

Title: Explorations in Quantum Information - Lecture 3

Date: Mar 16, 2011 09:00 AM

URL: <http://pirsa.org/11030014>

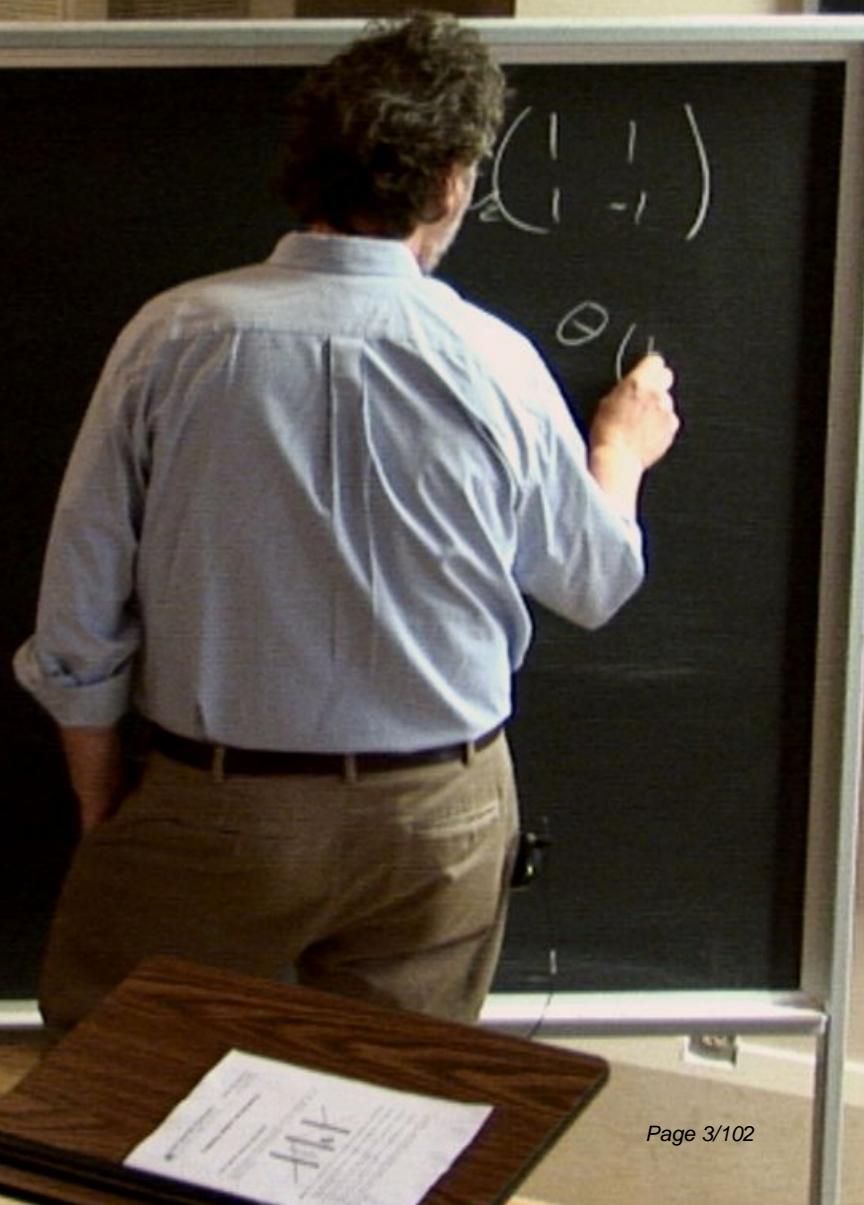
Abstract:



perimeter scholars
international

From
Grains of
Pollen to
Evidence
for Atoms

How
Big Is A
Molecule?



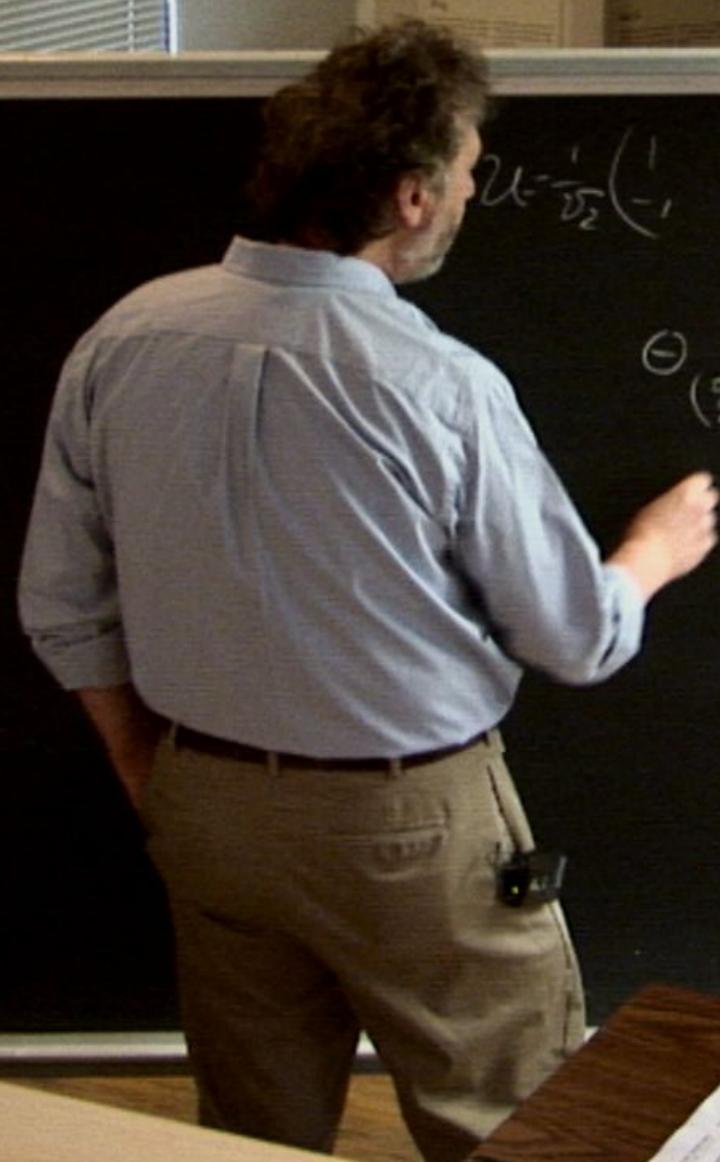
From
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How
Big Is A
Molecule?

$$U = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \quad U = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\Theta \left(\begin{smallmatrix} 0 \\ \Delta \end{smallmatrix} \right)$$

$$\Theta \left(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix} \right)$$



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$$U - \frac{1}{2\pi} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \quad U - \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

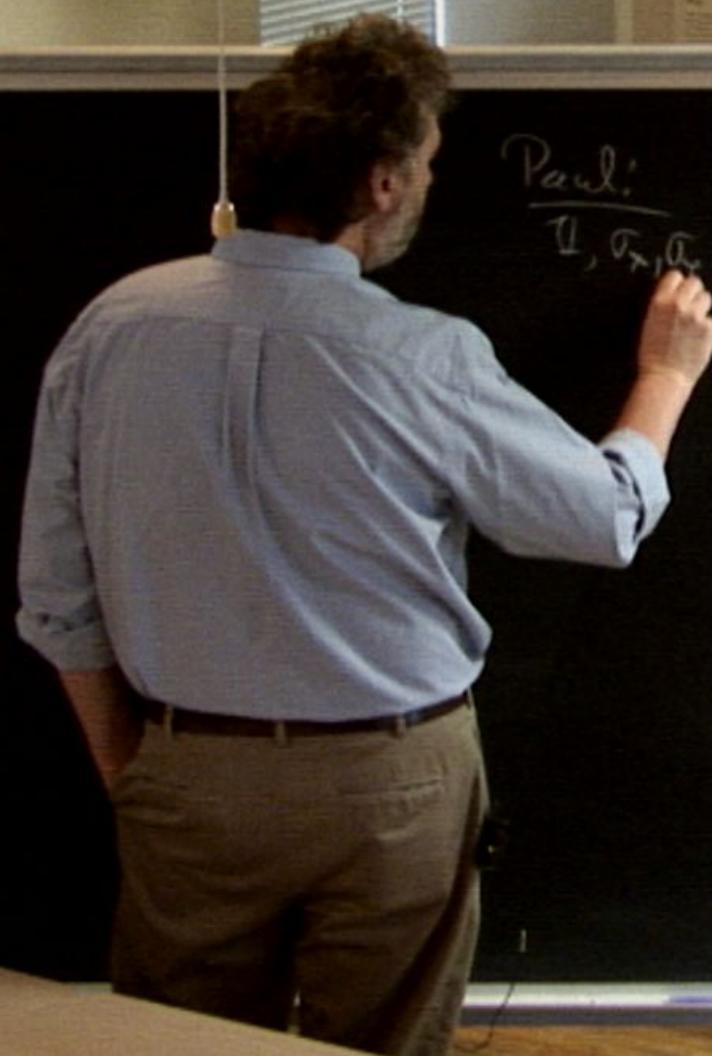
$$\Theta_{\left(\frac{\sigma}{\delta}\right)}$$

$$\Theta_{\left(\frac{1}{\delta}\right)}$$



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$$\frac{\text{Pauli:}}{\mathcal{U}, \sigma_x, \sigma_y} \quad \mathcal{U} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \quad \mathcal{U} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\Theta_{\left(\frac{\alpha}{\lambda}\right)}$$

$$\Theta_{\left(\frac{1}{\lambda}\right)}$$

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$$\frac{P_{\text{out}}}{P_{\text{in}}, \tau_x, \tau_y, P_{\text{in}}} = U - \frac{1}{D_2} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \quad U = \frac{1}{D_2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\Theta \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\Theta \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

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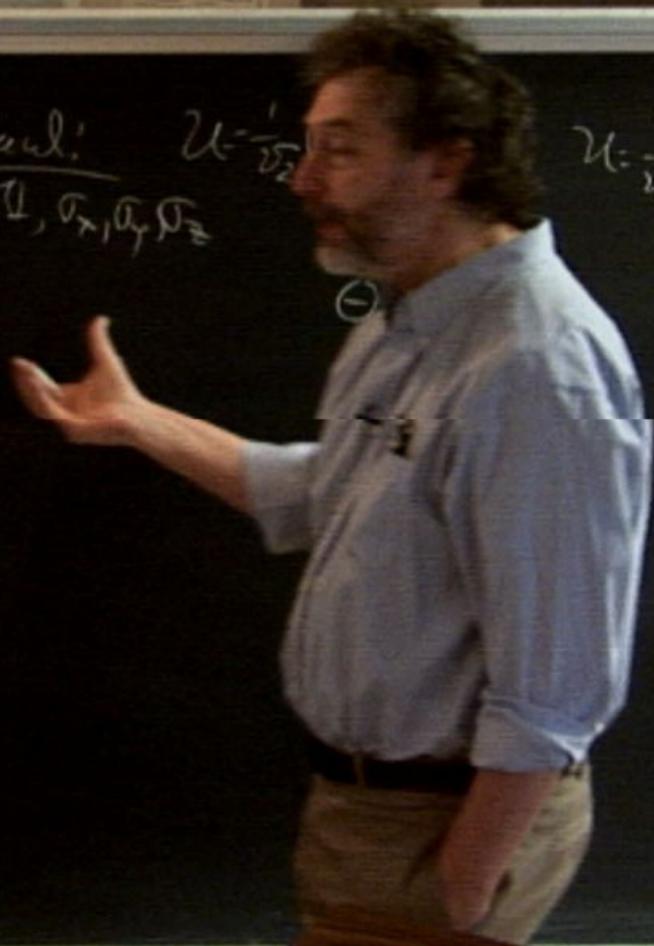
How
Big Is A
Molecule?

Trans.

$$\begin{array}{l} | \uparrow > \langle \downarrow | \\ | \uparrow > \langle \downarrow | \\ | \downarrow > \langle \uparrow | \\ | \downarrow > \langle \uparrow | \end{array}$$

Pauli: $\mathcal{U} = \frac{1}{\sqrt{2}} (\begin{matrix} 1 & 1 \\ 1 & -1 \end{matrix})$

$\Theta(\frac{1}{2})$



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Pauli: $\psi = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ $\psi = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$\Theta \left(\frac{\phi}{\lambda} \right)$ $\Theta \left(\frac{\phi}{\lambda} \right)$

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Trans.

$$| \uparrow \rangle \langle \downarrow |$$

$$| \uparrow \rangle \langle \uparrow |$$

$$| \downarrow \rangle \langle \downarrow |$$

$$| \downarrow \rangle \langle \uparrow |$$

Paul:

$$\frac{1}{\sqrt{2}}, \sigma_x, \sigma_y, \sigma_z$$

$$| + \rangle$$

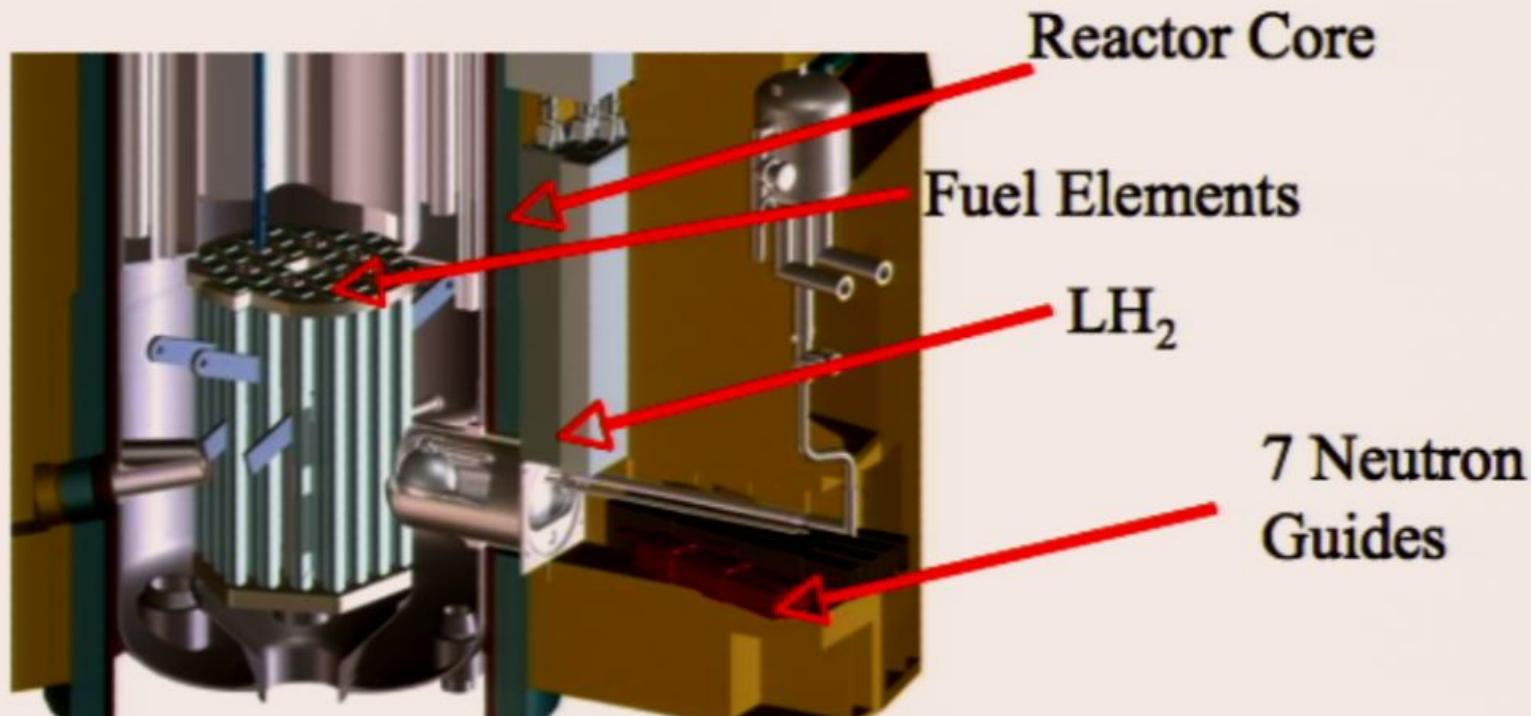
$$| - \rangle$$

$$\Theta(| \downarrow \rangle)$$



Nearly all neutron interferometry is performed with a reactor outfitted with a cold moderator as the neutron source.

The reactor produces neutrons via fission and a part of the production is directed at a cold moderator of hydrogen rich material. The slowed down neutrons are then directed into neutron guides that carry them to the various experiments on the guide hall floor. The neutron flux is approximately 10^{14} neutrons/cm 2 /sec. Note this flux is that integrated over the spread in neutron momenta.



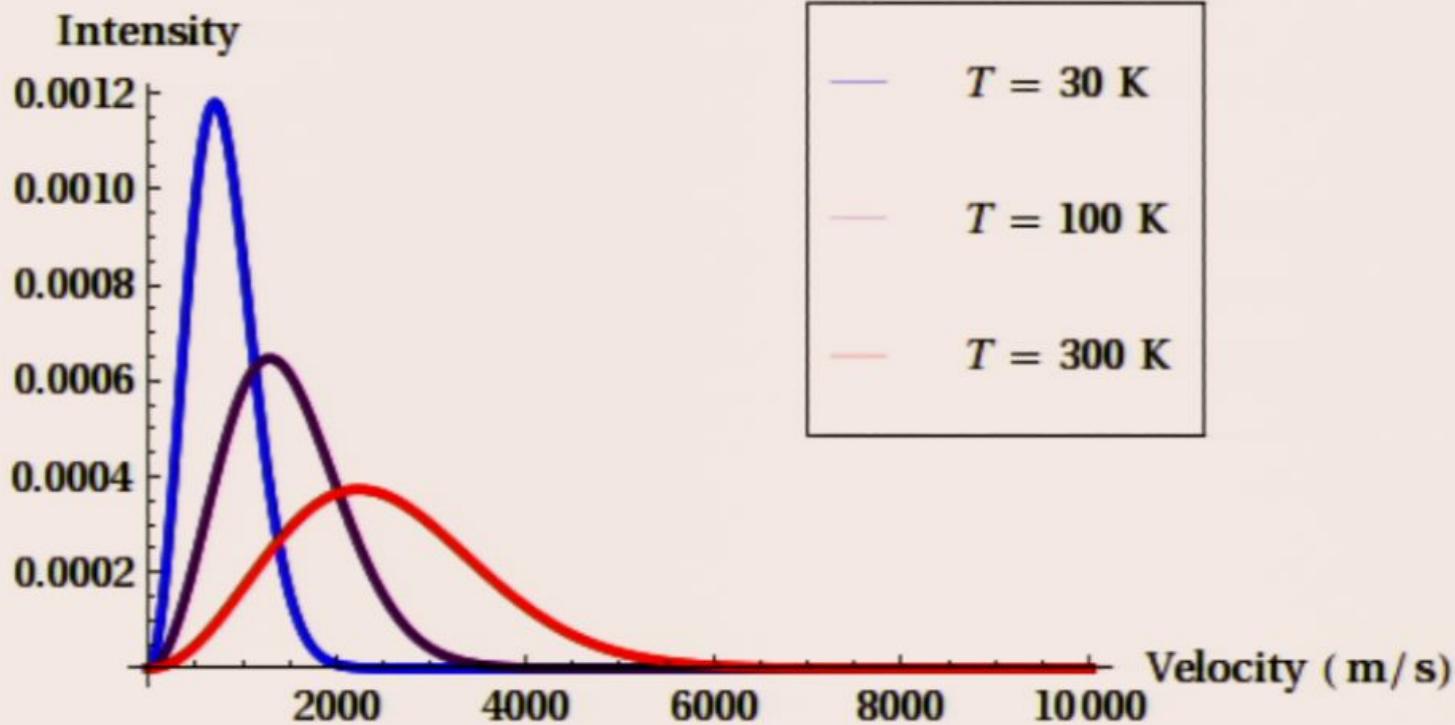
Recall that the de Broglie wavelength is Plank's constant divided by the momentum.

For neutron interferometry we will use a single crystal of Si as a beam splitter. We arrange the crystal to see Bragg scattering in the Laue geometry. A typical wavelength is 2.2 Å.

Note that the velocity is much too slow to require relativistic corrections.

```
wavelength = 2.2 * 10^-10; (* m *)
```

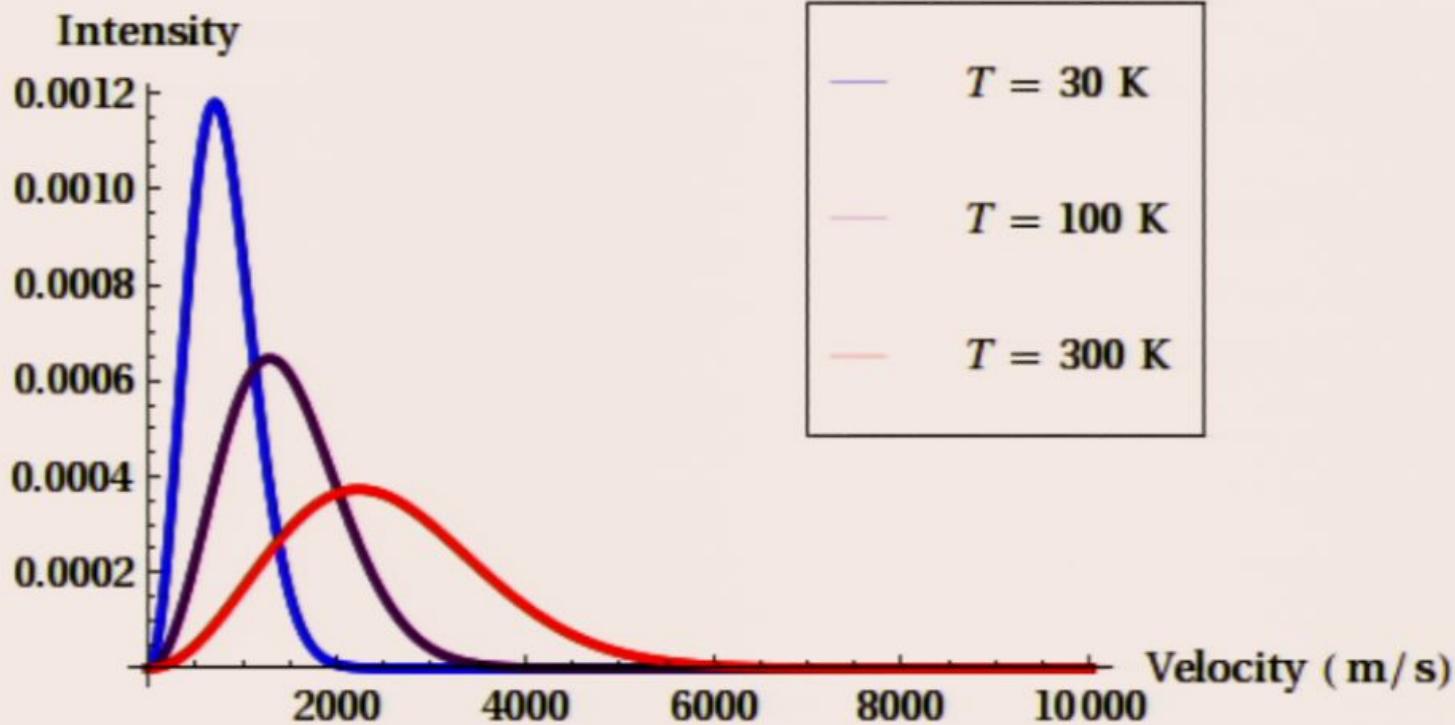
```
LegendShadow -> None,  
LegendSize -> 0.6]
```



Since the de Broglie wavelength of a neutron is a function of its velocity, we can express the Maxwell-Boltzmann velocity profile above as a profile over wavelengths instead:

```
Plot[{  
  f[h / (mass \[Lambda] 10^-10), 30],  
  f[h / (mass \[Lambda] 10^-10), 100]  
}, {\[Lambda], 0.1, 20},
```

```
LegendShadow -> None,  
LegendSize -> 0.6]
```



Since the de Broglie wavelength of a neutron is a function of its velocity, we can express the Maxwell-Boltzmann velocity profile above as a profile over wavelengths instead:

```
Plot[{  
  f[h / (mass λ 10^-10), 30],  
  f[h / (mass λ 10^-10), 100]  
}, {λ, 0.1, 20},
```

So the neutron velocity depends on the potential. There are three simple potentials (nuclear, magnetic and gravitation).

Potential		Phase Shift
Nuclear	$\frac{2\pi\hbar^2}{m} b_c \delta(r)$	$-N b_c \lambda D$
Magnetic	$-\mu \cdot B(r)$	$\pm \mu B m \lambda D / (2\pi\hbar^2)$
Gravitational	$m g \cdot r$	$m^2 g \lambda A \sin(\alpha) / (2\pi\hbar)^2$

where we have used the following variables:

B magnetic field strength

g gravitational acceleration

A normal area enclosed by the coherent beams

α angle between the horizontal and the area A

b_c coherent cross - section

N atom density

D path length in the material

m mass of the neutron

λ neutron wavelength

All this assumes of course that the capture cross-section is negligible. Typically polished silica plates are used as phase shifters. For 2.2 Å neutrons 40 μm of silica introduces a π phase shift.

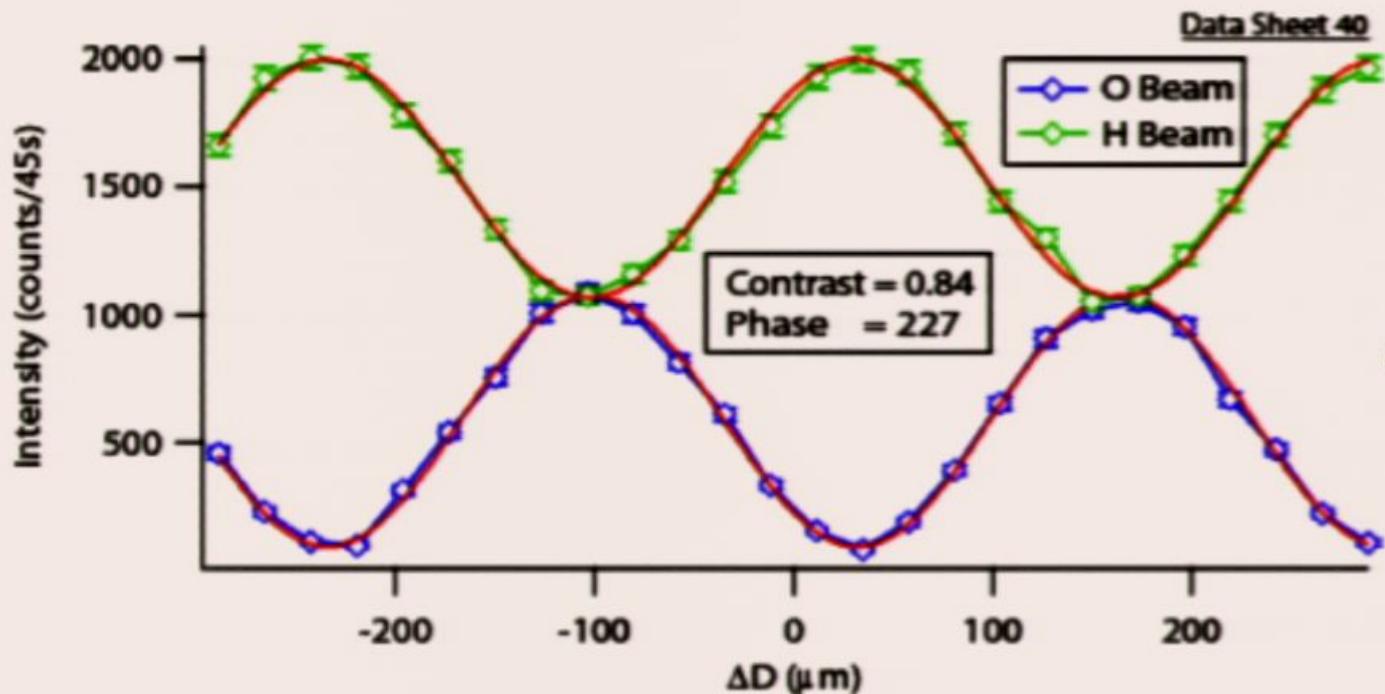
- PROBLEM 6: instead of mapping κ_x to a qubit we could have mapped the paths to a qubit (see the following figure). Give the propagators for the three blades under this mapping. Does it matter which description we use?



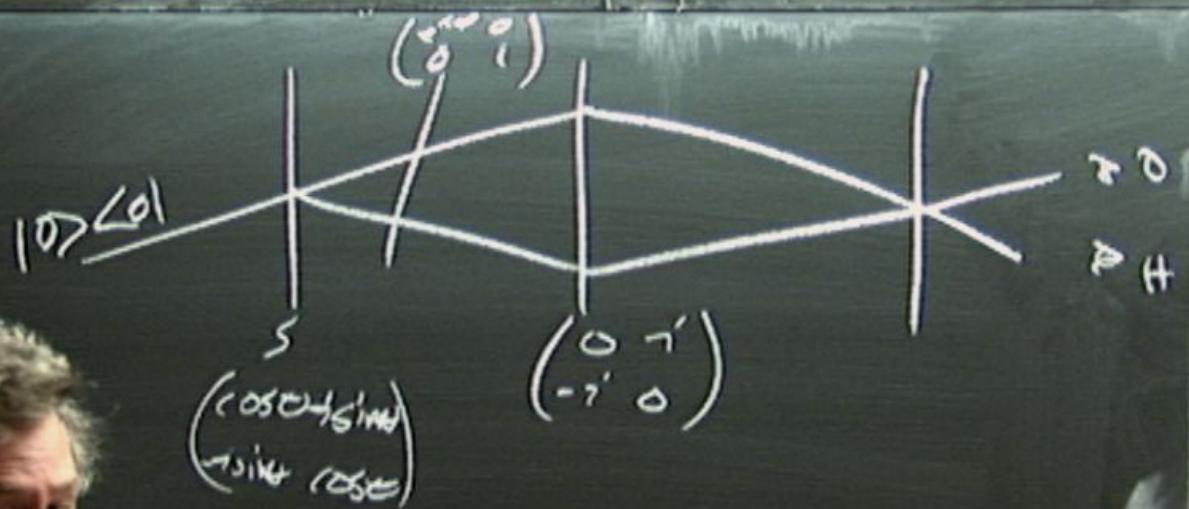
```
Simplify[M1O[a] + M1H[a]]
```

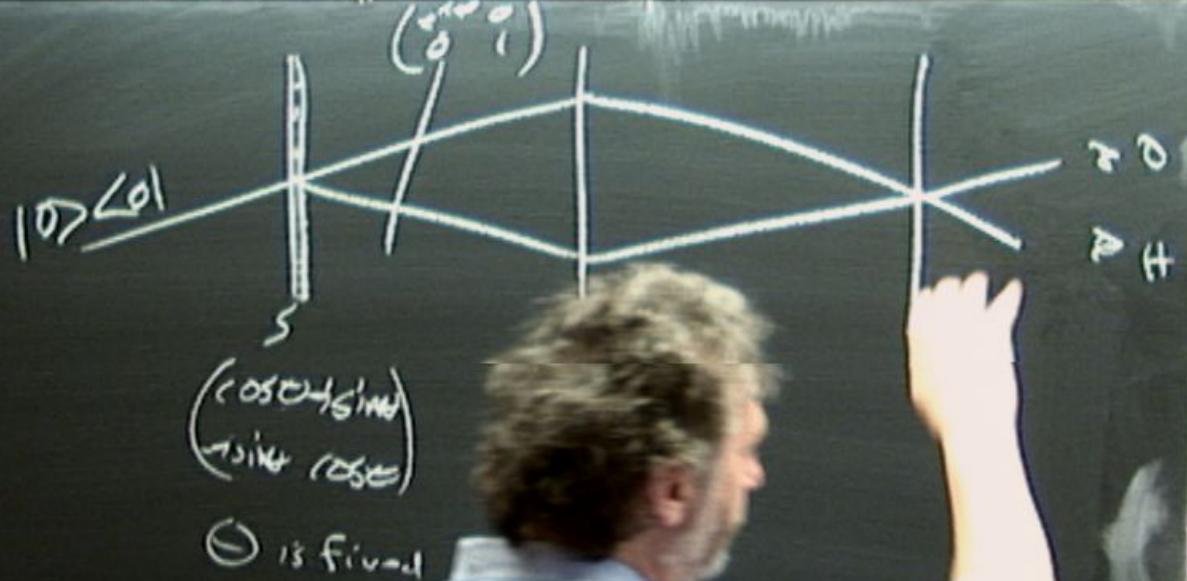
```
1
```

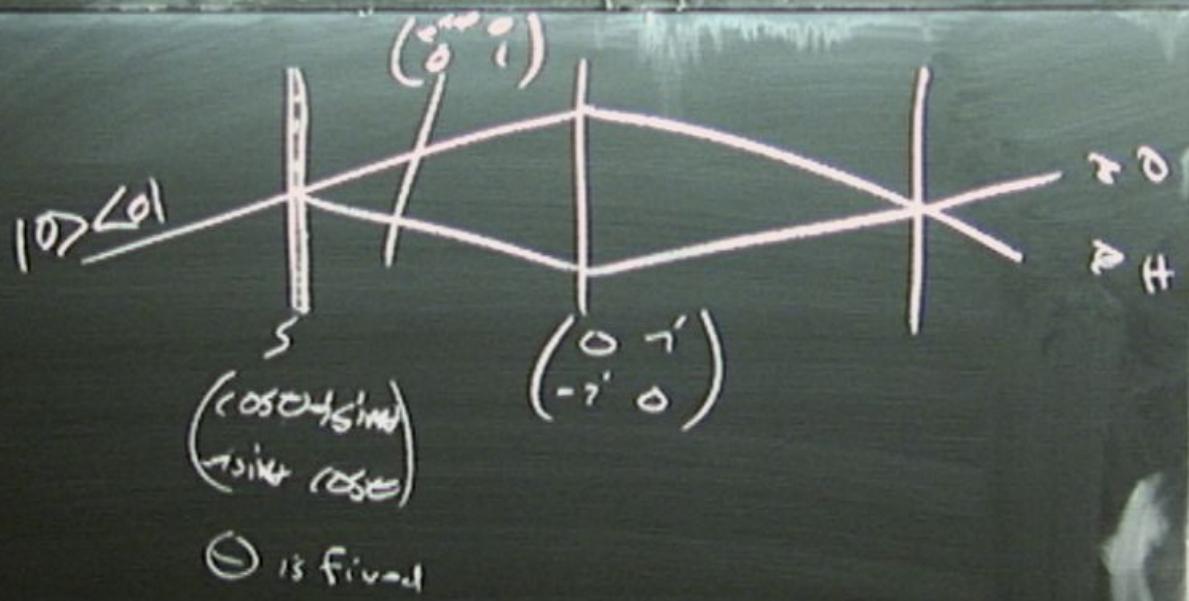
- Problem 11: Here is a set of experimental data. The horizontal axis is given in terms of the difference in path length of silica blades placed in the two paths. What width of silica corresponds to a π phase shift in this experiment? Suggest a few possible reasons for the differences between the experiment and theory. We will explore some of these next.

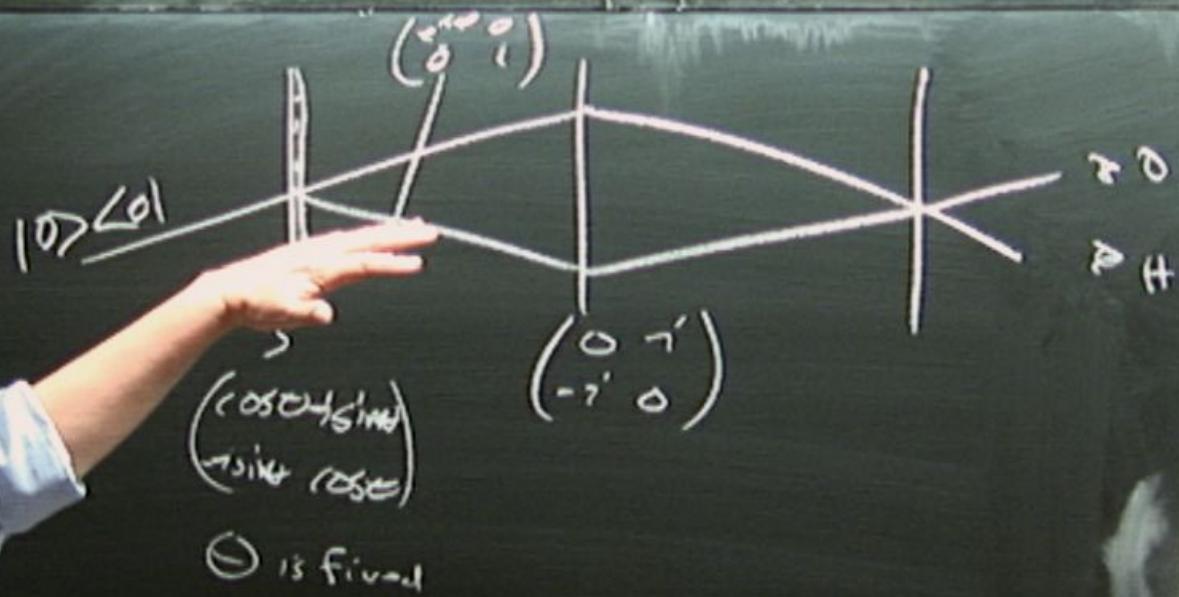


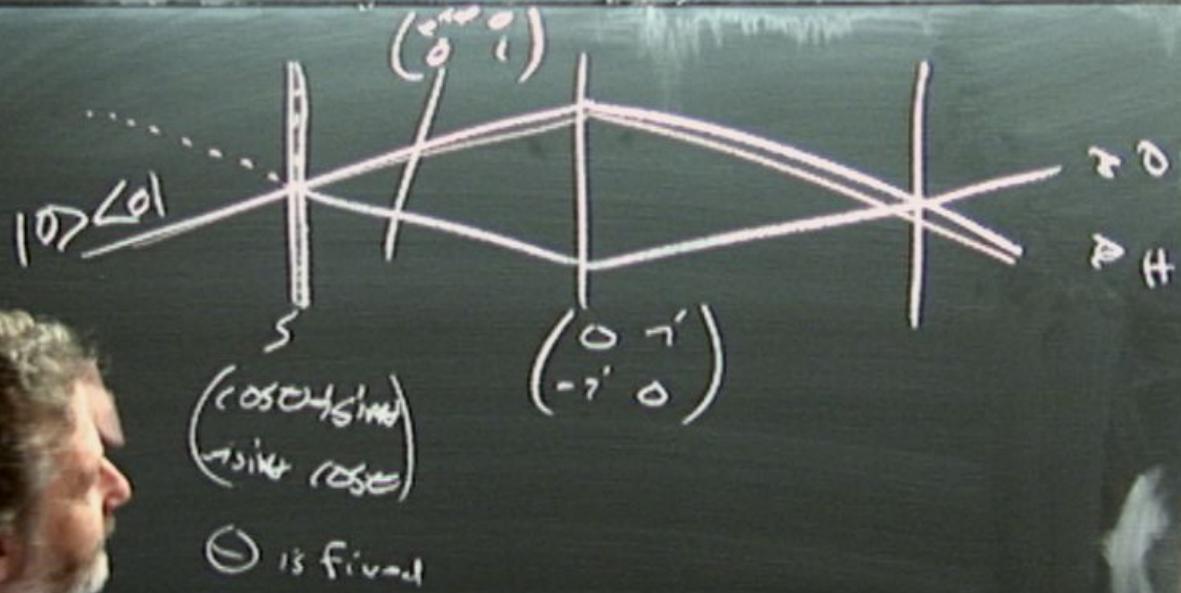
Note, all of the data on neutron interferometry was collected by Dr. Dmitry Pushin with the NIST setup.

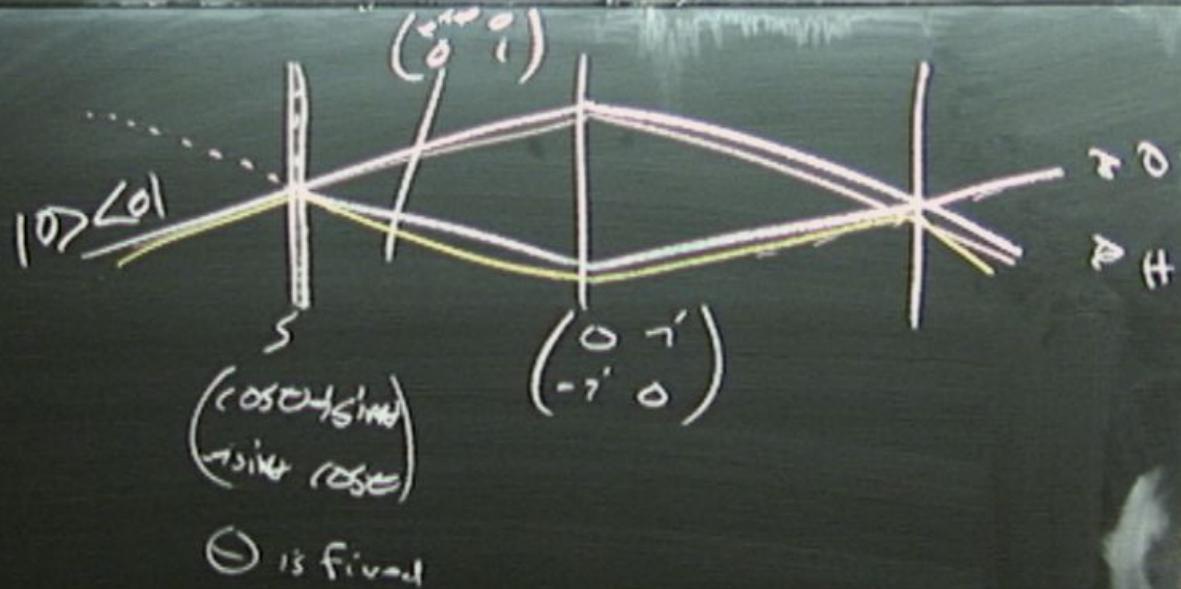


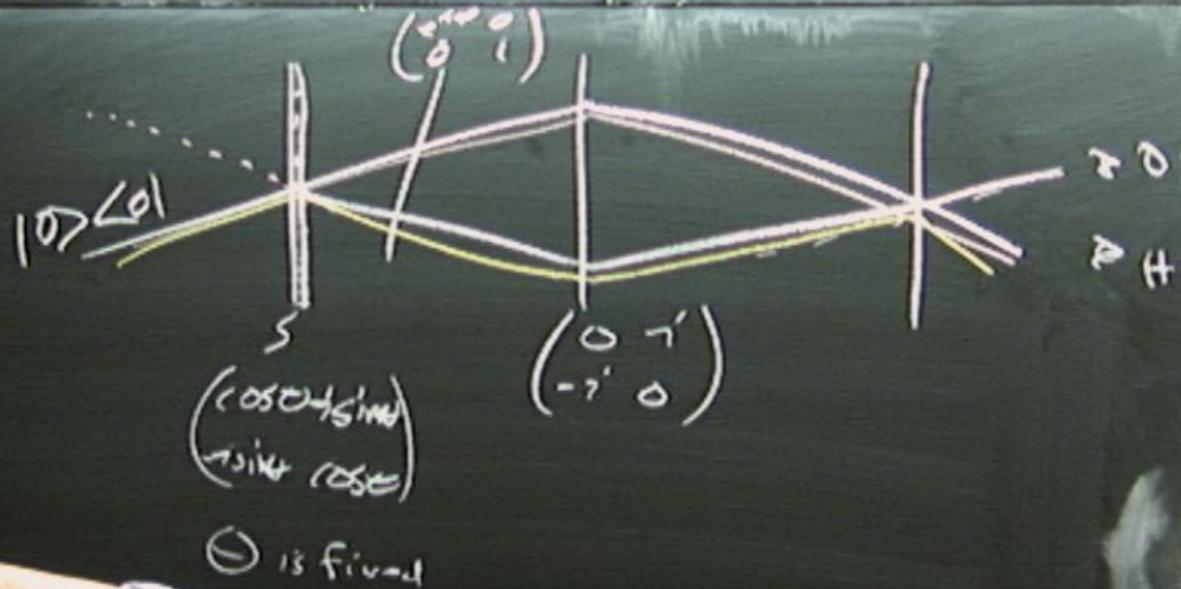












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$$S_{\text{out}} = P \bar{U}_{\text{ideal}} S_{\text{in}} \bar{U}_{\text{ideal}}^{-1} + (1-P)$$

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$$S_{out} = P \bar{U}_{ideal} S_{in} \bar{U}_{ideal}^{-1} + (1-P)$$

$$\bar{U}_{ideal} = e^{-\frac{\hbar^2}{2m} \nabla_x^2}$$



$$S_{out} = P U_{ideal} S_{in} U_{ideal}^{-1}$$

$$+ (1-P) U_{error} S_{in} U_{error}^{-1}$$

$$U_{error} = P^{-1} \nabla_x \nabla_x$$

$$S_{out} = P U_{ideal} S_{in} U_{ideal}^{-1}$$

$$+ (1-P) U_{err} S_{in} U_{err}^{-1}$$

$$U_{err} = e^{-j\frac{\pi}{2}\sigma_x}$$

$$S_{out} = P U_{ideal} S_{in} U_{ideal}^{-1} + (1-P) U_{error} S_{in} U_{error}^{-1}$$

$$U_{error} = e^{-j\frac{\pi}{2}\sigma_x}$$

```
. Usample[a_] := {{1, 0}, {0, Exp[I a]}};

. Usampleinv[a_] := {{1, 0}, {0, Exp[-I a]}};
```

Experiments

blade, no spin

state is the neutron in the k-x up path

```
. in = Exp[
```

Third experiment is to assume that the blades are real, we have a phase flag and look at the contrast

```
. res3[t_, a_] := Ublade[t].Um.Uphase[a].Ublade[t].in.Ubladeinv[t].Uphaseinv[a].Uminv.Ubladeinv[t]
```

- Problem 12: What is the purity of $\text{res3}[t,a]$? and what does this tell us?

Look at the O and H beam contrast

```
. M3O[t_, a_] := Tr[Exp . res3[t, a]]
```

that the transmission coefficient = $\sin^2[t]$ in the expressions above. So since we want to index the plot by the transmission coefficient we run T from 0 to 1 and recompute the angle as $\text{ArcSin}[\text{Sqrt}[T]]$

```
. Plot3D[M3O[ArcSin[Sqrt[T]], a], {a, 0, 2π}, {T, 0, 1}, {AxesLabel ->
 {"phase (Radians)", "transmission coefficient", "O-beam intensity"}}]
```

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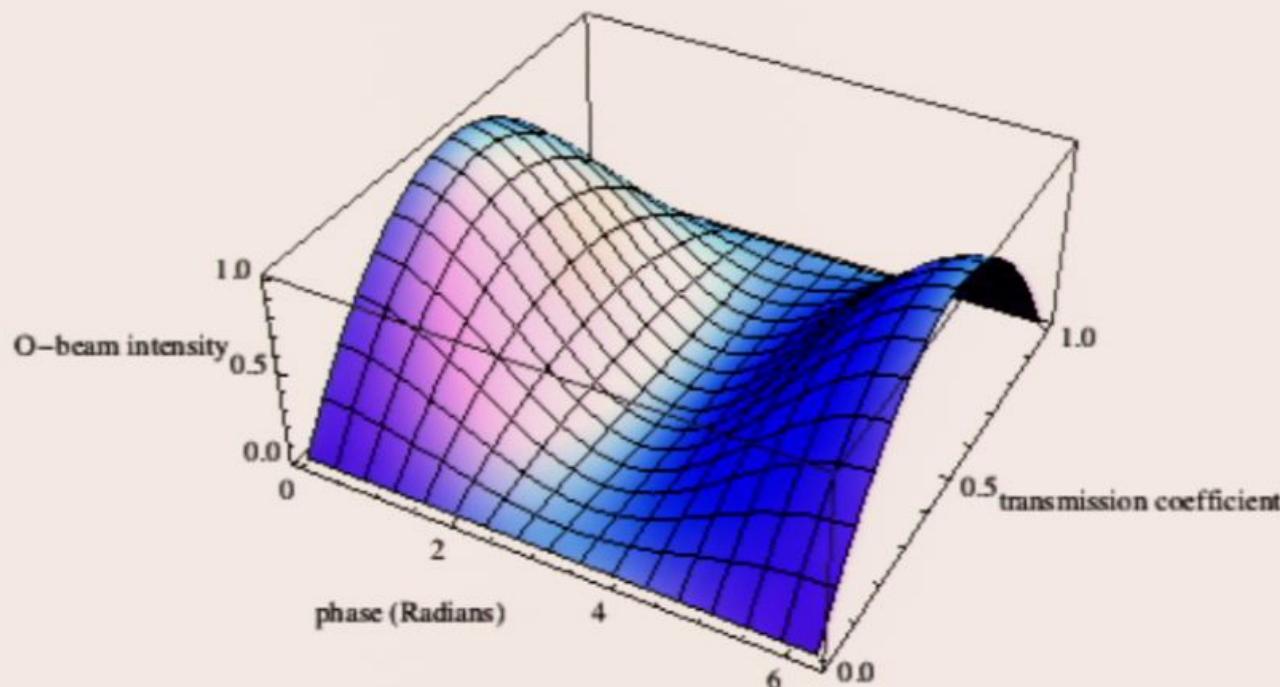
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```
Plot3D[M3O[ArcSin[Sqrt[T]], a], {a, 0, 2 \pi}, {T, 0, 1}, {AxesLabel -> {"phase (Radians)", "transmission coefficient", "O-beam intensity"}}]
```

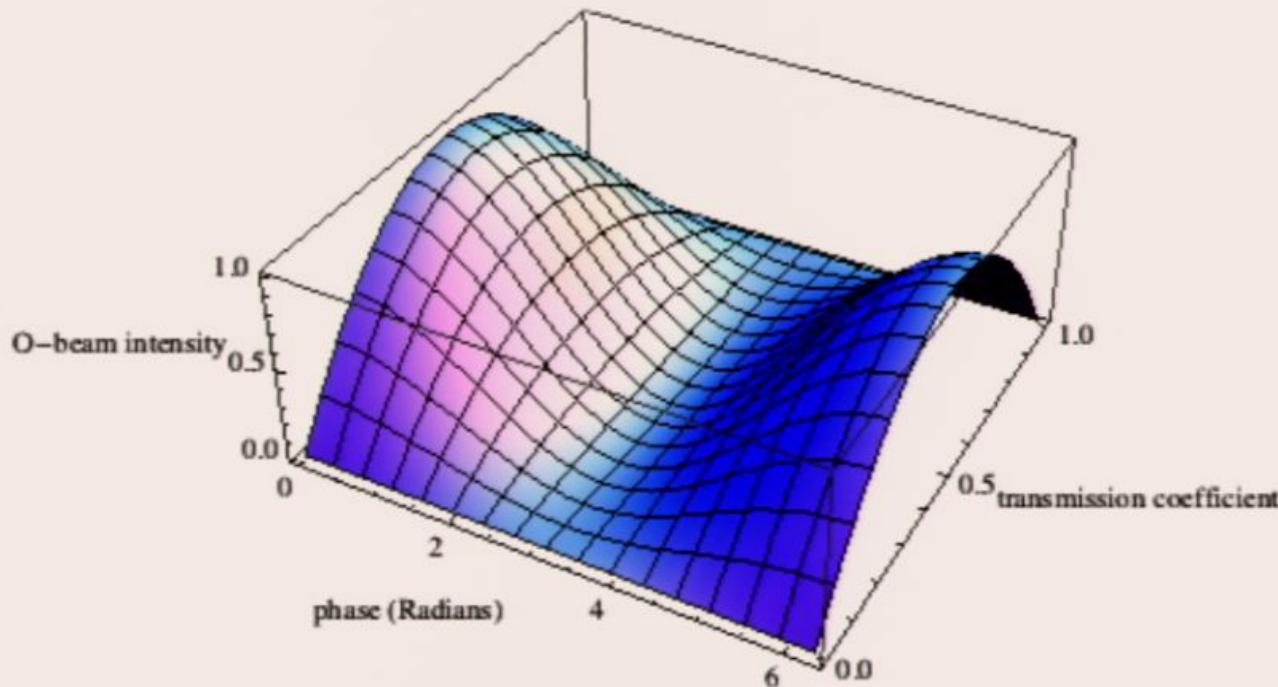


```
M3H[t_, a_] := Tr[Exp . res3[t, a]]
```

```
Plot3D[M3H[ArcSin[Sqrt[T]], a], {a, 0, 2 \pi}, {T, 0, 1}, {AxesLabel -> {"phase (Radians)", "transmission coefficient", "H-beam intensity"}}]
```

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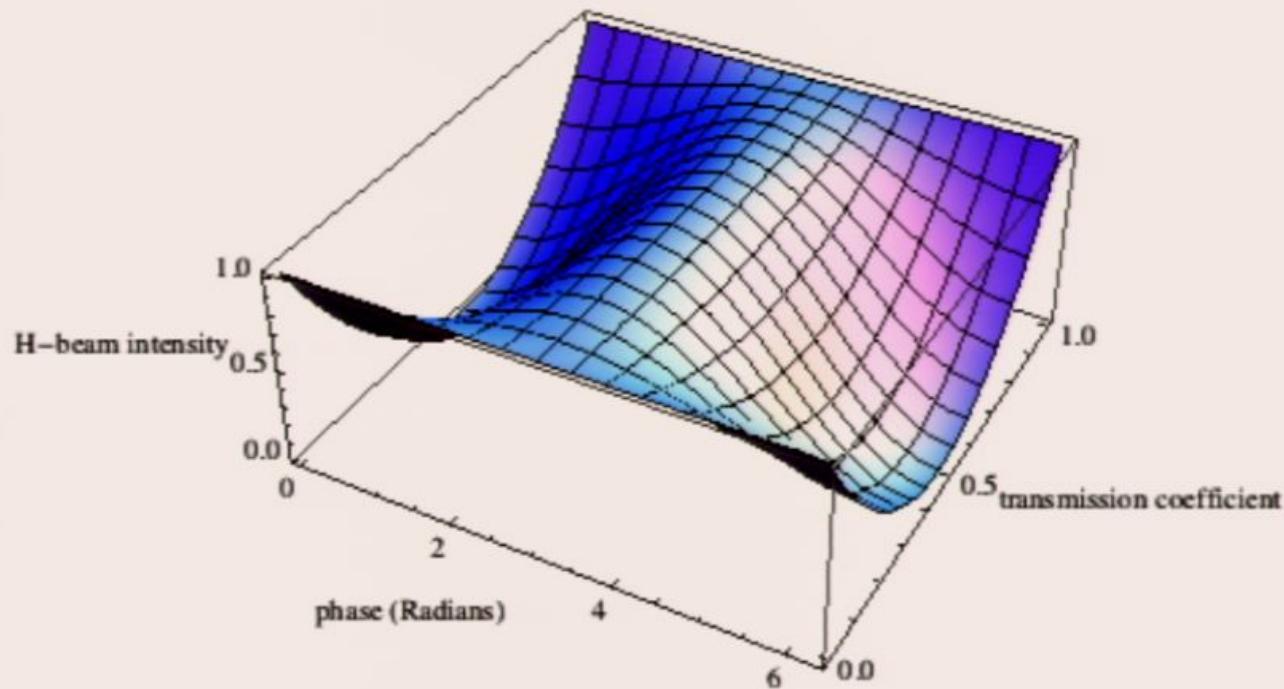
```
Plot3D[M3O[ArcSin[Sqrt[T]], a], {a, 0, 2π}, {T, 0, 1}, {AxesLabel -> {"phase (Radians)", "transmission coefficient", "O-beam intensity"}}]
```



```
M3H[t_, a_] := Tr[Ezm . res3[t, a]]
```

```
Plot3D[M3H[ArcSin[Sqrt[T]], a], {a, 0, 2π}, {T, 0, 1}, {AxesLabel -> {"phase (Radians)", "transmission coefficient", "H-beam intensity"}}]
```

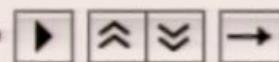
```
{ "phase (Radians)", "transmission coefficient", "H-beam intensity"}]
```



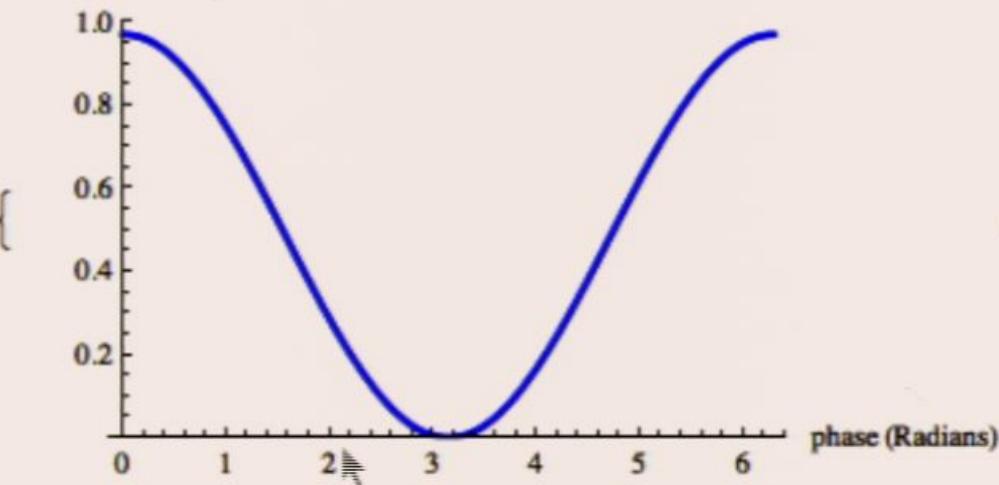
that when the transmission coefficient is either 0 or 1 there is no interference.

```
Animate[{Plot[M3O[ArcSin[Sqrt[T]]], a], {a, 0, 2 π},  
{AxesLabel -> {"phase (Radians)", "O-beam intensity"},  
PlotStyle -> {RGBColor[0, 0, 1], Thickness[0.01]}, PlotRange -> {0, 1}}],  
Plot[M3H[ArcSin[Sqrt[T]]], a], {a, 0, 2 π},  
{AxesLabel -> {"phase (Radians)", "H-beam intensity"},  
PlotStyle -> {RGBColor[1, 0, 0], Thickness[0.01]}, PlotRange -> {0, 1}}]],  
{T, 0, 1}, AnimationRunning -> False]
```

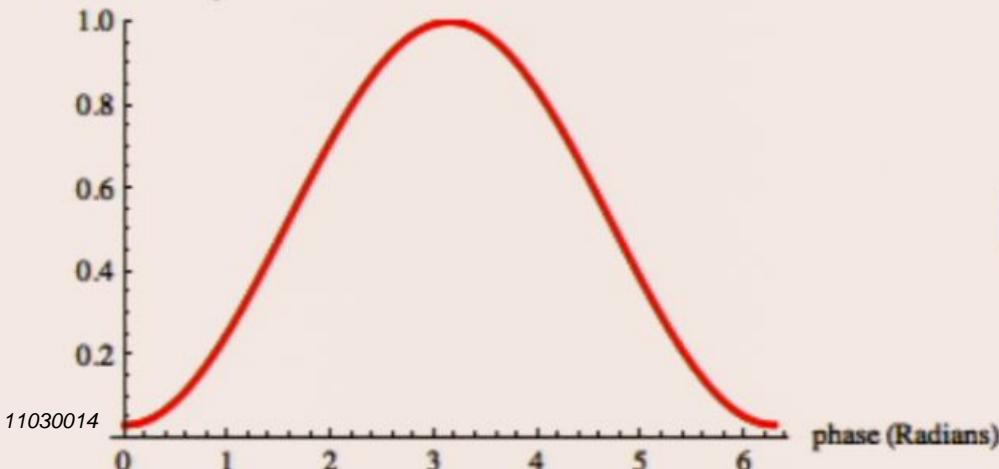
T



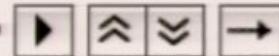
O-beam intensity



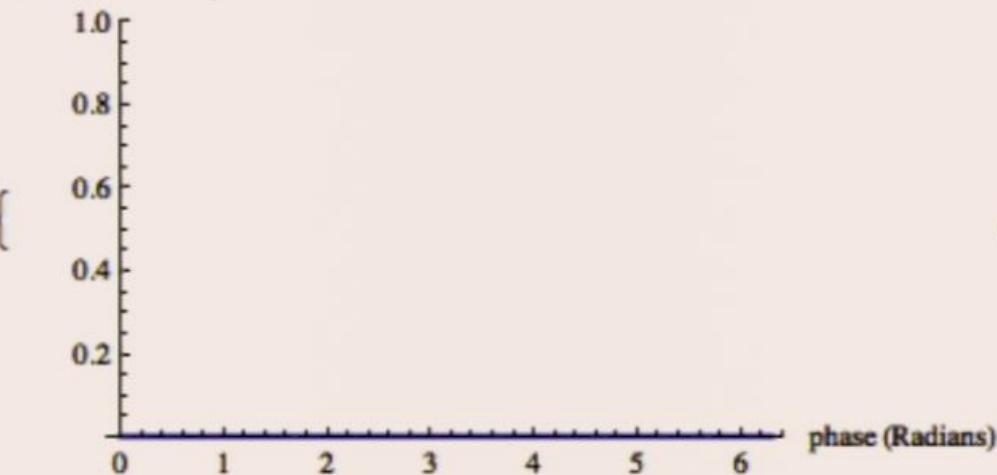
H-beam intensity



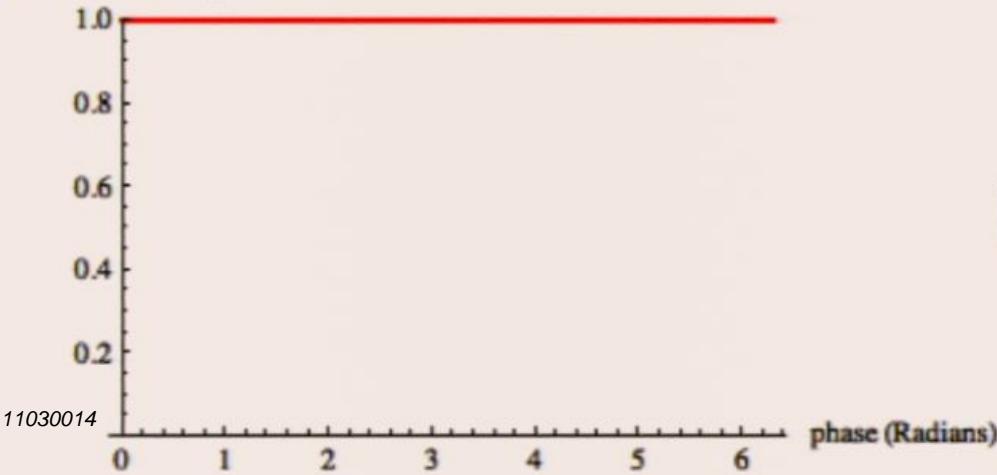
2 3 4 5 6 7 8



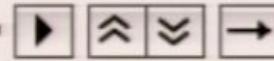
O-beam intensity



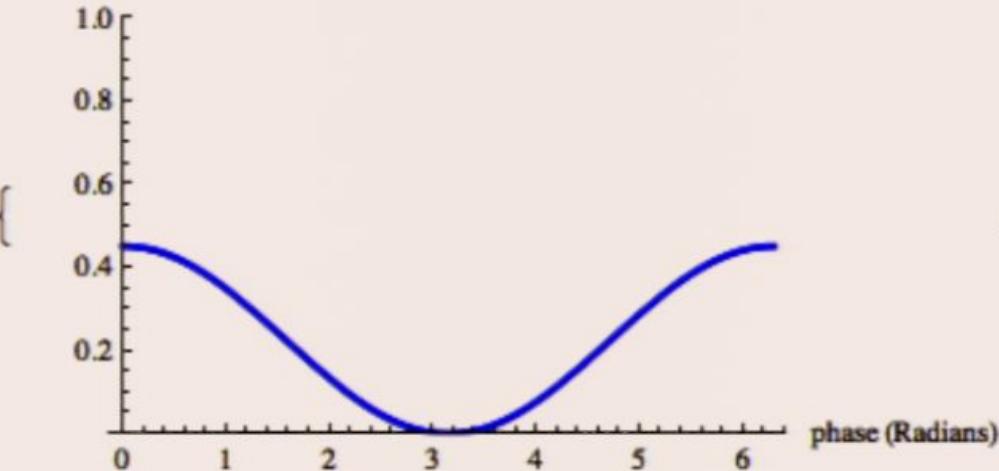
H-beam intensity



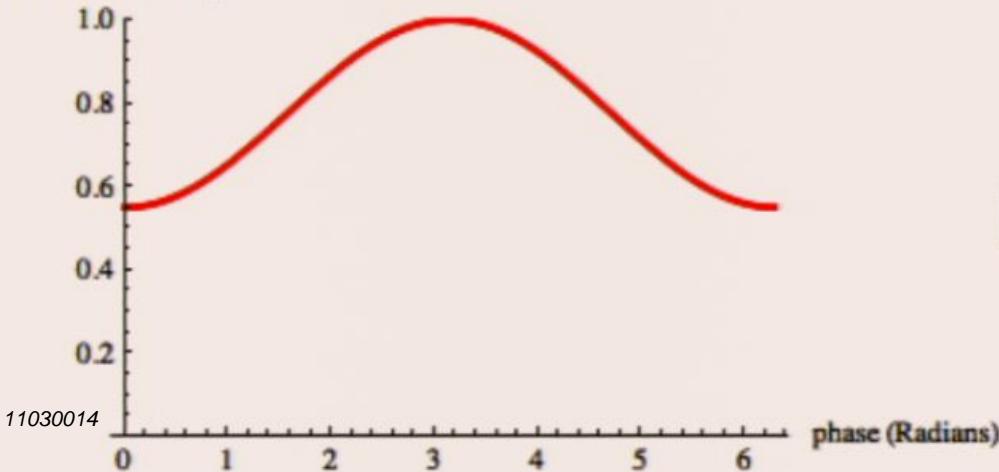
T



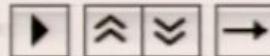
O-beam intensity



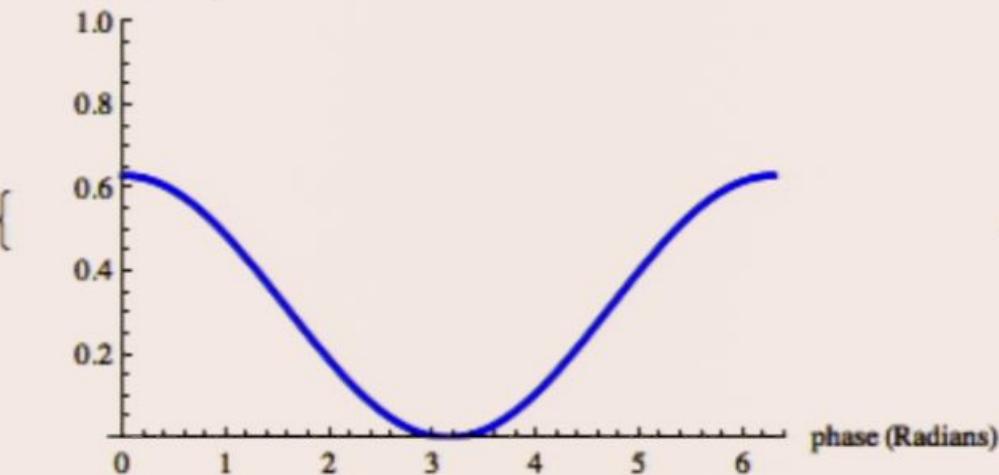
H-beam intensity



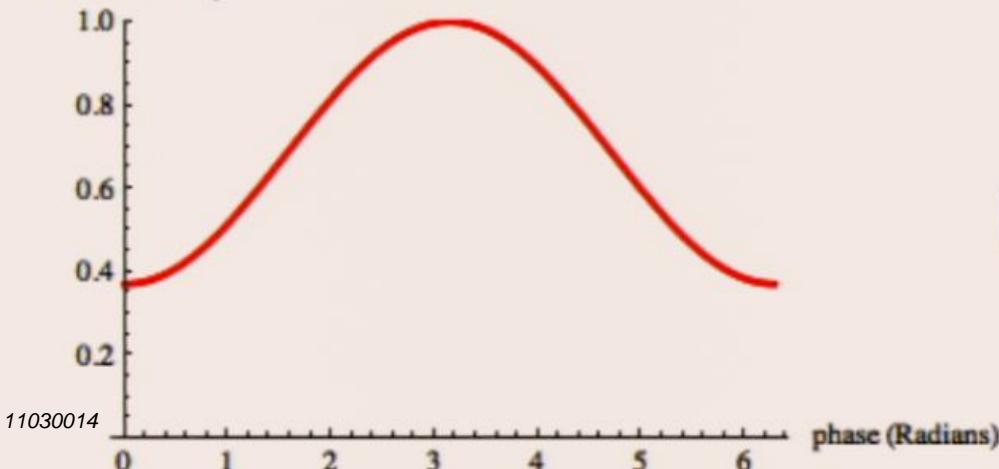
T



O-beam intensity



H-beam intensity



Minimize
Zoom

Magnification

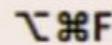
✓ Show Ruler
Show Toolbar

Stack Windows

Tile Windows Wide

Tile Windows Tall

Full Screen



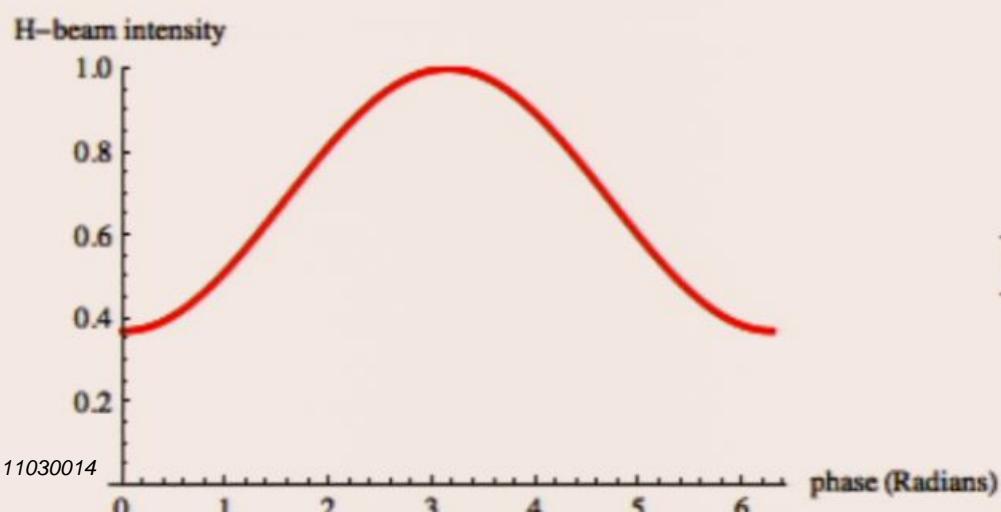
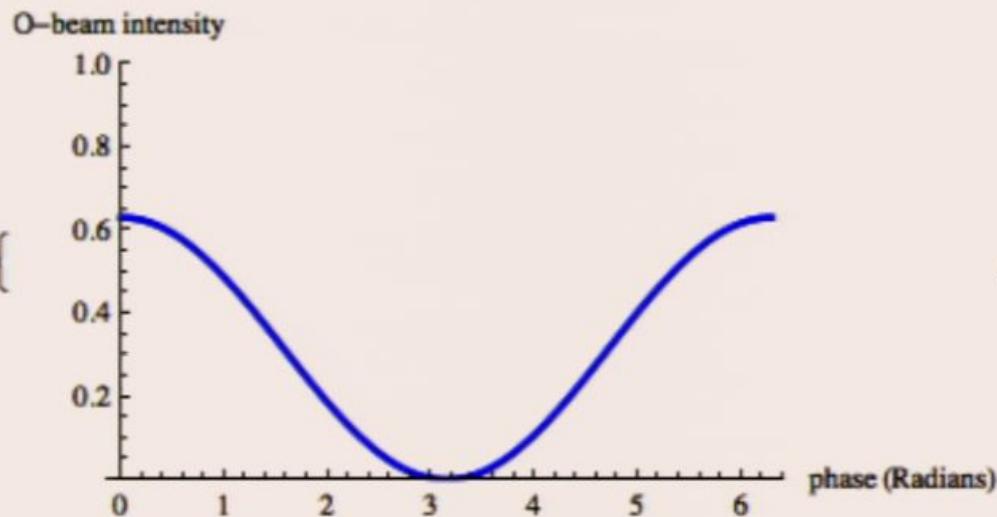
Bring All to Front

Messages

QE Lec 1 (Modified).nb

QE Lec 3 Choi.nb

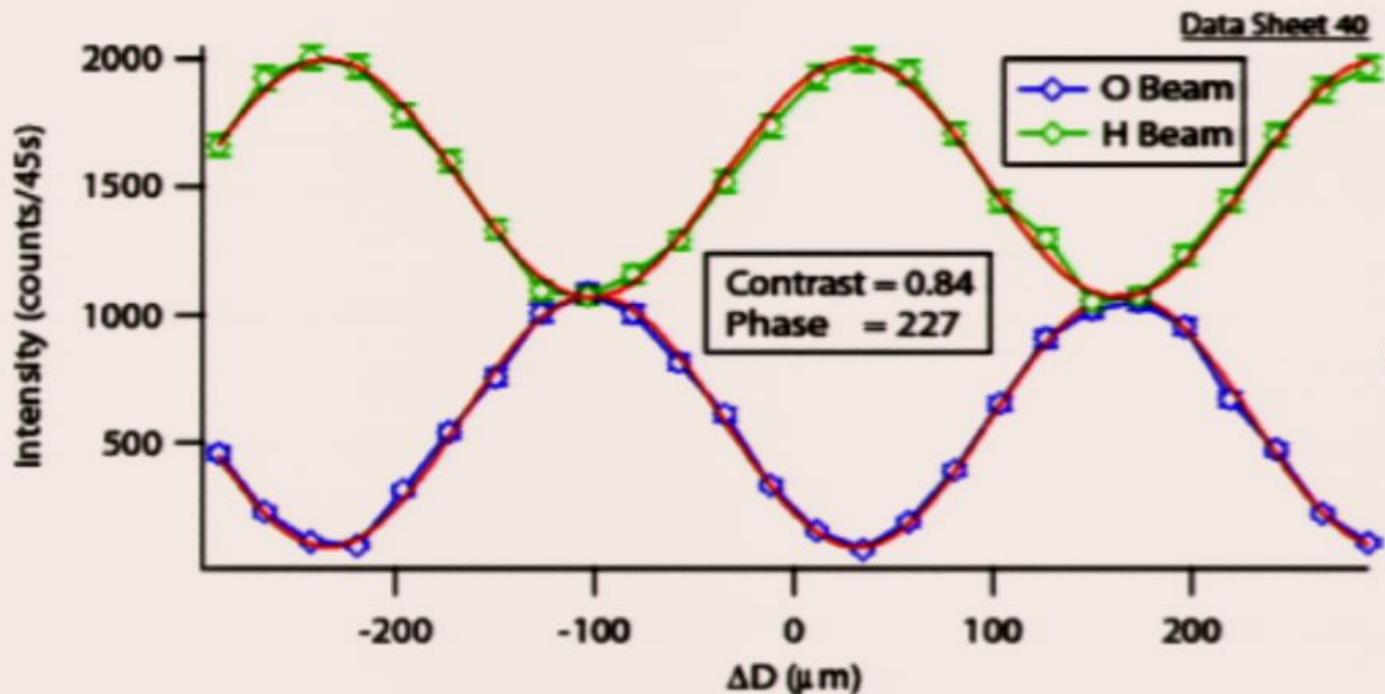
✓ QE Lec 3.nb



```
Simplify[M1O[a] + M1H[a]]
```

```
1
```

- Problem 11: Here is a set of experimental data. The horizontal axis is given in terms of the difference in path length of silica blades placed in the two paths. What width of silica corresponds to a π phase shift in this experiment? Suggest a few possible reasons for the differences between the experiment and theory. We will explore some of these next.

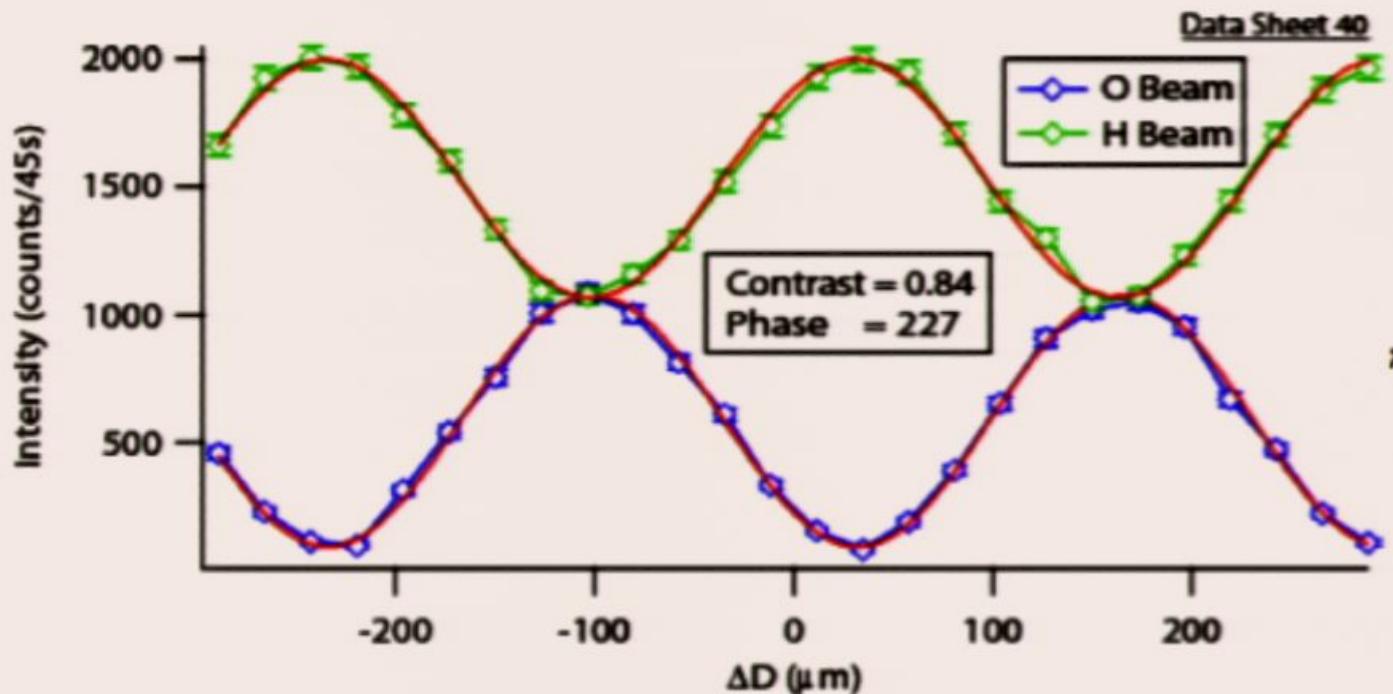


Note, all of the data on neutron interferometry was collected by Dr. Dmitry Pushin with the NIST setup.

Simplify[M1O[a] + M1H[a]]

1

- Problem 11: Here is a set of experimental data. The horizontal axis is given in terms of the difference in path length placed in the two paths. What width of silica corresponds to a π phase shift in this experiment? Suggest a reason for the differences between the experiment and theory. We will explore some of these next.



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Bring All to Front

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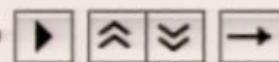
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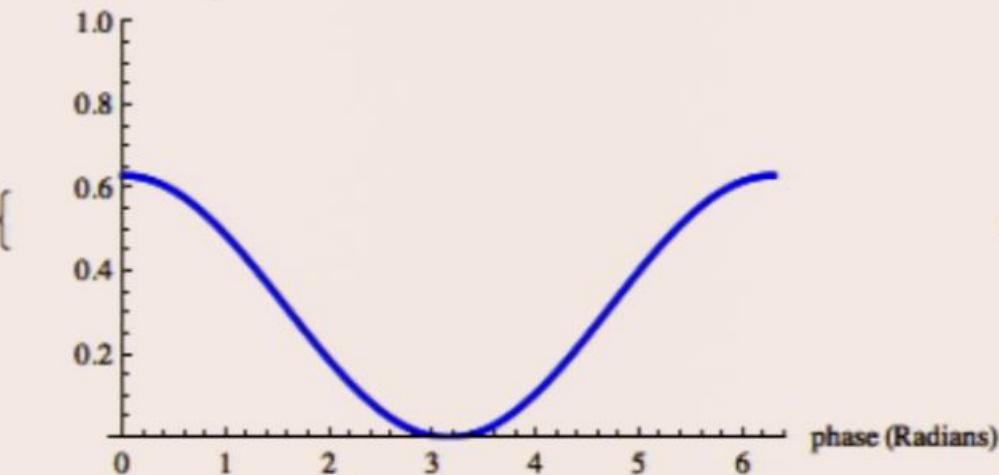
QE Lec 3.nb

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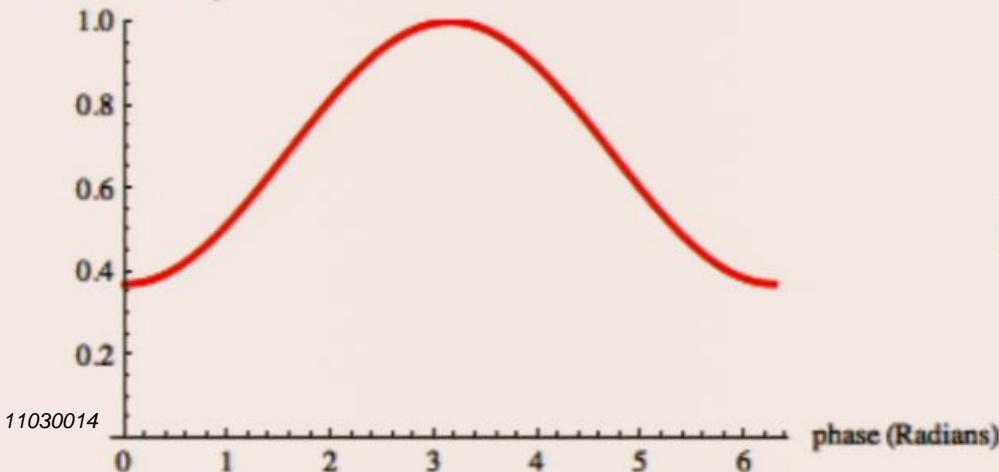
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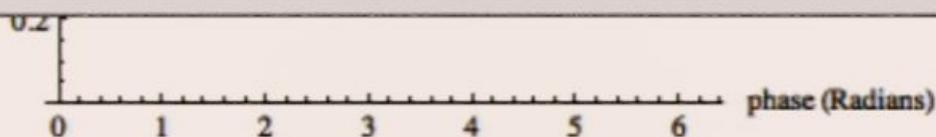


O-beam intensity



H-beam intensity





that the O-beam has zero offset in the absence of dark counts. The H-beam is always reduced from 1. that at $t=0$ or 1 all neutrons end up in the H detector. This is not the correct physics. Look at the action of the e blade where we still assume everything is perfect. Fix this. What you should see is that if the entire beam is nitted no neutrons hit the detector. If the entire beam is reflected than all neutrons arrive at the H detector.

pected the sum of the O and H-beam intensities is still 1 and is independent of the transmission probability.

- `Simplify[M3O[t, a] + M3H[t, a]]`

$$e^{-ia} \left((1 + e^{ia})^2 \text{Conjugate}[\cos[t]] \text{Conjugate}[\sin[t]] \cos[t] \sin[t] - \text{Conjugate}[\sin[t]]^2 (\cos[t]^2 - e^{ia} \sin[t]^2) + \text{Conjugate}[\cos[t]]^2 (e^{ia} \cos[t]^2 - e^{2ia} \sin[t]^2) \right)$$

$$e^{-ia} \left((1 + e^{ia})^2 \text{Conjugate}[\cos[t]] \text{Conjugate}[\sin[t]] \cos[t] \sin[t] - \text{Conjugate}[\sin[t]]^2 (\cos[t]^2 - e^{ia} \sin[t]^2) + \text{Conjugate}[\cos[t]]^2 (e^{ia} \cos[t]^2 - e^{2ia} \sin[t]^2) \right)$$

1

- Problem 13: `res[t,a]` remains a valid density matrix, why? Show where it fits on and in the Bloch sphere.

How to interpret the outcome

that when $T=0$ or when $T=1$ the neutrons end up at the H detector. So it appears that the outcome is a combination of two possibilities

```
res4[p_, a_] := p Ubladeq[π/4].Um.Uphase[a].Ubladeq[π/4].in.
```

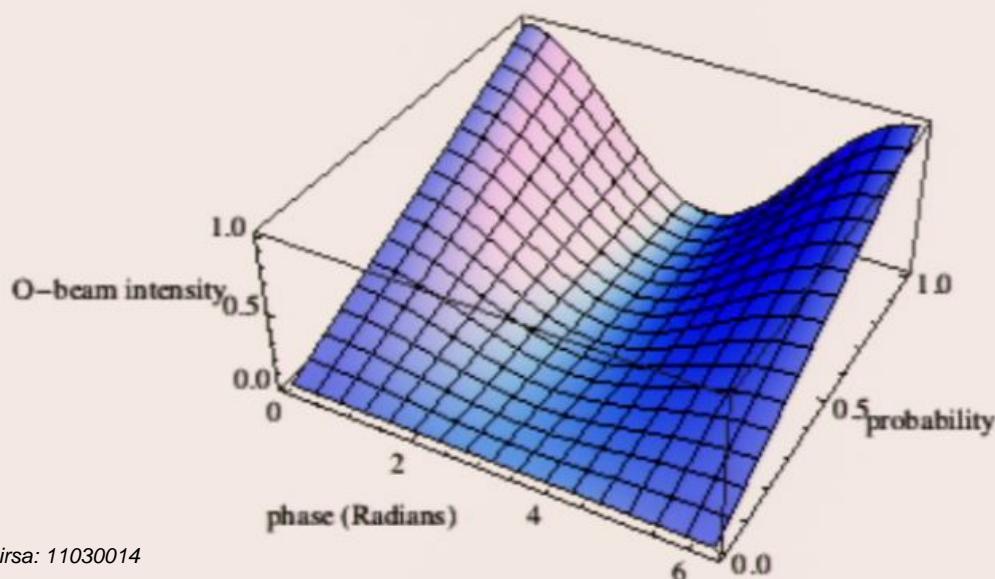
1

- Problem 13: $\text{res}[t,a]$ remains a valid density matrix, why? Show where it fits on and in the Bloch sphere.

How to interpret the outcome

that when $T=0$ or when $T=1$ the neutrons end up at the H detector. So it appears that the outcome is a combination of two possibilities

```
res4[p_, a_] := p Ubladeg[\pi/4].Um.Uphase[a].Ubladeg[\pi/4].in.  
Ubladeginv[\pi/4].Uphaseinv[a].Uminv.Ubladeginv[\pi/4] + (1 - p) Ezm  
M4O[p_, a_] := Tr[Exp . res4[p, a]]  
Plot3D[M4O[p, a], {a, 0, 2 \pi}, {p, 0, 1},  
{AxesLabel \rightarrow {"phase (Radians)", "probability", "O-beam intensity"}]}
```



```

L 2 3 4 5 6 7 8
  (Cos[t] - e^i a Sin[t]) / Conjugate(Cos[t]) (e^-i a Cos[t] - e^i a Sin[t])

$$e^{-ia} \left( (1 + e^{ia})^2 \text{Conjugate}[\text{Cos}[t]] \text{Conjugate}[\text{Sin}[t]] \text{Cos}[t] \text{Sin}[t] - \text{Conjugate}[\text{Sin}[t]]^2 \right.$$


$$\left. (\text{Cos}[t]^2 - e^{ia} \text{Sin}[t]^2) + \text{Conjugate}[\text{Cos}[t]]^2 (e^{ia} \text{Cos}[t]^2 - e^{2ia} \text{Sin}[t]^2) \right)$$

1

```

- Problem 13: $\text{res}[t,a]$ remains a valid density matrix, why? Show where it fits on and in the Bloch sphere.

w to interpret the outcome

that when $T=0$ or when $T=1$ the neutrons end up at the H detector. So it appears that the outcome is a combination of two possibilities

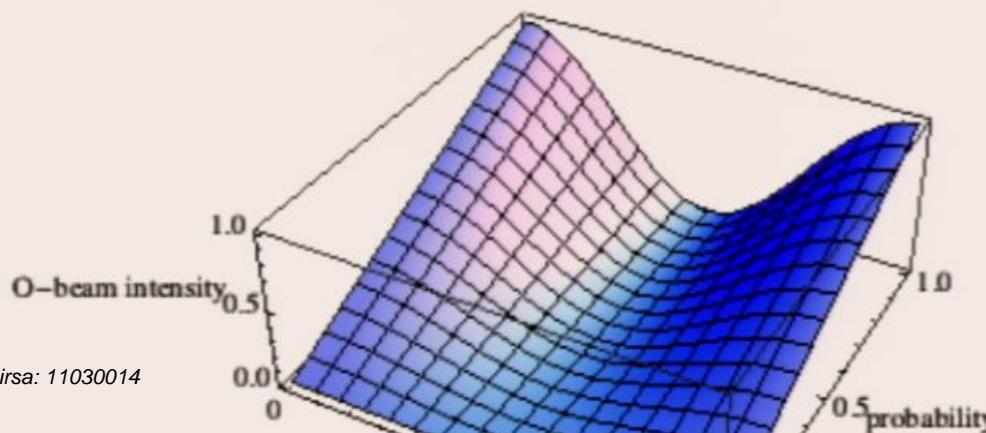
```

res4[p_, a_] := p Ubladeg[π/4].Um.Uphase[a].Ubladeg[π/4].in.
Ubladeginv[π/4].Uphaseinv[a].Uminv.Ubladeginv[π/4] + (1-p) Ezm

M4O[p_, a_] := Tr[Exp . res4[p, a]]

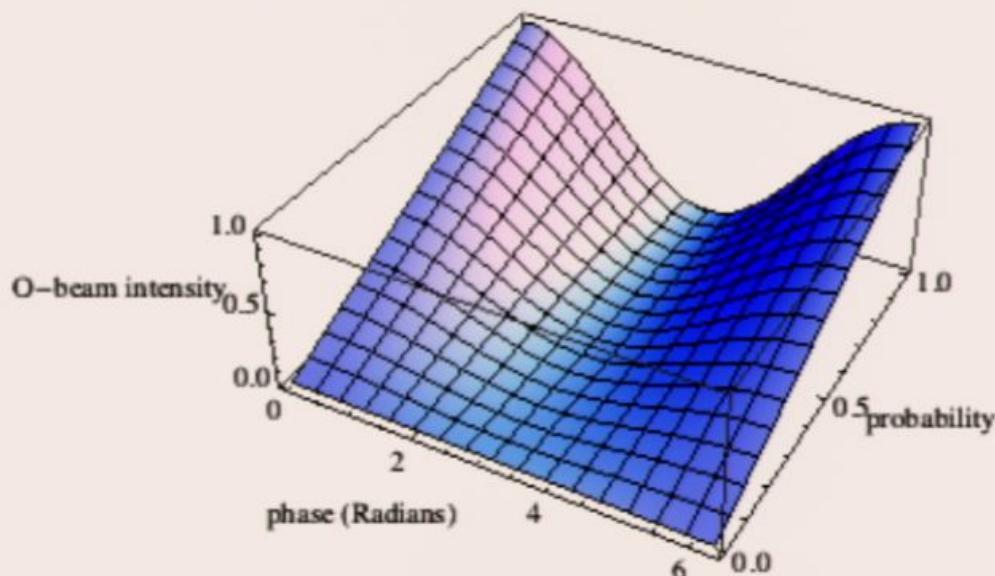
Plot3D[M4O[p, a], {a, 0, 2 π}, {p, 0, 1},
{AxesLabel → {"phase (Radians)", "probability", "O-beam intensity"}}]

```



TWO POSSIBILITIES

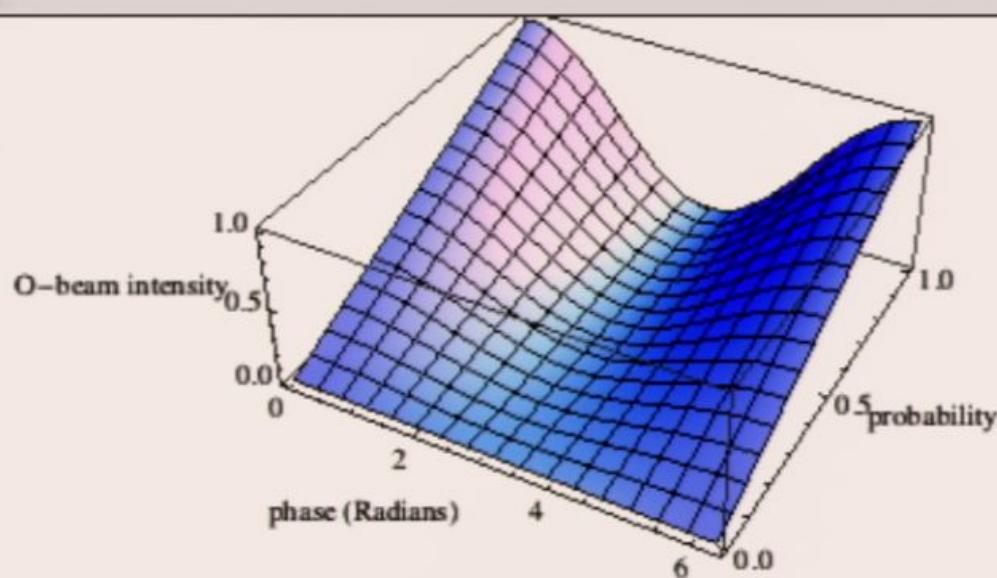
```
res4[p_, a_] := p Ubladeg[\pi/4].Um.Uphase[a].Ubladeg[\pi/4].in.  
Ubladeginv[\pi/4].Uphaseinv[a].Uminv.Ubladeginv[\pi/4] + (1-p) Ezm  
M4O[p_, a_] := Tr[Ezp . res4[p, a]]  
Plot3D[M4O[p, a], {a, 0, 2 \pi}, {p, 0, 1},  
{AxesLabel \rightarrow {"phase (Radians)", "probability", "O-beam intensity"}]}
```



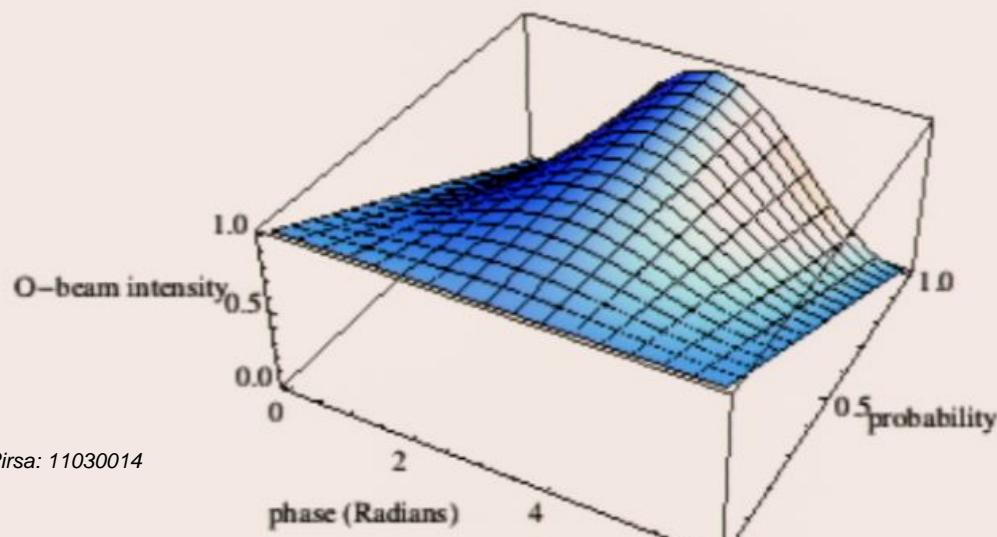
```
M4H[p_, a_] := Tr[Ezm . res4[p, a]]  
Plot3D[M4H[p, a], {a, 0, 2 \pi}, {p, 0, 1},  
{AxesLabel \rightarrow {"phase (Radians)", "probability", "O-beam intensity"}]}
```

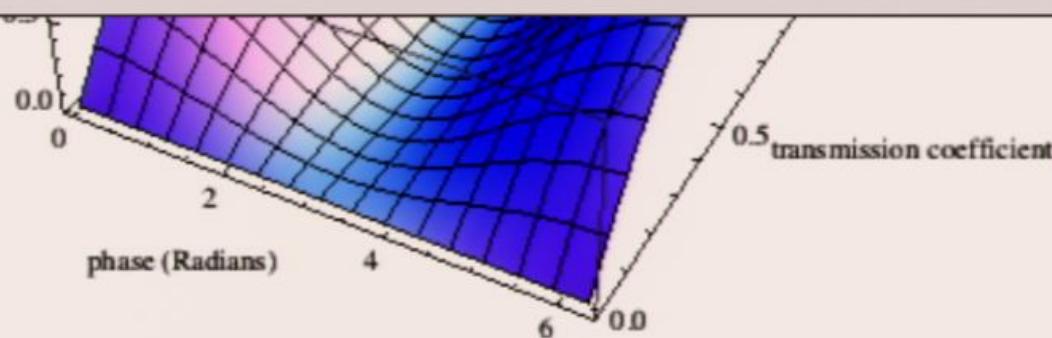
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

QE Lec 3.nb

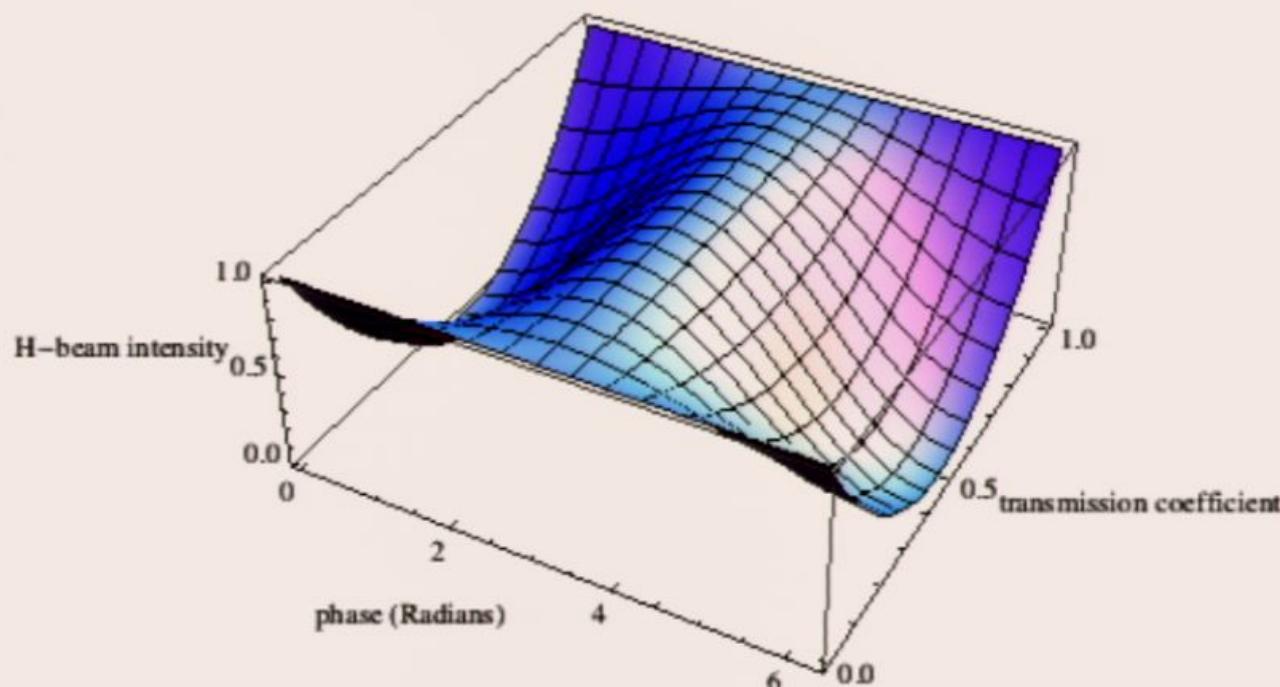


```
M4H[p_, a_] := Tr[Ezm . res4[p, a]]  
Plot3D[M4H[p, a], {a, 0, 2 \pi}, {p, 0, 1},  
{AxesLabel \rightarrow {"phase (Radians)", "probability", "O-beam intensity"}}]
```



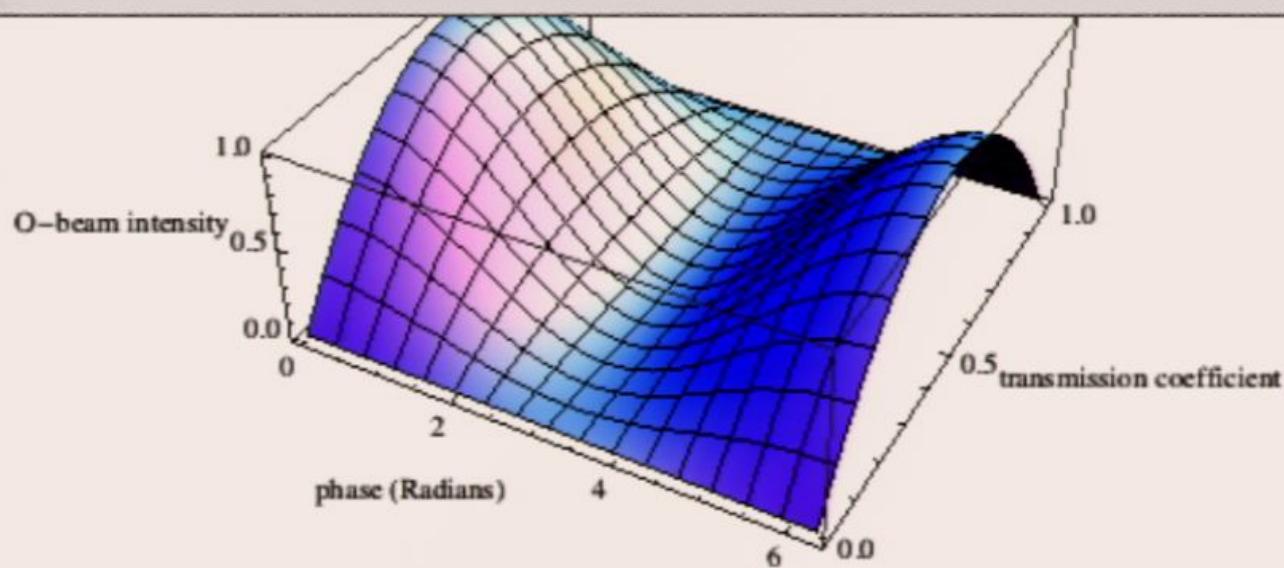


```
M3H[t_, a_] := Tr[Ezm . res3[t, a]]  
Plot3D[M3H[ArcSin[Sqrt[T]], a], {a, 0, 2π}, {T, 0, 1}, {AxesLabel ->  
{"phase (Radians)", "transmission coefficient", "H-beam intensity"}}]
```

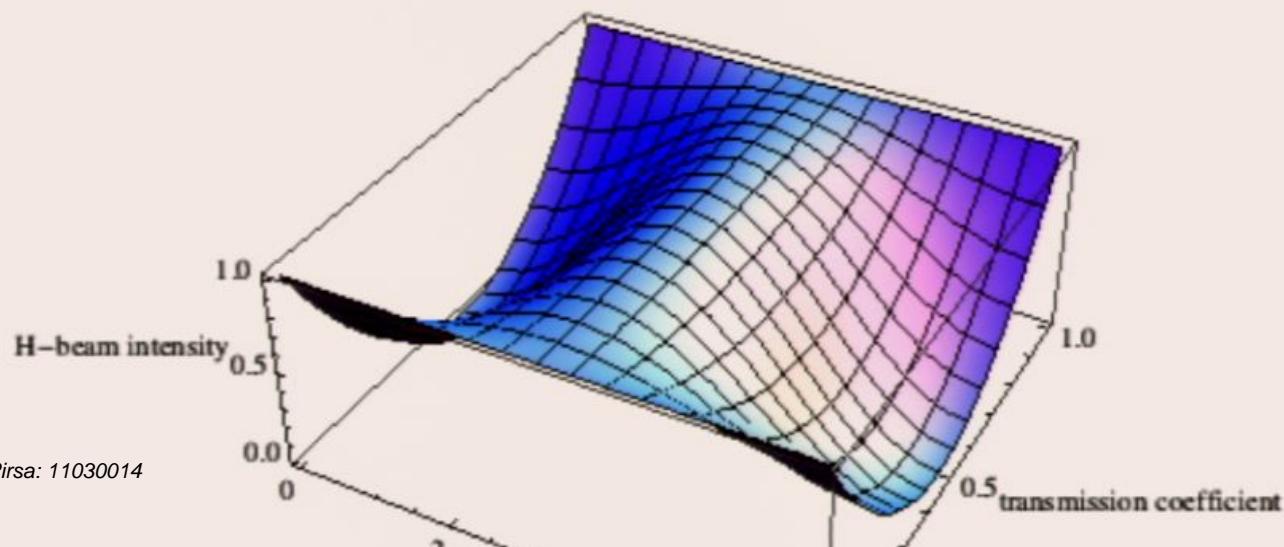


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

QE Lec 3.nb

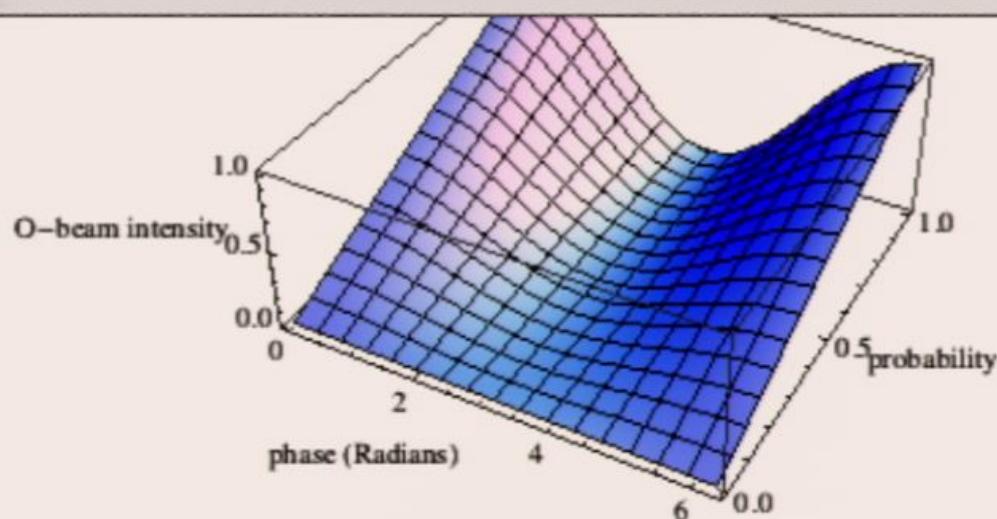


```
M3H[t_, a_] := Tr[Ezm . res3[t, a]]  
Plot3D[M3H[ArcSin[Sqrt[T]], a], {a, 0, 2π}, {T, 0, 1}, {AxesLabel →  
{"phase (Radians)", "transmission coefficient", "H-beam intensity"}}]
```

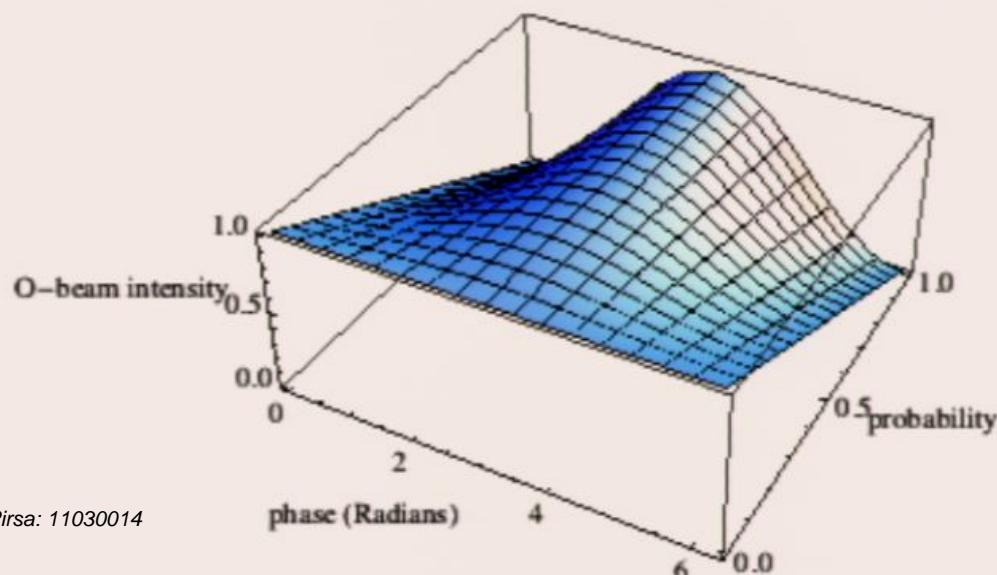


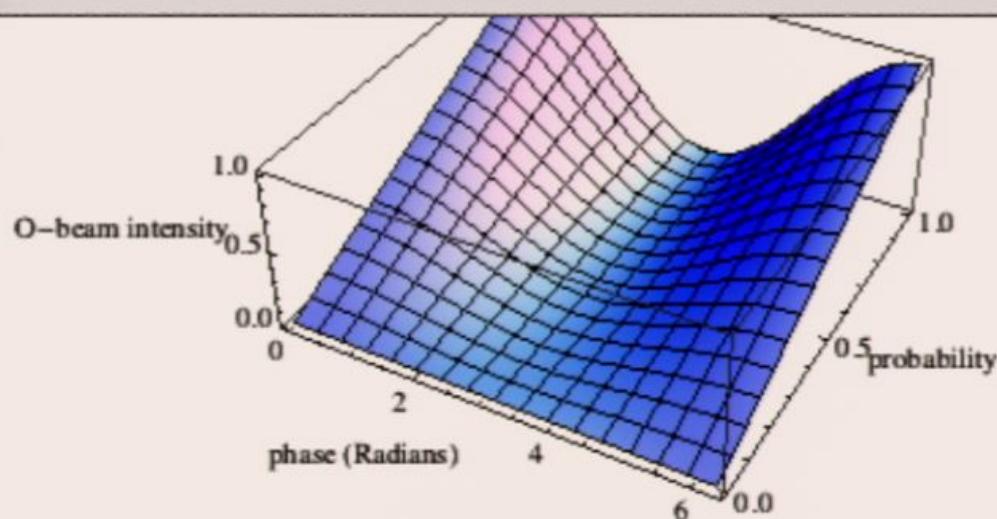
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

QE Lec 3.nb

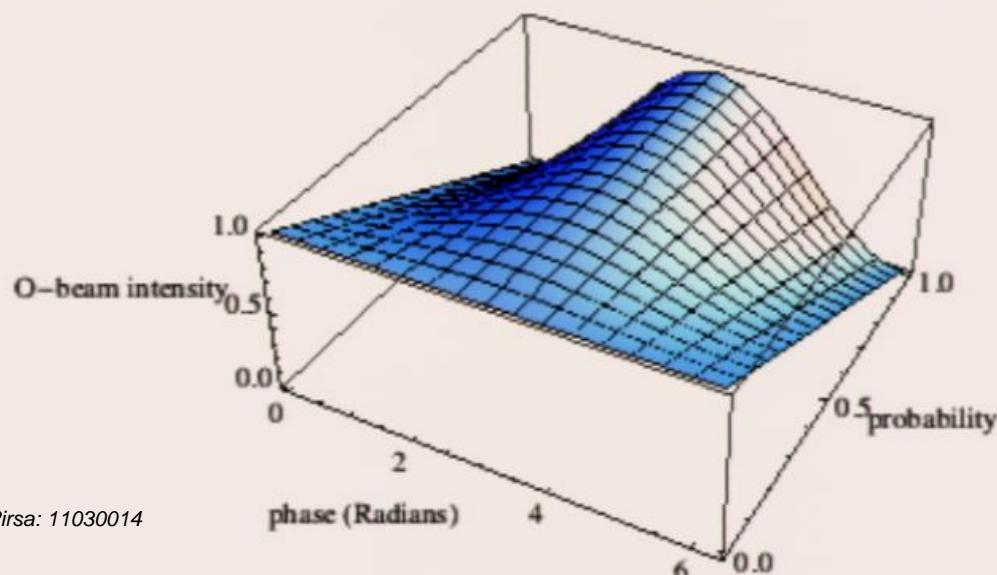


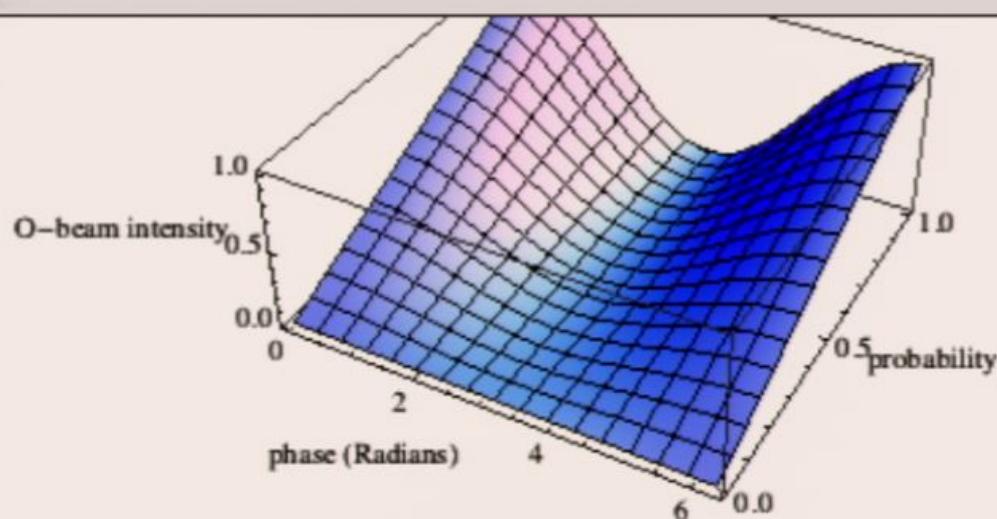
```
M4H[p_, a_] := Tr[Exm . res4[p, a]]  
Plot3D[M4H[p, a], {a, 0, 2 π}, {p, 0, 1},  
{AxesLabel -> {"phase (Radians)", "probability", "O-beam intensity"}}]
```



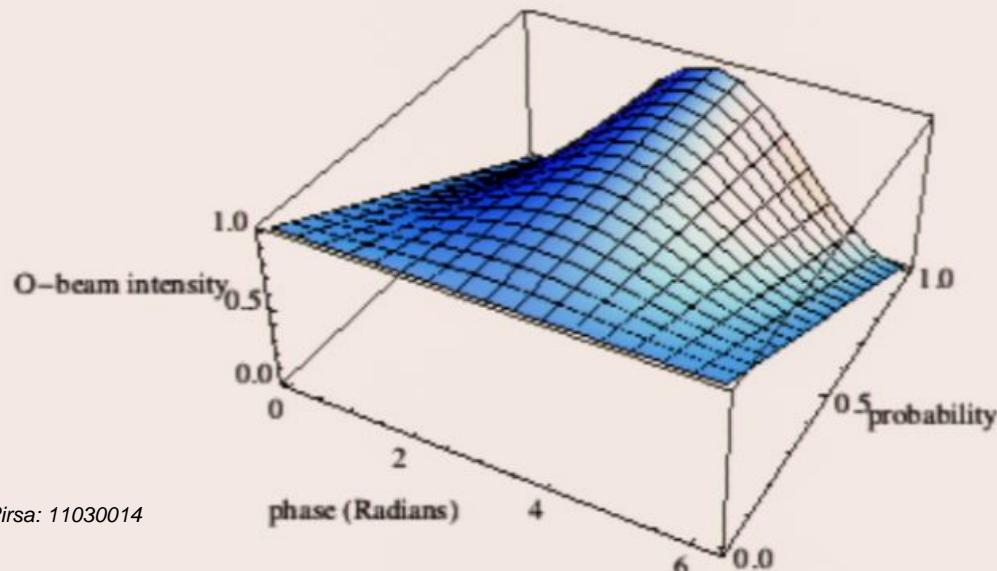


```
M4H[p_, a_] := Tr[Exm . res4[p, a]]  
Plot3D[M4H[p, a], {a, 0, 2 \pi}, {p, 0, 1},  
{AxesLabel \rightarrow {"phase (Radians)", "probability", "O-beam intensity"}}]
```





```
M4H[p_, a_] := Tr[Ecm . res4[p, a]]  
Plot3D[M4H[p, a], {a, 0, 2 \pi}, {p, 0, 1},  
{AxesLabel \rightarrow {"phase (Radians)", "probability", "O-beam intensity"}}]
```



$$S_{out} = P Z$$

$$+ (1-P) Z$$

$$U_{err} =$$



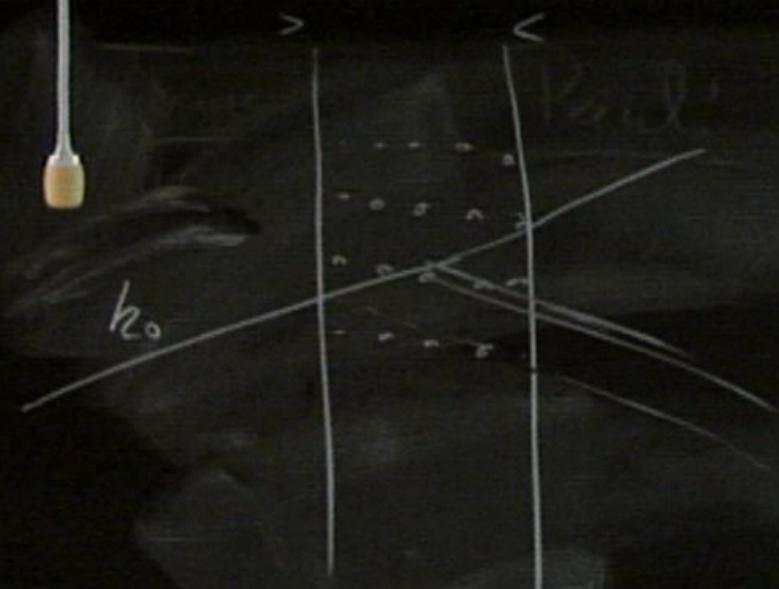


$$S_{out} = P \cdot 2$$

$$+ (1-P) \cdot 2$$

$$\theta_{err} =$$





$$S_{out} = P \cdot 2 + (1-P) \cdot 0$$

$$U_{out} =$$

evidence
for Atoms

Big Is A
Molecule?

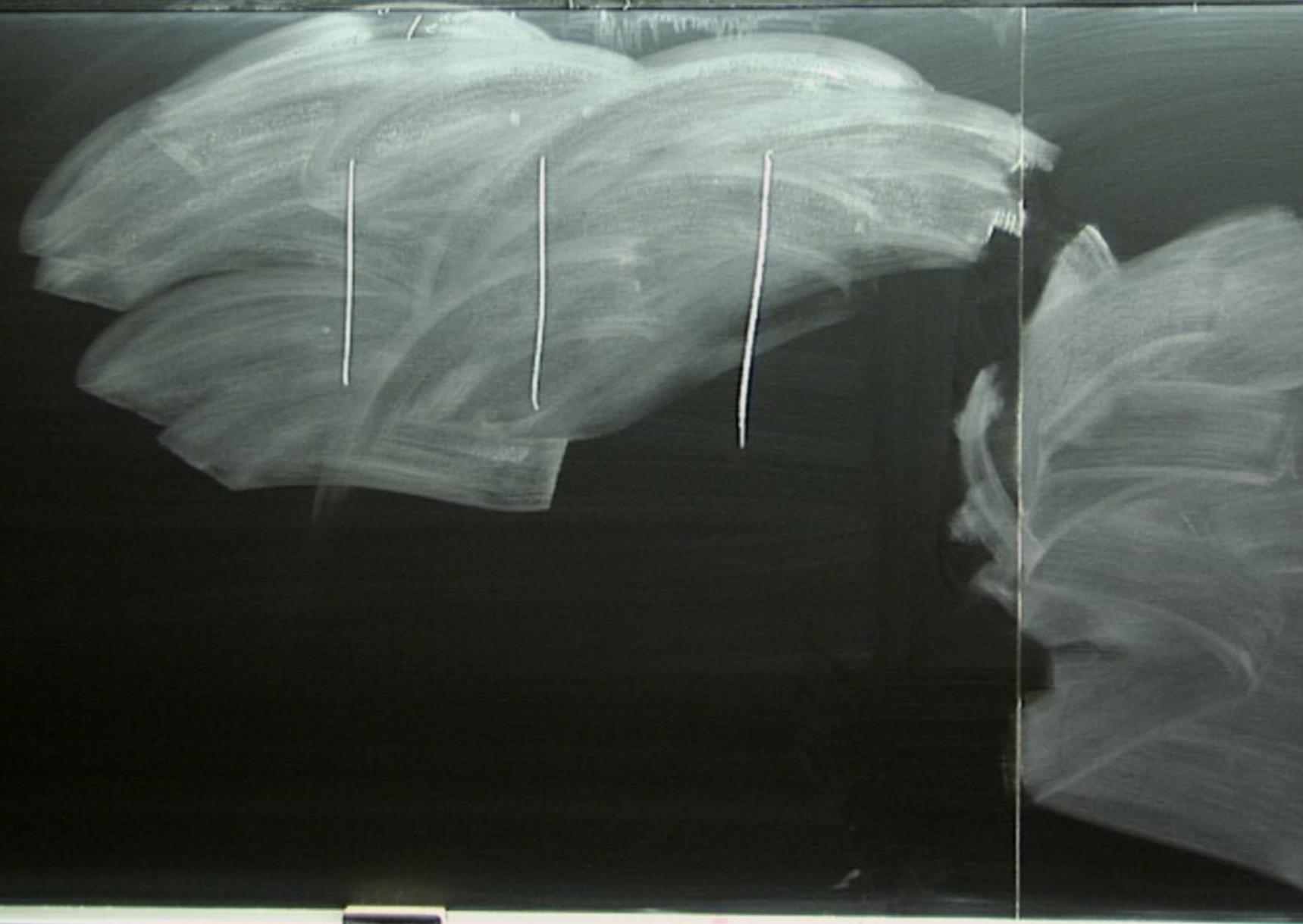
$$S_{\text{out}} = P \bar{U}_{\text{id}} S_{\text{in}} \bar{U}_{\text{id}}^\dagger$$

$$+ (1-P) \bar{U}_{\text{swp}} S_{\text{in}} \bar{U}_{\text{swp}}^\dagger$$

$$\bar{U}_{\text{swp}} = e^{-i \frac{\theta}{2} \sigma_x}$$

U_{swp}

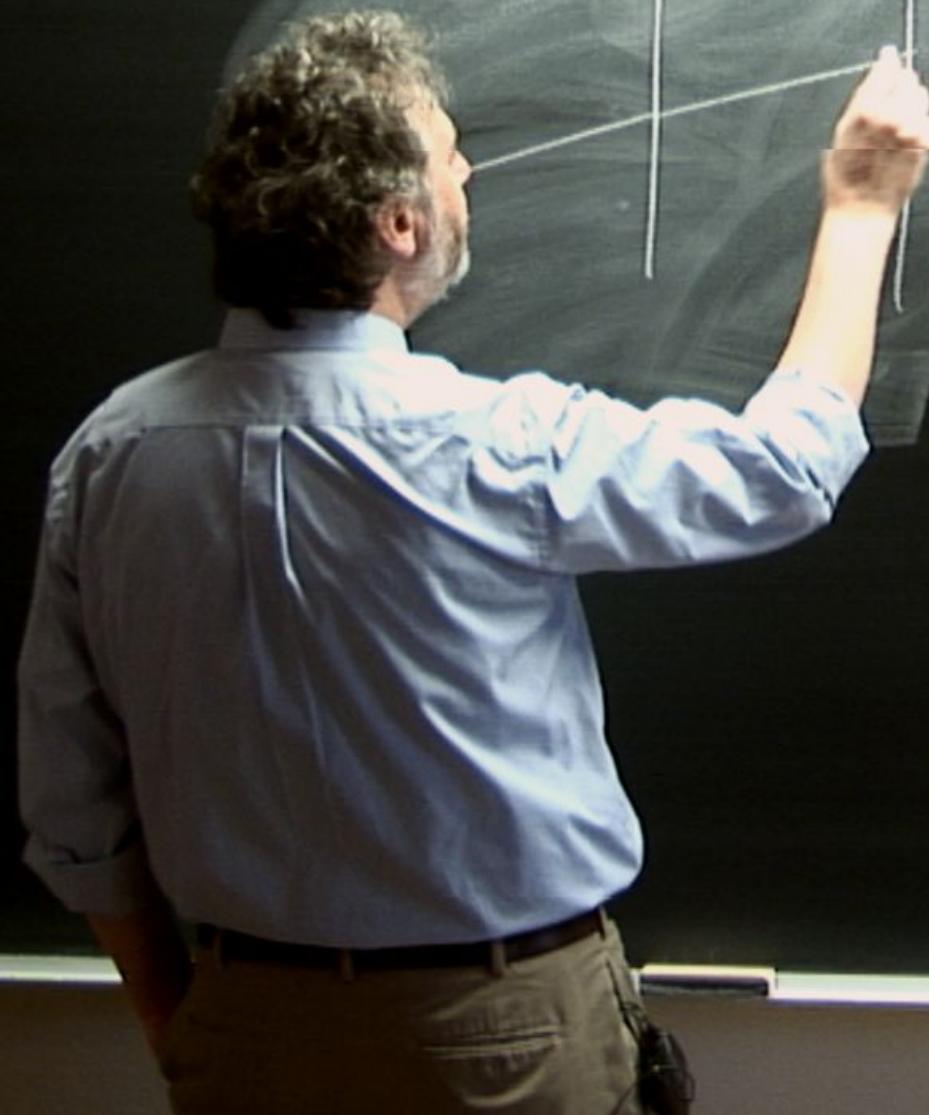




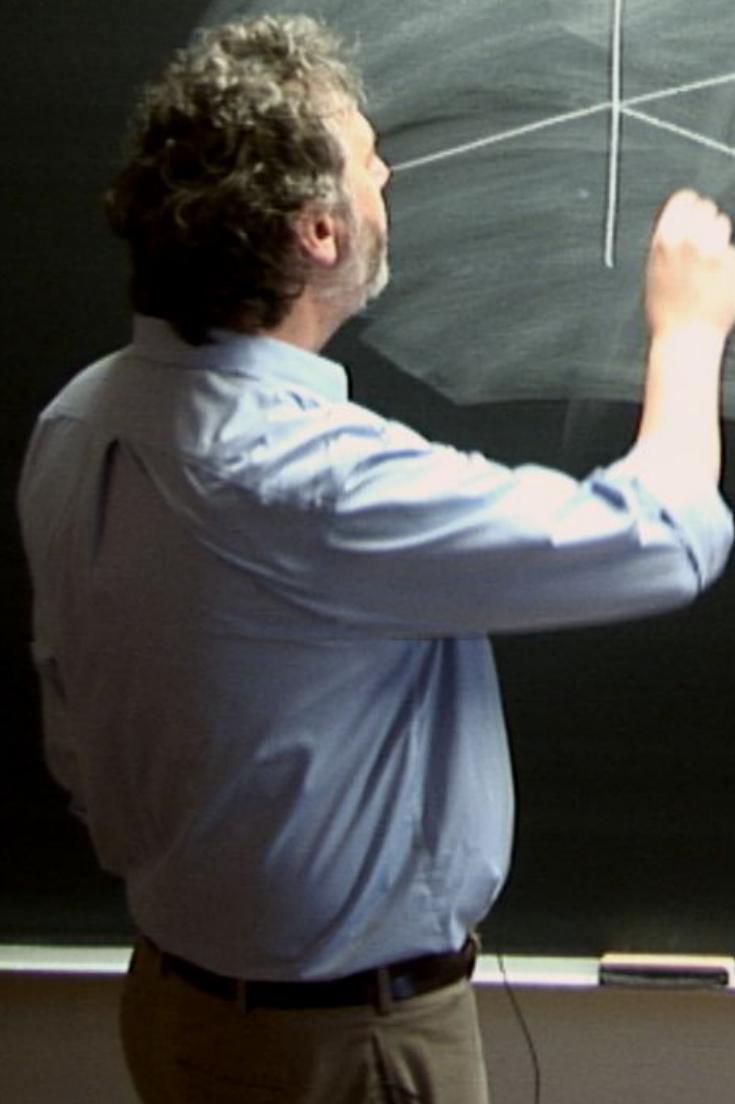
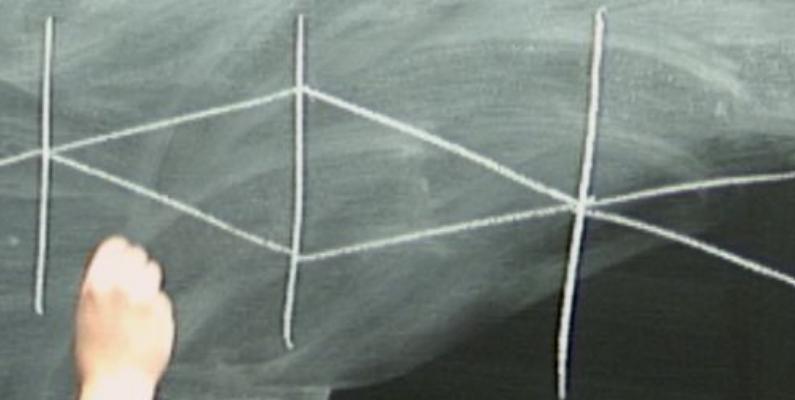
$$V = \gamma B_0 M_e$$



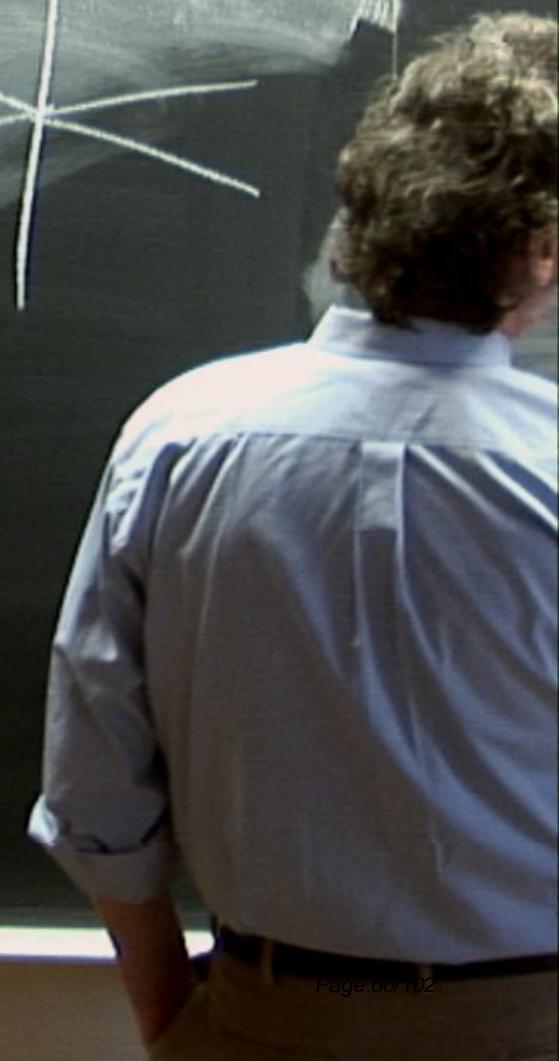
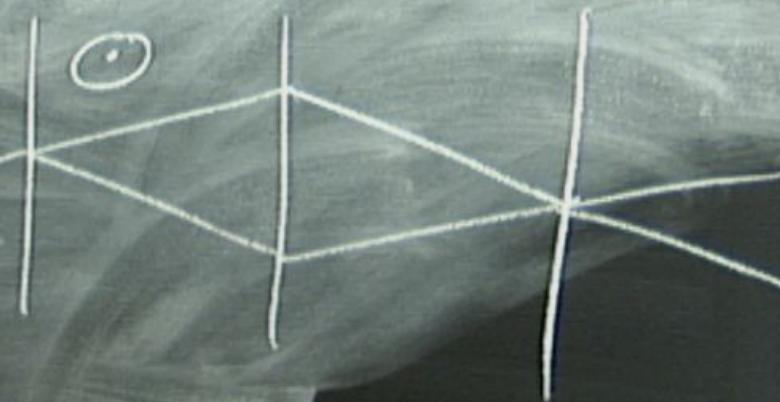
$$V = \gamma B_0 \Omega z$$

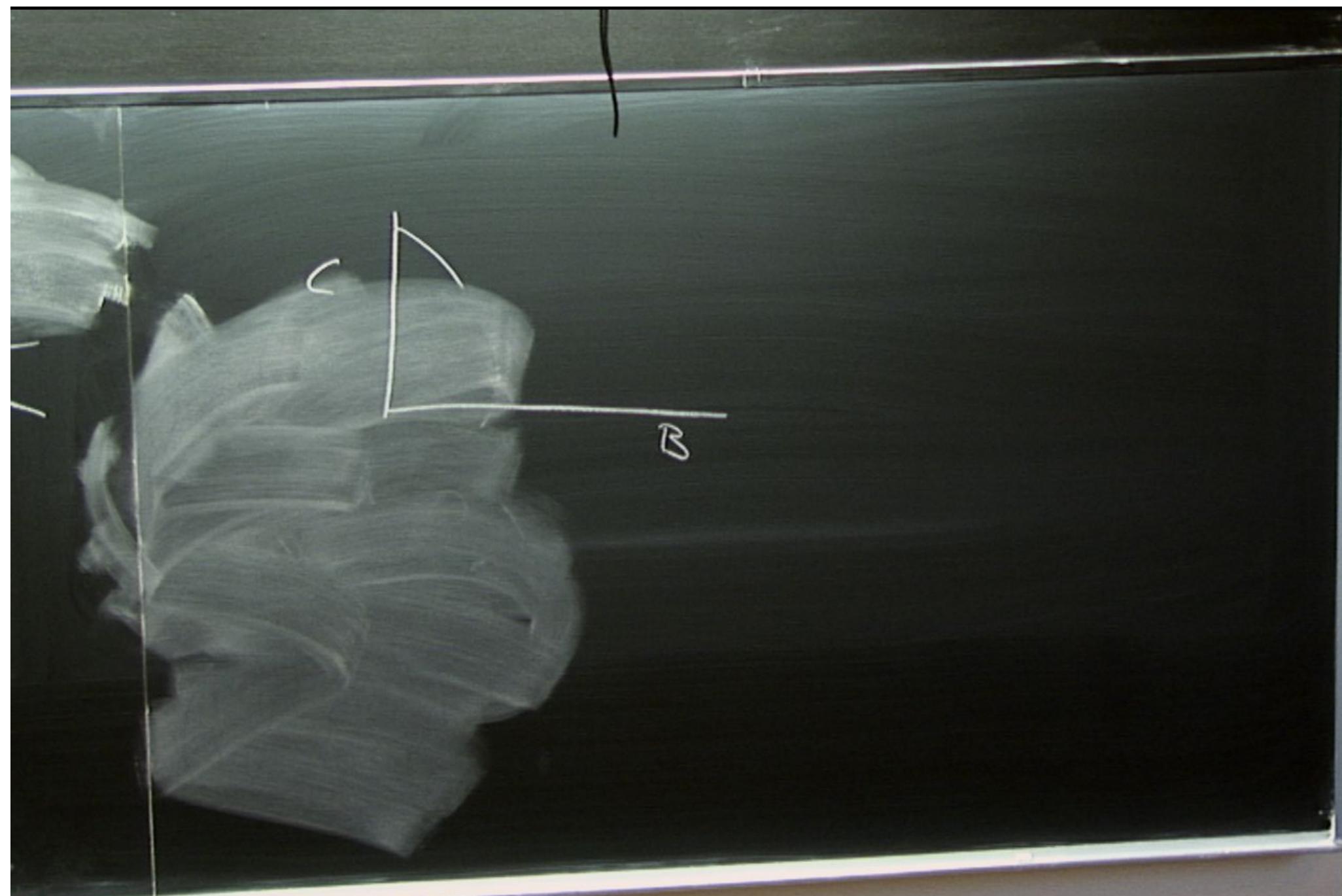


$$V = \gamma B_0 \Omega \epsilon$$



$$V = \gamma B_0 D_x$$

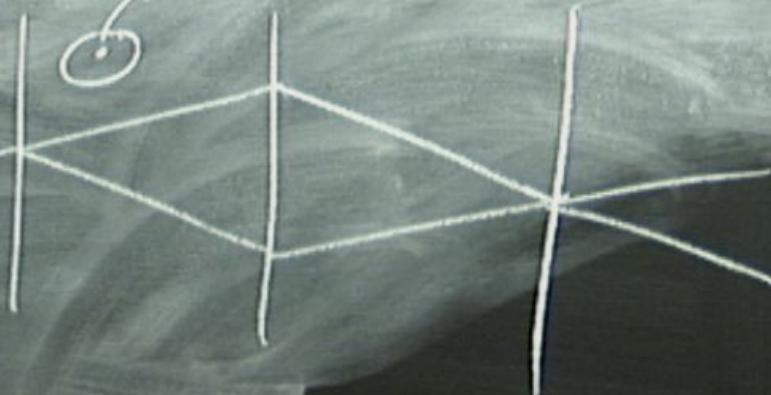




$$V = \gamma B_0 \partial z$$

$B_0 \hat{z}$

$10^3 < 1$

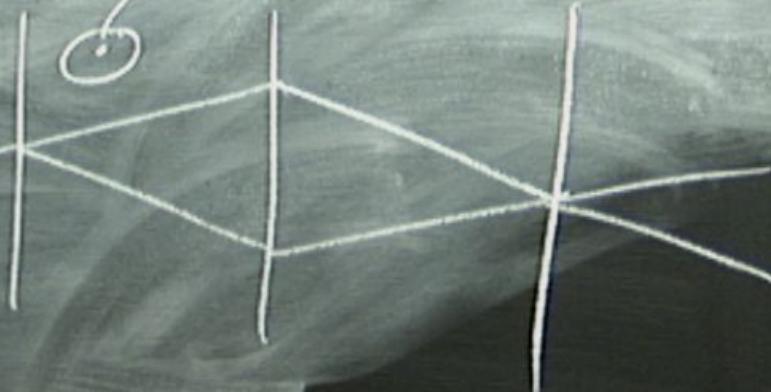


$$V = \gamma B_0 \Delta \hat{z}$$

$B_0 \hat{z}$

$10 > \angle 01$

$10 > \angle 01$



$$V = \gamma B_0 \Omega \hat{z}$$

$B_0 \hat{z}$

$|0\rangle\langle 0|$

$$|0\rangle\langle 0| \cdot (|1\rangle\langle 1| + |1\rangle\langle 1|)$$



$$V = \gamma B_0 D \hat{z}$$

$B_0 \hat{z}$

$|10\rangle \langle 01|$

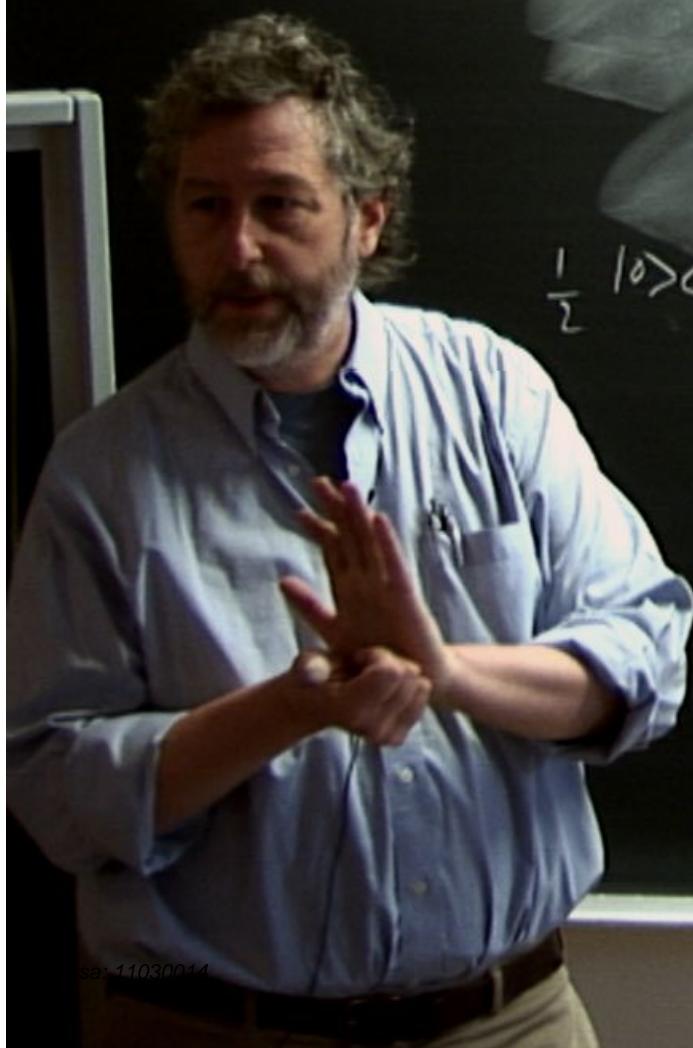
$$\frac{1}{2} |10\rangle \langle 01 | (|11\rangle \langle 11| + |11\rangle \langle 11|)$$

$$V = \gamma B_0 \partial_x$$

$B_0 \hat{z}$

$|10\rangle \langle 01|$

$$\frac{1}{2} |10\rangle \langle 01 | (|11\rangle \langle 11| + |11\rangle \langle 11|)$$



$V = \gamma B_0 \hat{\epsilon}$

Uhrwerk

$|0\rangle\langle 0|$

$\frac{1}{2} |0\rangle\langle 0|(|1\rangle\langle 1| + |1\rangle\langle 1|)$

$$V = \gamma B_0 \Omega \Sigma$$
$$B_0 \hat{z}$$
$$1/2 |10\rangle\langle 01| (|11\rangle\langle 11| + |11\rangle\langle 11|)$$

$V = \gamma B_0 \partial z$

$B_0 \hat{z}$

$|10\rangle \langle 01|$

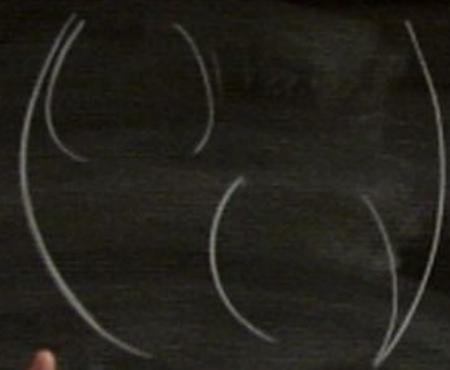
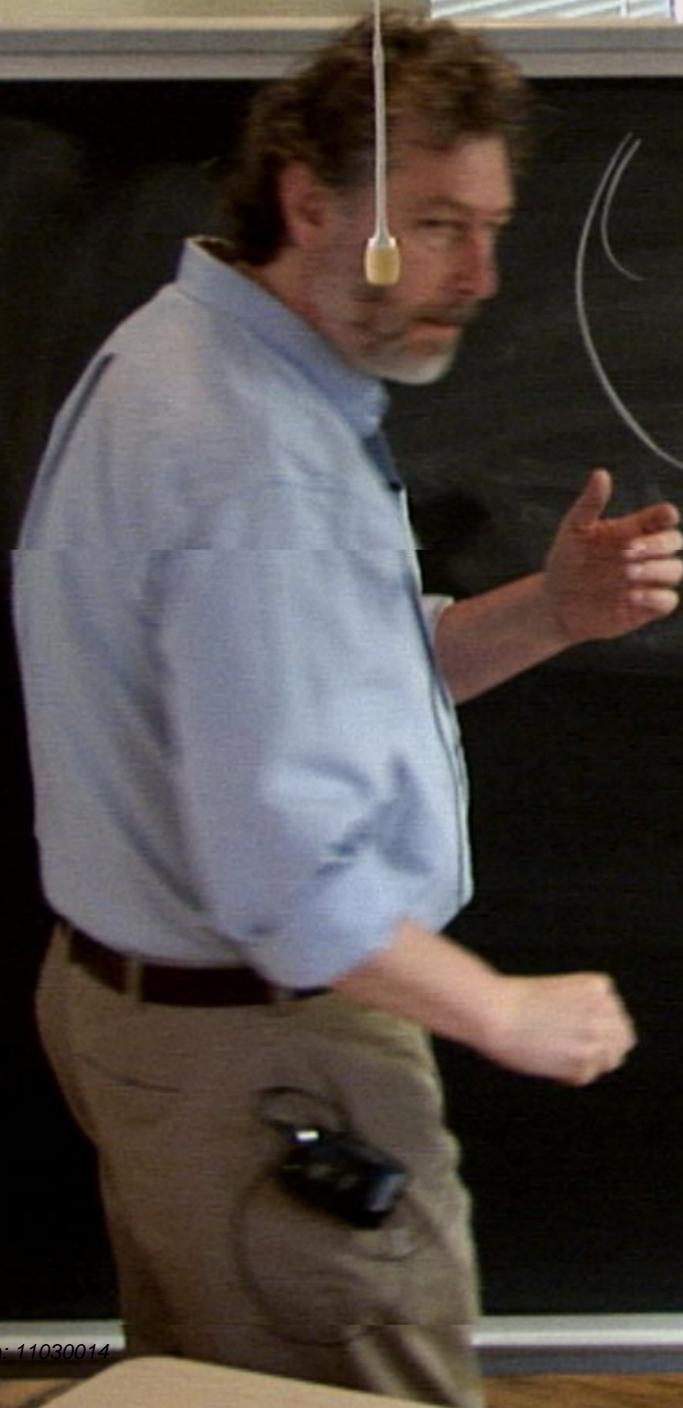
$\frac{1}{2} |10\rangle \langle 01 | (|11\rangle \langle 11| + |11\rangle \langle 11|)$

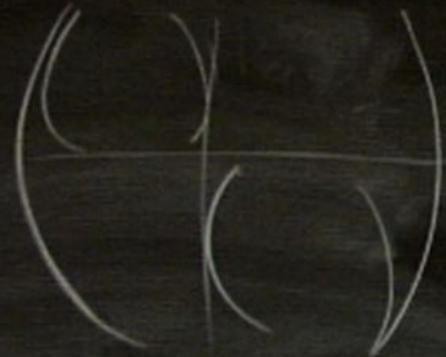
$$S_{out} = P U_{ideal} S_{in} \mathcal{U}$$

$$+ (1-P) U_{real} S_{in} \mathcal{U}$$

$$U_{real} = \tilde{C}^{-1} \frac{\partial}{\partial z} \nabla_x$$

Chap 2



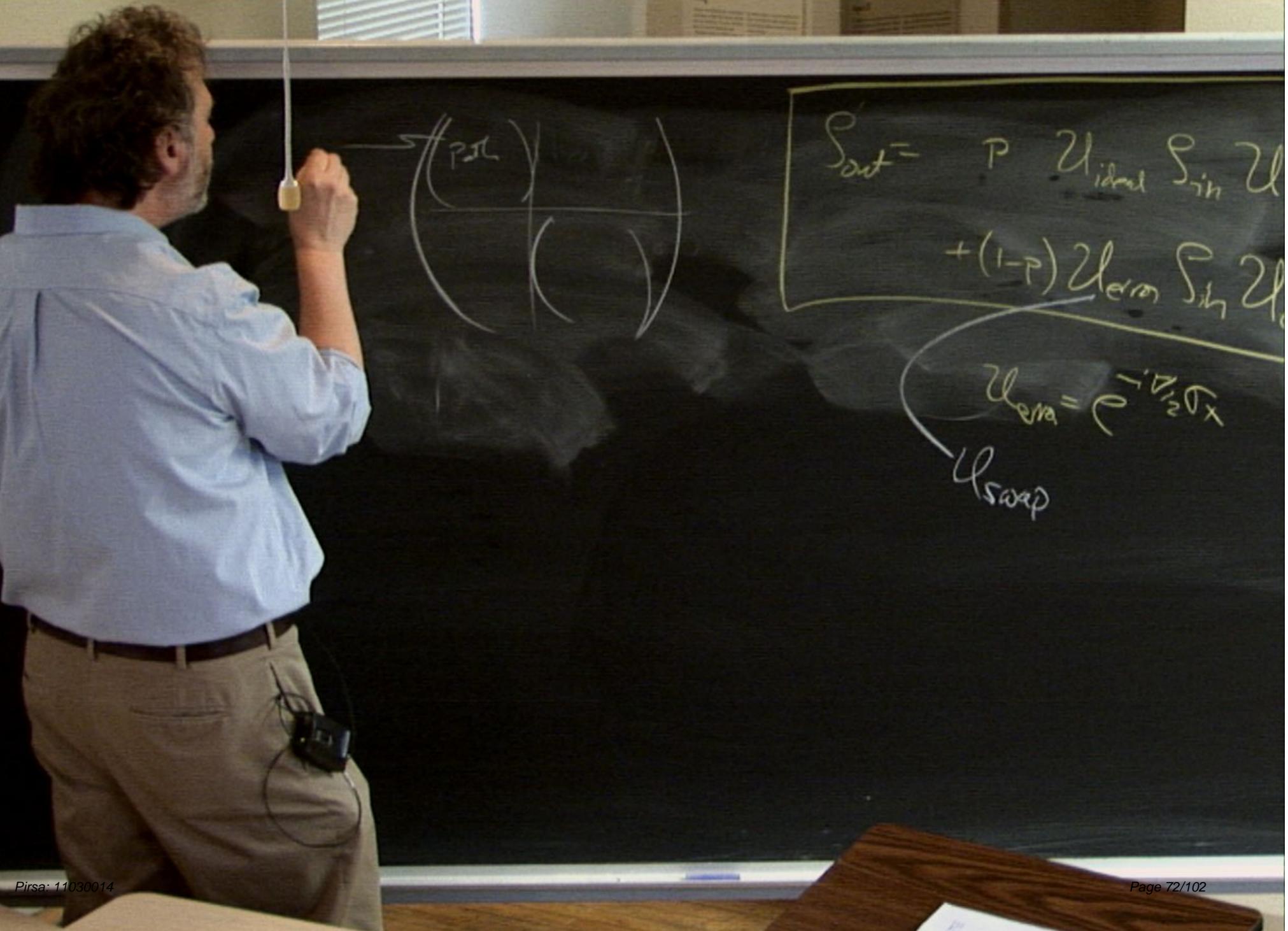


$$S_{\text{out}} = P U_{\text{ideal}} S_{\text{in}} U + (1-P) U_{\text{env}} S_{\text{in}} U$$

$$U_{\text{env}} = e^{-i \vec{\nabla}_z \cdot \vec{r}_x}$$

U_{swap}





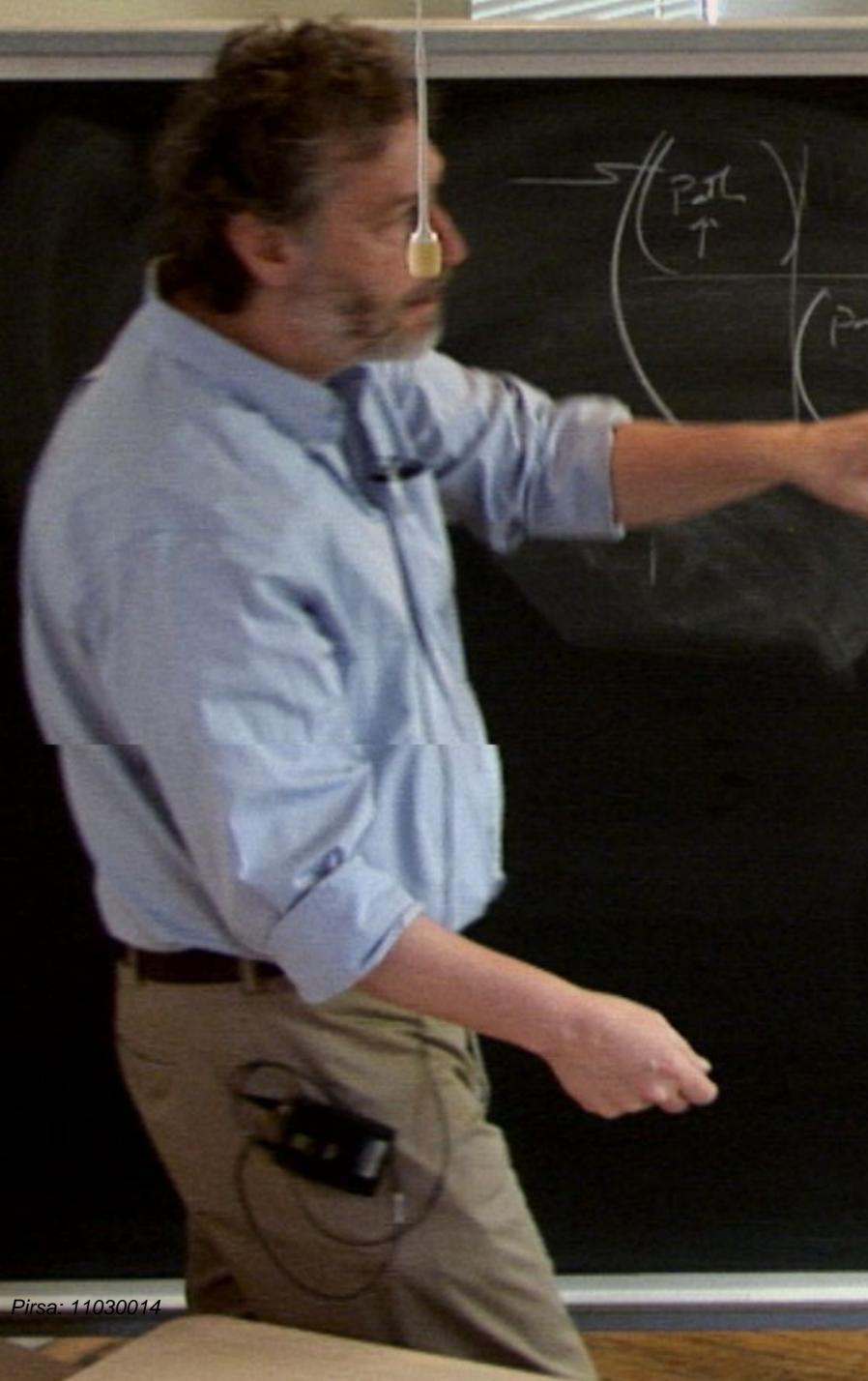
$$\begin{pmatrix} P_{SL} \\ \Gamma \end{pmatrix} \quad \begin{pmatrix} P_{SR} \\ \Gamma \end{pmatrix}$$

$$S_{out} = P U_{ideal} S_{in} U$$

$$+ (1-P) U_{error} S_{in} U$$

$$U_{error} = e^{-j\frac{\pi}{2}\sigma_x}$$

U_{swap}



$$\left(\begin{array}{c} p_{SL} \\ \uparrow \\ p_{SR} \\ \downarrow \end{array} \right)$$

$$S_{out} = P U_{ideal} S_{in} U + (1-P) U_{error} S_{in} U$$

$$U_{error} = e^{-j\frac{\pi}{2}\sigma_x}$$

U_{swap}

Uodos II

$$V = \gamma B_0 \Delta \Sigma$$

$B_0 \hat{z}$

$|10\rangle \langle 01|$

$$\frac{1}{2} |10\rangle \langle 01 | (|11\rangle \langle 11| + |11\rangle \langle 11|)$$

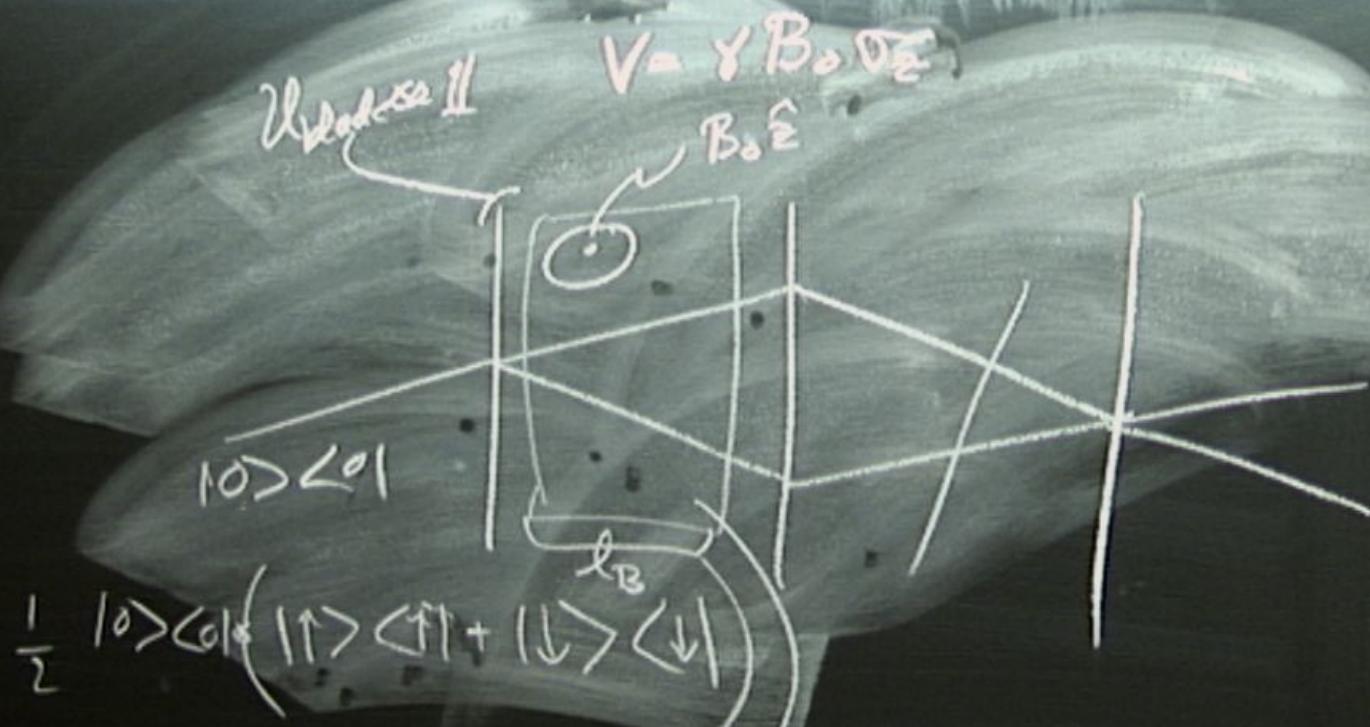
$V = \gamma B_0 \Omega z$

Under \ll

$B_0 \hat{z}$

$|10\rangle \langle 01|$

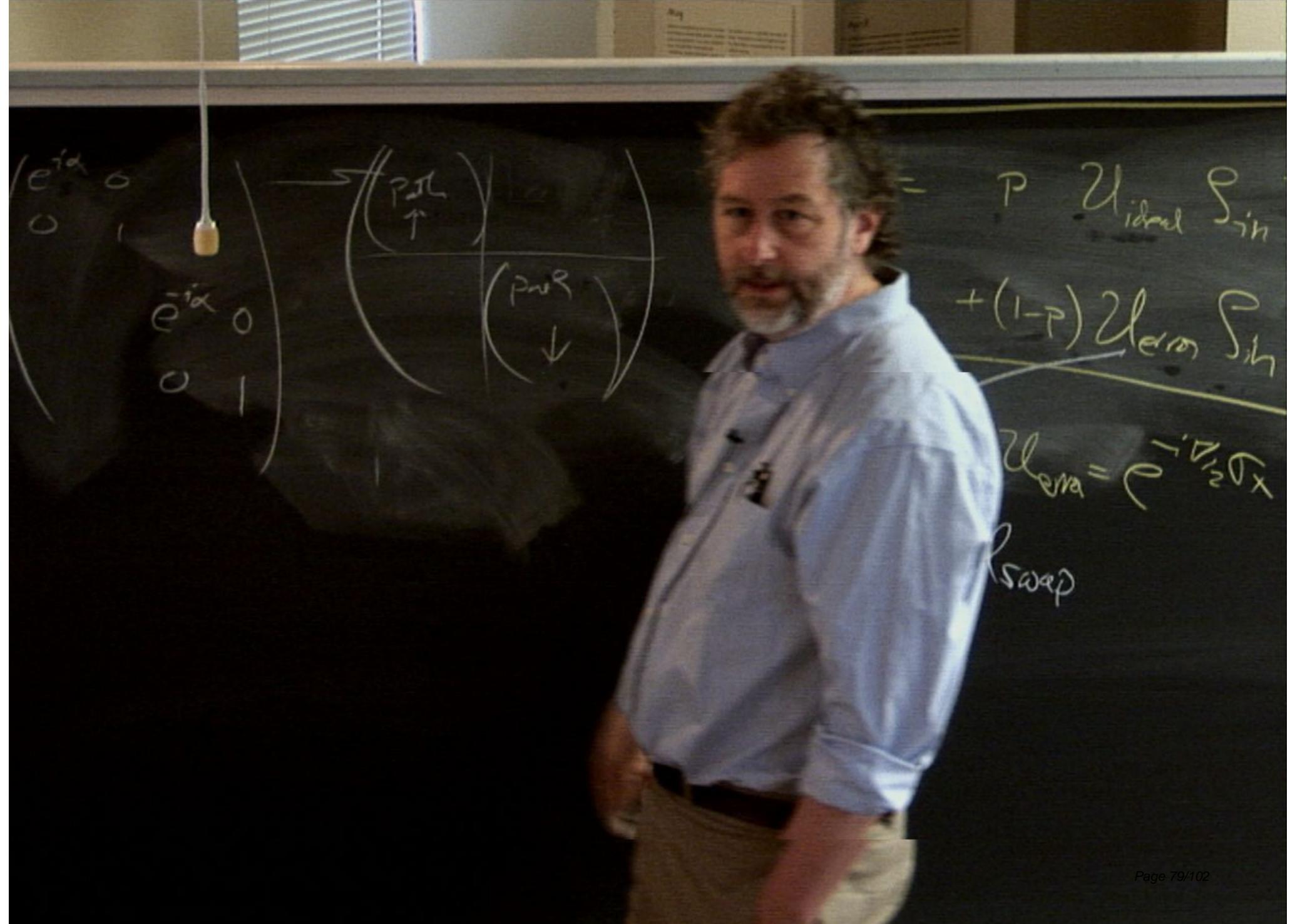
$\frac{1}{2} |10\rangle \langle 01 (|11\rangle \langle 11 + |1\downarrow\rangle \langle 1\downarrow|)$

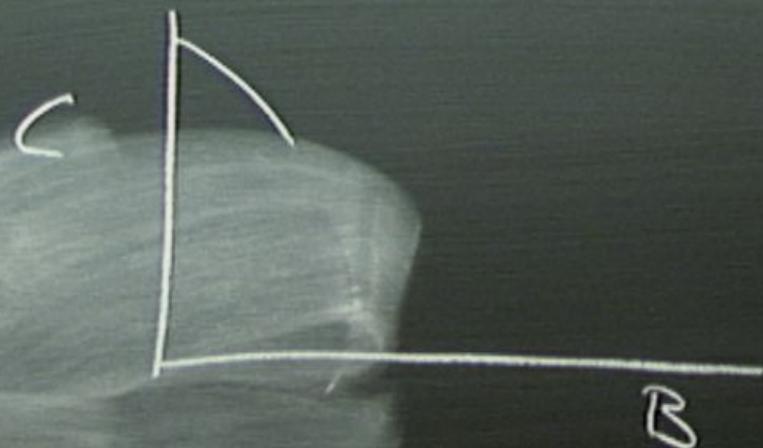
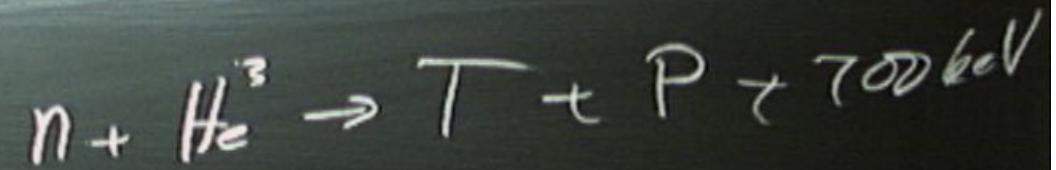


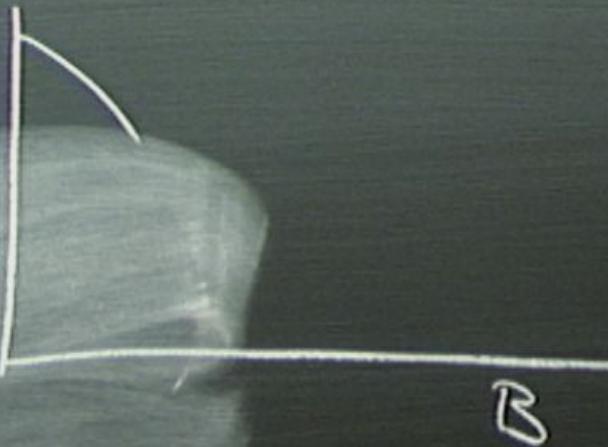
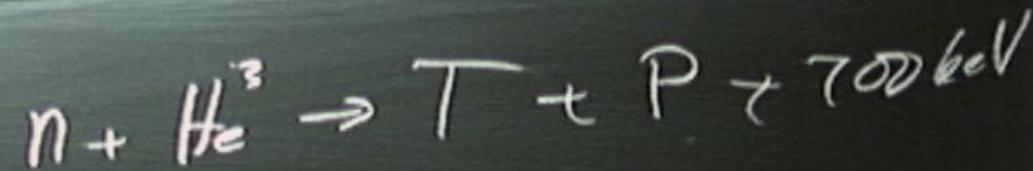
spin dependent
phase

$$\begin{pmatrix} e^{i\alpha} & 0 \\ 0 & 1 \end{pmatrix} \xrightarrow{\quad} \begin{pmatrix} P_{\text{IL}} \\ \uparrow \end{pmatrix} \quad \begin{pmatrix} P_{\text{IR}} \\ \downarrow \end{pmatrix}$$

$$\left[S_{\text{out}} \right]$$







```
L
```

```
M50[a_, b_] := (M50up[a, b] + M50down[a, b])/Sqrt[2]
```

```
Animate[
```

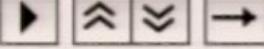
```
  Show[Plot[M50[a, b], {a, 0, 2 \pi}, {AxesLabel \rightarrow {"phase (Radians)", "intensity"}},
```

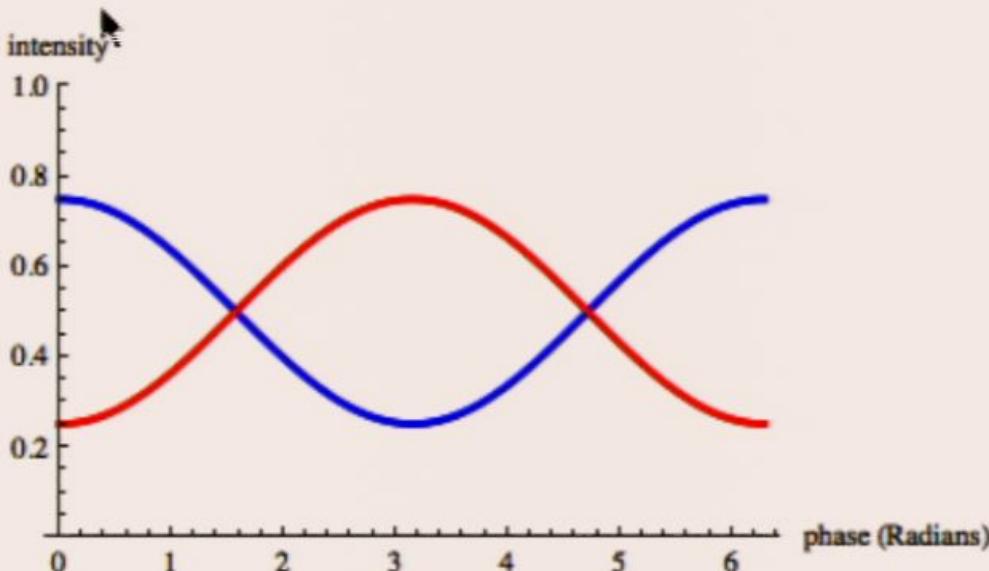
```
    PlotStyle \rightarrow {RGBColor[0, 0, 1], Thickness[0.01]}, PlotRange \rightarrow {0, 1}]],
```

```
  Plot[M5H[a, b], {a, 0, 2 \pi}, {AxesLabel \rightarrow {"phase (Radians)", "intensity"}},
```

```
    PlotStyle \rightarrow {RGBColor[1, 0, 0], Thickness[0.01]}, PlotRange \rightarrow {0, 1}]]],
```

```
  {b, 0, 2 \pi}, AnimationRunning \rightarrow False]
```

b  

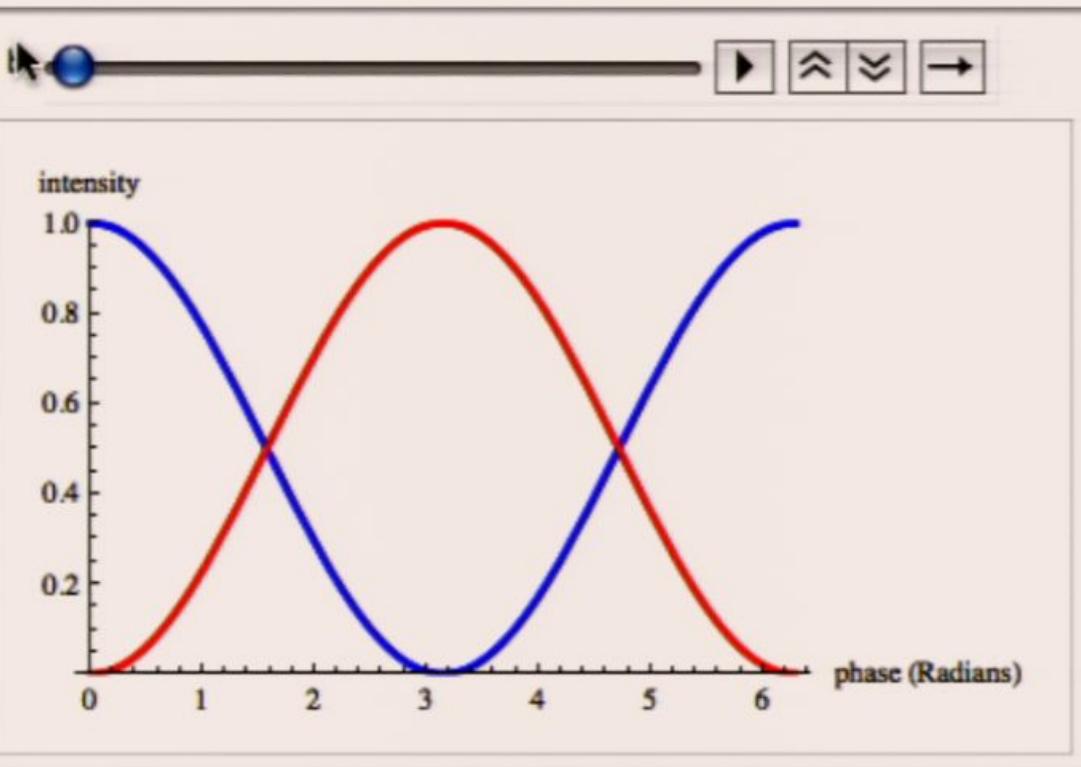


plot what happens to the two spin states then the total picture is easier to see. Of course we can do this experiment by either preparing the neutrons in a particular state or by using a spin polarized detector. Does it matter which state?

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

QE Lec 3.nb

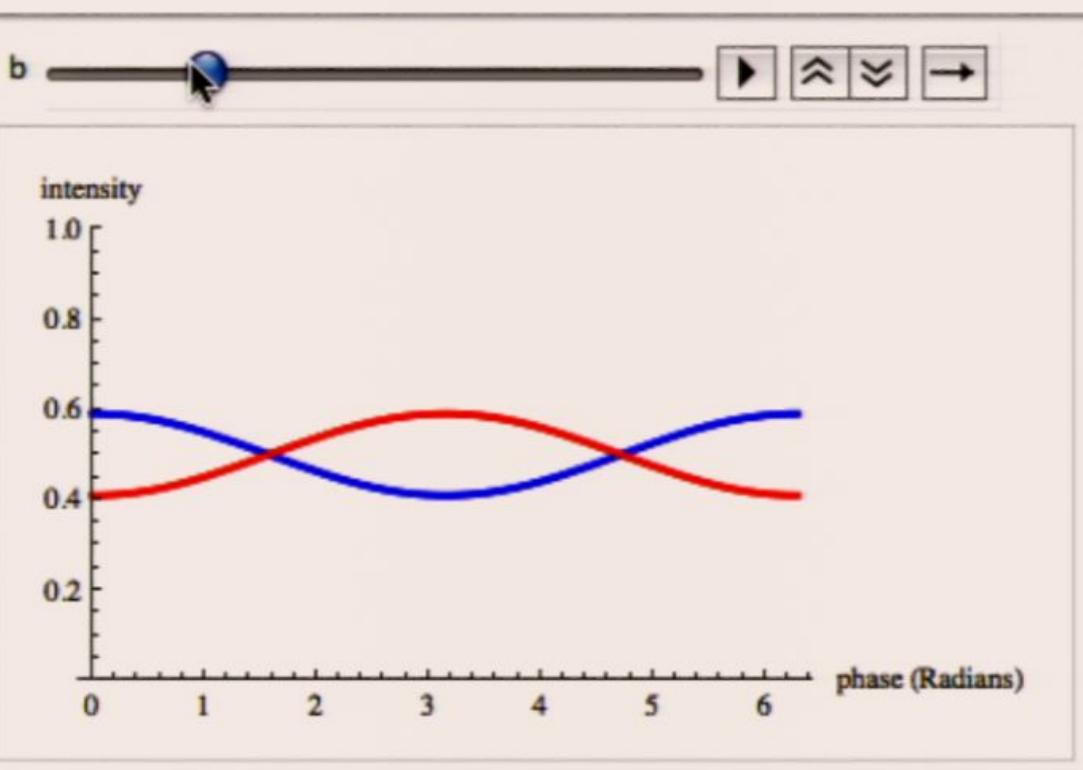
```
M50[a_, b_] := (m50up[a, b] + m50down[a, b])/2
Animate[
 Show[Plot[M50[a, b], {a, 0, 2 \pi}, {AxesLabel \rightarrow {"phase (Radians)", "intensity"}},
 PlotStyle \rightarrow {RGBColor[0, 0, 1], Thickness[0.01]}, PlotRange \rightarrow {0, 1}]],
 Plot[M5H[a, b], {a, 0, 2 \pi}, {AxesLabel \rightarrow {"phase (Radians)", "intensity"}},
 PlotStyle \rightarrow {RGBColor[1, 0, 0], Thickness[0.01]}, PlotRange \rightarrow {0, 1}]],
 {b, 0, 2 \pi}, AnimationRunning \rightarrow False]
```



plot what happens to the two spin states then the total picture is easier to see. Of course we can do this experiment by either preparing the neutrons in a particular state or by using a spin polarized detector. Does it matter which state?

```
M5O[a_, b_] := (m5up[a, b] + m5down[a, b])/2

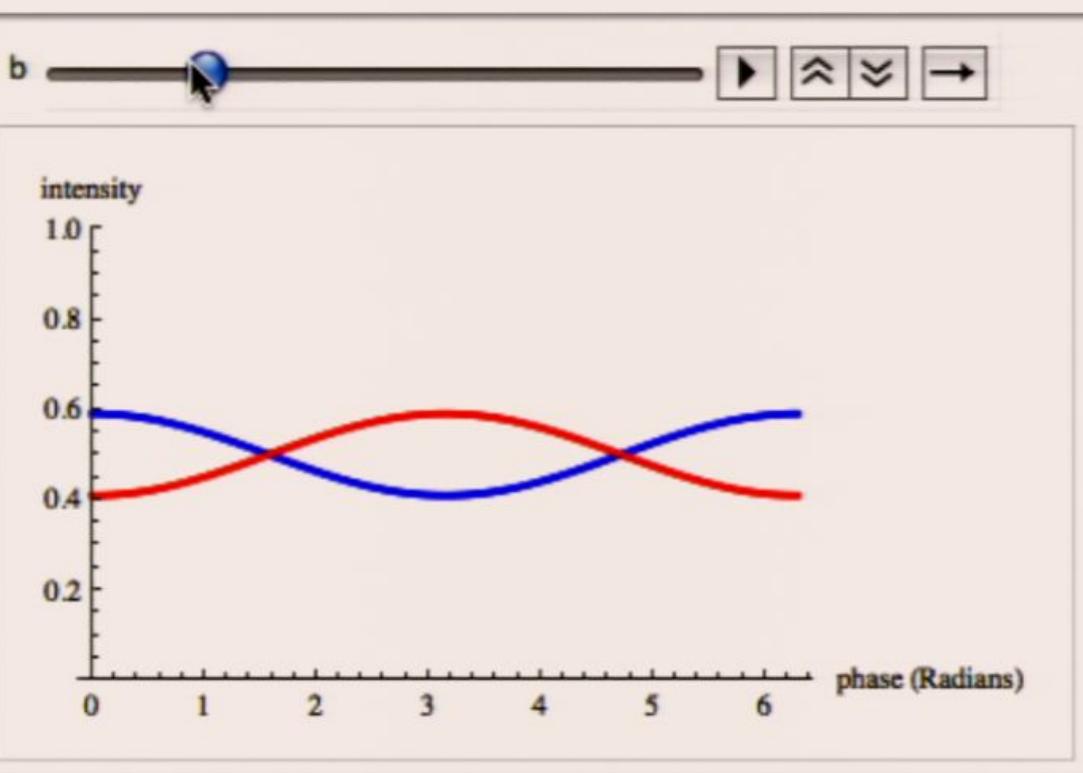
Animate[
 Show[Plot[M5O[a, b], {a, 0, 2 π}, {AxesLabel → {"phase (Radians)", "intensity"}},
 PlotStyle → {RGBColor[0, 0, 1], Thickness[0.01]}, PlotRange → {0, 1}],
 Plot[M5H[a, b], {a, 0, 2 π}, {AxesLabel → {"phase (Radians)", "intensity"}},
 PlotStyle → {RGBColor[1, 0, 0], Thickness[0.01]}, PlotRange → {0, 1}]],
 {b, 0, 2 π}, AnimationRunning → False]
```



plot what happens to the two spin states then the total picture is easier to see. Of course we can do this experiment by either preparing the neutrons in a particular state or by using a spin polarized detector. Does it matter which state?

```
M50[a_, b_] := (m50up[a, b] + m50down[a, b])/2

Animate[
 Show[Plot[M50[a, b], {a, 0, 2 π}, {AxesLabel → {"phase (Radians)", "intensity"}},
 PlotStyle → {RGBColor[0, 0, 1], Thickness[0.01]}, PlotRange → {0, 1}]],
 Plot[M5H[a, b], {a, 0, 2 π}, {AxesLabel → {"phase (Radians)", "intensity"}},
 PlotStyle → {RGBColor[1, 0, 0], Thickness[0.01]}, PlotRange → {0, 1}]],
 {b, 0, 2 π}, AnimationRunning → False]
```



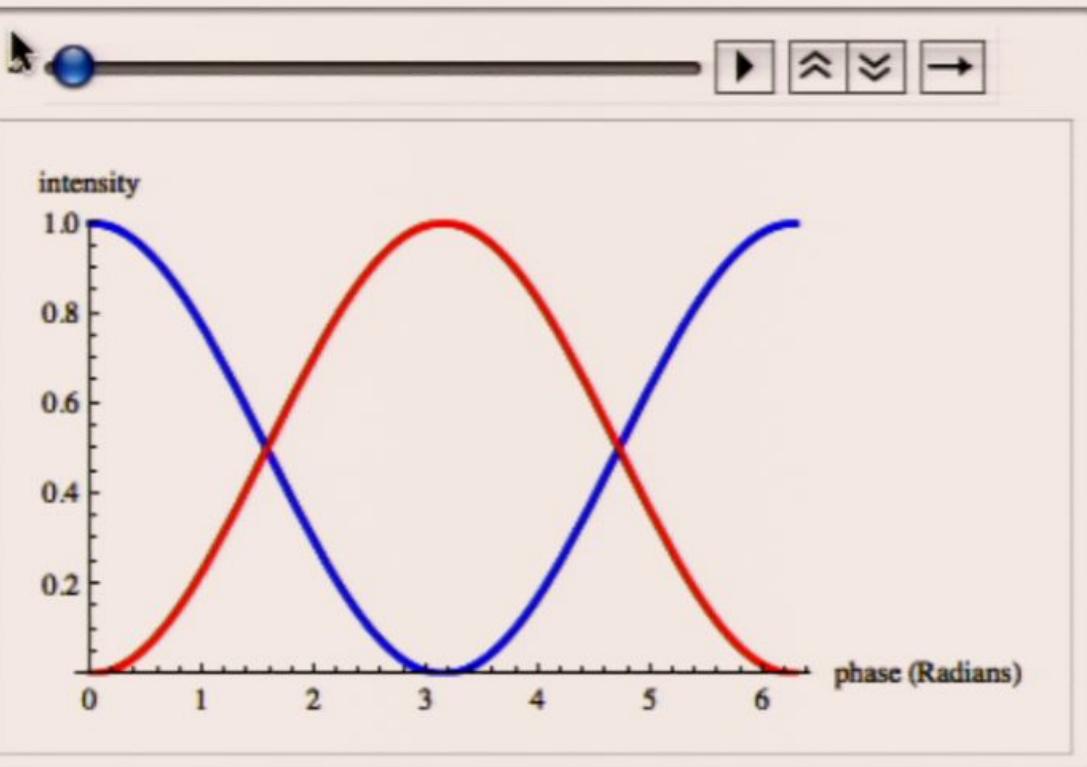
plot what happens to the two spin states then the total picture is easier to see. Of course we can do this experiment by either preparing the neutrons in a particular state or by using a spin polarized detector. Does it matter which state?

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

QE Lec 3.nb

```
M50[a_, b_] := (m50up[a, b] + m50down[a, b])/2

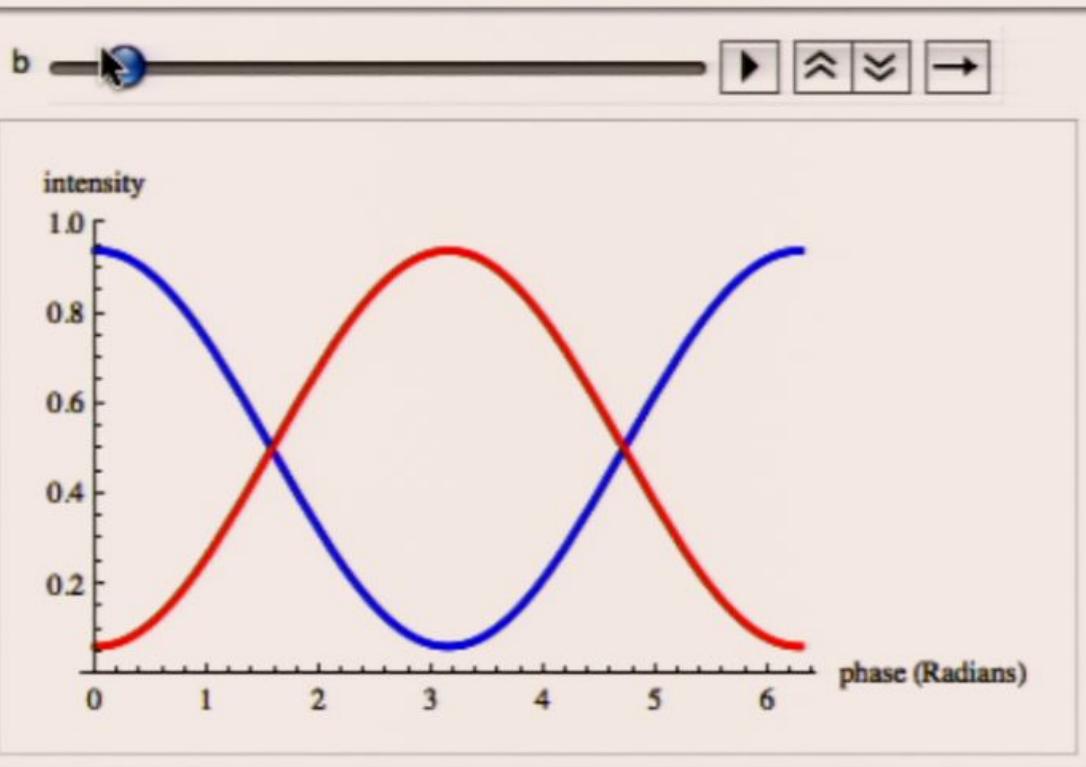
Animate[
 Show[Plot[M50[a, b], {a, 0, 2 \pi}, {AxesLabel \rightarrow {"phase (Radians)", "intensity"}},
 PlotStyle \rightarrow {RGBColor[0, 0, 1], Thickness[0.01]}, PlotRange \rightarrow {0, 1}]],
 Plot[M5H[a, b], {a, 0, 2 \pi}, {AxesLabel \rightarrow {"phase (Radians)", "intensity"}},
 PlotStyle \rightarrow {RGBColor[1, 0, 0], Thickness[0.01]}, PlotRange \rightarrow {0, 1}]],
 {b, 0, 2 \pi}, AnimationRunning \rightarrow False]
```



plot what happens to the two spin states then the total picture is easier to see. Of course we can do this experiment by either preparing the neutrons in a particular state or by using a spin polarized detector. Does it matter which state?

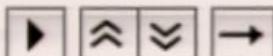
```
M50[a_, b_] := (m50up[a, b] + m50down[a, b])/2

Animate[
 Show[Plot[M50[a, b], {a, 0, 2 π}, {AxesLabel → {"phase (Radians)", "intensity"}},
 PlotStyle → {RGBColor[0, 0, 1], Thickness[0.01]}, PlotRange → {0, 1}],
 Plot[M5H[a, b], {a, 0, 2 π}, {AxesLabel → {"phase (Radians)", "intensity"}},
 PlotStyle → {RGBColor[1, 0, 0], Thickness[0.01]}, PlotRange → {0, 1}]],
 {b, 0, 2 π}, AnimationRunning → False]
```

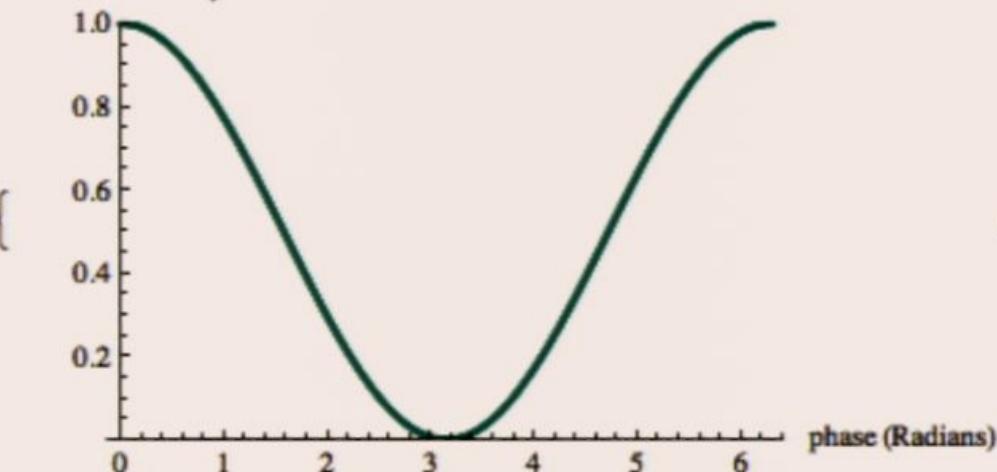


plot what happens to the two spin states then the total picture is easier to see. Of course we can do this experiment by either preparing the neutrons in a particular state or by using a spin polarized detector. Does it matter which state?

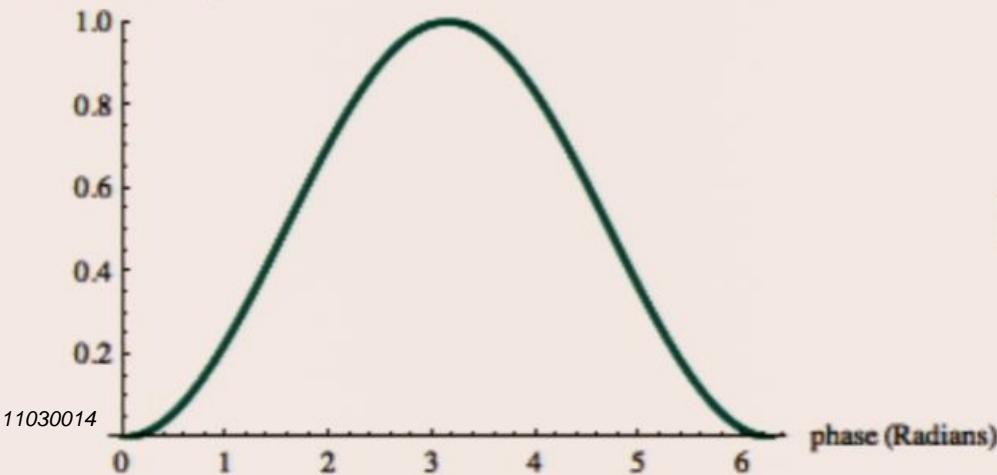
b



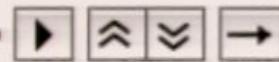
O-beam intensity



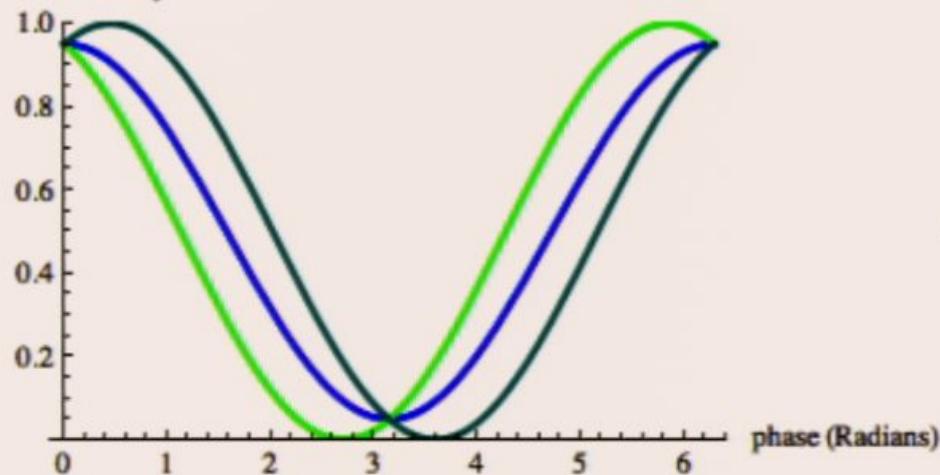
H-beam intensity



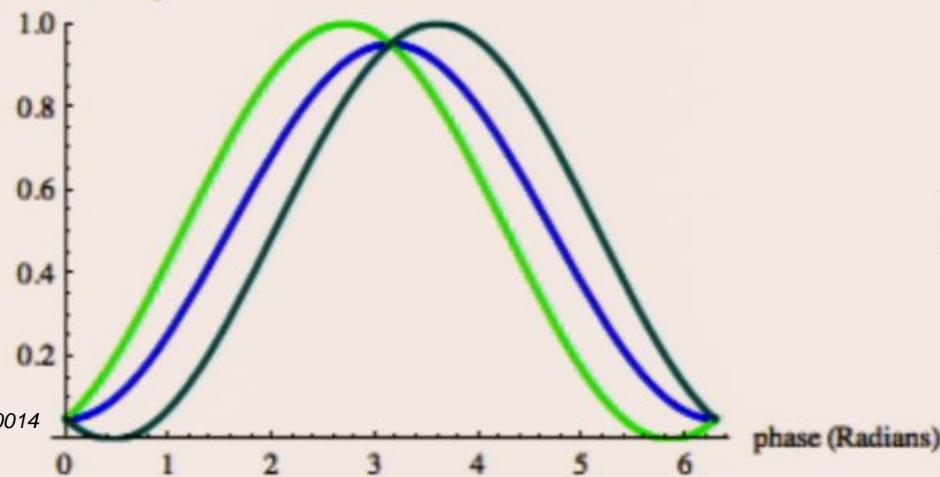
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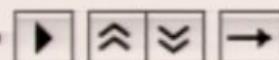
O-beam intensity



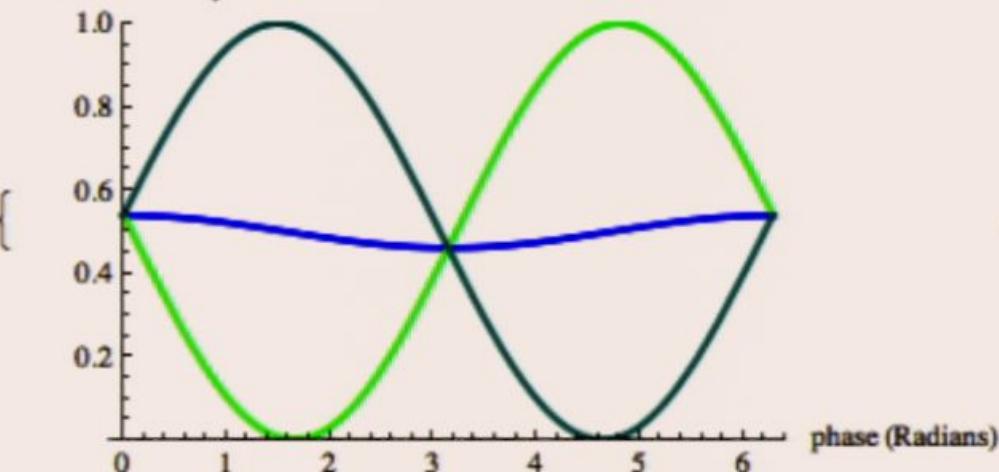
H-beam intensity



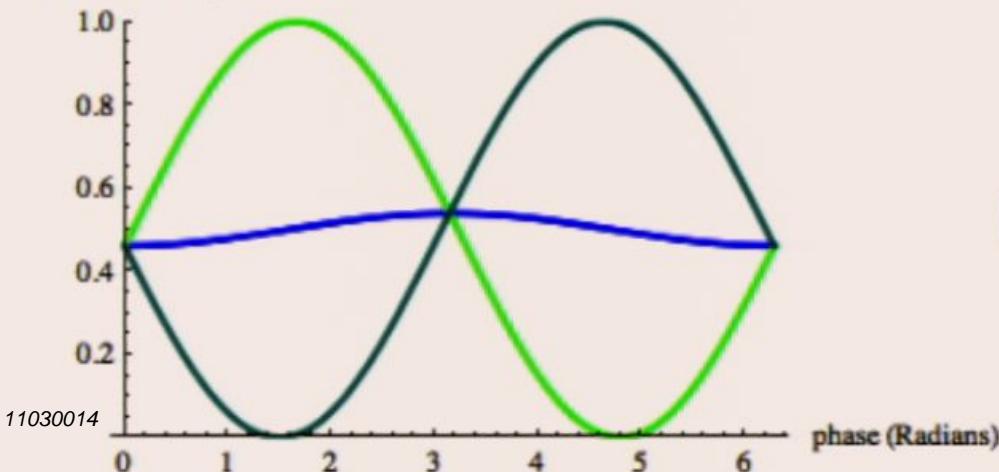
b



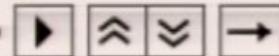
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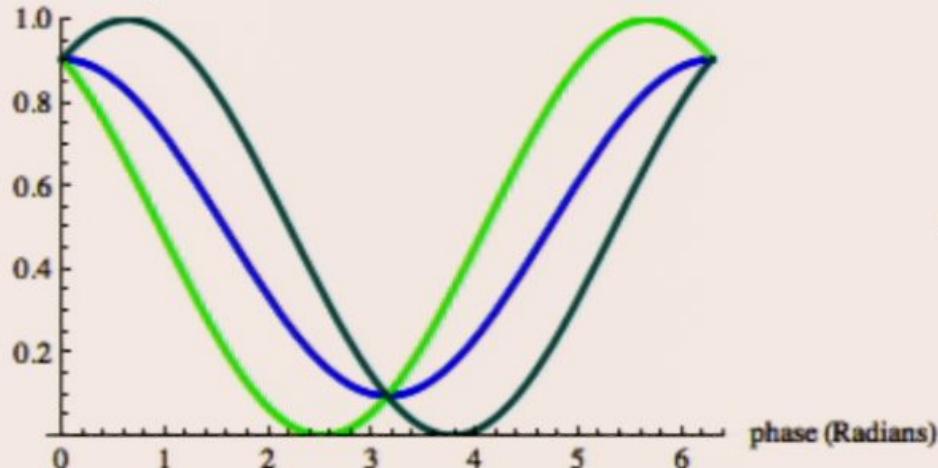
H-beam intensity



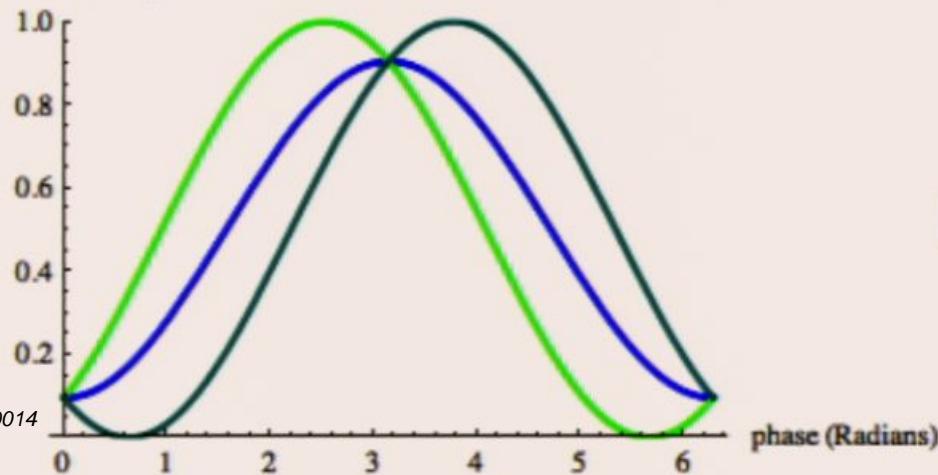
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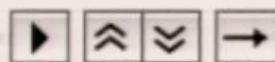
O-beam intensity



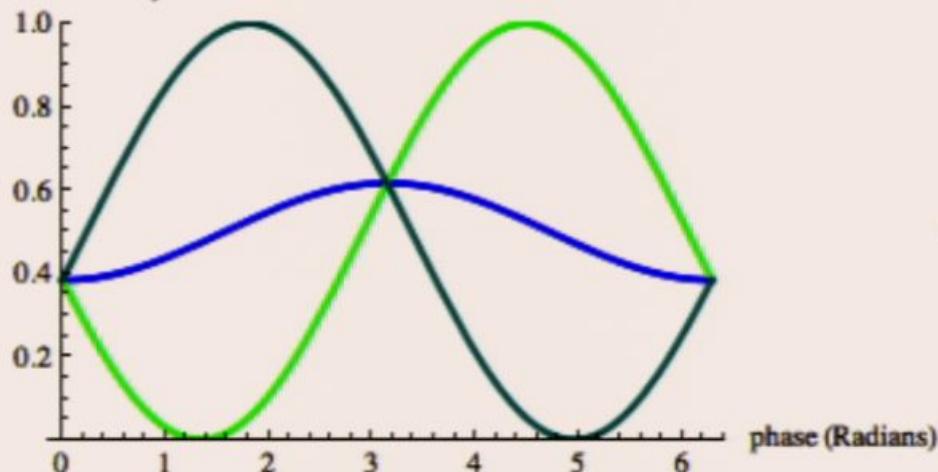
H-beam intensity



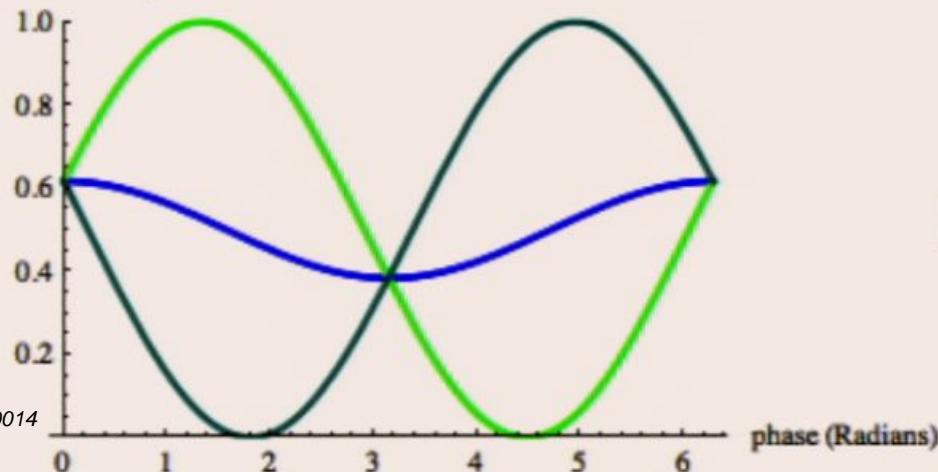
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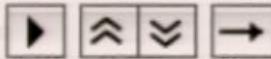
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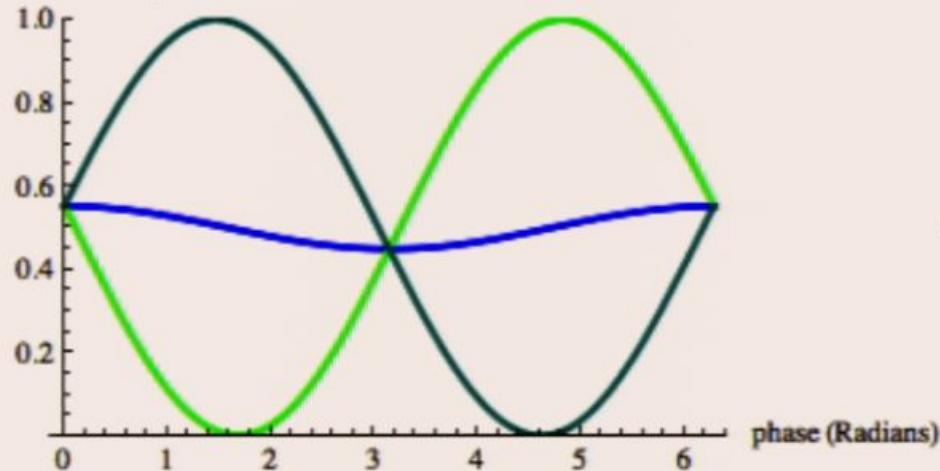
H-beam intensity



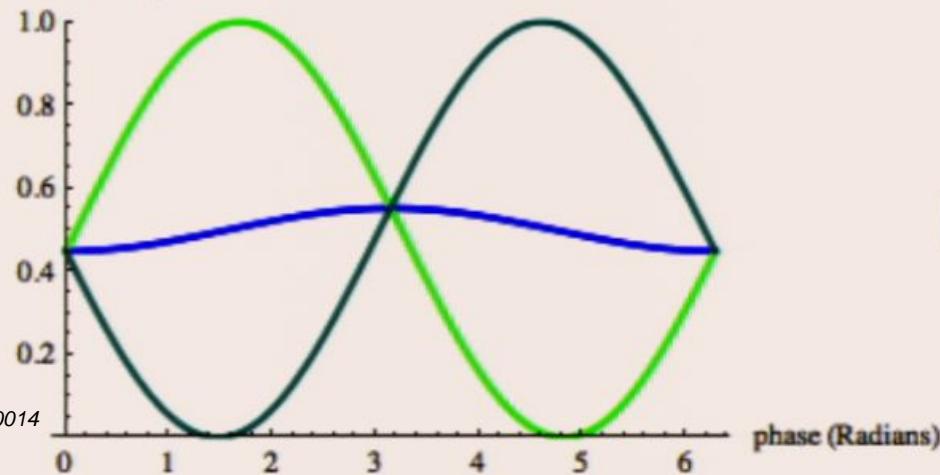
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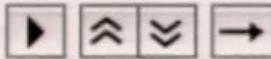
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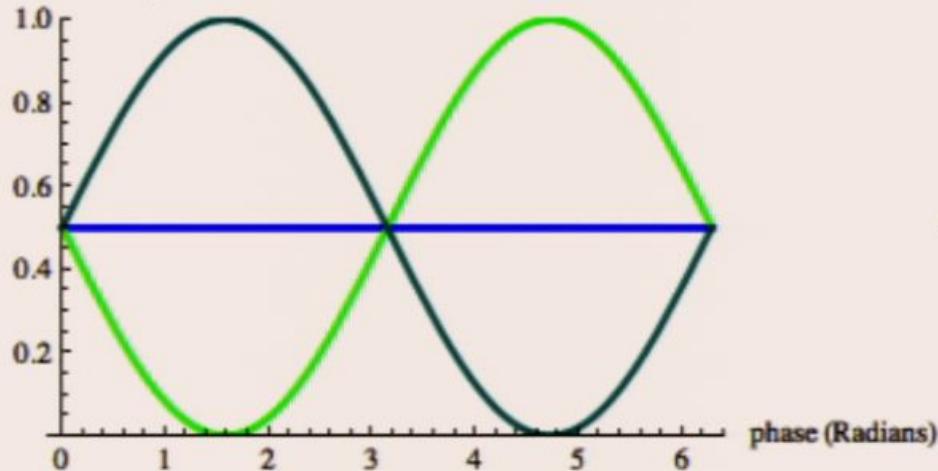
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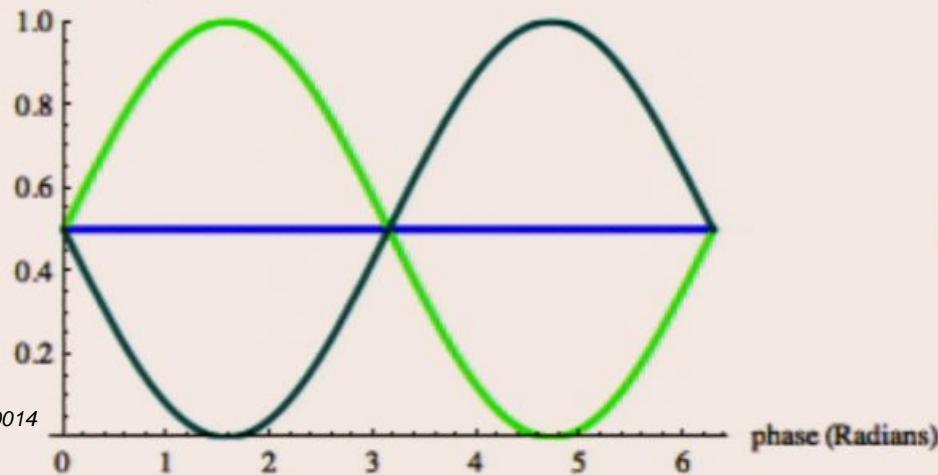
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O-beam intensity



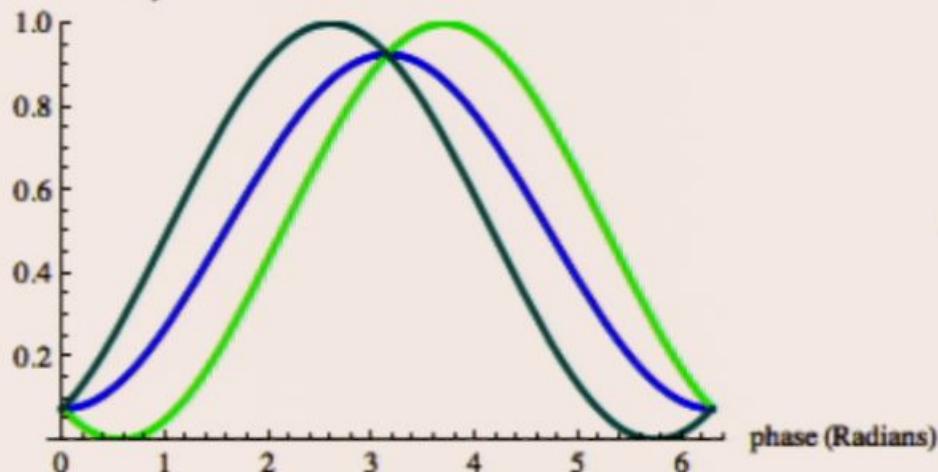
H-beam intensity



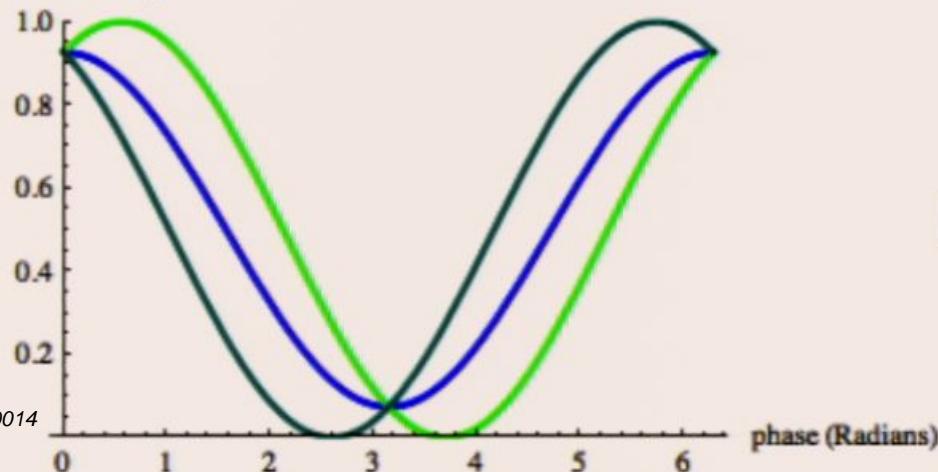
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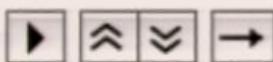
O-beam intensity



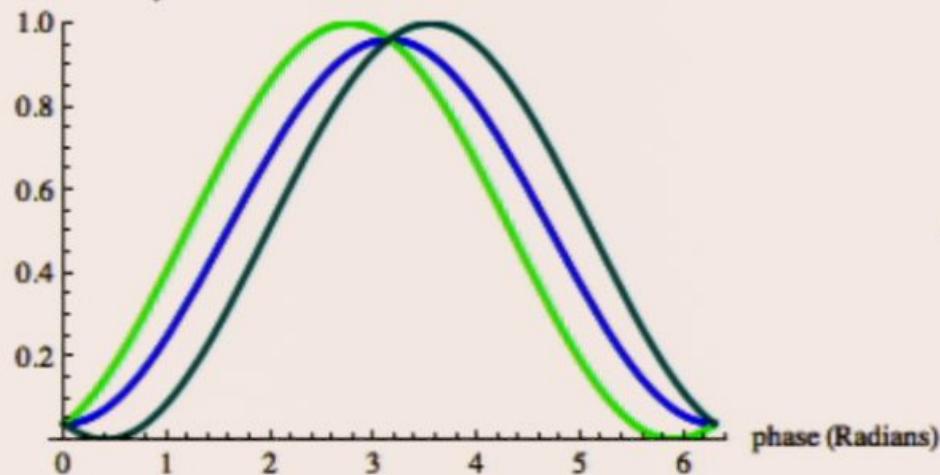
H-beam intensity



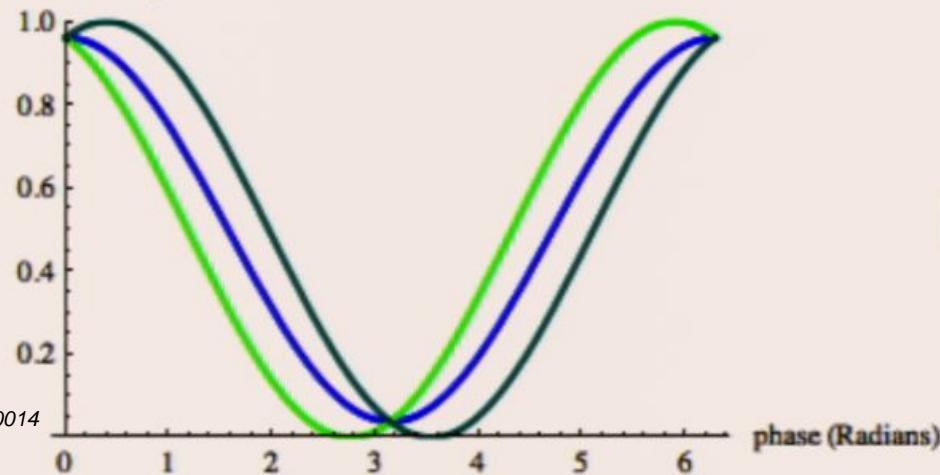
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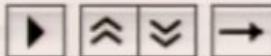
O-beam intensity



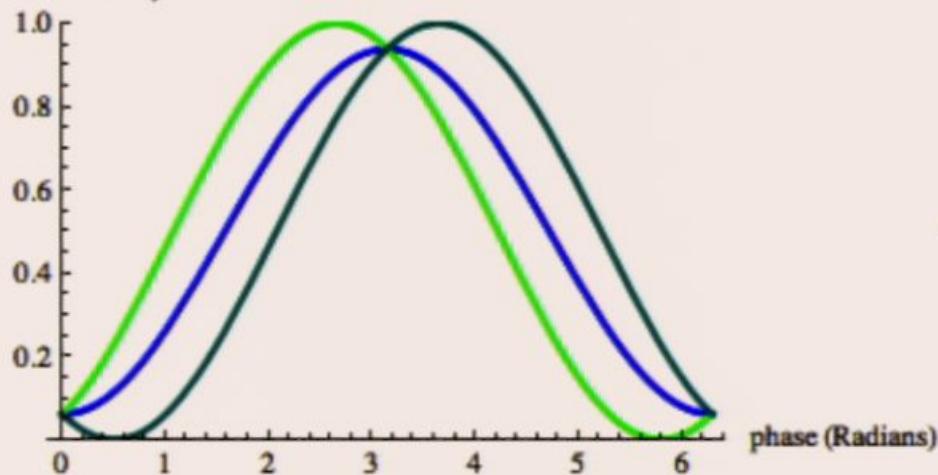
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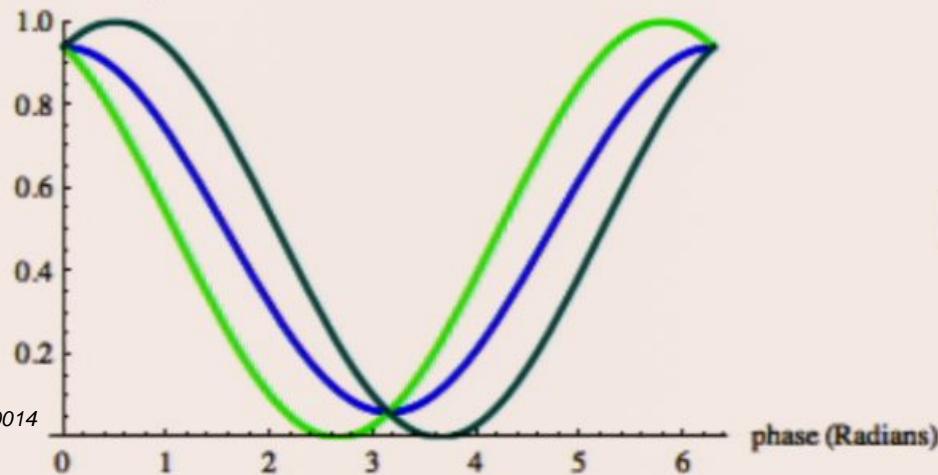
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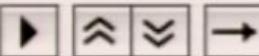
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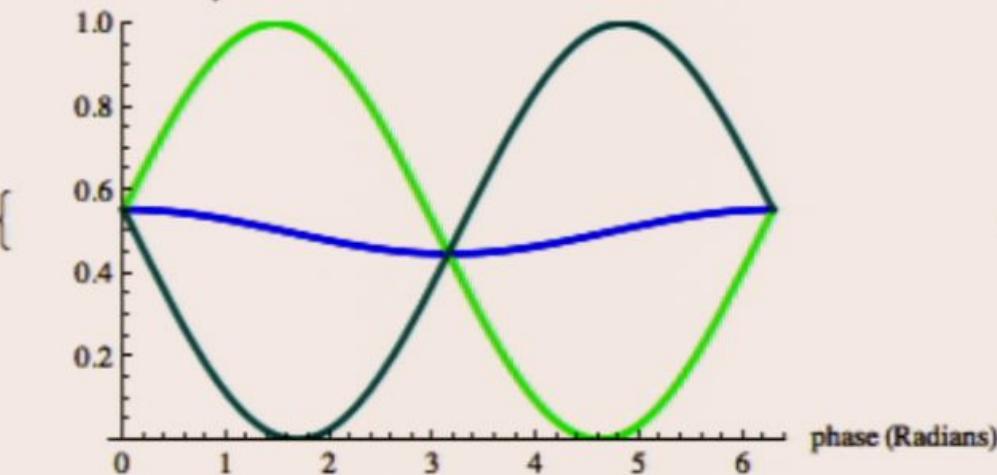
H-beam intensity



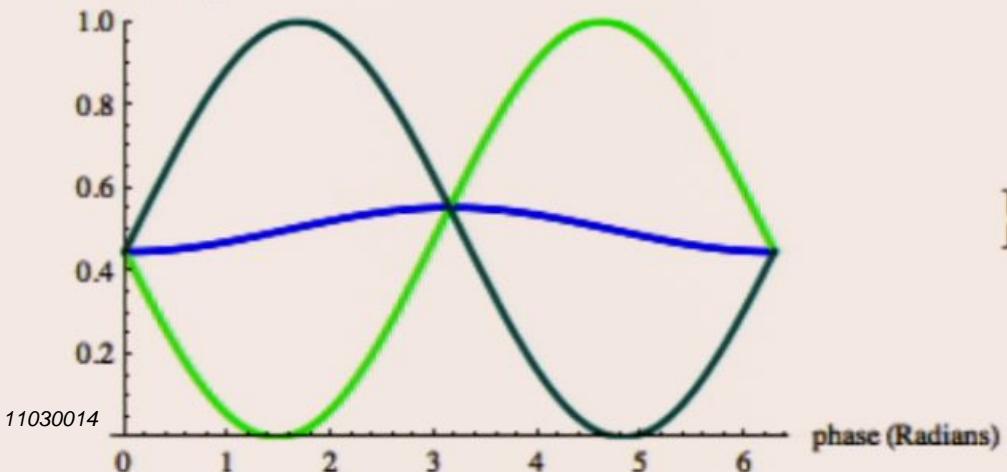
b



O-beam intensity



H-beam intensity



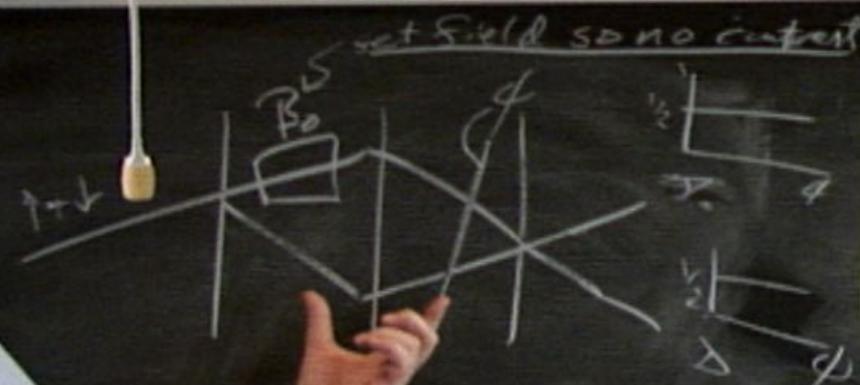
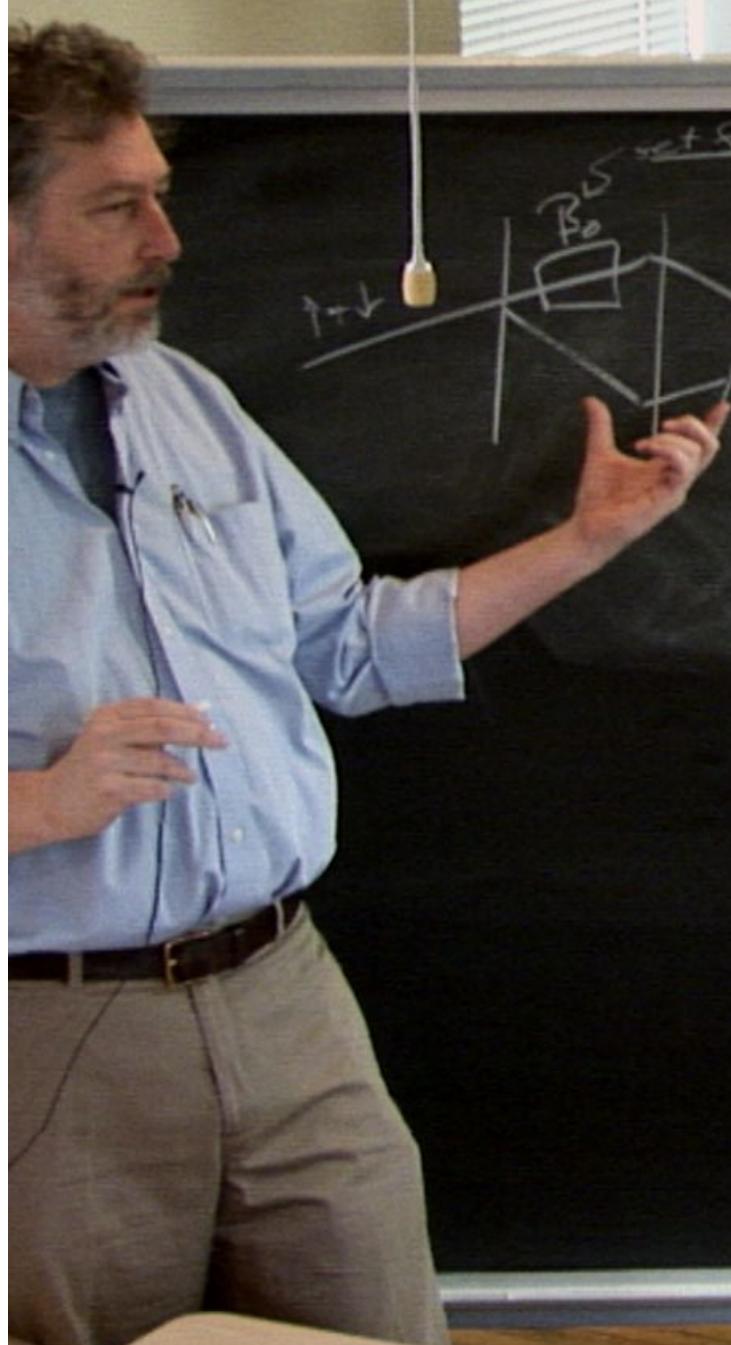


$$S_{\text{out}} = P U_{\text{ideal}} S_{\text{in}} U_{\text{ideal}}^{-1}$$

$$+ (1-P) U_{\text{error}} S_{\text{in}} U_{\text{error}}^{-1}$$

$$U_{\text{error}} = \tilde{P}^{-1} \nabla_x \tilde{\sigma}_x$$

U_{error}

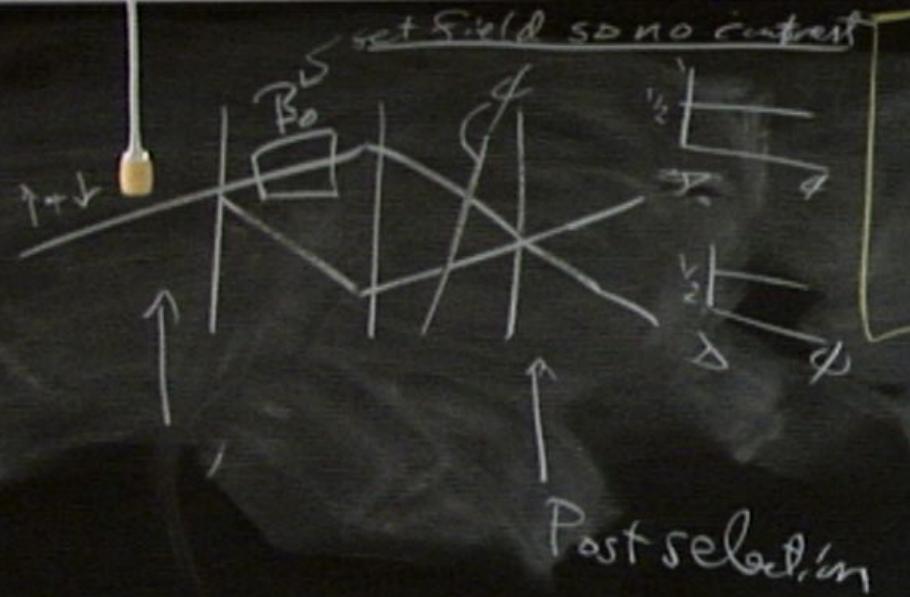


$$S_{out} = P \mathcal{U}_{ideal} S_{in} \mathcal{U}_{ideal}'$$

$$+ (1-P) \mathcal{U}_{error} S_{in} \mathcal{U}_{error}'$$

$$\mathcal{U}_{error} = \bar{P}^{-1} \sigma_x^2 \sigma_x$$

\mathcal{U}_{error}



$$S_{out} = P U_{ideal} S_{in} U_{ideal}^\dagger$$

$$+ (1-P) U_{error} S_{in} U_{error}^\dagger$$

$$U_{error} = \tilde{P}^{\dagger} \tilde{D}_{\frac{1}{2}} \tilde{D}_X$$

U_{swap}

