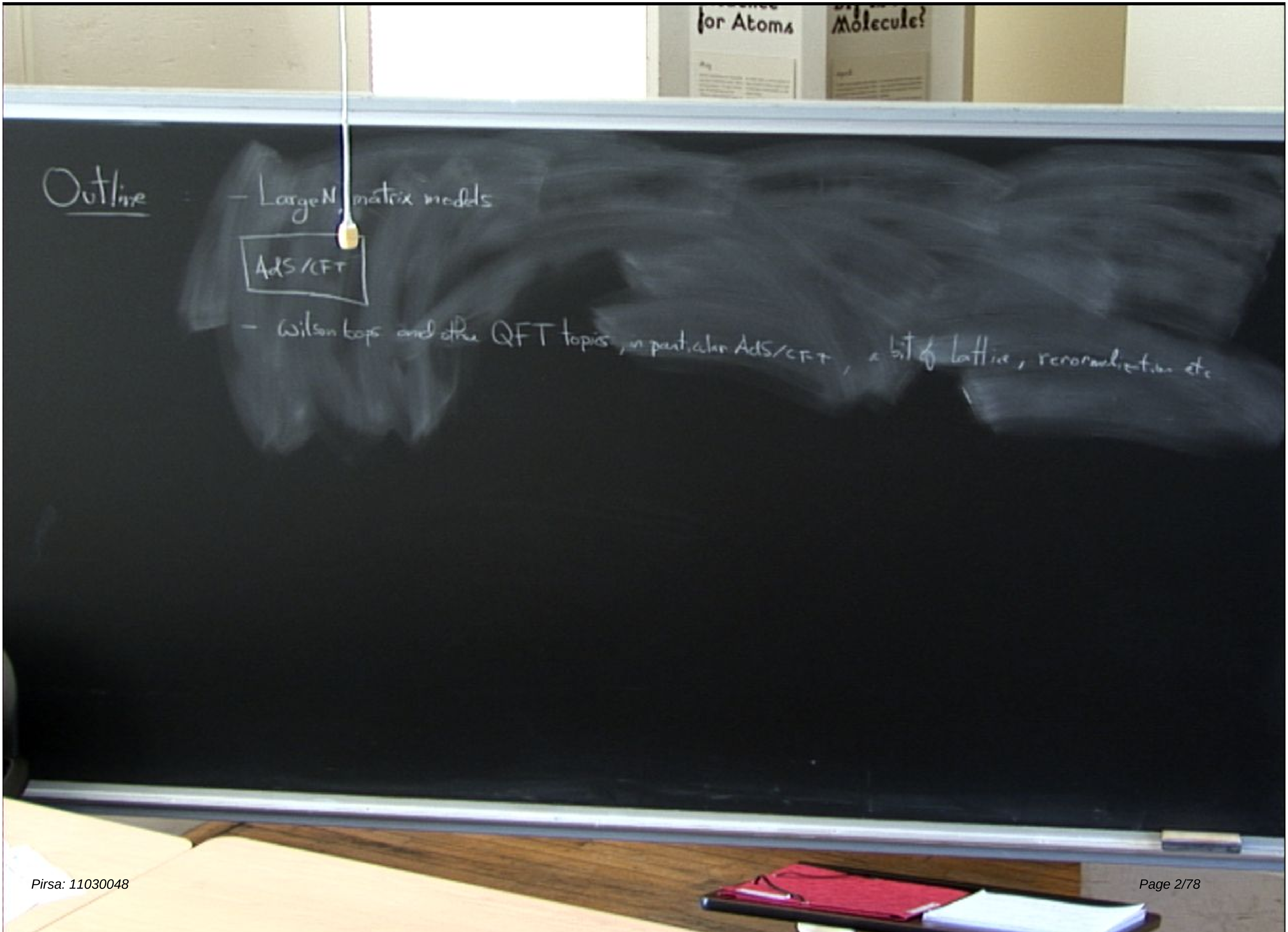
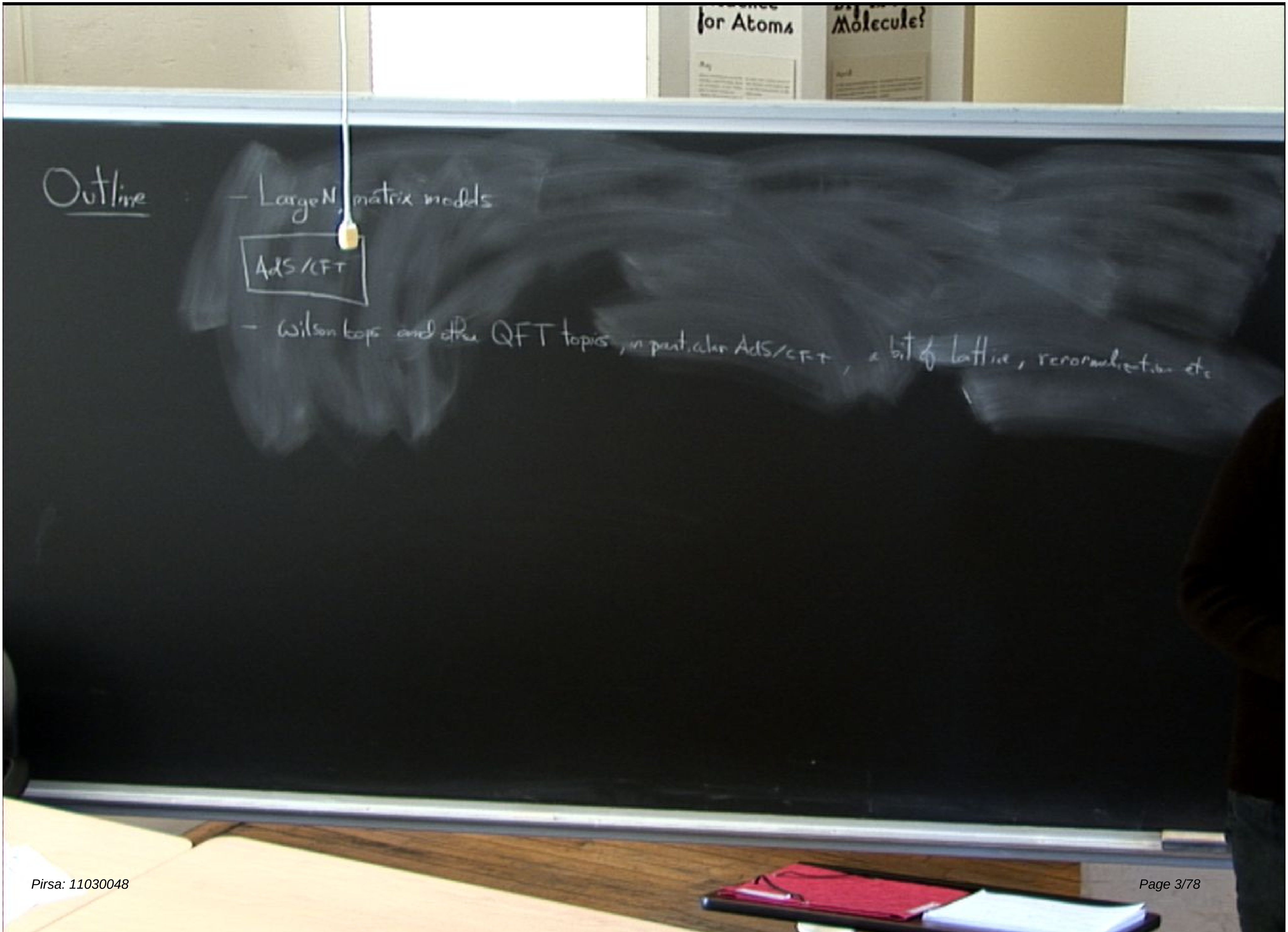
Title: Explorations in String Theory - Lecture 1

Date: Mar 14, 2011 11:30 AM

URL: http://pirsa.org/11030048

Abstract:


for Atoms
Molecule?
Outline
– Large N matrix models
AdS/CFT
– Wilson loops and other QFT topics, in particular AdS/CFT, a bit of lattice, renormalization etc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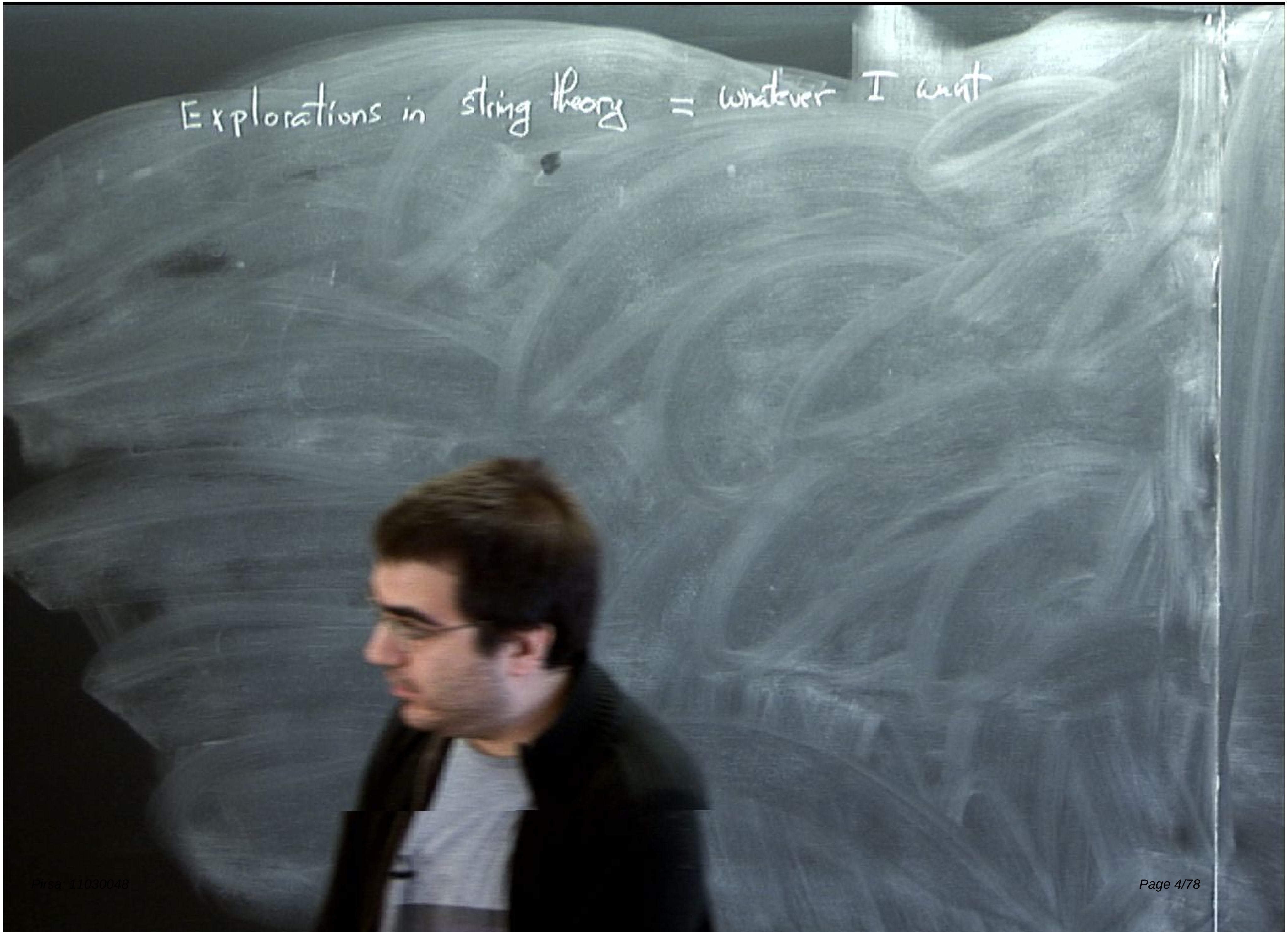
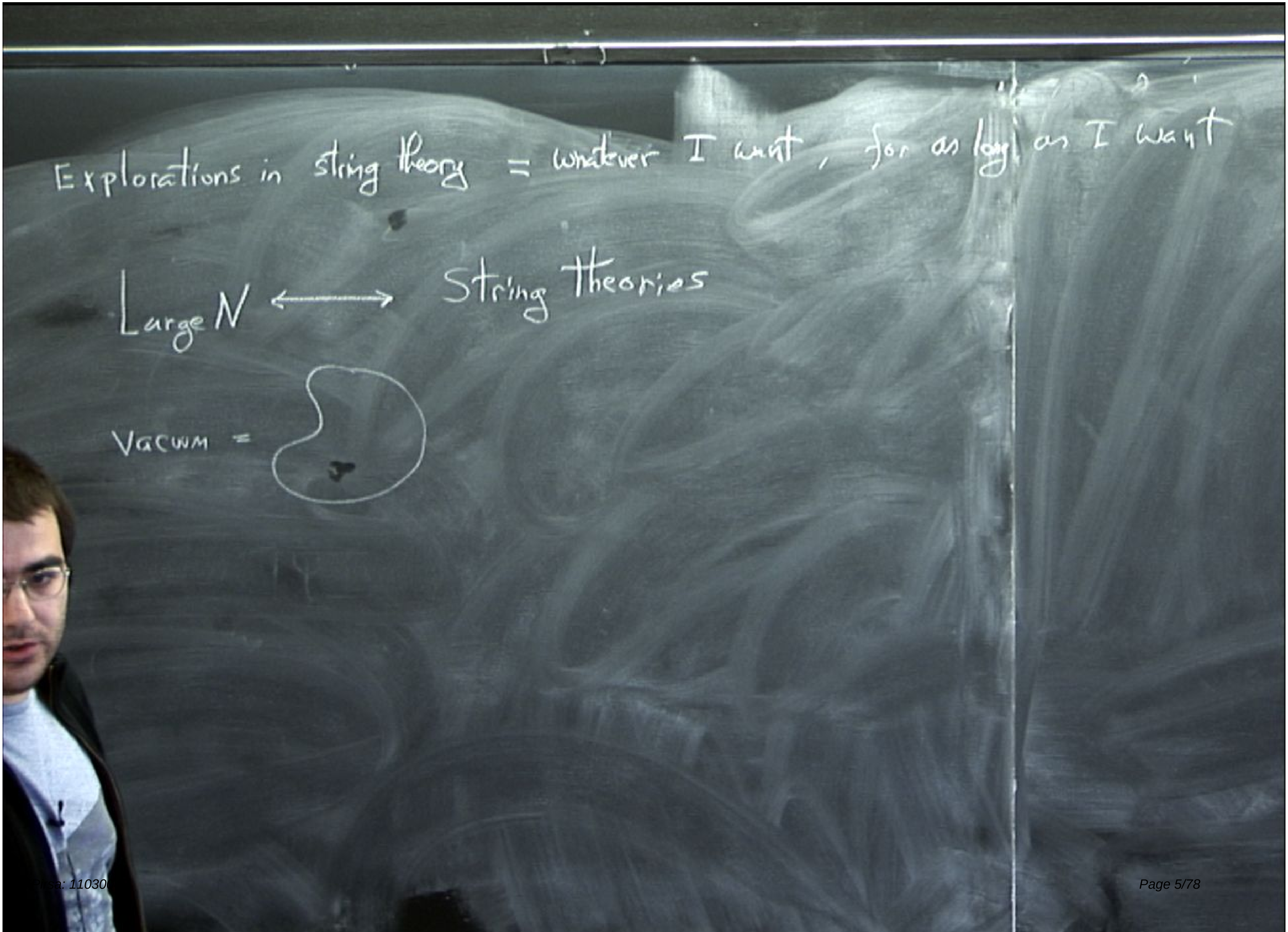
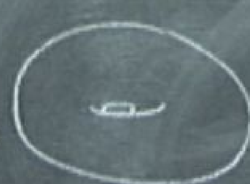

Explorations in string theory = whatever I want


Explorations in string theory = whatever I want, for as long as I want
Large N ⟷ String theories
Vacuum =

Explorations in string theory = whatever I want, for as long as I want

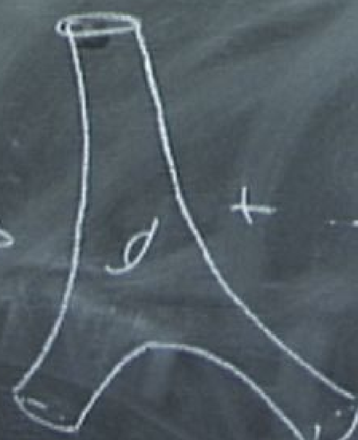
Large N $\longleftrightarrow$ String theories

Vacuum =  $+ g_s$  $+ g_s^2$  $+ \cdots$

Explorations in string theory = whatever I want, for as long as I want

Large N $\longleftrightarrow$ String theories

Vacuum =  $+ g_s$  $+ g_s^2$  $+ \cdots$

3 strings scattering =  $+ g_s$  $+ \cdots$

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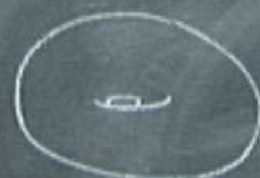
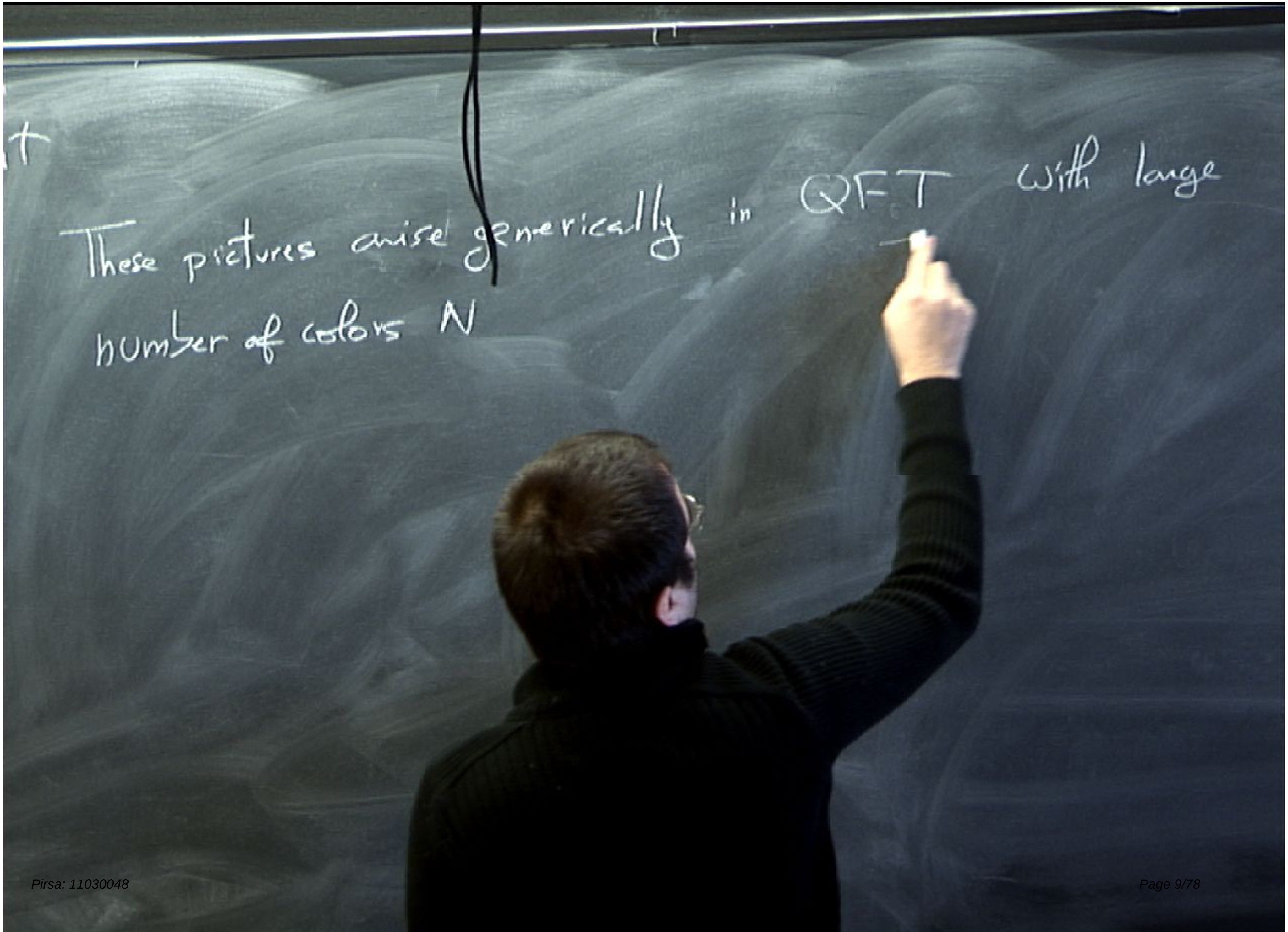
Large N $\longleftrightarrow$ String theories

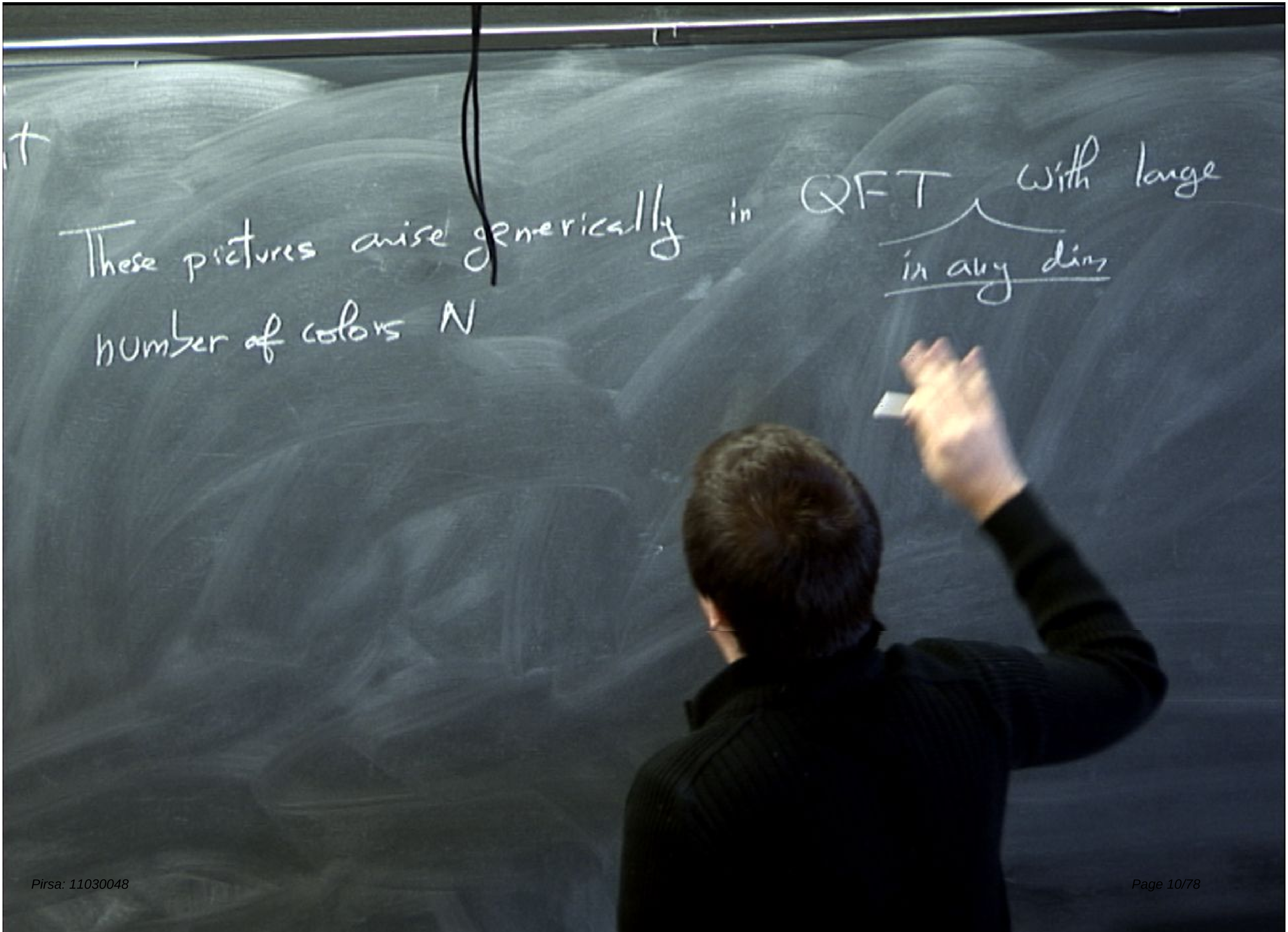
Vacuum =  $+ g_s$  $+ g_s^2$  $+ \cdots$

3 strings scattering =  $+ g_s$  $+ \cdots$

$\leftarrow g_s$ effects

$\leftarrow$ string tension


These pictures arise generically in QFT with large
number of colors N


These pictures arise generically in QFT with large
in any dim
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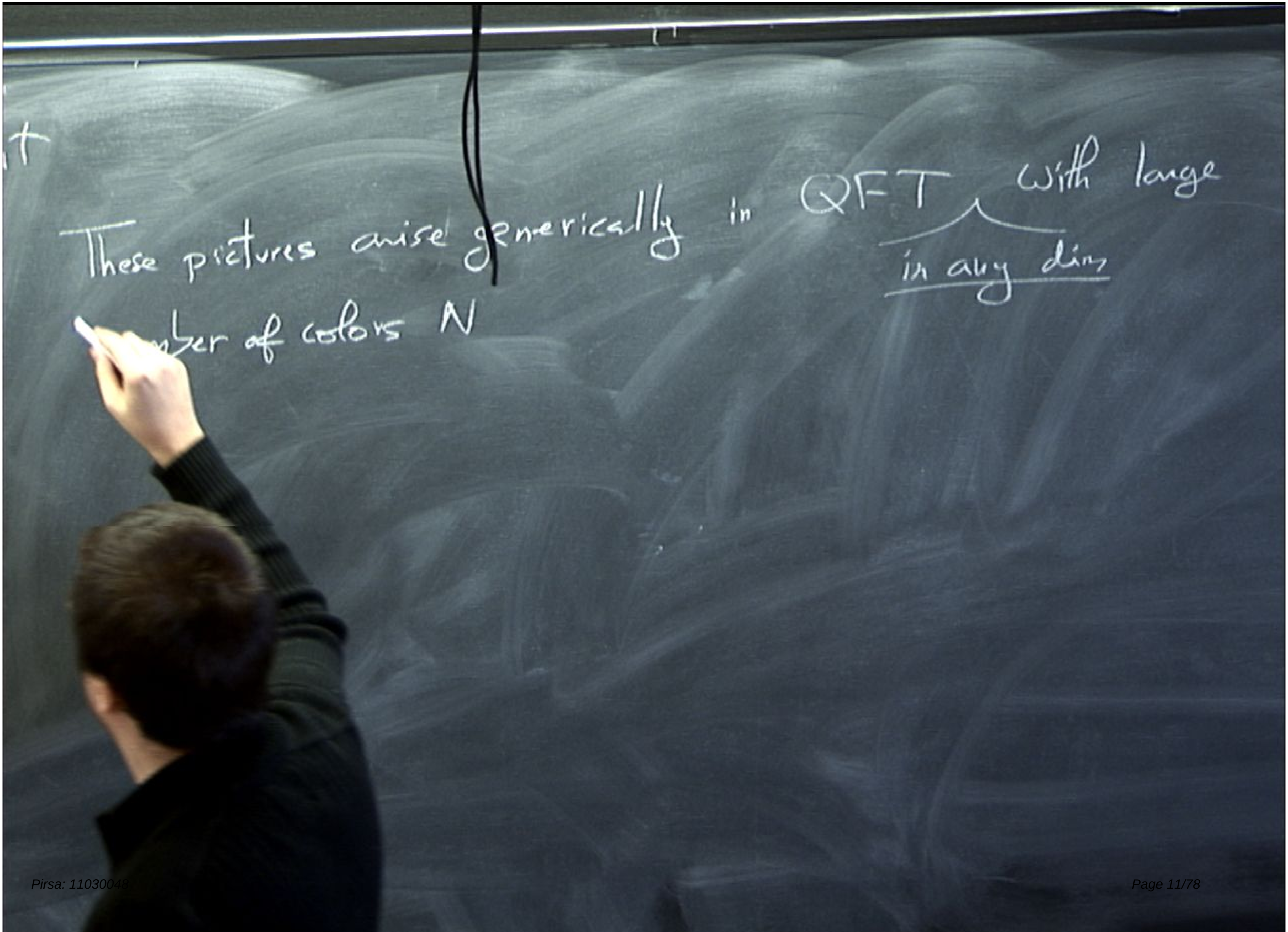
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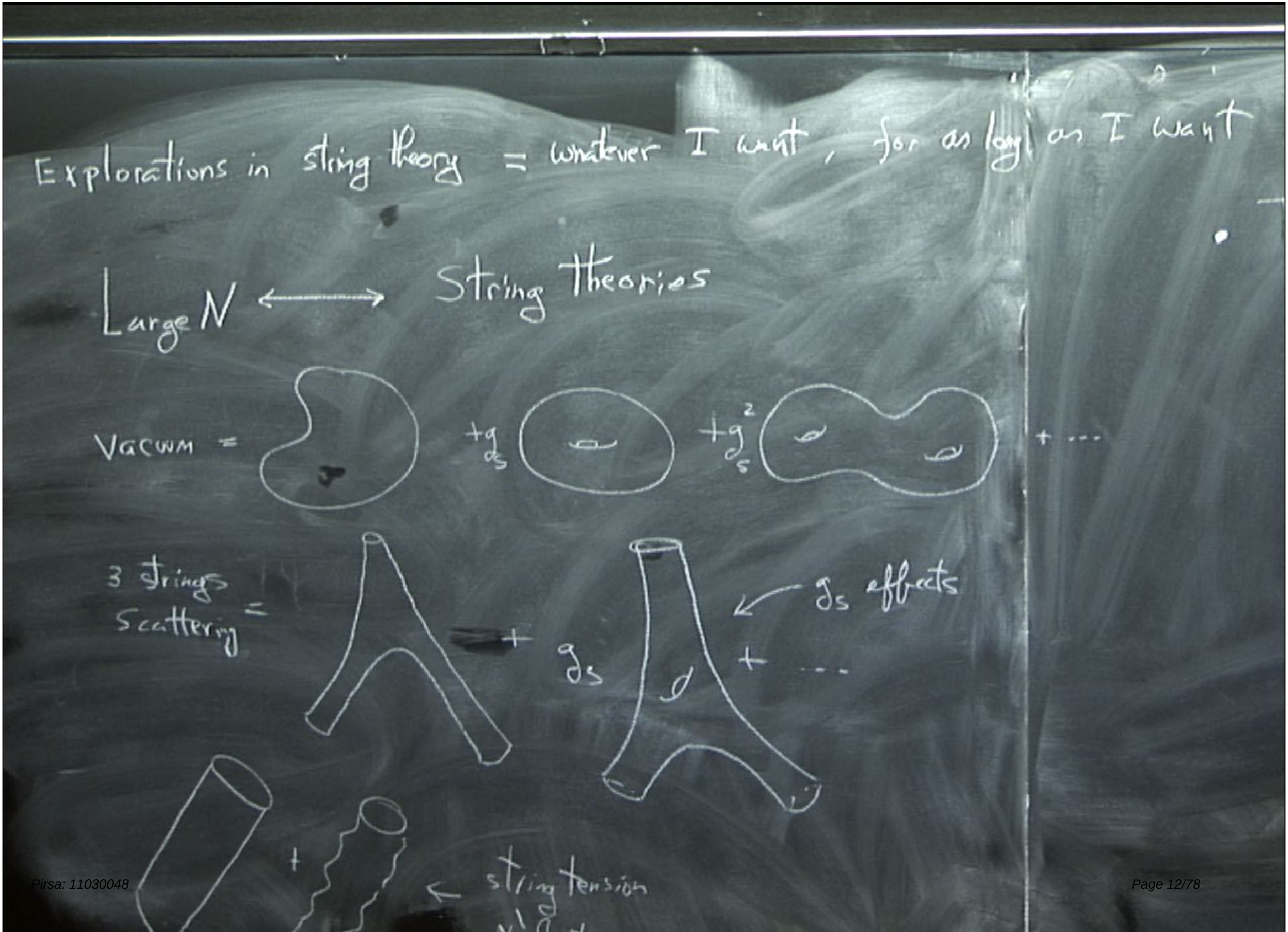
Explorations in string theory = whatever I want, for as long as I want
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nt

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$d=0$ Quantum gauge theory $\equiv$ matrix models

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- These pictures arise generically in $\underbrace{\text{QFT}}_{\text{in any dim}}$ with large number of colors N

$d=0$ Quantum gauge theory $\equiv$ matrix models

$$Z = \int \mathcal{D} \overset{\text{N×N matrices}}{M} \exp\left(-\frac{1}{g_s^2}\right.$$

want

- These pictures arise generically in $\underbrace{\text{QFT}}_{\text{in any dim}}$ with large number of colors N

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$d=0$ Quantum gauge theory $\equiv$ matrix models

NxN matrices

tyadic

$$Z = \int \underbrace{\mathcal{D}M} \exp\Big(-\frac{1}{2g_s}\mathrm{Tr}\big(M^2 + \overbrace{\tfrac{1}{3}t_3 M^3 + \tfrac{1}{4}t_4 M^4 + \ldots}^{V(M)}\big)\Big)$$

Haar measure $\mathcal{D}M = \prod_{i} dM_{ii}$

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$d=0$ Quantum gauge theory $\equiv$ matrix models

$N\times N$ matrices

kinetic

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d = 0 Quantum gauge theory $\equiv$ matrix models

$$Z = \int \mathcal{D} \overset{N\times N,\ U(N)\ \text{matrices}}{M} \exp\Big(-\frac{1}{2g_s}\mathrm{Tr}\big(\overset{\text{kinetic}}{M^2} + \overbrace{\tfrac{1}{3}t_3 M^3 + \tfrac{1}{4}t_4 M^4 + \cdots}^{V(M)}\big)\Big)$$

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$d=0$ Quantum gauge theory $\equiv$ matrix models

$$Z = \int \underbrace{\mathcal{D}M}_{\text{Haar measure}}^{N\times N,\ U(N) \text{ matrices}} \exp\Big(-\frac{1}{2g_s}\mathrm{Tr}\big(M^2 + \overbrace{\tfrac{1}{3}t_3 M^3 + \tfrac{1}{4}t_4 M^4 + \cdots}^{V(M)}\big)\Big)$$

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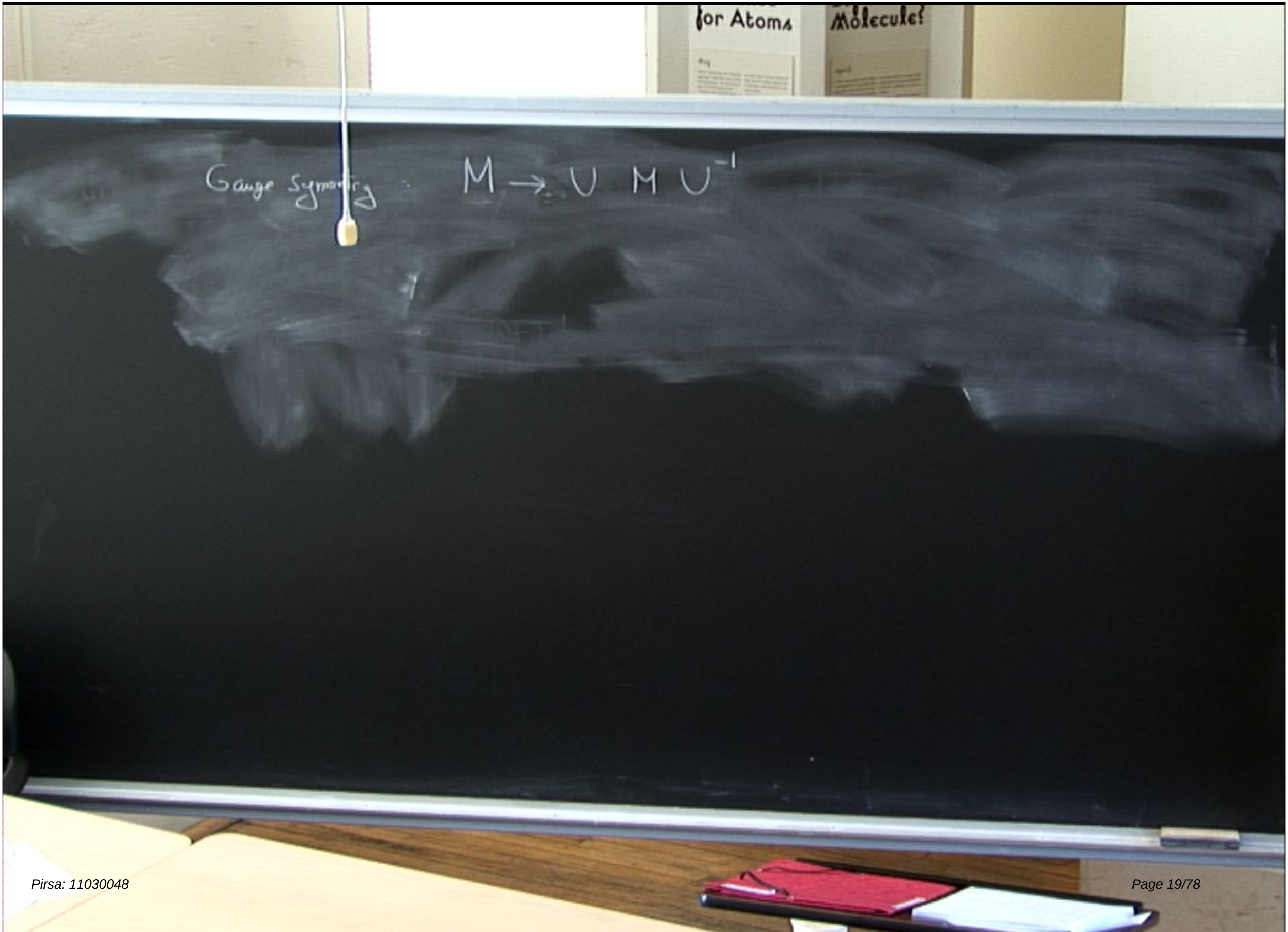
Haar measure $\mathcal{D}M = \frac{1}{\mathrm{Vol}(U(N))} \prod_i dM_{ii} \prod_{i<j} d\,\mathrm{Re}(M_{ij})\, d\,\mathrm{Im}(M_{ij})$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Molecule?
Gauge Symmetry : $M \to U M U^{-1}$

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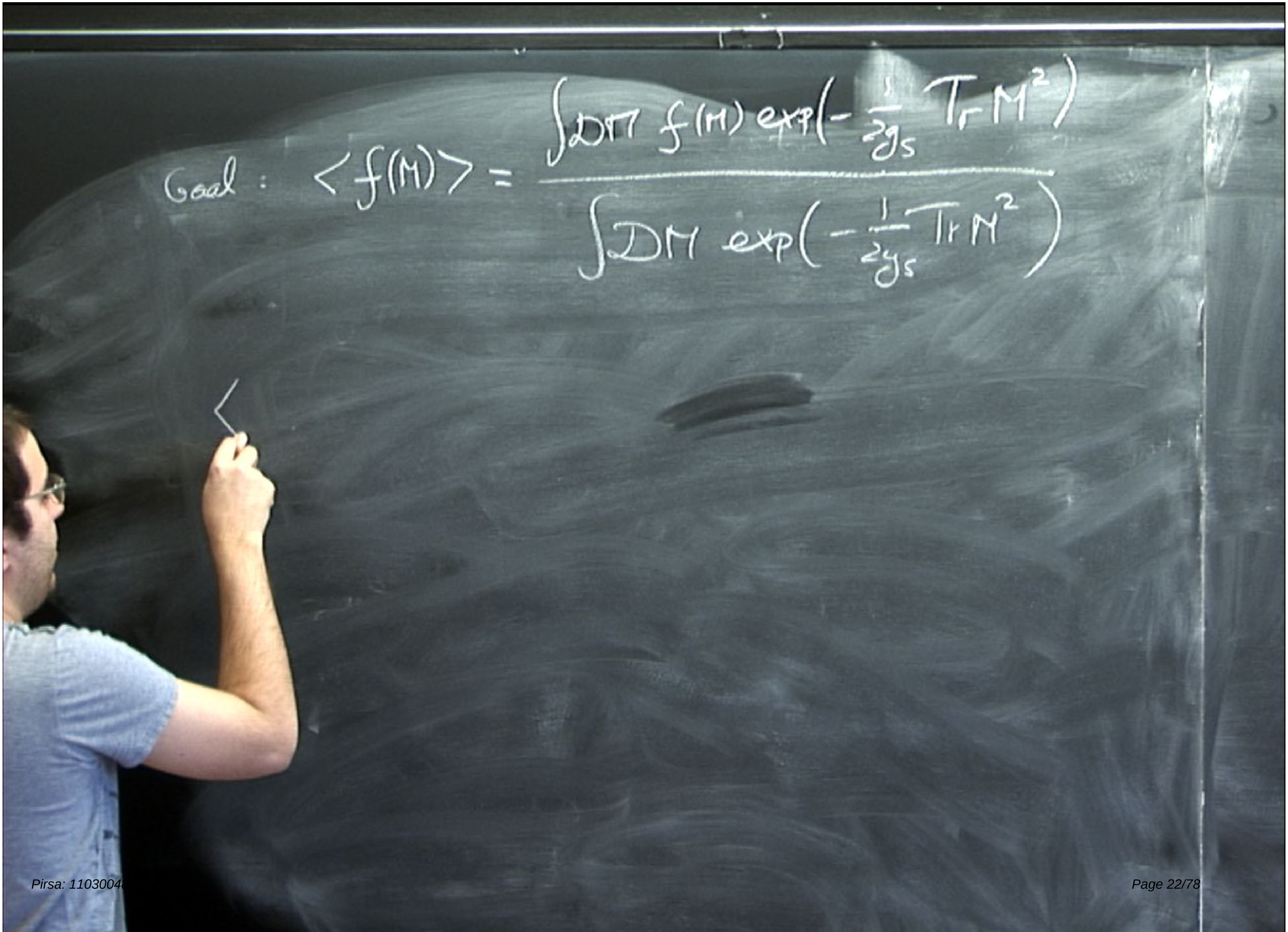
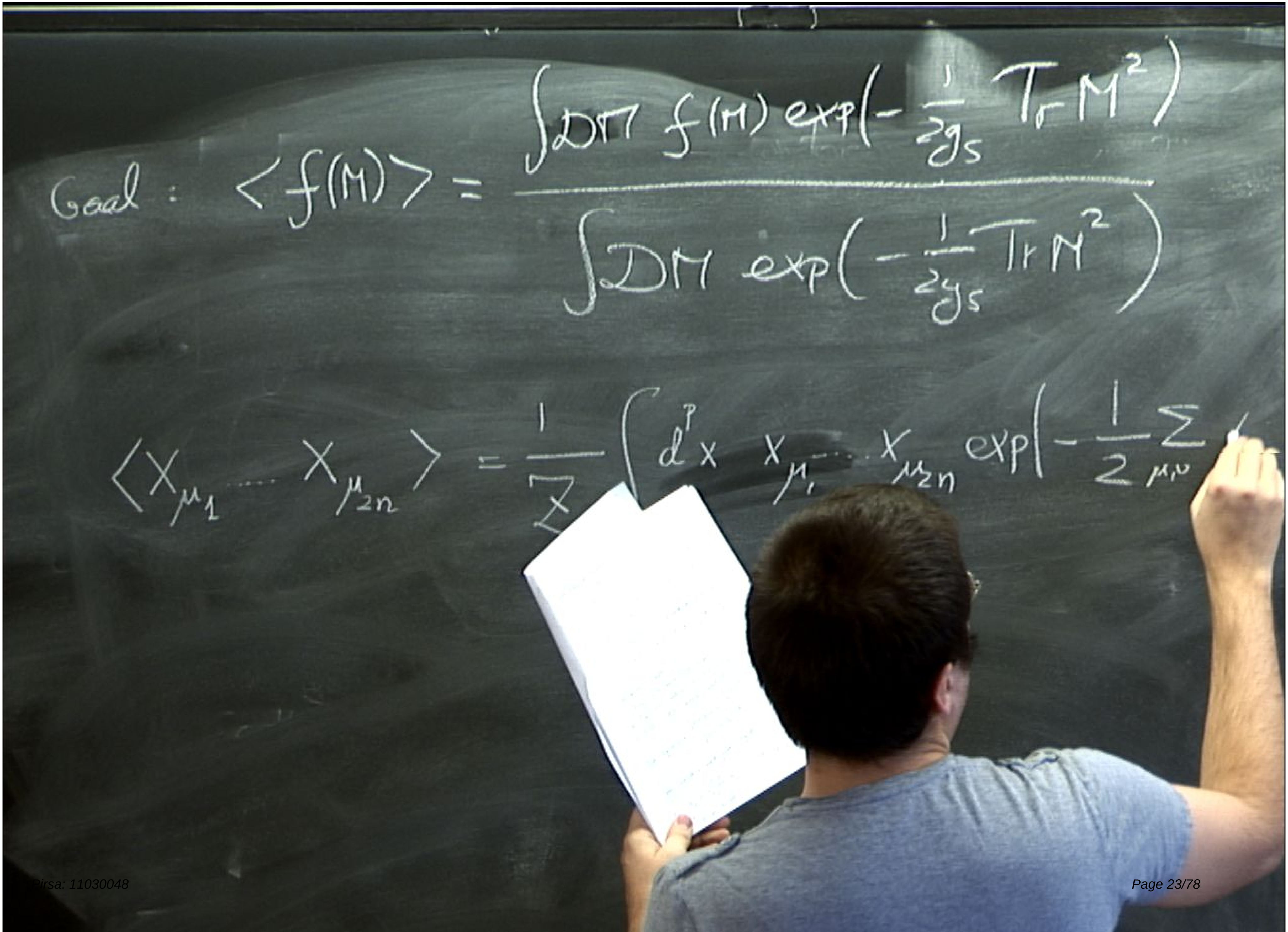
— Averages on the Gaussian MM

Gauge Symmetry : $M \to U M U^{-1}$

- Averages on the Gaussian MM
- Cubic MM & 2D QG
- 2 MM $(\mathcal{D}M \to \mathcal{D}A\,\mathcal{D}B)$ & 2D random ising model


Goal : $\langle f(M) \rangle = \dfrac{\int \mathcal{D}M \, f(M) \exp\left(-\frac{1}{2g_s} \mathrm{Tr}\, M^2\right)}{\int \mathcal{D}M \exp\left(-\frac{1}{2g_s} \mathrm{Tr}\, M^2\right)}$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$\langle X_{\mu_1} \cdots X_{\mu_{2n}} \rangle = \frac{1}{Z} \int d^p x \; x_{\mu_1} \cdots x_{\mu_{2n}} \exp\Big(-\frac{1}{2} \sum_{\mu,\nu}$

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$\langle X$

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$$\langle X_{\mu_1} X_{\mu_2} \rangle = A^{-1}_{\mu_1, \mu_2}$$

$$\langle X_{\mu_1} \cdots X_{\mu_{2n}} \rangle = \sum_{\text{pairings}} \prod_{i=1}^{n} \langle X_{\sigma_{2i-1}} X_{\sigma_{2i}} \rangle$$

$$XXXX + XXXX$$

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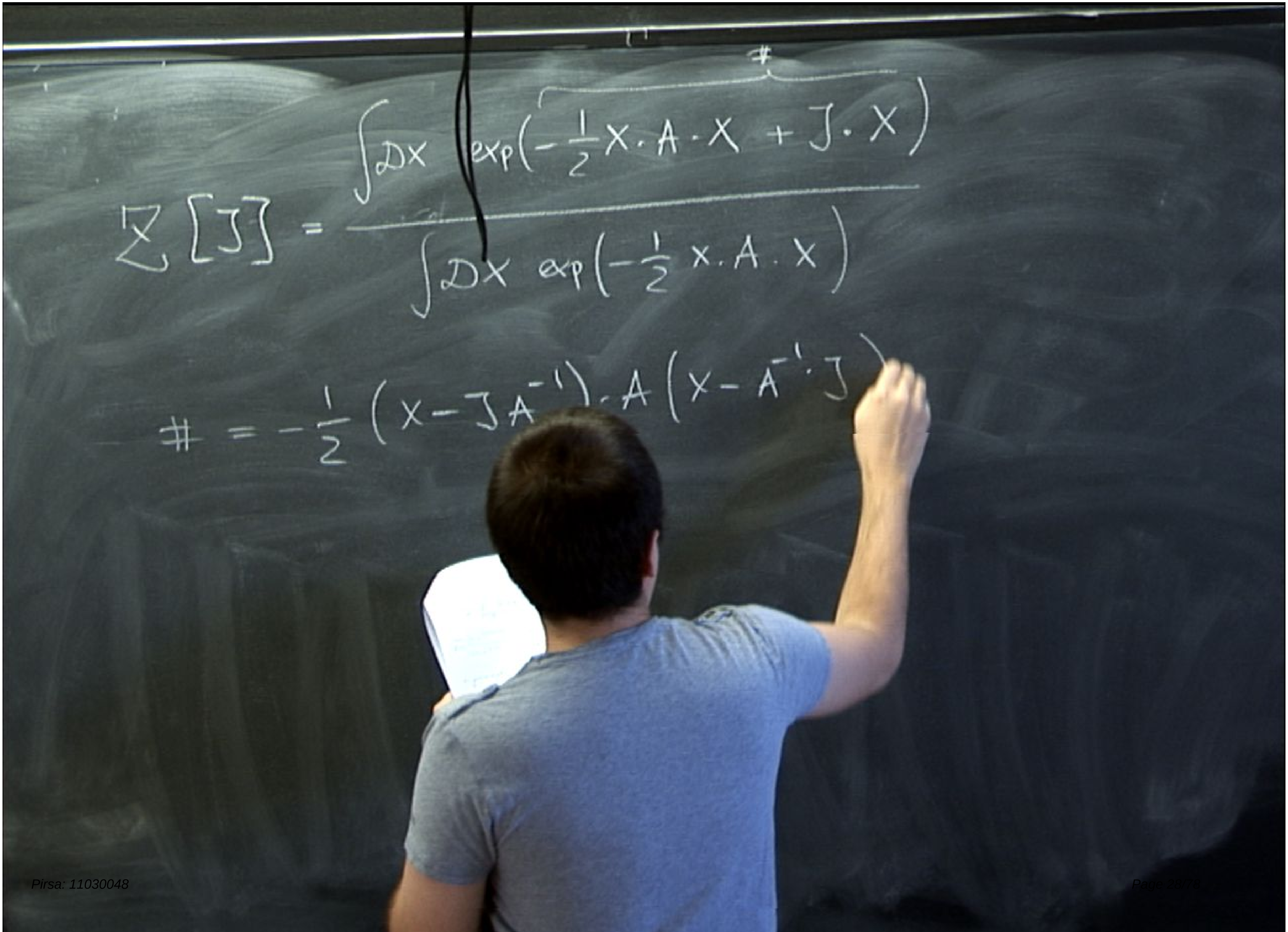
e.g. $XXXX + XXXX + XXXX$


$Z[J] = \dfrac{\int \mathcal{D}X \exp\big(\overbrace{-\frac{1}{2} X \cdot A \cdot X + J \cdot X}^{\#}\big)}{\int \mathcal{D}X \exp\big(-\frac{1}{2} X \cdot A \cdot X\big)}$
$\# = -\frac{1}{2}(X - J A^{-1}) \cdot A (X - A^{-1} \cdot J)$

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$$\# = \quad (X - J A^{-1}) \cdot A \left(X - A^{-1} \cdot J\right) + \tfrac{1}{2} J \cdot A^{-1} \cdot J$$

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$$\# = -\frac{1}{2}\left(x - J A^{-1}\right) \cdot A \left(x - A^{-1} \cdot J\right) + \frac{1}{2} J \cdot A^{-1} \cdot J$$

$$Z[J] = \frac{\int \mathcal{D}X \, \exp(\overbrace{-\frac{1}{2} X \cdot A \cdot X + J \cdot X}^{\#})}{\int \mathcal{D}X \, \exp(-\frac{1}{2} X \cdot A \cdot X)}$$

$$\# = -\frac{1}{2}(\underbrace{X - JA^{-1}}_{\tilde{X}}) \cdot A (X - A^{-1} \cdot J) + \frac{1}{2} J \cdot A^{-1} \cdot J$$

$$Z[J] = \exp\left(\frac{1}{2} J \cdot A^{-1} J\right)$$

Goal: $\langle f(M)\rangle = \dfrac{\int DM\, f(M) \exp\left(-\frac{1}{2g_s} \mathrm{Tr}\, M^2\right)}{\int DM \exp\left(-\frac{1}{2g_s} \mathrm{Tr}\, M^2\right)}$

$$\langle X_{\mu_1} \cdots X_{\mu_{2n}}\rangle = \frac{1}{Z}\int d^N x\; x_{\mu_1} \cdots x_{\mu_{2n}} \exp\left(-\frac{1}{2}\sum_{\mu\nu} x_\mu A_{\mu\nu} x_\nu\right)$$

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$$\langle X_{\mu_1} \cdots X_{\mu_{2n}}\rangle = \sum_{\text{pairings}} \prod_{i=1}^{n} \langle X_{\sigma_{2i-1}} X_{\sigma_{2i}}\rangle \longleftarrow \text{obtained by } \left.\frac{\delta^n}{\delta J^n} Z[J]\right|_{J=0}$$

e.g. $XXXX + XXXX + XXXX$

$$Z[J] = \frac{\int DX \exp\left(-\frac{1}{2} X\cdot A\cdot X + J\cdot X\right)}{\int DX \exp\left(-\frac{1}{2} X\cdot A\cdot X\right)}$$

$$\# = -\frac{1}{2}\big(\underbrace{X - J A^{-1}}_{\tilde{X}}\big)\cdot A\left(X - A^{-1} J\right) + \frac{1}{2} J\, A^{-1} J$$

$$Z[J] = \exp\left(\frac{1}{2} J\, A^{-1} J\right)$$

The gaussian integral is a particular example

$$Z[J] = \frac{\int \mathcal{D}X \, \exp\Big(\overbrace{-\frac{1}{2} X\cdot A\cdot X + J\cdot X}^{\#}\Big)}{\int \mathcal{D}X \, \exp\left(-\frac{1}{2} X\cdot A\cdot X\right)}$$

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ed by $\left.\frac{\delta^{2n}}{\delta J^{2n}} Z[J]\right|_{J=0}$

The gaussian integral is a particular example $\frac{1}{2}\mathrm{Tr}\,M^2 = \frac{1}{2}\sum_i M_{ii}^2 + \sum_{i<j} (\ldots$

$$Z[J] = \frac{\int \mathcal{D}X \, \exp\Big(\overbrace{-\frac{1}{2} X \cdot A \cdot X + J \cdot X}^{\#}\Big)}{\int \mathcal{D}X \, \exp\left(-\frac{1}{2} X \cdot A \cdot X\right)}$$

$$\# = -\frac{1}{2}\big(\underbrace{X - J A^{-1}}_{\tilde{X}}\big) \cdot A \left(X - A^{-1} \cdot J\right) + \frac{1}{2} J \cdot A^{-1} \cdot J$$

$$Z[J] = \exp\left(\frac{1}{2} J \cdot A^{-1} J\right)$$

We can now compute

$$\langle M_{ij} M_{kl} \rangle$$

ined by $\left.\frac{\delta^{2n}}{\delta J^{2n}} Z[J]\right|_{J=0}$

The gaussian integral is a particular example $\frac{1}{2} \mathrm{Tr}\, M^2 = \frac{1}{2} \sum_i M_{ii}^2 + \sum_{i<j} \left(\mathrm{Re}(M_{ij})^2 + (\mathrm{Im}(M_{ij}))^2\right)$

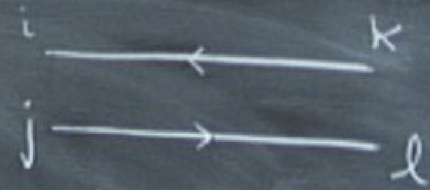
We find

$$\langle M_{ij} M_{k\ell} \rangle = g_s \, \delta_{ik} \, \delta_{j\ell}$$

random

We find

$$\langle M_{ij} M_{lk} \rangle = g_s \, \delta_{ik} \, \delta_{jl} = g_s$$

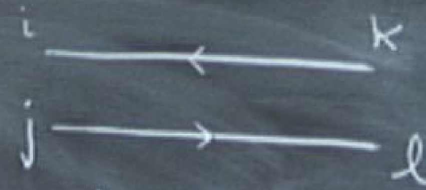


Exercise: Work out the details

double line notation on fat graphs

We find

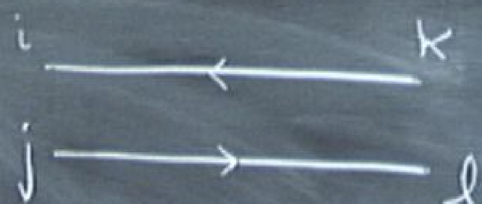
$$\langle M_{ij} M_{lk} \rangle = g_s \, \delta_{ik} \, \delta_{jl} = g_s$$



Exercise : Work out the details

double line notation on f.t graphs

$$g_s \, \delta_{ik} \, \delta_{j\ell} = g_s$$



double line notation on fat graphs

thin graph

fat

the details

$$f(M) = \mathrm{Tr}\, M^4 = \sum_{ijkl}^{N} M_{ij} M_{jk} M_{kl} M_{li}$$

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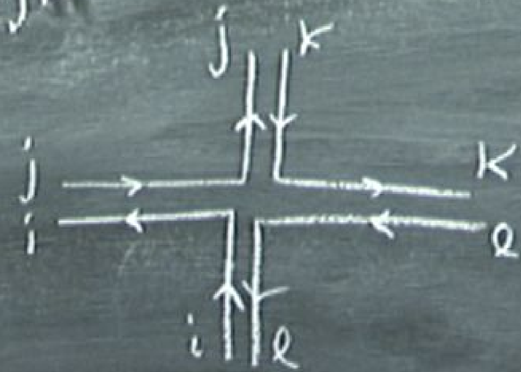
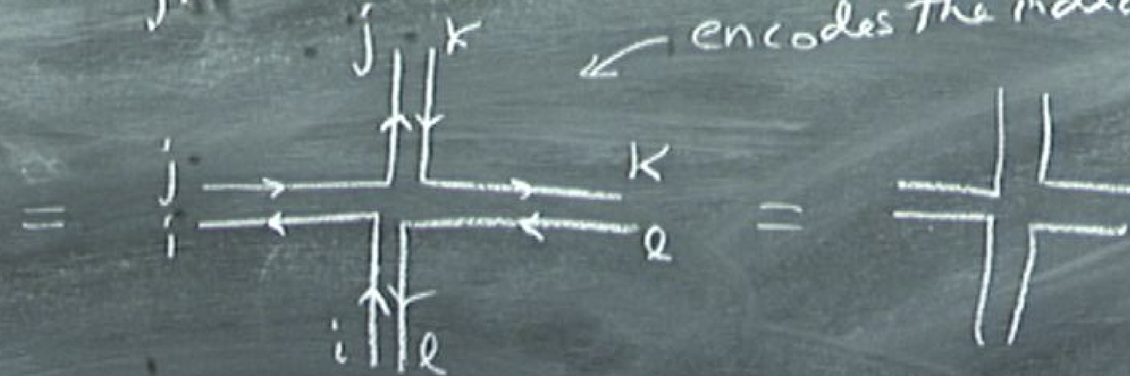
$=$

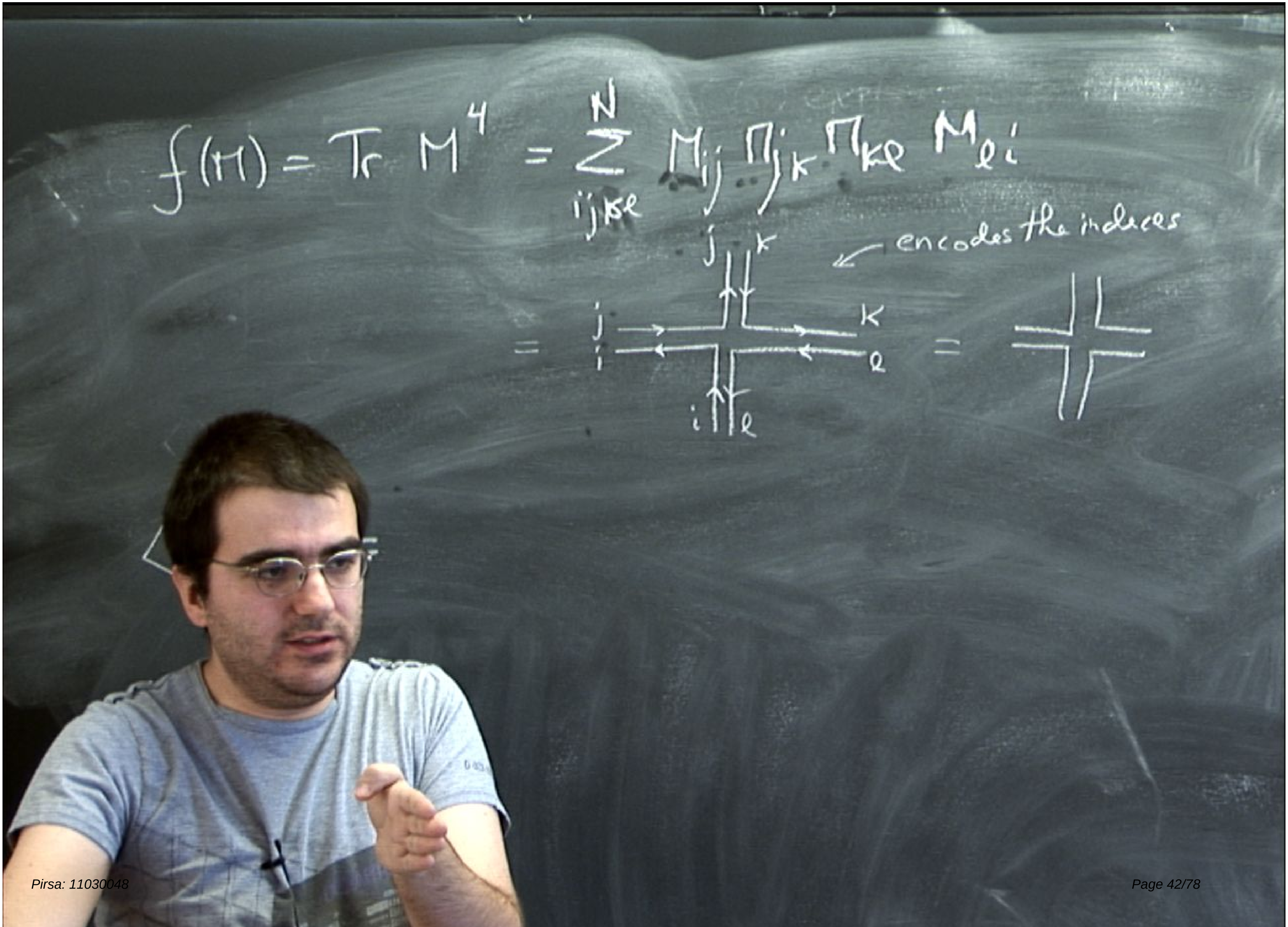


$$f(M) = \mathrm{Tr}\, M^4 = \sum_{ijkl}^{N} M_{ij} M_{jk} M_{kl} M_{li}$$

encodes the indices



$$\langle \mathrm{Tr}\, M^4 \rangle =$$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j
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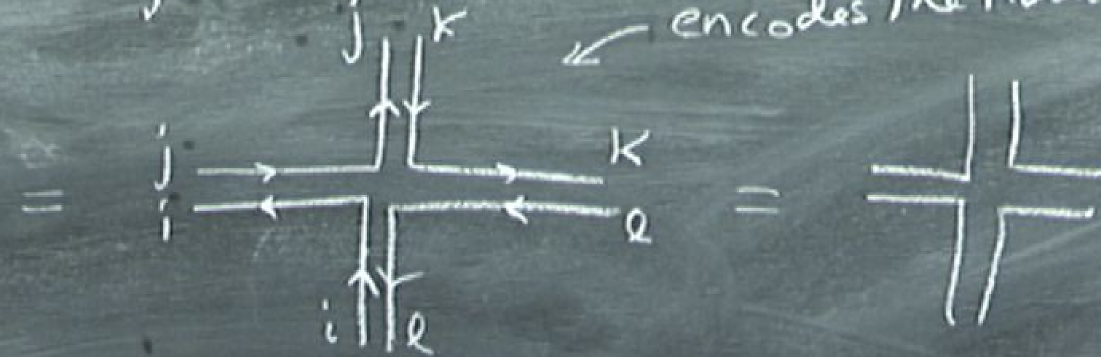

$$\langle \mathrm{Tr}\, M^4 \rangle = \sum_{ijkl} \langle M_{ij} M_{jk} \rangle \langle M_{kl} M_{li} \rangle$$

$$+ \sum_{ijkl} \langle M_{ij} M_{kl} \rangle \langle M_{jk} M_{li} \rangle$$

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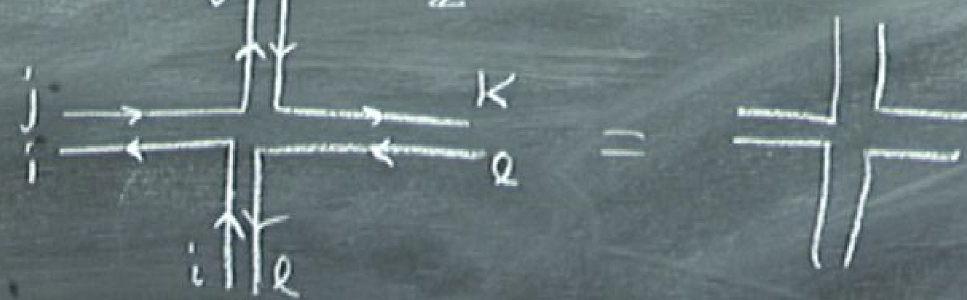
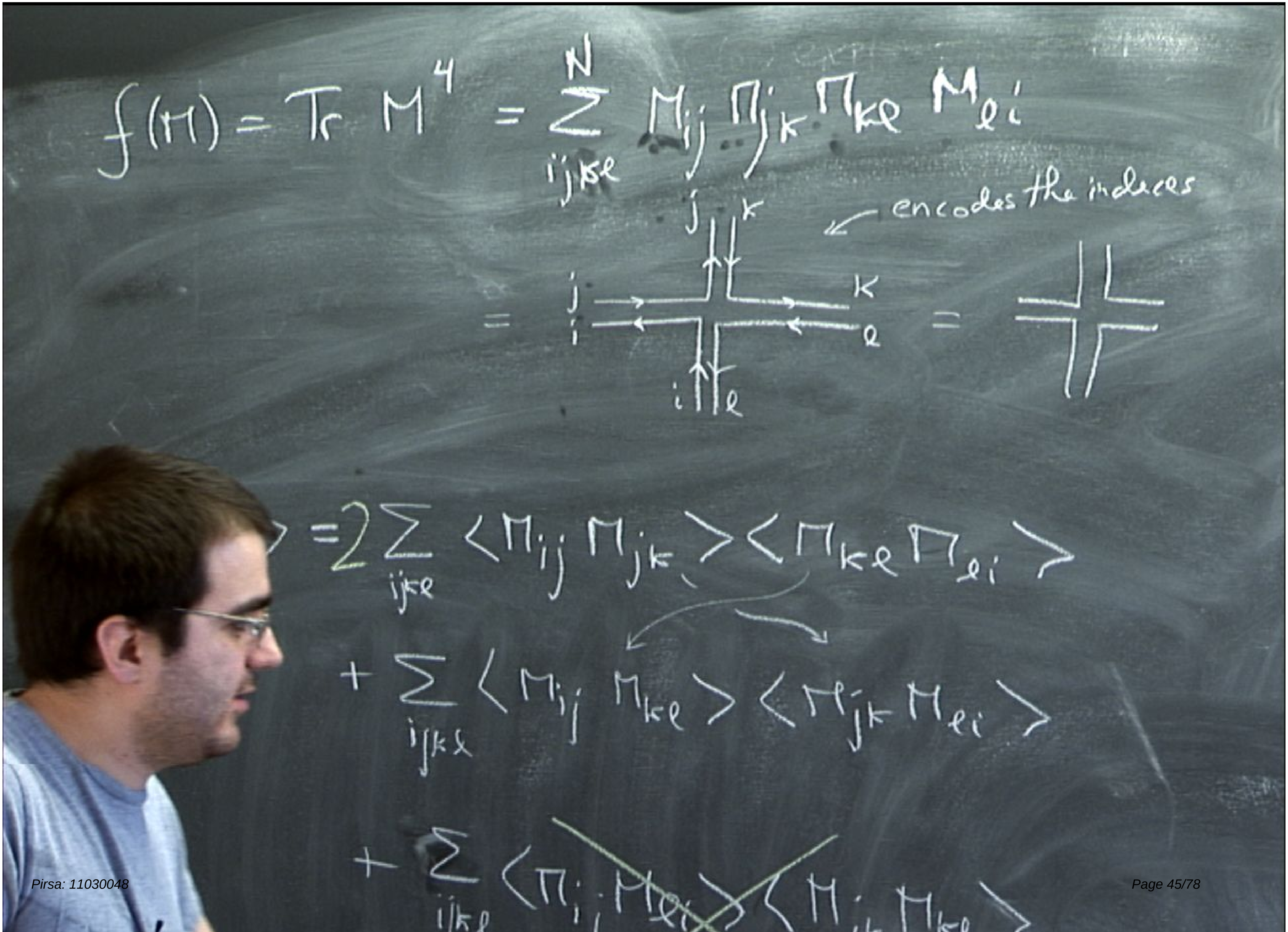


$$\langle \mathrm{Tr}\, M^4 \rangle = 2 \sum_{ijkl} \langle M_{ij} M_{jk} \rangle \langle M_{kl} M_{li} \rangle$$

$$+ \sum_{ijkl} \langle M_{ij} M_{kl} \rangle \langle M_{jk} M_{li} \rangle$$

$$+ \sum_{ijkl} \cancel{\langle M_{ij} M_{li} \rangle \langle M_{jk} M_{kl} \rangle}$$


$f(M) = \mathrm{Tr}\, M^4 = \sum_{ijkl}^{N} M_{ij} M_{jk} M_{kl} M_{li}$
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$= 2\sum_{ijkl} \langle M_{ij} M_{jk} \rangle \langle M_{kl} M_{li} \rangle$
$+ \sum_{ijkl} \langle M_{ij} M_{kl} \rangle \langle M_{jk} M_{li} \rangle$
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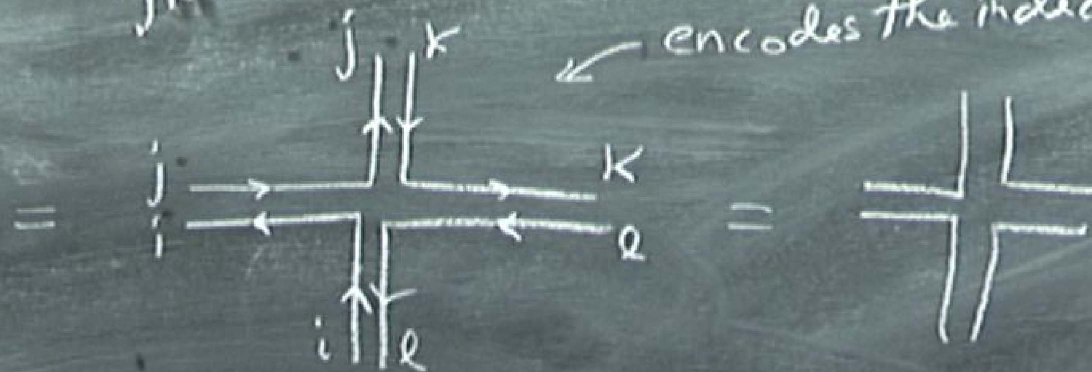
$$f(M) = \mathrm{Tr}\, M^4 = \sum_{ijkl}^{N} M_{ij} M_{jk} M_{kl} M_{li}$$

encodes the indices



$$\langle \mathrm{Tr}\, M^4 \rangle = 2\sum_{ijkl} \langle M_{ij} M_{jk} \rangle \langle M_{kl} M_{li} \rangle$$

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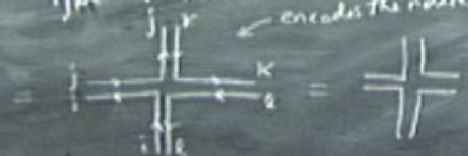
$$+ \sum_{ijkl} \cancel{\langle M_{ij} M_{li} \rangle \langle M_{jk} M_{kl} \rangle}$$

$$f(M) = \mathrm{Tr}\, M^4 = \sum_{ijkl}^{N} M_{ij} M_{jk} M_{kl} M_{li}$$

encodes the indices



$$\langle \mathrm{Tr}\, M^4 \rangle = 2 \sum_{ijkl} \langle M_{ij} M_{jk} \rangle \langle M_{kl} M_{li} \rangle$$

$$+ \sum_{ijkl} \langle M_{ij} M_{kl} \rangle \langle M_{jk} M_{li} \rangle$$

$$+ \sum_{ijkl} \cancel{\langle M_{ij} M_{kl} \rangle \langle M_{jk} M_{kl} \rangle}$$

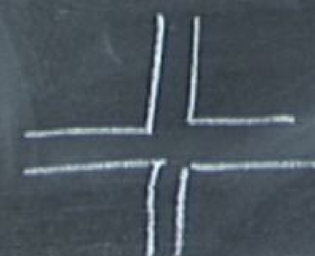
$$= 2 g_s^2 \sum \delta_{ik} \delta_{jj} \delta_{ki} \delta_{ll}$$

$$+ g_s^2 \sum \delta_{il} \delta_{kj} \delta_{ji} \delta_{kl}$$

$$= 2 g_s^2 N^3 + g_s N$$

$$= 2 g_s^2 \sum \delta_{ik}\, \delta_{jj}\, \delta_{ki}\, \delta_{\ell\ell}$$

$$+ g_s^2 \sum \delta_{i\ell}\, \delta_{kj}\, \delta_{ji}\, \delta_{k\ell}$$

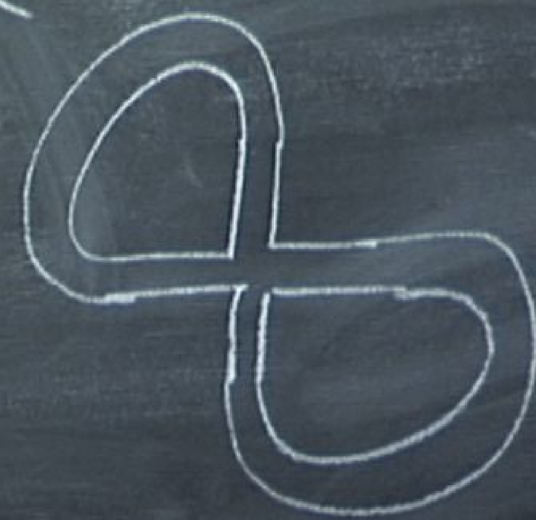
$$= \underbrace{2 g_s^2 N^3}_{\text{important one}} + g_s N = $$ 

if $N \to \infty$

$$= 2 g_s^2 \sum \delta_{ik}\, \delta_{jj}\, \delta_{ki}\, \delta_{ll}$$

$$+ g_s^2 \sum \delta_{il}\, \delta_{kj}\, \delta_{ji}\, \delta_{kl}$$

$$= \underbrace{2 g_s^2 N^3}_{\text{if } N \to \infty} + g_s N \qquad = 2$$



important one

$$= 2 g_s^2 \sum \delta_{ik} \delta_{jj} \delta_{ki} \delta_{\ell\ell}$$

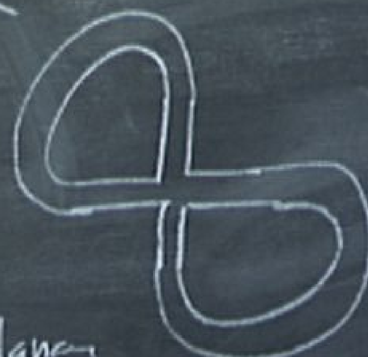
$$+ g_s^2 \sum \delta_{i\ell} \delta_{kj} \delta_{ji} \delta_{k\ell}$$

$$= \underbrace{2 g_s^2 N^3}_{\text{if } N \to \infty} + g_s N \qquad = 2$$

$\to$ important one

$$= 2g_s^2 \sum \delta_{ik}\, \delta_{jj}\, \delta_{ki}\, \delta_{\ell\ell}$$

$$+ g_s^2 \sum \delta_{i\ell}\, \delta_{kj}\, \delta_{ji}\, \delta_{k\ell}$$

$$= \underbrace{2g_s^2 N^3}_{\text{important one}} + g_s N \quad = 2$$  $+$ 

if $N \to \infty$ — important one

non-planar diags.

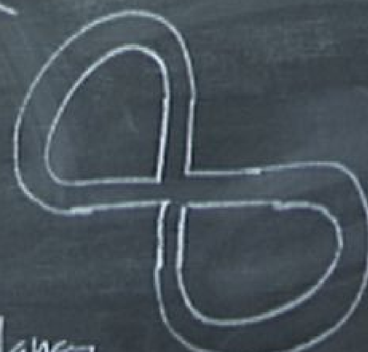
$= 2$  $+$  $\leftarrow \frac{1}{N^2}$ compared with 

planar diag

$$= 2g_s^2 \sum \delta_{ik}\,\delta_{jj}\,\delta_{ki}\,\delta_{\ell\ell}$$

$$+ g_s^2 \sum \delta_{i\ell}\,\delta_{kj}\,\delta_{ji}\,\delta_{k\ell}$$

$$= \underbrace{2 g_s^2 N^3}_{\text{if } N\to\infty} + g_s N \quad = 2$$



$+$

important one

non-planar diags.

$= 2$

$+$

$\frac{1}{N^2}$ compared with

planar diag

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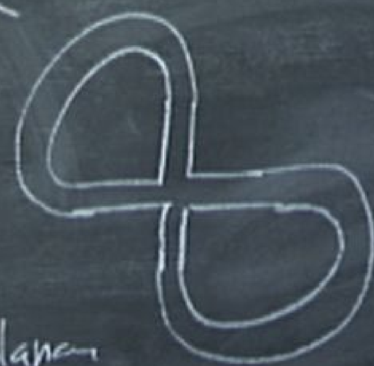
Exercise : $f(M) = \mathrm{tr}\, M^3\, \mathrm{tr}\, M^3$

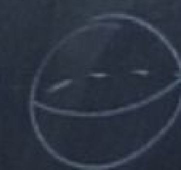
$$= 2 g_s^2 \sum \delta_{ik}\,\delta_{jj}\,\delta_{ki}\,\delta_{ll}$$

$$+ g_s^2 \sum \delta_{il}\,\delta_{kj}\,\delta_{ji}\,\delta_{kl}$$

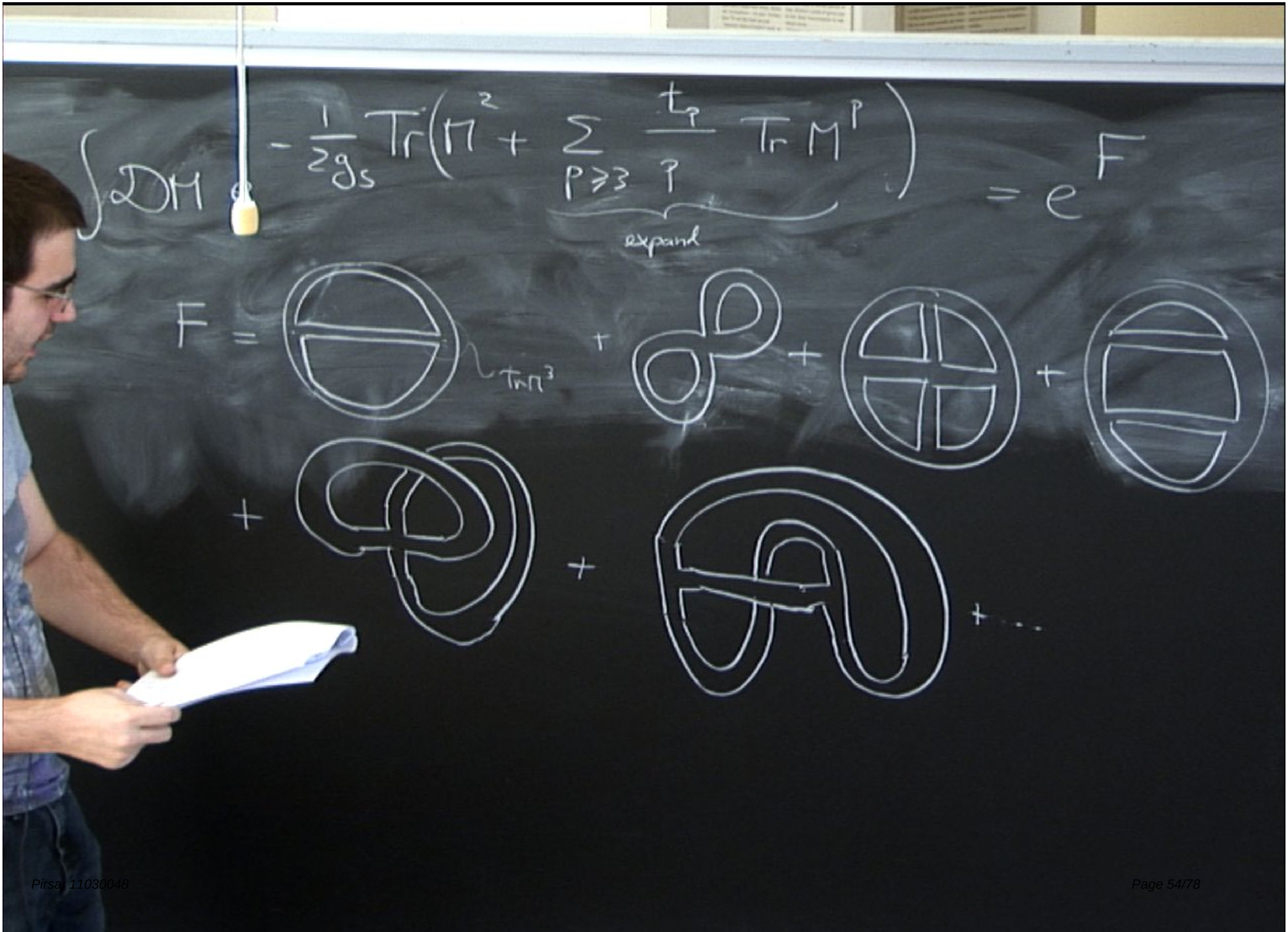
$$= \underbrace{2 g_s^2 N^3}_{\text{important one}} + g_s N \quad = 2$$  $+$ 

if $N \to \infty$

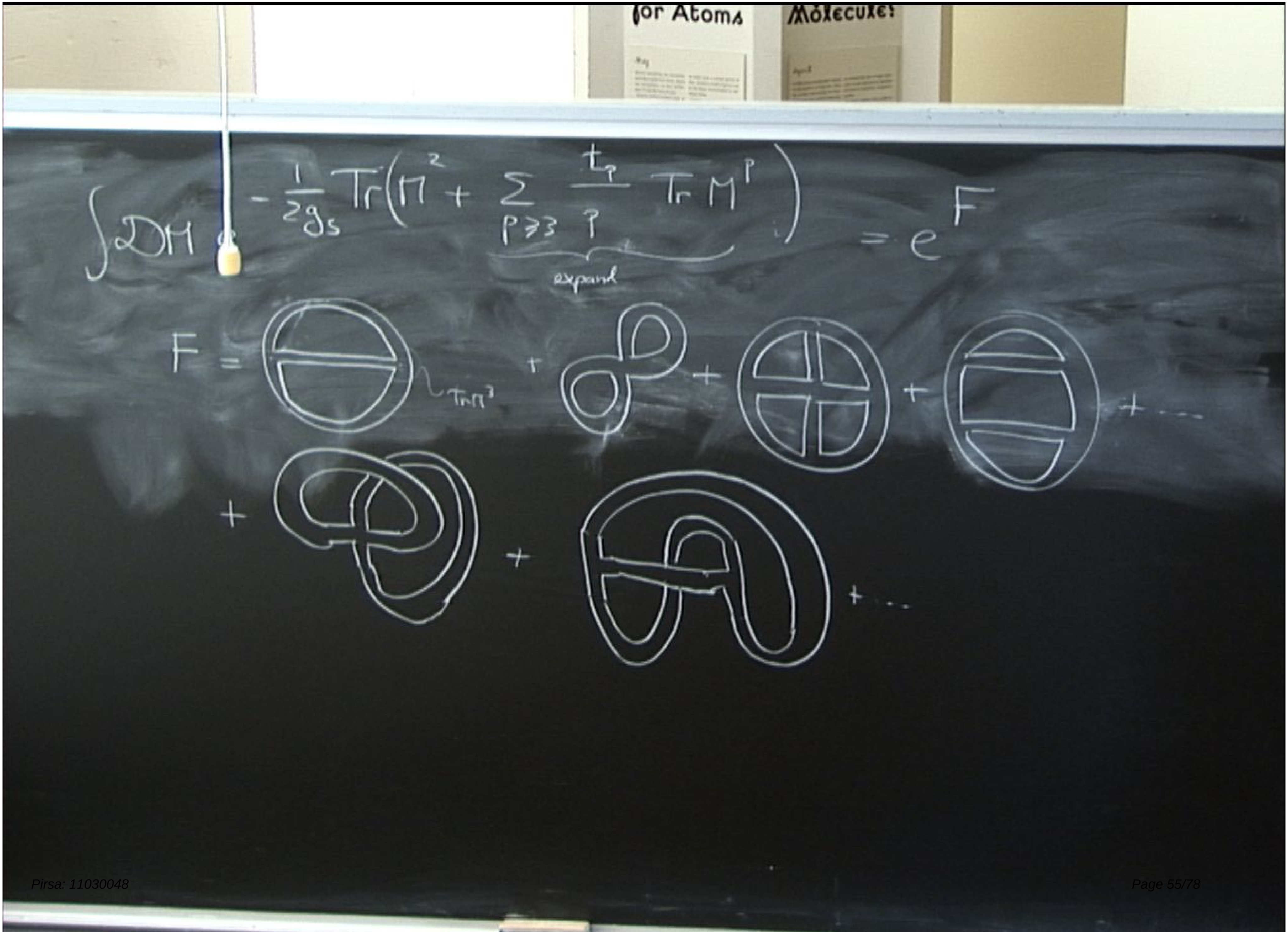
non-planar diags.

$= 2$  $+$  $\leftarrow \frac{1}{N^2}$ compared with 

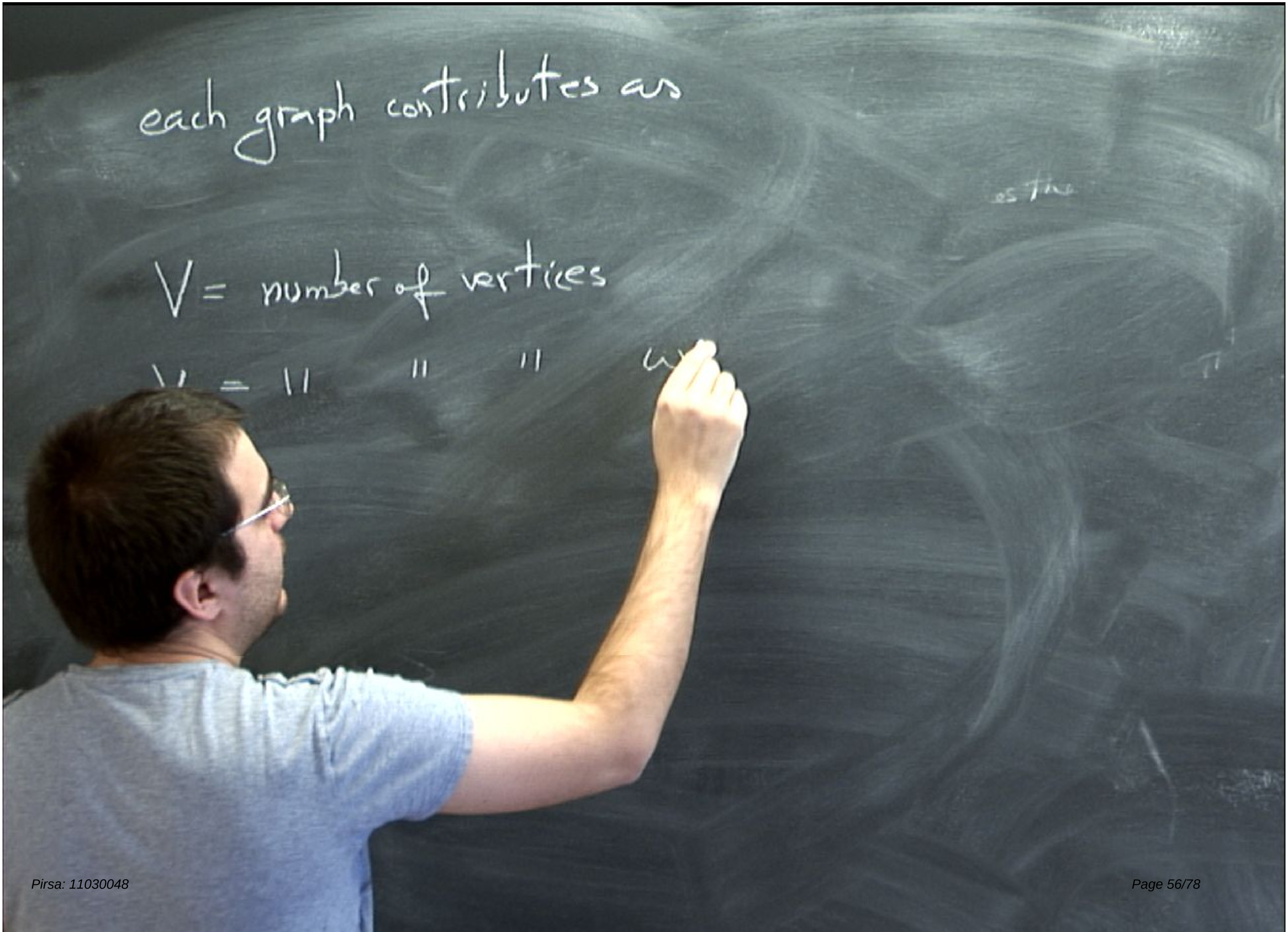
planar diag



$\int \mathcal{D}M \, e^{-\frac{1}{2g_s}\mathrm{Tr}\left(M^2 + \sum_{p\geq 3} \frac{t_p}{p} \mathrm{Tr}\, M^p\right)} = e^F$
expand
$F =$
$\mathrm{Tr}\, M^3$
$+ \cdots$



for Atoms
Molecules
$\int \mathcal{D}M\, e^{-\frac{1}{2g_s}\mathrm{Tr}\left(M^2 + \sum_{p\geq 3} \frac{t_p}{p} \mathrm{Tr}\, M^p\right)} = e^F$
expand
F =
$\mathrm{Tr}M^3$
+ + + + ...
+ + + ...


each graph contributes as
V = number of vertices

each graph contributes as

$V =$ number of vertices

$V_p =$ " " " with p legs

each graph contributes as

es the

$V =$ number of vertices

$V_p =$ " " " with p legs

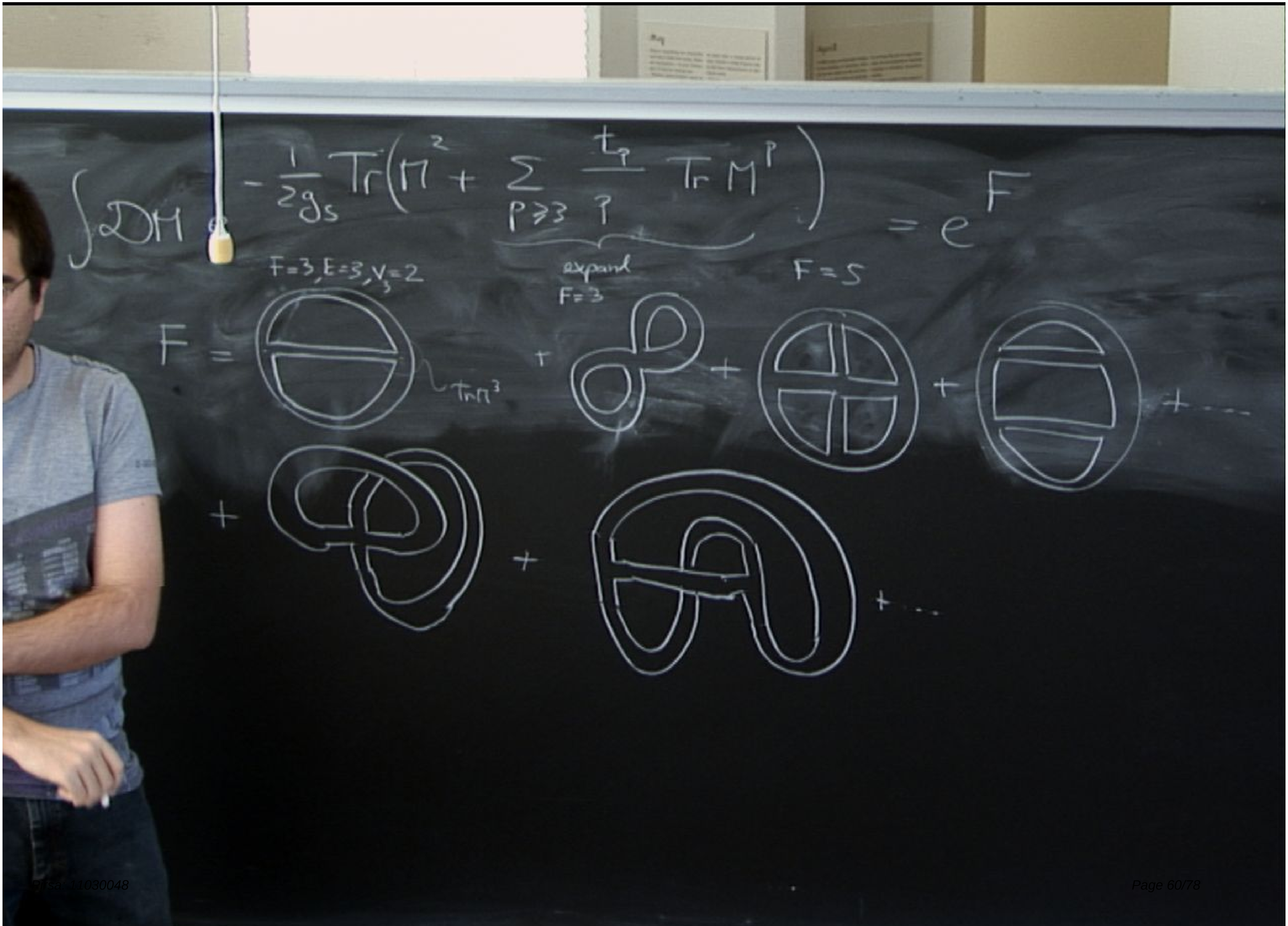
each graph contributes as

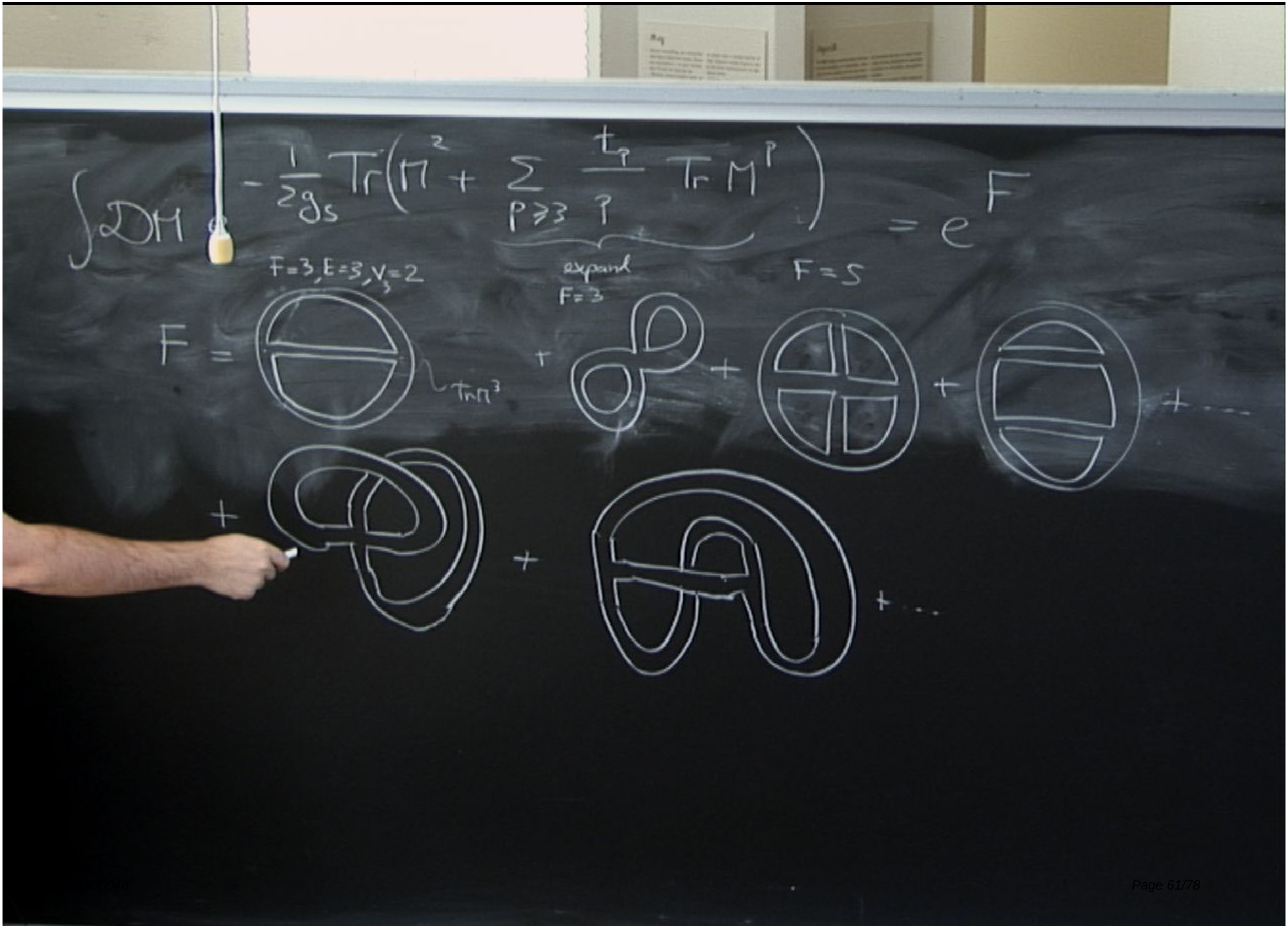
$V =$ number of vertices

$V_p =$ " " " with p legs

$E =$ number of edges

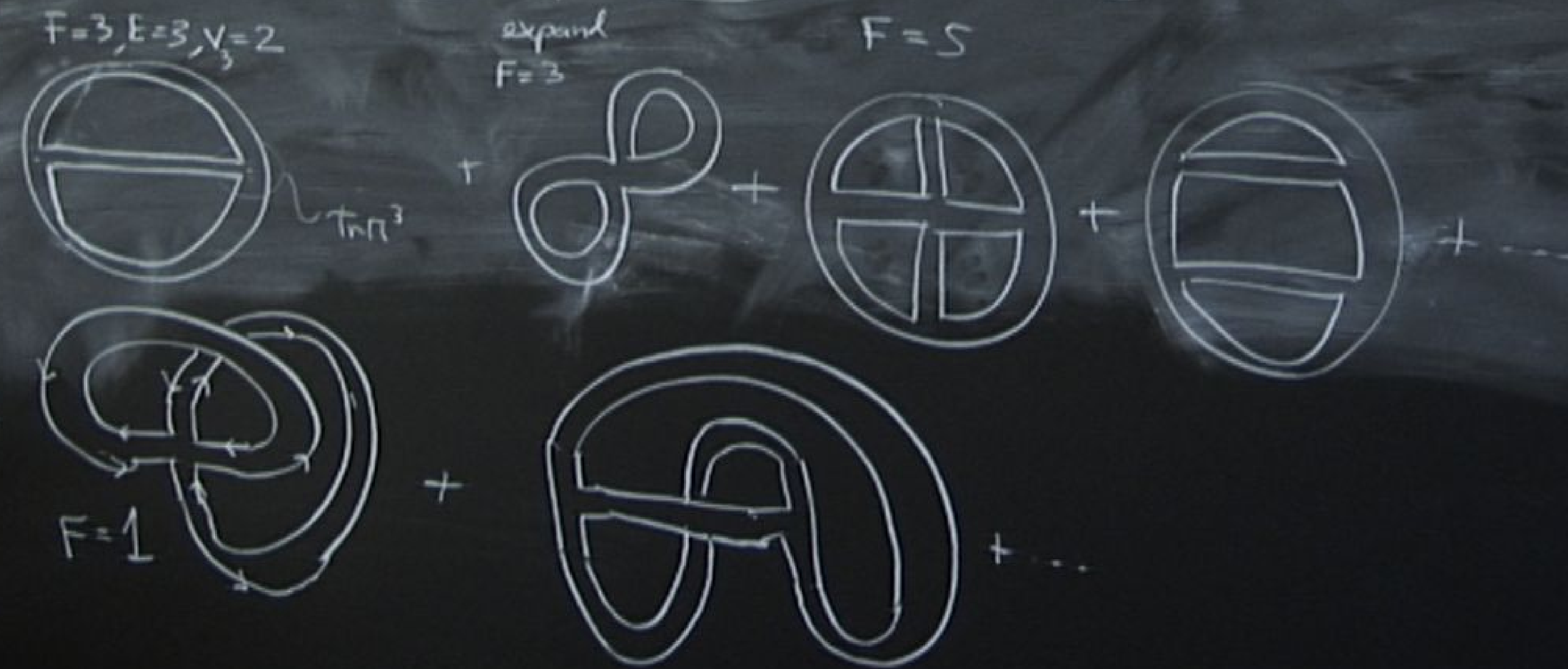
$F =$ number of color loops, i.e. number of faces

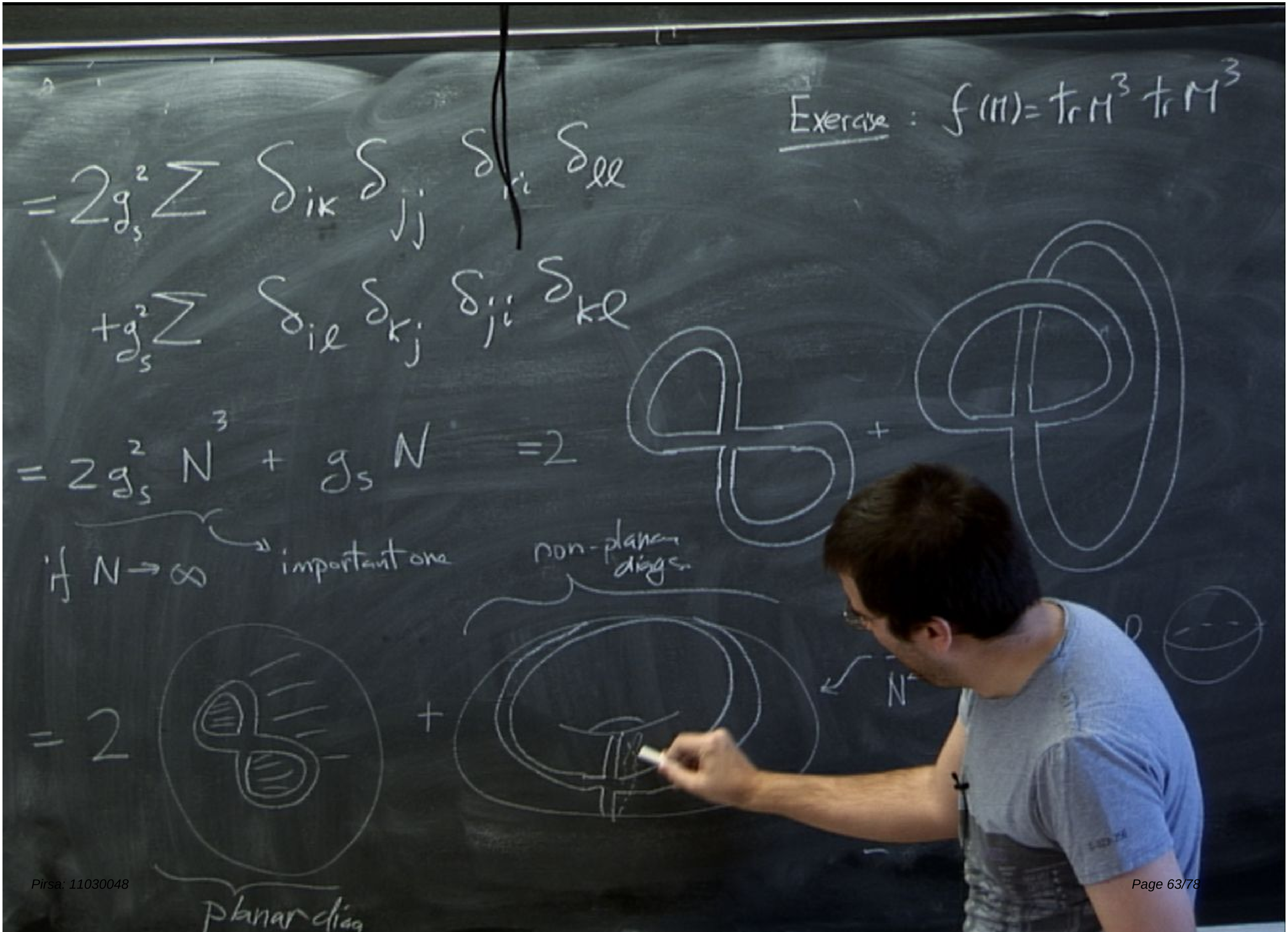

$\int \mathcal{D}M\, e^{-\frac{1}{2g_s}\mathrm{Tr}\left(M^2 + \sum_{p\geq 3} \frac{t_p}{p} \mathrm{Tr}\, M^p\right)} = e^F$
expand
$F=3, E=3, V_3=2$
$F=3$
$F=5$
$\mathrm{Tr}\, M^3$
$F =$
$+ \dots$


$\int \mathcal{D}M \, e^{-\frac{1}{2g_s}\mathrm{Tr}\left(M^2 + \sum_{p\geq 3} \frac{t_p}{p} \mathrm{Tr}\, M^p\right)} = e^F$
expand
F=3, E=3, V_3=2
F=3
F=5
F =
$\mathrm{Tr}\, M^3$
+ ...

$$\int \mathcal{D}M \; e^{-\frac{1}{2g_s}\mathrm{Tr}\left(M^2 + \underbrace{\sum_{p\geq 3} \frac{t_p}{p}\,\mathrm{Tr}\,M^p}_{\text{expand}}\right)} = e^F$$

$F =$





Exercise : $f(M) = \mathrm{tr}\, M^3 + \mathrm{tr}\, M^3$
$= 2g_s^2 \sum \delta_{ik} \delta_{jj} \delta_{ki} \delta_{ll}$
$+ g_s^2 \sum \delta_{il} \delta_{kj} \delta_{ji} \delta_{kl}$
$= 2g_s^2 N^3 + g_s N$
$= 2$
$+$
if $N \to \infty$
important one
non-planar diags
N
$= 2$
$+$
planar diag

Exercise : $f(M) = \mathrm{tr}\, M^3\, \mathrm{tr}\, M^3$

$$= 2g_s^2 \sum \delta_{ik}\, \delta_{jj}\, \delta_{ki}\, \delta_{\ell\ell}$$

$$+ g_s^2 \sum \delta_{i\ell}\, \delta_{kj}\, \delta_{ji}\, \delta_{k\ell}$$

$$= \underbrace{2g_s^2 N^3}_{\text{if } N\to\infty} + g_s N \quad = 2$$

+

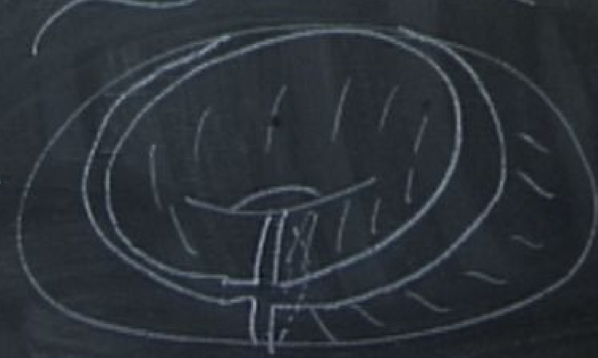
important one

non-planar diags.

$F=3$

$= 2$

+



$\frac{1}{N^2}$ compared with

$F=1$

planar diag

Pirsa: 11030048

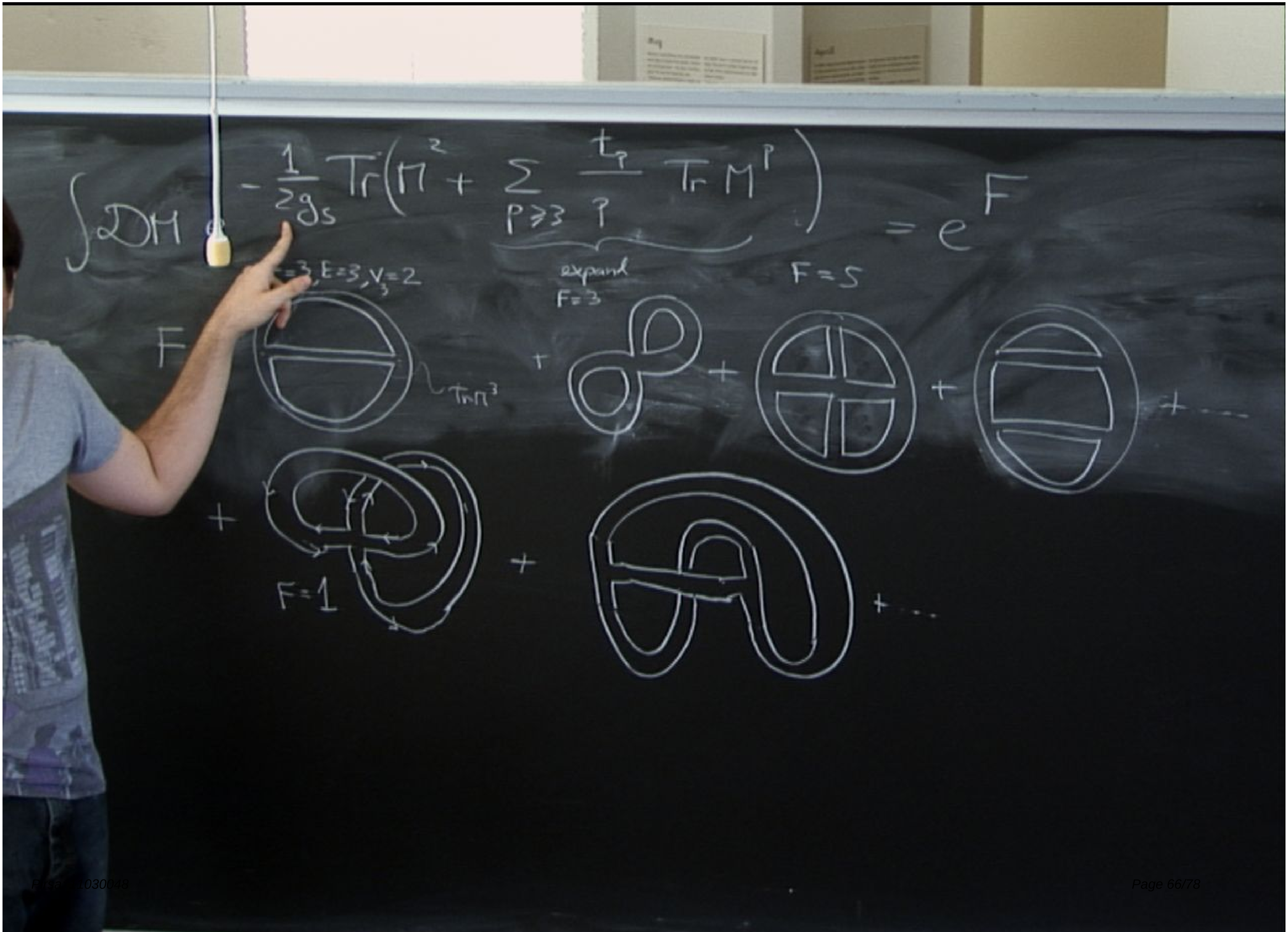
each graph contributes as g_s^E

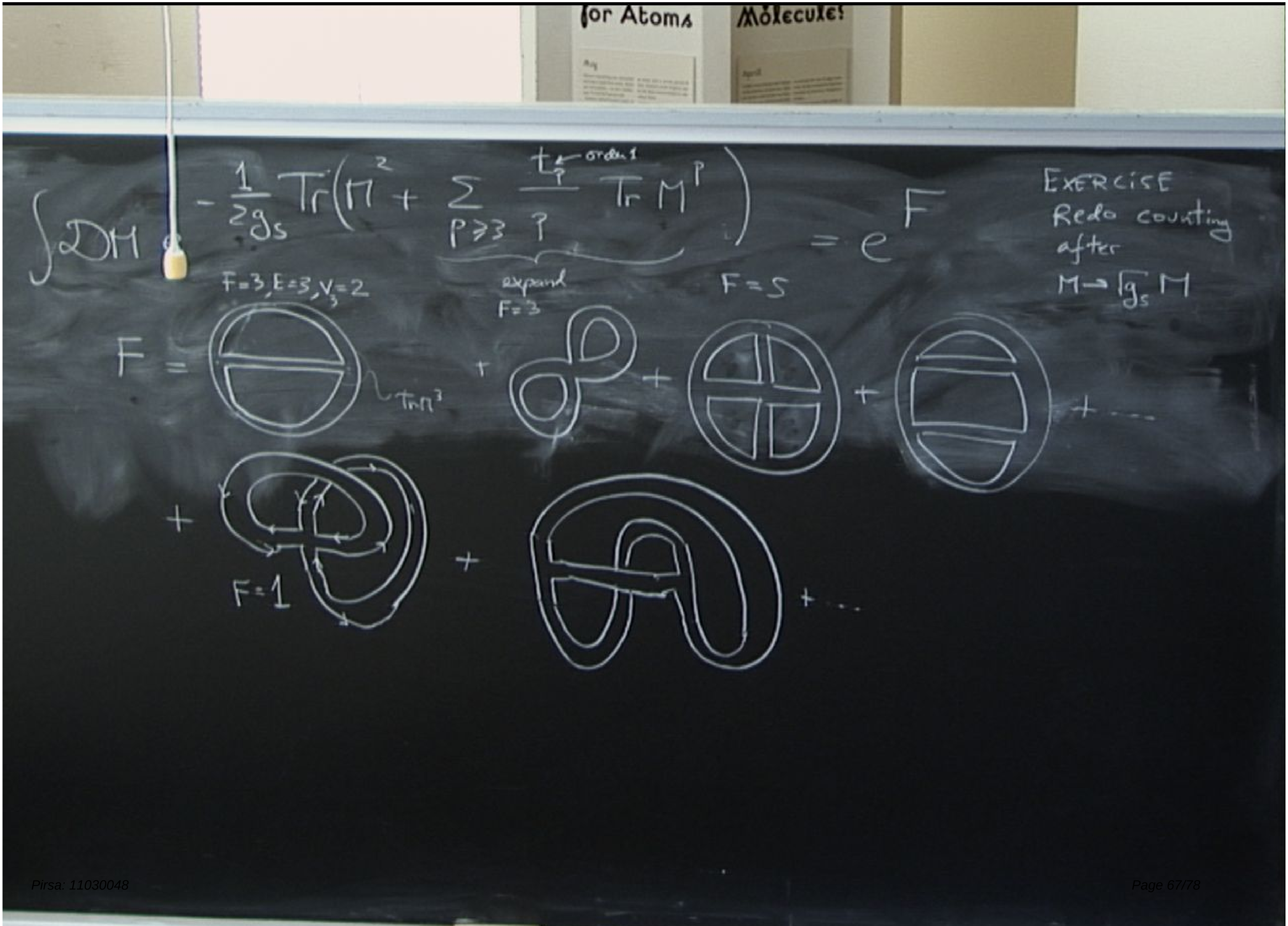
$V =$ number of vertices

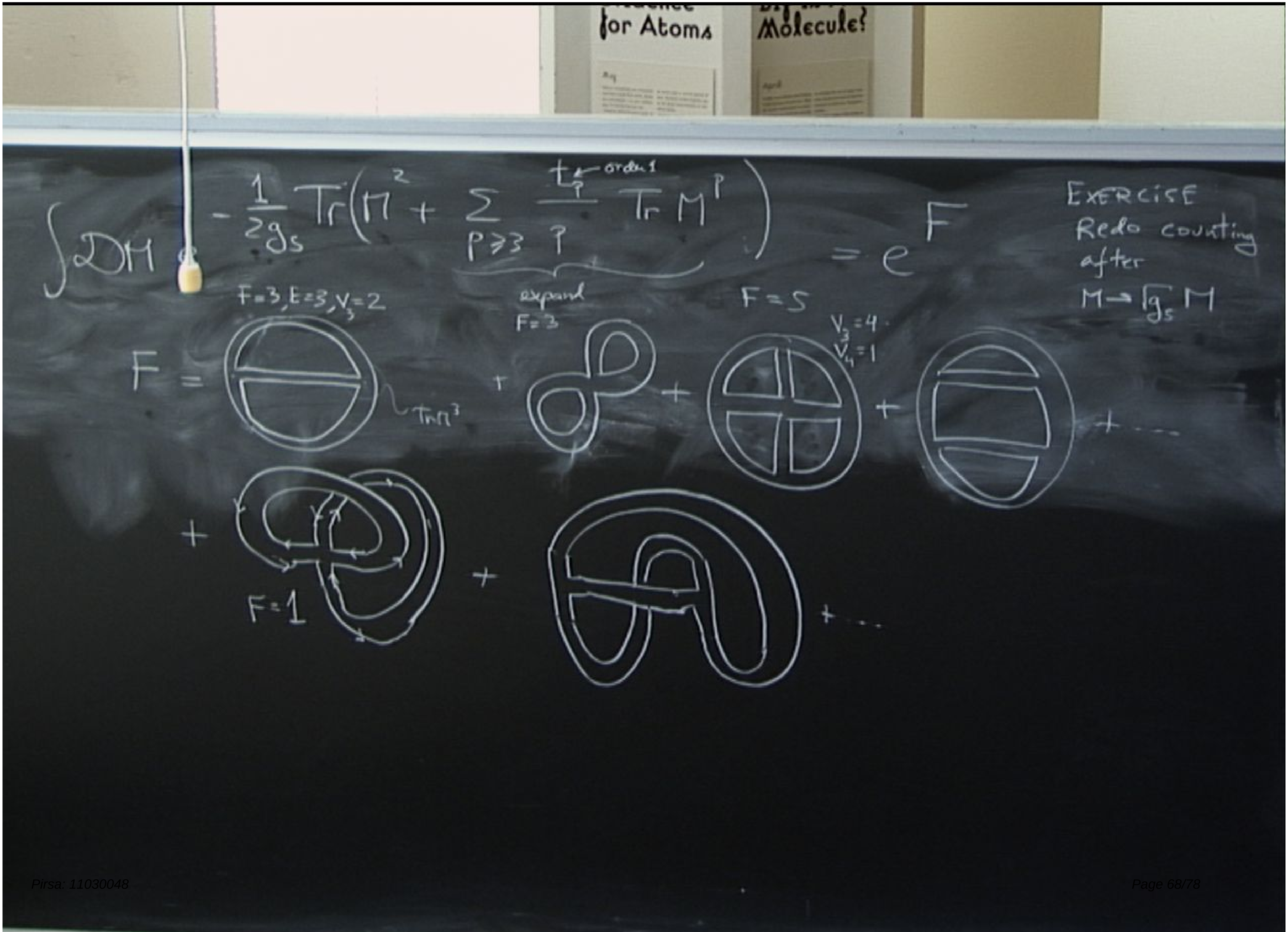
$V_p =$ " " " with p legs

$E =$ number of edges

$F =$ number of color loops, i.e. number of faces


∫𝒟M e^(−1/(2g_s) Tr(M² + Σ_{p≥3} t_p/p Tr M^p)) = e^F
F=3, E=3, V_3=2
expand
F=3
F=5
F
Tr M³
F=1


for Atoms
Molecules
$\int \mathcal{D}M \, e^{-\frac{1}{2g_s}\mathrm{Tr}\left(M^2 + \sum_{p\geq 3} \frac{t_p}{p} \mathrm{Tr}\, M^p\right)} = e^F$
order 1
expand
F=3
F=3, E=3, V_3=2
F=5
EXERCISE
Redo counting
after
$M \to \sqrt{g_s}\, M$
F =
$\mathrm{Tr}\, M^3$
F=1


for Atoms
Molecule?
$\int \mathcal{D}M \, e^{-\frac{1}{2g_s} \mathrm{Tr}\left(M^2 + \sum_{p\geq 3} \frac{t_p}{p} \mathrm{Tr} M^p\right)} = e^F$
order 1
expand
EXERCISE
Redo counting
after
$M \to \sqrt{g_s} M$
$F=3, E=3, V_3=2$
$F = \ldots + \ldots + \ldots + \ldots + \ldots$
$\mathrm{Tr} M^3$
$F=3$
$F=5$
$V_3=4$
$V_4=1$
$+ \ldots + \ldots$
$F=1$

each graph contributes as $g_s^{E} \left(\frac{1}{g_s}\right)^{V} N^{F} \underbrace{\prod_{p=3}^{\infty} t_p^{V_p}}_{O(1)}$

V = number of vertices

V_p = " " " with p legs

E = number of edges

F = number of color loops, i.e. number of faces each graph contributes as $\displaystyle g_s^{E} \left(\frac{1}{g_s}\right)^{V} N^{F} \underbrace{\prod_{p=3}^{\infty} t_p^{V_p}}_{\mathcal{O}(1)}$

$V =$ number of vertices

$V_p =$ " " " with p legs

$E =$ number of edges

$F =$ number of color loops, i.e. number of faces

each graph contributes as $g_s^{E}\left(\frac{1}{g_s}\right)^{V} N^{F} \prod_{p=3}^{\infty} t_p^{V_p} = g_s^{E-V-F} (g_s N)^{F}$

$\underbrace{}_{(7.13)}$

V = number of vertices

V_p = " " " with p legs

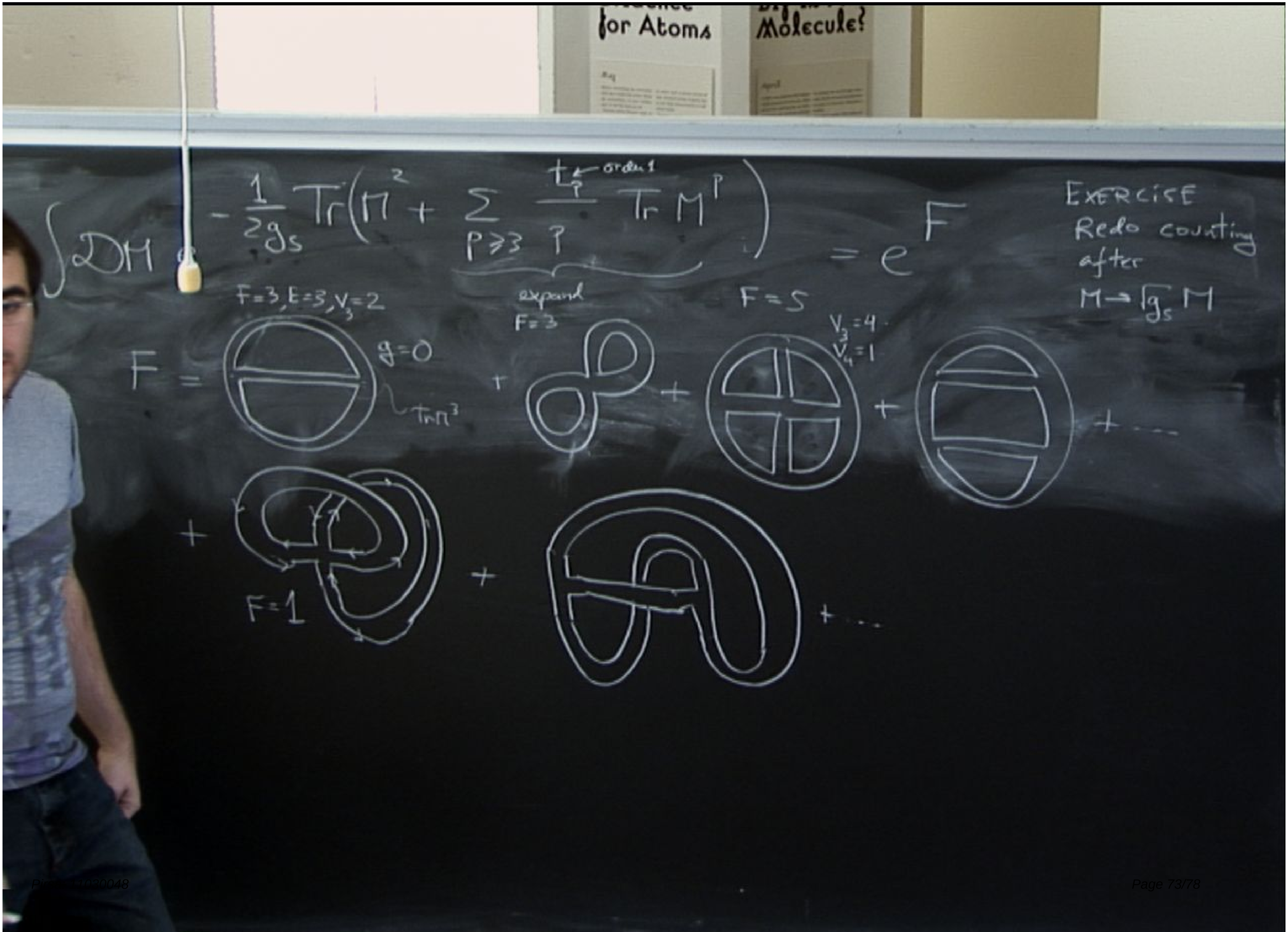
E = number of edges

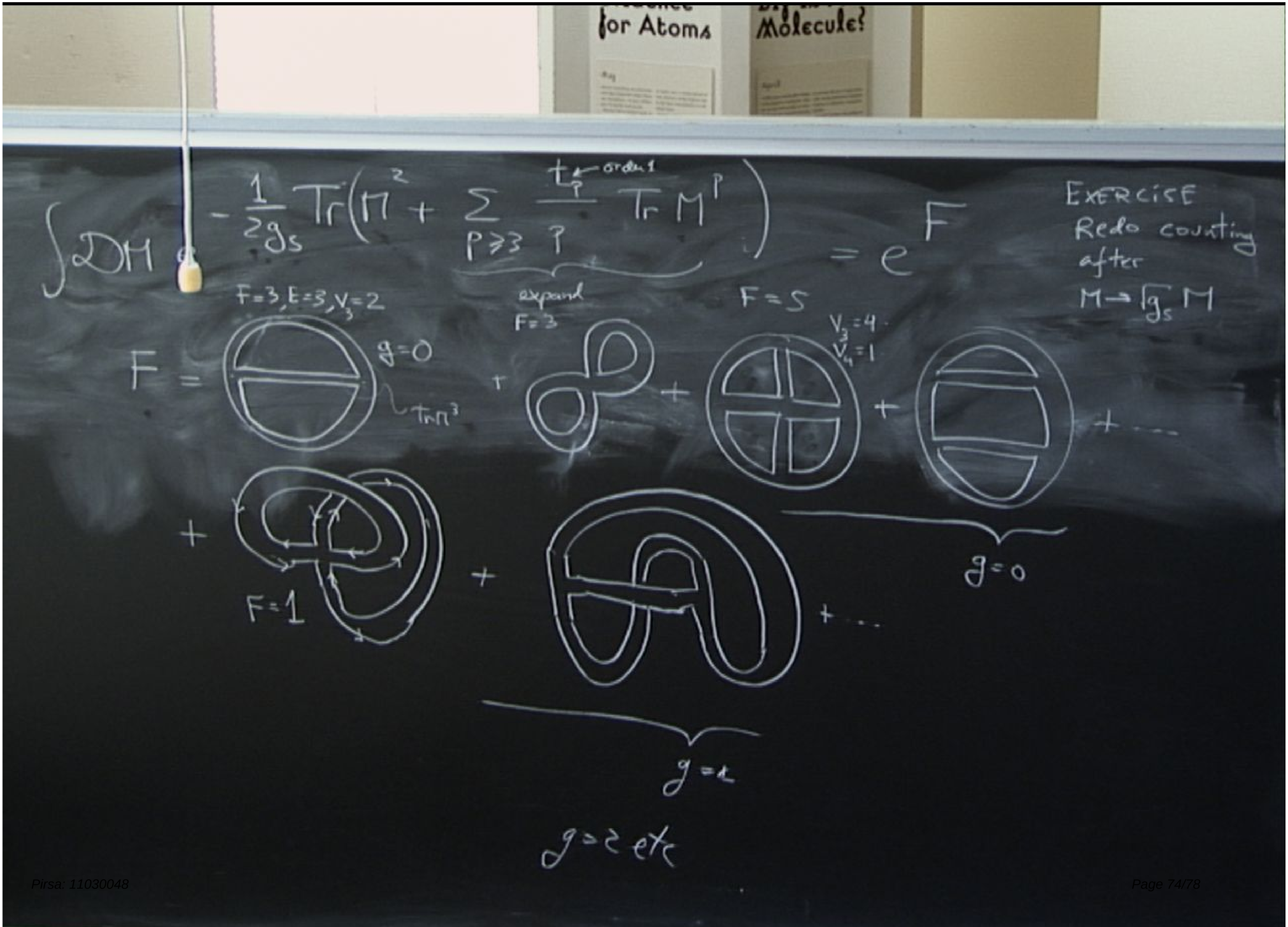
F = number of color loops, i.e. number of faces

$$= g_s^{E-V-F} (g_s N)^F \; (\cdot \mathcal{O}(1))$$

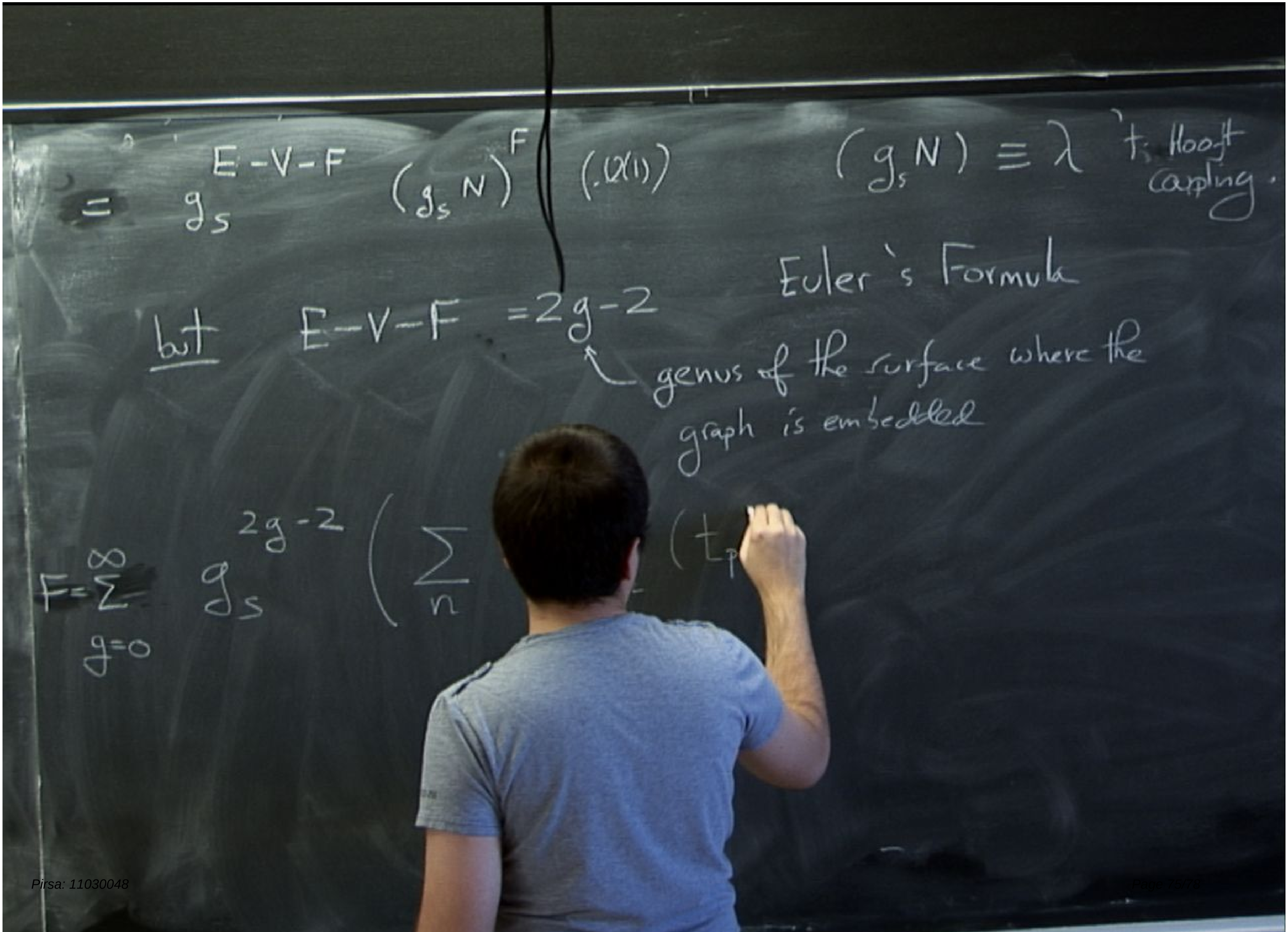
$\underline{\text{but}}$ $E-V-F = 2g-2$ Euler's Formula

genus of the surface where the graph is embedded


for Atoms
Molecule?
∫ DM e^(- 1/(2g_s) Tr(M² + Σ_{p≥3} t_p/p Tr M^p)) = e^F
order 1
expand
EXERCISE
Redo counting after
M → √g_s M
F=3, E=3, V_3=2
g=0
Tr M³
F=3
F=5
V_3=4
V_4=1
F =
F=1



for Atoms
Molecule?
$\int \mathcal{D}M \, e^{-\frac{1}{2g_s}\mathrm{Tr}\left(M^2 + \sum_{p\geq 3} \frac{t_p}{p} \mathrm{Tr}\, M^p\right)} = e^F$
order 1
expand
EXERCISE
Redo counting
after
$M \to \sqrt{g_s}\, M$
$F=3, E=3, V_3=2$
$F=3$
$F=5$
$V_3=4$
$V_4=1$
$F =$
$g=0$
$\mathrm{Tr}\, M^3$
$F=1$
$g=0$
$g=1$
$g=2$ etc


= g_s^{E-V-F} $(g_s N)^F$ $(O(1))$
$(g_s N) \equiv \lambda$ 't Hooft coupling.
Euler's Formula
but $E-V-F = 2g-2$
genus of the surface where the graph is embedded
$F = \sum_{g=0}^{\infty} g_s^{2g-2} \left(\sum_n \right.$ $(t_p$

$$= g_s^{E-V-F} \, (g_s N)^F \, (\cdot \mathcal{U}(1))$$

$$(g_s N) \equiv \lambda$$ 't Hooft coupling.

Euler's Formula

but $E-V-F = 2g-2$

genus of the surface where the graph is embedded

$$F = \sum_{g=0}^{\infty} g_s^{2g-2} \underbrace{\left(\sum_n \lambda^n \mathcal{F}_{n,g}(t_p) \right)}_{\mathcal{F}_{n,g}(\lambda; t_p)}$$

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$$= g_s^{E-V-F} \, (g_s N)^F \, (\mathcal{O}(1))$$

$(g_s N) \equiv \lambda$ 't Hooft coupling.

Euler's Formula

<u>but</u> $E-V-F = 2g-2$

genus of the surface where the graph is embedded

sum of all torus diags

$$\sum_{g=0}^{\infty} g_s^{2g-2} \underbrace{\left(\sum_n \lambda^n \mathcal{F}_{n,g}(t_p) \right)}_{\mathcal{F}_{n,g}(\lambda; t_p)} = \quad + g_s \quad + g_s^2 \quad + \dots$$

g_s expansion

$= \sum_{\text{all planar diagrams}}$

$\longleftrightarrow$ string coupling

string tension expansion

$$= g_s^{E-V-F} \, (g_s N)^F \quad (\lambda(1))$$

$(g_s N) \equiv \lambda$ 't Hooft coupling.

Euler's Formula

but $E-V-F = 2g-2$

g — genus of the surface where the graph is embedded

$$F = \sum_{g=0}^{\infty} g_s^{2g-2} \underbrace{\left(\sum_n \lambda^n \mathcal{F}_{n,g}(t_p) \right)}_{\mathcal{F}_{n,g}(\lambda; t_p)} = \quad + \; g_s \quad + \; g_s^2 \quad + \ldots$$

sum of all torus diags

$= \sum$ all planar diagrams

g_s expansion

string tension expansion

$g_s \longleftrightarrow$ string coupling

$g_s N \longleftrightarrow$ string tension