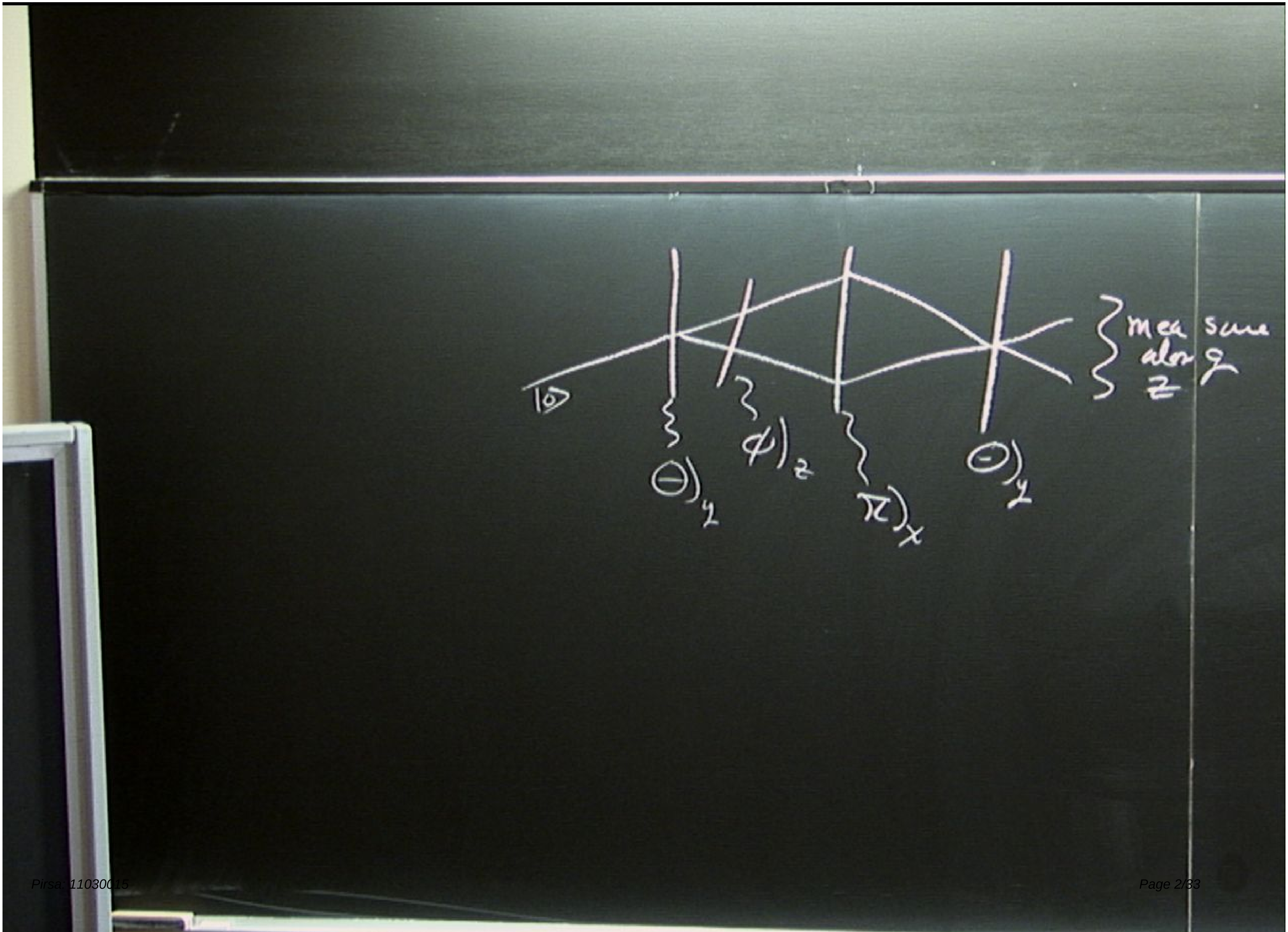
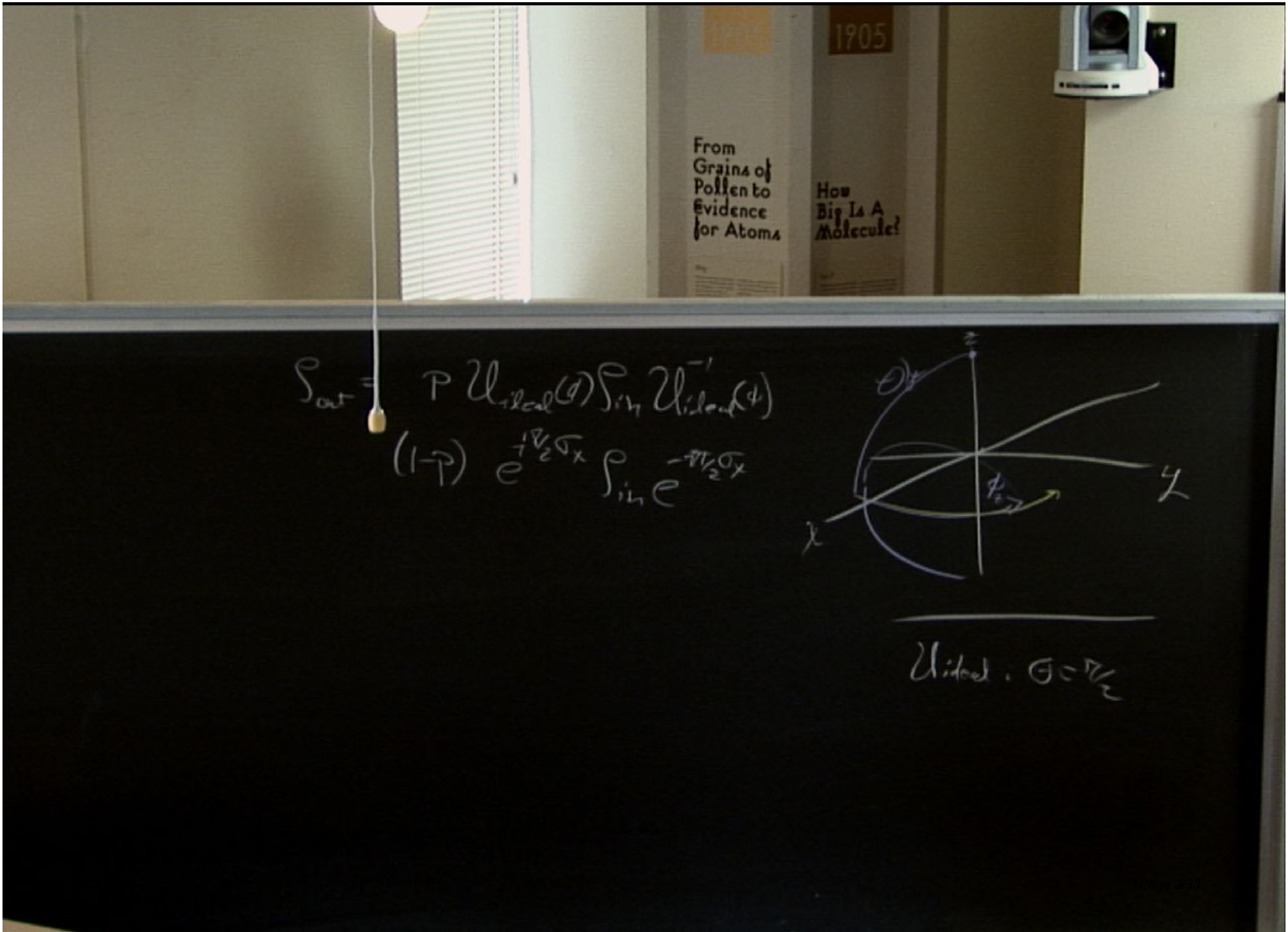
Title: Explorations in Quantum Information - Lecture 4

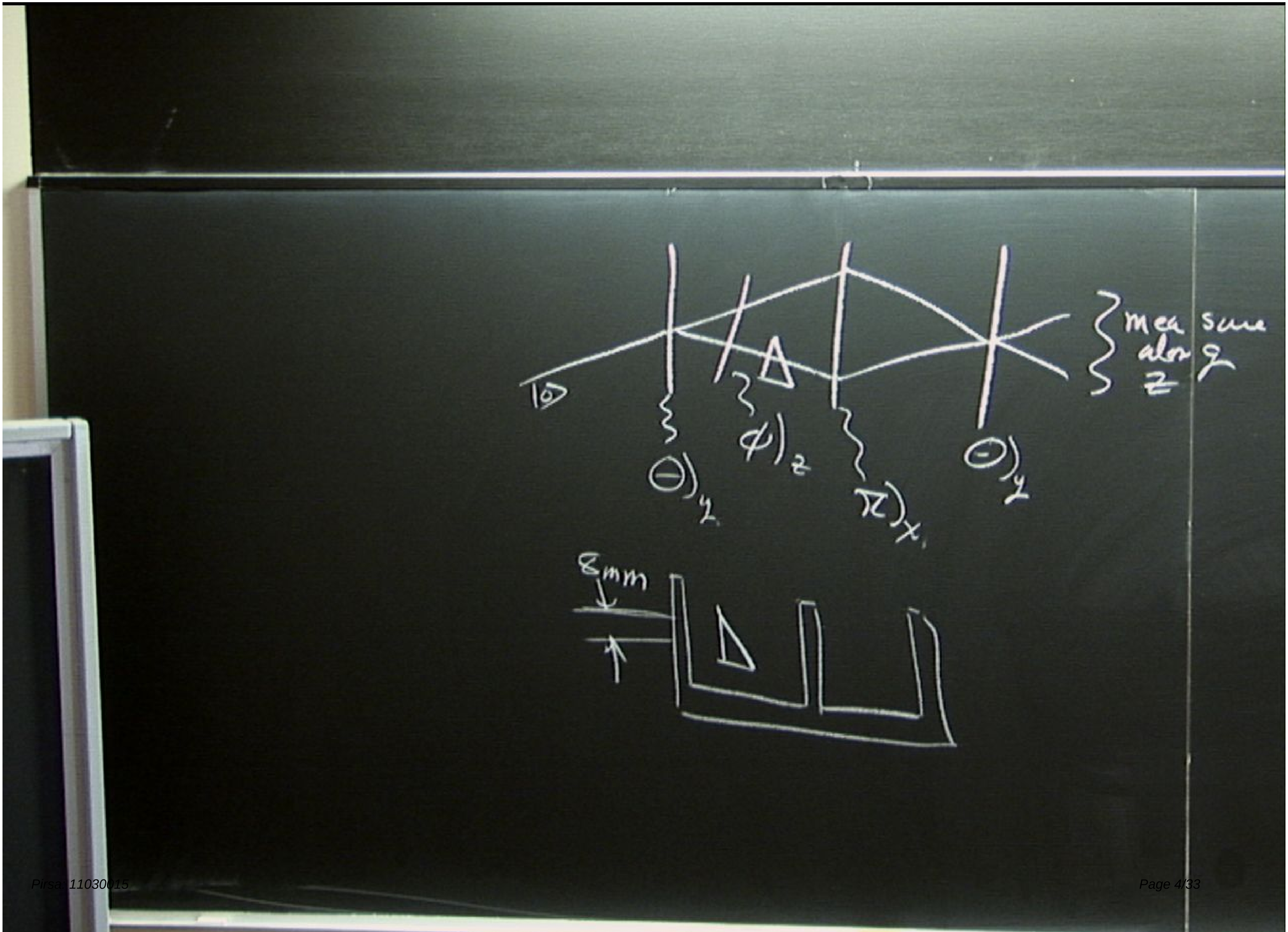
Date: Mar 17, 2011 09:00 AM

URL: http://pirsa.org/11030015

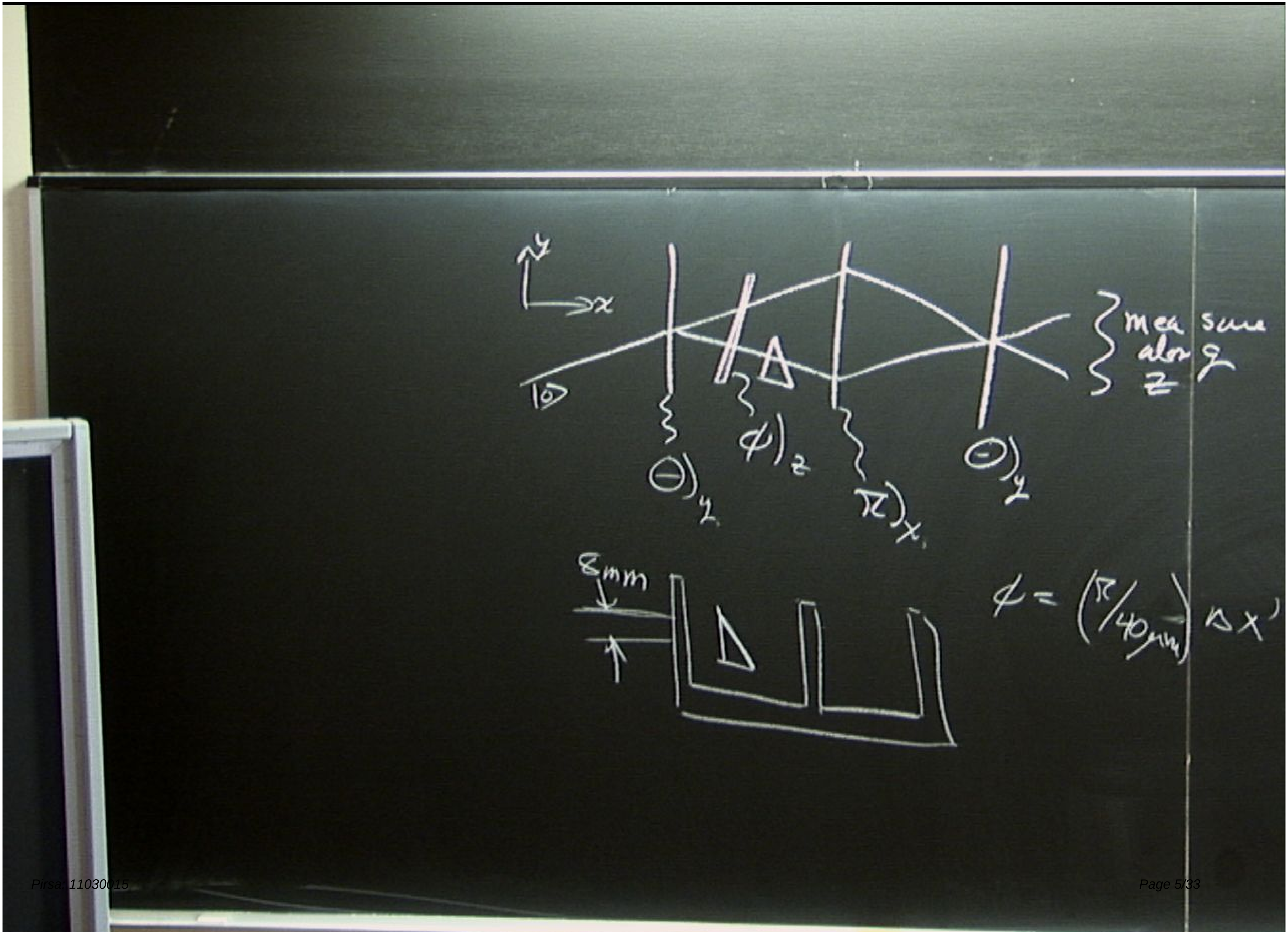
Abstract:

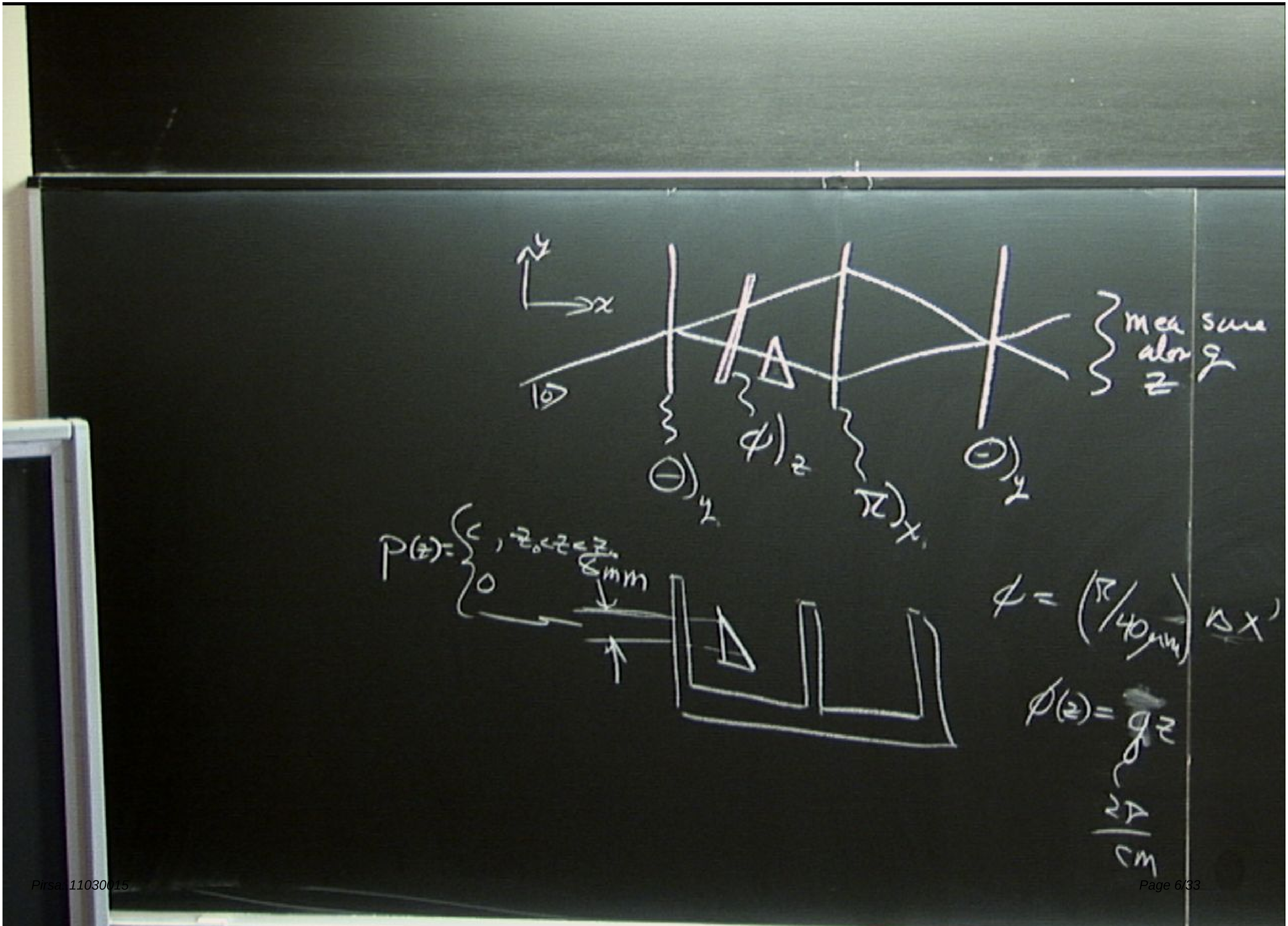

|0⟩
θ)y
φ)z
π)x
θ)y
measure
along
z


1905
From Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
ρ_out = p U_ideal(φ) ρ_in U_ideal^{-1}(φ)
(1-p) e^{-iπ/2 σx} ρ_in e^{-iπ/2 σx}
z
θ
x
y
φ_z
U_ideal . θ = π/2



|0⟩
θ)y
φ)z
π)x
θ)y
measure along z
8mm


y
x
|0⟩
measure along z
θ)y
φ)z
π)x
θ)y
8mm
φ = (π/40μm) Δx'


y
x
measure along z
Θ)y
φ)z
π)x
Θ)y
P(z)= c , z0<z<z1
0
8mm
φ = (π/40μm) Δx'
φ(z)= gz
2π/cm

$$I_0 = \int_{-z_0}^{z_0} P(z)\,\mathrm{Tr}\left\{ |0\rangle\langle 0|\; \rho_{out}(g z) \right\} dz$$

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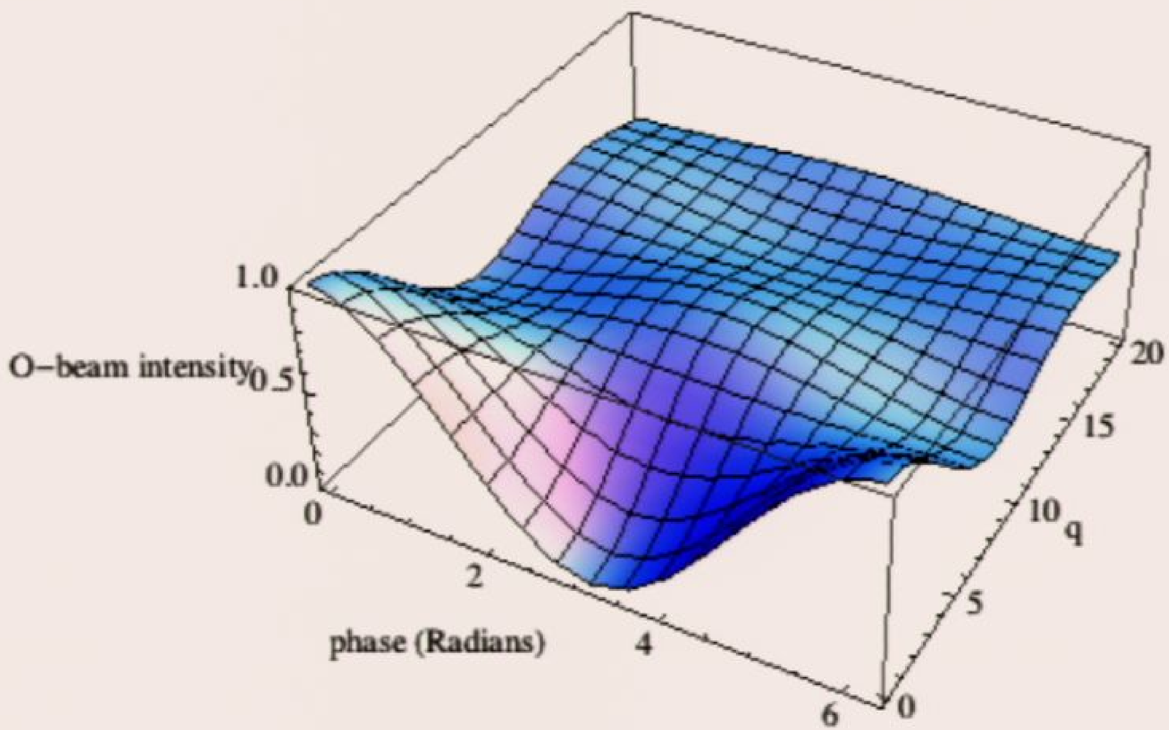
QE Lec 4 (Modified).nb

Note that the q dependence is a sinc function (the Fourier transform of a top-hat function).

In[51]:=
```
Plot3D[M6O[q, a], {a, 0, 2 π}, {q, 0, 20},
 {AxesLabel → { "phase (Radians)", "q", "O-beam intensity"},
  PlotRange → {0, 1}}]
```

Out[51]=



In[52]:=
```
M6H[q_,  a_] := Integrate[Tr[Ezm . res6[q,z,a]],{z,-0.5,0.5}]
```

In[53]:=
```
M6H[q, a]
```

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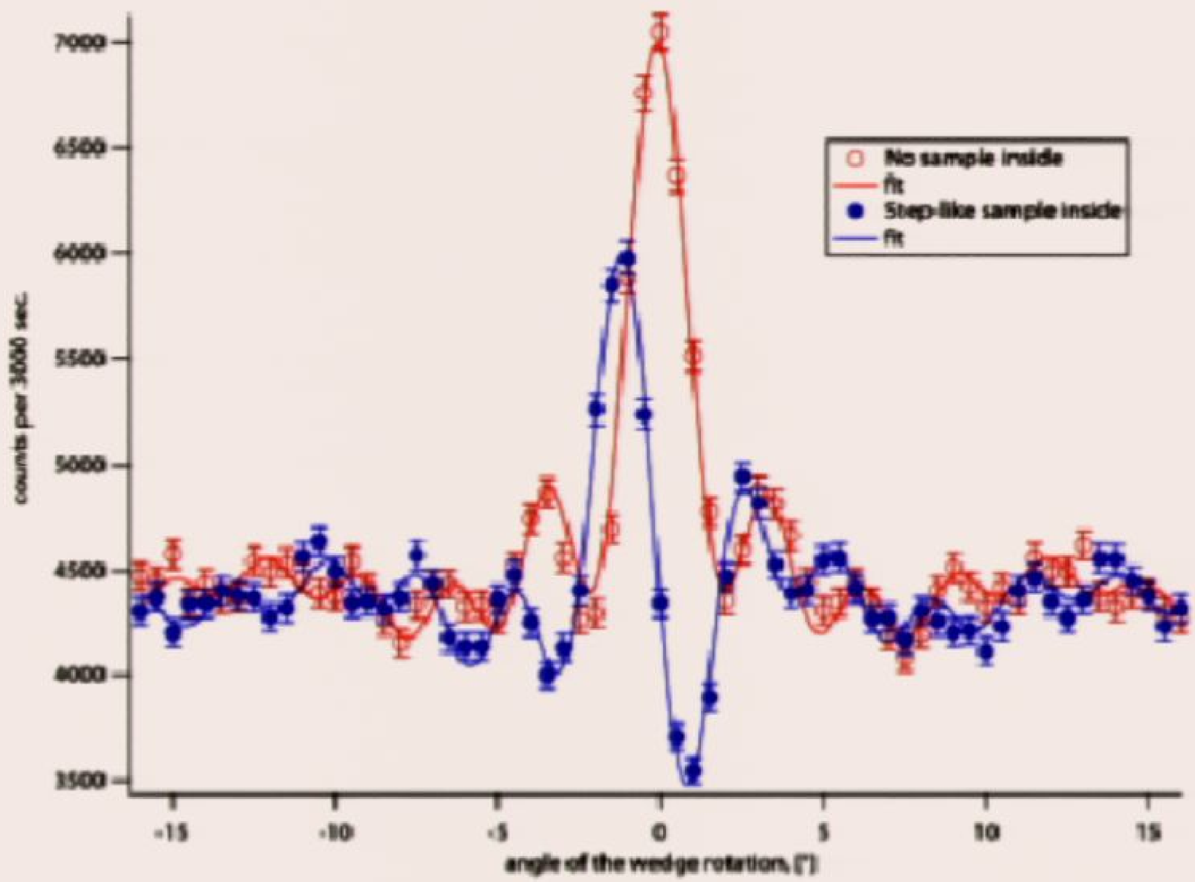
QE Lec 4 (Modified).nb

q

In[56]:=



;

Above are two data sets showing the O-beam intensity versus *q* for

1. an empty interferometer. So the shape should approximate a sinc function

2. a step sample.

These were acquired as we were developing imaging methods for neutron interferometry. Typically we use

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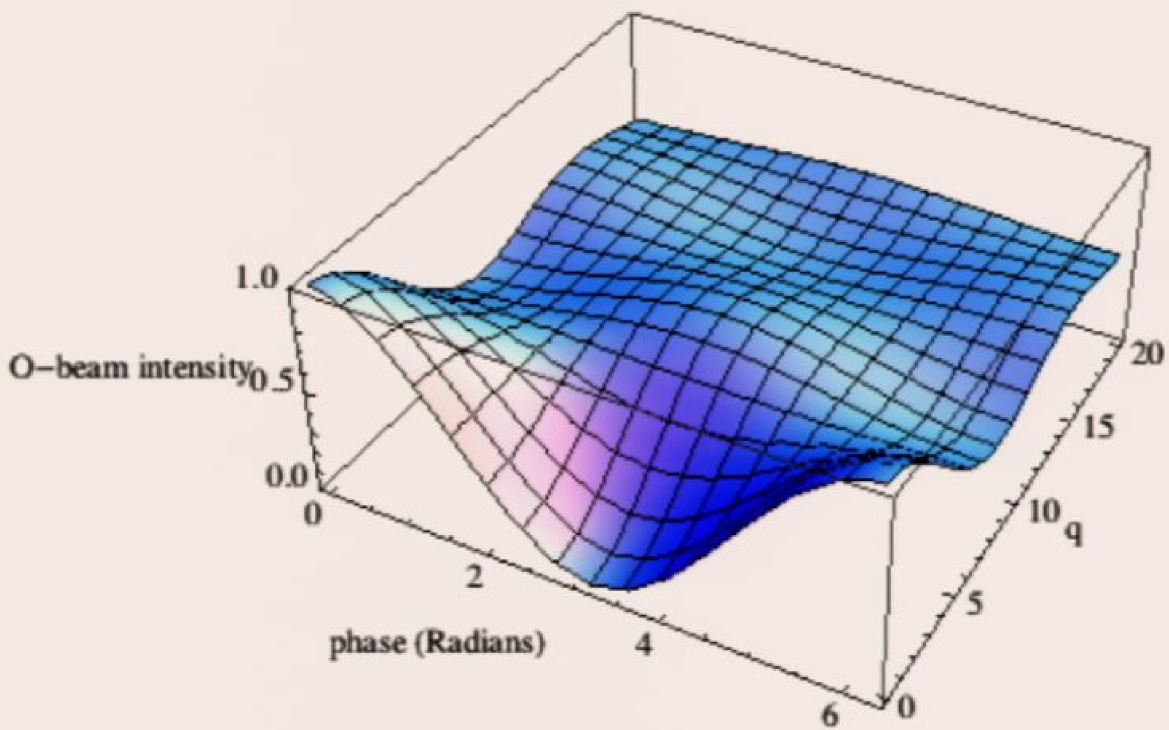
In[50]:= M60[q, a]

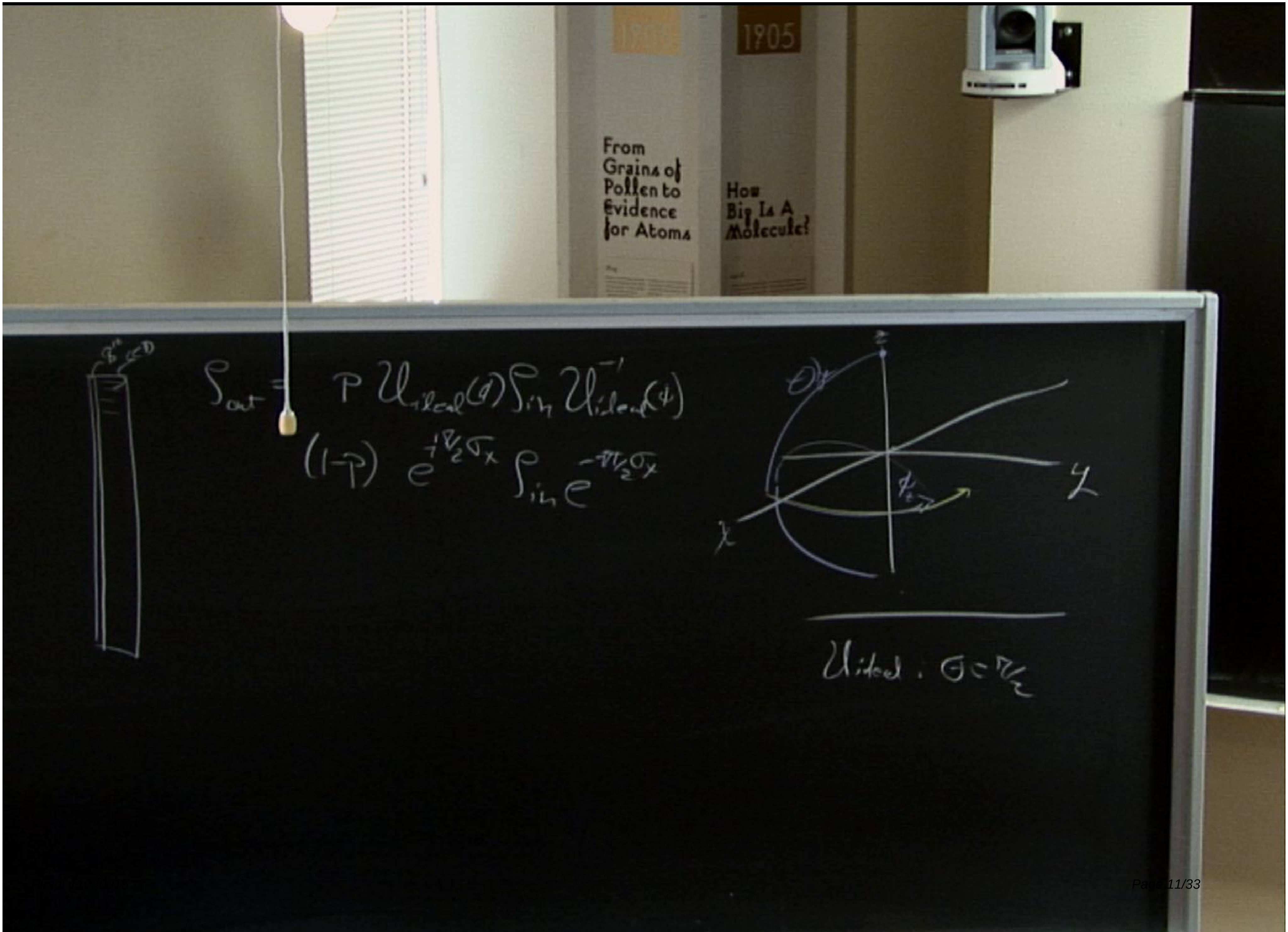
Out[50]= $0.5 + \frac{\text{Cos}[a]\,\text{Sin}[0.5\,q]}{q}$

Note that the *q* dependence is a sinc function (the Fourier transform of a top-hat function).

```
In[51]:= Plot3D[M60[q, a], {a, 0, 2 π}, {q, 0, 20},
  {AxesLabel → { "phase (Radians)", "q", "O-beam intensity"},
   PlotRange → {0, 1}}]
```

Out[51]=

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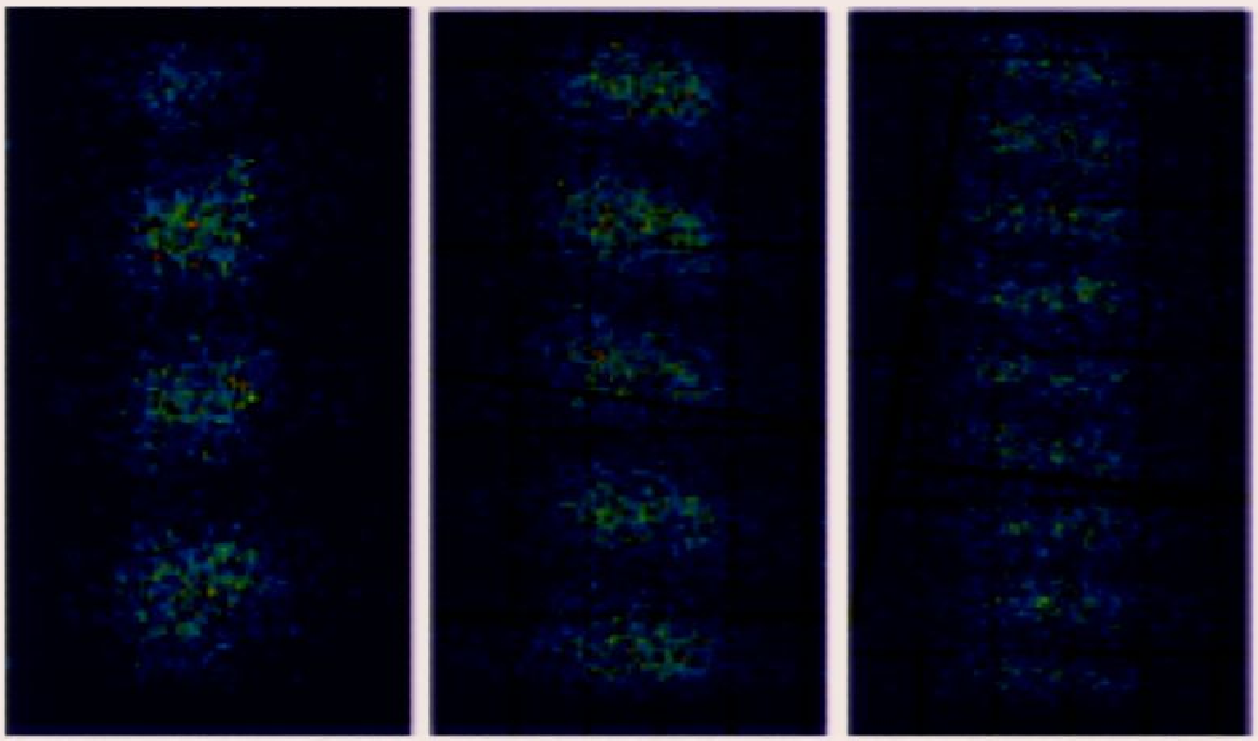
QE Lec 4 (Modified).nb

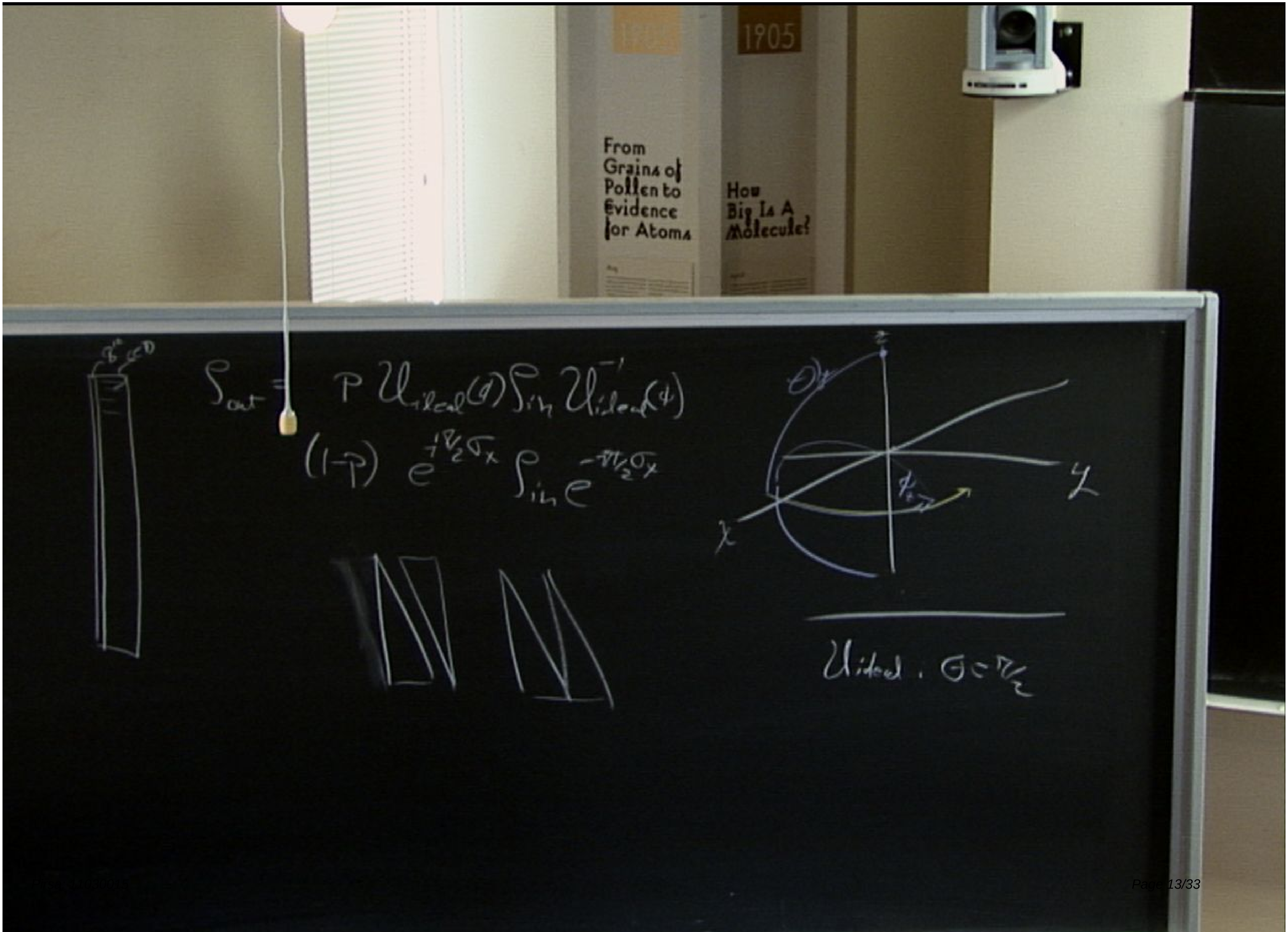
Clearly, the interferometer actually retains the desired contrast, it is just that the contrast curves from the various spatial locations are shifted in phase and add incoherently. For larger wave-numbers, this results in

In[62]:=



Measurements from a position sensitive detector showing the fringes and the beam profile. Note that the position sensitive detector has low quantum efficiency. The neutron is converted to light in a scintillator which is then collected in a CCD. If the scintillator is thick, then the quantum efficiency is increased but at a cost of resolution. The scintillation event acts as a point source for photons.

Notice that as the position sensitive detector is moved from one path to the other that the fringes are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$\rho_{out} = p\, U_{ideal}(\phi)\, \rho_{in}\, U_{ideal}^{-1}(\phi)$
$(1-p)\, e^{i\pi/2 \sigma_x} \rho_{in} e^{-i\pi/2 \sigma_x}$
z
y
x
$U_{ideal}: \sigma = \pi/2$

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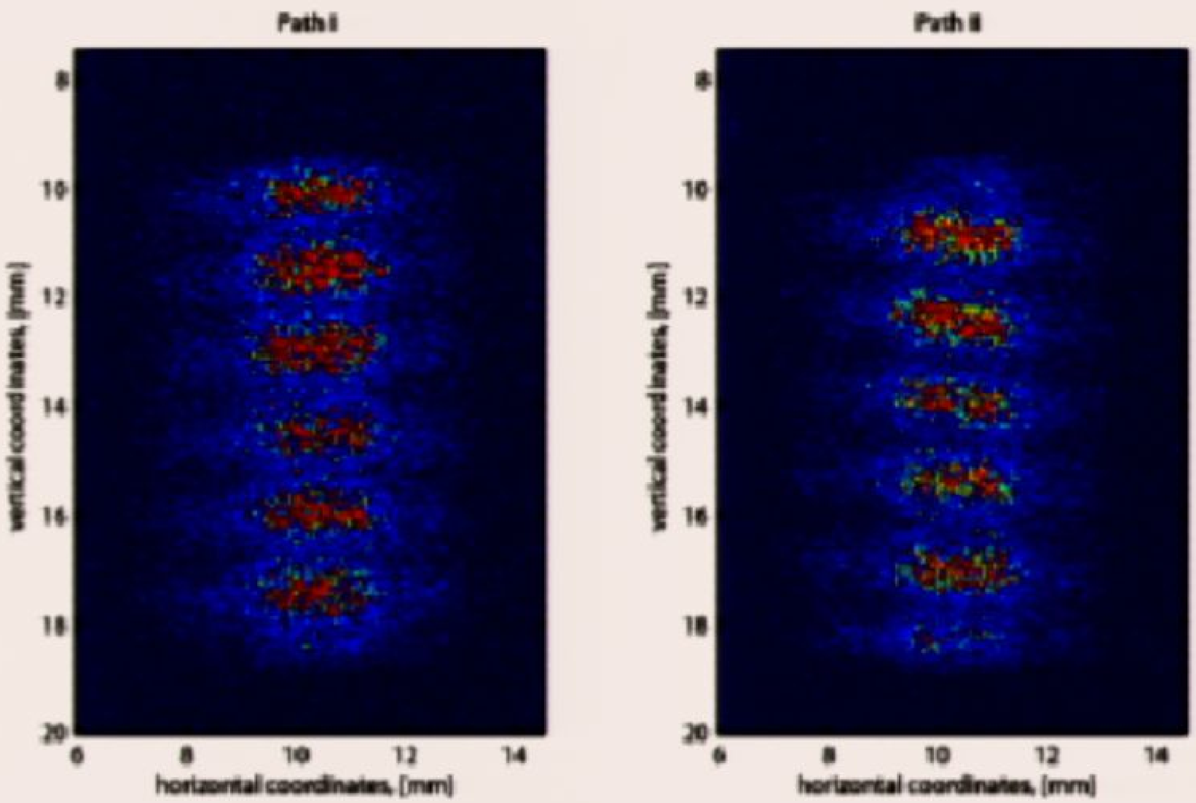
QE Lec 4 (Modified).nb

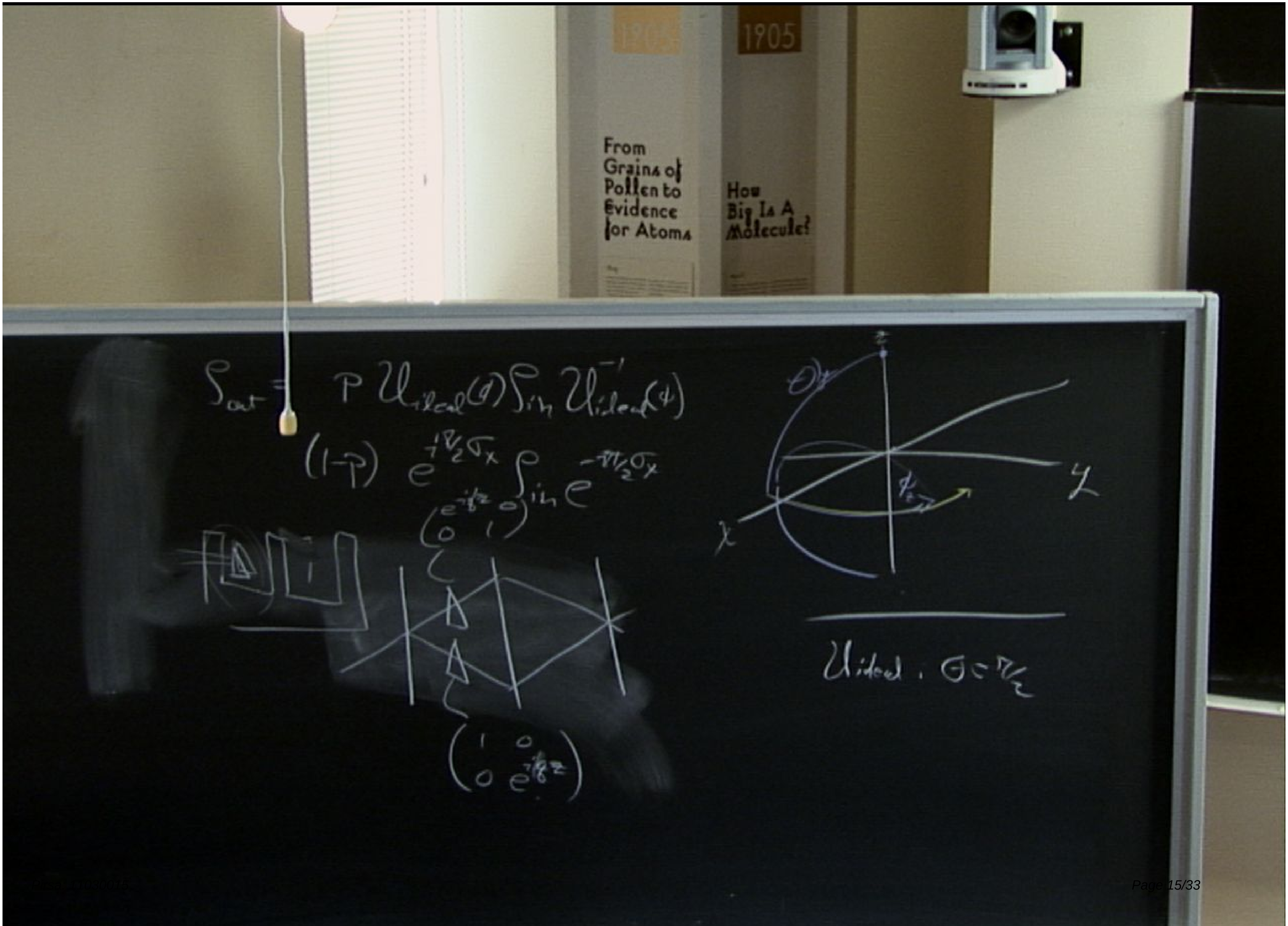
In[63]:=



- **Problem 20:**

Why are the fringes tilted, and why are the O and H fringes complementary. How can you show from this

- **Eighth Experiment: Refocusing the Phase Distribution**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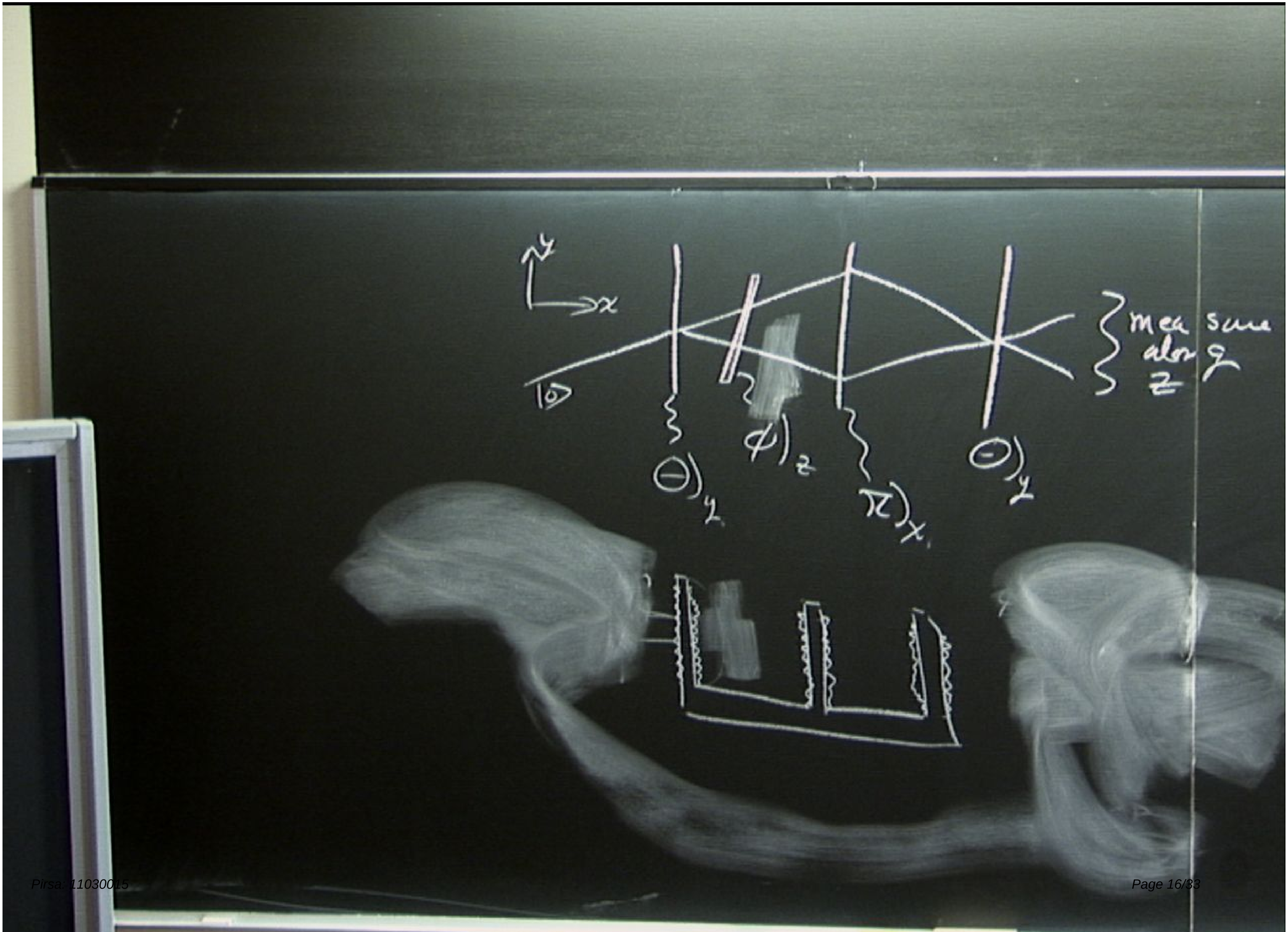
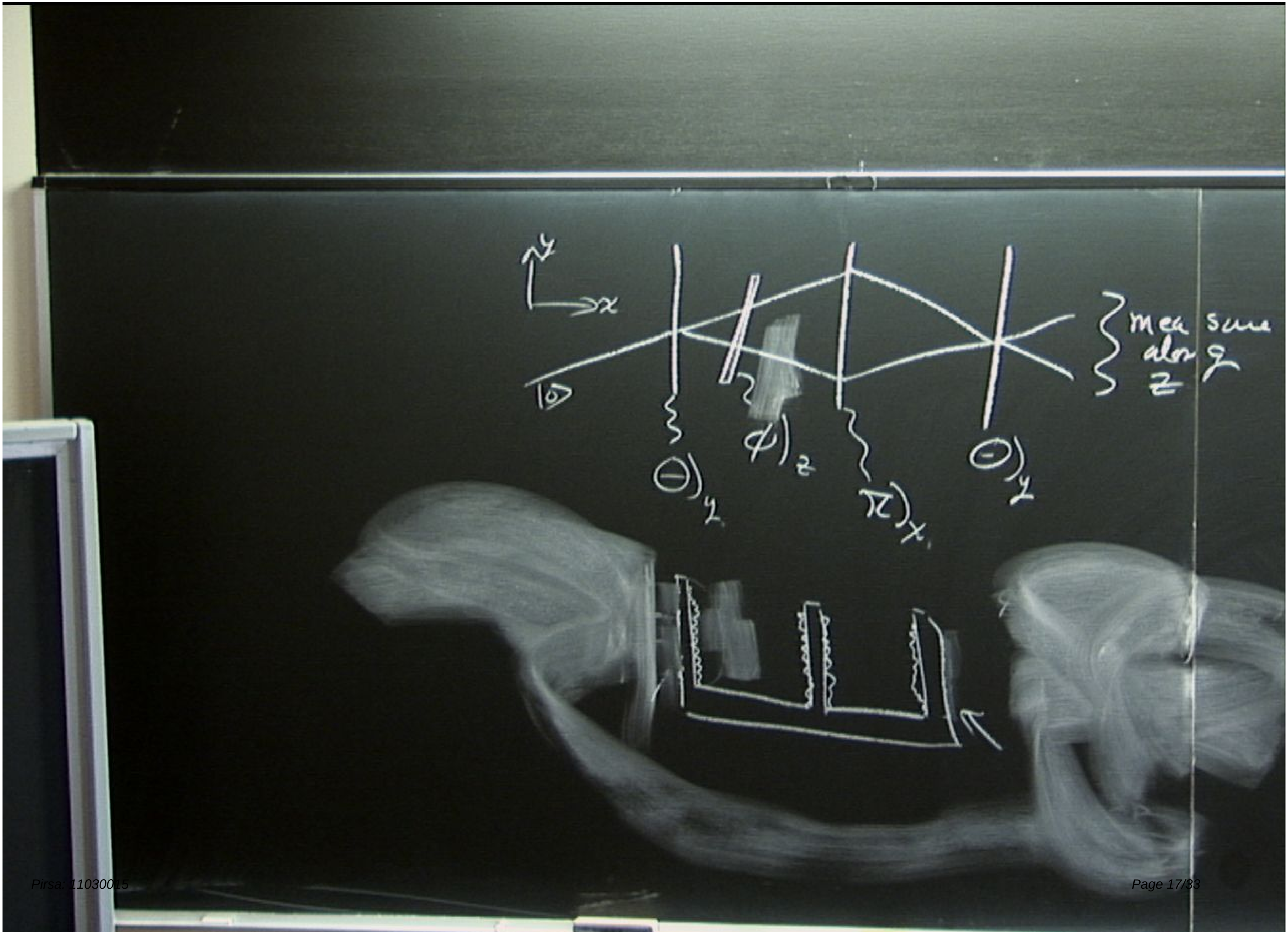
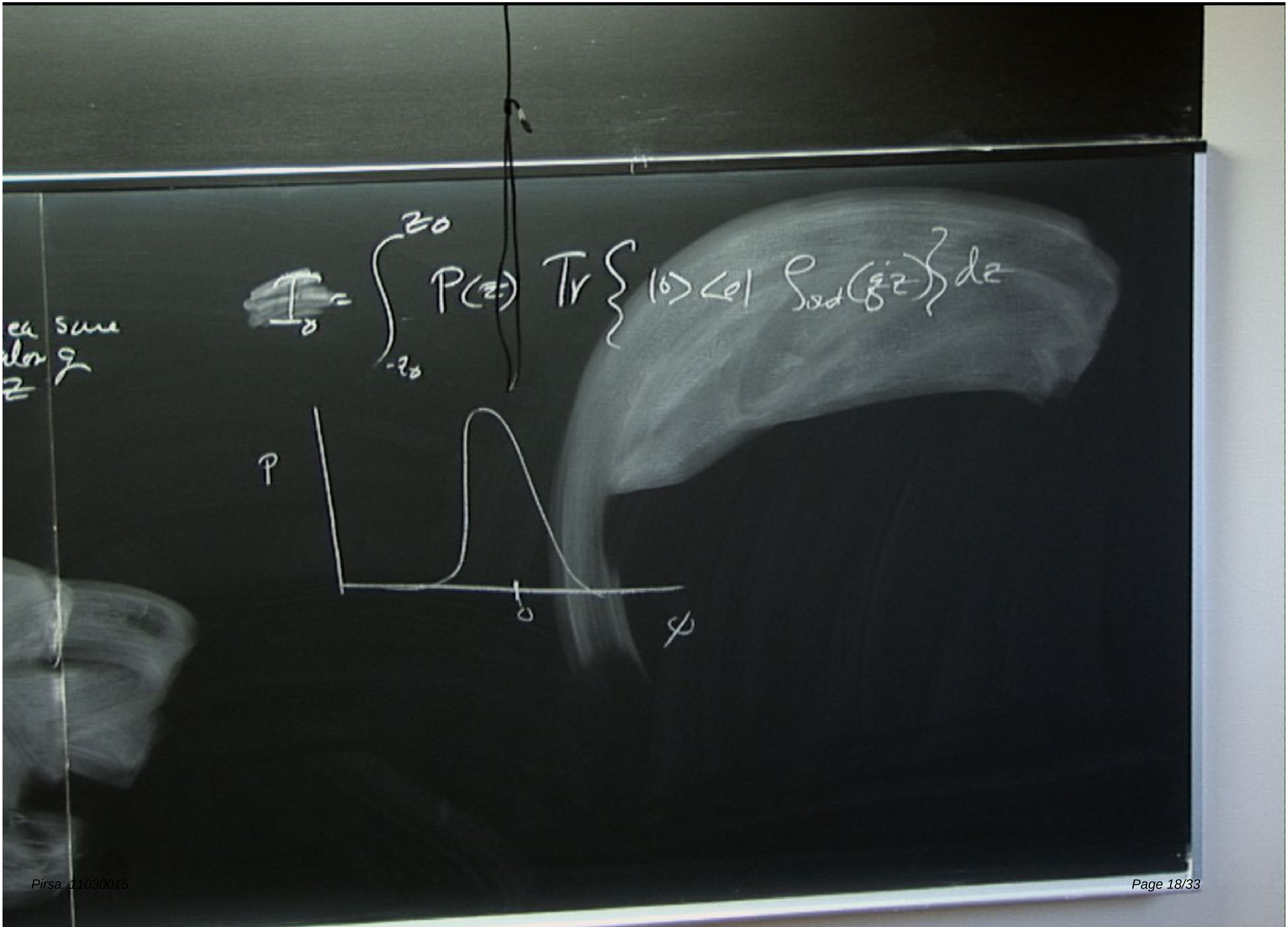
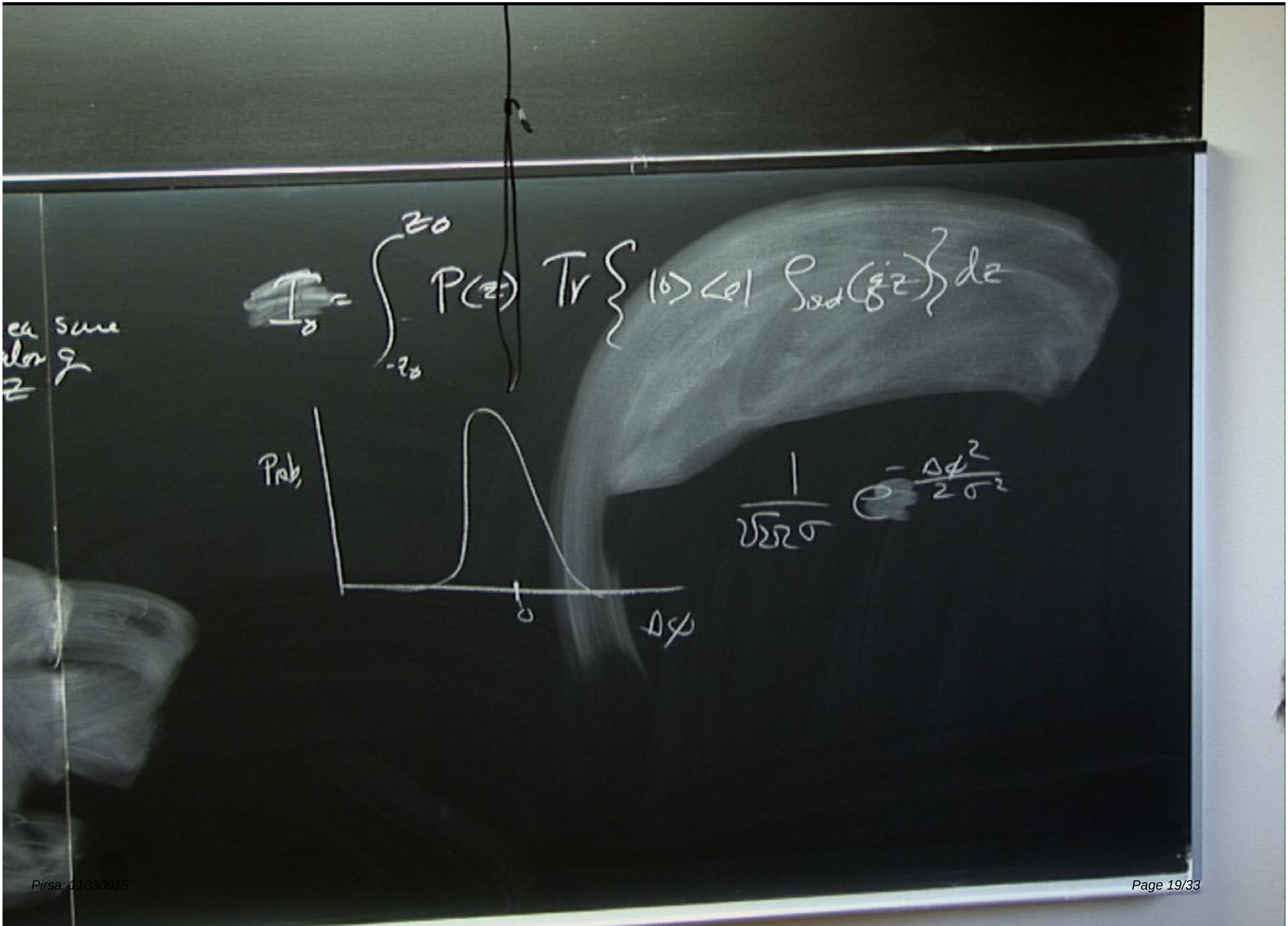
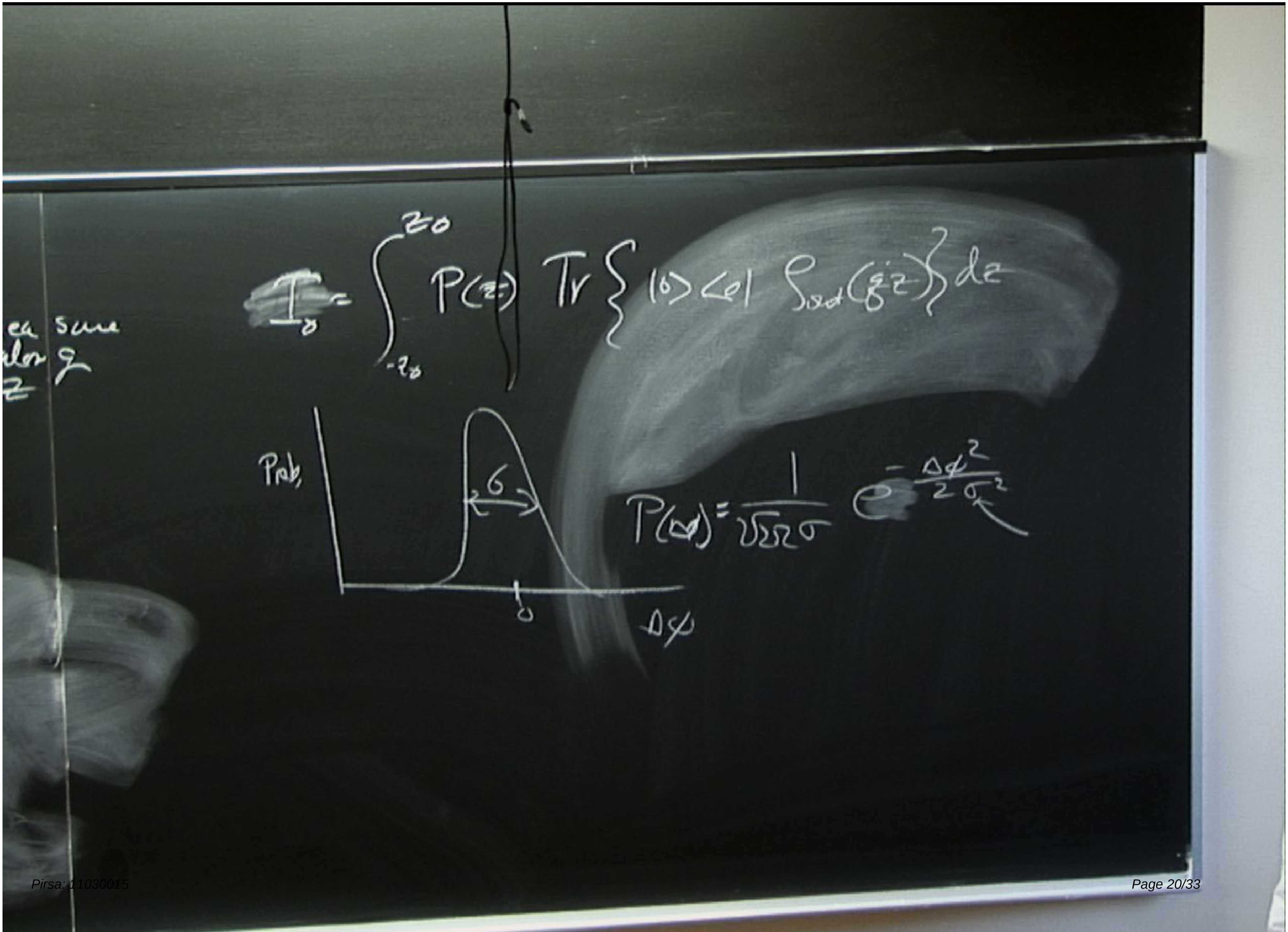

y
x
|0⟩
measure along z
Θ)y
φ)z
π)x
Θ)y


y
x
|0⟩
θ)y
φ)z
π)x
θ)y
measure along z


I_δ = ∫_{-z_0}^{z_0} P(z) Tr{ |0⟩⟨0| ρ_red(gz) } dz
P
0
φ


$\int_{-z_0}^{z_0} P(z)\, \mathrm{Tr}\{ |0\rangle\langle\phi|\, \rho_{sd}(gz) \}\, dz$
Prob.
0
Δϕ
$\frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{\Delta\phi^2}{2\sigma^2}}$



$I_0 = \int_{-z_0}^{z_0} P(z)\, Tr\{|0\rangle\langle 0|\, \rho_{red}(gz)\}\, dz$
Prob.
σ
$P(\Delta\phi) = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{\Delta\phi^2}{2\sigma^2}}$
0
Δφ

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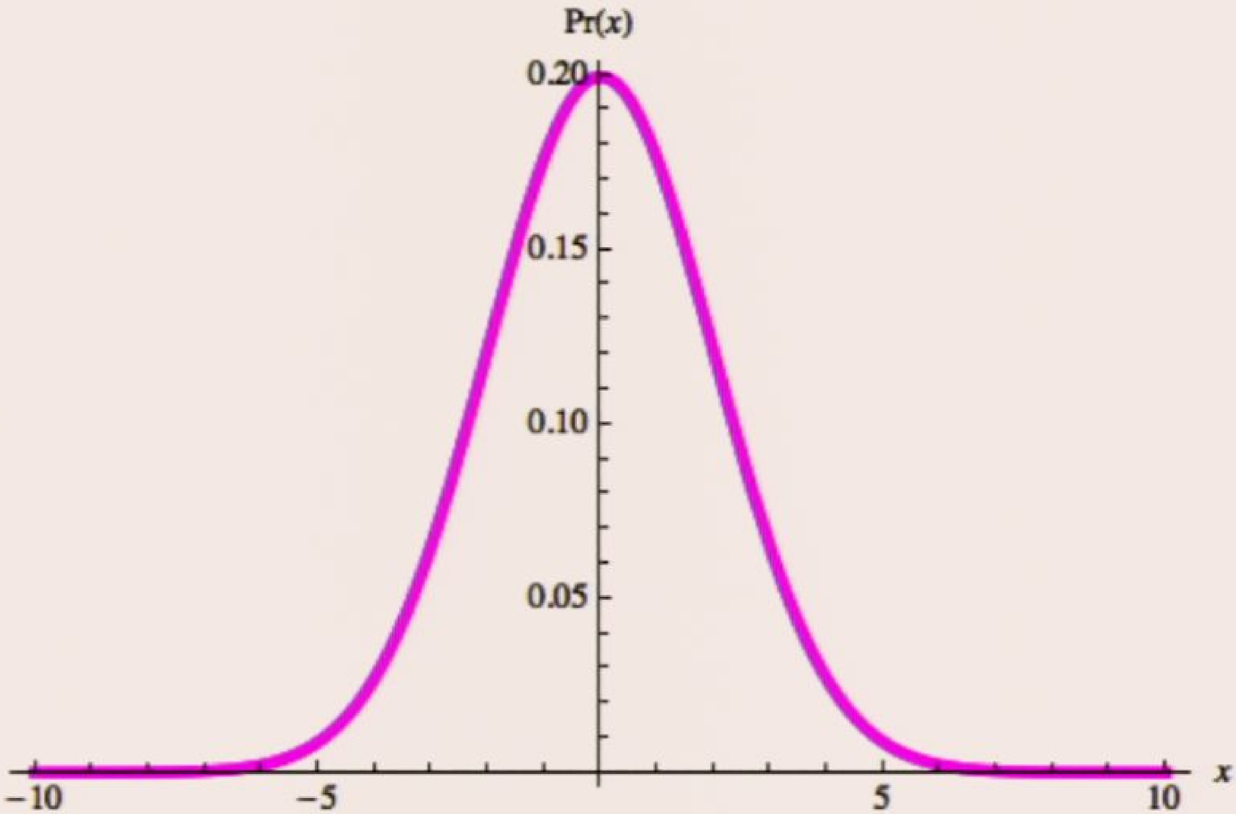
QE Lec 4 (Modified).nb

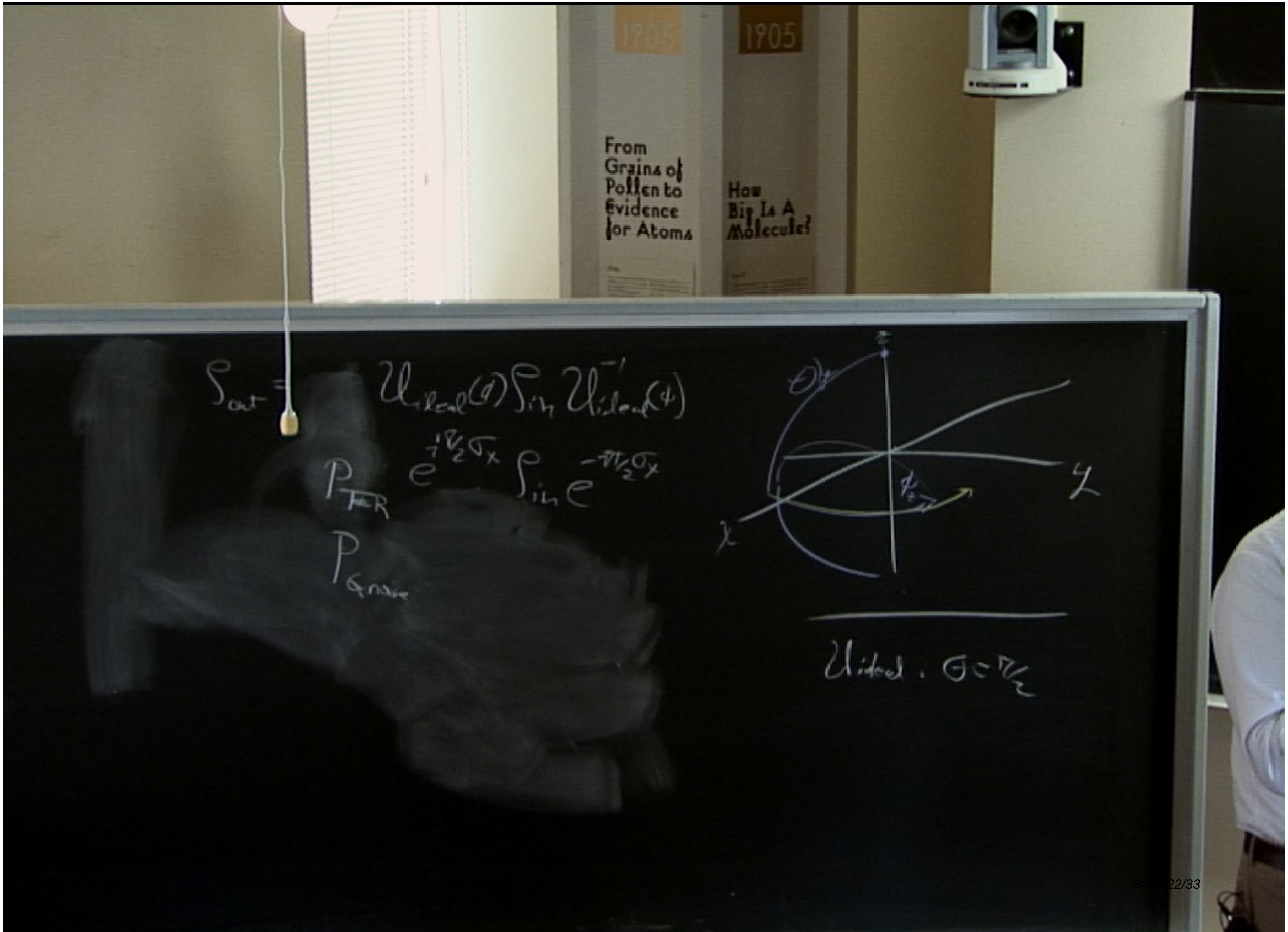
In[71]:=
```
nd[x_,sd_] := e^(...)/(Sqrt[2π] sd);
```

In[72]:=
```
Plot[nd[x, 2], {x, -10, 10},
 PlotStyle → {RGBColor[1, 0, 1], Thickness[0.01]}, AxesLabel → {x, Pr[x]}]
```

Out[72]=



In[73]:=
```
M90[a_, r_] = ∫ nd[x,r] Tr[Ezp . res9[a, x]] dx
```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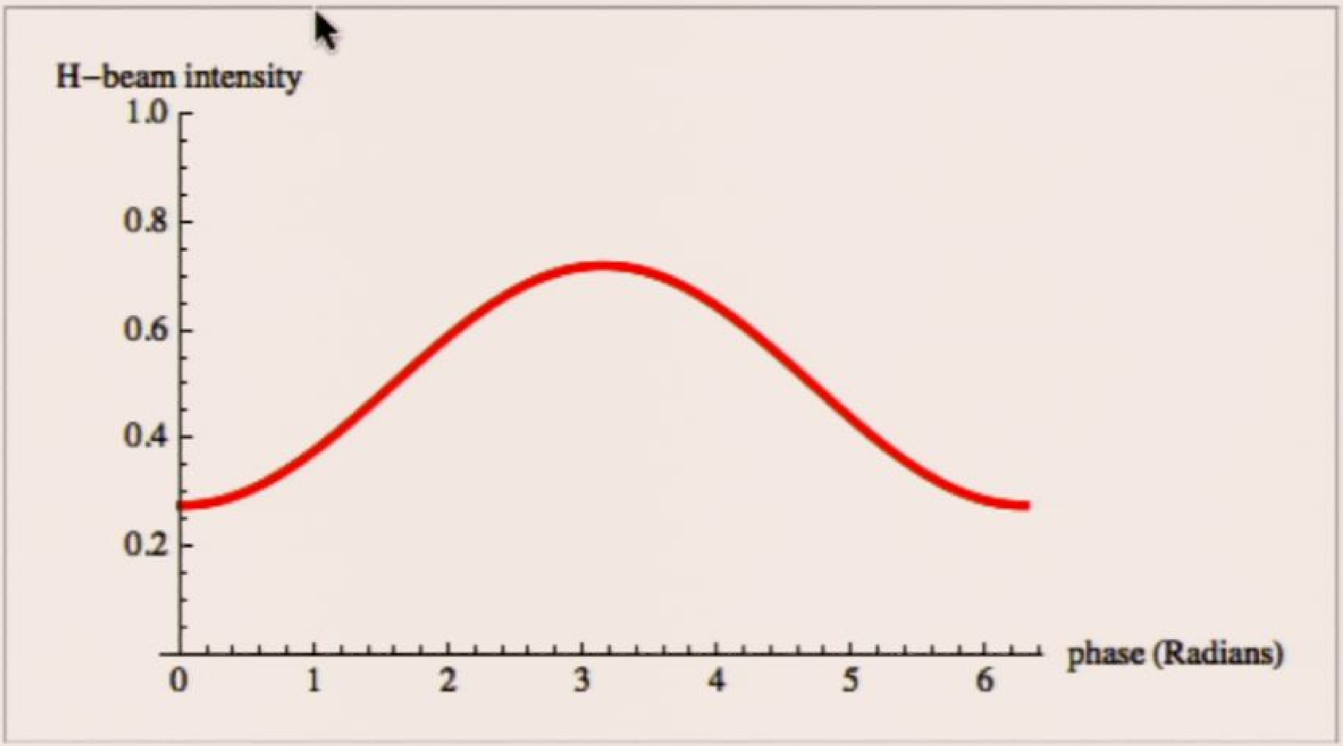
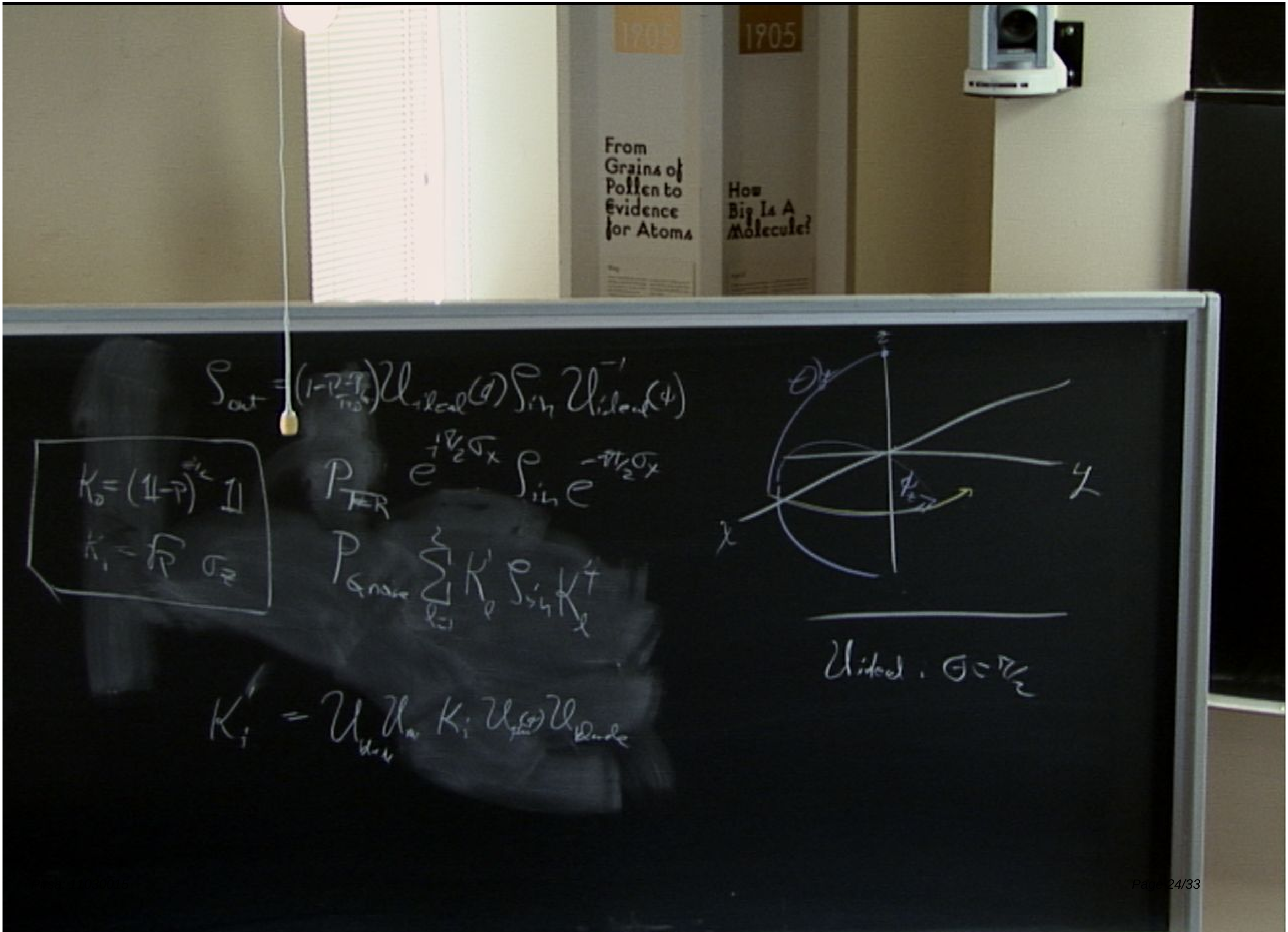
In[76]:=
```
ListAnimate[
 Table[Plot[M9H[a, r], {a, 0, 2 π},
   {AxesLabel → {"phase (Radians)", "H-beam intensity"},
    PlotStyle → {RGBColor[1, 0, 0], Thickness[0.01]},
    PlotRange → {0, 1}}], {r, 0, π, π / 32}], AnimationRunning → False]
```

Out[76]=



• **Problem 22:**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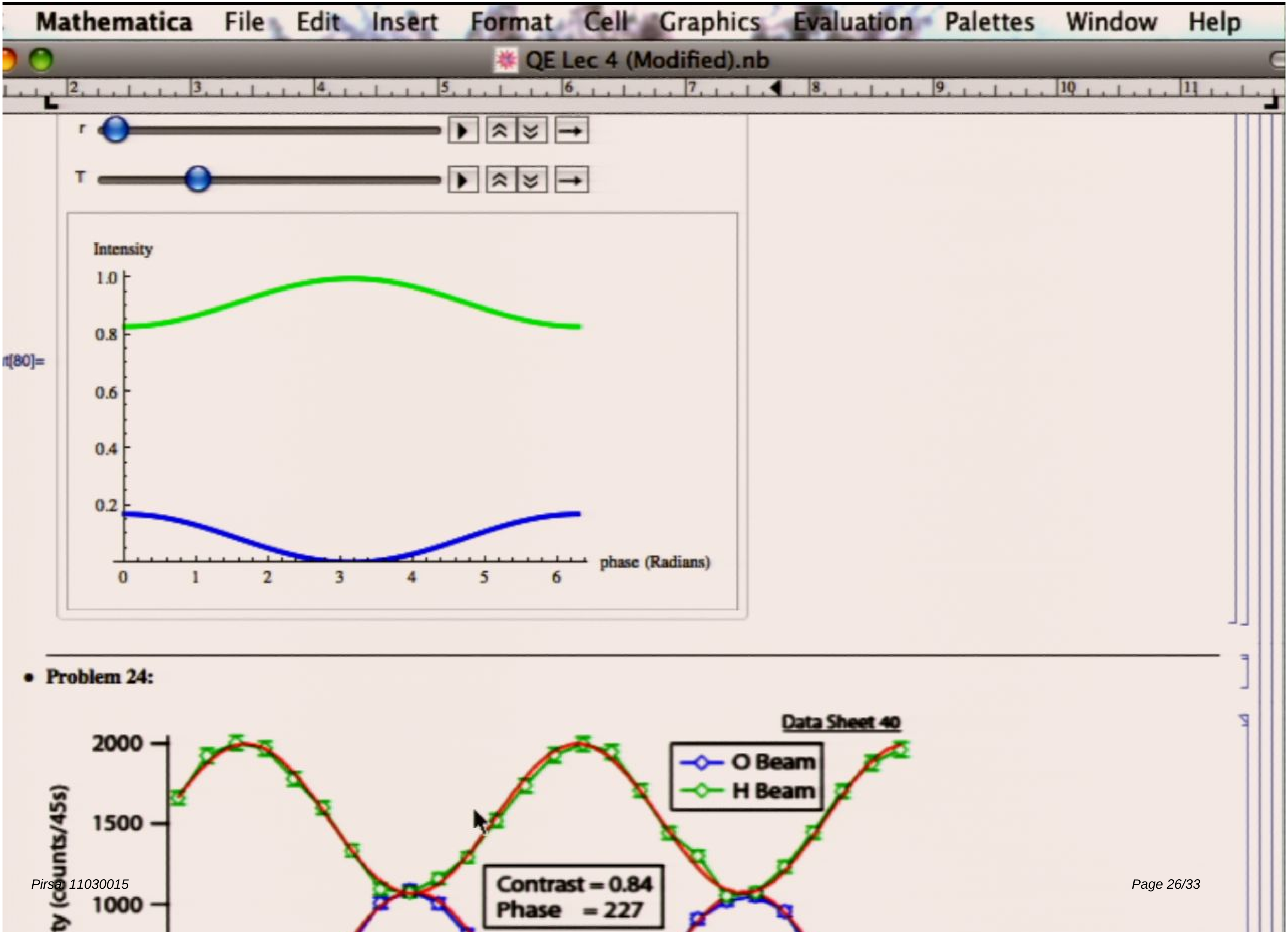
Out[78]= $\frac{1}{4}\left(2\,\text{Erf}\left[\frac{3}{\sqrt{2}}\right]+e^{-\frac{r^2}{2}}\,\text{Cos}[a]\,\left(\text{Erf}\left[\frac{3-i\,r}{\sqrt{2}}\right]+\text{Erf}\left[\frac{3+i\,r}{\sqrt{2}}\right]\right)\right)\,\text{Sin}[2\,T]^2$

In[79]:= $\text{M10H}[a\_, r\_, T\_] = \int_{-3r}^{3r} \text{nd}[x, r]\ \text{Tr}[\text{Ezm}\ .\ \text{res10}[a, x, T]]\ dx$

Out[79]= $\frac{1}{4}\left((3+\text{Cos}[4\,T])\,\text{Erf}\left[\frac{3}{\sqrt{2}}\right]-e^{-\frac{r^2}{2}}\,\text{Cos}[a]\,\left(\text{Erf}\left[\frac{3-i\,r}{\sqrt{2}}\right]+\text{Erf}\left[\frac{3+i\,r}{\sqrt{2}}\right]\right)\,\text{Sin}[2\,T]^2\right)$

```
In[80]:= Animate[Plot[{
    M10O[a, r, T],
    M10H[a, r, T]
   },
   {a, 0, 2 π},
   AxesLabel → {"phase (Radians)", "Intensity"},
   PlotStyle → {
     Directive[RGBColor[0, 0, 1], Thickness[0.01]],
     Directive[RGBColor[0, 1, 0], Thickness[0.01]]
   },
   PlotRange → {0, 1.02}
```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QE Lec 4 (Modified).nb
r
T
Intensity
1.0
0.8
0.6
0.4
0.2
0 1 2 3 4 5 6
phase (Radians)
Problem 24:
Data Sheet 40
O Beam
H Beam
2000
1500
1000
Contrast = 0.84
Phase = 227

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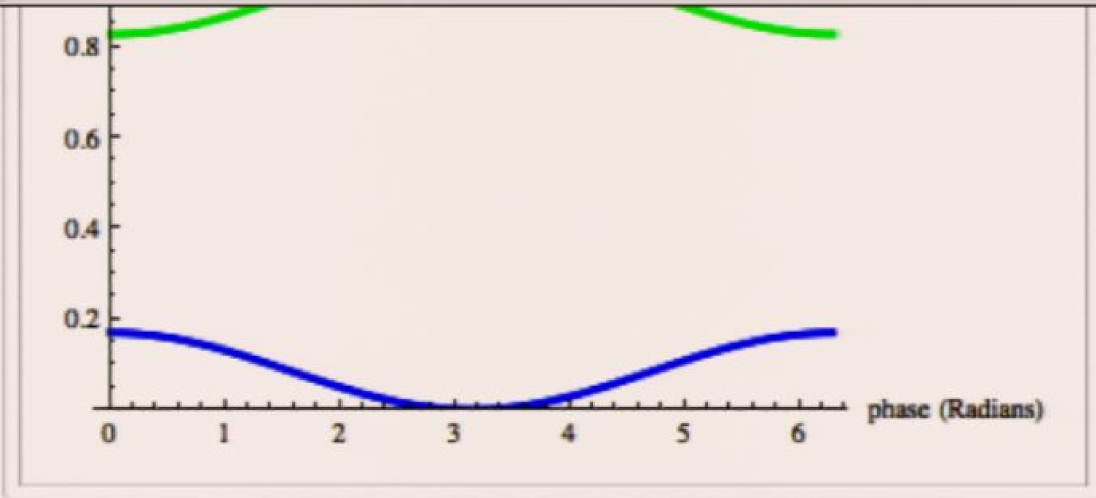
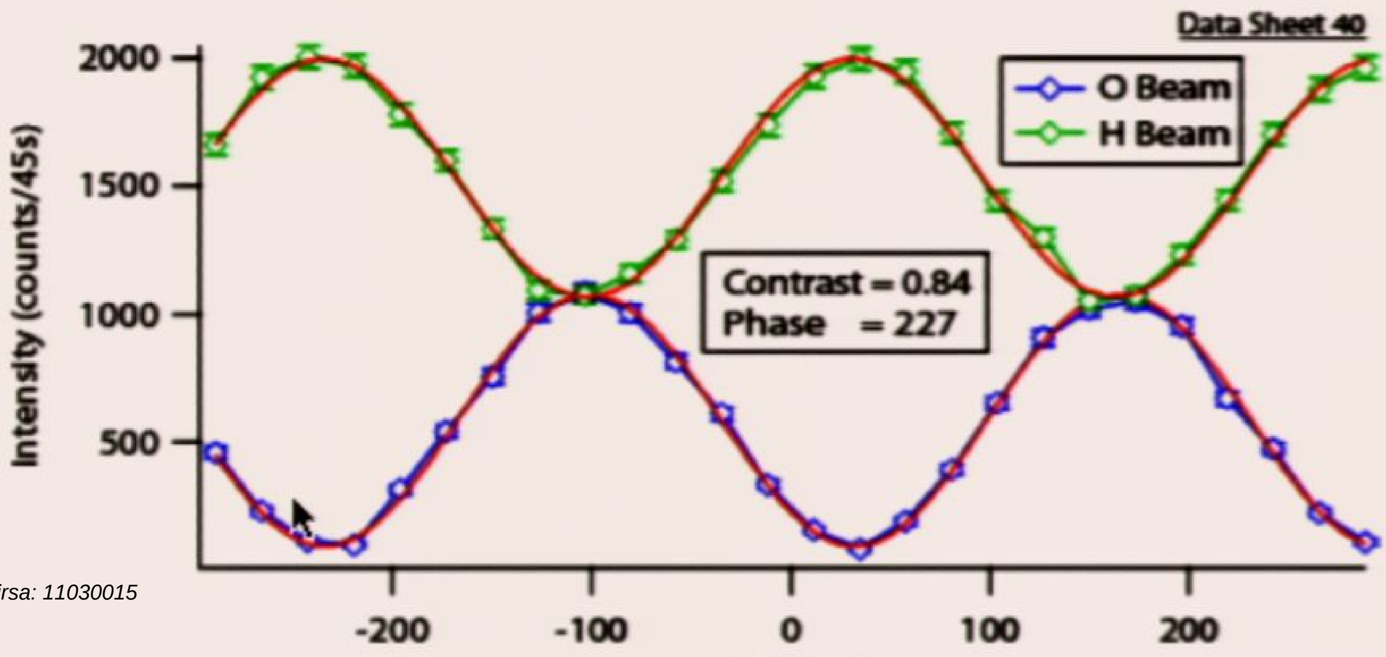
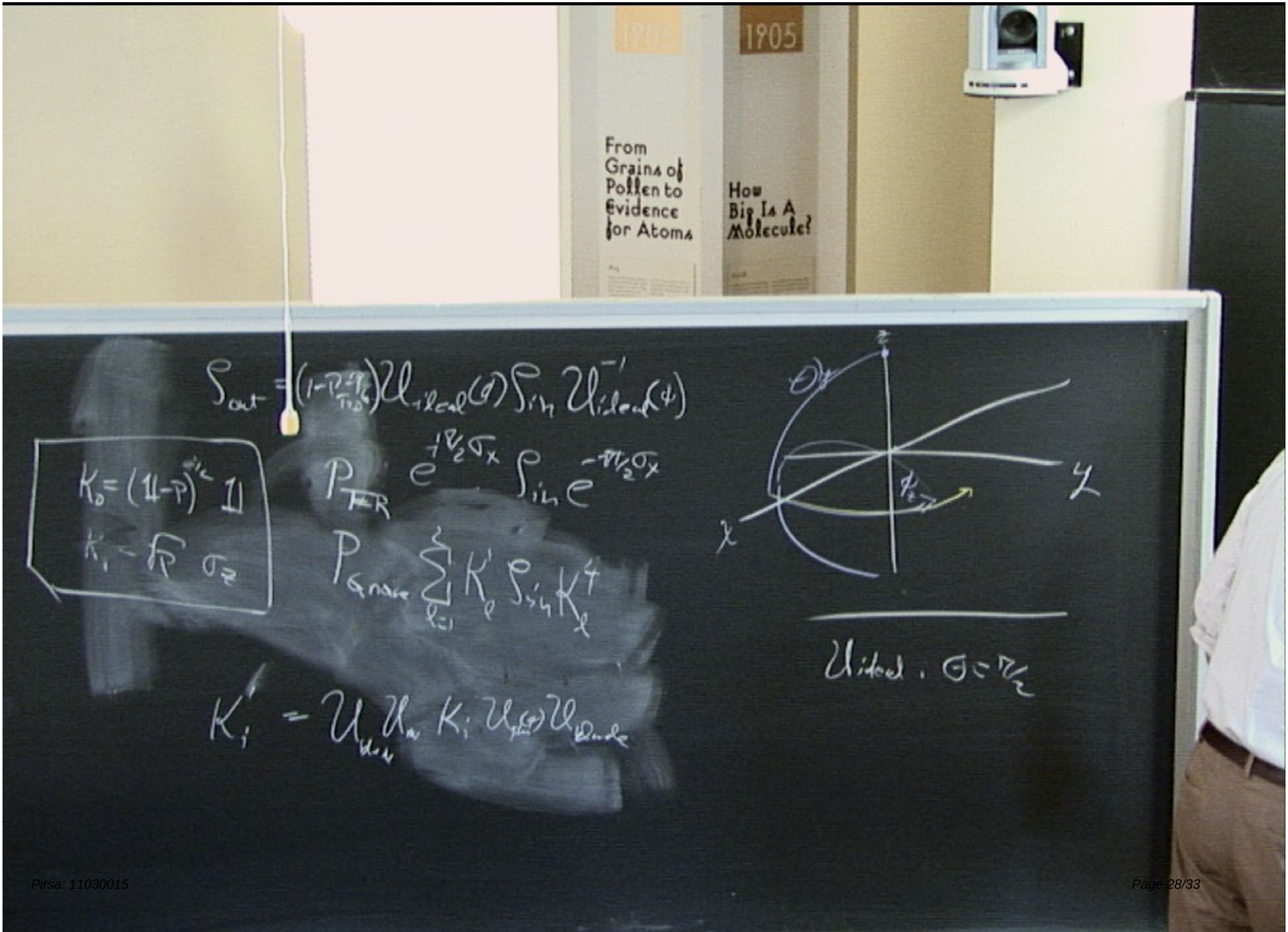
QE Lec 4 (Modified).nb

t[80]=



- **Problem 24:**

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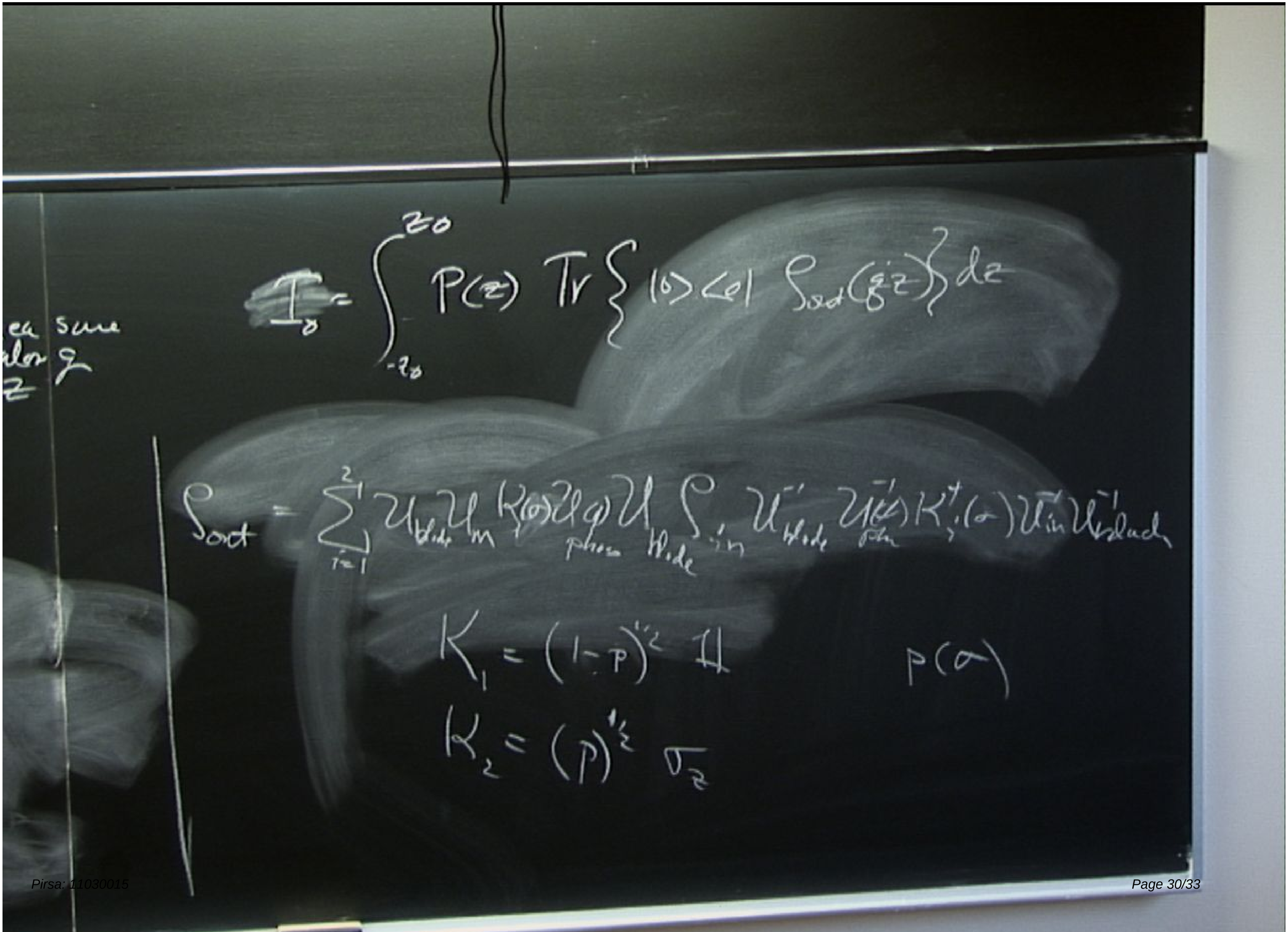
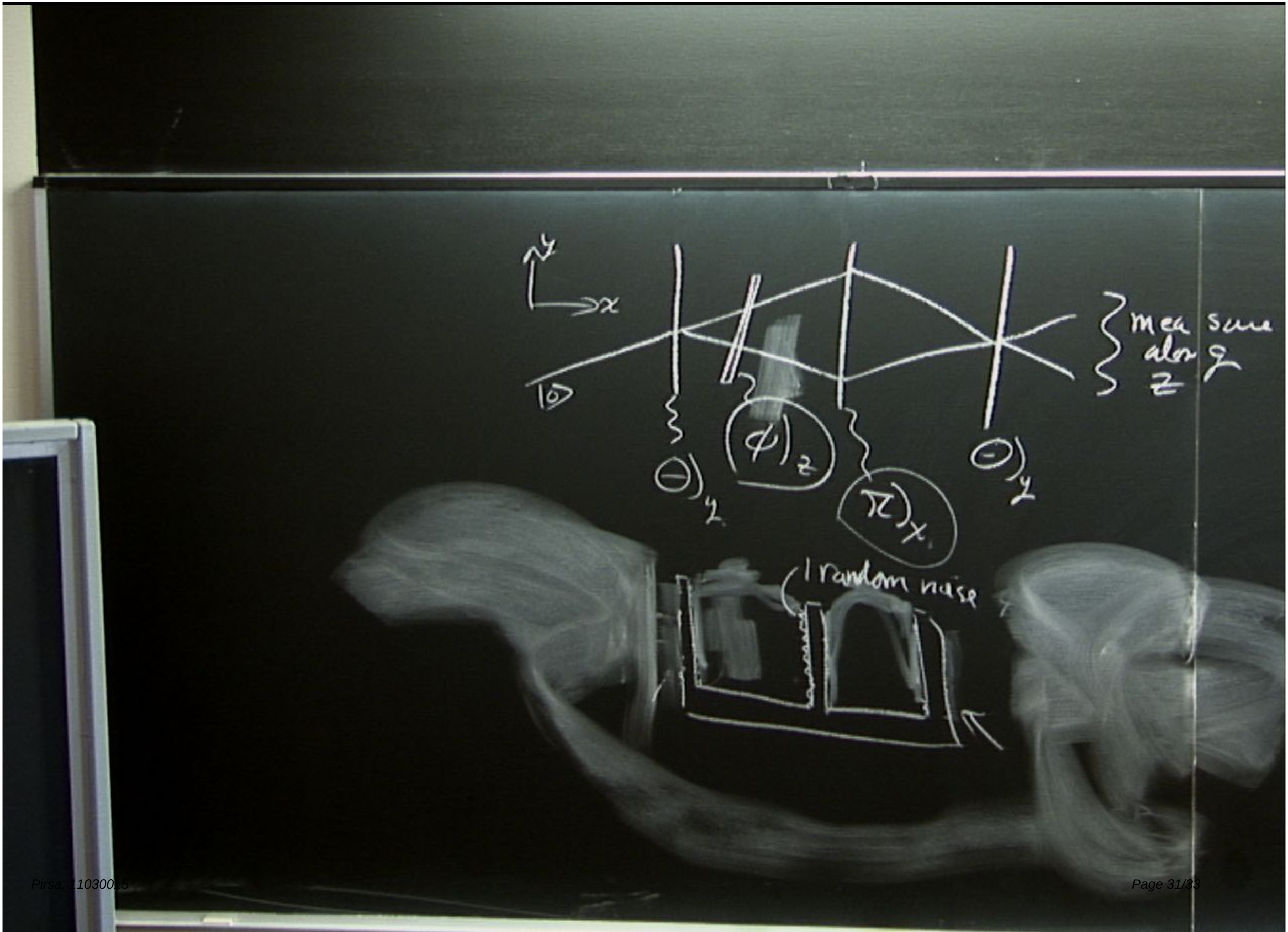
...ea sure
...alon g
... z

$$I_g = \int_{-z_0}^{z_0} P(z)\, Tr\{ |0\rangle\langle 0|\, \rho_{out}(g z) \}\, dz$$

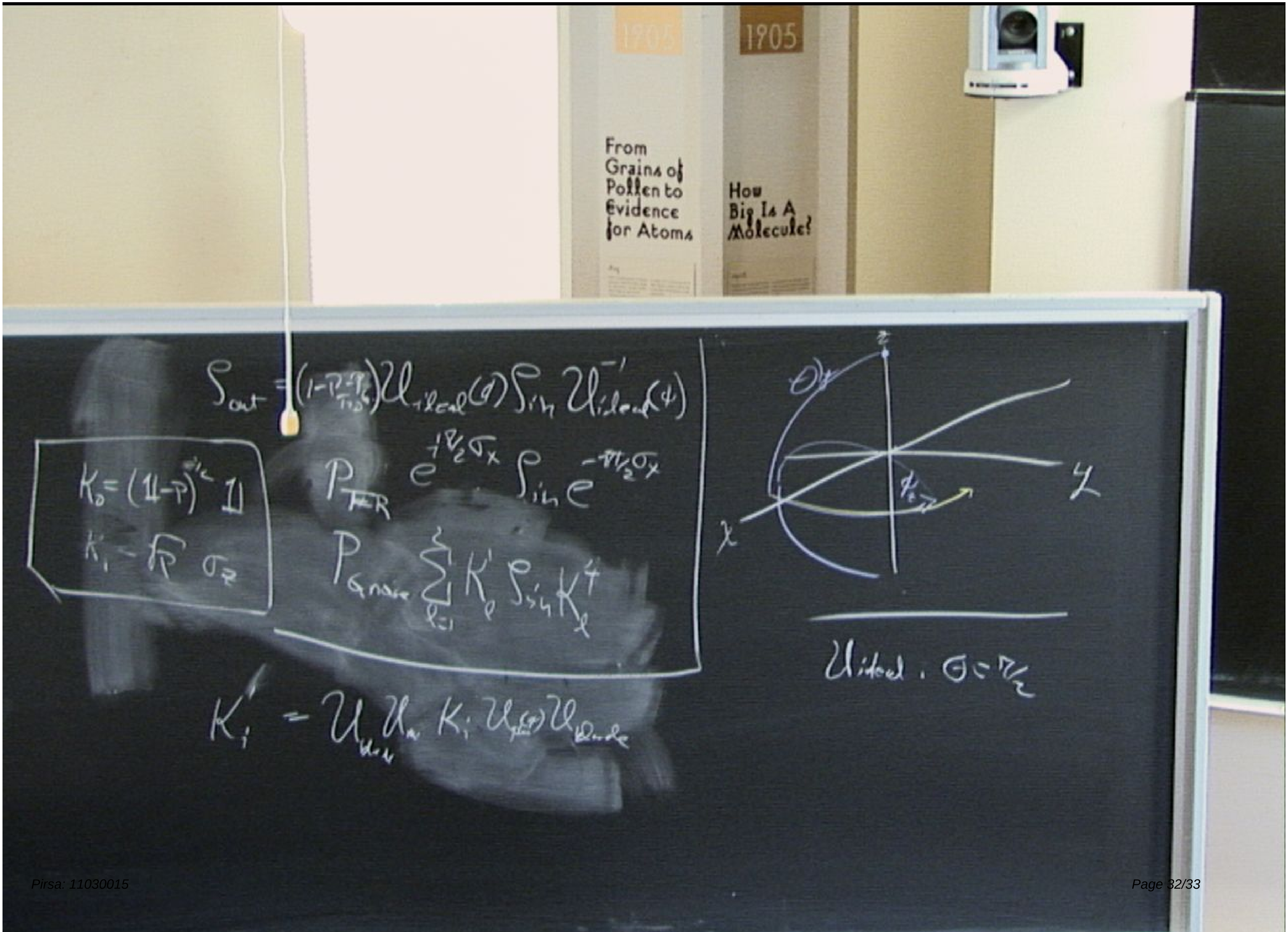
$$\rho_{out} = \sum_{i=1}^{2} U_{blade} U_m K_i(\sigma) U(g) U_{phase}\, U_{blade}\, \rho_{in}\, U^{-1}_{blade}\, U^{-1}_{phase}(g)\, K_i^{\dagger}(\sigma)\, U^{-1}_{m}\, U^{-1}_{blade}$$

$$K_1 = (1-$$


I_0 = ∫_{-z_0}^{z_0} P(z) Tr{ |0⟩⟨0| ρ_out(gz) } dz
K_1 = (1-p)^{1/2} 1
K_2 = (p)^{1/2} σ_z
p(σ)



y
x
|0⟩
|Θ⟩y
|φ⟩z
|π⟩x
|Θ⟩y
measure along z
random noise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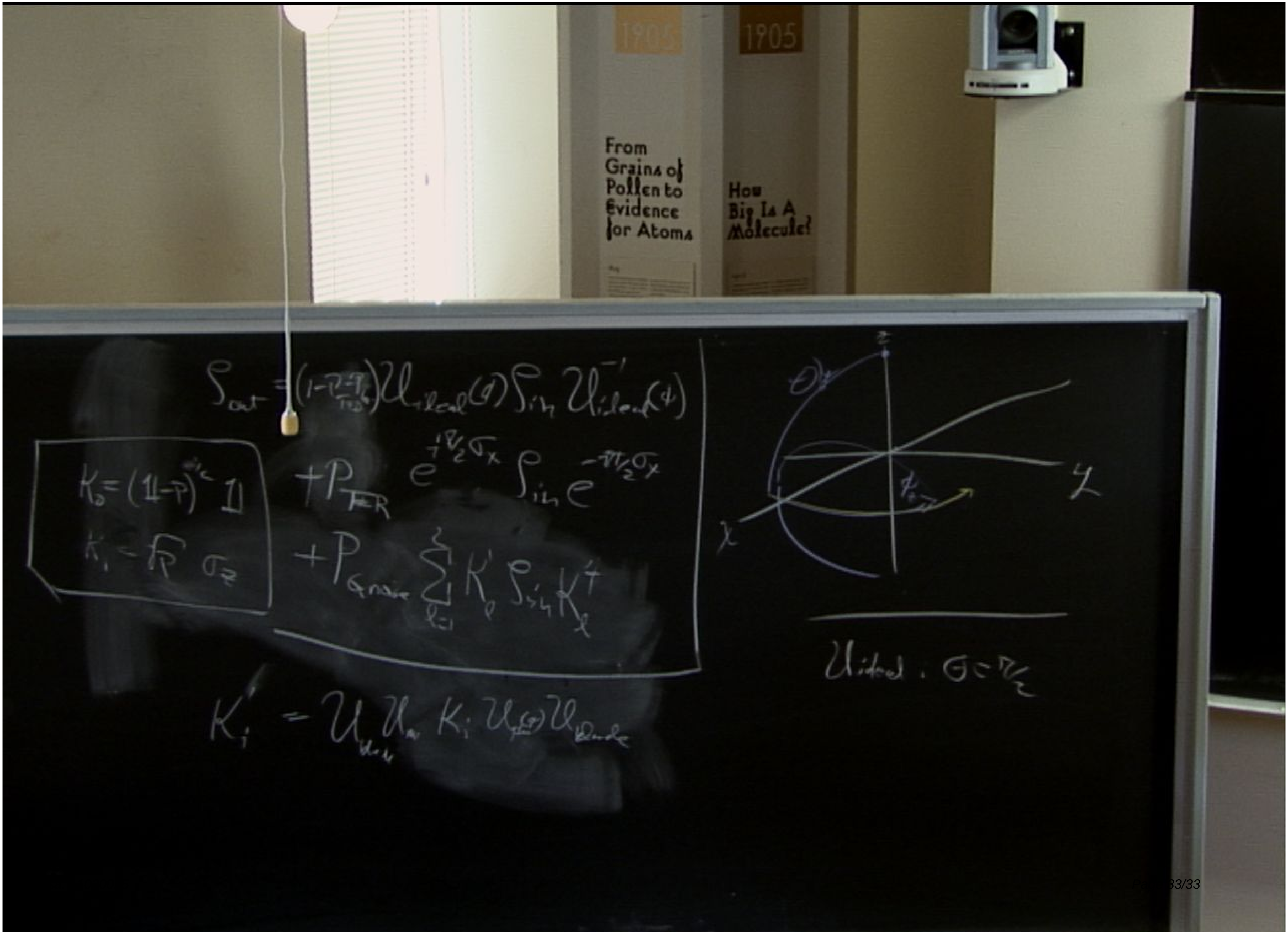
1905
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