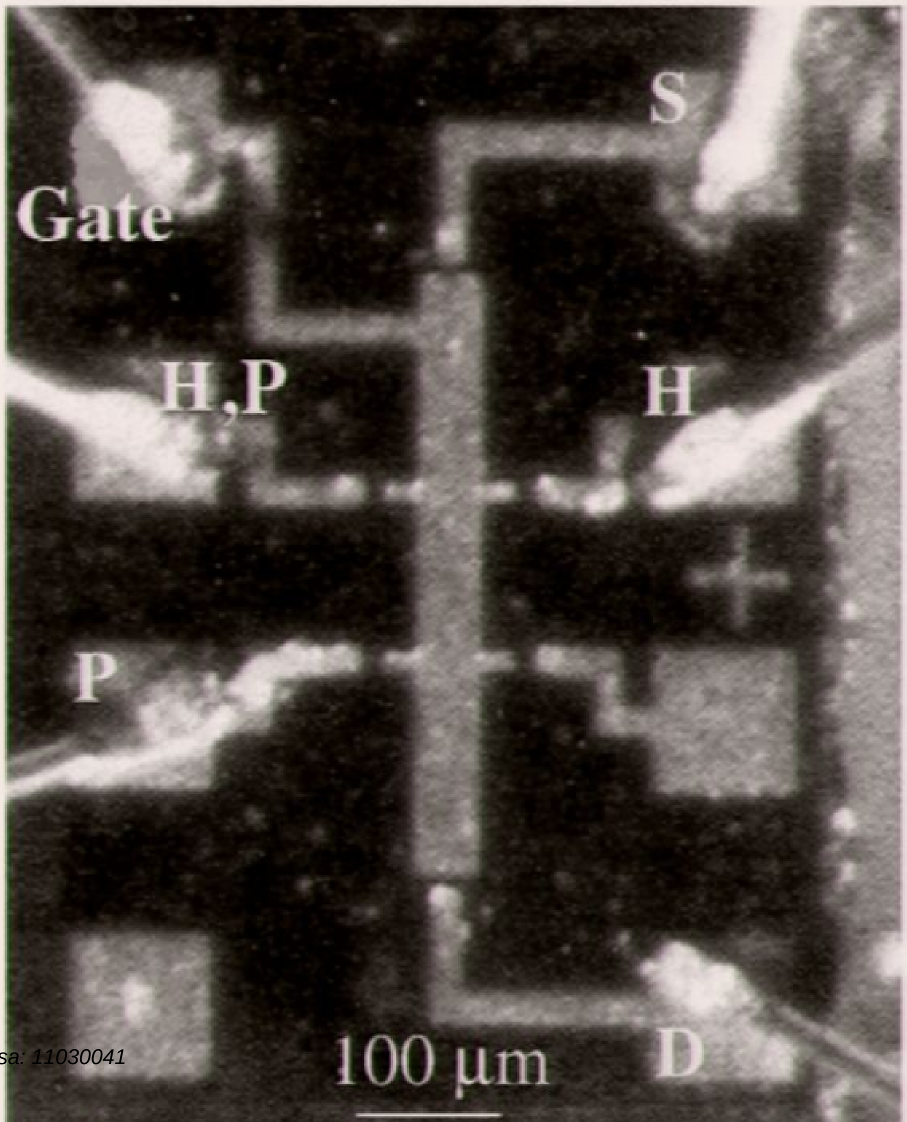
Title: Explorations in Condensed Matter - Lecture 8

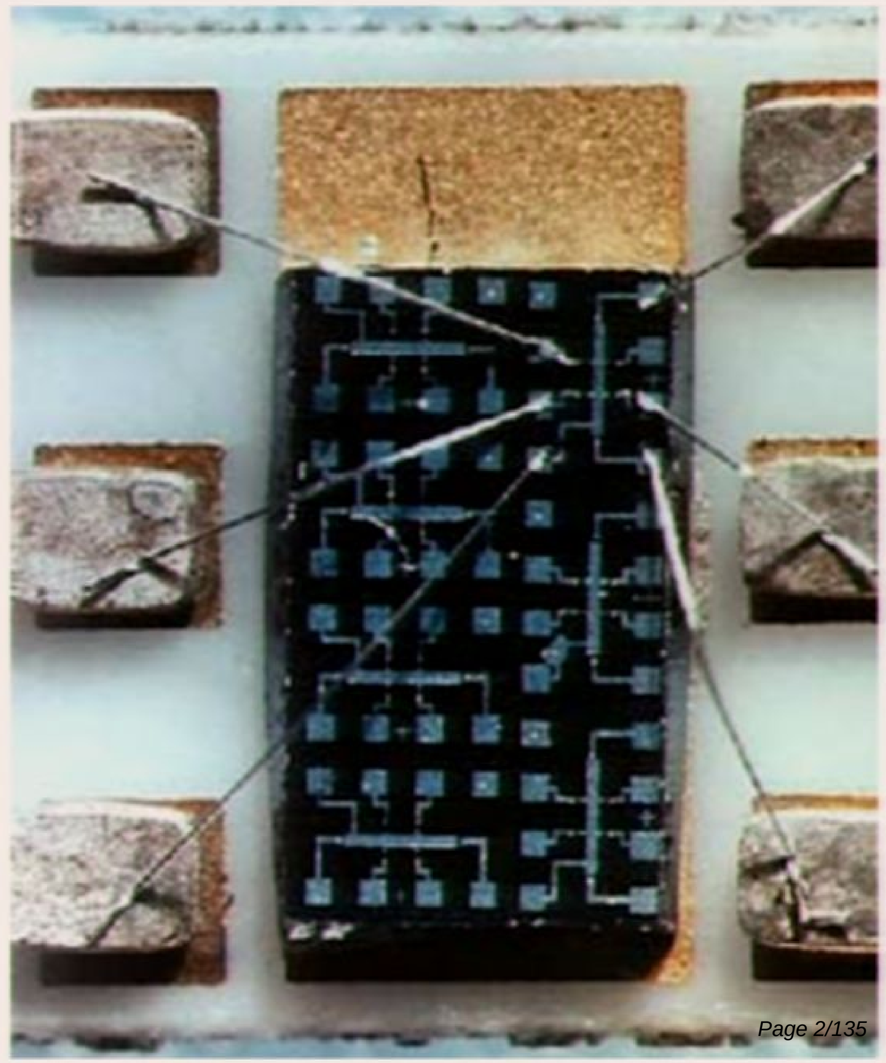
Date: Mar 23, 2011 10:15 AM

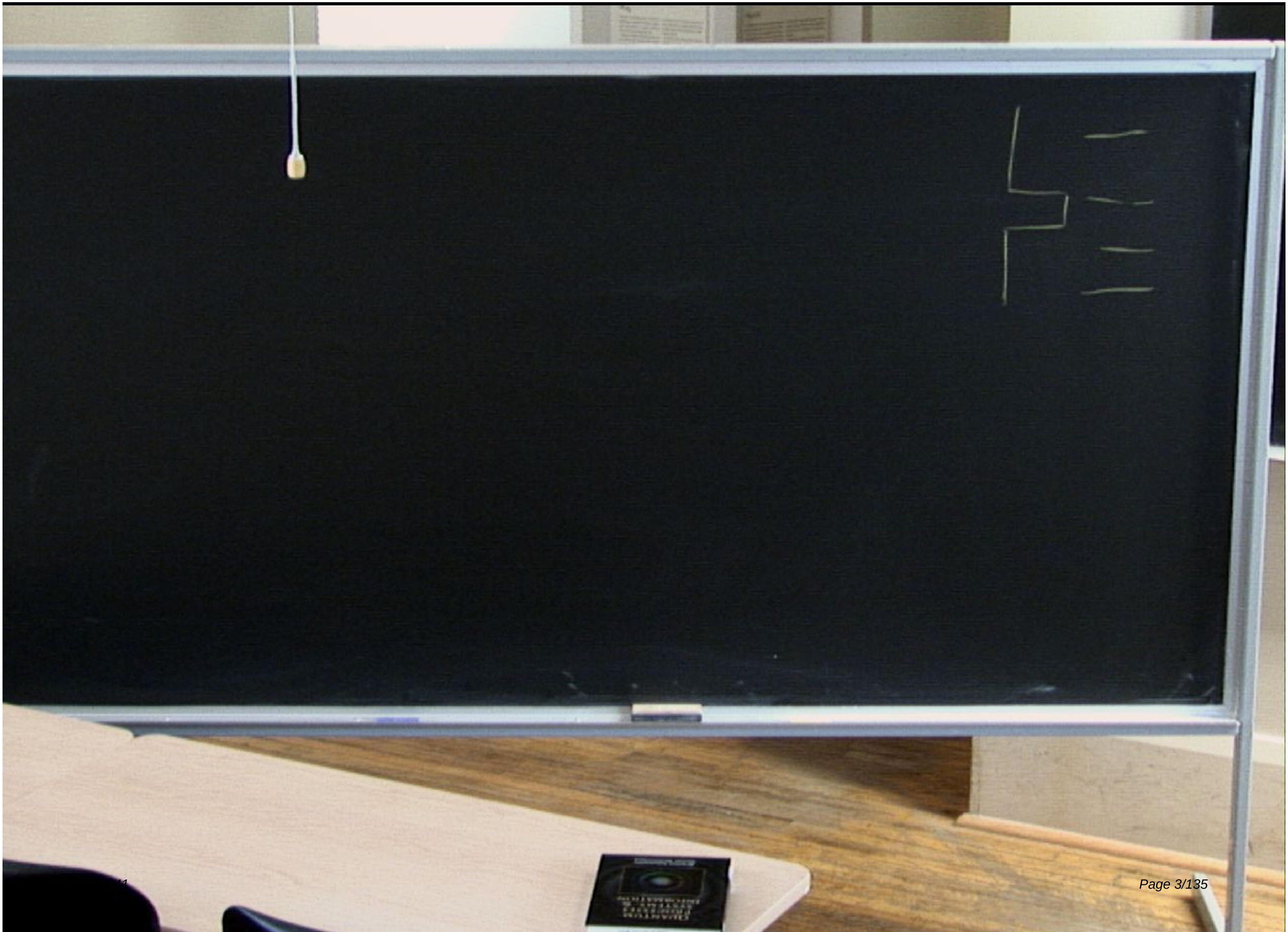
URL: http://pirsa.org/11030041

Abstract:

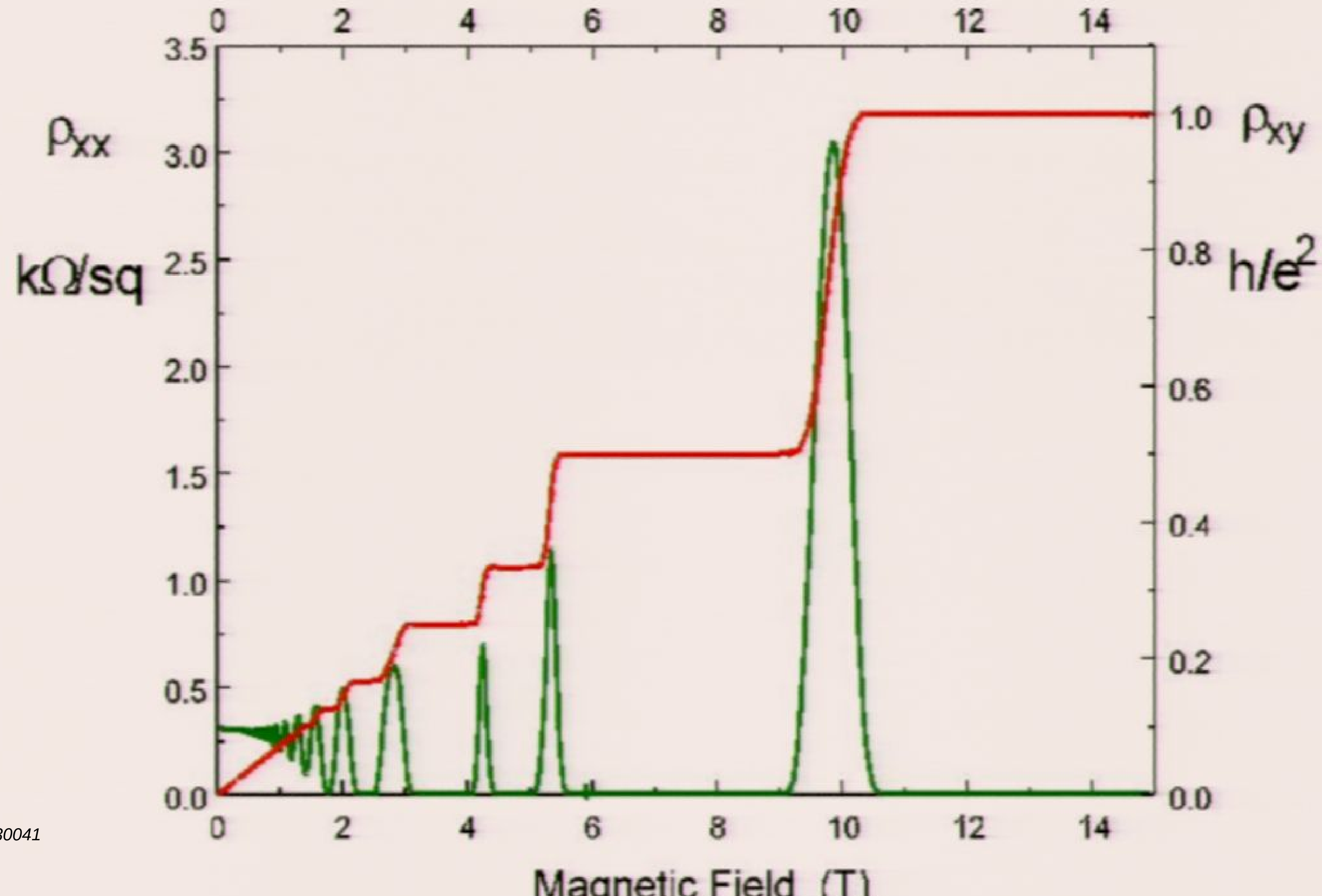
Samples



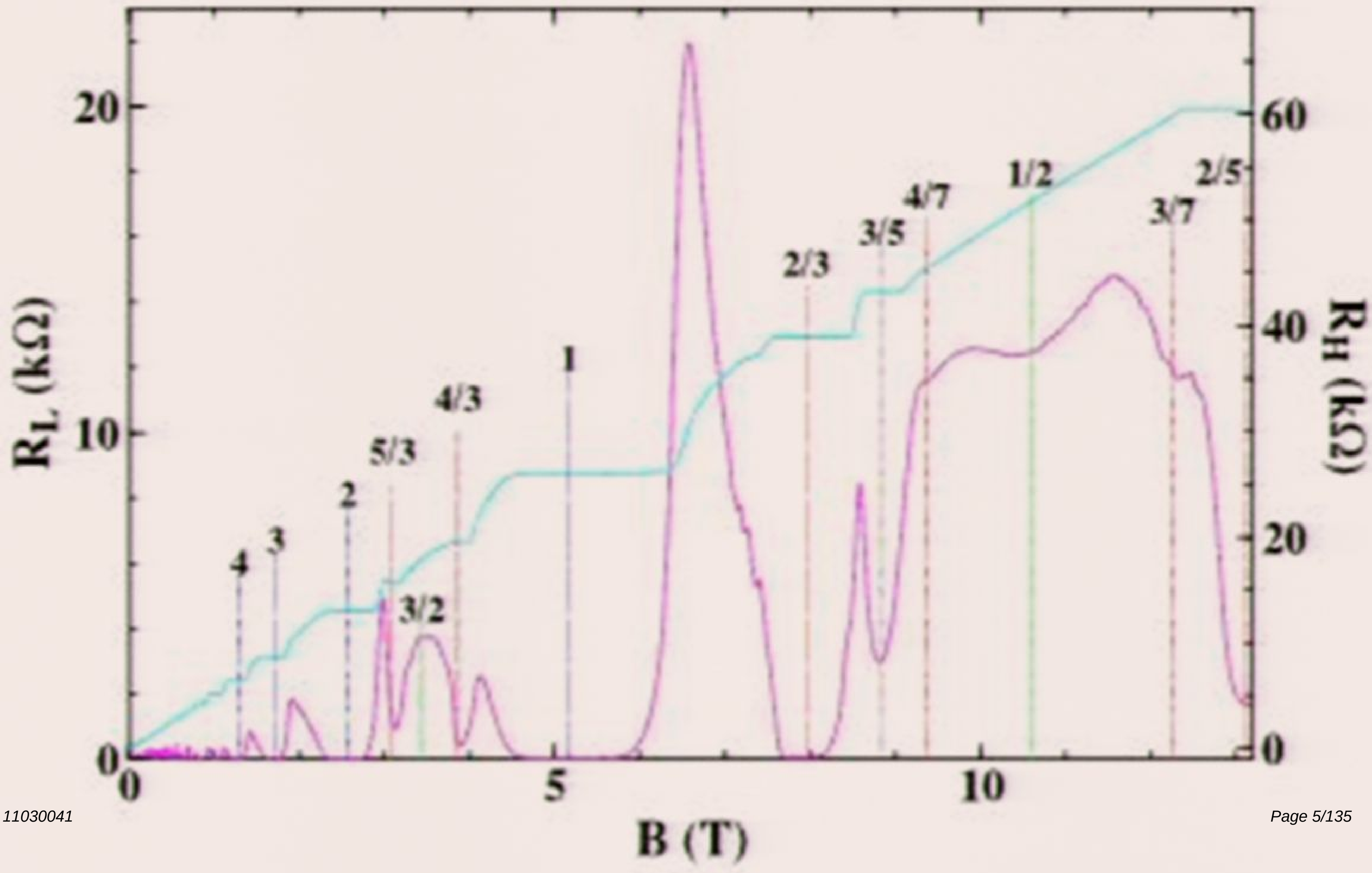




Resistivity Integer QHE



Integer and Fractional Quantum Hall Effects



QH ferromagnetism

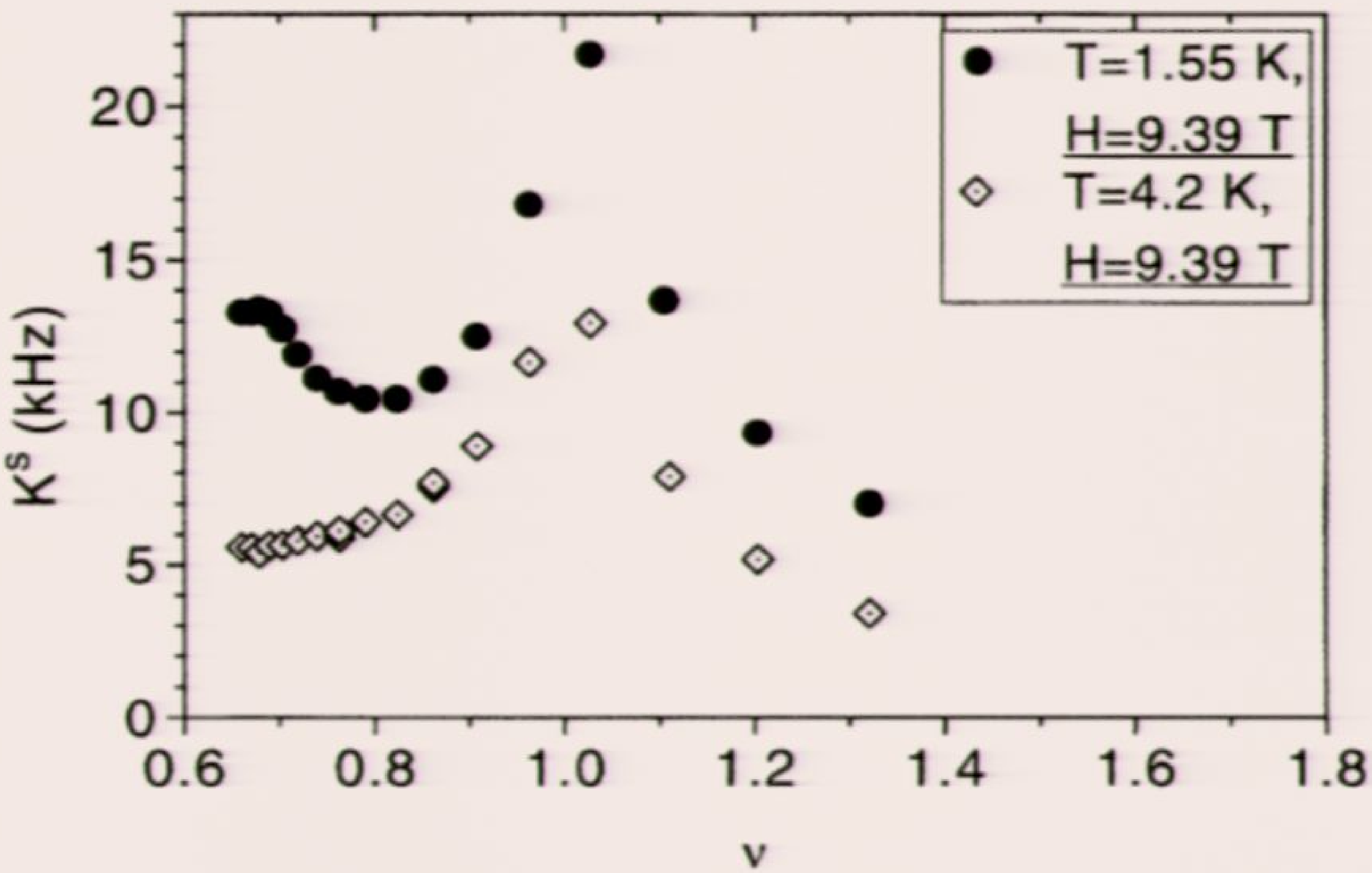
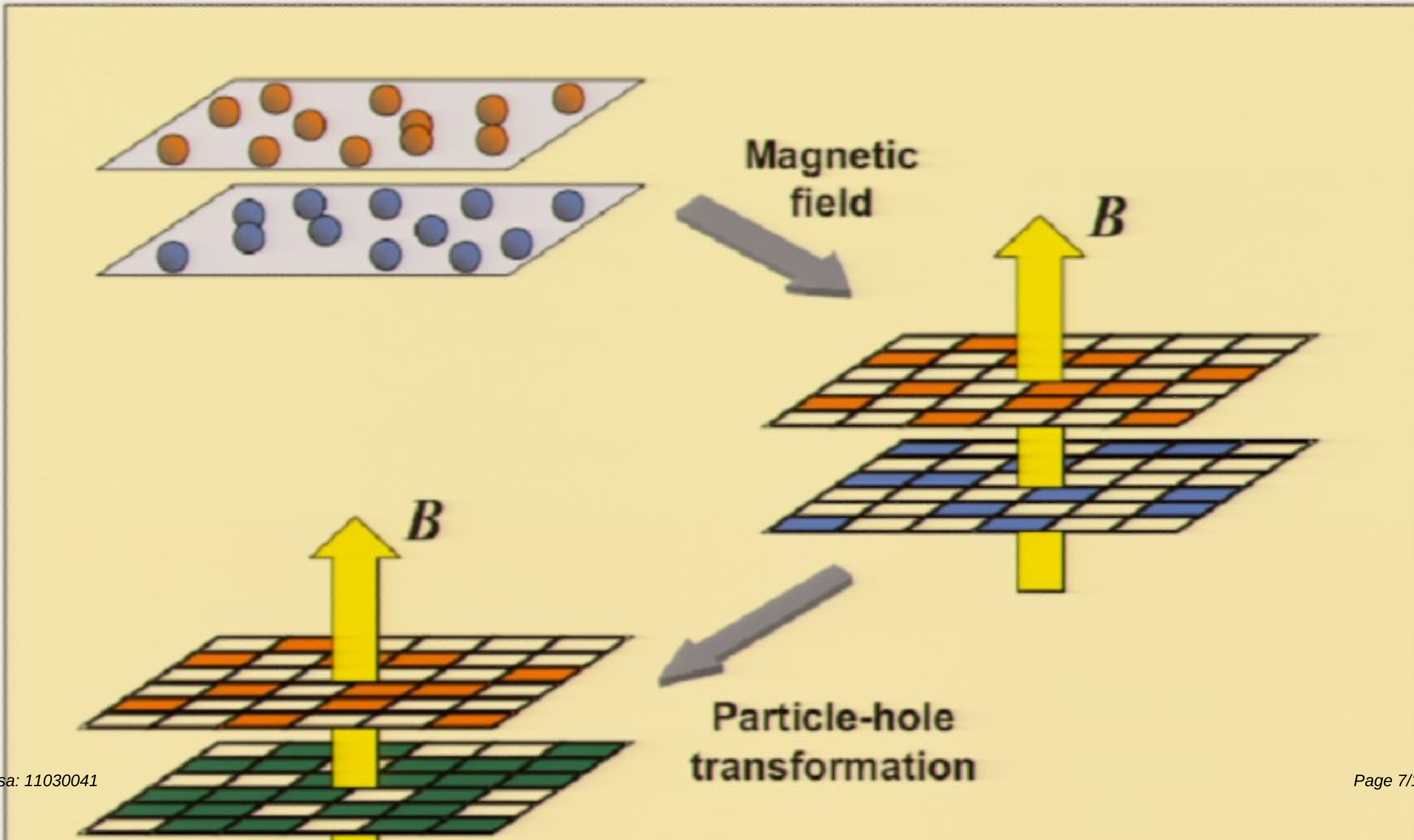
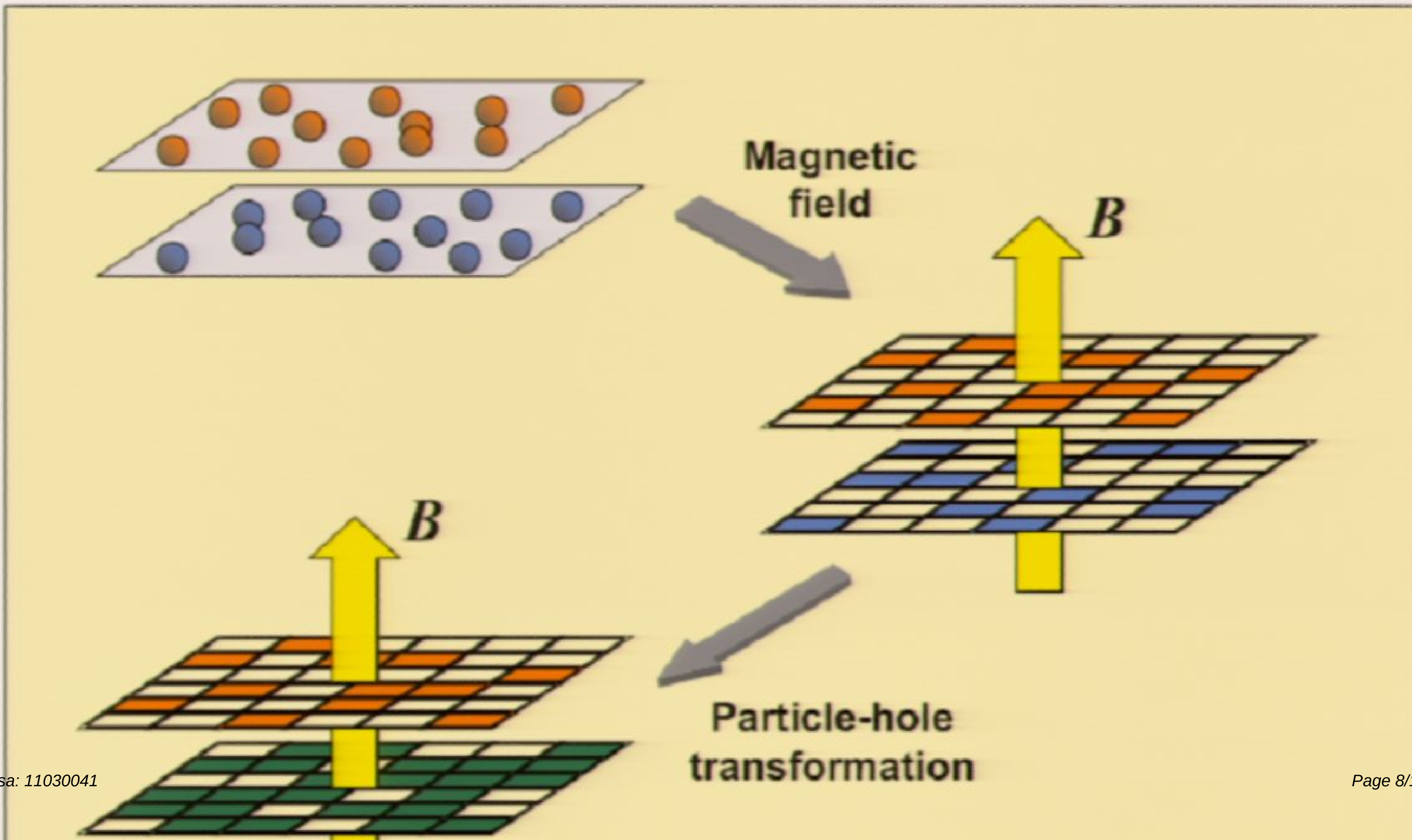


FIG. 5. Dependence of K_s on filling factor ν for $B = 9.39$ T, at $T = 4.2$ (open diamonds) and 1.55 K (filled circles).

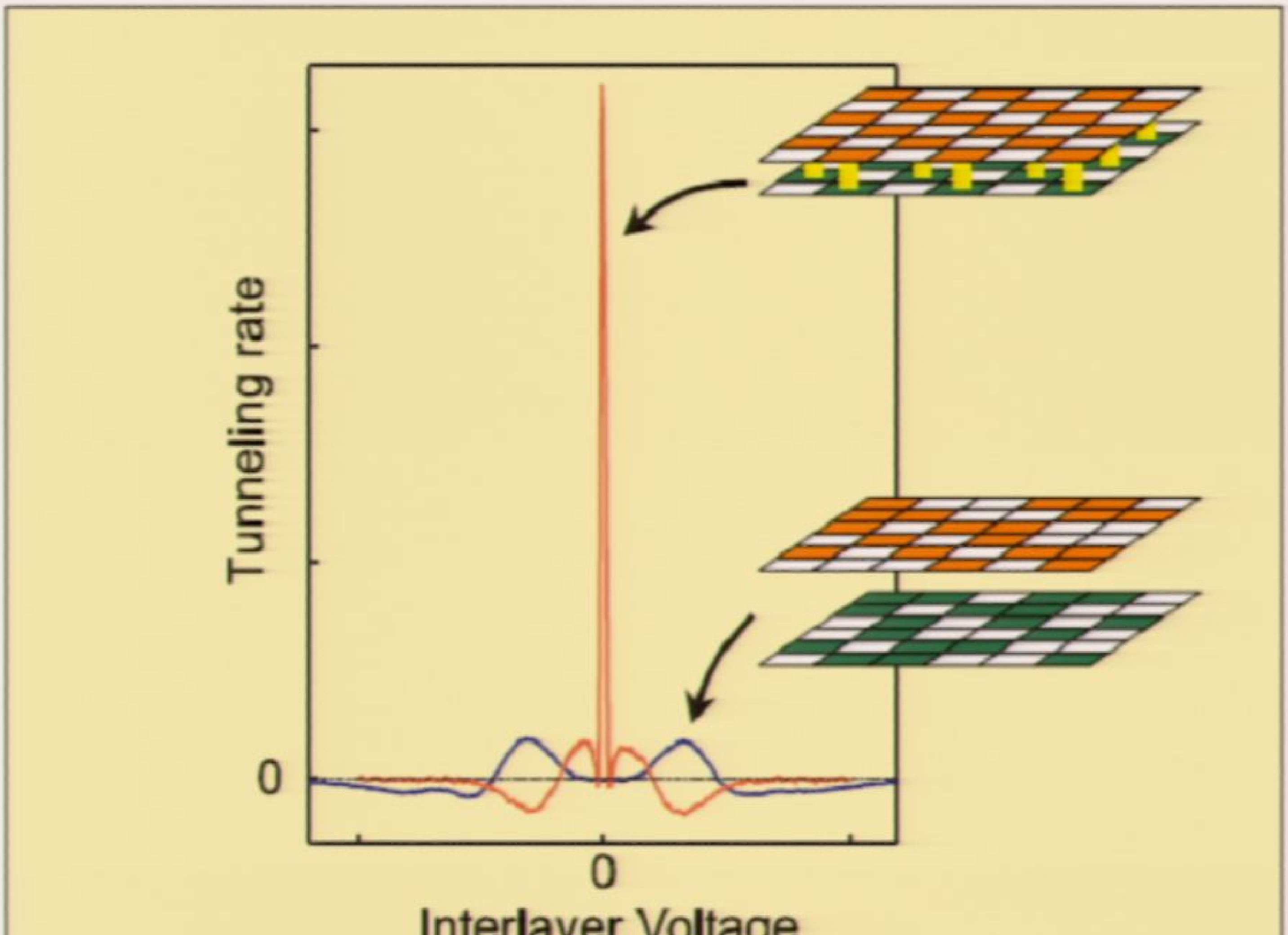
QH bilayers – excitonic condensate



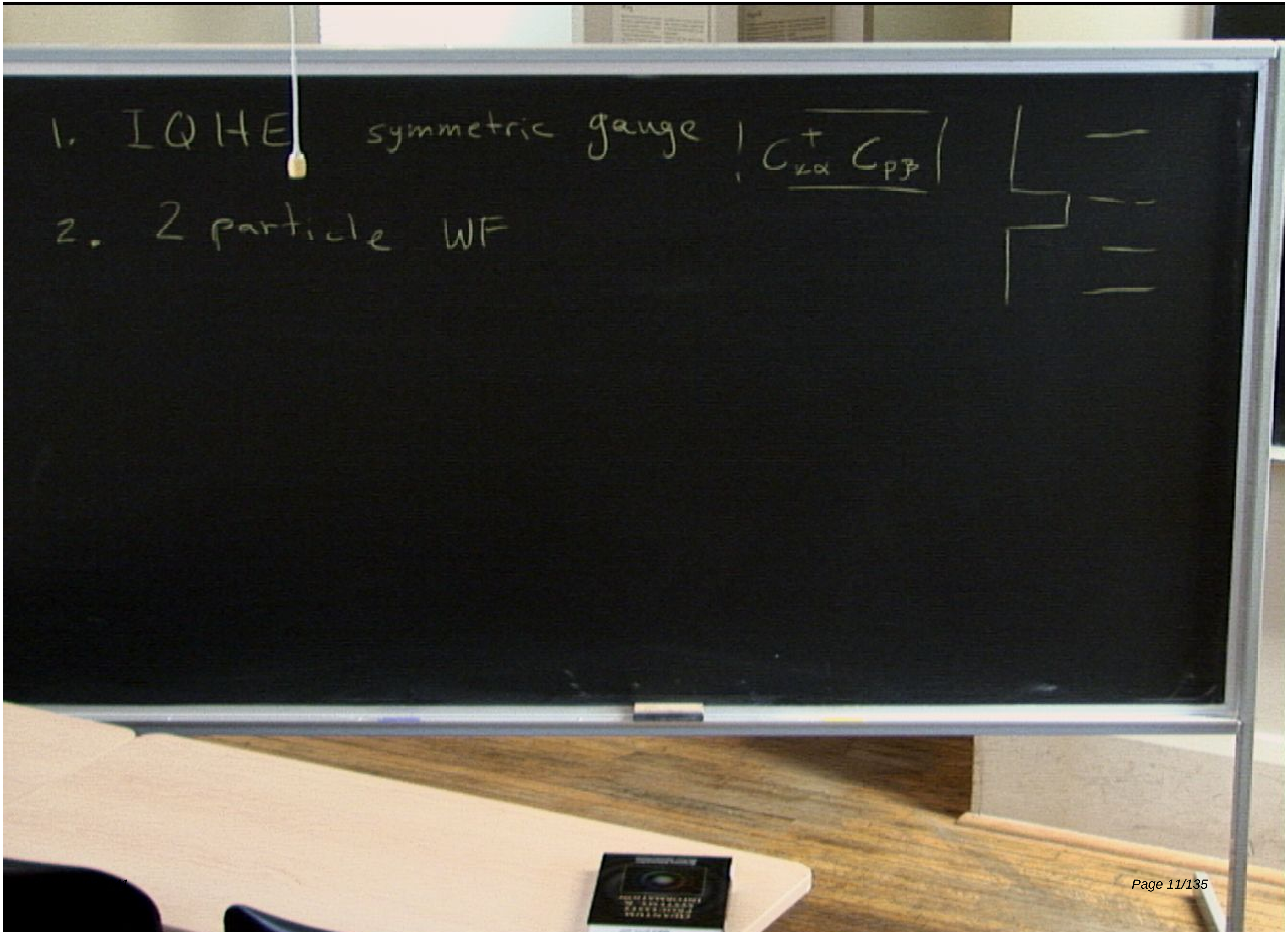
QH bilayers – excitonic condensate

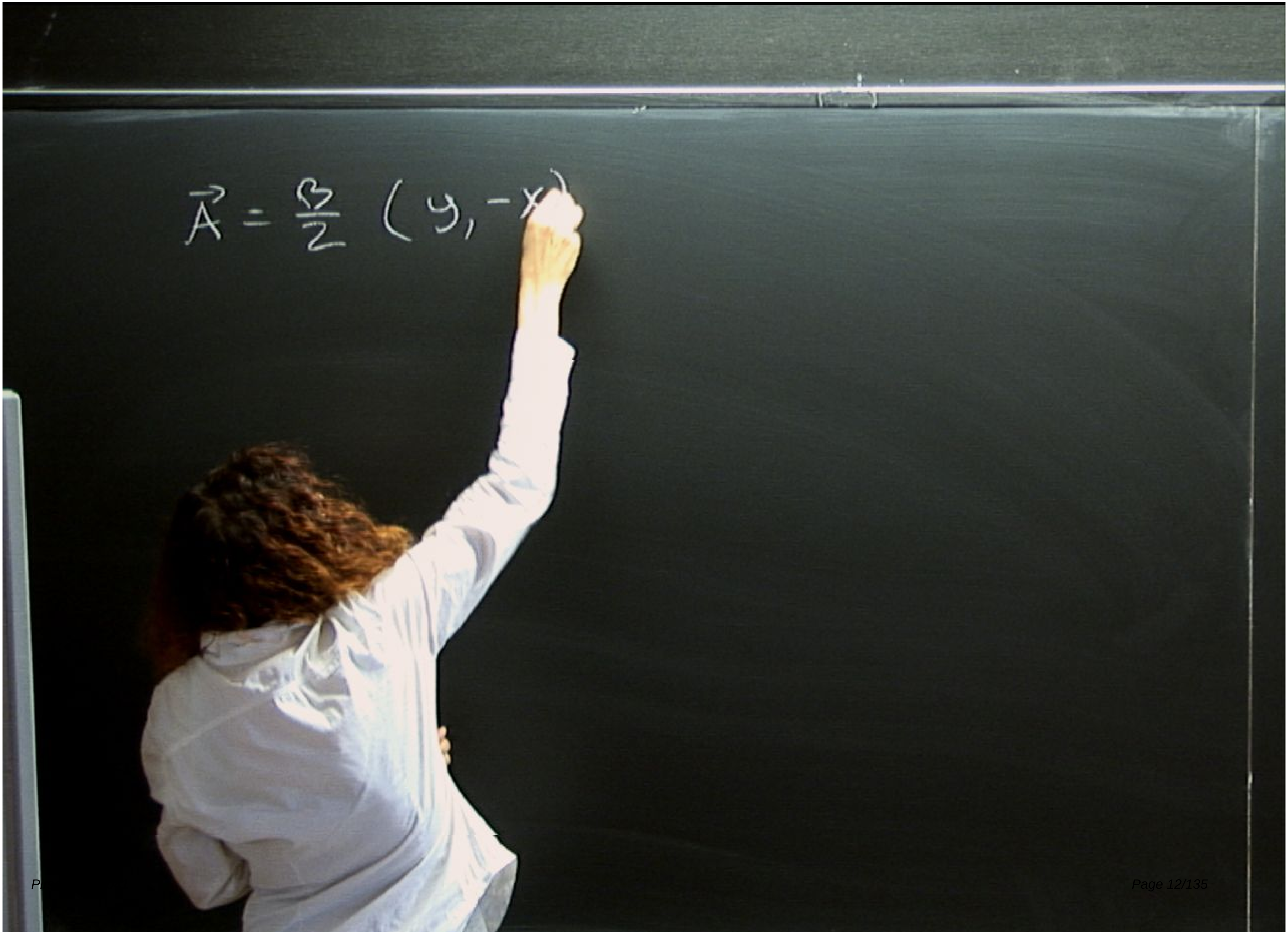


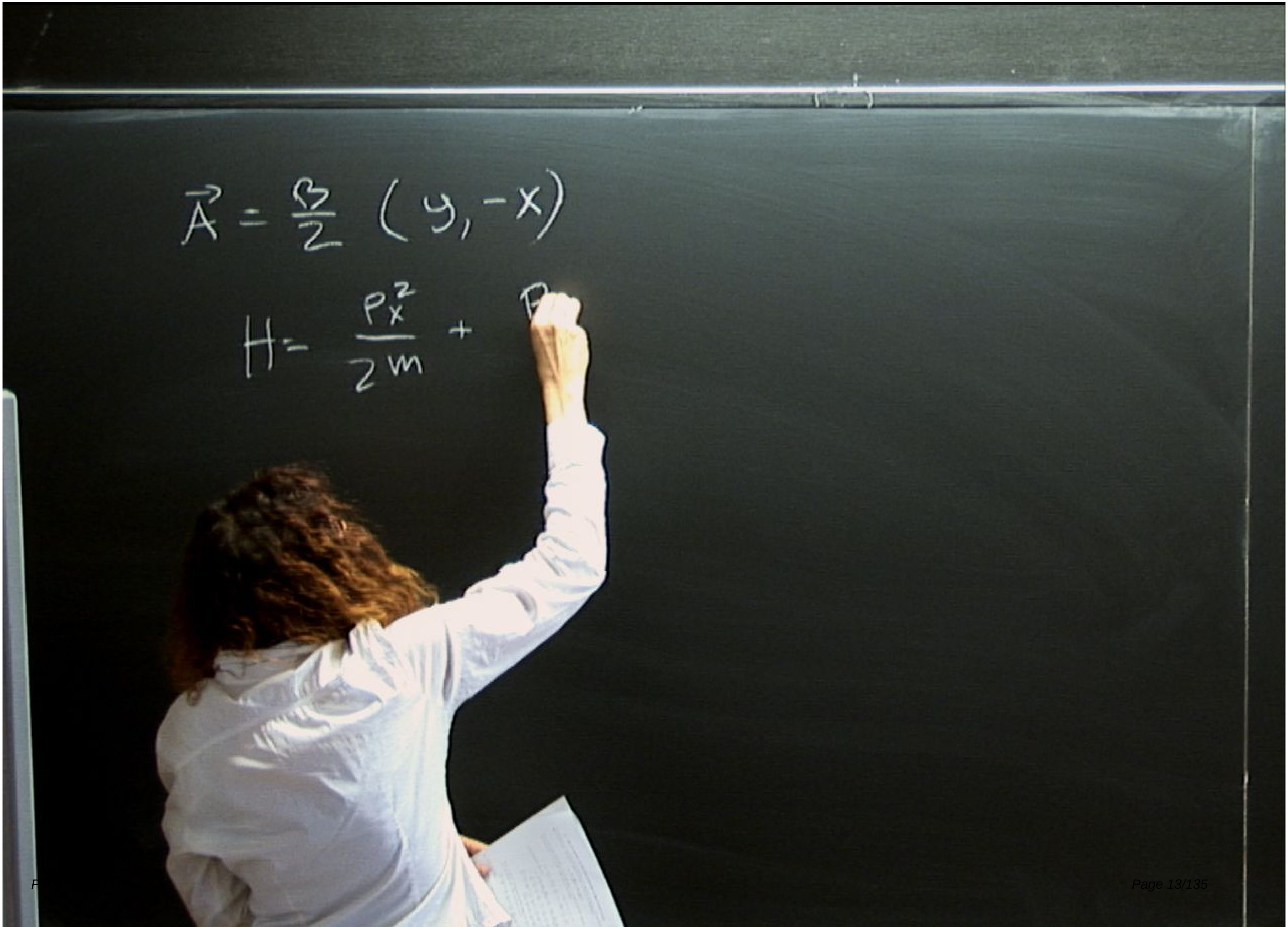
QH bilayers – excitonic condensate

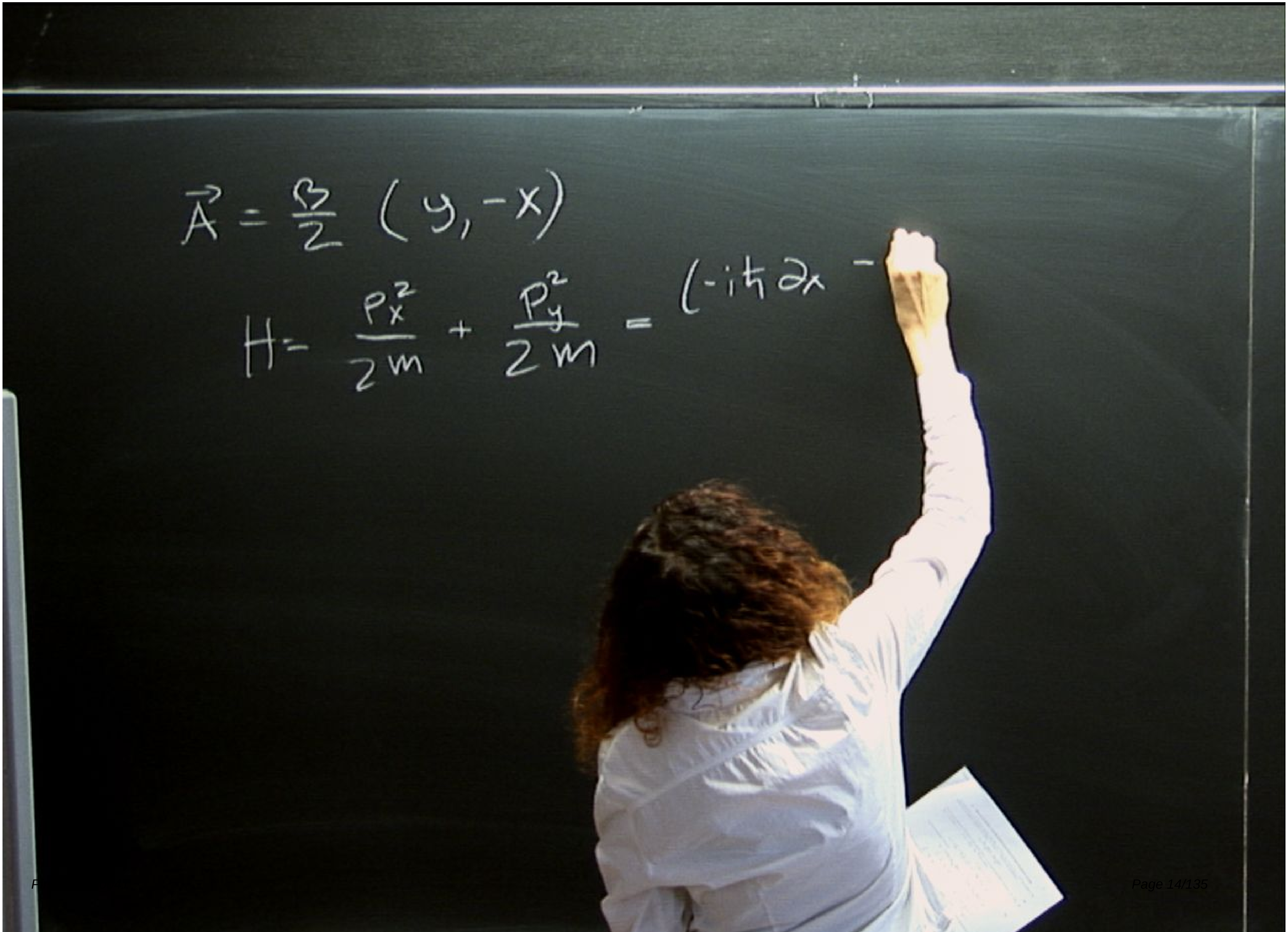


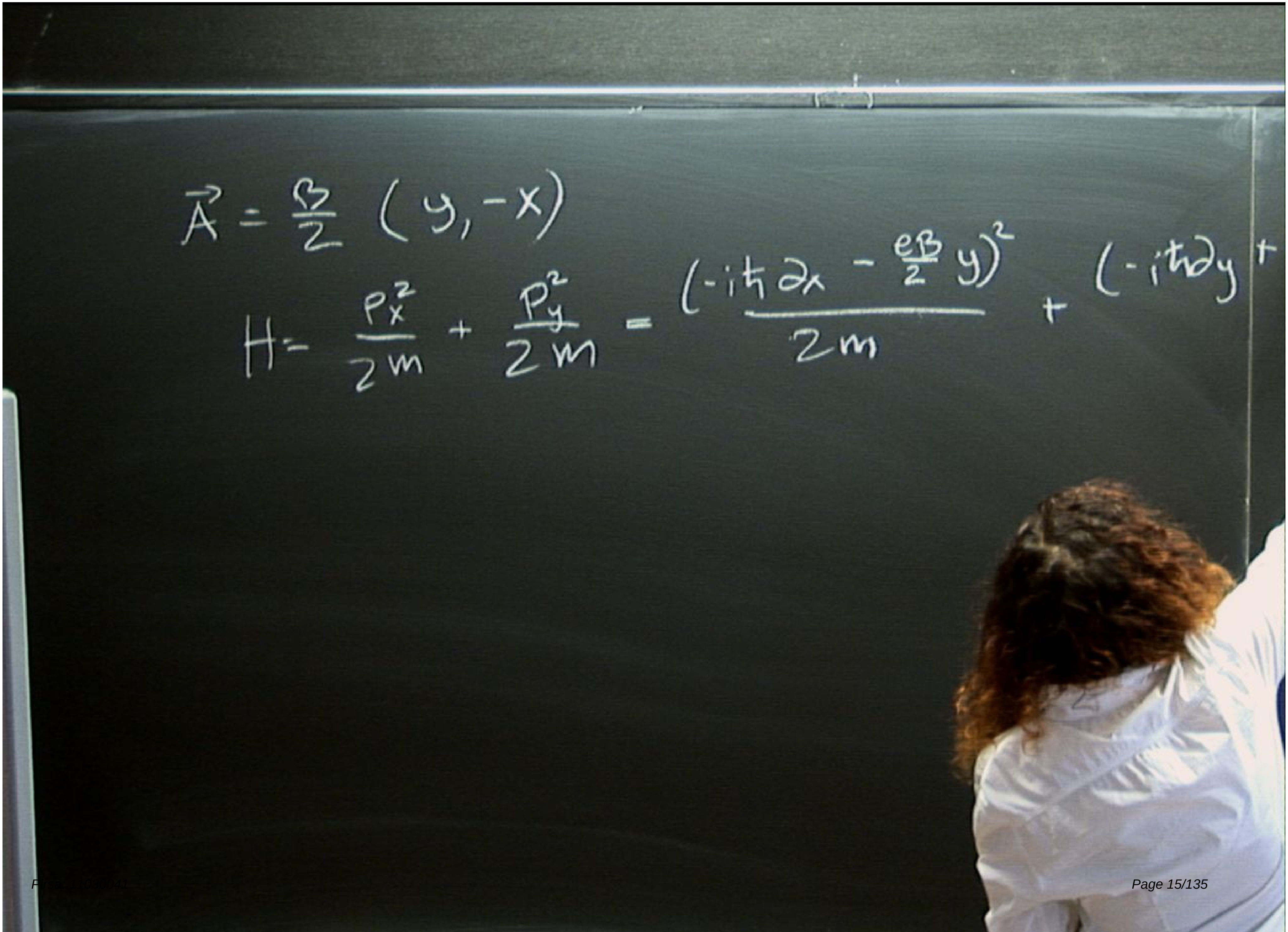
$$\left| \overline{\underline{C^{\dagger}_{k\alpha} C_{p\beta}}} \right|$$


1. IQHE symmetric gauge
$\overline{C^{\dagger}_{\kappa\alpha} C_{p\beta}}$
2. 2 particle WF


$\vec{A} = \frac{B}{2} (y, -x)$


$\vec{A} = \frac{B}{2}(y, -x)$
$H = \frac{p_x^2}{2m} +$


$\vec{A} = \frac{B}{2}(y, -x)$
$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} = (-i\hbar\partial_x -$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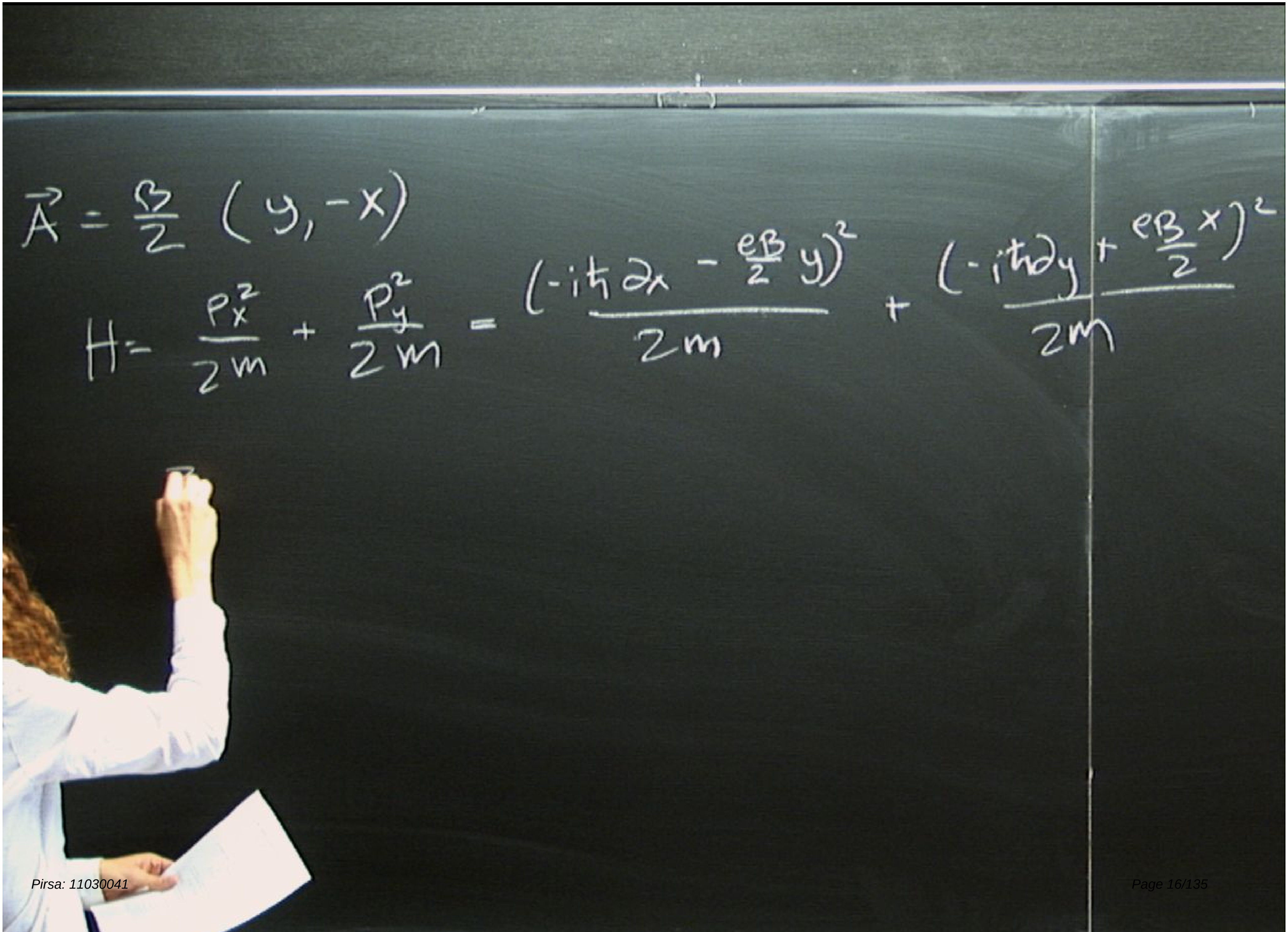
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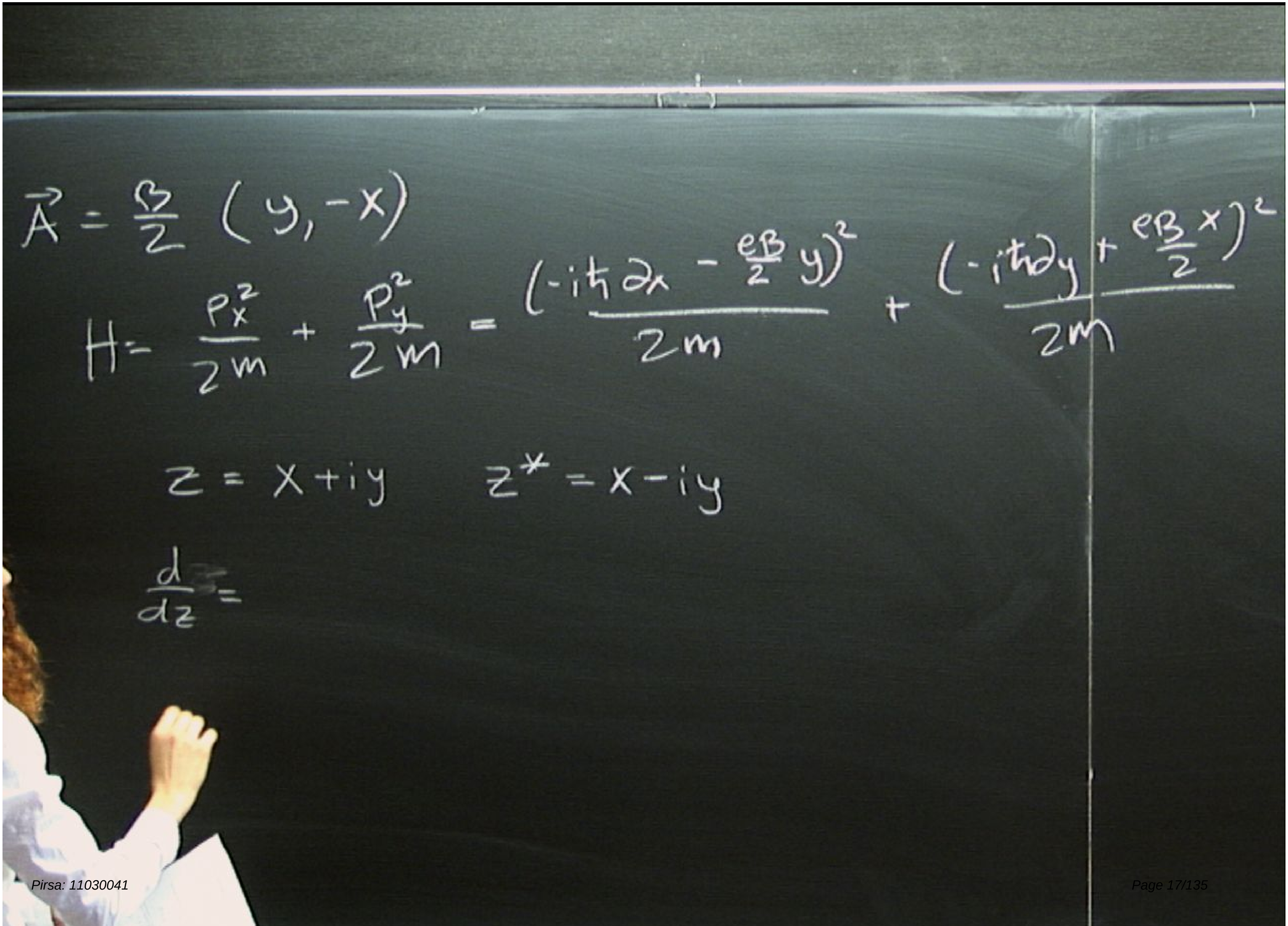
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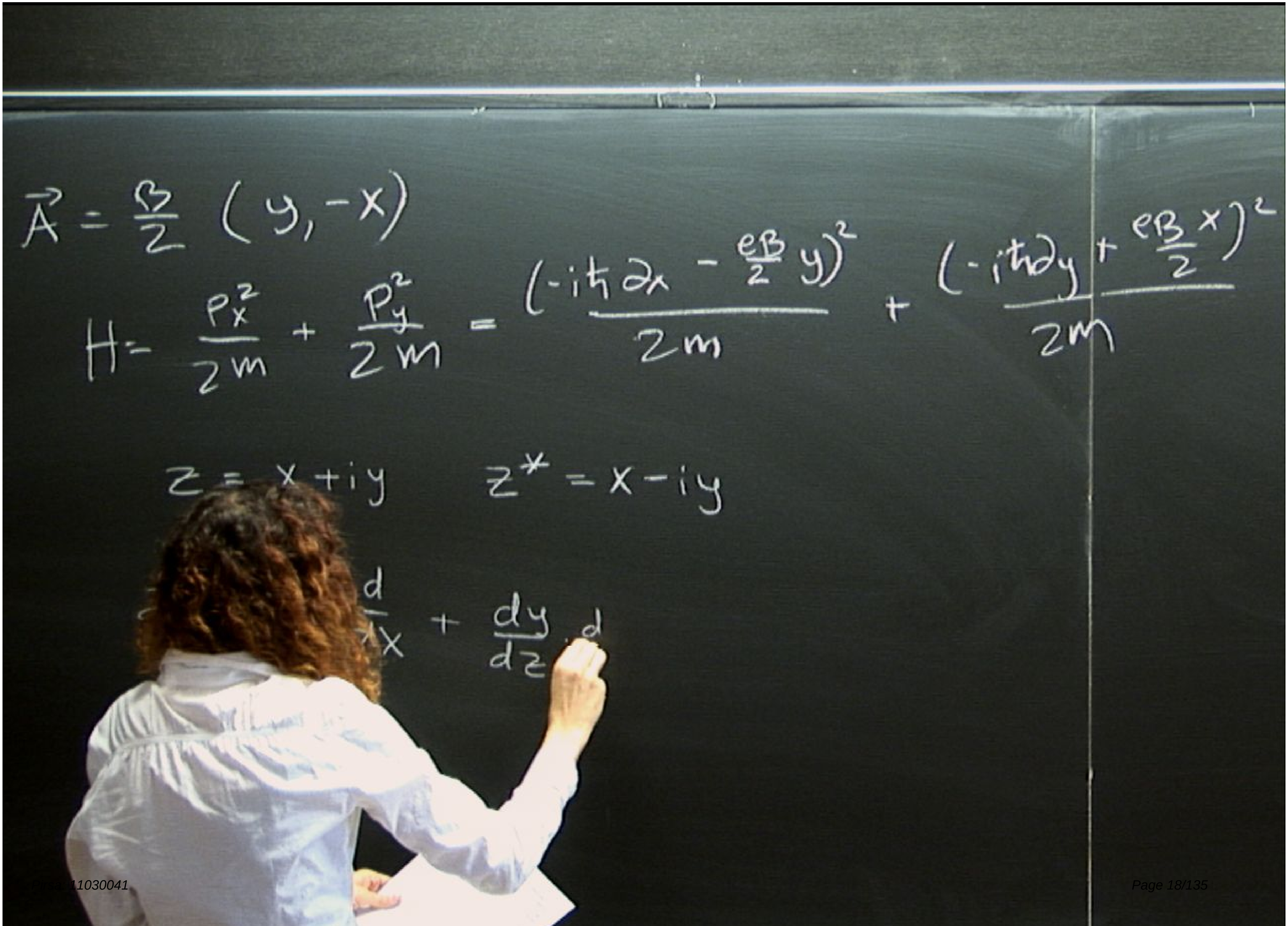
$z = x + iy \qquad z^* = x - iy$
$\frac{d}{dz} =$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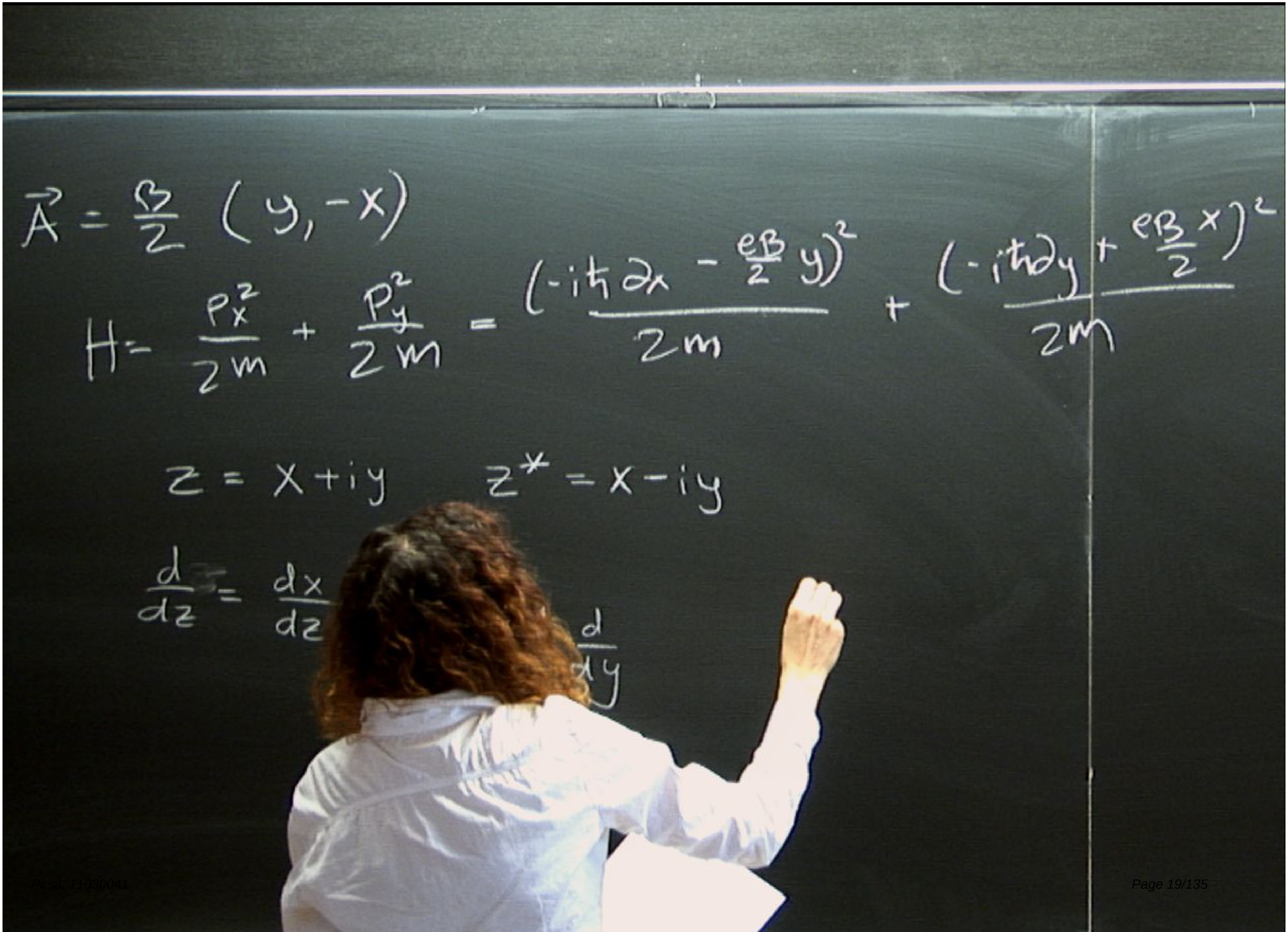
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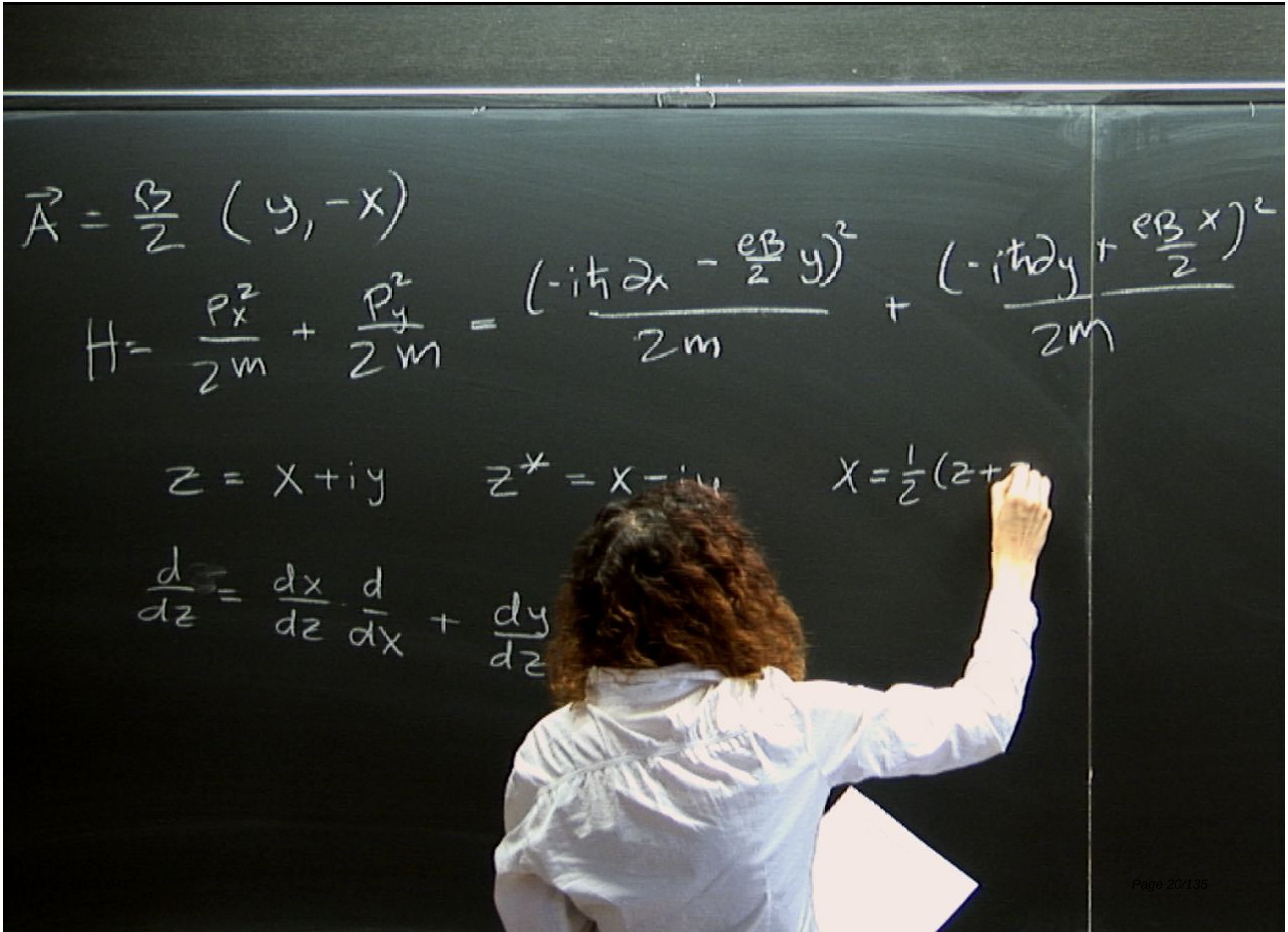
$\frac{d}{dz} = \frac{dx}{dz}$
$\frac{d}{dy}$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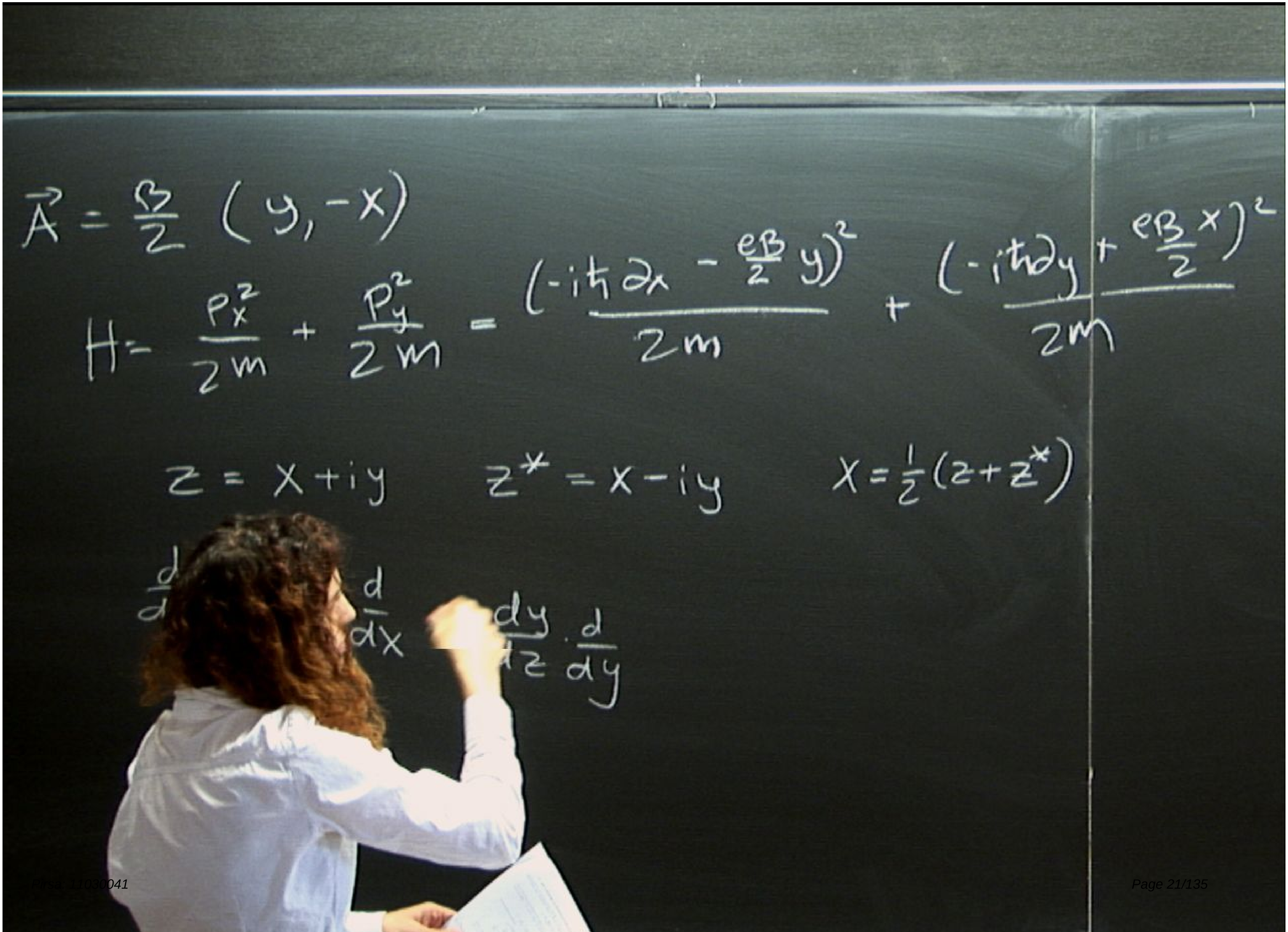
z = x + iy
z* = x - iy
x = 1/2 (z +
d/dz = dx/dz d/dx + dy/dz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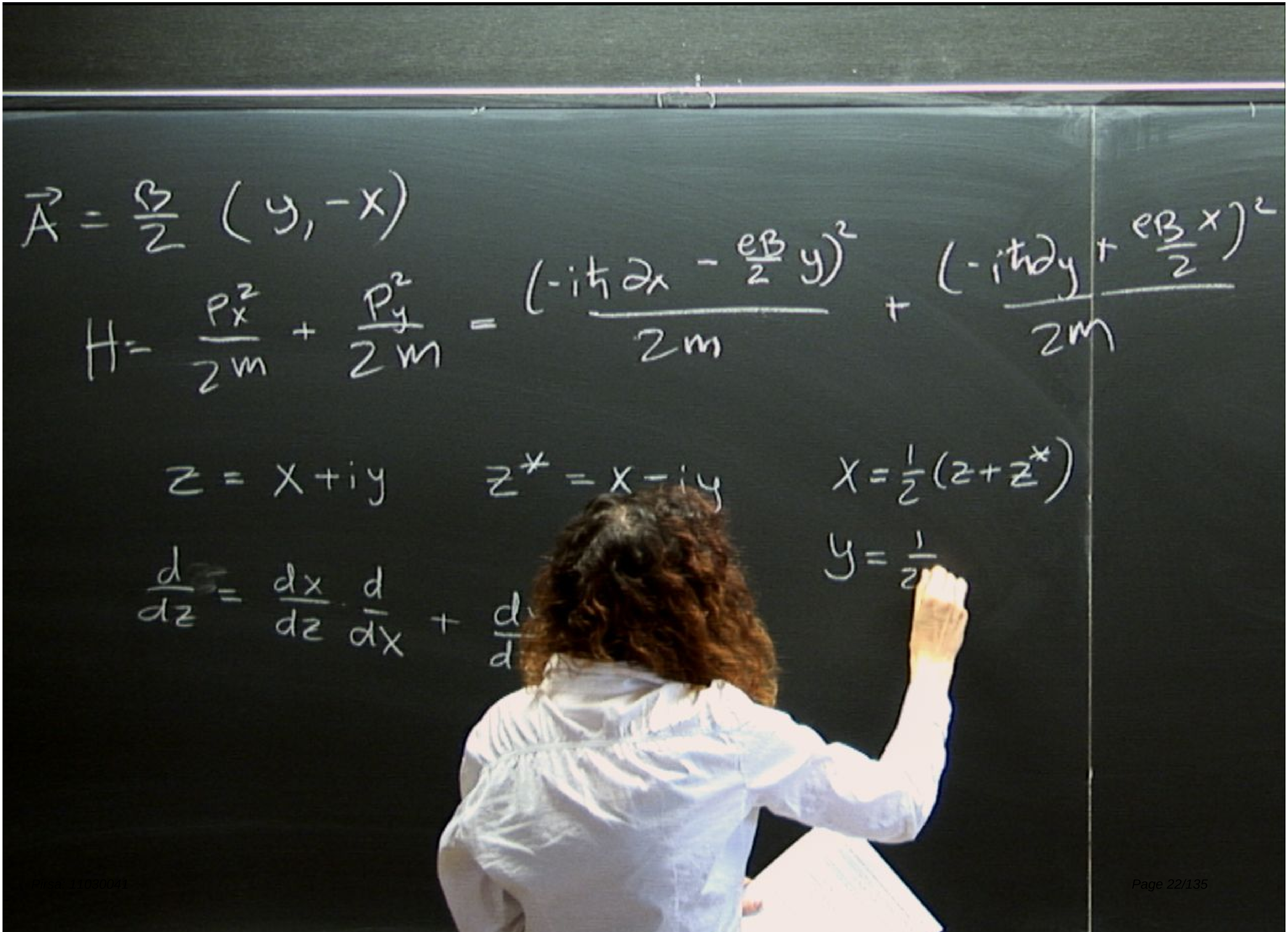

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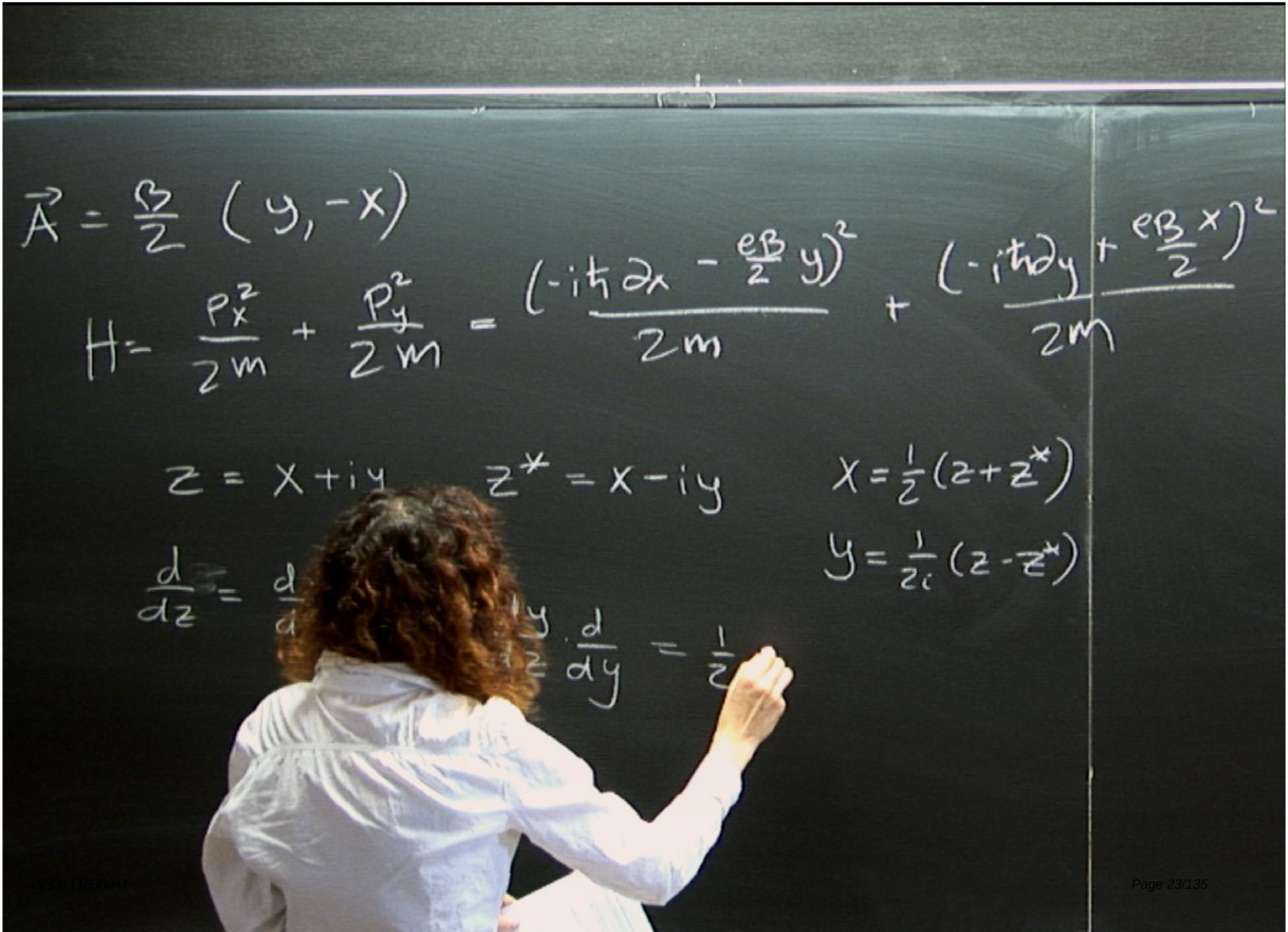
z* = x - iy
x = 1/2 (z + z*)
y = 1/2
d/dz = dx/dz d/dx + d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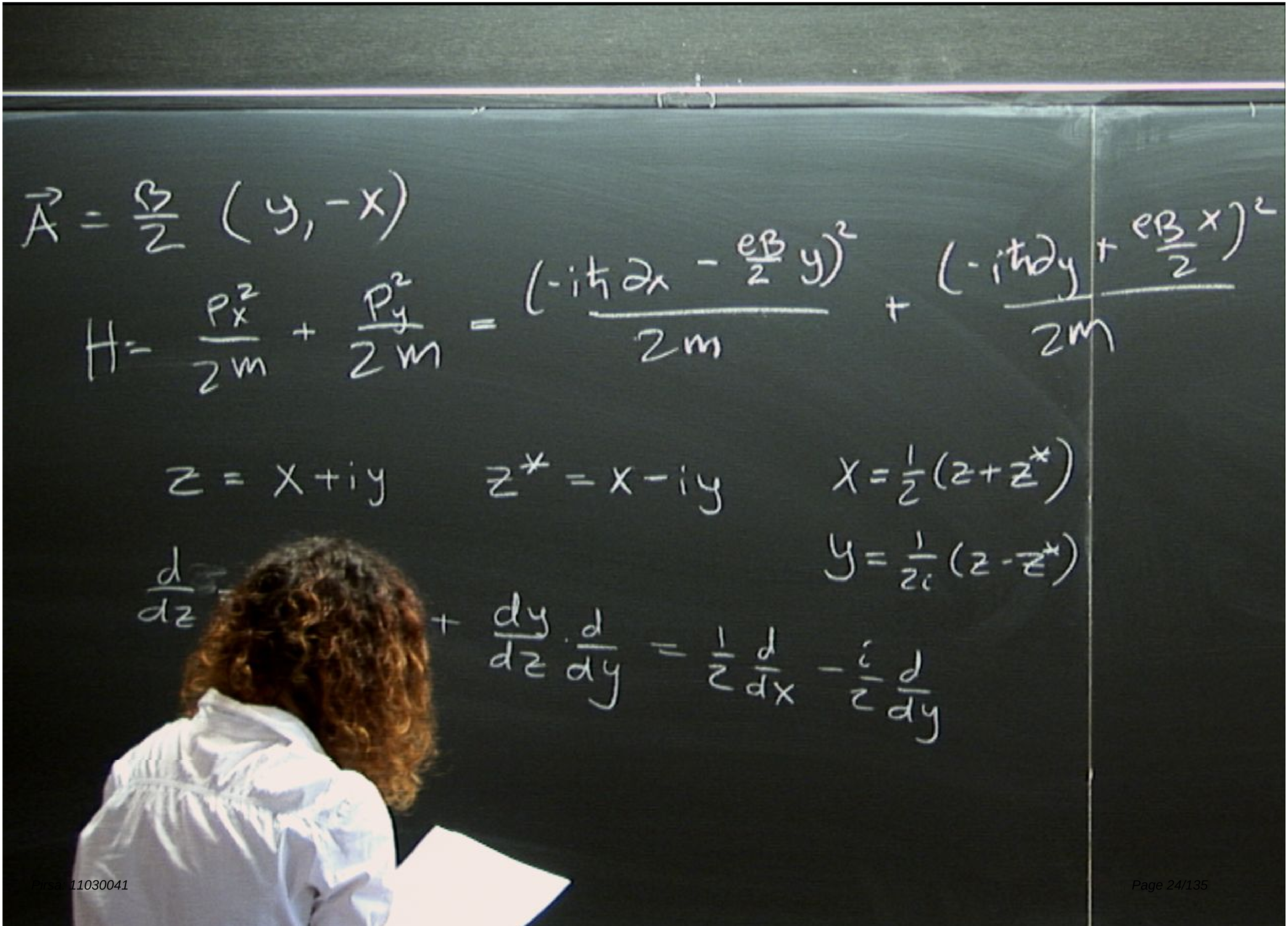
$z = x + iy$
$z^* = x - iy$
$x = \frac{1}{2}(z + z^*)$
$y = \frac{1}{2i}(z - z^*)$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$y = \frac{1}{2i}(z - z^*)$
$\frac{d}{dz}$
$+ \frac{dy}{dz}\frac{d}{dy} = \frac{1}{2}\frac{d}{dx} - \frac{i}{2}\frac{d}{dy}$

$$\vec{A} = \frac{B}{2}(y, -x)$$

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$$z = x + iy \qquad z^* = x - iy \qquad x = \frac{1}{2}(z + z^*)$$

$$y = \frac{1}{2i}(z - z^*)$$

$$\frac{d}{dz} = \frac{dx}{dz}\cdot\frac{d}{dx} + \frac{dy}{dz}\cdot\frac{d}{dy} = \frac{1}{2}\frac{d}{dx} - \frac{i}{2}\frac{d}{dy}$$

$$\frac{d}{dz}$$

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$$= \frac{dx}{dz}\frac{d}{dx} + \frac{dy}{dz}\frac{d}{dy} = \frac{1}{2}\frac{d}{dx} - \frac{i}{2}\frac{d}{dy}$$

$$z^* = \frac{1}{2}\Big(\frac{d}{d}$$

$$\vec{A} = \frac{B}{2}(y, -x)$$

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} = \frac{\left(-i\hbar\partial_x - \frac{eB}{2}y\right)^2}{2m} + \frac{\left(-i\hbar\partial_y + \frac{eB}{2}x\right)^2}{2m}$$

$$z = x + iy \qquad z^* = x - iy \qquad x = \frac{1}{2}(z + z^*)$$

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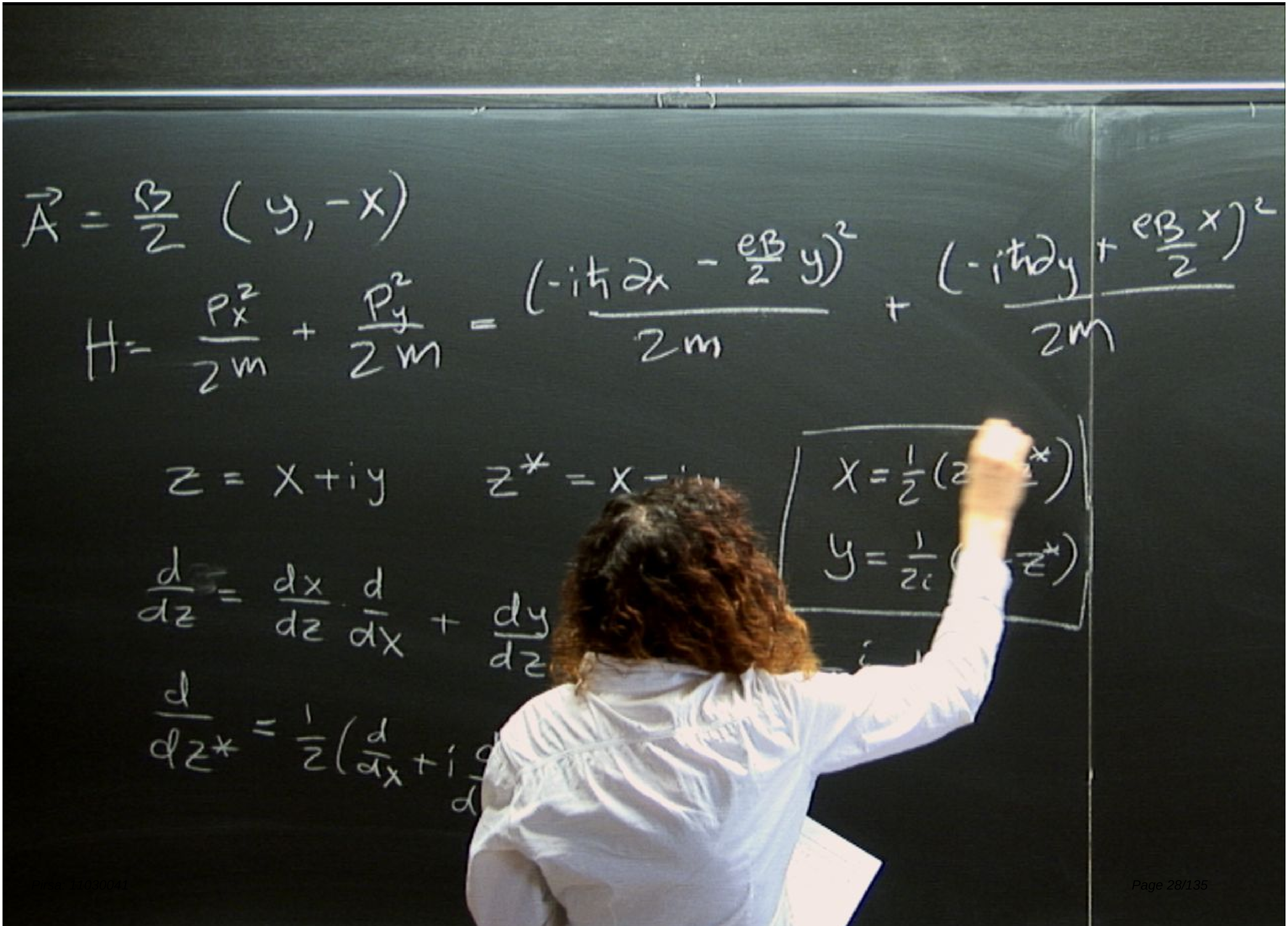
$$\frac{d}{dz} = \frac{dx}{dz}\frac{d}{dx} + \frac{dy}{dz}\frac{d}{dy} = \frac{1}{2}\frac{d}{dx} - \frac{i}{2}\frac{d}{dy}$$

$$\frac{d}{dz^*} = \frac{1}{2}\left(\frac{d}{dx} + i\frac{d}{dy}\right)$$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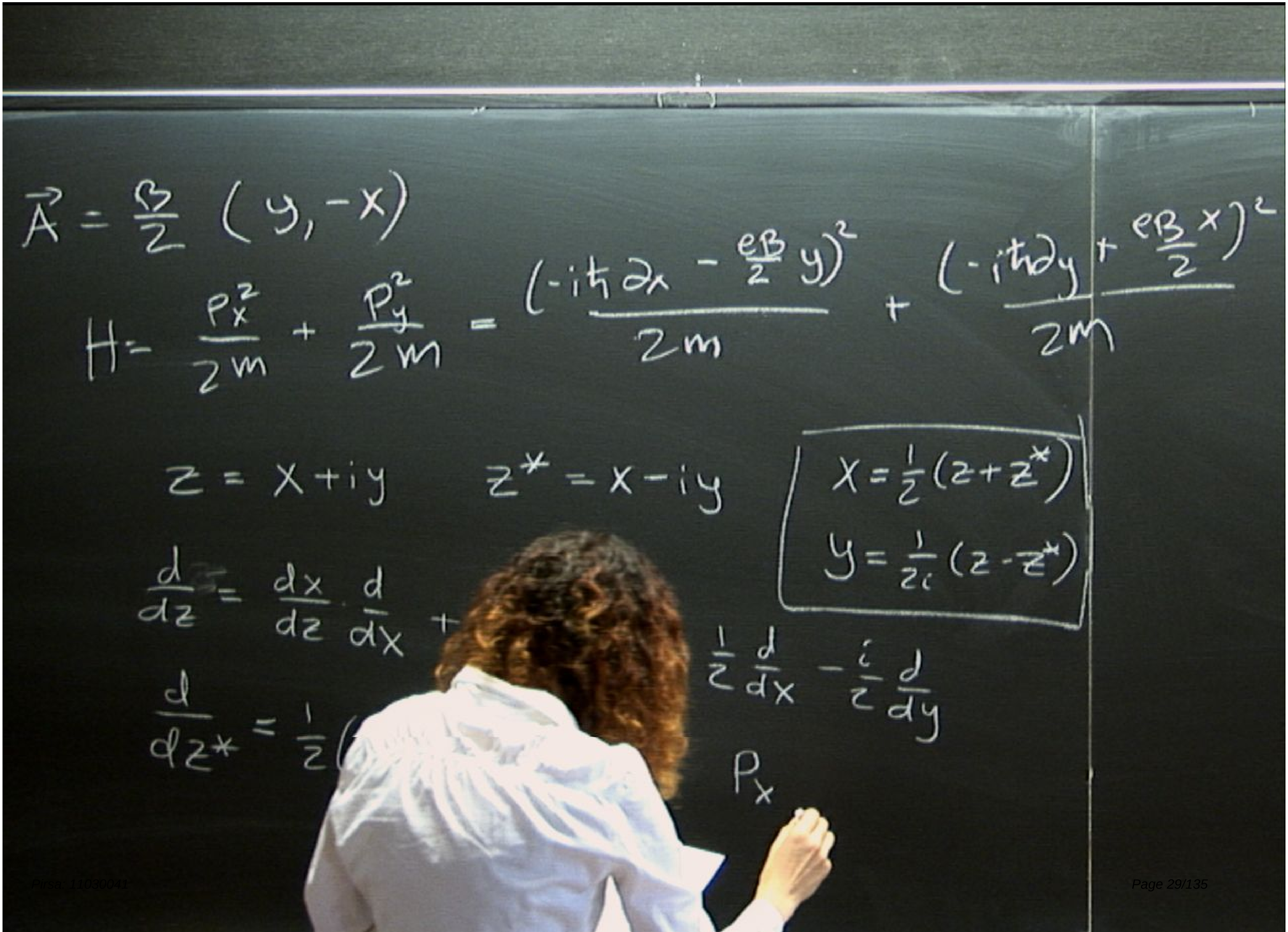
z = x + iy
z* = x -
x = 1/2 (z
y = 1/2i (
d/dz = dx/dz d/dx + dy/dz
d/dz* = 1/2 (d/dx + i



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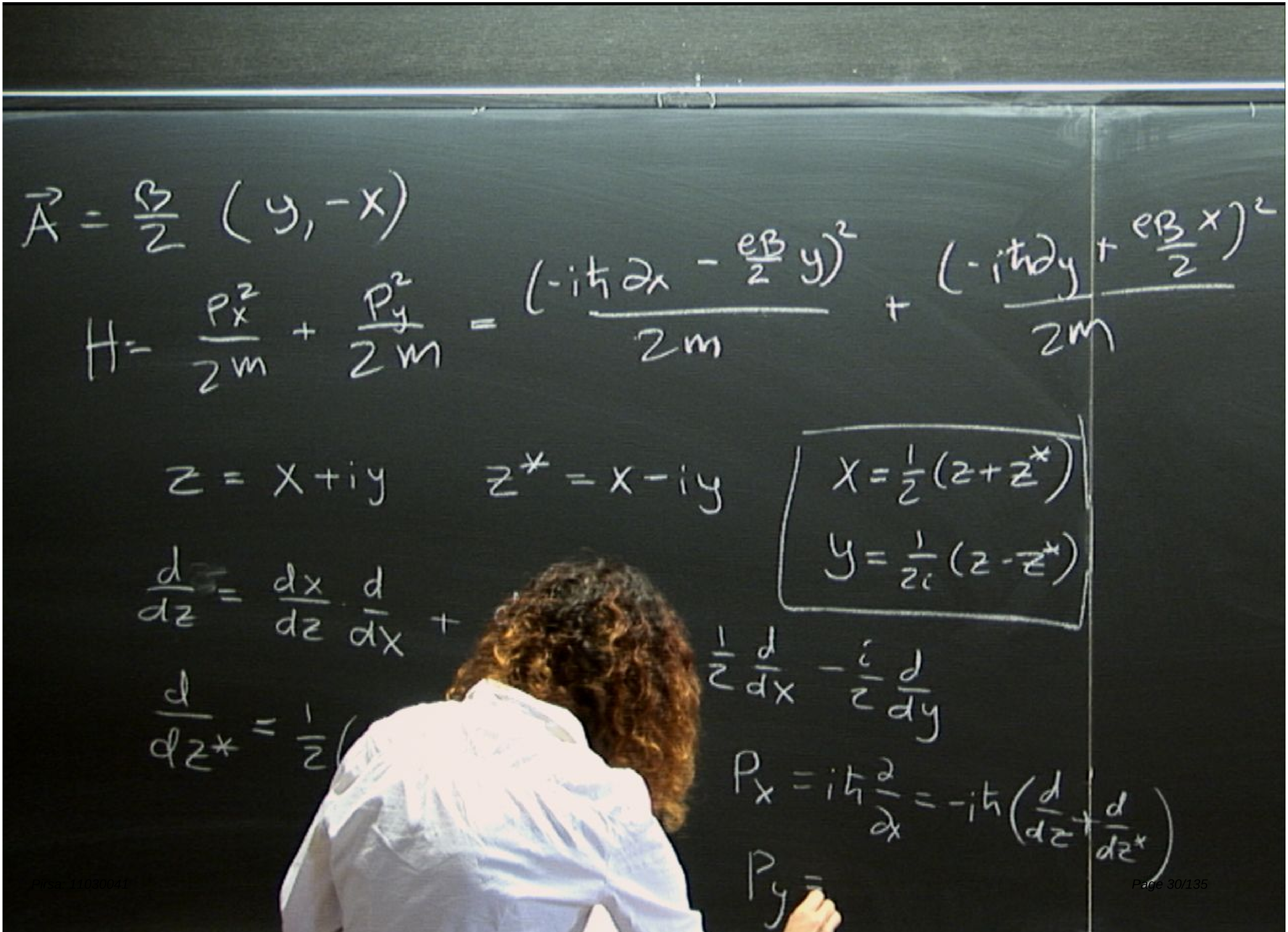
$x = \frac{1}{2}(z + z^*)$
$y = \frac{1}{2i}(z - z^*)$
$\frac{d}{dz} = \frac{dx}{dz}\frac{d}{dx} +$
$\frac{1}{2}\frac{d}{dx} - \frac{i}{2}\frac{d}{dy}$
$\frac{d}{dz^*} = \frac{1}{2}($
P_x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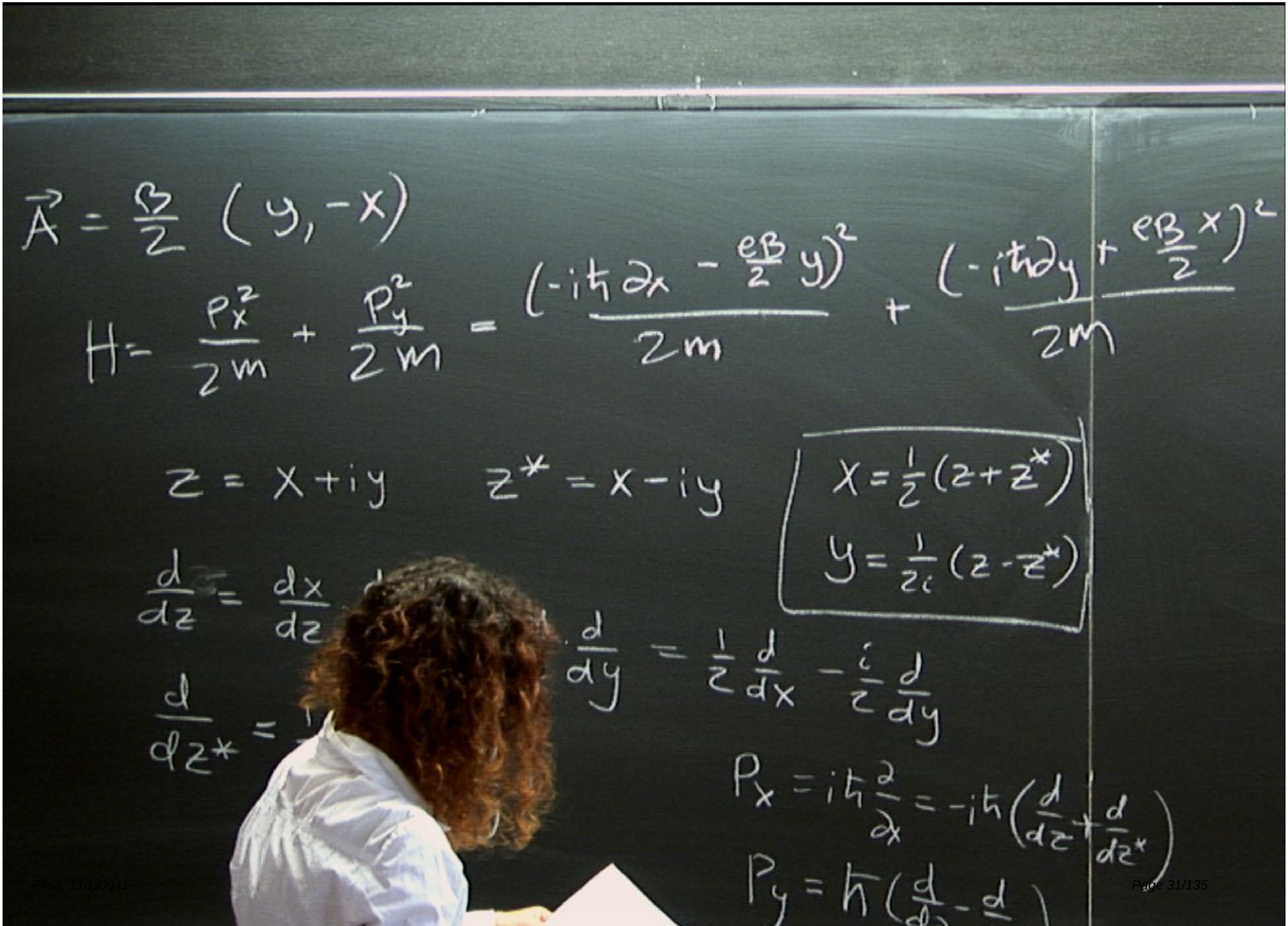
$\frac{1}{2}\frac{d}{dx} - \frac{i}{2}\frac{d}{dy}$
$\frac{d}{dz^*} = \frac{1}{2}($
$P_x = i\hbar\frac{\partial}{\partial x} = -i\hbar\left(\frac{d}{dz} + \frac{d}{dz^*}\right)$
$P_y =$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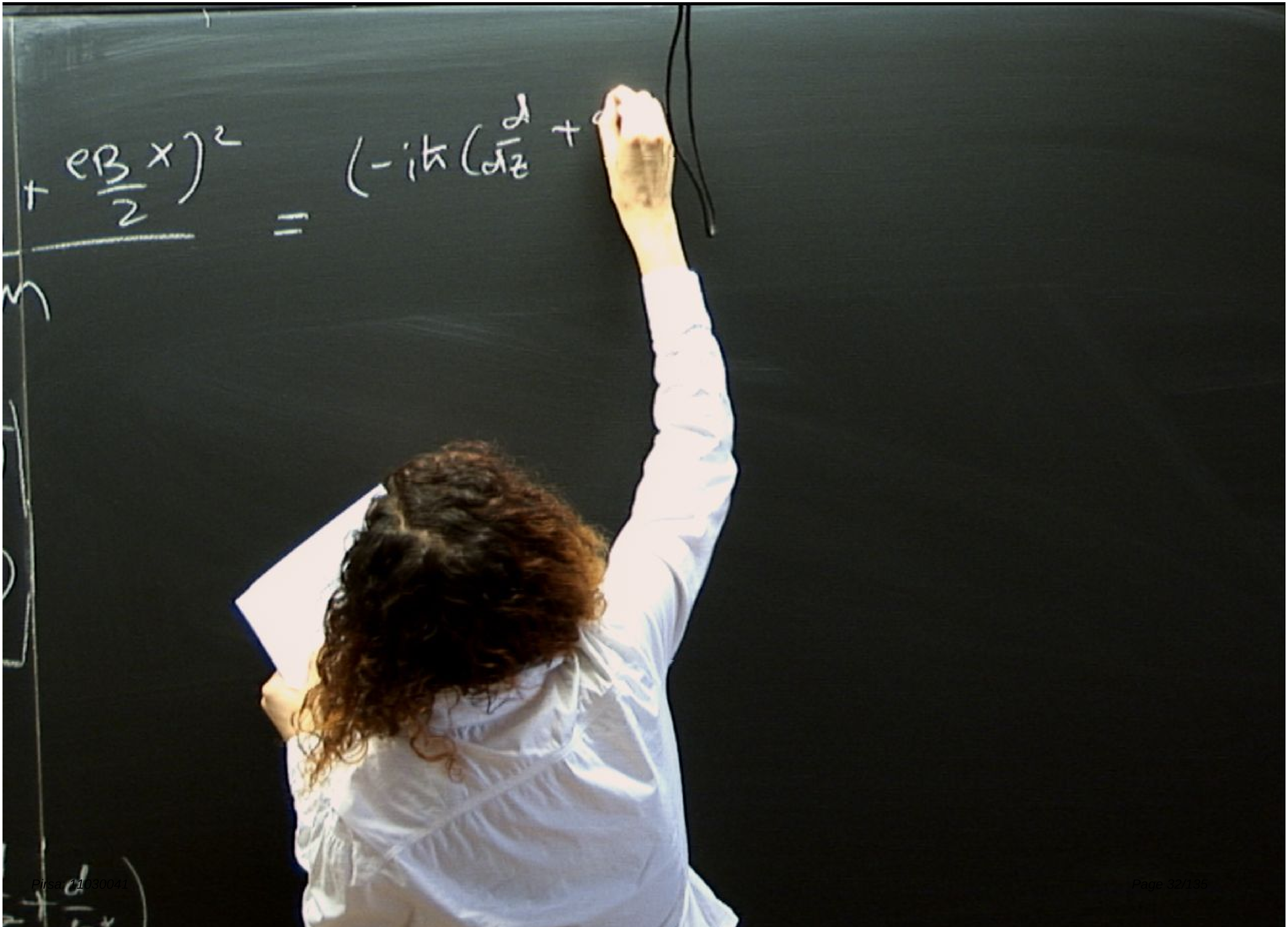
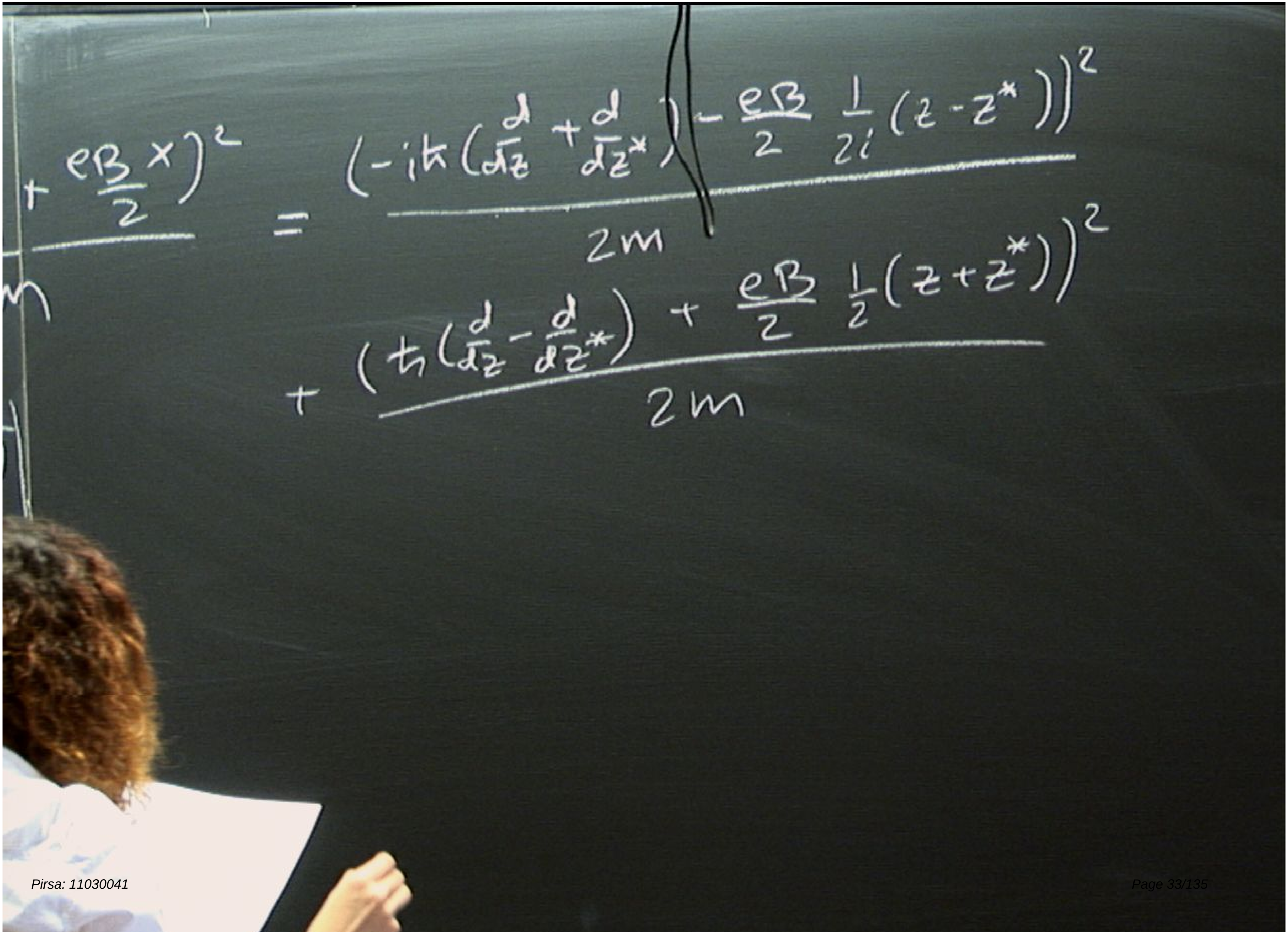
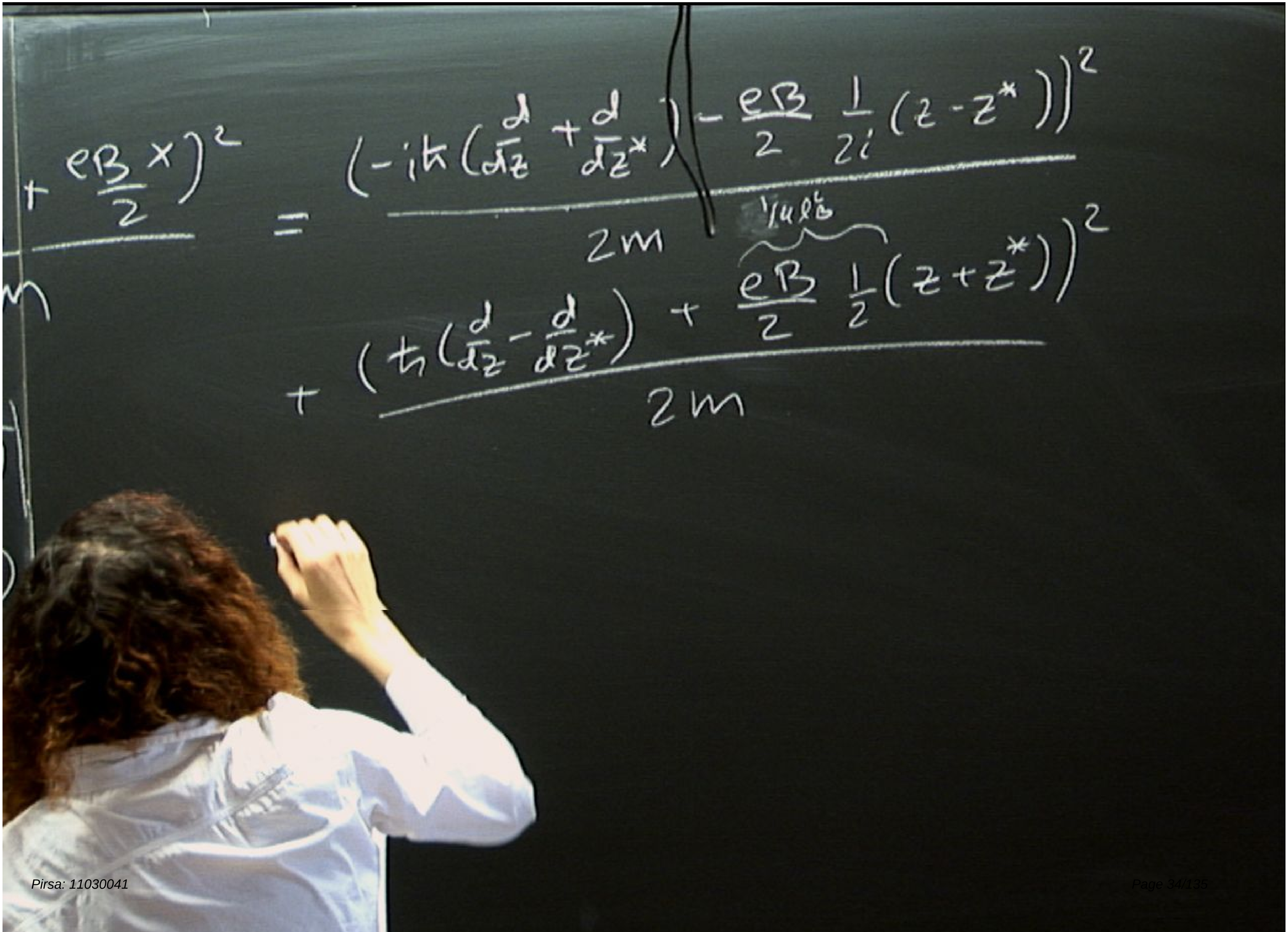
z = x + iy
z* = x - iy
x = 1/2 (z + z*)
y = 1/2i (z - z*)
d/dz = dx/dz
d/dy = 1/2 d/dx - i/2 d/dy
d/dz* =
Px = iħ ∂/∂x = -iħ (d/dz + d/dz*)
Py = ħ (d/dz - d/





$+ \frac{eB}{2} x)^2$
$= \frac{\left(-i\hbar\left(\frac{d}{dz} + \frac{d}{dz^*}\right) - \frac{eB}{2}\frac{1}{2i}(z - z^*)\right)^2}{2m}$
$+ \frac{\left(\hbar\left(\frac{d}{dz} - \frac{d}{dz^*}\right) + \frac{eB}{2}\frac{1}{2}(z + z^*)\right)^2}{2m}$


$\frac{(\ldots + \frac{eB}{2}x)^2}{\ldots m} = \frac{\left(-i\hbar\left(\frac{d}{dz} + \frac{d}{dz^*}\right) - \frac{eB}{2}\frac{1}{2i}(z - z^*)\right)^2}{2m}$
$1/4\ell_B^2$
$+ \frac{\left(\hbar\left(\frac{d}{dz} - \frac{d}{dz^*}\right) + \frac{eB}{2}\frac{1}{2}(z + z^*)\right)^2}{2m}$

$$\frac{\left(\ldots + \frac{eB}{2}x\right)^2}{\ldots m} = \frac{\left(-i\hbar\left(\frac{d}{dz} + \frac{d}{dz^*}\right) - \frac{eB}{2}\frac{1}{2i}(z - z^*)\right)^2}{2m}$$

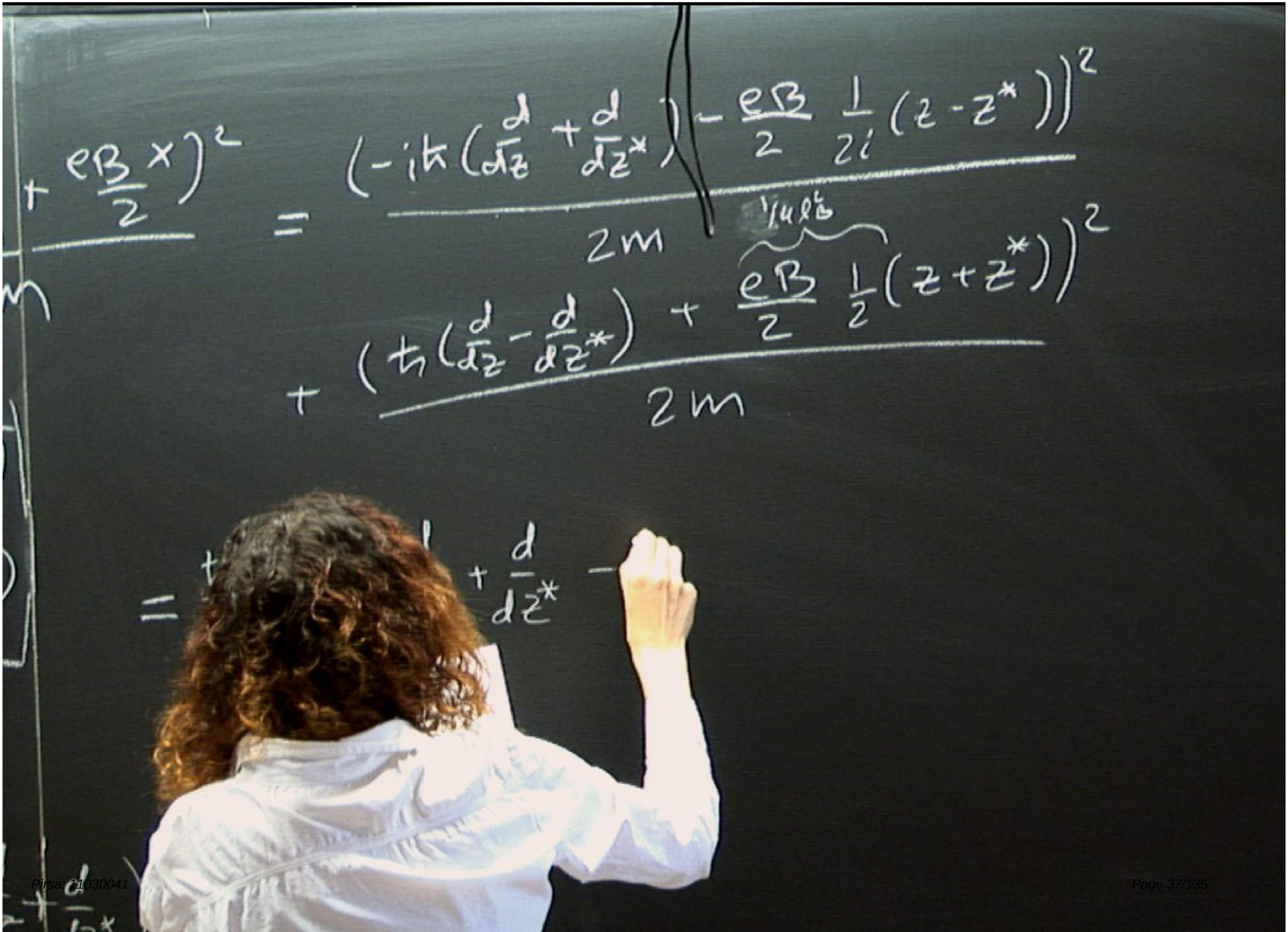
$$+ \frac{\left(\hbar\left(\frac{d}{dz} - \frac{d}{dz^*}\right) + \overbrace{\frac{eB}{2}}^{1/4\ell_B^2}\frac{1}{2}(z + z^*)\right)^2}{2m}$$

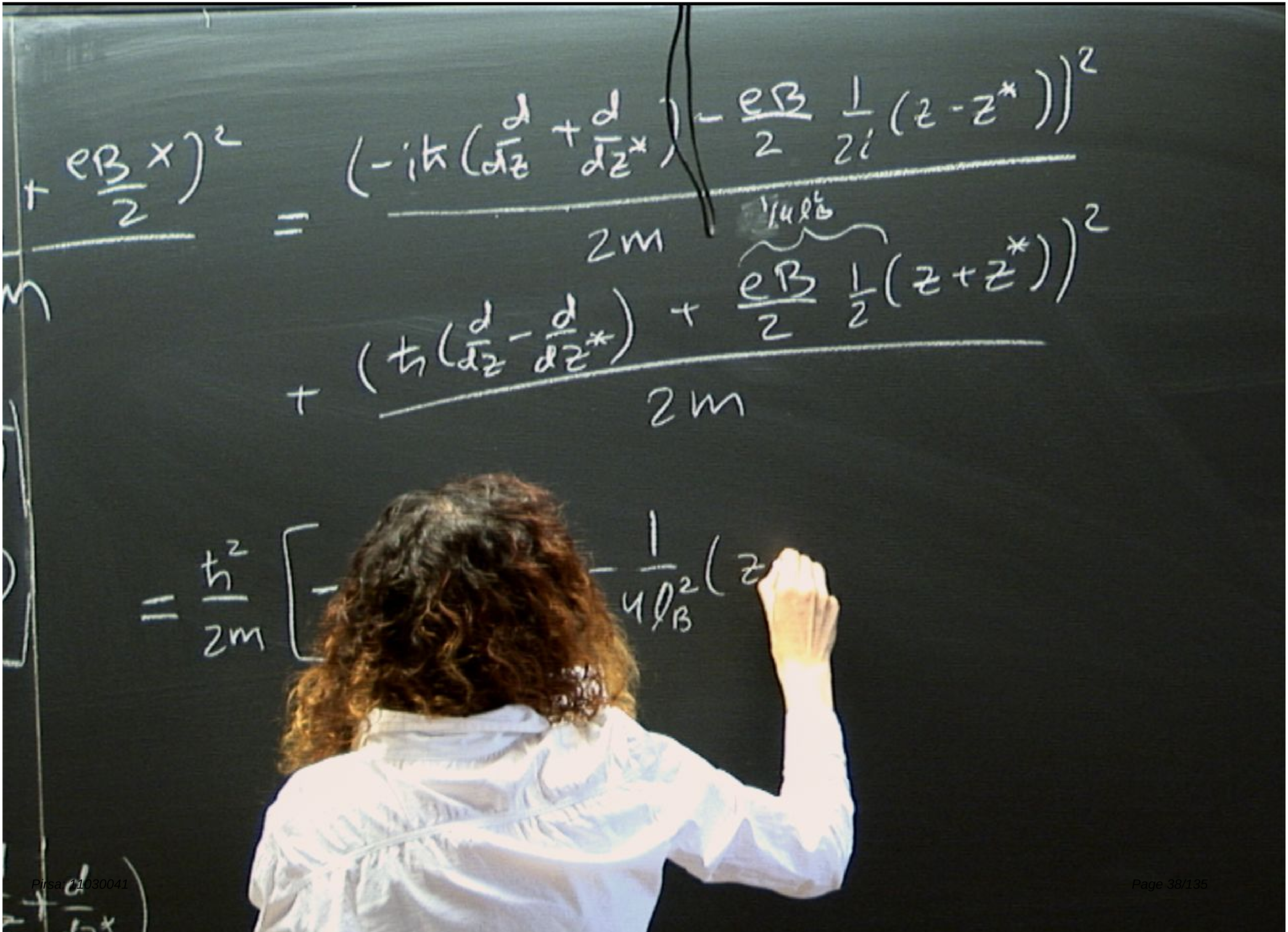
$$= \frac{\hbar^2}{2m}$$

$$\frac{\left(\ldots + \frac{eB}{2}x\right)^2}{\ldots m} = \frac{\left(-i\hbar\left(\frac{d}{dz}+\frac{d}{dz^*}\right) - \frac{eB}{2}\frac{1}{2i}(z-z^*)\right)^2}{2m}$$

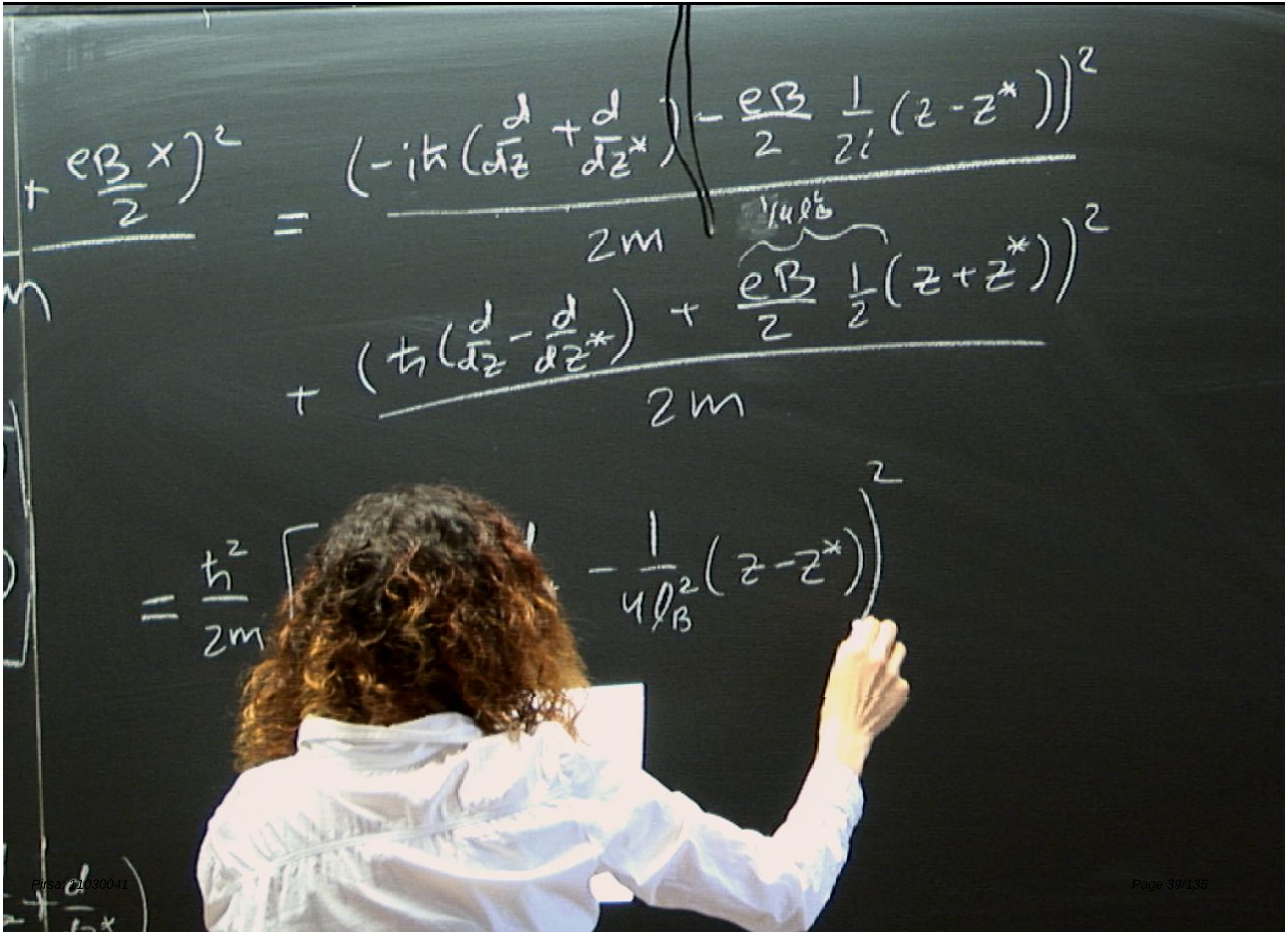
$$+ \frac{\left(\hbar\left(\frac{d}{dz}-\frac{d}{dz^*}\right) + \overbrace{\frac{eB}{2}\frac{1}{2}}^{1/4\ell_B^2}(z+z^*)\right)^2}{2m}$$

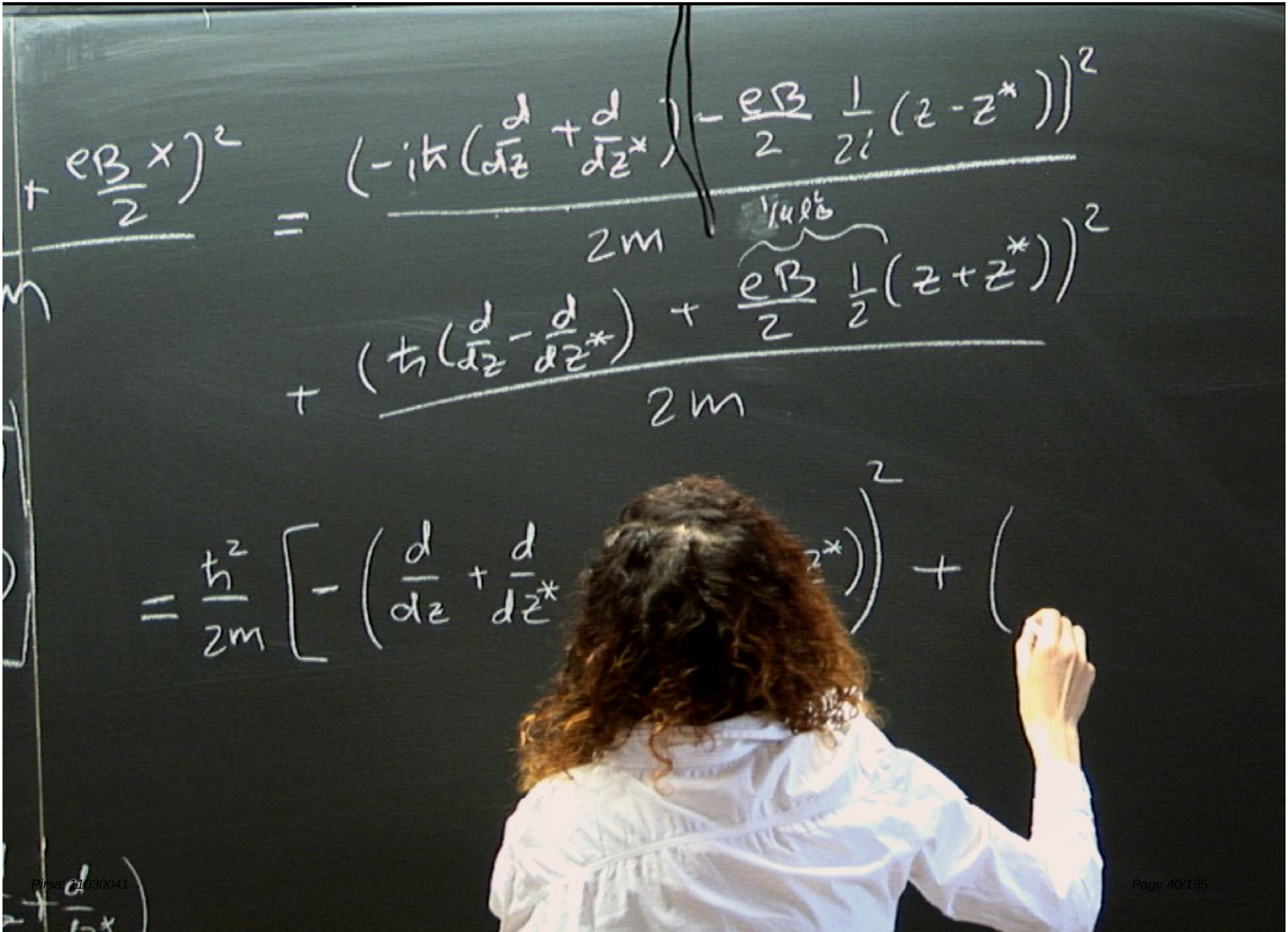
$$\left[-\left(\frac{d}{dz} + \ldots\right.\right.$$


$\frac{(\ldots + \frac{eB}{2}x)^2}{\ldots m} = \frac{\left(-i\hbar\left(\frac{d}{dz} + \frac{d}{dz^*}\right) - \frac{eB}{2}\frac{1}{2i}(z - z^*)\right)^2}{2m}$
$+ \frac{\left(\hbar\left(\frac{d}{dz} - \frac{d}{dz^*}\right) + \frac{eB}{2}\frac{1}{2}(z + z^*)\right)^2}{2m}$
$= \ldots + \frac{d}{dz^*} - \ldots$



+ eB/2 x)^2
= (-iħ(d/dz + d/dz*) - eB/2 1/2i (z - z*))^2 / 2m
1/4ℓ_B^2
+ (ħ(d/dz - d/dz*) + eB/2 1/2 (z + z*))^2 / 2m
= ħ^2/2m [- 1/4ℓ_B^2 (z


$$= \frac{\left(-i\hbar\left(\frac{d}{dz}+\frac{d}{dz^*}\right) - \frac{eB}{2}\frac{1}{2i}(z-z^*)\right)^2}{2m}$$
$$+ \frac{\left(\hbar\left(\frac{d}{dz}-\frac{d}{dz^*}\right) + \frac{eB}{2}\frac{1}{2}(z+z^*)\right)^2}{2m}$$
$$= \frac{\hbar^2}{2m}\left[\ldots - \frac{1}{4\ell_B^2}(z-z^*)\right)^2$$


+ eB/2 x)^2
= (-iħ(d/dz + d/dz*) - eB/2 1/2i (z - z*))^2 / 2m
1/4 l_B^2
+ (ħ(d/dz - d/dz*) + eB/2 1/2 (z + z*))^2 / 2m
= ħ^2/2m [- (d/dz + d/dz*
z*))^2 + (

$$\ldots)^2 = \frac{\left(-i\hbar\left(\frac{d}{dz}+\frac{d}{dz^*}\right) - \frac{eB}{2}\frac{1}{2i}(z-z^*)\right)^2}{2m}$$

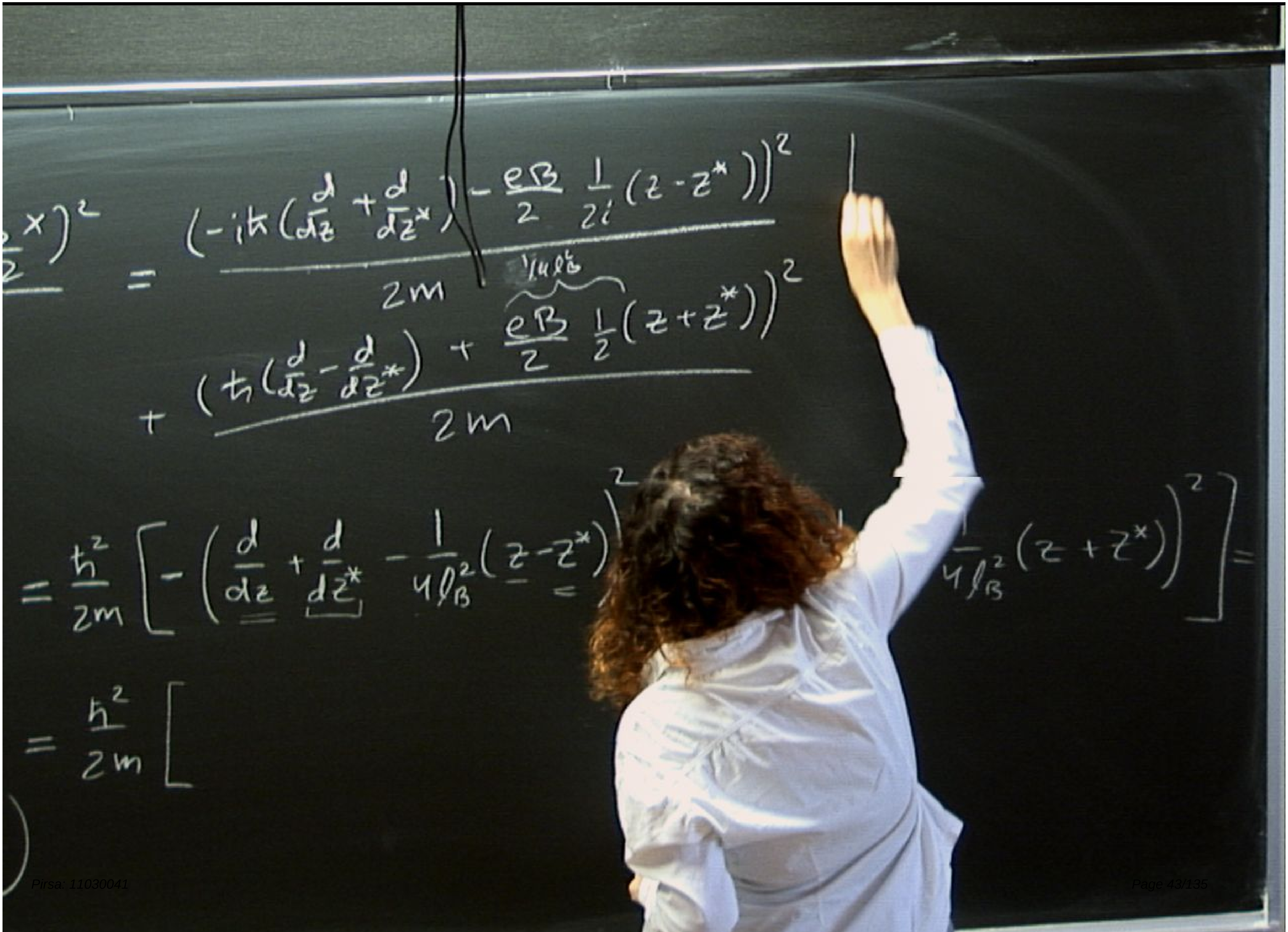
$$+ \frac{\left(\hbar\left(\frac{d}{dz}-\frac{d}{dz^*}\right) + \overbrace{\frac{eB}{2}\frac{1}{2}}^{1/4\ell_B^2}(z+z^*)\right)^2}{2m}$$

$$\hbar^2 \ldots + \frac{d}{dz^*} - \frac{1}{4\ell_B^2}(z-z^*)\Big)^2 + \left(\frac{d}{dz} - \frac{d}{dz^*} + \frac{1}{4\ell_B^2}(z \ldots\right.$$

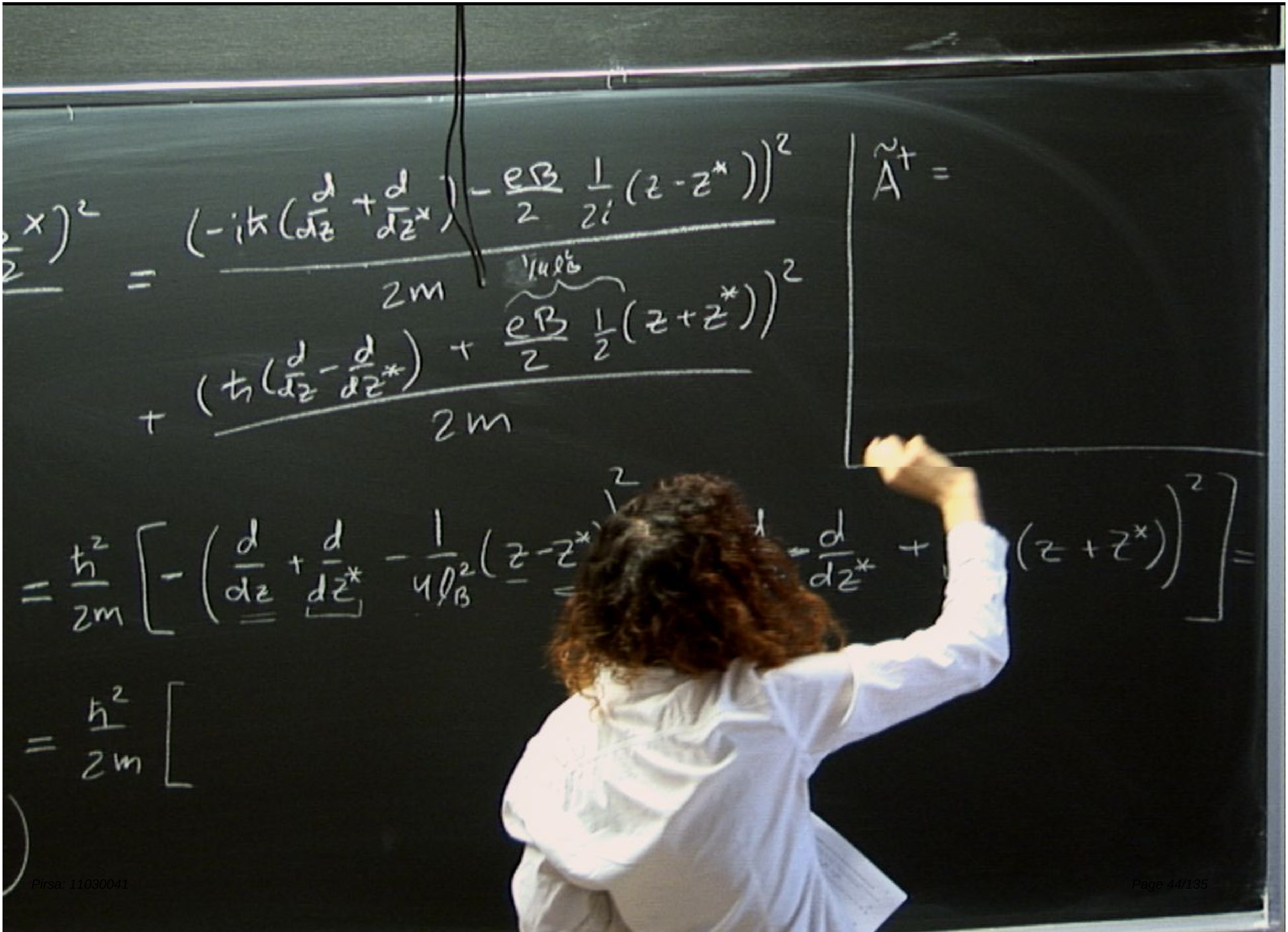
$$\frac{\left(-i\hbar\left(\frac{d}{dz}+\frac{d}{dz^*}\right)-\frac{eB}{2}\frac{1}{2i}(z-z^*)\right)^2}{2m}$$

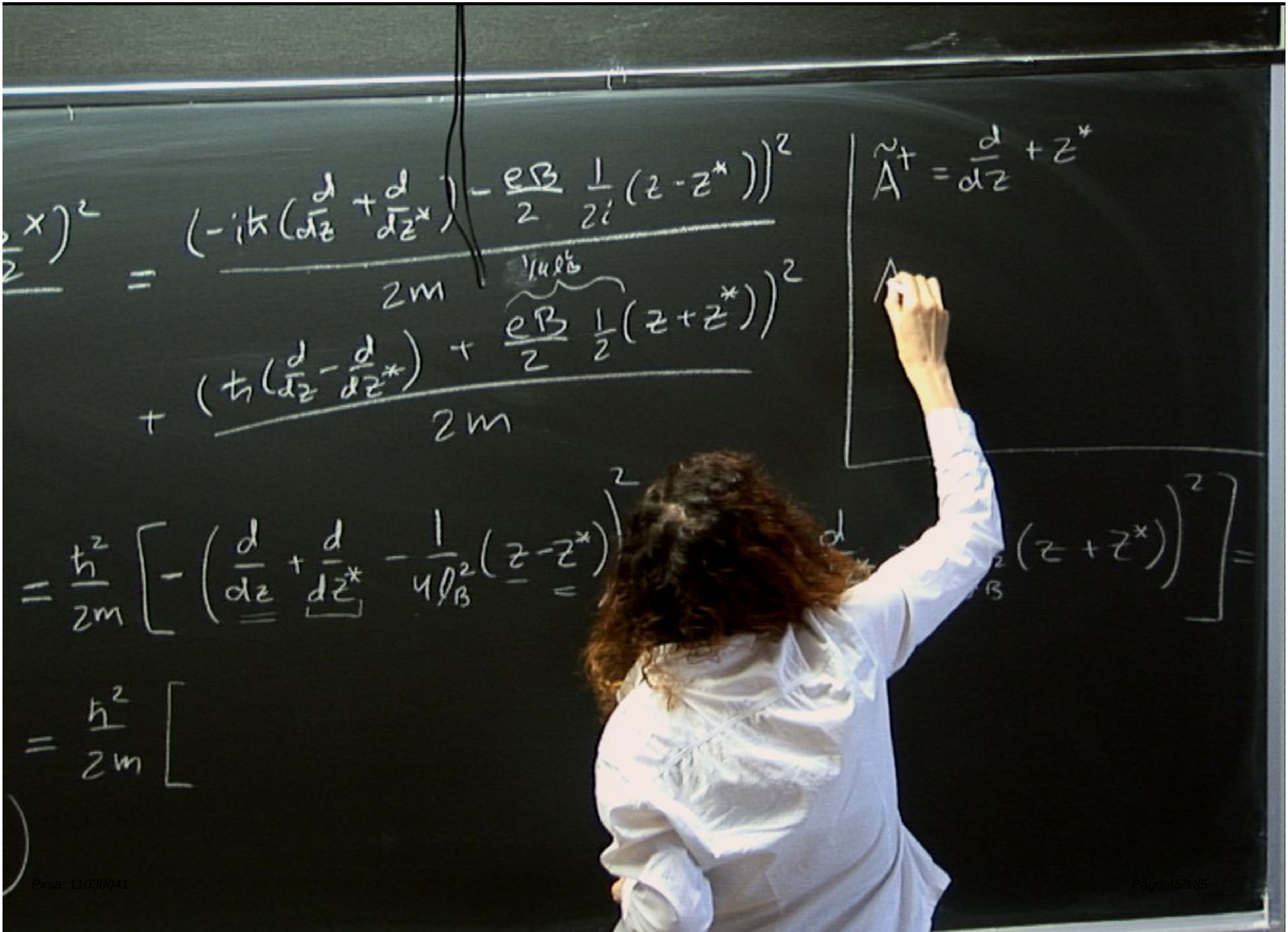
$$\frac{\left(\hbar\left(\frac{d}{dz}-\frac{d}{dz^*}\right)+\overbrace{\frac{eB}{2}\frac{1}{2}}^{1/4\ell_B^2}(z+z^*)\right)^2}{2m}$$

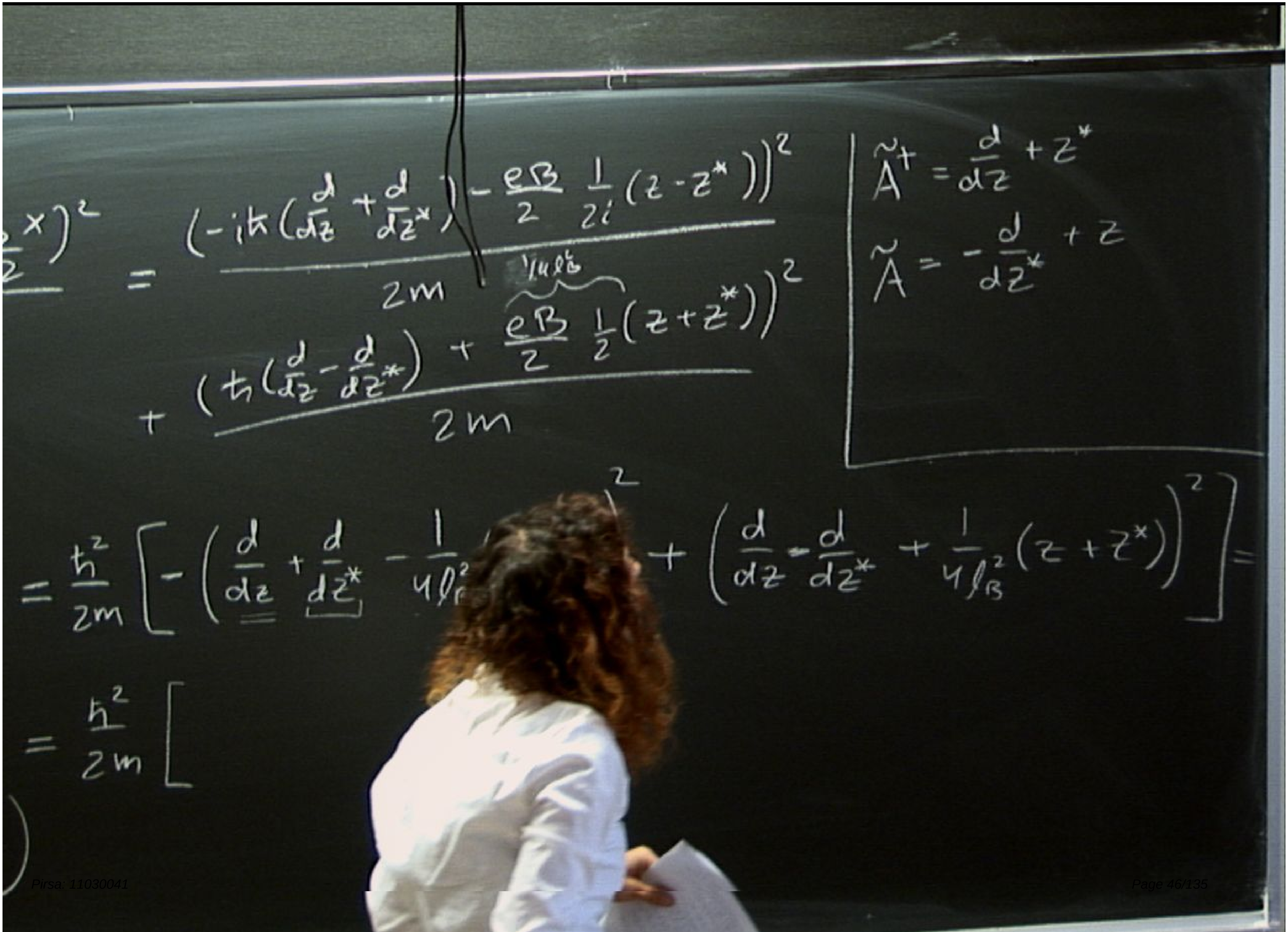
$$-\left(\frac{d}{dz}+\frac{d}{dz^*}-\frac{1}{4\ell_B^2}(z-z^*)\right)^2+\left(\frac{d}{dz}-\frac{d}{dz^*}+\frac{1}{4\ell_B^2}(z+z^*)\right)^2$$

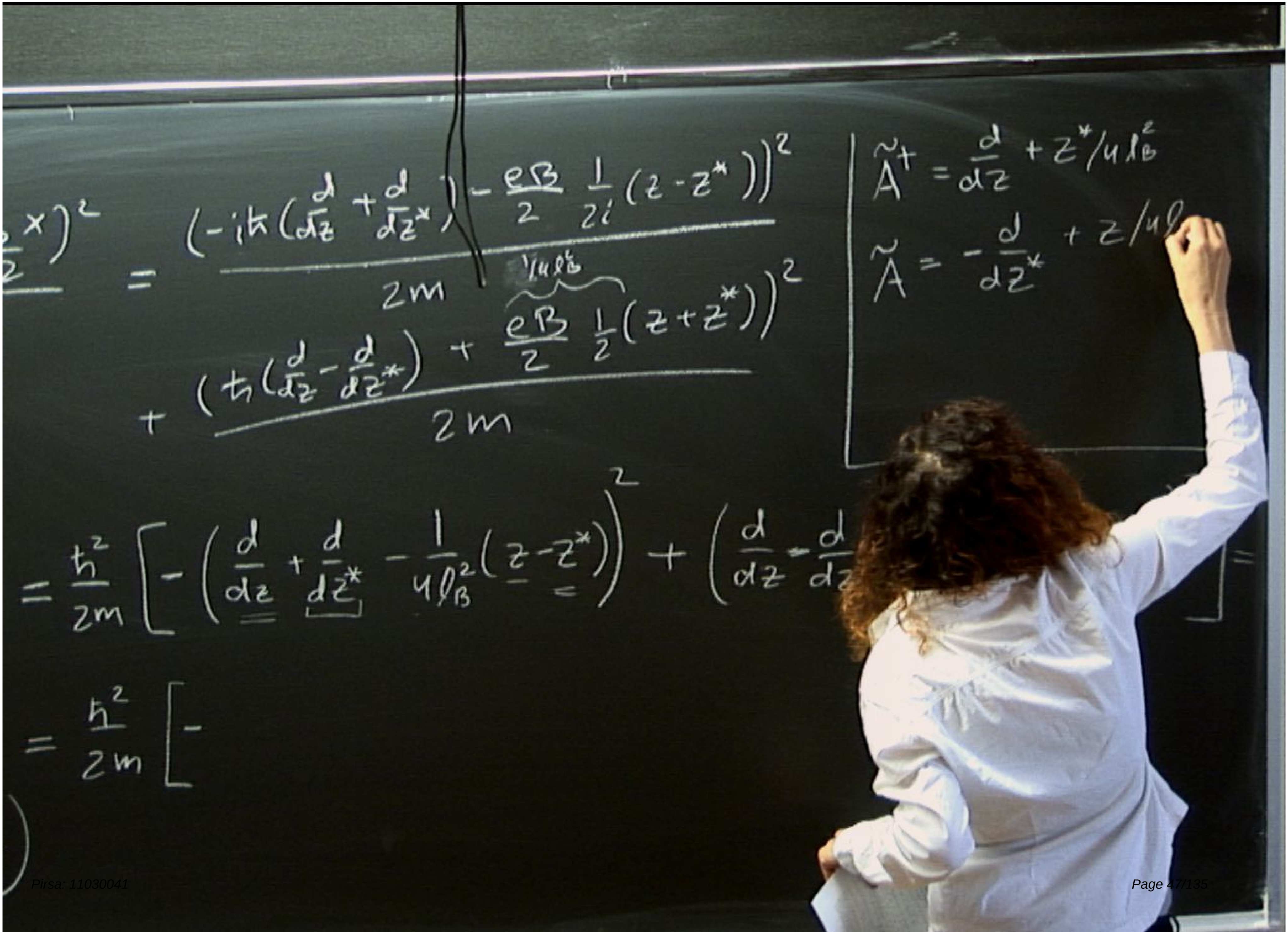


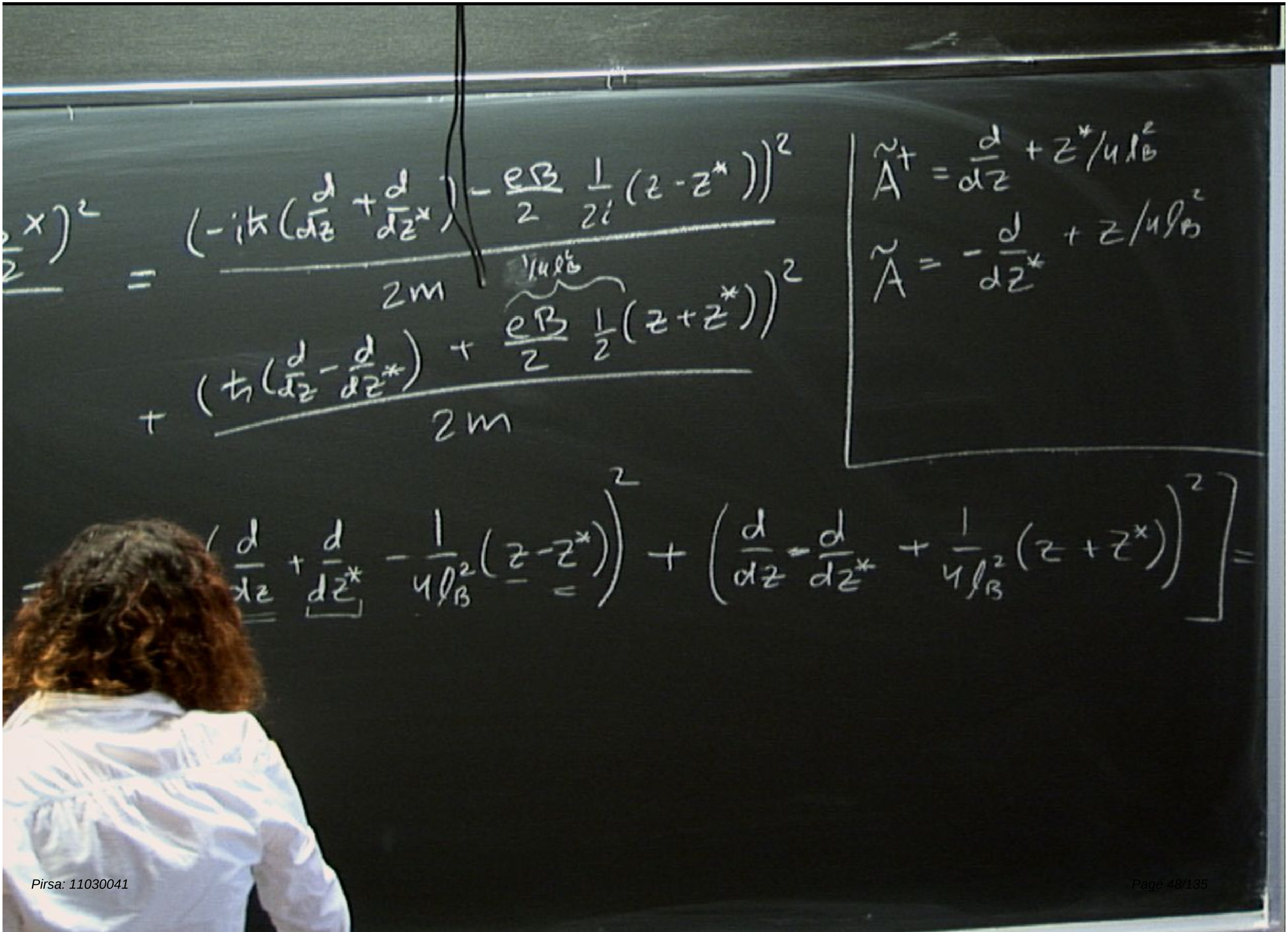
= (-iħ(d/dz + d/dz*) - eB/2 1/2i (z - z*))^2 / 2m
1/4ℓ_B^2
+ (ħ(d/dz - d/dz*) + eB/2 1/2 (z + z*))^2 / 2m
= ħ^2/2m [-(d/dz + d/dz* - 1/4ℓ_B^2 (z - z*))^2
1/4ℓ_B^2 (z + z*))^2] =
= ħ^2/2m [




$= \frac{\left(-i\hbar\left(\frac{d}{dz}+\frac{d}{dz^*}\right)-\frac{eB}{2}\frac{1}{2i}(z-z^*)\right)^2}{2m}$
$+ \frac{\left(\hbar\left(\frac{d}{dz}-\frac{d}{dz^*}\right)+\frac{eB}{2}\frac{1}{2}(z+z^*)\right)^2}{2m}$
$1/4\ell_B^2$
$\tilde{A}^+ = \frac{d}{dz} + z^*$
$= \frac{\hbar^2}{2m}\left[-\left(\frac{d}{dz}+\frac{d}{dz^*}-\frac{1}{4\ell_B^2}(z-z^*)\right)^2 \ldots (z+z^*)\right)^2\right] =$
$= \frac{\hbar^2}{2m}\left[\right.$


$= \frac{\left(-i\hbar\left(\frac{d}{dz}+\frac{d}{dz^*}\right)-\frac{eB}{2}\frac{1}{2i}(z-z^*)\right)^2}{2m}$
$+ \frac{\left(\hbar\left(\frac{d}{dz}-\frac{d}{dz^*}\right)+\frac{eB}{2}\frac{1}{2}(z+z^*)\right)^2}{2m}$
$1/4\ell_B^2$
$\tilde{A}^+ = \frac{d}{dz} + z^*$
$\tilde{A} = -\frac{d}{dz^*} + z$
$= \frac{\hbar^2}{2m}\left[-\left(\frac{d}{dz}+\frac{d}{dz^*}-\frac{1}{4\ell_B^2}\right)^2 + \left(\frac{d}{dz}-\frac{d}{dz^*}+\frac{1}{4\ell_B^2}(z+z^*)\right)^2\right] =$
$= \frac{\hbar^2}{2m}\Big[$


(-iħ(d/dz + d/dz*) - eB/2 1/2i (z - z*))^2 / 2m
+ (ħ(d/dz - d/dz*) + eB/2 1/2 (z + z*))^2 / 2m
1/4ℓ_B^2
Ã^+ = d/dz + z*/4ℓ_B^2
Ã = - d/dz* + z/4ℓ
= ħ^2/2m [- (d/dz + d/dz* - 1/4ℓ_B^2 (z - z*))^2 + (d/dz - d/dz
= ħ^2/2m [-


= (-iħ(d/dz + d/dz*) - eB/2 · 1/2i (z - z*))^2 / 2m
+ (ħ(d/dz - d/dz*) + eB/2 · 1/2 (z + z*))^2 / 2m
1/4ℓ_B^2
Ã† = d/dz + z*/4ℓ_B^2
Ã = - d/dz* + z/4ℓ_B^2
(d/dz + d/dz* - 1/4ℓ_B^2 (z - z*))^2 + (d/dz - d/dz* + 1/4ℓ_B^2 (z + z*))^2] =

$$\ldots x)^2 = \frac{\left(-i\hbar\left(\frac{d}{dz}+\frac{d}{dz^*}\right) - \frac{eB}{2}\frac{1}{2i}(z-z^*)\right)^2}{2m} + \frac{\left(\hbar\left(\frac{d}{dz}-\frac{d}{dz^*}\right) + \overbrace{\frac{eB}{2}}^{1/4l_B^2}\frac{1}{2}(z+z^*)\right)^2}{2m}$$

$$\tilde{A}^+ = \frac{d}{dz} + z^*/4l_B^2$$

$$\tilde{A} = -\frac{d}{dz^*} + z/4l_B^2$$

$$\ldots\hbar^2\left[-\left(\frac{d}{dz}+\frac{d}{dz^*} - \frac{1}{4l_B^2}(z-z^*)\right)^2 + \left(\frac{d}{dz}-\frac{d}{dz^*} + \frac{1}{4l_B^2}(z+z^*)\right)^2\right] =$$

$$\left[-\left(A\right.\right.$$

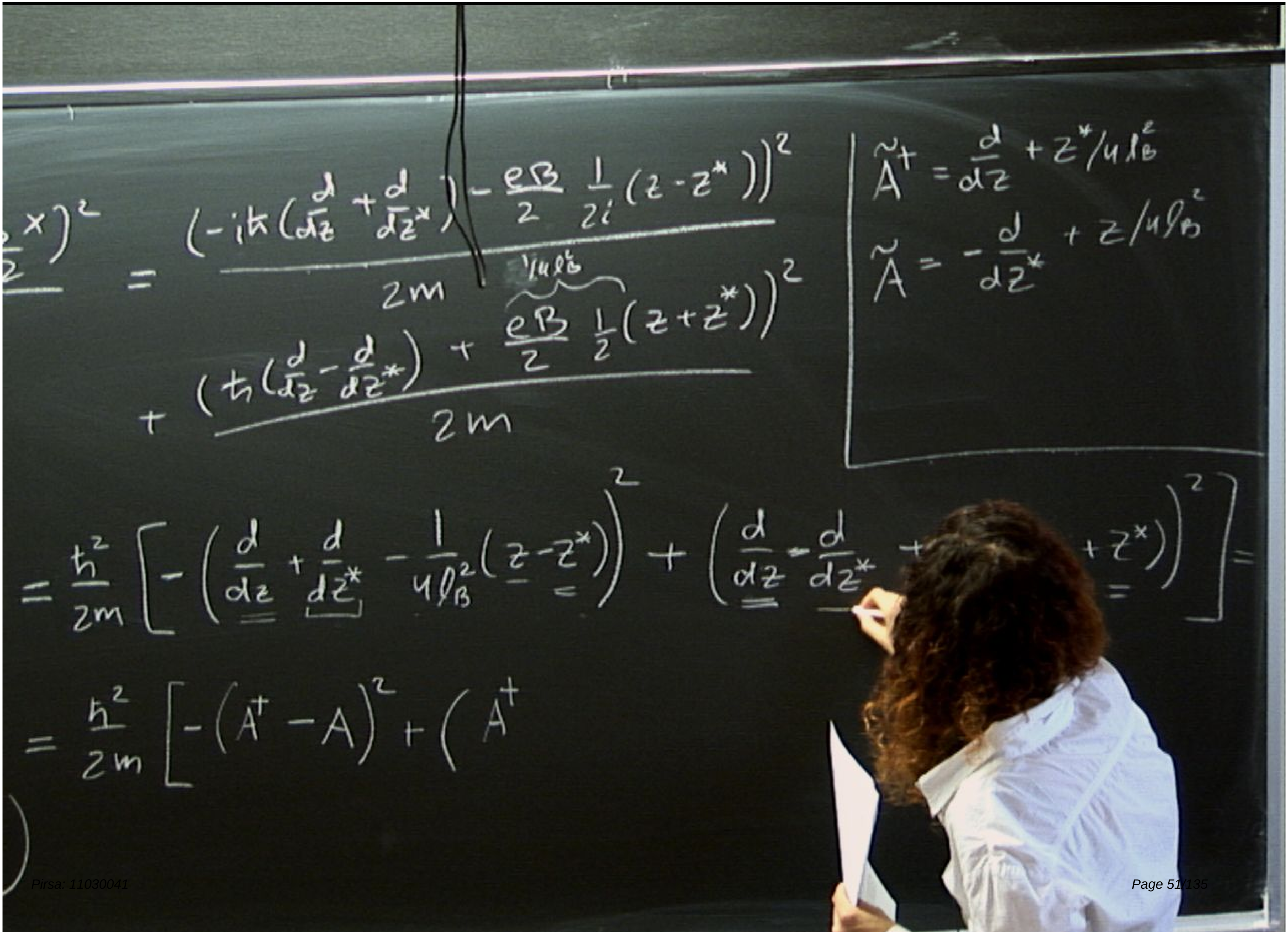
$$\ldots x)^2 = \frac{\left(-i\hbar\left(\frac{d}{dz}+\frac{d}{dz^*}\right) - \frac{eB}{2}\frac{1}{2i}(z-z^*)\right)^2}{2m} + \frac{\left(\hbar\left(\frac{d}{dz}-\frac{d}{dz^*}\right) + \overbrace{\frac{eB}{2}}^{1/4\ell_B^2}\frac{1}{2}(z+z^*)\right)^2}{2m}$$

$$\tilde{A}^\dagger = \frac{d}{dz} + z^*/4\ell_B^2$$

$$\tilde{A} = -\frac{d}{dz^*} + z/4\ell_B^2$$

$$= \frac{\hbar^2}{2m}\left[-\left(\frac{d}{dz}+\frac{d}{dz^*} - \frac{1}{4\ell_B^2}(z-z^*)\right)^2 + \left(\frac{d}{dz}-\frac{d}{dz^*} + \ldots z^*)\right)^2\right] =$$

$$= \frac{\hbar^2}{2m}\left[-(A^\dagger - A)^2 + (\right.$$



= (−iħ(d/dz + d/dz*) − eB/2 · 1/2i (z − z*))² / 2m
+ (ħ(d/dz − d/dz*) + eB/2 · 1/2 (z + z*))² / 2m
Ã⁺ = d/dz + z*/4l_B²
Ã = − d/dz* + z/4l_B²
= ħ²/2m [− (d/dz + d/dz* − 1/4l_B² (z − z*))² + (d/dz − d/dz* + … + z*))²] =
= ħ²/2m [− (A⁺ − A)² + (A⁺

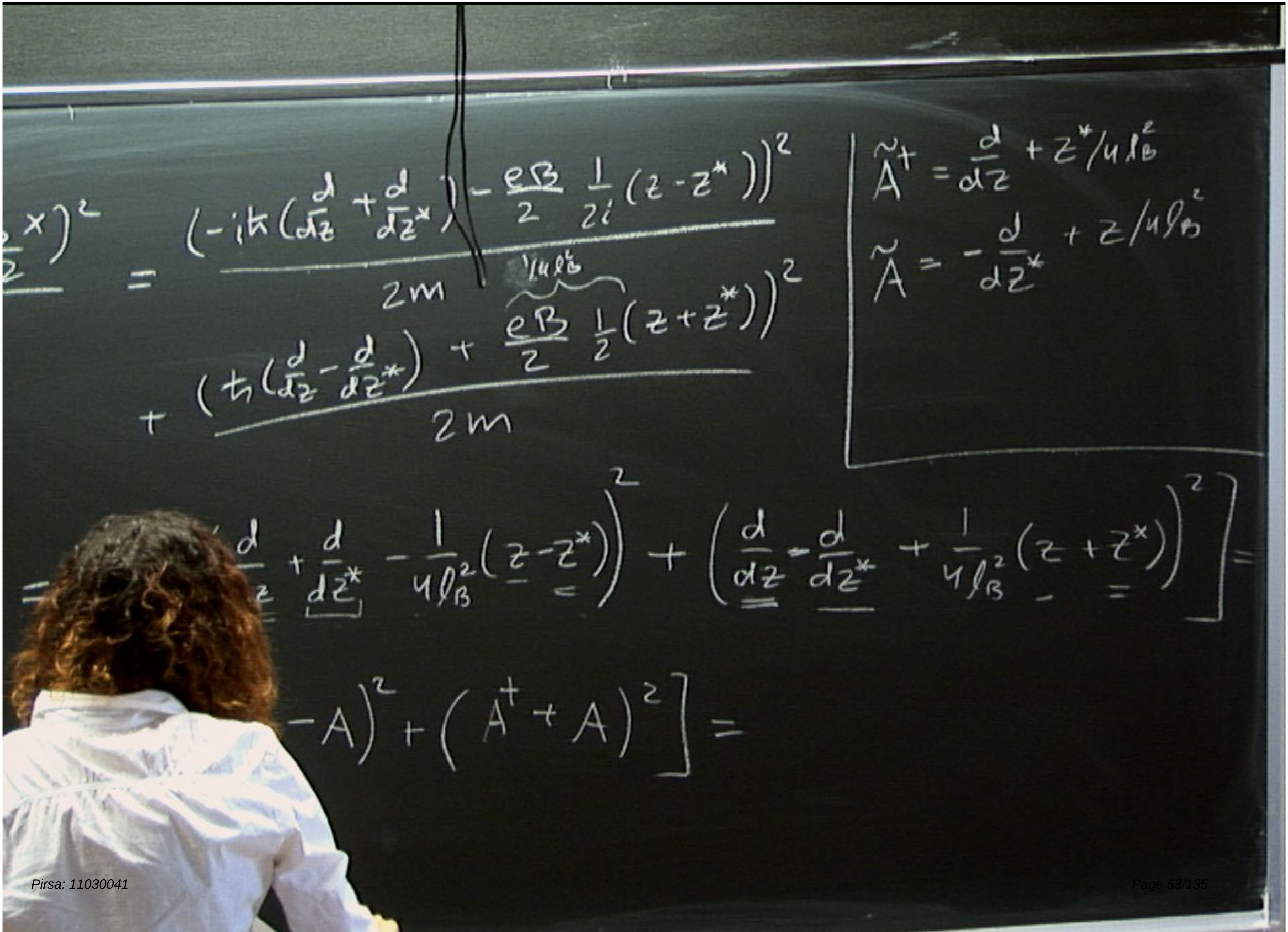
$$\ldots x)^2 = \frac{\left(-i\hbar\left(\frac{d}{dz}+\frac{d}{dz^*}\right)-\frac{eB}{2}\frac{1}{2i}(z-z^*)\right)^2}{2m} + \frac{\left(\hbar\left(\frac{d}{dz}-\frac{d}{dz^*}\right)+\underbrace{\frac{eB}{2}}_{1/4\ell_B^2}\frac{1}{2}(z+z^*)\right)^2}{2m}$$

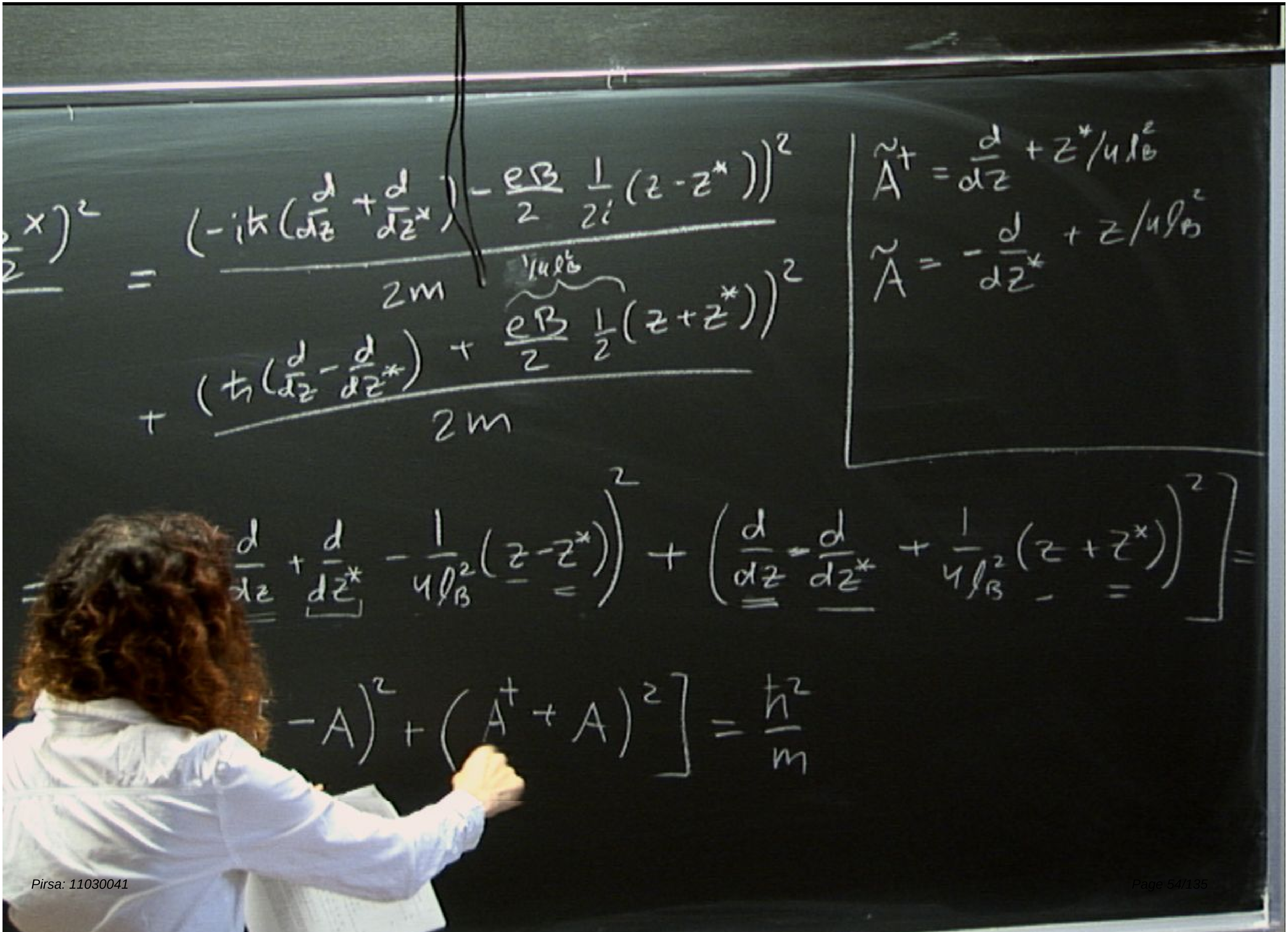
$$\tilde{A}^+ = \frac{d}{dz} + z^*/4\ell_B^2$$

$$\tilde{A} = -\frac{d}{dz^*} + z/4\ell_B^2$$

$$\frac{\hbar^2}{\ldots}\left[-\left(\frac{d}{dz}+\frac{d}{dz^*}-\frac{1}{4\ell_B^2}(z-z^*)\right)^2+\left(\frac{d}{dz}-\frac{d}{dz^*}+\frac{1}{4\ell_B^2}(z+z^*)\right)^2\right]=$$

$$\left[-(A^+-A)^2+(A^++A)^2\right]$$


$\tilde{A}^{+} = \frac{d}{dz} + z^{*}/4\ell_B^2$
$\tilde{A} = -\frac{d}{dz^{*}} + z/4\ell_B^2$
$= \frac{\left(-i\hbar\left(\frac{d}{dz} + \frac{d}{dz^{*}}\right) - \frac{eB}{2}\frac{1}{2i}(z - z^{*})\right)^2}{2m}$
$1/4\ell_B^2$
$+ \frac{\left(\hbar\left(\frac{d}{dz} - \frac{d}{dz^{*}}\right) + \frac{eB}{2}\frac{1}{2}(z + z^{*})\right)^2}{2m}$
$\frac{d}{dz^{*}} - \frac{1}{4\ell_B^2}(z - z^{*})\Big)^2 + \left(\frac{d}{dz} - \frac{d}{dz^{*}} + \frac{1}{4\ell_B^2}(z + z^{*})\right)^2\Big] =$
$- A)^2 + (A^{+} + A)^2\Big] =$



$= \frac{\left(-i\hbar\left(\frac{d}{dz}+\frac{d}{dz^*}\right) - \frac{eB}{2}\frac{1}{2i}(z-z^*)\right)^2}{2m}$
$+ \frac{\left(\hbar\left(\frac{d}{dz}-\frac{d}{dz^*}\right) + \frac{eB}{2}\frac{1}{2}(z+z^*)\right)^2}{2m}$
$\tilde{A}^+ = \frac{d}{dz} + z^*/4l_B^2$
$\tilde{A} = -\frac{d}{dz^*} + z/4l_B^2$

$$\ldots x)^2 \;=\; \frac{\left(-i\hbar\left(\frac{d}{dz}+\frac{d}{dz^*}\right) - \frac{eB}{2}\,\frac{1}{2i}(z-z^*)\right)^2}{2m} + \frac{\left(\hbar\left(\frac{d}{dz}-\frac{d}{dz^*}\right) + \overbrace{\frac{eB}{2}\,\frac{1}{2}}^{1/4l_B^2}(z+z^*)\right)^2}{2m}$$

$$\tilde{A}^+ = \frac{d}{dz} + z^*/4l_B^2$$

$$\tilde{A} = -\frac{d}{dz^*} + z/4l_B^2$$

$$= \frac{\hbar^2}{2m}\left[-\left(\frac{d}{dz}+\frac{d}{dz^*} - \frac{1}{4l_B^2}(z-z^*)\right)^2 + \left(\frac{d}{dz}-\frac{d}{dz^*} + \frac{1}{4l_B}\ldots\right)^2\right] =$$

$$= \frac{\hbar^2}{2m}\left[-(A^+ - A)^2 + (A^+ + A)^2\right] = \frac{\hbar^2}{m}\left[A^+A + A^+\ldots\right.$$

$$\ldots x)^2}{\ldots z} = \frac{\left(-i\hbar\left(\frac{d}{dz}+\frac{d}{dz^*}\right)-\frac{eB}{2}\frac{1}{2i}(z-z^*)\right)^2}{2m}$$

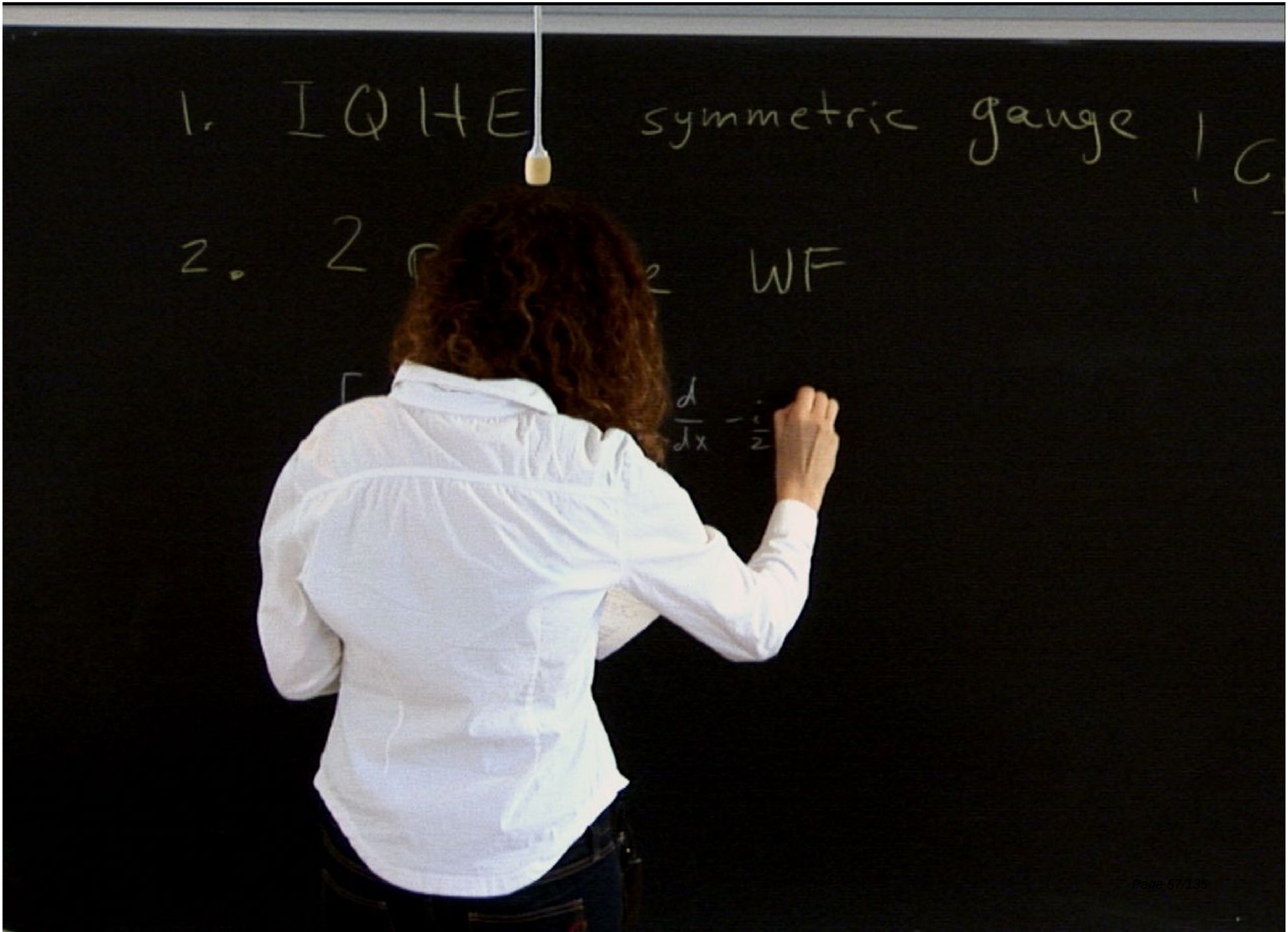
$$+\frac{\left(\hbar\left(\frac{d}{dz}-\frac{d}{dz^*}\right)+\underbrace{\frac{eB}{2}}_{1/4\ell_B^2}\frac{1}{2}(z+z^*)\right)^2}{2m}$$

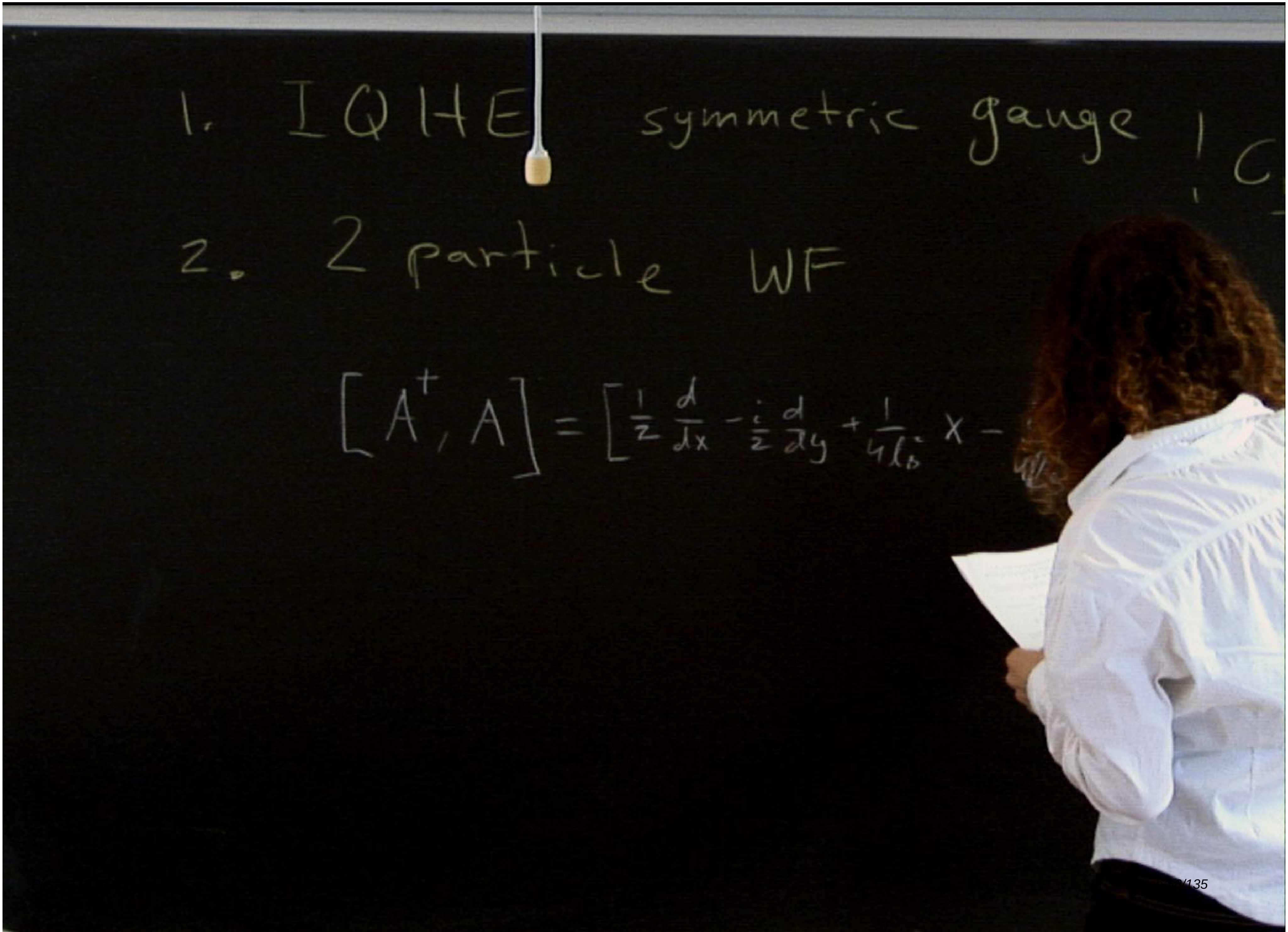
$$\tilde{A}^+ = \frac{d}{dz} + z^*/4\ell_B^2$$

$$\tilde{A} = -\frac{d}{dz^*} + z/4\ell_B^2$$

$$= \frac{\hbar^2}{2m}\left[-\left(\frac{d}{dz}+\frac{d}{dz^*}-\frac{1}{4\ell_B^2}(z-z^*)\right)^2+\left(\frac{d}{dz}-\frac{d}{dz^*}+\frac{1}{4\ell_B^2}(z+z^*)\right)^2\right] =$$

$$= \frac{\hbar^2}{2m}\left[-(A^+-A)^2+(A^++A)^2\right] = \frac{\hbar^2}{m}\left[\tilde{A}^+\tilde{A}+\tilde{A}^+\tilde{A}\right]$$


1. IQHE symmetric gauge
2. 2
WF

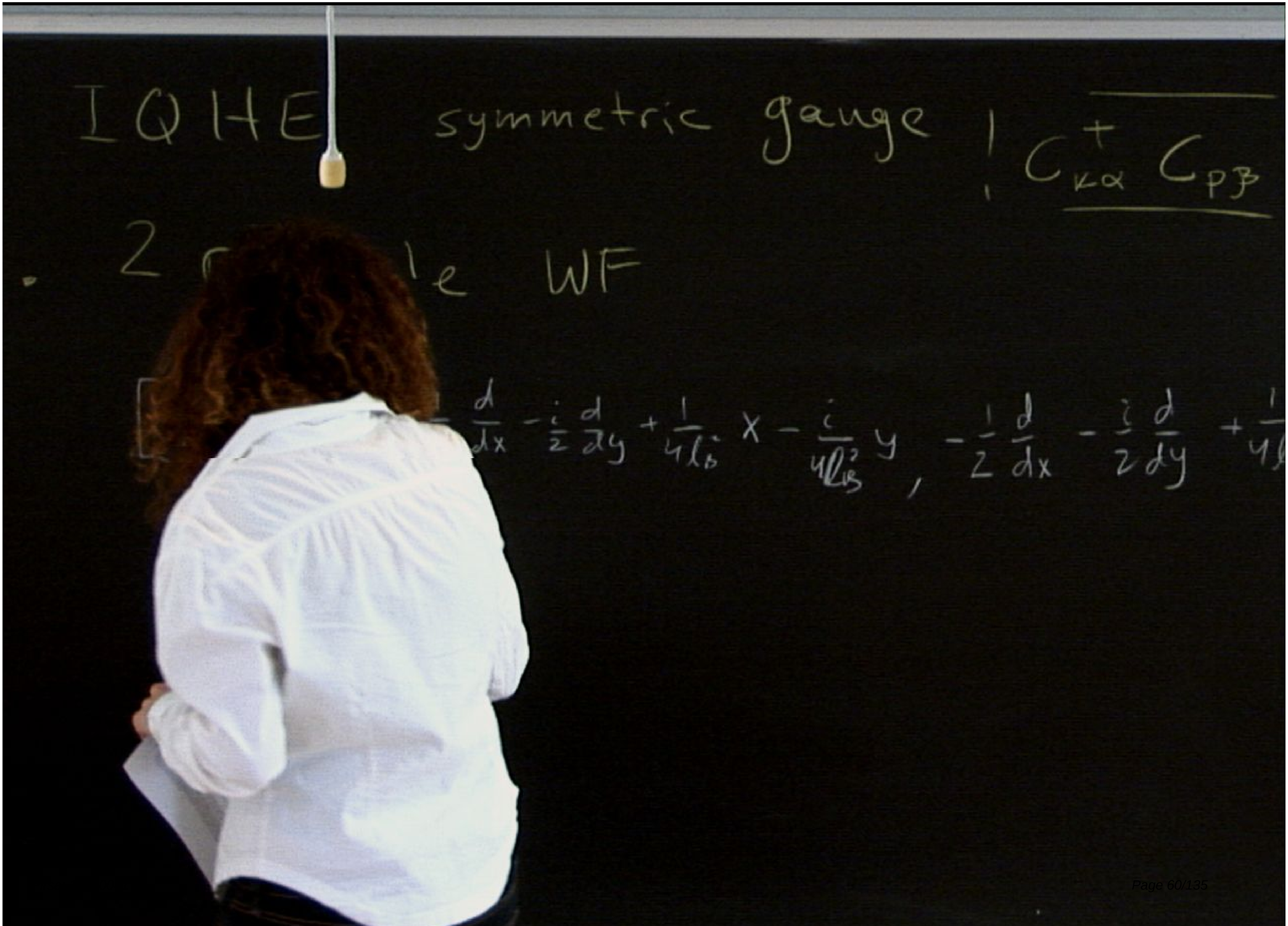

1. IQHE symmetric gauge
2. 2 particle WF
$[A^{+}, A] = [\frac{1}{2}\frac{d}{dx} - \frac{i}{2}\frac{d}{dy} + \frac{1}{4l_b^2}x -$

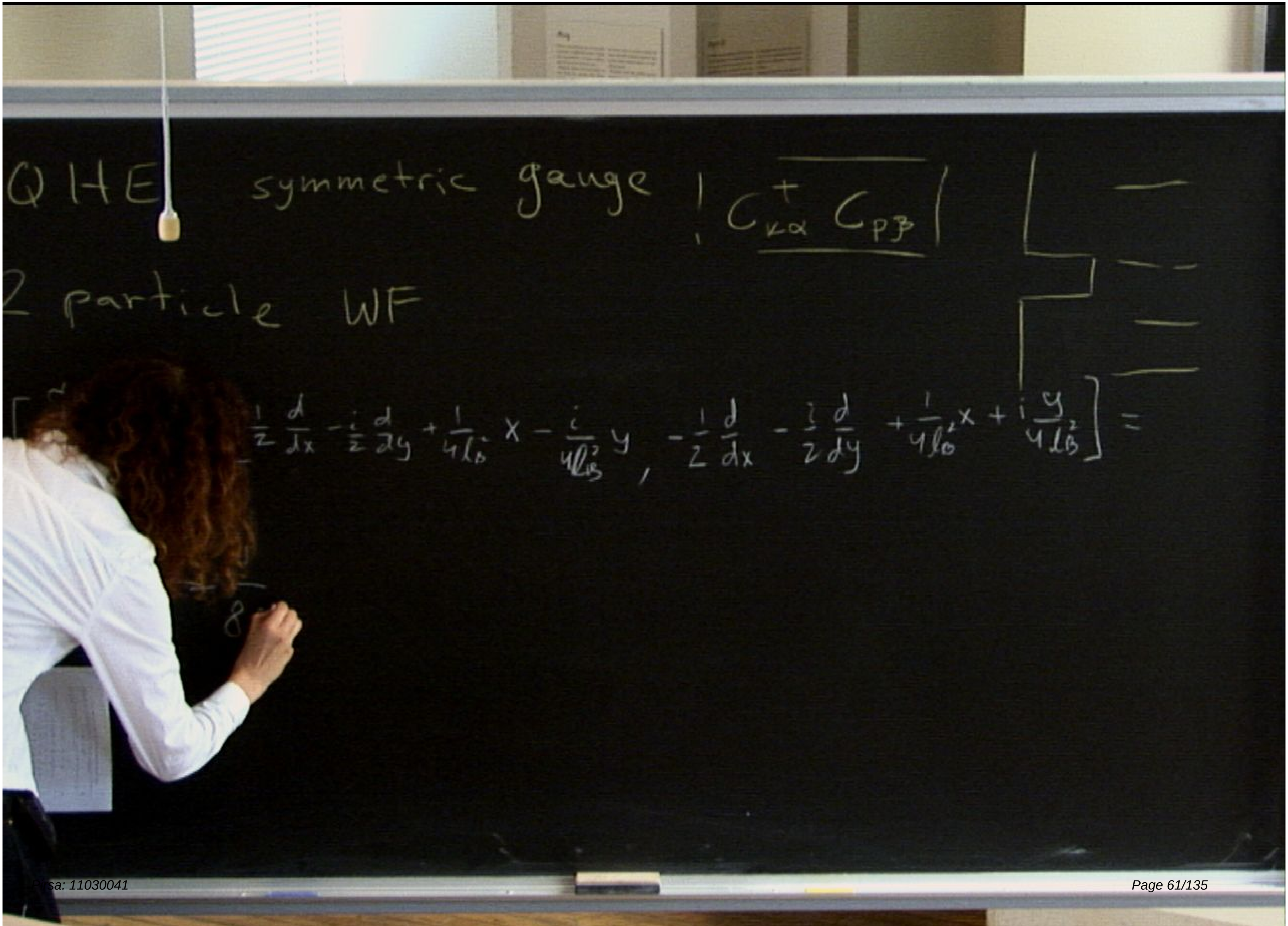
IQHE symmetric gauge

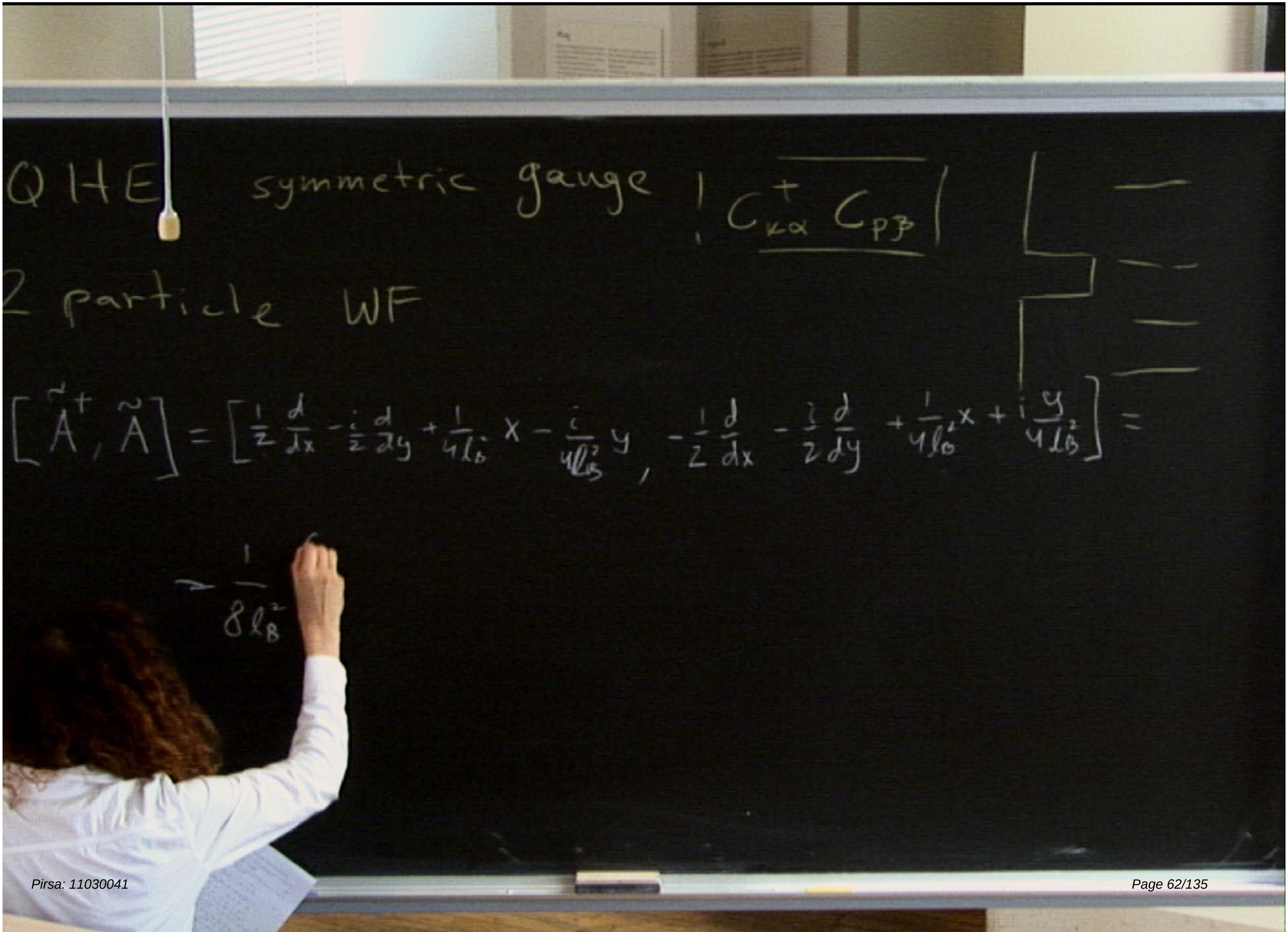
$c^{\dagger}_{k\alpha} c_{p\beta}$

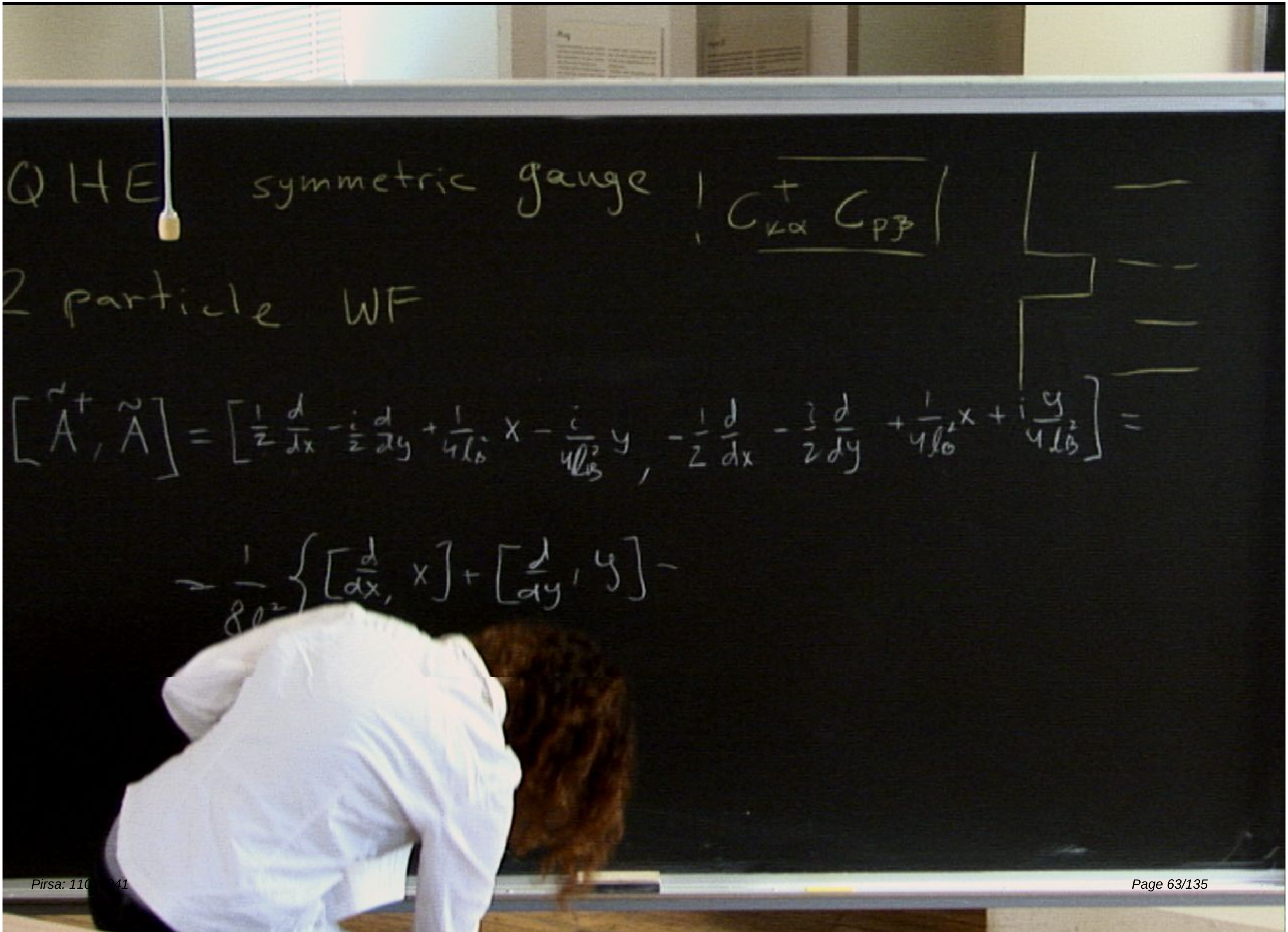
- 2 particle WF

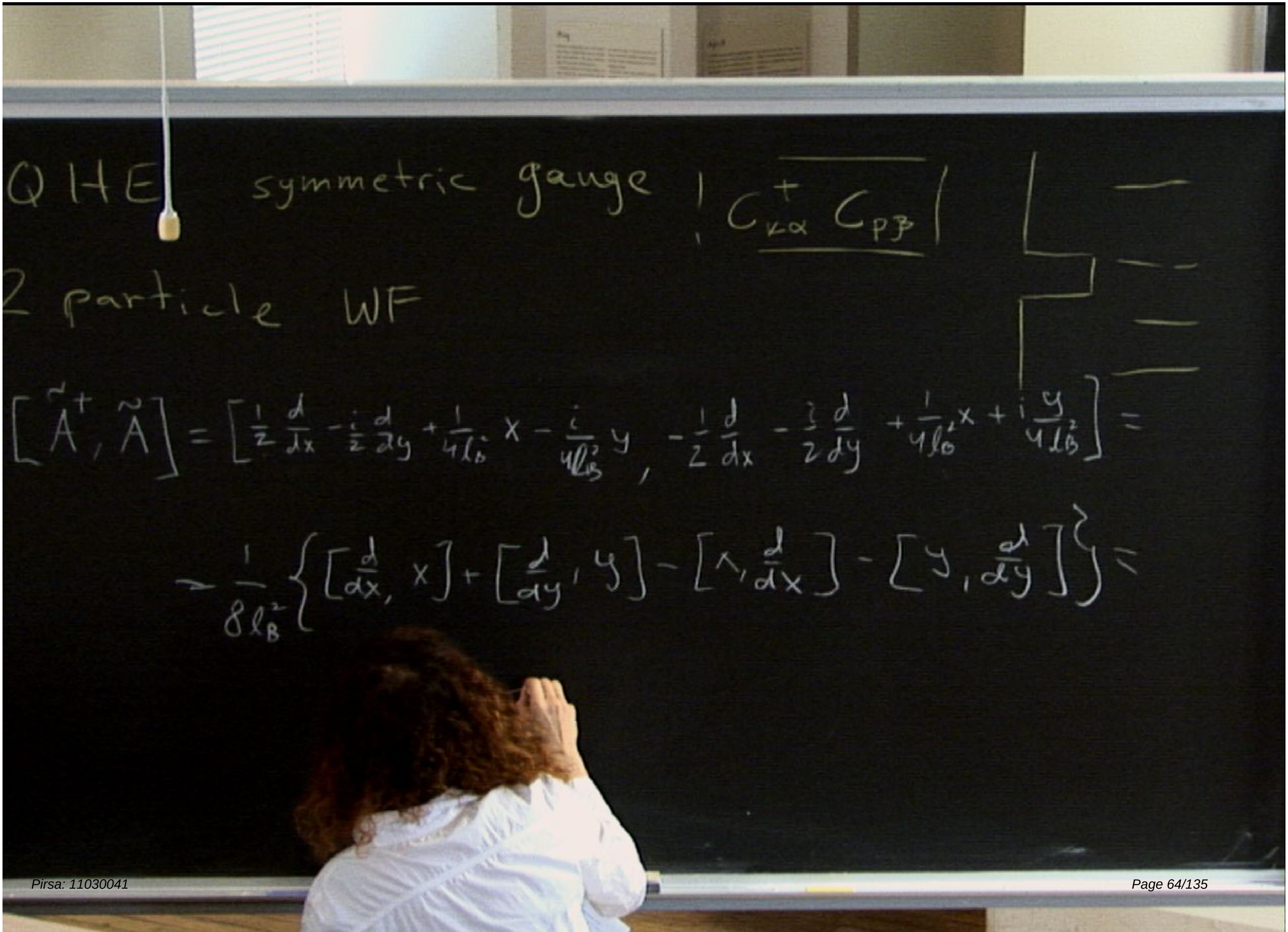
$$[A^{\dagger}, A] = \left[\frac{1}{2}\frac{d}{dx} - \frac{i}{2}\frac{d}{dy} + \frac{1}{4l_B^2}x - \frac{i}{4l_B^2}y\ ,\ -\frac{1}{2}\frac{d}{dx}\right.$$


IQHE
symmetric gauge
$C^{\dagger}_{k\alpha} C_{p\beta}$
WF
$\frac{d}{dx} - \frac{i}{2}\frac{d}{dy} + \frac{1}{4l_B^2} x - \frac{i}{4l_B^2} y , -\frac{1}{2}\frac{d}{dx} - \frac{i}{2}\frac{d}{dy} +$


QHE symmetric gauge
2 particle WF


QHE symmetric gauge
2 particle WF


QHE symmetric gauge
2 particle WF


QHE symmetric gauge
$C^{+}_{k\alpha} C_{p\beta}$
2 particle WF
$[\tilde{A}^{+}, \tilde{A}] = [\frac{1}{2}\frac{d}{dx} - \frac{i}{2}\frac{d}{dy} + \frac{1}{4l_B^2}x - \frac{i}{4l_B^2}y, -\frac{1}{2}\frac{d}{dx} - \frac{i}{2}\frac{d}{dy} + \frac{1}{4l_B^2}x + i\frac{y}{4l_B^2}] =$
$= \frac{1}{8l_B^2}\{[\frac{d}{dx}, x] + [\frac{d}{dy}, y] - [x, \frac{d}{dx}] - [y, \frac{d}{dy}]\} =$

QHE symmetric gauge $\overline{c^{\dagger}_{k\alpha}\, c_{p\beta}}$

2 particle WF

$$[\tilde{A}^{\dagger}, \tilde{A}] = \left[\frac{1}{2}\frac{d}{dx} - \frac{i}{2}\frac{d}{dy} + \frac{1}{4l_B^2}x - \frac{i}{4l_B^2}y, -\frac{1}{2}\frac{d}{dx} - \frac{i}{2}\frac{d}{dy} + \frac{1}{4l_B^2}x + i\frac{y}{4l_B^2}\right] =$$

$$= \frac{1}{8l_B^2}\left\{\left[\frac{d}{dx}, x\right] + \left[\frac{d}{dy}, y\right] - \left[x, \frac{d}{dx}\right] - \left[y, \frac{d}{dy}\right]\right\} =$$

$$= \frac{1}{2l_B^2}$$

$$\ldots x)^2 \,/\, 2 \ldots = \frac{\left(-i\hbar\left(\frac{d}{dz}+\frac{d}{dz^*}\right) - \frac{eB}{2}\frac{1}{2i}(z-z^*)\right)^2}{2m} + \frac{\left(\hbar\left(\frac{d}{dz}-\frac{d}{dz^*}\right) + \overbrace{\frac{eB}{2}}^{1/4l_B^2}\frac{1}{2}(z+z^*)\right)^2}{2m}$$

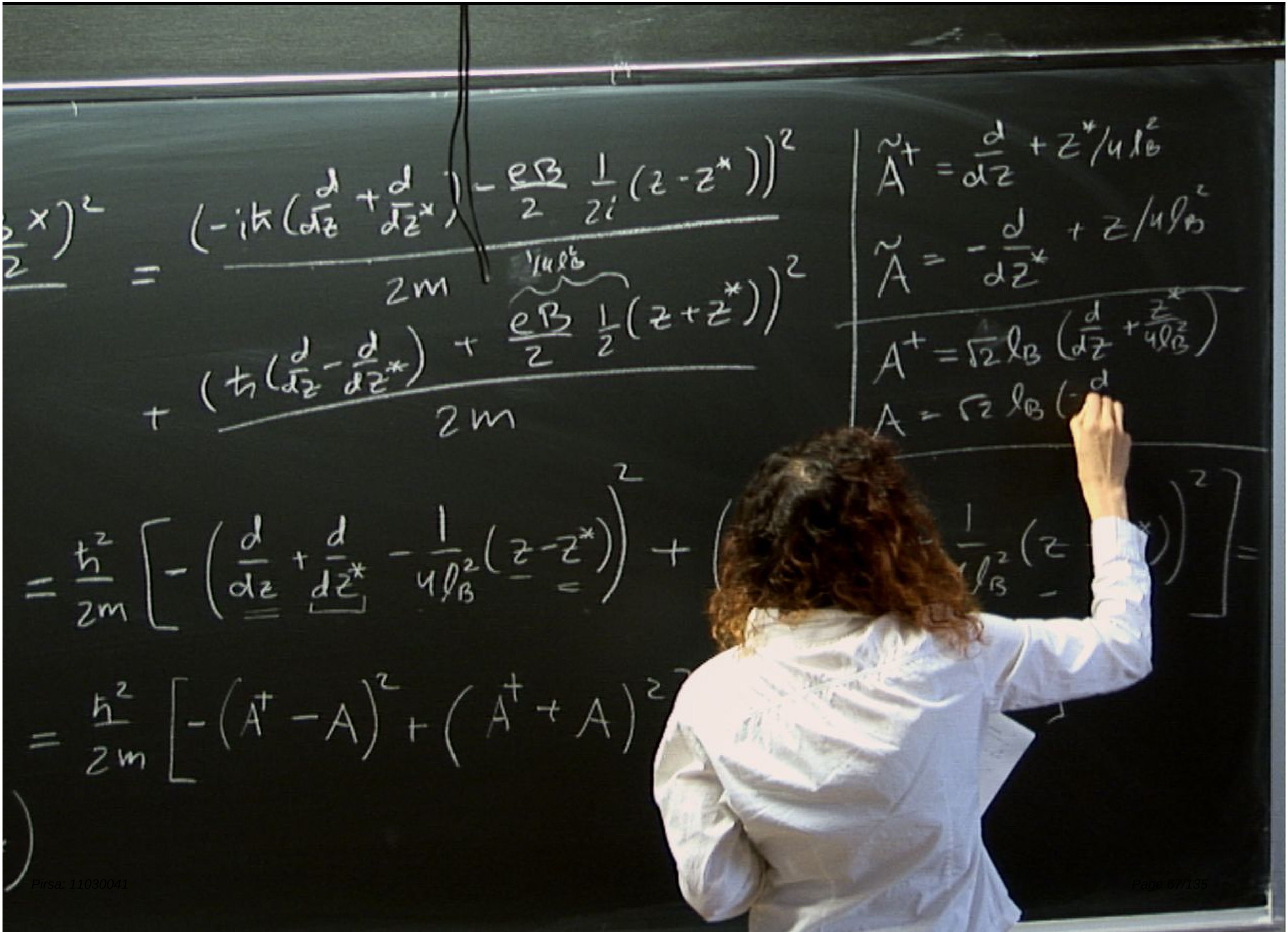
$$= \frac{\hbar^2}{2m}\left[-\left(\frac{d}{dz}+\frac{d}{dz^*} - \frac{1}{4l_B^2}(z-z^*)\right)^2 + \left(\frac{d}{dz} - \ldots (z+z^*)\right)^2 \ldots\right]$$

$$= \frac{\hbar^2}{2m}\left[-\left(A^\dagger - A\right)^2 + \left(A^\dagger + A\right)^2\right] = \frac{\hbar \ldots}{\ldots}$$

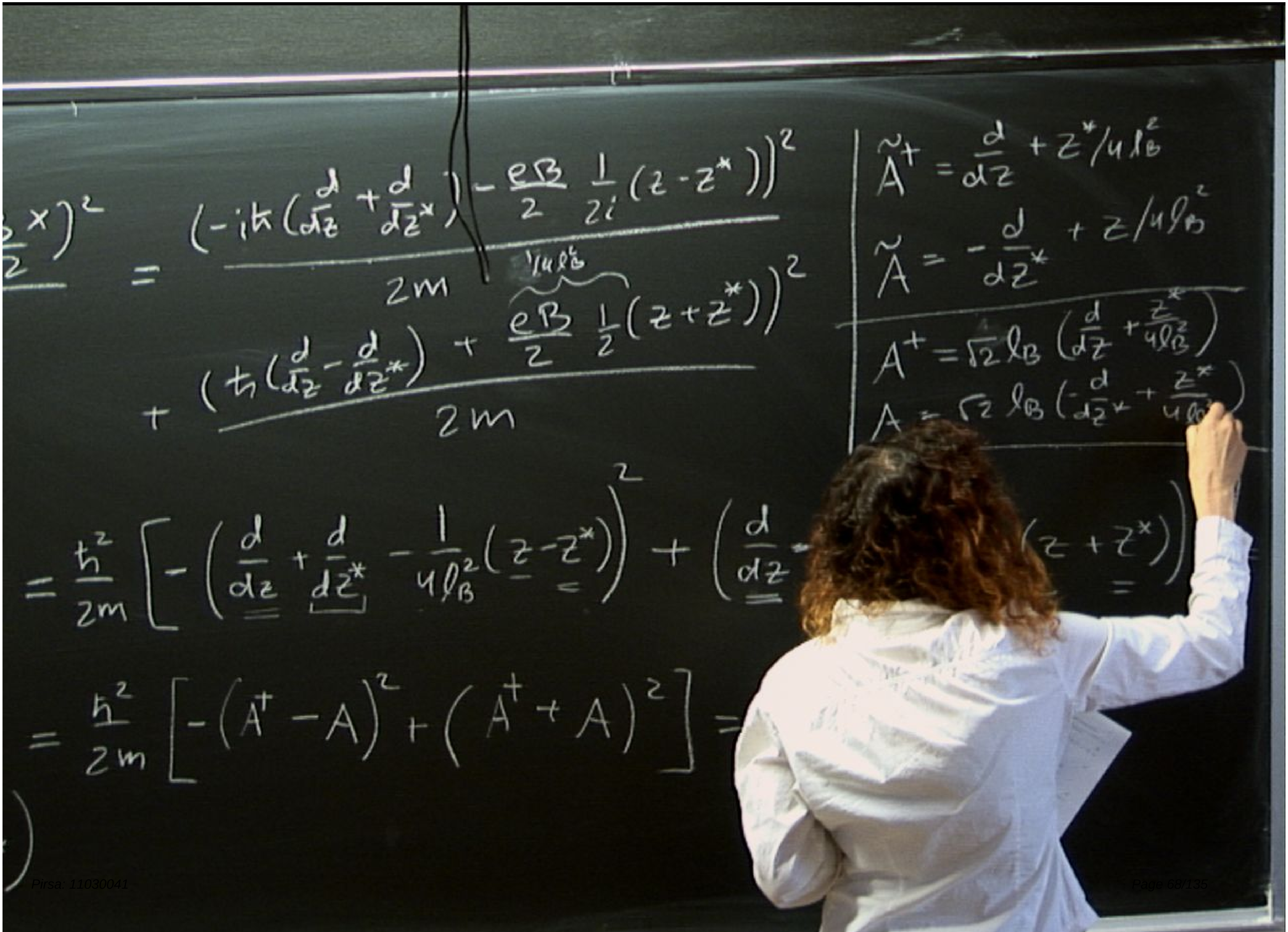
$$\tilde{A}^\dagger = \frac{d}{dz} + z^*/4l_B^2$$

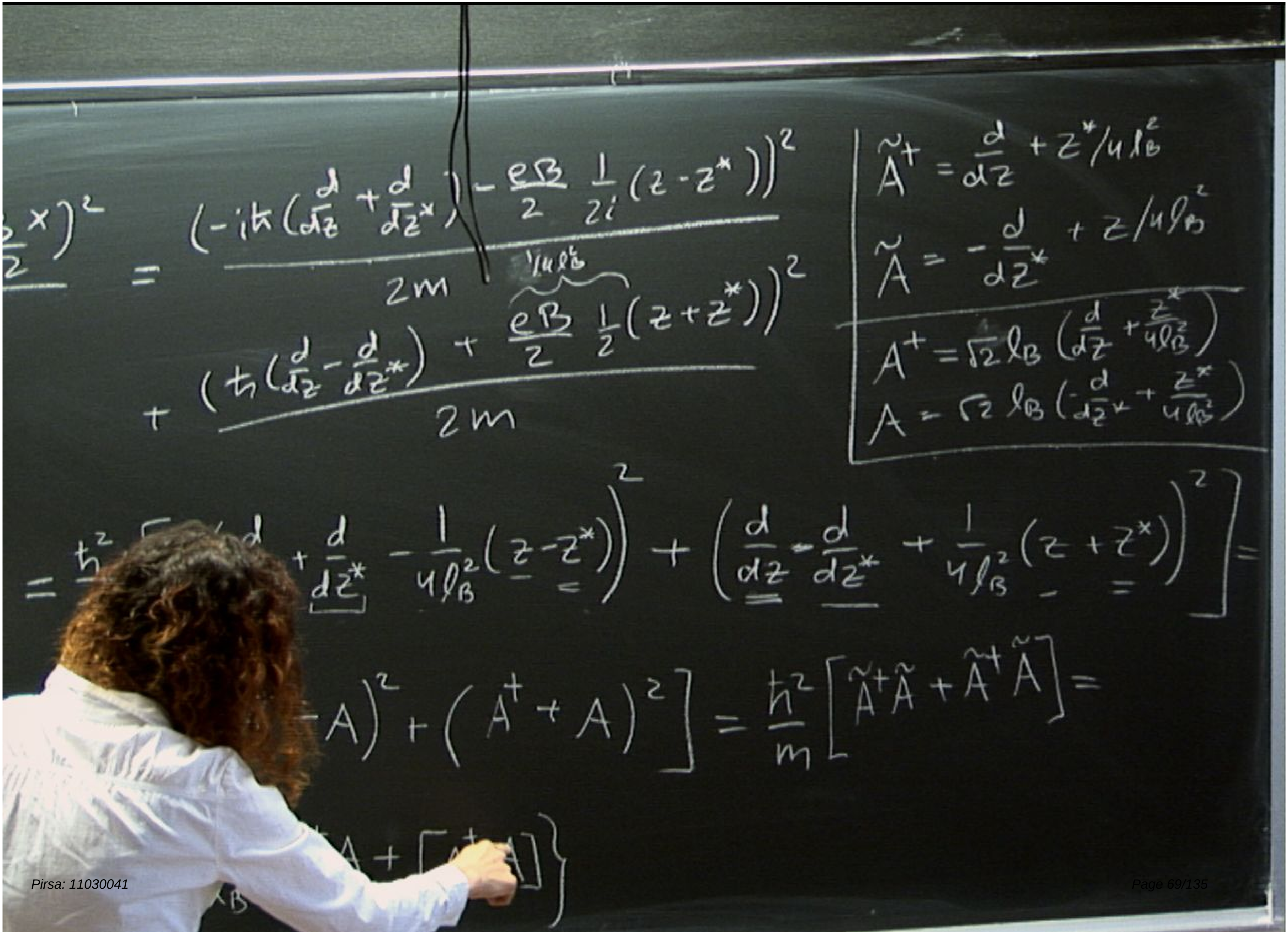
$$\tilde{A} = -\frac{d}{dz^*} + z/4l_B^2$$

$$A^\dagger = \sqrt{2}\,l_B\left(\frac{d}{dz} + \frac{z^*}{4\ldots}\right.$$

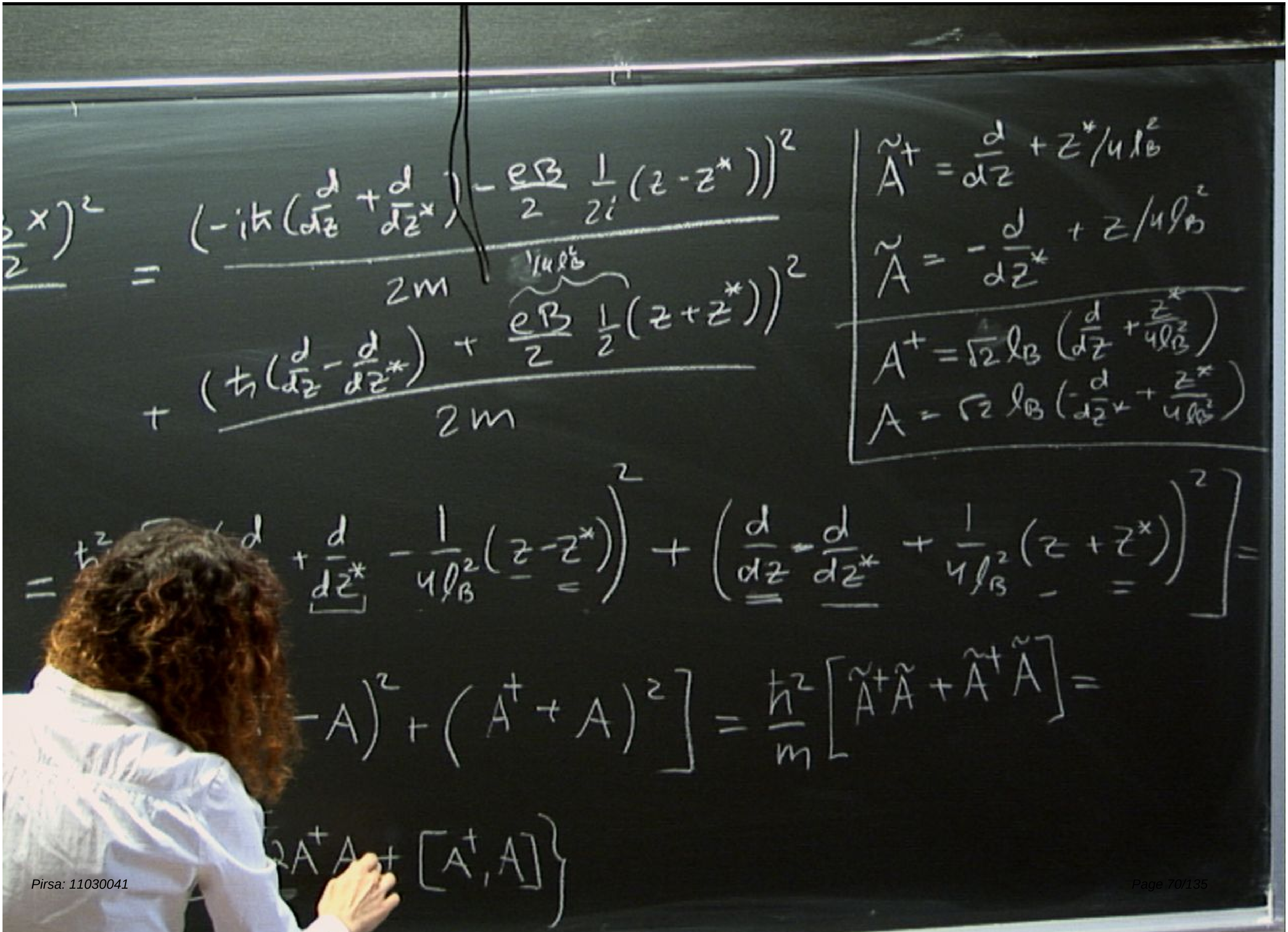


$\tilde{A}^+ = \frac{d}{dz} + z^*/4l_B^2$
$\tilde{A} = -\frac{d}{dz^*} + z/4l_B^2$
$A^+ = \sqrt{2}\, l_B \left(\frac{d}{dz} + \frac{z^*}{4l_B^2}\right)$
$A = \sqrt{2}\, l_B \left(-\frac{d}{\ }\right.$
$= \frac{\left(-i\hbar\left(\frac{d}{dz} + \frac{d}{dz^*}\right) - \frac{eB}{2}\frac{1}{2i}(z - z^*)\right)^2}{2m} + \frac{\left(\hbar\left(\frac{d}{dz} - \frac{d}{dz^*}\right) + \frac{eB}{2}\frac{1}{2}(z + z^*)\right)^2}{2m}$
$= \frac{\hbar^2}{2m}\left[-\left(\frac{d}{dz} + \frac{d}{dz^*} - \frac{1}{4l_B^2}(z - z^*)\right)^2 + \right.$
$= \frac{\hbar^2}{2m}\left[-(A^+ - A)^2 + (A^+ + A)^2\right.$


= (-iħ(d/dz + d/dz*) - eB/2 · 1/2i (z - z*))² / 2m
+ (ħ(d/dz - d/dz*) + eB/2 · 1/2 (z + z*))² / 2m
1/4l_B²
= ħ²/2m [-(d/dz + d/dz* - 1/4l_B² (z - z*))² + (d/dz ... (z + z*))²
= ħ²/2m [-(A† - A)² + (A† + A)²] =
Ã† = d/dz + z*/4l_B²
Ã = -d/dz* + z/4l_B²
A† = √2 l_B (d/dz + z*/4l_B²)
A = √2 l_B (-d/dz* + z*/4l_B²)



Ã⁺ = d/dz + z*/4l_B²
Ã = − d/dz* + z/4l_B²
A⁺ = √2 l_B (d/dz + z*/4l_B²)
A = √2 l_B (− d/dz* + z/4l_B²)
2m
2m
= h²/m [Ã⁺Ã + Ã⁺Ã] =


= (−iħ(d/dz + d/dz*) − eB/2 · 1/2i (z − z*))² / 2m
+ (ħ(d/dz − d/dz*) + eB/2 · 1/2 (z + z*))² / 2m
1/4ℓ_B²
Ã⁺ = d/dz + z*/4ℓ_B²
Ã = −d/dz* + z/4ℓ_B²
A⁺ = √2 ℓ_B (d/dz + z*/4ℓ_B²)
A = √2 ℓ_B (−d/dz* + z*/4ℓ_B²)
= ħ² [(d/dz + d/dz* − 1/4ℓ_B² (z − z*))² + (d/dz − d/dz* + 1/4ℓ_B² (z + z*))²] =
−A)² + (A⁺ + A)²] = ħ²/m [Ã⁺Ã + Ã⁺Ã] =
2A⁺A + [A⁺, A]}

$$\ldots x)^2 = \frac{\left(-i\hbar\left(\frac{d}{dz}+\frac{d}{dz^*}\right)-\frac{eB}{2}\frac{1}{2i}(z-z^*)\right)^2}{2m} + \frac{\left(\hbar\left(\frac{d}{dz}-\frac{d}{dz^*}\right)+\overbrace{\frac{eB}{2}}^{1/4\ell_B^2}\frac{1}{2}(z+z^*)\right)^2}{2m}$$

$$\tilde{A}^+ = \frac{d}{dz} + z^*/4\ell_B^2$$

$$\tilde{A} = -\frac{d}{dz^*} + z/4\ell_B^2$$

$$A^+ = \sqrt{2}\,\ell_B\left(\frac{d}{dz}+\frac{z^*}{4\ell_B^2}\right)$$

$$A = \sqrt{2}\,\ell_B\left(-\frac{d}{dz^*}+\frac{z^*}{4\ell_B^2}\right)$$

$$= \frac{\hbar^2}{2m}\left[-\left(\frac{d}{dz}+\frac{d}{dz^*}-\frac{1}{4\ell_B^2}(z-z^*)\right)^2+\left(\frac{d}{dz}-\frac{d}{dz^*}+\frac{1}{4\ell_B^2}(z+z^*)\right)^2\right]=$$

$$= \frac{\hbar^2}{2m}\left[-(A^+-A)^2+(A^++A)^2\right] = \frac{\hbar^2}{m}\left[\tilde{A}^+\tilde{A}+\tilde{A}^+\tilde{A}\right]=$$

$$= \frac{\hbar^2}{m}\frac{1}{2\ell_B^2}\left\{2A^+A + [A^+, A]\right\}$$

$$\hbar\left(\frac{d}{dz}+\frac{d}{dz^*}\right)-\frac{eB}{2}\frac{1}{2i}(z-z^*)\Big)^2 \Big/ 2m$$

$$\left(\frac{d}{dz}-\frac{d}{dz^*}\right)+\overbrace{\frac{eB}{2}}^{1/4\ell_B^2}\frac{1}{2}(z+z^*)\Big)^2 \Big/ 2m$$

$$\tilde{A}^+ = \frac{d}{dz} + z^*/4\ell_B^2$$

$$\tilde{A} = -\frac{d}{dz^*} + z/4\ell_B^2$$

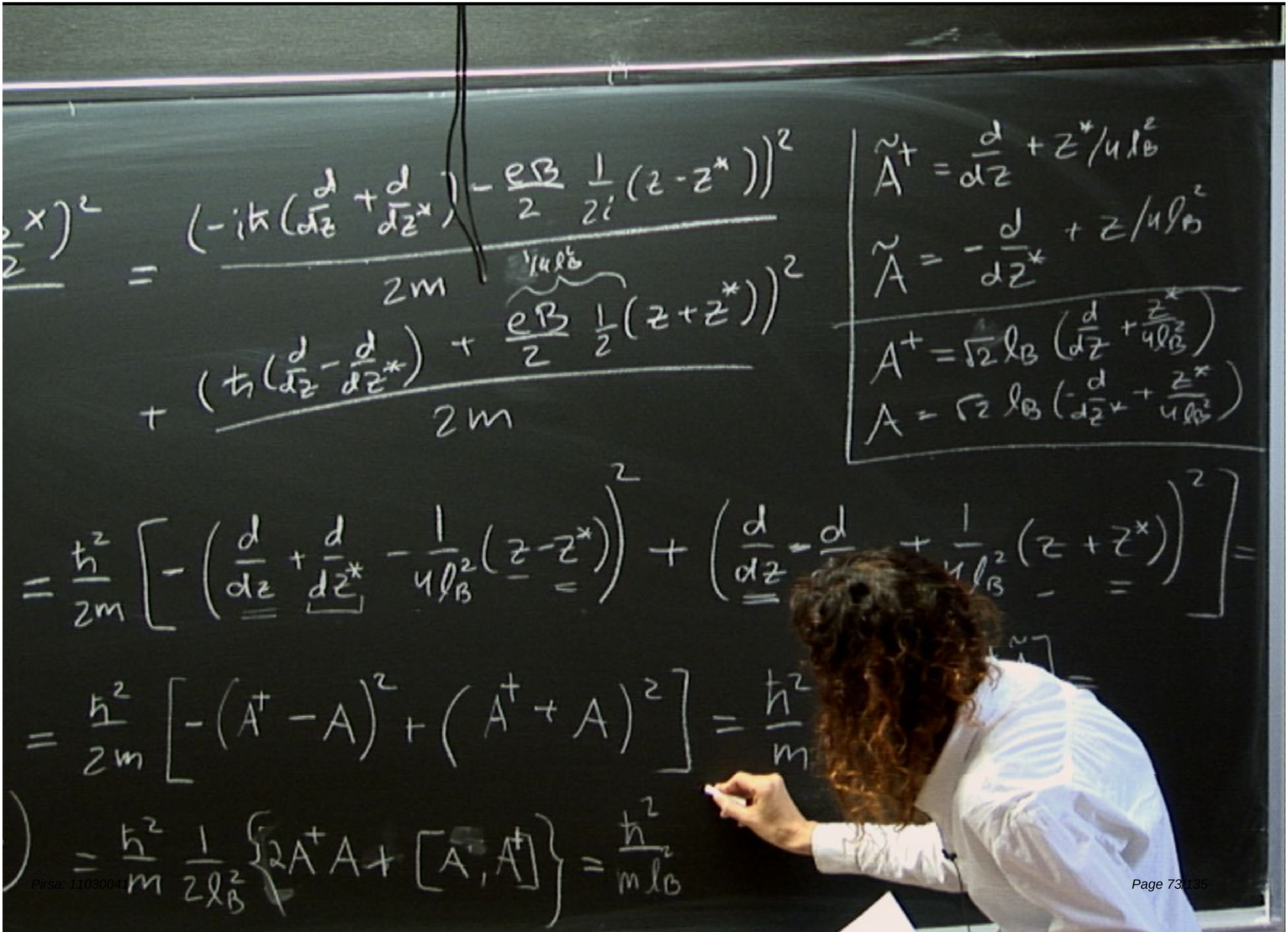
$$A^+ = \sqrt{2}\,\ell_B\left(\frac{d}{dz} + \frac{z^*}{4\ell_B^2}\right)$$

$$A = \sqrt{2}\,\ell_B\left(-\frac{d}{dz^*} + \frac{z^*}{4\ell_B^2}\right)$$

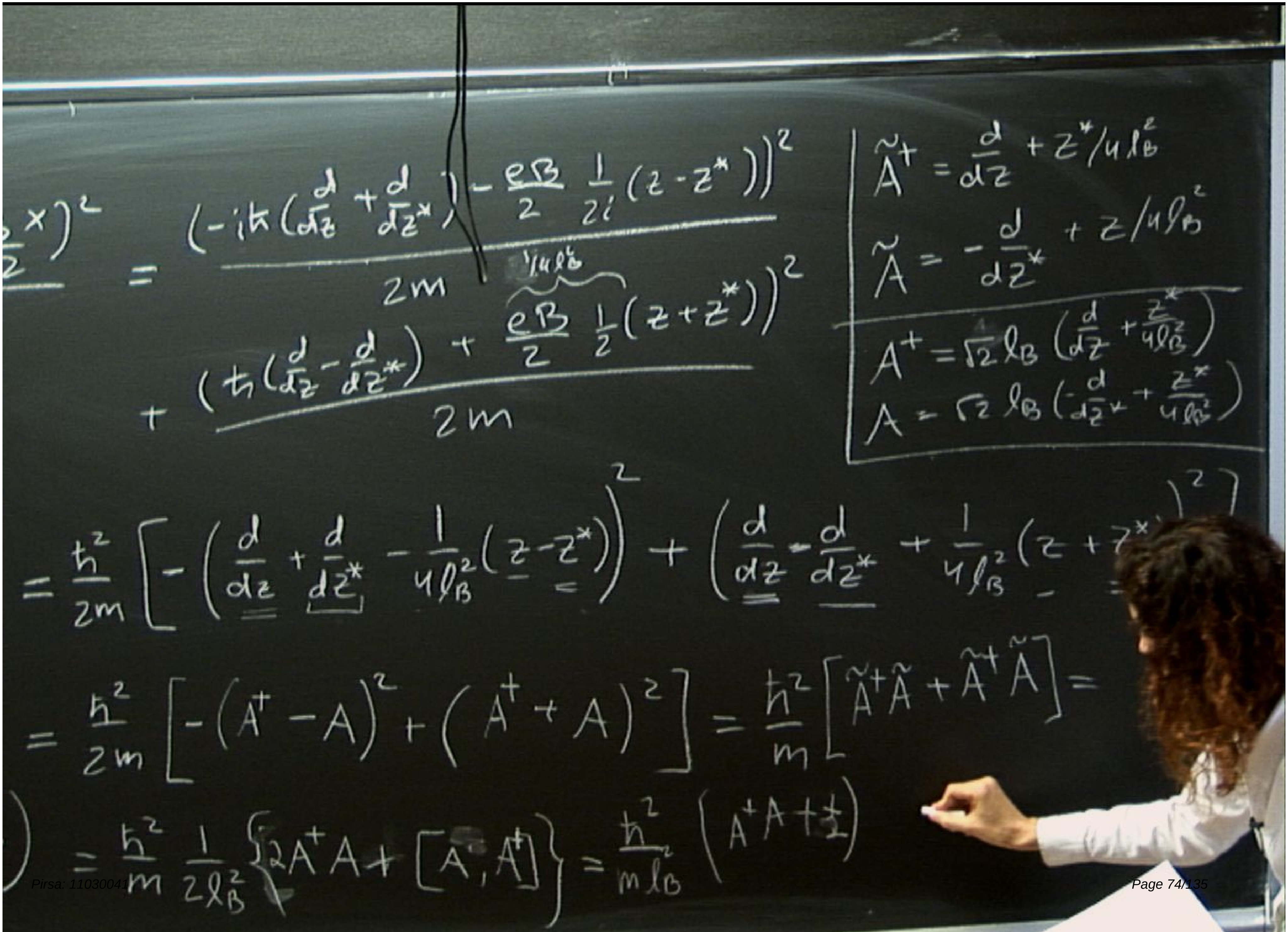
$$\frac{d}{dz^*} - \frac{1}{4\ell_B^2}(z-z^*)\Big)^2 + \left(\frac{d}{dz} - \frac{d}{dz^*} + \frac{1}{4\ell_B^2}(z+z^*)\right)^2\Big] =$$

$$A)^2 + (A^+ + A)^2\Big] = \frac{\hbar^2}{m}\left[\tilde{A}^+\tilde{A} + \tilde{A}^+\tilde{A}\right] =$$

$$A \; [A, A^+]\}$$



$\tilde{A}^{+} = \frac{d}{dz} + z^{*}/4 l_B^2$
$\tilde{A} = -\frac{d}{dz^{*}} + z/4 l_B^2$
$A^{+} = \sqrt{2}\, l_B \left(\frac{d}{dz} + \frac{z^{*}}{4 l_B^2}\right)$
$A = \sqrt{2}\, l_B \left(-\frac{d}{dz^{*}} + \frac{z^{*}}{4 l_B^2}\right)$
$= \frac{\left(-i\hbar\left(\frac{d}{dz} + \frac{d}{dz^{*}}\right) - \frac{eB}{2}\frac{1}{2i}(z - z^{*})\right)^2}{2m} + \frac{\left(\hbar\left(\frac{d}{dz} - \frac{d}{dz^{*}}\right) + \frac{eB}{2}\frac{1}{2}(z + z^{*})\right)^2}{2m}$
$= \frac{\hbar^2}{2m}\left[-\left(\frac{d}{dz} + \frac{d}{dz^{*}} - \frac{1}{4 l_B^2}(z - z^{*})\right)^2 + \left(\frac{d}{dz} - \frac{d}{dz^{*}} + \frac{1}{4 l_B^2}(z + z^{*})\right)^2\right] =$
$= \frac{\hbar^2}{2m}\left[-(A^{+} - A)^2 + (A^{+} + A)^2\right] = \frac{\hbar^2}{m}$
$= \frac{\hbar^2}{m}\frac{1}{2 l_B^2}\left\{2A^{+}A + [A, A^{+}]\right\} = \frac{\hbar^2}{m l_B^2}$



Ã⁺ = d/dz + z*/4l_B²
Ã = −d/dz* + z/4l_B²
A⁺ = √2 l_B (d/dz + z*/4l_B²)
A = √2 l_B (−d/dz* + z/4l_B²)
= (−iħ(d/dz + d/dz*) − eB/2 · 1/2i (z − z*))² / 2m
+ (ħ(d/dz − d/dz*) + eB/2 · 1/2 (z + z*))² / 2m
= ħ²/2m [−(d/dz + d/dz* − 1/4l_B² (z − z*))² + (d/dz − d/dz* + 1/4l_B² (z + z*))²]
= ħ²/2m [−(A⁺ − A)² + (A⁺ + A)²] = ħ²/m [Ã⁺Ã + Ã⁺Ã] =
= ħ²/m · 1/2l_B² {2A⁺A + [A, A⁺]} = ħ²/ml_B² (A⁺A + 1/2)

$$\ldots z^*)^2 = \frac{\left(-i\hbar\left(\frac{d}{dz}+\frac{d}{dz^*}\right) - \frac{eB}{2}\frac{1}{2i}(z-z^*)\right)^2}{2m} + \frac{\left(\hbar\left(\frac{d}{dz}-\frac{d}{dz^*}\right) + \overbrace{\frac{eB}{2}}^{1/4l_B^2}\frac{1}{2}(z+z^*)\right)^2}{2m}$$

$$\tilde{A}^\dagger = \frac{d}{dz} + z^*/4l_B^2$$

$$\tilde{A} = -\frac{d}{dz^*} + z/4l_B^2$$

$$A^\dagger = \sqrt{2}\, l_B\left(\frac{d}{dz} + \frac{z^*}{4l_B^2}\right)$$

$$A = \sqrt{2}\, l_B\left(-\frac{d}{dz^*} + \frac{z^*}{4l_B^2}\right)$$

$$= \frac{\hbar^2}{2m}\left[-\left(\frac{d}{dz}+\frac{d}{dz^*}-\frac{1}{4l_B^2}(z-z^*)\right)^2 + \left(\frac{d}{dz}-\frac{d}{dz^*}+\frac{1}{4l_B^2}(z+z^*)\right)^2\right] =$$

$$= \frac{\hbar^2}{2m}\left[-(A^\dagger - A)^2 + (A^\dagger + A)^2\right] = \frac{\hbar^2}{m}\left[\tilde{A}^\dagger\tilde{A} + \tilde{A}^\dagger\tilde{A}\right]$$

$$= \frac{\hbar^2}{m}\frac{1}{2l_B^2}\left\{2A^\dagger A + [A, A^\dagger]\right\} = \frac{\hbar^2}{m l_B^2}\left(A^\dagger A + \frac{1}{2}\right) = \hbar\omega_c\left(A^\dagger \ldots\right.$$

$$\ldots x)^2 = \frac{\left(-i\hbar\left(\frac{d}{dz}+\frac{d}{dz^*}\right) - \frac{eB}{2}\frac{1}{2i}(z-z^*)\right)^2}{2m} + \frac{\left(\hbar\left(\frac{d}{dz}-\frac{d}{dz^*}\right) + \frac{eB}{2}\frac{1}{2}(z+z^*)\right)^2}{2m}$$

$$\tilde{A}^+ = \frac{d}{dz} + z^*/4l_B^2$$

$$\tilde{A} = -\frac{d}{dz^*} + z/4l_B^2$$

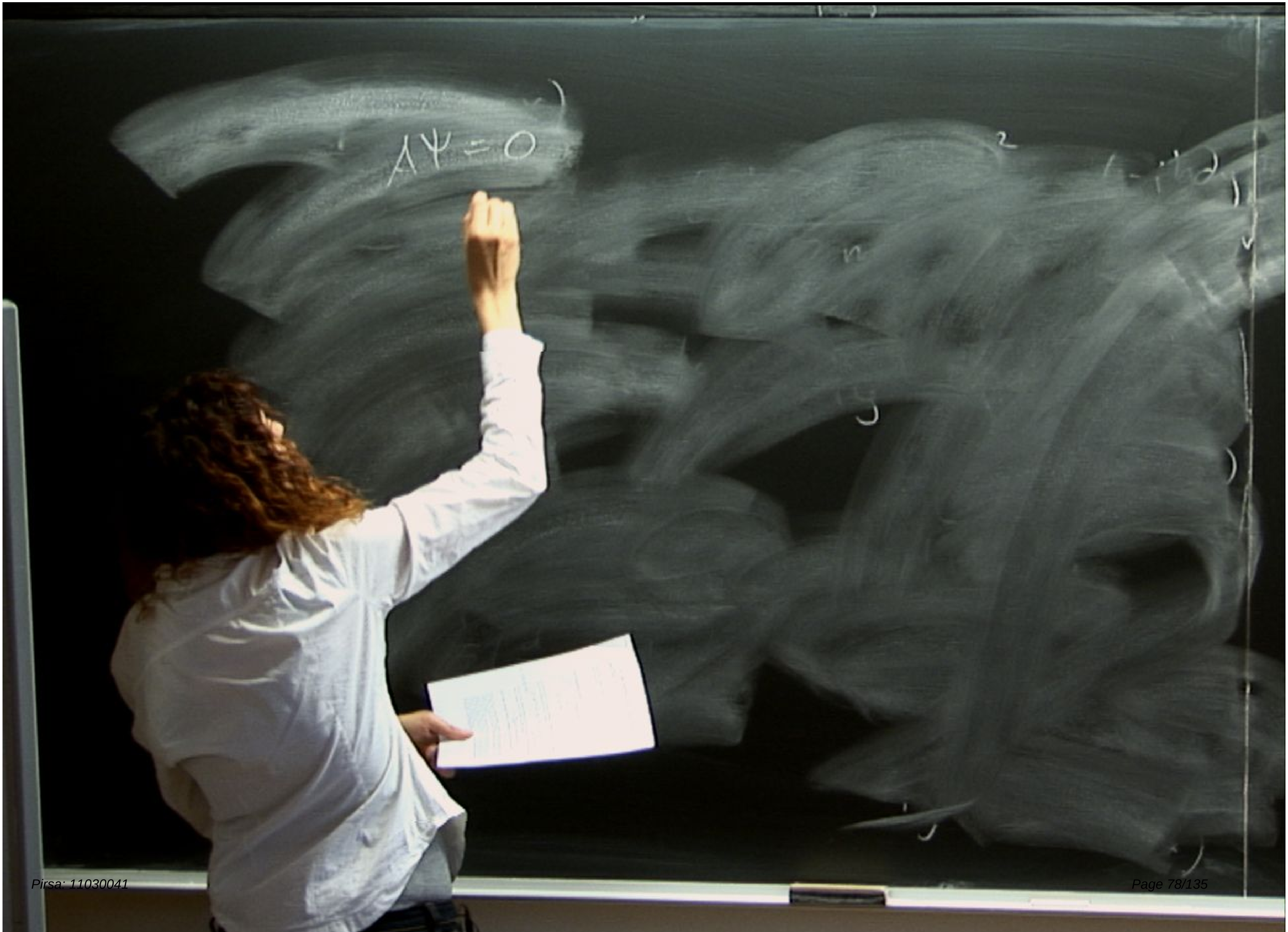
$$A^+ = \sqrt{2}\,l_B\left(\frac{d}{dz} + \frac{z^*}{4l_B^2}\right)$$

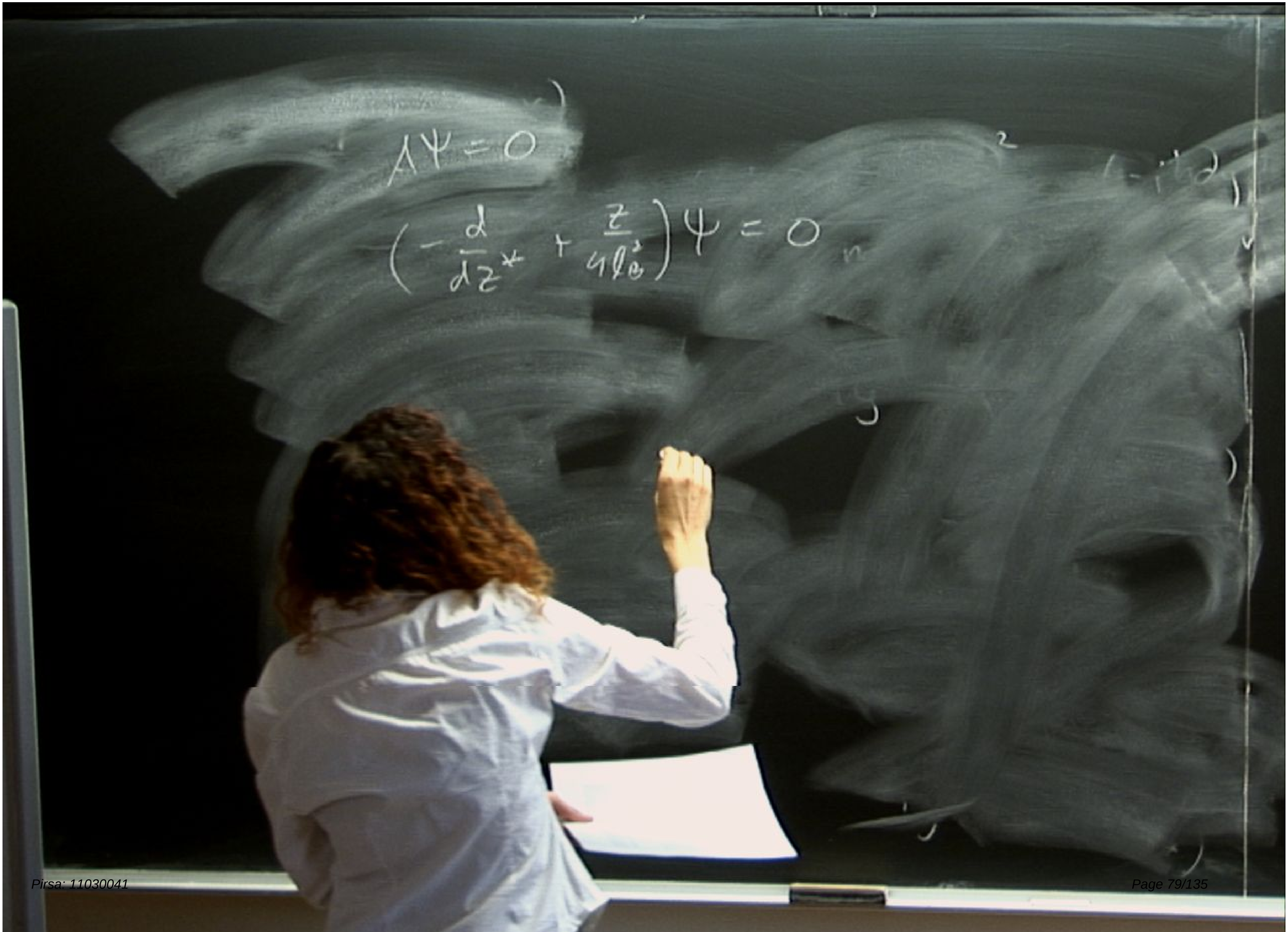
$$A = \sqrt{2}\,l_B\left(-\frac{d}{dz^*} + \frac{z^*}{4l_B^2}\right)$$

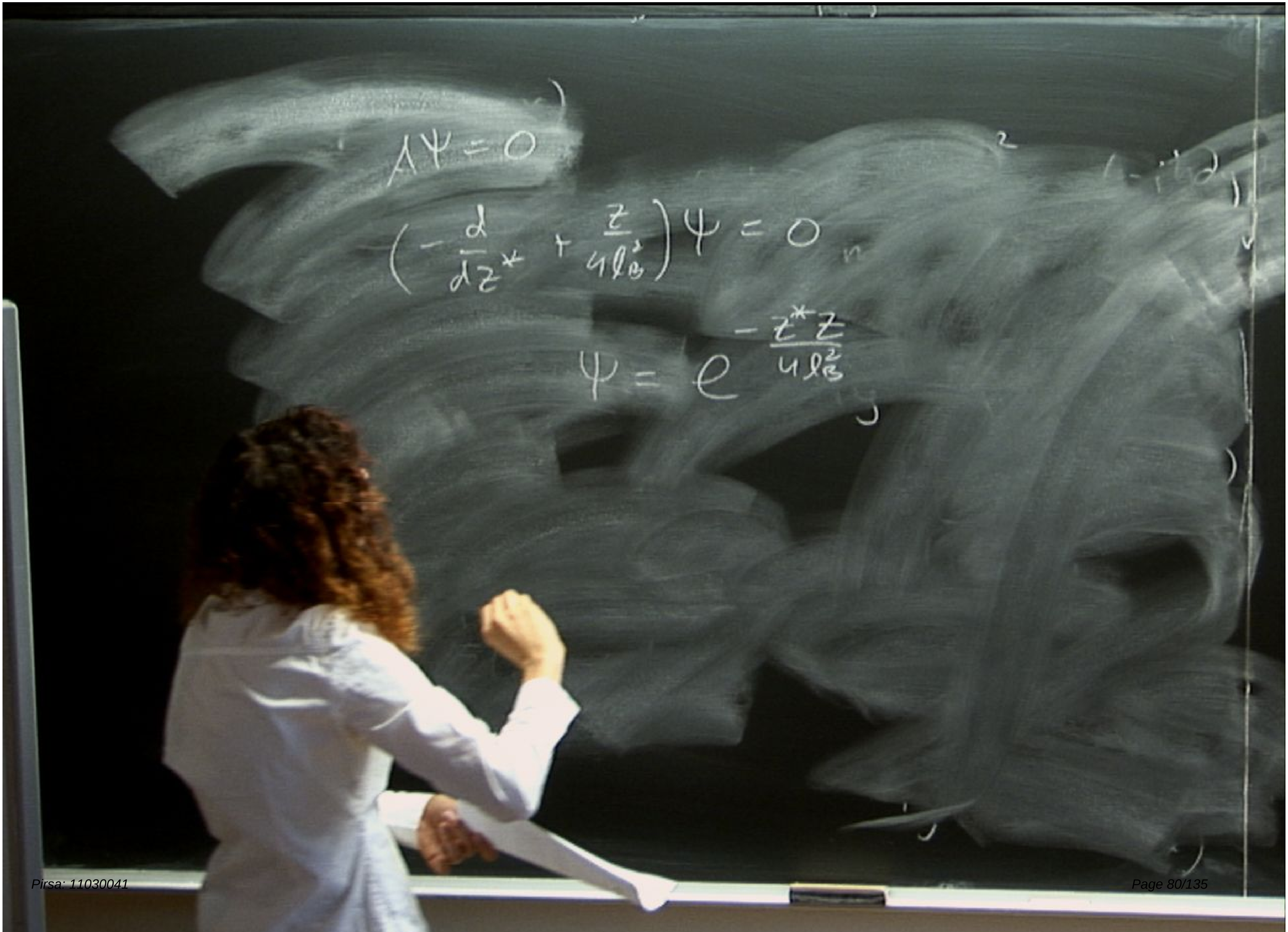
$$= \frac{\hbar^2}{2m}\left[-\left(\frac{d}{dz}+\frac{d}{dz^*} - \frac{1}{4l_B^2}(z-z^*)\right)^2 + \left(\frac{d}{dz}-\frac{d}{dz^*} + \frac{1}{4l_B^2}(z+z^*)\right)^2\right] =$$

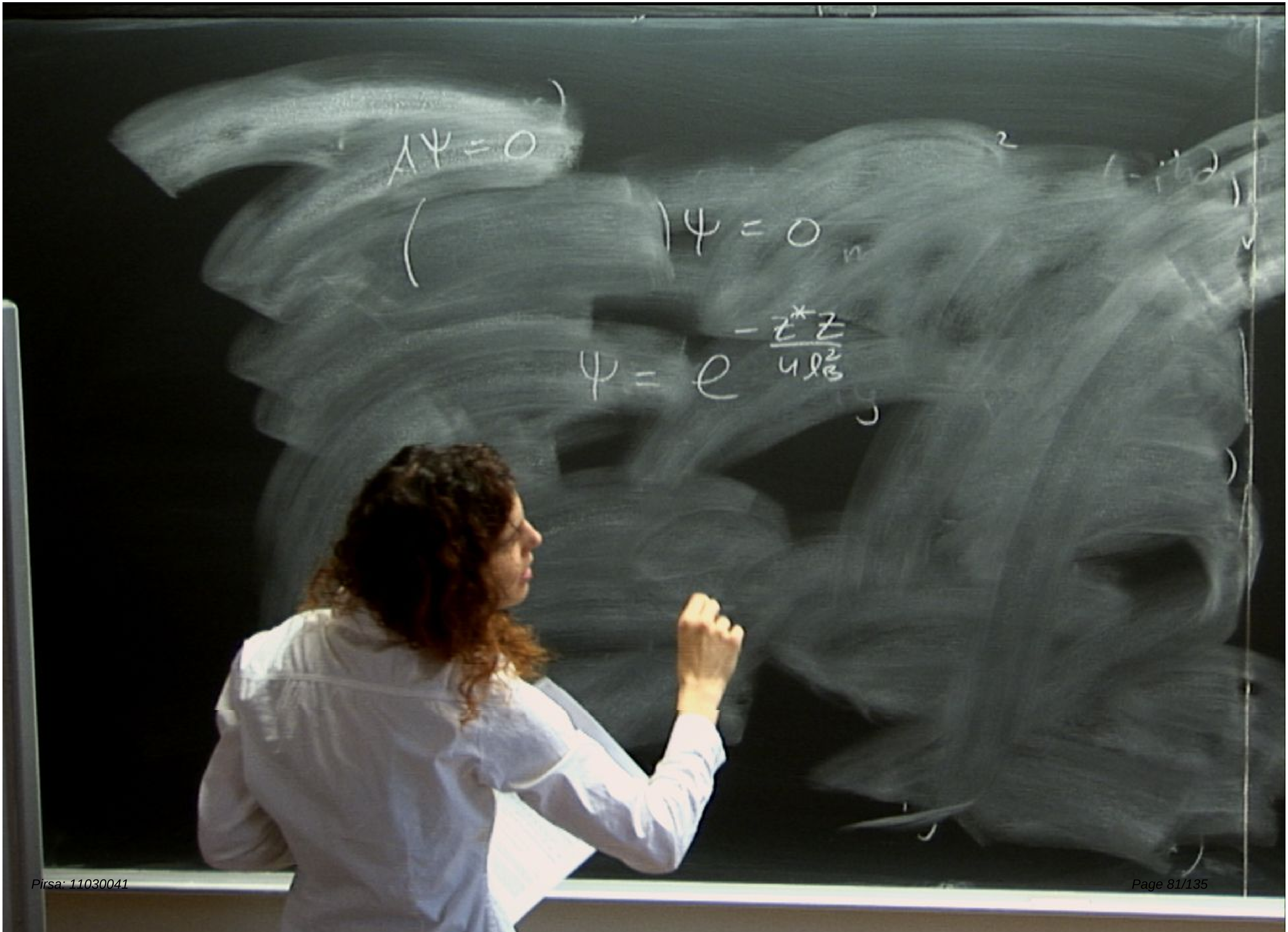
$$= \frac{\hbar^2}{2m}\left[-(A^+ - A)^2 + (A^+ + A)^2\right] = \frac{\hbar^2}{m}\left[\tilde{A}^+\tilde{A} + \tilde{A}^+\tilde{A}\right] =$$

$$= \frac{\hbar^2}{m}\frac{1}{2l_B^2}\left\{2A^+A + [A, A^+]\right\} = \frac{\hbar^2}{m l_B^2}\left(A^+A + \frac{1}{2}\right) = \hbar\omega_c\left(A^+A + \frac{1}{2}\right)$$

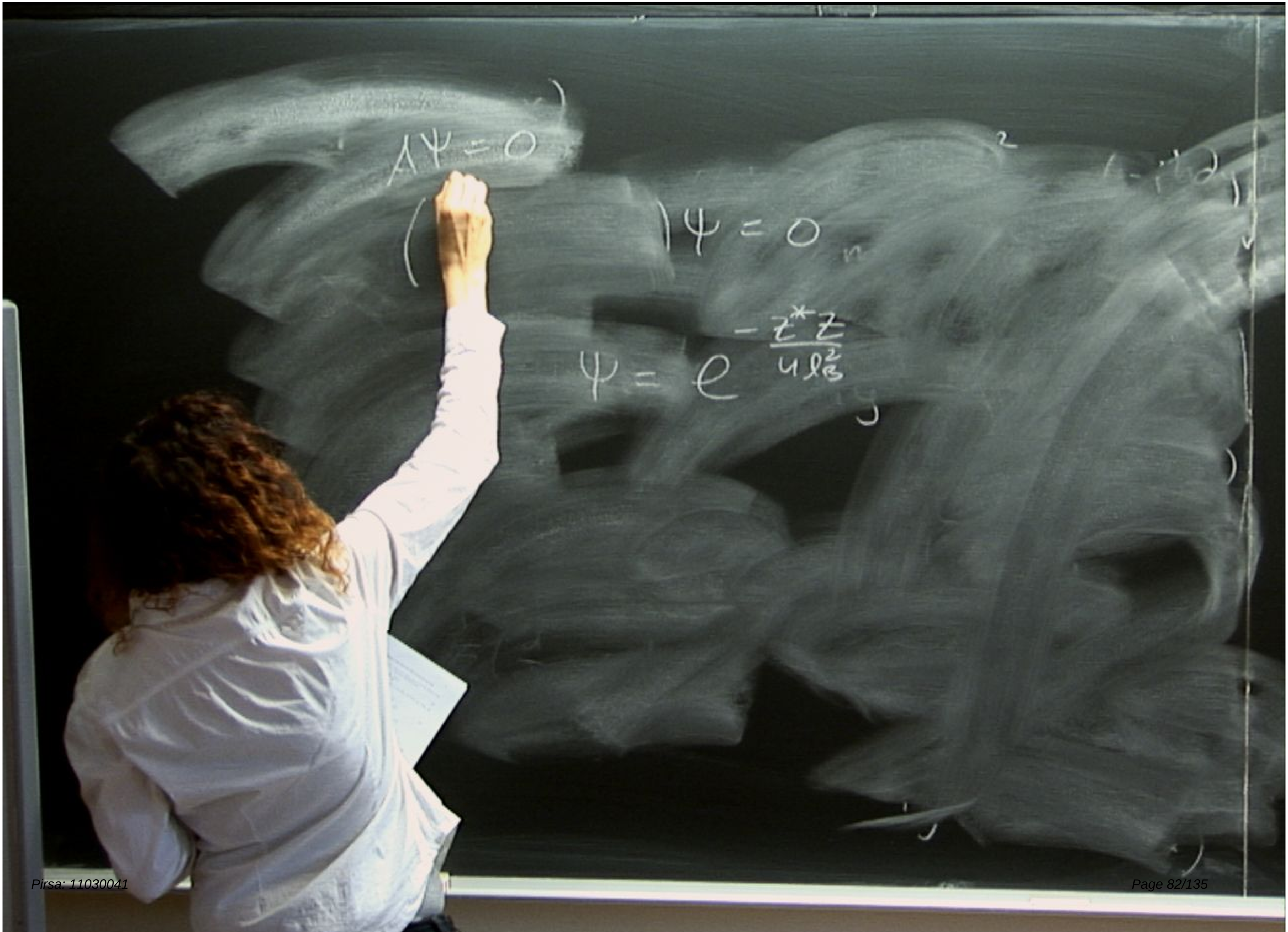

$\Lambda\Psi = 0$


ΛΨ = 0
(−d/dz* + z/4ℓ_B²)Ψ = 0

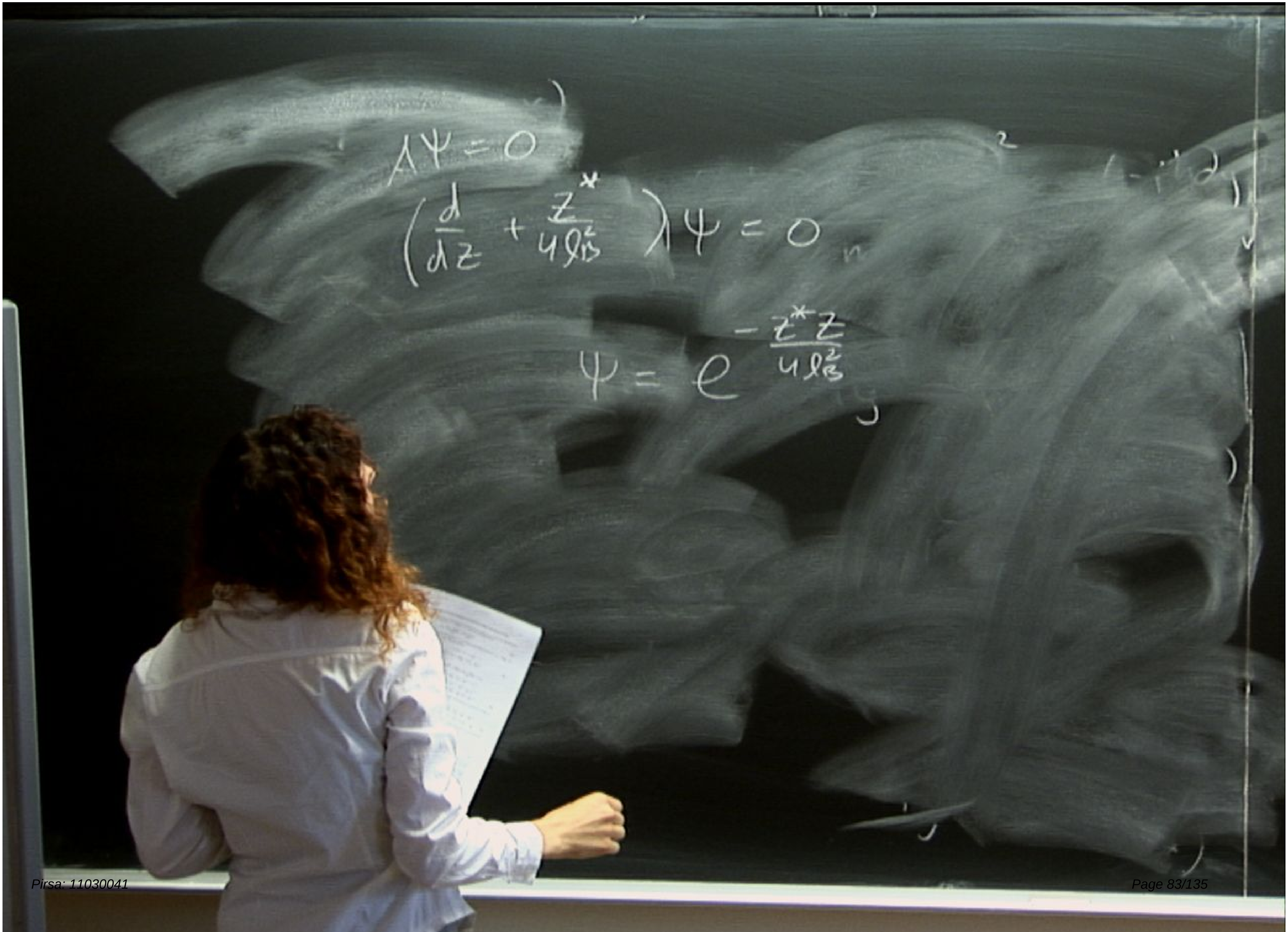

$\Lambda\Psi = 0$
$\left(-\frac{d}{dz^*} + \frac{z}{4l_B^2}\right)\Psi = 0$
$\Psi = e^{-\frac{z^* z}{4l_B^2}}$

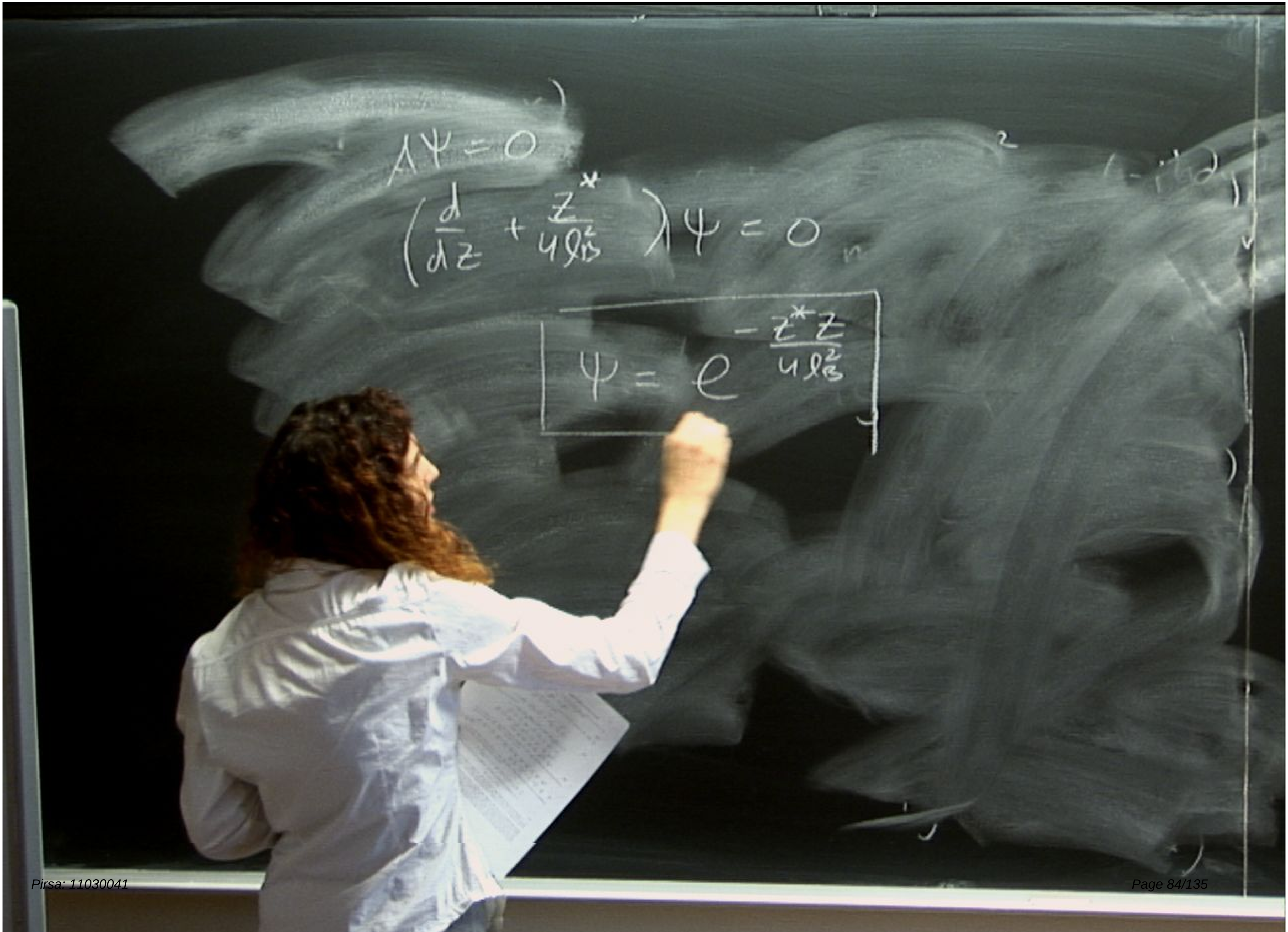


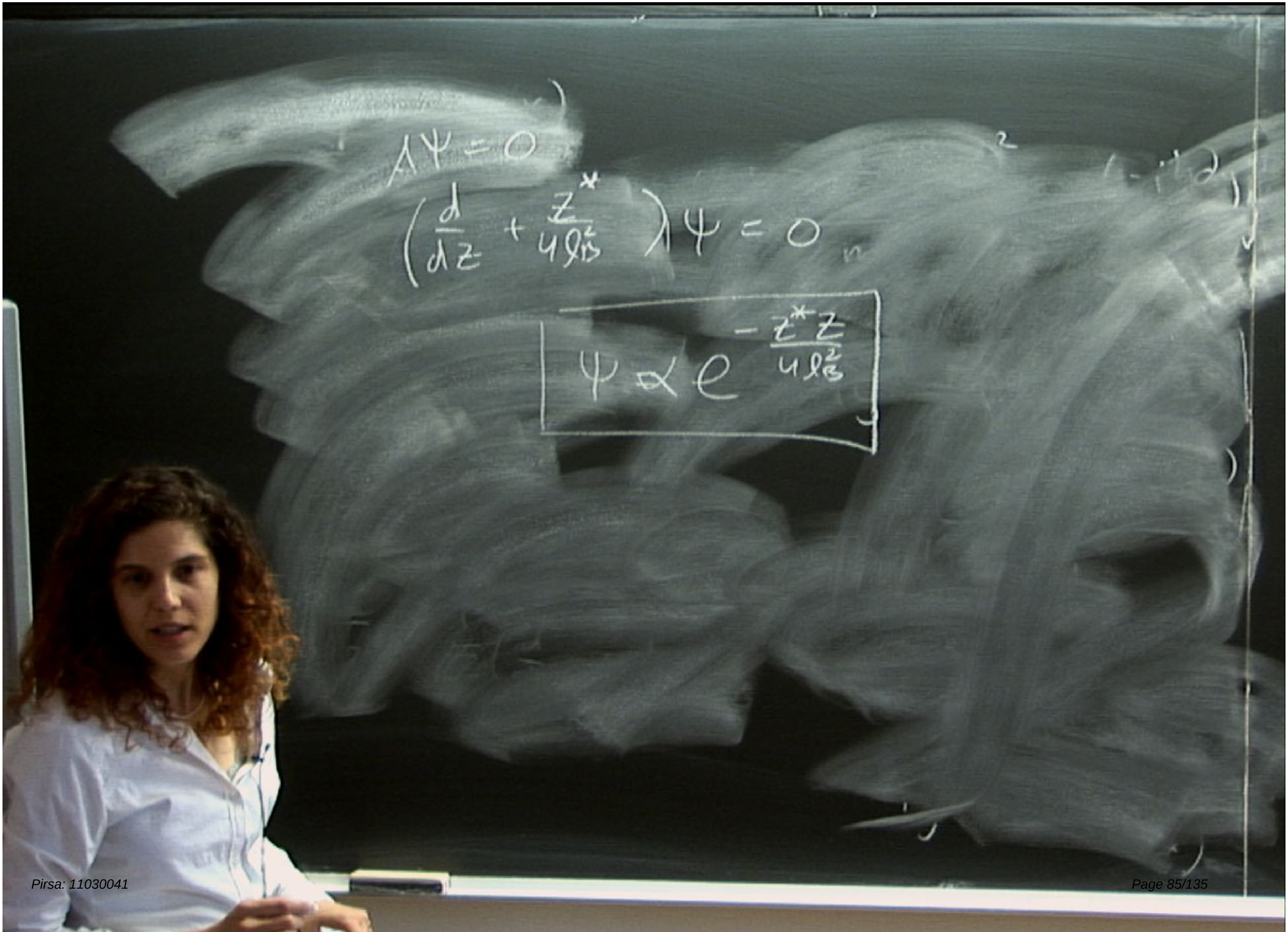
$\Lambda\Psi = 0$
$(\quad)\Psi = 0$
$\Psi = e^{-\frac{z^* z}{4 l_B^2}}$

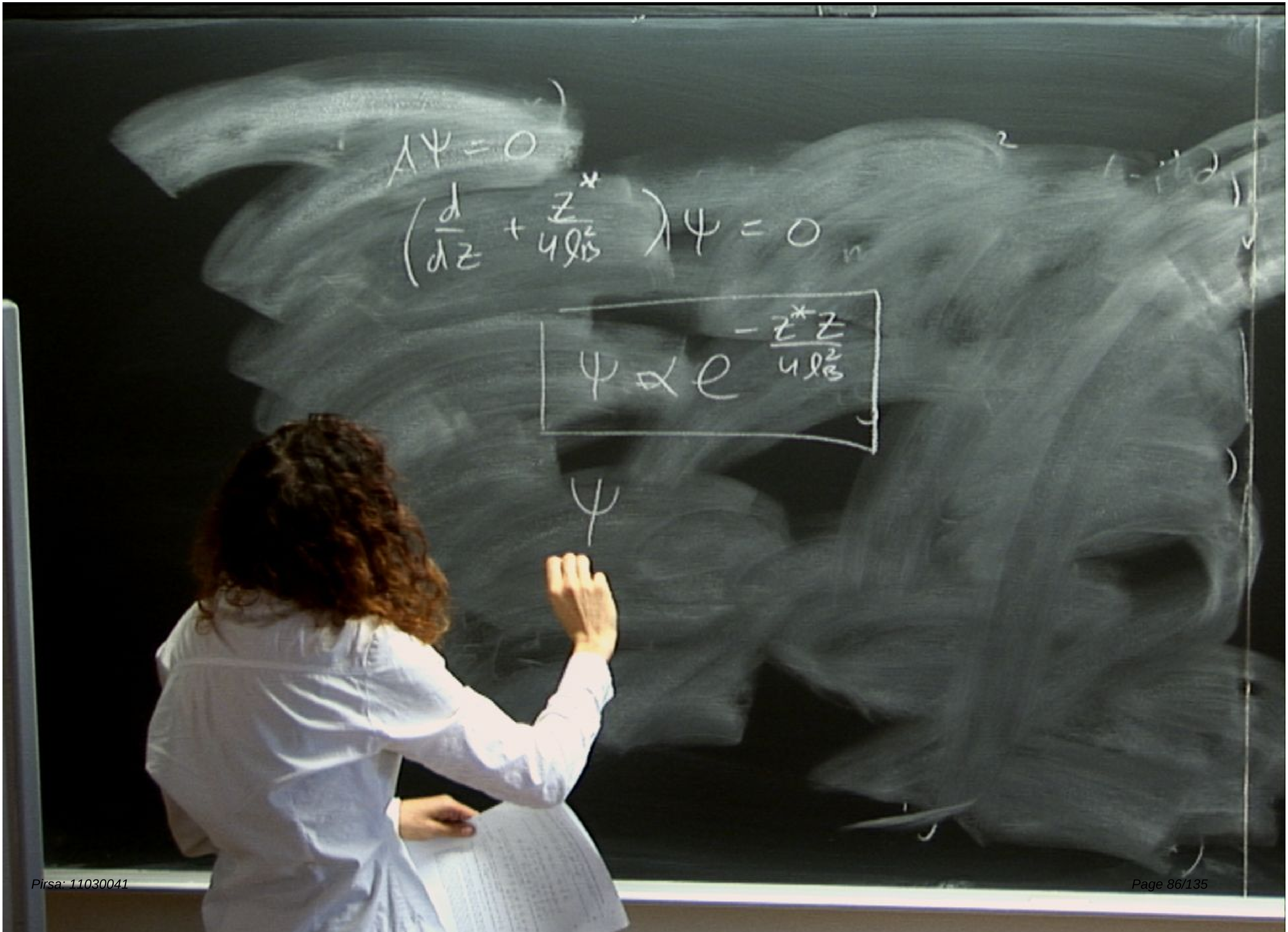


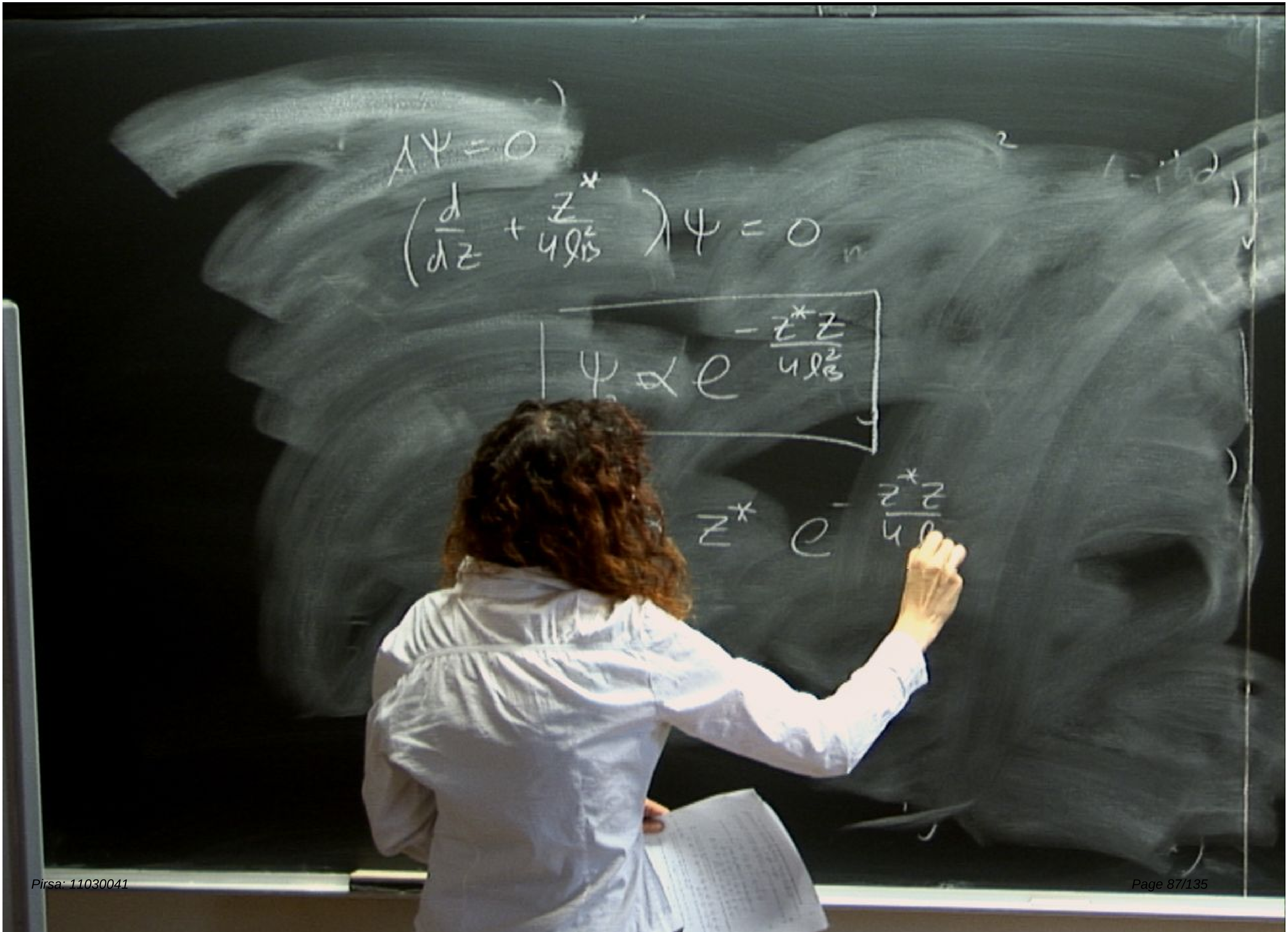
ΛΨ = 0
Ψ = 0
Ψ = e^{-z*z / 4l_B^2}


$\lambda\Psi = 0$
$\left(\frac{d}{dz} + \frac{z^*}{4l_B^2}\right)\lambda\Psi = 0$
$\Psi = e^{-\frac{z^* z}{4l_B^2}}$

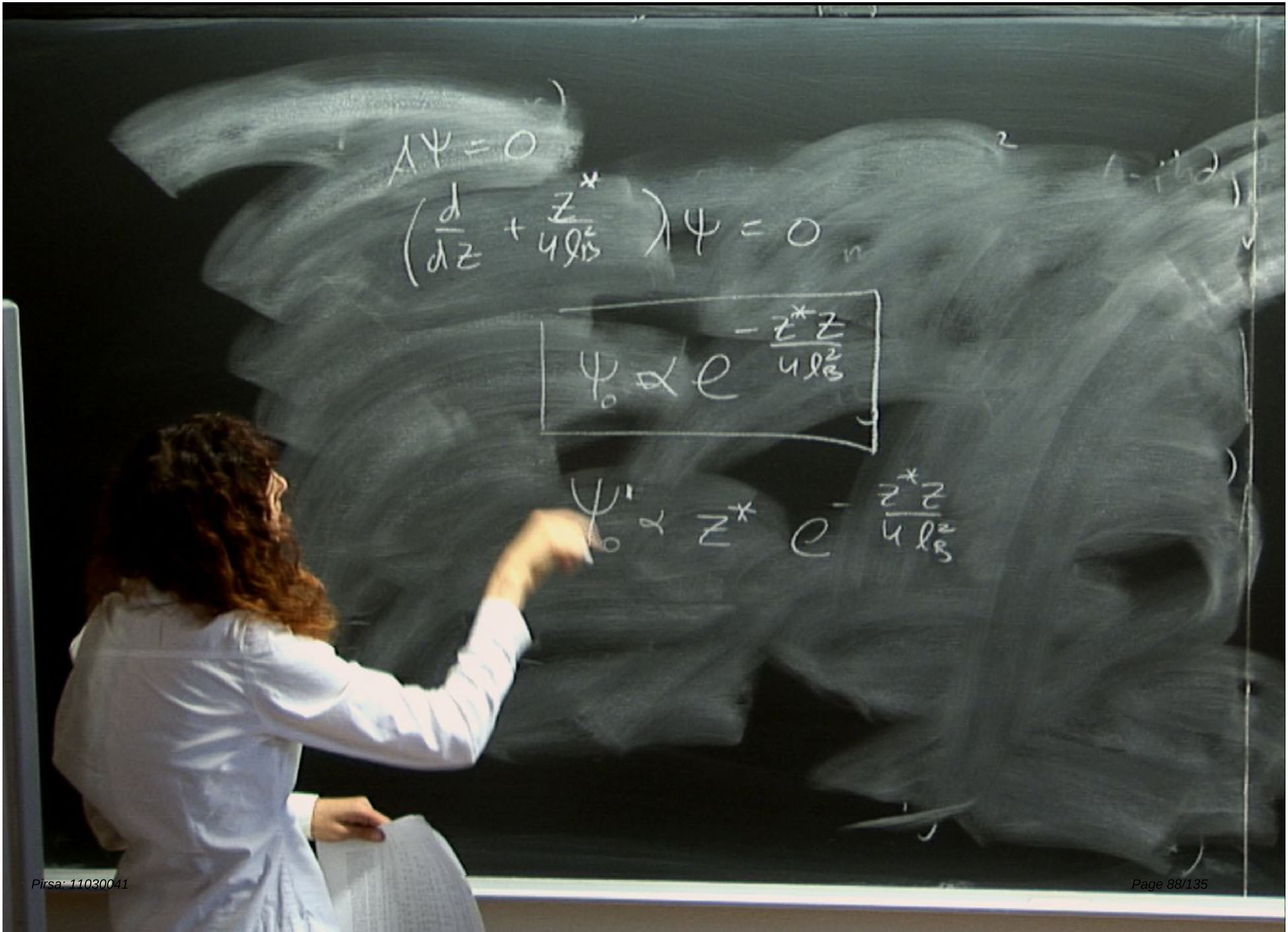

$A\Psi = 0$
$\left(\frac{d}{dz} + \frac{z^*}{4 l_B^2}\right) A\Psi = 0$
$\Psi = e^{-\frac{z^* z}{4 l_B^2}}$

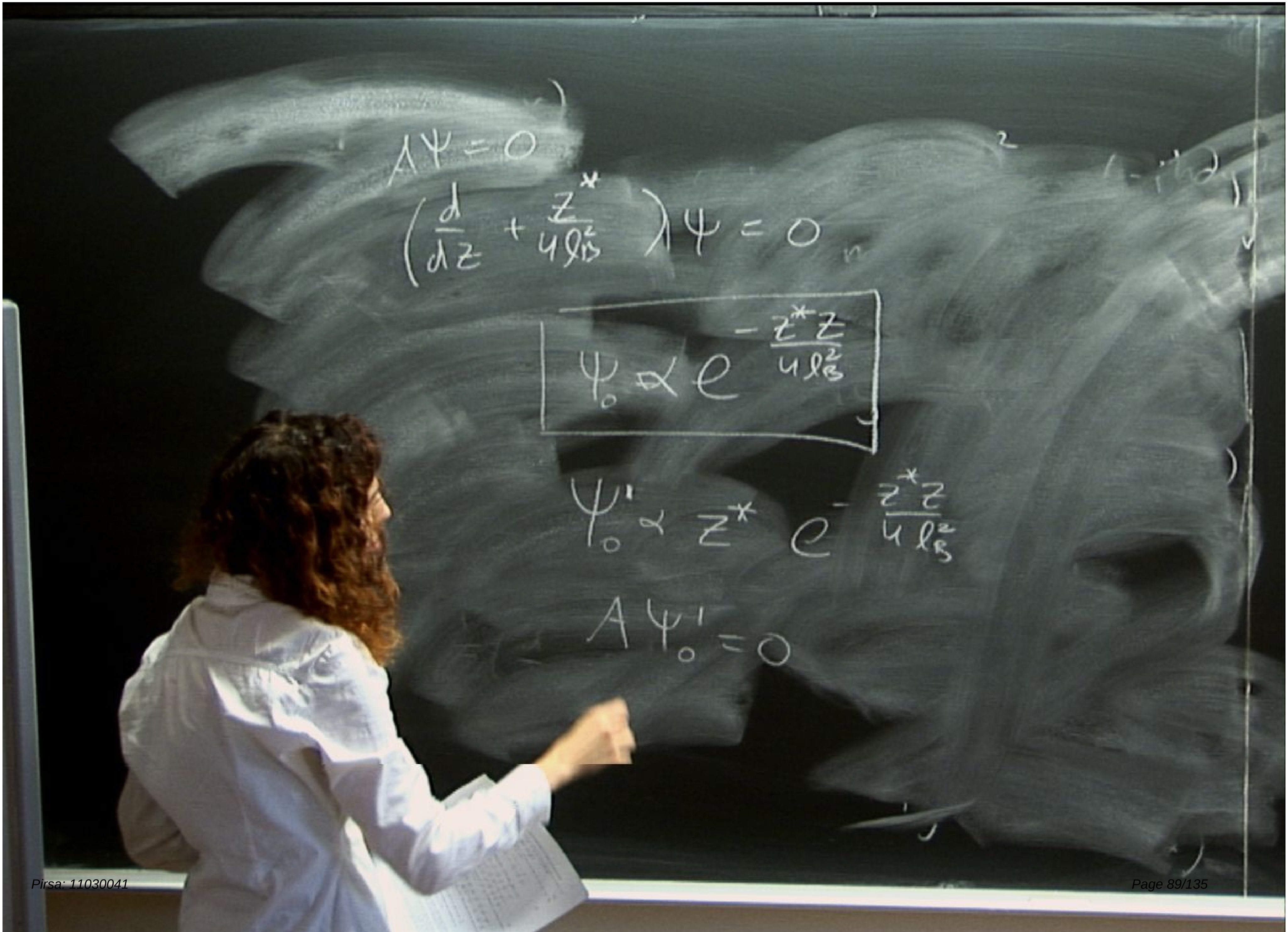

$\Lambda\Psi = 0$
$\left(\frac{d}{dz} + \frac{z^*}{4\ell_B^2}\right)\Psi = 0$
$\Psi \propto e^{-\frac{z^* z}{4\ell_B^2}}$

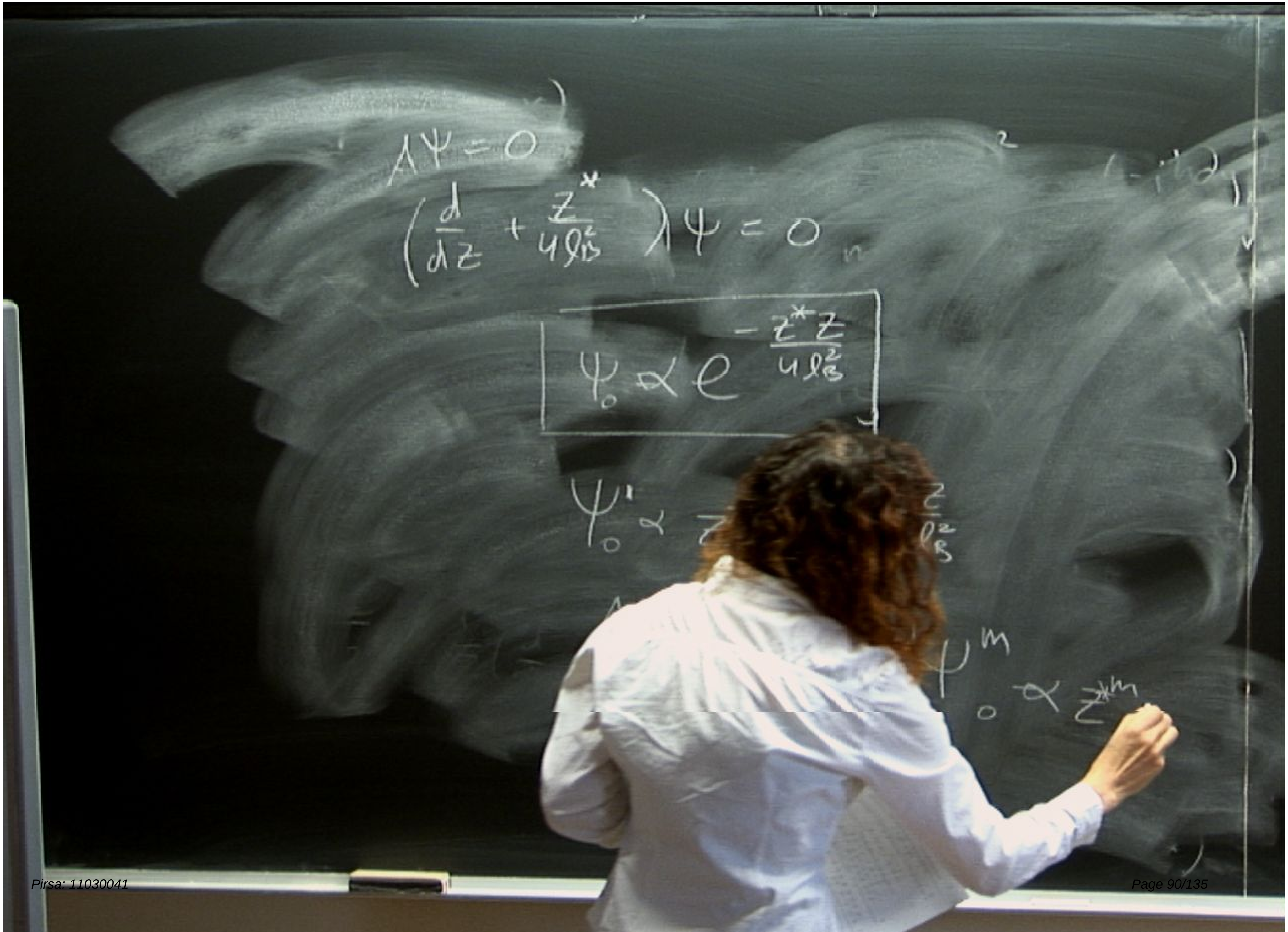

ΛΨ = 0
(d/dz + z*/4l_B^2) ΛΨ = 0
Ψ ∝ e^(−z*z/4l_B^2)
Ψ

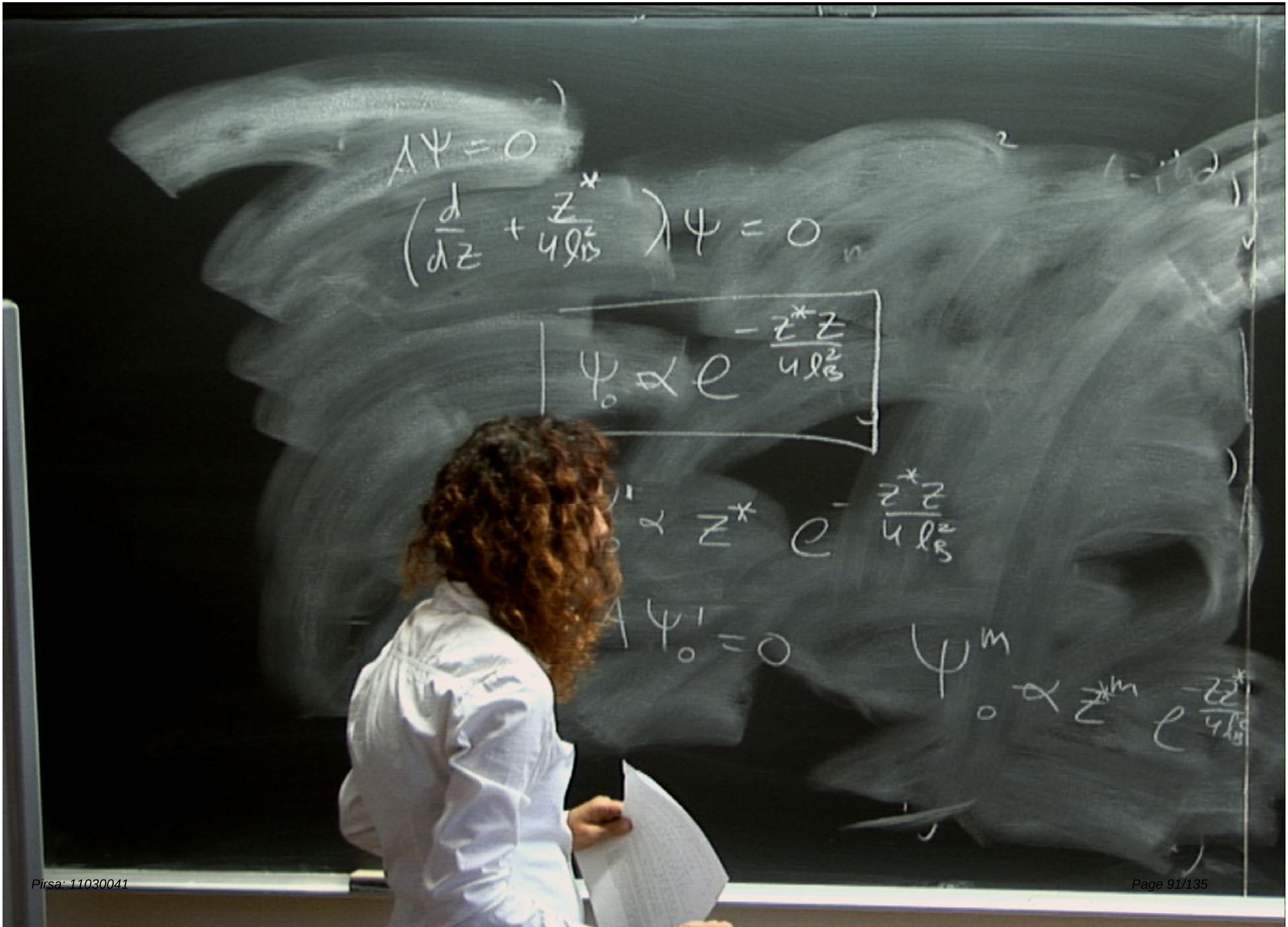


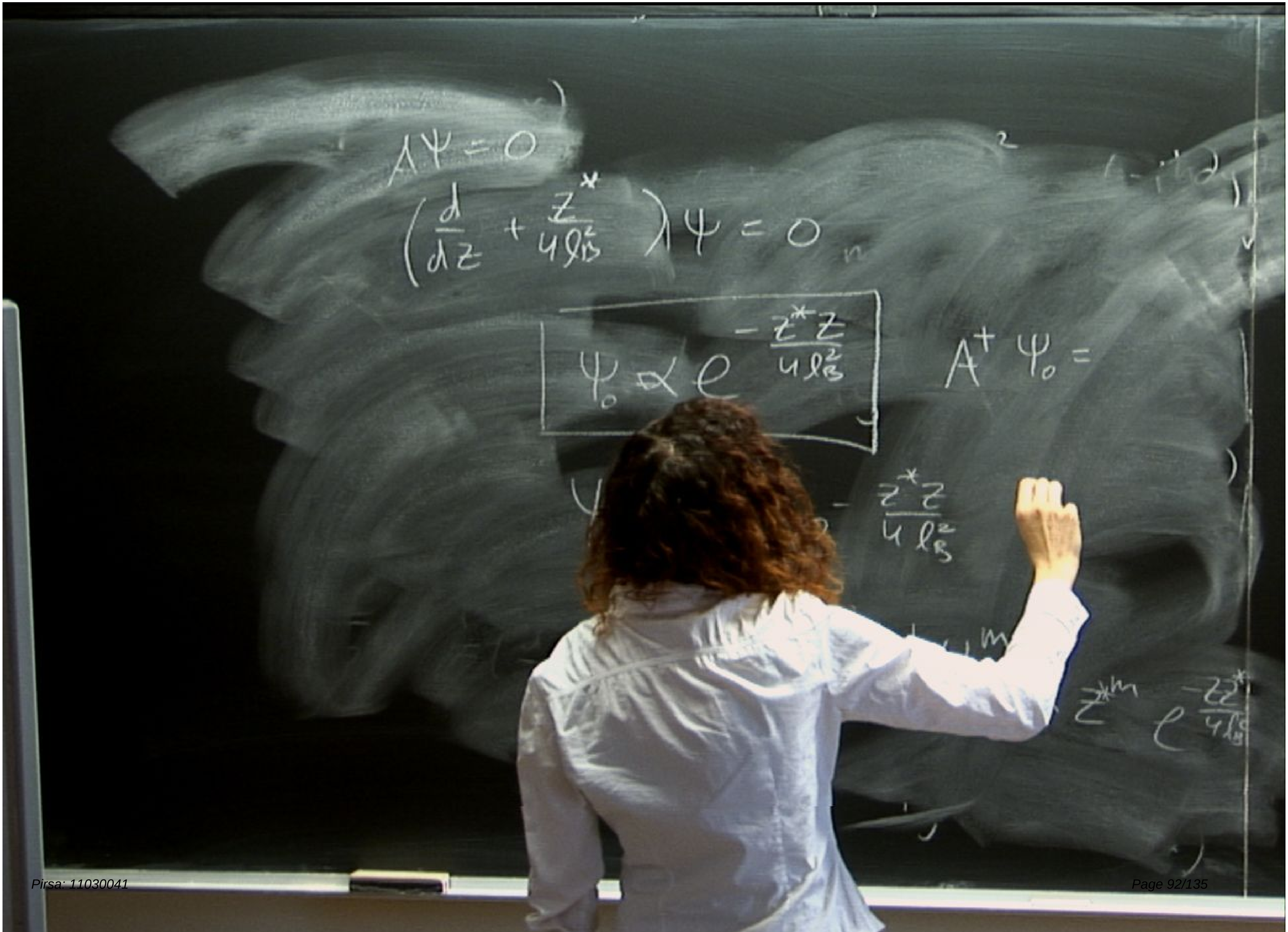
$\Lambda\Psi = 0$
$\left(\frac{d}{dz} + \frac{z^*}{4l_B^2}\right)\Lambda\Psi = 0$
$\Psi \propto e^{-\frac{z^*z}{4l_B^2}}$
$z^* e^{-\frac{z^*z}{4l}}$

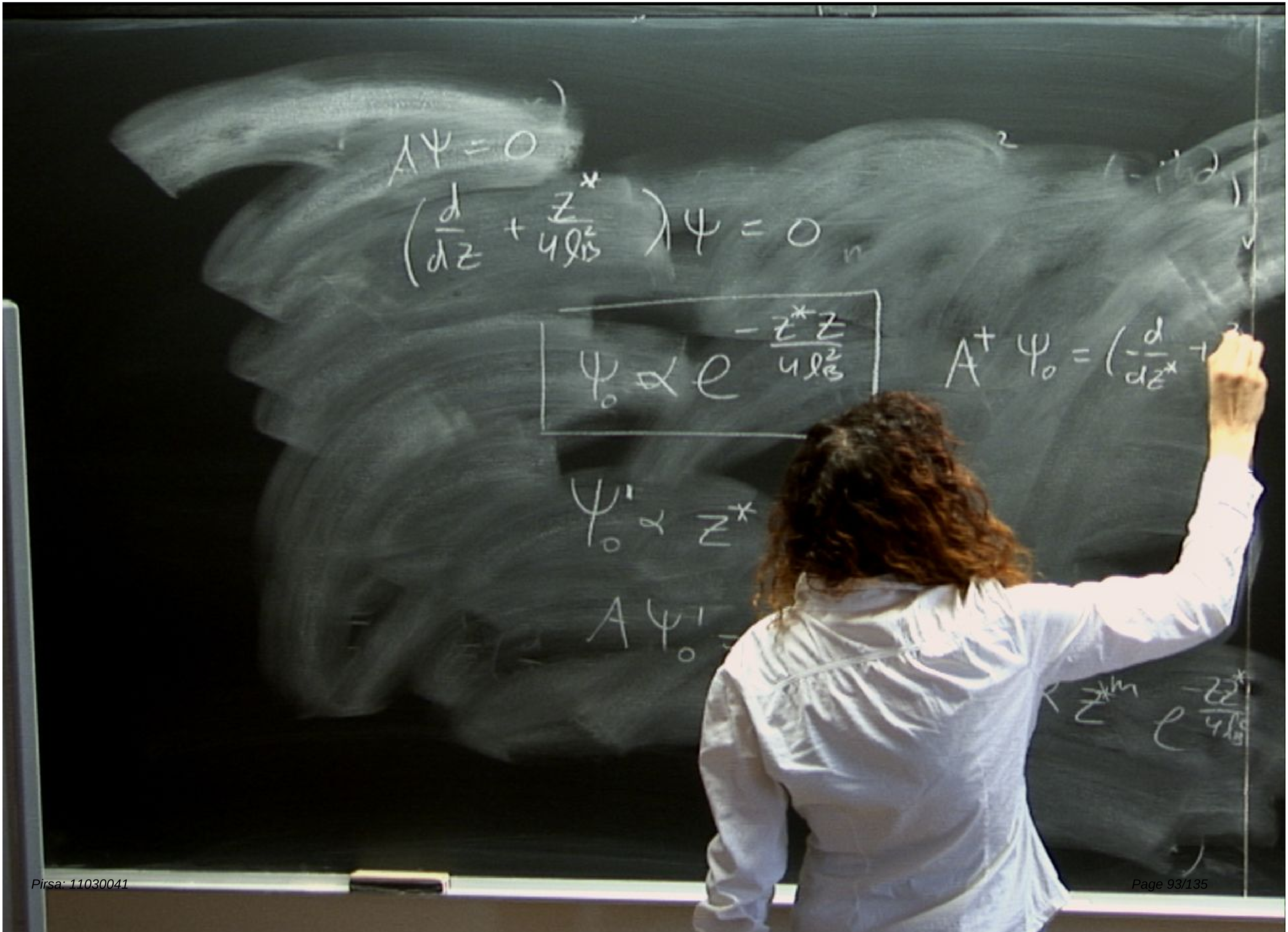

$\lambda\Psi = 0$
$\left(\frac{d}{dz} + \frac{z^*}{4\ell_B^2}\right)\lambda\Psi = 0$
$\Psi_0 \propto e^{-\frac{z^* z}{4\ell_B^2}}$
$\Psi_0' \propto z^* \, e^{-\frac{z^* z}{4\ell_B^2}}$

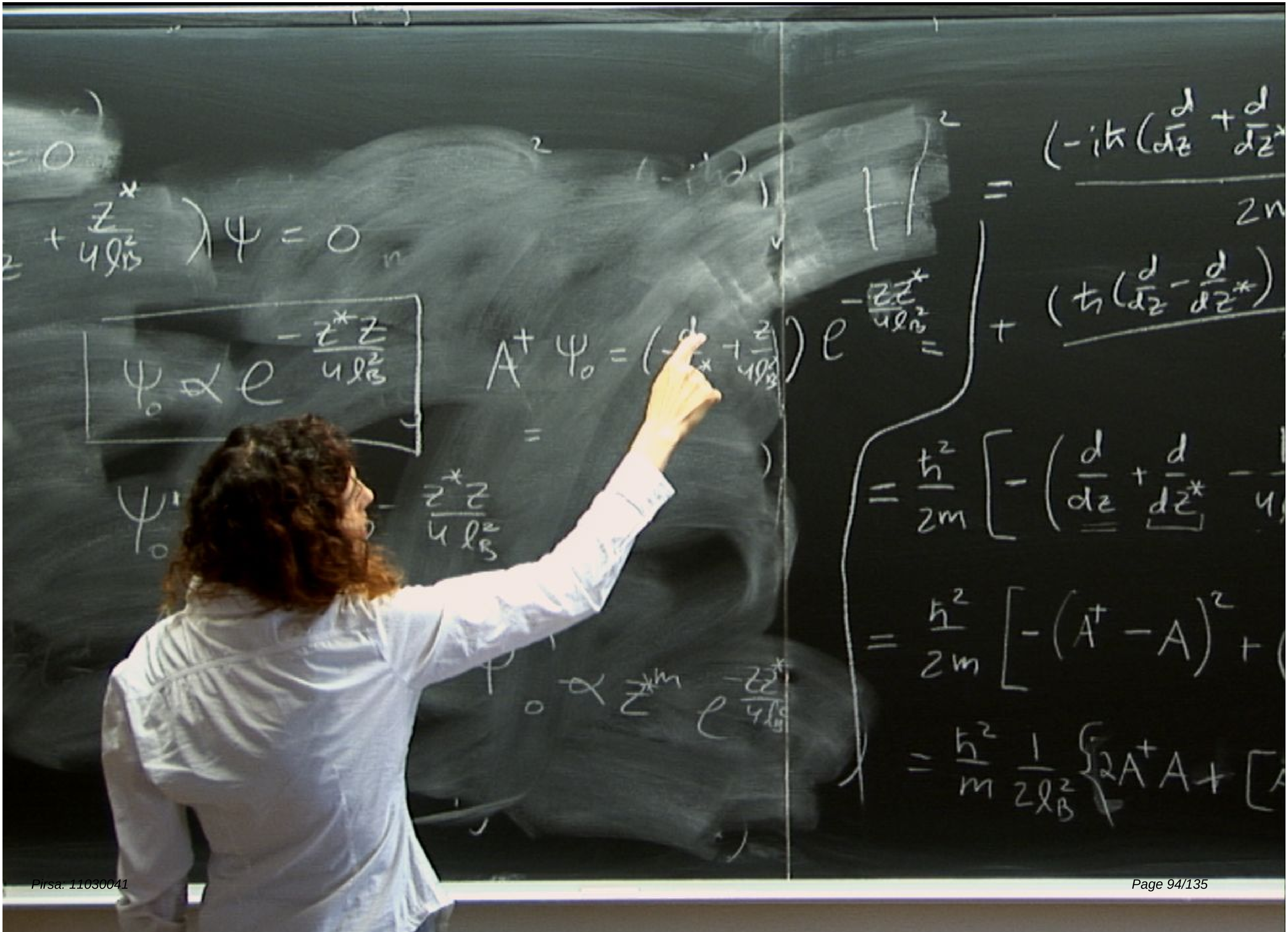

$A\Psi = 0$
$\left(\frac{d}{dz} + \frac{z^*}{4\ell_B^2}\right) A\Psi = 0$
$\Psi_0 \propto e^{-\frac{z^* z}{4\ell_B^2}}$
$\Psi'_0 \propto z^* e^{-\frac{z^* z}{4\ell_B^2}}$
$A\Psi'_0 = 0$

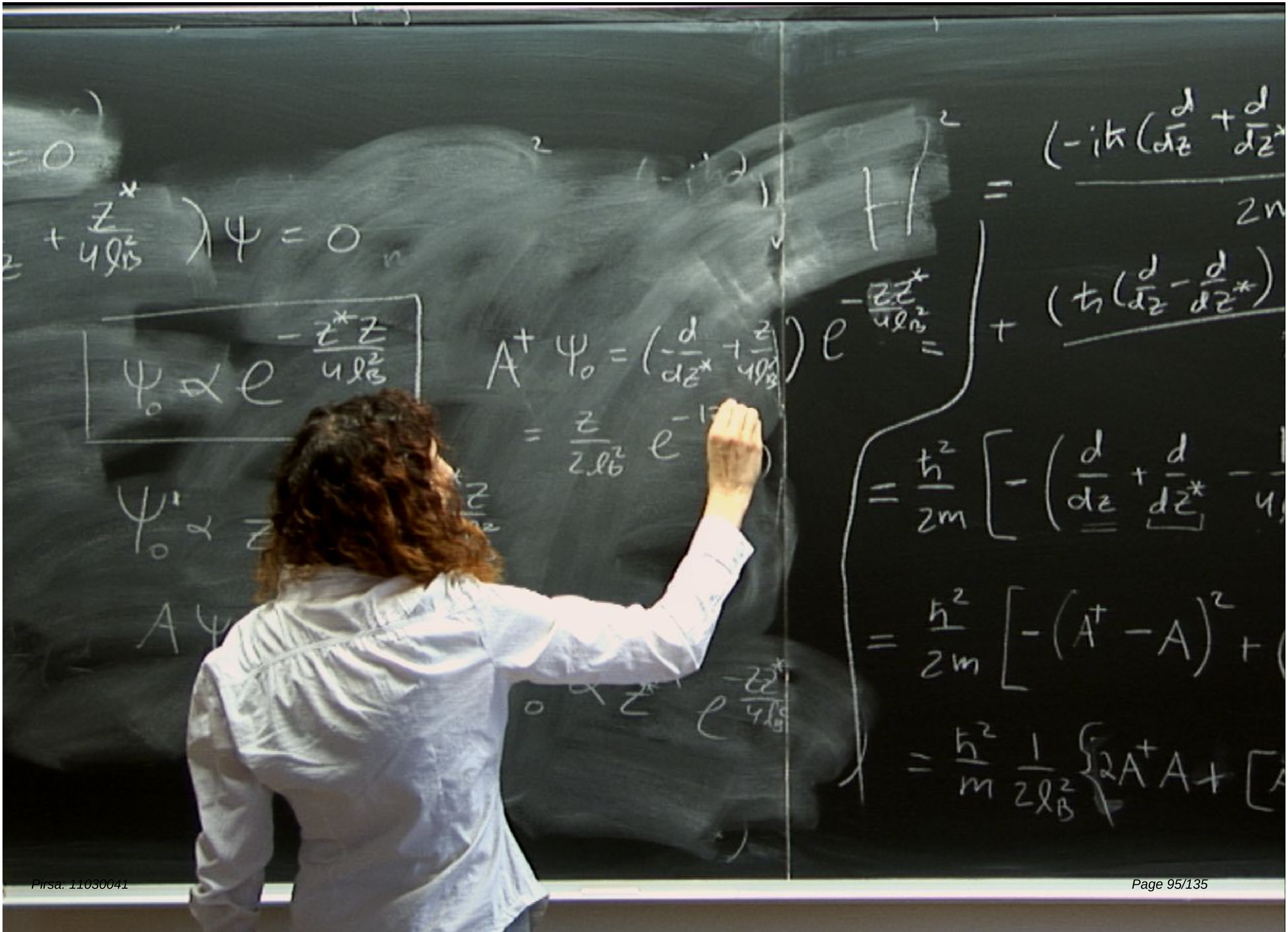

$A\Psi = 0$
$\left(\frac{d}{dz} + \frac{z^*}{4\ell_B^2}\right)\Psi = 0$
$\Psi_0 \propto e^{-\frac{z^* z}{4\ell_B^2}}$
$\Psi_0^m \propto z^{*m}$

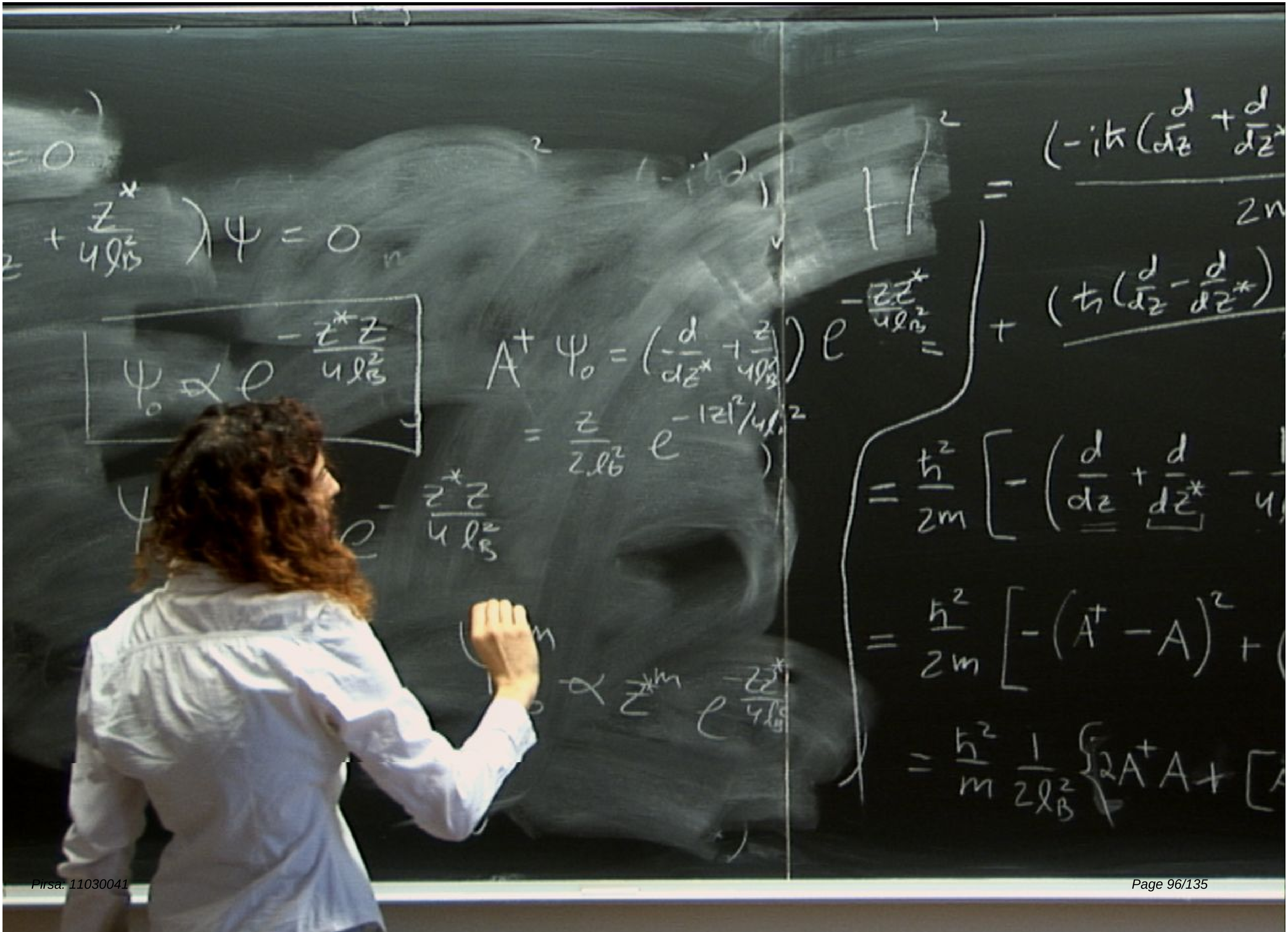

$\Lambda\Psi = 0$
$\left(\frac{d}{dz} + \frac{z^*}{4l_B^2}\right)\Lambda\Psi = 0$
$\Psi_0 \propto e^{-\frac{z^* z}{4l_B^2}}$
$\propto z^* e^{-\frac{z^* z}{4l_B^2}}$
$\Lambda\Psi_0' = 0$
$\Psi_0^m \propto z^{*m} e^{-\frac{zz^*}{4l_B^2}}$


AΨ = 0
(d/dz + z*/4l_B^2) Ψ = 0
Ψ_0 ∝ e^(-z*z/4l_B^2)
A^+ Ψ_0 =


AΨ = 0
(d/dz + z*/4l_B^2) Ψ = 0
Ψ_0 ∝ e^(-z*z/4l_B^2)
A^+ Ψ_0 = (-d/dz*
Ψ_0' ∝ z*
AΨ_0' =
z*^m e^(-zz*/4l_B^2)







$$= 0$$

$$\left(\ldots + \frac{z^*}{4\ell_B^2} \right) \psi = 0$$

$$\boxed{\psi_0 \propto e^{-\frac{z^* z}{4\ell_B^2}}}$$

$$A^+ \psi_0 = \left(-\frac{d}{dz^*} + \frac{z}{4\ell_B^2} \right) e^{-\frac{z z^*}{4\ell_B^2}}$$

$$= \frac{z}{2\ell_B^2} e^{-|z|^2/4\ell_B^2}$$

$$\psi_0^1 \propto z^* e^{-\frac{z^* z}{4\ell_B^2}}$$

$$A \psi_0^1 = 0$$

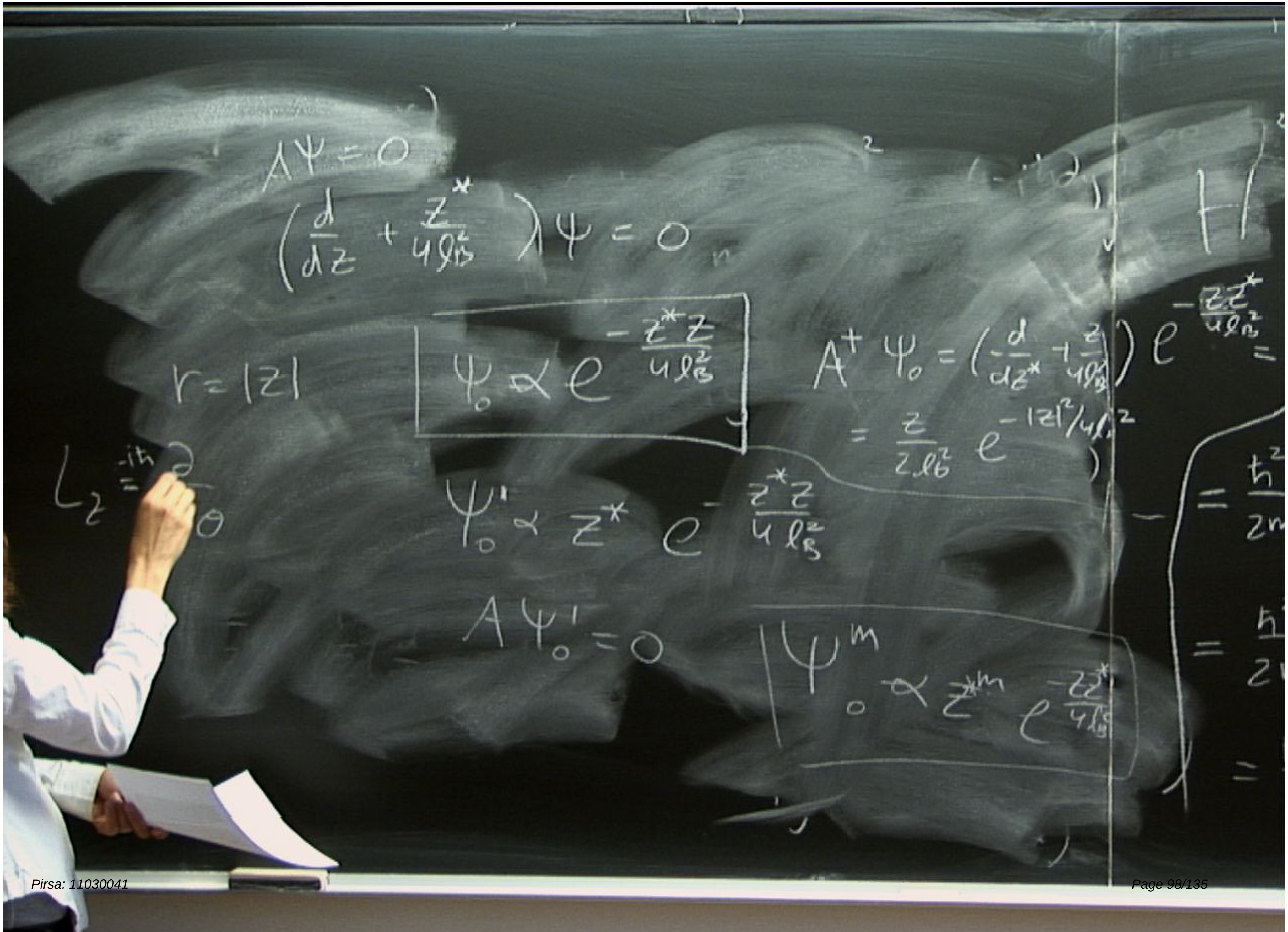
$$\psi_0^m \propto z^{*m} e^{-\frac{z z^*}{4\ell_B^2}}$$

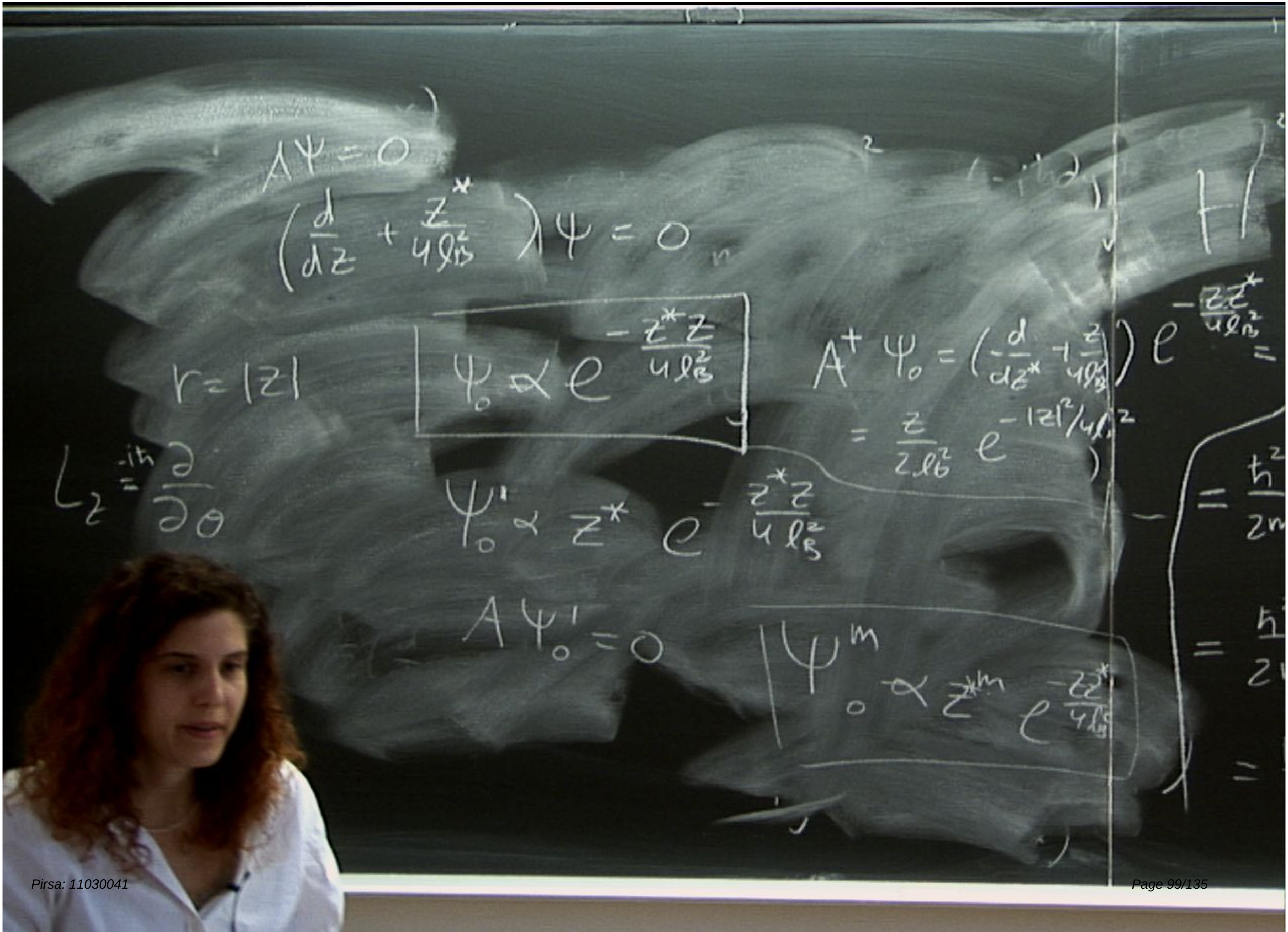
$$H = \frac{\left(-i\hbar\left(\frac{d}{dz} + \frac{d}{dz^*}\right) \ldots\right)^2}{2m} + \frac{\left(\hbar\left(\frac{d}{dz} - \frac{d}{dz^*}\right) \ldots\right)^2}{\ldots}$$

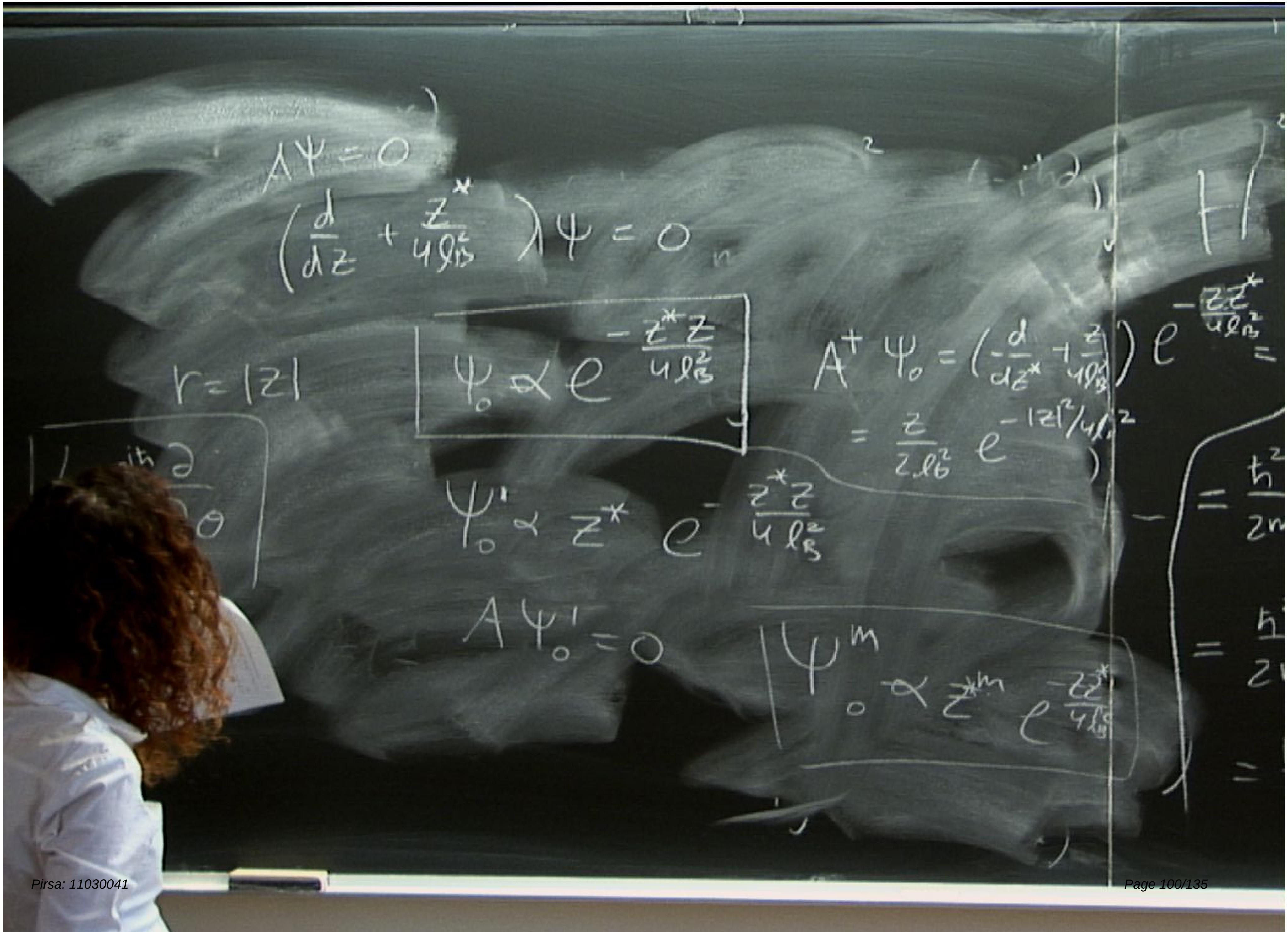
$$= \frac{\hbar^2}{2m} \left[-\left(\frac{d}{dz} + \frac{d}{dz^*} - \ldots \right) \ldots \right.$$

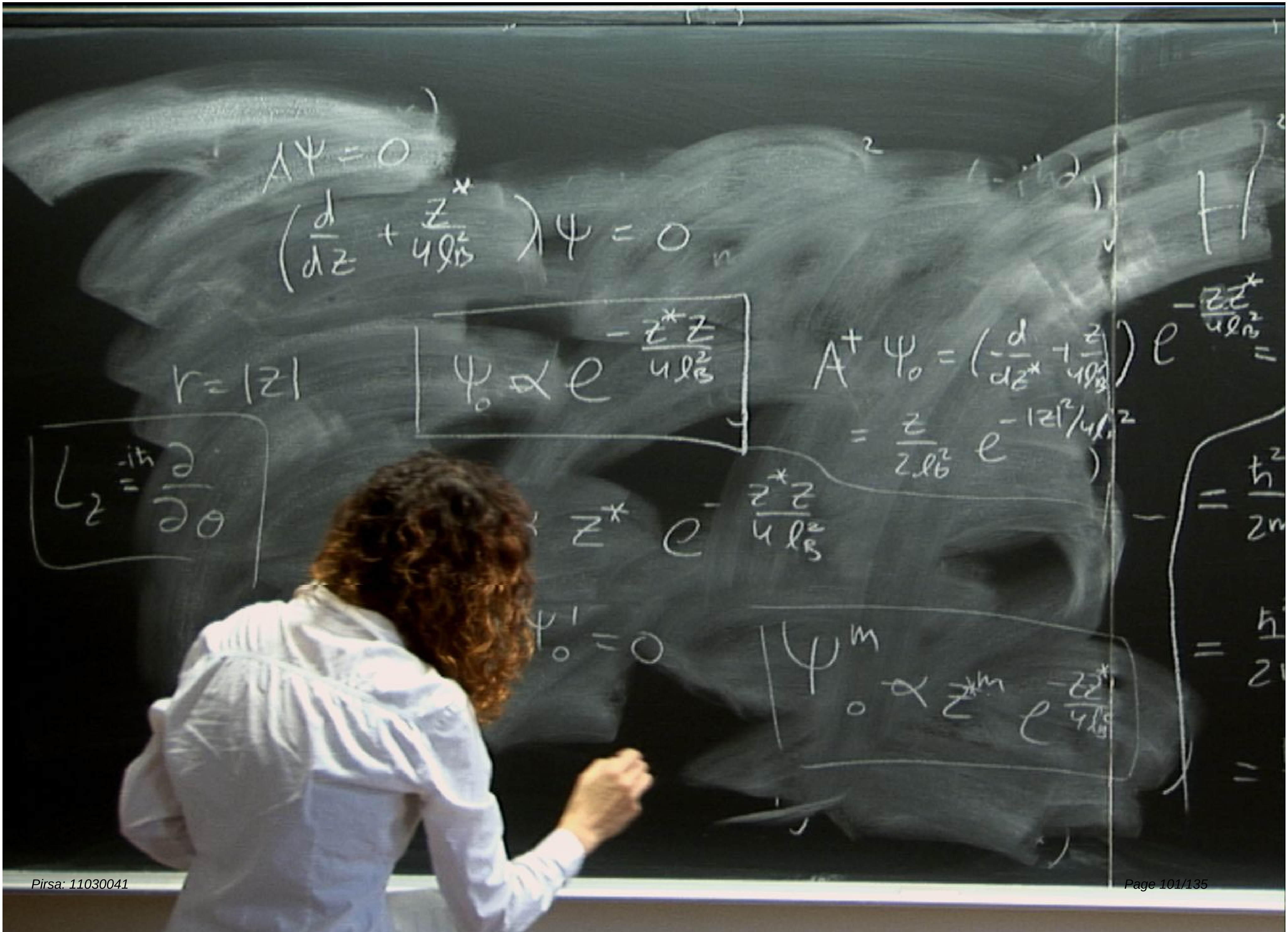
$$= \frac{\hbar^2}{2m} \left[-\left(A^+ - A \right)^2 + \ldots \right.$$

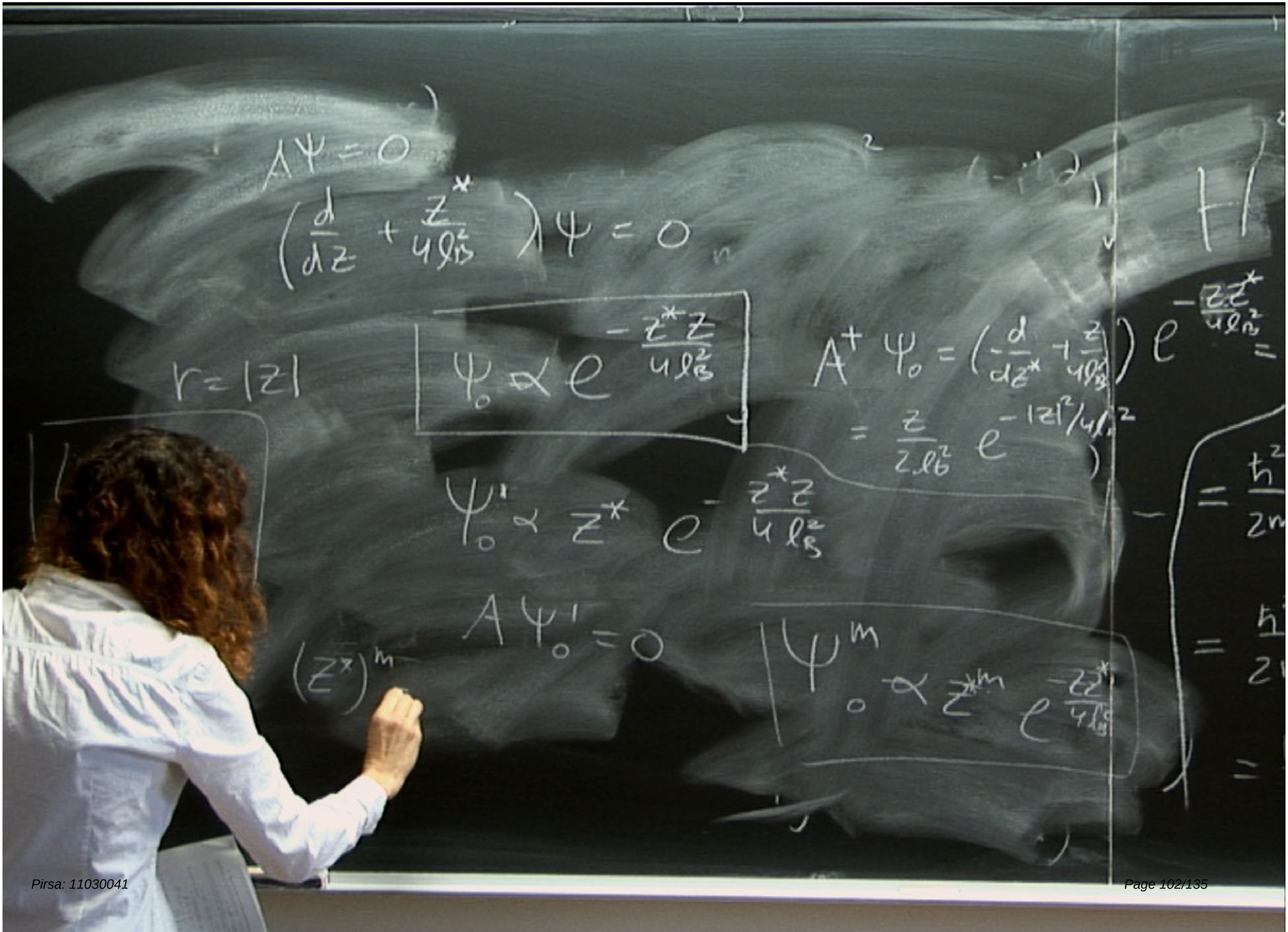
$$= \frac{\hbar^2}{m} \frac{1}{2\ell_B^2} \left\{ 2A^+A + [A \ldots \right.$$

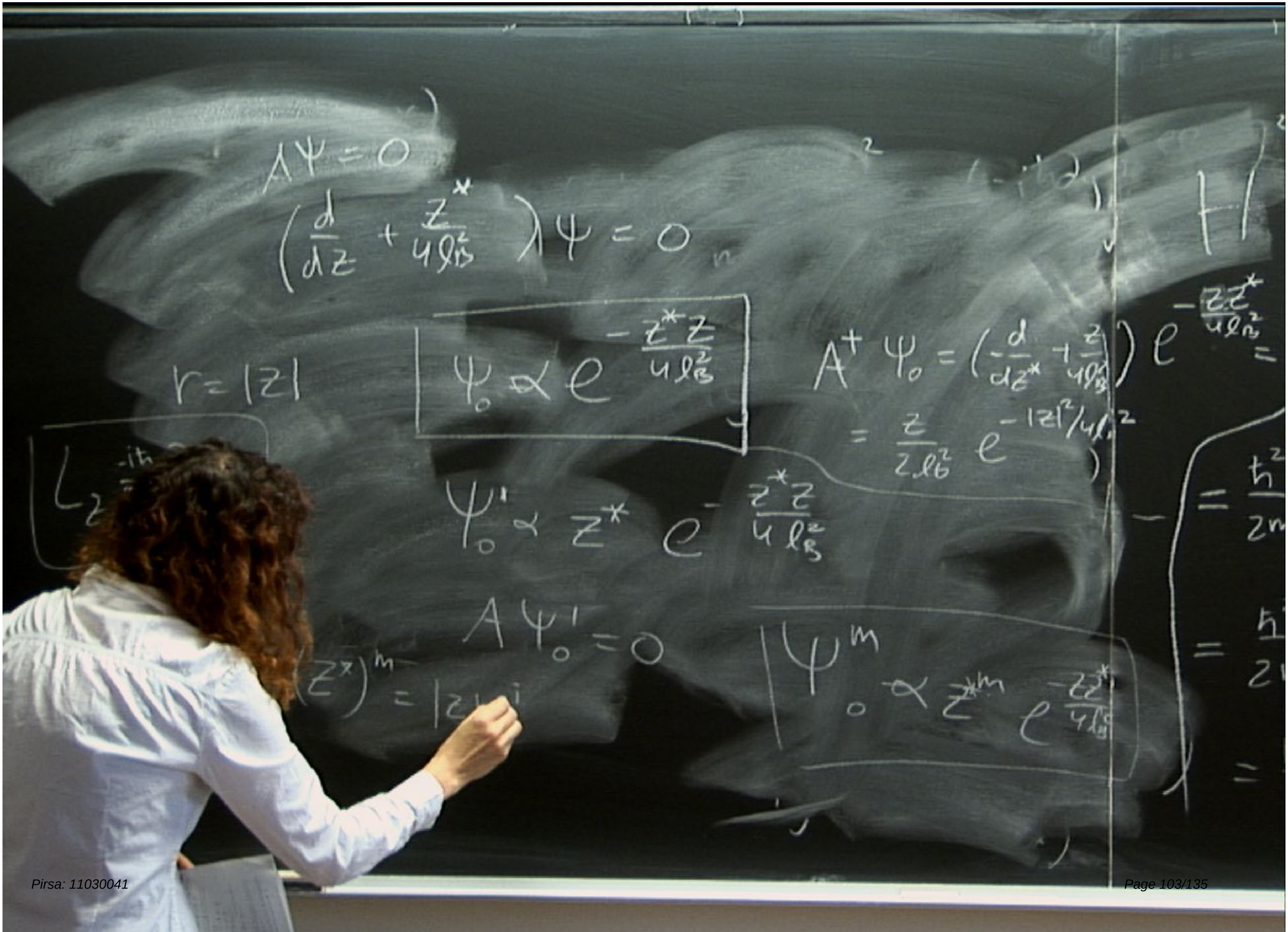

AΨ = 0
(d/dz + z*/4ℓ_B²) Ψ = 0
r = |z|
Ψ₀ ∝ e^(−z*z/4ℓ_B²)
A⁺ Ψ₀ = (−d/dz* + z/4ℓ_B²) e^(−zz*/4ℓ_B²)
= z/2ℓ_B² e^(−|z|²/4ℓ_B²)
L_z = −iħ ∂/∂θ
Ψ₀′ ∝ z* e^(−z*z/4ℓ_B²)
AΨ₀′ = 0
Ψ₀^m ∝ z*^m e^(−zz*/4ℓ_B²)

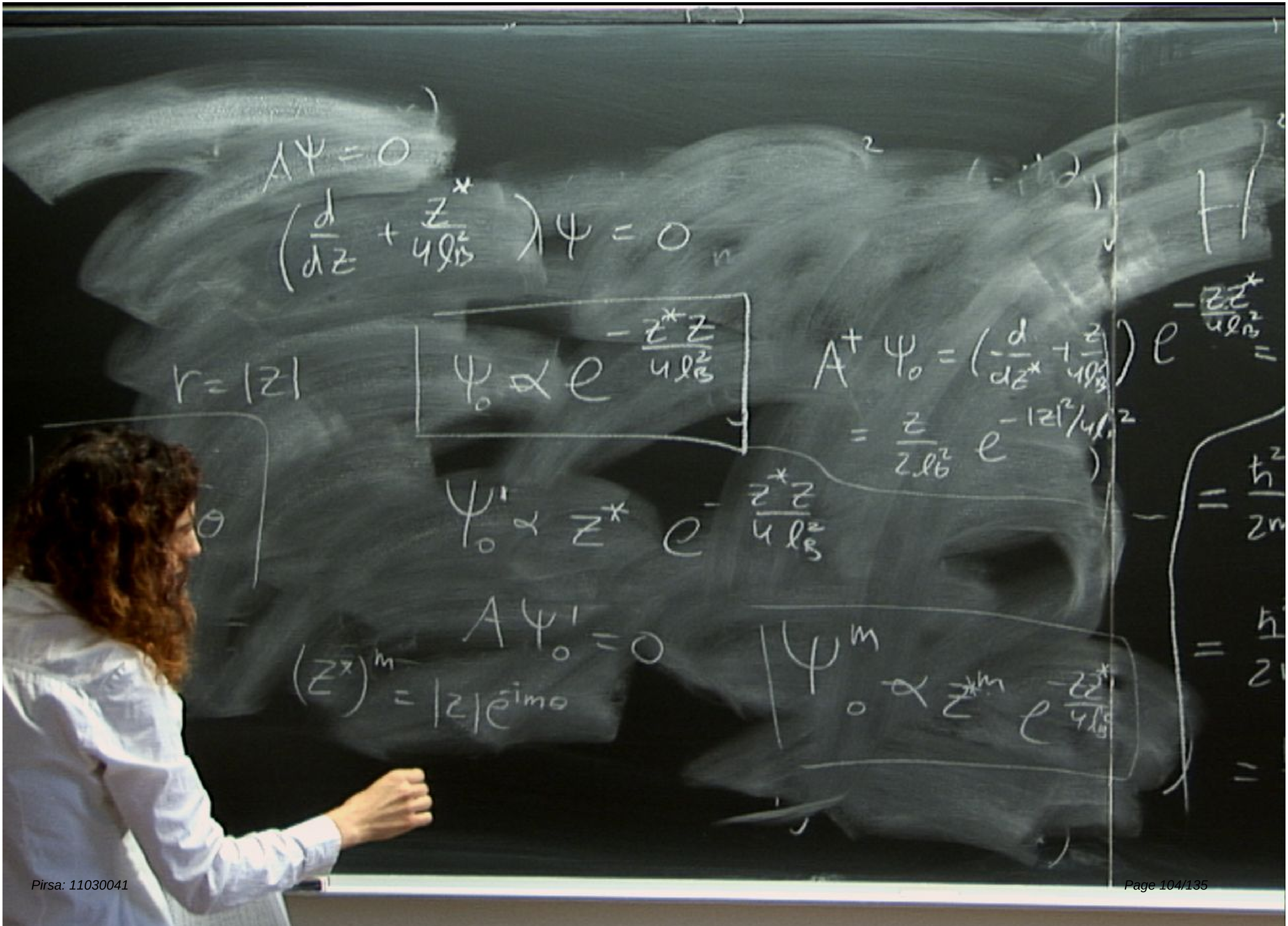

AΨ = 0
(d/dz + z*/4ℓ_B²) Ψ = 0
r = |z|
Ψ₀ ∝ e^(−z*z/4ℓ_B²)
L_z = −iħ ∂/∂θ
Ψ'₀ ∝ z* e^(−z*z/4ℓ_B²)
AΨ'₀ = 0
Ψ^m₀ ∝ z*^m e^(−zz*/4ℓ_B²)
A⁺Ψ₀ = (−d/dz* + z/4ℓ_B²) e^(−zz*/4ℓ_B²)
= z/2ℓ_B² e^(−|z|²/4ℓ_B²)

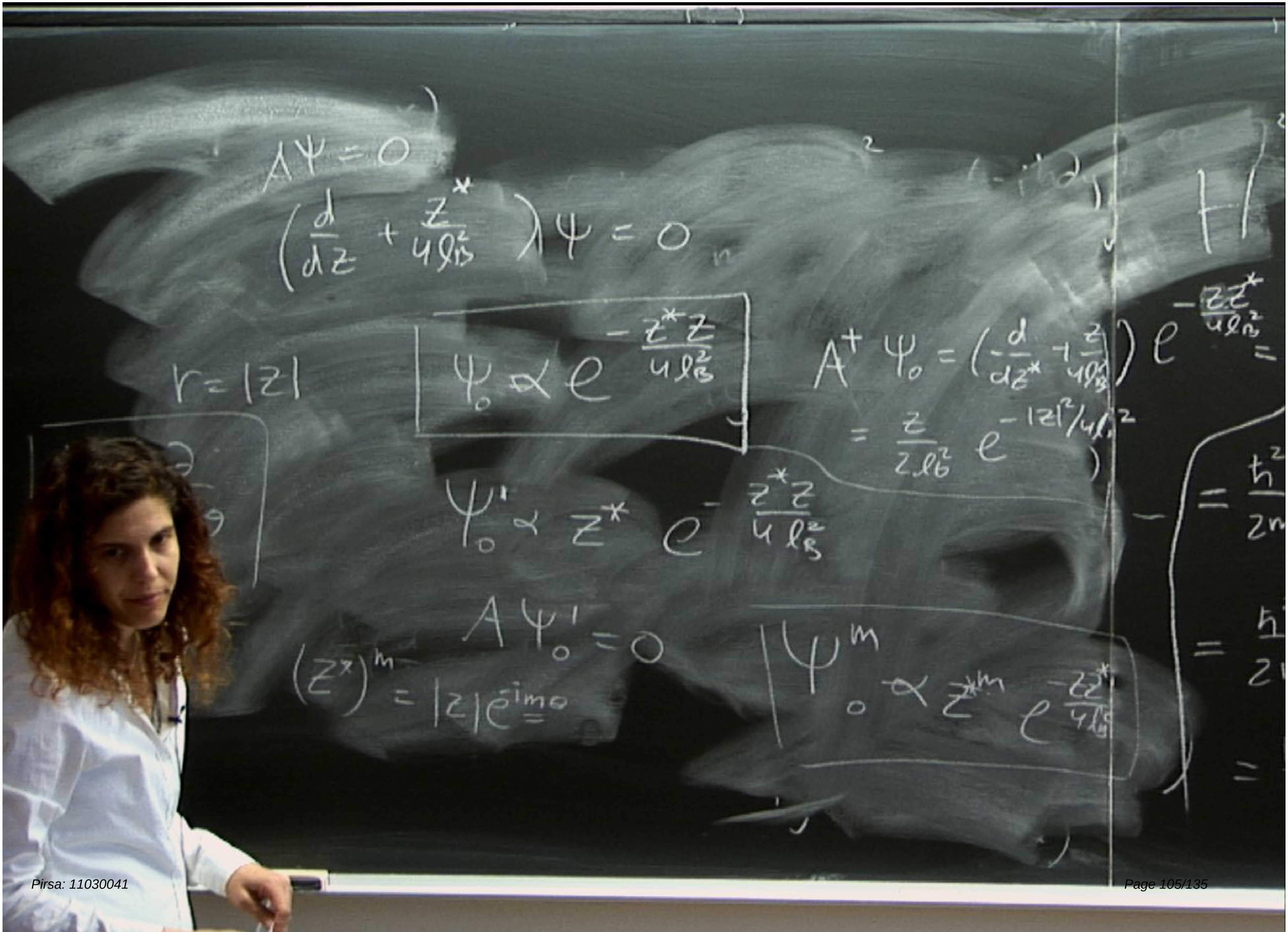

AΨ = 0
(d/dz + z*/4ℓ_B²) Ψ = 0
r = |z|
Ψ₀ ∝ e^(−z*z/4ℓ_B²)
A⁺ Ψ₀ = (−d/dz* + z/4ℓ_B²) e^(−zz*/4ℓ_B²)
= z/2ℓ_B² e^(−|z|²/4ℓ_B²)
Ψ₀' ∝ z* e^(−z*z/4ℓ_B²)
AΨ₀' = 0
Ψ₀^m ∝ z*^m e^(−zz*/4ℓ_B²)


AΨ = 0
r = |z|

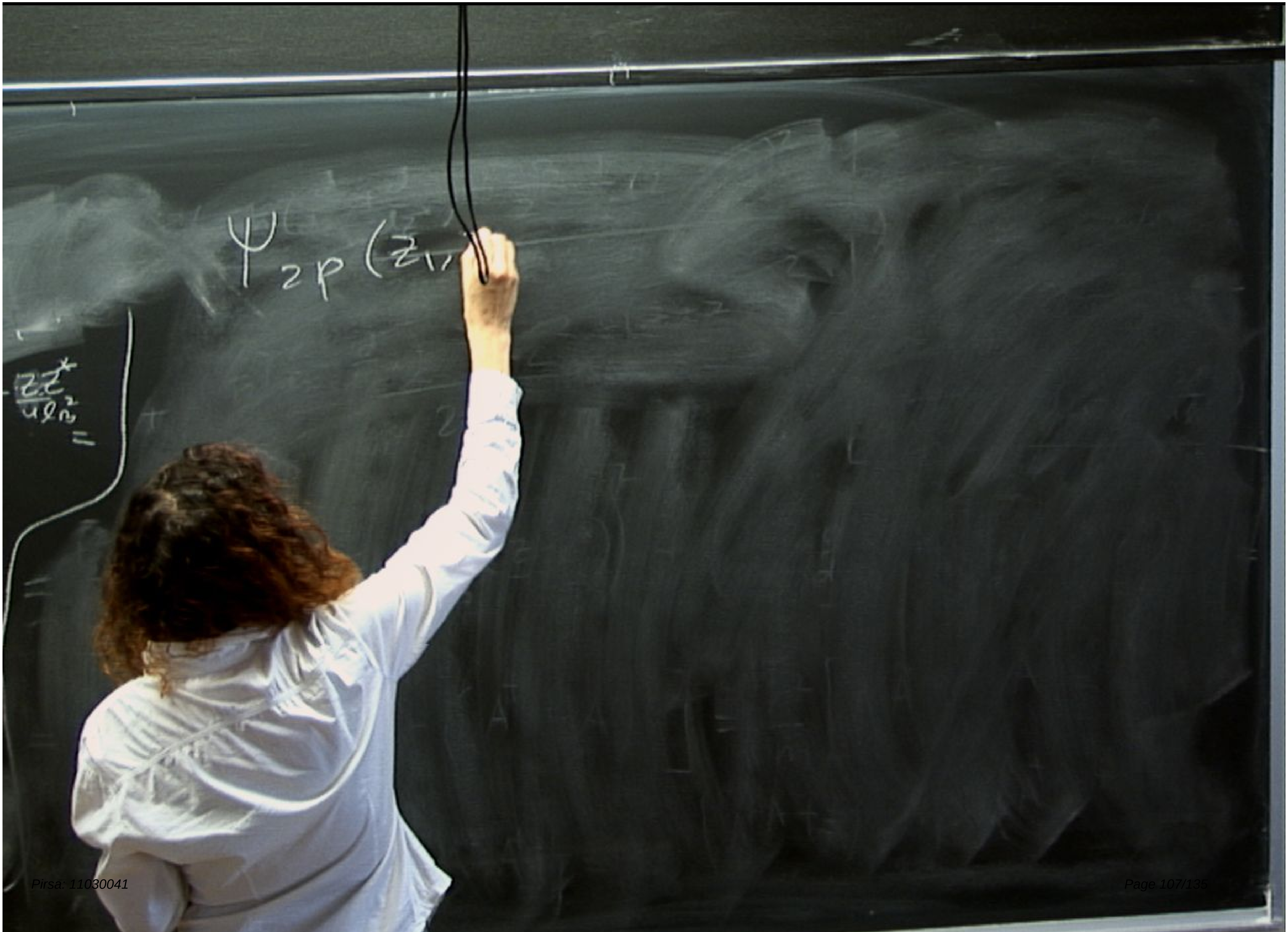

AΨ = 0
(d/dz + z*/4l_B²) Ψ = 0
r = |z|
Ψ₀ ∝ e^(−z*z/4l_B²)
A⁺ Ψ₀ = (−d/dz* + z/4l_B²) e^(−zz*/4l_B²)
= z/2l_B² e^(−|z|²/4l_B²)
Ψ₀¹ ∝ z* e^(−z*z/4l_B²)
AΨ₀¹ = 0
(z*)^m
Ψ₀^m ∝ z*^m e^(−zz*/4l_B²)

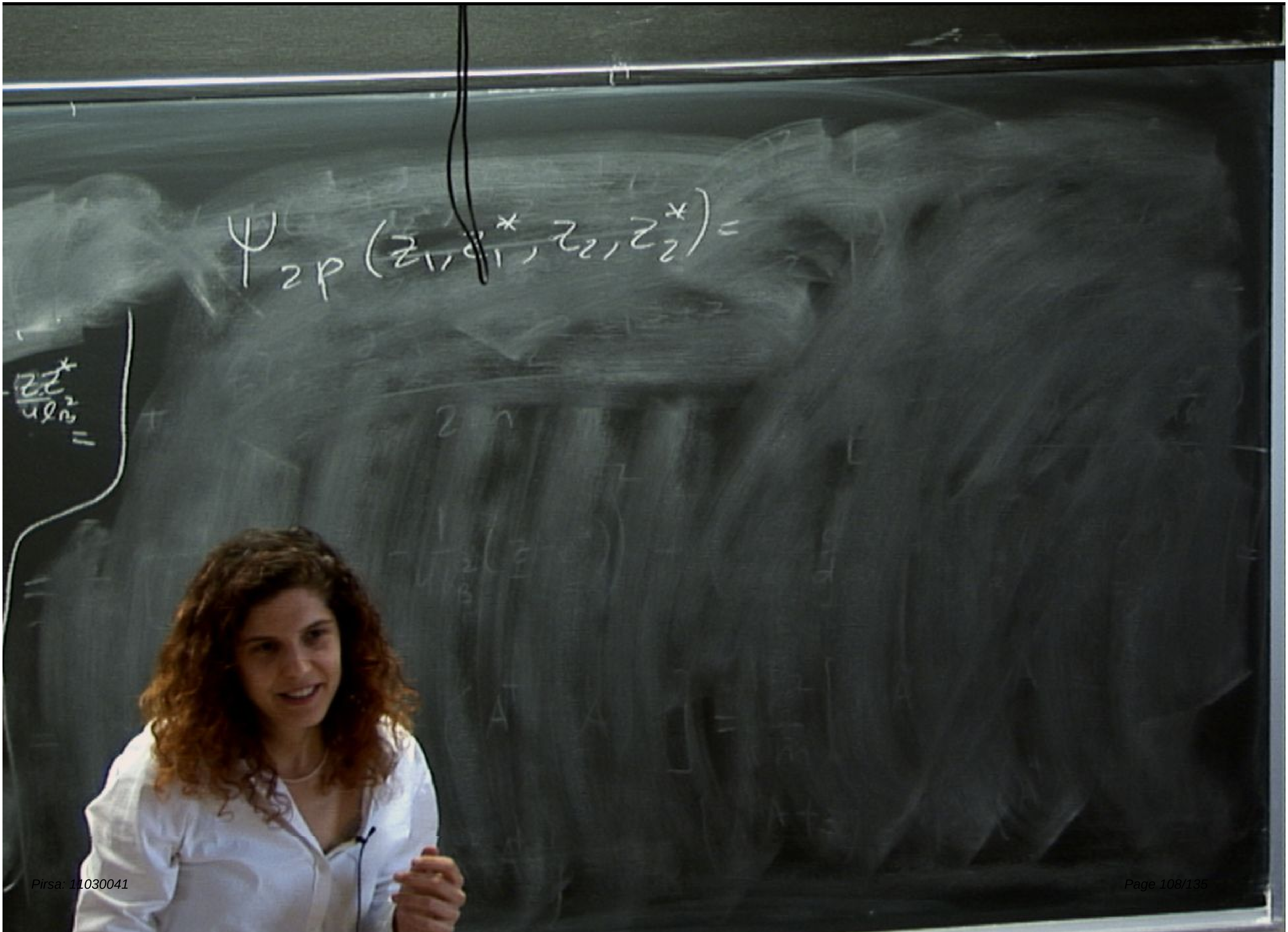

AΨ = 0
(d/dz + z*/4l_B^2) Ψ = 0
r = |z|
Ψ_0 ∝ e^(-z*z/4l_B^2)
A^+ Ψ_0 = (-d/dz* + z/4l_B^2) e^(-zz*/4l_B^2)
= z/2l_B^2 e^(-|z|^2/4l_B^2)
Ψ'_0 ∝ z* e^(-z*z/4l_B^2)
AΨ'_0 = 0
Ψ^m_0 ∝ z*^m e^(-zz*/4l_B^2)

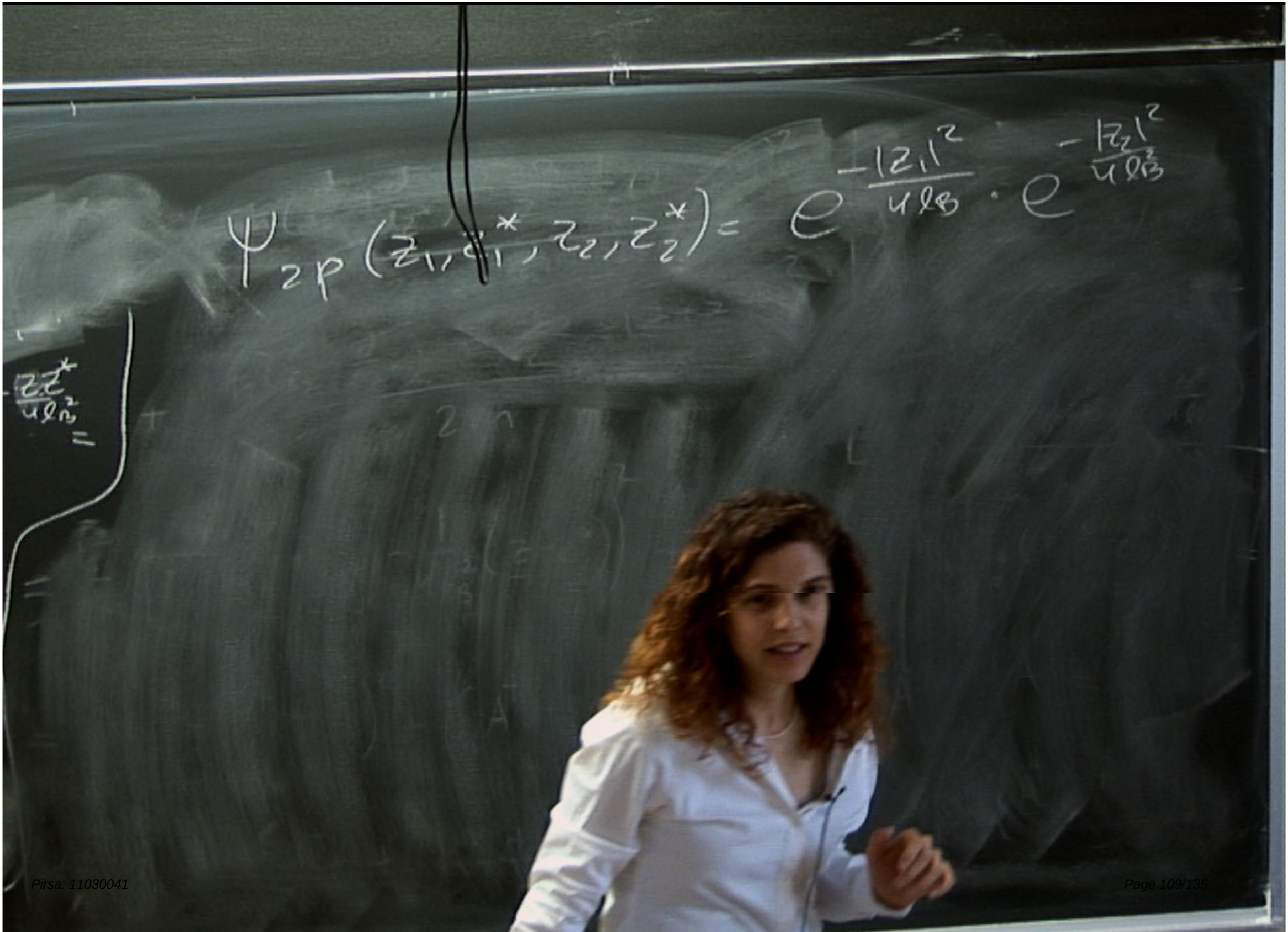

AΨ = 0
(d/dz + z*/4ℓ_B²) Ψ = 0
r = |z|
Ψ₀ ∝ e^(−z*z/4ℓ_B²)
A⁺ Ψ₀ = (−d/dz* + z/4ℓ_B²) e^(−zz*/4ℓ_B²)
= z/2ℓ_B² e^(−|z|²/4ℓ_B²)
Ψ₀' ∝ z* e^(−z*z/4ℓ_B²)
AΨ₀' = 0
(z*)^m = |z| e^(−imθ)
Ψ₀^m ∝ z*^m e^(−zz*/4ℓ_B²)

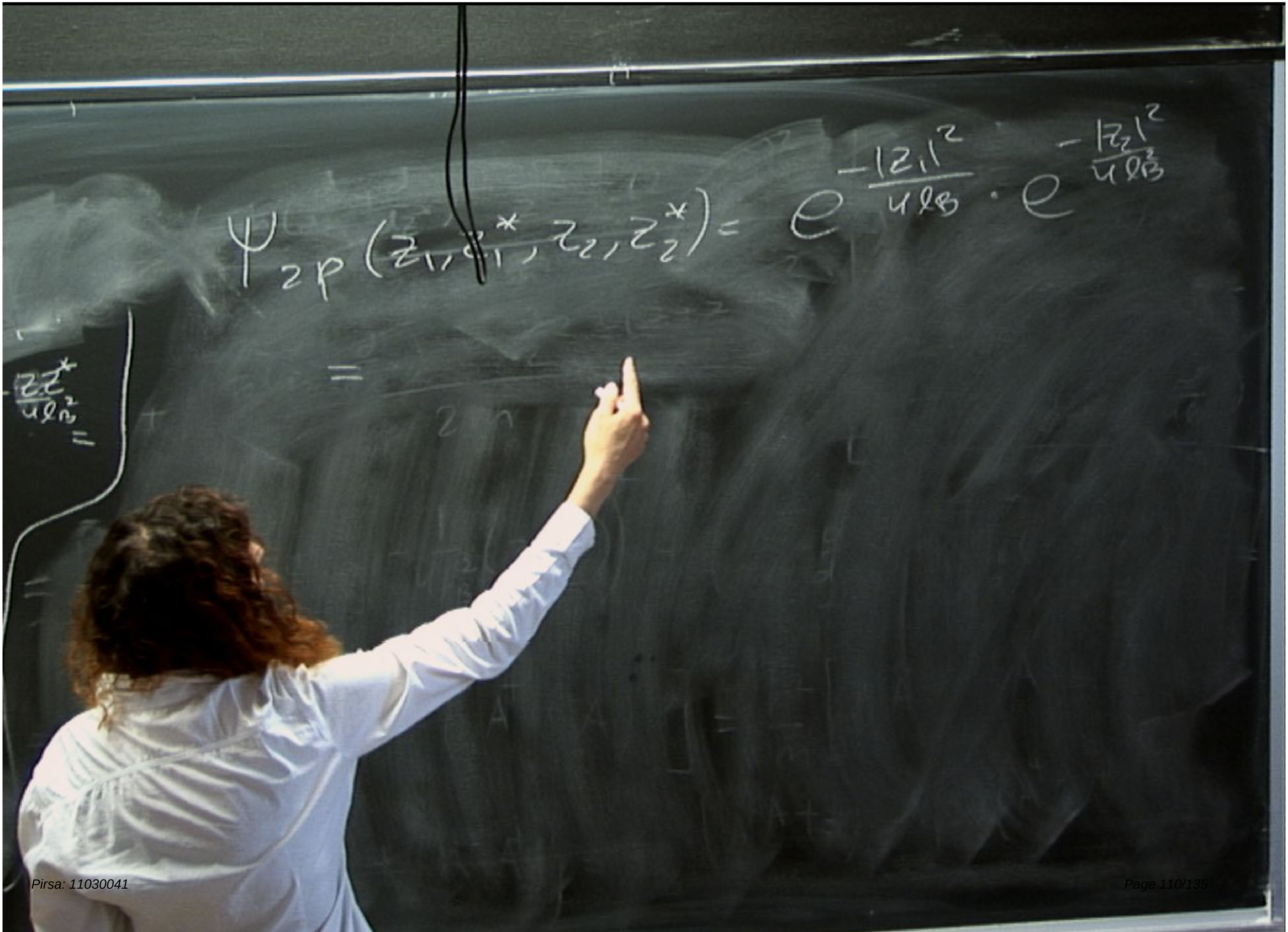


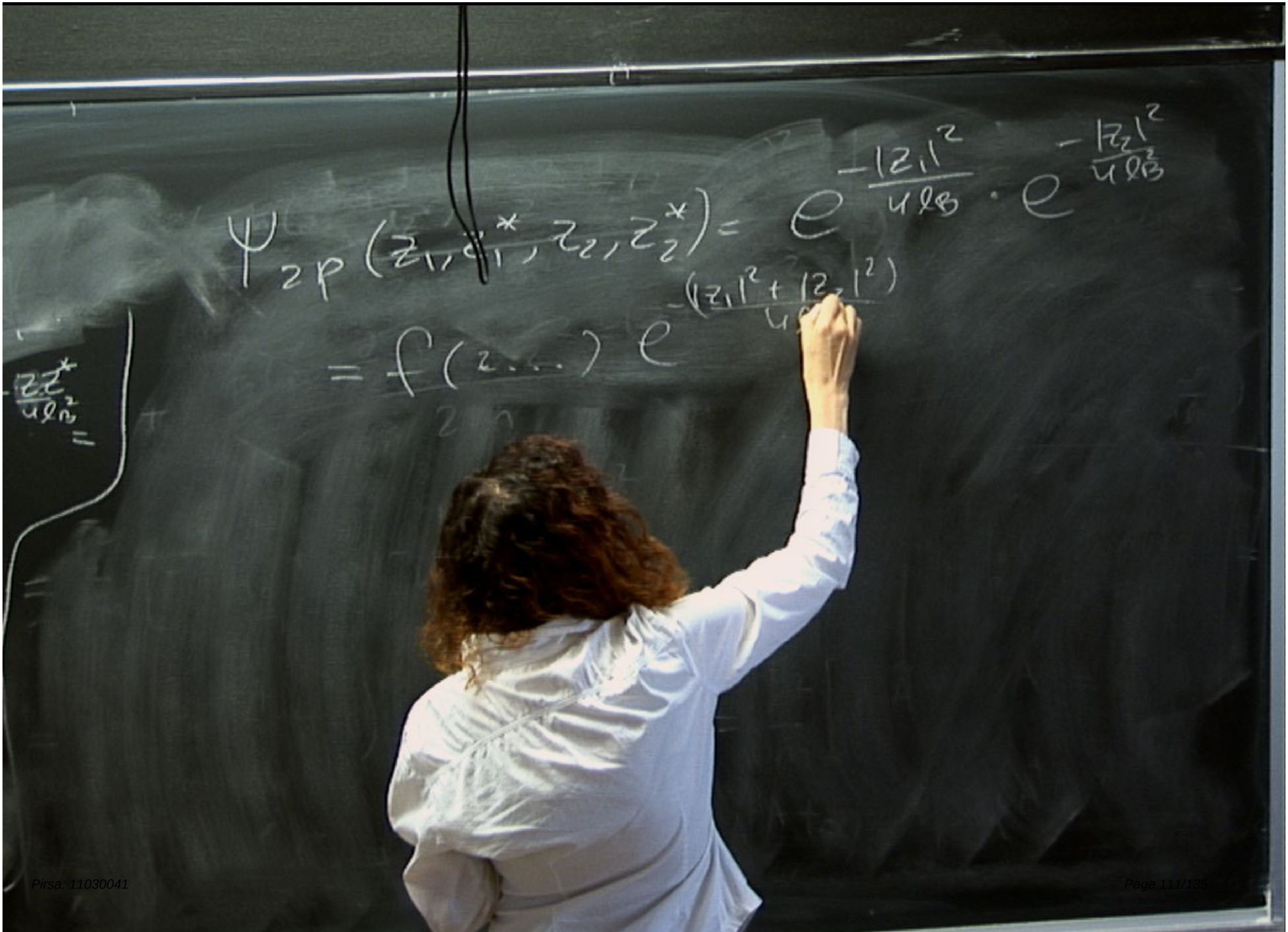
$A\Psi = 0$
$\left(\frac{d}{dz} + \frac{z^*}{4\ell_B^2}\right)\Psi = 0$
$r = |z|$
$\Psi_0 \propto e^{-\frac{z^* z}{4\ell_B^2}}$
$A^\dagger \Psi_0 = \left(-\frac{d}{dz^*} + \frac{z}{4\ell_B^2}\right) e^{-\frac{zz^*}{4\ell_B^2}}$
$= \frac{z}{2\ell_B^2} e^{-|z|^2/4\ell_B^2}$
$\Psi_0' \propto z^* e^{-\frac{z^* z}{4\ell_B^2}}$
$A\Psi_0' = 0$
$(z^*)^m = |z| e^{-im\theta}$
$\Psi_0^m \propto z^{*m} e^{-\frac{zz^*}{4\ell_B^2}}$


Ψ2p (z1,


Ψ2p (z1, z1*, z2, z2*) =


$\Psi_{2p}(z_1, z_1^*, z_2, z_2^*) = e^{-\frac{|z_1|^2}{4\ell_B}} \cdot e^{-\frac{|z_2|^2}{4\ell_B^2}}$

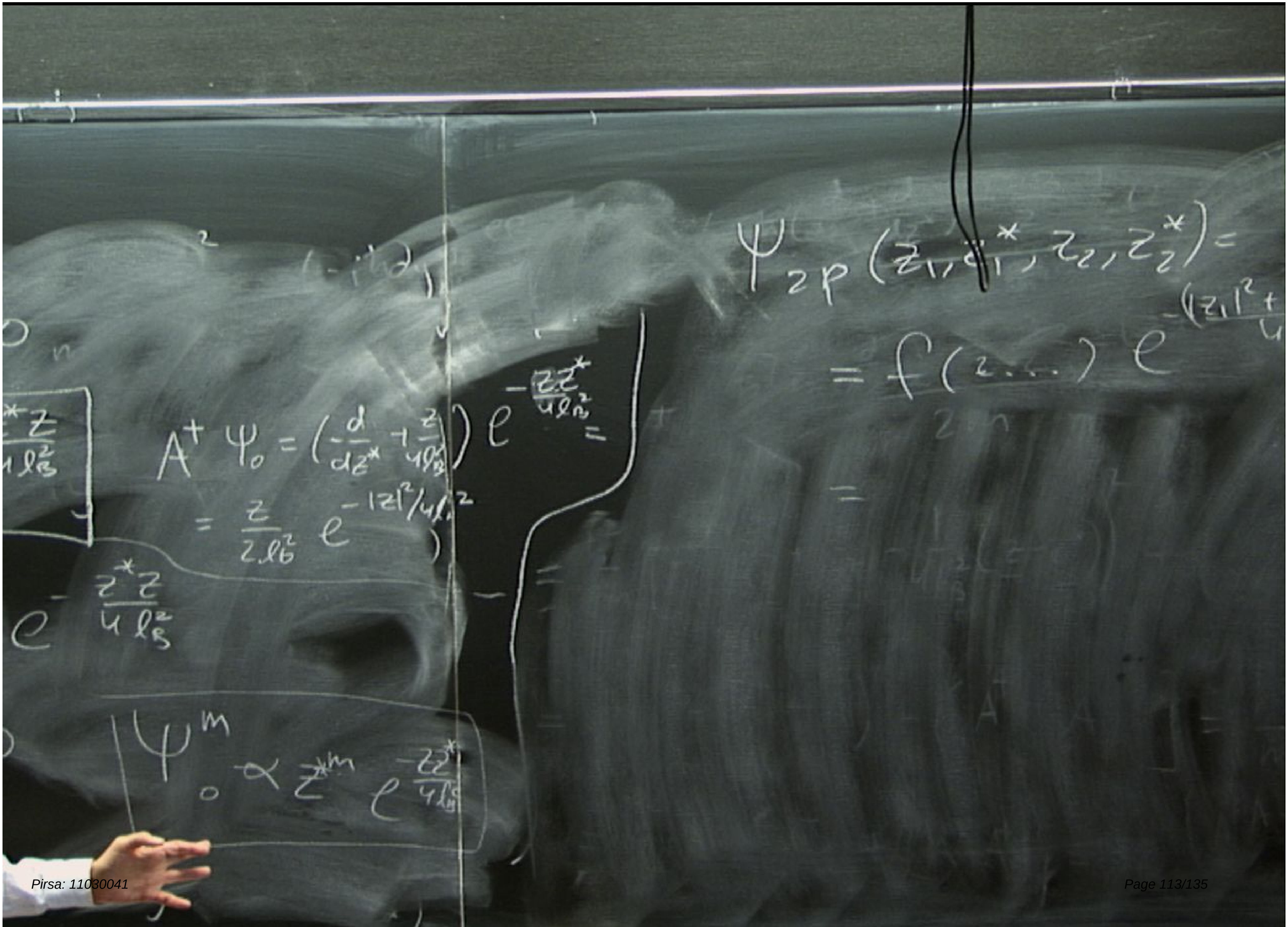

Ψ2p(z1, z1*, z2, z2*) = e^(-|z1|²/4lB) · e^(-|z2|²/4lB)


Ψ2p(z1, z1*, z2, z2*) = e^(-|z1|²/4lB) · e^(-|z2|²/4lB²)
= f(z...) e^(-(|z1|² + |z2|²)/4l...)

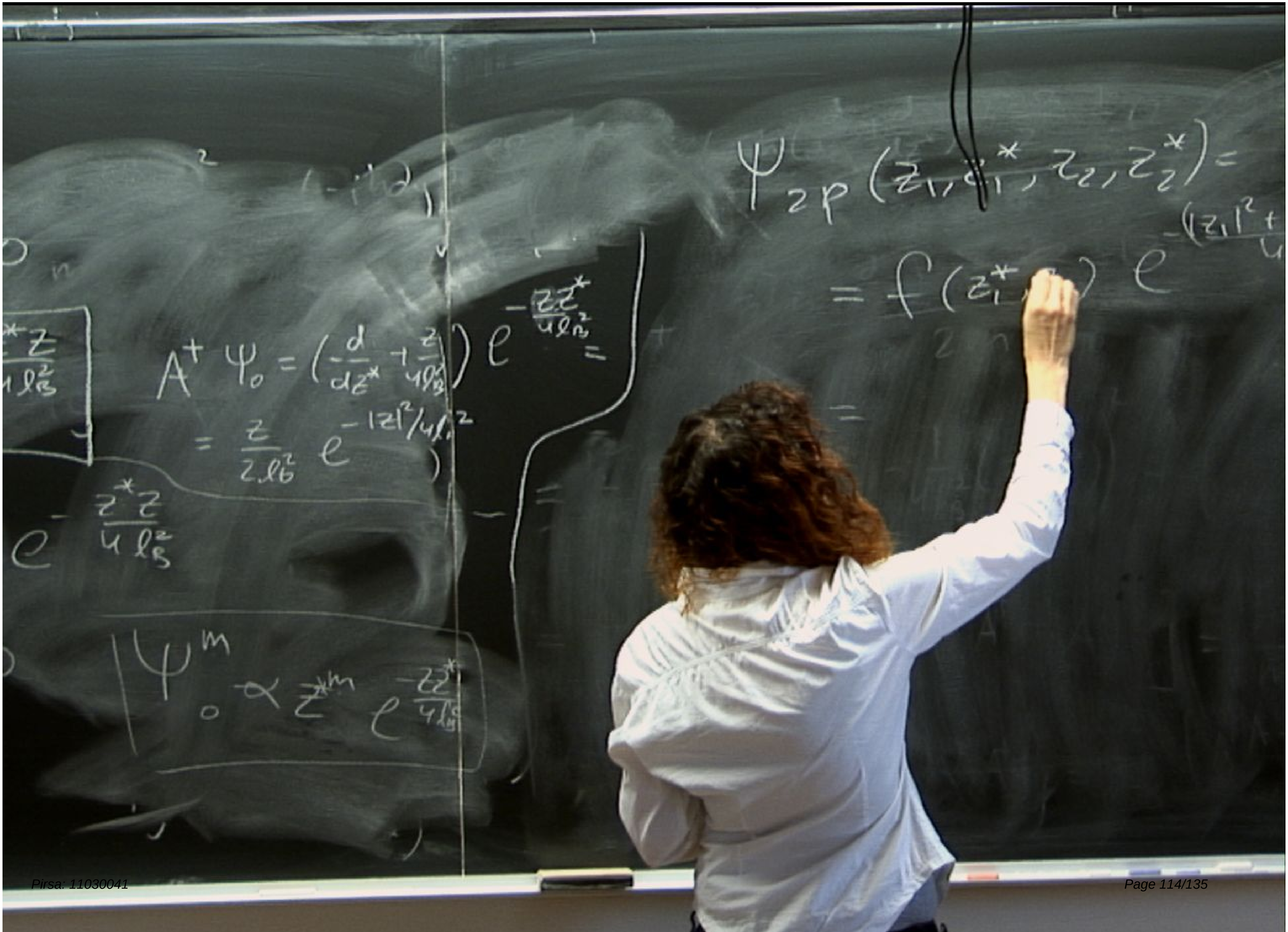
$$\Psi_{2p}(z_1, z_1^*, z_2, z_2^*) = e^{-\frac{|z_1|^2}{4\ell_B}} \cdot e^{-\frac{|z_2|^2}{4\ell_B^2}}$$

$$= f(z \ldots)\, e^{-\frac{(|z_1|^2 + |z_2|^2)}{4\ell_B^2}}$$

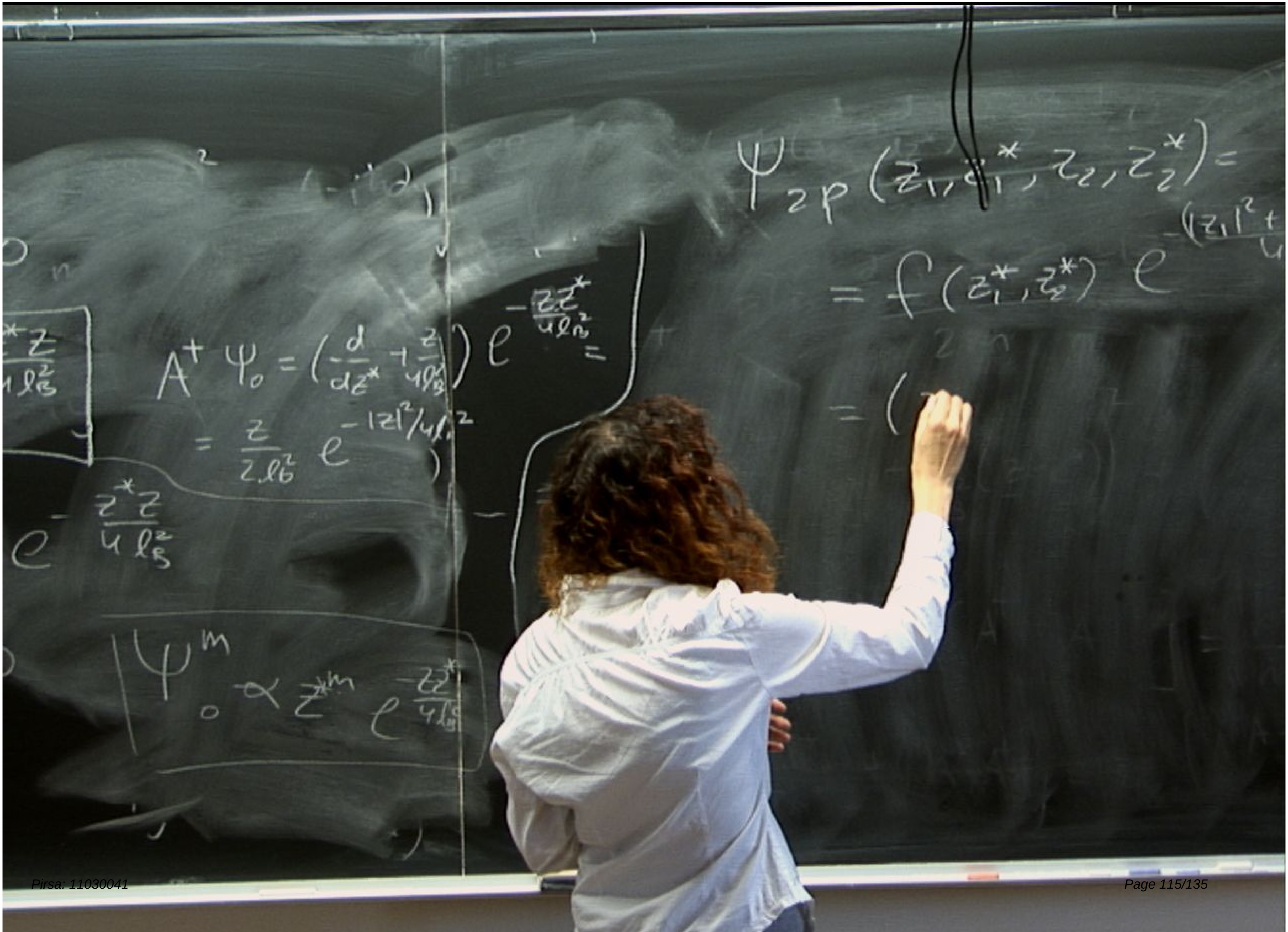
$$-\frac{zz^*}{4\ell_B^2} =$$

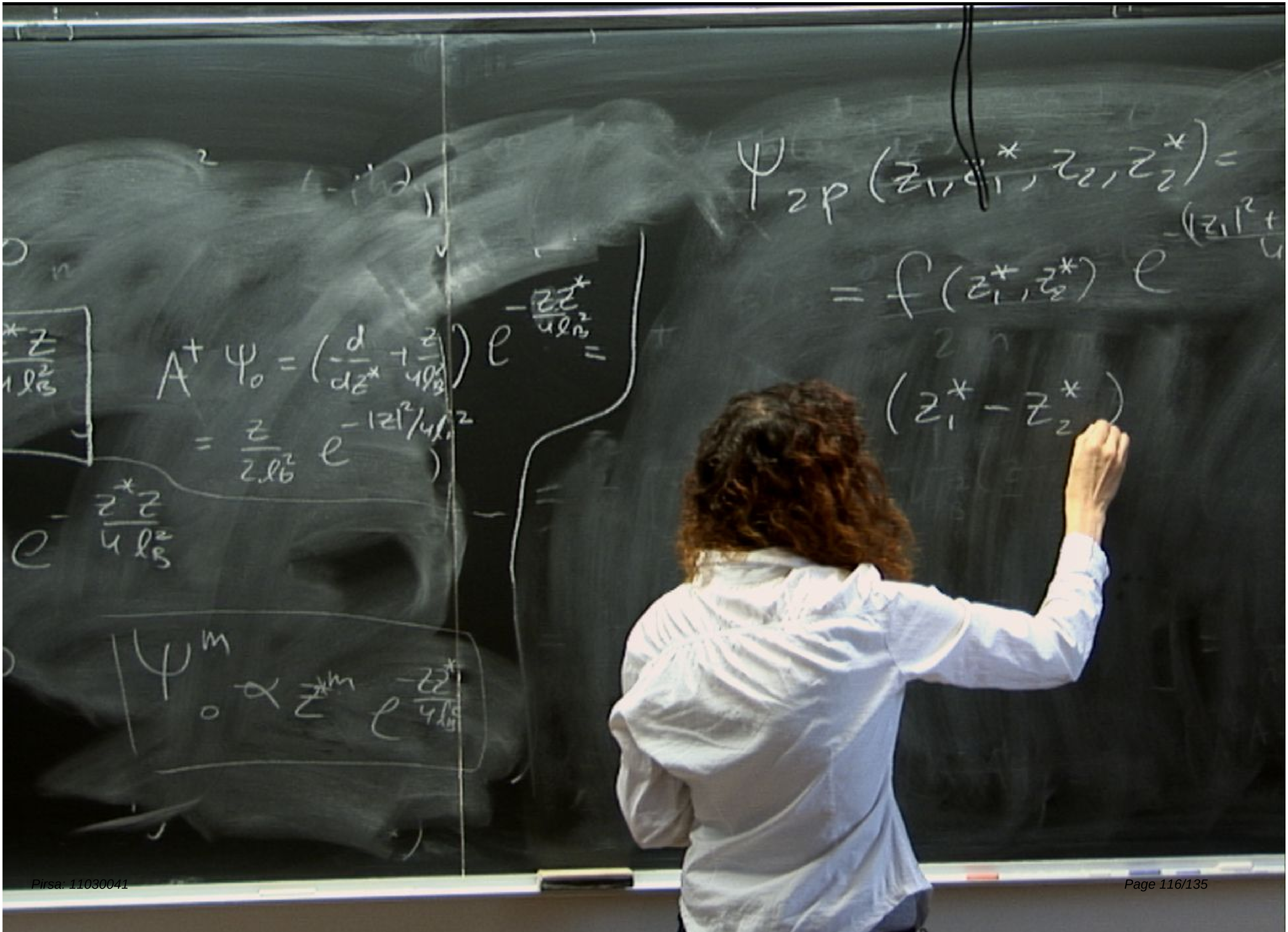


$A^{+}\Psi_0 = \left(-\frac{d}{dz^*} + \frac{z}{4\ell_B^2}\right) e^{-\frac{zz^*}{4\ell_B^2}} =$
$= \frac{z}{2\ell_B^2} e^{-|z|^2/4\ell_B^2}$
$e^{-\frac{z^* z}{4\ell_B^2}}$
$\Psi_0^m \propto z^{*m} e^{-\frac{zz^*}{4\ell_B^2}}$
$\Psi_{2p}(z_1, z_1^*, z_2, z_2^*) =$
$= f(z\ldots)\, e^{-\frac{(|z_1|^2+}{4}}$

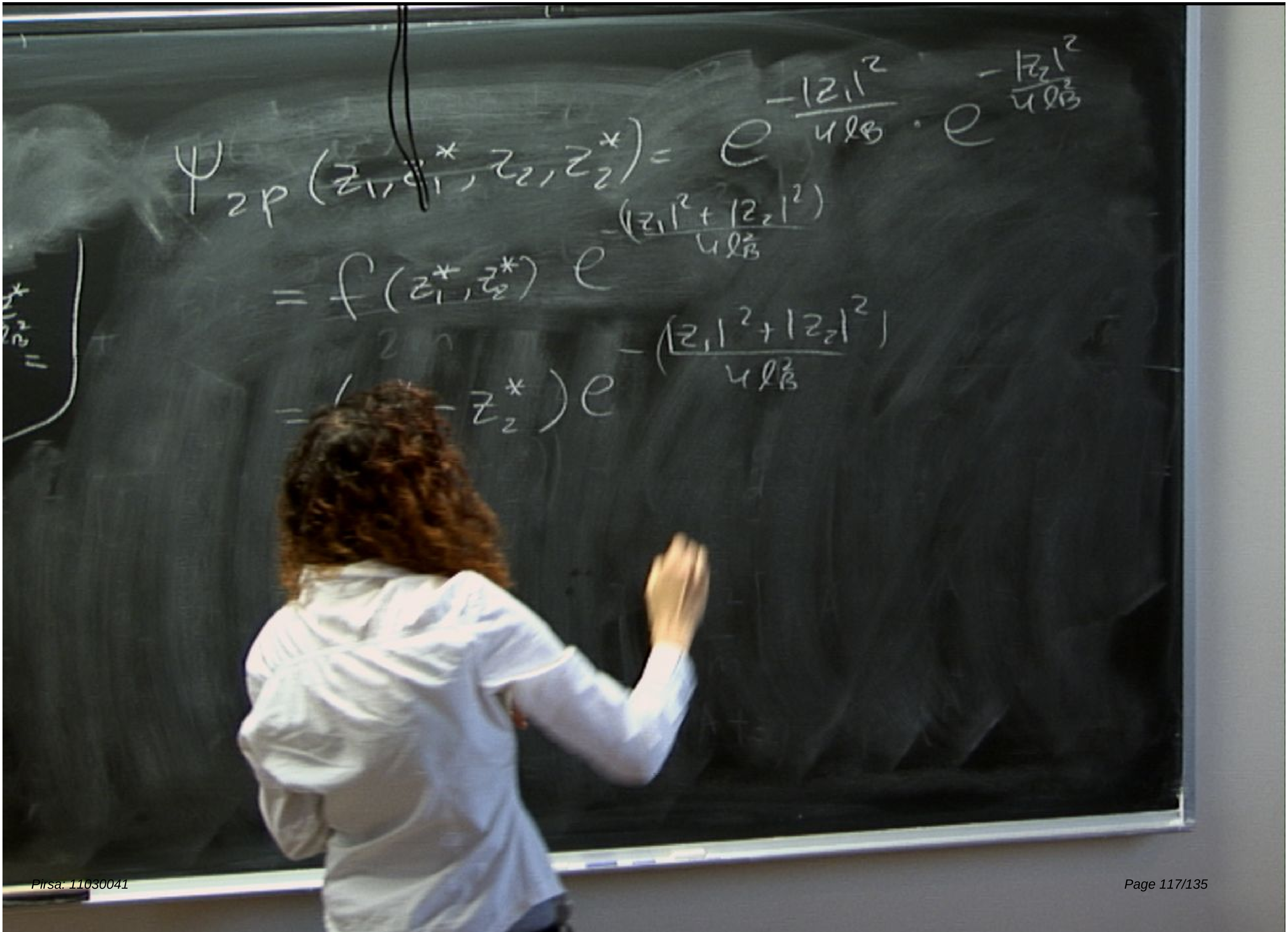


A+ Ψ0 = (-d/dz* + z/4lB²) e^(-zz*/4lB²) =
= z/2lB² e^(-|z|²/4lB²)
Ψ0^m ∝ z*^m e^(-zz*/4lB²)
Ψ2p(z1, z1*, z2, z2*) =

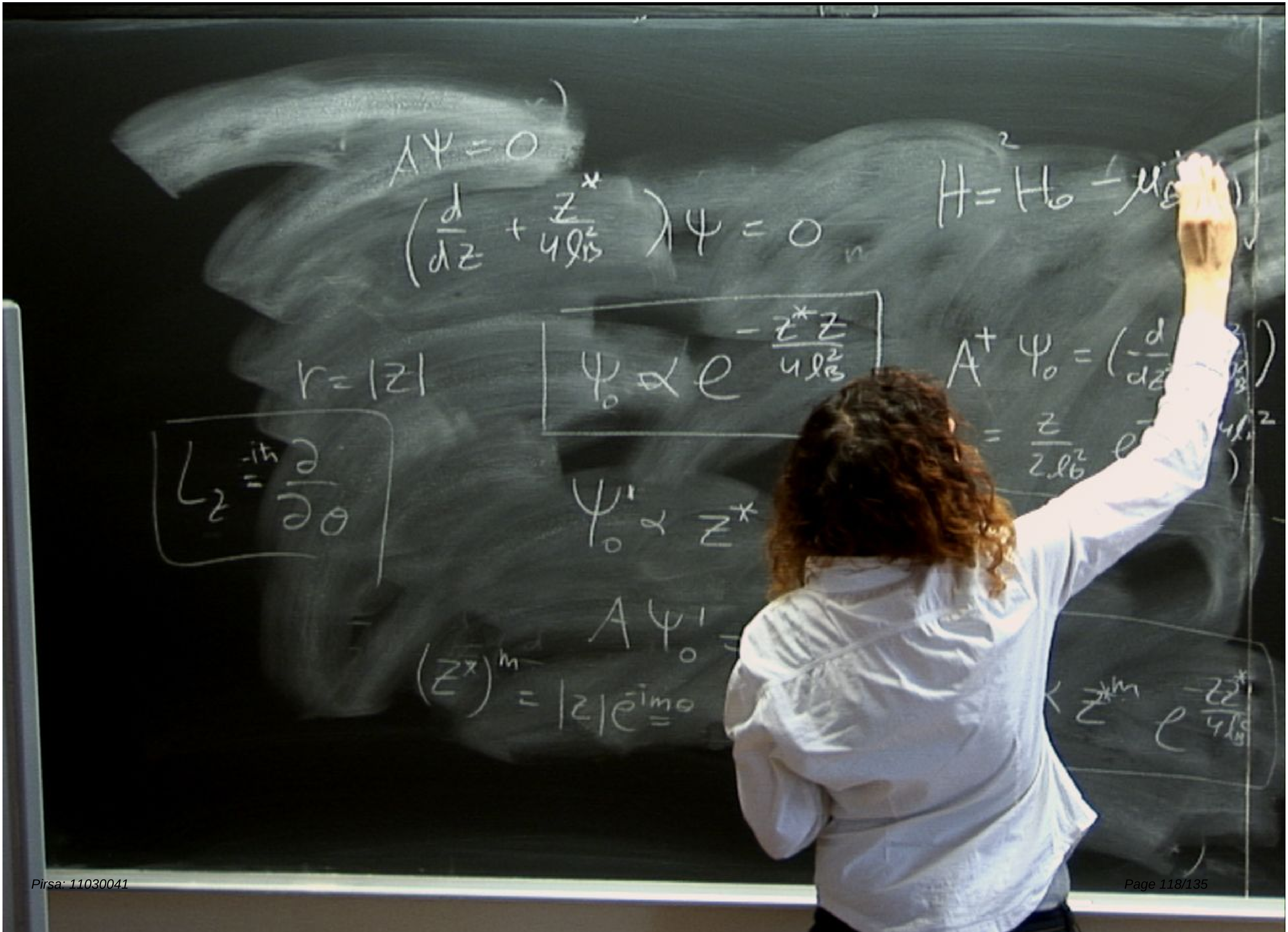

Ψ_2p (z_1, z_1^*, z_2, z_2^*) =
= f (z_1^*, z_2^*) e
A^+ Ψ_0 = (-d/dz^* + z/4l_B^2) e^(-zz^*/4l_B^2) =
= z/2l_B^2 e^(-|z|^2/4l_B^2)
Ψ^m_0 ∝ z^*m e^(-zz^*/4l_B^2)



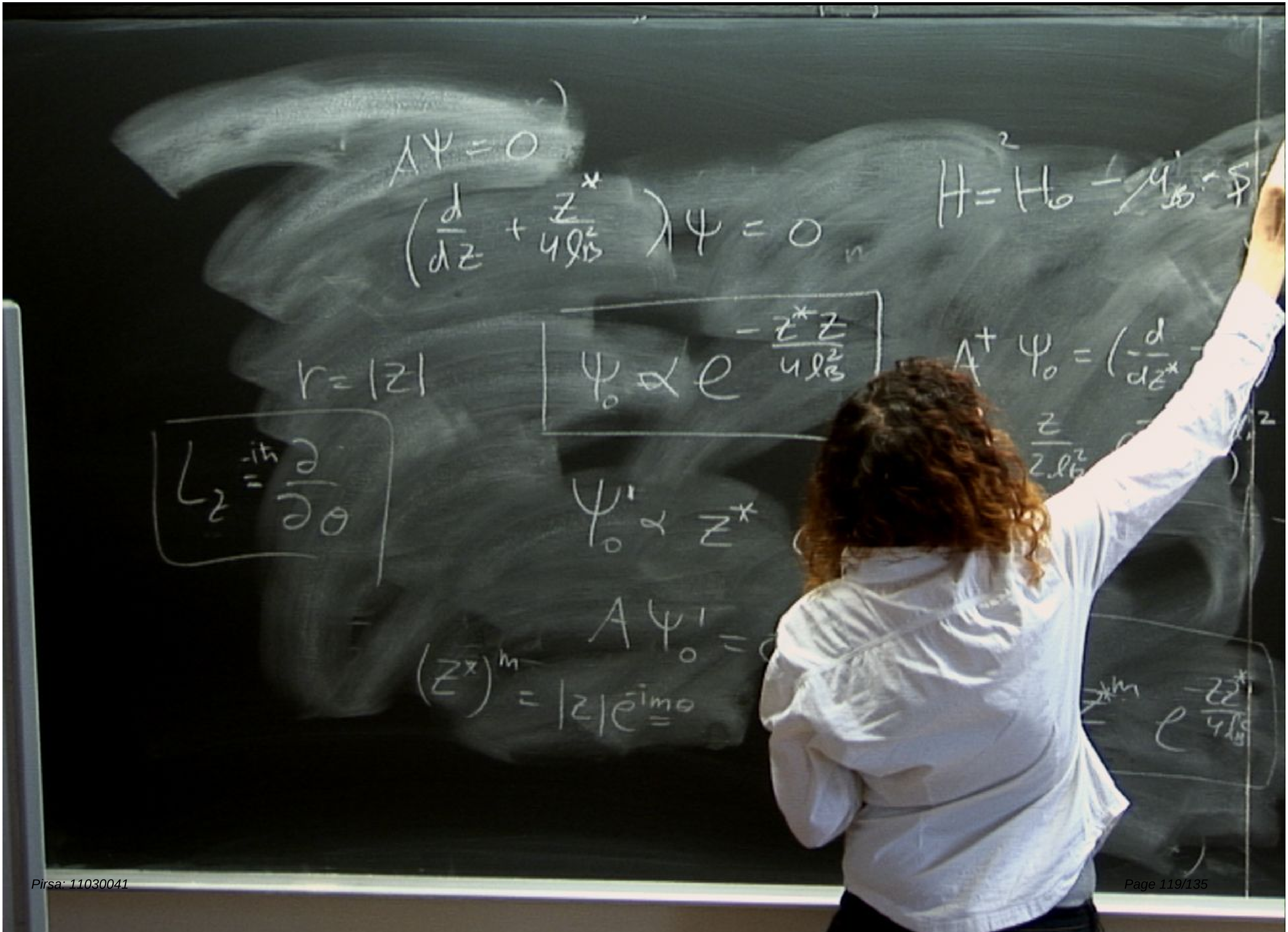
Ψ2p (z1, z1*, z2, z2*) =
= f (z1*, z2*) e
(z1* − z2*)
A+ Ψ0 = (−d/dz* + z/4lB2) e

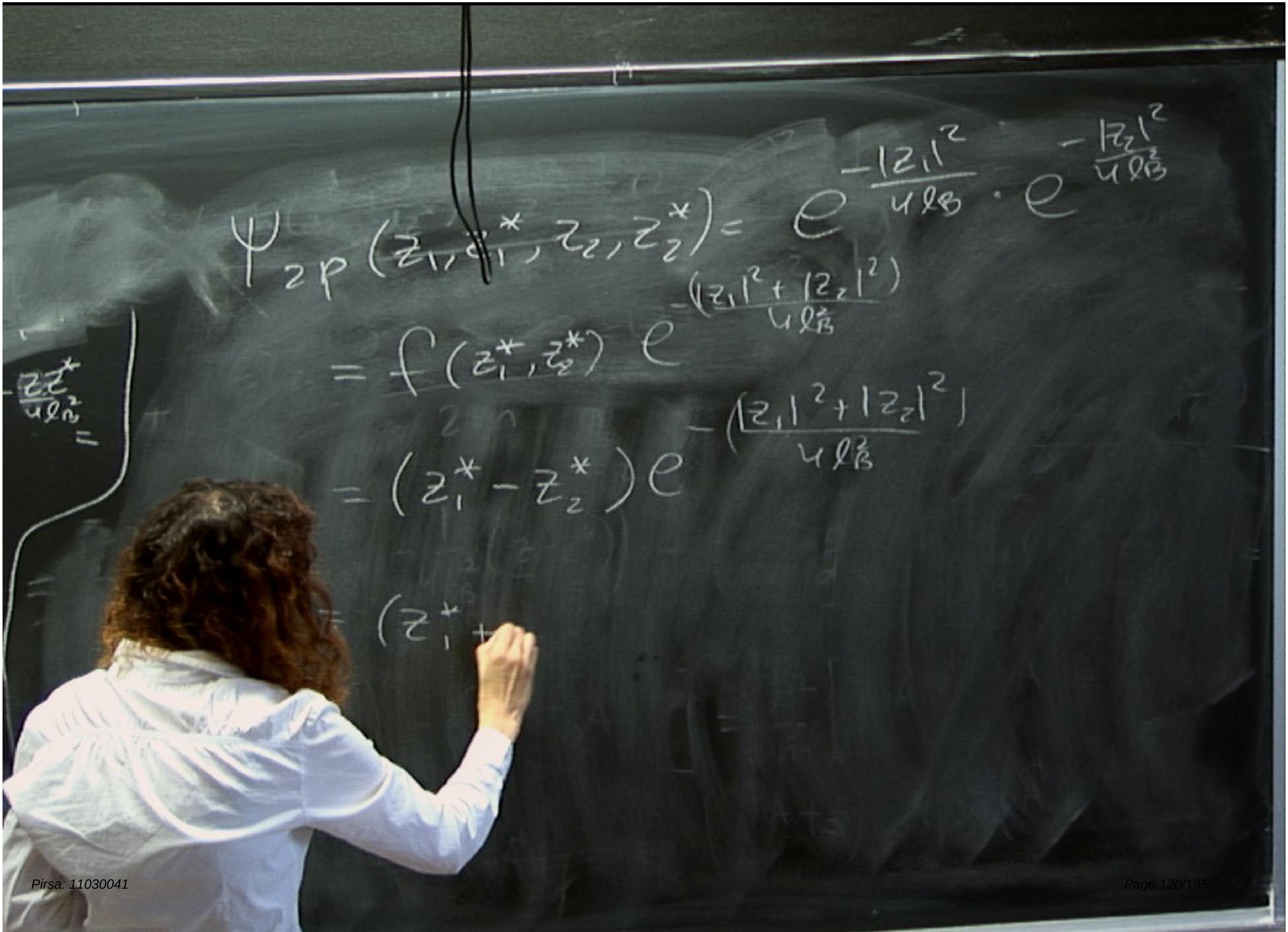


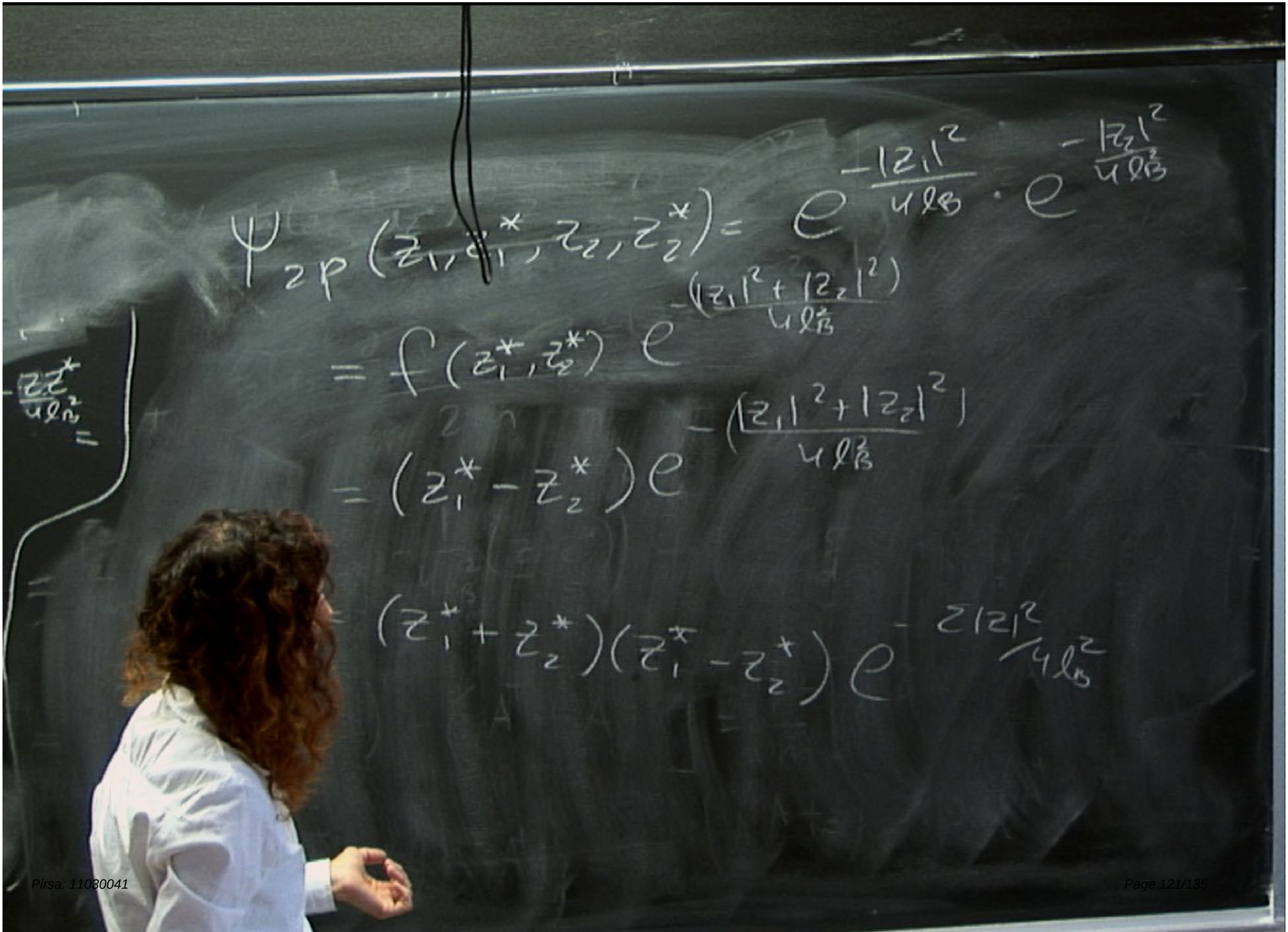
$\Psi_{2p}(z_1, z_1^*, z_2, z_2^*) = e^{-\frac{|z_1|^2}{4\ell_B^2}} \cdot e^{-\frac{|z_2|^2}{4\ell_B^2}}$
$= f(z_1^*, z_2^*)\, e^{-\frac{(|z_1|^2 + |z_2|^2)}{4\ell_B^2}}$



AΨ = 0
(d/dz + z*/4l_B²) Ψ = 0
H = H₀ − μ
r = |z|
Ψ₀ ∝ e^(−z*z/4l_B²)
A⁺ Ψ₀ = (−d/dz
= z/2l_B² e
L_z = −iħ ∂/∂θ
Ψ₀' ∝ z*
AΨ₀' =
(z*)^m = |z| e^(−imθ)
z*^m e^(−zz*/4l_B²)


AΨ = 0
r = |z|
Ψ₀' ∝ z*
AΨ₀' = 0


$\Psi_{2p}(z_1, z_1^*, z_2, z_2^*) = e^{-\frac{|z_1|^2}{4\ell_B^2}} \cdot e^{-\frac{|z_2|^2}{4\ell_B^2}}$
$= f(z_1^*, z_2^*)\, e^{-\frac{(|z_1|^2 + |z_2|^2)}{4\ell_B^2}}$
$= (z_1^* - z_2^*)\, e^{-\frac{(|z_1|^2 + |z_2|^2)}{4\ell_B^2}}$
$= (z_1^* +$
$-\frac{z z^*}{4\ell_B^2}$


$\Psi_{2p}(z_1, z_1^*, z_2, z_2^*) = e^{-\frac{|z_1|^2}{4\ell_B^2}} \cdot e^{-\frac{|z_2|^2}{4\ell_B^2}}$
$= f(z_1^*, z_2^*)\, e^{-\frac{(|z_1|^2 + |z_2|^2)}{4\ell_B^2}}$
$= (z_1^* - z_2^*)\, e^{-\frac{(|z_1|^2 + |z_2|^2)}{4\ell_B^2}}$
$= (z_1^* + z_2^*)(z_1^* - z_2^*)\, e^{-\sum |z|^2 / 4\ell_B^2}$
$-\frac{zz^*}{4\ell_B^2}$

$$-\frac{zz^*}{4\ell_B^2} =$$

$$\psi_{2p}(z_1, z_1^*, z_2, z_2^*) = e^{-\frac{|z_1|^2}{4\ell_B^2}} \cdot e^{-\frac{|z_2|^2}{4\ell_B^2}}$$

$$= f(z_1^*, z_2^*)\, e^{-\frac{(|z_1|^2 + |z_2|^2)}{4\ell_B^2}}$$

$$= (z_1^* - z_2^*)\, e^{-\frac{(|z_1|^2 + |z_2|^2)}{4\ell_B^2}}$$

$$= (z_1^* + z_2^*)(z_1^* - z_2^*)\, e^{-\sum |z_i|^2 / 4\ell_B^2}$$

$$V(|r|)$$

$$e^{-\frac{zz^*}{4\ell_B^2}} =$$

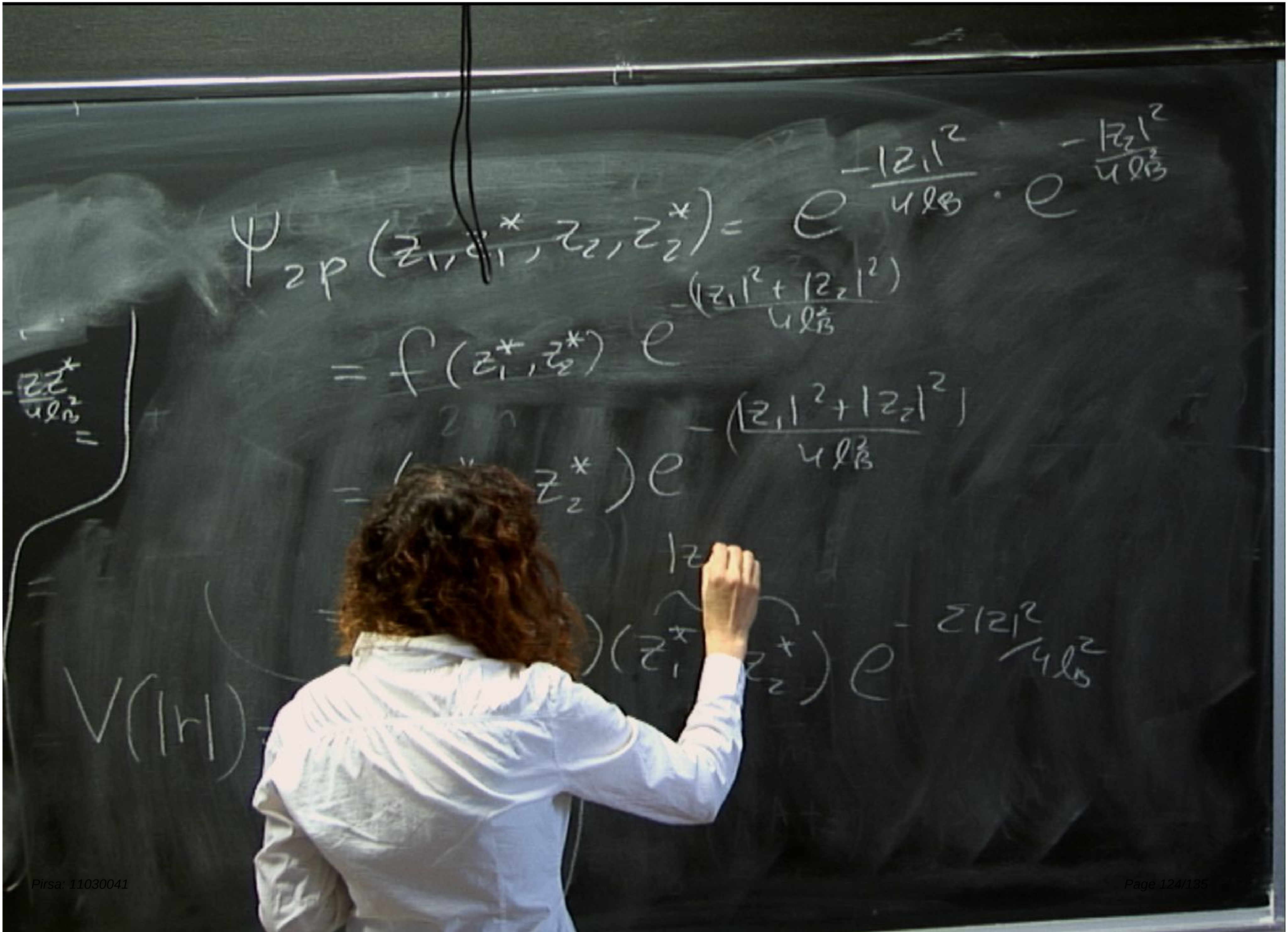
$$\Psi_{2p}(z_1, z_1^*, z_2, z_2^*) = e^{-\frac{|z_1|^2}{4\ell_B^2}} \cdot e^{-\frac{|z_2|^2}{4\ell_B^2}}$$

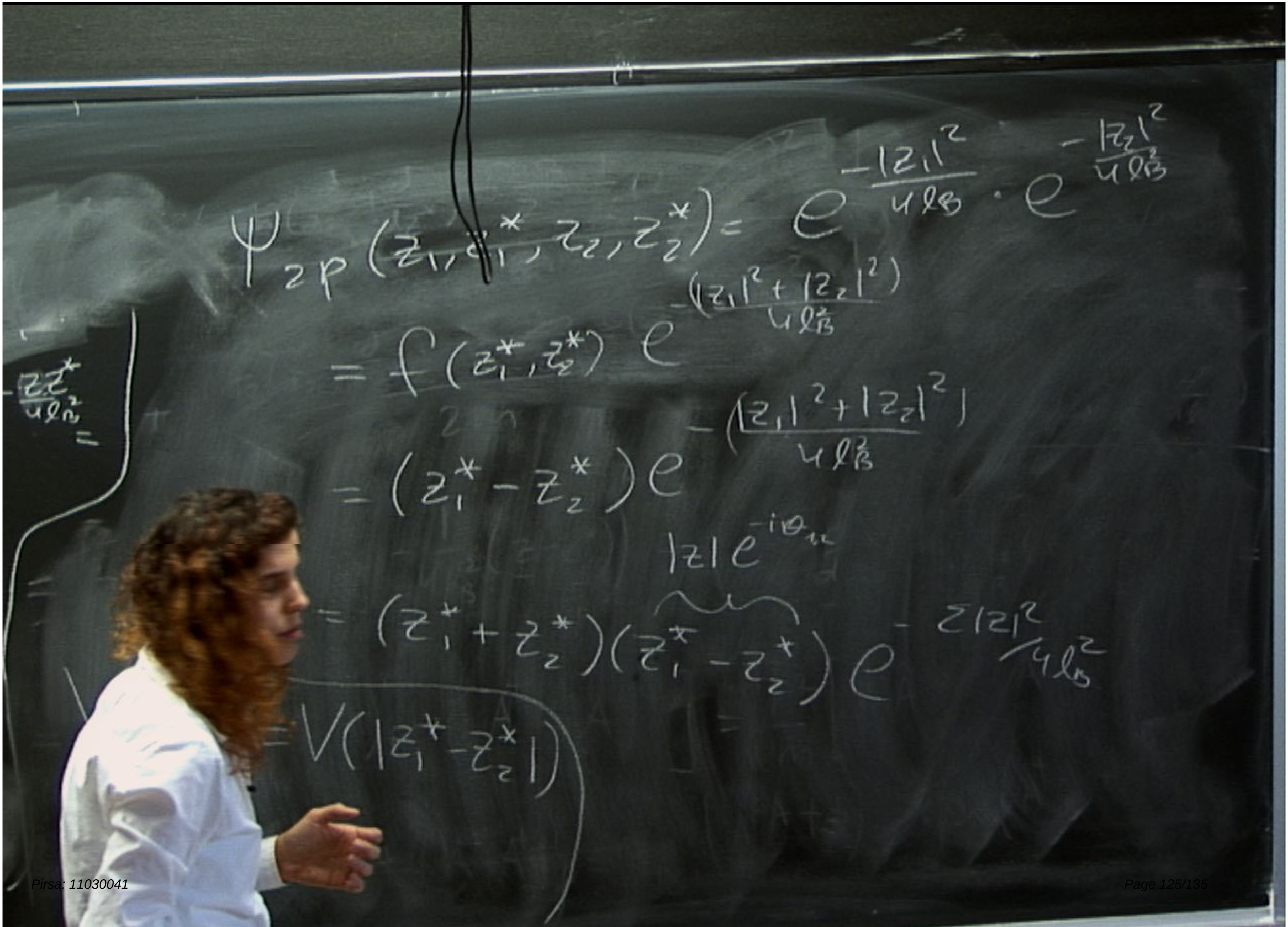
$$= f(z_1^*, z_2^*)\, e^{-\frac{(|z_1|^2+|z_2|^2)}{4\ell_B^2}}$$

$$= (z_1^* - z_2^*)\, e^{-\frac{(|z_1|^2+|z_2|^2)}{4\ell_B^2}}$$

$$= (z_1^* + z_2^*)(z_1^* - z_2^*)\, e^{-\Sigma |z|^2 / 4\ell_B^2}$$

$$= V(|z_1^* - z_2^*|)$$


Ψ2p(z1, z1*, z2, z2*) = e^(−|z1|²/4ℓB²) · e^(−|z2|²/4ℓB²)
= f(z1*, z2*) e^(−(|z1|² + |z2|²)/4ℓB²)
V(|r|) =


$\Psi_{2p}(z_1, z_1^*, z_2, z_2^*) = e^{-\frac{|z_1|^2}{4\ell_B}} \cdot e^{-\frac{|z_2|^2}{4\ell_B^2}}$
$= f(z_1^*, z_2^*)\, e^{-\frac{(|z_1|^2+|z_2|^2)}{4\ell_B^2}}$
$= (z_1^* - z_2^*)\, e^{-\frac{(|z_1|^2+|z_2|^2)}{4\ell_B^2}}$
$|z| e^{-i\theta_{12}}$
$= (z_1^* + z_2^*)(z_1^* - z_2^*)\, e^{-\sum |z|^2 / 4\ell_B^2}$
$V(|z_1^* - z_2^*|)$

$$-\frac{z z^*}{4\ell_B^2} =$$

$$\Psi_{2p}(z_1, z_1^*, z_2, z_2^*) = e^{-\frac{|z_1|^2}{4\ell_B^2}} \cdot e^{-\frac{|z_2|^2}{4\ell_B^2}}$$

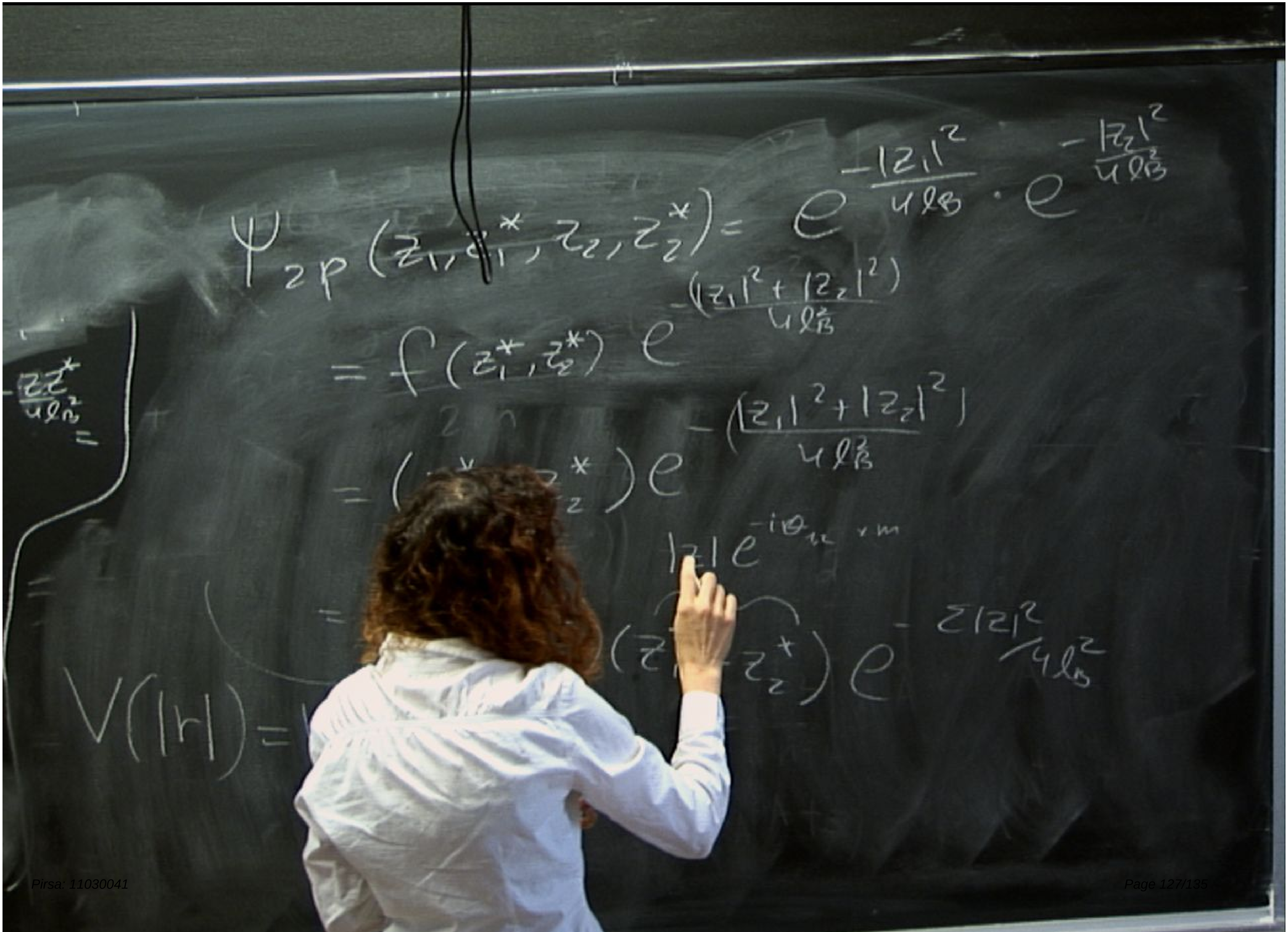
$$= f(z_1^*, z_2^*)\, e^{-\frac{(|z_1|^2+|z_2|^2)}{4\ell_B^2}}$$

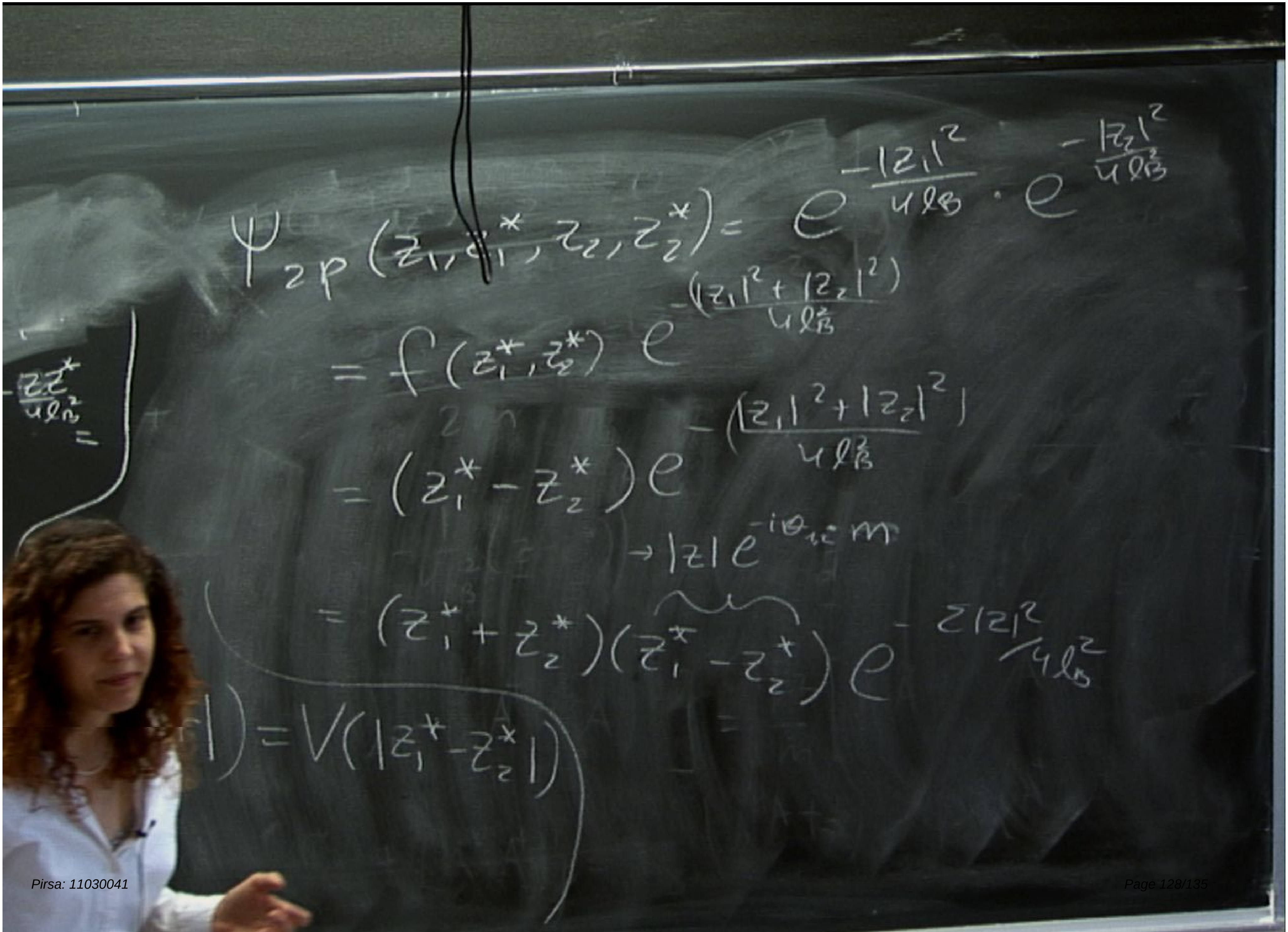
$$= (z_1^* - z_2^*)\, e^{-\frac{(|z_1|^2+|z_2|^2)}{4\ell_B^2}}$$

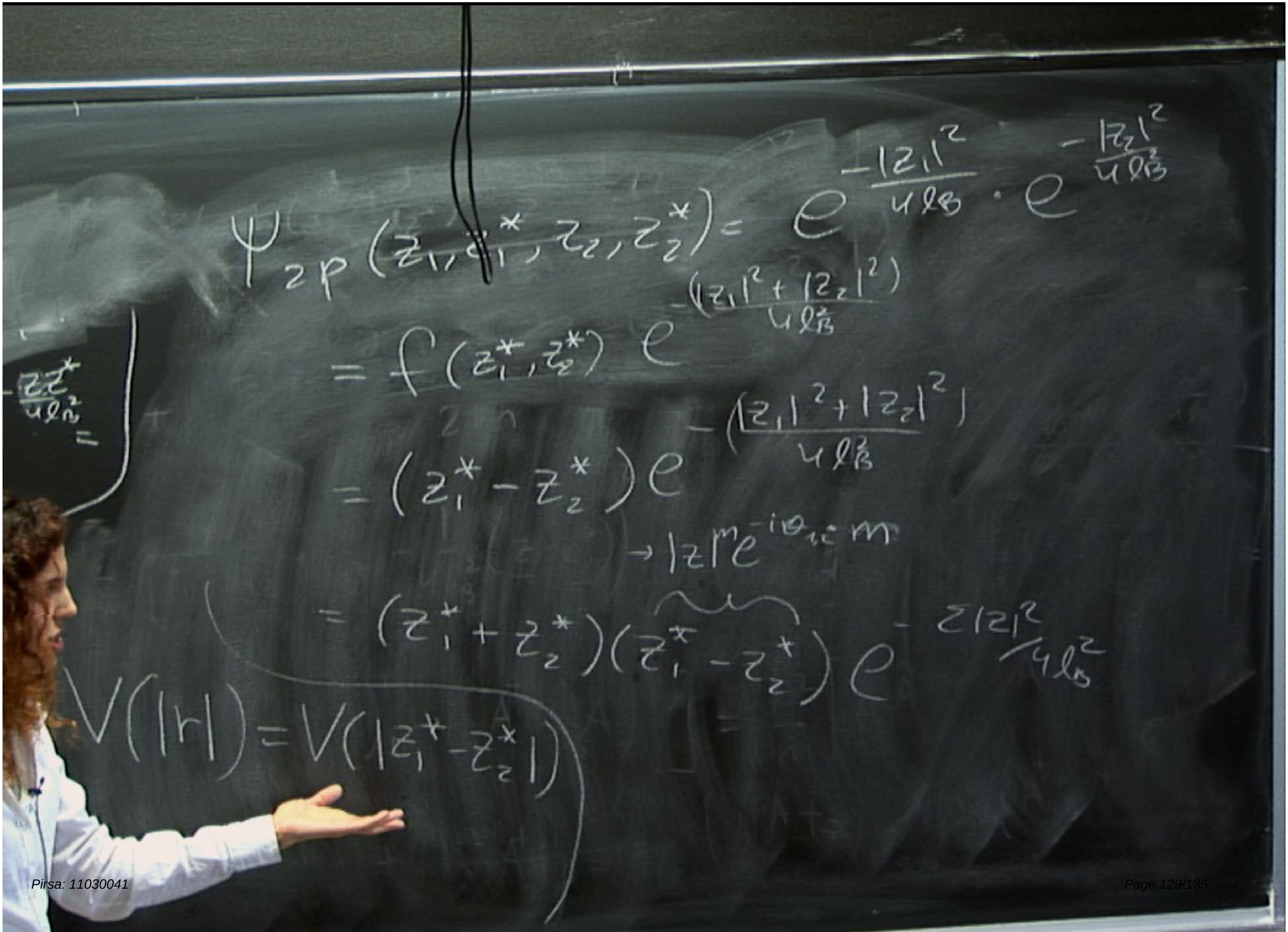
$$|z|\, e^{-i\theta_{12}}$$

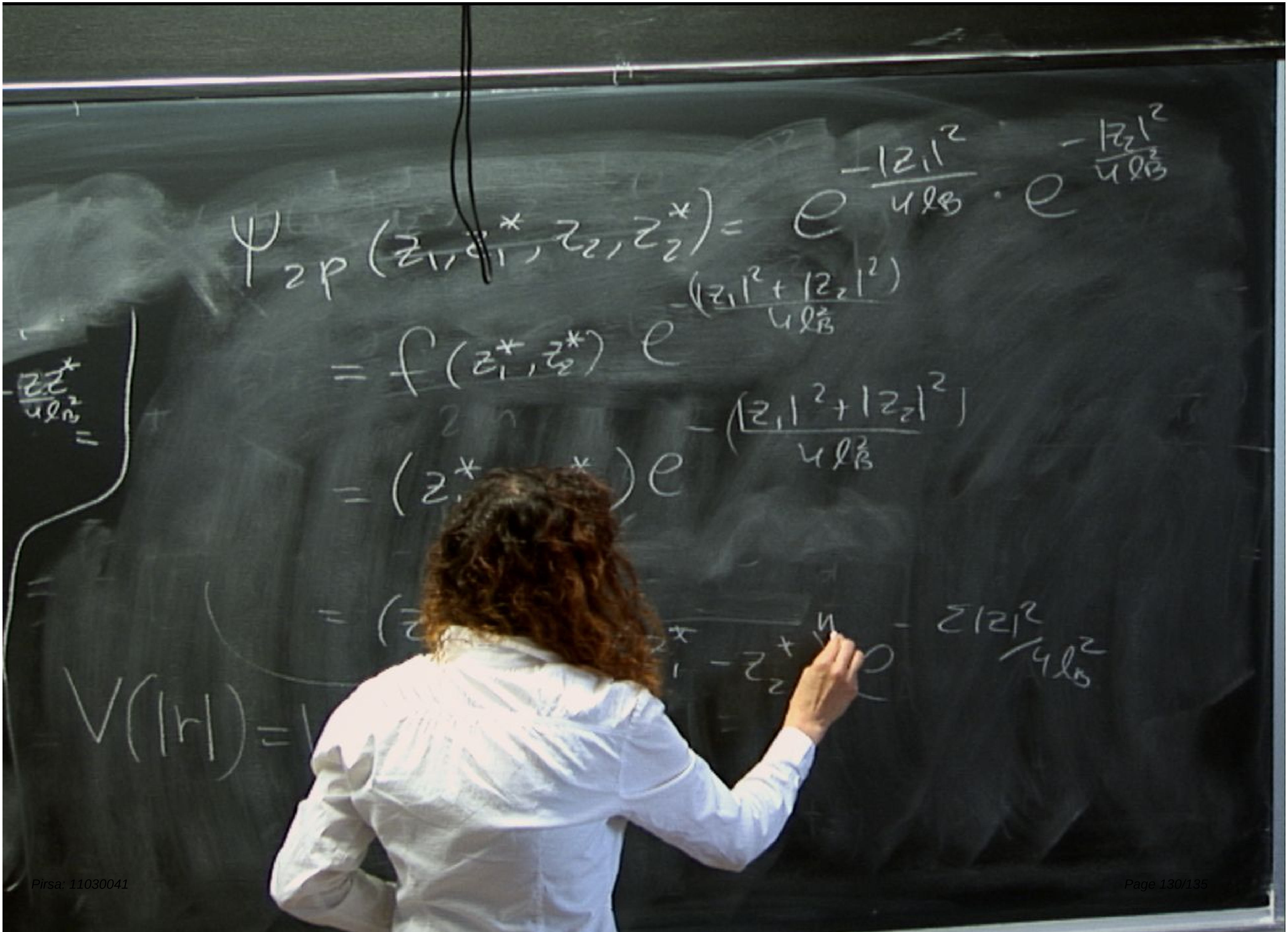
$$= (z_1^* + z_2^*)\underbrace{(z_1^* - z_2^*)}\, e^{-\sum |z_i|^2 / 4\ell_B^2}$$

$$V(|r|) = V(|z_1^* - z_2^*|)$$


$\Psi_{2p}(z_1, z_1^*, z_2, z_2^*) = e^{-\frac{|z_1|^2}{4\ell_B}} \cdot e^{-\frac{|z_2|^2}{4\ell_B^2}}$
$= f(z_1^*, z_2^*)\, e^{-\frac{(|z_1|^2+|z_2|^2)}{4\ell_B^2}}$
$e^{-\frac{(|z_1|^2+|z_2|^2)}{4\ell_B^2}}$
$|z|\, e^{-i\theta_{12} \times m}$
$(z_1^* - z_2^*)\, e^{-\frac{\Sigma |z_i|^2}{4\ell_B^2}}$
$V(|r|) =$


Ψ_2p(z_1, z_1*, z_2, z_2*) = e^(−|z_1|²/4l_B²) · e^(−|z_2|²/4l_B²)
= f(z_1*, z_2*) e^(−(|z_1|² + |z_2|²)/4l_B²)
= (z_1* − z_2*) e^(−(|z_1|² + |z_2|²)/4l_B²)
→ |z| e^(−iθ_12 m)
= (z_1* + z_2*)(z_1* − z_2*) e^(−Σ|z|²/4l_B²)
= V(|z_1* − z_2*|)


$\Psi_{2p}(z_1, z_1^*, z_2, z_2^*) = e^{-\frac{|z_1|^2}{4 l_B^2}} \cdot e^{-\frac{|z_2|^2}{4 l_B^2}}$
$= f(z_1^*, z_2^*)\, e^{-\frac{(|z_1|^2 + |z_2|^2)}{4 l_B^2}}$
$= (z_1^* - z_2^*)\, e^{-\frac{(|z_1|^2 + |z_2|^2)}{4 l_B^2}}$
$\rightarrow |z|^m e^{-i\theta_{12} m}$
$= (z_1^* + z_2^*)(z_1^* - z_2^*)\, e^{-\Sigma |z|^2 / 4 l_B^2}$
$V(|r|) = V(|z_1^* - z_2^*|)$
$-\frac{z z^*}{4 l_B^2}$ =



$\Psi_{2p}(z_1, z_1^*, z_2, z_2^*) = e^{-\frac{|z_1|^2}{4\ell_B^2}} \cdot e^{-\frac{|z_2|^2}{4\ell_B^2}}$
V(|r|) =

$-\frac{zz^*}{4\ell_B^2} =$

$$\Psi_{2p}(z_1, z_1^*, z_2, z_2^*) = e^{-\frac{|z_1|^2}{4\ell_B}} \cdot e^{-\frac{|z_2|^2}{4\ell_B^2}}$$

$$= f(z_1^*, z_2^*)\, e^{-\frac{(|z_1|^2+|z_2|^2)}{4\ell_B^2}}$$

$$= (z_1^* - z_2^*)\, e^{-\frac{(|z_1|^2+|z_2|^2)}{4\ell_B^2}}$$

$$= (z_1^* + z_2^*)(z_1^* - z_2^*)^m\, e^{-\sum |z|^2 / 4\ell_B^2}$$

$$V(|r|) = V(|z_1^* - z_2^*|)$$

$$\Psi_{2p}(z_1, z_1^*, z_2, z_2^*) = e^{-\frac{|z_1|^2}{4\ell_B^2}} \cdot e^{-\frac{|z_2|^2}{4\ell_B^2}}$$

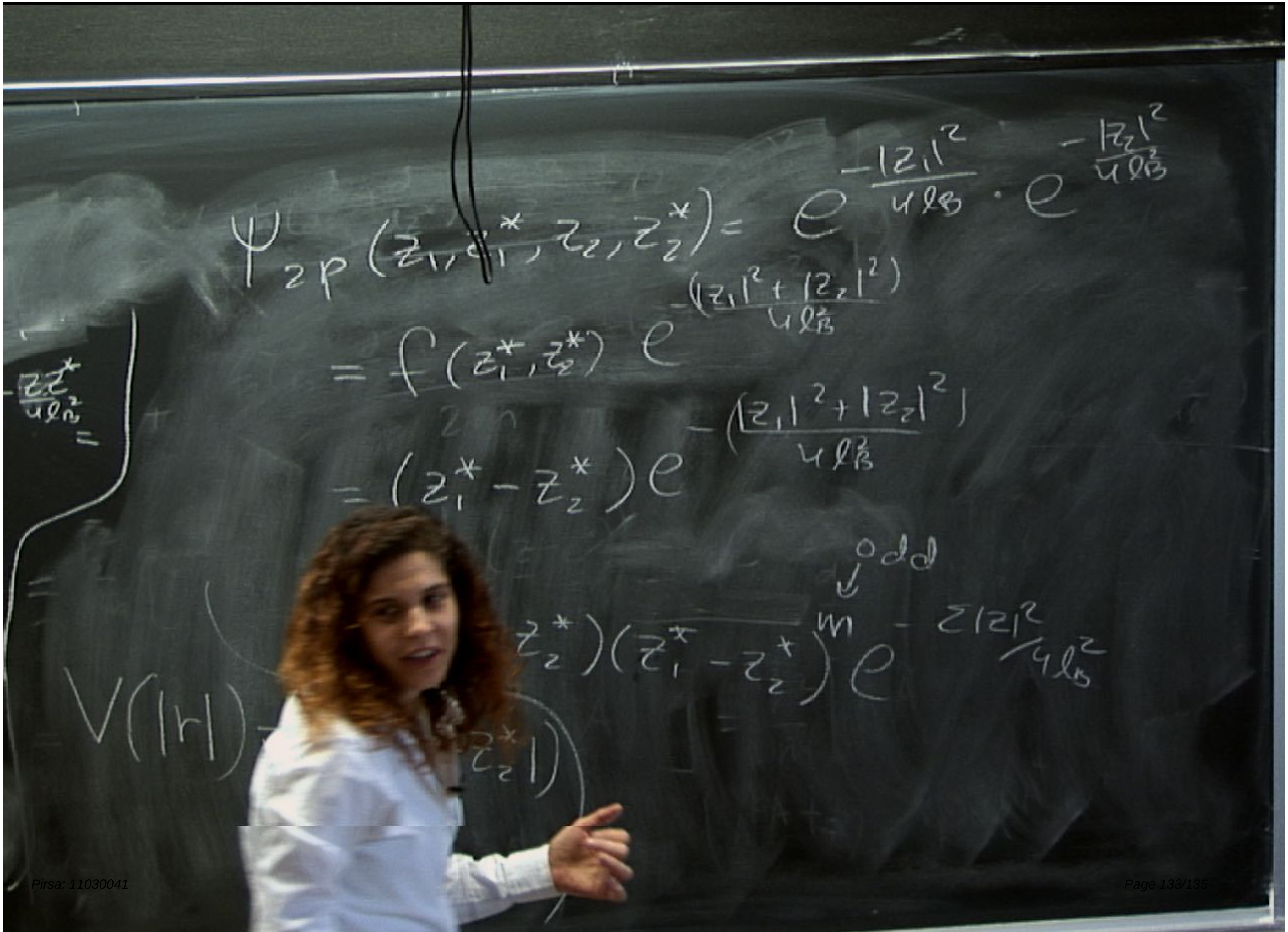
$$= f(z_1^*, z_2^*)\, e^{-\frac{(|z_1|^2 + |z_2|^2)}{4\ell_B^2}}$$

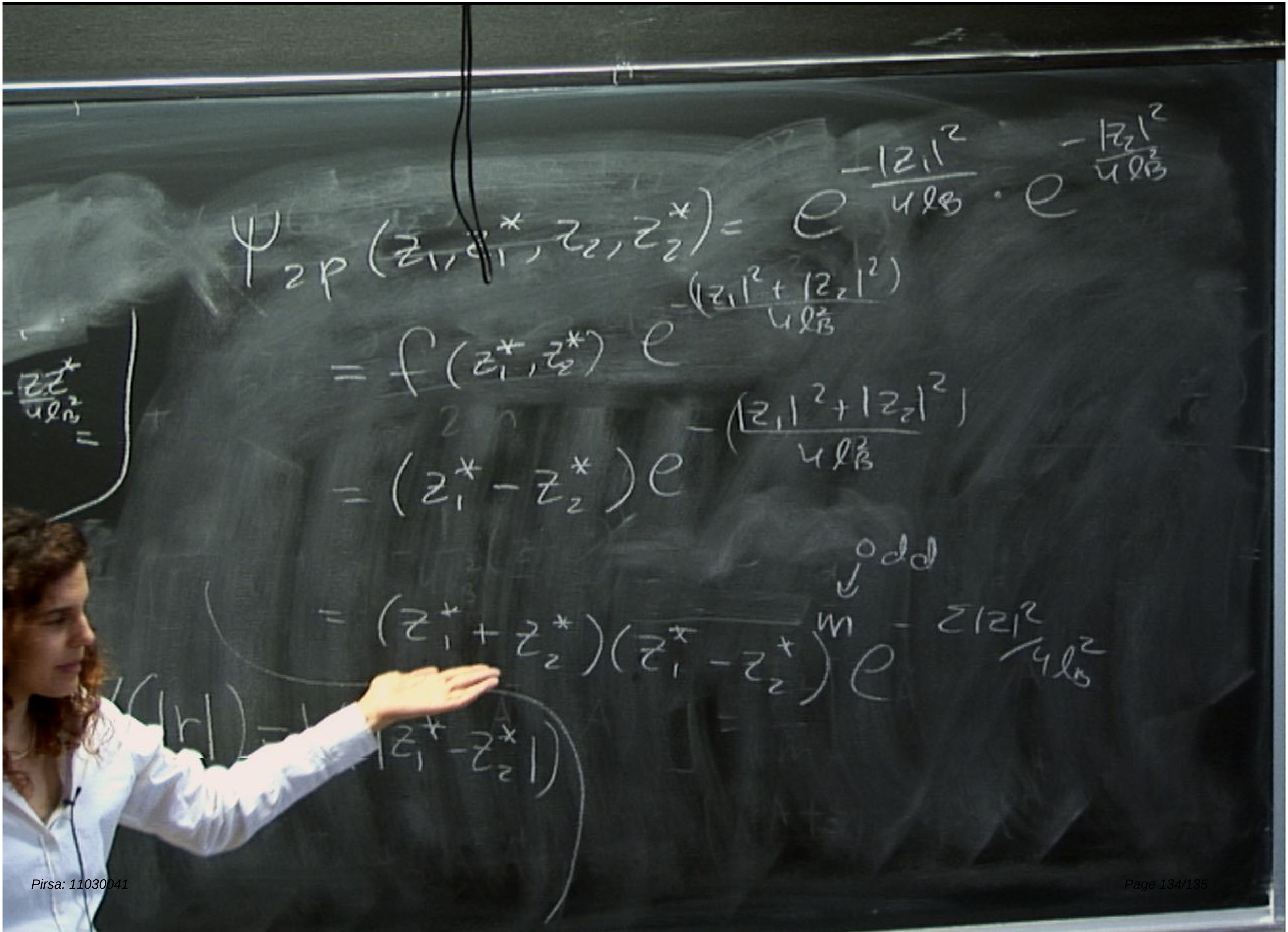
$$= (z_1^* - z_2^*)\, e^{-\frac{(|z_1|^2 + |z_2|^2)}{4\ell_B^2}}$$

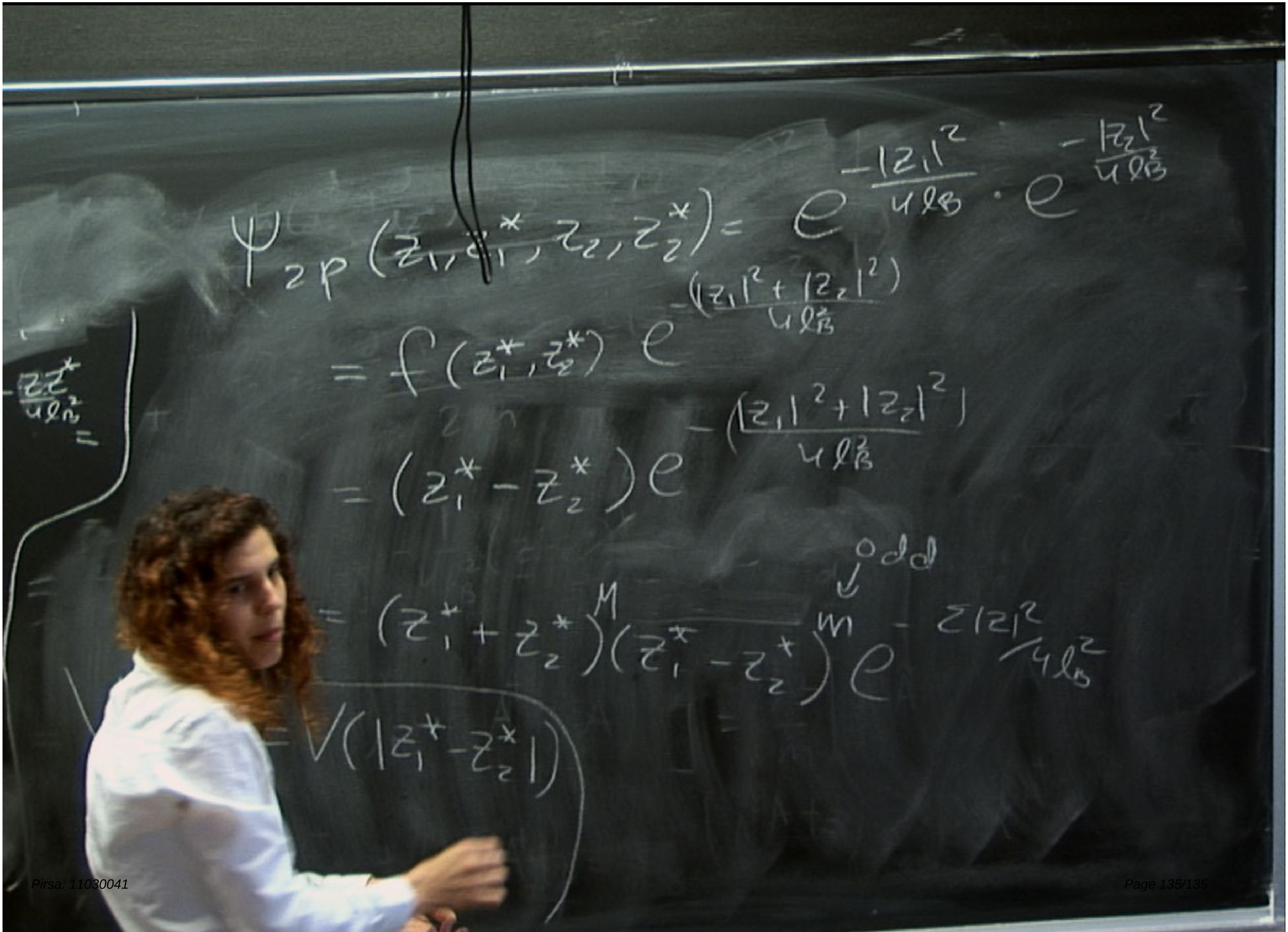
$$= (z_1^* + z_2^*)(z_1^* - z_2^*)^m e^{-\frac{\Sigma |z|^2}{4\ell_B^2}}$$

$$-\frac{zz^*}{4\ell_B^2} =$$

$$V(|r|) = V(|z_1^* - z_2^*|)$$


ψ2p(z1, z1*, z2, z2*) = e^(−|z1|²/4ℓB²) · e^(−|z2|²/4ℓB²)
= f(z1*, z2*) e^(−(|z1|² + |z2|²)/4ℓB²)
= (z1* − z2*) e^(−(|z1|² + |z2|²)/4ℓB²)
odd
V(|r|)


$\Psi_{2p}(z_1, z_1^*, z_2, z_2^*) = e^{-\frac{|z_1|^2}{4\ell_B}} \cdot e^{-\frac{|z_2|^2}{4\ell_B^2}}$
$= f(z_1^*, z_2^*)\, e^{-\frac{(|z_1|^2+|z_2|^2)}{4\ell_B^2}}$
$= (z_1^* - z_2^*)\, e^{-\frac{(|z_1|^2+|z_2|^2)}{4\ell_B^2}}$
odd
$= (z_1^* + z_2^*)(z_1^* - z_2^*)^m\, e^{-\Sigma|z|^2/4\ell_B^2}$
$V(|r|)$
$|z_1^* - z_2^*|)$



Ψ_2p(z_1, z_1^*, z_2, z_2^*) = e^{-|z_1|^2/4ℓ_B} · e^{-|z_2|^2/4ℓ_B^2}
= f(z_1^*, z_2^*) e^{-(|z_1|^2 + |z_2|^2)/4ℓ_B^2}
= (z_1^* - z_2^*) e^{-(|z_1|^2 + |z_2|^2)/4ℓ_B^2}
odd
= (z_1^* + z_2^*)^M (z_1^* - z_2^*)^m e^{-Σ|z|^2/4ℓ_B^2}
= V(|z_1^* - z_2^*|)