

Title: Explorations in Condensed Matter - Lecture 5

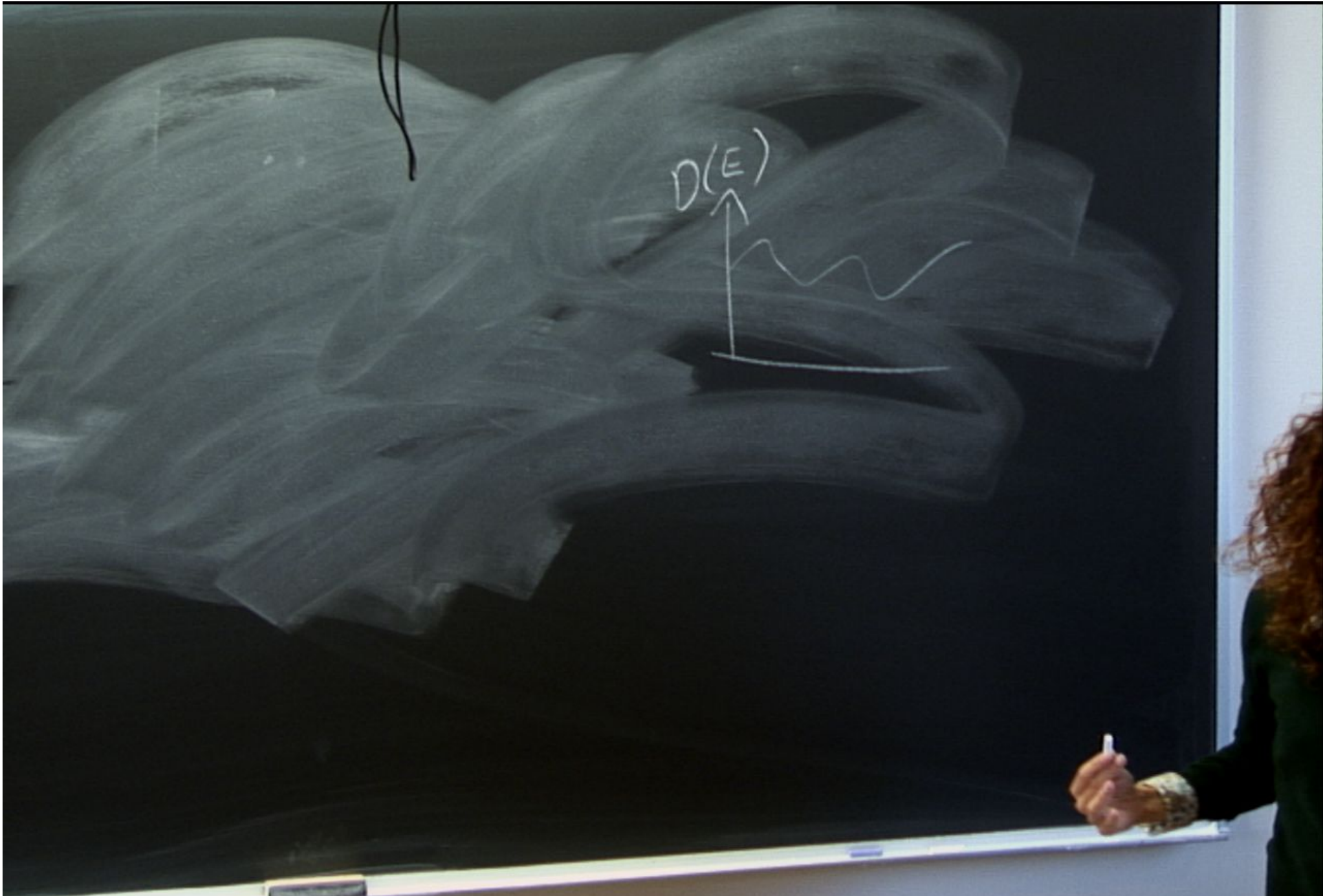
Date: Mar 18, 2011 10:15 AM

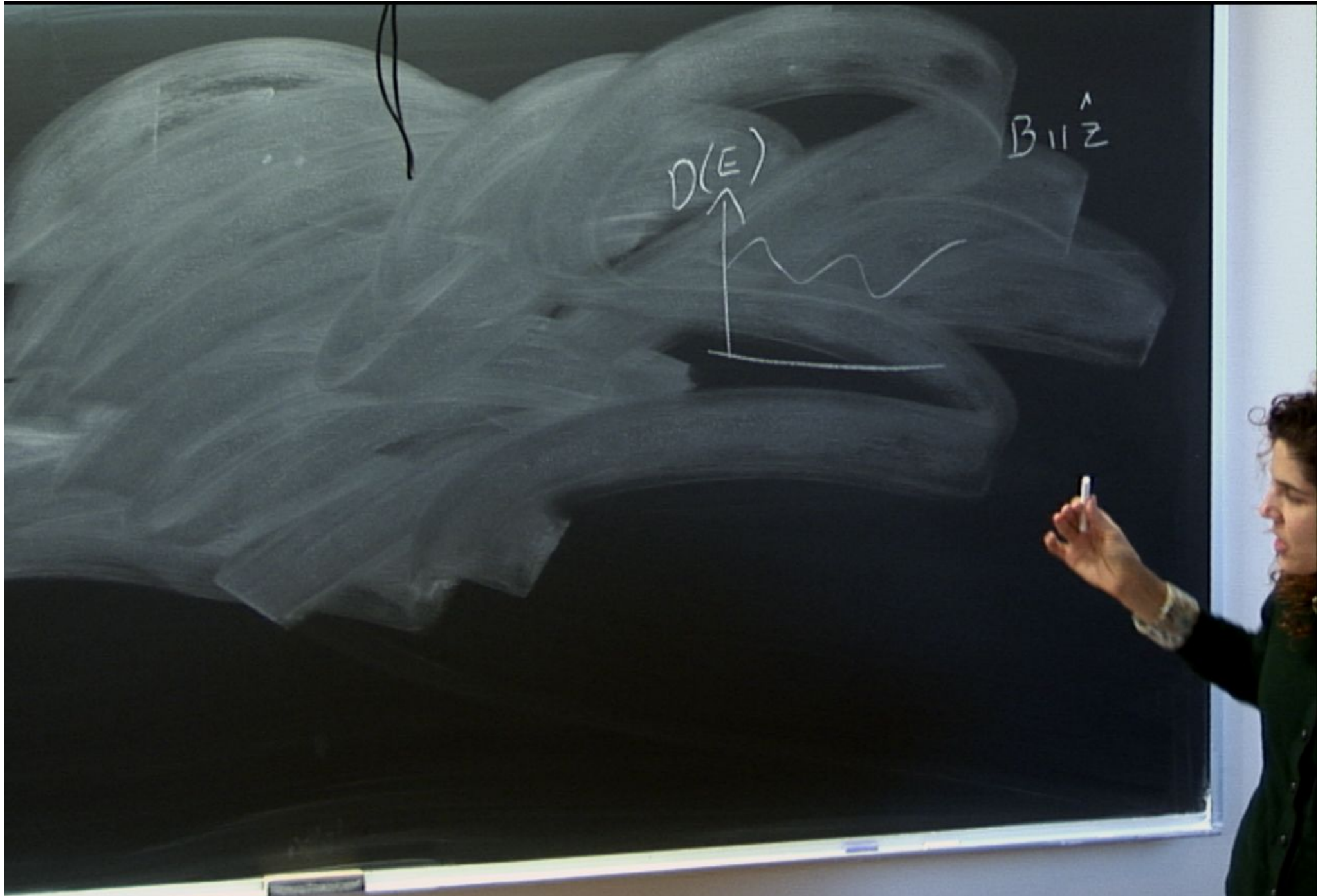
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Abstract:

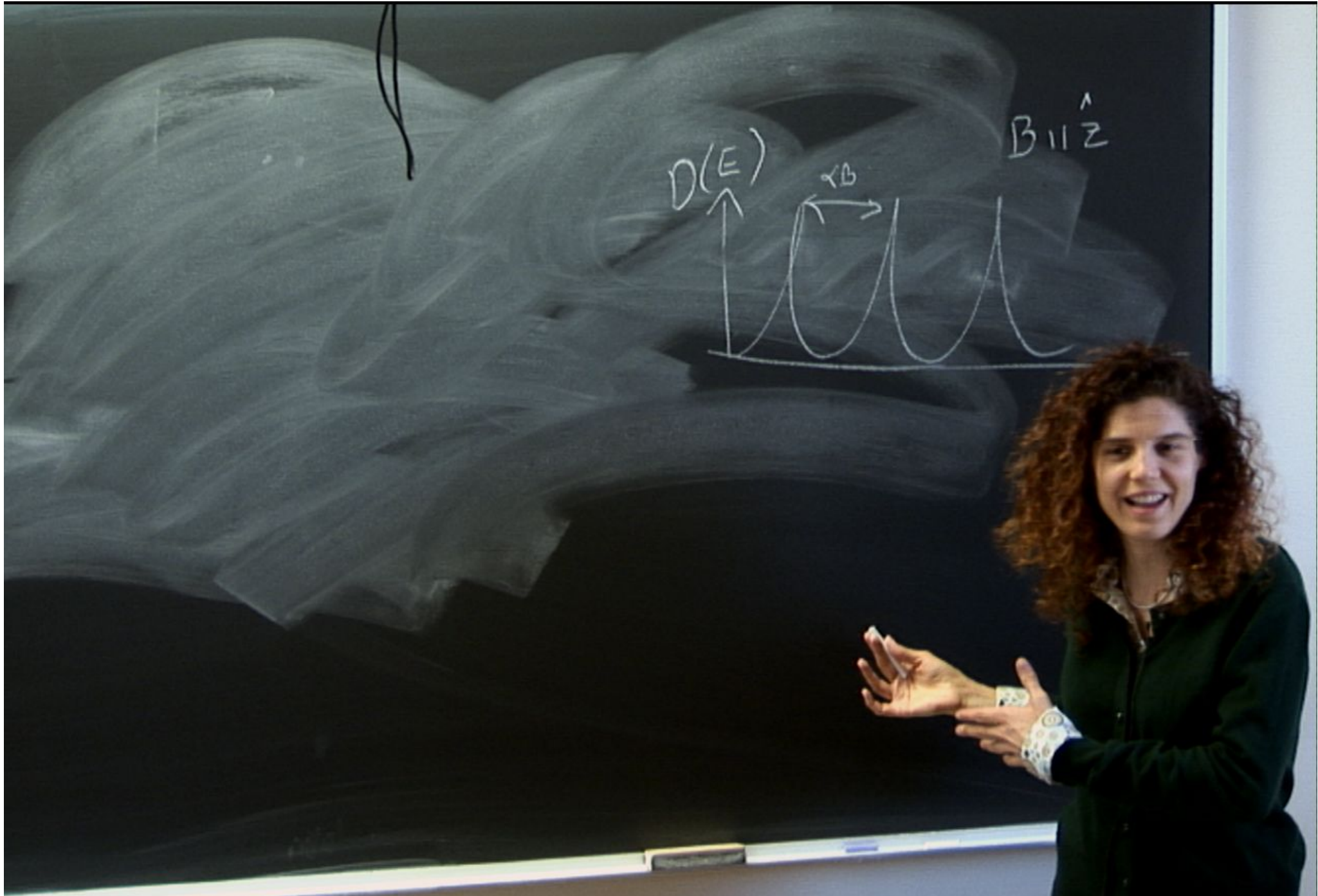


perimeter scholars
INTERNATIONAL









$$\Psi \rightarrow e^{i\varphi} \Psi$$

$D(E)$

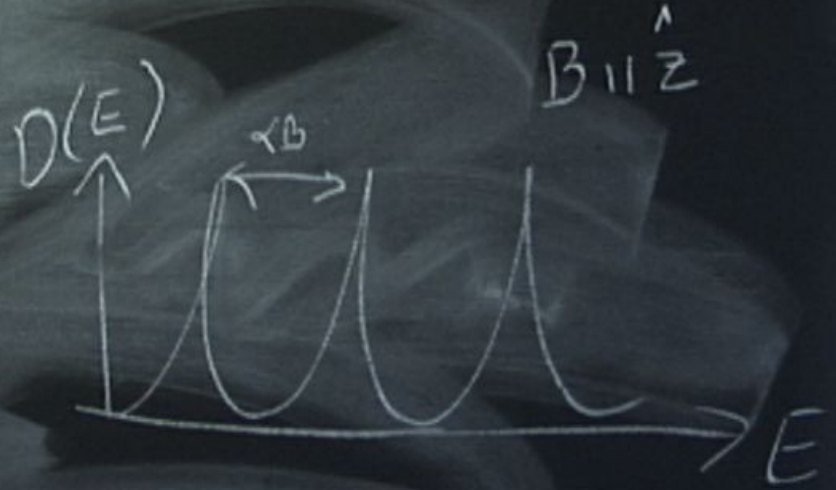
χ_B

$\hat{B} \parallel \hat{z}$



$$\psi \rightarrow e^{i\varphi} \psi$$

$$= \psi^\dagger M \psi$$



$$\Psi \rightarrow e^{i\varphi(\vec{r})} \Psi$$

$$H = \Psi^\dagger \underbrace{M}_{\partial} \Psi$$

$D(E)$



$$\Psi \rightarrow e^{i\psi(\vec{r})} \Psi$$

$$H = \Psi^\dagger \underbrace{M}_{\partial_\mu} \Psi$$

$$c_i^\dagger c_j \rightarrow e^{i(\phi_j - \phi_i)} \equiv e$$

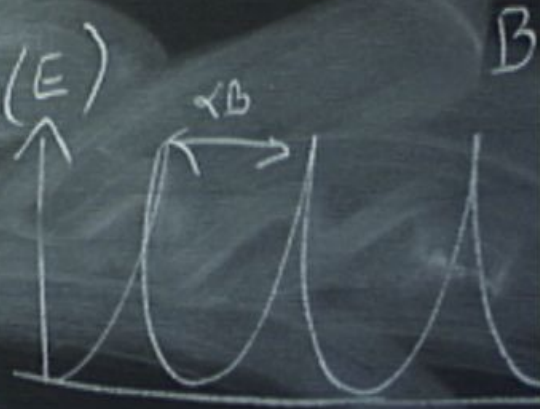


$$\Psi \rightarrow e^{i\varphi(\vec{r})} \Psi$$

$$H = \Psi^\dagger \underbrace{M}_{\partial_\mu} \Psi$$

$$c_i^\dagger c_j \rightarrow e^{i(\varphi_j - \varphi_i)} \equiv e^{i \int \vec{A} \cdot d\vec{r}}$$

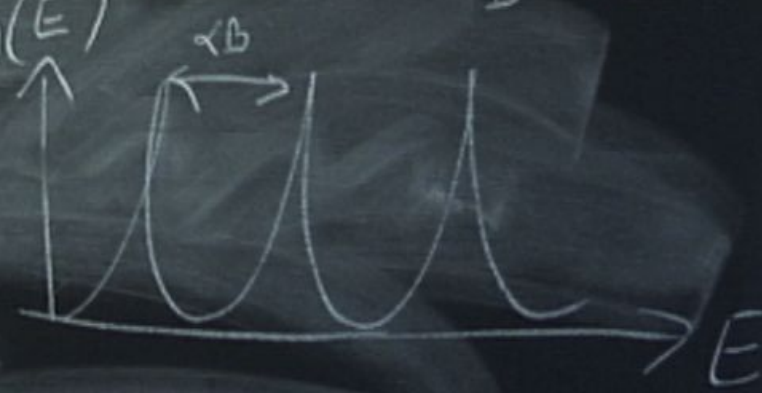
$D(E)$



$$\Psi \rightarrow e^{i\varphi(\vec{r})} \Psi$$

$$\Psi^\dagger \underbrace{M}_{\frac{\partial}{\partial x}} \Psi$$

$$D(E)$$



$$c_j \rightarrow e^{i(\phi_j - \varphi_i)} \equiv e^{i \int \vec{A} \cdot d\vec{r}}$$

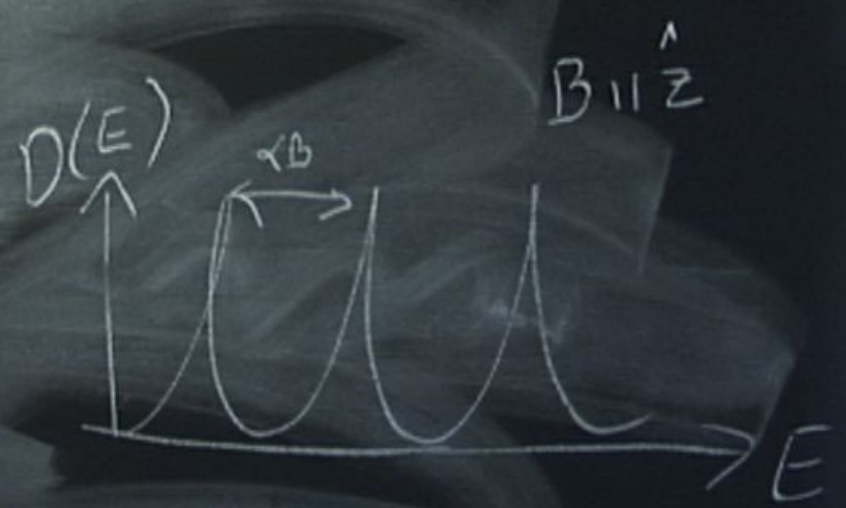
$$c_i^\dagger c_j$$

$$k \rightarrow e^{i\psi(\vec{r})} \psi$$

$$\psi^\dagger \underbrace{M}_{\frac{\partial}{\partial x}} \psi$$

$$c_i^\dagger c_j \rightarrow e^{i(\phi_j - \phi_i)} \equiv e^{i \int \vec{A} \cdot d\vec{r}}$$

$$c_i^\dagger c_j \rightarrow t e^{iA_{ij}} c_i^\dagger c_j$$

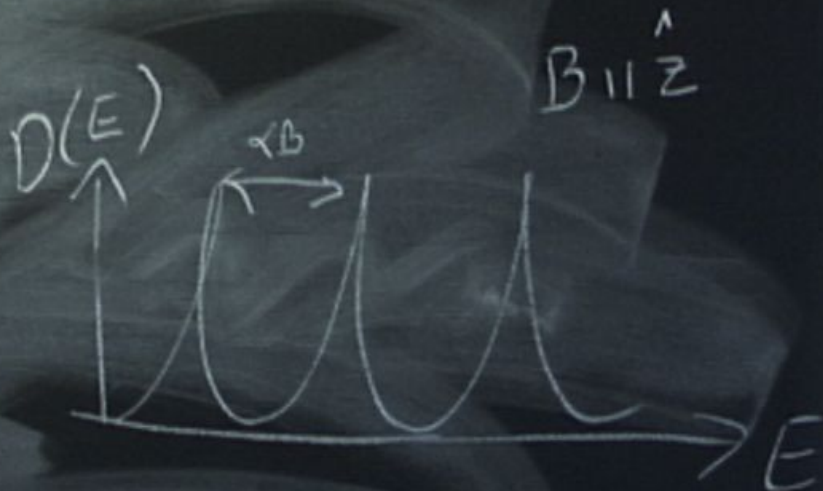


$$\Psi \rightarrow e^{i\varphi(\vec{r})} \Psi$$

$$H = \Psi^\dagger \underbrace{M}_{\frac{\partial \mu}{\partial x}} \Psi$$

$$c_i^\dagger c_j \rightarrow e^{i(\varphi_j - \varphi_i)} \equiv e^{i \int \vec{A} \cdot d\vec{r}}$$

$$t c_i^\dagger c_j \rightarrow t e^{i A_{ij}} c_i^\dagger c_j$$



t deforms

t deforms

$$t_{ij} \rightarrow t_0 + \gamma t \cdot (\vec{u}_i - \vec{u}_j)$$

t deforms

$$t_{ij} \rightarrow t_0 + \gamma \vec{\sigma}_{ij} \cdot (\vec{u}_i - \vec{u}_j)$$

t deforms

$$t_{ij} \rightarrow t_0 + \gamma \tau_{ij} (\vec{u}_i - \vec{u}_j)$$

$$f(d) \equiv f(d) =$$

t deforms

$$t_{ij} \rightarrow t_0 + \gamma \underbrace{\vec{\sigma}_{ij}}_{\text{stress}} (\vec{u}_i - \vec{u}_j)$$

$$f(\vec{d}) \equiv f(d) = f_0 + f_1$$

t deforms

$$t_{ij} \rightarrow t_0 + \gamma \tau_{ij} (\vec{u}_i - \vec{u}_j)$$

$$f(\vec{d}) \equiv f(d) = f_0 + f_1$$



t deforms

$$t_{ij} \rightarrow t_0 + \gamma \vec{c}_{ij} \cdot (\vec{u}_i - \vec{u}_j)$$

$$f(d) = f(d) = f_0 + f_1$$



$$d = |\vec{c} + \vec{u}_i - \vec{u}_j|$$

t deforms

$$t_{ij} \rightarrow t_0 + \gamma \tau_{ij} (\vec{u}_i - \vec{u}_j)$$

$$f(|\vec{d}|) \equiv f(d) = f_0 + f_1$$



$$|\vec{u}_i - \vec{u}_j| \equiv \sqrt{\tau^2 + (u_i - u_j)^2 + 2\tau \cdot (u_i - u_j)}$$

t deforms

$$t_{ij} \rightarrow t_0 + \gamma \tau_{ij} (\vec{u}_i - \vec{u}_j)$$

$$f(d) \equiv f(d) = f_0 + f_1$$

$$d = |\vec{\tau} + \vec{u}_i - \vec{u}_j| = \sqrt{\tau^2 + (u_i - u_j)^2 + 2\vec{\tau} \cdot (\vec{u}_i - \vec{u}_j)}$$

$$\approx |\tau| + \frac{\vec{\tau} \cdot (u_i - u_j)}{|\tau|}$$

t deforms

$$t_{ij} \rightarrow t_0 + \gamma \tau_{ij} (\vec{u}_i - \vec{u}_j)$$

$$f(d) = f_0 + f_1$$

$$d = |\vec{c} + \vec{u}_i - \vec{u}_j| = \sqrt{\tau^2 + (u_i - u_j)^2 + 2\vec{\tau} \cdot (\vec{u}_i - \vec{u}_j)}$$

$$\approx |\tau| + \frac{\vec{\tau} \cdot (u_i - u_j)}{|\tau|}$$

t deforms

$$t_{ij} \rightarrow t_0 + \gamma \tau_{ij} (\vec{u}_{iA} - \vec{u}_{jB})$$

$$|\vec{d}| = f(d) = f_0 + f_1$$

$$d = |\vec{u}_i - \vec{u}_j| = \sqrt{\tau^2 + (u_i - u_j)^2 + 2\tau \cdot (\vec{u}_i - \vec{u}_j)}$$

$$\approx |\tau| + \frac{\tau \cdot (u_i - u_j)}{|\tau|}$$

u_i

$$u_i \rightarrow \vec{u}(\vec{r})$$

$$^2 + (u_i - u_j)^2 + 2\vec{\tau} \cdot (\vec{u}_i - \vec{u}_j)$$

$$\frac{1 + \vec{\tau} \cdot (u_i - u_j)}{|\tau|}$$

$$u_i \rightarrow \vec{u}(\vec{r})$$

$$u_i - u_j =$$

$$^2 + (u_i - u_j)^2$$

$$+ |\vec{r} \cdot (u_i - u_j)|$$

$$u_i \rightarrow \vec{u}(\vec{r})$$

$$u_i - u_j = R_i + \frac{\partial u}{\partial r} R_i$$

$$^2 + (u_i - u_j)^2 + 2\vec{\tau} \cdot (\vec{u}_i - \vec{u}_j)$$

$$\frac{1 + \vec{\tau} \cdot (u_i - u_j)}{|\tau|}$$

$$u_i \rightarrow \vec{u}(\vec{r})$$

$$u_i - u_j = (\vec{\tau} \cdot \nabla) u$$

$$^2 + (u_i - u_j)^2 + \dots$$

$$= \frac{|\vec{\tau} \cdot (u_i - u_j)|}{|\vec{\tau}|}$$

$$u_i \rightarrow \vec{u}(\vec{r})$$

$$u_i - u_j = (\vec{\tau} \cdot \vec{\nabla}) \vec{u} =$$

$$\tau_x \partial_x u_x + \tau_y \partial_y u_y + \tau_z \partial_z u_z$$

$$\sqrt{(\tau_x \partial_x u_x + \tau_y \partial_y u_y + \tau_z \partial_z u_z)^2 + (u_i - u_j)^2} + \tau \cdot (u_i - u_j)$$

$$\frac{|\tau \cdot (u_i - u_j)|}{|\tau|}$$

$$u_i \rightarrow \vec{u}(\vec{r})$$

$$u_i - u_j = (\vec{\tau} \cdot \vec{\nabla}) \vec{u} = \begin{pmatrix} \tau_x \partial_x u_x + \tau_y \partial_y u_x \\ \tau_x \partial_x u_y + \tau_y \partial_y u_y \end{pmatrix}$$



$$u_i \rightarrow \vec{u}(\vec{r})$$

$$u_i - u_j = (\vec{\tau} \cdot \vec{\nabla}) \vec{u} = \begin{pmatrix} \tau_x \partial_x u_x + \tau_y \partial_y u_x \\ \tau_x \partial_x u_y + \tau_y \partial_y u_y \end{pmatrix}$$

$$\sqrt{r^2 + (u_i - u_j)^2} + \vec{\tau} \cdot (\vec{u}_i - \vec{u}_j)$$

$$\frac{r^2 + \vec{\tau} \cdot (u_i - u_j)}{|\tau|}$$

$$u_i \rightarrow \vec{u}(\vec{r})$$

$$u_i - u_j = (\vec{\tau} \cdot \vec{\nabla}) \vec{u} = \begin{pmatrix} \tau_x \partial_x u_x + \tau_y \partial_y u_x \\ \tau_x \partial_x u_y + \tau_y \partial_y u_y \end{pmatrix}$$

$$\Rightarrow \delta t = \delta \vec{\tau} \cdot (\vec{\tau} \cdot \vec{\nabla}) \vec{u}$$

$$u_i - u_j)^2 + 2\vec{\tau}$$

$$\frac{\vec{\tau} \cdot (u_i - u_j)}{|\tau|}$$

$$u_i \rightarrow \vec{u}(\vec{r})$$

$$u_i - u_j = (\vec{\tau} \cdot \vec{\nabla}) \vec{u} = \begin{pmatrix} \tau_x \partial_x u_x + \tau_y \partial_y u_x \\ \tau_x \partial_x u_y + \tau_y \partial_y u_y \end{pmatrix}$$

$$\Rightarrow \delta t = \delta \vec{\tau} \cdot (\vec{\tau} \cdot \vec{\nabla}) \vec{u}$$

$$u_i - u_j)^2 + \dots$$

$$\frac{\vec{\tau} \cdot (u_i - u_j)}{|\tau|}$$

$$\tau_\mu \partial_\nu u_\mu$$

$$u_i \rightarrow \vec{u}(\vec{r})$$

$$u_i - u_j = (\vec{\tau} \cdot \vec{\nabla}) \vec{u} = \begin{pmatrix} \tau_x \partial_x u_x + \tau_y \partial_y u_x \\ \tau_x \partial_x u_y + \tau_y \partial_y u_y \end{pmatrix}$$

$$\Rightarrow \delta t = \gamma \vec{\tau} \cdot (\vec{\tau} \cdot \vec{\nabla}) \vec{u}$$

$$= \gamma \tau_\mu \tau_\nu \partial_\nu u_\mu$$

$$u_i \rightarrow \vec{u}(\vec{r})$$

$$u_i - u_j = (\vec{\tau} \cdot \vec{\nabla}) \vec{u} = \begin{pmatrix} \tau_x \partial_x u_x + \tau_y \partial_y u_x \\ \tau_x \partial_x u_y + \tau_y \partial_y u_y \end{pmatrix}$$

$$\Rightarrow \delta t = \gamma \vec{\tau} \cdot (\vec{\tau} \cdot \vec{\nabla}) \vec{u}$$

$$u_i - u_j)^2 + 2\vec{\tau} \cdot (u_i - u_j) = \gamma \tau_\mu \tau_\nu \partial_\nu u_\mu$$

$$\frac{\vec{\tau} \cdot (u_i - u_j)}{|\tau|}$$

$$\rightarrow \vec{u}(\vec{r})$$

$$u_i - u_j = (\vec{\tau} \cdot \vec{\nabla}) \vec{u} = \begin{pmatrix} \tau_x \partial_x u_x + \tau_y \partial_y u_x \\ \tau_x \partial_x u_y + \tau_y \partial_y u_y \end{pmatrix}$$

$$u(\vec{r}) \approx e^{i\vec{k} \cdot \vec{r}} u(\vec{r})$$

$$\Rightarrow \delta t = \delta \vec{\tau} \cdot (\vec{\tau} \cdot \vec{\nabla}) \vec{u}$$

$$\vec{\tau} \cdot (u_i - u_j) = \delta \tau_\mu \tau_\nu \partial_\nu u_\mu$$

$$\vec{u}(\vec{r})$$

$$u_i - u_j = (\vec{\tau} \cdot \vec{\nabla}) \vec{u} = \begin{pmatrix} \tau_x \partial_x u_x + \tau_y \partial_y u_x \\ \tau_x \partial_x u_y + \tau_y \partial_y u_y \end{pmatrix}$$

$$\Rightarrow \delta t = \gamma \vec{\tau} \cdot (\vec{\tau} \cdot \vec{\nabla}) \vec{u}$$

$$\vec{\tau} \cdot (\vec{u}_i - \vec{u}_j) = \gamma \tau_\mu \tau_\nu \partial_\nu u_\mu$$

$$\psi(\vec{r}) \approx e^{i\vec{k} \cdot \vec{r}} \chi(\vec{r})$$

$$\rightarrow \vec{u}(\vec{r})$$

$$u_i - u_j = (\vec{\tau} \cdot \vec{\nabla}) \vec{u} = \begin{pmatrix} \tau_x \partial_x u_x + \tau_y \partial_y u_x \\ \tau_x \partial_x u_y + \tau_y \partial_y u_y \end{pmatrix}$$

$$\Rightarrow \delta t = \delta \vec{\tau} \cdot (\vec{\tau} \cdot \vec{\nabla}) \vec{u}$$

$$\overline{\vec{\tau} \cdot (\vec{u}_i - \vec{u}_j)} = \delta \tau_\mu \tau_\nu \partial_\nu u_\mu$$

$$\psi_1(\vec{r}) \approx e^{i\vec{k} \cdot \vec{r}} \chi(\vec{r})$$

$$\downarrow$$

$$e^{i\vec{L} \cdot \vec{r}} = e^{i(\vec{k} + \vec{q}) \cdot \vec{r}}$$



$$\vec{\nabla} \cdot \vec{u} = \begin{pmatrix} \tau_x \partial_x u_x + \tau_y \partial_y u_x \\ \tau_x \partial_x u_y + \tau_y \partial_y u_y \end{pmatrix}$$

$$= \delta \vec{\tau} \cdot (\vec{\tau} \cdot \vec{\nabla}) \vec{u}$$

$$= \delta \tau_\mu \tau_\nu \partial_\nu u_\mu$$

$$\psi(r) \approx e^{i\mathbf{k} \cdot \mathbf{r}} \chi(r)$$

$$\downarrow$$

$$e^{i\mathbf{k} \cdot \mathbf{r}} = e^{i(\mathbf{k} + \mathbf{q}) \cdot \mathbf{r}}$$



$$\vec{u}(\vec{r})$$

$$u_i - u_j = (\vec{\tau} \cdot \vec{\nabla}) \vec{u} = \begin{pmatrix} \tau_x \partial_x u_x + \tau_y \partial_y u_x \\ \tau_x \partial_x u_y + \tau_y \partial_y u_y \end{pmatrix}$$

$$\Rightarrow \delta t = \delta \vec{\tau} \cdot (\vec{\tau} \cdot \vec{\nabla}) \vec{u}$$

$$\vec{\tau} \cdot (u_i - u_j) = \delta \tau_\mu \tau_\nu \partial_\nu u_\mu$$

$$\sum_{\vec{r}} t \vec{c}_j \rightarrow E_0 + \sum_{\vec{r}} \delta t(\vec{r}, i) e^{i\vec{k} \cdot \vec{r}}$$

$$\psi_1(\vec{r}) \approx e^{i\vec{k} \cdot \vec{r}} \chi(\vec{r})$$

$$i(\vec{k} + \vec{q}) \cdot \vec{r}$$

$$\vec{u}(\vec{r})$$

$$u_i - u_j = (\vec{\tau} \cdot \vec{\nabla}) \vec{u} = \begin{pmatrix} \tau_x \partial_x u_x + \tau_y \partial_y u_x \\ \tau_x \partial_x u_y + \tau_y \partial_y u_y \end{pmatrix}$$

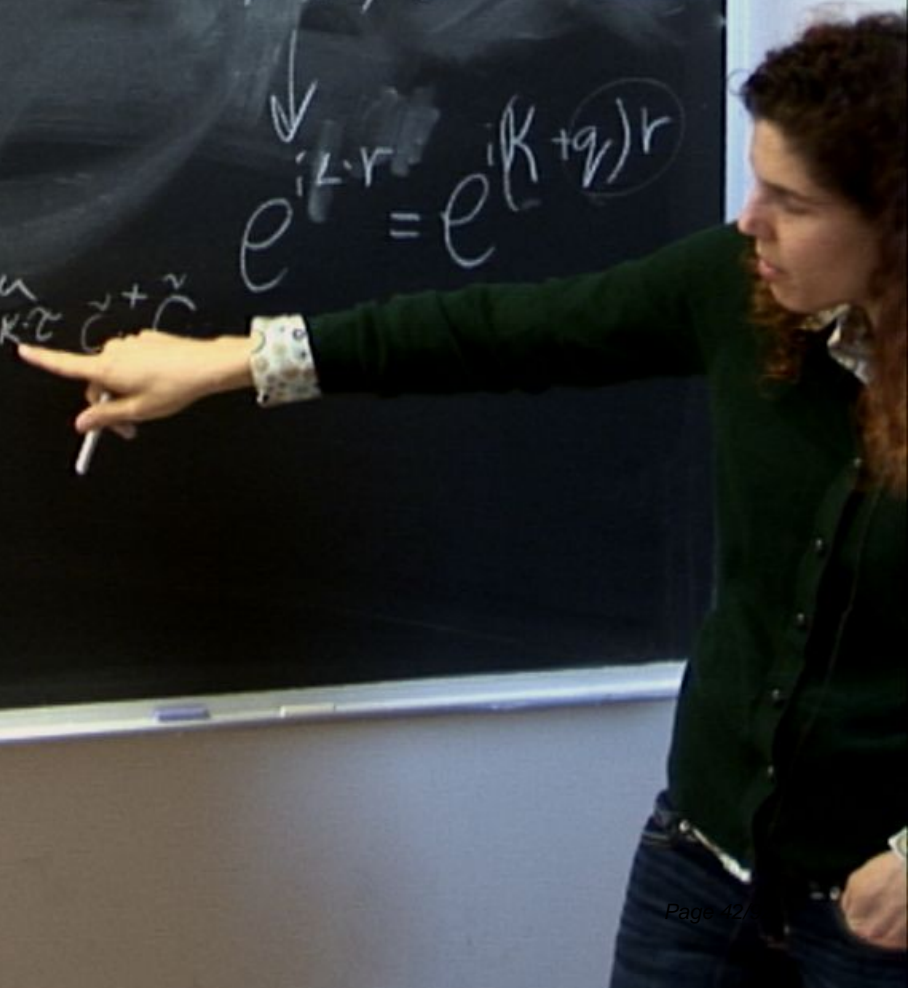
$$\Rightarrow \delta t = \gamma \vec{\tau} \cdot (\vec{\tau} \cdot \vec{\nabla}) \vec{u}$$

$$\vec{\tau} \cdot (u_i - u_j) = \gamma \tau_\mu \tau_\nu \partial_\nu u_\mu$$

$$\sum_{\vec{r}} t c_{\vec{r}} \rightarrow E_0 + \sum_{\vec{r}} \delta t(\vec{r}, i) e^{i\vec{k} \cdot \vec{r} + \vec{c} + \vec{v}}$$

$$\psi_1(\vec{r}) \approx e^{i\vec{k} \cdot \vec{r}} \chi(\vec{r})$$

$$e^{i\vec{k} \cdot \vec{r}} = e^{i(\vec{k} + \vec{q}) \cdot \vec{r}}$$



t deforms

$$\sum_{\mathcal{R}} \begin{pmatrix} \tau_x \\ \tau_y \end{pmatrix}.$$

t deforms

$$\sum_{\vec{r}} \begin{pmatrix} \tau_x \\ \tau_y \end{pmatrix} \cdot \begin{pmatrix} \tau_x u_{xx} + \tau_y u_{xy} \\ \tau_x u_{xy} + \tau_y u_{yy} \end{pmatrix} \cdot e^{i\vec{k} \cdot \vec{r}}$$

$$u_{\mu\nu} = \frac{\partial u_{\mu}}{\partial x_{\nu}}$$

t deforms

$$\sum_{\vec{r}} \begin{pmatrix} \tau_x \\ \tau_y \end{pmatrix} \cdot \begin{pmatrix} \tau_x u_{xx} + \tau_y u_{xy} \\ \tau_x u_{xy} + \tau_y u_{yy} \end{pmatrix} \cdot e^{i\vec{k} \cdot \vec{r}}$$

$$u_{\mu\nu} = \frac{\partial u_{\mu}}{\partial x^{\nu}}$$

$$u = \nabla f$$

$$u_{\mu} = \partial_{\mu} f$$

$$u_{\mu\nu} = \frac{\partial^2 f}{\partial x^{\mu} \partial x^{\nu}}$$

t deforms

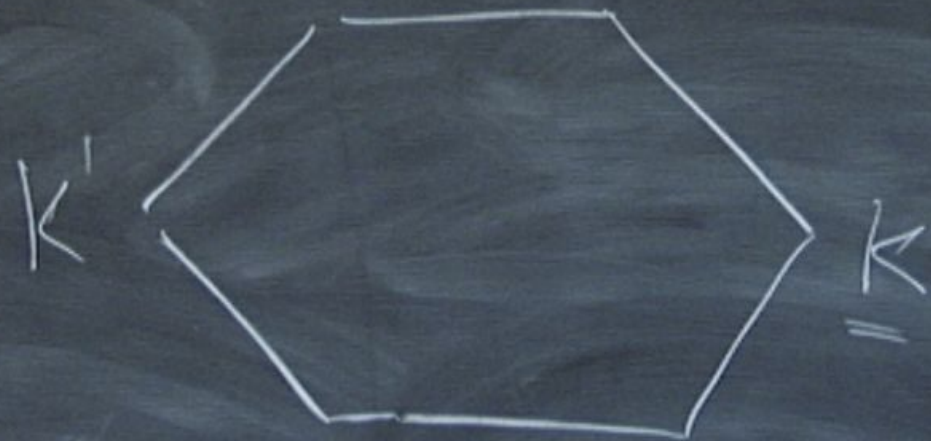
$$\sum_{\vec{\tau}} \begin{pmatrix} \tau_x \\ \tau_y \end{pmatrix} \cdot \begin{pmatrix} \tau_x u_{xx} + \tau_y u_{xy} \\ \tau_x u_{xy} + \tau_y u_{yy} \end{pmatrix} \cdot e^{i\vec{k} \cdot \vec{r}}$$

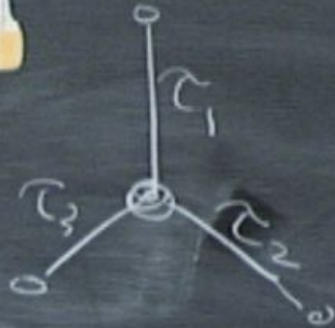
$$u_{\mu\nu} = \frac{\partial u_{\mu}}{\partial x^{\nu}}$$

$$u = \nabla f$$

$$u_{\mu} = \partial_{\mu} f$$

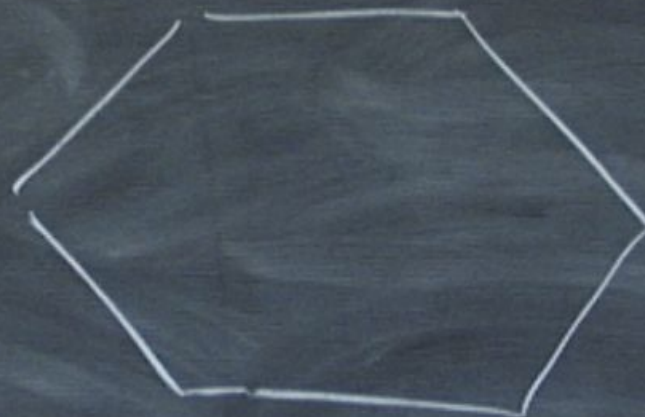
$$u_{\mu\nu} = \frac{\partial^2 f}{\partial x^{\mu} \partial x^{\nu}}$$

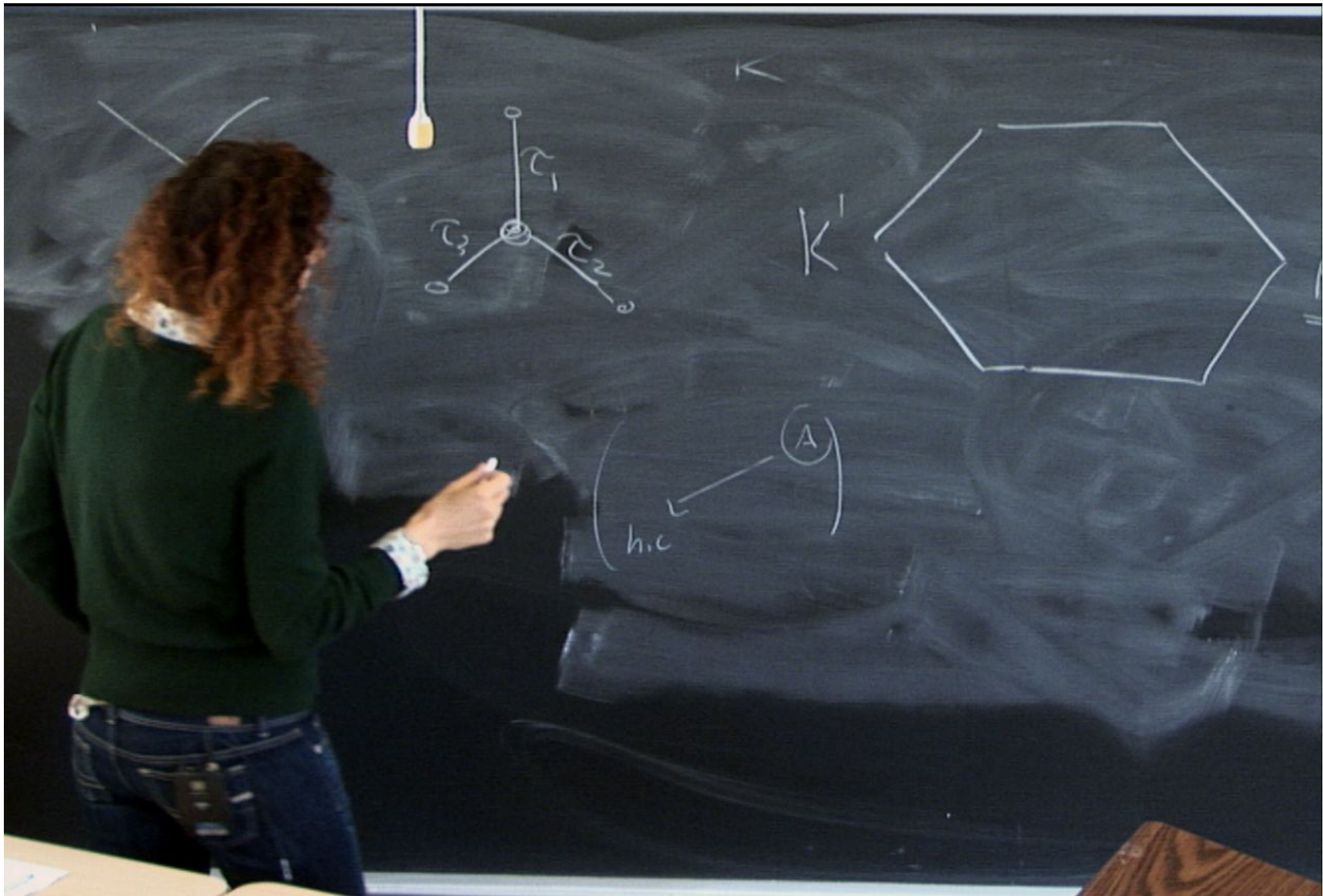


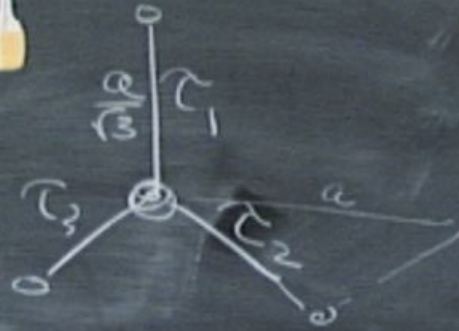


K

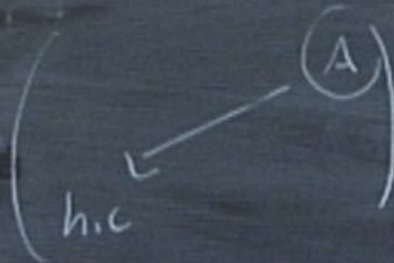
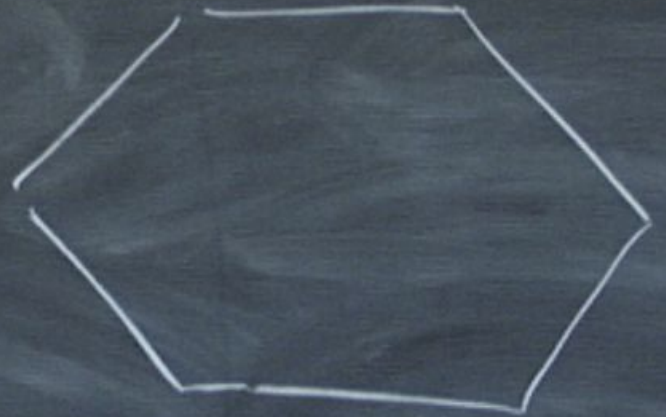
K'







K'



t deforms

$$\sum_{\vec{k}} \begin{pmatrix} \tau_x \\ \tau_y \end{pmatrix} \cdot \begin{pmatrix} \tau_x u_{xx} + \tau_y u_{xy} \\ \tau_x u_{xy} + \tau_y u_{yy} \end{pmatrix} \cdot e^{i\vec{k} \cdot \vec{r}}$$

$$\tau_1 = \frac{1}{\sqrt{3}} \hat{y}$$

$$\tau_2 = \frac{1}{2} \hat{x} - \frac{1}{2\sqrt{3}} \hat{y}$$

$$\tau_3 = -\frac{1}{2} \hat{x} - \frac{1}{2\sqrt{3}} \hat{y}$$

m_s

$(u_{xy}, \tau_y u_{yy})$

$e^{i\vec{k}\cdot\vec{r}}$

$$\tau_1 = \frac{1}{\sqrt{3}} \hat{y}$$

$$\tau_2 = \frac{1}{2} \hat{x} - \frac{1}{2\sqrt{3}} \hat{y}$$

$$\tau_3 = -\frac{1}{2} \hat{x} - \frac{1}{2\sqrt{3}} \hat{y}$$

$e^{i\vec{k}\cdot\vec{r}}$

$$K = \left(\frac{4\pi}{3}, 0 \right)$$

$$\begin{pmatrix} u_{xy} \\ u_{yy} \end{pmatrix} \cdot e^{i\vec{k} \cdot \vec{r}}$$

$$\tau_1 = \frac{1}{\sqrt{3}} \hat{y}$$

$$\tau_2 = \frac{1}{2} \hat{x} - \frac{1}{2\sqrt{3}} \hat{y}$$

$$\tau_3 = -\frac{1}{2} \hat{x} - \frac{1}{2\sqrt{3}} \hat{y}$$

$$e^{i\vec{k} \cdot \vec{r}}$$

$$e^{2\pi i / 3}$$

$$e^{-\frac{2\pi i}{3}}$$

$$K = \left(\frac{5\pi}{3} \right)$$

t deforms

$$\sum_{\vec{r}} \begin{pmatrix} \tau_x \\ \tau_y \end{pmatrix} \cdot \begin{pmatrix} \tau_x u_{xx} + \tau_y u_{xy} \\ \tau_x u_{xy} + \tau_y u_{yy} \end{pmatrix} \cdot e^{i\vec{k} \cdot \vec{r}}$$

$$\begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\tau_1 = \sqrt{3}$$

$$\tau_2 = \frac{1}{2}$$

$$\tau_3 = -\frac{1}{2}$$

t deforms

$$\sum_{\vec{k}} \begin{pmatrix} \tau_x \\ \tau_y \end{pmatrix} \cdot \begin{pmatrix} \tau_x u_{xx} + \tau_y u_{xy} \\ \tau_x u_{xy} + \tau_y u_{yy} \end{pmatrix} \cdot e^{i\vec{k} \cdot \vec{r}}$$

$$\left[\frac{1}{3} u_{yy} \right]$$

$$\tau_1 = \sqrt{3}$$

$$\tau_2 = \frac{1}{2}$$

$$\tau_3 = -\frac{1}{2}$$

t deforms

$$\sum_{\vec{k}} \begin{pmatrix} \tau_x \\ \tau_y \end{pmatrix} \cdot \begin{pmatrix} \tau_x u_{xx} + \tau_y u_{xy} \\ \tau_x u_{xy} + \tau_y u_{yy} \end{pmatrix} \cdot e^{i\vec{k} \cdot \vec{r}}$$

$$\left[\frac{1}{3} u_{yy} + \frac{1}{2} \left(\frac{1}{2} u_{xx} - \frac{\sqrt{3}}{2} u_{xy} \right) \right] + \frac{\sqrt{3}}{2} \left(\frac{1}{2} u_{xx} \right)$$

$$\tau_1 = \sqrt{3}$$

$$\tau_2 = \frac{1}{2}$$

$$\tau_3 = -\frac{1}{2}$$

t deforms

$$\sum_{\tau} \begin{pmatrix} \tau_x \\ \tau_y \end{pmatrix} \cdot \begin{pmatrix} \tau_x u_{xx} + \tau_y u_{xy} \\ \tau_x u_{xy} + \tau_y u_{yy} \end{pmatrix} \cdot e^{i\vec{k} \cdot \vec{r}}$$

$$\left[\frac{1}{3} u_{yy} + \frac{1}{2} \left(\frac{1}{2} u_{xx} - \frac{\sqrt{3}}{2} u_{xy} \right) \right] e^{\frac{2\pi i}{3}}$$

$$+ \frac{\sqrt{3}}{2} \left(\frac{1}{2} u_{xy} - \frac{\sqrt{3}}{2} u_{yy} \right) e^{-\frac{2\pi i}{3}}$$

$$+ \left(-\frac{1}{2}\right) \left(-\frac{1}{2} u_{xx} - \frac{\sqrt{3}}{2} u_{xy}\right) e^{-\frac{2\pi i}{3}}$$

$$\tau_1 = \frac{1}{\sqrt{3}} \hat{y}$$

$$\tau_2 = \frac{1}{2} \hat{x} - \frac{1}{2\sqrt{3}} \hat{y}$$

$$\tau_3 = -\frac{1}{2} \hat{x} - \frac{1}{2\sqrt{3}} \hat{y}$$

$$e^{i\vec{k} \cdot \vec{r}}$$

$$1$$

$$e^{2\pi i/3}$$

$$e^{-2\pi i/3}$$

t deforms

$$\sum_{\tau} \begin{pmatrix} \tau_x \\ \tau_y \end{pmatrix} \cdot \begin{pmatrix} \tau_x u_{xx} + \tau_y u_{xy} \\ \tau_x u_{xy} + \tau_y u_{yy} \end{pmatrix} \cdot e^{i\vec{k} \cdot \vec{r}}$$

$$\left[\frac{1}{3} u_{yy} + \frac{1}{2} \left(\frac{1}{2} u_{xx} - \frac{1}{2\sqrt{3}} u_{xy} \right) \right] e^{\frac{2\pi i}{3}}$$

$$- \frac{1}{2\sqrt{3}} \left(\frac{1}{2} u_{xy} - \frac{1}{2\sqrt{3}} u_{yy} \right) e^{-\frac{2\pi i}{3}}$$

$$\tau_1 = \frac{1}{\sqrt{3}} \hat{y}$$

$$\tau_2 = \frac{1}{2} \hat{x} - \frac{1}{2\sqrt{3}} \hat{y}$$

$$\tau_3 = -\frac{1}{2} \hat{x} - \frac{1}{2\sqrt{3}} \hat{y}$$

$$e^{i\vec{k} \cdot \vec{r}}$$

$$e^{2\pi i/3}$$

$$e^{-\frac{2\pi i}{3}}$$

$$\left(\frac{1}{2} u_{xx} - \frac{1}{2\sqrt{3}} u_{xy} \right) e^{-\frac{2\pi i}{3}}$$

t deforms

$$\sum_{\tau} \begin{pmatrix} \tau_x \\ \tau_y \end{pmatrix} \cdot \begin{pmatrix} \tau_x u_{xx} + \tau_y u_{xy} \\ \tau_x u_{xy} + \tau_y u_{yy} \end{pmatrix} e^{i\vec{k} \cdot \vec{r}}$$

$$\tau_1 = \frac{1}{\sqrt{3}} \hat{y}$$

$$\tau_2 = \frac{1}{2} \hat{x} - \frac{1}{2\sqrt{3}} \hat{y}$$

$$\tau_3 = -\frac{1}{2} \hat{x} - \frac{1}{2\sqrt{3}} \hat{y}$$

$$e^{i\vec{k} \cdot \vec{r}}$$

$$e^{2\pi i/3}$$

$$e^{-2\pi i/3}$$

$$K = \begin{pmatrix} \frac{4\pi}{3} & 0 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} \tau_x & 2x \\ \tau_y & 2y \end{pmatrix}$$

$$\left[\frac{1}{3} u_{yy} + \frac{1}{2} \left(\frac{1}{2} u_{xx} - \frac{1}{2\sqrt{3}} u_{xy} \right) e^{\frac{2\pi i}{3}} \right.$$

$$\left. \frac{1}{3} \left(\frac{1}{2} u_{xy} - \frac{1}{2\sqrt{3}} u_{yy} \right) e^{-\frac{2\pi i}{3}} \right.$$

$$\left. + \left(-\frac{1}{2}\right) \left(-\frac{1}{2} u_{xx} - \frac{1}{2\sqrt{3}} u_{xy}\right) e^{-\frac{2\pi i}{3}} - \frac{1}{2\sqrt{3}} \left(-\frac{1}{2} u_{xy} - \frac{1}{2\sqrt{3}} u_{yy}\right) \right]$$

t deforms

$$\sum_{\tau} \begin{pmatrix} \tau_x \\ \tau_y \end{pmatrix} \cdot \begin{pmatrix} \tau_x u_{xx} + \tau_y u_{xy} \\ \tau_x u_{xy} + \tau_y u_{yy} \end{pmatrix} \cdot e^{i\vec{k} \cdot \vec{r}}$$

$$\tau_1 = \frac{1}{\sqrt{3}} \hat{y}$$

$$\tau_2 = \frac{1}{2} \hat{x} - \frac{1}{2\sqrt{3}} \hat{y}$$

$$\tau_3 = -\frac{1}{2} \hat{x} - \frac{1}{2\sqrt{3}} \hat{y}$$

$$e^{i\vec{k} \cdot \vec{r}}$$

$$e^{2\pi i/3}$$

$$e^{-2\pi i/3}$$

$$K = \begin{pmatrix} \frac{4\pi}{3} \\ 0 \end{pmatrix} = \begin{pmatrix} \tau_x 2\pi \\ \tau_y 2\pi \end{pmatrix}$$

$$\left[\frac{1}{3} u_{yy} + \frac{1}{2} \left(\frac{1}{2} u_{xx} - \frac{1}{2\sqrt{3}} u_{xy} \right) \right] e^{\frac{2\pi i}{3}}$$

$$- \frac{1}{2\sqrt{3}} \left(\frac{1}{2} u_{xy} - \frac{1}{2\sqrt{3}} u_{yy} \right) e^{\frac{2\pi i}{3}} + \left(-\frac{1}{2} \right) \left(-\frac{1}{2} u_{xx} - \frac{1}{2\sqrt{3}} u_{xy} \right) e^{\frac{2\pi i}{3}}$$

$$- \frac{1}{2\sqrt{3}} \left(-\frac{1}{2} u_{xy} - \frac{1}{2\sqrt{3}} u_{yy} \right) e^{-\frac{2\pi i}{3}}$$

$$= u_{yy} \left(\frac{1}{3} + \frac{1}{12} \left(e^{\frac{2\pi i}{3}} + e^{-\frac{2\pi i}{3}} \right) \right)$$

t deforms

$$\sum_{\tau} \begin{pmatrix} \tau_x \\ \tau_y \end{pmatrix} \cdot \begin{pmatrix} \tau_x u_{xx} + \tau_y u_{xy} \\ \tau_x u_{xy} + \tau_y u_{yy} \end{pmatrix} e^{i\vec{k} \cdot \vec{r}}$$

$$\tau_1 = \frac{1}{\sqrt{3}} \hat{y}$$

$$\tau_2 = \frac{1}{2} \hat{x} - \frac{1}{2\sqrt{3}} \hat{y}$$

$$\tau_3 = -\frac{1}{2} \hat{x} - \frac{1}{2\sqrt{3}} \hat{y}$$

$$e^{i\vec{k} \cdot \vec{r}}$$

$$e^{2\pi i/3}$$

$$e^{-2\pi i/3}$$

$$K = \begin{pmatrix} \frac{4\pi}{3} & 0 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} \tau_x & 2\tau_x \\ \tau_y & 2\tau_y \end{pmatrix}$$

$$\left[\frac{1}{3} u_{yy} + \frac{1}{2} \left(\frac{1}{2} u_{xx} - \frac{1}{2\sqrt{3}} u_{xy} \right) \right] e^{\frac{2\pi i}{3}}$$

$$- \frac{1}{2} \left(\frac{1}{2} u_{xy} - \frac{1}{2\sqrt{3}} u_{yy} \right) e^{\frac{2\pi i}{3}}$$

$$+ \left(-\frac{1}{2} \right) \left(-\frac{1}{2} u_{xx} - \frac{1}{2\sqrt{3}} u_{xy} \right) e^{-\frac{2\pi i}{3}}$$

$$- \frac{1}{2\sqrt{3}} \left(-\frac{1}{2} u_{xy} - \frac{1}{2\sqrt{3}} u_{yy} \right) e^{-\frac{2\pi i}{3}}$$

$$+ \frac{1}{12} \left(e^{\frac{2\pi i}{3}} + e^{-\frac{2\pi i}{3}} \right)$$

t deforms

$$\sum_{\tau} \begin{pmatrix} \tau_x \\ \tau_y \end{pmatrix} \cdot \begin{pmatrix} \tau_x u_{xx} + \tau_y u_{xy} \\ \tau_x u_{xy} + \tau_y u_{yy} \end{pmatrix} \cdot e^{i\vec{k} \cdot \vec{r}}$$

$$\tau_1 = \frac{1}{\sqrt{3}} \hat{y}$$

$$\tau_2 = \frac{1}{2} \hat{x} - \frac{1}{2\sqrt{3}} \hat{y}$$

$$\tau_3 = -\frac{1}{2} \hat{x} - \frac{1}{2\sqrt{3}} \hat{y}$$

$$\left[\frac{1}{3} u_{yy} + \frac{1}{2} \left(\frac{1}{2} u_{xx} - \frac{1}{2\sqrt{3}} u_{xy} \right) e^{\frac{2\pi i}{3}} \right.$$

$$\left. u_{xy} - \frac{1}{2\sqrt{3}} u_{yy} \right) e^{\frac{2\pi i}{3}} + \left(-\frac{1}{2} \right) \left(-\frac{1}{2} u_{xx} - \frac{1}{2\sqrt{3}} u_{xy} \right)$$

$$\left(\frac{1}{3} + \frac{1}{12} \left(e^{\frac{2\pi i}{3}} + e^{-\frac{2\pi i}{3}} \right) \right)$$

$$\left(\frac{1}{2} \cos\left(\frac{2\pi}{3}\right) \right) + u_{xy} \left(-\frac{1}{4\sqrt{3}} 2i \sin\left(\frac{2\pi}{3}\right) \right)$$

t deforms

$$\sum_{\tau} \begin{pmatrix} \tau_x \\ \tau_y \end{pmatrix} \cdot \begin{pmatrix} \tau_x u_{xx} + \tau_y u_{xy} \\ \tau_x u_{xy} + \tau_y u_{yy} \end{pmatrix} \cdot e^{i\vec{k} \cdot \vec{r}}$$

$$\tau_1 = \frac{1}{\sqrt{3}} \hat{y}$$

$$\tau_2 = \frac{1}{2} \hat{x} - \frac{1}{2\sqrt{3}} \hat{y}$$

$$\tau_3 = -\frac{1}{2} \hat{x} - \frac{1}{2\sqrt{3}} \hat{y}$$

$$\left[\frac{1}{3} u_{yy} + \frac{1}{2} \left(\frac{1}{2} u_{xx} - \frac{1}{2\sqrt{3}} u_{xy} \right) \right] e^{\frac{2\pi i}{3}}$$

$$- \frac{1}{2\sqrt{3}} \left(\frac{1}{2} u_{xy} - \frac{1}{2\sqrt{3}} u_{yy} \right) e^{\frac{2\pi i}{3}} + \left(-\frac{1}{2} \right) \left(-\frac{1}{2} u_{xx} - \frac{1}{2\sqrt{3}} u_{xy} \right)$$

$$= u_{yy} \left(\frac{1}{3} + \frac{1}{12} \left(e^{\frac{2\pi i}{3}} + e^{-\frac{2\pi i}{3}} \right) \right) + u_{xx} \left(\frac{1}{2} \cos\left(\frac{2\pi}{3}\right) \right) + u_{xy} \left(-\frac{1}{4\sqrt{3}} 2i \sin\left(\frac{2\pi}{3}\right) \right)$$

$$e^{ikz}$$

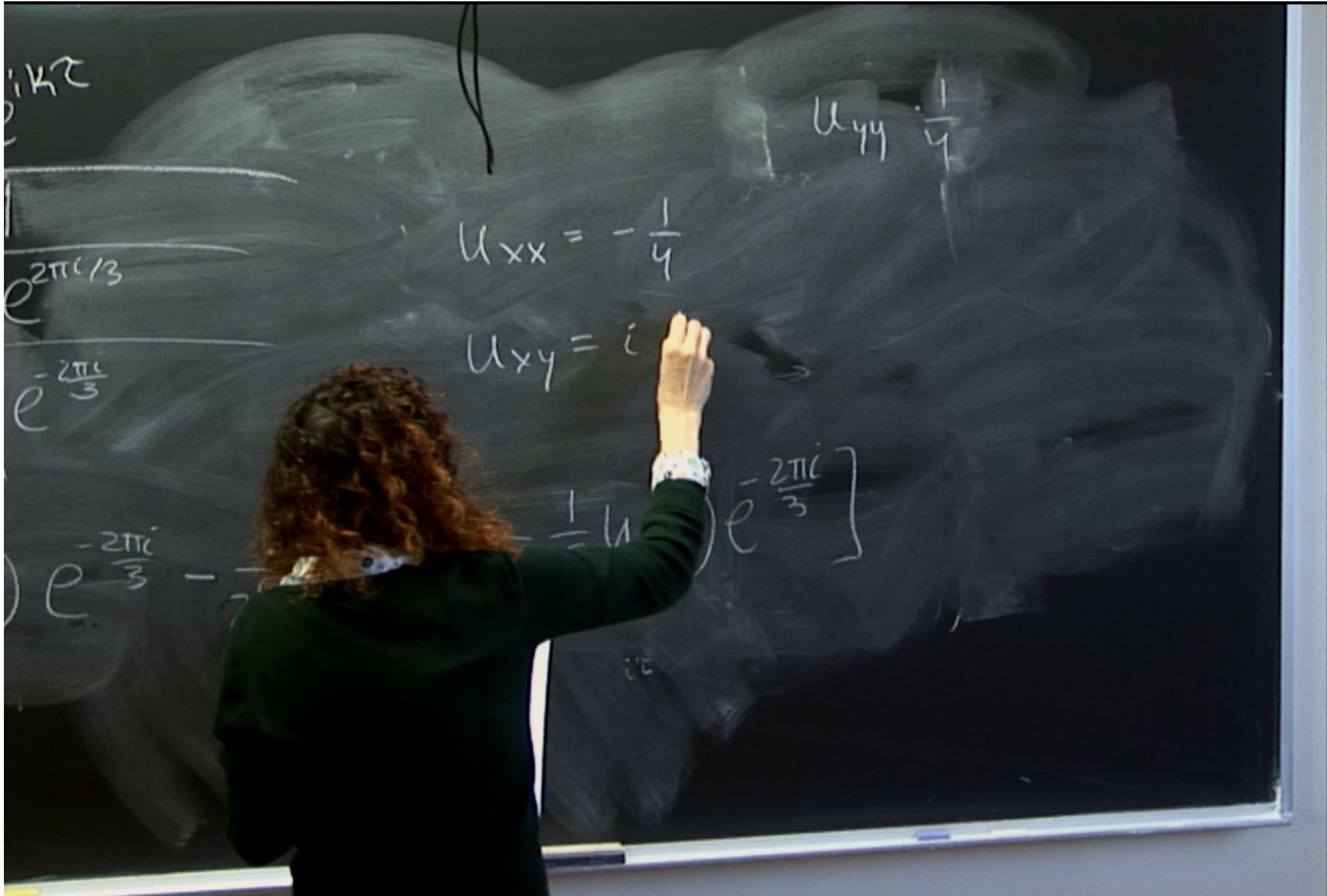
$$e^{2\pi i/3}$$

$$e^{-2\pi i/3}$$

$$u_{yy} \left(\frac{1}{3} - \frac{1}{6} \right) = u_{yy} \cdot \frac{1}{6}$$

$$u_{xx} =$$

$$u_{xy} e^{-\frac{2\pi i}{3}} - \frac{1}{2\sqrt{3}} \left(-\frac{1}{2} u_{xy} - \frac{1}{2\sqrt{3}} u_{yy} \right) e^{-\frac{2\pi i}{3}}$$



$i k z$

$e^{2\pi i / 3}$

$e^{-2\pi i / 3}$

$e^{-2\pi i / 3}$

$$u_{yy} = \frac{1}{4}$$

$$u_{xx} = -\frac{1}{4}$$

$$u_{xy} = -i \frac{1}{2}$$

$$e^{-2\pi i / 3} - \frac{1}{2\sqrt{3}} \left(-\frac{1}{2} u_{xy} - \frac{1}{2\sqrt{3}} u_{yy} \right) e^{-2\pi i / 3}$$

$i k z$

$e^{2\pi i / 3}$

$e^{-2\pi i / 3}$

$e^{-2\pi i / 3}$

$u_{xx} = -\frac{1}{4}$

$u_{xy} (-i \frac{1}{2})$

$u_{yy} \frac{1}{4}$

$\psi^\dagger (\nabla \cdot \hat{b}) \psi$

$-i\hbar (\partial_x - i\partial_y)$

$u_{xx} - u_{yy} = A_x$

$\frac{1}{2\sqrt{3}} \left(-\frac{1}{2} u_{xy} - \frac{1}{2\sqrt{3}} u_{yy} \right) e^{-\frac{2\pi i}{3}}$

$i k z$

$e^{2\pi i / 3}$

$e^{-2\pi i / 3}$

$e^{-2\pi i / 3}$

$$-\frac{1}{2\sqrt{3}} \left(-\frac{1}{2} u_{xy} - \frac{1}{2\sqrt{3}} u_{yy} \right) e^{-\frac{2\pi i}{3}}$$

$$u_{yy} = \frac{1}{4}$$

$$u_{xx} = -\frac{1}{4}$$

$$u_{xy} = -i \frac{1}{2}$$

$$\psi^\dagger (\nabla \cdot \hat{b}) \psi$$

$$-i\hbar (\partial_x - i\partial_y)$$

$$u_{xx} - u_{yy} = A_x$$

$$2i u_{xy} = A_y$$

$i k z$

$e^{2\pi i / 3}$

$e^{-2\pi i / 3}$

$$u_{yy} = \frac{1}{4}$$

$$u_{xx} = -\frac{1}{4}$$

$$u_{xy} = -i \frac{1}{2}$$

$$\psi^\dagger (\nabla \cdot \mathbf{b}) \psi$$

$$-i\hbar(\partial_x - i\partial_y)$$

$$e^{-2\pi i / 3} - \frac{1}{2\sqrt{3}} \left(-\frac{1}{2} u_{xy} - \frac{1}{2\sqrt{3}} u_{yy} \right) e^{-\frac{2\pi i}{3}}$$

$$u_{xx} - u_{yy} = A_x$$

$$2i u_{xy} = A_y$$

$$u_r = c r^2 \sin(3\theta)$$

$$u_\theta = c r^2 \cos(3\theta)$$

u_r

$$u_r = c r^2 \sin(3\theta)$$

$$u_\theta = c r^2 \cos(3\theta)$$

$$\begin{pmatrix} u_x \\ u_y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$u_r = cr^2 \sin(3\theta)$$

$$u_\theta = cr^2 \cos(3\theta)$$

$$\begin{pmatrix} u_x \\ u_y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} u_r \\ u_\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta \sin(3\theta) - \sin\theta \cos(3\theta) \\ \sin\theta \cos(3\theta) + \cos\theta \sin(3\theta) \end{pmatrix}$$

 u_{xx}
 u_{xy}

$$u_r = cr^2 \sin(3\theta)$$

$$u_\theta = cr^2 \cos(3\theta)$$

$$\begin{pmatrix} u_x \\ u_y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} u_r \\ u_\theta \end{pmatrix} = cr^2 \begin{pmatrix} \cos\theta \sin 3\theta - \sin\theta \cos 3\theta \\ \sin\theta \sin 3\theta + \cos\theta \cos 3\theta \end{pmatrix}$$

$$2yx$$

$$r^2 2 \sin\theta \cos\theta$$

$$-\sin(\theta - 3\theta)$$

$$\cos\theta \sin 3\theta - \sin\theta \cos 3\theta$$

$$\sin\theta \sin 3\theta + \cos\theta \cos 3\theta$$

$$\cos(3\theta)$$

$$\cos(\theta)$$

u_{xx}

u_{xy}

$$2yx$$

$$r^2 2 \sin\theta \cos\theta$$

$$-\sin(\theta - 30)$$

$$\begin{pmatrix} \cos\theta \sin 30 - \sin\theta \cos 30 \\ \sin\theta \sin 30 + \cos\theta \cos 30 \end{pmatrix} = \vec{A} = c \left(\begin{array}{l} \cos(30 - \theta) \\ \sin(30 - \theta) \end{array} \right)$$

$$\cos(30 - \theta) = \cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$xr^2 \rightarrow -yz$$

$$u_{xx} = -\frac{1}{4}$$

$$u_{xy} = -i \frac{1}{2}$$

$$u_{yy} = \frac{1}{4}$$

$$\psi(\nabla \cdot \vec{b})$$

$$-i\hbar(\partial_x)$$

$$xx^{-1}$$

$$ik$$

$$2yx$$

$$r^2 \sin\theta \cos\theta$$

$$= \sin(\theta - 30^\circ)$$

$$\cos\theta \sin 30^\circ - \sin\theta \cos 30^\circ$$

$$\sin\theta \sin(30^\circ) + \cos\theta \cos(30^\circ)$$

$$\cos(30^\circ - \theta) = \cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$xr^2 \rightarrow x^2 - y^2$$

$$u_{xx} = -\frac{1}{4}$$

$$u_{xy} = -i \frac{1}{2}$$

$$u_{yy} = \frac{1}{4}$$

$$\psi(\nabla \cdot \theta)$$

$$-i\hbar(\partial)$$

$$\vec{u} = c(2xy, x^2 - y^2)$$

$$u_{xx} = 1$$

$$2iy$$

$$\lambda xy \left(-i \frac{\partial}{\partial z} \right)$$

$$-i\hbar(\partial_x - i\partial_y)$$

$$\vec{u} = c(2xy, x^2 - y^2)$$

$$u_{xx} = 2cy$$

$$u_{yy} = -2cy$$

$$A_x = 4cy$$

$$= A_x$$

$$z(\theta) = \tilde{c} \cos^2 \theta - \sin^2 \theta$$

$$xr^2 \rightarrow x^2 - y^2$$

$$x y (-i \frac{z}{z}) \rightarrow$$

$$\vec{u} = c (2xy, x^2 - y^2)$$
$$\left. \begin{aligned} u_{xx} &= 2cy \\ u_{yy} &= -2cy \end{aligned} \right\} A_x = 4cy$$

$$-it h (\partial_x - i \partial_y)$$

$$\begin{aligned} u_{xx} - u_{yy} &= A_x \\ 2i u_{xy} &= A_y \end{aligned}$$

$$2i u_{xy} = 2i c x$$

$$u_{xy}(-i \frac{z}{2}) \rightarrow$$

$$-i h (\partial_x - i \partial_y)$$

$$\vec{u} = c (2xy, x^2 - y^2)$$

$$\left. \begin{aligned} u_{xx} &= 2cy \\ u_{yy} &= -2cy \end{aligned} \right\} A_x = 4cy$$

$$\begin{aligned} u_{xx} - u_{yy} &= A_x \\ 2i u_{xy} &= A_y \end{aligned}$$

$$2i u_{xy} = 2i c x$$

$\frac{\partial}{\partial y} - \frac{\partial A_y}{2}$

$$u_{xy} \left(-i \frac{1}{z} \right)$$

$$\cos(30)$$

$$\vec{u} = c(2xy, x^2 - y^2)$$

$$\left. \begin{aligned} u_{xx} &= 2cy \\ u_{yy} &= -2cy \end{aligned} \right\} A_x = 4cy$$

$$-it(\partial_x - i\partial_y)$$

$$u_{xx} - u_{yy} = A_x$$

$$2iu_{xy} = A_y$$

$$2iu_{xy} = 2icx$$

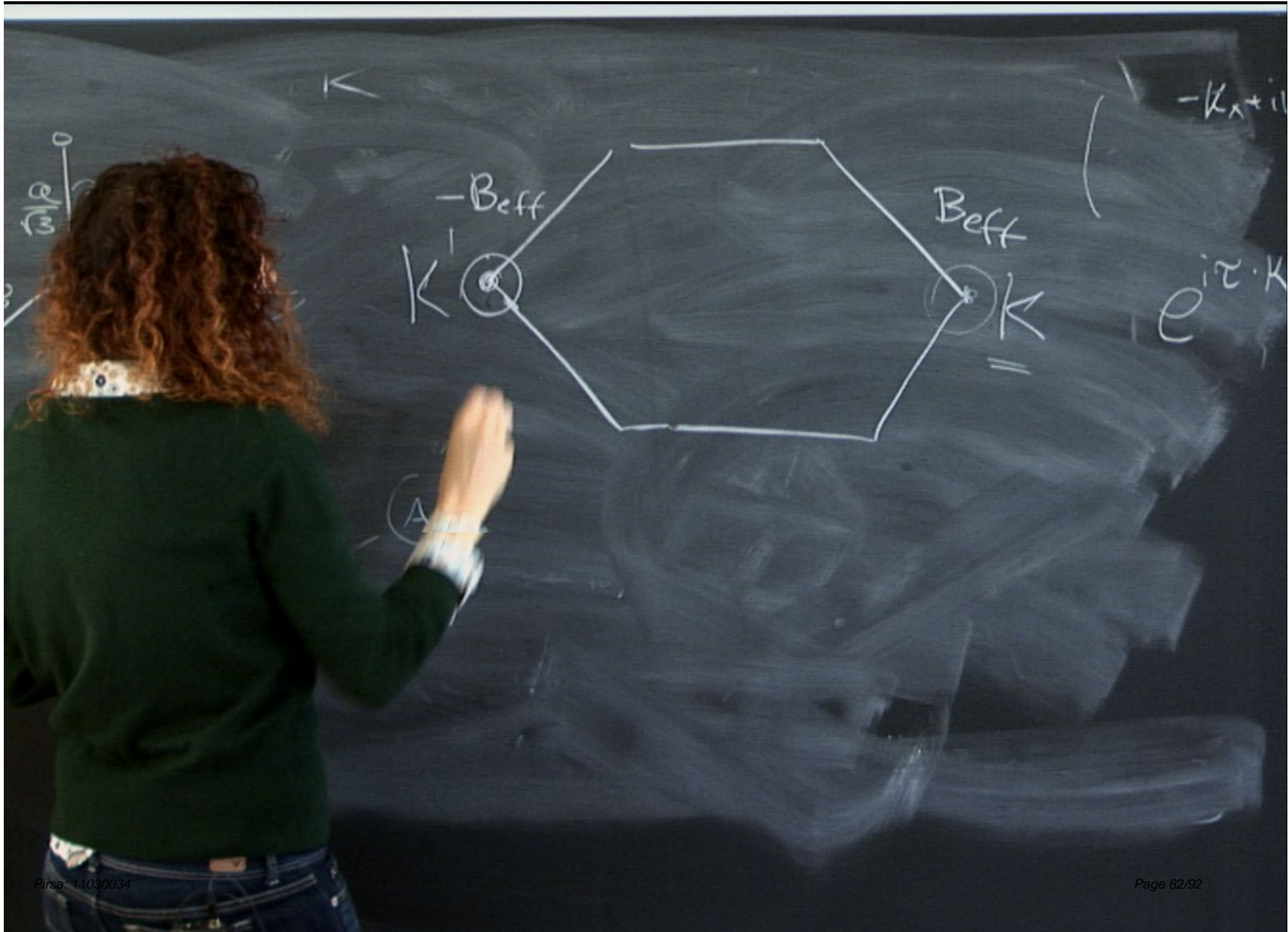
$$\frac{\partial A_x}{\partial y} - \frac{\partial A_y}{\partial x} = 2c$$

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} u_r \\ u_\theta \end{pmatrix} = cr^2 \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

$$C_{k\sigma}^+ |\phi\rangle$$

$$-k_x + i\nu y$$





K

$-Beff$

$Beff$

K'

K

$-k_x + i$

$e^{i\tau \cdot K}$

A



$$= c r^2 \sin(3\theta)$$

$$= c r^2 \cos(3\theta)$$

$$A = (0, -x) B$$

$$e^{i\mathbf{k} \cdot \mathbf{x} + i\omega t}$$

$$\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

$$\begin{pmatrix} u_r \\ u_\theta \end{pmatrix} = c r^2$$

$$\begin{pmatrix} \cos\theta \sin(3\theta) \\ \sin\theta \sin(3\theta) \end{pmatrix}$$

$$\ominus c_k^+ |\phi\rangle$$

$$= \downarrow c r^2 \sin(3\theta)$$

$$= c r^2 \cos(3\theta)$$

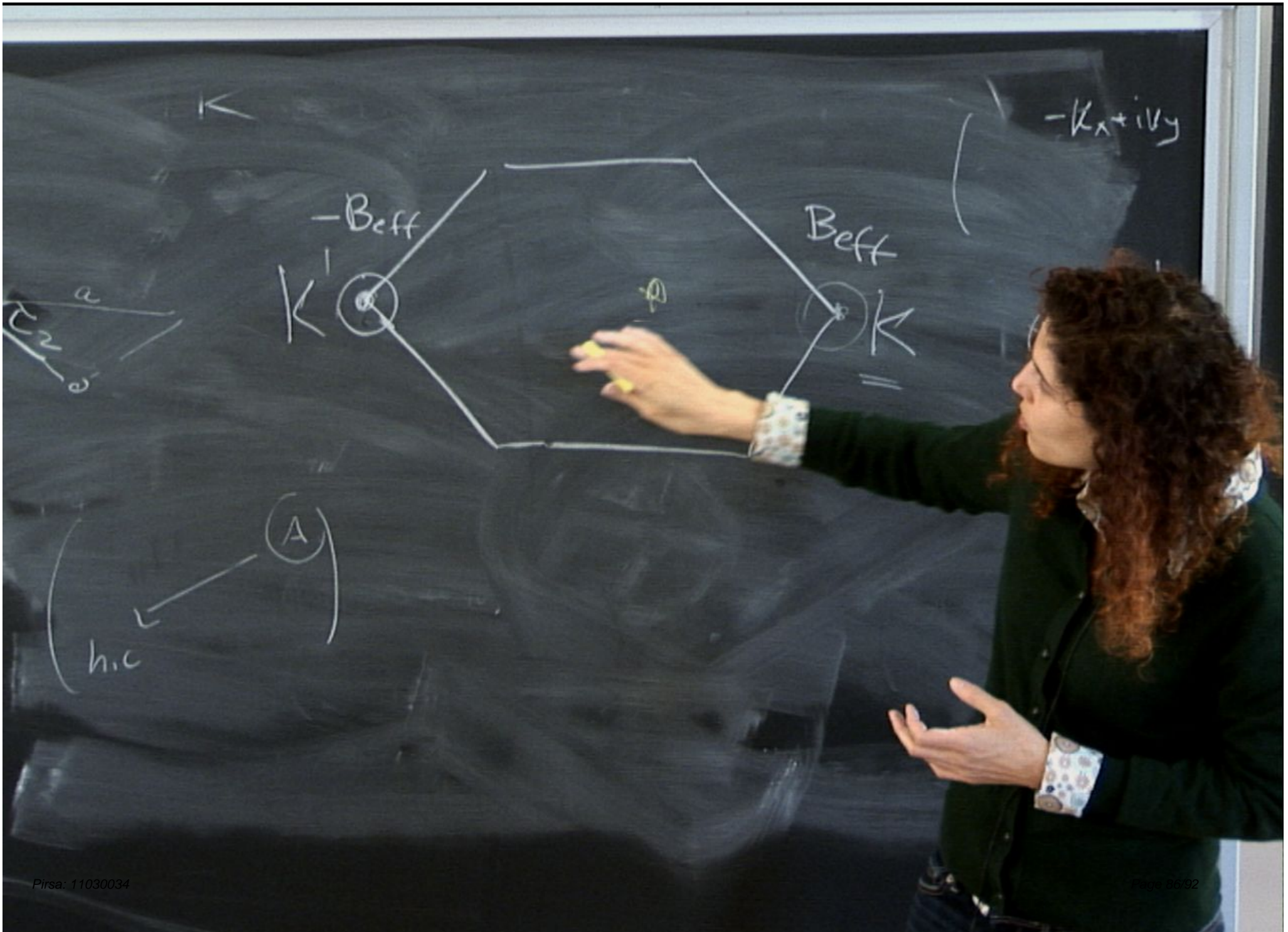
$$A = (0, -x) B$$

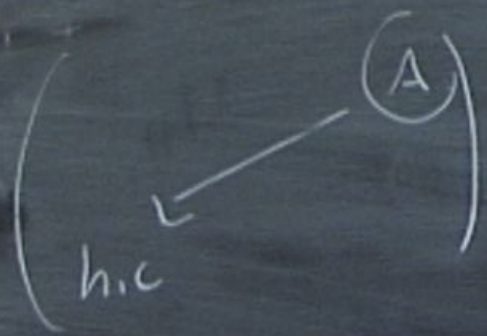
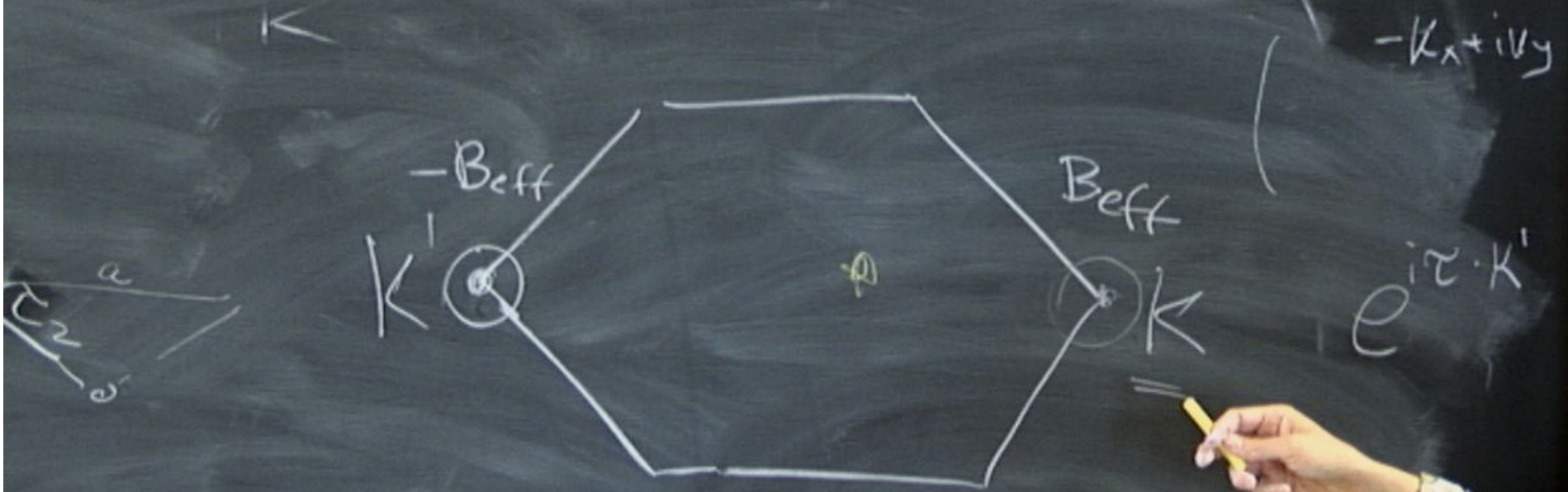
$$e^{i\mathbf{k} \cdot \mathbf{x} + i\hbar \omega}$$

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} u_r \\ u_\theta \end{pmatrix} = c r^2 \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

$$\hat{\Theta} c_{k_0}^+ |\phi\rangle$$

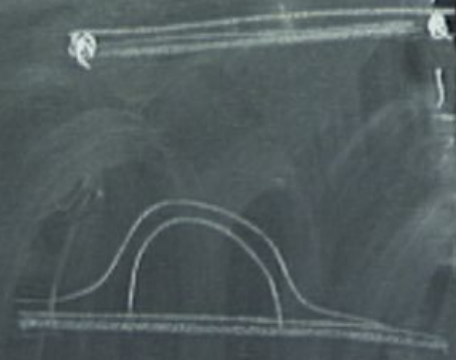
$$\begin{aligned} & 2y) \\ & r^2 2 \sin \\ & - \sin \\ & \cos\theta \sin \\ & \sin\theta \sin(3 \end{aligned}$$





$$r^2 \sin(3\theta)$$
$$r^2 \cos(3\theta)$$

$$\cos\theta$$
$$-\sin\theta$$
$$\cos\theta$$



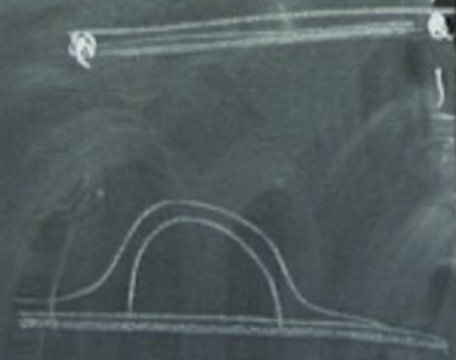
$$u_{xx}$$
$$u_{xy}$$

$$\frac{\partial A_x}{\partial y}$$

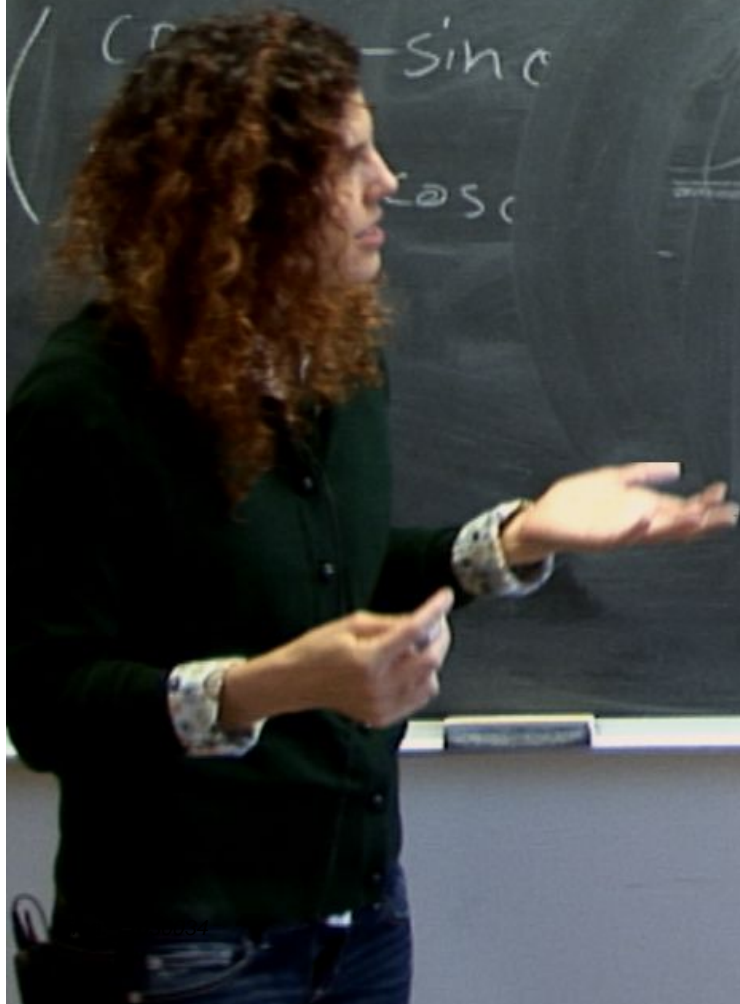


$$r^2 \sin(3\theta)$$
$$r^2 \cos(3\theta)$$

$$(\cos - \sin)$$
$$(\cos)$$



$$u_{xx}$$
$$u_{xy}$$
$$\frac{\partial A_x}{\partial y}$$



$$r^2 \sin(3\theta)$$
$$r^2 \cos(3\theta)$$

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

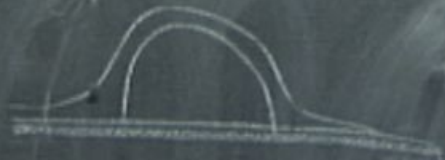


$$\frac{\partial^2 u}{\partial x^2}$$
$$\frac{\partial^2 u}{\partial y^2}$$
$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2}$$



$$r^2 \sin(3\theta)$$
$$r^2 \cos(3\theta)$$

$$(\cos - \sin)$$
$$(\cos + \sin)$$



$$u_{xx}$$

$$u_{xy}$$

$$\frac{\partial^2 u}{\partial x^2}$$

$$r^2 \sin(3\theta)$$
$$r^2 \cos(3\theta)$$

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$



$$\frac{u_{xx}}{u_{xy}}$$
$$\frac{\partial^2 A_x}{\partial y^2}$$