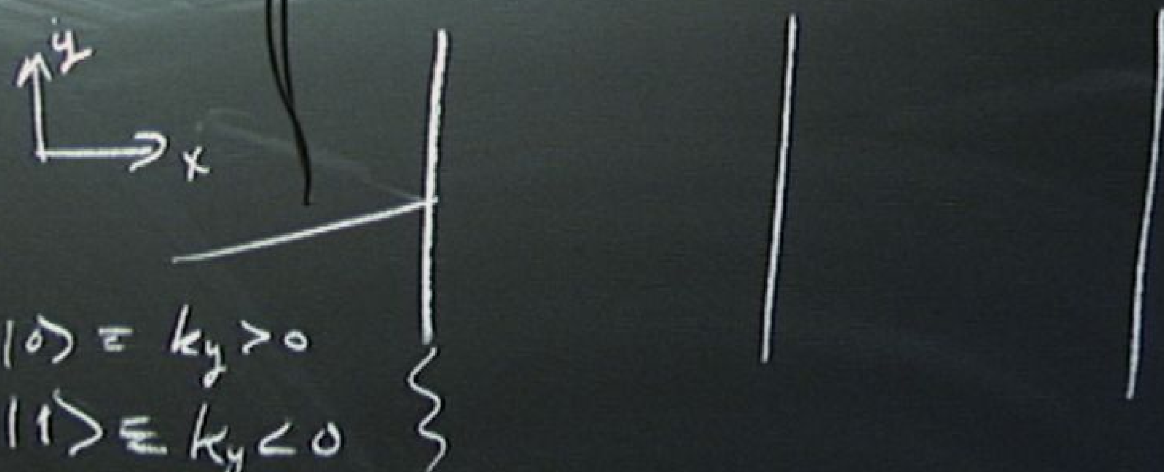
Title: Explorations in Quantum Information - Lecture 2

Date: Mar 15, 2011 09:00 AM

URL: http://pirsa.org/11030013

Abstract:

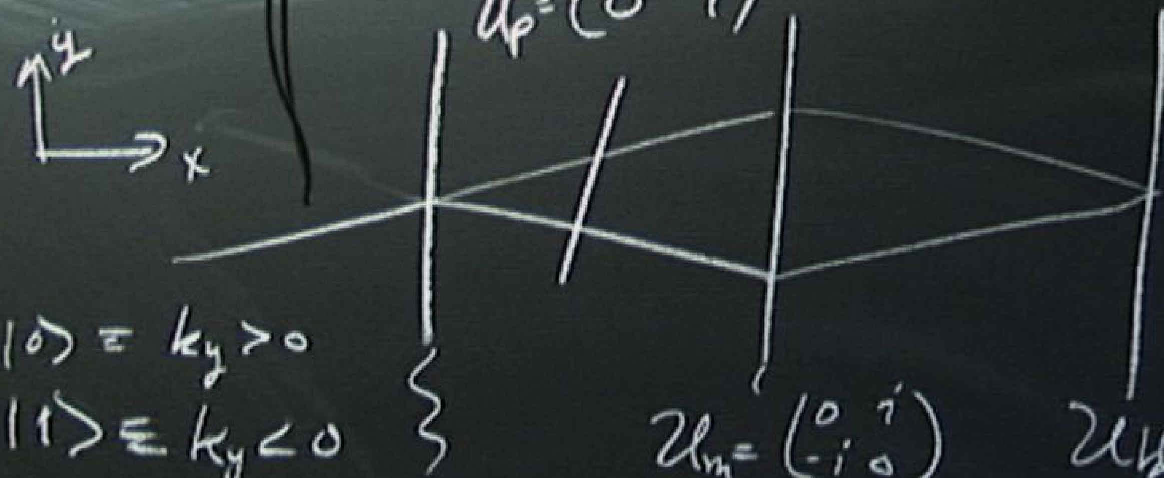


$|0\rangle \equiv k_y > 0$

$|1\rangle \equiv k_y < 0$

$$U_{Node} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

$\mathcal{U}_p = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{pmatrix}$



$|0\rangle \equiv k_y > 0$

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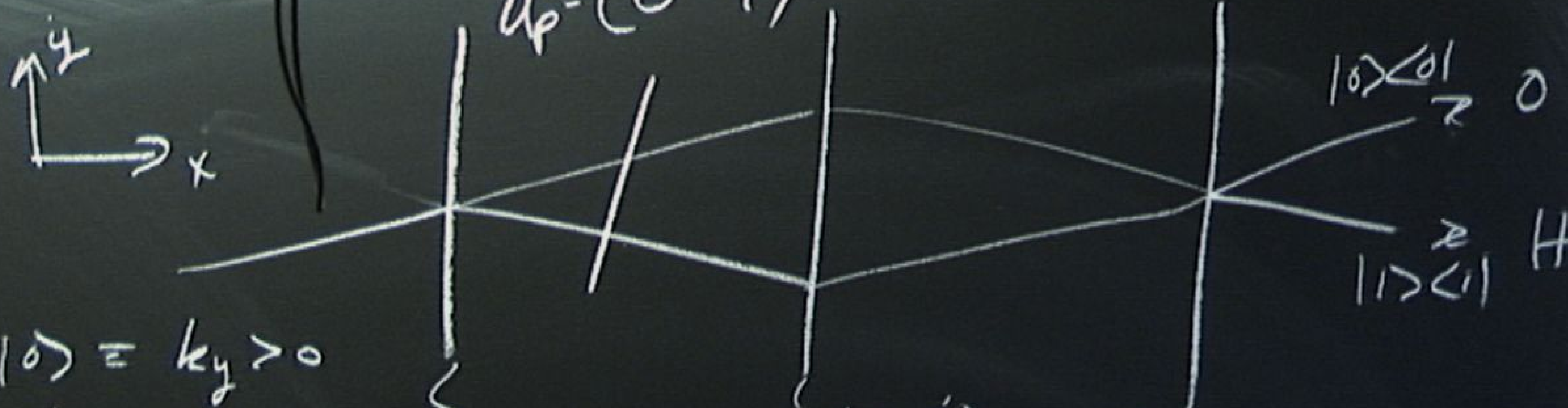
$\mathcal{U}_m = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$ $\mathcal{U}_{blade}$

$\mathcal{U}_{blade} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

$$\mathcal{U}_\phi = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{pmatrix}$$



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$$\mathcal{U}_{blade} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

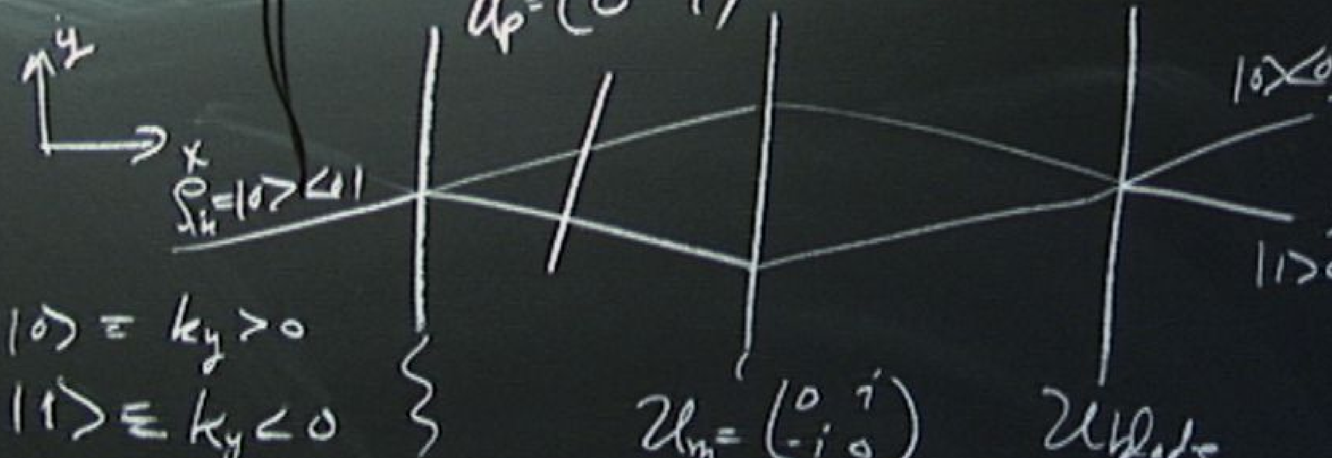
$$n + He^3 \rightarrow T + P + 700\ keV$$

5333 barn

100 fm^2

700 keV

$U_\phi = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{pmatrix}$



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$$\rho_{out} = U_{blade} U_P(\phi) U_m U_{blade} \rho_{in} U^{-1} \ldots = \frac{1}{2} \begin{pmatrix} 1+\cos\phi & i\sin\phi \\ -i\sin\phi & 1-\cos\phi \end{pmatrix}$$

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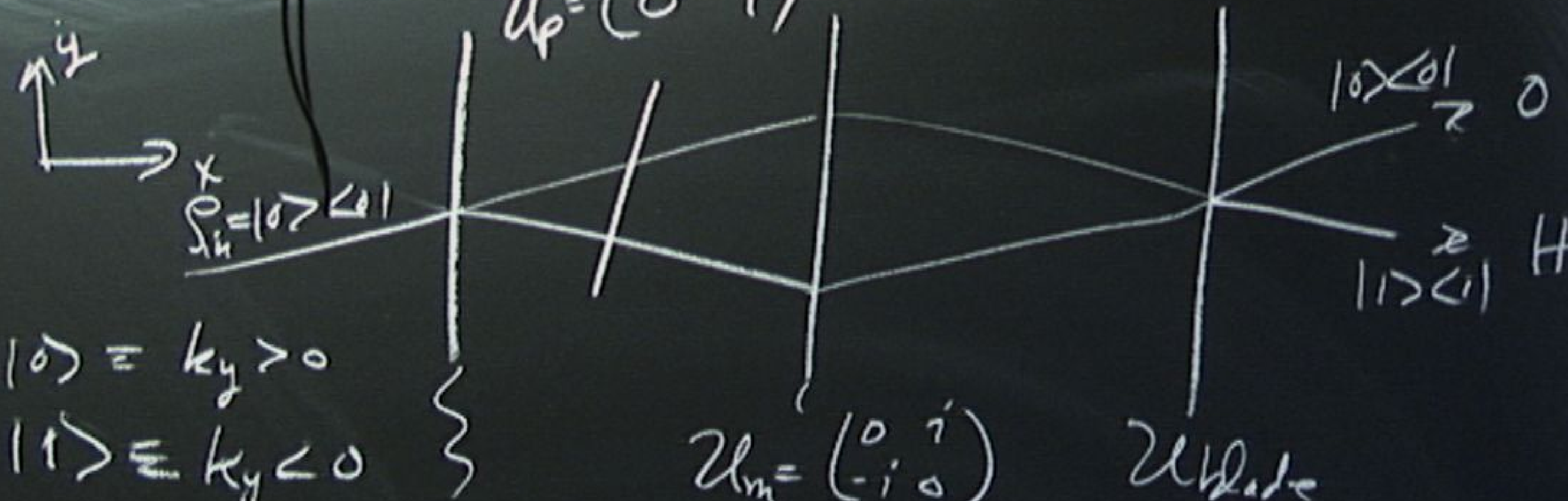

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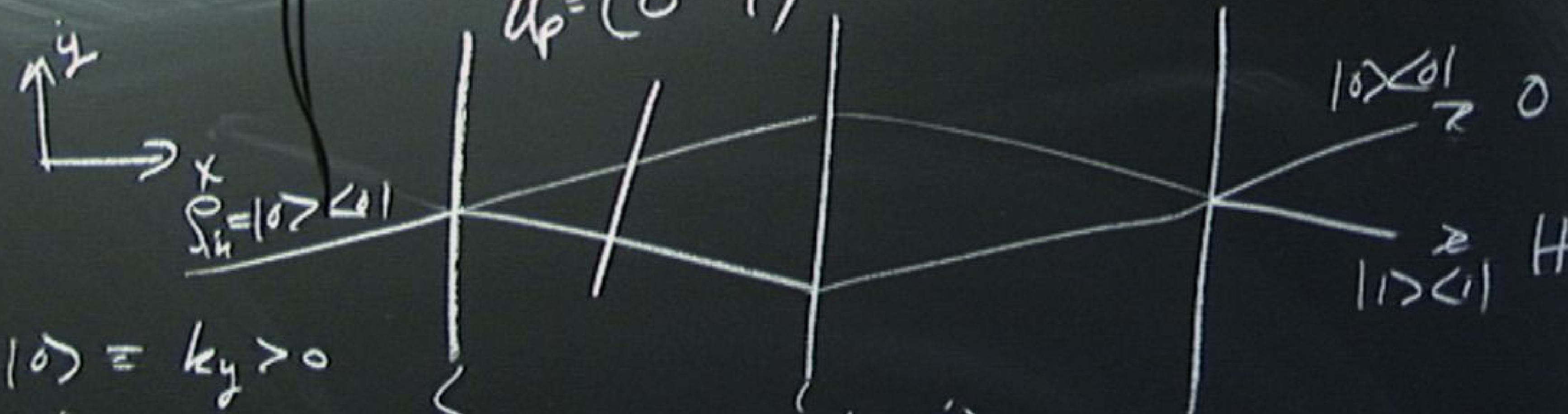

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$$\mathcal{U}_{blade} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

Open System

$$\rho_{out} = \int P(z)\, \mathcal{U}(z,t)\, \rho_{in}(z)\, \mathcal{U}^{\dagger}(z,t)\, dz$$

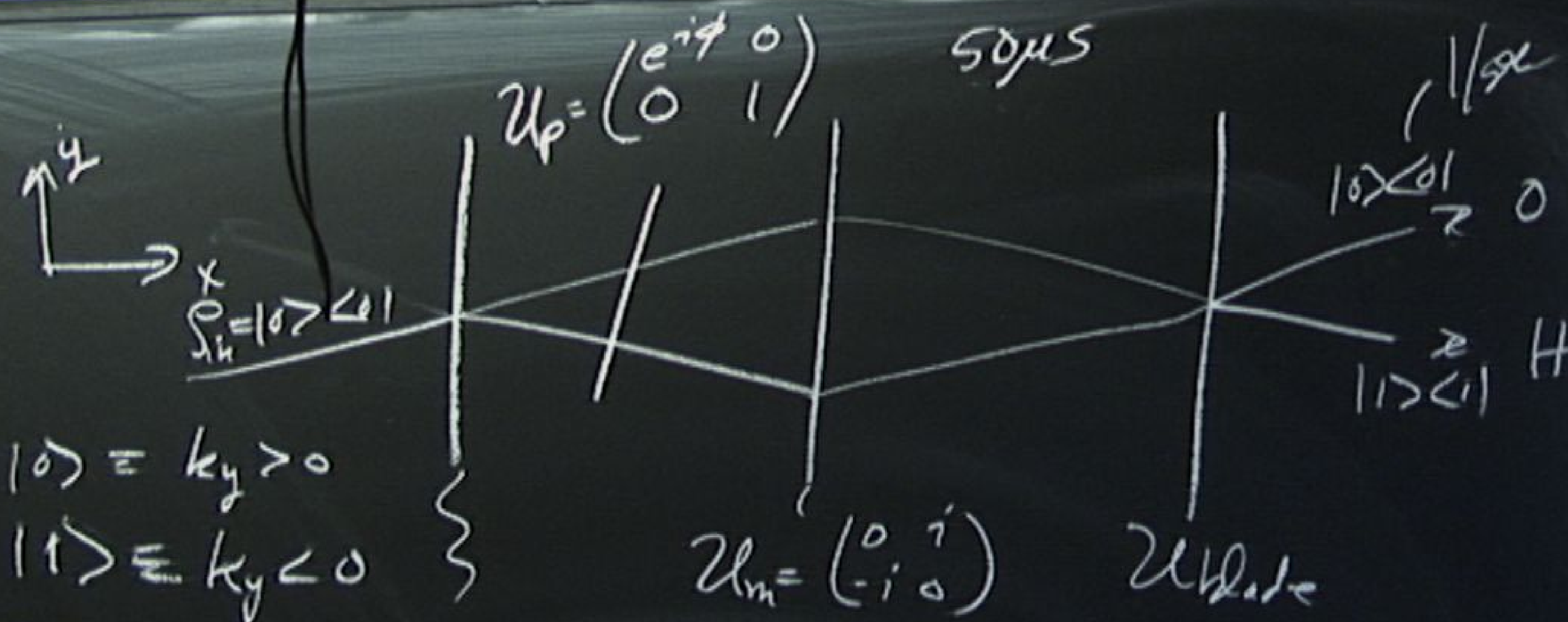
$\uparrow$

Classical probability dist.

Open System

$$\rho_{out} = \int P(z)\, \mathcal{U}(z,t)\, \rho_{in}(z)\, \mathcal{U}^{-1}(z,t)\, dz$$

$\uparrow$ Classical probability dist.



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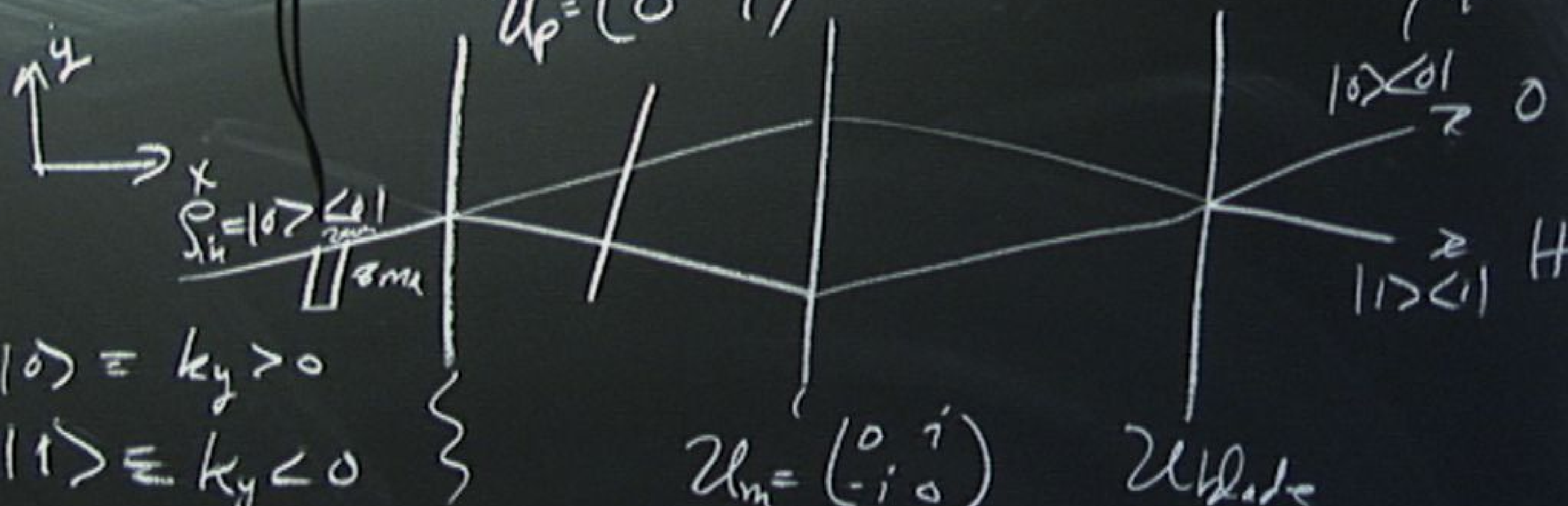
$$U_p = \begin{pmatrix} e^{-i\phi} & 0 \\ 0 & 1 \end{pmatrix}$$

50 μs



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$$U_{blade} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

Open System

$$\rho_{out} = \int P(z)\, \mathcal{U}(z,t)\, \rho_{in}(z)\, \mathcal{U}^{-1}(z,t)\, dz$$

$P(z)$ ↑ Classical probability dist.

$$\rho_{in}(z) = \rho_{in}$$

Open System

$$\rho_{out} = \int P(z)\, \mathcal{U}(z,t)\, \rho_{in}(z)\, \mathcal{U}^{-1}(z,t)\, dz$$

$\uparrow$ Classical probability dist.

required $\Rightarrow$ $\rho_{in}(z) = \rho_{in}$

$$\rho_{out} = \sum_i K_i \rho K_i^{+}$$

Open System

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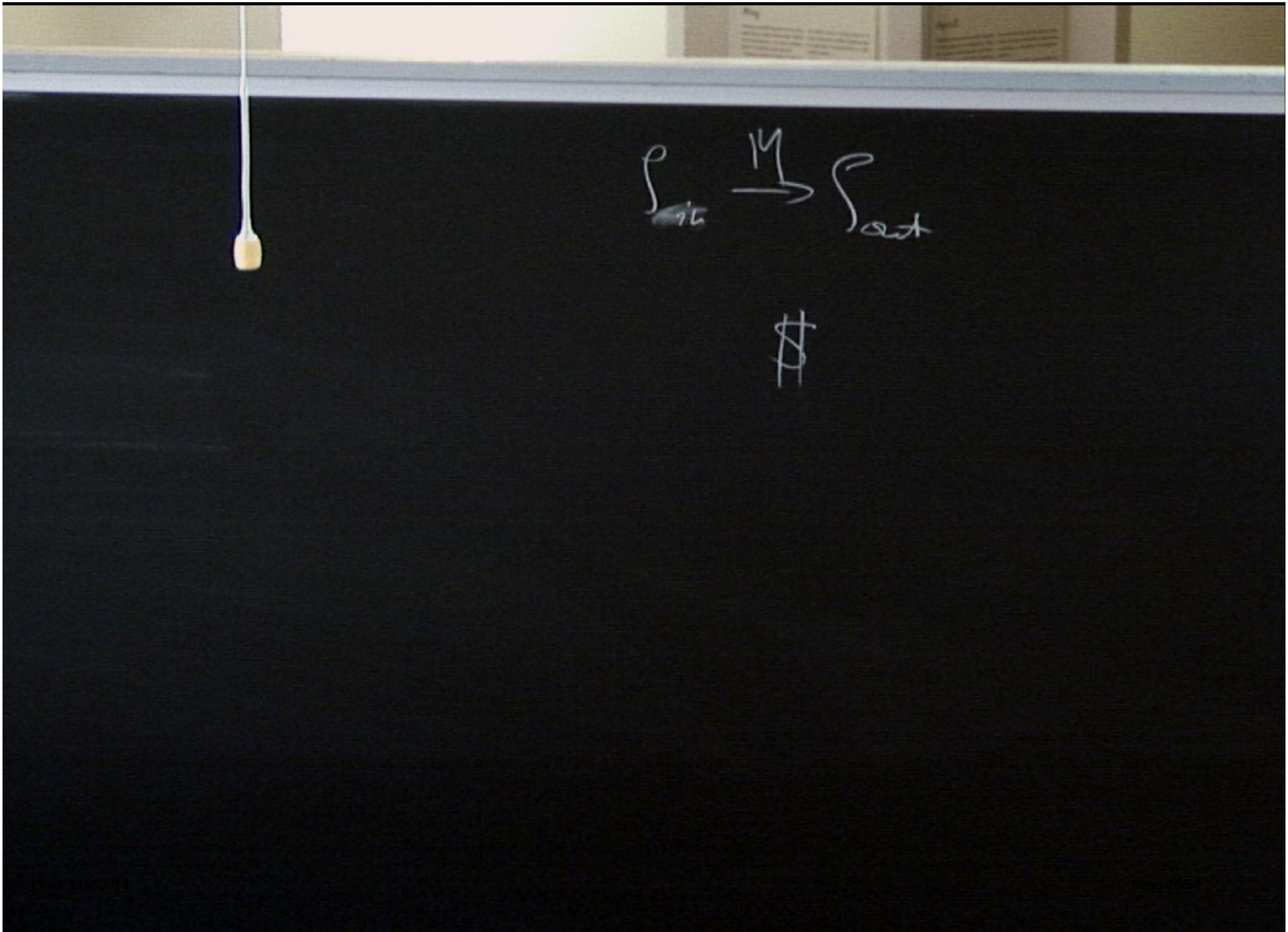
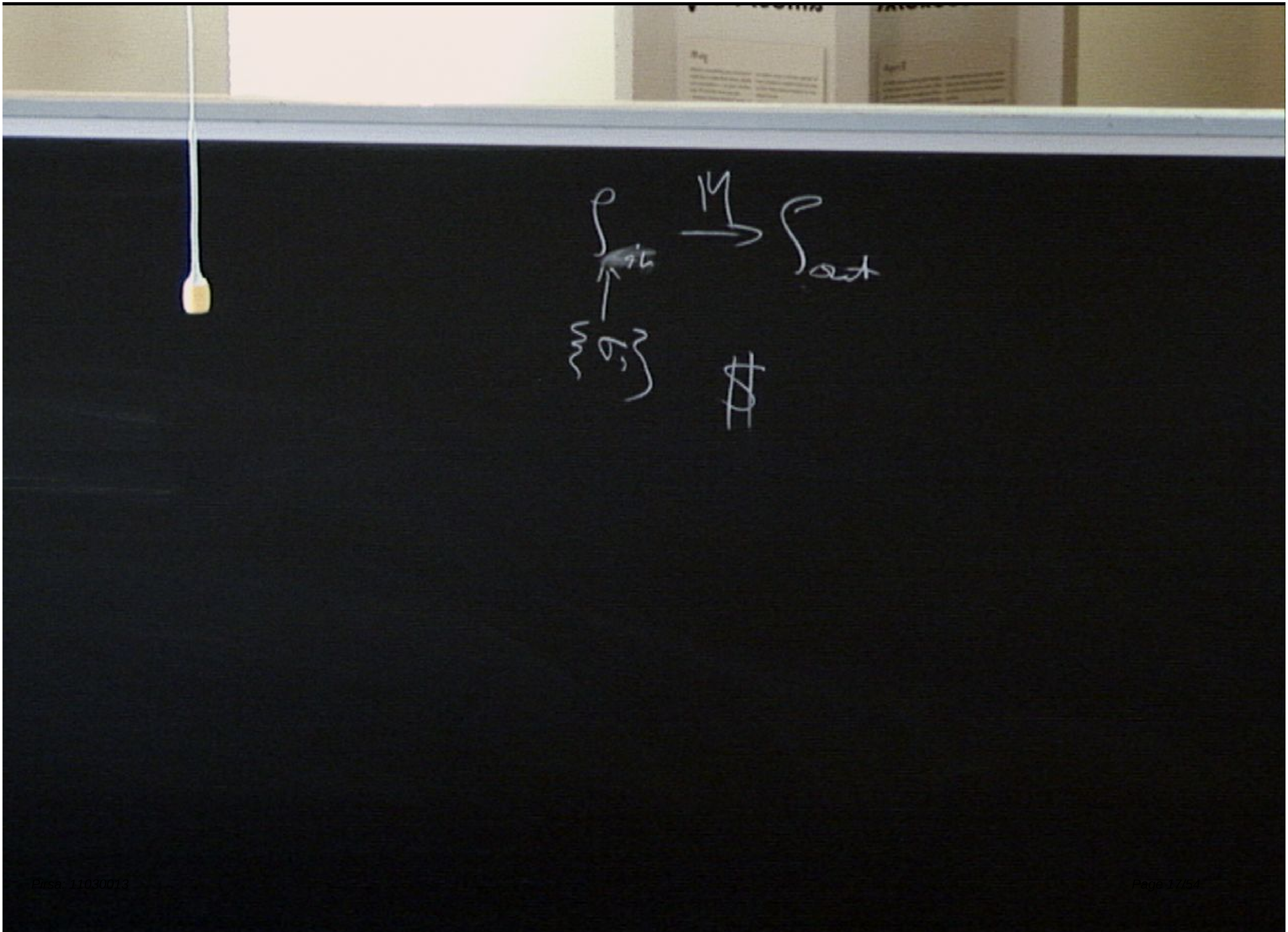
$\uparrow$ Classical probability dist.

required $\Rightarrow \rho_{in}(z) = \rho_{in}$

$$\rho_{out} = \sum_{i}^{n^2} K_i\, \rho_{in}\, K_i^{\dagger}$$


$\rho_{in} \xrightarrow{M} \rho_{out}$
$


$\rho_{in} \xrightarrow{M} \rho_{out}$
$\{0,1\}$
$

Open System

$$\rho_{out} = \int P(z)\, \mathcal{U}(z,t)\, \rho_{in}(z)\, \mathcal{U}^{-1}(z,t)\, dz$$

$P(z)$: Classical probability dist.

required $\Longrightarrow \rho_{in}(z) = \rho_{in}$

$$\rho_{out} = \sum_{i}^{n^2} K_i \rho_{in} K_i^{+}$$

$$\sum K_i^{+} K_i = \mathbb{1}$$

Open System

$$\rho_{out} = \int P(z)\; \mathcal{U}(z,t)\, \rho_{in}(z)\, \mathcal{U}^{-1}(z,t)\; dz$$

Classical probability dist.

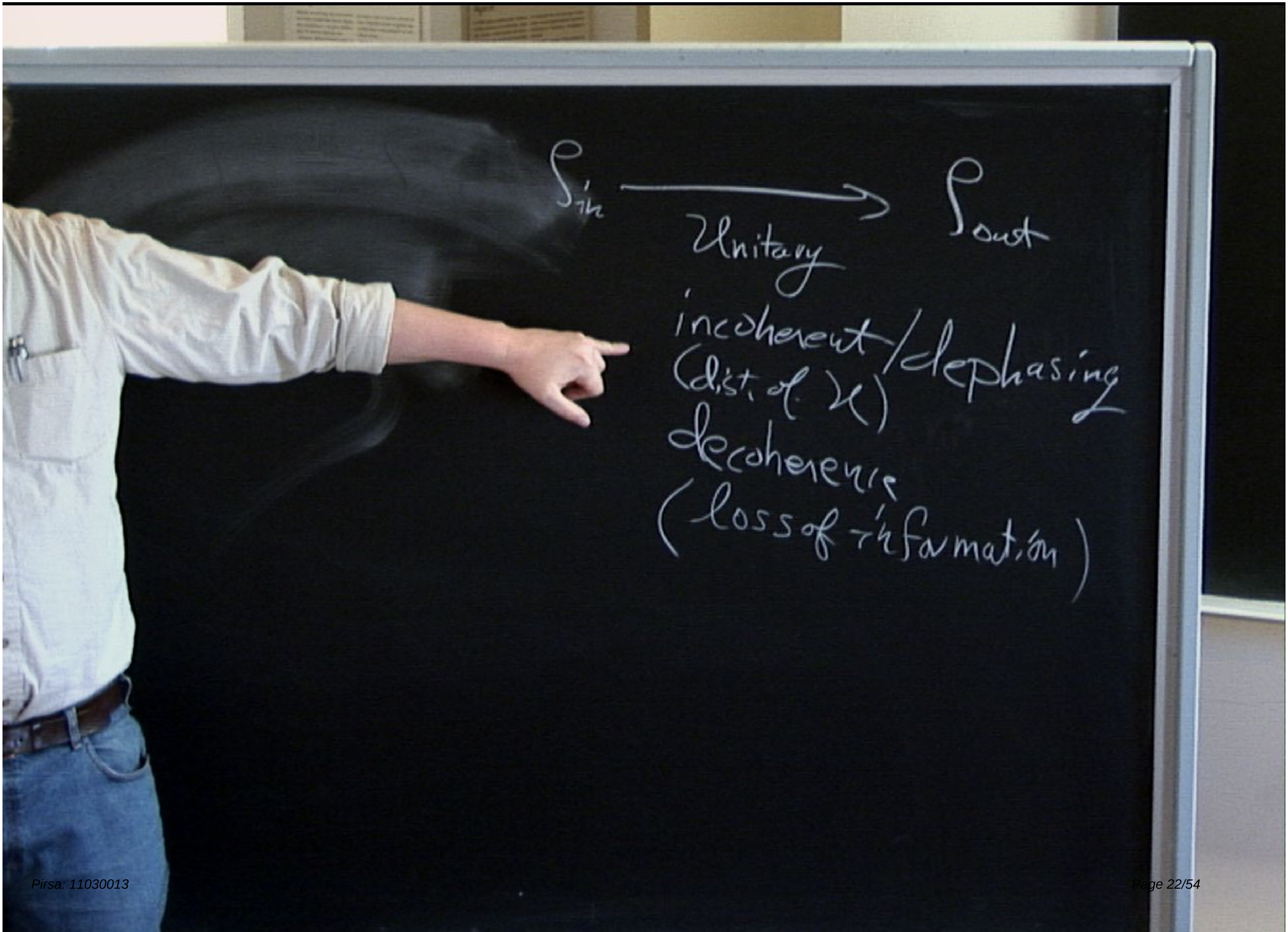
required $\Longrightarrow$ $\rho_{in}(z) = \rho_{in}$

$$\rho_{out} = \sum_i^{n^2} \overbrace{K_i}^{\sqrt{p_i}\, U_i} \rho_{in} K_i^{\dagger}$$

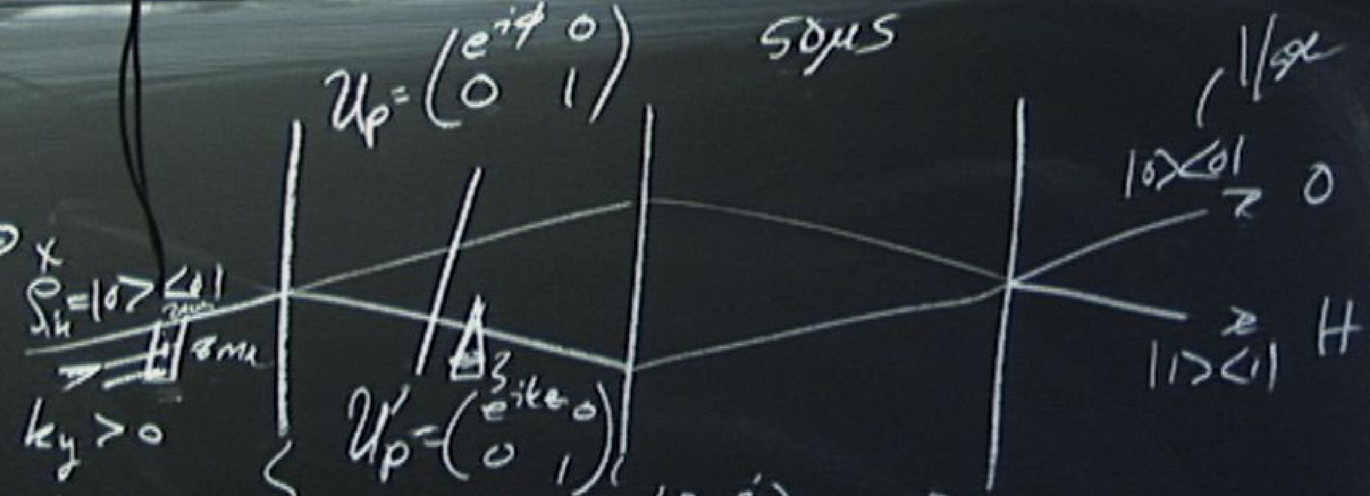
$$\sum K_i^{\dagger} K_i = \mathbb{I}$$


From Grains of Pollen to Evidence for Atoms
How Big Is A Molecule?
$\rho_{in} \xrightarrow[\text{Unitary}]{} \rho_{out}$

$$\rho_{in} \xrightarrow[\text{incoherent}]{\text{Unitary}} \rho_{out}$$


ρ_in → ρ_out
Unitary
incoherent/dephasing
(dist. of. 𝓗)
decoherence
(loss of information)

y, x



$|0\rangle \equiv k_y > 0$

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$$\rho_{out} = \mathcal{U}_{blade}\,\mathcal{U}_m\,\mathcal{U}_p\,\mathcal{U}_{blade}\,\rho_{in}\,\mathcal{U}^{-1} \ldots = \frac{1}{2}\begin{pmatrix} 1+\cos\phi & i\sin\phi \\ -i\sin\phi & 1-\cos\phi \end{pmatrix}$$

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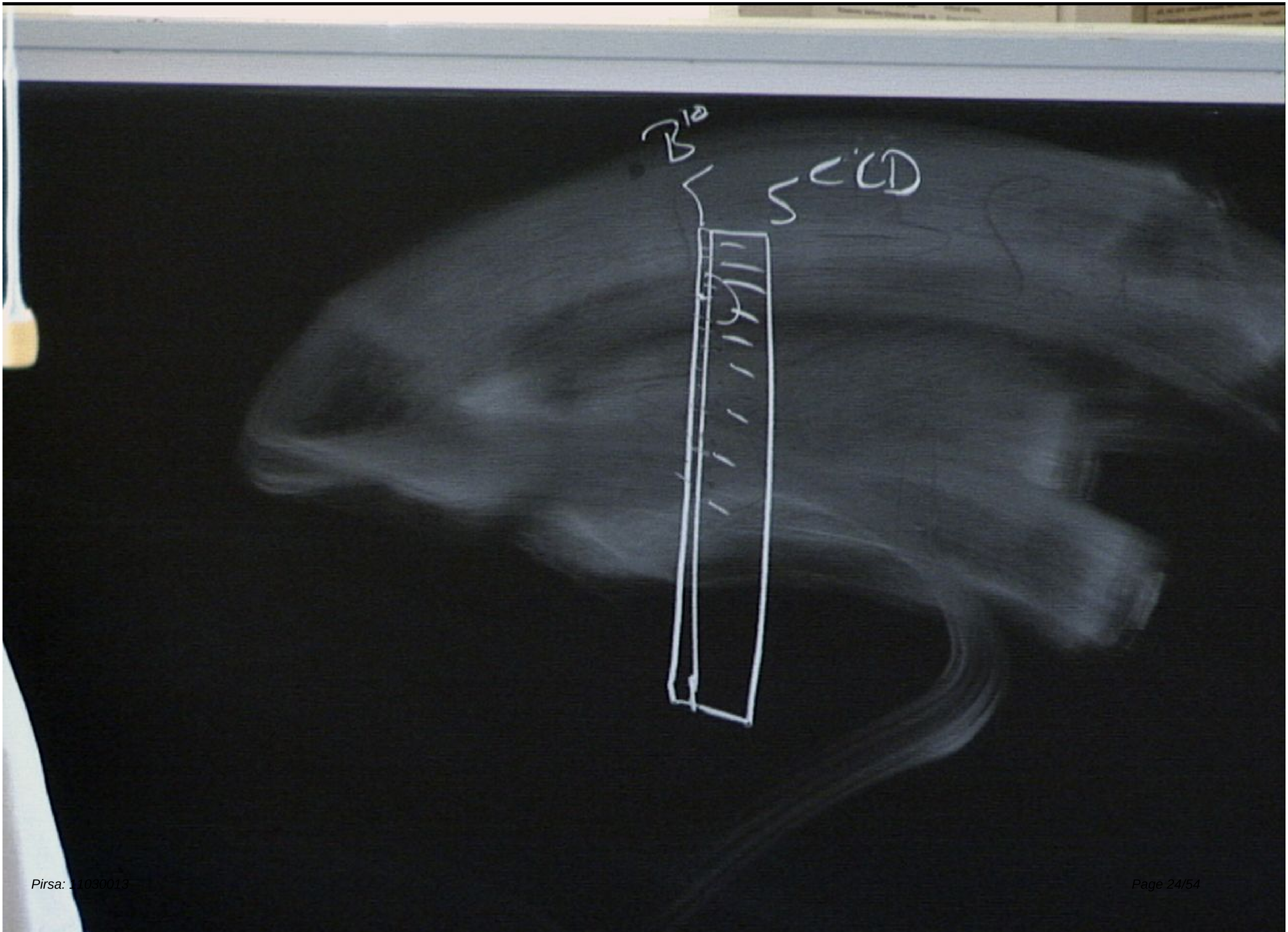
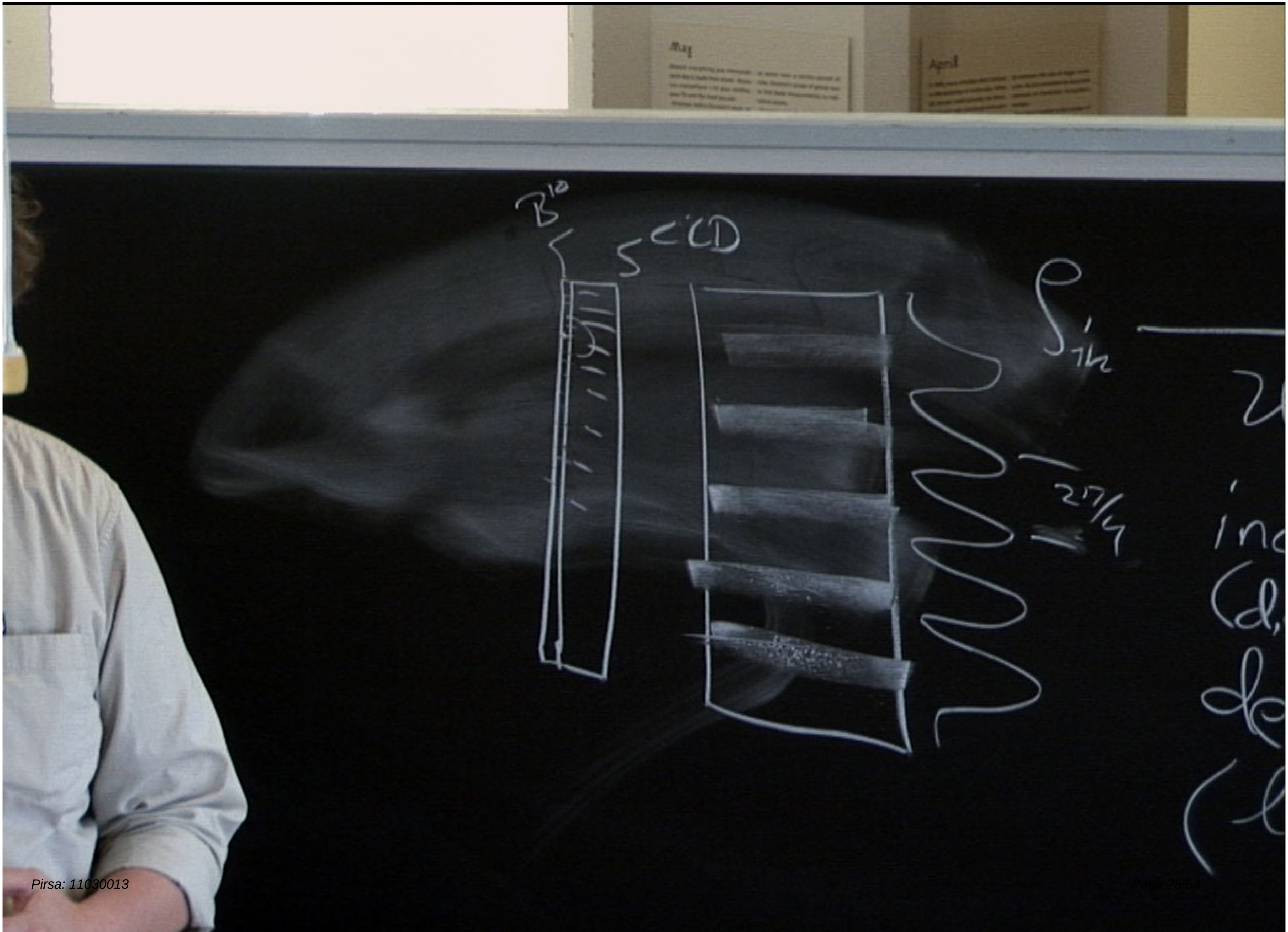
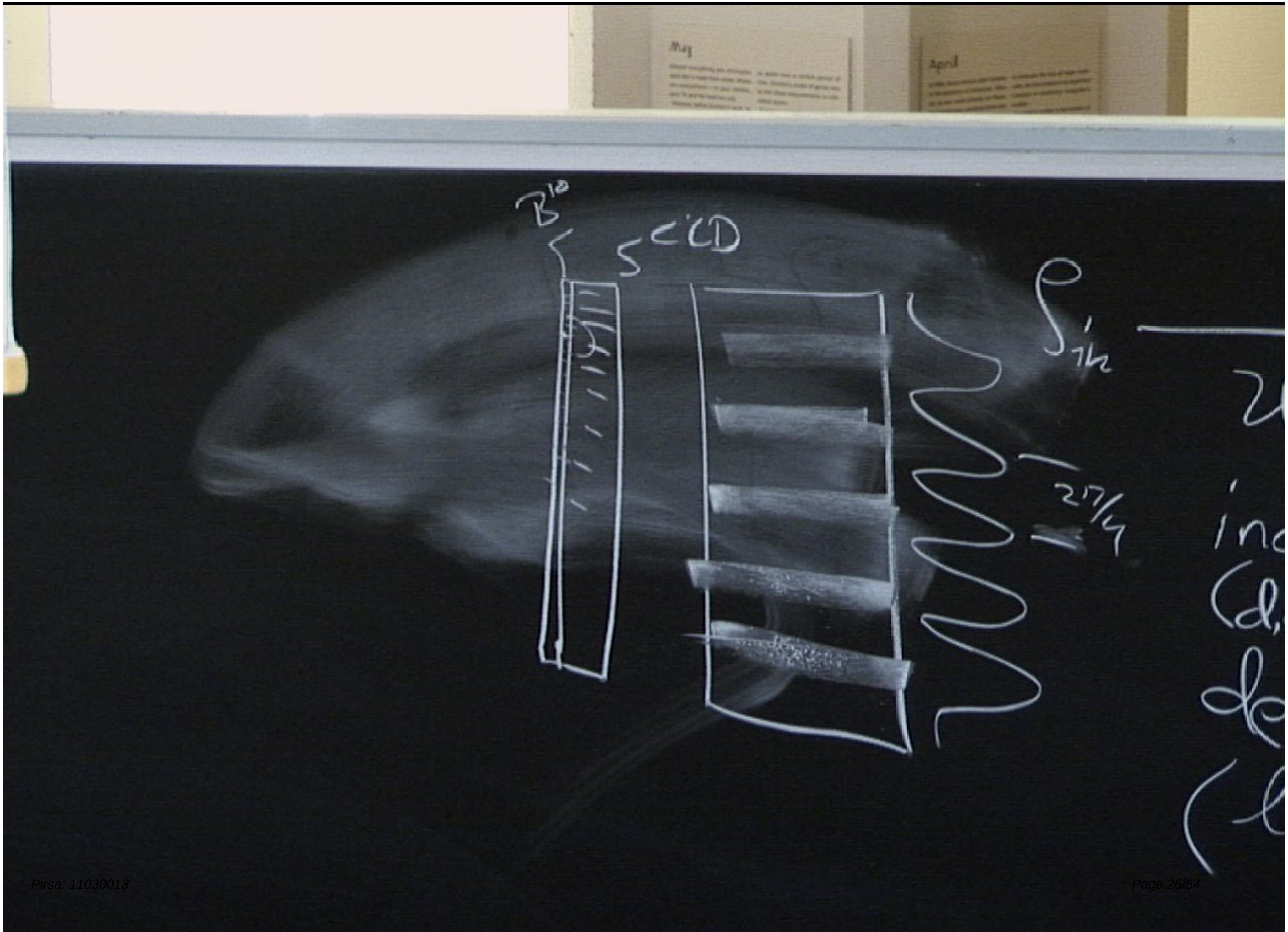
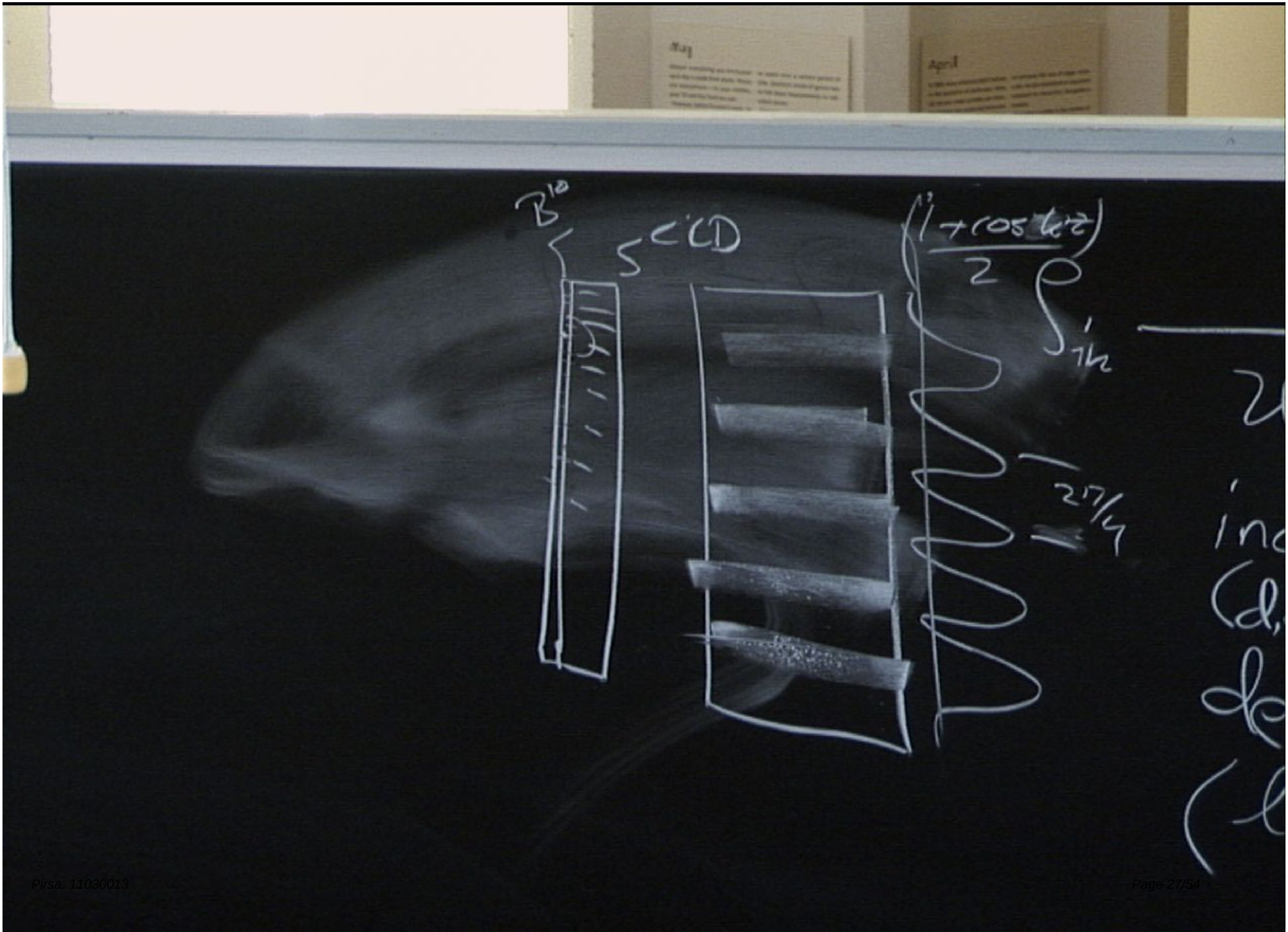
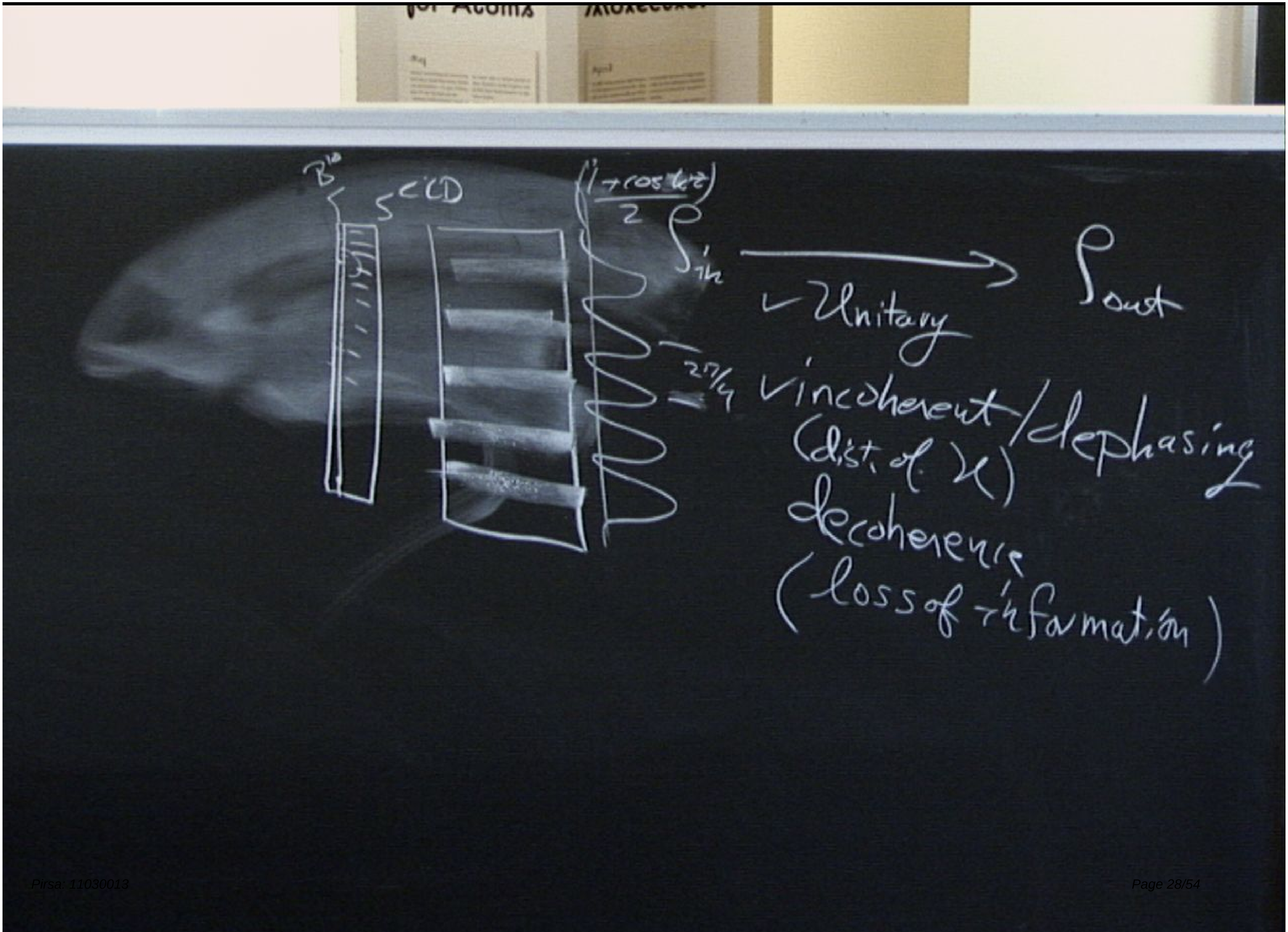

B10
5 C'CD


B
27/4




B10
CCD
1 + cos kz
2



(1+cos kz)/2
ρ in
ρ out
Unitary
incoherent/dephasing
(dist. of
decoherence
(loss of information)

$\int \rho_{in}(z)\, \mathcal{U}^{\dagger}(z,t)\, dz$

...obability dist.

$\mathcal{H} = \omega \sigma_z$

$\sum p_i \mathcal{U}_i$

$\sum K_i \rho_{in} K_i^{\dagger}$

$\sum K_i^{\dagger} K_i = \mathbb{1}$

$\rho_{in} = |0\rangle\langle 0|$

$\mathcal{U}_p = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{pmatrix}$

$\mathcal{U}_p' = \begin{pmatrix} e^{ik\sigma} & 0 \\ 0 & 1 \end{pmatrix}$

$\mathcal{U}_m = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$

$\rho_{in}(z)\, \mathcal{U}^{-1}(z,t)\; dz$

obability dist.

$p_i\, \mathcal{U}_i$

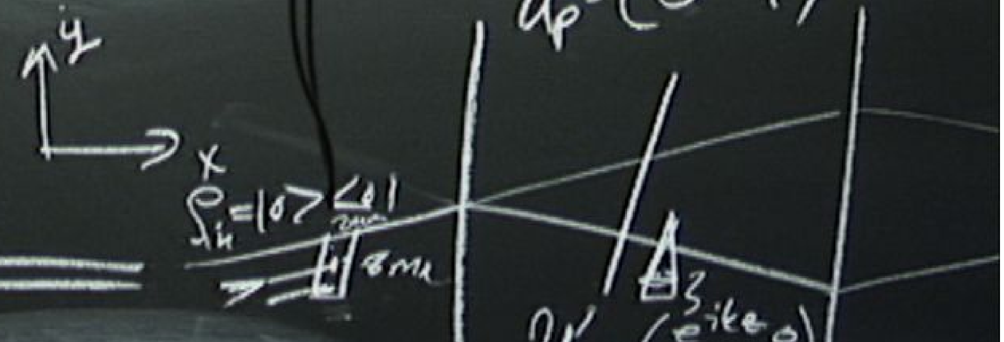
$K_i\, \rho_{in}\, K_i^\dagger$

$K_i^\dagger K_i = \mathbb{1}$

$\mathcal{H} = \omega\, \sigma_z$

$K_0 = \sqrt{1-e^{-t/t_1}}\; \mathbb{1}$

$K_1 =$



$\rho_{in} = |0\rangle\langle 0|$

$\mathcal{U}_p = \begin{pmatrix} e^{-i\phi} & 0 \\ 0 & 1 \end{pmatrix}$

$\mathcal{U}_p' = \begin{pmatrix} e^{ik\epsilon} & 0 \\ 0 & 1 \end{pmatrix}$

$\mathcal{U}_m = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$

$\rho_{in}(z)\, \mathcal{U}^{-1}(z,t)\ dz$

$\mathcal{H} = \omega \sigma_z$

$K_0 = \sqrt{1-e^{-t/T_1}}\ \mathbb{1}$

$K_1 = \sqrt{e^{-t/T_1}}\ \sigma_x$

robability dist.

$p_i\, \mathcal{U}_i$

$K_i\, \rho_{in}\, K_i^{\dagger}$

$K_i^{\dagger} K_i = \mathbb{1}$

$\mathcal{U}_p = \begin{pmatrix} e^{-i\phi} & 0 \\ 0 & 1 \end{pmatrix}$

$\mathcal{U}_p' = \begin{pmatrix} e^{ik\theta} & 0 \\ 0 & 1 \end{pmatrix}$

$\mathcal{U}_m = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$

$\rho_{in} = |0\rangle\langle 0|$

$$(t)\,\rho_{in}(z)\,\mathcal{U}^{-1}(z,t)\;dz$$

probability dist.

$\sqrt{p_i}\,\mathcal{U}_i$

$$K_i\,\rho_{in}\,K_i^{\dagger}$$

$$K_i^{\dagger} K_i = \mathbb{1}$$

$$\mathcal{H} = \omega\,\sigma_z$$

$$\boxed{\begin{aligned} K_0 &= \sqrt{1-e^{-t/T_1}}\;\mathbb{1} \\ K_1 &= \sqrt{e^{-t/T_1}}\;\sigma_x \end{aligned}}$$

y, x

$\rho_{in} = |0\rangle\langle 0|$

$$\mathcal{U}_p = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{pmatrix}$$

Δz

$$\mathcal{U}_p' = \begin{pmatrix} e^{ik\theta} & 0 \\ 0 & 1 \end{pmatrix}$$

$\mathcal{U}_m = ($

$$t)\, \rho_{in}(z)\, \mathcal{U}^{-1}(z,t)\, dz$$

probability dist.

$$\sum p_i\, \mathcal{U}_i$$

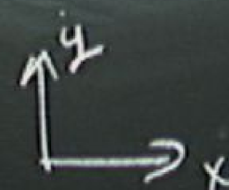
$$K_i\, \rho_{in}\, K_i^\dagger$$

$$K_i^\dagger K_i = \mathbb{1}$$

$$\mathcal{H} = \omega\, \sigma_z$$

$$K_0 = \sqrt{1 - \frac{t}{T_1}}\; \mathbb{1}$$

$$K_1 = \sqrt{\frac{t}{T_1}}\; \sigma_x$$

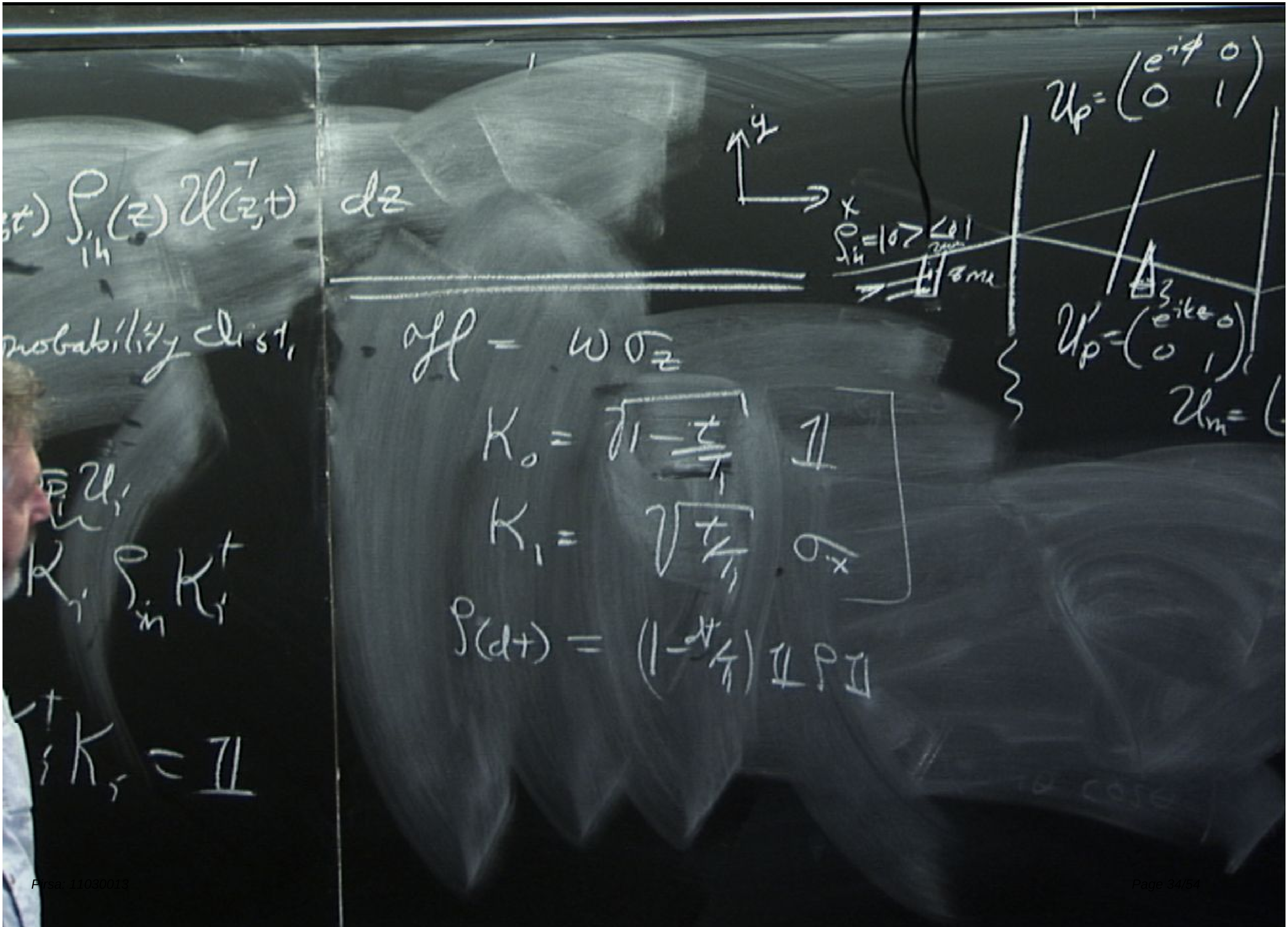


$$\mathcal{U}_p = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{pmatrix}$$

$$\rho_{in} = |0\rangle\langle 0|$$

$$\mathcal{U}_p' = \begin{pmatrix} e^{ik\phi_0} & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathcal{U}_m = ($$



$\rho_{in}(z)\,\mathcal{U}^{-1}(z,t)\; dz$
probability dist.
$\mathcal{H} = \omega\,\sigma_z$
$K_0 = \sqrt{1 - t/T_1}\;\mathbb{1}$
$K_1 = \sqrt{t/T_1}\;\sigma_x$
$\rho(dt) = (1 - dt/T_1)\,\mathbb{1}\,\rho\,\mathbb{1}$
$K_i\,\rho_{in}\,K_i^{\dagger}$
$K_i^{\dagger} K_i = \mathbb{1}$
$\rho_{in} = |0\rangle\langle 0|$
$\mathcal{U}_p = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{pmatrix}$
$\mathcal{U}'_p = \begin{pmatrix} e^{ik\epsilon} & 0 \\ 0 & 1 \end{pmatrix}$
$\mathcal{U}_m =$

$$\ldots,t)\, \rho_{in}(z)\, \mathcal{U}^{-1}(z,t)\; dz$$

probability dist.

$$\sqrt{p_i}\, \mathcal{U}_i$$

$$K_i\, \rho_{in}\, K_i^{\dagger}$$

$$K_i^{\dagger} K_i = \mathbb{1}$$

$$\mathcal{H} = \omega\, \sigma_z$$

$$\boxed{\begin{aligned} K_0 &= \sqrt{1 - t/T_1}\;\mathbb{1} \\ K_1 &= \sqrt{t/T_1}\;\sigma_x \end{aligned}}$$

$$\rho(dt) = \left(1 - dt/T_1\right) \mathbb{1}\rho\mathbb{1} + t/T_1\, \sigma_x \rho\, \sigma_x$$

y

x

$\rho_{in} = |0\rangle\langle 0|$

2mm

8mm

Δz

$$\mathcal{U}_\phi = \begin{pmatrix} e^{-i\phi} & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathcal{U}'_\phi = \begin{pmatrix} e^{ikz} & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathcal{U}_m = ($$

$\ell(z,t)\ dz$

dist.

$\mathcal{H} = \omega \sigma_z$

$$\boxed{\begin{aligned} K_0 &= \sqrt{1 - t/\tau_1}\ \mathbb{1} \\ K_1 &= \sqrt{t/\tau_1}\ \sigma_x \end{aligned}}$$

$$\rho(dt) = (1 - dt/\tau_1)\, \mathbb{1} \rho \mathbb{1} + dt/\tau_1\, \sigma_x \rho \sigma_x$$

y, x

$\rho_{in} = |0\rangle\langle 0|$

$U_\phi = \begin{pmatrix} e^{-i\phi} & 0 \\ 0 & 1 \end{pmatrix}$

$U'_\phi = \begin{pmatrix} e^{ik\theta} & 0 \\ 0 & 1 \end{pmatrix}$

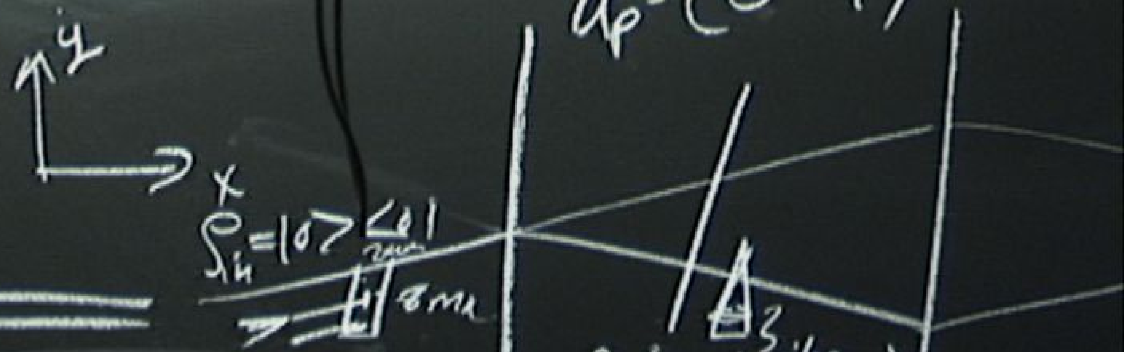
$U_m = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$

$K_1^\dagger$

$\mathbb{1}$

$\mathcal{U}_p = \begin{pmatrix} e^{i\varphi} & 0 \\ 0 & 1 \end{pmatrix}$

$U^{-1}(z,t) \quad dz$



$\mathcal{U}_p' = \begin{pmatrix} e^{ik\Delta z} & 0 \\ 0 & 1 \end{pmatrix}$

$\mathcal{U}_m = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$

dist.

$\mathcal{H} = \omega \sigma_z$

$K_0 = \sqrt{1 - \frac{t}{T_1}} \; \mathbb{1}$

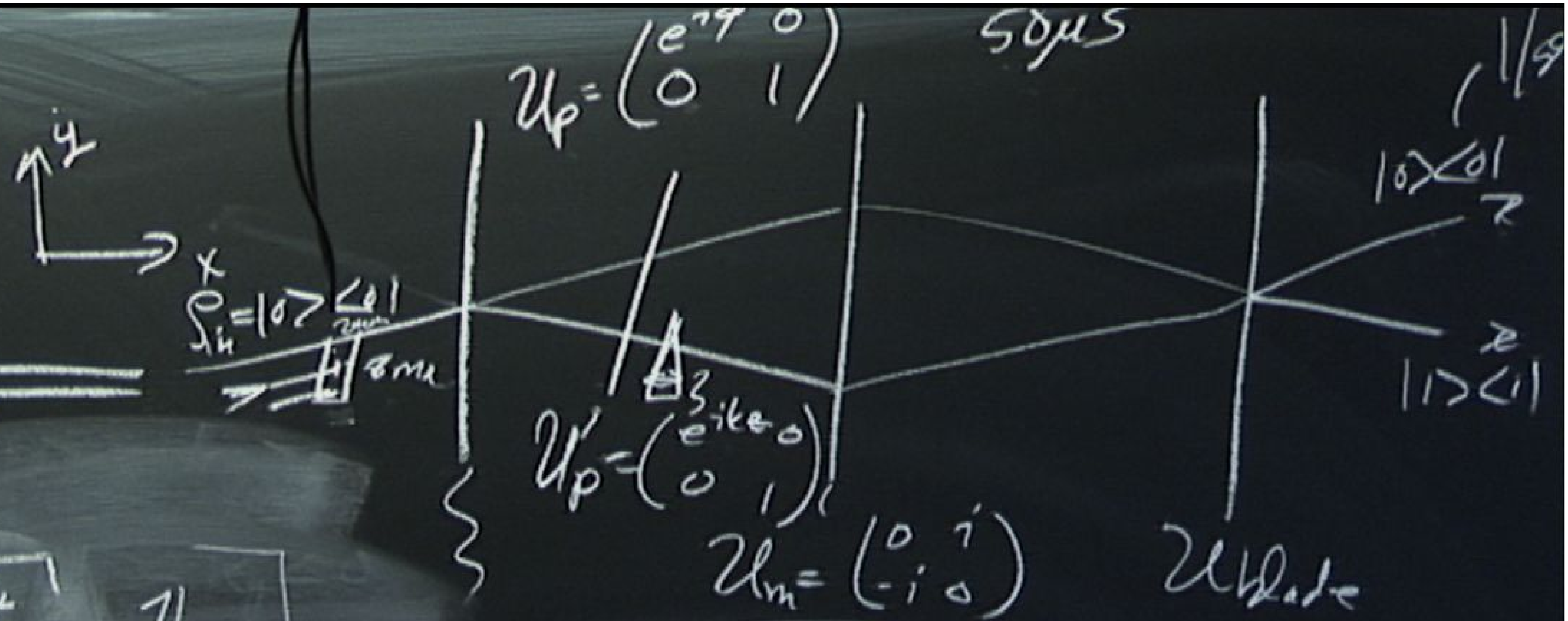
$K_1 = \sqrt{t/T_1} \; \sigma_x$

K_1^{+}

$\mathbb{1}$

$\rho(dt) = (1 - dt/T_1)\, \mathbb{1}\rho\mathbb{1} + dt/T_1\, \sigma_x \rho \sigma_x$

$$\frac{d\rho}{dt} = \frac{1}{T_1}\left[\sigma_x \rho \sigma_x - \rho\right]$$



$\omega \sigma_z$

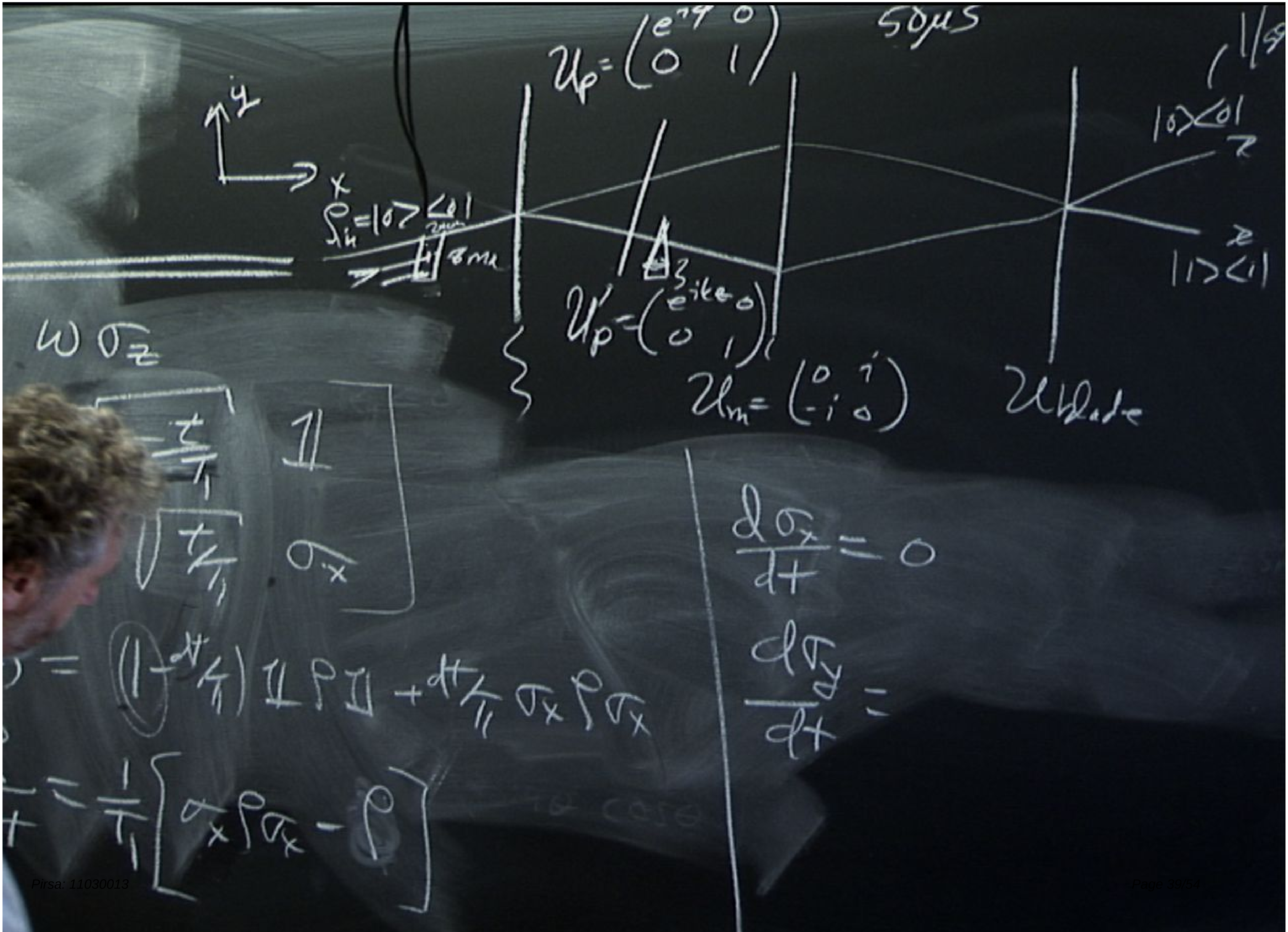
$$= \sqrt{1 - t/T_1}\; \mathbb{1}$$

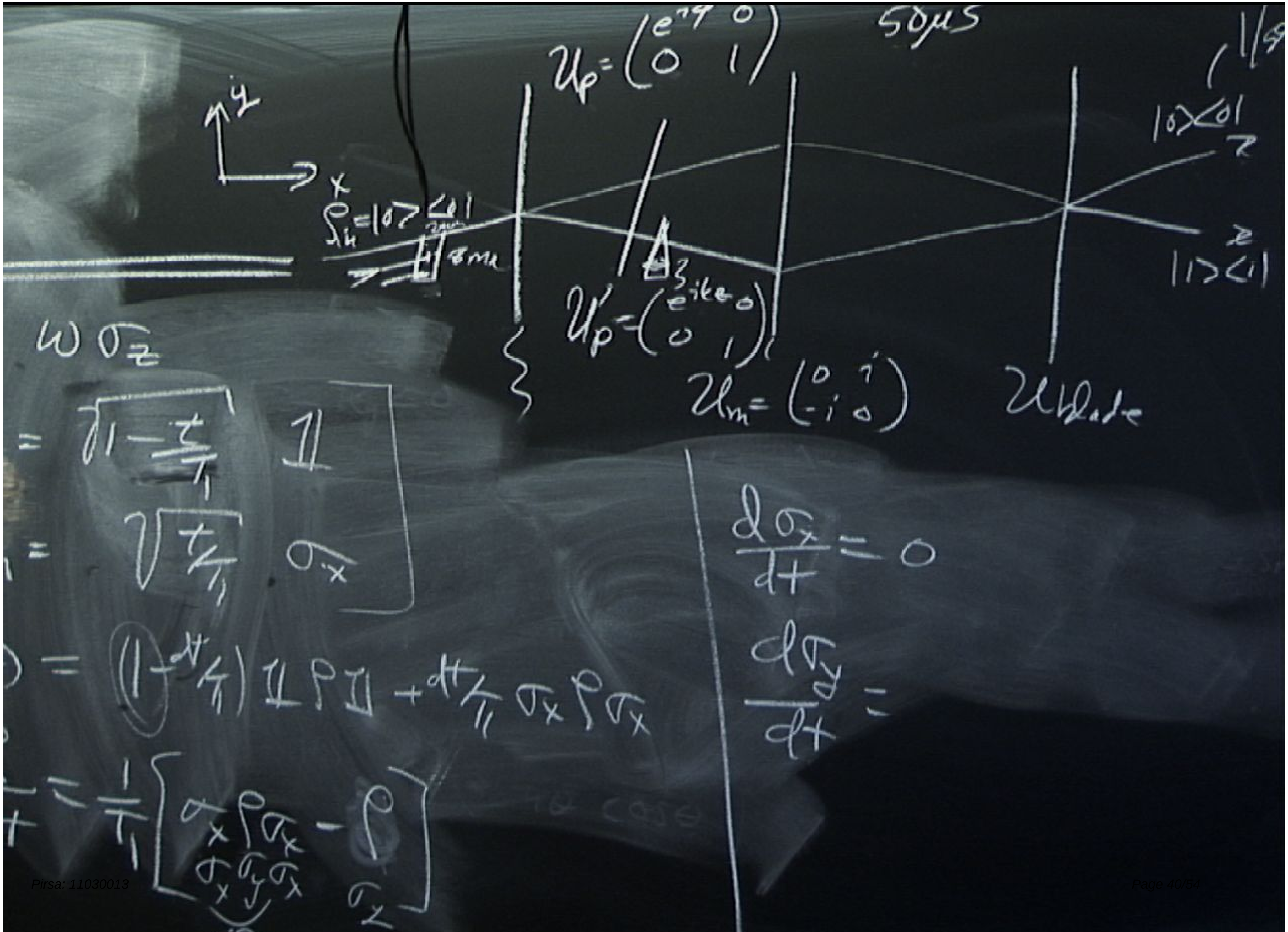
$$= \sqrt{t/T_1}\; \sigma_x$$

$$) = (1 - dt/T_1)\, \mathbb{1} \rho \mathbb{1} + dt/T_1\, \sigma_x \rho \sigma_x$$

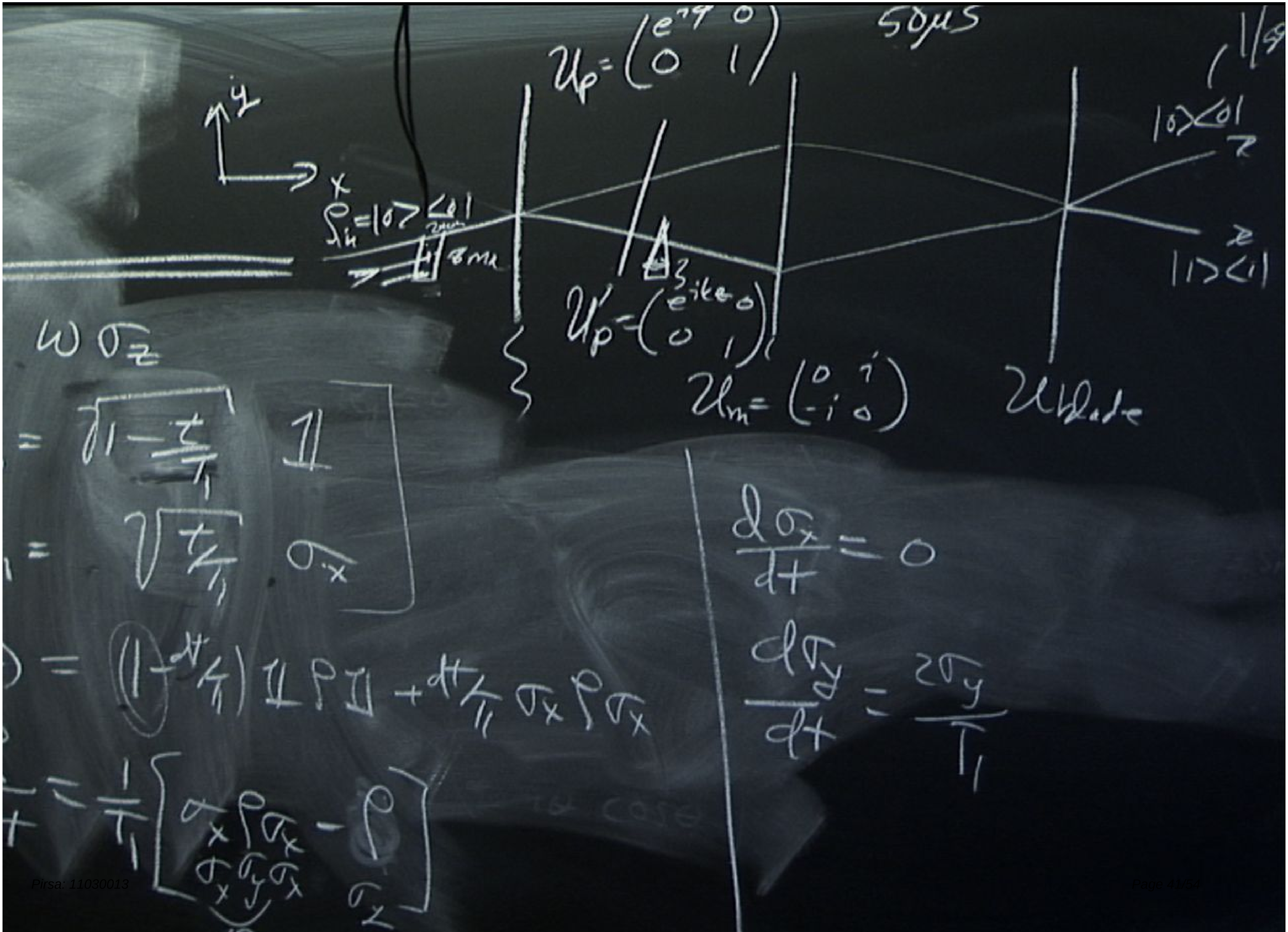
$$\frac{d\rho}{dt} = \frac{1}{T_1}\left[\sigma_x \rho \sigma_x - \rho\right]$$

$$\frac{d\sigma_x}{dt} = 0$$


50μs
$U_\phi = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{pmatrix}$
$\rho_{in} = |0\rangle\langle 0|$
8mm
$U'_\phi = \begin{pmatrix} e^{ik\xi} & 0 \\ 0 & 1 \end{pmatrix}$
$U_m = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$
$|0\rangle\langle 0|$
$|1\rangle\langle 1|$
$\omega \sigma_z$
$\frac{d\sigma_x}{dt} = 0$
$\frac{d\sigma_y}{dt} =$
$= (1 - dt/T_1)\, \mathbb{1} \rho \mathbb{1} + dt/T_1\, \sigma_x \rho \sigma_x$
$= \frac{1}{T_1}\left[\sigma_x \rho \sigma_x - \rho\right]$



50μs
$U_p = \begin{pmatrix} e^{i\varphi} & 0 \\ 0 & 1 \end{pmatrix}$
y
x
$\rho_{in} = |0\rangle\langle 0|$
8mu
$|0\rangle\langle 0|$
z
z
$|1\rangle\langle 1|$
$U_p' = \begin{pmatrix} e^{ikz} & 0 \\ 0 & 1 \end{pmatrix}$
$\omega\sigma_z$
$U_m = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$
U_{blade}
$= \sqrt{1 - t/T_1}\ \mathbb{1}$
$= \sqrt{t/T_1}\ \sigma_x$
$= (1 - dt/T_1)\ \mathbb{1}\rho\mathbb{1} + dt/T_1\ \sigma_x\rho\sigma_x$
$= \frac{1}{T_1}\left[\sigma_x\rho\sigma_x - \rho\right]$
$\sigma_x\sigma_y\sigma_x$
σ_z
$\frac{d\sigma_x}{dt} = 0$
$\frac{d\sigma_y}{dt} =$


50μs
$U_p = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{pmatrix}$
$\rho_{in} = |0\rangle\langle 0|$
$|0\rangle\langle 0|$
$|1\rangle\langle 1|$
$U_p' = \begin{pmatrix} e^{ik\epsilon} & 0 \\ 0 & 1 \end{pmatrix}$
$U_m = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$
U_{blade}
$\omega \sigma_z$
$= \sqrt{1 - t/T_1}\, \mathbb{1}$
$= \sqrt{t/T_1}\, \sigma_x$
$= (1 - dt/T_1)\, \mathbb{1} \rho \mathbb{1} + dt/T_1\, \sigma_x \rho \sigma_x$
$= \frac{1}{T_1} [\sigma_x \rho \sigma_x - \rho]$
$\frac{d\sigma_x}{dt} = 0$
$\frac{d\sigma_y}{dt} = \frac{2\sigma_y}{T_1}$

$$\mathcal{U}_m = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$$

$$\boxed{\begin{matrix} \frac{t}{T_1} & \mathbb{1} \\ \frac{t}{T_1} & \sigma_x \end{matrix}}$$

$$\left(1 - \frac{dt}{T_1}\right) \mathbb{1} \rho \mathbb{1} + \frac{dt}{T_1} \sigma_x \rho \sigma_x$$

$$\left[\sigma_x \rho \sigma_x - \rho\right]$$

$$\sigma_x \sigma_y \sigma_x \qquad \sigma_z$$

$$i\sigma_z$$

$$\sigma_x$$

no case

$$\frac{d\sigma_x}{dt} = 0$$

$$\frac{d\sigma_y}{dt} = \frac{2\sigma_y}{T_1}$$

$$\frac{d\sigma_z}{dt} = \frac{2\sigma_z}{T_1}$$

$\rho_{in} = |0\rangle\langle 0|$

$U_p' = \begin{pmatrix} e^{ik\theta} & 0 \\ 0 & 1 \end{pmatrix}$

$U_m = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$ $\quad U_{blade}$

$|1\rangle\langle 1|$ $\quad H$

$\mathbb{1}$

σ_x

$\mathbb{1}\rho\mathbb{1} + {}^{dt}/_{T_1}\, \sigma_x \rho \sigma_x$

$\rho\sigma_x - \rho$

$\sigma_y\sigma_x \quad \sigma_x$

Bloch's Eqn

$$\frac{d\langle\sigma_x\rangle}{dt} = 0$$

$$\frac{d\langle\sigma_y\rangle}{dt} = \frac{2\langle\sigma_y\rangle}{T_1}$$

$$\frac{d\langle\sigma_z\rangle}{dt} = \frac{2\langle\sigma_z\rangle}{T_1}$$

$U_p' = \begin{pmatrix} e^{ik\theta} & 0 \\ 0 & 1 \end{pmatrix}$

$U_m = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$ U_{blade}

$\mathbb{1}$

σ_x

$) \mathbb{1} \rho \mathbb{1} + {}^{dt}/_{T_1} \sigma_x \rho \sigma_x$

$\rho \sigma_x - \rho$

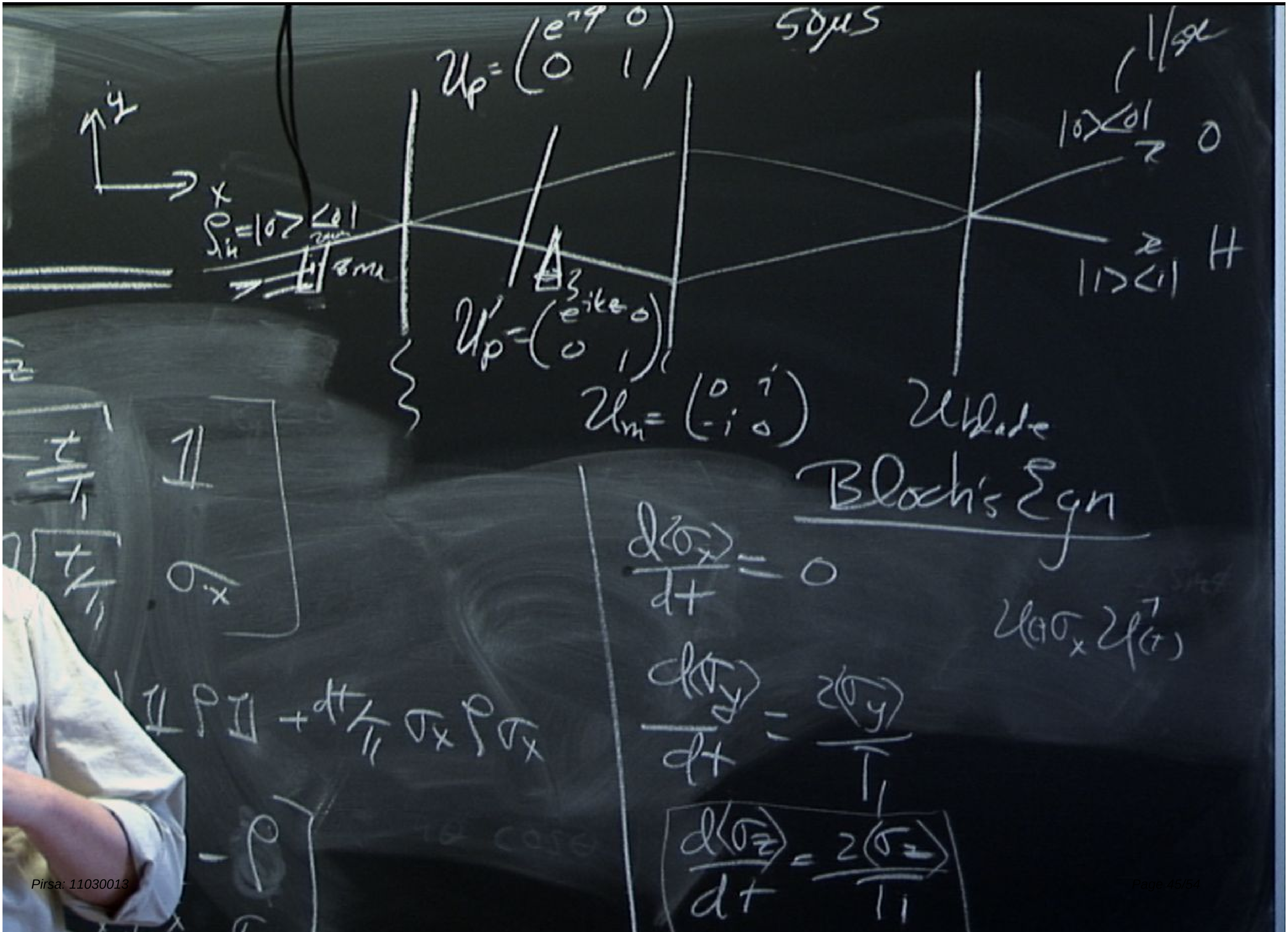
$\sigma_y \sigma_x \quad \sigma_z$

<u>Bloch's Eqn</u>

$$\frac{d\langle\sigma_x\rangle}{dt} = 0$$

$$\frac{d\langle\sigma_y\rangle}{dt} = \frac{2\langle\sigma_y\rangle}{T_1}$$

$$\boxed{\frac{d\langle\sigma_z\rangle}{dt} = \frac{2\langle\sigma_z\rangle}{T_1}}$$


50μs
y
x
O
H
|0⟩⟨0|
|1⟩⟨1|
Bloch's Eqn
d⟨σx⟩/dt = 0
d⟨σy⟩/dt = 2⟨σy⟩/T1
d⟨σz⟩/dt = 2⟨σz⟩/T1
U(t)σx U†(t)

$50\mu s$

$U_p = \begin{pmatrix} e^{i\varphi} & 0 \\ 0 & 1 \end{pmatrix}$

$1/\sqrt{2}$

y, x

$\rho_{in} = |0\rangle\langle 0|$

$|0\rangle\langle 0|$ $\; 0$

$|1\rangle\langle 1|$ $\; H$

Δz

$U_p' = \begin{pmatrix} e^{ik z_0} & 0 \\ 0 & 1 \end{pmatrix}$

$U_m = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$ $\qquad U_{Hade}$

$\frac{t}{T_1}$ $\quad \mathbb{1}$

$\frac{t}{T_1}$ $\quad \sigma_x$

$(1 - dt/T_1)\, \mathbb{1}\rho\mathbb{1} + dt/T_1\, \sigma_x \rho \sigma_x$

$[\sigma_x \rho \sigma_x - \rho$

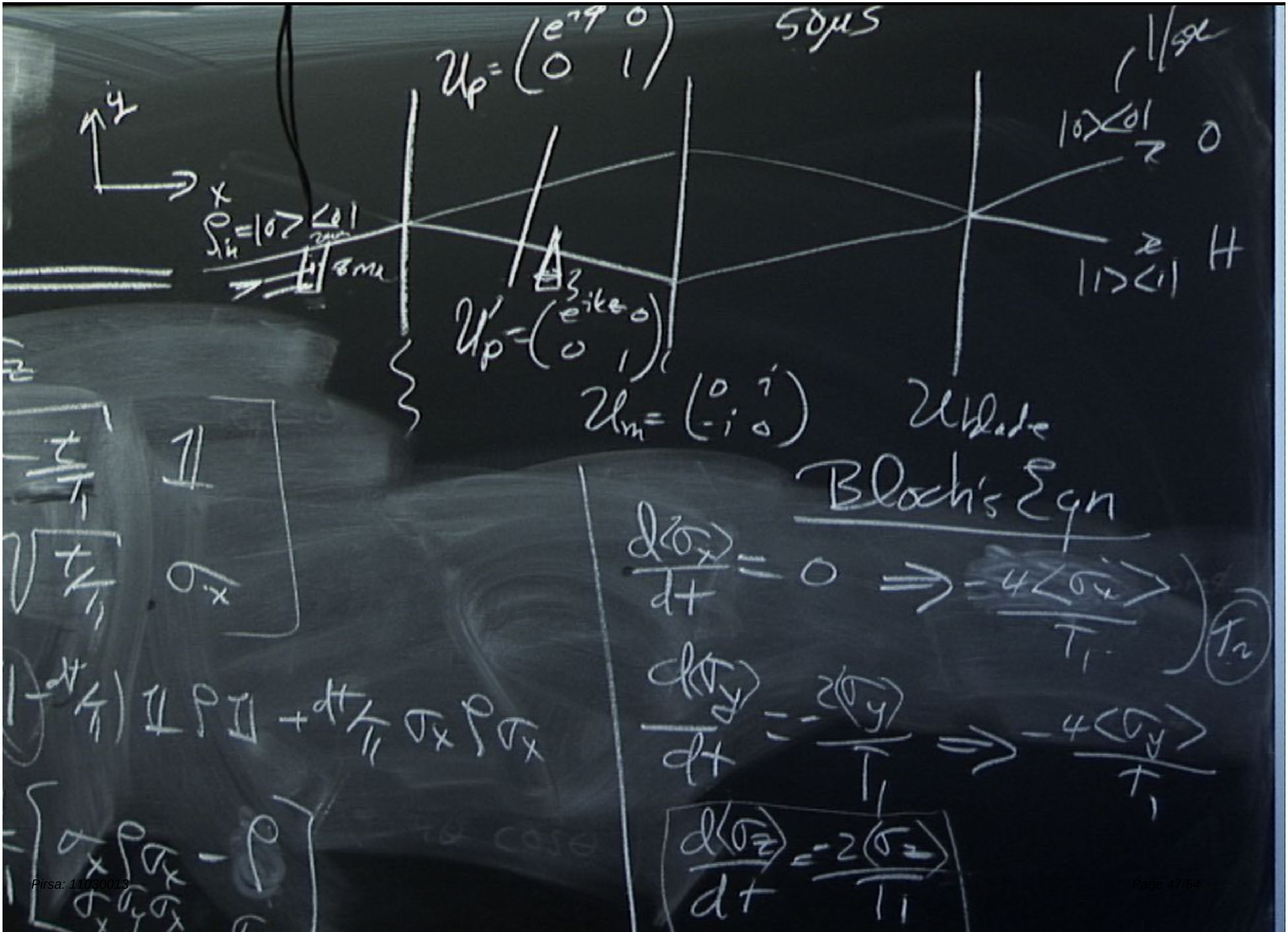
$\sigma_x \sigma_y \sigma_x$

Bloch's Eqn

$$\frac{d\langle\sigma_x\rangle}{dt} = 0 \Rightarrow -\frac{4\langle\sigma_x\rangle}{T_1}$$

$$\frac{d\langle\sigma_y\rangle}{dt} = -\frac{2\langle\sigma_y\rangle}{T_1} \Rightarrow -\frac{4\langle\sigma_y\rangle}{T_1}$$

$$\boxed{\frac{d\langle\sigma_z\rangle}{dt} = -\frac{2\langle\sigma_z\rangle}{T_1}}$$


50μs
Bloch's Eqn
H
0
x
y

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QE Lec 2 (Modified).nb

▪ Decoherence: z-axis noise

Note that this is the standard dephasing picture, with T_2 the rate of decoherence and t the time that the decoherence acts for.
If $t = 0$, then only the first operator is present and it is proportional to the identity. On the other hand, if T_2 , then both operators are present with equal probability.
Both operators commute with σ_z, so there is no relaxation of the z magnetization, even in the presence of a Zeeman frequency.
It is important to note that if the quantization axis is not along z then there would be T_1 relaxation.

$$\text{KT2[1,t\_,T2\_]} := \sqrt{\frac{1+e^{-t/T2}}{2}}\ \text{PauliMatrix[0];}$$

$$\text{KT2[2,t\_,T2\_]} := \sqrt{\frac{1-e^{-t/T2}}{2}}\ \text{PauliMatrix[3];}$$

Note that the above operators should be read as the square-root of a probability times a jump operator. Clearly the probabilities add to 1. The probabilities appear as square-roots since the operators sandwich the density matrix when applied.

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QE Lec 2 (Modified).nb

Notice that pure T_2 relaxation (that is, relaxation with $\omega = 0$) is diagonal and leads to attenuation of the σ_x and σ_y terms.

```
Sdec[0, 0.001, 0.2] // Chop // MatrixForm
```

$$\begin{pmatrix} 1. & 0 & 0 & 0 \\ 0 & 0.995012 & 0 & 0 \\ 0 & 0 & 0.995012 & 0 \\ 0 & 0 & 0 & 1. \end{pmatrix}$$

Adding precession mixes σ_x and σ_y, but the attenuation is exactly that seen in the case of pure T_2.

```
Sdec[100, 0.001, 0.2] // Chop // MatrixForm
```

$$\begin{pmatrix} 1. & 0 & 0 & 0 \\ 0 & 0.990042 & -0.0993355 & 0 \\ 0 & 0.0993355 & 0.990042 & 0 \\ 0 & 0 & 0 & 1. \end{pmatrix}$$

```
Sqrt[(Sdec[100, 0.001, 0.2][[2, 2]])^2 +
   (Sdec[100, 0.001, 0.2][[2, 3]])^2] // N
```

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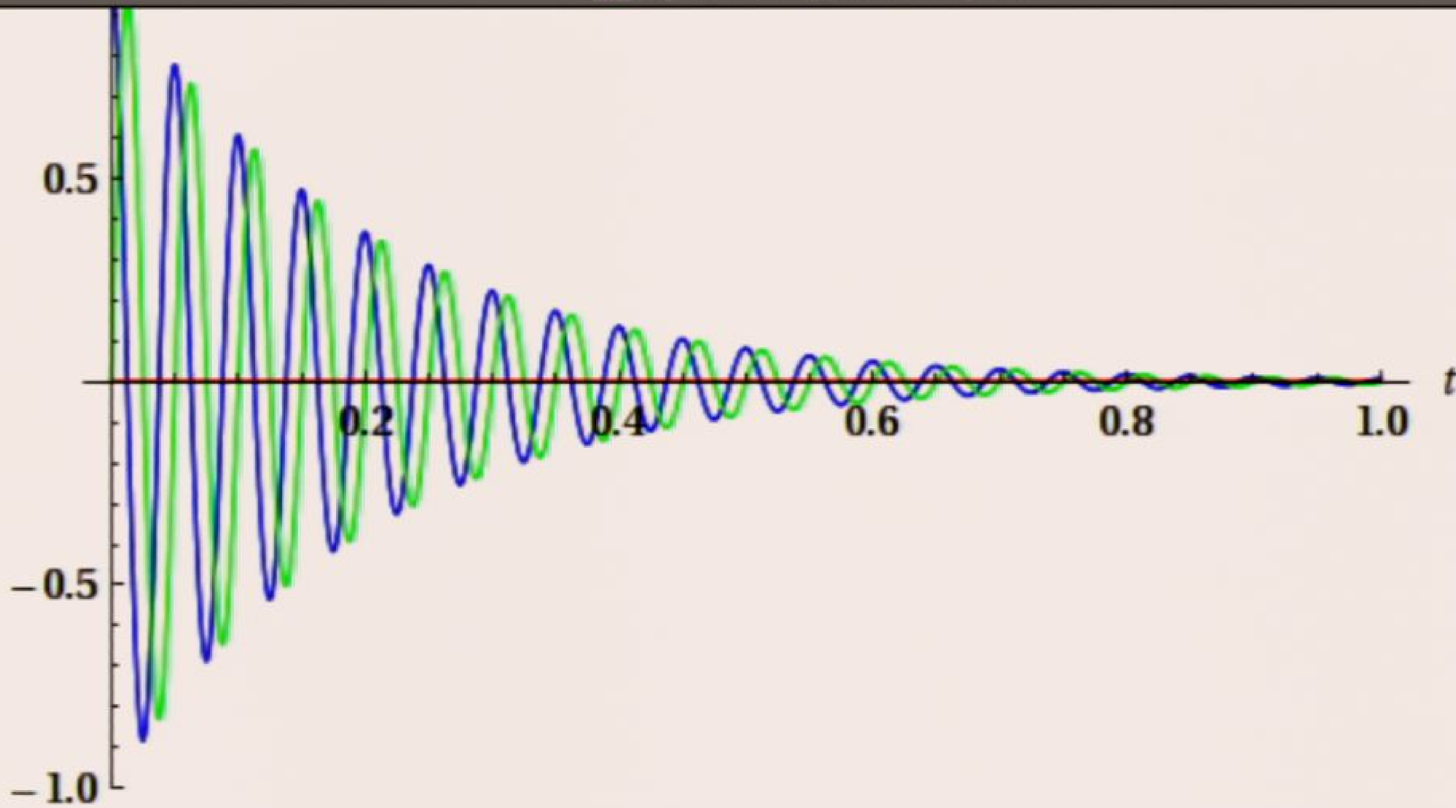
QE Lec 2 (Modified).nb



▪ depolarizing x&y evolution via super-operator

Now the Kraus operators and the system propagator do not commute at all times and we must compose the evolution from small steps.

```
pmdep[0]   =  {0,1,0,1};
pmdep[n_] := pmdep[n] = Sdepxy[2 π 25, 0.001, 0.2] . pmdep[
```

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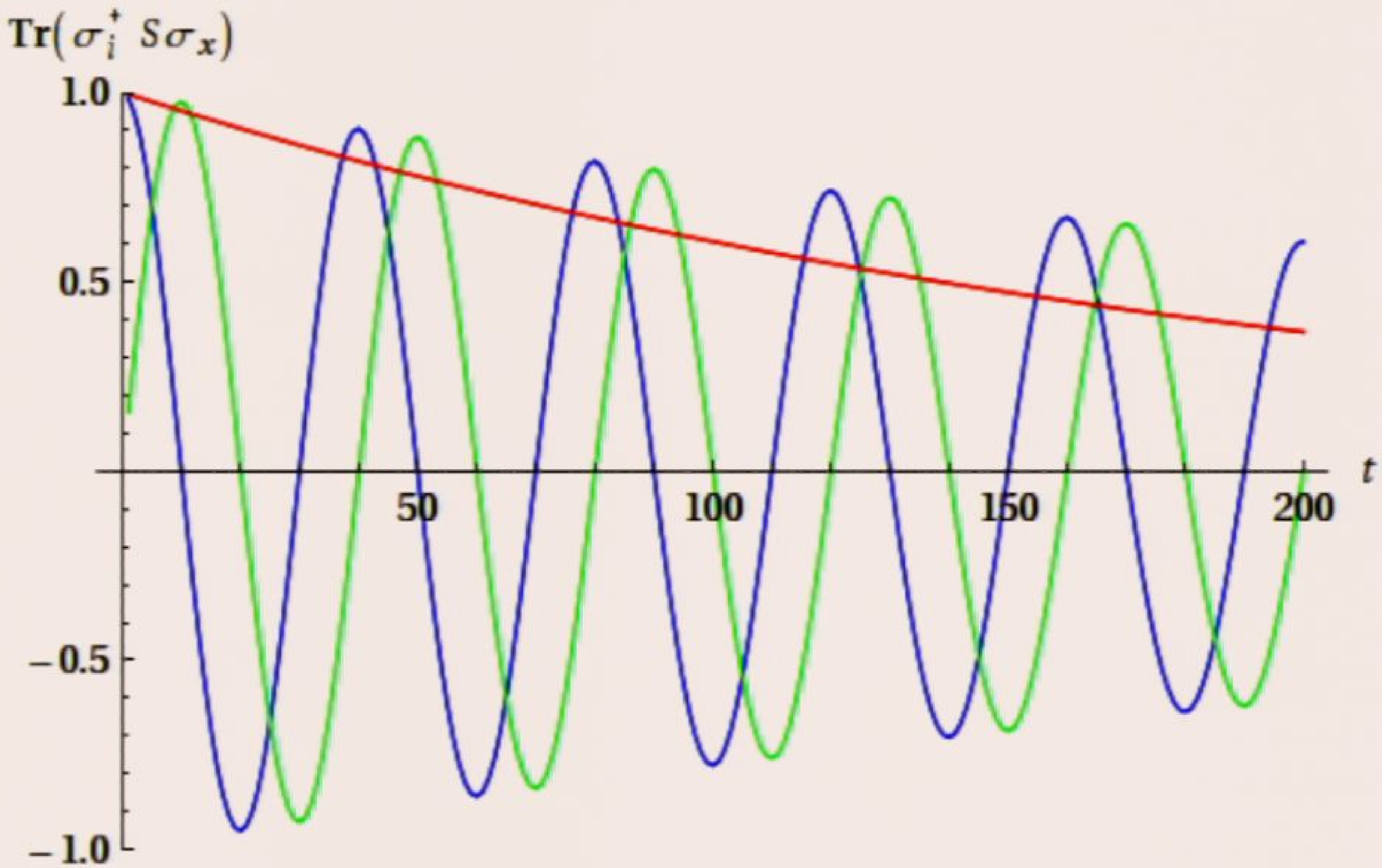
QE Lec 2 (Modified).nb

```
    Directive[RGBColor[1, 0, 0], Thickness[0.003]],
   }
  ]
 ]
```



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QE Lec 2 Choi.nb

```
ElemMat[3] := {{0, 0}, {1, 0}};
ElemMat[4] := {{0, 0}, {0, 1}};
```

Transformation Matrix: Pauli to Transition

```
BasisTrans := {{1, 0, 0, 1}, {0, 1, i, 0}, {0, 1, -i, 0}, {1, 0, 0, -1}}
```

Matrix

```
Choi[mat_] := {{mat[[1, 1]], mat[[3, 1]], mat[[1, 3]], mat[[3, 3]]},
               {mat[[2, 1]], mat[[4, 1]], mat[[2, 3]], mat[[3, 4]]},
               {mat[[1, 2]], mat[[3, 2]], mat[[1, 4]], mat[[4, 3]]},
               {mat[[2, 2]], mat[[4, 2]], mat[[2, 4]], mat[[4, 4]]}};
```

us Representation

lculate the Kraus representation we will first transform the superoperator to the transition basis

```
SupOpTrans = BasisTrans . S . Inverse[BasisTrans];
```

we calculate the Choi matrix

```
ChoiS = Choi[SupOpTrans];
```

raus operators are given by the eigenvectors of the Choi matrix

$$\texttt{KrausOps} = \sqrt{2}\ \texttt{Eigenvectors[ChoiS];}$$

robabilities associated with each Kraus operator are given by the eigenvalues of the Choi matrix

$$\texttt{KrausProbs} = \frac{1}{2}\ \texttt{Eigenvalues[ChoiS];}$$

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

QE Lec 2 Choi.nb

(raus operators are given by the eigenvectors of the Choi matrix

```
KrausOps = √2 Eigenvectors[ChoiS];
```

robabilities associated with each Kraus operator are given by the eigenvalues of the Choi matrix

```
KrausProbs = 1/2 Eigenvalues[ChoiS];
```

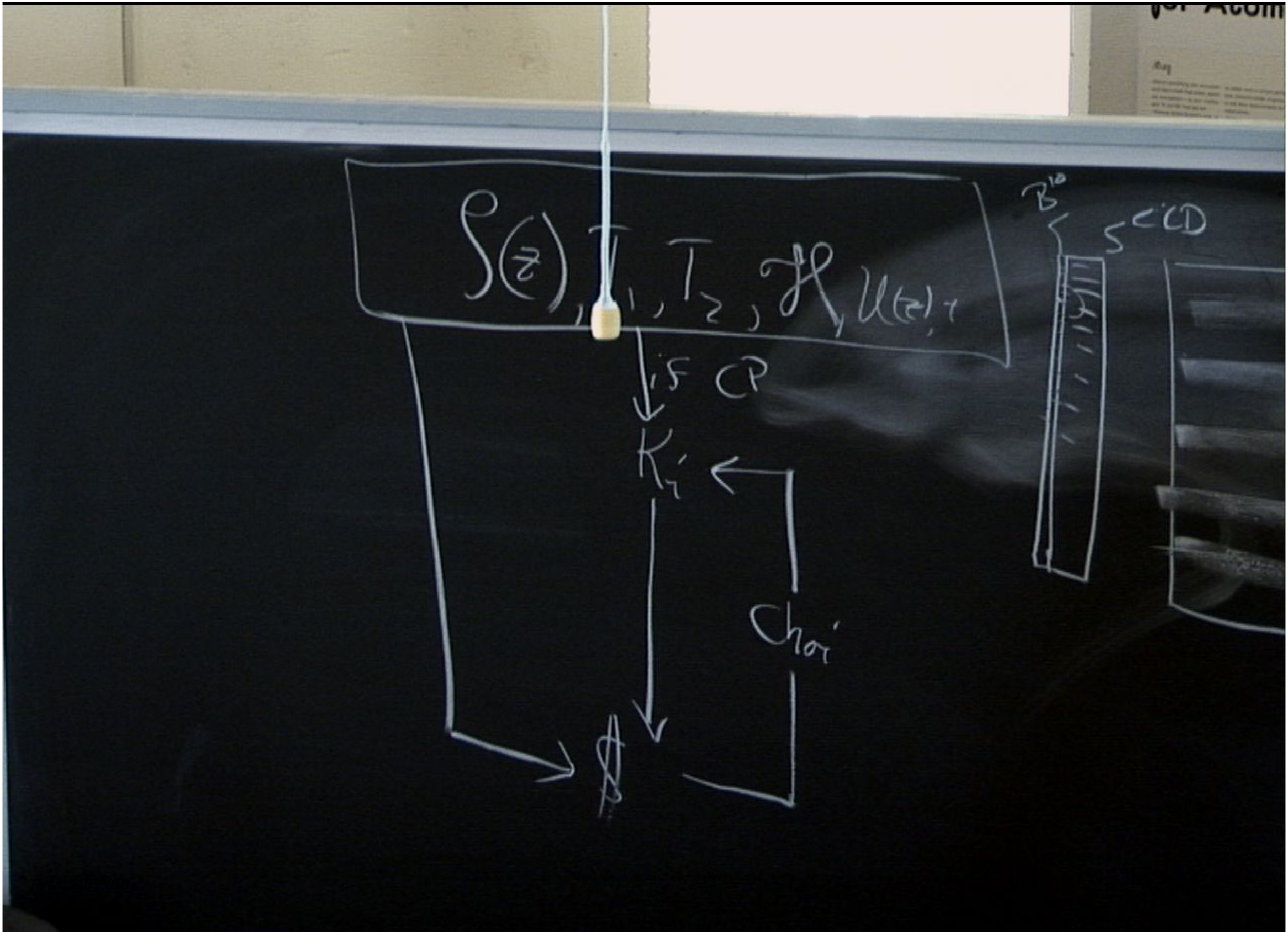
```
MatrixForm[KrausProbs]
```

$$\begin{pmatrix} 0.925 \\ 0.025 \\ 0.025 \\ 0.025 \end{pmatrix}$$

```
MatrixForm[KrausOps]
```

$$\begin{pmatrix} 1. & 0. & 0. & 1. \\ 0. & 1.41421 & 0. & 0. \\ 0. & 0. & 1.41421 & 0. \\ 1. & 0. & 0. & -1. \end{pmatrix}$$

: see as we expect that the Kraus sum for a model of only relaxation with T1 = T2 is an isotropic noise model.
irst row corresponds to the identity matrix, the four to σ_z, and the second and third to symmetric combinations of x
.


ρ(t), T1, T2, H, U(t), τ
if CP
Ki
Choi
$