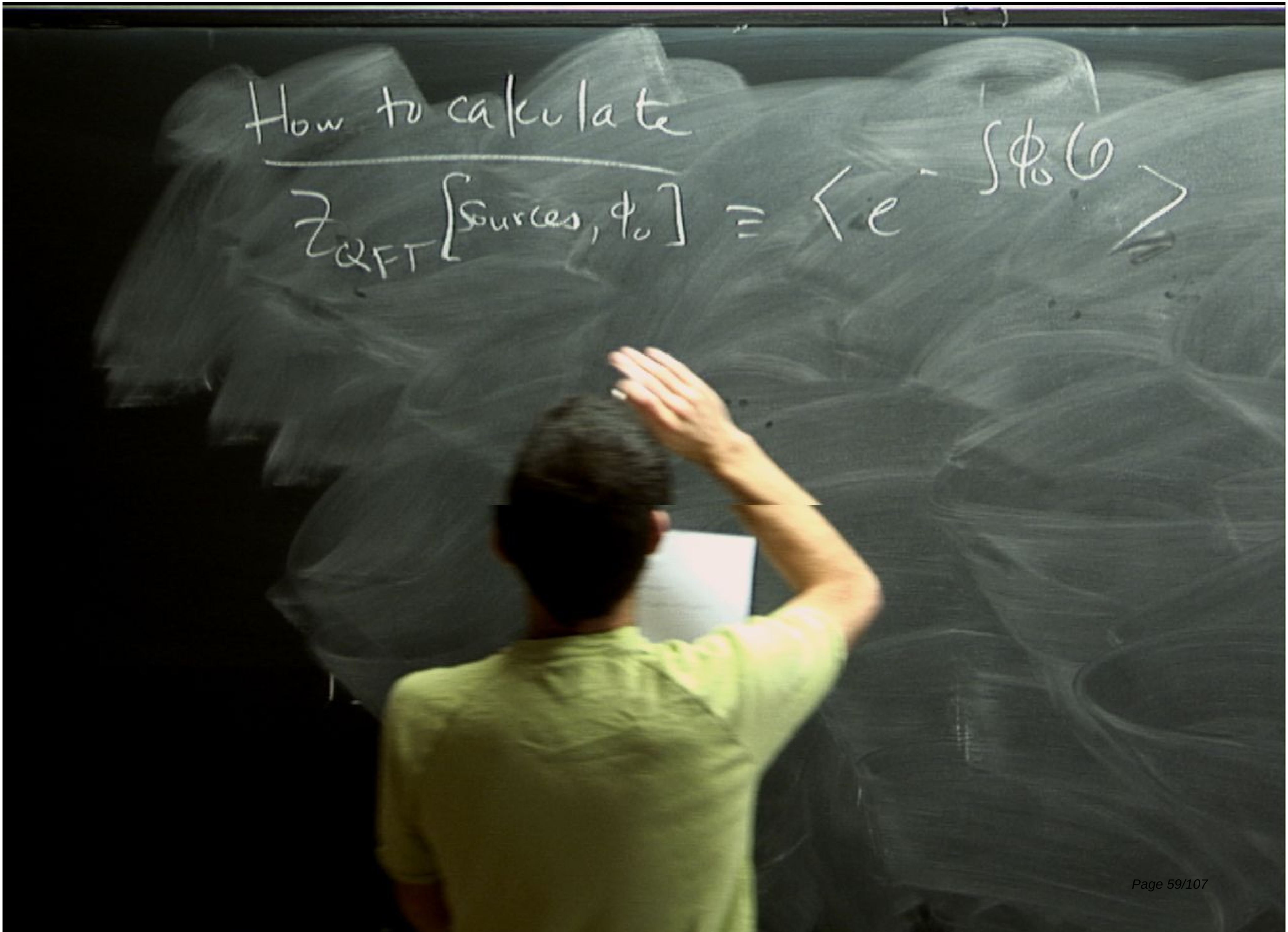


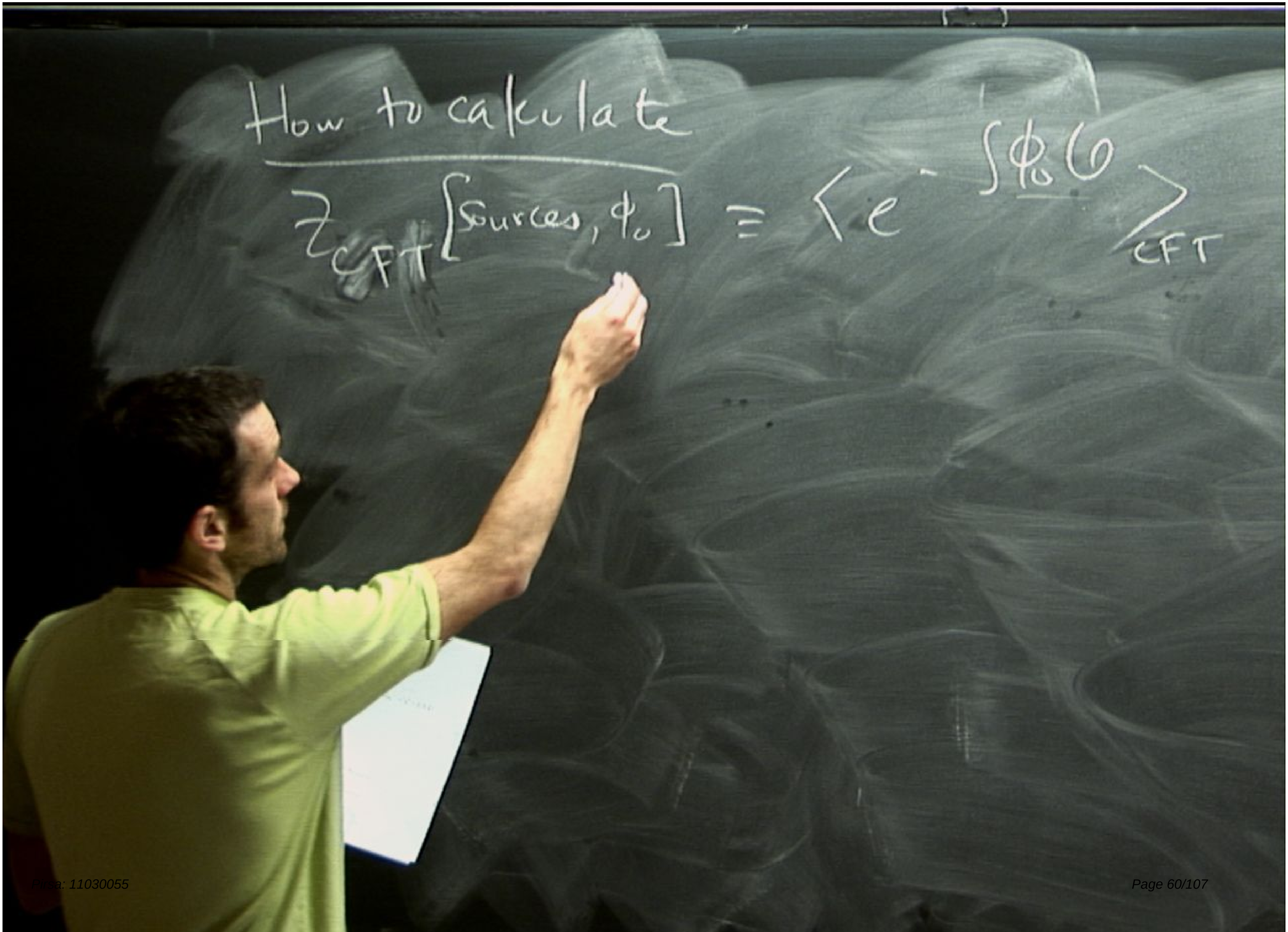
Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
Excited closed string state
many QFT (non-BPS)
whereas
simple gravity dual.
at large λ large N
strings
gravity mode

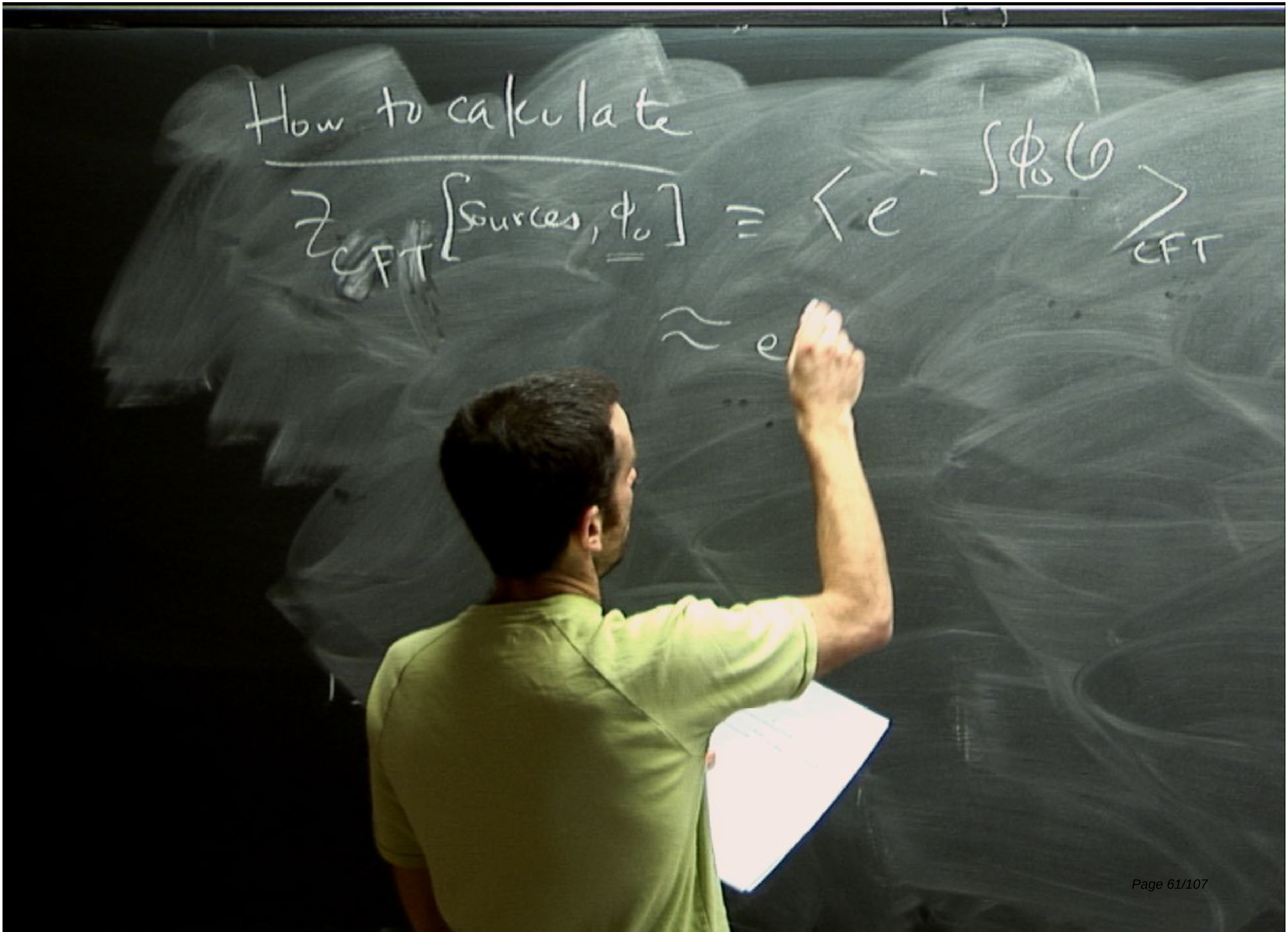

Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
excited closed string states ⟷ many QFT (non-BPS)
whereas
simple gravity dual
at large λ
large N

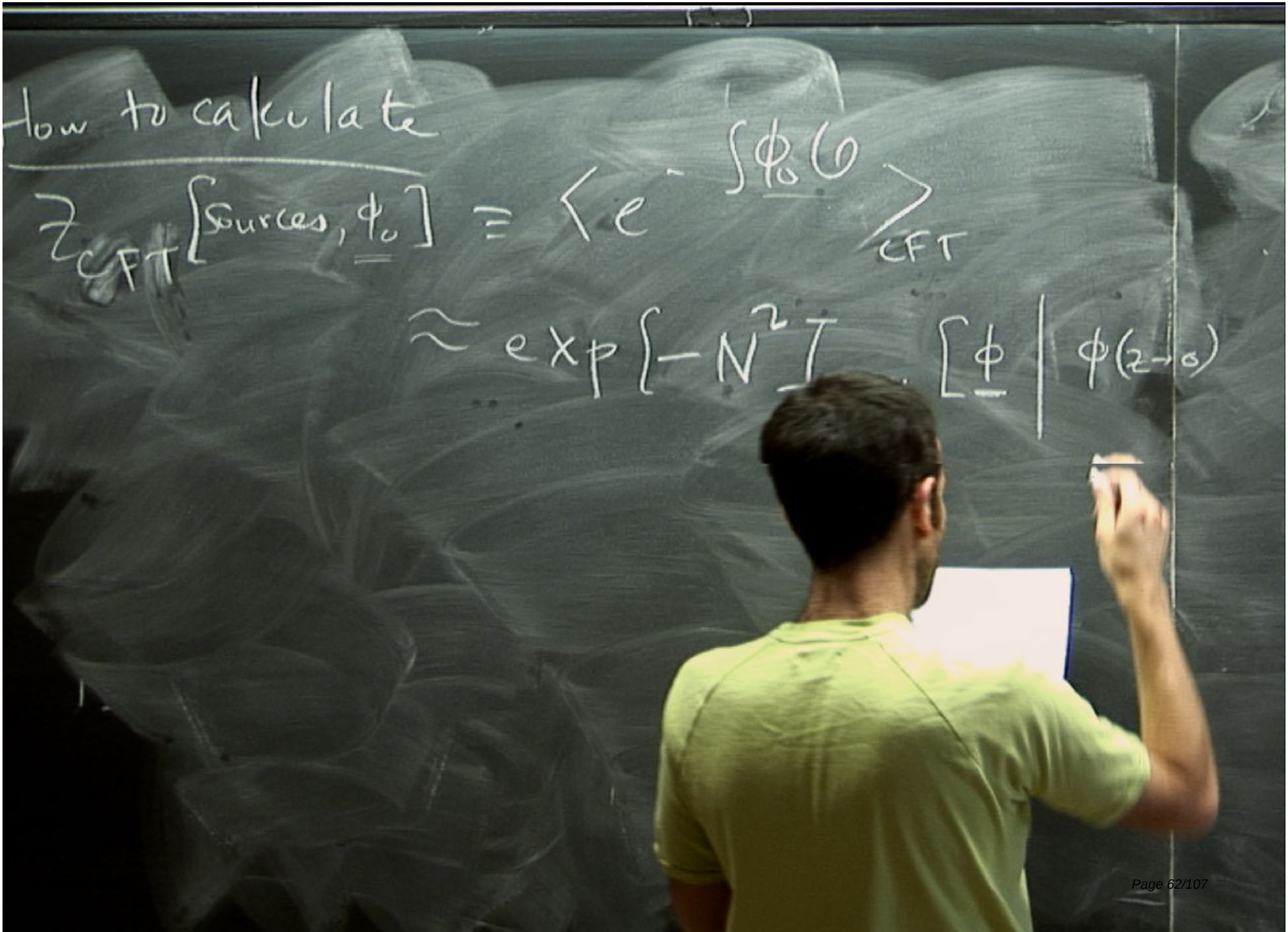

Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
Excited closed string states ⟷ many QFT (non-BPS)
whereas
simple gravity dual.
at large λ large N
N large: bulk is classical strings
λ large: strings decouple

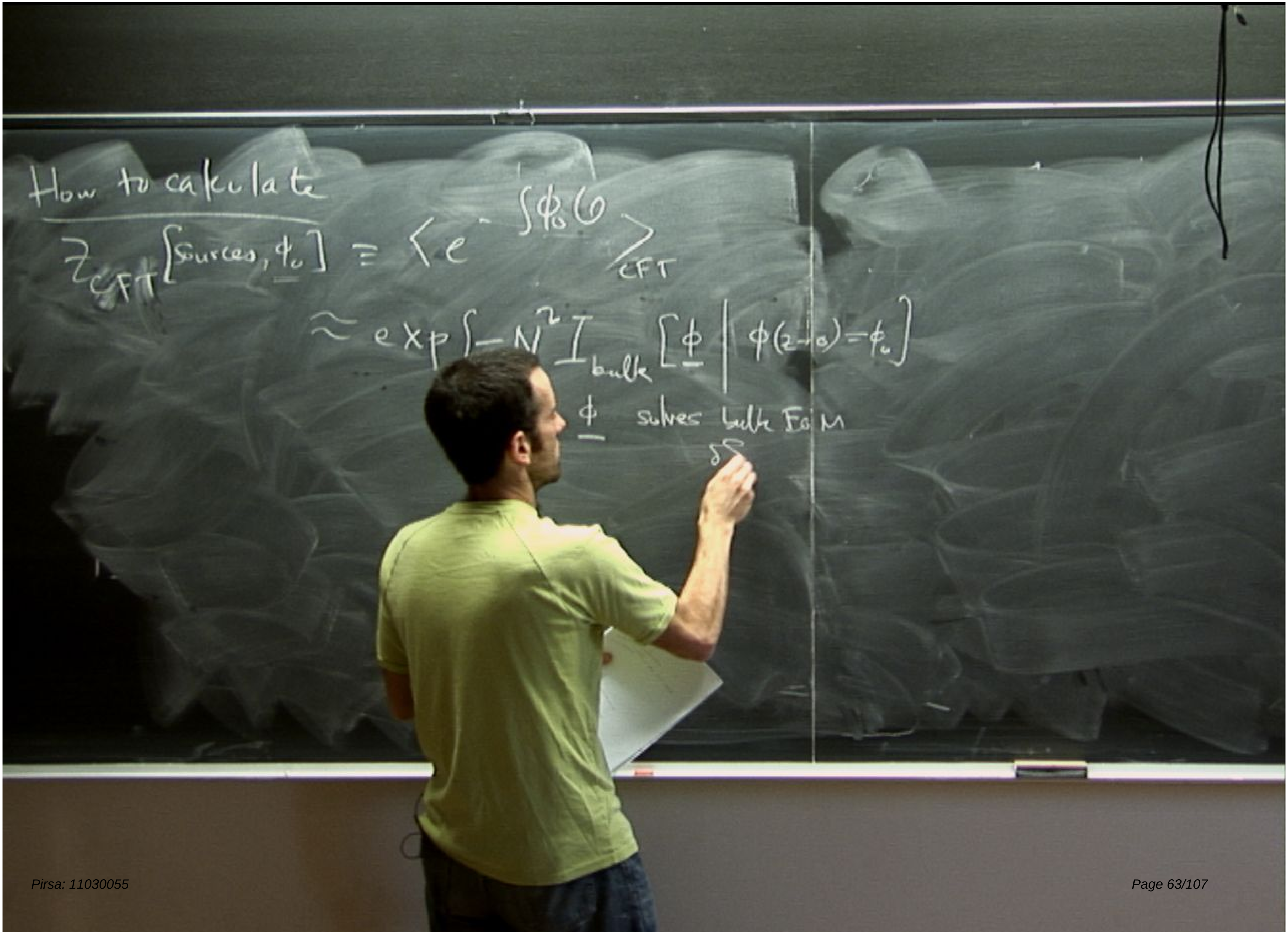

Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
Excited closed string states ⟷ many QFT (non-BPS)
whereas $M^2_{\text{sugra mode}} \sim \frac{1}{L^2_{AdS}} \longleftrightarrow \Delta_{SUGRA} \sim N^0 \lambda^0$
$M^2_{\text{string state}} \sim \frac{1}{\alpha'} = \frac{\sqrt{\lambda}}{L^2_{AdS}} \longleftrightarrow \Delta_{str} \sim N^0 \lambda^{1/4}$
$M_{D\text{-brane}} \sim \frac{1}{g_s} \longleftrightarrow \Delta_{Dbrane} \sim N^1 \lambda^{\#}$
simple gravity dual
at large λ
large N
N large: bulk is classical strings
λ large: strings decouple
D-brane
strings
gravity mode


How to calculate
$Z_{QFT}[\text{sources}, \phi_0] \equiv \langle e^{-\int \phi_0 \mathcal{O}} \rangle$


How to calculate
$Z_{CFT}[\text{sources}, \phi_0] \equiv \langle e^{-\int \phi_0 \mathcal{O}} \rangle_{CFT}$


How to calculate
Z_CFT [sources, φ₀] ≡ ⟨e^(−∫φ₀O)⟩_CFT
≈ e


How to calculate
$Z_{CFT}[\text{sources}, \phi_0] \equiv \langle e^{-\int \phi_0 \mathcal{O}} \rangle_{CFT}$
$\approx \exp\{-N^2 I \; [\phi \,|\, \phi(z\to 0)$


How to calculate
$Z_{CFT}[\text{sources}, \phi_0] = \langle e^{-\int \phi_0 \mathcal{O}} \rangle_{CFT}$
$\approx \exp\{-N^2 I_{bulk}[\phi \mid \phi(z \to 0) = \phi_0]$
ϕ solves bulk EoM
δS

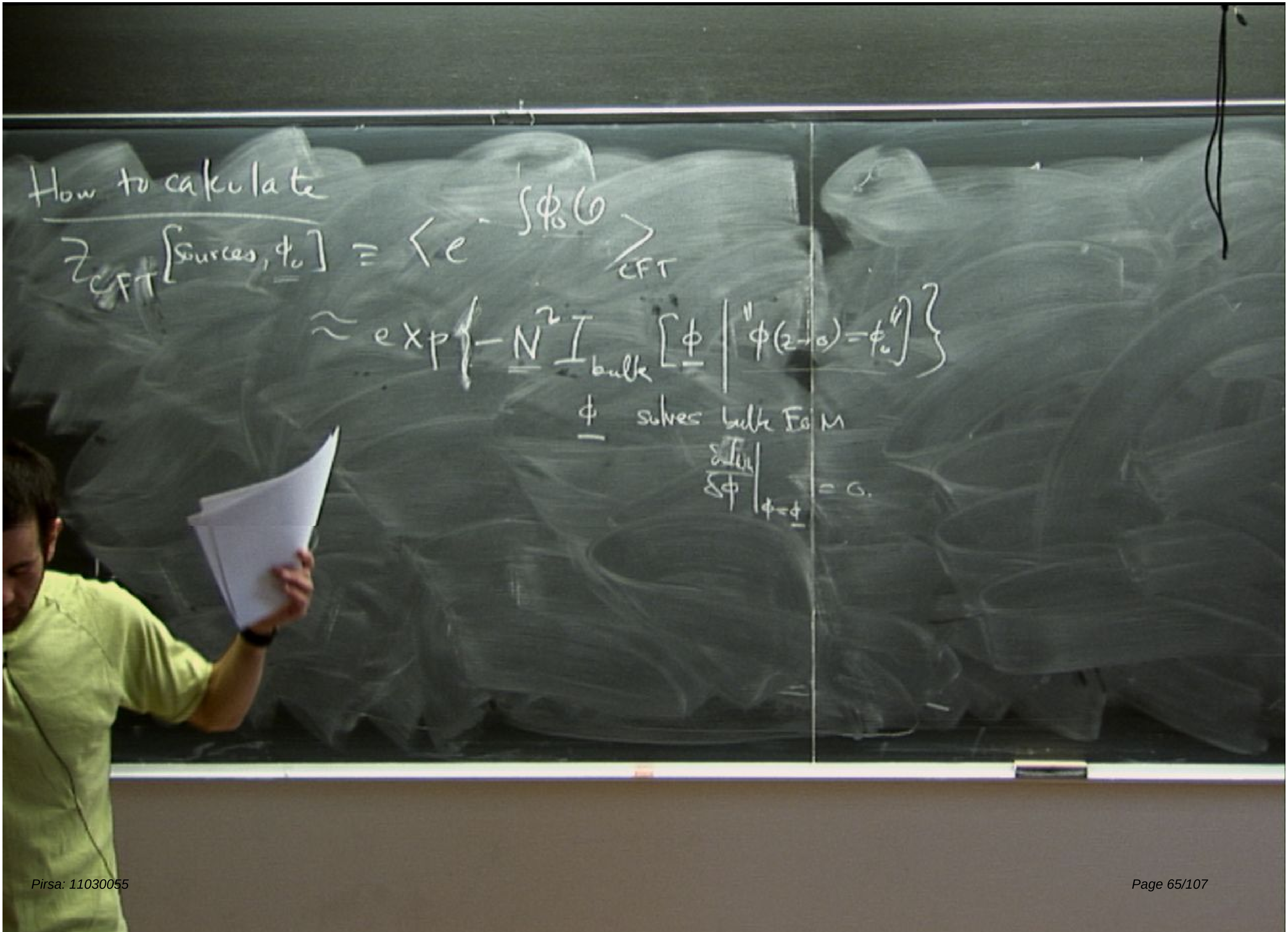
How to calculate

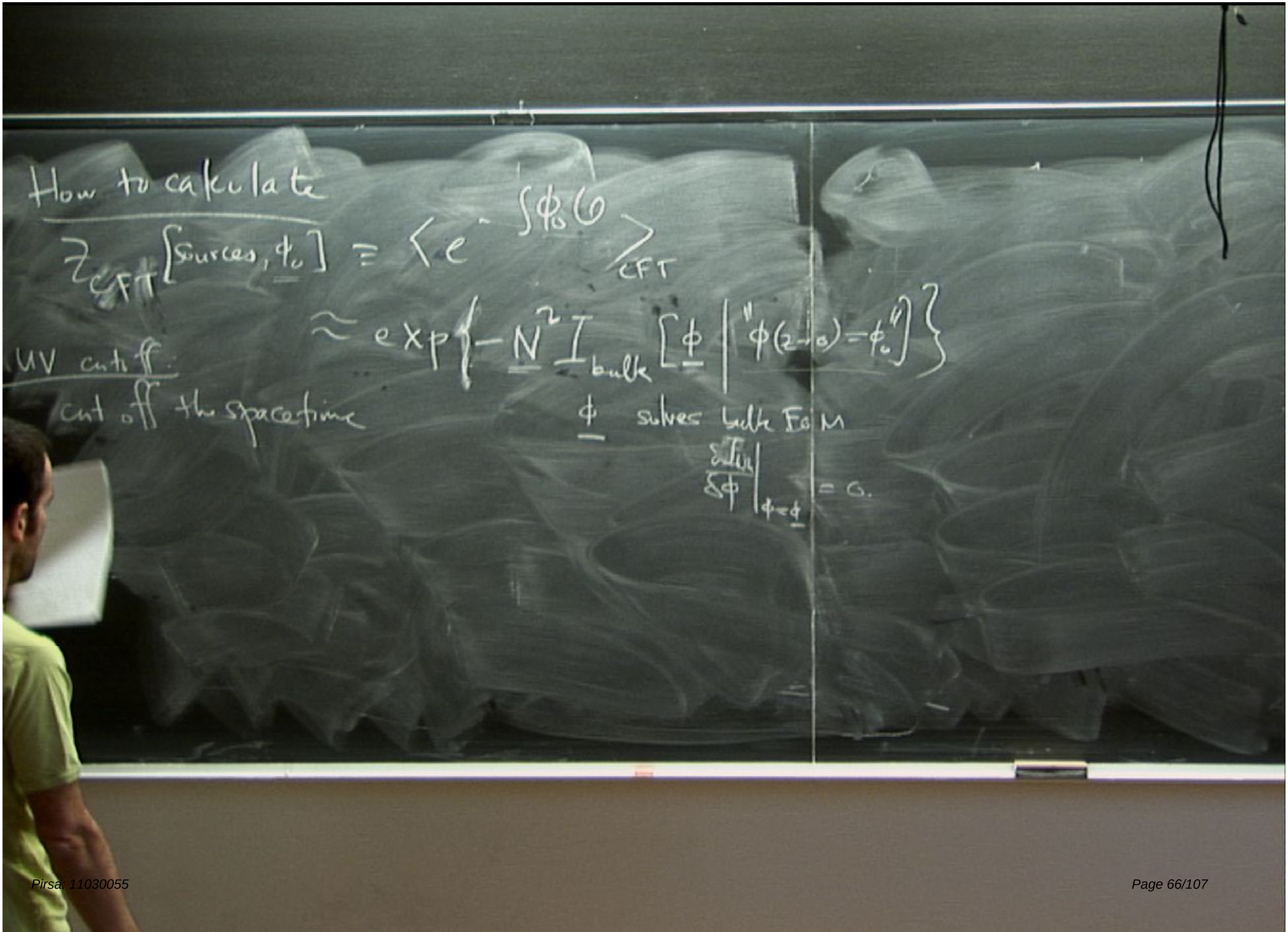
$$Z_{CFT}[\text{sources}, \underline{\phi_0}] \equiv \left\langle e^{-\int \underline{\phi_0}\,\mathcal{O}} \right\rangle_{CFT}$$

$$\approx \exp\left\{-\underline{N}^2 I_{bulk}\left[\underline{\phi} \,\middle|\, \phi(z\to 0) = \phi_0\right]\right.$$

$\underline{\phi}$ solves bulk EoM

$$\left.\frac{\delta I_{bulk}}{\delta\phi}\right|_{\phi=\underline{\phi}} = 0.$$


How to calculate
$Z_{CFT}[\text{sources}, \phi_0] = \langle e^{-\int \phi_0 \mathcal{O}} \rangle_{CFT}$
$\approx \exp\{-N^2 I_{bulk}[\underline{\phi} \,|\, \text{"}\phi(z\to 0) = \phi_0\text{"}]\}$
$\underline{\phi}$ solves bulk EoM
$\frac{\delta I_{bulk}}{\delta \phi}\Big|_{\phi=\underline{\phi}} = 0.$



How to calculate
$Z_{CFT}[\text{sources}, \phi_0] = \langle e^{\int \phi_0 \mathcal{O}} \rangle_{CFT}$
$\approx \exp\{-N^2 I_{bulk}[\underline{\phi} \,|\, \text{"}\phi(z\to 0) = \phi_0\text{"}]\}$
UV cutoff:
cut off the spacetime
$\underline{\phi}$ solves bulk EoM
$\left.\frac{\delta I_{bulk}}{\delta \phi}\right|_{\phi=\underline{\phi}} = 0.$

How to calculate

$$Z_{CFT}[\text{sources}, \phi_0] = \left\langle e^{-\int \phi_0 \mathcal{O}} \right\rangle_{CFT}$$

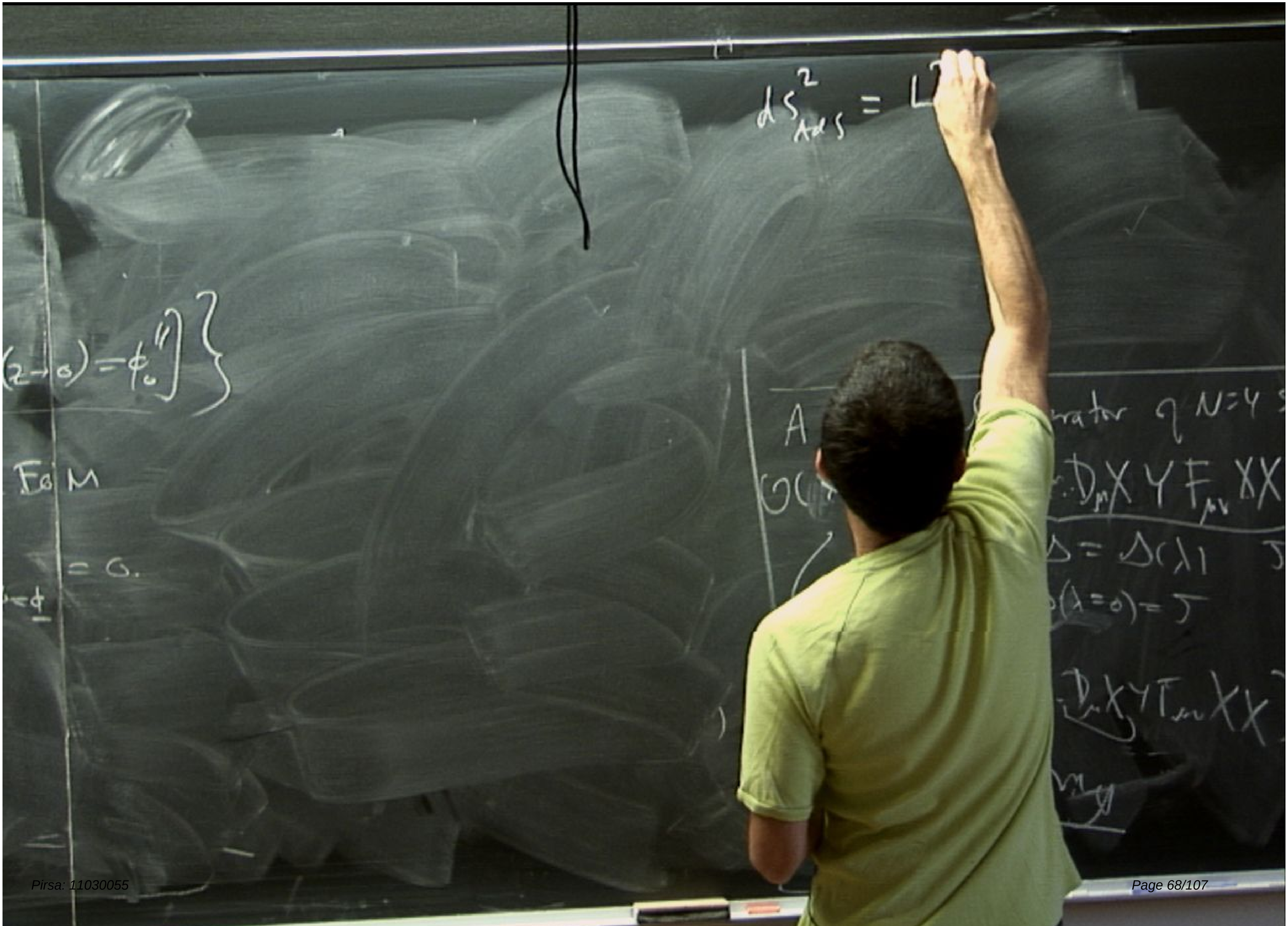
$$\approx \exp\left\{ -N^2 I_{bulk}\left[\underline{\phi} \,\middle|\, ``\phi(z\to 0) = \phi_0" \right] \right\}$$

UV cutoff:

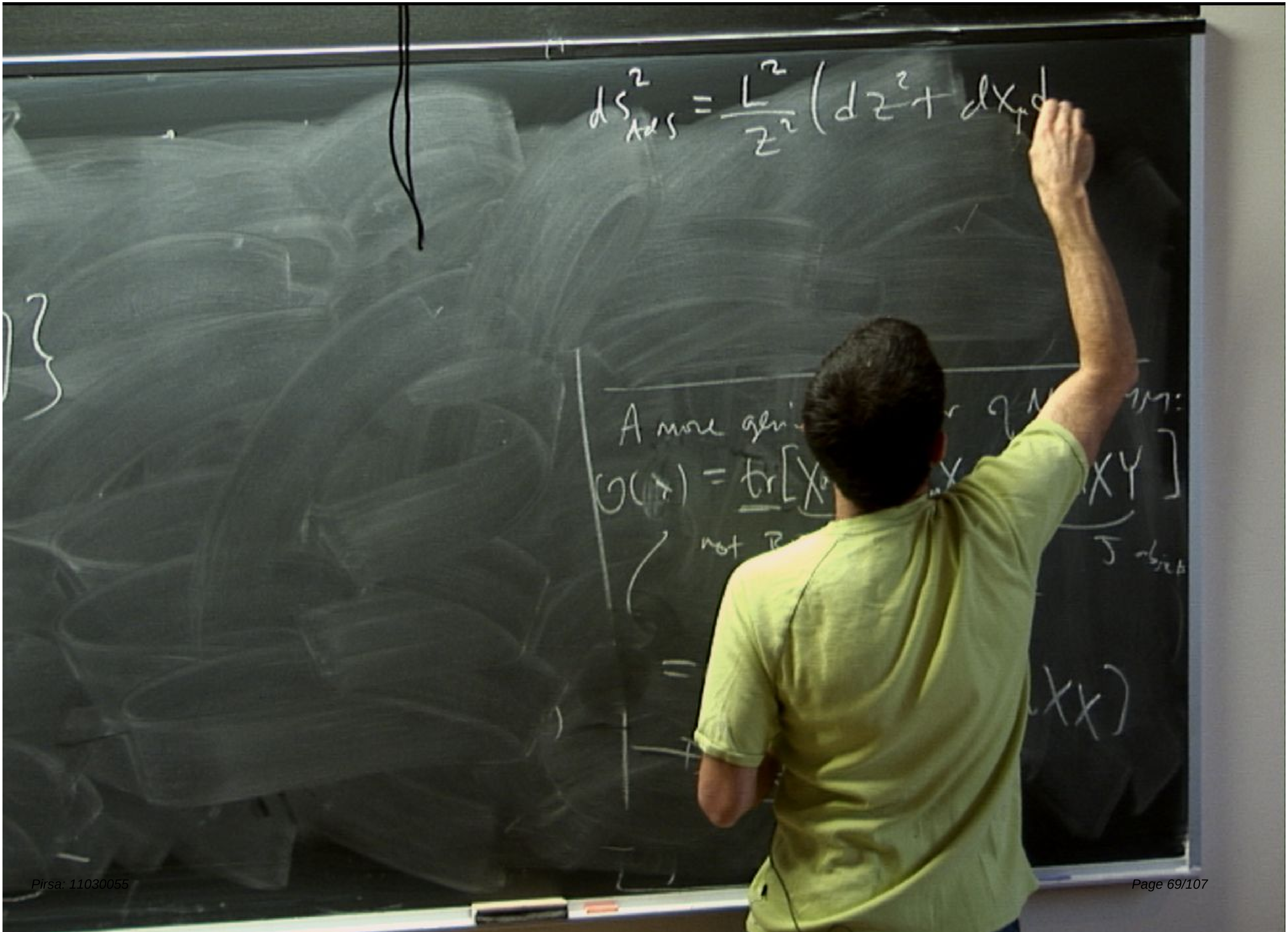
cut off the spacetime at $z > \epsilon$

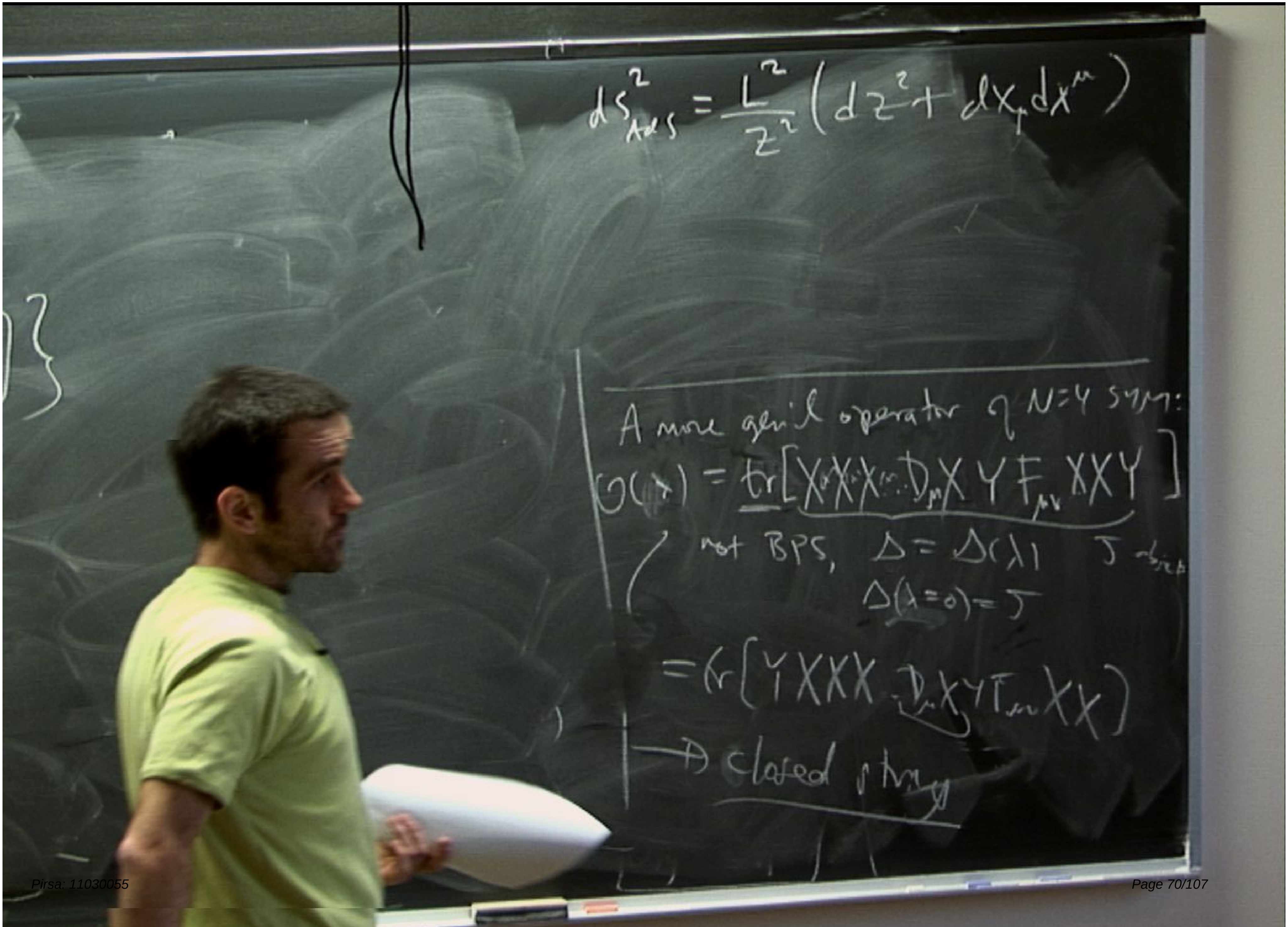
$\underline{\phi}$ solves bulk EoM

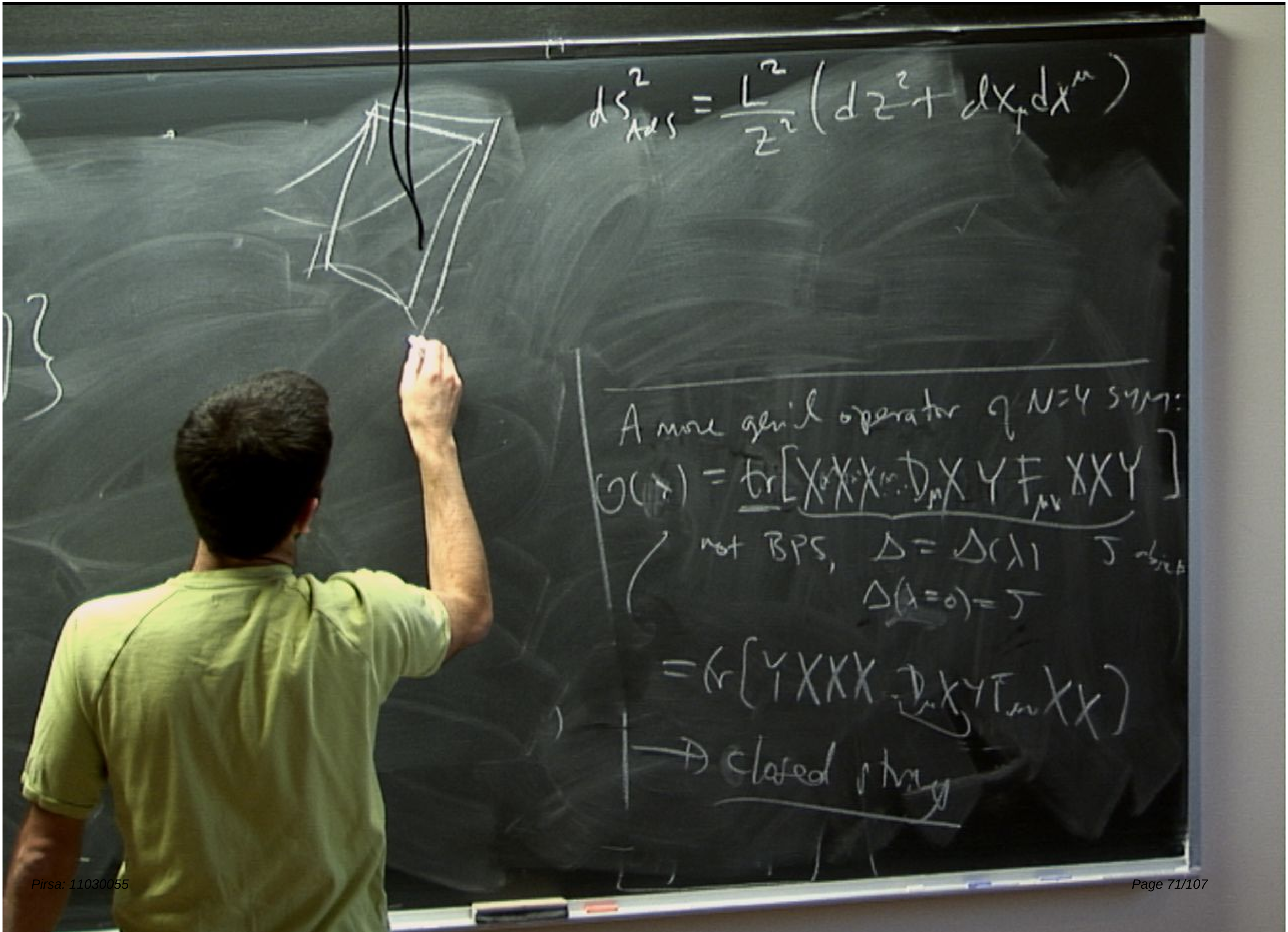
$$\left.\frac{\delta I_{bulk}}{\delta \phi}\right|_{\phi = \underline{\phi}} = 0.$$



$ds^2_{AdS} = L$
EoM


$ds^2_{AdS} = \frac{L^2}{z^2}(dz^2 + dx_\mu d$


ds²_AdS = L²/z² (dz² + dx_μ dx^μ)
A more gen'l operator of N=4 SYM:
O(λ) = tr[XXX...D_μ X Y F_μν XXY]
not BPS, Δ = Δ(λ)   J indep
Δ(λ=0) = J
= tr[YXXX...D_μ X Y F_μν XX]
→ closed string


$ds^2_{AdS} = \frac{L^2}{z^2}\left(dz^2 + dx_\mu dx^\mu\right)$
A more gen'l operator of N=4 SYM:
$\mathcal{O}(\lambda) = \mathrm{tr}[XXX \cdots D_\mu X Y F_{\mu\nu} XXY]$
not BPS, $\Delta = \Delta(\lambda)$
$\Delta(\lambda=0) = J$
$= \mathrm{tr}[YXXX \cdots D_\mu X Y F_{\mu\nu} XX)$
→ closed string

$$ds^2_{AdS} = \frac{L^2}{z^2}\left(dz^2 + dx_\mu dx^\mu\right)$$

A more gen'l operator of $\mathcal{N}=4$ SYM:

$$\mathcal{O}(\lambda) = \mathrm{tr}\left[XXX\cdots D_\mu X\, Y\, F_{\mu\nu}\, XXY\right]$$

not BPS, $\Delta = \Delta(\lambda)$ $\quad J$

$$\Delta(\lambda=0) = J$$

$$= \mathrm{tr}\left[YXXX\cdots D_\mu X\, Y\, F_{\mu\nu}\, XX\right]$$

$\rightarrow$ closed string

How to calculate

$$Z_{CFT}[\text{sources}, \phi_0] \equiv \left\langle e^{-\int \phi_0 \mathcal{O}} \right\rangle_{CFT}$$

$$\approx \exp\left\{-N^2 I_{bulk}\left[\underline{\phi} \,\middle|\, \text{"}\phi(z=\epsilon) = \phi_0\text{"}\right]\right\}$$

UV cutoff:

cut off the spacetime at $z > \epsilon$

$\underline{\phi}$ solves bulk EoM

$$\left.\frac{\delta I_{bulk}}{\delta \phi}\right|_{\phi = \underline{\phi}} = 0.$$

How to calculate

$$Z_{CFT}[\text{sources}, \underline{\phi_0}] \equiv \left\langle e^{-\int \phi_0 \mathcal{O}} \right\rangle_{CFT}$$

$$\approx \exp\left\{ -\underline{N}^2 \underline{I}_{bulk}\left[\underline{\phi} \,\middle|\, \text{"}\phi(\underline{z} = \epsilon) = \phi_0\text{"}\right] \right\}$$

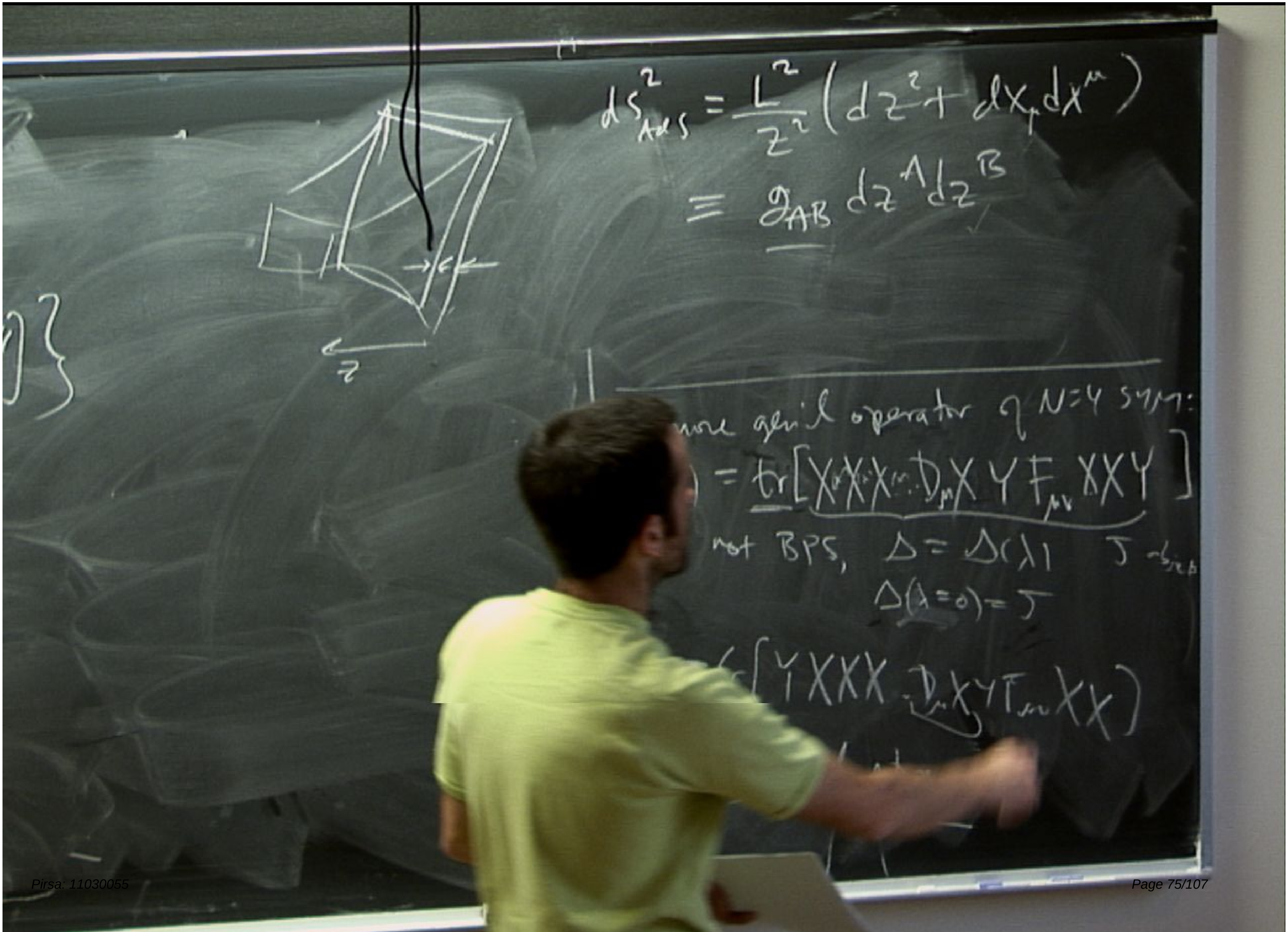
UV cutoff:

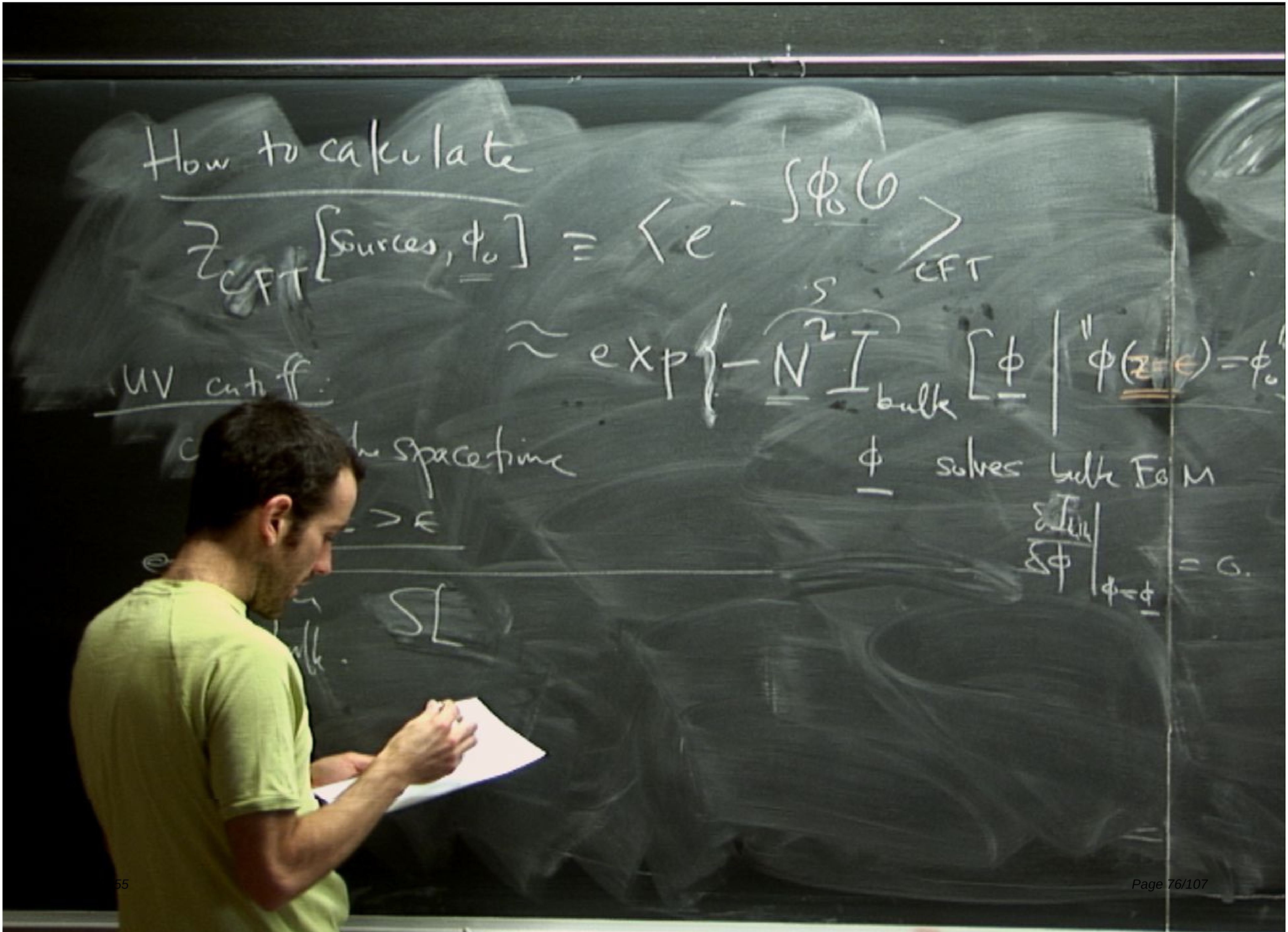
cut off the spacetime at $z > \epsilon$

$\underline{\phi}$ solves bulk EoM

$$\left.\frac{\delta I_{bulk}}{\delta \phi}\right|_{\phi = \underline{\phi}} = 0.$$

eg: scalar in bulk.


$ds^2_{AdS} = \frac{L^2}{z^2}(dz^2 + dx_\mu dx^\mu)$
$\equiv g_{AB}\, dz^A dz^B$
ϵ
z
more gen'l operator of N=4 SYM:
$= \mathrm{tr}[XXX \cdots D_\mu X Y F_{\mu\nu} XXY]$
not BPS, $\Delta = \Delta(\lambda)$
$\Delta(\lambda=0) = J$
$YXXX \cdots D_\mu X Y F_{\mu\nu} XX)$


How to calculate
$Z_{CFT}[\text{sources}, \phi_0] \equiv \langle e^{\int \phi_0 \mathcal{O}} \rangle_{CFT}$
$\approx \exp\{-\overbrace{N^2 I}^{S}_{bulk}[\phi \mid "\phi(z=\epsilon)=\phi_0"]$
UV cutoff:
spacetime
ϕ solves bulk EoM
$\left.\frac{\delta I_{bulk}}{\delta \phi}\right|_{\phi=\phi} = 0.$
SL

How to calculate

$$Z_{CFT}[\text{sources}, \phi_0] \equiv \left\langle e^{-\int \phi_0 \mathcal{O}} \right\rangle_{CFT}$$

$$\approx \exp\left\{ -\overbrace{N^2 I}^{S}_{bulk}\left[\underline{\phi} \;\middle|\; \text{"}\phi(z=\epsilon) = \phi_0\text{"}\right]\right.$$

UV cutoff:

cut off the spacetime at $z > \epsilon$

$\underline{\phi}$ solves bulk EoM

$$\left.\frac{\delta I_{bulk}}{\delta \phi}\right|_{\phi = \underline{\phi}} = 0.$$

eg: scalar in bulk.

$$S[\phi] = -\frac{k}{2} \int d^{d+1}x \sqrt{g} \left[g^{AB} \partial_A \phi \partial_B \phi + m^2 \phi^2 + b\phi^3 + \dots \right]$$

How to calculate

$$Z_{CFT}[\text{sources}, \underline{\phi_0}] \equiv \left\langle e^{-\int \underline{\phi_0}\, \mathcal{O}} \right\rangle_{CFT}$$

$$\approx \exp\left\{ -\overbrace{\underline{N}^2 \underline{I}}^{S}_{\text{bulk}}\left[\underline{\phi} \,\middle|\, {}^{\shortparallel}\phi(\underline{z=\epsilon}) = \phi_0{}^{\shortparallel} \right. \right.$$

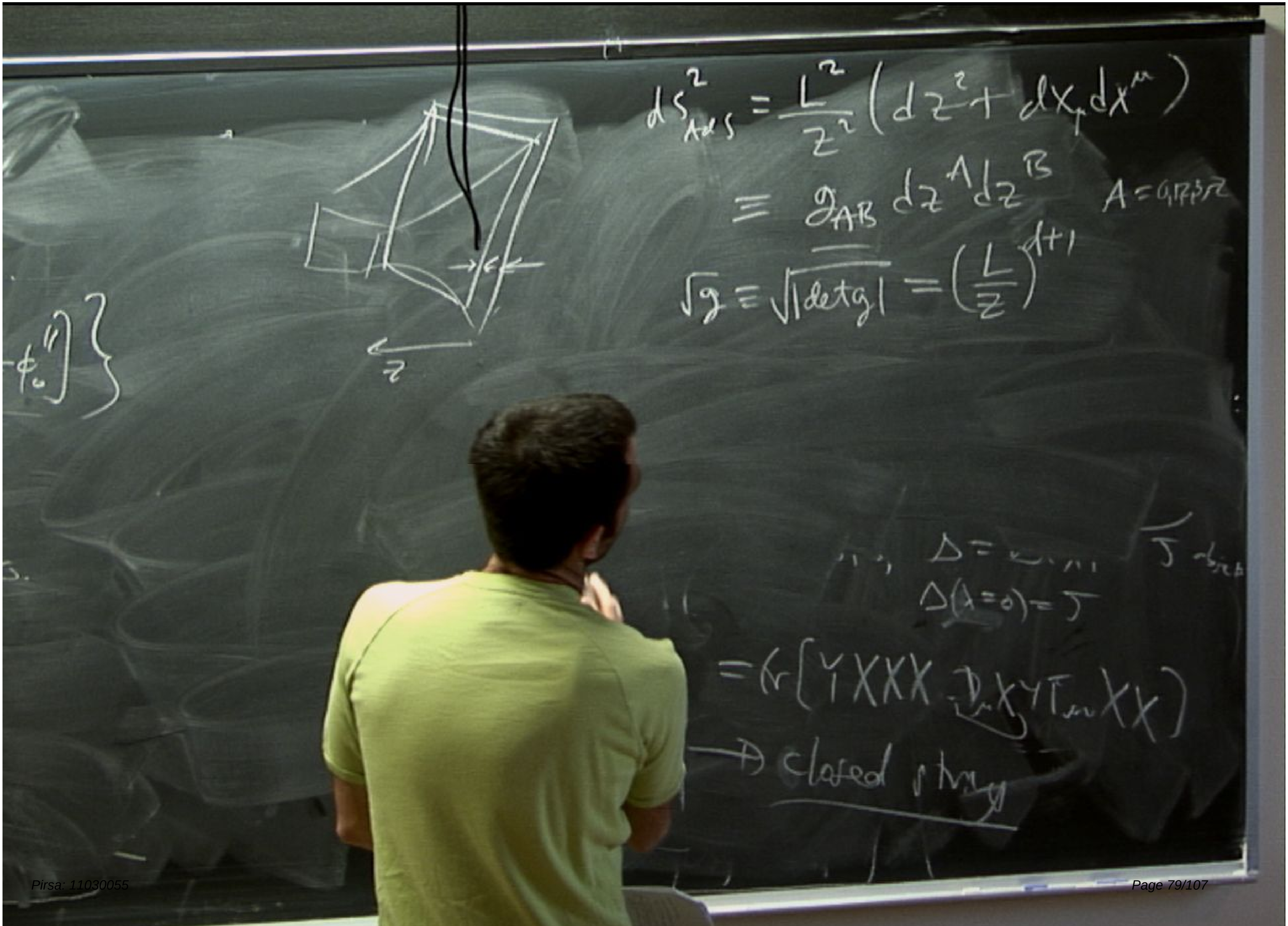
UV cutoff:
cut off the spacetime
at $z > \epsilon$

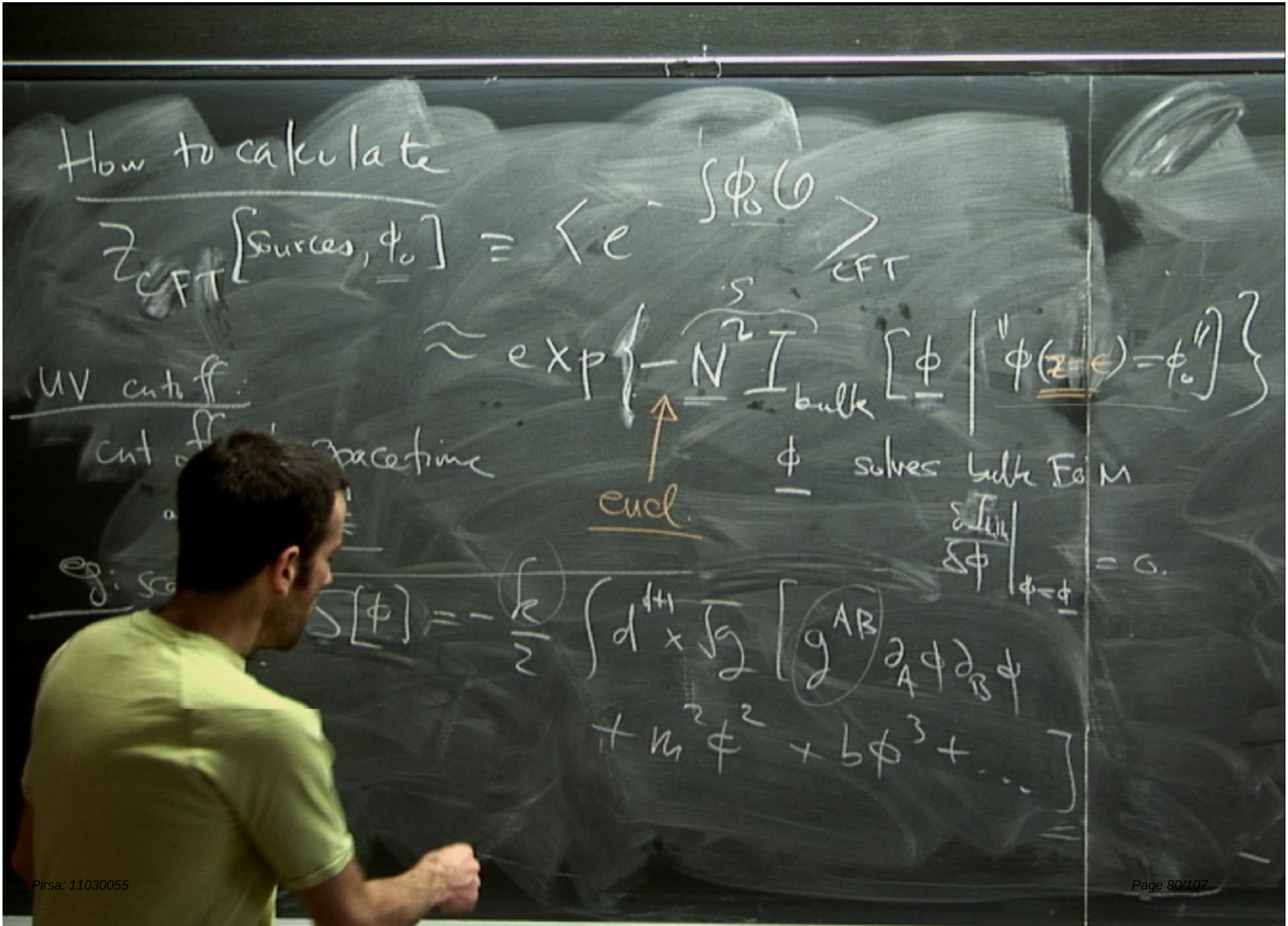
$\underline{\phi}$ solves bulk EoM

$$\left.\frac{\delta I_{\text{bulk}}}{\delta \phi}\right|_{\phi = \underline{\phi}} = 0.$$

eg: scalar
in bulk.

$$S[\phi] = -\frac{k}{2} \int d^{d+1}x \sqrt{g} \left[g^{AB} \partial_A \phi \partial_B \phi + m^2 \phi^2 + b\phi^3 + \ldots \right]$$


$ds^2_{AdS} = \frac{L^2}{z^2}(dz^2 + dx_\mu dx^\mu)$
$\equiv g_{AB}\, dz^A dz^B$
$A = 0,1,2,3,z$
$\sqrt{g} \equiv \sqrt{|\det g|} = \left(\frac{L}{z}\right)^{d+1}$
z
$\Delta(\lambda=0)=J$
$= \mathrm{tr}\{YXXX\, D_\mu X\, Y\, T_{\mu\nu} XX)$
→ closed string


How to calculate
$Z_{CFT}[\text{sources}, \phi_0] \equiv \langle e^{-\int \phi_0 \mathcal{O}} \rangle_{CFT}$
$\approx \exp\{-N^2 I_{bulk}[\underline{\phi} \mid \text{"}\phi(z=\epsilon)=\phi_0\text{"}]\}$
eucl.
$\underline{\phi}$ solves bulk EoM
$\left.\frac{\delta I_{bulk}}{\delta\phi}\right|_{\phi=\underline{\phi}} = 0.$
UV cutoff:
cut off spacetime
eg:
$S[\phi] = -\frac{k}{2}\int d^{d+1}x\sqrt{g}\,[g^{AB}\partial_A\phi\partial_B\phi + m^2\phi^2 + b\phi^3 + \dots]$

How to calculate

$$Z_{CFT}[\text{sources}, \phi_0] \equiv \left\langle e^{-\int \phi_0 \mathcal{O}} \right\rangle_{CFT}$$

$$\approx \exp\left\{ -\overbrace{N^2 I_{bulk}}^{S} [\underline{\phi} \,|\, \text{"}\phi(z=\epsilon) = \phi_0\text{"}] \right\}$$

eucl.

$\underline{\phi}$ solves bulk EoM

$$\left.\frac{\delta I_{bulk}}{\delta \phi}\right|_{\phi=\underline{\phi}} = 0.$$

UV cutoff:

cut off the spacetime at $z > \epsilon$

eg: scalar in bulk.

$$S[\phi] = -\frac{k}{2} \int d^{d+1}x \sqrt{g} \left[g^{AB} \partial_A \phi \partial_B \phi + m^2 \phi^2 + b\phi^3 + \dots \right]$$

$\boxed{k \sim N^2}$ saddle: AdS

How to calculate

$$Z_{CFT}[\text{sources}, \phi_0] \equiv \langle e^{-\int \phi_0 \mathcal{O}} \rangle_{CFT}$$

$$\simeq \exp\{-\overbrace{N^2 I}^{S}_{bulk}[\underline{\phi} \mid \text{"}\phi(z=\epsilon)=\phi_0\text{"}]\}$$

eucl.

$\underline{\phi}$ solves bulk EoM

$$\frac{\delta I_{bulk}}{\delta\phi}\Big|_{\phi=\underline{\phi}} = 0.$$

UV cutoff:

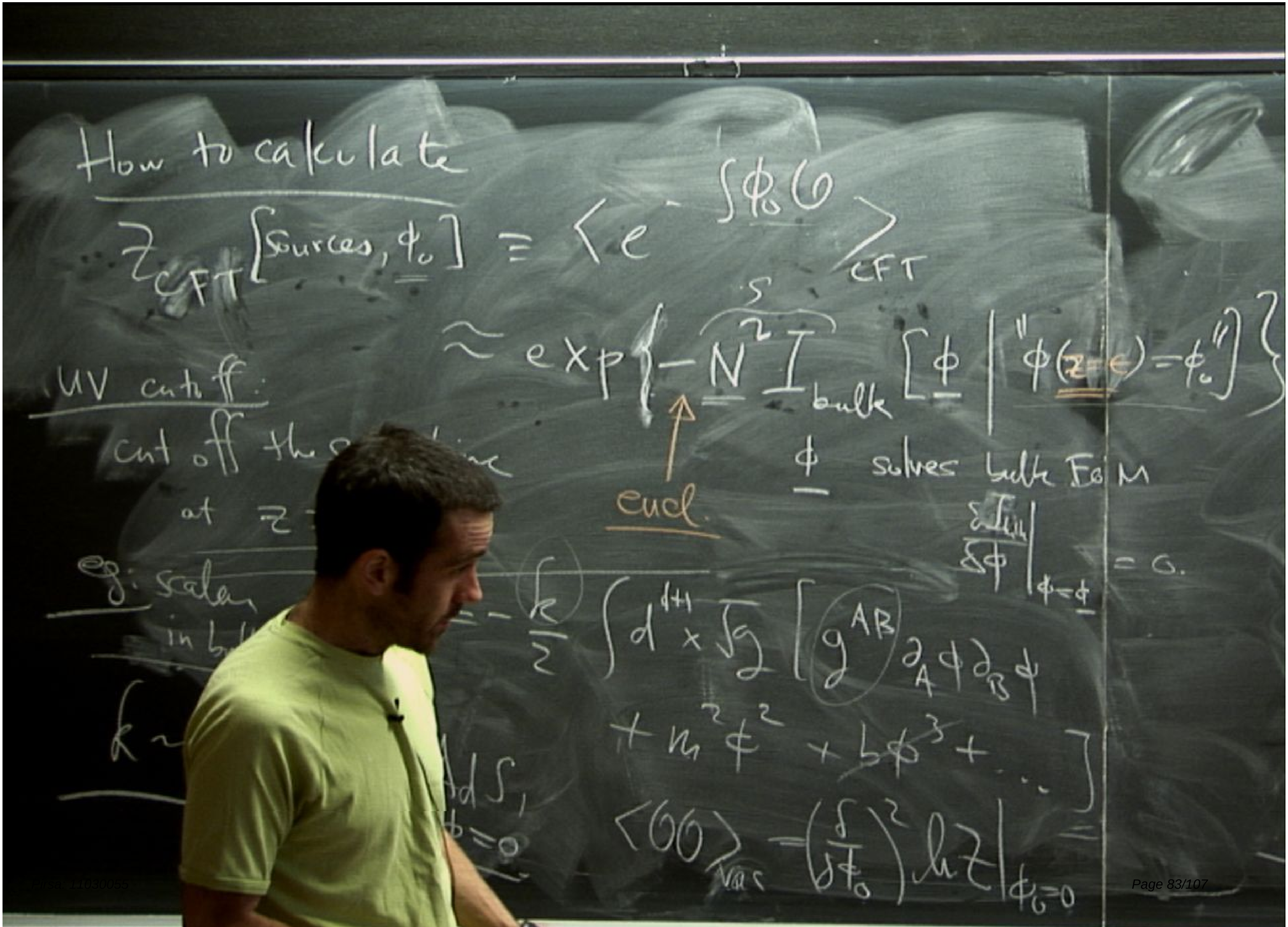
cut off the spacetime at $z > \epsilon$

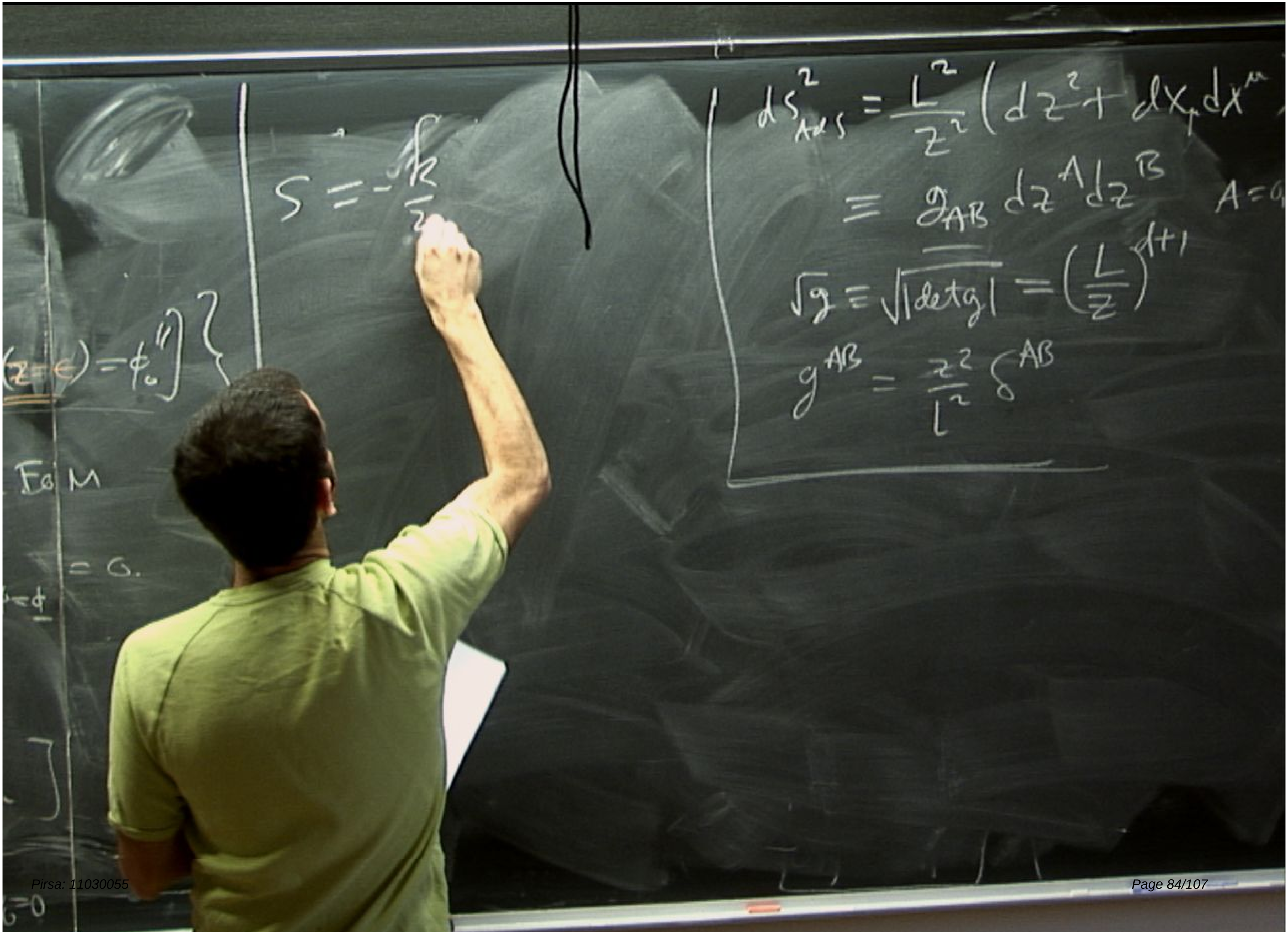
eg: scalar in bulk.

$$S[\phi] = -\frac{k}{2}\int d^{d+1}x\sqrt{g}\left[g^{AB}\partial_A\phi\,\partial_B\phi + m^2\phi^2 + b\phi^3 + \dots\right]$$

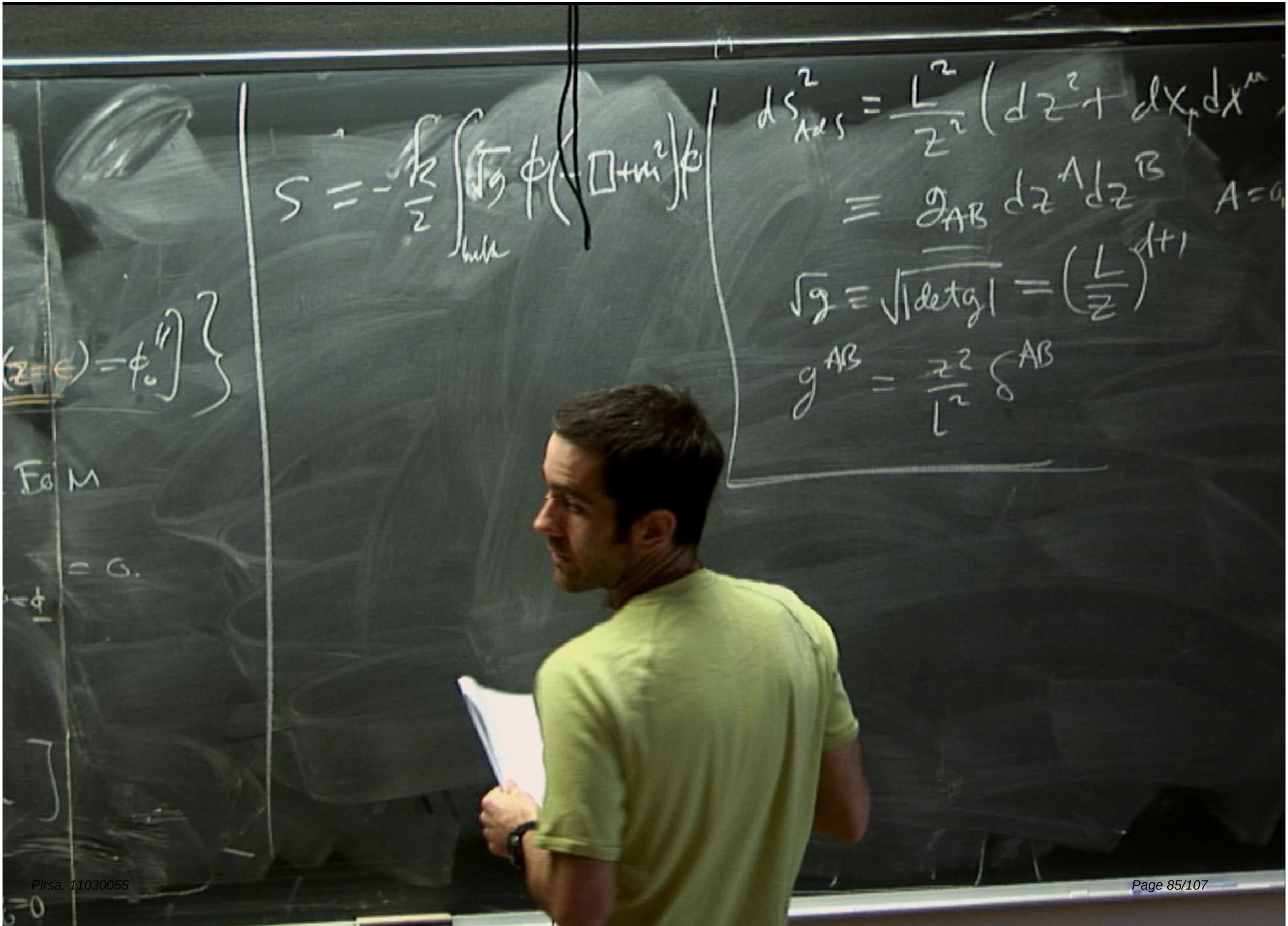
$\boxed{k \sim N^2}$

saddle: AdS, $\phi = 0$


How to calculate
Z CFT [sources, φ₀] ≡ ⟨ e^{-∫φ₀O} ⟩ CFT
≈ exp{ −N² I bulk [φ | "φ(z=ε) = φ₀"] }
UV cutoff:
cut off the
at z =
eucl.
φ solves bulk EoM
δI bulk / δφ | φ=φ = 0.
eg: scalar
in bulk
k/2 ∫d^{d+1}x √g [g^{AB} ∂_A φ ∂_B φ
+ m² φ² + b φ³ + ...]
⟨OO⟩ vac = (δ/δφ₀)² ln Z | φ₀=0



$S = -\int \frac{k}{2}$
$ds^2_{AdS} = \frac{L^2}{z^2}\left(dz^2 + dx_\mu dx^\mu\right)$
$\equiv g_{AB}\, dz^A dz^B$
$A =$
$\sqrt{g} \equiv \sqrt{|\det g|} = \left(\frac{L}{z}\right)^{d+1}$
$g^{AB} = \frac{z^2}{L^2}\delta^{AB}$
EoM
$= 0.$


$S = -\frac{k}{2}\int_{bulk}\sqrt{g}\,\phi(-\Box+m^2)\phi$
$ds^2_{AdS} = \frac{L^2}{z^2}(dz^2 + dx_\mu dx^\mu$
$\equiv g_{AB}\, dz^A dz^B$
$\sqrt{g} \equiv \sqrt{|\det g|} = \left(\frac{L}{z}\right)^{d+1}$
$g^{AB} = \frac{z^2}{L^2}\delta^{AB}$
EoM

How to calculate

$$Z_{CFT}[\text{sources}, \phi_0] \equiv \left\langle e^{-\int \phi_0 \mathcal{O}} \right\rangle_{CFT}$$

$$\approx \exp\{-N^2 I_{bulk}[\underline{\phi} \mid \text{"}\phi(z=\epsilon) = \phi_0\text{"}]$$

eucl

$\underline{\phi}$ solves bulk EoM

$$\left.\frac{\delta I_{bulk}}{\delta \phi}\right|_{\phi=\underline{\phi}} = 0.$$

UV cutoff:

cut off the spacetime at $z > \epsilon$

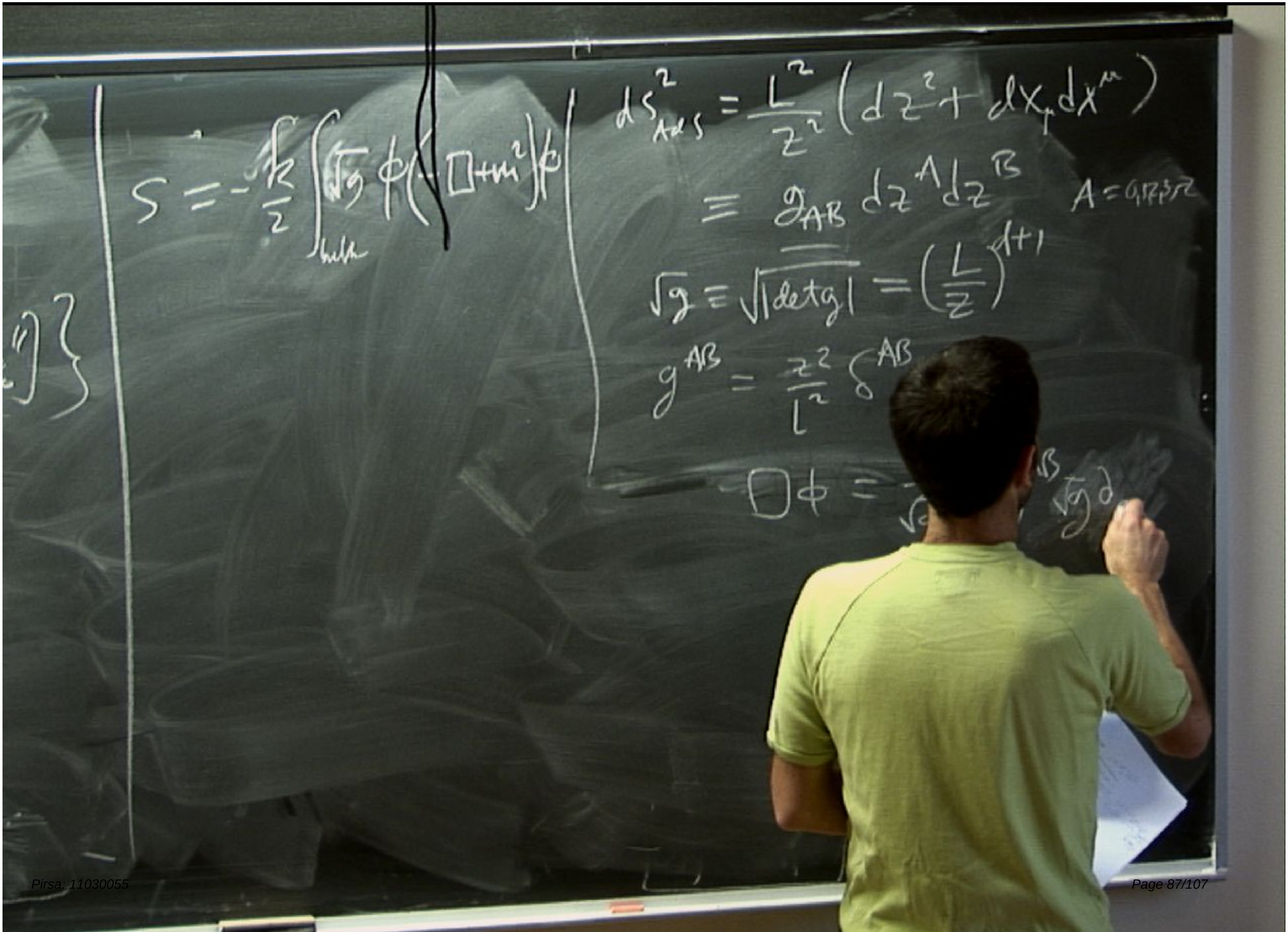
eg: scalar in bulk.

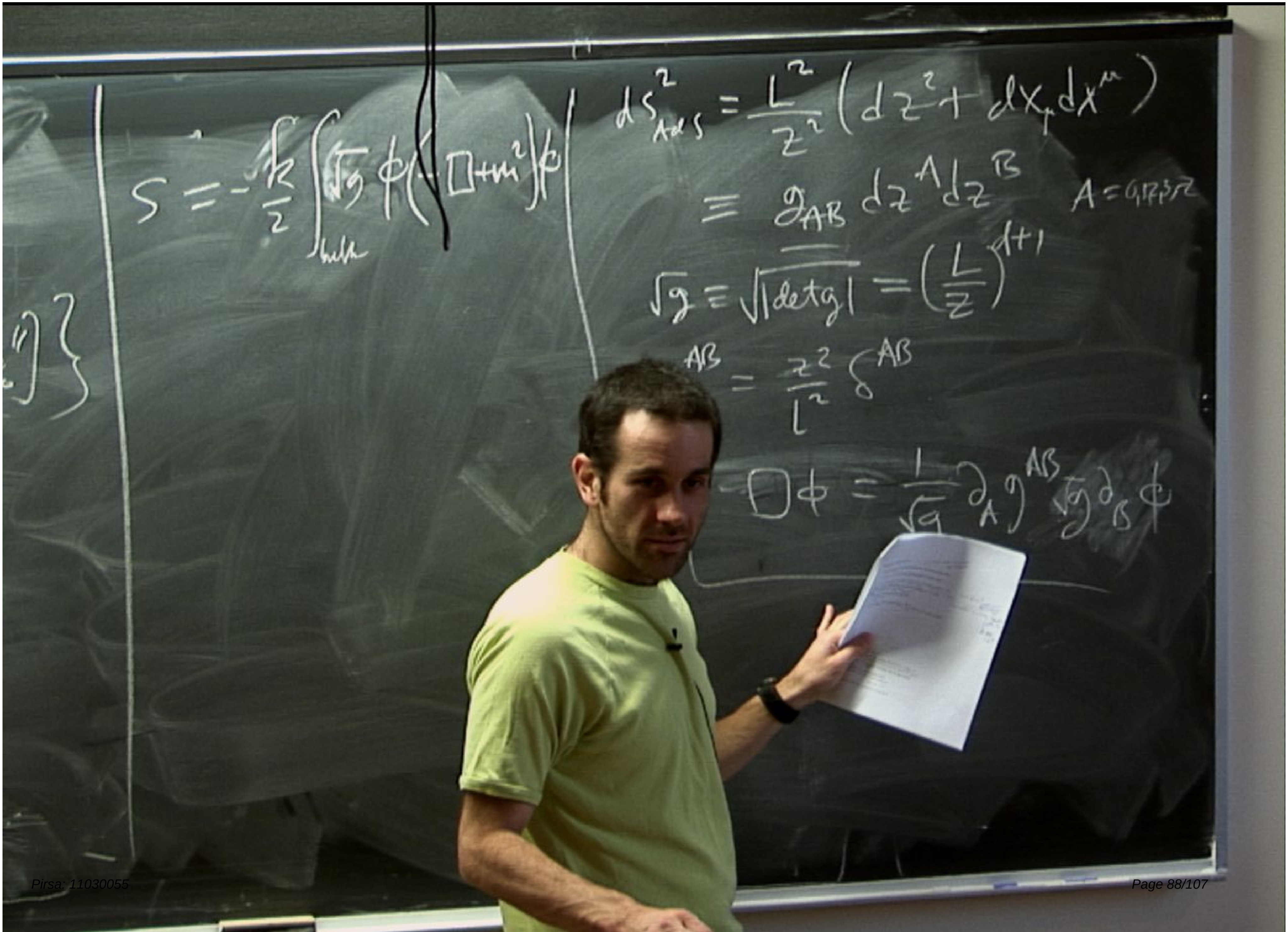
$$S[\phi] = -\frac{k}{2} \int d^{d+1} \dots g^{AB} \partial_A \phi \partial_B \phi \dots b\phi^3 + \dots]$$

$k \sim N^2$

saddle: AdS, $\phi = 0$

$$\dots)^2 \ln Z|_{\phi_0 = 0}$$


$S = -\frac{1}{2}\int_{bulk}\sqrt{g}\,\phi(-\Box+m^2)\phi$
$ds^2_{AdS} = \frac{L^2}{z^2}(dz^2 + dx_\mu dx^\mu)$
$\equiv g_{AB}\,dz^A dz^B$
$A = 0,1,2,3,z$
$\sqrt{g} \equiv \sqrt{|\det g|} = \left(\frac{L}{z}\right)^{d+1}$
$g^{AB} = \frac{z^2}{L^2}\delta^{AB}$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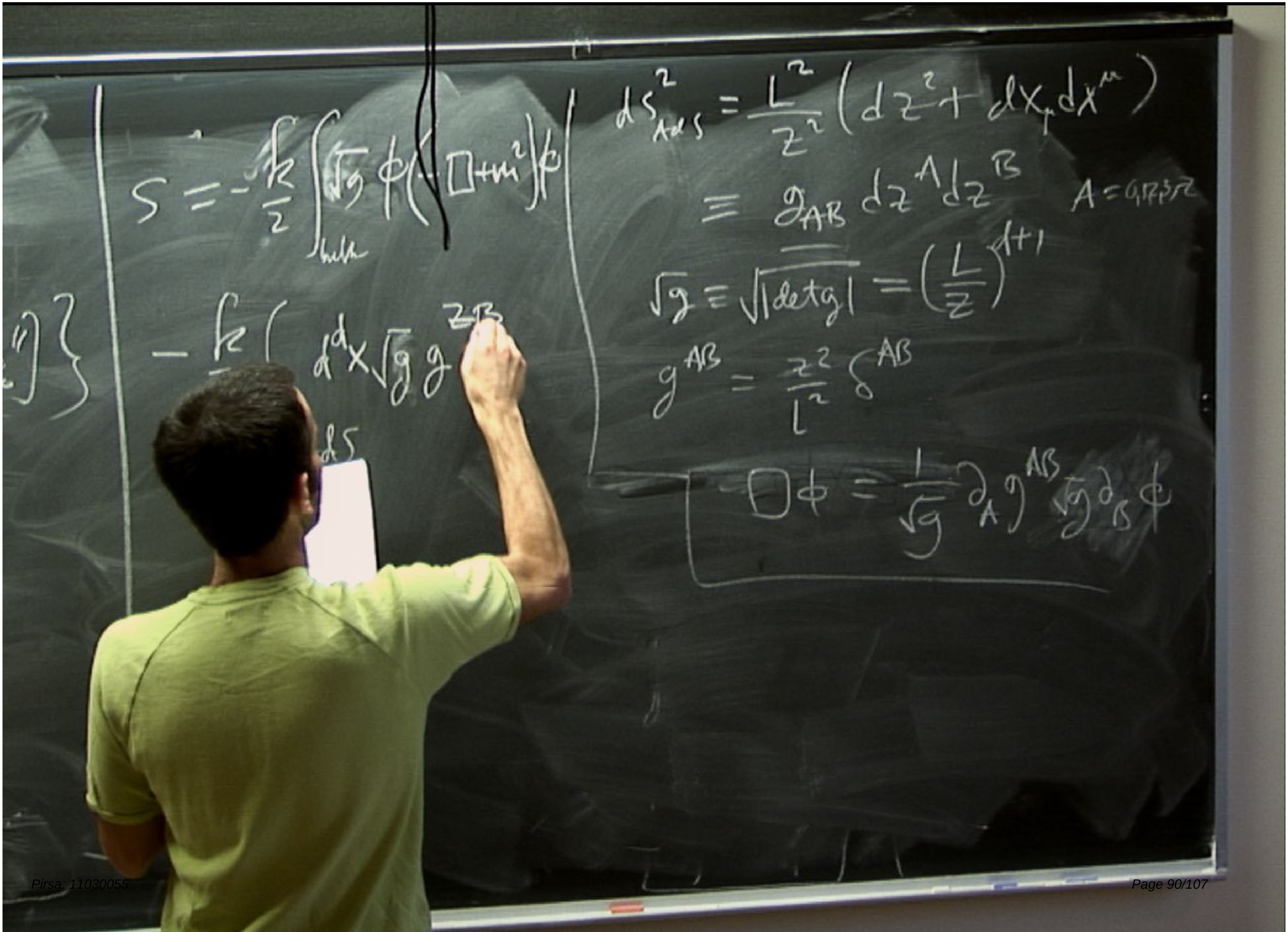
$S = -\frac{k}{2}\int_{bulk}\sqrt{g}\,\phi(-\Box+m^2)\phi$
$-k\int d^dx\sqrt{g}\,g^{zB}$
$ds^2_{AdS} = \frac{L^2}{z^2}(dz^2 + dx_\mu dx^\mu)$
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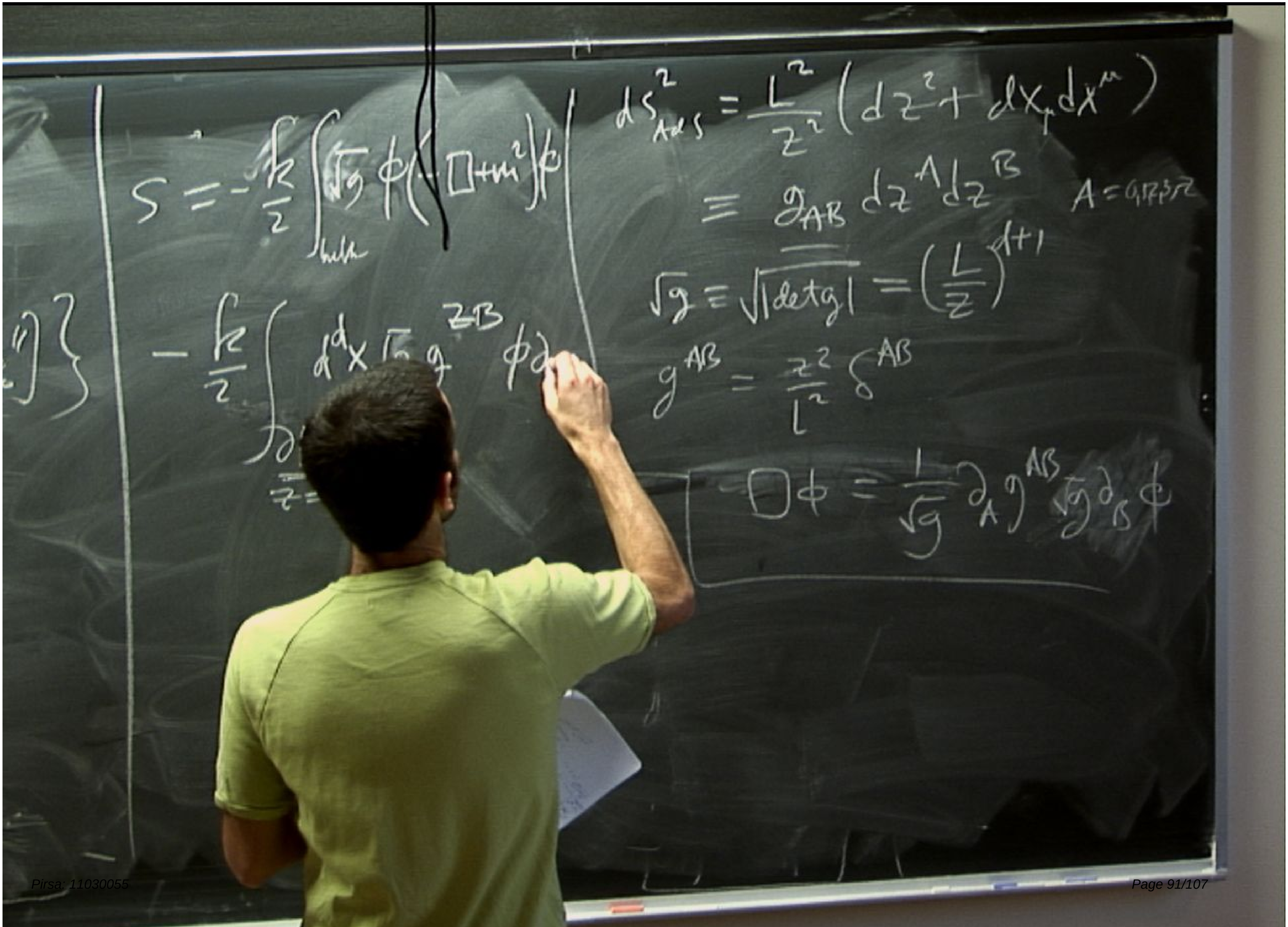
ds²_AdS = L²/z² (dz² + dx_μ dx^μ)
≡ g_AB dz^A dz^B
A = 0,1,2,3,z
√g ≡ √|det g| = (L/z)^(d+1)
g^AB = z²/L² δ^AB
□φ = 1/√g ∂_A g^AB √g ∂_B φ
S = - k/2 ∫_bulk √g φ(-□+m²)φ
- k/2 ∫ d^d x √γ g^zB φ∂

$$S = -\frac{k}{2}\int_{bulk} \sqrt{g}\,\phi\left(-\Box + m^2\right)\phi$$

$$-\frac{k}{2}\int_{\partial AdS,\ z=\epsilon} d^dx \sqrt{g}\, g^{zB}\, \phi\,\partial_B\phi$$

$$ds^2_{AdS} = \frac{L^2}{z^2}\left(dz^2 + dx_\mu dx^\mu\right)$$

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$$\boxed{\Box\phi = \frac{1}{\sqrt{g}}\partial_A g^{AB}\sqrt{g}\,\partial_B\phi}$$

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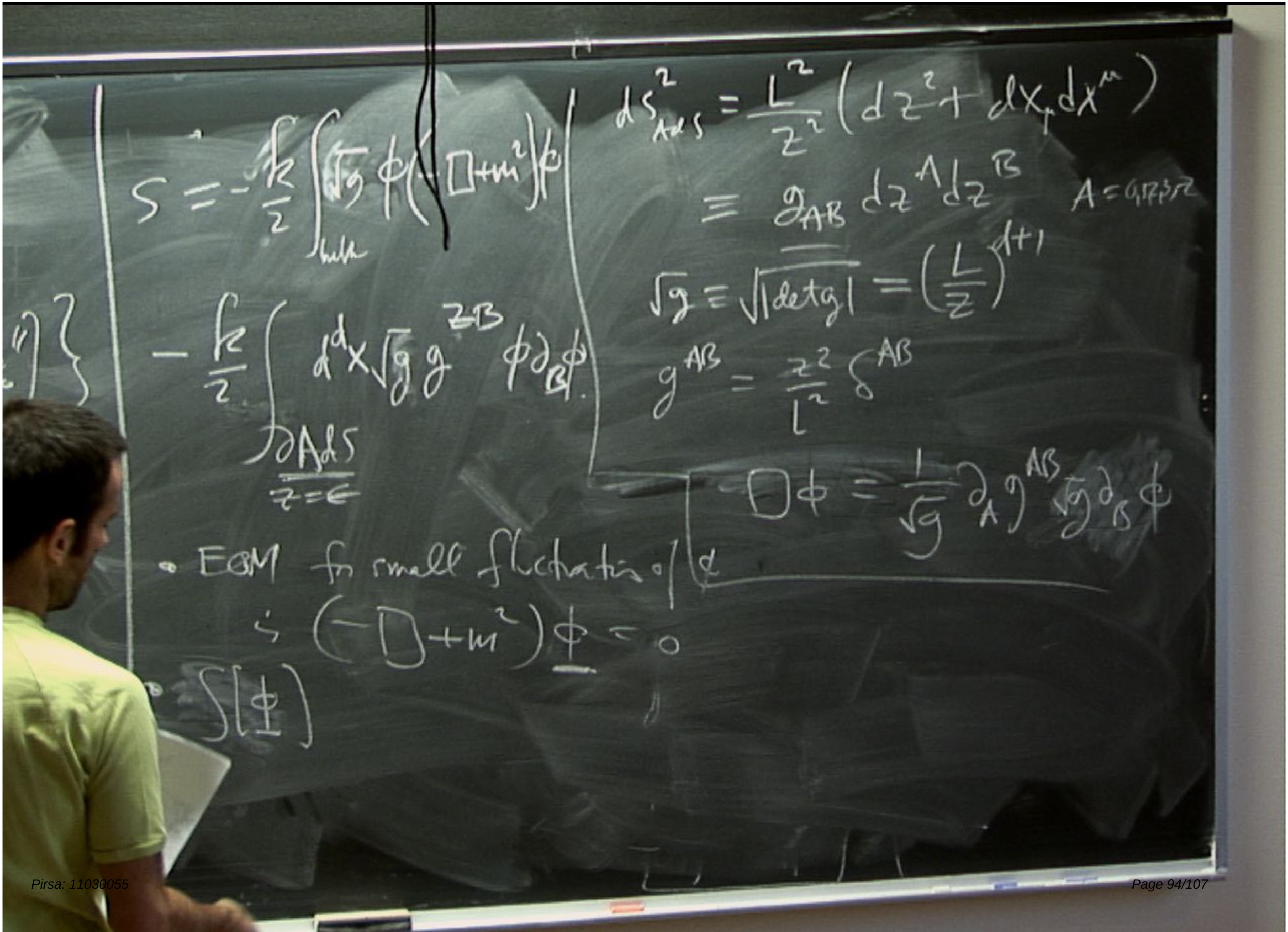
$$\boxed{\Box\phi = \frac{1}{\sqrt{g}}\,\partial_A\, g^{AB}\sqrt{g}\,\partial_B\phi}$$

- EOM for small fluctuations of ϕ is $\left(-\Box+m^2\right)\underline{\phi} = 0$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• EOM for small fluctuations of ϕ
$(-\Box+m^2)\phi = 0$
• $S[\Phi]$

$$S = -\frac{k}{2}\int_{\text{bulk}} \sqrt{g}\,\phi(-\Box + m^2)\phi$$

$$-\frac{k}{2}\int_{\substack{\partial AdS \\ z=\epsilon}} d^d x \sqrt{g}\, g^{zB}\, \phi\,\partial_B \phi.$$

$$ds^2_{AdS} = \frac{L^2}{z^2}\left(dz^2 + dx_\mu dx^\mu\right)$$

$$\equiv g_{AB}\, dz^A dz^B \qquad A = 0,1,2,3,z$$

$$\sqrt{g} \equiv \sqrt{|\det g|} = \left(\frac{L}{z}\right)^{d+1}$$

$$g^{AB} = \frac{z^2}{L^2}\delta^{AB}$$

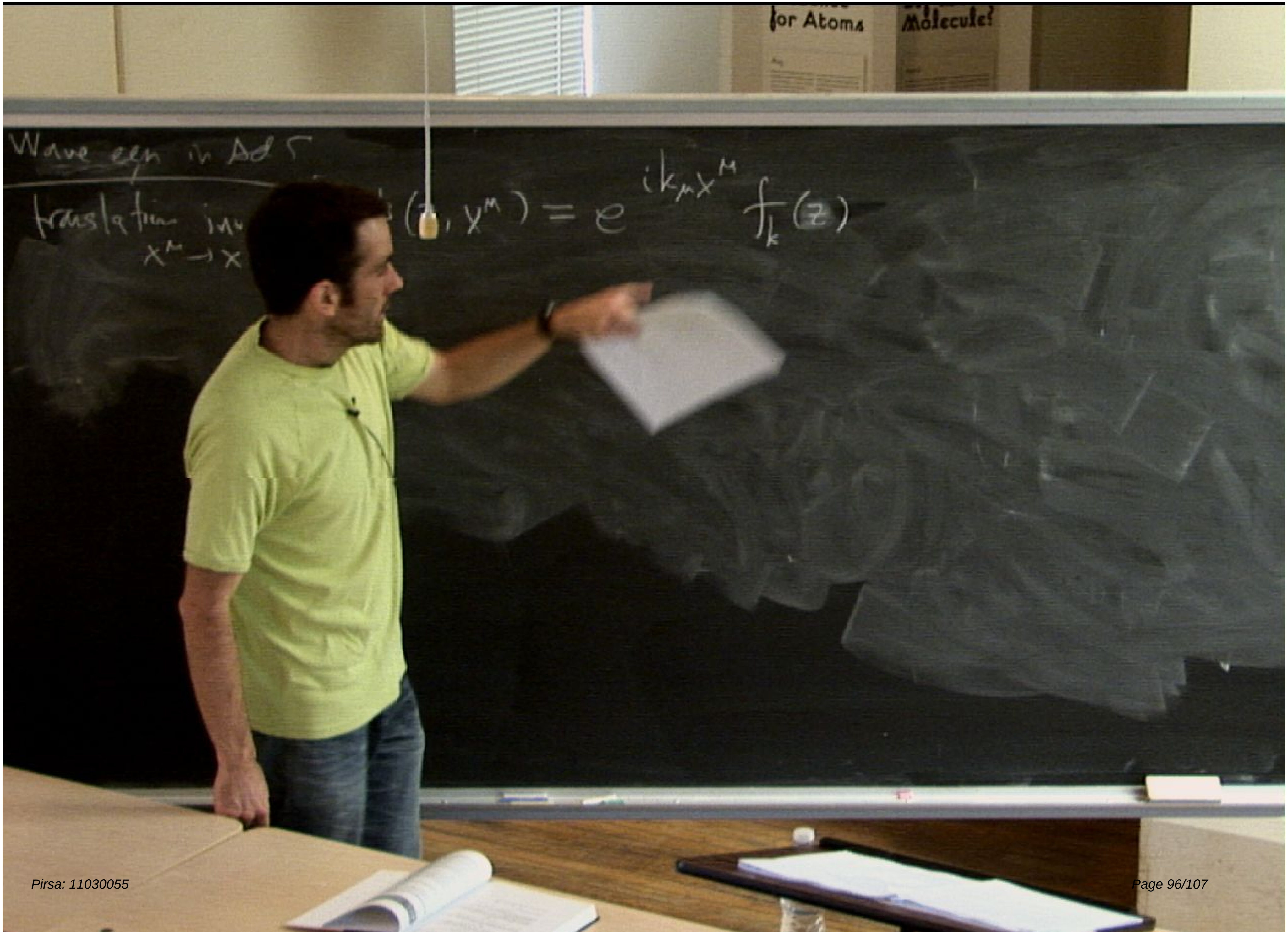
$$\Box\phi = \frac{1}{\sqrt{g}}\partial_A g^{AB}\sqrt{g}\,\partial_B\phi$$

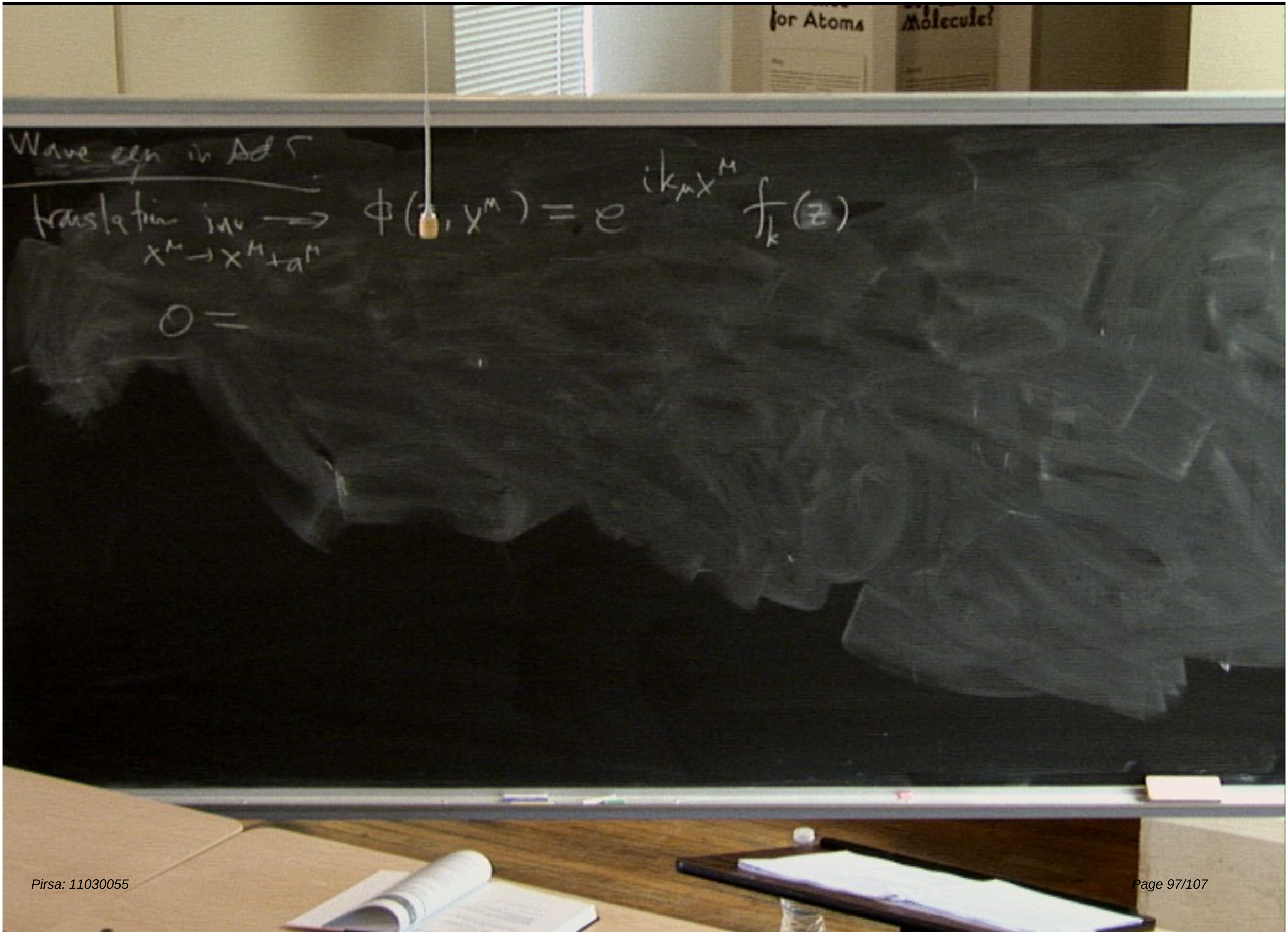
- EOM for small fluctuations of ϕ

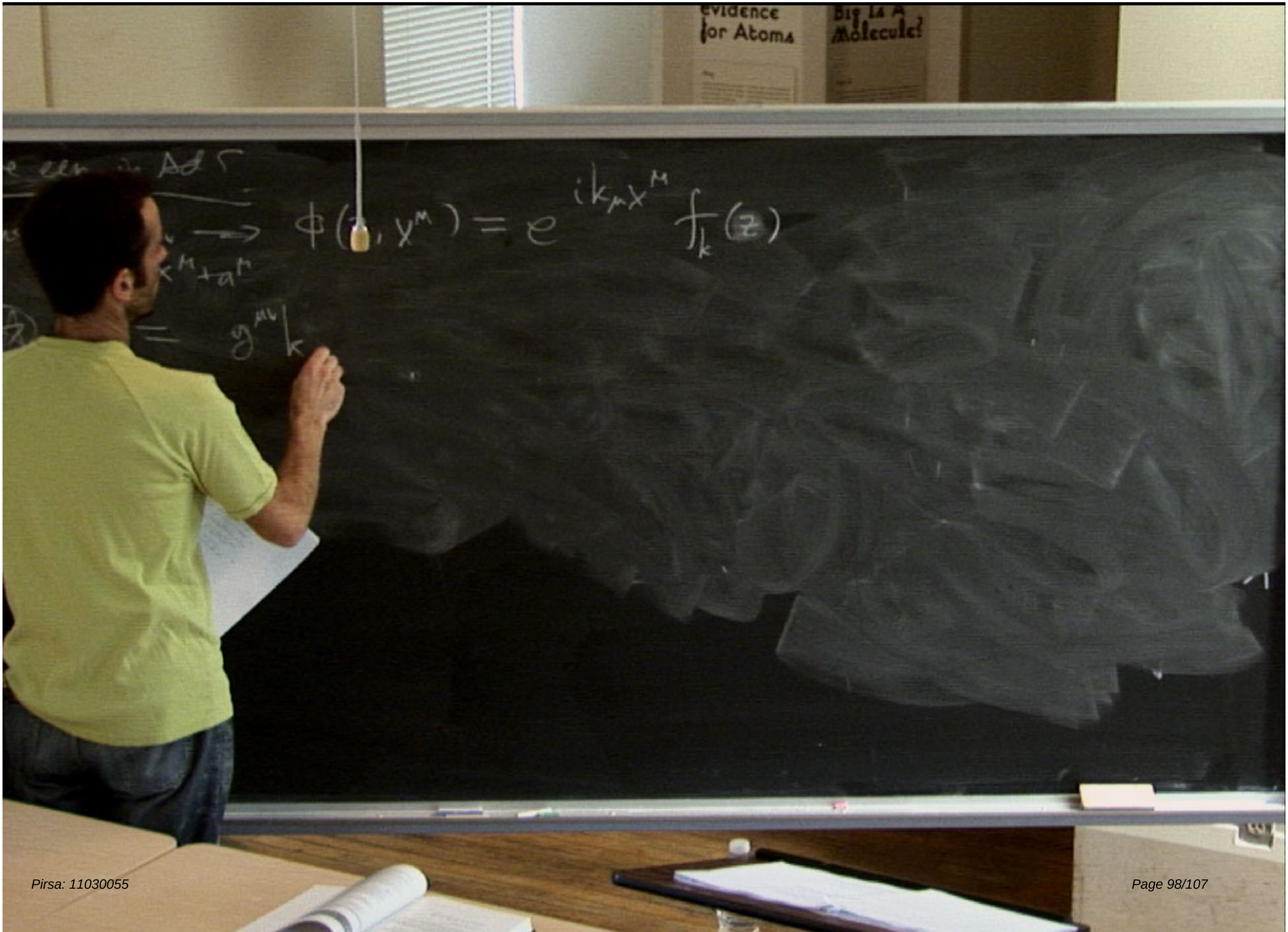
$$\text{:}\ (-\Box + m^2)\phi = 0$$

- $S[\Phi] =$ bdy term.

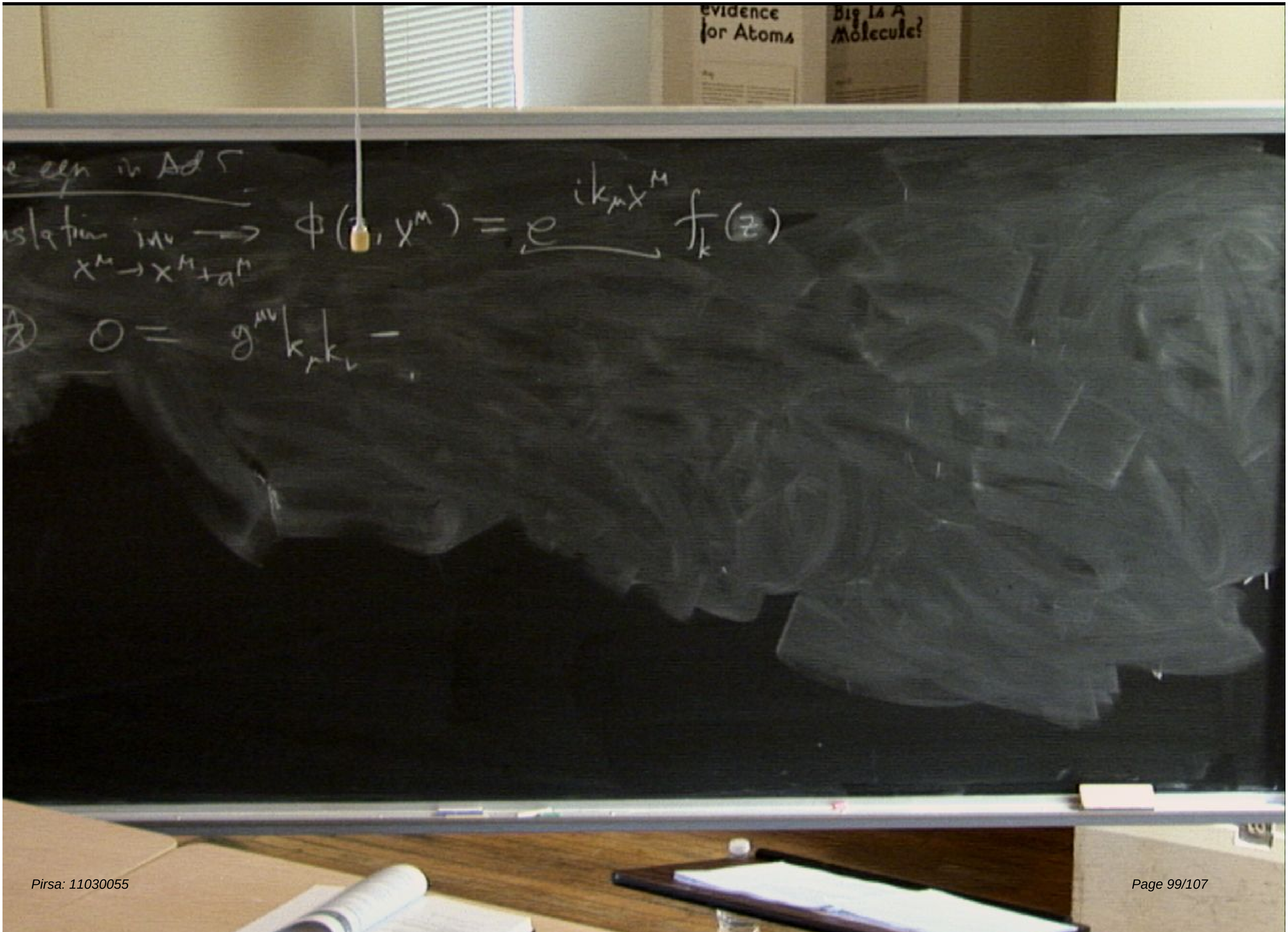
$$\left(Z = e^{-S[\Phi]}\right)$$


for Atoms
Molecule?
Wave eqn in AdS
translation inv
$(z, x^\mu) = e^{ik_\mu x^\mu} f_k(z)$


for Atoms
Molecules
Wave eqn in AdS
translation inv ⟹ φ(z, x^μ) = e^{ik_μ x^μ} f_k(z)
x^μ → x^μ + a^μ
O =



Evidence for Atoms
Big Is A Molecule?
$\phi(z, x^\mu) = e^{ik_\mu x^\mu} f_k(z)$


Evidence for Atoms
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e eqn in AdS
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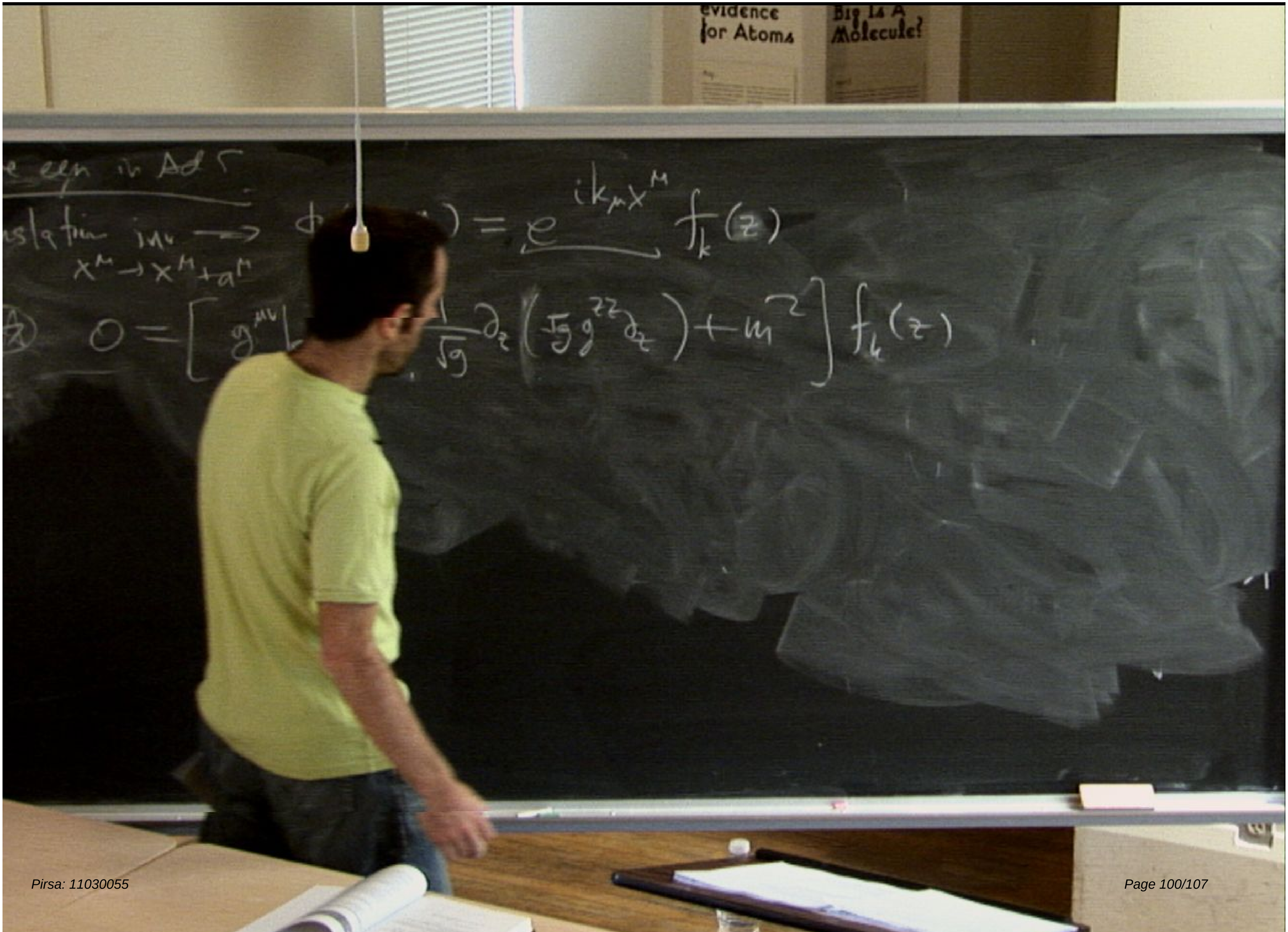
0 = g^{μν} k_μ k_ν −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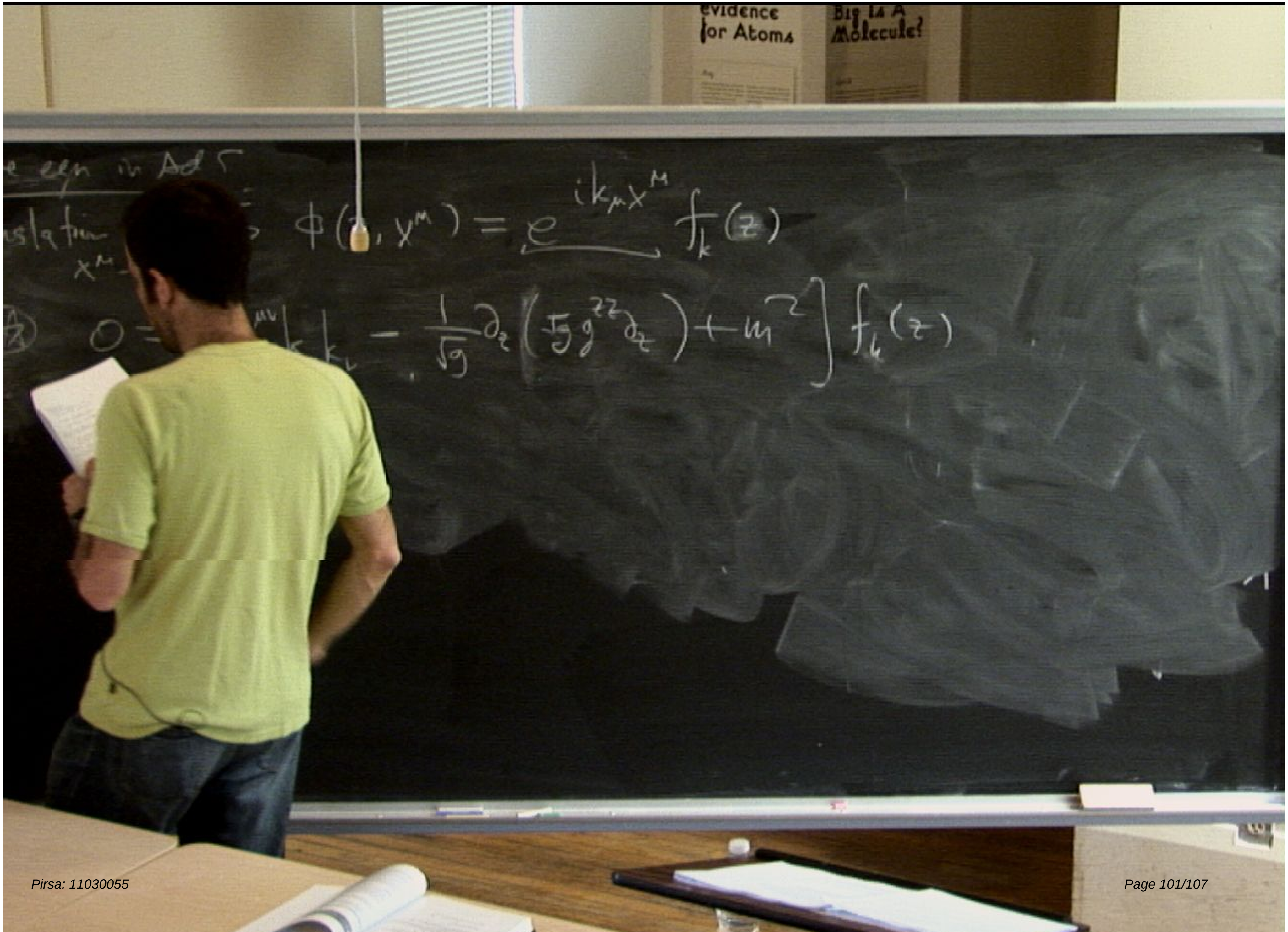
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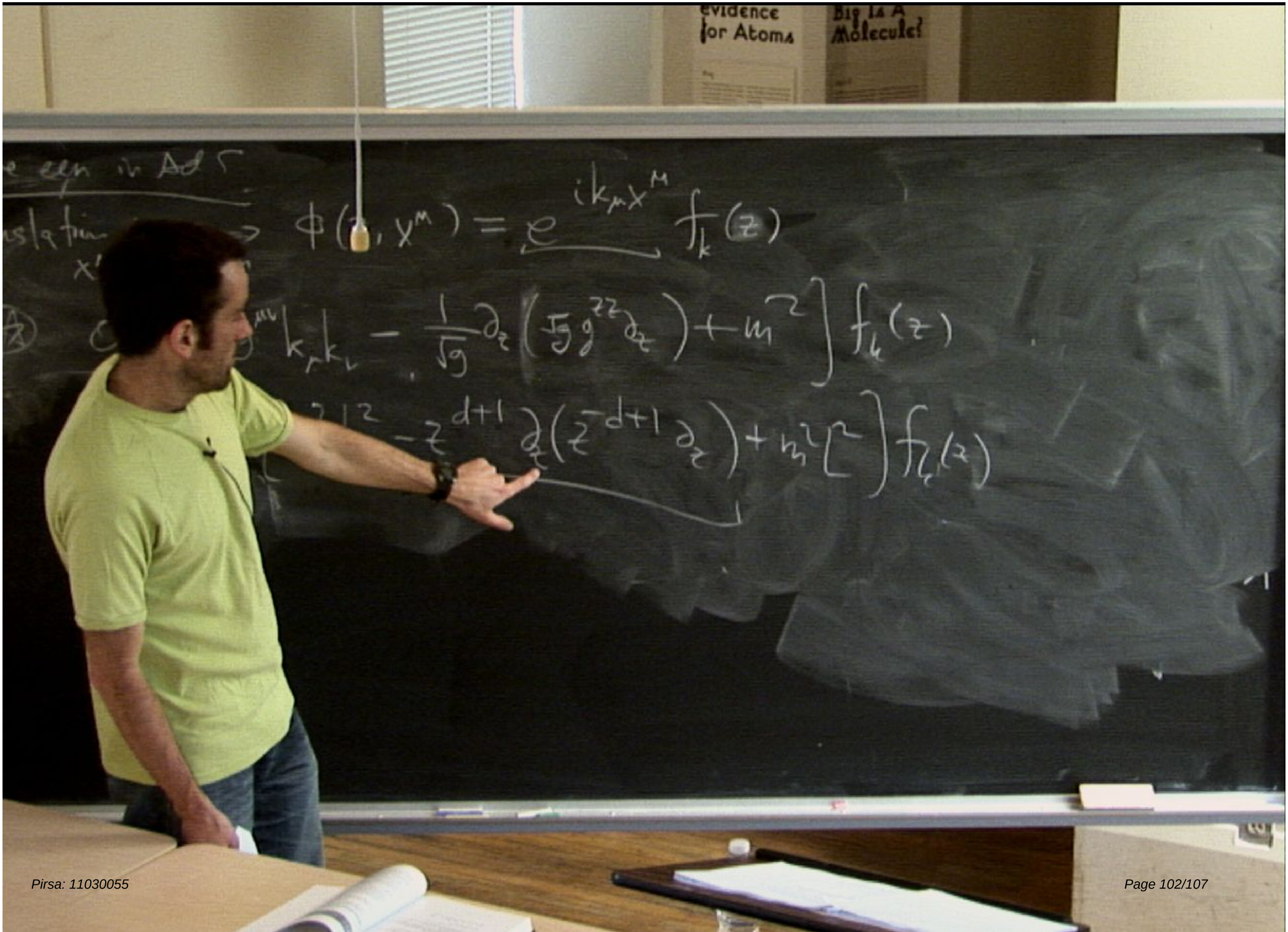
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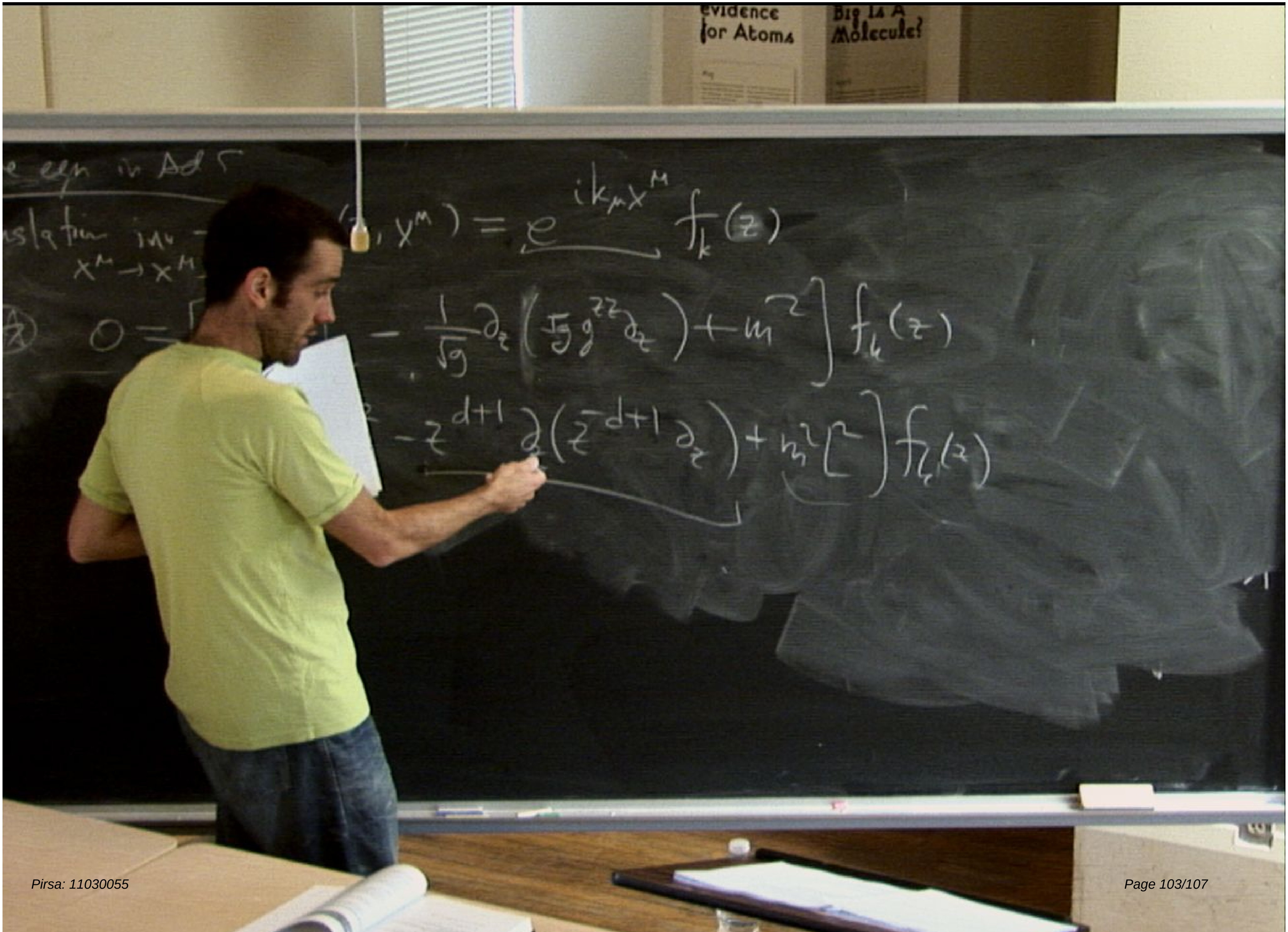
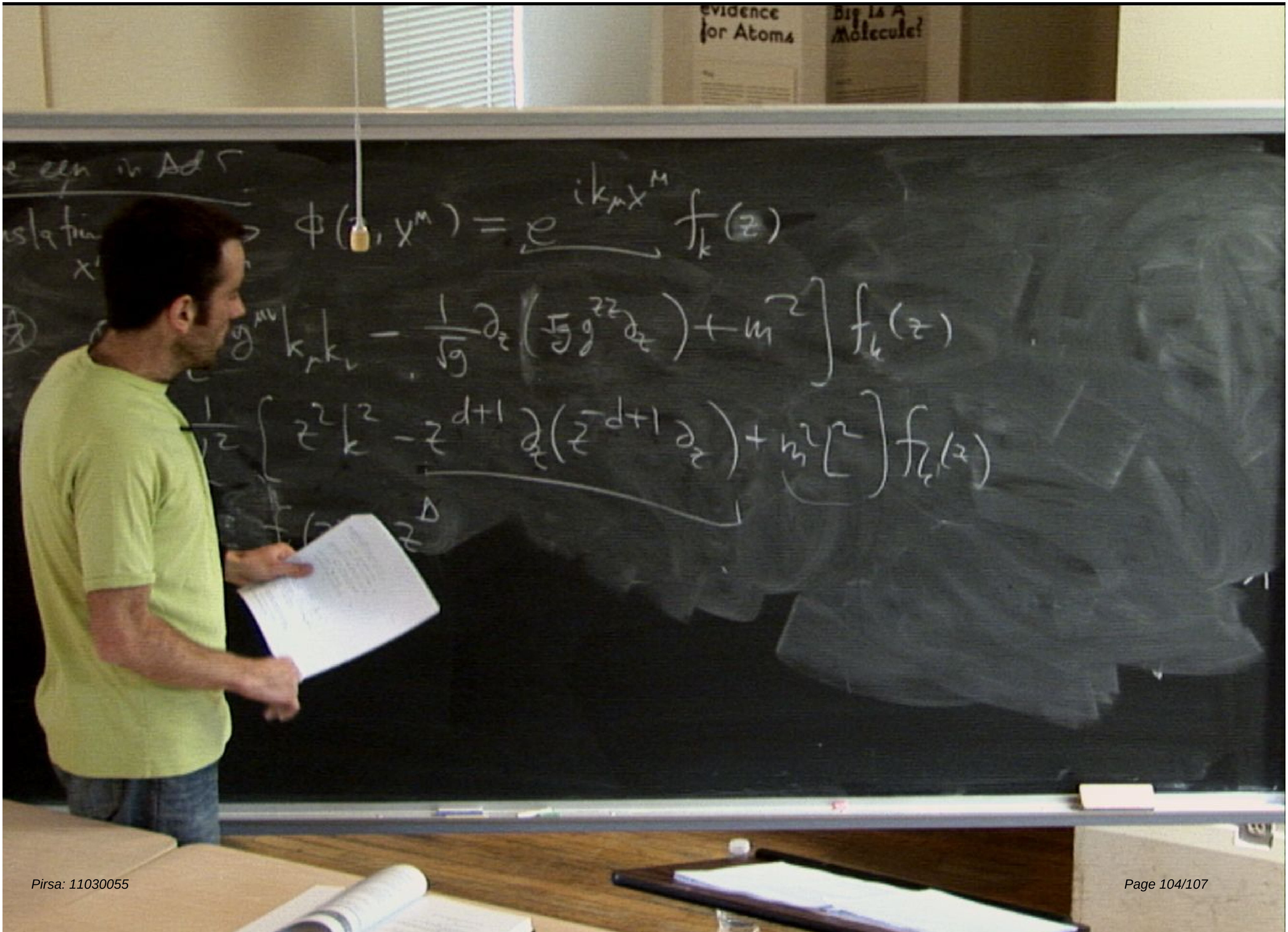
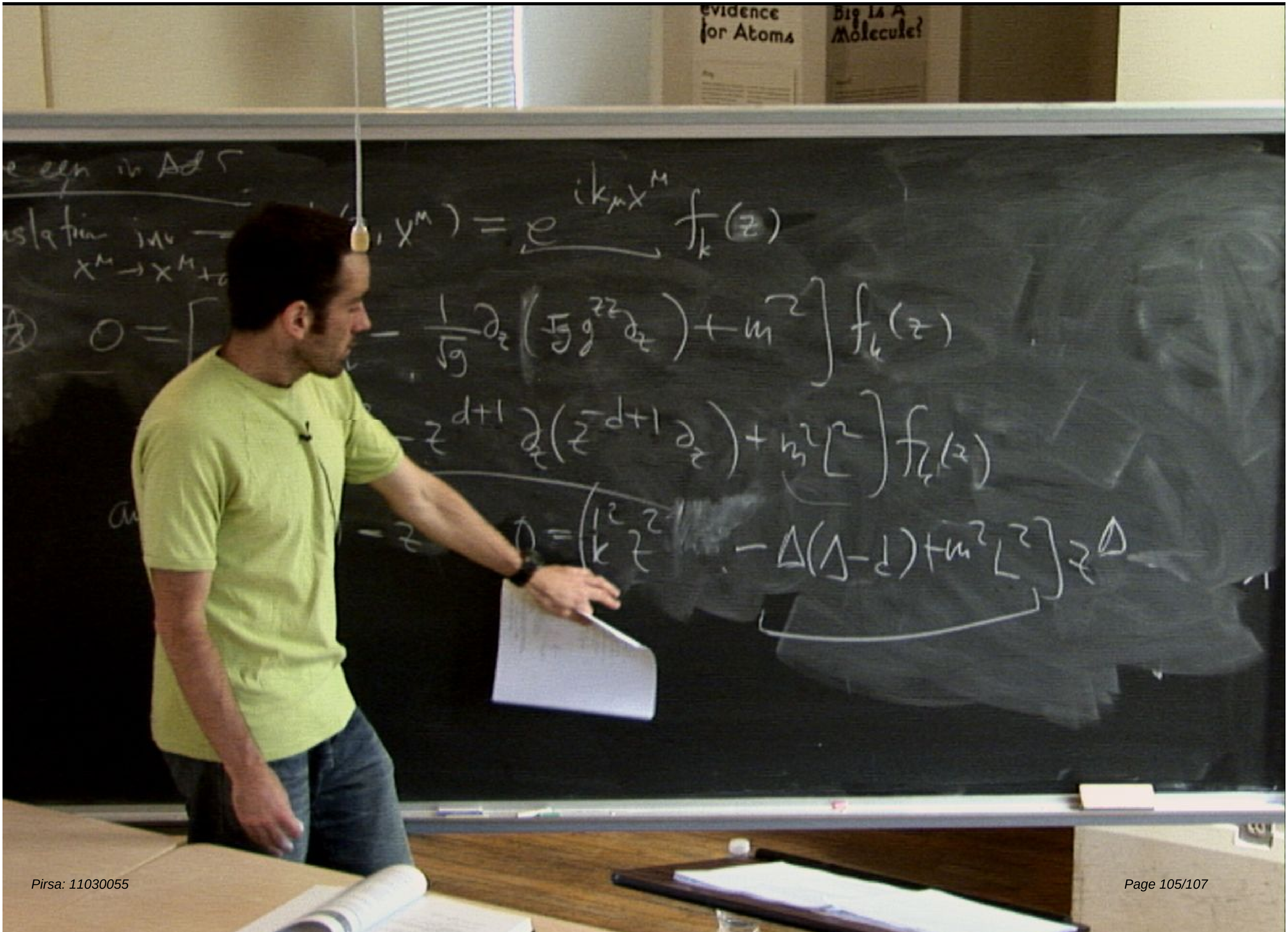


evidence
for Atoms
Big Is A
Molecule?
e eqn in AdS
$= e^{ik_\mu x^\mu} f_k(z)$
$x^\mu \to x^\mu$
$0 = [- \frac{1}{\sqrt{g}} \partial_z (\sqrt{g} g^{zz} \partial_z) + m^2] f_k(z)$
$- z^{d+1} \partial (z^{-d+1} \partial_z) + m^2 L^2] f_k(z)$


evidence for Atoms
Big Is A Molecule?
e eqn in AdS
$\phi(z, x^\mu) = e^{ik_\mu x^\mu} f_k(z)$
$[g^{\mu\nu}k_\mu k_\nu - \frac{1}{\sqrt{g}}\partial_z(\sqrt{g}\, g^{zz}\partial_z) + m^2] f_k(z)$
$\frac{1}{L^2}[z^2k^2 - z^{d+1}\partial_z(z^{-d+1}\partial_z) + m^2L^2] f_k(z)$
z^Δ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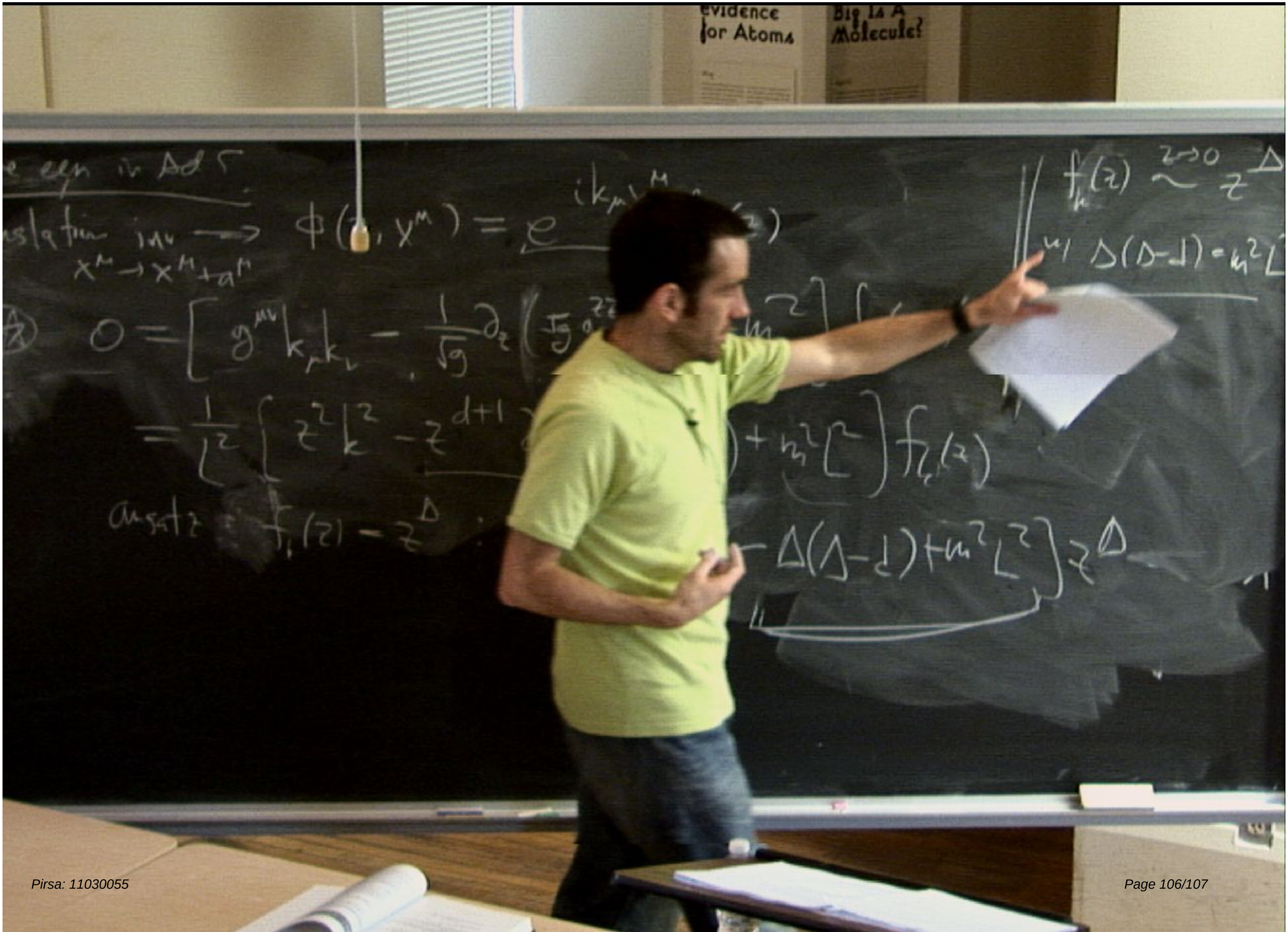

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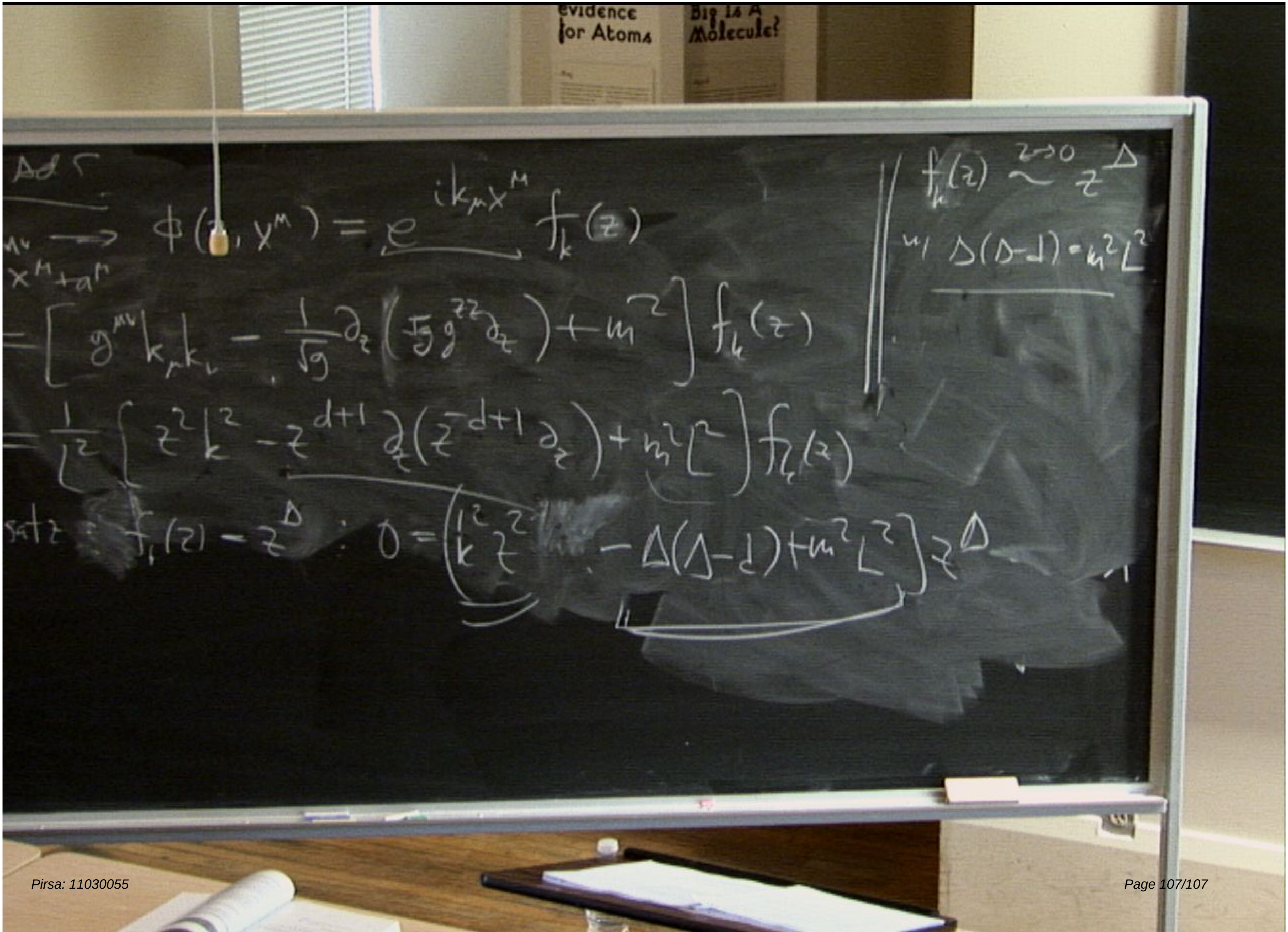
evidence
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Big Is A
Molecule?
AdS
$\phi(z, x^\mu) = e^{ik_\mu x^\mu} f_k(z)$
$x^\mu + a^\mu$
$= \left[g^{\mu\nu} k_\mu k_\nu - \frac{1}{\sqrt{g}} \partial_z \left(\sqrt{g}\, g^{zz} \partial_z \right) + m^2 \right] f_k(z)$
$= \frac{1}{L^2} \left[z^2 k^2 - z^{d+1} \partial_z \left(z^{-d+1} \partial_z \right) + m^2 L^2 \right] f_k(z)$
$f_k(z) \sim z^\Delta$: $0 = \left[k^2 z^2 - \Delta(\Delta - d) + m^2 L^2 \right] z^\Delta$
$f_k(z) \overset{z \to 0}{\sim} z^\Delta$
$\Delta(\Delta - d) = m^2 L^2$