

Title: Explorations in Quantum Information - Lecture 7

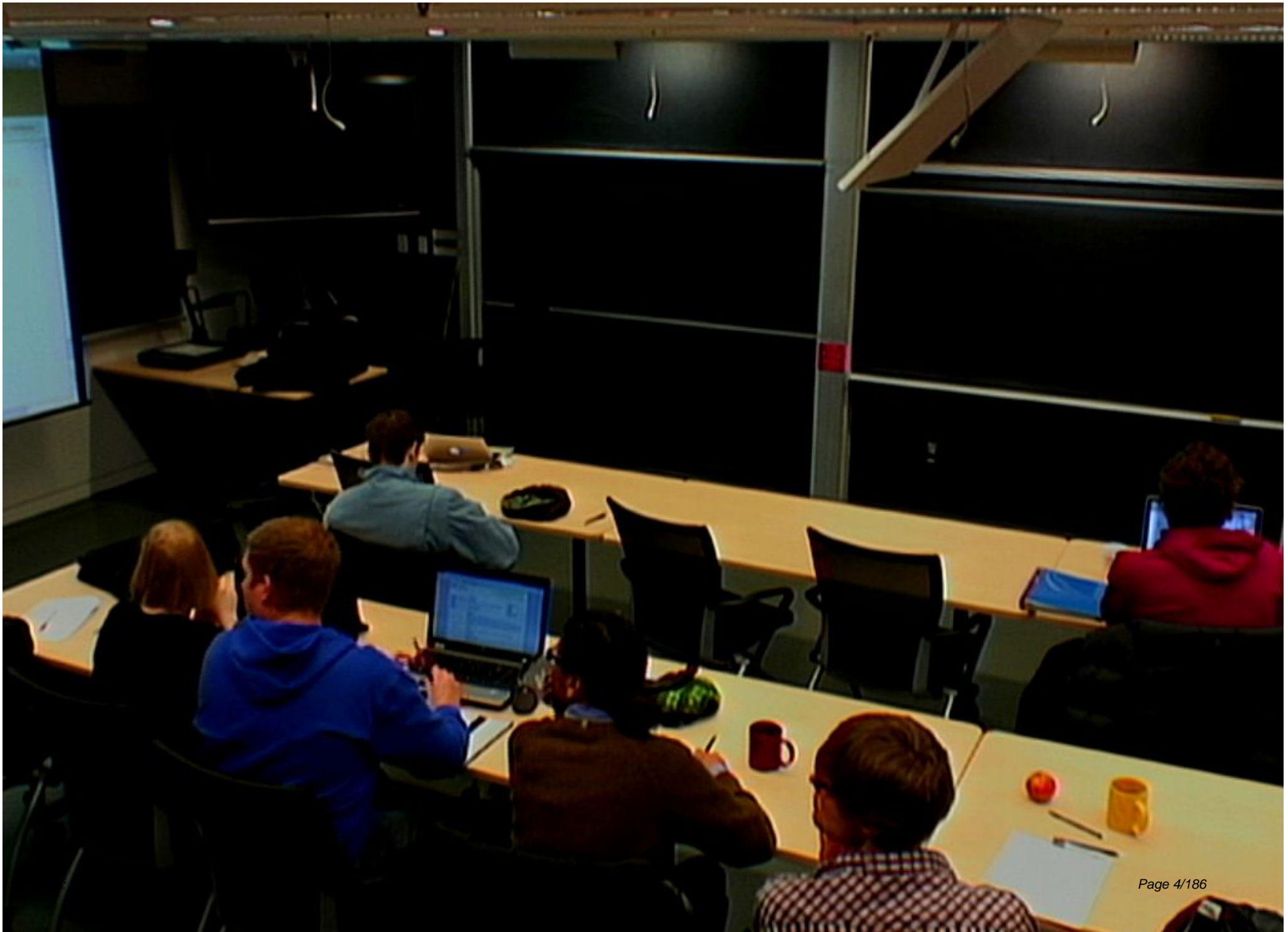
Date: Mar 23, 2011 09:00 AM

URL: <http://pirsa.org/11030020>

Abstract:





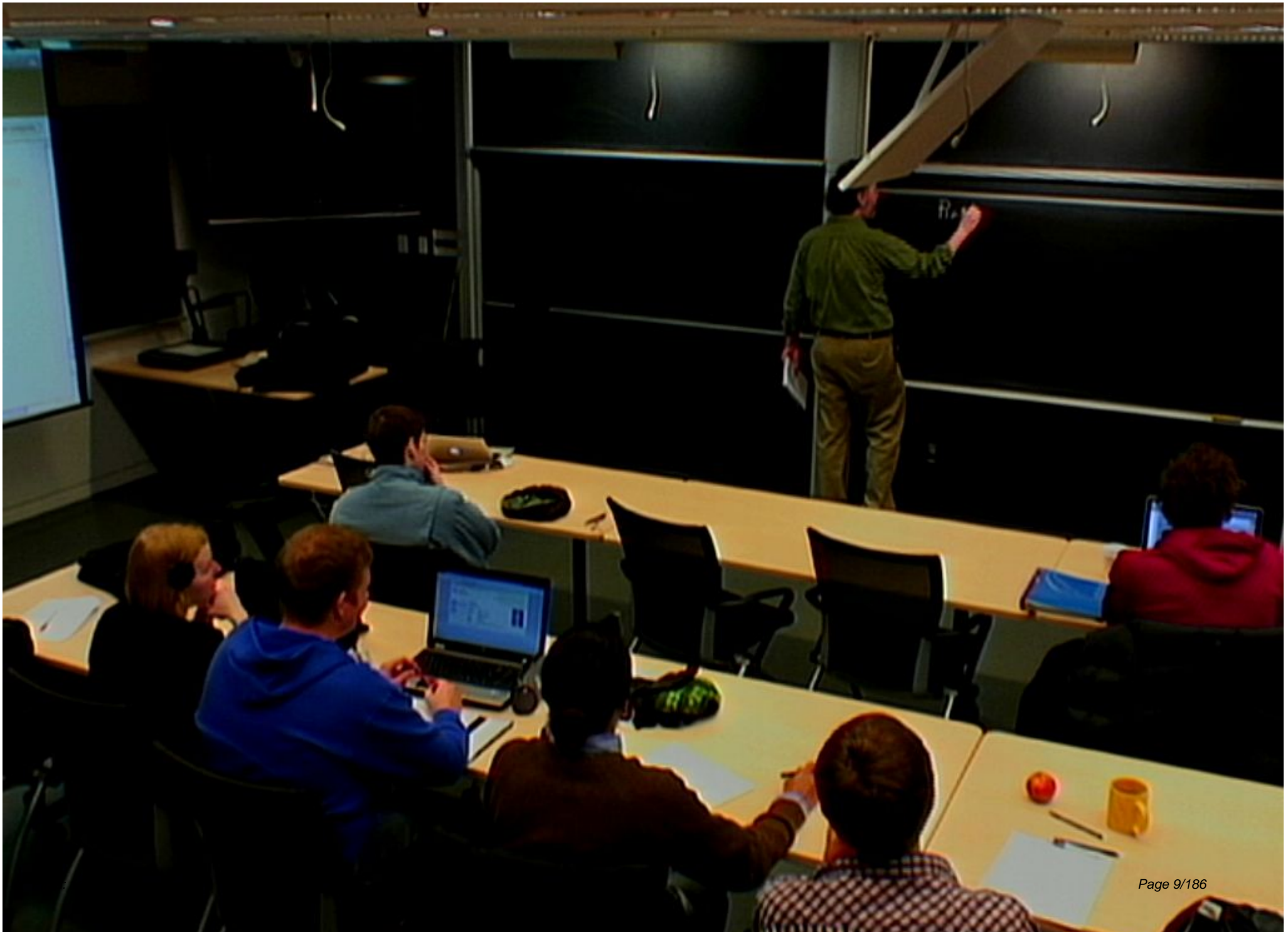


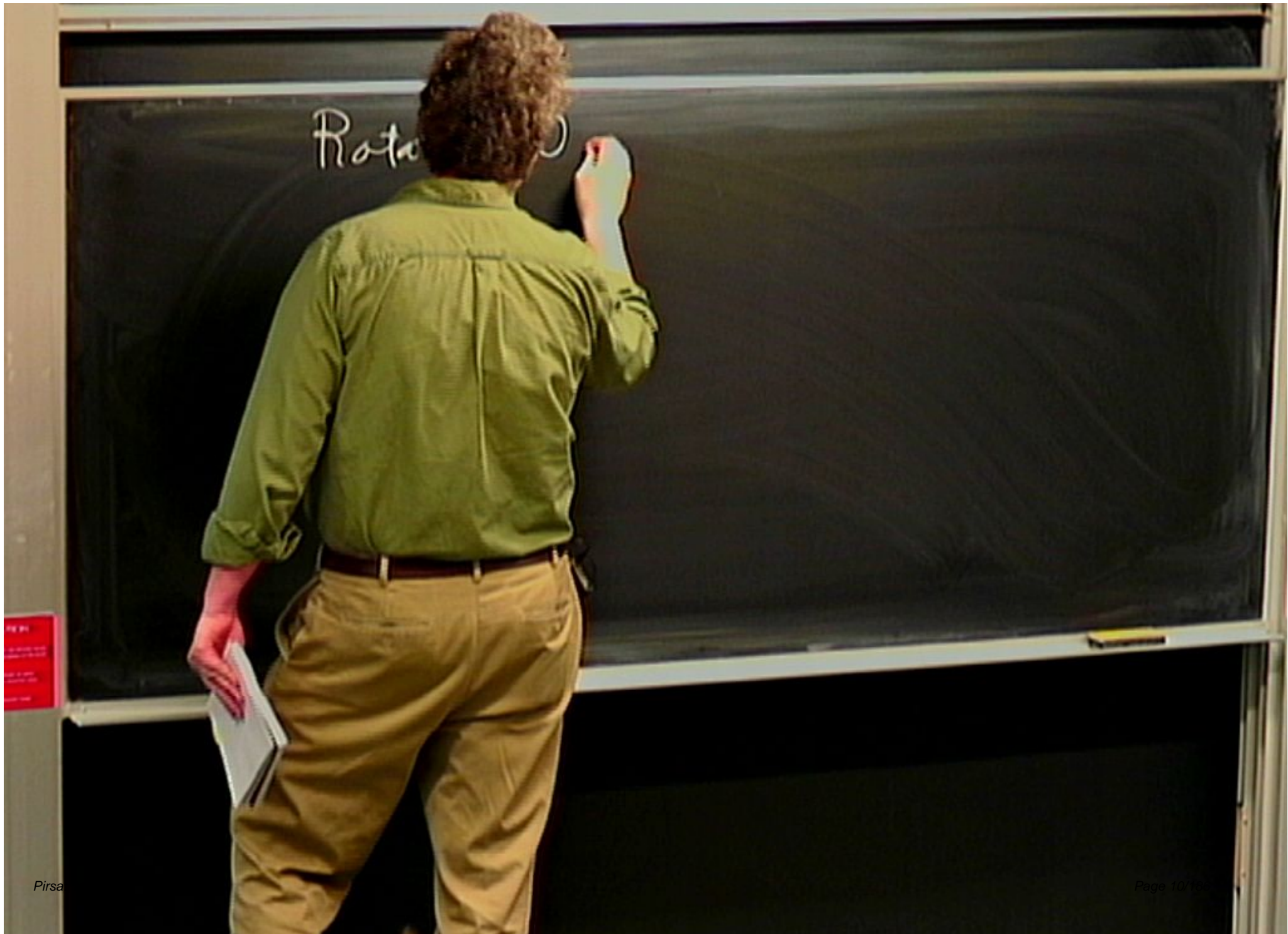












Rata

Rotating Wave

Rotating Wave Approximation

Rotating Wave Approximation

$$\mathcal{H} = \frac{\omega_0}{2} \sigma_x$$

Rotating Wave Approxim'n.

$$\mathcal{H} = \frac{\omega_0}{2} \sigma_z \quad ; \quad \mathcal{H}(t) = \omega_1 \cos(\omega_+ t) \sigma_x$$





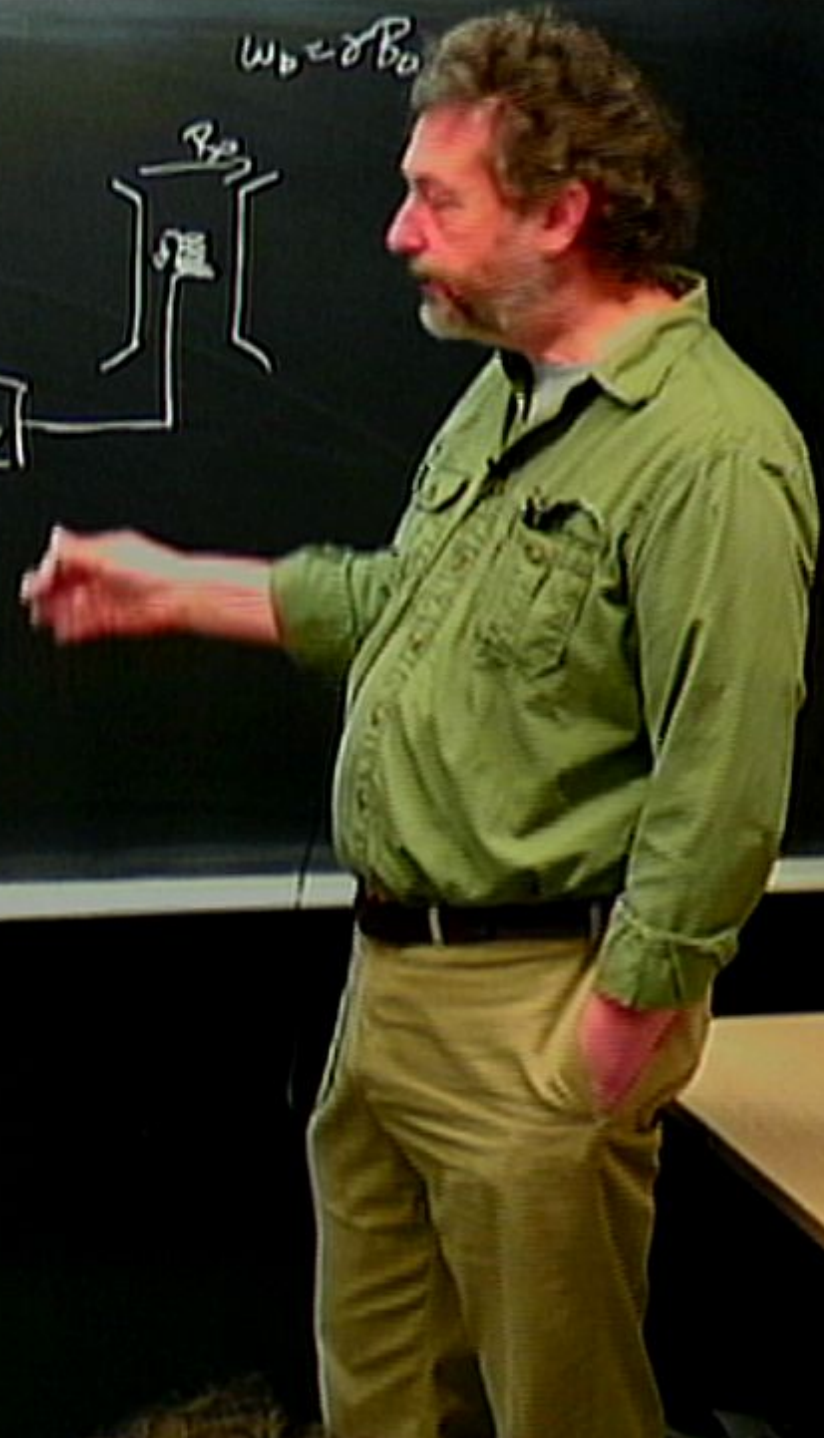
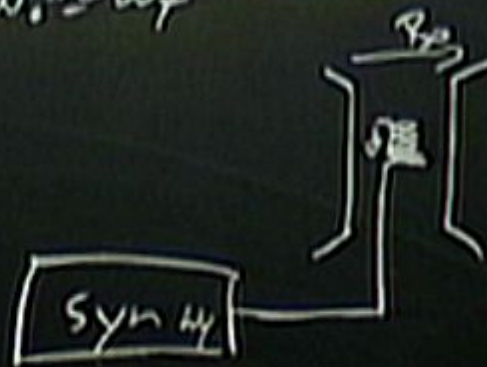
$$\omega_1 \ll \omega_0 \approx \omega_2$$





$$\omega \ll \omega_0 \approx \omega_f$$

$$\omega_0 \approx \sigma B_0$$



$$\mathcal{H} = \frac{\omega_0}{2} \sigma_z ; \mathcal{H}(t) = \omega_1 \cos(\omega_1 t) \sigma_x ; \frac{d\rho}{dt} = -i[\mathcal{H}, \rho]$$

$$\mathcal{H} = \frac{\omega_0}{2} \sigma_x ; \quad \mathcal{H}(t) = \omega_1 \cos(\omega_1 t) \sigma_x ; \quad \frac{d\rho}{dt} = -i[\mathcal{H}, \rho]$$

$$\rho = \mathcal{U}(t) \rho \mathcal{U}^\dagger(t)$$

$$\mathcal{H}_0 = \frac{\omega_0}{2} \sigma_x \quad ; \quad \mathcal{H}(t) = \omega_1 \cos(\omega_1 t) \sigma_x \quad ; \quad \frac{d\rho}{dt} = -i[\mathcal{H}, \rho]$$

$$\rho = \mathcal{U}(t) \rho \mathcal{U}^\dagger(t)$$

$$\mathcal{H}_0 = \frac{\omega_0}{2} \sigma_x ; \quad \mathcal{H}(t) = \omega_1 \cos(\omega_1 t) \sigma_x ; \quad \frac{d\rho}{dt} = -i[\mathcal{H}, \rho]$$

$$\rho = \mathcal{U}(t) \rho \mathcal{U}^\dagger(t)$$

$\frac{3}{2} \sigma$

$$\psi(t) = \omega_1 \cos(\omega_1 t) \sigma_x$$
$$\psi_1(t) = \frac{3}{2} \sigma$$

$$\frac{dP}{dt} = -i[\mathcal{H}, \rho]$$
$$\rho = \mathcal{U}(t) \rho \mathcal{U}^\dagger(t)$$

$\frac{3}{2} \sigma_x$

$$\mathcal{H}(t) = \omega_1 \cos(\omega_1 t) \sigma_x \quad ; \quad \frac{d\rho}{dt} = -i[\mathcal{H}, \rho]$$
$$\mathcal{H}_1(t) = \frac{\omega_1}{2} \left\{ e^{i\frac{\omega_1 t}{2} \sigma_z} \sigma_x e^{-i\frac{\omega_1 t}{2} \sigma_z} + e^{-i\frac{\omega_1 t}{2} \sigma_z} \sigma_x e^{i\frac{\omega_1 t}{2} \sigma_z} \right\} \quad | \rho = \mathcal{U}(t) \rho \mathcal{U}^\dagger(t)$$

$$H_0 = \frac{\omega_0}{2} \sigma_x \quad ; \quad H(t) = \omega_1 \cos(\omega_1 t) \sigma_x \quad ; \quad \frac{dP}{dt} = -i [H, P]$$

$$H_1(t) = \frac{\omega_1}{2} \left\{ \underbrace{e^{i\frac{\omega_1 t}{2} \sigma_x} \sigma_x e^{-i\frac{\omega_1 t}{2} \sigma_x}}_{\text{rotates}} + \underbrace{e^{-i\frac{\omega_1 t}{2} \sigma_x} \sigma_x e^{i\frac{\omega_1 t}{2} \sigma_x}}_{\text{counter rotates}} \right\} \quad \Bigg| \quad P = U(t) P U^\dagger(t)$$



$$\omega \ll \omega_0 \approx \omega_p$$

Syn ω



ω_0

$$U_1(t) = e^{-i \frac{H_1 t}{\hbar}}$$

(Part 1)

(Part 2)

dP

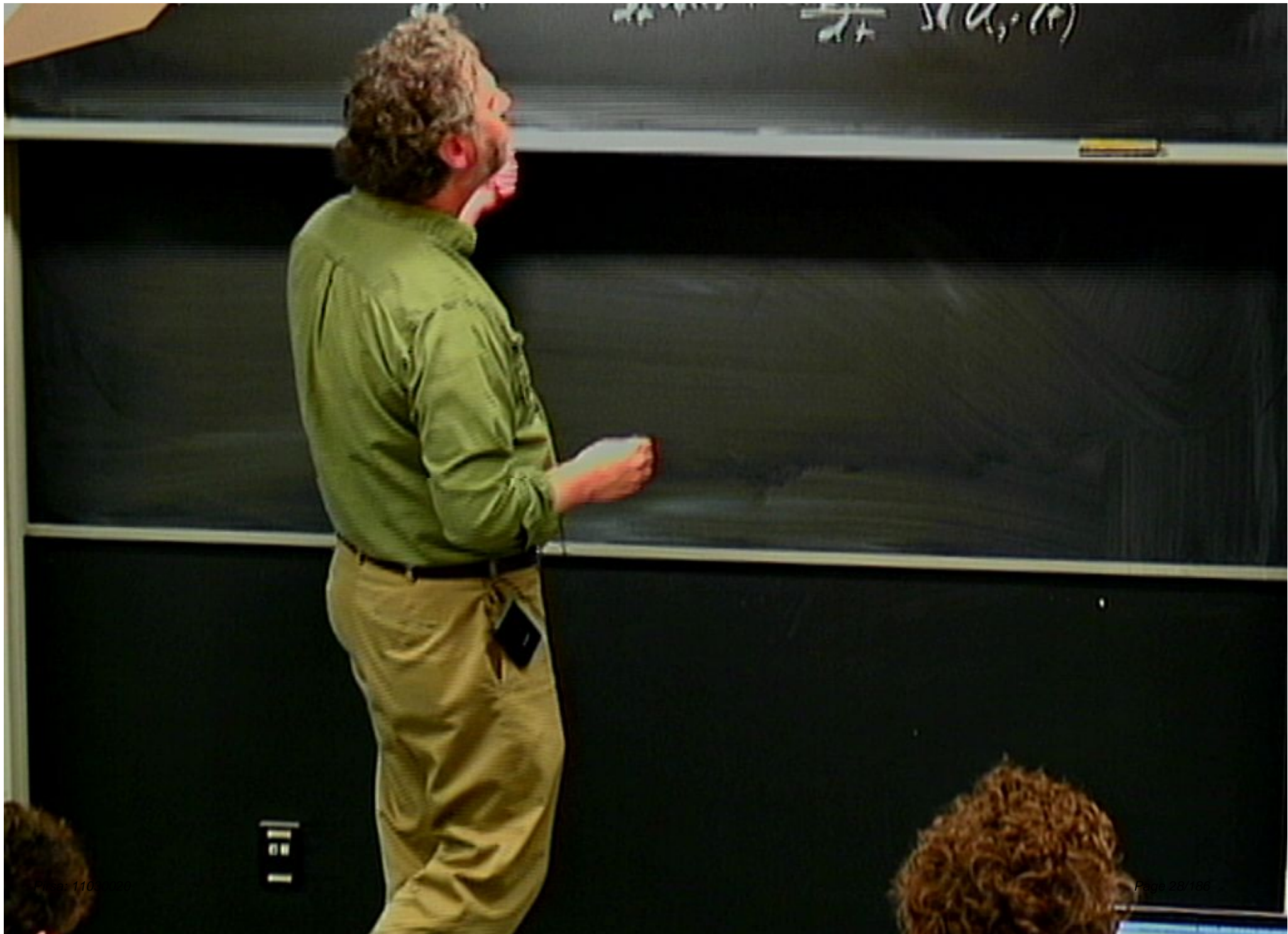
$$U_1(t) = e^{-i\frac{H_0}{\hbar}t}$$

$$\tilde{P} = U_1(t) P U_1^\dagger(t)$$

rotations

counter-rotations

111



$\frac{1}{2+}$ $S(u_2 = (r))$

1 = 1



u

$$P = U_1(t) P U_1^{-1}(t)$$

$$P = U(t) P U^{-1}(t)$$

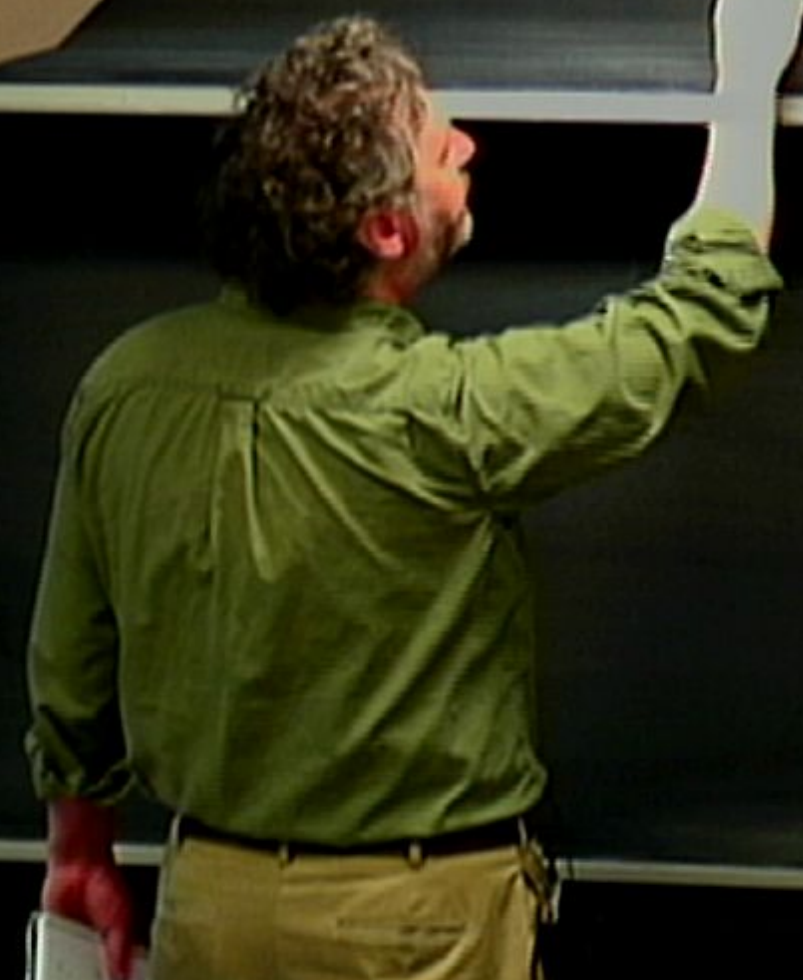
constant matrix

$$U(t) P \frac{dU^{-1}}{dt} + U_1(t) \frac{dP}{dt} U_1^{-1}(t) + \frac{dU_1^{-1}(t)}{dt} P U_1(t)$$

$$P = U(t) P U^{-1}(t)$$

$\underbrace{e^{-\int \sigma dt}}_{\text{constant}}$

$$U(t) P \frac{dU^{-1}}{dt} + U(t) \frac{dP}{dt} U^{-1}(t) + \frac{dU(t)}{dt} P U^{-1}(t)$$



u_2

$$\vec{P} = u_1(t) P u_2(t)$$

$$e^{\frac{\partial \sigma}{\partial t}} \Big|_{S = u_1(t), u_2(t)}$$

constant velocity

$$u_1(t) P \frac{d u_2(t)}{dt} + u_2(t) \frac{d u_1(t)}{dt} + \frac{d u_1(t)}{dt} P u_2(t)$$

$u_1(t) P u_2(t)$

\mathcal{H}

$$\mathcal{H}(t) = \omega_1 \cos(\omega_1 t) \sigma_x \quad ; \quad \frac{dP}{dt} = -i[\mathcal{H}, P]$$

$$\tilde{\mathcal{H}}_1(t) = \frac{\omega_1}{2} \left\{ \underbrace{e^{i\frac{\omega_1 t}{2}} \sigma_x e^{-i\frac{\omega_1 t}{2}}}_{\text{rotates}} + \underbrace{e^{-i\frac{\omega_1 t}{2}} \sigma_x e^{i\frac{\omega_1 t}{2}}}_{\text{counter-rotates}} \right\} P = U(t) P U^\dagger(t)$$

$$: \tilde{P} = U_1(t) P U_1^\dagger(t)$$

$$U_1(t) P \frac{dU_1^\dagger}{dt} + U_1(t) \frac{dP}{dt} U_1^\dagger(t) + \frac{dU_1(t)}{dt} P U_1^\dagger(t)$$

Rotating Wave Approximation

$$H_i = \frac{\omega_i}{2} \sigma_x \quad ; \quad H(t) = \omega_i \cos(\omega_i t) \sigma_x \quad ; \quad \frac{dP}{dt} = -i [H, P]$$

$$H_i(t) = \frac{\omega_i}{2} \left\{ \underbrace{e^{i\omega_i t} \sigma_x}_{\text{rotating}} + \underbrace{e^{-i\omega_i t} \sigma_x}_{\text{counter rotating}} \right\} \quad \Bigg| \quad P = U(t) P U^\dagger(t)$$

$$: \quad \tilde{P} = U_i(t) P U_i^\dagger(t)$$

$$U_i(t) = e^{-i\omega_i t \sigma_x}$$

dP

$$U_i(t) P \frac{dU_i^\dagger}{dt} + U_i(t) \frac{dP}{dt} U_i^\dagger(t) + \frac{dU_i(t)}{dt} P U_i^\dagger(t)$$

$\underbrace{\quad}_{i[H, P]} \quad \underbrace{\quad}_{[H, P]} \quad \underbrace{\quad}_{[H, P]}$

Rotating Wave Approximation

$$\frac{\omega_1}{2} \sigma_x : \mathcal{H}(t) = \underbrace{\omega_1 \cos(\omega_1 t)}_{\text{rotating}} \sigma_x ; \frac{dP}{dt} = -i [\mathcal{H}, P]$$

$$H_1 = \frac{\omega_1}{2} \sigma_x$$

$$\mathcal{H}_1(t) = \frac{\omega_1}{2} \left\{ \underbrace{e^{i\frac{\omega_1 t}{2} \sigma_z} \sigma_x e^{-i\frac{\omega_1 t}{2} \sigma_z}}_{\text{rotating}} + \underbrace{e^{-i\frac{\omega_1 t}{2} \sigma_z} \sigma_x e^{i\frac{\omega_1 t}{2} \sigma_z}}_{\text{counter rotating}} \right\} P = U(t) P U^\dagger(t)$$

$$U_1(t) = e^{-i\frac{\omega_1 t}{2} \sigma_z}$$

$$P = U_1(t) P U_1^\dagger(t)$$

$$\frac{dP}{dt} = U_1(t) P \underbrace{\frac{dU_1^\dagger}{dt}}_{i\frac{\omega_1}{2} \sigma_z} + U_1(t) \underbrace{\frac{dP}{dt} U_1^\dagger(t)}_{[\mathcal{H}, P]} + \frac{dU_1(t)}{dt} P U_1^\dagger(t)$$

$u_i =$

$$\tilde{P} = U_i(t) P U_i^{-1}(t)$$

Constant matrix

$$P \frac{dU_i^{-1}}{dt} + U_i(t) \frac{dP}{dt} U_i^{-1}(t) + \frac{dU_i(t)}{dt} P U_i^{-1}(t)$$

$$\frac{d\tilde{P}}{dt} = +U_i(t) P U_i^{-1}(t) \mathcal{H}_i - i \mathcal{H}_i U_i P U_i^{-1} + U_i(t) [\mathcal{H}_i, P] U_i^{-1}$$

$$U_2 =$$

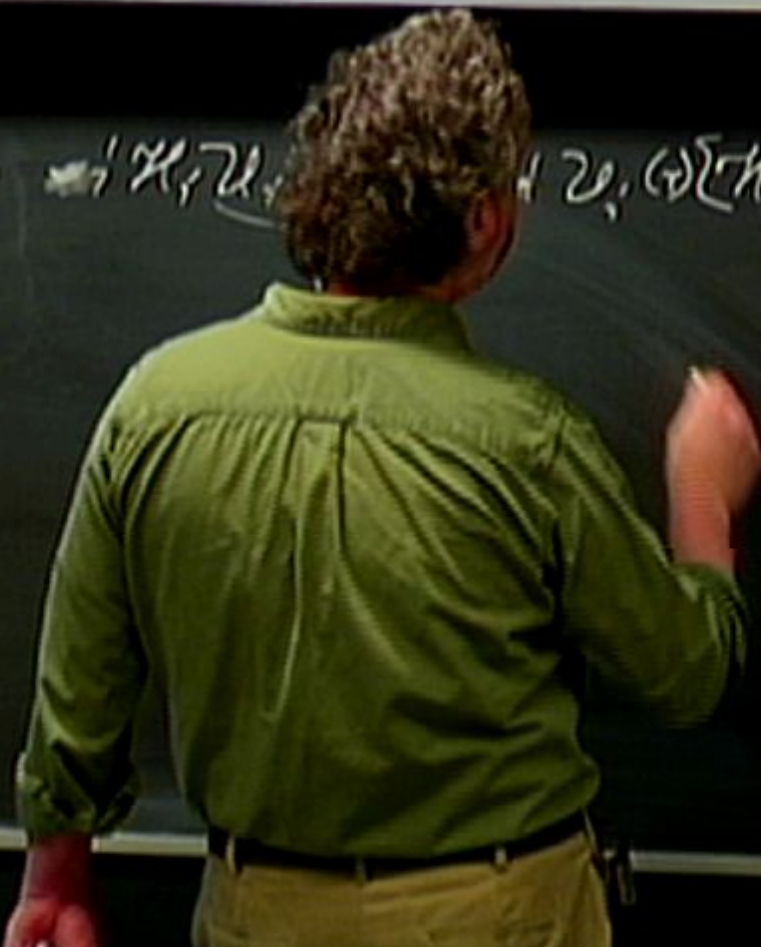
$$\tilde{P} = U_1(t) P U_1^{-1}(t)$$

constant matrix

$$P \frac{dU_1^{-1}}{dt} + U_1(t) \frac{dP}{dt} U_1^{-1}(t) + \frac{dU_1(t)}{dt} P U_1^{-1}(t)$$

$\underbrace{\hspace{10em}}_{+iH_1 U_1^{-1}}$

$$\frac{d\tilde{P}}{dt} = +i \underbrace{U_1(t) P U_1^{-1}(t)}_{\tilde{P}} \mathcal{H}_1 - i \mathcal{H}_1 U_1(t) P U_1^{-1}(t) + U_1(t) [i\mathcal{H}_1, P] U_1^{-1}(t)$$



\mathcal{H}

$$\mathcal{H}(t) = \omega_1 \cos(\omega_1 t) \sigma_x ; \frac{dP}{dt} = -i[\mathcal{H}, P]$$

$$\tilde{\mathcal{H}}_1(t) = \frac{\omega_1}{2} \left\{ \underbrace{e^{i\frac{\omega_1}{2}t} \sigma_x e^{-i\frac{\omega_1}{2}t}}_{\text{rotates}} + \underbrace{e^{-i\frac{\omega_1}{2}t} \sigma_x e^{i\frac{\omega_1}{2}t}}_{\text{counterrotates}} \right\} P = U(t) P U^\dagger(t)$$

$$: \tilde{P} = U_1(t) P U_1^\dagger(t)$$

$$\tilde{\mathcal{H}} = U_1 \mathcal{H} U_1^\dagger$$

$$\frac{d\tilde{P}}{dt} = U_1(t) P \underbrace{\frac{dU_1^\dagger}{dt}}_{+i\mathcal{H}_1 U_1^\dagger} + U_1(t) \underbrace{\frac{dP}{dt}}_{-i[\tilde{\mathcal{H}}, \tilde{P}]} U_1^\dagger(t) + \underbrace{\frac{dU_1(t)}{dt}}_{-i\mathcal{H}_1 U_1} P U_1^\dagger(t)$$

$$u_i^{-1} =$$

$$\vec{P} = U_i(t) P U_i^{-1}(t)$$

constant!

$$\vec{H} = U_i^{-1} P U_i$$

$$+ P \frac{dU_i^{-1}}{dt} + U_i^{-1} \frac{dP}{dt} U_i + \frac{dU_i^{-1}}{dt} P U_i^{-1}$$

$\underbrace{\hspace{10em}}_{+i\hbar H_i U_i^{-1}}$

$$\frac{d\vec{P}}{dt} = +i U_i^{-1} P U_i^{-1} \vec{H}_i - U_i^{-1} \vec{H}_i U_i^{-1} P U_i^{-1}$$

$u_i =$

$\tilde{P} = U_i(t) P U_i^{-1}(t)$

constraint

$\tilde{H} = U_i H U_i^{-1}$

$$i \underbrace{P \frac{dU_i^{-1}}{dt}}_{+iH_i U_i^{-1}} + U_i(t) \underbrace{\frac{dP U_i^{-1}(t)}{dt}}_{\text{constraint}} + \underbrace{\frac{dU_i(t)}{dt}}_{-iH_i U_i} P U_i^{-1}(t)$$

$$\frac{d\tilde{P}}{dt} = +i \underbrace{U_i(t) P U_i^{-1}(t)}_{\tilde{P}} \tilde{H}_i - i \underbrace{U_i^{-1}(t) \tilde{H}_i U_i(t)}_{\tilde{P}} - i [\tilde{H}_i, \tilde{P}]$$

$u_i =$

$$\tilde{P} = U_i(t) P U_i^{-1}(t)$$

Constant

$$\tilde{H} = U_i \tilde{H} P U_i^{-1}$$

$$i P \frac{dU_i^{-1}}{dt} + U_i(t) \frac{dP}{dt} U_i^{-1}(t) + \frac{dU_i(t)}{dt} P U_i^{-1}(t)$$

$\underbrace{\hspace{10em}}_{+iH_i U_i^{-1}} \quad \underbrace{\hspace{10em}}_{\text{Constant}} \quad \underbrace{\hspace{10em}}_{-iH_i U_i}$

$$\frac{d\tilde{P}}{dt} = +i U_i(t) P U_i^{-1}(t) \tilde{H} - i \tilde{H} U_i(t) P U_i^{-1}(t) + U_i(t) \frac{dP}{dt} U_i^{-1}(t) - i [\tilde{H}, \tilde{P}]$$

$$\frac{d\tilde{P}}{dt} = -i [\tilde{H}, \tilde{P}]$$

Rotating Wave Approximation

$$\begin{aligned}
 \mathcal{H}_0 &= \frac{\omega_0}{2} \sigma_z & ; & \quad \mathcal{H}(t) = \underbrace{\omega_1 \cos(\omega_1 t)}_{\text{rotates}} \sigma_x & ; \quad \frac{d\rho}{dt} = -i[\mathcal{H}, \rho] \\
 \mathcal{H}_1 &= \frac{\omega_1}{2} \sigma_x & & \quad \mathcal{H}_1(t) = \frac{\omega_1}{2} \left\{ \underbrace{e^{i\frac{\omega_1 t}{2} \sigma_z} \sigma_x e^{-i\frac{\omega_1 t}{2} \sigma_z}}_{\text{rotates}} + \underbrace{e^{-i\frac{\omega_1 t}{2} \sigma_z} \sigma_x e^{i\frac{\omega_1 t}{2} \sigma_z}}_{\text{counter rotates}} \right\} & \quad \rho = \mathcal{U}(t) \rho \mathcal{U}^\dagger(t) \\
 \mathcal{U}_1(t) &= e^{-i\frac{\omega_1 t}{2} \sigma_z} & ; & \quad \tilde{\rho} = \mathcal{U}_1(t) \rho \mathcal{U}_1^\dagger(t)
 \end{aligned}$$

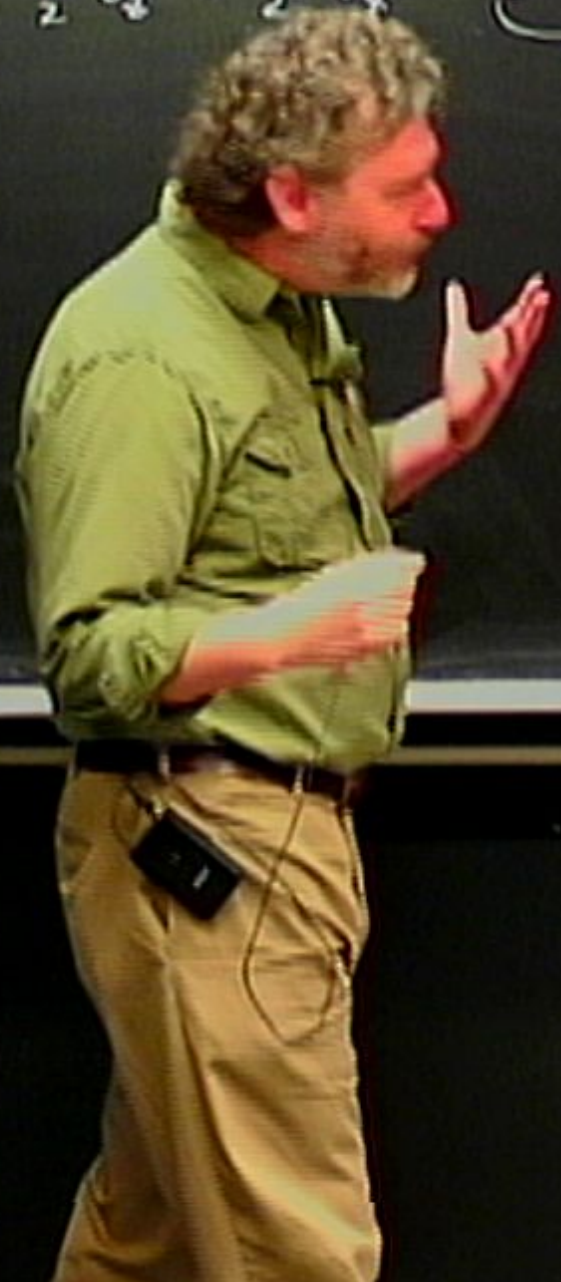
$$\frac{d\tilde{\rho}}{dt} = \mathcal{U}_1(t) \rho \underbrace{\frac{d\mathcal{U}_1^\dagger}{dt}}_{+i\mathcal{H}_1 \mathcal{U}_1^\dagger} + \mathcal{U}_1(t) \underbrace{\frac{d\rho}{dt}}_{-i[\mathcal{H}, \rho]} \mathcal{U}_1^\dagger(t) + \underbrace{\frac{d\mathcal{U}_1(t)}{dt}}_{-i\mathcal{H}_1 \mathcal{U}_1} \rho \mathcal{U}_1^\dagger(t)$$

$$M^T = \begin{matrix} 3 & 0 & 9 \\ 2 & 1 & 2 \end{matrix}$$

$$H = \frac{3}{2} \sigma_z + \frac{3}{2} \sigma_x$$

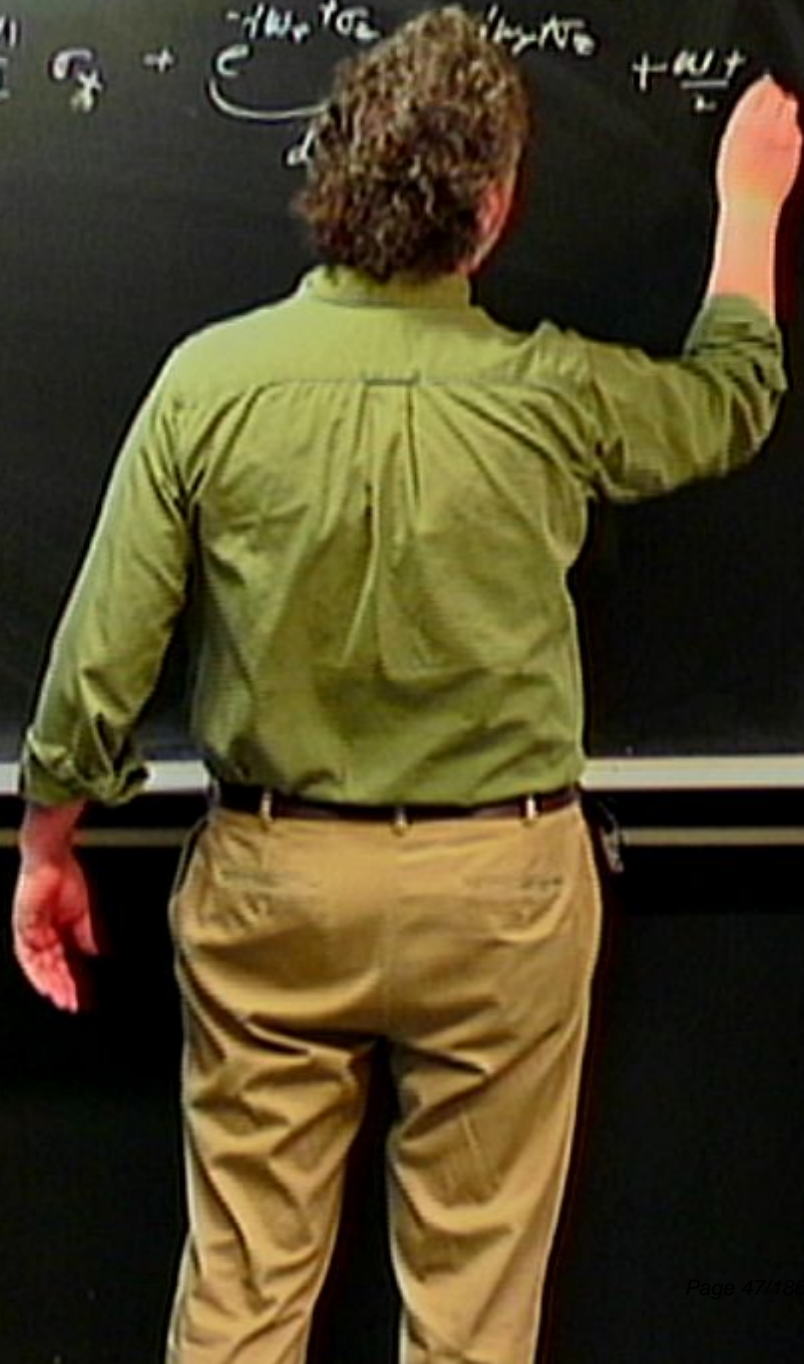
$$\Psi = \frac{\omega_0}{2} \sigma_y + e^{-i\omega_0 t} \sigma_x e^{i\omega_0 t}$$

$$\hat{H} = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + \underbrace{e^{-i\omega_0 t} \sigma_x e^{i\omega_0 t}}_{\text{doubly rotating}}$$



$$\hat{H} = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + \underbrace{e^{-i\omega_0 t \sigma_z} \sigma_x e^{i\omega_0 t \sigma_z}}_{\text{dynamically}}$$

$$H = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + \frac{-i\omega_2 \sigma_z + i\omega_2 \sigma_x}{2} + \frac{\omega_1 + \omega_2}{2}$$



$$\mathcal{H} = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + c \left(\frac{-i\omega_+ \sigma_z}{\sigma_x} + \frac{i\omega_- \sigma_z}{\sigma_x} \right) + \frac{\omega_+}{2} \sigma_x$$

$\underbrace{\hspace{10em}}_{\text{by identity}}$



$$\begin{aligned}
 \mu_T &= \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + \underbrace{e^{-i\omega_0 t} \sigma_x}_{\text{daily return}} = \frac{\omega_1}{2} \sigma_x \\
 &= \frac{\omega_0 - \omega_1}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x
 \end{aligned}$$



$$\mathcal{H} = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + \underbrace{e^{-i\omega_0 t \sigma_z} \frac{i\omega_1 \tau \sigma_x}{2} e^{i\omega_0 t \sigma_z}}_{\text{doubly rotating}} = \frac{\omega_1}{2} \sigma_x$$

$$\mathcal{H}^2 = \frac{\omega_0 - \omega_1}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x$$

$$M_T = \frac{w_0}{2} \sigma_x^2 + \frac{w_1}{2} \sigma_x^2 + \underbrace{e^{-i w_0 t_0} \sigma_x e^{i w_1 t_0}}_{\text{daily return}} \frac{w_1 + \sigma_x}{2}$$

$$M_T = \frac{w_0 - w_1}{2} \sigma_x^2 + \frac{w_1}{2} \sigma_x^2$$

H_0

$$H = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + \underbrace{e^{-i\omega_0 t} \sigma_x e^{i\omega_0 t}}_{\text{doubly rotating}} = \frac{\omega_1}{2} \sigma_x$$

$$H = \frac{\omega_0 - \omega_1}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x$$

H_{ISS}

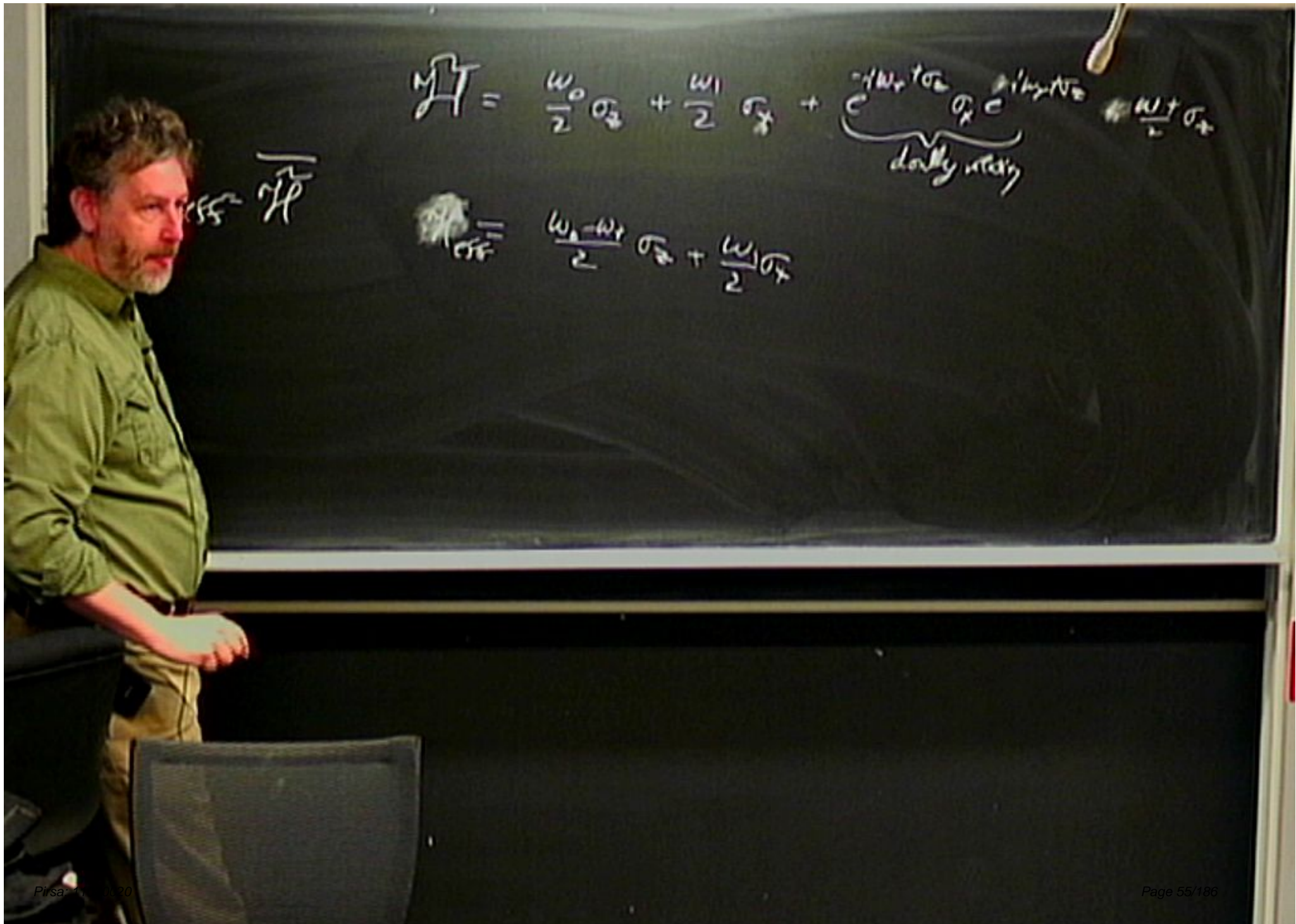
$$H = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + \underbrace{e^{-i\omega_0 t} \sigma_x e^{i\omega_0 t}}_{\text{dobby stuff}} = \frac{\omega_1}{2} \sigma_x$$

$$H = \frac{\omega_0 - \omega_1}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x$$

$\mathcal{H}_{ISS} = \overline{\mathcal{H}}$

$$\mathcal{H} = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + \underbrace{e^{-i\omega_1 t \sigma_z} e^{i\omega_1 t \sigma_z}}_{\text{doubly unitary}} \frac{\omega_1 + \sigma_x}{2}$$

$$\mathcal{H} = \frac{\omega_0 - \omega_1}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x$$



$$H = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + \underbrace{\left(\frac{-i\omega_2 \sigma_z}{2} + \frac{i\omega_2 \sigma_z}{2} \right)}_{\text{doubly degenerate}} + \frac{\omega_1 + \omega_2}{2} \sigma_x$$

$$\text{CSF} \rightarrow \overline{H}$$

$$\text{CSF} = \frac{\omega_0 - \omega_1}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x$$

$$H = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + \underbrace{c \frac{-i\omega_1 \sigma_z}{2} + c \frac{i\omega_1 \sigma_z}{2}}_{\text{doubly anti}} + \frac{\omega_1}{2} \sigma_x$$

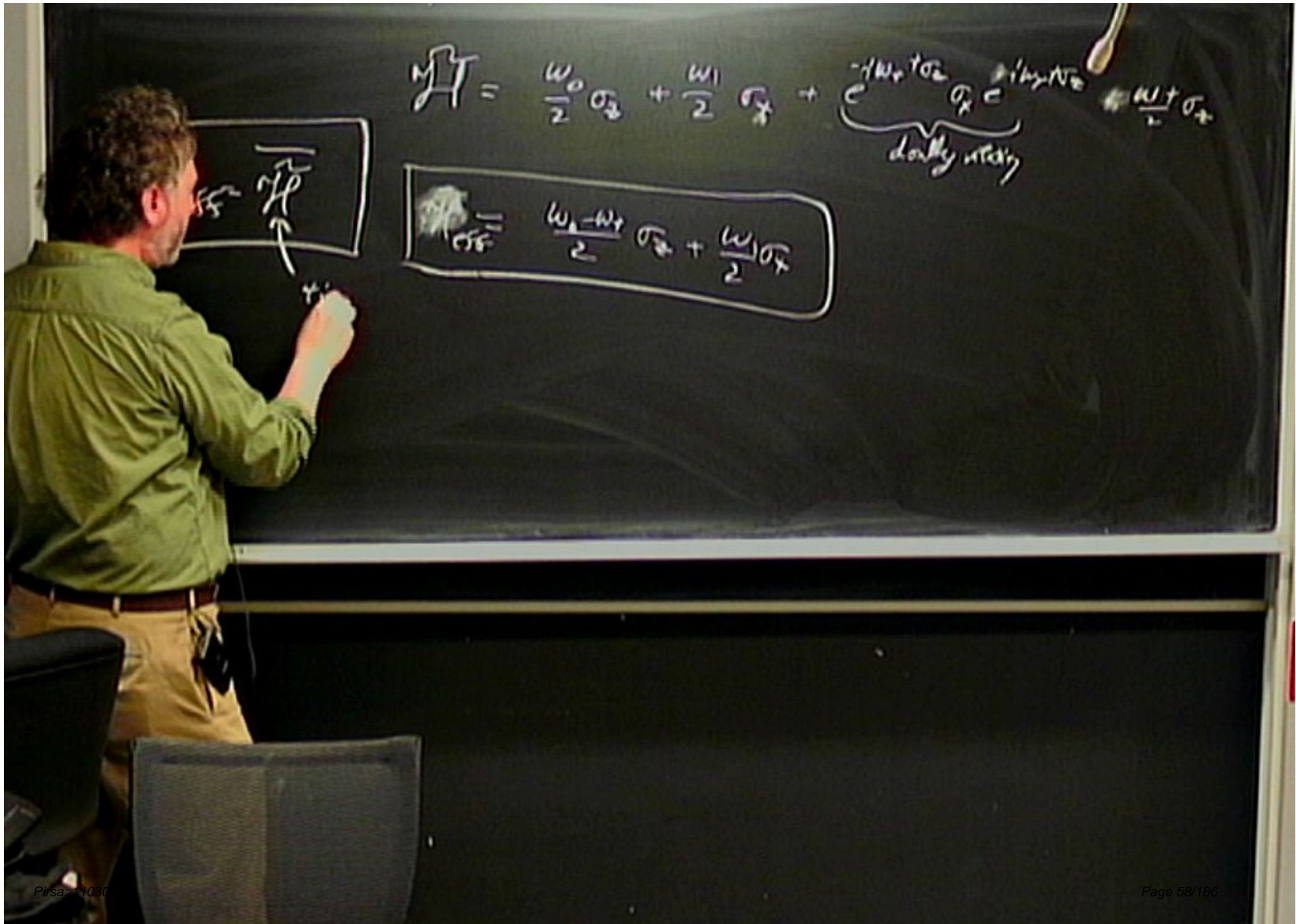
H

$$\text{CFE} = \frac{\omega_0 - \omega_1}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x$$

$$\mathcal{H} = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + \underbrace{\left(\frac{-i\omega_2 \sigma_z}{2} + \frac{i\omega_2 \sigma_z}{2} \right)}_{\text{doublet}} + \frac{\omega_1 + \omega_2}{2} \sigma_x$$

$$\mathcal{H}_{\text{eff}} = \overline{\mathcal{H}}$$

$$\mathcal{H}_{\text{eff}} = \frac{\omega_0 - \omega_1}{2} \sigma_z + \frac{\omega_1 + \omega_2}{2} \sigma_x$$



$$H = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + \underbrace{e^{-i\omega_0 t} \sigma_x e^{i\omega_0 t}}_{\text{doubly acting}} = \frac{\omega_1}{2} \sigma_x$$

$$\langle H \rangle = \langle \psi | H | \psi \rangle$$

$$\langle H \rangle = \frac{\omega_0 - \omega_1}{2} \langle \sigma_z \rangle + \frac{\omega_1}{2} \langle \sigma_x \rangle$$

$$H = \frac{w_0}{2} \sigma_z + \frac{w_1}{2} \sigma_x + \underbrace{\left(\frac{-w_0 \tau \sigma_z}{2} + \frac{w_1 \tau \sigma_x}{2} \right)}_{\text{daily utility}} + \frac{w_1}{2} \sigma_x$$

H_{eff}

$$H_{\text{eff}} = \frac{w_0 - w_1 \tau}{2} \sigma_z + \frac{w_1}{2} \sigma_x$$

independent



$$H = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + \underbrace{c \frac{-i\hbar \omega_0 \sigma_z}{2} + i\hbar \omega_1 \sigma_x}_{\text{doubly acting}} + \frac{\omega_2}{2} \sigma_x$$

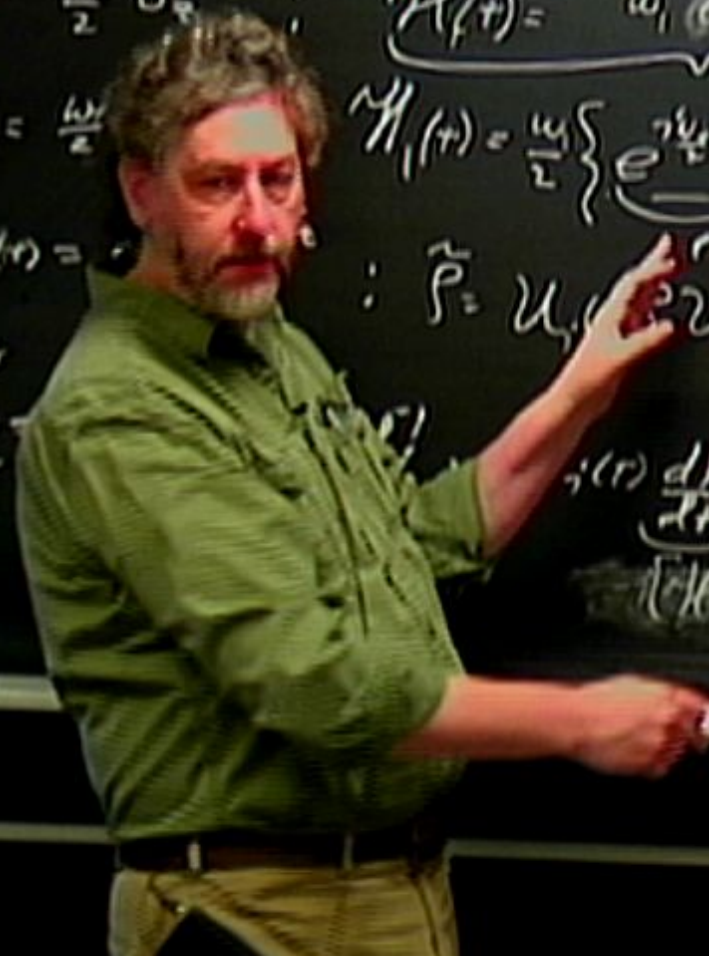
$$H_{\text{eff}} = \overline{H}$$

time independent

$$H_{\text{eff}} = \frac{\omega_0 - \omega_1}{2} \sigma_z + \frac{\omega_1 \omega_2}{2} \sigma_x$$

Rotating Wave Approximation

$\sigma_0 = \frac{\hbar \omega_0}{2} \sigma_x$; $\mathcal{H}(t) = \omega_1 \cos(\omega_1 t) \sigma_x$; $\frac{d\rho}{dt} = -i[\mathcal{H}, \rho]$
 $\mathcal{H}_1 = \frac{\hbar \omega_1}{2} \sigma_x$; $\mathcal{H}_1(t) = \frac{\hbar \omega_1}{2} \left\{ \underbrace{e^{i\frac{\omega_1 t}{2} \sigma_x} \sigma_x e^{-i\frac{\omega_1 t}{2} \sigma_x}}_{\text{rotates}} + \underbrace{e^{-i\frac{\omega_1 t}{2} \sigma_x} \sigma_x e^{i\frac{\omega_1 t}{2} \sigma_x}}_{\text{counter rotates}} \right\}$; $\rho = \mathcal{U}(t) \rho \mathcal{U}^\dagger(t)$
 $\mathcal{U}_1(t) = e^{-i\frac{\omega_1 t}{2} \sigma_x}$; $\tilde{\rho} = \mathcal{U}_1^\dagger(t) \rho \mathcal{U}_1(t)$
 $\frac{d\tilde{\rho}}{dt} = -i[\tilde{\mathcal{H}}, \tilde{\rho}]$; $\tilde{\mathcal{H}} = \mathcal{U}_1^\dagger \mathcal{H} \mathcal{U}_1$
 $-i(t) \frac{d\rho}{dt} \mathcal{U}_1^\dagger(t) + \frac{d\mathcal{U}_1^\dagger(t)}{dt} \rho \mathcal{U}_1(t)$; $-i\tilde{\mathcal{H}}\tilde{\rho}$



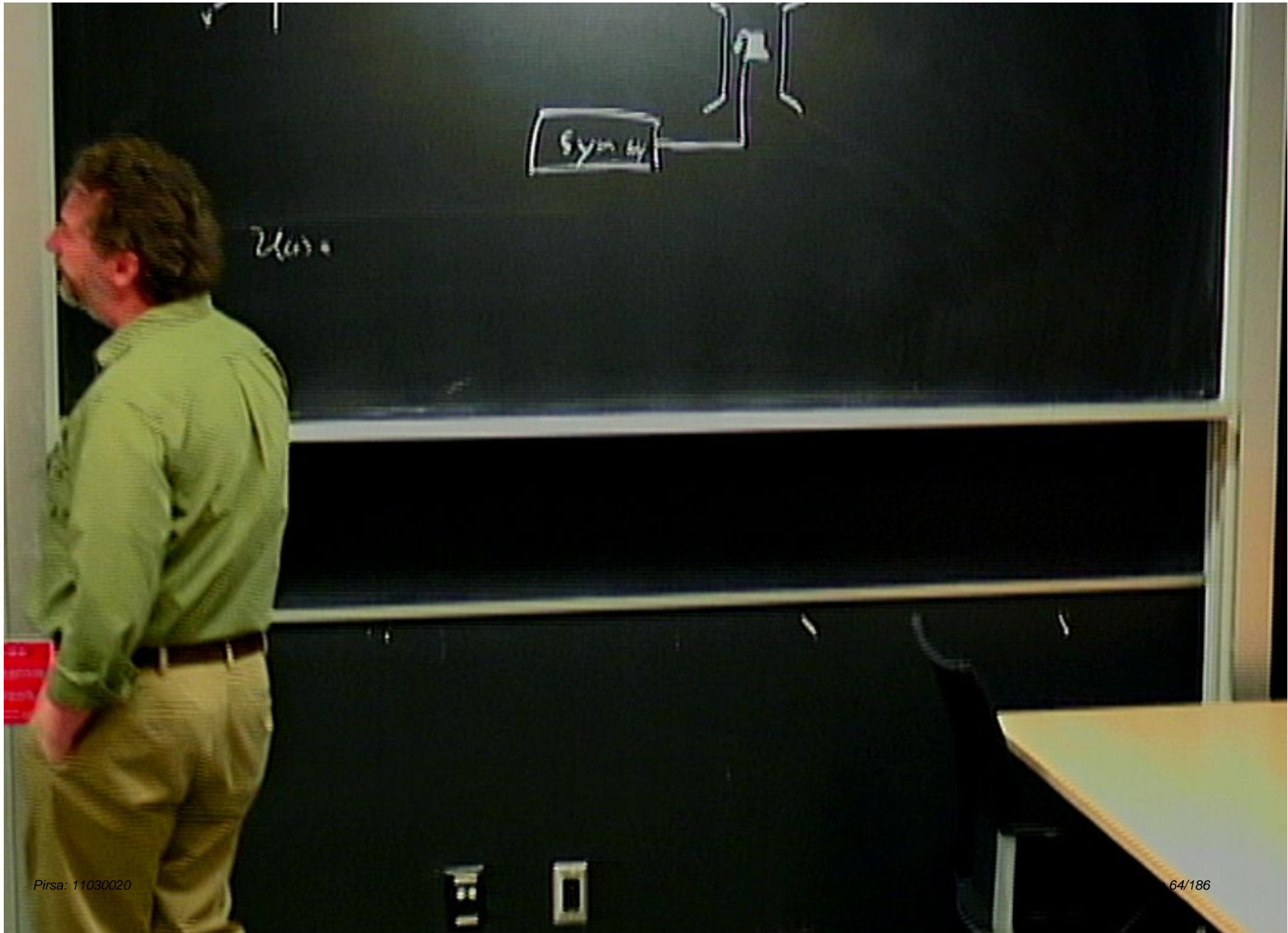
Rotating Wave Approximation

$\mathcal{H}_0 = \frac{\omega_0}{2} \sigma_x$
 $\mathcal{H}_1 = \frac{\omega_1}{2} \left\{ e^{i\omega_1 t} \sigma_x + e^{-i\omega_1 t} \sigma_x \right\}$
 $\mathcal{H}(t) = \mathcal{H}_0 + \mathcal{H}_1$
 $\vec{P} = \mathcal{U}_1(t) \vec{P} \mathcal{U}_1^{-1}(t)$
 $\frac{d\vec{P}}{dt} = -i[\mathcal{H}, \vec{P}]$
 $\vec{P} = \mathcal{U}(t) \vec{P} \mathcal{U}^{-1}(t)$
 $\mathcal{H} = \mathcal{U}_1 \mathcal{H}_0 \mathcal{U}_1^{-1}$
 $\frac{d\vec{P}}{dt} = \frac{d\mathcal{U}_1^{-1}(t)}{dt} \vec{P} \mathcal{U}_1(t) + \mathcal{U}_1^{-1}(t) \frac{d\vec{P}}{dt} \mathcal{U}_1(t) + \mathcal{U}_1^{-1}(t) \mathcal{H}_0 \mathcal{U}_1(t) - \mathcal{U}_1^{-1}(t) \mathcal{H}_1 \mathcal{U}_1(t)$



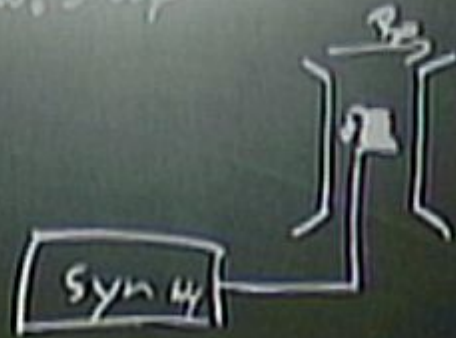
Rotating Wave Approximation

$\omega_0 = \omega_0 \sigma_x$; $\mathcal{H}(t) = \omega_1 \cos(\omega_1 t) \sigma_x$; $\frac{d\rho}{dt} = -i[\mathcal{H}, \rho]$
 $\mathcal{H}_1(t) = \frac{\omega_1}{2} \left\{ e^{i\frac{\omega_1 t}{2} \sigma_x} \sigma_x e^{-i\frac{\omega_1 t}{2} \sigma_x} + e^{-i\frac{\omega_1 t}{2} \sigma_x} \sigma_x e^{i\frac{\omega_1 t}{2} \sigma_x} \right\}$; $\rho = \mathcal{U}(t) \rho \mathcal{U}^\dagger(t)$
 $\mathcal{P} = \mathcal{U}_1(t)$; $\mathcal{H} = \mathcal{U}_1 \mathcal{H} \mathcal{U}_1^\dagger$
 $\mathcal{U}_1(t) \frac{d\rho \mathcal{U}_1^\dagger(t)}{dt} + \frac{d\mathcal{U}_1(t)}{dt} \rho \mathcal{U}_1^\dagger(t)$
 $\mathcal{U}_1(t) \frac{d\rho \mathcal{U}_1^\dagger(t)}{dt} = -i[\mathcal{H}, \rho]$; $\frac{d\mathcal{U}_1(t)}{dt} = -i\mathcal{H}_1 \mathcal{U}_1$

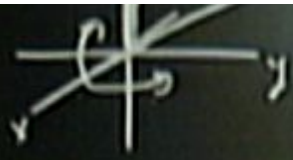




$w_0 = 3w_p$



~~$C_1 w_0 + C_2$~~



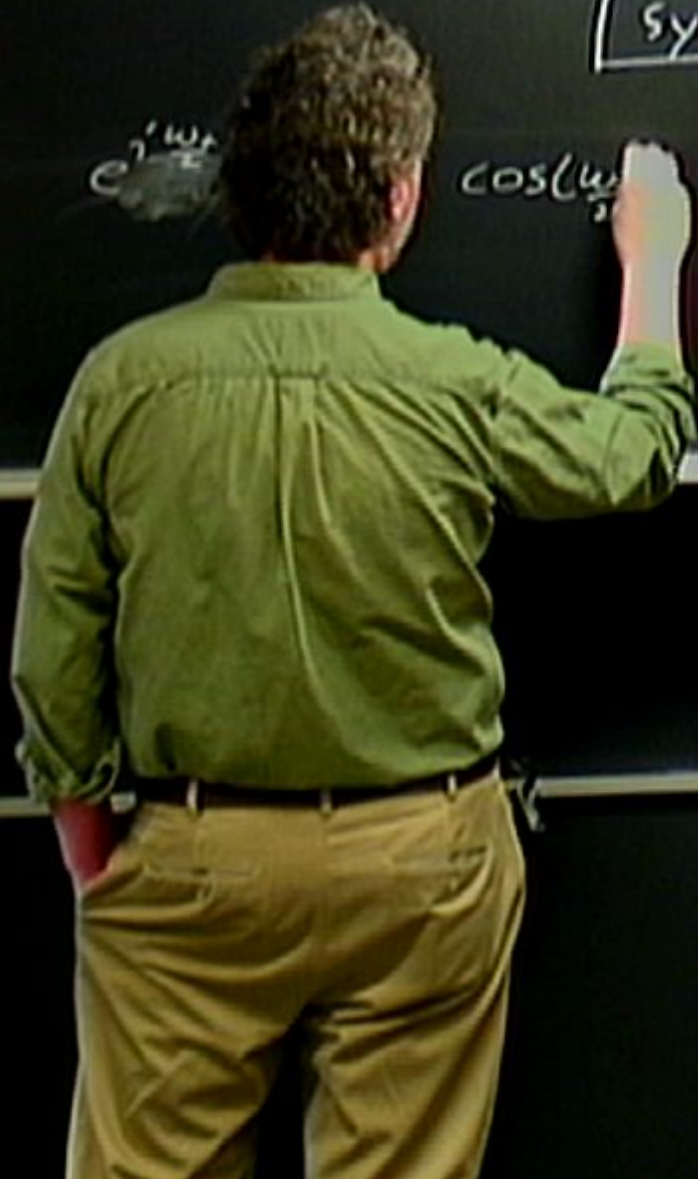
$w_1 \cos w_0 t + w_2 \sin w_0 t$



Syn H

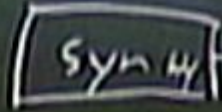
$e^{j\omega t}$

$\cos(\omega t)$



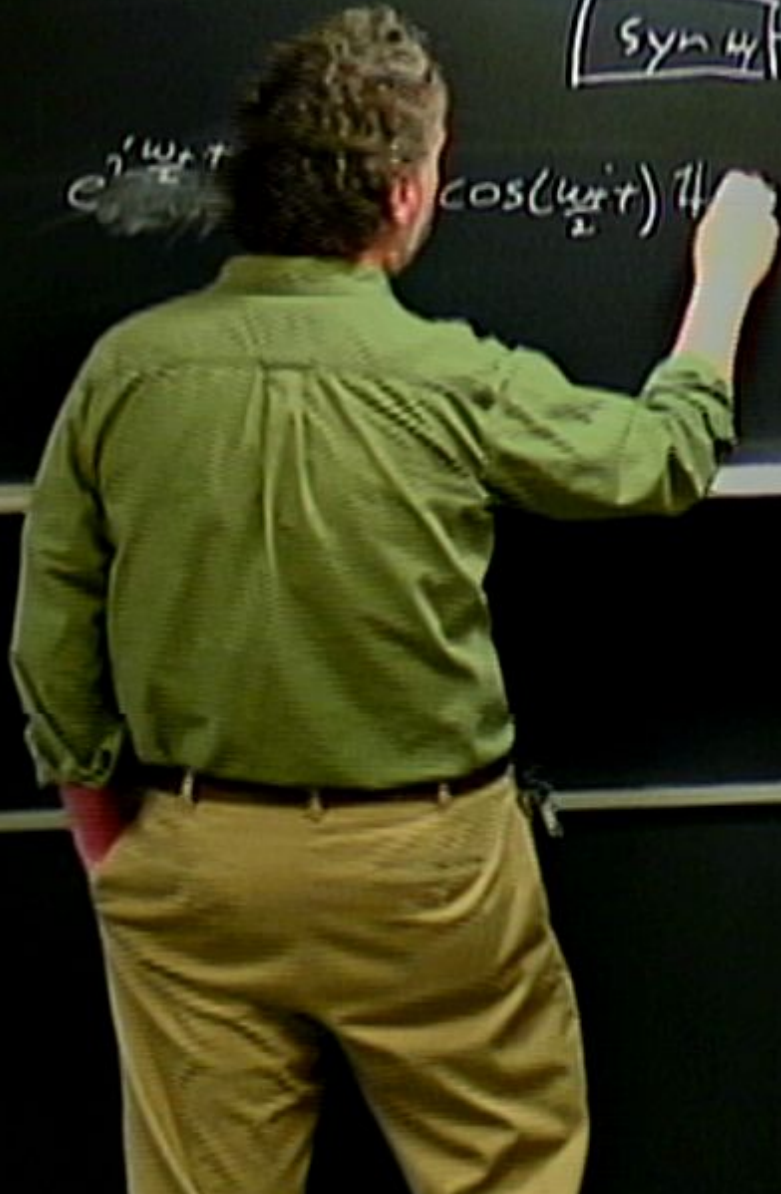


$$\omega_c = \omega_c = \omega_c$$



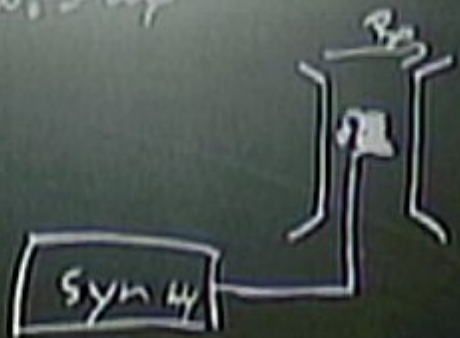
$$e^{j\omega_c t}$$

$$\cos(\frac{\omega_c t}{2})$$





$\omega_1 \approx \omega_2 \approx \omega_0$



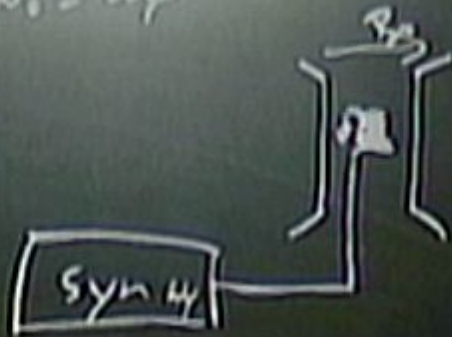
~~$e^{j(\omega_1 t + \phi)}$~~

$$e^{j(\frac{\omega_1 + \omega_2}{2} t)} \cos(\frac{\omega_1 - \omega_2}{2} t) \approx \cos(\frac{\omega_1 - \omega_2}{2} t) \approx 1$$





$\omega_1 \approx \omega_0 \approx \omega_p$

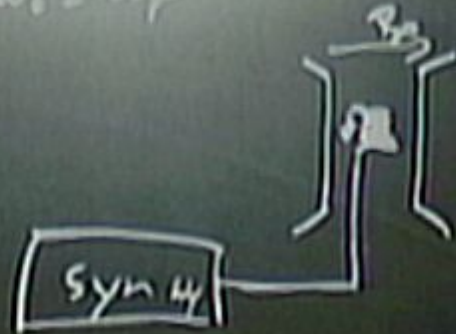


$e^{j(\omega_1 t + \theta)}$

$$\rightarrow \cos\left(\frac{\omega_1 t}{2}\right) \frac{1}{2} + j \sin\left(\frac{\omega_1 t}{2}\right) \frac{1}{2}$$



$\omega = \omega_0 = 2\pi f$



~~$e^{j(\omega t + \phi)}$~~

$$\cos\left(\frac{\omega t}{2}\right) \hat{u} + j \sin\left(\frac{\omega t}{2}\right) \hat{v}$$

Syn 44

~~$e^{j(\omega t + \sigma)}$~~

$$\cos\left(\frac{\omega t}{2}\right) \mathbb{1} + j \sin\left(\frac{\omega t}{2}\right) \sigma$$

$$\hat{H} = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + e^{-i\omega_0 t} \sigma_x e^{i\omega_0 t} = \frac{\omega_0 + \omega_1}{2} \sigma_x$$

$$\hat{H}_{eff} = \hat{H}$$

time independent

$$\hat{H}_{eff} = \frac{\omega_0 - \omega_1}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x$$

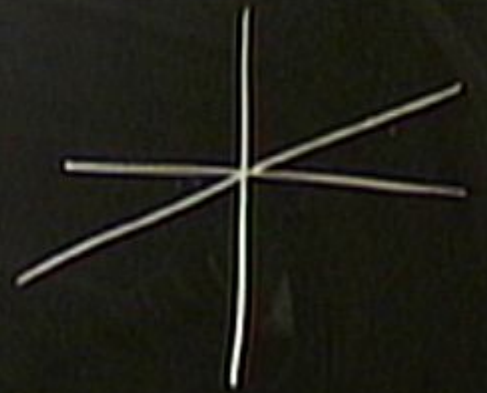


$$\Psi = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + \underbrace{e^{-i\omega_0 t} \sigma_x e^{i\omega_0 t}}_{\text{doubly acting}} = \frac{\omega_1}{2} \sigma_x$$

Ψ

$$\Psi_{\text{eff}} = \frac{\omega_0 - \omega_1}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x$$

time independent

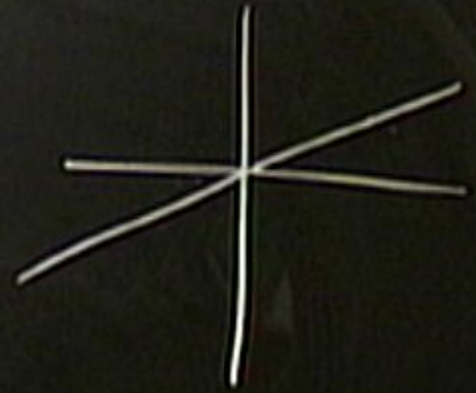


$$M_T = \frac{w_0}{2} \sigma_x^2 + \frac{w_1}{2} \sigma_x^2 + \underbrace{e^{-i w_0 t \sigma_x} e^{i w_1 t \sigma_x}}_{\text{daily return}} \approx \frac{w_1 + \sigma_x}{2}$$

y

$$CFR = \frac{w_0 - w_1}{2} \sigma_x^2 + \frac{w_1 \sigma_x}{2}$$

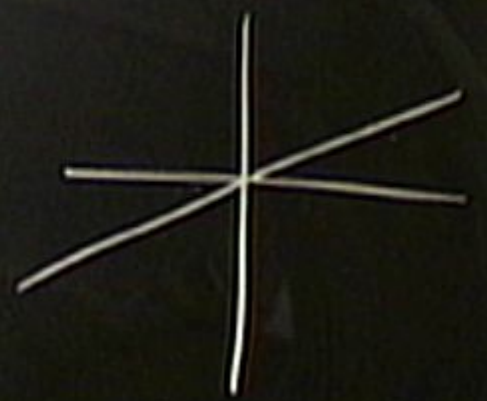
are independent



$$\mathcal{H} = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + \underbrace{e^{-i\omega_0 t} \sigma_x e^{i\omega_0 t}}_{\text{dotly mady}} + \frac{\omega_1}{2} \sigma_x$$

\mathcal{H}_{eff}

$$\mathcal{H}_{\text{eff}} = \frac{\omega_0 - \omega_1}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x$$



$$\hat{H} = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + e^{-i\omega_0 t} \sigma_x e^{i\omega_0 t} = \frac{\omega_0 + \omega_1}{2} \sigma_x$$

$$\hat{H}_{eff} = \hat{H}$$

time independent

$$\hat{H}_{eff} = \frac{\omega_0 - \omega_1}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x$$

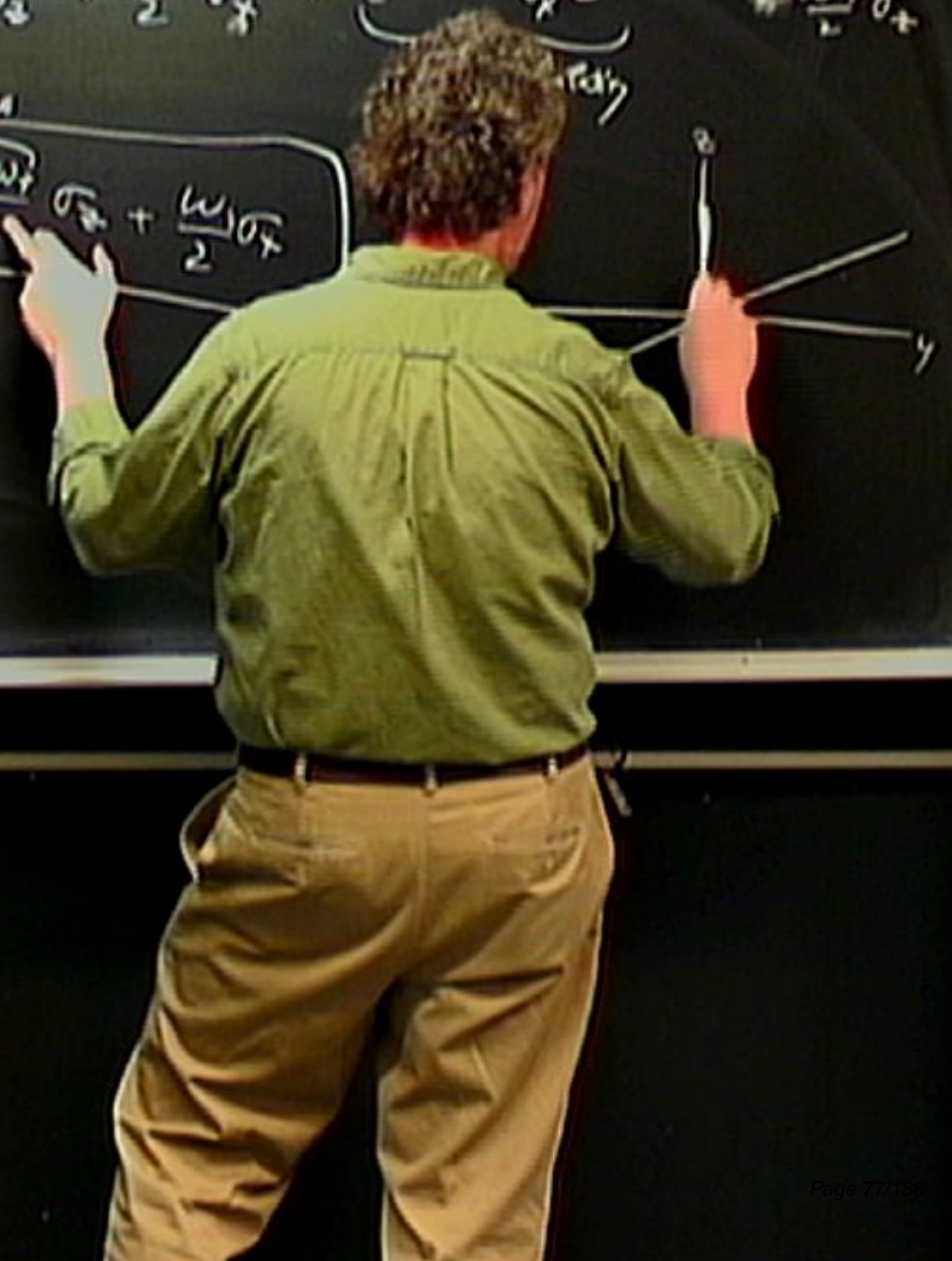


$$\hat{H} = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + \left(\frac{-i\omega_2 \tau \sigma_z}{2} + \frac{i\omega_2 \tau \sigma_z}{2} \right) \frac{\omega_1 + \sigma_x}{2}$$

$$\hat{H}_{\text{eff}} = \hat{H}$$

time independent

$$\hat{H}_{\text{eff}} = \frac{\omega_0 - \omega_1}{2} \sigma_z + \frac{\omega_1 \sigma_x}{2}$$

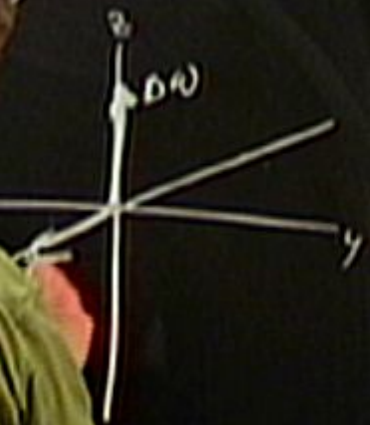


$$\hat{H} = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + e^{-i\omega_1 t} \sigma_x e^{i\omega_1 t} = \frac{\omega_0 + \omega_1}{2} \sigma_z$$

$$\hat{H}_{eff} = \hat{H}$$

time independent

$$\hat{H}_{eff} = \frac{\omega_0 - \omega_1}{2} \sigma_z + \frac{\omega_1 \sigma_x}{2}$$



$$\hat{H} = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + e^{-i\omega_0 t} \sigma_x e^{i\omega_0 t} = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x$$

$$\hat{H}_{eff} = \hat{H}$$

time independent

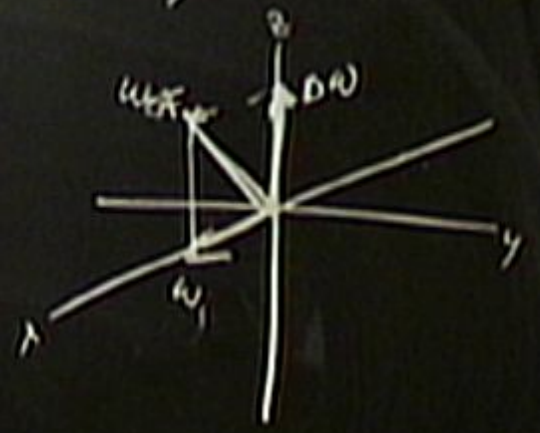
$$\hat{H}_{eff} = \frac{\omega_0 - \omega_1}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x$$



$$\mathcal{H} = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + \underbrace{e^{-i\omega_0 t} \sigma_x e^{i\omega_0 t}}_{\text{dobby unit}} = \frac{\omega_0}{2} \sigma_z$$

$$\mathcal{H}_{\text{eff}} = \mathcal{H}$$

$$\frac{\omega_0 - \omega_1}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x$$

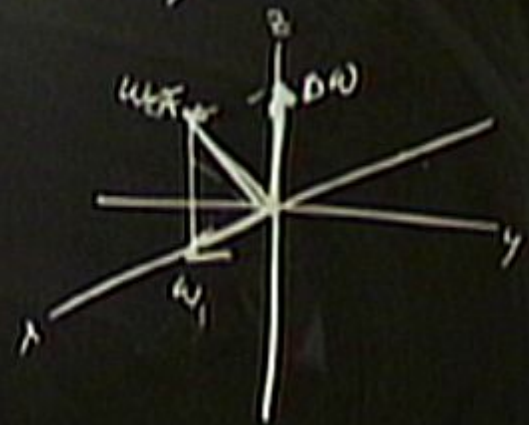


$$\mathcal{H} = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + \underbrace{e^{-i\omega_0 t} \sigma_z e^{i\omega_0 t}}_{\text{double rotation}} = \frac{\omega}{2} \sigma_x$$

$$\mathcal{H}_{\text{eff}} = \overline{\mathcal{H}}$$

$$\frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x$$

$$\omega_{\text{eff}} = \sqrt{\omega_0^2 + \omega_1^2}$$



$$\hat{H} = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + e^{-i\omega_0 t} \sigma_z e^{i\omega_0 t} = \frac{\omega_0 + \omega_1}{2} \sigma_z$$

$$\hat{H}_{\text{eff}} = \overline{\hat{H}}$$

time independent

$$\hat{H}_{\text{eff}} = \frac{\omega_0 - \omega_1}{2} \sigma_z + \frac{\omega_1}{2}$$

ω_{eff}



My utility

$$\frac{d\tilde{\rho}}{dt} = +\underbrace{U(t)\tilde{\rho}U^\dagger(t)}_{\tilde{\rho}} - i\underbrace{[H, \tilde{\rho}]}_{\tilde{\rho}} + \underbrace{U(t)[H, \rho]U^\dagger(t)}_{-i[\tilde{H}, \tilde{\rho}]}$$

$$\boxed{\frac{d\tilde{\rho}}{dt} = -i[\tilde{H}, \tilde{\rho}]}$$

$$U(t) = e^{-i\int_0^t H(t') dt'}$$

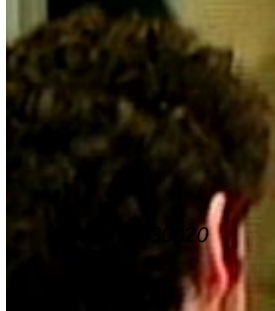
$\frac{d\psi}{dt}$

$$i\hbar \frac{d\psi}{dt} = H\psi \Rightarrow i\hbar \frac{d}{dt} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$$\frac{d}{dt} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$$\psi(t) = e^{-\frac{i}{\hbar} H_0 t} \psi(0)$$

$$= e^{-i H_0 t / \hbar}$$



$$U(t) \psi(0) = e^{-i \int_0^t H(t') dt'} \psi(0)$$

$$[H, P]$$

$$U(t) = e^{-i \int_0^t H(t') dt'}$$

change Hamiltonian



101

$$\bar{\Psi}(t) = \mathcal{H}^{(0)} + \mathcal{H}^{(1)} + \dots$$

$$\frac{d\vec{p}}{dt} = +i(\psi, \hat{p}) \psi' - \hat{p}$$

$$+i(\psi, \hat{p}) \psi' - \hat{p} = -i[\hat{H}, \vec{p}]$$

$$\frac{d\vec{p}}{dt} = -i[\hat{H}, \vec{p}]$$

$$U(t) = e^{-i\int_0^t \hat{H}(t') dt'}$$

↙ average Hamiltonian

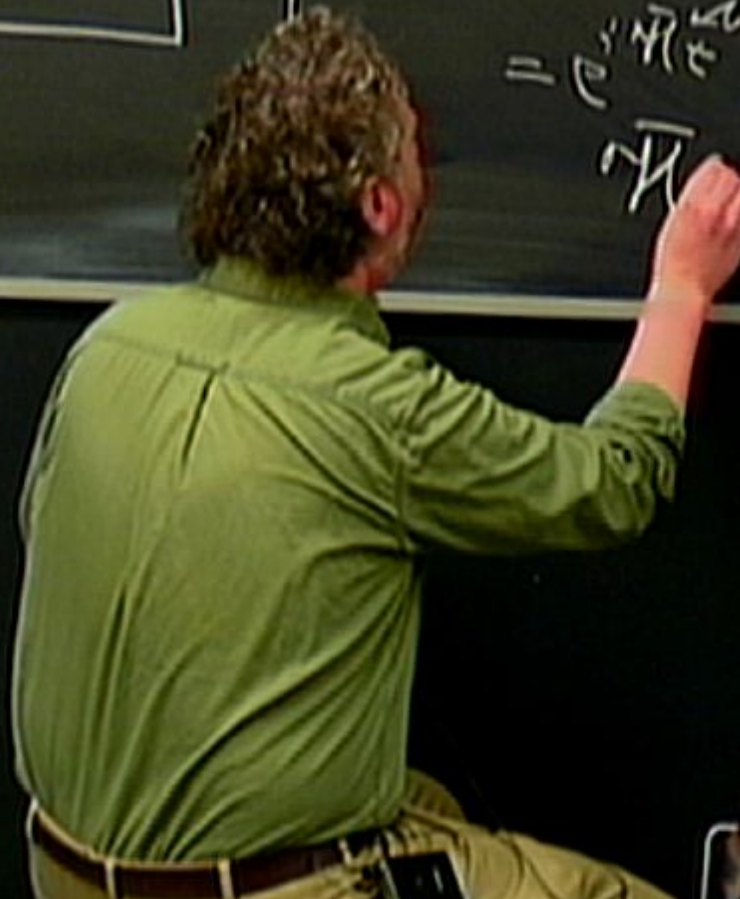
$$\frac{d\tilde{\rho}}{dt} = +i \underbrace{U(t) \tilde{\rho} U^\dagger(t)}_{\tilde{\rho}} - i \underbrace{U(t) \tilde{\rho} U^\dagger(t)}_{\tilde{\rho}} + U(t) [H, \tilde{\rho}] U^\dagger(t) - i [\tilde{H}, \tilde{\rho}]$$

$$\boxed{\frac{d\tilde{\rho}}{dt} = -i [\tilde{H} - H_0, \tilde{\rho}]}$$

$$U(t) = e^{-i \int_0^t H_0 dt}$$

$$= e^{-i H_0 t} \text{ average Hamiltonian}$$

$$\tilde{H}$$



$$\frac{d\tilde{\rho}}{dt} = + \underbrace{U(t) \tilde{\rho} U^\dagger(t)}_{\tilde{\rho}} - i \underbrace{[H, \tilde{\rho}]}_{\tilde{\rho}} + \underbrace{U(t) [H, \rho] U^\dagger(t)}_{-i [\tilde{H}, \tilde{\rho}]}$$

$$\boxed{\frac{d\tilde{\rho}}{dt} = -i [\tilde{H}, \tilde{\rho}]}$$

$$U(t) = e^{-i \int_0^t H(t') dt'}$$

\swarrow *change Hamiltonian*
 \searrow *Hamiltonian*
 \tilde{H}

$$\bar{H} = H^{(0)} + H^{(1)} + \dots$$

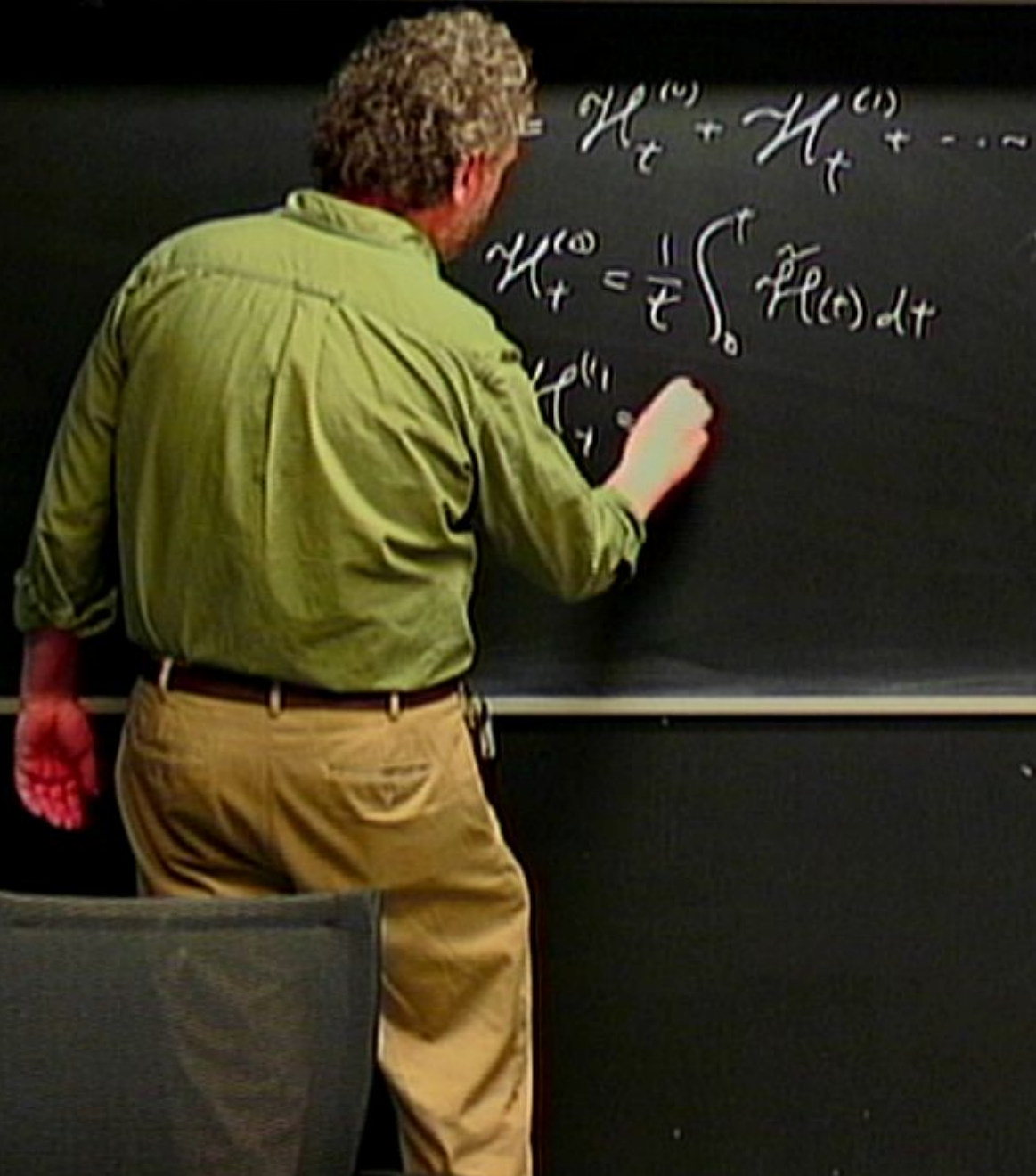
$$\bar{K}_t = K_t^{(0)} + K_t^{(1)} + \dots$$

$$\bar{H}_t = \mathcal{H}_t^{(0)} + \mathcal{H}_t^{(1)} + \dots$$

$$\mathcal{H}_t^{(0)} = \frac{1}{T} \int_0^T H(t) dt$$

$$= \mathcal{K}_t^{(0)} + \mathcal{K}_t^{(1)} + \dots$$

$$\mathcal{K}_t^{(0)} = \frac{1}{T} \int_0^T \tilde{F}(t) dt$$



$$\bar{\psi}(t) = \psi_t^{(0)} + \psi_t^{(1)} + \dots$$

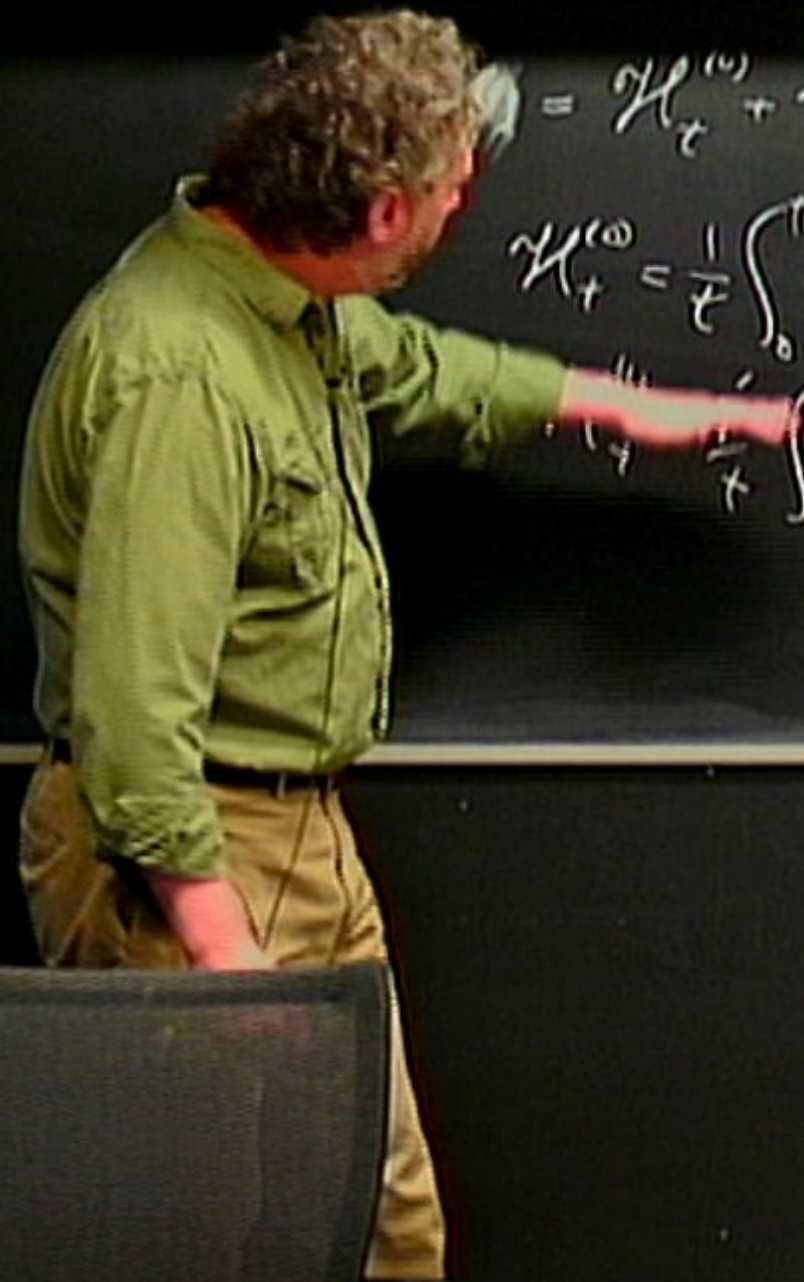
$$\psi_t^{(0)} = \frac{1}{T} \int_0^T \tilde{\psi}(t) dt$$

$$\psi_t^{(1)} = \frac{1}{T} \int_0^T dt_i \int_0^{t_i} [\mathcal{H}(t_i), \psi(t_i)] dt_i$$

$$= \mathcal{K}_t^{(0)} + \mathcal{K}_t^{(1)} + \dots$$

$$\mathcal{K}_t^{(0)} = \frac{1}{t} \int_0^t \tilde{H}(t) dt$$

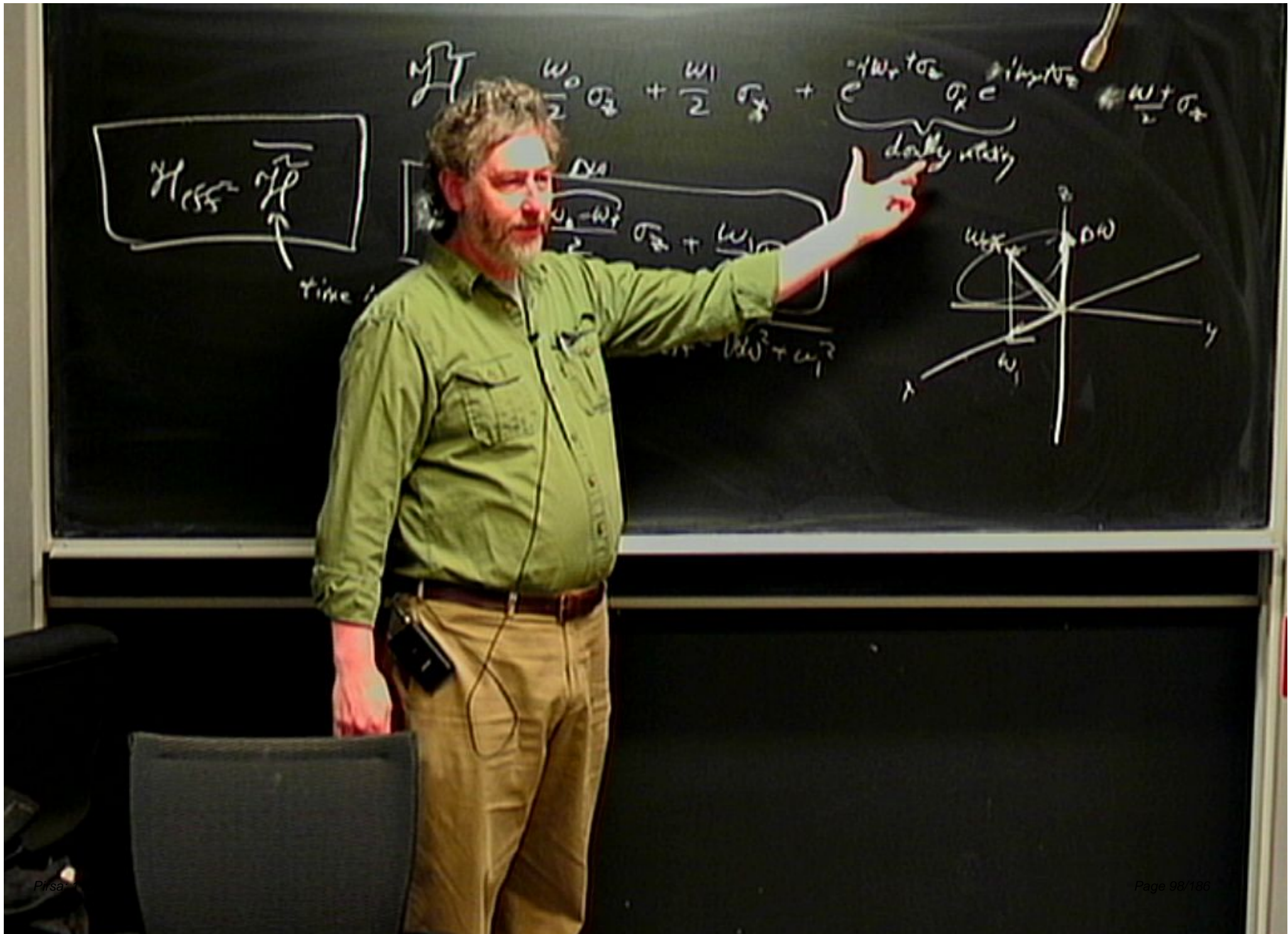
$$\mathcal{K}_t^{(1)} = \frac{1}{t} \int_0^t dt_1 \int_0^{t_1} [\tilde{H}(t_1), \tilde{H}(t_1)] dt_1$$



$$\tilde{\mathcal{H}}(t) = \tilde{\mathcal{H}}_t^{(0)} + \tilde{\mathcal{H}}_t^{(1)} + \dots$$

$$\tilde{\mathcal{H}}_t^{(0)} = \frac{1}{T} \int_0^T \tilde{\mathcal{H}}(t) dt$$

$$\tilde{\mathcal{H}}_t^{(1)} = \frac{1}{T} \int_0^T dt_1 \int_0^{t_1} [\tilde{\mathcal{H}}(t_1), \tilde{\mathcal{H}}(t)] dt$$

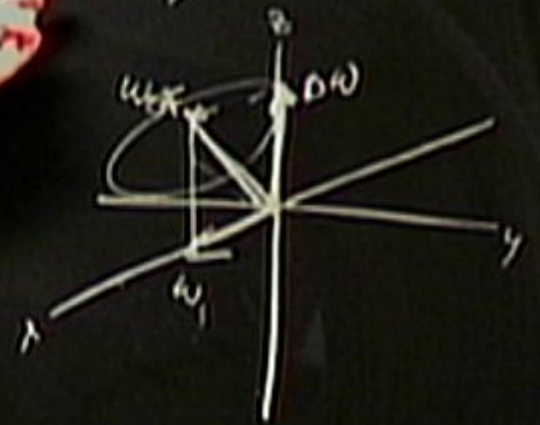


$$H = \frac{W_0}{2} \sigma_z + \frac{W_1}{2} \sigma_x + \underbrace{-4W_1 \sigma_x}_{\text{doubly deg.}} + iW_2 \sigma_y = \frac{W}{2} \sigma_x$$

$$H_{\text{eff}} = \overline{H}$$

time

$$H_1 = \frac{W_1}{2} \sigma_x + \frac{W_2}{2} \sigma_y$$



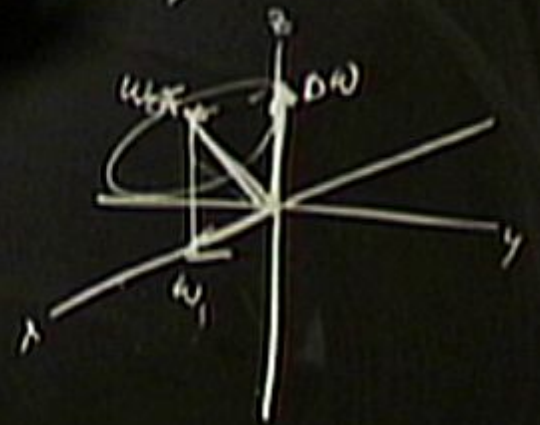
$$\hat{H} = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + \left(\frac{-i\omega_r \sigma_z}{2} + \frac{i\omega_r \sigma_z}{2} \right) \sigma_x \approx \frac{\omega_0 + \omega_1}{2} \sigma_z$$

$$\hat{H}_{\text{eff}} = \hat{H}$$

time independent

$$\hat{H}_{\text{eff}} = \hat{H}$$

$$E = \sqrt{\omega_0^2 + \omega_1^2}$$



$\tilde{H}(t)$

$$\tilde{H}(t) = \tilde{H}_t^{(0)} + \tilde{H}_t^{(1)} + \dots$$

$$\tilde{H}_t^{(0)} = \frac{1}{T} \int_0^T \tilde{H}(t) dt$$

$$\tilde{H}_t^{(1)} = \frac{1}{T} \int_0^T dt_1 \int_0^{t_1} [\tilde{H}(t_1), \tilde{H}(t)] dt$$

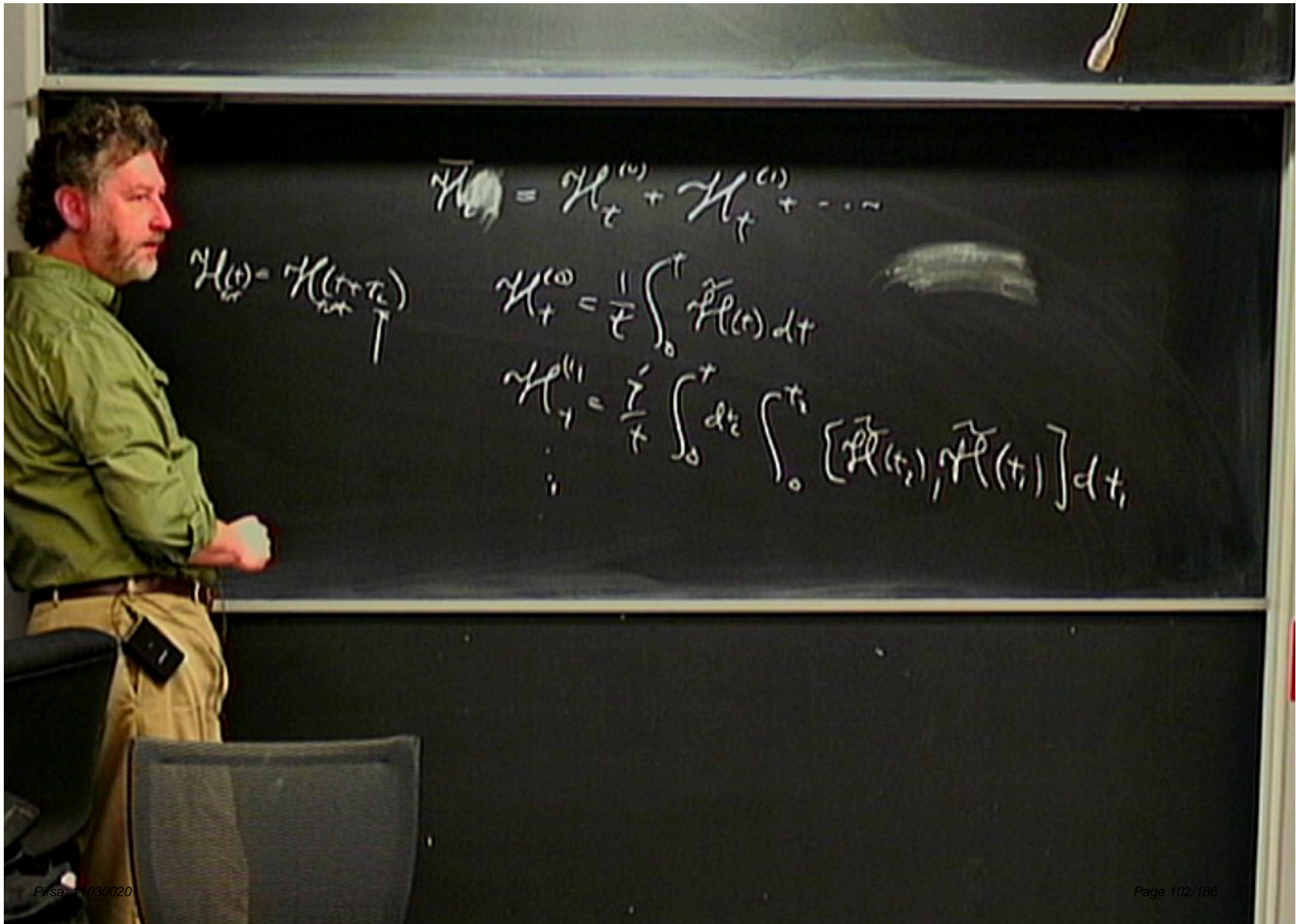


$$\tilde{H}_t = \tilde{H}_t^{(0)} + \tilde{H}_t^{(1)} + \dots$$

$$\tilde{H}_t^{(0)} = \tilde{H}_{t+\tau}$$

$$\tilde{H}_t^{(0)} = \frac{1}{T} \int_0^T \tilde{H}(t) dt$$

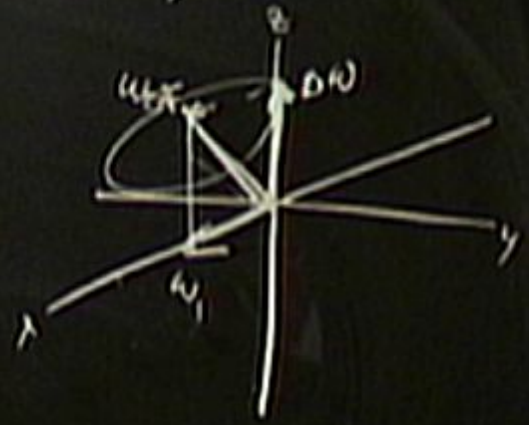
$$\tilde{H}_t^{(1)} = \frac{1}{T} \int_0^T dt_1 \int_0^{\tau_1} [\tilde{H}(t_1), \tilde{H}(t_1)] dt_1$$



$$\mathcal{H}_{\text{eff}} = \frac{\mathcal{H}}{\hbar}$$

$$\frac{\omega_0}{2} \sigma_x + \frac{\omega_1}{2} \sigma_y + \underbrace{e^{-i\omega_0 t} \sigma_x}_{\text{driving term}} + i\hbar \gamma \sigma_x$$

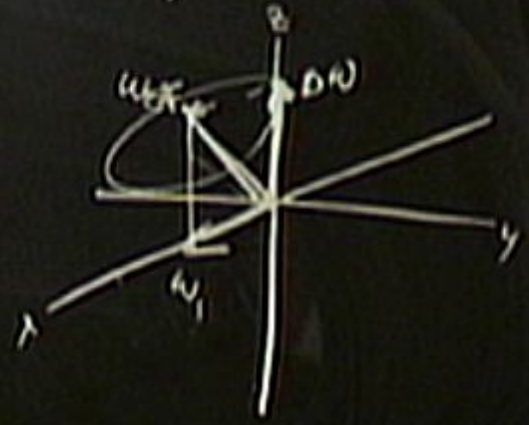
$$\omega_{\text{eff}} = \sqrt{\omega_0^2 + \omega_1^2}$$



$$\hat{H} = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + \underbrace{e^{-i\omega_0 t} \sigma_x e^{i\omega_0 t}}_{\text{daily mean}} = \frac{\omega_0 + \sigma_x}{2}$$

$$\omega_{\text{eff}} = \frac{\omega_0 - \omega_1}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x$$

$$\omega_{\text{eff}} = \sqrt{\omega_0^2 + \omega_1^2}$$



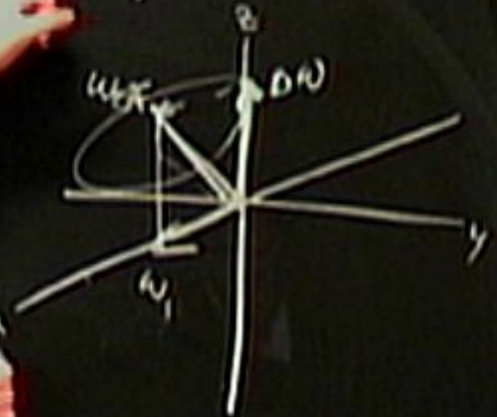
$$\mathcal{H}_{\text{CSF}} = \overline{\mathcal{H}}$$

time independent

$$\mathcal{H} = \frac{\omega_0}{2} \sigma_z + \omega_1 \sigma_x$$

$$\mathcal{H}_{\text{CSF}} = \frac{\omega_+ - \omega_-}{2} \sigma_x$$

$-\frac{\omega_+ + \omega_-}{2} \sigma_z + \frac{\omega_+ - \omega_-}{2} \sigma_x$
 dotly rising



$$\mathcal{H} = \frac{p_x^2}{2m} + \frac{1}{2} m \omega^2 x^2 + \frac{\hbar \omega}{2} \left(\frac{1}{2} \left(\frac{p_x}{\hbar} + i m \omega x \right) \left(\frac{1}{2} \left(\frac{p_x}{\hbar} - i m \omega x \right) \right) \right) = \frac{p_x^2}{2m} + \frac{1}{2} m \omega^2 x^2$$

$$\mathcal{H}_{eff} = \mathcal{H}$$

time indep

$$\sigma_x + \frac{\hbar \omega}{2} \sigma_x = \sqrt{\hbar^2 \omega^2 + \omega_1^2}$$



$$\mathcal{H}_{\text{eff}} = \overline{\mathcal{H}}$$

time independent

$$\mathcal{H} = \frac{p^2}{2m} + V(x)$$

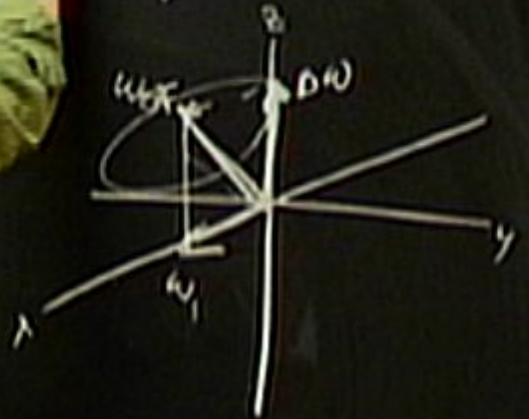
$$\mathcal{H}_{\text{eff}} = \frac{p^2}{2m} + \overline{V(x)}$$

$$\sigma_y \rightarrow \sigma_x$$

$$+ \frac{1}{2} \omega_1^2 x^2$$

$$+ \frac{1}{2} \omega_2^2 y^2$$

$$+ \frac{1}{2} \omega_3^2 z^2$$



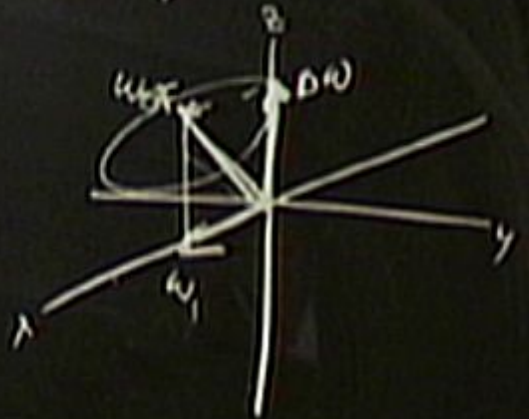
$$H_{\text{eff}} = \overline{H}$$

τ_{in}

$$\frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + \underbrace{e^{-i\omega_0 \tau} \sigma_x e^{i\omega_0 \tau}}_{\text{doubly rotating}} \approx \frac{\omega_0 + \omega_1}{2} \sigma_z$$

$$\frac{\omega_0 - \omega_1}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x$$

$$\omega_{\text{eff}} = \sqrt{\omega_0^2 + \omega_1^2}$$



at times t_1 and t_2 .

```
Ht1 = Ht /. {t -> t1};
Ht2 = Ht /. {t -> t2};
```

The first order term in the magnus expansion is given by

```
Hb1 =
Simplify[
  (-I / 2 / tc)
  Integrate[Integrate[comm[Ht2, Ht1], {t1, 0, t2}],
    {t2, 0, tc}], Element[{w1, w0}, Reals]];
Hb1 // MatrixForm
```

$$\begin{pmatrix} \frac{w_1^2}{8 w r f} & 0 \\ 0 & -\frac{w_1^2}{8 w r f} \end{pmatrix}$$

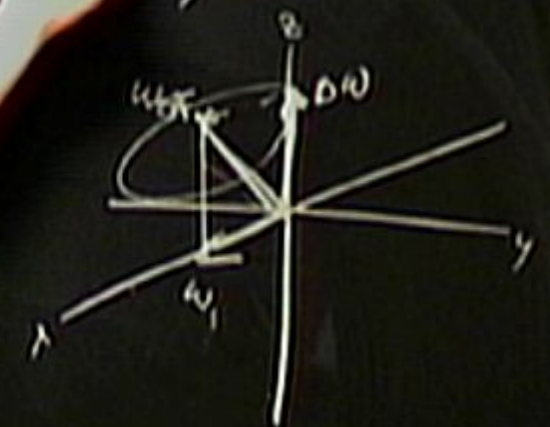
$$H = \frac{p^2}{2m} + \frac{1}{2} m \omega^2 x^2 + \dots$$

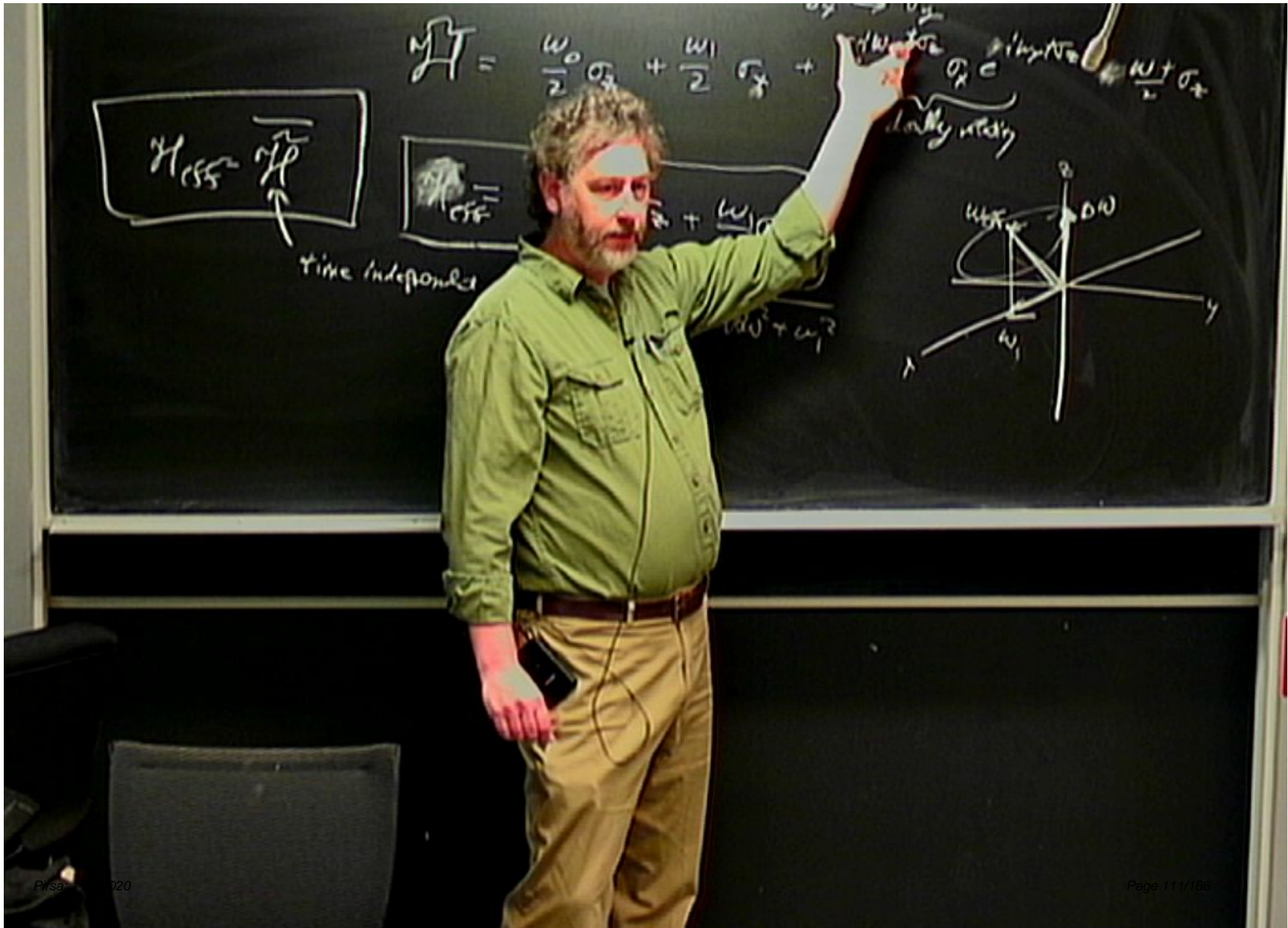
$$H_{eff} = \overline{H}$$

time independent

$$H_{eff} = \dots$$

$$+ \frac{m \omega^2}{2} x^2$$



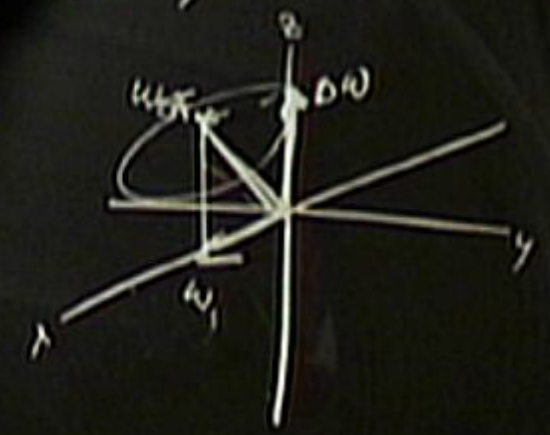


$$H = \frac{\omega_0}{2} q^2 + \frac{\omega_1}{2} p^2 + \dots$$

$$H_{CSF} = H_P$$

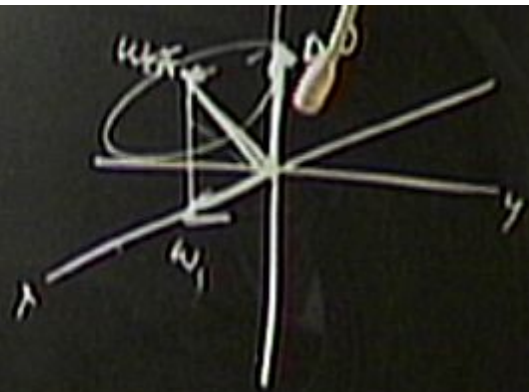
time independent

$$H_{CSF} = H_P$$



$$\frac{\omega_A - \omega_B}{2} \sigma_x + \frac{\omega_B}{2} \sigma_y$$

time intervals



$$\omega_{\text{eff}} = \sqrt{\omega^2 + \omega_1^2}$$

$\mathcal{H}^{(1)}$

$$\frac{\gamma}{\tau} \int_0^{\tau} dt_2 \int_0^{t_2} [\tilde{\mathcal{H}}(t_2), \tilde{\mathcal{H}}(t_1)] dt_1$$

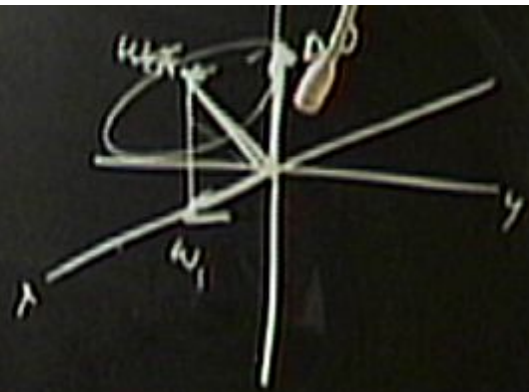
$$\left[\frac{\omega_A - \omega_B}{2} \sigma_x + \frac{\omega_B}{2} \sigma_y \right]$$

time independent

$$\omega_{\text{eff}} = \sqrt{\omega^2 + \omega_1^2}$$

$$\mathcal{H}^{(1)} =$$

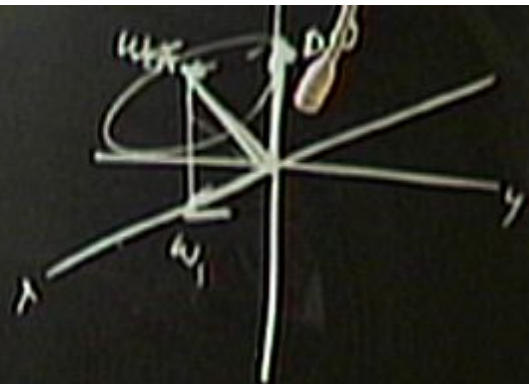
$$\sigma_x$$



$$\int_0^{2\pi} [\vec{r}(t_1), \vec{r}(t_1)] d(t_1)$$

$$\frac{\omega_1 - \omega_2}{2} \sigma_x + \frac{\omega_1 + \omega_2}{2} \sigma_y$$

time independent



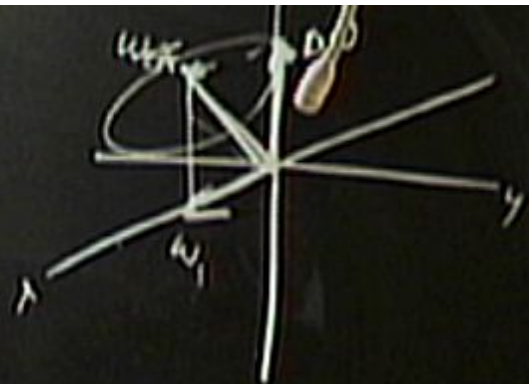
$$\omega_{\text{CSF}} = \sqrt{\omega^2 + \omega_1^2}$$

$$\frac{\omega_1^2}{8\omega_1} \sigma_x$$

$$\frac{1}{T} \int_0^T dt_i \int_0^{t_i} [\vec{F}(t_1), \vec{F}(t_1)] dt_1$$

$$\frac{\omega_1 - \omega_2}{2} \sigma_x + \frac{\omega_1 + \omega_2}{2} \sigma_y$$

time independent



$$\omega_{CSF} = \sqrt{\omega_1^2 + \omega_2^2}$$

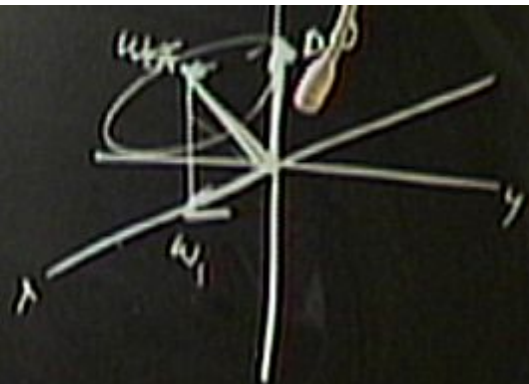
$$\mathcal{H}^{(1)} = \frac{\omega_1^2}{8\omega_1} \sigma_x$$

$$\int_0^{t_1} [\vec{H}(t_1), \vec{H}(t_1)] dt_1$$



time independent

$$\frac{\omega_1 - \omega_2}{2} \sigma_x + \frac{\omega_1 + \omega_2}{2} \sigma_y$$



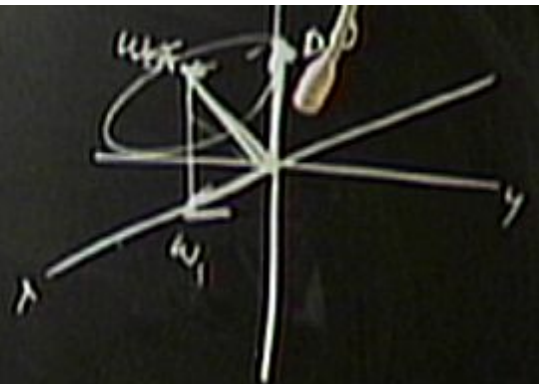
$$\omega_{\text{eff}} = \sqrt{\omega^2 + \omega_1^2}$$

$$\tilde{H}^{(1)} = \frac{\omega_1^2}{8\omega t_0} \sigma_x$$

$$\tilde{H}^{(1)} = \frac{1}{t} \int_0^t dt_1 \int_0^{t_1} [\tilde{H}(t_1), \tilde{H}(t_1)] dt_1$$



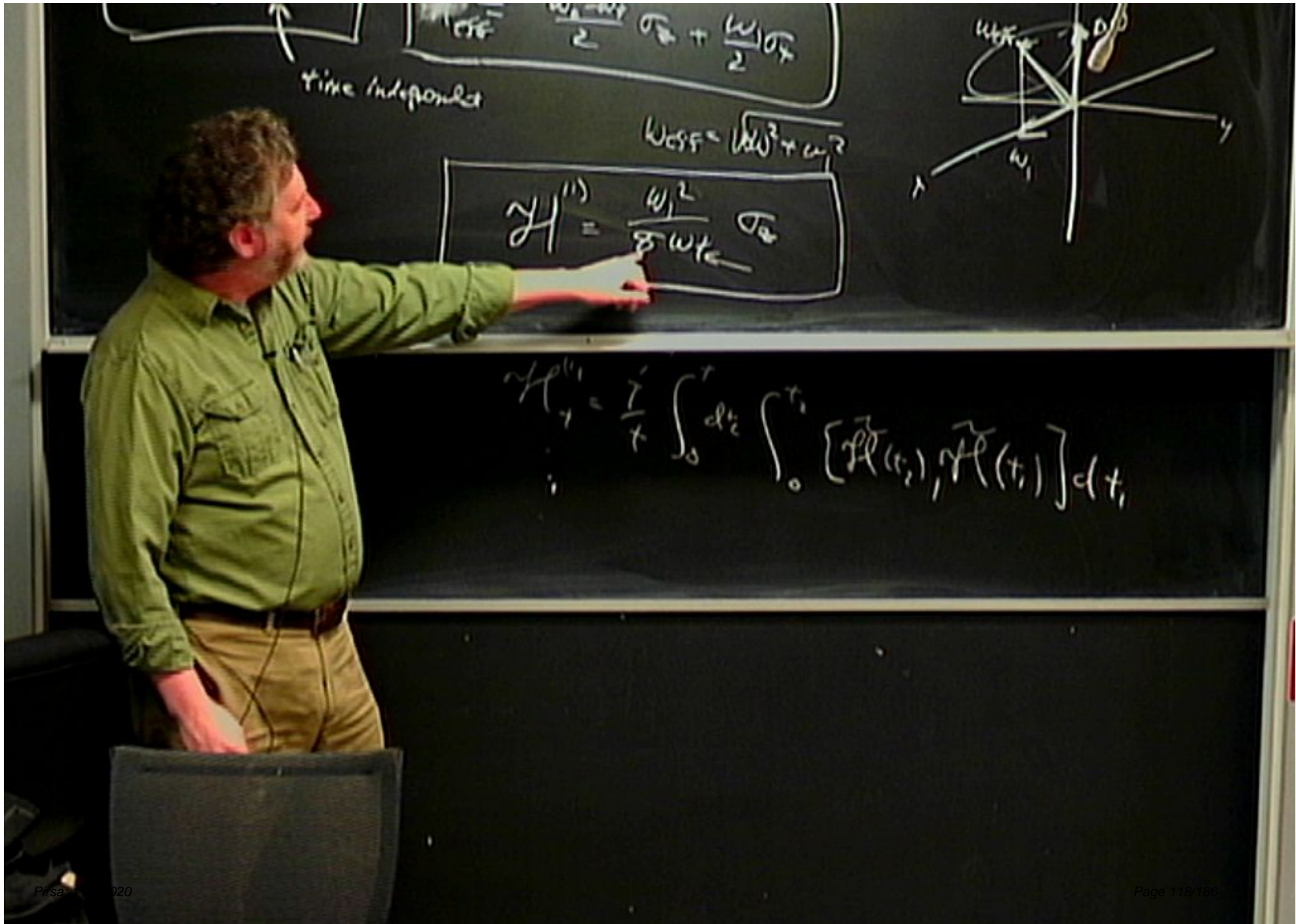
time independent $\frac{\omega_1^2}{2} \sigma_z + \frac{\omega_2}{2} \sigma_x$

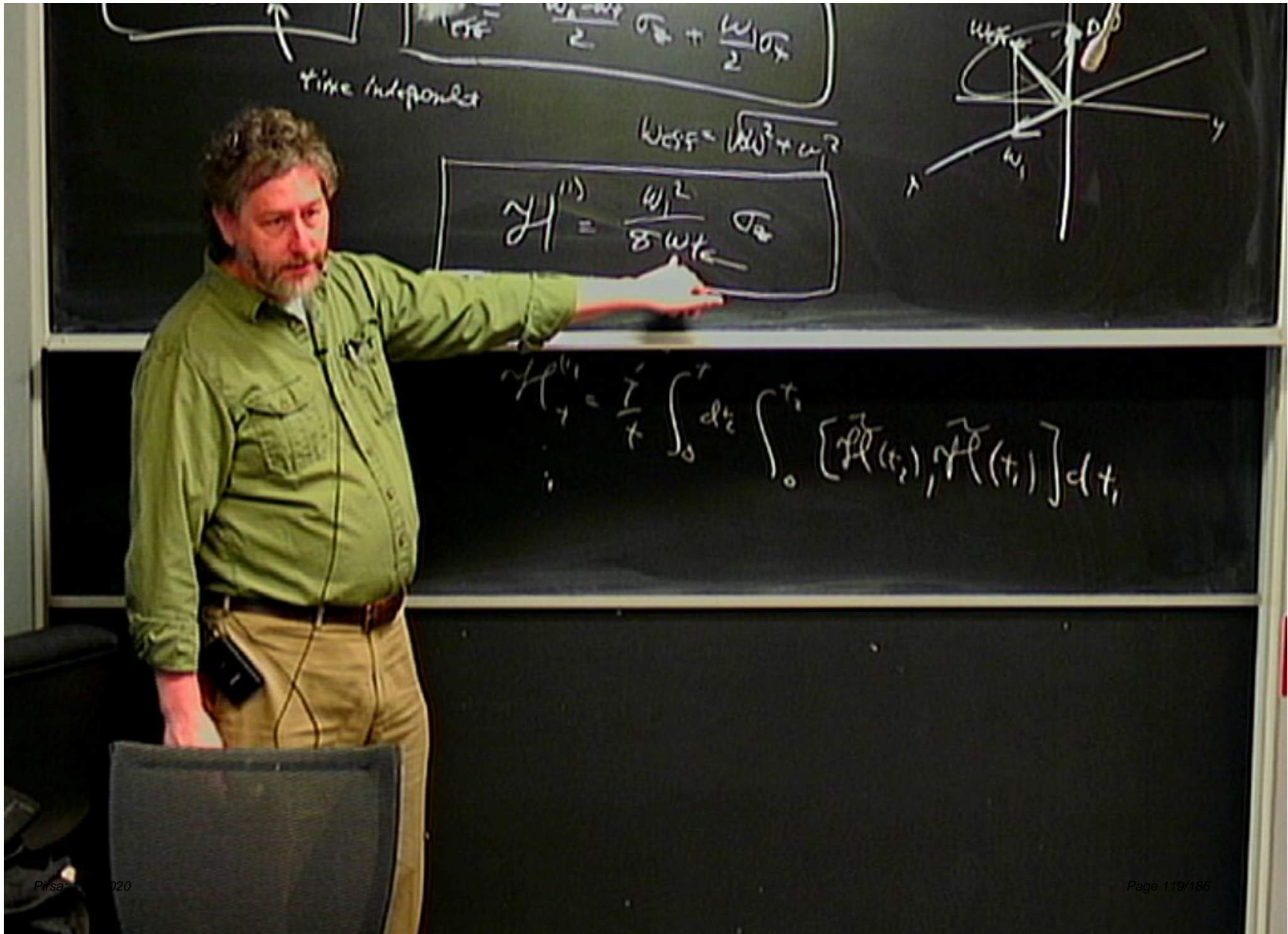


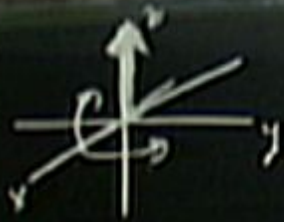
$$\omega_{\text{eff}} = \sqrt{\omega_1^2 + \omega_2^2}$$

$$\tilde{H}^{(1)} = \frac{\omega_1^2}{8\omega_2} \sigma_z$$

$$\tilde{H}^{(1)} = \frac{i}{\tau} \int_0^\tau dt_2 \int_0^{t_2} [\tilde{H}(t_1), \tilde{H}(t_1)] dt_1$$

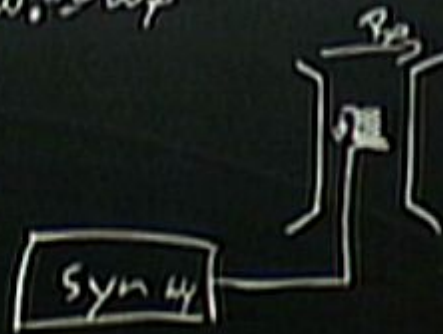






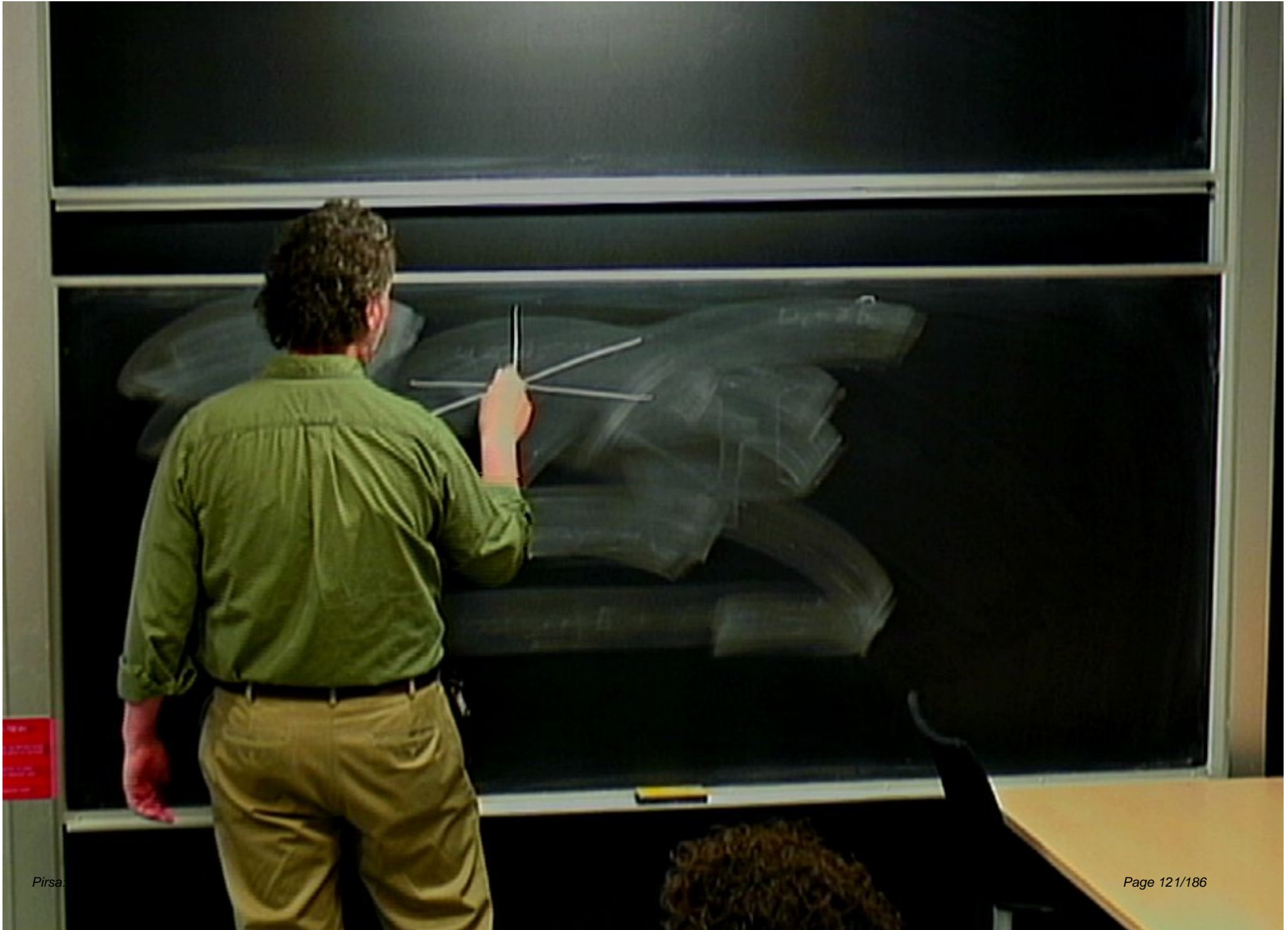
$$\omega \ll \omega_0 \Rightarrow \omega_p$$

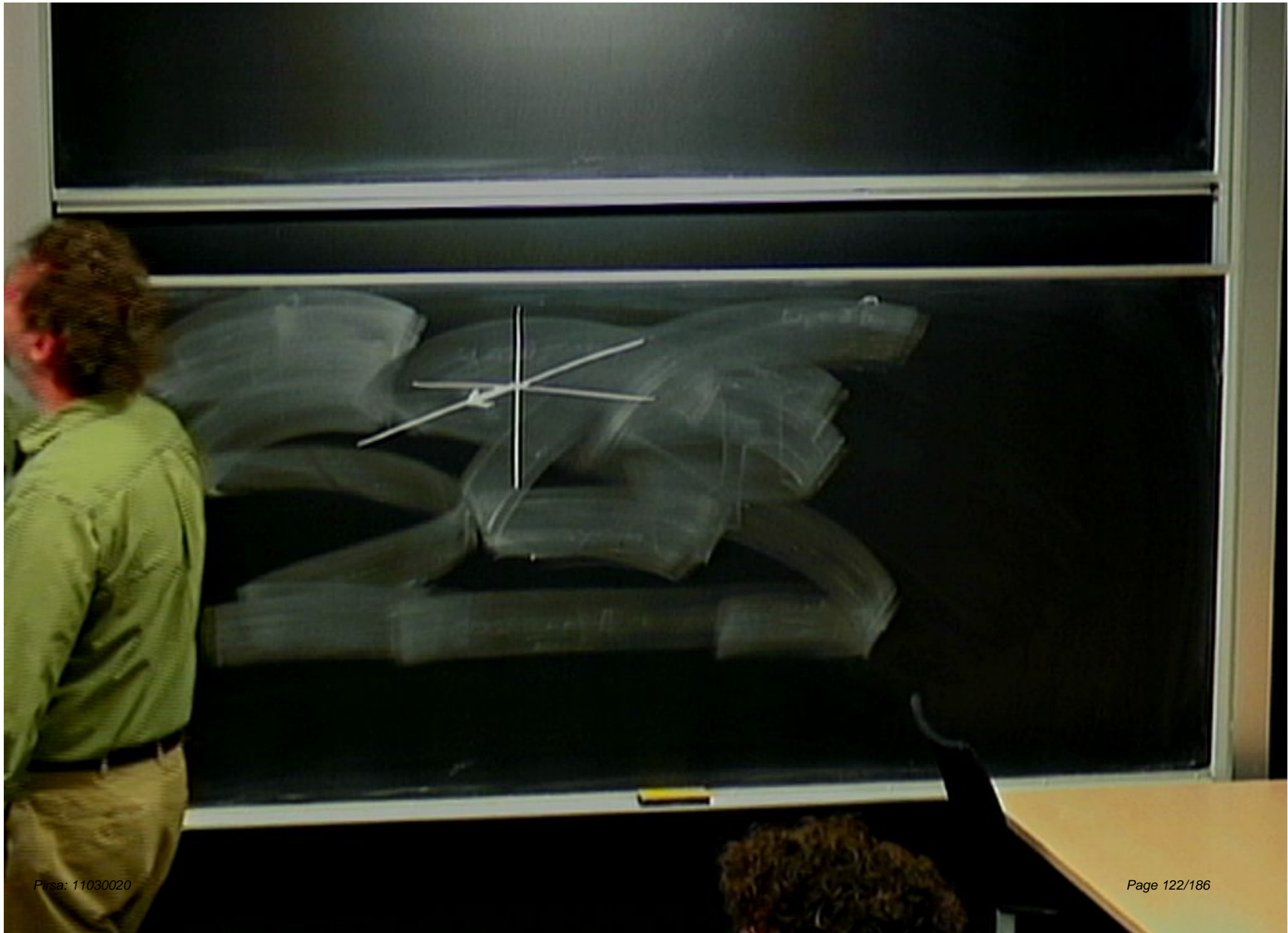
$$\omega_0 \approx \delta B_0$$

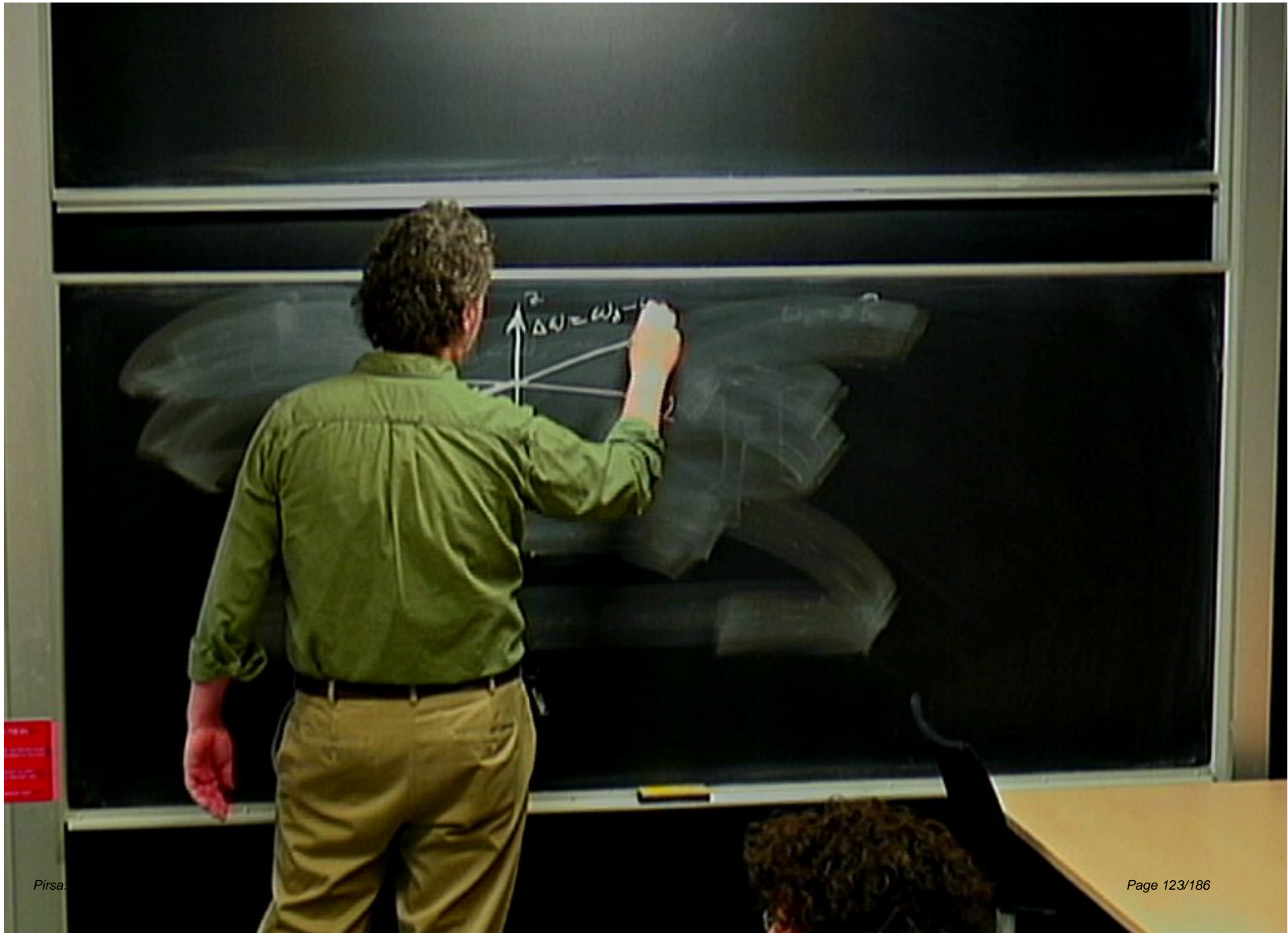


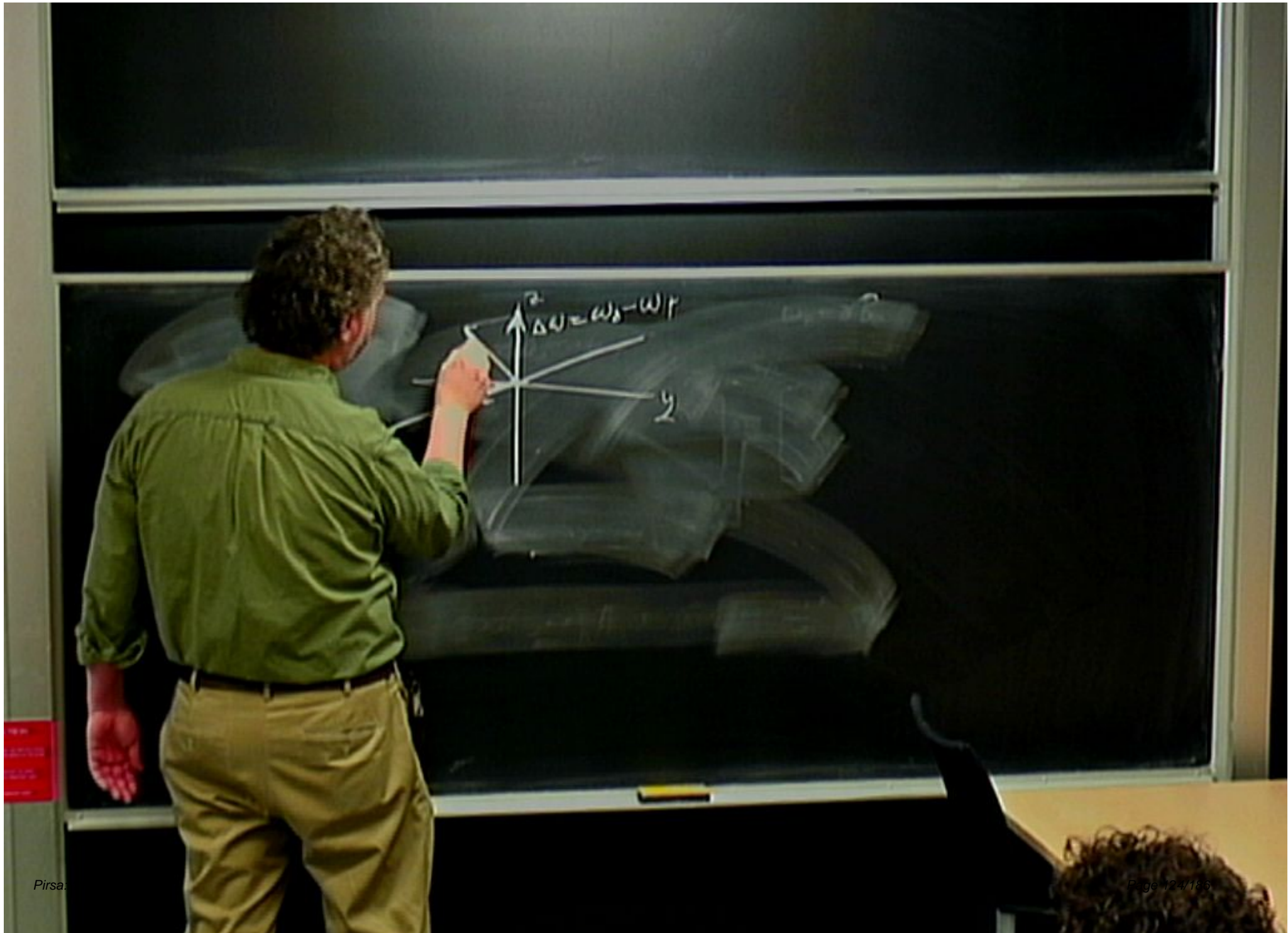
~~$$e^{i(\omega_p t + \phi)}$$~~

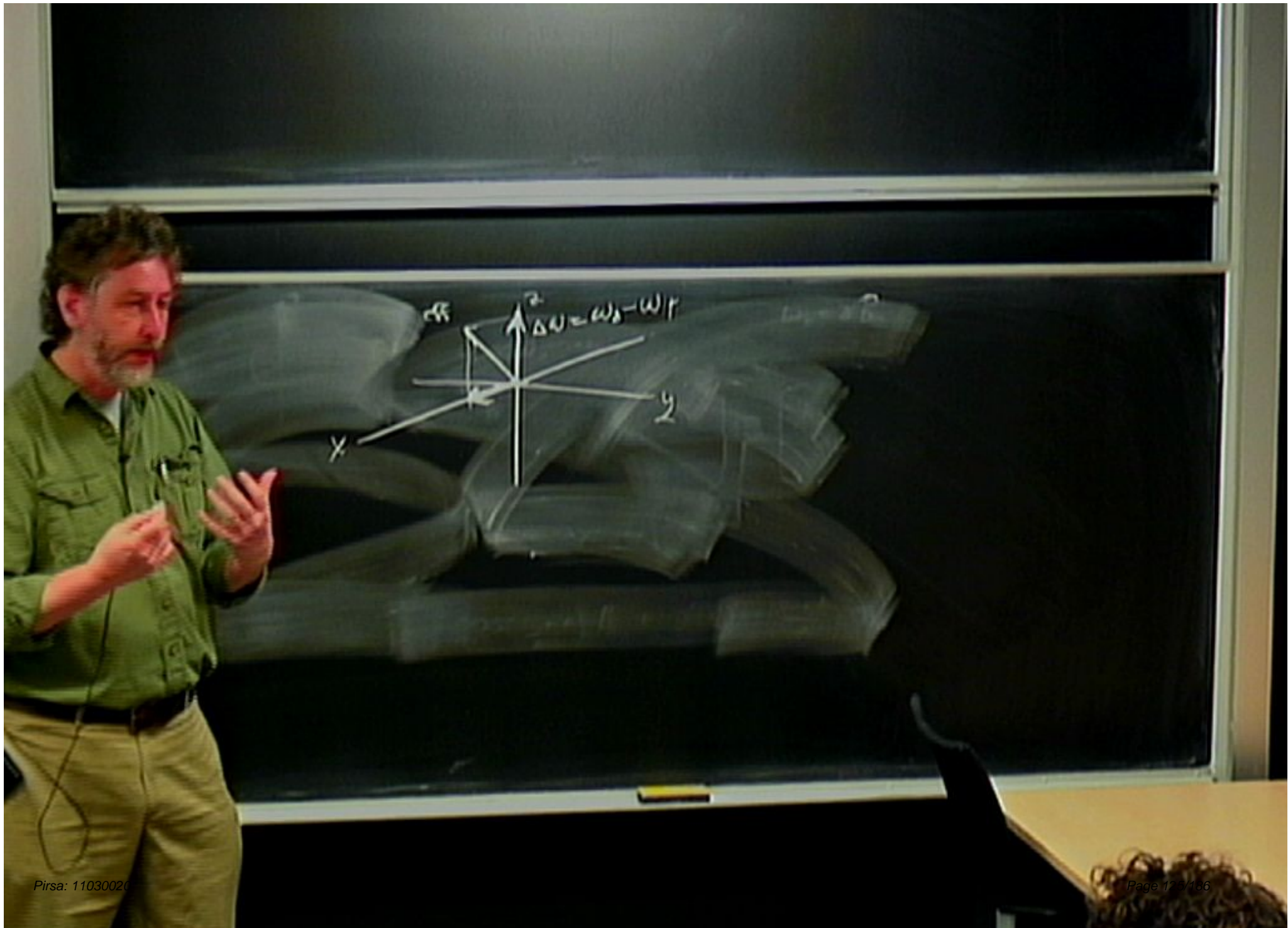
$$\cos\left(\frac{\omega_p t}{2}\right) \mathbb{1}_+ - i \sin\left(\frac{\omega_p t}{2}\right) \mathbb{1}_-$$

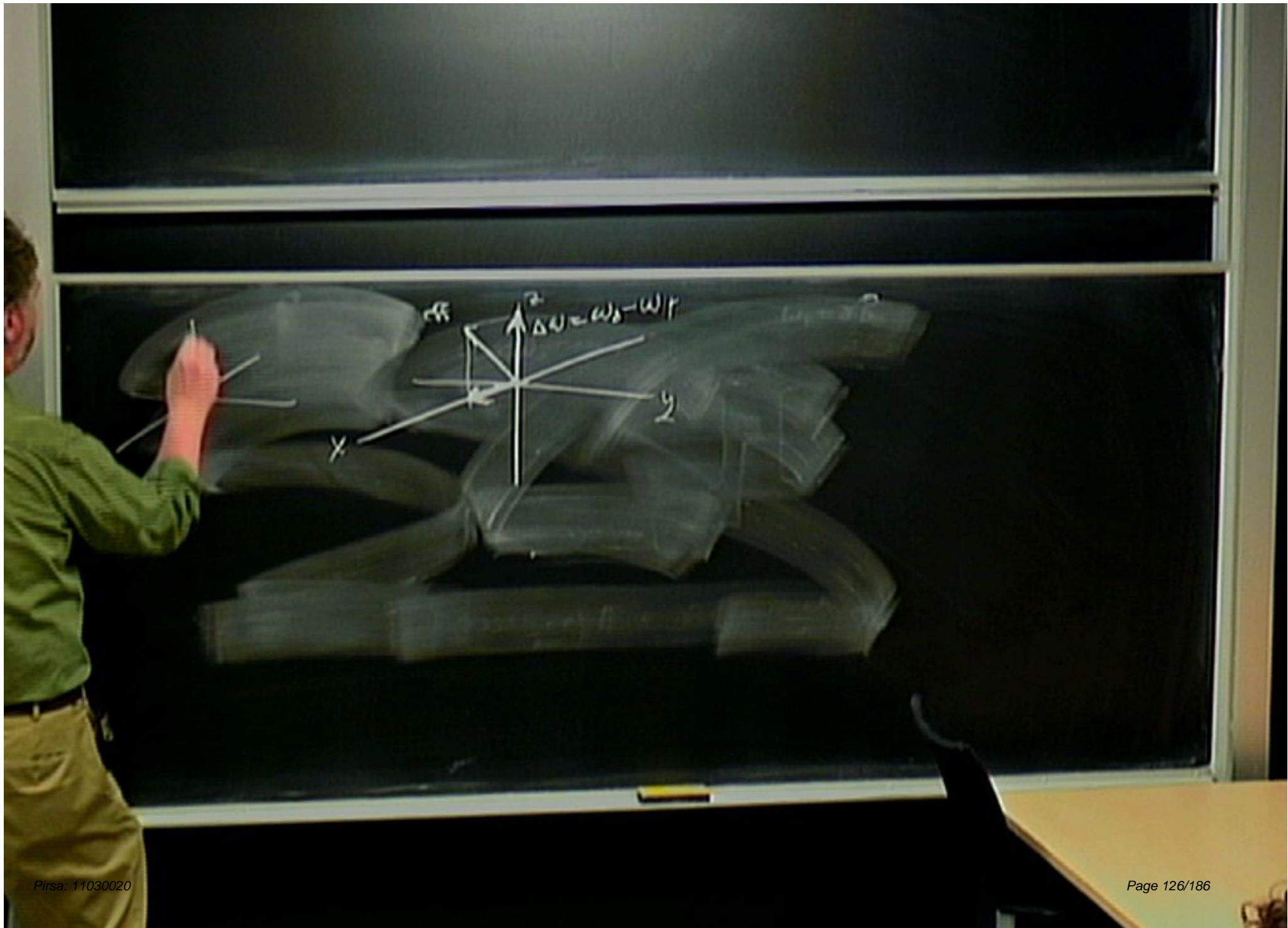


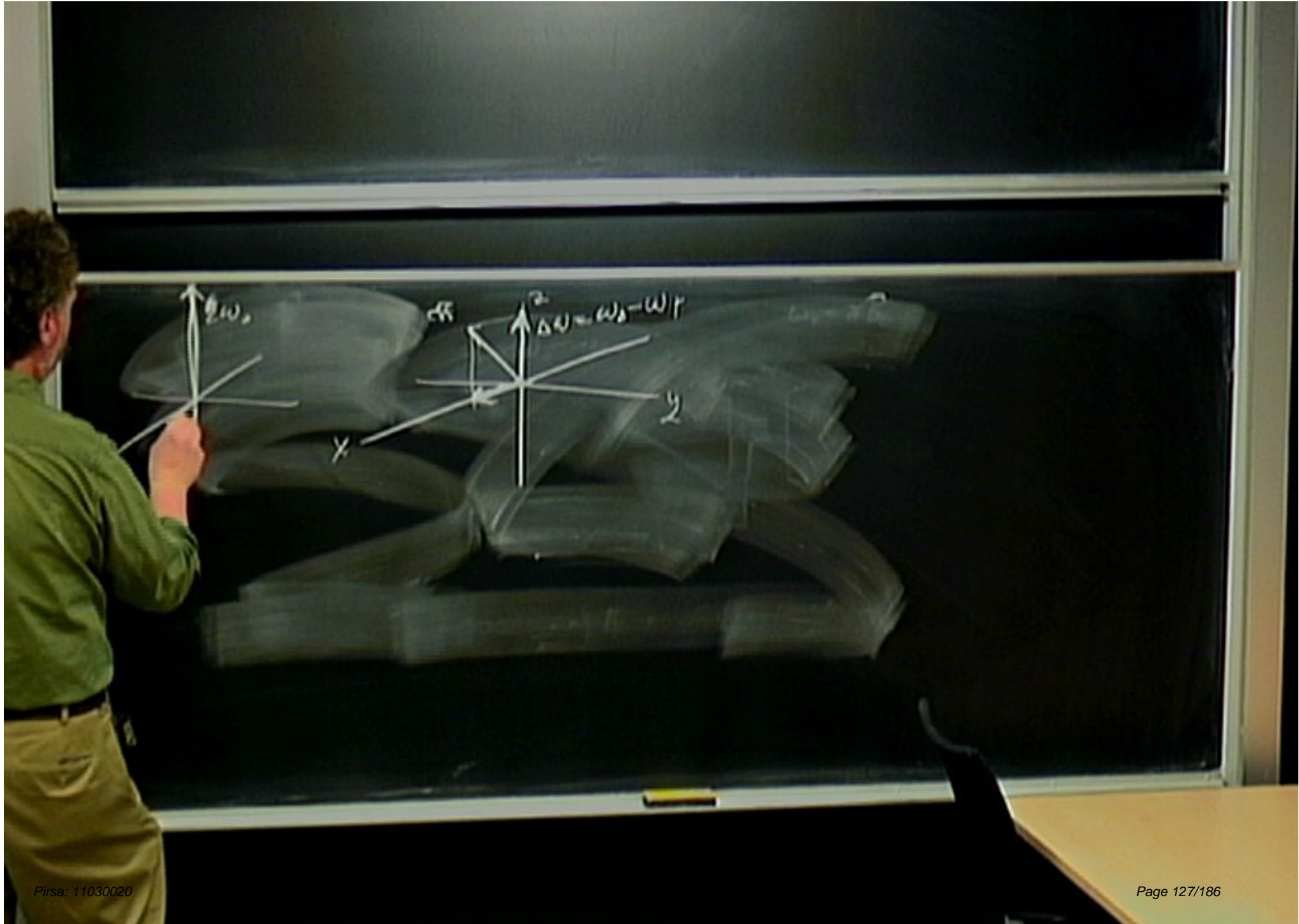


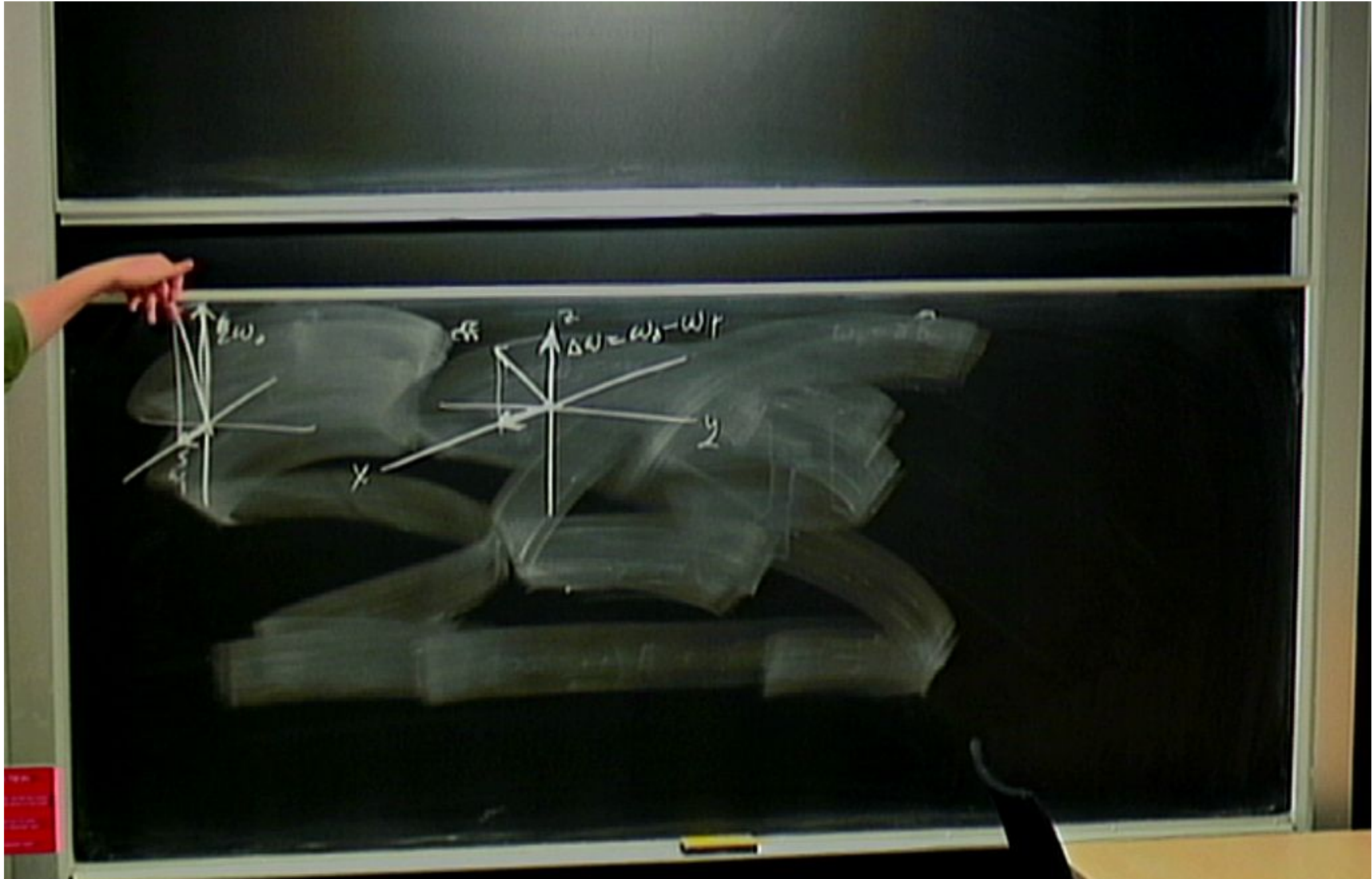


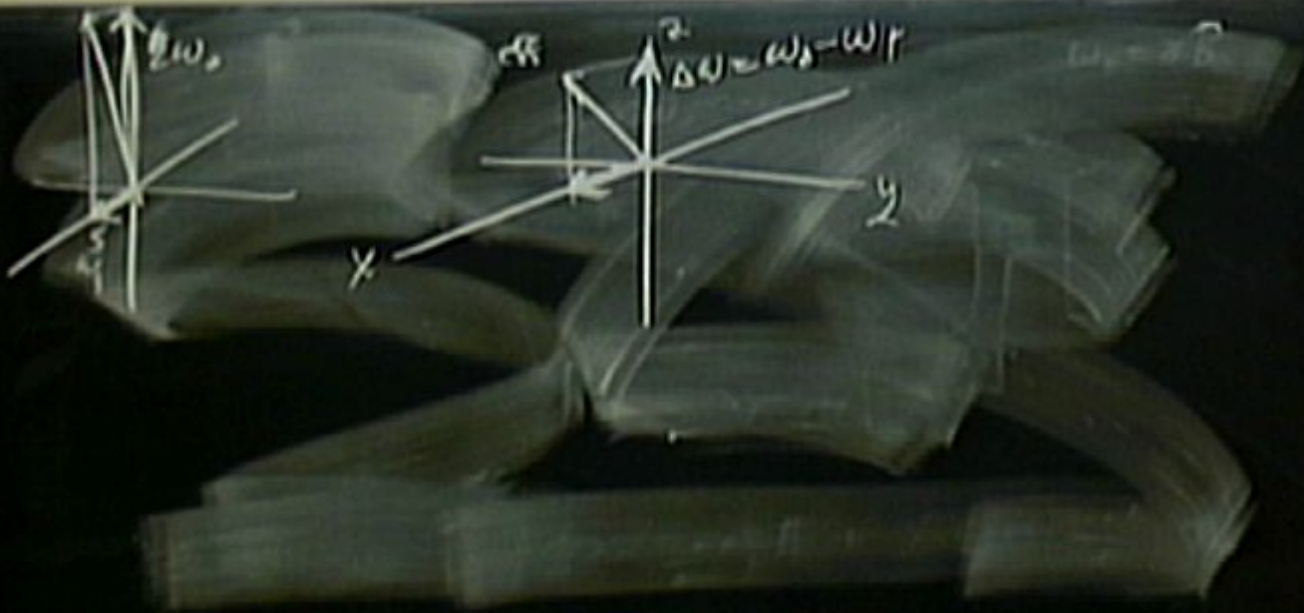












$$\tilde{H}_t = \mathcal{H}_t^{(0)} + \mathcal{H}_t^{(1)} + \dots$$

$$\mathcal{H}_t^{(0)} = \frac{1}{T} \int_0^T \tilde{H}(t) dt$$

$$\mathcal{H}_t^{(1)} = \frac{1}{T} \int_0^T dt_i \int_0^{t_i} [\tilde{H}(t_2), \tilde{H}(t_1)] dt_1$$

$$\tilde{H}(t) = \mathcal{H}_t^{(0)} + \mathcal{H}_t^{(1)} + \dots$$

$$\mathcal{H}_t^{(0)} = \mathcal{H}\left(\frac{t+\tau_c}{T}\right)$$

$$\mathcal{H}_t^{(1)} = \frac{1}{T} \int_0^T \tilde{H}(t) dt$$

$$\mathcal{H}_t^{(1)} = \frac{1}{T} \int_0^T dt_1 \int_0^{t_1} [\tilde{H}(t_2), \tilde{H}(t_1)] dt_2$$

$$\frac{d\vec{S}}{dt} = +\underbrace{U_1(t)}_{\vec{S}} \underbrace{[H_1, \vec{S}]}_{\vec{S}} U_1^\dagger + \underbrace{U_2(t)}_{\vec{S}} [H_2, \vec{S}] U_2^\dagger - [\vec{H}, \vec{S}]$$

$$\frac{d\vec{S}}{dt} = -\gamma \vec{S} \times \vec{H}$$

$$U(t) = e^{-i \int_0^t H(t') dt'}$$

average Hamiltonian

$$= e^{-i \bar{H} t}$$

Rotating Wave Approximation

$$\mathcal{H} : \mathcal{H}(t) = \omega_1 \cos(\omega_1 t) \sigma_x ; \frac{d\rho}{dt} = -i[\mathcal{H}, \rho]$$

$$\mathcal{H}_I(t) = \frac{\omega_1}{2} \left\{ \underbrace{e^{i\frac{\omega_1 t}{2} \sigma_z} \sigma_x e^{-i\frac{\omega_1 t}{2} \sigma_z}}_{\text{rotates}} \cdot \underbrace{e^{-i\omega_1 t \sigma_z}}_{\text{counter rotates}} \right\} \rho = \mathcal{U}(t) \rho \mathcal{U}^\dagger(t)$$

$$\tilde{\rho} = \mathcal{U}_I(t) \rho \mathcal{U}_I^\dagger(t) \quad \tilde{\mathcal{H}} = \mathcal{U}_I \mathcal{H} \mathcal{U}_I^\dagger$$

$$\frac{d\tilde{\rho}}{dt} + \mathcal{U}_I(t) \frac{d\rho}{dt} \mathcal{U}_I^\dagger(t) + \frac{d\mathcal{U}_I(t)}{dt} \rho \mathcal{U}_I^\dagger(t)$$

$$\sigma^2 = \sigma_w^2 \tau_c$$

$$\bar{H}_t = H_t^{(0)} + H_t^{(1)} + \dots$$

$$H_t^{(0)} = H(t, \tau_c)$$

$$H_t^{(1)} = \frac{1}{T} \int_0^T \bar{H}(t) dt$$

$$H_t^{(2)} = \frac{1}{T} \int_0^T dt_i \int_0^{\tau_i} [\bar{H}(t_i), \bar{H}(t_i)] dt_i$$

$$\delta t = \delta \omega t_c$$

$$\bar{\Psi}_t = \Psi_t^{(0)} + \Psi_t^{(1)} + \dots$$

$$\Psi_t^{(0)} = \Psi(t, t_0)$$

$$\Psi_t^{(1)} = \frac{1}{T} \int_0^t \bar{\Psi}(t_1) dt_1$$

$$\Psi_t^{(2)} = \frac{1}{T^2} \int_0^t dt_2 \int_0^{t_2} [\bar{\Psi}(t_2), \bar{\Psi}(t_1)] dt_1$$

Rot Wave Approximation

$$\begin{aligned}
 \mathcal{H}_0 &= \frac{\omega_0}{2} \sigma_x & \mathcal{H}(t) &= \underbrace{\omega_1 \cos(\omega_1 t)}_{\text{rotating}} \sigma_x ; \frac{dP}{dt} \approx -i [\mathcal{H}, P] \\
 \mathcal{H}_1 &= \frac{\omega_1}{2} \sigma_y & & \underbrace{e^{-i \sigma_x t}}_{\text{counter rotating}} \Big| P = U(t) P U^\dagger(t) \\
 U_1(t) &= U_1(t) P U_1^\dagger(t) & & \underline{\mathcal{H} = U_1 \mathcal{H} U_1^\dagger} \\
 \frac{dP}{dt} &= U_1(t) \frac{dP U_1^\dagger(t)}{dt} + \frac{dU_1(t)}{dt} P U_1^\dagger(t) \\
 &= \underbrace{[\mathcal{H}, P]}_{-i \mathcal{H}_1 U_1}
 \end{aligned}$$

Rotating Wave Approximation

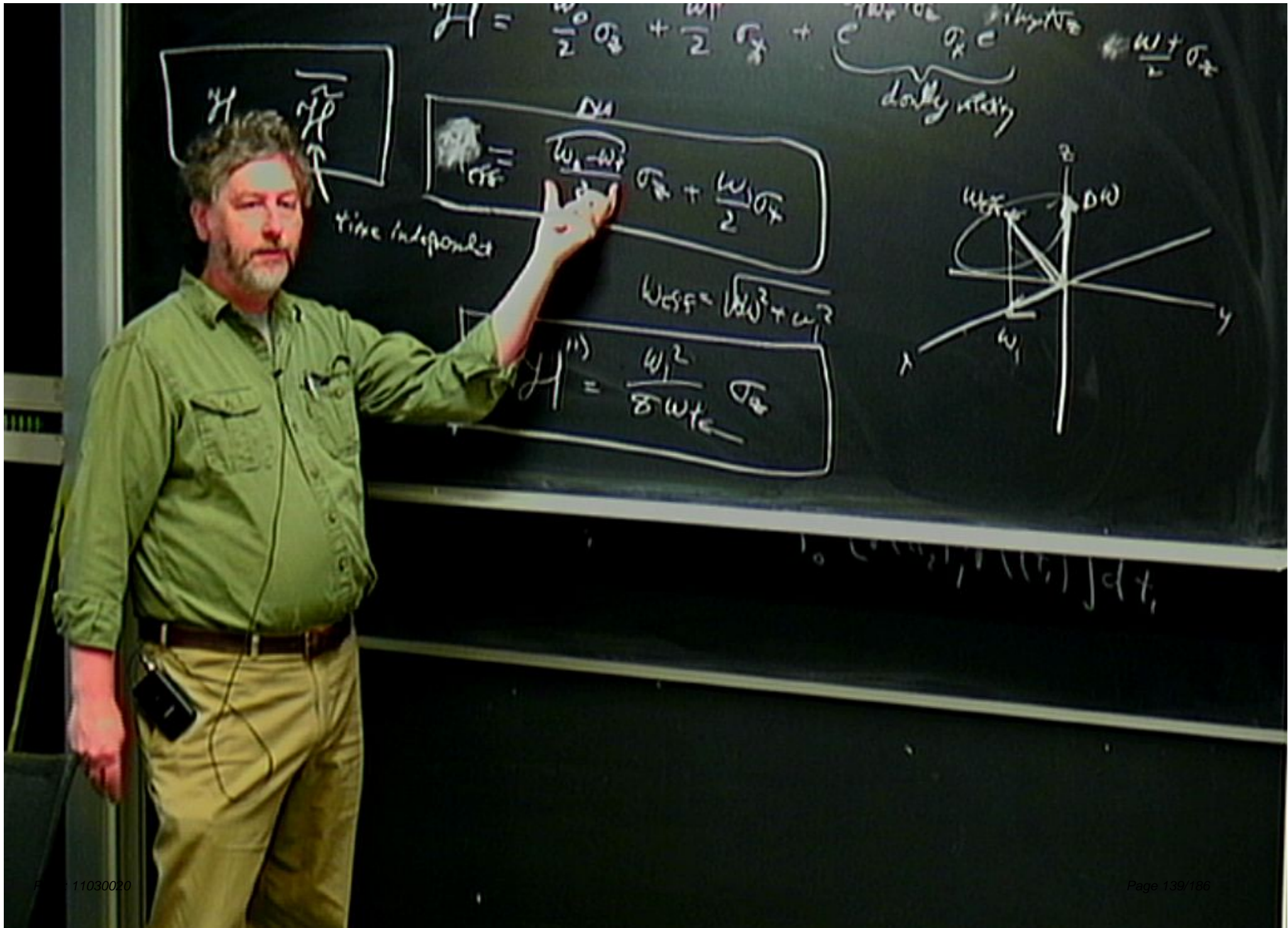
$$H_0 = \frac{\omega_0}{2} \sigma_z \quad ; \quad H(t) = \underbrace{\omega_1 \cos(\omega_1 t)}_{\text{rotates}} \sigma_x \quad ; \quad \frac{dP}{dt} = -i[H, P]$$

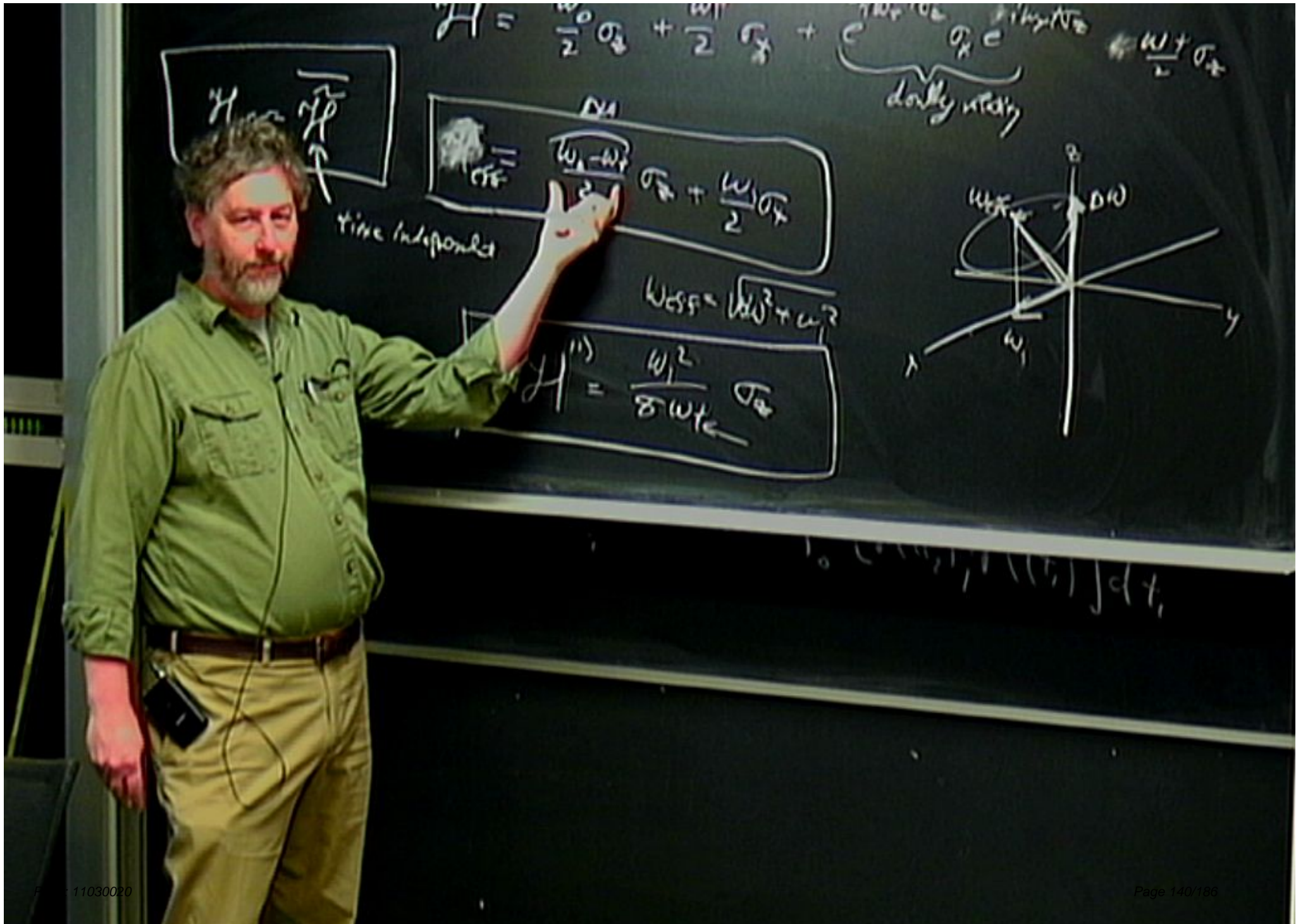
$$H_1(t) = \frac{\omega_1}{2} \left\{ \underbrace{e^{i\frac{\omega_1 t}{2} \sigma_z} \sigma_x e^{-i\frac{\omega_1 t}{2} \sigma_z}}_{\text{rotates}} + \underbrace{e^{-i\frac{\omega_1 t}{2} \sigma_z}}_{\text{counter rotates}} \right\} P = U(t) P U^\dagger(t)$$

$$P = e^{-i\frac{\omega_1 t}{2} \sigma_z} \quad ; \quad \tilde{P} = U_1(t) P U_1^\dagger(t) \quad \quad \quad \tilde{H} = U_1 H U_1^\dagger$$

$$\frac{d\tilde{P}}{dt} = U_1(t) P \frac{dU_1^\dagger}{dt} + U_1(t) \frac{dP}{dt} U_1^\dagger(t) + \frac{dU_1(t)}{dt} P U_1^\dagger(t)$$

$$\underbrace{\quad}_{i[H_1, \tilde{P}]} \quad \quad \quad \underbrace{\quad}_{[H, P]} \quad \quad \quad \underbrace{\quad}_{-i[H_1, \tilde{P}]}$$





$$H = \frac{\omega_0^2}{2} \sigma_z + \frac{\omega_1}{2} \sigma_x + \underbrace{\dots}_{\text{doubly degenerate}} \sigma_x \approx \frac{\omega + \sigma_x}{2}$$

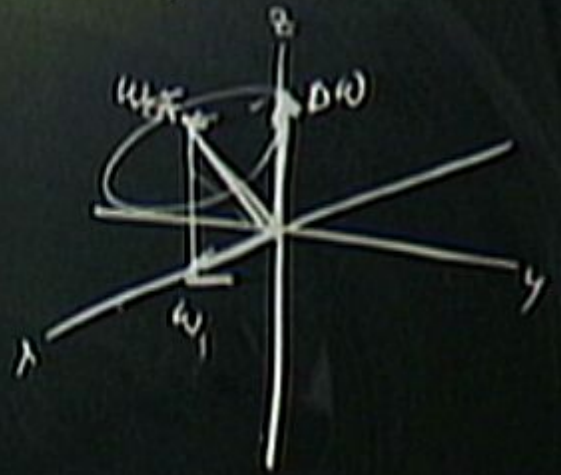
$$H_{\text{eff}} = \frac{H}{\hbar}$$

$$H_{\text{eff}} = \frac{\omega_0 - \omega_1}{2} \sigma_z + \frac{\omega_1 \sigma_x}{2}$$

time independent

$$\omega_{\text{eff}} = \sqrt{\omega_0^2 + \omega_1^2}$$

$$H^{(1)} = \frac{\omega_1^2}{8\omega_0} \sigma_z$$



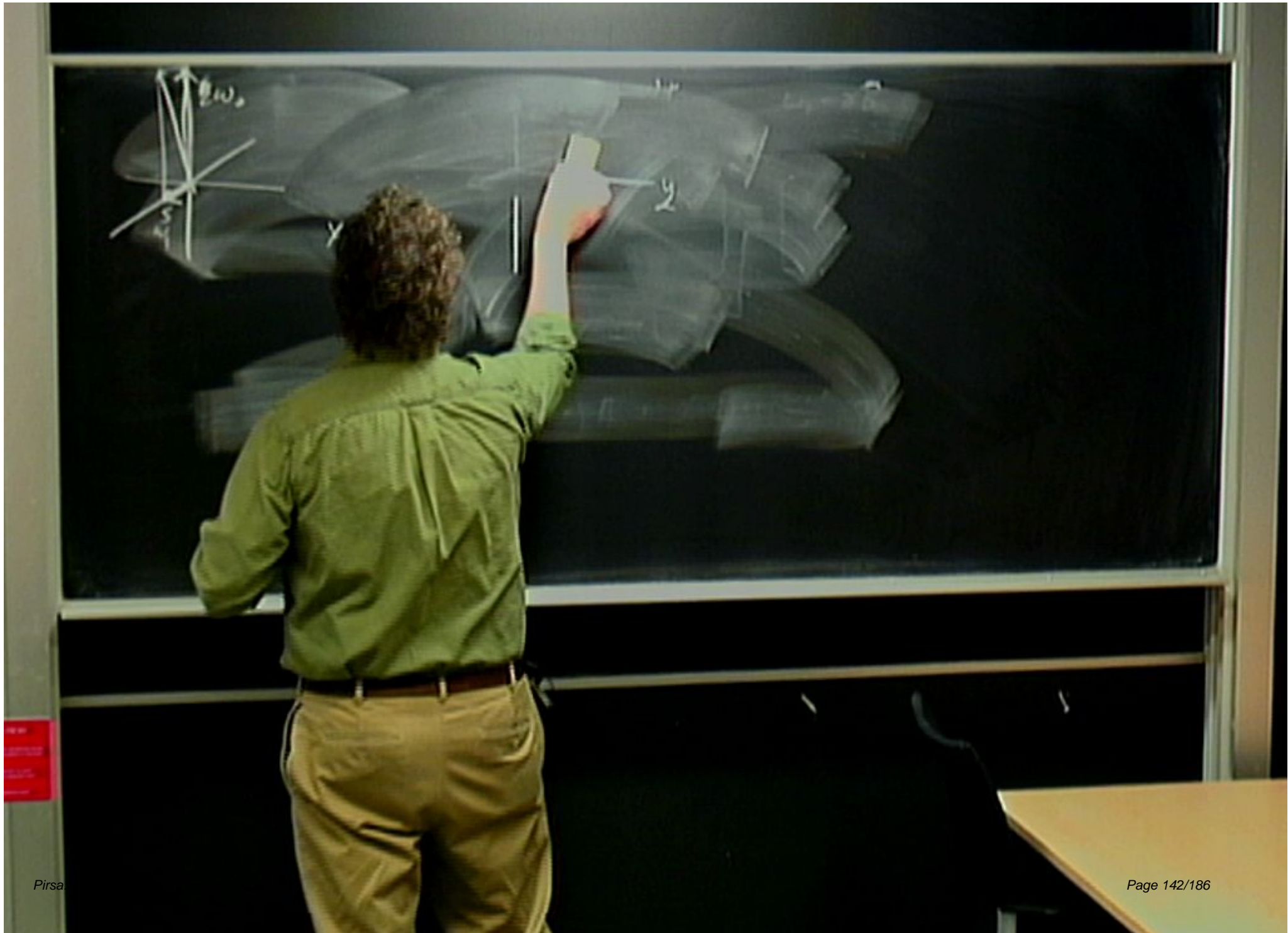
$$\sigma^2 = \delta \omega t_c$$

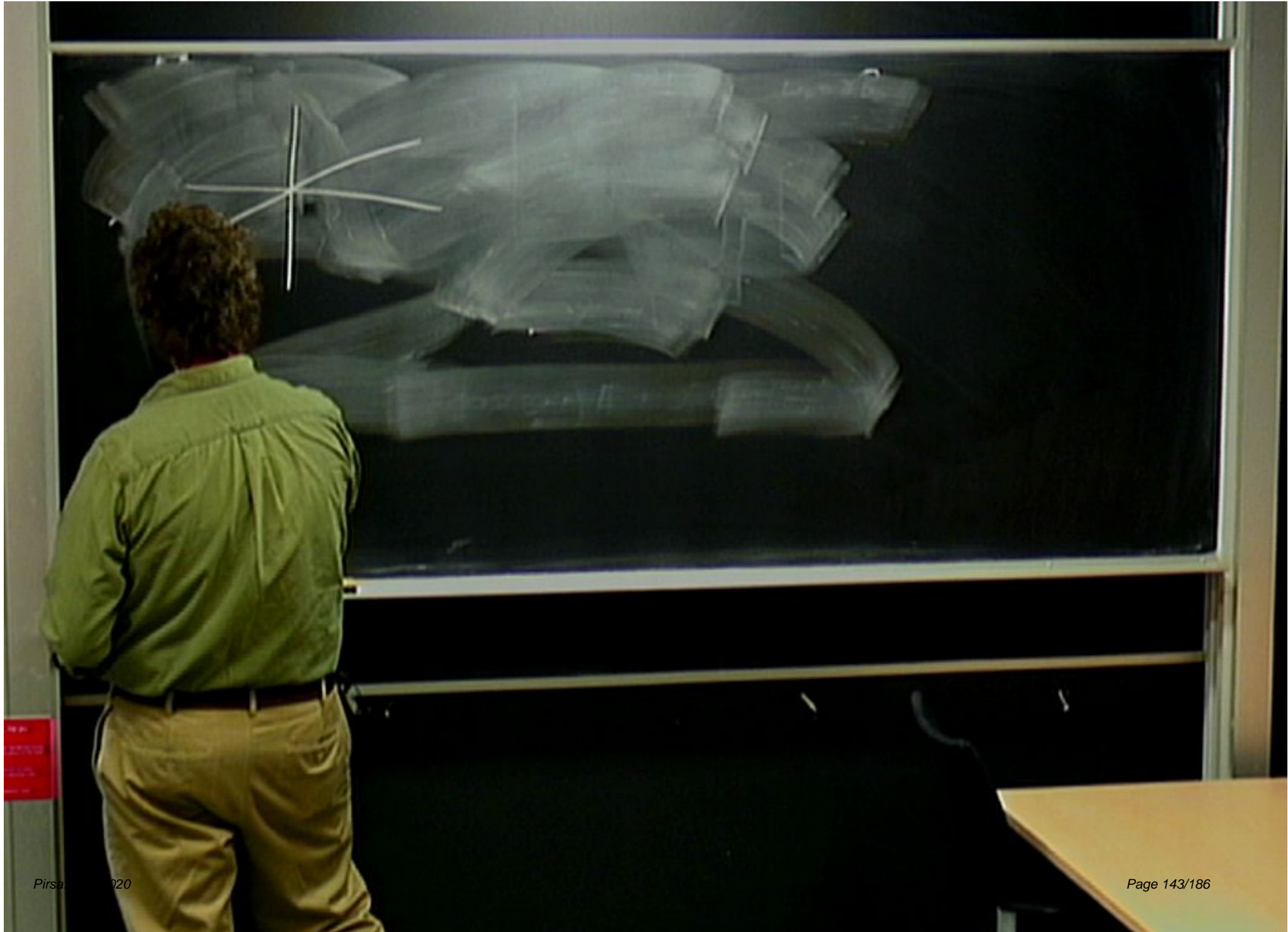
$$\bar{\Psi}_t = \Psi_t^{(0)} + \Psi_t^{(1)} + \dots$$

$$\Psi_t^{(0)} = \Psi\left(\frac{t-t_c}{\tau}\right)$$

$$\Psi_t^{(1)} = \frac{1}{\tau} \int_0^t \bar{\Psi}(t_1) dt_1$$

$$\Psi_t^{(2)} = \frac{1}{\tau} \int_0^t dt_1 \int_0^{t_1} [\bar{\Psi}(t_2), \bar{\Psi}(t_1)] dt_2$$





lub frame

\mathbb{R}^4

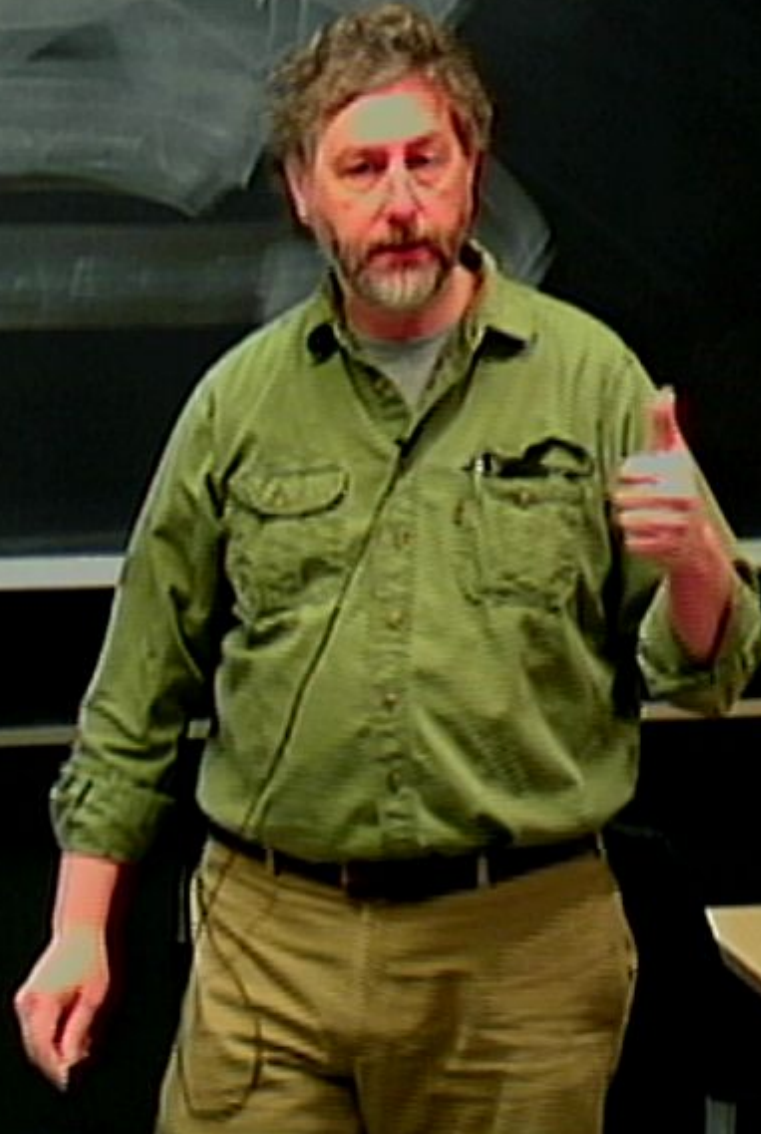
$$\Delta U = 0$$

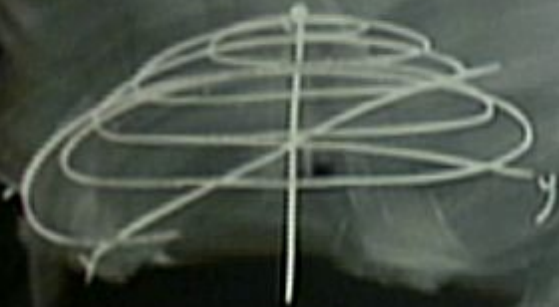


lub frame

$$\mathcal{H}_4 \quad \Delta U = 0$$

$$\beta = (1 + \sqrt{5})/2$$





lab frame

\mathcal{H}_+

$$\Delta U = 0$$

$$S = (1 + \frac{v^2}{c^2})/2$$

$$\tilde{\mathcal{H}} = \mathcal{H}_t^{(0)} + \mathcal{H}_t^{(1)} + \dots$$

$$\mathcal{H}_t^{(0)} = \mathcal{H}\left(\frac{t+\tau_c}{\tau_c}\right)$$

$$\mathcal{H}_t^{(0)} = \frac{1}{T} \int_0^T \tilde{\mathcal{H}}(t) dt$$

$$\mathcal{H}_t^{(1)} = \frac{\gamma}{T} \int_0^T dt_2 \int_0^{t_2} [\tilde{\mathcal{H}}(t_2), \tilde{\mathcal{H}}(t_1)] dt_1$$

Rotating Wave Approximation

$$\mathcal{H}_0 = \frac{\omega_0}{2} \sigma_z \quad ; \quad \mathcal{H}(t) = \omega_1 \cos(\omega_+ t) \sigma_x$$

$$\mathcal{H}_1 = \frac{\omega_+}{2} \sigma_x$$

$$\mathcal{H}_1(t) = \frac{\omega_+}{2} \left\{ e^{i\frac{\omega_+}{2}t} \sigma_x + e^{-i\frac{\omega_+}{2}t} \sigma_x \right\}$$

rotates

$$U_+(t) = e^{-i\frac{\omega_+}{2}t} \sigma_x$$

$$: \tilde{\rho} = U_+(t) \rho U_+^\dagger(t)$$

$$\frac{d\tilde{\rho}}{dt} = U_+(t) \rho \frac{dU_+^\dagger}{dt} + U_+(t) \frac{d\rho}{dt} U_+^\dagger(t) + \frac{dU_+(t)}{dt} \rho U_+^\dagger(t)$$

$(+i\frac{\omega_+}{2}\sigma_x \tilde{\rho})$ $(\tilde{\rho}, \mathcal{H}_1)$ $(-i\frac{\omega_+}{2}\sigma_x \tilde{\rho})$

$[\mathcal{H}, \rho]$

$\rho U_+^\dagger(t)$

$U_+^\dagger(t)$



lub frame

\mathcal{H}_+

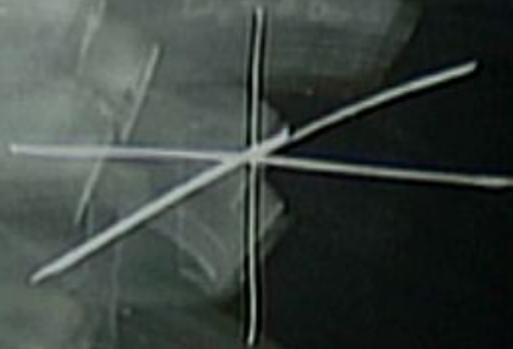
$$\Delta W = 0$$

$$S = (1 + \mathbb{E})/2$$

lub frame

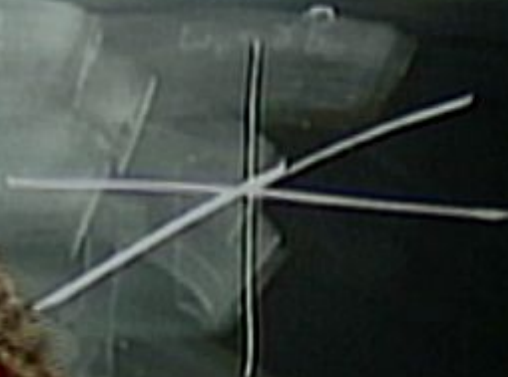
\mathbb{H}_4

$$S = (1+)$$





$R_2(\omega_0 t)$
 \rightarrow



lab frame

\mathcal{H}_4 $\Delta U = 0$

$$J = (I + \mathcal{E})/2$$



$R_2(\omega t)$



lab frame

$$\mathcal{H}_+$$

$$\Delta U = 0$$

$$S = (1 + \mathcal{E})/2$$



$R_z(\omega t)$

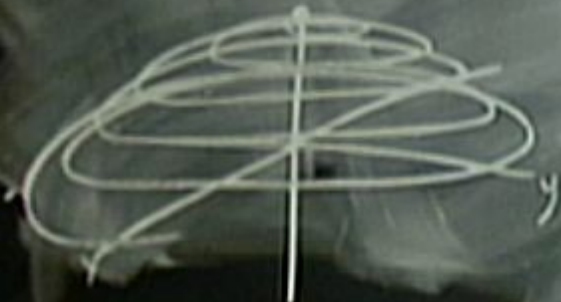


lab frame

$$\mathcal{H}_I \quad \omega = 0$$

$$S = (I + \sigma_z)/2$$



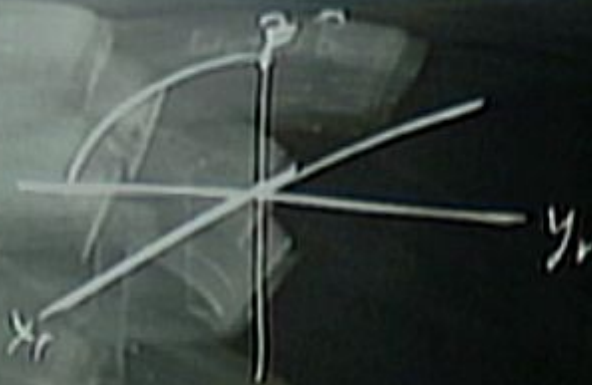


local frame

$$\mathcal{H}_+ \quad \Delta u = 0$$

$$S = (1 + \mathcal{E})/2$$

$R_z(\omega t)$

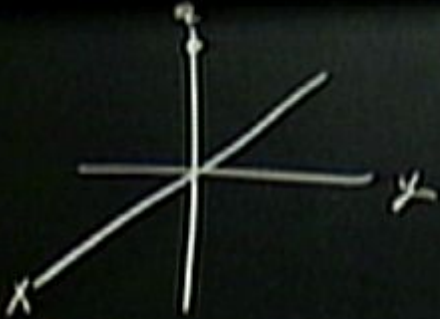


$$\mathcal{H}_4 \quad \Delta U = 0$$
$$P = (1 + \Phi)/2$$

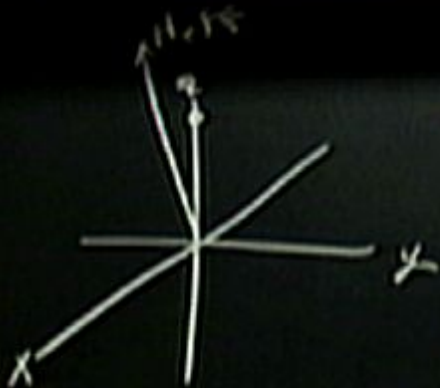




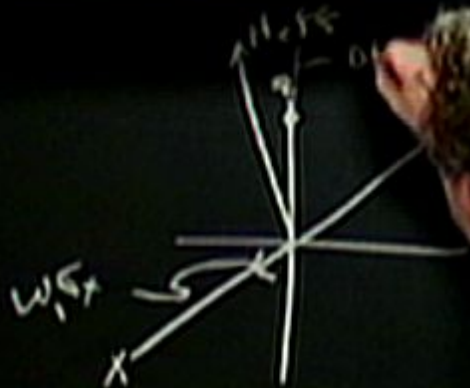
$$\rho = (1 + \Phi) / 2$$



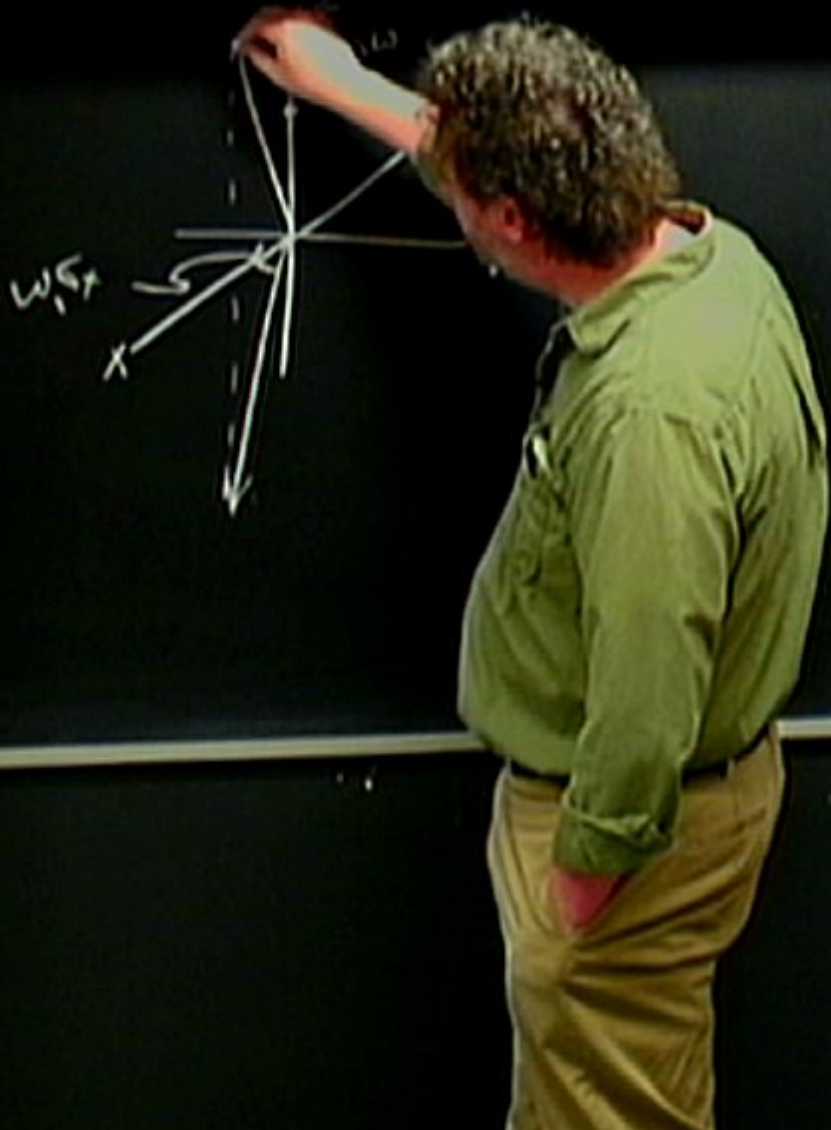
$$\rho = (1 + \Phi) / 2$$



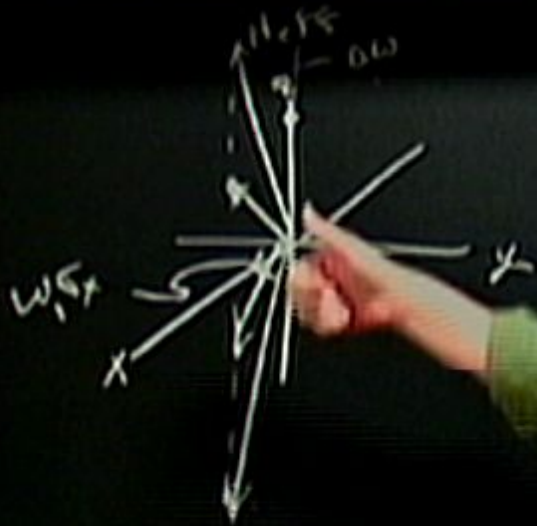
$$\rho = (1 + \Phi) / 2$$



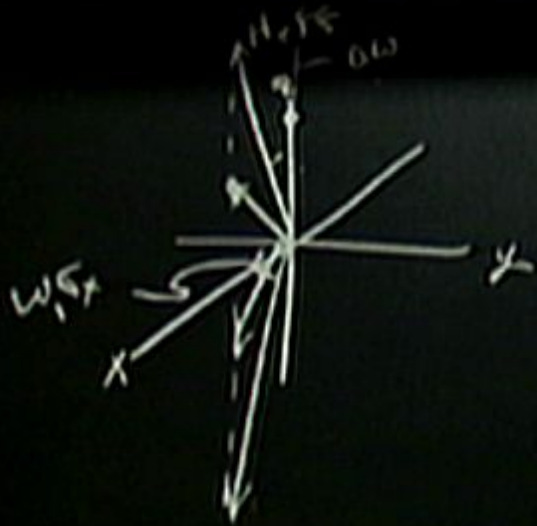
$$\rho = (1 + \Phi) / 2$$



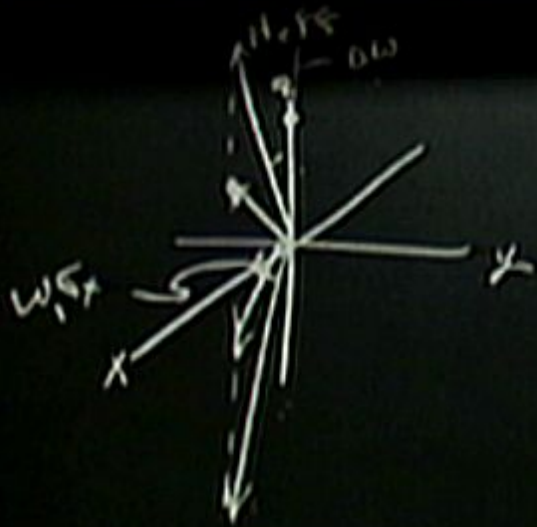
$$\rho = (1 + \Phi)/2$$



$$\rho = (1 + \Phi)/2$$

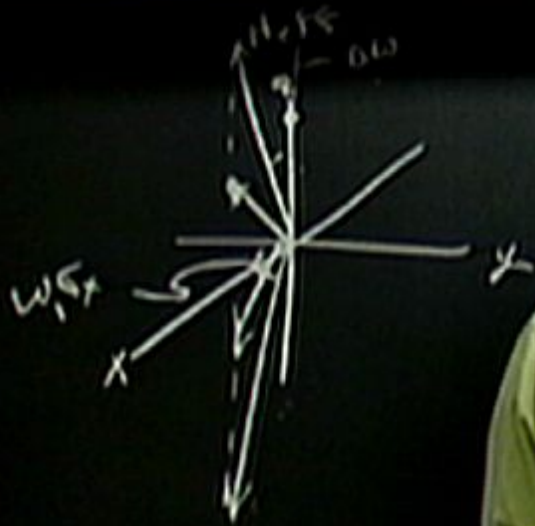


$$\rho = (1 + \mathcal{E})/2$$



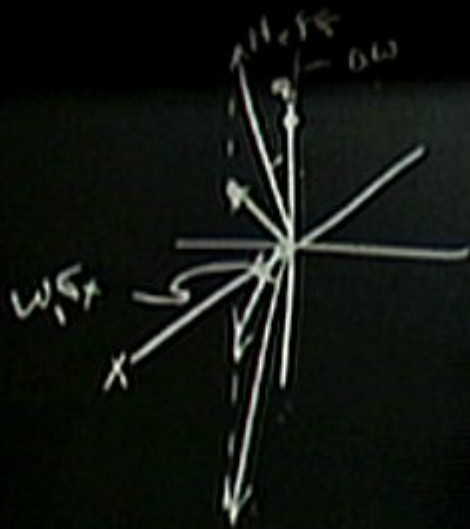
Mess

$$\rho = (1 + \langle \sigma \rangle) / 2$$



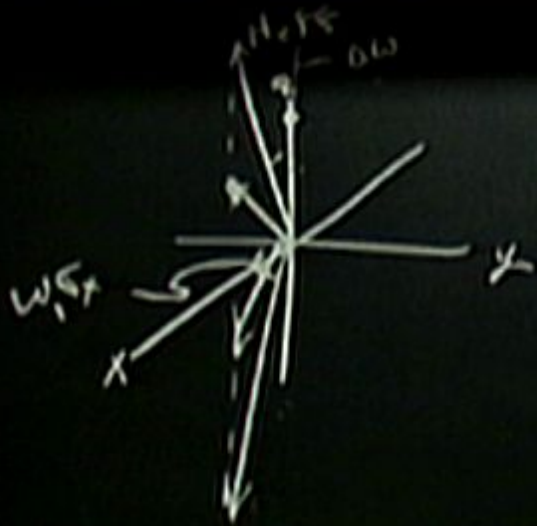
mess

$$\rho = (1 + \langle \sigma_x \rangle) / 2$$



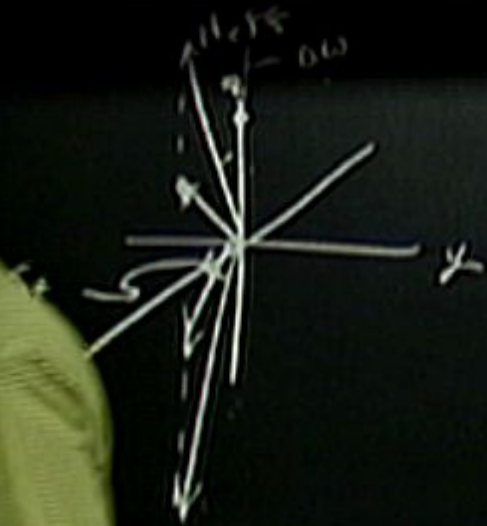
$\frac{1}{2}(1 + \sigma_x)$ $\frac{1}{2}(1 + \sigma_y)$

$$\rho = (1 + \mathcal{E})/2$$



$e^{-i\gamma_5 \mathcal{M} \mathcal{E}}$
 \rightarrow
 $\rho + \frac{3}{2} \mathcal{M} \mathcal{E}$

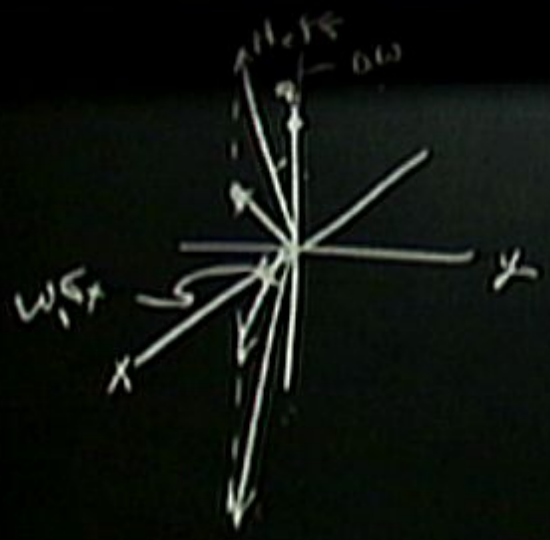
$$\rho = (1 + \mathcal{E})/2$$



$e^{+i\phi} \text{ Messung } e^{-i\phi}$
 \downarrow
 ρ $\frac{\mathcal{E}}{2}$

$$\Delta U = 0$$

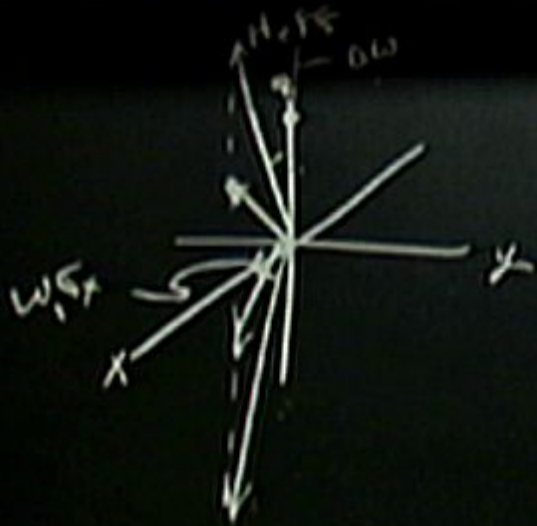
$$\rho = (1 + \Phi)/2$$



$e^{-i\psi} \text{Messung}$



$$\rho = (1 + \sigma_z)/2$$



$e^{-i\phi} \text{Messung } e^{-i\phi}$

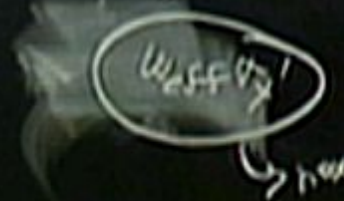
W_{eff}

$\rightarrow h_{eff}$

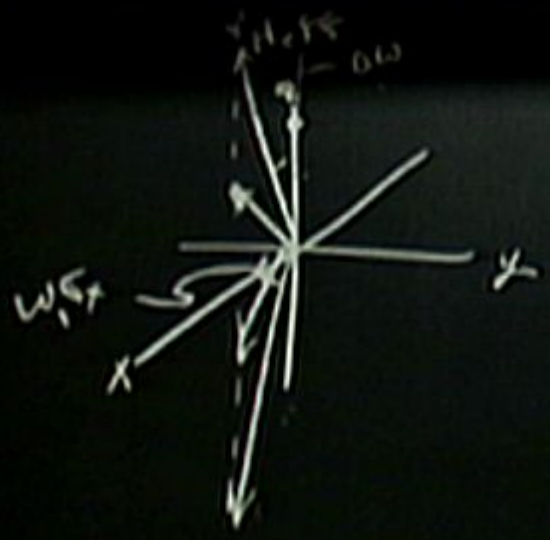
$$\rho = (1 + \Phi)/2$$



$e^{-i\gamma_0 t}$ Messung $e^{-i\gamma_0 t}$



$$\rho = (1 + \sigma_z)/2$$



$e^{-i\gamma_0 \sigma_z} \text{Messung}$

↓

$W_{eff} = \hbar \omega$

→ $\hbar \omega$

$$\rho = (1 + \Phi)/2$$

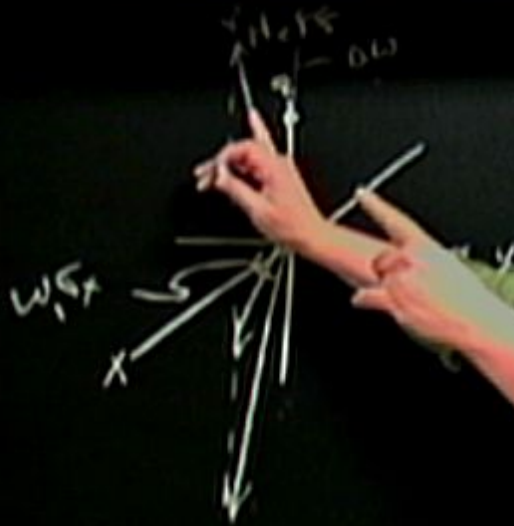


$$e^{-i \leftarrow \Phi}$$

$$w_{eff} = \sqrt{w_i^2 + \Delta w^2}$$

next

$$\rho = (1 + \Phi)/2$$

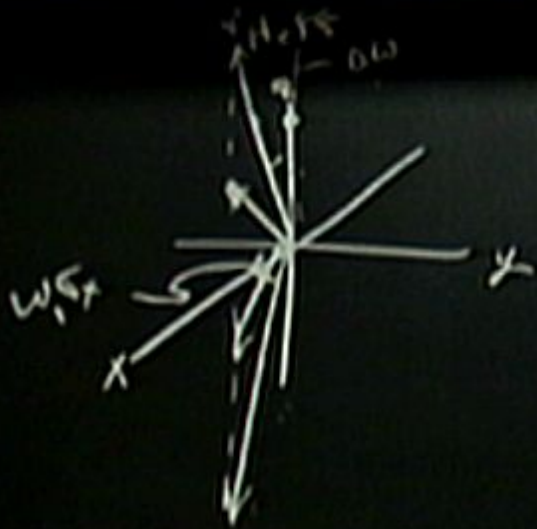


408 Messing $e^{-i \cdot \Delta \theta}$

$w_{eff} \Delta \theta$
→ next

$$w_{eff} = \sqrt{w_i^2 + \Delta w^2}$$

$$\rho = (1 + \mathcal{E})/2$$



$e^{+i\omega_0 t} \text{Messung } e^{-i\omega_0 t}$

$w_{eff} \tau$
 \hookrightarrow next

$$w_{eff} = \sqrt{\omega_0^2 + \Delta \omega^2}$$

Mathematica Version Advisory

This notebook was created in an earlier version of *Mathematica*.

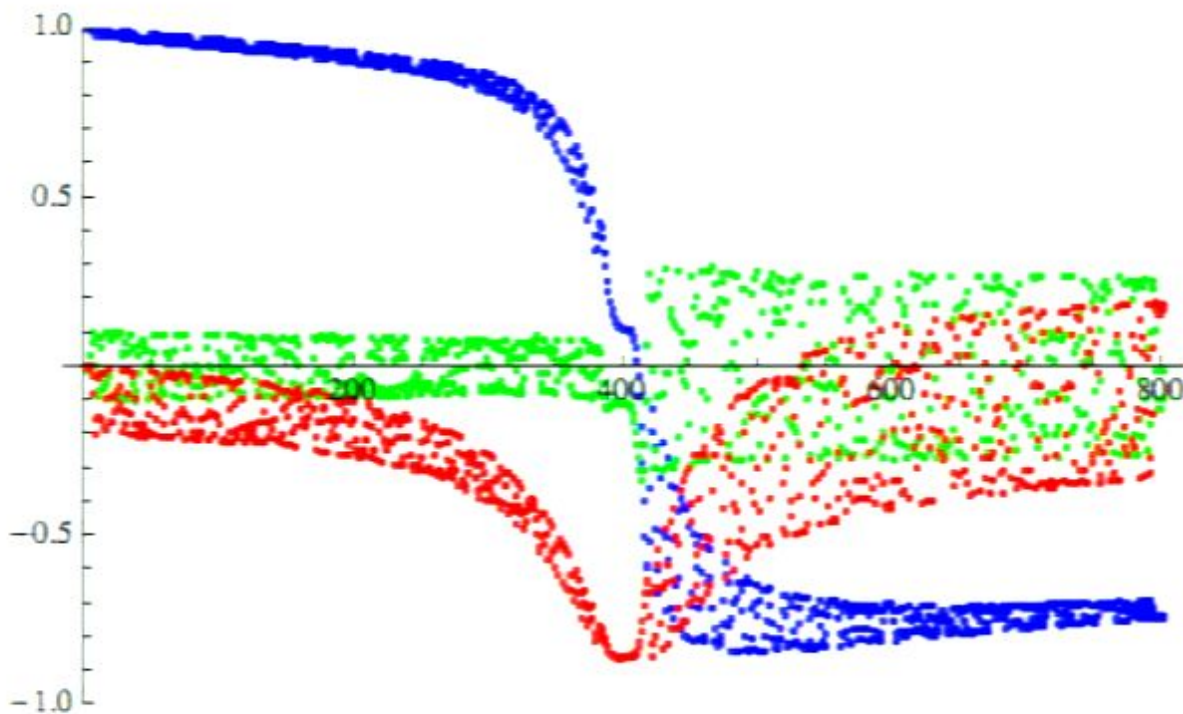
Most notebooks run without change. This tool scans for possible issues and suggests changes.

Scan for possible issues

Do not scan this notebook

Never scan notebooks

```
ListPlot[Table[Ad3[n][[1]][[2]], {n, 0, 800}],
  PlotStyle -> {RGBColor[0, 1, 0], Thickness[0.01]}],
ListPlot[Table[Ad3[n][[1]][[1]], {n, 0, 800}],
  {PlotRange -> {-1, 1}, PlotStyle -> {RGBColor[1, 0, 0], Thickness[0.01]}}]]
```



Mathematica Version Advisory

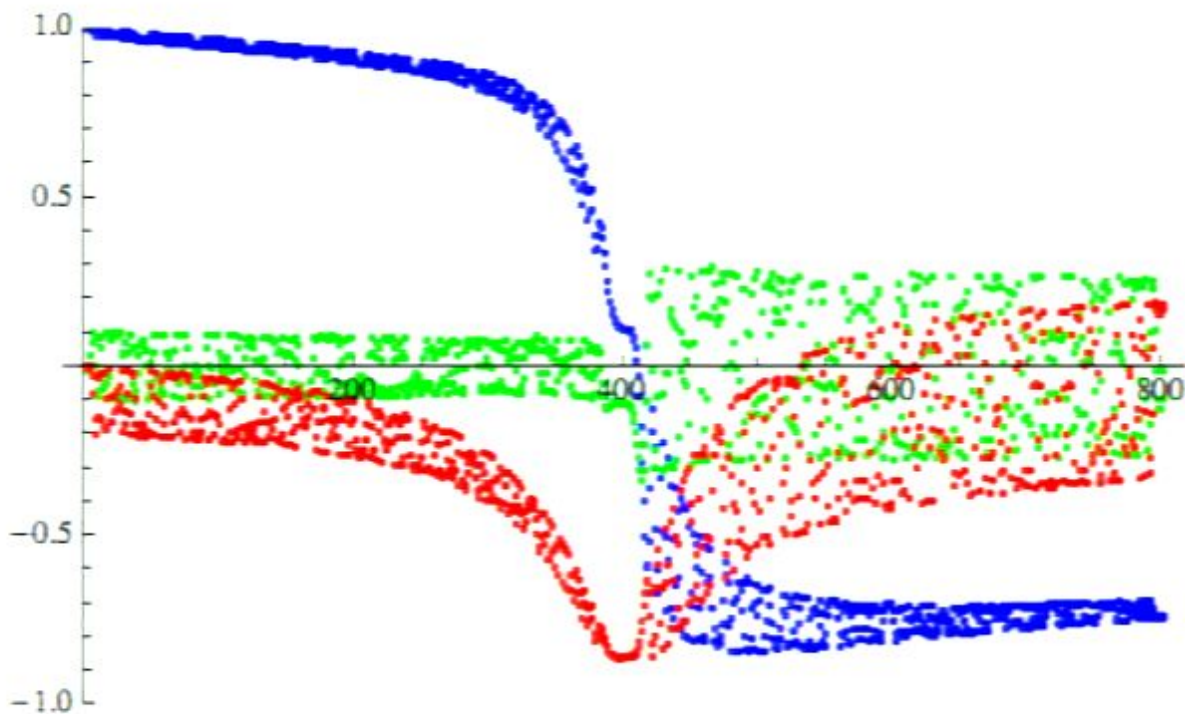
This notebook was created in an earlier version of *Mathematica*.
 Most notebooks run without change. This tool scans for possible issues and suggests changes.

Scan for possible issues

Do not scan this notebook

Never scan notebooks

```
ListPlot[Table[Ad3[n][[1]][[2]], {n, 0, 800}],
  PlotStyle -> {RGBColor[0, 1, 0], Thickness[0.01]}],
ListPlot[Table[Ad3[n][[1]][[1]], {n, 0, 800}],
  {PlotRange -> {-1, 1}, PlotStyle -> {RGBColor[1, 0, 0], Thickness[0.01]}}]]
```



$$\mathcal{H}(t) = \mathcal{H}(t + \tau_c)$$

$$\mathcal{H}(t) = \mathcal{H}_t^{(0)} + \dots$$

$$\mathcal{H}_t^{(0)} = \dots$$

$$\mathcal{H}_t^{(1)}$$

$$\vdots$$

Magnus Expansion

$$\int \mathcal{H}(t, \dots) dt$$

$$\mathcal{H}(t) = \mathcal{H}(t + \tau_c)$$

$$\tilde{\mathcal{H}} = \mathcal{H}$$

... Magnus Expansion

~~BCH~~

$$\int [\tilde{\mathcal{H}}(t_2), \tilde{\mathcal{H}}(t_1)] dt_1$$

$$\tilde{H}(t) = \tilde{H}(t + \tau_c)$$

$$\tilde{H}(t) = \tilde{H}^{(0)} + \dots$$

Magnus Expansion

BCH
Wilcox

$$\tilde{H}(t) dt$$

$$\int_0^{t_2} [\tilde{H}(t_2), \tilde{H}(t_1)] dt_1$$

$$\mathcal{H}_t^{(+)} = \mathcal{H}_{t+\tau_c}^{(+)}$$

$$\mathcal{H}_t = \mathcal{H}_t^{(0)} + \mathcal{H}_t^{(1)}$$

$$\mathcal{H}_t^{(0)} = \frac{1}{2} \mathcal{H}_t$$

$$\mathcal{H}_t^{(1)}$$

Magnus Expansion

BCA, Trotter
Wilcox

$\int dt$

$$\tilde{U}(t) = \mathcal{U}_t^{(0)} + \mathcal{U}_t^{(1)} + \dots$$

Magnus Expansion

$$\mathcal{U}_t^{(0)} = \mathcal{U}_{t+\tau_c}^{(0)}$$

$$\mathcal{U}_t^{(0)} = \frac{1}{i} \int_0^t \tilde{H}(t) dt$$

$$\mathcal{U}_t^{(1)} = \frac{1}{i} \int_0^t dt_2 \int_0^{t_2}$$

$$[\tilde{H}(t_2), \tilde{H}(t_1)] dt_1$$

BCA, Trotter
Wilcox

$$\tilde{U}(t) = \mathcal{H}_t^{(0)} + \mathcal{H}_t^{(1)} + \dots$$

Magnus Expansion

\mathcal{H}_t

$$\mathcal{H}_t^{(0)} = \frac{1}{T} \int_0^T \tilde{\mathcal{H}}(t) dt$$

BCA, Trotter
Wilcox

$$\int_0^{t_2} [\tilde{\mathcal{H}}(t_2), \tilde{\mathcal{H}}(t_1)] dt_1$$

$$\tilde{U}(t) = \mathcal{U}_t^{(0)} + \mathcal{U}_t^{(1)} + \dots \quad \text{Magnus Expansion}$$

$$\mathcal{H}(t) = \mathcal{H}(t + \tau_c)$$

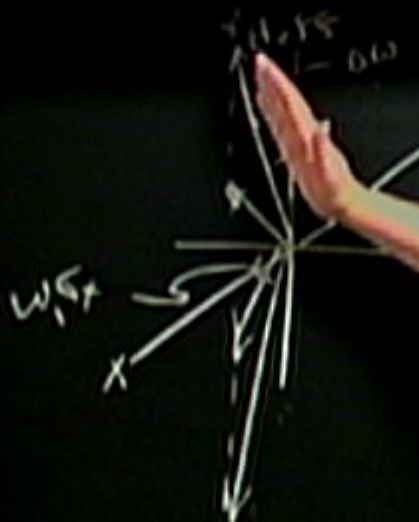
$$\mathcal{U}_t^{(0)} = \frac{1}{i} \int_0^t \tilde{\mathcal{H}}(t') dt'$$

$$\mathcal{U}_t^{(1)} = \frac{1}{i} \int_0^t dt' \int_0^{t'} [\tilde{\mathcal{H}}(t_2), \tilde{\mathcal{H}}(t_1)] dt_1$$

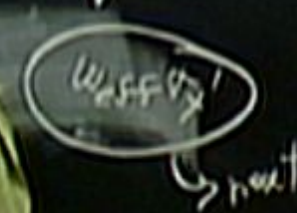
BCA, Trotter
Wilcox

$$\Delta W = 0$$

$$\rho = (1 + \sigma_z)/2$$

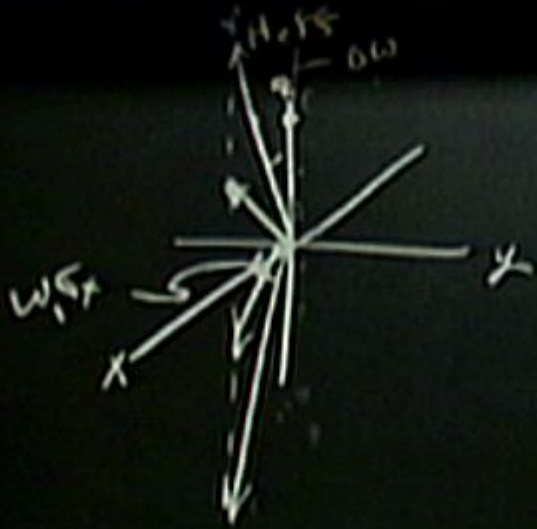


$$e^{i\omega_{eff} t} \text{ Mess } e^{-i\omega_{eff} t}$$

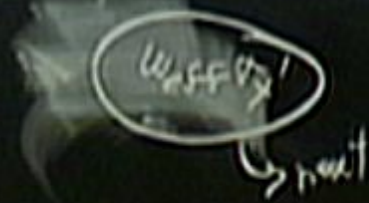


$$\omega_{eff} = \sqrt{\omega_1^2 + \Delta\omega^2}$$

$$\rho = (1 + \sigma_z)/2$$



$e^{-i\omega_{SM}t}$ Messung $e^{-i\omega_{eff}t}$



$$\omega_{eff} = \sqrt{\omega_1^2 + \Delta\omega^2}$$

Mathematica Version Advisory

This notebook was created in an earlier version of *Mathematica*.

Most notebooks run without change. This tool scans for possible issues and suggests changes.

Scan for possible issues

Do not scan this notebook

Never scan notebooks

```
ListPlot[Table[Ad3[n][[1]][[2]], {n, 0, 800}],
  PlotStyle -> {RGBColor[0, 1, 0], Thickness[0.01]}],
ListPlot[Table[Ad3[n][[1]][[1]], {n, 0, 800}],
  {PlotRange -> {-1, 1}, PlotStyle -> {RGBColor[1, 0, 0], Thickness[0.01]}}]]
```

