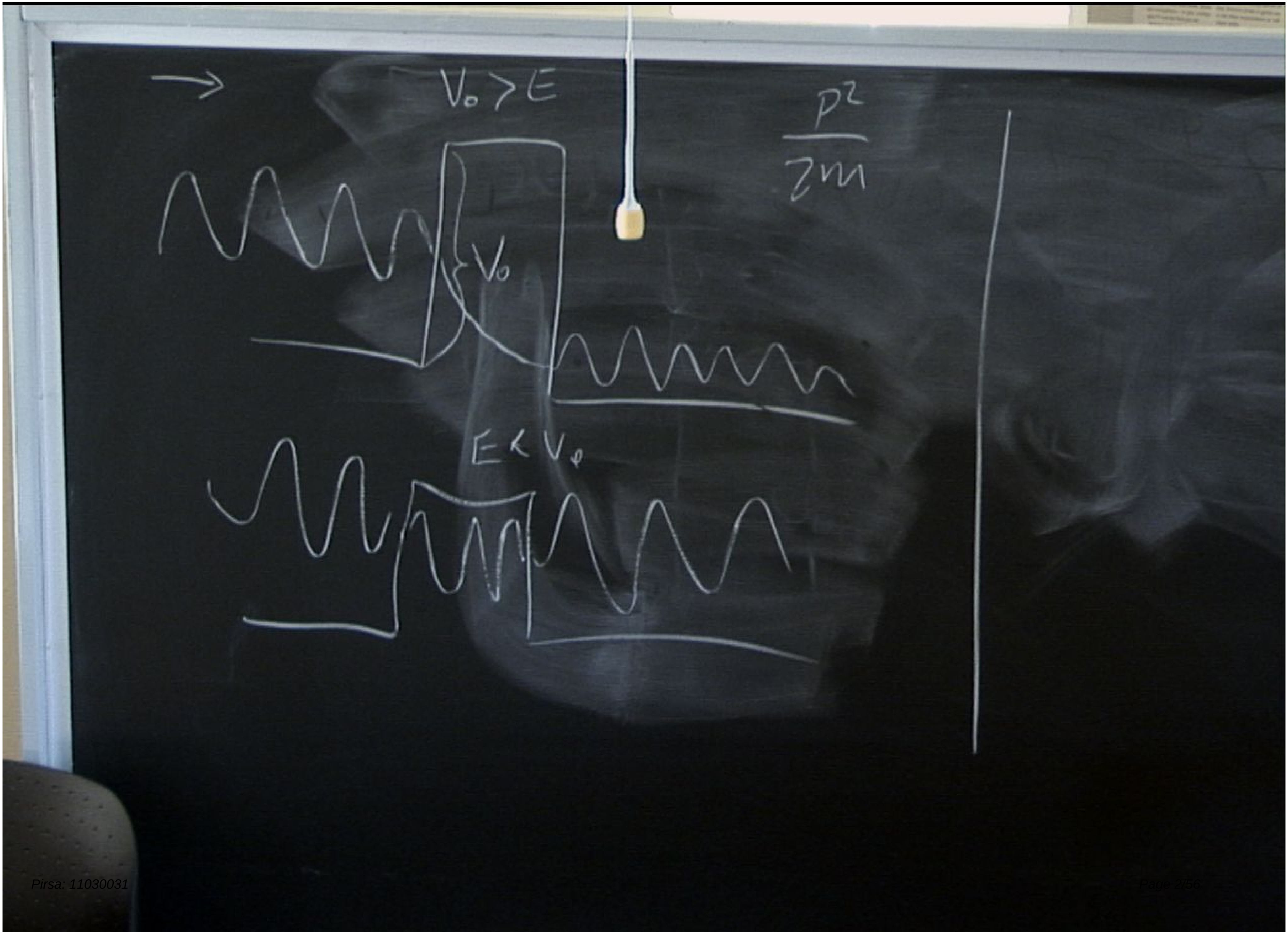
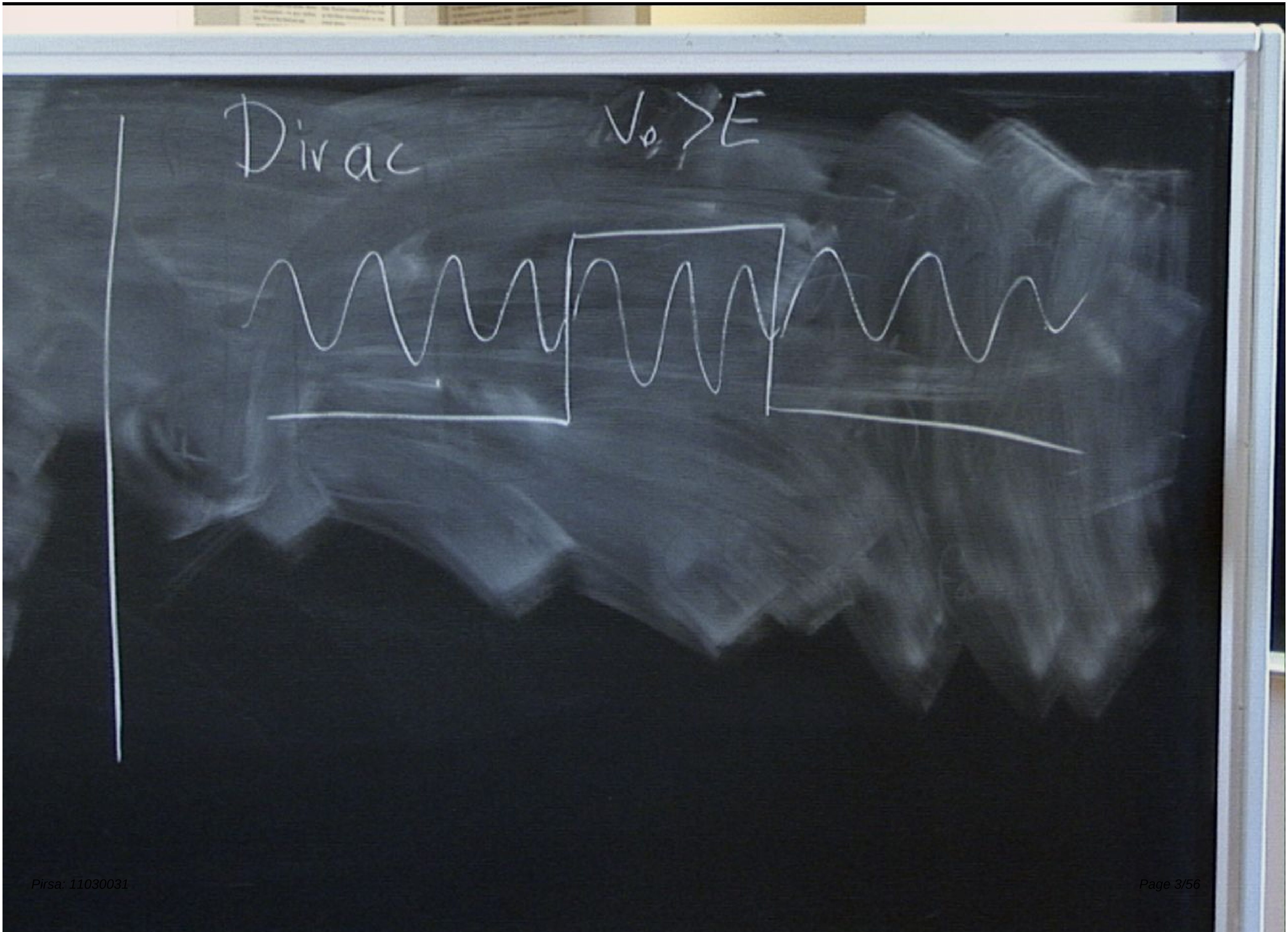
Title: Explorations in Condensed Matter - Lecture 2

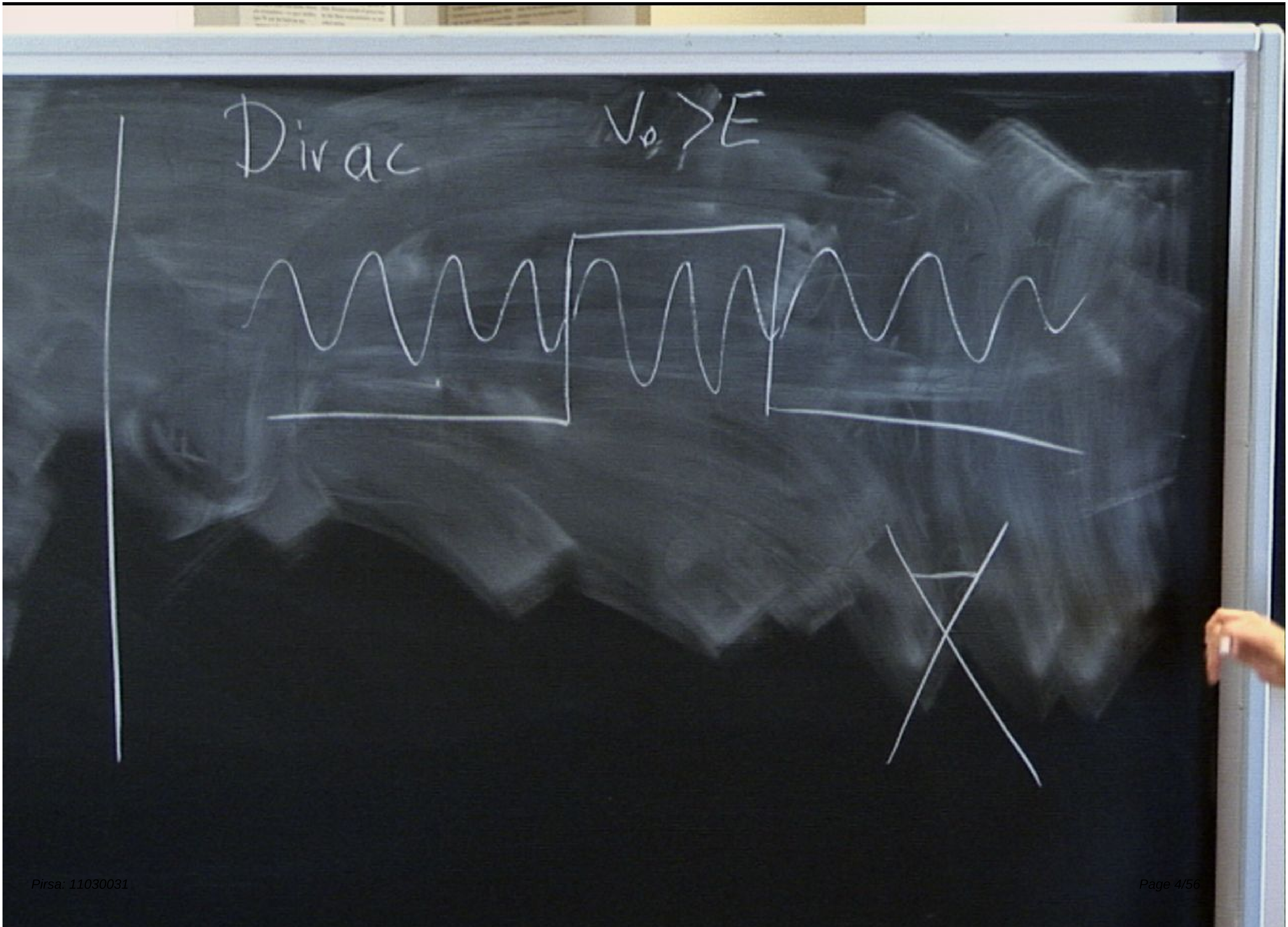
Date: Mar 15, 2011 10:15 AM

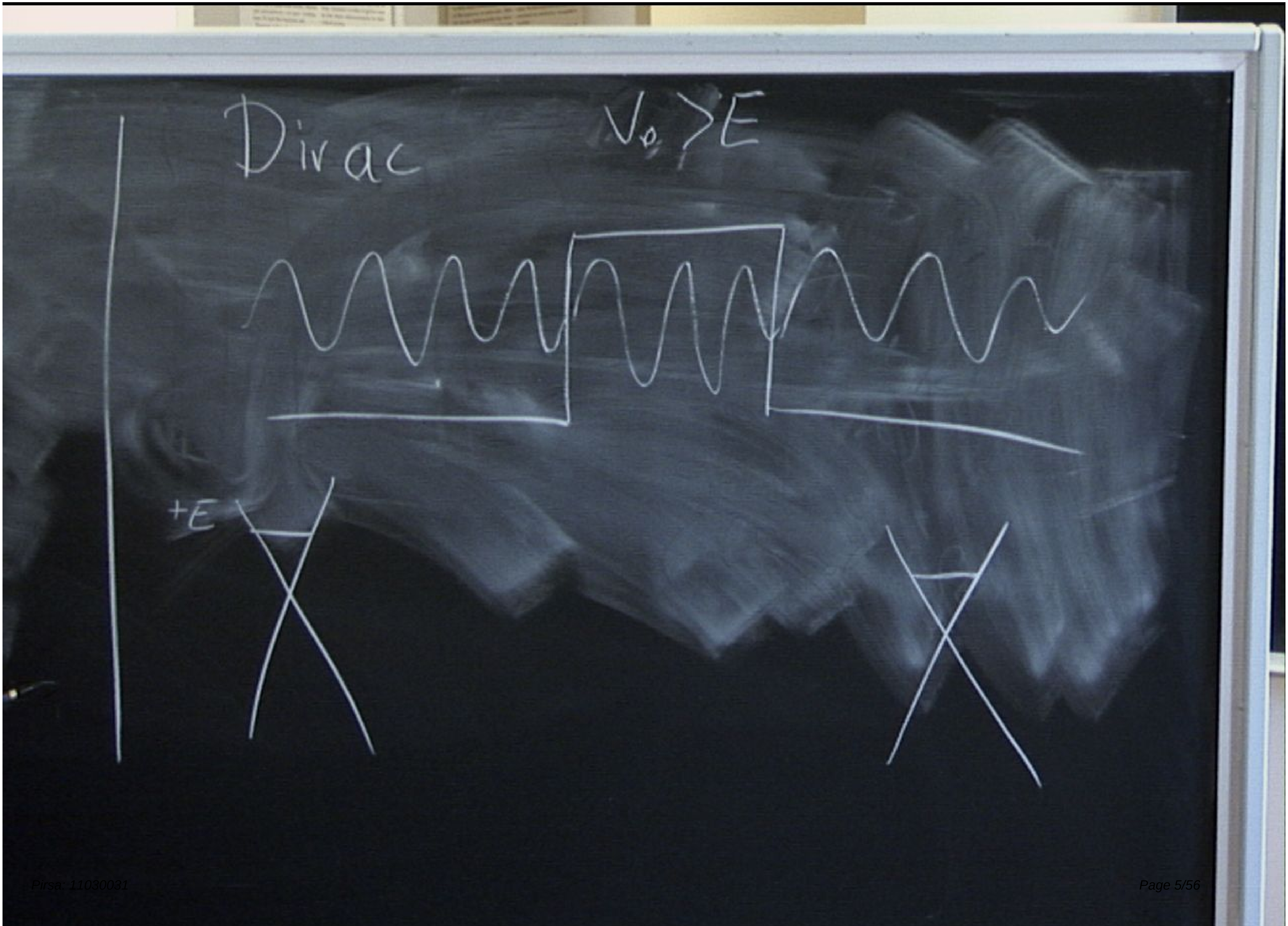
URL: http://pirsa.org/11030031

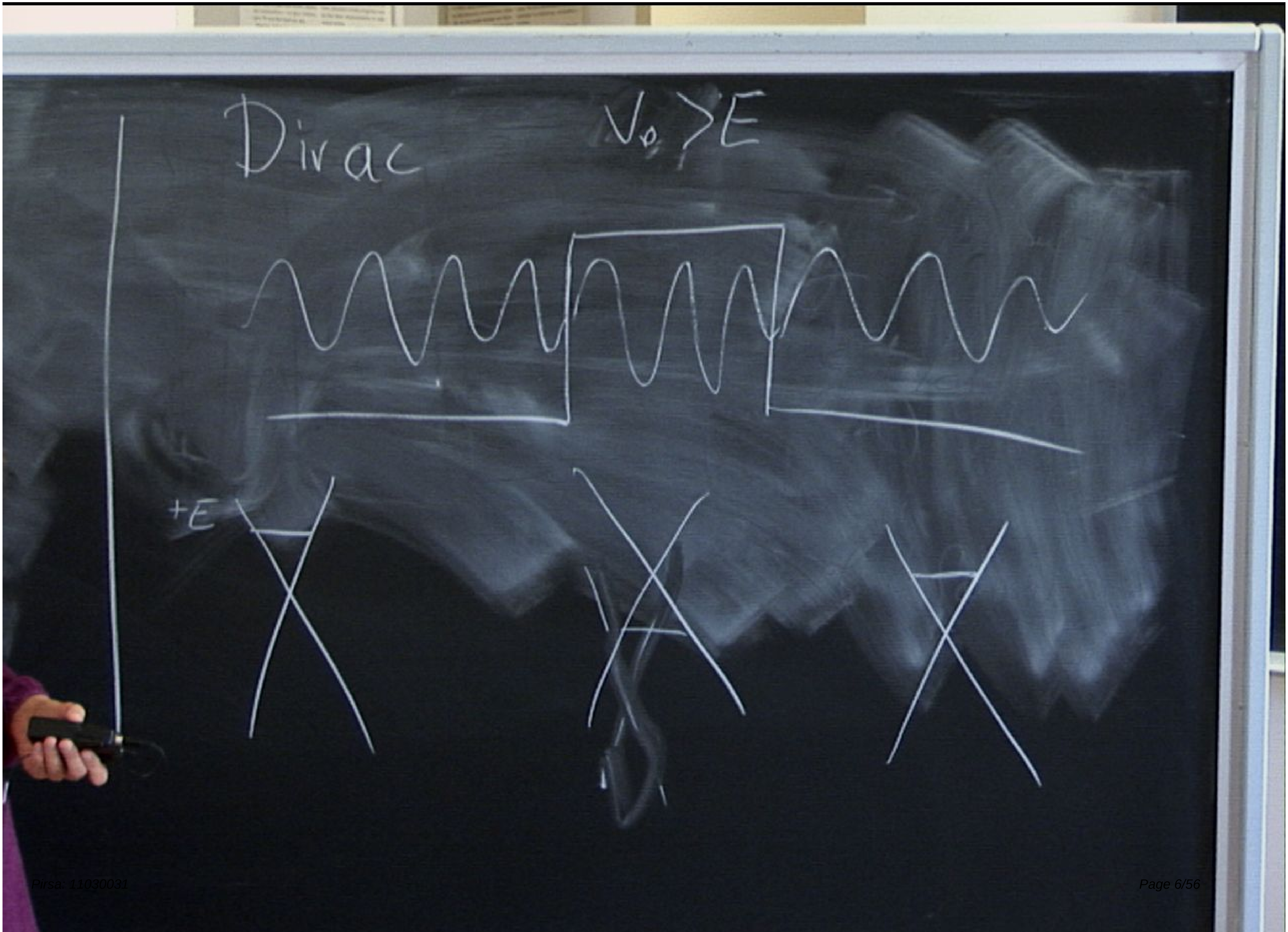
Abstract:


V0 > E
p2/2m
V0
E < V0


Dirac
$V_0 > E$


Dirac
$V_0 > E$


Dirac
$V_0 > E$
+E


Dirac
$V_0 > E$
+E

$$\begin{pmatrix} & k_x + ik_y \\ k_x - ik_y & \end{pmatrix} \xrightarrow{\text{real}} \begin{pmatrix} 0 & -i\hbar\partial_x + \partial_y \\ -i\hbar\partial_x - \partial_y & 0 \end{pmatrix}$$

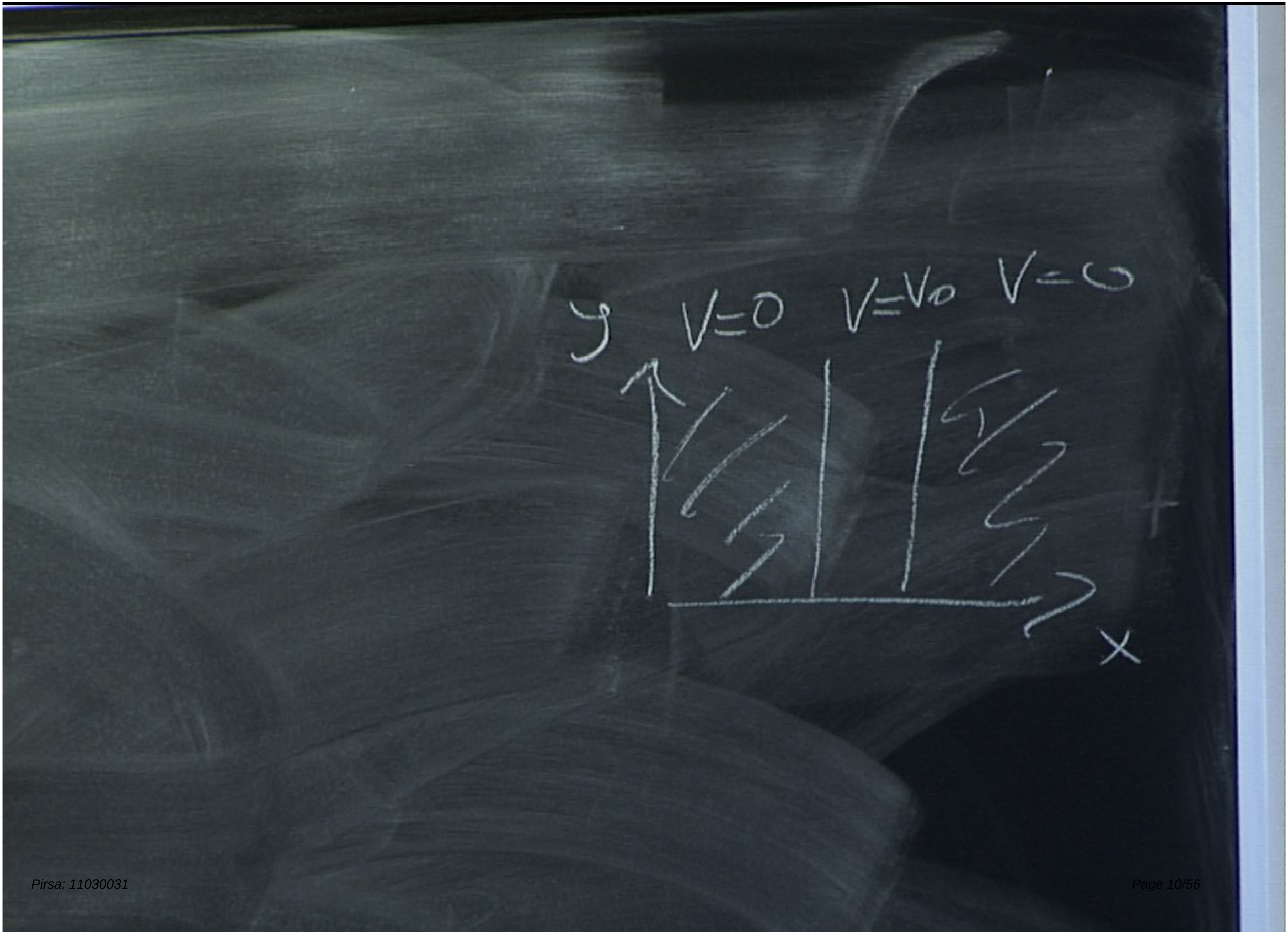
$$\begin{pmatrix} & k_x + ik_y \\ k_x - ik_y & \end{pmatrix} \xrightarrow{\text{real}} \begin{pmatrix} 0 & -i\hbar\partial_x + \hbar\partial_y \\ -i\hbar\partial_x - \hbar\partial_y & 0 \end{pmatrix}$$

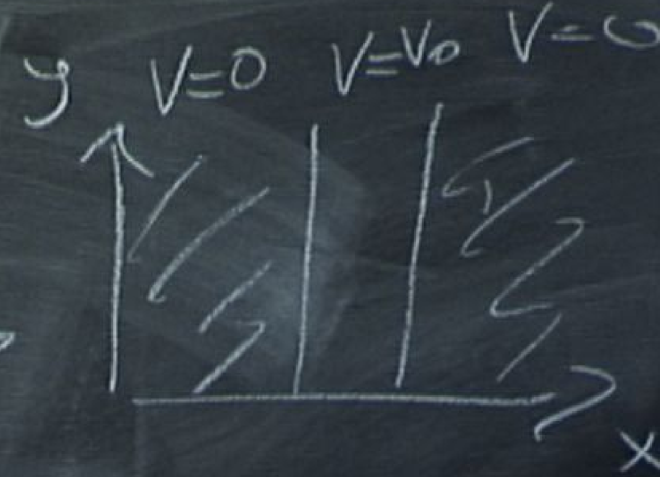
$$\Psi_{(\pm E)} = \begin{pmatrix} 1 \\ \pm e^{i\varphi_k} \end{pmatrix} e^{ik_x x + ik_y y}$$

$$\begin{pmatrix} & k_x + i k_y \\ k_x - i k_y & \end{pmatrix} \xrightarrow{\text{real}} \begin{pmatrix} 0 & -i\hbar\partial_x + \hbar\partial_y \\ -i\hbar\partial_x - \hbar\partial_y & 0 \end{pmatrix}$$

$$\Psi_{(\pm E)} = \begin{pmatrix} 1 \\ \pm e^{i\varphi_k} \end{pmatrix} e^{i k_x x + i k_y y}$$

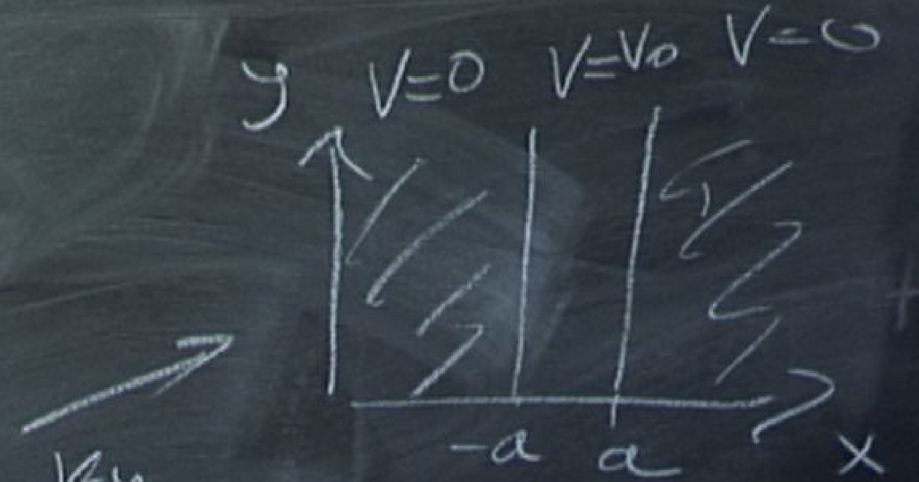
$$\varphi_k = \mathrm{Arg}(k)$$


y
V=0
V=V0
V=0
x



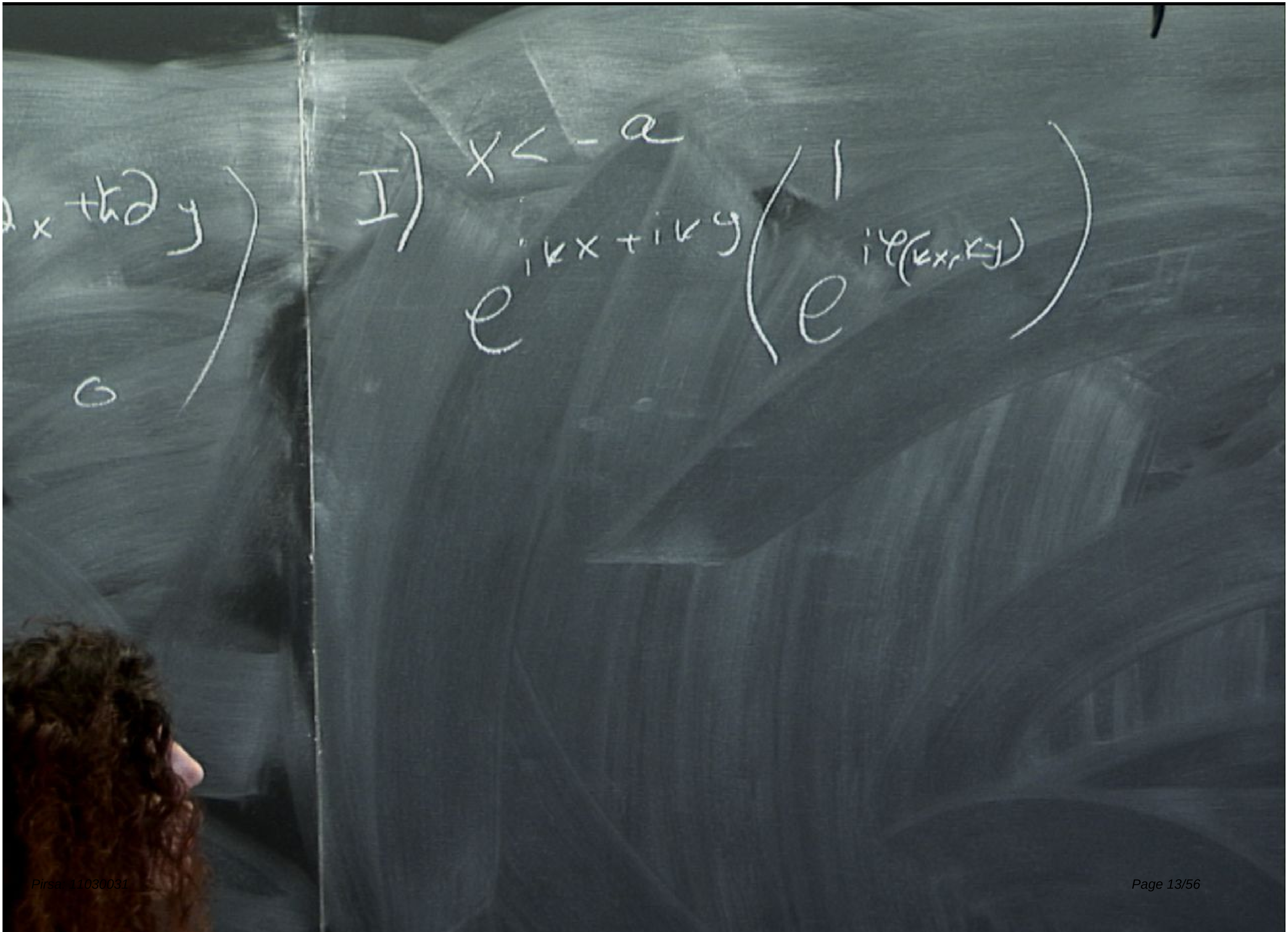
k_x, k_y

$$E = \sqrt{k_x^2 + k_y^2}$$
$$= |k|$$



k_x, k_y

$$E = \sqrt{k_x^2 + k_y^2} = |k|$$


∂x + iħ∂y
0
I) x < -a
e^{ikx+iky} (1, e^{iφ(kx,ky)})

$\partial_y)$

I) $x < -a$

$$e^{ik_x x + ik_y y}\begin{pmatrix} 1 \\ e^{i\varphi(k_x, k_y)} \end{pmatrix} + r e^{ik_y y - ik_x x}\begin{pmatrix} 1 \\ e^{i\varphi(-k_x, k_y)} \end{pmatrix}$$

III) $x > a$

$$t\, e^{ik_x x + ik_y y}$$

$\partial_y \Big)$

I) $x < -a$

$$e^{ikx+iky}\begin{pmatrix}1\\ e^{i\varphi(k_x,k_y)}\end{pmatrix} + r\,e^{ik_y y - ik_x x}\begin{pmatrix}1\\ e^{i\varphi(-k_x,\ldots}\end{pmatrix}$$

III) $x > a$

$$t\,e^{ik_x x + ik_y y}\begin{pmatrix}1\\ e^{i\varphi(k_x,k_y)}\end{pmatrix}$$

∂_y)

I) $x < -a$

$$e^{ikx+iky}\begin{pmatrix}1\\ e^{i\varphi(k_x,k_y)}\end{pmatrix} + r\, e^{ik_y y - ik_x x}\begin{pmatrix}1\\ e^{i\varphi(-k_x,k_y)}\end{pmatrix}$$

II) $x > a$

$$t\, e^{ik_x x + ik_y y}\begin{pmatrix}1\\ e^{i\varphi(k_x,k_y)}\end{pmatrix}$$

III) $-a < x < a$

I) $e^{ik_x x + ik_y y}\begin{pmatrix}1\\ e^{i\varphi(k_x,k_y)}\end{pmatrix} + r e^{\dots}\begin{pmatrix}\dots\\ e^{i\varphi(-k_x, \dots}\end{pmatrix}$

II) $x > a$

$$t\, e^{ik_x x + ik_y y}\begin{pmatrix}1\\ e^{i\varphi(k_x,k_y)}\end{pmatrix}$$

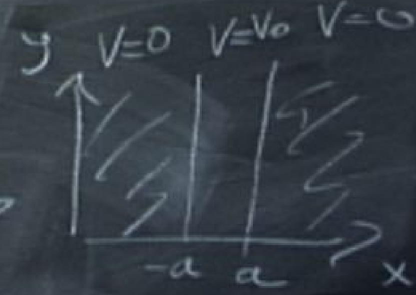
III) $-a < x < a$

$$\alpha\, e^{iq_x x + ik_y}\begin{pmatrix}1\\ e^{i\varphi(q_x,k_y)}\end{pmatrix} +$$

$$\sqrt{q_x^2 + k_y^2} = E - V_0$$

I) $x < -a$

$$e^{ik_x x + ik_y y}\begin{pmatrix}1\\ e^{i\varphi(k_x,k_y)}\end{pmatrix} + r\, e^{ik_y y - ik_x x}\begin{pmatrix}1\\ e^{i\varphi(-k_x,k_y)}\end{pmatrix}$$

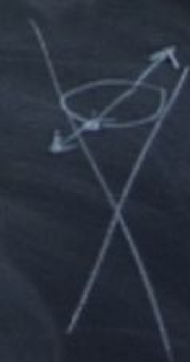


k_x, k_y

$E = \sqrt{k_x^2 + k_y^2} = |k|$

II) $x > a$

$$t\, e^{ik_x x + ik_y y}\begin{pmatrix}1\\ e^{i\varphi(k_x,k_y)}\end{pmatrix}$$



III) $-a < x < a$

$$\alpha\, e^{iq_x x + ik_y y}\begin{pmatrix}1\\ e^{i\varphi(q_x,k_y)}\end{pmatrix} + \beta\, e^{-iq_x x + ik_y y}\begin{pmatrix}1\\ e^{i\varphi(-q_x,k_y)}\end{pmatrix}$$

$$\sqrt{q_x^2 + k_y^2} = E - V_0$$

Katsnelson et al.

$$\begin{pmatrix} & k_x + i k_y \\ k_x - i k_y & \end{pmatrix} \xrightarrow{\text{real}} \begin{pmatrix} 0 & -i\hbar\partial_x + \hbar\partial_y \\ -i\hbar\partial_x - \hbar\partial_y & 0 \end{pmatrix}$$

1. $k_y = 0$
2. $E - V_0 < 0$

$$\Psi_{(\pm E)} = \begin{pmatrix} 1 \\ \pm e^{i\varphi_k} \end{pmatrix} e^{i k_x x + \underline{i k_y y}}$$

$\varphi_k = \mathrm{Arg}(k)$

I) $x < -a$

$e^{i k_x x + i\ldots}$

III) $x > a$

$t\, e^{i\ldots}$

II) $-a < \ldots$

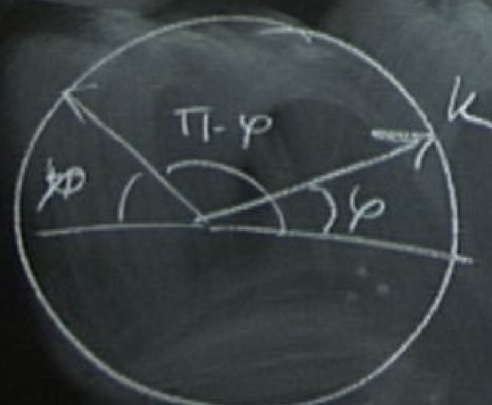
$\alpha e^{i\ldots}$

Katsnelson et al.

$$\begin{pmatrix} & k_x + i k_y \\ k_x - i k_y & \end{pmatrix} \xrightarrow{\text{real}} \begin{pmatrix} 0 & -i\hbar\partial_x + \hbar\partial_y \\ -i\hbar\partial_x - \hbar\partial_y & 0 \end{pmatrix}$$

I)

III)



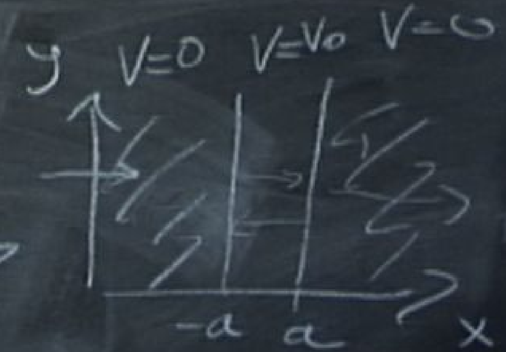
1. $k_y = 0$
2. $E - V_0 < 0$

$$\varphi(k_x, 0) \rightarrow \varphi(-k_x, 0)$$
$$= -\varphi(k_x, 0) + \pi$$

III

$$e^{ik_y y}\begin{pmatrix}1\\ e^{i\varphi(k_x,k_y)}\end{pmatrix} + r\, e^{ik_y y - ik_x x}\begin{pmatrix}1\\ e^{i\varphi(-k_x,k_y)}\end{pmatrix}$$

(-1)



k_x, k_y

$$E = \sqrt{k_x^2 + k_y^2} = |k|$$

$$e^{ik_x x + ik_y y}\begin{pmatrix}1\\ e^{i\varphi(k_x,k_y)}\end{pmatrix}$$

$< x < a$

$$e^{iq_x x + ik_y}\begin{pmatrix}1\\ -e^{i\varphi(q_x,k_y)}\end{pmatrix} + \beta\, e^{-iq_x x + ik_y y}\begin{pmatrix}1\\ -e^{i\varphi(-q_x,k_y)}\end{pmatrix}$$

$+1$

Katsnelson et al.

$-a)\quad \begin{pmatrix}1\\1\end{pmatrix} e^{-ika} + \begin{pmatrix}1\\-1\end{pmatrix} e^{ika}$

$= t\, e^{-ik_x a} \begin{pmatrix}1\\1\end{pmatrix}$

$a)\quad t\, e^{ika} = \alpha e^{iqa} \begin{pmatrix}1\\-1\end{pmatrix}$

$+ \beta e^{-iqa} \begin{pmatrix}1\\1\end{pmatrix}$

I) $x < -a$

$e^{ik_x x + ik_y \ldots}$

II) $x > a$

$t\, e^{ik_x x \ldots}$

III) $-a < x \ldots$

$\alpha e^{iq_x x \ldots}$

$\sqrt{q_x^2 + k_y^2} = E - V \ldots$

Katsne…

$$-a)\quad \begin{pmatrix}1\\1\end{pmatrix} e^{-ika} + r\begin{pmatrix}1\\-1\end{pmatrix} e^{ika}$$

$$= t\, e^{-ika} \begin{pmatrix}1\\1\end{pmatrix}$$

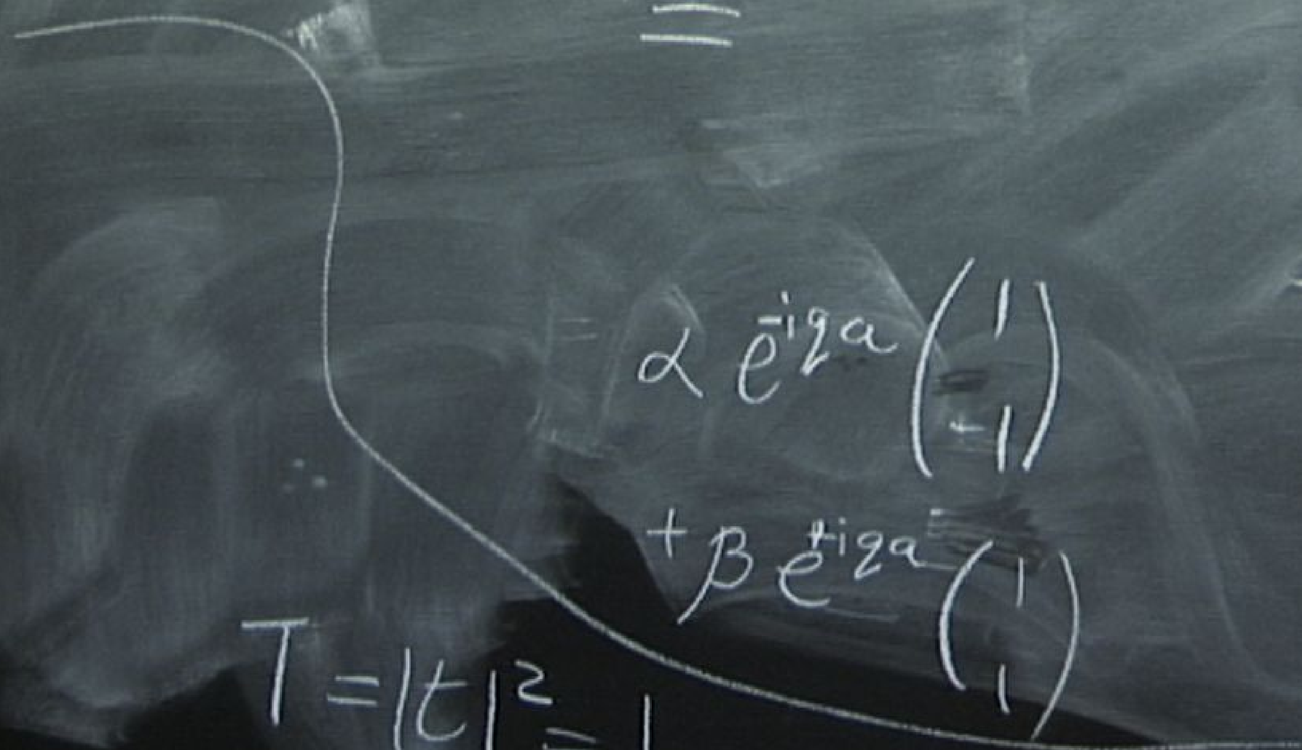
$$a)\quad t\, e^{ika} = \alpha\, e^{iqa} \begin{pmatrix}1\\-1\end{pmatrix} + \beta\, e^{-iqa} \begin{pmatrix}1\\1\end{pmatrix}$$

$$T = |t|^2 = 1$$

$$|r|^2 = 0$$

Katsnel

$-a)\ \begin{pmatrix}1\\1\end{pmatrix}e^{-ika} + \begin{pmatrix}1\\-1\end{pmatrix}e^{ika}\ )$



$=$

$\alpha e^{-iqa}\begin{pmatrix}1\\-1\end{pmatrix}$

$+\beta e^{+iqa}\begin{pmatrix}1\\1\end{pmatrix}$

$T = |t|^2 = 1$

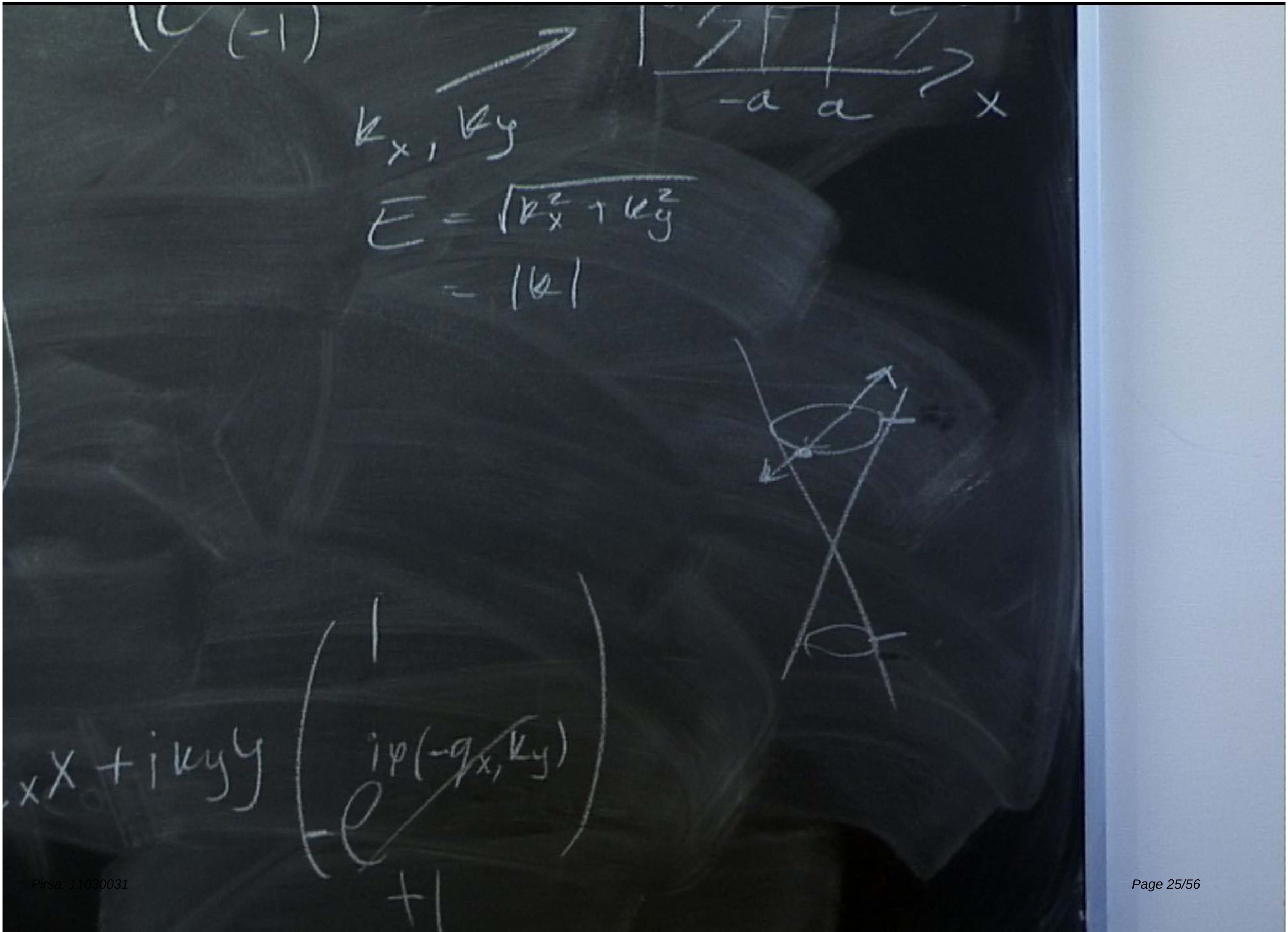
$|r|^2 = 0$

I

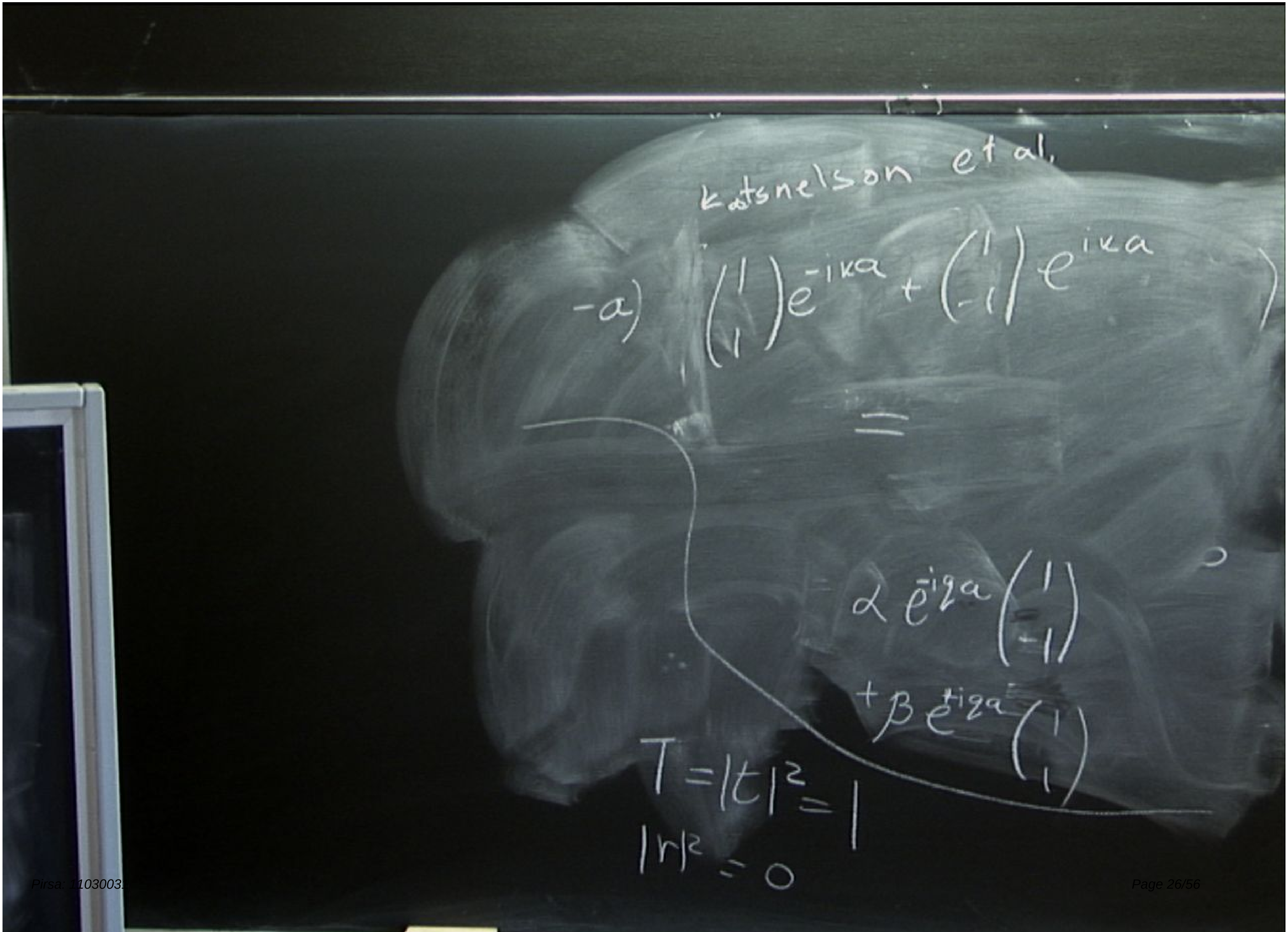
II

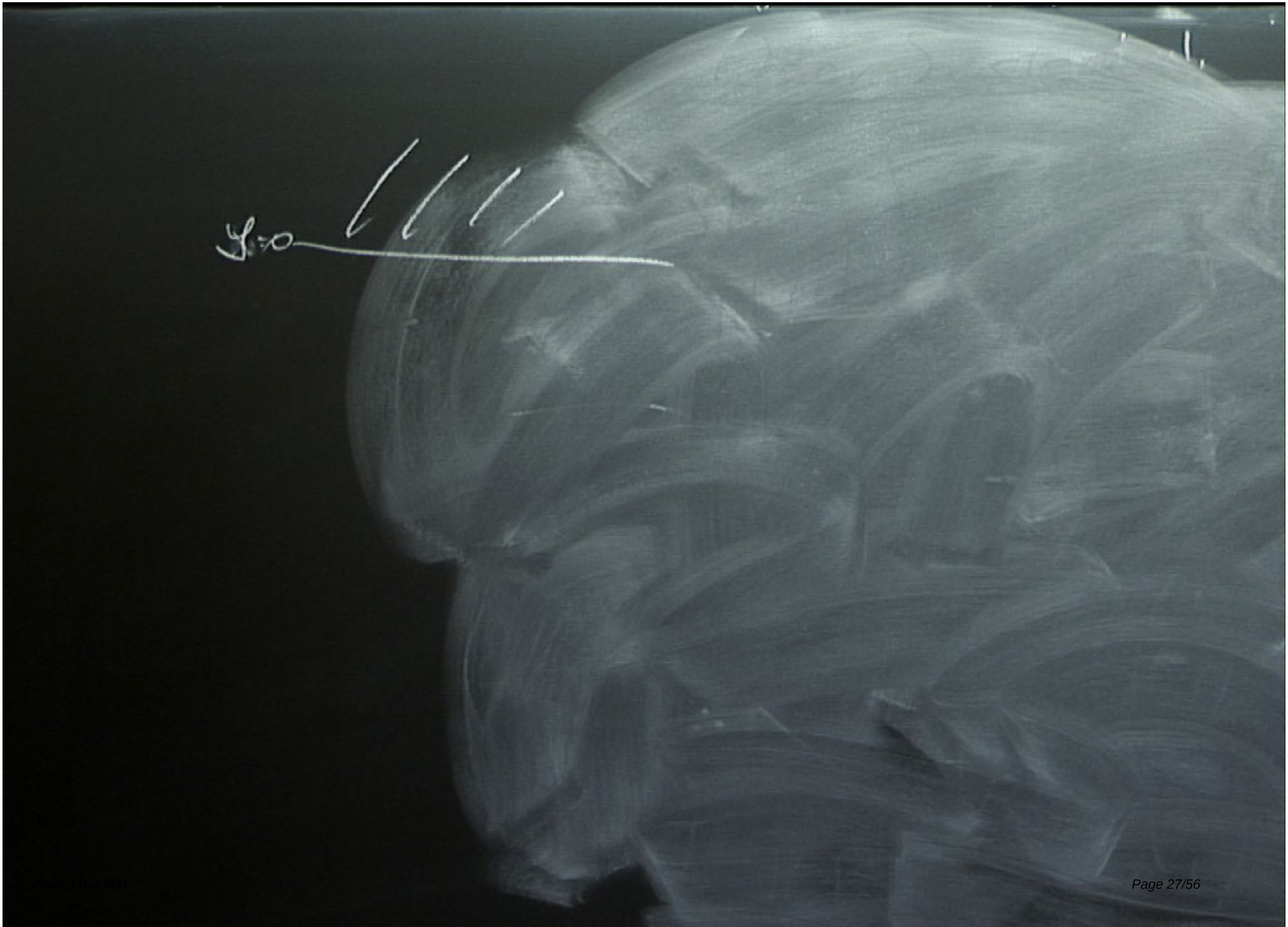
III

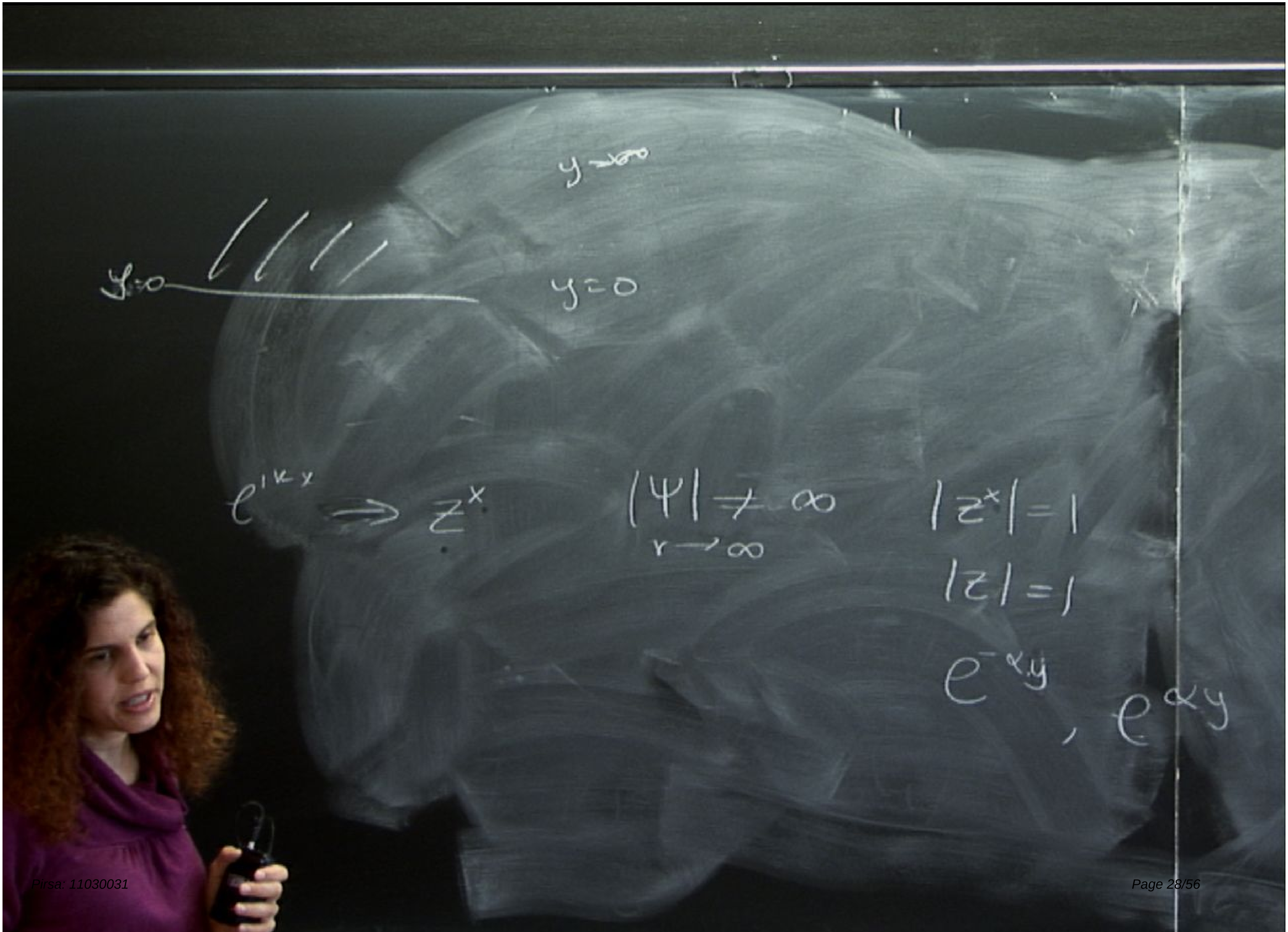
$\sqrt{\varepsilon_x}$



(-1)
-a a x
kx, ky
E = √(kx² + ky²)
= |k|
x X + i ky y
e^{iφ(-qx, ky)}
+1


Katsnelson et al.
-a)
(1, 1) e^{-ika} + (1, -1) e^{ika}
=
α e^{-iqa} (1, -1)
+β e^{+iqa} (1, 1)
T = |t|² = 1
|r|² = 0




y→∞
y=0
y=0
e^{ikx} ⇒ z^x
|Ψ| ≠ ∞
r→∞
|z^x|=1
|z|=1
e^{-αy}, e^{αy}

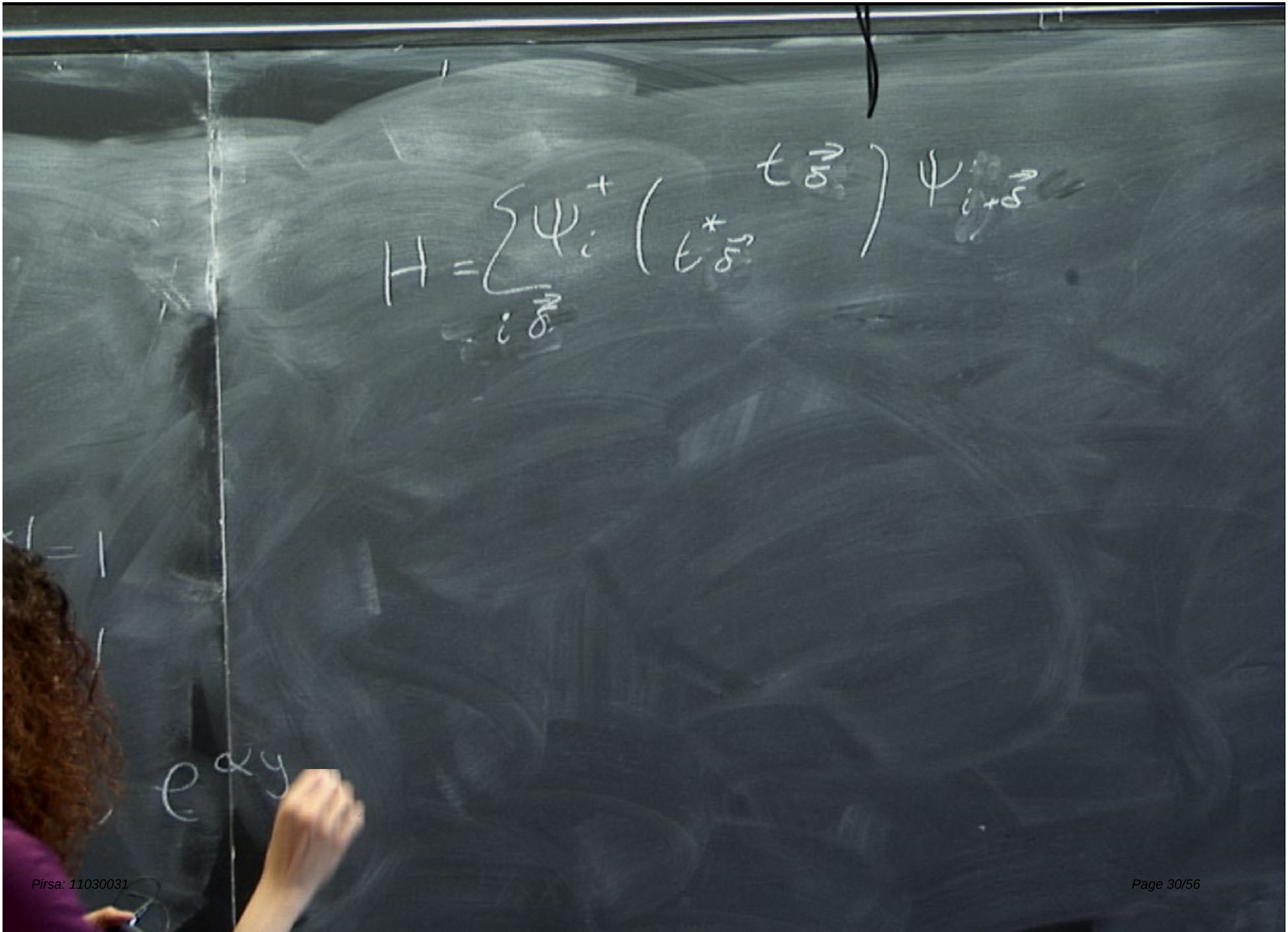
$$H = \sum_{\langle ij \rangle} \psi_i^{+} \begin{pmatrix} & t_{ij} \\ t_{ij}^{*} & \end{pmatrix} \psi_j$$

$|=1$

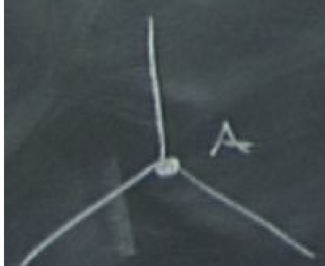
$|=1$

αy

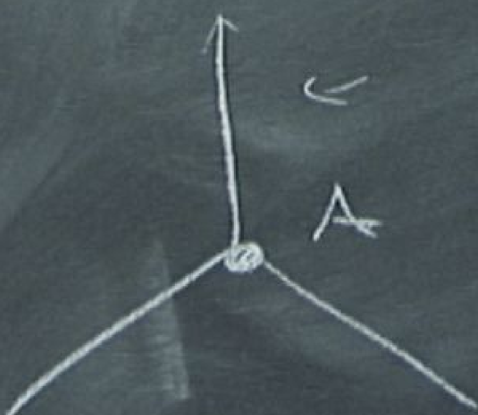
$, e^{\alpha y}$


$H = \sum_{i\vec{\delta}} \psi_i^{+} \begin{pmatrix} & t_{\vec{\delta}} \\ t^*_{\vec{\delta}} & \end{pmatrix} \psi_{i+\vec{\delta}}$
$e^{\alpha y}$

$$H = \sum_{i,\vec{\delta}} \psi_i^{+} \begin{pmatrix} & t_{\vec{\delta}} \\ t^{*}_{\vec{\delta}} & \end{pmatrix} \psi_{i+\vec{\delta}} = \sum_{\substack{k_x, i_y \\ \vec{\delta}}} \psi_{k_x, i_y} \begin{pmatrix} & t_k(\vec{\delta}_y) \\ * & \end{pmatrix} \psi_{k_x, i_y+\delta_y}$$



$$H = \sum_{i,\vec{\delta}} \psi_i^{+} \begin{pmatrix} & t_{\vec{\delta}} \\ t^{*}_{\vec{\delta}} & \end{pmatrix} \psi_{i+\vec{\delta}} = \sum_{k_x, \ldots}$$



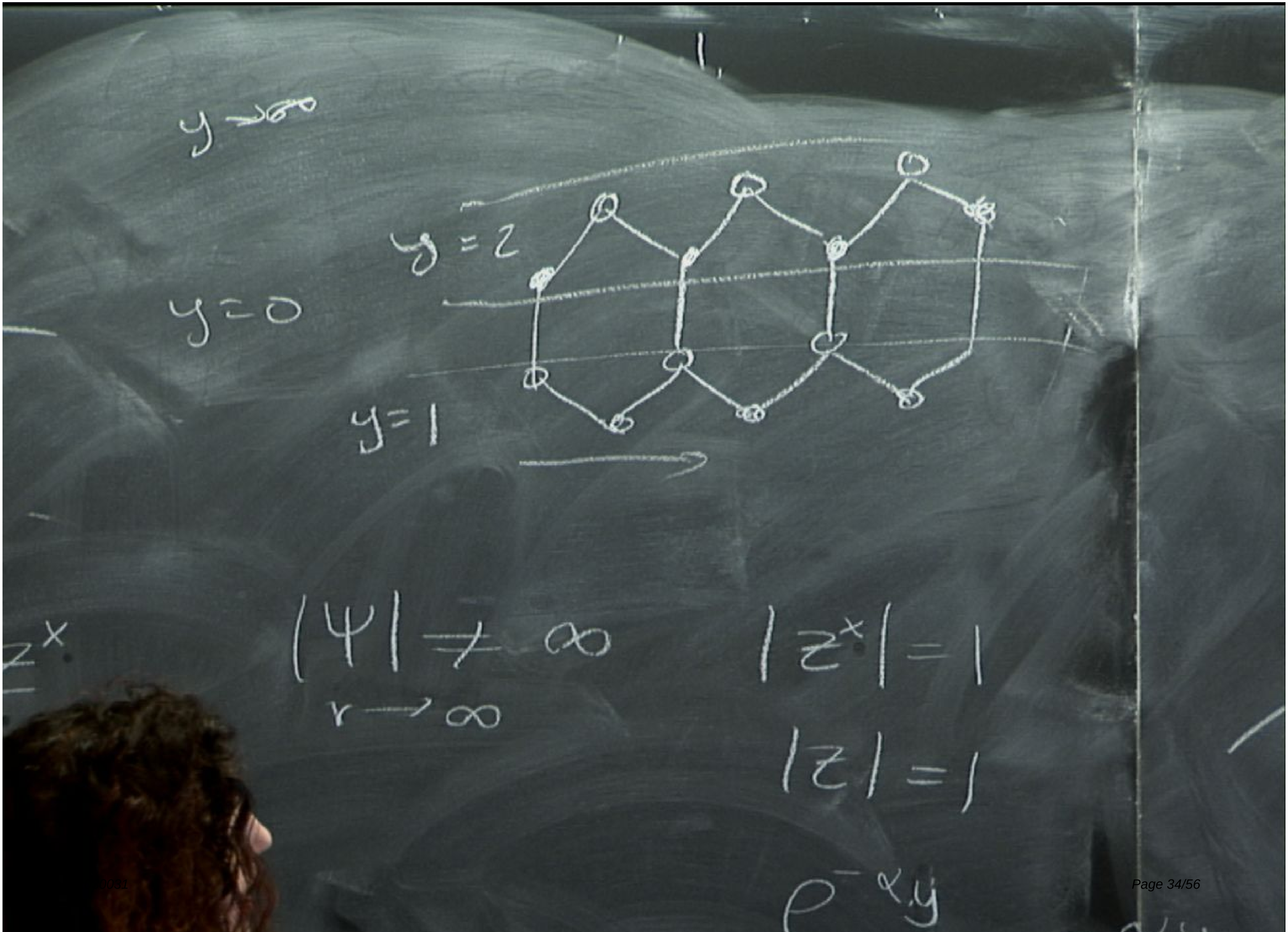
$$\sum_{\vec{\delta}_y} t_{k,\vec{\delta}_y} = R^{+}$$

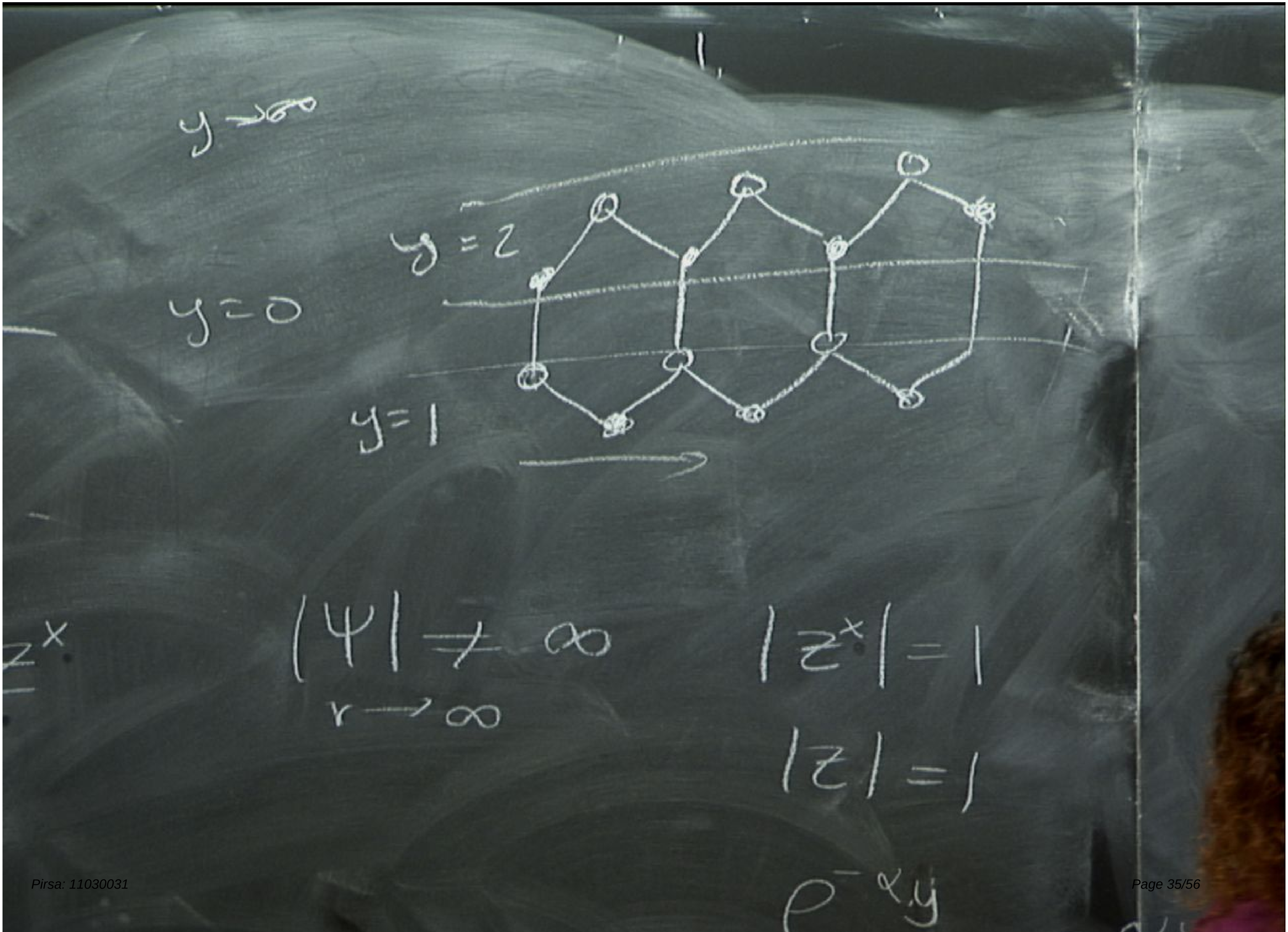
xy

$$\psi_{i+\vec{\delta}} = \sum_{\substack{k_x, i_y \\ \vec{\delta}}} \psi_{k_x, i_y} \left(* \quad t_k(\vec{\delta}_y) \right) \psi_{k_x, i_y + \delta_y}$$

$\times$

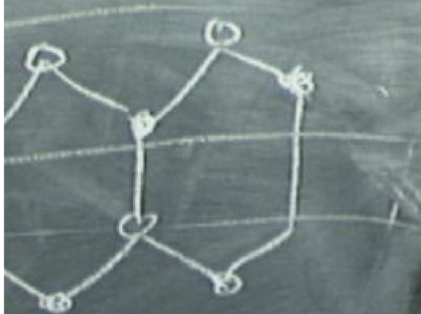
R^+


y → ∞
y = 2
y = 0
y = 1
|Ψ| ≠ ∞
r → ∞
|z^x| = 1
|z| = 1
e^{-αy}



y → ∞
y = 2
y = 0
y = 1
|Ψ| ≠ ∞
r → ∞
|z^x| = 1
|z| = 1
e^{-αy}

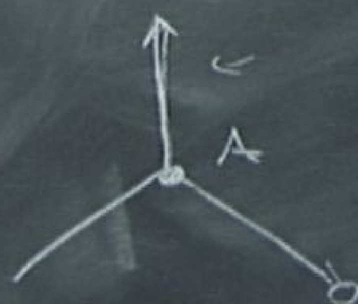
$$H = \sum_{i,\vec{\delta}} \psi_i^\dagger \begin{pmatrix} & t_{\vec{\delta}} \\ t^*_{\vec{\delta}} & \end{pmatrix} \psi_{i+\vec{\delta}} = \sum_{k_x, i_y, \vec{\delta}} \psi_{k_x, \ldots}$$



$|z^*| = 1$

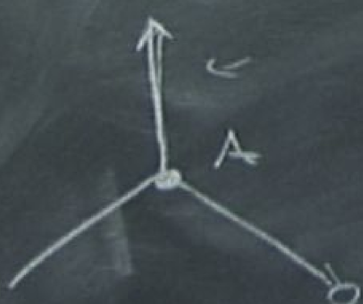
$|z| = 1$

$e^{-\alpha y}, \; e^{\alpha y}$



$$\sum_{\vec{\delta}_y} t_{k,\vec{\delta}_y} = R^+ + e^{i\frac{k_x}{2}a}$$

$$H = \sum_{i,\vec{\delta}} \psi_i^{\dagger} \begin{pmatrix} & t_{\vec{\delta}} \\ t^{*}_{\vec{\delta}} & \end{pmatrix} \psi_{i+\vec{\delta}} = \sum_{k_x, i_y, \vec{\delta}} \psi_{k_x, i_y} \begin{pmatrix} & t_k(\vec{\delta}_y) \\ * & \end{pmatrix}$$

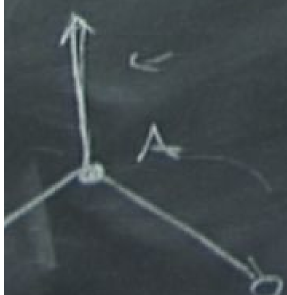


$$\sum_{\vec{\delta}_y} t_{k,\vec{\delta}_y} = R^{\dagger} + e^{i\frac{k_x a}{2}} + e^{-i\frac{k_x a}{2}}$$

$$= R^{\dagger} + 2\cos\left(\frac{k_x}{2}\right)$$

$$H = \sum_{i,\vec{\delta}} \psi_i^{\dagger} \begin{pmatrix} & t_{\vec{\delta}} \\ t^{*}_{\vec{\delta}} & \end{pmatrix} \psi_{i+\vec{\delta}} = \sum_{k_x, i_y} \psi_{k_x, i_y} \begin{pmatrix} & t_k(\vec{\delta}_y) \\ * & \end{pmatrix} \psi_{k_x, i_y+\delta_y}$$

$$\psi(x,y) = \frac{1}{\sqrt{N}} \sum_k e^{i k \cdot x} \psi(y) \qquad R^{\dagger}\psi(y) = \psi(y+1)$$

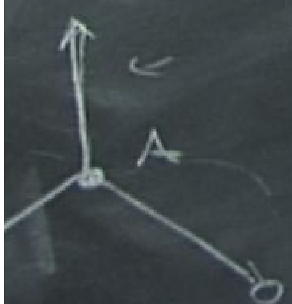


$$\sum_{\vec{\delta}_y} t_{k,\vec{\delta}_y} = R^{\dagger} + e^{i\frac{k_x a}{2}} + e^{-i\frac{k_x a}{2}}$$

$$= R^{\dagger} + 2\cos\left(\frac{k_x}{2}\right)$$

$$H = \sum_{i,\vec{\delta}} \Psi_i^{\dagger} \begin{pmatrix} & t_{\vec{\delta}} \\ t^{*}_{\vec{\delta}} & \end{pmatrix} \Psi_{i+\vec{\delta}} = \sum_{k_x, i_y} \Psi_{k_x, i_y} \begin{pmatrix} & t_k(\vec{\delta}_y) \\ * & \end{pmatrix} \Psi_{k_x, i_y + \delta_y}$$

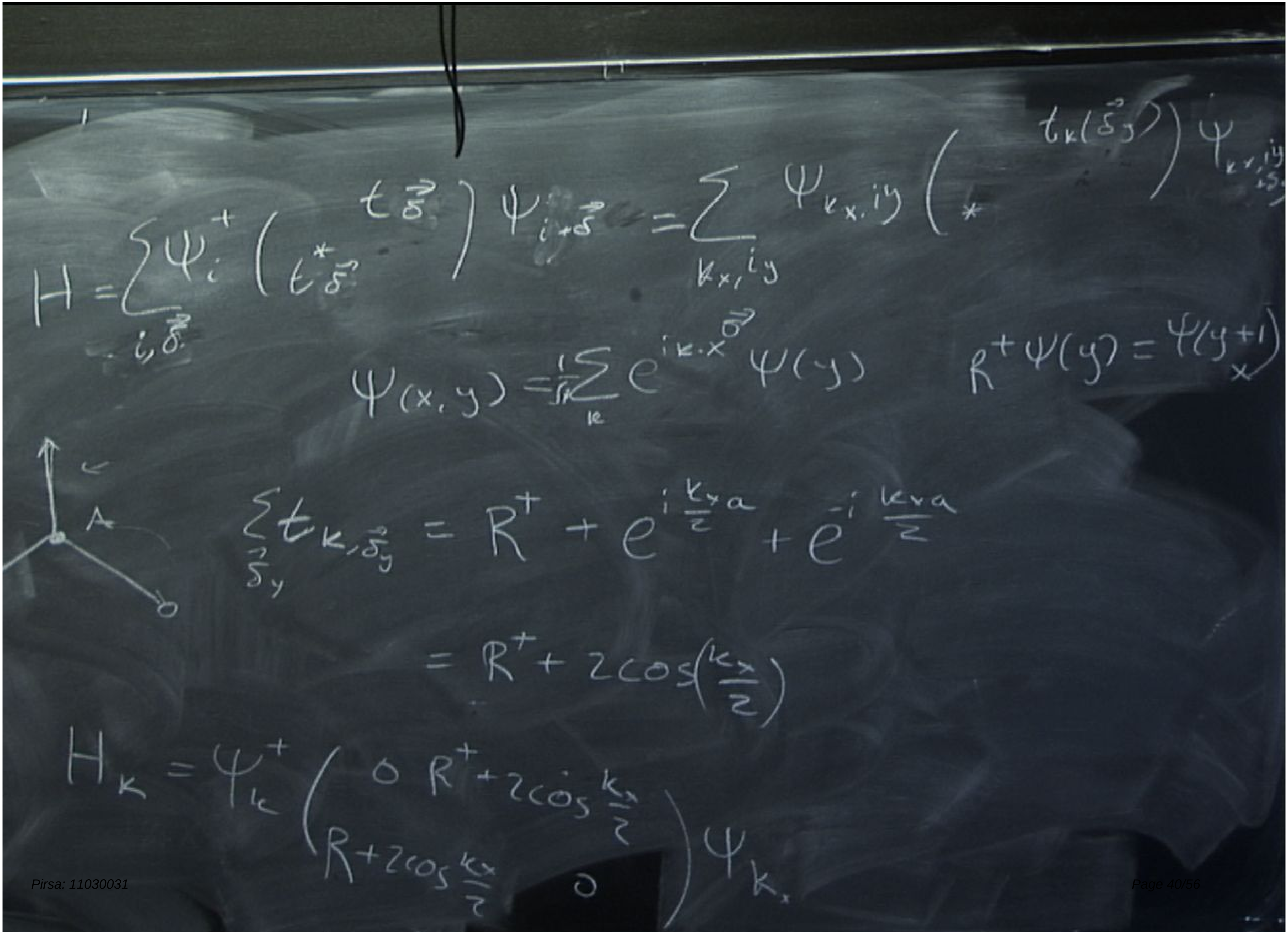
$$\Psi(x,y) = \frac{1}{\sqrt{N}} \sum_{k} e^{ik\cdot x^{\vec{\delta}}} \Psi(y) \qquad R^{\dagger}\Psi(y) = \Psi(y+1)$$



$$\sum_{\vec{\delta}_y} t_{k,\vec{\delta}_y} = R^{\dagger} + e^{i\frac{k_x a}{2}} + e^{-i\frac{k_x a}{2}}$$

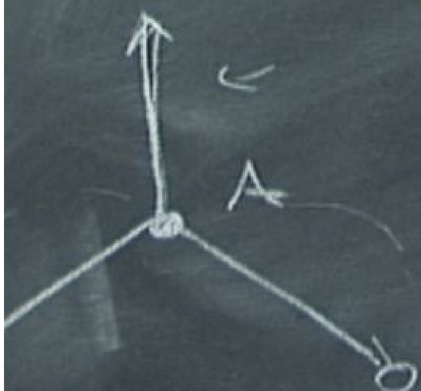
$$= R^{\dagger} + 2\cos\left(\frac{k_x}{2}\right)$$

$$H_k = \Psi_k^{\dagger} \begin{pmatrix} & R^{\dagger} + 2\cos\frac{k_x}{2} \\ R & \end{pmatrix}$$


H = Σ_{i,δ} ψ_i^+ (t_δ ; t*_δ) ψ_{i+δ} = Σ_{kx, iy} ψ_{kx, iy} (* t_k(δ_y)) ψ_{kx, iy+δ_y}
ψ(x, y) = 1/√N Σ_k e^{ik·x} ψ(y)
R^+ ψ(y) = ψ(y+1)
Σ_{δ_y} t_{k, δ_y} = R^+ + e^{i kx a/2} + e^{-i kx a/2}
= R^+ + 2cos(kx/2)
H_k = ψ_k^+ (0   R^+ + 2cos kx/2 ; R + 2cos kx/2   0) ψ_k

$$H = \sum_{i,\vec{\delta}} \Psi_i \left(t^*_{\vec{\delta}} \quad \right) \qquad k_x, i_y$$

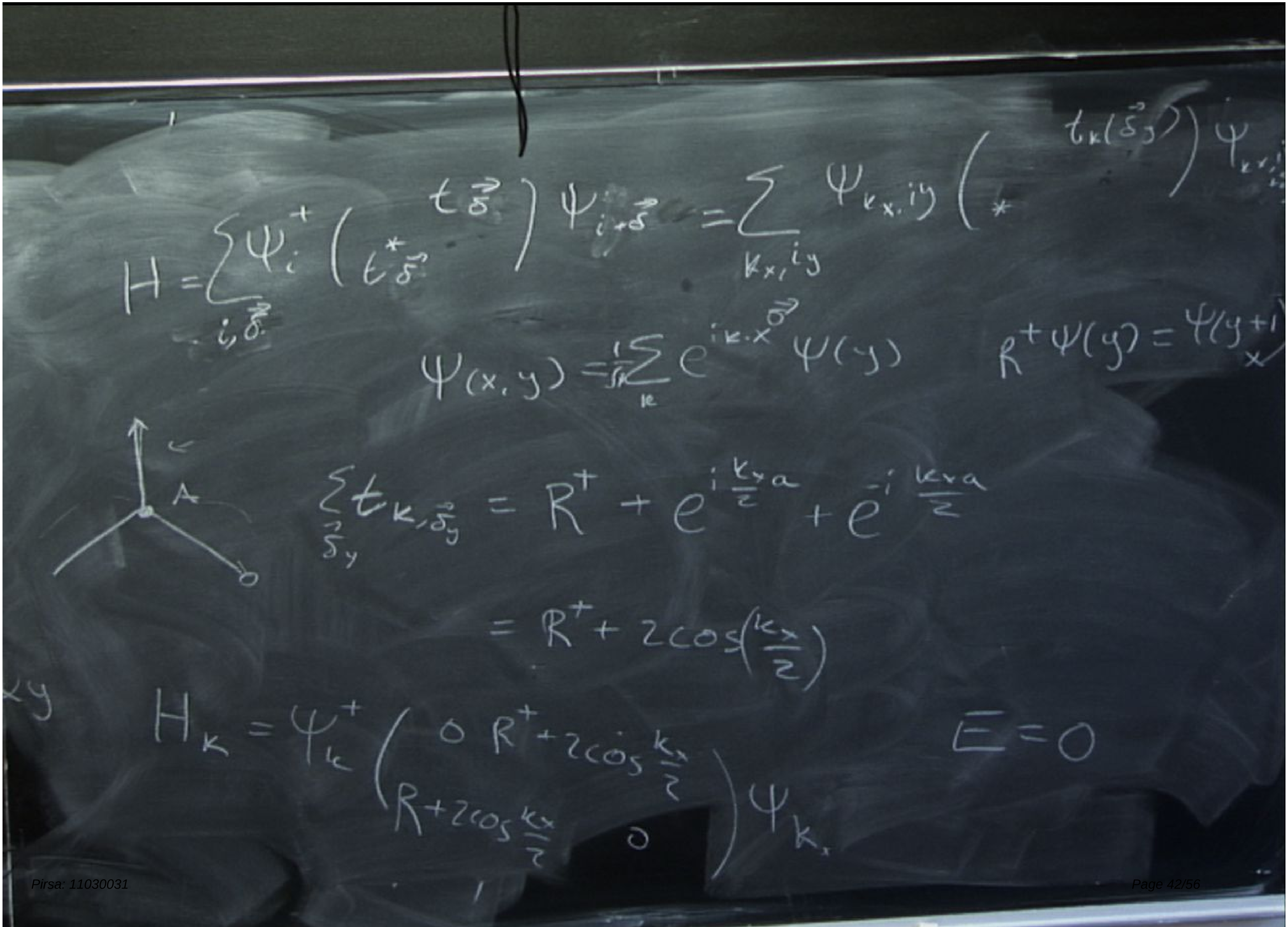
$$\Psi(x,y) = \frac{1}{\sqrt{N}} \sum_{k} e^{i k \cdot x_{\vec{\delta}}} \psi(y)$$

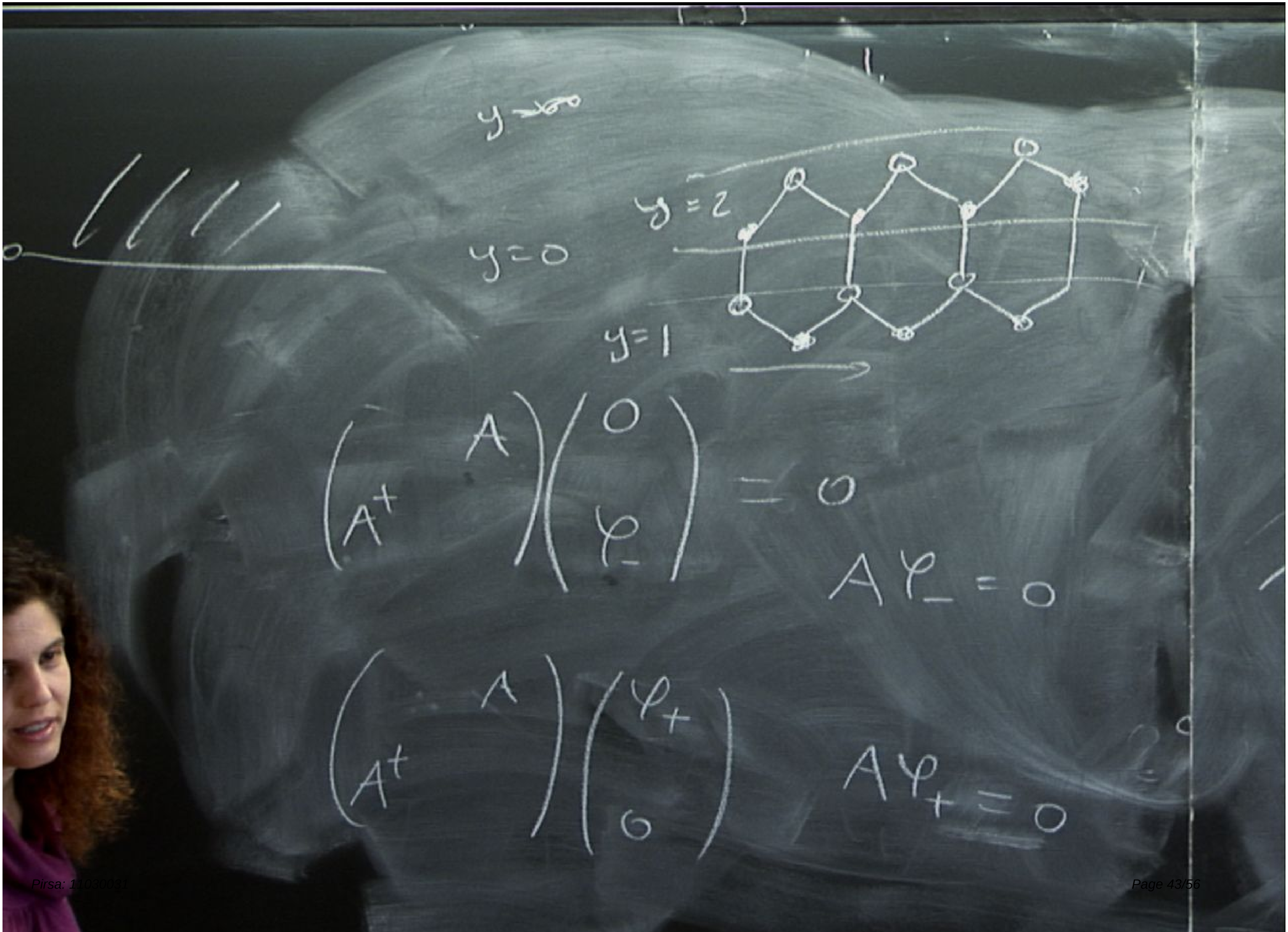


$$\sum_{\vec{\delta}_y} t_{k,\vec{\delta}_y} = R^{\dagger} + e^{i\frac{k_x a}{2}} + e^{-i\frac{k_x a}{2}}$$

$$= R^{\dagger} + 2\cos\left(\frac{k_x}{2}\right)$$

$$H_k = \Psi_k^{\dagger} \begin{pmatrix} 0 & R^{\dagger} + 2\cos\frac{k_x}{2} \\ R + 2\cos\frac{k_x}{2} & 0 \end{pmatrix} \Psi_{k_x}$$

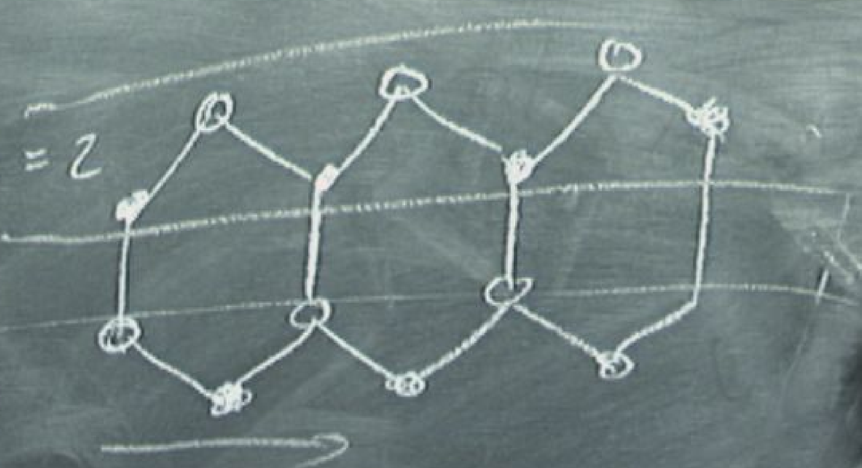

H = Σ Ψ_i^+ (t_δ ; t*_δ) Ψ_{i+δ} = Σ Ψ_{kx, iy} (* t_k(δ_y)) Ψ
i, δ
kx, iy
Ψ(x, y) = 1/√N Σ_k e^{ik·x} Ψ(y)
R^+ Ψ(y) = Ψ(y+1)
A
Σ_{δ_y} t_{k, δ_y} = R^+ + e^{i k_x a / 2} + e^{-i k_x a / 2}
= R^+ + 2cos(k_x / 2)
H_k = Ψ_k^+ (0 R^+ + 2cos k_x/2 ; R + 2cos k_x/2 0) Ψ_{k_x}
E = 0


y→∞
y=2
y=0
y=1
A
A+
0
φ-
= 0
Aφ- = 0
A
A+
φ+
0
Aφ+ = 0

$y \to \infty$

$y = 2$

$y = 0$

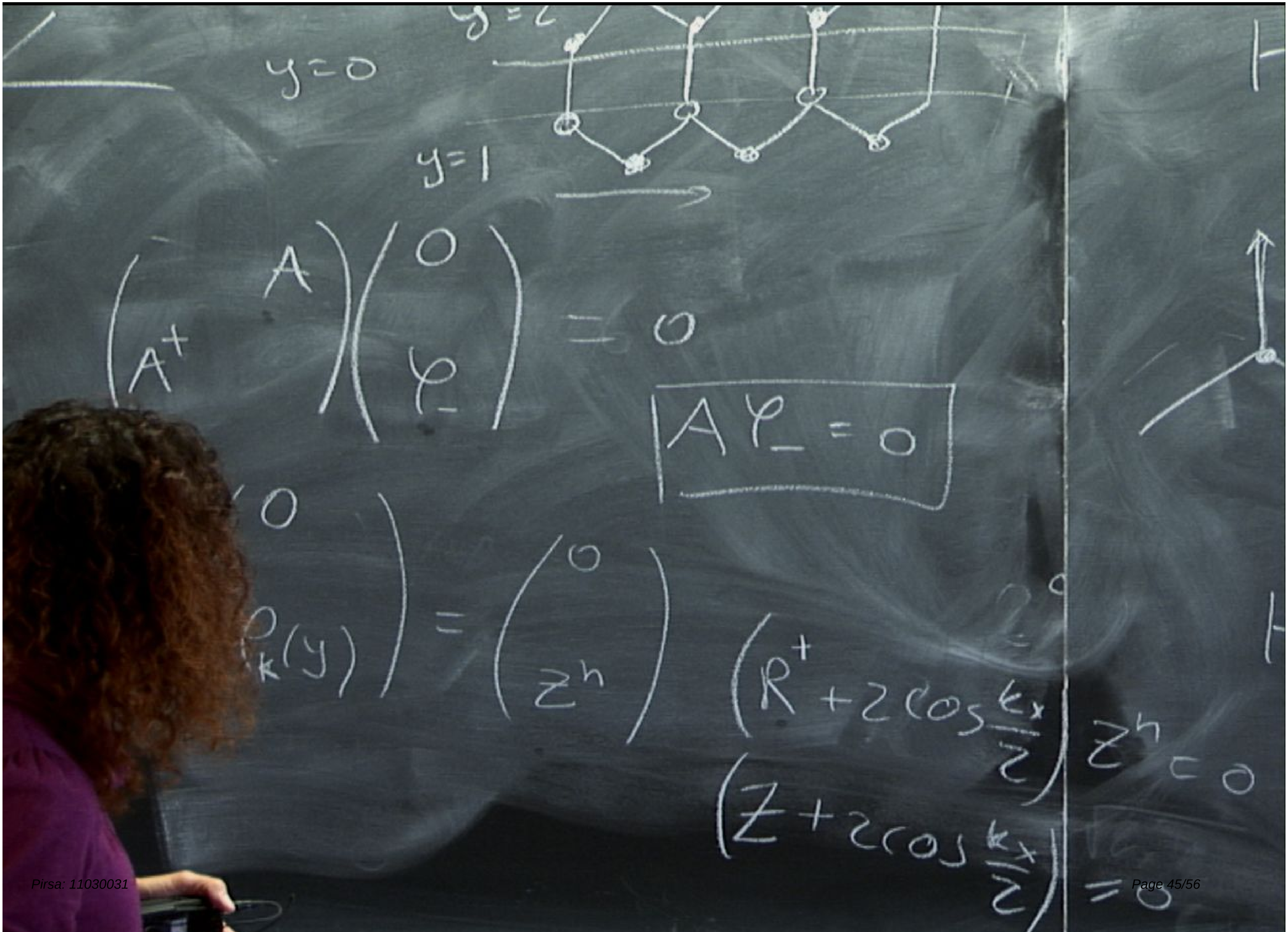


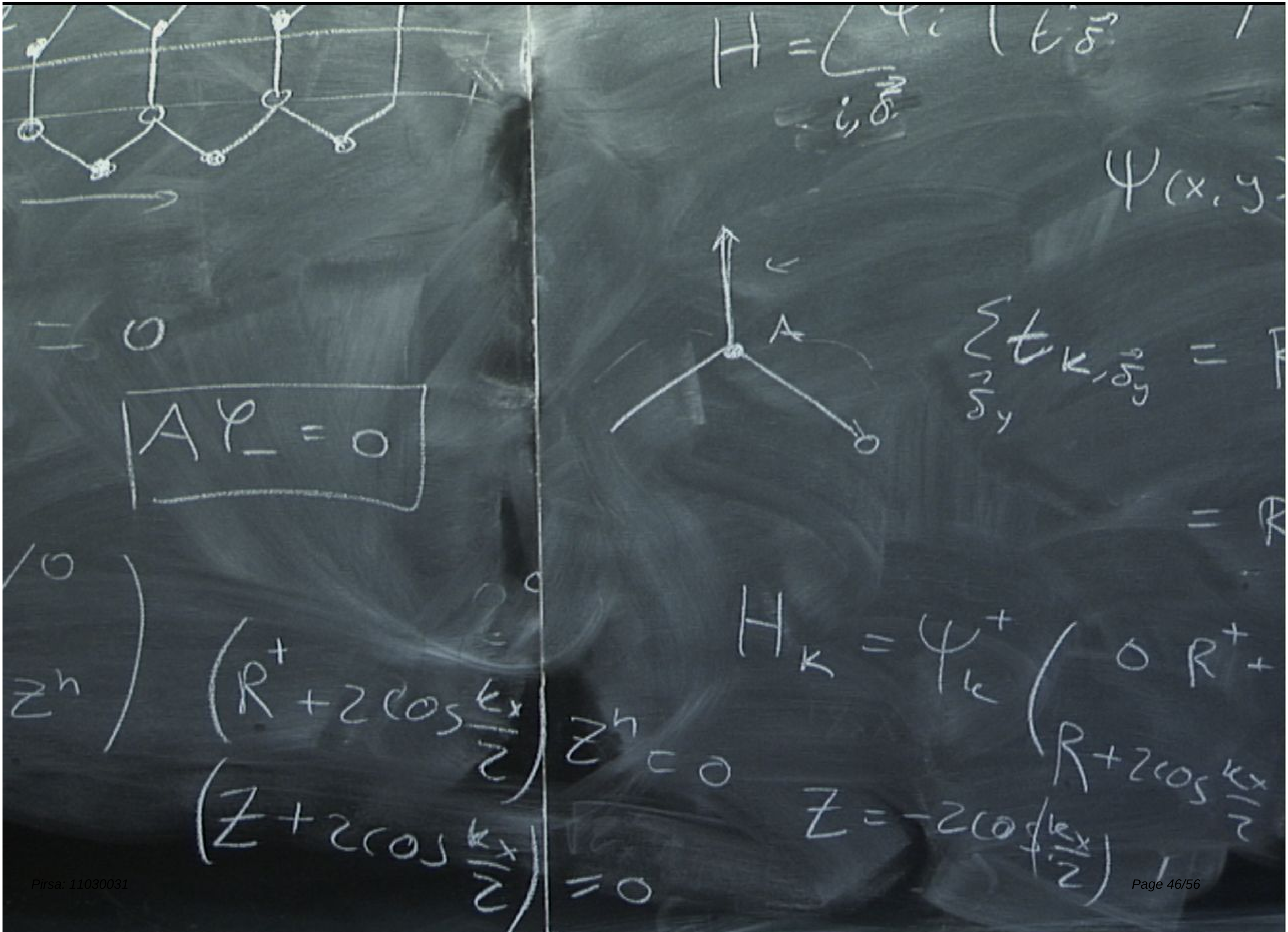
$y = 1$

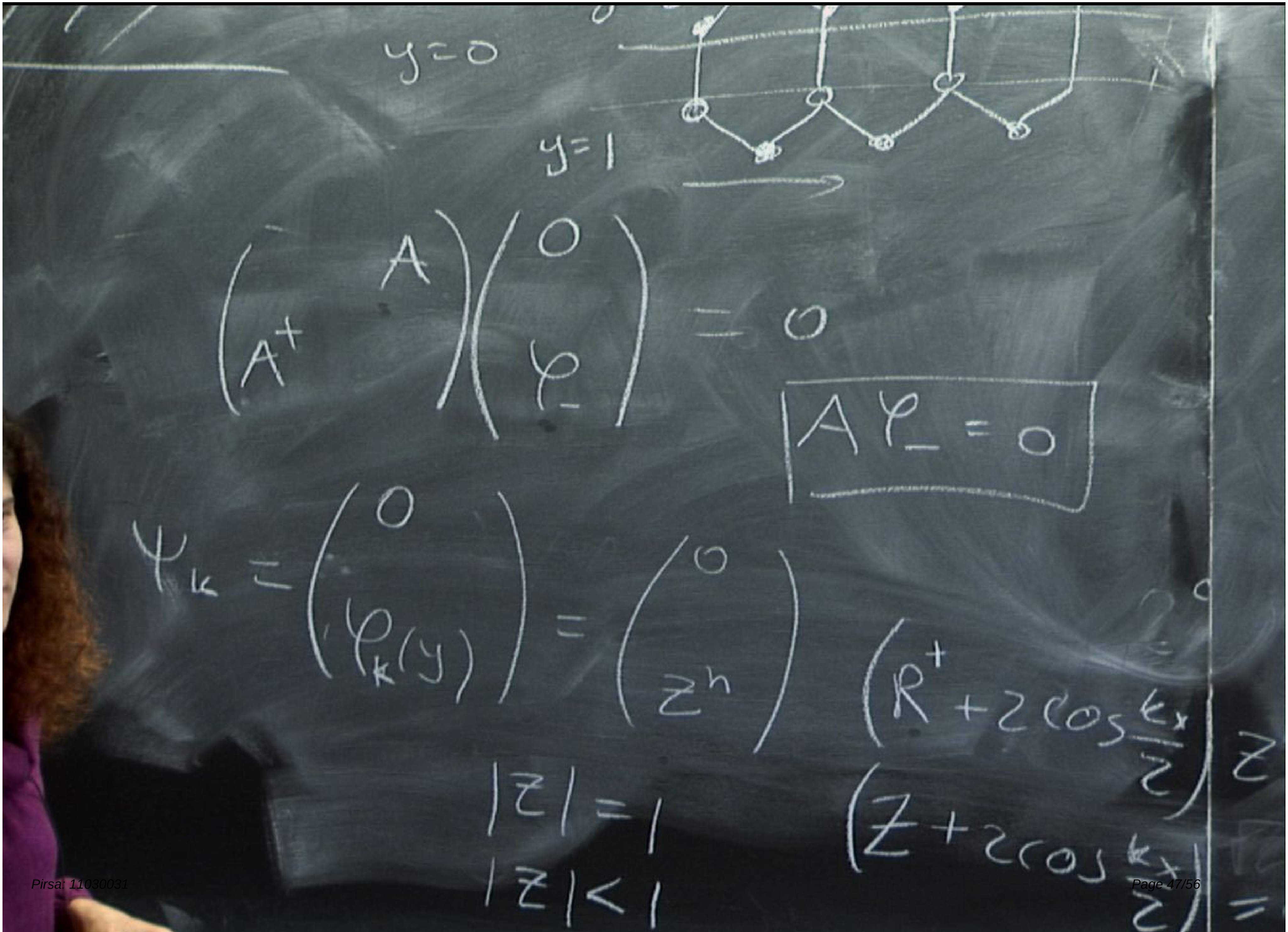
$$\begin{pmatrix} & A \\ A^+ & \end{pmatrix}\begin{pmatrix} 0 \\ \varphi_- \end{pmatrix} = 0$$

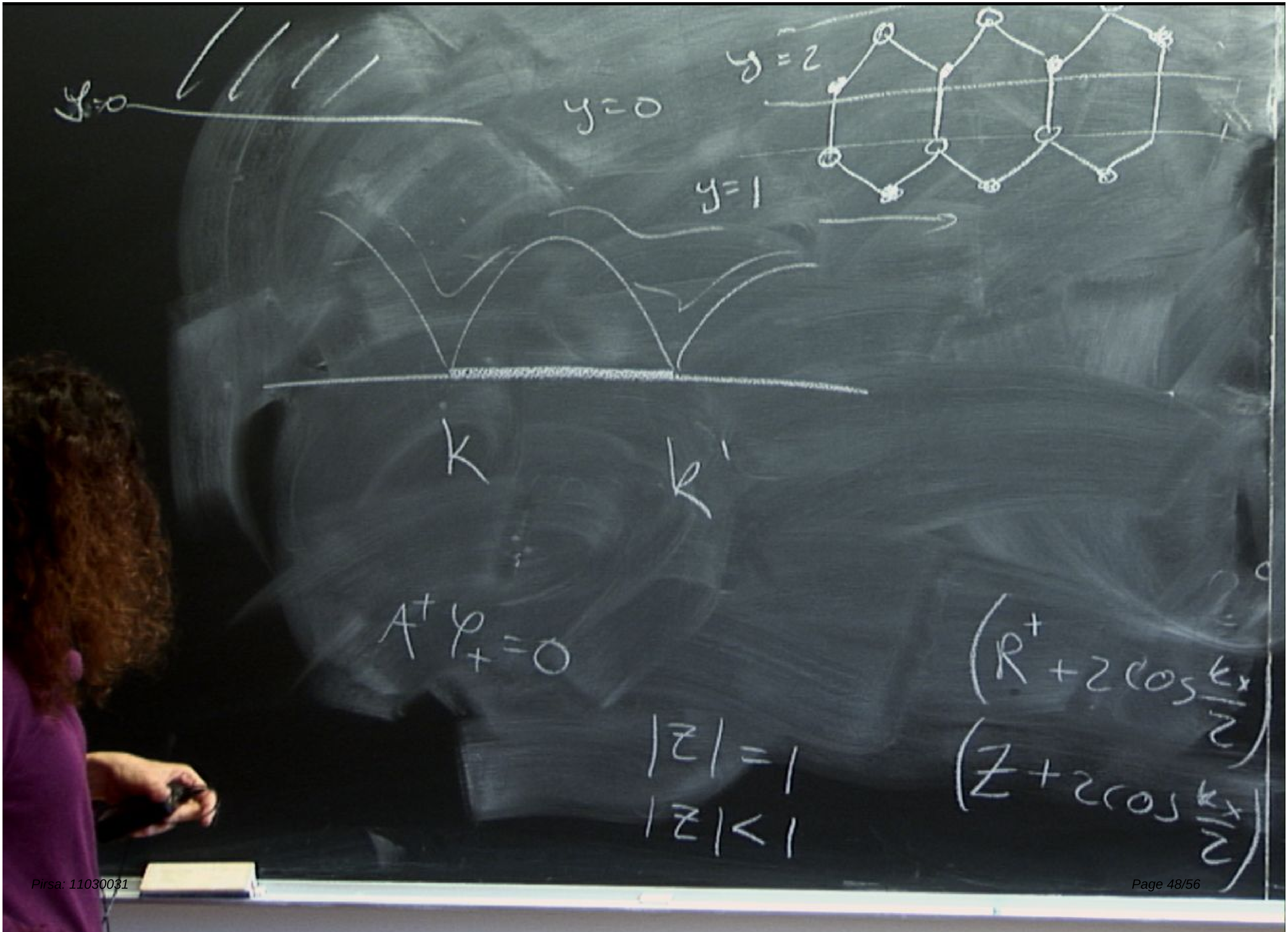
$$\boxed{A\varphi_- = 0}$$

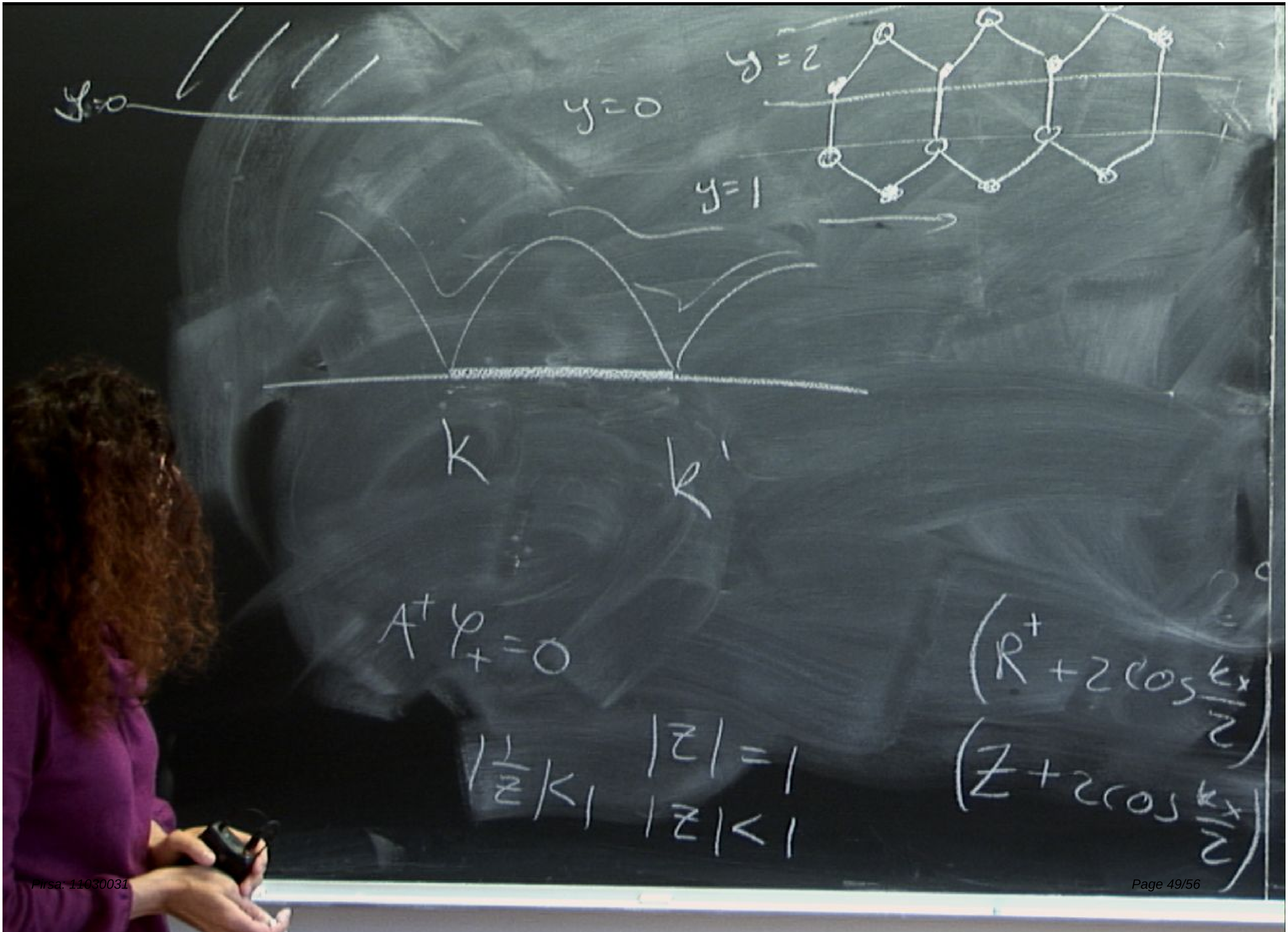
$$\Psi_k = \begin{pmatrix} 0 \\ \varphi_k(y) \end{pmatrix} = \begin{pmatrix} 0 \\ z^n \end{pmatrix}$$

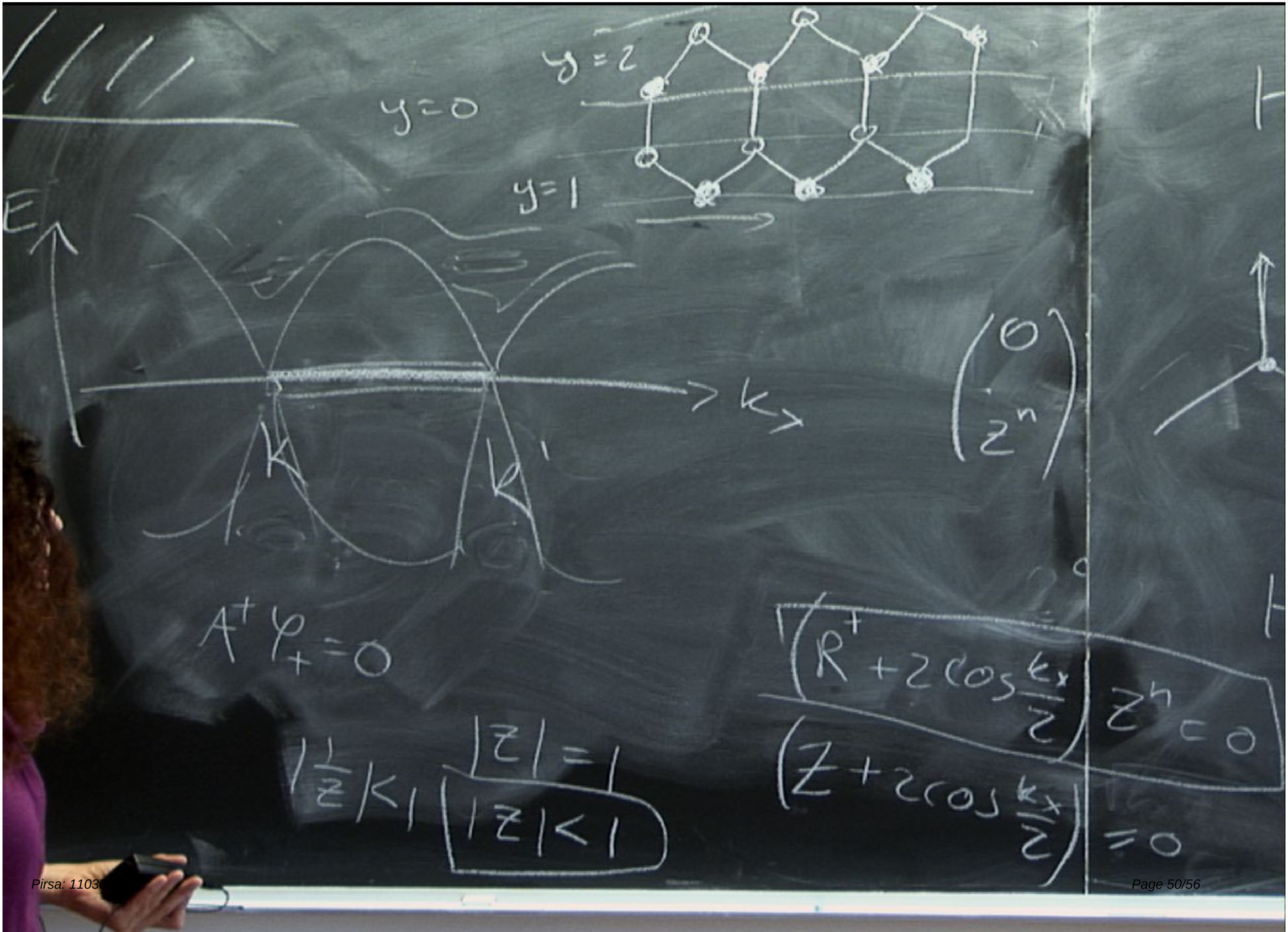

y=0
y=1
A
A+
0
φ_-
= 0
$A\varphi_- = 0$
0
= 0
z^n
$(R^+ + 2\cos\frac{k_x}{2})z^n = 0$
$(z + 2\cos\frac{k_x}{2}) = 0$


H =
i, δ
Ψ(x, y
A
= 0
AΨ_ = 0
Σ t k, δy = R
δy
= R
z^n
H_k = Ψ_k^+ (0 R^+ +
R + 2cos kx/2
(R^+ + 2cos kx/2) z^n = 0
(Z + 2cos kx/2) = 0
Z = −2cos(kx/2)

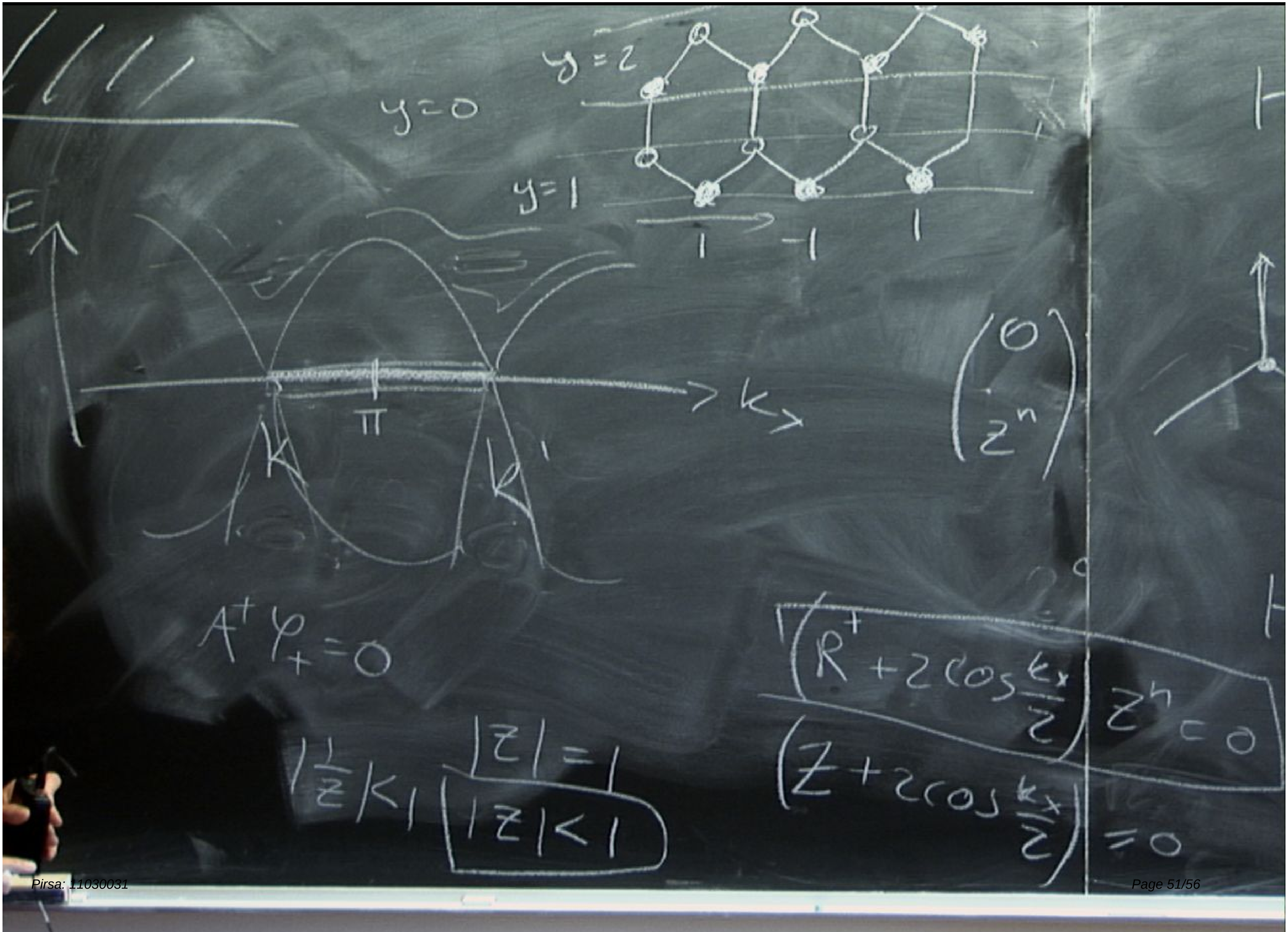

y=0
y=1
AΨ_ = 0
|Z| = 1
|Z| < 1

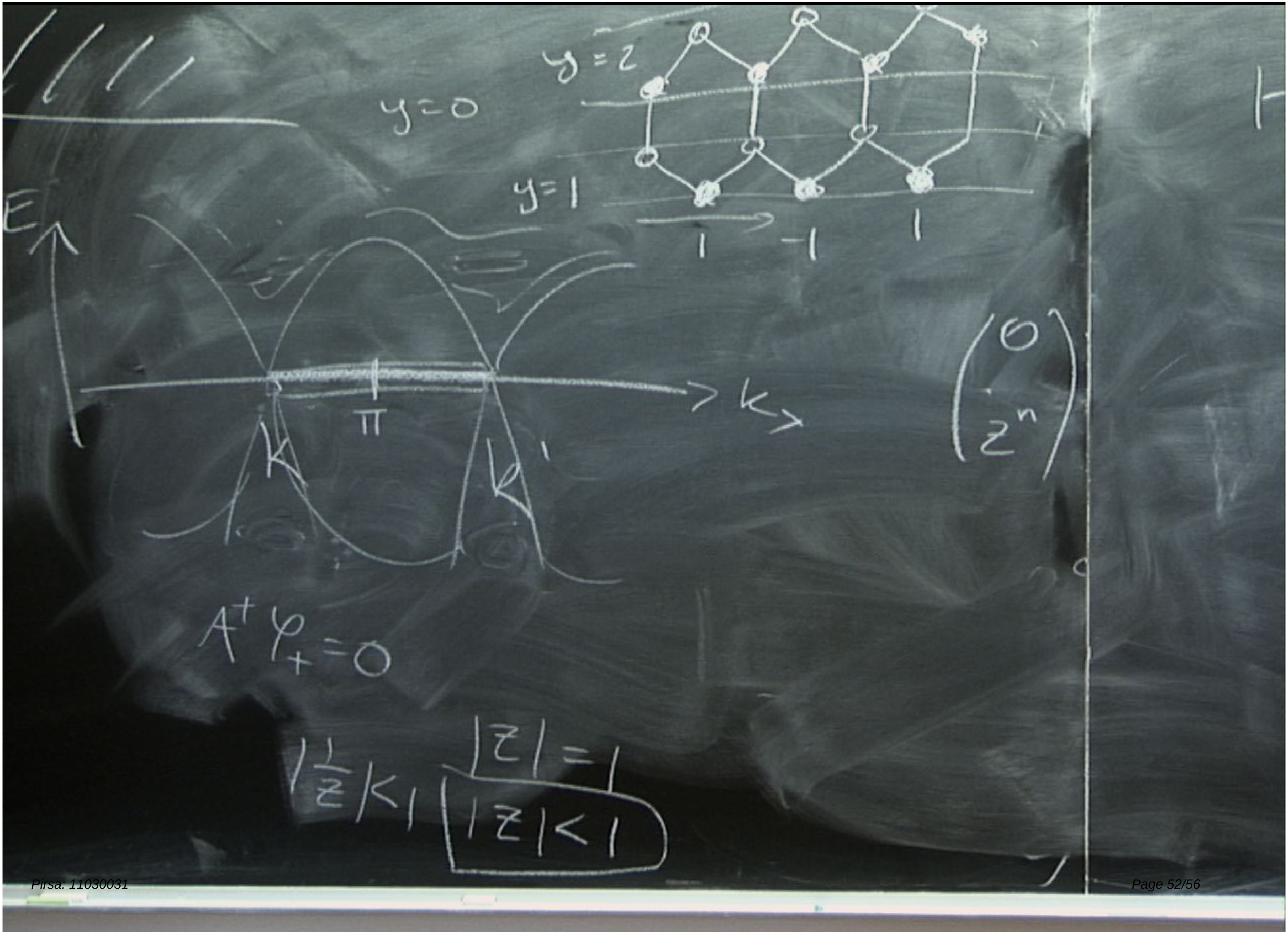

y=0
y=0
y=2
y=1
K
k'
A+ φ+ = 0
|z| = 1
|z| < 1
(R+ + 2 cos kx/2
(z + 2 cos kx/2)


y=0
y=0
y=2
y=1
K
k'
A+ φ+ = 0
|Z| = 1
|1/Z| < 1
|Z| < 1
(R+ + 2cos kx/2
(Z + 2cos kx/2)



y=2
y=0
y=1
E
k_x
K
K'
A+ φ+ = 0
|1/z|<1
|z|=1
|z|<1
(R+ + 2cos kx/2) z^n = 0
(z + 2cos kx/2) = 0

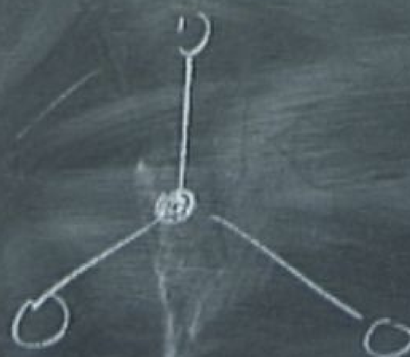

y=2
y=0
y=1
1 -1 1
E
k_x
π
K
K'
(0, Z^n)
A^+ φ_+ = 0
|Z| = 1
|1/Z| < 1
|Z| < 1
(R^+ + 2cos k_x/2) Z^n = 0
(Z + 2cos k_x/2) = 0

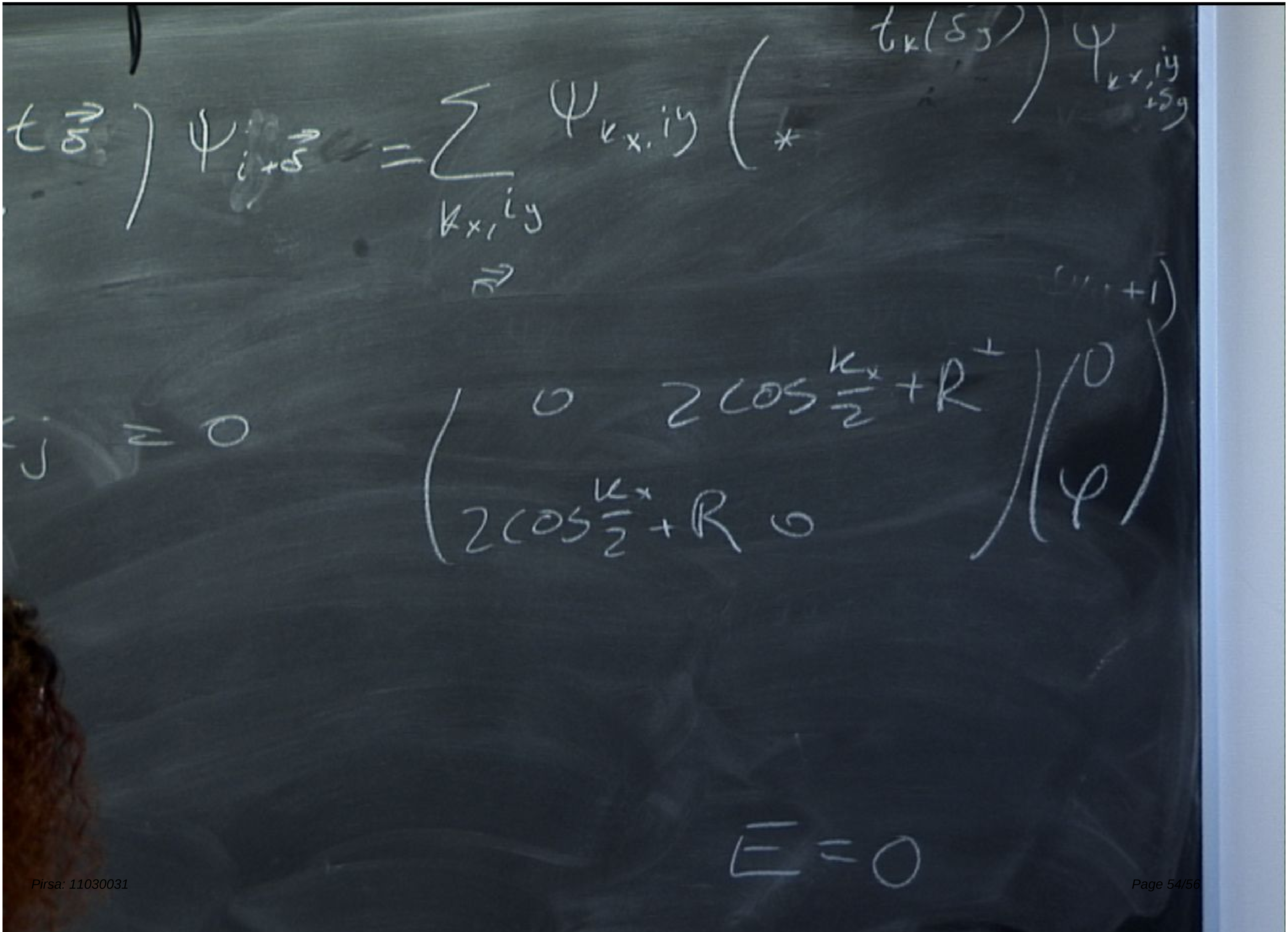

y=2
y=0
y=1
1 -1 1
E
π
K
K'
k_y
A⁺ φ₊ = 0
|z| = 1
|1/z| < 1
|z| < 1

$$H = \sum_{i,\vec{\delta}} \Psi_i \left(t^*_{\vec{\delta}} \right) \Psi_{i+\delta}$$

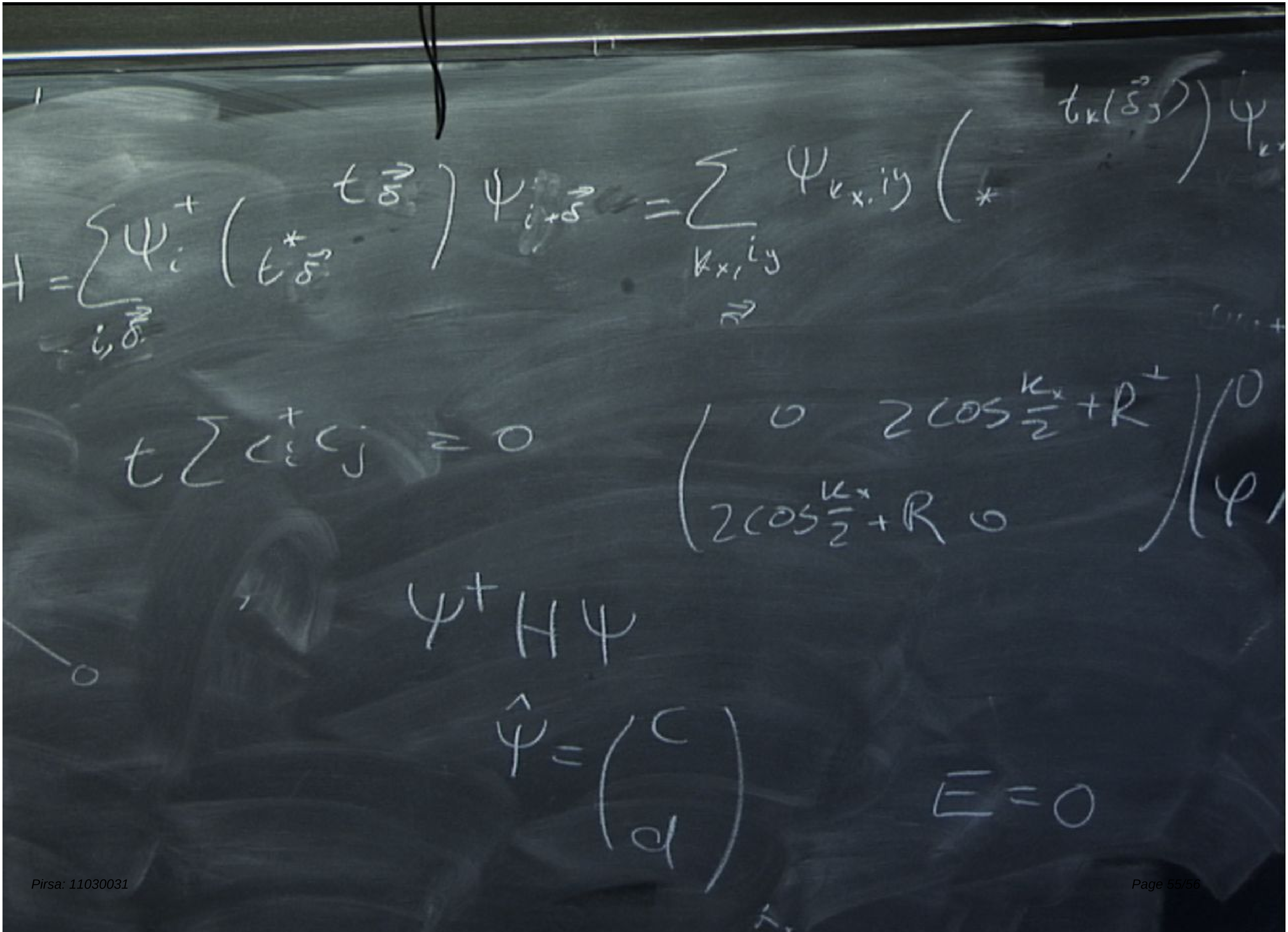
$$t \sum c_i^\dagger c_j = 0$$

$$\begin{pmatrix} 0 \\ z^n \end{pmatrix}$$





= Σ Ψ
= 0
0 2cos kx/2 + R+
2cos kx/2 + R 0
E = 0



$H = \sum_{i,\vec{\delta}} \psi_i^\dagger \begin{pmatrix} & t_{\vec{\delta}} \\ t^*_{\vec{\delta}} & \end{pmatrix} \psi_{i+\vec{\delta}} = \sum_{k_x, i_y, \vec{\delta}} \psi_{k_x, i_y} \begin{pmatrix} & t_k(\vec{\delta}_y) \\ * & \end{pmatrix} \psi_k$
$t \sum c_i^\dagger c_j = 0$
$\begin{pmatrix} 0 & 2\cos\frac{k_x}{2} + R^\dagger \\ 2\cos\frac{k_x}{2} + R & 0 \end{pmatrix}$
$\psi^\dagger H \psi$
$\hat{\psi} = \begin{pmatrix} c \\ d \end{pmatrix}$
$E = 0$

$$H = \sum_{i,\vec{\delta}} \psi_i^{\dagger} \begin{pmatrix} & t_{\vec{\delta}} \\ t^{*}_{\vec{\delta}} & \end{pmatrix} \psi_{i+\vec{\delta}} = \sum_{k_x, i_y, \vec{\delta}} \psi_{k_x, i_y} \begin{pmatrix} & t_k(\vec{\delta}_y) \\ * & \end{pmatrix} \psi_{k_x}$$

$$t \sum c_i^{\dagger} c_j = 0$$

$$\begin{pmatrix} 0 & 2\cos\frac{k_x}{2} + R^{\dagger} \\ 2\cos\frac{k_x}{2} + R & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \varphi \end{pmatrix}$$

$$\psi^{\dagger} H \psi \qquad H\chi = 0$$

$$\hat{\psi} = \begin{pmatrix} c \\ d \end{pmatrix} \qquad E = 0$$