Title: Explorations in Condensed Matter - Lecture 7

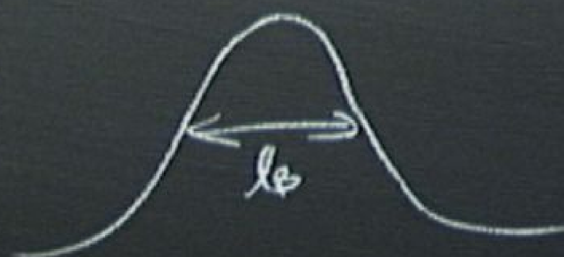
Date: Mar 22, 2011 10:15 AM

URL: http://pirsa.org/11030040

Abstract:

1. Edge states - all of the current carried by them

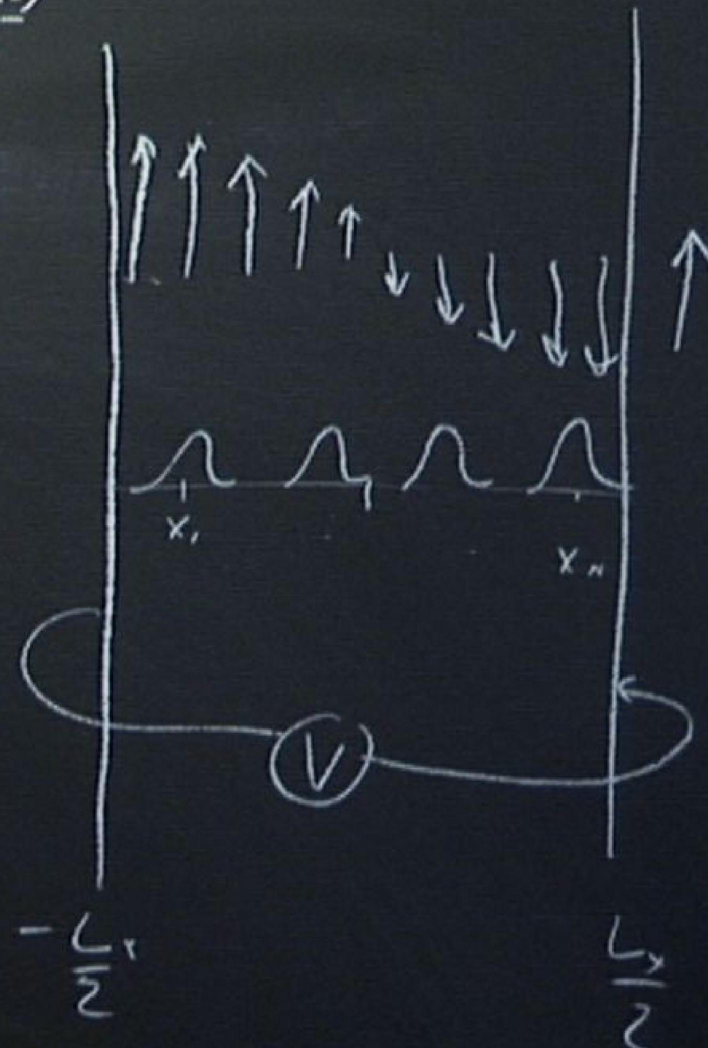
$$\psi = e^{ik_y y} H_n(x - x_0)\, e^{-\frac{(x-x_0)^2}{2 l_B^2}}$$



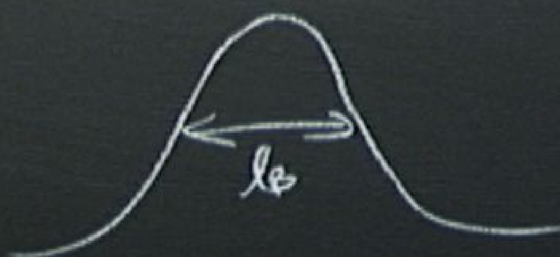
$$x_0 = k_y l_B^2$$

$$x_0 \rightarrow x_0 + \Delta x$$

$$\Delta x = \frac{eE}{B^2}$$



$$\psi = e^{ik_y y} H_n(x - x_0)\, e^{-\frac{(x-x_0)^2}{2 l_B^2}}$$

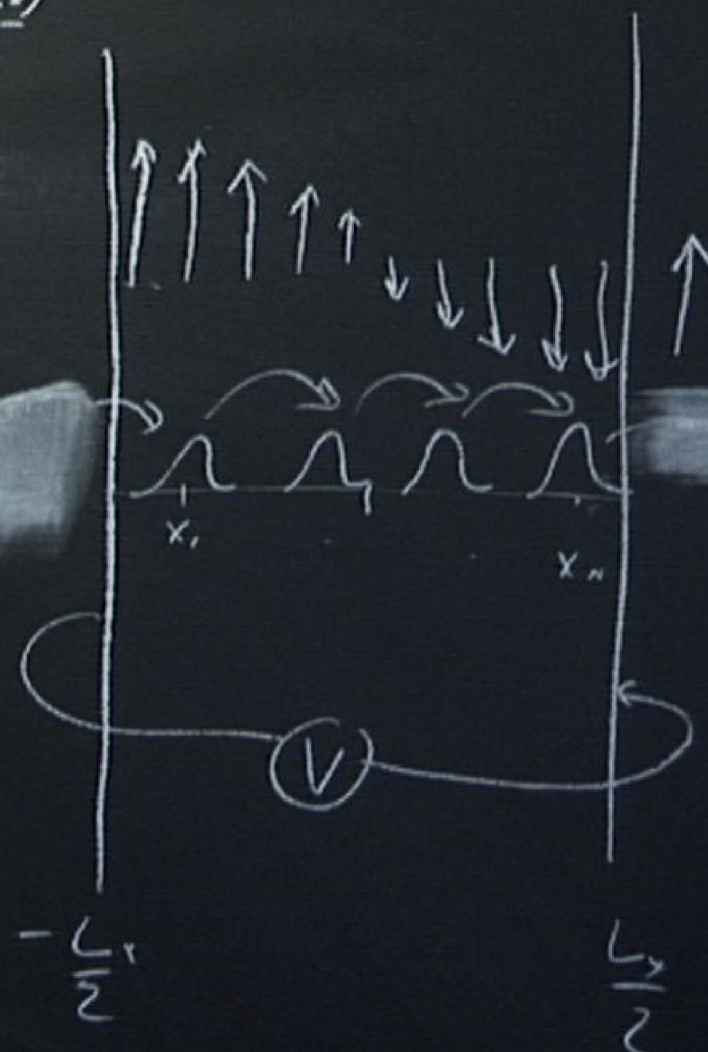


$$\sigma_{xx} = 0$$

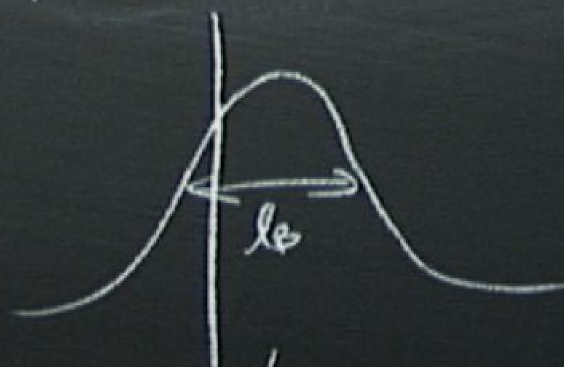
$$x_0 = \underline{\underline{k_y l_B^2}}$$

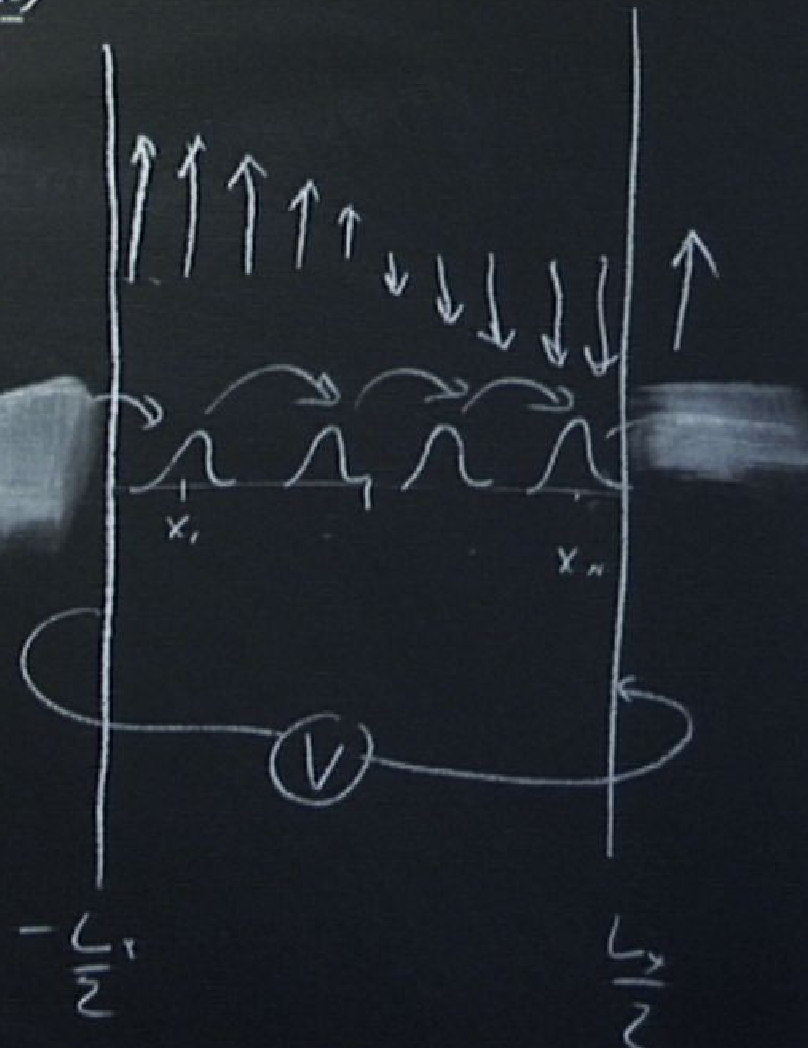
$$x_0 \to x_0 + \Delta x$$

$$\Delta x = \frac{eE}{B^2}$$



$$\Psi = e^{iky} H_n(x-x_0)\, e^{-\frac{(x-x_0)^2}{2l_B^2}}$$



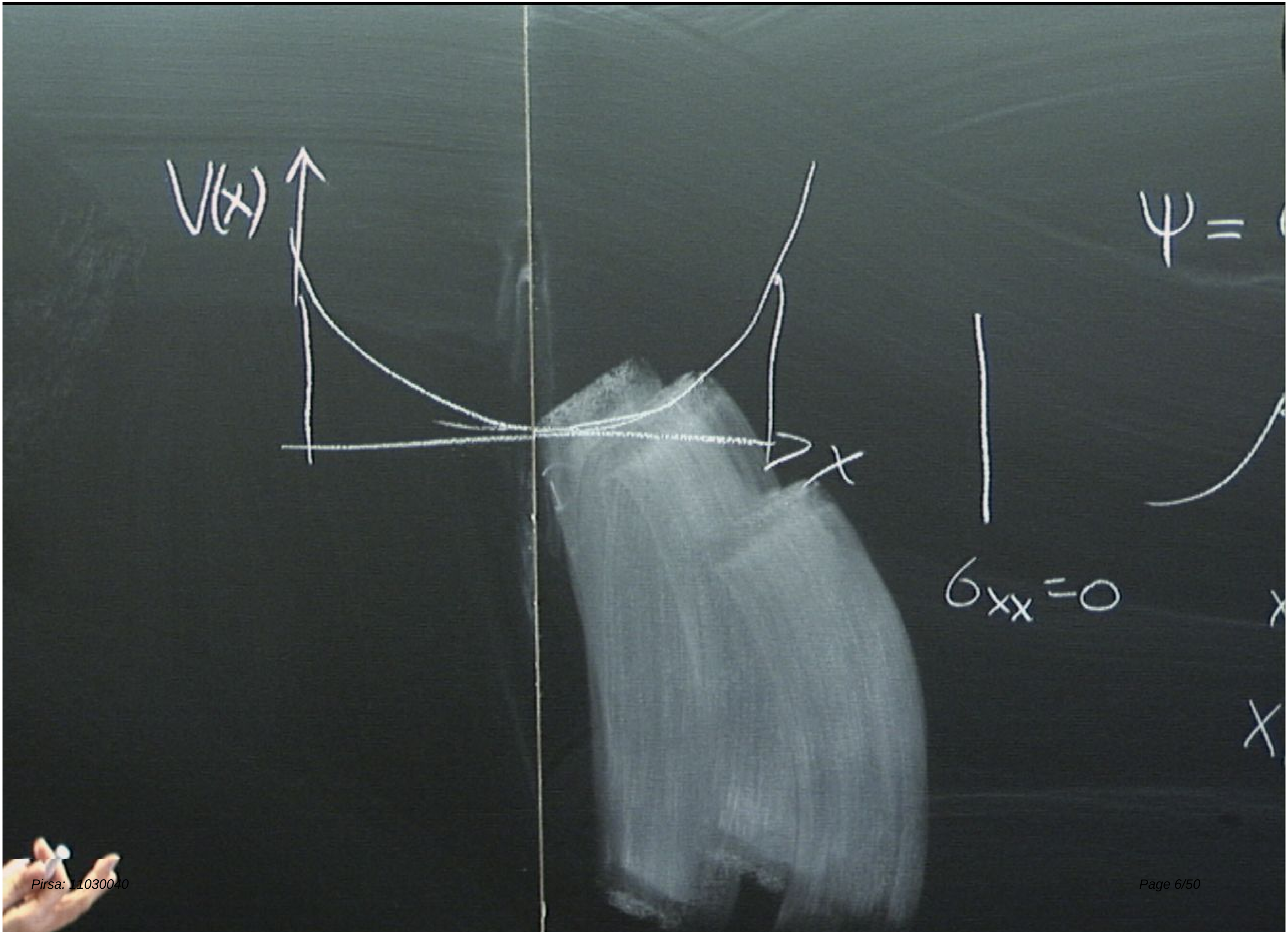


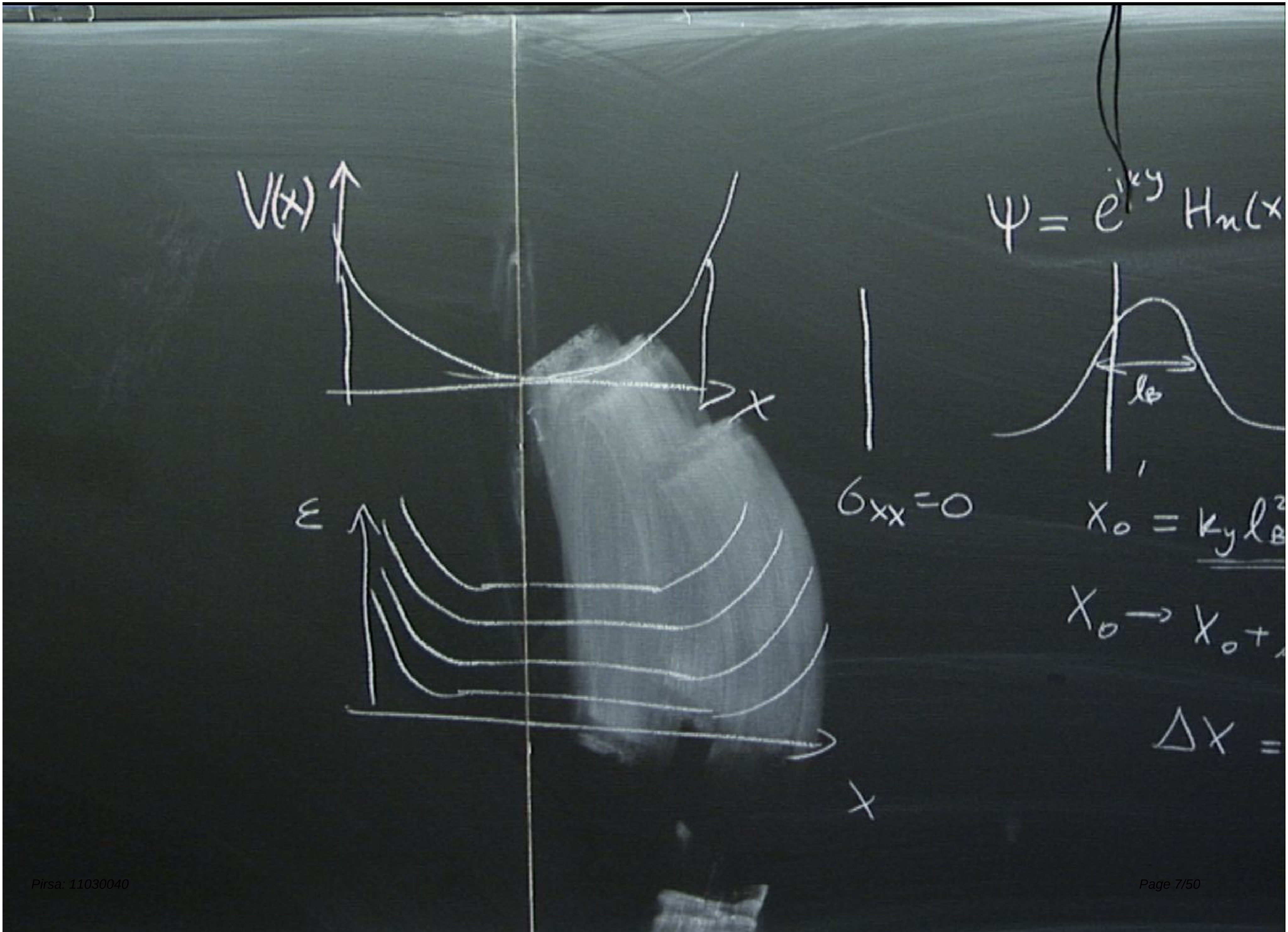
$\sigma_{xx}=0$

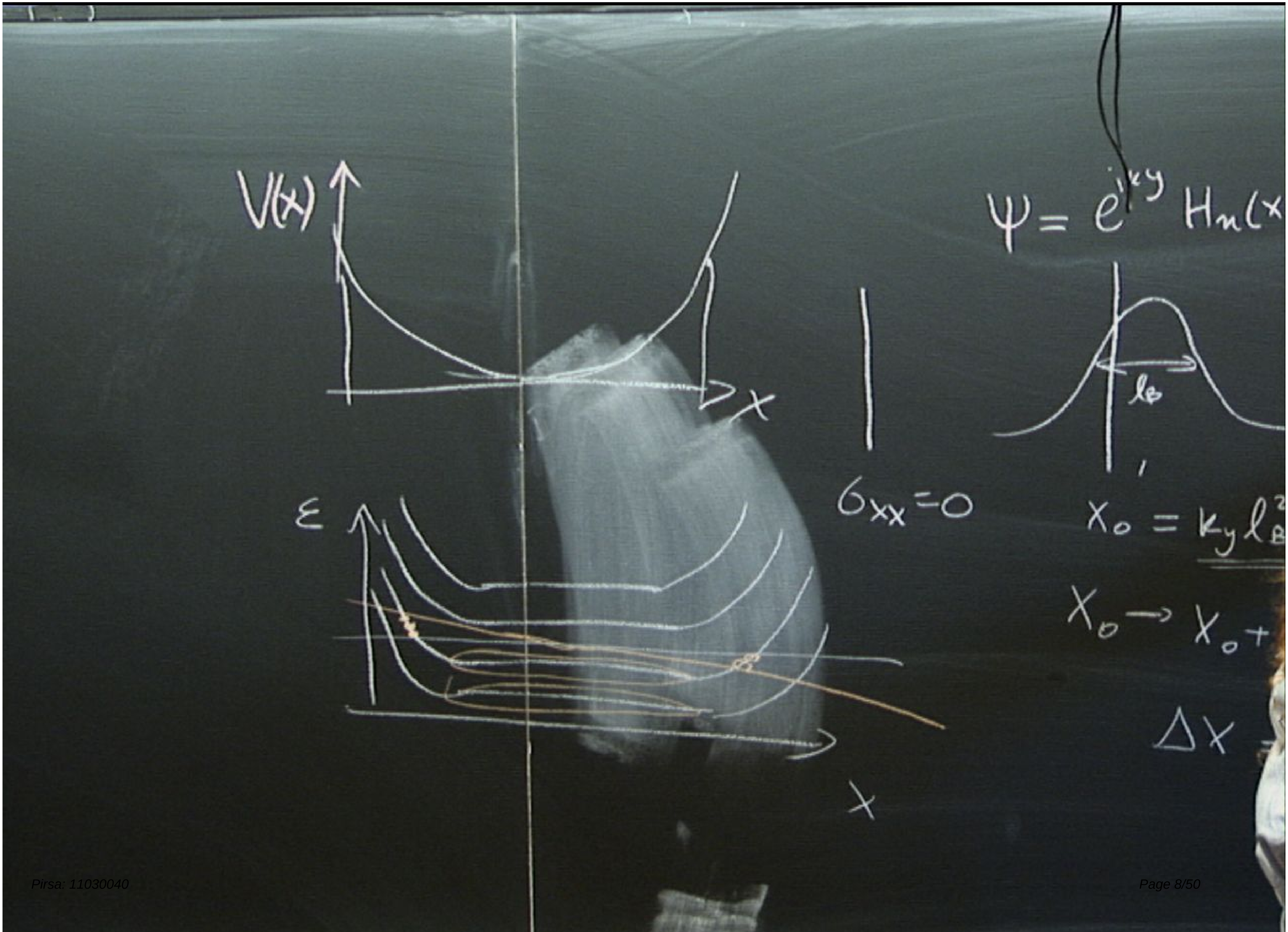
$$x_0 = k_y l_B^2$$

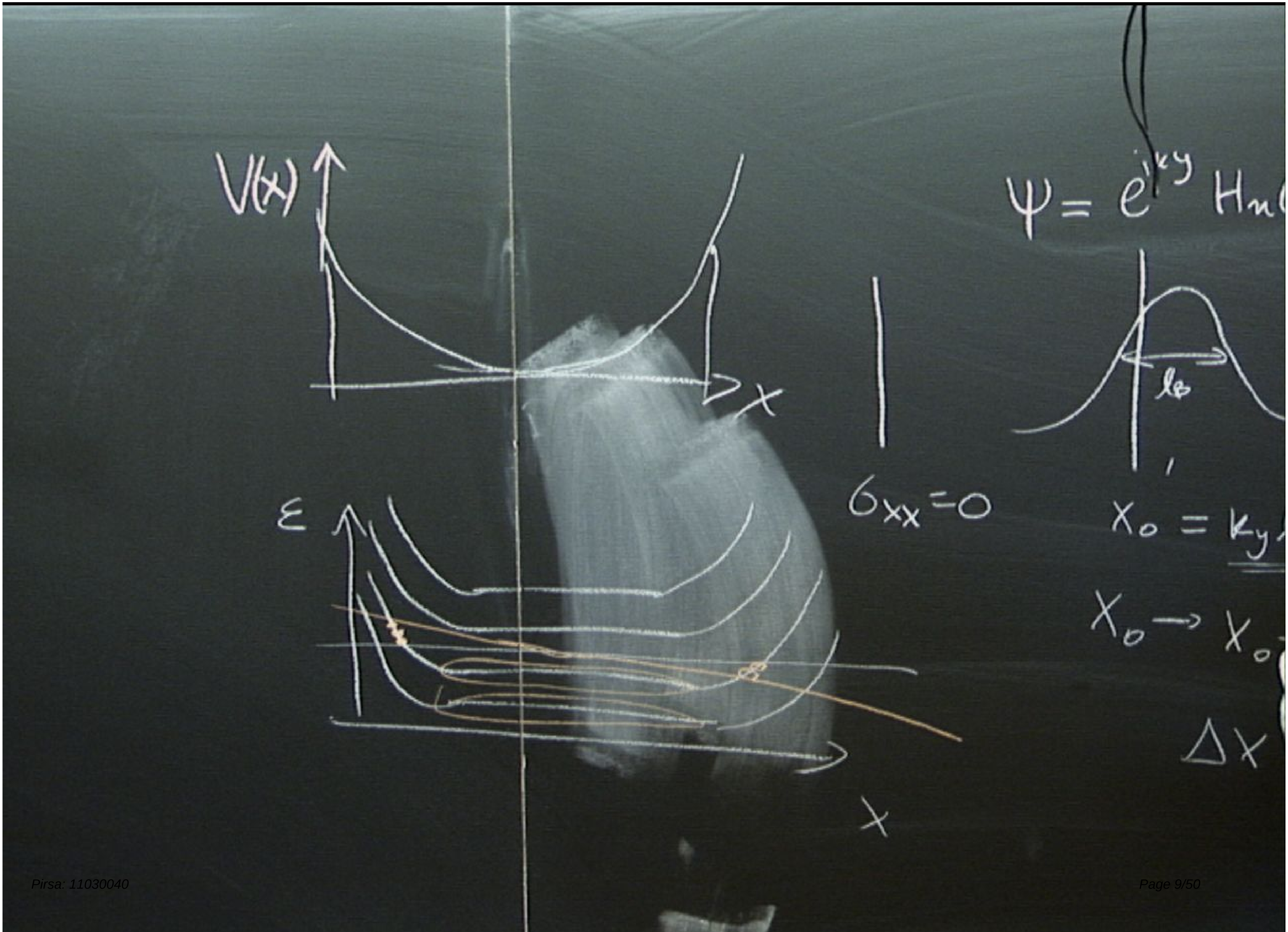
$$x_0 \to x_0 + \Delta x$$

$$\Delta x = \frac{eE}{B^2}$$


V(x)
x
Ψ =
σxx=0

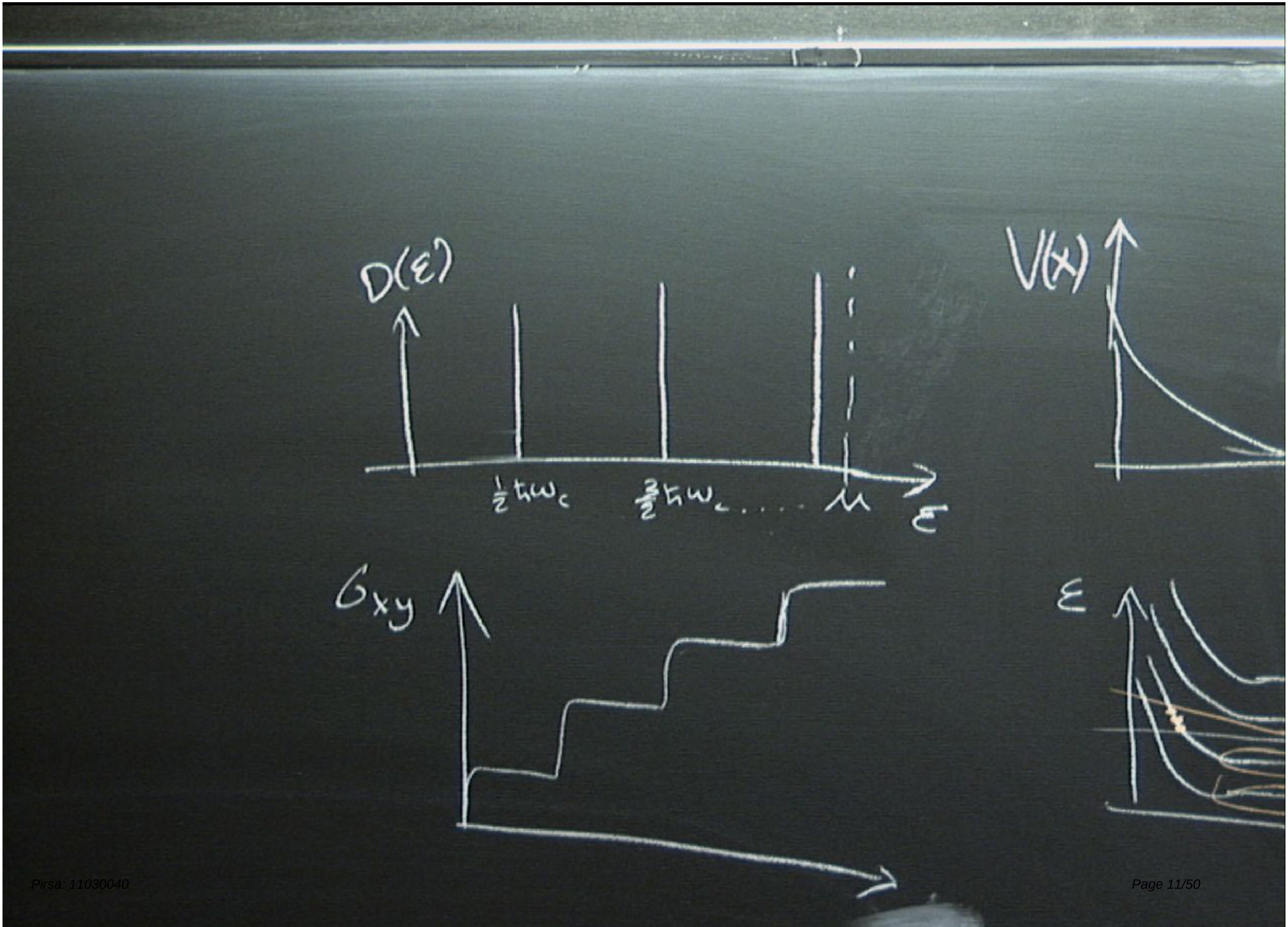

V(x)
x
Ψ = e^{iky} H_n(x
l_B
σ_xx = 0
X_0 = k_y l_B^2
X_0 → X_0 +
ΔX =
ε
x

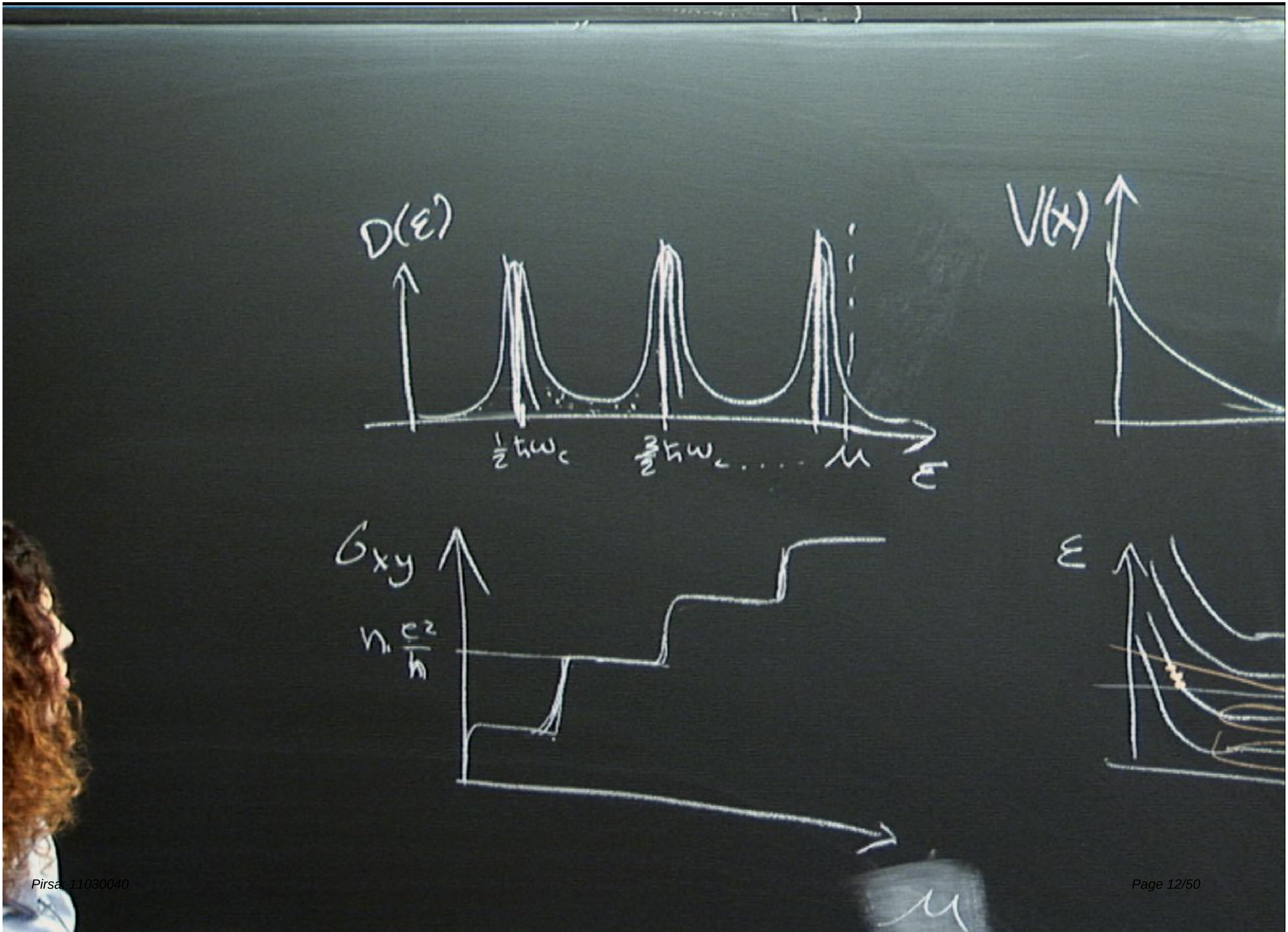

V(x)
x
Ψ = e^{iky} H_n(x
l_B
σ_xx = 0
X_0 = k_y l_B^2
X_0 → X_0 +
ΔX
ε
x


V(x)
x
ε
x
σxx = 0
Ψ = e^{iky} Hn
lB
x0 = ky
x0 → x0
Δx

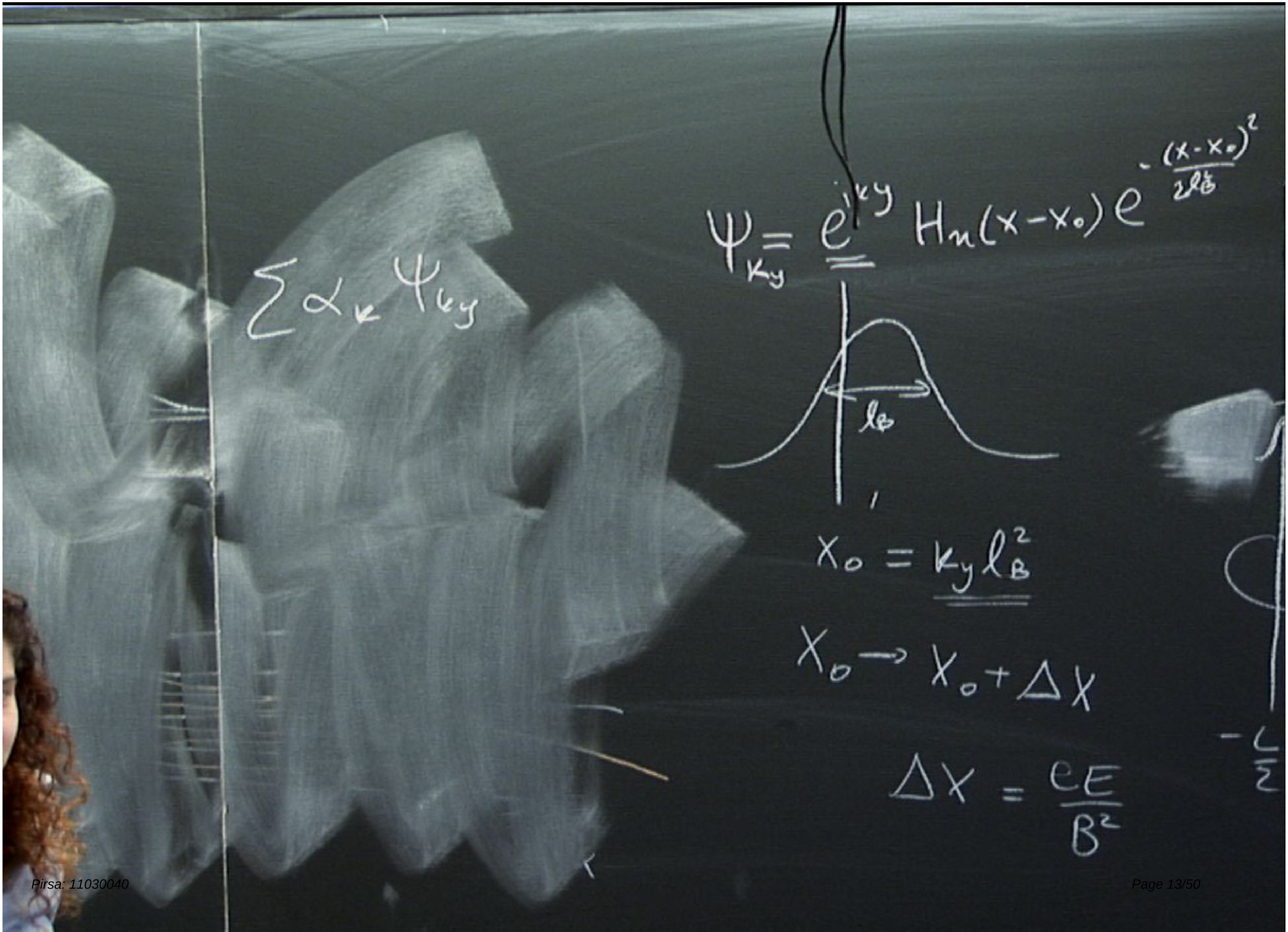
1. Edge states - all of the current carried by them

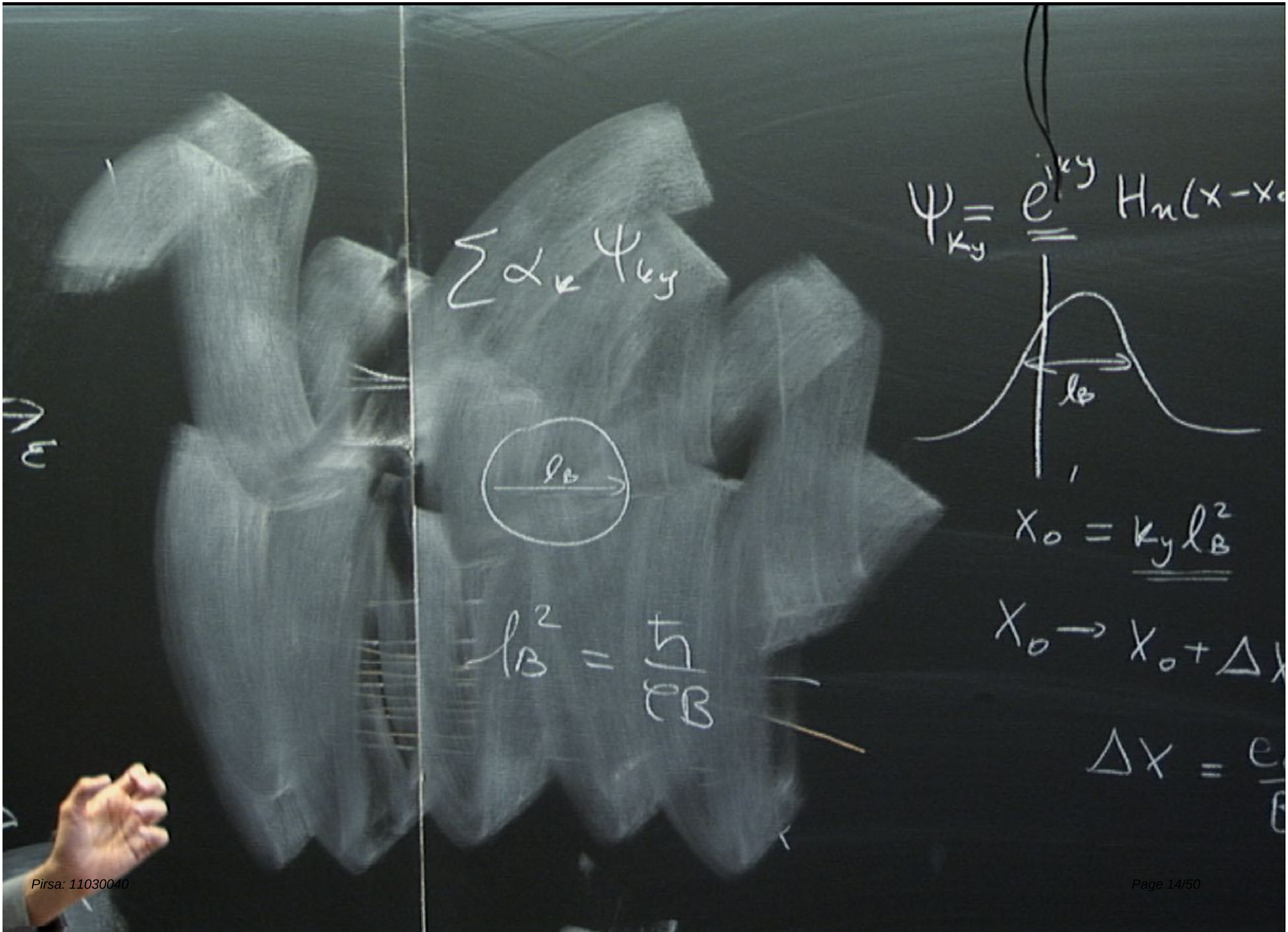
2. Disorder


D(ε)
½ ħω_c
³⁄₂ ħω_c .....
μ
ε
V(x)
σ_xy
ε

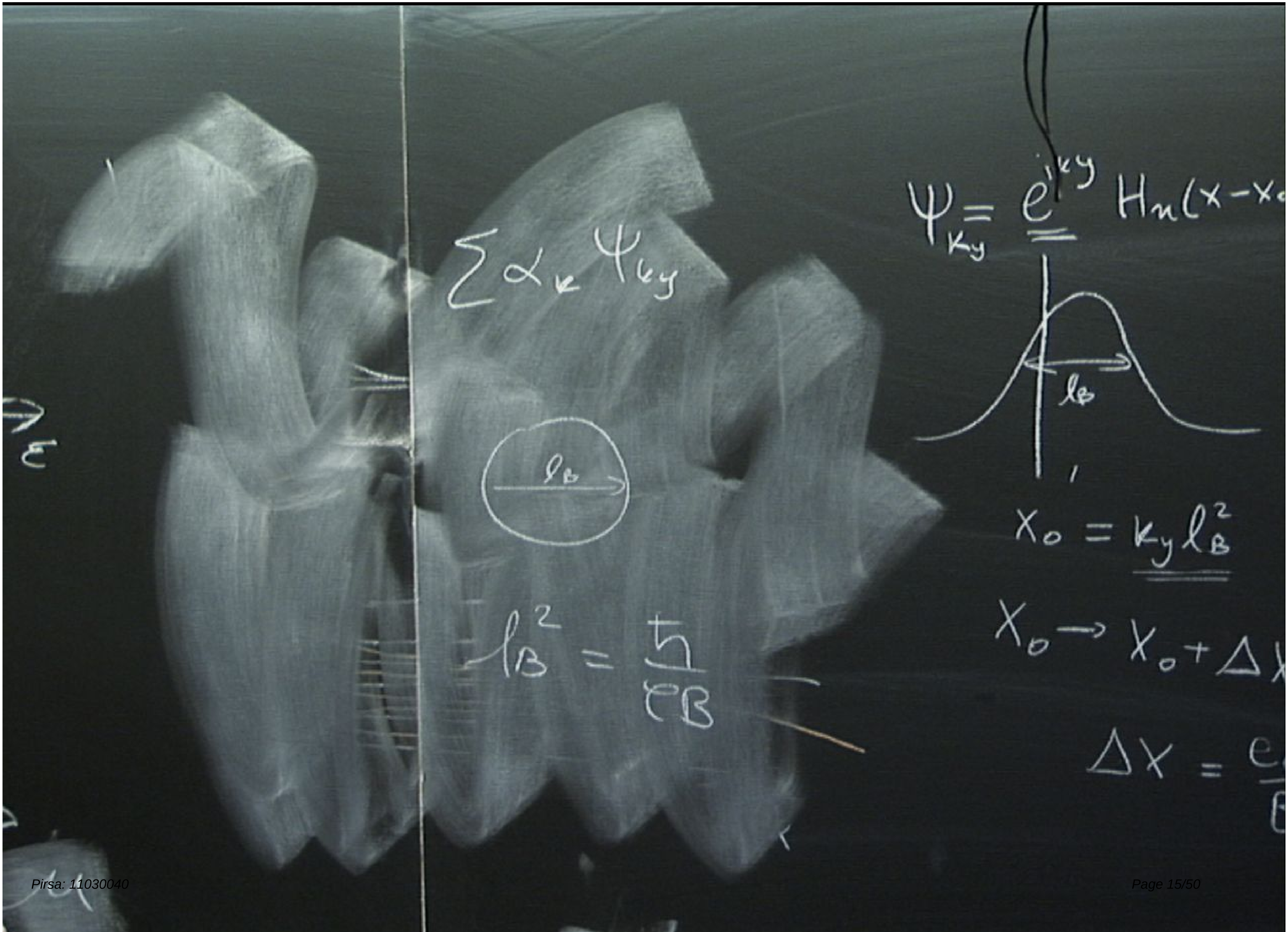


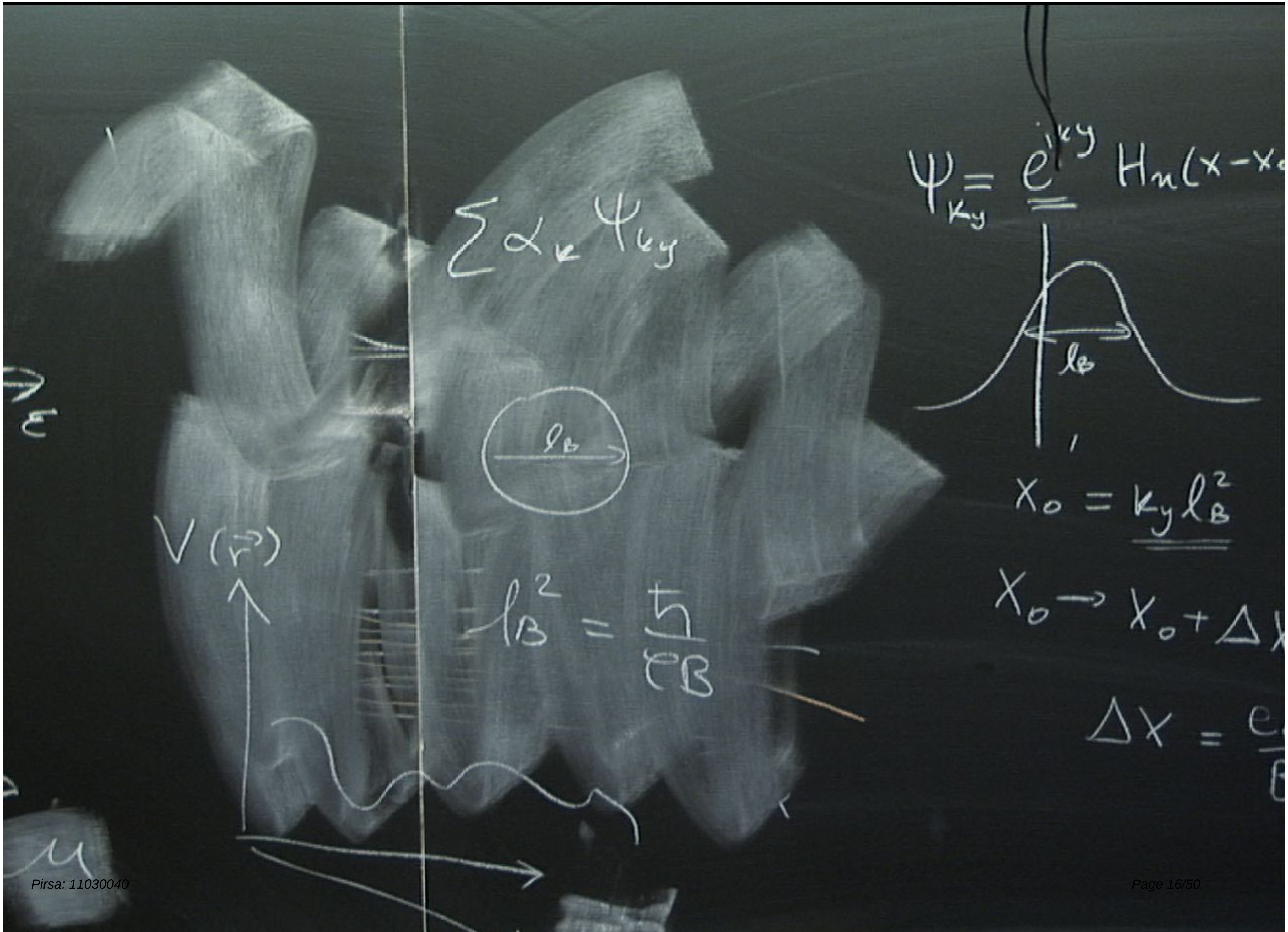
D(ε)
½ ħω_c
3/2 ħω_c .....
μ
ε
V(x)
σ_xy
n e²/h
ε
μ


$\Psi_{k_y} = e^{ik_y y} H_n(x-x_0) e^{-\frac{(x-x_0)^2}{2l_B^2}}$
$\sum \alpha_k \Psi_{k_y}$
l_B
$x_0 = k_y l_B^2$
$X_0 \rightarrow X_0 + \Delta X$
$\Delta X = \frac{eE}{B^2}$
$-\frac{L}{2}$

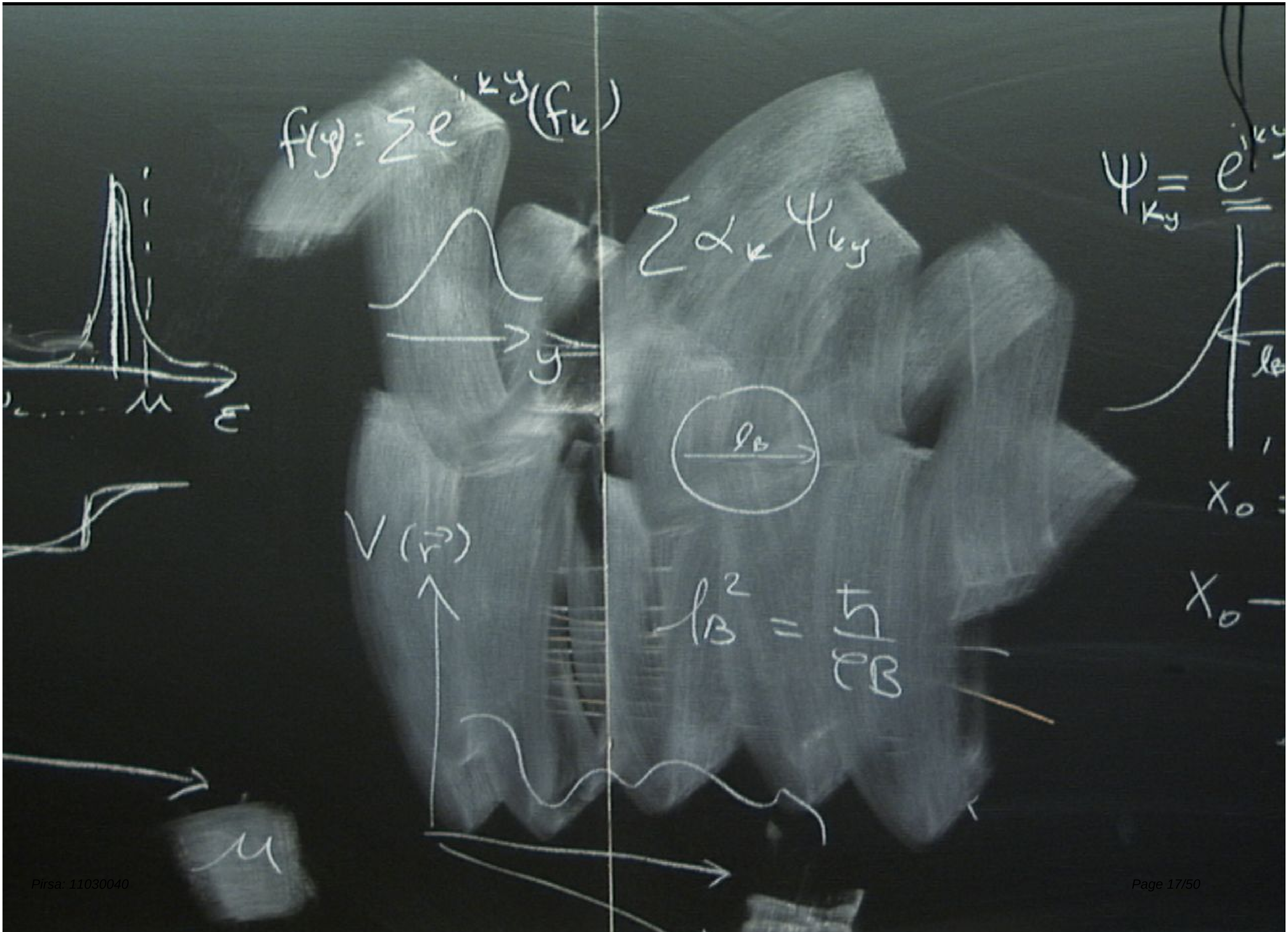


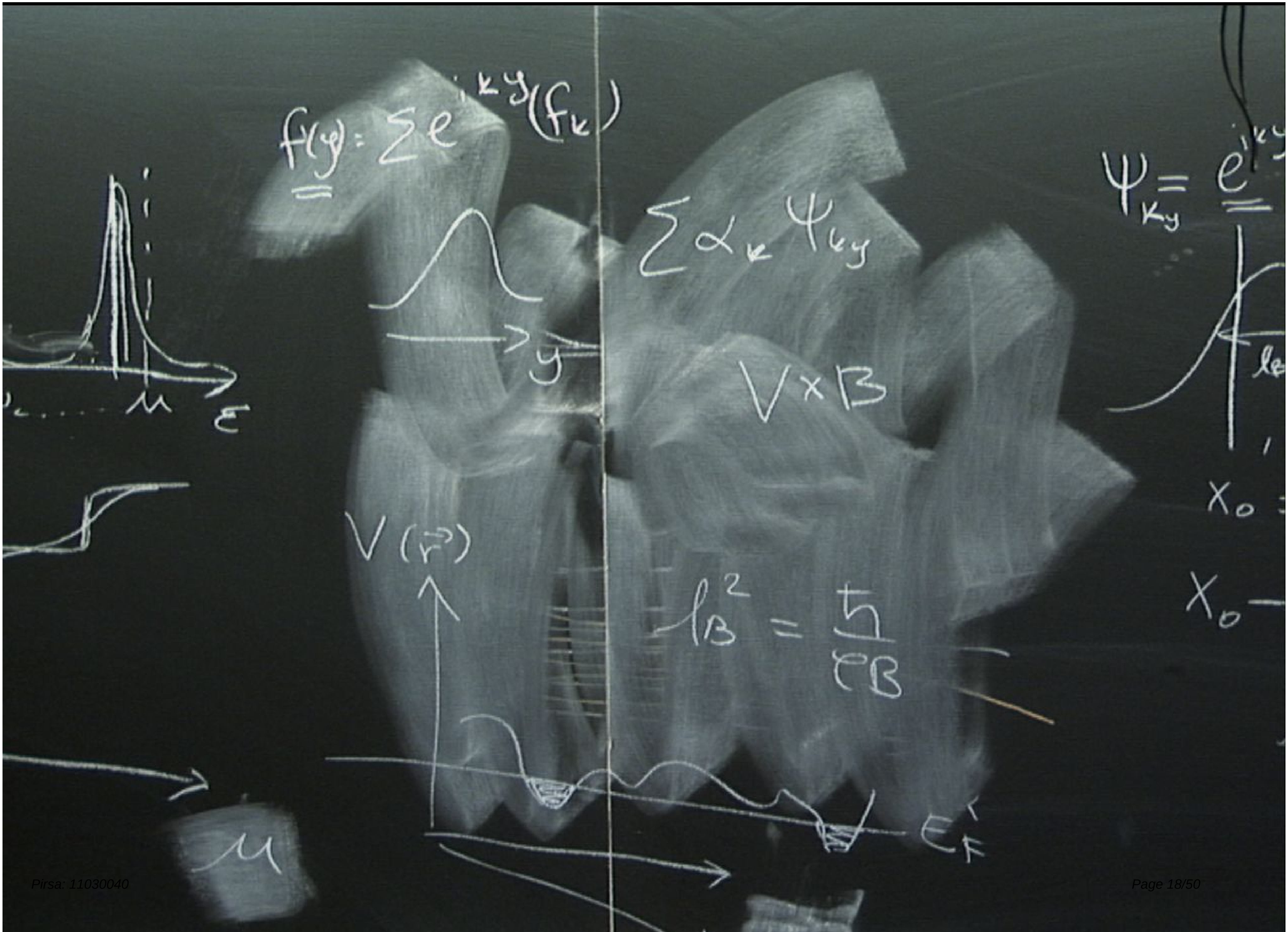
$\Psi_{k_y} = e^{iky} H_n(x-x$
$\sum \alpha_k \Psi_{k_y}$
l_B
l_B
$x_0 = k_y l_B^2$
$x_0 \to x_0 + \Delta x$
$l_B^2 = \frac{\hbar}{eB}$
$\Delta x =$

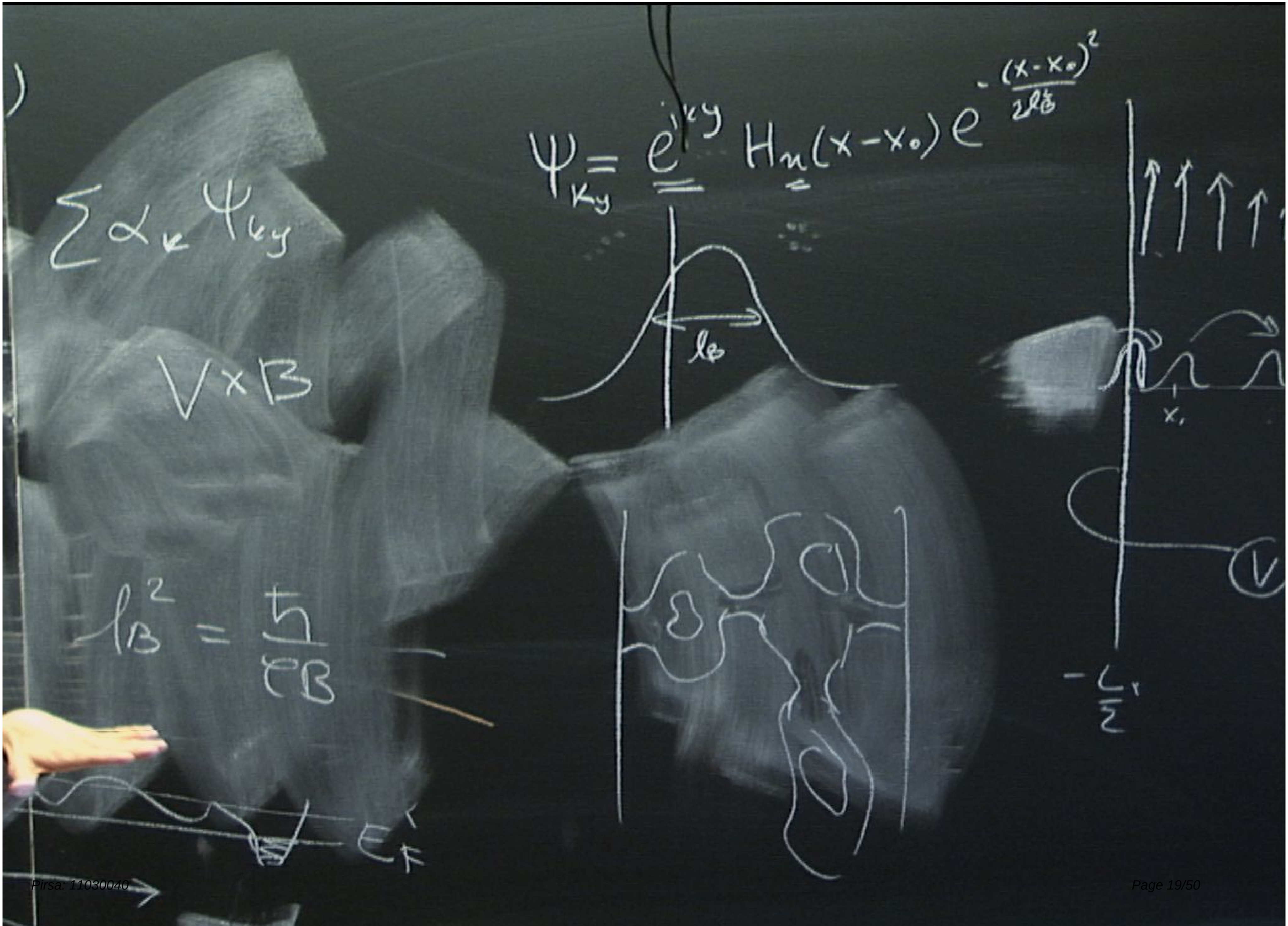

$\Psi_{k_y} = e^{iky} H_n(x-x$
$\sum \alpha_k \Psi_{k_y}$
l_B
l_B
$x_0 = k_y l_B^2$
$x_0 \rightarrow x_0 + \Delta x$
$\Delta x =$
$l_B^2 = \frac{\hbar}{eB}$

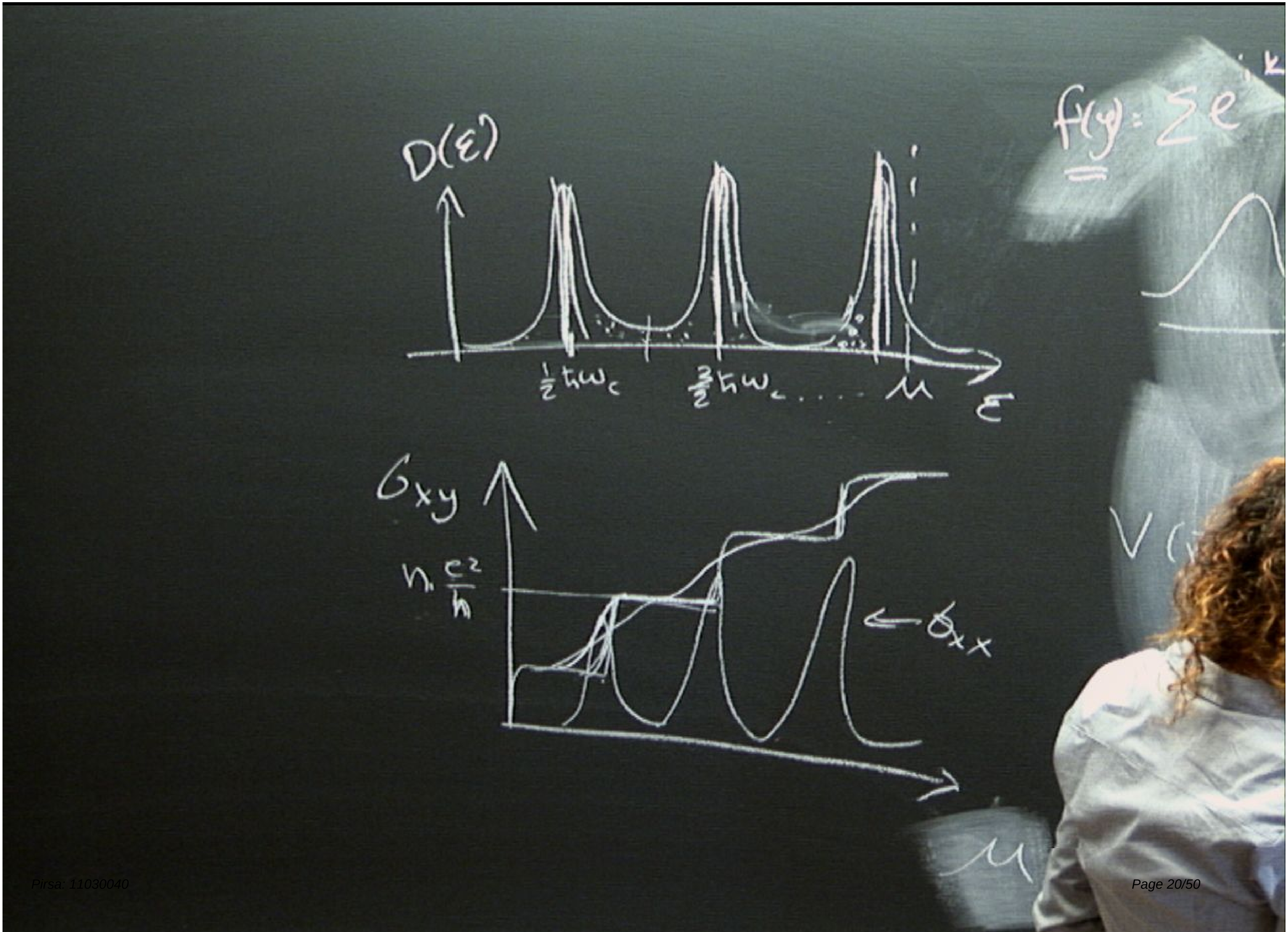


$\Psi_{k_y} = e^{iky} H_n(x - x$
$\sum \alpha_k \Psi_{k_y}$
l_B
l_B
$x_0 = k_y l_B^2$
$V(\vec{r})$
$l_B^2 = \frac{\hbar}{eB}$
$x_0 \to x_0 + \Delta x$
$\Delta x =$
μ


f(y) = Σ e^{iky}(f_k)
Σ α_k ψ_{ky}
ψ_{ky} = e
y
μ
E
l_B
V(r⃗)
l_B² = ħ/eB
x_0
μ


f(y) = Σ e^{iky}(f_k)
Σ α_k Ψ_ky
V × B
Ψ_ky = e^{iky}
V(r⃗)
l_B^2 = ħ/eB
μ
E
y
x_0
E_F


$\Psi_{k_y} = e^{iky} H_n(x-x_0) e^{-\frac{(x-x_0)^2}{2l_B^2}}$
$\sum \alpha_k \Psi_{k_y}$
$V \times B$
l_B
$l_B^2 = \frac{\hbar}{eB}$
x_1
$-\frac{L_x}{2}$
V
E_F


D(ε)
½ ħω_c
3/2 ħω_c .....
μ
ε
σ_xy
n e²/h
← σ_xx
f(y) = Σe
μ

$f_k)$

$$\sum \alpha_k \psi_{k_y}$$

$$\nabla \times B$$

$$l_B^2 = \frac{\hbar}{eB}$$

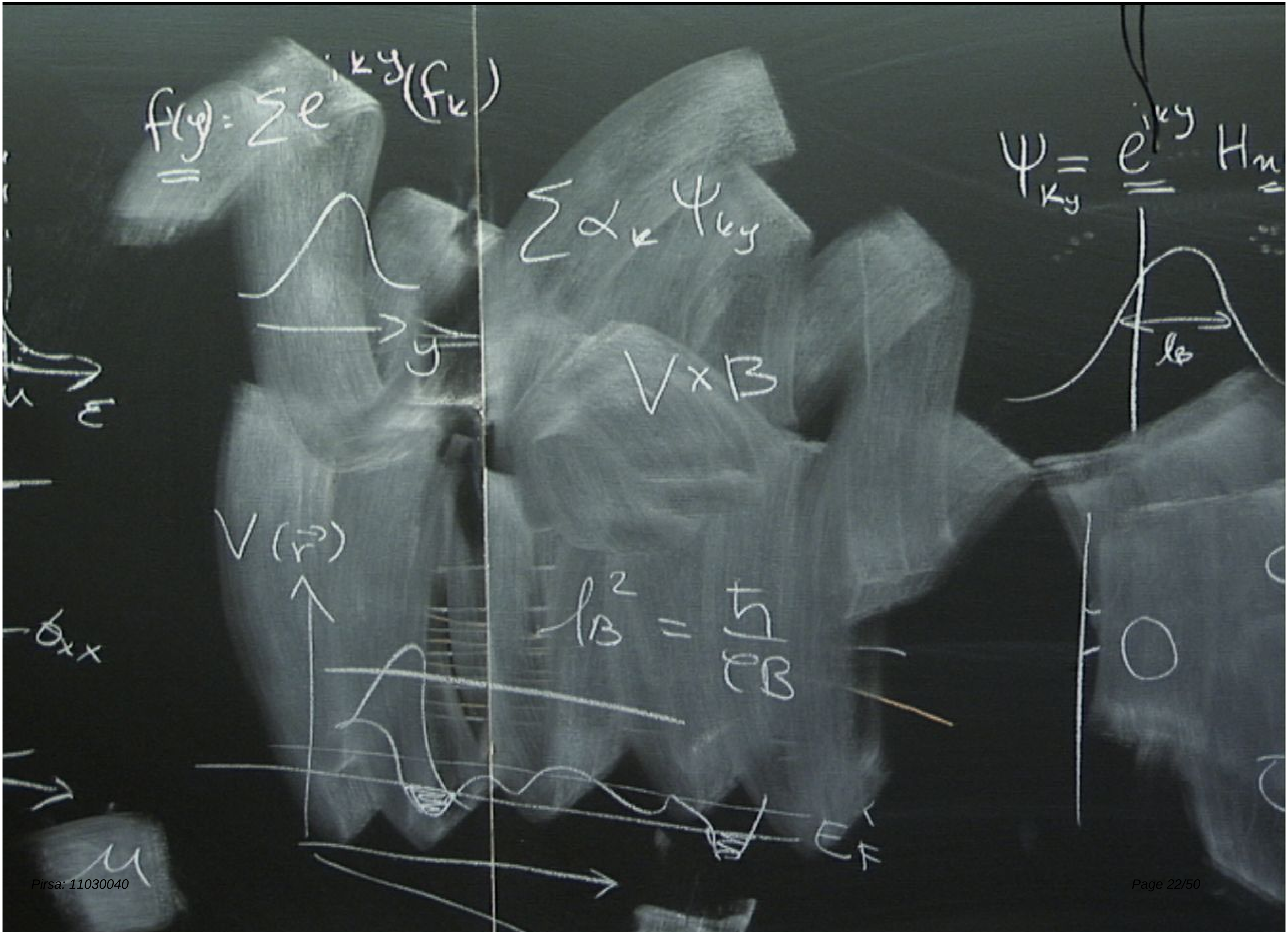
$$\psi_{k_y} = e^{ik_y y} H_n(x - x_0)\, e^{-\frac{(x-x_0)^2}{2 l_B^2}}$$

l_B

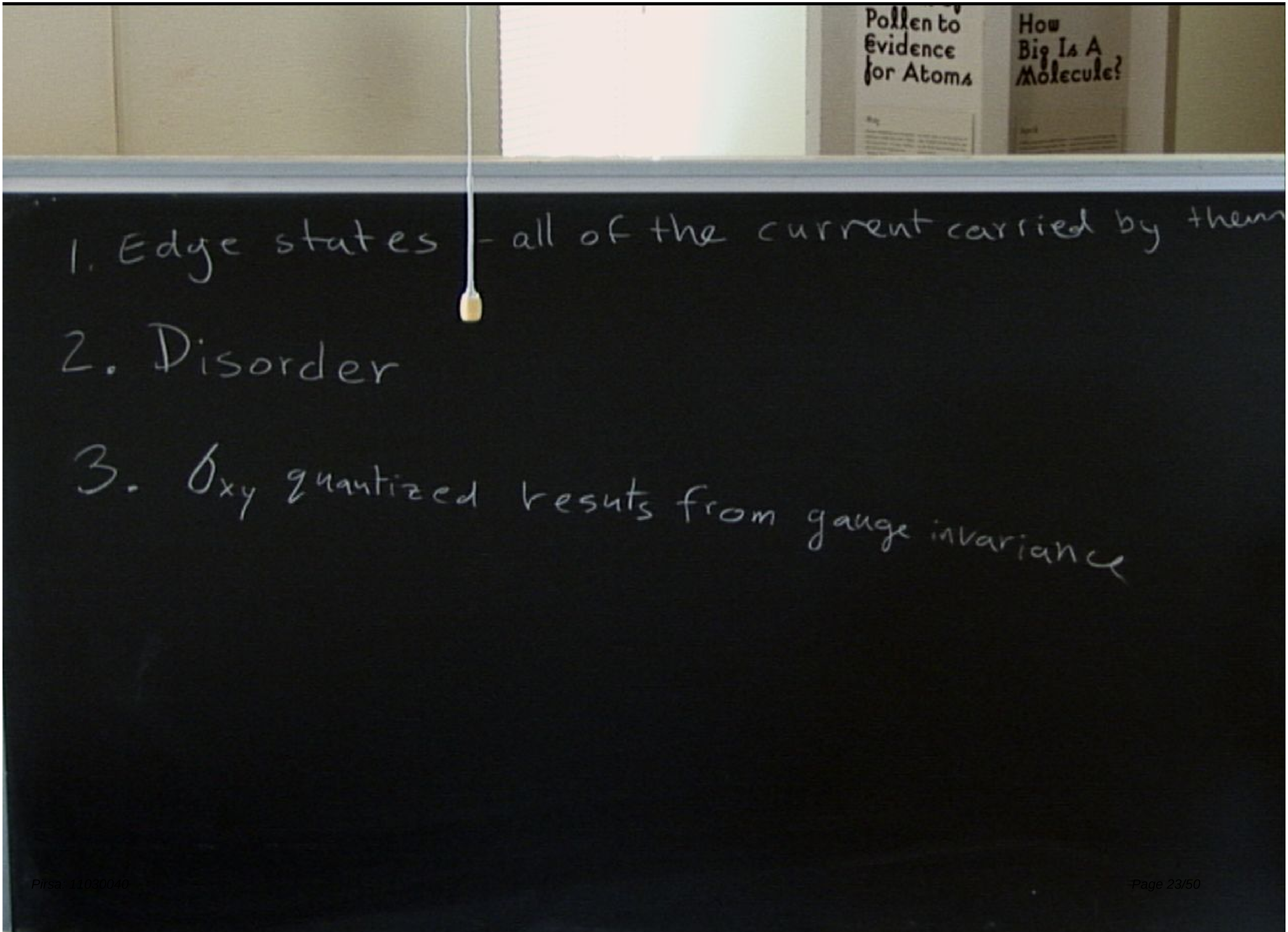
x_1

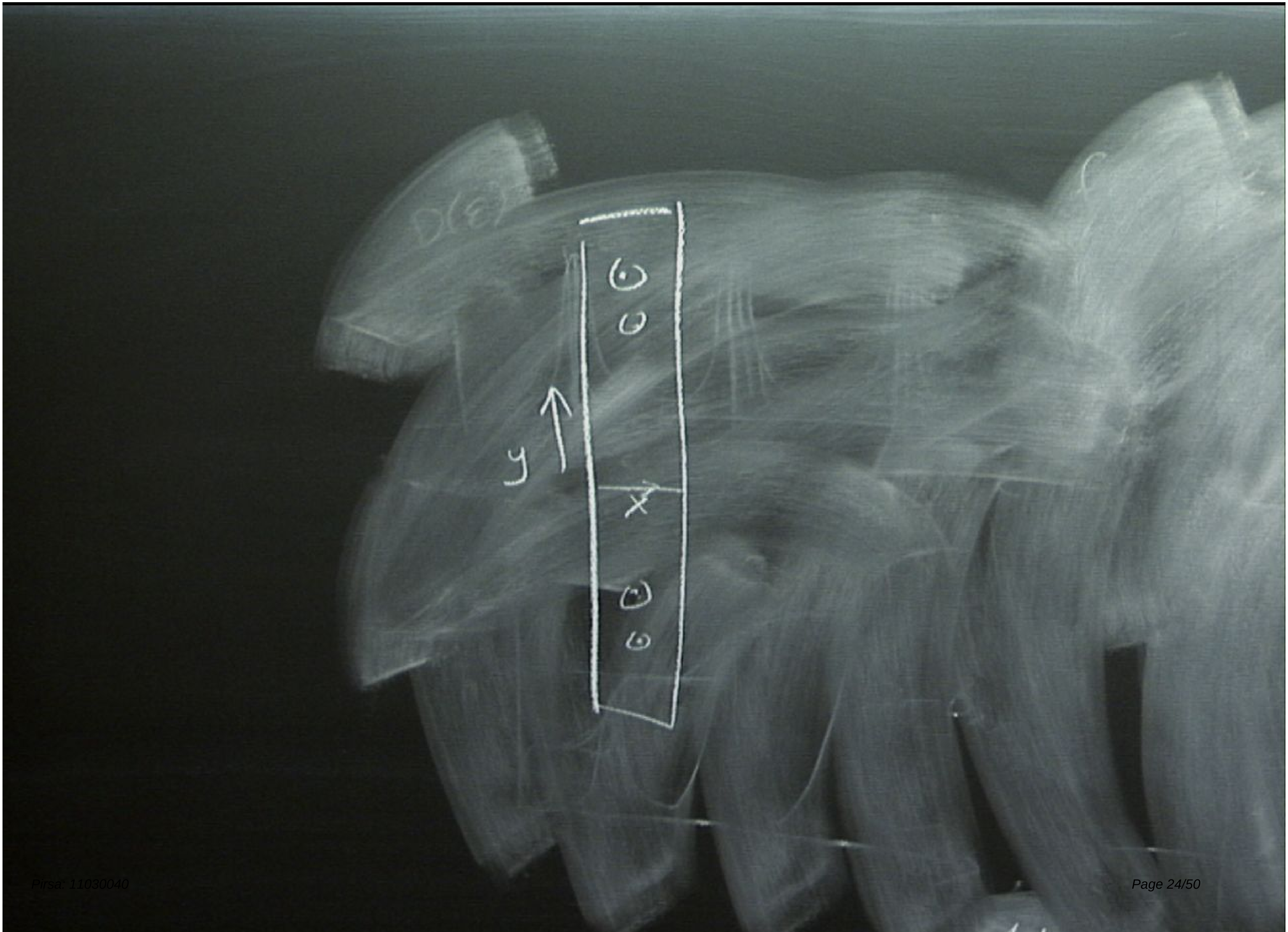
$-\frac{L_x}{2}$

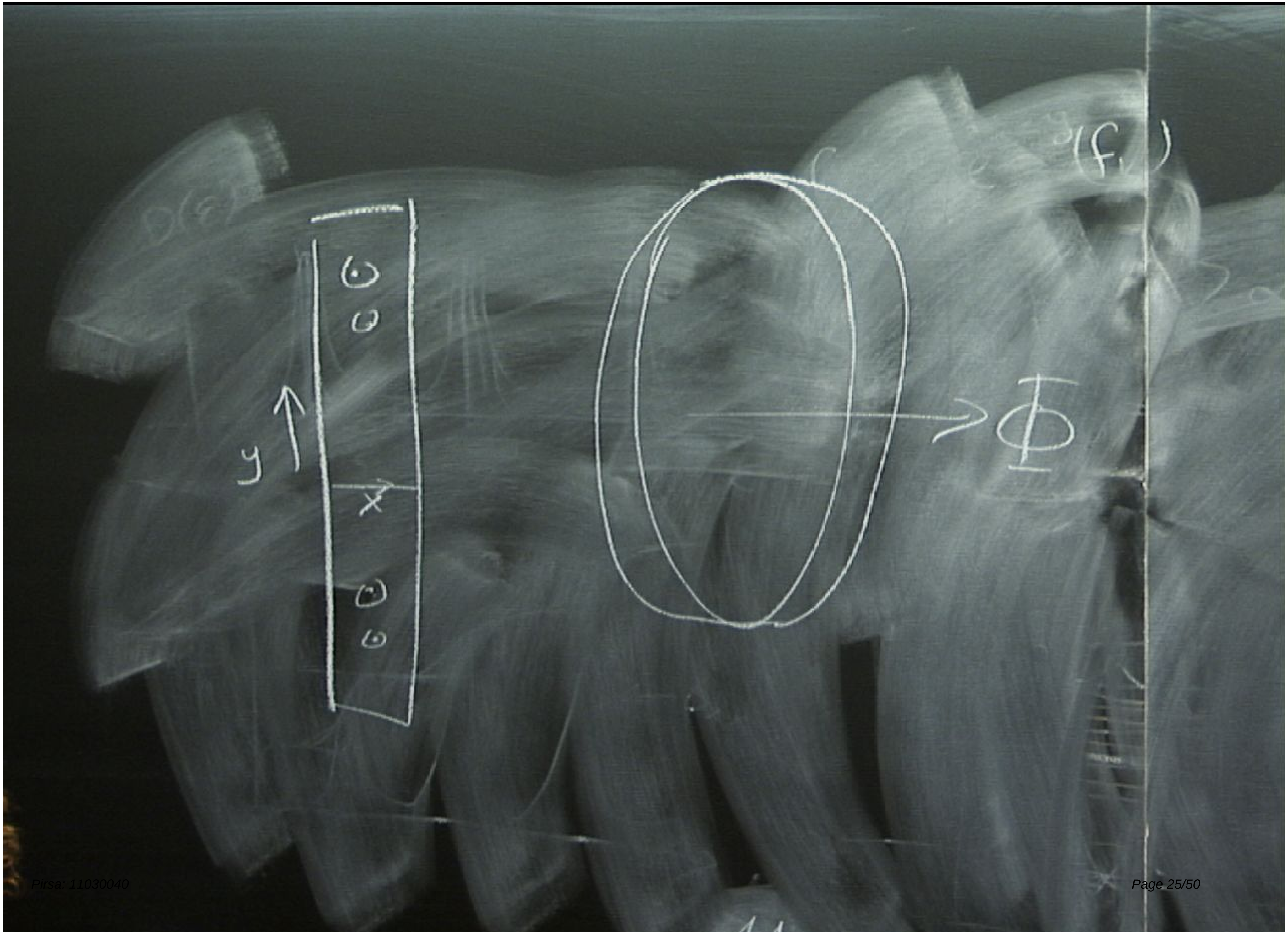
E_F

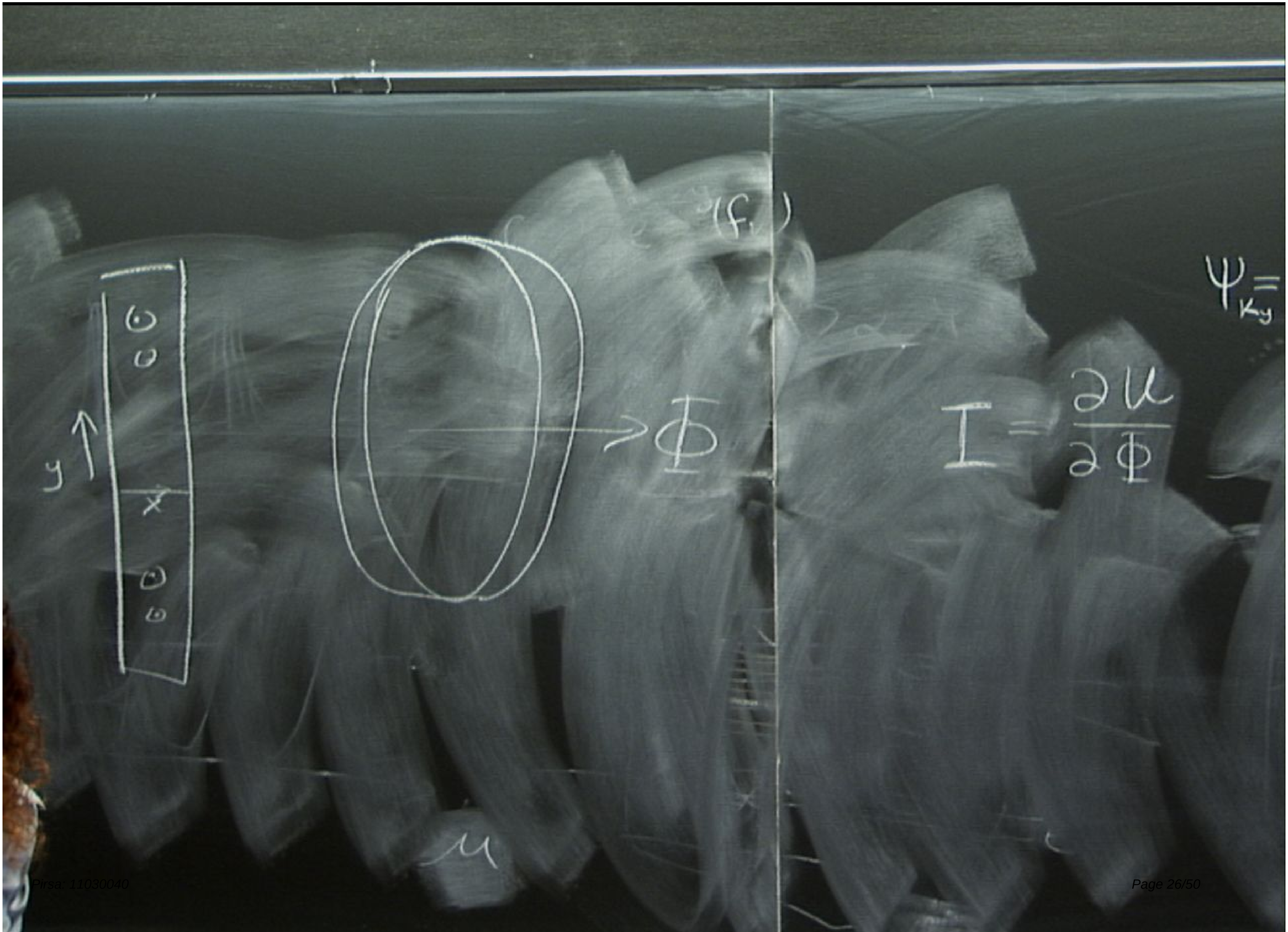


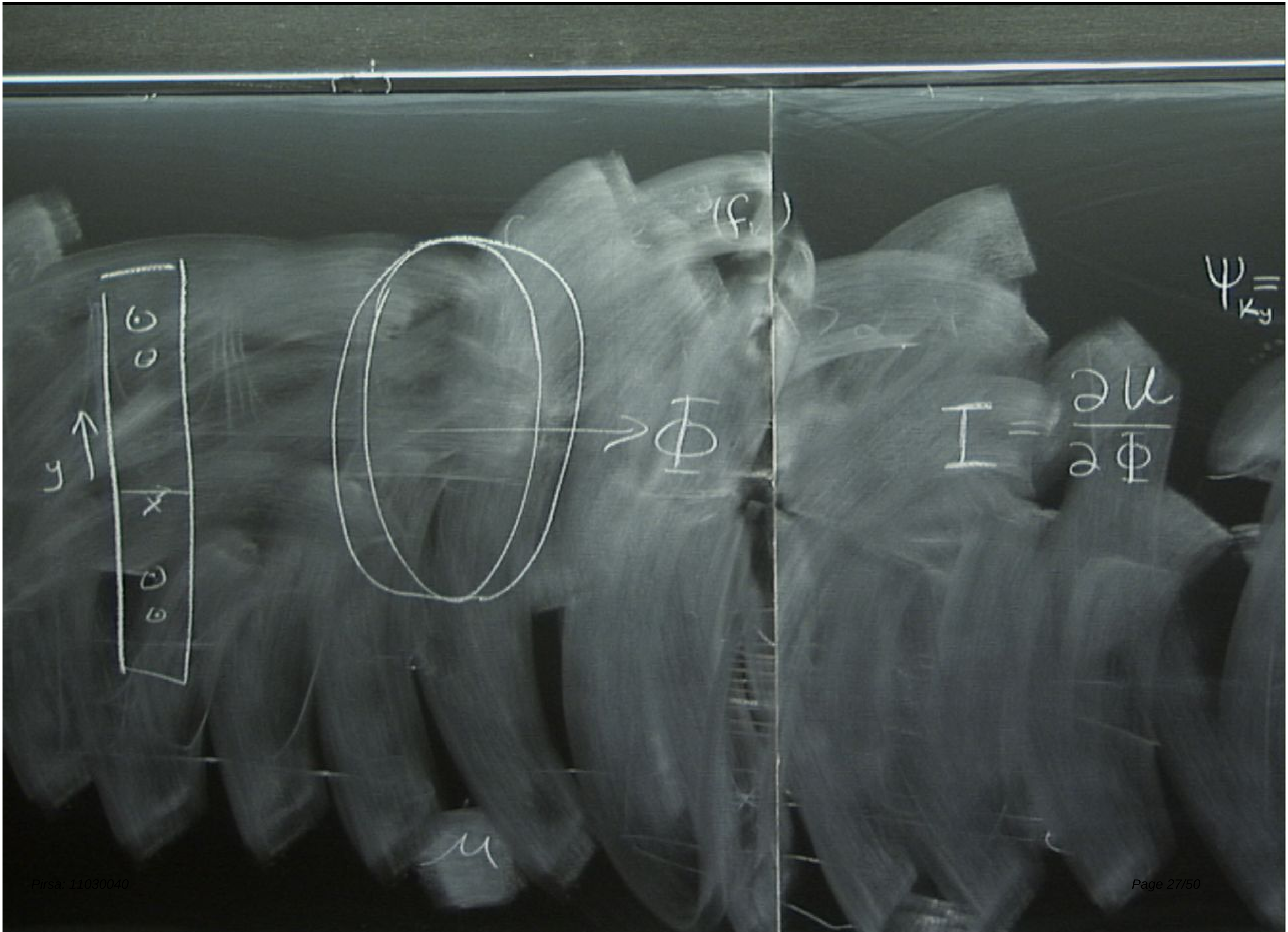
f(y) = Σ e^{iky}(f_k)
Σ α_k Ψ_ky
y
V × B
Ψ_ky = e^{iky} H_n
l_B
E
σ_xx
V(r)
l_B^2 = ħ/eB
E_F
μ

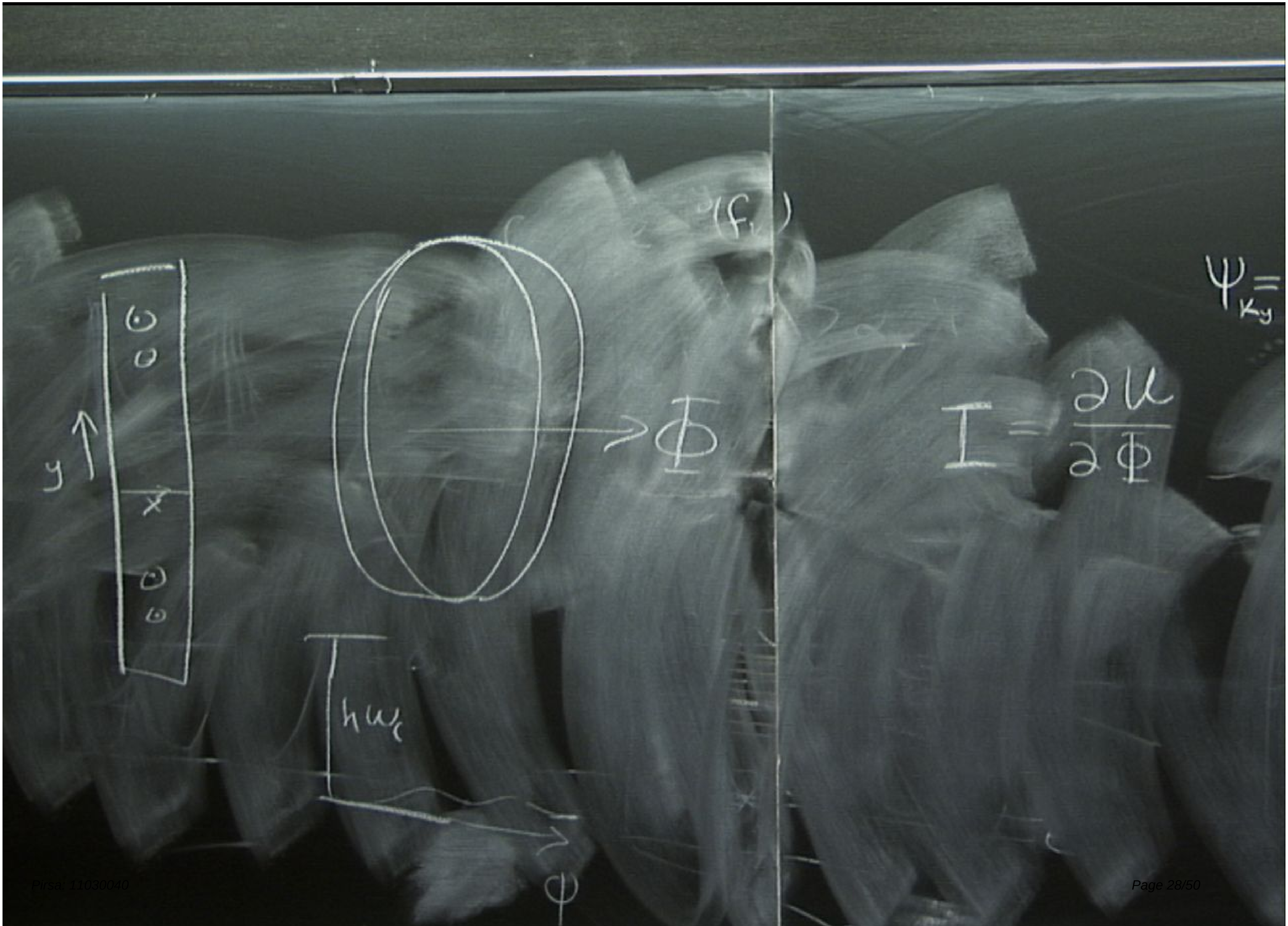

Pollen to Evidence for Atoms
How Big Is A Molecule?
1. Edge states - all of the current carried by them
2. Disorder
3. σ_{xy} quantized resuts from gauge invariance


D(E)
y
x


y
x
Φ


y
x
Φ
I = ∂U/∂Φ
Ψ_Ky =
u


y
x
Φ
$I = \frac{\partial u}{\partial \Phi}$
$\Psi_{k_y} =$
u


y
x
ħω_c
Φ
φ
I = ∂U/∂Φ
Ψ_{k_y} =

$$\Psi_{k_y} = e^{iky} H_n(x-x_0)\, e^{-\frac{(x-x_0)^2}{2\ell_B}}$$

$$\rightarrow \Phi$$

$$I = \frac{\partial U}{\partial \Phi}$$

$$A_\theta = \frac{\Phi}{L}$$

$$\Psi_{k_y} = e^{iky} H_n(x-x_0)\, e^{-\frac{(x-x_0)^2}{2\ell_B^2}}$$

$$I = \frac{\partial U}{\partial \Phi}$$

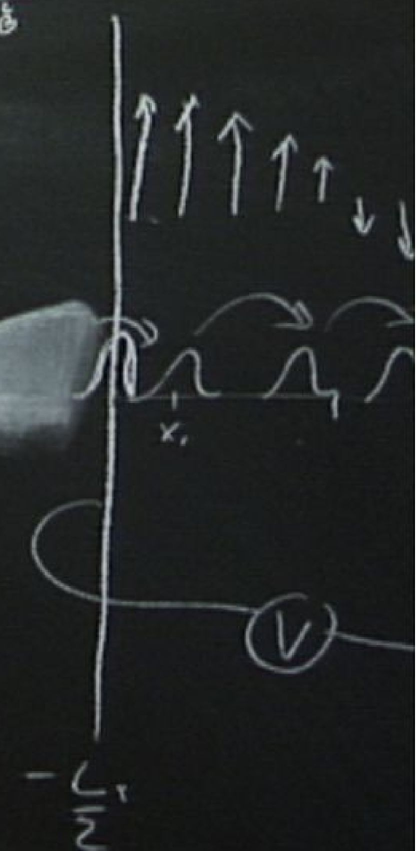
$$A_\theta = \frac{\Phi}{L_y}$$

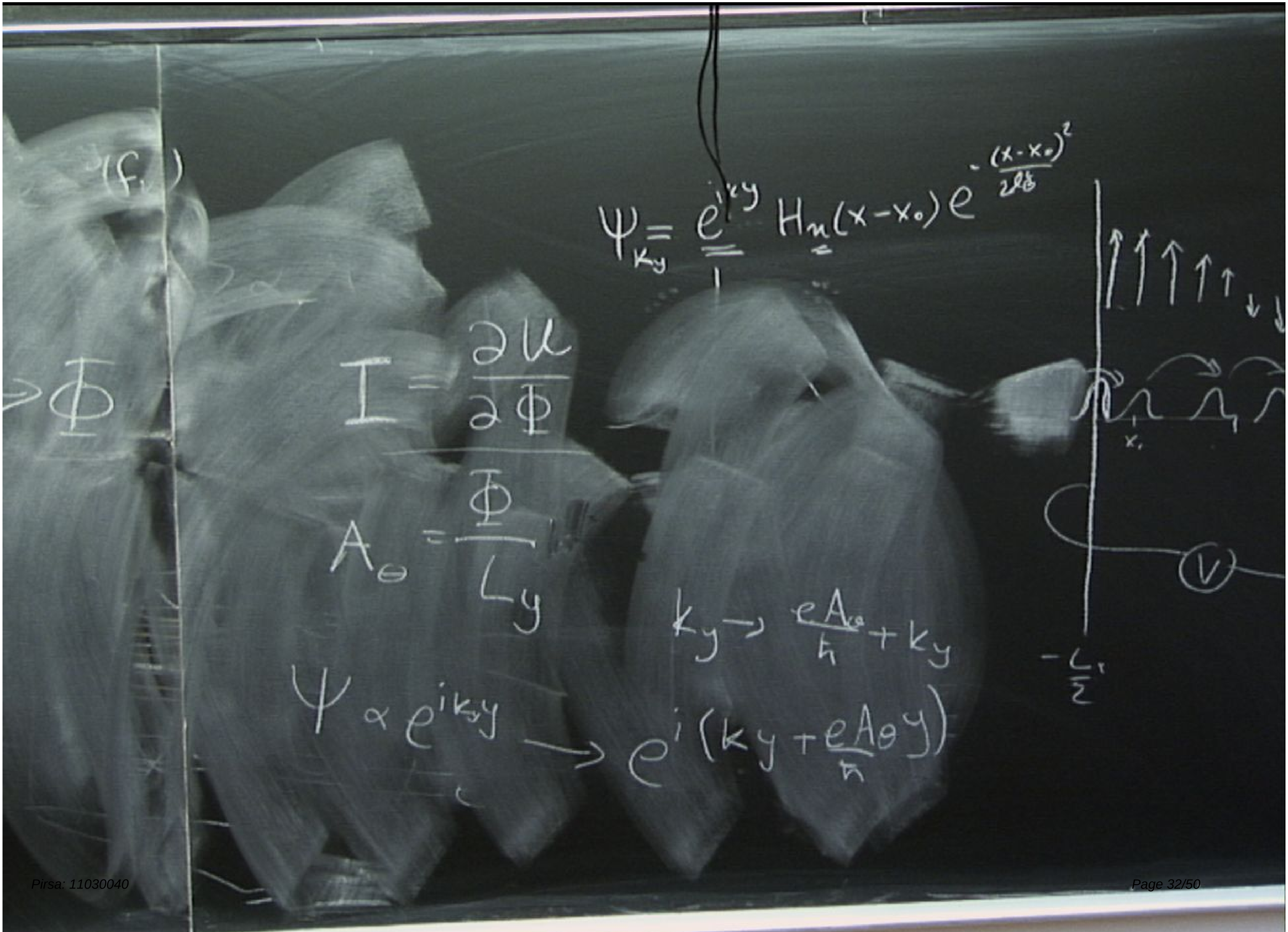
$$\Psi_{k_y} = e^{iky} H_n(x-x_0)\, e^{-\frac{(x-x_0)^2}{2\ell_B^2}}$$

$$I = \frac{\partial U}{\partial \Phi}$$

$$A_\theta = \frac{\Phi}{L_y}$$

$$\Psi \propto e^{ik_y y} \longrightarrow e^{i\left(ky + \frac{eA_\theta y}{\hbar}\right)}$$




$\Psi_{k_y} = e^{iky} H_n(x-x_0) e^{-\frac{(x-x_0)^2}{2l_B^2}}$
$I = \frac{\partial U}{\partial \Phi}$
$A_\theta = \frac{\Phi}{L_y}$
$k_y \to \frac{eA_\theta}{\hbar} + k_y$
$\Psi \propto e^{ik_y y} \longrightarrow e^{i(k_y + \frac{eA_\theta}{\hbar} y)}$
x_0
$-\frac{L_x}{2}$
V

$$\psi_{k_y} = e^{iky} H_n(x-x_0)\, e^{-\frac{(x-x_0)^2}{2\ell_B^2}}$$

$$I = \frac{\partial U}{\partial \Phi}$$

$$A_\theta = \frac{\Phi}{L_y}$$

$$k_y \to \frac{eA_\theta}{\hbar} + k_y$$

$$\psi \propto e^{ik_y y} \longrightarrow e^{i(ky + \frac{eA_\theta}{\hbar} y)}$$

$$e^{i\frac{eA_\theta}{\hbar}(y)} = e^{i\frac{eA_\theta}{\hbar}(y+L)}$$

$$\Psi_{k_y} = e^{iky} H_n(x-x_0)\, e^{-\frac{(x-x_0)^2}{2\ell_B^2}}$$

$$e^{i\frac{eA_\theta}{\hbar}(y)} = e^{i\frac{eA_\theta}{\hbar}(y+L_y)}$$

$$I = \frac{\partial U}{\partial \Phi}$$

$$A_\theta = \frac{\Phi}{L_y}$$

$$k_y \to \frac{eA_\theta}{\hbar} + k_y$$

$$\Psi \propto e^{ik_y y} \longrightarrow e^{i(ky + \frac{eA_\theta}{\hbar} y)}$$

$$\Psi_{k_y} = e^{iky} H_n(x-x_0)\, e^{-\frac{(x-x_0)^2}{2\ell_B^2}}$$

$$I = \frac{\partial U}{\partial \Phi}$$

$$A_\theta = \frac{\Phi}{L_y}$$

$$k_y \to \frac{eA_\theta}{\hbar} + k_y$$

$$\psi \propto e^{ik_y y} \longrightarrow e^{i\left(k y + \frac{eA_\theta}{\hbar} y\right)}$$

$$e^{i\frac{eA_\theta}{\hbar}(y)} = e^{i\frac{eA_\theta}{\hbar}(y+L_y)}$$

$$\frac{eA_\theta}{\hbar} L_y = 2\pi \cdot \tilde{n}$$

$$\Psi_{k_y} = e^{iky} H_n(x-x_0)\, e^{-\frac{(x-x_0)^2}{2\ell_B^2}}$$

$$I = \frac{\partial U}{\partial \Phi}$$

$$A_\theta = \frac{\Phi}{L_y}$$

$$k_y \to \frac{eA_\theta}{\hbar} + k_y$$

$$\psi \propto e^{ik_y y} \longrightarrow e^{i\left(k_y + \frac{eA_\theta}{\hbar} y\right)}$$

$$e^{i\frac{eA_\theta}{\hbar}(y)} = e^{i\frac{eA_\theta}{\hbar}(y+L_y)}$$

$$\frac{eA_\theta}{\hbar} L_y = 2\pi \cdot \tilde{n}$$

$$\frac{e\Phi}{\hbar \cancel{L_y}} \cancel{L_y} = 2\pi n$$

$$\Phi = \frac{2\pi\hbar}{e} n = \Phi_0 \cdot n$$

$$\Psi_{k_y} = e^{iky} H_n(x-x_0)\, e^{-\frac{(x-x_0)^2}{2\ell_B^2}}$$

$$I = \frac{\partial U}{\partial \Phi}$$

$$A_\theta = \frac{\Phi}{L_y}$$

$$k_y \to \frac{eA_\theta}{\hbar} + k_y$$

$$\Psi \propto e^{ik_y y} \longrightarrow e^{i\left(k_y + \frac{eA_\theta}{\hbar} y\right)}$$

$$e^{i\frac{eA_\theta}{\hbar}(y)} = e^{i\frac{eA_\theta}{\hbar}(y+L_y)}$$

$$\frac{eA_\theta}{\hbar} L_y = 2\pi \cdot \tilde{n}$$

$$\frac{e\Phi}{\hbar \cancel{L_y}} \cancel{L_y} = 2\pi n$$

$$\Phi = \frac{2\pi\hbar}{e} n = \Phi_0 \cdot n$$

$$\Psi_{k_y} = e^{ik_y y} H_n(x-x_0)\, e^{-\frac{(x-x_0)^2}{2\ell_B^2}}$$

$$\boxed{I = \frac{\partial U}{\partial \Phi}}$$

$$A_\theta = \frac{\Phi}{L_y}$$

$$k_y \to \frac{eA_\theta}{\hbar} + k_y$$

$$\Psi \propto e^{ik_y y} \longrightarrow e^{i\left(k_y y + \frac{eA_\theta}{\hbar} y\right)}$$

$$\Psi_{k_y} = e^{ik_y y} H_n(x - x_0)\, e^{-\frac{(x-x_0)^2}{2l_B^2}}$$

$$\boxed{I = \frac{\partial U}{\partial \Phi}}$$

$$x_0 = l_B^2 k_y$$

$$e^{i\frac{eA_\theta}{\hbar}(\ldots}$$

$$A_\theta = \frac{\Phi}{L_y}$$

$$\frac{eA_\theta}{\hbar} L_y$$

$$k_y \to \frac{eA_\theta}{\hbar} + k_y$$

$$\frac{e\Phi}{\hbar \cancel{L_y}} \cancel{L_y}$$

$$\Psi \propto e^{ik_y y} \longrightarrow e^{i(k_y + \frac{eA_\theta}{\hbar} y)}$$

$$\phi = \frac{2\pi\hbar}{e}$$

$$\Psi_{k_y} = e^{ik_y y} H_n(x-x_0)\, e^{-\frac{(x-x_0)^2}{2l_B^2}}$$

$$\boxed{I = \frac{\partial U}{\partial \Phi}}$$

$$A_\theta = \frac{\Phi}{L_y}$$

$$X_0 = l_B^2 k_y$$

$$E_n \to E_n + eEX_0 + \dots$$

$$k_y \to \frac{eA_\theta}{\hbar} + k_y$$

$$\Psi \propto e^{ik_y y} \longrightarrow e^{i\left(k_y + \frac{eA_\theta}{\hbar}y\right)}$$

$$e^{i\frac{eA_\theta}{\hbar}(\dots}$$

$$\frac{eA_\theta}{\hbar}L_y$$

$$\frac{e\Phi}{\hbar \cancel{L_y}}$$

$$\Phi = \frac{2\pi\hbar}{e}$$

$$\Psi_{k_y} = e^{ik_y y} H_n(x-x_0)\, e^{-\frac{(x-x_0)^2}{2l_B^2}}$$

$$\boxed{I = \frac{\partial U}{\partial \Phi}}$$

$$X_0 = l_B^2 k_y$$

$$A_\theta = \frac{\Phi}{L_y}$$

$$E_n \to E_n + eEX_0 + \frac{eE}{B^2}$$

$$k_y \to \frac{eA_\theta}{\hbar} + k_y$$

$$\Psi \propto e^{ik_y y} \longrightarrow e^{i\left(k_y + \frac{eA_\theta}{\hbar}\right) y}$$

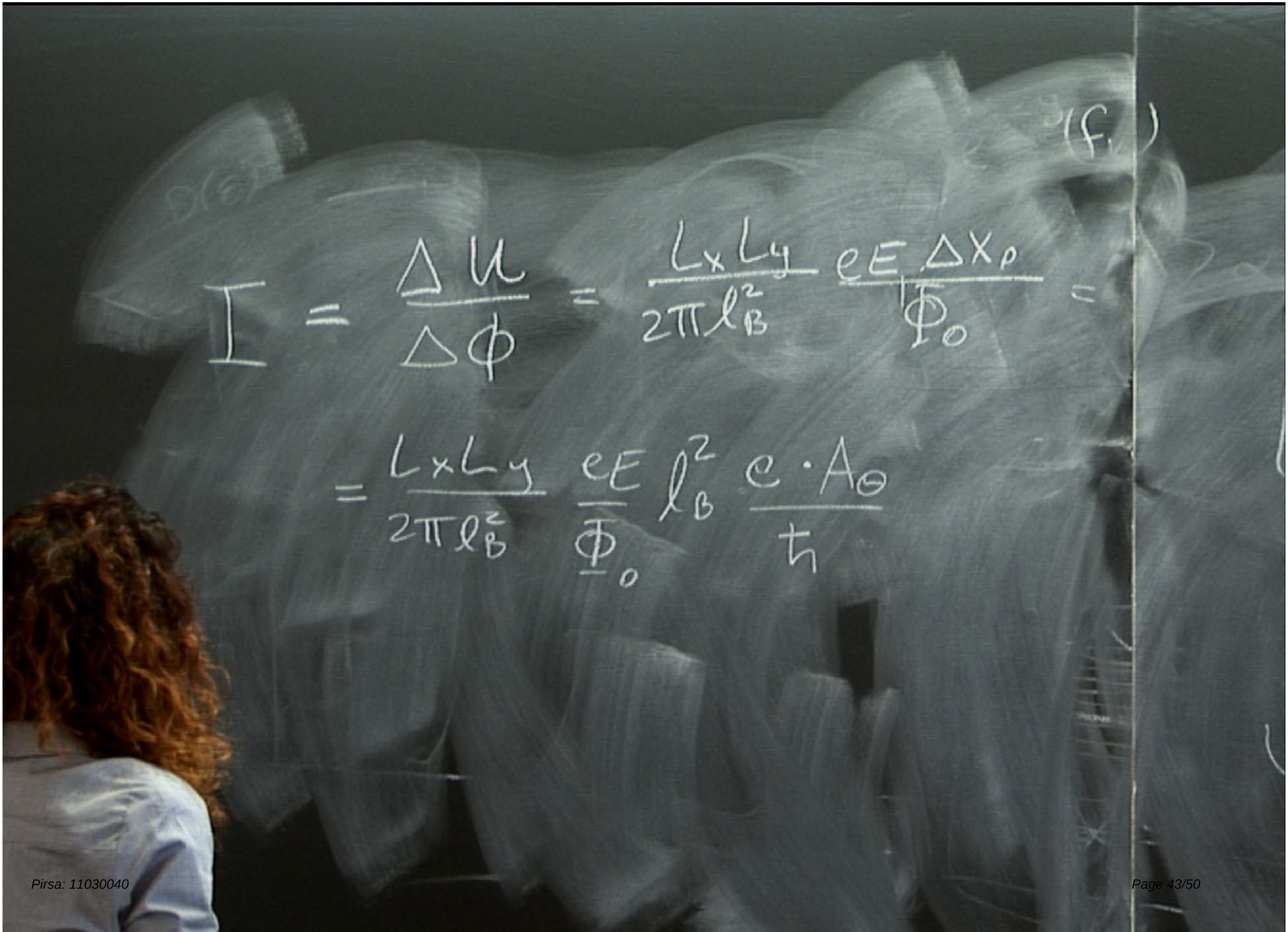
$$e^{i\frac{eA_\theta}{\hbar}(y}$$

$$\frac{eA_\theta}{\hbar} L_y$$

$$\frac{e\Phi}{\hbar \cancel{k_y}}$$

$$\phi = \frac{2\pi\hbar}{e}$$

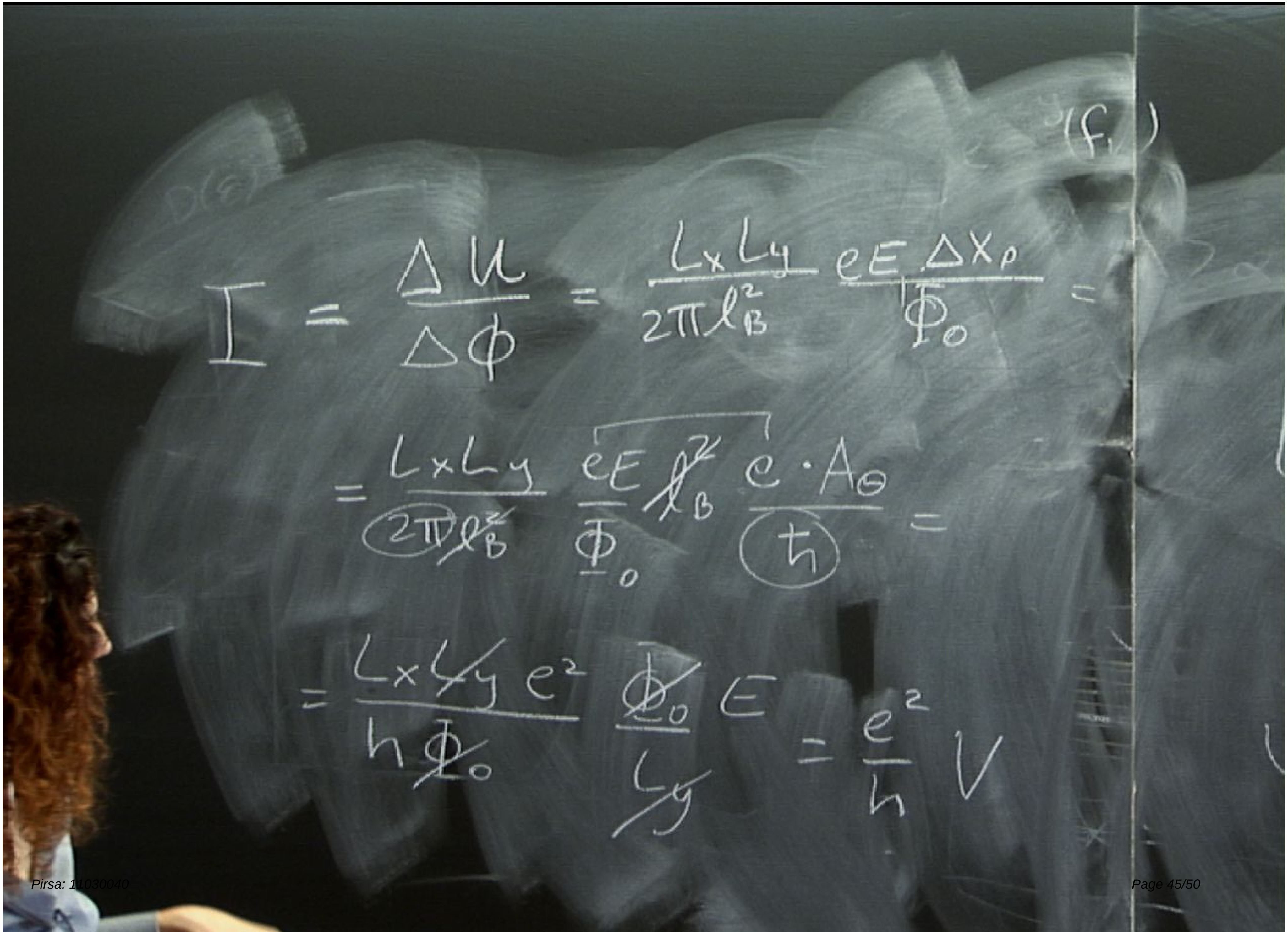
$$I = \frac{\Delta U}{\Delta \phi} = \frac{L_x L_y}{2\pi \ell_B^2}$$

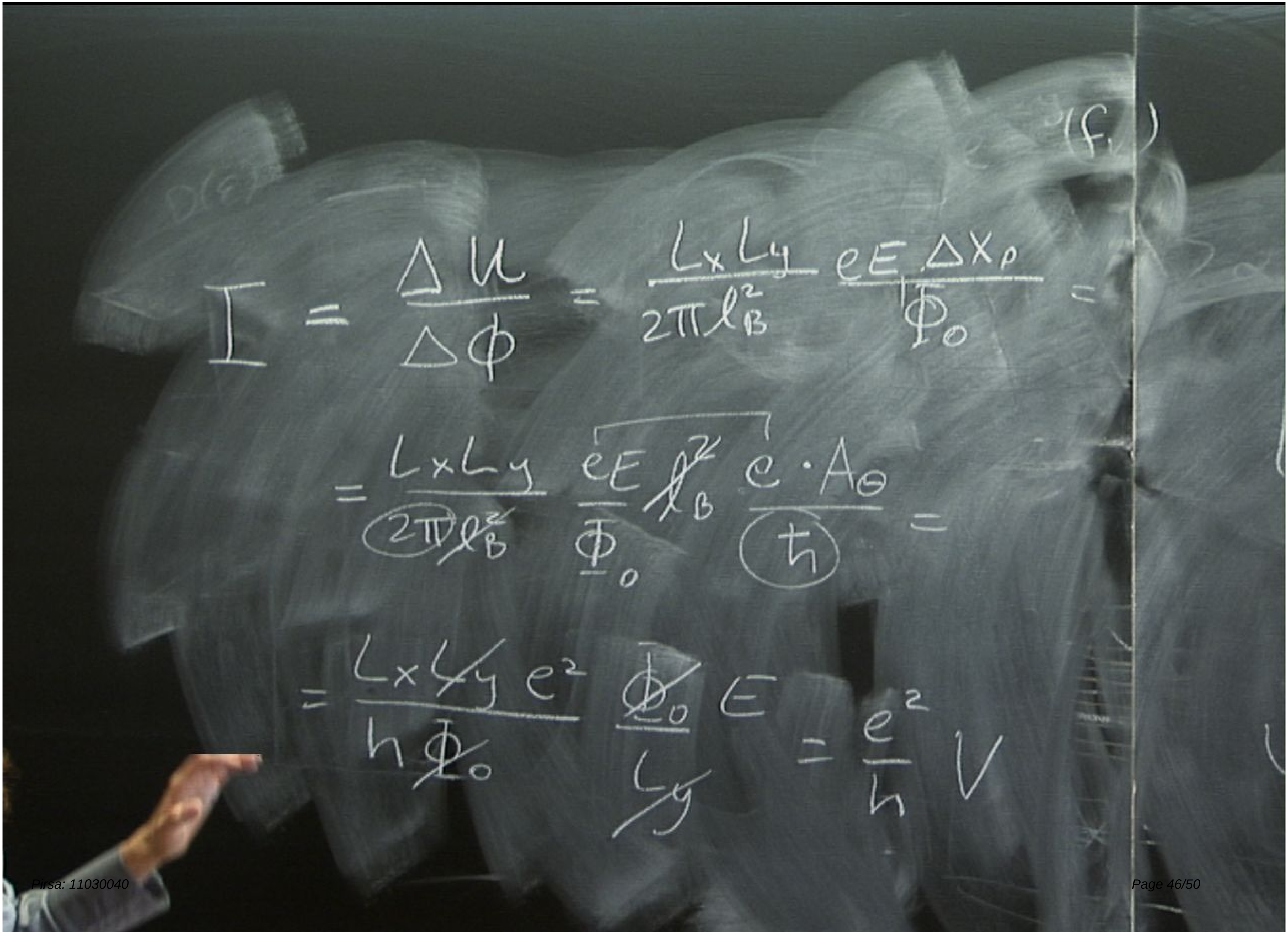

$I = \frac{\Delta U}{\Delta \phi} = \frac{L_x L_y}{2\pi \ell_B^2} \frac{eE\,\Delta x_0}{\Phi_0} =$
$= \frac{L_x L_y}{2\pi \ell_B^2} \frac{eE}{\Phi_0} \ell_B^2 \frac{e \cdot A_\theta}{\hbar}$

$$I = \frac{\Delta U}{\Delta \phi} = \frac{L_x L_y}{2\pi \ell_B^2} \frac{eE\,\Delta x_0}{\Phi_0} =$$

$$= \frac{L_x L_y}{2\pi \cancel{\ell_B^2}} \frac{eE\,\cancel{\ell_B^2}}{\Phi_0} \frac{e \cdot A_\theta}{\hbar} =$$

$$= \frac{L_x L_y e^2}{h \Phi_0} \frac{\Phi_0}{L_y}$$

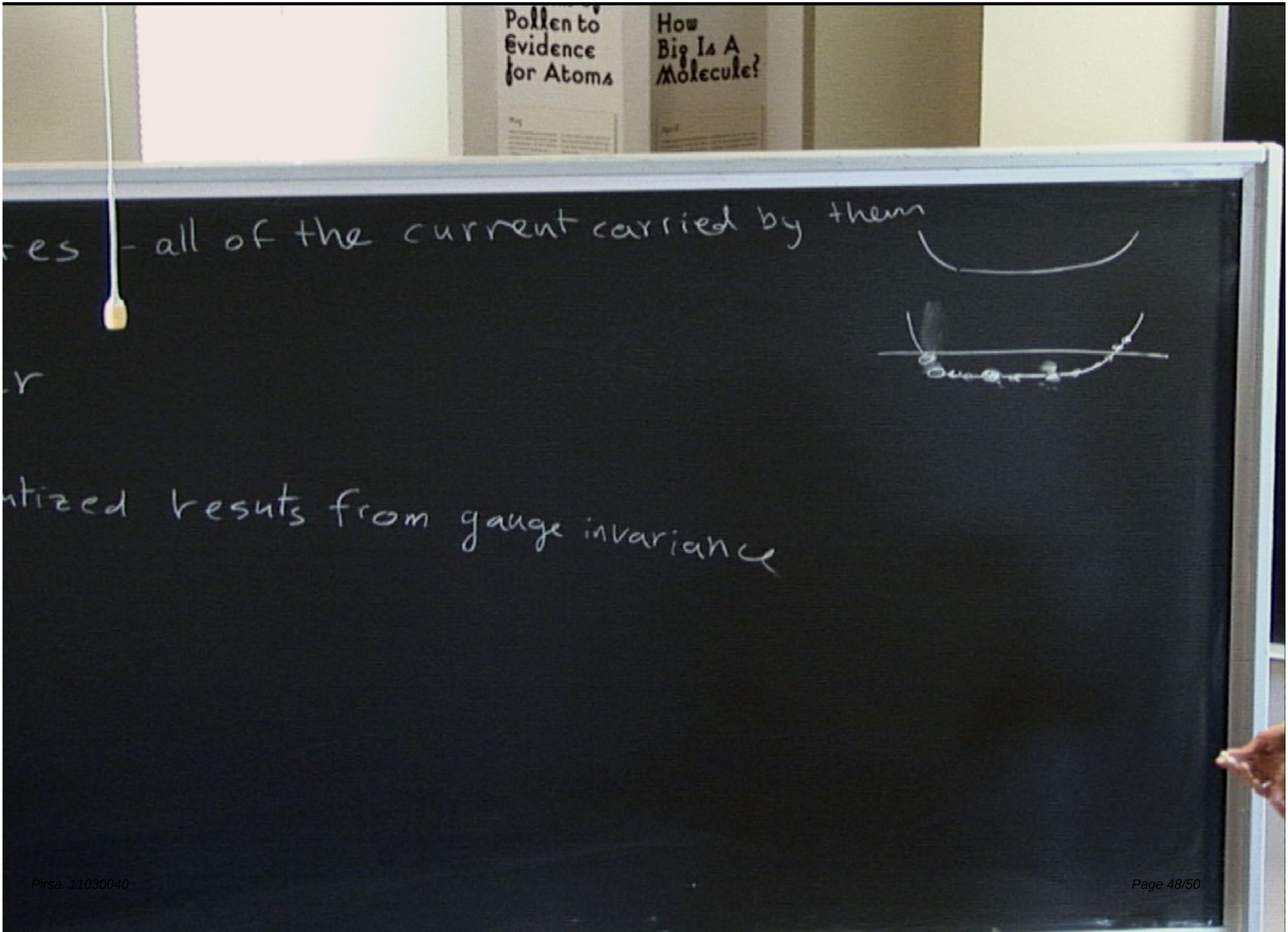

$I = \frac{\Delta U}{\Delta \phi} = \frac{L_x L_y}{2\pi \ell_B^2} \frac{eE\Delta x_0}{\Phi_0} =$
$= \frac{L_x L_y}{2\pi \cancel{\ell_B^2}} \frac{eE}{\Phi_0} \cancel{\ell_B^2} \frac{e \cdot A_\theta}{\hbar} =$
$= \frac{L_x \cancel{L_y} e^2}{h \cancel{\Phi_0}} \frac{\cancel{\Phi_0}}{\cancel{L_y}} E = \frac{e^2}{h} V$

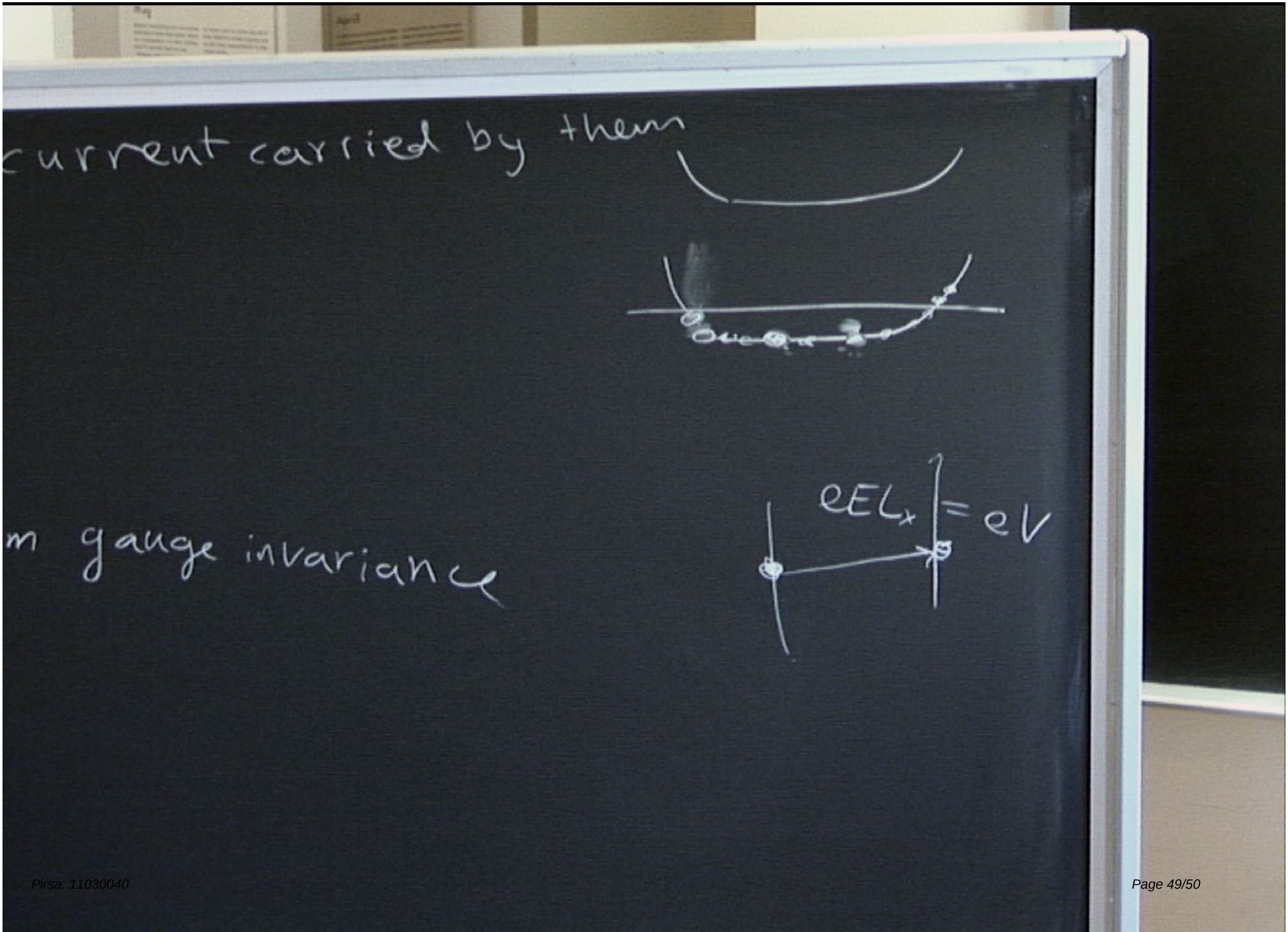


$I = \frac{\Delta U}{\Delta \phi} = \frac{L_x L_y}{2\pi \ell_B^2} \frac{eE\,\Delta x_p}{\Phi_0} =$
$= \frac{L_x L_y}{2\pi \ell_B^2} \frac{eE\,\ell_B^2}{\Phi_0} \frac{e \cdot A_\theta}{\hbar} =$
$= \frac{L_x \cancel{L_y} e^2}{h \cancel{\Phi_0}} \frac{\cancel{\Phi_0}}{\cancel{L_y}} E = \frac{e^2}{h} V$

$$I = \frac{\Delta U}{\Delta \phi} =$$

$$I = \frac{L_x \cancel{L_y} e^2}{h \cancel{\Phi_0}} \, \frac{\cancel{\Phi_0}}{\cancel{L_y}} E = \frac{e^2}{h} V$$


Pollen to
Evidence
for Atoms
How
Big Is A
Molecule?
es - all of the current carried by them
r
ntized results from gauge invariance


current carried by them
m gauge invariance
eEL_x = eV

$$I = \frac{\Delta u}{\Delta \phi} = \frac{\tilde{n} e V}{h/e} = \tilde{n}\left(\frac{e^2}{h}\right) V$$

$$I = \frac{L_x \cancel{L_y}\, e^2}{h \cancel{\Phi_0}} \, \frac{\cancel{\Phi_0}}{\cancel{L_y}} \, E = \frac{e^2}{h} V$$