

Title: Explorations in Condensed Matter - Lecture 1

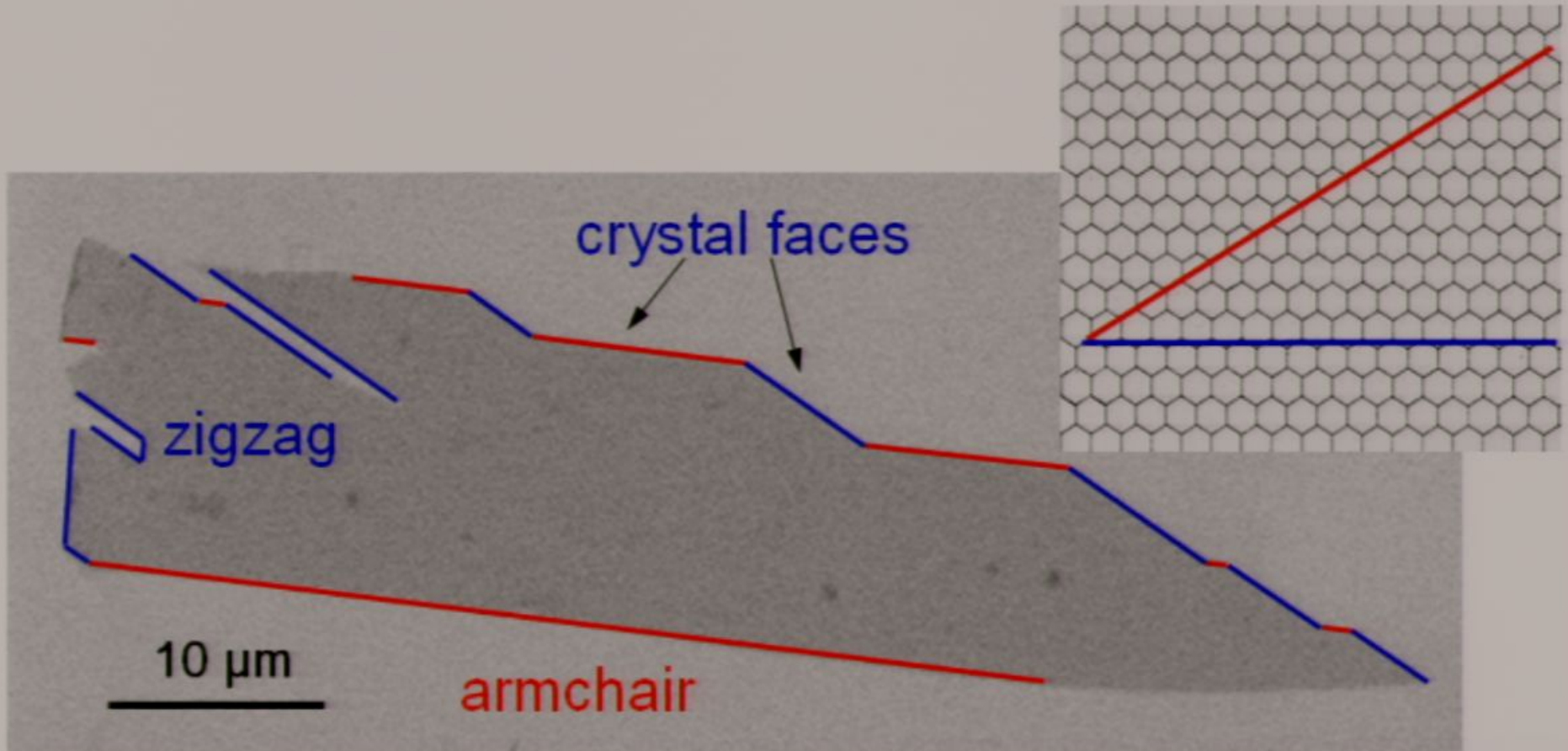
Date: Mar 14, 2011 10:15 AM

URL: <http://pirsa.org/11030030>

Abstract:

First Crystals

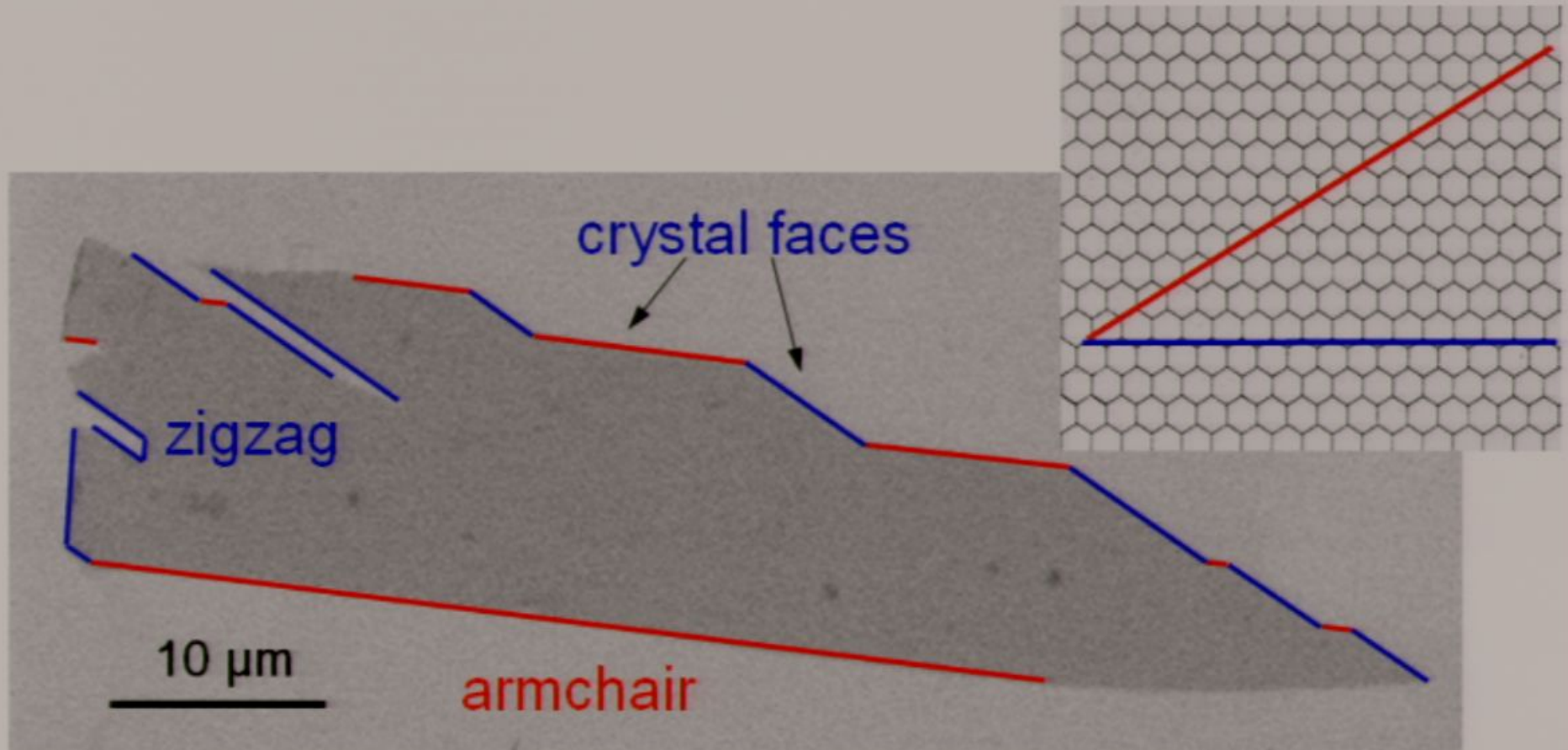
Geim and Novoselov 2004



not just flakes

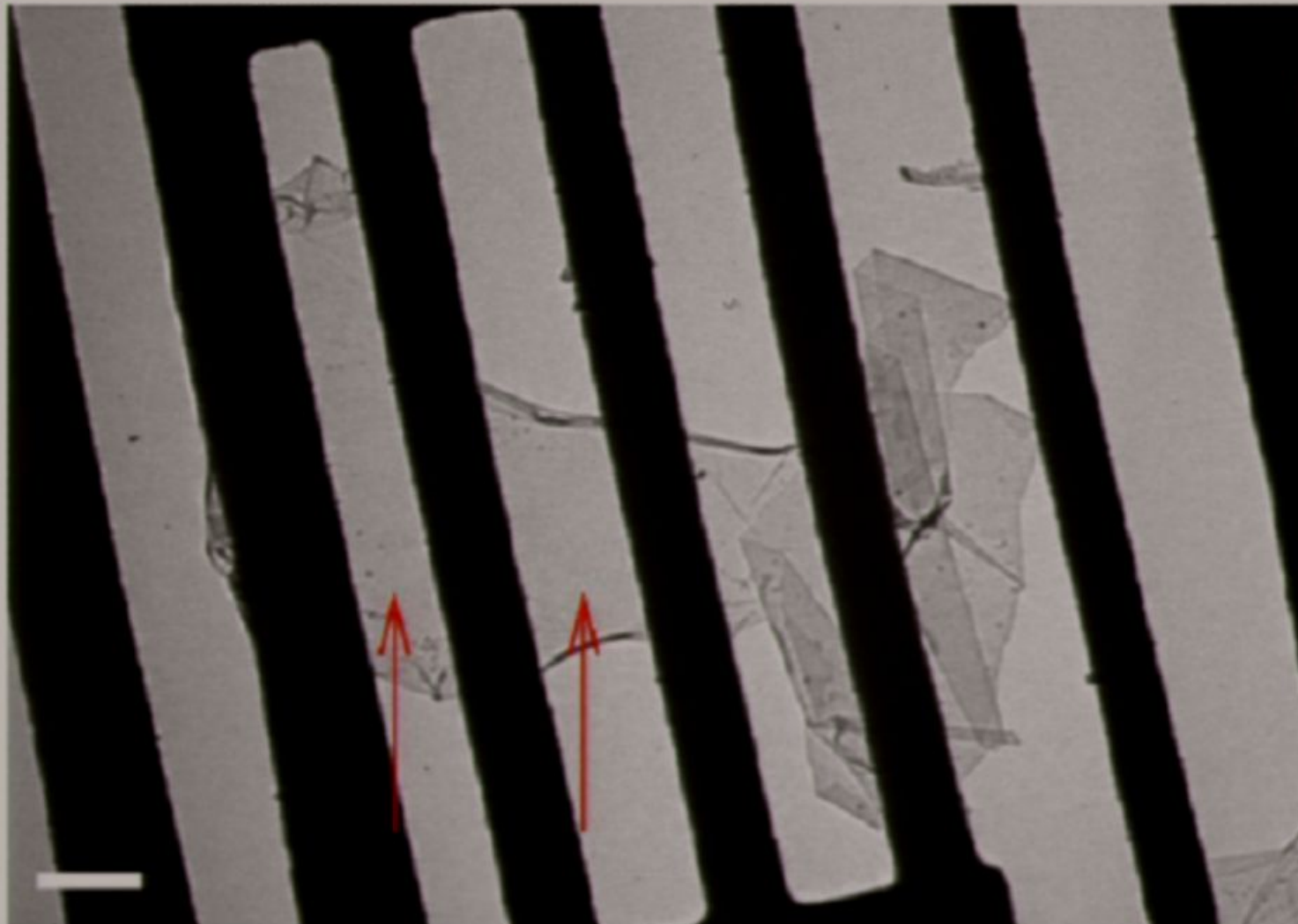
First Crystals

Geim and Novoselov 2004

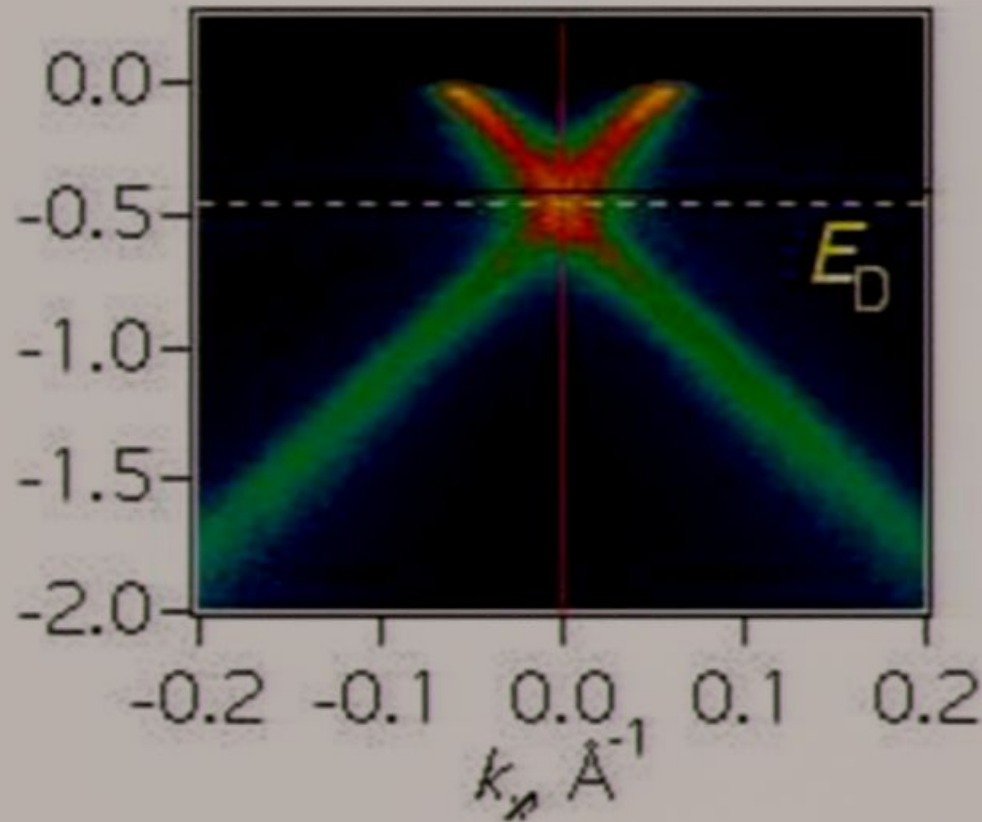


not just flakes

Suspended graphene

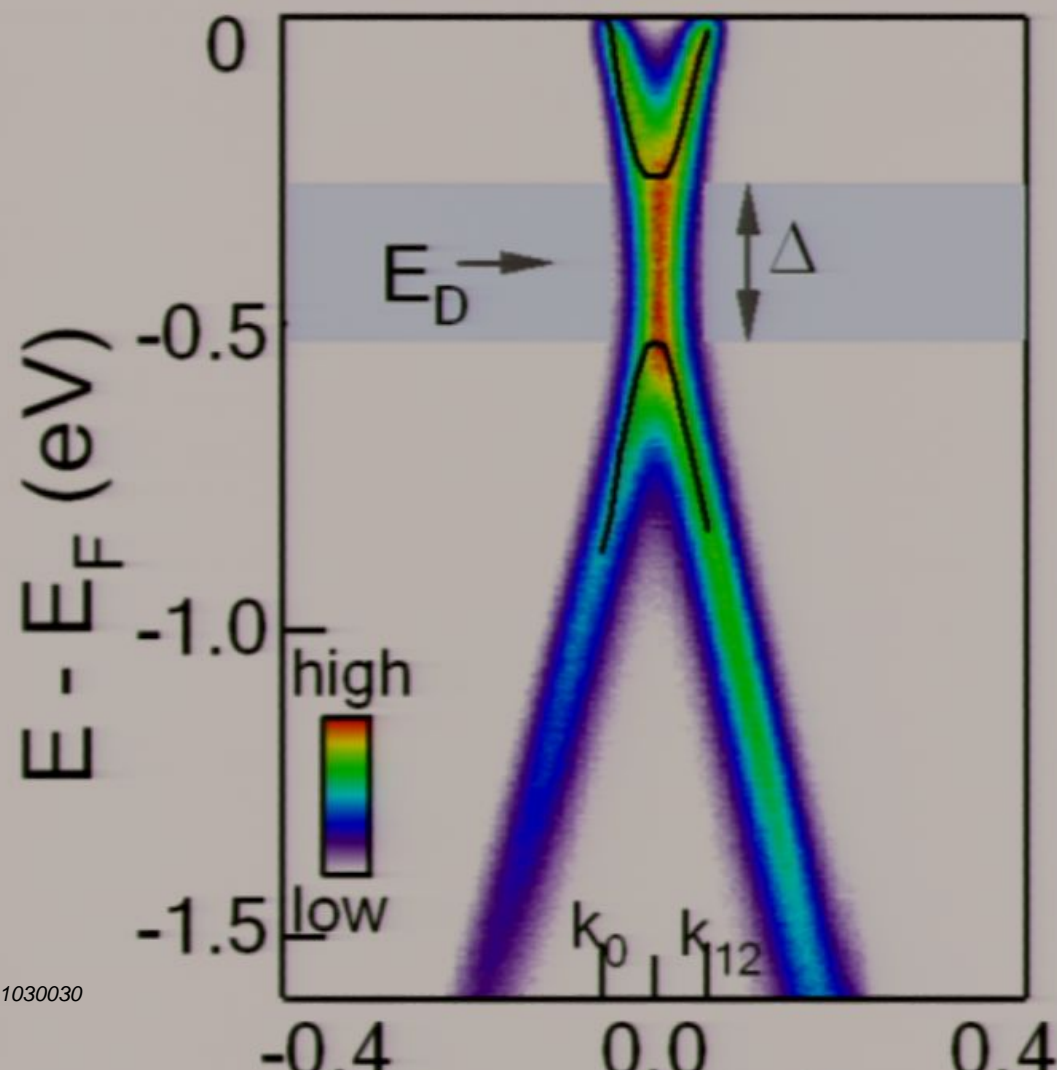


Single Layer – Dirac cone dispersion



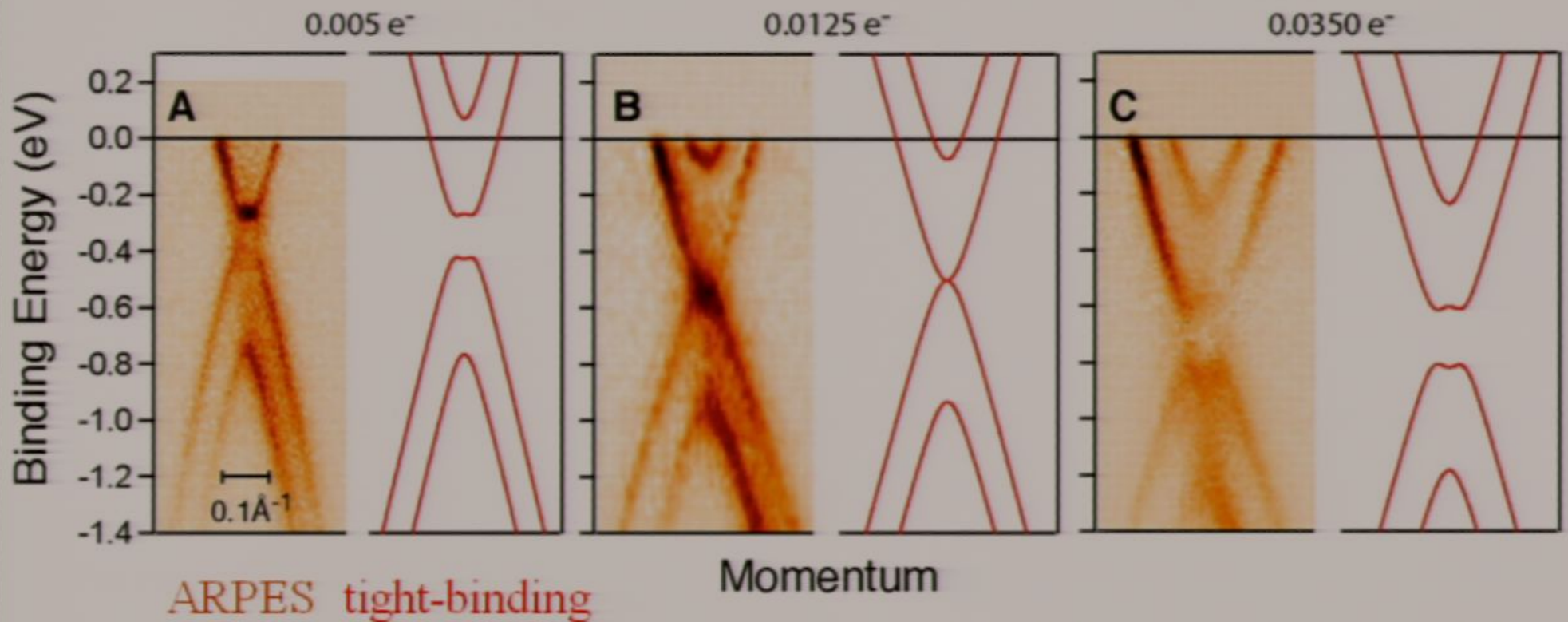
Varchon *et. al*, Phys. Rev. Lett. 2007

Induced gap



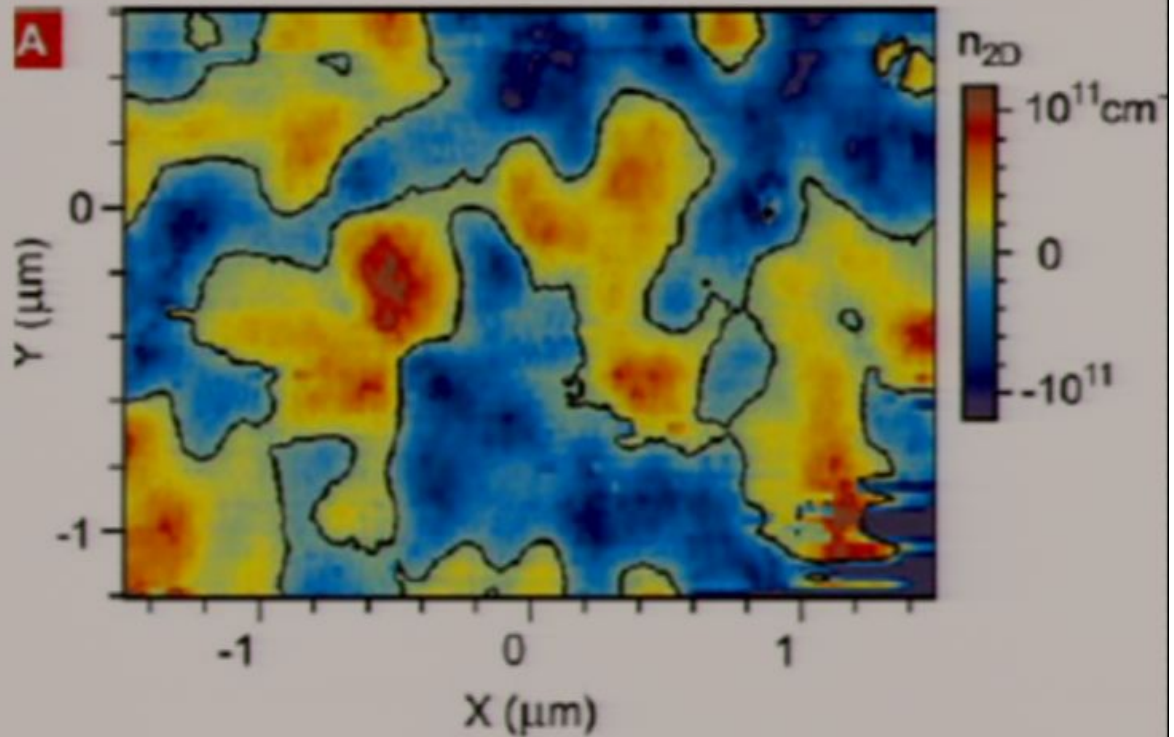
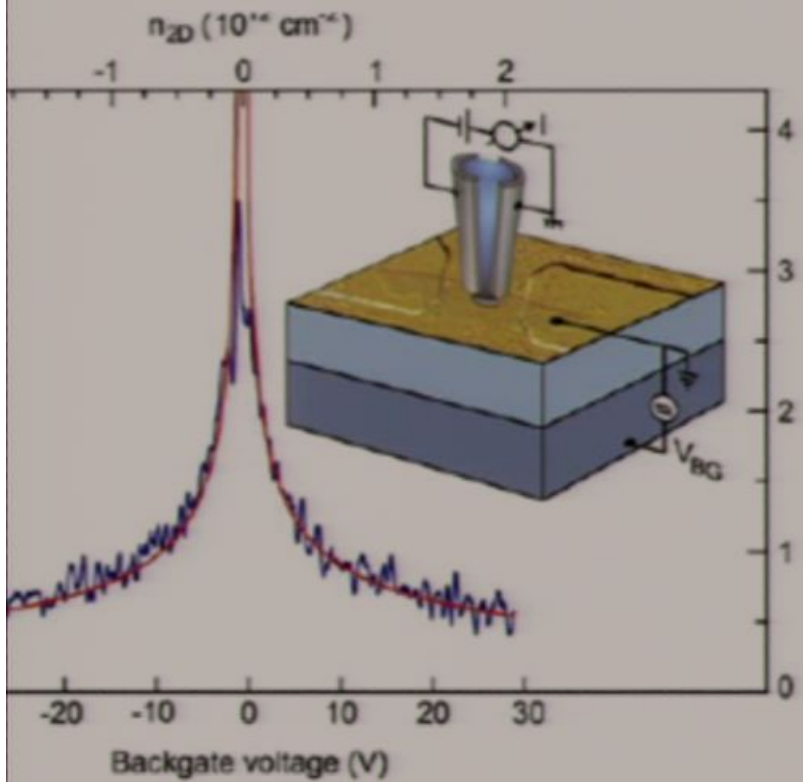
Zhou *et al.*
Nature Materials **6**, 770 (2007)

Graphene Bilayer – ARPES



Ohta et al. Science **313**, 951 (2006)

Electron-hole puddles



Please wait...

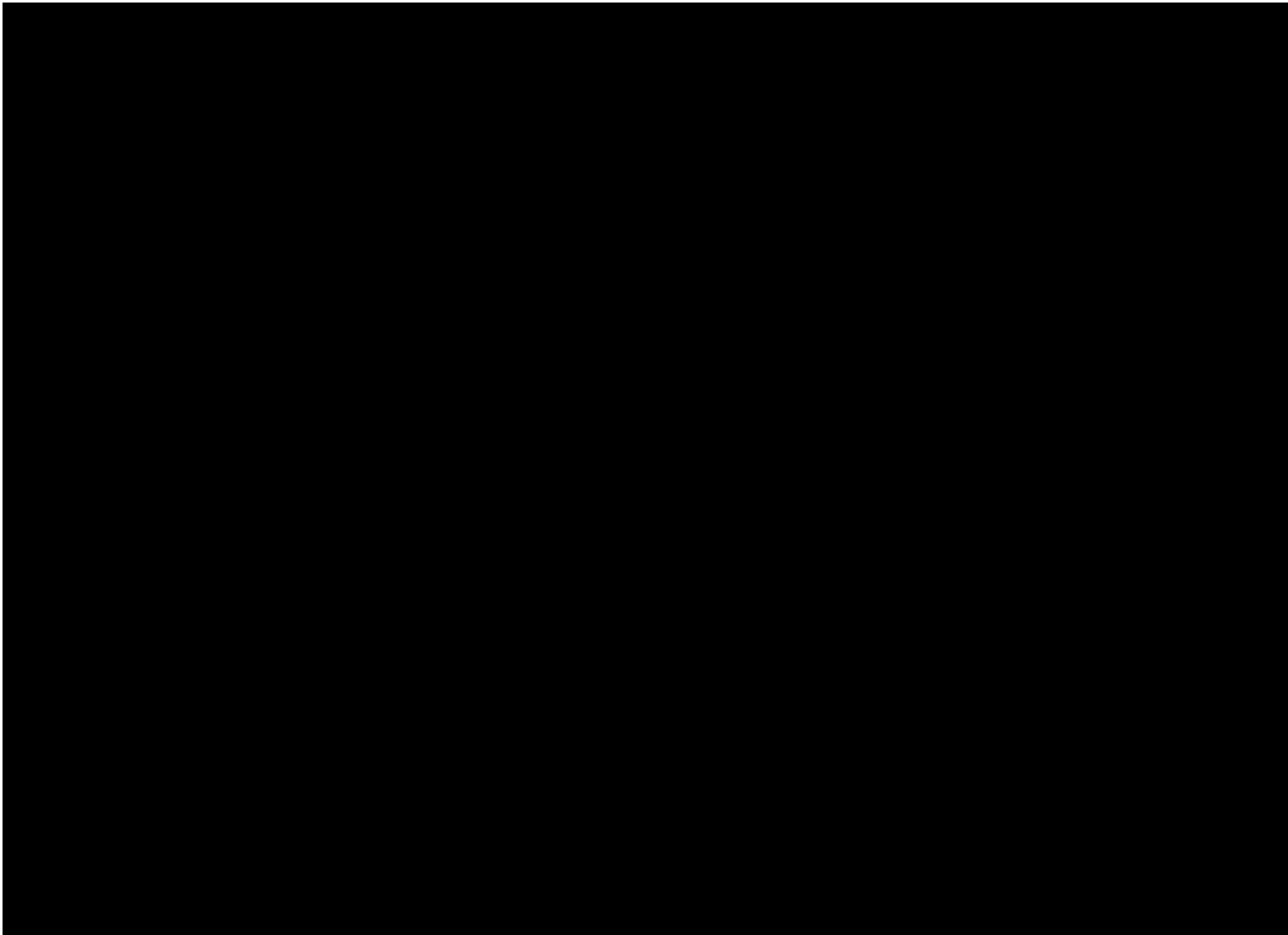


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Preparing to stand by...



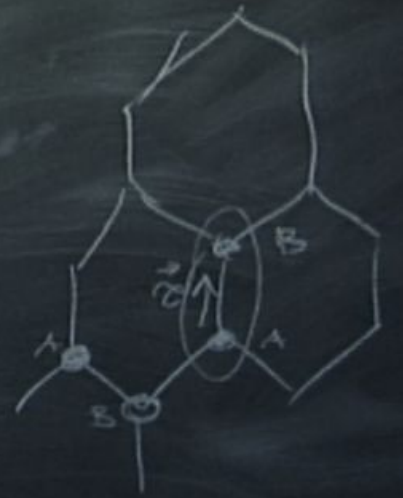
$$H = -t \sum_{\langle ij \rangle} c_i^+ c_j$$

$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j$$

$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j = t \sum_{\langle ij \rangle} c_i^\dagger d_{i+\delta} + \text{h.c.} = -t \sum_{\langle ij \rangle} \psi_i (t_{ij}^* \psi_j)$$

$$s + h.c. = -t \sum_{\langle ij \rangle} \psi_i^\dagger (t_{ij}) \psi_j$$

$$\psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$



$$H = -t \sum_{\langle ij \rangle_b} c_i^\dagger c_j = t \sum_{\langle ij \rangle_b} c_i^\dagger d_{i+\delta} + h.c. = -t \sum_{\langle ij \rangle} \psi_i (c_{ij}^* \cdot t_{ij})$$

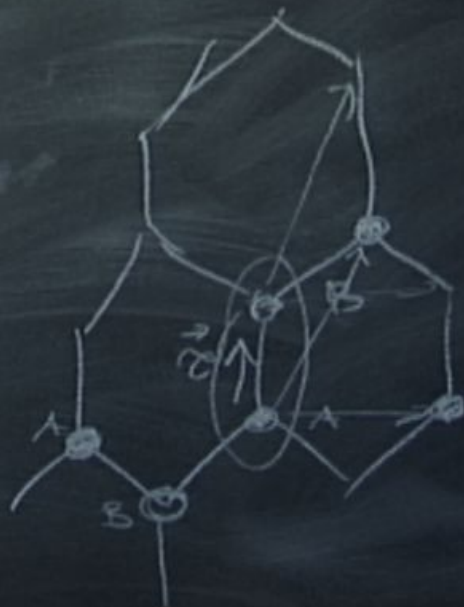
$$[H, \hat{T}] = 0$$

$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j = t \sum_{\langle ij \rangle} c_i^\dagger d_{i+\delta} + h.c. = -t \sum_{\langle ij \rangle} \psi_i (c_{ij}^* t_{ij})$$

$$[H, \hat{T}] = 0$$

$$\begin{pmatrix} t_{ij} \\ t_{ij}^* \end{pmatrix} \psi_j$$

$$\psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$



$$t \sum_{i \rightarrow j} c_i^\dagger d_{i+\sigma} + h.c. = -t \sum_{\langle i,j \rangle} \psi_i (t_{ij}^* \psi_j)$$

$$\psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$

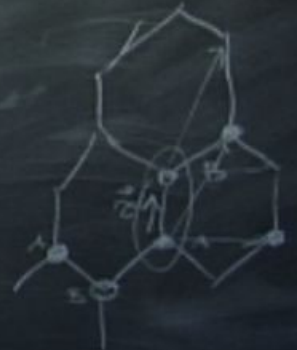
$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j = t \sum_{\langle ij \rangle} c_i^\dagger d_{ij} + h.c. = -t \sum_{\langle ij \rangle} \psi_i^\dagger (d_{ij}) \psi_j$$

$$\psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$

$$[H, \hat{T}] = 0$$

$$\psi_i^{(k)} = e^{ik \cdot r_i} u(\vec{r})$$

$$\Psi(\vec{r}) = \sum_k e^{-ik \cdot \vec{r}} \psi_k$$



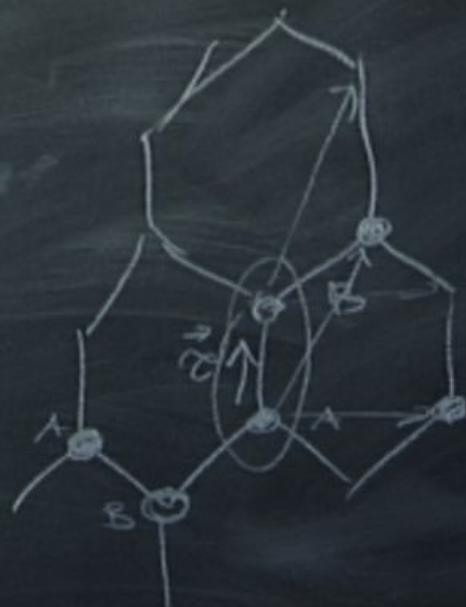
$$\begin{pmatrix} t_{ij} \\ \vdots \\ t_{ij}^* \end{pmatrix} \psi_j$$

$$\psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$

$$\psi_i^{(k)} = e^{i\mathbf{k} \cdot \mathbf{r}_i} u(\mathbf{r})$$

$$\psi(\mathbf{r}) = \sum_{\mathbf{k}} e^{-i\mathbf{k} \cdot \mathbf{r}} \psi_{\mathbf{k}}$$

$$\psi^\dagger(\mathbf{r}_i) \psi(\mathbf{r}_j) \approx \sum_{\mathbf{p}, \mathbf{k}} e^{i(\mathbf{p}-\mathbf{k}) \cdot \mathbf{r}_i - i\mathbf{k} \cdot \mathbf{r}_j}$$

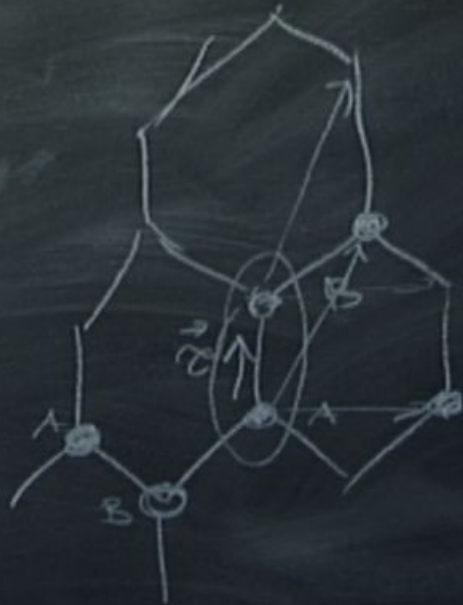


$$t_{ij} \psi_j$$

$$\psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$

$$\psi_i^{(k)} = e^{i\mathbf{k} \cdot \mathbf{r}_i} u(\mathbf{r})$$

$$\psi(\mathbf{r}) = \sum_k e^{-i\mathbf{k} \cdot \mathbf{r}} \psi_k$$



$$\psi^\dagger(\mathbf{r}_i) \psi(\mathbf{r}_j) \approx \sum_{p, k} e^{i(p-k) \cdot \mathbf{r}_i - i\mathbf{k} \cdot \mathbf{d}_{ij}} \psi_p^\dagger \psi_k$$

$\delta(\mathbf{k}-\mathbf{p})$

$$t_{ij} \psi_j$$

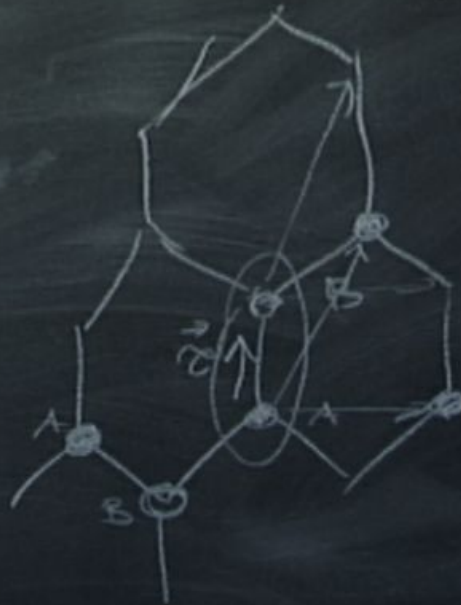
$$\psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$

$$\psi_i^{(k)} = e^{i\mathbf{k} \cdot \mathbf{r}_i} u(\mathbf{r})$$

$$\psi(\mathbf{r}) = \sum_{\mathbf{k}} e^{-i\mathbf{k} \cdot \mathbf{r}} \psi_{\mathbf{k}}$$

$$\psi^+(\mathbf{r}_i) \psi(\mathbf{r}_j) \approx \sum_{\mathbf{p}, \mathbf{k}} e^{i(\mathbf{p}-\mathbf{k}) \cdot \mathbf{r}_i} e^{-i\mathbf{k} \cdot \mathbf{r}_j} \psi_{\mathbf{p}}^+ \psi_{\mathbf{k}}$$

$\delta(\mathbf{k}-\mathbf{p})$



$$t_{ij} \psi_j =$$

$$\psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$

$$\psi_i^{(k)} = e^{i\mathbf{k} \cdot \mathbf{r}_i} u(\mathbf{r}_i)$$

$$\psi(\mathbf{r}) = \sum_{\mathbf{k}} e^{-i\mathbf{k} \cdot \mathbf{r}} \psi_{\mathbf{k}}$$



$$\psi^\dagger(\mathbf{r}_i) \psi(\mathbf{r}_j) = \sum_{\mathbf{p}, \mathbf{k}} e^{i(\mathbf{p}-\mathbf{k}) \cdot \mathbf{r}_i} e^{-i\mathbf{k} \cdot \mathbf{r}_j} = \psi_{\mathbf{p}}^\dagger \psi_{\mathbf{k}}$$

$\delta(\mathbf{k}-\mathbf{p})$

$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j = t \sum_{i, \delta} c_i^\dagger d_{i+\delta} + h.c. = -t \sum_{\langle ij \rangle} \psi_i^\dagger (t_{ij}^* \psi_j)$$

$$[H, \hat{T}] = 0 = -t \sum_k \psi_k^\dagger (\gamma_k^* \psi_k)$$

$$\gamma_k = \sum_{\alpha \rightarrow \beta} e^{i\vec{k} \cdot \vec{\delta}} = e^{i\frac{k_x}{\sqrt{3}}} + e^{-i\frac{k_x}{\sqrt{3}}} = 2 \cos\left(\frac{k_x}{\sqrt{3}}\right)$$

$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j = t \sum_{\langle i, \delta \rangle} c_i^\dagger d_{i+\delta} + \text{h.c.} = -t \sum_{\langle ij \rangle} \psi_i^\dagger (t_{ij}^* \psi_j)$$

$$[H, \hat{T}] = 0 = -t \sum_k \psi_k^\dagger (\gamma_k^* \psi_k)$$

$$\gamma_k = \sum_{a \rightarrow b} e^{i\vec{k} \cdot \vec{\delta}} = e^{i\frac{k_x}{\sqrt{3}}} + e^{-i\frac{k_x}{\sqrt{3}}} = 2 \cos\left(\frac{k_x}{\sqrt{3}}\right)$$

$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j = t \sum_{\langle i, \delta \rangle} c_i^\dagger d_{i+\delta} + \text{h.c.} = -t \sum_{\langle ij \rangle} \psi_i^\dagger (t_{ij}^* \psi_j)$$

$$[H, \hat{T}] = 0 = -t \sum_{\mathbf{k}} \psi_{\mathbf{k}}^\dagger (\gamma_{\mathbf{k}}^* \psi_{\mathbf{k}})$$

$$\gamma_{\mathbf{k}} = \sum_{\alpha \rightarrow \beta} e^{i\mathbf{k} \cdot \delta} = e^{i\frac{k_x}{\sqrt{3}}} + e^{-i\frac{k_y}{2\sqrt{3}}} 2\cos\left(\frac{k_x}{2}\right)$$

$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j = t \sum_{\substack{i,j \\ \delta}} c_i^\dagger d_{i+\delta} + h.c. = -t \sum_{\langle ij \rangle} \psi_i^\dagger (t_{ij}^* \psi_j)$$

$$[H, \hat{T}] = 0 = -t \sum_k \psi_k^\dagger (\delta_k^* \psi_k)$$

$$\hat{H}^2 = \begin{pmatrix} |\delta_k|^2 & \\ & |\delta_k|^2 \end{pmatrix}$$

$$\delta_k = \sum_{\alpha \rightarrow \beta} e^{i\vec{k} \cdot \vec{\delta}} = e^{i\frac{k_x}{\sqrt{3}}} + e^{-i\frac{k_y}{2\sqrt{3}}} = 2 \cos\left(\frac{k_x}{2}\right)$$

$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_{j\sigma} = t \sum_{\substack{i \rightarrow j \\ i, j}} c_i^\dagger d_{i\sigma} + h.c. = -t \sum_{\langle ij \rangle} \psi_i^\dagger (t_{ij}^* \psi_j)$$

$$[H, \hat{T}] = 0 = -t \sum_k \psi_k^\dagger (\gamma_k^* \psi_k)$$

$$\hat{H}^2 = \begin{pmatrix} |\gamma_k|^2 \\ |\gamma_k|^2 \end{pmatrix}$$

$$E_k = \pm |\gamma_k|$$

$$\gamma_k = \sum_{\sigma \rightarrow \sigma'} e^{i\vec{k} \cdot \vec{\sigma}} = e^{i\frac{\sqrt{3}k_x}{3}} + e^{-i\frac{\sqrt{3}k_y}{2\sqrt{3}}} = 2 \cos\left(\frac{k_x}{2}\right)$$

$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j = t \sum_{\vec{i}, \vec{j}} c_i^\dagger d_{i+\vec{s}} + h.c. = -t \sum_{\langle ij \rangle} \psi_i^\dagger (t_{ij}^* \psi_j)$$

$$[H, \hat{T}] = 0 = -t \sum_k \psi_k^\dagger (\gamma_k^* \psi_k)$$

$$\gamma_k = \sum_{\vec{a} \rightarrow \vec{b}} e^{i\vec{k} \cdot \vec{a}} = e^{i\frac{k_x}{\sqrt{3}}} + e^{-i\frac{k_x}{2\sqrt{3}}} = 2 \cos\left(\frac{k_x}{2}\right)$$

$$H^2 = \begin{pmatrix} |\gamma_k|^2 & \\ & |\gamma_k|^2 \end{pmatrix}$$

$$E_k = \pm |\gamma_k|$$

$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_{j\sigma} = t \sum_{\substack{\langle ij \rangle \\ i \rightarrow j}} c_i^\dagger d_{i+\delta} + h.c. = -t \sum_{\langle ij \rangle} \psi_i^\dagger (t_{ij}^* \psi_j)$$

$$[H, \hat{T}] = 0 = -t \sum_k \psi_k^\dagger (\gamma_k^* \psi_k)$$

$$\hat{H}^2 = \begin{pmatrix} |\gamma_k|^2 \\ |\gamma_k|^2 \end{pmatrix}$$

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$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_{j\sigma} = t \sum_{\substack{i \rightarrow j \\ i, j}} c_i^\dagger d_{i\sigma} + h.c. = -t \sum_{\langle ij \rangle} \psi_i^\dagger (t_{ij}^* \psi_j)$$

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$$\hat{H}^2 = \begin{pmatrix} |\gamma_k|^2 \\ |\gamma_k|^2 \end{pmatrix}$$

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$$\gamma_k = \sum_{\alpha \rightarrow \beta} e^{i\vec{k} \cdot \vec{\delta}} = e^{i\frac{\vec{k} \cdot \vec{a}}{3}} + e^{-i\frac{\vec{k} \cdot \vec{a}}{3}} = 2 \cos\left(\frac{k_x a}{3}\right)$$

$$\Psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$



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$$\Psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$

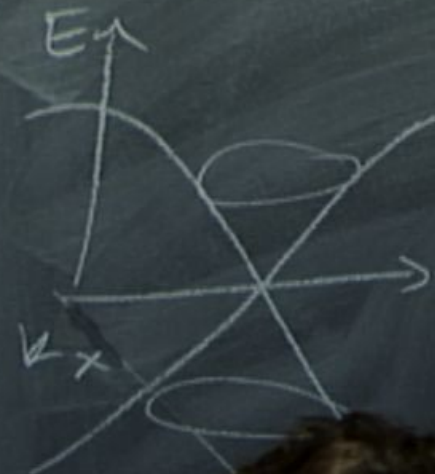
$$\left(\frac{2\pi}{3}, \frac{2\pi}{\sqrt{3}} \right)$$



$$\left(\frac{4\pi}{3}, 0 \right)$$

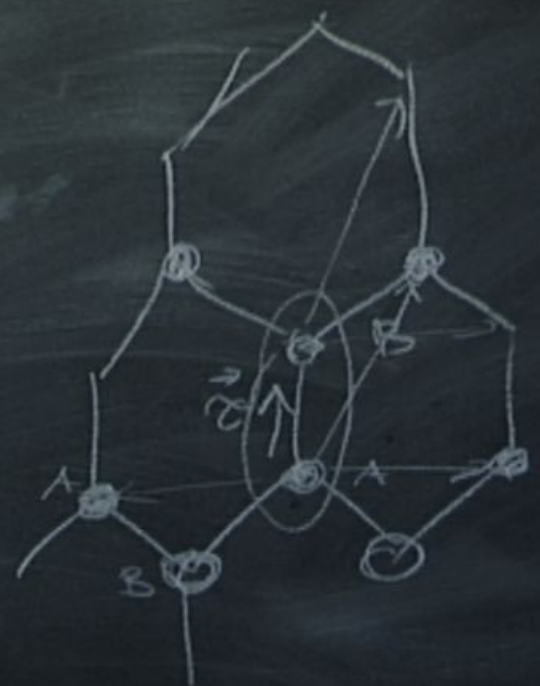
$$t_{ij} \psi_j =$$

$$\psi_i = \begin{pmatrix} C_i \\ d_i \end{pmatrix}$$



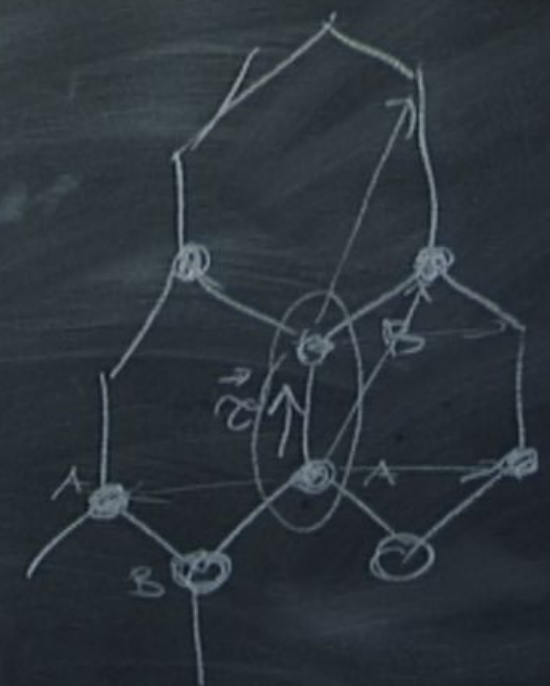
$$\left(\frac{2\pi}{3}, \frac{2\pi}{\sqrt{3}} \right)$$

$$\left(\frac{4\pi}{3}, 0 \right)$$



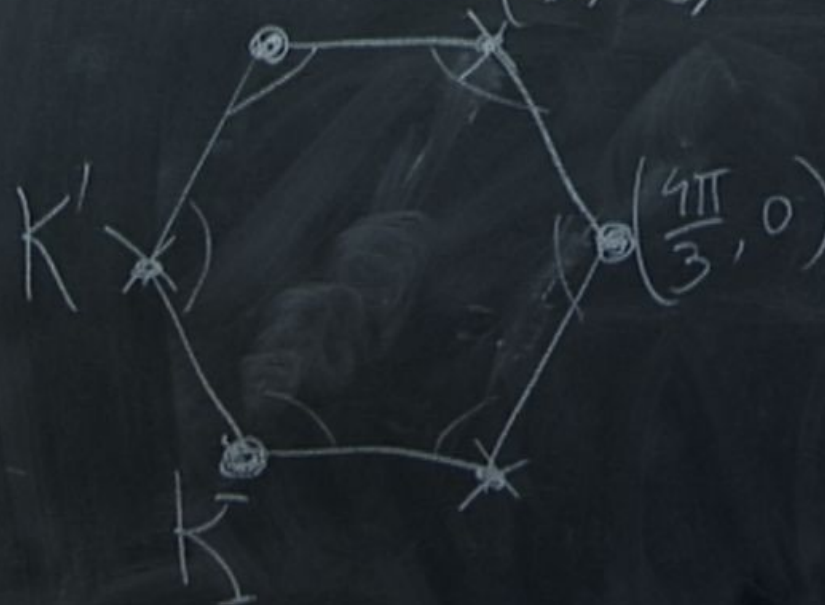
$$t_{ij} \psi_j = \psi_i$$

$$\psi_i = \begin{pmatrix} C_i \\ D_i \end{pmatrix}$$



$$\left(\frac{2\pi}{3}, \frac{2\pi}{\sqrt{3}} \right)$$

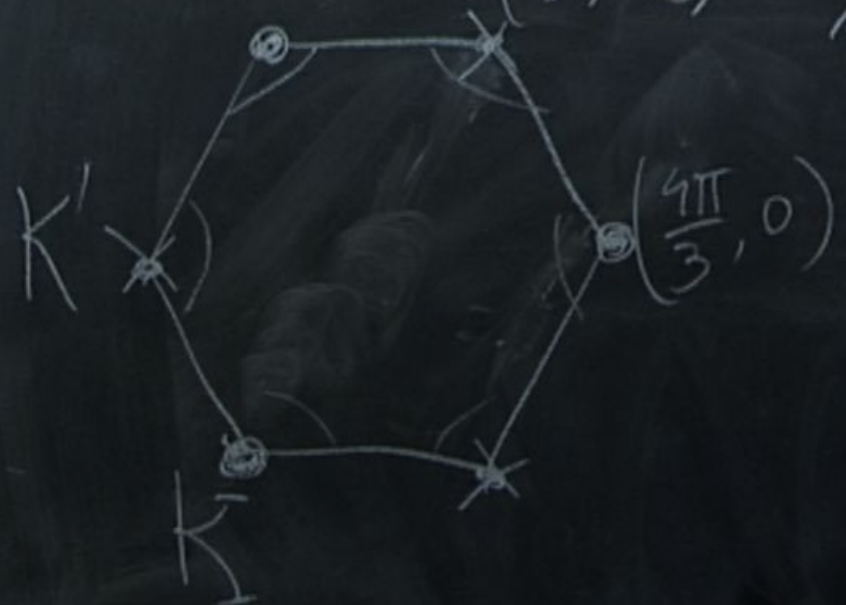
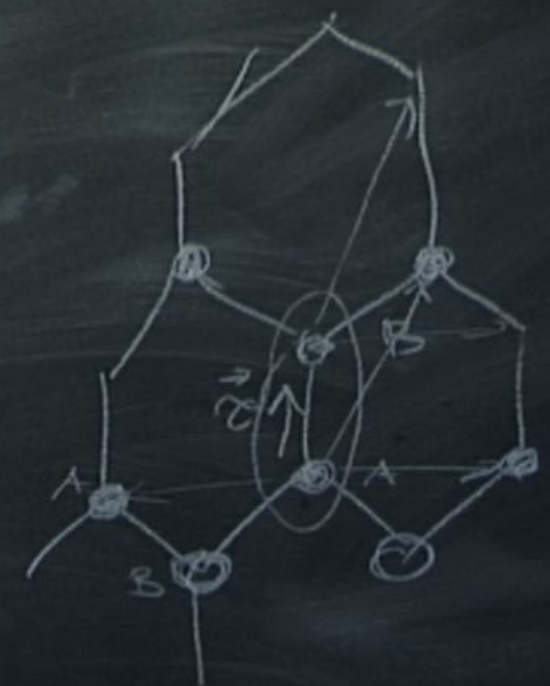
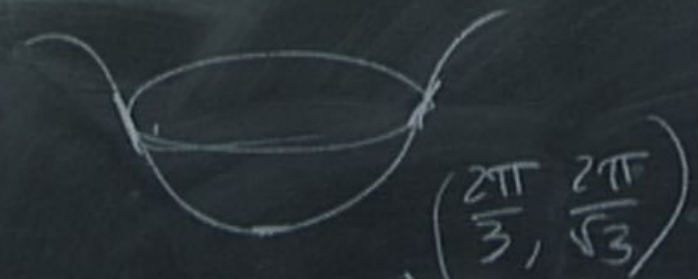
$$\left(\frac{4\pi}{3}, 0 \right)$$



$$\cos\left(\frac{k_x}{2}\right)$$

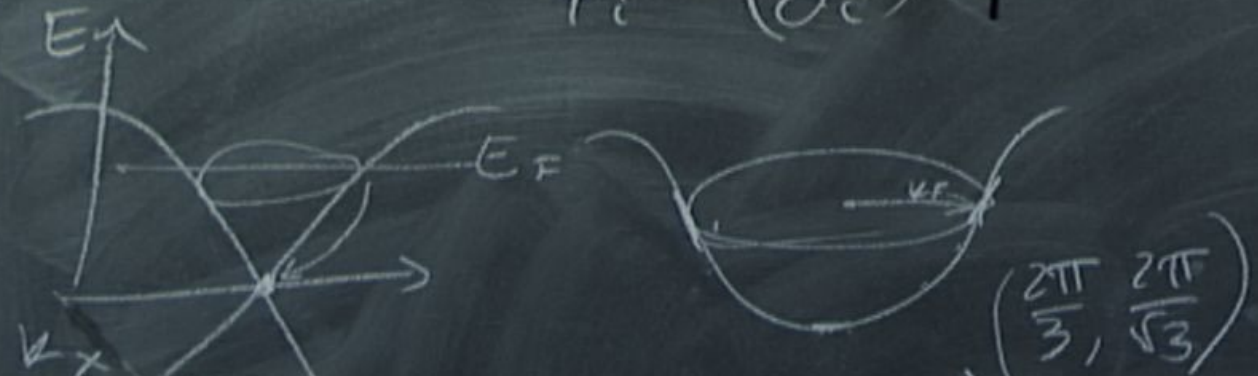
$$L_{ij}^* \Psi_j = E_i \Psi_i$$

$$\Psi_i = \begin{pmatrix} C_i \\ D_i \end{pmatrix}$$

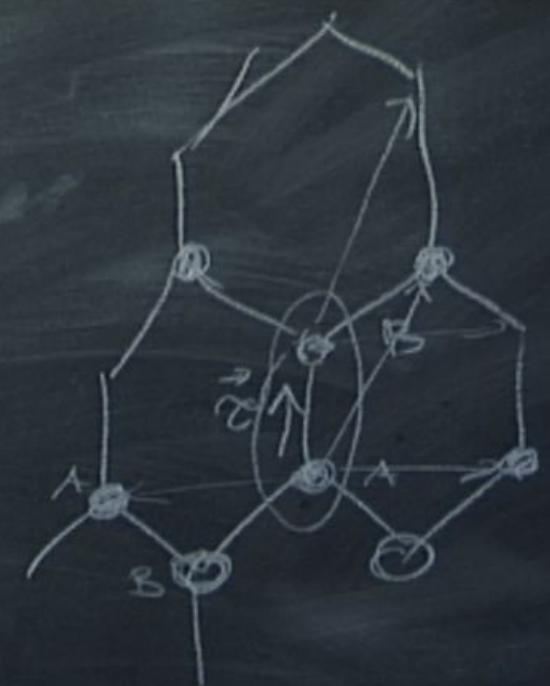


$t_{ij} \psi_j =$

$\psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$



$\left(\frac{2\pi}{3}, \frac{2\pi}{\sqrt{3}} \right)$

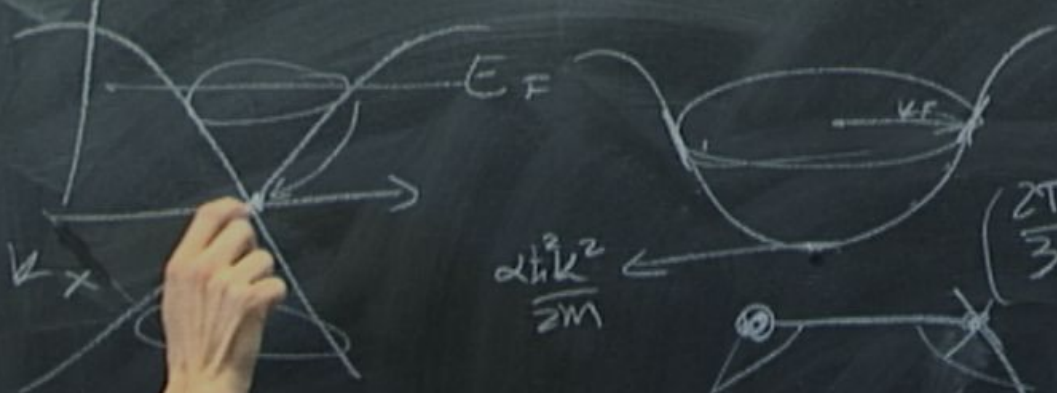


$\cos\left(\frac{k_x}{2}\right)$

$$t_{ij} \psi_j =$$

$$\psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$

$E \uparrow$



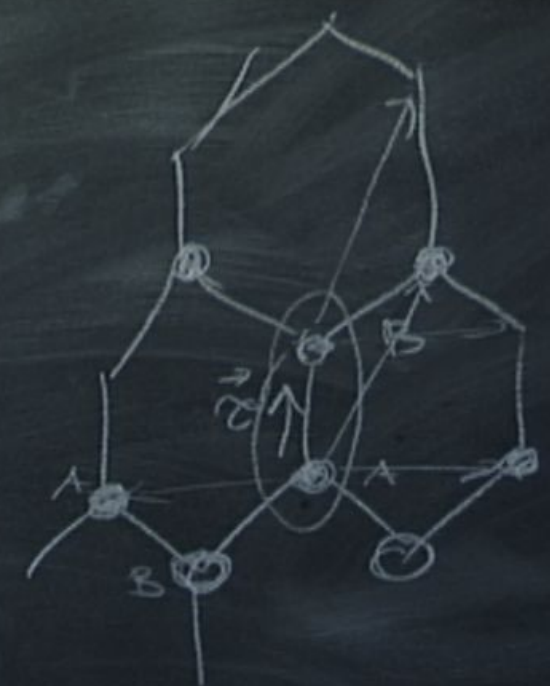
$$\frac{\hbar^2 k^2}{2m}$$

$$\left(\frac{2\pi}{3}, \frac{2\pi}{\sqrt{3}} \right)$$

$$\left(\frac{4\pi}{3}, 0 \right)$$

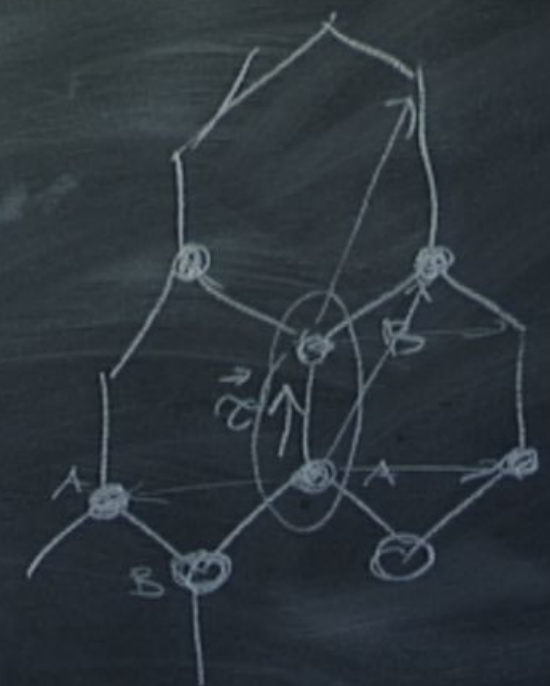
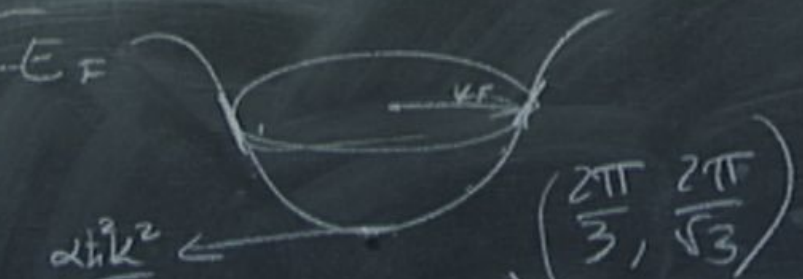
K'

K''



$$t_{ij} \psi_j =$$

$$\psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$

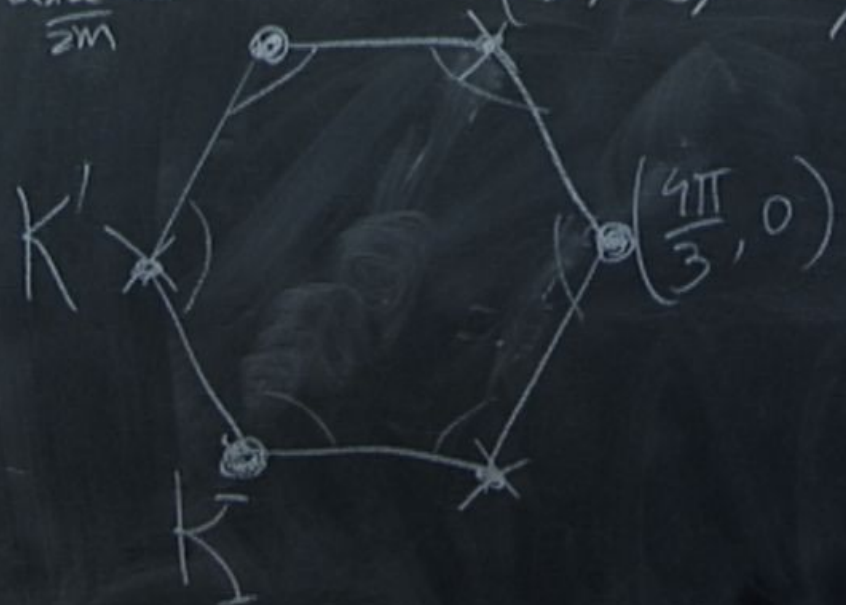


$$\frac{\hbar^2 k^2}{2m}$$

$$\left(\frac{2\pi}{3}, \frac{2\pi}{\sqrt{3}} \right)$$

$$\left(\frac{4\pi}{3}, 0 \right)$$

$$\cos\left(\frac{k_x}{2}\right)$$



$$H = -t \sum_{\langle ij \rangle b} c_i^\dagger c_{j\sigma} = t \sum_{i,j} c_i^\dagger d_{i\sigma} + h.c. = -t \sum_{\langle ij \rangle} \psi_i^\dagger (t_{ij}^\sigma) \psi_j =$$

$$[H, \hat{T}] = 0$$

$$= -t \sum_k \psi_k^\dagger (\delta_k^\sigma) \psi_k$$

$$\hat{H}^2 = (|\delta_k|^2)$$

$$= e^{i\mathbf{k} \cdot \delta} = e^{i\frac{k_x}{3}} + e^{-i\frac{k_x}{3}} = 2 \cos\left(\frac{k_x}{3}\right)$$

$$E_k = \frac{v_F^2}{3} \left(1 + 2 \cos\left(\frac{2\pi}{3}\right) \right) = 0 \Rightarrow \delta_q = (q_x + iq_y) \mathbf{V}$$



$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j = t \sum_{\langle ij \rangle} c_i^\dagger d_{i+\delta} + h.c. = -t \sum_{\langle ij \rangle} \psi_i^\dagger (t_{ij}^* \psi_j + t_{ij} \psi_j^\dagger)$$

$$[H, \hat{T}] = 0$$

$$= -t \sum_{\mathbf{k}} \psi_{\mathbf{k}}^\dagger (\delta_{\mathbf{k}}^* \psi_{\mathbf{k}} + \delta_{\mathbf{k}} \psi_{\mathbf{k}}^\dagger) \Rightarrow -t \sum_{\mathbf{k}} \psi_{\mathbf{k}}^\dagger (\mathbf{k} \cdot \mathbf{v}) \psi_{\mathbf{k}}$$

$$\hat{H}^2 = \begin{pmatrix} |\gamma_{\mathbf{k}}|^2 \\ |\gamma_{\mathbf{k}}|^2 \end{pmatrix}$$

$$E_{\mathbf{k}} = \pm |\gamma_{\mathbf{k}}|$$

$$\gamma_{\mathbf{k}} = \sum_{\alpha \rightarrow \beta} e^{i\mathbf{k} \cdot \boldsymbol{\delta}_{\alpha\beta}} = e^{i\frac{k_x}{3}} + e^{-i\frac{k_x}{3}}$$

$$= 2 \cos\left(\frac{k_x}{3}\right)$$

$$\gamma_{\mathbf{k}=\frac{4\pi}{3}, 0} = 1 + 2 \cos\left(\frac{2\pi}{3}\right) = 0 \Rightarrow \delta_{\mathbf{q}} =$$



$$H = -t \sum_{\langle ij \rangle b} c_i^\dagger c_{j\sigma} = t \sum_{\langle ij \rangle b} c_i^\dagger d_{i\sigma} + h.c. = -t \sum_{\langle ij \rangle} \psi_i^\dagger (t_{ij}) \psi_j =$$

$$[H, \hat{T}] = 0$$

$$= -t \sum_k \psi_k^\dagger (\gamma_k^* \psi_k)$$

$$\Rightarrow -t \sum_k \psi_k^\dagger (v \cdot \hat{p}) \psi_k$$

$$\hat{H}^2 = \begin{pmatrix} |\gamma_k|^2 \\ |\gamma_k|^2 \end{pmatrix}$$

$$E_k = \pm |\gamma_k|$$

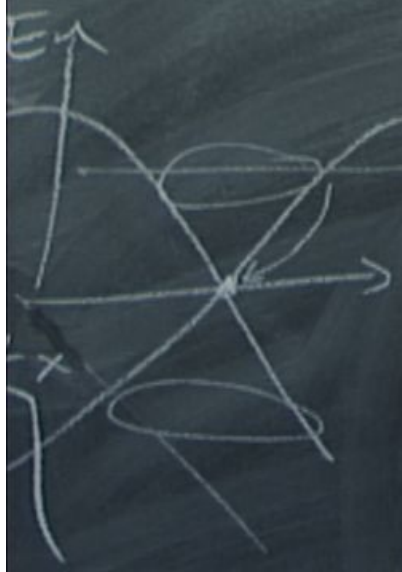
$$\gamma_k = \sum_{\sigma \rightarrow} e^{i\vec{k} \cdot \vec{\sigma}} = e^{i\frac{\sqrt{3}k_x}{3}} + e^{-i\frac{\sqrt{3}k_y}{3}} = 2 \cos\left(\frac{k_x}{2}\right)$$

$$\gamma_{k=(\frac{4\pi}{3}, 0)} = 1 + 2 \cos\left(\frac{2\pi}{3}\right) = 0 \Rightarrow \gamma_q = (q_x + i q_y) v$$



$$\langle i|j \rangle \Psi_j =$$

$$\Psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$



$$\left(\frac{2\pi}{3}, \frac{2\pi}{\sqrt{3}} \right)$$

$$K' \left(\frac{4\pi}{3}, 0 \right)$$

$$\left(\frac{k_x}{2} \right)$$

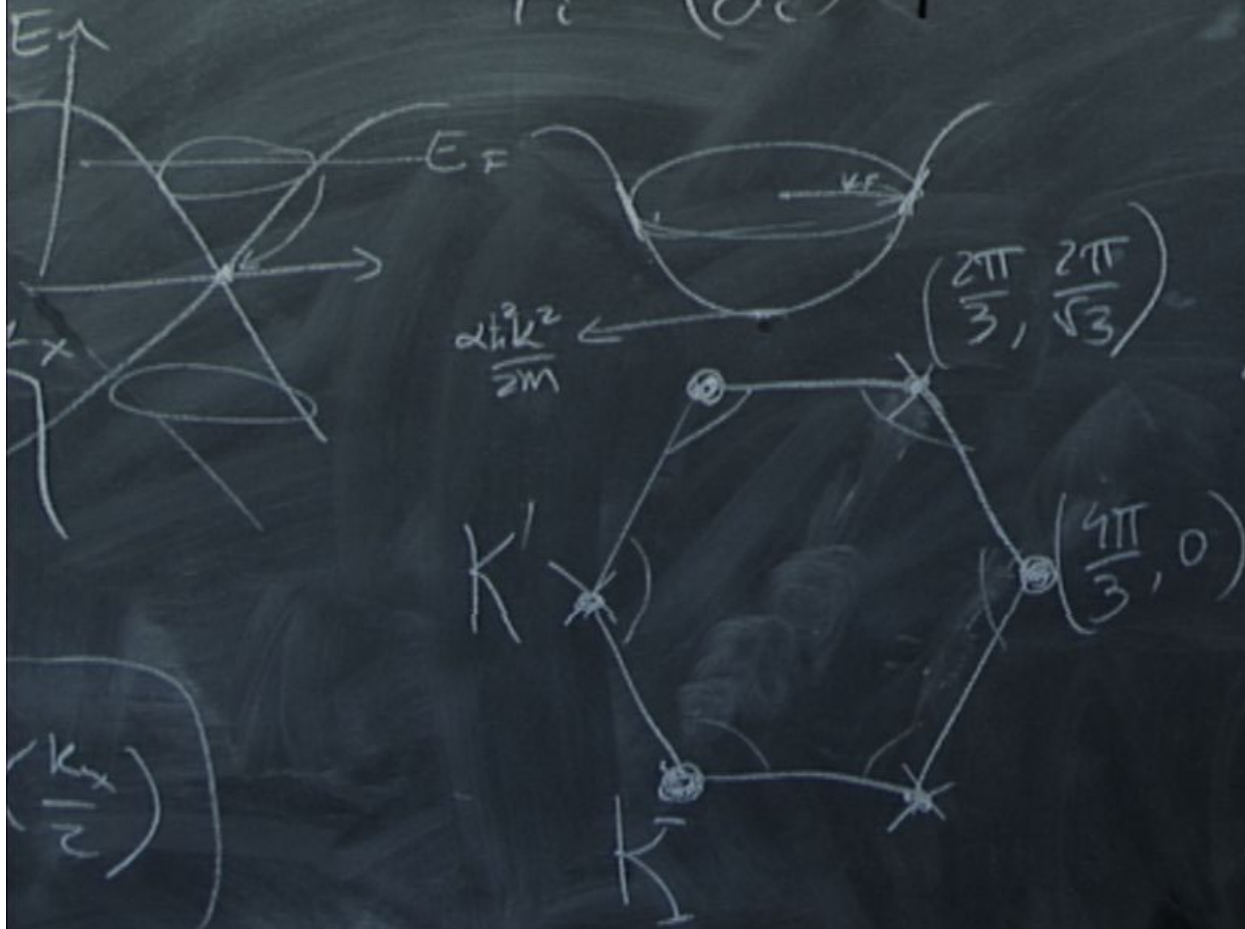


$$H_K^{LE} = \sum \Psi_k^+ (\delta_{xk}) \Psi_k$$

$$H_K^{LE} = \sum \Psi_k (-\delta_{xk})$$

$$(c_{ij}) \psi_j =$$

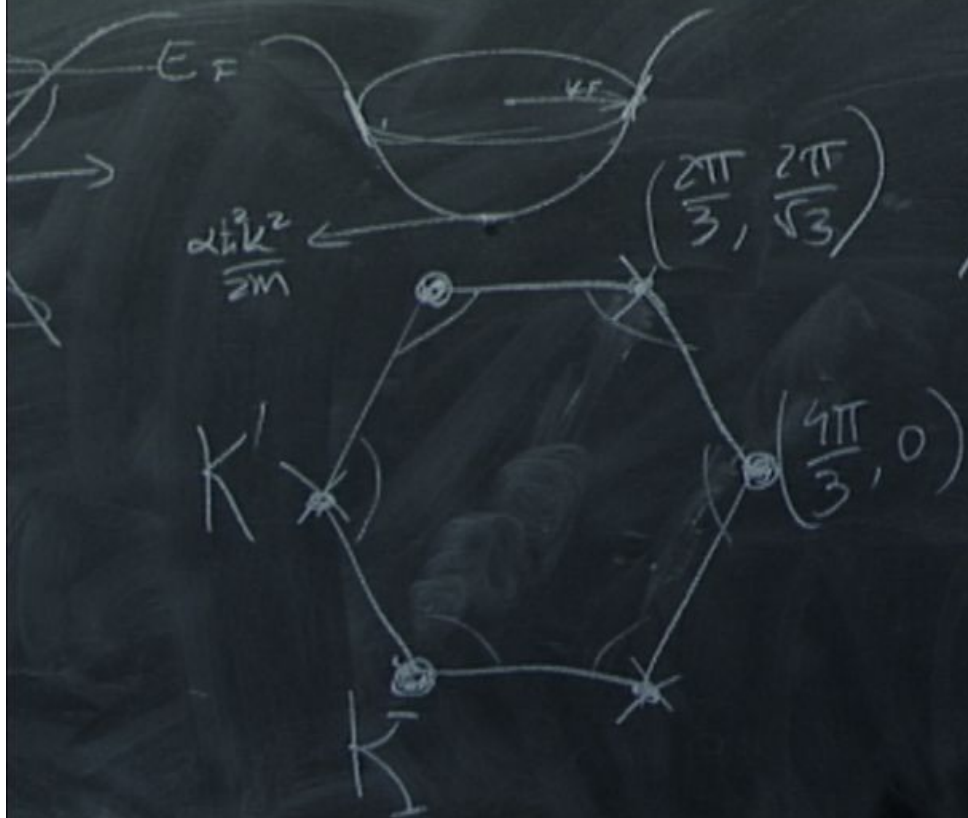
$$\psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$



$$H_K^{LE} = \sum \psi_k^+ (\sigma \cdot k) \psi_k$$

$$H_K^{LE} = \sum \psi_k (-\sigma_x k_x +$$

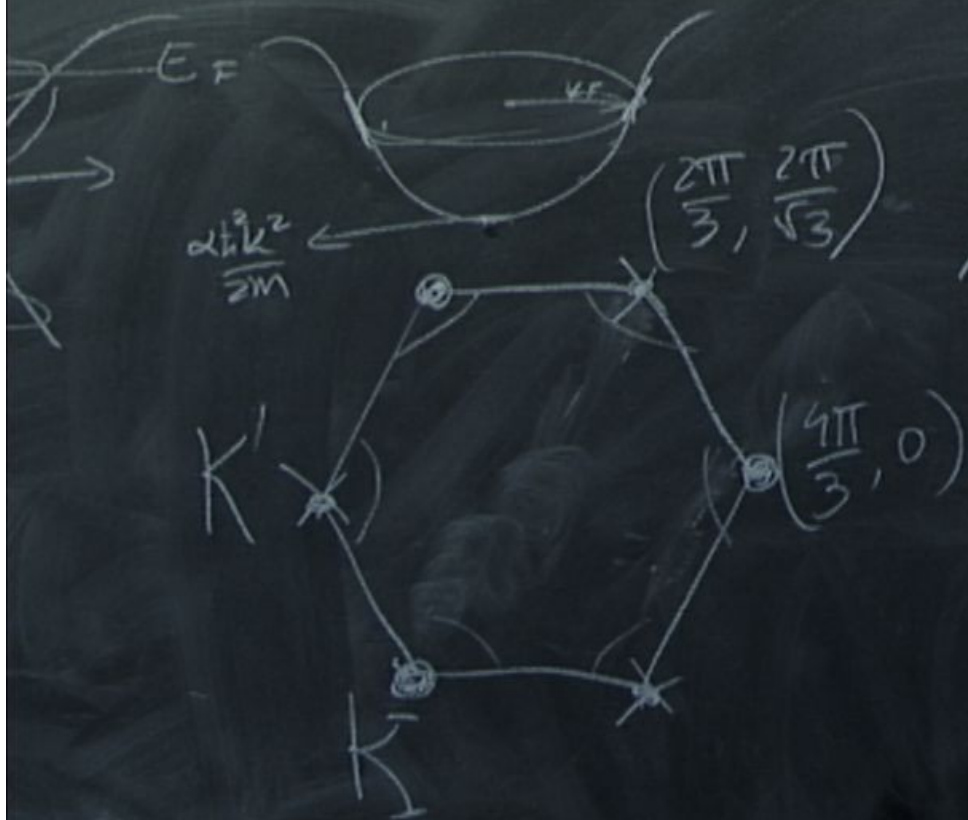
$$\Psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$



$$H|_K^{LE} = \sum \Psi_k^+ (\sigma \cdot k) \Psi_k$$

$$H|_{K'}^{LE} = \sum \Psi_k^+ (-\sigma_x k_x + \sigma_y k_y) \Psi_k$$

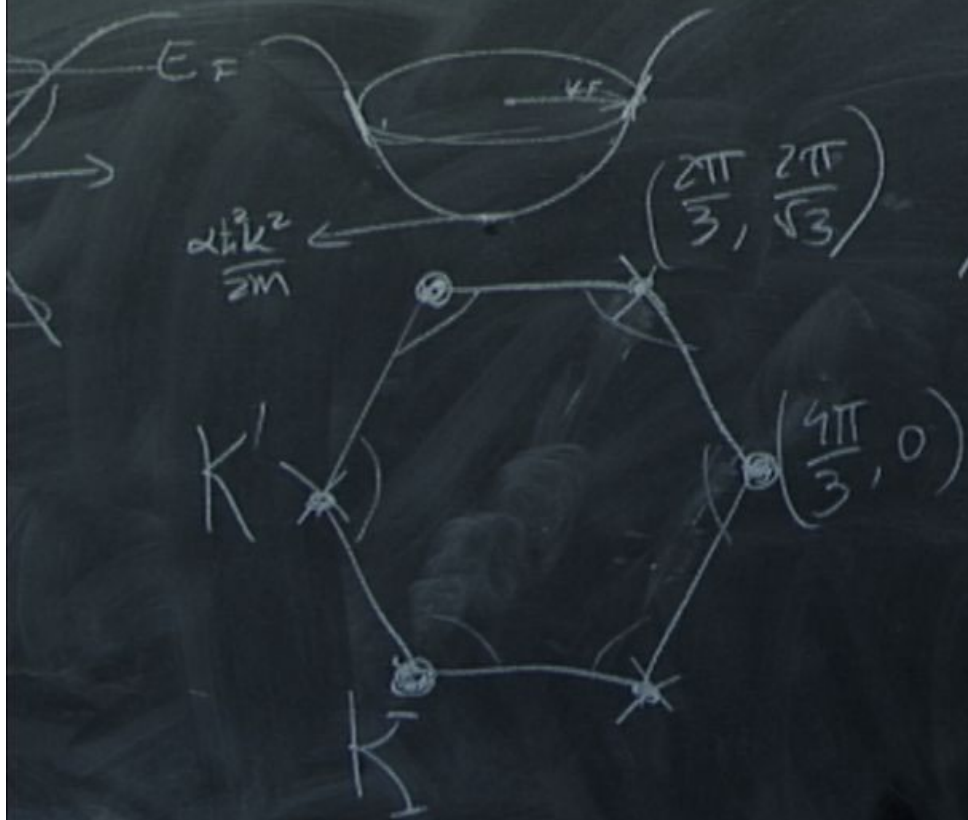
$$\Psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$



$$H_K^{LE} = \sum \Psi_k^+ (\sigma \cdot k) \Psi_k$$

$$H_{K'}^{LE} = \sum \Psi_k^+ (-\sigma_x k_x + \sigma_y k_y) \Psi_k$$

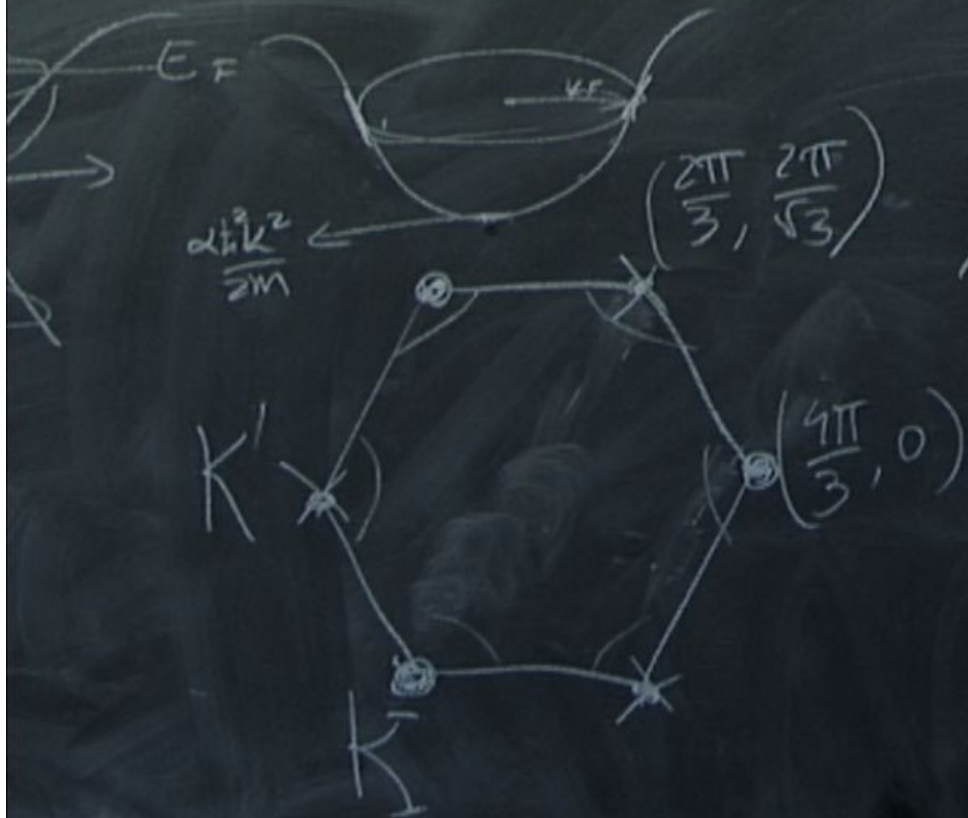
$$\Psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$



$$H|_K^{LE} = \sum \Psi_k^+ (\sigma \cdot k) \Psi_k$$

$$H|_{K'}^{LE} = \sum \Psi_k^+ (-\sigma_x k_x + \sigma_y k_y) \Psi_k$$

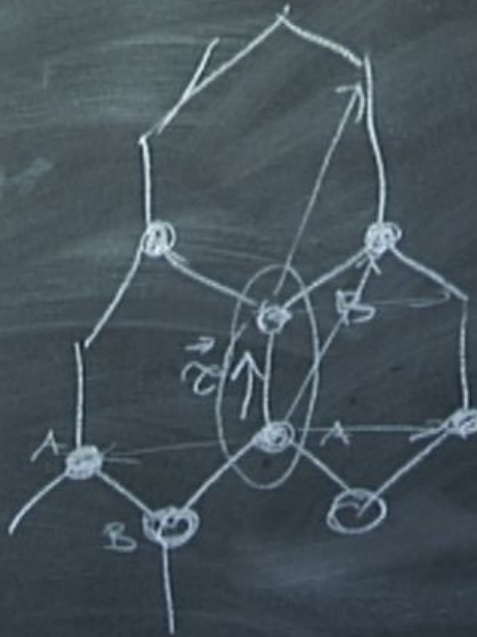
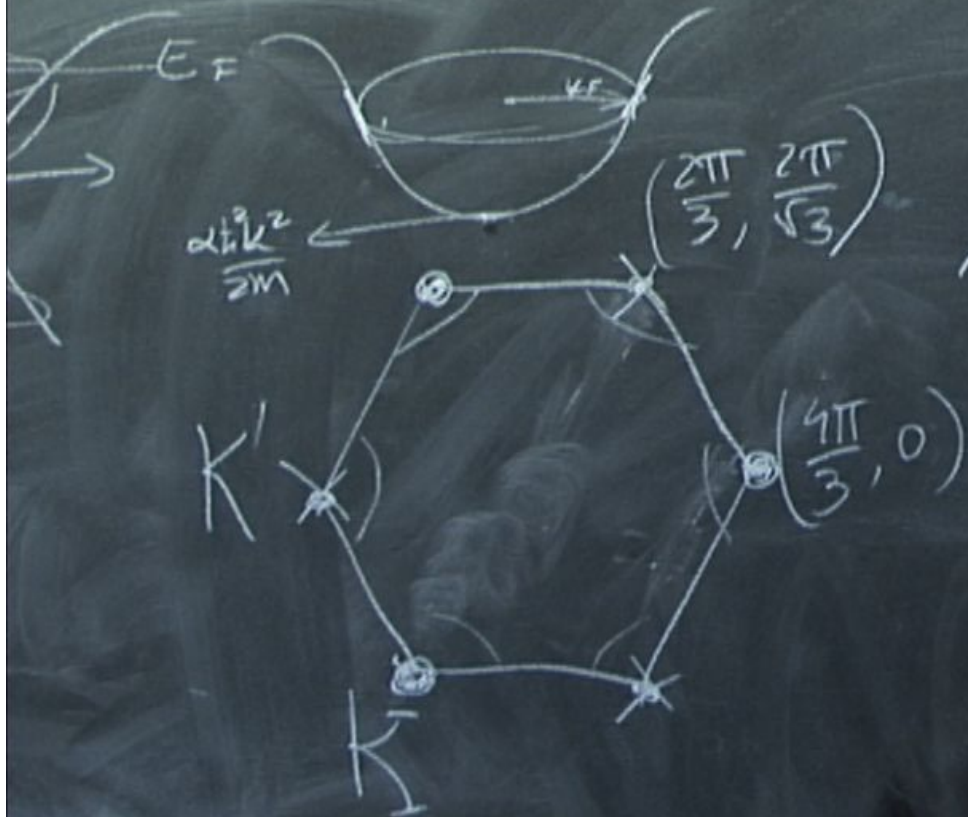
$$\Psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$



$$H_K^{LE} = \sum \Psi_k^+ (\sigma \cdot k) \Psi_k$$

$$H_{K'}^{LE} = \sum \Psi_k^+ (-\sigma_x k_x + \sigma_y k_y) \Psi_k$$

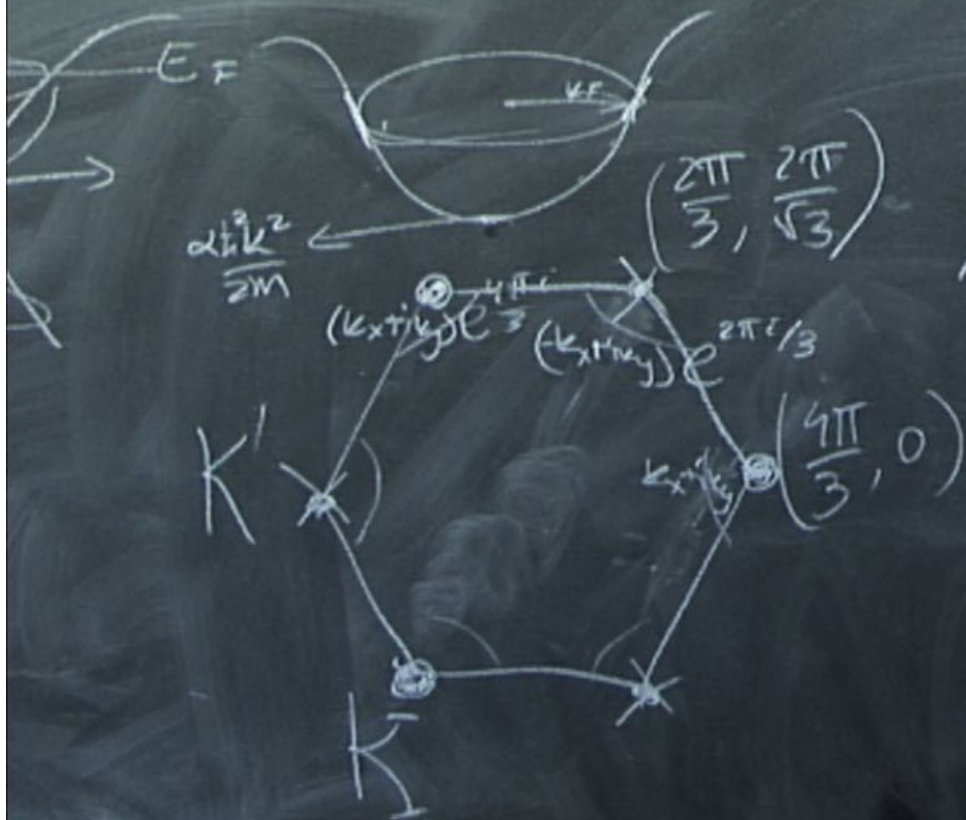
$$\Psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$



$$H|_K^{LE} = \sum \Psi_k^+ (\sigma \cdot k) \Psi_k$$

$$H|_{K'}^{LE} = \sum \Psi_k^+ (-\sigma_x k_x + \sigma_y k_y) \Psi_k$$

$$\Psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$



$$H|K\rangle^{LE} = \sum \Psi_k^+ (\sigma \cdot k) \Psi_k \quad [V=1]$$

$$H|K'\rangle^{LE} = \sum \Psi_k^+ (-\sigma_x k_x + \sigma_y k_y) \Psi_k$$

$$\Psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$



$$\vec{a} = (a_x, a_y)$$

$$H|K\rangle^{LE} = \sum \Psi_k^+ (\vec{a} \cdot \vec{k}) \Psi_k \quad [V=1]$$

$$H|K'\rangle^{LE} = \sum \Psi_k^+ (-a_x k_x + a_y k_y) \Psi_k$$

$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j = t \sum_{i, \delta} c_i^\dagger d_{i+\delta} + h.c. = -t \sum_{\langle ij \rangle} \psi_i^\dagger (\psi_{ij}^*)$$

$$[H, \hat{T}] = 0 \quad \psi_{\mathbf{k}+\mathbf{k}}(\vec{r}) = e^{i\mathbf{k} \cdot \vec{r}} \begin{pmatrix} 1 \\ e^{i\varphi_{\mathbf{k}}} \end{pmatrix}$$

$$\varphi_{\mathbf{k}} = \text{Arg}(c_{\mathbf{k}})$$



$$|c_{\mathbf{k}}|^2$$

$$|c_{\mathbf{k}}|^2$$

$$k = \pm |c_{\mathbf{k}}|$$

$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j = t \sum_{\langle i, \delta \rangle} c_i^\dagger d_{i+\delta} + h.c. = -t \sum_{\langle ij \rangle} \psi_i^\dagger (\psi_{ij}^*)$$

$$[H, \hat{T}] = 0 \quad \psi_{\mathbf{k}+\mathbf{k}}(\vec{r}) = e^{i\vec{k} \cdot \vec{r}} \begin{pmatrix} 1 \\ e^{i\varphi_{\mathbf{k}}} \end{pmatrix}$$

$$\varphi_{\mathbf{k}} = \text{Arg}(*)$$



$$e^{i\varphi_{\mathbf{k}}} = \frac{k_x + i k_y}{|\mathbf{k}|}$$

$$\left(\begin{array}{c} |\gamma_{\mathbf{k}}|^2 \\ |\gamma_{\mathbf{k}}|^2 \end{array} \right) \\ \mathbf{k} = \pm |\gamma_{\mathbf{k}}|$$

$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j = t \sum_{i, \delta} c_i^\dagger d_{i+\delta} + h.c. = -t \sum_{\langle ij \rangle} \psi_i^\dagger \psi_j$$

$$[H, \hat{T}] = 0$$

$$\psi_{\mathbf{k}+\mathbf{k}'}^{\pm E}(\vec{r}) = e^{i\vec{k} \cdot \vec{r}} \begin{pmatrix} 1 \\ \pm e^{i\varphi_{\mathbf{k}}} \end{pmatrix}$$

$$\varphi_{\mathbf{k}} = \text{Arg}(*)$$



$$e^{i\varphi_{\mathbf{k}}} = \frac{k_x + i k_y}{|k|}$$

$$\begin{aligned} & |\delta_{\mathbf{k}}|^2 \\ & |\delta_{\mathbf{k}}|^2 \\ & = \pm |\delta_{\mathbf{k}}| \end{aligned}$$

$$H = -t \sum_{\langle ij \rangle b} c_i^\dagger c_{j\sigma} = t \sum_{i, \sigma} c_i^\dagger d_{i+\sigma} + h.c. = -t \sum_{\langle ij \rangle} \psi_i^\dagger (\psi_{ij}^*)$$

$$\psi_{k+\kappa} \propto \begin{pmatrix} 1 \\ \pm e^{-i\varphi_\kappa} \end{pmatrix}$$

$$\psi_{k+\kappa}(\vec{r}) = e^{i\vec{k} \cdot \vec{r}} \begin{pmatrix} 1 \\ \pm e^{i\varphi_\kappa} \end{pmatrix}$$

$$\varphi_\kappa = \text{Arg}(*)$$



$$e^{i\varphi_\kappa} = \frac{k_x + i k_y}{|k|}$$

$$\hat{H}^2 = \begin{pmatrix} |\gamma_\kappa|^2 & \\ & |\gamma_\kappa|^2 \end{pmatrix}$$

$$E_\kappa = \pm |\gamma_\kappa|$$

$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j = t \sum_{i, \delta} c_i^\dagger d_{i+\delta} + h.c. = -t \sum_{\langle ij \rangle} \psi_i^\dagger (\psi_{ij}^*)$$

$$\psi_{\mathbf{k}+\mathbf{k}} \propto \begin{pmatrix} 1 \\ \pm e^{-i\varphi_{\mathbf{k}}} \end{pmatrix}$$

$$\psi_{\mathbf{k}+\mathbf{k}}^{\pm E}(\vec{r}) = e^{i\vec{k} \cdot \vec{r}} \begin{pmatrix} 1 \\ \pm e^{i\varphi_{\mathbf{k}}} \end{pmatrix}$$

$$\varphi_{\mathbf{k}} = \text{Arg}(*)$$



$$e^{i\varphi_{\mathbf{k}}} = \frac{k_x + i k_y}{|\mathbf{k}|}$$

$$\hat{H}^2 = \begin{pmatrix} |\gamma_{\mathbf{k}}|^2 & \\ & |\gamma_{\mathbf{k}}|^2 \end{pmatrix}$$

$$E_{\mathbf{k}} = \pm |\gamma_{\mathbf{k}}|$$

$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j = t \sum_{i, \delta} c_i^\dagger d_{i+\delta} + h.c. = -t \sum_{\langle ij \rangle} \psi_i^\dagger (\psi_{ij}^*)$$

$$\psi_{\mathbf{k}+\mathbf{k}} \propto \begin{pmatrix} 1 \\ \pm e^{-i\varphi_{\mathbf{k}}} \end{pmatrix}$$

$$\psi_{\mathbf{k}+\mathbf{k}}^{\pm E}(\vec{r}) = e^{i\vec{k} \cdot \vec{r}} \begin{pmatrix} 1 \\ \pm e^{i\varphi_{\mathbf{k}}} \end{pmatrix}$$

$$\varphi_{\mathbf{k}} = \text{Arg}(k)$$



$$e^{i\varphi_{\mathbf{k}}} = \frac{k_x + i k_y}{|k|}$$

$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j = t \sum_{\substack{i,j \\ i \neq j}} c_i^\dagger d_{i+\hat{s}} + h.c. = -t \sum_{\langle ij \rangle} \psi_i^\dagger (\psi_j^*)$$

$$\psi_{\vec{k}+\vec{k}} \propto \begin{pmatrix} 1 \\ \pm e^{-i\varphi_{\vec{k}}} \end{pmatrix}$$

$$\psi_{\vec{k}+\vec{k}}^{\pm E}(\vec{r}) = e^{i\vec{k} \cdot \vec{r}} \begin{pmatrix} 1 \\ \pm e^{i\varphi_{\vec{k}}} \end{pmatrix}$$

$$\varphi_{\vec{k}} = A_1(k_x)$$

$$"S_x" = \langle \psi | \sigma_x | \psi \rangle$$

$$= \begin{pmatrix} 1 & e^{-i\varphi_{\vec{k}}} \end{pmatrix}$$

$$e^{i\varphi_{\vec{k}}} = \frac{k_x + i k_y}{|k|}$$

$$\begin{pmatrix} 1 & e^{-i\varphi_{\vec{k}}} \end{pmatrix} \begin{pmatrix} e^{i\varphi_{\vec{k}}} \\ 1 \end{pmatrix}$$

$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j = t \sum_{\substack{i,j \\ i \neq j}} c_i^\dagger d_{i+\delta} + h.c. = -t \sum_{\langle ij \rangle} \psi_i^\dagger (\psi_j + \psi_j^\dagger)$$

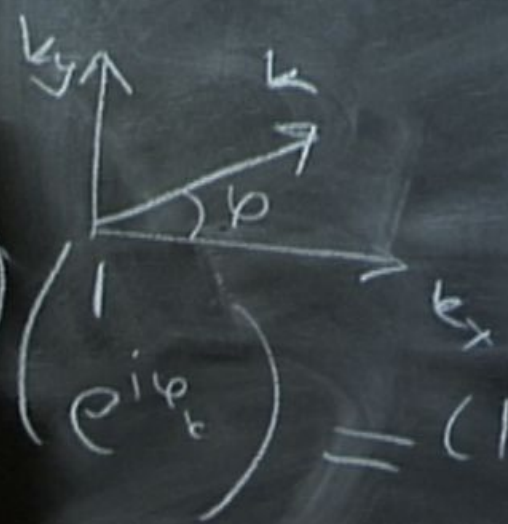
$$\psi_{k+\pi} \propto \begin{pmatrix} 1 \\ \pm e^{-i\varphi_k} \end{pmatrix}$$

$$\psi_{k+\pi}^{\pm E}(\vec{r}) = e^{i\vec{k} \cdot \vec{r}} \begin{pmatrix} 1 \\ \pm e^{i\varphi_k} \end{pmatrix}$$

$$\varphi_k = \text{Arg}(k)$$

$$\langle S_x \rangle = \langle \psi | \sigma_x | \psi \rangle$$

$$= \frac{1}{2} \begin{pmatrix} 1 & e^{-i\varphi_k} \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$



$$e^{i\varphi_k} = \frac{k_x + i k_y}{|k|}$$

$$= \begin{pmatrix} 1 & e^{-i\varphi_k} \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ e^{i\varphi_k} \end{pmatrix}$$

$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j = t \sum_{i \rightarrow j} c_i^\dagger d_{i+\delta} + h.c. = -t \sum_{\langle ij \rangle} \psi_i^\dagger (\psi_j^*)$$

$$\psi_{\mathbf{k}+\mathbf{k}} \propto \begin{pmatrix} 1 \\ \pm e^{-i\varphi_{\mathbf{k}}} \end{pmatrix}$$

$$\psi_{\mathbf{k}+\mathbf{k}}^{\pm E}(\vec{r}) = e^{i\vec{k} \cdot \vec{r}} \begin{pmatrix} 1 \\ \pm e^{i\varphi_{\mathbf{k}}} \end{pmatrix}$$

$$\varphi_{\mathbf{k}} = \text{Arg}(\mathbf{k})$$



$$e^{i\varphi_{\mathbf{k}}} = \frac{k_x + i k_y}{|\mathbf{k}|}$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ e^{i\varphi_{\mathbf{k}}} \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} 1 & e^{-i\varphi_{\mathbf{k}}} \\ e^{i\varphi_{\mathbf{k}}} & 1 \end{pmatrix} \begin{pmatrix} e^{i\varphi_{\mathbf{k}}} \\ 1 \end{pmatrix}$$

"S_x" $\langle \mathbf{k} | \psi \rangle$

$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j = t \sum_{\substack{i \rightarrow j \\ i, j \in \Lambda}} c_i^\dagger d_{i+\delta} + h.c. = -t \sum_{\langle ij \rangle} \psi_i^\dagger \psi_j$$

$$\psi_{k+\pi} \propto \begin{pmatrix} 1 \\ \pm e^{-i\varphi_k} \end{pmatrix}$$

$$\psi_{k+\pi}^{\pm E}(\vec{r}) = e^{i\vec{k} \cdot \vec{r}} \begin{pmatrix} 1 \\ \pm e^{i\varphi_k} \end{pmatrix}$$

$$\varphi_k = \text{Arg}(k)$$

$$"s_x" = \langle \psi | \sigma_x | \psi \rangle$$

$$= \frac{1}{2} \begin{pmatrix} 1 & e^{-i\varphi_k} \\ 1 & 1 \end{pmatrix}$$



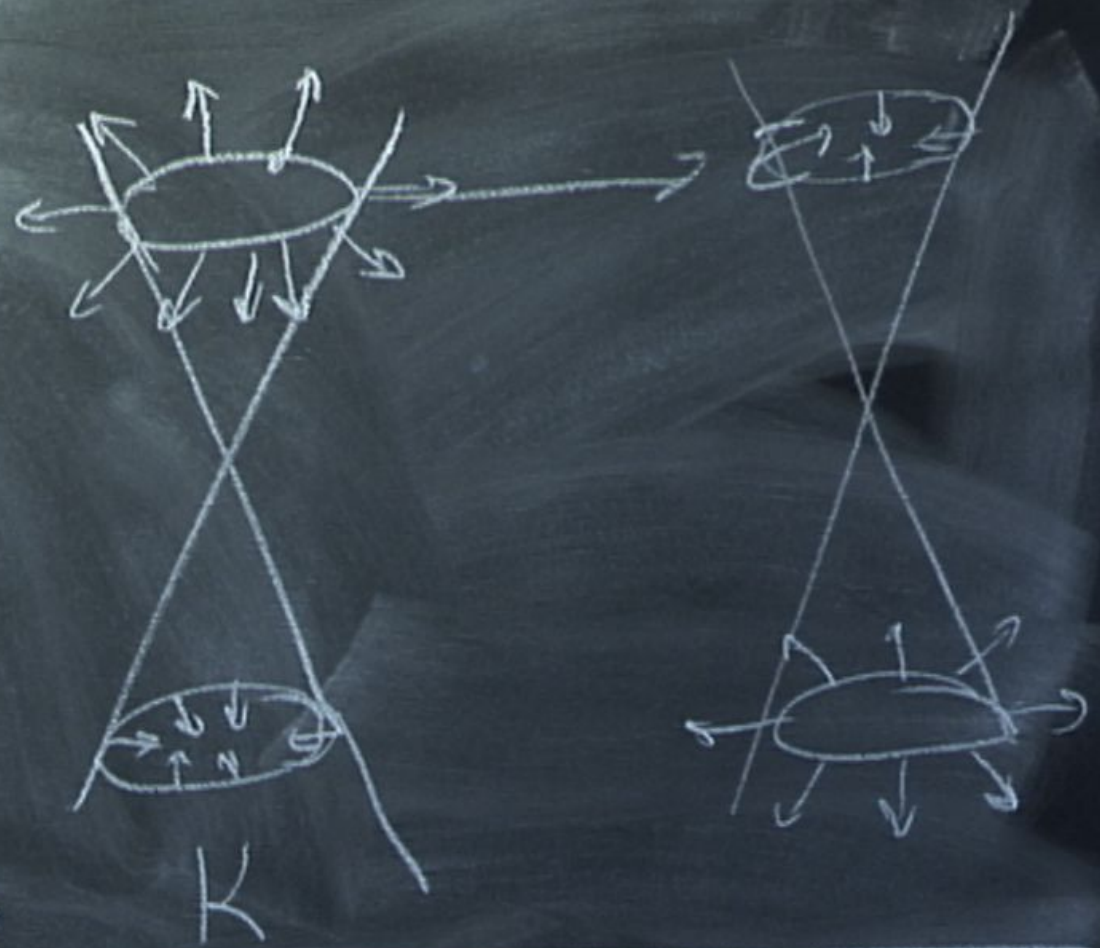
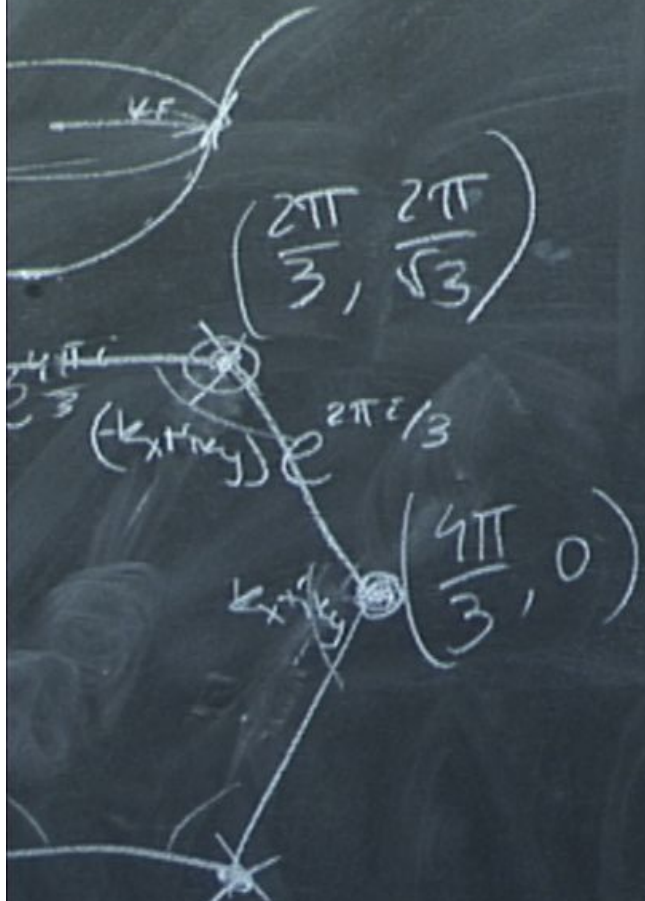
$$e^{i\varphi_k} = \frac{k_x + i k_y}{|k|}$$

$$"s_y" = \frac{k_y}{|k|}$$

$$= \frac{1}{2} \begin{pmatrix} 1 & e^{-i\varphi_k} \\ e^{i\varphi_k} & 1 \end{pmatrix}$$

$$= \cos \varphi_k = \frac{k_x}{|k|}$$

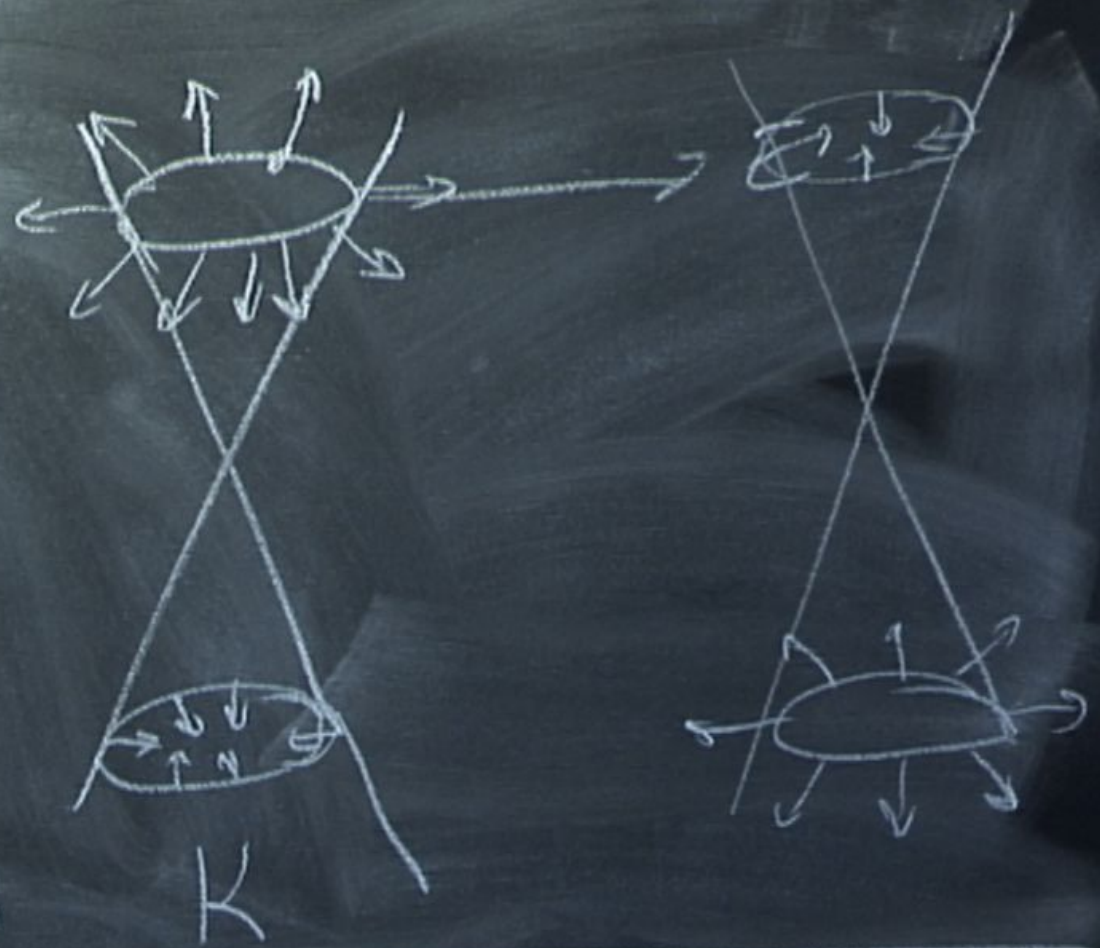
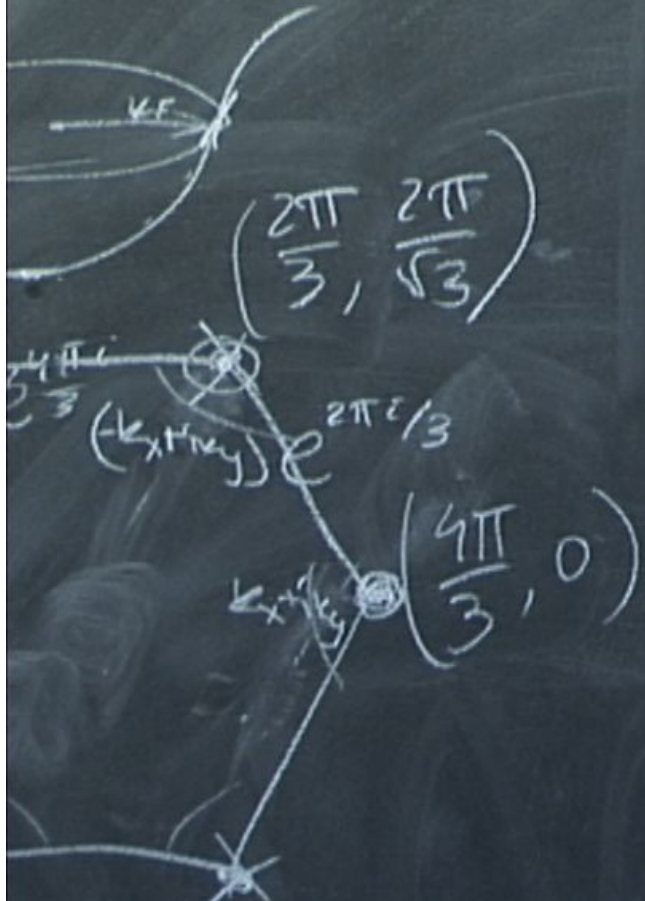
(i)
(ii)



$$H|K\rangle^{LE} = \sum_k \Psi_k^\dagger (\vec{\sigma} \cdot \vec{k}) \Psi_k \quad [V=1]$$

$$H|K\rangle^{LE} = \sum_k \Psi_k^\dagger (-\sigma_x k_x + \sigma_y k_y) \Psi_k$$

(c)
(d)

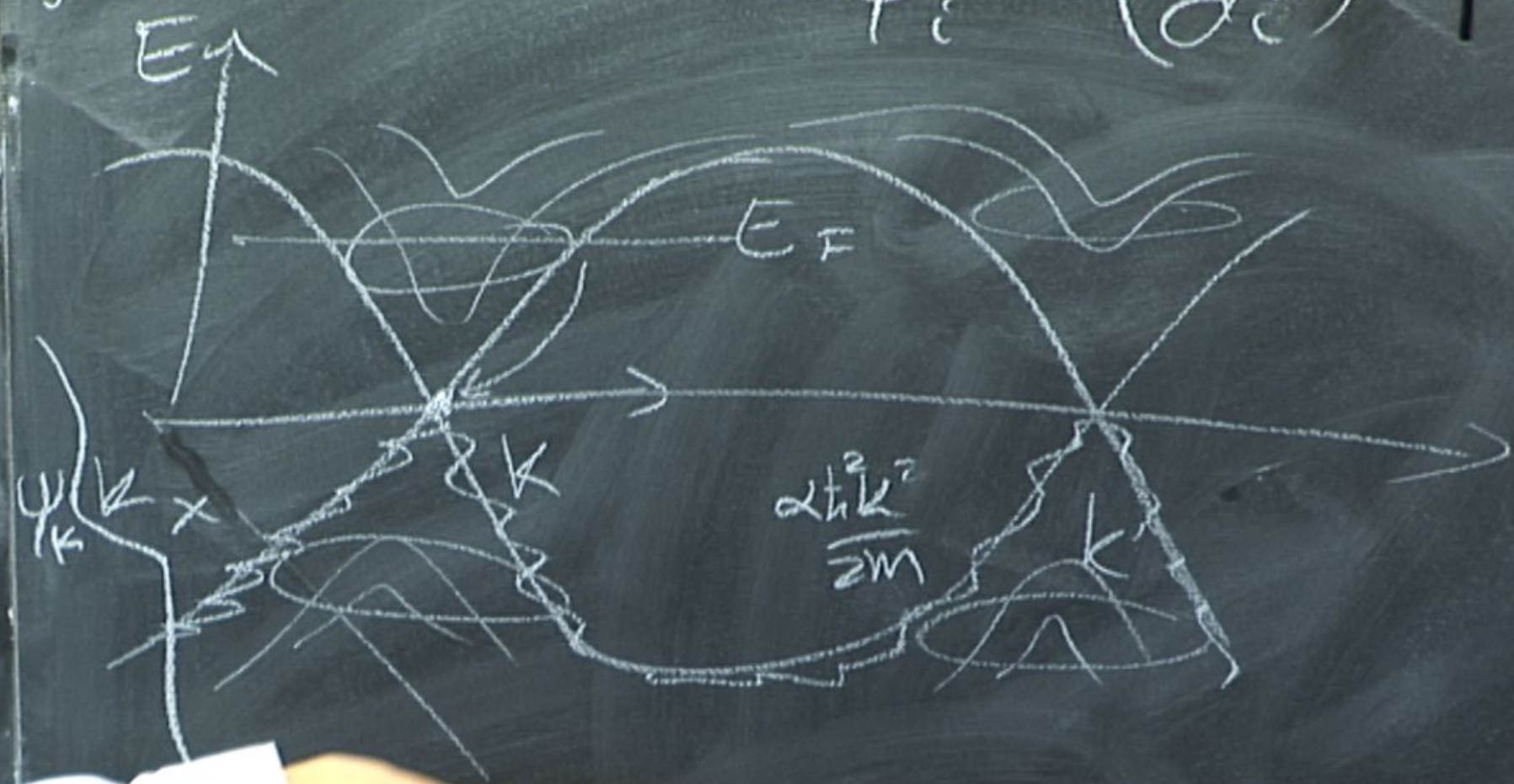


$$H|K\rangle^{LE} = \sum \Psi_k^\dagger (\vec{\sigma} \cdot \vec{k}) \Psi_k \quad [V=1]$$

$$H|B1\rangle^{LE} = \sum \Psi_k^\dagger (-\sigma_x k_x + \sigma_y k_y) \Psi_k$$

$$\begin{pmatrix} t_{ij} \\ t_{ij}^* \end{pmatrix} \psi_j =$$

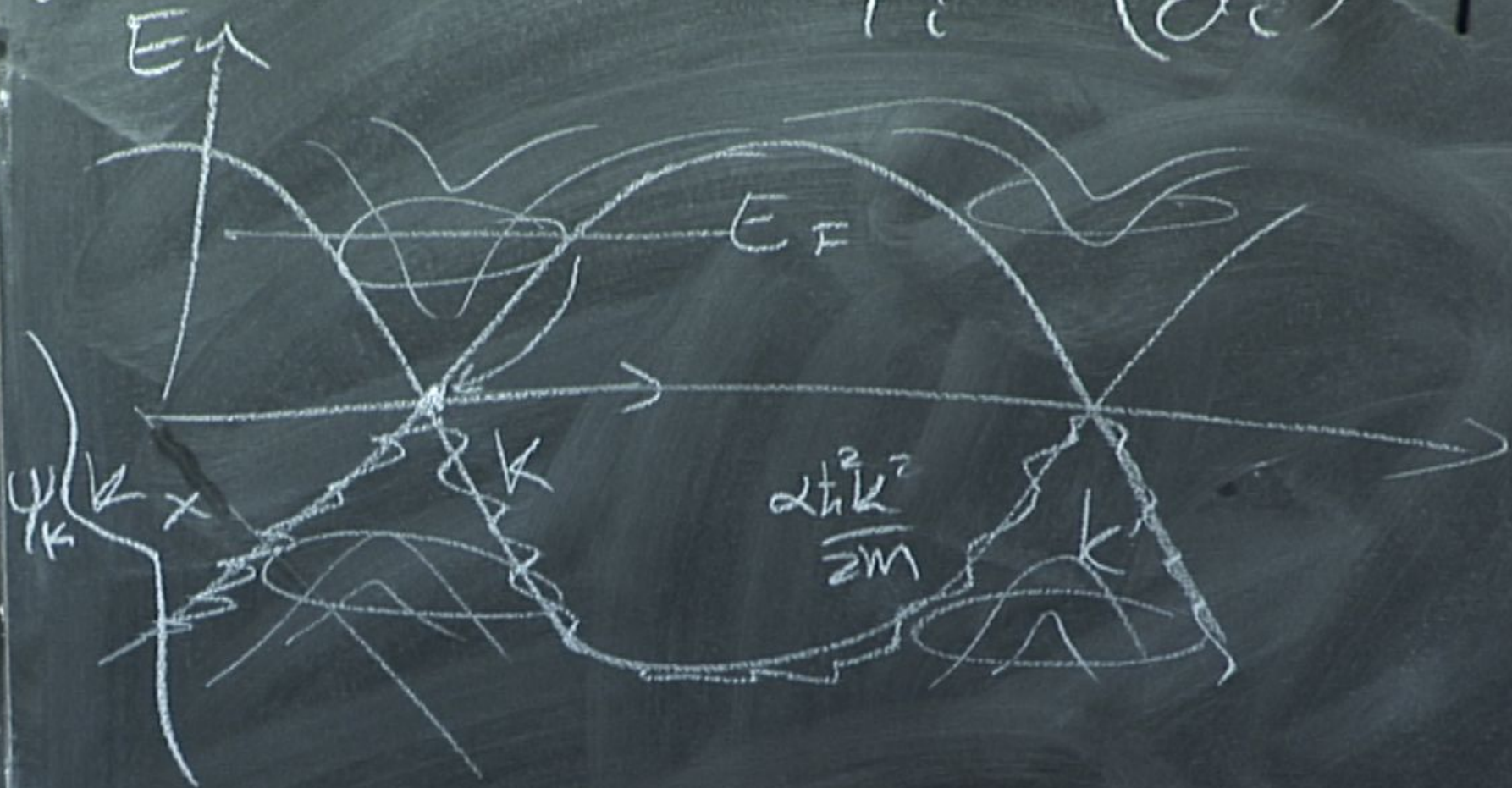
$$\psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$



$$\frac{\hbar^2 k^2}{2m}$$

$$\begin{pmatrix} t_{ij} \\ t_{ij}^* \end{pmatrix} \psi_j =$$

$$\psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$



$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j = t \sum_{\langle ij \rangle} c_i^\dagger d_{i+1} + h.c. = -t \sum_{\langle ij \rangle} \psi_i^\dagger(t_{ij}) \psi_j$$

$$\psi_{k+\pi} \propto \begin{pmatrix} 1 \\ e^{-i\varphi_k} \end{pmatrix}$$

$$\psi_{k\pi}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} \begin{pmatrix} 1 \\ \pm e^{i\varphi_k} \end{pmatrix} \quad 14\text{eV}$$

$$\varphi_k = \text{Arg}(k)$$

$$s_x = \langle \psi | \sigma_x | \psi \rangle$$

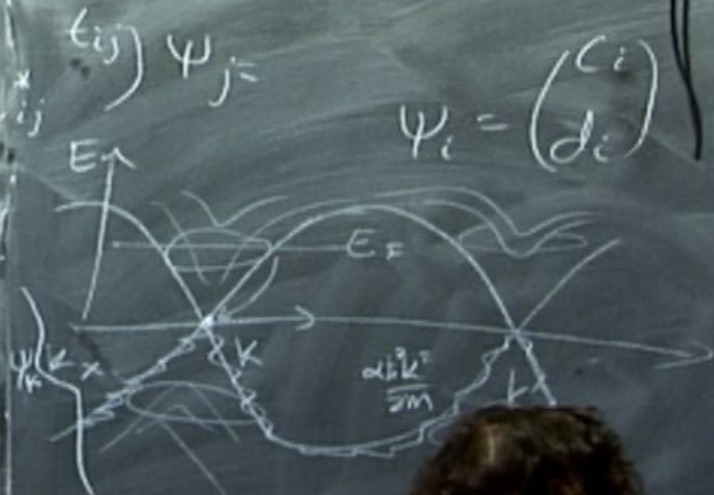
$$= \frac{1}{2} \begin{pmatrix} 1 & e^{-i\varphi_k} \\ \bar{2} & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$s_y = \frac{k_y}{|k|}$$



$$e^{i\varphi_k} = \frac{k_x + i k_y}{|k|}$$

$$= \frac{1}{2} \begin{pmatrix} 1 & e^{i\varphi_k} \\ 1 & 1 \end{pmatrix} \begin{pmatrix} e^{i\varphi_k} \\ 1 \end{pmatrix} = \cos \varphi_k = \frac{k_x}{|k|}$$



$$\psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$

$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j = t \sum_{\langle ij \rangle} c_i^\dagger d_{i+\delta} + h.c. = -t \sum_{\langle ij \rangle} \psi_i^\dagger (t_{ij}) \psi_j$$

$$\psi_{k'+k} \propto \begin{pmatrix} 1 \\ e^{-i\varphi_k} \end{pmatrix}$$

$$\psi_{k+k}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} \begin{pmatrix} 1 \\ \pm e^{i\varphi_k} \end{pmatrix} 14\text{eV}$$

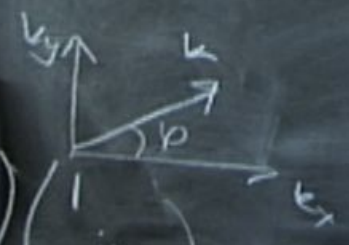
$$\psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$



$$\varphi_k = \text{Arg}(k)$$

$$s_x = \langle \psi | \sigma_x | \psi \rangle$$

$$= \frac{1}{2} \begin{pmatrix} 1 & e^{-i\varphi_k} \\ e^{i\varphi_k} & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$



$$e^{i\varphi_k} = \frac{k_x + i k_y}{|k|}$$

$$s_y = \frac{k_y}{|k|}$$

$$= \cos \varphi_k$$

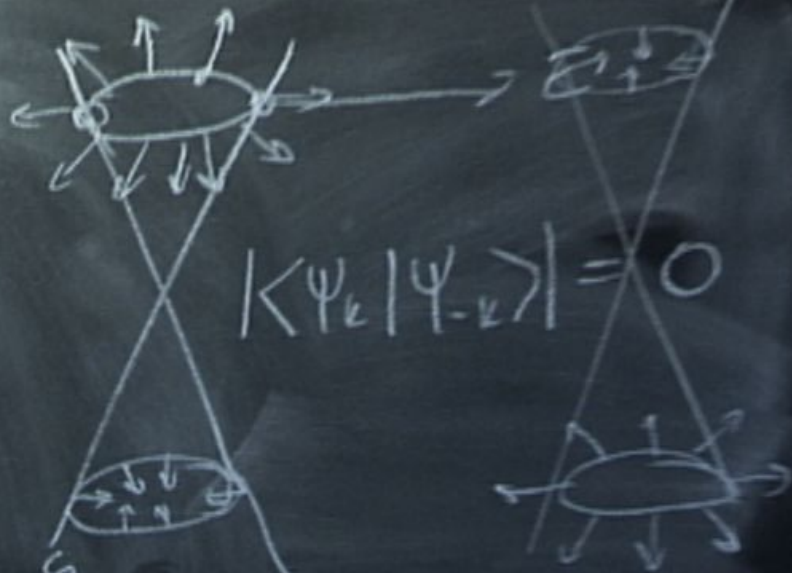
$$= (z_x + i z_y) V$$

$$\psi_j =$$

$$\psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$



$$v = \frac{\sqrt{3}}{2} v_a \approx 10^6 \text{ m/s}$$



$$\langle \psi_k | \psi_{-k} \rangle = 0$$

$$H|K\rangle^{LE} = \sum \psi_k^+ (\vec{\sigma} \cdot \vec{k}) \psi_k \quad |v=1\rangle$$

$$H|K'\rangle^{LE} = \sum \psi_k^+ (-\sigma_x k_x + \sigma_y k_y) \psi_k$$

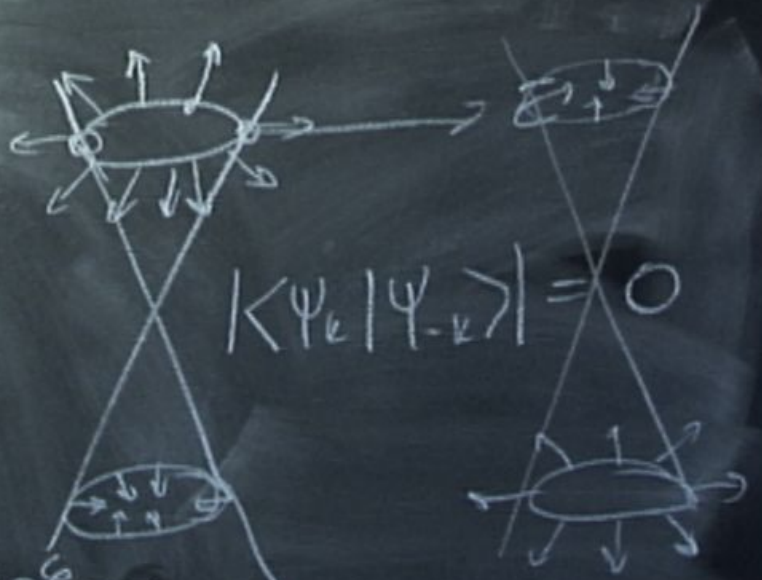
$$\gamma_g = (g_x + i g_y) V$$

$$\Psi_j =$$

$$\Psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$



$$v = \frac{\sqrt{3}}{2} ta \approx 10^6 \text{ m/s}$$



$$H_K^{LE} = \sum \Psi_k^+ (\vec{\sigma} \cdot \vec{k}) \Psi_k \quad [V=1]$$

$$H_{K'}^{LE} = \sum \Psi_k^+ (-\sigma_x k_x + \sigma_y k_y) \Psi_k$$

$$\chi_2 = (q_x + i q_y) V$$

$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j = t \sum_{\langle ij \rangle} c_i^\dagger d_{i+1} + h.c. = -t \sum_{\langle ij \rangle} \psi_i^\dagger (t_{ij}) \psi_j$$

$$\psi_{k+\pi} \propto \begin{pmatrix} 1 \\ -ie^{-ik_x} \end{pmatrix}$$

$$\psi_{k\pi}(\vec{r}) = e^{i\vec{r}\cdot\vec{k}} \begin{pmatrix} 1 \\ \pm e^{i\phi_k} \end{pmatrix} \quad 14 \text{ eV}$$

$$\phi_k = \text{Arg}(z)$$

$$s_x = \langle \psi | \sigma_x | \psi \rangle$$

$$= \frac{1}{2} \begin{pmatrix} 1 & e^{-i\phi_k} \\ 1 & 1 \end{pmatrix}$$

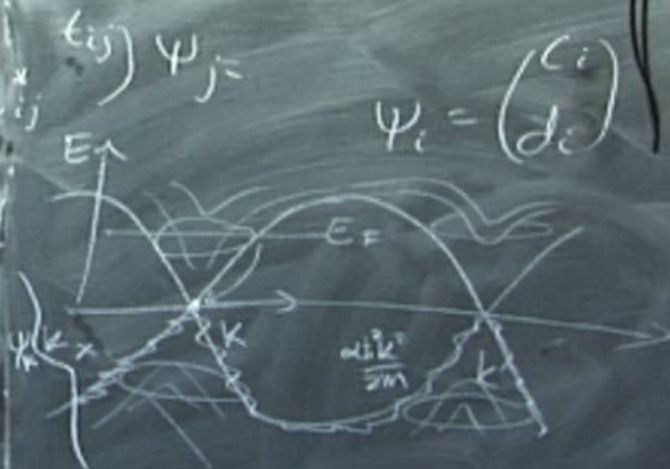
$$s_y = \frac{k_y}{|k|}$$



$$e^{i\phi_k} = \frac{k_x + i k_y}{|k|}$$

$$= \frac{1}{2} \begin{pmatrix} 1 & e^{i\phi_k} \\ 1 & 1 \end{pmatrix} \begin{pmatrix} e^{i\phi_k} \\ 1 \end{pmatrix} = \cos \phi_k = \frac{k_x}{|k|}$$

$$\delta g = (g_x + i g_y) V$$

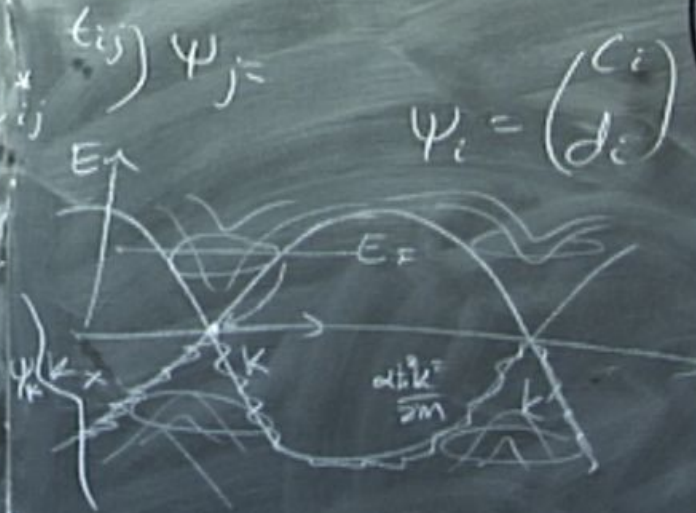


$$\psi_i = \begin{pmatrix} c_i \\ d_i \end{pmatrix}$$

$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j = t \sum_{\langle ij \rangle} c_i^\dagger d_{i+1} + h.c. = -t \sum_{\langle ij \rangle} \psi_i^\dagger (t_{ij}) \psi_j$$

$$\psi_{k+k} \propto \begin{pmatrix} 1 \\ \pm e^{-i\varphi_k} \end{pmatrix}$$

$$\psi_{k+e}(\vec{r}) = e^{i\vec{r} \cdot \vec{k}} \begin{pmatrix} 1 \\ \pm e^{i\varphi_k} \end{pmatrix} \quad 14 \text{ eV}$$



$$s_x = \langle \psi | \sigma_x | \psi \rangle$$

$$= \frac{1}{2} \begin{pmatrix} 1 & e^{-i\varphi_k} \\ 1 & 1 \end{pmatrix}$$

$$s_y = \frac{k_y}{|k|}$$

$$\varphi_k = \text{Arg}(k)$$



$$e^{i\varphi_k} = \frac{k_x + i k_y}{|k|}$$

$$= \frac{1}{2} \begin{pmatrix} 1 & e^{i\varphi_k} \\ 1 & 1 \end{pmatrix}$$

$$= \cos \varphi_k = \frac{k_x}{|k|}$$

$$\delta g = (z_x + i z_y) V$$