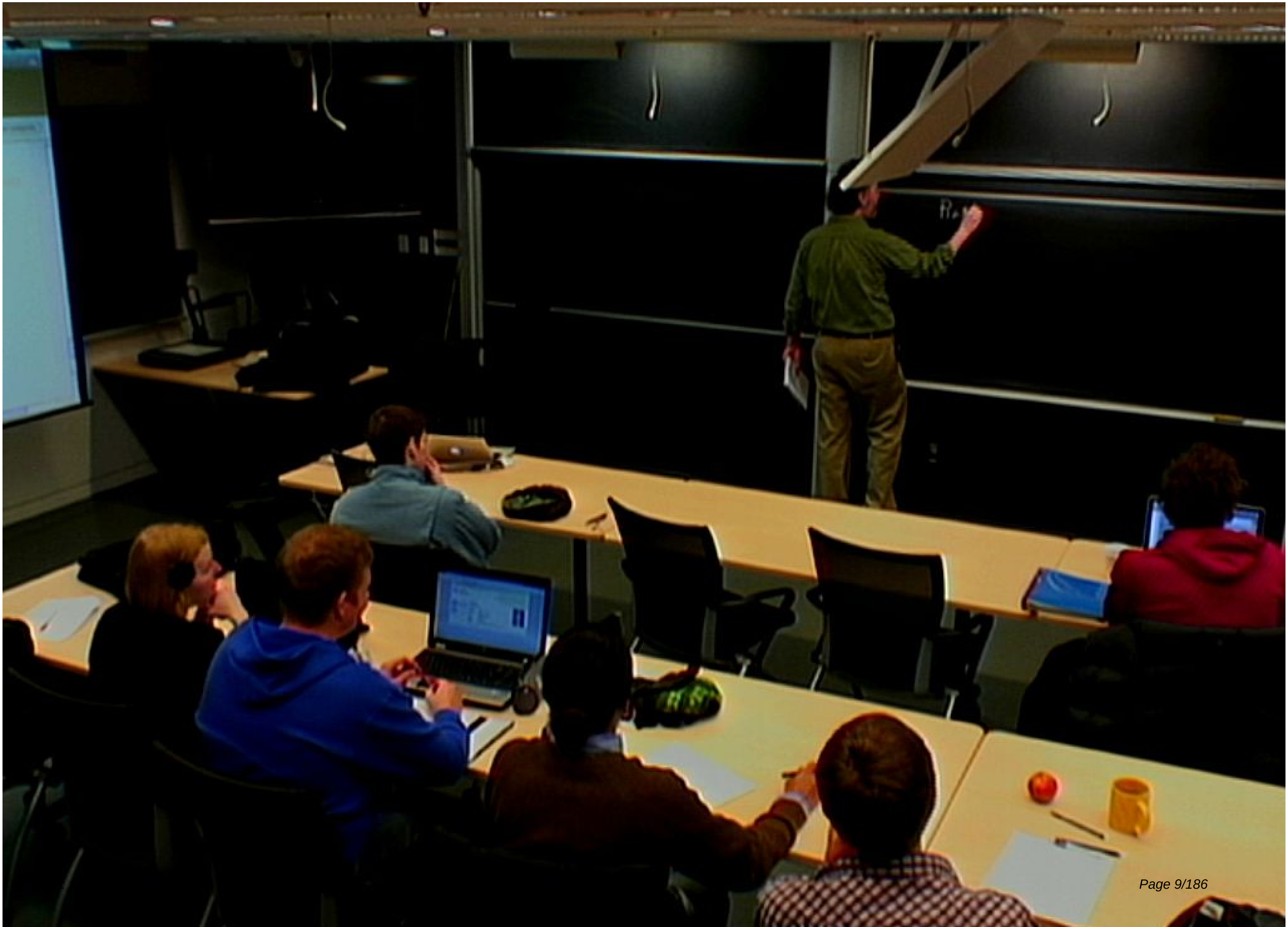
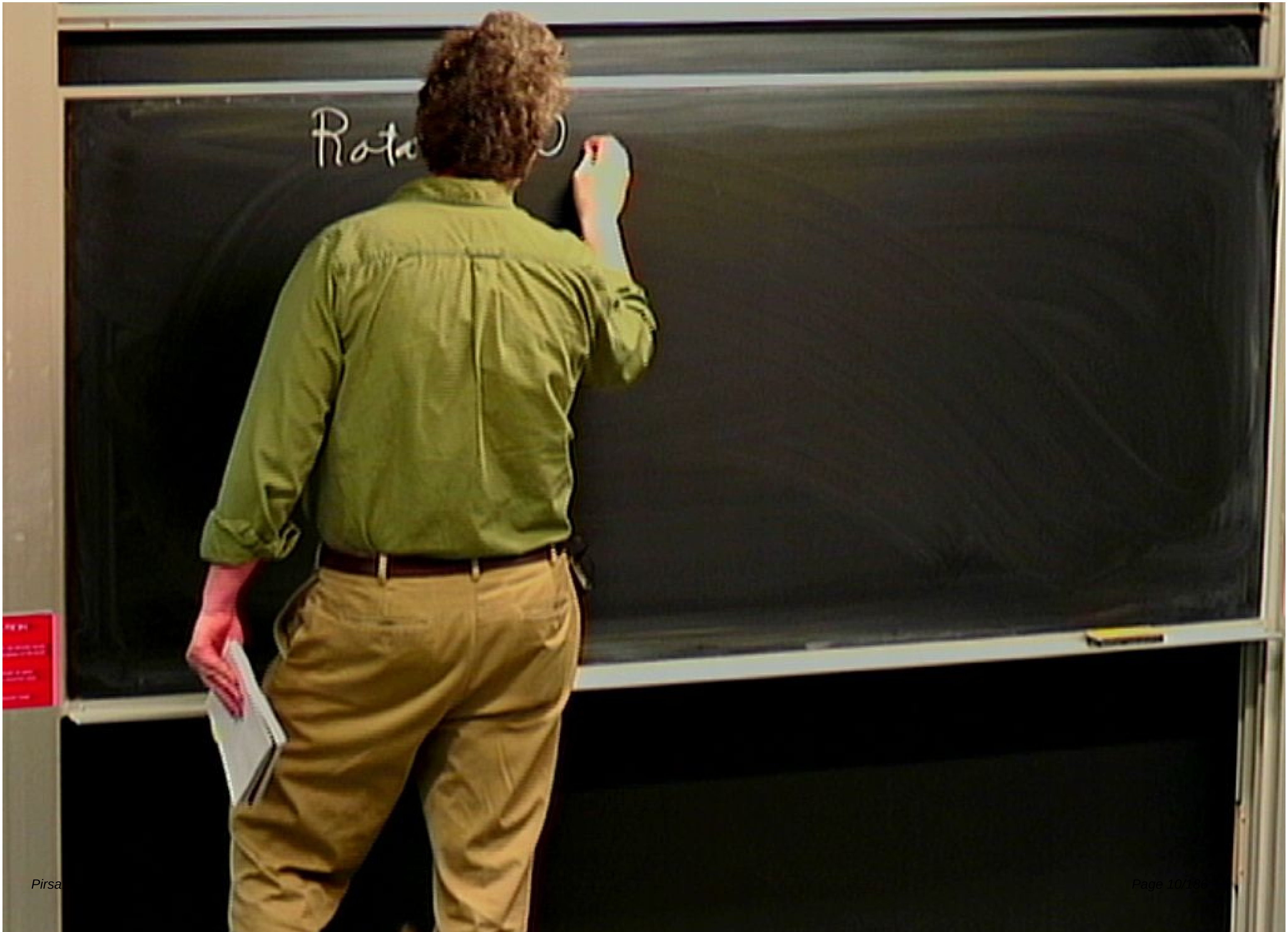
Title: Explorations in Quantum Information - Lecture 7

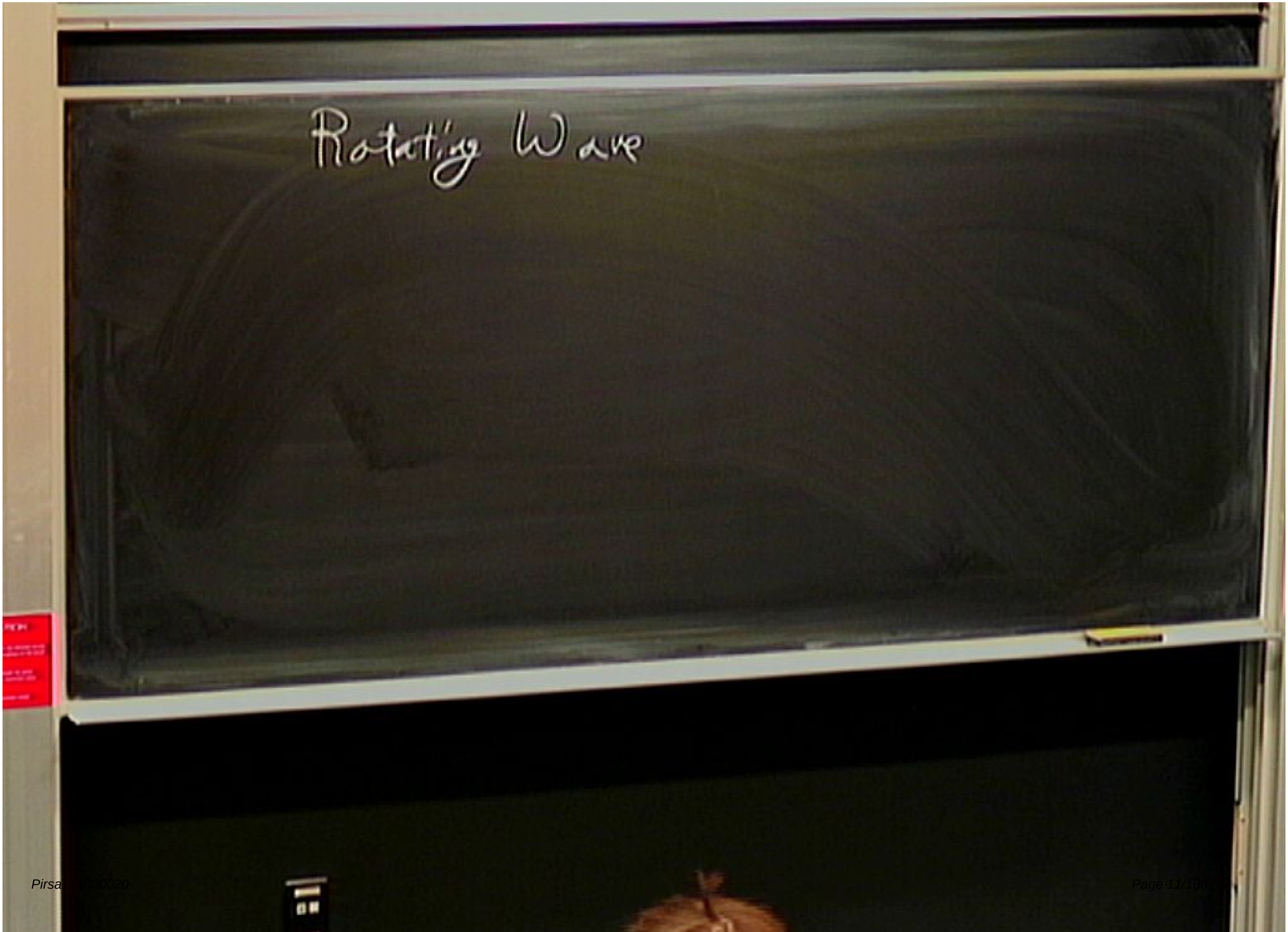
Date: Mar 23, 2011 09:00 AM

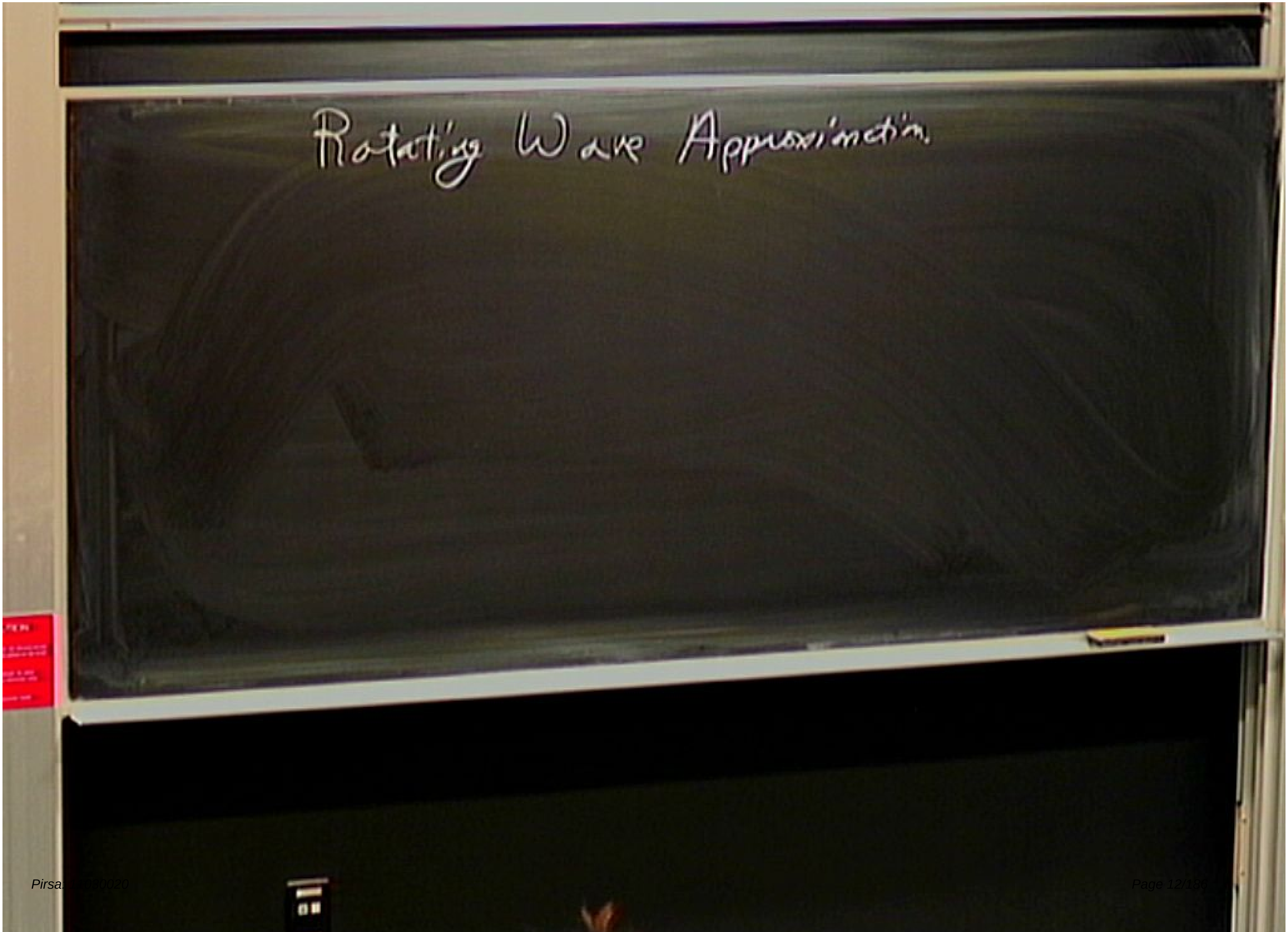
URL: http://pirsa.org/11030020

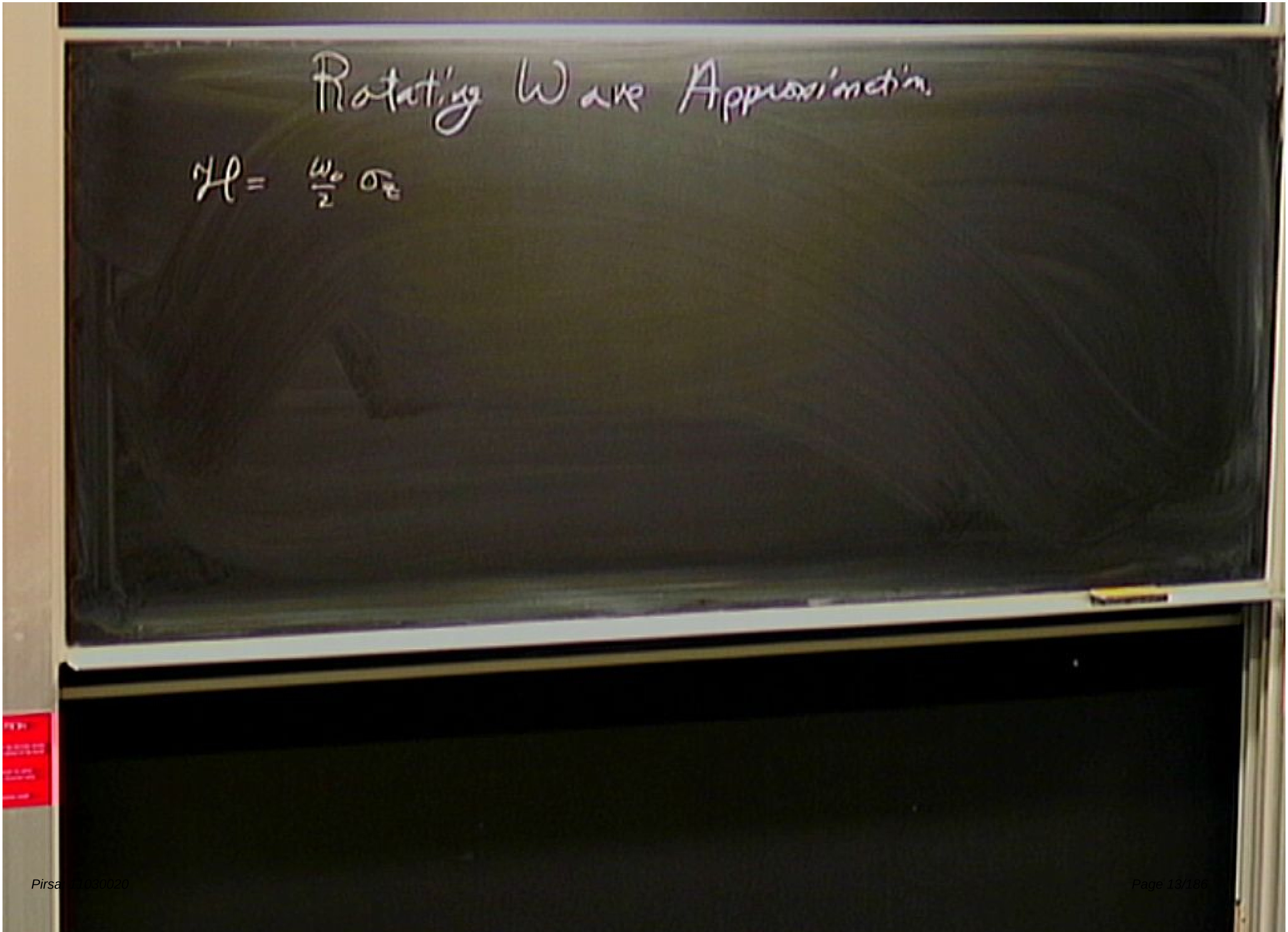
Abstract:





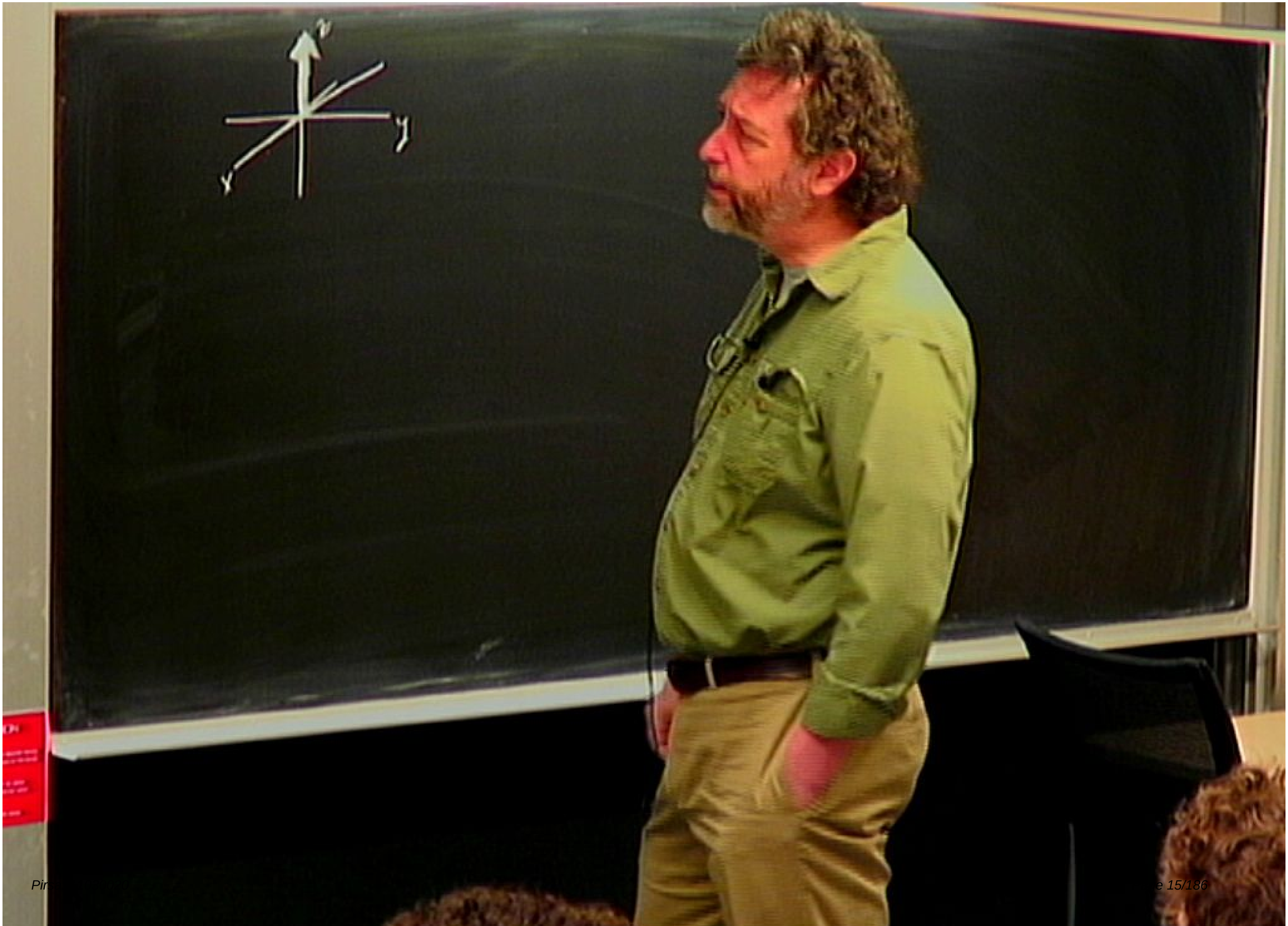

Rotating Wave

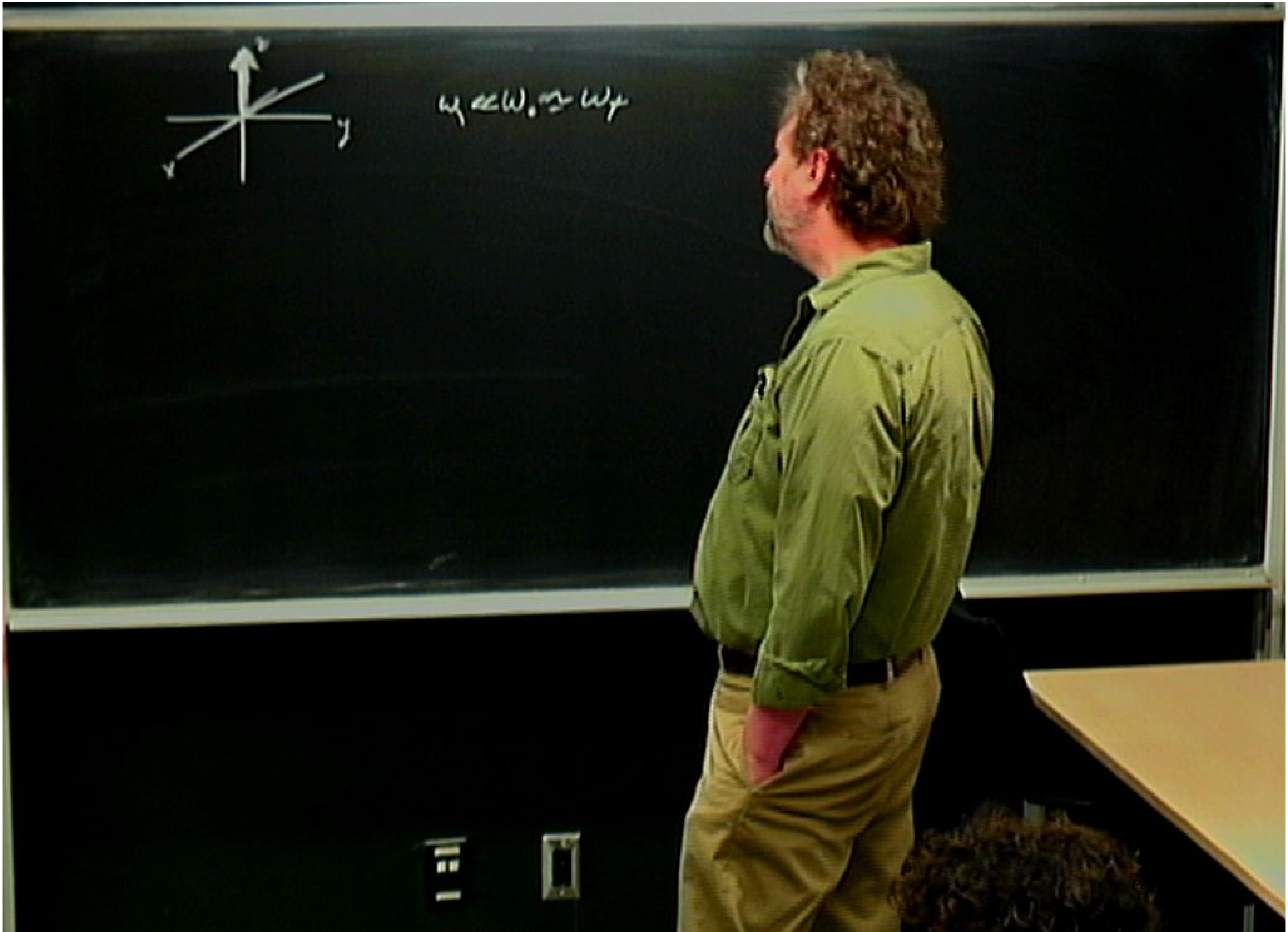

Rotating Wave Approximation

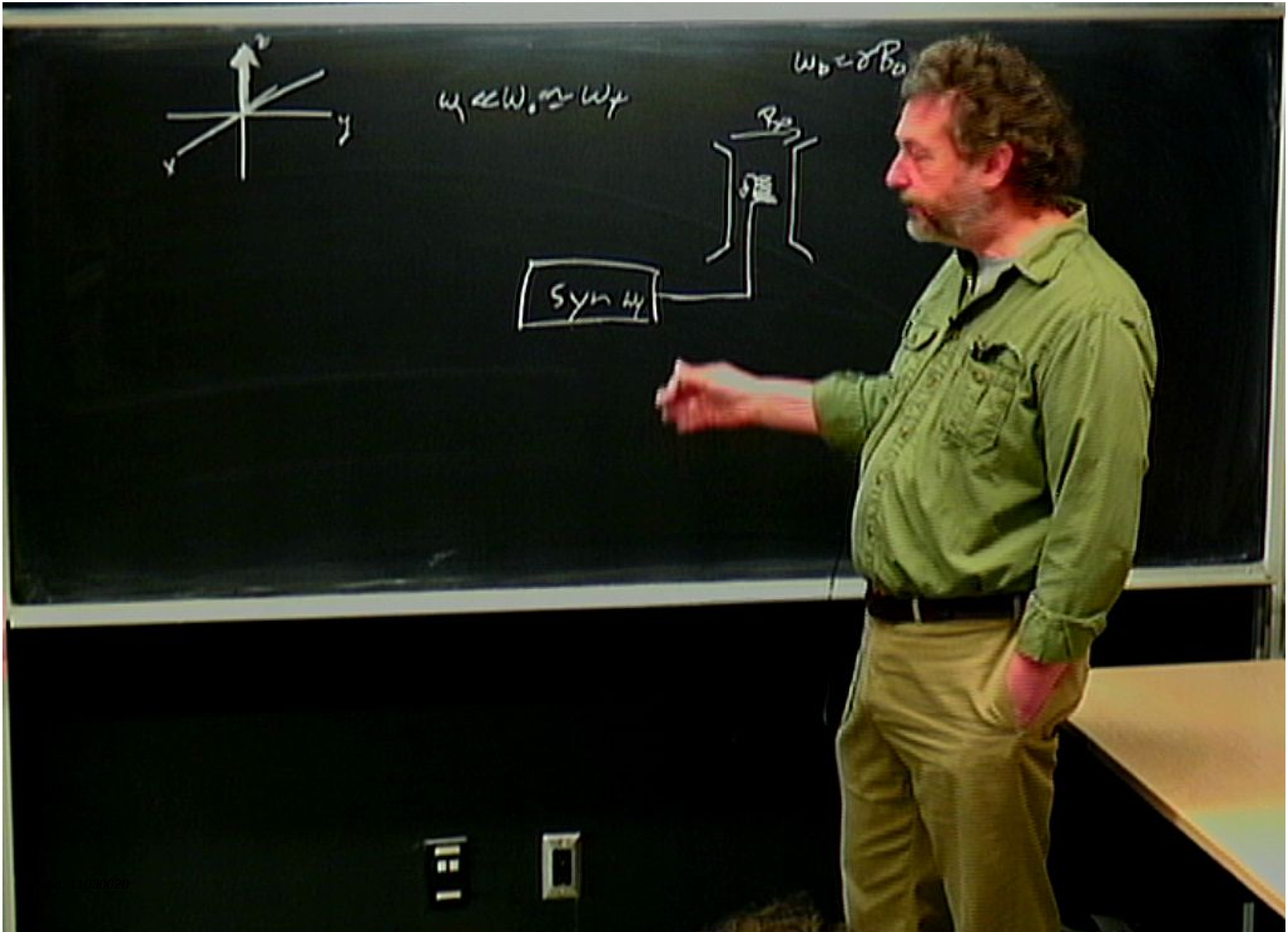

Rotating Wave Approximation
$\mathcal{H} = \frac{\omega_0}{2} \sigma_z$

Rotating Wave Approximation

$$\mathcal{H} = \frac{\omega_0}{2}\sigma_z \quad ; \quad \mathcal{H}_1(t) = \omega_1 \cos(\omega_+ t)\,\sigma_x$$

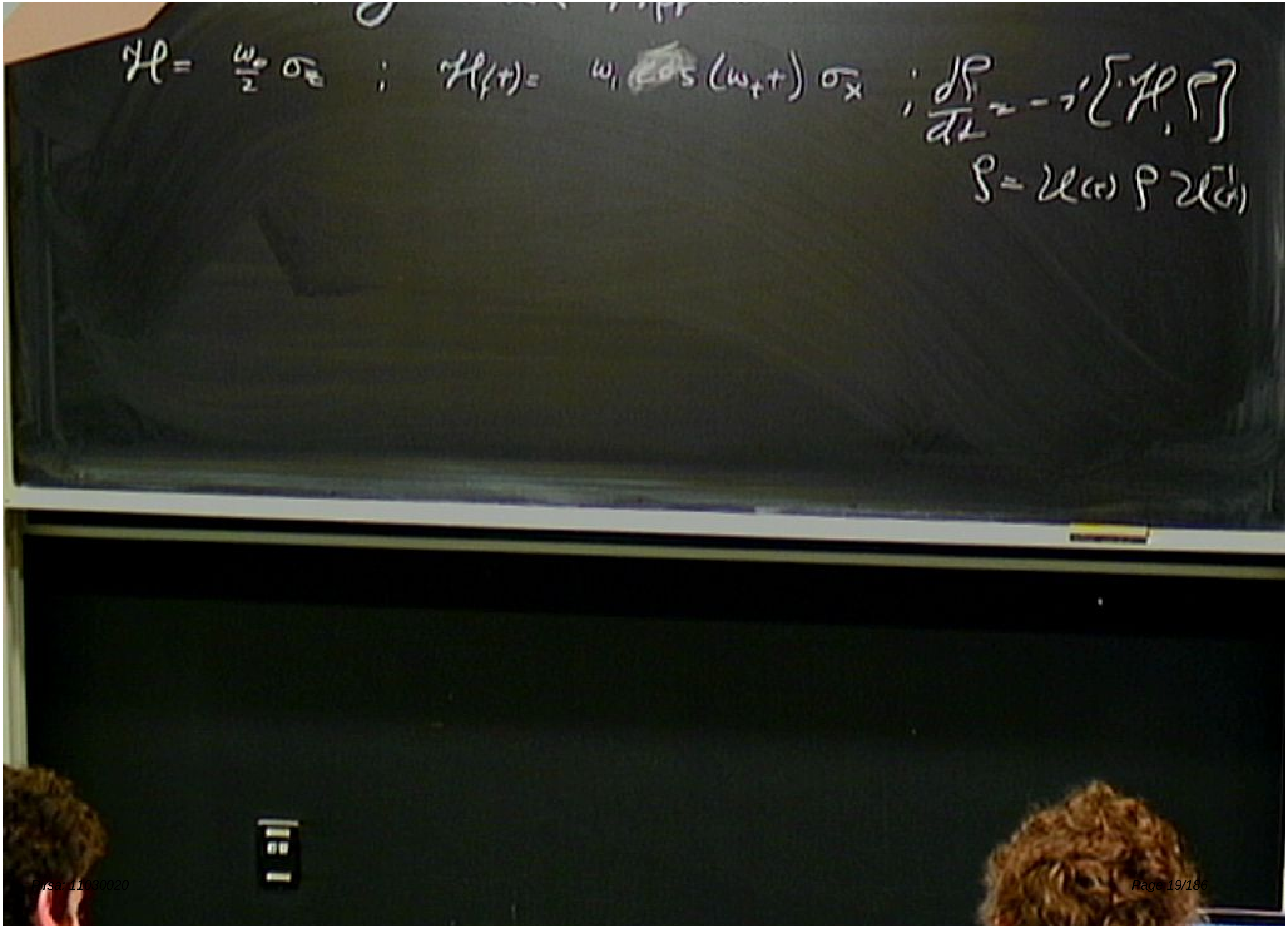

z
y
x

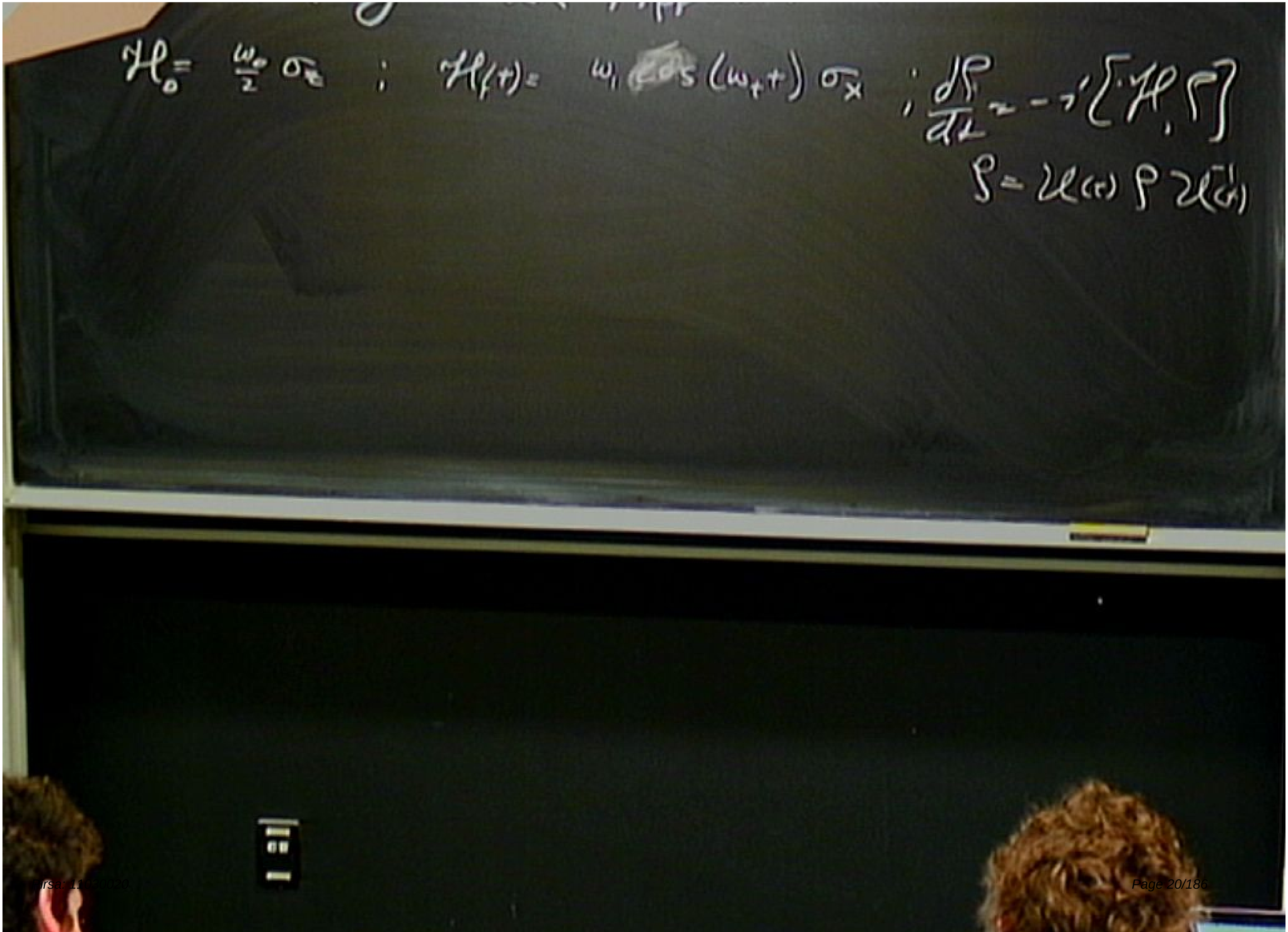


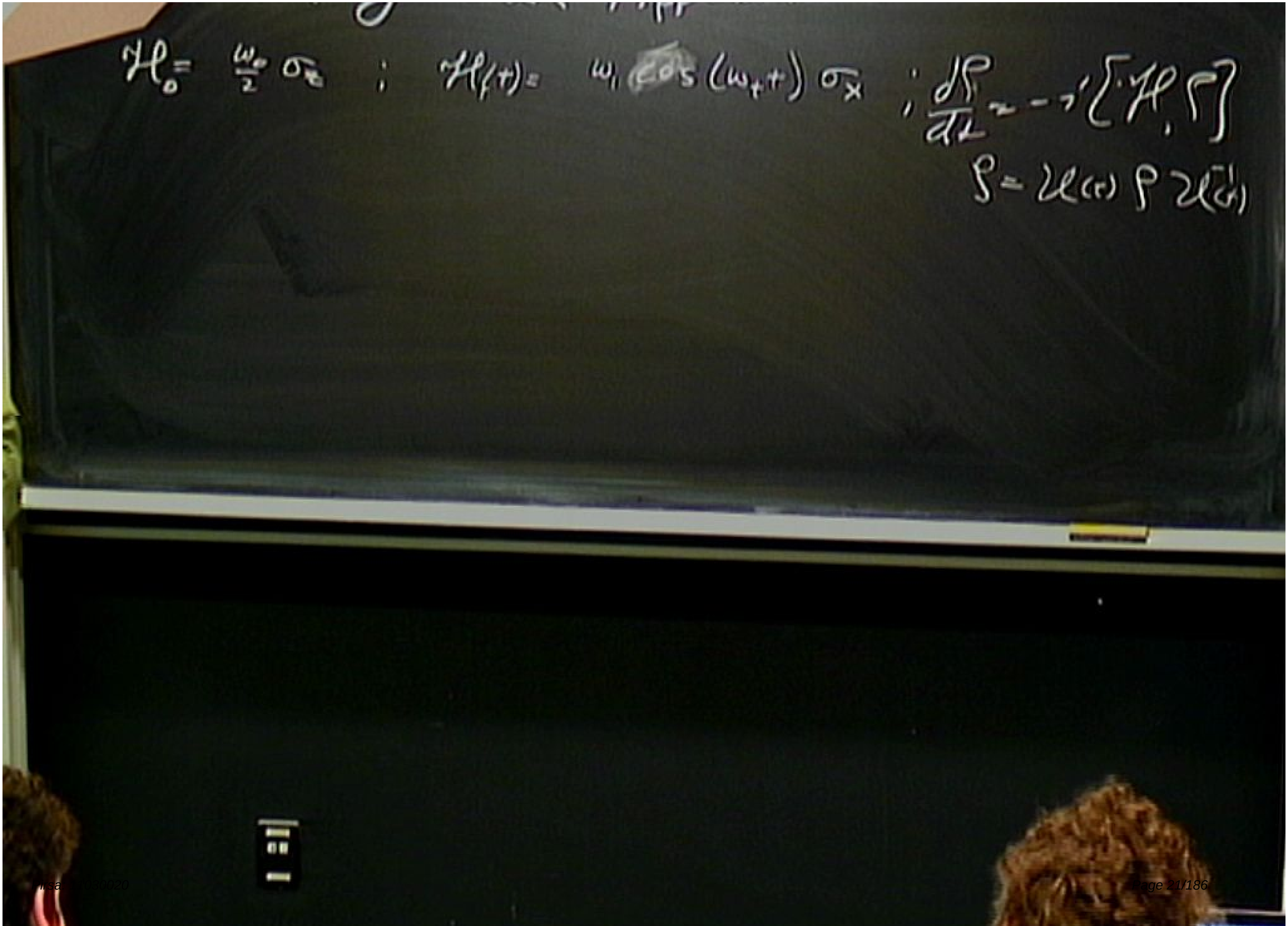


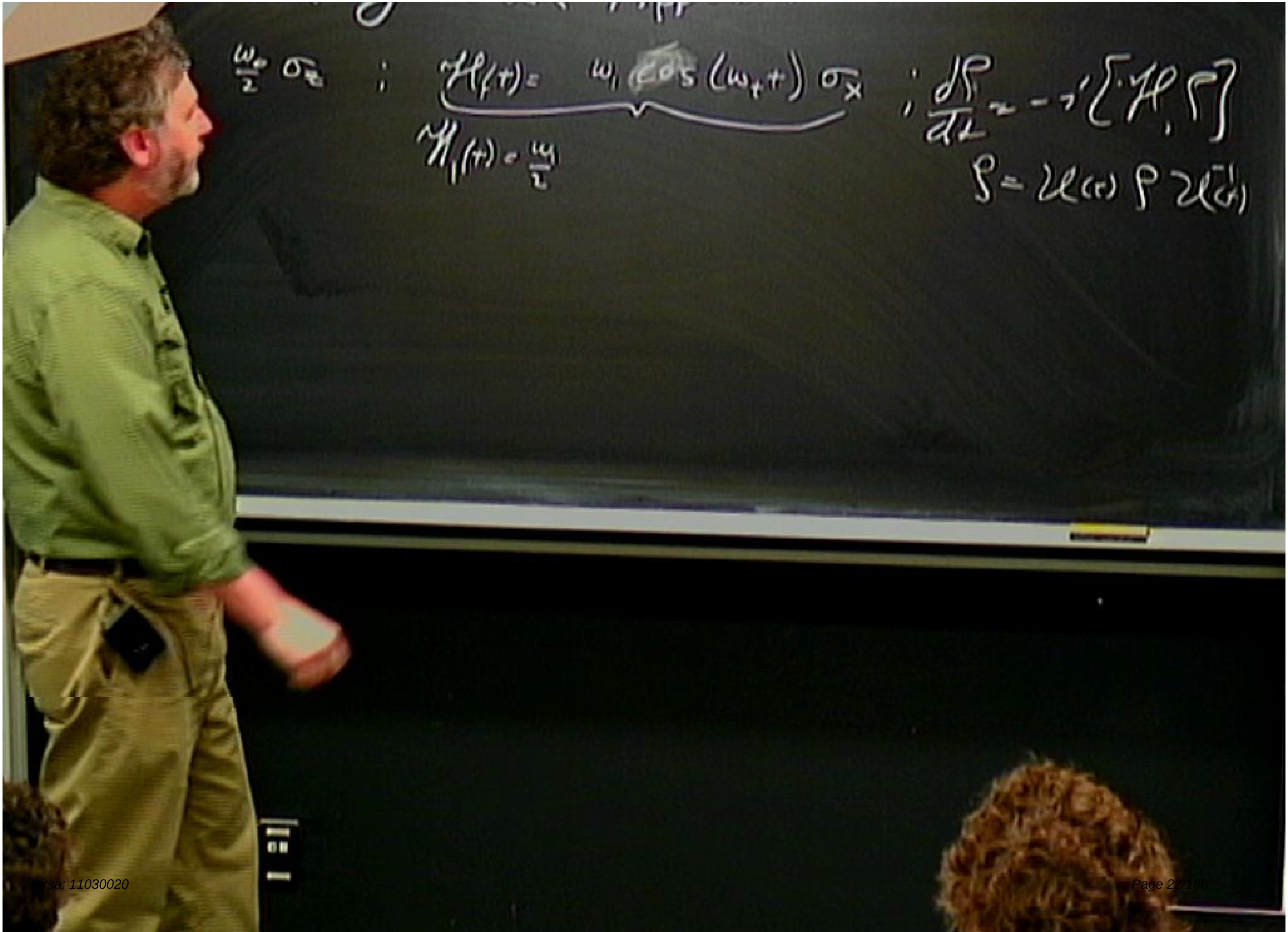
y
x
ω0 = γB0
Syn

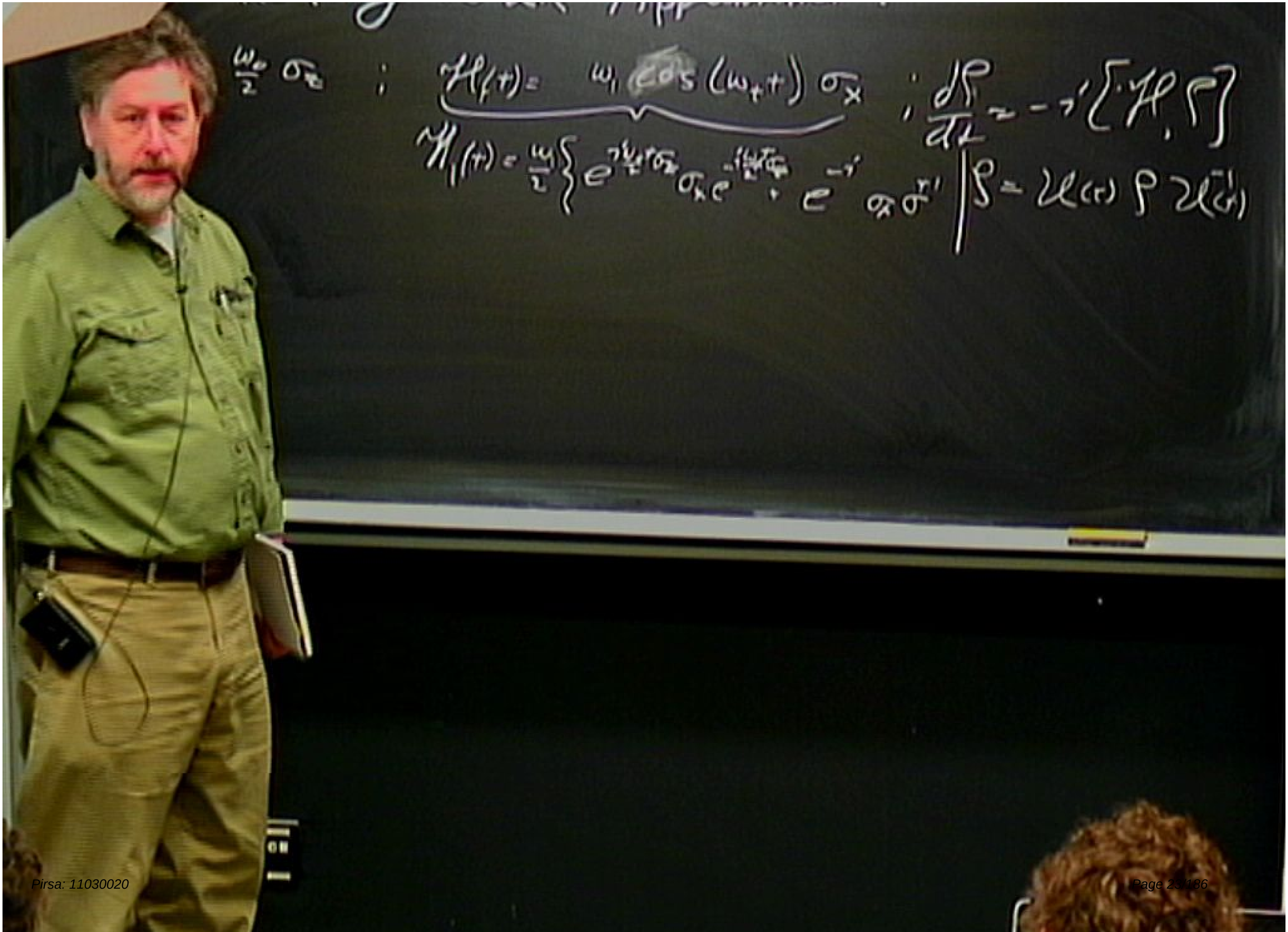
$\mathcal{H} = \frac{\omega_0}{2}\sigma_z \; ; \; \mathcal{H}_1(t) = \omega_1 \cos(\omega_t t)\,\sigma_x \; ; \; \frac{d\rho}{dt} = -i[\mathcal{H}, \rho]$


$\mathcal{H} = \frac{\omega_0}{2} \sigma_z$; $\mathcal{H}(t) = \omega_1 \cos(\omega_1 t) \sigma_x$; $\frac{d\rho}{dt} = -i[\mathcal{H}, \rho]$
$\rho = \mathcal{U}(t) \rho \mathcal{U}(t)^{-1}$


$\mathcal{H}_0 = \frac{\omega_0}{2}\sigma_z$; $\mathcal{H}_1(t) = \omega_1 \cos(\omega_f t)\,\sigma_x$; $\frac{d\rho}{dt} = -i[\mathcal{H}, \rho]$
$\tilde{\rho} = \mathcal{U}(t)\,\rho\,\mathcal{U}^{\dagger}(t)$


$\mathcal{H}_0 = \frac{\omega_0}{2}\sigma_z$; $\mathcal{H}_f(t) = \omega_1 \cos(\omega_f t)\,\sigma_x$; $\frac{d\rho}{dt} = -i[\mathcal{H}, \rho]$
$\rho = \mathcal{U}(t)\,\rho\,\mathcal{U}^{-1}(t)$

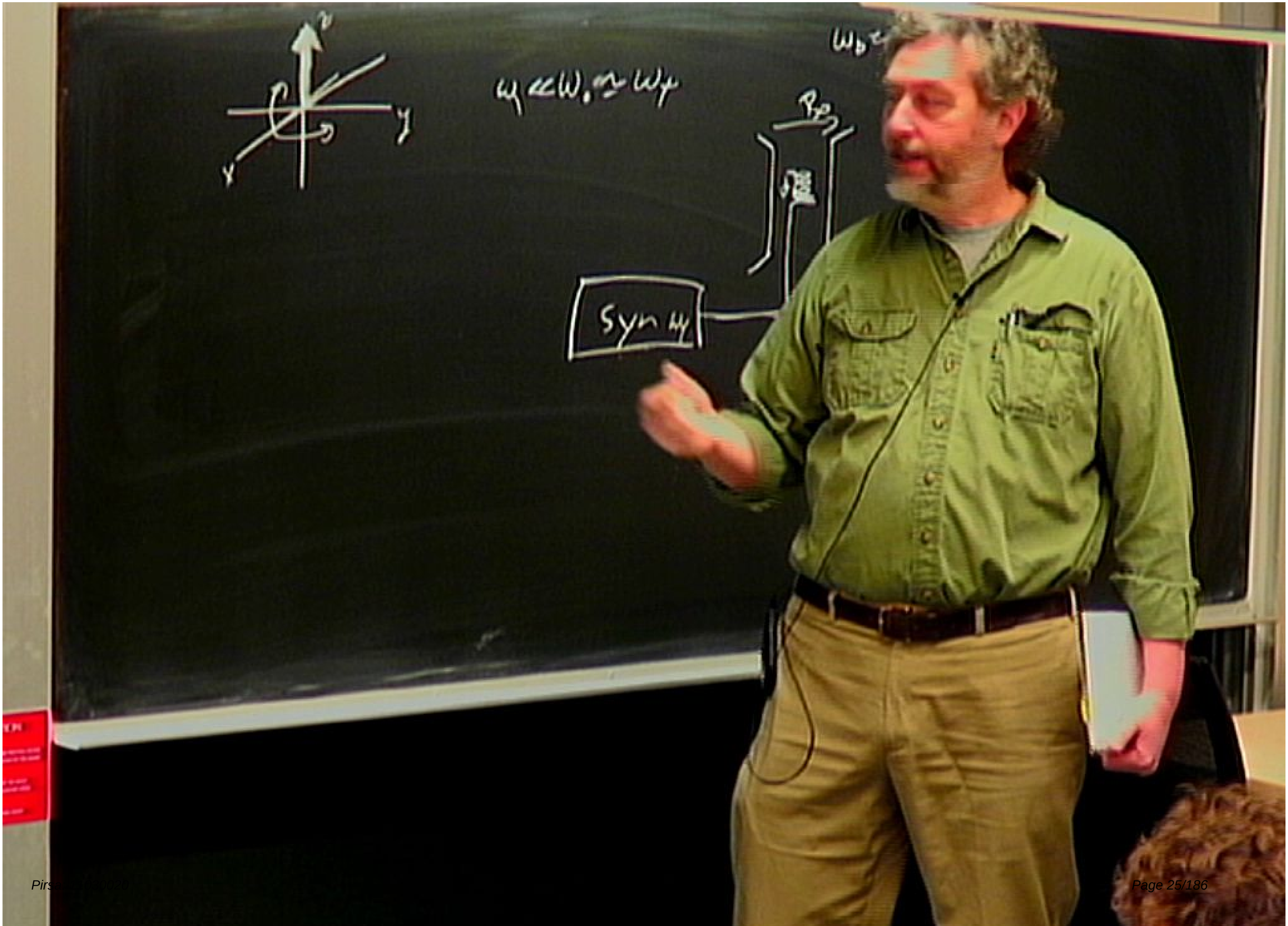

ω0/2 σz ; H(t) = ω1 cos (ωr t) σx ; dρ/dt = -i [H, ρ]
H1(t) = ω1/2
ρ = U(t) ρ U†(t)

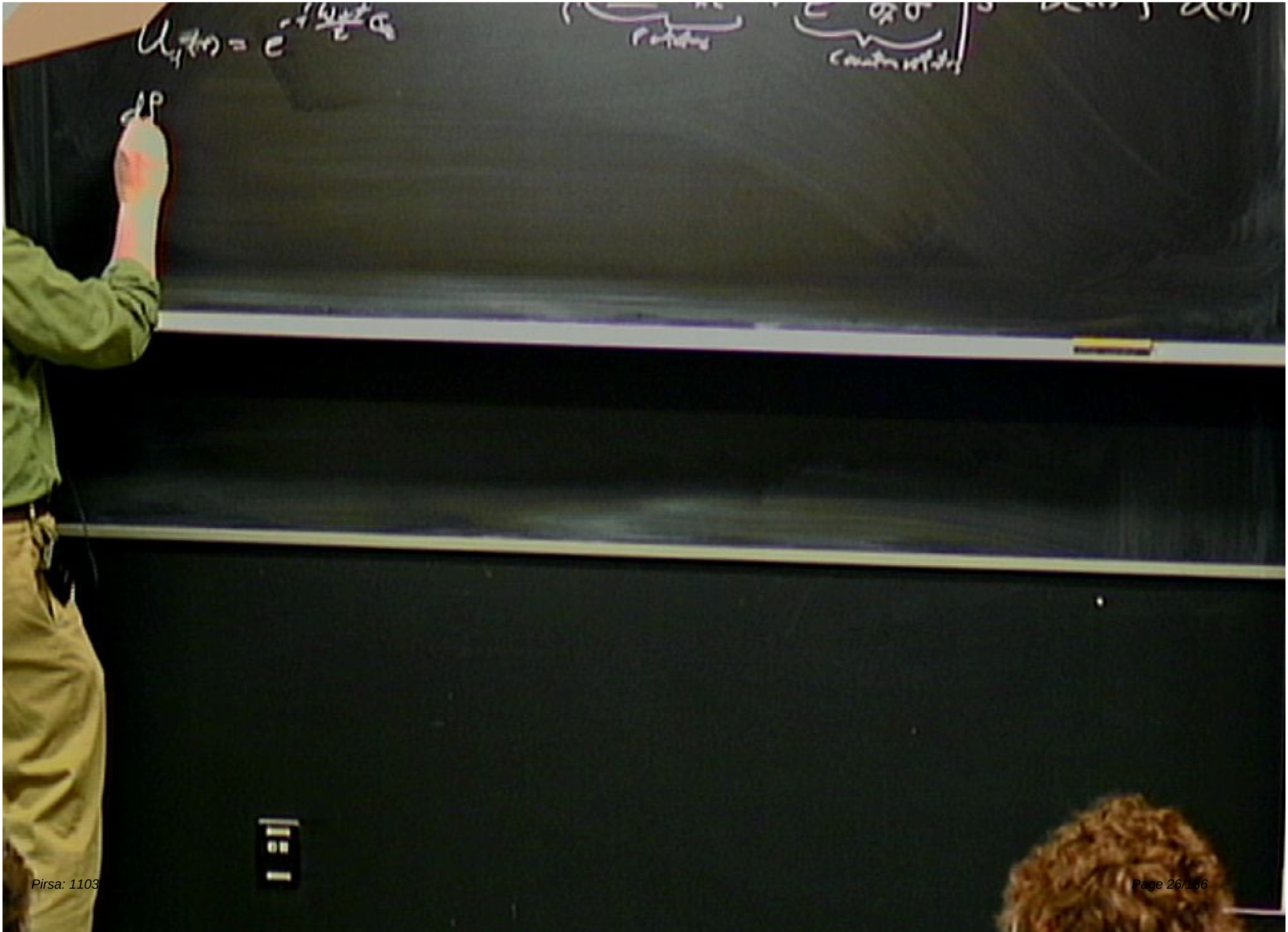

$\frac{\omega_0}{2}\sigma_z$; $\mathcal{H}(t) = \omega_1 \cos(\omega_r t)\,\sigma_x$; $\frac{d\rho}{dt} = -i[\mathcal{H}, \rho]$
$\rho = \mathcal{U}(t)\,\rho\,\mathcal{U}^{-1}(t)$

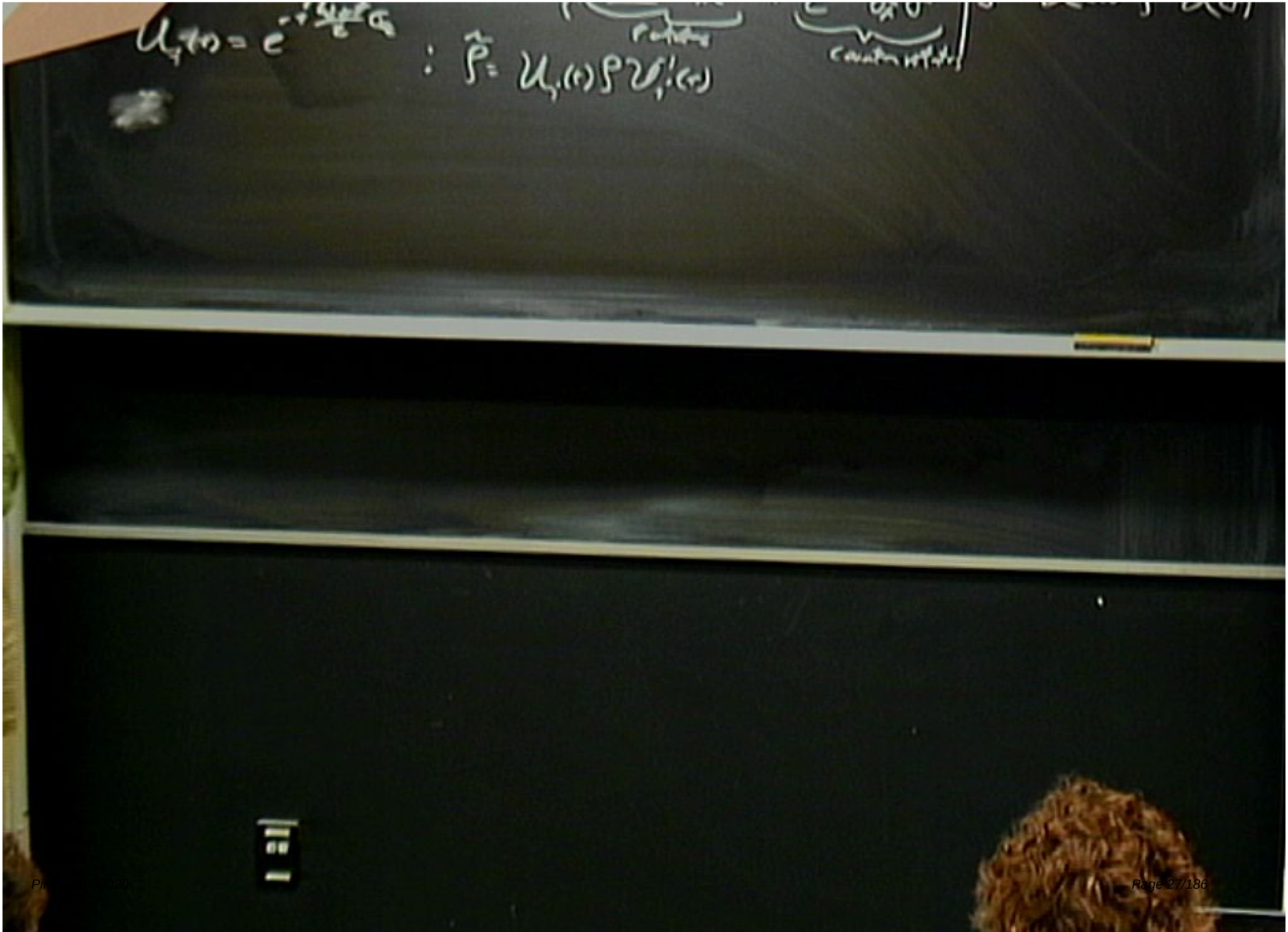
$$\mathcal{H}_0 = \frac{\omega_0}{2}\sigma_z \; ; \; \mathcal{H}(t) = \omega_1 \cos(\omega_r t)\,\sigma_x \; ; \; \frac{d\rho}{dt} = -i[\mathcal{H}, \rho]$$

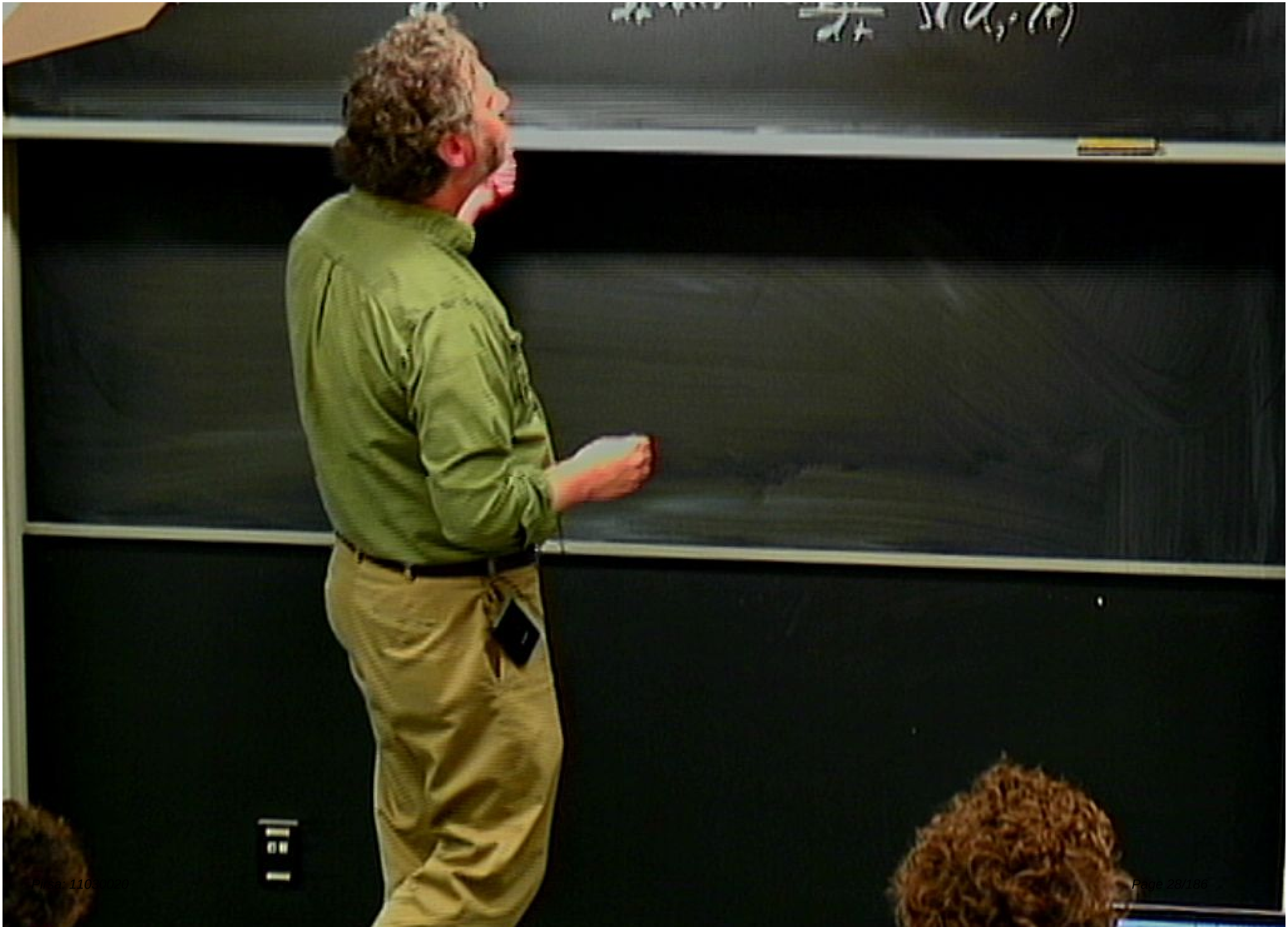
$$\mathcal{H}_1(t) = \frac{\omega_1}{2}\Big\{ \underbrace{e^{i\frac{\omega_r t}{2}\sigma_z}\sigma_x e^{-i\frac{\omega_r t}{2}\sigma_z}}_{\text{rotating}} + \underbrace{e^{-i\ldots}\sigma_x \sigma^{i}}_{\text{counter rotating}}$$

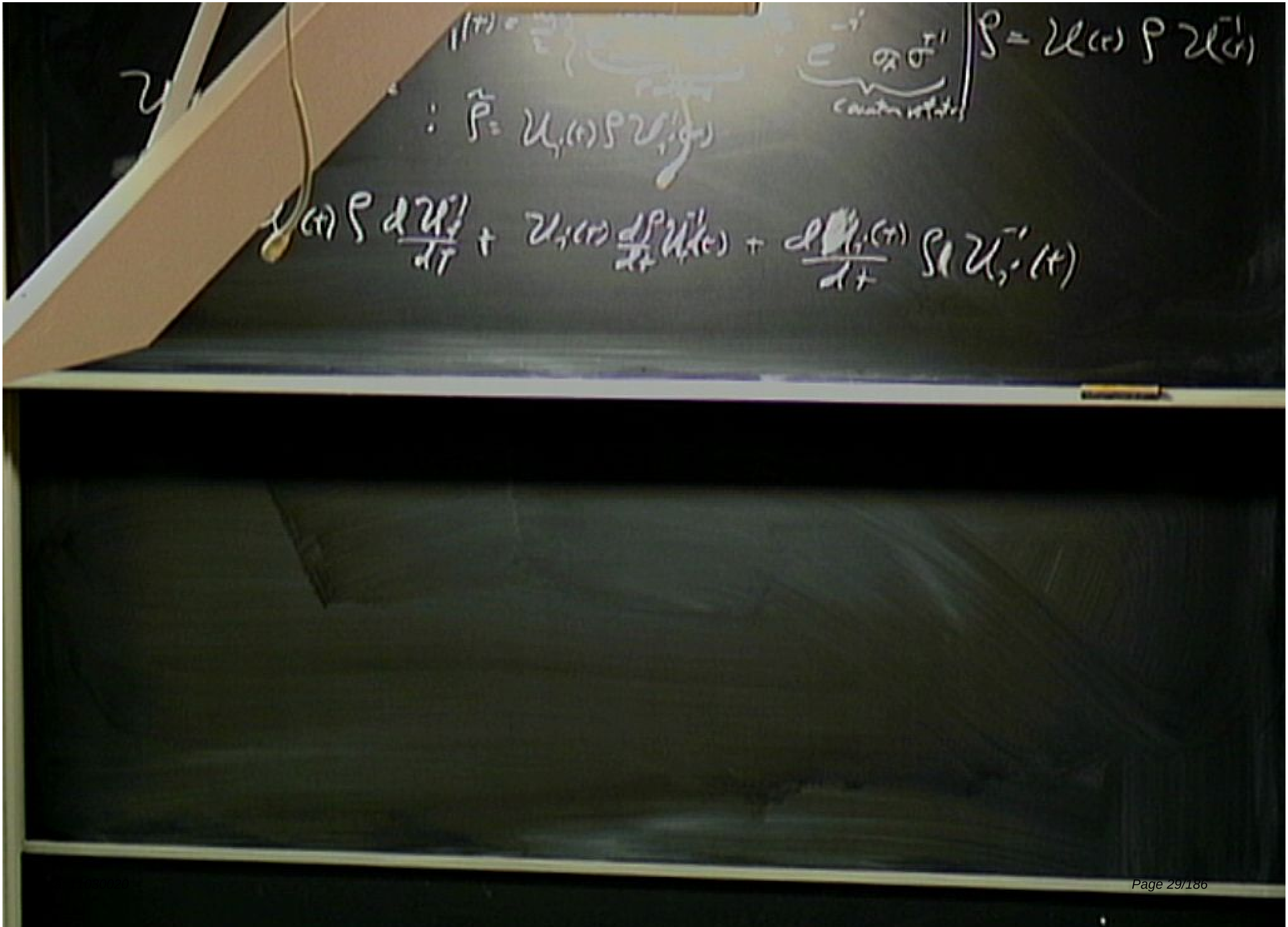
$$\tilde{\rho} = \mathcal{U}(t)\,\rho\,\mathcal{U}^{-1}(t)$$


x
y
Syn

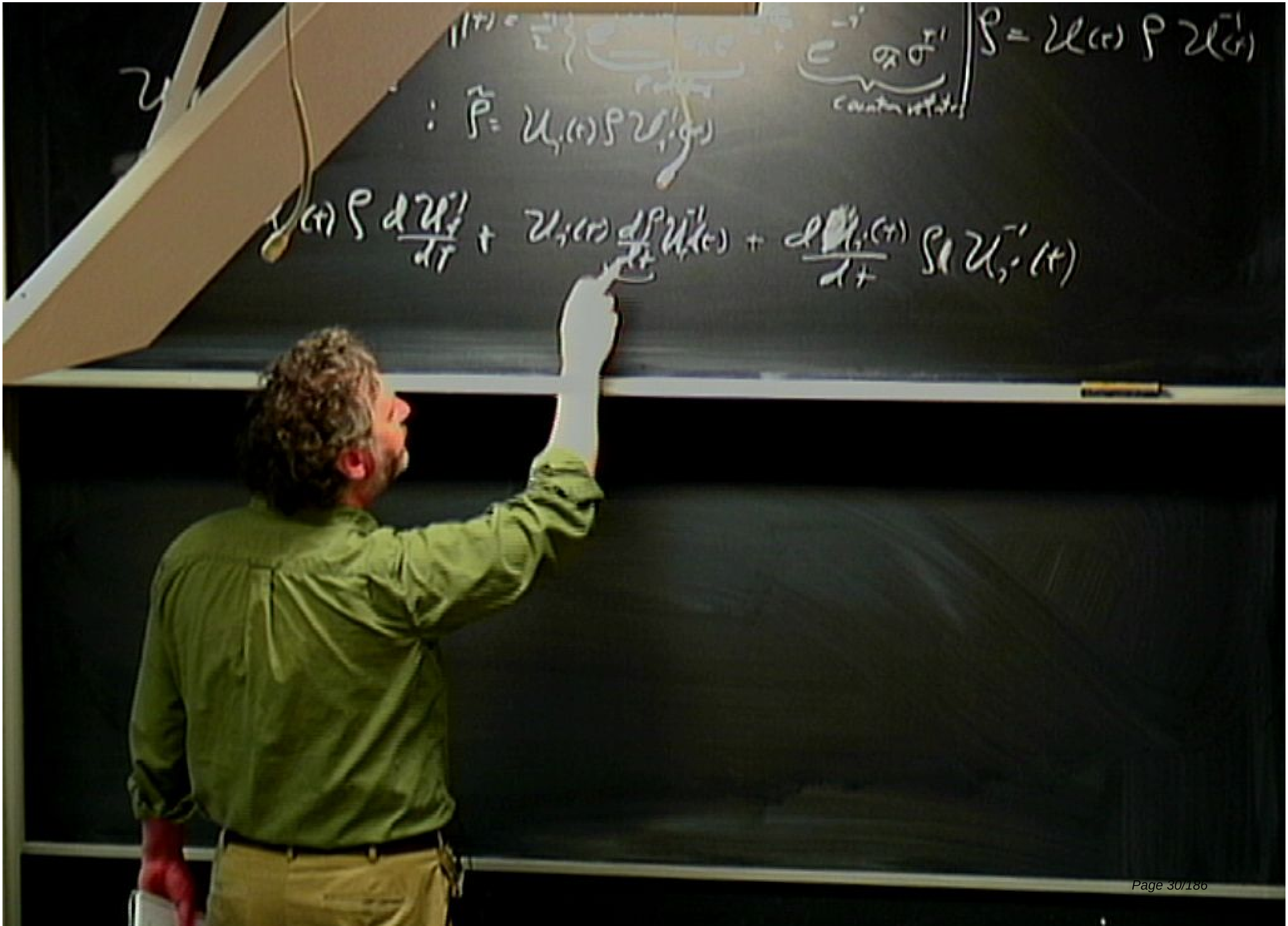



ρ̃ = U₁(t) ρ U₁†(t)





$\rho = \mathcal{U}(t)\,\rho\,\mathcal{U}^{-1}(t)$
$\tilde{\rho} = \mathcal{U}_1(t)\,\rho\,\mathcal{U}_1^{-1}(t)$



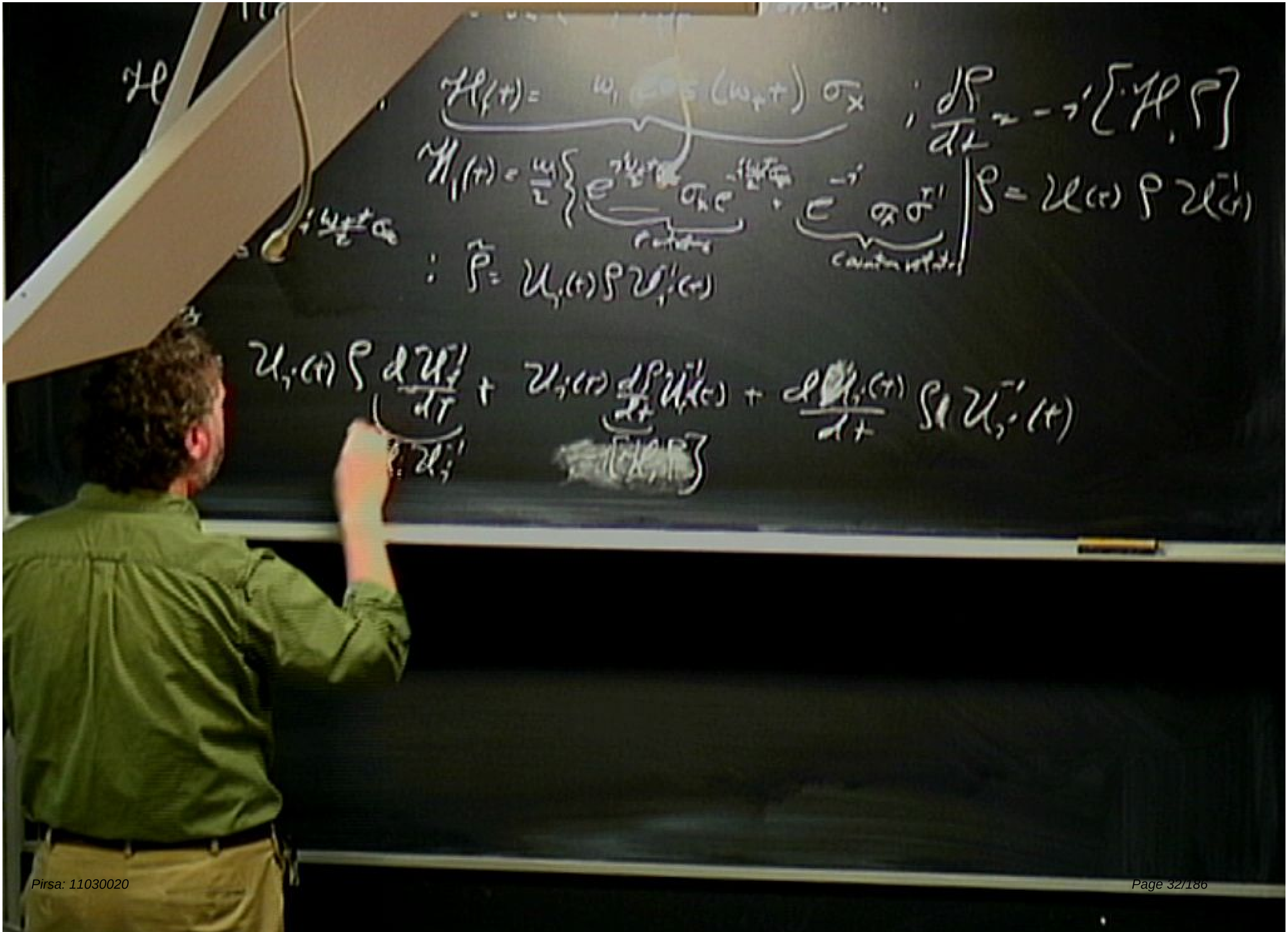
$$\rho = U(t)\,\rho\,U(t)$$

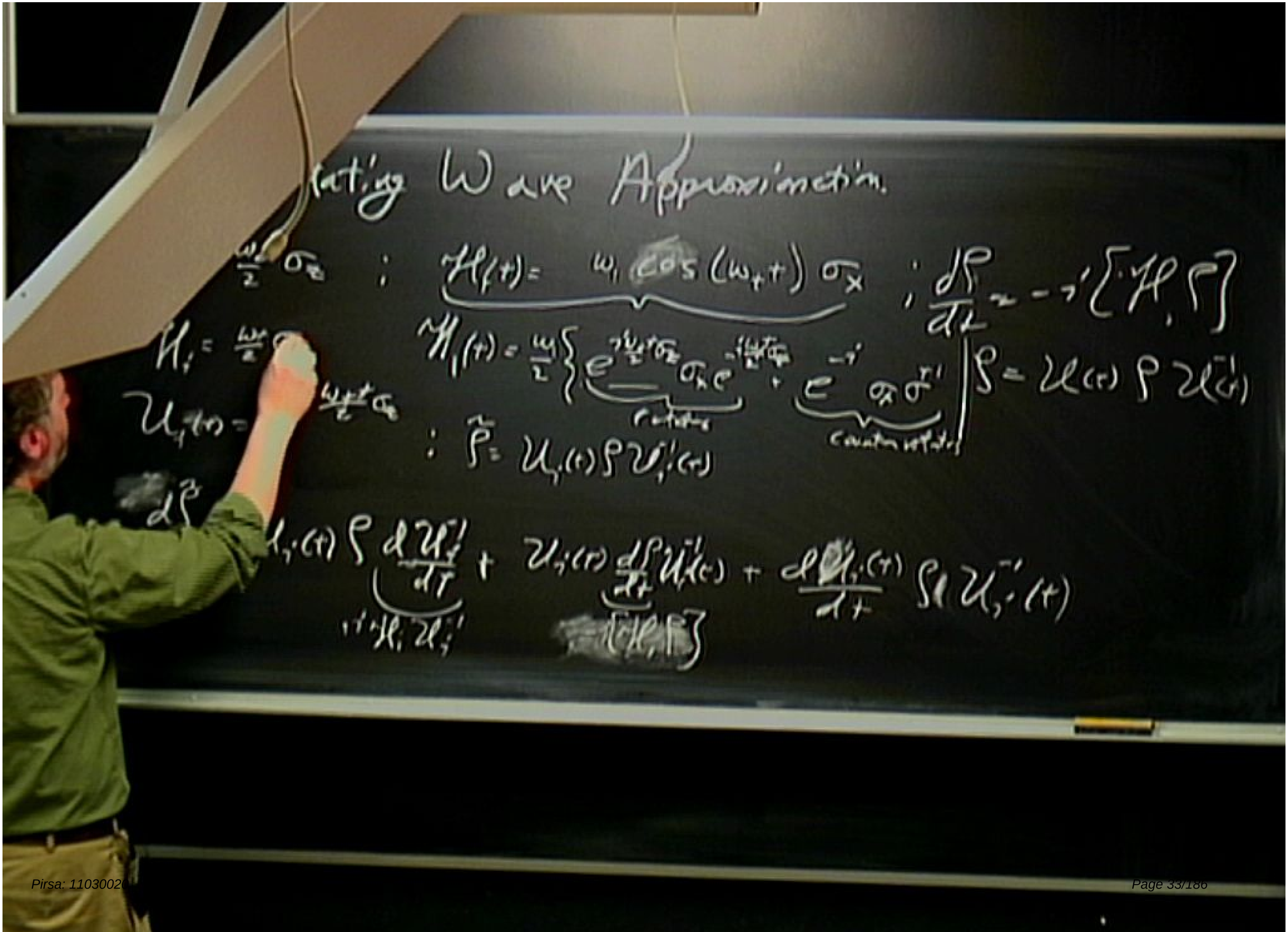
coordinate rotation

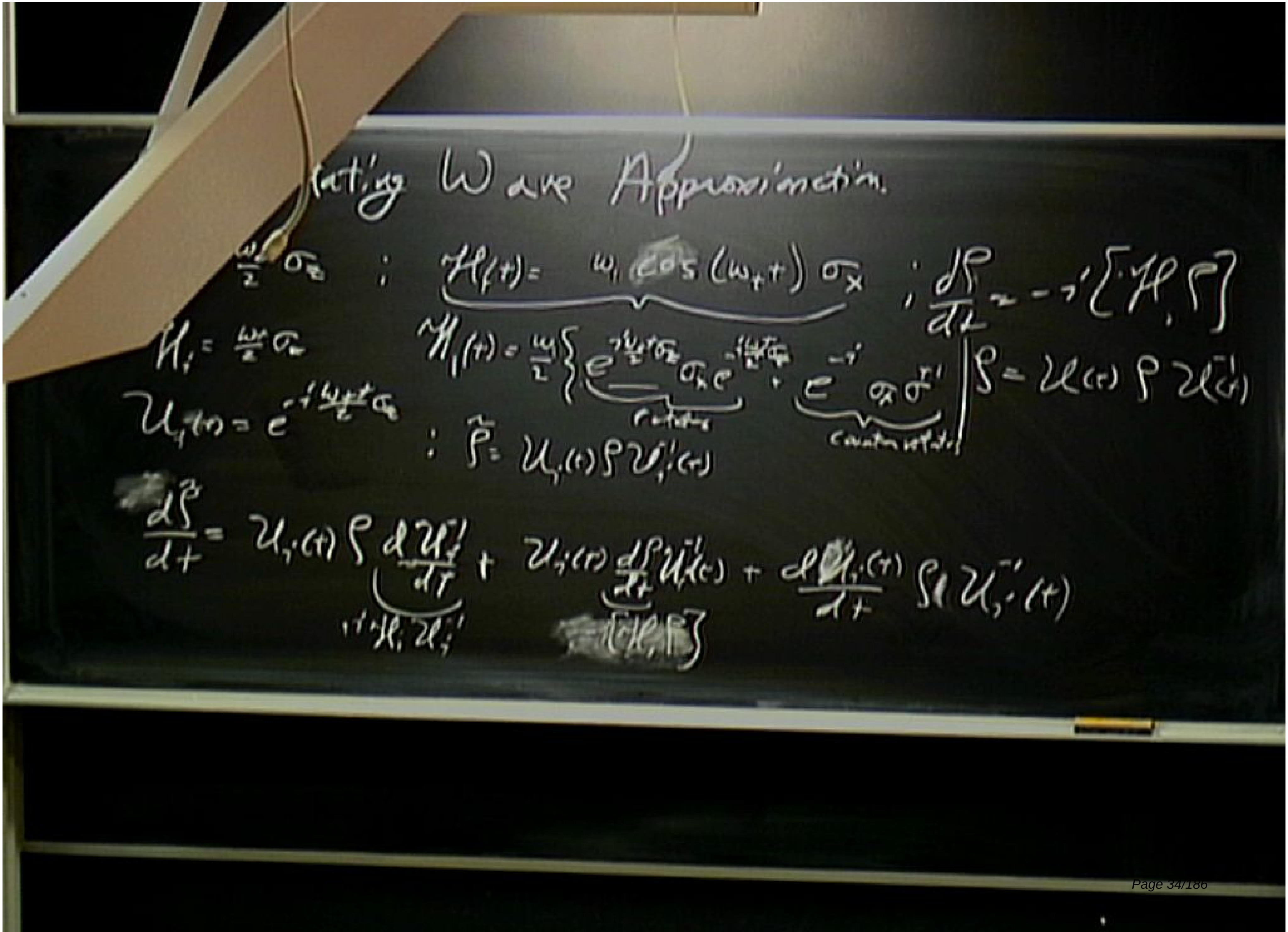
$$: \quad \tilde{\rho} = U_1(t)\,\rho\,U_1^{-1}(t)$$

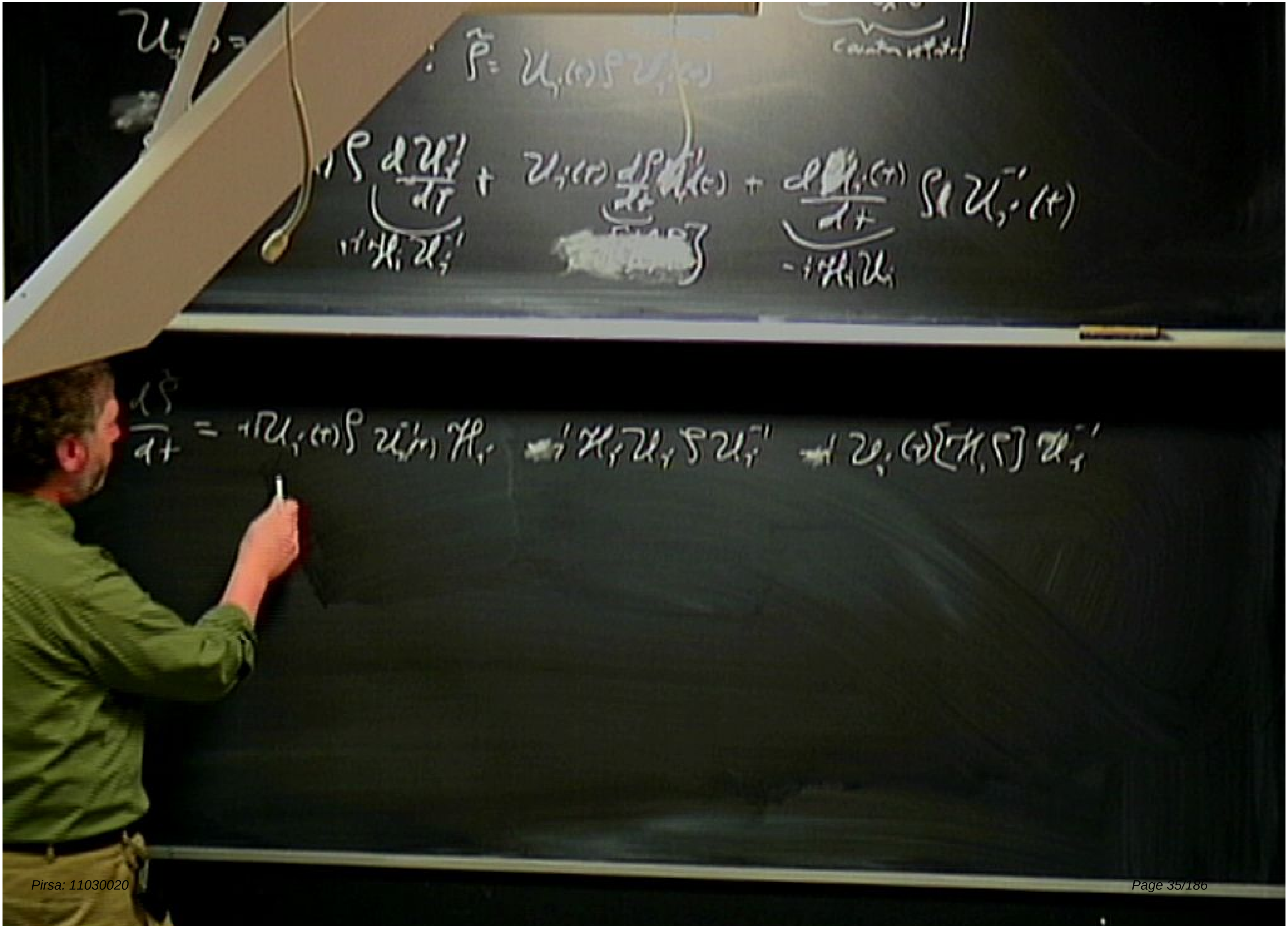
$$\ldots(t)\,\rho\,\frac{dU_1^{-1}}{dt} + U_1(t)\,\frac{d}{dt}U_1^{-1}(t) + \frac{dU_1(t)}{dt}\,\rho\,U_1^{-1}(t)$$

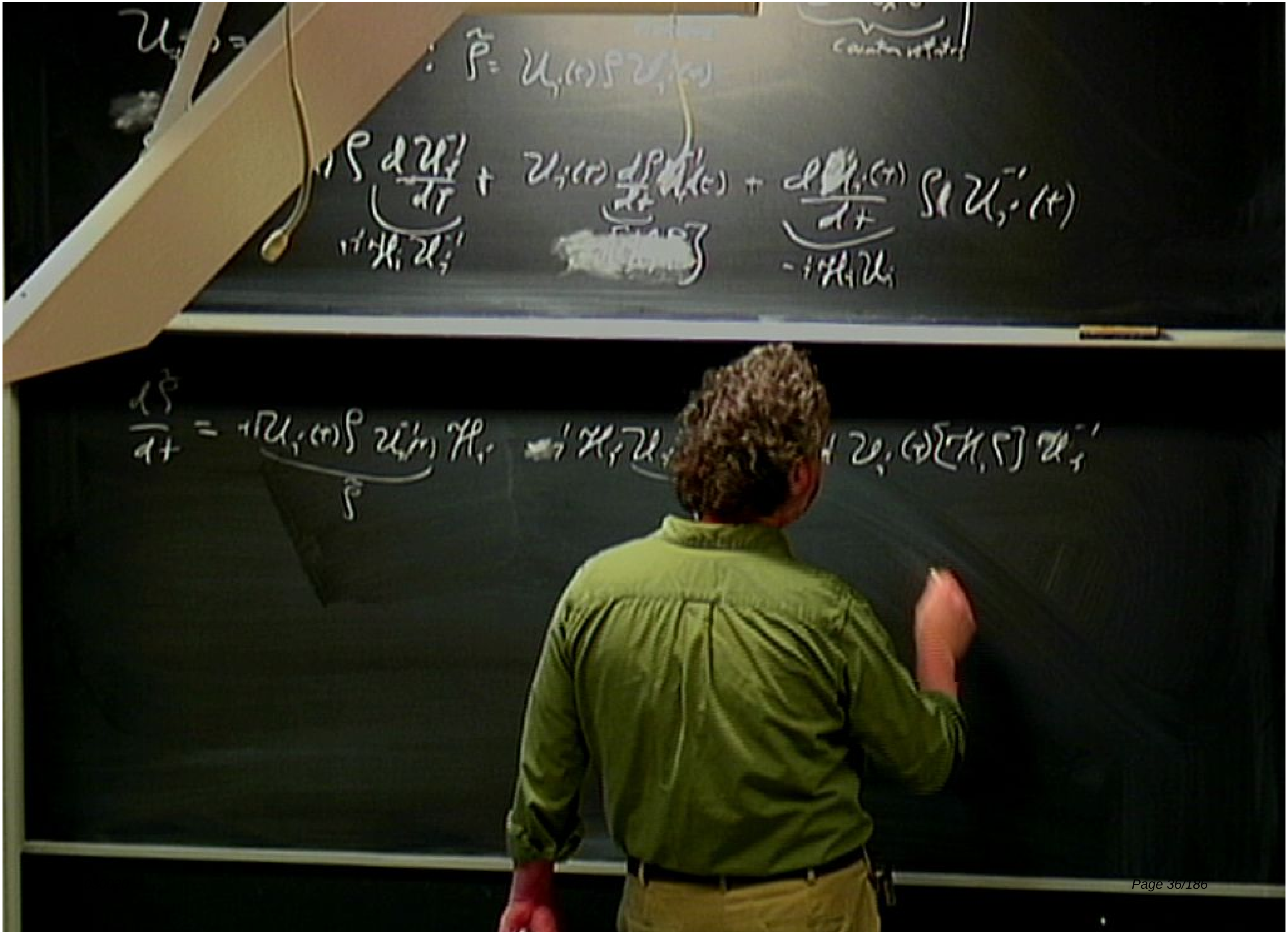
$$U(t)\,\rho\,U(t)$$

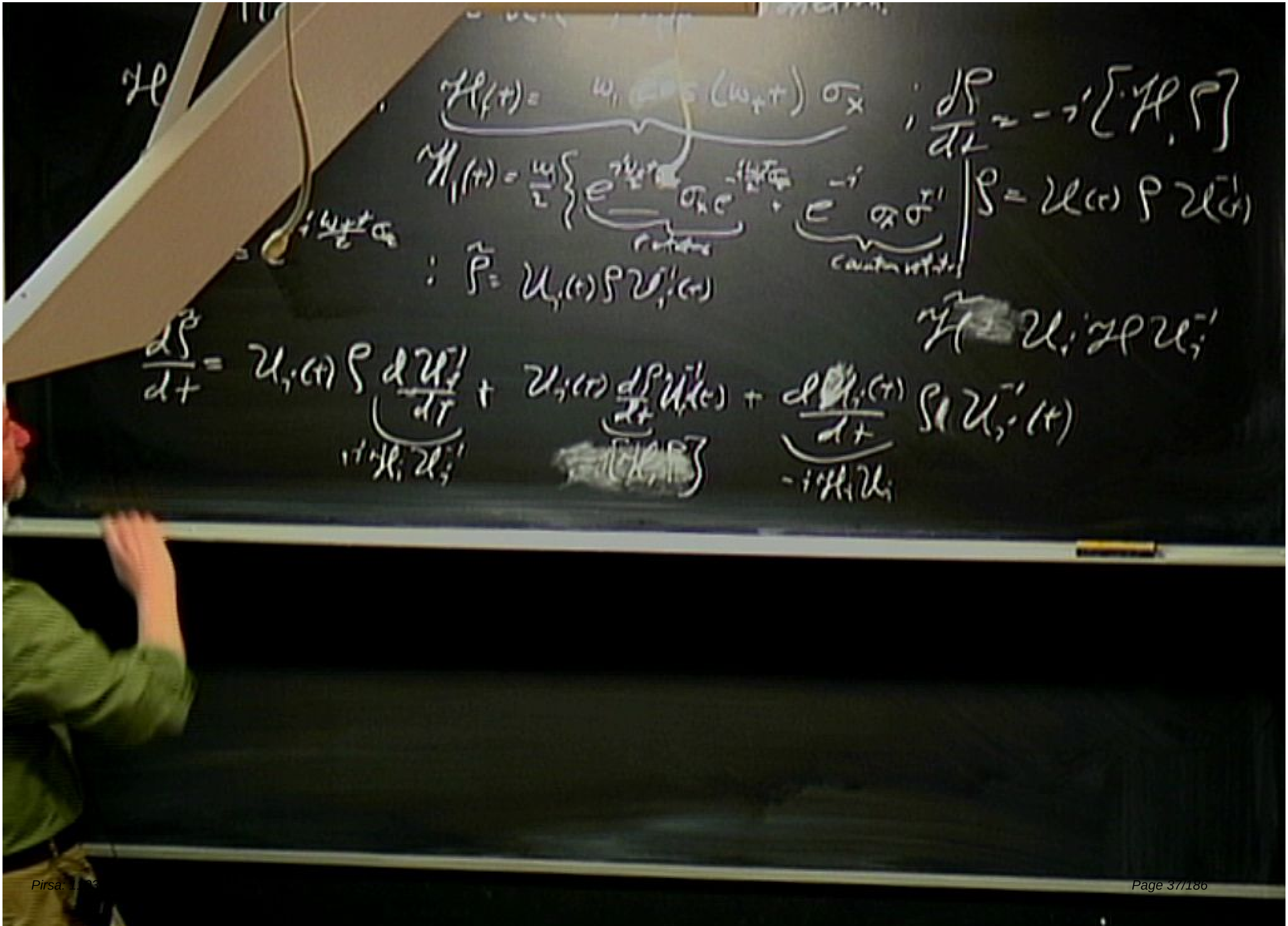


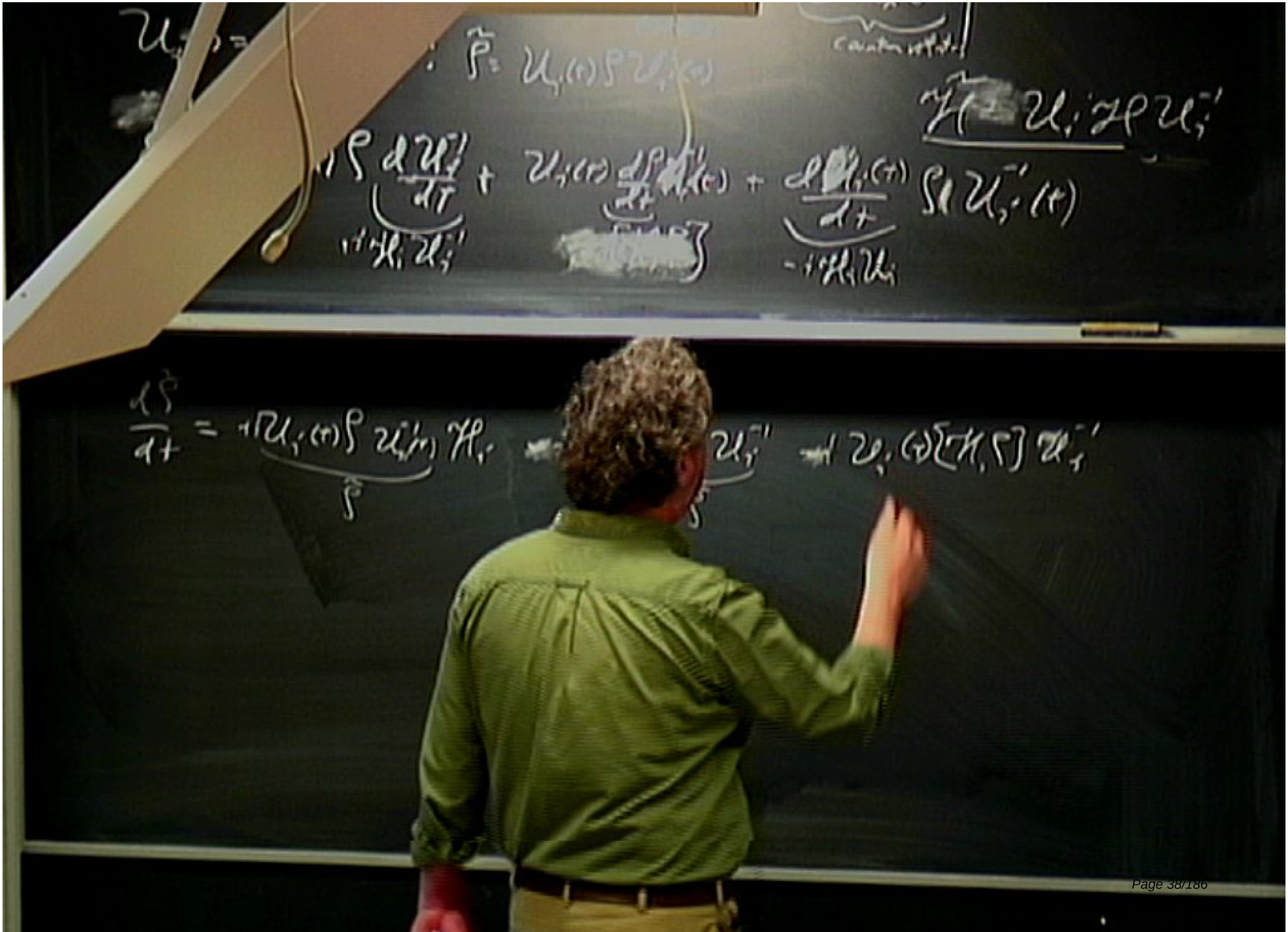

Wave Approximation

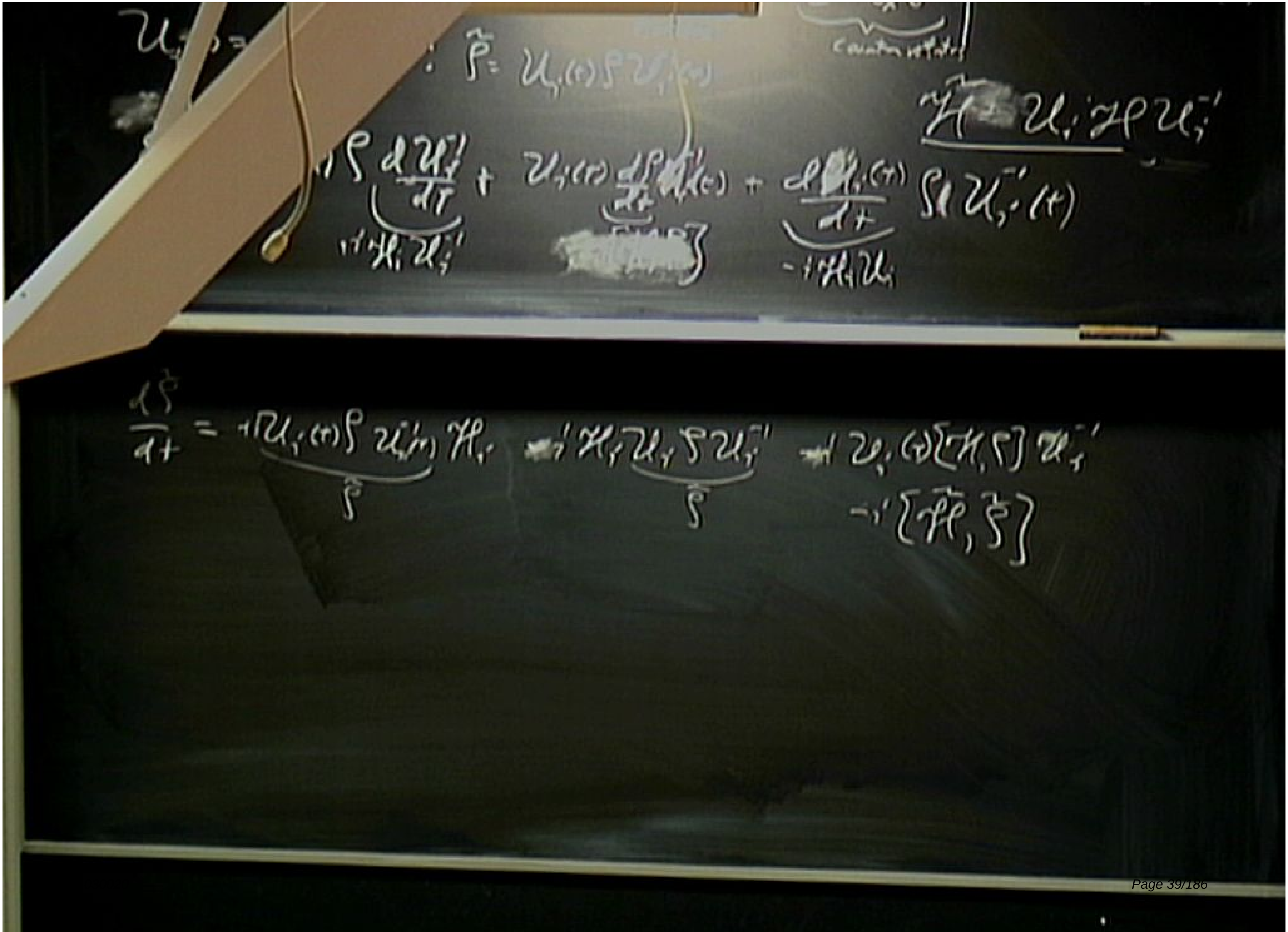

Wave Approximation
Rotating
Counter rotating










ρ̃ = U_i(t) ρ U_i^{-1}(t)
H̃ = U_i H U_i^{-1}
dρ̃/dt = -i[H̃, ρ̃]

$$\tilde{\rho} = U_i(t)\,\rho\,U_i^{-1}(t)$$

(counter-rotate)

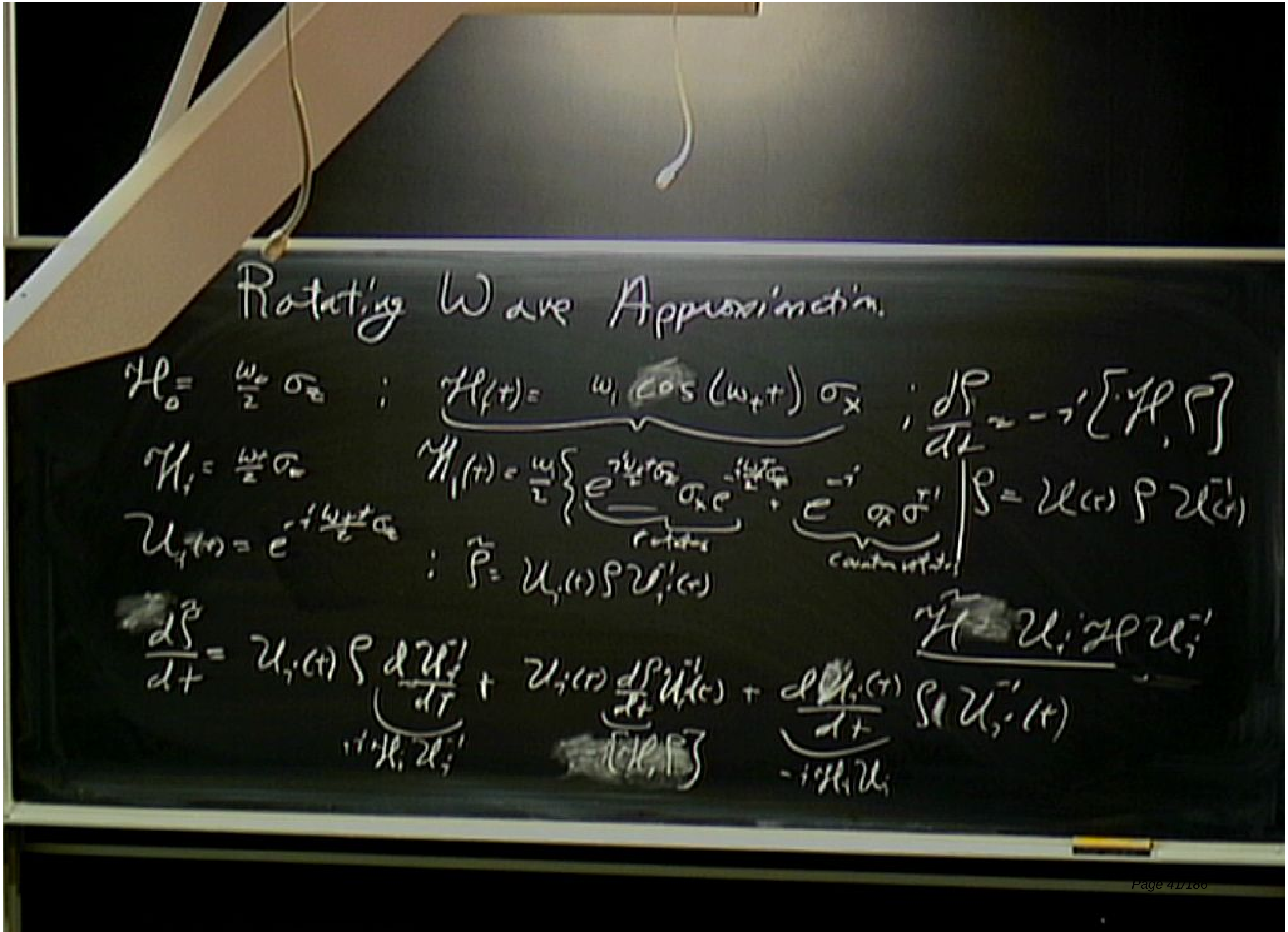
$$\tilde{H} = U_i H U_i^{-1}$$

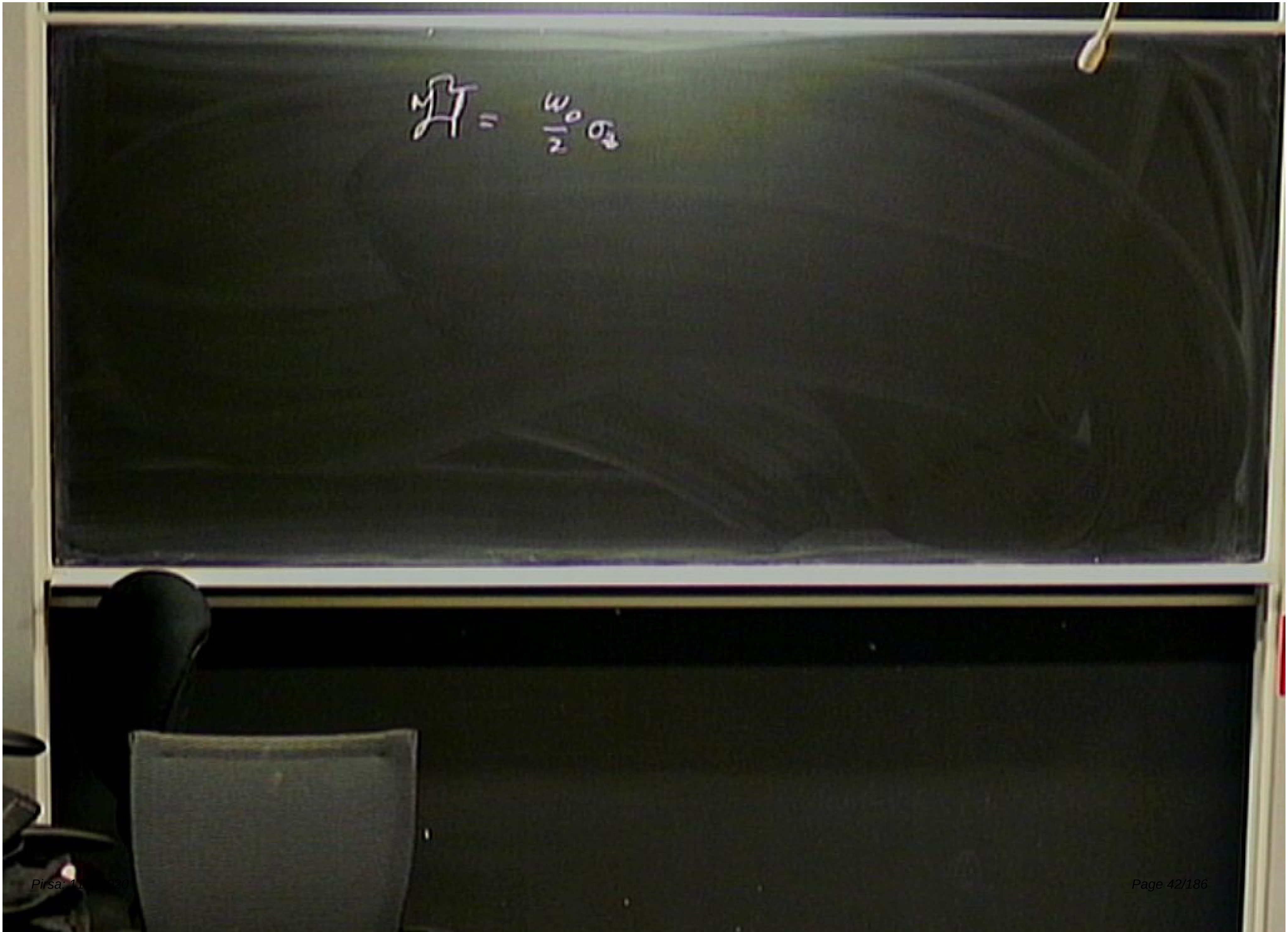
$$U_i(t)\,\rho\,\underbrace{\frac{dU_i^{-1}}{dt}}_{iH_i U_i^{-1}} + U_i(t)\,\frac{d\rho}{dt}\,U_i^{-1}(t) + \underbrace{\frac{dU_i(t)}{dt}}_{-iH_i U_i}\,\rho\,U_i^{-1}(t)$$

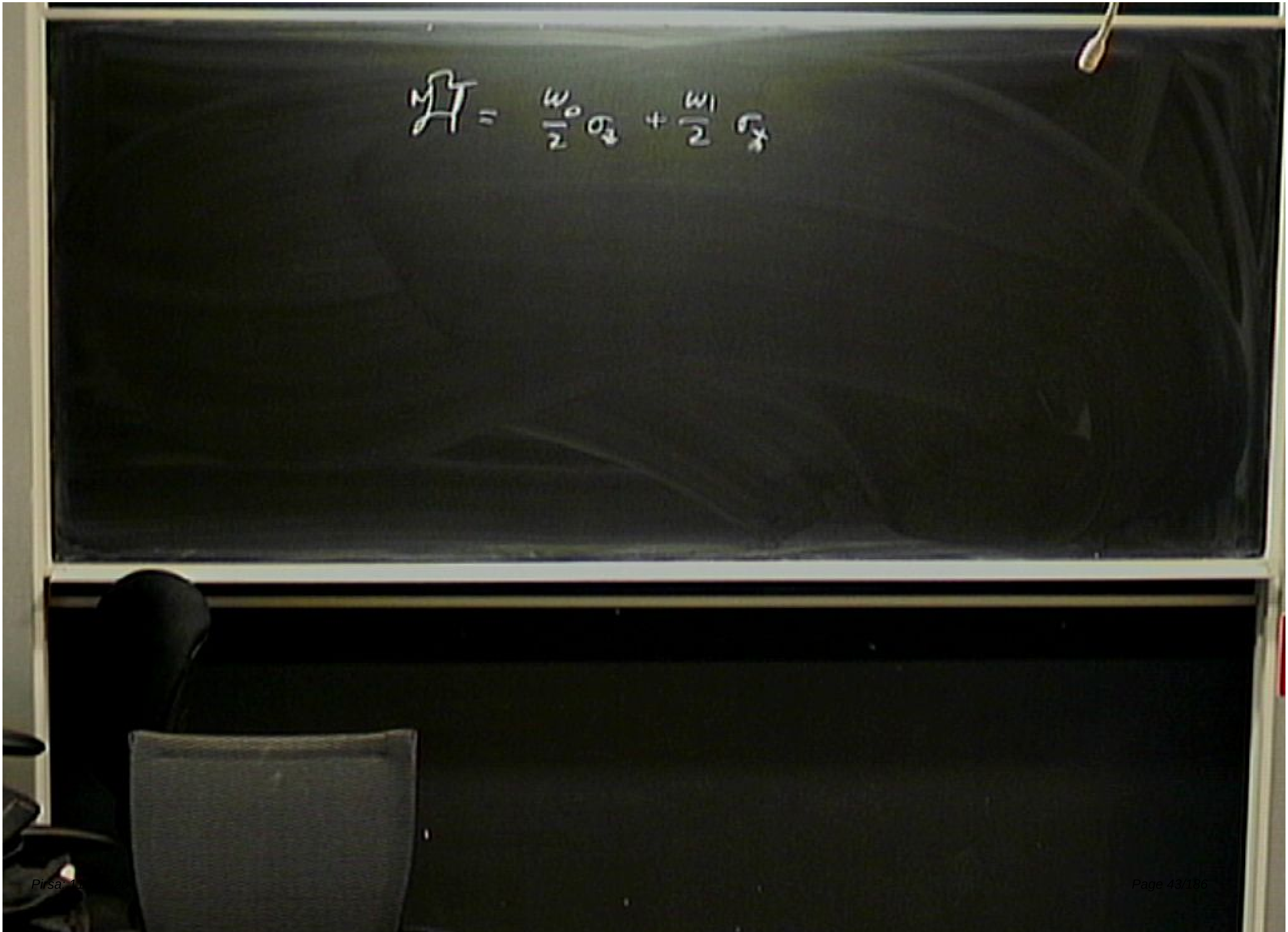
$$\frac{d\tilde{\rho}}{dt} = i\underbrace{U_i(t)\,\rho\,U_i^{-1}(t)}_{\tilde{\rho}}H_i - iH_i\underbrace{U_i\,\rho\,U_i^{-1}}_{\tilde{\rho}} - iU_i(t)[H,\rho]U_i^{-1}$$

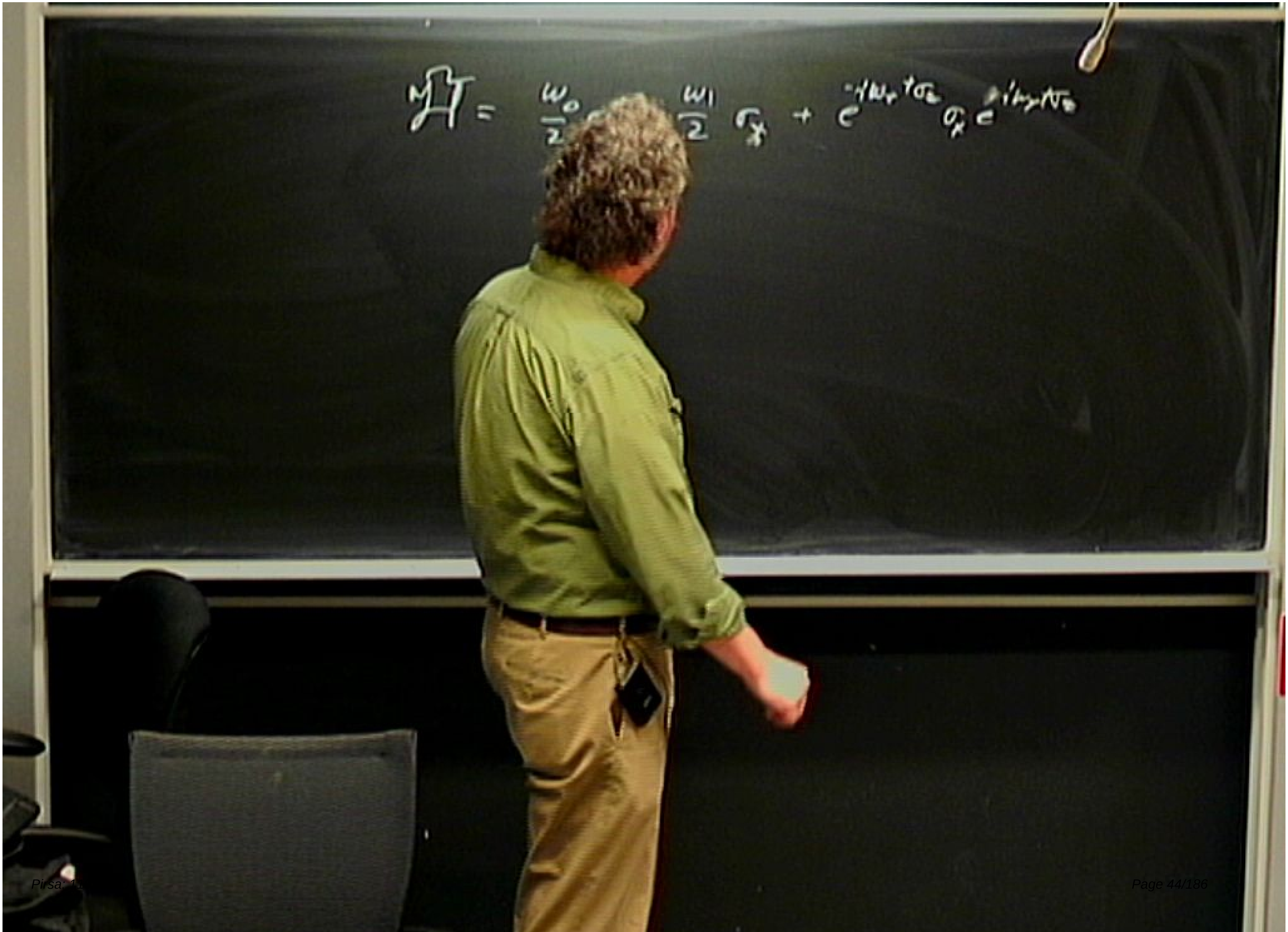
$$= -i[\tilde{H},\tilde{\rho}]$$

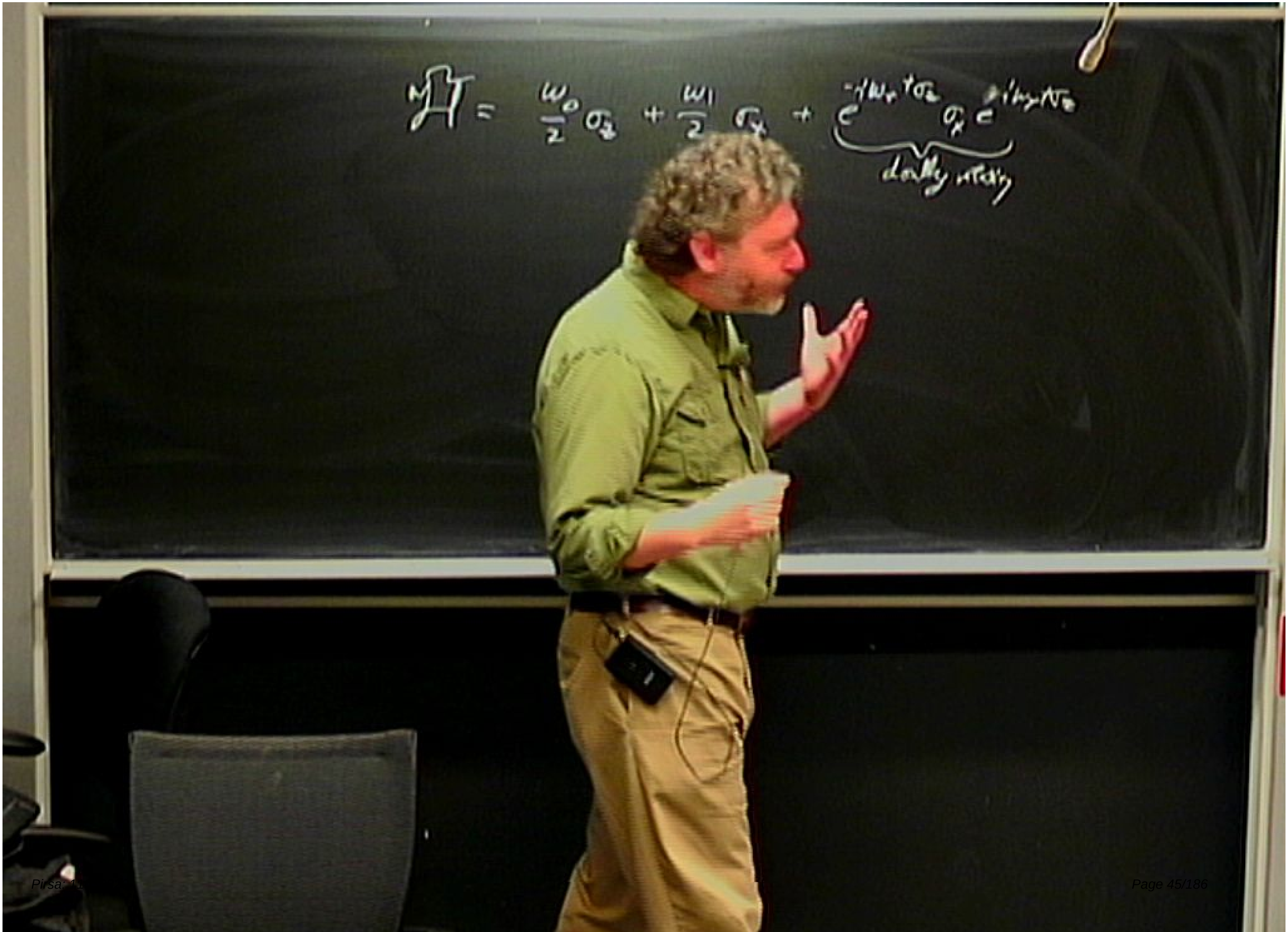
$$\boxed{\frac{d\tilde{\rho}}{dt} = -i[\tilde{H} - H_i, \tilde{\rho}]}$$


Rotating Wave Approximation
$\mathcal{H}_0 = \frac{\omega_0}{2}\sigma_z$; $\mathcal{H}_1(t) = \omega_1 \cos(\omega_1 t)\,\sigma_x$; $\frac{d\rho}{dt} = -i[\mathcal{H},\rho]$
$\mathcal{H}_1 = \frac{\omega_0}{2}\sigma_z$
$\mathcal{H}_1(t) = \frac{\omega_1}{2}\{ e^{i\frac{\omega_1 t}{2}\sigma_z}\sigma_x e^{-i\frac{\omega_1 t}{2}\sigma_z} + e^{-i}\sigma_x\sigma^{i} \}$
rotating
counter rotating
$\rho = \mathcal{U}(t)\,\rho\,\mathcal{U}^{-1}(t)$
$\mathcal{U}_1(t) = e^{-i\frac{\omega_1 t}{2}\sigma_z}$; $\tilde{\rho} = \mathcal{U}_1(t)\,\rho\,\mathcal{U}_1^{-1}(t)$
$\tilde{\mathcal{H}} = \mathcal{U}_1 \mathcal{H} \mathcal{U}_1^{-1}$
$\frac{d\tilde{\rho}}{dt} = \mathcal{U}_1(t)\,\rho\,\frac{d\mathcal{U}_1^{-1}}{dt} + \mathcal{U}_1(t)\frac{d\rho}{dt}\mathcal{U}_1^{-1}(t) + \frac{d\mathcal{U}_1(t)}{dt}\rho\,\mathcal{U}_1^{-1}(t)$
$i\mathcal{H}_1\mathcal{U}_1^{-1}$
$[\mathcal{H},\rho]$
$-i\mathcal{H}_1\mathcal{U}_1$


$\mathcal{H} = \frac{\omega_0}{2}\sigma_z$

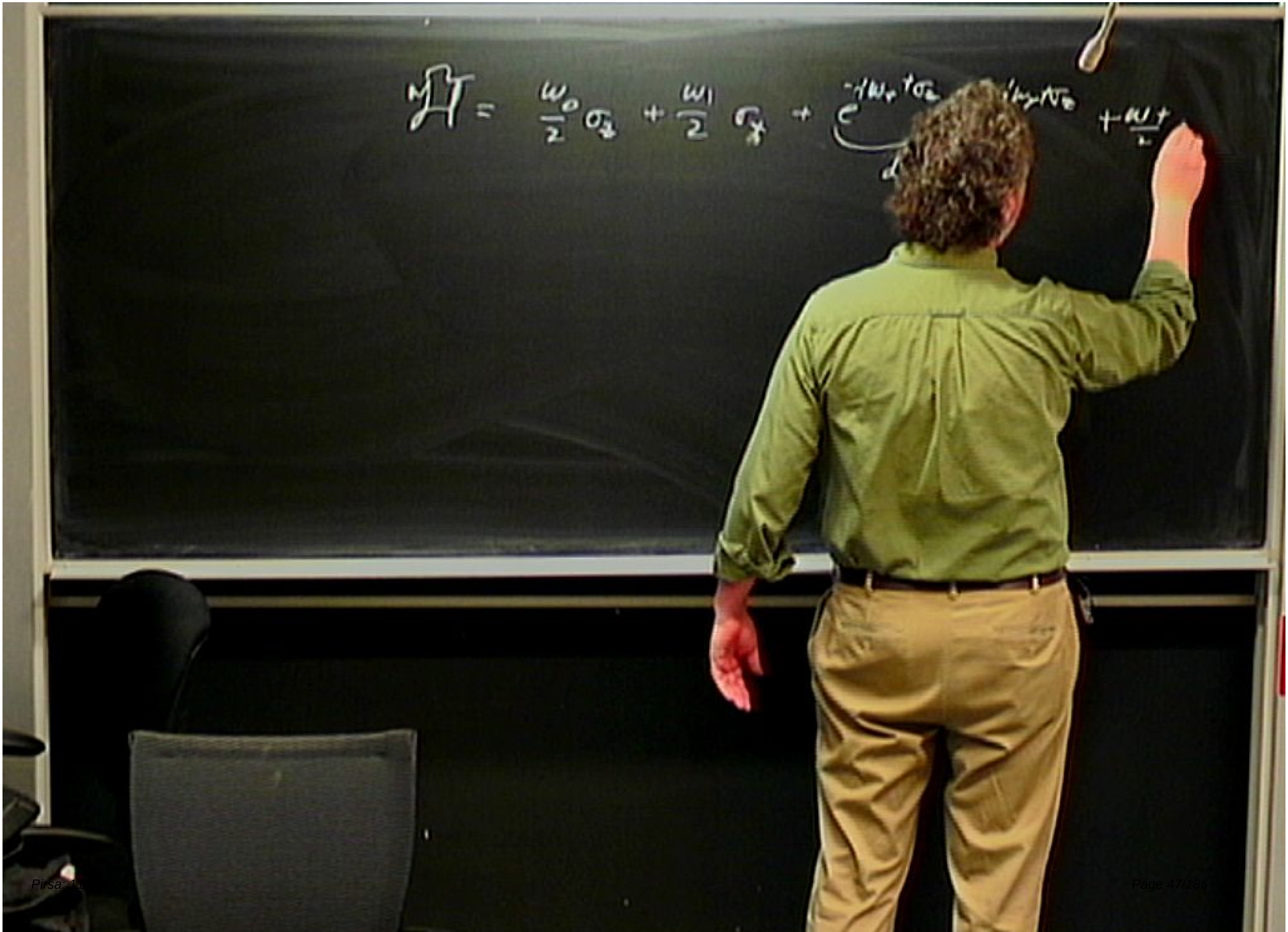

$\mathcal{H} = \frac{\omega_0}{2}\sigma_z + \frac{\omega_1}{2}\sigma_x$

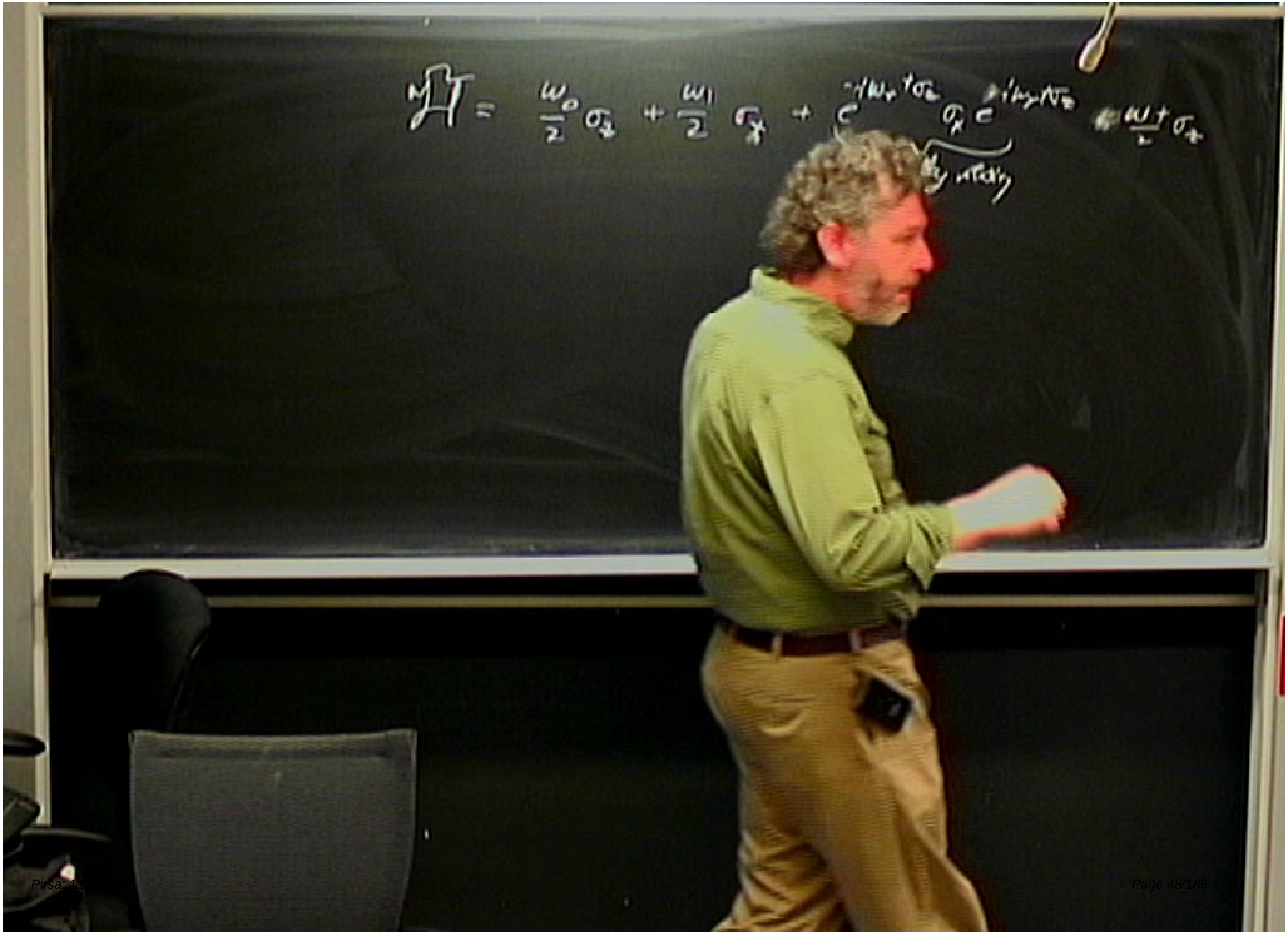


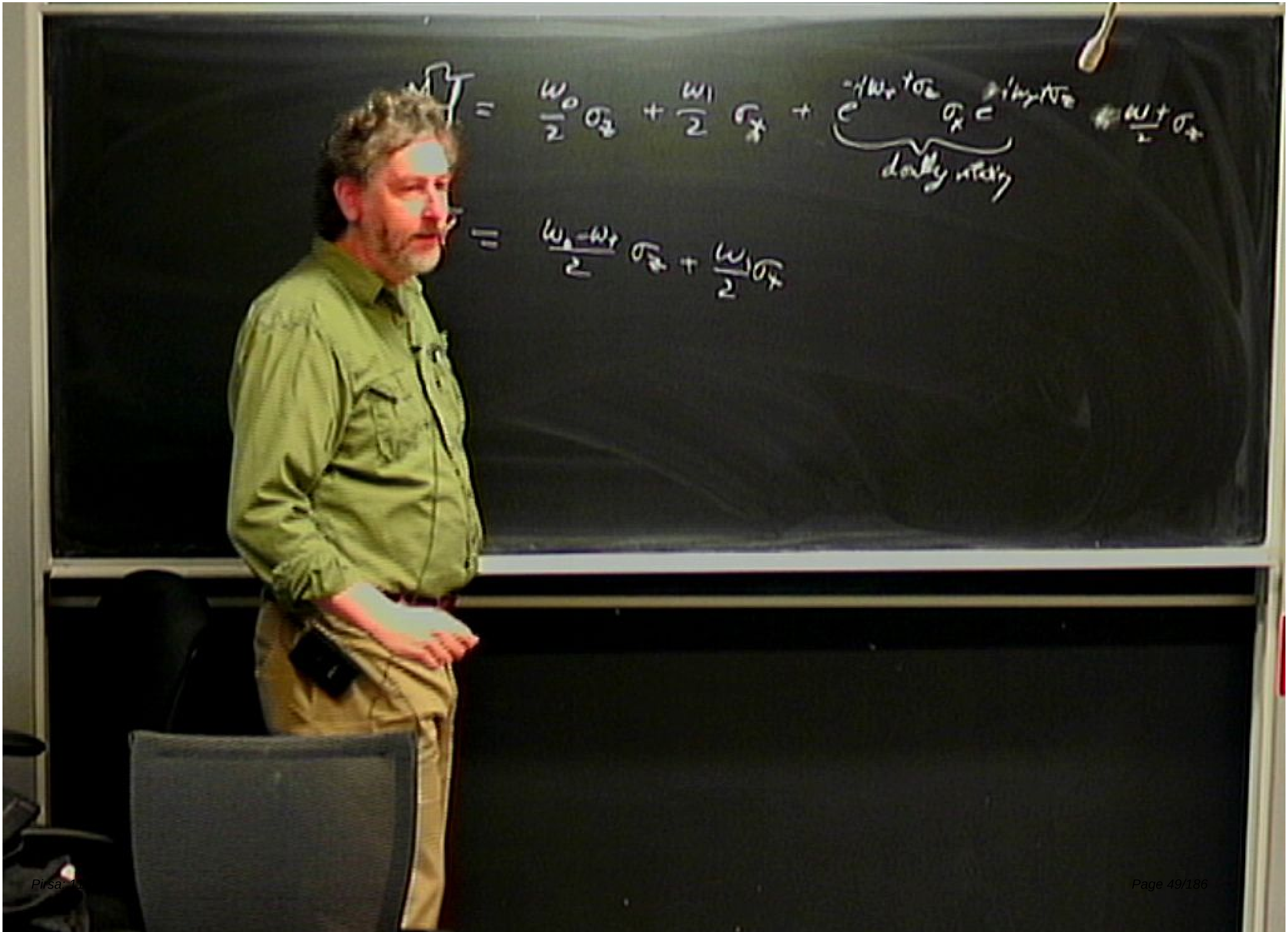


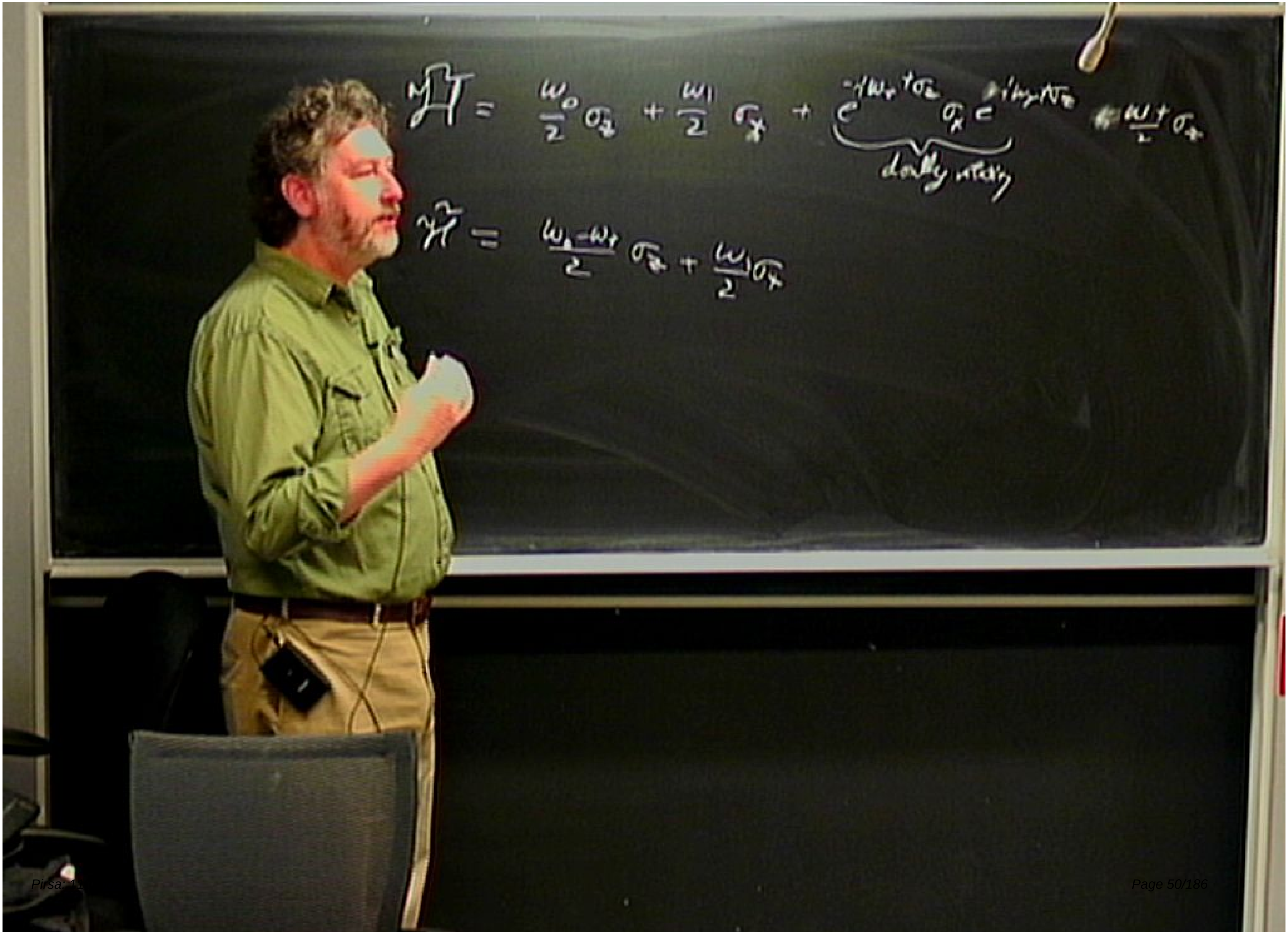
$\mathcal{H} = \frac{\omega_0}{2}\sigma_z + \frac{\omega_1}{2}\sigma_x +$

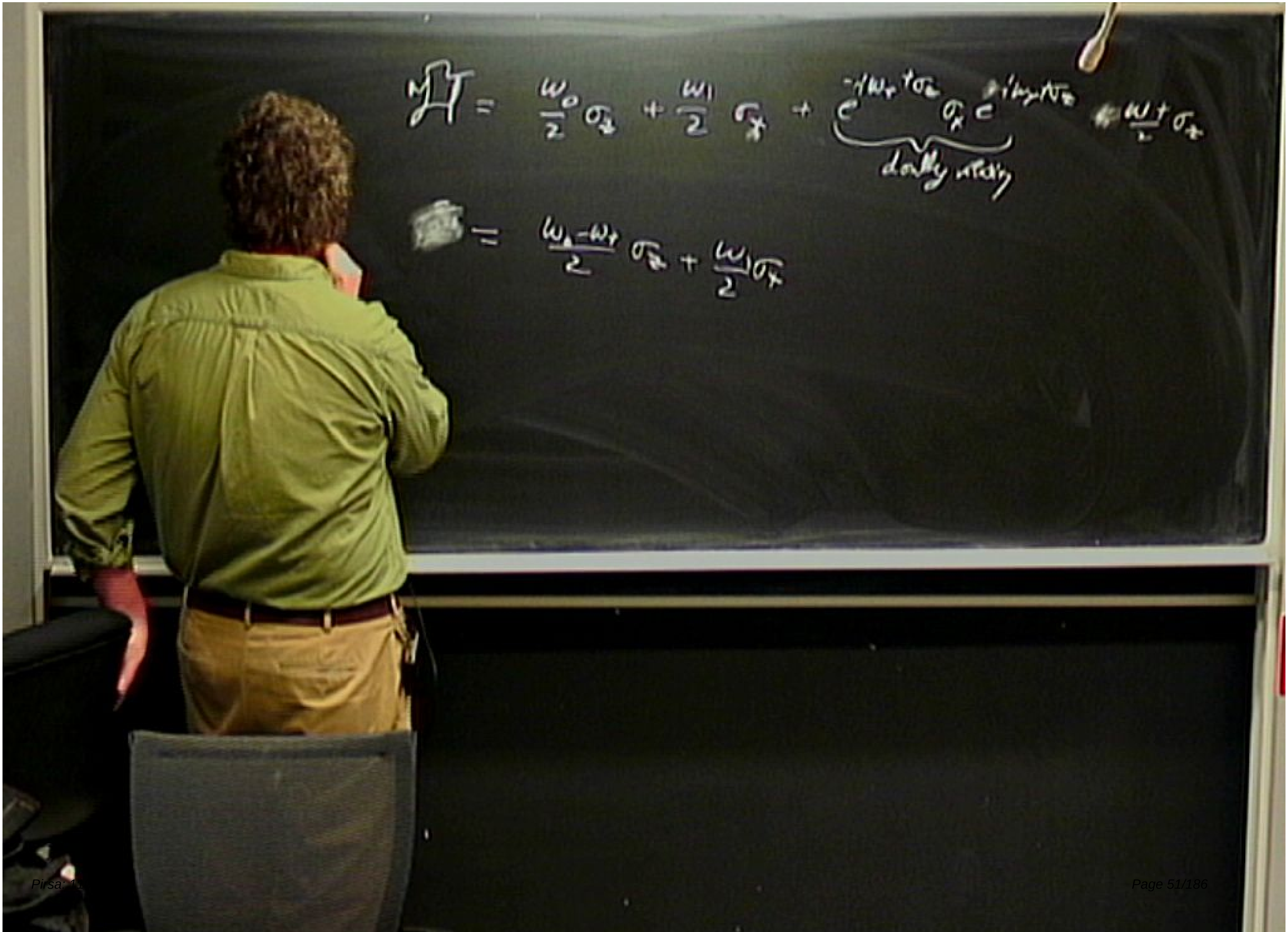
$$\mathcal{H} = \frac{\omega_0}{2}\sigma_z + \frac{\omega_1}{2}\sigma_x + \underbrace{e^{-i\omega_r t\sigma_z}\sigma_x e^{i\omega_r t\sigma_z}}_{\text{locally rotating}}$$

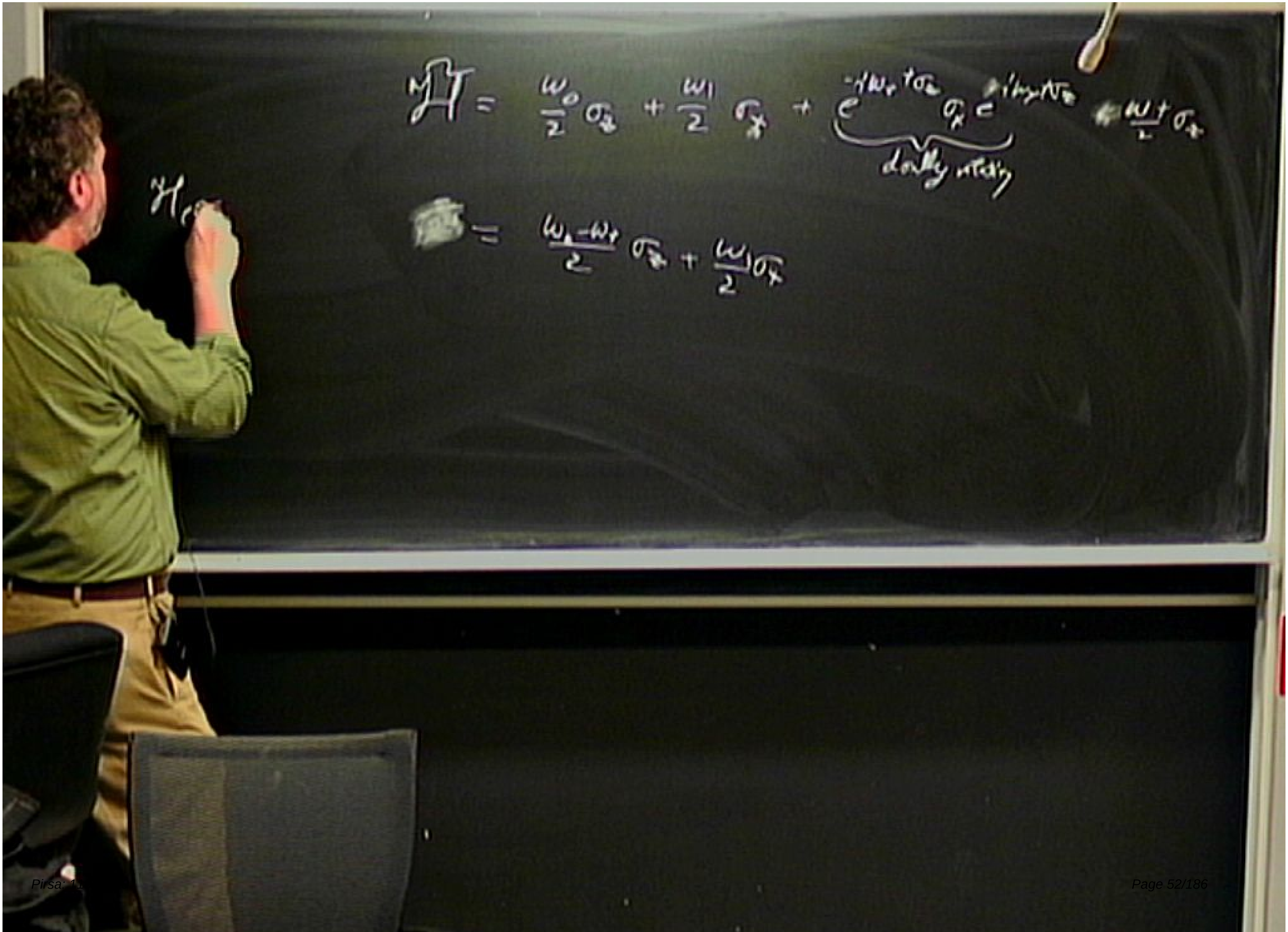


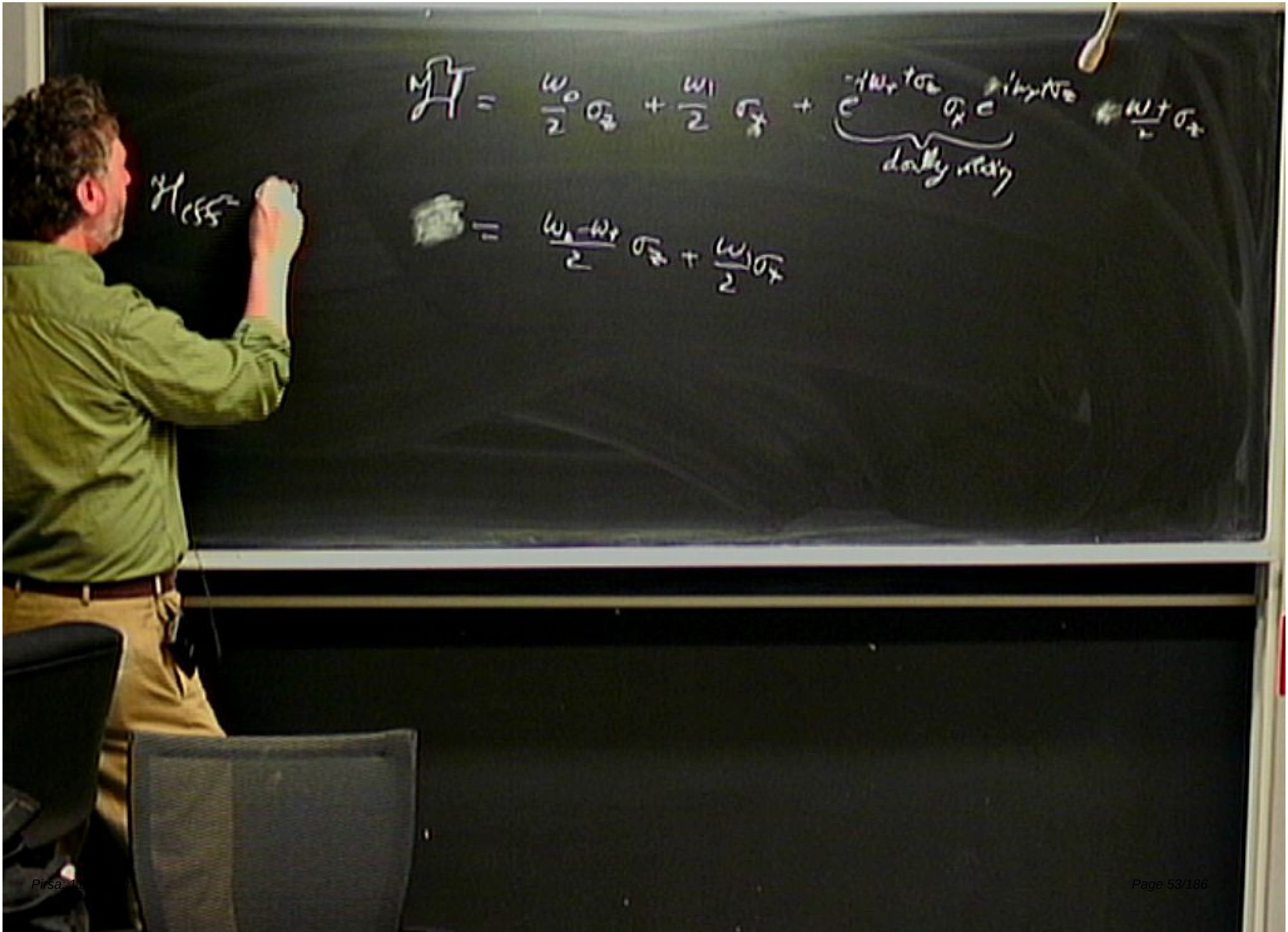


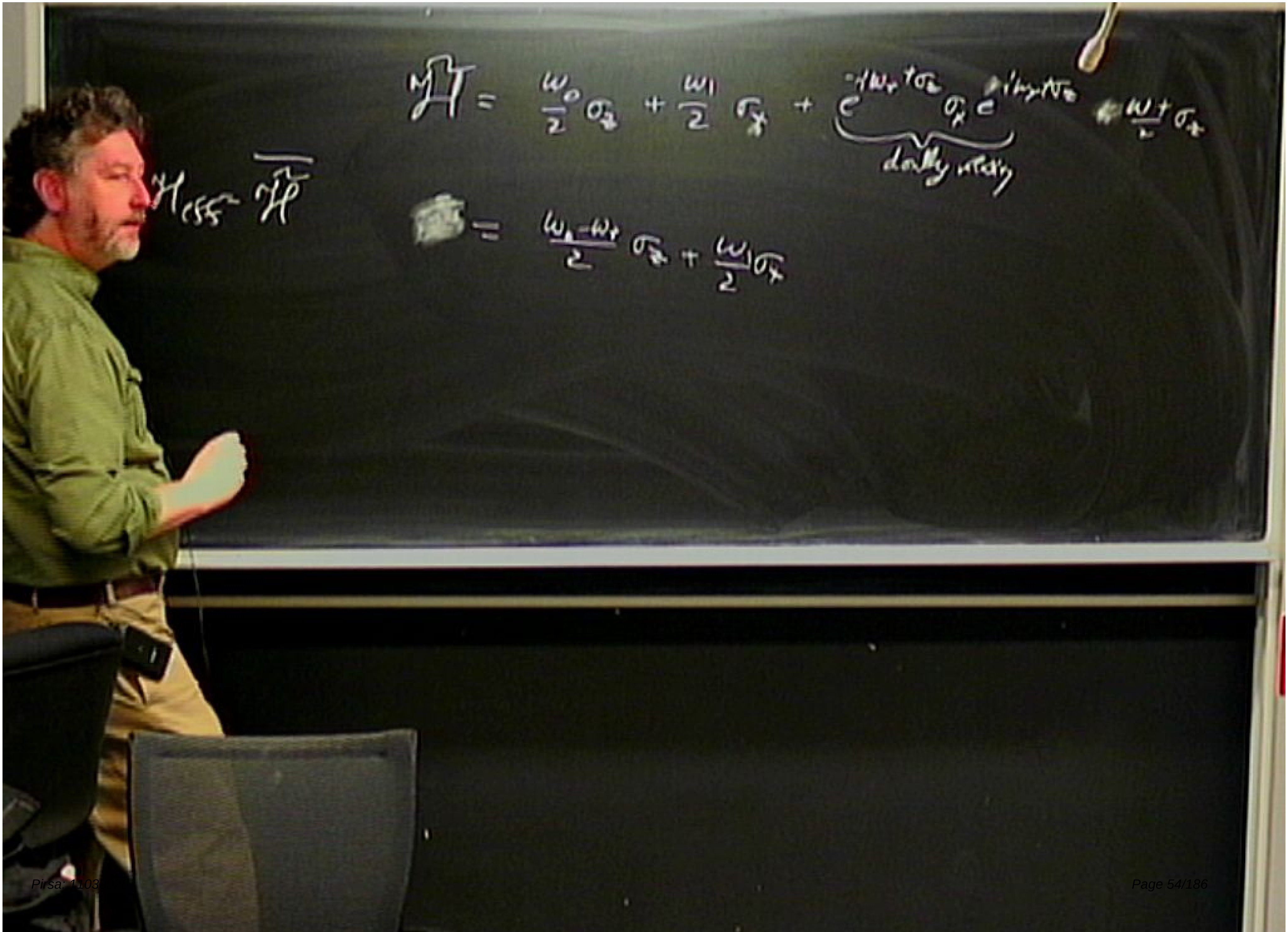


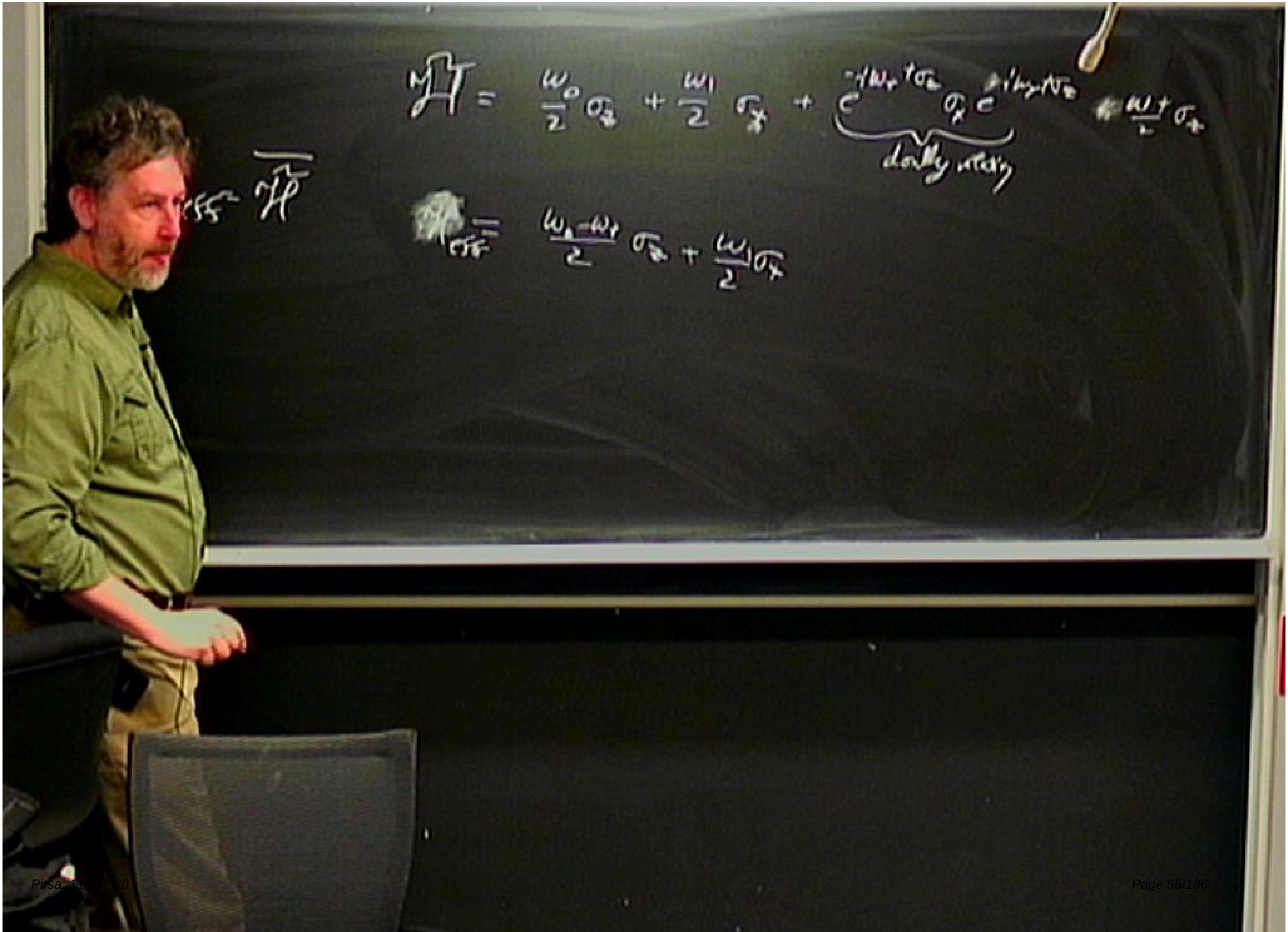


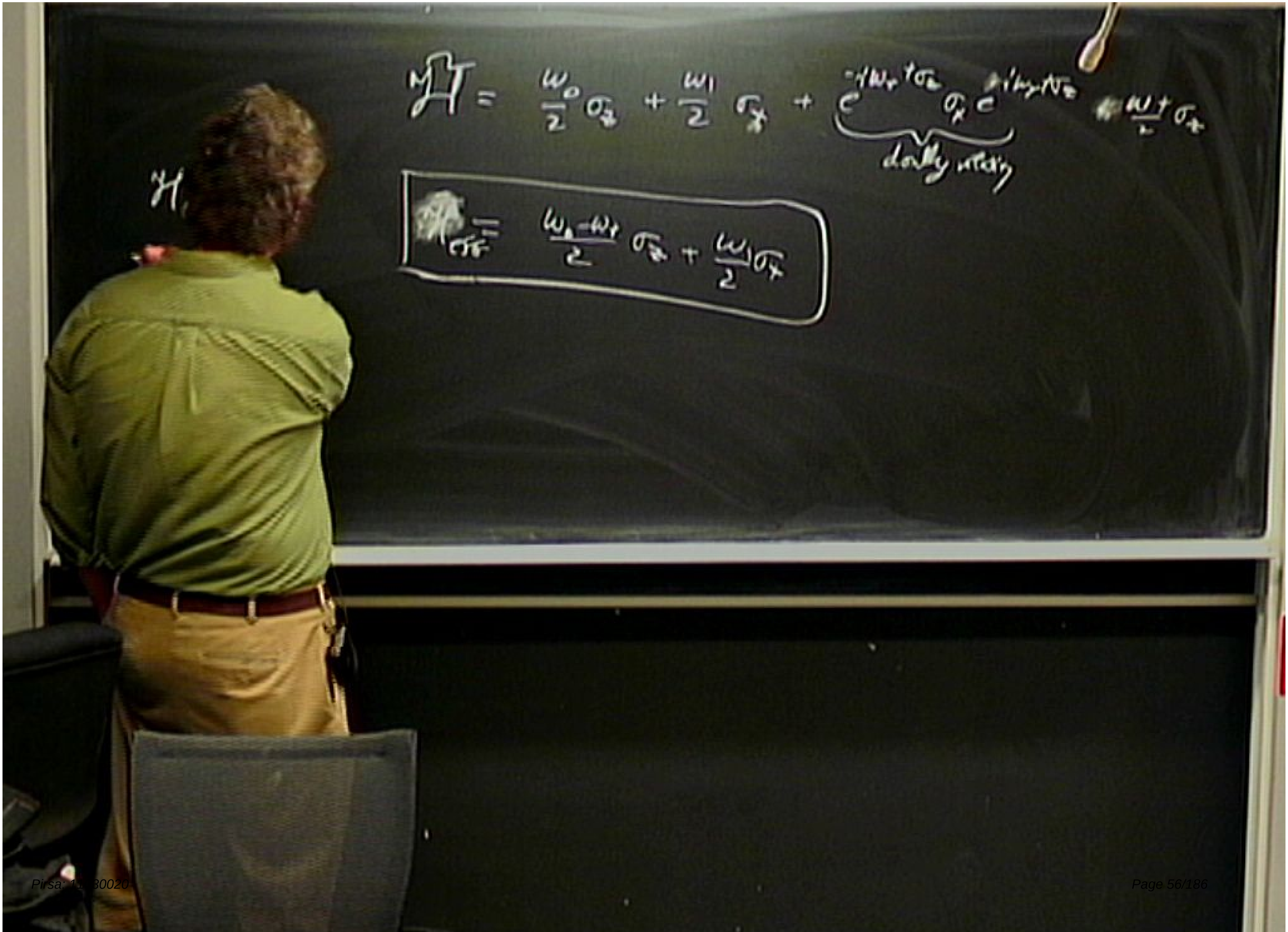

= (ω_0 − ω_r)/2 σ_z + ω_1/2 σ_x

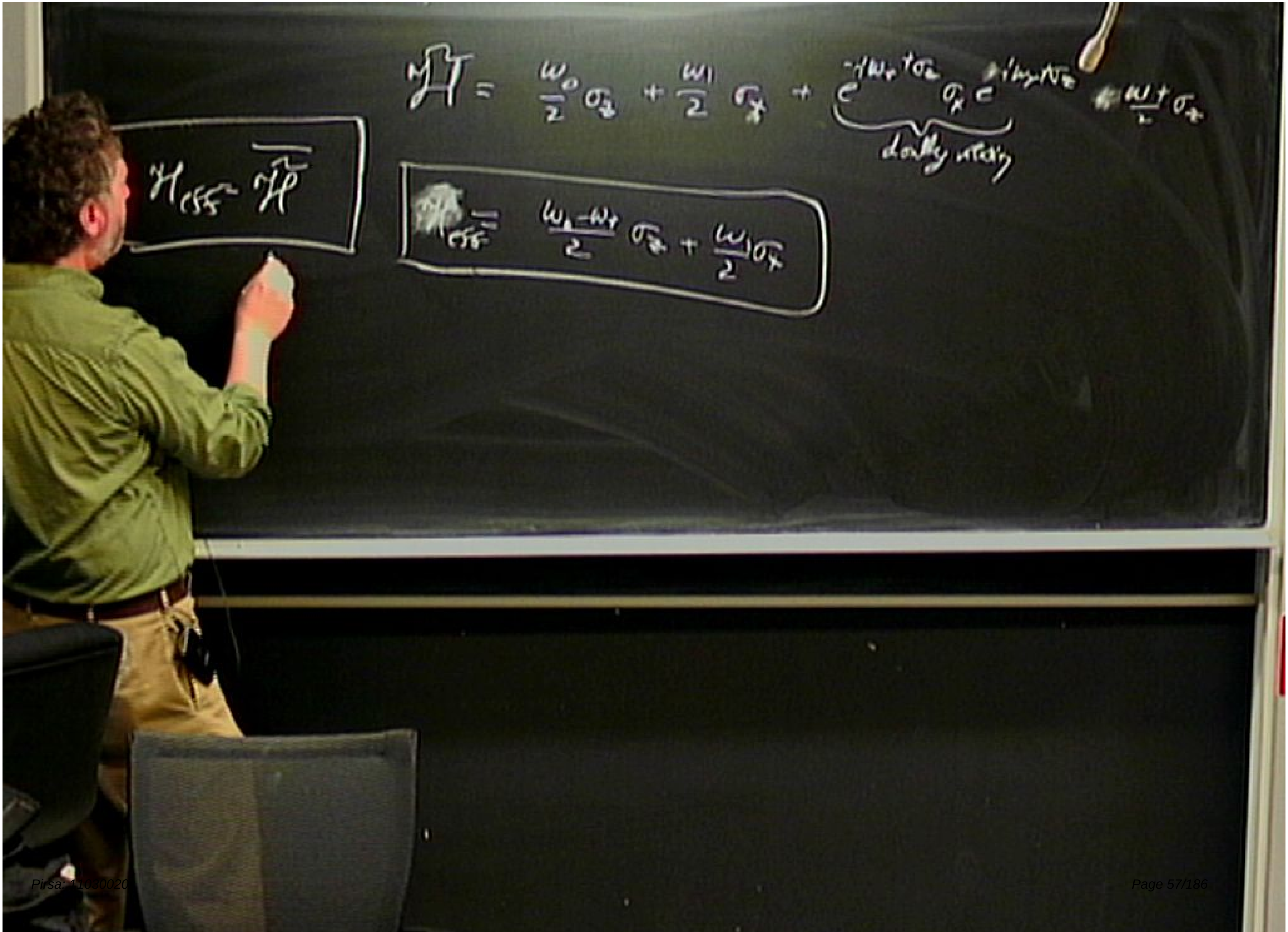

$= \frac{\omega_0-\omega_r}{2}\sigma_z + \frac{\omega_1}{2}\sigma_x$

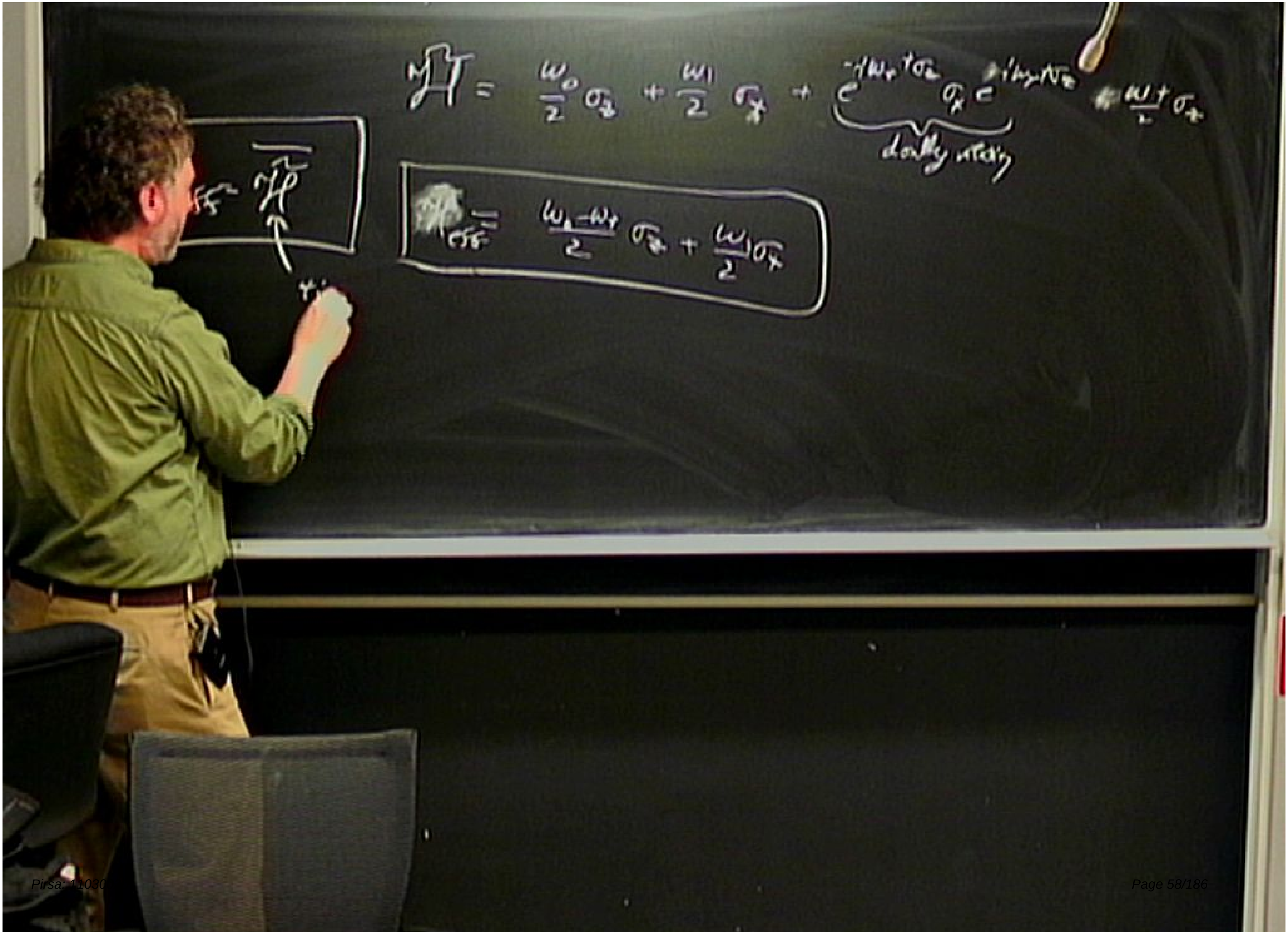


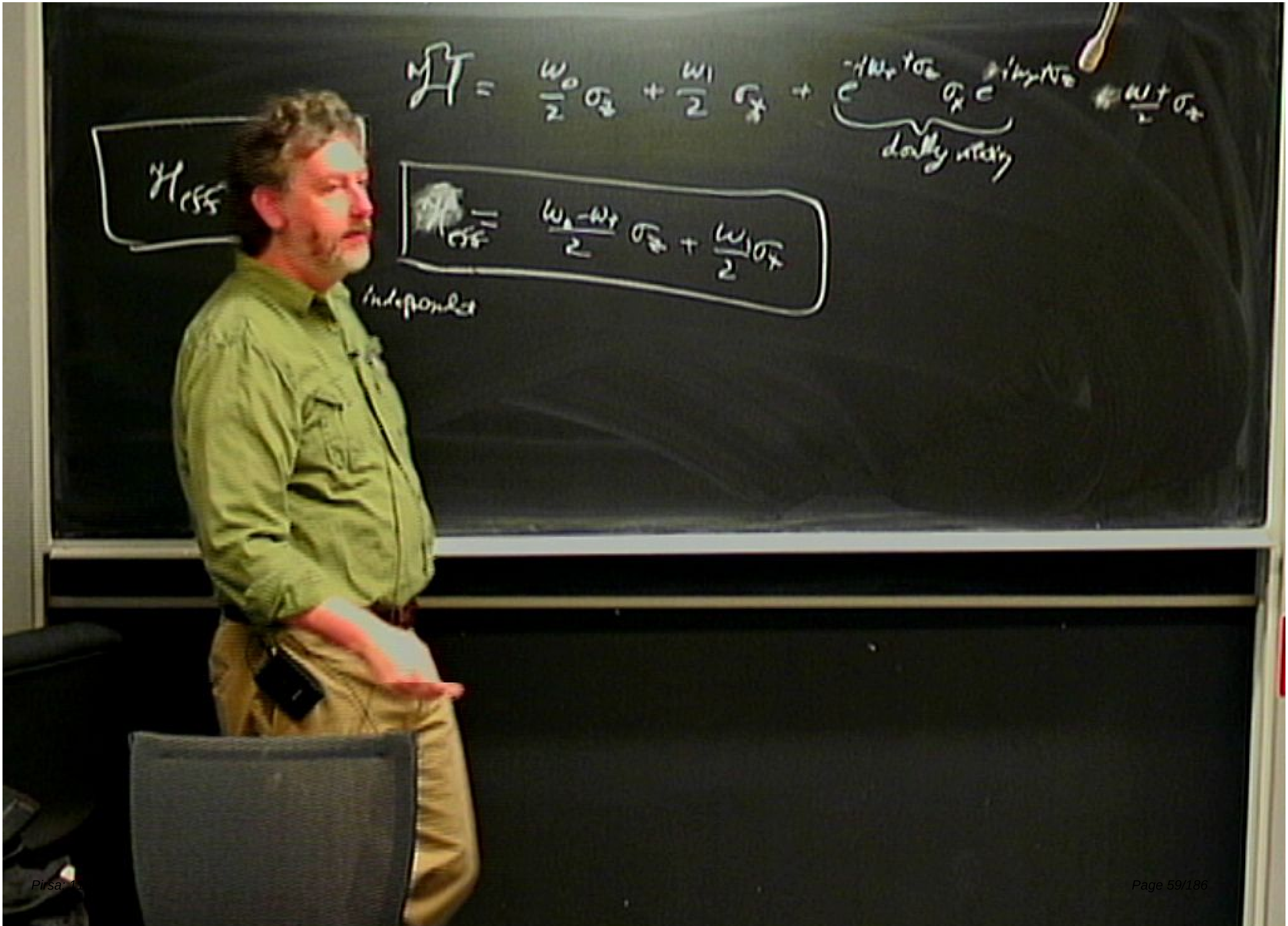


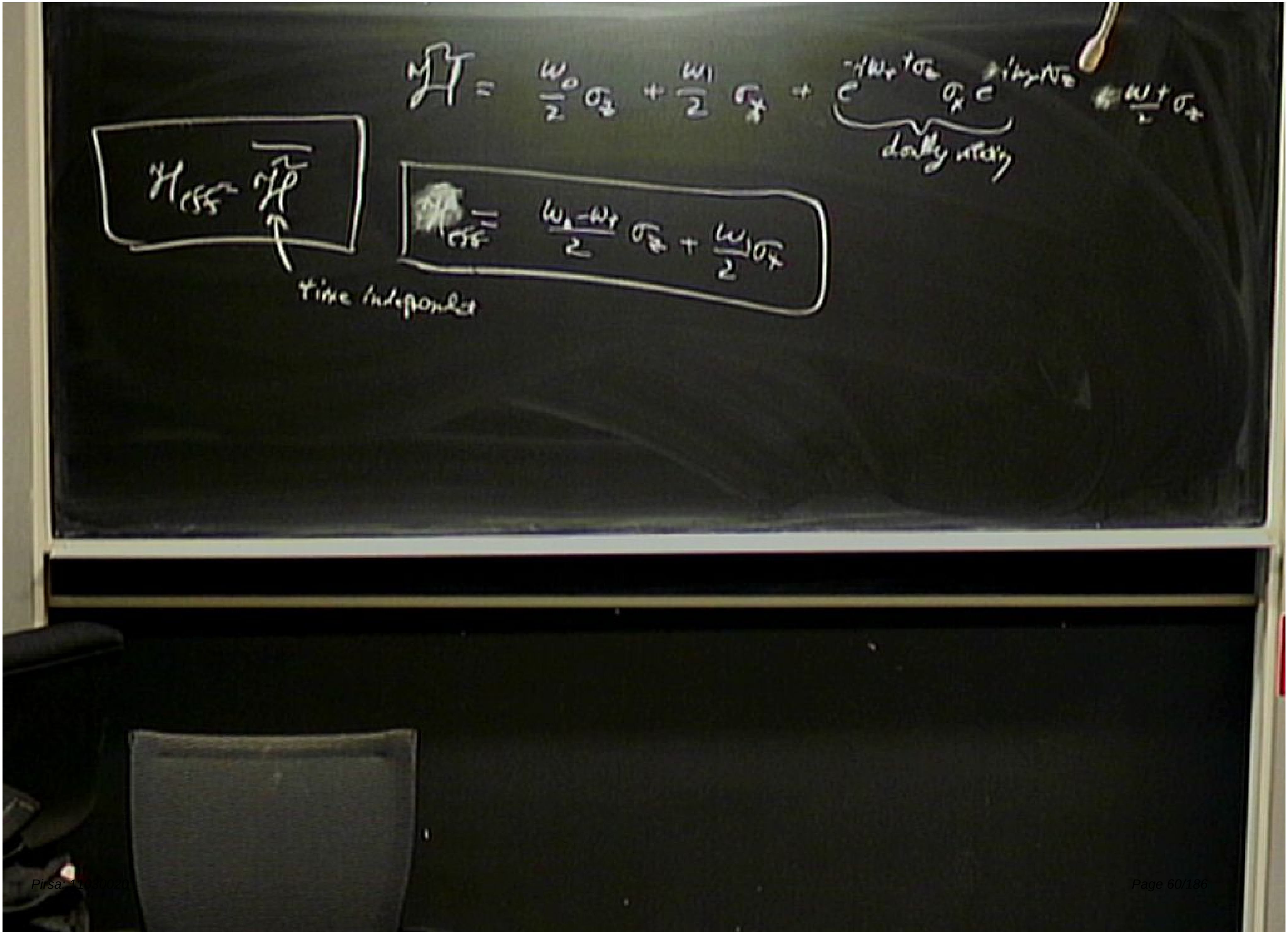


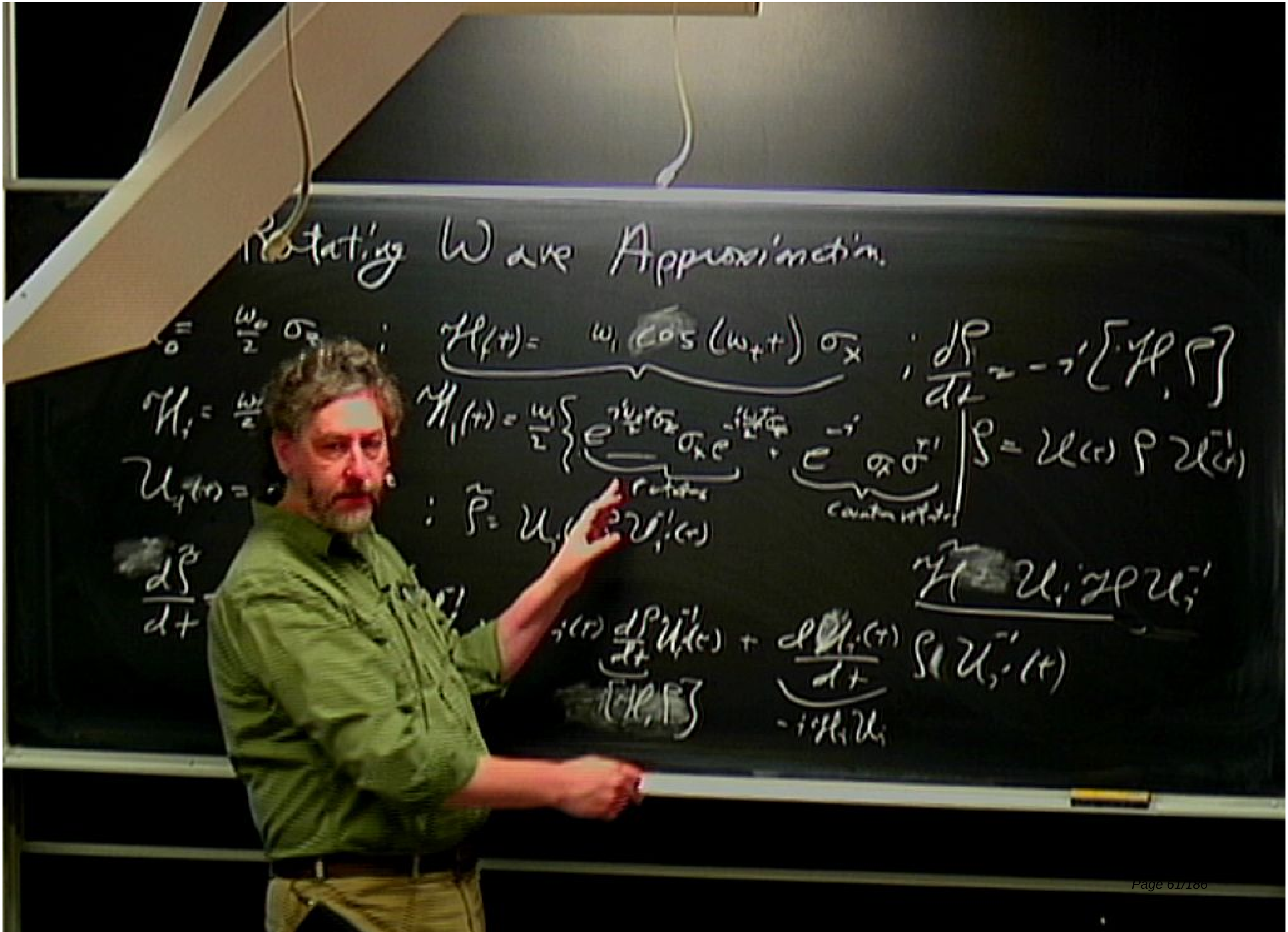


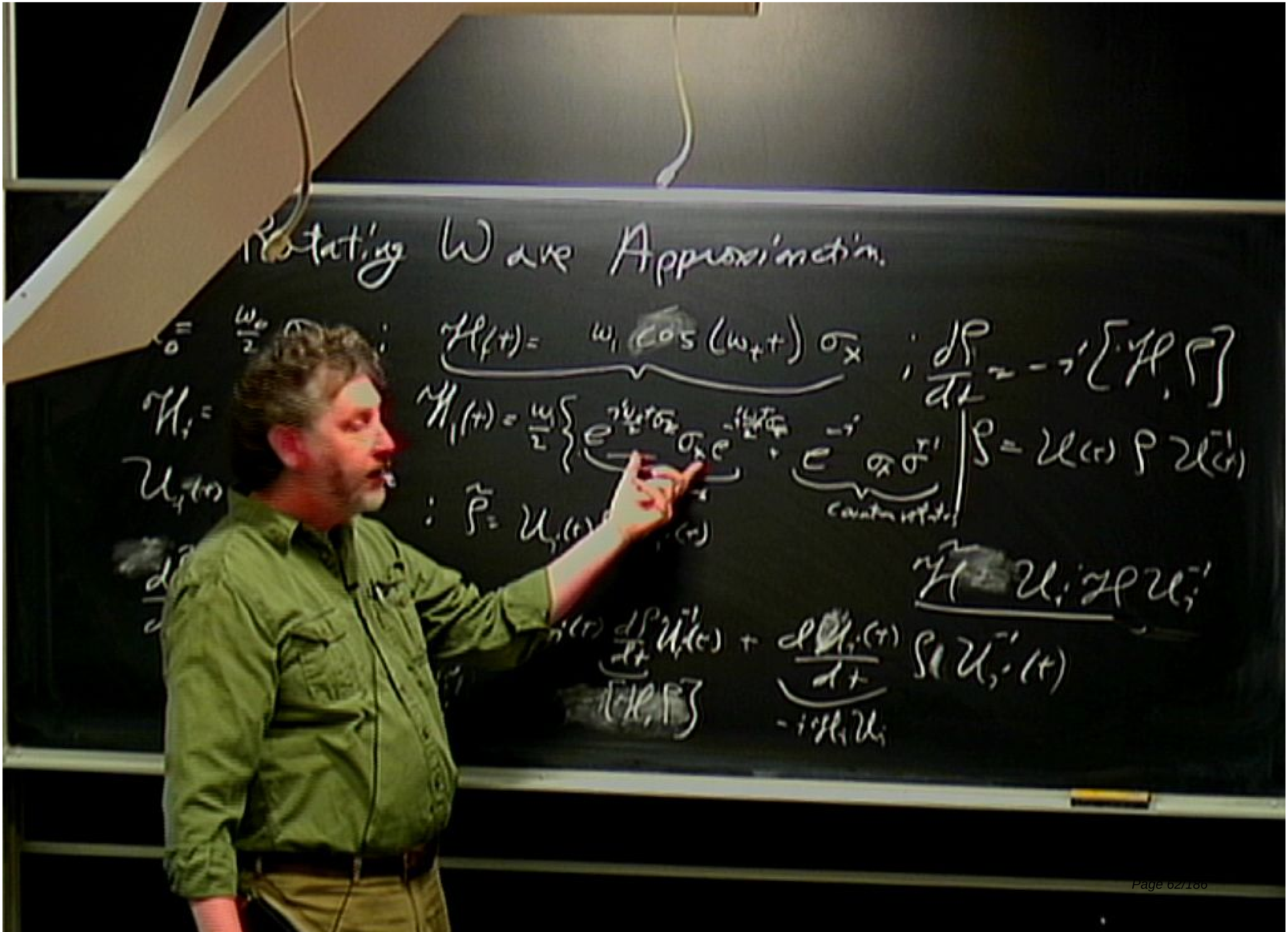


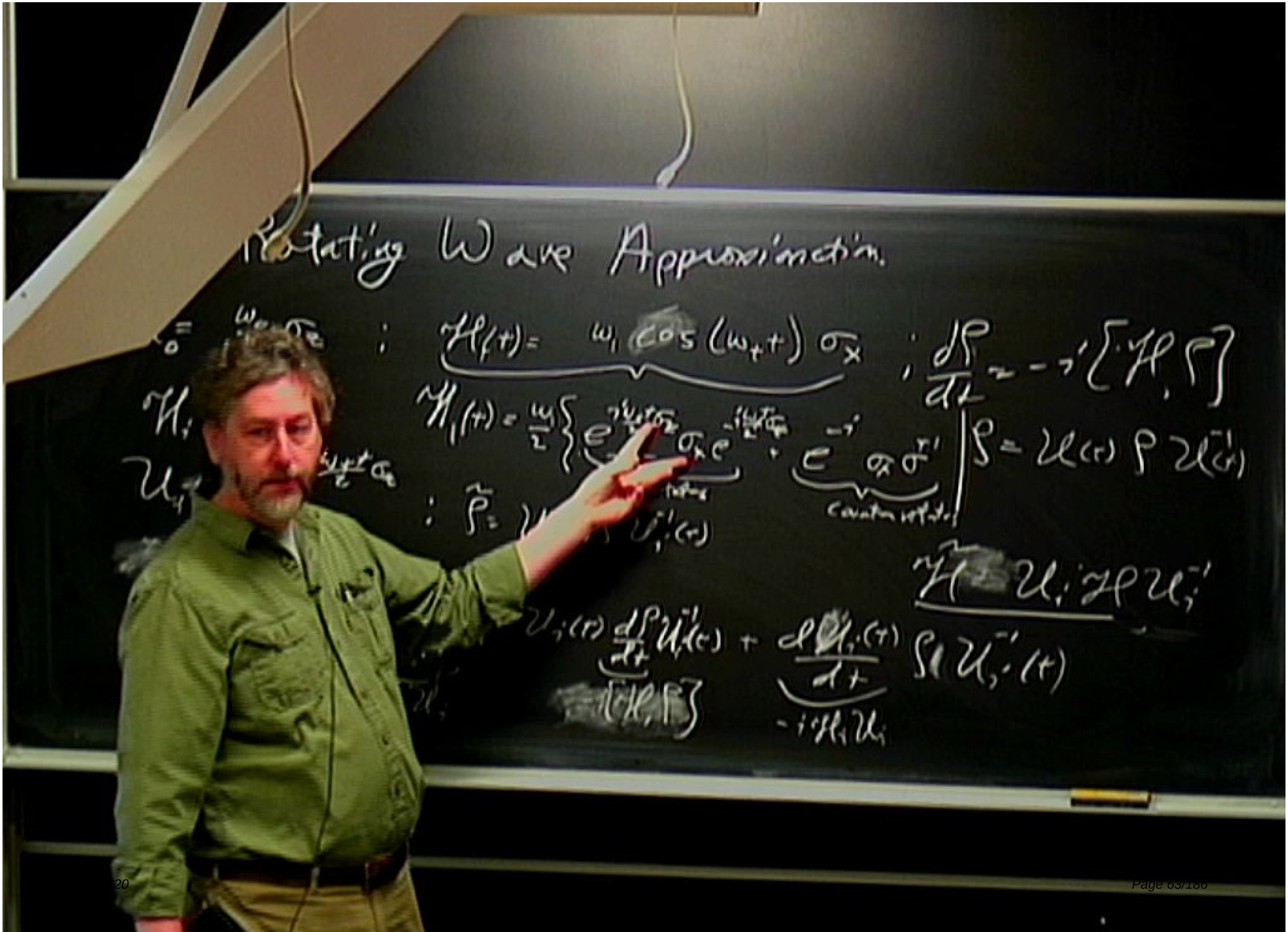


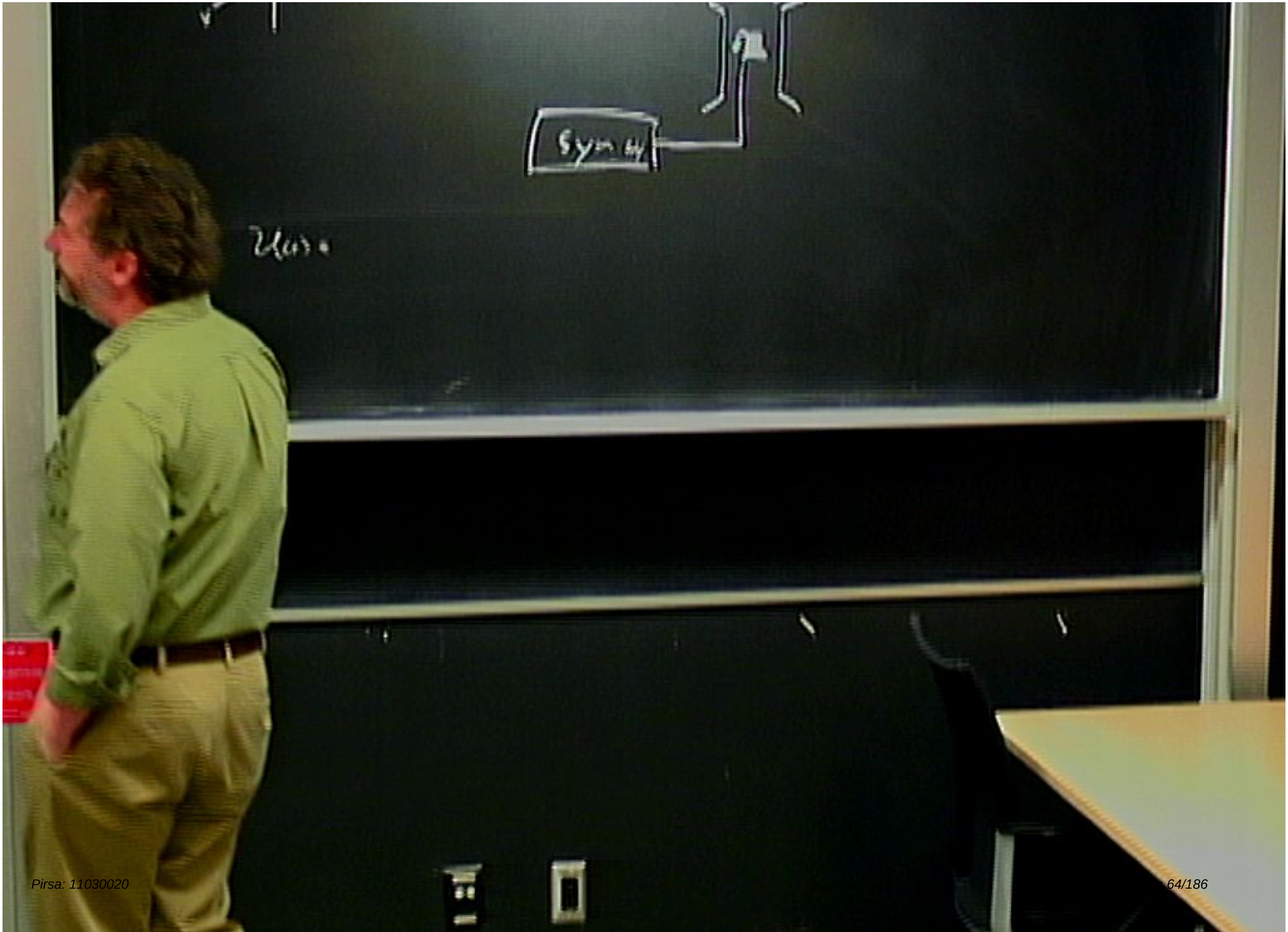


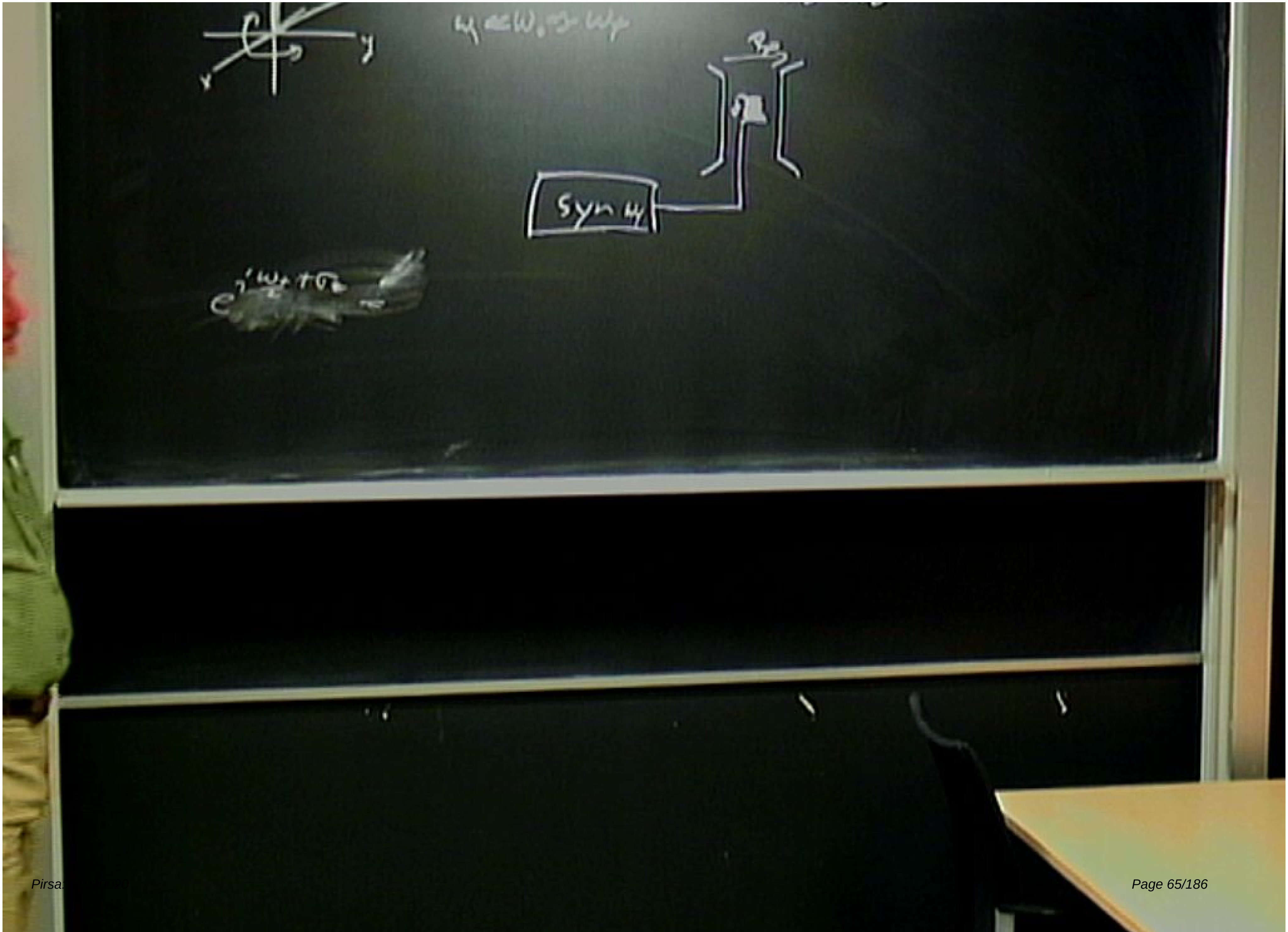

time independent

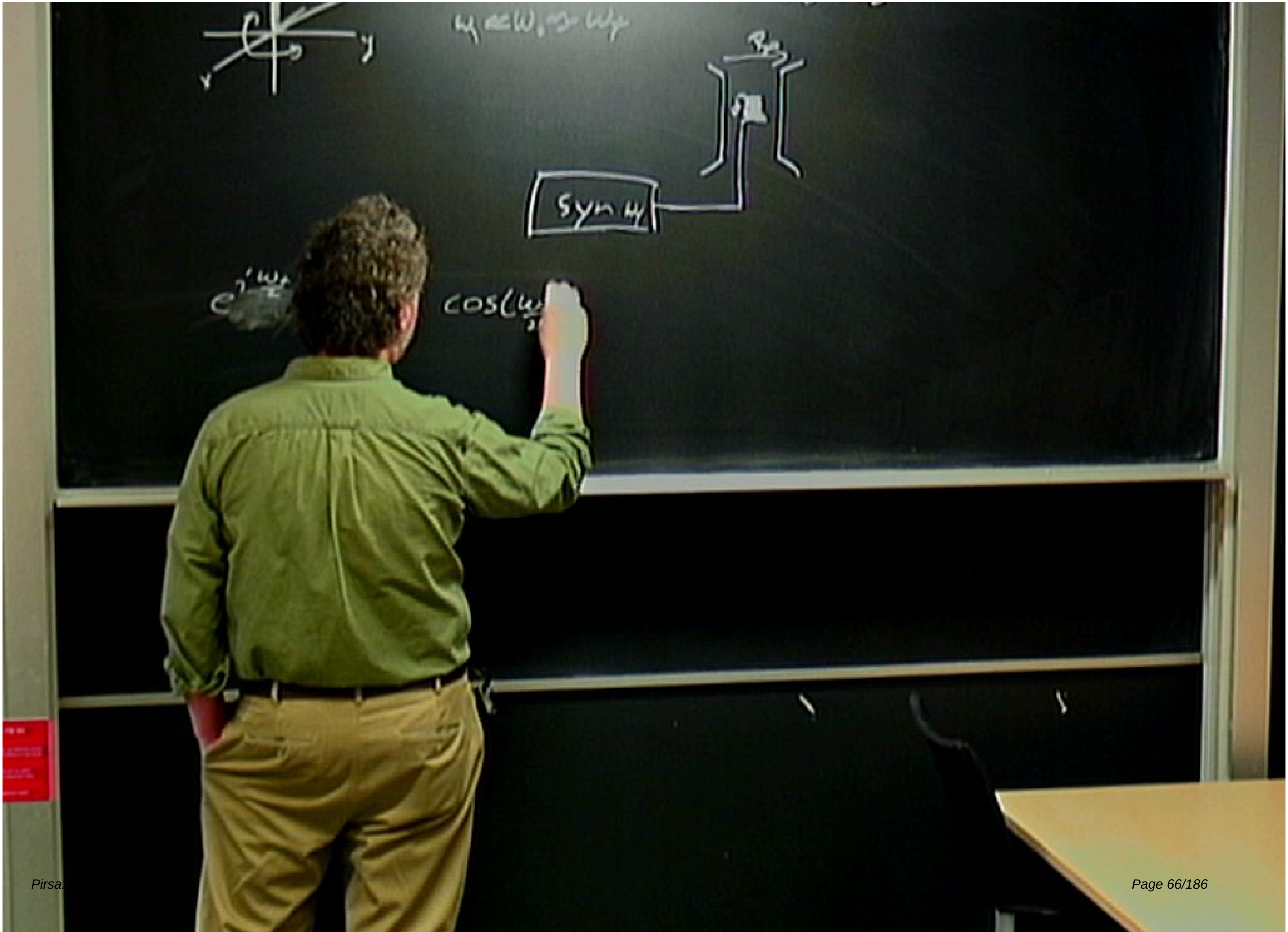

Rotating Wave Approximation

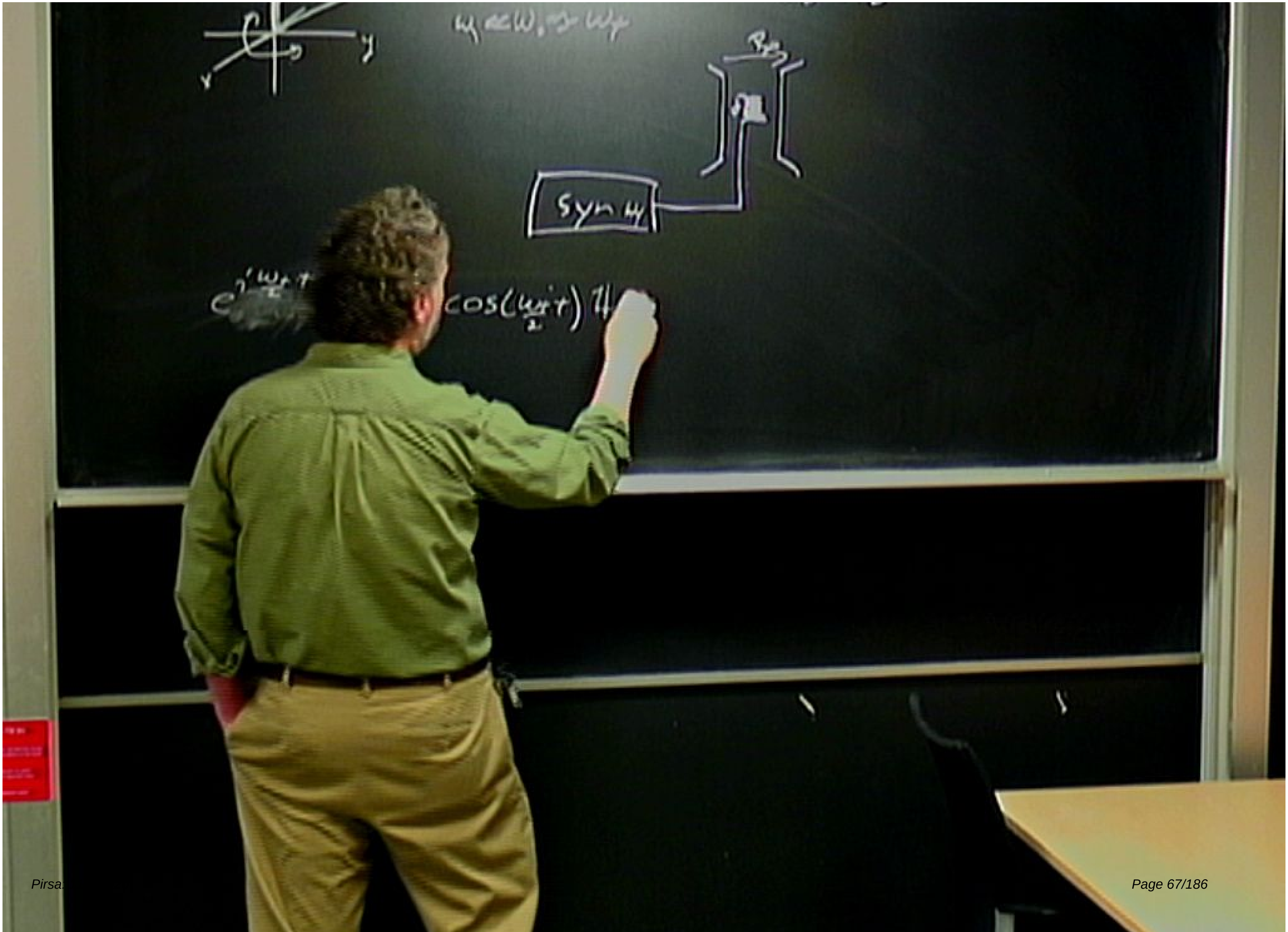

Rotating Wave Approximation

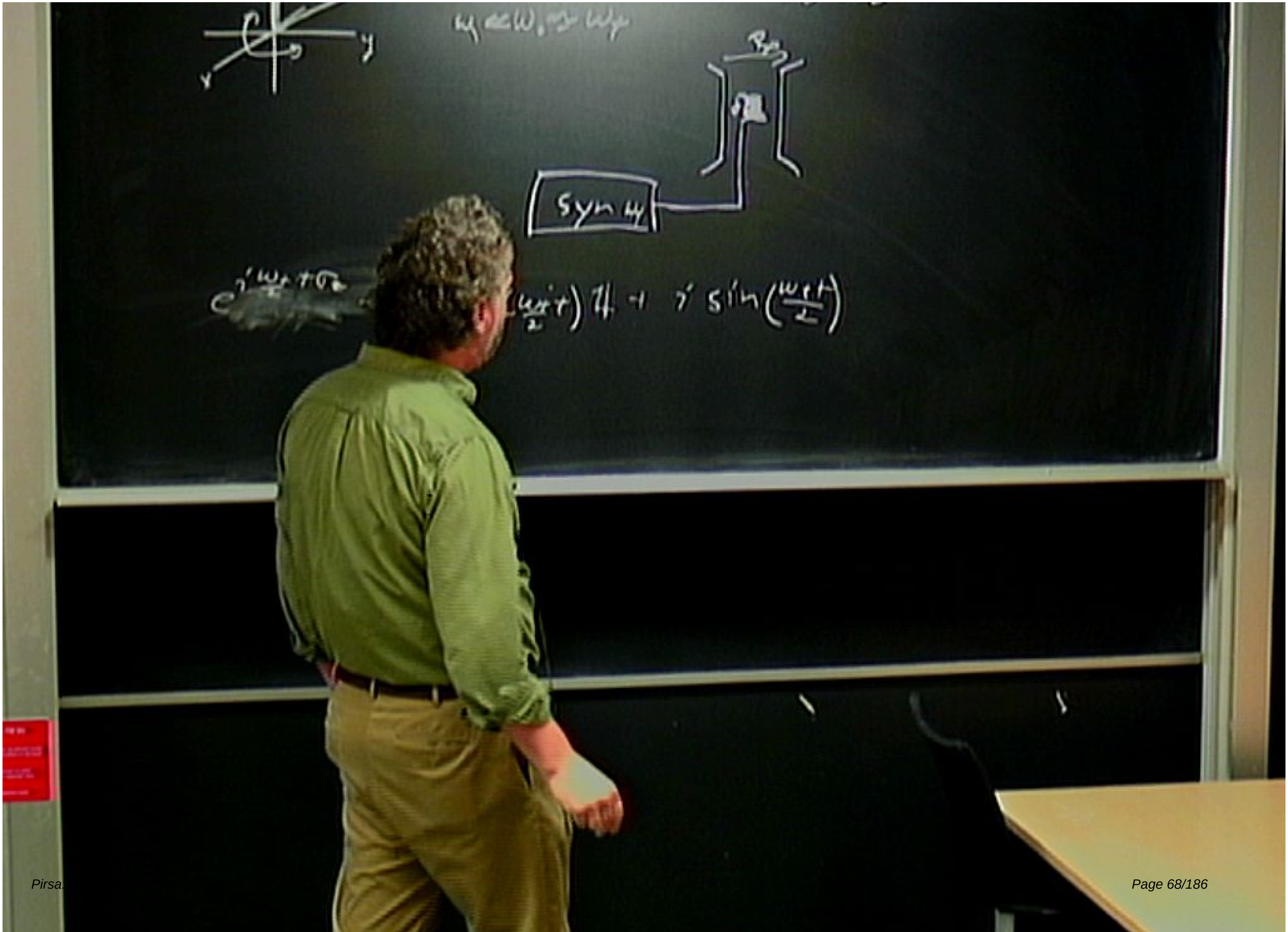

Rotating Wave Approximation

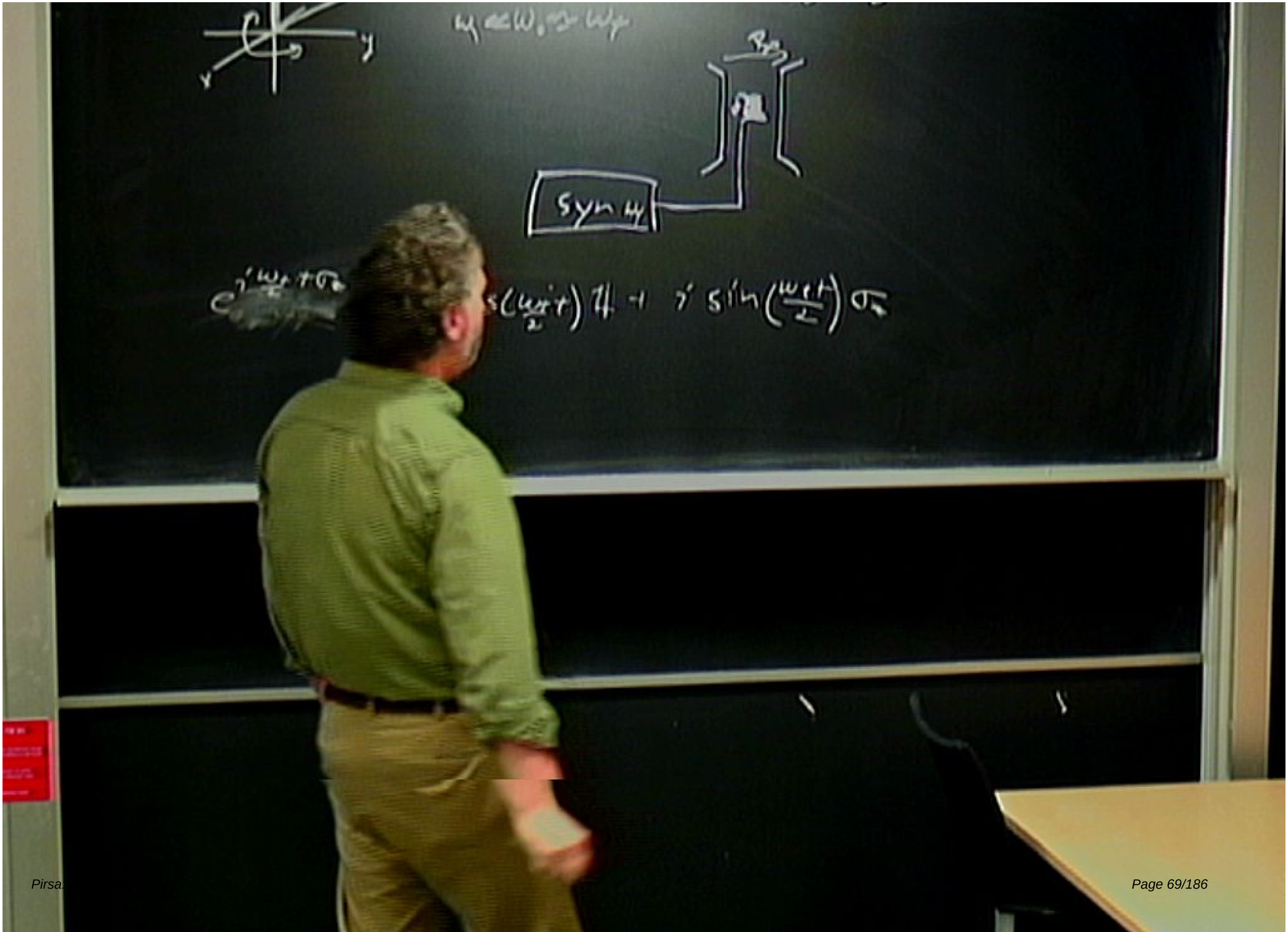


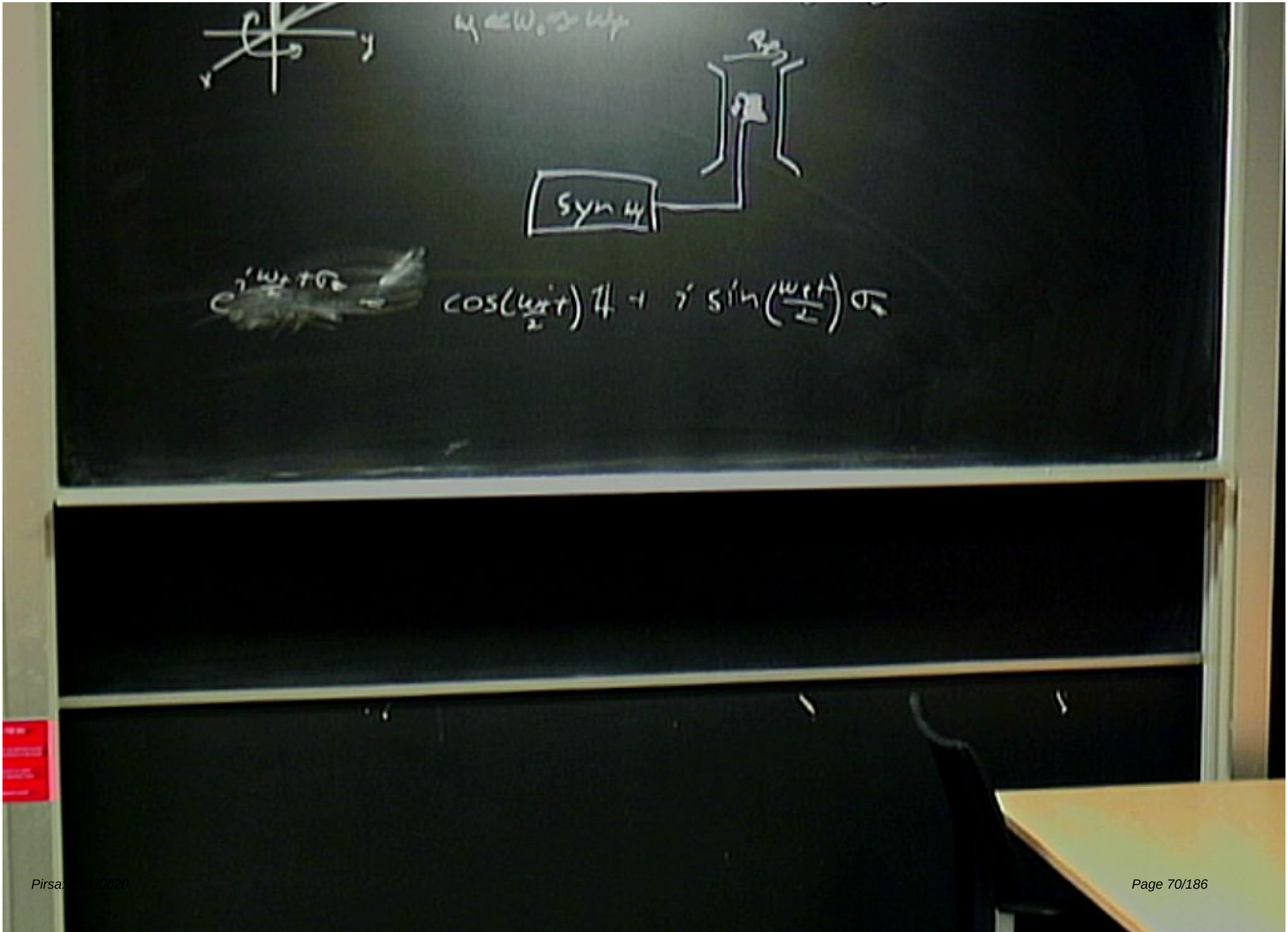


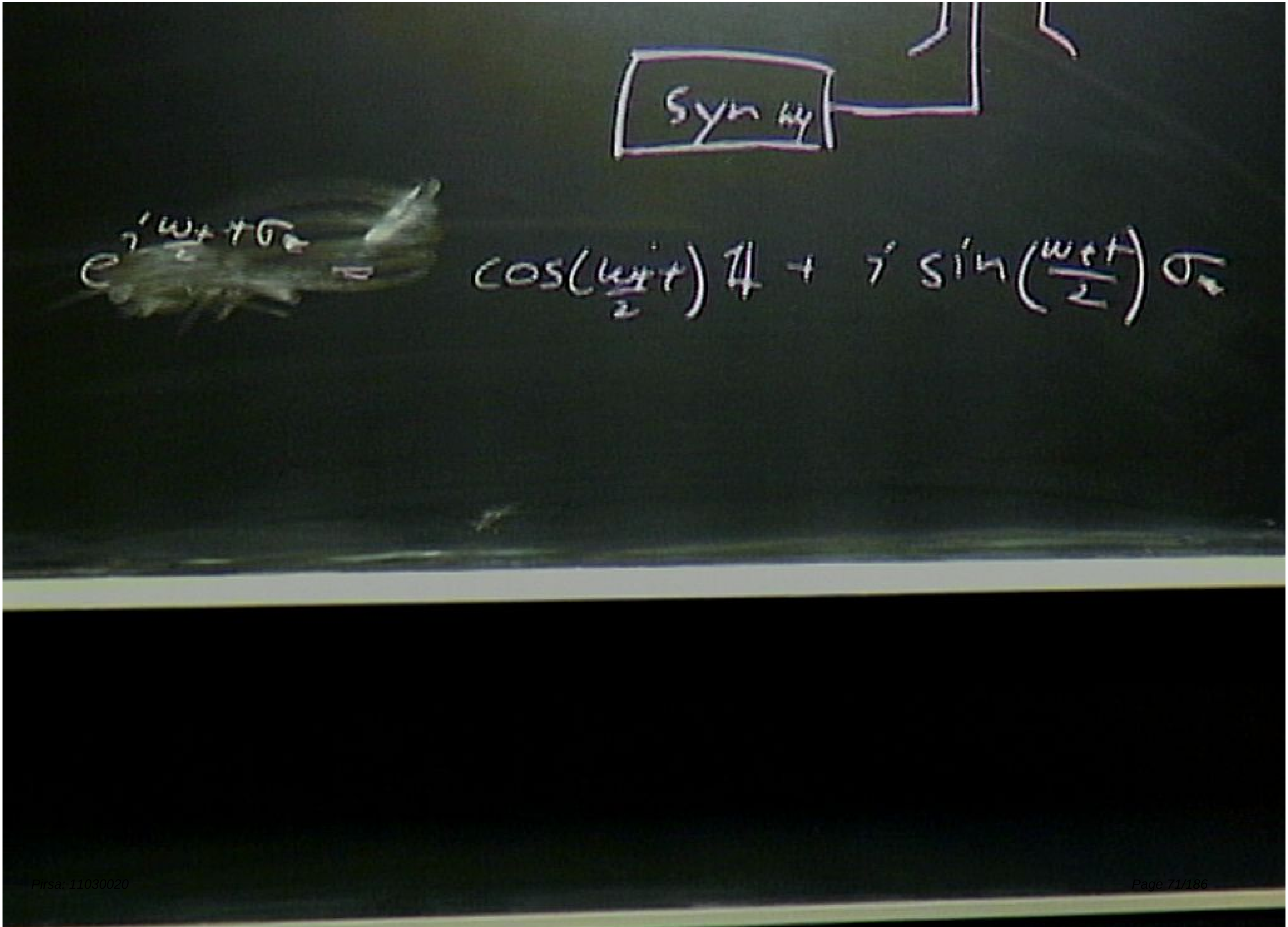

Syn ω_f
cos(

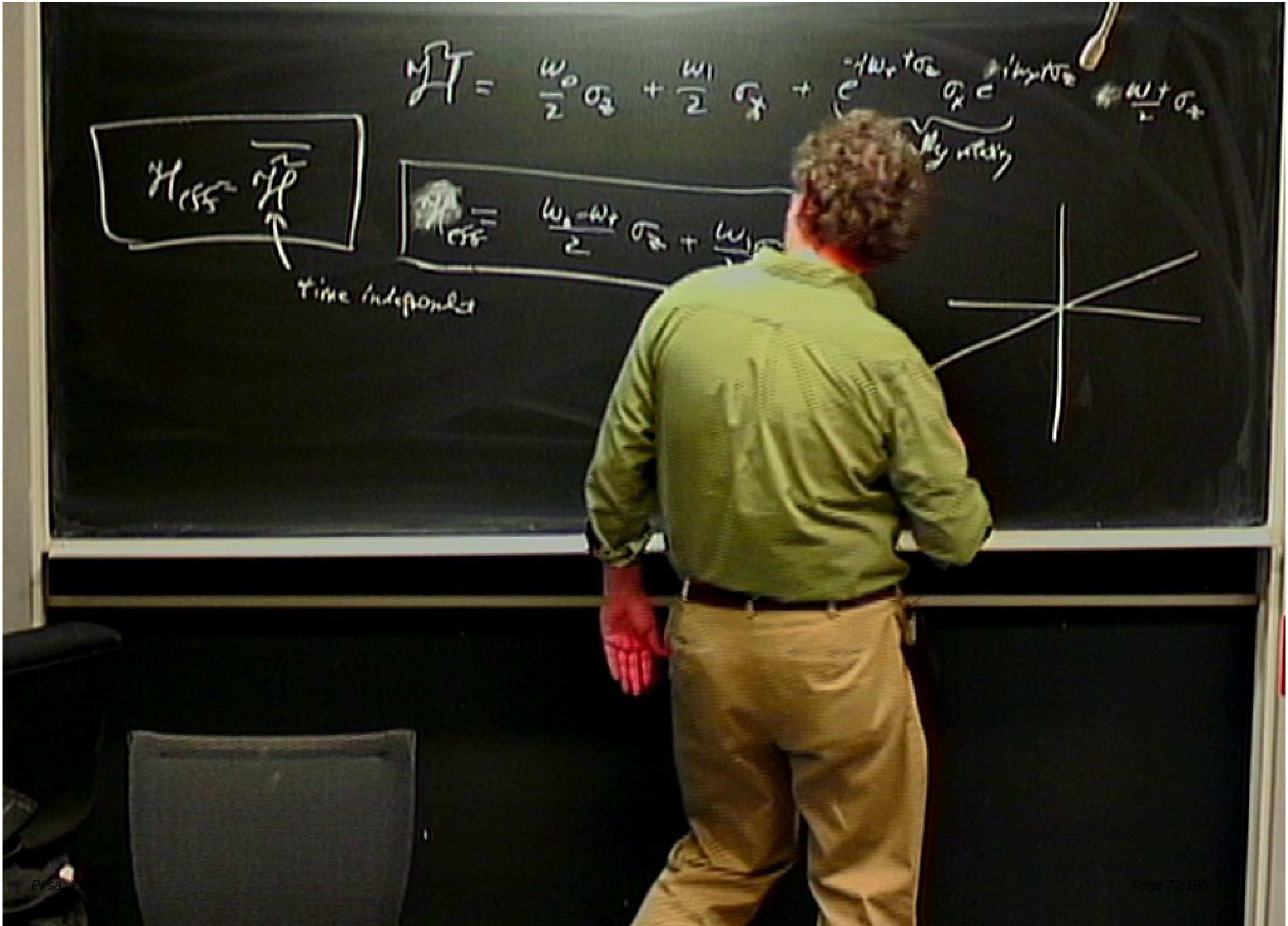


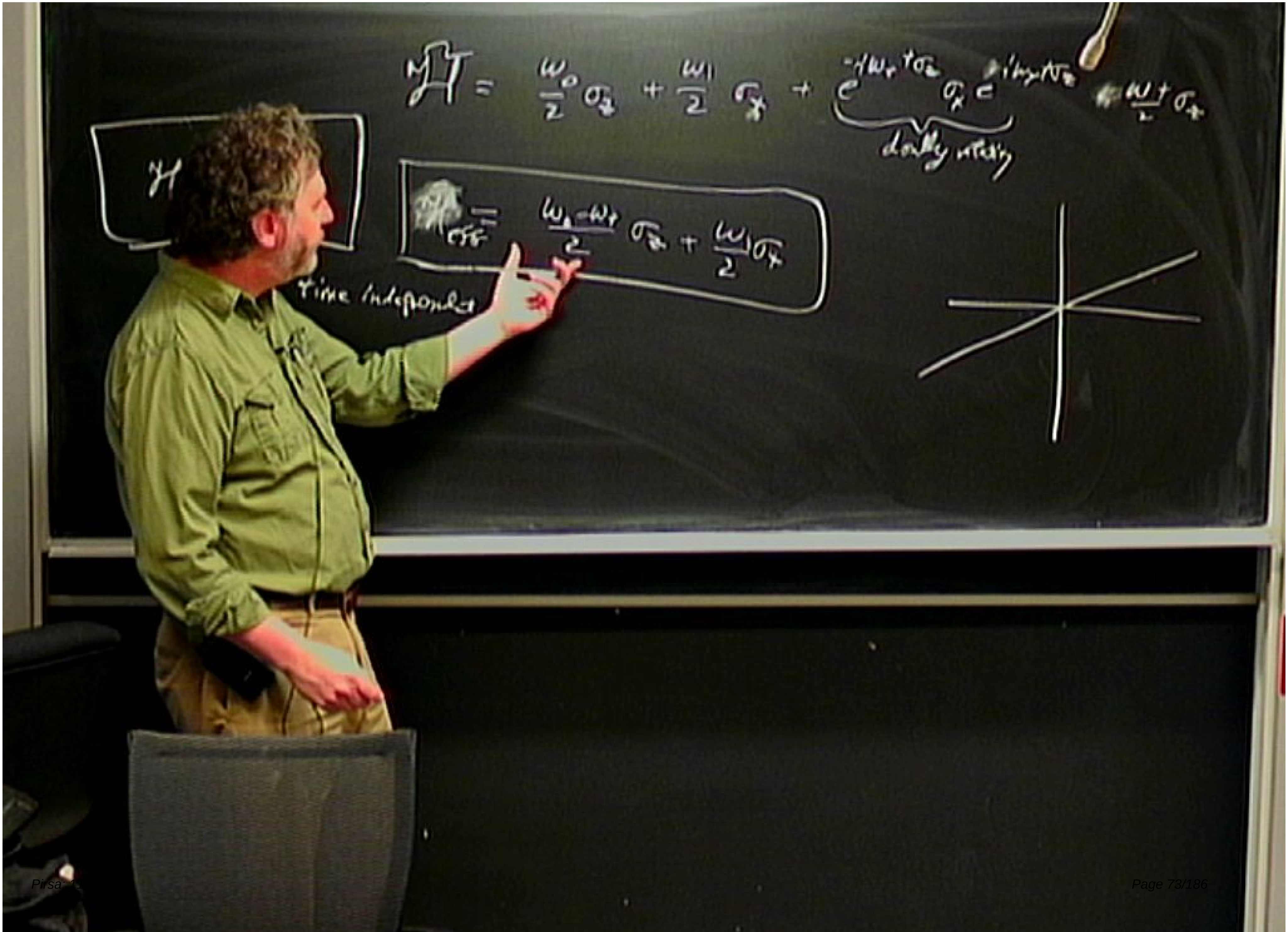


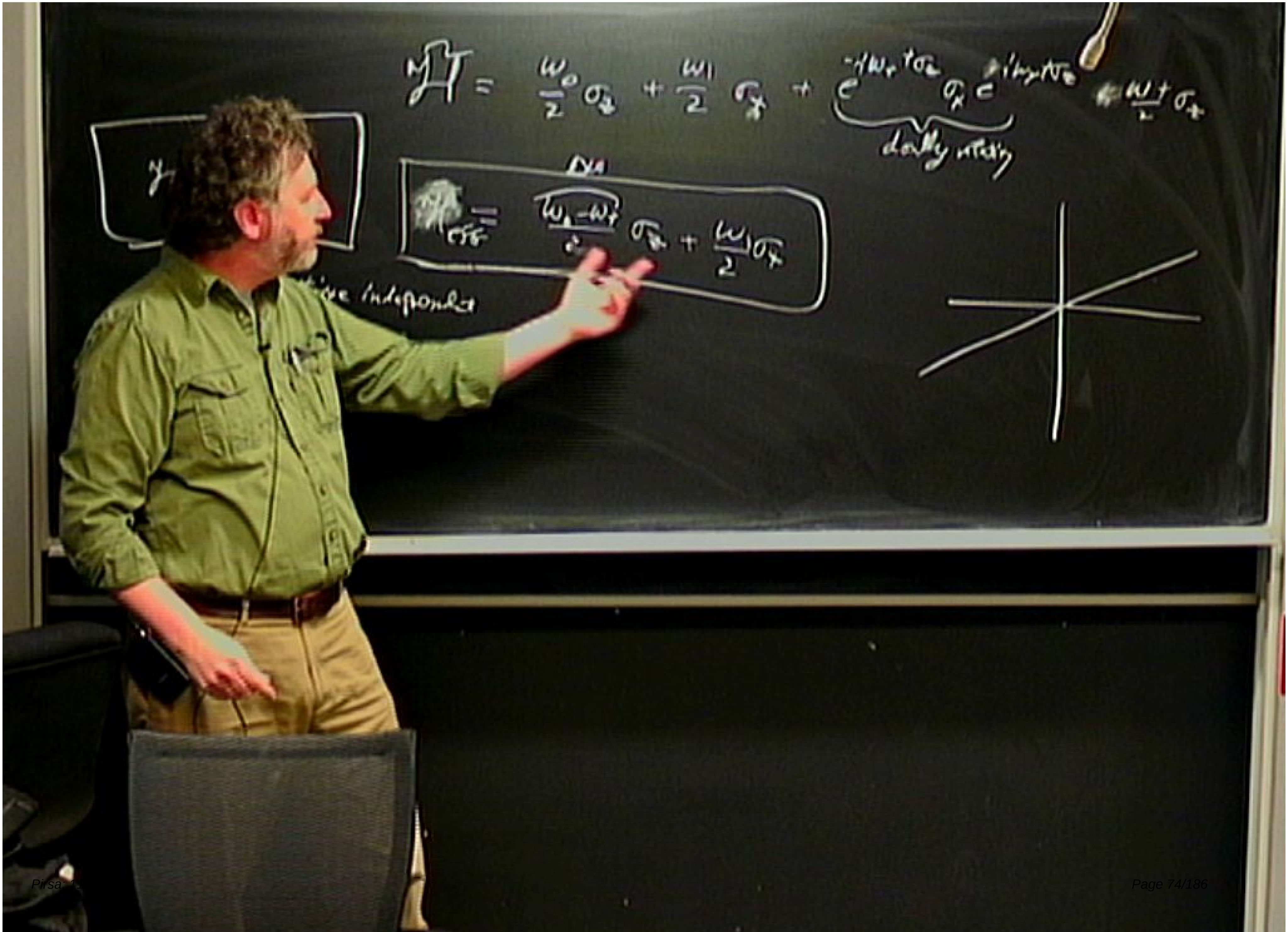


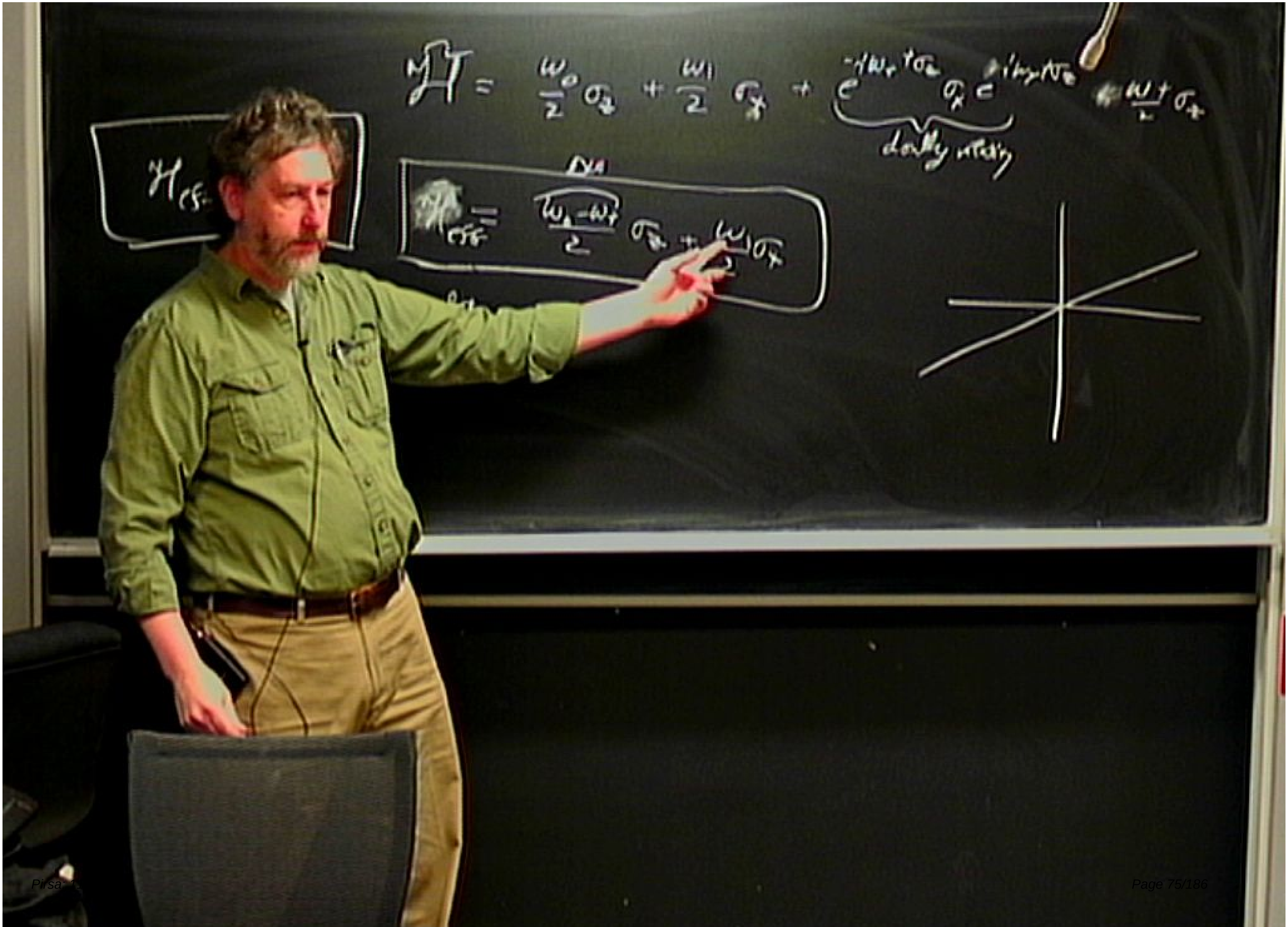

y
x
Syn

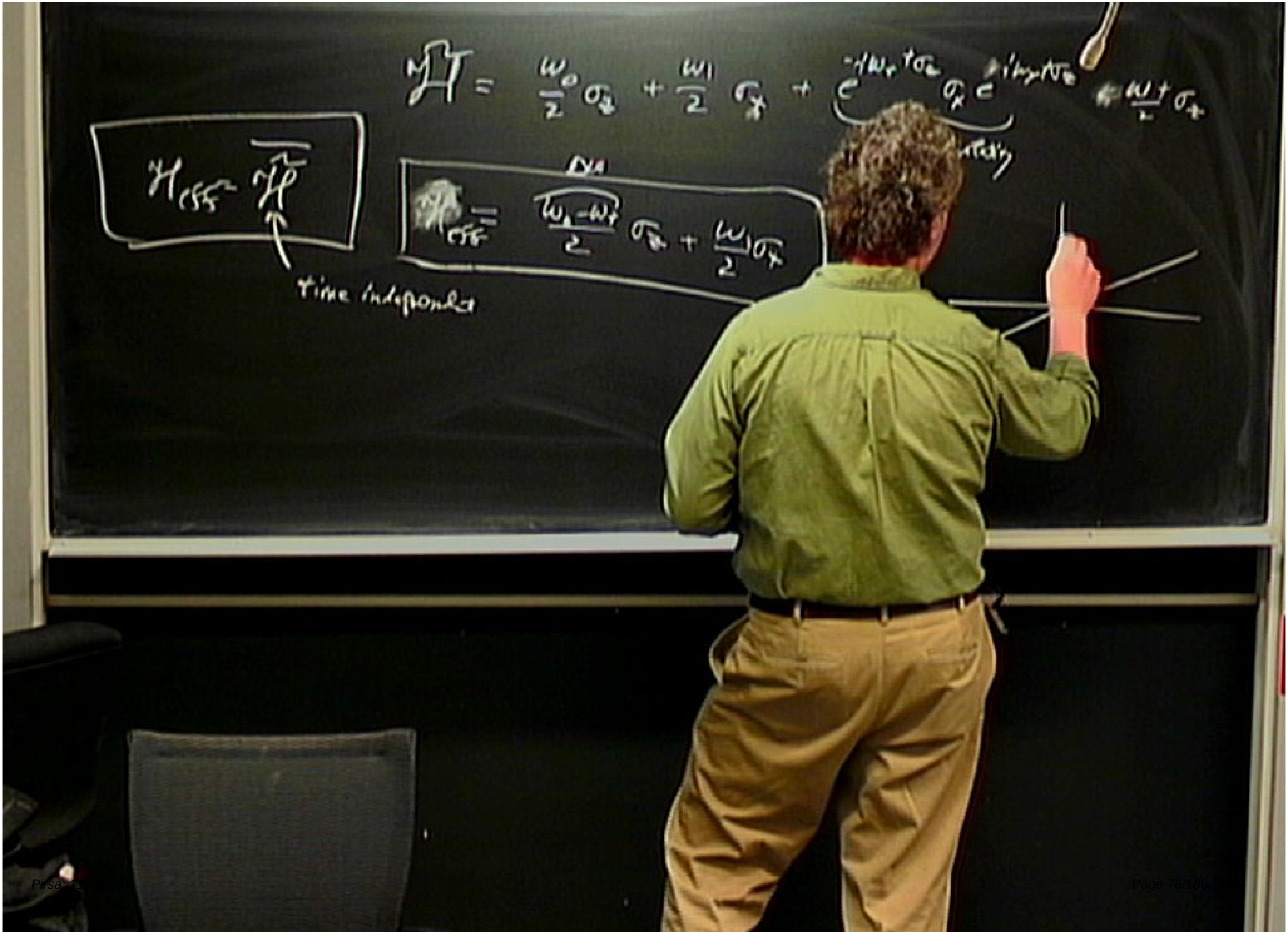

cos(ωt/2) 𝟙 + i sin(ωt/2) σz

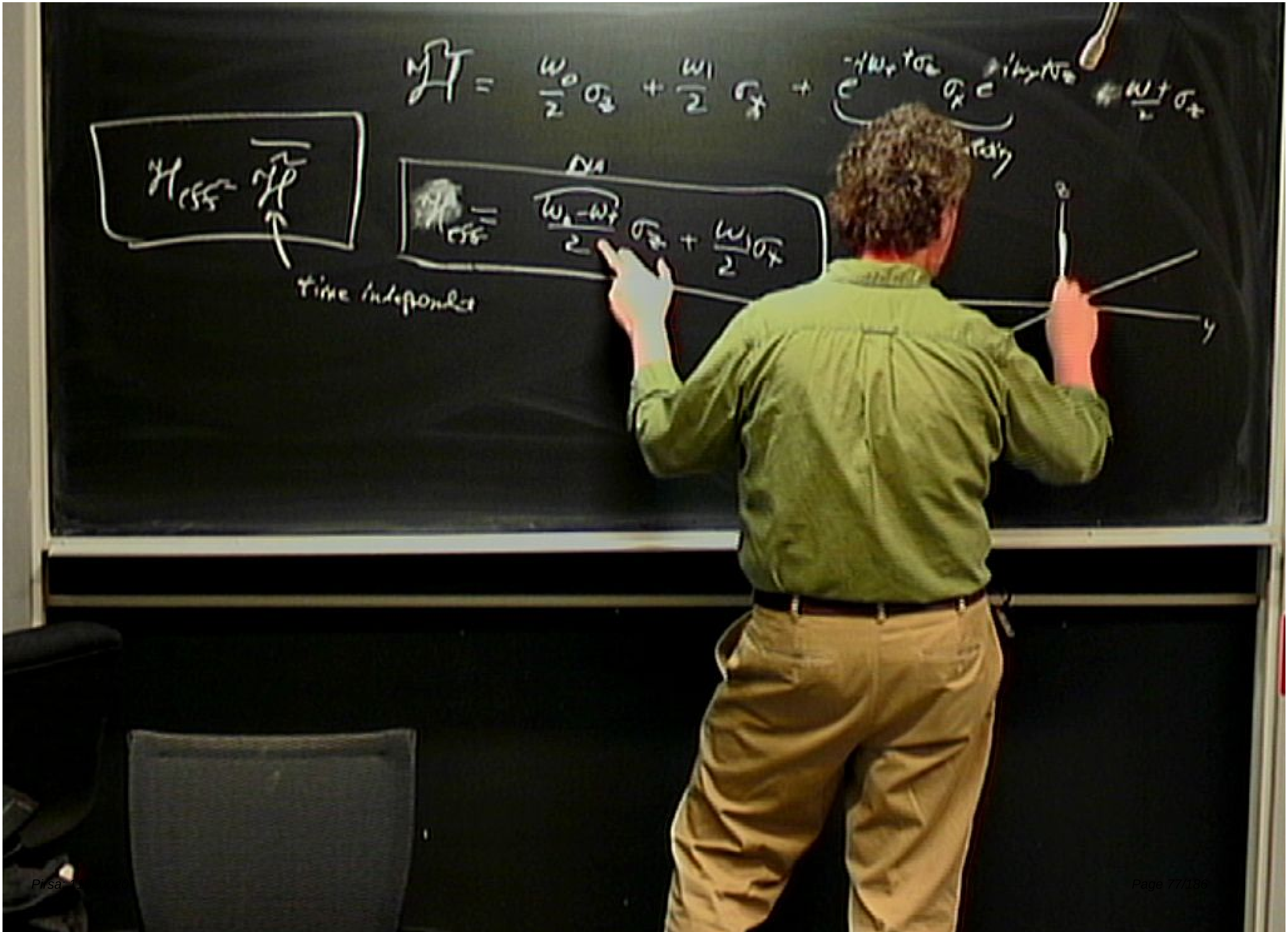

time independent







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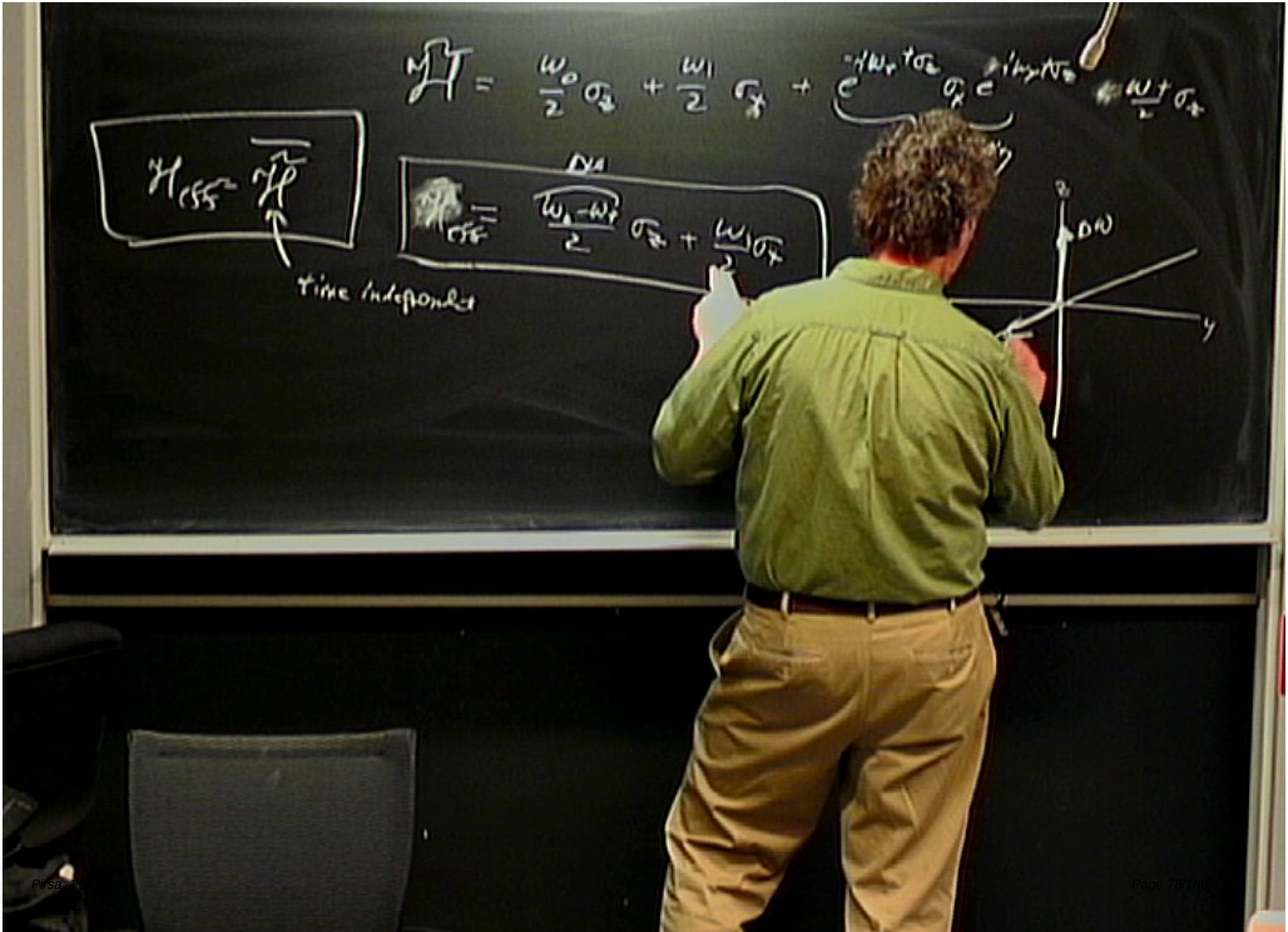

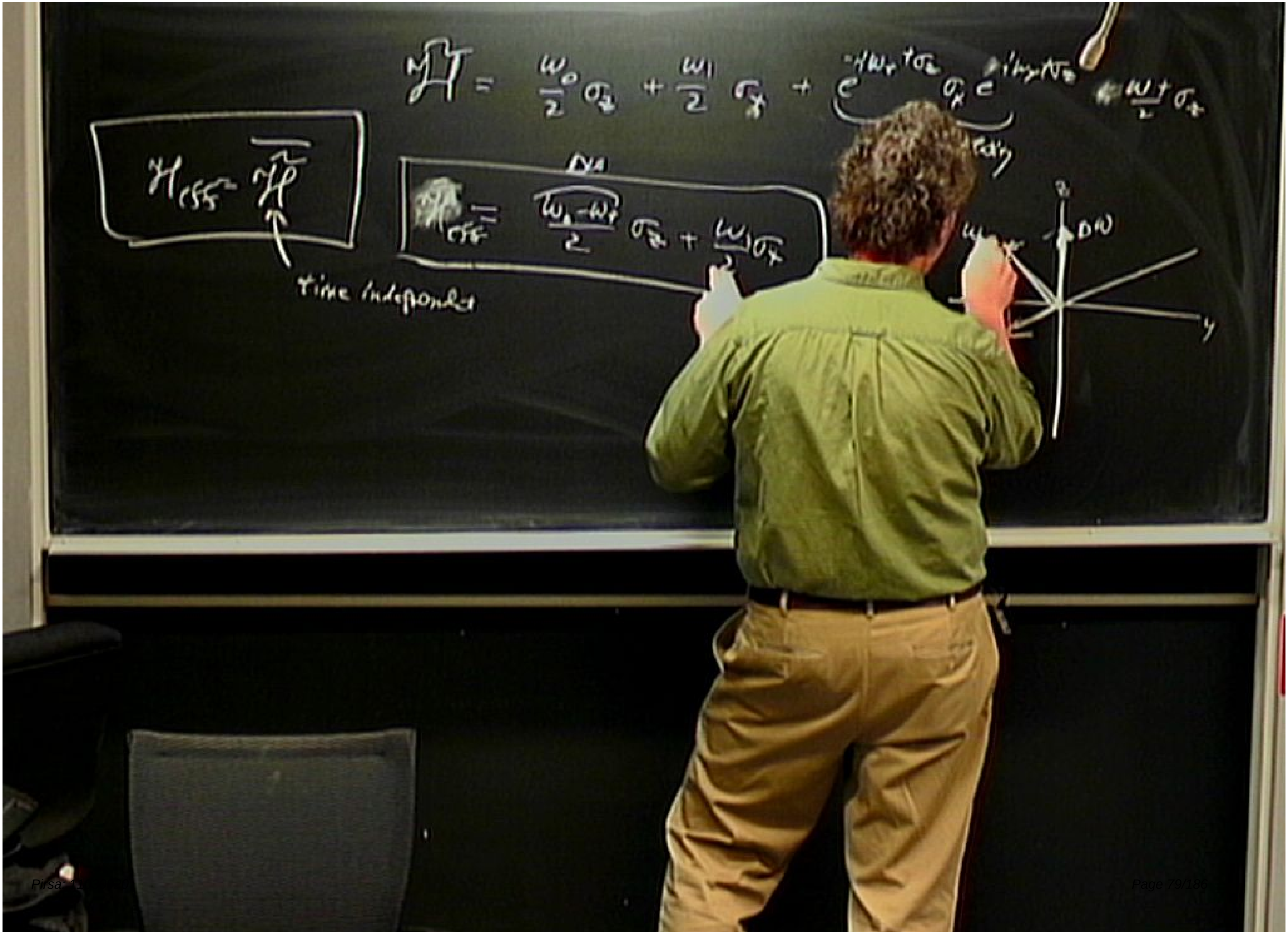
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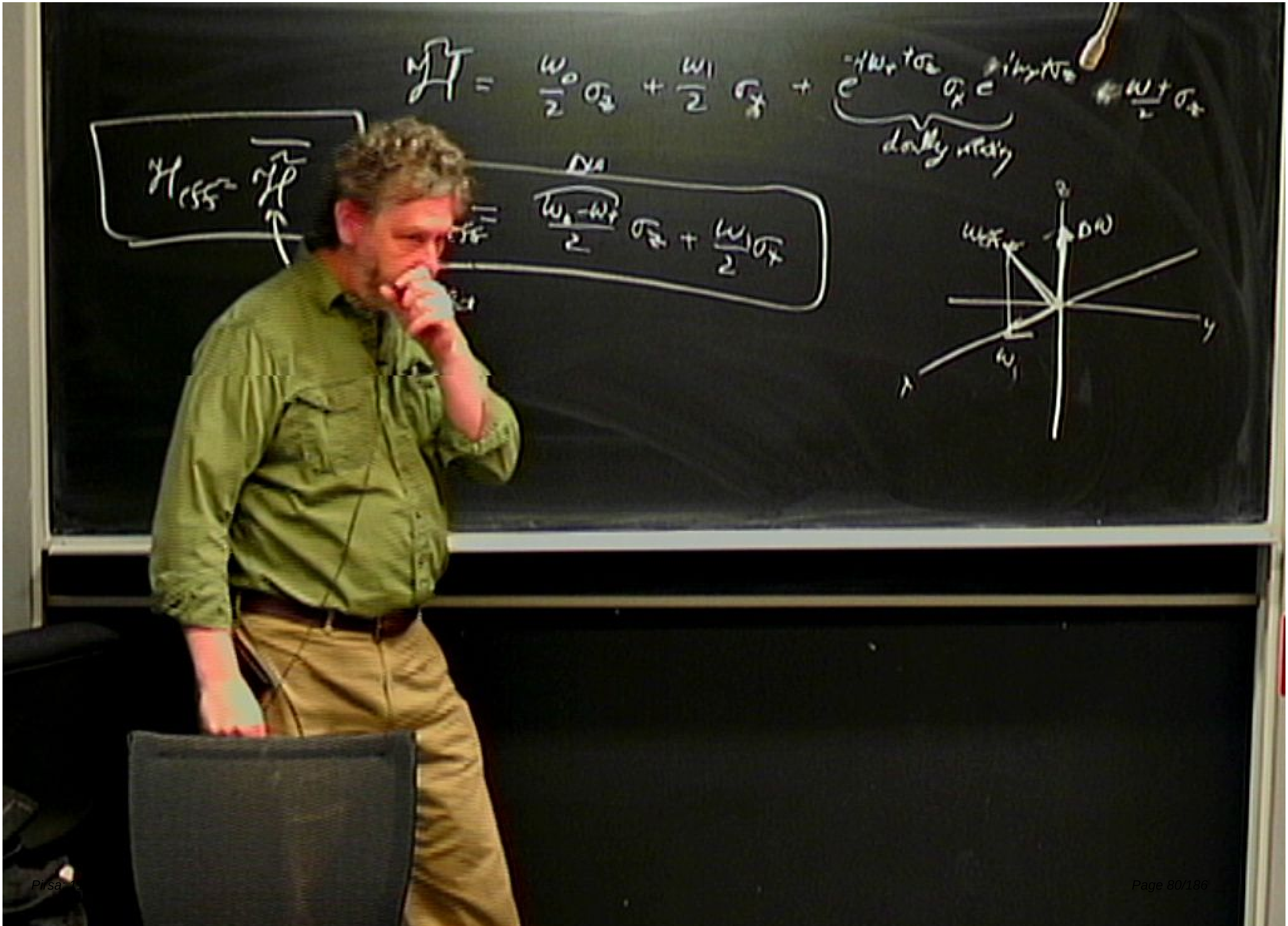
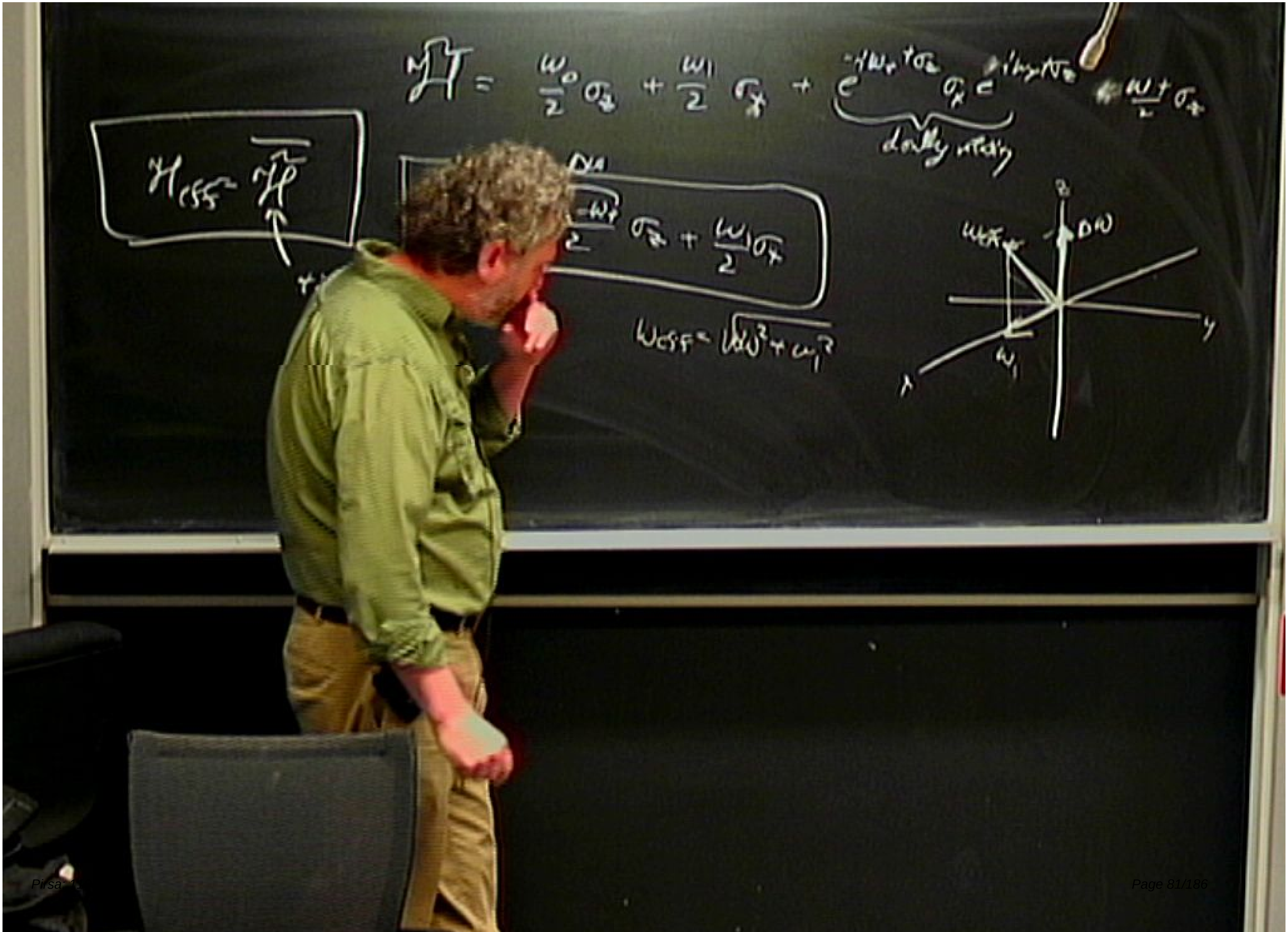
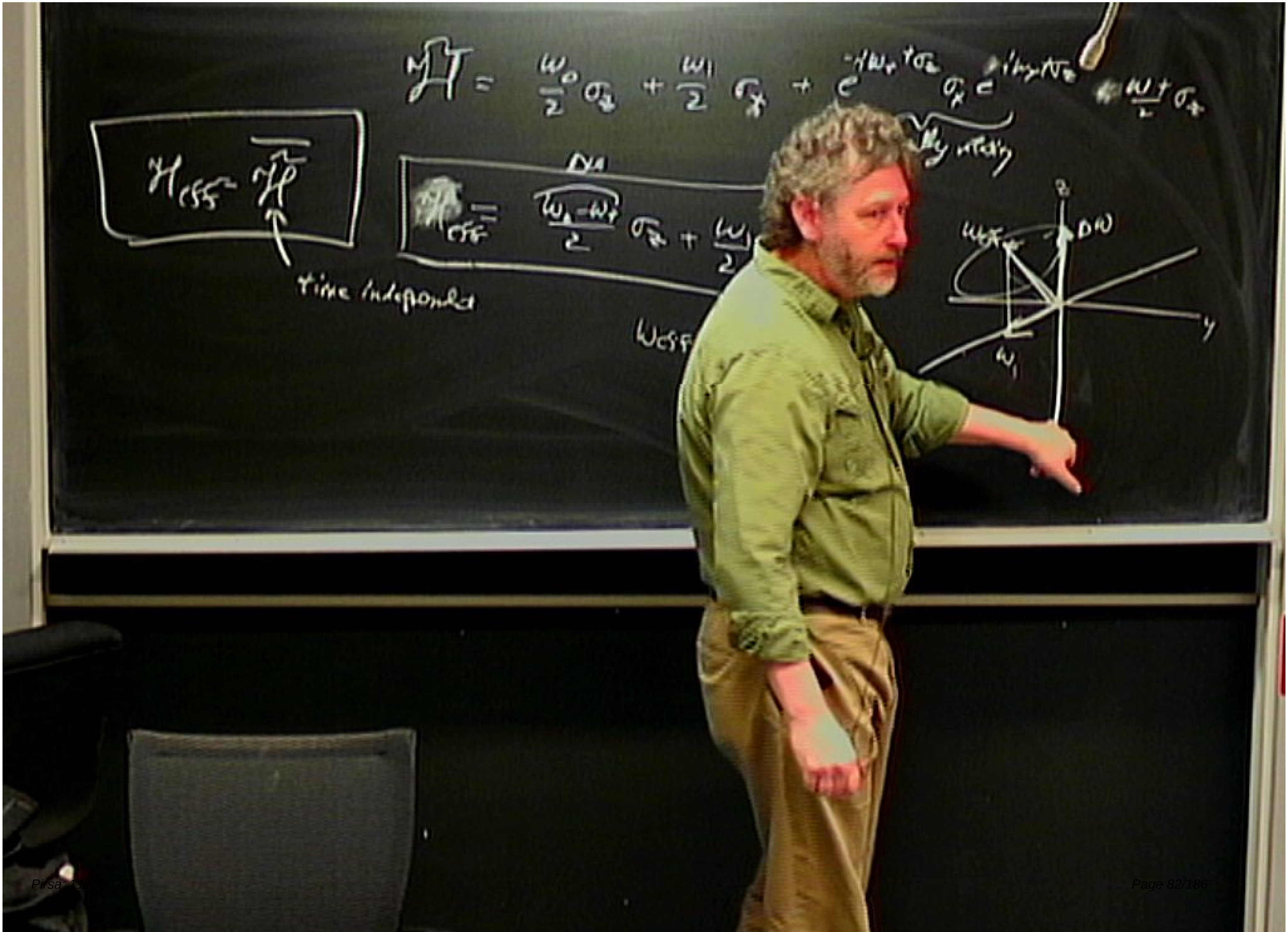
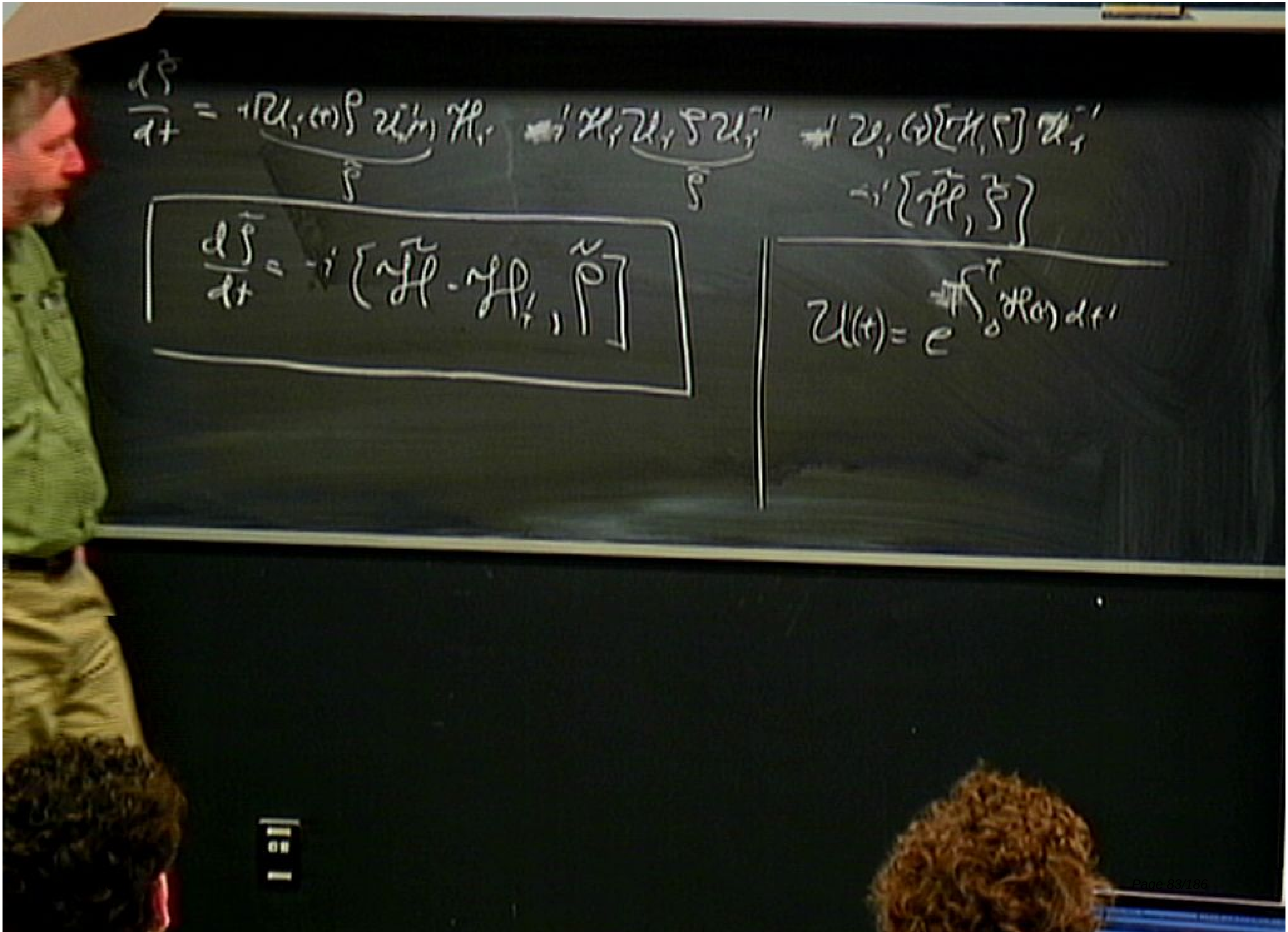
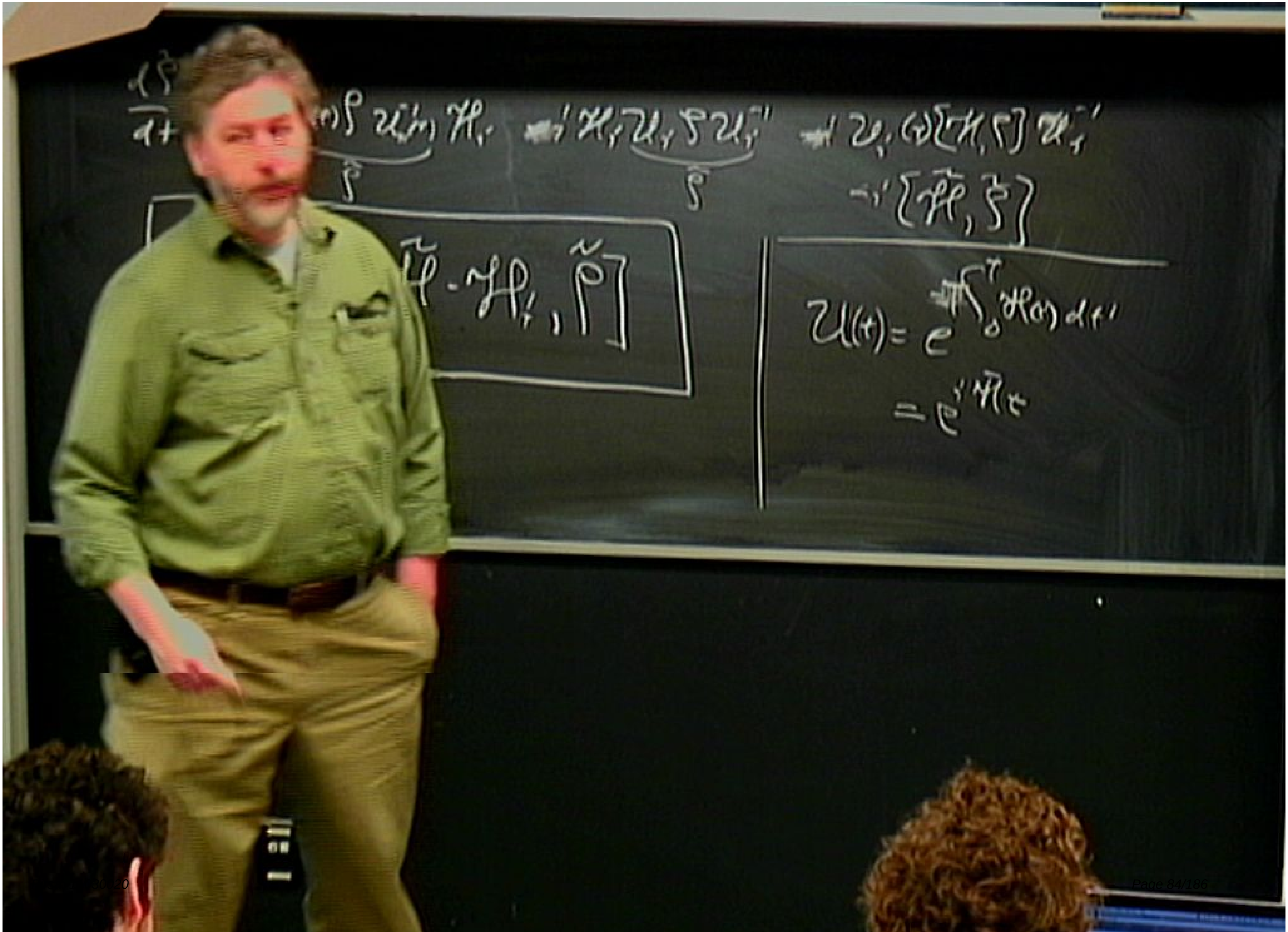
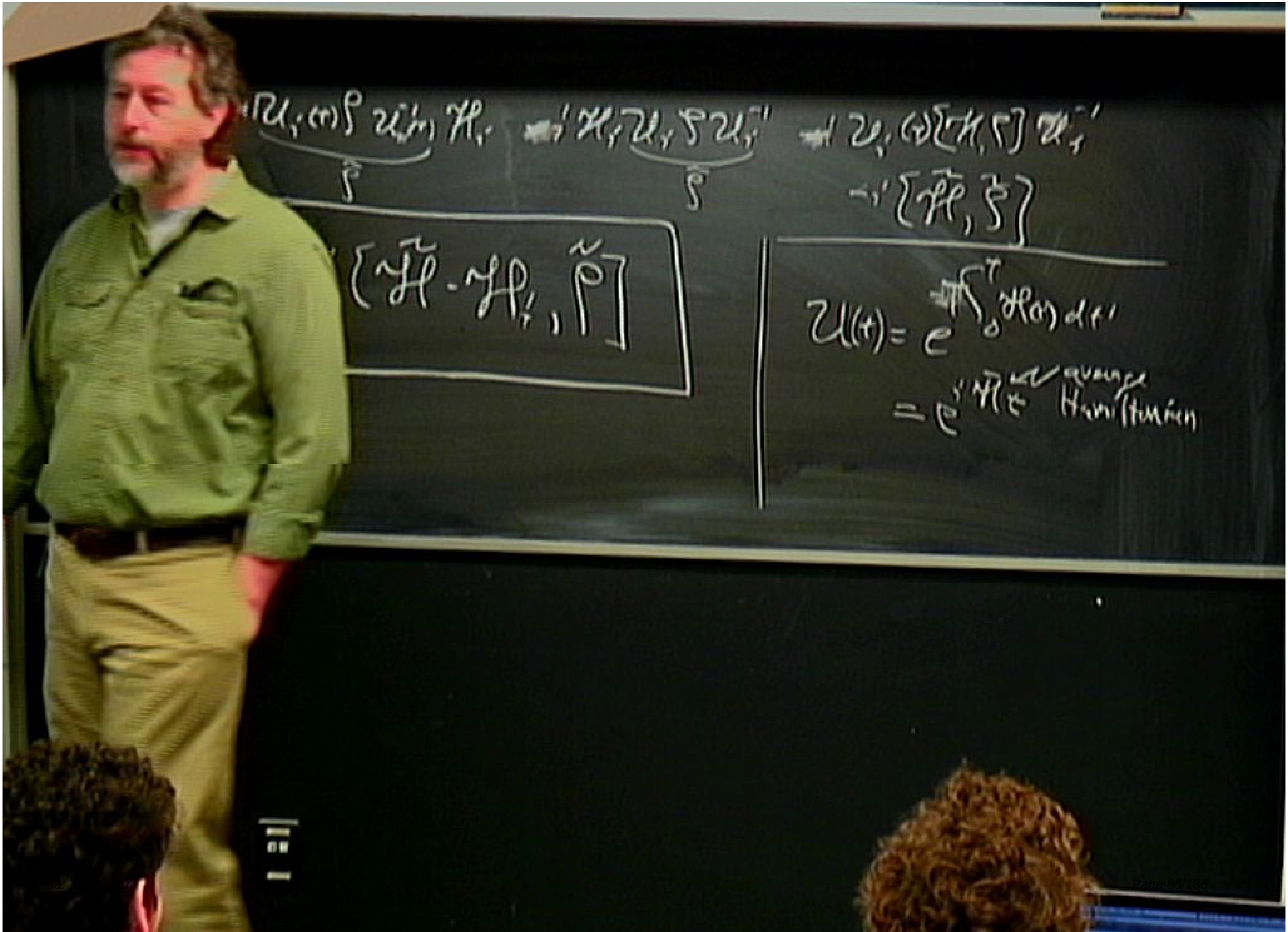


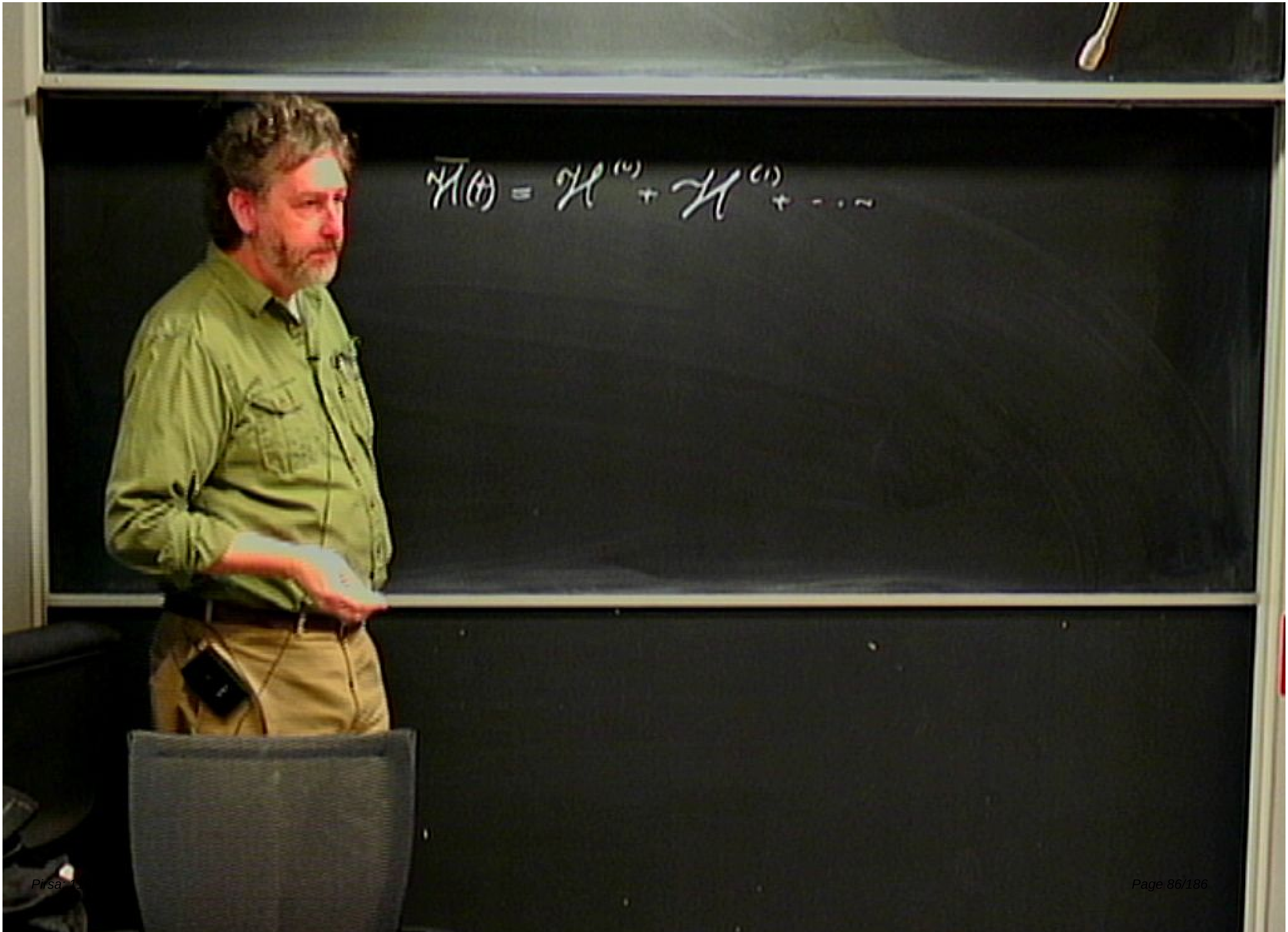


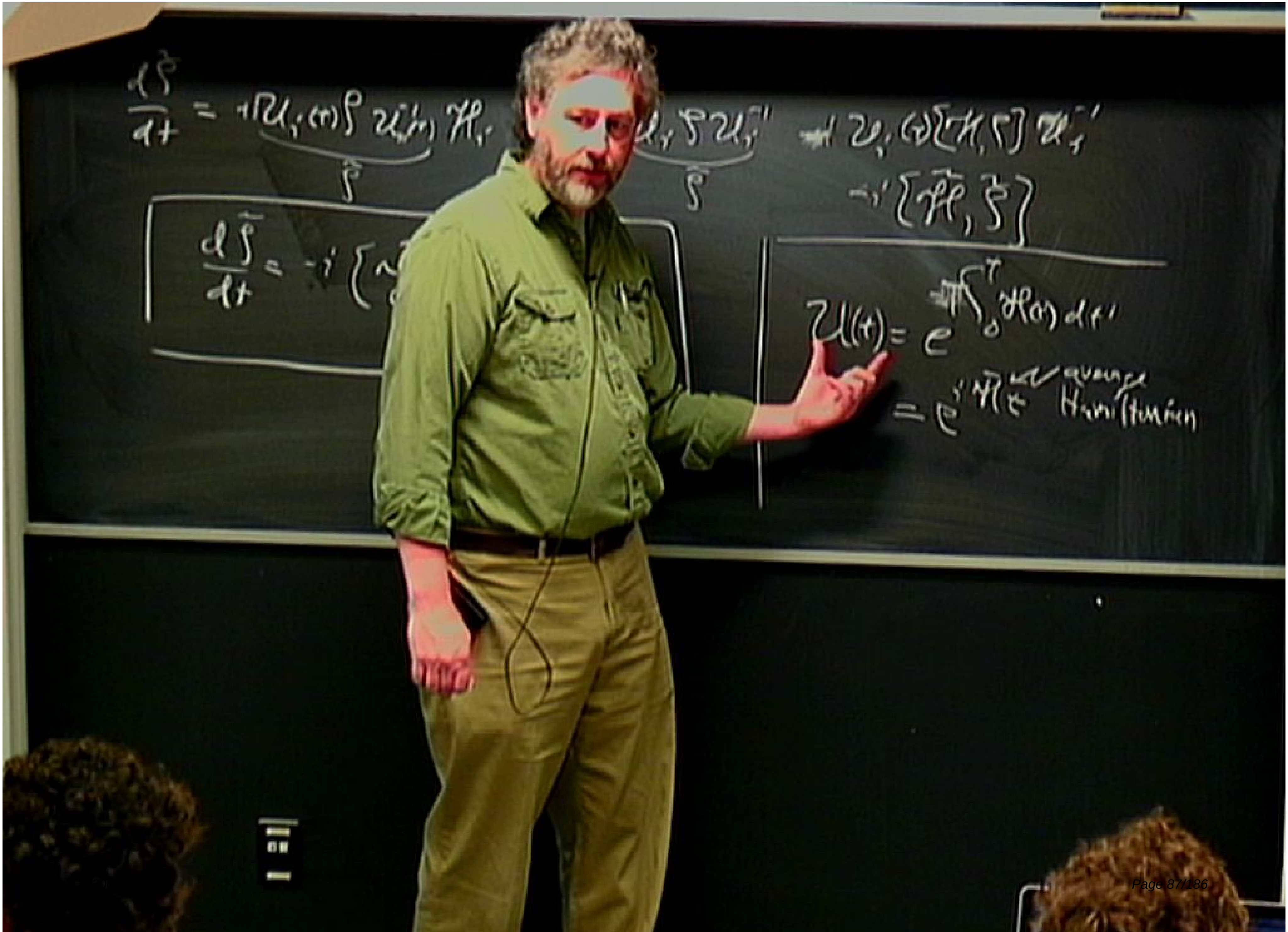


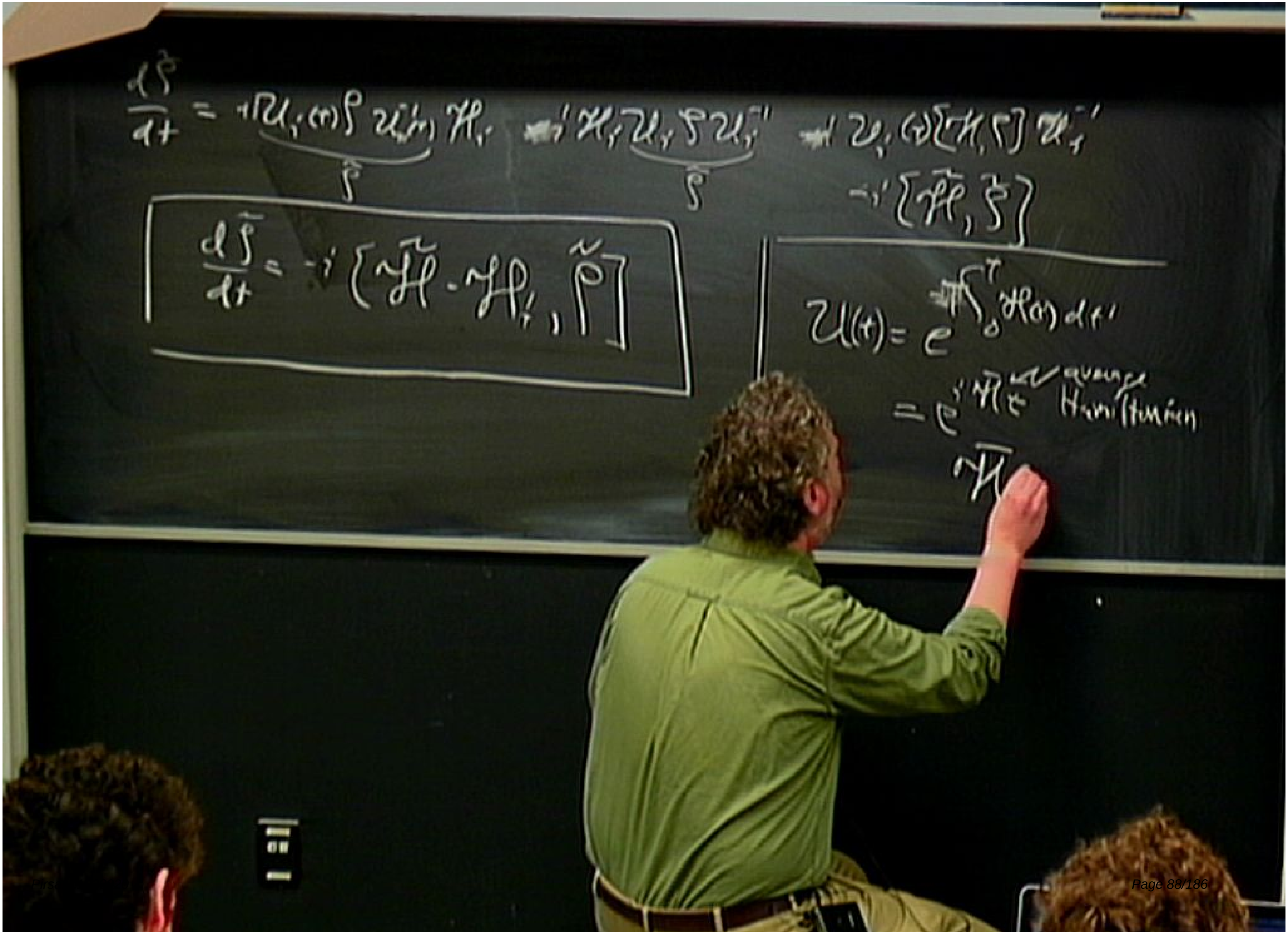

dρ/dt = i U_1(t) ρ U_1^†(t) H_1 − i H_1 U_1 ρ U_1^† = i U_1(t) [H_1, ρ] U_1^†
= −i [H̃, ρ̃]
dρ̃/dt = −i [H̃ − H_1, ρ̃]
U(t) = e^{−i ∫_0^t H(t') dt'}

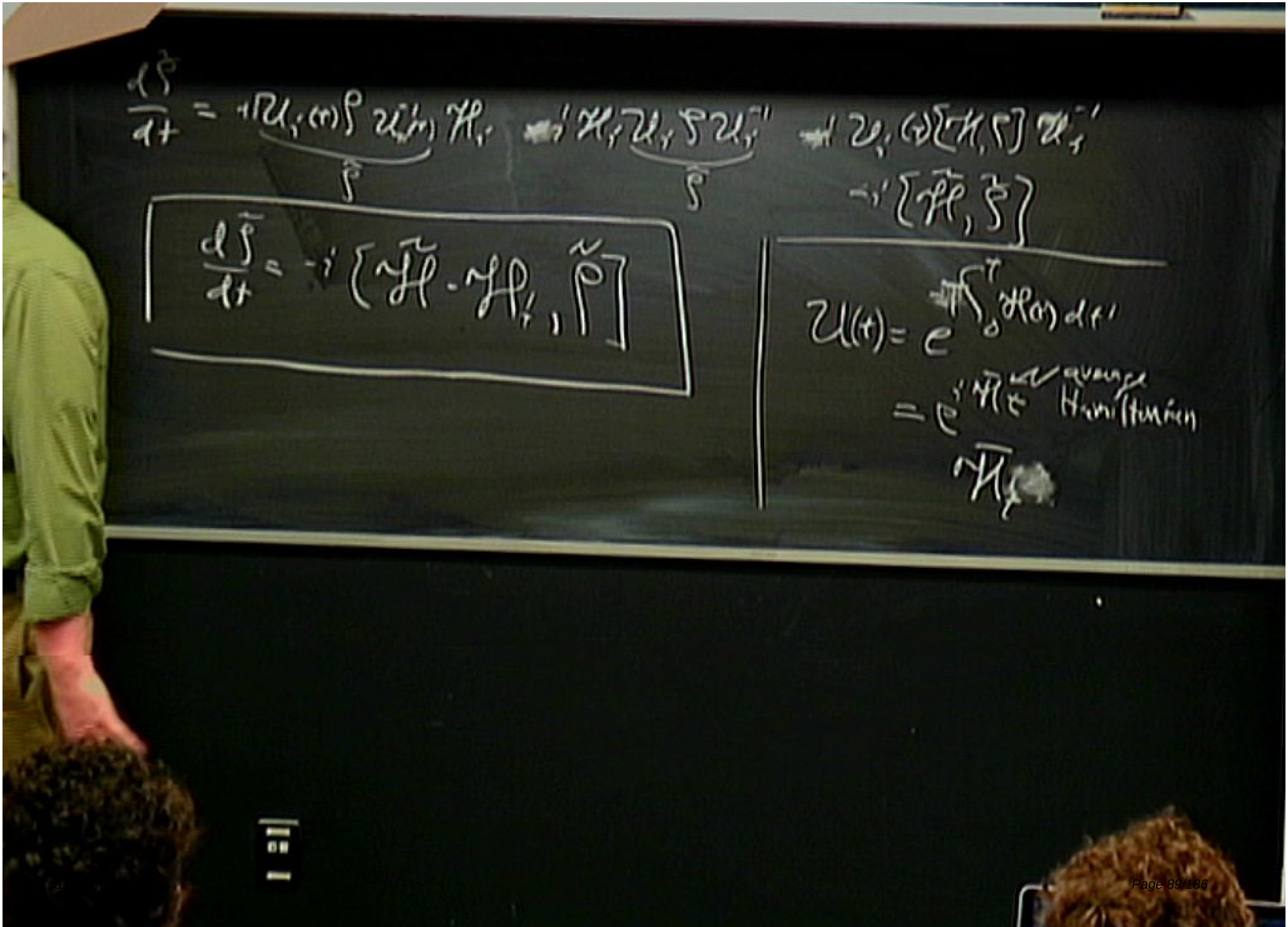


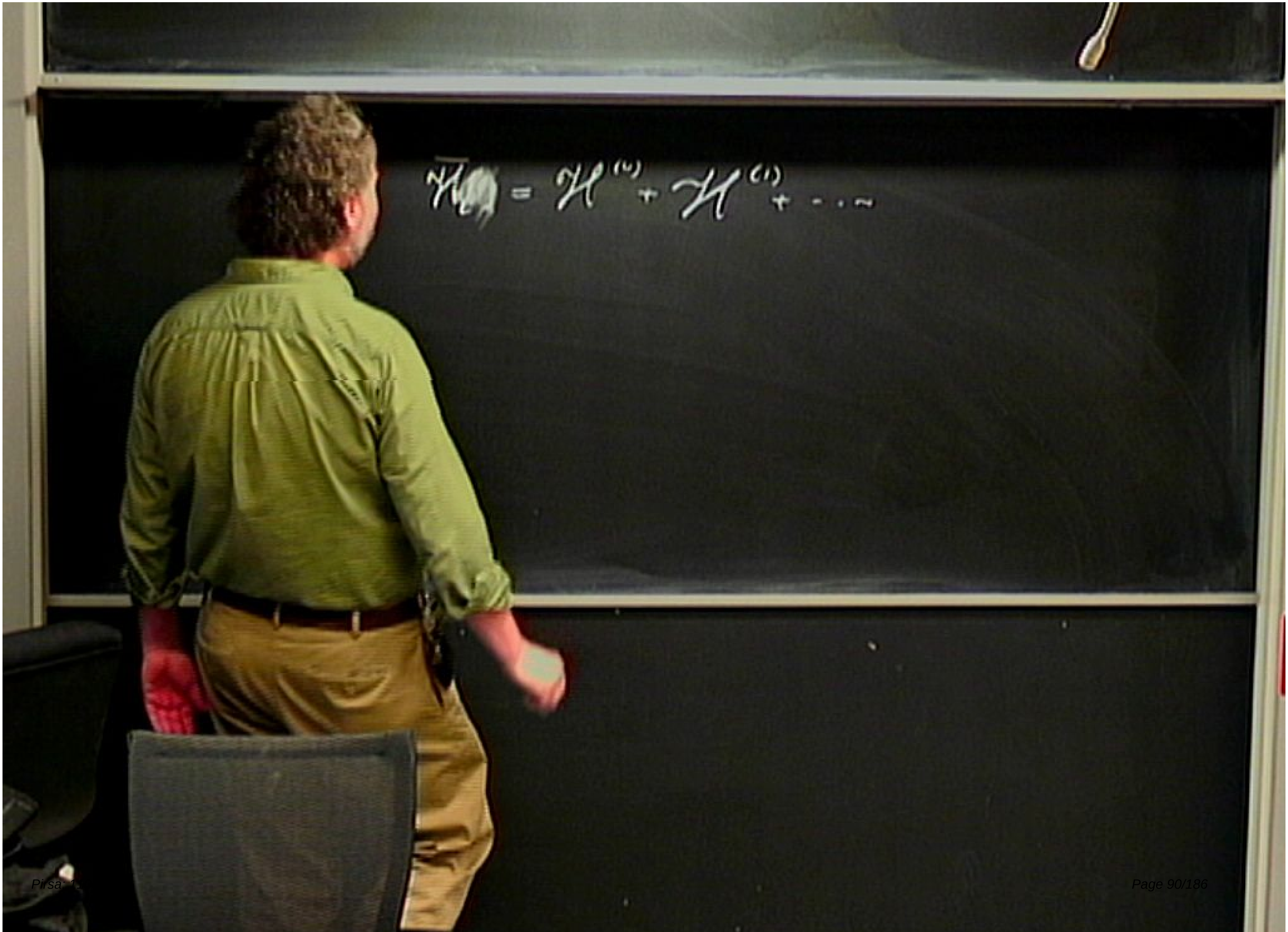

average Hamiltonian

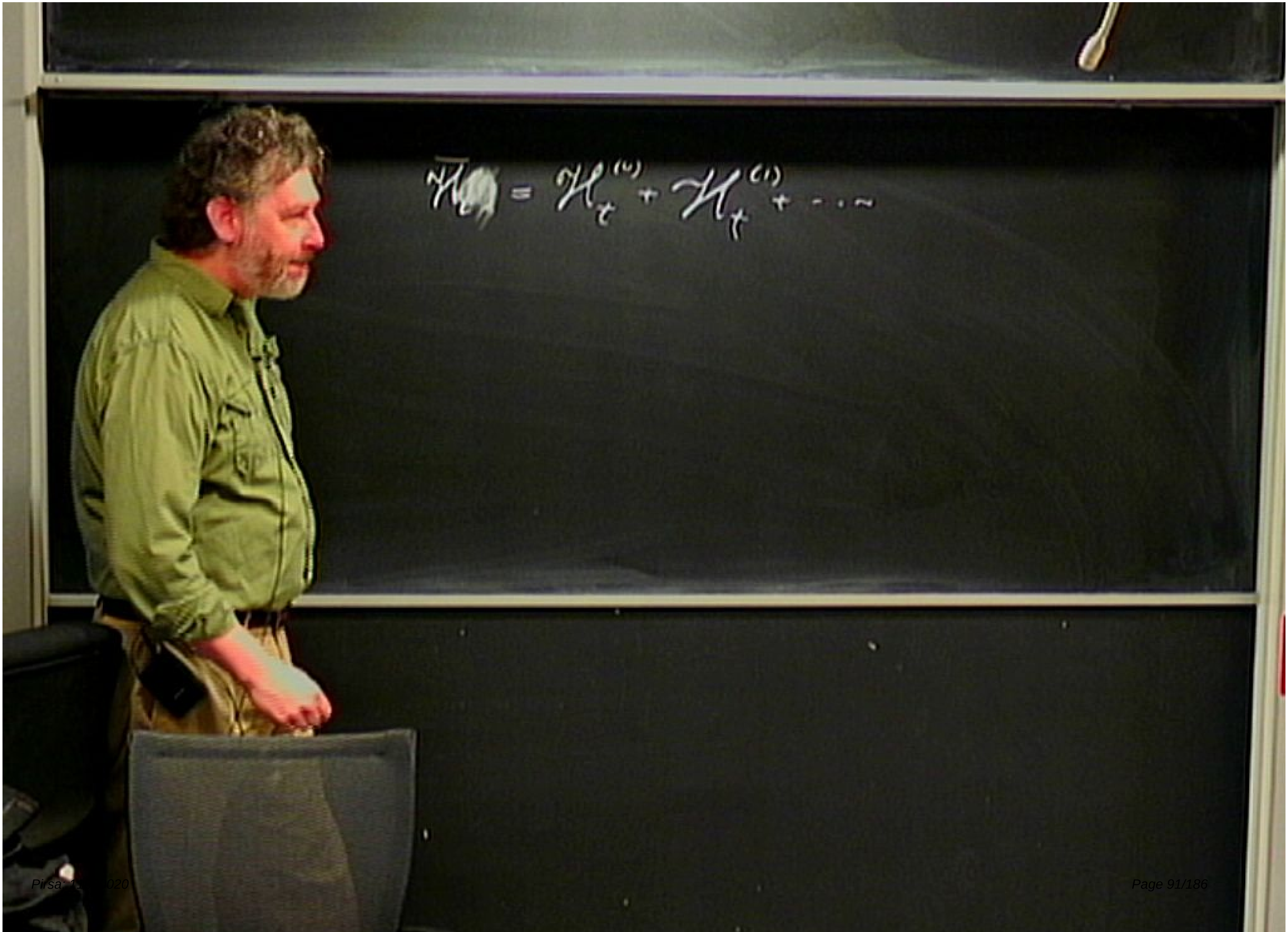

$\overline{\mathcal{H}}(t) = \mathcal{H}^{(0)} + \mathcal{H}^{(1)} + \cdots$

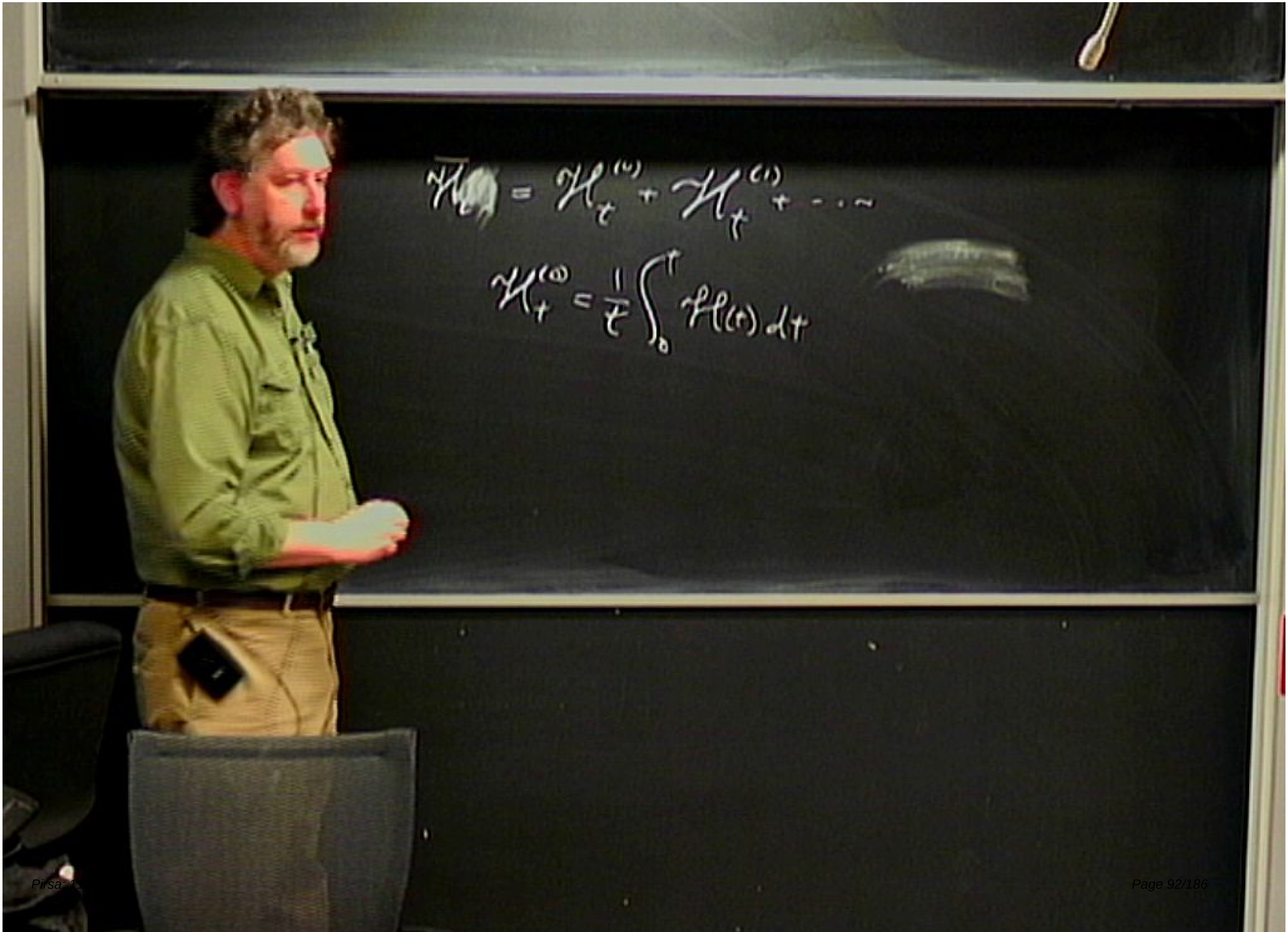

average
Hamiltonian

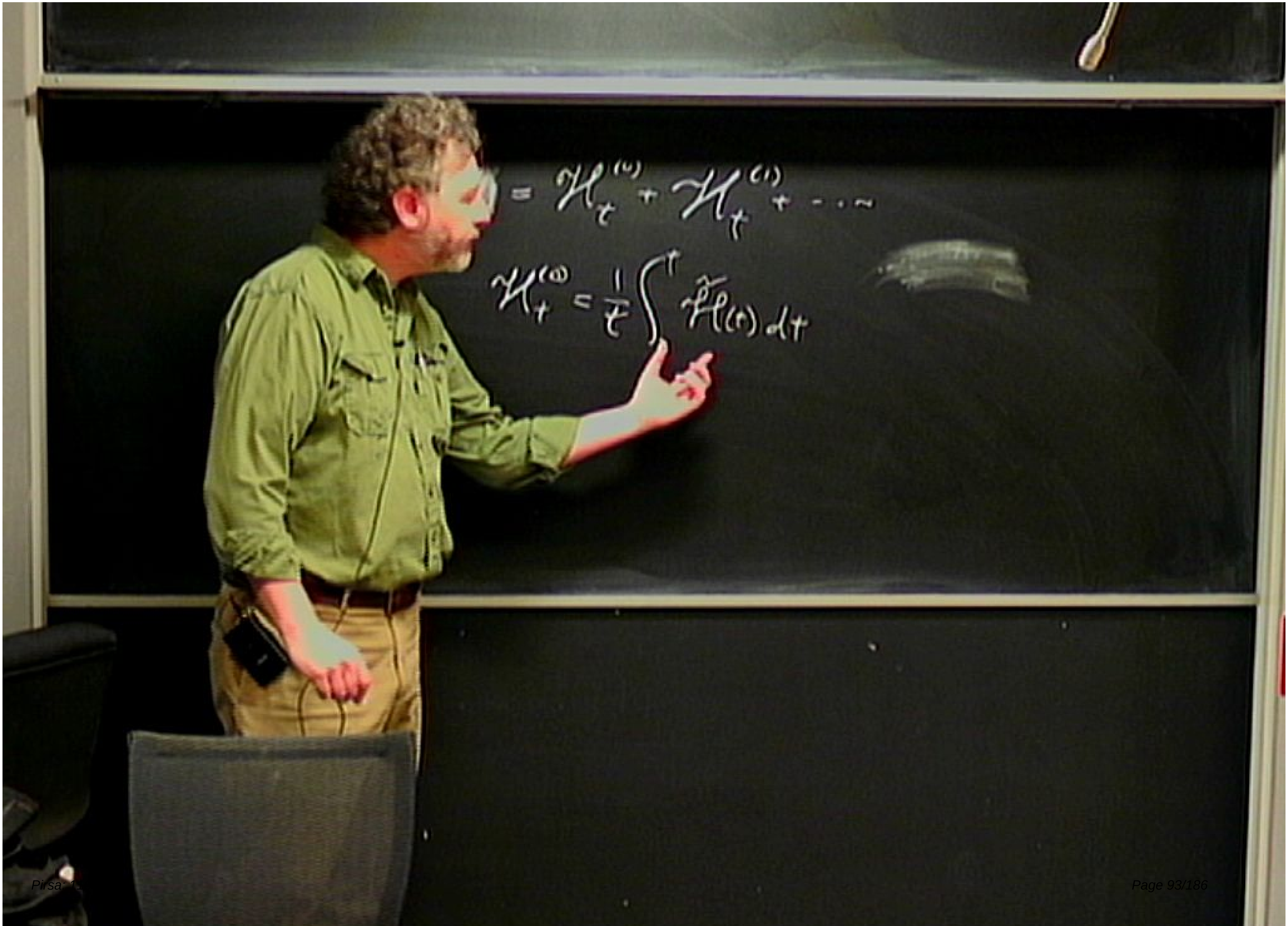

average
Hamiltonian


average
Hamiltonian

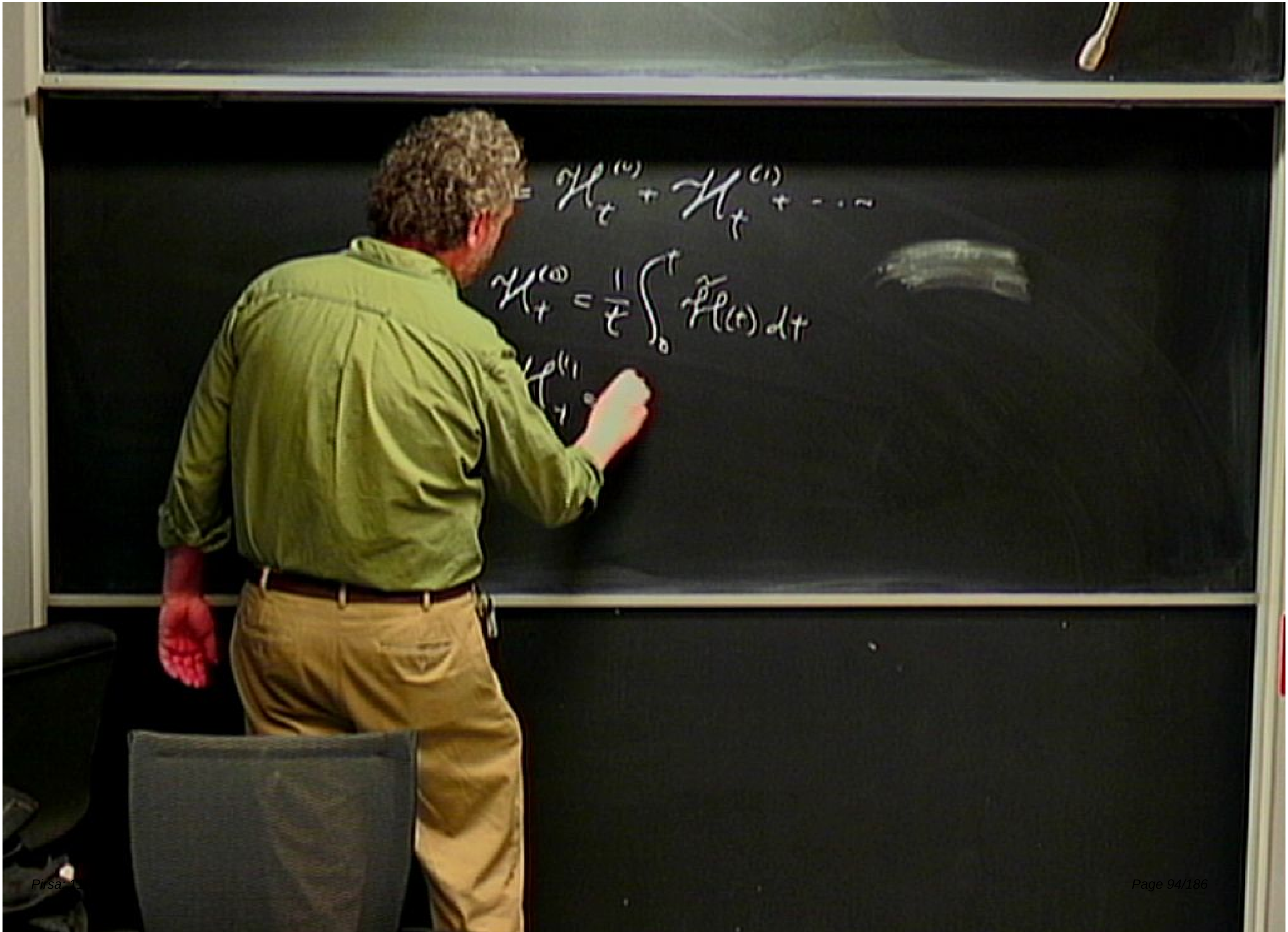

= H^(0) + H^(1) + . . .

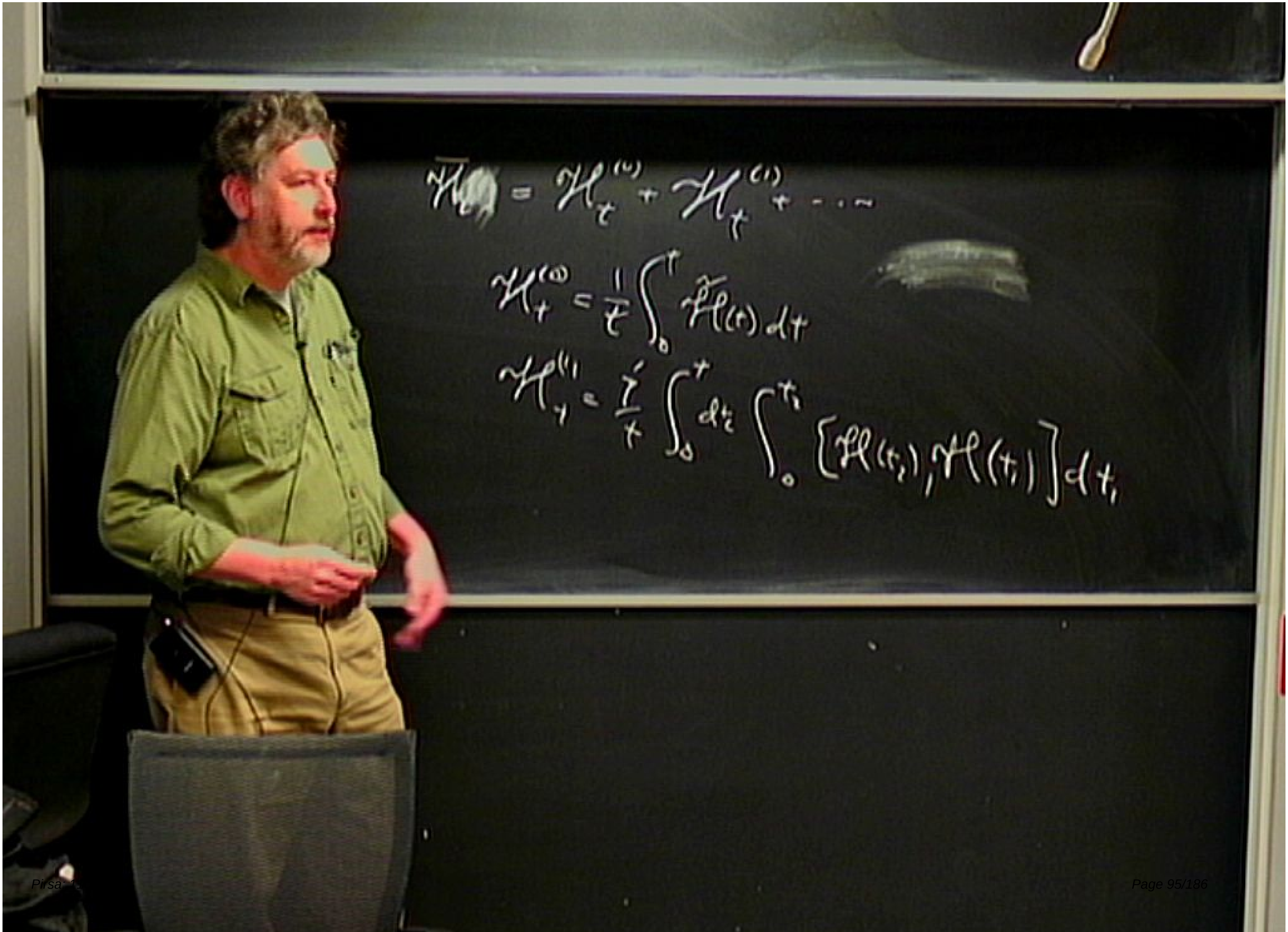

$\tilde{\mathcal{H}}_t = \mathcal{H}_t^{(0)} + \mathcal{H}_t^{(1)} + \cdots$


$\bar{\mathcal{H}}_t = \mathcal{H}_t^{(0)} + \mathcal{H}_t^{(1)} + \cdots$
$\mathcal{H}_t^{(0)} = \frac{1}{t}\int_0^t \mathcal{H}(t)\,dt$

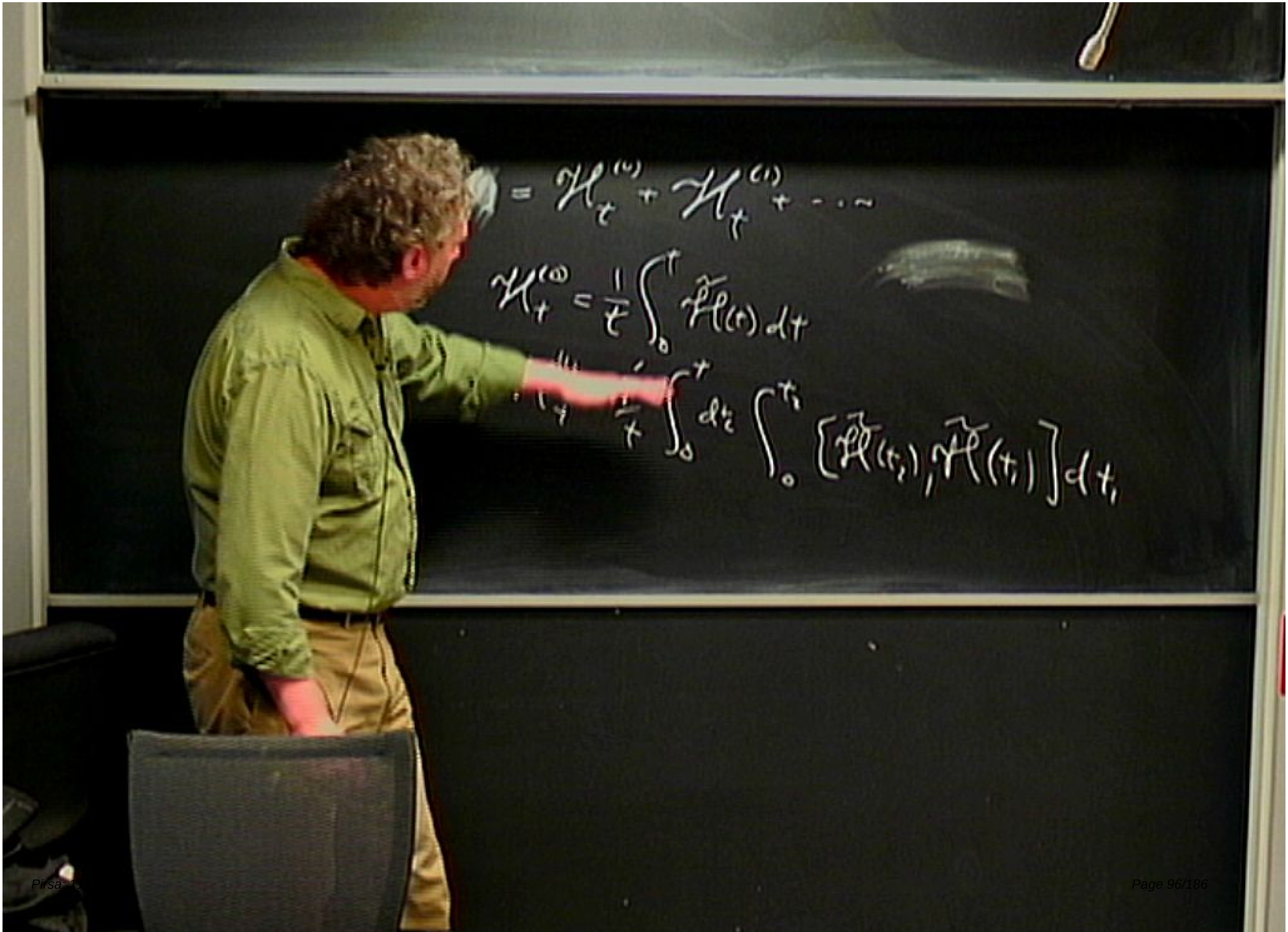


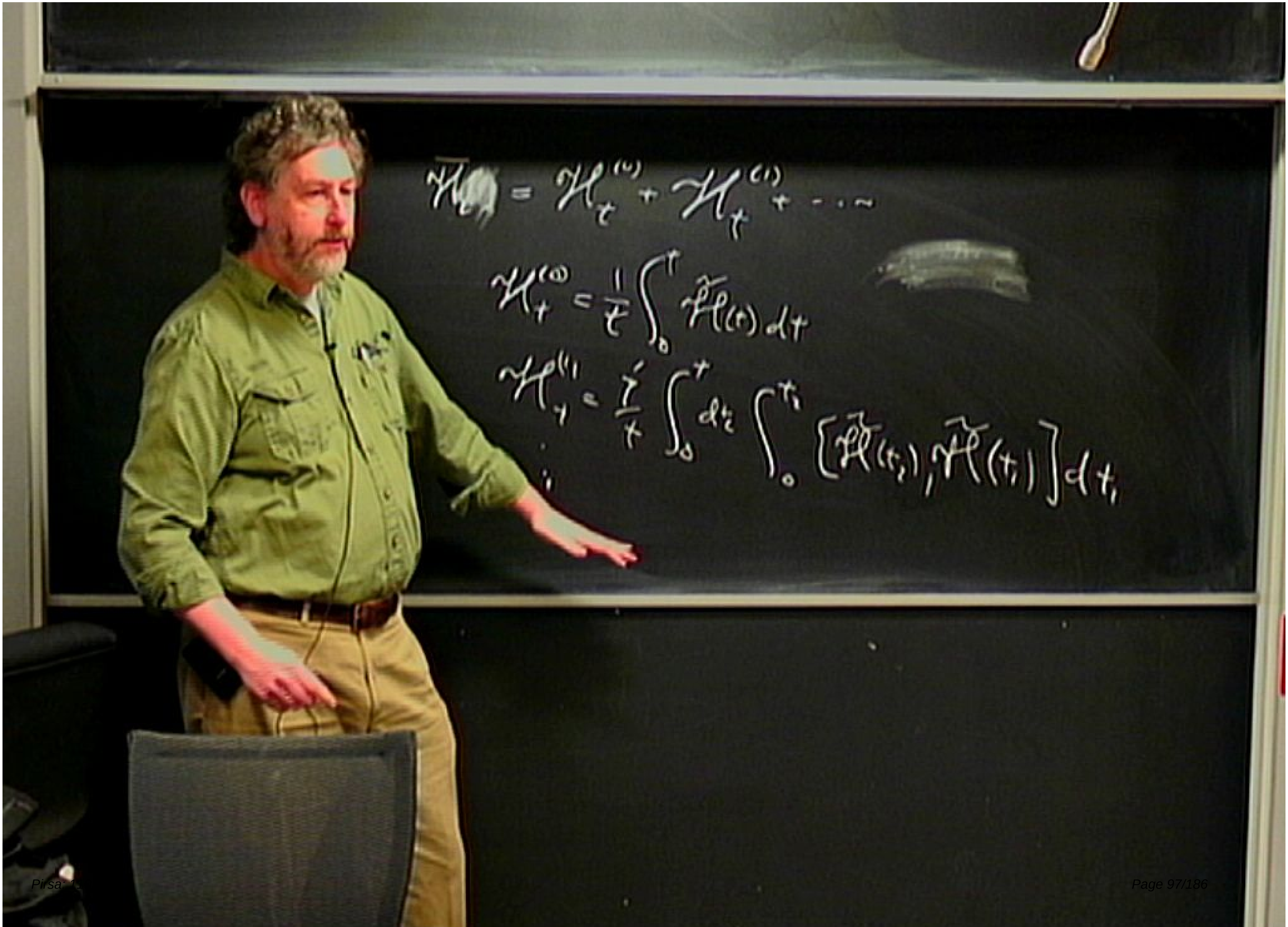
$= \mathcal{H}_t^{(0)} + \mathcal{H}_t^{(1)} + \cdots$
$\mathcal{H}_t^{(0)} = \frac{1}{t}\int^t \tilde{\mathcal{H}}(t)\, dt$

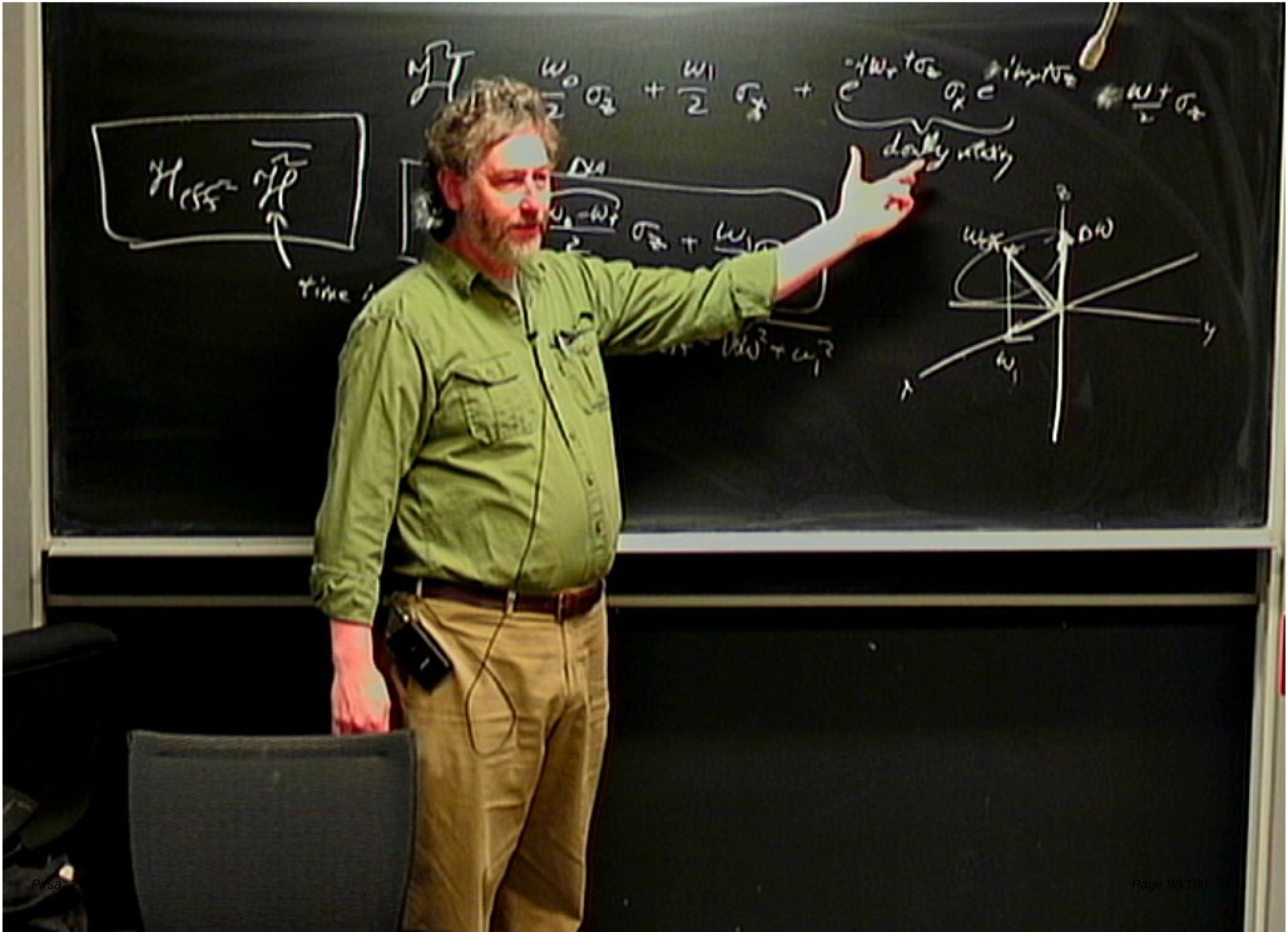

= $\mathcal{H}_t^{(0)} + \mathcal{H}_t^{(1)} + \cdots$
$\mathcal{H}_t^{(0)} = \frac{1}{t}\int_0^t \tilde{\mathcal{H}}(t)\,dt$
$\mathcal{H}_t^{(1)} =$

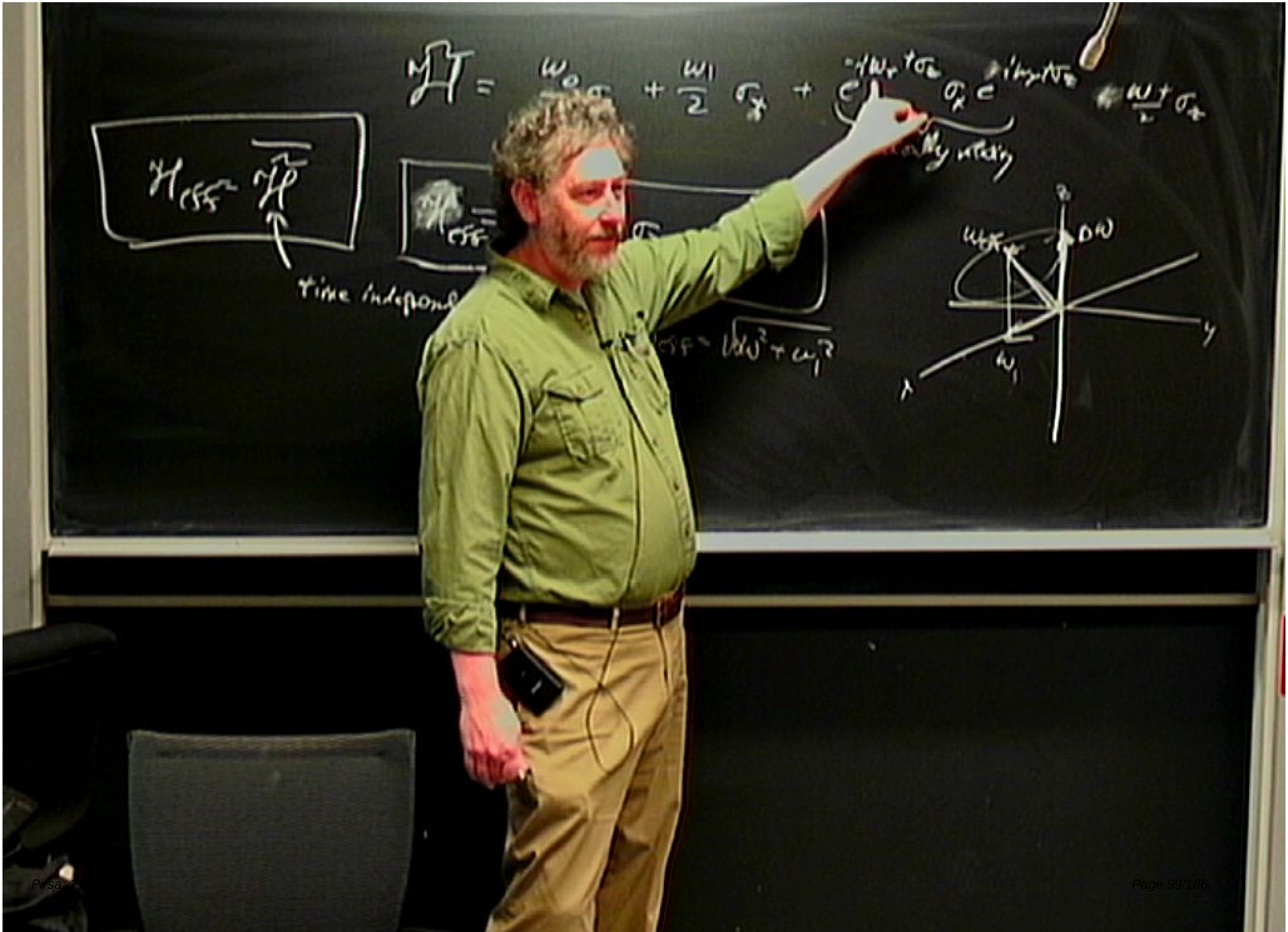


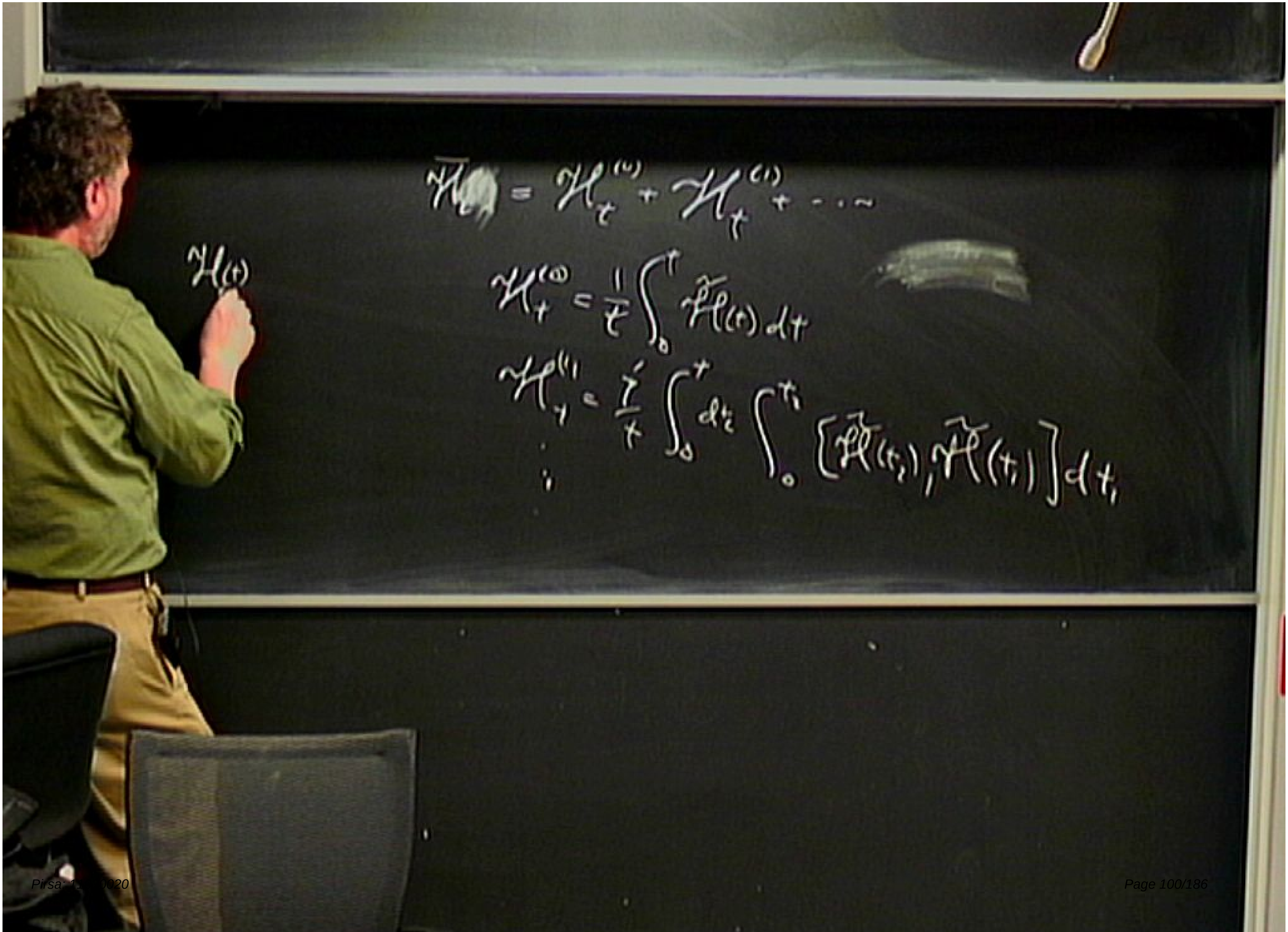
$\bar{\mathcal{H}}_t = \mathcal{H}_t^{(0)} + \mathcal{H}_t^{(1)} + \cdots$
$\mathcal{H}_t^{(0)} = \frac{1}{t}\int_0^t \tilde{\mathcal{H}}(t)\,dt$
$\mathcal{H}_t^{(1)} = \frac{i}{t}\int_0^t dt_2 \int_0^{t_2} [\mathcal{H}(t_2), \mathcal{H}(t_1)]\,dt_1$

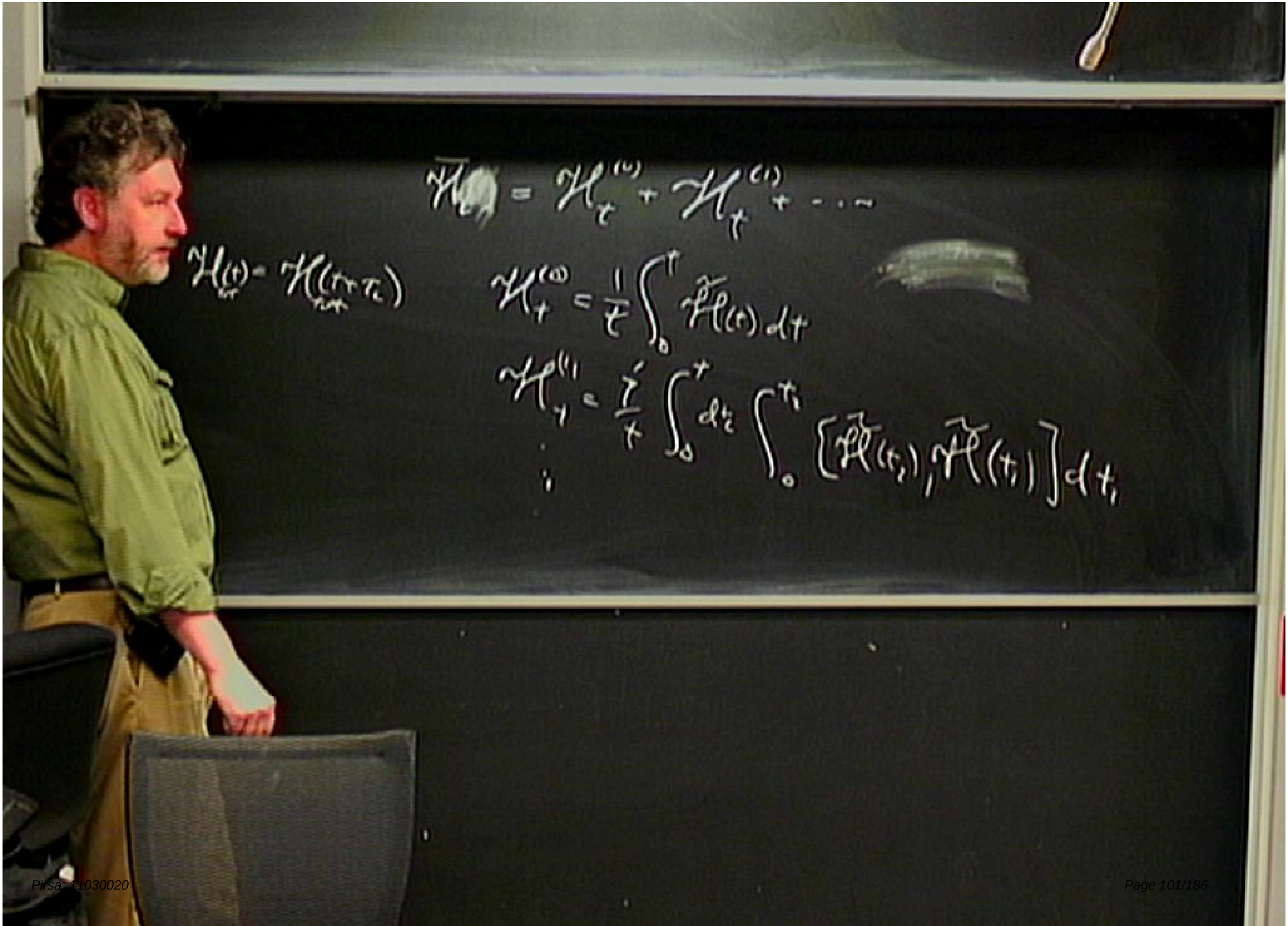

= $\mathcal{H}_t^{(0)} + \mathcal{H}_t^{(1)} + \cdots$
$\mathcal{H}_t^{(0)} = \frac{1}{t}\int_0^t \tilde{\mathcal{H}}(t)\,dt$
$\int_0^t dt_2 \int_0^{t_2} [\tilde{\mathcal{H}}(t_2), \tilde{\mathcal{H}}(t_1)]\,dt_1$

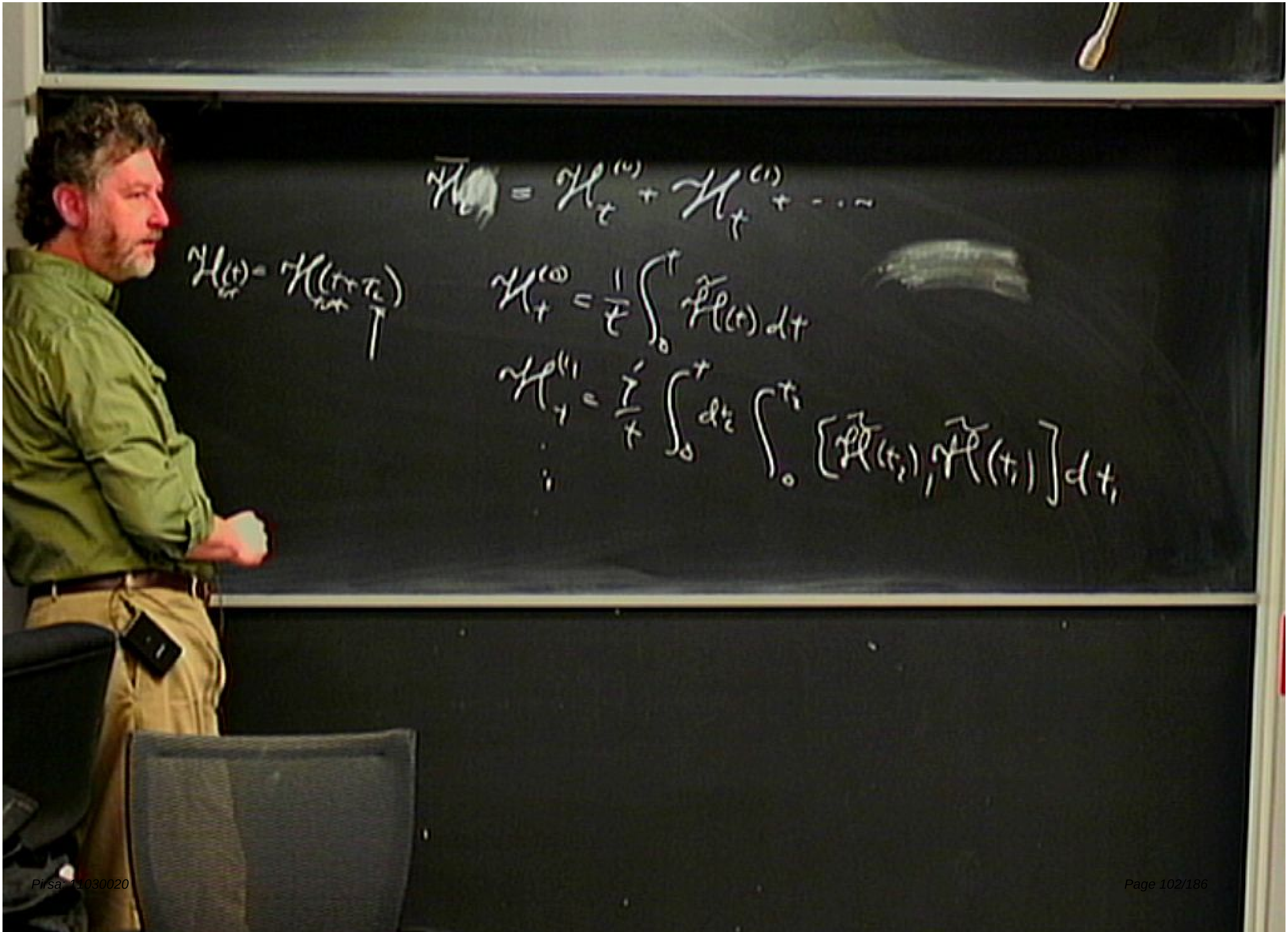


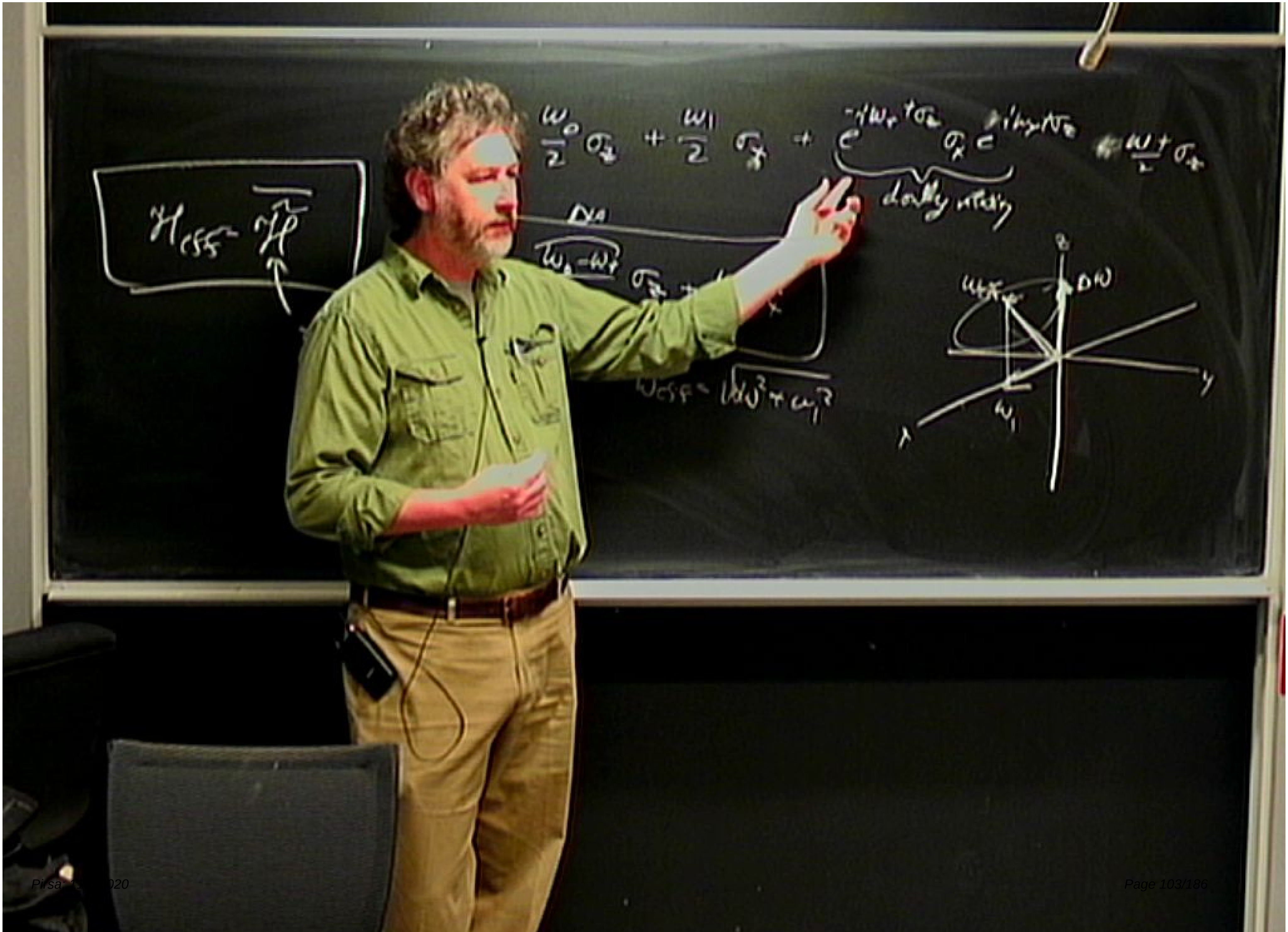


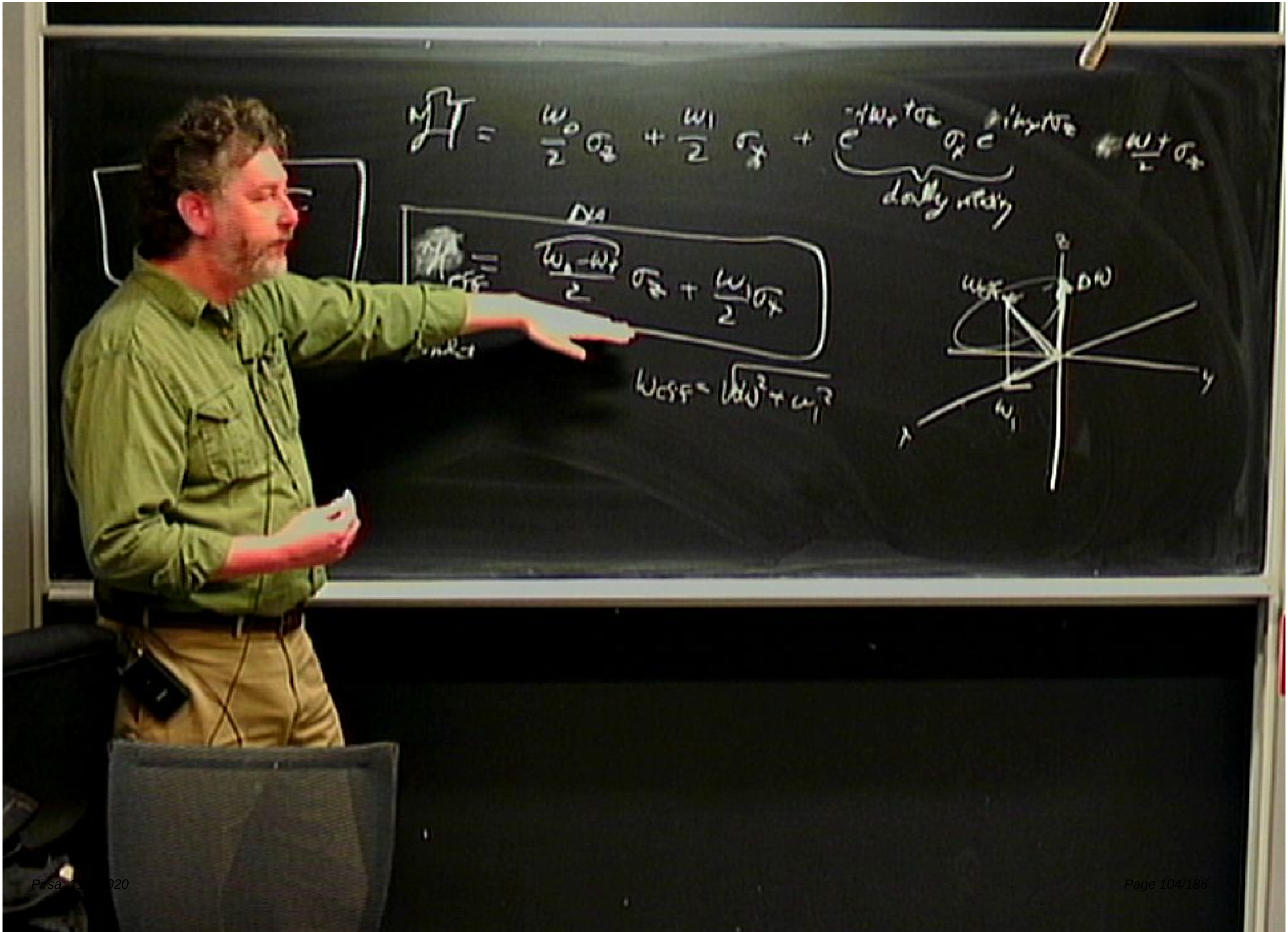


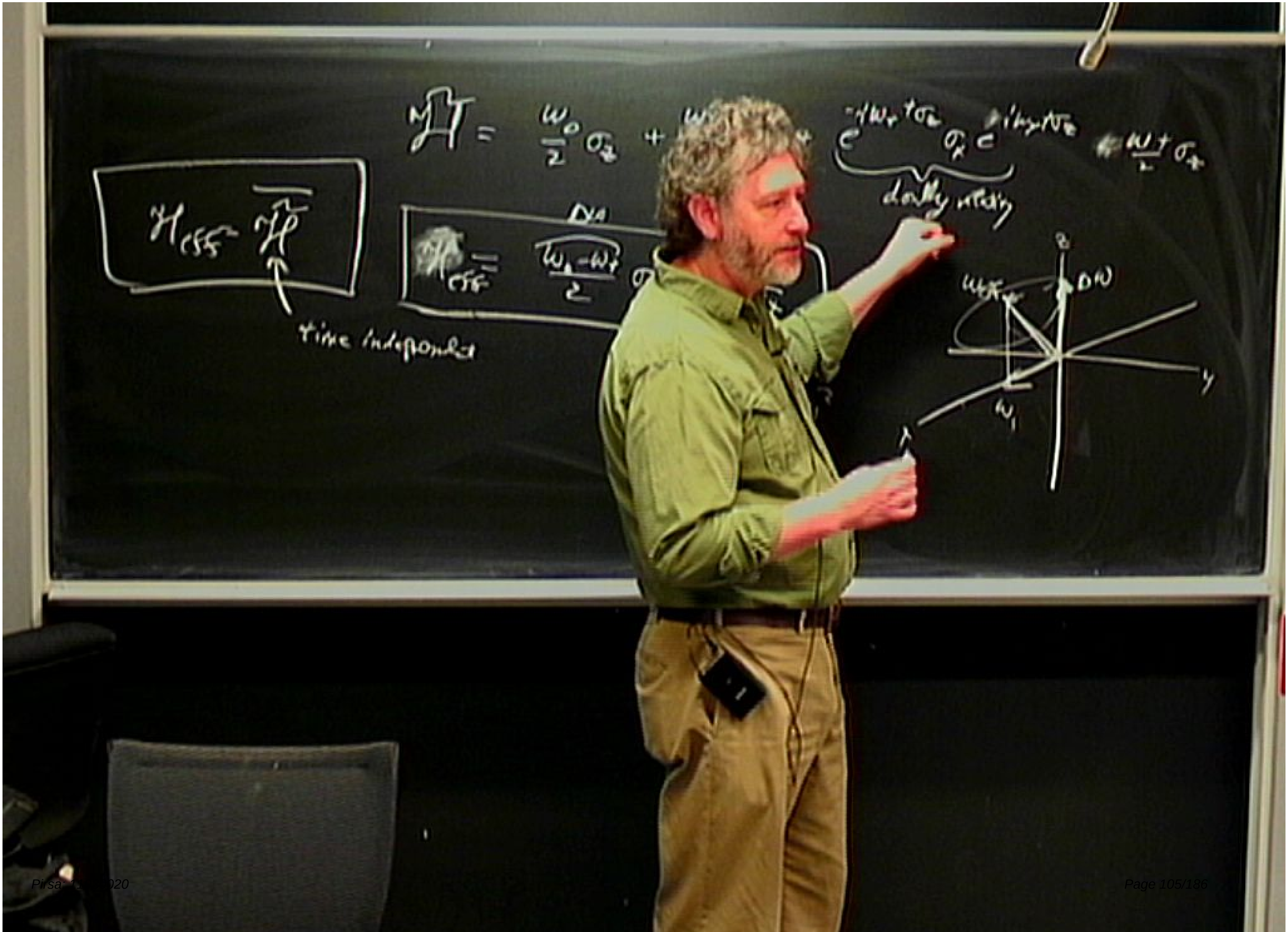

$\bar{\mathcal{H}}_t = \mathcal{H}_t^{(0)} + \mathcal{H}_t^{(1)} + \cdots$
$\mathcal{H}(t)$
$\mathcal{H}_t^{(0)} = \frac{1}{t}\int_0^t \tilde{\mathcal{H}}(t)\,dt$
$\mathcal{H}_t^{(1)} = \frac{i}{t}\int_0^t dt_2 \int_0^{t_2} [\tilde{\mathcal{H}}(t_2), \tilde{\mathcal{H}}(t_1)]\,dt_1$


$\bar{\mathcal{H}}_t = \mathcal{H}_t^{(0)} + \mathcal{H}_t^{(1)} + \cdots$
$\mathcal{H}(t) = \mathcal{H}(t + t_c)$
$\mathcal{H}_t^{(0)} = \frac{1}{t}\int_0^t \tilde{\mathcal{H}}(t)\,dt$
$\mathcal{H}_t^{(1)} = \frac{i}{t}\int_0^t dt_2 \int_0^{t_2} [\tilde{\mathcal{H}}(t_2), \tilde{\mathcal{H}}(t_1)]\,dt_1$


$\bar{\mathcal{H}}_t = \mathcal{H}_t^{(0)} + \mathcal{H}_t^{(1)} + \cdots$
$\mathcal{H}(t) = \mathcal{H}(t + t_c)$
$\mathcal{H}_t^{(0)} = \frac{1}{t}\int_0^t \tilde{\mathcal{H}}(t)\,dt$
$\mathcal{H}_t^{(1)} = \frac{i}{t}\int_0^t dt_2 \int_0^{t_2} [\tilde{\mathcal{H}}(t_2), \tilde{\mathcal{H}}(t_1)]\,dt_1$



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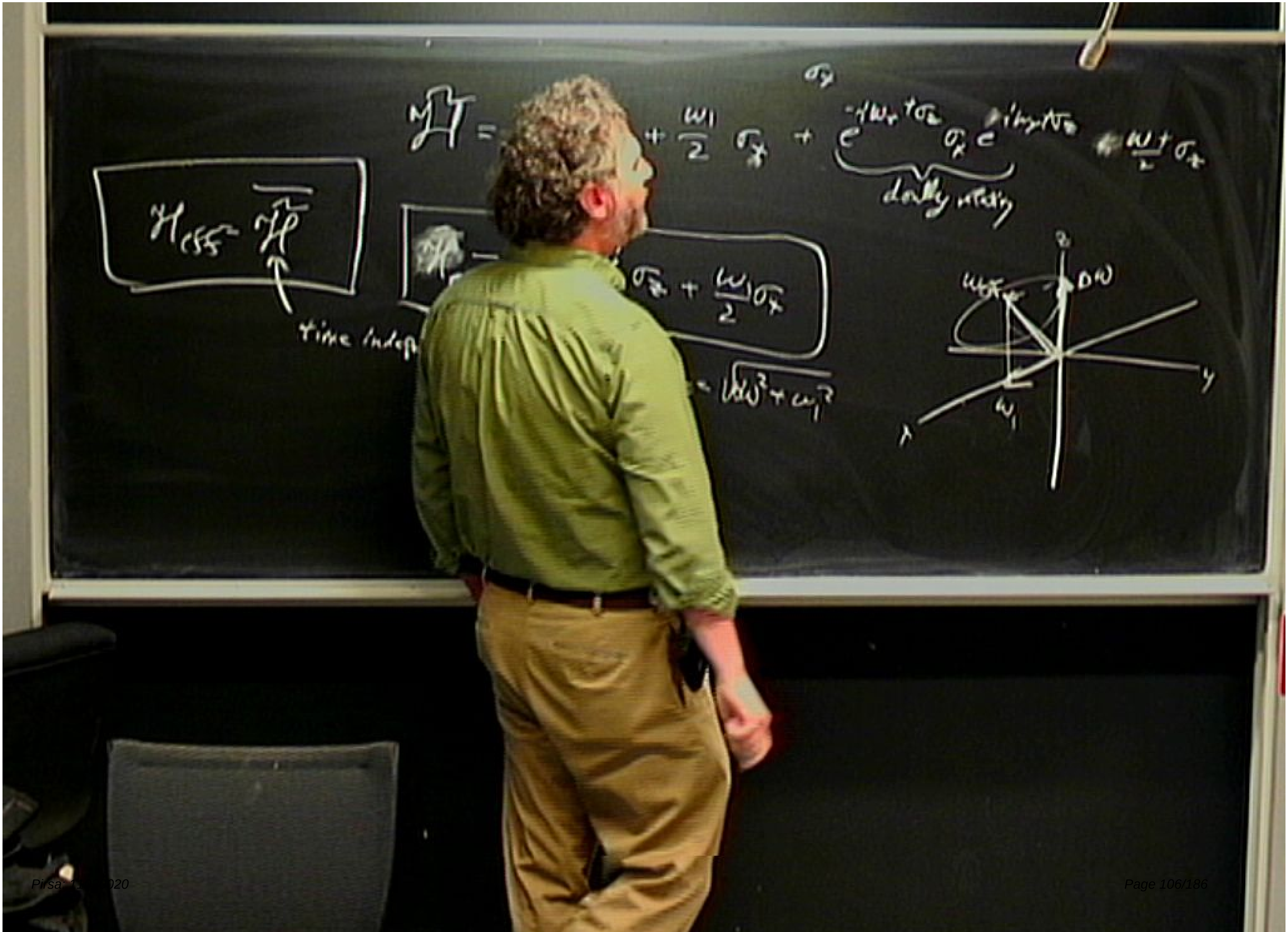
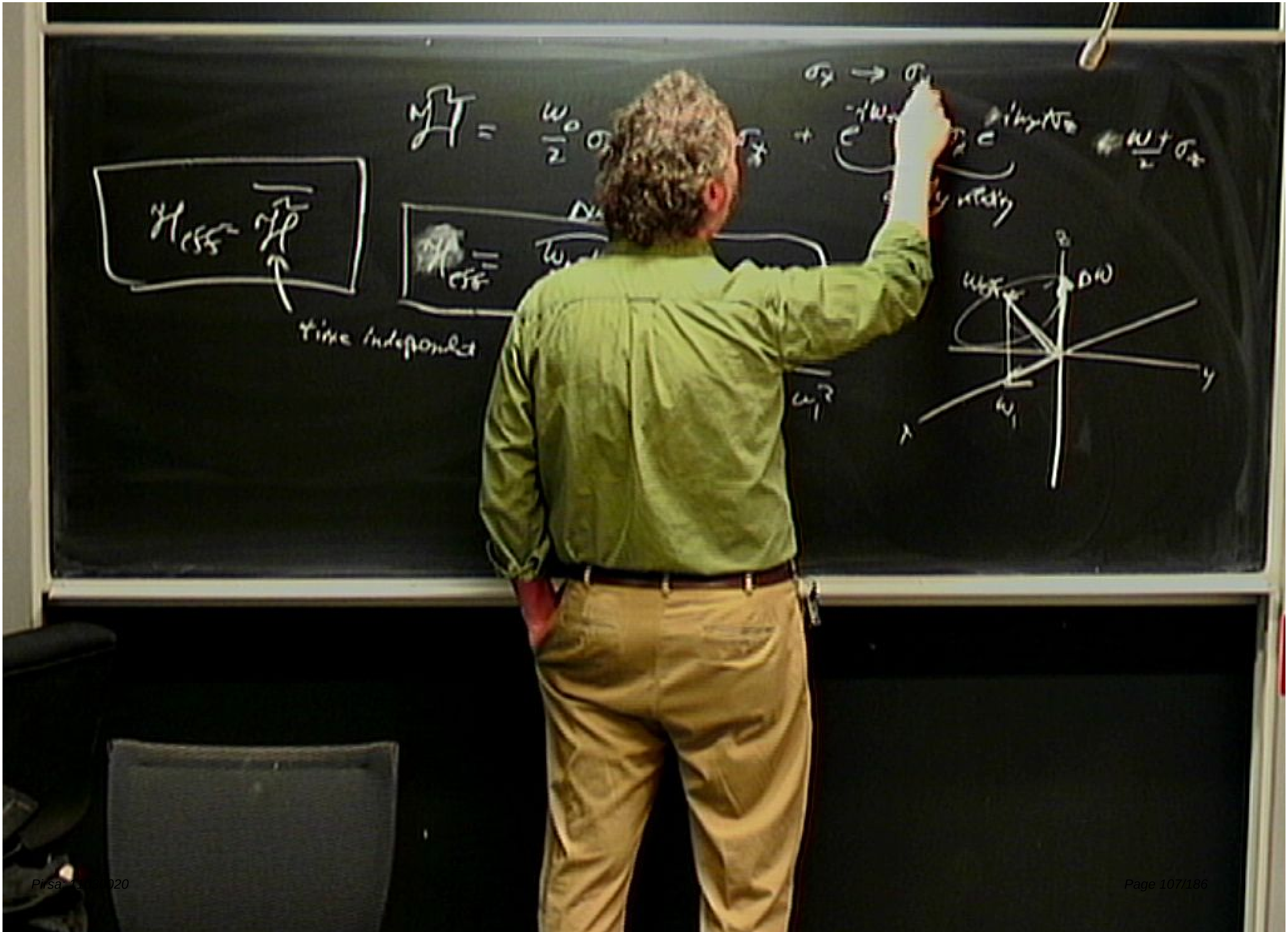
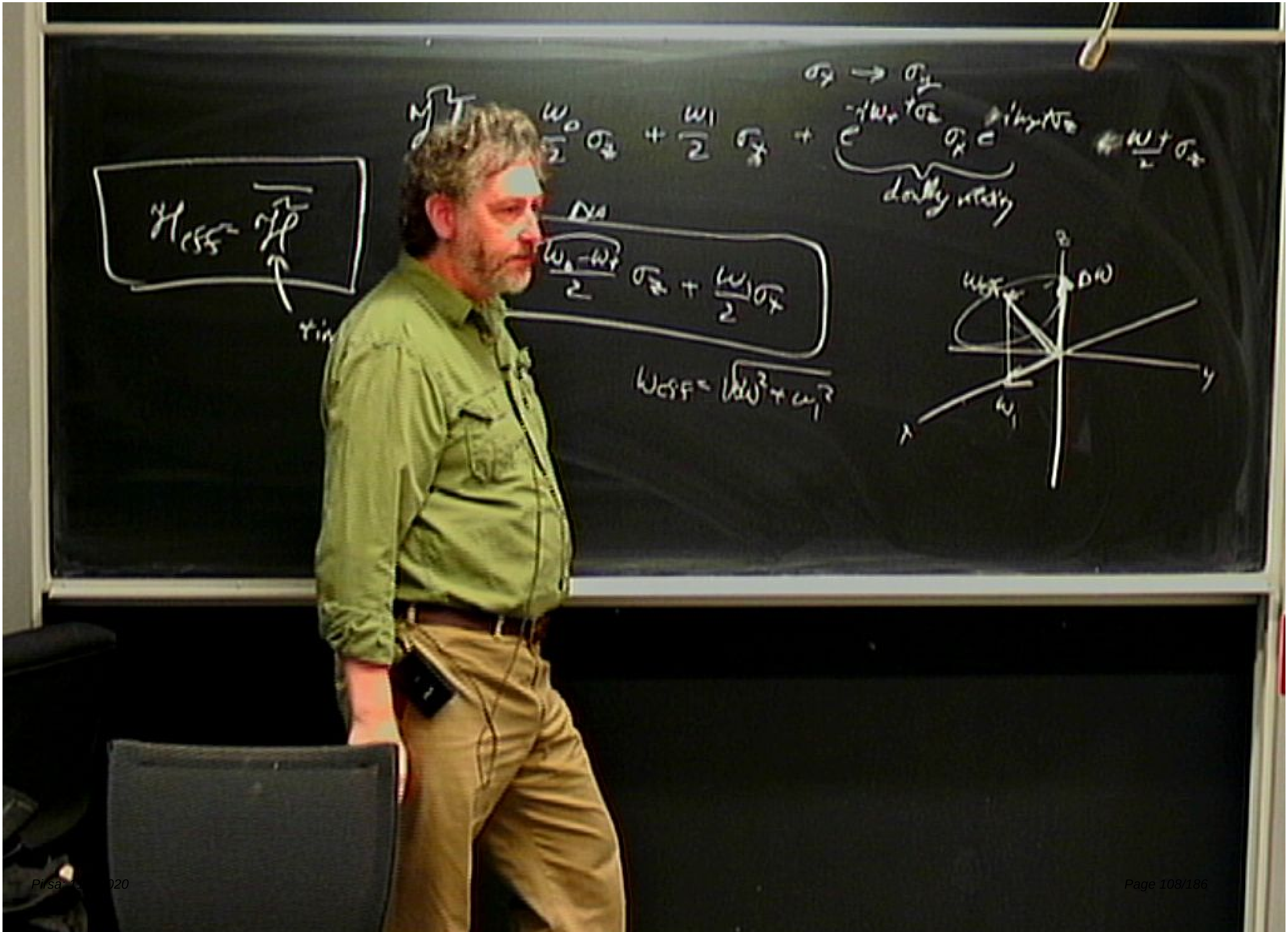

time indep
dually rotating





Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Bloch Siegert shift.nb

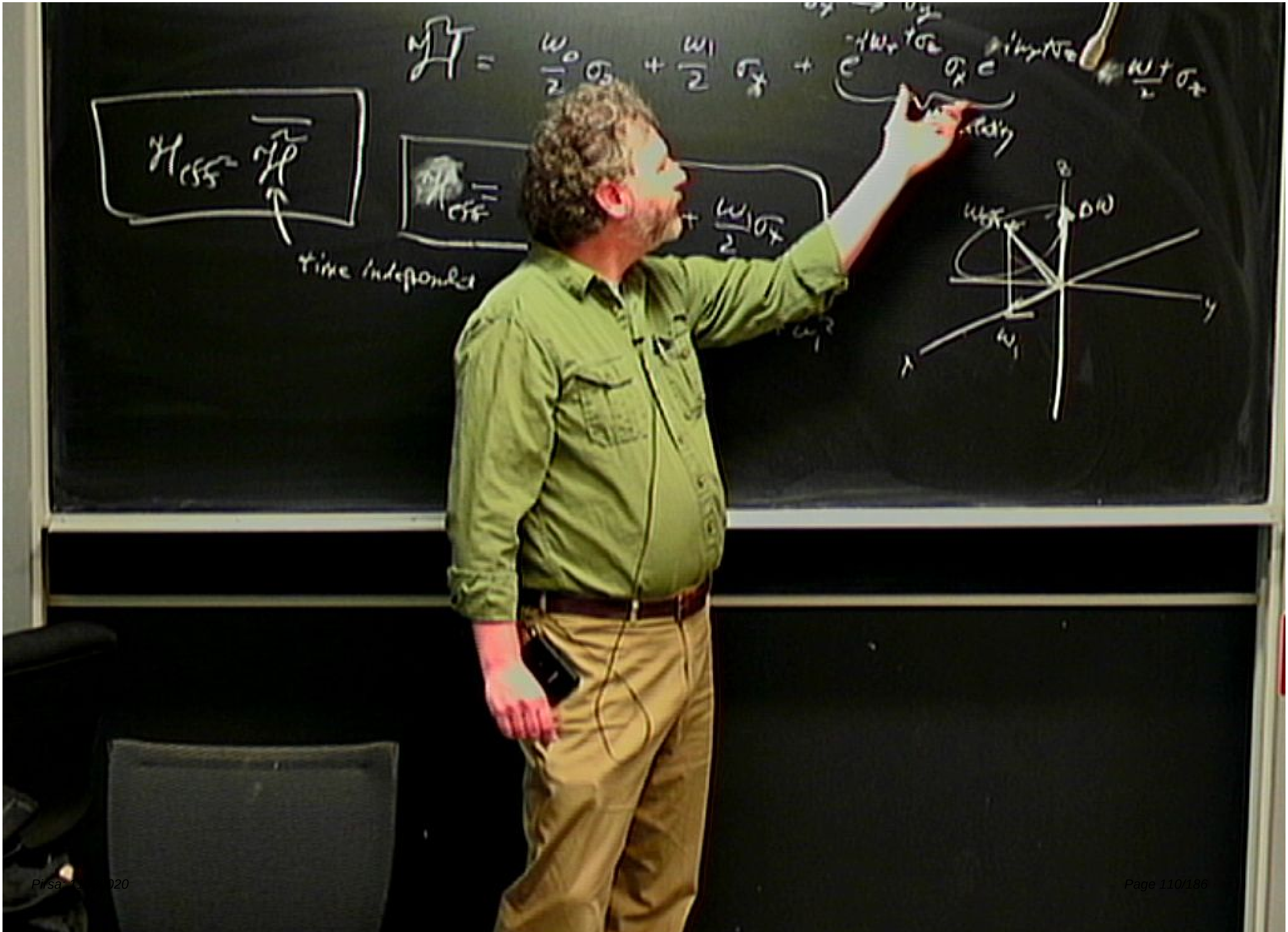
at times t1 and t2.

```
Ht1 = Ht /. {t → t1};
Ht2 = Ht /. {t → t2};
```

The first order term in the magnus expansion is given by

```
Hb1 =
 Simplify[
  (-I / 2 / tc)
   Integrate[Integrate[comm[Ht2, Ht1], {t1, 0, t2}],
    {t2, 0, tc}], Element[{w1, w0}, Reals]];
Hb1 // MatrixForm
```

$$\begin{pmatrix} \frac{w1^2}{8\,wrf} & 0 \\ 0 & -\frac{w1^2}{8\,wrf} \end{pmatrix}$$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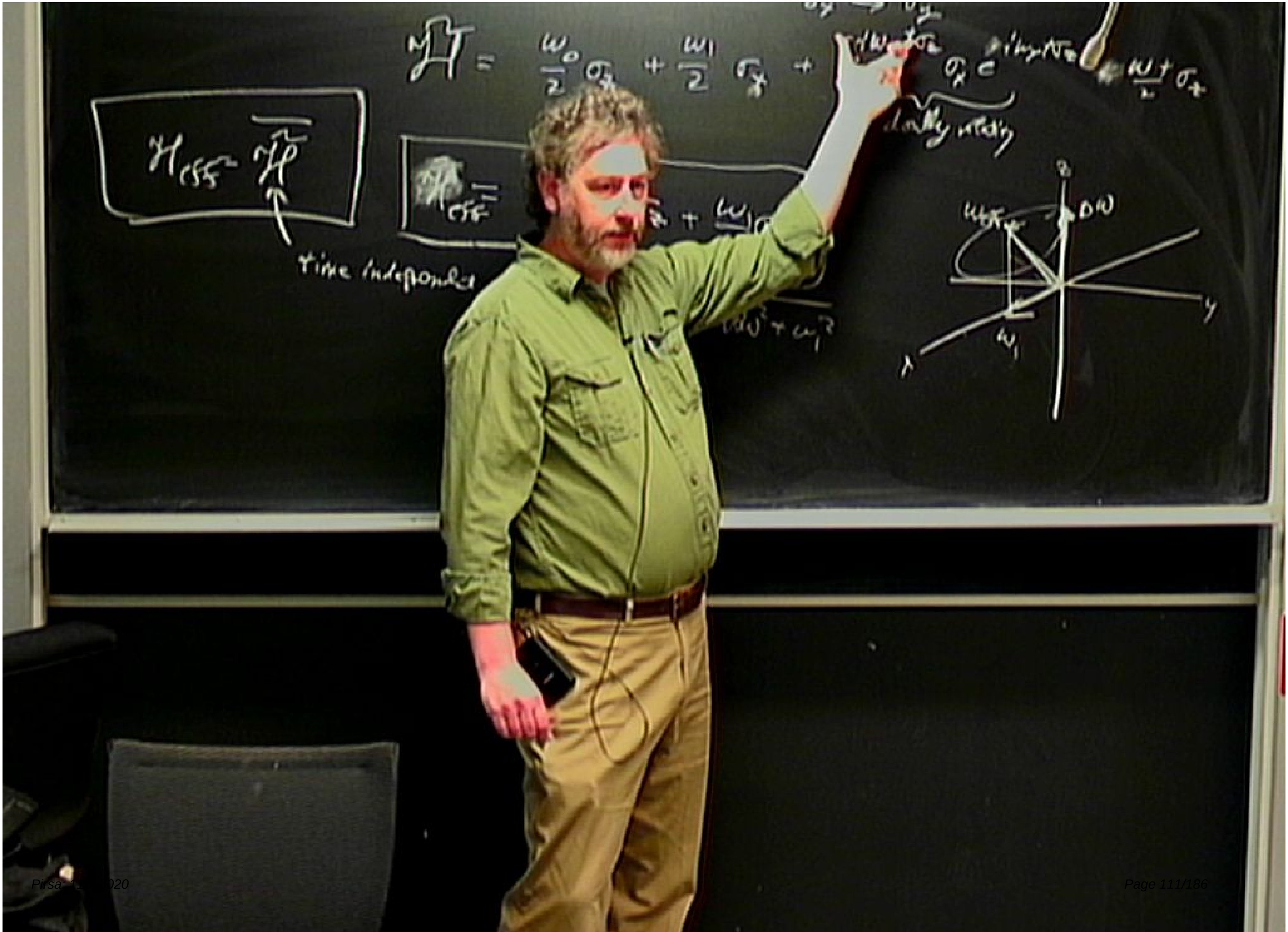
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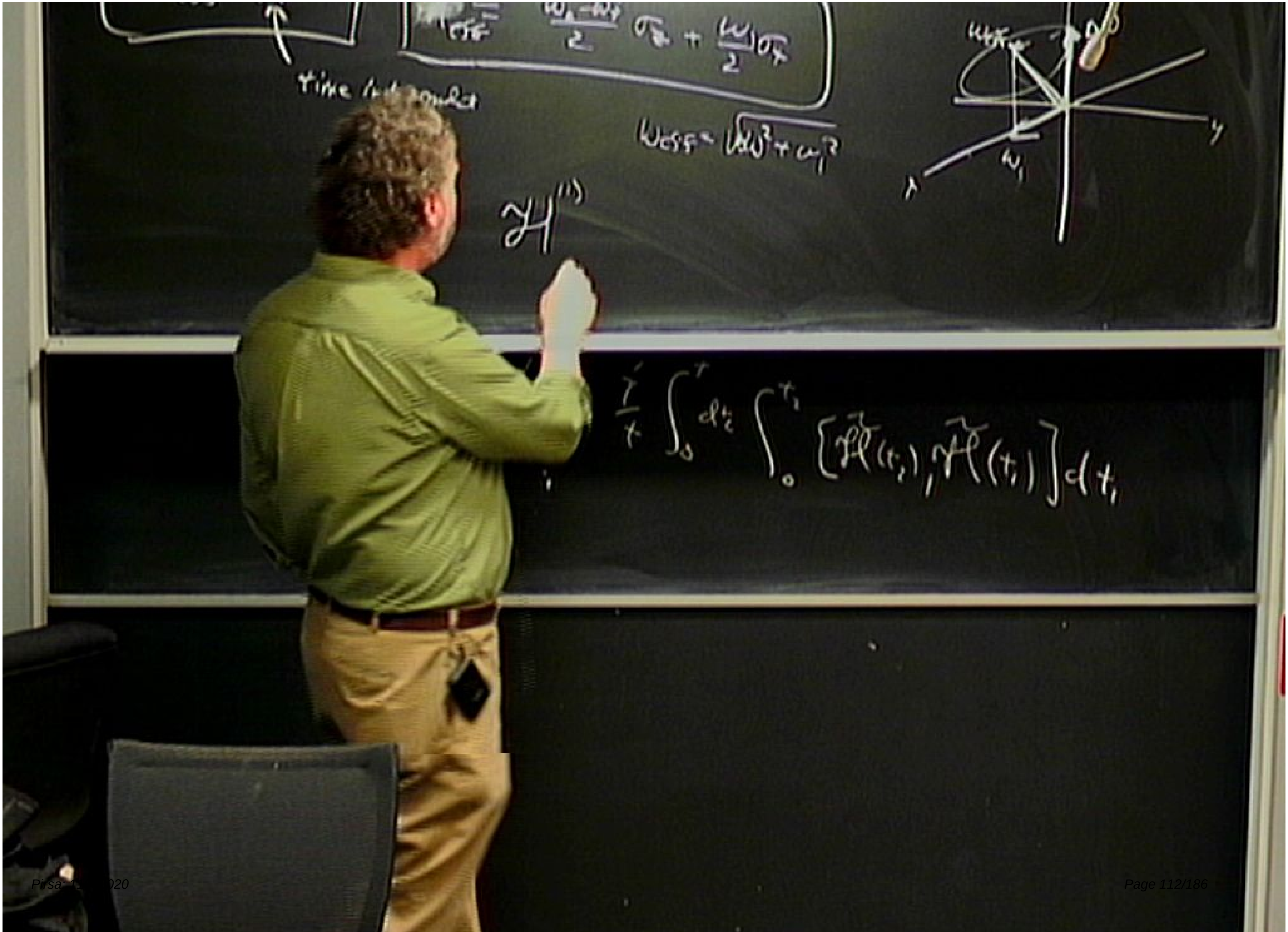

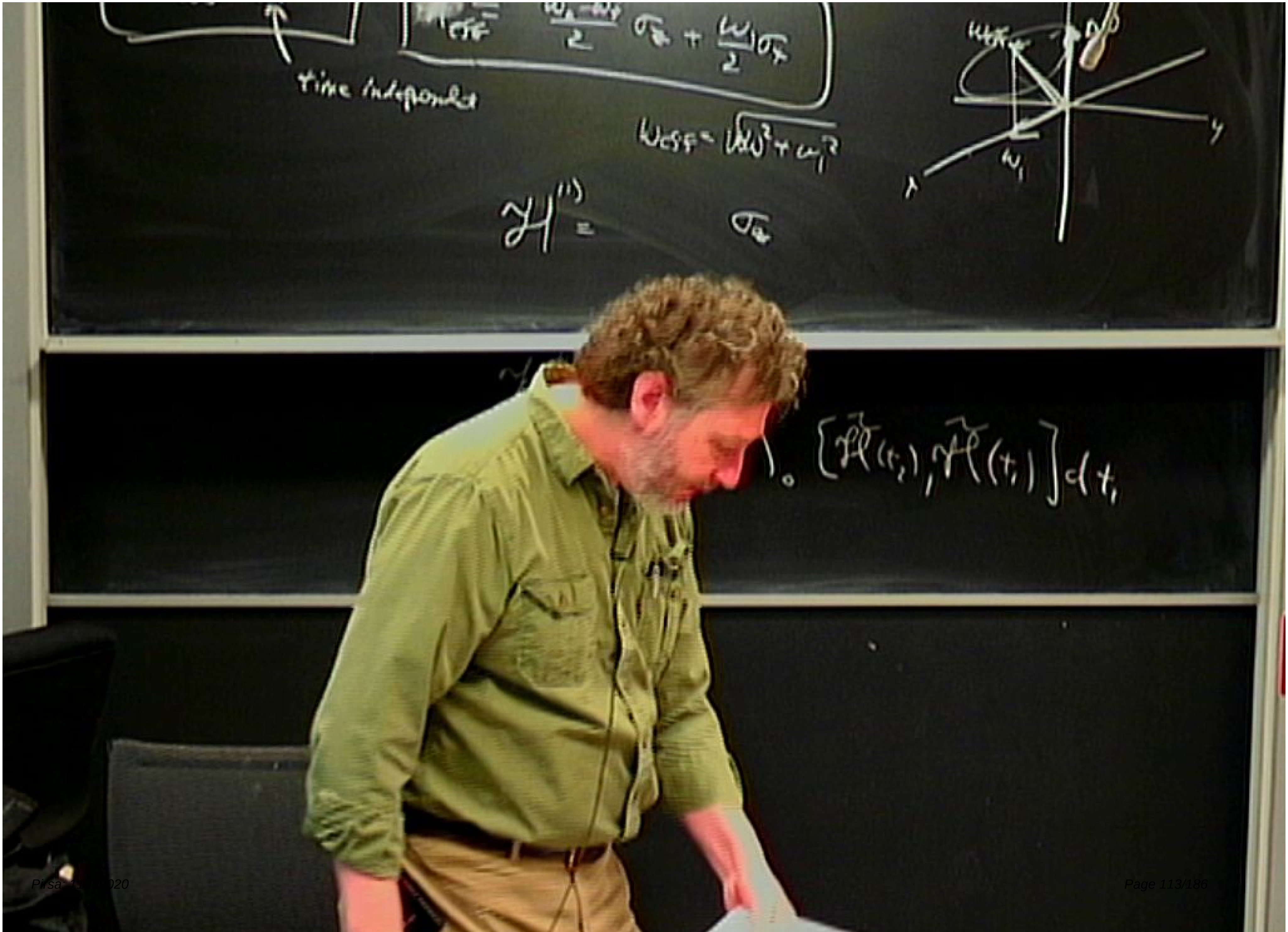

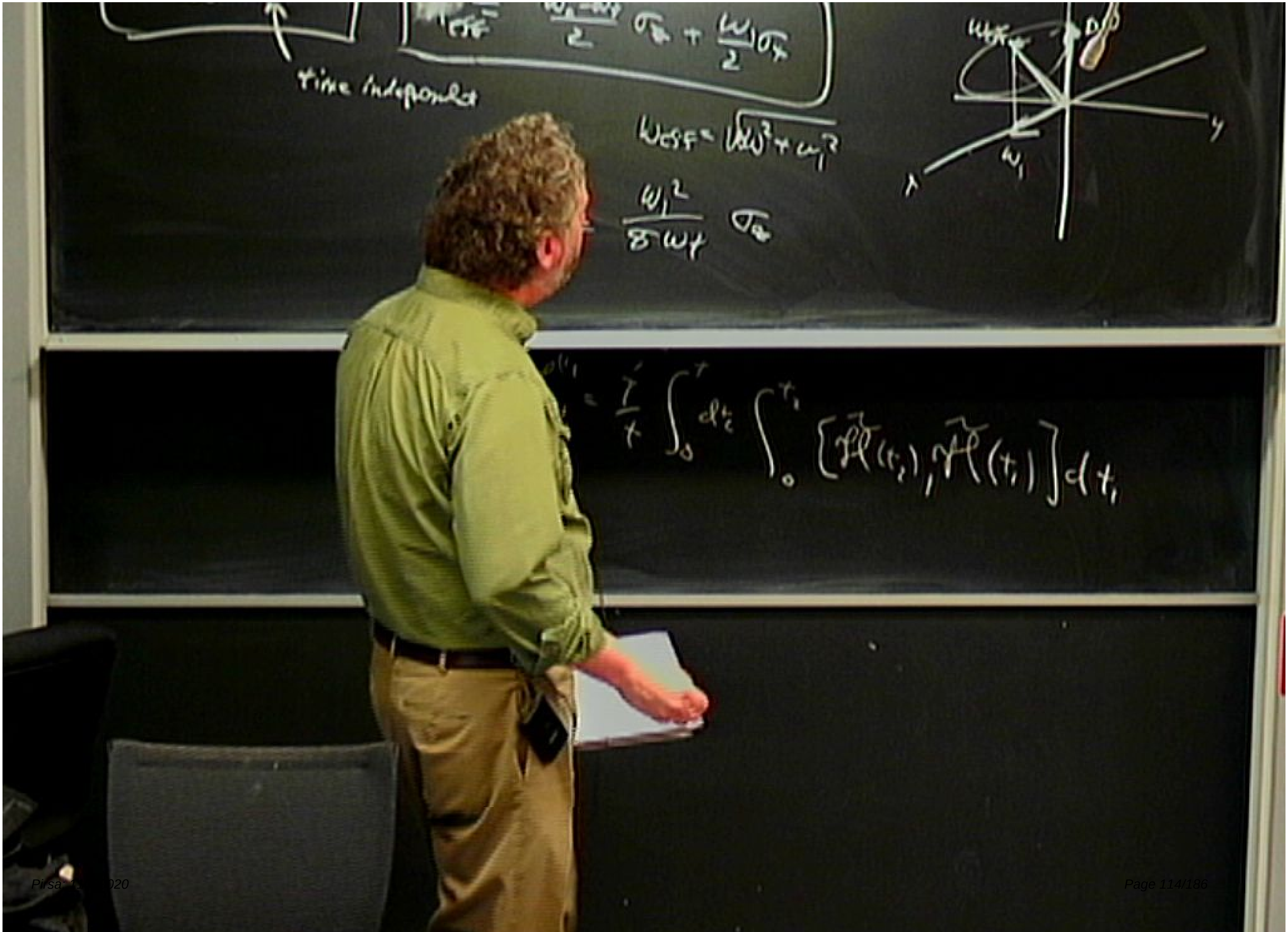
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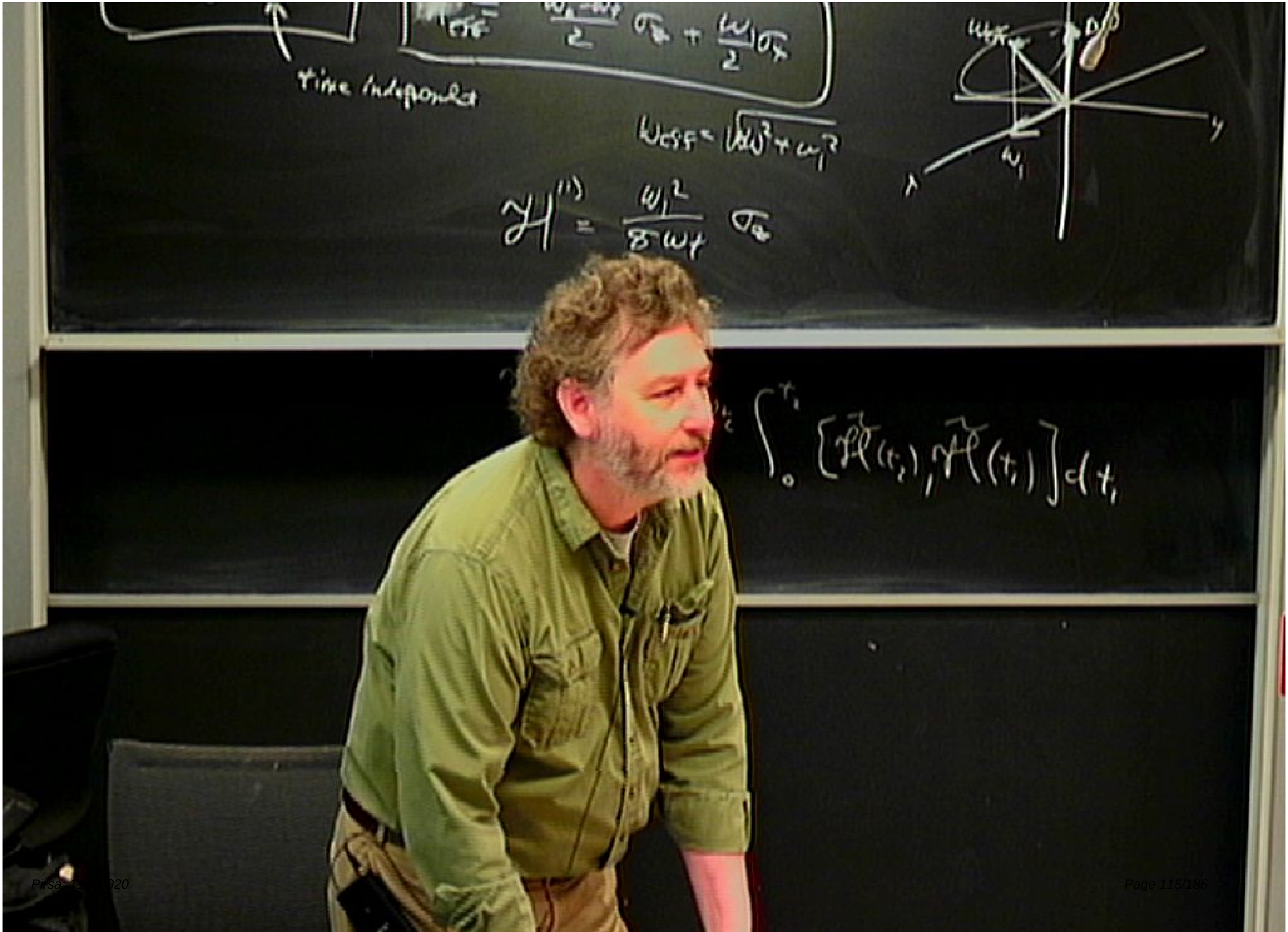
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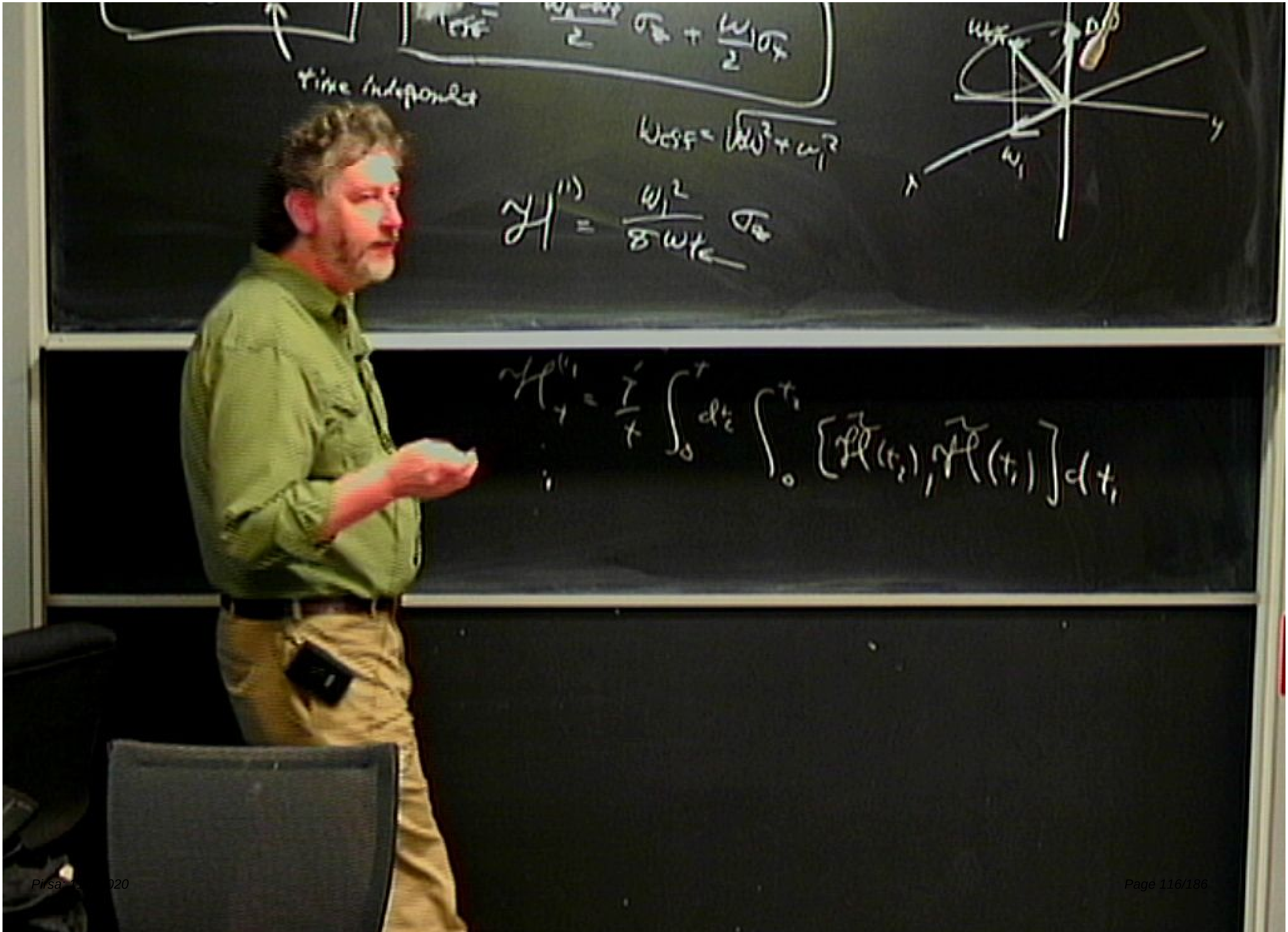
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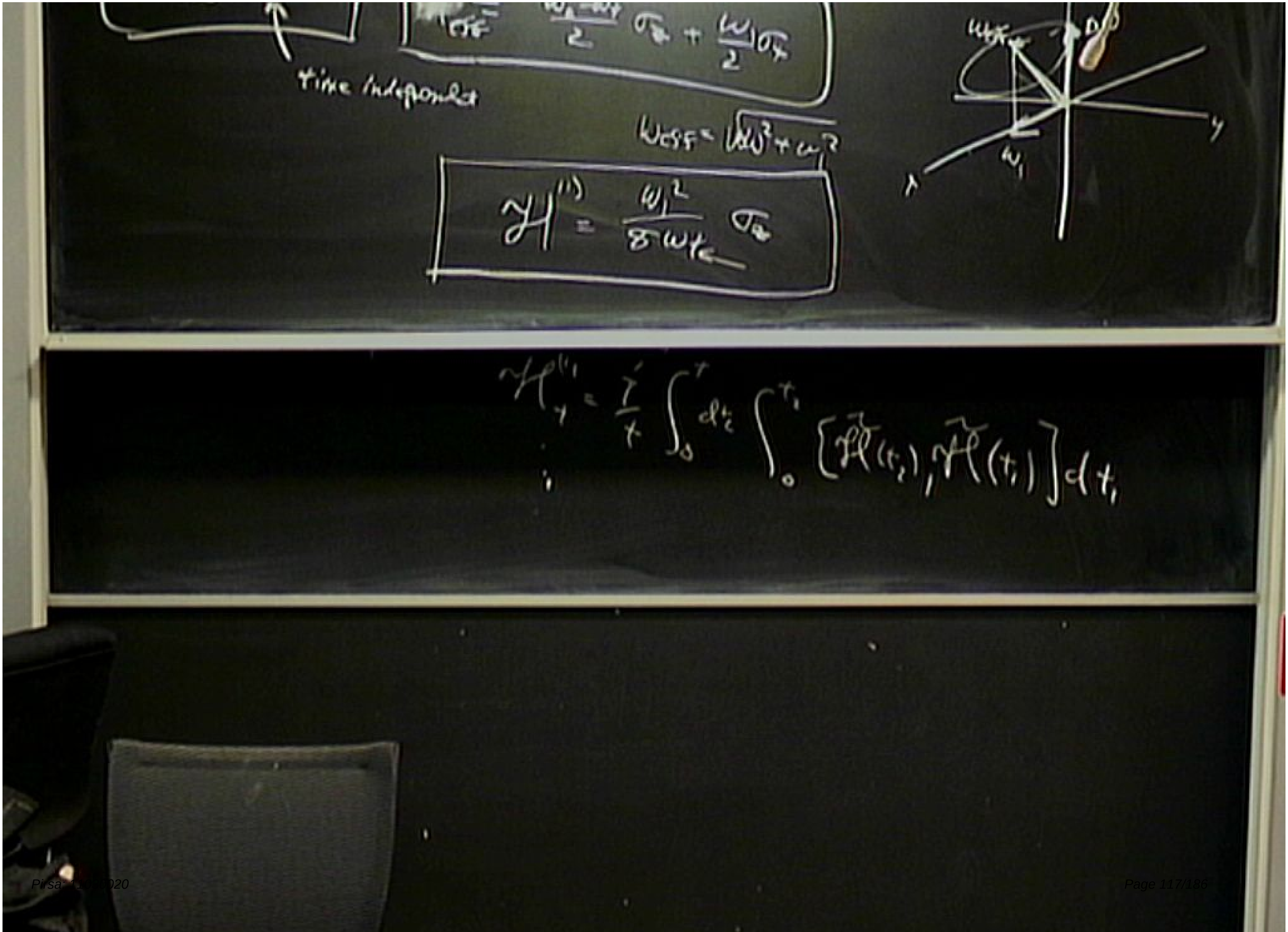
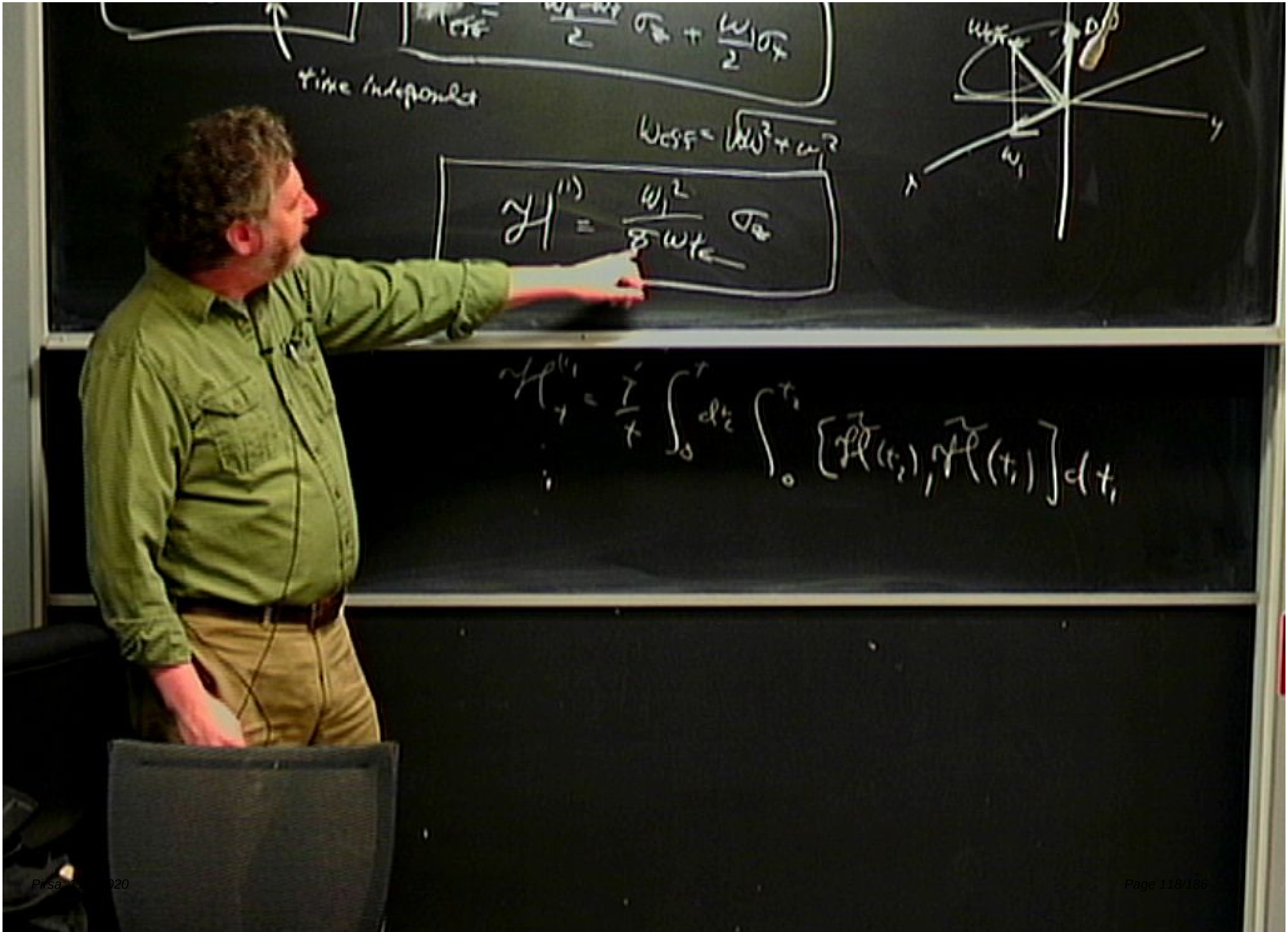

time independent
ω_eff = √(Δω² + ω₁²)
H^(1) = ω₁² / (8 ω_eff) σ_z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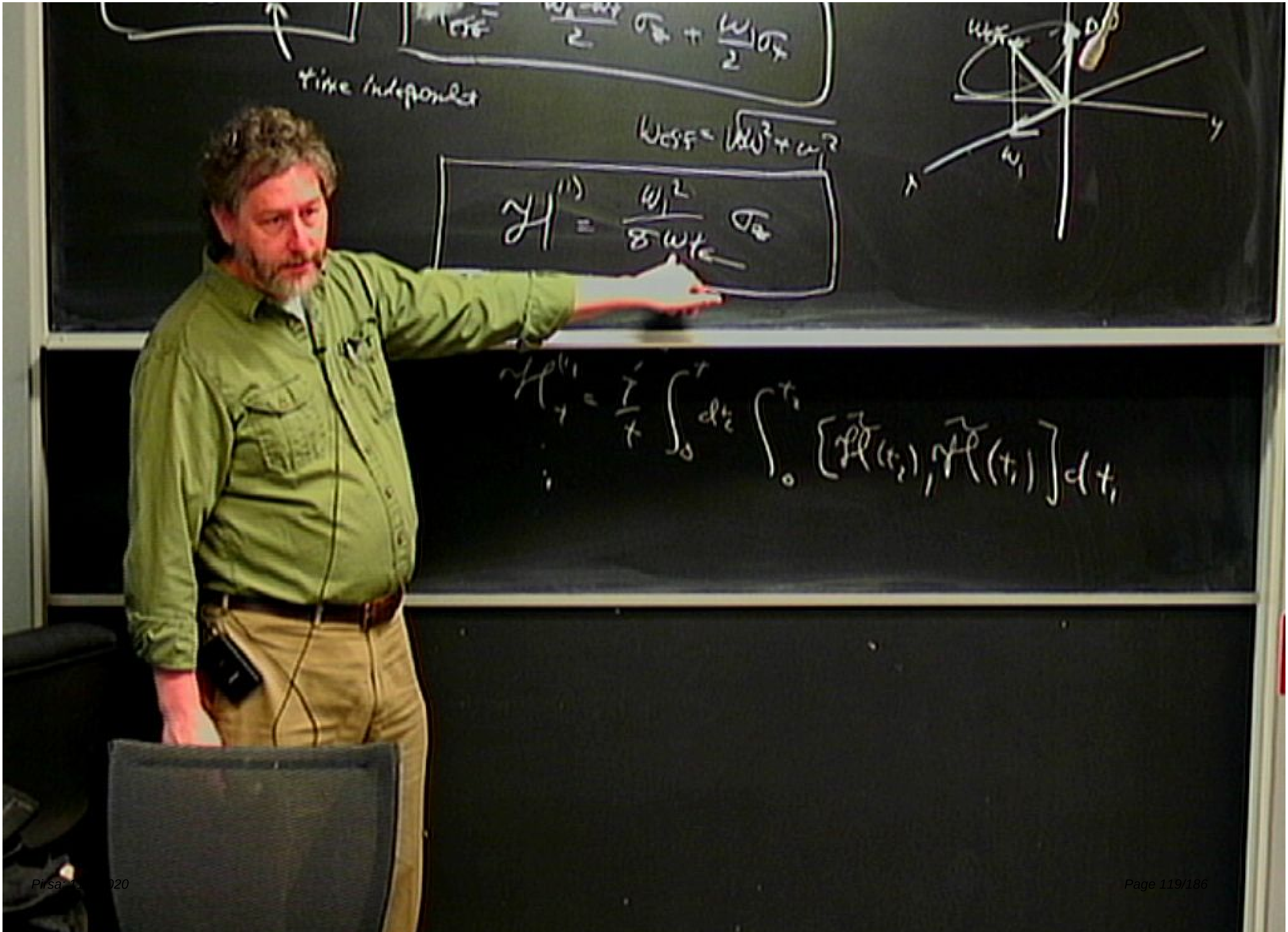
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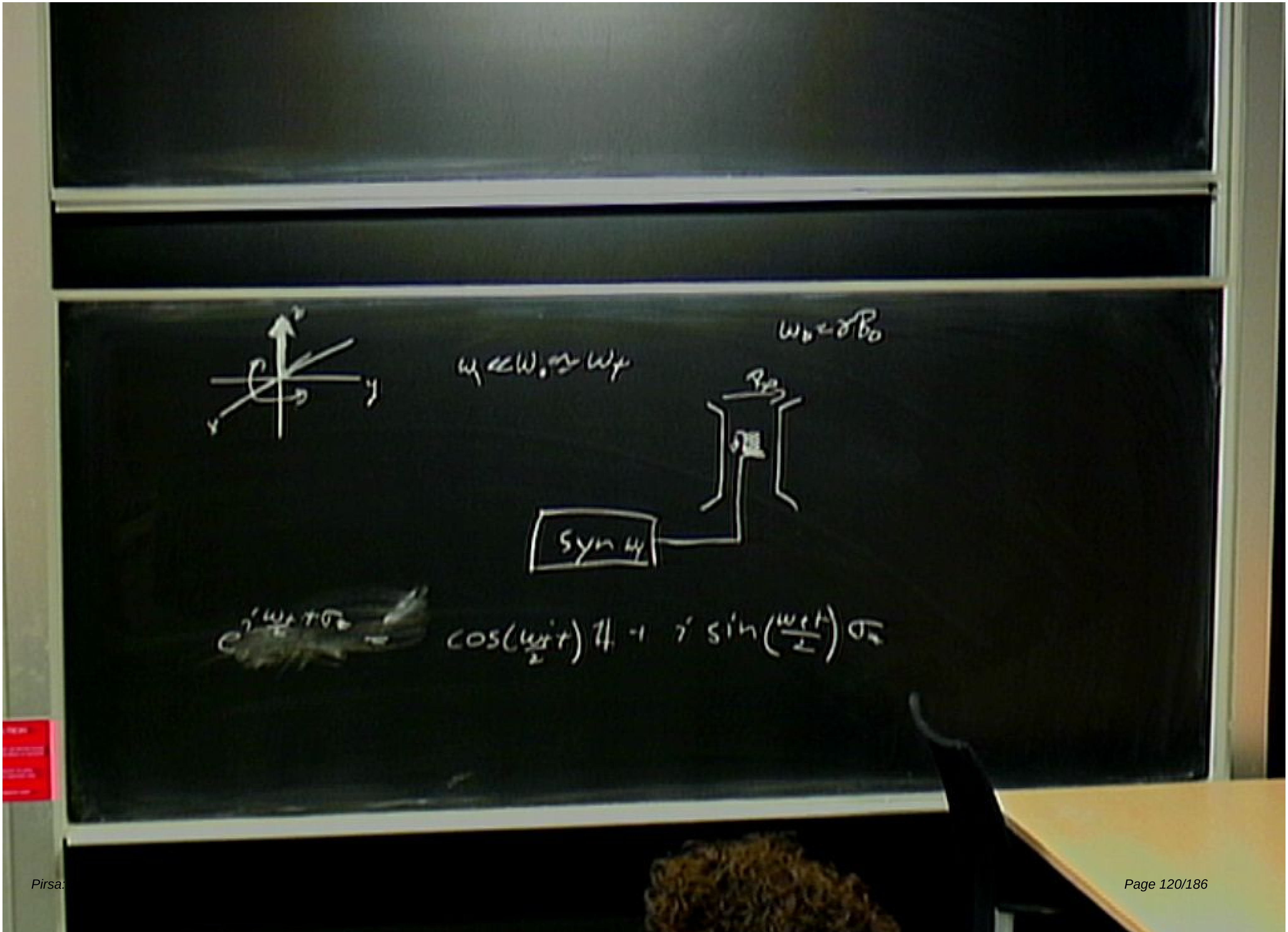
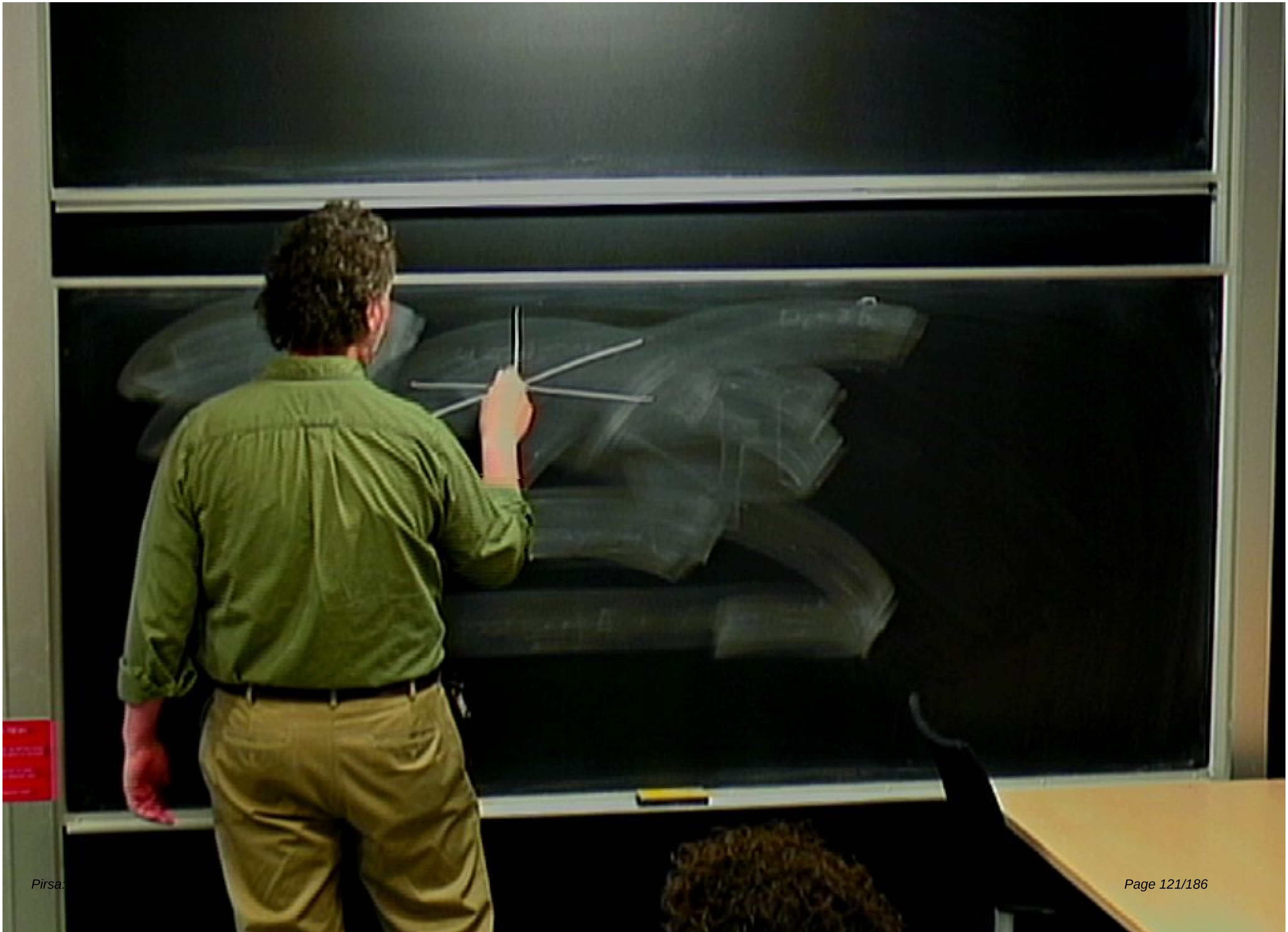
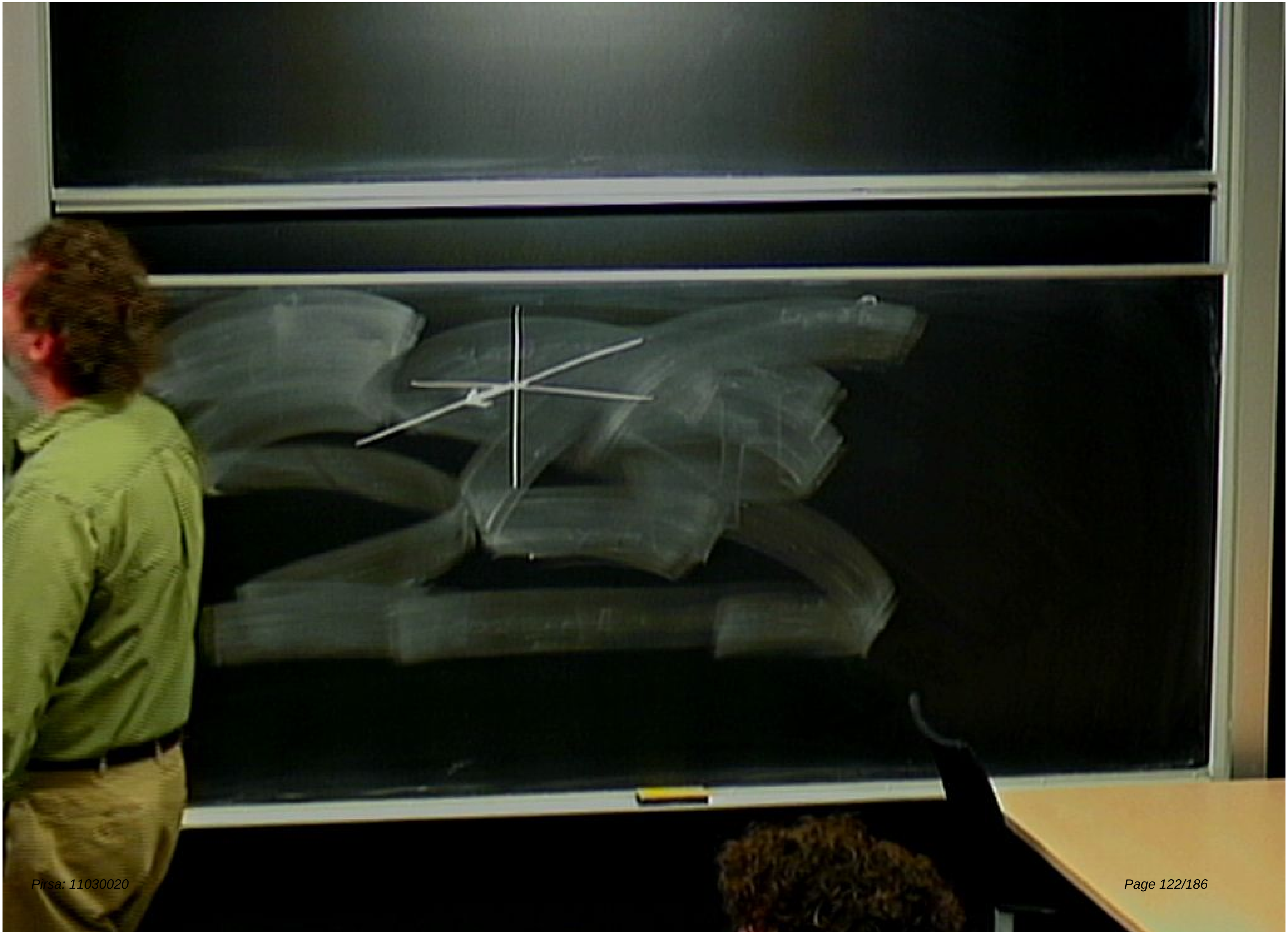
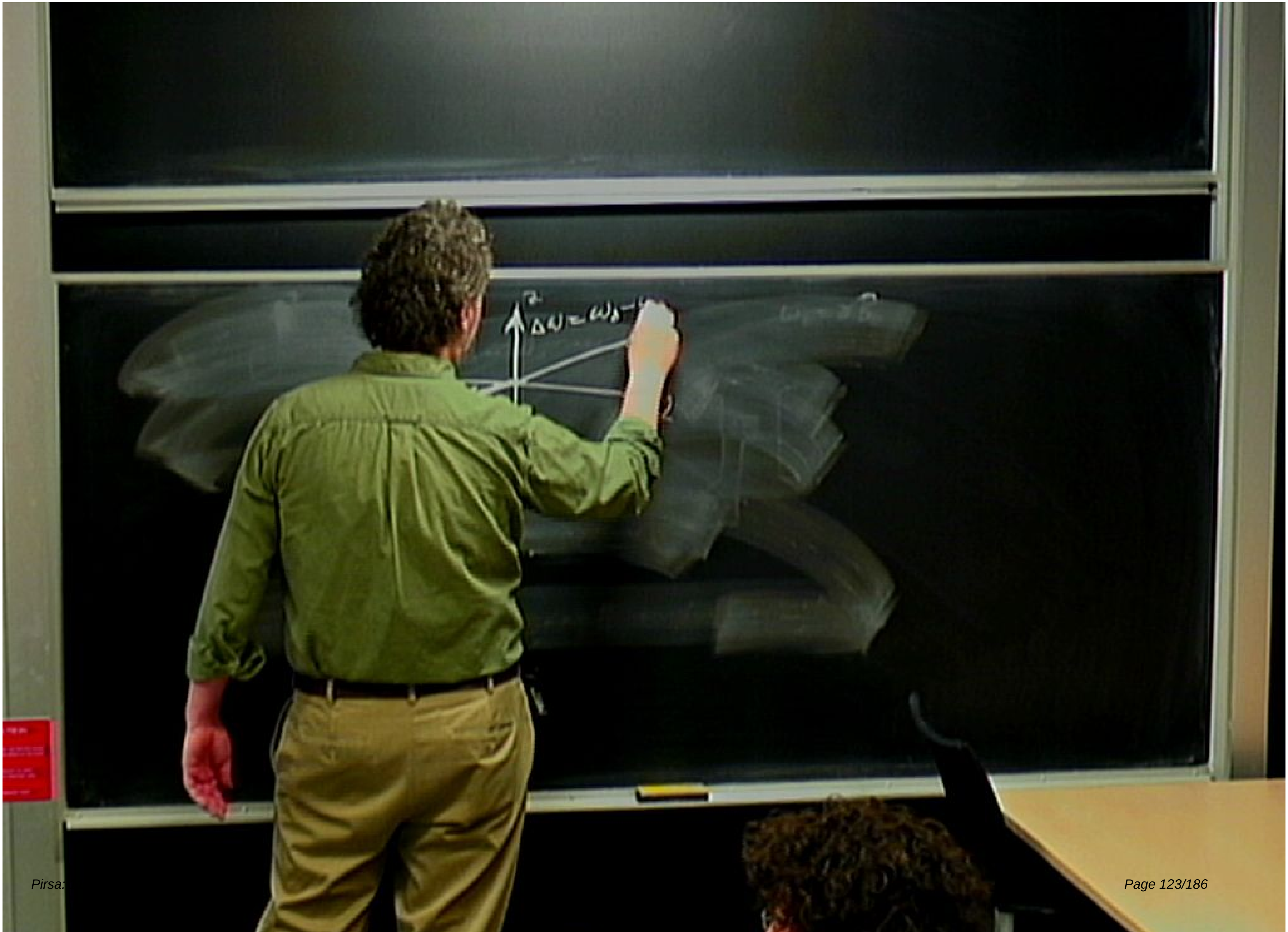
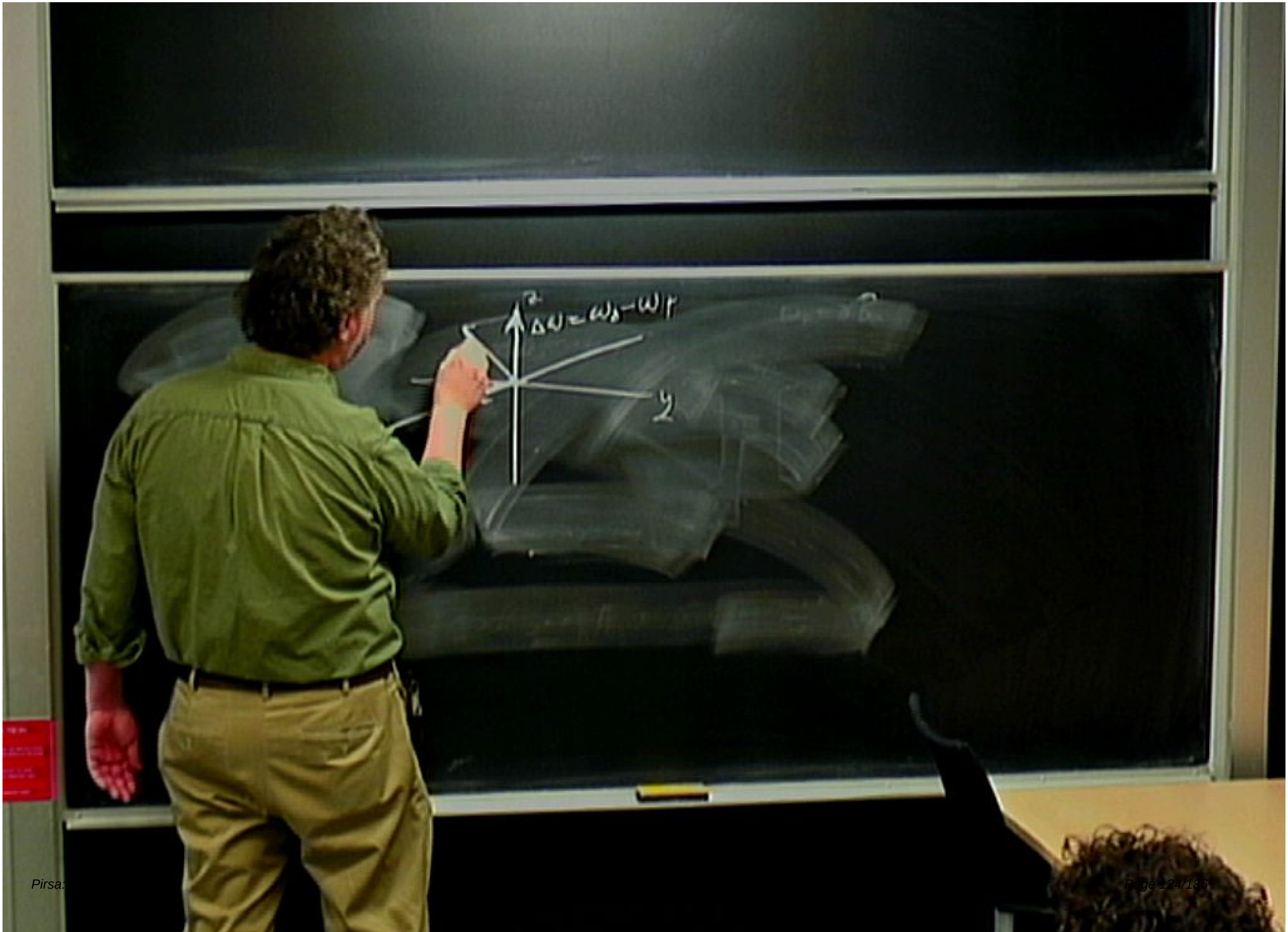
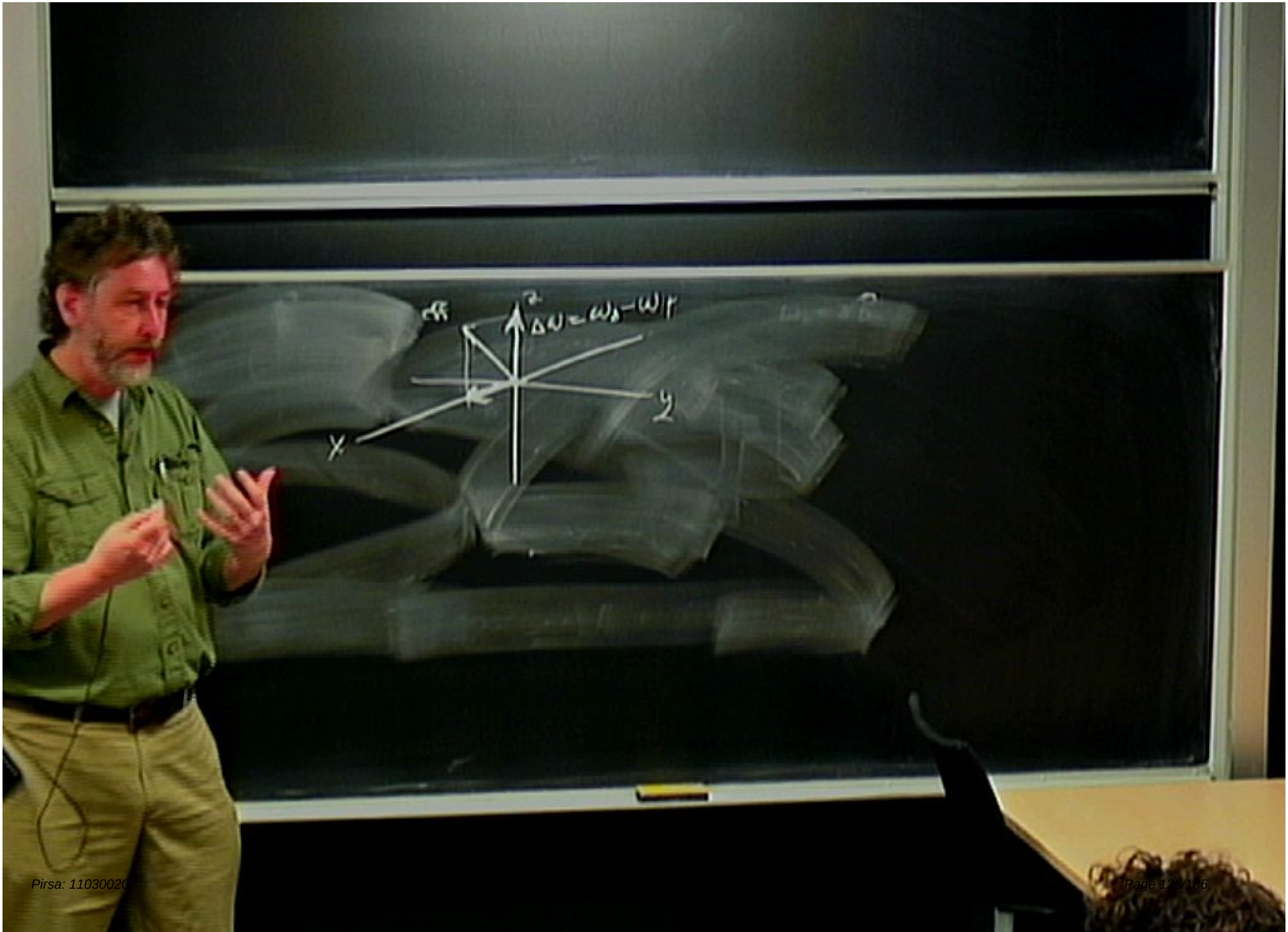


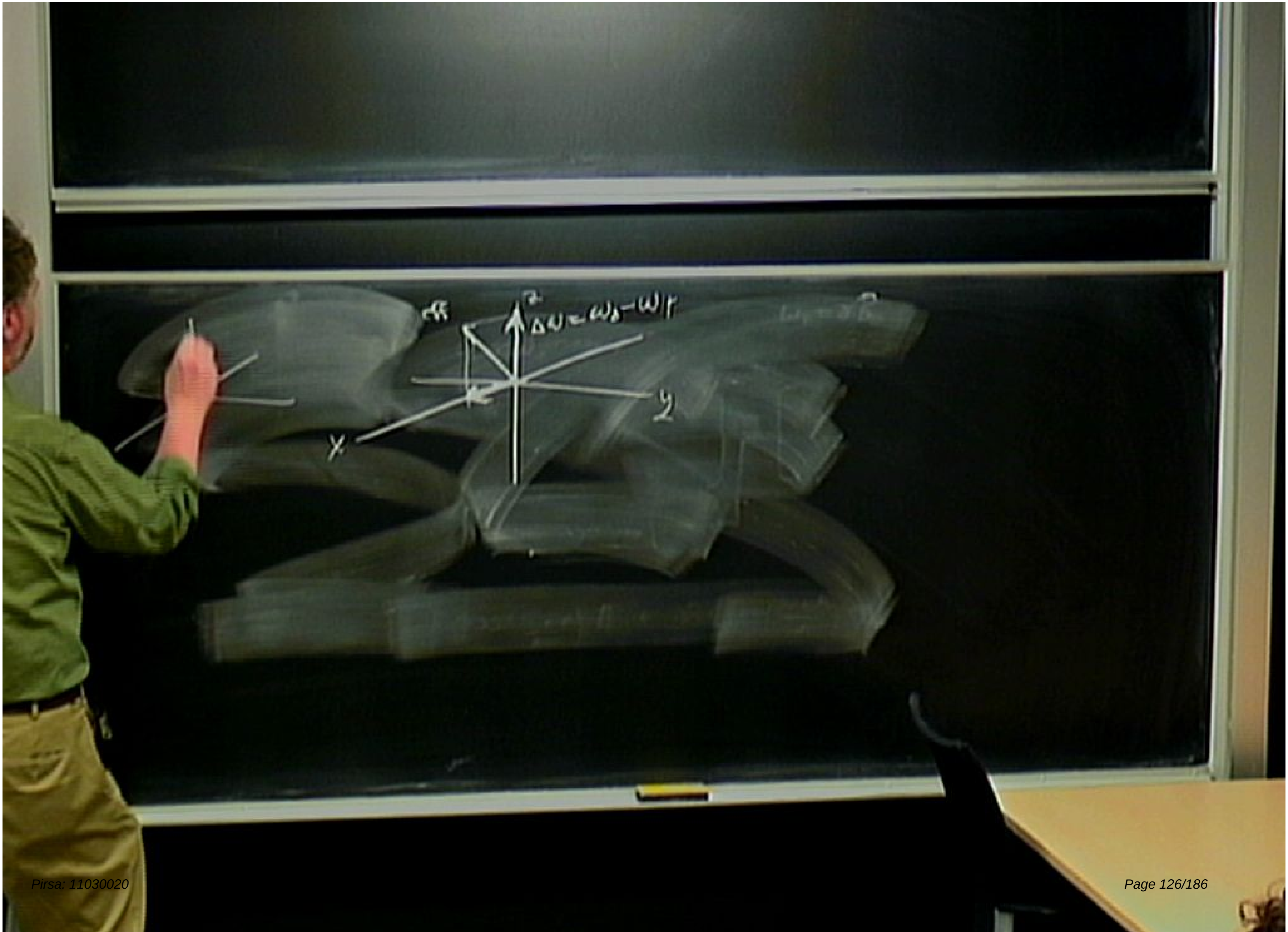


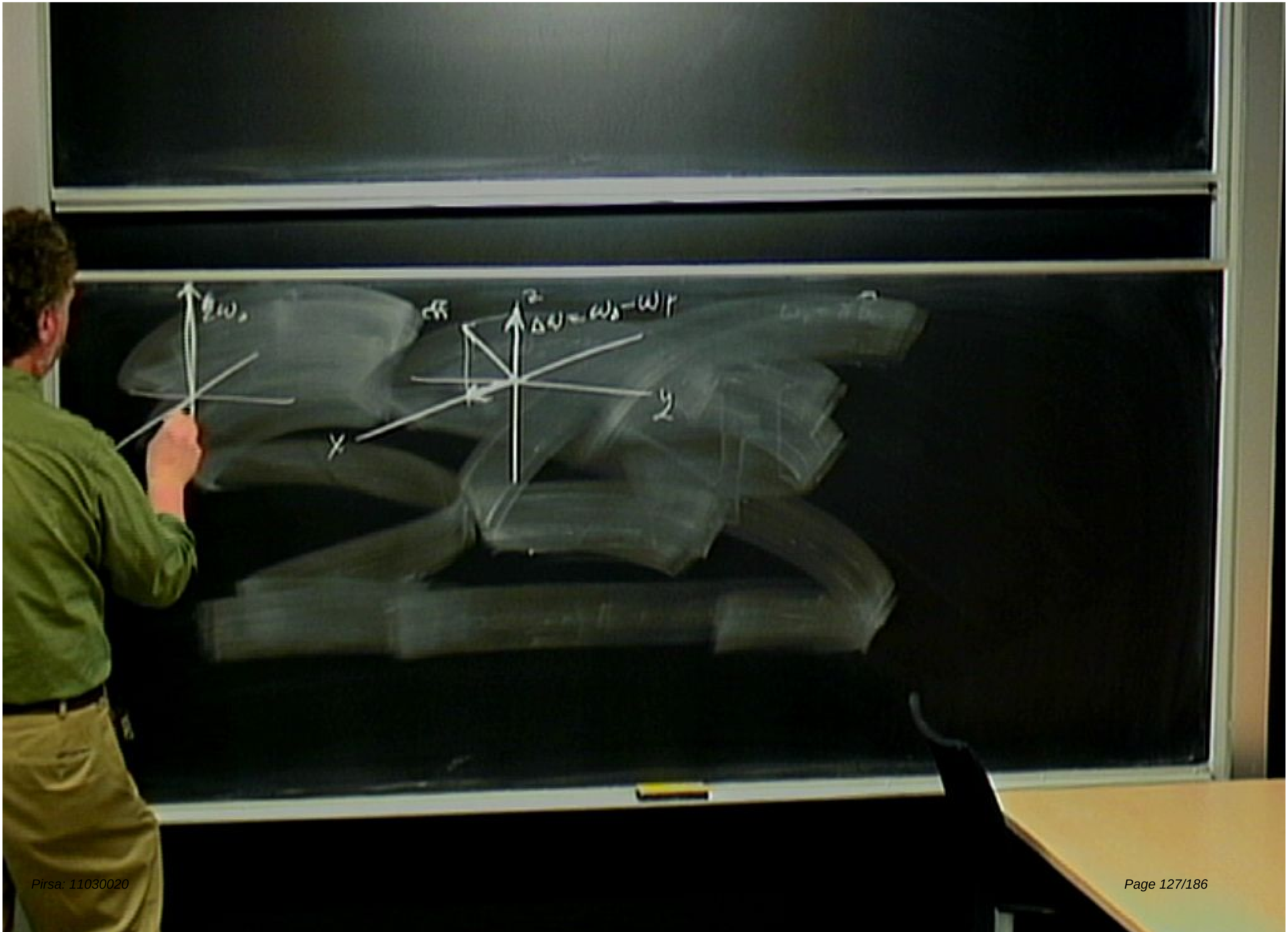


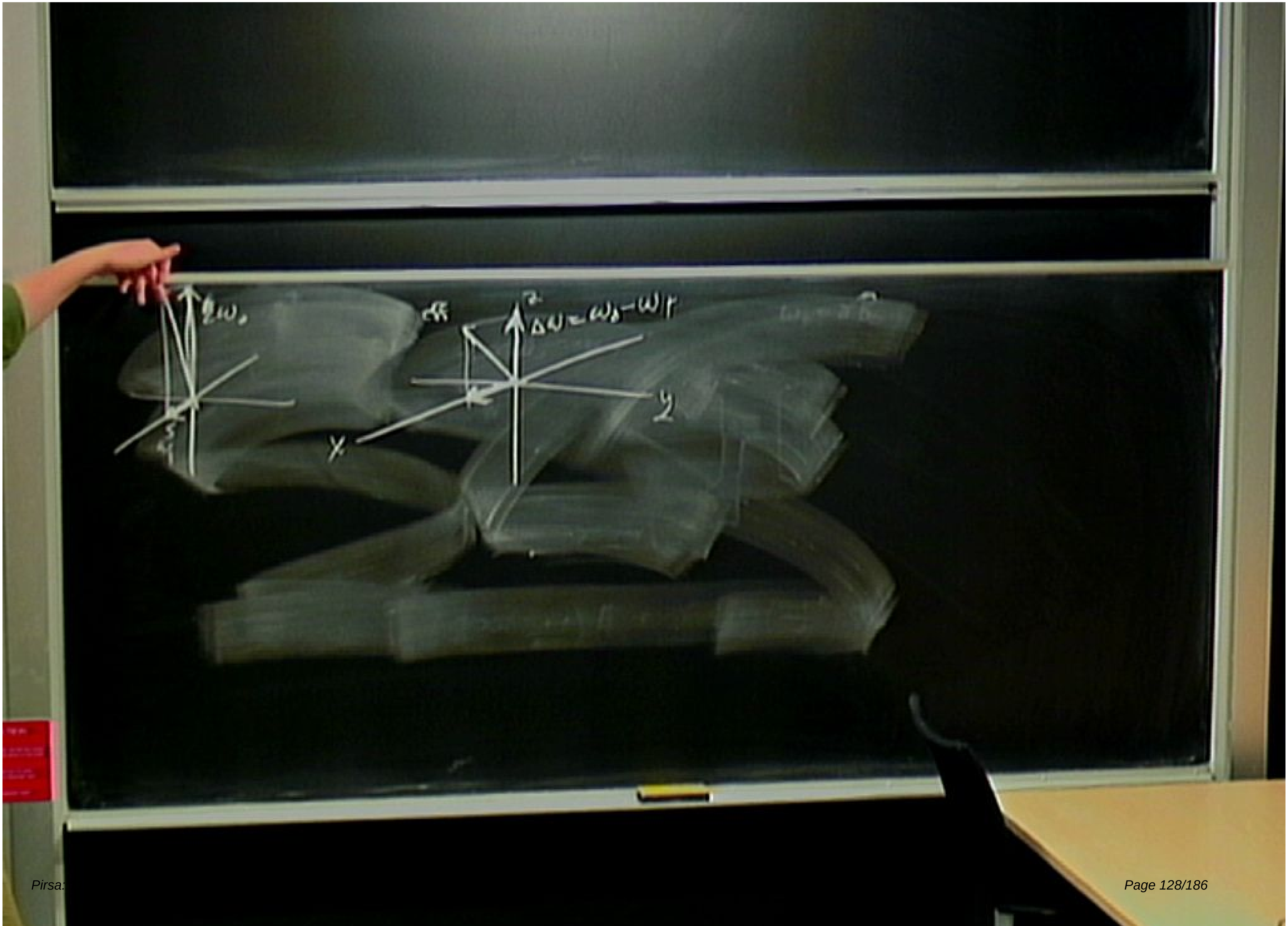



z
Δω = ω_s − ω_r
y

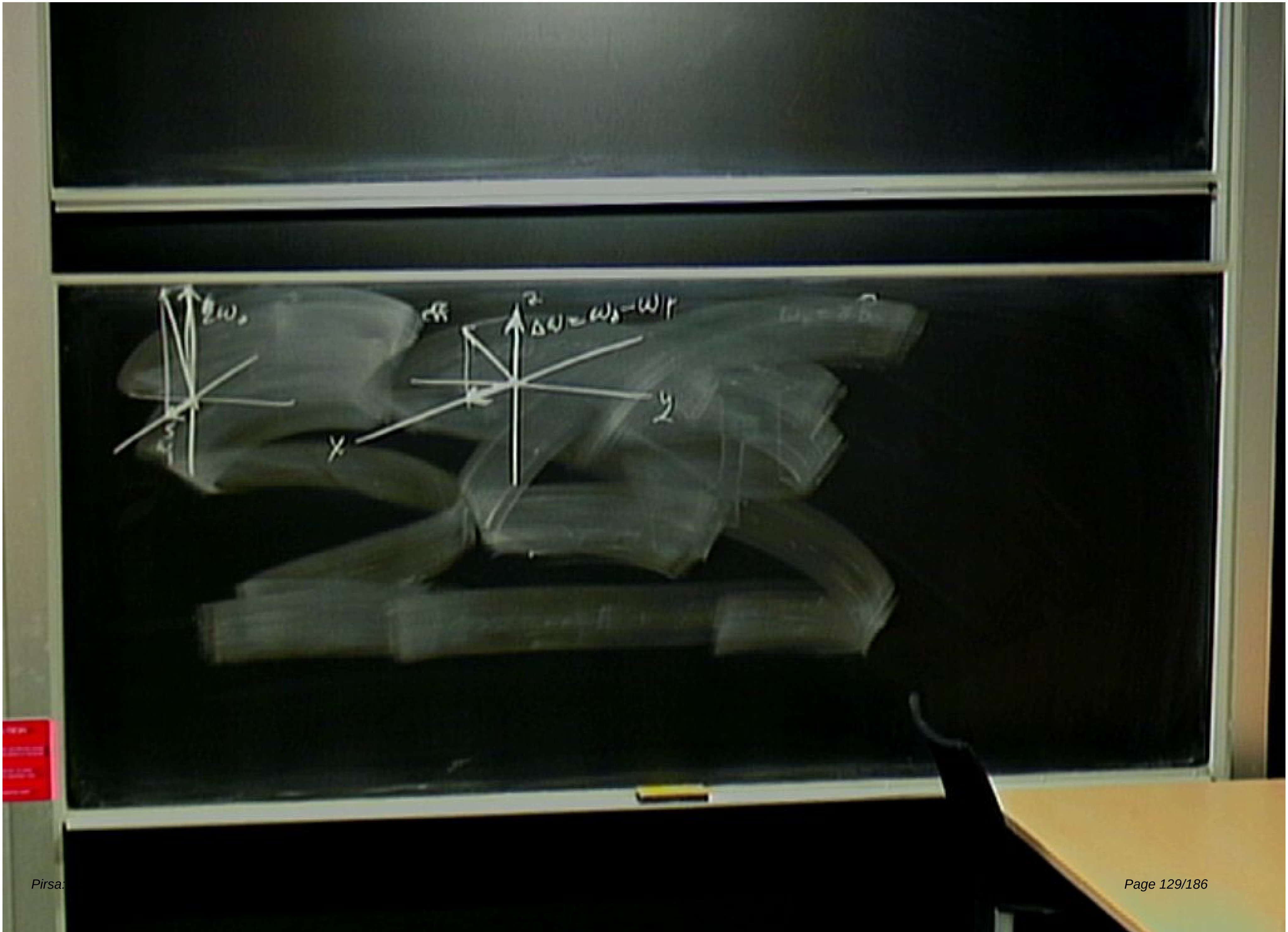

x
y
z

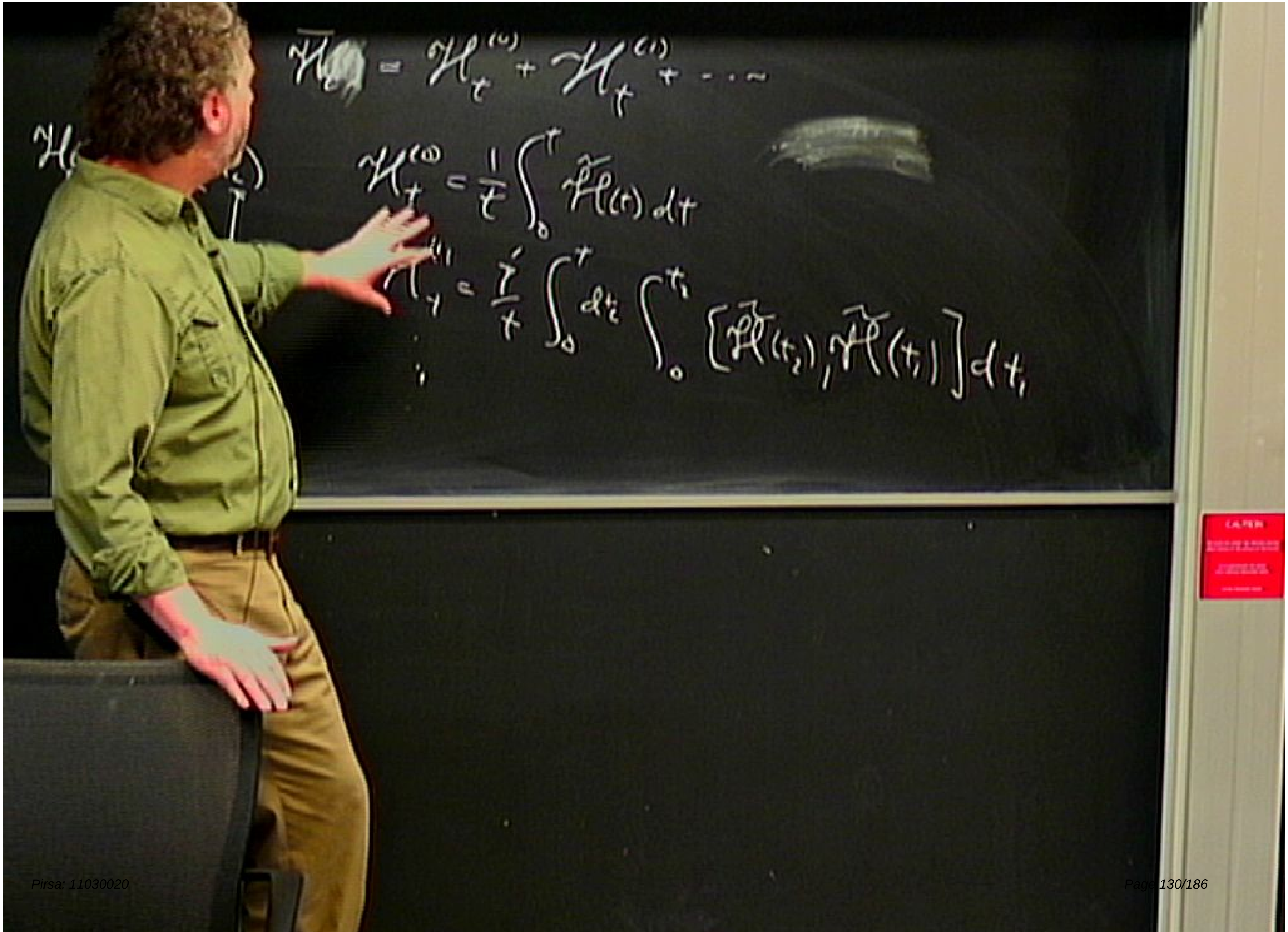

x
y


x
y



2ω₀
Δω = ω₀ − ω_r
x
y


x
y
z


$\bar{\mathcal{H}}_t = \mathcal{H}_t^{(0)} + \mathcal{H}_t^{(1)} + \cdots$
$\mathcal{H}_t^{(0)} = \frac{1}{t}\int_0^t \tilde{\mathcal{H}}(t)\,dt$
$\mathcal{H}_t^{(1)} = \frac{-i}{t}\int_0^t dt_2 \int_0^{t_2} [\tilde{\mathcal{H}}(t_2), \tilde{\mathcal{H}}(t_1)]\,dt_1$

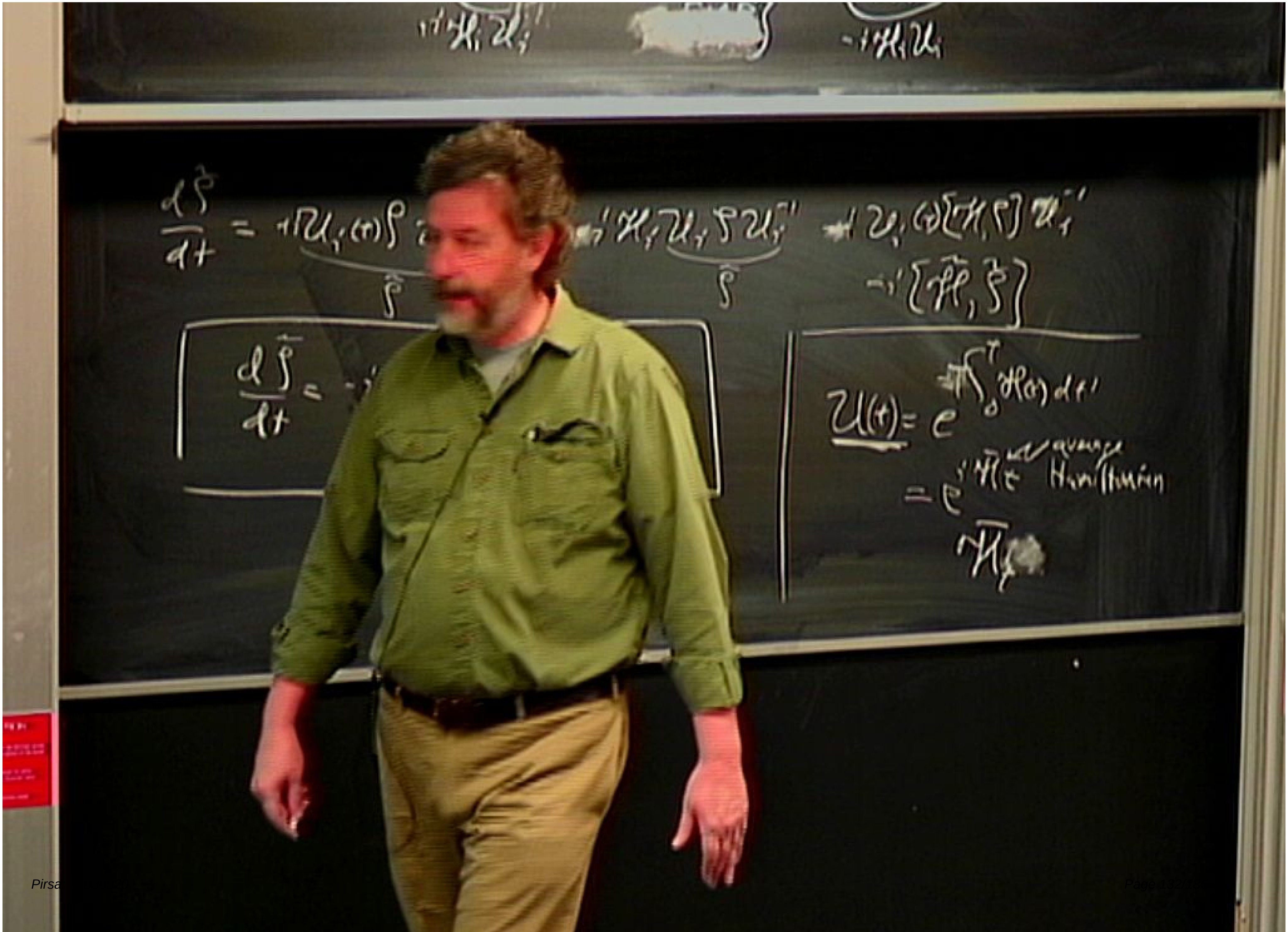
$$\bar{\mathcal{H}}_t = \mathcal{H}_t^{(0)} + \mathcal{H}_t^{(1)} + \cdots$$

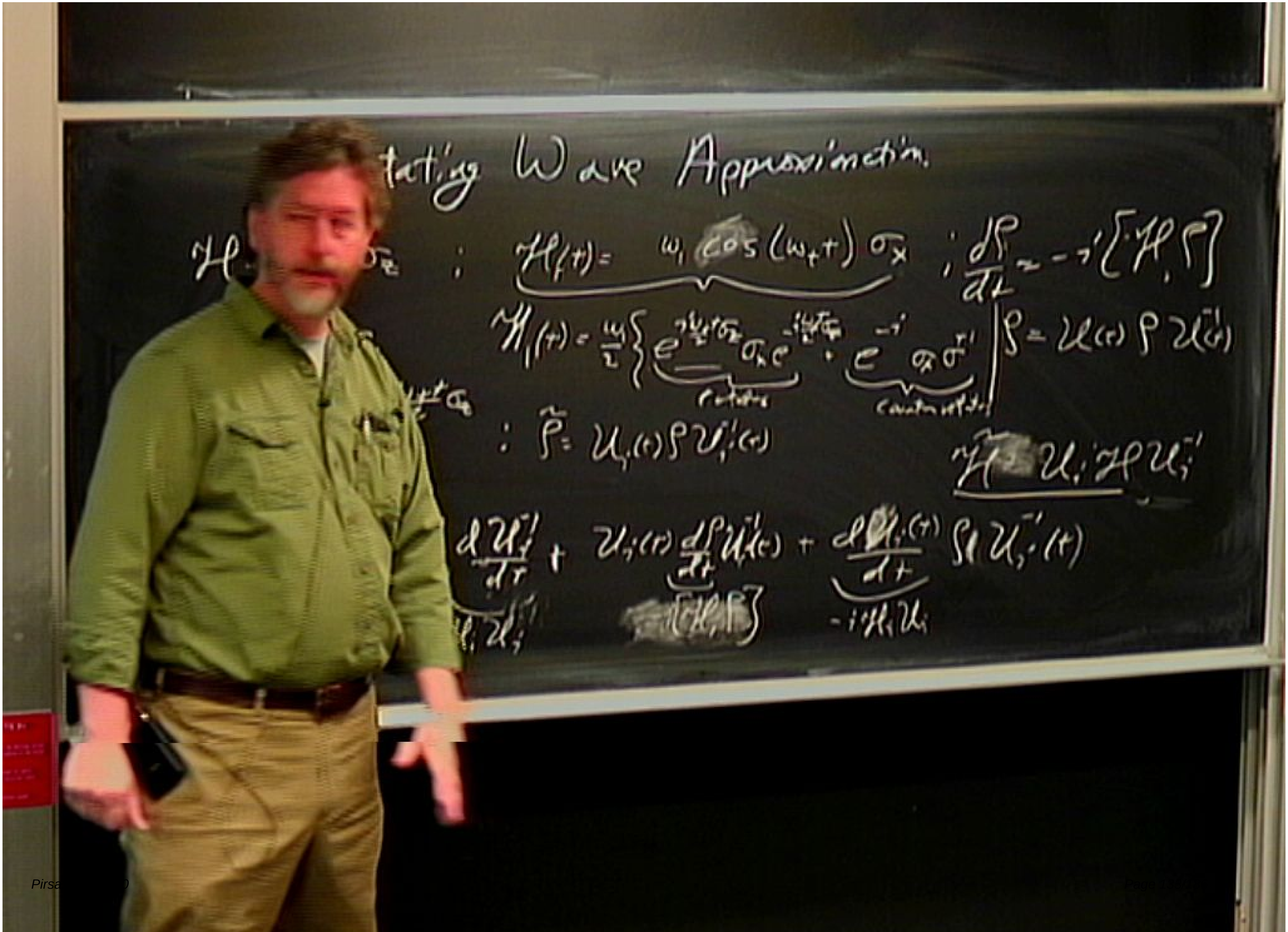
$$\mathcal{H}(t) = \mathcal{H}(t + t_c)$$

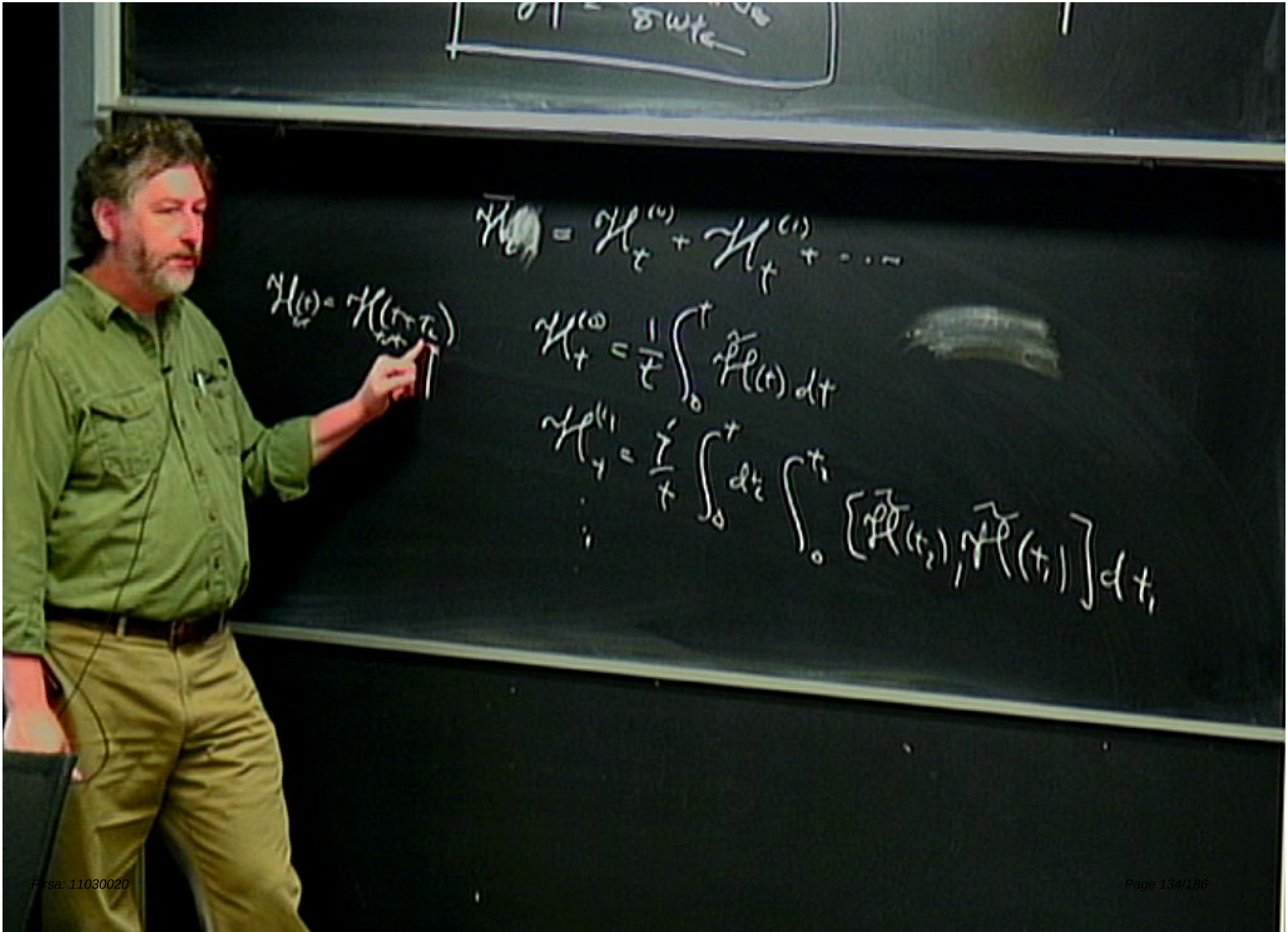
$$\mathcal{H}_t^{(0)} = \frac{1}{t}\int_0^t \tilde{\mathcal{H}}(t)\,dt$$

$$\mathcal{H}_t^{(1)} = \frac{-i}{t}\int_0^t dt_2 \int_0^{t_2} \left[\tilde{\mathcal{H}}(t_2), \tilde{\mathcal{H}}(t_1)\right] dt_1$$

$\vdots$


average
Hamiltonian


Wave Approximation



$$\bar{\mathcal{H}}_t = \mathcal{H}_t^{(0)} + \mathcal{H}_t^{(1)} + \cdots$$

$$\mathcal{H}_{int}(t) = \mathcal{H}_{int}(t + t_c)$$

$$\mathcal{H}_t^{(0)} = \frac{1}{t}\int_0^t \tilde{\mathcal{H}}(t)\,dt$$

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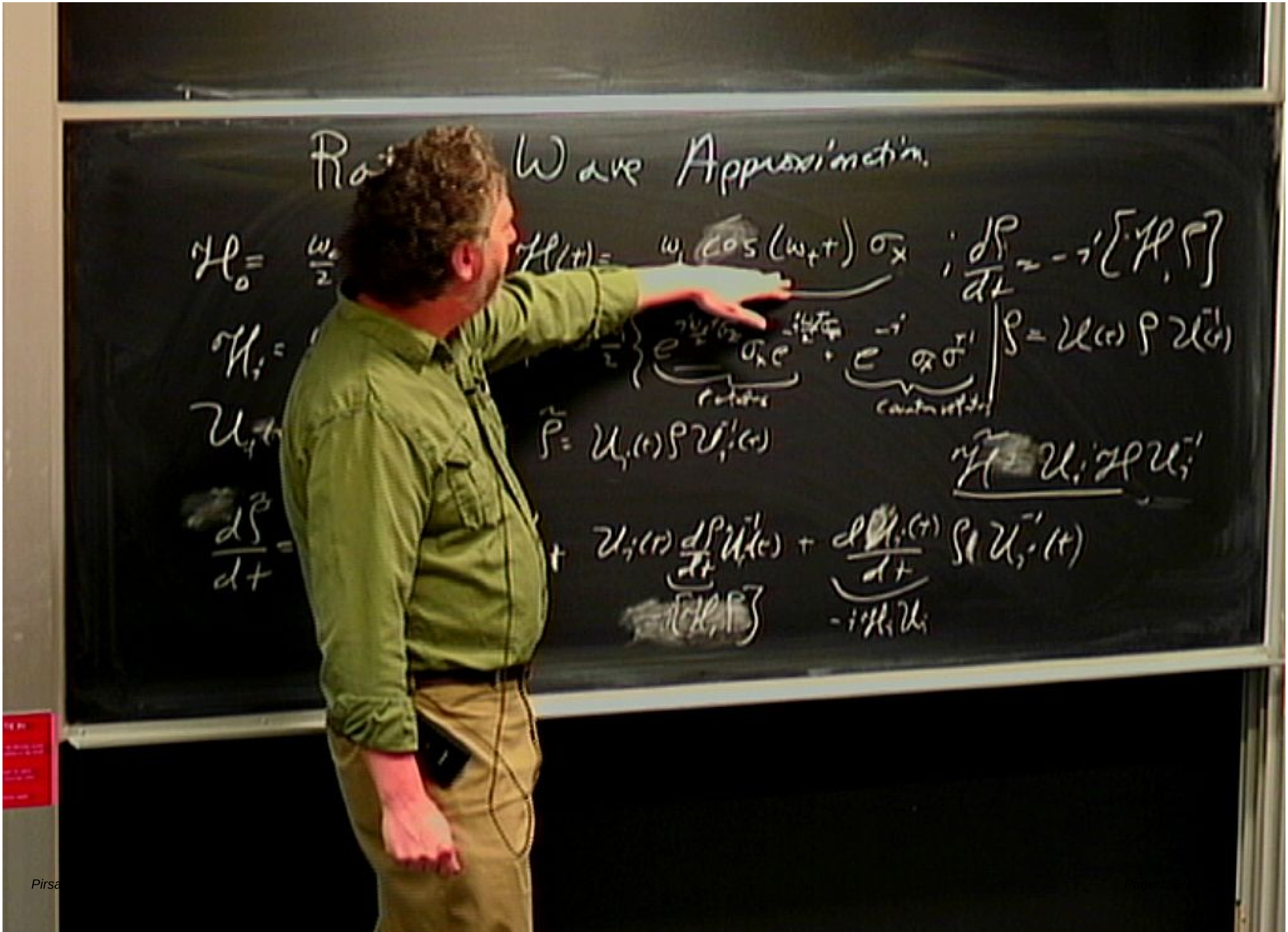
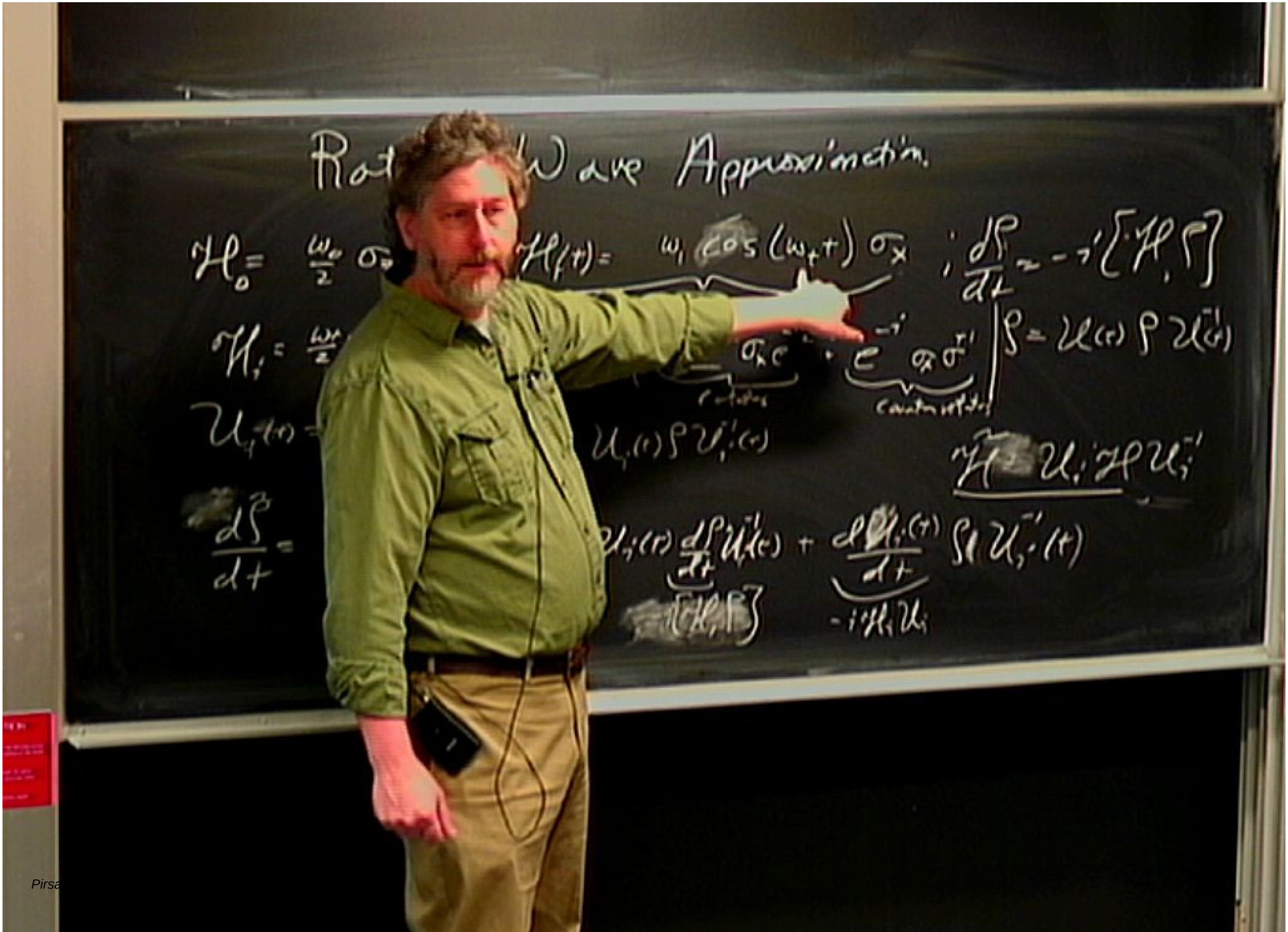
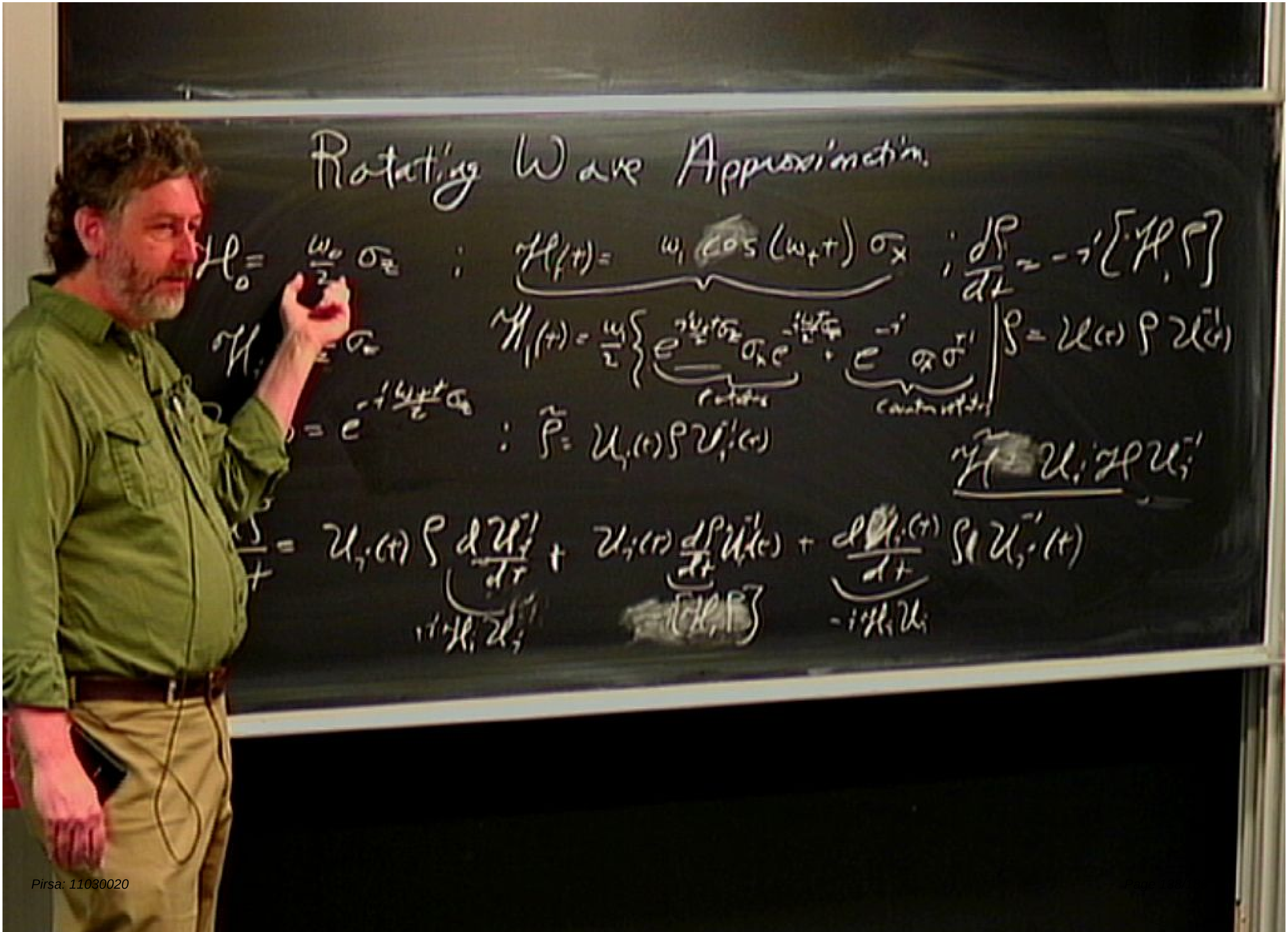
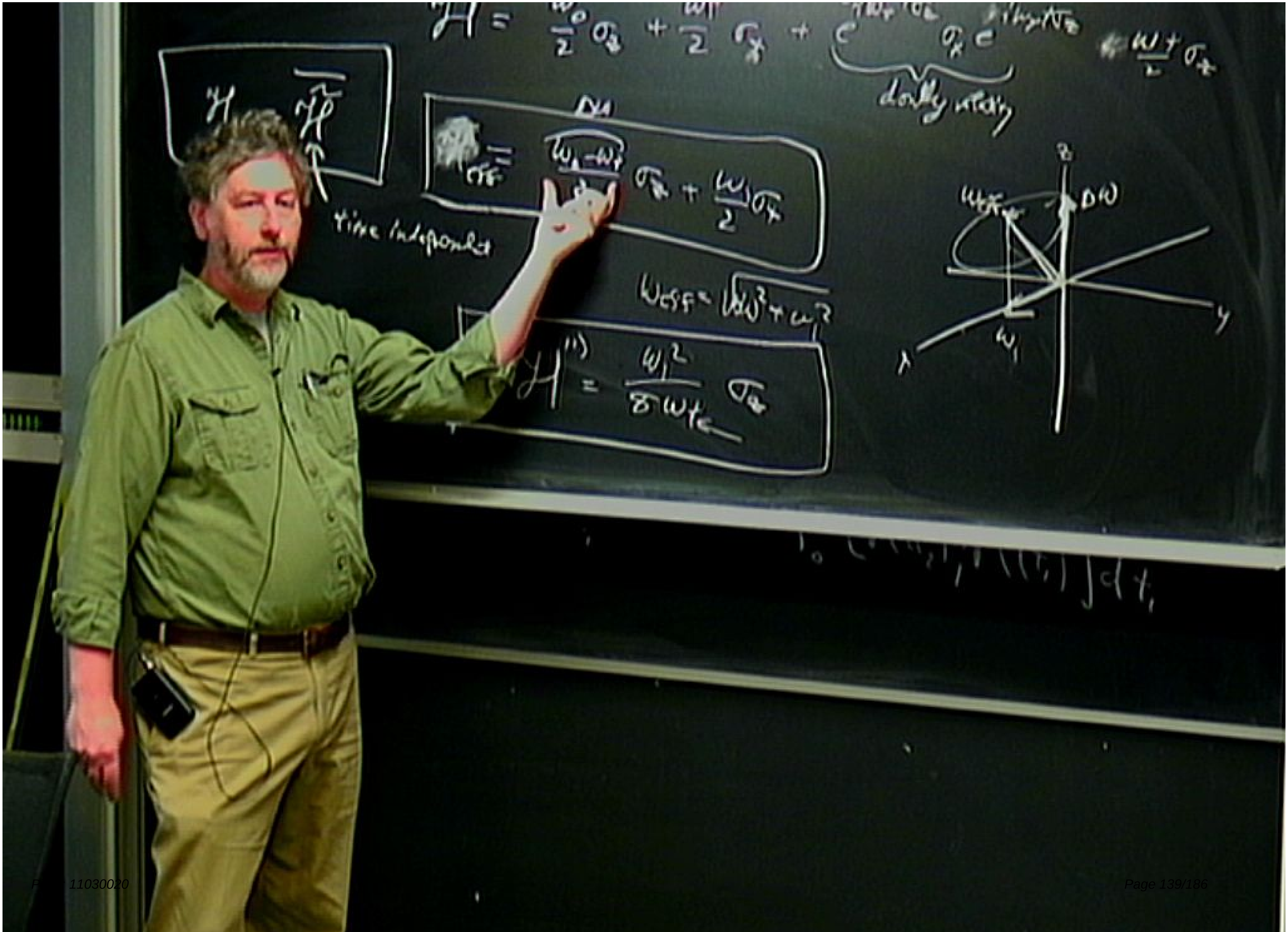
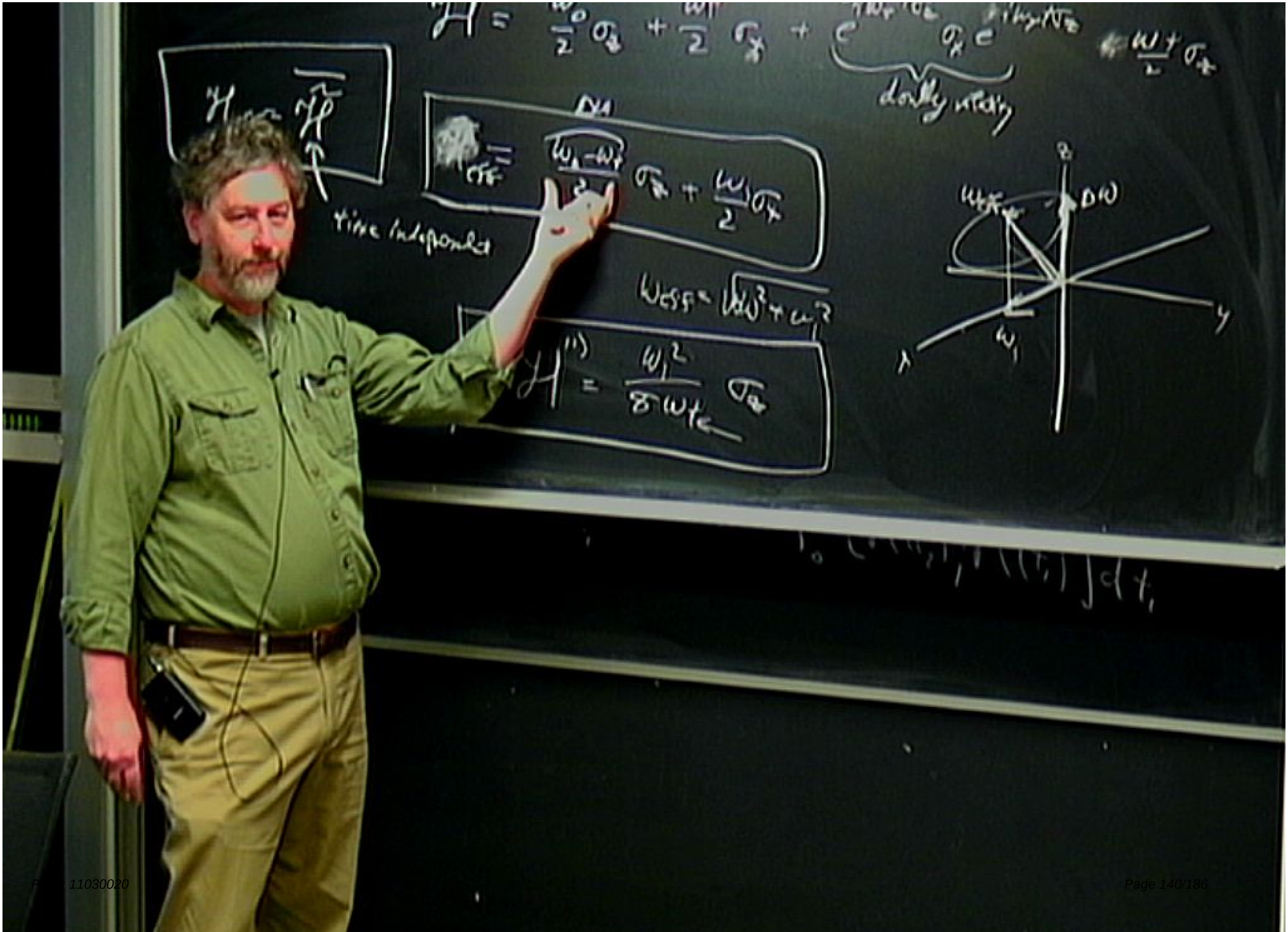
$\vdots$


Wave Approximation
rotating
counter rotating
Pirsa


Wave Approximation


Rotating Wave Approximation





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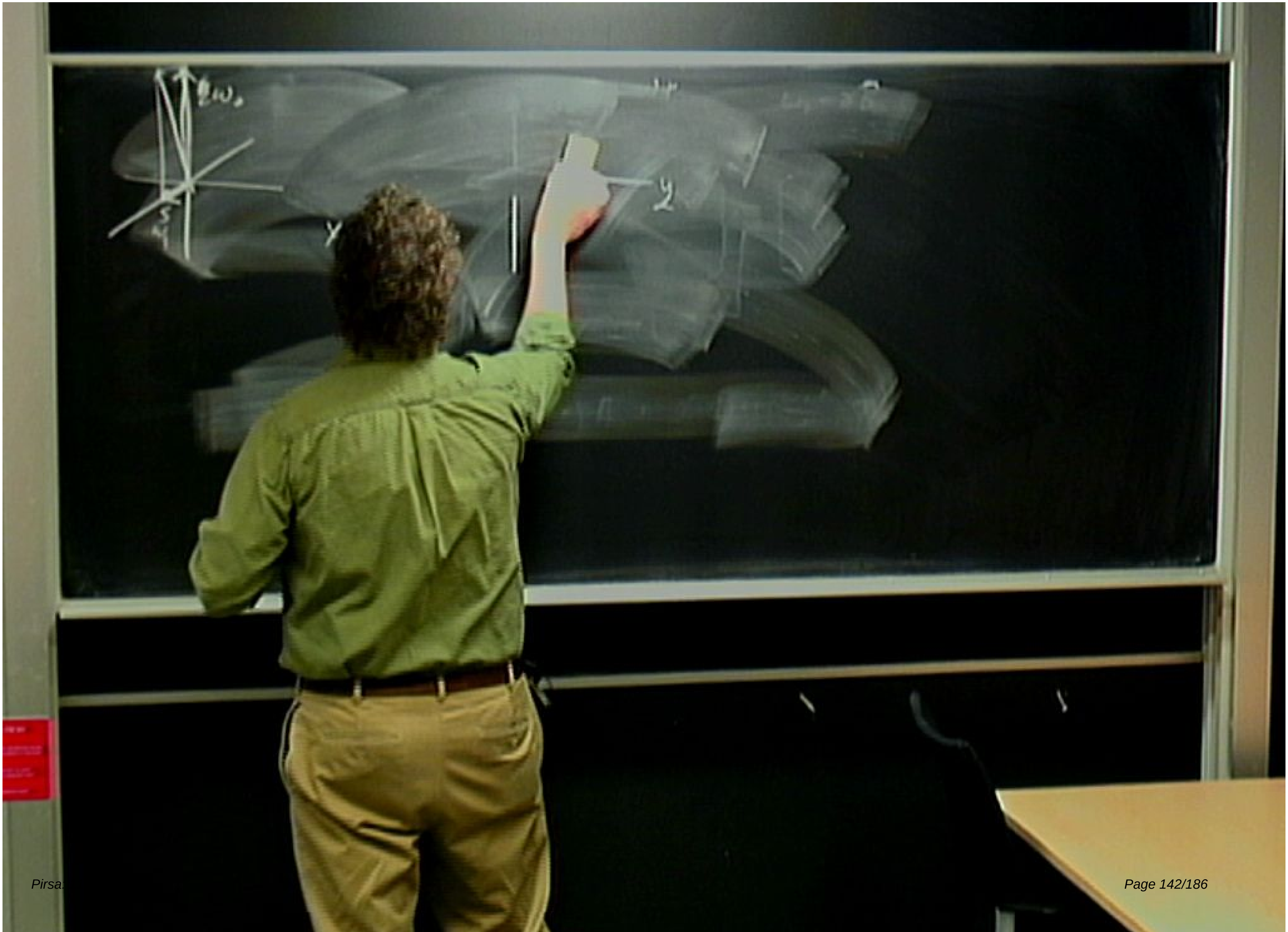
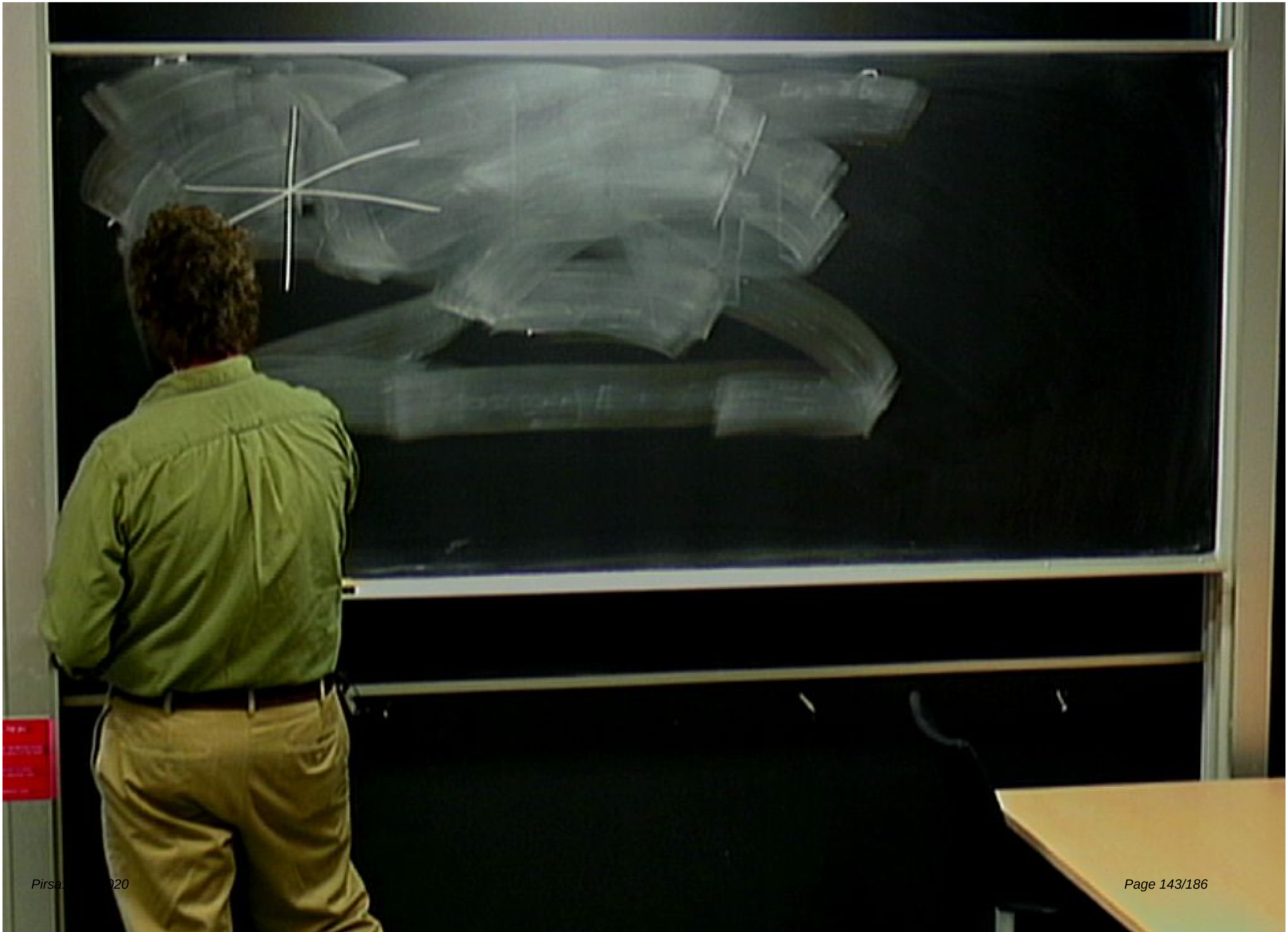
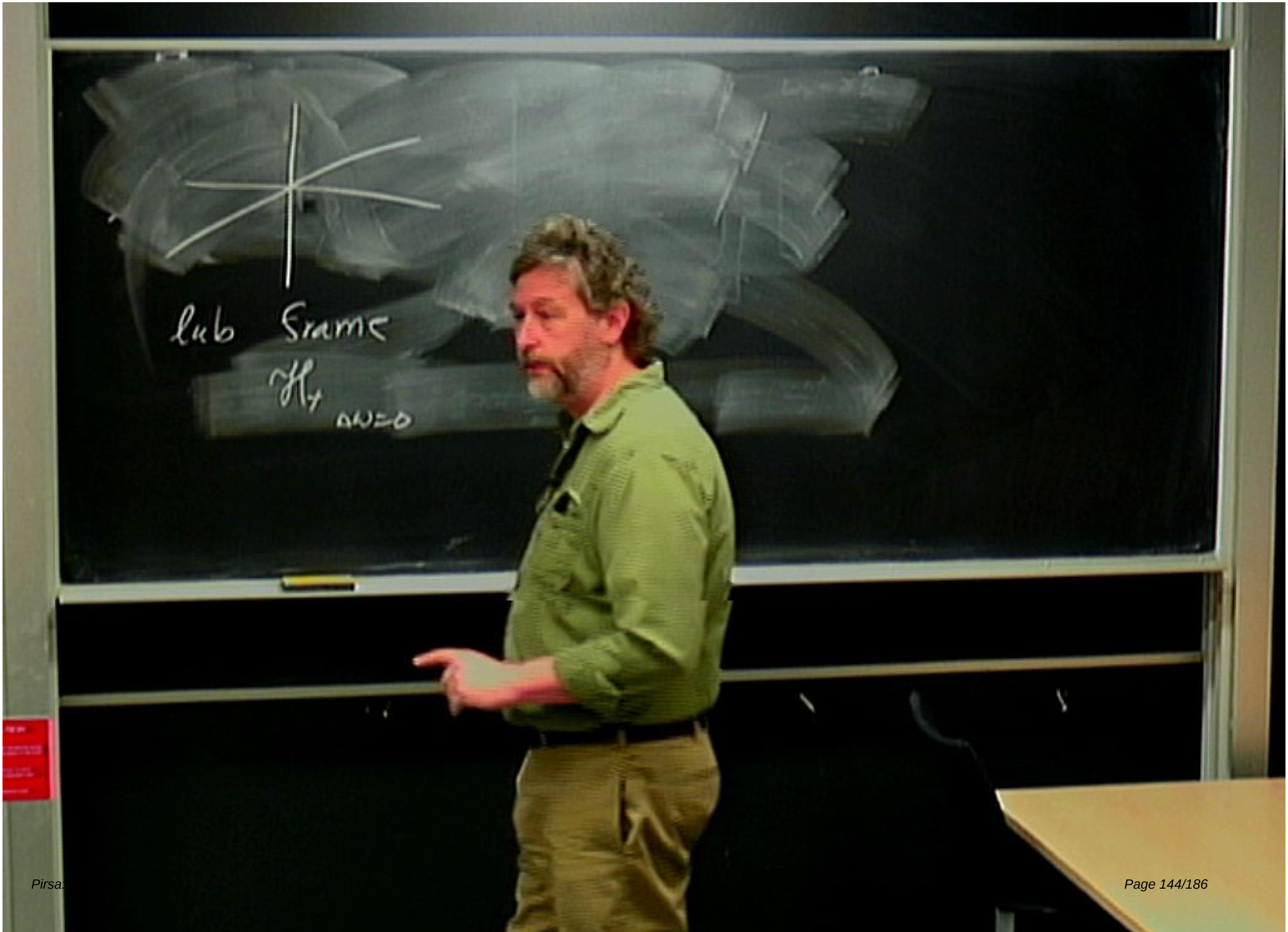
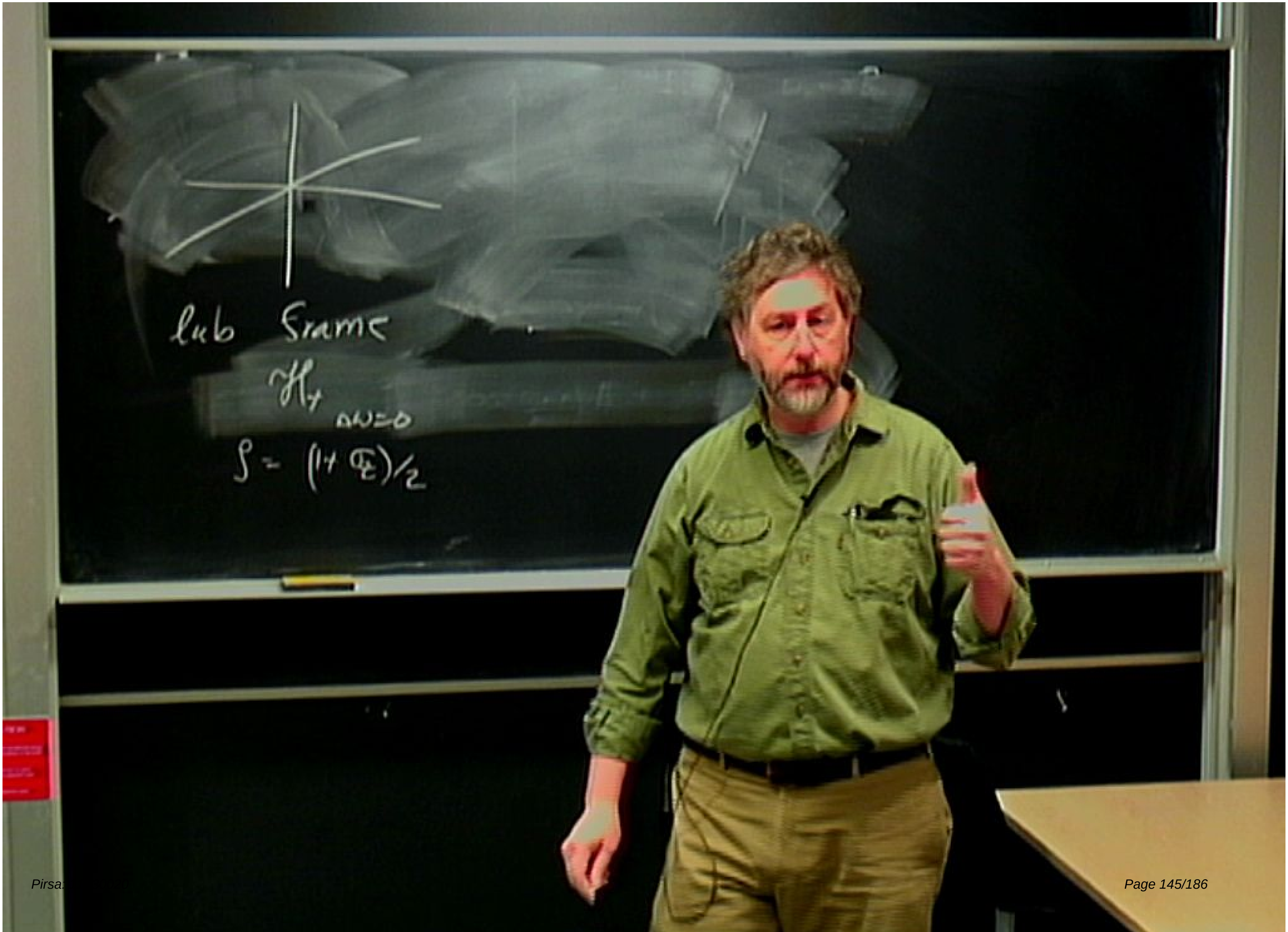





lab frame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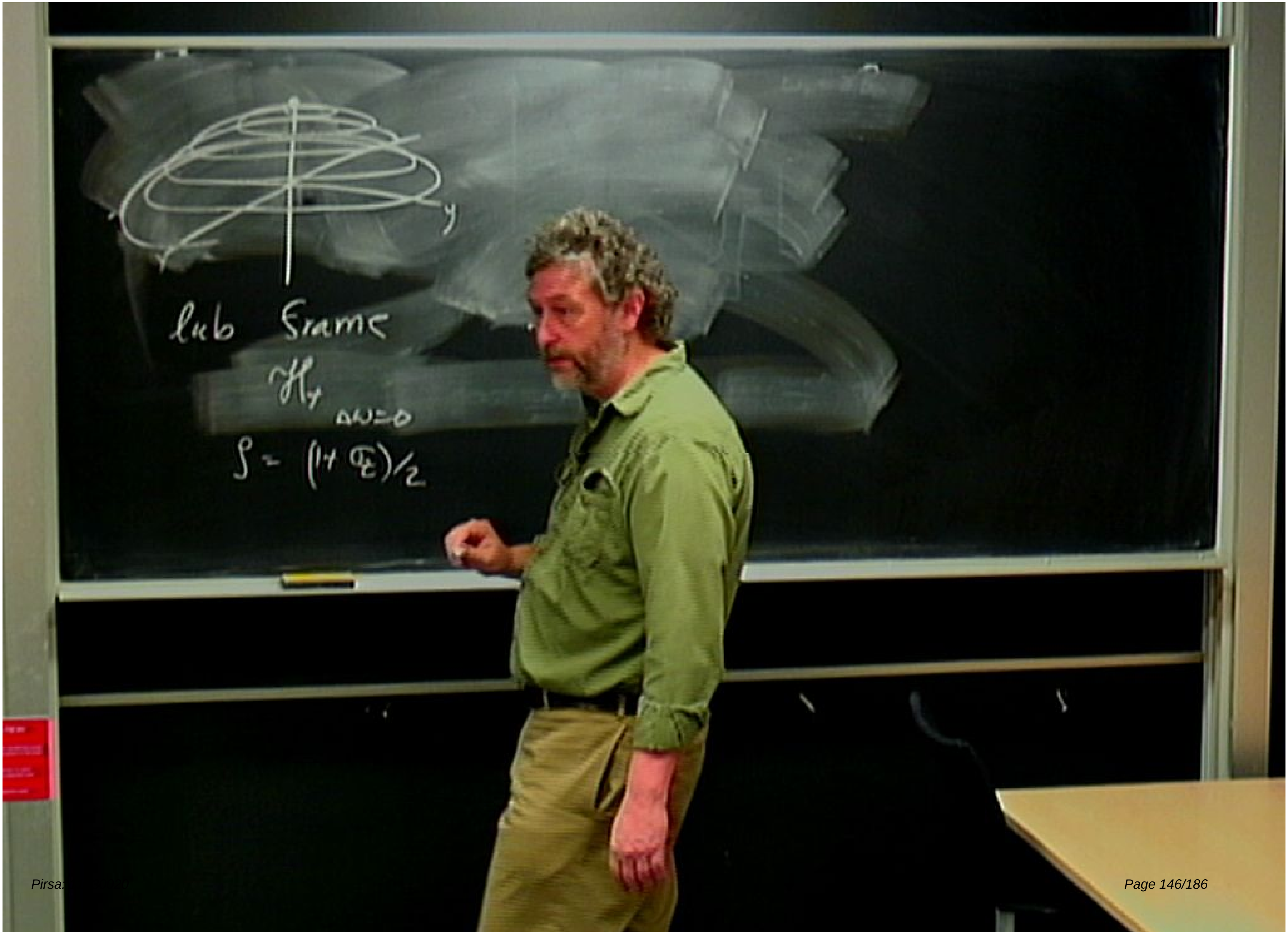
lab frame

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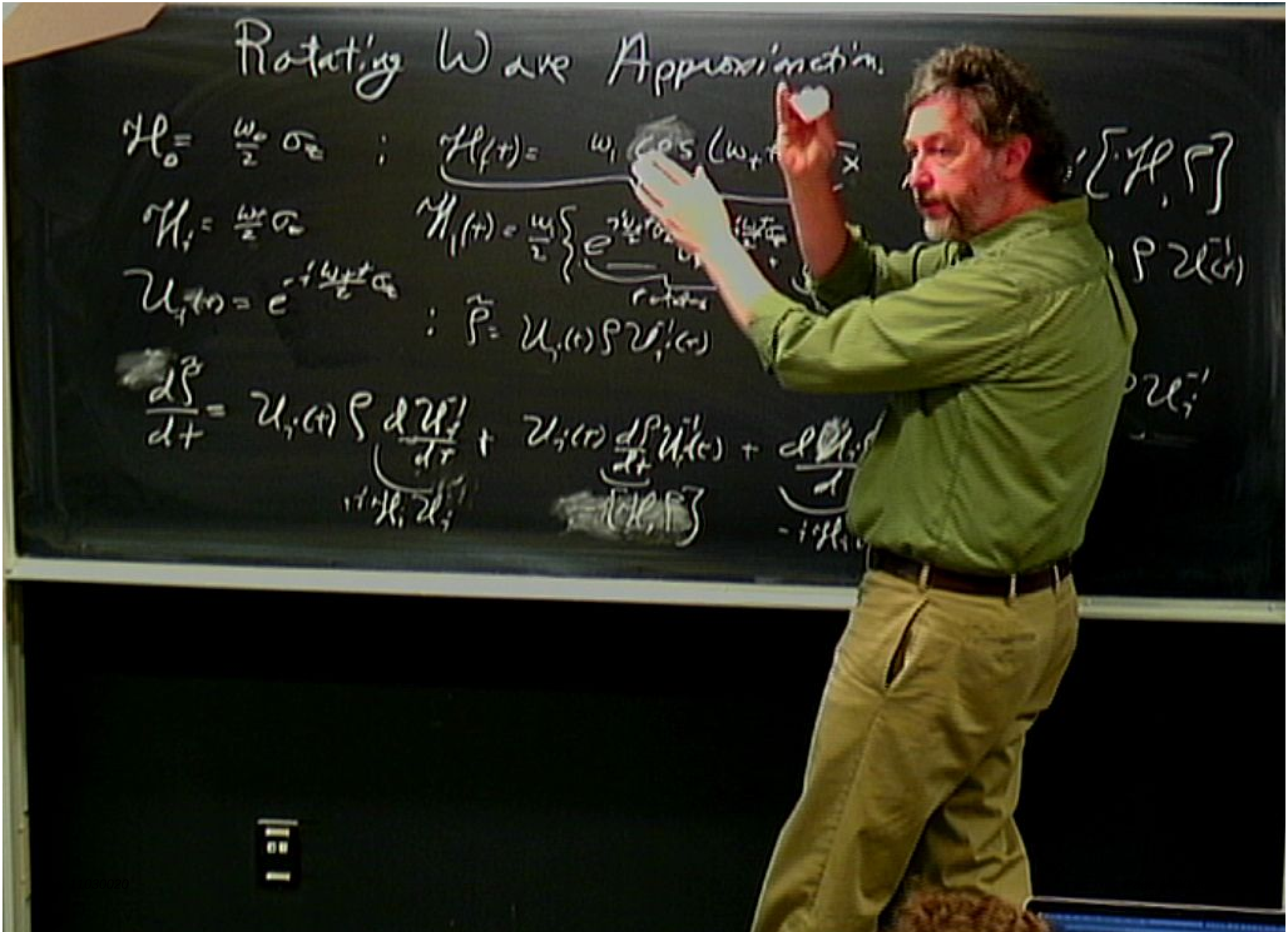
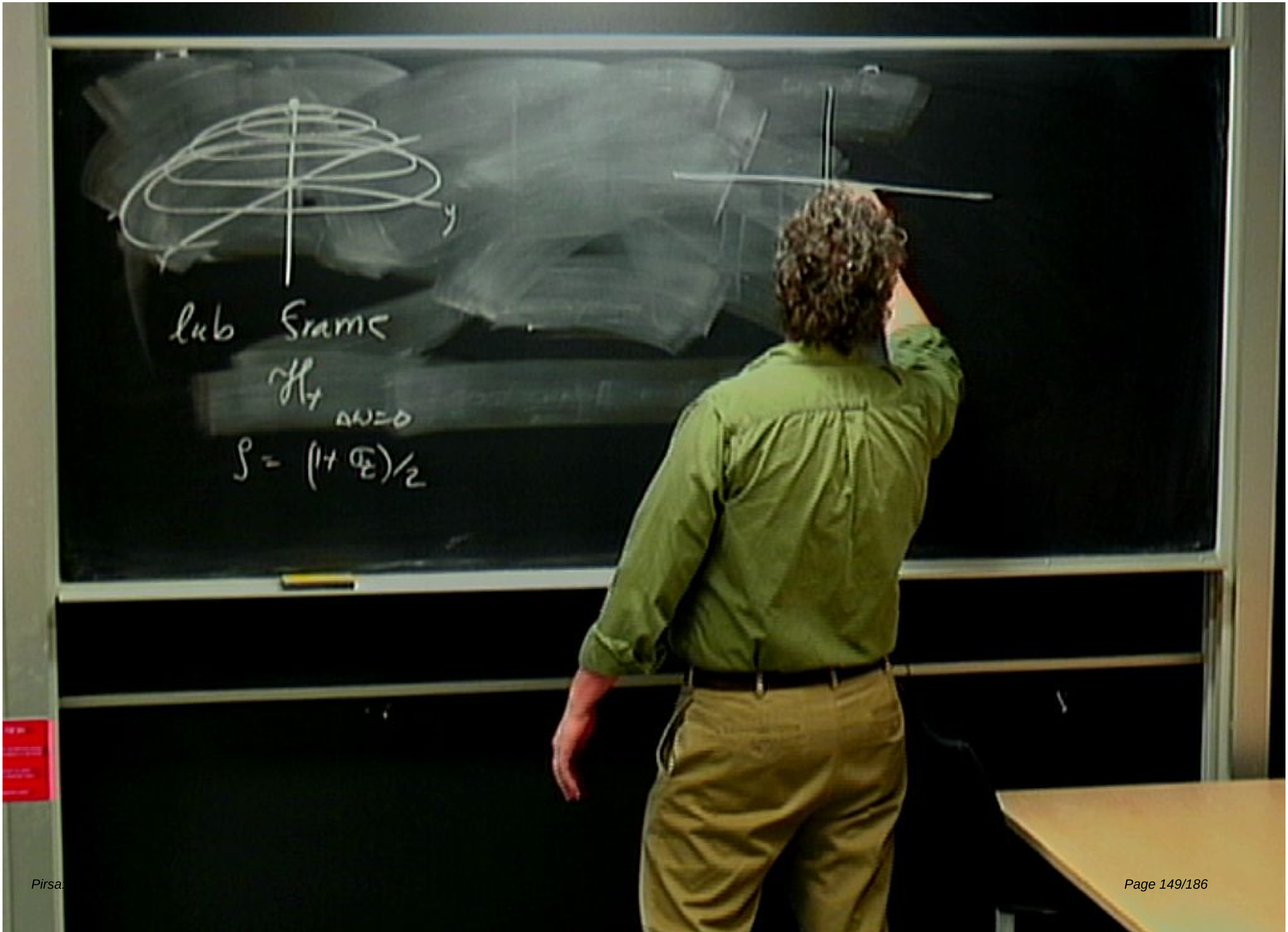
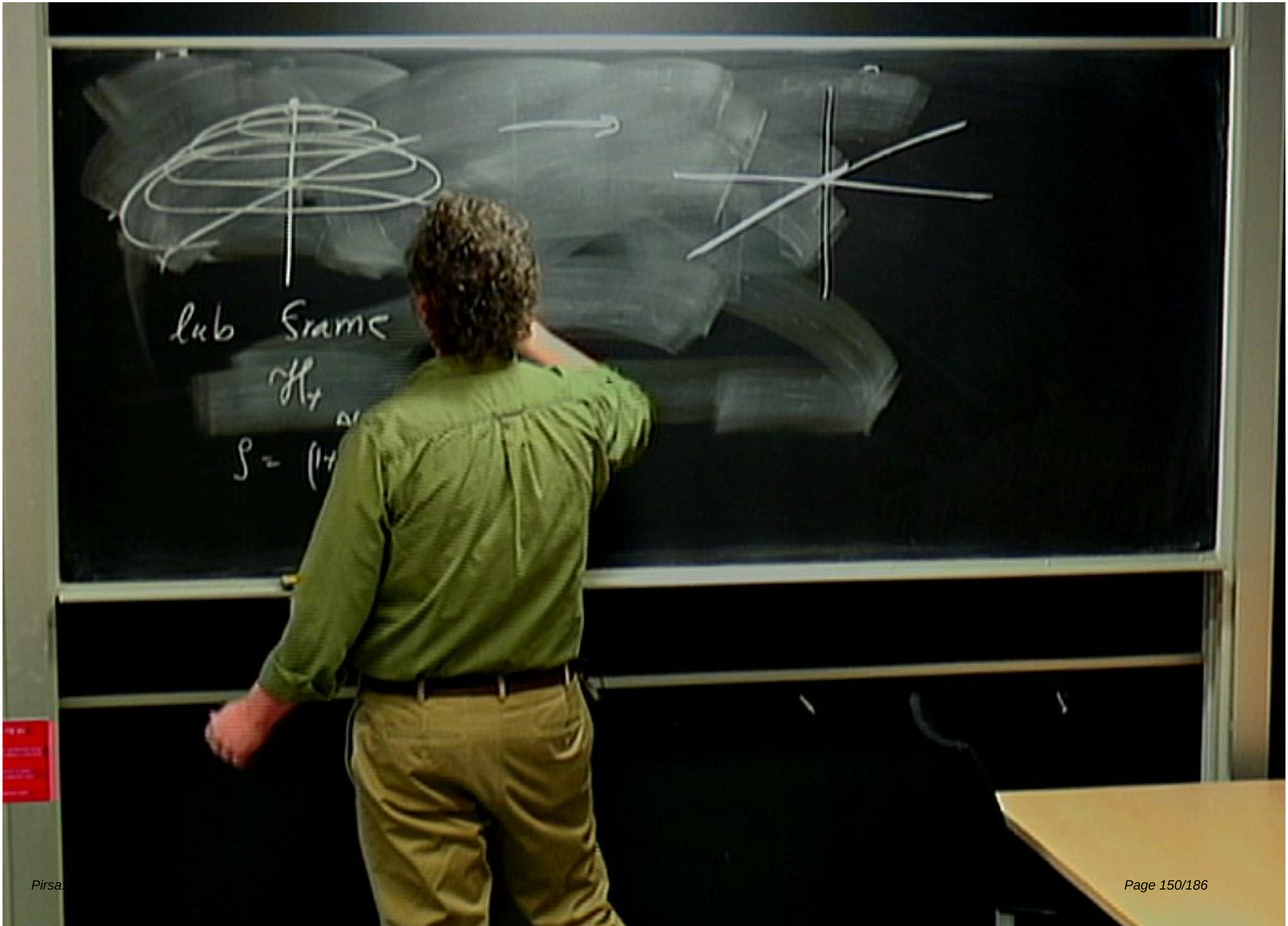
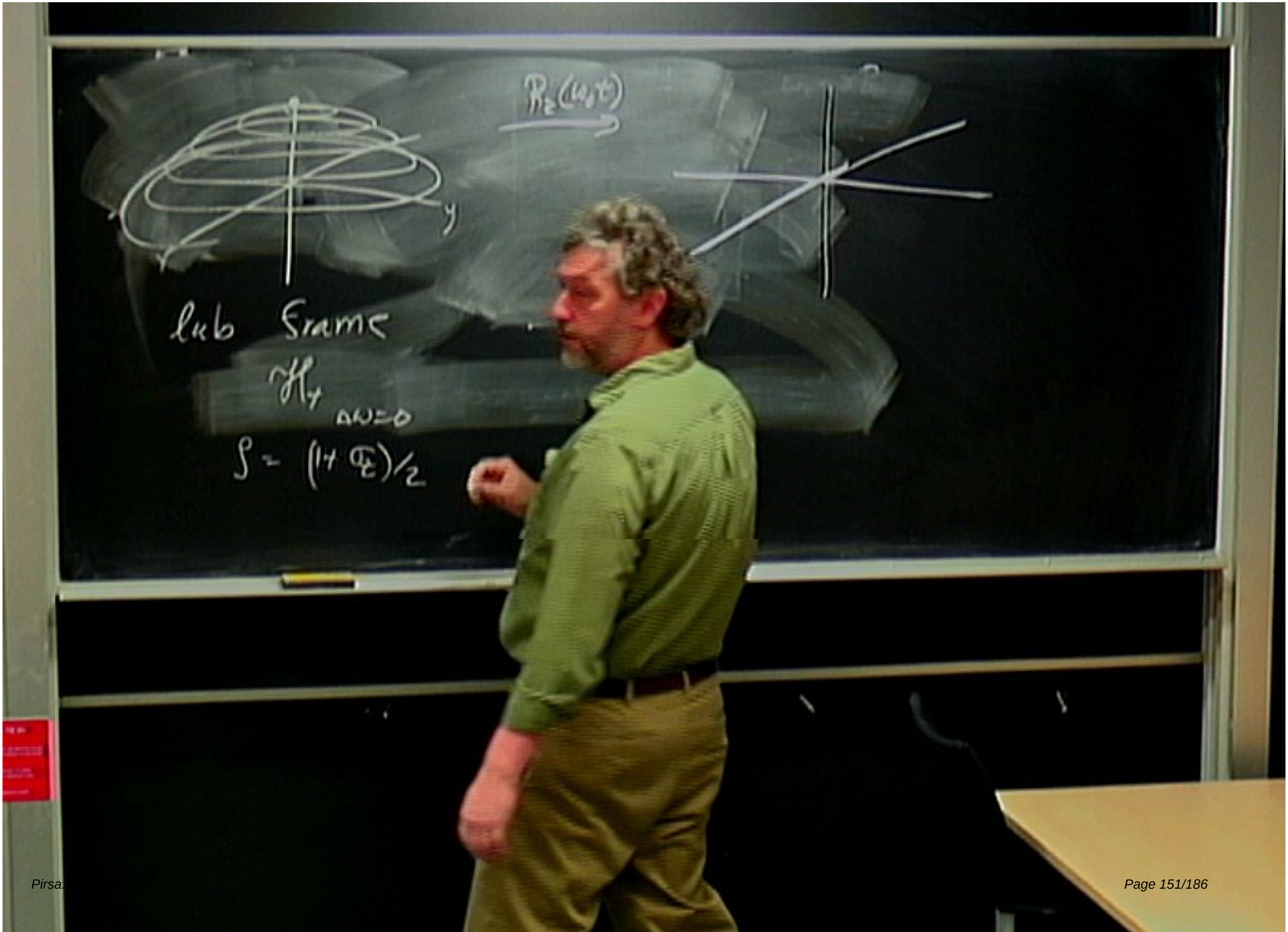
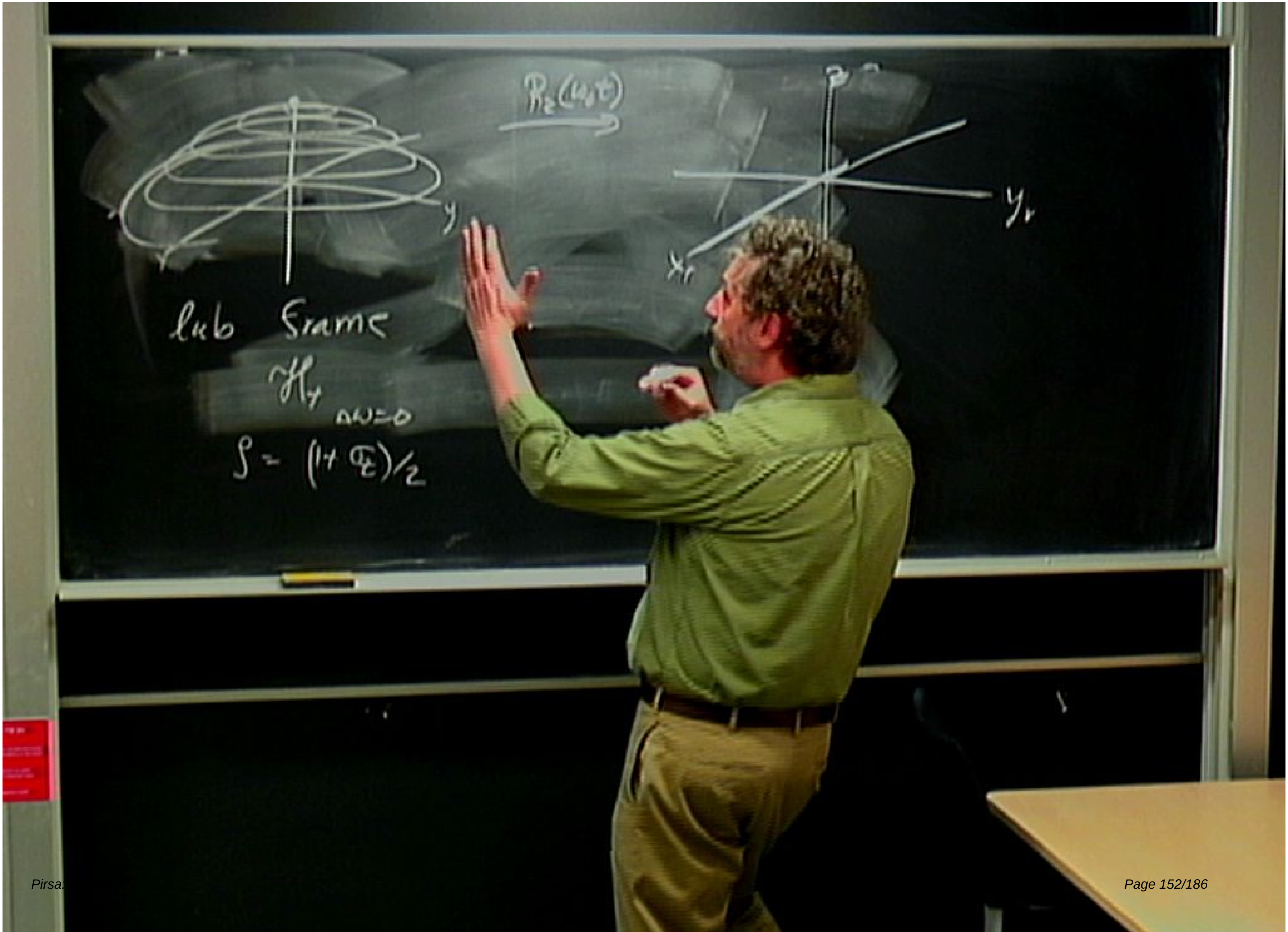

Rotating Wave Approximation

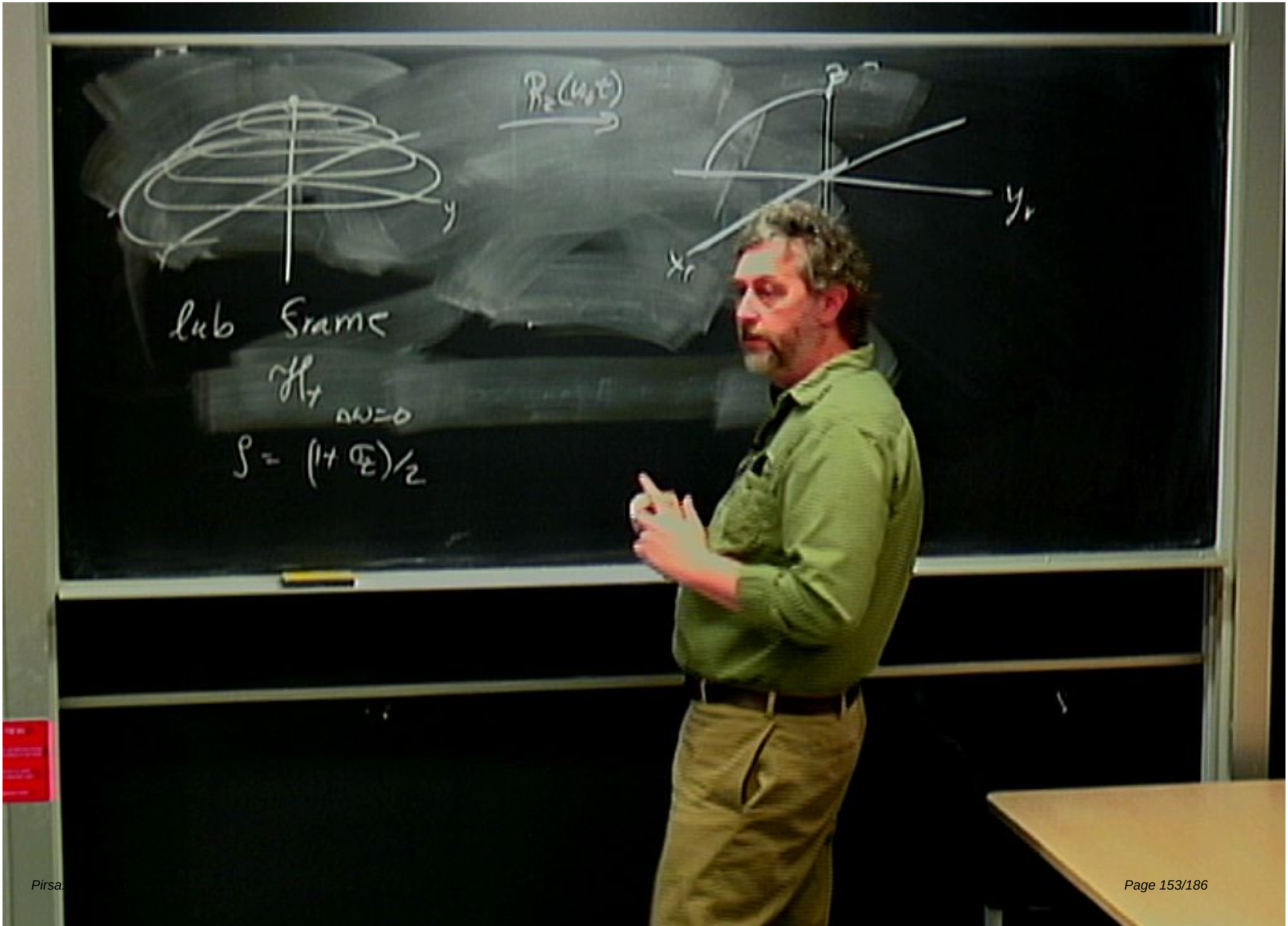

y
lab frame
$\mathcal{H}_y$
$\Delta w = 0$
$\rho = (1 + \sigma_z)/2$

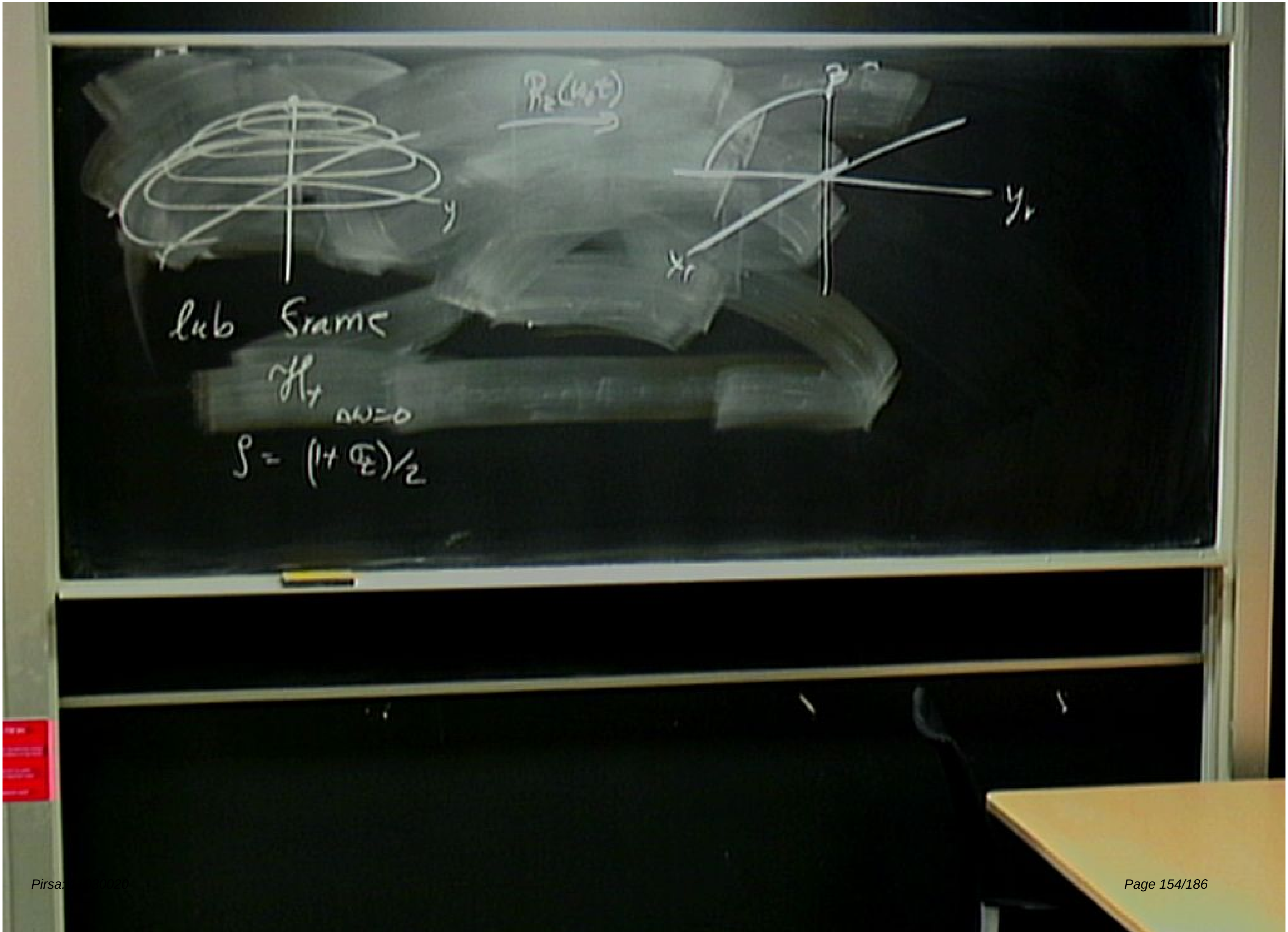

lab frame


lab frame

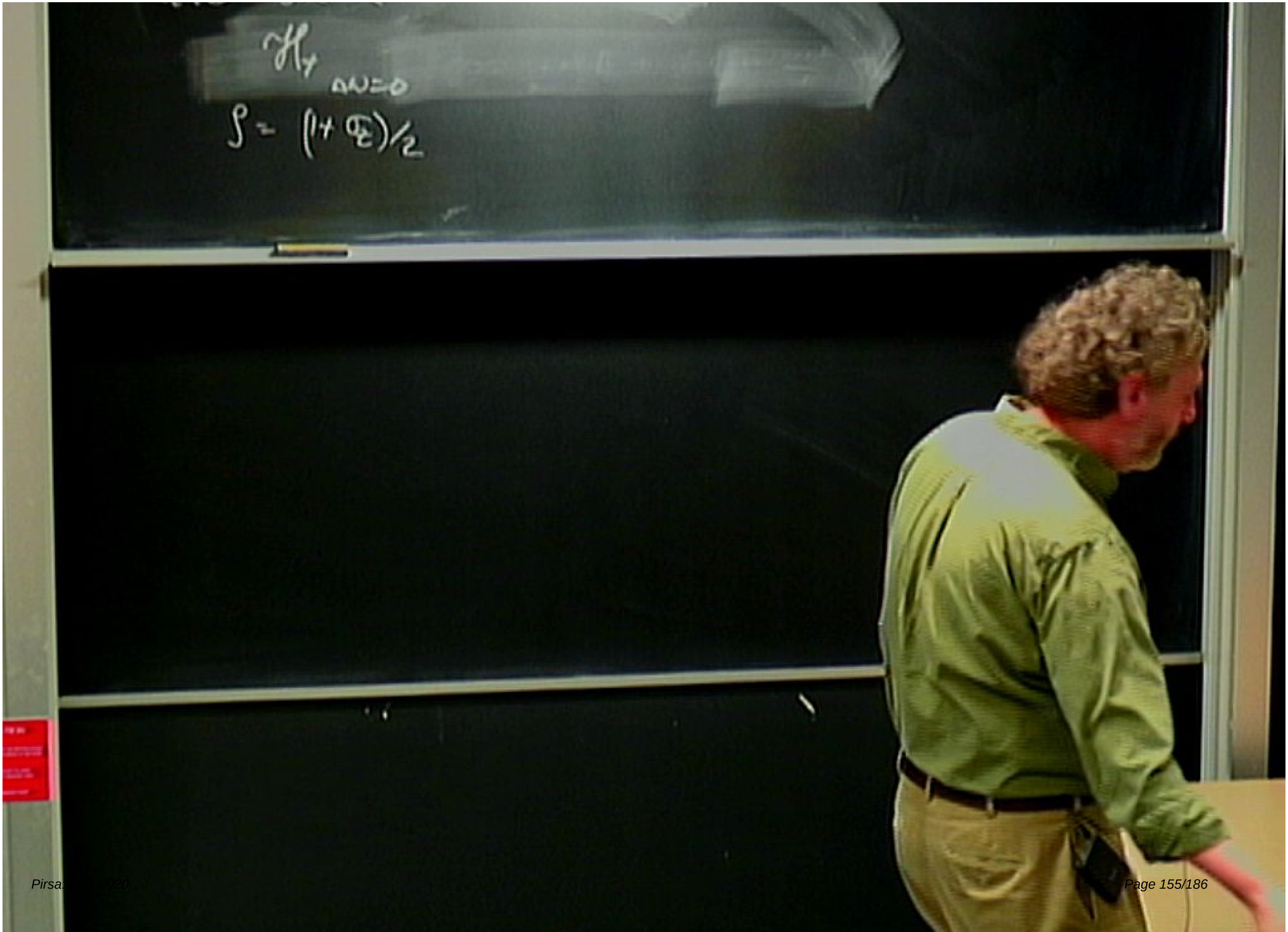


$R_z(\omega_0 t)$
lab frame
$\tilde{H}_{y}$
$\Delta W = 0$
$\rho = (1 + \sigma_z)/2$
y
y_r
x_r

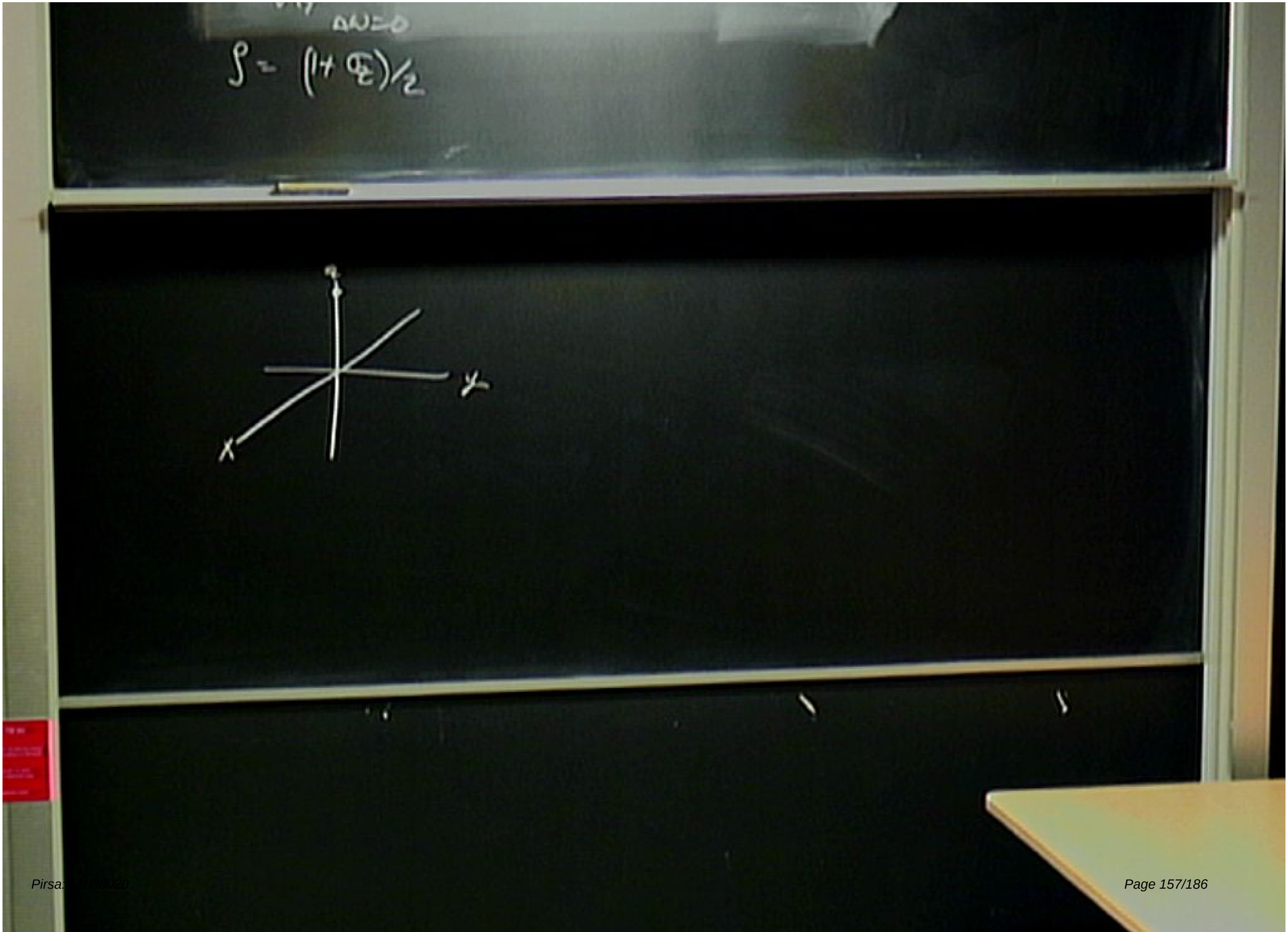

$R_z(\omega_0 t)$
y
y_r
x_r
lab frame
$\Delta W = 0$
$\rho = (1+\sigma_z)/2$



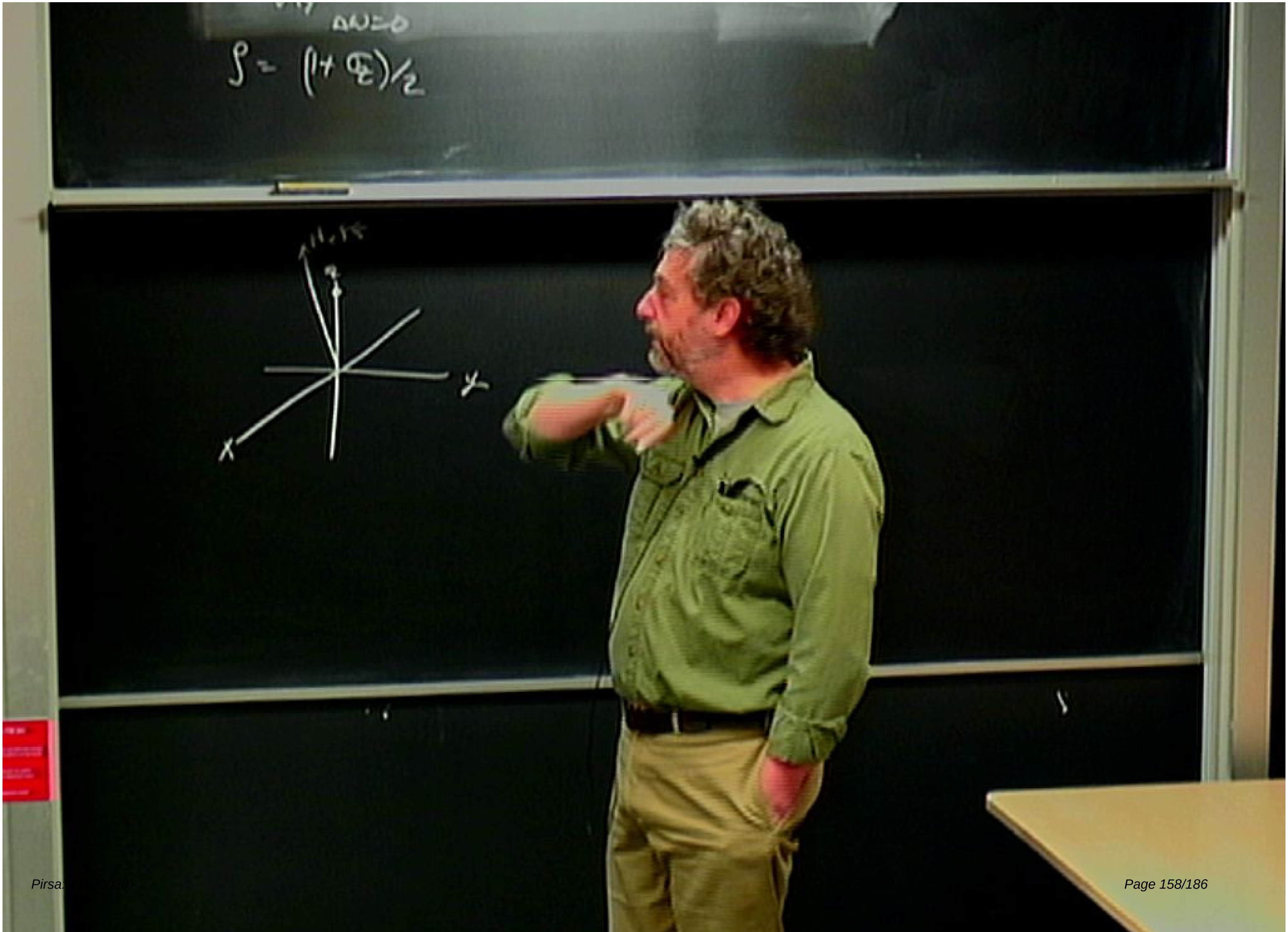
$R_z(\omega_0 t)$
y
y_r
x_r
lab frame
H_t
$\Delta\omega=0$
$\rho = (1+\sigma_z)/2$

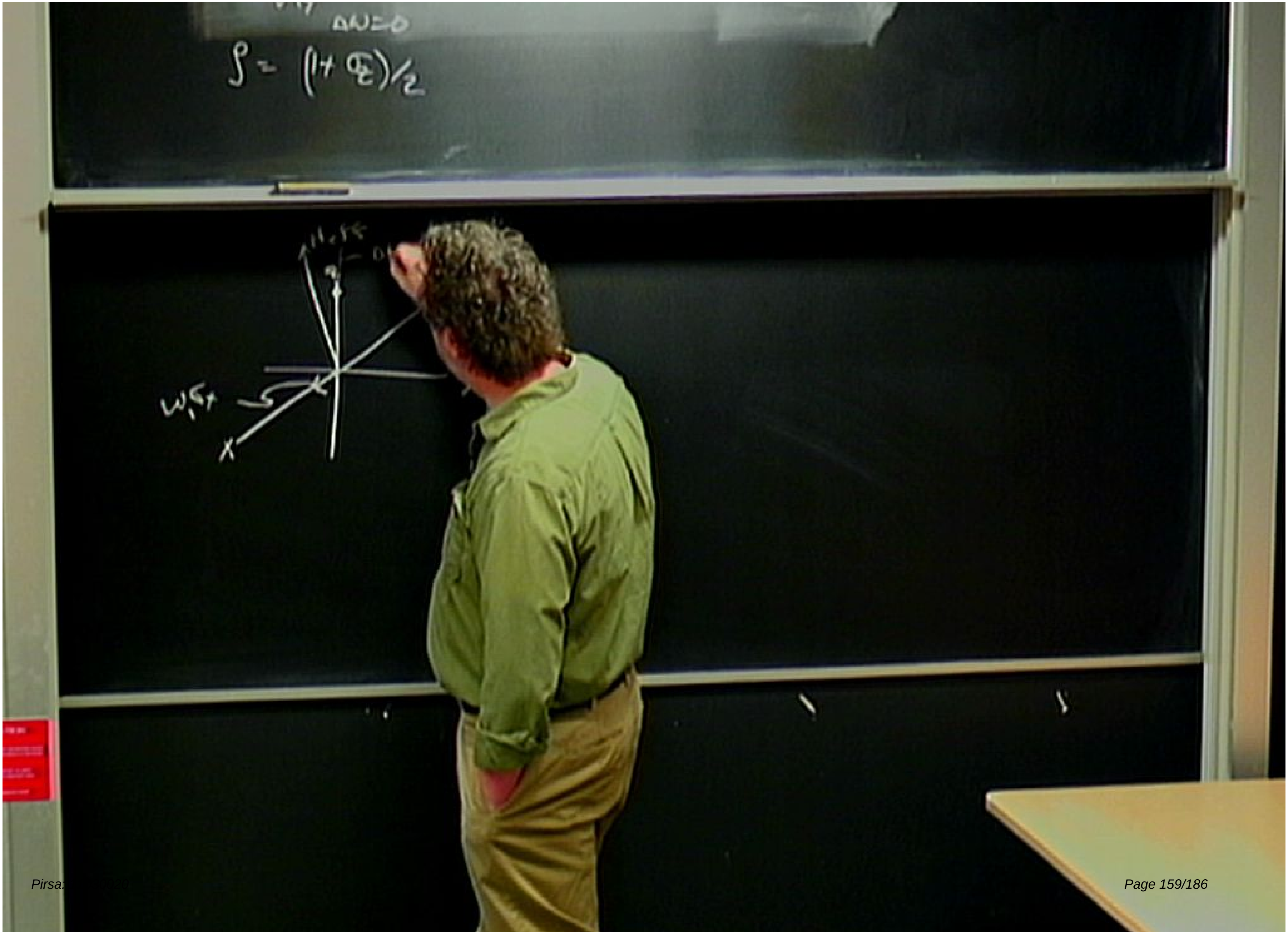


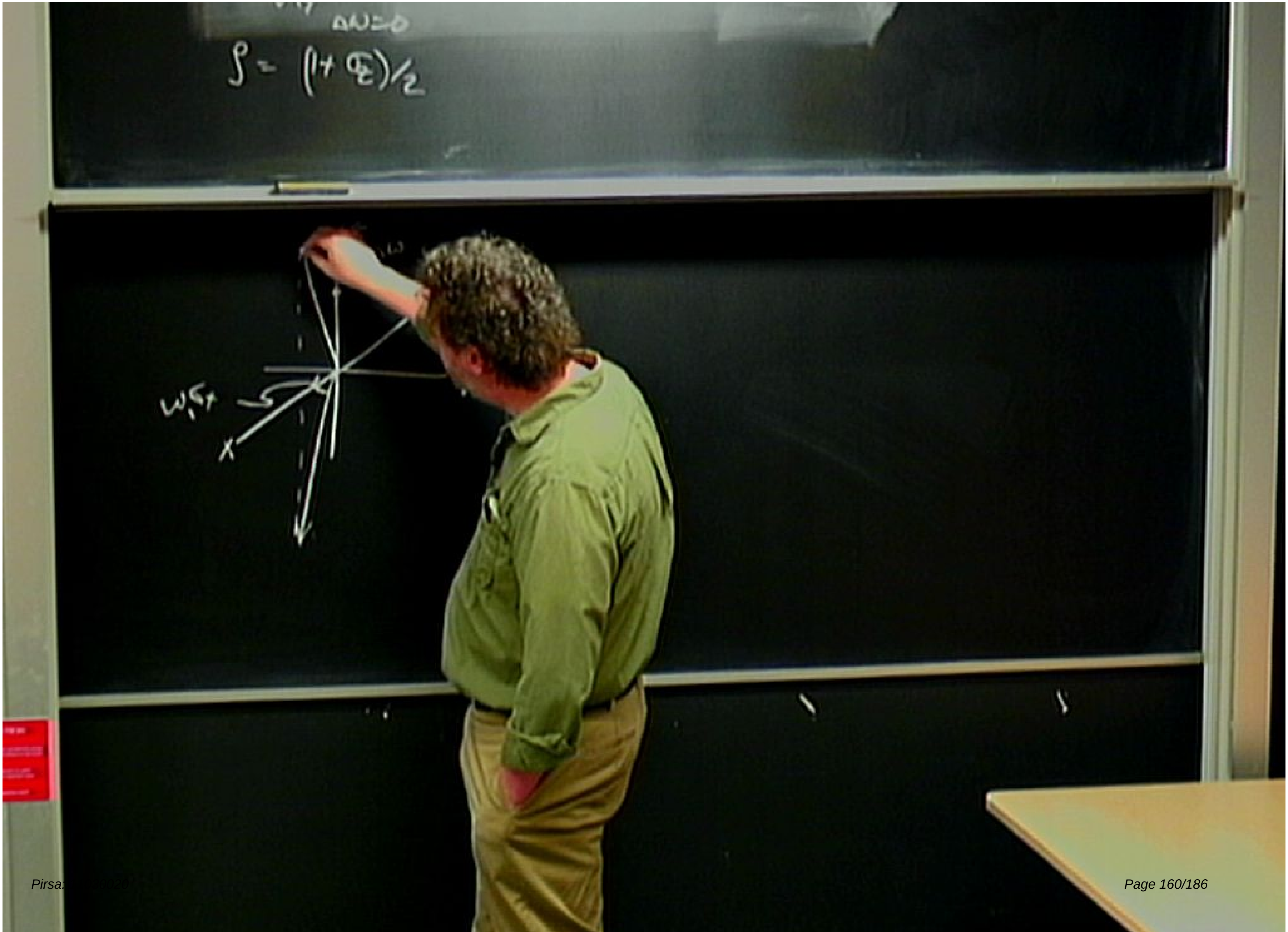
$\mathcal{H}_4$
ΔN=0
ρ = (1+ σ_z)/2

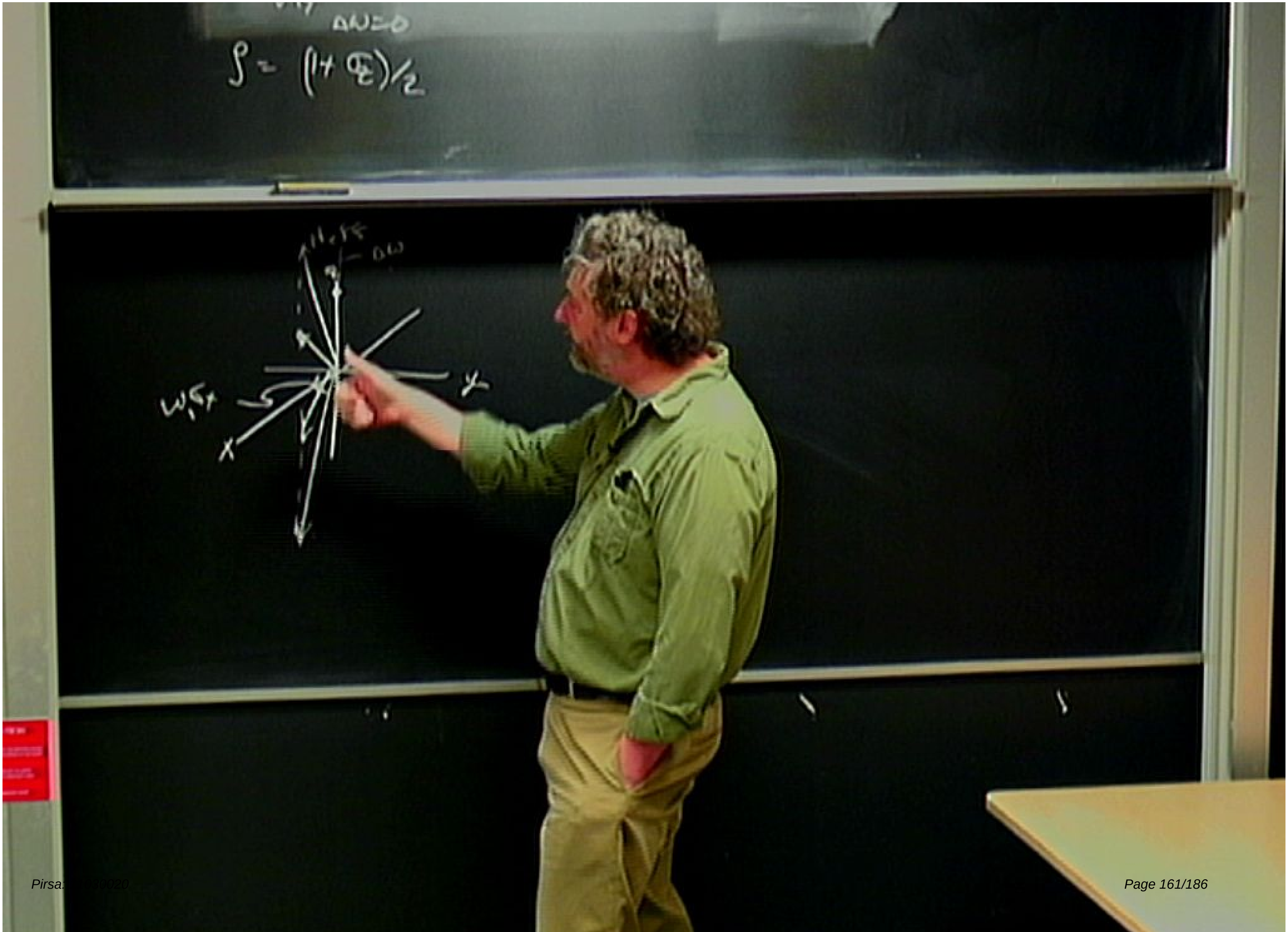


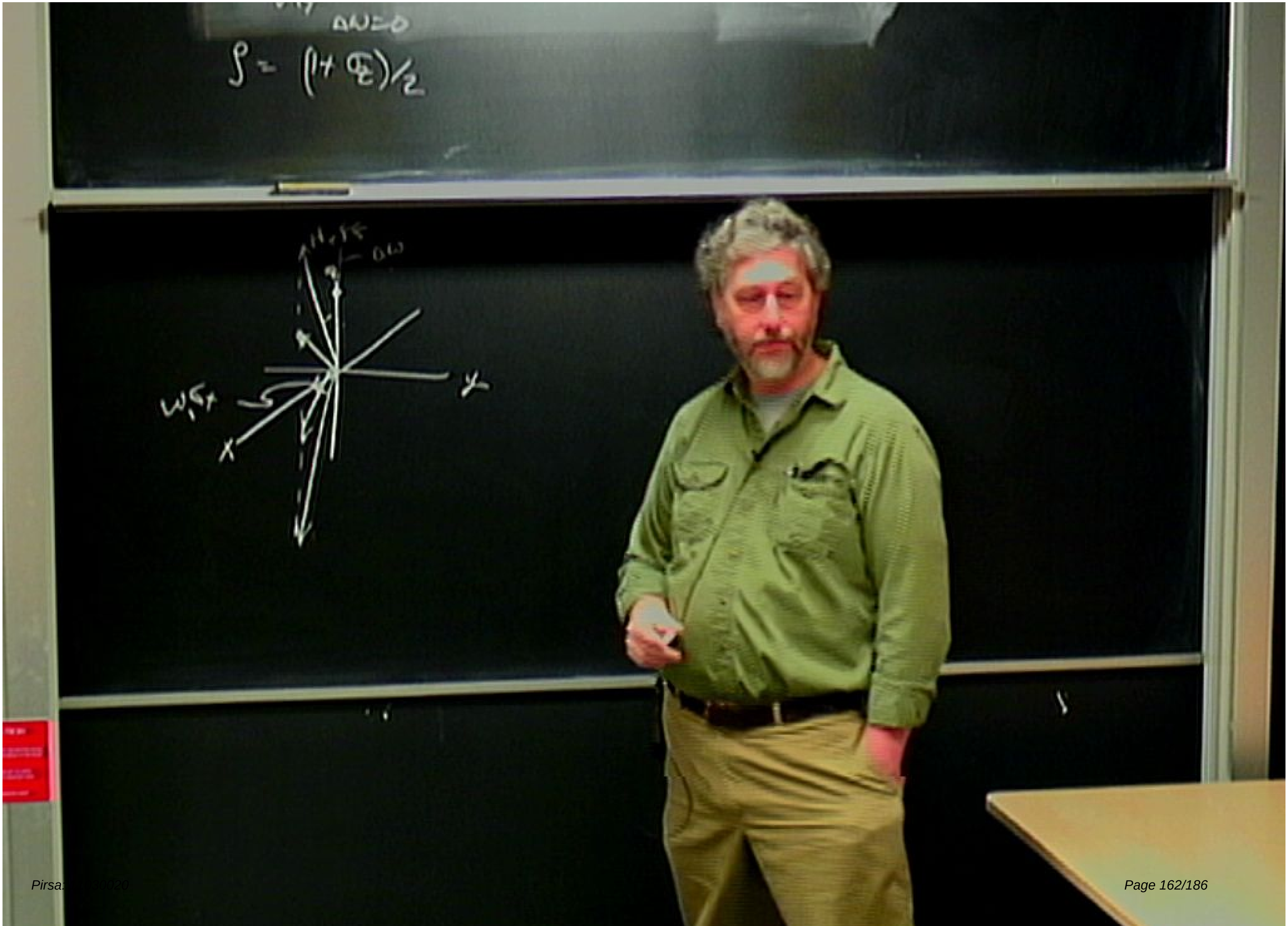
ΔN=0
x
y

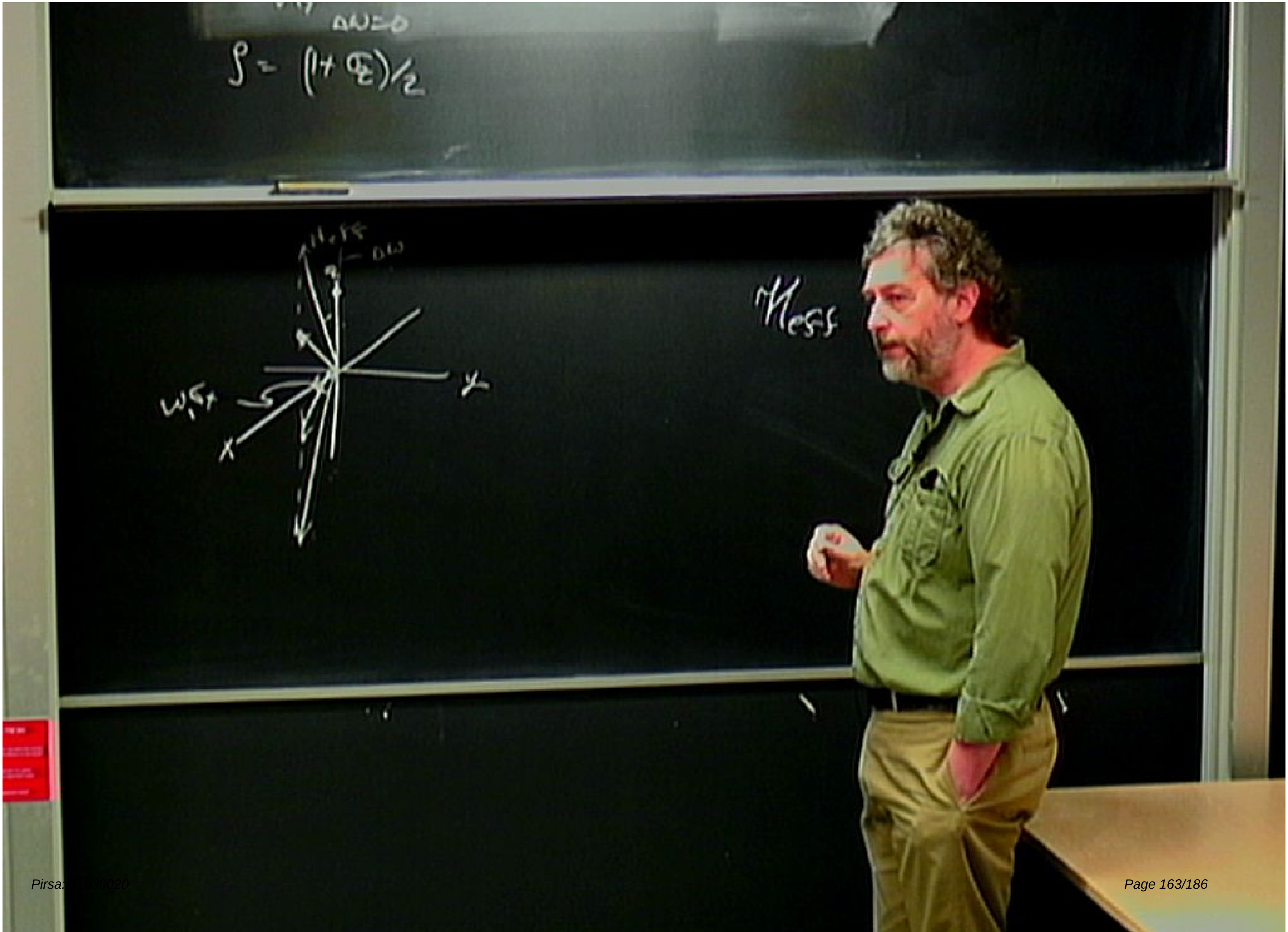


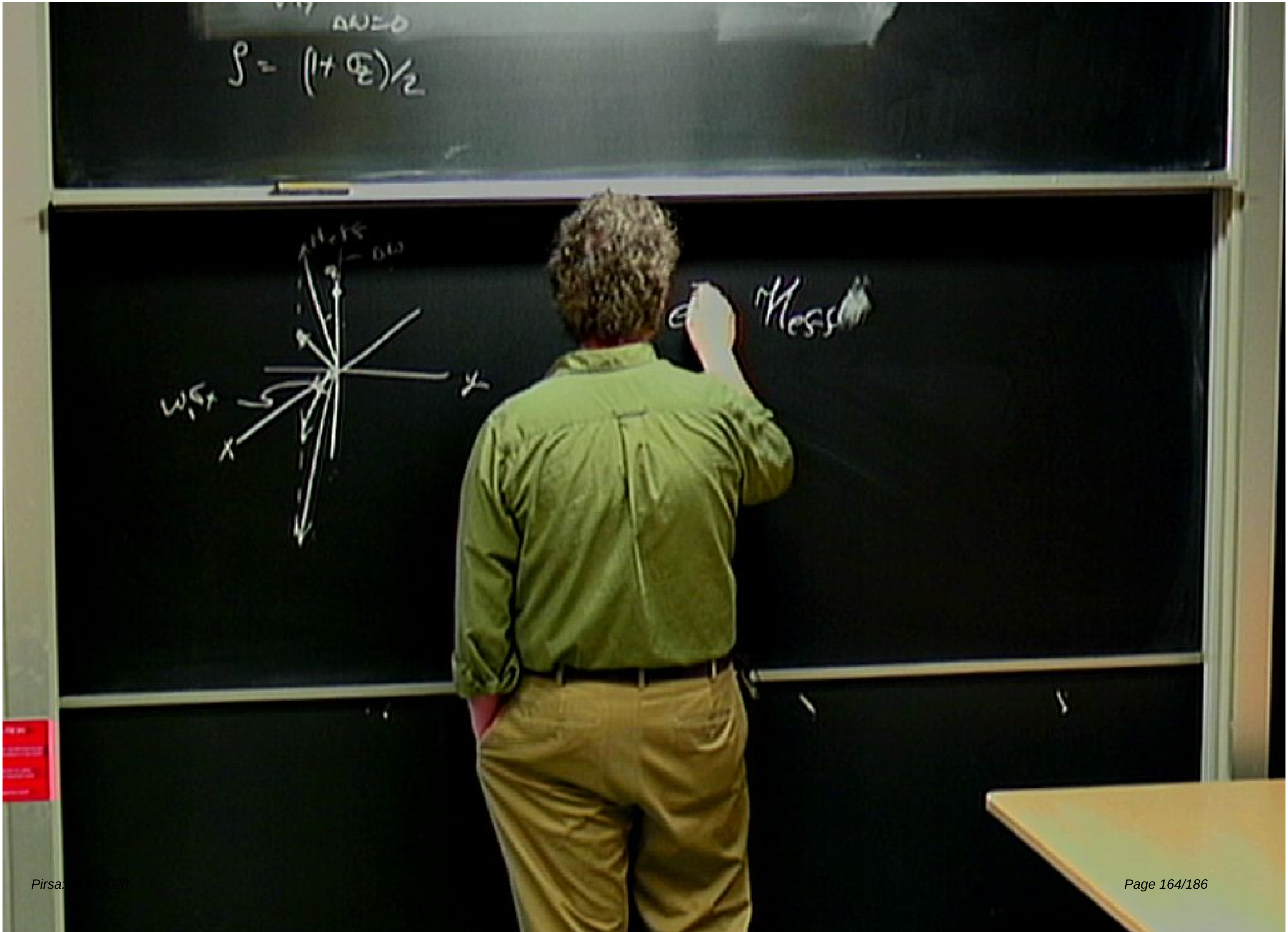

ΔN=0
ρ = (1+ σ·E)/2

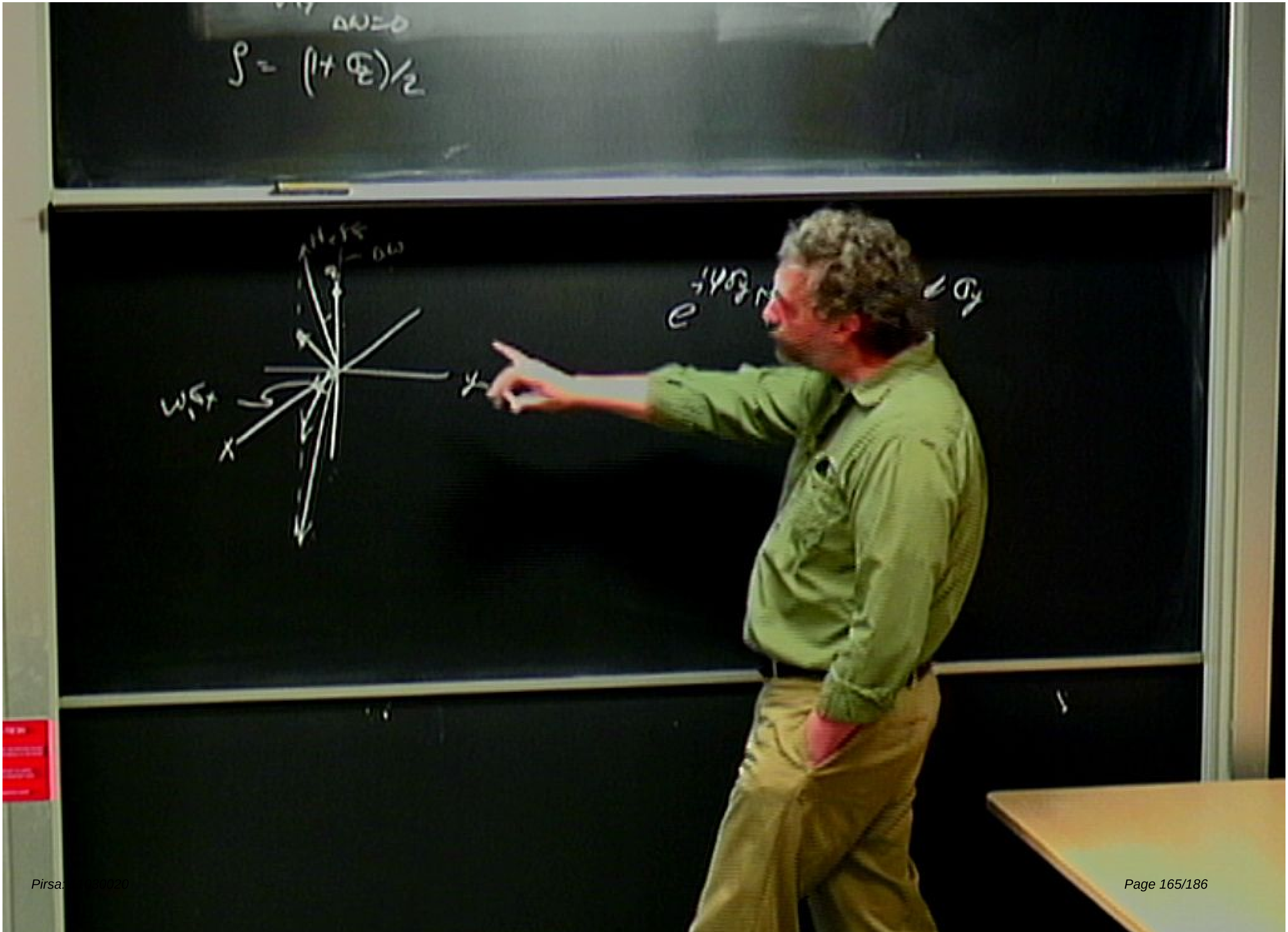

ΔN=0
ρ = (1+ σE)/2
x

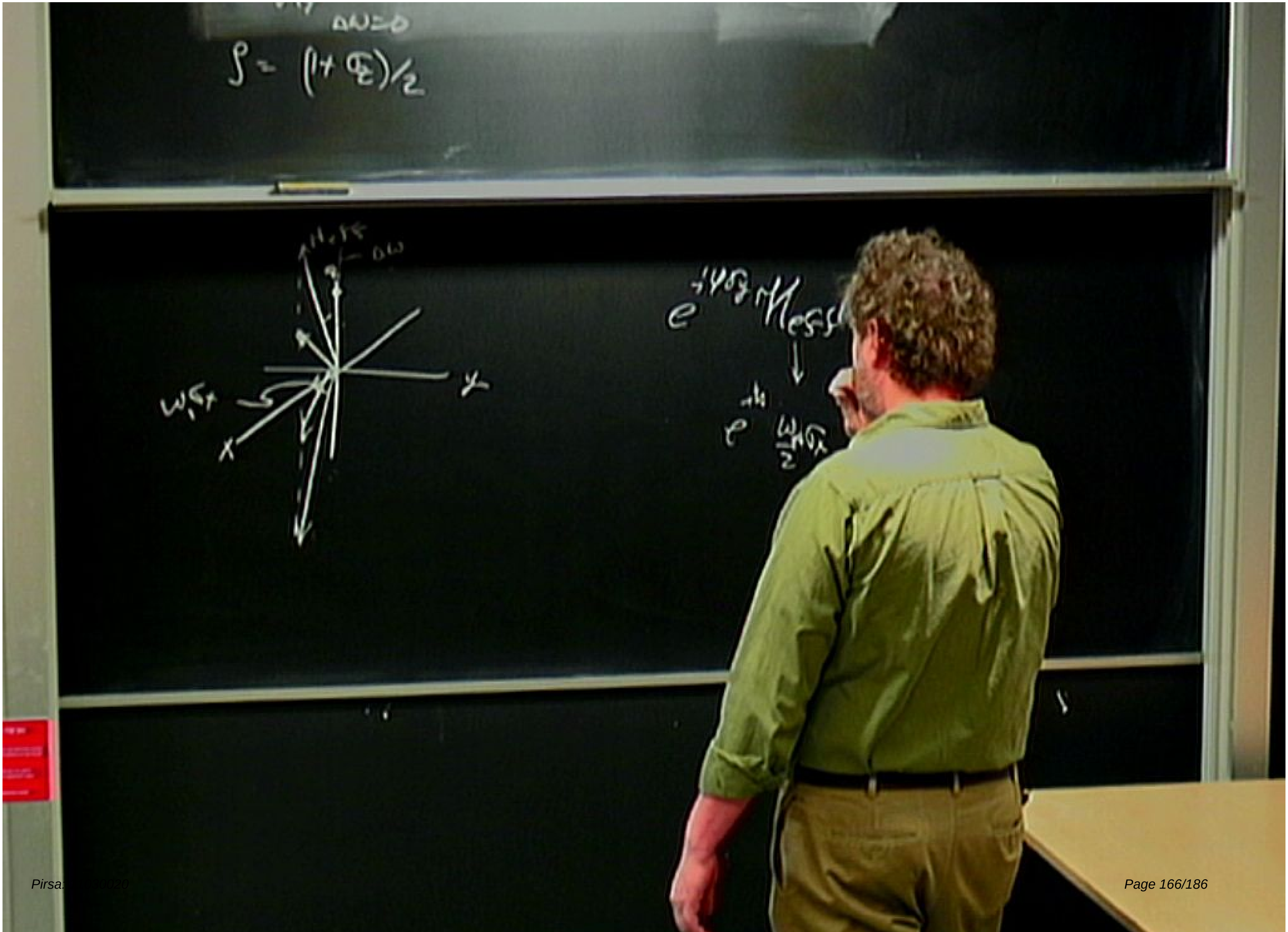

ΔN=0

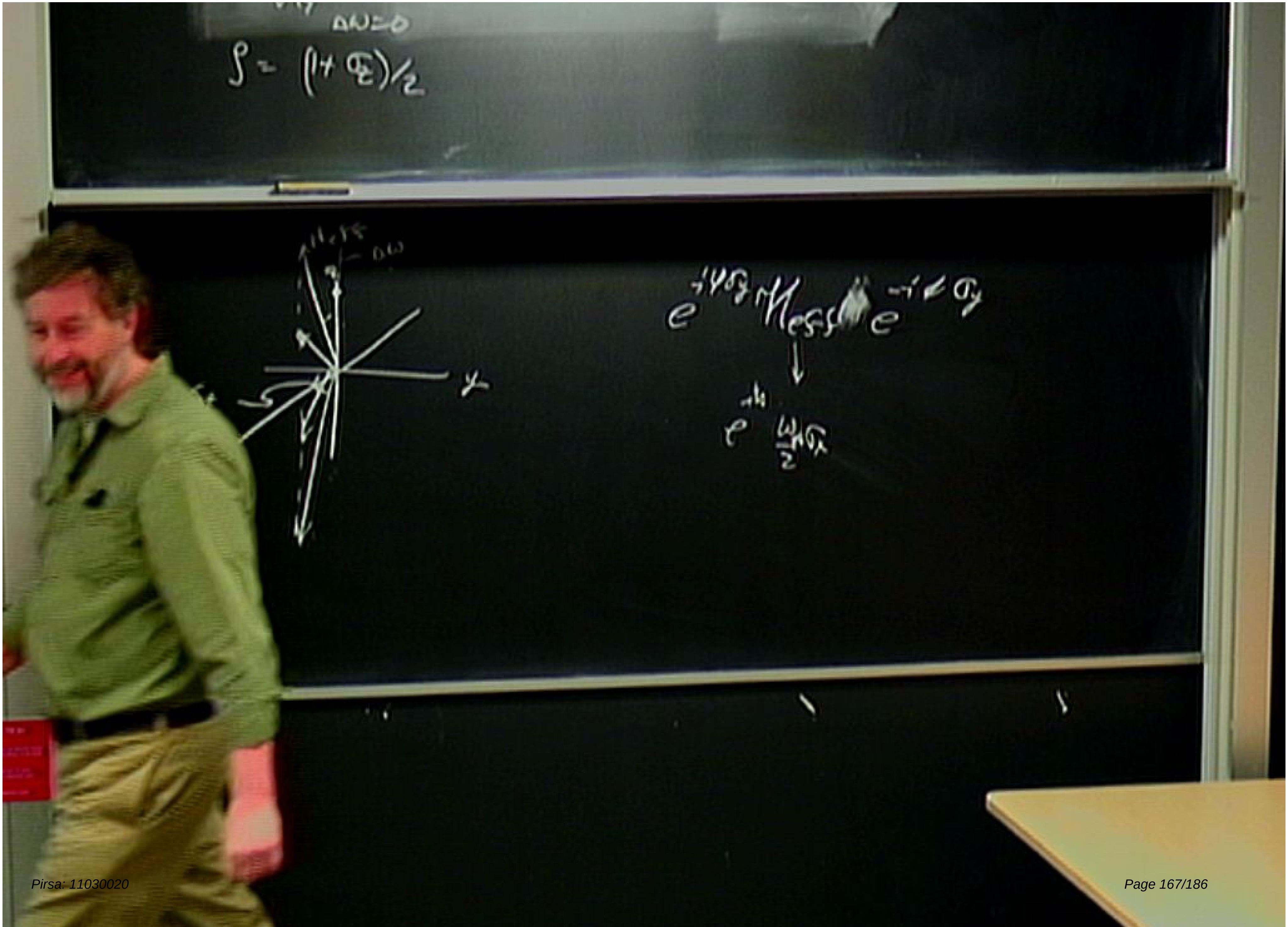

ΔN=0
x
y

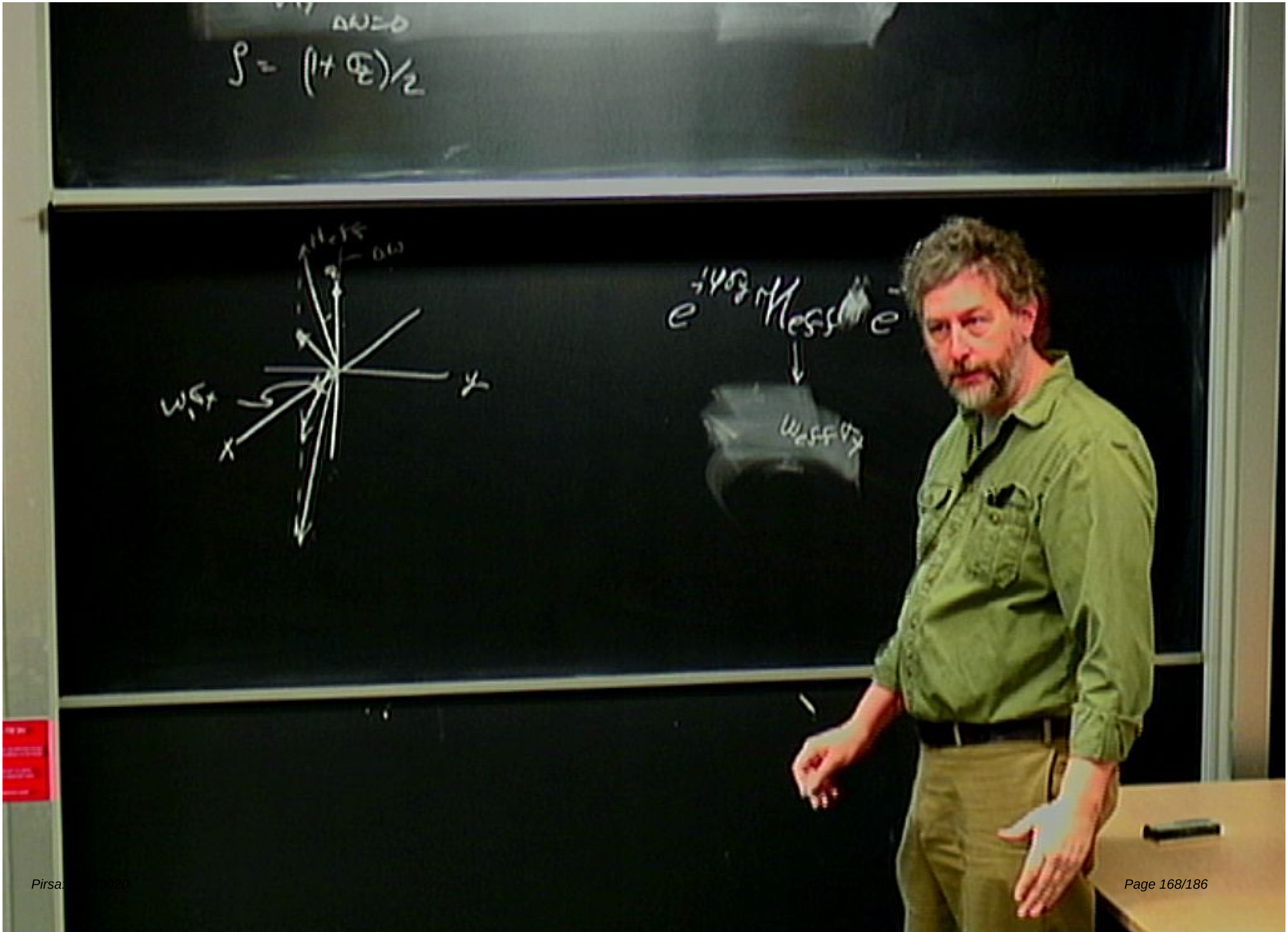


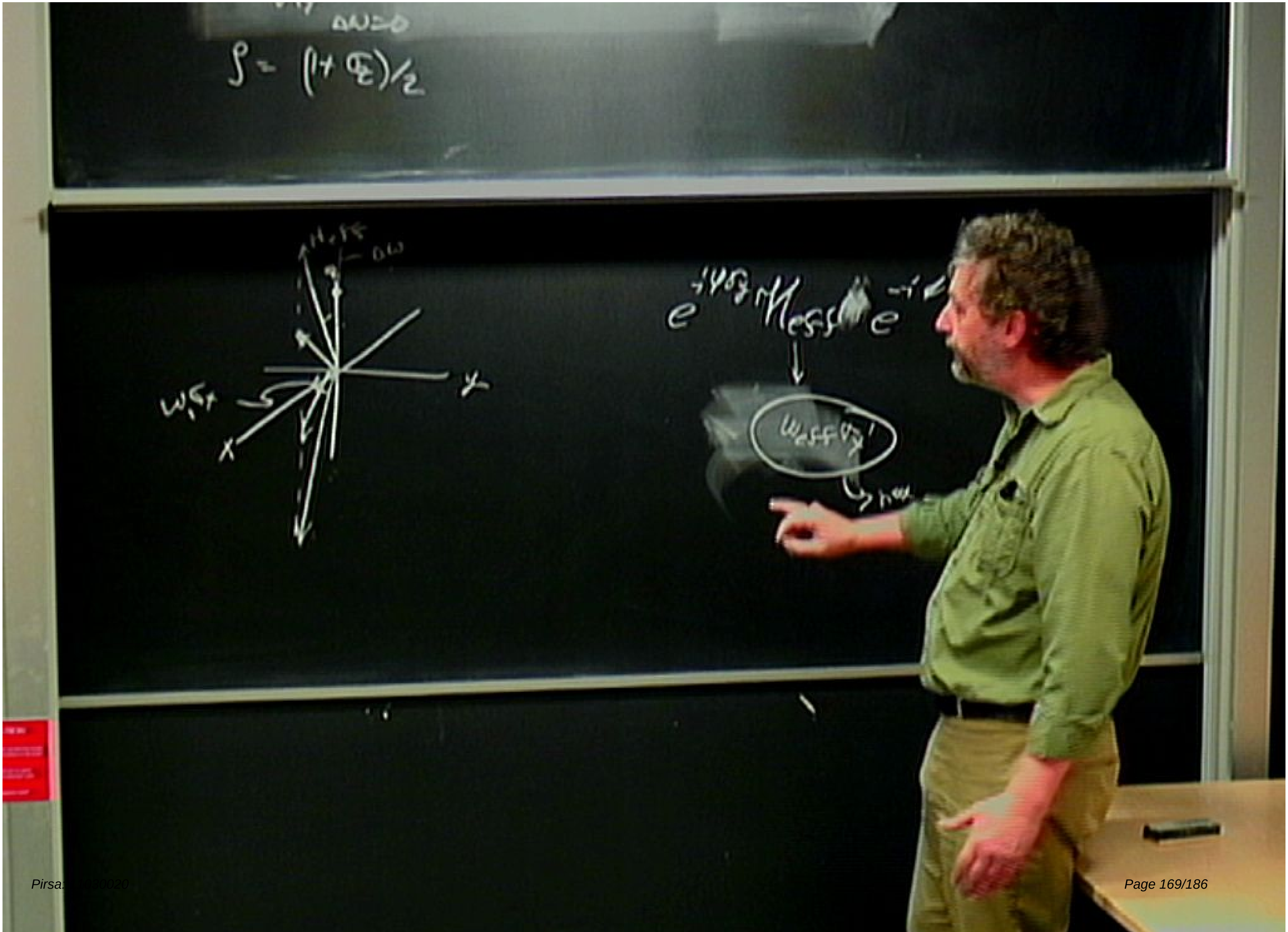




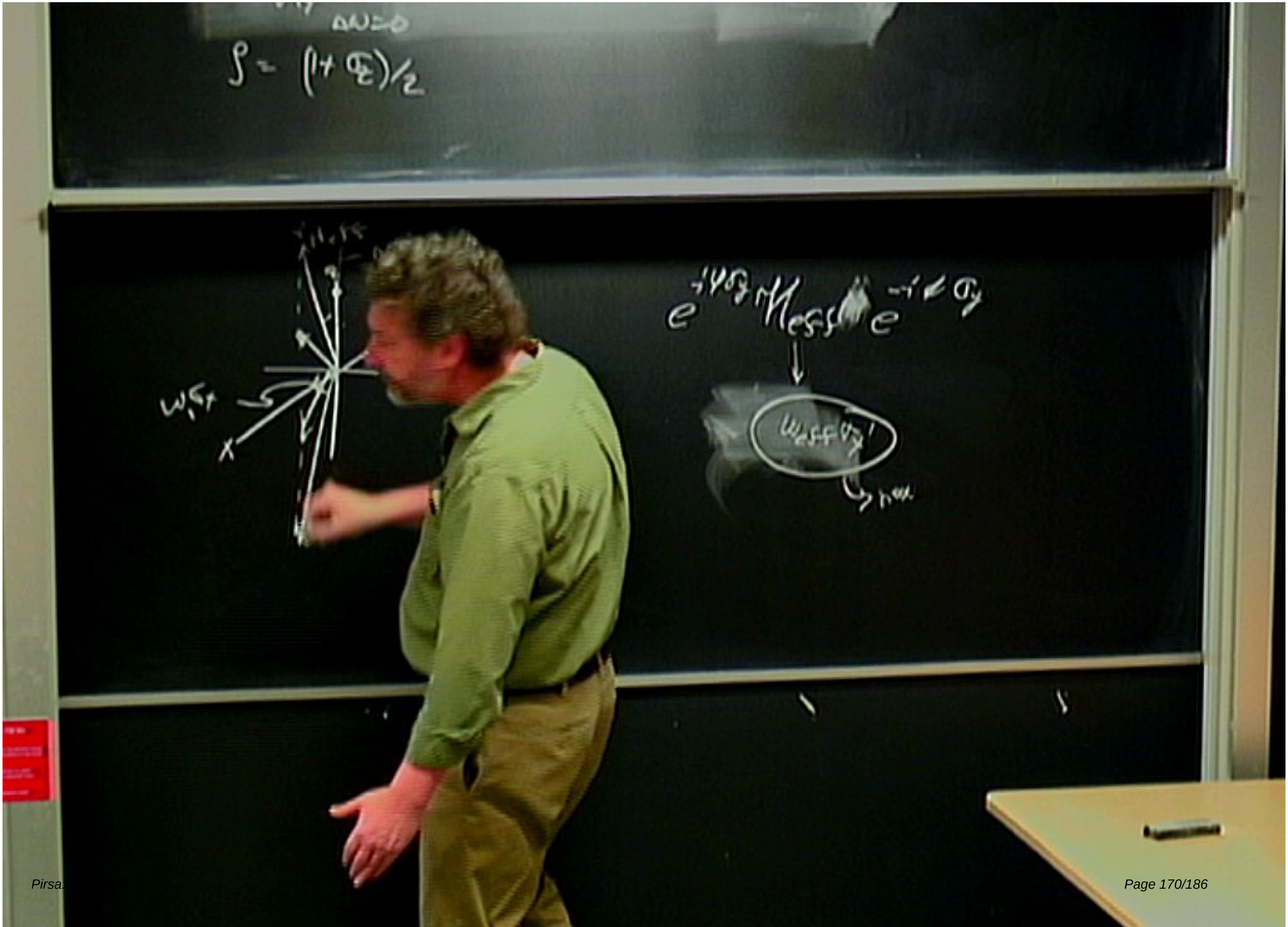

ΔN=0
ρ = (1+σz)/2

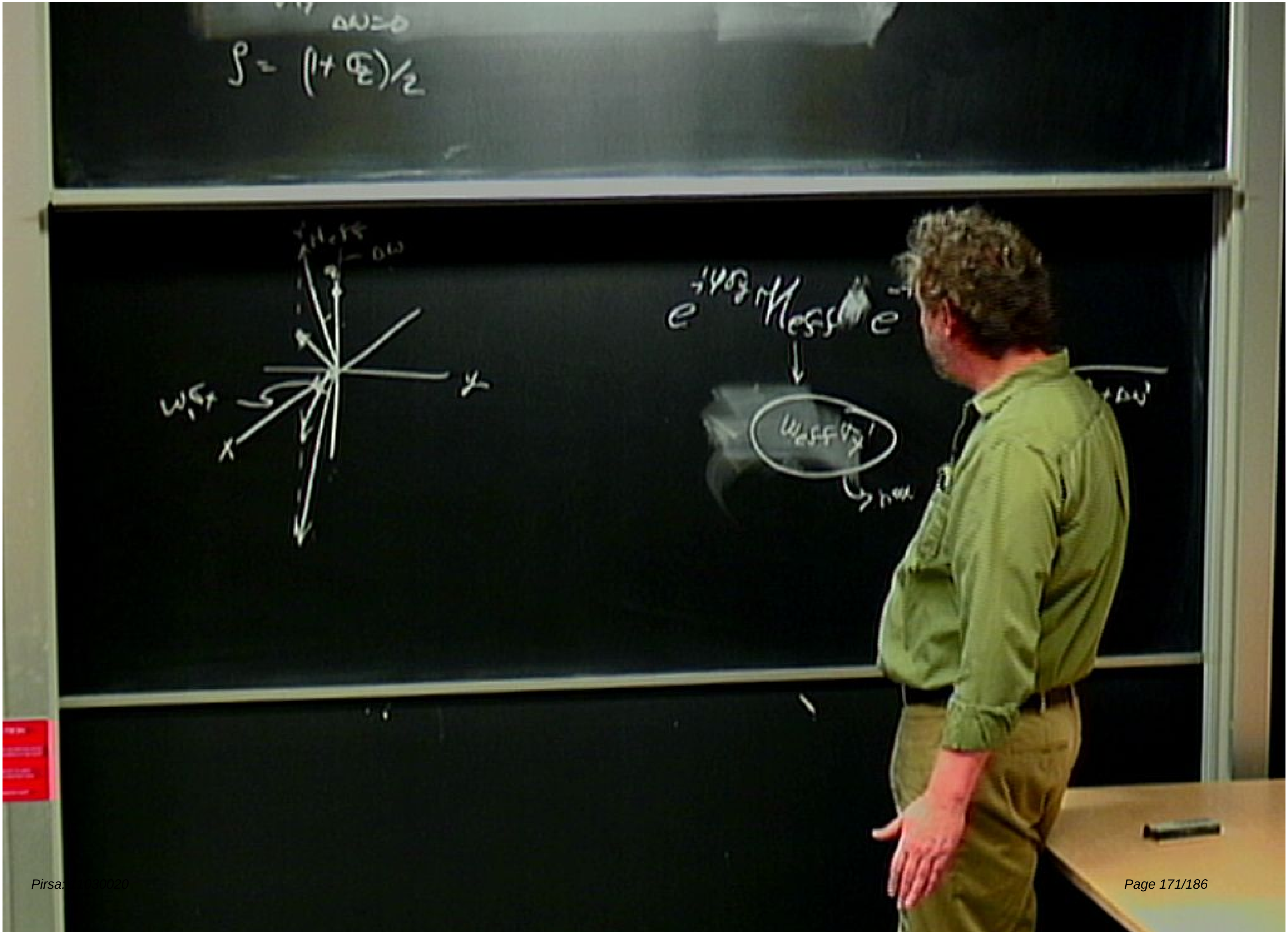


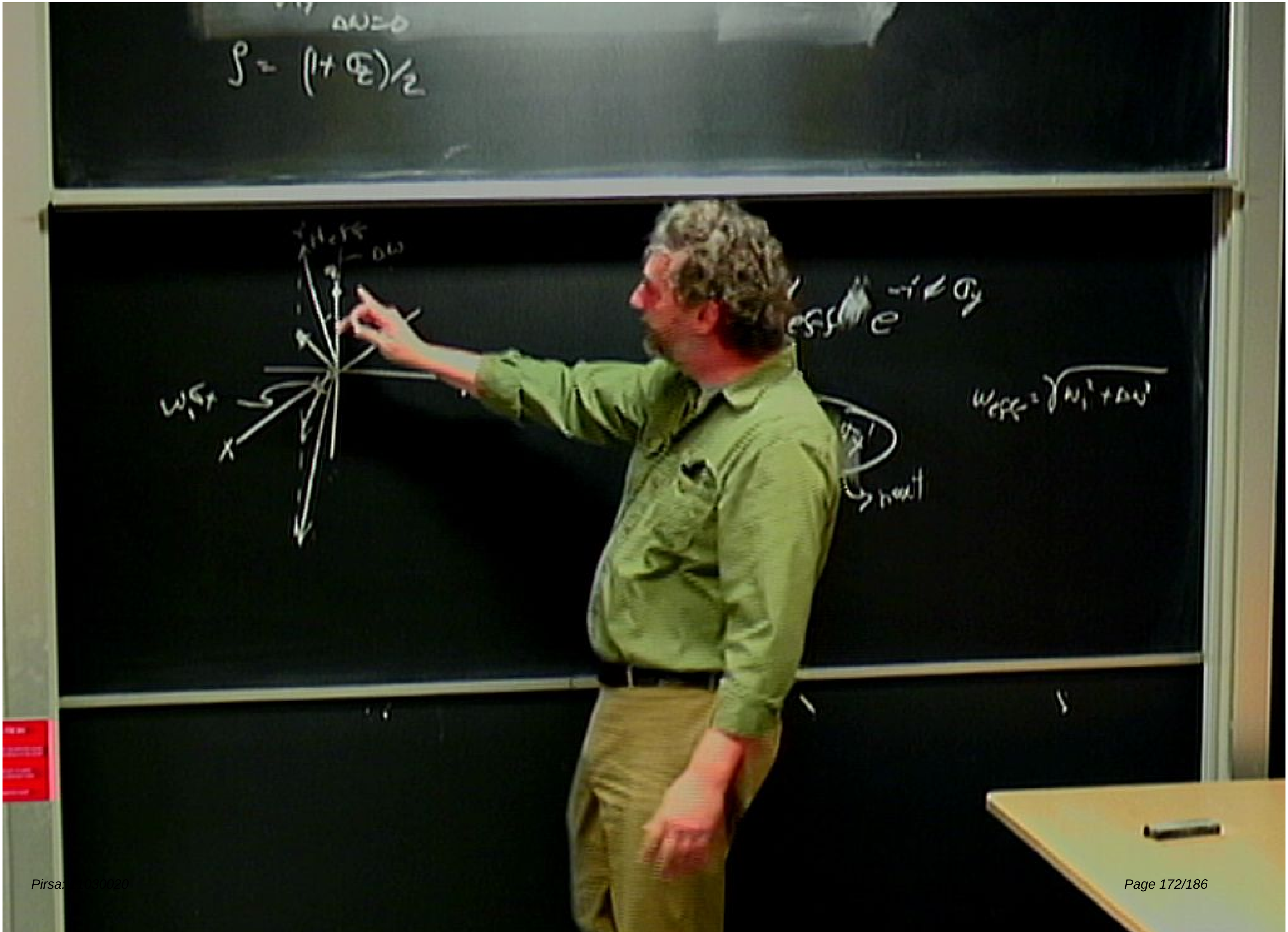




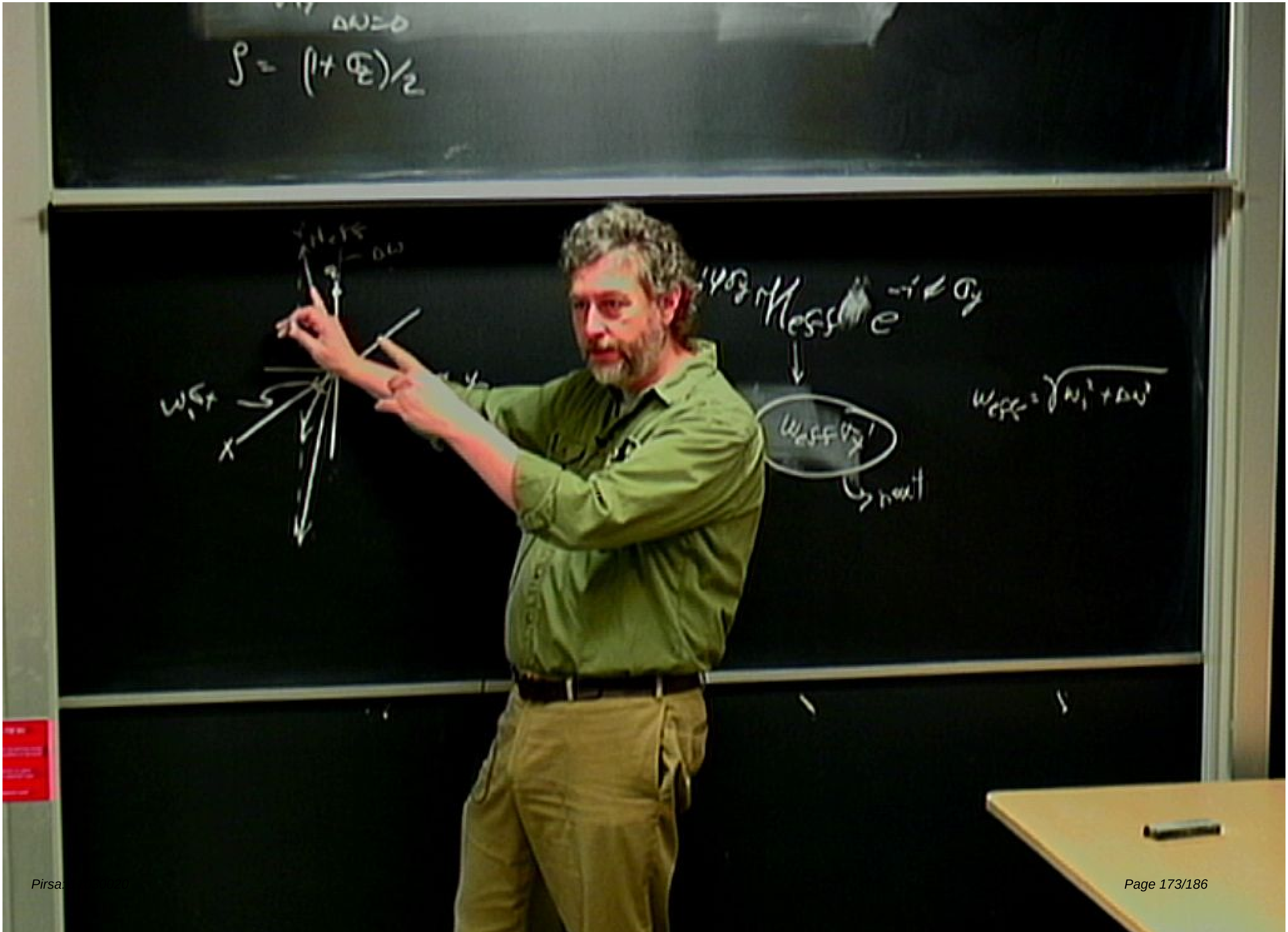
ΔN=0
ρ = (1+σz)/2
x
x

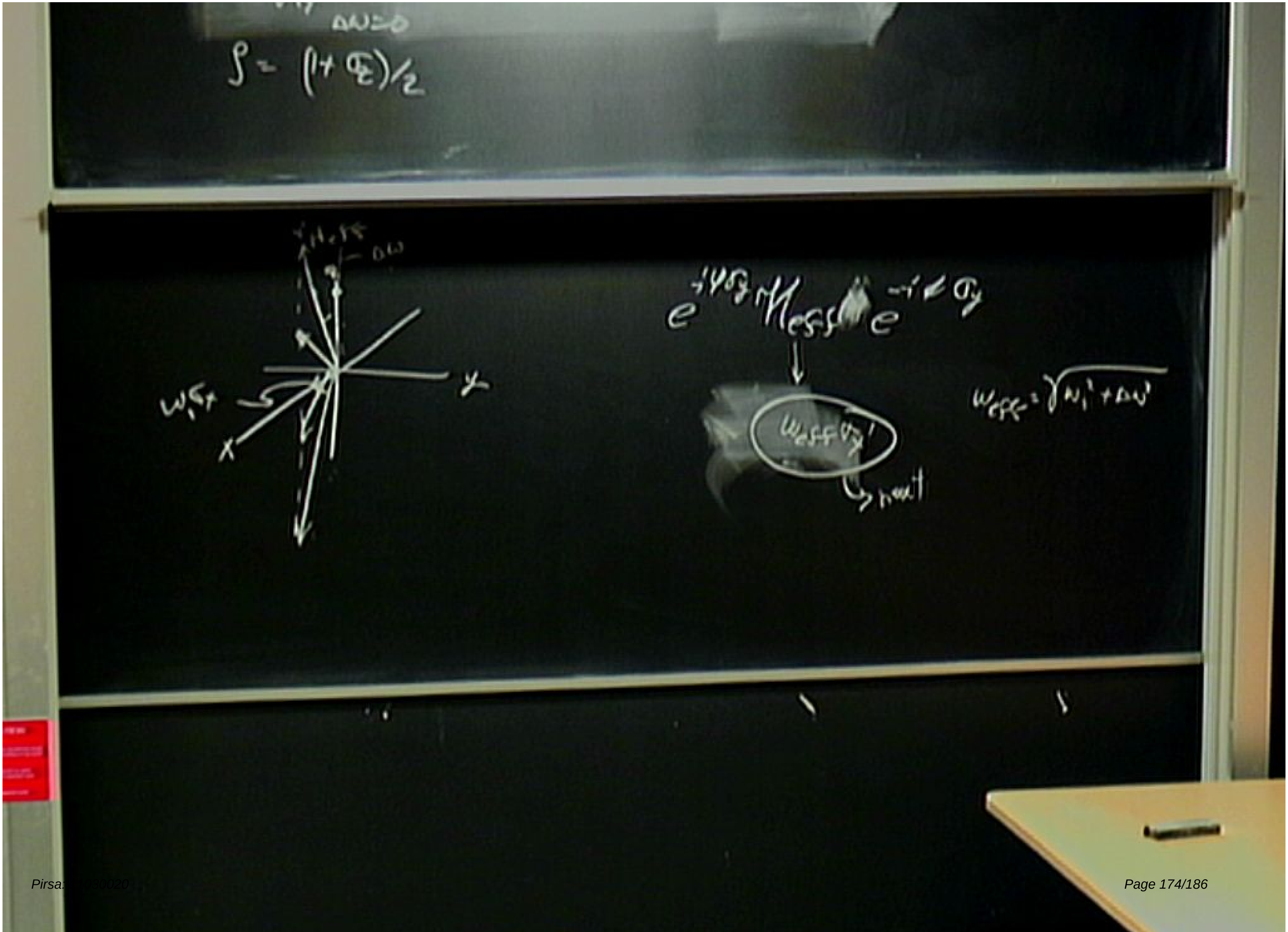

ρ = (1+ σz)/2





$\rho = (1+\sigma_z)/2$
$w_{eff} = \sqrt{w_1^2 + \Delta w^2}$


ΔW=0
ρ = (1+σz)/2


$\rho = (1+\sigma_z)/2$
x

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  PlotStyle → {RGBColor[0, 1, 0], Thickness[0.01]}],
ListPlot[Table[Ad3[n][[1]][[1]], {n, 0, 800}],
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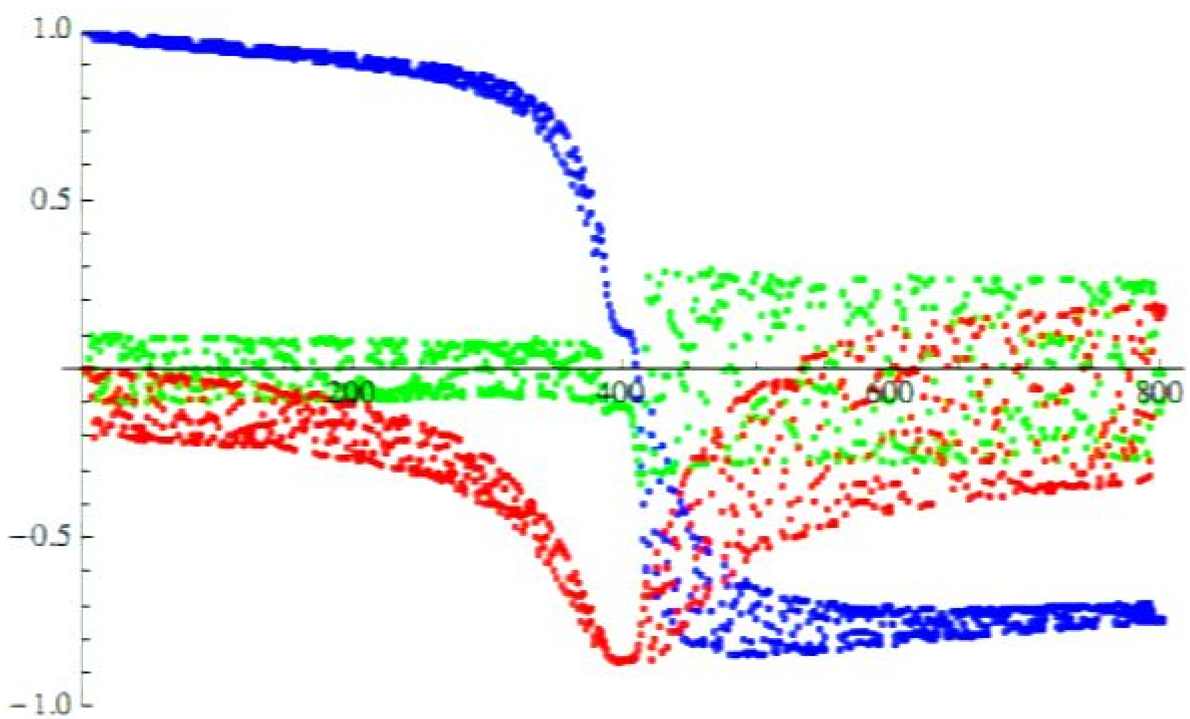

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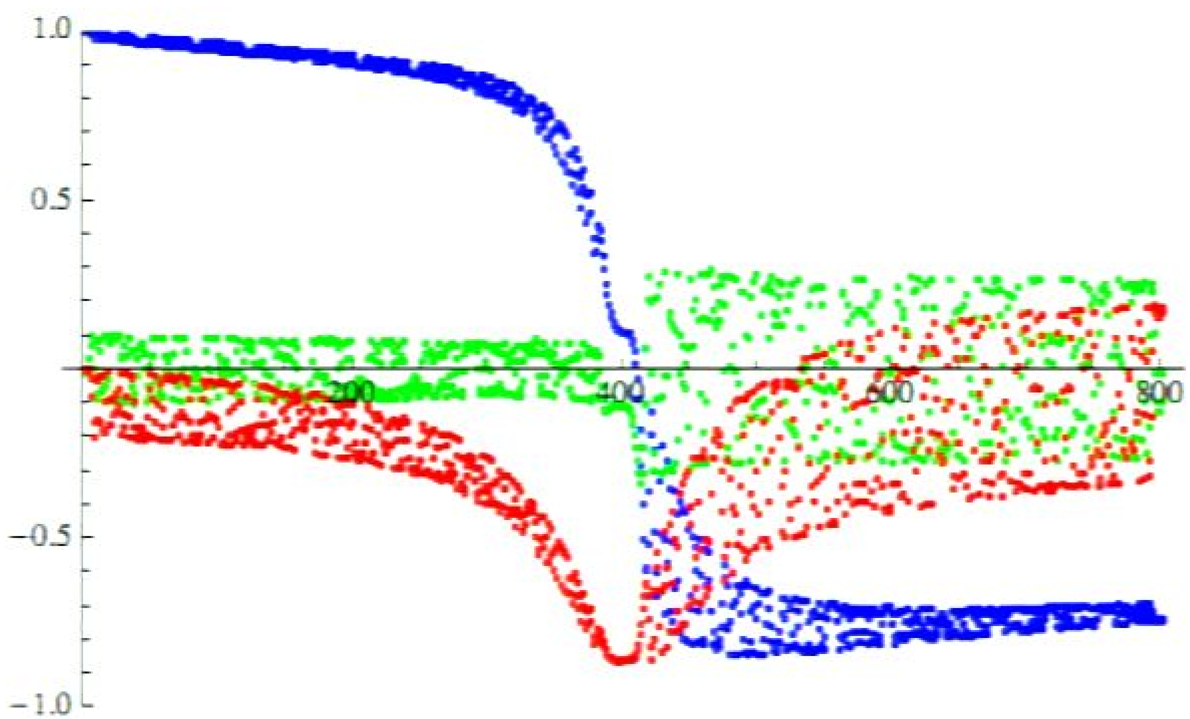
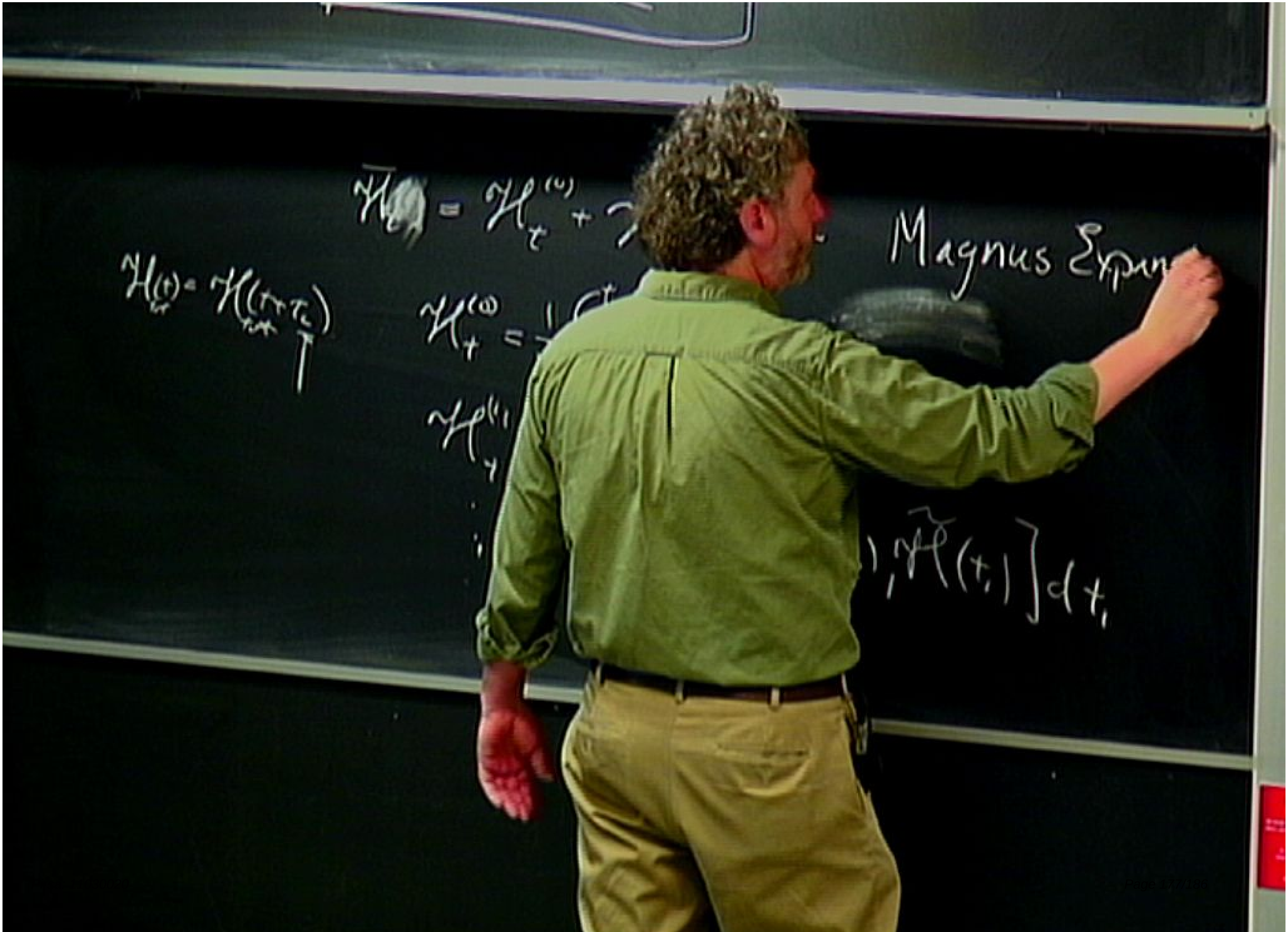
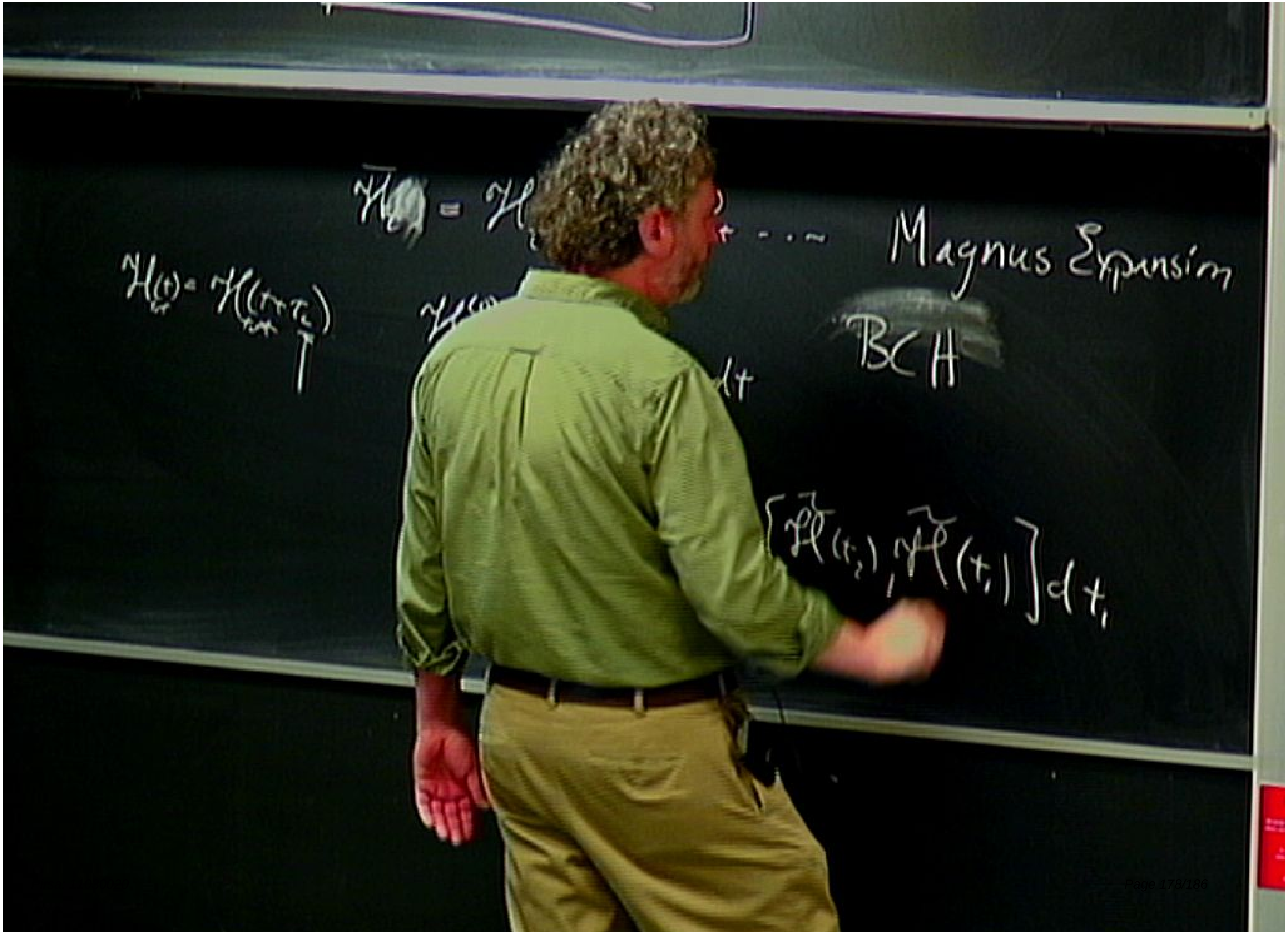
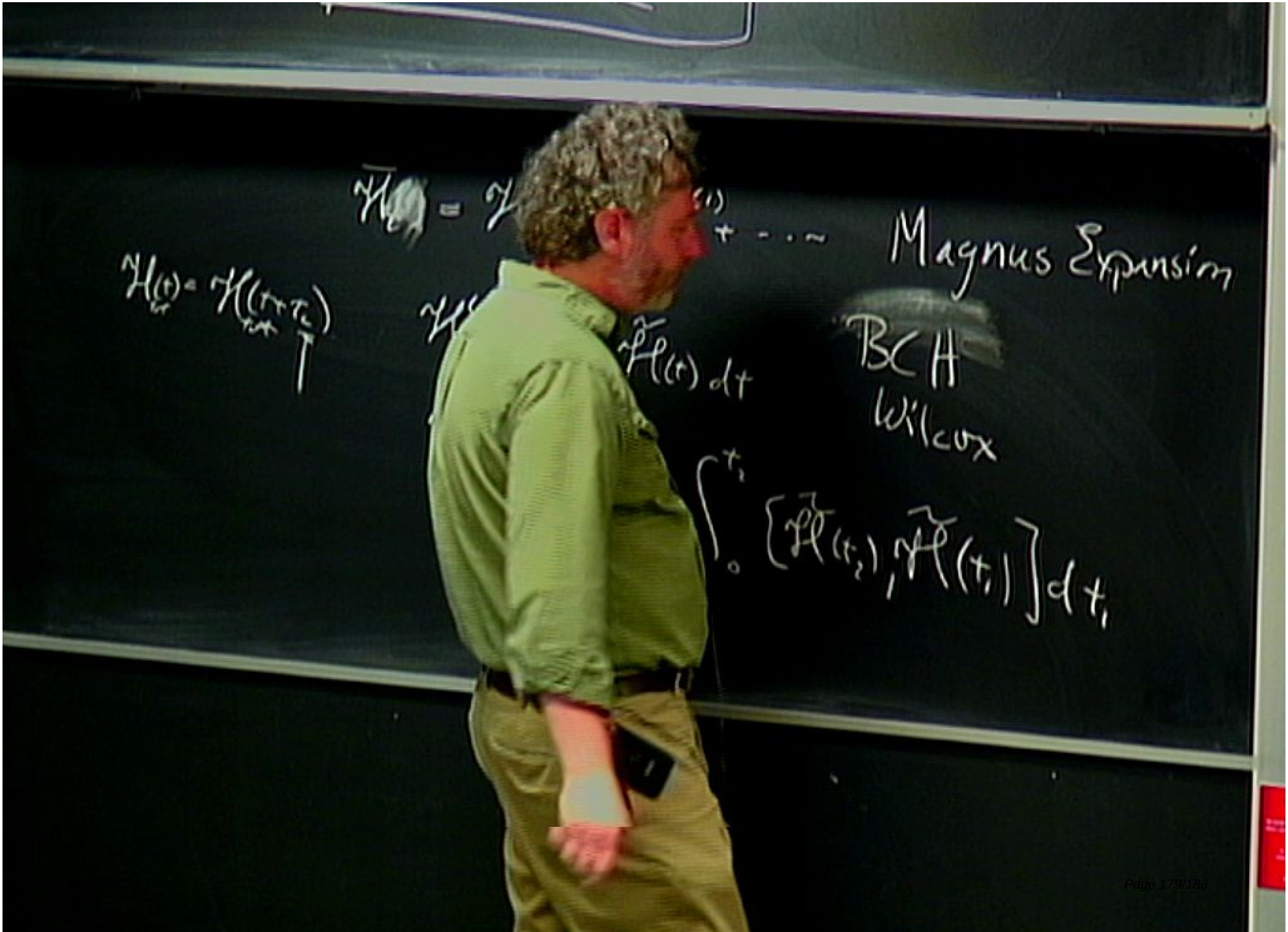



Magnus


Magnus Expansim
BCH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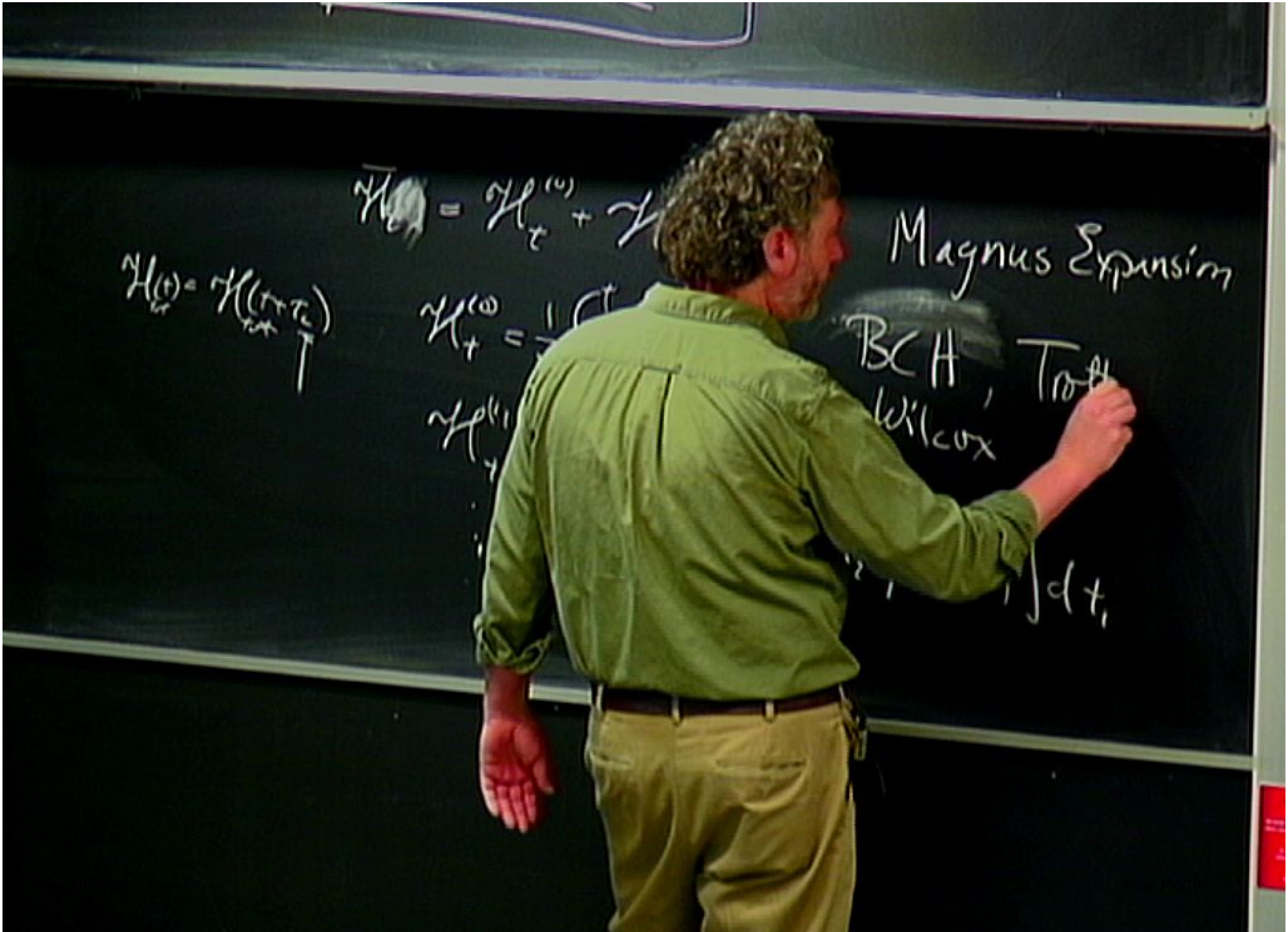
BCH
Wilcox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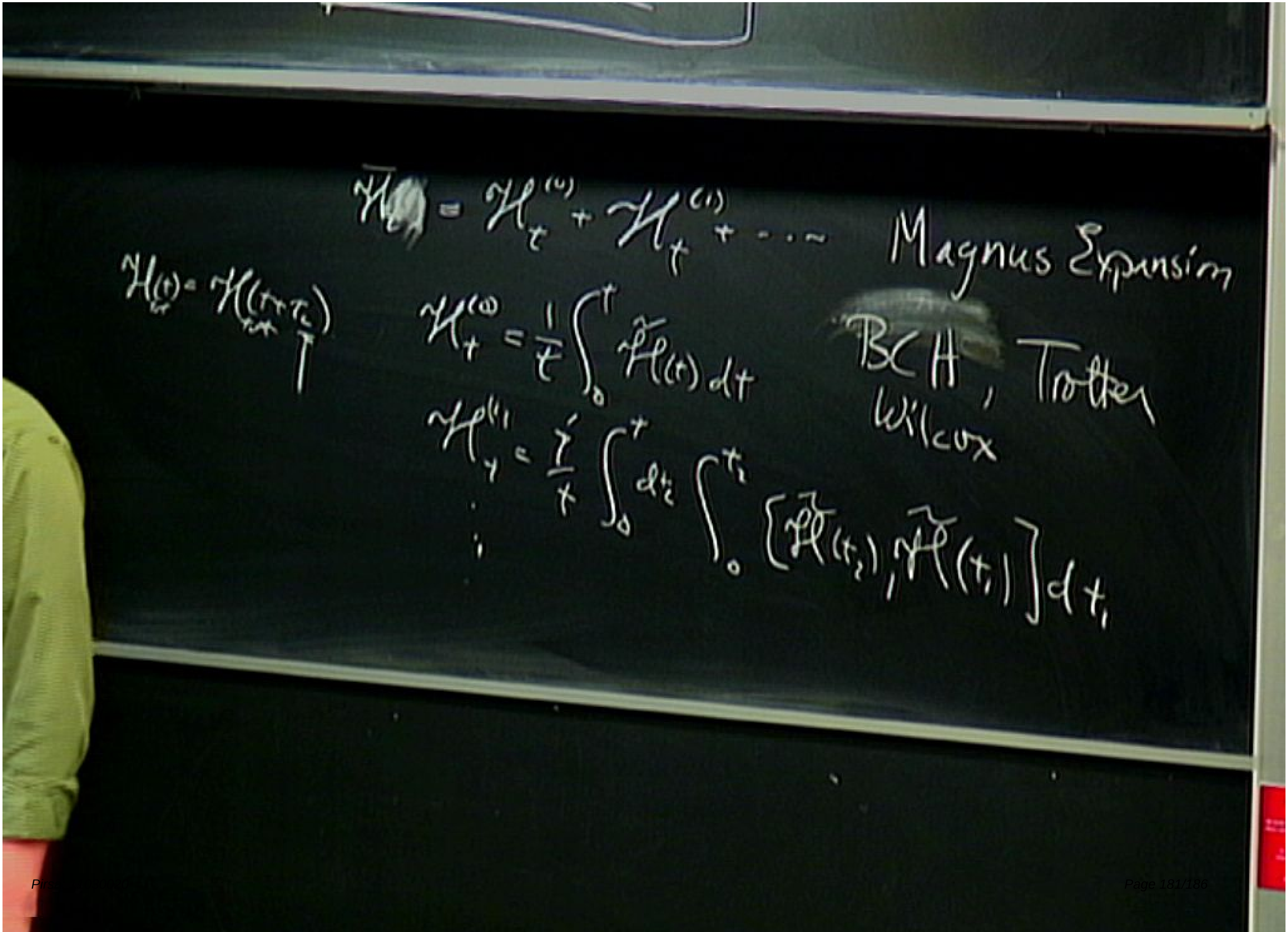
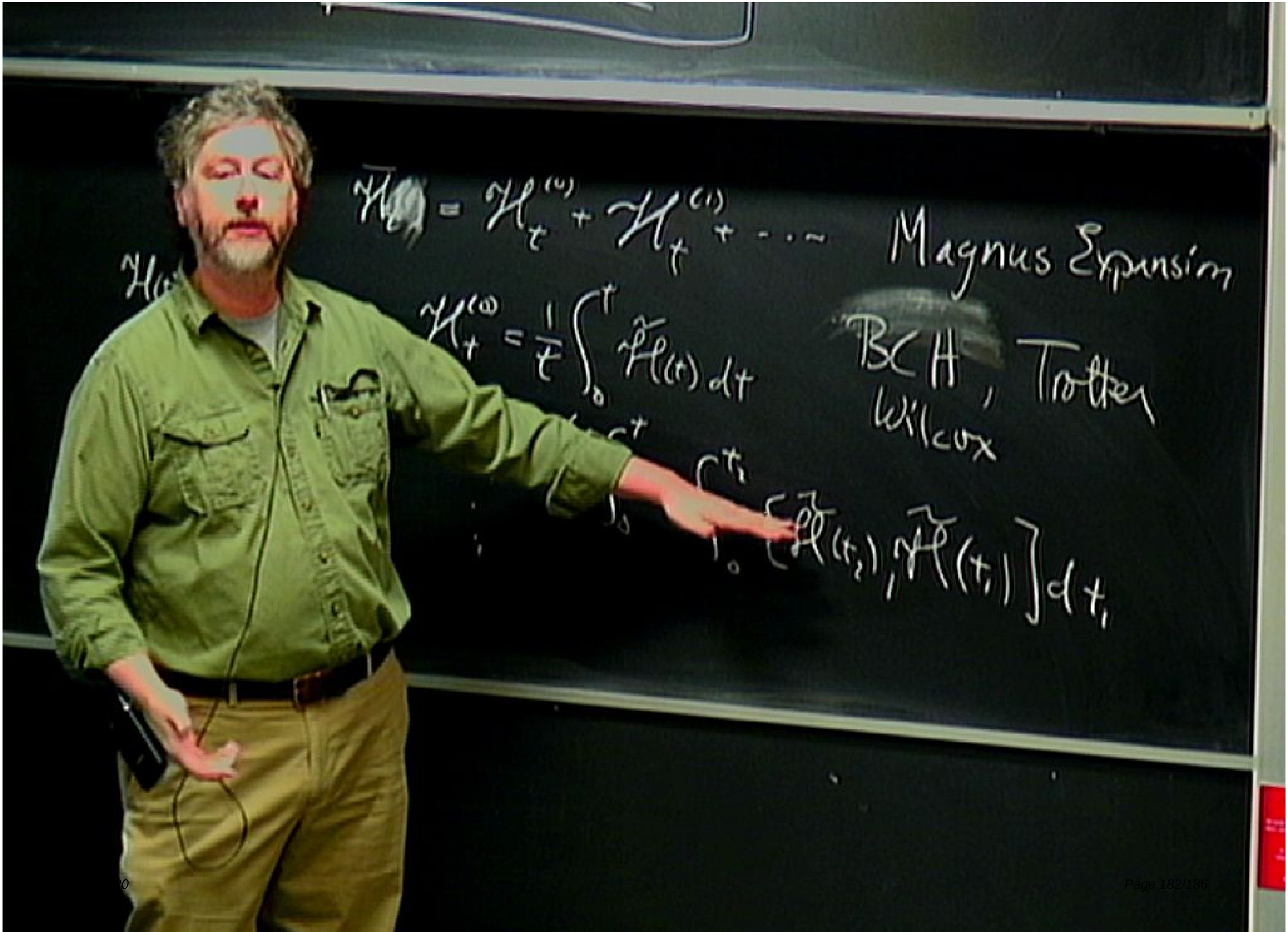

Magnus Expansion
BCH, Trotter
Wilcox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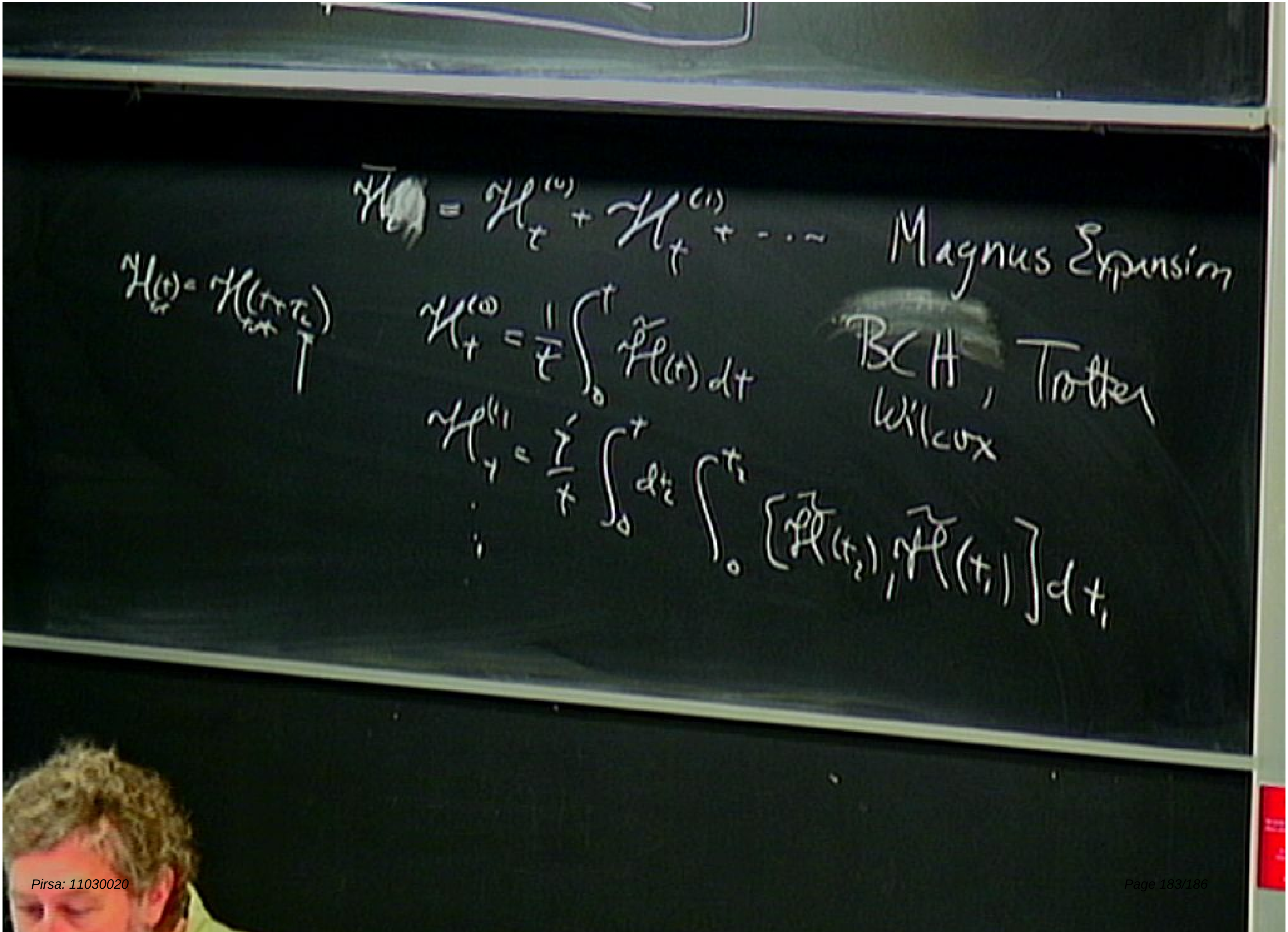

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BCH, Trotter
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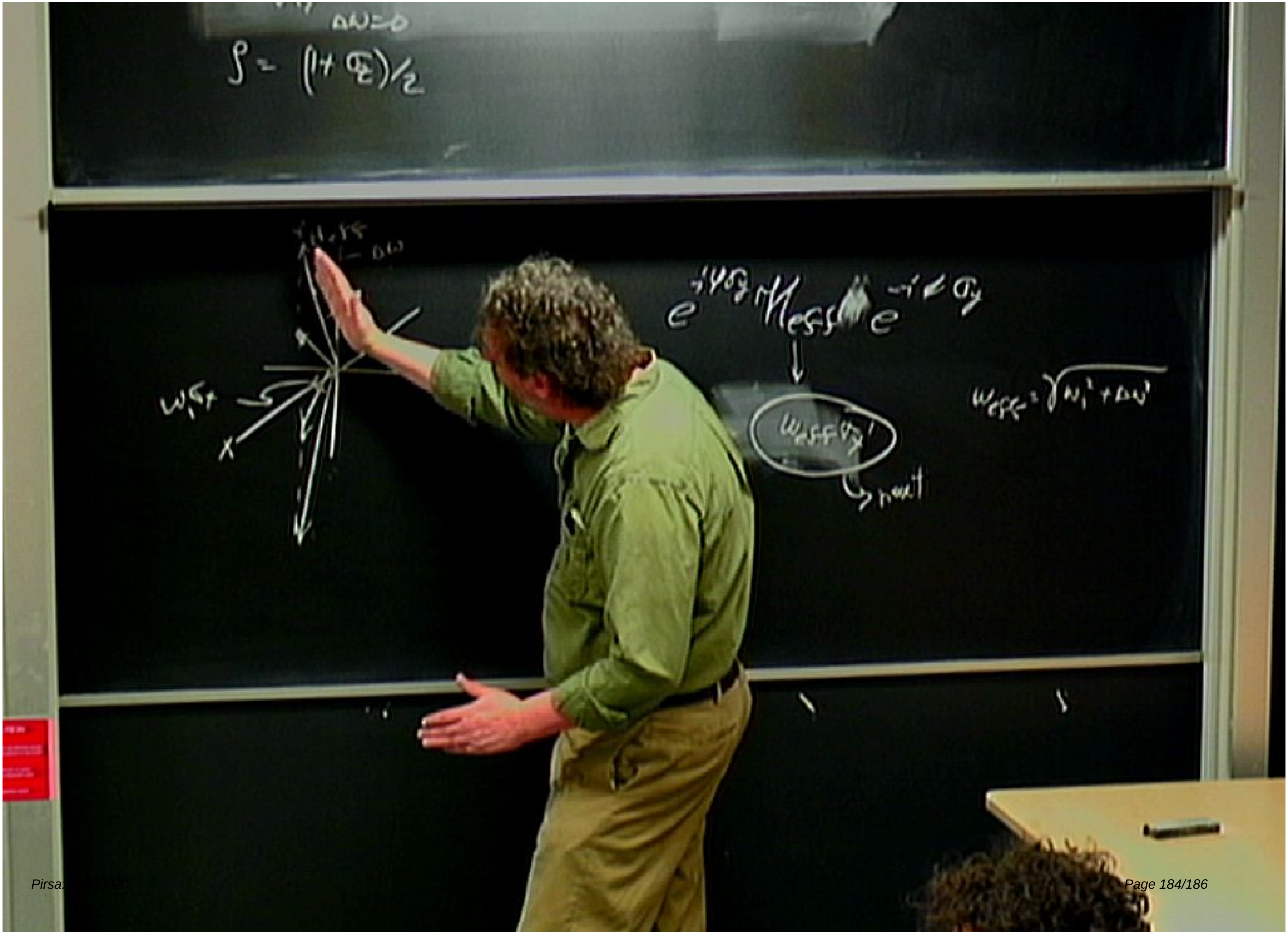

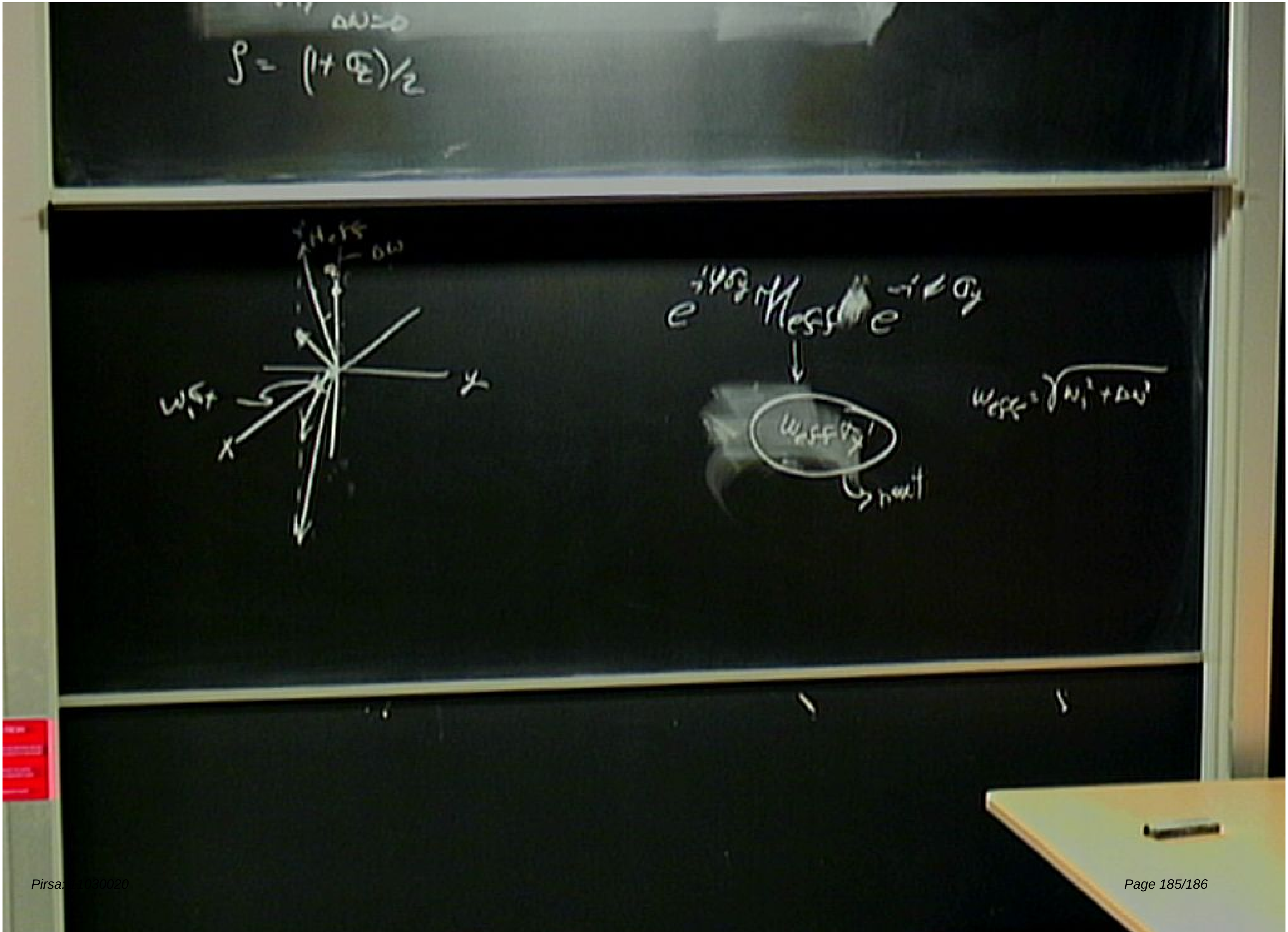

ΔW=0
ρ = (1+σz)/2
x
y
ωeff = √(ω1² + Δω²)
next

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