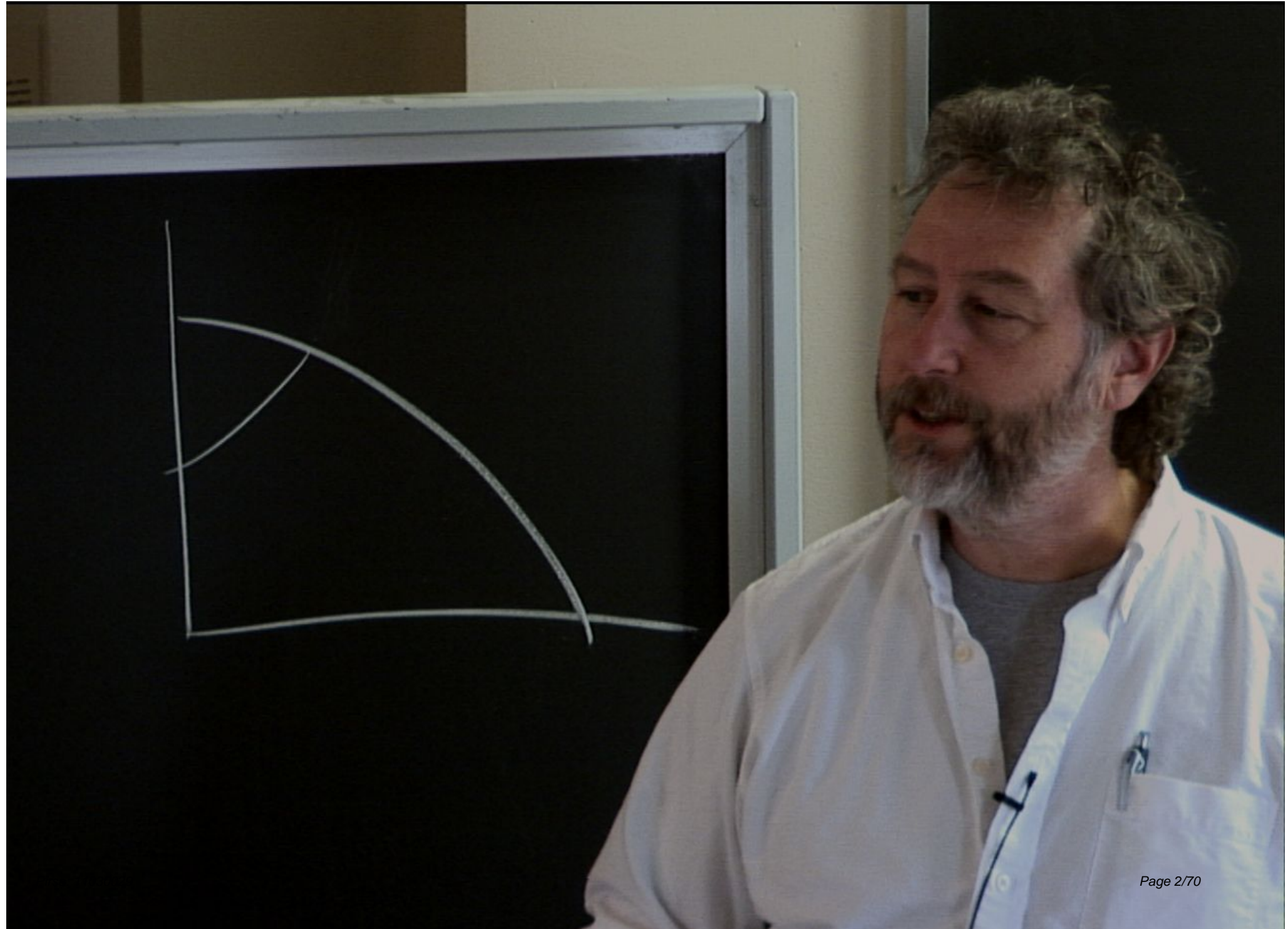


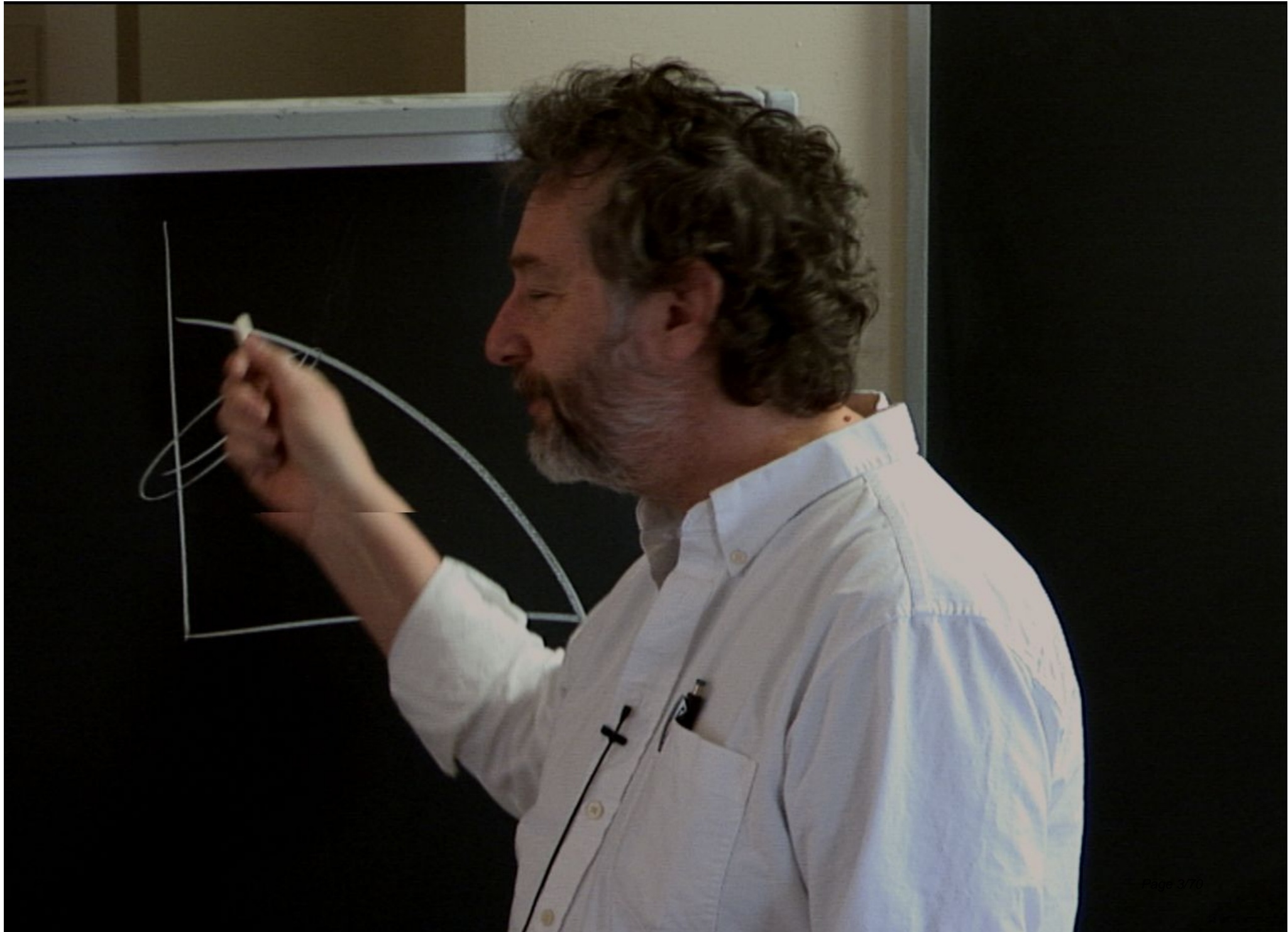
Title: Explorations in Quantum Information - Lecture 14

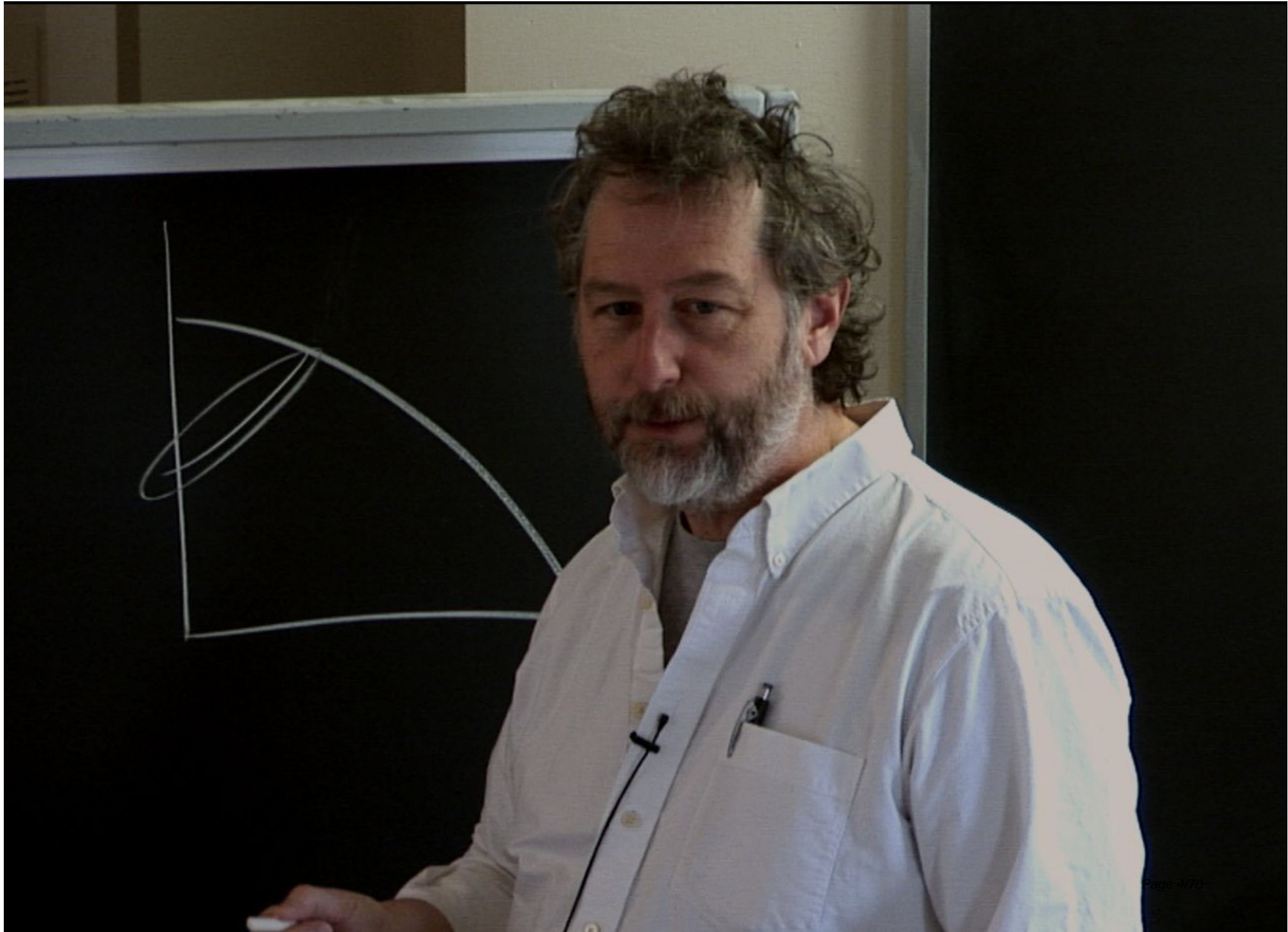
Date: Apr 01, 2011 09:00 AM

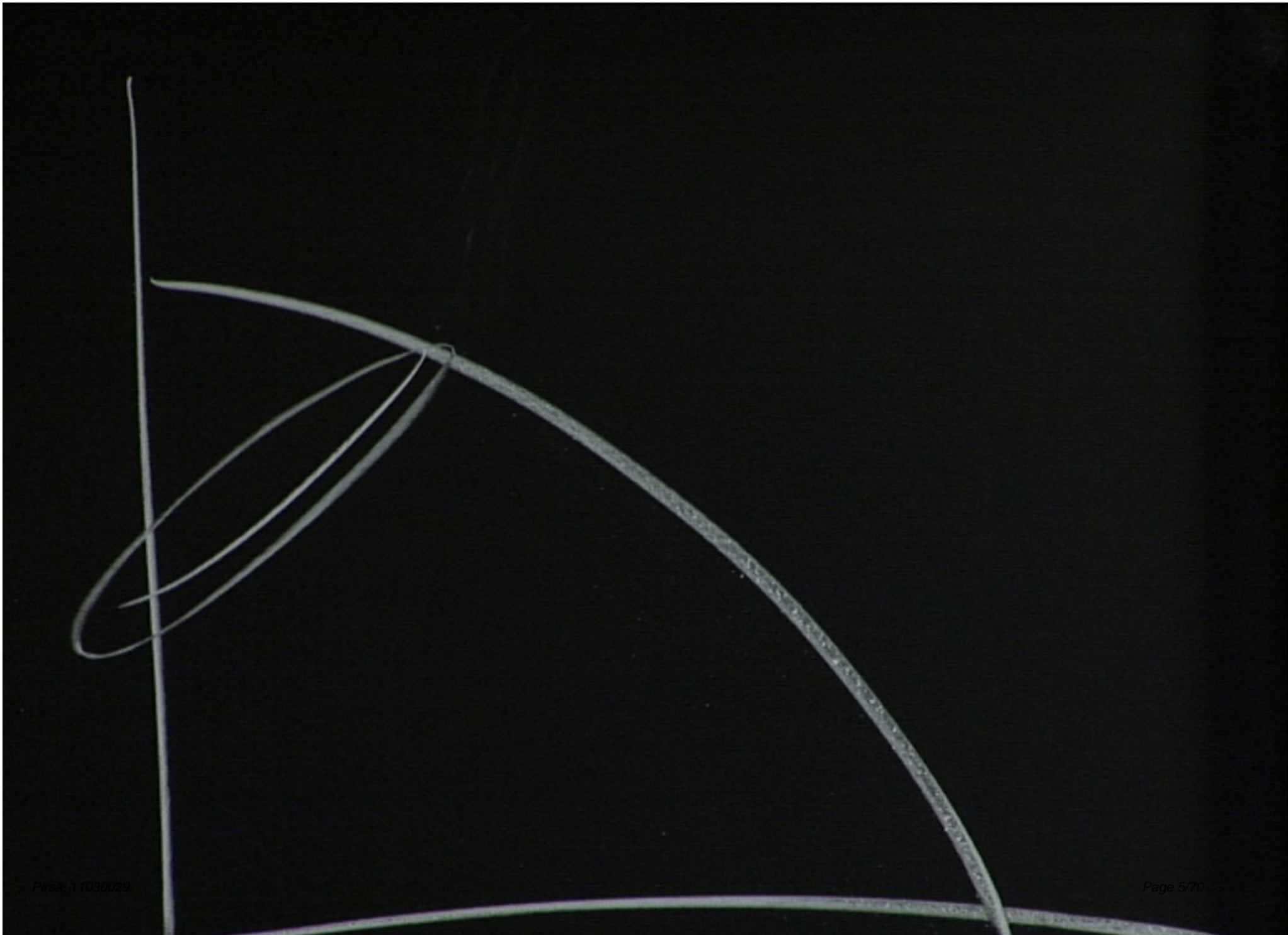
URL: <http://pirsa.org/11030029>

Abstract:









Liquid State NMR

Liquid State NMR

ensemble of 10^{17}
molecules

interconnected set of
atoms



Liquid State NMR

ensemble of 10^{17}
molecules

interconnected set of
atoms \approx 1 molecule

no - molecular / molecular
interactions

density matrix
highly mixed

Liquid State NMR

$$\mathcal{H} = \sum_i \omega_i \sigma_z^i + \sum_{i,j} \omega_{ij} \sigma_z^i \sigma_z^j$$

ensemble of 10^{17}
molecules

interconnected set of
atoms $\{1 \text{ molecule}\}$

no-molecular/molecular
interactions

density matrix
highly mixed

Liquid State NMR

$$\mathcal{H} = \sum_i^{\text{mols.}} \omega_i \sigma_z^i + \sum_{i \neq j}^{\text{mols.}} \omega_{ij} \sigma_z^i \sigma_z^j$$

ensemble of 10^{17}
molecules

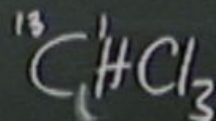
interconnected set of
atoms $\{1 \text{ molecule}\}$

no-molecular/molecular
interactions,

density matrix
highly mixed,

Liquid State NMR

ensemble of 10^{17}
molecules



$$\mathcal{H} = \sum_i^{\text{mols.}} \omega_i \sigma_z^i + \sum_{i \neq j}^{\text{mols.}} \omega_{ij} \sigma_z^i \sigma_z^j$$

interconnected set of
atoms $\{i, j, \dots\}$ molecules

no - molecular/molecular
interaction

density matrix
highly mixed.

Liquid State NMR

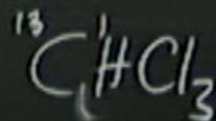
$$\mathcal{H} = \sum_I^{\text{mols.}} \omega_I \sigma_z^I + \sum_{I \neq J}^{\text{mols.}} \omega_{IJ} \sigma_z^I \sigma_z^J$$

ensemble of 10^{17}
molecules

interconnected set of
atoms ξ 1 molecule

no - molecular/molecular
interaction

density matrix
highly mixed.



Liquid State NMR

$$\mathcal{H} = \sum_i^{\text{mole.}} \omega_i \sigma_z^i + \sum_{i \neq j}^{\text{mole.}} \omega_{ij} \sigma_z^i \sigma_z^j$$

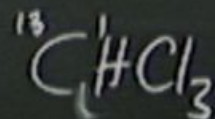
↑
Fermi

ensemble of 10^{17}
molecules

interconnected set of
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Liquid State NMR

$$\mathcal{H} = \sum_i^{\text{mols.}} \omega_i \sigma_z^i + \sum_{i \neq j}^{\text{mols.}} \omega_{ij} \sigma_z^i \sigma_z^j$$

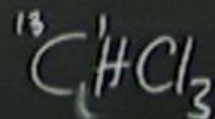
↑
Fermi

ensemble of 10^{17}
molecules

interconnected set of
atoms ξ 1 molecule

no - molecular/molecular
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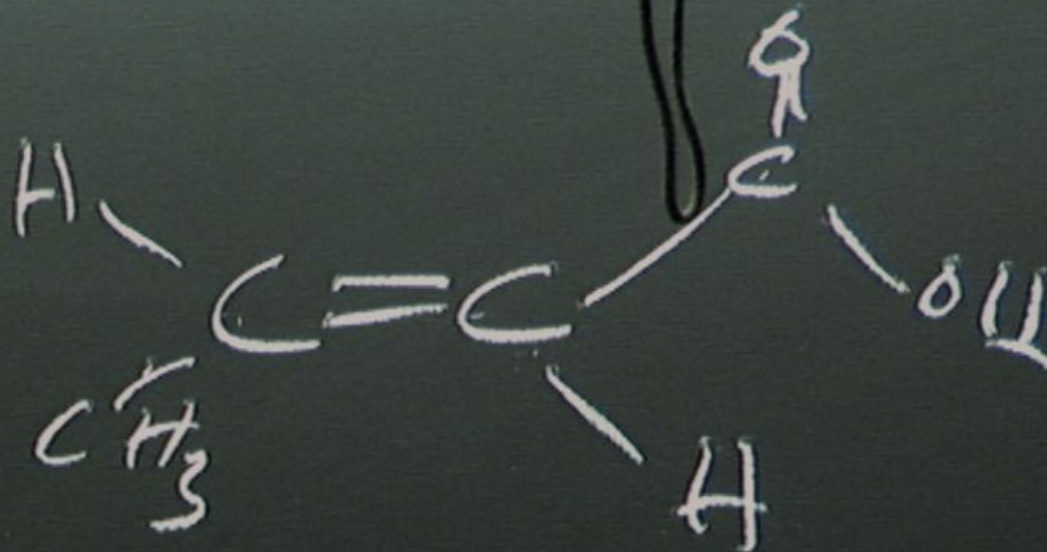
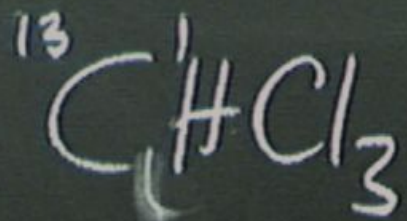
density matrix
highly mixed



$$\Delta W(\sigma_+^1 - \sigma_+^2) + W_3(\sigma_+^1 + \sigma_+^2)$$

$$\sigma_+ \sigma_- + \sigma_- \sigma_+ + \sigma_+ \sigma_+$$





Liquid State NMR

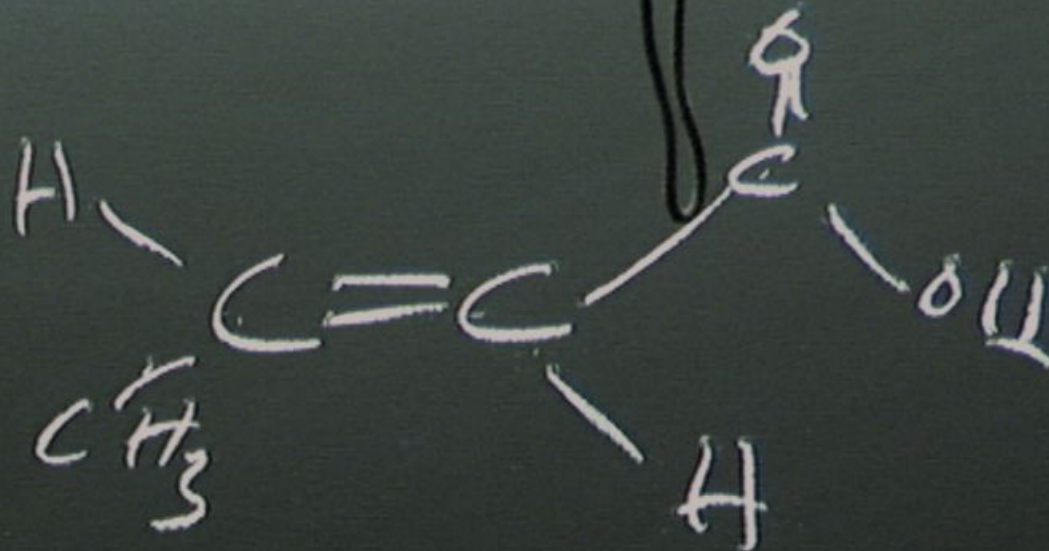
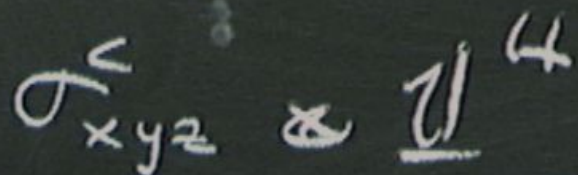
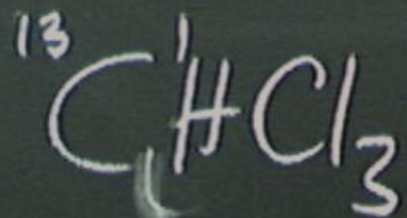
ensemble of 10^{17} molecules.

$$\mathcal{H} = \sum_{i \in \text{mols.}} \omega_i \sigma_z^i + \sum_{i \neq j \in \text{mols.}} \omega_{ij} \sigma_z^i \sigma_z^j$$

↑ chemistry ↑ Fermi

interconnected atoms
 no-molecular calculation.
 density
 $i \in \{\text{molecules}\}$

$$\mathcal{H}_{\text{cont}} = \omega_i \sigma_x^i$$



Liquid State NMR

ensemble of 10^{17} molecules.

$$\mathcal{H} = \sum_i^{\text{mols.}} \omega_i \sigma_z^i + \sum_{i \neq j}^{\text{mols.}} \omega_{ij} \sigma_z^i \sigma_z^j$$

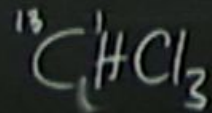
↑ chemistry ↑ Fermi

interconnected set of atoms $\{i, \text{molecules}\}$
 no - molecular/molecular interaction.

$$\mathcal{H}_{\text{cont}} = \omega_i \sigma_x^i \quad i \in \{\text{molecules}\}$$

density matrix highly mixed.

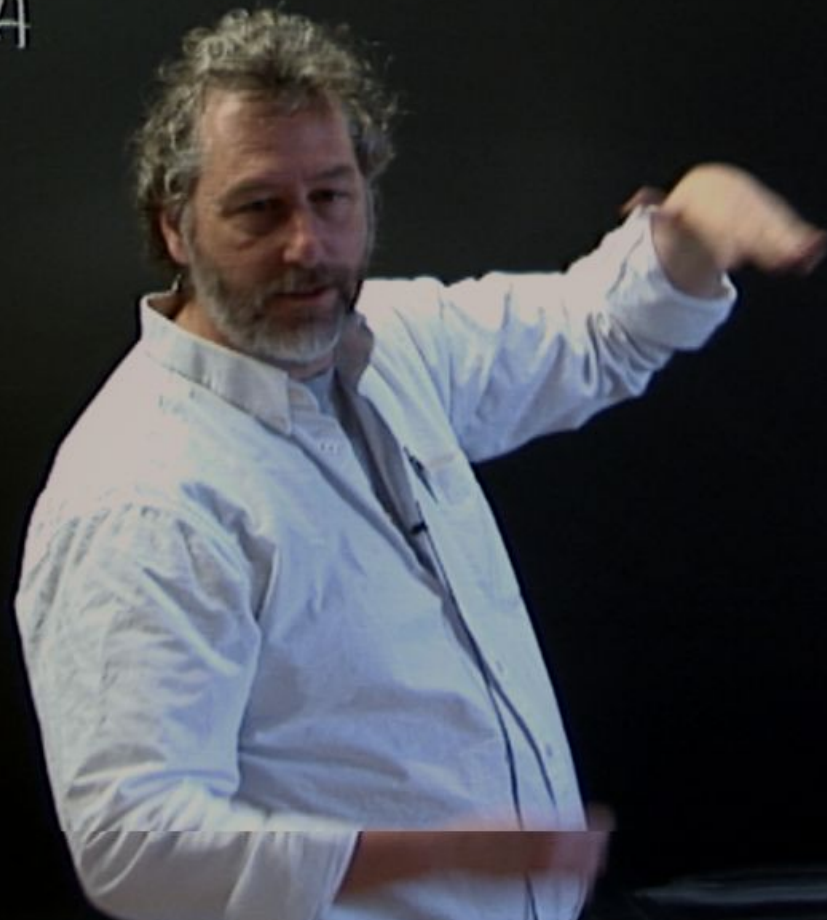
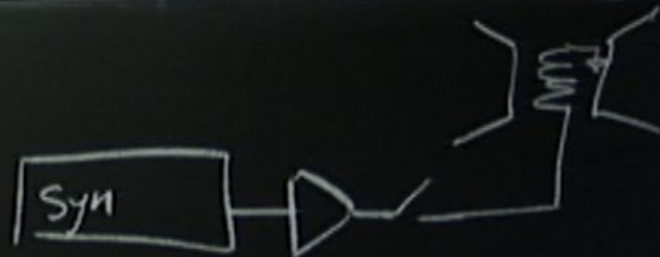
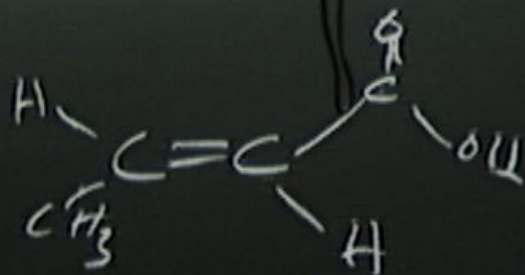
$$e^{-\beta \mathcal{H}} = e^{-\beta \sum_i \omega_i \sigma_x^i} e^{-\beta \sum_{i \neq j} \omega_{ij} \sigma_z^i \sigma_z^j} e^{-\beta \sum_{i \neq j} \omega_{ij} \sigma_x^i \sigma_x^j}$$

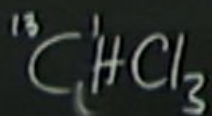


$$\sigma_{xy}^c \propto \uparrow \text{H}$$

$$\sigma_{xy}^+ \propto \uparrow \text{C}$$

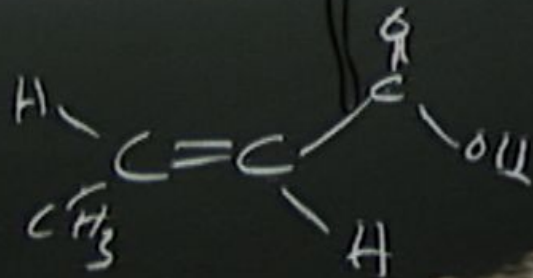
$$\sigma_{xy}^c \propto \sigma_{xy}^+$$





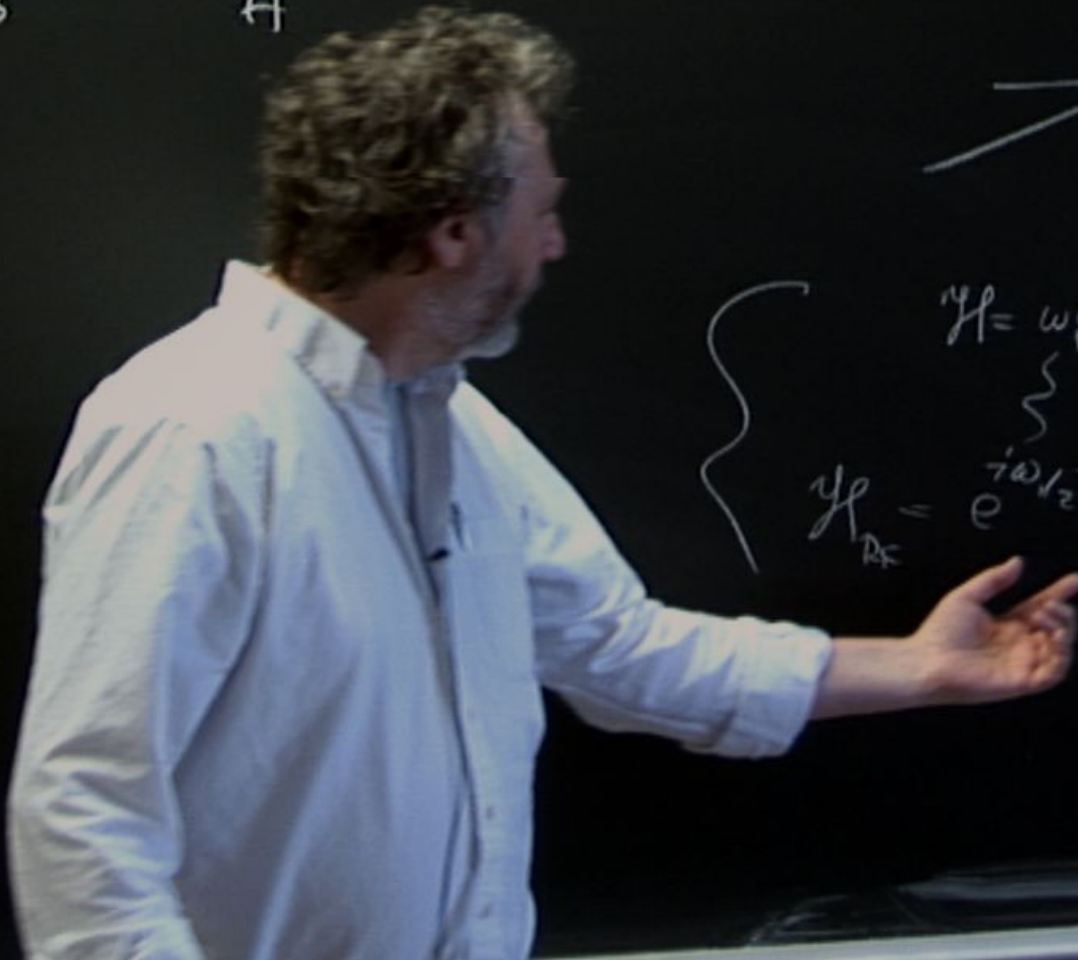
$$\sigma_{\text{CH}_2}^{\text{C}} \approx \sigma_{\text{CH}_2}^{\text{H}}$$

$$\sigma_{\text{CH}_2}^{\text{C}} \approx \sigma_{\text{CH}_2}^{\text{H}}$$



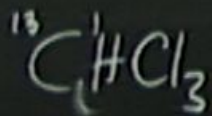
$$H = \omega_1 \sigma_1 + \omega_2 \sigma_2$$

$$U_{RF} = e^{i\omega_1 t \sigma_1} e^{i\omega_2 t \sigma_2}$$



$$\mathcal{H} = \omega_1 \sigma_z^1 + \omega_2 \sigma_z^2$$

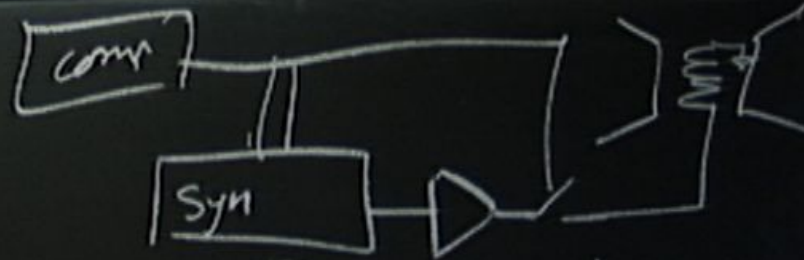
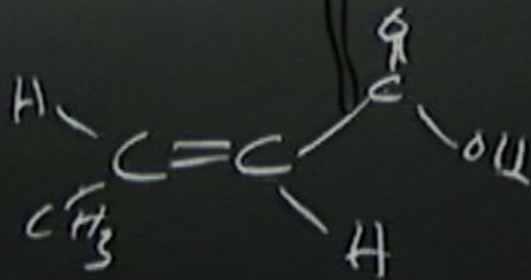
$$\mathcal{H}_{RF} = e^{-i\omega_1/2 + \sigma_z^1} \omega_R \sigma_x e^{-i\omega_2/2 + \sigma_z^2}$$



$$\sigma_{xy}^{\text{C}} \approx \sigma_{xy}^{\text{H}}$$

$$\sigma_{xy}^{\text{H}} \approx \sigma_{xy}^{\text{C}}$$

$$\sigma_{xy}^{\text{C}} \approx \sigma_{xy}^{\text{H}}$$



Liquid State NMR

ensemble of 10^{17}
molecules

$$\mathcal{H}_{int} = \sum_i^{mols.} \omega_i \sigma_z^i + \sum_{i \neq j}^{mols.} \omega_{ij} \sigma_z^i \sigma_z^j$$

↑ chemistry ↑ Fermi

interconnected set of
atoms ξ 1 molecule

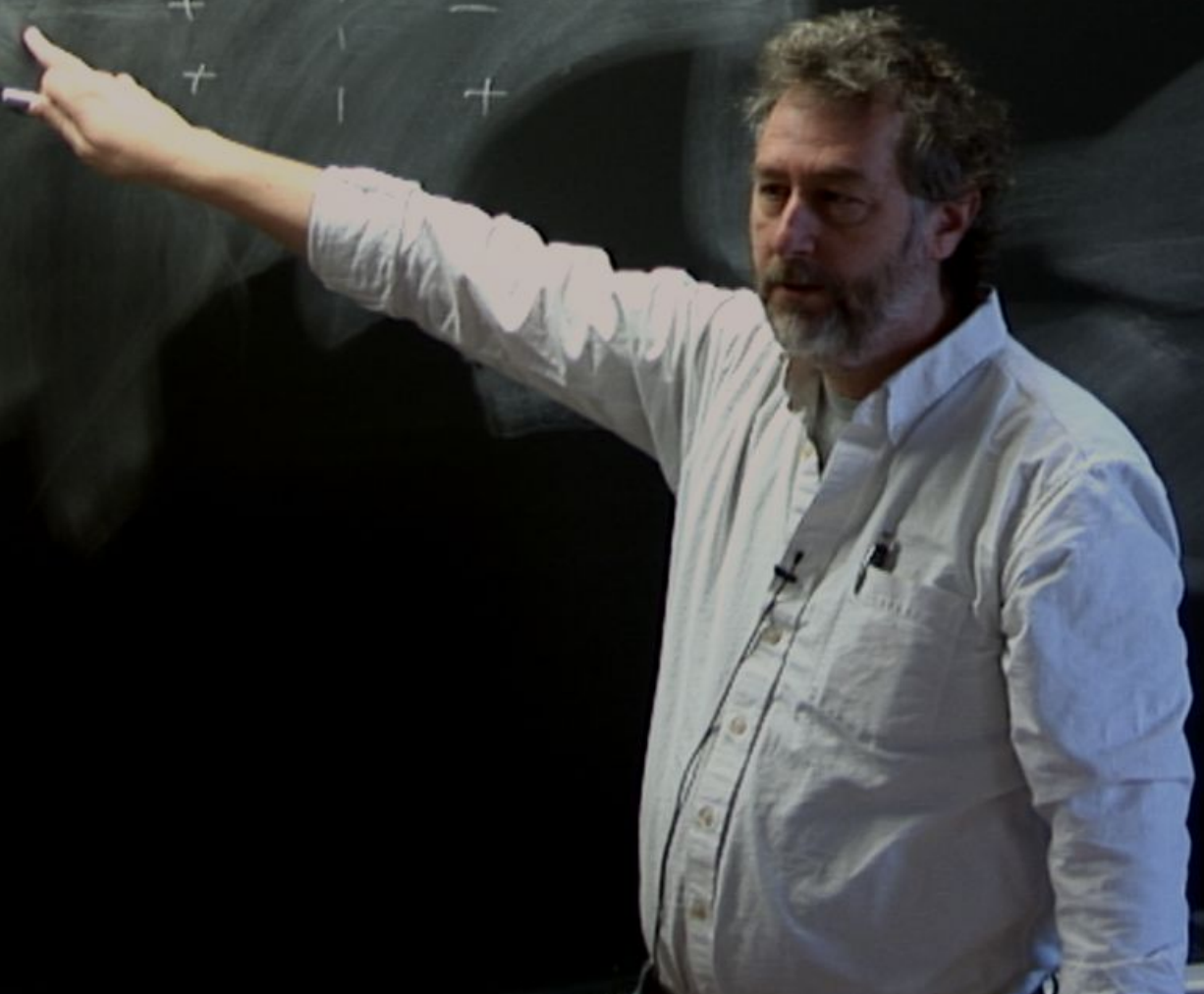
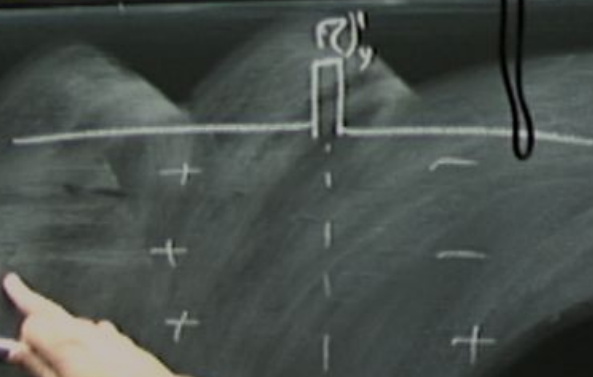
no-molecular/mo
interact

density matrix
highly mixed.

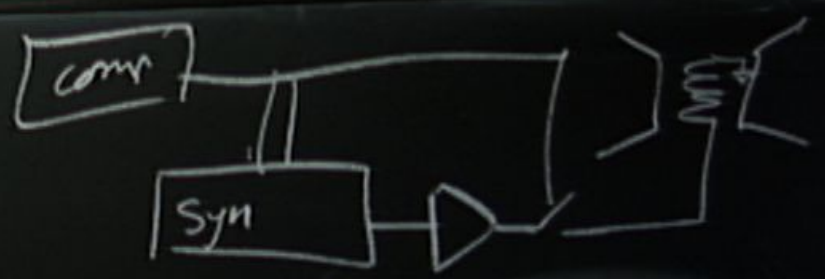
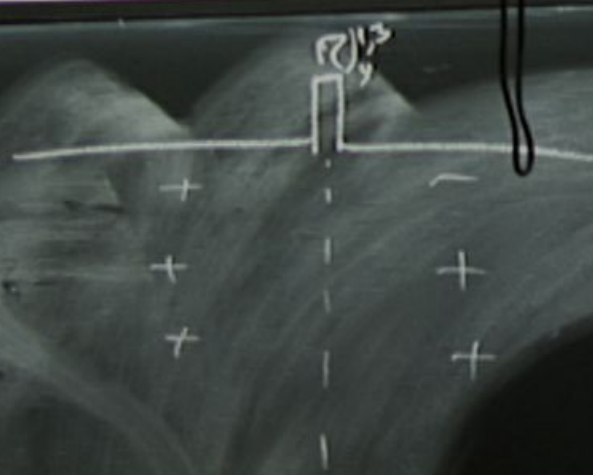
$$\mathcal{H}_{cont} = \omega_i \sigma_x^i \quad i \in \{molecules\}$$

$$\sigma_x^i \sigma_x^j \quad \sigma_y^i \sigma_y^j \quad \sigma_z^i \sigma_z^j$$

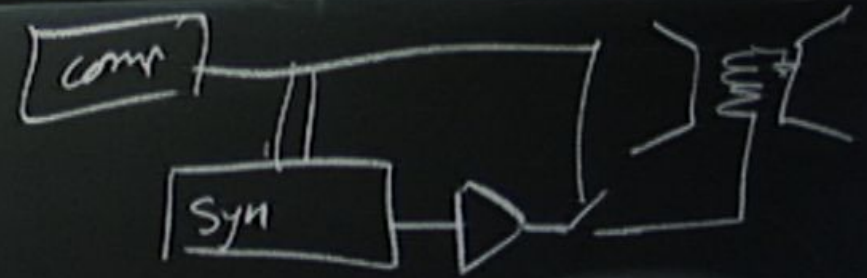
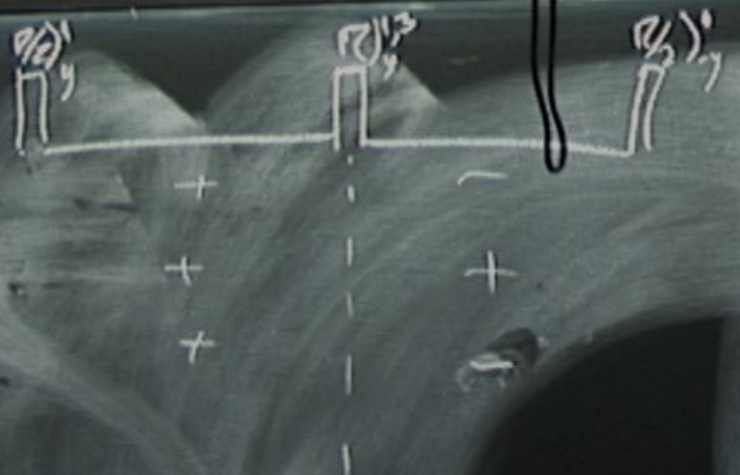
σ_2^1	σ_2^2		+		-
σ_2^1	σ_2^3		+		-
σ_2^2	σ_2^4		+		+



$\sigma_2^1 \sigma_2^2$
 $\sigma_2^1 \sigma_2^3$
 $\sigma_2^2 \sigma_2^3$



$\sigma_2^1 \sigma_2^2$
 $\sigma_2^1 \sigma_2^3$
 $\sigma_2^2 \sigma_2^3$



Liquid State NMR

ensemble of 10^{17} molecules

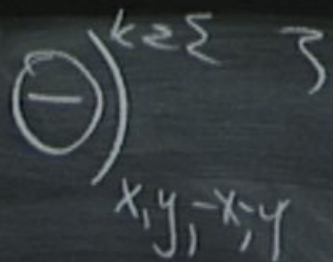
$$\mathcal{H}_{int} = \sum_i^{mols.} \omega_i \sigma_z^i + \sum_{i \neq j}^{mols.} \omega_{ij} \sigma_z^i \sigma_z^j$$

↑ chemical shift ↑ Fermi

interconnected set of atoms $\{i, \text{molecules}\}$
 no-molecular/molecular interaction

density matrix highly mixed

$$\mathcal{H}_{cont} = \omega_i \sigma_x^i \quad i \in \{\text{molecules}\}$$



Liquid State NMR

ensemble of 10^{17} molecules.

$$H_{int} = \sum_i^{mols.} \omega_i \sigma_z^i + \sum_{i \neq j}^{mols.} \omega_{ij} \sigma_z^i \sigma_z^j$$

↑ chemical shift ↑ Fermi

interconnected set of atoms $\{i, \text{molecules}\}$

no-molecular/molecular interaction.

density matrix highly mixed.

$$H_{cont} = \omega_i \sigma_x^i \quad i \in \{\text{molecules}\}$$

$$\left(\begin{array}{c} \ominus \\ \oplus \end{array} \right)_{x,y, -x,-y}^{k \in \Sigma}$$

$$\sigma_x^i \sigma_x^j$$

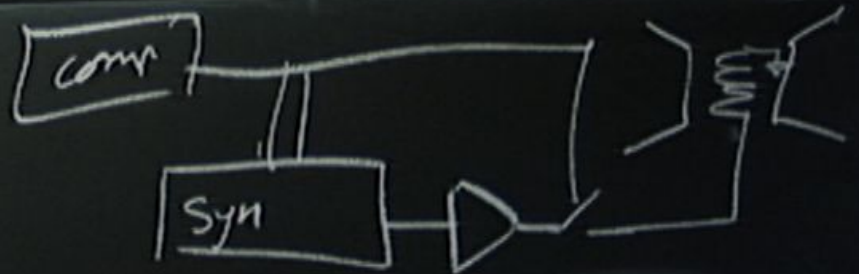
$$U = \sigma_x E_- + U E_+$$
$$E_{\pm} = \frac{1}{\sqrt{2}} (1 \pm \sigma_z)$$



$$U = \sigma_x^A E_-^B + \mathbb{1} E_+^B$$

$$E_{\pm} = \frac{1}{2} (\mathbb{1} \pm \sigma_z)$$

$$(E_{\pm})^2 = E_{\pm} ; E_+ E_- = 0$$



$$U = \sigma_x^A E_-^B + \mathbb{1} E_+^B$$

$$E_+ = \frac{1}{2}(\mathbb{1} + \sigma_z)$$

$$(E_+)^2 = E_+ ; E_+ E_- = 0$$

$$e^{A E_+} = e^A E_+ + E_-$$

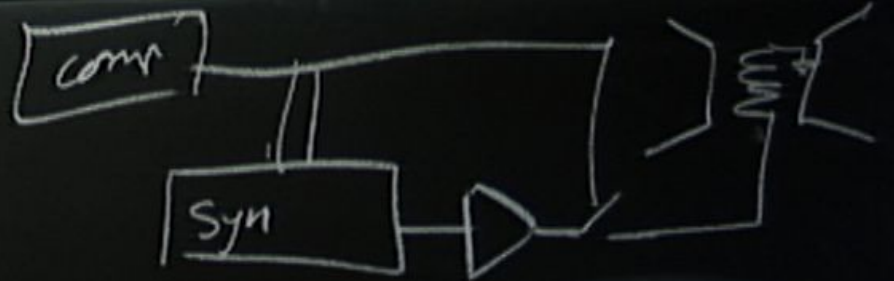


$$U = \sigma_x^A E_-^B + \mathbb{1} E_+^B$$

$$E_+ = \frac{1}{2}(\mathbb{1} + \sigma_z)$$

$$(E_+)^2 = E_+ ; E_+ E_- = 0$$

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$$U = \sigma_x^A E_-^B + \mathbb{1} E_+^B$$

$$E_+ = \frac{1}{2} (\mathbb{1} + \sigma_z)$$

$$(E_+)^2 = E_+ ; E_+ E_- = 0$$

$$e^{A E_+} = e^A E_+ + E_- ; \dots = 0$$



$$U = a_x^A E_-^B + \mathbb{1} E_+^B$$

$$E_+ = \frac{1}{2}(\mathbb{1} + \sigma_z)$$

$$(E_+)^2 = E_+ ; E_+ E_- = 0$$

$$e^{A E_+} = e^A E_+ + E_- ; [A, E_+] = 0$$



$$U = \sigma_x^A E_-^B + \mathbb{1} E_+^B$$

$$E_+ = \frac{1}{2}(\mathbb{1} + \sigma_z)$$

$$(E_+)^2 = E_+ ; E_+ E_- = 0$$

$$e^{A E_+} = e^A E_+ + E_- ; [A, E_+] = 0$$

$$e^{i\frac{\pi}{4}} e^{-i\frac{\pi}{2}\sigma_x} e^{-i\frac{\pi}{4}\mathbb{1}\sigma_z} e^{i\sigma_x\sigma_z\frac{\pi}{4}}$$



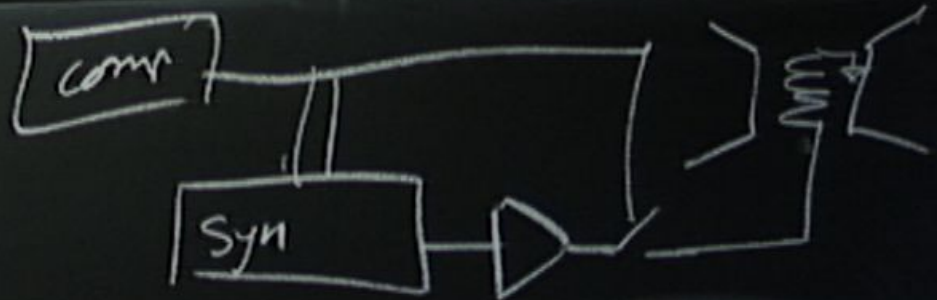
$$U = \sigma_x^A E_-^B + \mathbb{1} E_+^B$$

$$E_+ = \frac{1}{2}(\mathbb{1} + \sigma_z)$$

$$(E_+)^2 = E_+ ; E_+ E_- = 0$$

$$e^{A E_+} = e^A E_+ + E_- ; [A, E_+] = 0$$

$$U = e^{i\pi/4} e^{-i\pi/2 \sigma_x \mathbb{1}} e^{-i\pi/4 \mathbb{1} \sigma_z} e^{i\sigma_x \sigma_z \pi/4}$$



Liquid State NMR

ensemble of 10^{17} molecules

$$\mathcal{H}_{int} = \sum_i^{mole.} \omega_i \sigma_z^i + \sum_{i \neq j}^{mole.} \omega_{ij} \sigma_z^i \sigma_z^j$$

↑ chemical shift ↑ Fermi

interconnected set of atoms $\{i, j, \dots\}$ molecules
 no - molecular/molecular interaction

$$\mathcal{H}_{cont} = \omega_i \sigma_x^i \quad i \in \{molecules\}$$

density matrix highly mixed

$$\left(\begin{array}{c} \ominus \\ \oplus \end{array} \right)_{x,y, -x,-y}^{k \in \Sigma}$$

$$\sigma_z^i \sigma_z^j$$

$$U = \sigma_x^A E_-^B + \mathbb{1} E_+^B$$

$$E_+ = \frac{1}{2}(\mathbb{1} + \sigma_z)$$

$$(E_+)^2 = E_+ ; E_+ E_- = 0$$

$$e^{A E_+} = e^A E_+ + E_- ; [A, E_+] = -0$$

$$U = e^{i\pi/4} \underbrace{e^{-i\pi/2 \sigma_x \mathbb{1}}}_{\left(\frac{\pi}{2}\right)_x} \underbrace{e^{-i\pi/4 \mathbb{1} \sigma_z}}_{\left(\frac{\pi}{2}\right)_z} \underbrace{e^{+i\sigma_x \sigma_z \pi/4}}_{\left(\frac{\pi}{2}\right)_y}$$

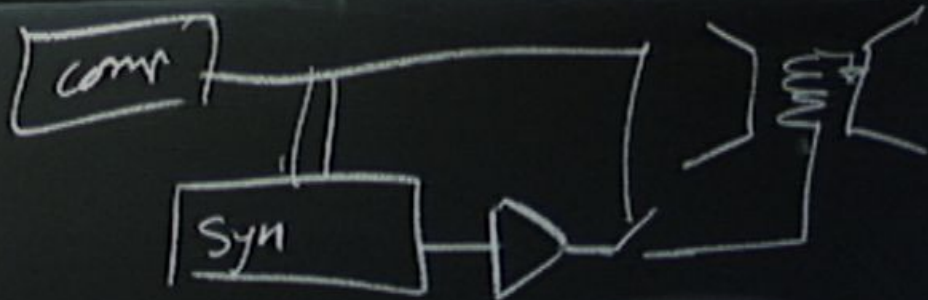
$$\left(\frac{\pi}{2}\right)_x$$

$$\left(\frac{\pi}{2}\right)_z$$

$$\left(\frac{\pi}{2}\right)_y$$

$$\left(\frac{\pi}{2}\right)_{yz}$$

$$\left(\frac{\pi}{2}\right)_{-y}$$



$$U = \sigma_x^A E_-^B + \mathbb{1} E_+^B$$

$$E_+ = \frac{1}{2}(\mathbb{1} + \sigma_z)$$

$$(E_+)^2 = E_+ ; E_+ E_- = 0$$

$$e^{A E_+} = e^A E_+ + E_- ; [A, E_+] = 0$$

$$U = e^{i\pi/4} e^{-i\pi/2 \sigma_x} e^{-i\pi/4 \mathbb{1} \sigma_z} e^{i\sigma_x \sigma_z \pi/4}$$

$$\left(\frac{\pi}{2}\right)_x$$

$$\left(\frac{\pi}{2}\right)_z$$

$$\left(\frac{\pi}{2}\right)_y$$

$$\left(\frac{\pi}{2}\right)_{zz}$$

$$\left(\frac{\pi}{2}\right)_{yy}$$



Liquid State NMR

ensemble of 10^{17} molecules

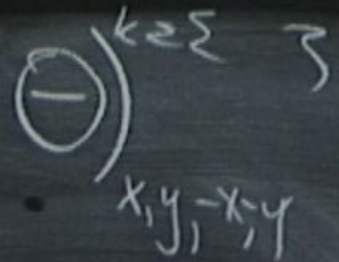
$$\mathcal{H}_{int} = \sum_i^{mole.} \omega_i \sigma_z^i + \sum_{i \neq j}^{mole.} \omega_{ij} \sigma_z^i \sigma_z^j$$

↑ chemical shift ↑ Fermi

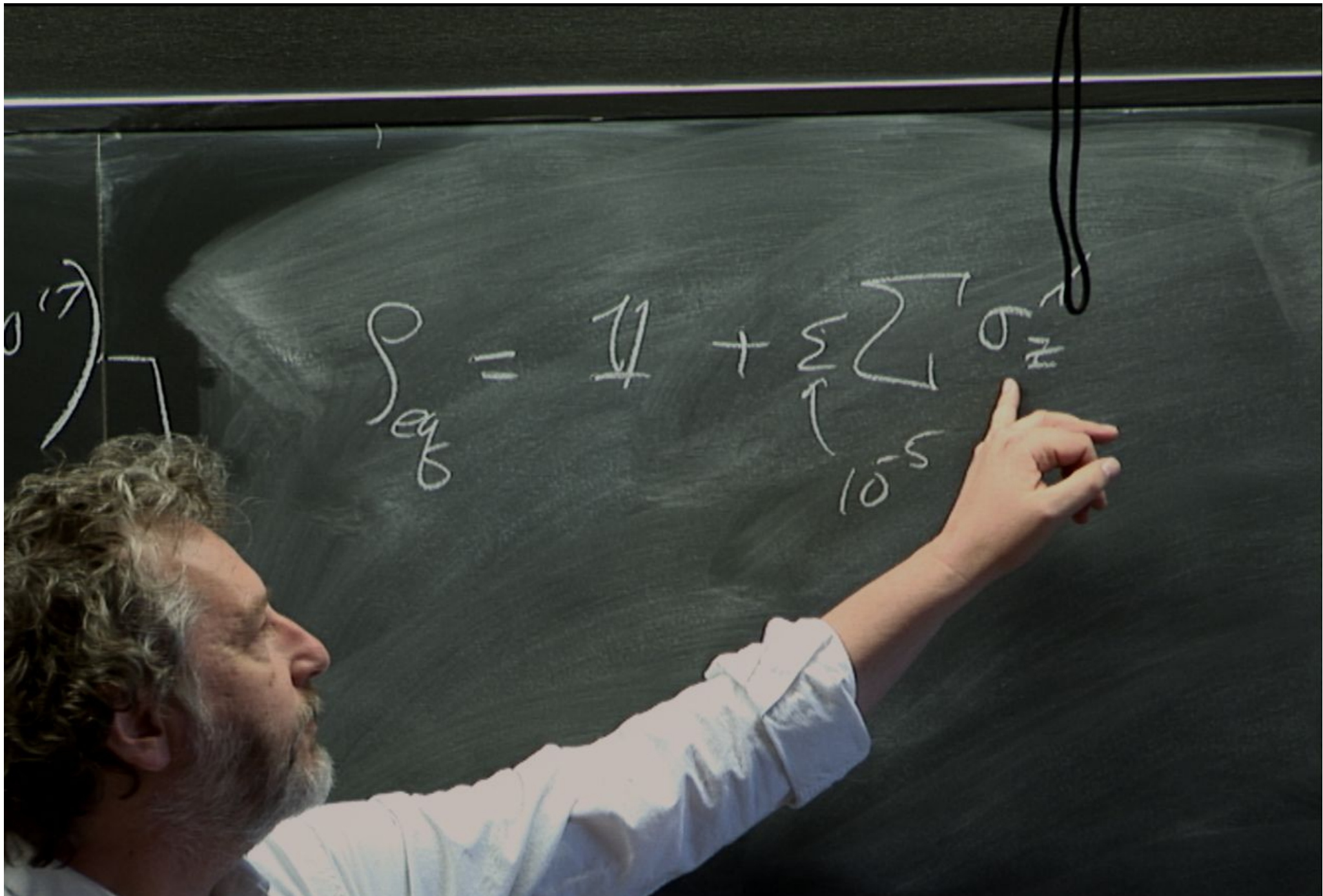
interconnected set of atoms $\{i, \text{molecules}\}$
 no - molecular / molecular interaction

density matrix highly mixed

$$\mathcal{H}_{cont} = \omega_i \sigma_x^i \quad i \in \{\text{molecules}\}$$



$$\rho \sigma_z^i \sigma_z^j$$



$$\rho_{eq} = \mathbb{1} + \sum_{\omega} \rho_{\omega}$$

\uparrow
 10^{-5}

$$\rho_{\text{eq}} = \mathbb{1} + \sum_{\uparrow} \sum_{\downarrow} \sigma_z^{\uparrow} \sigma_z^{\downarrow}$$

10^{-5}

$$\rho = |00\rangle\langle 00|$$

$$= \mathbb{1} + \sigma_z^{\uparrow} + \sigma_z^{\downarrow} + \sigma_z^{\uparrow} \sigma_z^{\downarrow}$$

$$\rho_{\text{eq}} = \mathbb{1} + \sum_{\substack{\uparrow \\ 10^{-5}}} \sigma_z^2$$

$$\rho = |00\rangle\langle 00|$$

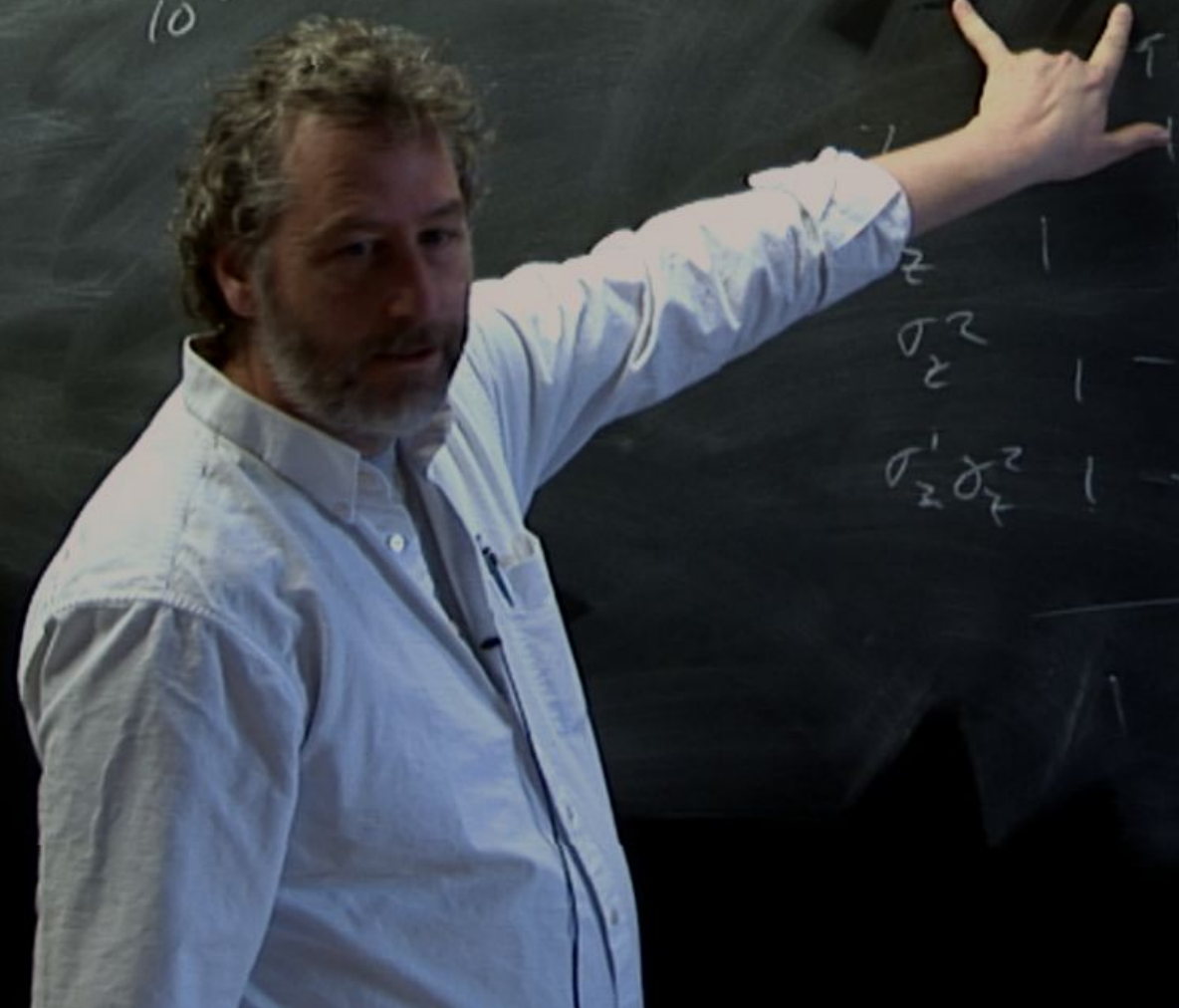
$$= \mathbb{1} + \sigma_z^1 + \sigma_z^2 + \sigma_z^1 \sigma_z^2$$

$$\rho = |00\rangle\langle 00|$$

$$= |1\rangle\langle 1| + \sigma_z^1 + \sigma_z^2 + \sigma_z^1 \sigma_z^2$$

$$\rho_{\text{eq}} = \mathbb{1} + \sum_{\substack{\uparrow \\ 10^5}} \sigma_z^2$$

	$\uparrow\downarrow$	$\downarrow\uparrow$	$\downarrow\downarrow$
$\uparrow\downarrow$	1	1	1
$\downarrow\uparrow$	1	1	1
$\downarrow\downarrow$	1	1	1
σ_z^1	1	-1	1
σ_z^2	1	-1	1
$\sigma_z^1 \sigma_z^2$	1	1	-1



1 0 0 0

$$\rho = 1000 > (\infty)$$

$$\rho_{eq} = 1 + \sum_{\substack{\uparrow \\ 10^{-5}}} \sum_{\substack{\uparrow \\ H}} \sigma_{\substack{\uparrow \\ H}}^2$$

$$= 1 + \sigma_z^1 + \sigma_z^2 + \sigma_z^1 \sigma_z^2$$

	↑↑	↑L	↓↑	↓↓
1	1	1	1	1
σ_z	1	1	-1	-1
σ_z^2	1	-1	1	-1
$\sigma_z^1 \sigma_z^2$	1	-1	-1	1

1 0 0 0

$$\rho = |000\rangle\langle 000|$$

$$\rho_{\text{eq}} = \mathbb{1} + \sum_{\sigma_z} \sigma_z \otimes \sigma_z \otimes \sigma_z$$

\uparrow
 10^{-5}

$$\rho_{\text{TPP}} = (1-\eta)\mathbb{1} + \eta|00\rangle\langle 00|$$

$$= \mathbb{1} + \sigma_z^1 + \sigma_z^2 + \sigma_z^1 \sigma_z^2$$

	↑↑	↑↓	↓↑	↓↓
1	1	1	1	1
σ_z^1	1	1	-1	-1
σ_z^2	1	-1	1	-1
$\sigma_z^1 \sigma_z^2$	1	-1	-1	1

1	0	0	0
---	---	---	---

$$\rho = |000\rangle\langle 000|$$

$$\rho_{\text{eq}} = \mathbb{1} + \sum_{\sigma_z} \sigma_z$$

10^{-5}

$$= \mathbb{1} + \sigma_z^1 + \sigma_z^2 + \sigma_z^1 \sigma_z^2$$

$$\rho_{\text{PP}} = (1-\eta) \mathbb{1} + \eta |00\rangle\langle 00|$$

$\frac{M}{L}$

	$\uparrow\uparrow$	$\uparrow\downarrow$	$\downarrow\uparrow$	$\downarrow\downarrow$
$\mathbb{1}$	1	1	1	1
σ_z^1	1	1	-1	-1
σ_z^2	1	-1	1	-1
$\sigma_z^1 \sigma_z^2$	1	-1	-1	1

0 0 0

$$\rho = |00\rangle\langle 00|$$

$$\rho_{\text{eq}} = \mathbb{1} + \epsilon \sum_{\sigma \neq \mathbb{1}} \sigma$$

non-Unitary

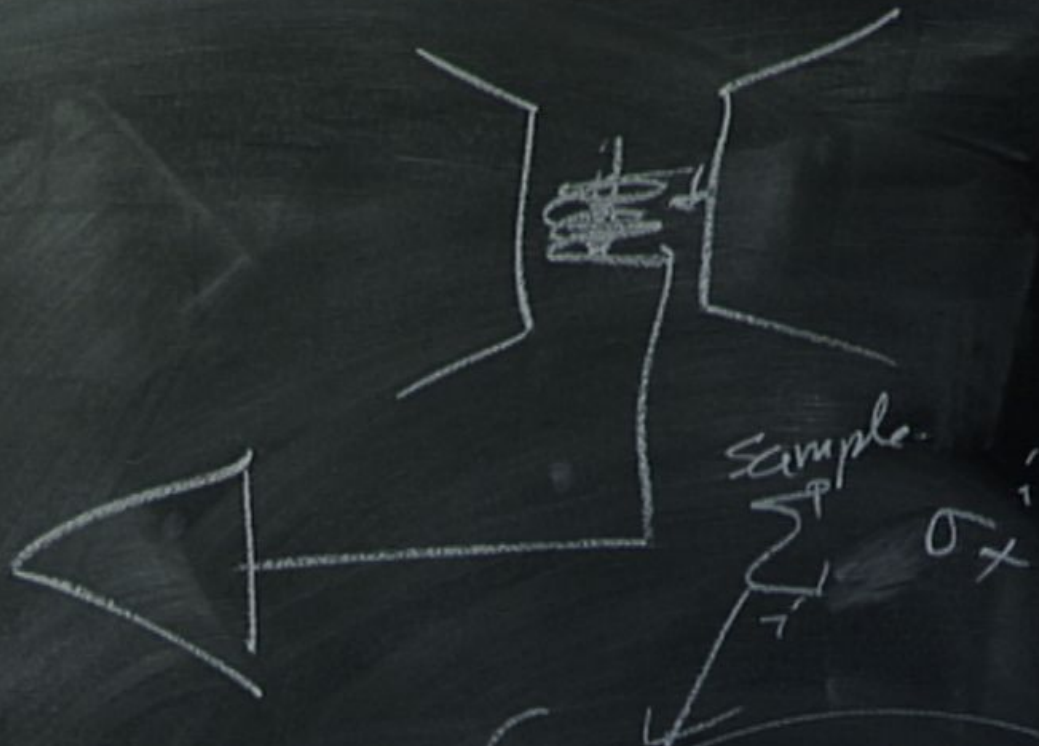
$$\rho_{\text{TPP}} = (1-\eta) \mathbb{1} + \eta |00\rangle\langle 00|$$

$\eta/5$

$$= \mathbb{1} + \sigma_z^1 + \sigma_z^2 + \sigma_z^1 \sigma_z^2$$

	$\uparrow\uparrow$	$\uparrow\downarrow$	$\downarrow\uparrow$	$\downarrow\downarrow$
$\mathbb{1}$	1	1	1	1
σ_z^1	1	1	-1	-1
σ_z^2	1	-1	1	-1
$\sigma_z^1 \sigma_z^2$	1	-1	-1	1

0 0 0



$$\eta \left(\frac{-d}{dt} \right) \frac{M(t) \cdot B_{coil}}{I B_{coil}} dt$$

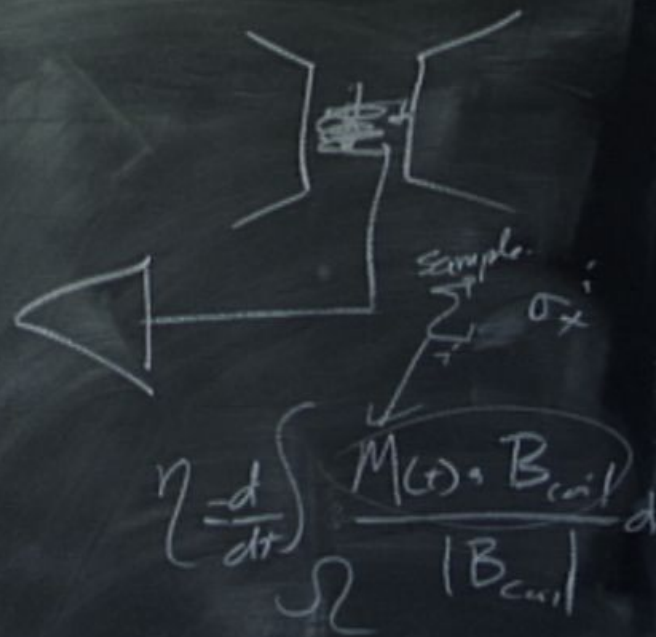
Ω

$$|4\rangle = \alpha|0\rangle + \beta|1\rangle$$

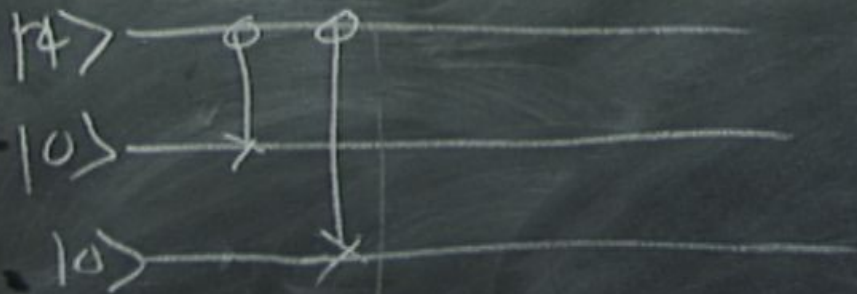
$$|4\rangle \longrightarrow$$

$$|0\rangle \longrightarrow$$

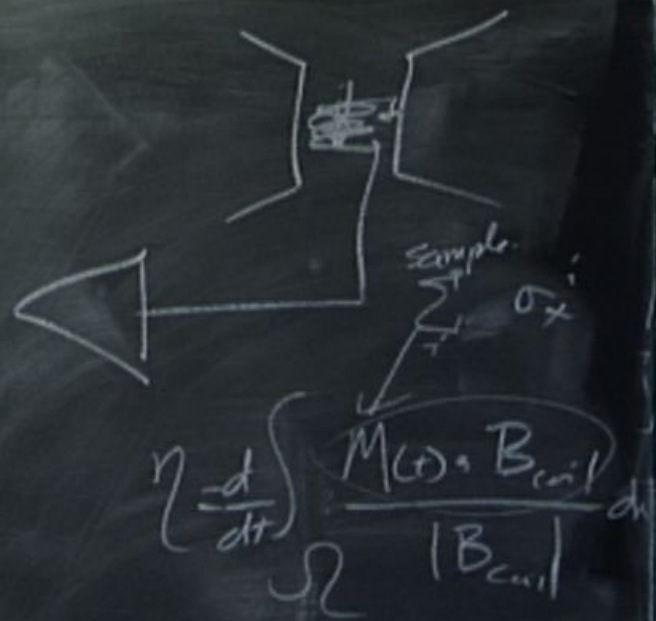
$$|0\rangle \longrightarrow$$



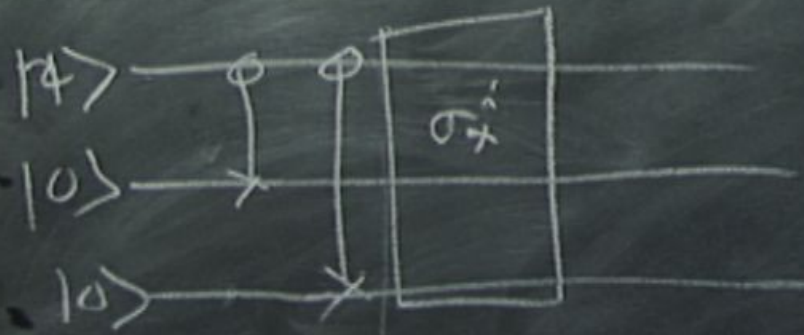
$$|4\rangle = \alpha|0\rangle + \beta|1\rangle$$



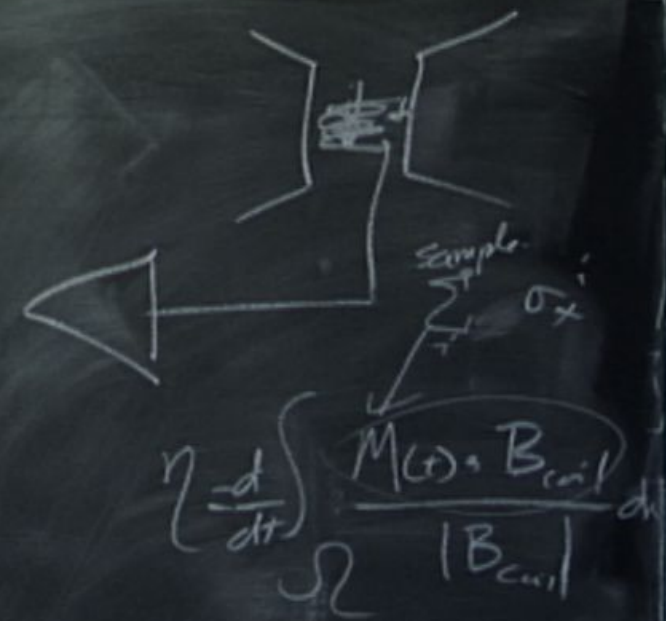
$$\alpha|000\rangle + \beta|111\rangle$$



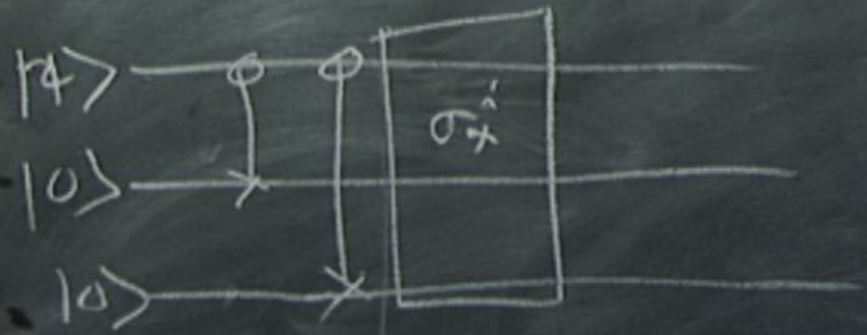
$$|4\rangle = \alpha|0\rangle + \beta|1\rangle$$



$$\alpha|520\rangle + \beta|111\rangle$$



$$|7\rangle = \alpha|10\rangle + \beta|11\rangle$$

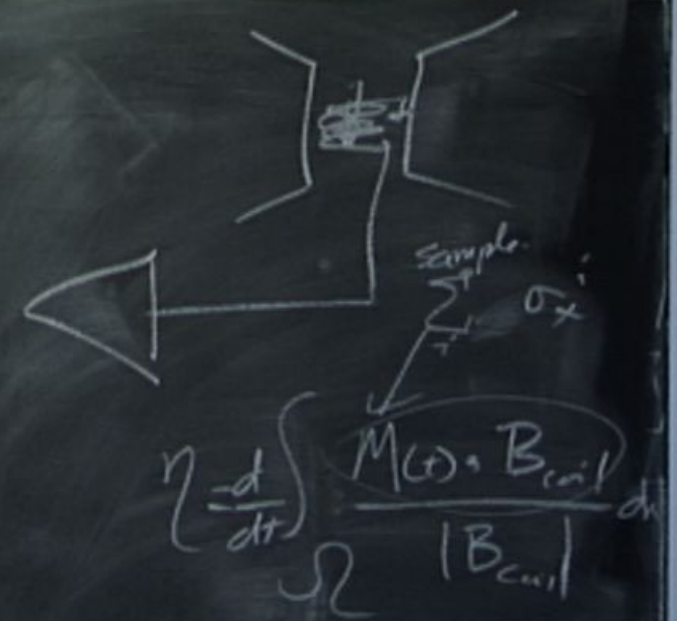


$$\alpha|000\rangle + \beta|111\rangle$$

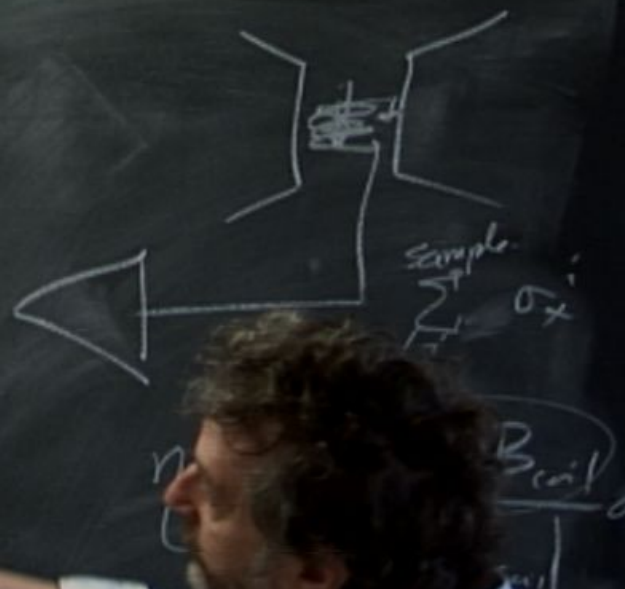
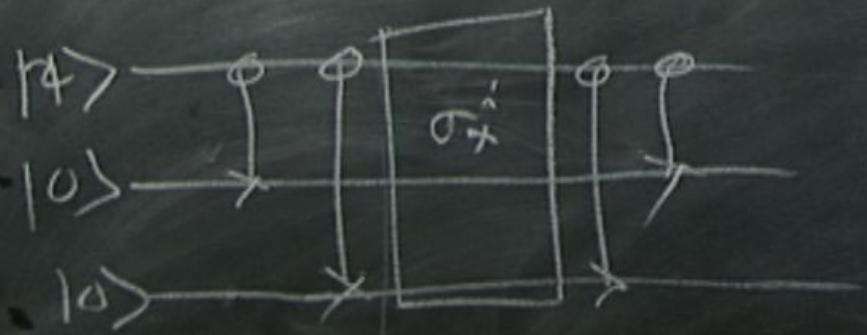
$$\alpha|100\rangle + \beta|011\rangle$$

$$\alpha|010\rangle + \beta|101\rangle$$

$$\alpha|001\rangle + \beta|110\rangle$$



$$|4\rangle = \alpha|10\rangle + \beta|11\rangle$$



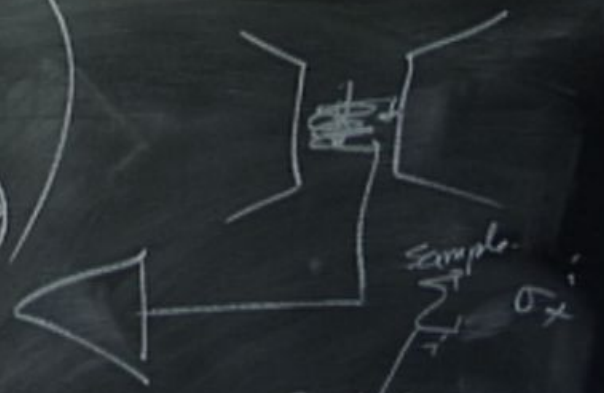
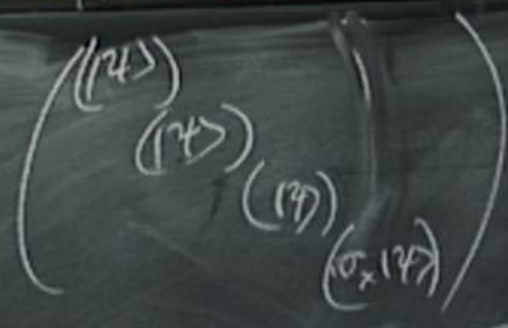
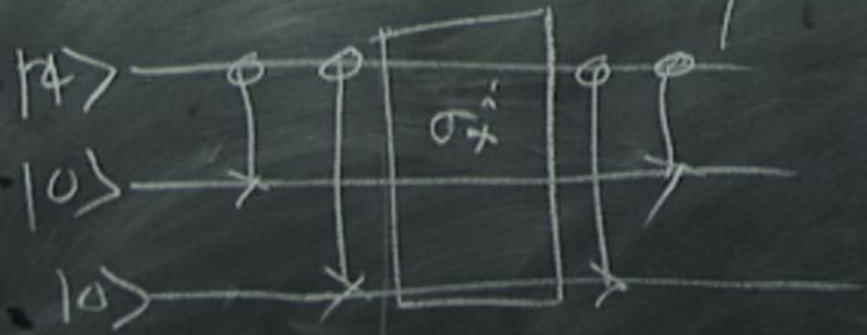
$$\alpha|000\rangle + \beta|111\rangle \rightarrow (\alpha|0\rangle + \beta|1\rangle) \otimes |00\rangle$$

$$\alpha|100\rangle + \beta|011\rangle \rightarrow (\alpha|1\rangle + \beta|0\rangle) \otimes |11\rangle$$

$$\alpha|010\rangle + \beta|101\rangle \rightarrow (\alpha|0\rangle + \beta|1\rangle) \otimes |10\rangle$$

$$\alpha|001\rangle + \beta|110\rangle \rightarrow (\alpha|0\rangle + \beta|1\rangle) \otimes |01\rangle$$

$$|14\rangle = \alpha|10\rangle + \beta|11\rangle$$



$$\eta \left(\frac{-d}{dt} \right) \frac{M(t) \cdot B_{coil}}{B_{coil}}$$

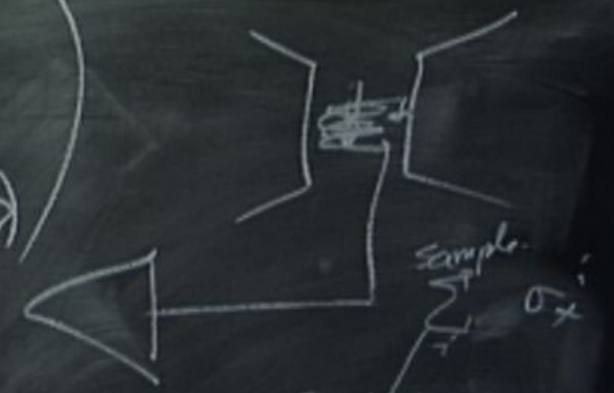
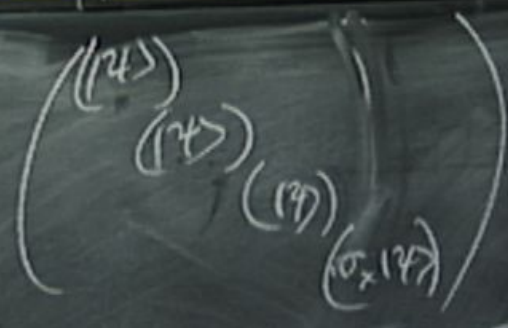
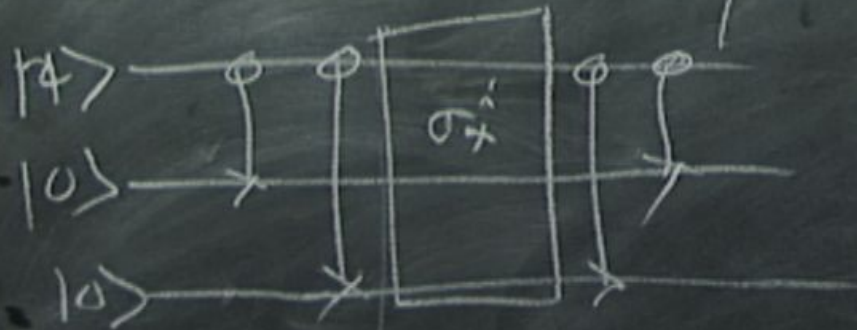
$$\alpha|000\rangle + \beta|111\rangle \rightarrow (\alpha|0\rangle + \beta|1\rangle) \otimes |00\rangle$$

$$\alpha|100\rangle + \beta|011\rangle \rightarrow (\alpha|1\rangle + \beta|0\rangle) \otimes |11\rangle$$

$$\alpha|010\rangle + \beta|101\rangle \rightarrow (\alpha|0\rangle + \beta|1\rangle) \otimes |10\rangle$$

$$\alpha|001\rangle + \beta|110\rangle \rightarrow (\alpha|0\rangle + \beta|1\rangle) \otimes |01\rangle$$

$$|74\rangle = \alpha|10\rangle + \beta|11\rangle$$



$$\eta \frac{-d}{dt} \frac{M(t) \cdot B_{coil}}{B_{coil}}$$

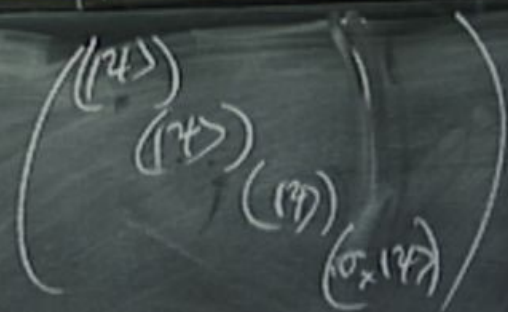
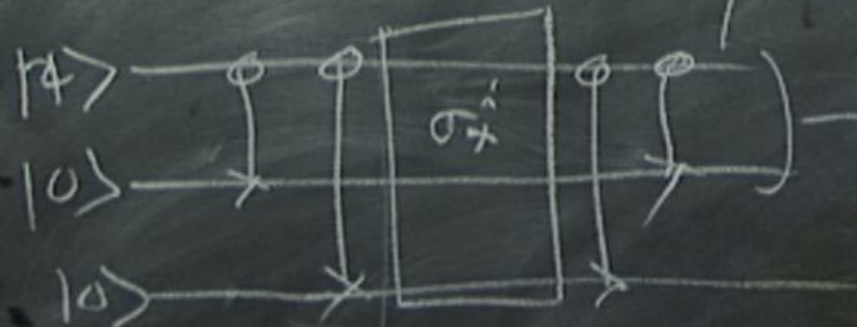
$$\alpha|520\rangle + \beta|111\rangle \rightarrow (\alpha|10\rangle + \beta|11\rangle) \otimes |00\rangle$$

$$\alpha|1100\rangle + \beta|1011\rangle \rightarrow (\alpha|11\rangle + \beta|10\rangle) \otimes |11\rangle$$

$$\alpha|1010\rangle + \beta|1101\rangle \rightarrow (\alpha|10\rangle + \beta|11\rangle) \otimes |10\rangle$$

$$\alpha|1001\rangle + \beta|1110\rangle \rightarrow (\alpha|10\rangle + \beta|11\rangle) \otimes |01\rangle$$

$$|14\rangle = \alpha|10\rangle + \beta|11\rangle$$



$$\eta = \frac{d}{dt} \left(\frac{M(t) \cdot B_{coil}}{B_{coil}} \right)$$

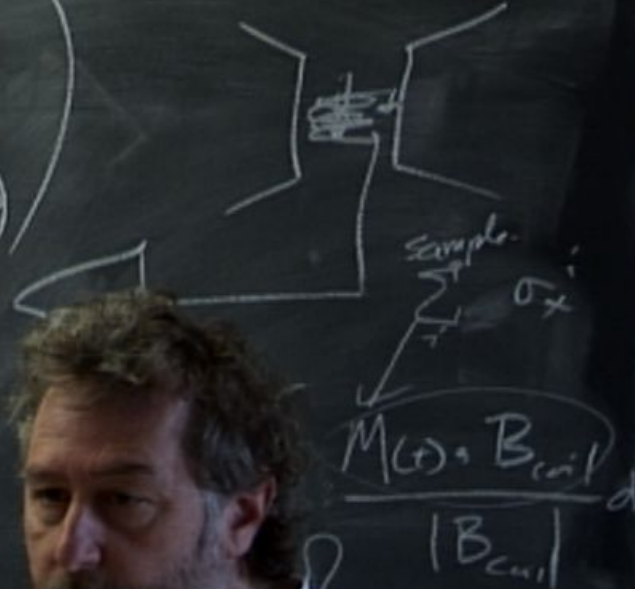
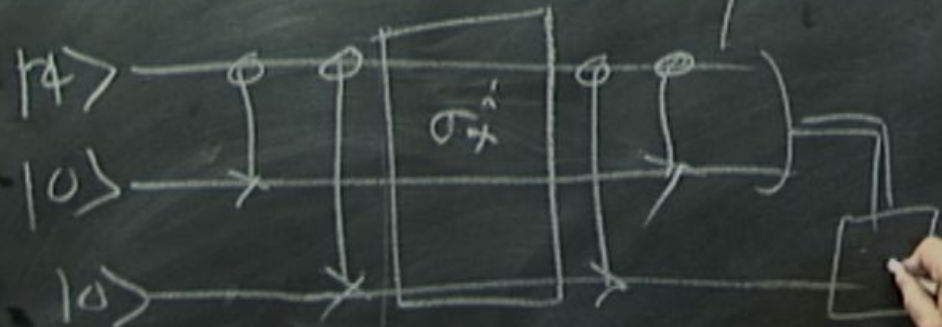
$$\alpha|000\rangle + \beta|111\rangle \rightarrow (\alpha|0\rangle + \beta|1\rangle) \otimes |00\rangle$$

$$\alpha|100\rangle + \beta|011\rangle \rightarrow (\alpha|1\rangle + \beta|0\rangle) \otimes |11\rangle$$

$$\alpha|010\rangle + \beta|101\rangle \rightarrow (\alpha|0\rangle + \beta|1\rangle) \otimes |10\rangle$$

$$\alpha|001\rangle + \beta|110\rangle \rightarrow (\alpha|0\rangle + \beta|1\rangle) \otimes |01\rangle$$

$$|7\rangle = \alpha|10\rangle + \beta|11\rangle$$



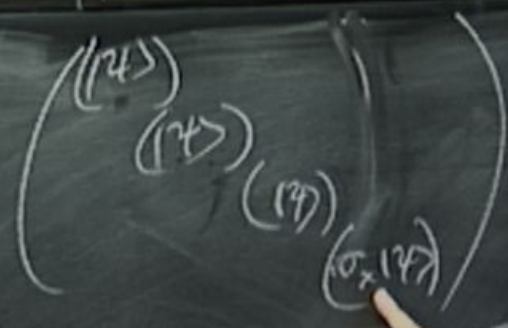
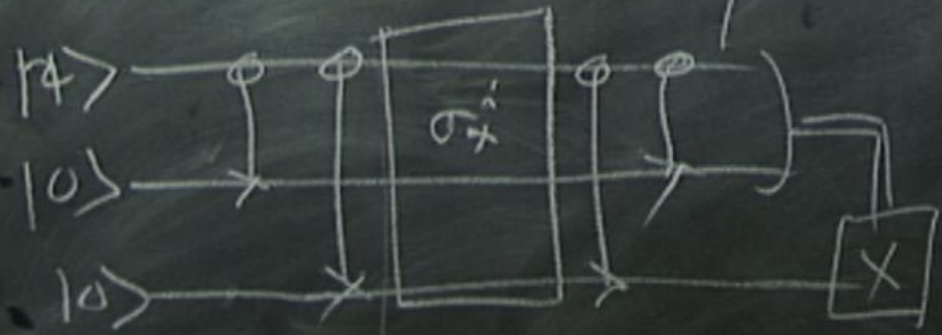
$$\alpha|520\rangle + \beta|111\rangle \rightarrow (\alpha|10\rangle + \beta|11\rangle) \otimes |00\rangle$$

$$\alpha|100\rangle + \beta|101\rangle \rightarrow (\alpha|11\rangle + \beta|10\rangle) \otimes |11\rangle$$

$$\alpha|1010\rangle + \beta|101\rangle \rightarrow (\alpha|10\rangle + \beta|11\rangle) \otimes |10\rangle$$

$$\alpha|001\rangle + \beta|110\rangle \rightarrow (\alpha|10\rangle + \beta|11\rangle) \otimes |01\rangle$$

$$|74\rangle = \alpha|10\rangle + \beta|11\rangle$$



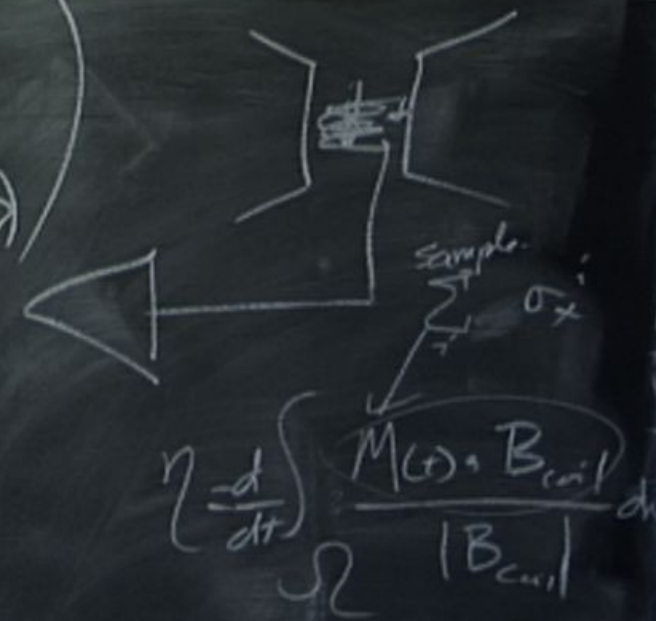
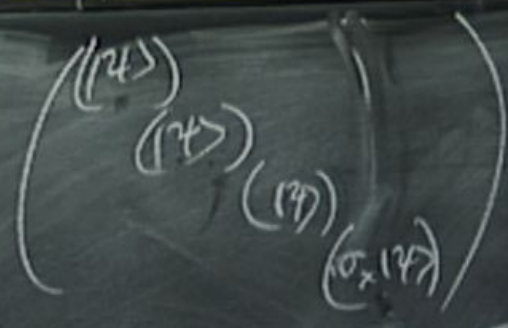
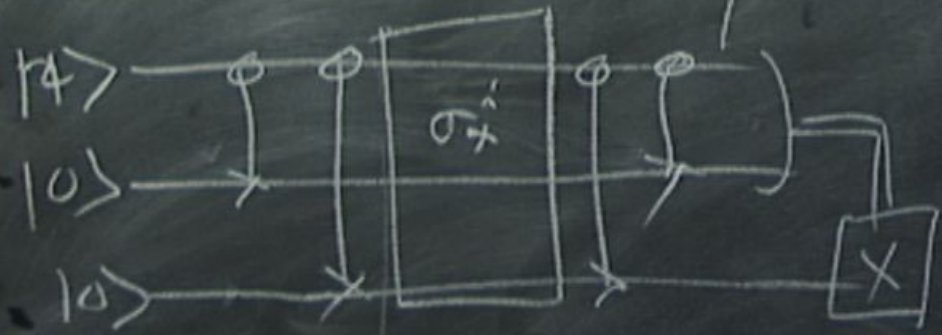
$$\alpha|000\rangle + \beta|111\rangle \rightarrow (\alpha|0\rangle + \beta|1\rangle) \otimes |00\rangle$$

$$\alpha|100\rangle + \beta|011\rangle \rightarrow (\alpha|1\rangle + \beta|0\rangle) \otimes |11\rangle$$

$$\alpha|010\rangle + \beta|101\rangle \rightarrow (\alpha|0\rangle + \beta|1\rangle) \otimes |10\rangle$$

$$\alpha|001\rangle + \beta|110\rangle \rightarrow (\alpha|0\rangle + \beta|1\rangle) \otimes |01\rangle$$

$$|7\rangle = \alpha|10\rangle + \beta|11\rangle$$



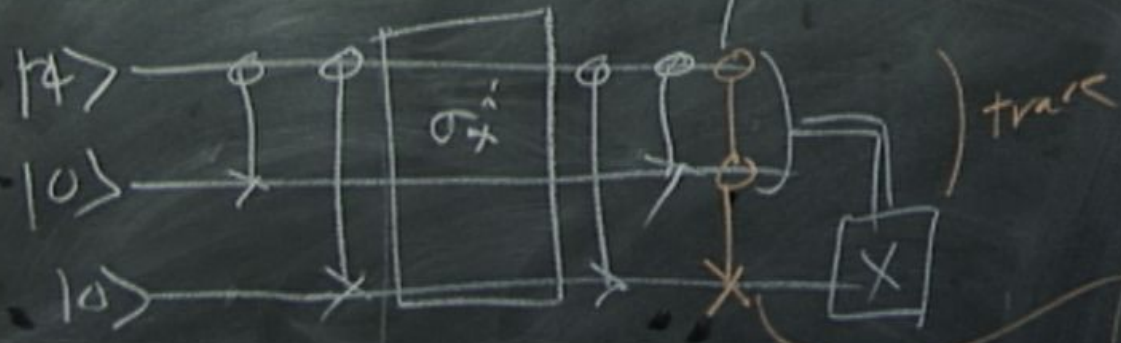
$$\alpha|520\rangle + \beta|111\rangle \rightarrow (\alpha|0\rangle + \beta|1\rangle) \otimes |00\rangle$$

$$\alpha|100\rangle + \beta|011\rangle \rightarrow (\alpha|1\rangle + \beta|0\rangle) \otimes |11\rangle$$

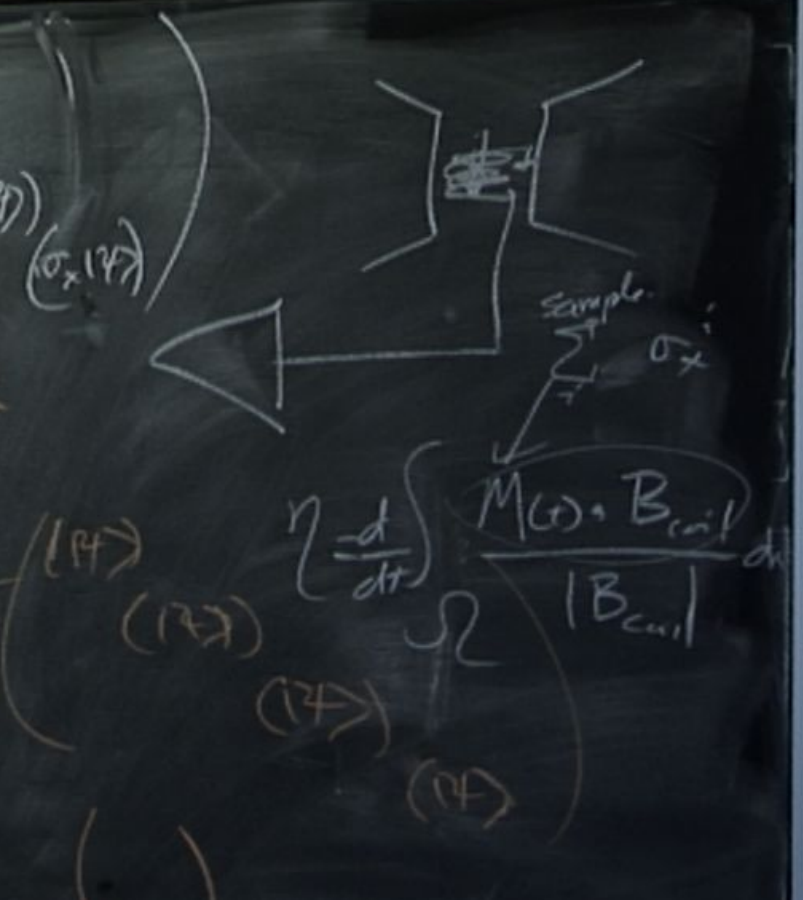
$$\alpha|010\rangle + \beta|101\rangle \rightarrow (\alpha|0\rangle + \beta|1\rangle) \otimes |10\rangle$$

$$\alpha|001\rangle + \beta|110\rangle \rightarrow (\alpha|0\rangle + \beta|1\rangle) \otimes |01\rangle$$

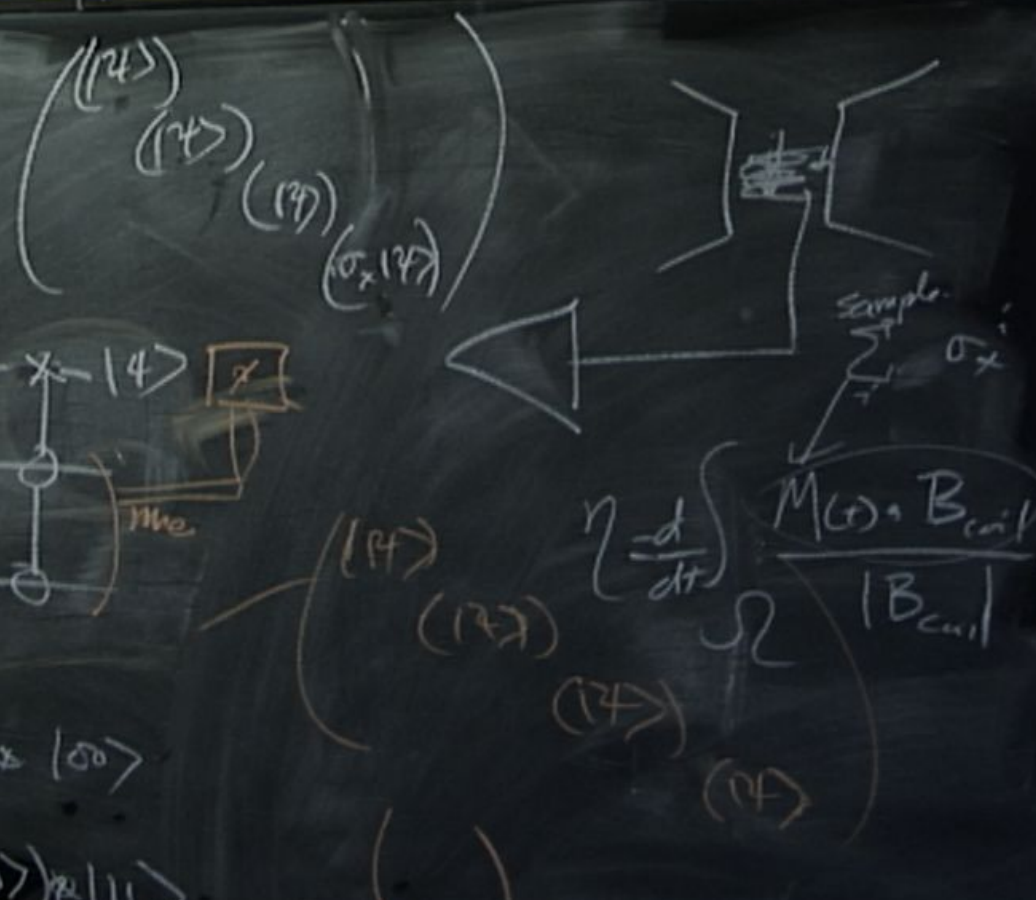
$$|14\rangle = \alpha|10\rangle + \beta|11\rangle$$



$$\begin{aligned} \alpha|100\rangle + \beta|111\rangle &\rightarrow (\alpha|10\rangle + \beta|11\rangle) \otimes |00\rangle \\ \alpha|110\rangle + \beta|101\rangle &\rightarrow (\alpha|11\rangle + \beta|10\rangle) \otimes |11\rangle \\ \alpha|1010\rangle + \beta|1101\rangle &\rightarrow (\alpha|10\rangle + \beta|11\rangle) \otimes |10\rangle \\ \alpha|001\rangle + \beta|110\rangle &\rightarrow (\alpha|10\rangle + \beta|11\rangle) \otimes |01\rangle \end{aligned}$$



$$|7\rangle = \alpha|10\rangle + \beta|11\rangle$$



$$\alpha|500\rangle + \beta|111\rangle \rightarrow (\alpha|0\rangle + \beta|1\rangle) \otimes |00\rangle$$

$$\alpha|100\rangle + \beta|011\rangle \rightarrow (\alpha|1\rangle + \beta|0\rangle) \otimes |11\rangle$$

$$\alpha|010\rangle + \beta|101\rangle \rightarrow (\alpha|0\rangle + \beta|1\rangle) \otimes |10\rangle$$

$$\alpha|001\rangle + \beta|110\rangle \rightarrow (\alpha|0\rangle + \beta|1\rangle) \otimes |01\rangle$$

Grains of
Pollen to
Evidence
for Atoms

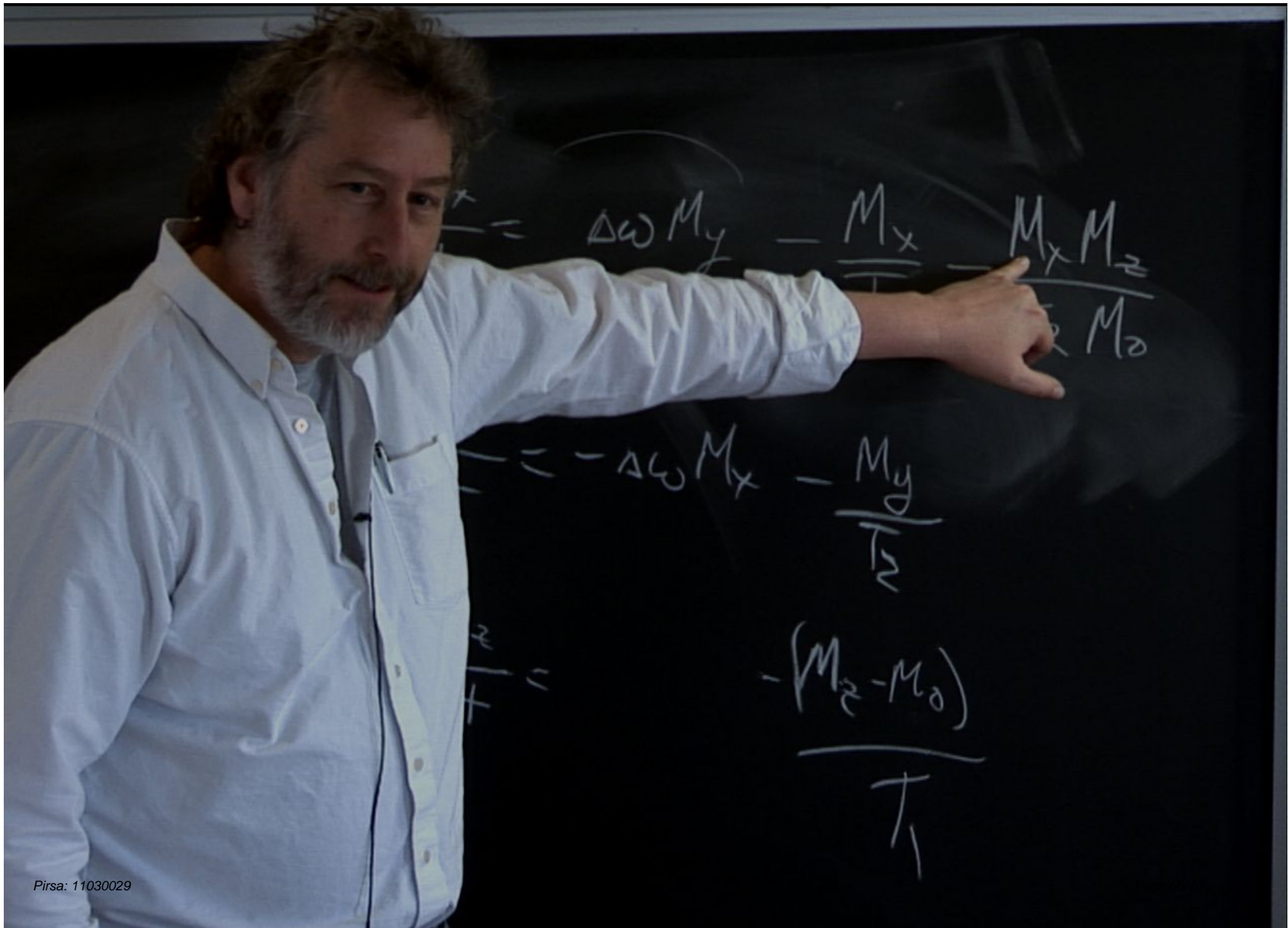
How
Big Is A
Molecule?

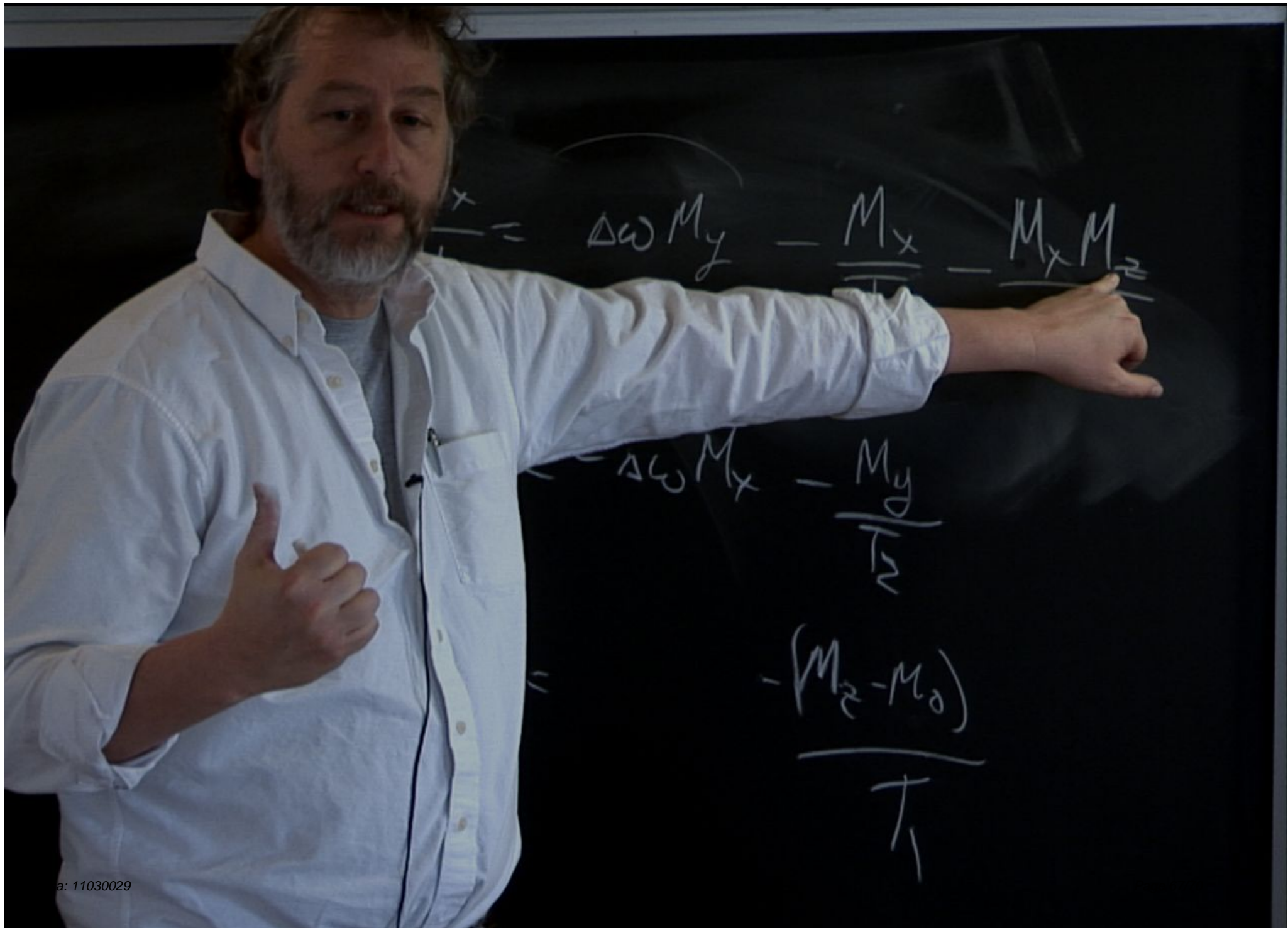
$$\frac{dM_x}{dt} =$$

$$\frac{dM_x}{dt} = \Delta\omega M_y - \frac{M_x}{T_2}$$

$$\frac{dM_y}{dt} = -\Delta\omega M_x - \frac{M_y}{T_2}$$

$$\frac{dM_z}{dt} = \frac{-(M_z - M_0)}{T_1}$$





$$\frac{\partial E}{\partial \omega} = \Delta \omega M_y - \frac{M_x}{T} - \frac{M_x M_z}{T}$$

$$\frac{\partial E}{\partial \omega} = \Delta \omega M_x - \frac{M_y}{T}$$

$$\frac{(M_z - M_0)}{T}$$

$$\frac{dM_x}{dt} = \Delta\omega M_y - \frac{M_x}{T_2} - \frac{M_x M_z}{T_R M_0}$$

$$\frac{dM_y}{dt} = -\Delta\omega M_x - \frac{M_y}{T_2} - \frac{M_y M_z}{T_R M_0}$$

$$\frac{dM_z}{dt} = \frac{-(M_z - M_0)}{T_1} + \frac{M_x^2 + M_y^2}{T_R M_0}$$

Radiation Damping
Super Radiation

$$\frac{dM_x}{dt} = \Delta\omega M_y - \frac{M_x}{T_2} - \frac{M_x M_z}{T_R M_0}$$

$$\frac{dM_y}{dt} = -\Delta\omega M_x - \frac{M_y}{T_2} - \frac{M_y M_z}{T_R M_0}$$

$$\frac{dM_z}{dt} = -\frac{(M_z - M_0)}{T_1} + \frac{M_x^2 + M_y^2}{T_R M_0}$$

Radiation Damping
Super Radiation

$$\tau_R = \frac{1}{f Q M_0}$$

↑ ↑
Filling

$$\frac{dM_x}{dt} = \Delta\omega M_y - \frac{M_x}{T_2} - \frac{M_x M_z}{\tau_R M_0}$$

$$\frac{dM_y}{dt} = -\Delta\omega M_x - \frac{M_y}{T_2} - \frac{M_y M_z}{\tau_R M_0}$$

$$\frac{dM_z}{dt} = \frac{-(M_z - M_0)}{T_1} + \frac{M_x^2 + M_y^2}{\tau_R M_0}$$