Title: Explorations in Condensed Matter - Lecture 4

Date: Mar 17, 2011 10:15 AM

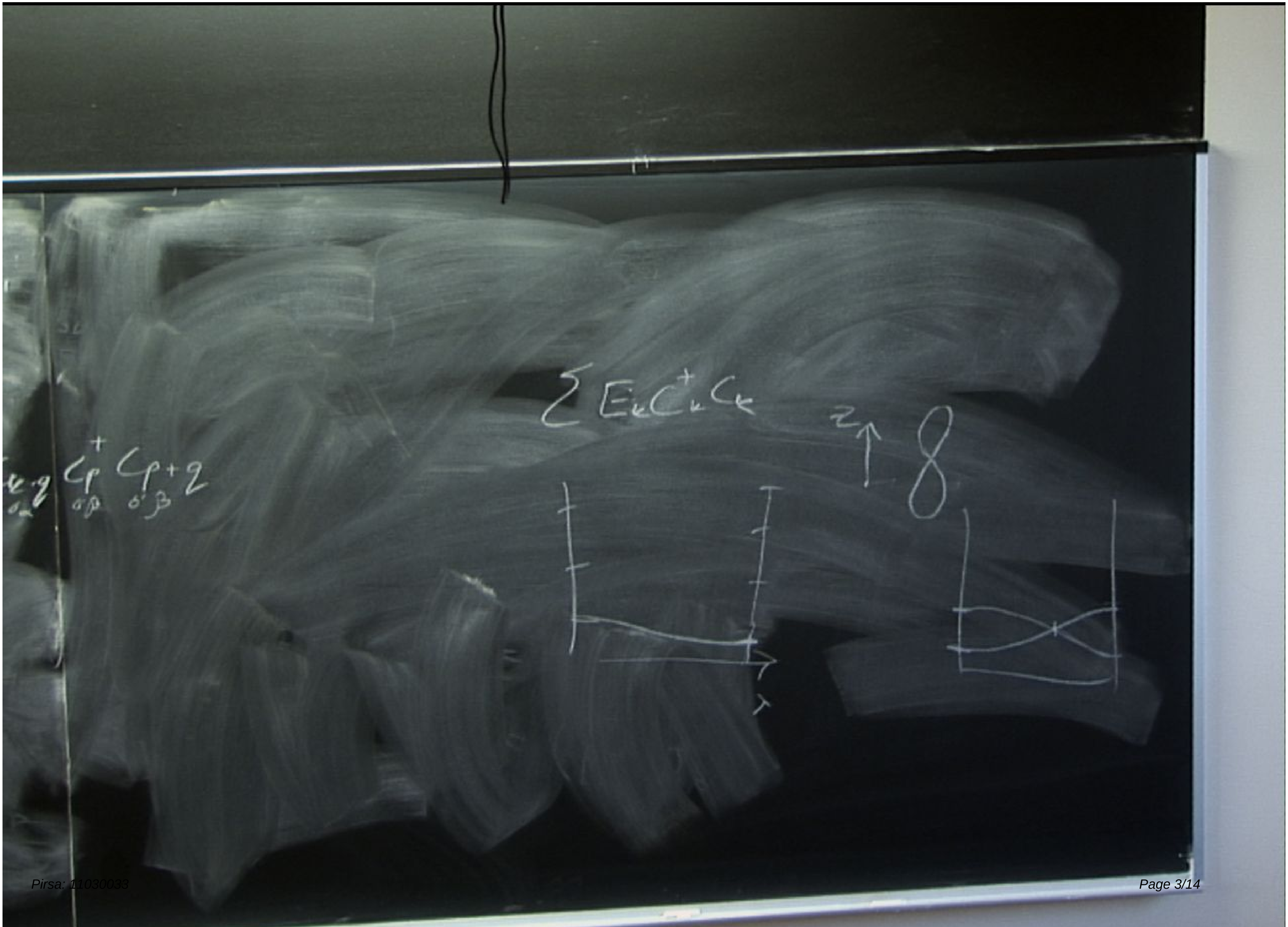
URL: http://pirsa.org/11030033

Abstract:

$$H = H_0 + \int dr\, dr'\, V(r-r')\, n_r n_{r'}$$

$$\rightarrow \sum_q V_q n_q n_{-q} = \sum_{kqp} V_q c^+_{k\sigma\alpha} c_{k-q\,\sigma\alpha} c^+_{p\sigma'\beta} c_{p+q\,\sigma'\beta}$$

$$\sum_{\substack{k\sigma \\ \alpha=A,B}} c^+_{k\sigma\alpha} c_{k-q\,\sigma\alpha}$$


$\sum E_k C_k^+ C_k$

$$H = H_0 + \int dr\, dr'\, V(r-r')\, n_r n_{r'}$$

$$E_{ex} = -\sum_{kk'q}$$

$$\rightarrow \sum_q V_q n_q n_{-q} = \sum_{kqp} V_q\, c^{\dagger}_{k\,\sigma\alpha}\, c_{k-q\,\sigma\alpha}\, c^{\dagger}_{p\,\sigma'\beta}\, c_{p+q\,\sigma'\beta}$$

$$\sum_{\substack{k\sigma \\ \alpha=A,B}} c^{\dagger}_{k\sigma\alpha} c_{k-q\,\sigma\alpha}$$

$$E_{int} = \langle c^{\dagger} c\, c^{\dagger} c \rangle$$

$$\langle c^{\dagger}_k c_{k-q} \rangle_0 \rightarrow \delta(q=0)$$

$$E_{ex} = -\sum_{\substack{k k' \\ q, \sigma\sigma' \\ \alpha\beta}} V(k-k') \underbrace{\langle c^\dagger_{k\sigma\alpha} c_{k'+q\,\sigma'\beta} \rangle}_{\substack{\delta_{k,k'+q} \\ \delta_{\sigma\sigma'} \\ n_k}} \langle c^\dagger_{k'\sigma'\beta} c_{k-q\,\sigma\alpha} \rangle$$

$$c_{k-q\,\sigma\alpha} \; c^\dagger_{p\,\sigma\beta} \, c_{p+q\,\sigma'\beta}$$

$$\langle c^\dagger c \, c^\dagger c \rangle$$

$$\langle c^\dagger_k c_{k-q} \rangle_0 \longrightarrow \delta(q=0)$$

$$E_{ex} = -\sum_{\substack{kk' \\ q,\sigma\sigma' \\ \alpha\beta}} V(k-k') \underbrace{\langle c^\dagger_{k\sigma\alpha} c_{k'+q\,\sigma\beta} \rangle}_{\substack{\delta_{k,k'+q} \\ \delta_{\sigma\sigma'} \\ n_k}} \langle c^\dagger_{k'\sigma'\beta}\, c_{k-q\,\sigma\alpha} \rangle$$

$$c_{k-q\,\sigma\alpha}\; c^\dagger_{p\,\sigma\beta}\, c_{p+q\,\sigma'\beta}$$

$$\langle c^\dagger c\, c^\dagger c \rangle$$

$$\xrightarrow{FL} -\sum V(k-k')\, n_{k\sigma} n_{k'\sigma}$$

$$\langle c^\dagger_k c_{k-q} \rangle_0 \longrightarrow \delta(q=0)$$

$$H = H_0 + \int dr\, dr'\, V(r-r')\, n_r\, n_{r'}$$

$$H_0 \to \sum \varepsilon_k c_k^{+} c_k$$

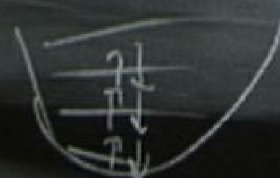
$$\to \sum_q V_q\, n_q\, n_{-q} = \sum_{kqp} V_q\, c^{+}_{k\,\sigma\alpha}\, c_{k-q\,\sigma\alpha}\, c^{+}_{p\,\sigma'\beta}\, c_{p+q\,\sigma'\beta}$$

$$n_q = \sum_{\substack{k\sigma \\ \alpha = A,B}} c^{+}_{k\sigma\alpha}\, c_{k-q\,\sigma\alpha}$$

$$E_{int} = \langle c^{+} c\, c^{+} c \rangle$$

$$\langle c^{+}_k\, c_{k-q} \rangle \to$$

E



$$E_{ex} = -\sum_{\substack{k k' \\ q, \sigma\sigma' \\ \alpha\beta}} V(k-k') \underbrace{\langle c^\dagger_{k\sigma\alpha} c_{k'+q\,\sigma\beta} \rangle}_{\substack{\delta_{k,k'+q} \\ \delta_{\sigma\sigma'} \\ n_k}} \langle c^\dagger_{k'\sigma'\beta}\, c_{k-q\,\sigma\alpha} \rangle$$

$n = n_\uparrow + n_\downarrow$

$n_\uparrow \propto k_{F\uparrow}^2$

$c_{k-q\,\sigma\alpha}\; c^\dagger_{p\,\sigma\beta}\; c_{p+q\,\sigma'\beta}$

$\langle c^\dagger\, c\, c^\dagger\, c \rangle$

FL

$$\rightsquigarrow -\sum V(k-k')\, n_{k\sigma} n_{k'\sigma} \longrightarrow -\underline{\left(n_\uparrow^{3/2} + n_\downarrow^{3/2}\right)}$$

$$\frac{1}{k_{F\uparrow}}\, k_{F\uparrow}^4 \rightsquigarrow n_\uparrow^{3/2}$$

I) $n^{3/2}$ $\quad n_\uparrow = n$

II) $\left(\frac{n}{2}\right)^{3/2} \cdot 2 = \frac{n^{3/2}}{\sqrt{2}}$ $\quad n_\uparrow = n_\downarrow = \frac{n}{2}$

$$\langle c^\dagger_k\, c_{k-q} \rangle \longrightarrow \delta(q=0)$$

$$H = H_0 + \int dr\, dr'\, V(r-r')\, n_r n_{r'}$$

$$H_0 \to \sum \varepsilon_k c_k^{+} c_k$$

$$\to \sum_q V_q n_q n_{-q} = \sum_{kqp} V_q\, c^{+}_{k\,\sigma\alpha}\, c_{k-q\,\sigma\alpha}\, c^{+}_{p\,\sigma'\beta}\, c_{p+q\,\sigma'\beta}$$

$$\sum_{\substack{k\sigma \\ \alpha=A,B}} c^{+}_{k\alpha} c_{k-q\,\sigma\alpha}$$

$$E_{ex} = -\sum_{\substack{kk' \\ q,\sigma\sigma' \\ \alpha\beta}} V(k-k')\, \langle \underbrace{c^{\dagger}_{k\sigma\alpha}\, c_{k'+q\,\sigma'\beta}}_{\substack{\delta_{k,k'+q} \\ \delta_{\sigma\sigma'} \\ n_k}} \rangle \langle c^{\dagger}_{k'\sigma'\beta}\, c_{k-q\,\sigma\alpha} \rangle$$

$$c_{k-q\,\sigma\alpha}\; c^{\dagger}_{p\,\sigma\beta}\; c_{p+q\,\sigma'\beta}$$

$$\langle c^{\dagger}_{k\sigma\alpha}\, c_{k\sigma\beta} \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{i\varphi_k} \\ e^{-i\varphi_k} & 1 \end{pmatrix} \Theta(\mu - E^{s}_{k}),$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ e^{i\varphi_k} \end{pmatrix}$$

$$E_{ex} = -\sum_{\substack{kk' \\ \sigma}} V(k-k')\, n_\sigma^s(k)\, n_\sigma^{s'}(k') \cdot \frac{1}{2} \times \mathrm{Tr}\left[\underbrace{\begin{pmatrix} 1 & s e^{i\varphi_k} \\ s e^{-i\varphi_k} & 1 \end{pmatrix} \begin{pmatrix} 1 & s' e^{i\varphi_{k'}} \\ s' e^{-i\varphi_{k'}} & 1 \end{pmatrix}}_{} \right]$$

$$= -\sum_{\substack{kk' \\ \sigma}} \underbrace{\tilde{V}_{kk'}^{ss'}}\, n_\sigma^s\, n_\sigma^{s'}$$

$$1 + e^{i\varphi_k - \varphi_{k'}} + e^{-i\varphi_k + i\varphi_{k'}} + 1$$

$$\boxed{V(k-k')\left(1 + ss'\cos(\varphi_k - \varphi_{k'})\right)}$$

$$E_{ex} = -\sum_{\substack{kk' \\ q,\ \alpha\beta}}^{\sigma\delta} V(\underbrace{k-k'}_{q}) \langle$$

$$\sum_{\alpha\beta} M_{\alpha\beta} M_{\beta\alpha}$$

$$= \frac{1}{\sqrt{2}} \Big(s$$

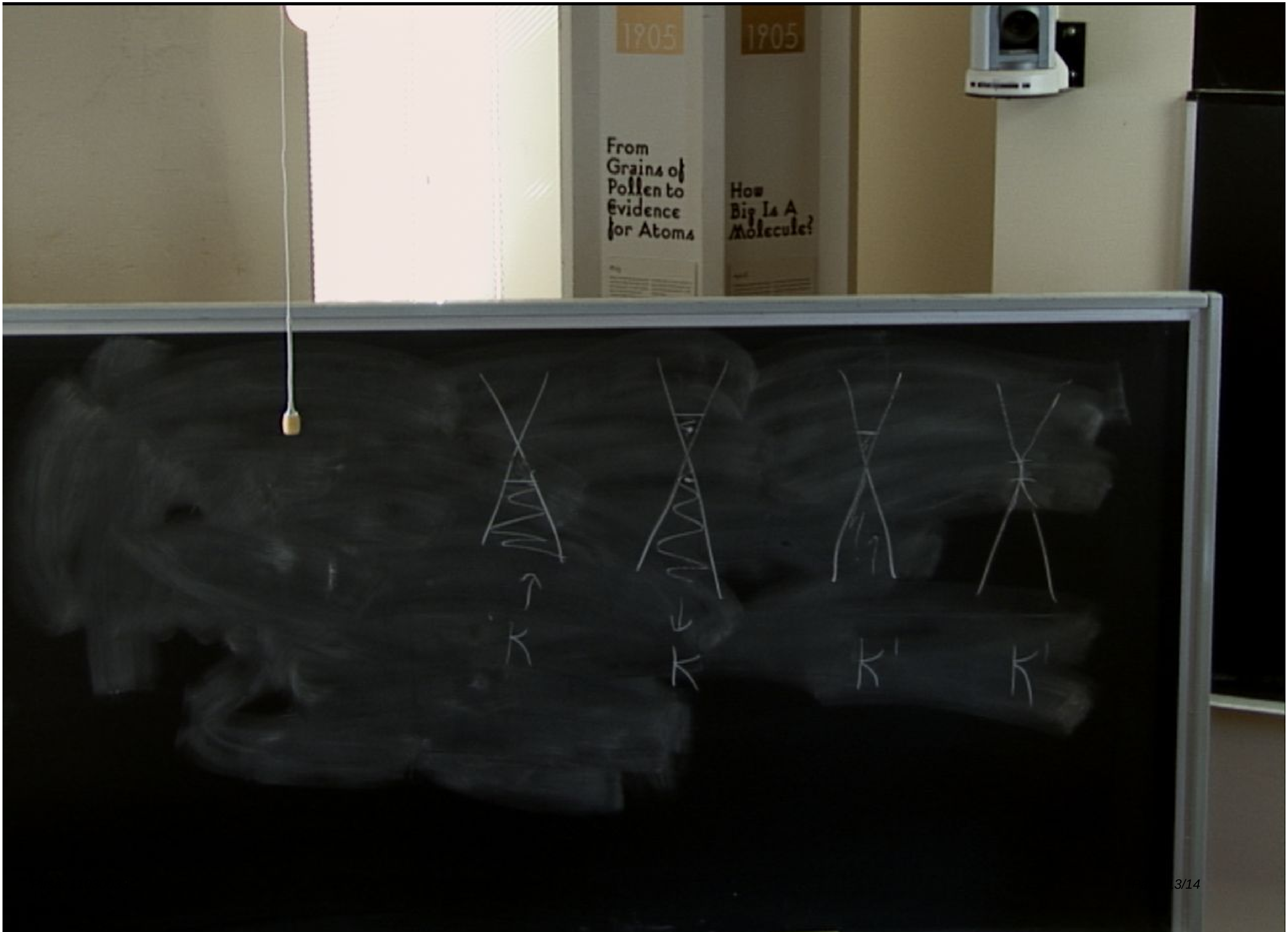
$$\Psi = \begin{pmatrix} 1 \\ s e^{i\varphi_k} \end{pmatrix}$$

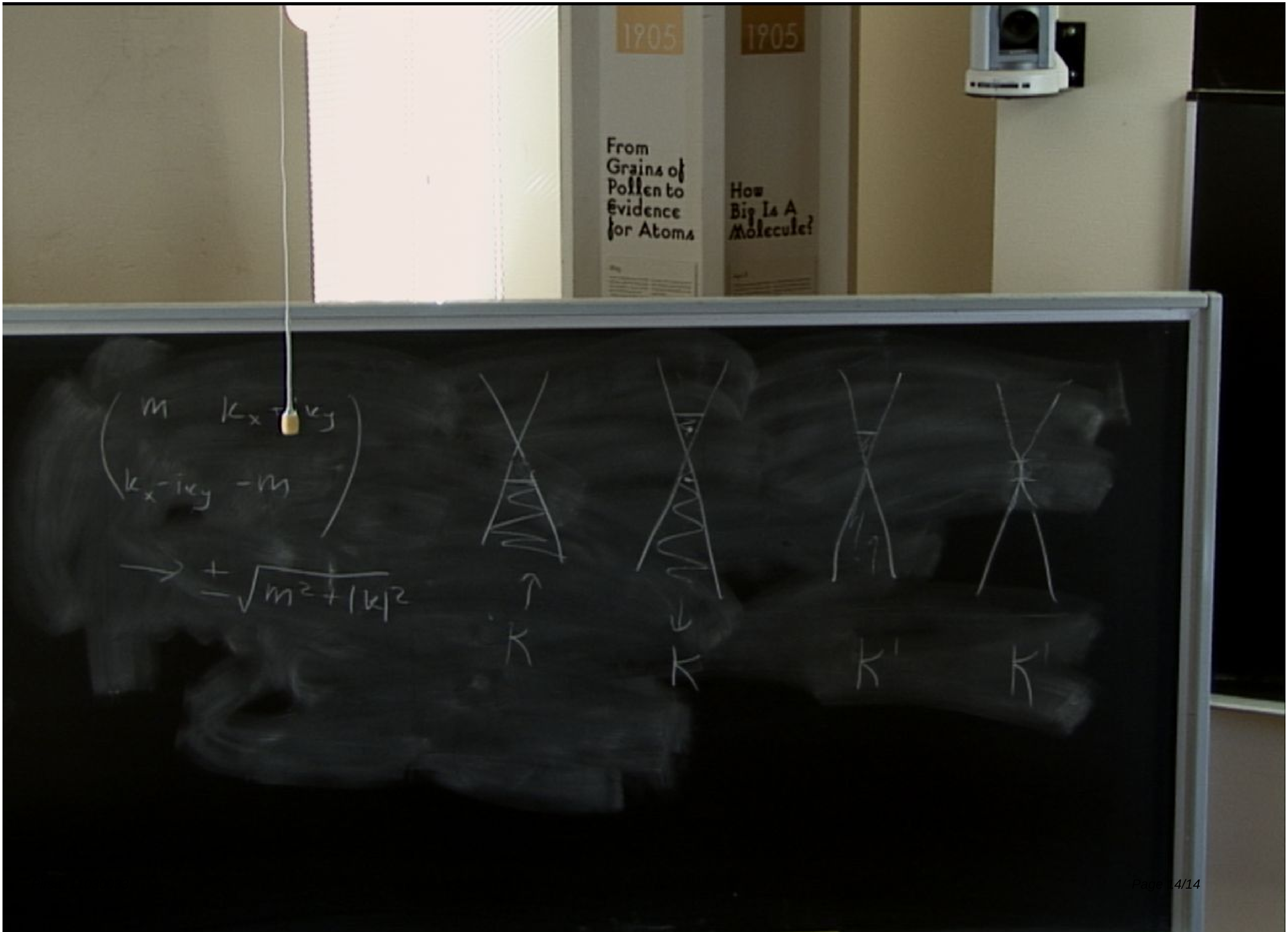
$$E_{ex} = -\sum_{kk'\sigma} V(k-k')\, n_\sigma^{s}(k)\, n_\sigma^{s'}(k') \cdot \frac{1}{2} \times \mathrm{Tr}\left[\underbrace{\begin{pmatrix} 1 & s e^{i\varphi_k} \\ s e^{-i\varphi_k} & 1 \end{pmatrix}\begin{pmatrix} 1 & s' e^{i\varphi_{k'}} \\ s' e^{-i\varphi_{k'}} & 1 \end{pmatrix}}\right]$$

$$1 + e^{i(\varphi_k - \varphi_{k'})} + e^{-i\varphi_k \ldots}$$

$$= -\sum_{kk'\sigma} \underbrace{\tilde{V}_{kk'}^{ss'}}\, n_\sigma^{s}\, n_\sigma^{s'}$$

$$\frac{2\pi e^2}{|k-k'|} \leftarrow \boxed{V(k-k')\left(1 + ss' \cos(\varphi_k - \varphi_{k'})\right)}$$


1905
1905
From Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
K
K
K'
K'


1905
1905
From Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
K
K
K'
K'