

Title: Explorations in String Theory - Lecture 6

Date: Mar 21, 2011 11:30 AM

URL: <http://pirsa.org/11030055>

Abstract:

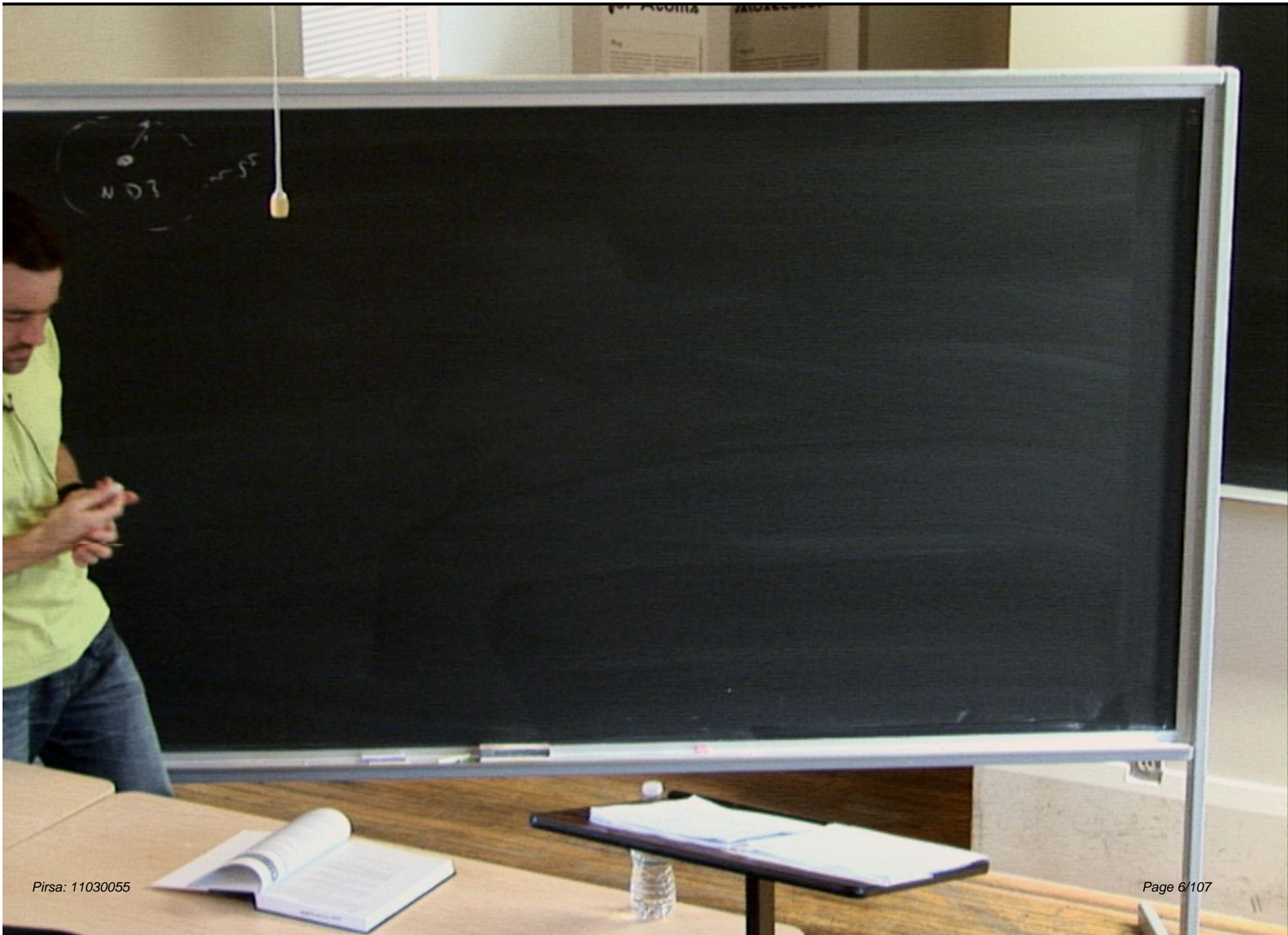


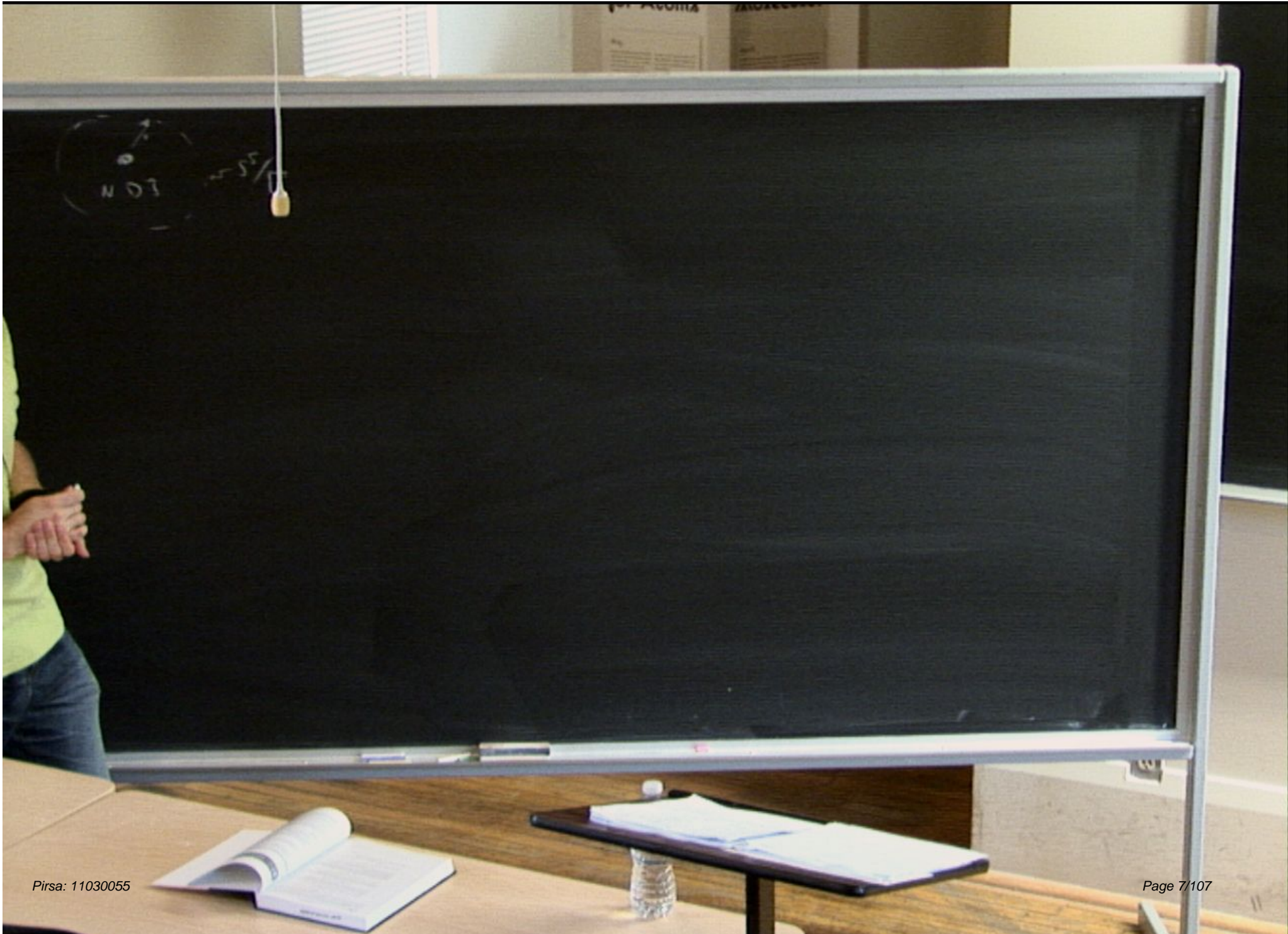
perimeter scholars  
INTERNATIONAL



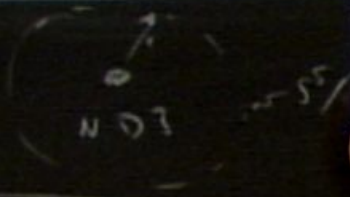








Basic checks of AdS/CFT for  $N=4$  SYM



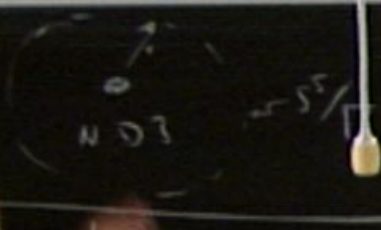


Basic checks of AdS/CFT for  $N=4$  SYM

$AdS_5 \times S^5$

$N=4$  SYM

isometries:  $SO(4,2) \times SO(6)$



Basic checks of AdS/CFT for  $N=4$  SYM.

$\boxed{\text{IIB on } \text{AdS}_5 \times S^5}$   
isometries  $SO(4,2) \times SO(6)$

$\boxed{N=4 \text{ SYM}}$   
 $SO(4,2) \times SO(6)$

conformal group  $U$   
 $SO(3,1) \times \text{dilations}$   
also:  $K^\mu$   $x^\mu \rightarrow \lambda x^\mu$

Symmetry

Basic checks of AdS/CFT for  $N=4$  SYM

IIB on  $AdS_5 \times S^5$   
isometries:  $SO(4,2) \times SO(6)$

$N=4$  SYM  
 $SO(4,2) \times SO(6)$

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 $SO(3,1) \times$  dilatations  
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Basic checks of AdS/CFT for  $N=4$  SYM

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$N=4$  SYM

$SO(4,2) \times SO(6)$

conformal gp U

$SO(3,1) \times$  dilatations

also:  $K^\mu$

$\left. \begin{matrix} \uparrow \\ \uparrow \\ \uparrow \end{matrix} \right\} \begin{matrix} R \\ \mathbb{R} \\ \mathbb{C} \end{matrix}$   
 $X^i = 1..6$   
 $\mathbb{C}$



Basic checks of AdS/CFT for  $N=4$  SYM

IIB on  $AdS_5 \times S^5$

isometries:  $SO(4,2) \times SO(6)$

2 supercharges

$N=4$  SYM  $\cong SU(4)$

$SO(4,2) \times SO(6)$

conformal gp U

$SO(3,1) \times$  dilatations

also:  $K^\mu$

$16 Q_i$

$16 S = [K, Q]$

$X^i, i=1..6$   
 $\in \mathfrak{so}(6)$   
supercharges  
 $\cong \mathfrak{su}(4)$

symmetry

Basic checks of AdS/CFT for  $N=4$  SYM

IIB on  $AdS_5 \times S^5$   
 isometries:  $SO(4,2) \times SO(6)$

32 supercharges

$SL(2, \mathbb{R})$  S-duality

$$\tau \equiv \frac{t + i\epsilon}{g_s} + \dots \xrightarrow{KR} \frac{q\tau + s}{c\tau + d}$$

$$g_s \rightarrow \frac{1}{g_s}$$

$N=4$  SYM  $\cong SU(4)$

$SO(4,2) \times SO(6)$

conformal gp  $U$   
 $SO(3,1) \times$  dilatations  
 also:  $K^\mu$   
 $X^\mu \rightarrow \lambda X^\mu$   
 $X^i = 1..6$   
 $\in \mathbb{C}$   
 supercharges  $\in 4$

$$\left\{ \begin{array}{l} 16 Q's \\ 16 S = [K, Q] \end{array} \right.$$

Basic checks of AdS/CFT for  $N=4$  SYM

$\text{IIB on } \text{AdS}_5 \times S^5$

isometries:  $SO(4,2) \times SO(6)$

32 supercharges

$SL(2, \mathbb{R})$  S-duality

$$\tau \equiv \frac{t}{g_s} + \frac{C(0)}{g_s} \xrightarrow{KR} \frac{q\tau + s}{c\tau + d}$$

$$g_s \rightarrow \frac{1}{g_s}$$

$N=4$  SYM  $\cong SU(4)$

$SO(4,2) \times SO(6)$

conformal gp U  $\rightarrow$   $SO(3,1) \times$  dilatations

also:  $K^\mu$

$$\left\{ \begin{array}{l} 16 Q's \\ 16 S = [K, Q] \end{array} \right.$$

$$\tau = \frac{c\tau + d}{g_s^2} + \theta$$

$x^i (i=1..6)$   
 $\in SU(6)$   
 supercharges  $\rightarrow 4$

Basic checks of AdS/CFT for  $N=4$  SYM

$SO(2,4) \times SO(6)$   
 symmetries:  $SO(4,2) \times SO(6)$

$N=4$  SYM  $\cong SU(4)$   
 $SO(4,2) \times SO(6)$

symmetry

$$S \Rightarrow \frac{\partial}{\partial t} (F \wedge F)$$

supercharges

$L(2,2)$   $S$ -duality

$$\frac{2}{g_s} + \binom{0}{KR} \rightarrow \frac{9\tau + 5}{c\tau + d}$$

$$g_s \rightarrow \frac{1}{g_s}$$

conformal group  $U$   
 $SO(3,1) \times$  dilatations  
 also:  $K^\mu$

$X^i, i=1..6$   
 $\in \mathfrak{so}(6)$   
 supercharges  
 $\mathfrak{psu}(4)$

$$\left. \begin{array}{l} 16 Q_i \\ 16 S = [K, Q] \end{array} \right\} \tau = \frac{c\tau}{g_s^2} + \theta \rightarrow \frac{9\tau + 5}{c\tau + d}$$



② perturbation: which reps.

② perturbation: which reps. appear spectrum?

claim: all linearized SUGRA  
perturbations

↔ some  $N=4$  ops.

② perturbation: which reps. appear spectrum?

claim: all linearized SUGRA  
perturbations

↔ some  $N=4$  ops.

field on  $AdS_5 \times S^5$   
 $(x, y)$

$\mathcal{F}(x, y)$

② perturbation: which reps. appear spectrum?

claim: all linearized SUGRA perturbations  $\longleftrightarrow$  some  $N=4$  ops.

field on  $AdS_5 \times S^5$   
(x, y)

$$\Phi(x, y) = \sum_{\{i_1 \dots i_\ell\}} T_{i_1 \dots i_\ell}(x) \underline{y^{i_1} \dots y^{i_\ell}}$$

$y^i = 1 \dots 6$

Sym., traceless

unit sph.  
 $\sum_{i=1}^6 (y^i)^2 = 1$

② perturbation: which reps. appear spectrum?

claim: all linearized SUGRA perturbations  $\longleftrightarrow$  some  $N=4$  ops.

field on  $AdS_5 \times S^5$

$$\Phi(x, y) = \sum_{\{i_1 \dots i_\ell\}} \left[ \begin{array}{c} T_{i_1 \dots i_\ell}(x) \\ \text{Sym, traceless} \end{array} \right] \underbrace{y^{i_1} \dots y^{i_\ell}}_{\substack{\text{unit sph.} \\ \sum_{i=1}^5 (y^i)^2 = 1}}$$

$y^i = 1 \dots 6$

field in  $AdS_5$

$$T_{i_1 \dots i_\ell}(x)$$

② perturbation: which reps. appear spectrum?

claim: all linearized SUGRA perturbations  $\longleftrightarrow$  some  $N=4$  ops.

field on  $AdS_5 \times S^5$   
 $(x, y)$

$$\Phi(x, y) = \sum_{\{i_1 \dots i_4\}} \left[ \begin{array}{c} T \\ i_1 \dots i_4 \end{array} (x) \right] \underline{y^{i_1} \dots y^{i_4}}$$

Sym., traceless

field in  $AdS_5$

unit sph.  
 $\sum_{i=1}^5 (y^i)^2 = 1$

$$m_{AdS_5}^2(T) \sim \frac{p}{L^2}$$

② perturbation: which reps. appear spectrum?

claim: all linearized SUGRA perturbations  $\longleftrightarrow$  some  $N=4$  ops.

field on  $AdS_5 \times S^5$   
(x, y)

$$\Phi(x, y) = \sum_{\{i_1 \dots i_4\}} \left[ \begin{array}{c} T \\ i_1 \dots i_4 \end{array} (x) \right] y^{i_1} \dots y^{i_4}$$

Sym., traceless

field in  $AdS_5$

unit sph.  
 $\sum_{i=1}^5 (y^i)^2 = 1$

$y^i = 1 \dots 6$

$$m_{S^5}(T) \sim \left( \frac{\rho}{L} \right)^2$$

② perturbation: which reps. appear spectrum?

claim: all linearized SUGRA perturbations  $\longleftrightarrow$  some  $N=4$  ops.

field on  $AdS_5 \times S^5$   
(x, y)

$$\Phi(x, y) = \sum_{\{i_1 \dots i_4\}} \left[ T_{i_1 \dots i_4}(x) \right] \underline{y^{i_1} \dots y^{i_4}}$$

$y^i = 1 \dots 6$

Sym., traceless  
field in  $AdS_5$

unit spt  
 $\sum_{i=1}^6 (y^i)^2 = 1$

$$m_{S^5}^2(T) \sim \left( \frac{\rho}{L} \right)^2$$



... appear spectrum?

SUGRA  
perturbations

Some  $N=4$  opr.

$$\sum_{i_1 \dots i_n} \left[ \begin{array}{c} T \\ \dots \\ T \end{array} \right] (x) \quad \underline{\underline{y^{i_1} \dots y^{i_n}}}$$

Sym., traceless  
field in  $AdS_5$

$$M_{S^2}(\mathbb{T}) \sim$$

$$\mathcal{O}(x^{M03}) = \left[ \begin{array}{c} T \\ \dots \\ T \end{array} \right]_{i_1 \dots i_n} \text{tr} X_{(x_1} \dots X_{i_n}(x)$$

appear spectrum?

SUGRA  $\longleftrightarrow$  Some  $N=4$  opr.  
variations

$$\sum_{i=1}^n T_{i_1 \dots i_n}(x) \left| \begin{array}{c} y^{i_1} \dots y^{i_n} \\ \hline \text{unit sph.} \\ \sum_{i=1}^n (y^i)^2 = 1 \end{array} \right|$$

Sym., traces  
field in  $AdS_5$

$$T_{i_1 \dots i_n}(x) \left[ \begin{array}{c} \int_{\mathcal{D}} m(T) \sim \left( \frac{\mathcal{L}}{L} \right)^2 \end{array} \right]$$

$$t_{i_1 \dots i_n} \dots X^{i_n}(+)$$

appear spectrum?

SUGRA  $\longleftrightarrow$  Some  $N=4$  opr.  
 variations

$$\sum_{i=1}^n \left[ \begin{array}{c} T \\ \dots \\ i_1 \dots i_n \end{array} (x) \right] \left( y^{i_1} \dots y^{i_n} \right)$$

unit sph.  
 $\sum_{i=1}^n (y^i)^2 = 1$

Sym., traces

AdS<sub>5</sub>

$$M(T) \sim \left( \frac{\mathcal{L}}{L} \right)^2$$

$$\mathcal{O}(x^{M \pm 3}) = \left[ \dots \right] \text{tr} X^{i_1}(x) \dots X^{i_n}(x)$$

appear spectrum?

SUGRA  $\longleftrightarrow$  Some  $N=4$  opr.  
 variations

$\sum_{i=1}^l T_{i_1 \dots i_l}(x) \underbrace{(y^{i_1} \dots y^{i_l})}_{\substack{\text{unit sph.} \\ \sum_{i=1}^l (y^i)^2 = 1}}$   
 Sym, traceless  
 field in  $AdS_5$

$$M(T) \sim \left( \frac{L}{\rho} \right)^2$$

$$\mathcal{O}(x^{M03}) = \underbrace{\sum_{i=1}^l \text{tr} X_{(x)}^{i_1} \dots X_{(x)}^{i_l}}_{\Delta \sim l}$$

② perturbation: which reps. appear spectrum?

claim: all linearized SUGRA perturbations  $\longleftrightarrow$  some  $N=4$  ops.

field on  $AdS_5 \times S^5$

$$\Phi(x, y) = \sum_{\substack{? i_1 \dots i_5 \\ \text{Sym, traceless}}} T_{i_1 \dots i_5}(x) \underbrace{(y^{i_1} \dots y^{i_5})}_{\substack{\text{unit sph.} \\ \sum_{i=1}^5 (y^i)^2 = 1}} y^{i_6}$$

field in  $AdS_5$

$$\Delta(\Delta - d) = m^2 L^2$$

$$m_{S^5}^2(T) \sim \left( \frac{\ell}{L} \right)^2$$

② perturbation: which reps. appear spectrum?

claim: all linearized SUGRA perturbations  $\longleftrightarrow$  some  $N=4$  ops.

field on  $AdS_5 \times S^5$   
(x, y)

$$\Phi(x, y) = \sum_{\{i_1, \dots, i_5\}} \left[ \begin{array}{c} T \\ L_{i_1} \dots i_5 \end{array} (x) \right] \left( \underbrace{y^{i_1} \dots y^{i_5}}_{\text{unit sph. } \sum_{i=1}^5 (y^i)^2 = 1} \right)$$

Sym, traceless  
field in  $AdS_5$

$$\Delta = m^2 L^2$$

$$\Delta^2 \sim k^2 L^2$$



② perturbation: which reps. appear spectrum?

claim: all linearized SUGRA perturbations  $\longleftrightarrow$  some  $N=4$  opr.

field on  $AdS_5 \times S^5$

$$\Phi(x, y) = \sum_{i_1, \dots, i_5} \left[ T_{i_1, \dots, i_5}(x) \right] \left( y^{i_1} \dots y^{i_5} \right)$$

$i=1 \dots 6$

Sym., traceless  
field in  $AdS_5$

unit sph.  
 $\sum_{i=1}^5 (y^i)^2 = 1$

$$\Delta(\Delta - d) = m^2 L^2$$

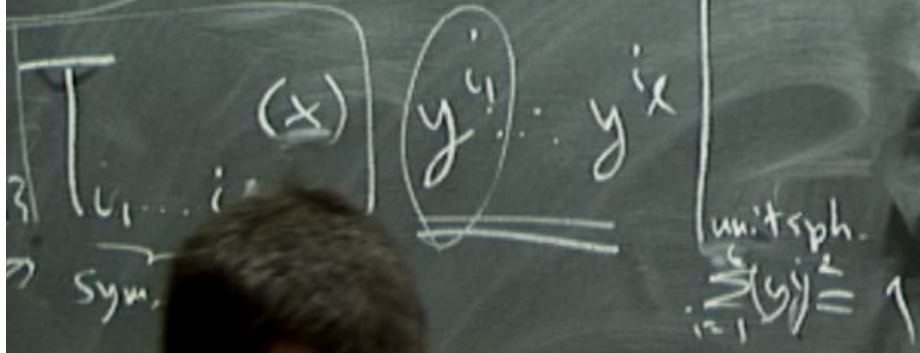
$$\Delta^2 \sim k^2 L^2$$

$$m_{S^5}^2(T) \sim \left( \frac{\ell}{L} \right)^2$$

appear spectrum!

G-RA  
tions

Some  $N=4$  opr.



$$X_{(x^i)}^{(y^i)} \in \mathbb{R}^{3 \times 1}$$

$i=1 \dots n$   
 $a, b = 1 \dots n$

Sometimes:  $\{Q, \mathcal{O}\} = 0.$



$$\mathcal{O}(x^{MO-3}) = \int_{\mathcal{A}_{(1 \dots n)}} \text{tr} X_{(x^i)}^{(y^i)} X_{(y^i)}^{(x^i)}$$



mm!

Some  $N=4$  opr.

$x$ )  $y^i$   $y^x$

unit sph.

$$\sum_{i=1}^n (y^i)^2 = 1$$



$$X^i \in \mathbb{R}^3!$$

$i=1 \dots n$

$a, b = 1 \dots n$

Sometimes:  $\{Q, G\} = 0$ . "BPS"

$$G(x^{M0.3}) = \sum_{i=1}^n t^i X^i(x) \dots X^i(x)$$

$\Delta \sim l$

(2) 3- $\mu$  für  $\sigma$   
 BPS up

$$X^i \begin{matrix} \swarrow \\ \downarrow \\ \searrow \end{matrix} \begin{matrix} j=1 \dots n \\ (x^j) \\ \uparrow \\ a, b=1 \dots n \end{matrix} \in \mathbb{R}^{2,1}$$

Sometimes:  $\{Q, \mathcal{O}\} = \text{PS}''$

$$\begin{matrix} \rightarrow \\ \rightarrow \end{matrix} \begin{matrix} \mathcal{O}(x^{MO,3}) \\ T \\ \sim \end{matrix} \begin{matrix} T \\ \dots \\ \dots \end{matrix}$$

$$X^i \begin{matrix} \swarrow \\ \searrow \end{matrix} \begin{matrix} j=1 \dots l \\ a,b \end{matrix} \in \mathbb{R}^{3 \times 1}$$

$\uparrow a, b = 1 \dots n$

Sometimes:  $\{Q, \mathcal{O}\} = 0$ . "BPS"

$$\mathcal{O}(x^{M03}) = \prod_{i=1 \dots l} \text{tr} X_{(x)}^i \dots X_{(x)}^l$$

$\uparrow$

(2) 3-pt for  
BPS ops

$$(\lambda = 0)$$

$$X_{(x^M)}^{i_j} \in \mathbb{R}^{2 \times 1} \quad j=1 \dots L$$

$\uparrow$   
 $a, b = 1 \dots N$

Sometimes:  $\{Q, \Theta\} = 0$ . "BPS"

$$\Theta(x^{M \times 3}) = \sum_{i=1}^L \text{tr} X_{(x^1 \dots x^L)}^{i_j}$$

$$C(\lambda = \infty) = C(\lambda = 0)$$



$$X_{(x^M)}^{i_j} \in \mathbb{R}^{2|1}$$

$i, j = 1 \dots 6$

$a, b = 1 \dots N$

Sometimes:  $\{Q, \mathcal{O}\} = 0$ . "BPS"

$$\mathcal{O}(x^{M03}) = \prod_{i=1}^6 \text{tr} X_{(x^1)}^{i_1} \dots X_{(x^6)}^{i_6}$$

(2) 3- $\mathcal{A}$  for  $\mathcal{O}$   
BPS ops

$$C(\lambda=\infty) = C(\lambda=0)$$

A more gen'l operator  $\mathcal{O}$  of  $N$   
 $\mathcal{O}(x) = \text{tr}[X X X \dots]$

$$X^i = \begin{pmatrix} X^i \\ (X^i) \end{pmatrix} \in \mathbb{R}^{2|1}$$

$i = 1 \dots L$

$a, b = 1 \dots N$

$$X = X^1$$

$$Y = X^2$$

$$\vdots$$

Sometimes:  $\{Q, \mathcal{G}\} = 0$ . ("BPS")

$$\mathcal{G}(X^{MO3}) = \int_{\Delta_{1 \dots L}} \text{tr} X^i_{(x)} \dots X^i_{(x)}$$

(2) 3- $\rightarrow$  for  $\forall$  of  
BPS ops

$$C(\lambda = \infty) = C(\lambda = 0)$$

A more gen'l operator of  $N$   
 $\mathcal{G}(X) = \text{tr}[X X X \dots D_N X Y]$

$$X^{(i)} \in \mathbb{R}^{2 \times 1}$$

$i = 1, \dots, 6$

$(X^{(i)})$

$a, b$

$(a, b = 1, \dots, 1)$

$$X = X^1$$

$$Y = X^2$$

$$\vdots$$

② 3- $\lambda$  for  $\sigma_b$   
BPS ops

$$C(\lambda = \infty) = C(\lambda = 0)$$

A more gen'l operator of  $N$   
 $G(x) = \text{tr}[X X X \dots]_{p \times p} Y F_{p \times p}$

Sometimes:  $\{ Q, G \}$  "BPS"

$$G(x^{M \times 3})$$

$N=4$  opr.

$y^i x^j$

$$X^i = \begin{pmatrix} X^1 \\ \vdots \\ X^i \\ \vdots \\ X^N \end{pmatrix} \in \mathbb{R}^{31}$$

$i=1 \dots 6$

$a, b = 1 \dots N$

$$X = X^1$$

$$Y = X^2$$

sometimes:  $\{Q, G\} = 0$  "BPS"

$$\mathcal{G}(X^{M03}) = \sum_{i=1}^4 \text{tr} X_{(x)}^i \dots X_{(+)}^i$$

② 3-rt for  $\sigma^c$   
BPS ops

$$C(\lambda=\infty) = C(\lambda=0)$$

A more gen'l operator of  $N=4$  sym:  
 $\mathcal{G}(X) = \text{tr} [X X X \cdot D_{\mu} X Y F_{\mu\nu} X X Y]$



"chiral")  
 $N=4$  opr.

$y^i x^j$   
 unit sph.  
 $\sum_{i=1}^4 (y^i)^2 = 1$

$X^i \in \mathbb{R}^5$   
 $(X^i)$   
 $i=1, \dots, 6$   
 $a, b = 1, \dots, N$

$X = X^1$   
 $Y = X^2$

Sometimes:  $\{Q, G\} = 0$  "BPS"

$$G(X^{M03}) = \int_{\mathcal{A}} \text{tr} X^i X^j X^k X^l$$

3-pt fun<sup>c</sup> of  
 BPS ops

$$C(\lambda \rightarrow \infty) = C(\lambda = 0)$$

A more general operator of  $N=4$  sym:  
 $G(X) = \dots [D_{\mu\nu} X Y F_{\mu\nu} X X Y]$   
 $\Delta = \Delta(\lambda)$

"chiral")  
 $N=4$  opr.

$y^i x^j$   
 unit sph.  
 $\sum_{i=1}^4 (y^i)^2 = 1$

$X^i \in \mathbb{R}^3$   
 $(X^i) \in \mathbb{R}^3$   
 $a, b = 1 \dots N$

$X = X^1$   
 $Y = X^2$

Sometimes:  $\{Q, G\} = 0$

$G(X^{M03}) = \dots$   
 $\Delta \sim l$

② 3-pt for  $\sigma$   
 BPS ops

$$C(\lambda=0) = C(\lambda=0)$$

A more gen'l operator of  $N=4$  sym:  
 $G(X) = \text{tr} [X X X \cdot D_{\mu} X Y F_{\mu\nu} X X Y]$   
 not BPS,  $\Delta = \Delta(\lambda)$   $J = 6$   
 $\Delta(\lambda=0) = 5$

"chiral")  
 $N=4$  opr.

$y^i x^j$   
 unit sph.  
 $\sum_{i=1}^6 (y^i)^2 = 1$

$X^i \in \mathbb{R}^3$   
 $i=1,2,3$   
 $a,b=1,2,3$

$X = X^1$   
 $Y = X^2$

Sometimes  $\{Q, G\} = 0$  "BPS"

$\mathcal{O}(x^{M03}) = \prod_{i=1,2,3} \text{tr} X_{(x)}^{i_1} \dots X_{(x)}^{i_3}$

$\Delta \sim l$

② 3-pt for  $\mathcal{O}$   
 BPS ops

$C(\lambda=0) = C(\lambda=\infty)$

A more gen'l operator of  $N=4$  sym:  
 $\mathcal{O}(x) = \text{tr} [X_{\mu\nu} X_{\rho\sigma} Y_{\mu\rho} Y_{\nu\sigma}]$   
 not BPS,  $\Delta = \Delta(\lambda)$   $\Delta(\lambda=0) = 5$

$$X^i \in \mathbb{R}^{2,1} \quad i=1, \dots, 6$$

at

$$a, b = 1, \dots, n$$

$$X = X^1$$

$$Y = X^2$$

$$\vdots$$

Sometimes:  $\{Q, \mathcal{G}\} = 0$  "BPS"

$$\mathcal{G}(X^{M0,3}) = \int_{\mathcal{A}} \text{tr} X^i X^i$$

$\mathcal{A} \sim l$

② 3- $\alpha$  for  $\sigma$   
BPS ops

$$C(\lambda=\infty) = C(\lambda=0)$$

A more gen'l operator of  $N=4$

$$\mathcal{G}(X) = \text{tr} [X X X \cdot \mathcal{D}_\mu X Y F_{\mu\nu} X]$$

not BPS,  $\Delta = \Delta(\lambda)$   
 $\Delta(\lambda=0) = 5$

$$= \text{tr} [Y X X X \cdot \mathcal{D}_\mu X Y F_{\mu\nu} X]$$

$\rightarrow$  closed string

2) opr.

stroph.  $(y) = 1$

$X^i \in \mathbb{R}^{2,1}$   $i=1, \dots, 6$

$X^i = X^i(x)$

$a, b \in \{1, \dots, 6\}$

$X = X^1$

$Y = X^2$

Sometimes:  $\{Q, \mathcal{O}\} = 0$  "BPS"

$\mathcal{O}(x^{M+3}) = \int_{\mathcal{A}} \text{tr} X^i(x) \dots X^i(x)$

$\mathcal{A} = \{1, \dots, l\}$

$\Delta \sim l$

3) 3-pt for  $\sigma_b$  BPS ops

$C(\lambda = \infty) = C(\lambda = 0)$

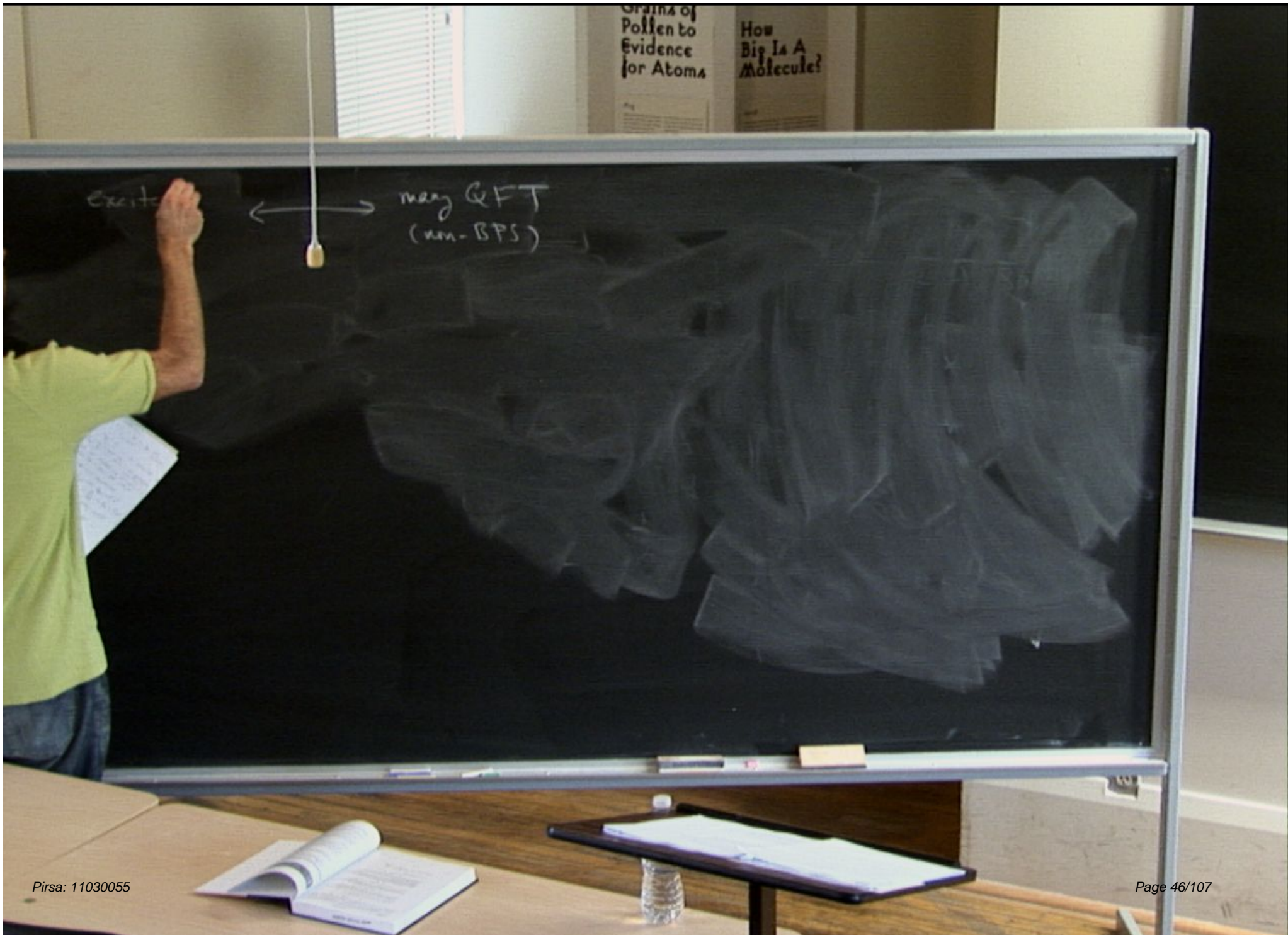
A more gen'l operator of  $N=4$  sym

$\mathcal{O}(x) = \text{tr} [X_{\mu\nu} X_{\rho\sigma} D_{\mu\nu} X_{\rho\sigma} F_{\mu\nu} X_{\rho\sigma}]$

not BPS,  $\Delta = \Delta(\lambda) \rightarrow 5$   
 $\Delta(\lambda=0) = 5$

$= \text{tr} [Y_{\mu\nu} X_{\rho\sigma} D_{\mu\nu} X_{\rho\sigma} F_{\mu\nu} X_{\rho\sigma}]$

$\rightarrow$  closed string



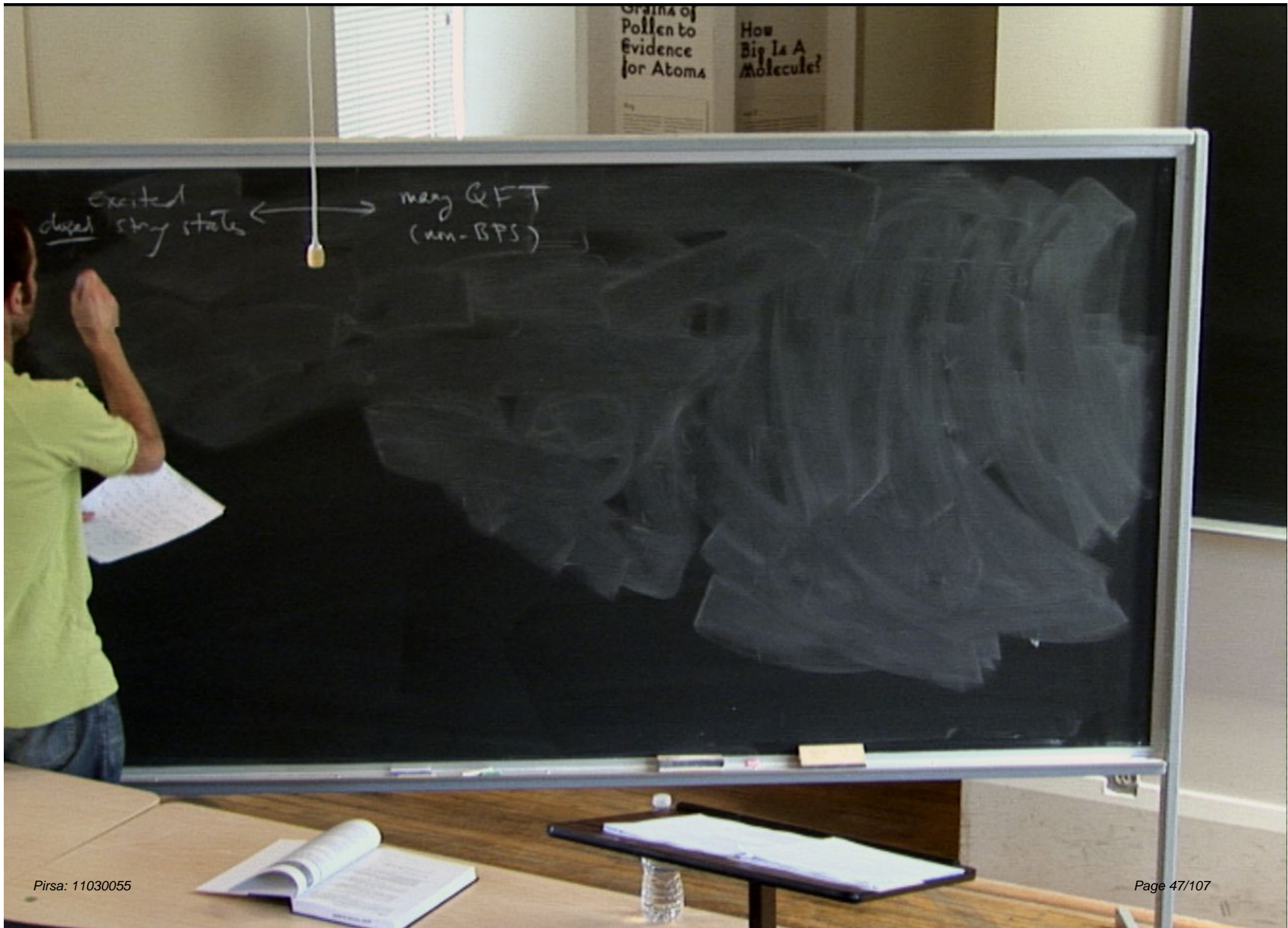
Grains of  
Pollen to  
Evidence  
for Atoms

How  
Big Is A  
Molecule?

Excite



may QFT  
(non-BPS)



Grains of  
Pollen to  
Evidence  
for Atoms

How  
Big is a  
Molecule?

excited  
closed string states



many QFT  
(non-BPS)

Grains of  
Pollen to  
Evidence  
for Atoms

How  
Big Is A  
Molecule?

excited  
closed string states ← → many QFT  
(non-BPS)



Grains of  
Pollen to  
Evidence  
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How  
Big Is A  
Molecule?

Excited  
closed string states  $\longleftrightarrow$  many QFT  
(non-BPS)

whereas  $M_{\text{super}}^2 \sim \frac{1}{\alpha'} \sim \frac{1}{L_{\text{MS}}^2}$   $\longleftrightarrow \Delta_{\text{SUGRA}} \sim N^2 \lambda$

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How  
Big Is A  
Molecule?

Excited  
state

many QFT  
(non-BPS)

$M_{\text{super}}$   
state

$$\sim \frac{1}{\Lambda^2}$$

$$\Delta_{\text{SUGRA}} \sim N^0 \lambda$$

$M_{\text{sing}}$   
state

$$\sim \frac{1}{\Lambda^4}$$

Grains of  
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for Atoms

How  
Big Is A  
Molecule?

Excited  
doped string states

many QFT  
(non-BPS)

whereas

↑  
M<sub>super</sub>  
nodes

$$\sim \frac{1}{L_{MS}} z$$

$$\longleftrightarrow \Delta_{S=GR} \sim N^0 \lambda^0$$

↑  
M<sub>long</sub>  
state

$$\sim \frac{1}{Q^1} = \frac{\sqrt{\lambda}}{L_{MS} z}$$

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How  
Big Is A  
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Excited  
closed string states  $\longleftrightarrow$  many QFT  
(non-BPS)

whereas  $M_{\text{super}} \sim \frac{1}{L_{\text{MS}}} \sim \frac{1}{\alpha'} = \frac{\sqrt{\lambda}}{L_{\text{MS}}}$   $\longleftrightarrow \Delta_{\text{str}}$

$M_{\text{string state}} \sim \frac{1}{\alpha'} = \frac{\sqrt{\lambda}}{L_{\text{MS}}}$   $\longleftrightarrow \Delta_{\text{str}}$

$\Delta_{\text{string}} \sim N^2 \lambda$

$\Delta_{\text{str}}$

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closed string states

where  $\lambda \alpha'^2$

$M_{\text{super}}^2 \sim \frac{1}{\alpha'^2}$

$M_{\text{str}}^2 \sim \frac{1}{\alpha'^2} = \frac{\sqrt{\lambda}}{L_{\text{MS}}^2}$

many QFT  
(non-BPS)

$\Delta_{\text{SUGRA}} \sim N^0 \lambda^0$

$\Delta_{\text{str}} \sim N^0 \lambda^{1/2}$

simple gravity dual

gravity  
mode

$N^1$

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How  
Big Is A  
Molecule?

Excited  
closed string states

whereas

$M_{\text{super}}^2 \sim \frac{1}{\alpha'} z$

$M_{\text{shy}}^2 \sim \frac{1}{\alpha'} z = \frac{\sqrt{\lambda}}{L_{\text{MS}}}$

$= \lambda \alpha'^2$

many QFT  
(non-BPS)

$\Delta_{\text{SUGRA}} \sim N$

$\frac{1}{\sqrt{\lambda}}$

simple gravity dual

gravity well

$\Delta \sim N^{3/4}$



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How  
Big Is A  
Molecule?

Excited closed string states  $\longleftrightarrow$  many QFT (non-BPS)

whereas  $\lambda \alpha'^2$

$\frac{1}{L_{MS}^2} \longleftrightarrow \Delta_{S-GRA} \sim N^0 \lambda^0$

$\frac{1}{\alpha'} = \frac{\sqrt{\lambda}}{L_{MS}^2} \longleftrightarrow \Delta_{str} \sim N^0 \lambda^{1/2}$

simple gravity dual at large  $\lambda$  large  $N$

Plane  $\Delta$

strings  $\Delta_{str} \sim N^0 \lambda^{1/2}$

gravity well  $\Delta_{str} \sim N^0 \lambda^0$

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How Big Is A Molecule?

excited closed string states ← → many QFT (non-BPS)

whereas  $\lambda \alpha'^2$

M-super state  $\sim \frac{1}{L_{MS}^2}$  ↔  $\Delta_{super} \sim N^0 \lambda^0$

M-string state  $\sim \frac{1}{\alpha'^2} = \frac{\sqrt{\lambda}}{L_{MS}^2}$  ↔  $\Delta_{str} \sim N^0 \lambda^{1/2}$

M-D-brane  $\sim \frac{1}{g_s^2}$  ↔  $\Delta_{D-brane} \sim N^2 \lambda^{\#}$

simple consistency dual at large  $\lambda$  large  $N$



Grains of  
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How  
Big Is A  
Molecule?

Excited  
doped string states

many QFT  
(non-BPS)

whereas

$M_{\text{super}} \sim \frac{1}{L_{\text{AdS}}^2}$

$M_{\text{string state}} \sim \frac{1}{\alpha'} = \frac{\sqrt{\lambda}}{L_{\text{AdS}}^2}$

$M_{\text{D-brane}} \sim \frac{1}{g_s}$

$\Delta_{\text{string}} \sim \sqrt{\lambda}$

$\Delta_{\text{D-brane}} \sim \frac{1}{g_s}$

simple gravity dual  
at  
large  $\lambda$   
large  $N$

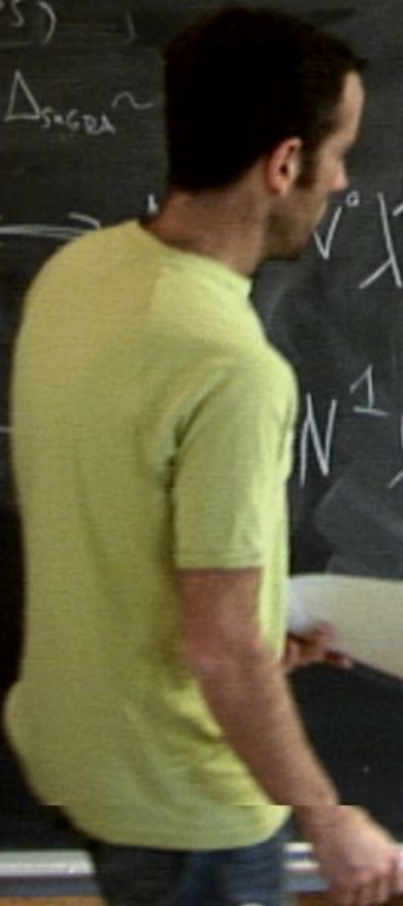
$N$  large: bulk is classical strings

$\lambda$  large: strings decouple

Plane  $\Delta_{\text{D-brane}} \sim \frac{1}{g_s}$

strings  $\Delta_{\text{string}} \sim \sqrt{\lambda}$

gravity rule  $\Delta_{\text{D-brane}} \sim \frac{1}{g_s}$



Grains of  
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How  
Big Is A  
Molecule?

Excited  
dual string states

whereas

$M_{\text{super}} \sim \frac{1}{L_{\text{MS}}^2}$

$M_{\text{str}} \sim \frac{1}{\alpha'} = \frac{\sqrt{\lambda}}{L_{\text{MS}}^2}$

$M_{\text{D-brane}} \sim \frac{1}{g_s}$

many QFT  
(non-BPS)

$\Delta_{\text{SUGRA}} \sim N^0 \lambda^0$

$\Delta_{\text{str}} \sim N^0 \lambda^{1/2}$

$\Delta_{\text{D-brane}} \sim N^1 \lambda^1$

simple gravity dual  
at  
large  $\lambda$   
large  $N$

$N$  large: bulk is classical strings

$\lambda$  large: strings decouple

above  $\Delta \sim N^0 \lambda^0$

strings  $\Delta \sim N^0 \lambda^{1/2}$

gravity rule  $\Delta \sim N^1 \lambda^1$

How to calculate

$$Z_{\text{QFT}}[\text{sources}, \phi_0] \equiv \langle e^{-\int \phi_0 \mathcal{L}} \rangle$$

How to calculate

$$Z_{\text{CFT}}[\text{sources}, \phi_0] \equiv \left\langle e^{-\int \phi_0 \mathcal{L}} \right\rangle_{\text{CFT}}$$

How to calculate

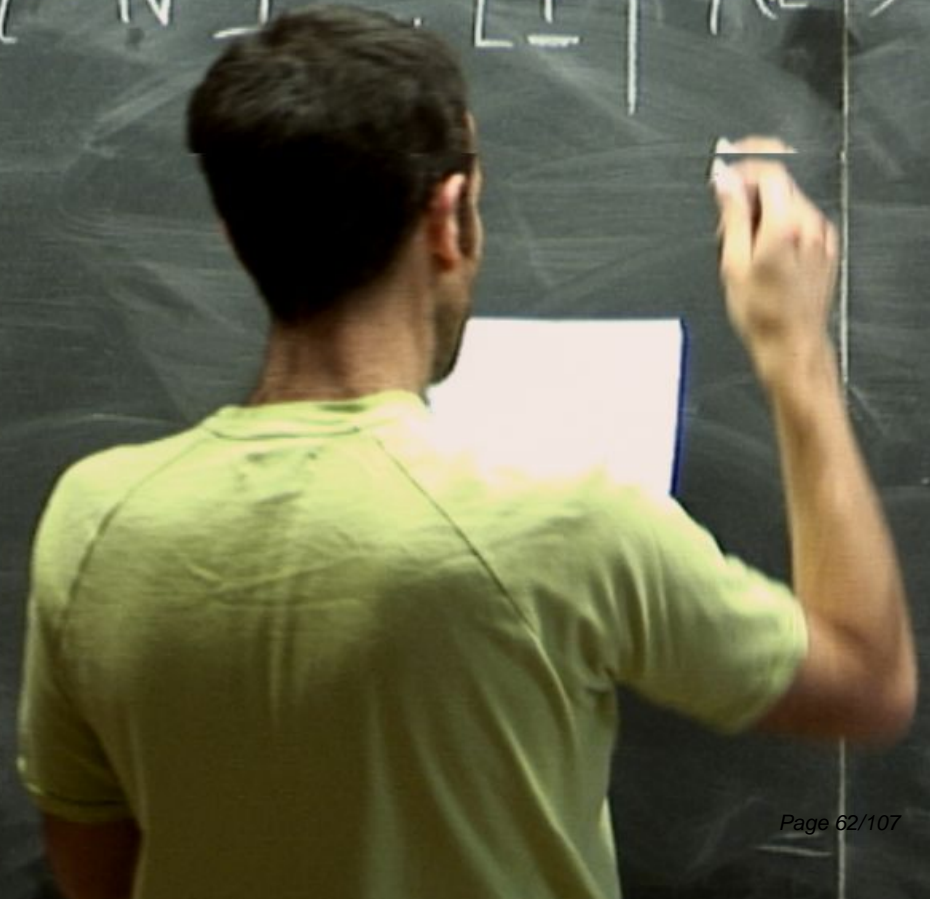
$$Z_{\text{CFT}}[\text{sources}, \phi_0] \equiv \left\langle e^{-\int \phi_0 \mathcal{L}} \right\rangle_{\text{CFT}}$$

$$\approx e$$

how to calculate

$$Z_{\text{CFT}}[\text{sources}, \underline{\phi}_0] \equiv \left\langle e^{-\int \phi_0 \mathcal{L}} \right\rangle_{\text{CFT}}$$

$$\approx \exp[-N^2 I] \cdot \left[ \underline{\phi} \mid \phi(z \rightarrow 0) \right]$$



How to calculate

$$Z_{\text{CFT}}[\text{sources}, \phi_0] \equiv \left\langle e^{-\int \phi_0 \mathcal{L}} \right\rangle_{\text{CFT}}$$

$$\approx \exp \left[ -N^2 \mathcal{I}_{\text{bulk}} \left[ \underline{\phi} \mid \phi(z \rightarrow 0) = \phi_0 \right] \right]$$

$\underline{\phi}$  solves bulk EoM

How to calculate

$$Z_{\text{CFT}}[\text{sources}, \phi_0] \equiv \left\langle e^{-\int \phi_0 \mathcal{O}} \right\rangle_{\text{CFT}}$$

$$\approx \exp\left[-N^2 I_{\text{bulk}}[\phi \mid \phi(z \rightarrow \infty) = \phi_0]\right]$$

$\phi$  solves bulk EoM

$$\left. \frac{\delta I_{\text{bulk}}}{\delta \phi} \right|_{\phi = \phi} = 0.$$



How to calculate

$$Z_{\text{CFT}}[\text{sources}, \phi_0] \equiv \left\langle e^{-\int \phi_0 \mathcal{L}} \right\rangle_{\text{CFT}}$$

$$\approx \exp\left\{-N^2 \int_{\text{bulk}} \left[ \phi \mid \phi(z \rightarrow 0) = \phi_0 \right] \right\}$$

$\phi$  solves bulk EoM

$$\left. \frac{\delta I_{\text{bulk}}}{\delta \phi} \right|_{\phi=\phi} = 0.$$

How to calculate

$$Z_{\text{CFT}}[\text{sources}, \phi_0] \equiv \left\langle e^{-\int \phi_0 \mathcal{O}} \right\rangle_{\text{CFT}}$$

$$\approx \exp\left\{-N^2 I_{\text{bulk}}[\phi \mid \phi(z \rightarrow 0) = \phi_0]\right\}$$

UV cutoff:

cut off the spacetime

$\phi$  solves bulk EoM

$$\left. \frac{\delta I_{\text{bulk}}}{\delta \phi} \right|_{\phi=\phi_0} = 0.$$

How to calculate

$$Z_{\text{CFT}}[\text{sources}, \phi_0] \equiv \langle e^{-\int \phi_0 \mathcal{O}} \rangle_{\text{CFT}}$$

$$\approx \exp\left\{-N^2 \int_{\text{bulk}} [\phi \mid \phi(z \rightarrow 0) = \phi_0]\right\}$$

UV cutoff:

cut off the spacetime  
at  $z > \epsilon$

$\phi$  solves bulk EoM

$$\left. \frac{\delta I_{\text{bulk}}}{\delta \phi} \right|_{\phi=\phi_0} = 0.$$

$$ds^2_{AdS} = L^2$$

$$(z \rightarrow 0) = \left. \begin{matrix} \dots \\ \dots \end{matrix} \right\}$$

Eq M

$$= 0.$$

A generator of  $N=4$  S  
 $D_{\mu\nu} X Y F_{\mu\nu} X X$   
 $\Delta = \Delta(\lambda) \quad J$   
 $\rho(\lambda=0) = J$   
 $D_{\mu\nu} X Y F_{\mu\nu} X X$   
 $m, q$

$$ds_{AdS}^2 = \frac{L^2}{z^2} (dz^2 + dx_\mu dx^\mu)$$

A more general form for  $g_{MN}$  is:

$$G(X) = \text{tr}[X^T Y X Y]$$

not  $\mathbb{R}$  5 objects

$$ds_{\text{AdS}}^2 = \frac{L^2}{z^2} (dz^2 + dx_\mu dx^\mu)$$

A more genil operator of  $N=4$  SYM:

$$\mathcal{O}(\lambda) = \text{tr} [X^\mu X^\nu \dots D_\mu X^\rho Y F_{\rho\nu} X X Y]$$

not BPS,  $\Delta = \Delta(\lambda)$   $\mathcal{J}$ -object  
 $\Delta(\lambda=0) = \mathcal{J}$

$$= \text{tr} [Y X X X \dots D_\mu X^\rho Y F_{\rho\nu} X X]$$

$\rightarrow$  closed string

$$ds_{AdS}^2 = \frac{L^2}{z^2} (dz^2 + dx_\mu dx^\mu)$$



A more gen'l operator of  $N=4$  SYM:

$$\mathcal{O}(\lambda) = \text{tr} [X^1 X^2 X^3 \dots D_\mu X^4 F_{\mu\nu} X^5 X^6 X^7 X^8]$$

not BPS,  $\Delta = \Delta(\lambda)$   $\mathcal{J}$ -invariant  
 $\Delta(\lambda=0) = \mathcal{J}$

$$= \text{tr} [Y X^1 X^2 X^3 \dots D_\mu X^4 F_{\mu\nu} X^5 X^6 X^7 X^8]$$

$\rightarrow$  closed string



$$ds_{AdS}^2 = \frac{L^2}{z^2} (dz^2 + dx_\mu dx^\mu)$$

A more genil operator of  $N=4$  SYM:

$$\mathcal{O}(\lambda) = \text{tr} [X^1 X^2 X^3 \underbrace{D_\mu X^4 F_{\mu\nu}}_{\text{object}} X^5 X^6 X^7]$$

not BPS,  $\Delta = \Delta(\lambda)$   $\mathcal{J}$ -object

$$\Delta(\lambda=0) = \mathcal{J}$$

$$= \text{tr} [Y X^1 X^2 X^3 \underbrace{D_\mu X^4 F_{\mu\nu}}_{\text{object}} X^5 X^6 X^7]$$

$\rightarrow$  closed string



How to calculate

$$Z_{\text{CFT}}[\text{sources}, \underline{\phi}_0] \equiv \langle e^{-\int \phi_0 \mathcal{O}} \rangle_{\text{CFT}}$$

UV cutoff:

cut off the spacetime  
at  $z > \epsilon$

$$\sim \exp\left\{-N^2 \int_{\text{bulk}} [\underline{\phi} \mid \underline{\phi}(z=\epsilon) = \underline{\phi}_0]\right\}$$

$\underline{\phi}$  solves bulk EoM

$$\left. \frac{\delta I_{\text{bulk}}}{\delta \underline{\phi}} \right|_{\underline{\phi} = \underline{\phi}} = 0.$$

How to calculate

$$Z_{\text{CFT}}[\text{sources}, \underline{\phi}_0] \equiv \langle e^{-\int \phi_0 \mathcal{O}} \rangle_{\text{CFT}}$$

$$\approx \exp\left\{-N^2 \underline{I}_{\text{bulk}}[\underline{\phi} \mid \underline{\phi}(z=\epsilon) = \underline{\phi}_0]\right\}$$

UV cutoff:

cut off the spacetime  
at  $z > \epsilon$

$\underline{\phi}$  solves bulk EoM

$$\left. \frac{\delta I_{\text{bulk}}}{\delta \phi} \right|_{\phi = \underline{\phi}} = 0.$$

eg. scalar  
in bulk.

$$ds_{AdS}^2 = \frac{L^2}{z^2} (dz^2 + dx_\mu dx^\mu)$$

$$\equiv \underline{g_{AB}} dz^A dz^B$$



more gen'l operator of  $N=4$  SYM:

$$= \text{tr} [X_\mu X_\nu \dots D_\mu X_\nu F_{\mu\nu} X X Y]$$

not BPS,  $\Delta = \Delta(\lambda) \quad J$ -scaling

$$\Delta(\lambda=0) = J$$

$$\text{tr} [Y X X X \dots D_\mu X_\nu F_{\mu\nu} X X]$$

How to calculate

$$Z_{\text{CFT}}[\text{sources}, \underline{\phi}_0] \equiv \langle e^{-\int \phi_0 \mathcal{O}} \rangle_{\text{CFT}}$$

$$\approx \exp\left\{-\frac{1}{2} N^2 I_{\text{bulk}}[\underline{\phi} \mid \phi(z=\epsilon) = \underline{\phi}_0]\right\}$$

UV cutoff

in the spacetime

$\epsilon$

$\underline{\phi}$  solves bulk EoM

$$\frac{\delta I_{\text{bulk}}}{\delta \phi} \Big|_{\phi = \underline{\phi}} = 0.$$

SL

How to calculate

$$Z_{\text{CFT}}[\text{sources}, \phi_0] \equiv \left\langle e^{-\int \phi_0 \mathcal{O}} \right\rangle_{\text{CFT}}$$

$$\sim \exp\left\{-\frac{1}{N^2} I_{\text{bulk}}[\phi] \Big|_{\phi(z=\epsilon)=\phi_0}\right\}$$

UV cutoff

cut off the spacetime  
at  $z > \epsilon$

$\phi$  solves bulk EoM

$$\frac{\delta I_{\text{bulk}}}{\delta \phi} \Big|_{\phi=\phi_0} = 0.$$

eg: scalar  
in bulk.

$$S[\phi] = -\frac{k}{2} \int d^{d+1}x \sqrt{g} \left[ g^{AB} \partial_A \phi \partial_B \phi + m^2 \phi^2 + b \phi^3 + \dots \right]$$

How to calculate

$$Z_{\text{CFT}}[\text{sources}, \phi_0] \equiv \left\langle e^{-\int \phi_0 \mathcal{O}} \right\rangle_{\text{CFT}}$$

$$\approx \exp \left\{ -\frac{1}{N^2} \int_{\text{bulk}} \left[ \phi \left| \phi(z=\epsilon) = \phi_0 \right. \right. \right.$$

UV cutoff:

cut off the spacetime  
at  $z > \epsilon$

$\phi$  solves bulk EoM

$$\left. \frac{\delta I_{\text{bulk}}}{\delta \phi} \right|_{\phi=\phi_0} = 0.$$

eg: scalar  
in bulk.

$$S[\phi] = -\frac{k}{2} \int d^{d+1}x \sqrt{g} \left[ g^{AB} \partial_A \phi \partial_B \phi + m^2 \phi^2 + b \phi^3 + \dots \right]$$



$$ds_{AdS}^2 = \frac{L^2}{z^2} (dz^2 + dx_\mu dx^\mu)$$

$$\equiv g_{AB} dz^A dz^B \quad A = 0, 1, \dots, D$$

$$\sqrt{g} = \sqrt{|\det g|} = \left(\frac{L}{z}\right)^{D+1}$$

$\left. \begin{matrix} \dots \\ \dots \end{matrix} \right\}$

$$\Delta = \dots \int \dots$$

$$\Delta(\lambda=0) = \dots$$

$$= \text{Tr} \left( \gamma \text{XXXX} \dots \right)$$

→ closed string

How to calculate

$$Z_{\text{CFT}}[\text{sources}, \phi_0] \equiv \langle e^{-\int \phi_0 \mathcal{O}} \rangle_{\text{CFT}}$$

$$\sim \exp\left\{-\frac{N^2}{2} I_{\text{bulk}}[\phi \mid \phi(z=\epsilon) = \phi_0]\right\}$$

UV cutoff:

cut off spacetime

eucl.

$\phi$  solves bulk EoM

$$\left. \frac{\delta I_{\text{bulk}}}{\delta \phi} \right|_{\phi=\phi} = 0.$$

eg: scalar

$$S[\phi] = -\frac{k}{2} \int d^{d+1}x \sqrt{g} \left[ g^{AB} \partial_A \phi \partial_B \phi + m^2 \phi^2 + b \phi^3 + \dots \right]$$



How to calculate

$$Z_{\text{CFT}}[\text{sources}, \phi_0] \equiv \langle e^{-\int \phi_0 \mathcal{O}} \rangle_{\text{CFT}}$$

$$\sim \exp \left\{ -\frac{N^2}{2} I_{\text{bulk}} \left[ \phi \mid \phi(z=\epsilon) = \phi_0 \right] \right\}$$

UV cutoff:

cut off the spacetime  
at  $z > \epsilon$

eucl.

$\phi$  solves bulk EoM

$$\frac{\delta I_{\text{bulk}}}{\delta \phi} \Big|_{\phi=\phi_0} = 0.$$

eg: scalar  
in bulk.

$$S[\phi] = -\frac{k}{2} \int d^{d+1}x \sqrt{g} \left[ g^{AB} \partial_A \phi \partial_B \phi + m^2 \phi^2 + b \phi^3 + \dots \right]$$

$k \sim N^2$   
saddle: AdS

How to calculate

$$Z_{\text{CFT}}[\text{sources}, \phi_0] \equiv \langle e^{-\int \phi_0 \mathcal{O}} \rangle_{\text{CFT}}$$

$$\sim \exp \left\{ -\frac{N^2}{2} I_{\text{bulk}} \left[ \phi \mid \phi(z=\epsilon) = \phi_0 \right] \right\}$$

UV cutoff:

cut off the spacetime  
at  $z > \epsilon$

eucl.

$\phi$  solves bulk EoM

$$\frac{\delta I_{\text{bulk}}}{\delta \phi} \Big|_{\phi=\phi_0} = 0.$$

eg: scalar  
in bulk.

$$S[\phi] = -\frac{k}{2} \int d^{d+1}x \sqrt{g} \left[ g^{AB} \partial_A \phi \partial_B \phi + m^2 \phi^2 + b \phi^3 + \dots \right]$$

$$k \sim N^2$$

saddle: AdS,  
 $\phi=0$

How to calculate

$$Z_{\text{CFT}}[\text{sources}, \phi_0] \equiv \langle e^{-\int \phi_0 \mathcal{O}} \rangle_{\text{CFT}}$$

$$\approx \exp \left\{ -\frac{N^2}{2} \int_{\text{bulk}} \left[ \phi \mid \phi(z=\epsilon) = \phi_0 \right] \right\}$$

UV cutoff

cut off the time  
at  $z = \epsilon$

eucl.

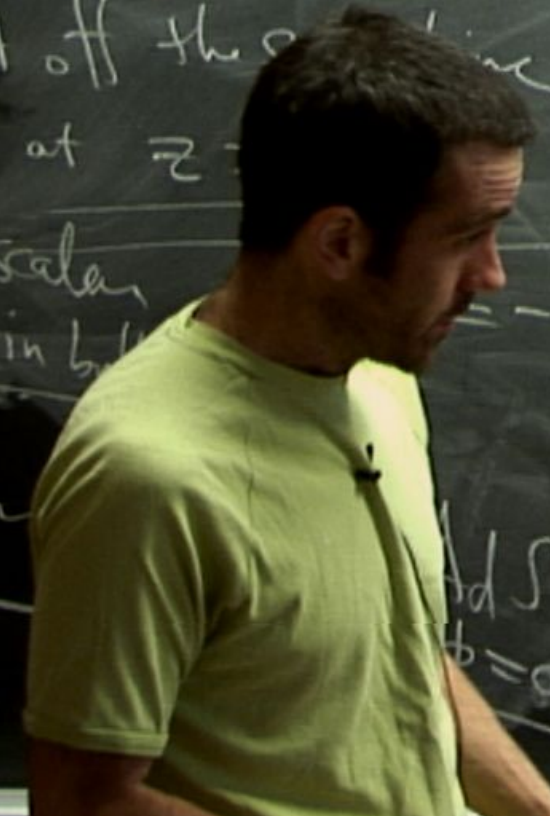
$\phi$  solves bulk EoM

$$\left. \frac{\delta I_{\text{bulk}}}{\delta \phi} \right|_{\phi=\phi_0} = 0.$$

eg: scalar  
in bulk

$$-\frac{k}{2} \int d^d x \sqrt{g} \left[ g^{AB} \partial_A \phi \partial_B \phi + m^2 \phi^2 + b \phi^3 + \dots \right]$$

$$\langle \mathcal{O} \rangle_{\text{vac}} = \left( \frac{\delta}{\delta \phi_0} \right) \ln Z \Big|_{\phi_0=0}$$



$$(z=e^{-\phi_0}) \left. \vphantom{z} \right\}$$

$$S = -\frac{f}{z}$$

$$\begin{aligned} ds_{AdS}^2 &= \frac{L^2}{z^2} (dz^2 + dx_\mu dx^\mu) \\ &\equiv g_{AB} dz^A dz^B \quad A=0 \\ \sqrt{g} &= \sqrt{|\det g|} = \left(\frac{L}{z}\right)^{d+1} \\ g^{AB} &= \frac{z^2}{L^2} \delta^{AB} \end{aligned}$$

Eq. M

= 0.

$$S = -\frac{ik}{2} \int_{\text{bulk}} \sqrt{|g|} \phi (\square + m^2) \phi$$

$$ds_{\text{AdS}}^2 = \frac{L^2}{z^2} (dz^2 + dx_\mu dx^\mu)$$

$$\equiv g_{AB} dz^A dz^B \quad A=0$$

$$\sqrt{g} = \sqrt{|\det g|} = \left(\frac{L}{z}\right)^{d+1}$$

$$g^{AB} = \frac{z^2}{L^2} \delta^{AB}$$

$$(z=\epsilon) = \phi_0 \Big\}$$

Eq M  
= 0  
= 0



How to calculate

$$Z_{\text{CFT}}[\text{sources}, \phi_0] \equiv \langle e^{-\int \phi_0 \mathcal{O}} \rangle_{\text{CFT}}$$

UV cutoff:

cut off the spacetime  
at  $z > \epsilon$

$$\sim \exp\left\{-\frac{N^2}{2} I_{\text{bulk}}[\phi] \Big|_{\phi(z=\epsilon)=\phi_0}\right\}$$

eucl

solves bulk EoM

$$\frac{\delta I_{\text{bulk}}}{\delta \phi} \Big|_{\phi=\phi_0} = 0.$$

eg: scalar  
in bulk.

$$S[\phi] = -\frac{k}{2} \int d^{d+1}x$$

$$g^{AB} \partial_A \phi \partial_B \phi$$

$$k \sim N^2$$

saddle: AdS,  
 $\phi=0$

$$b\phi^3 + \dots$$

$$S = -\frac{1}{2} \int_{\text{bulk}} \sqrt{|g|} \phi (\square + m^2) \phi$$

$$ds_{\text{AdS}}^2 = \frac{L^2}{z^2} (dz^2 + dx_\mu dx^\mu)$$

$$\equiv g_{AB} dz^A dz^B \quad A = 0, 1, \dots, D$$

$$\sqrt{|g|} = \sqrt{|\det g|} = \left(\frac{L}{z}\right)^{D+1}$$

$$g^{AB} = \frac{z^2}{L^2} \delta^{AB}$$

$$\square \phi = \frac{1}{\sqrt{|g|}} \partial_A (\sqrt{|g|} g^{AB} \partial_B \phi)$$

$$S = -\frac{1}{2} \int_{\text{bulk}} \sqrt{|g|} \phi (\square + m^2) \phi$$

$$ds_{\text{AdS}}^2 = \frac{L^2}{z^2} (dz^2 + dx_\mu dx^\mu)$$

$$\equiv g_{AB} dz^A dz^B \quad A = 0, 1, 2, 3, z$$

$$\sqrt{|g|} = \sqrt{|\det g|} = \left(\frac{L}{z}\right)^{d+1}$$

$$g_{AB} = \frac{z^2}{L^2} \delta^{AB}$$

$$\square \phi = \frac{1}{\sqrt{|g|}} \partial_A (g^{AB} \sqrt{|g|} \partial_B \phi)$$



$$S = -\frac{1}{2} \int_{\text{bulk}} \sqrt{g} \phi (\square + m^2) \phi$$

$$ds_{\text{AdS}}^2 = \frac{L^2}{z^2} (dz^2 + dx_\mu dx^\mu)$$

$$\equiv g_{AB} dz^A dz^B \quad A = 0, 1, 2, \dots, D$$

$$\sqrt{g} = \sqrt{|\det g|} = \left(\frac{L}{z}\right)^{D+1}$$

$$g^{AB} = \frac{z^2}{L^2} \delta^{AB}$$

$$\square \phi = \frac{1}{\sqrt{g}} \partial_A (g^{AB} \sqrt{g} \partial_B \phi)$$

$$S = -\frac{\hbar c}{2} \int_{\text{bulk}} \sqrt{|g|} \phi (\square + m^2) \phi$$

$$- \frac{\hbar c}{2} \int d^D x \sqrt{|g|} g^{\mu\nu} \partial_\mu \partial_\nu \phi$$

$$ds_{\text{AdS}}^2 = \frac{L^2}{z^2} (dz^2 + dx_\mu dx^\mu)$$

$$\equiv g_{AB} dz^A dz^B \quad A = 0, 1, \dots, D-2$$

$$\sqrt{|g|} = \sqrt{|\det g|} = \left(\frac{L}{z}\right)^{D+1}$$

$$g^{AB} = \frac{z^2}{L^2} \delta^{AB}$$

$$\square \phi = \frac{1}{\sqrt{|g|}} \partial_A (g^{AB} \sqrt{|g|} \partial_B \phi)$$

$$S = -\frac{\hbar c}{2} \int_{\text{bulk}} \sqrt{|g|} \phi (\square + m^2) \phi$$

$$-\frac{\hbar c}{2} \int d^D x \sqrt{|g|} \phi \square \phi$$

$$ds_{\text{AdS}}^2 = \frac{L^2}{z^2} (dz^2 + dx_\mu dx^\mu)$$

$$\equiv g_{AB} dz^A dz^B \quad A = 0, 1, \dots, D-2$$

$$\sqrt{|g|} = \sqrt{|\det g|} = \left(\frac{L}{z}\right)^{D+1}$$

$$g^{AB} = \frac{z^2}{L^2} \delta^{AB}$$

$$\square \phi = \frac{1}{\sqrt{|g|}} \partial_A (g^{AB} \sqrt{|g|} \partial_B \phi)$$

$$S = -\frac{\hbar c}{2} \int \sqrt{g} \phi (\square + m^2) \phi$$

$$- \frac{\hbar c}{2} \int d^d x \sqrt{g} g^{\alpha\beta} \phi \partial_\alpha \phi$$

$\frac{\delta S}{\delta \phi} = 0$

$$ds_{AdS}^2 = \frac{L^2}{z^2} (dz^2 + dx_\mu dx^\mu)$$

$$\equiv g_{AB} dz^A dz^B \quad A = 0, 1, \dots, d$$

$$\sqrt{g} = \sqrt{|\det g|} = \left(\frac{L}{z}\right)^{d+1}$$

$$g^{AB} = \frac{z^2}{L^2} \delta^{AB}$$

$$\square \phi = \frac{1}{\sqrt{g}} \partial_A (g^{AB} \sqrt{g} \partial_B \phi)$$

$$S = -\frac{\hbar c}{2} \int_{\text{bulk}} \sqrt{g} \phi (-\square + m^2) \phi$$

$$- \frac{\hbar c}{2} \int d^d x \sqrt{g} g^{\alpha\beta} \phi \partial_\beta \phi$$

$\frac{\partial S}{\partial g^{\alpha\beta}} = 0$

• EOM for small fluctuations of  $\phi$

$$\square \phi = 0$$

$$ds_{\text{AdS}}^2 = \frac{L^2}{z^2} (dz^2 + dx_\mu dx^\mu)$$

$$\equiv g_{AB} dz^A dz^B \quad A = 0, 1, \dots, d-2$$

$$\sqrt{g} = \sqrt{|\det g|} = \left(\frac{L}{z}\right)^{d+1}$$

$$g^{AB} = \frac{z^2}{L^2} \delta^{AB}$$

$$\square \phi = \frac{1}{\sqrt{g}} \partial_A (g^{AB} \sqrt{g} \partial_B \phi)$$

$$S = -\frac{\hbar c}{2} \int_{\text{bulk}} \sqrt{g} \phi (-\square + m^2) \phi$$

$$ds_{\text{AdS}}^2 = \frac{L^2}{z^2} (dz^2 + dx_\mu dx^\mu)$$

$$\equiv g_{AB} dz^A dz^B \quad A = 0, 1, 2, \dots, D$$

$$-\frac{\hbar c}{2} \int d^D x \sqrt{g} g^{\mu\nu} \phi \partial_\mu \partial_\nu \phi$$

$\frac{\partial \mathcal{L}}{\partial \phi} = 0$

$$\sqrt{g} = \sqrt{|\det g|} = \left(\frac{L}{z}\right)^{D+1}$$

$$g^{AB} = \frac{z^2}{L^2} \delta^{AB}$$

$$\square \phi = \frac{1}{\sqrt{g}} \partial_A (g^{AB} \sqrt{g} \partial_B \phi)$$

• EOM for small fluctuations of  $\phi$

$$\square \phi = 0$$

$S[\phi]$

$$S = -\frac{\hbar}{2} \int_{\text{bulk}} \sqrt{g} \phi (-\square + m^2) \phi$$

$$- \frac{\hbar}{2} \int d^d x \sqrt{g} g^{\alpha\beta} \phi \partial_\beta \phi$$

$\frac{\partial S}{\partial A_{\alpha\beta}} = \epsilon$

• EOM for small fluctuations of  $\phi$

$$(-\square + m^2) \phi = 0$$

•  $S[\phi] = \text{bdy term}$

$$(Z = e^{-S[\phi]})$$

$$ds_{\text{AdS}}^2 = \frac{L^2}{z^2} (dz^2 + dx_\mu dx^\mu)$$

$$\equiv g_{AB} dz^A dz^B \quad A = 0, 1, 2, \dots, d$$

$$\sqrt{g} = \sqrt{|\det g|} = \left(\frac{L}{z}\right)^{d+1}$$

$$g^{AB} = \frac{z^2}{L^2} \delta^{AB}$$

$$\square \phi = \frac{1}{\sqrt{g}} \partial_A (g^{AB} \sqrt{g} \partial_B \phi)$$

Wave een in Ad  $\tau$

translation into  
 $x^M \rightarrow x$

$$\psi(x^M) = e^{ik_M x^M} f_k(z)$$

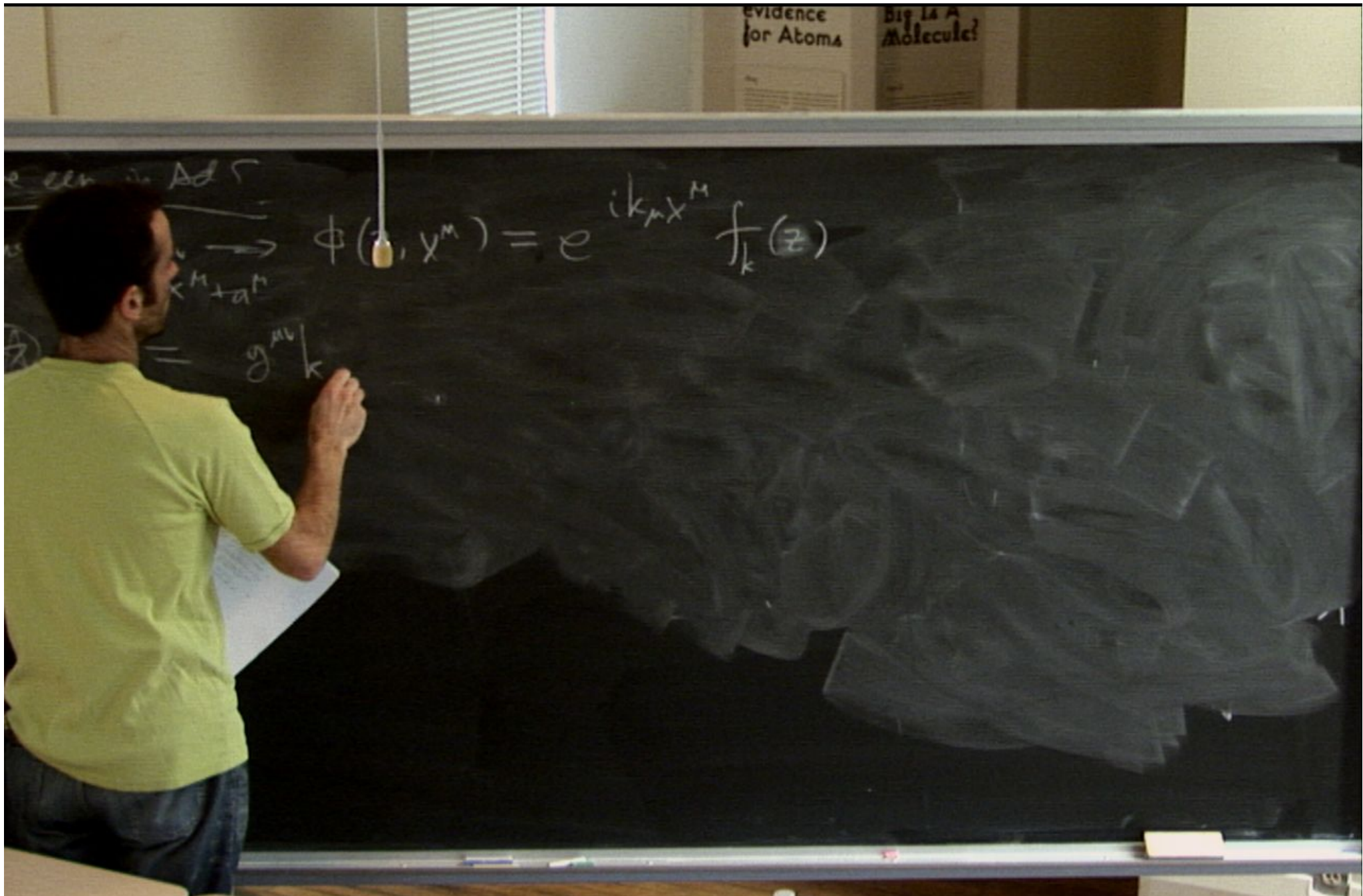


Wave een in Ad  $\Gamma$

translation inv  $\rightarrow$   
 $x^M \rightarrow x^M + a^M$

$$\phi(z, x^M) = e^{ik_M x^M} f_k(z)$$

$0 =$



Evidence for Atoms

Big Is A Molecule?

$\psi$  in  $\text{Ad } \Gamma$

$\rightarrow$   
 $k^M + a^M$

$=$   
 $g^{\mu\nu} / k$

$$\phi(\vec{r}, x^M) = e^{ik_M x^M} f_k(z)$$

evidence  
for Atoms

Big Is A  
Molecule?

seen in Ad  $\Gamma$

translation inv  $\rightarrow$   
 $x^M \rightarrow x^M + a^M$

$$\phi(x^M) = e^{ik_M x^M} f_k(z)$$

$$\textcircled{A} \quad 0 = g^{\mu\nu} k_\mu k_\nu$$

evidence  
for Atoms

Big Is A  
Molecule?

seen in Ad  $\Gamma$   
relation in  $\rightarrow \phi$   
 $x^M \rightarrow x^M + a^M$

$$\phi(x) = e^{ik_\mu x^M} f_k(z)$$

$$\star \quad 0 = \left[ g^{\mu\nu} \partial_\mu \partial_\nu + \frac{1}{\sqrt{g}} \partial_z \left( \sqrt{g} g^{zz} \partial_z \right) + m^2 \right] f_k(z)$$

evidence  
for Atoms

Big Is A  
Molecule?

relation  
 $x^M$

$$\phi(z, x^M) = e^{ik_\mu x^M} f_k(z)$$

$$0 = \left[ -k_\nu k_\nu - \frac{1}{\sqrt{g}} \partial_z \left( \sqrt{g} g^{zz} \partial_z \right) + m^2 \right] f_k(z)$$

evidence  
for Atoms

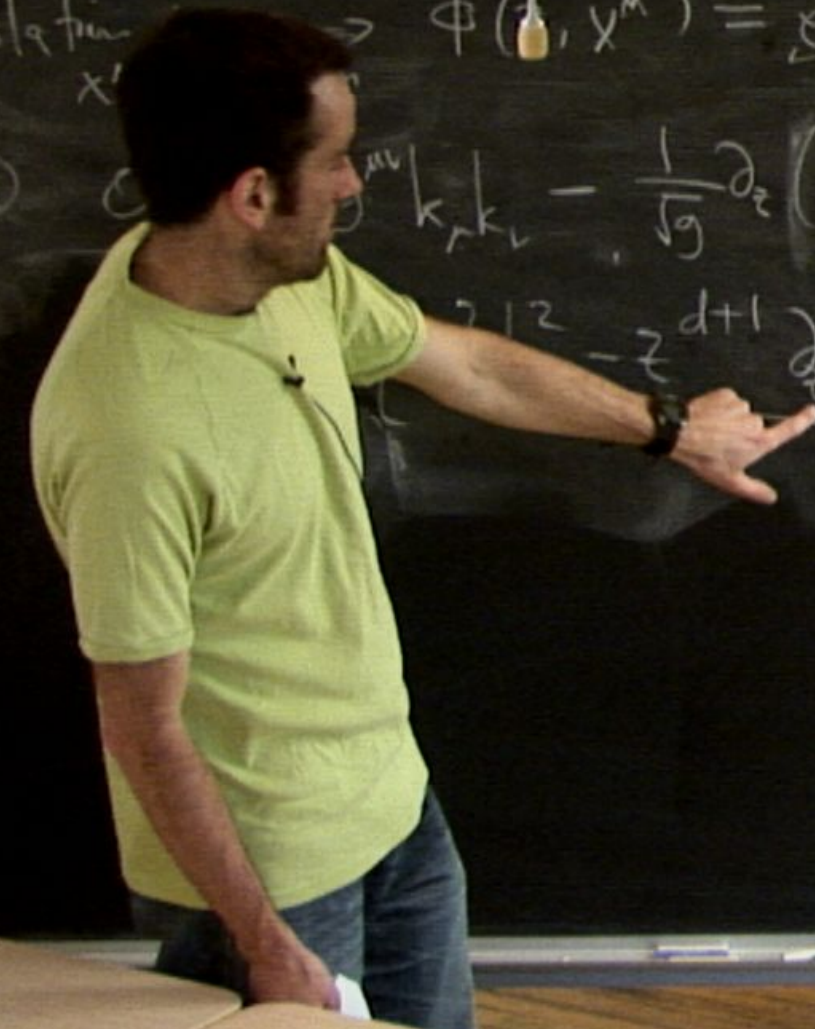
Big Is A  
Molecule?

relation  
 $x^M$

$$\phi(z, x^M) = e^{ik_\mu x^M} f_k(z)$$

$$\left[ k_\mu k_\nu - \frac{1}{\sqrt{g}} \partial_z (\sqrt{g} g^{zz} \partial_z) + m^2 \right] f_k(z)$$

$$-z^{d+1} \partial_z (z^{-d+1} \partial_z) + m^2 z^2 \left[ f_k(z) \right]$$



evidence  
for Atoms

Big Is A  
Molecule?

seen in Ad  
relation in  
 $x^M \rightarrow x^M$   
①  $0 = \dots$

$$\psi(x^M) = e^{ik_M x^M} f_k(z)$$

$$-\frac{1}{g} \partial_z (g^2 \partial_z) + m^2 \Big] f_k(z)$$

$$-z^{d+1} \partial_z (z^{-d+1} \partial_z) + m^2 \Big] f_k(z)$$

evidence  
for Atoms

Big Is A  
Molecule?

relation  
 $x^M$

$$\phi(z, x^M) = e^{ik_\mu x^M} f_k(z)$$

$$g^{\mu\nu} k_\mu k_\nu - \frac{1}{\sqrt{g}} \partial_z (\sqrt{g} g^{zz} \partial_z) + m^2 \Big] f_k(z)$$

$$\frac{1}{z^2} \left[ z^2 k^2 - z^{d+1} \partial_z (z^{-d+1} \partial_z) + m^2 z^2 \right] f_k(z)$$

$z^D$   
[Handwritten notes on a piece of paper]



evidence  
for Atoms

Big Is A  
Molecule?

seen in Ad  
relation inv  
 $x^M \rightarrow x^{M+1}$

$$\psi(x^M) = e^{ik_M x^M} f_k(z)$$

$$0 = \left[ -\frac{1}{\sqrt{g}} \partial_z \left( \sqrt{g} g^{zz} \partial_z \right) + m^2 \right] f_k(z)$$

$$-z^{d+1} \partial_z \left( z^{-d+1} \partial_z \right) + m^2 L^2 \left] f_k(z) \right.$$

$$-z^{\Delta} \left[ \frac{1}{k} z^2 \partial_z^2 - \Delta(\Delta-1) + m^2 L^2 \right] z^{\Delta}$$

evidence  
for Atoms

Big Is A  
Molecule?

invariance in AdS  
relativistic invariance  $\rightarrow$   
 $x^M \rightarrow x^M + a^M$

$$\phi(x, y^M) = e^{i(k_\mu x^\mu + \dots)}$$

$$f_k(z) \stackrel{z \rightarrow 0}{\sim} z^\Delta$$
$$\Delta(\Delta-1) = m^2 L^2$$

$$0 = \left[ g^{\mu\nu} k_\mu k_\nu - \frac{1}{\sqrt{g}} \partial_z \left( \sqrt{g} \partial_z \right) \right] f_k(z)$$
$$= \frac{1}{z^2} \left[ z^2 k^2 - z^{d+1} \partial_z \left( z^{-d} \partial_z \right) + m^2 L^2 \right] f_k(z)$$

Ansatz:  $f_k(z) = z^\Delta$

$$\left[ -\Delta(\Delta-1) + m^2 L^2 \right] z^\Delta$$

evidence  
for Atoms

Big is A  
Molecule?

$\Delta d \gamma$   
 $\nu \rightarrow$   
 $x^M + a^M$

$$\phi(z, y^M) = e^{ik_\mu x^M} f_k(z)$$

$$= \left[ g^{\mu\nu} k_\mu k_\nu - \frac{1}{\sqrt{g}} \partial_z (\sqrt{g} g^{zz} \partial_z) + m^2 \right] f_k(z)$$

$$= \frac{1}{z^2} \left[ z^2 k^2 - z^{d+1} \partial_z (z^{-d+1} \partial_z) + m^2 z^2 \right] f_k(z)$$

$$\text{set } z = f_k(z) - z^\Delta : 0 = \left( \frac{z^2}{k^2} - \Delta(\Delta-1) + m^2 z^2 \right) z^\Delta$$

$$\left. \begin{aligned} f_k(z) \xrightarrow{z \rightarrow 0} z^\Delta \\ \Delta(\Delta-1) = m^2 L^2 \end{aligned} \right\}$$