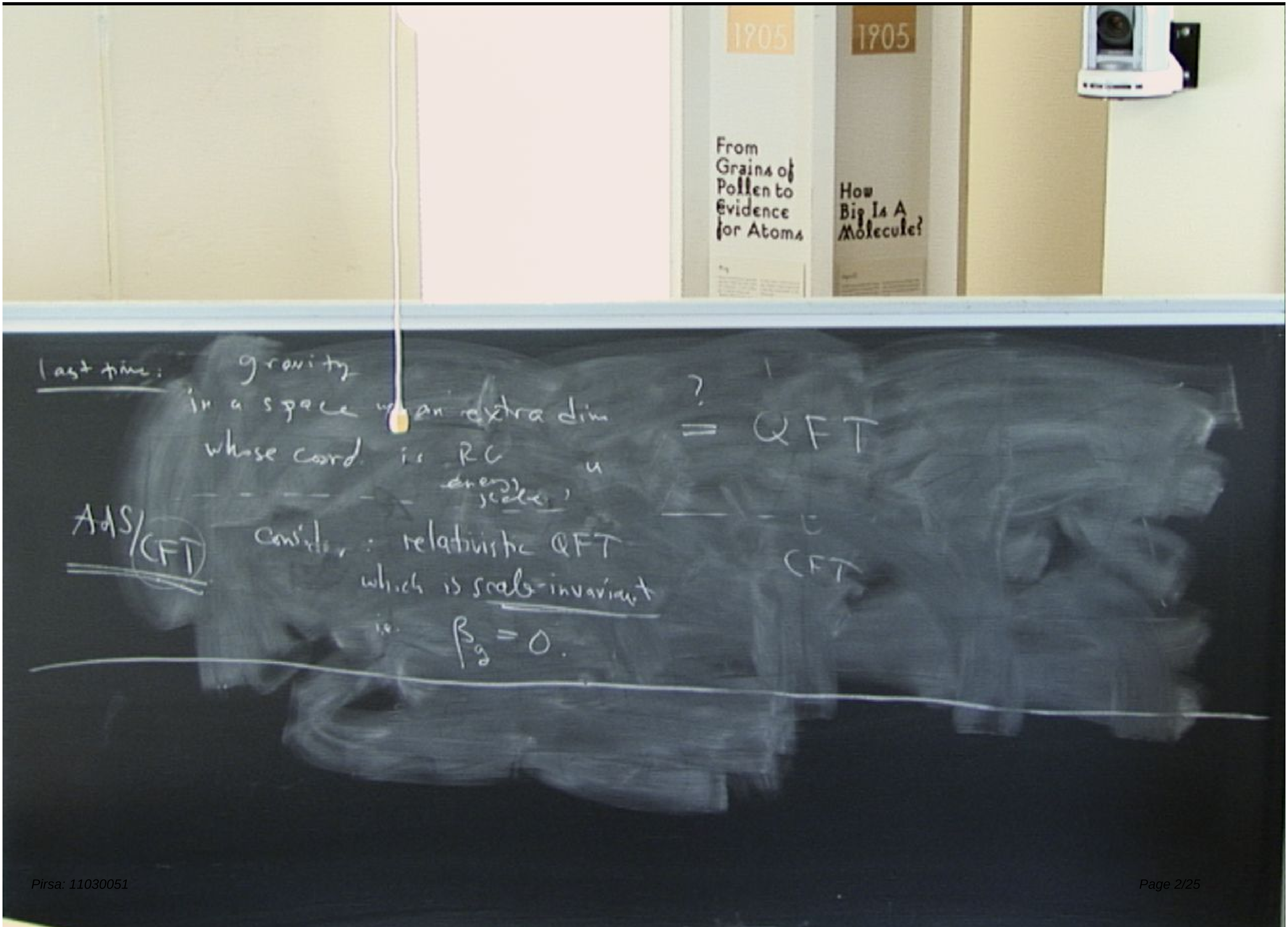
Title: Explorations in String Theory - Lecture 4

Date: Mar 17, 2011 11:30 AM

URL: http://pirsa.org/11030051

Abstract:


1905
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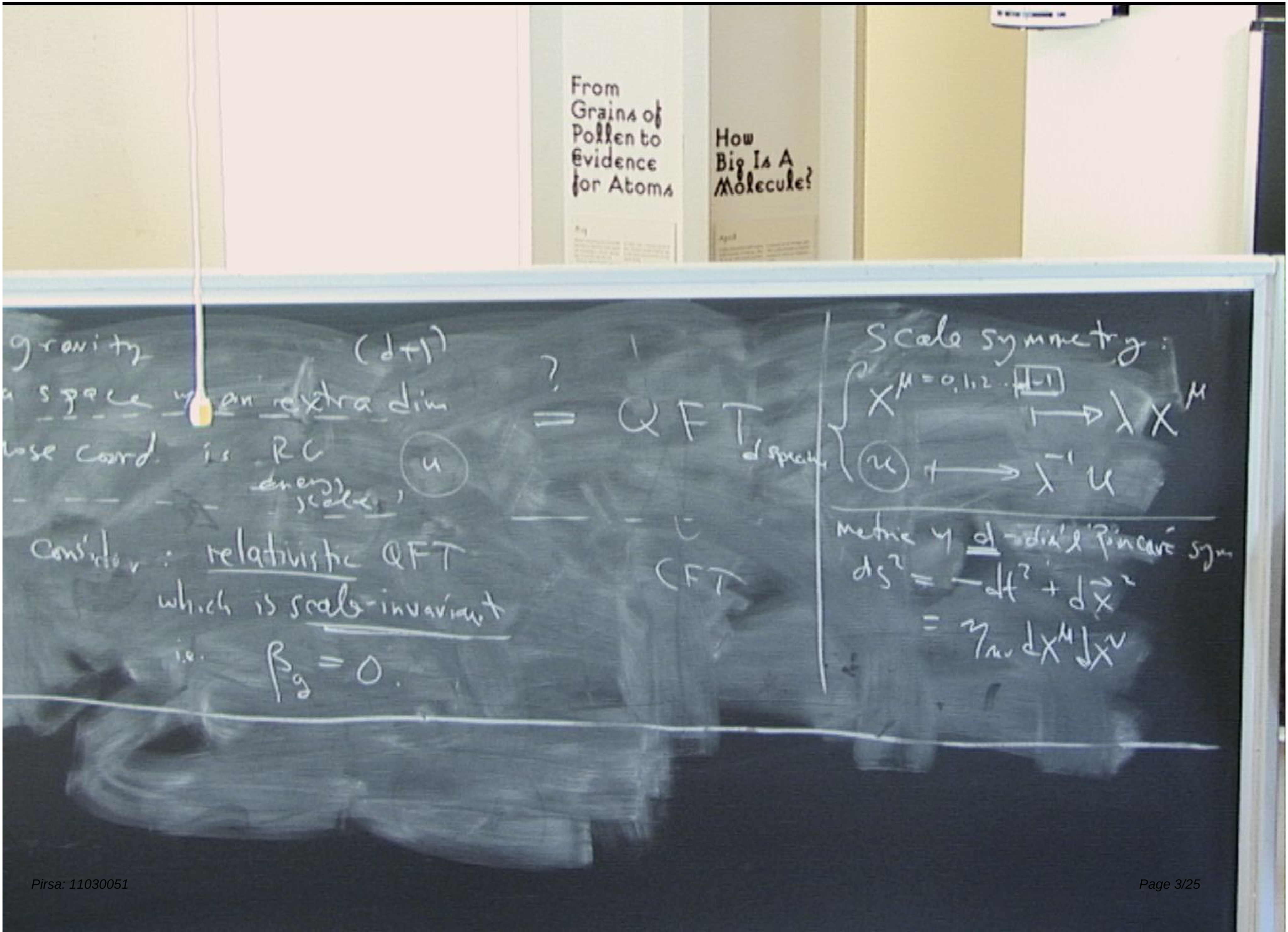
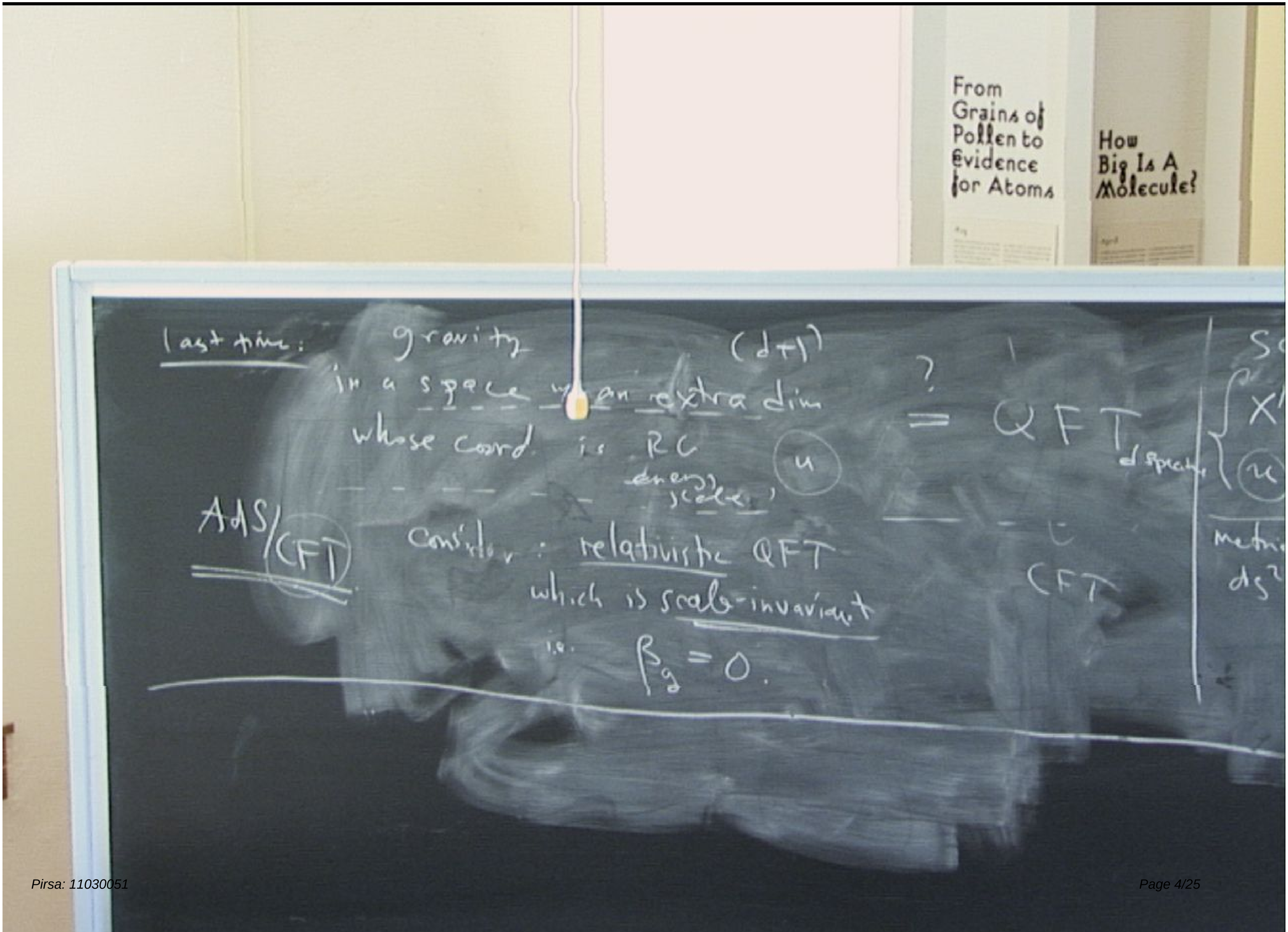
From Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
last time:
gravity
in a space w/ an extra dim
whose coord is RG
energy scale?
u
?
= QFT
AdS/CFT
Consider: relativistic QFT
which is scale-invariant
i.e. $\beta_g = 0$.
CFT


From Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
gravity (d+1)
space w/ an extra dim
whose coord. is RG energy scale
= QFT d space
CFT
Consider: relativistic QFT
which is scale-invariant
i.e. β_g = 0.
Scale symmetry:
x^μ=0,1,2...d-1 ⟼ λ x^μ
u ⟼ λ^-1 u
metric w/ d-dim'l Poincaré sym
ds² = -dt² + dx²
= η_μν dx^μ dx^ν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CFT
metric

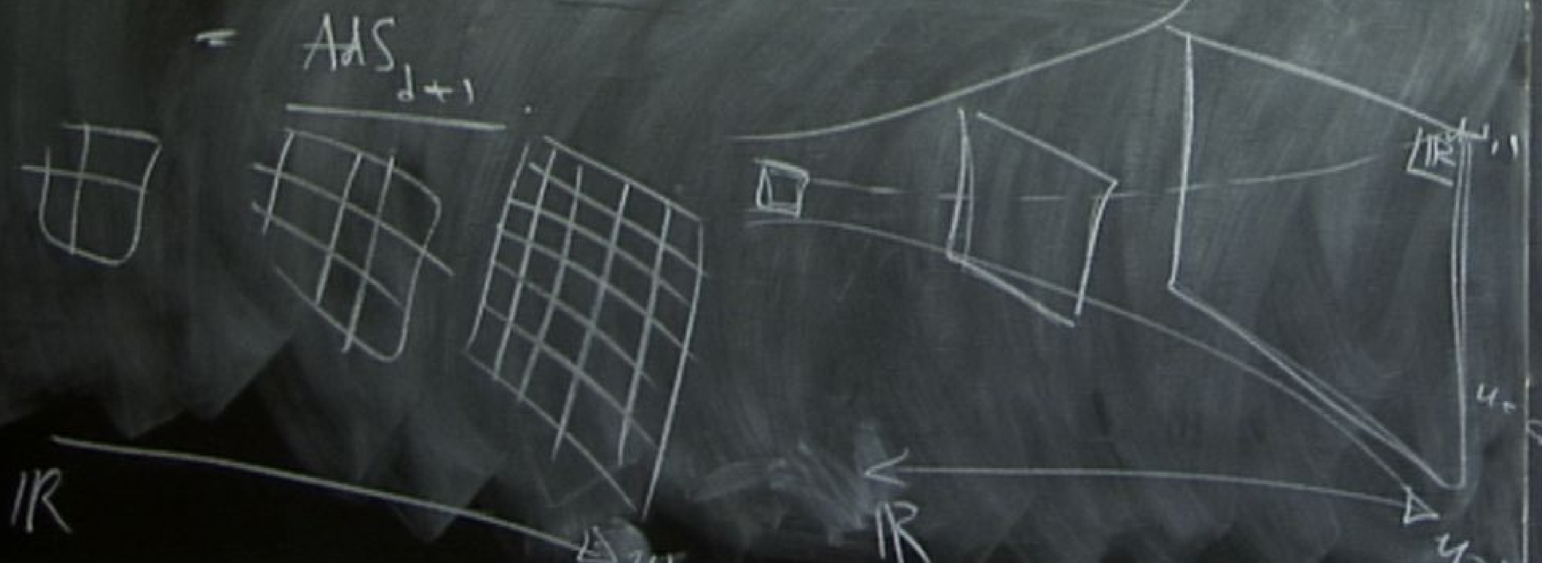
in $d+1$ dims

Most gen'l metric w/ d-dim'l Poincaré

and the scale sym. (★) $ds^2 \mapsto ds^2$

is $$ds^2 = \frac{u^2}{L^2}\left(-\eta_{\mu\nu}dx^\mu dx^\nu\right) + L^2\frac{du^2}{u^2}$$

$= AdS_{d+1}$



another useful coord: $z = \frac{L^2}{u}$ $\sim L^{\#}$

$[z] = \text{length}$

$[u] = \text{energy}$

$\boxed{\hbar = c = 1}$

$$ds^2_{AdS} = L^2\left(\frac{dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu}{z^2}\right).$$

Crucial refinement: " gravity in AdS_{d+1} $\stackrel{?}{=}$ CFT_d "

...dims

d-dim'l Poincaré

scale sym. (★) $\quad ds^2 \mapsto ds^2$

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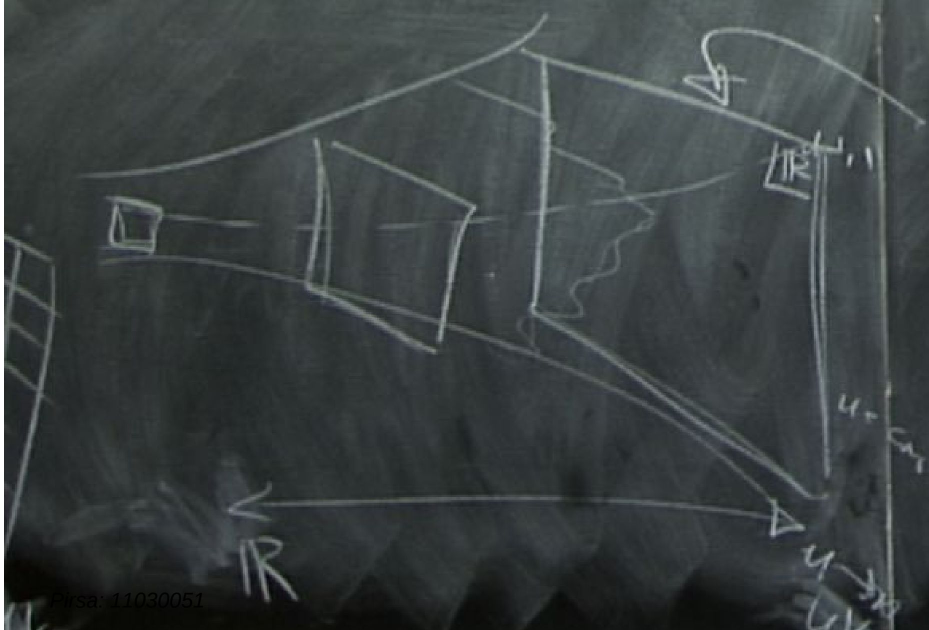

another useful coord: $z = \frac{L^2}{u} \sim L$ (#)

$$ds^2_{AdS} = L^2 \left(\frac{dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu}{z^2} \right).$$

Crucial refinement: " gravity in $\boxed{\text{asymptotically}}$ AdS_{d+1}

AdS has a boundary (at $u = \infty$, $z = 0$) $\Longrightarrow$ need B.C.'s

One of those B.C.'s is:

near $z \to 0$, geometry is AdS.

another useful coord: $z = \frac{L^2}{u} \sim l$ ħ

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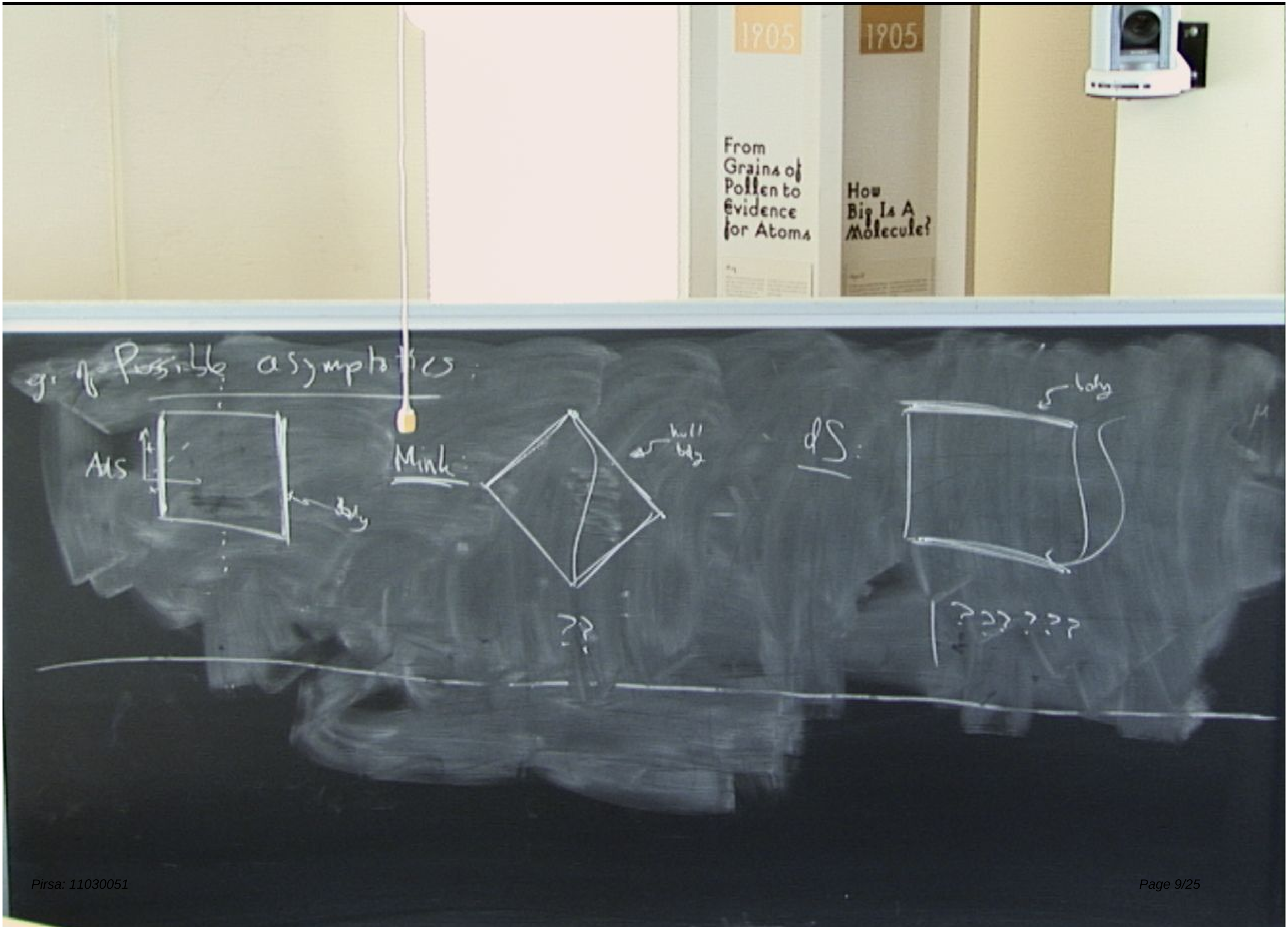
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How Big Is A Molecule?
eg. of Possible asymptotics:
AdS
bdy
Mink:
null bdy
??
dS:
bdy
??????

Preview of dictionary

"bulk" $\longleftrightarrow$ "boundary"

fields in the bulk $\longleftrightarrow$ operators in QFT

(eg $g_{\mu\nu} \longleftrightarrow T^{\mu\nu}$)

spin, charge $\longleftrightarrow$ spin charge.

MASS $\longleftrightarrow$ scaling dim Δ

eg for a scalar, $m^2 L^2 = \Delta(\Delta - d)$

simple bulk theory w/ few, fields light. $\longleftrightarrow$ CFT w/ few operators of low Δ

anot

ds^2_{AdS}

"AdS

Crucia

AdS h

(at

One of

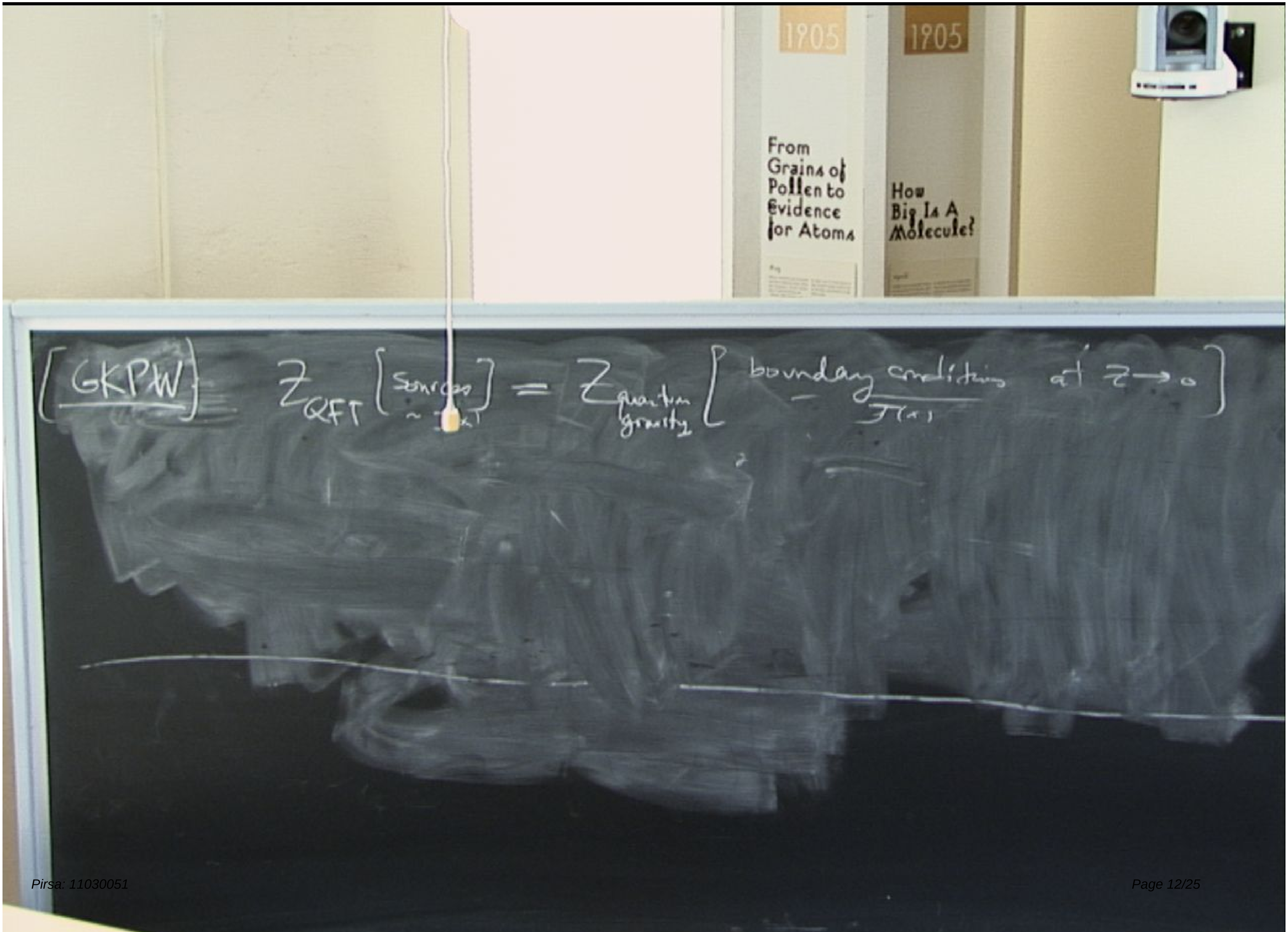
What to calculate: euclidean correlators of local ops.

$$\langle \mathcal{O}_1(x_1)\,\mathcal{O}_2(x_2)\cdots\mathcal{O}_n(x_n)\rangle$$

$$\mathcal{L}_{QFT}(x) \longrightarrow \mathcal{L}_{CFT}(x) + \sum_A \mathcal{O}_A(x)\, J^A(x) \equiv \mathcal{L} + \mathcal{L}_J$$

$$Z[J] \equiv \left\langle e^{-\int \mathcal{L}_J} \right\rangle_{CFT}$$

$\mathcal{L}_J$ is a UV perturbation.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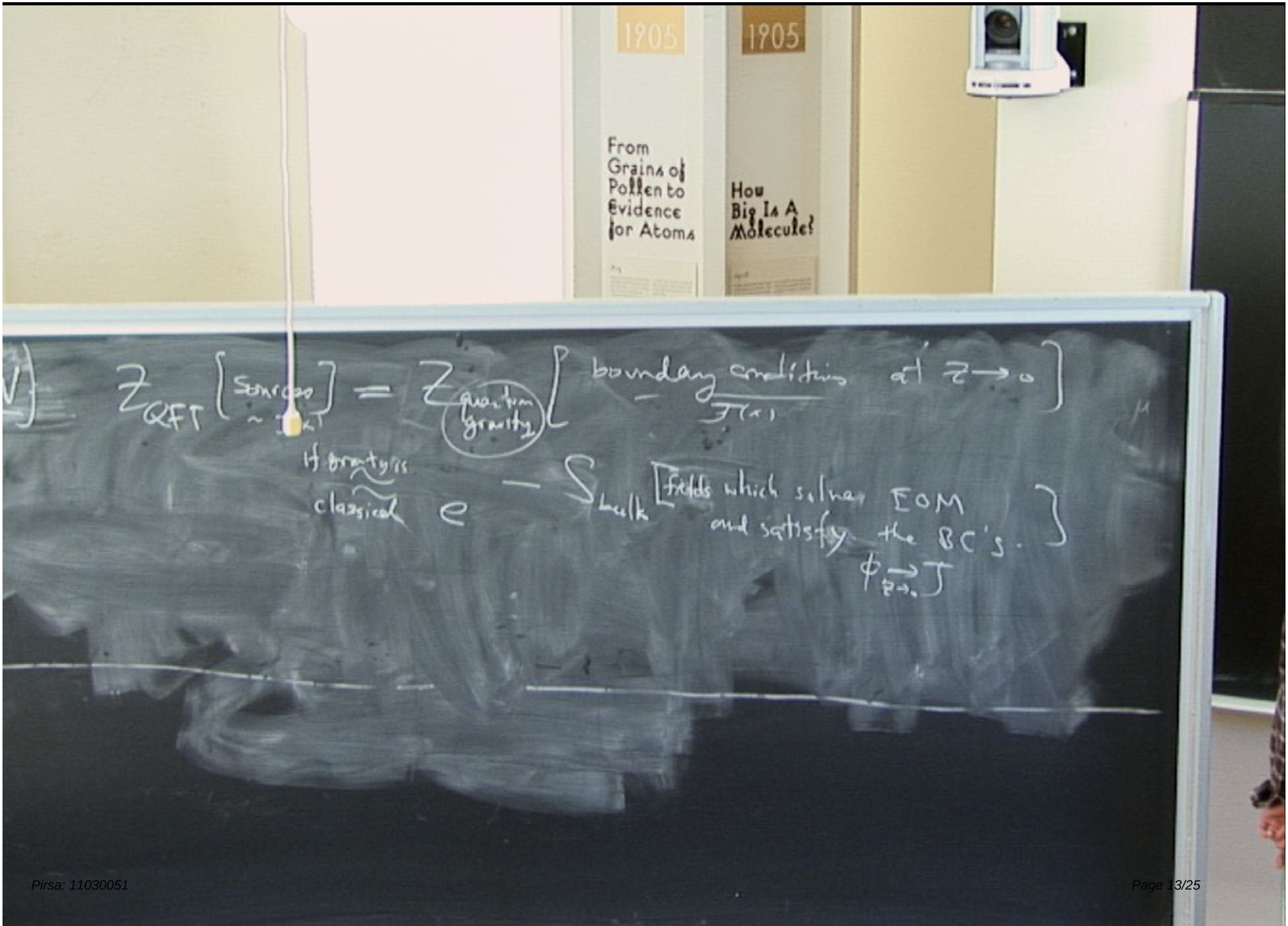
How Big Is A Molecule?
[GKPW]
Z_QFT [sources ~ J(x)] = Z_quantum gravity [boundary conditions at z → 0 J(x)]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How Big Is A Molecule?
Z_QFT [sources] = Z_quantum gravity [boundary conditions at z → 0]
J(x)
If gravity is classical e^{-S_bulk [fields which solve EOM and satisfy the BC's.]}
φ → J (z → 0)

What to calculate: euclidean correlators of local ops.

$$\langle O_1(x_1)\, O_2(x_2) \cdots O_n(x_n) \rangle$$

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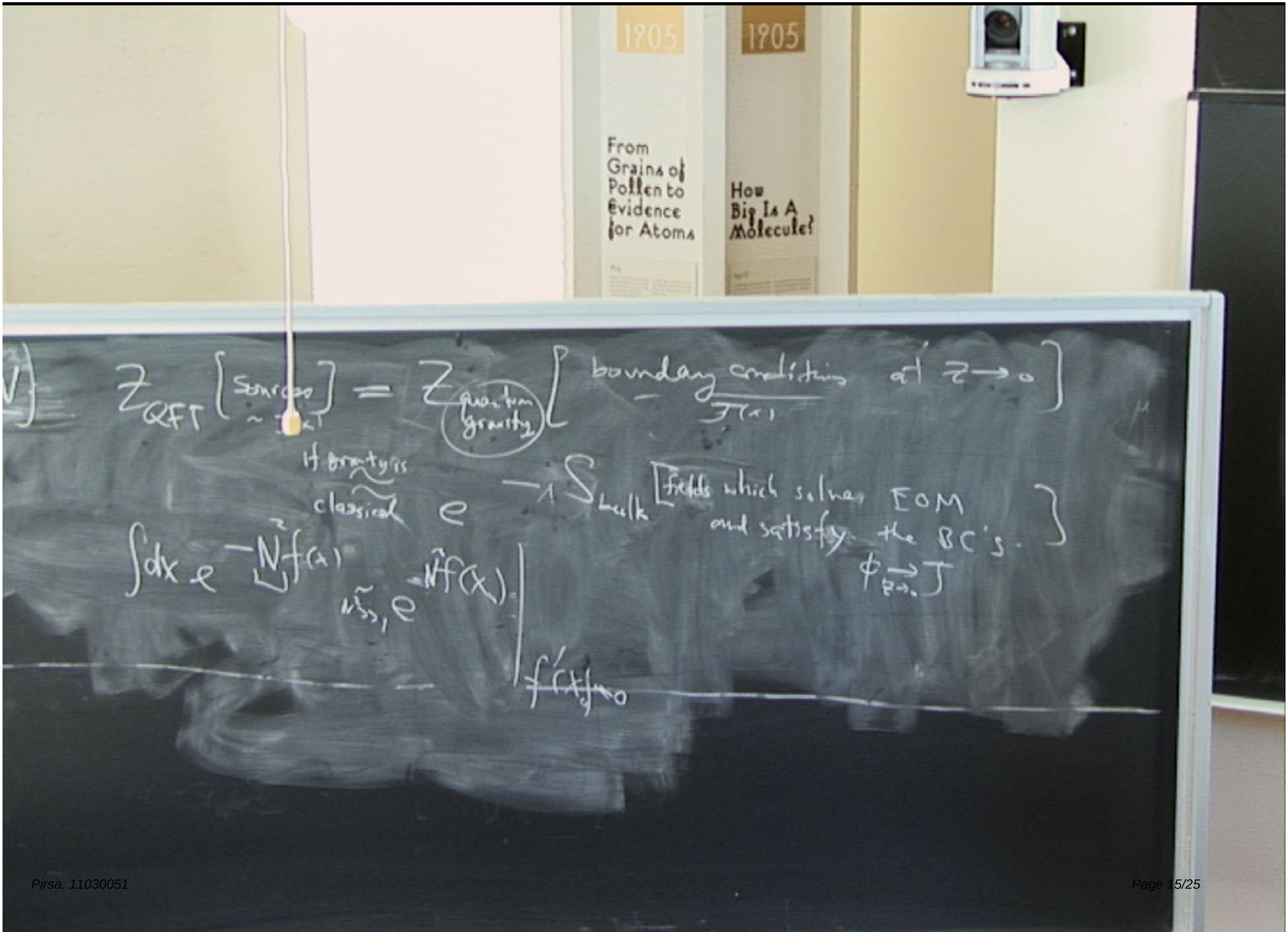
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If gravity is classical
e^{-Λ S_bulk [fields which solve EOM and satisfy the BC's.]}
φ → J (z→0)
∫dx e^{-N² f(x)}
N>>1, e^{-N² f(x_*)}
f'(x_*) = 0

Preview of dictionary

"bulk" $\longleftrightarrow$ "boundary"

fields in the bulk $\longleftrightarrow$ operators in QFT

(eg $g_{\mu\nu} \longleftrightarrow T^{\mu\nu}$)

ϕ_A $\quad \{\mathcal{O}^A\}$ $\quad \{\mathcal{O}^A\} = \{\phi, \ldots\}$

spin, charge $\longleftrightarrow$ spin charge

MASS $\longleftrightarrow$ scaling dim Δ

eg for a scalar, $m^2 L^2 = \Delta(\Delta - d)$

simple bulk theory $\longleftrightarrow$ CFT w/ few
w/ few, light fields $\quad$ operators of low Δ

What

$\mathcal{L}_{CFT}(x)$

$Z[$

Z_{IR}

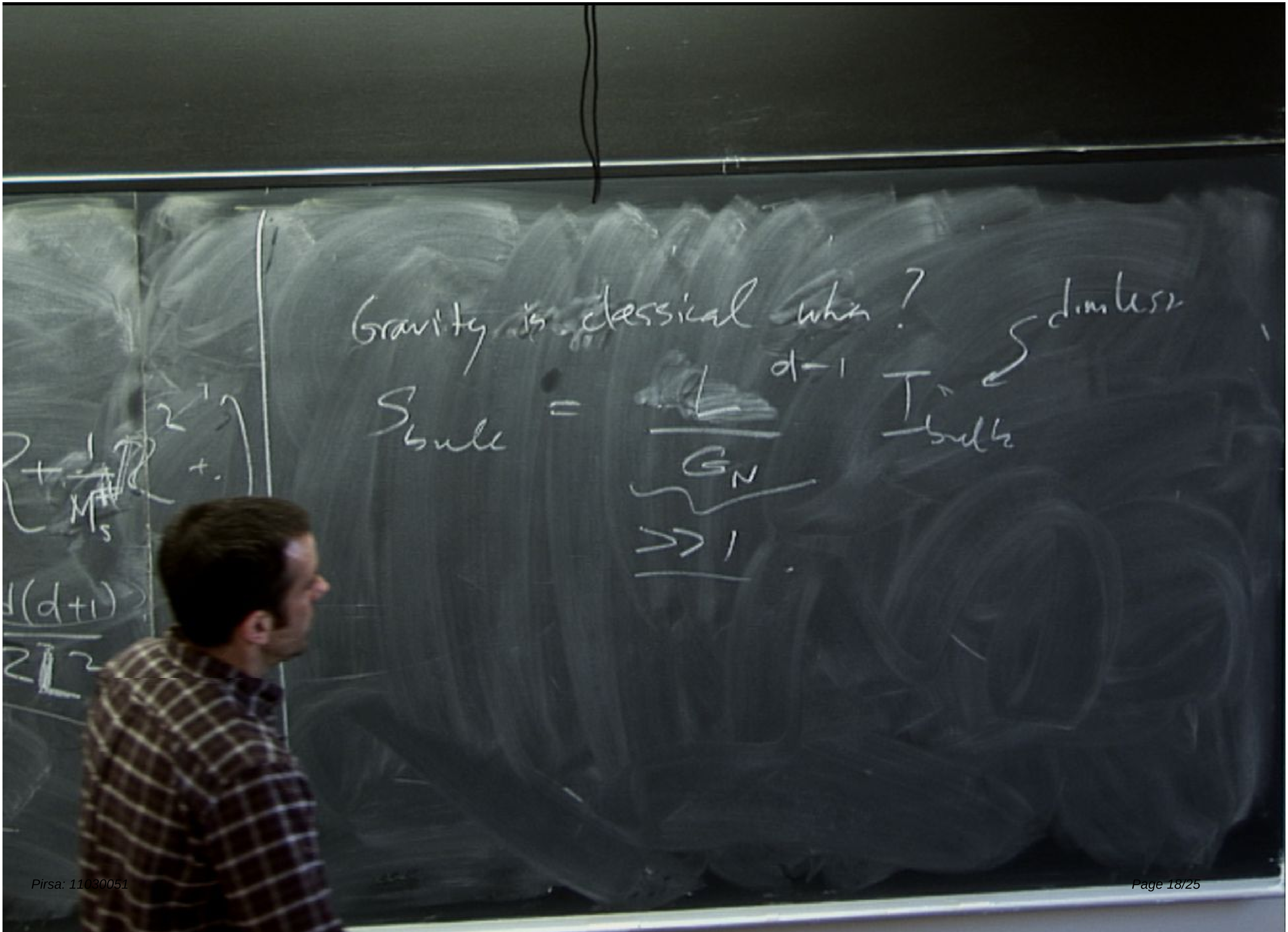
What's S_{bulk}?

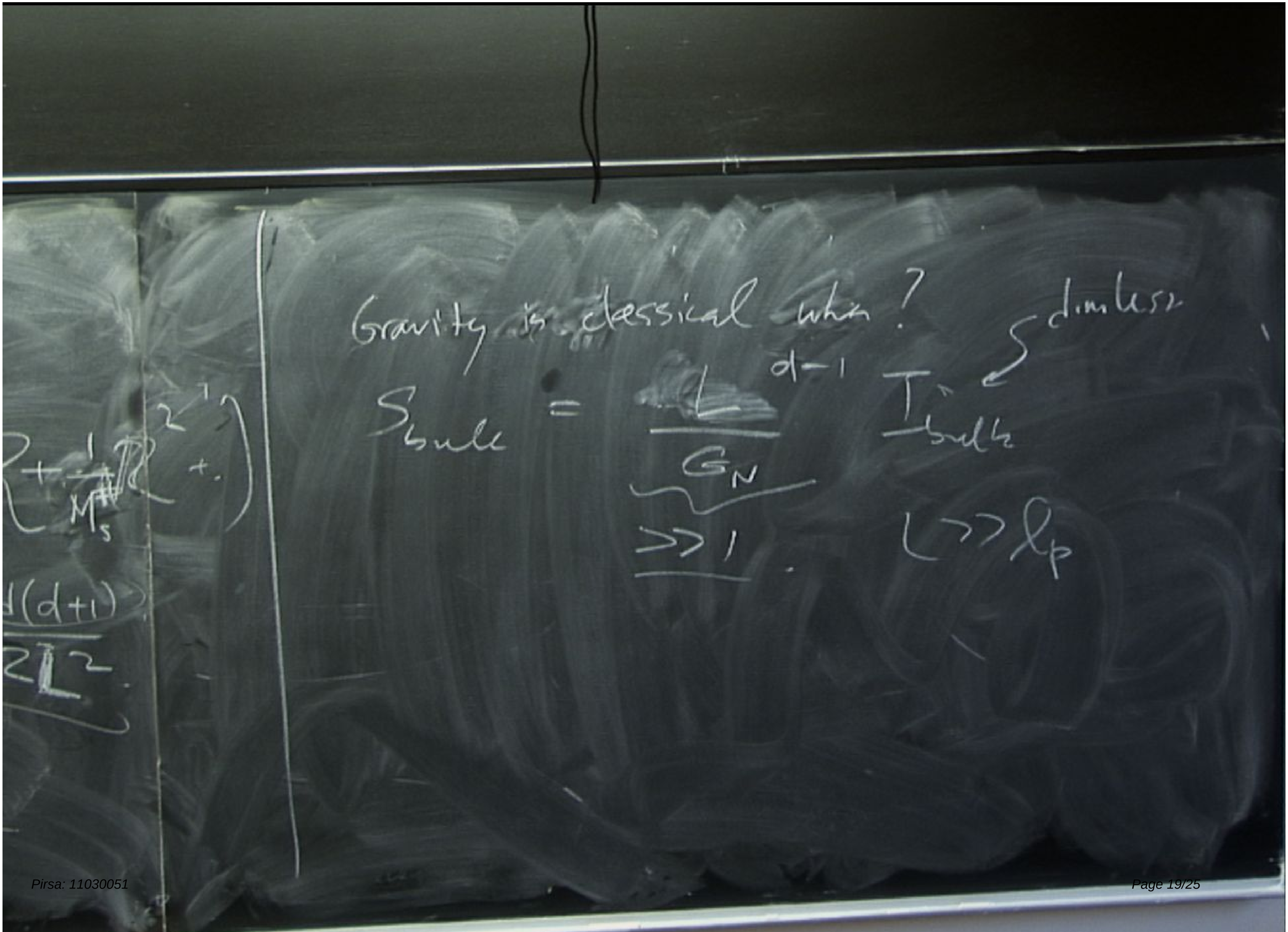
AdS solves the EOM of

$$S_{bulk} = \frac{1}{\# G_N}\int d^{d+1}x\,\sqrt{g}\left[-2\Lambda + R + \frac{1}{M_s^2}R^2 + \dots\right]$$

$$0 = \frac{\delta S}{\delta g_{\mu\nu}} \Leftarrow ds^2 = ds^2_{AdS} \text{ w/ } \Lambda = -\frac{d(d+1)}{L^2}$$

$$L \gg \frac{1}{M_s}$$


Gravity is classical when?
dimless
$S_{bulk} = \frac{L^{d-1}}{G_N} T_{bulk}$
$\gg 1$



Gravity is classical when?
S bulk = L d-1 / G N T bulk
dimless
>> 1
L >> l p
d(d+1)
2L 2

Gravity is classical when?

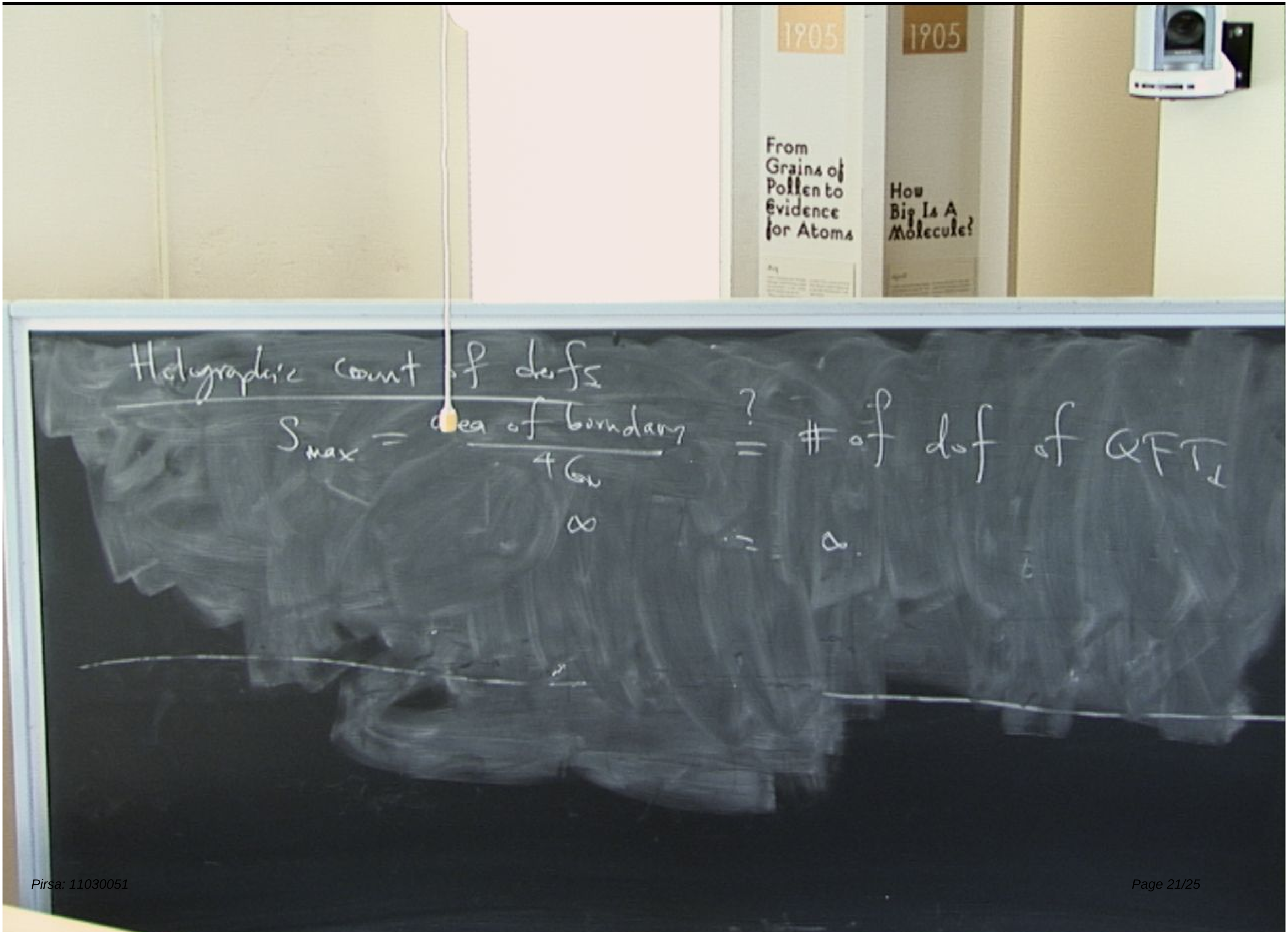
$$S_{bulk} = \underbrace{\frac{L^{d-1}}{G_N}}_{\gg 1} T_{bulk}$$

dimless

$L \gg l_p$

large AdS radius $\longleftrightarrow$ strong coupling of QFT

$L M_5 \gg 1$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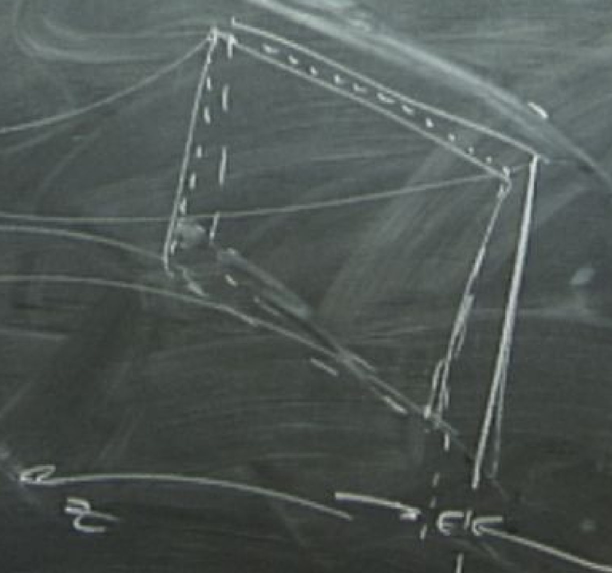
Holographic count of dofs
$S_{max} = \frac{\text{area of boundary}}{4G_N} \stackrel{?}{=} \#\text{ of dof of } QFT_d$
$\infty \quad = \quad a.$

$$ds^2 = \left(\frac{-dt^2 + d\vec{x}^2 + dz^2}{z^2}\right) L^2$$

IR cutoff: $x^i \in [0, R]$

UV cutoff: $z > \epsilon$

$$A = A_{\text{rea}}\begin{pmatrix} t = \text{const} \\ z = \epsilon \\ x^i \in [0,R] \end{pmatrix}$$



$A =$

$$) L^2$$

$$A = \int_{bdy} \sqrt{g}\Big|_{surf} d^{d-1}x = \int_0^R d\vec{x} \left(\frac{L}{z}\right)^{d-1}\Bigg|_{z=\epsilon}$$

$$= \left(\frac{R}{\epsilon}\right)^{d-1} L^{d-1}$$

$$\boxed{\begin{aligned} ds^2\Big|_{surf} &= d\vec{x}^2 \frac{L^2}{z^2} \\ \sqrt{g} &= \left(\frac{L}{z}\right)^{d-1} \end{aligned}}$$

$$S_{max} = \frac{A}{4G_N} \sim \left(\frac{R}{\epsilon}\right)^{d-1} \left(\frac{L^{d-1}}{G_N}\right)$$

Lessons: ① R-dependence works.

$$A = \int_{bdy} \sqrt{g}\Big|_{surf} d^{d-1}x = \int_0^R d\vec{x}\, \left(\frac{L}{z}\right)^{d-1}\Bigg|_{z=\epsilon}$$

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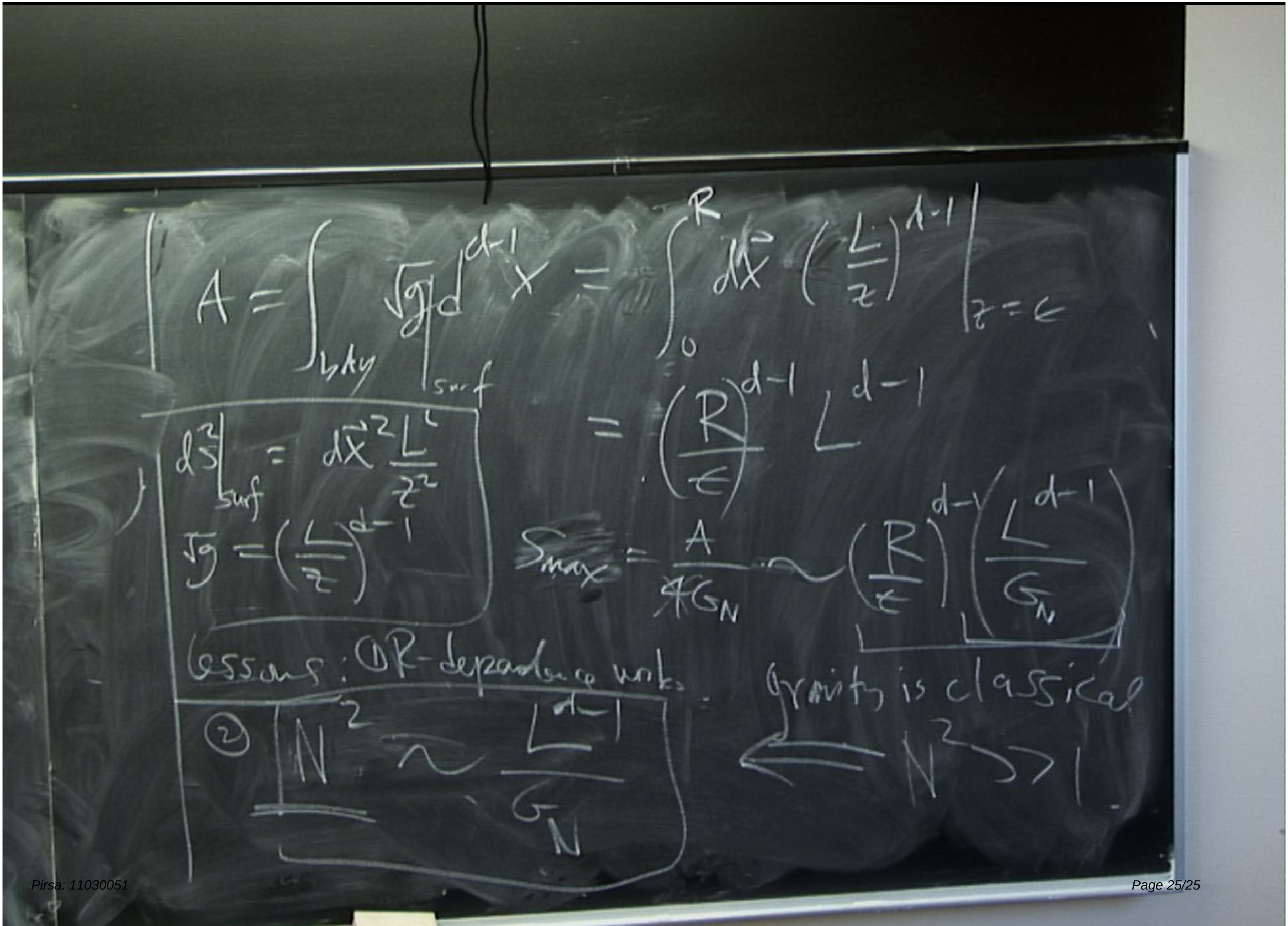
Lessons: (1) R-dependence works.

(2)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$= \left(\frac{R}{\epsilon}\right)^{d-1} L^{d-1}$
$ds^2\big|_{surf} = d\vec{x}^2 \frac{L^2}{z^2}$
$\sqrt{g} = \left(\frac{L}{z}\right)^{d-1}$
$S_{max} = \frac{A}{4G_N} \sim \left(\frac{R}{\epsilon}\right)^{d-1}\left(\frac{L^{d-1}}{G_N}\right)$
lessons: ① R-dependence works.
② $N^2 \sim \frac{L^{d-1}}{G_N}$
Gravity is classical $\Longleftarrow N^2 \gg 1$