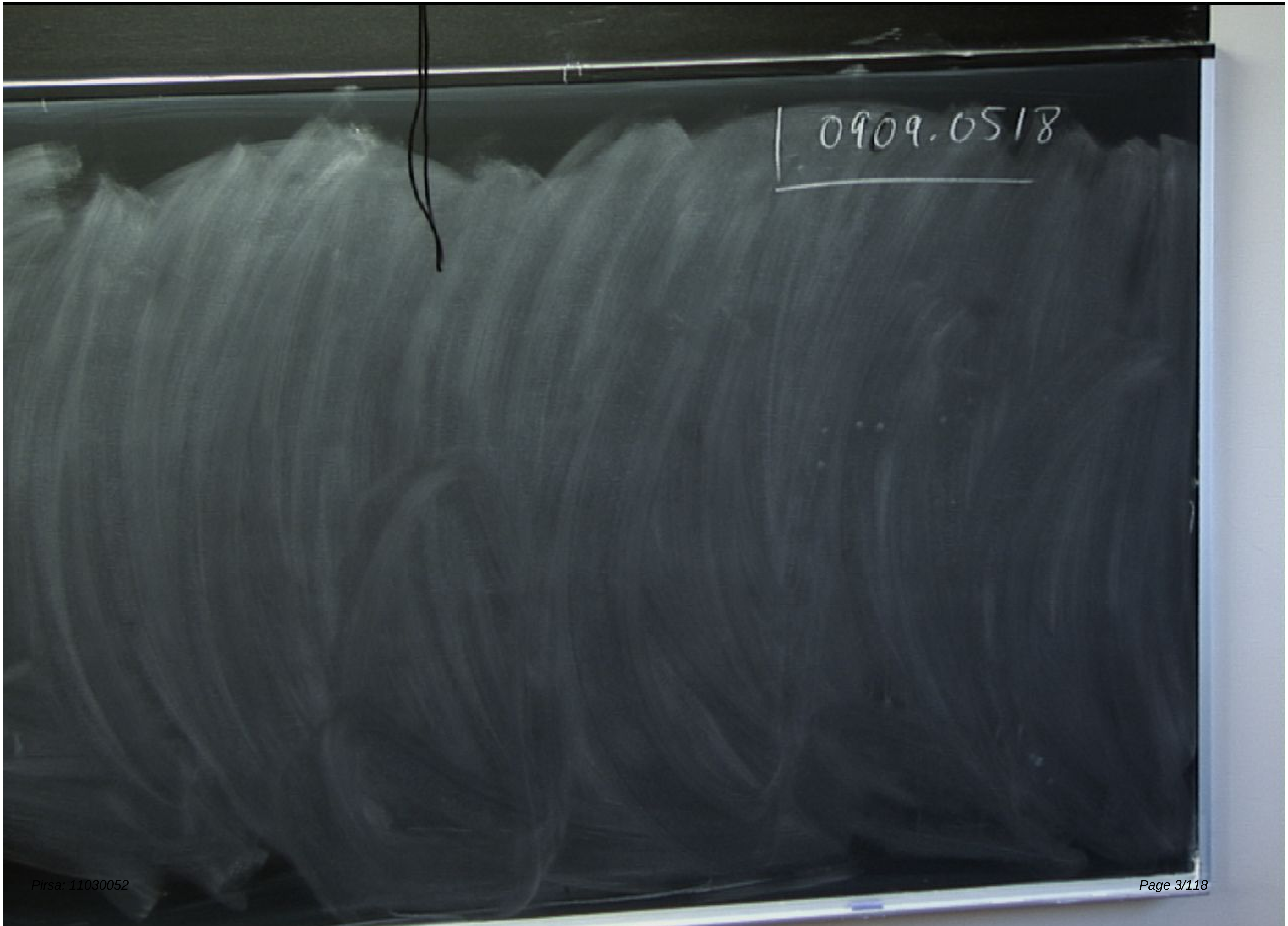
Title: Explorations in String Theory - Lecture 5

Date: Mar 18, 2011 11:30 AM

URL: http://pirsa.org/11030052

Abstract:

perimeter scholars
international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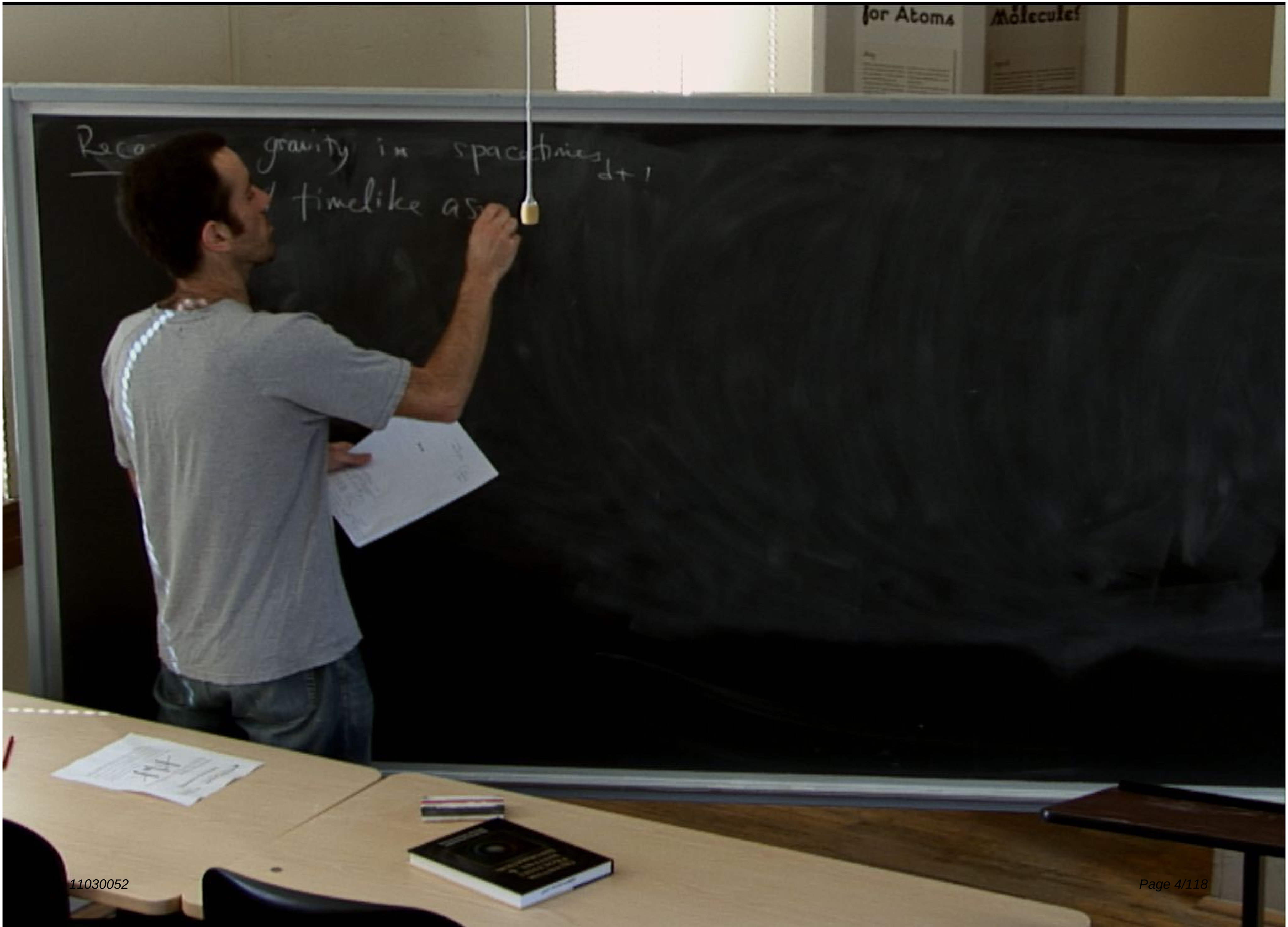
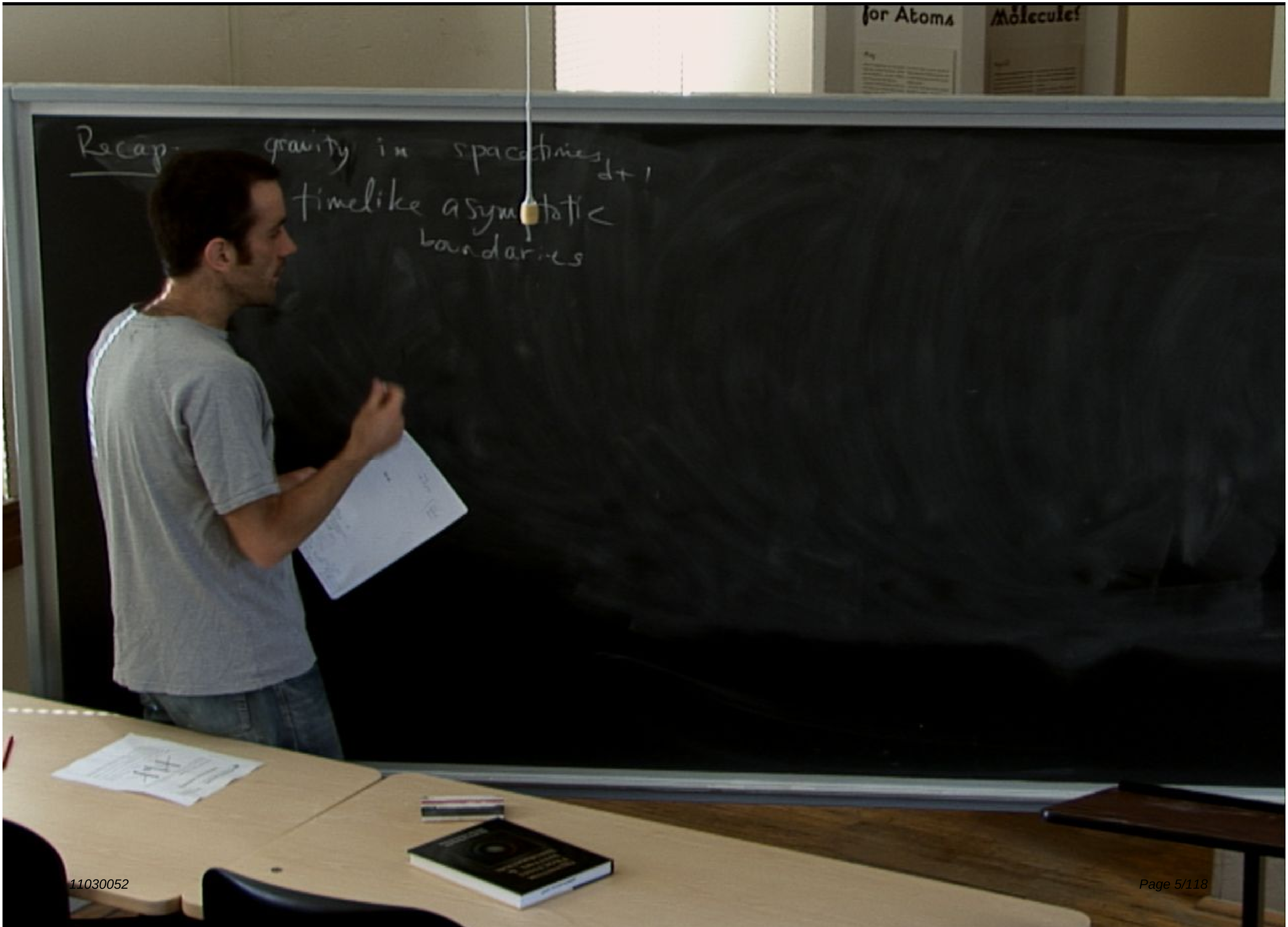
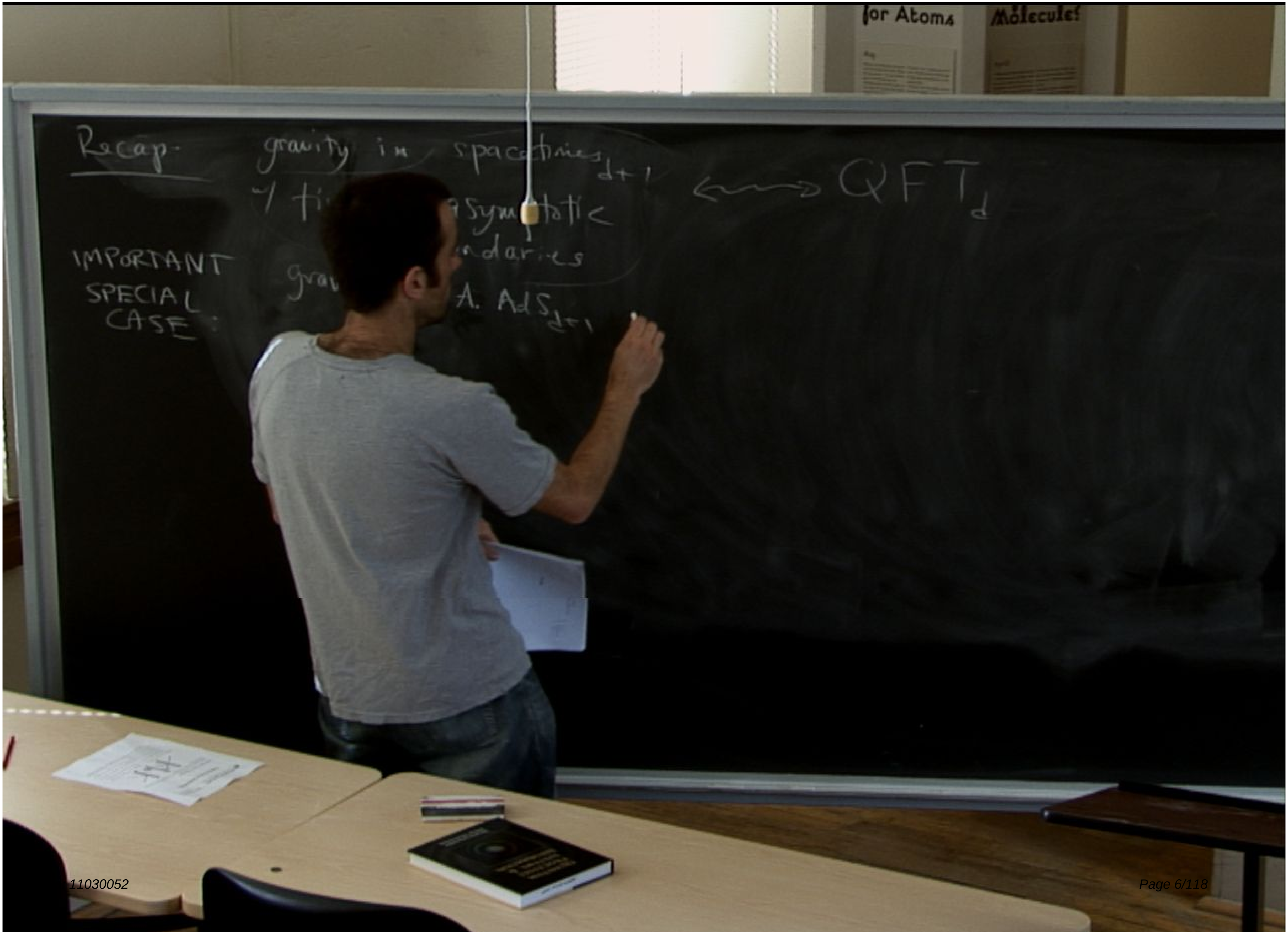
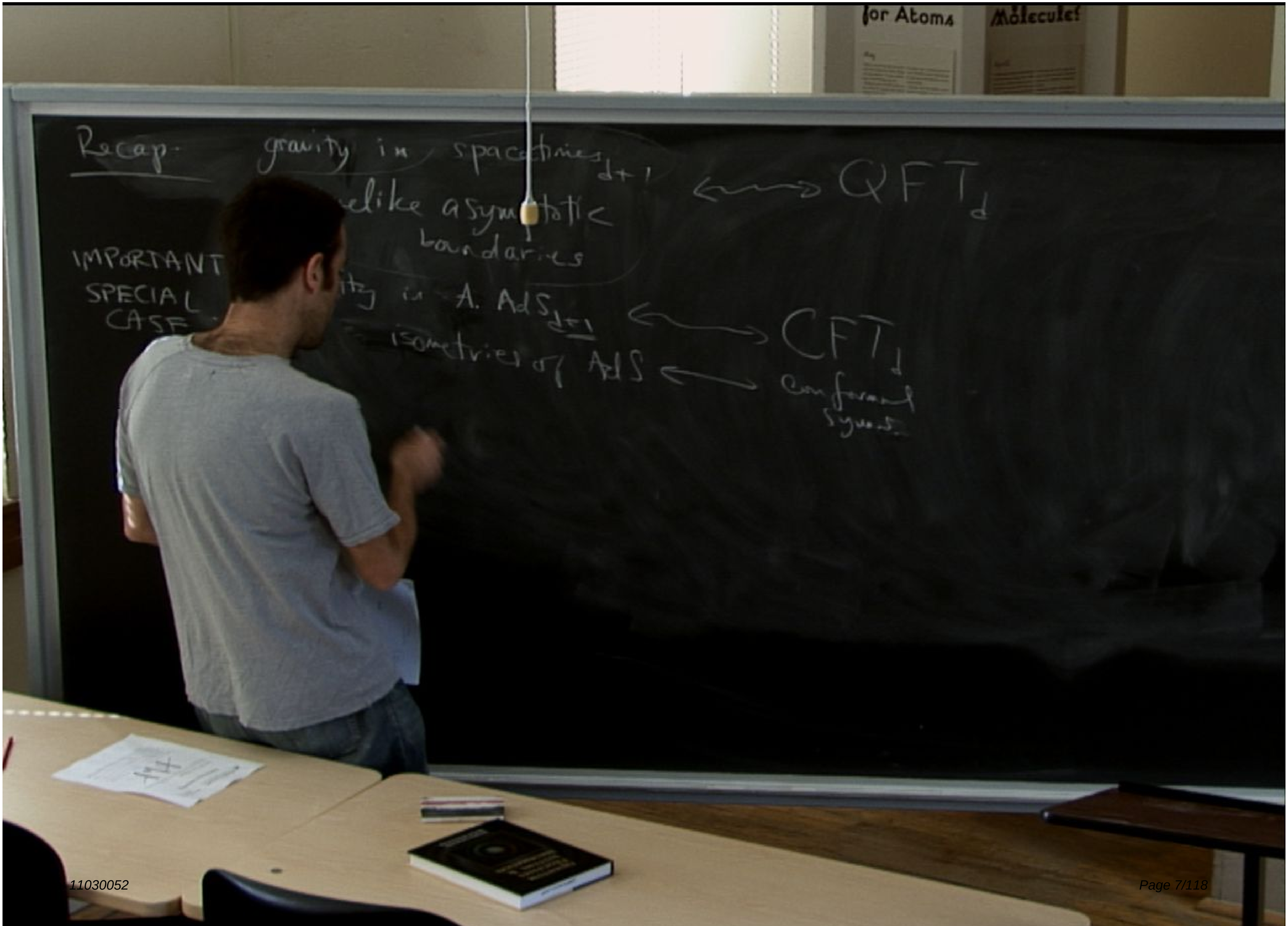
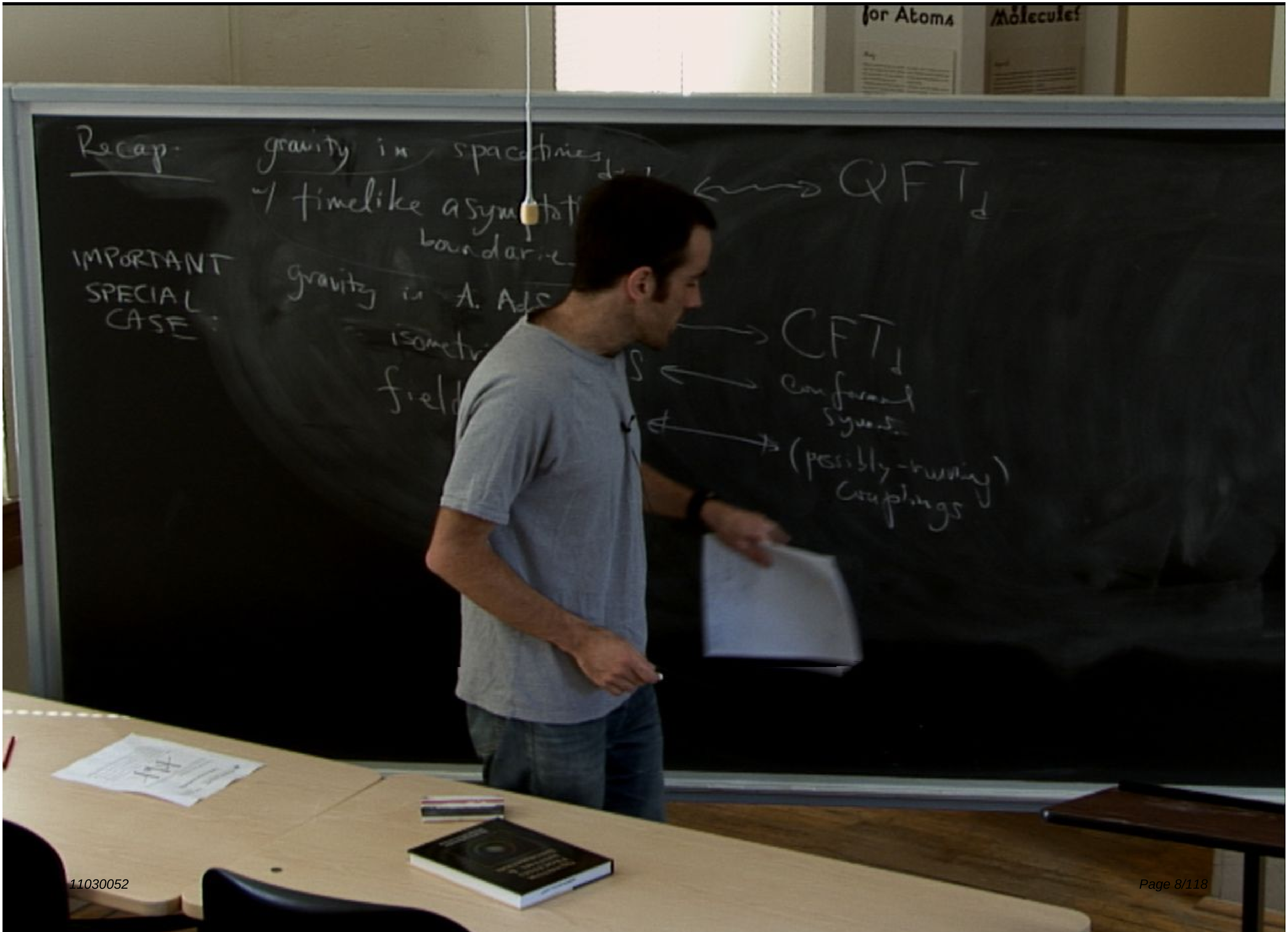
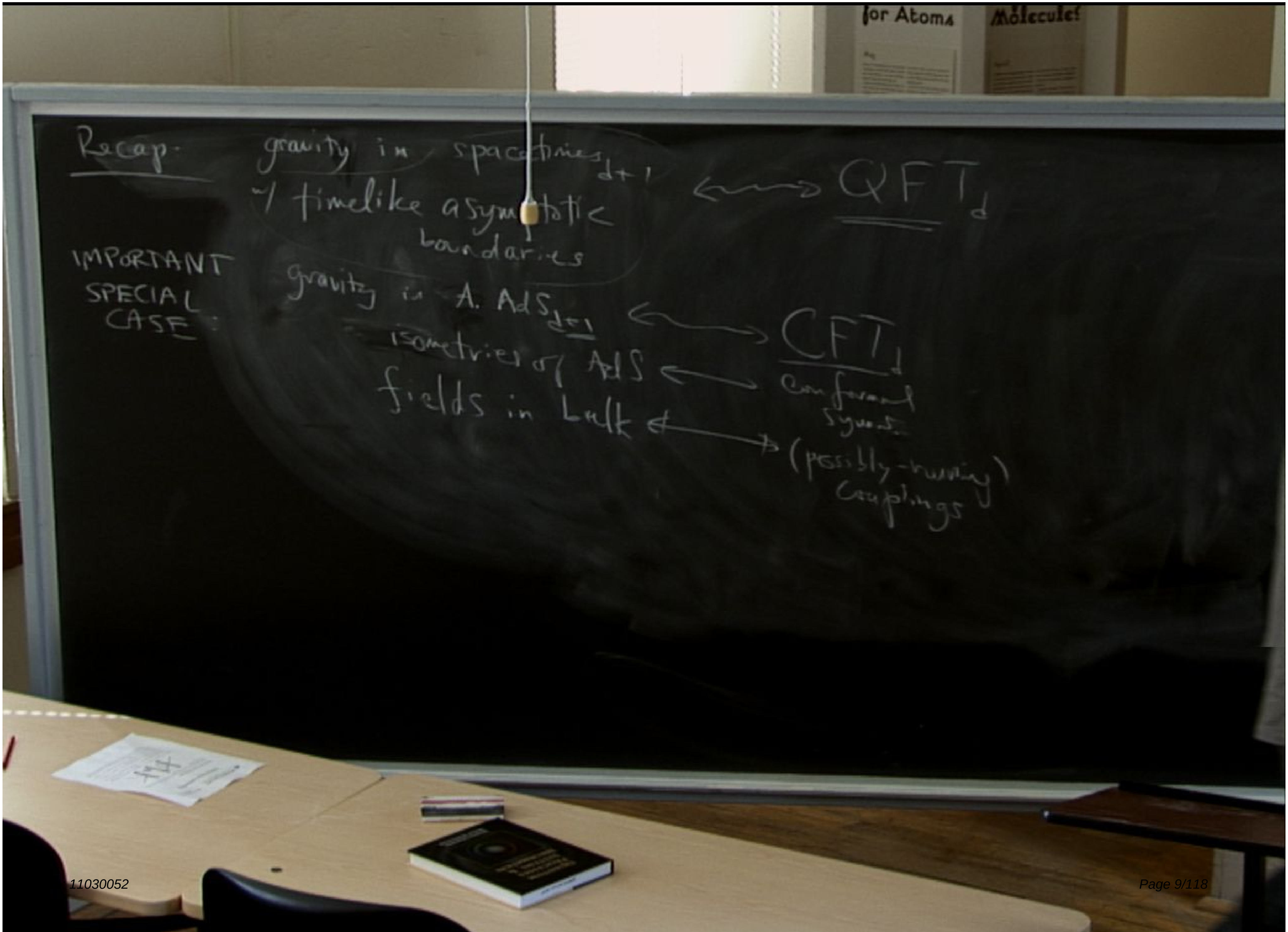

for Atoms
Molecules
gravity in spacetimes
d+1
timelike

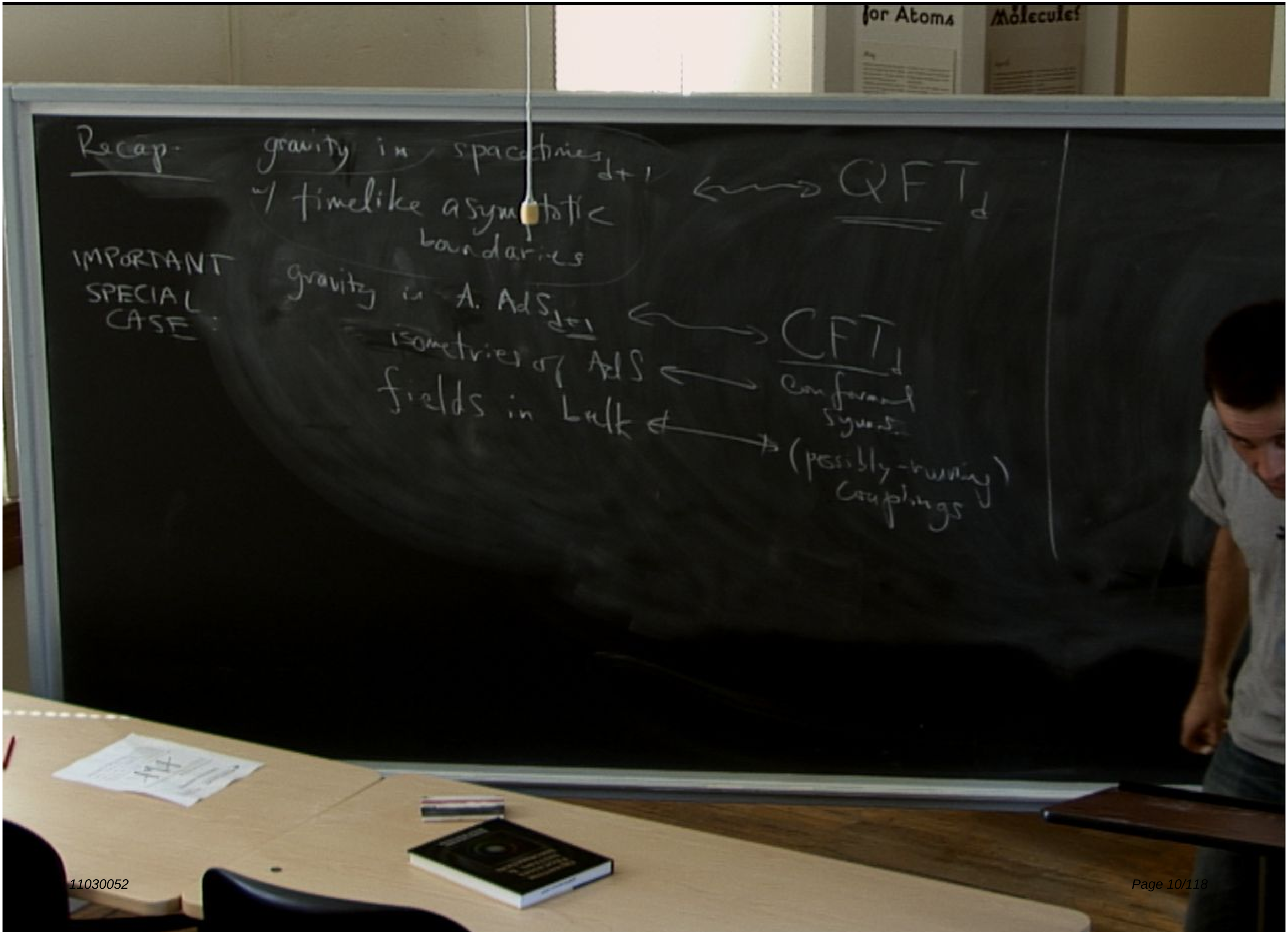

for Atoms
Molecules
Recap: gravity in spacetimes d+1
timelike asymptotic
boundaries


for Atoms
Molecules
Recap
gravity in spacetimes d+1
⟷ QFT d
asymptotic
IMPORTANT
SPECIAL
CASE:
A. AdS d+1


for Atoms
Molecules
Recap.
gravity in spacetimes d+1
elike asymptotic boundaries
QFT d
IMPORTANT
SPECIAL
CASE
ity in A. AdS d+1
CFT d
isometries of AdS
conformal symms


for Atoms
Molecules
Recap:
gravity in spacetimes
QFT
IMPORTANT
SPECIAL
CASE:
gravity in A. AdS
CFT
conformal
symms.
(possibly-running)
couplings


for Atoms
Molecules
Recap:
gravity in spacetimes d+1
w/ timelike asymptotic boundaries
QFT d
IMPORTANT SPECIAL CASE:
gravity in A. AdS d+1
CFT d
isometries of AdS
conformal symms.
fields in bulk
(possibly-running) couplings


Recap-
gravity in spacetimes d+1
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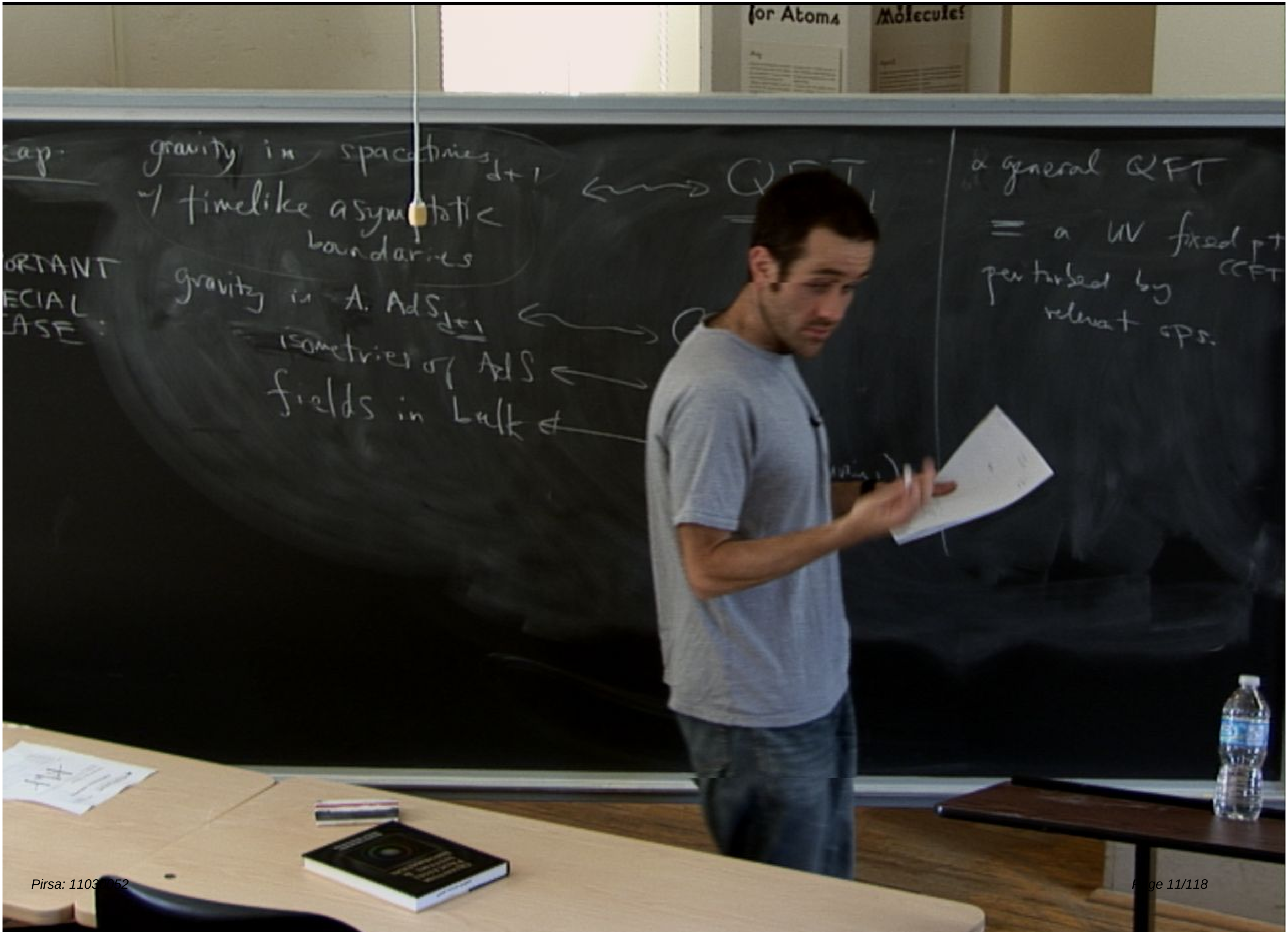
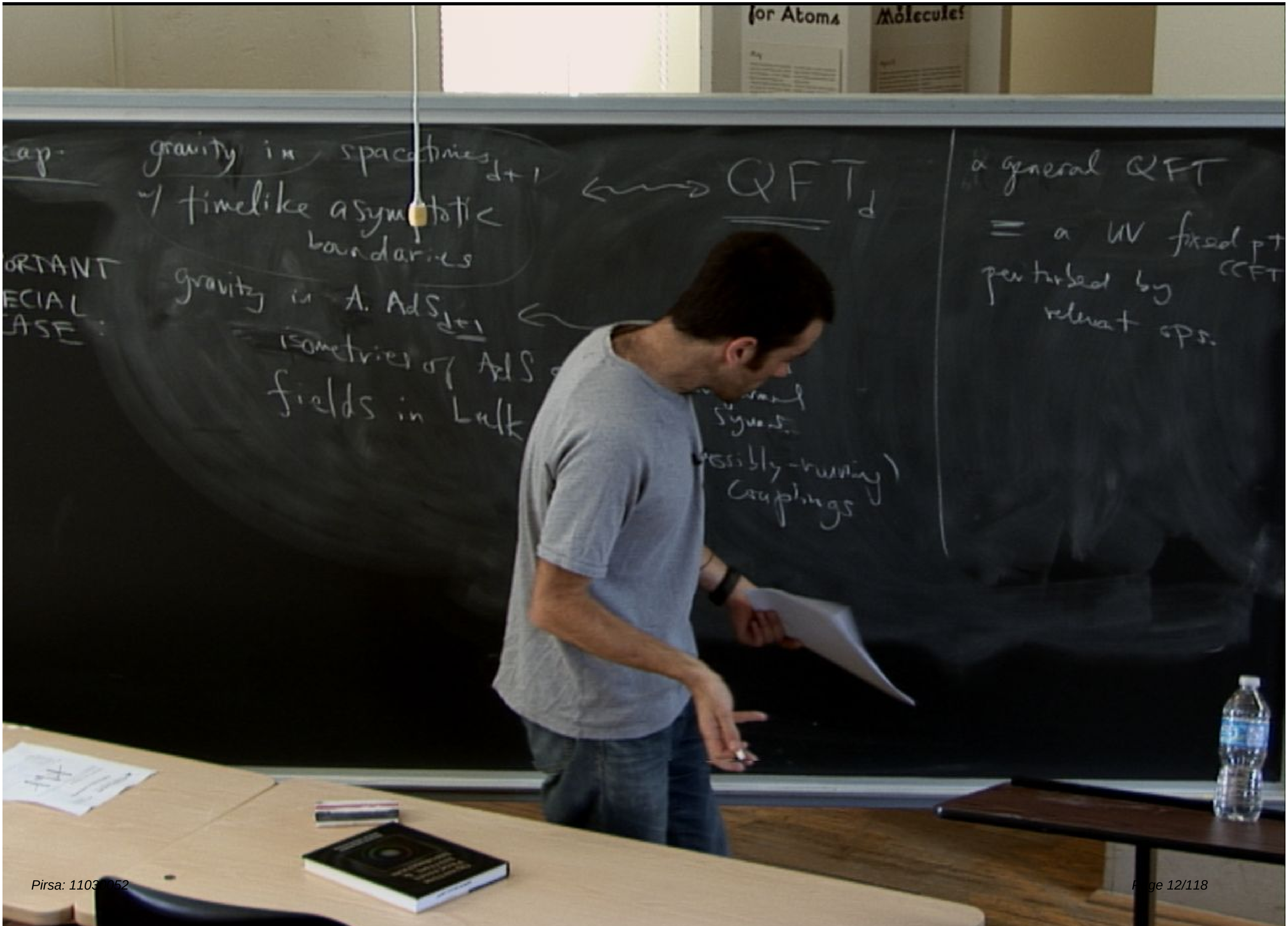
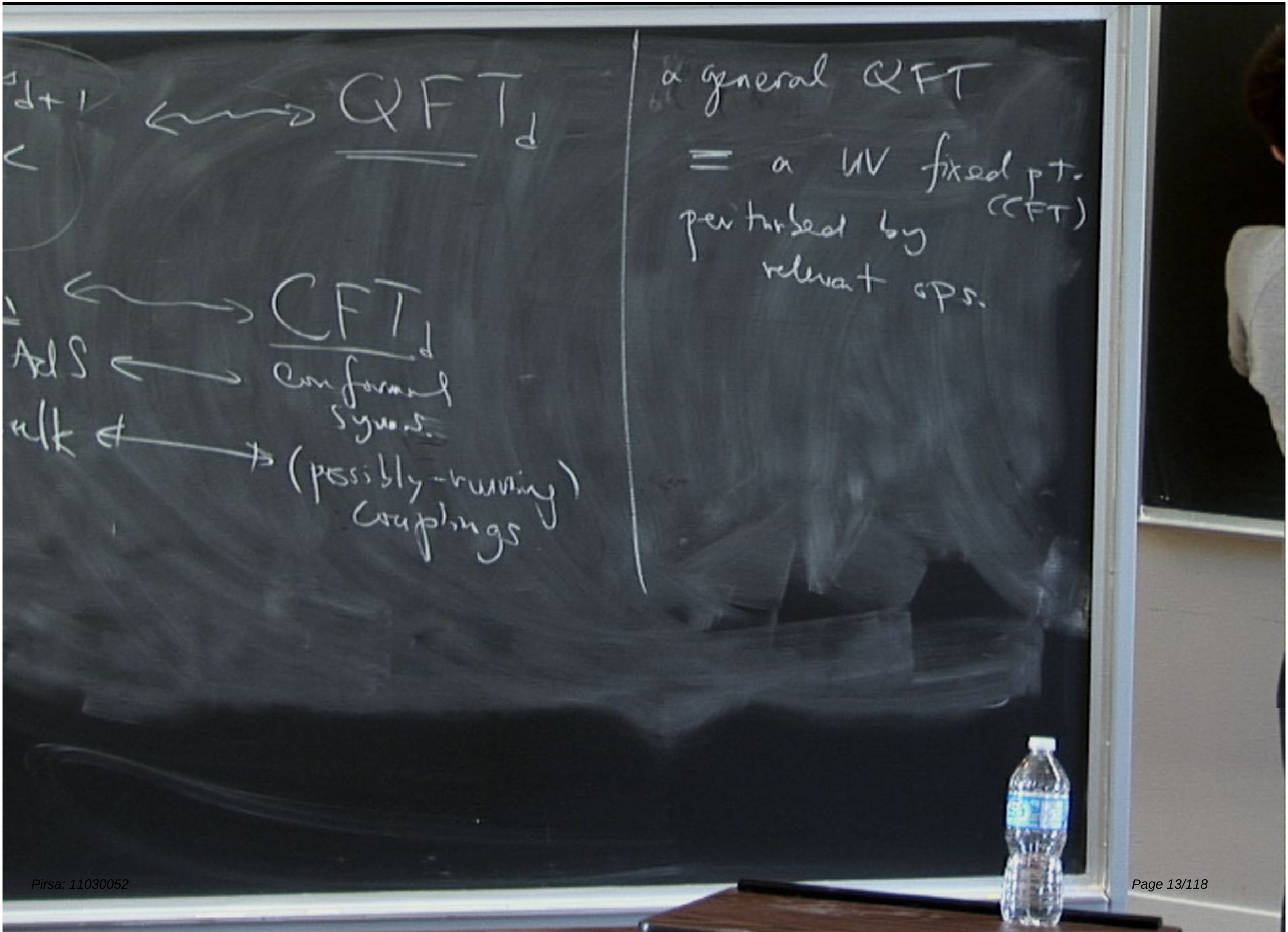
QFT d
IMPORTANT SPECIAL CASE:
gravity in A. AdS d+1
CFT d
isometries of AdS
conformal symms.
fields in bulk
(possibly-running) couplings
for Atoms
Molecules

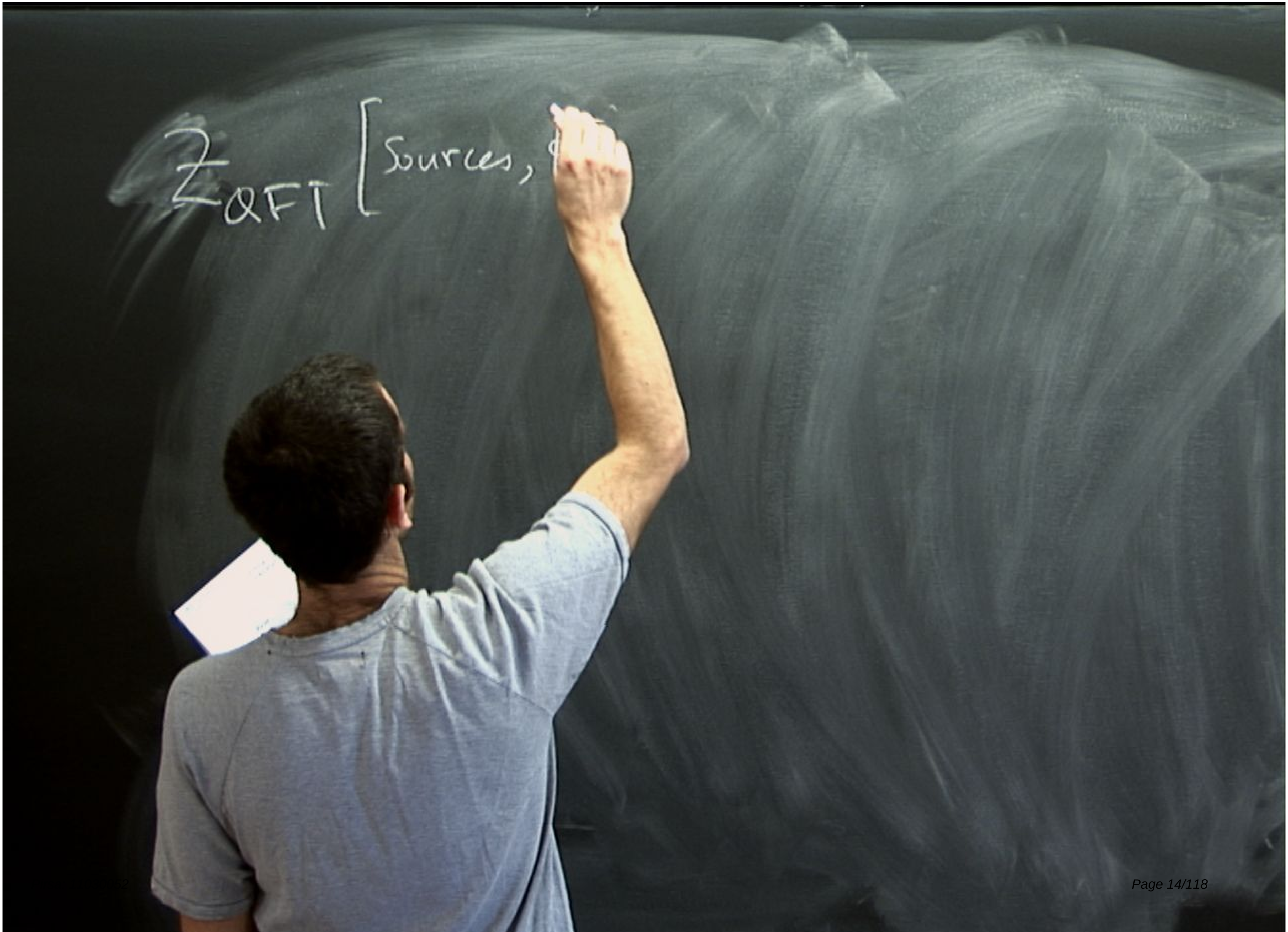


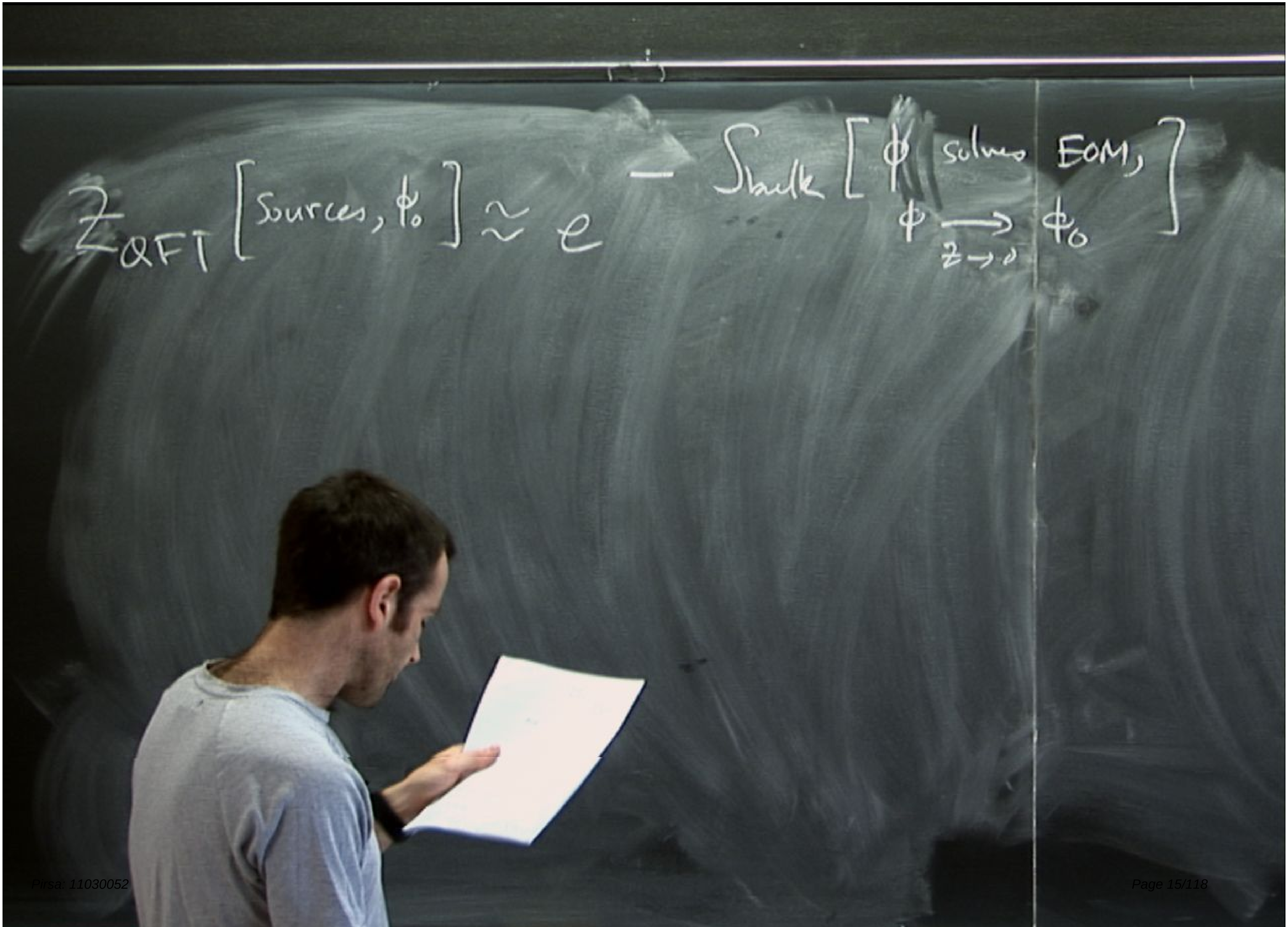
for Atoms
Molecules
gravity in spacetimes d+1 w/ timelike asymptotic boundaries
QFT
gravity in A. AdS d+1
isometries of AdS
fields in bulk
a general QFT
≡ a UV fixed pt (CFT) perturbed by relevant ops.

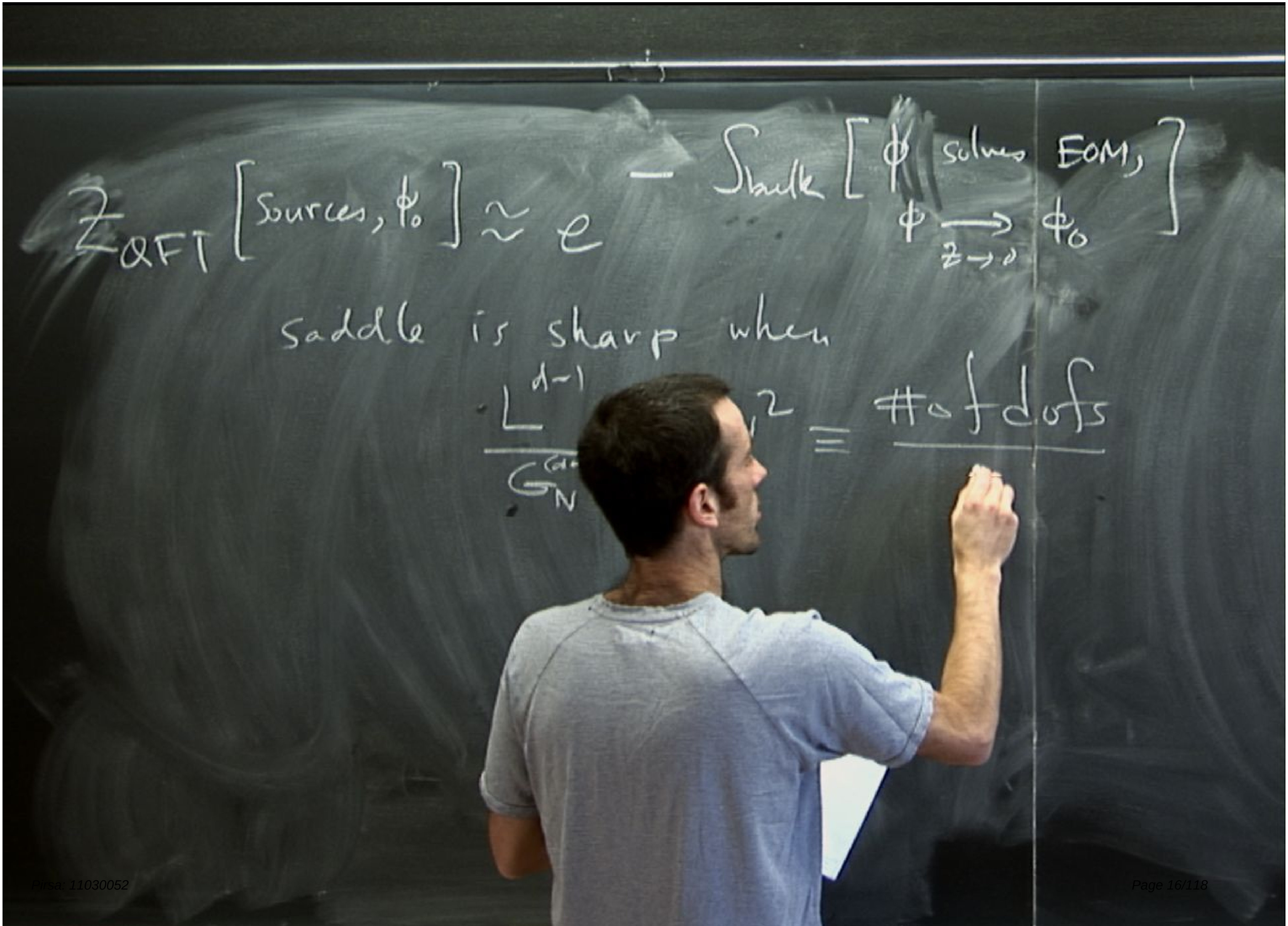


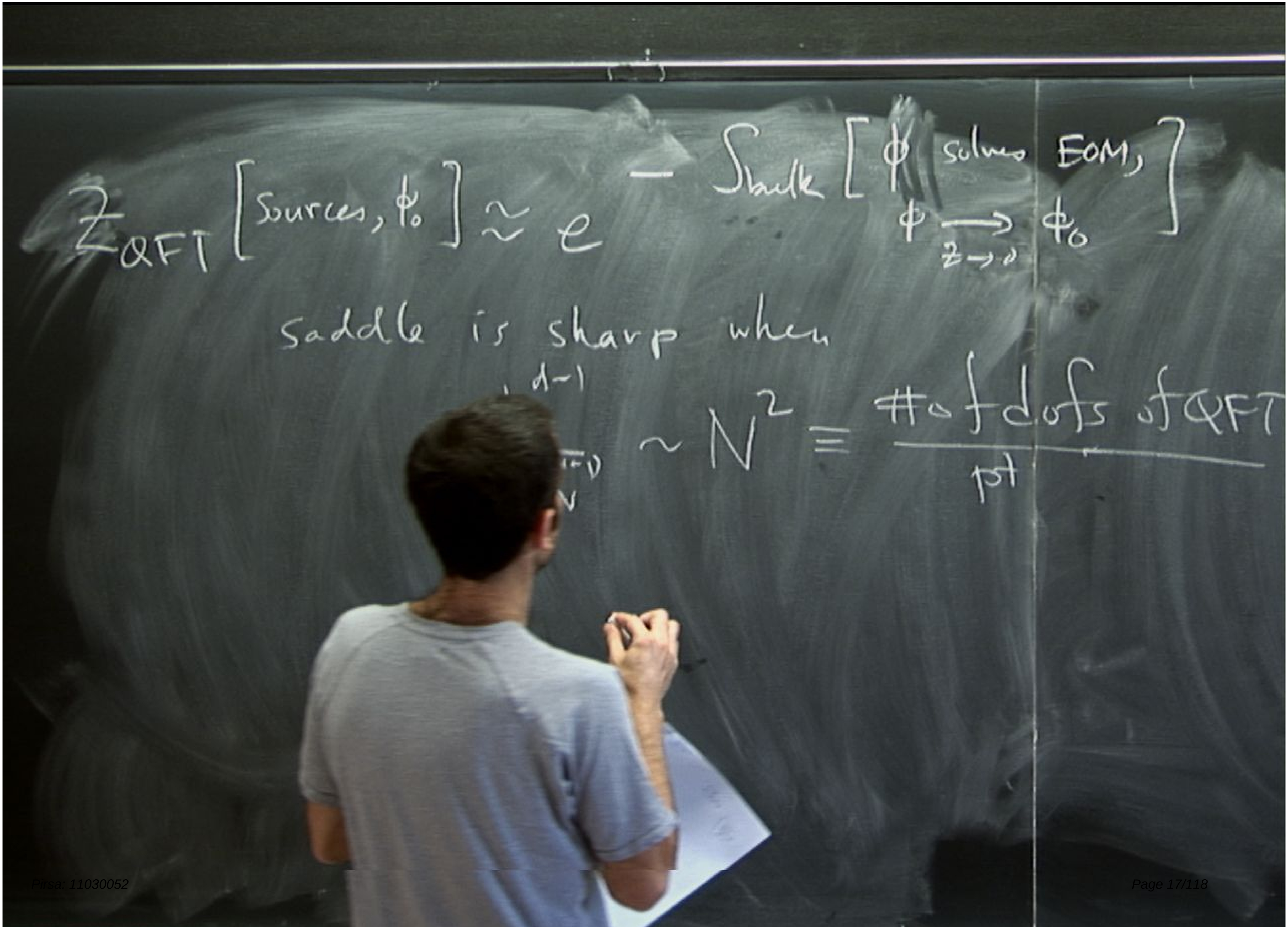
for Atoms
Molecules
gravity in spacetimes$_{d+1}$
w/ timelike asymptotic boundaries
$\longleftrightarrow$ QFT$_d$
gravity in A. AdS$_{d+1}$
isometries of AdS
fields in bulk
symm.
(possibly-running)
couplings
a general QFT
= a UV fixed pt (CFT)
perturbed by relevant ops.


d+1
QFT_d
CFT_d
AdS
conformal symms.
(possibly-running) couplings
a general QFT
≡ a UV fixed pt. (CFT)
perturbed by relevant ops.


Z_{QFT} [Sources,


$Z_{QFT}\left[\text{Sources}, \phi_0\right] \approx e^{-S_{bulk}\left[\phi \text{ solves EOM, } \phi \xrightarrow[z\to 0]{} \phi_0\right]}$


Z_{QFT} [Sources, ϕ_0] $\approx e^{-S_{bulk}}$ [ϕ solves EOM, $\phi \xrightarrow[z\to 0]{} \phi_0$]
saddle is sharp when
L^{d-1}
G_N
= # of dofs


Z_QFT [sources, φ_0] ≈ e^{− S_bulk [φ solves EOM, φ → φ_0 as z → 0]}
saddle is sharp when
d−1
~ N^2 ≡ # of dofs of QFT / pt

$$Z_{QFT}[\text{sources}, \phi_0] \simeq e^{-S_{bulk}\left[\phi \text{ solves EOM}, \ \phi \xrightarrow[z \to 0]{} \phi_0\right]}$$

saddle is sharp when

$$1 \ll \frac{L^{d-1}}{G_N^{(d+1)}} \sim N^2 \equiv \frac{\#\text{ of dofs of QFT}}{\text{pt}}$$

counting dofs.

$$\propto S_{bulk}$$

$$Z_{QFT}\left[\text{sources}, \phi_0\right] \approx e^{-S_{bulk}\left[\begin{array}{l}\phi \text{ solves EOM,} \\ \phi \xrightarrow[z\to 0]{} \phi_0\end{array}\right]}$$

saddle is sharp when

$$1 \ll \frac{L^{d-1}}{G_N^{(d+1)}} \sim N^2 \equiv \frac{\#\text{ of dofs of QFT}}{\text{pt}}$$

counting dofs.

$$\propto S_{bulk}$$

$$\int dx\, e^{-N^2 f(x)} \underset{N^2 \gg 1}{\sim} e^{-N^2 f(x_0)}\Big|_{f'(x_0)=0}$$

EOM,]
ϕ_0

Comments about N^2

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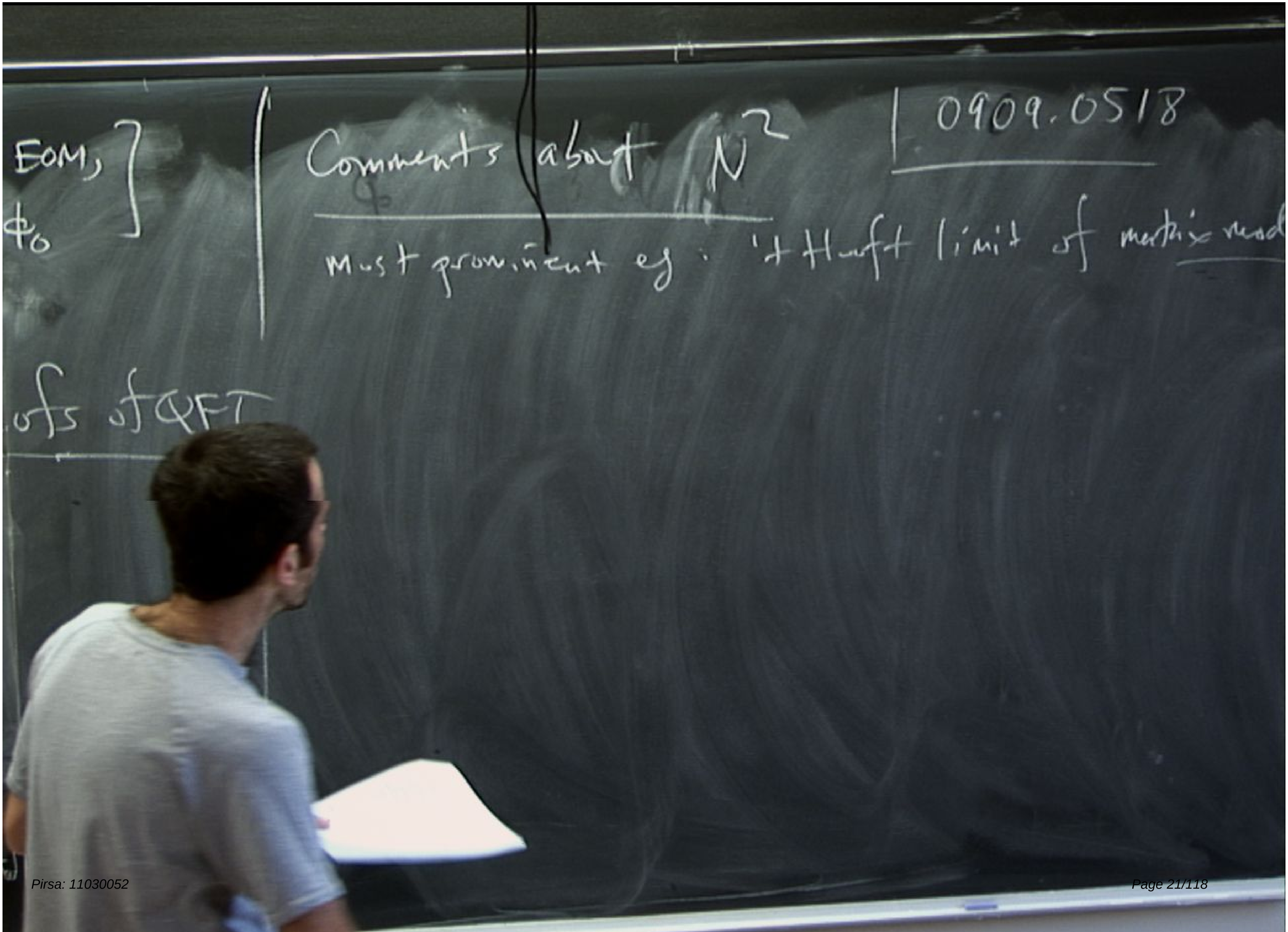
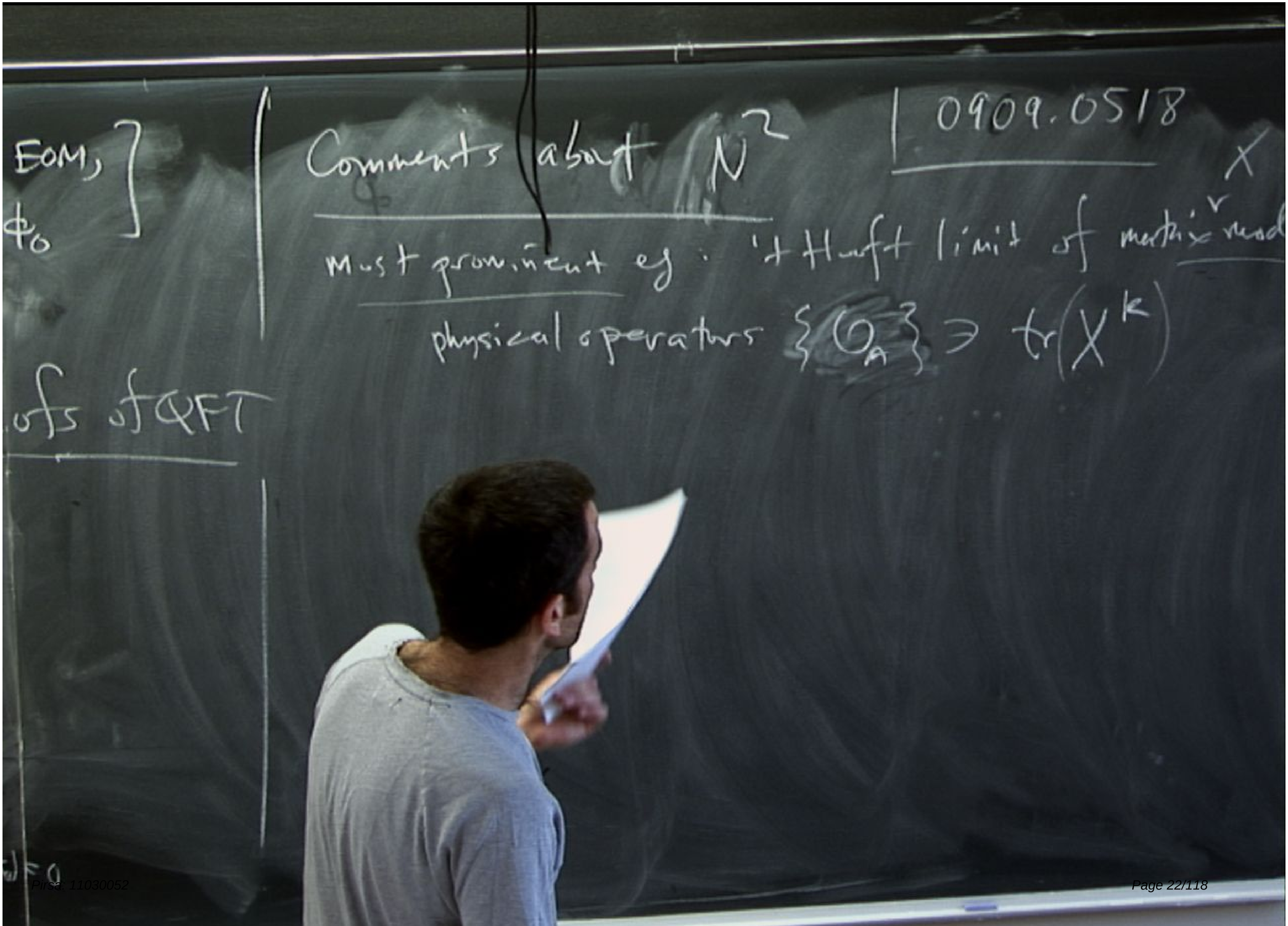
..ofs of QFT

$\neq 0$


EOM,
ϕ_0
Comments about N^2
0909.0518
Most prominent eg: 't Hooft limit of matrix mod
ofs of QFT


EOM,
Comments about N²
0909.0518
Must prominent eg: 't Hooft limit of matrix model
physical operators {O_A} ∋ tr(X^k)
ofs of QFT

RFT

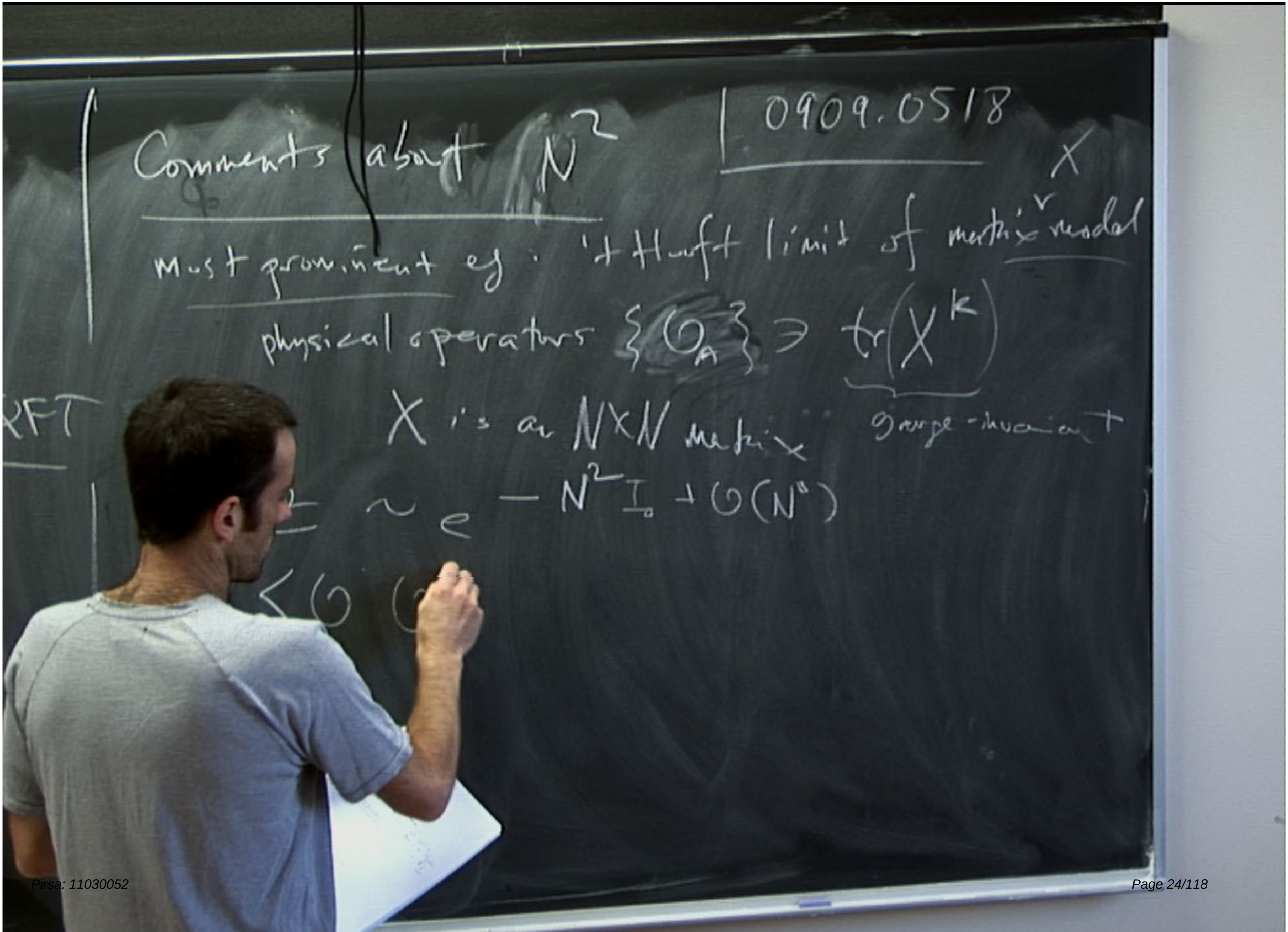
Comments about N^2

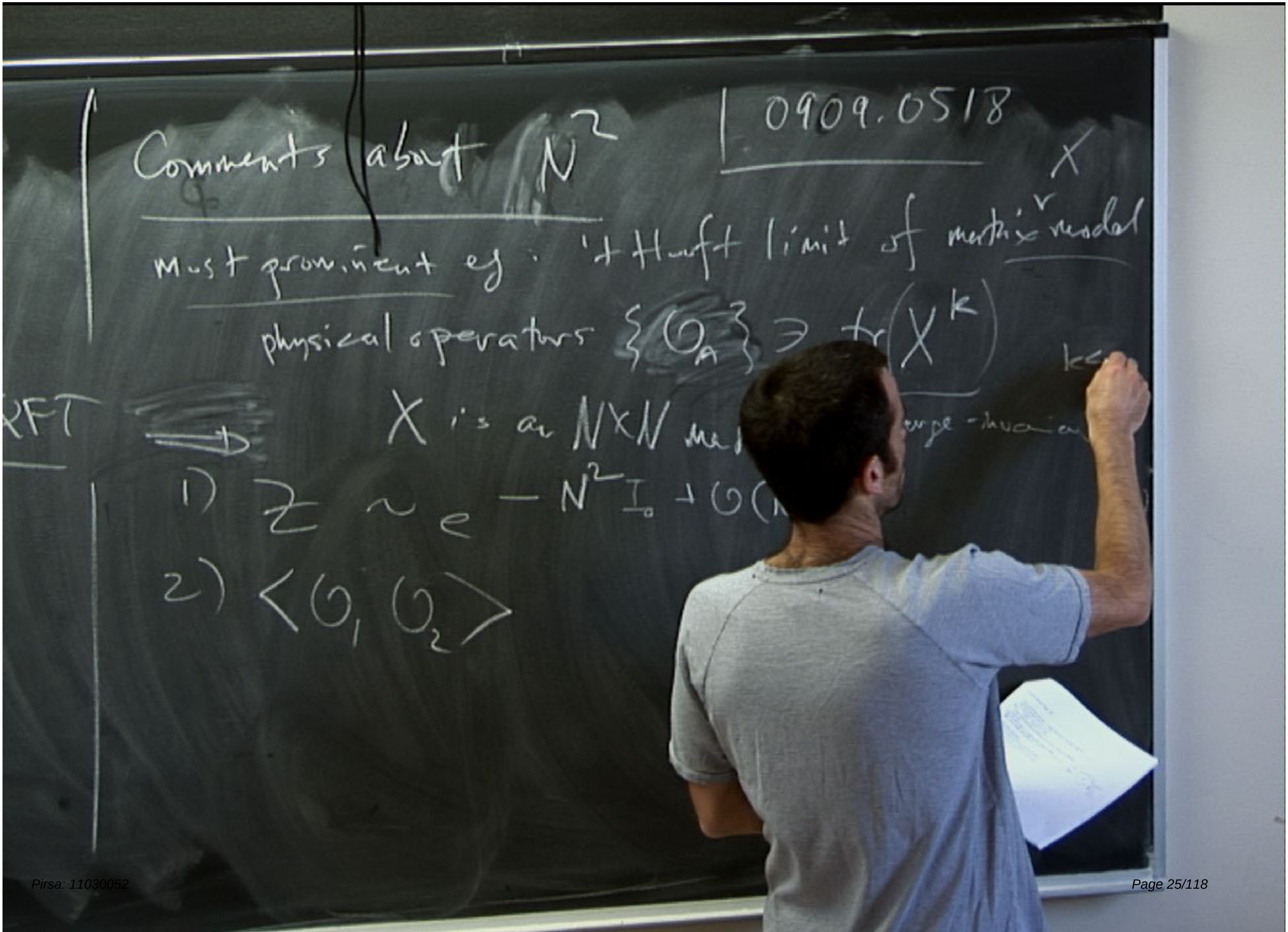
0909.0518

X

Most prominent eg: 't Hooft limit of matrix model

physical operators $\{\mathcal{O}_A\} \ni \underbrace{\mathrm{tr}(X^k)}_{\text{gauge-invariant}}$


Comments about N²
0909.0518
Most prominent eg: 't Hooft limit of matrix model
physical operators {O_A} ∋ tr(X^k)
gauge-invariant
X is an N×N matrix
~ e^(−N² I₀ + O(N⁰))



Comments about N²
0909.0518
Most prominent eg: 't Hooft limit of matrix model
physical operators {O_A} ∋ tr(X^k)
X is an N×N
1) Z ~ e^{−N² I₀ + O(
2) ⟨O₁ O₂⟩

0909.0518

Comments about N^2

X

Most prominent eg: 't Hooft limit of matrix model

physical operators $\{\mathcal{O}_A\} \ni \underbrace{\mathrm{tr}(X^k)}_{\text{gauge-invariant}}$ $k \ll N$.

X is an $N \times N$ matrix

$$Z \sim e^{-N^2 I_0 + \mathcal{O}(N^0)}$$

$$\langle \mathcal{O}_1 \mathcal{O}_2 \rangle \sim$$

QFT

Comments about N^2

0909.0518

Most prominent eg: 't Hooft limit of matrix model

physical operators $\{\mathcal{O}_A\} \ni \underbrace{\mathrm{tr}(X^k)}_{\text{gauge-invar...}}$ $k \ll N$.

X is an $N \times N$ matrix

1) $Z \sim e^{-N^2 I_0 + \mathcal{O}(N^0)}$

2) $\langle \mathcal{O}_1 \mathcal{O}_2 \rangle \sim \langle \mathcal{O}_1 \rangle \langle \mathcal{O}_2 \rangle + \mathcal{O}($

"large-N factoriz

Comments about N^2

0909.0518

Most prominent eg: 't Hooft limit of matrix model X

physical operators $\{\mathcal{O}_A\} \ni \underbrace{\mathrm{tr}(X^k)}_{\text{gauge-invariant}}$ $\quad k \ll N.$

X is an $N \times N$ matrix

1) $Z \sim e^{-N^2 I_0 + \mathcal{O}(N^0)}$

2) $\langle \mathcal{O}_1 \mathcal{O}_2 \rangle \simeq \langle \mathcal{O}_1 \rangle \langle \mathcal{O}_2 \rangle + \mathcal{O}\left(\frac{1}{N^2}\right)$

"large-N factorization"

Comments about N^2

| 0909.0518

Most prominent eg: 't Hooft limit of matrix model

physical operators $\{\mathcal{O}_A\} \ni \underbrace{\mathrm{tr}(X^k)}_{\text{gauge-invariant}}$ $k \ll N$.

X is an $N \times N$ matrix

1) $Z \sim e^{-N^2 I_0 + \mathcal{O}(N^0)}$

2) $\langle \mathcal{O}_1 \mathcal{O}_2 \rangle \simeq \langle \mathcal{O}_1 \rangle \langle \mathcal{O}_2 \rangle + \mathcal{O}\left(\frac{1}{N^2}\right)$

"large-N factorization"

Comments about N^2

0909.0518

Most prominent eg: 't Hooft limit of matrix model

physical operators $\{\mathcal{O}_A\} \ni \underbrace{\mathrm{tr}(X^k)}_{\text{gauge-invariant}}$ $\quad k \ll N$

QFT $\Longrightarrow$ X is an $N \times N$ matrix

1) $Z \sim e^{-N^2 I_0 + \mathcal{O}(N^0)}$

2) $\langle \mathcal{O}_1 \mathcal{O}_2 \rangle \simeq \langle \mathcal{O}_1 \rangle \langle \mathcal{O}_2 \rangle + \mathcal{O}\left(\frac{1}{N^2}\right)$ "large-N factorization"

$\Longrightarrow$ something is classical.

Comments about N^2

| 0909.0518

Most prominent eg: 't Hooft limit of matrix model X

physical operators $\{\mathcal{O}_A\} \ni \underbrace{\mathrm{tr}(X^k)}_{\text{gauge-invariant}}$ $k \ll N.$

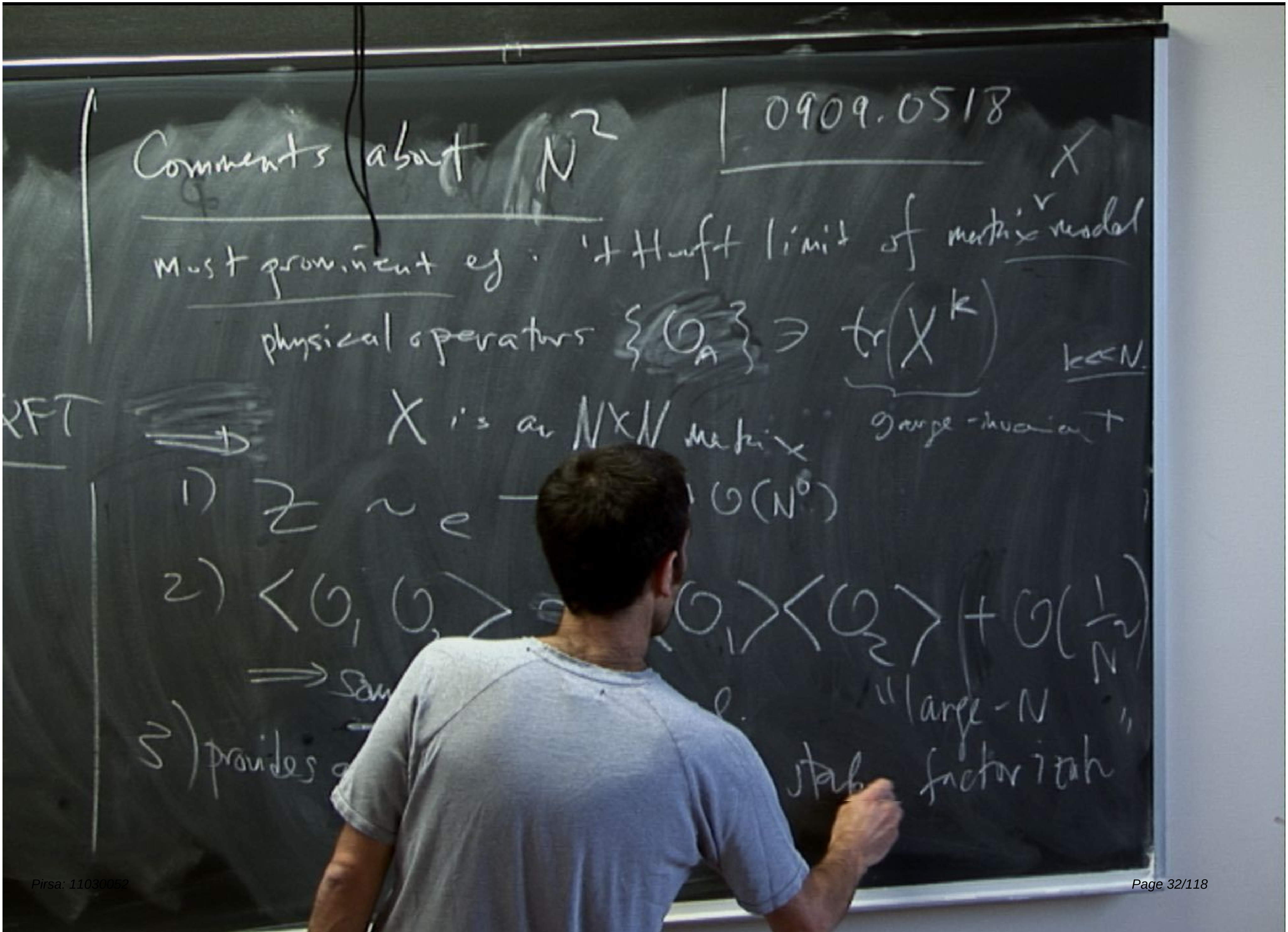
RFT $\Longrightarrow$ X is an $N \times N$ matrix

1) $Z \sim e^{-N^2 I_0 + \mathcal{O}(N^0)}$

2) $\langle \mathcal{O}_1 \mathcal{O}_2 \rangle \simeq \langle \mathcal{O}_1 \rangle \langle \mathcal{O}_2 \rangle + \mathcal{O}\left(\frac{1}{N^2}\right)$

$\Longrightarrow$ something is classical. "large-N factorization"

3)


Comments about N^2
0909.0518
Most prominent eg: 't Hooft limit of matrix model
physical operators $\{\mathcal{O}_A\} \ni \mathrm{tr}(X^k)$
$k \ll N$
gauge-invariant
X is an N×N matrix
1) $Z \sim e$ ... $\mathcal{O}(N^0)$
2) $\langle \mathcal{O}_1 \mathcal{O}_2 \rangle$ ... $\langle \mathcal{O}_1 \rangle \langle \mathcal{O}_2 \rangle + \mathcal{O}(\frac{1}{N^2})$
"large-N"
3) provides ... factorization

RFT

Comments about N^2

0909.0518

Most prominent eg: 't Hooft limit of matrix model

physical operators $\{\mathcal{O}_A\} \ni \underbrace{\mathrm{tr}\left(X^k\right)}_{\text{gauge-invariant}}$ $\quad k \ll N$.

X is an $N \times N$ matrix

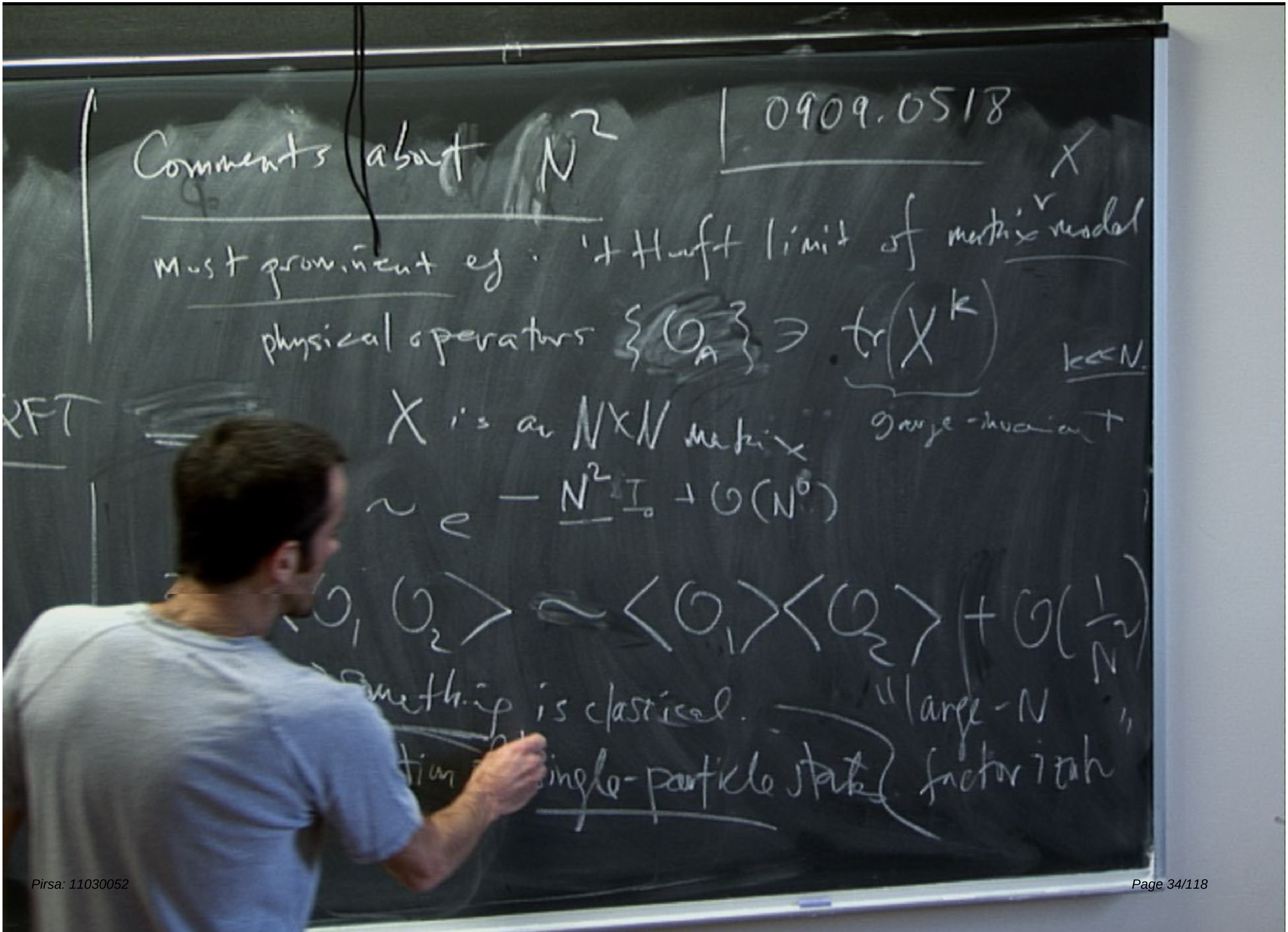
$\Longrightarrow$

1) $Z \sim e^{-N^2 I_0 + \mathcal{O}(N^0)}$

2) $\langle \mathcal{O}_1 \mathcal{O}_2 \rangle \sim \langle \mathcal{O}_1 \rangle \langle \mathcal{O}_2 \rangle + \mathcal{O}\left(\frac{1}{N^2}\right)$

$\Longrightarrow$ something is classical. "large-N factorization"

3) provides a notion of single-particle states


Comments about N^2
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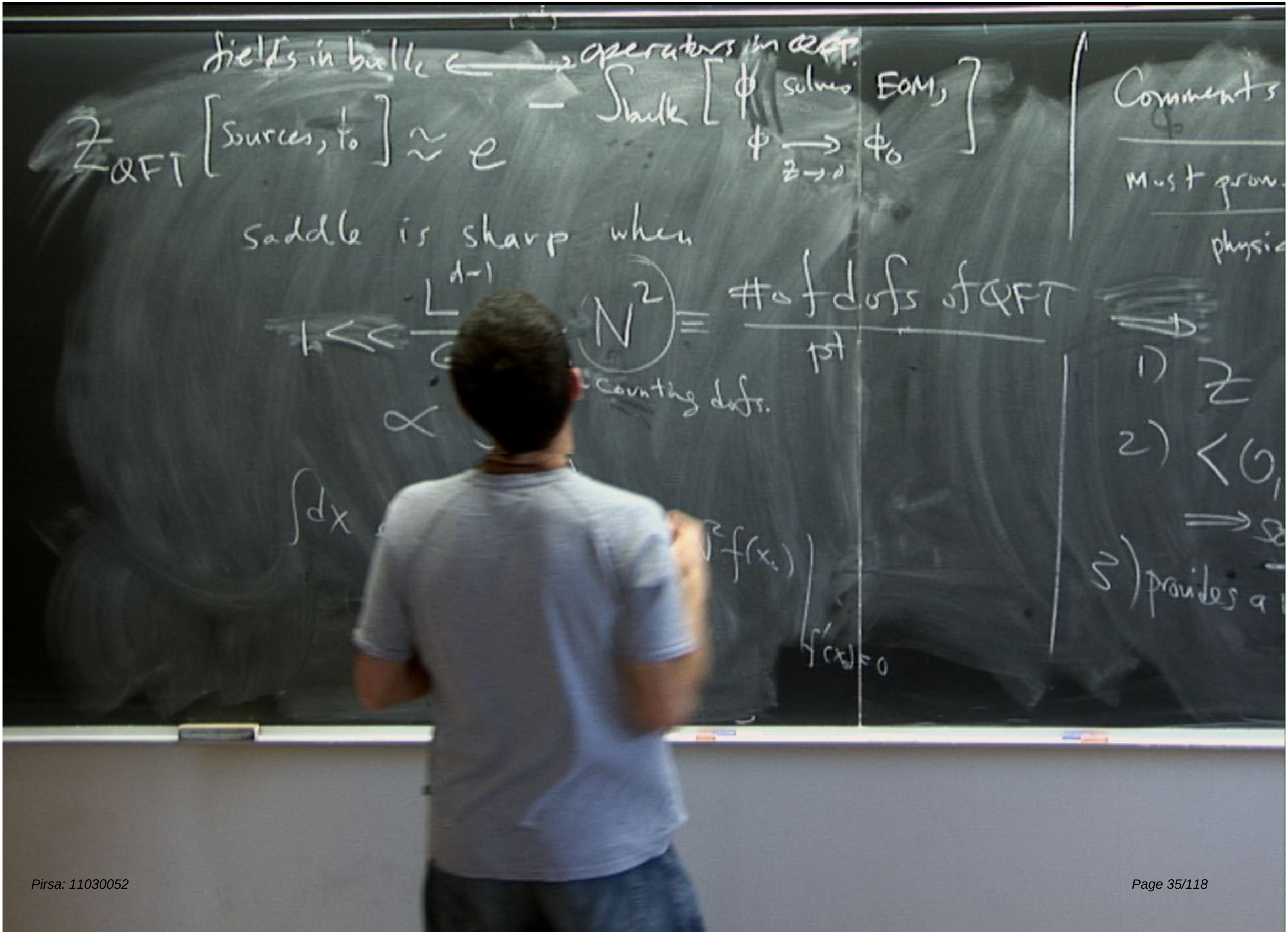
Most prominent eg: 't Hooft limit of matrix model
physical operators {O_A} ∋ tr(X^k)
k<<N.
X is an N×N matrix
gauge-invariant
~ e^{-N^2 I_0 + O(N^0)}
<O_1 O_2> ~ <O_1><O_2> + O(1/N^2)
something is classical.
"large-N factorization"
single-particle states


fields in bulk ⟷ operators in QFT.
Z_QFT [sources, φ0] ≈ e − S_bulk [φ solves EOM, φ → φ0, z → 0]
saddle is sharp when
1 << L^{d-1} N^2 = # of dofs of QFT / pt
counting dofs.
∫dx
f(x_i)
f'(x_i) = 0
Comments
Must
1) Z
2)
3) provides a

fields in bulk $\longleftrightarrow$ operators in QFT.

$$Z_{QFT}\left[\text{sources}, \phi_0\right] \simeq e^{-S_{bulk}\left[\begin{array}{l}\phi \text{ solves EOM,} \\ \phi \xrightarrow[z\to 0]{} \phi_0\end{array}\right]}$$

saddle is sharp when

$$1 << \frac{L^{d-1}}{G_N^{(d+1)}} \sim N^2 = \frac{\#\text{ of dofs of QFT}}{\text{pt}}$$

counting dofs.

$$\propto S_{bulk}$$

$$\int dx\, e^{-N^2 f(x)} \underset{N^2>>1}{\sim} e^{-N^2 f(x_0)}\Big|_{f'(x_0)=0}$$

Comments

Must prov...

physic...

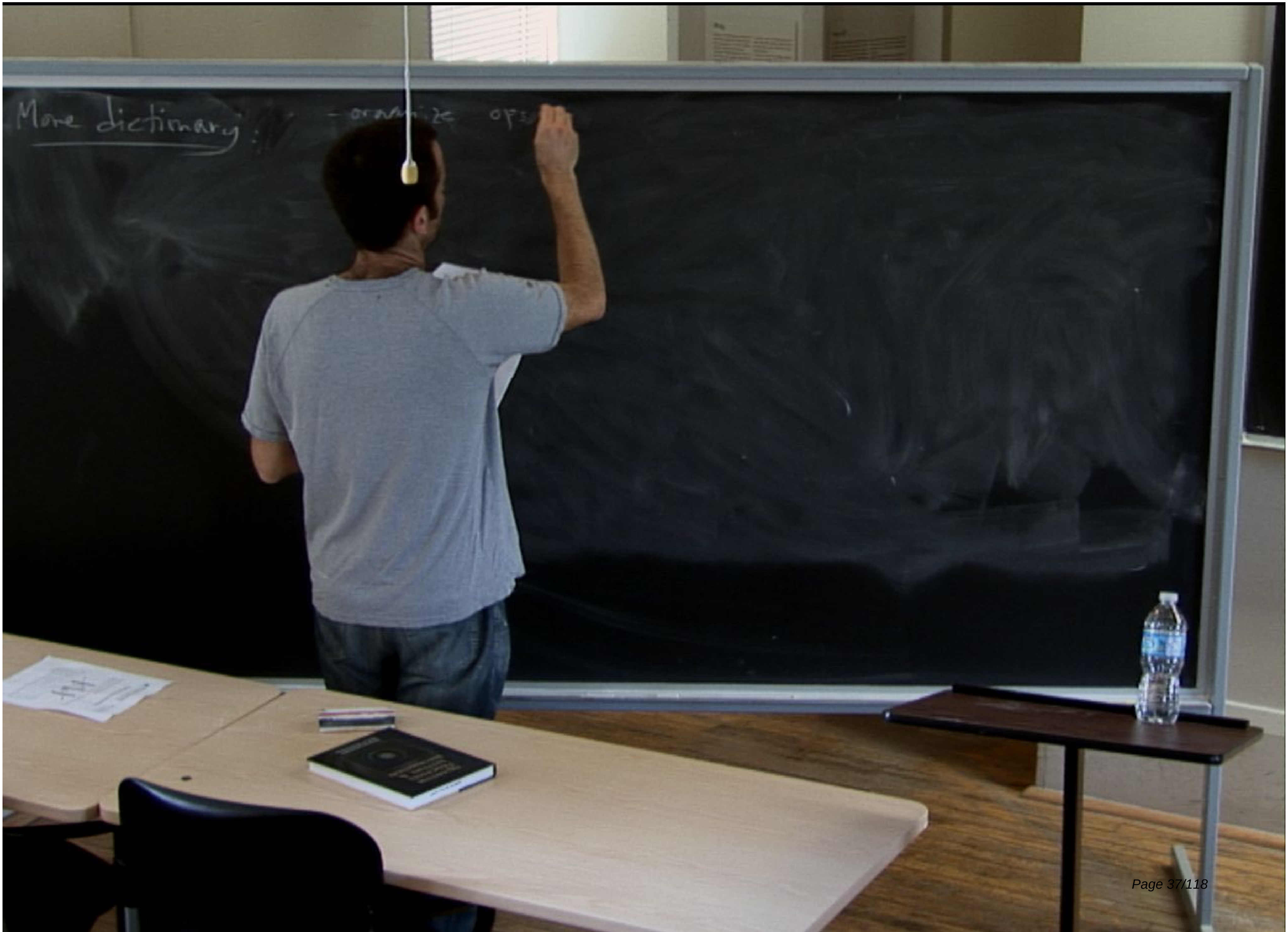
$\Longrightarrow$

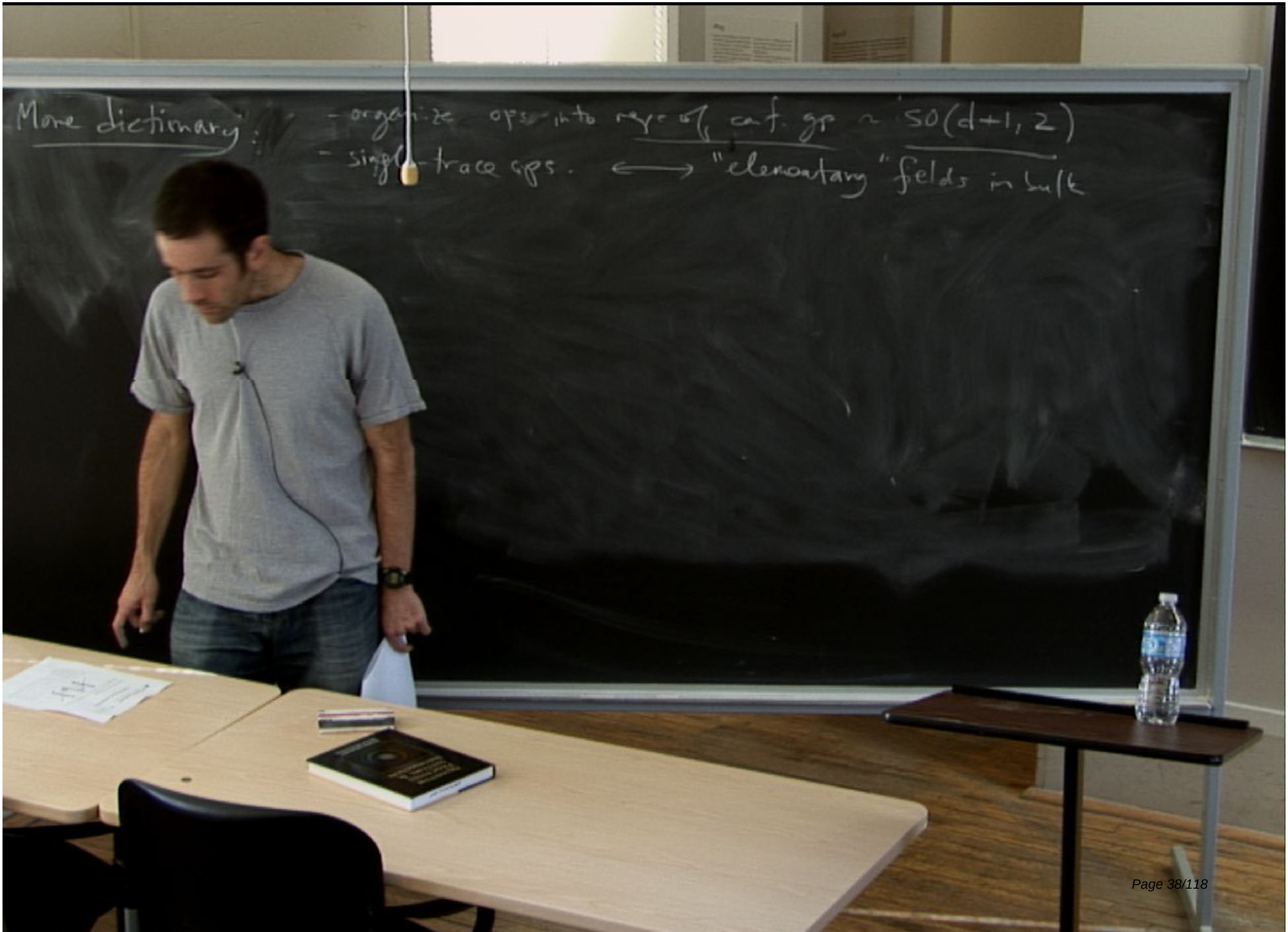
1) Z

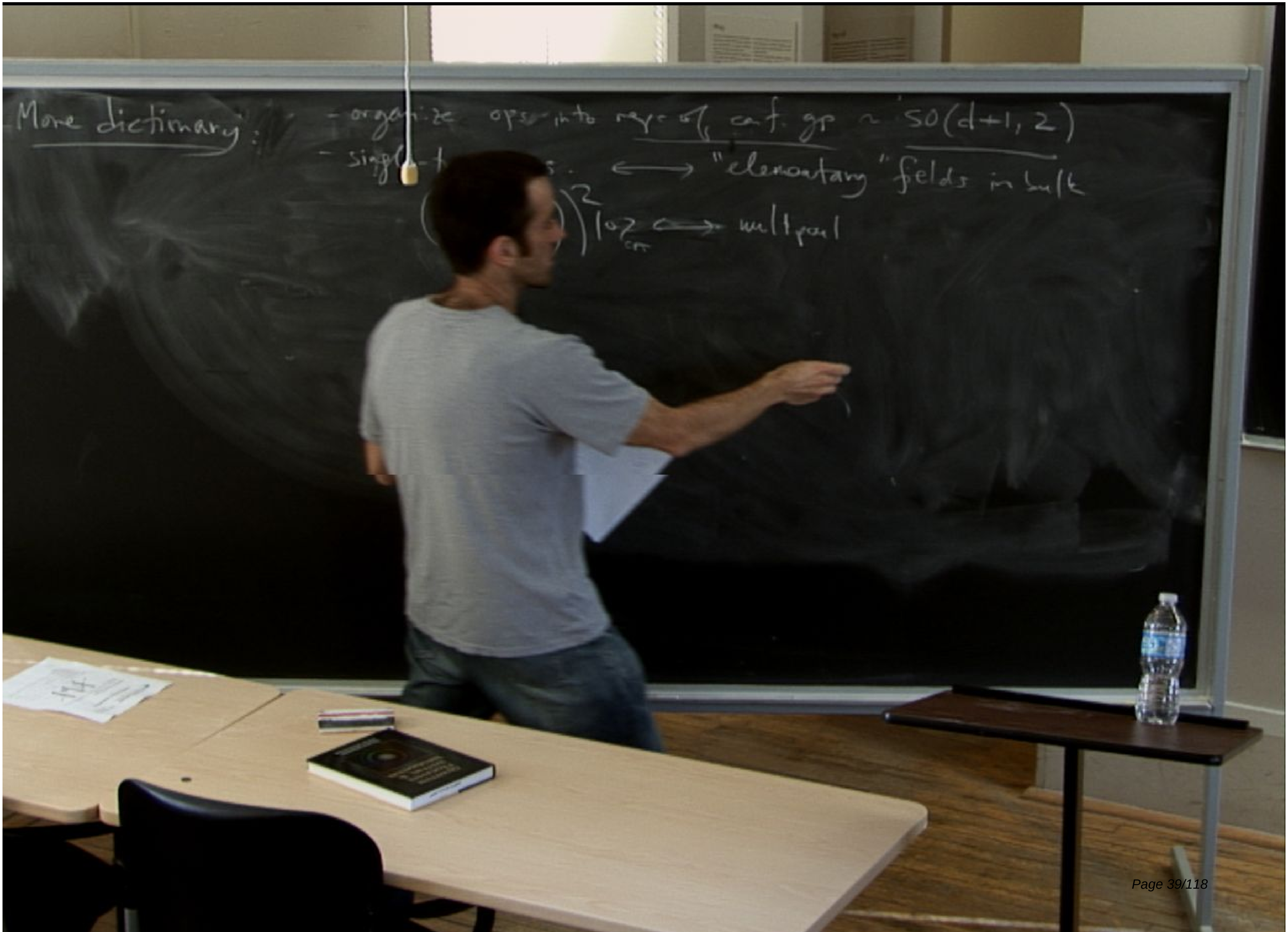
2) $\langle O_1$...

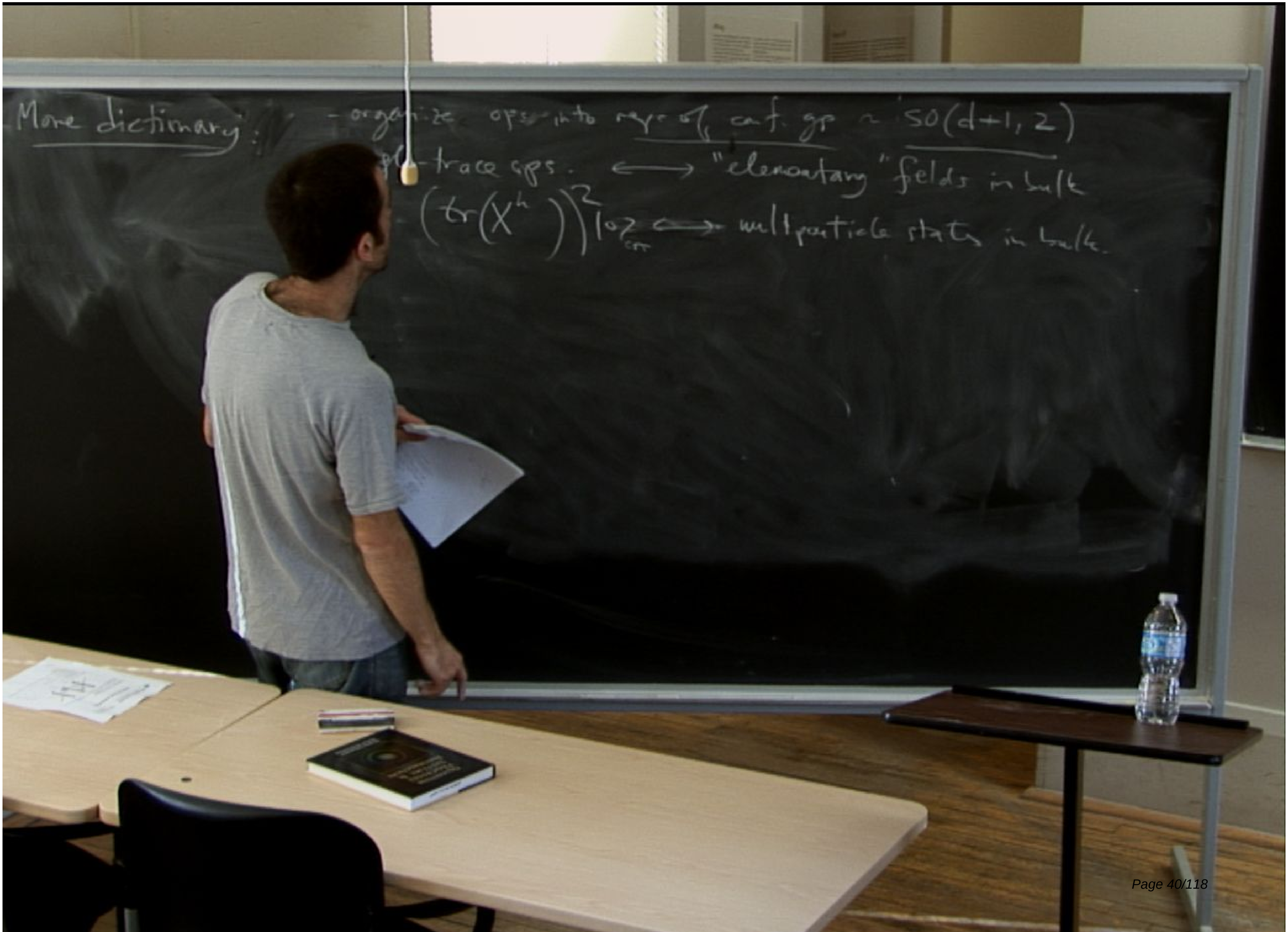
$\Longrightarrow$ ...

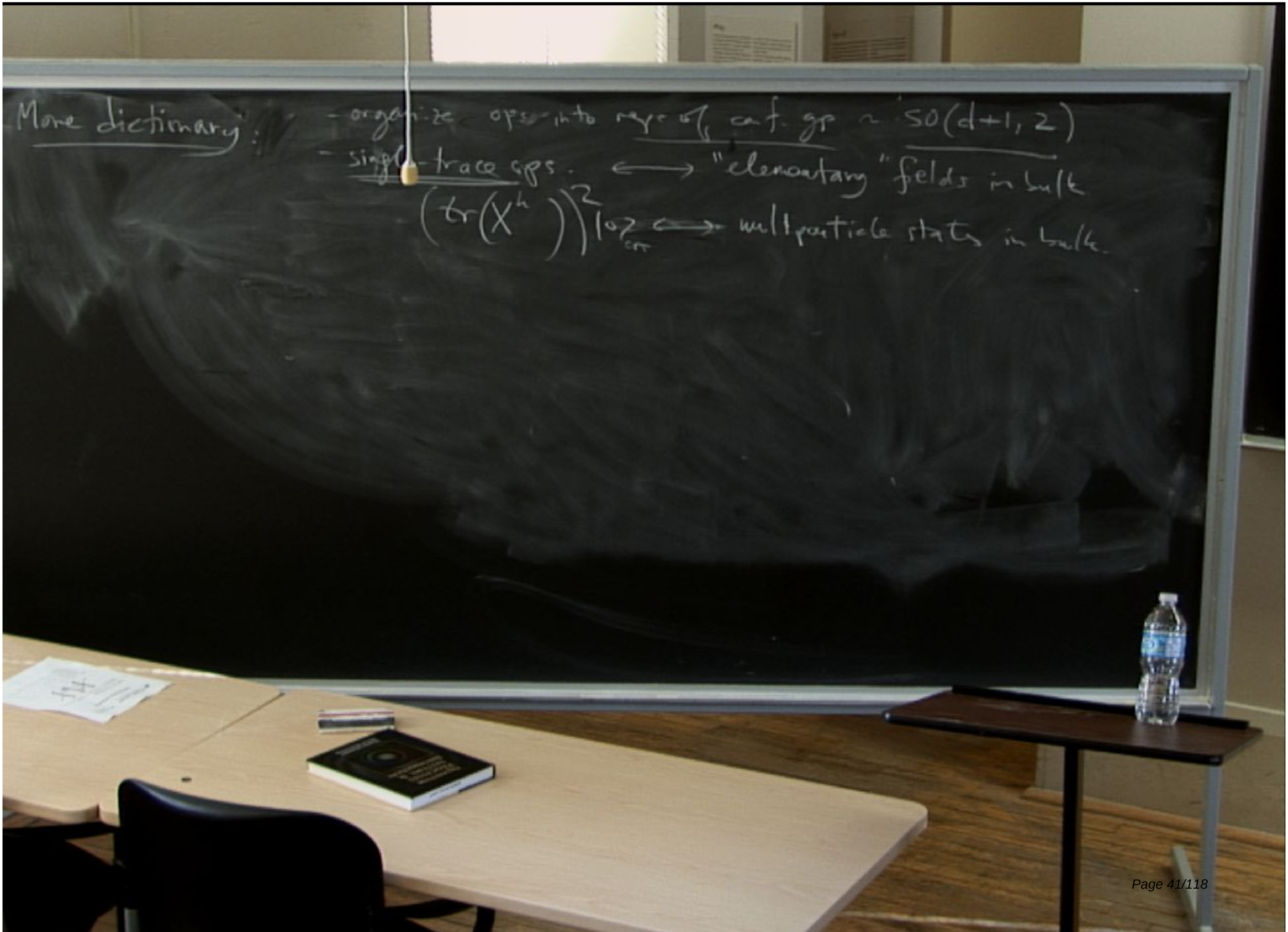
3) provides a ...


More dictionary:

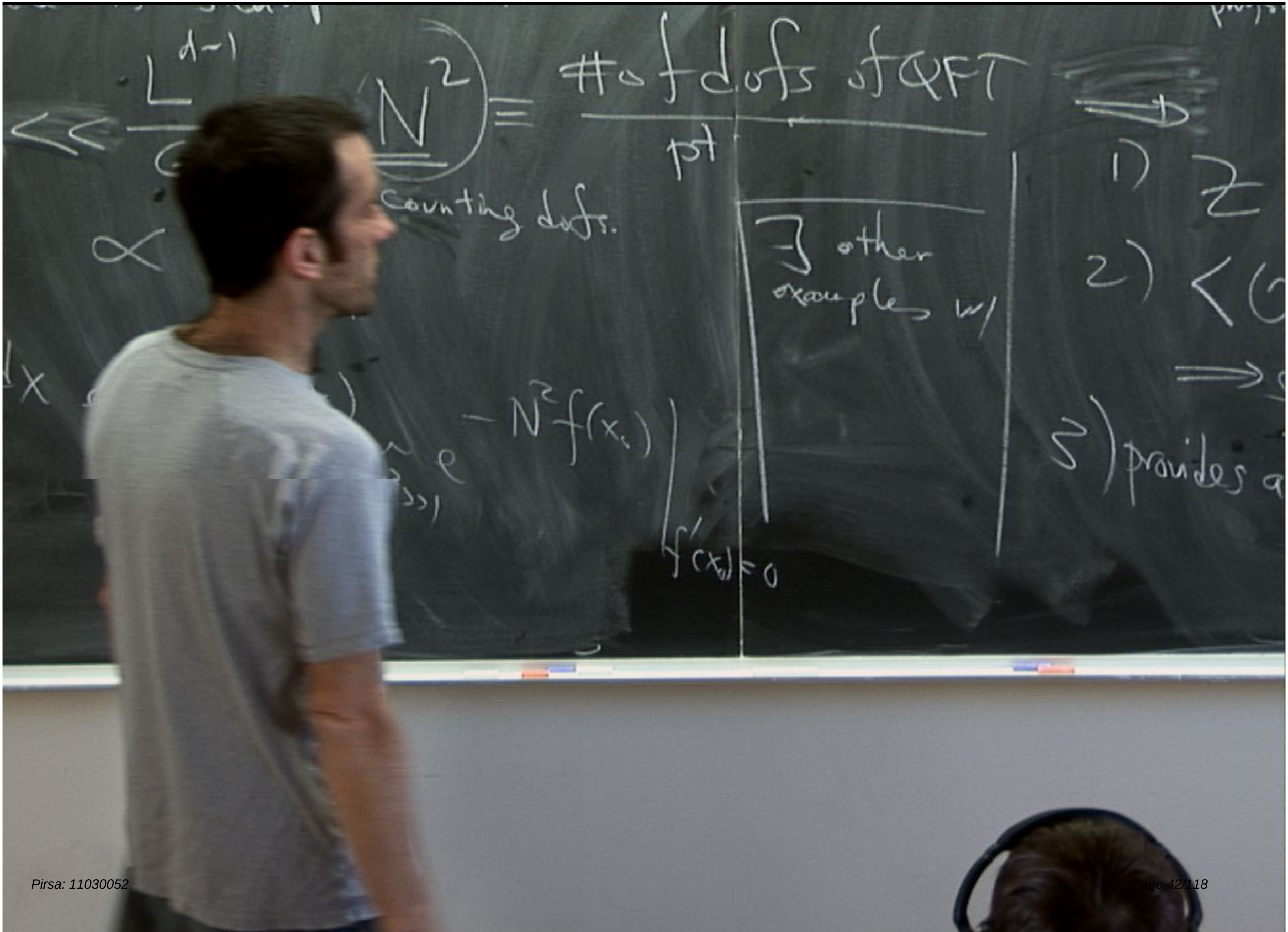

More dictionary:
- organize ops into reps of conf. gp ≈ SO(d+1, 2)
- single-trace ops. ⟷ "elementary" fields in bulk

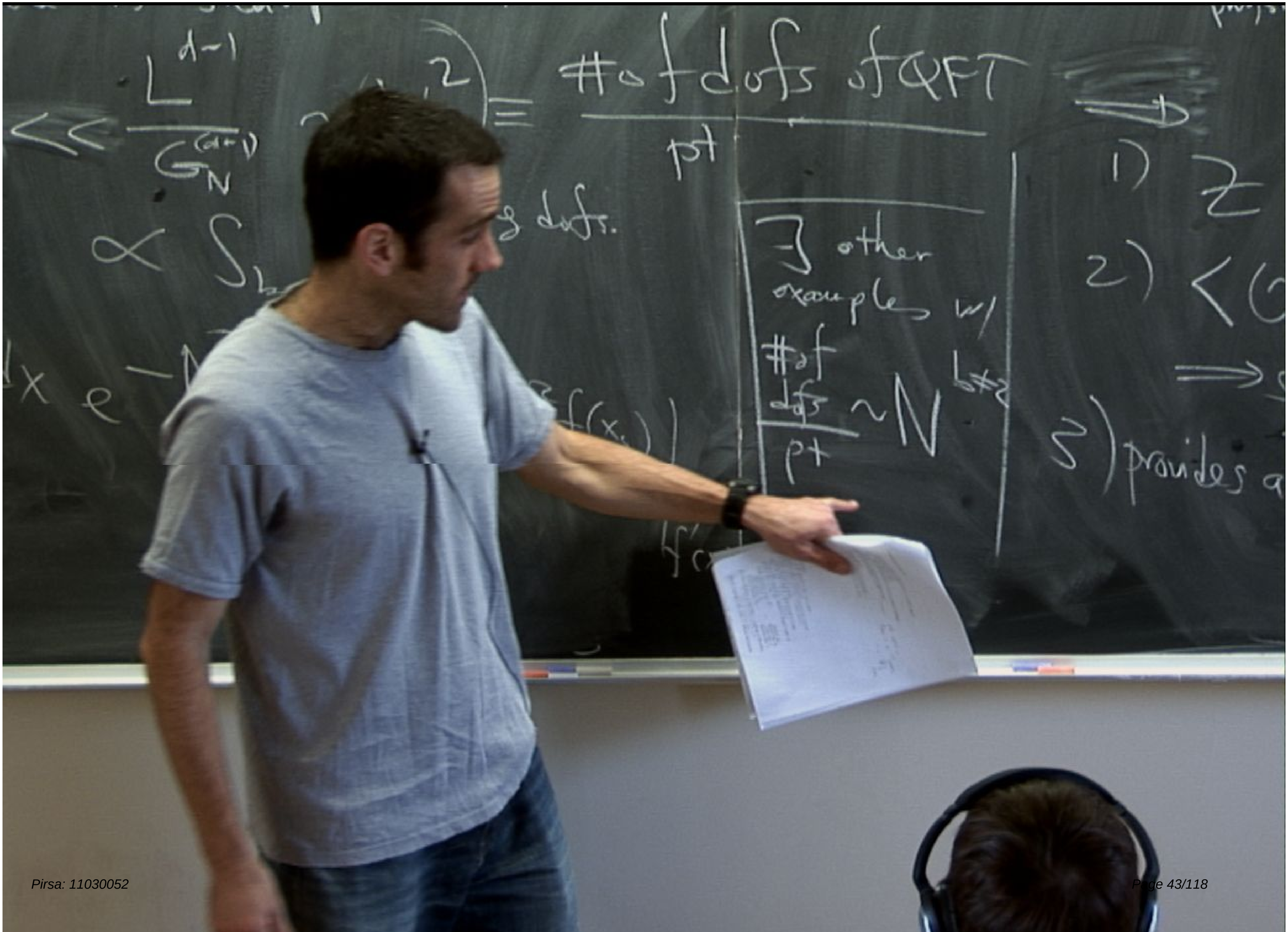

More dictionary:
- organize ops into reps of conf. gp ~ SO(d+1, 2)
"elementary" fields in bulk


More dictionary:
- organize ops into reps of conf. gp ≈ SO(d+1, 2)
trace ops. ⟷ "elementary" fields in bulk
⟷ multiparticle states in bulk.

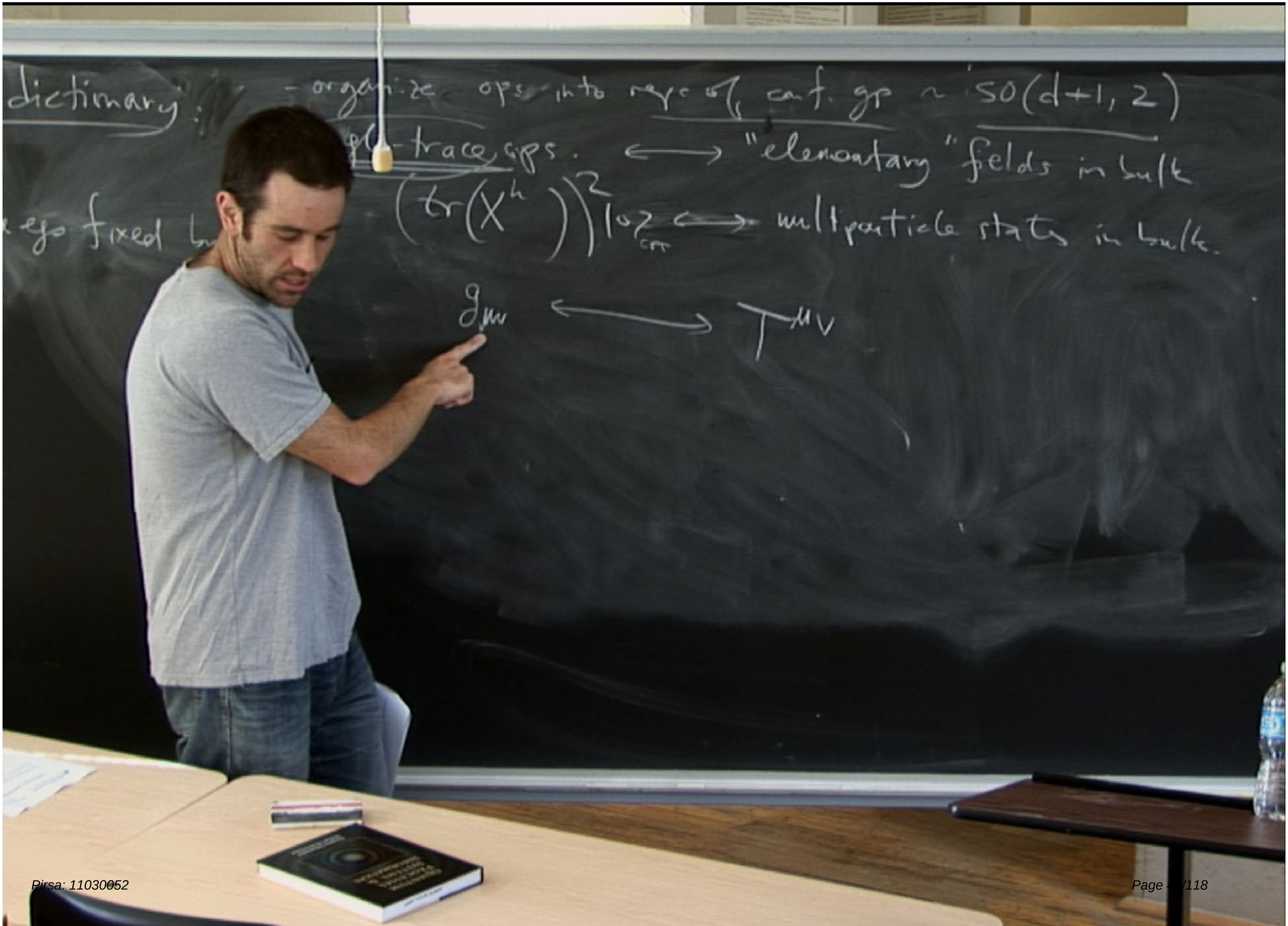


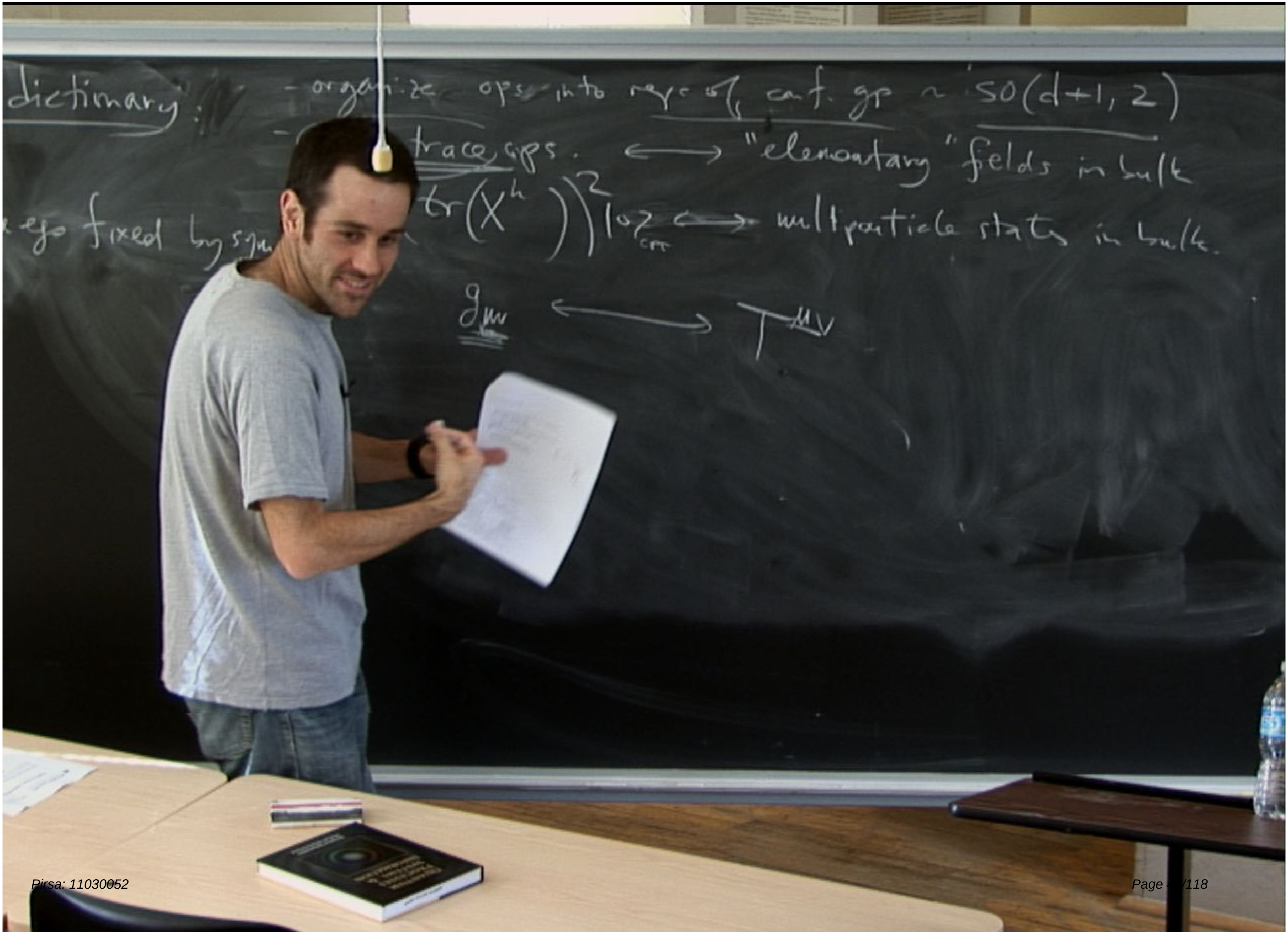
More dictionary:
- organize ops into reps of conf. gp ≃ SO(d+1, 2)
- single-trace ops. ⟷ "elementary" fields in bulk
(tr(X^n))^2 | op ⟷ multiparticle states in bulk.

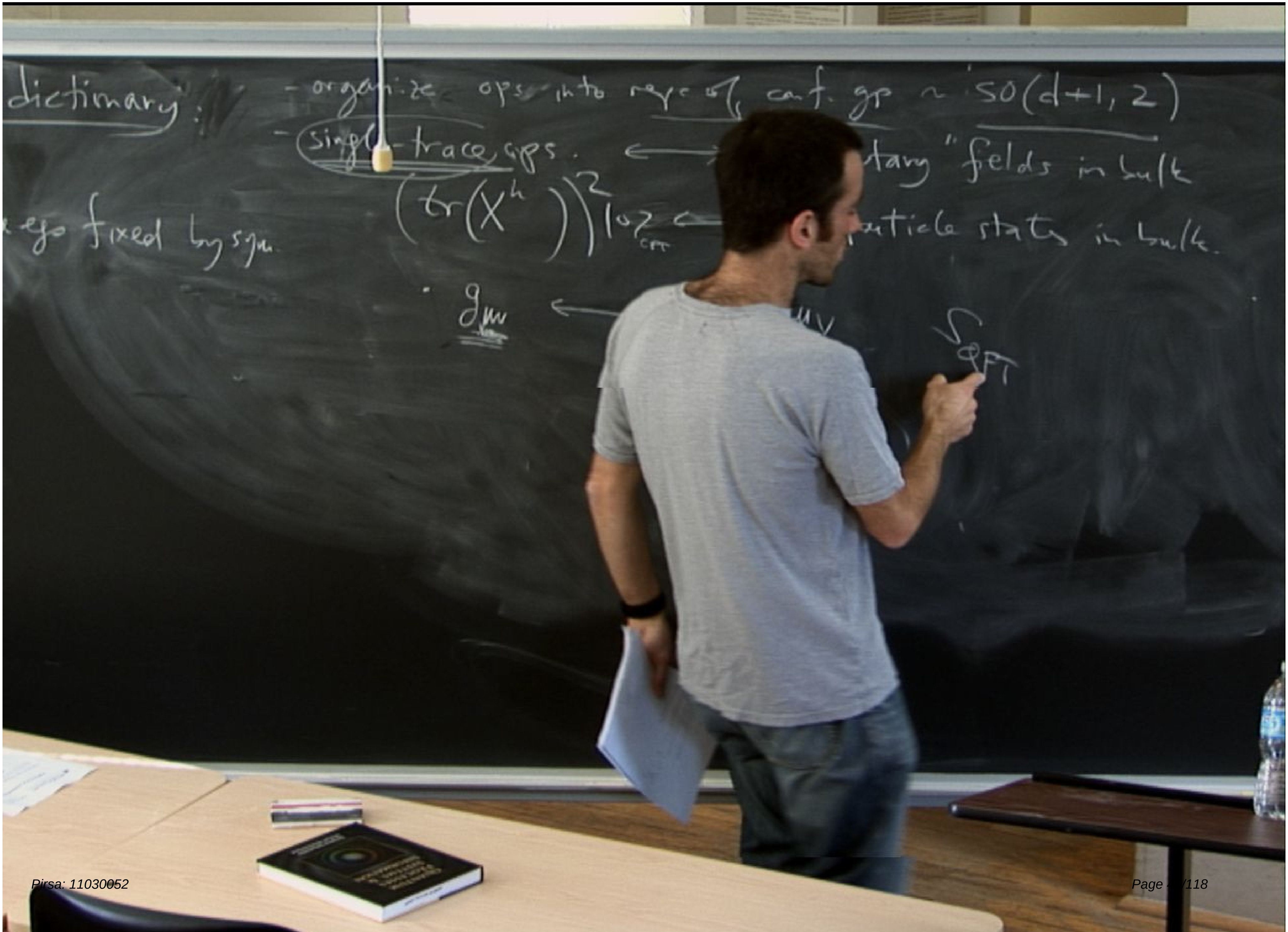

#of d.ofs of QFT
pt
counting d.ofs.
∃ other
examples w/
1)
2)
3) provides a

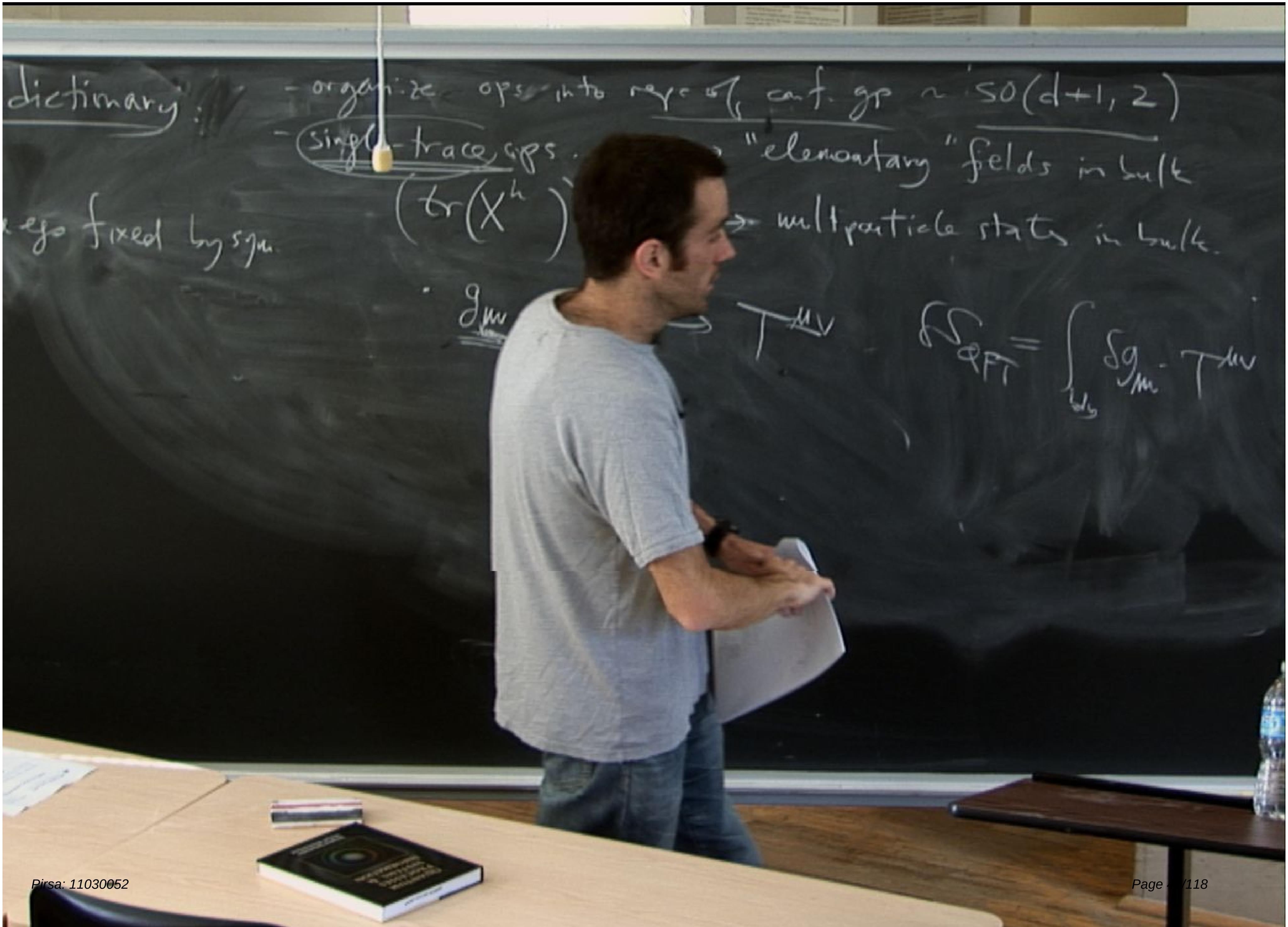


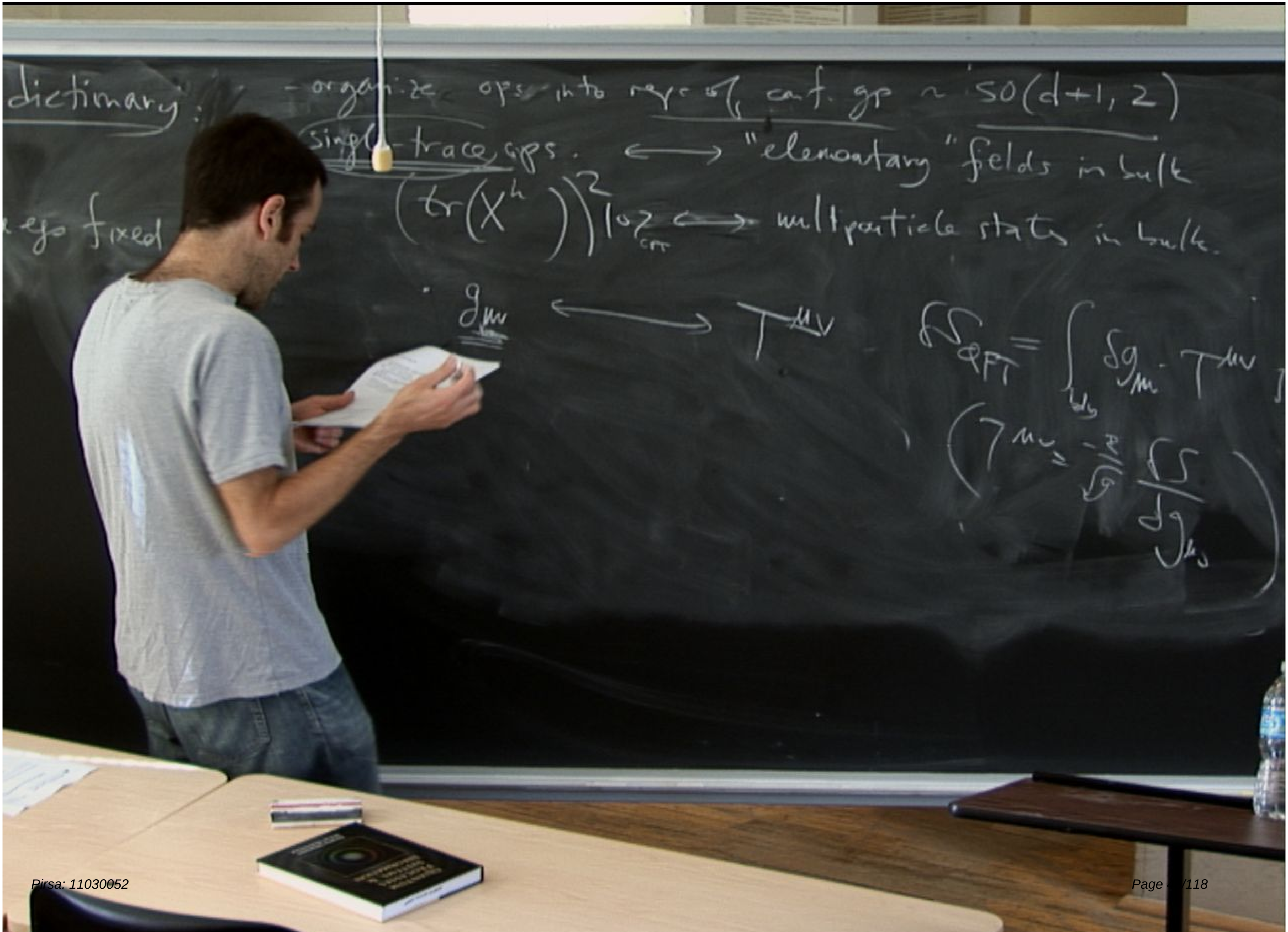
# of dofs of QFT
pt
∃ other
examples w/
of
dofs
pt
~ N
1)
2)
3) provides a

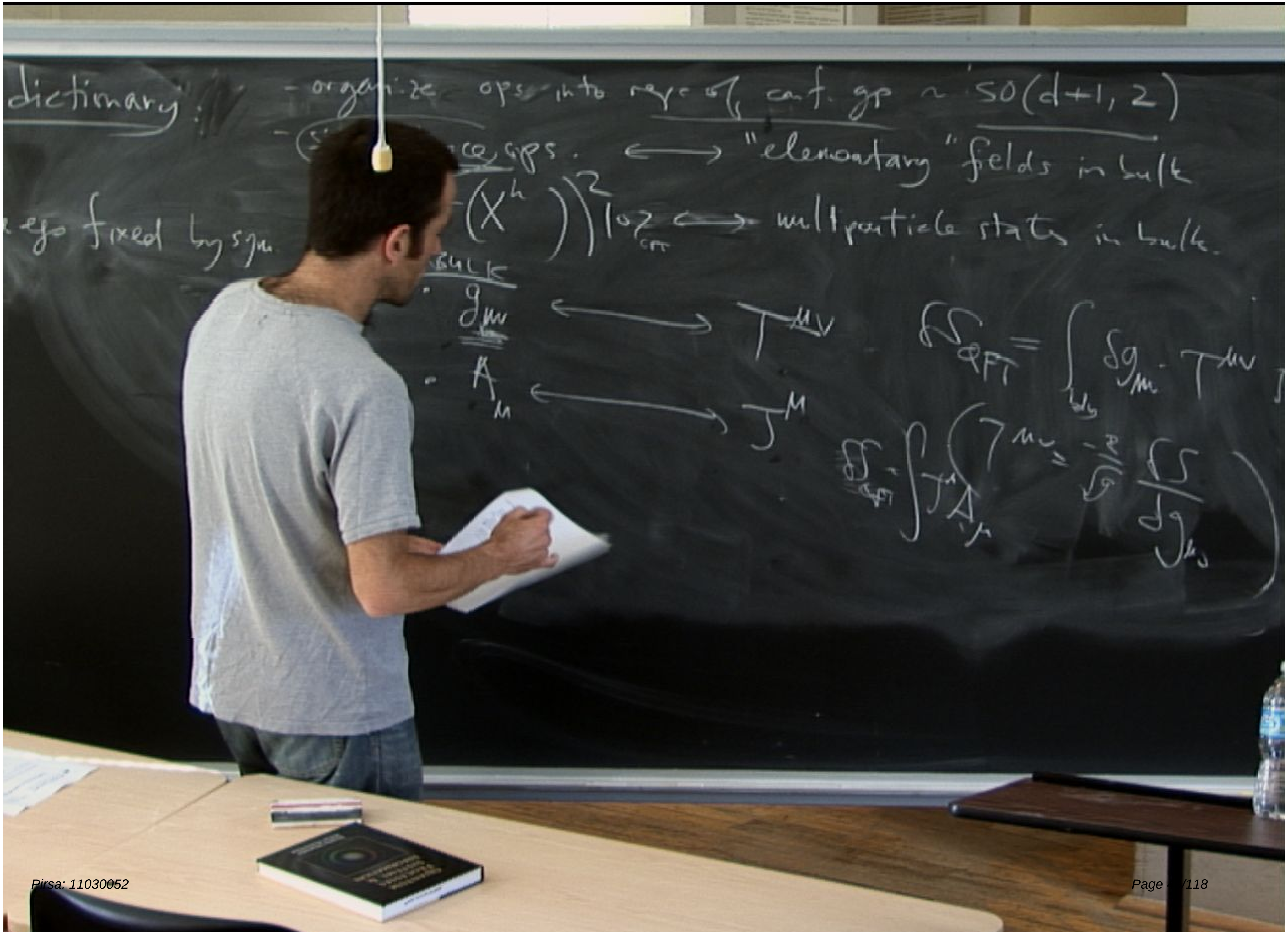

dictionary:
- organize ops into reps of conf. gp ~ SO(d+1, 2)
-trace ops. ⟷ "elementary" fields in bulk
(tr(X^n))^2 ⟷ multiparticle states in bulk.
g_μν ⟷ T^μν

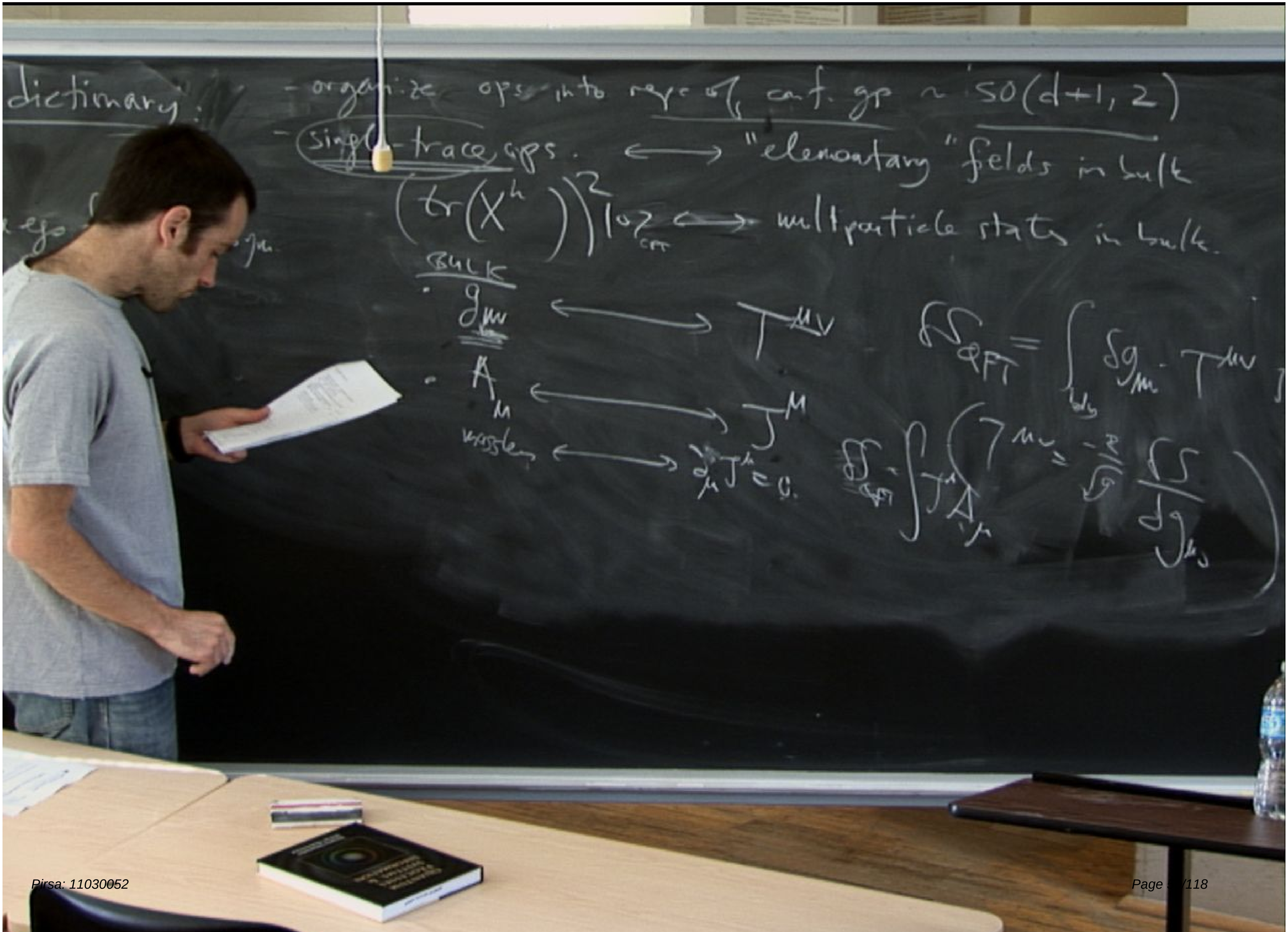

dictionary:
- organize ops into reps of conf. gp ~ SO(d+1, 2)
trace ops. ⟷ "elementary" fields in bulk
fixed by sym
tr(X^h))^2 |0>_CFT ⟷ multiparticle states in bulk
g_μν ⟷ T^μν

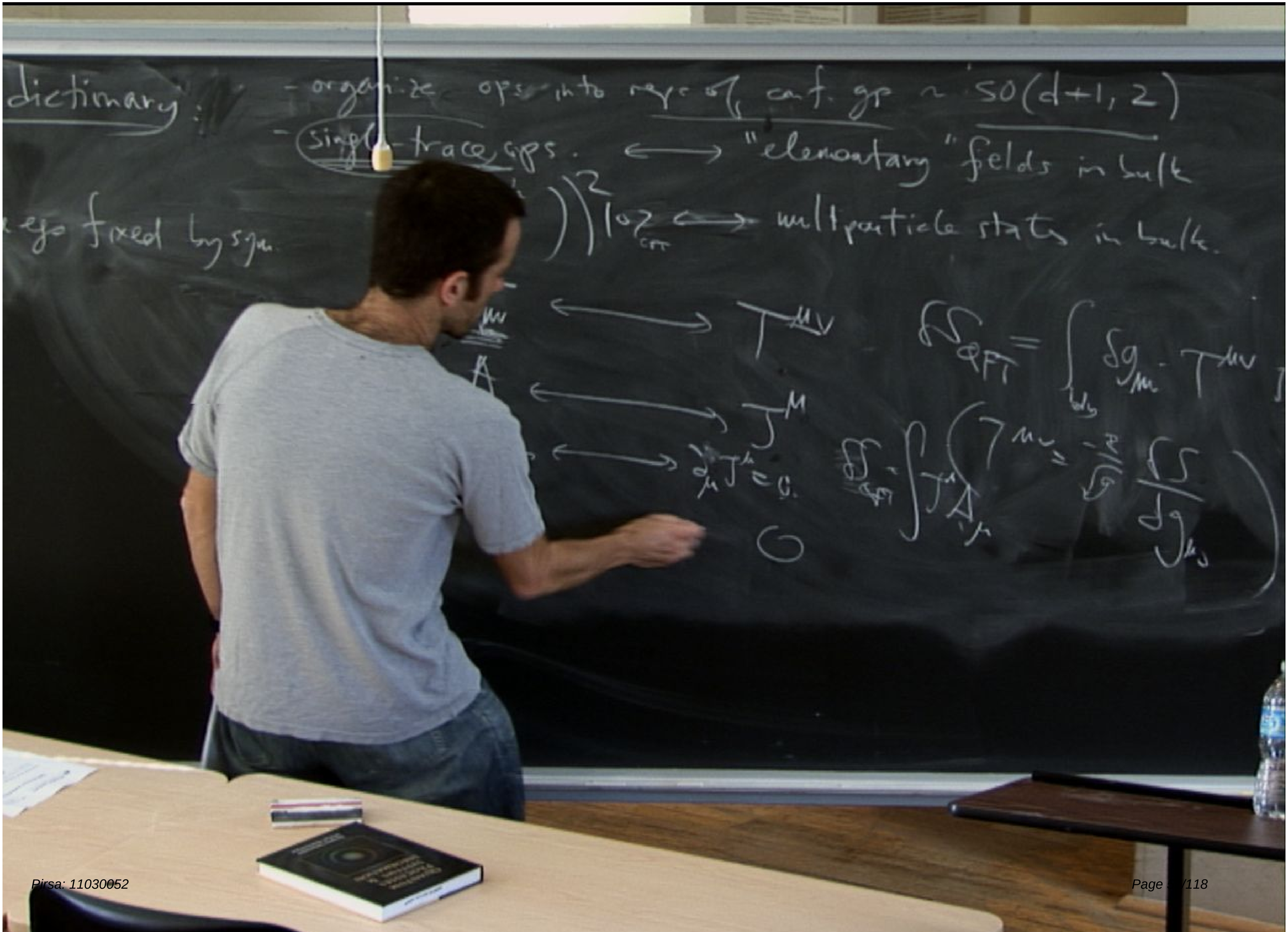

dictionary:
- organize ops into reps of conf. gp ~ SO(d+1, 2)
- single-trace ops.
"fields in bulk
fixed by sym.
particle states in bulk.

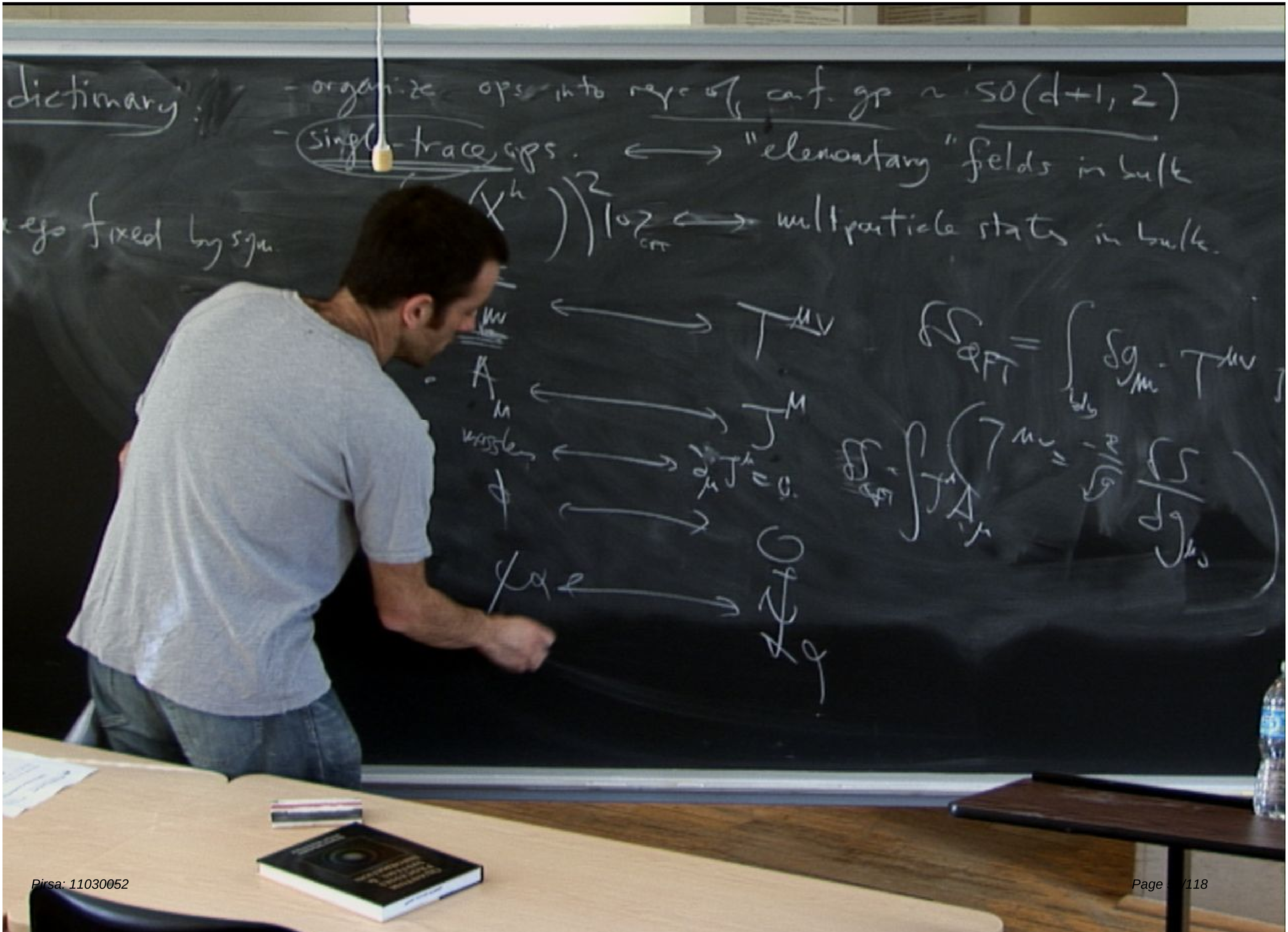

dictionary:
- organize ops into reps of conf. gp ~ SO(d+1, 2)
- single-trace ops
"elementary" fields in bulk
(tr(X^n))
⇒ multiparticle states in bulk
fixed by sym.
g_μν
T^μν
δS_QFT = ∫_bdy δg_μν T^μν


dictionary:
- organize ops into reps of conf. gp ~ SO(d+1, 2)
single-trace ops. ⟷ "elementary" fields in bulk
multiparticle states in bulk.
gp fixed


dictionary:
- organize ops into reps of conf. gp ~ SO(d+1, 2)
"elementary" fields in bulk
multiparticle states in bulk
fixed by sym
BULK
$g_{\mu\nu} \longleftrightarrow T^{\mu\nu}$
$A_\mu \longleftrightarrow J^\mu$


dictionary:
- organize ops into reps of conf. gp ~ SO(d+1, 2)
- single-trace ops. ⟷ "elementary" fields in bulk
(tr(X^n))^2 |0> CFT ⟷ multiparticle states in bulk.
BULK
g_μν ⟷ T^μν
δS_QFT = ∫_bdy δg_μν T^μν
A_μ ⟷ J^μ
∂_μ J^μ = 0.


dictionary:
- organize ops into reps of conf. gp ~ SO(d+1, 2)
- single-trace ops. ⟷ "elementary" fields in bulk
multiparticle states in bulk.
fixed by sym.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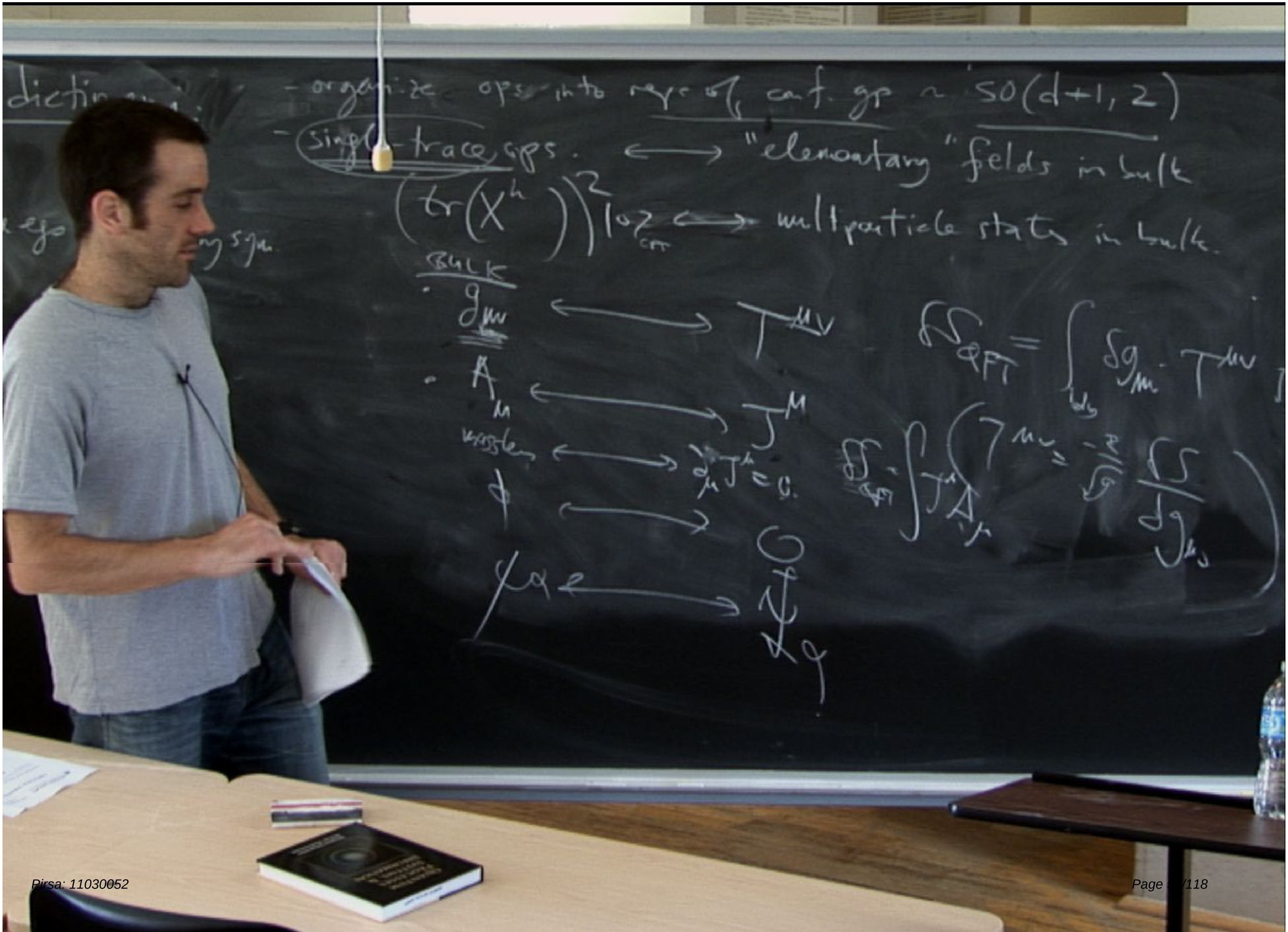
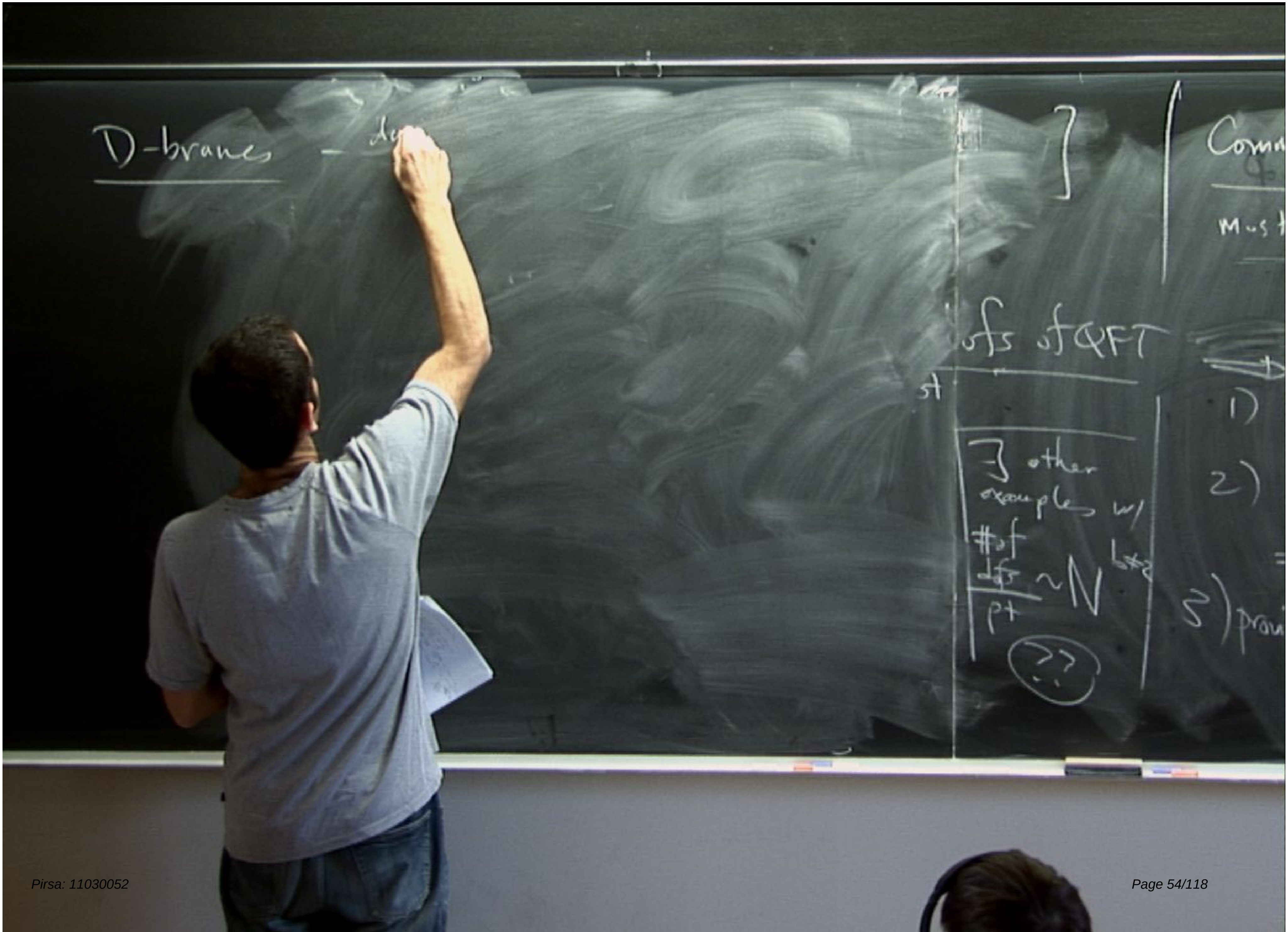
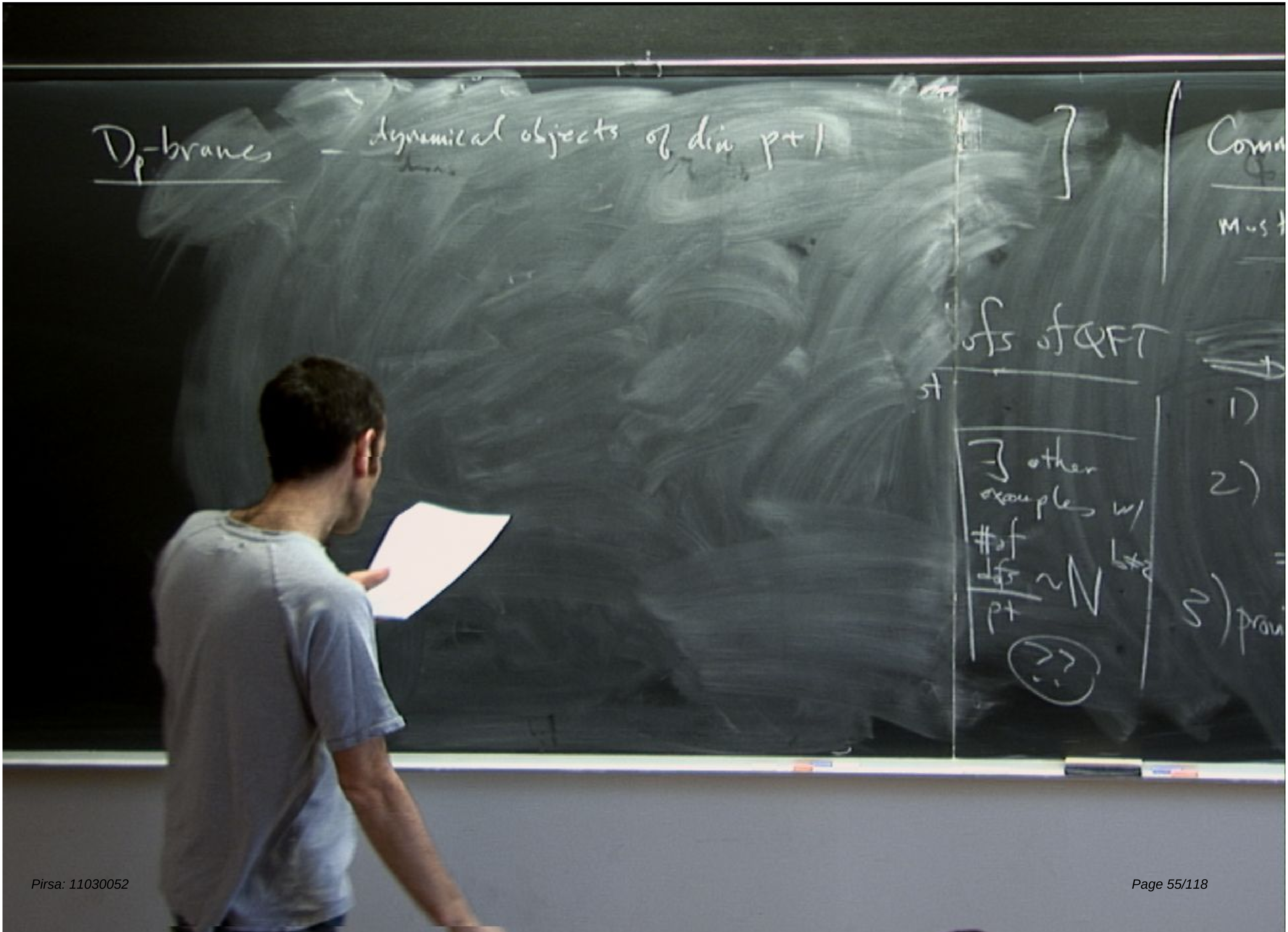
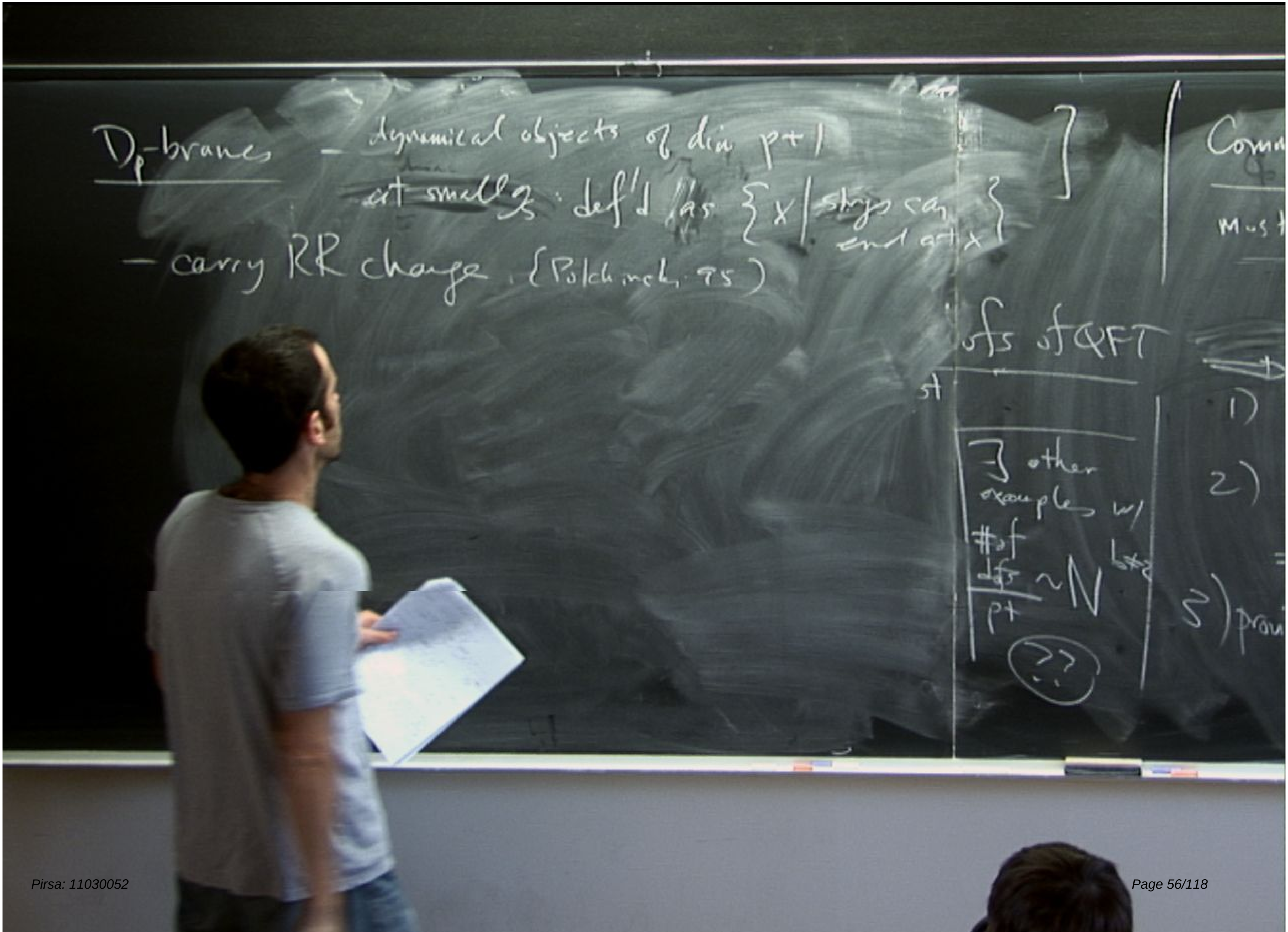
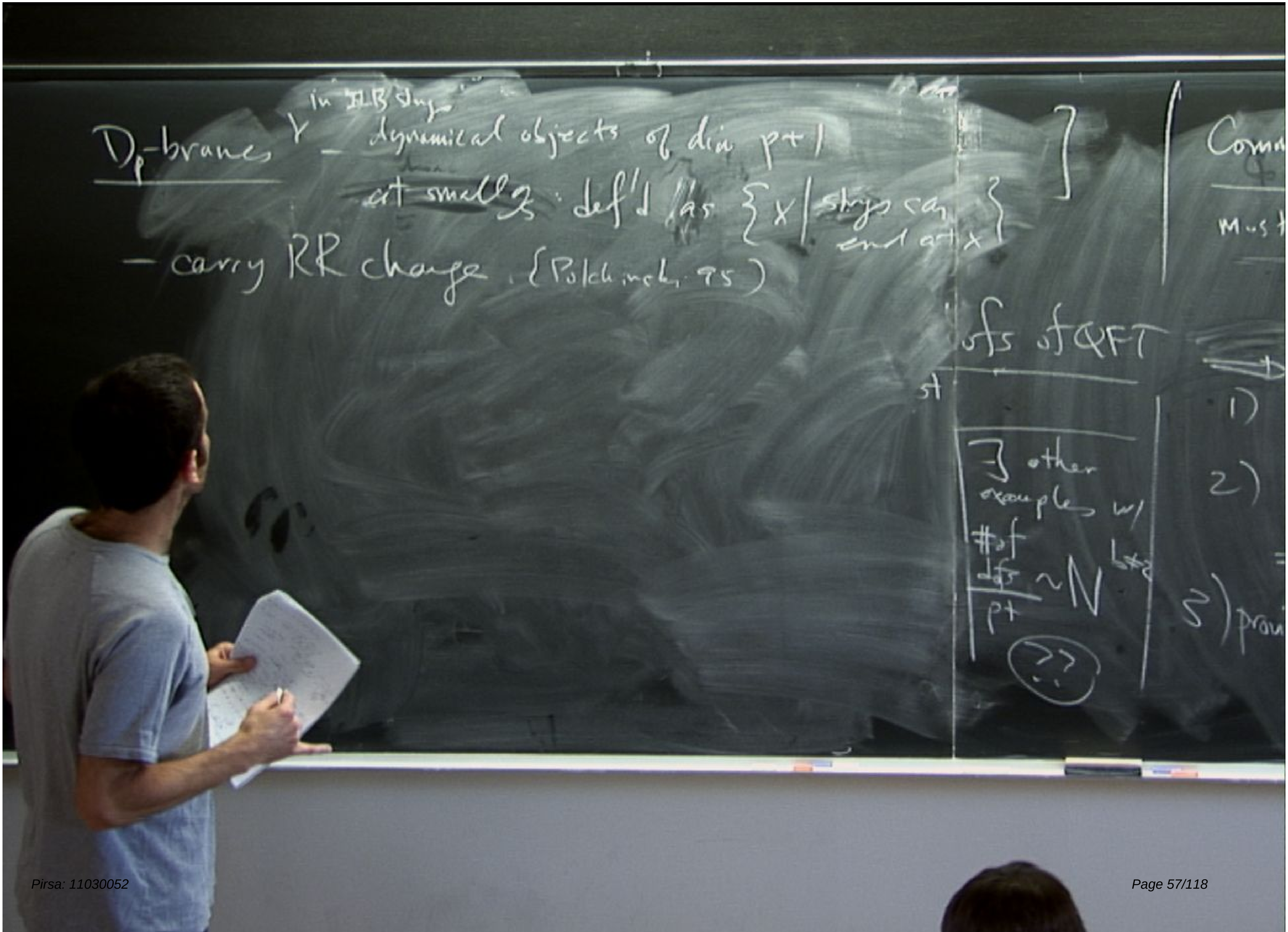
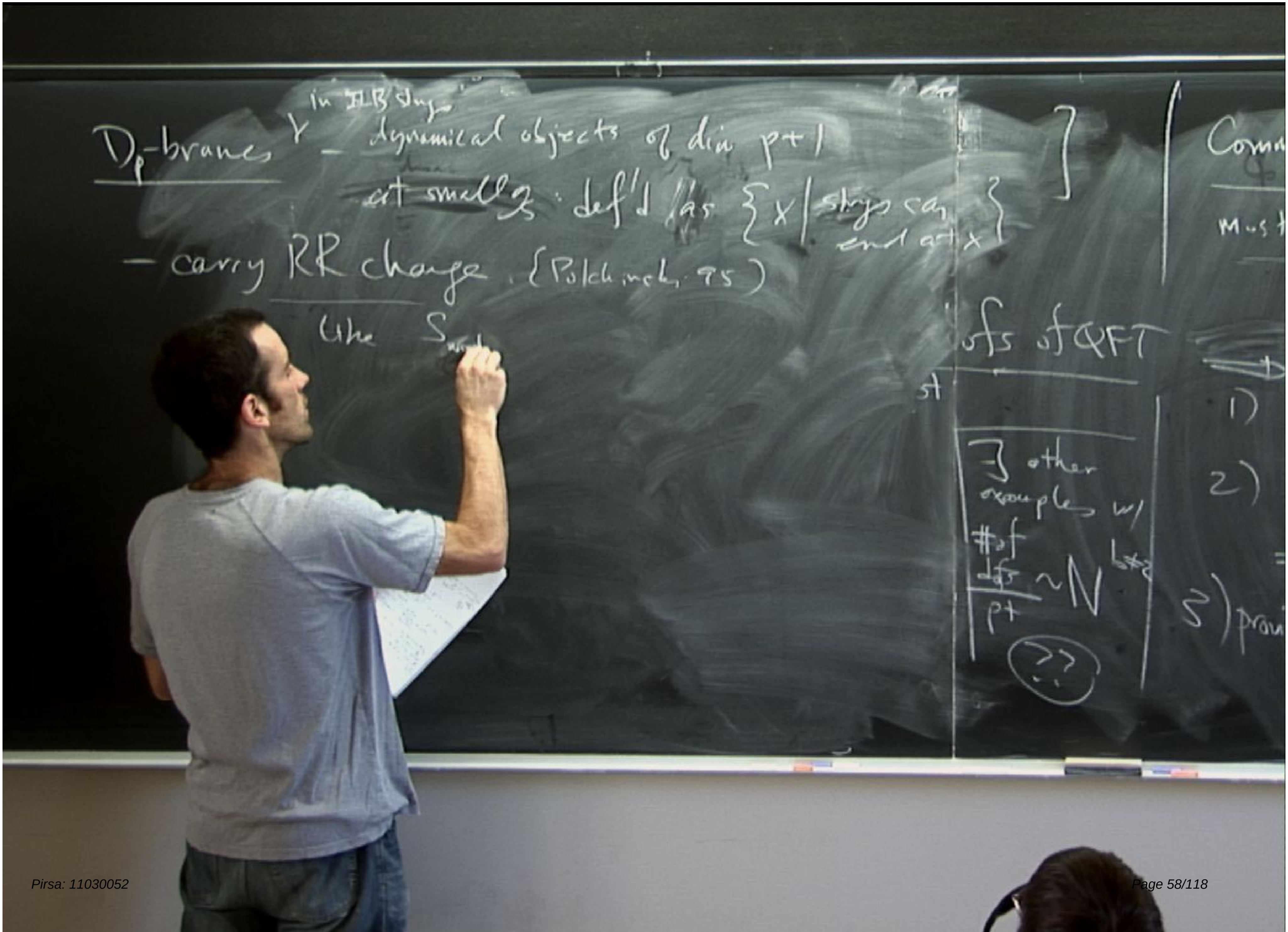

- organize ops into reps of conf. gp ~ SO(d+1, 2)
- single-trace ops. ⟷ "elementary" fields in bulk
multiparticle states in bulk.
BULK


D-branes
ofs of QFT
∃ other
examples w/
of
dfs
pt
~N
1)
2)
3) prov
Comm
Must


Dp-branes — dynamical objects of dim p+1
Comm
must
ofs of QFT
∃ other
examples w/
of
dofs ~ N
p+
1)
2)
3) prov


Dp-branes — dynamical objects of dim p+1
— at small gs : def'd as {x | strings can end at x}
— carry RR charge (Polchinski 95)
ofs of QFT
∃ other examples w/ #of dofs ~ N
??
1)
2)
3)


in IIB strings
Dp-branes — dynamical objects of dim p+1
at small gs : def'd as {x | strings can end at x}
— carry RR charge (Polchinski 95)
ofs of QFT
∃ other examples w/ # of dofs ~ N
pt
??
1)
2)
3) prov


Dp-branes
in IIB strings
dynamical objects of dim p+1
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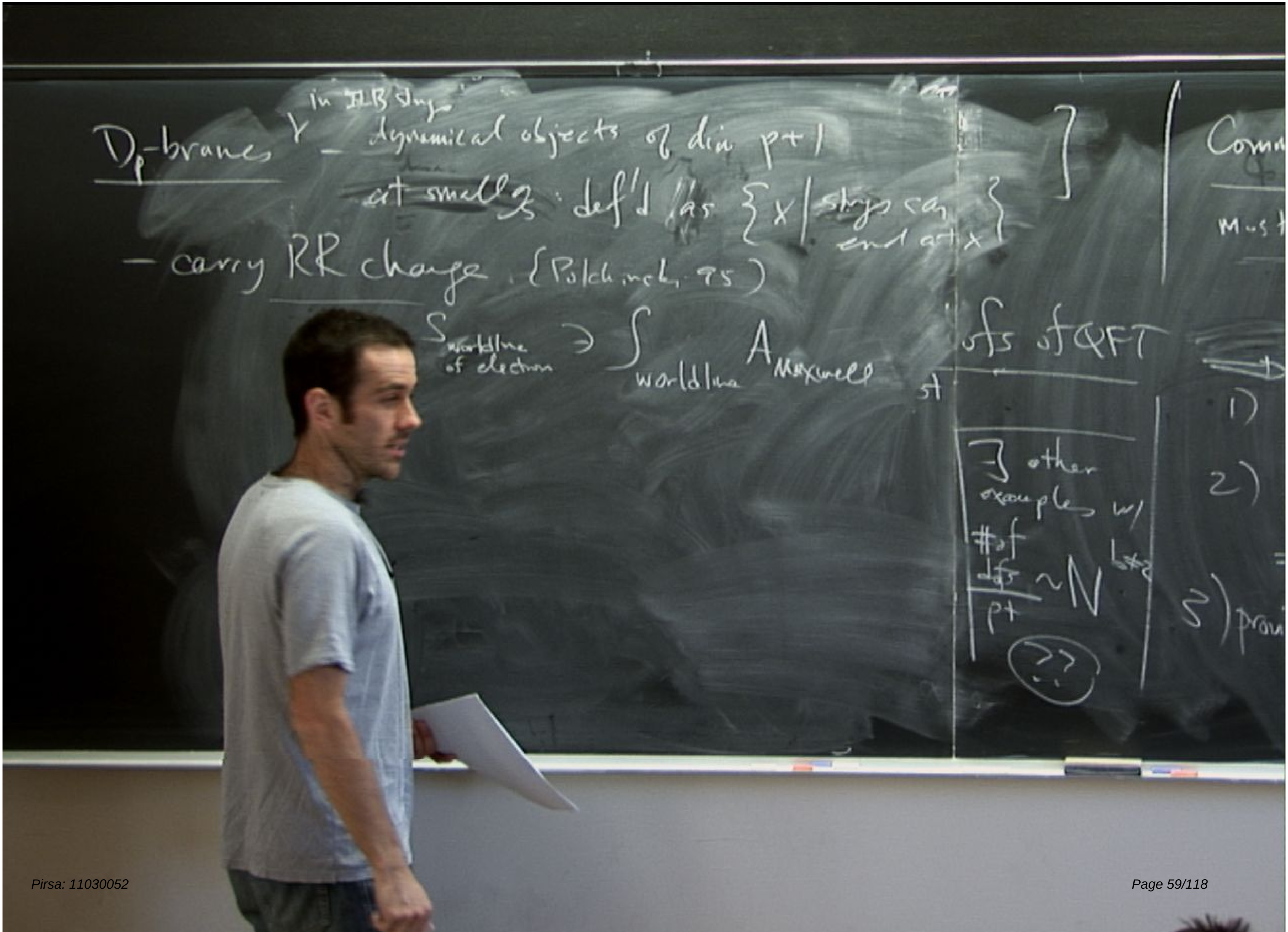
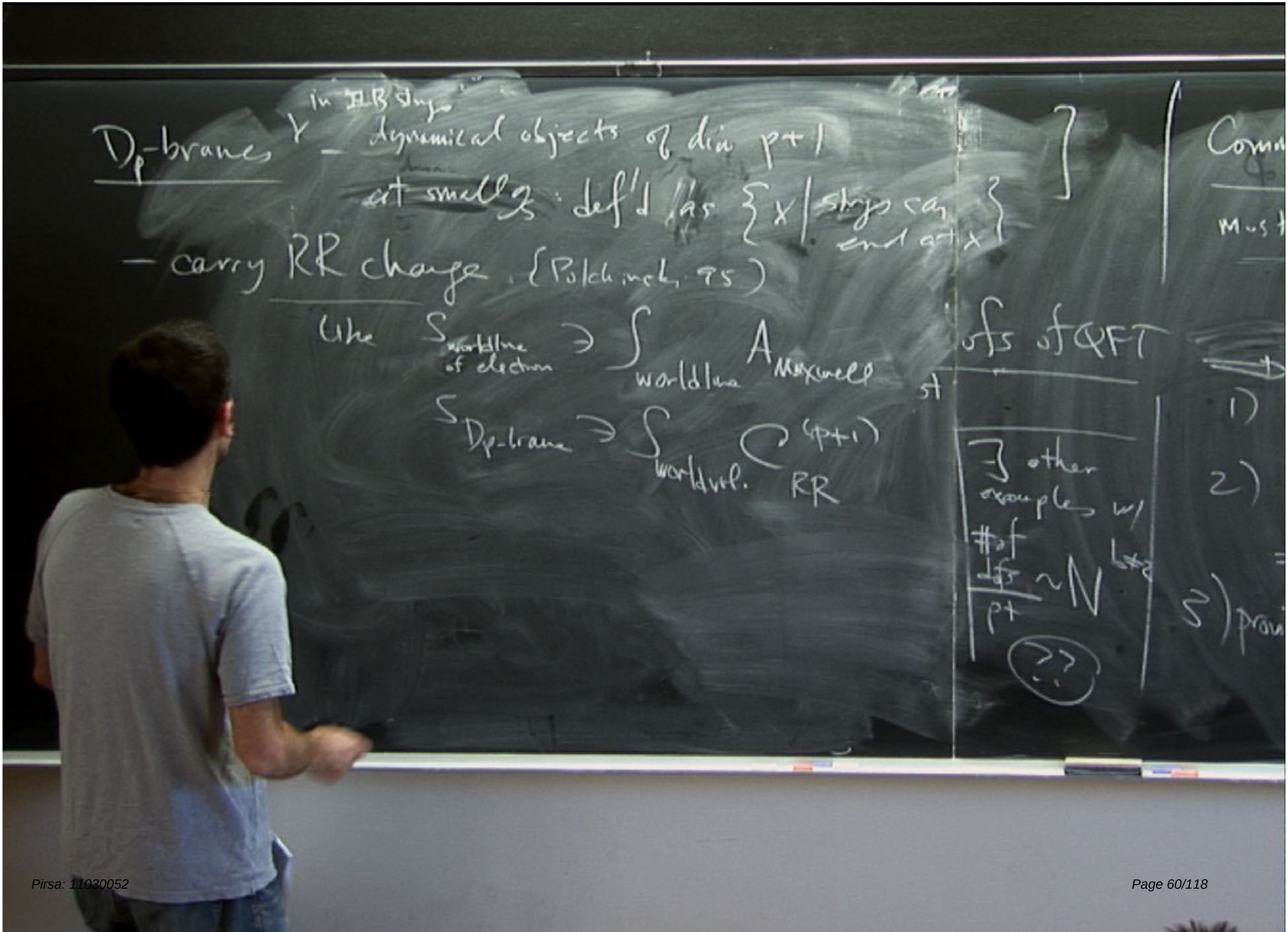
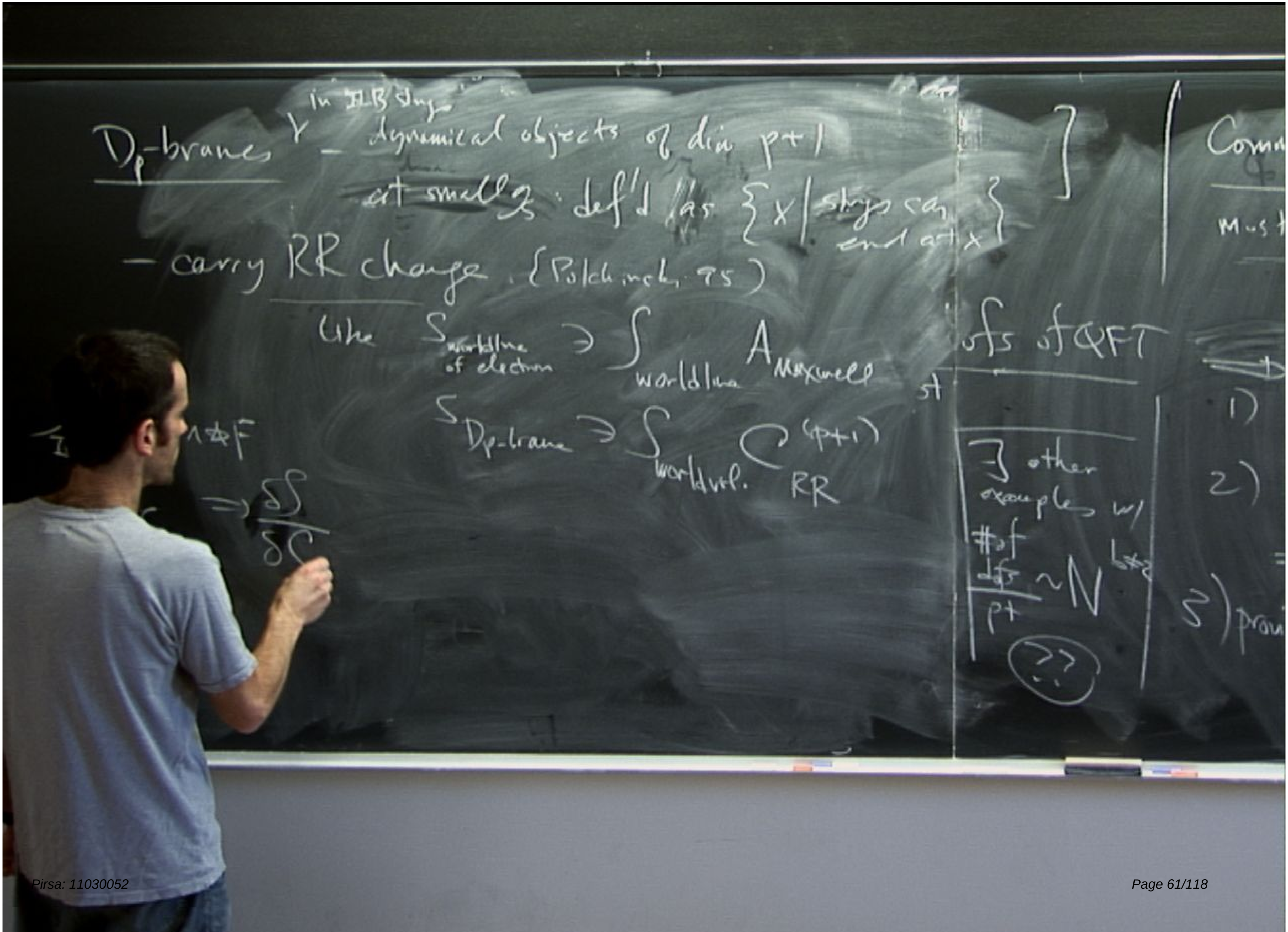
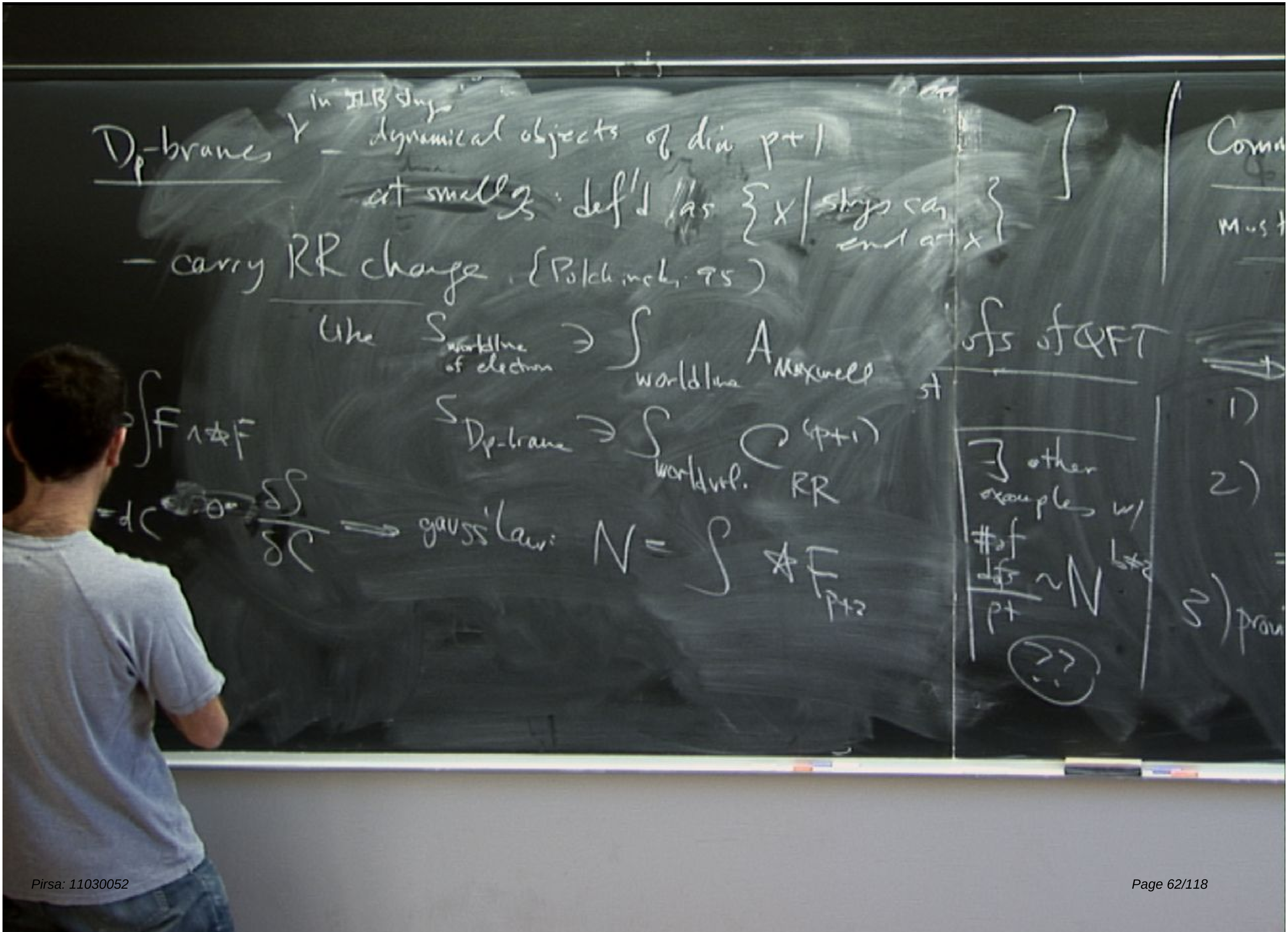
like
ofs of QFT
∃ other examples w/ # of dofs ~ N
??
Comm
must
1)
2)
3) prov



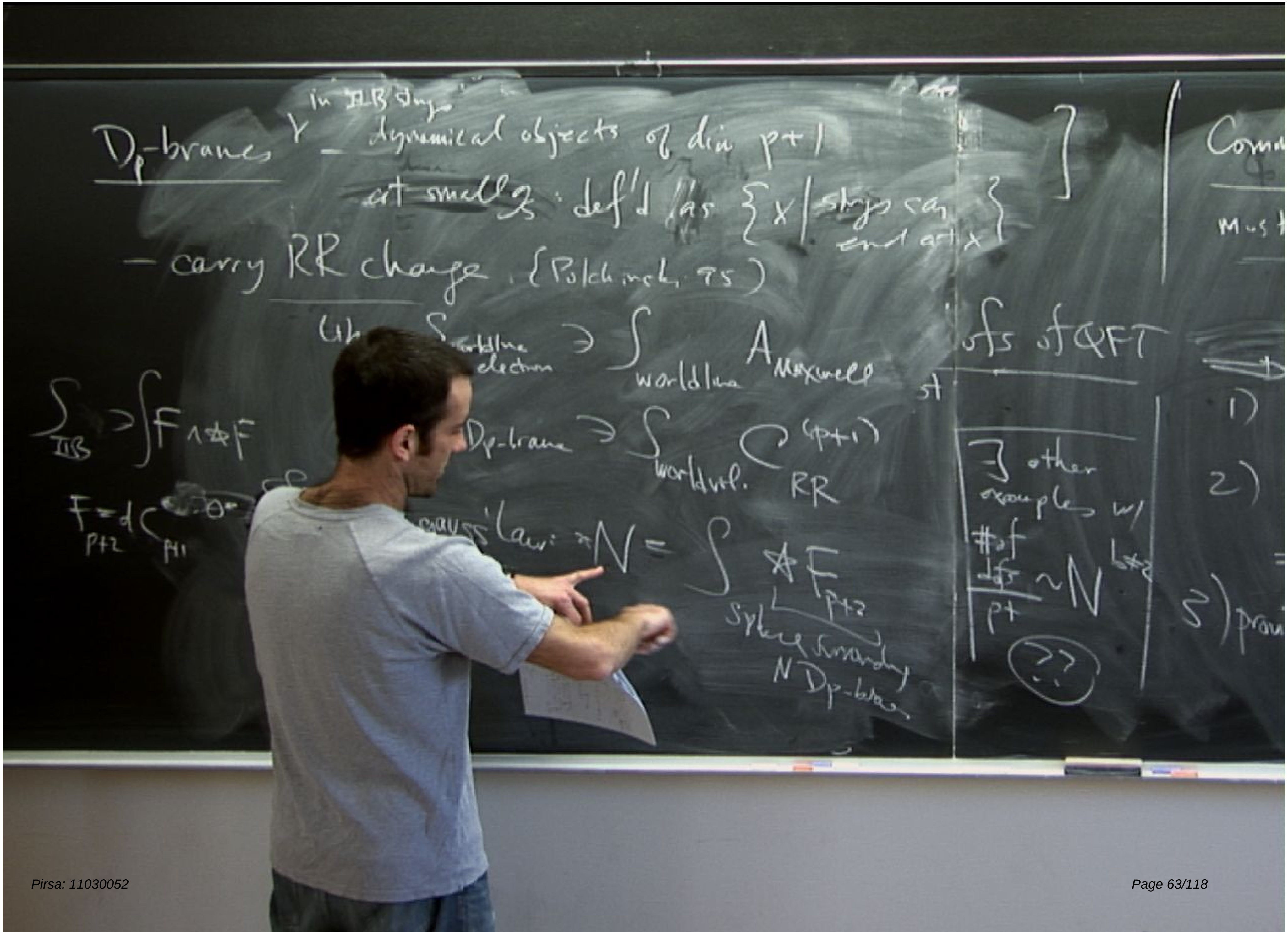
Dp-branes
in IIB strings
dynamical objects of dim p+1
at small gs: def'd as {x | strings can end at x}
carry RR charge (Polchinski 95)
S worldline of electron ∋ ∫ worldline A Maxwell
ofs of QFT
∃ other examples w/
# of dofs ~ N
??
1)
2)
3)

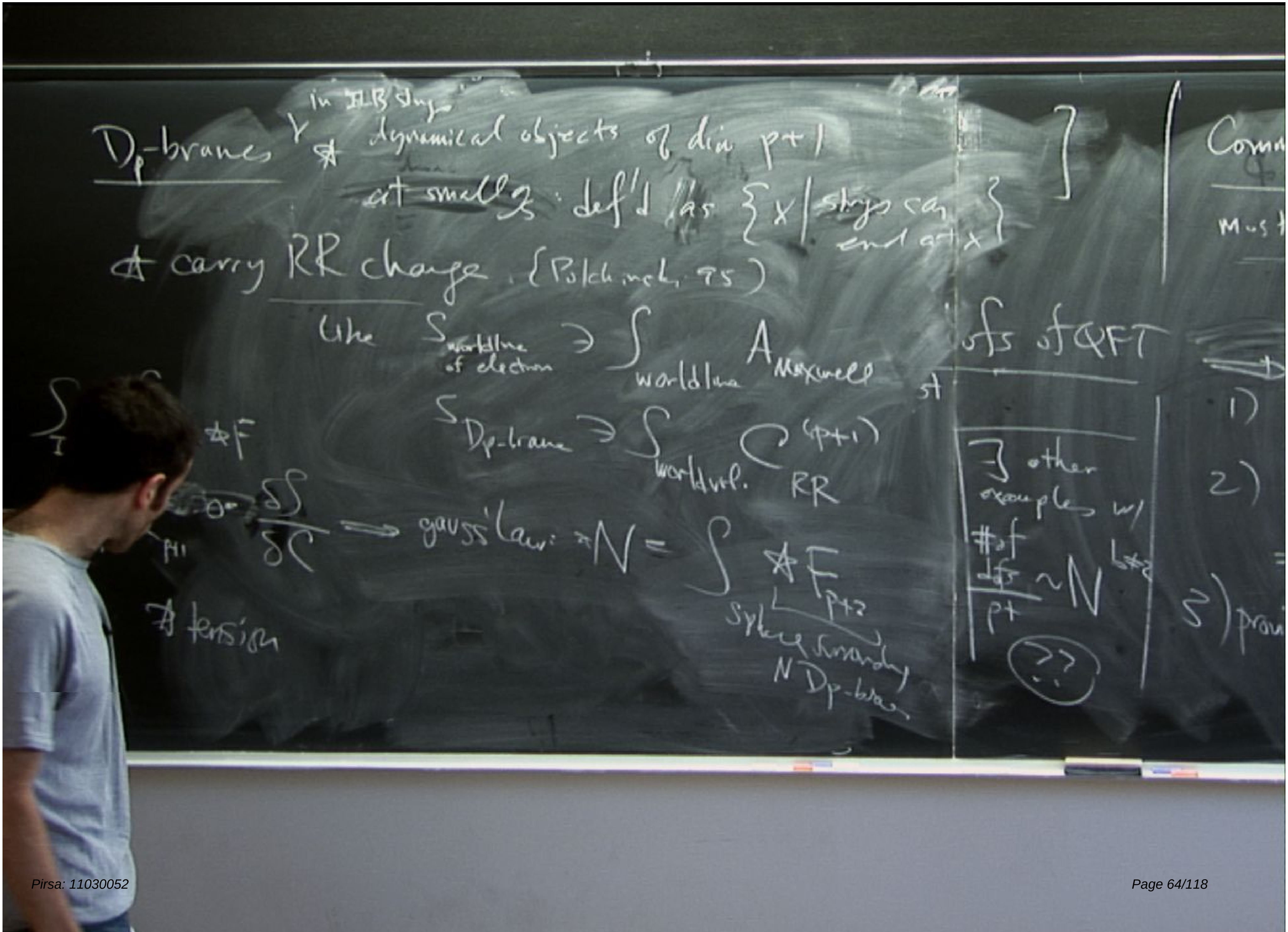

in IIB strings
Dp-branes — dynamical objects of dim p+1
at small gs : def'd as {x | strings can end at x}
— carry RR charge (Polchinski 95)
like S worldline of electron ∋ ∫ worldline A Maxwell
S Dp-brane ∋ ∫ worldvol. C (p+1) RR
dofs of QFT
∃ other examples w/ # of dofs / pt ~ N
??
Comm
must
1)
2)
3) provi


in IIB strings
Dp-branes – dynamical objects of dim p+1
– at small gs : def'd as { x | strings can end at x }
– carry RR charge. (Polchinski 95)
like S worldline of electron ∋ ∫ worldline A Maxwell
S Dp-brane ∋ ∫ worldvol. C (p+1) RR
⇒ δS / δC
ofs of QFT
∃ other examples w/ # of dofs per ~ N
??
Comm
must
1)
2)
3) prov



Dp-branes in IIB strings
dynamical objects of dim p+1
at small gs: def'd as {x | strings can end at x}
– carry RR charge (Polchinski 95)
like S worldline of electron ∋ ∫ worldline A Maxwell
S Dp-brane ∋ ∫ worldvol. C (p+1) RR
∫ F ∧ ⋆F
gauss' law: N = ∫ ⋆F p+2
dofs of QFT
∃ other examples w/
Comm
must
1)
2)
3) prov


in IIB str.
Dp-branes
- dynamical objects of dim p+1
- at small g_s: def'd as {x | strings can end at x}
- carry RR charge (Polchinski, 95)
worldline
electron
worldline
A Maxwell
ofs of QFT
Dp-brane
worldvol.
RR
(p+1)
F = d C
p+2
p+1
Gauss' law:
N = ∫ ⋆F
p+2
sphere surrounding N Dp-branes
∃ other examples w/
# of dofs ~ N
p+
??
Comm
must
1)
2)
3) prov


Dp-branes
in IIB strings
dynamical objects of dim p+1
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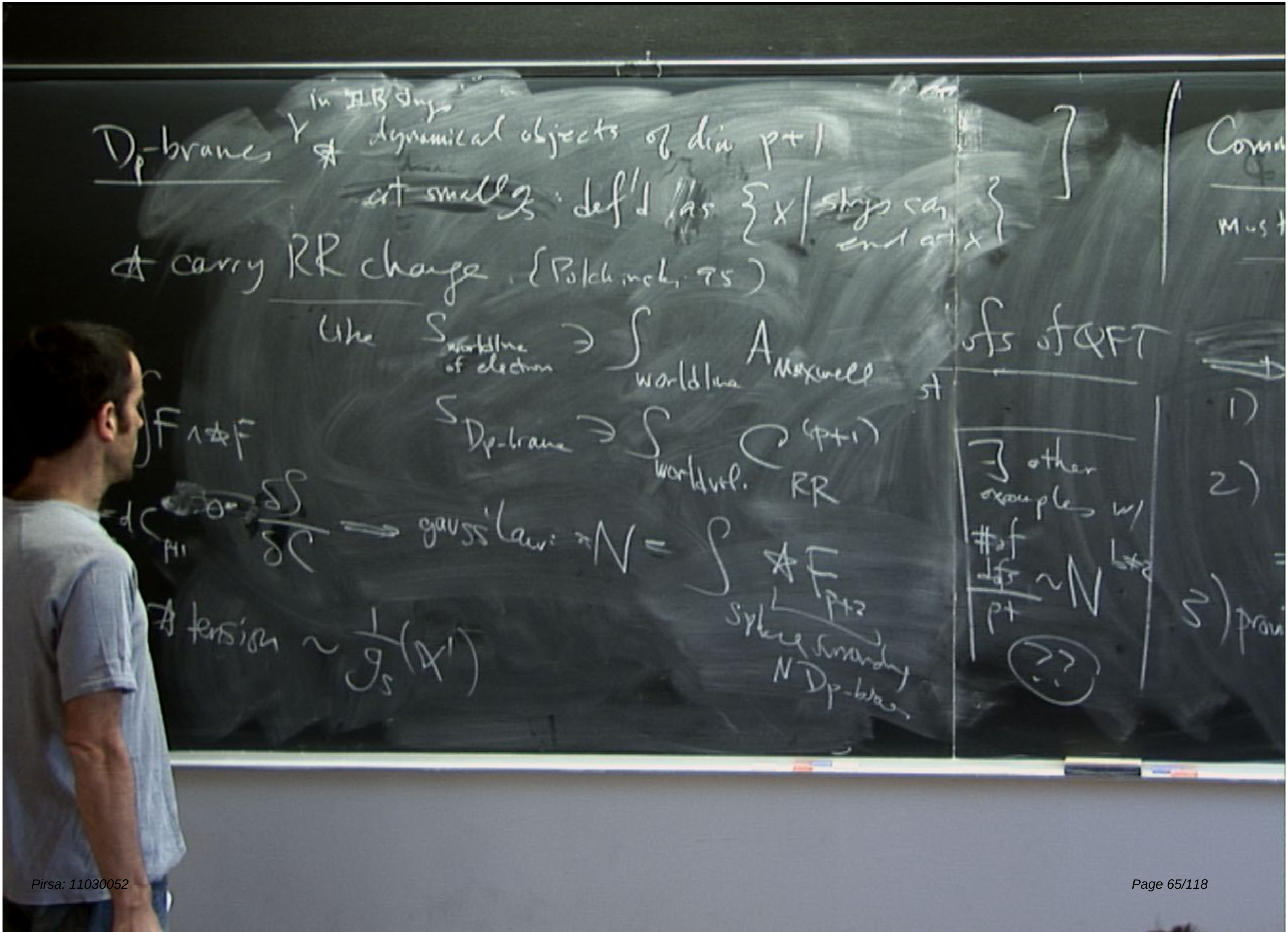
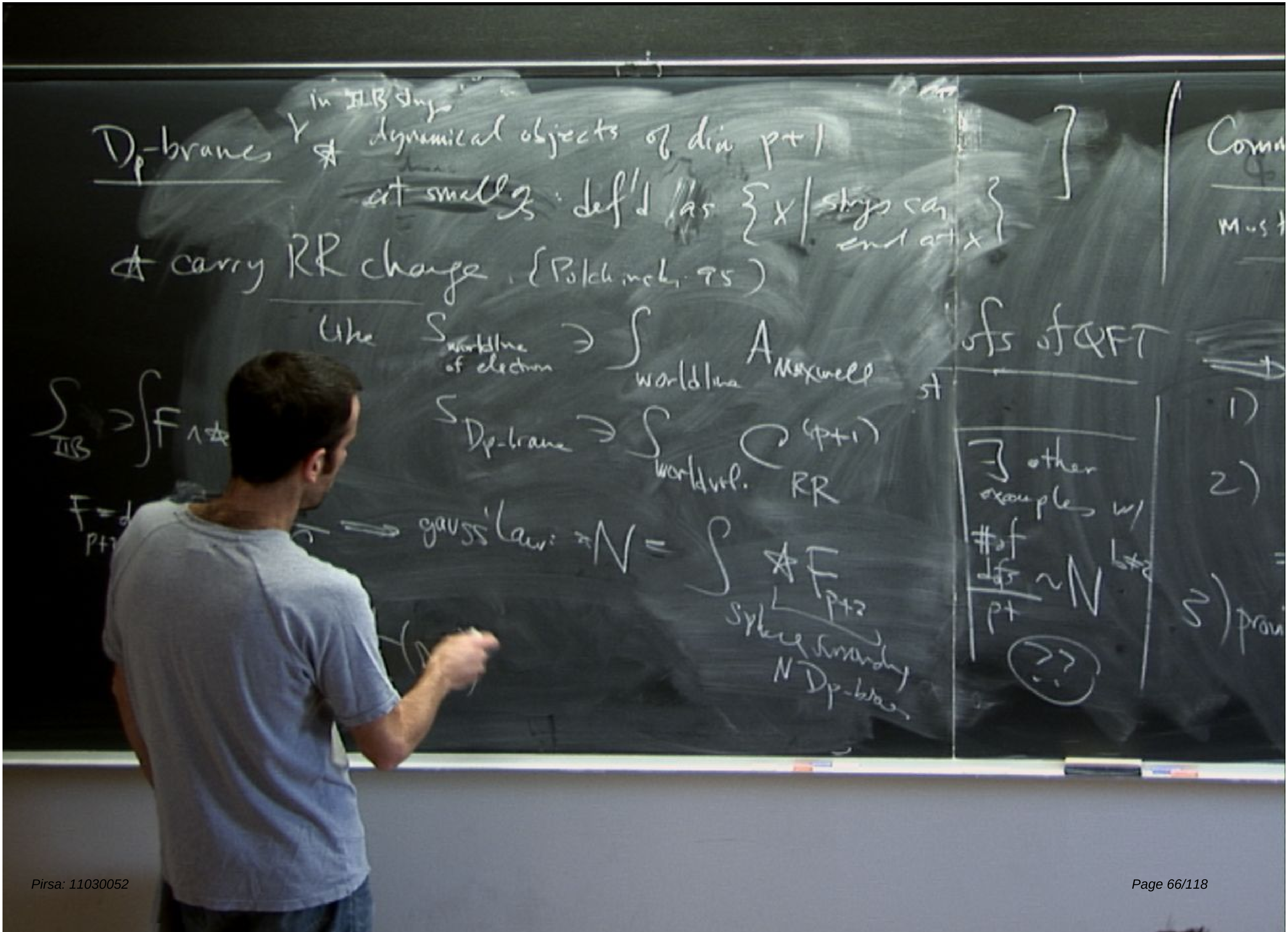
S Dp-brane ∋ ∫ worldvol. C (p+1) RR
gauss' law: N = ∫ ★F p+2
sphere surrounding N Dp-branes
tension
★F
δS/δC
ofs of QFT
∃ other examples w/ # of dofs ~ N
??
Comm
must
1)
2)
3) prov



Dp-branes
in IIB strings
dynamical objects of dim p+1
at small gs: def'd as {x | strings can end at x}
carry RR charge (Polchinski 95)
like S worldline of electron ∋ ∫ worldline A Maxwell
S Dp-brane ∋ ∫ worldvol. C RR (p+1)
∫F∧*F
gauss' law: N = ∫ *F p+2
sphere surrounding N Dp-branes
tension ~ 1/gs (α')
ofs of QFT
∃ other examples w/ # of dofs ~ N
??
1)
2)
3) prov
Comm
must


Dp-branes
in IIB strings
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S IIB ∋ ∫ F ∧ ★
S Dp-brane ∋ ∫ worldvol. C (p+1) RR
⟹ gauss' law: N = ∫ ★F p+2
sphere surrounding N Dp-branes
dofs of QFT
∃ other examples w/ # of dofs p+ ~ N
??
Comm
1)
2)
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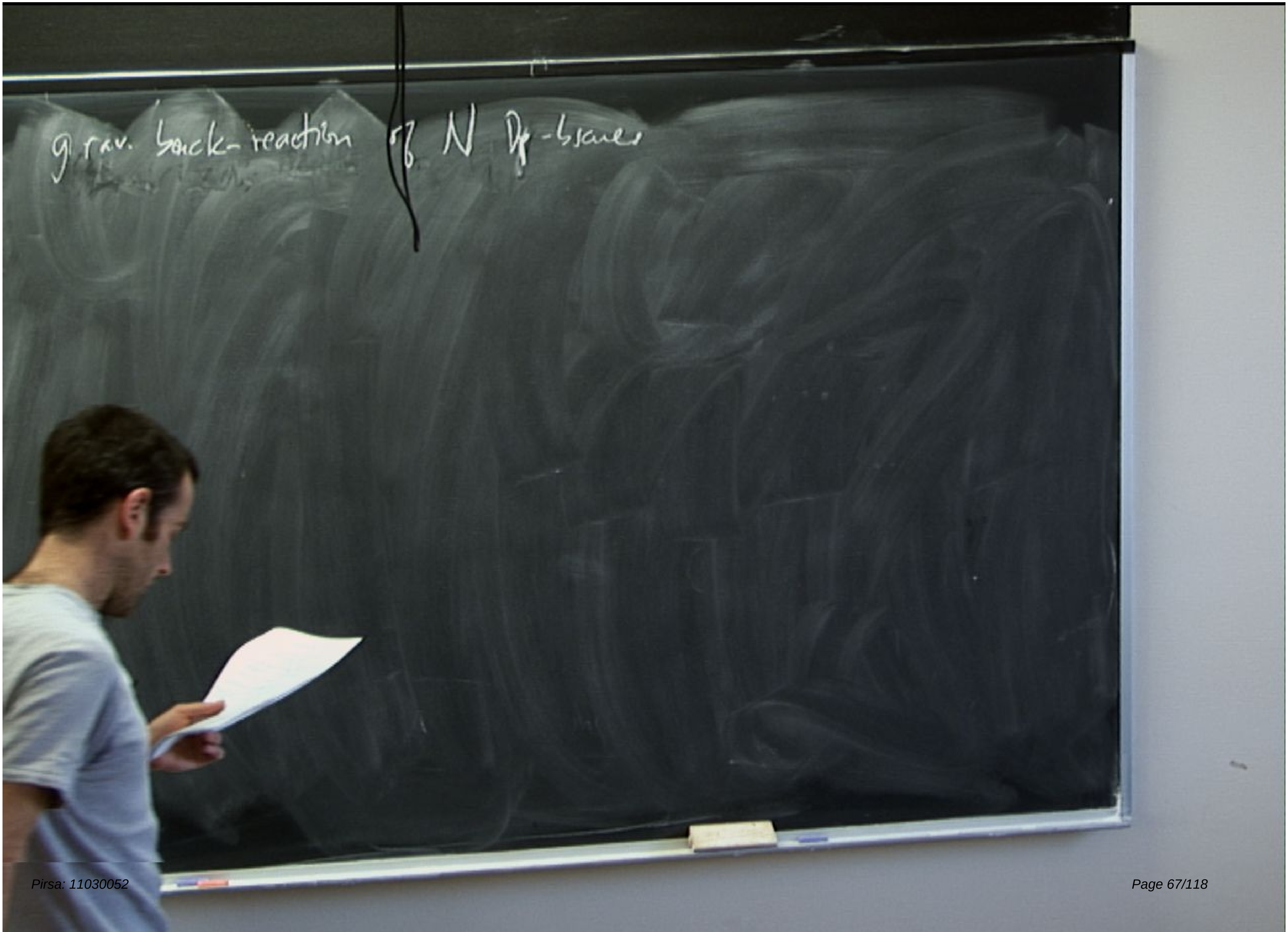
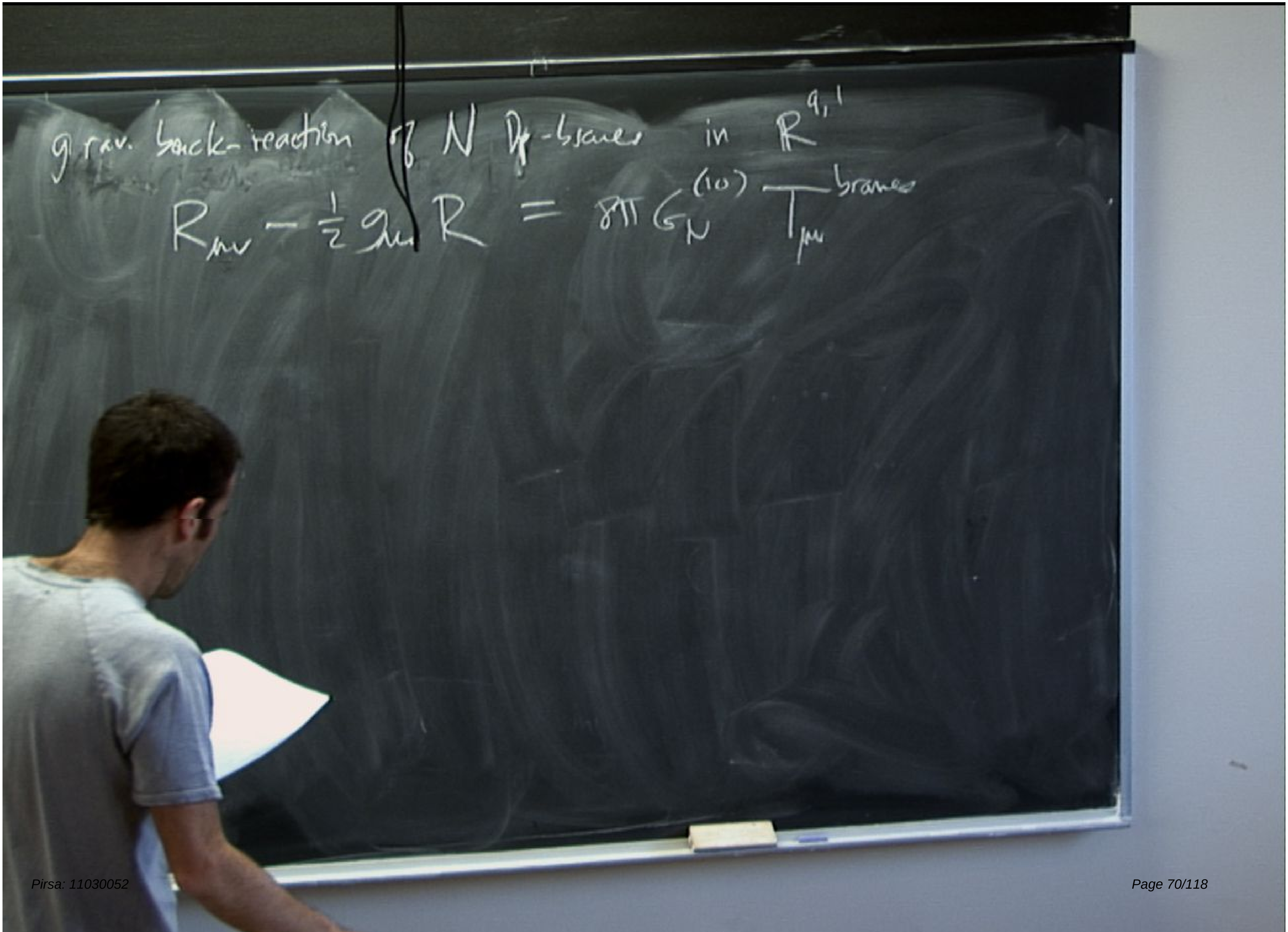
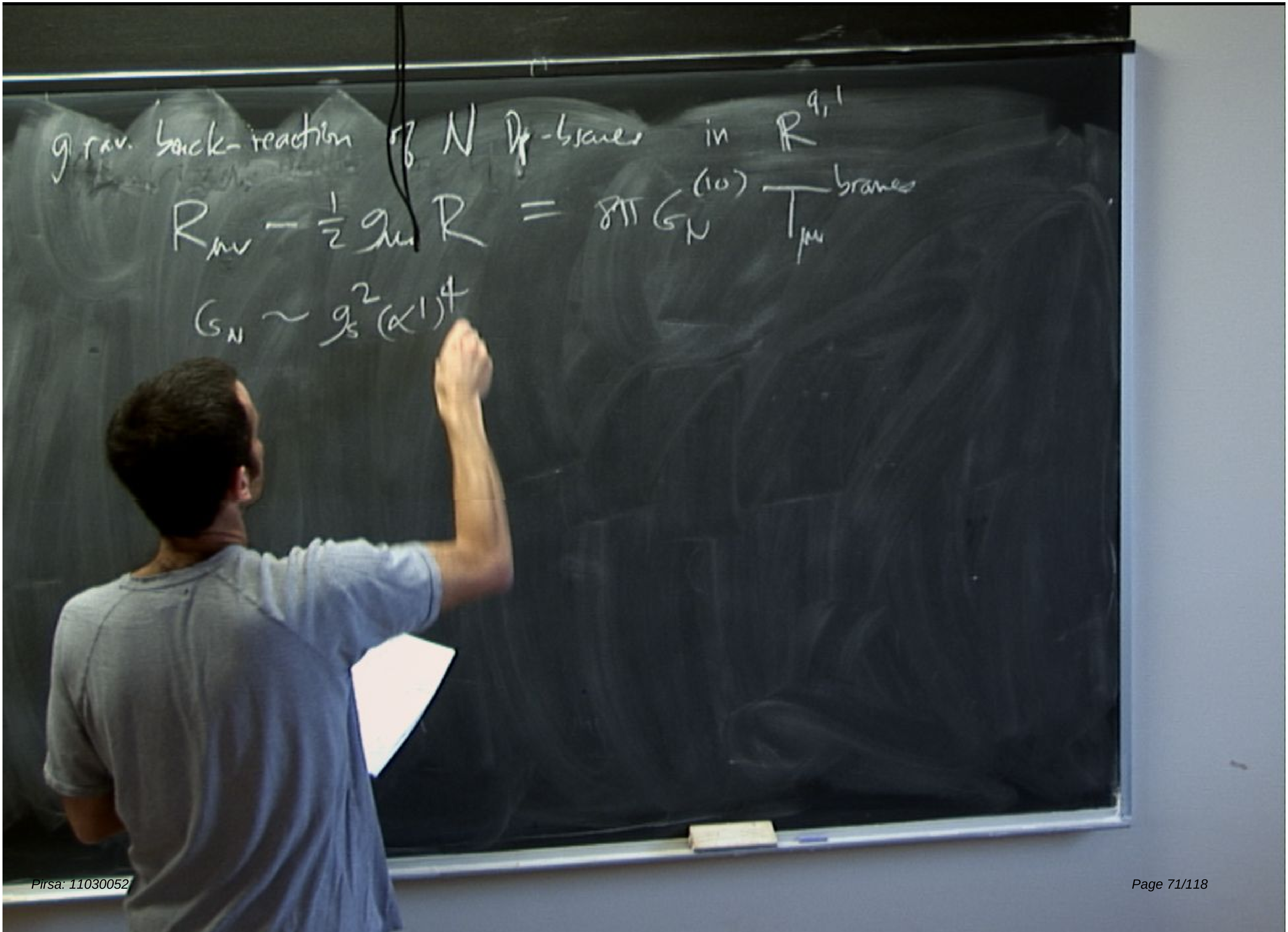

grav. back-reaction of N Dp-branes

grav. back-reaction of N Dp-branes in $\mathbb{R}^{9,1}$

grav. back-reaction of N Dp-branes in $\mathbb{R}^{9,1}$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

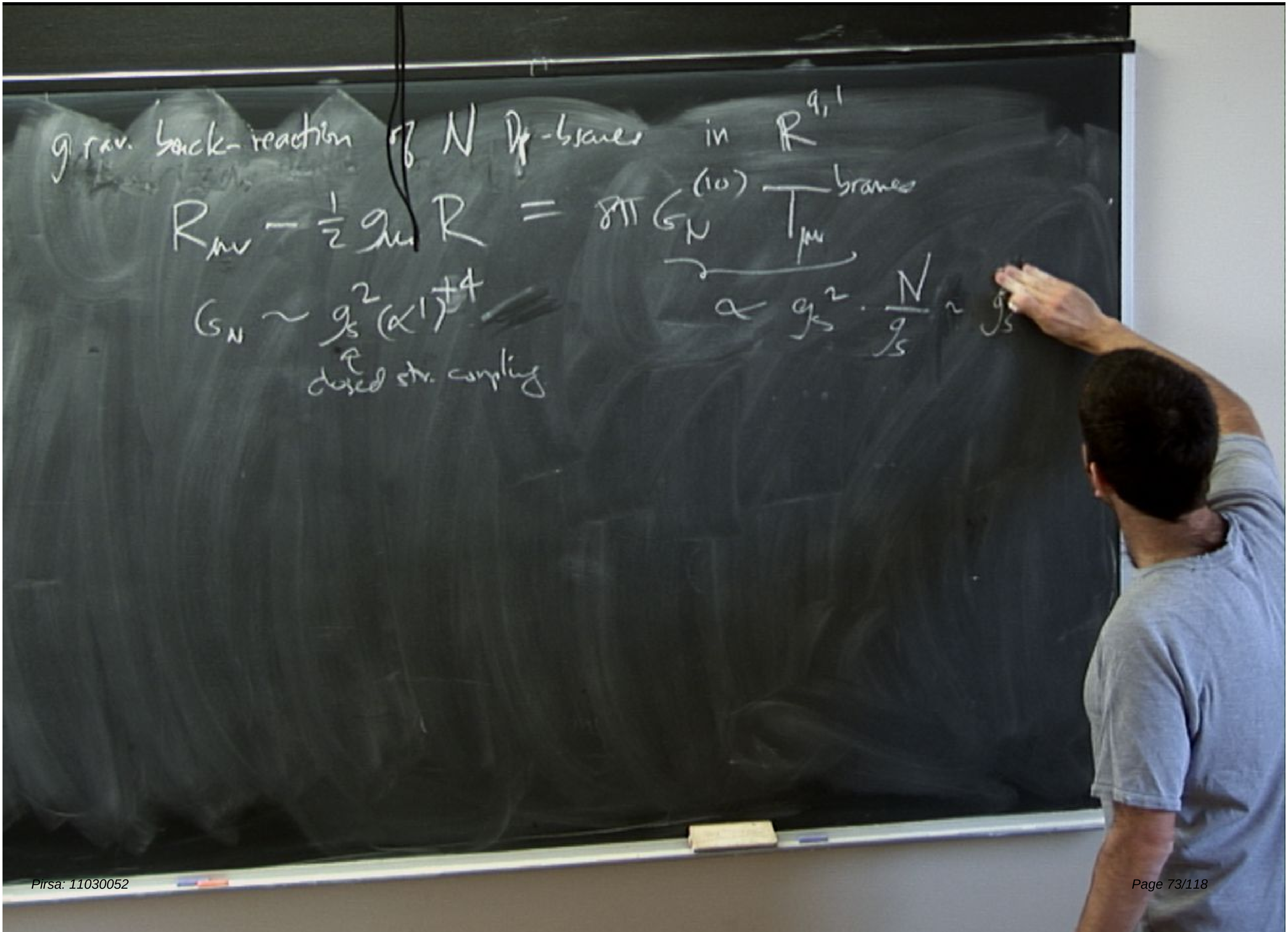

grav. back-reaction of N Dp-branes in $\mathbb{R}^{9,1}$
$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N^{(10)} T_{\mu\nu}^{\text{branes}}$


grav. back-reaction of N Dp-branes in $\mathbb{R}^{9,1}$
$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N^{(10)} T_{\mu\nu}^{\text{branes}}$
$G_N \sim g_s^2 (\alpha')^4$

grav. back-reaction of N Dp-branes in $\mathbb{R}^{9,1}$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N^{(10)} T_{\mu\nu}^{\text{branes}}$$

$$G_N \sim g_s^2 (\alpha')^4$$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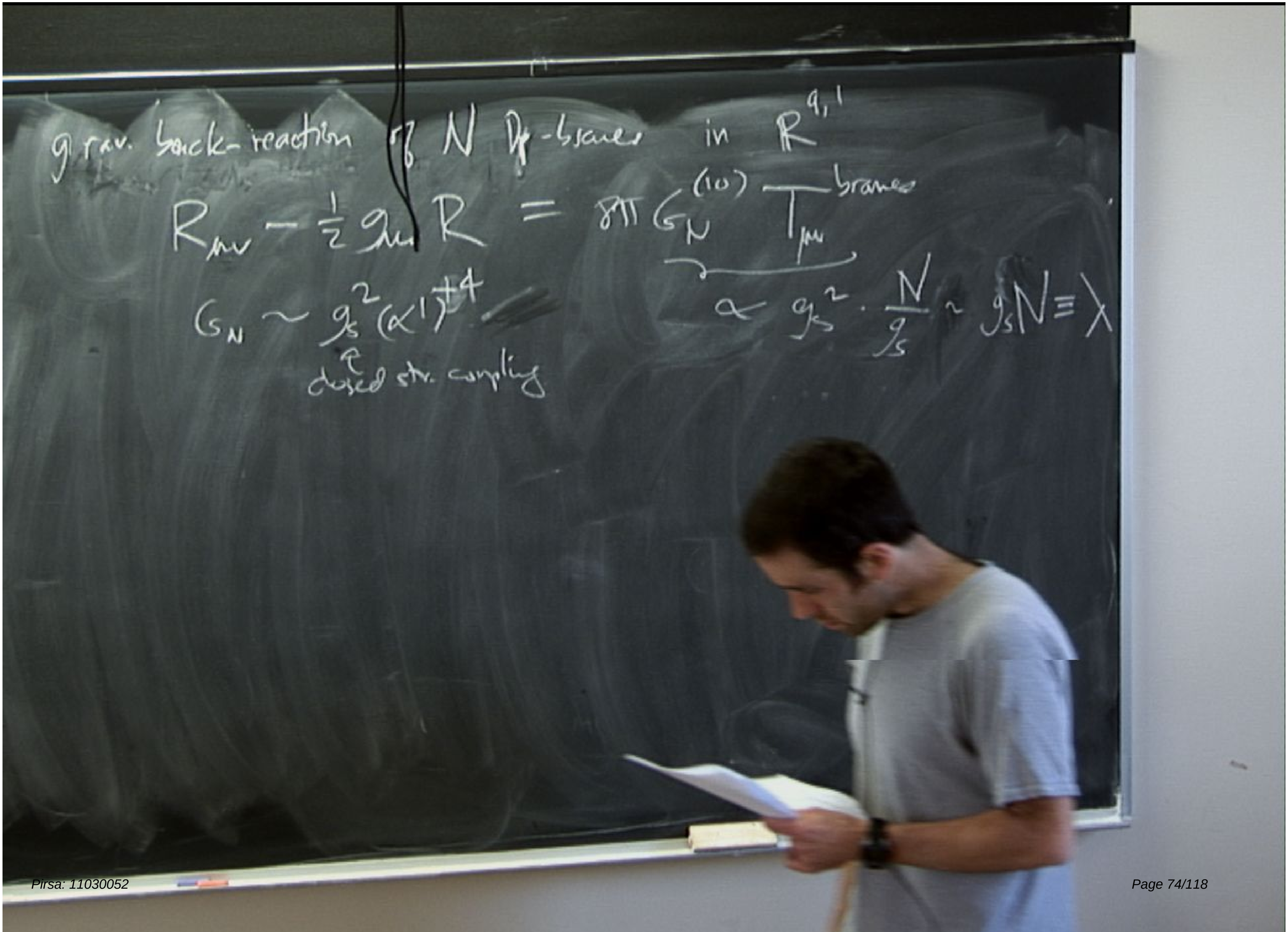
$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N^{(10)} T_{\mu\nu}^{\text{branes}}$
$G_N \sim g_s^2 (\alpha')^{+4}$
closed str. coupling
$\propto g_s^2 \cdot \frac{N}{g_s} \sim g_s$


grav. back-reaction of N Dp-branes in $\mathbb{R}^{9,1}$
$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \underbrace{8\pi G_N^{(10)} T_{\mu\nu}^{\text{branes}}}_{\propto g_s^2 \cdot \frac{N}{g_s} \sim g_s N \equiv \lambda}$
$G_N \sim g_s^2 (\alpha')^4$
closed str. coupling

grav. back-reaction of N Dp-branes in $\mathbb{R}^{9,1}$
(B.R.)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N^{(10)} \underbrace{T_{\mu\nu}^{\text{branes}}}_{\propto\, g_s^2 \cdot \frac{N}{g_s}}$$

$$G_N \sim g_s^2 (\alpha')^{+4}$$

closed str. coupling

at small λ: B.R. negligible

$p+1$

$\{x \mid$ strings can end at $x\}$

A_{Maxwell}

$(p+1)$

RR

F_p

grav. back-reaction of N Dp-branes in $\mathbb{R}^{9,1}$
(B.R.)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N^{(10)} \overset{\text{branes}}{T_{\mu\nu}}$$

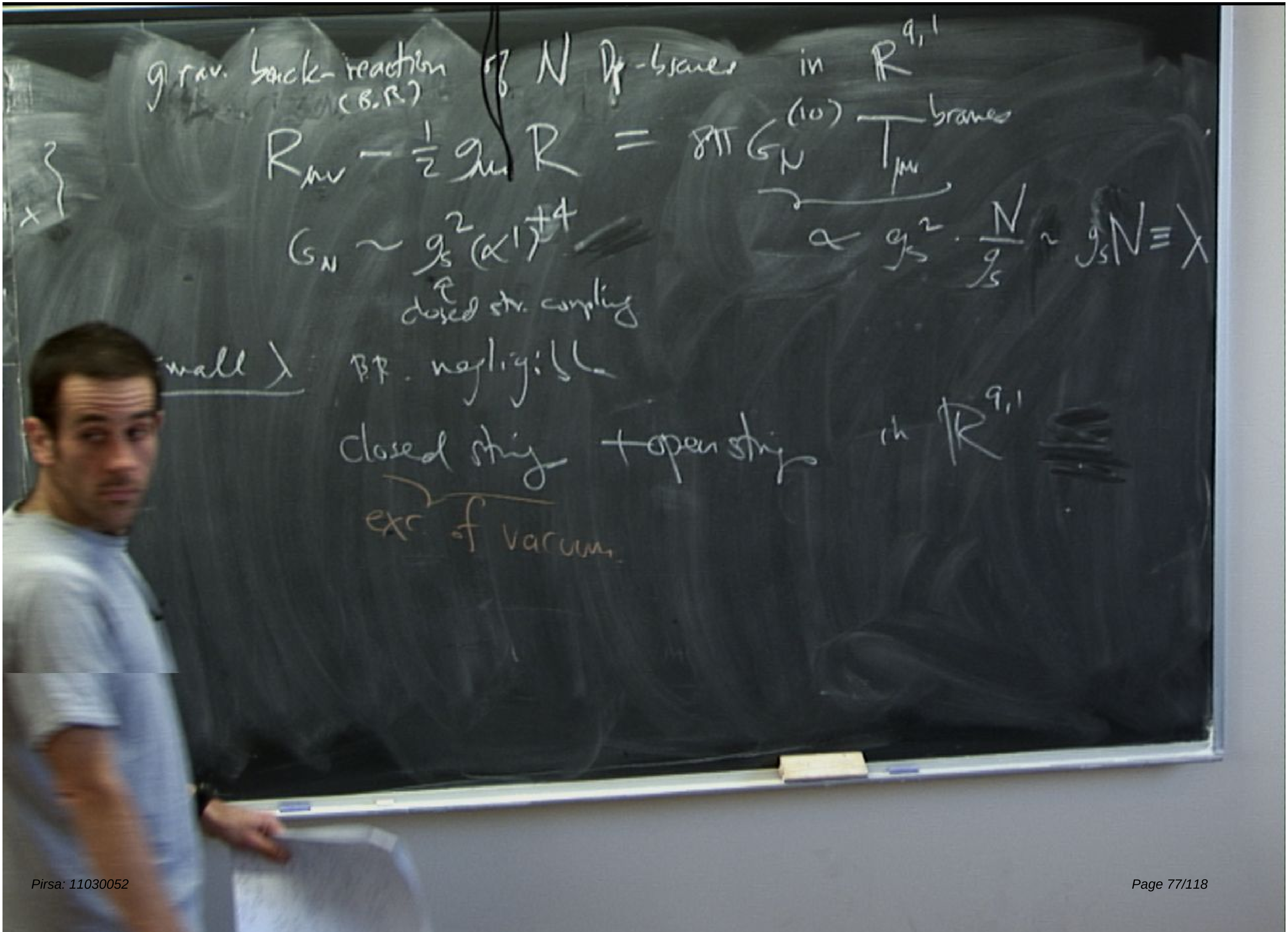
$$\underbrace{G_N^{(10)} T_{\mu\nu}}_{\propto\, g_s^2 \cdot \frac{N}{g_s} \sim g_s N \equiv \lambda}$$

$$G_N \sim \underset{\uparrow}{g_s^2} (\alpha')^{+4}$$
closed str. coupling

at small λ B.R. negligible

closed strings + open strings in $\mathbb{R}^{9,1}$

exc. of vacuum


grav. back-reaction of N Dp-branes in $\mathbb{R}^{9,1}$
(B.R.)
$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N^{(10)} T_{\mu\nu}^{\text{branes}}$
$G_N \sim g_s^2 (\alpha')^4$
closed str. coupling
$\propto g_s^2 \cdot \frac{N}{g_s} \sim g_s N \equiv \lambda$
small λ: B.R. negligible
closed string + open strings in $\mathbb{R}^{9,1}$
exc. of vacuum

grav. back-reaction (B.R.) of N Dp-branes in $\mathbb{R}^{9,1}$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi \underbrace{G_N^{(10)} T_{\mu\nu}^{\text{branes}}}_{\propto\, g_s^2 \cdot \frac{N}{g_s} \sim g_s N \equiv \lambda}$$

$G_N \sim g_s^2 (\alpha')^{+4}$

g_s: closed str. coupling

at small λ: B.R. negligible

$\underbrace{\text{closed string}}_{\text{exc. of vacuum}}$ + open strings in $\mathbb{R}^{9,1}$

at low E

$\simeq$ IIB sugra + α' corrections grav. back-reaction (B.R.) of N Dp-branes in $\mathbb{R}^{9,1}$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \underbrace{8\pi G_N^{(10)} T_{\mu\nu}^{\text{branes}}}_{\propto g_s^2 \cdot \frac{N}{g_s} \sim g_s N \equiv \lambda}$$

$G_N \sim g_s^2 (\alpha')^{+4}$

g_s: closed str. coupling

at small λ: B.R. negligible

$$\underbrace{\text{closed string}}_{\text{exc. of vacuum}} + \underbrace{\text{open string}}_{\text{exc. of D-brane}} \text{ in } \mathbb{R}^{9,1}$$

at low E

$\simeq$ IIB Sugra $+ \alpha'^{\#} R^{\#}$ corrections $\qquad \simeq$ YM gauge th.

grav. back-reaction of N D3-branes in $\mathbb{R}^{9,1}$
(B.R.)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi \underbrace{G_N^{(10)} T_{\mu\nu}^{\text{branes}}}_{\propto\, g_s^2 \cdot \frac{N}{g_s} \sim g_s N \equiv \lambda}$$

$G_N \sim g_s^2 (\alpha')^{+4}$

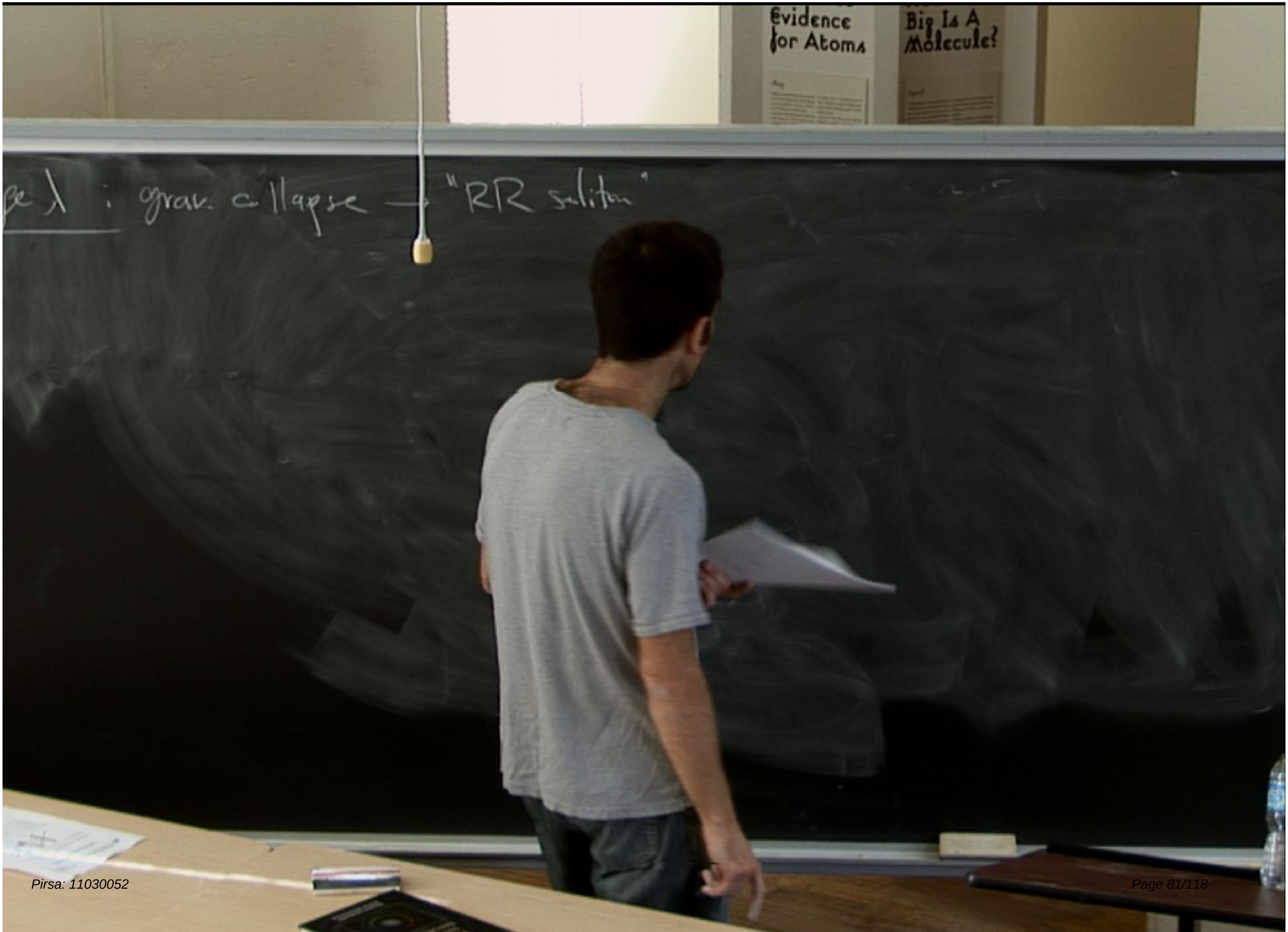
g_s ← closed str. coupling

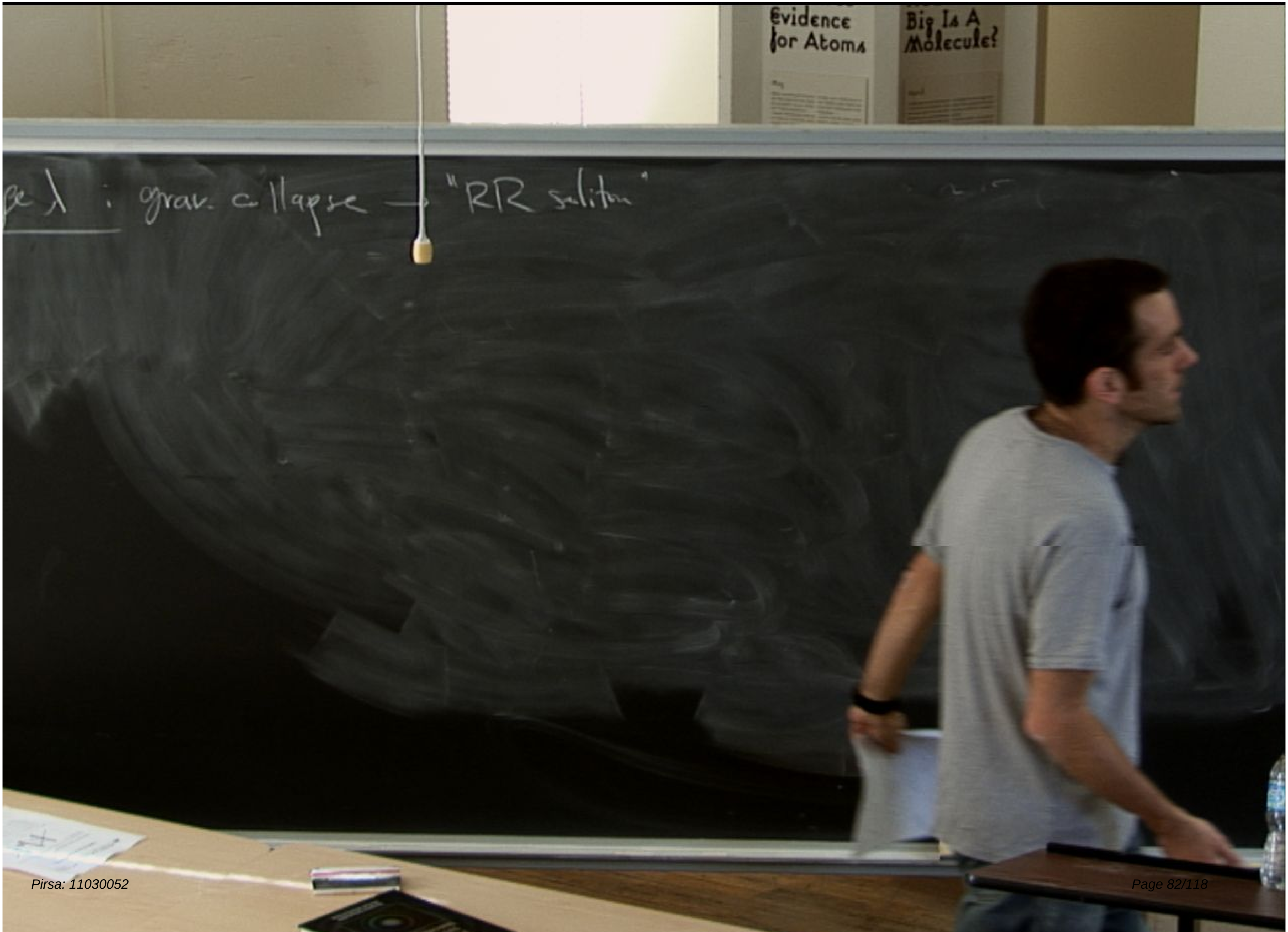
at small λ : B.R. negligible

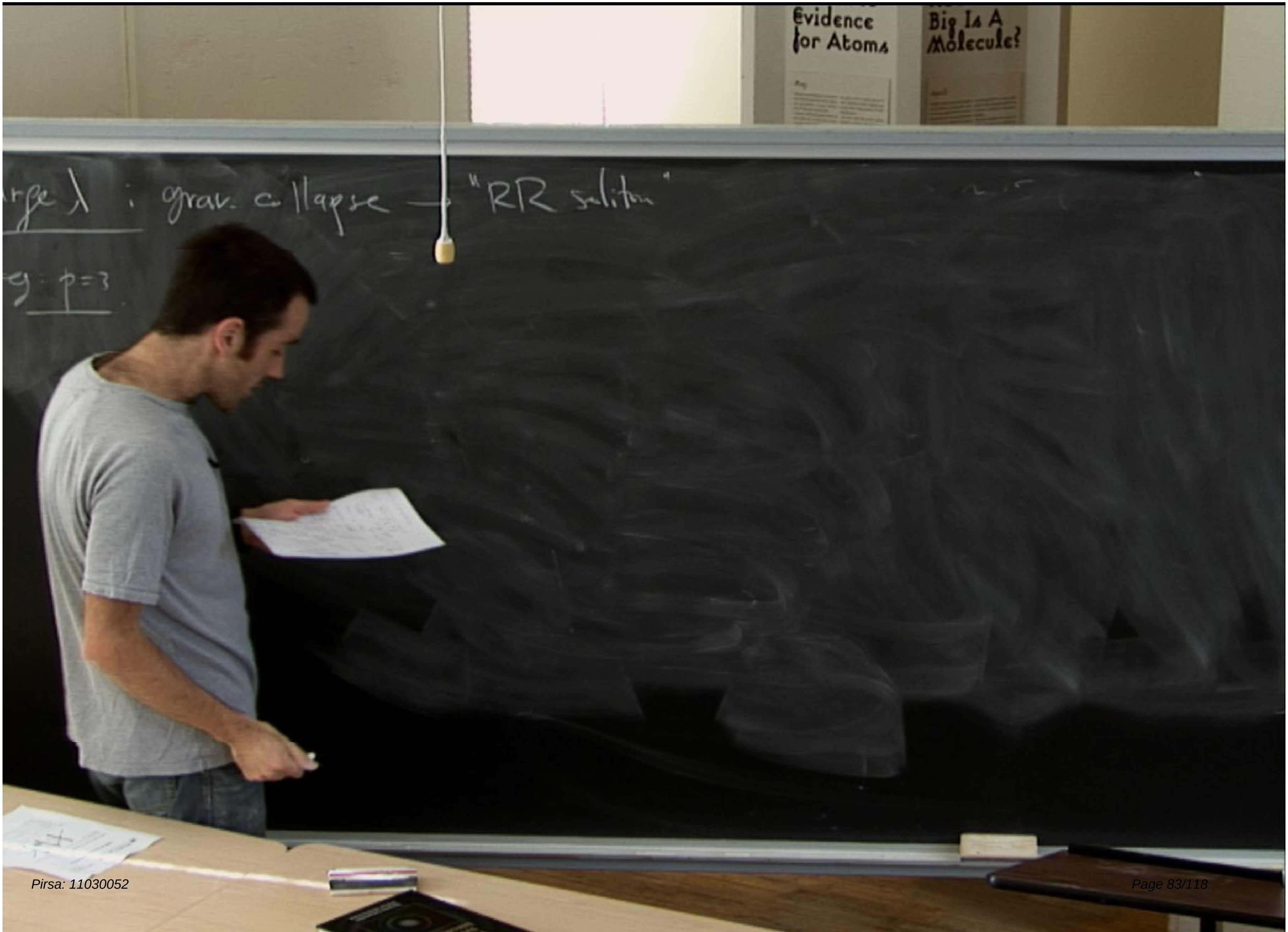
$\underbrace{\text{closed string}}_{\text{exc. of vacuum}} + \underbrace{\text{open string}}_{\text{exc. of D-brane}}$ in $\mathbb{R}^{9,1}$

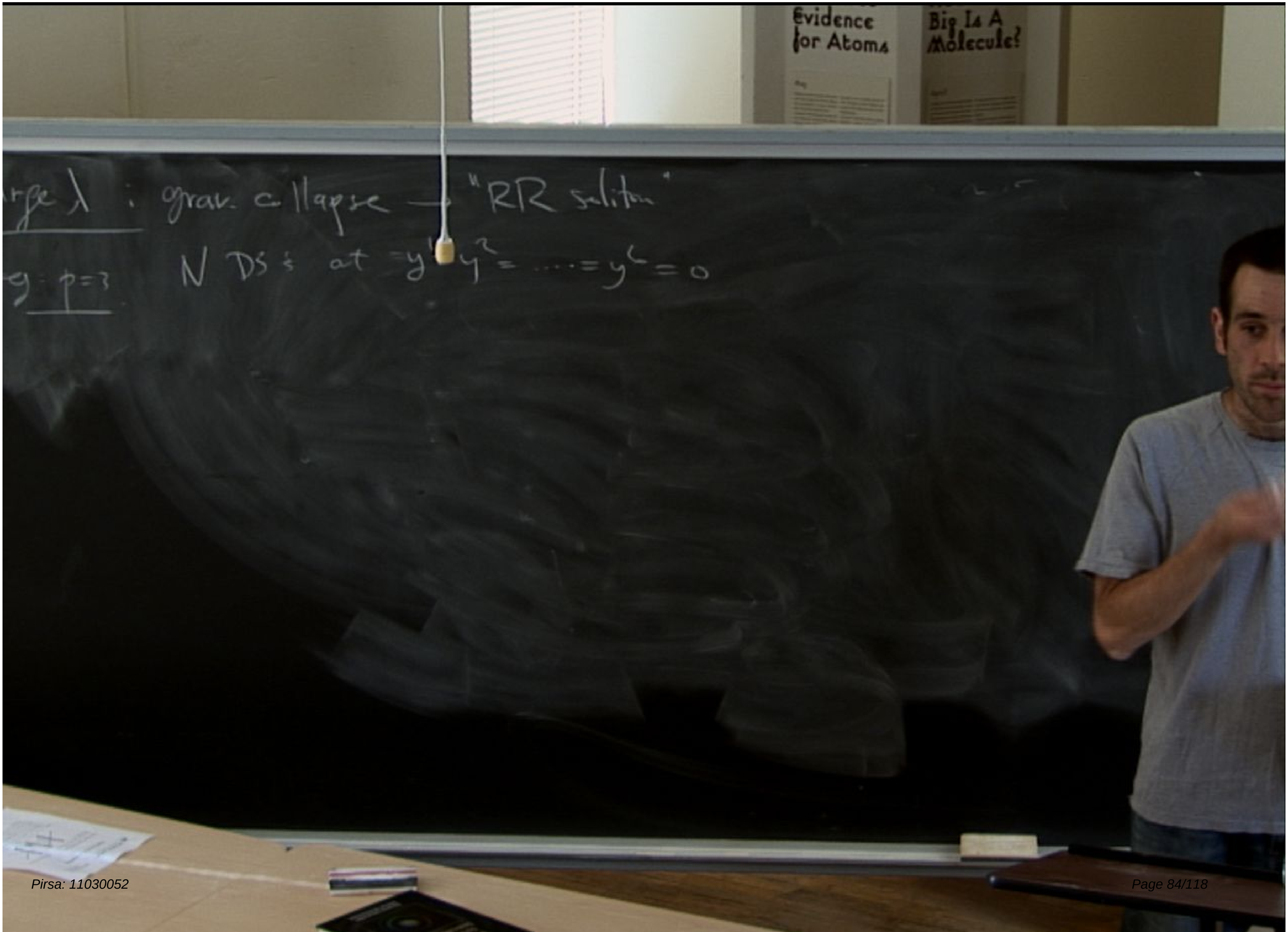
at low E

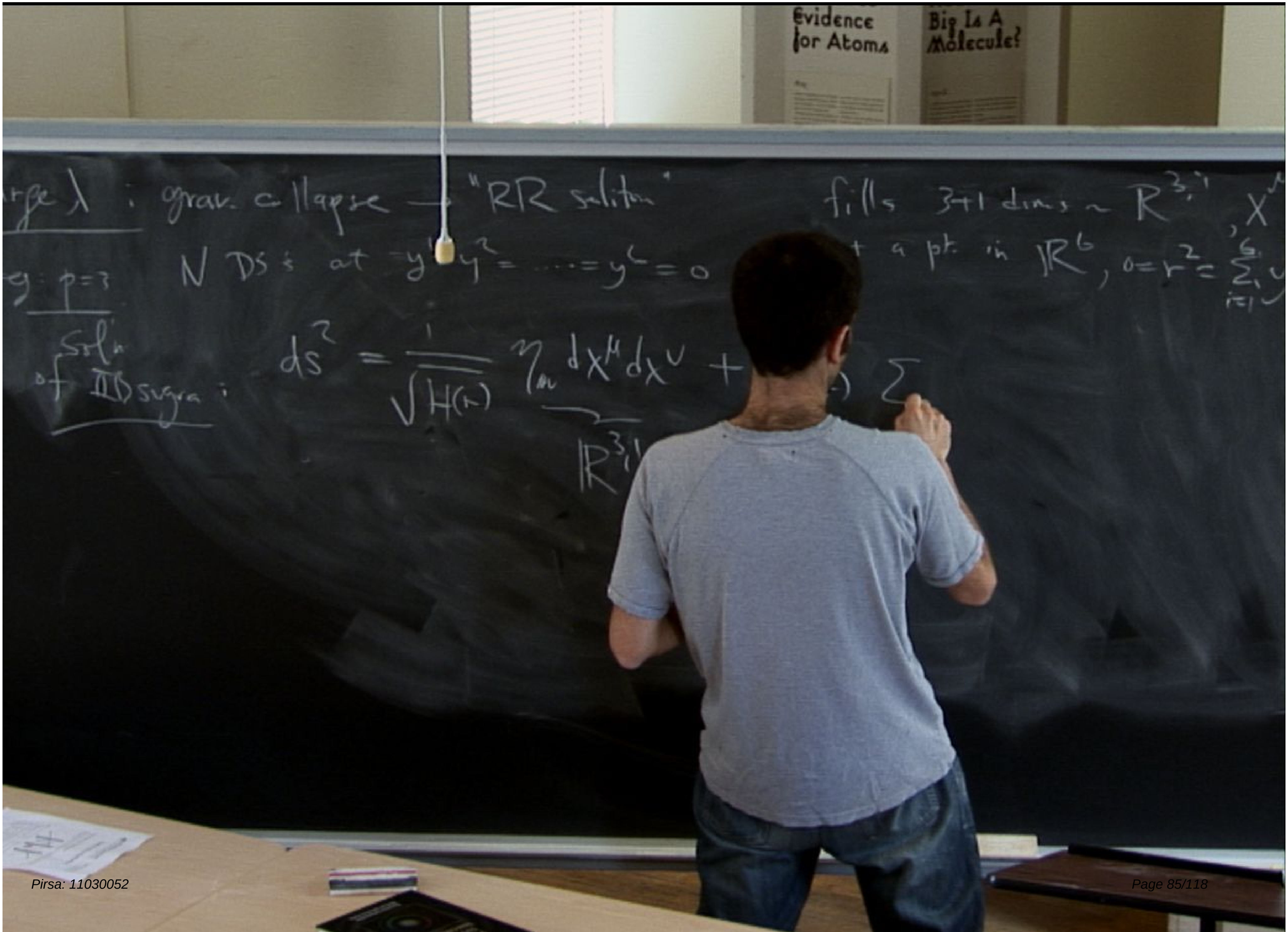
$\simeq$ IIB Sugra $+ \alpha'^{\#} R^{\#}$ corrections $\quad \simeq \underset{\text{gauge th}}{YM} + (\alpha')^2 F^4 \ldots$

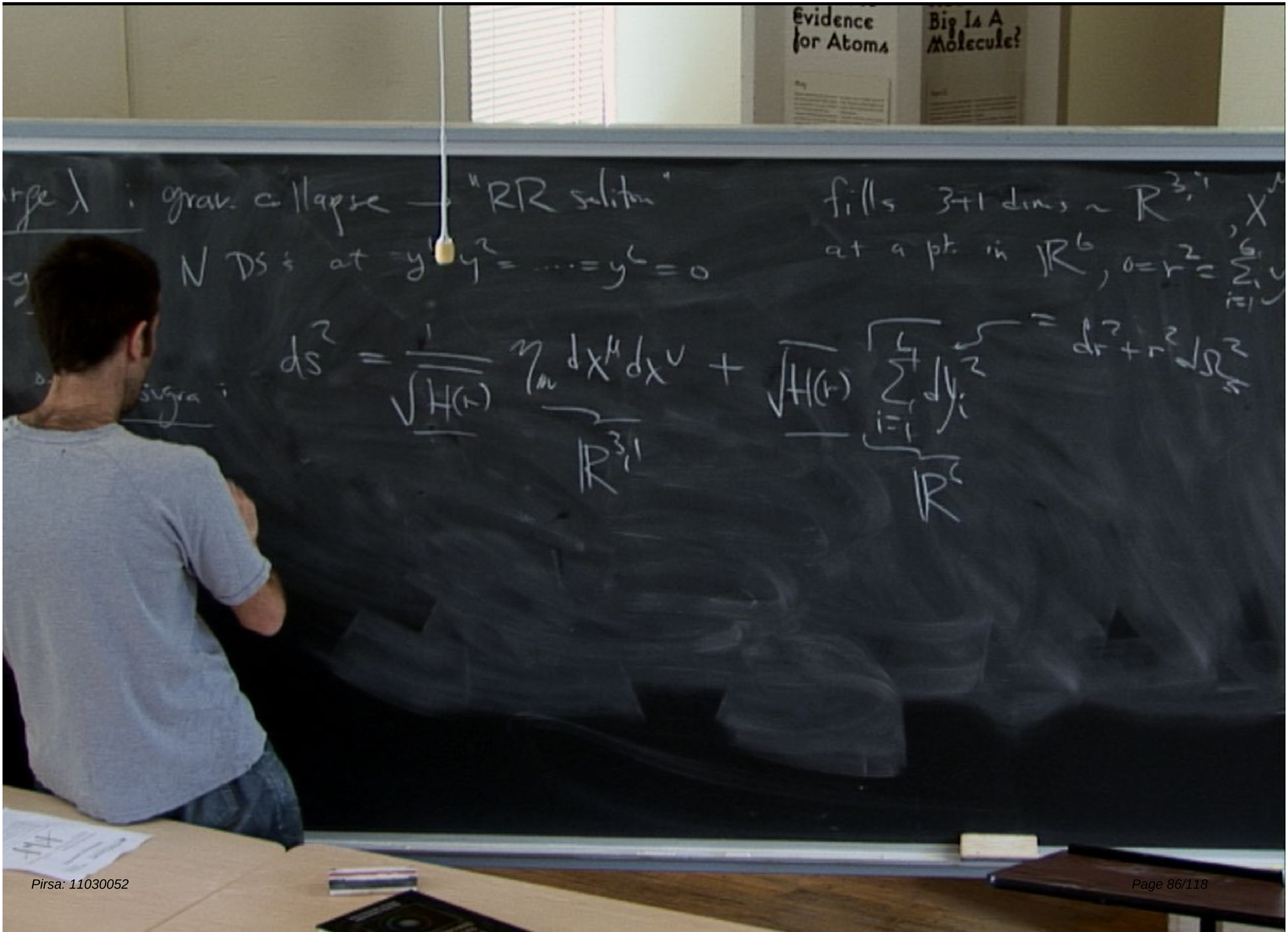

Evidence
for Atoms
Big Is A
Molecule?
λ : grav. collapse → "RR soliton"


Evidence for Atoms
Big Is A Molecule?
pe λ : grav. collapse → "RR soliton"


Evidence for Atoms
Big Is A Molecule?
grav. collapse → "RR soliton"
p=3


Evidence for Atoms
Big Is A Molecule?
rge λ : grav. collapse → "RR soliton"
g: p=3 N D5's at y^1 = y^2 = ... = y^6 = 0


Evidence for Atoms
Big Is A Molecule?
grav. collapse → "RR soliton"
fills 3+1 dims ~ $\mathbb{R}^{3,1}$
N D3's at $y^1 = y^2 = \cdots = y^6 = 0$
a pt. in $\mathbb{R}^6$, $0 = r^2 = \sum_{i=1}^{6}$
p=3
Sol'n of IIB sugra:
$ds^2 = \frac{1}{\sqrt{H(r)}} \eta_{\mu\nu} dx^\mu dx^\nu + \ldots \sum$
$\mathbb{R}^{3,1}$


Evidence for Atoms
Big Is A Molecule?
rge λ : grav. collapse → "RR soliton"
fills 3+1 dims ~ $\mathbb{R}^{3,1}$, X^μ
N Ds at $y^1 = y^2 = \cdots = y^6 = 0$
at a pt. in $\mathbb{R}^6$, $0 = r^2 = \sum_{i=1}^{6} y$
sugra :
$ds^2 = \underbrace{\frac{1}{\sqrt{H(r)}} \eta_{\mu\nu} dx^\mu dx^\nu}_{\mathbb{R}^{3,1}} + \sqrt{H(r)} \underbrace{\sum_{i=1}^{6} dy_i^2}_{\mathbb{R}^6}$
$= dr^2 + r^2 d\Omega_5^2$

rge λ : grav. collapse $\to$ "RR soliton" fills 3+1 dims ~ $\mathbb{R}^{3,1}$, X^μ

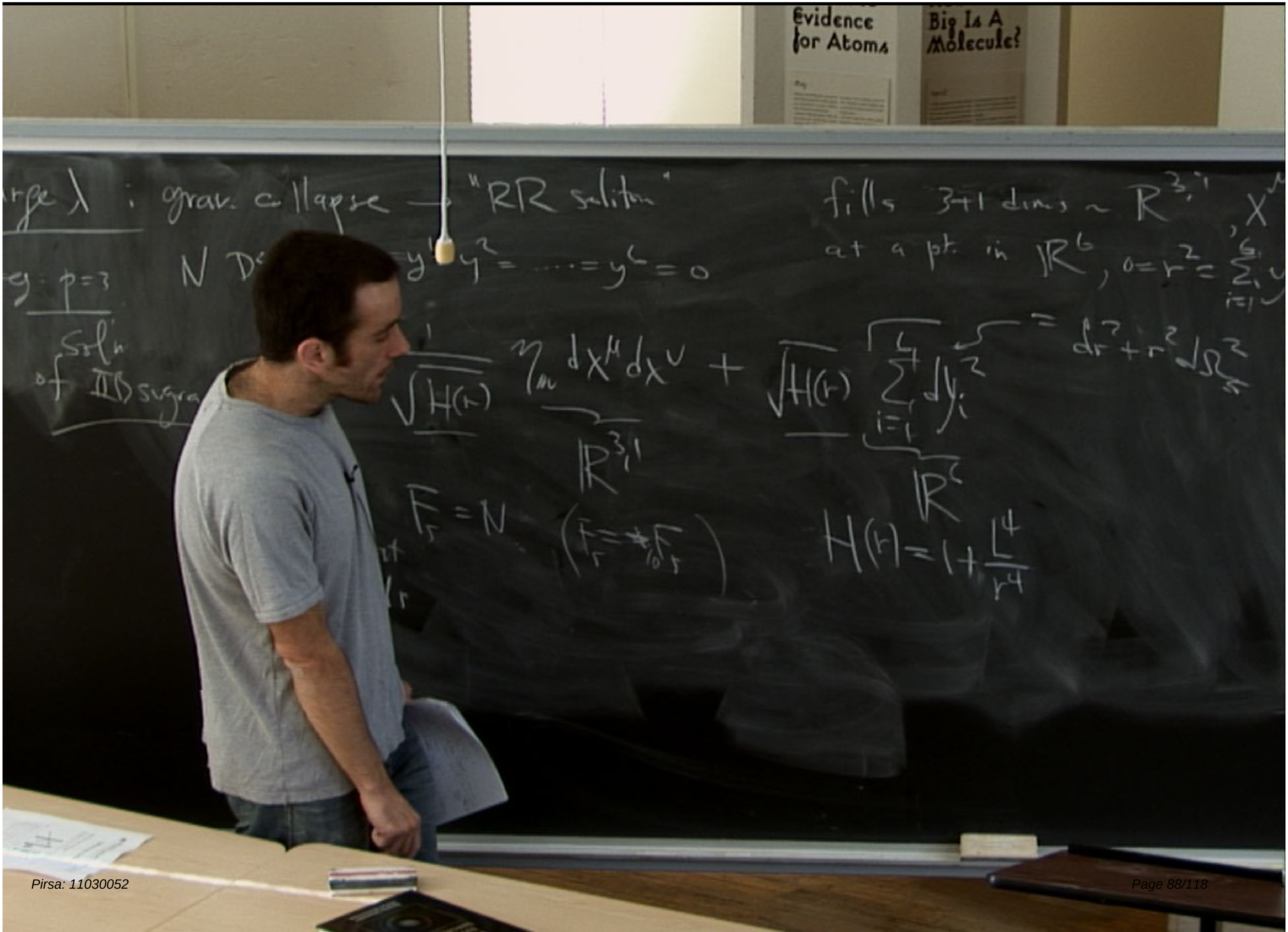
eg: $p=3$ N D3's at $y^1 = y^2 = \cdots = y^6 = 0$ at a pt. in $\mathbb{R}^6$, $0 = r^2 = \sum_{i=1}^{6} y_i^2$

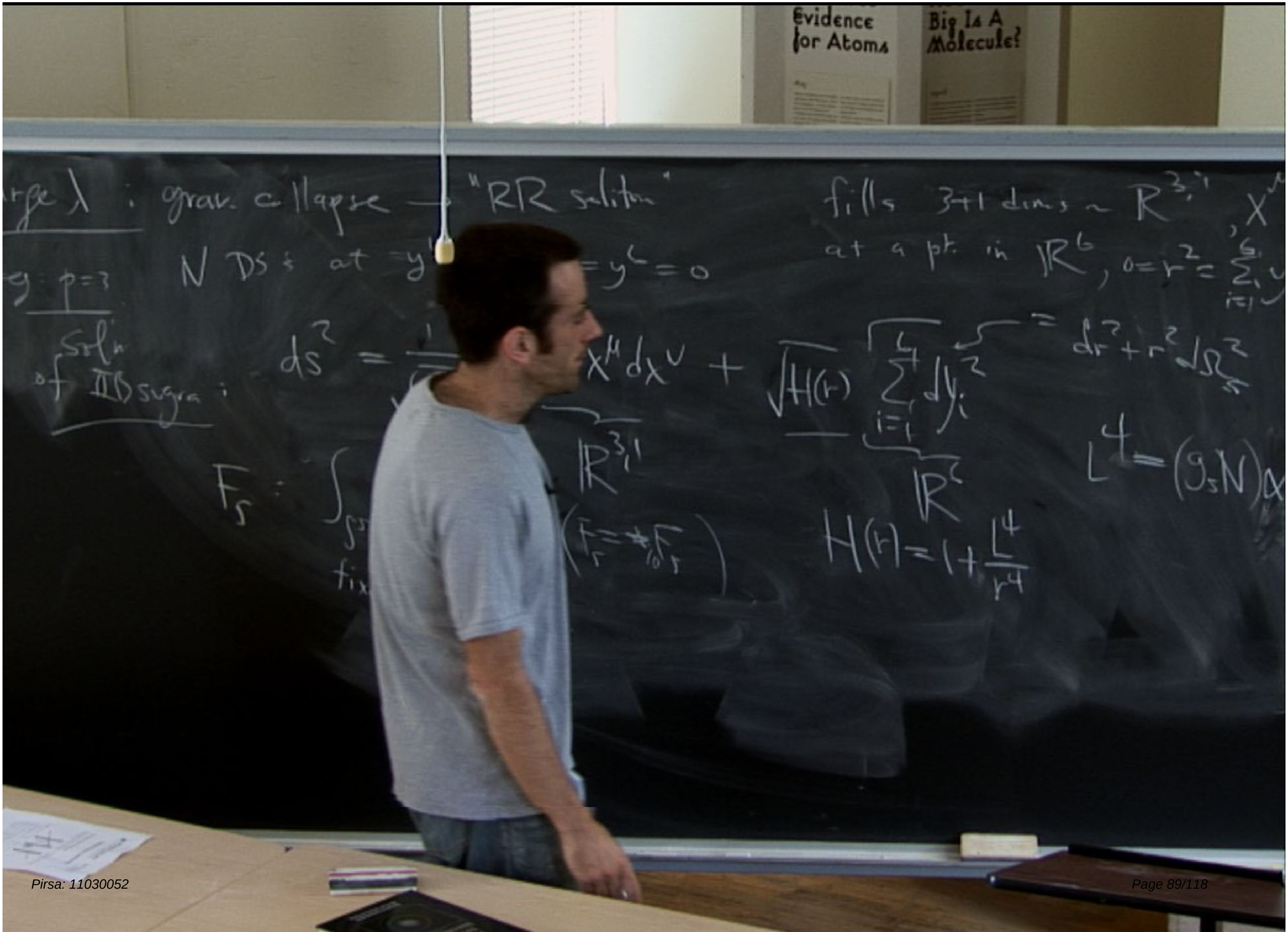
Sol'n of IIB sugra:

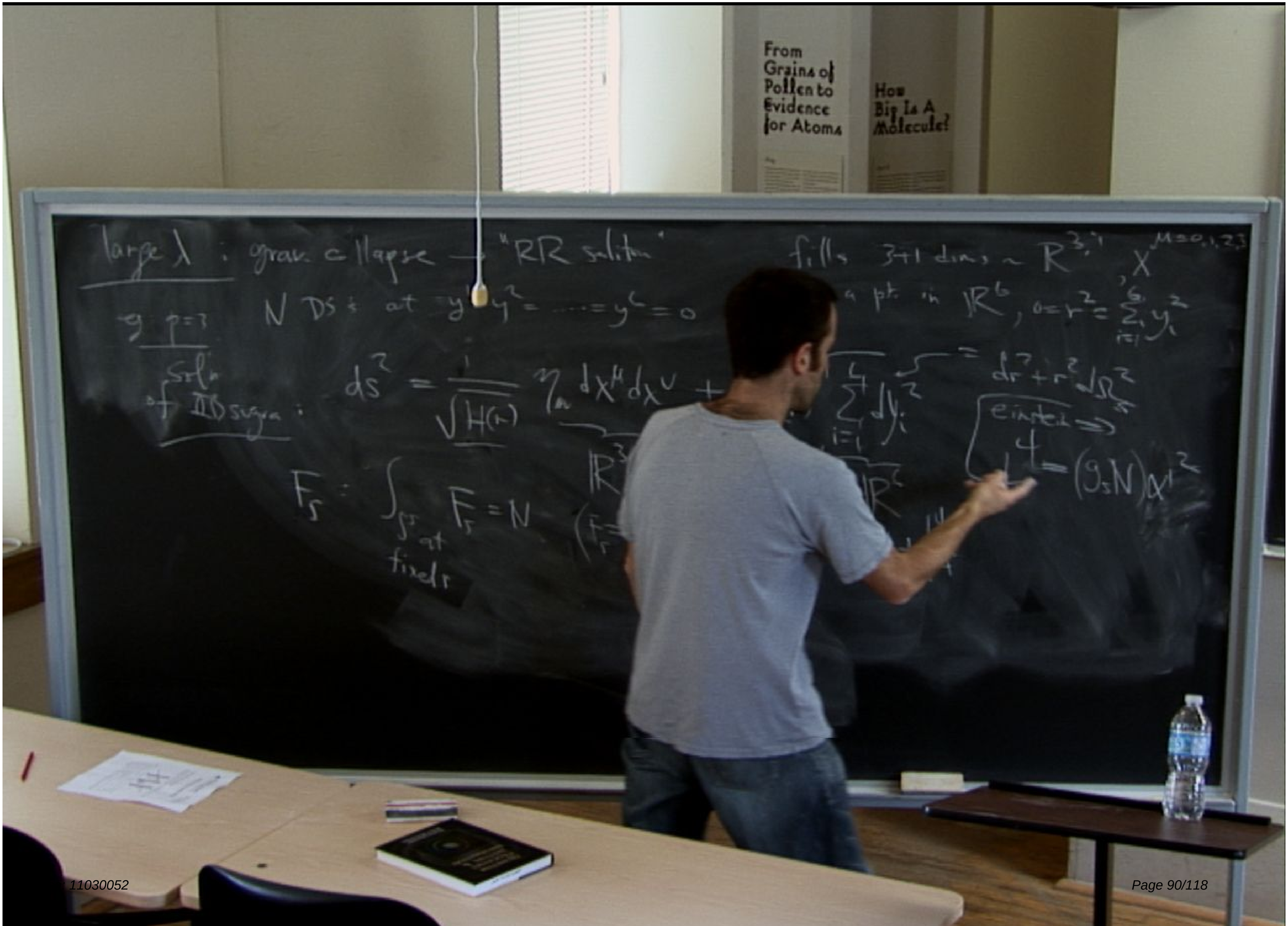
$$ds^2 = \frac{1}{\sqrt{H(r)}} \underbrace{\eta_{\mu\nu} dx^\mu dx^\nu}_{\mathbb{R}^{3,1}} + \sqrt{H(r)} \underbrace{\sum_{i=1}^{6} dy_i^2}_{\mathbb{R}^6}$$

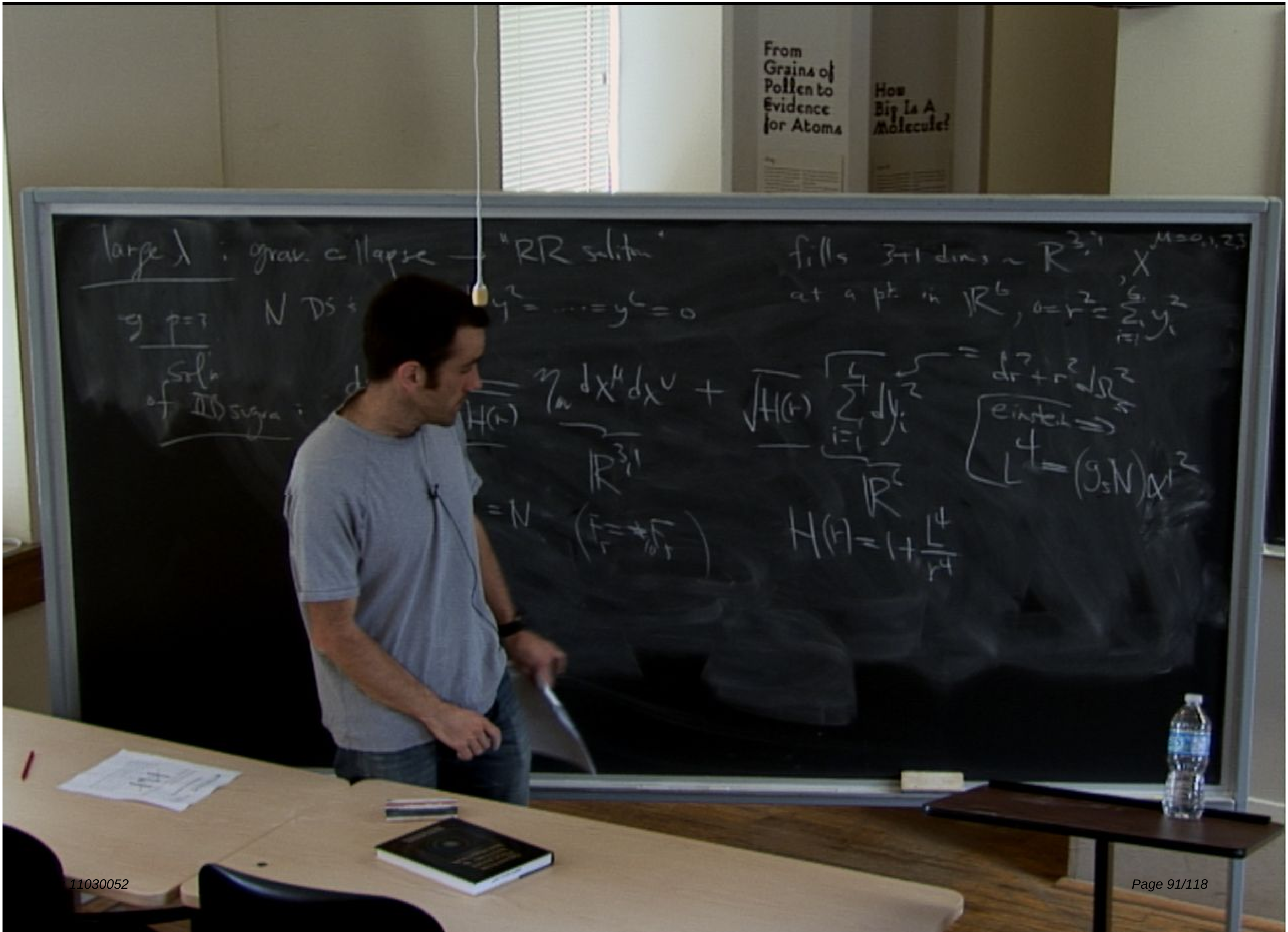
$$\sum_{i=1}^{6} dy_i^2 = dr^2 + r^2 d\Omega_5^2$$

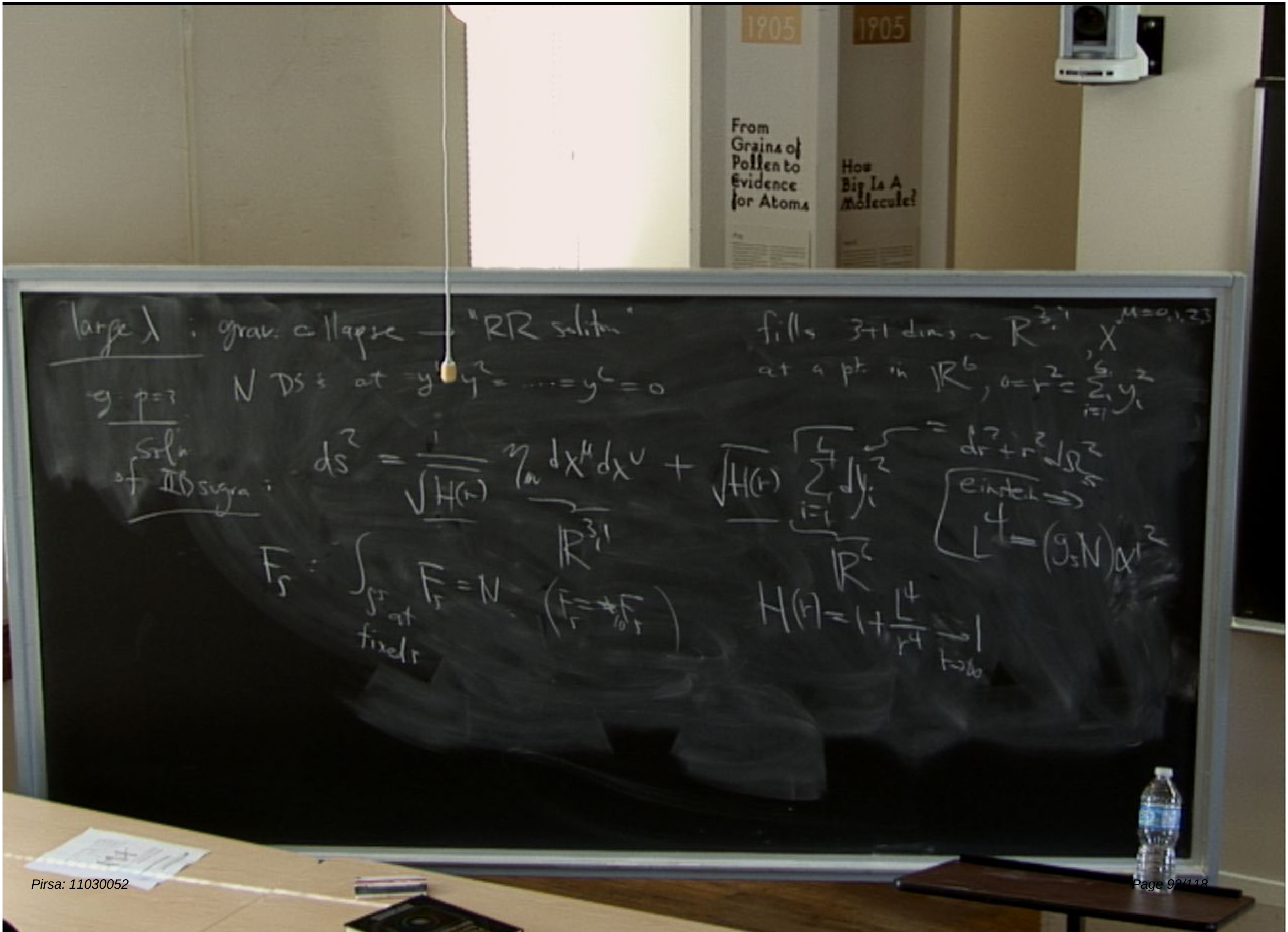
F_5 : $\int$

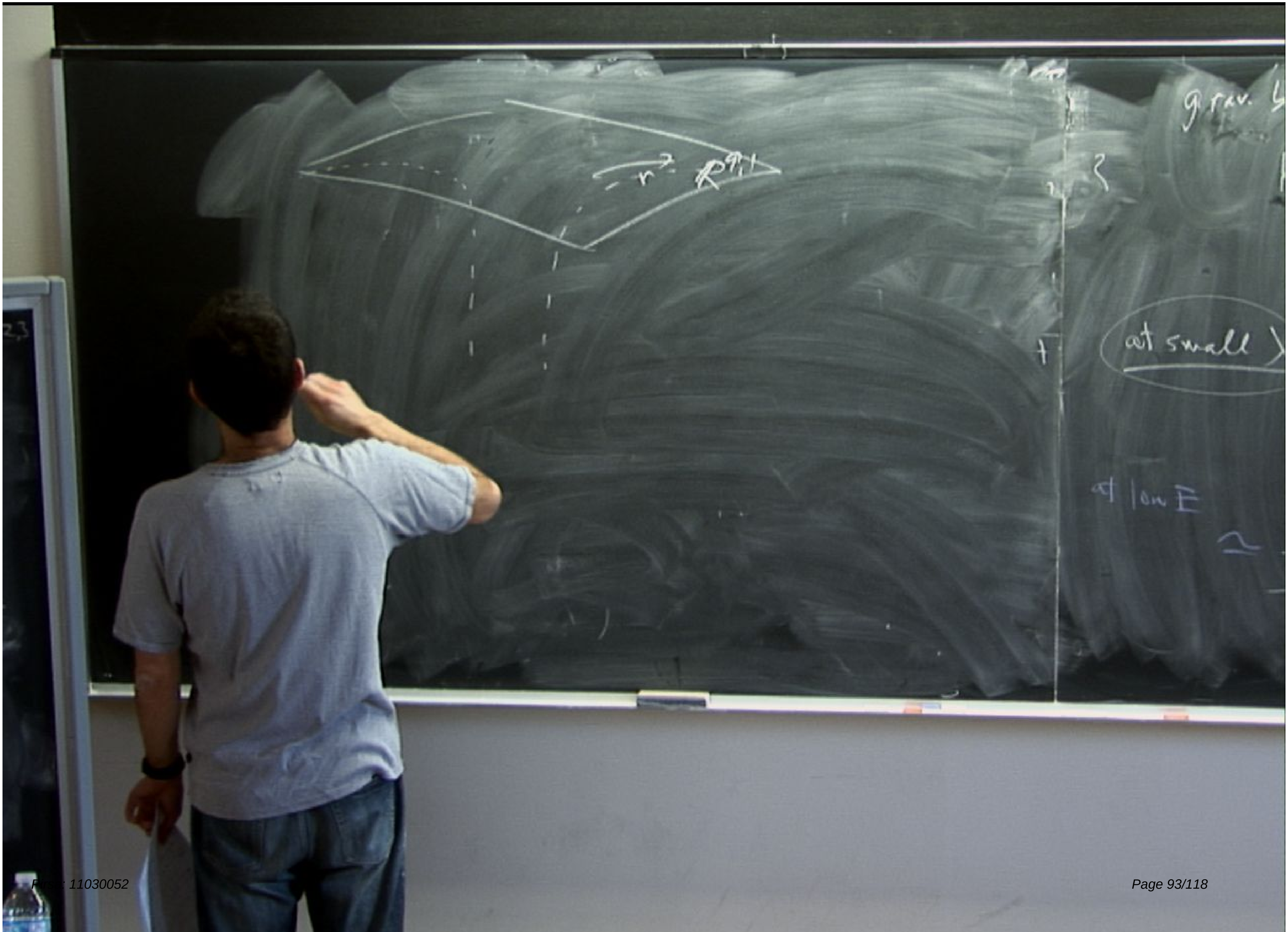

Evidence for Atoms
Big Is A Molecule?
grav. collapse → "RR soliton"
fills 3+1 dims ~ R^{3,1}, X^μ
N D
y^1 = y^2 = ... = y^6 = 0
at a pt. in R^6, 0 = r^2 = Σ_{i=1}^{6} y
e.g.: p=3
Sol^n of IIB sugra
1/√H(r) η_μν dx^μ dx^ν + √H(r) Σ_{i=1}^{6} dy_i^2
= dr^2 + r^2 dΩ_5^2
R^{3,1}
R^6
F_5 = N (F_5 = *_{10} F_5)
H(r) = 1 + L^4/r^4

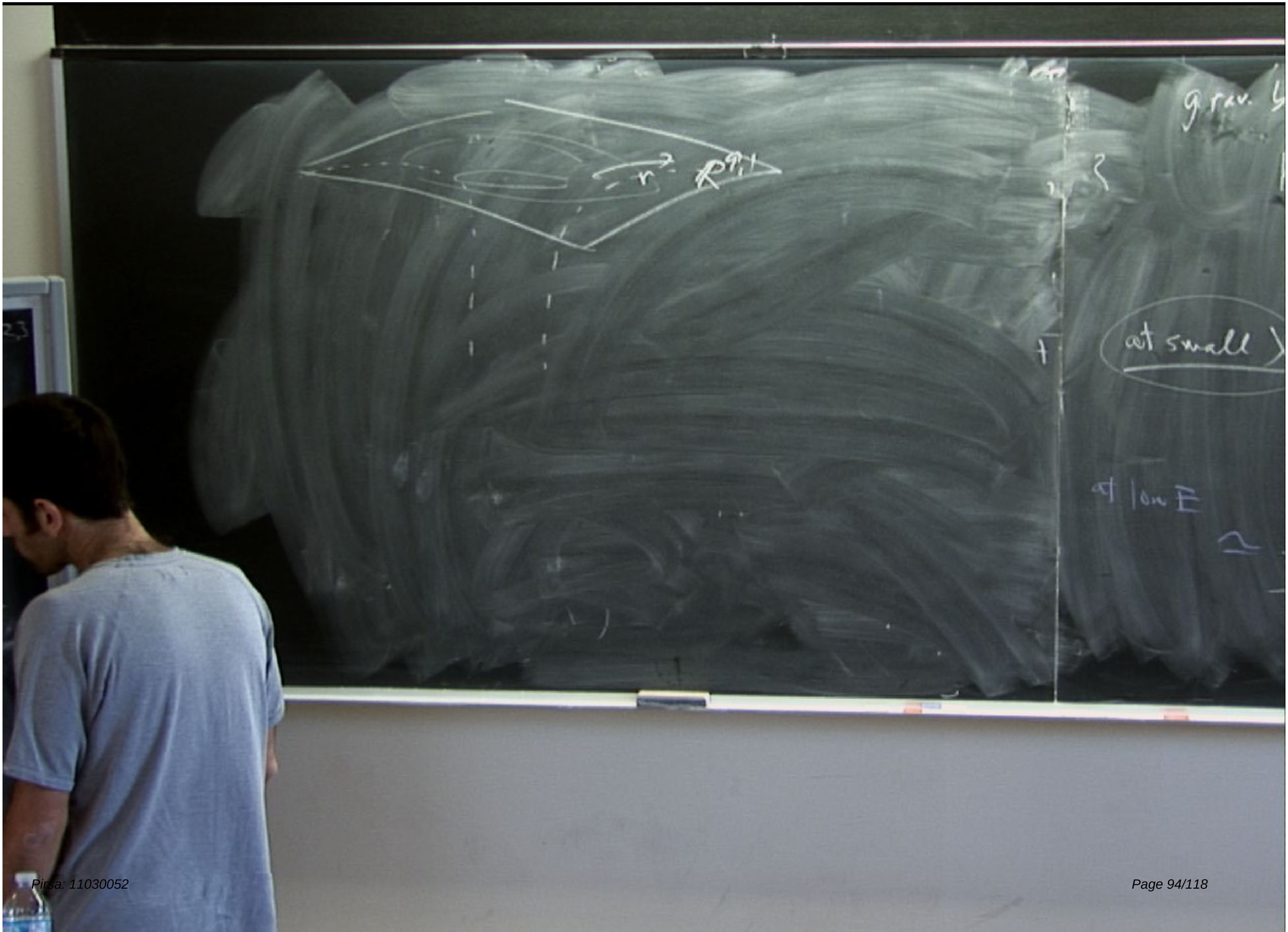

Evidence
for Atoms
Big Is A
Molecule?

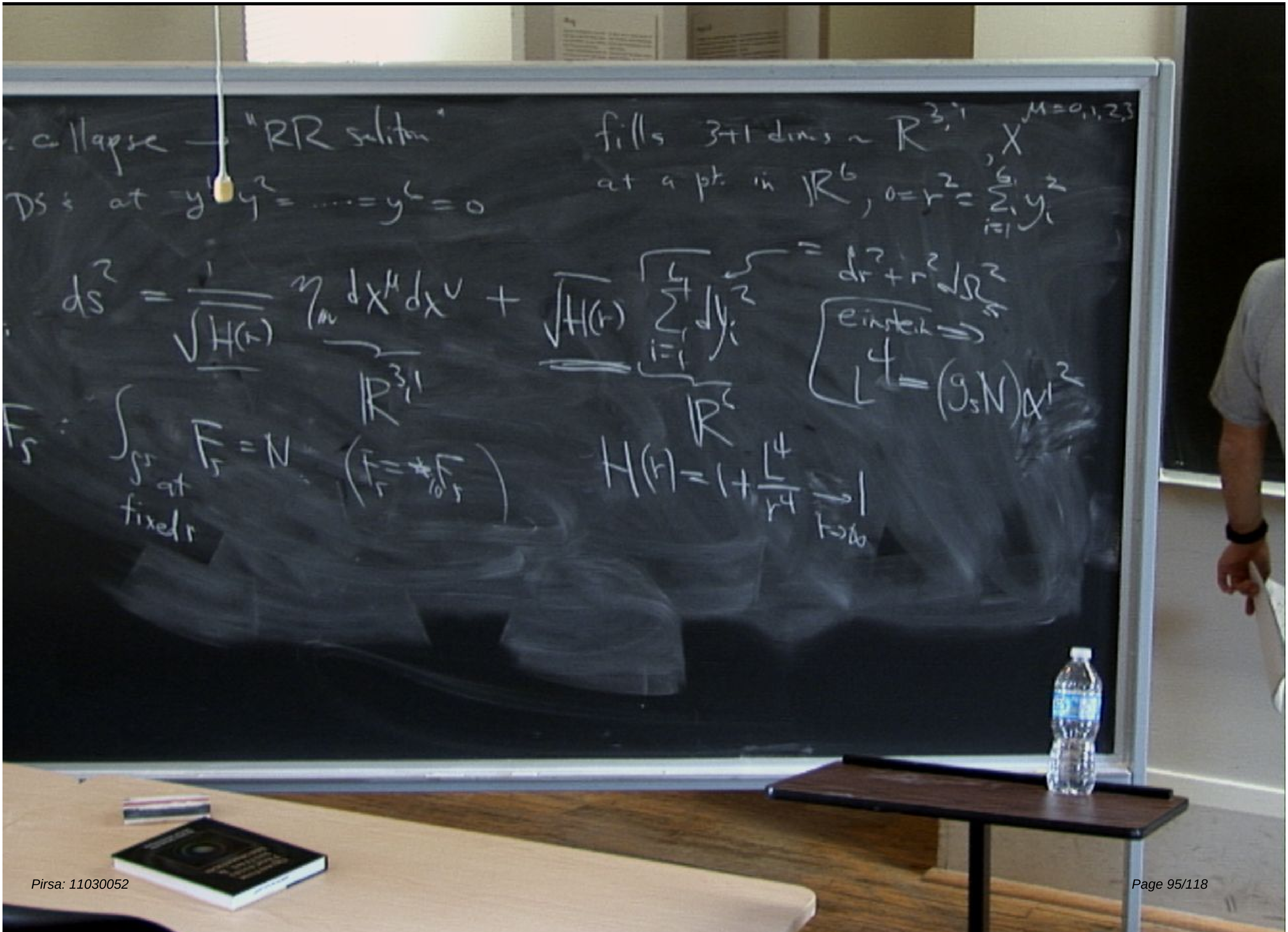

From Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
large λ : grav. collapse → "RR soliton"
fills 3+1 dims ~ R^{3,1}
einstein ⇒
fixed r


From Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
large λ : grav. collapse → "RR soliton"
fills 3+1 dims ~ $\mathbb{R}^{3,1}$, $X^{\mu=0,1,2,3}$
at a pt. in $\mathbb{R}^6$, $0=r^2=\sum_{i=1}^{6} y_i^2$
$H(r)=1+\frac{L^4}{r^4}$
einstein ⇒ $L^4=(g_sN)\alpha'^2$


1905
1905
From Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
large λ : grav. collapse → "RR soliton"
fills 3+1 dims ~ $\mathbb{R}^{3,1}$, $x^{\mu=0,1,2,3}$
eg $p=3$
N D3's at $y^1 = y^2 = \dots = y^6 = 0$
at a pt. in $\mathbb{R}^6$, $0 = r^2 = \sum_{i=1}^{6} y_i^2$
Soln of IIB sugra :
$ds^2 = \frac{1}{\sqrt{H(r)}} \underbrace{\eta_{\mu\nu} dx^\mu dx^\nu}_{\mathbb{R}^{3,1}} + \sqrt{H(r)} \underbrace{\sum_{i=1}^{6} dy_i^2}_{\mathbb{R}^6}$
$= dr^2 + r^2 d\Omega_5^2$
einstein ⇒ $L^4 = (g_s N)\alpha'^2$
F_5 : $\int_{S^5 \text{ at fixed } r} F_5 = N$ $(F_5 = * F_5)$
$H(r) = 1 + \frac{L^4}{r^4} \xrightarrow[r \to \infty]{} 1$


grav.
at small
at low E


$\mathbb{R}^{9,1}$
grav.
at small
at low E


collapse → "RR soliton"
fills 3+1 dims ~ $\mathbb{R}^{3,1}$, $X^{\mu=0,1,2,3}$
Ds at $y^1 = y^2 = \dots = y^6 = 0$
at a pt. in $\mathbb{R}^6$, $0 = r^2 = \sum_{i=1}^{6} y_i^2$
$ds^2 = \frac{1}{\sqrt{H(r)}} \eta_{\mu\nu} dx^\mu dx^\nu + \sqrt{H(r)} \sum_{i=1}^{6} dy_i^2$
$= dr^2 + r^2 d\Omega_5^2$
$\mathbb{R}^{3,1}$
$\mathbb{R}^6$
einstein ⇒
$L^4 = (g_s N) \alpha'^2$
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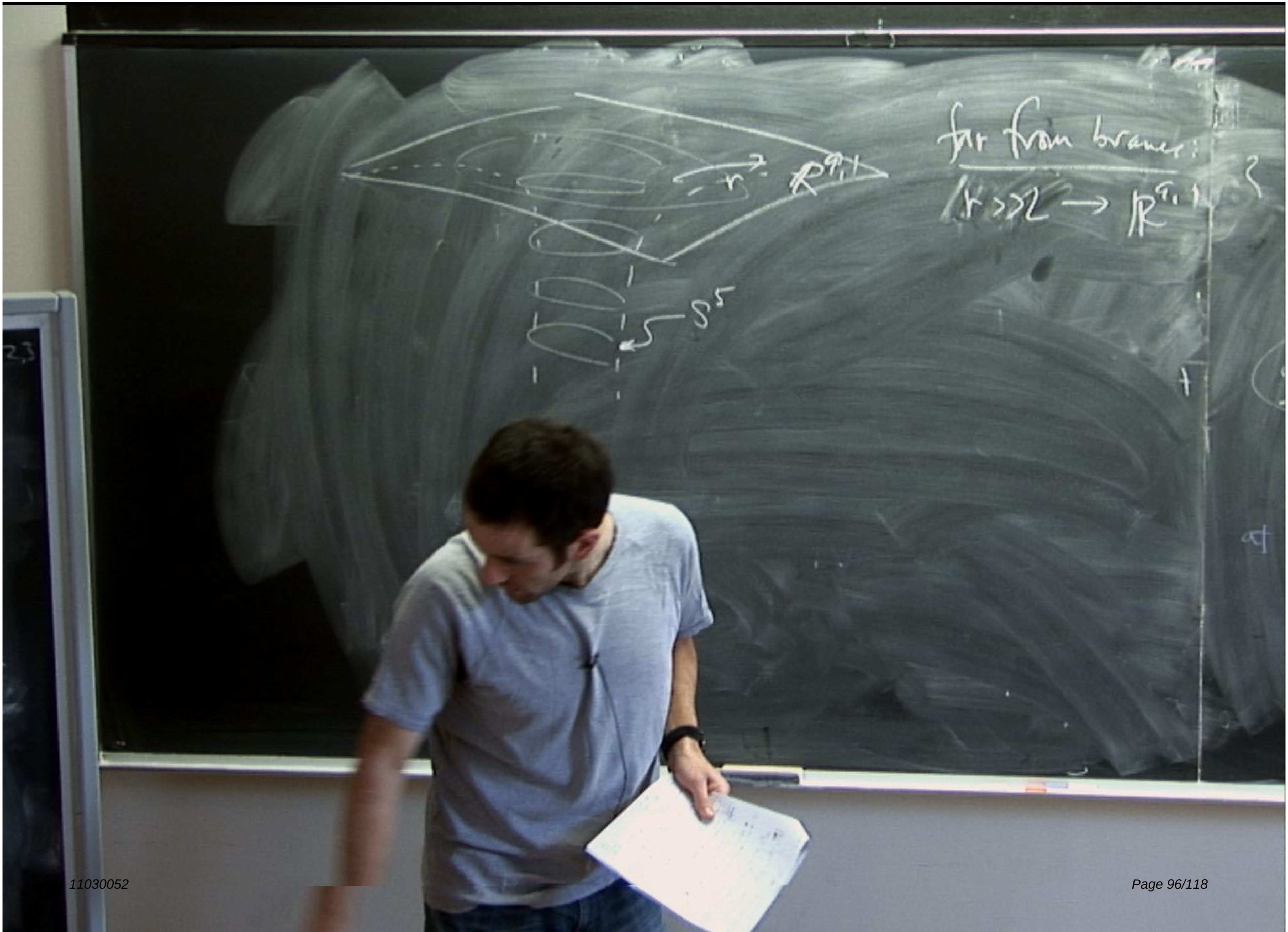
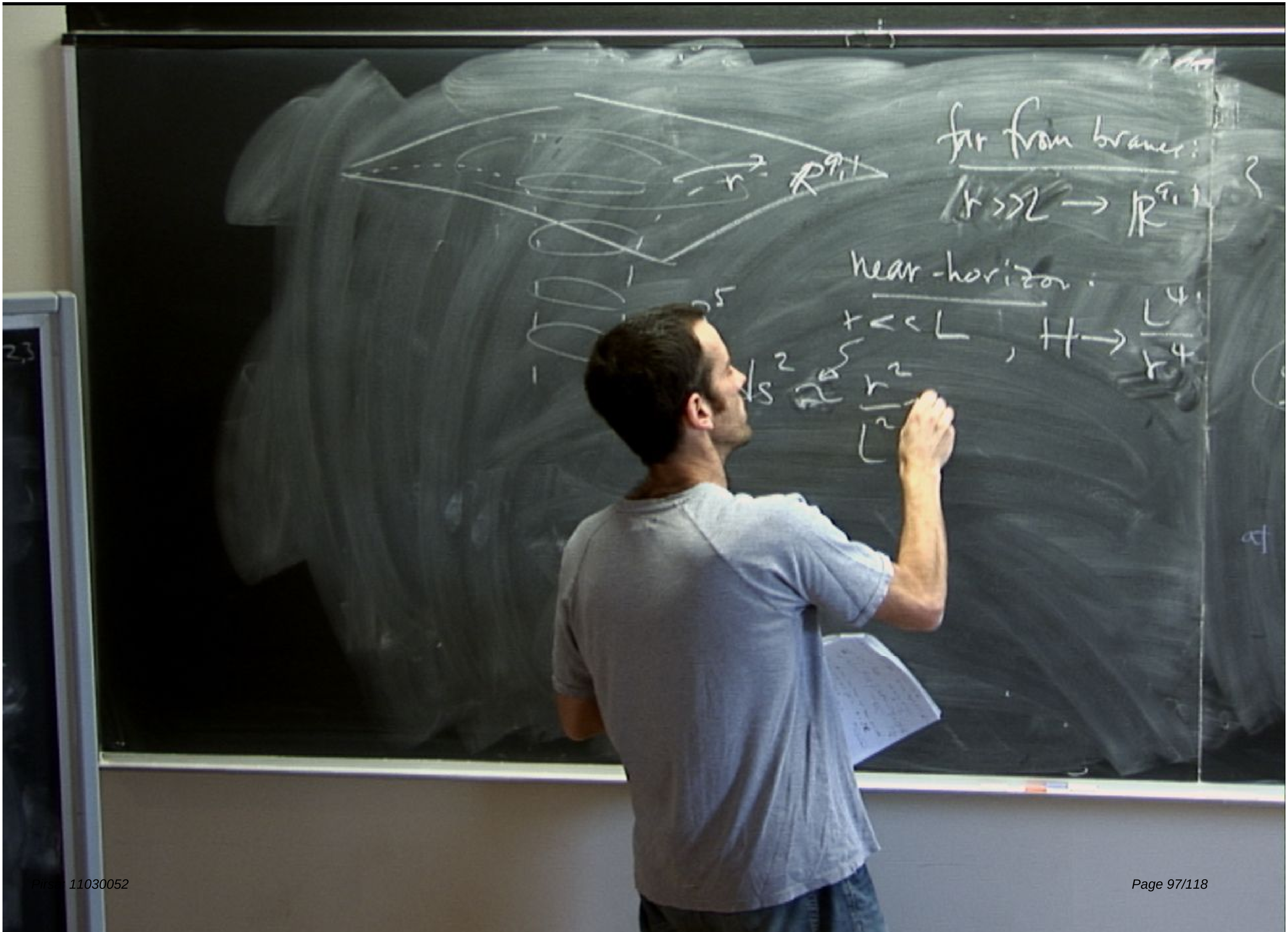

far from branes:
r >> L → R^9,1
R^9,1
S^5


far from branes:
r >> L → R^9,1
near-horizon:
r << L , H → L^4/r^4
ds^2 ≈ r^2/L^2
R^9,1

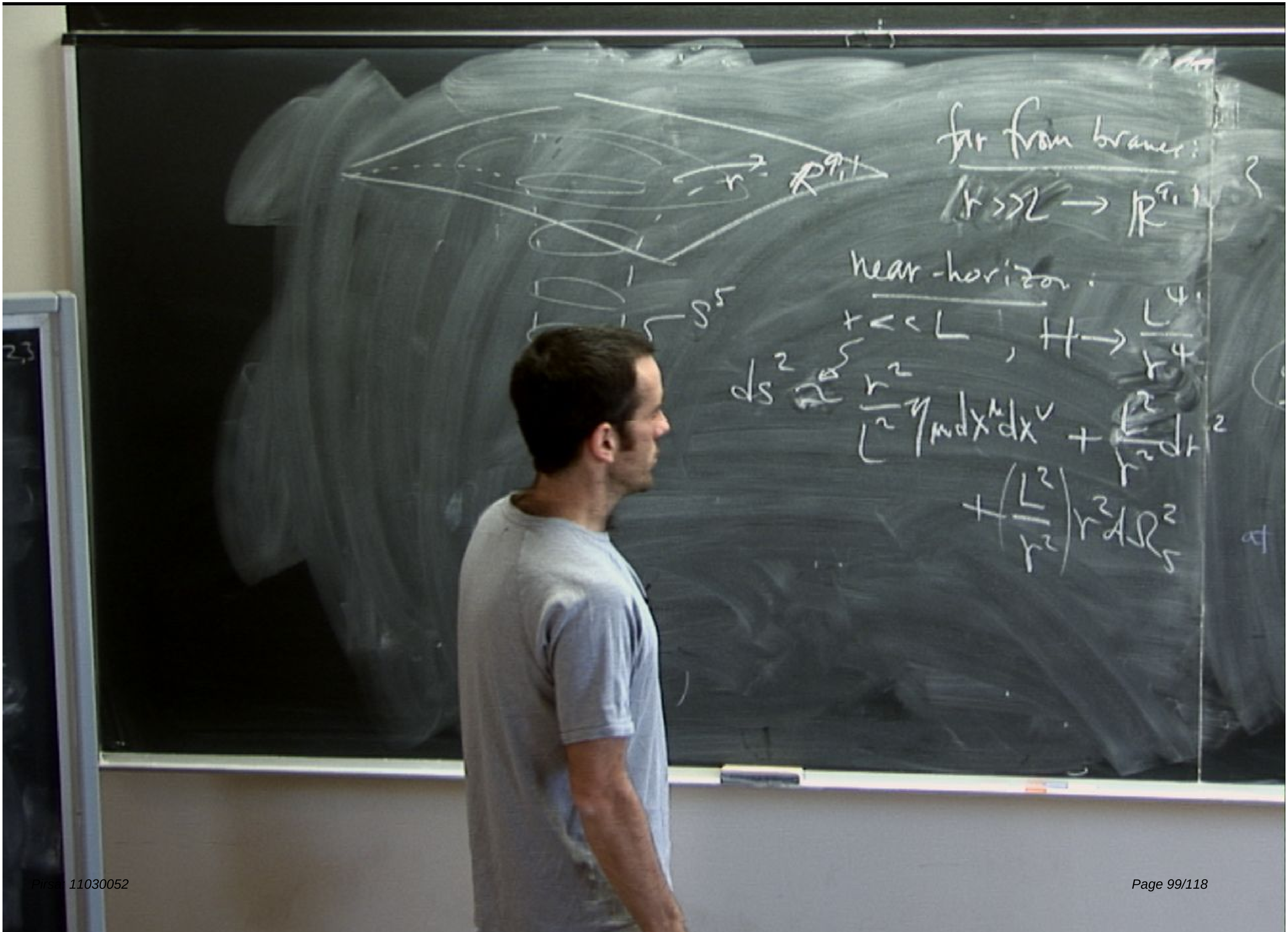
far from branes:

$r \gg L \longrightarrow \mathbb{R}^{9,1}$

near-horizon:

$r \ll L$, $H \longrightarrow \frac{L^4}{r^4}$

$$ds^2 \simeq \frac{r^2}{L^2}\eta_{\mu\nu}dx^\mu dx^\nu +$$



far from branes:
$r \gg L \rightarrow \mathbb{R}^{9,1}$
near-horizon:
$r \ll L$, $H \rightarrow \frac{L^4}{r^4}$
$ds^2 \simeq \frac{r^2}{L^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{L^2}{r^2}dr^2 + \left(\frac{L^2}{r^2}\right) r^2 d\Omega_5^2$
r
$\mathbb{R}^{9,1}$
S^5

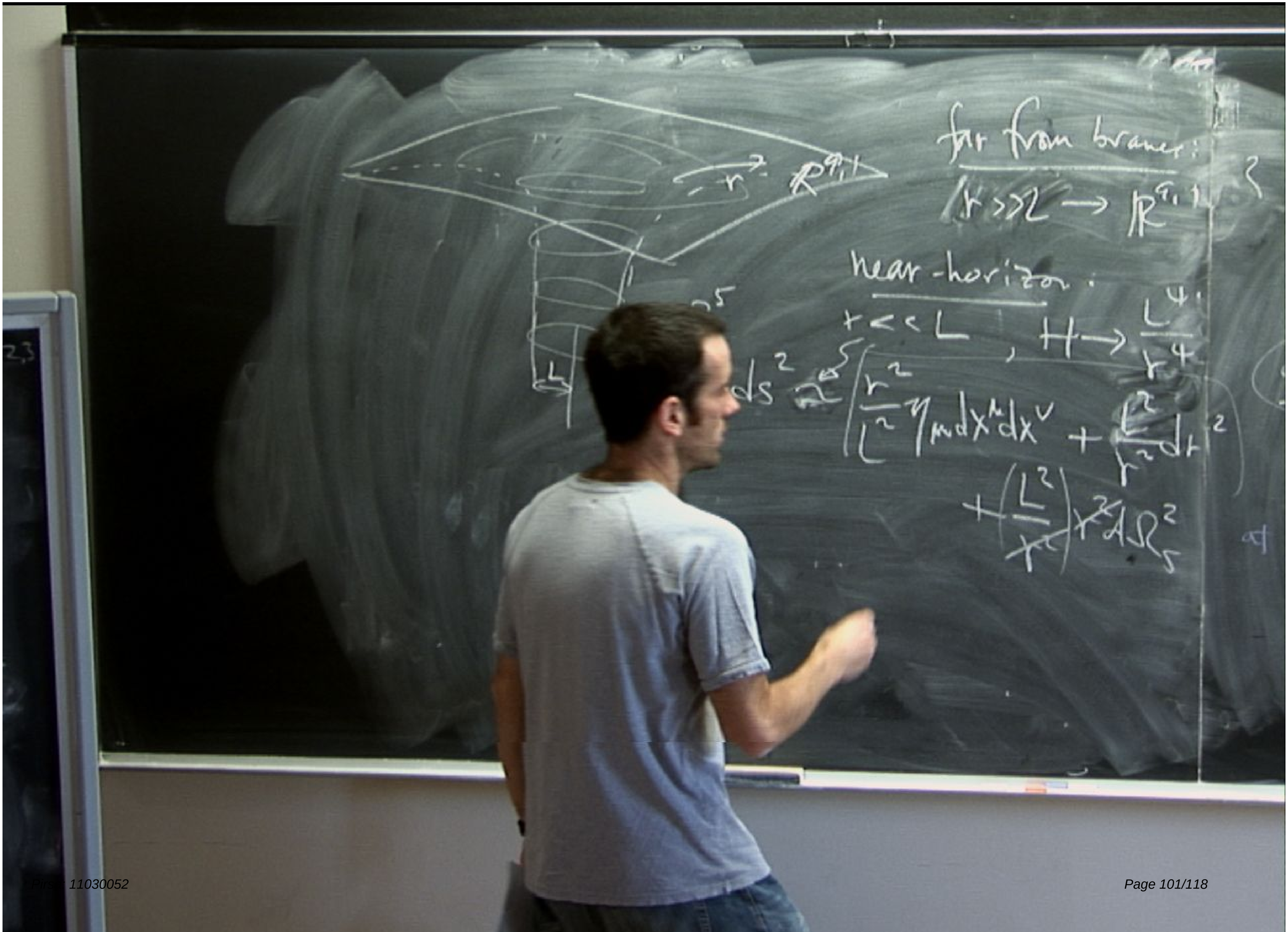
far from branes:

$r \gg L \to \mathbb{R}^{9,1}$

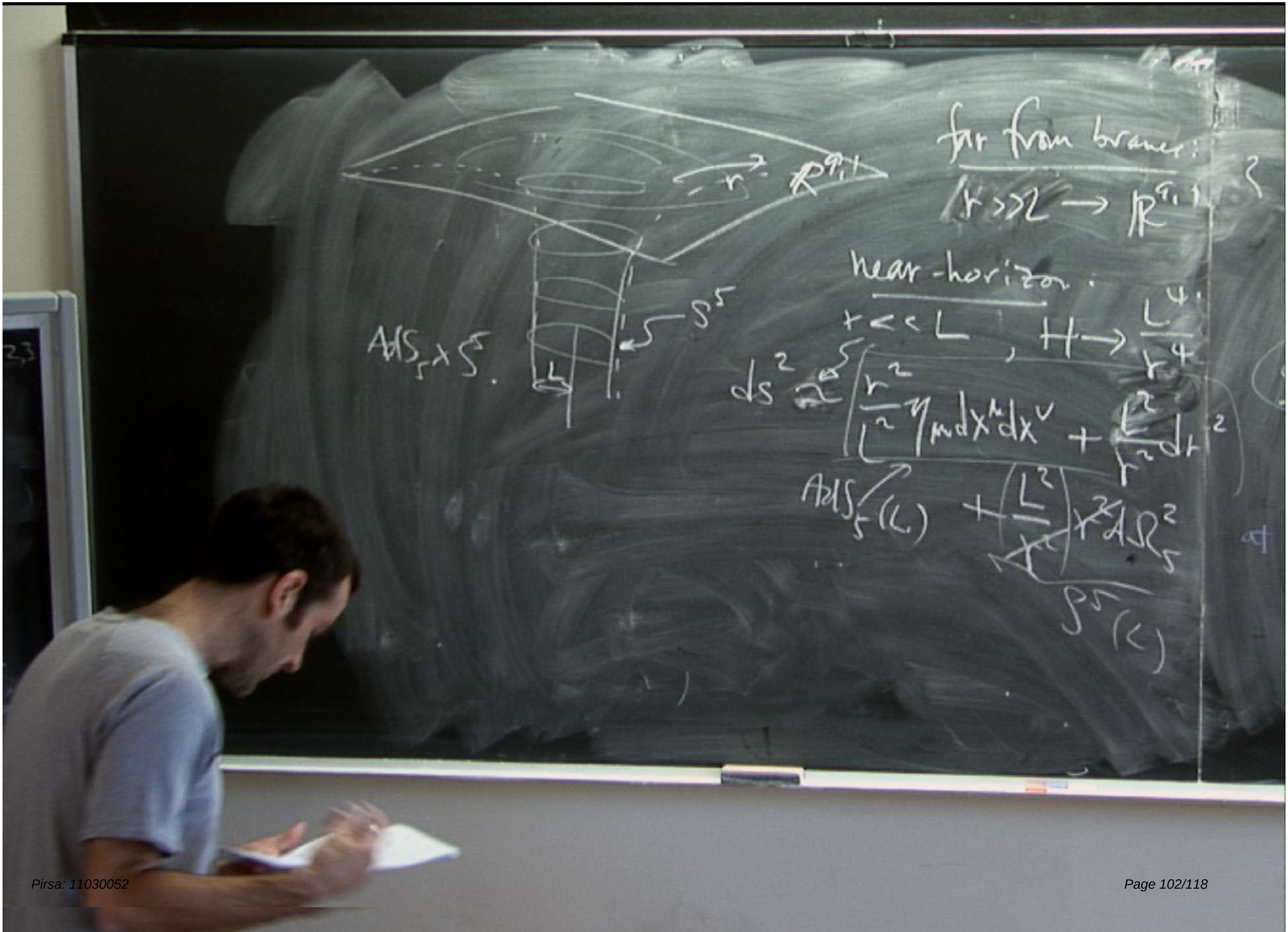
near-horizon:

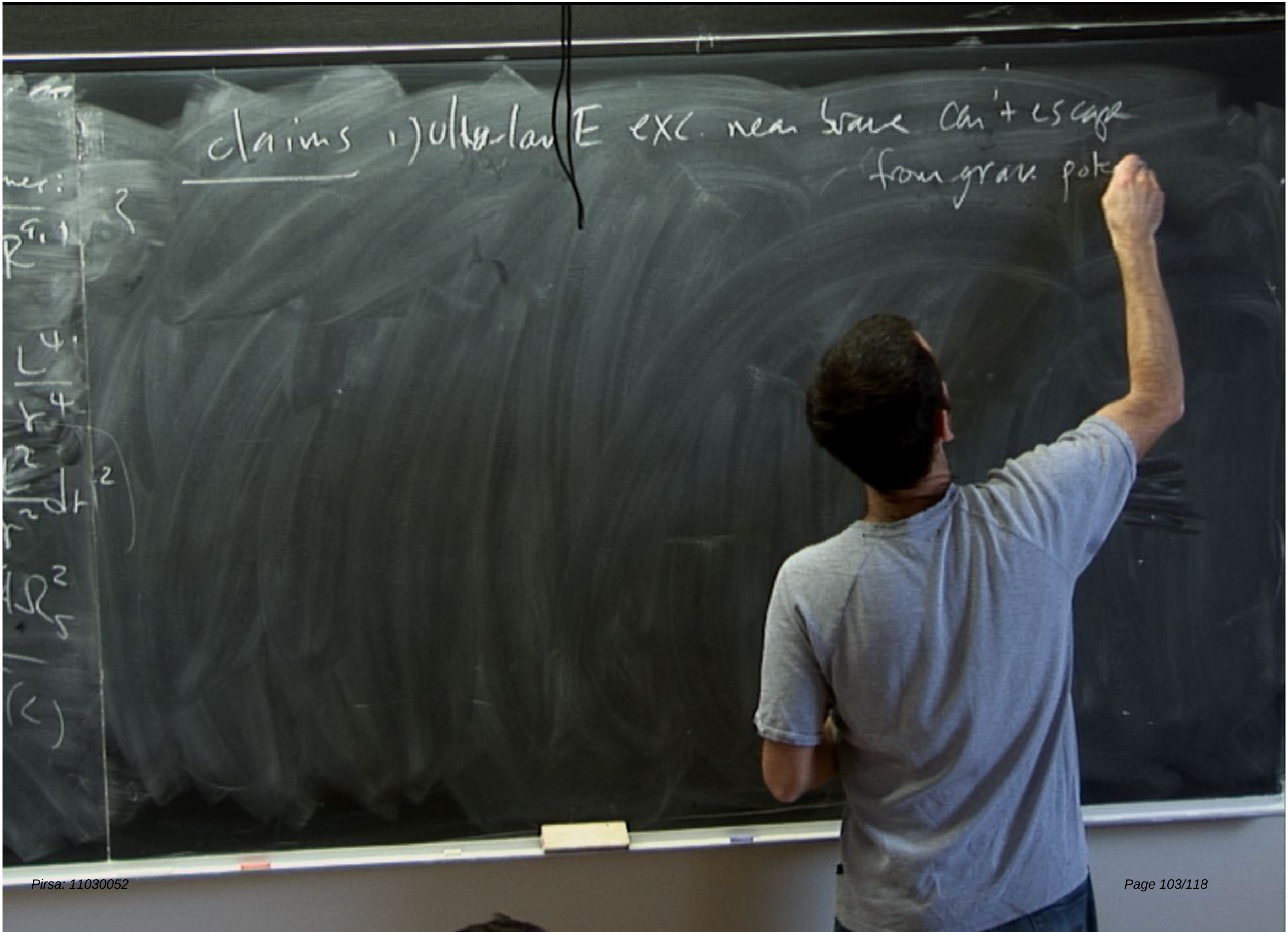
$r \ll L$, $H \to \frac{L^4}{r^4}$

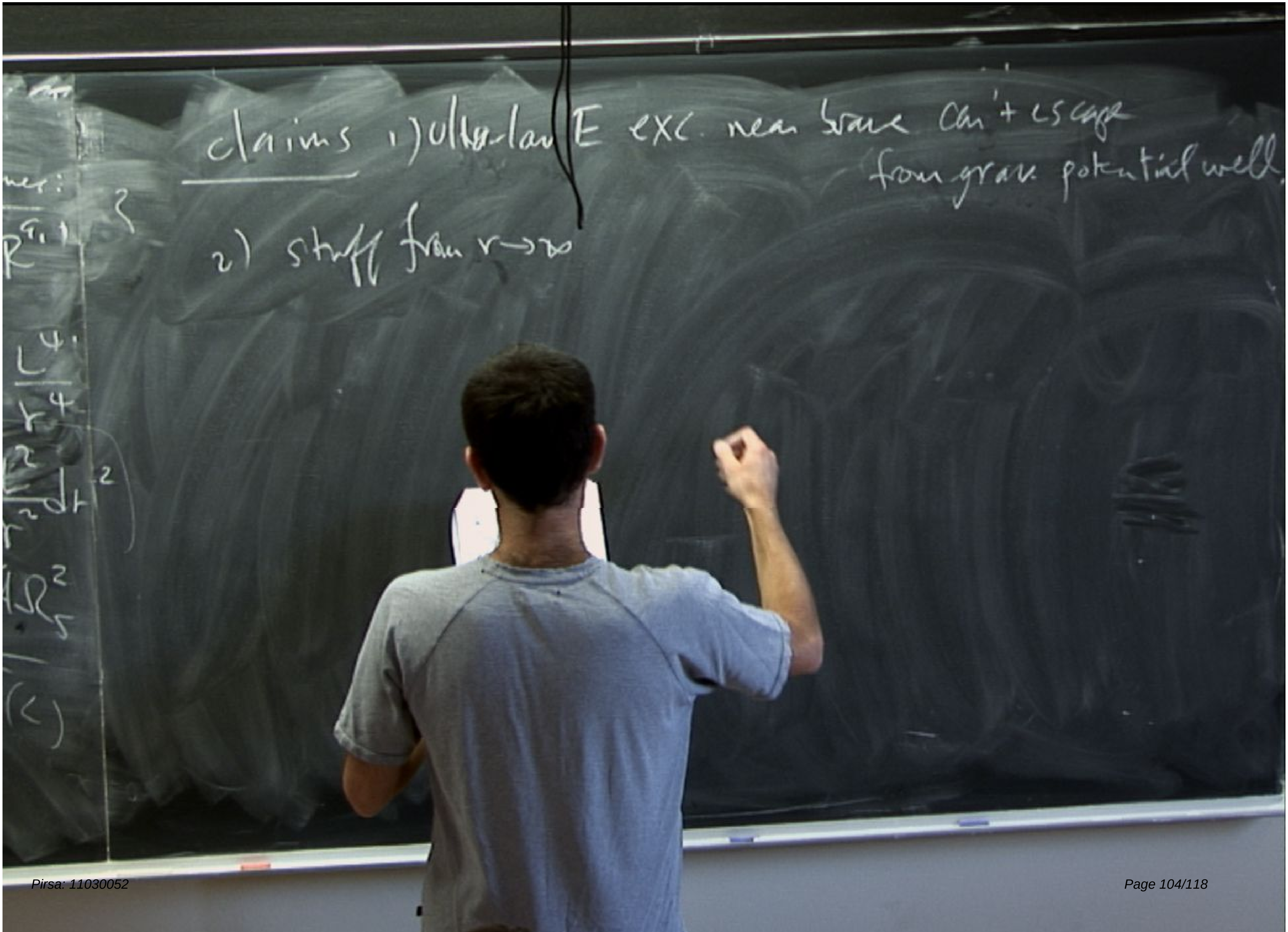
$$ds^2 \approx \frac{r^2}{L^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{L^2}{r^2}dr^2 + \left(\frac{L^2}{r^2}\right) r^2 d\Omega_5^2$$



r
$\mathbb{R}^{9,1}$
S^5
L
far from branes:
$r \gg L \longrightarrow \mathbb{R}^{9,1}$
near-horizon:
$r \ll L$, $H \to \frac{L^4}{r^4}$
$ds^2 \simeq \left(\frac{r^2}{L^2} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{L^2}{r^2} dr^2 + \left(\frac{L^2}{\cancel{r^2}}\right) \cancel{r^2} d\Omega_5^2 \right)$


far from branes:
r >> L → R^{9,1}
near-horizon:
r << L , H → L^4/r^4
ds^2 ≈ r^2/L^2 η_{μν} dx^μ dx^ν + L^2/r^2 dr^2
AdS_5(L) + L^2 dΩ_5^2
S^5(L)
AdS_5 × S^5
S^5
L
r
R^{9,1}


claims 1) ultra-low E exc. near brane can't escape
from grav. pot.


claims 1) ultra-low E exc. near brane can't escape
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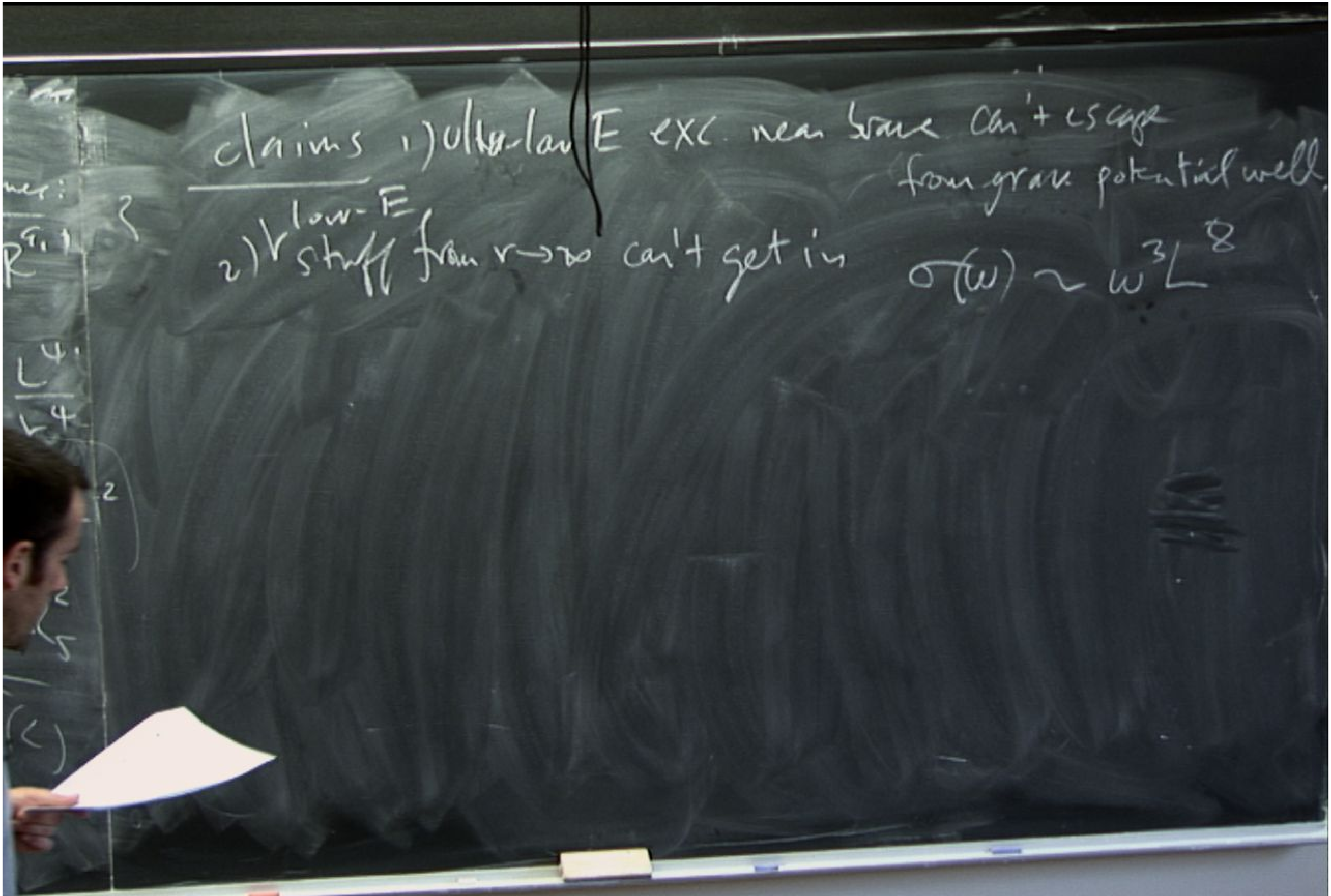
2) stuff from r→∞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σ(ω) ~ ω^3 L^8

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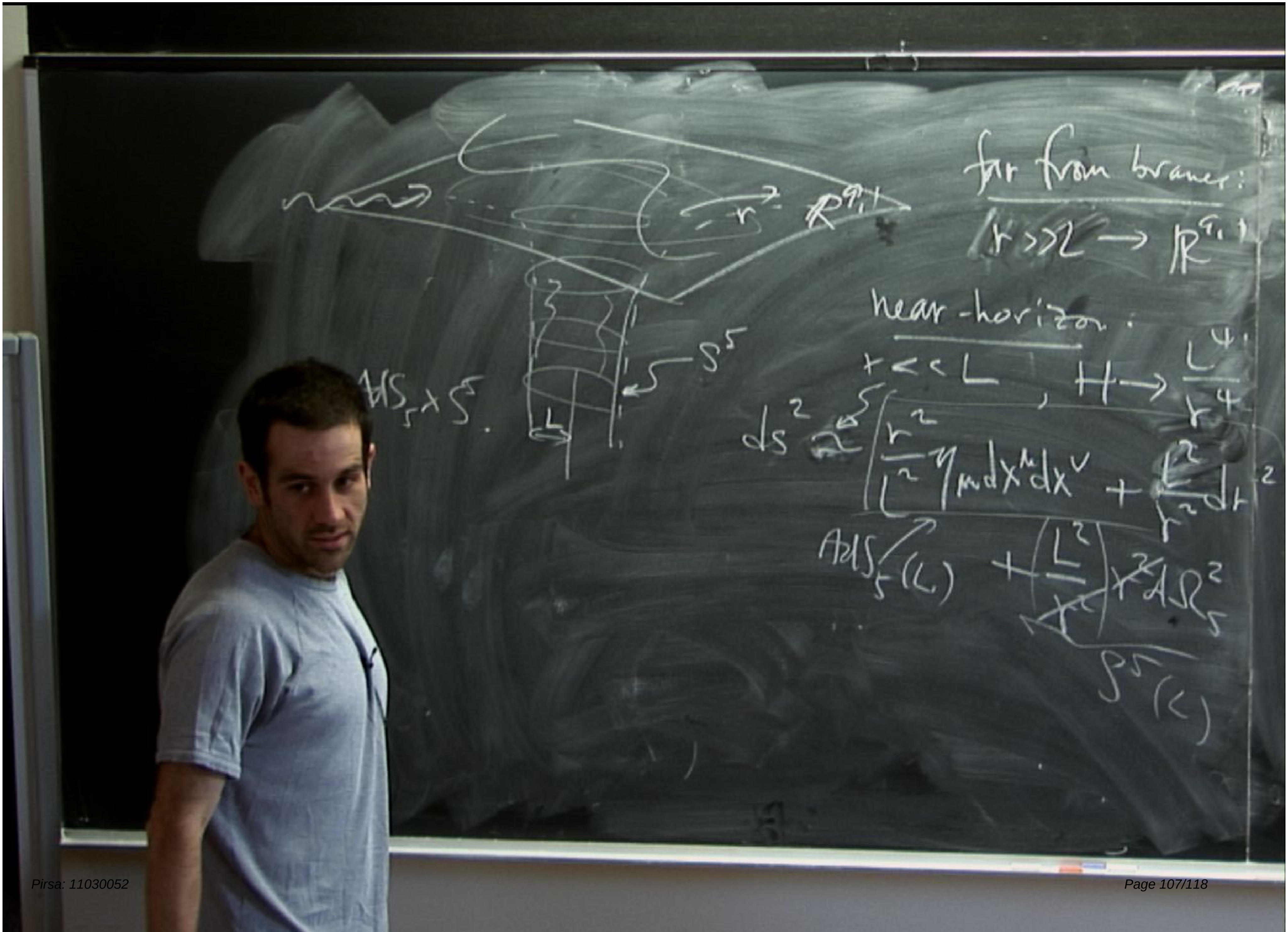
2) low-E stuff from $r \to \infty$ can't get in $\quad \sigma(\omega) \sim \omega^3 L^8$

idea: low $\omega \Rightarrow$ large λ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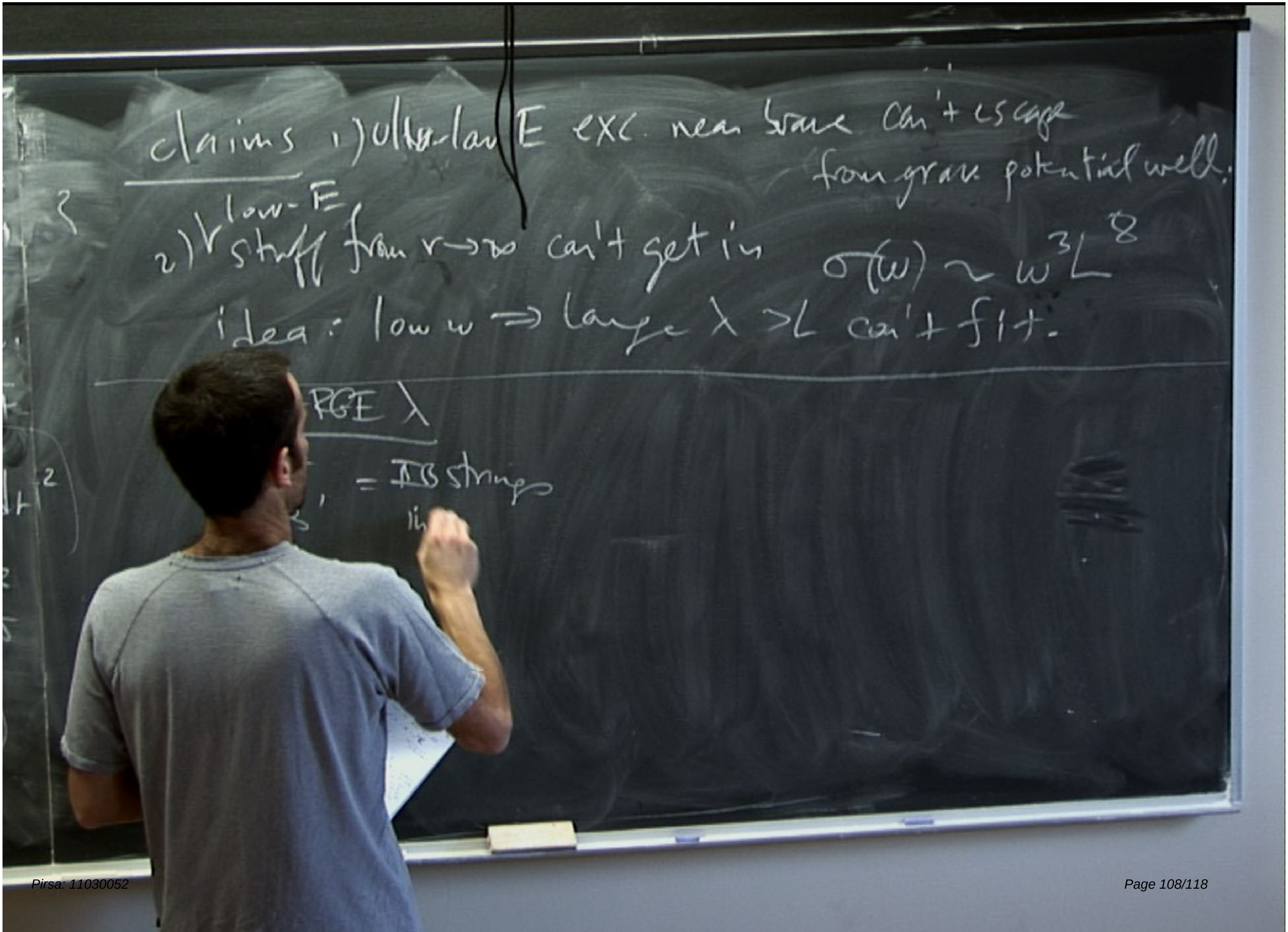
AdS_5 × S^5
S^5
AdS_5 (L)
S^5 (L)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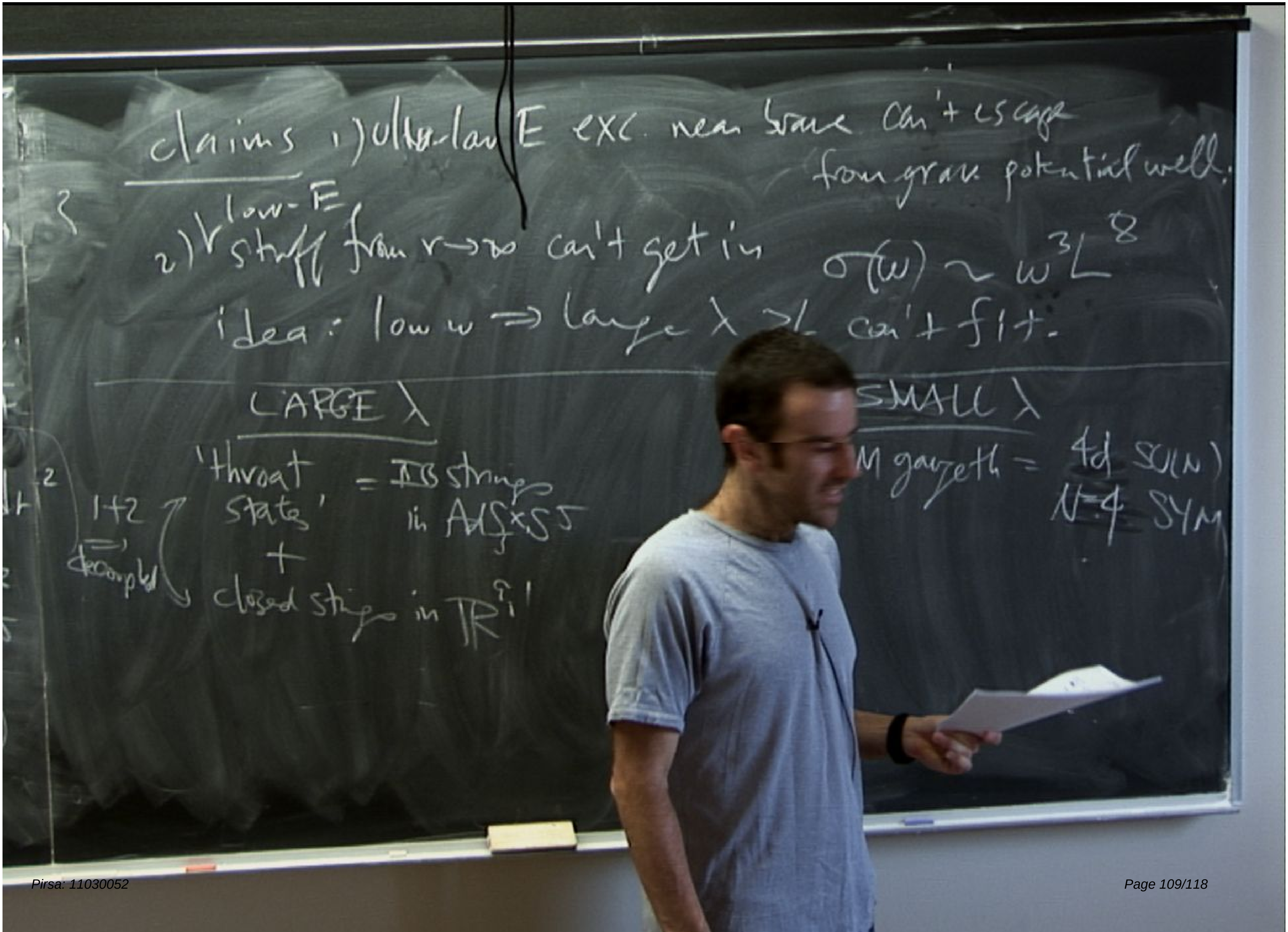
idea: low ω ⇒ large λ > L can't fit.
RGE λ
= IIB strings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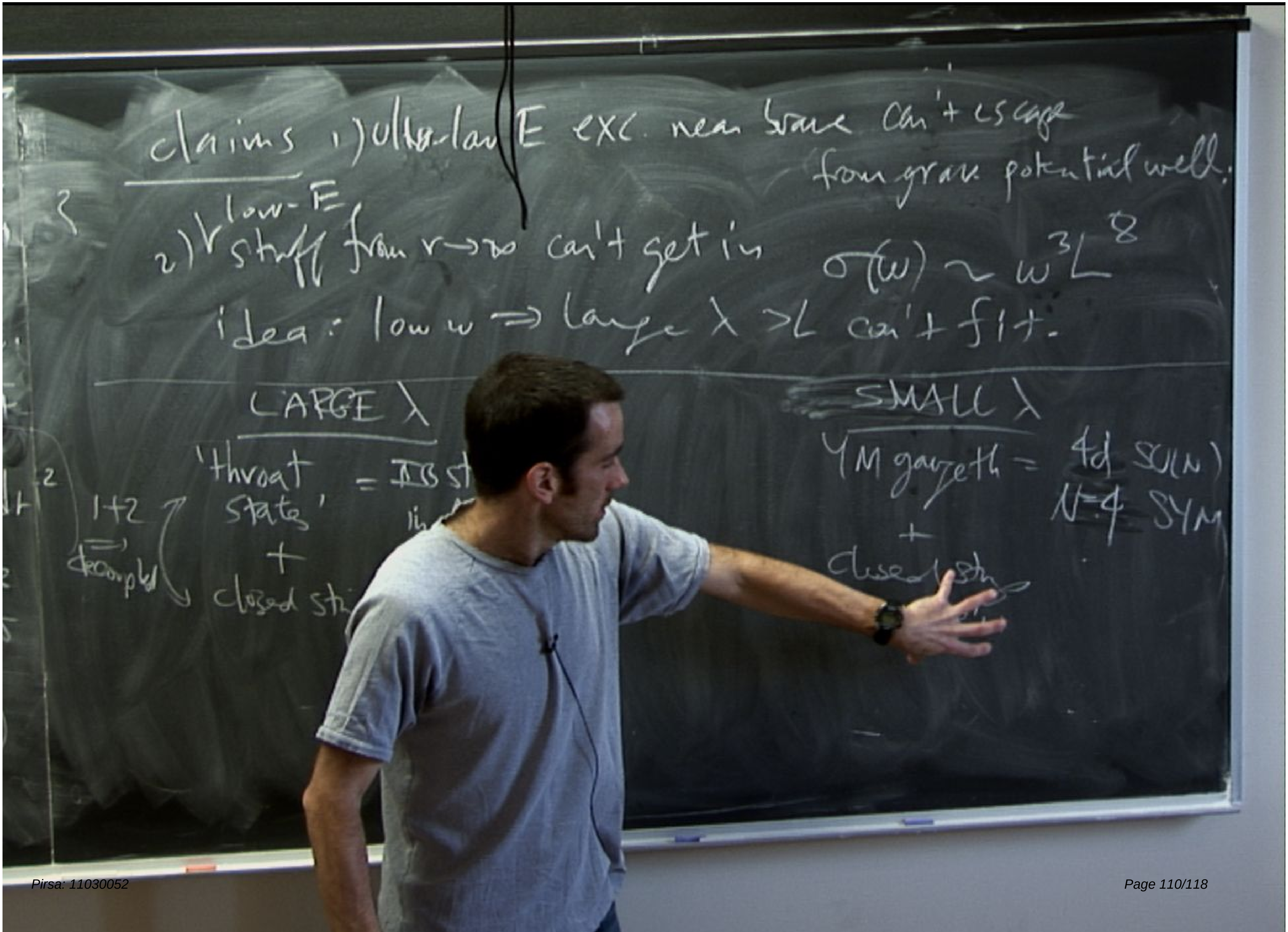
LARGE λ
SMALL λ
'throat states' = IIB strings in AdS5×S5
+
closed strings in R9,1
1+2 ⇒ decoupled
YM gauge th = 4d SU(N) N=4 SYM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'throat states' = IIB strings in $AdS_5 \times S^5$

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closed strings in $\mathbb{R}^{9,1}$

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SMALL λ

YM gauge th = 4d SU... $\mathcal{N}=4$ SY...

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coupled only by irrelev...

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[Maldacena 97] subtract " from BH S.

=

SMALL λ

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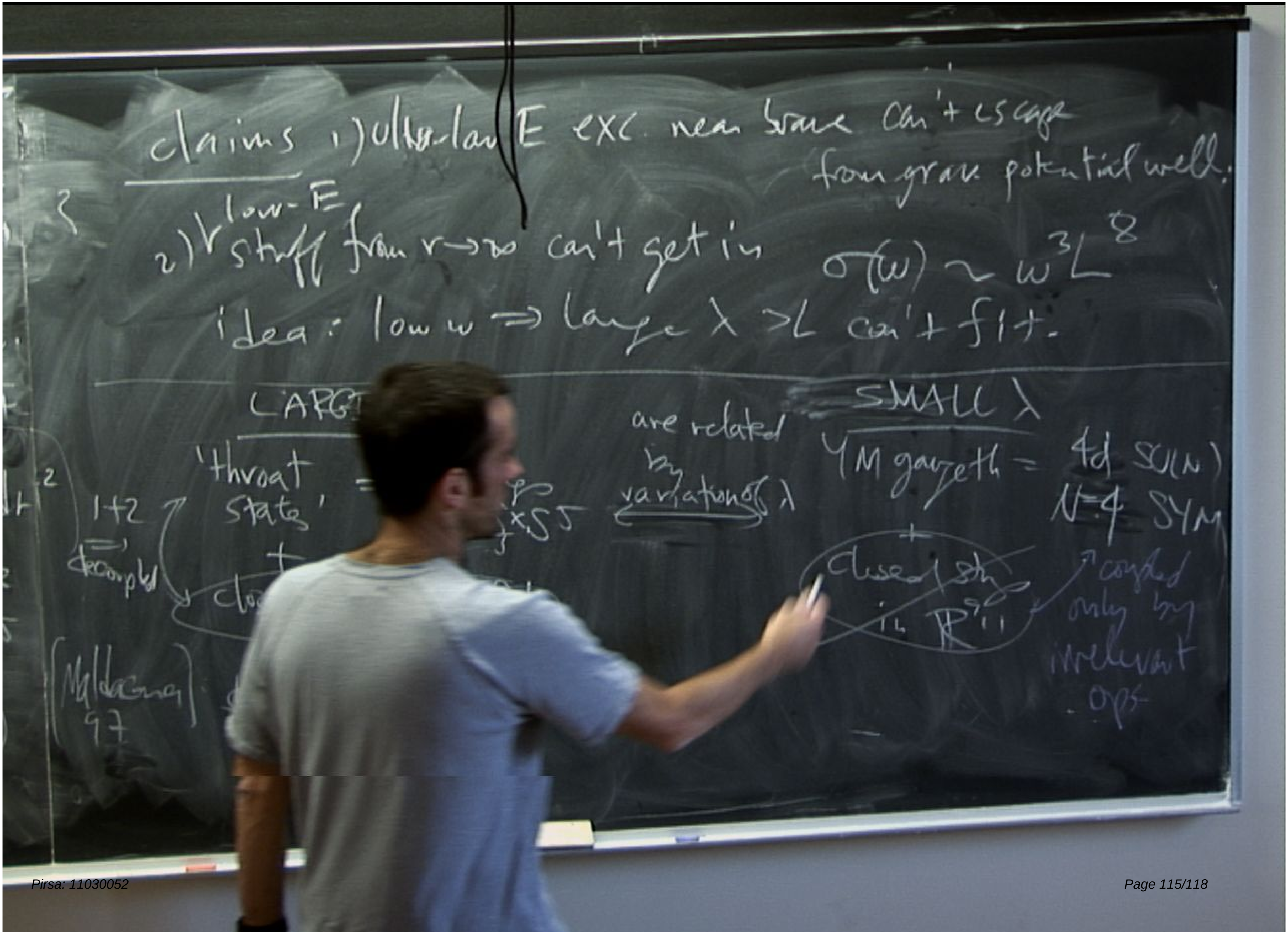
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SMALL λ
are related
by
variations of λ
'throat
state'
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closed strings
in R⁹'¹
coupled
only by
irrelevant
ops
1+2
⇒
decoupled
Maldacena
97

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LARGE λ

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+

closed strings in $\mathbb{R}^{9,1}$

1+2 $\Rightarrow$ decoupled

(Maldacena 97) subtract " from BH Σ

are related by variations of λ

$\equiv$ "dual"

SMALL λ

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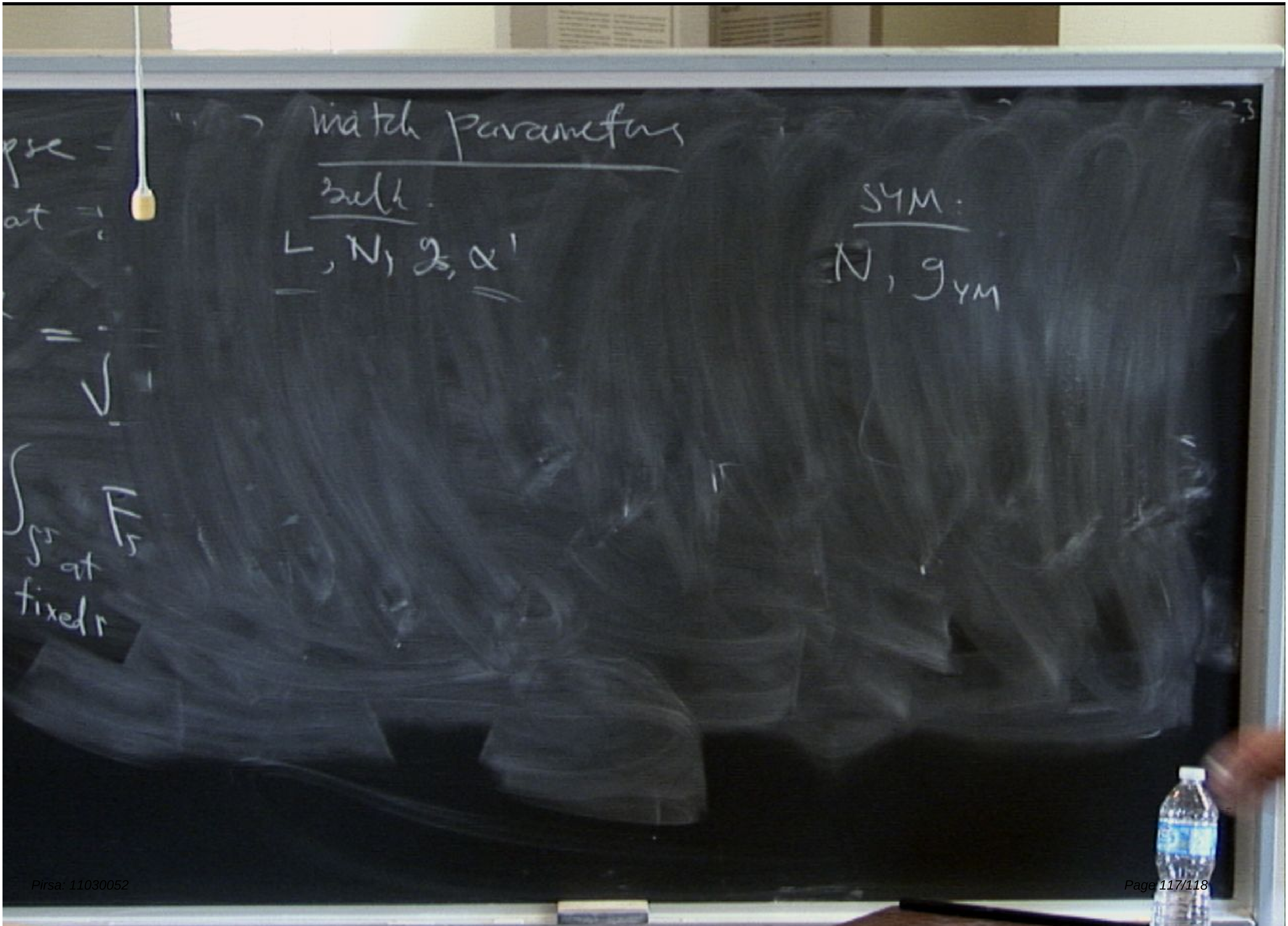
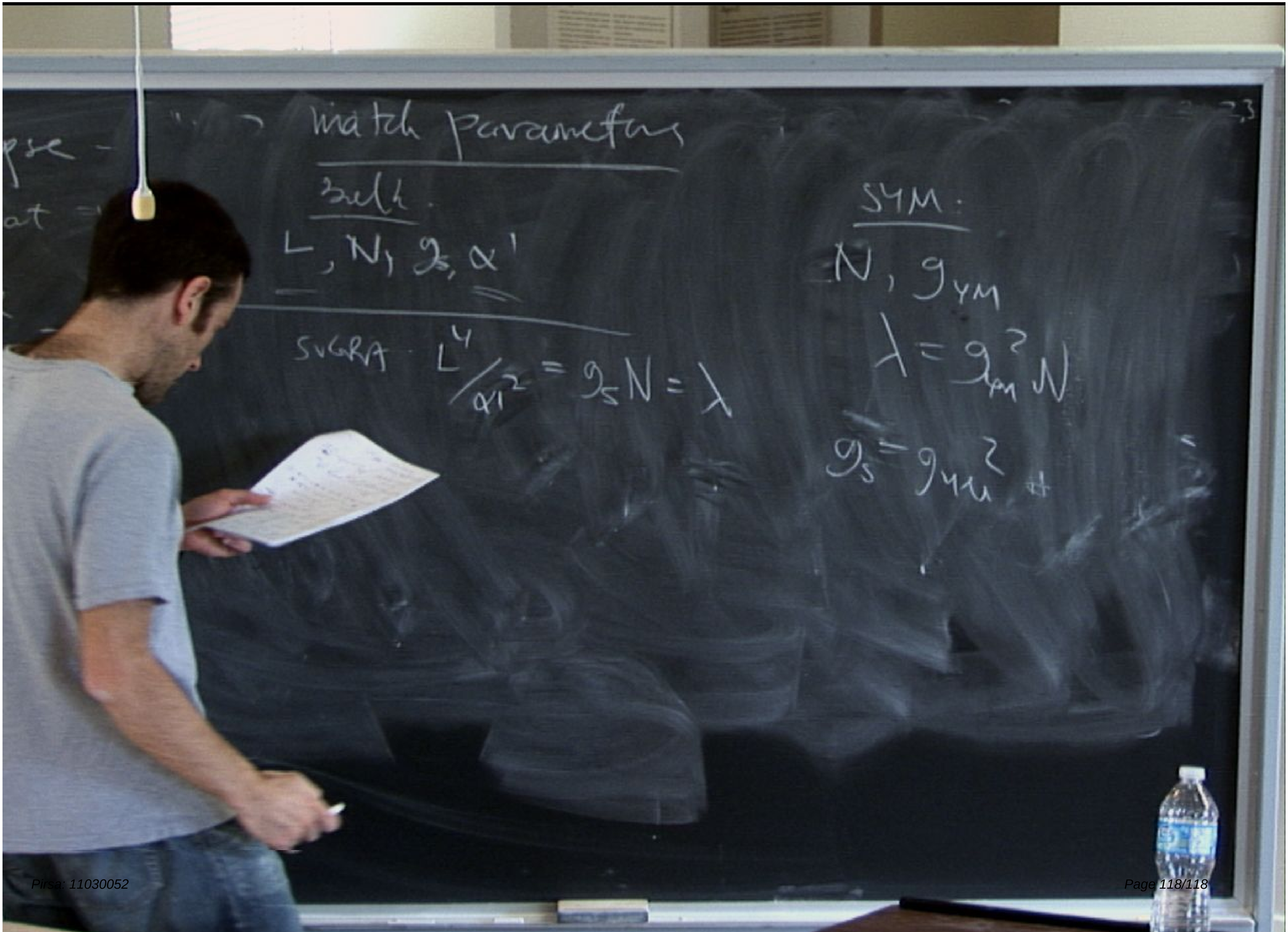
+

closed strings in $\mathbb{R}^{9,1}$

coupled only by irrelevant ops


match parameters
bulk:
L, N, g_s, α'
SYM:
N, g_{YM}
fixed r


match parameters
both
L, N, g_s, α'
SUGRA $L^4/\alpha'^2 = g_s N = \lambda$
SYM:
N, g_{YM}
$\lambda = g_{YM}^2 N$
$g_s = g_{YM}^2$