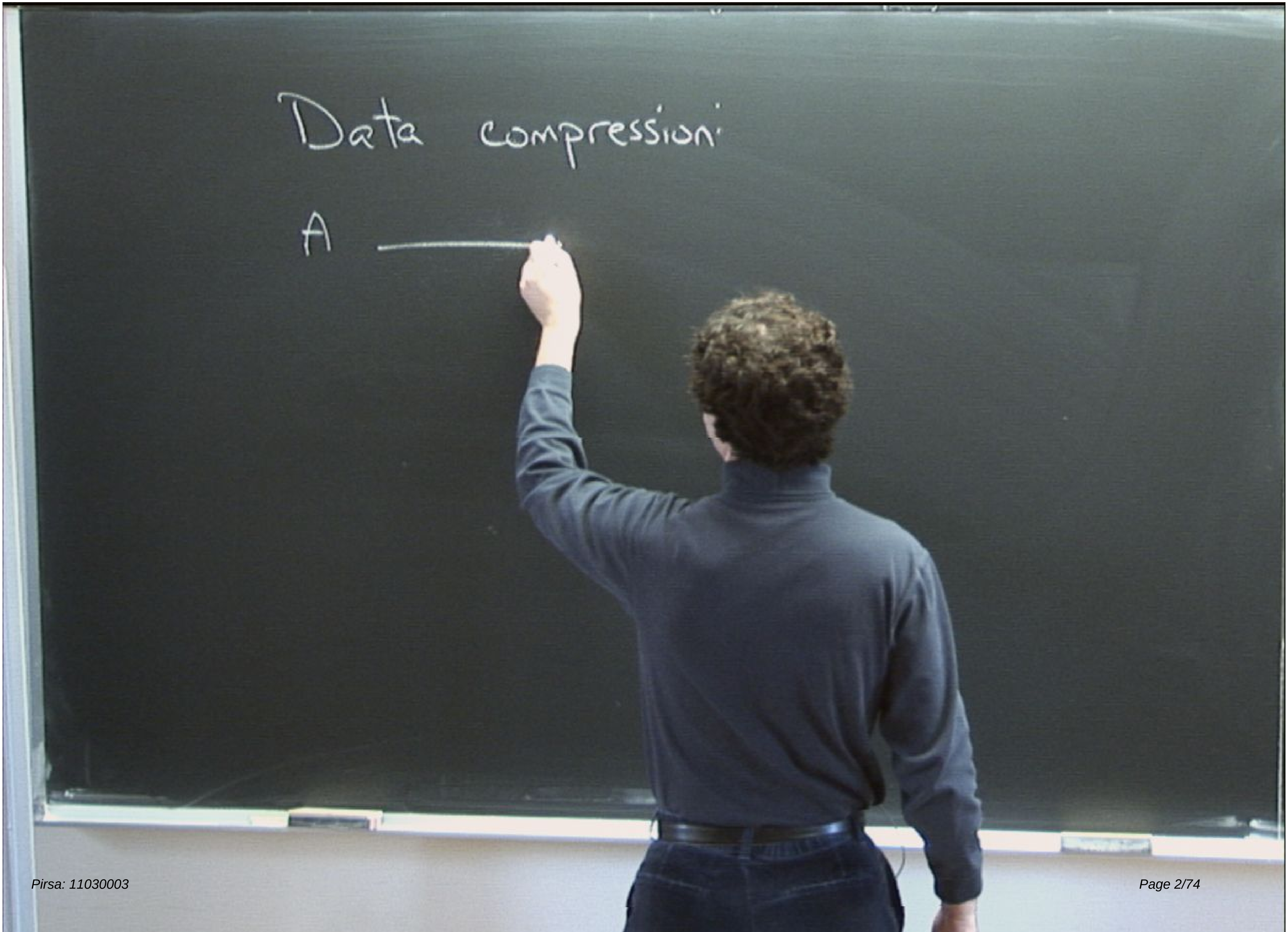
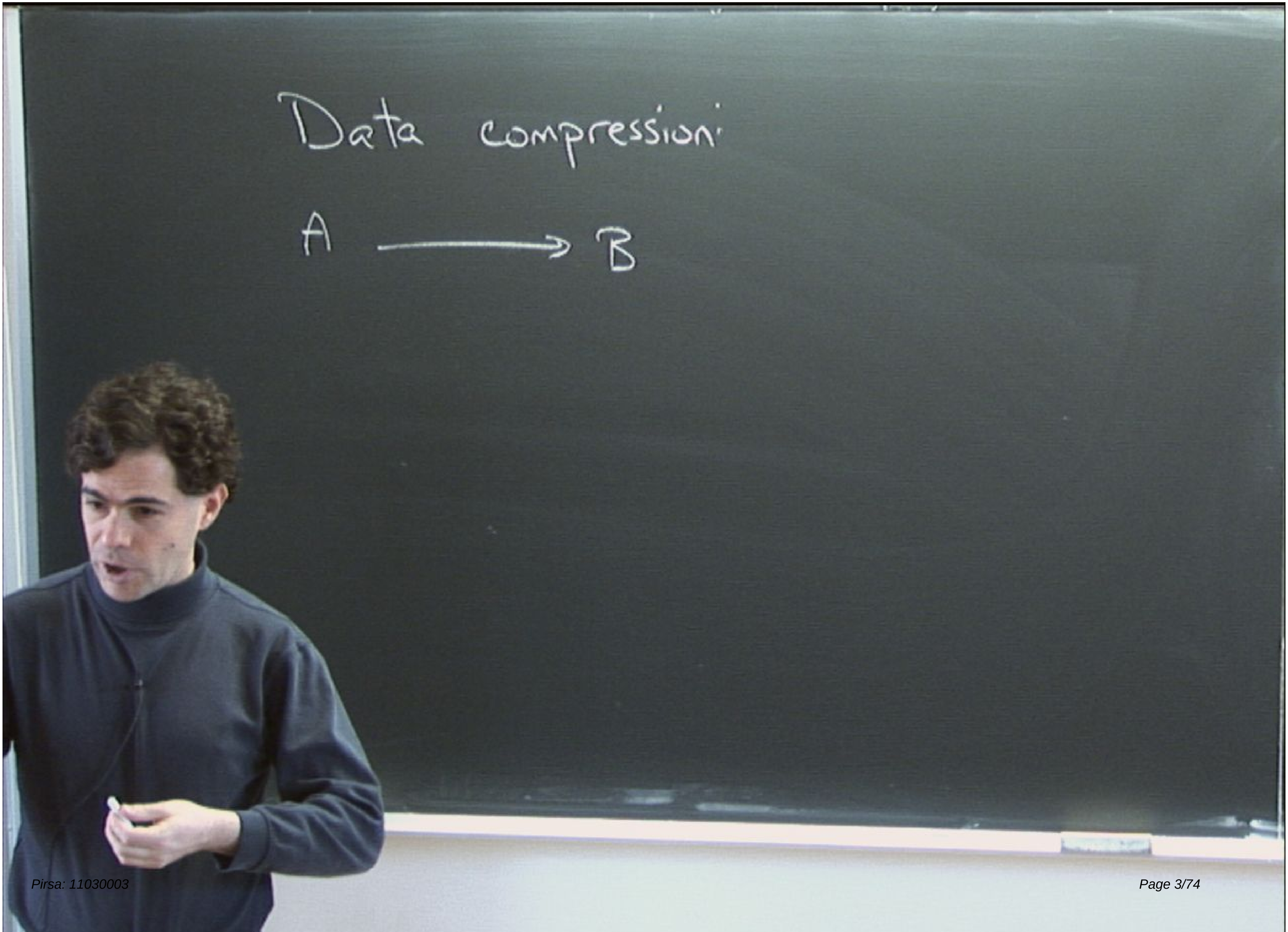
Title: Quantum Information Review - Lecture 14

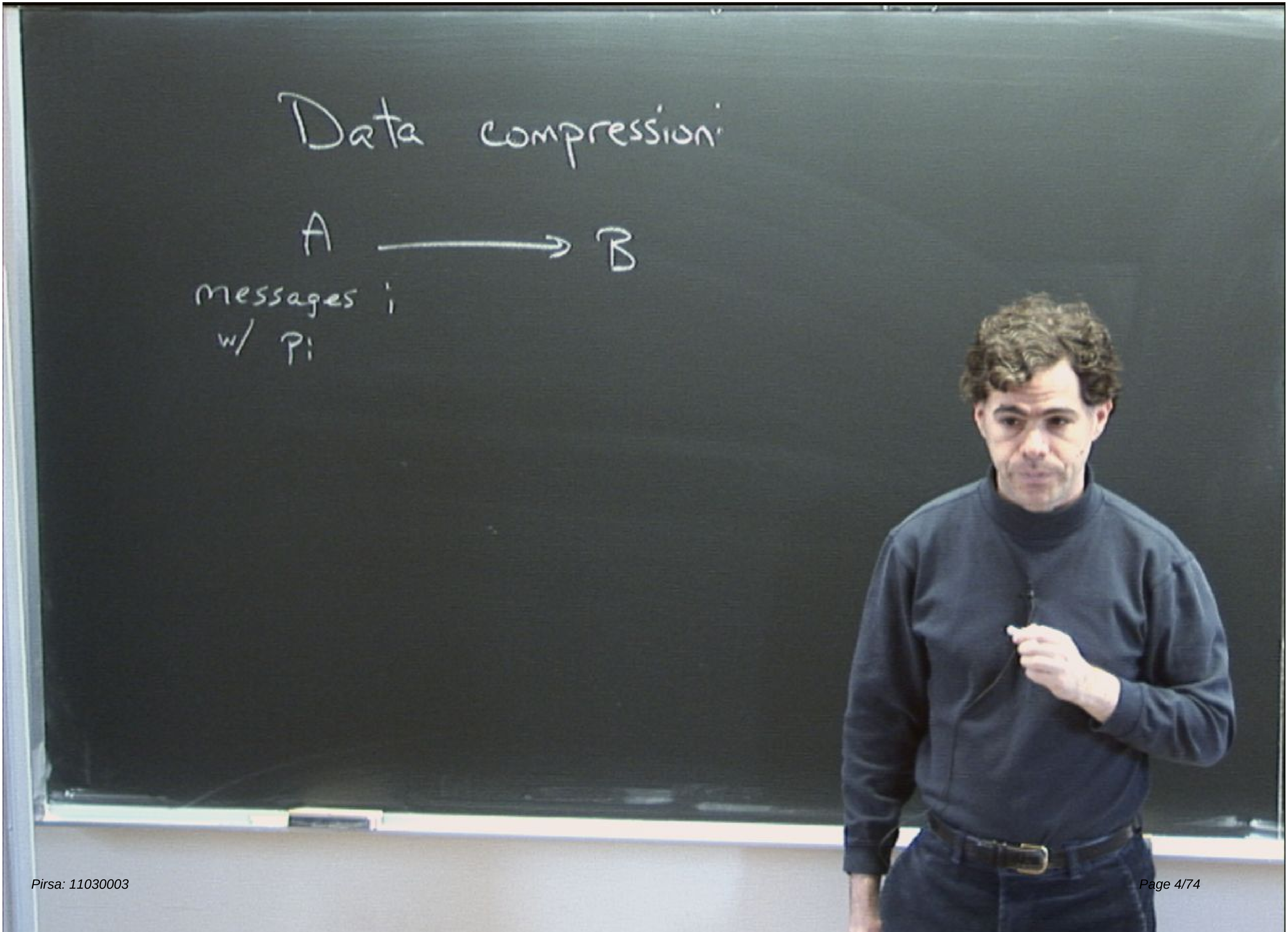
Date: Mar 04, 2011 09:00 AM

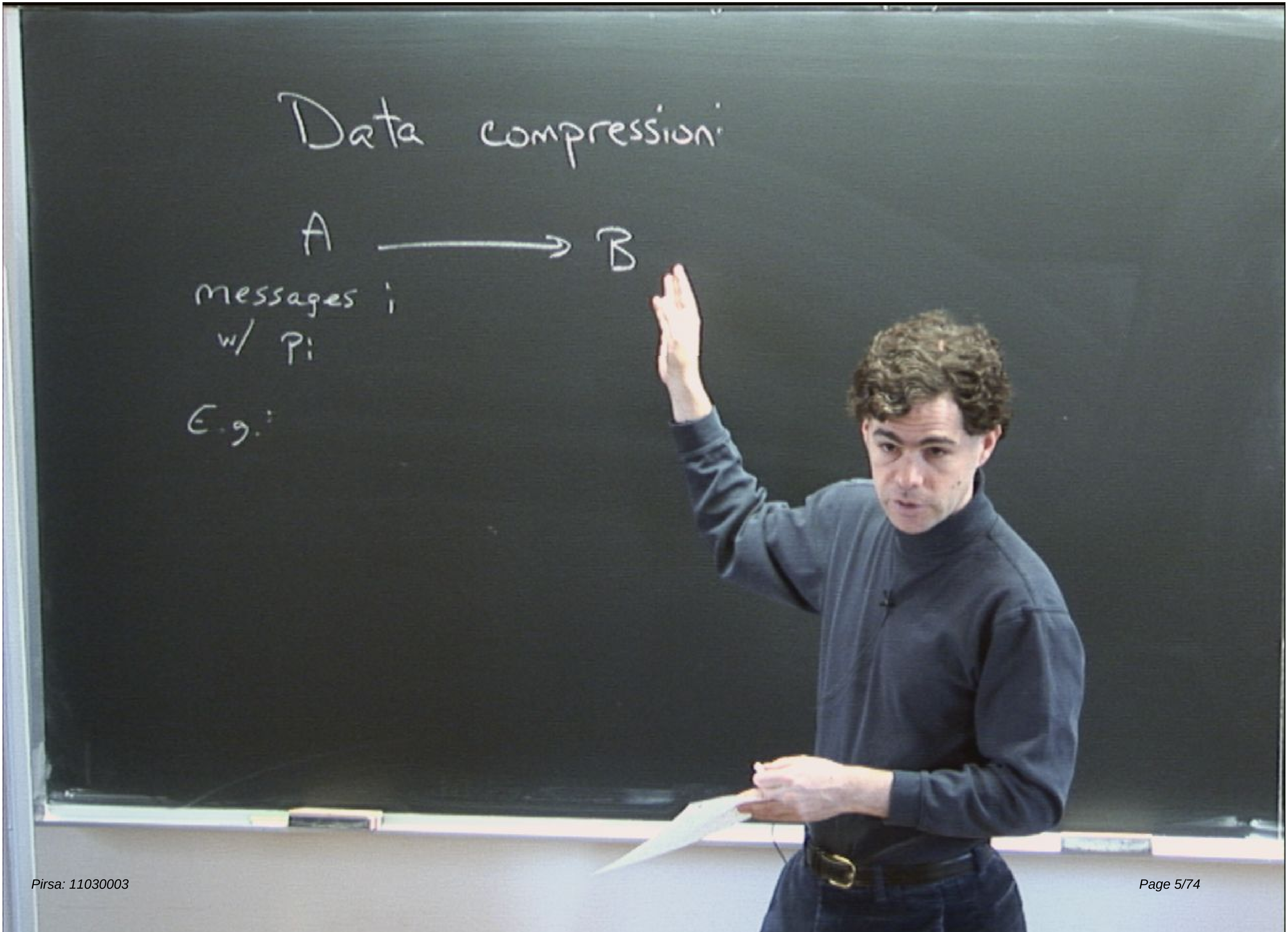
URL: http://pirsa.org/11030003

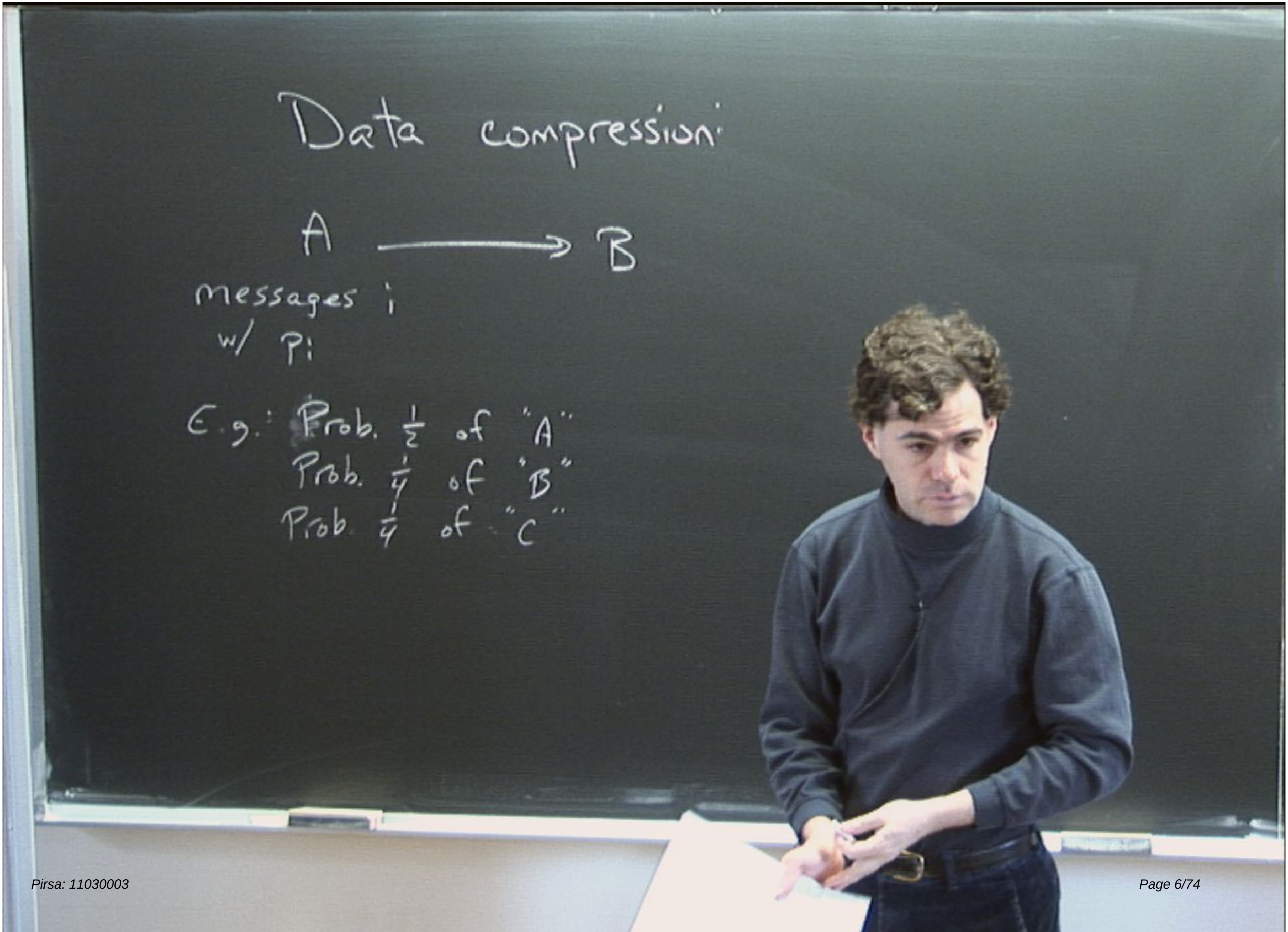
Abstract:

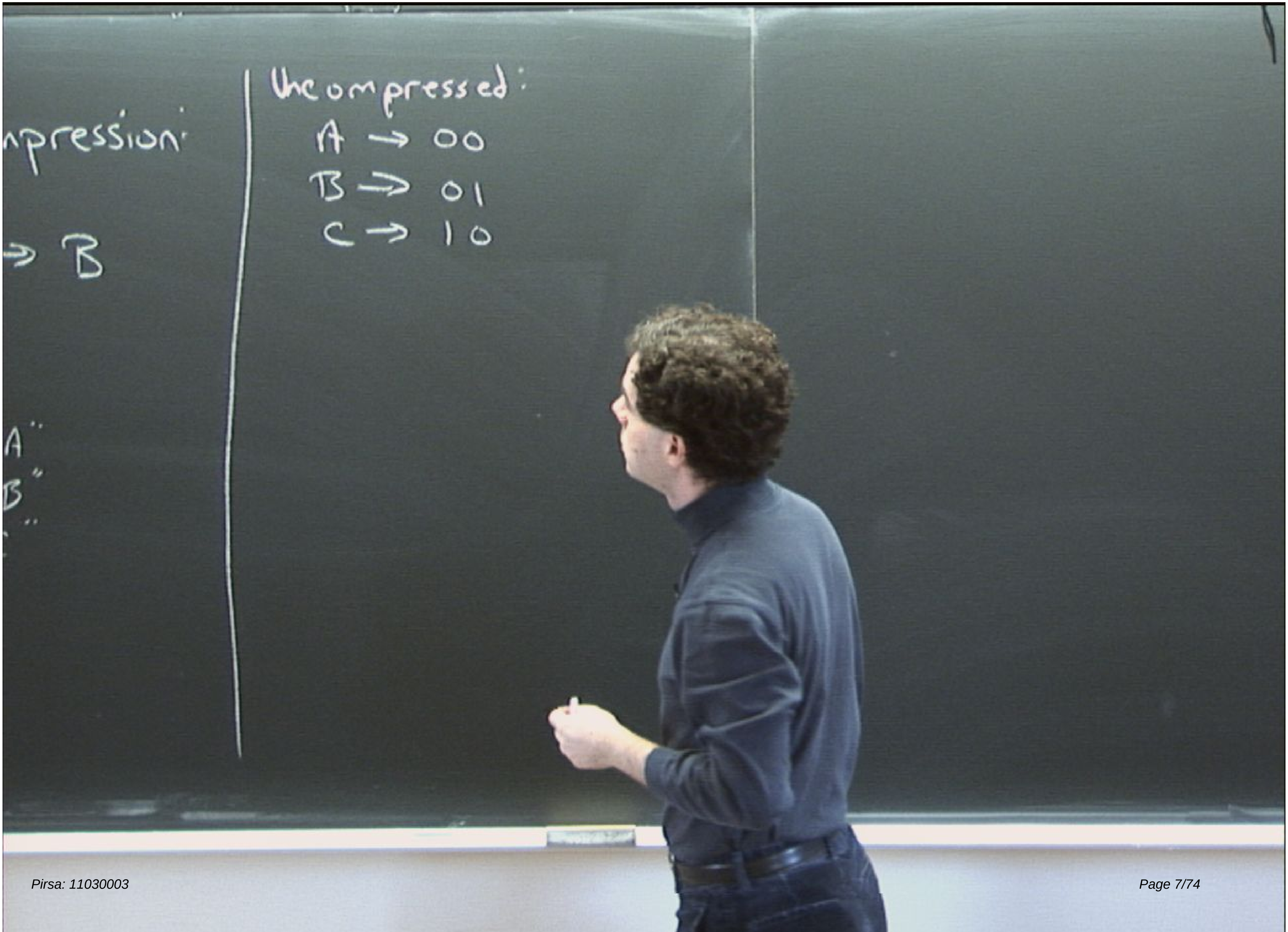

Data compression
A

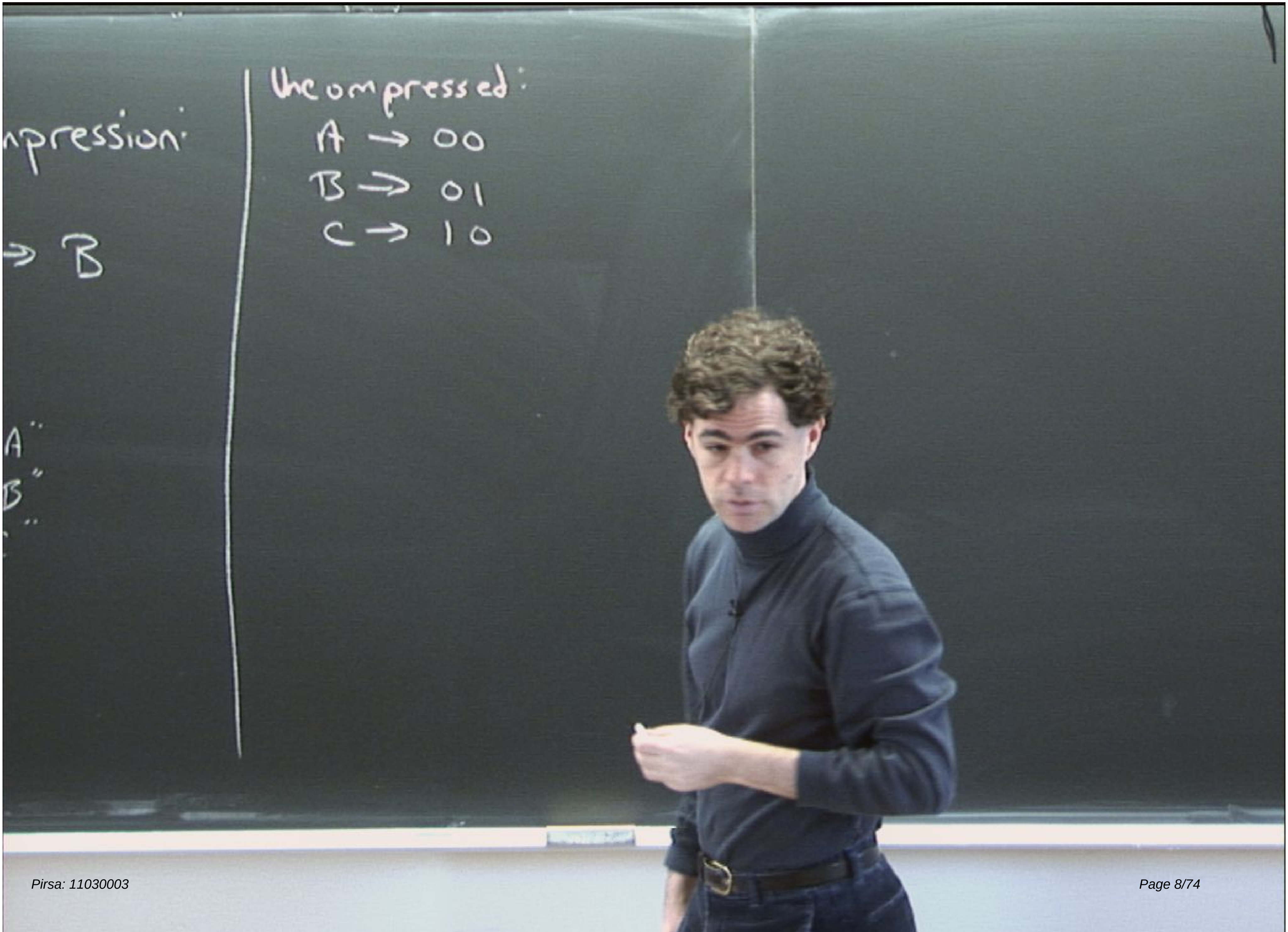

Data compression:
A ⟶ B


Data compression
A → B
messages i
w/ p_i


Data compression:
A → B
messages i
w/ p_i
E.g.:


Data compression:
A → B
messages i
w/ pi
E.g.: Prob. 1/2 of "A"
Prob. 1/4 of "B"
Prob. 1/4 of "C"


Uncompressed:
A → 00
B → 01
C → 10
⇒ B


Uncompressed:
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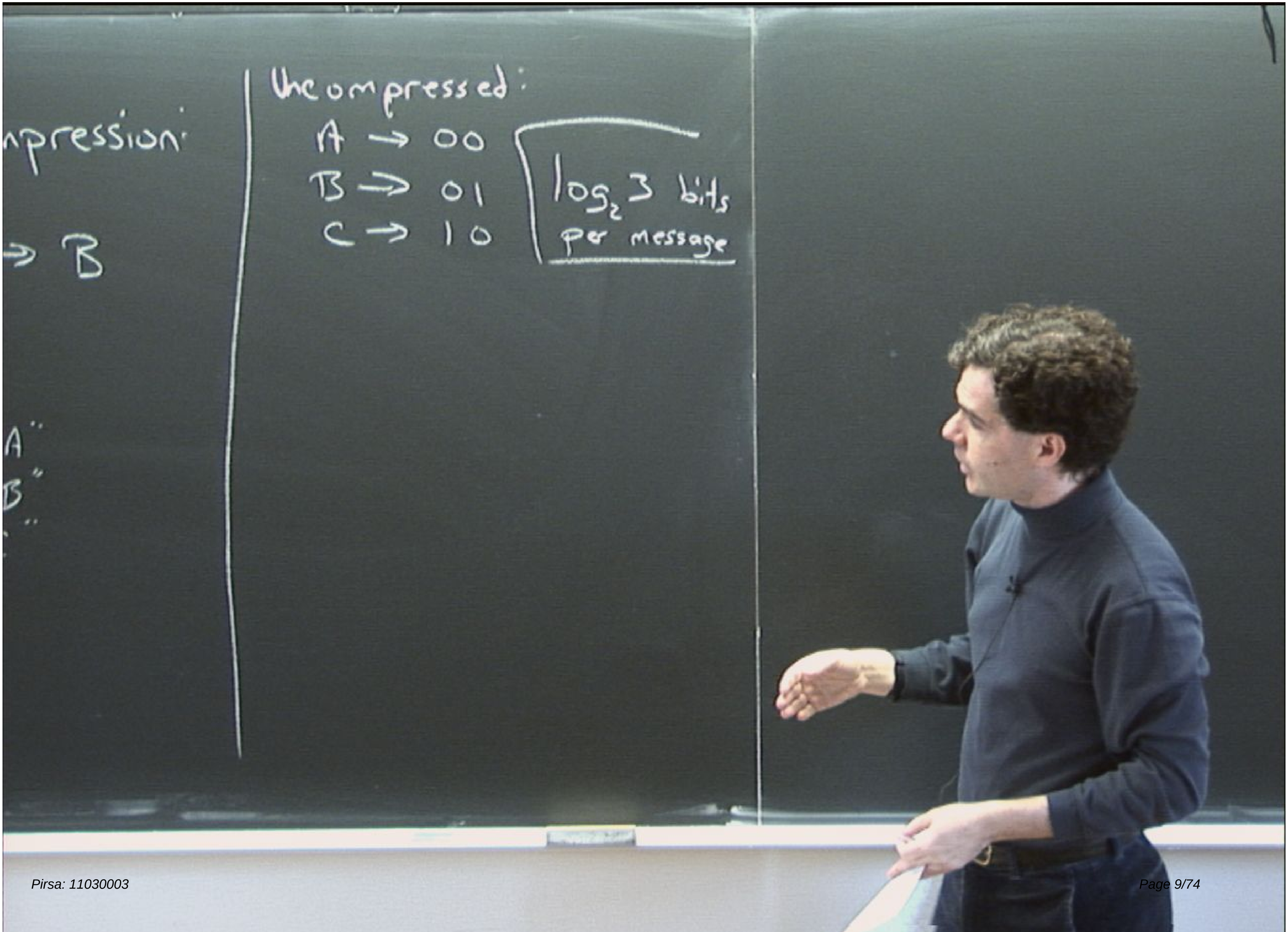
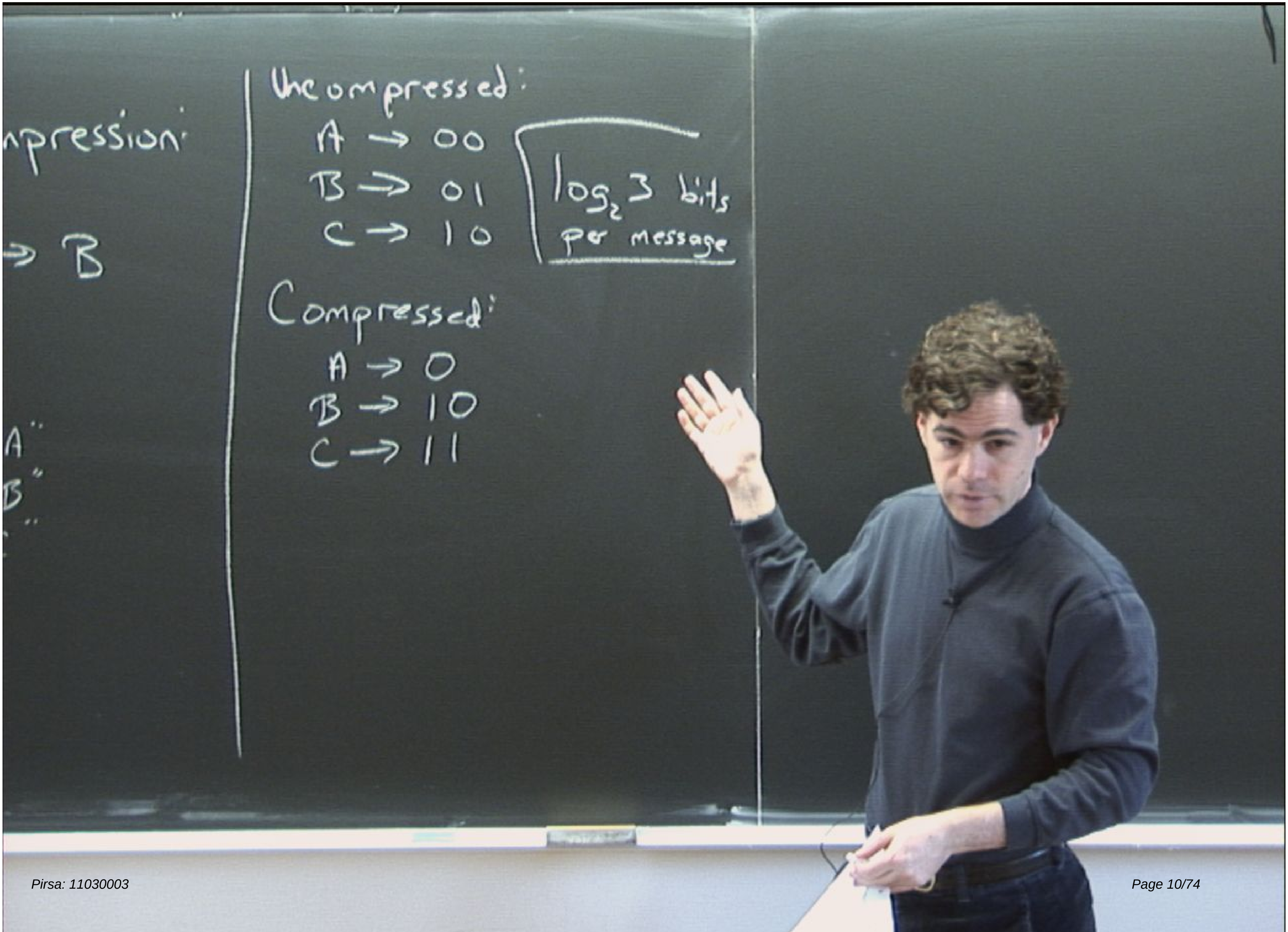
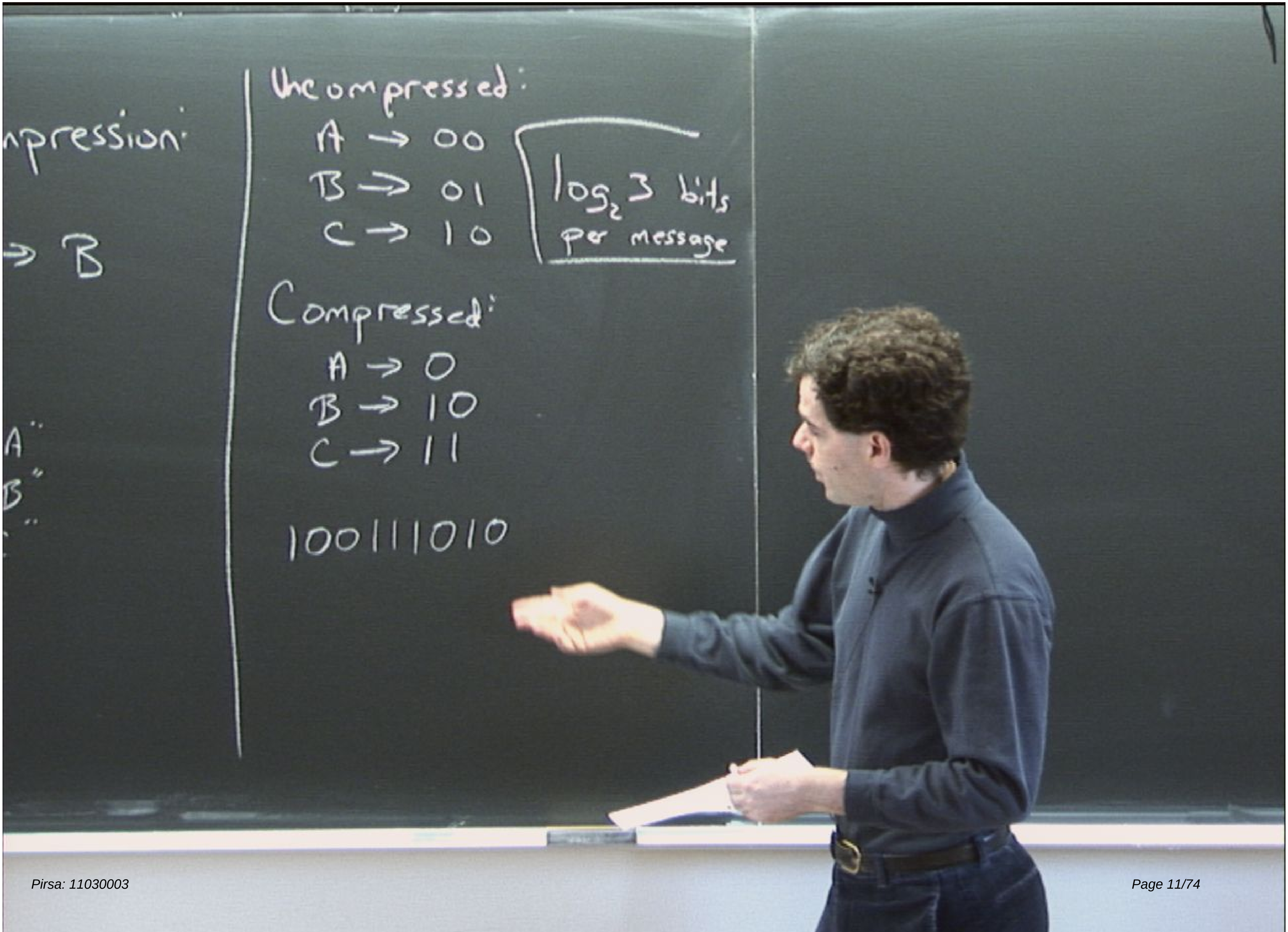

Uncompressed:
A → 00
B → 01
C → 10
$\log_2 3$ bits per message
⇒ B


Uncompressed:
A → 00
B → 01
C → 10
$\log_2 3$ bits per message
Compressed:
A → 0
B → 10
C → 11



Uncompressed:
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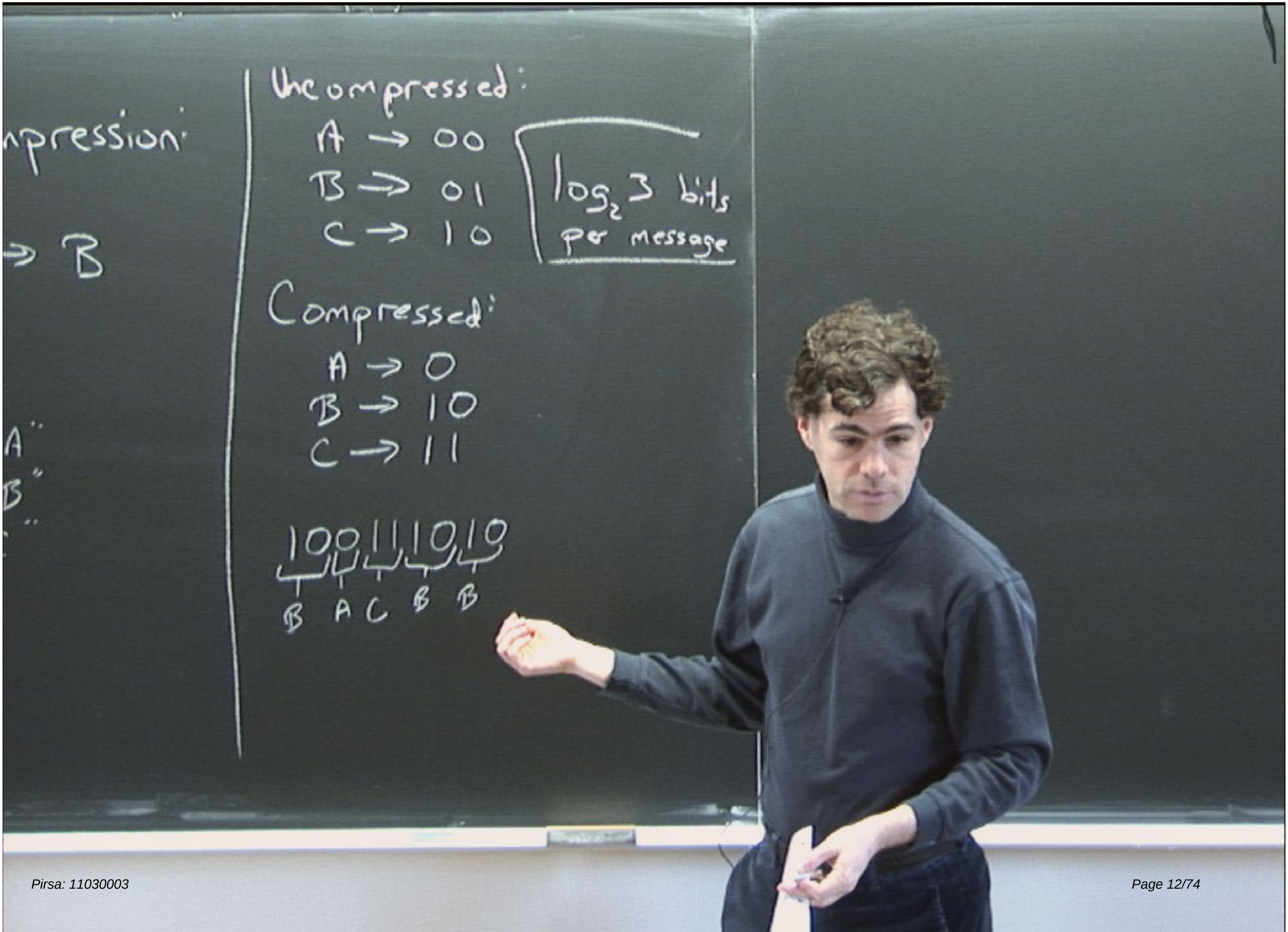
C → 10
log2 3 bits
per message
Compressed:
A → 0
B → 10
C → 11
100111010



Uncompressed:
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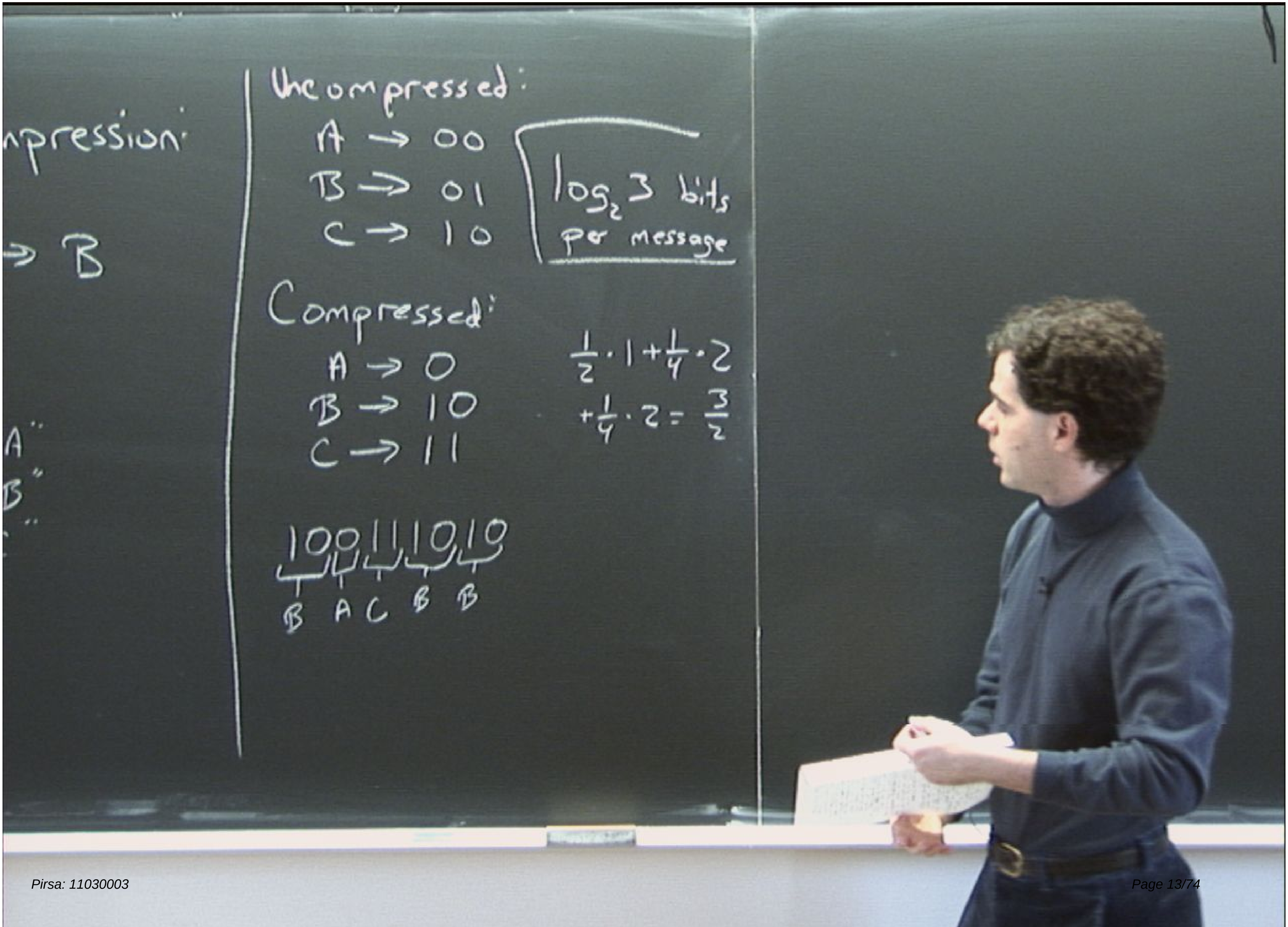
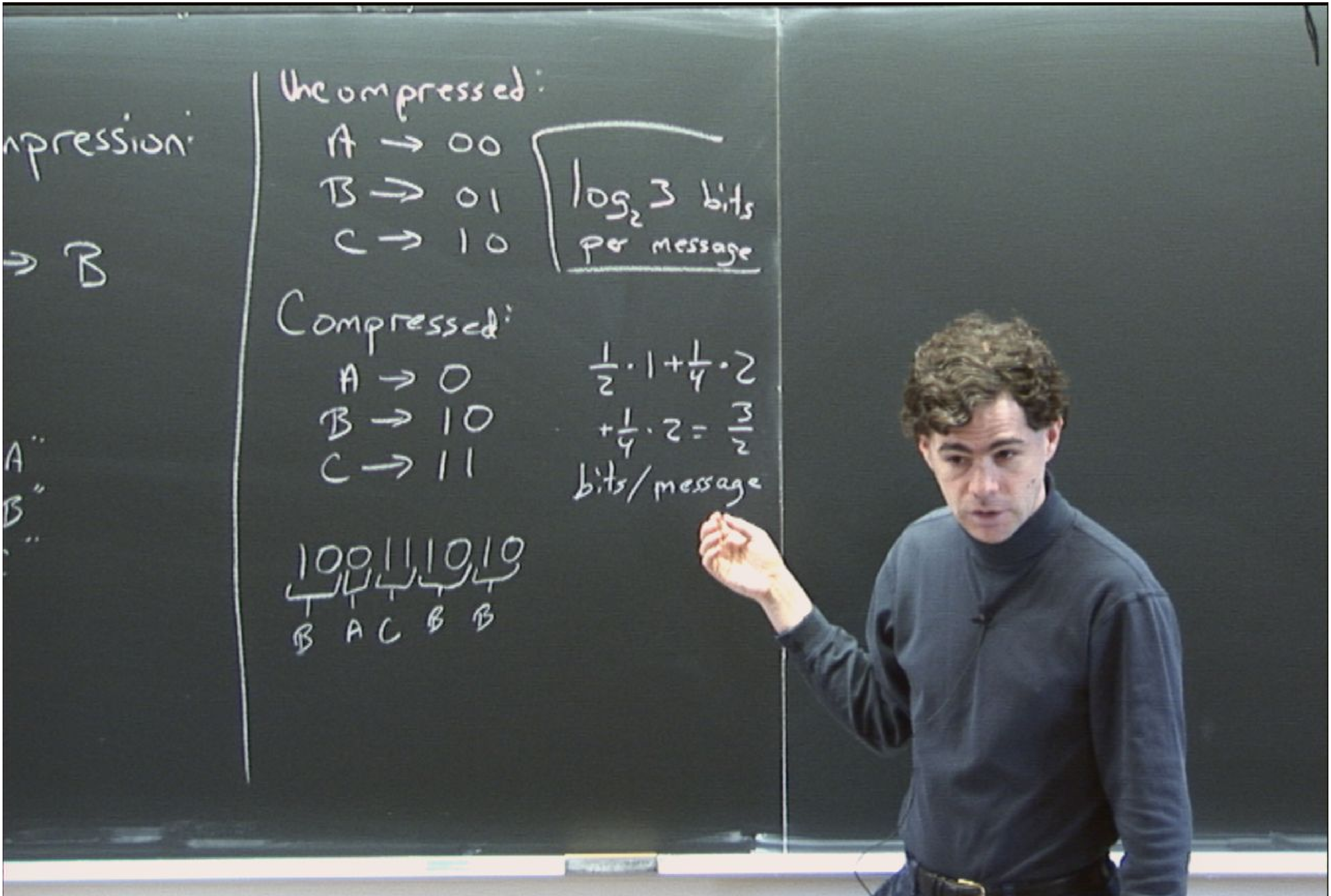
B → 01
C → 10
log2 3 bits per message
Compressed:
A → 0
B → 10
C → 11
10011110 10
B A C B B



Uncompressed:
A → 00
B → 01
C → 10
log₂ 3 bits per message
Compressed:
A → 0
B → 10
C → 11
1/2 · 1 + 1/4 · 2 + 1/4 · 2 = 3/2
10011110 10
B A C B B


npression:
⇒ B
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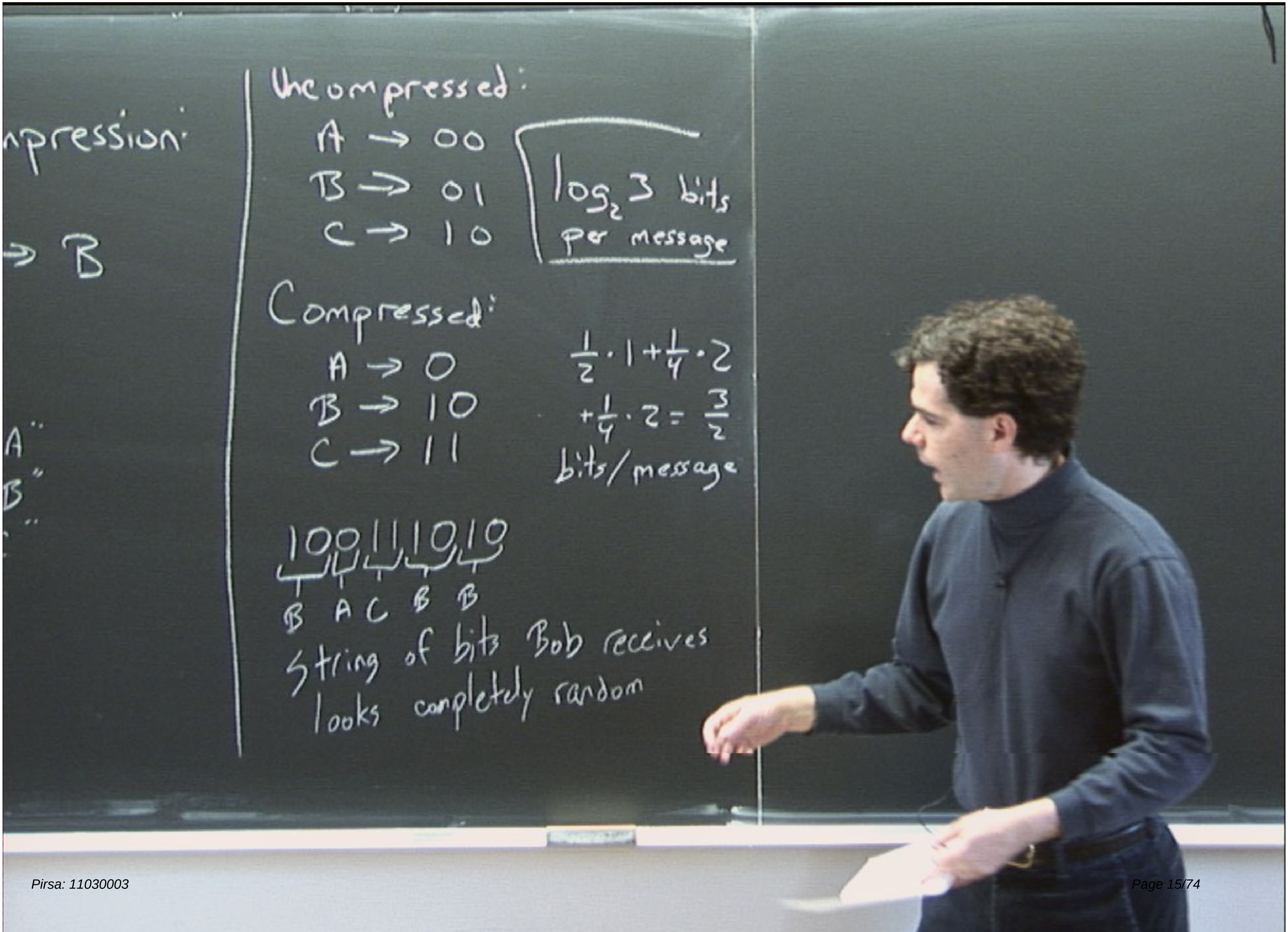
1001110101
B A C B B


npression:
⇒ B
A"
B"
Uncompressed:
A → 00
B → 01
C → 10
log2 3 bits per message
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1/2 · 1 + 1/4 · 2 + 1/4 · 2 = 3/2 bits/message
100111010
B A C B B
String of bits Bob receives looks completely random

npression:

$\Rightarrow B$

A"

B"

"

Uncompressed:

$A \rightarrow 00$

$B \Rightarrow 01$

$C \rightarrow 10$

$\boxed{\log_2 3 \text{ bits per message}}$

Compressed:

$A \rightarrow 0$

$B \rightarrow 10$

$C \rightarrow 11$

$\frac{1}{2}\cdot 1+\frac{1}{4}\cdot 2 + \frac{1}{4}\cdot 2 = \frac{3}{2}$ bits/message

$\underbrace{10}_{B}\underbrace{0}_{A}\underbrace{11}_{C}\underbrace{10}_{B}\underbrace{10}_{B}$

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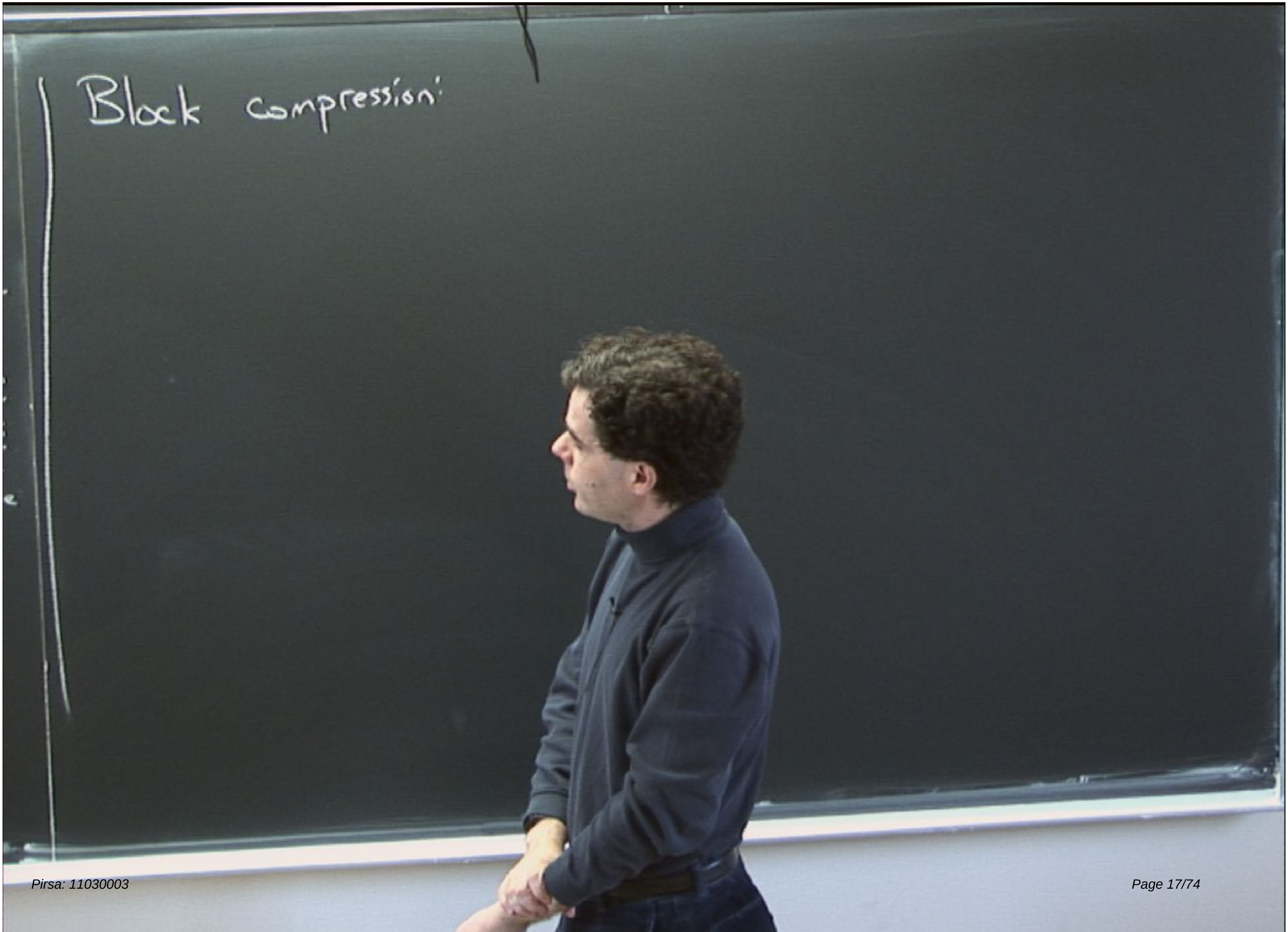
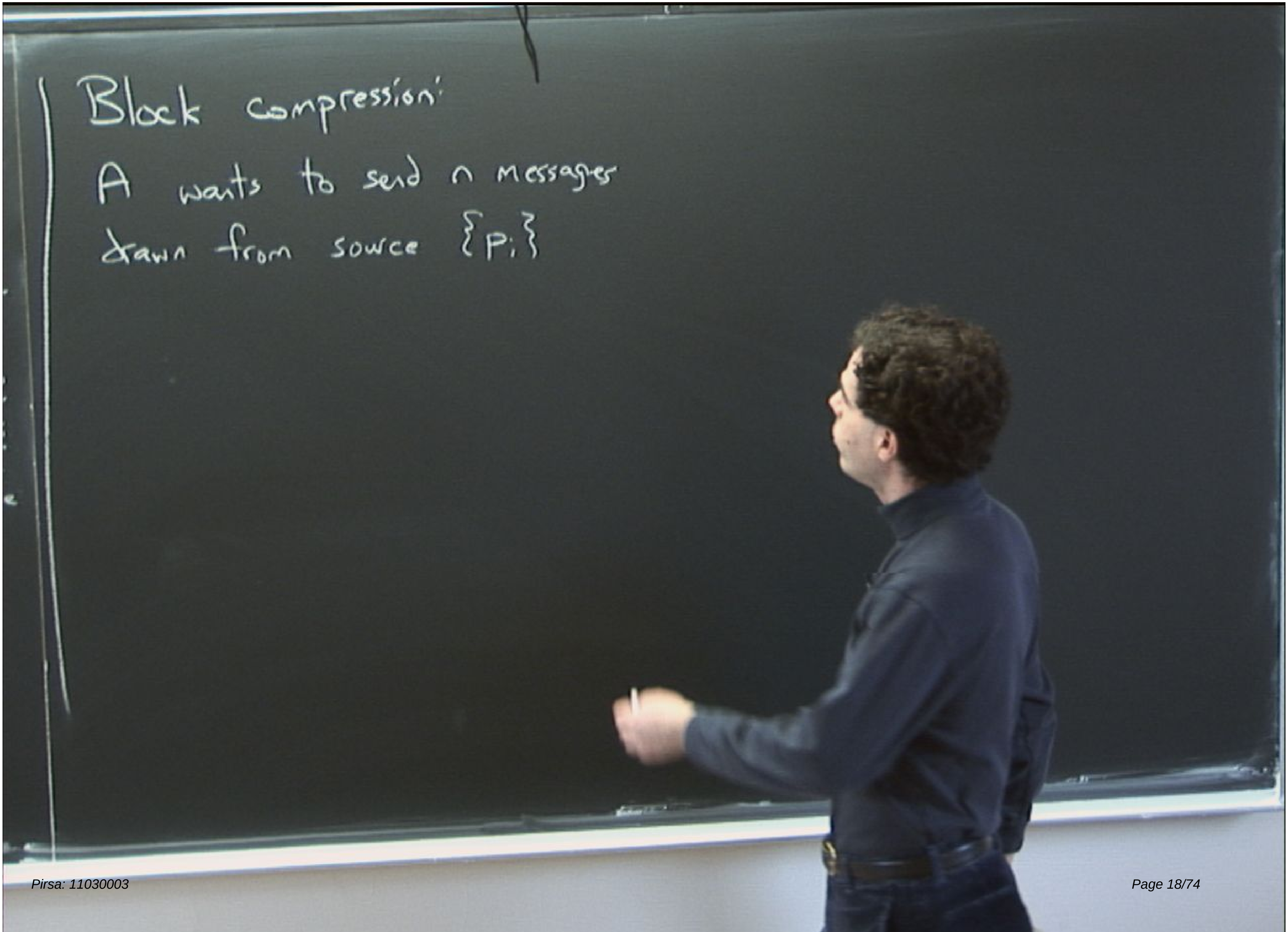

Block compression: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Make likely (strings of) messages into short strings to send to Bob, ignore unlikely sequences.

Pick "typical" set of messages so that prob. of getting a sequence of messages from the typical set $\to 1$ as $n \to \infty$

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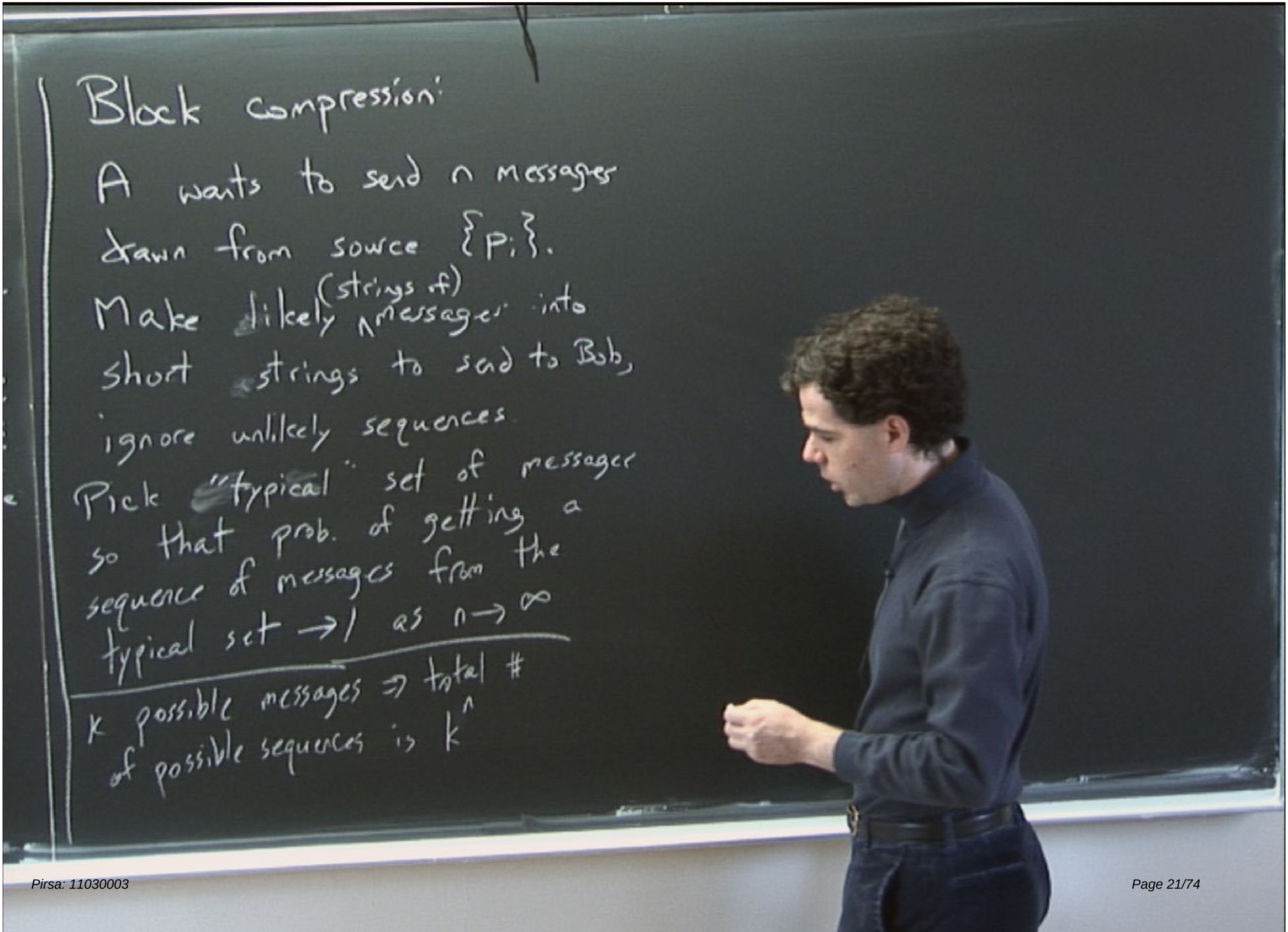
k possible messages $\Rightarrow$ total # of possible sequences is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k possible messages ⇒ total #
of possible sequences is k^n

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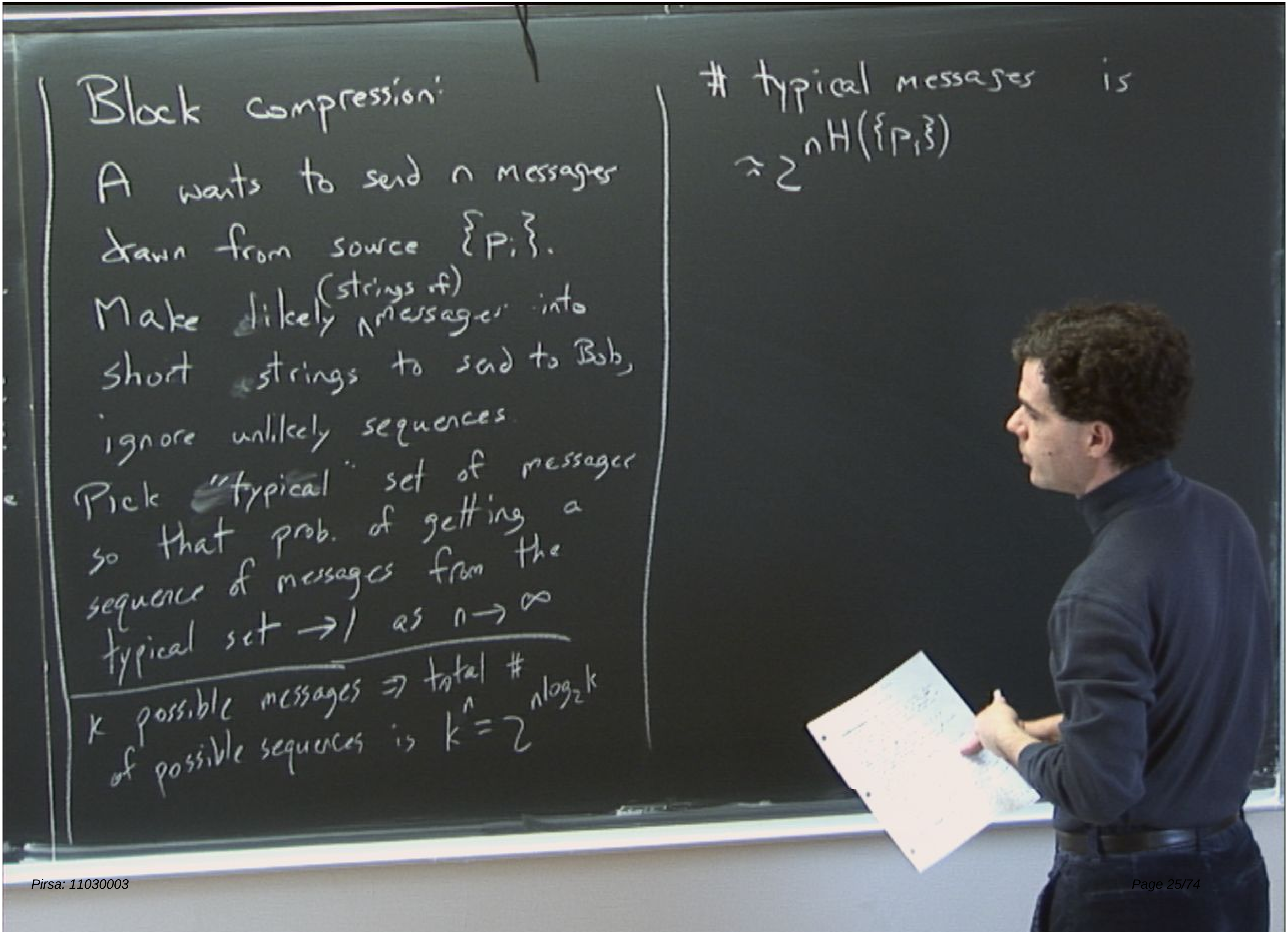
typical messages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$$H(X) = -\sum p_i \log_2 p_i \quad \text{"Shannon entropy"}$$

$X = \{ i$ w/ prob. p_i

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Shannon's source coding thm

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Shannon's source coding thm:

As $n \to \infty$, we can send n messages from source X with $nH(X)$ bits

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Shannon's source coding thm:

As $n \to \infty$, we can send n messages from source X with $nH(X)$ bits with fidelity $\to 1$ as $n \to \infty$. We cannot do better lock compression:

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i.i.d. = "independent, identically distributed" ck compression:

wants to send n messages

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k "typical" set of messages

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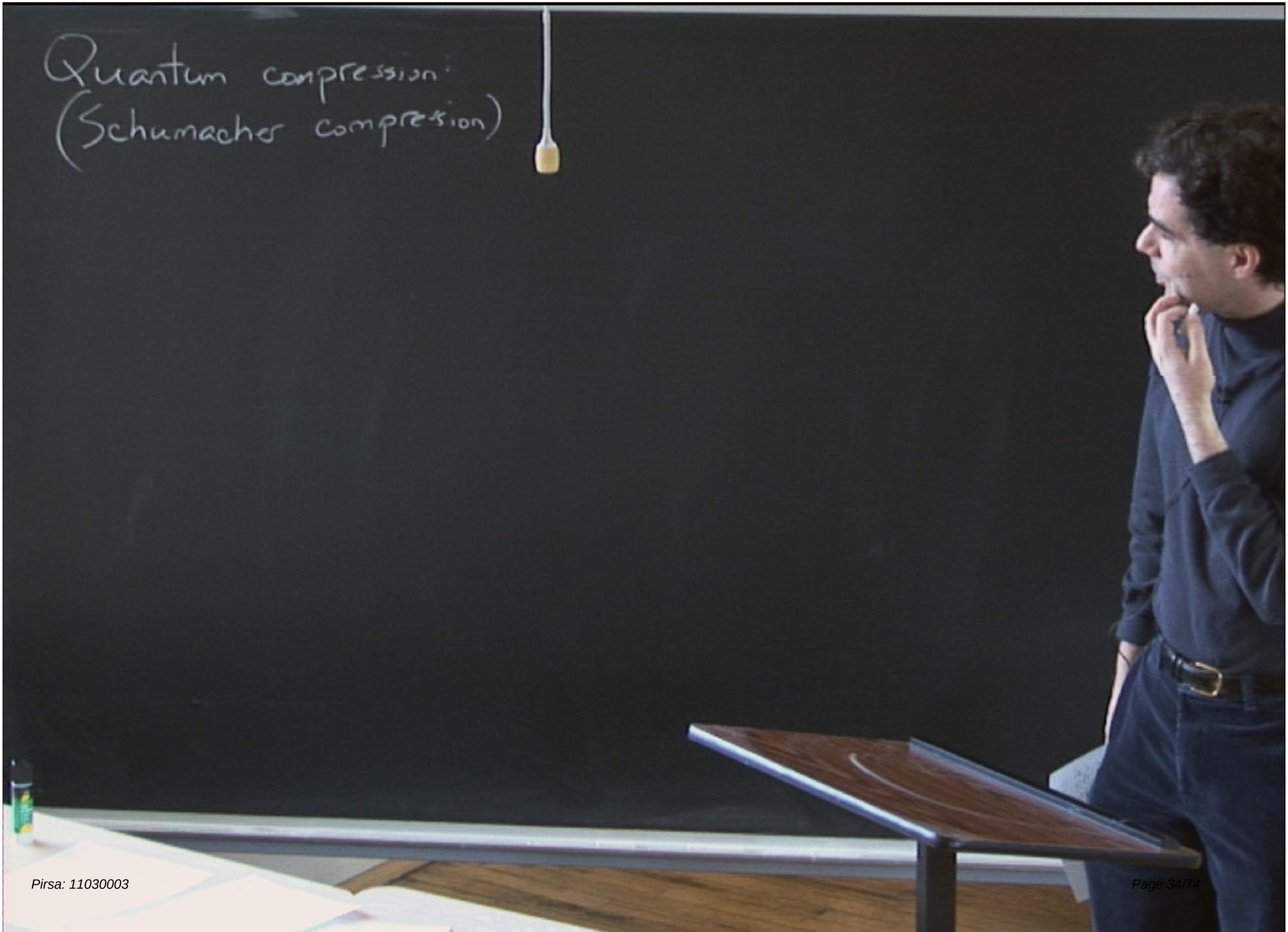
Quantum compression:
(Schumacher compression)


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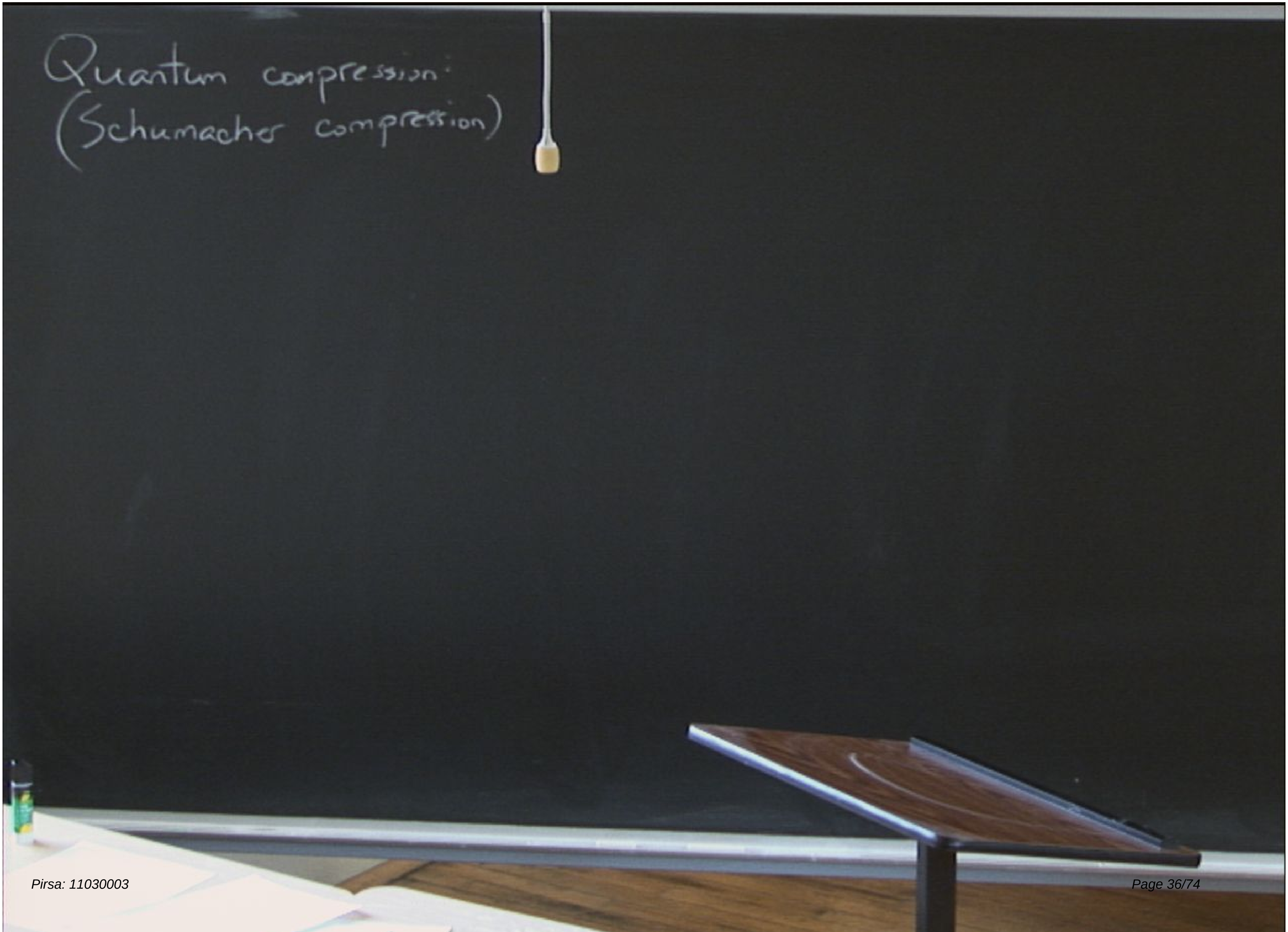
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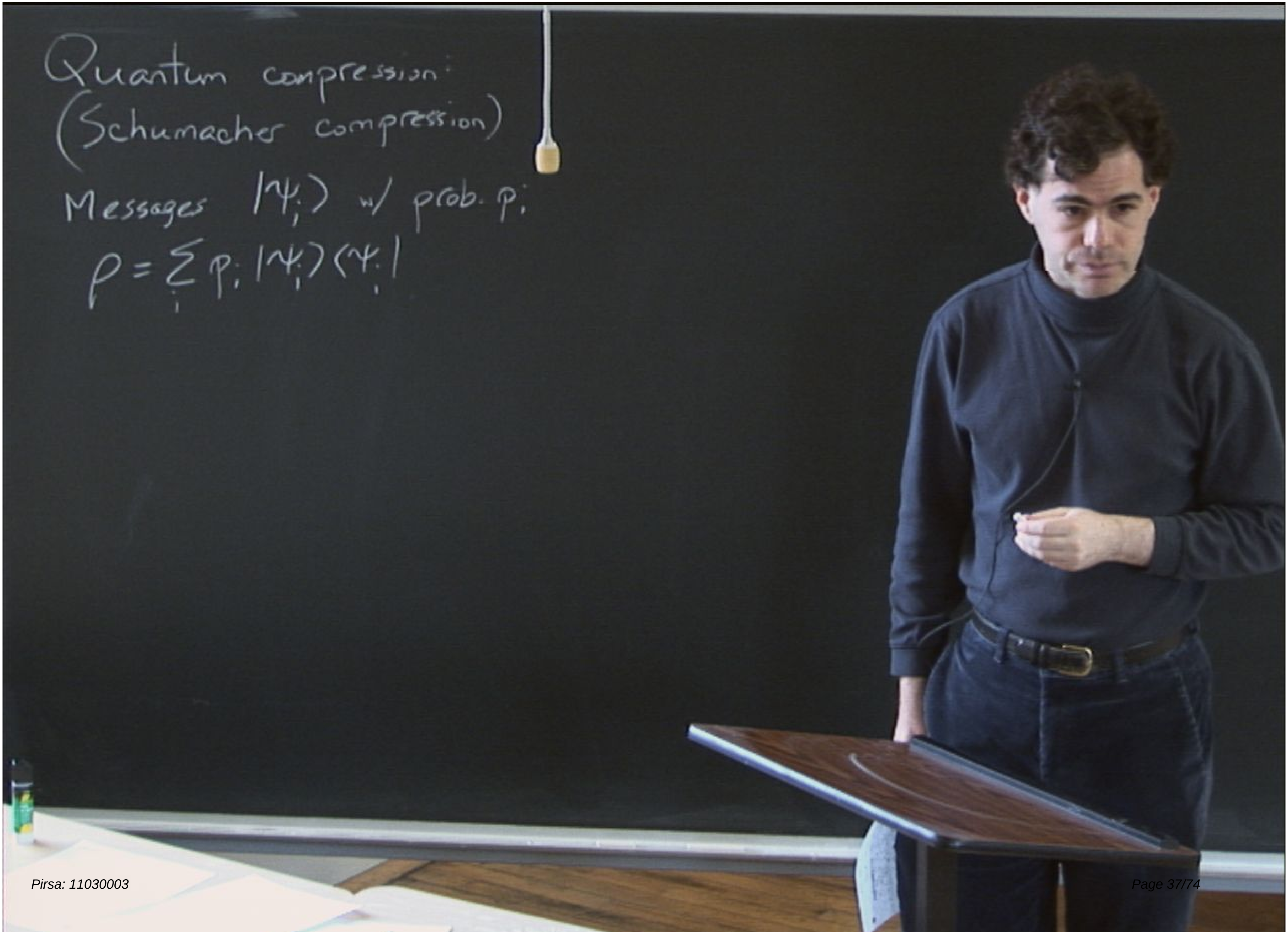
Quantum compression:
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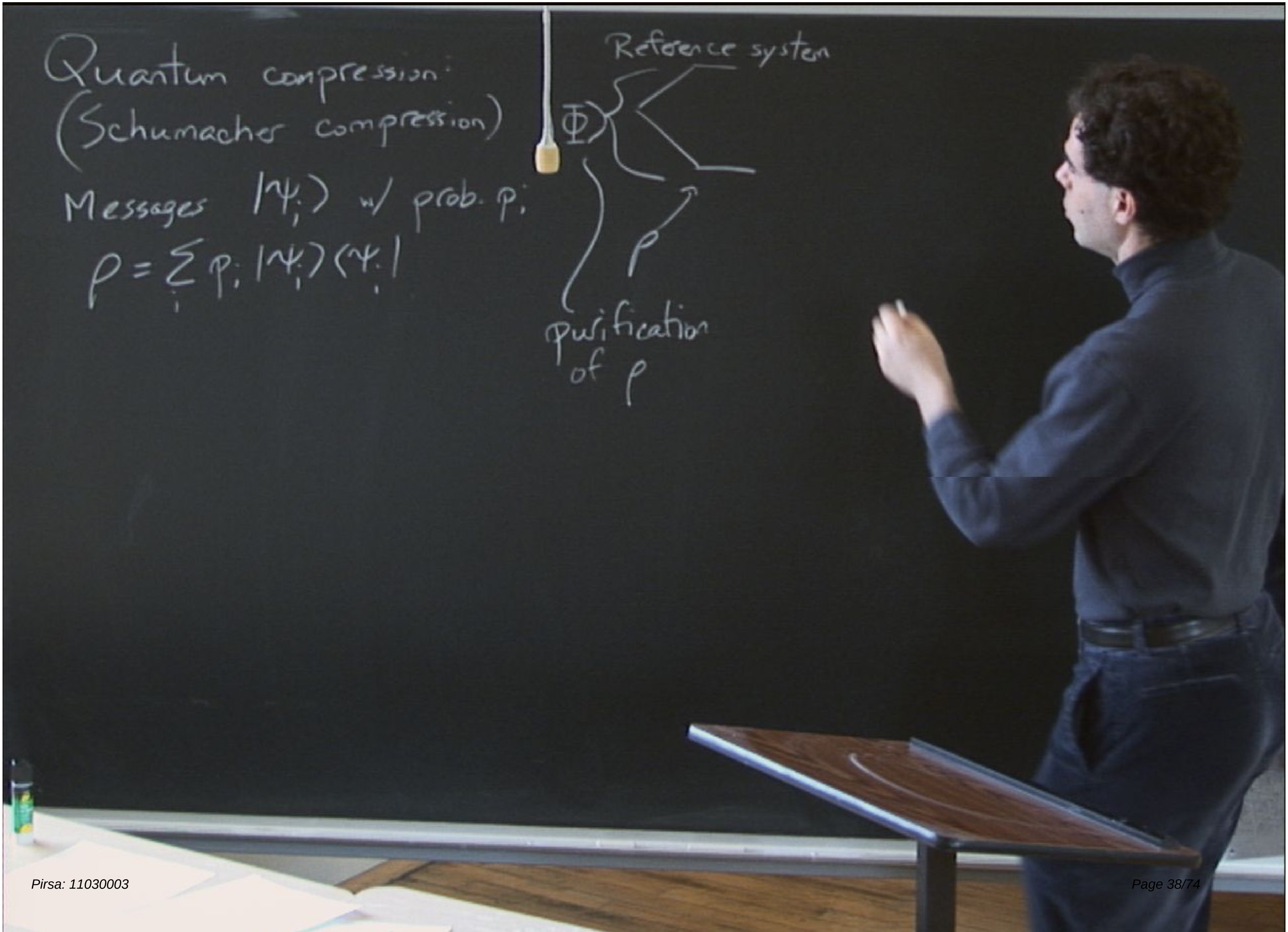
Messages $|\psi_i\rangle$ w/ prob. p_i
$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$



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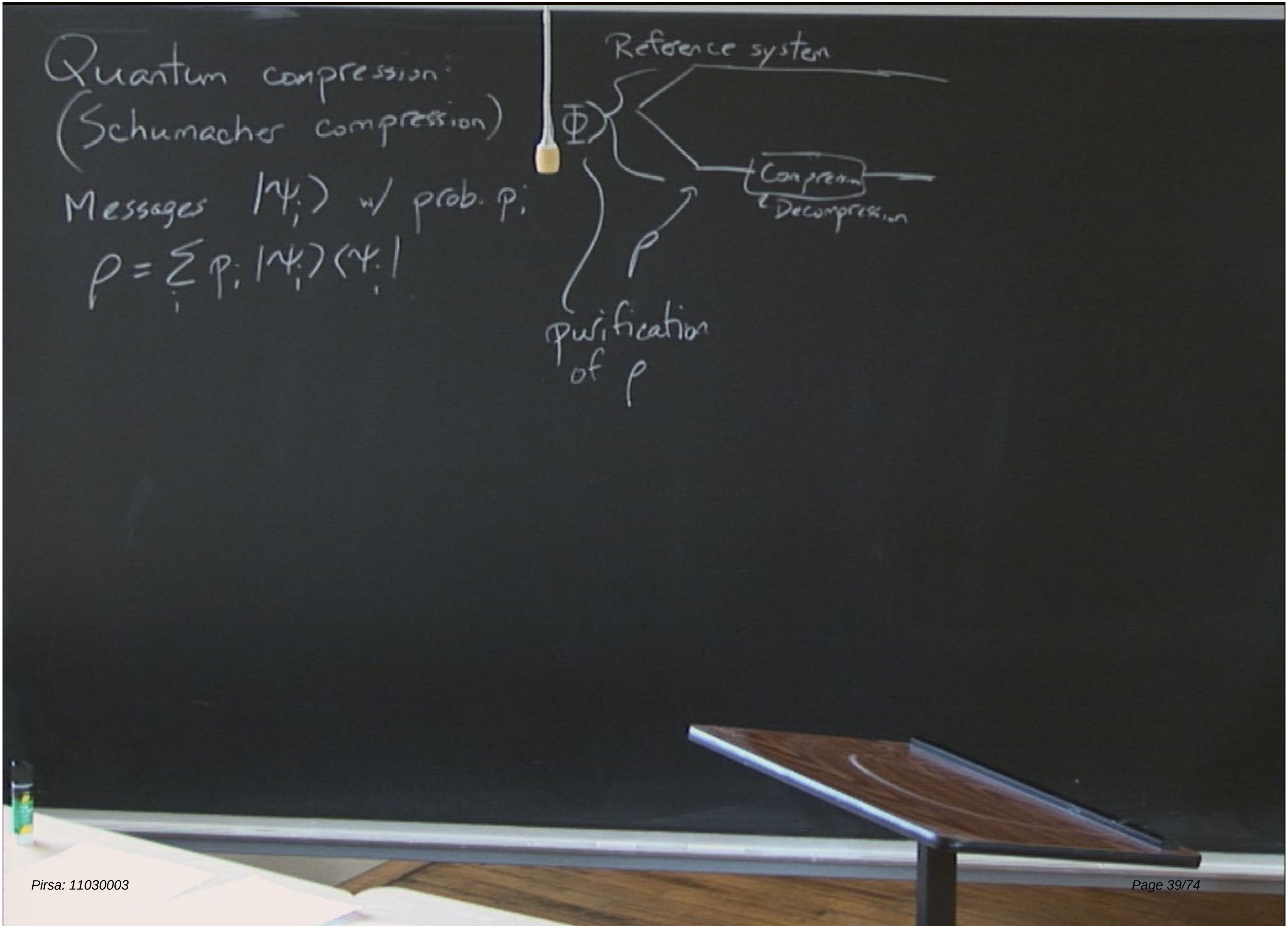
$|\Phi\rangle$
Reference system
ρ
purification of ρ



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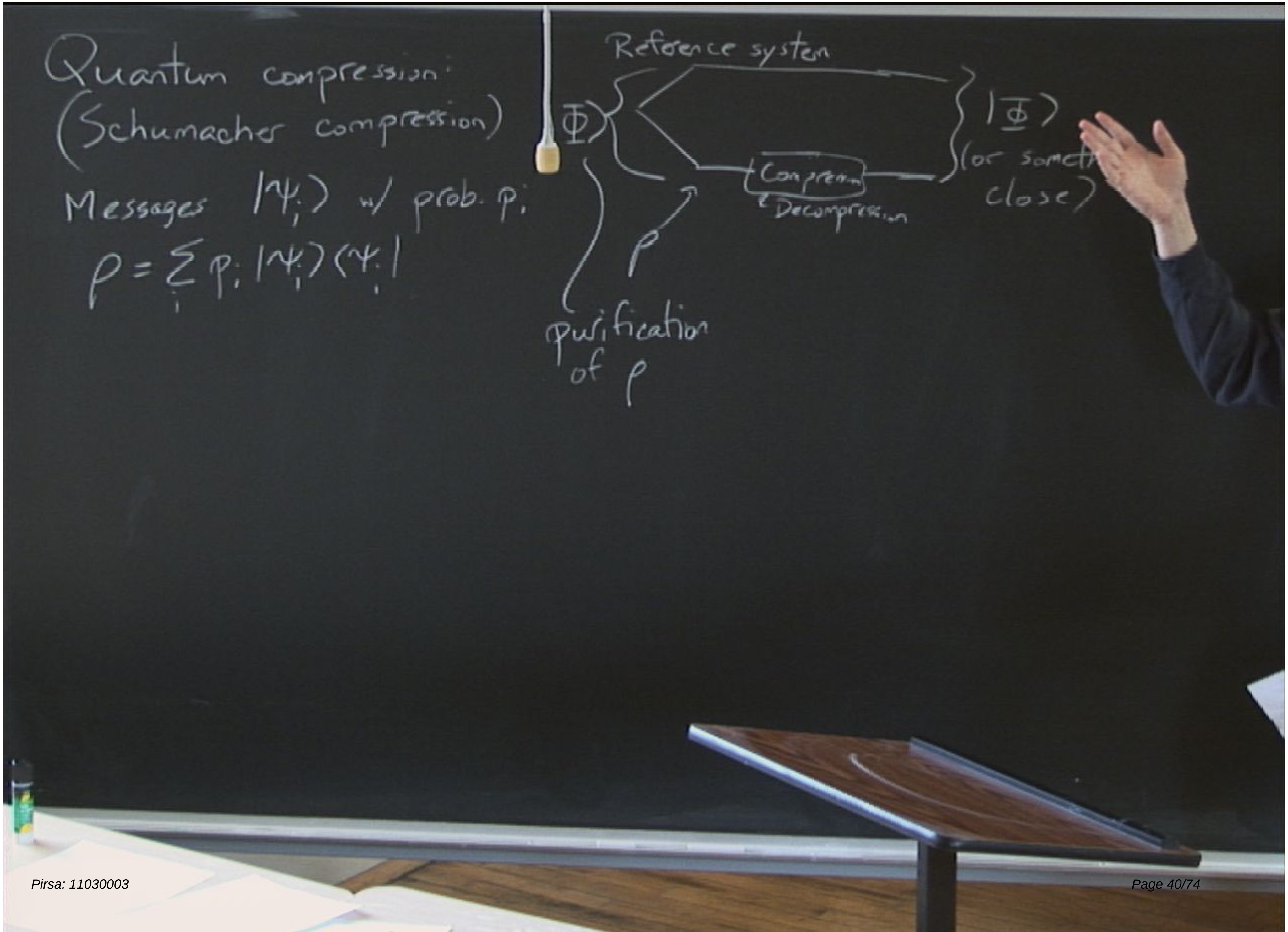
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Compression
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(or someth
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ρ
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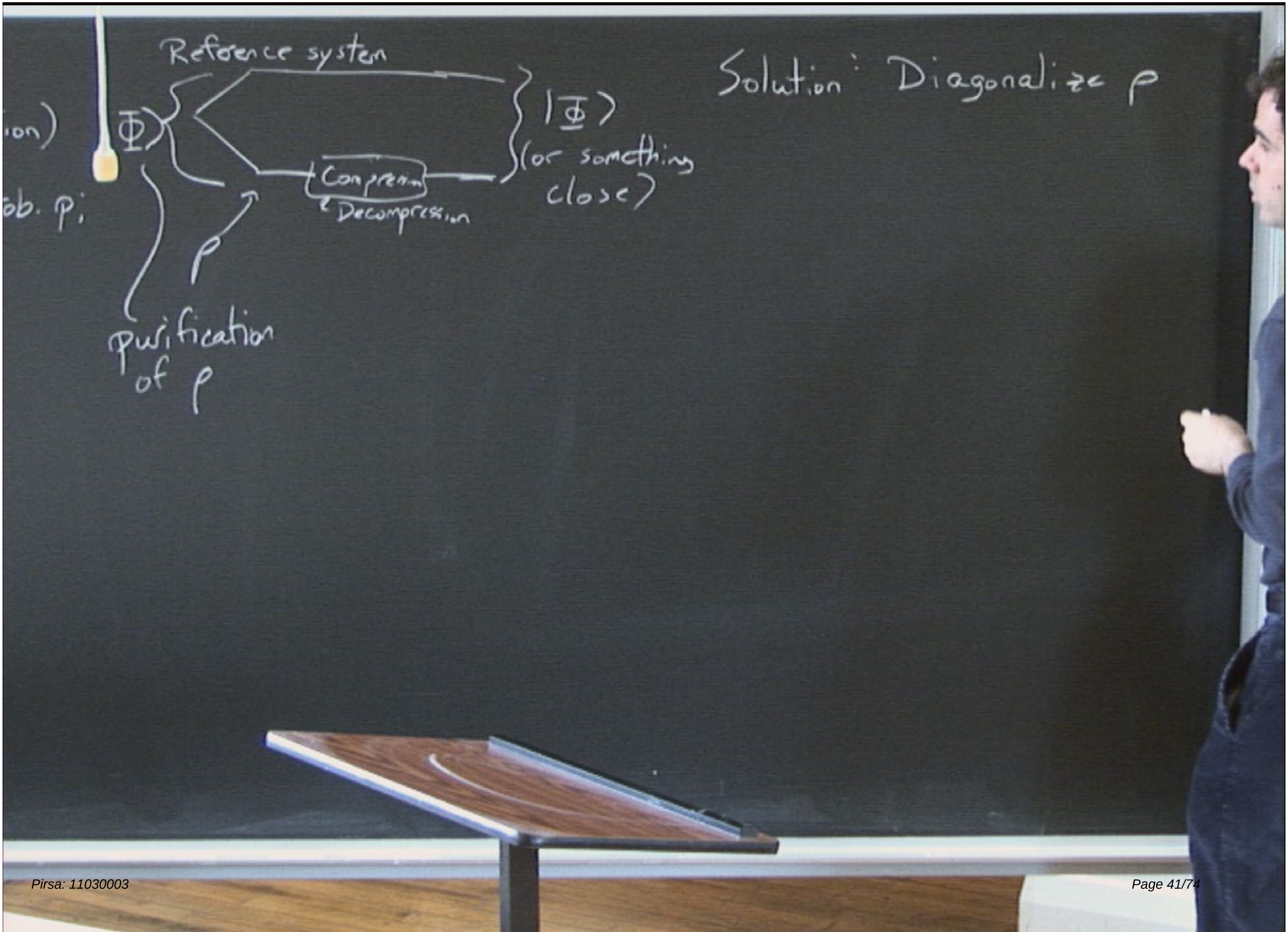
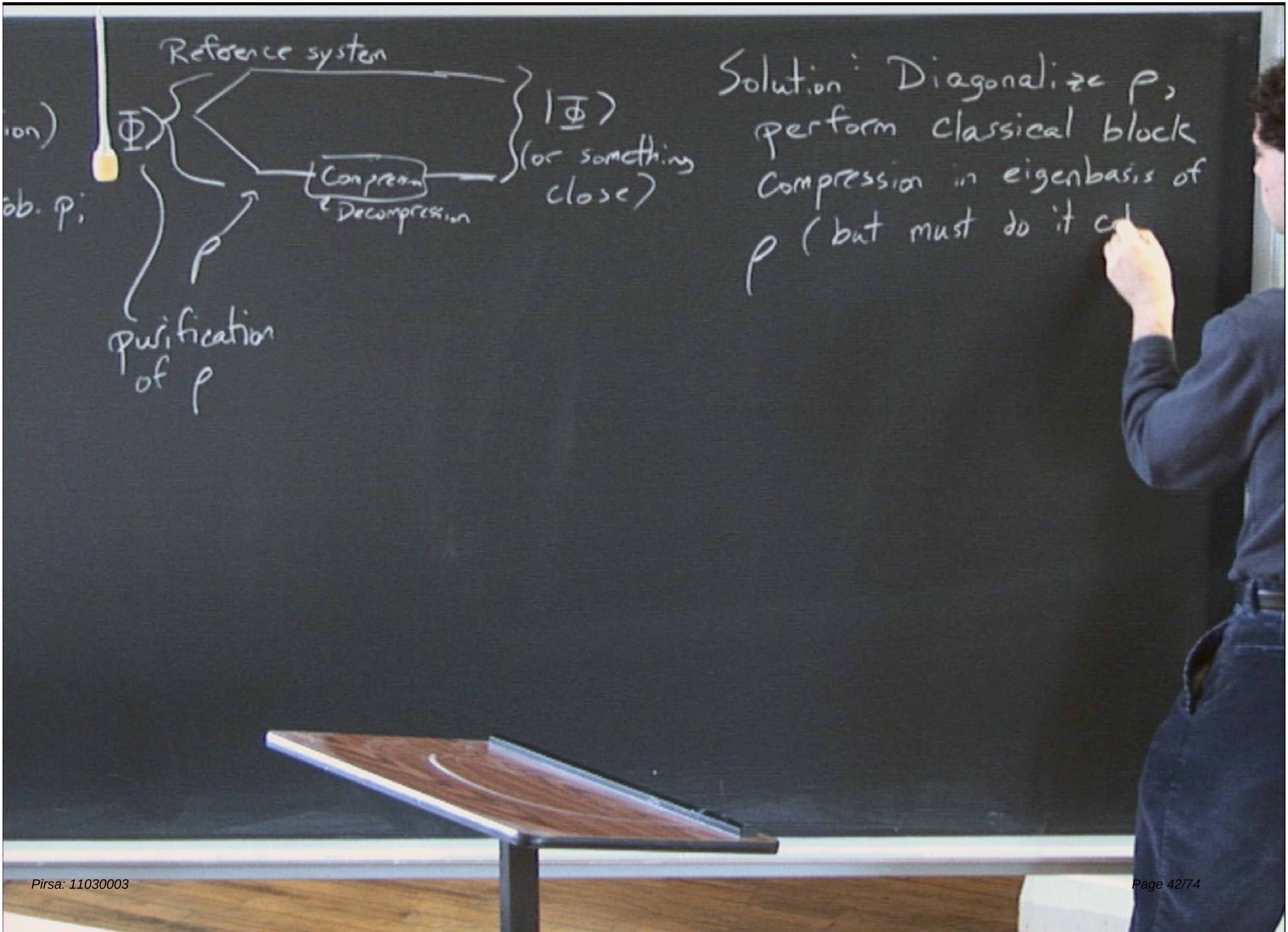

Reference system
|Φ⟩
(or something close)
Compression
& Decompression
ρ
purification of ρ
Solution: Diagonalize ρ


ion)
ob. ρ;
|Φ⟩
Reference system
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Decompression
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Solution: Diagonalize ρ,
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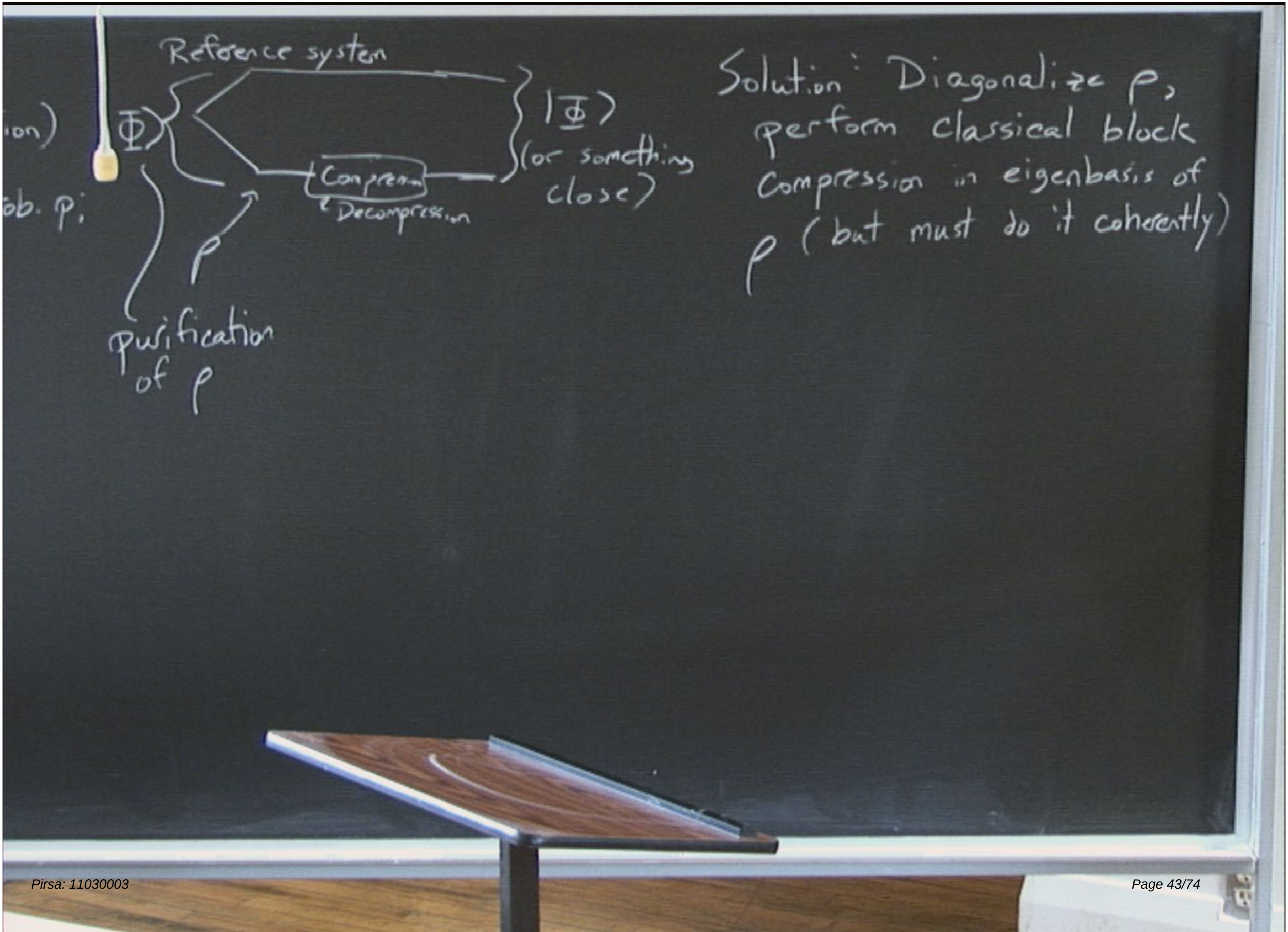
ion)
ob. p_i
Reference system
$|\Phi\rangle$
$|\Phi\rangle$
(or something close)
Compression
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ρ
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Solution: Diagonalize ρ, perform classical block compression in eigenbasis of ρ (but must do it coherently)

...ion)

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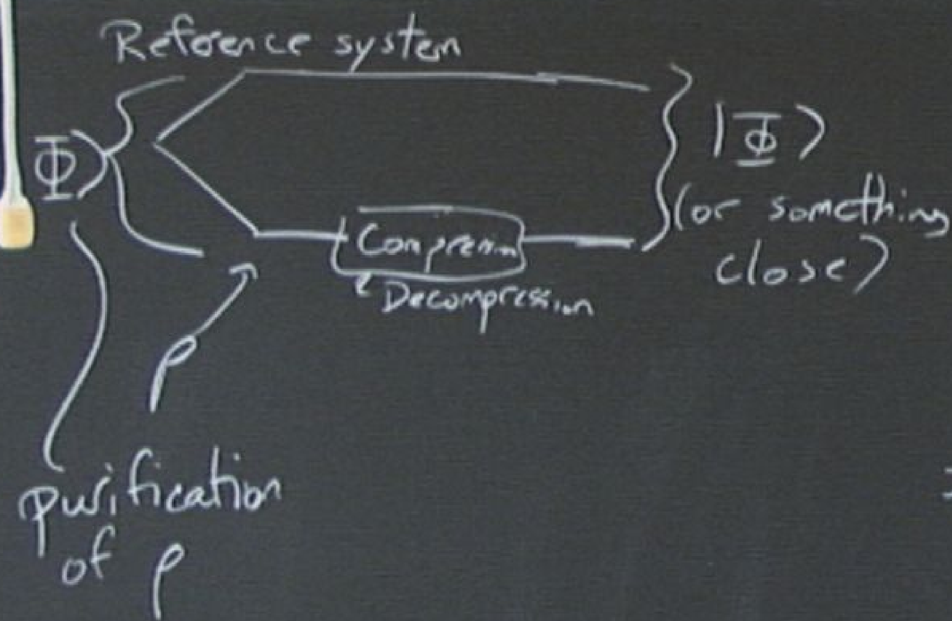
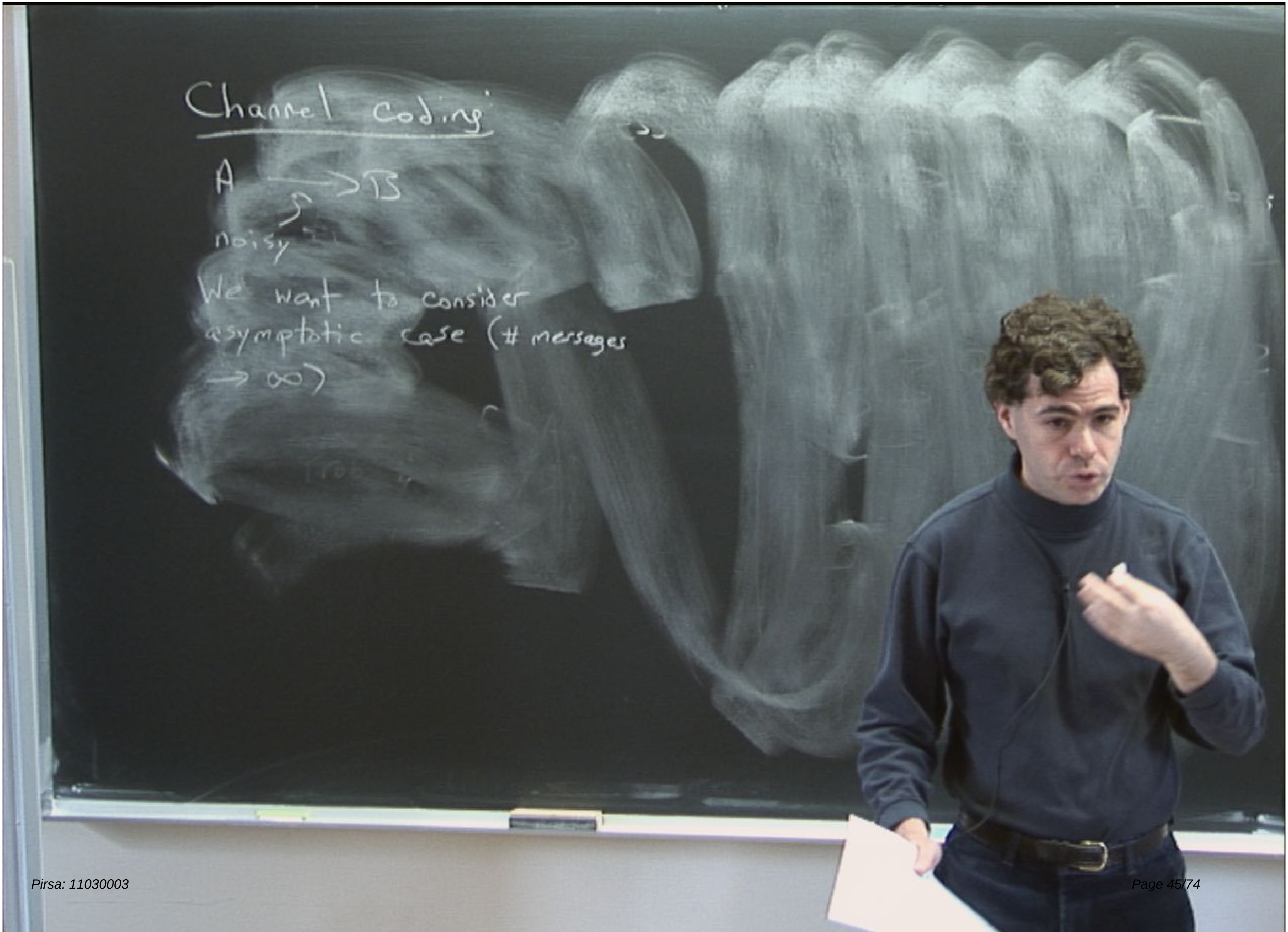
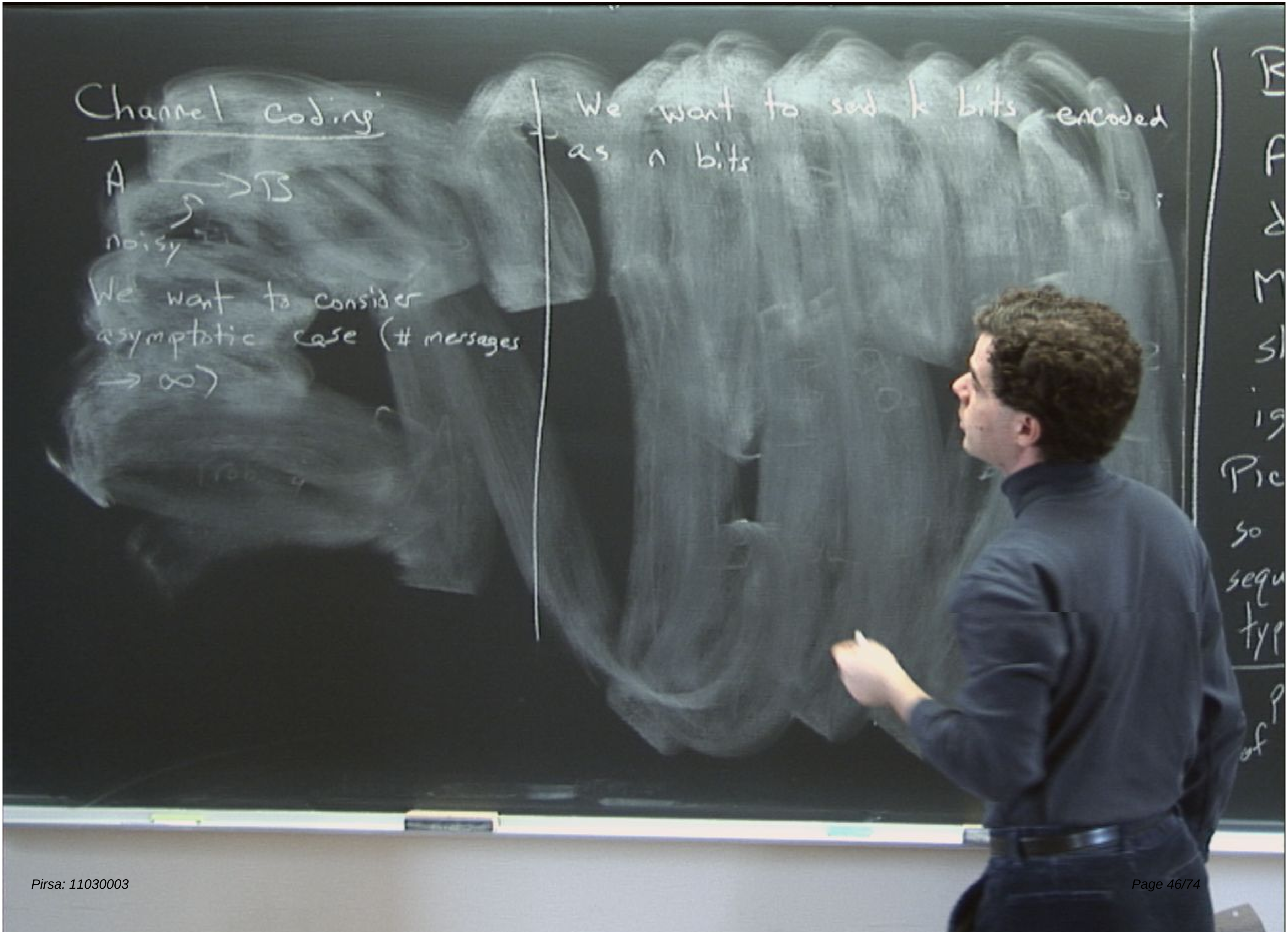


Solution: Diagonalize ρ, perform classical block compression in eigenbasis of ρ (but must do it coherently)

$\Rightarrow$ Compress n messages to $\approx n S(\rho)$ qubits w/ fidelity $\to 1$ as $n \to \infty$.


Channel coding:
A → B
noisy
We want to consider
asymptotic case (# messages
→ ∞)


Channel coding
A → B
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We want to consider
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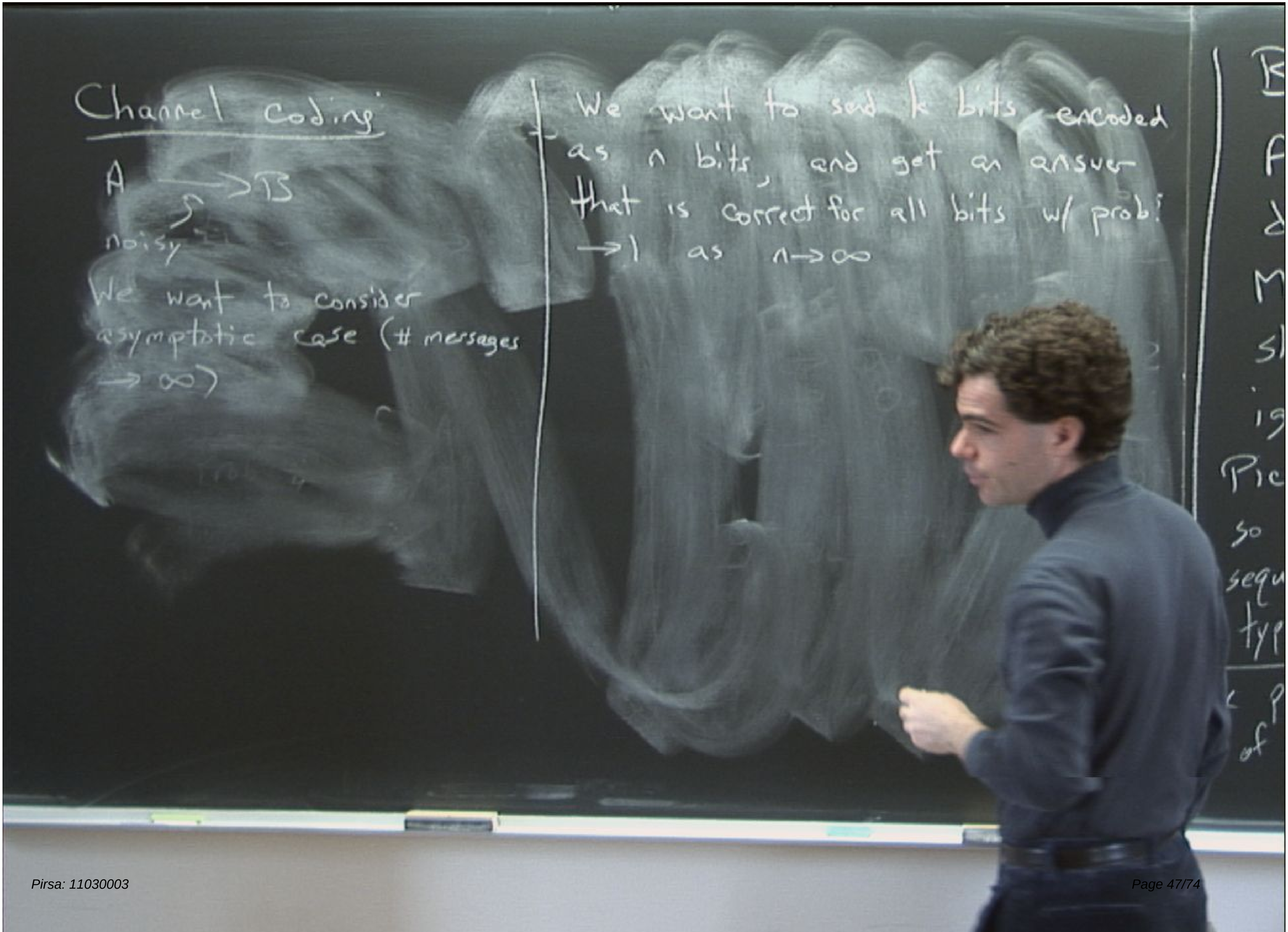
We want to send k bits encoded
as n bits


Channel Coding
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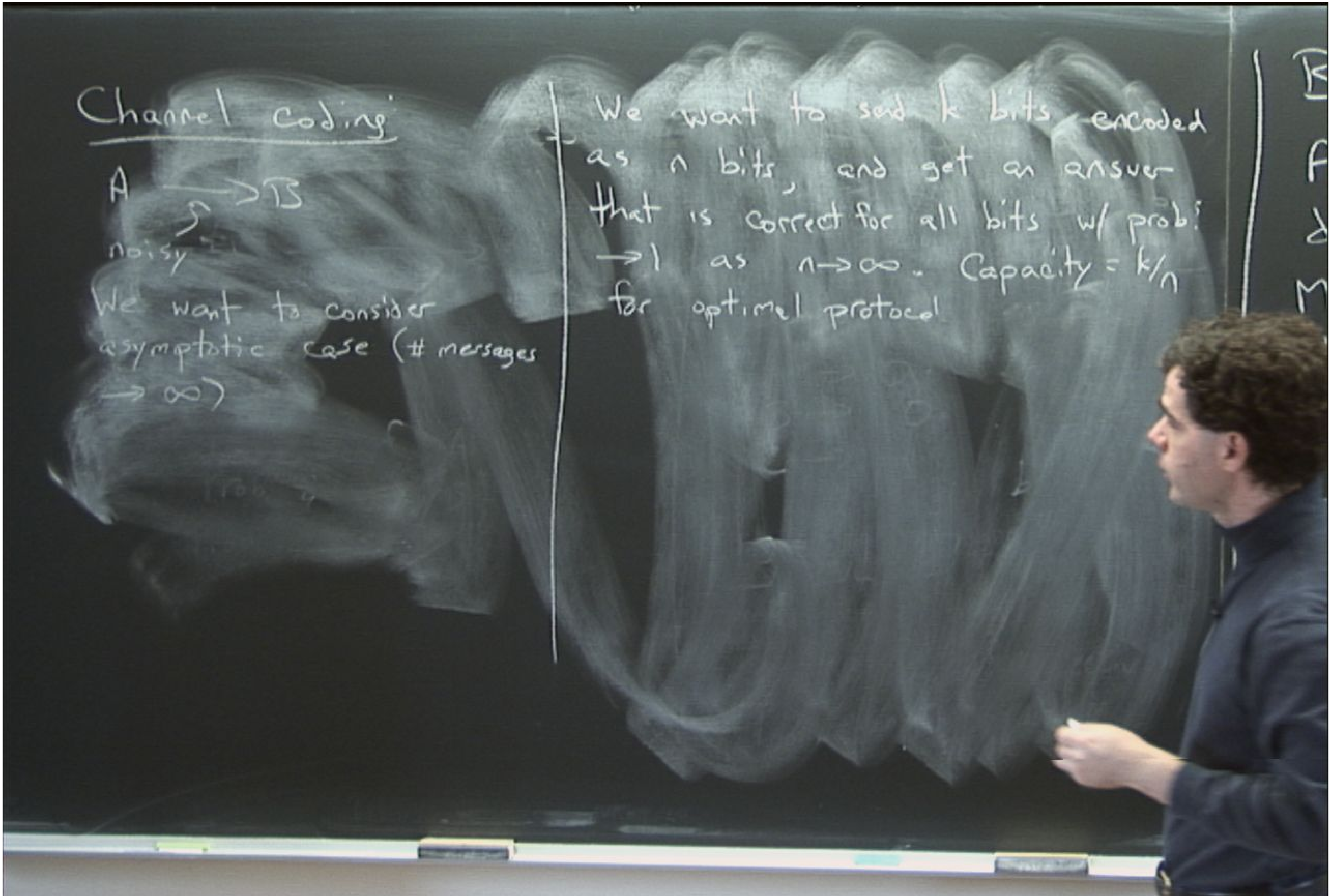
We want to consider asymptotic case (# messages → ∞)
We want to send k bits encoded as n bits, and get an answer that is correct for all bits w/ prob. → 1 as n → ∞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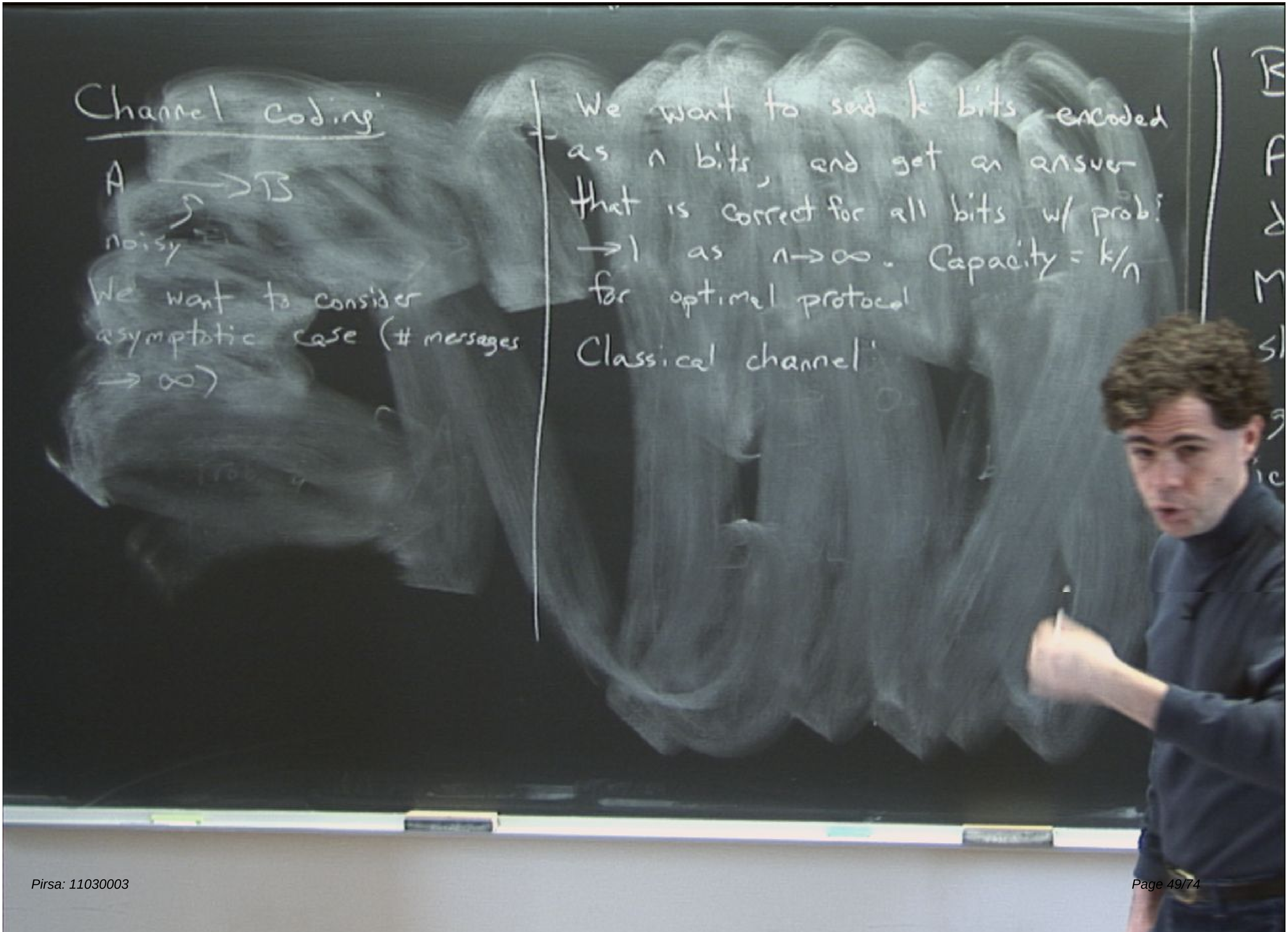
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Classical channel:

Channel Coding:

$A \longrightarrow B$

noisy

We want to consider asymptotic case (# messages $\rightarrow \infty$)

We want to send k bits encoded as n bits, and get an answer that is correct for all bits w/ prob. $\rightarrow 1$ as $n \rightarrow \infty$. Capacity $= k/n$ for optimal protocol

Classical channel: Put in bits

Channel Coding:

$A \longrightarrow B$

noisy

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Classical channel: Put in bits/states described random variable X, get out random variable Y.

Channel Coding:

$A \longrightarrow B$

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Classical channel: Put in bits/states described random variable X, get out random variable Y. Y correlated w/ X.

Mutual information

$$\underline{I}(X;Y) = H(X) + H(Y) - H(X,Y)$$

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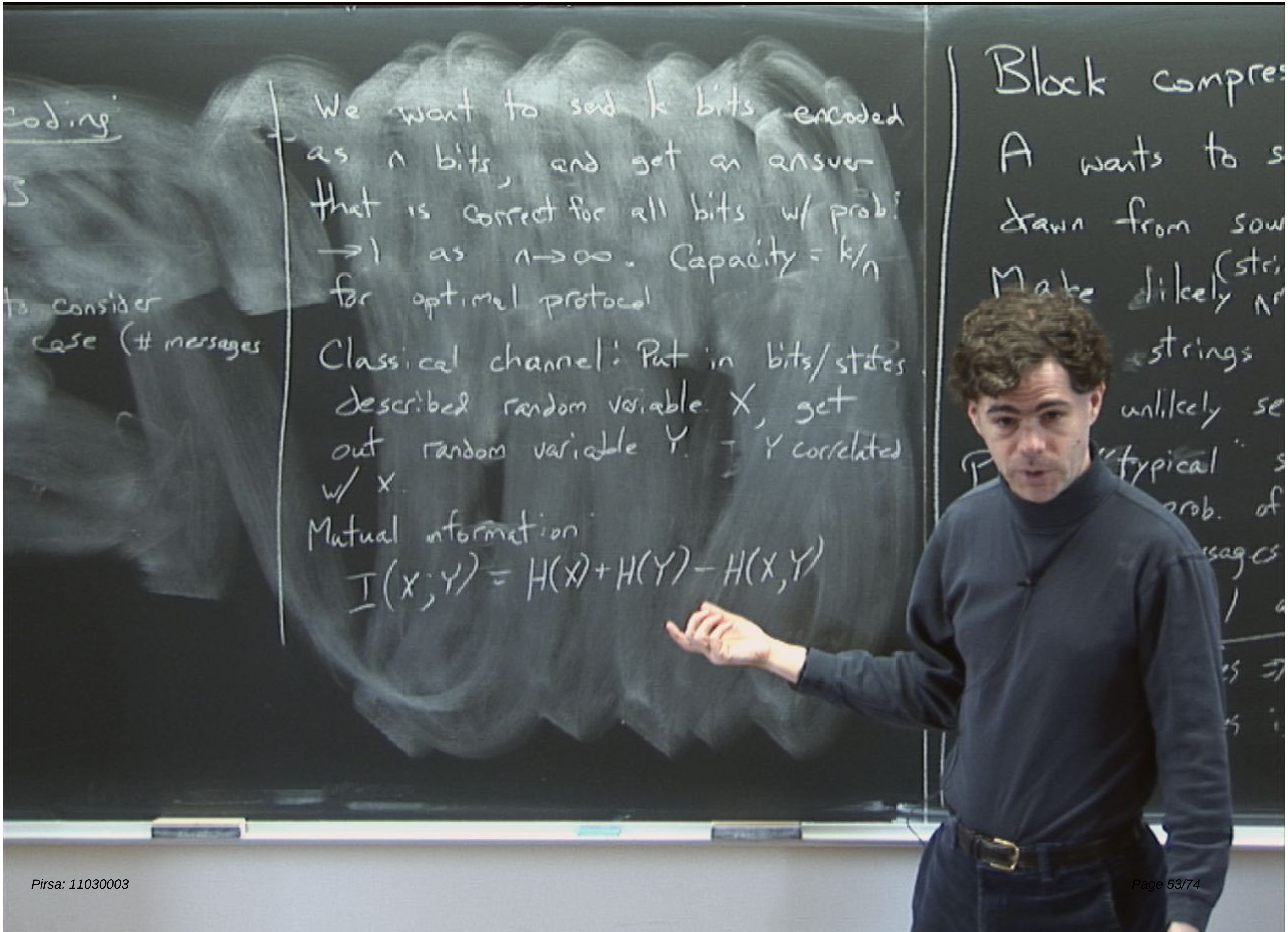
Coding
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that is correct for all bits w/ prob.
→ 1 as n → ∞. Capacity = k/n
for optimal protocol
Classical channel: Put in bits/states
described random variable X, get
out random variable Y. Y correlated
w/ X.
Mutual information:
$I(X;Y) = H(X) + H(Y) - H(X,Y)$
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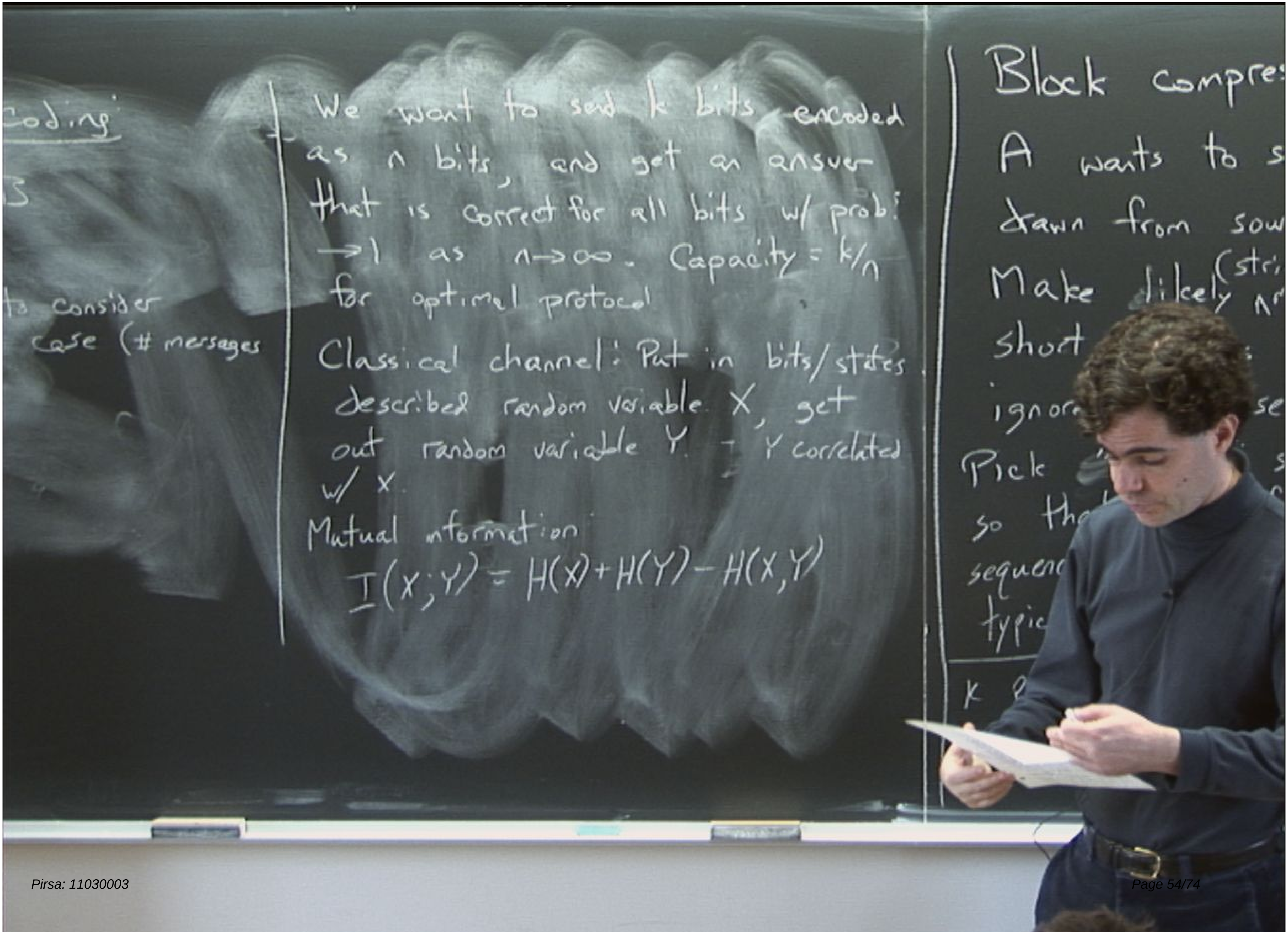
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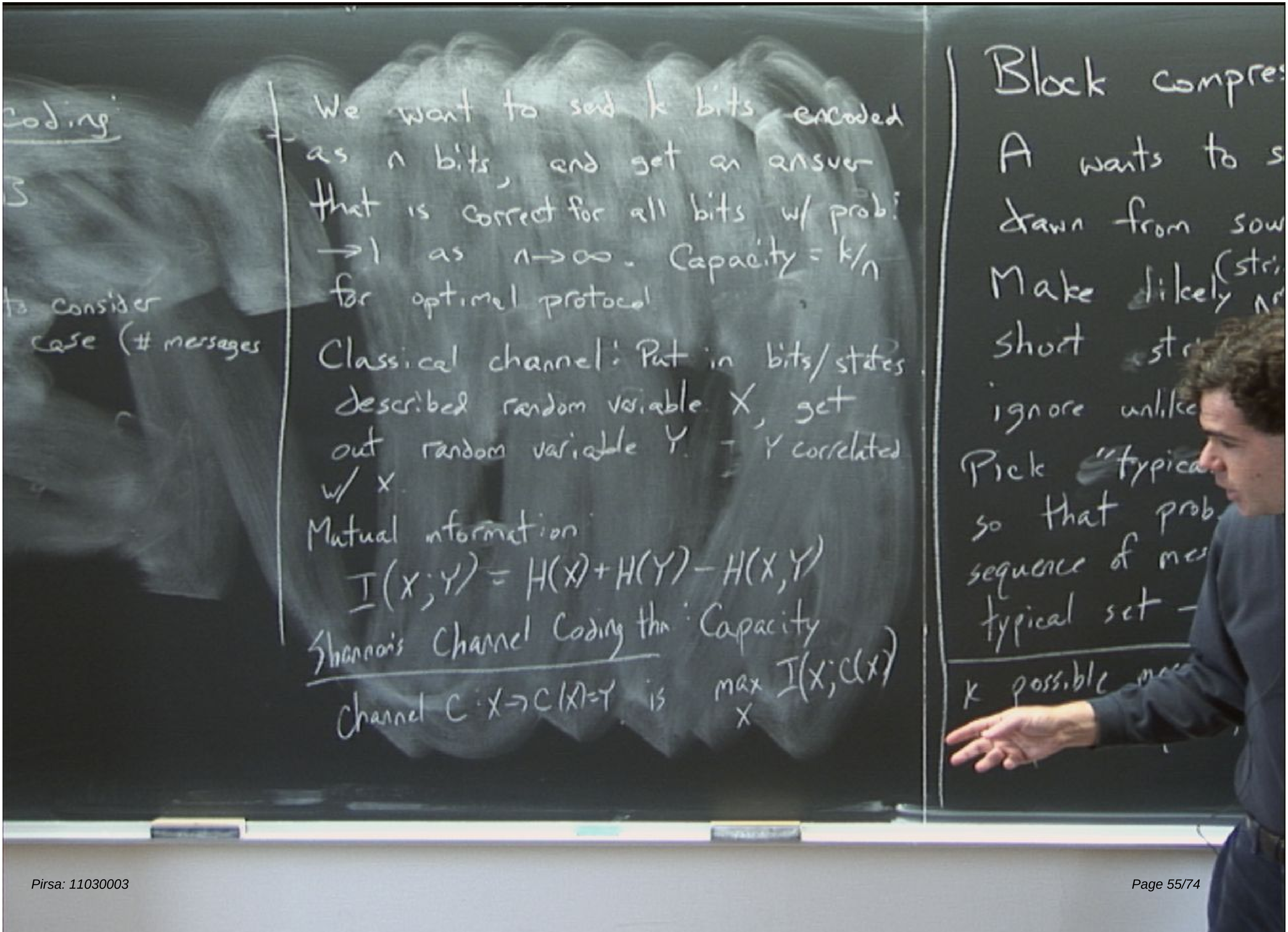
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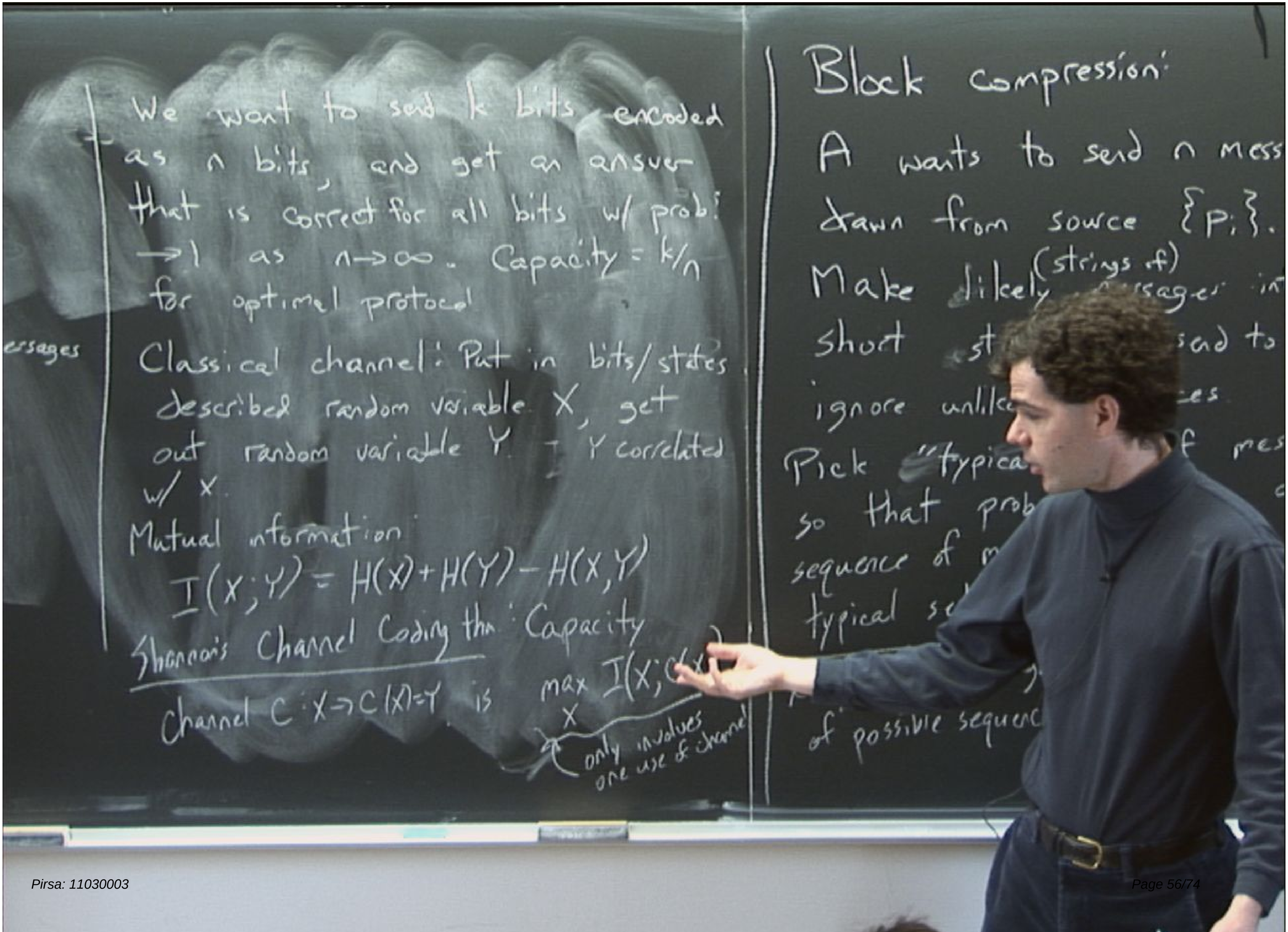
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Block compression:

A wants to send n mess
drawn from source $\{p_i\}$.
Make likely (strings of) messages in
short strings to send to
ignore unlikely sequences.
Pick "typical" set of mes
so that prob. of getting
sequence of messages from the
typical set $\to 1$ as $n \to \infty$

k possible messages $\Rightarrow$ total # of possible sequences is $k^n = 2^{n \log_2}$

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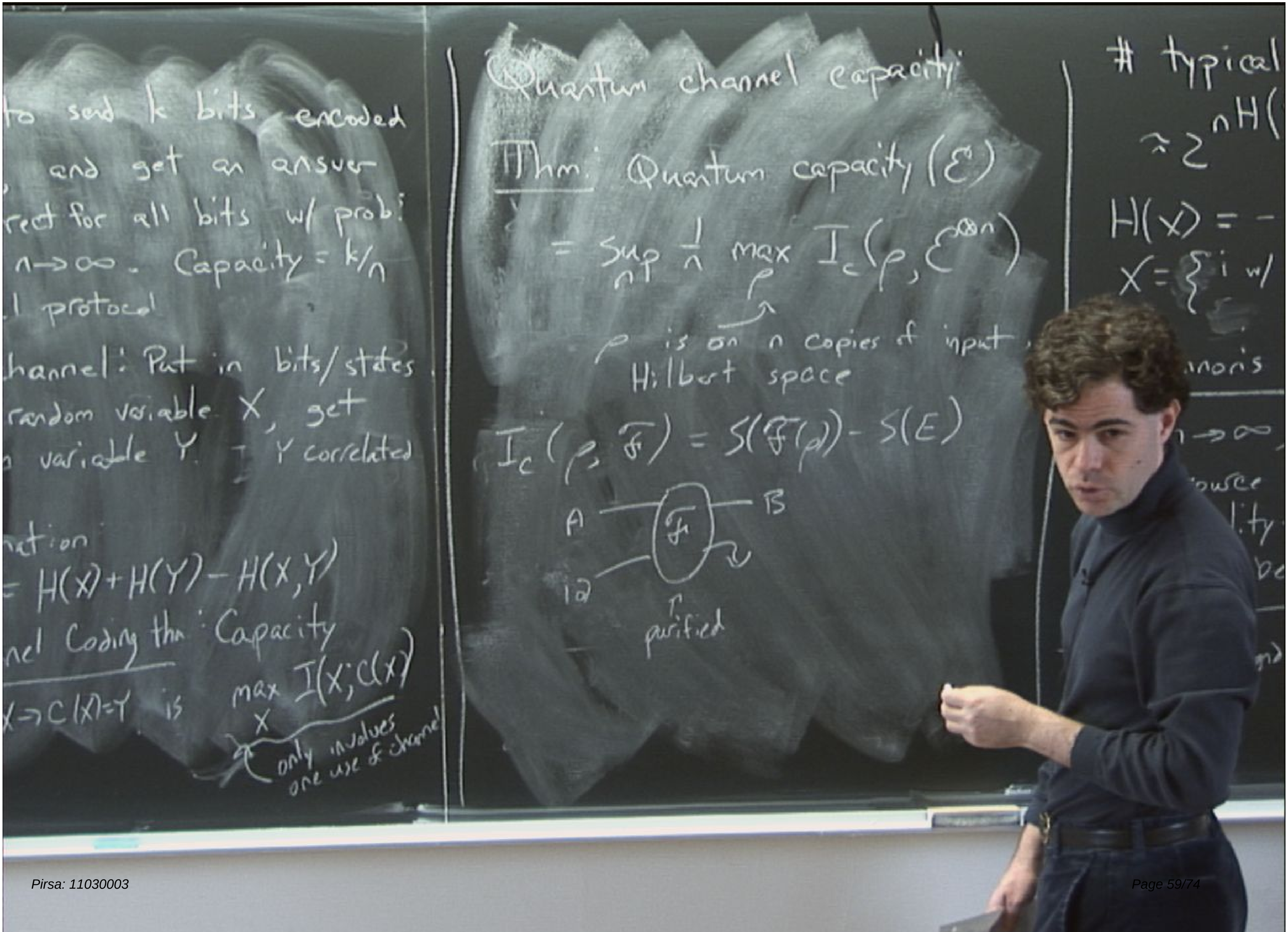
Quantum channel capacity: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Thm: Quantum capacity (E)
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Thm: Quantum capacity $(\mathcal{E})$

$$= \sup_n \frac{1}{n} \max_\rho I_c(\rho, \mathcal{E}^{\otimes n})$$

ρ is on n copies of input Hilbert space

$$I_c(\rho, \mathcal{F}) = S(\mathcal{F}(\rho)) - S(E)$$

$S(\mathcal{F}(\rho))$: entropy Bob receives

$S(E)$: entropy of environment

A, $|\psi\rangle$ purified, $\mathcal{F}$, B, E

typical

$\approx 2^{nH(}$

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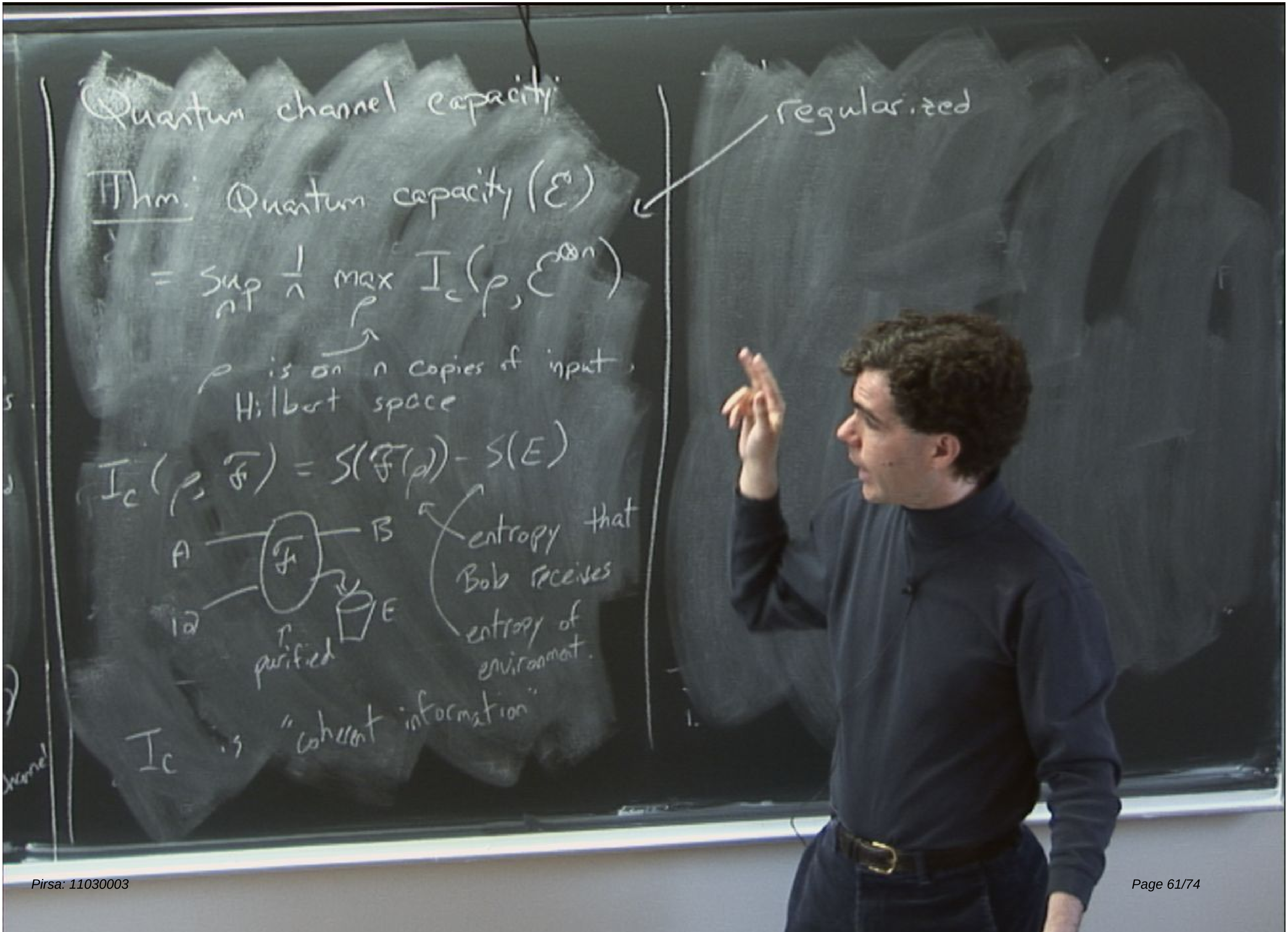

Quantum channel capacity:
Thm: Quantum capacity$(\mathcal{E})$
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I_c is "coherent information"

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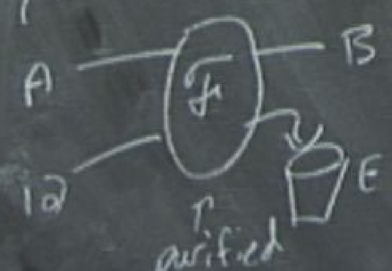

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Classical mutual info additive

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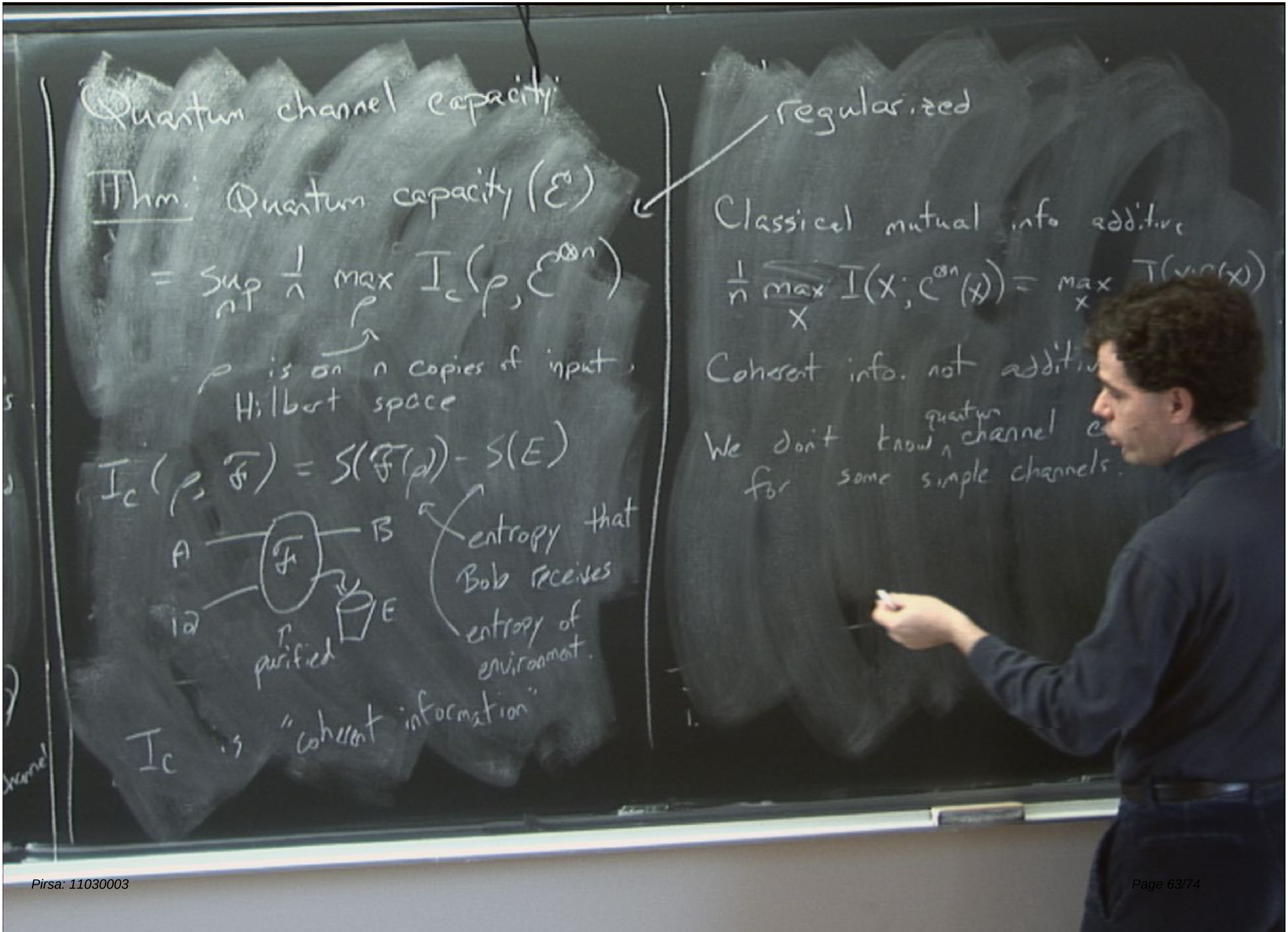
Coherent info. not additive



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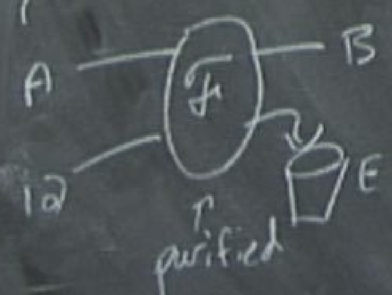

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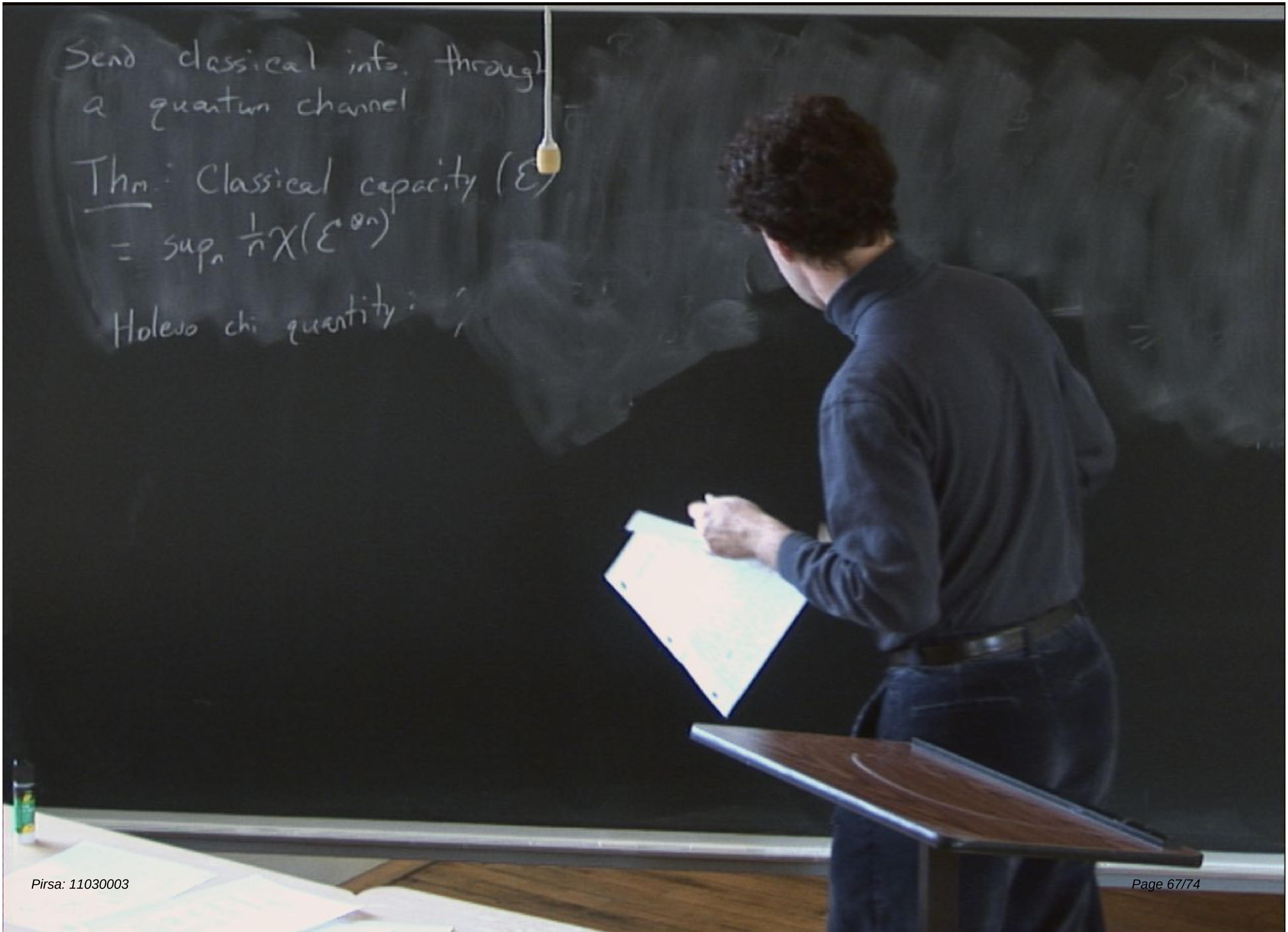
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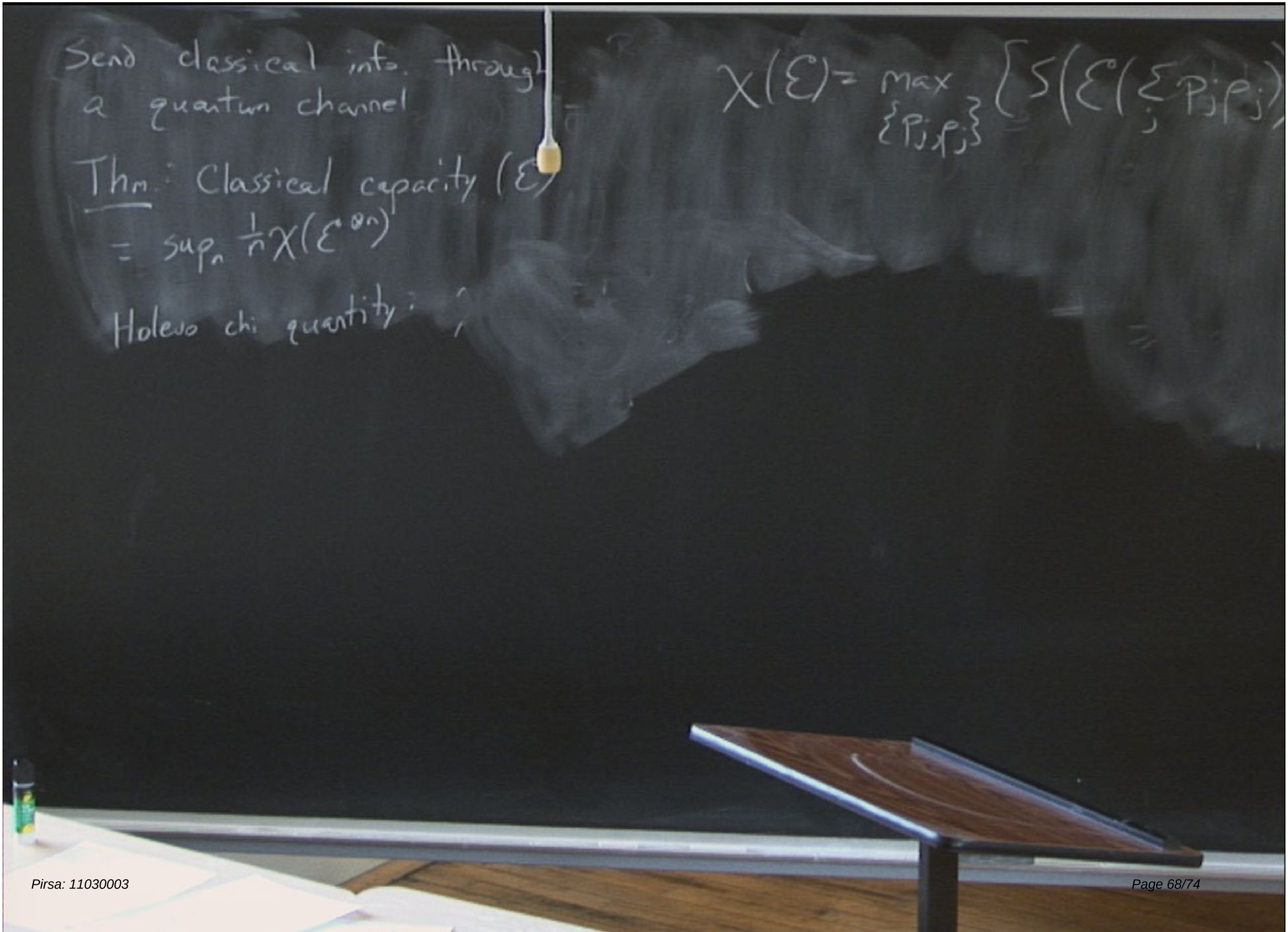
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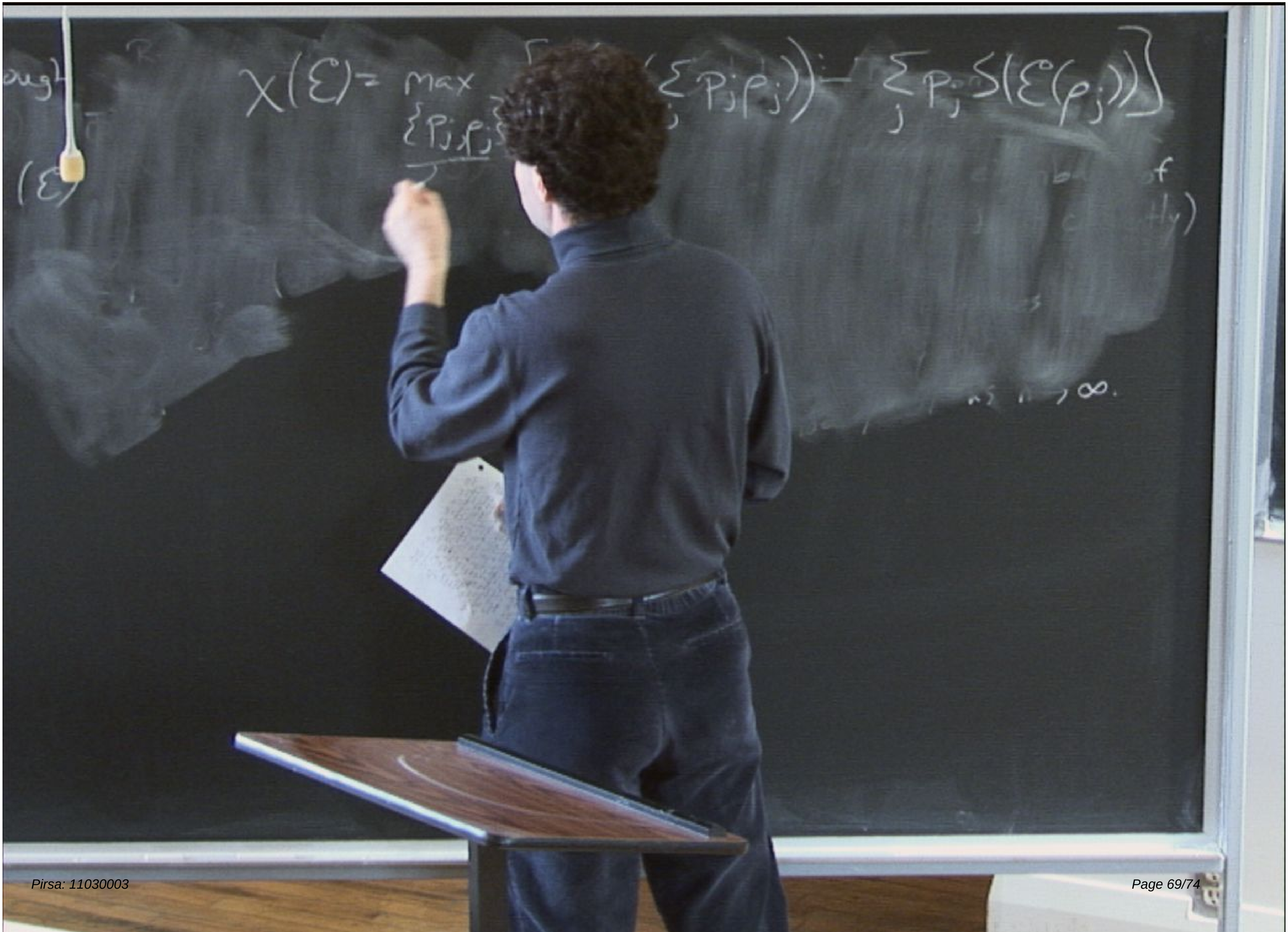
χ(ℰ)= max
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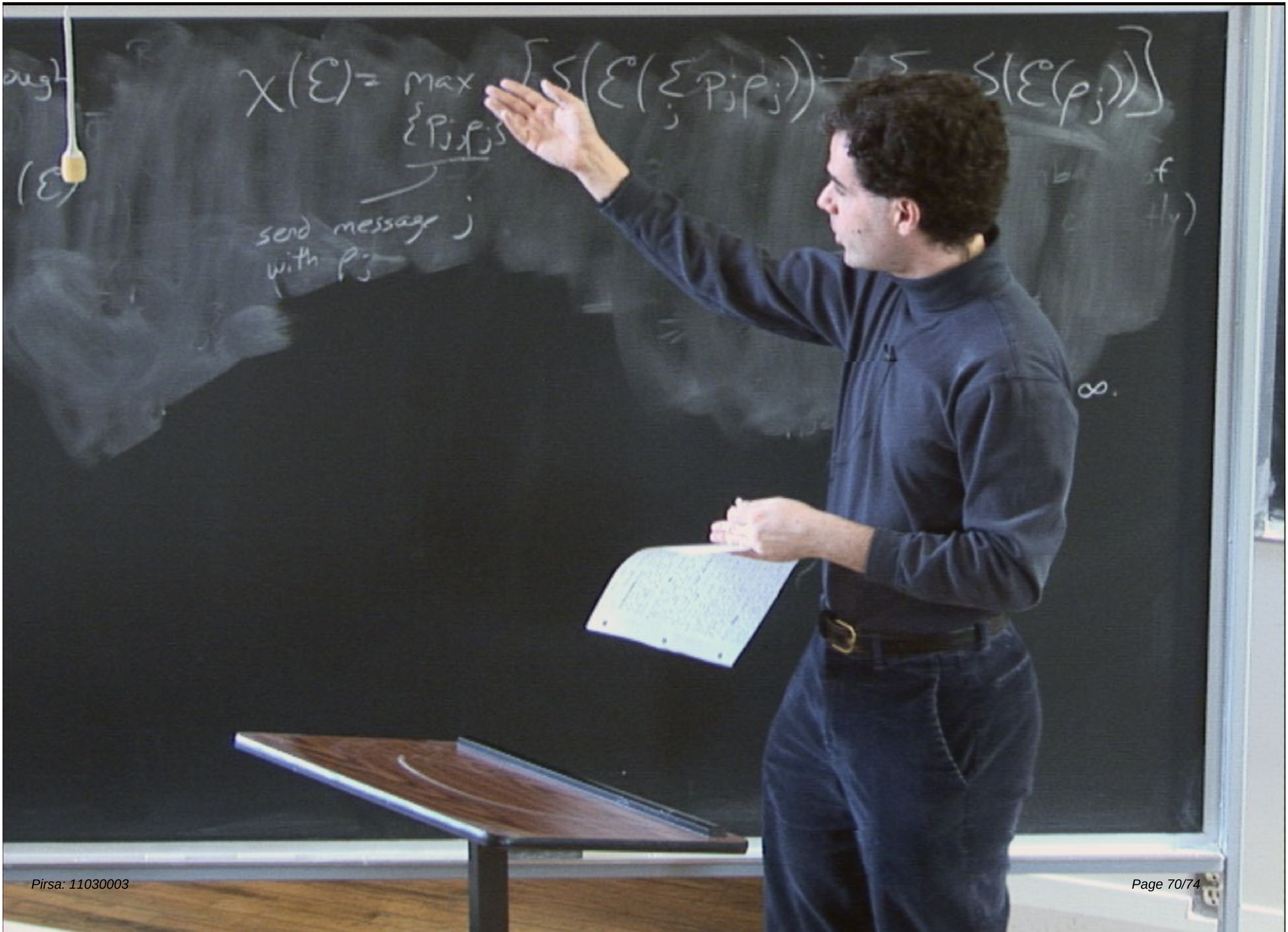
∞.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as $n \to \infty$.

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