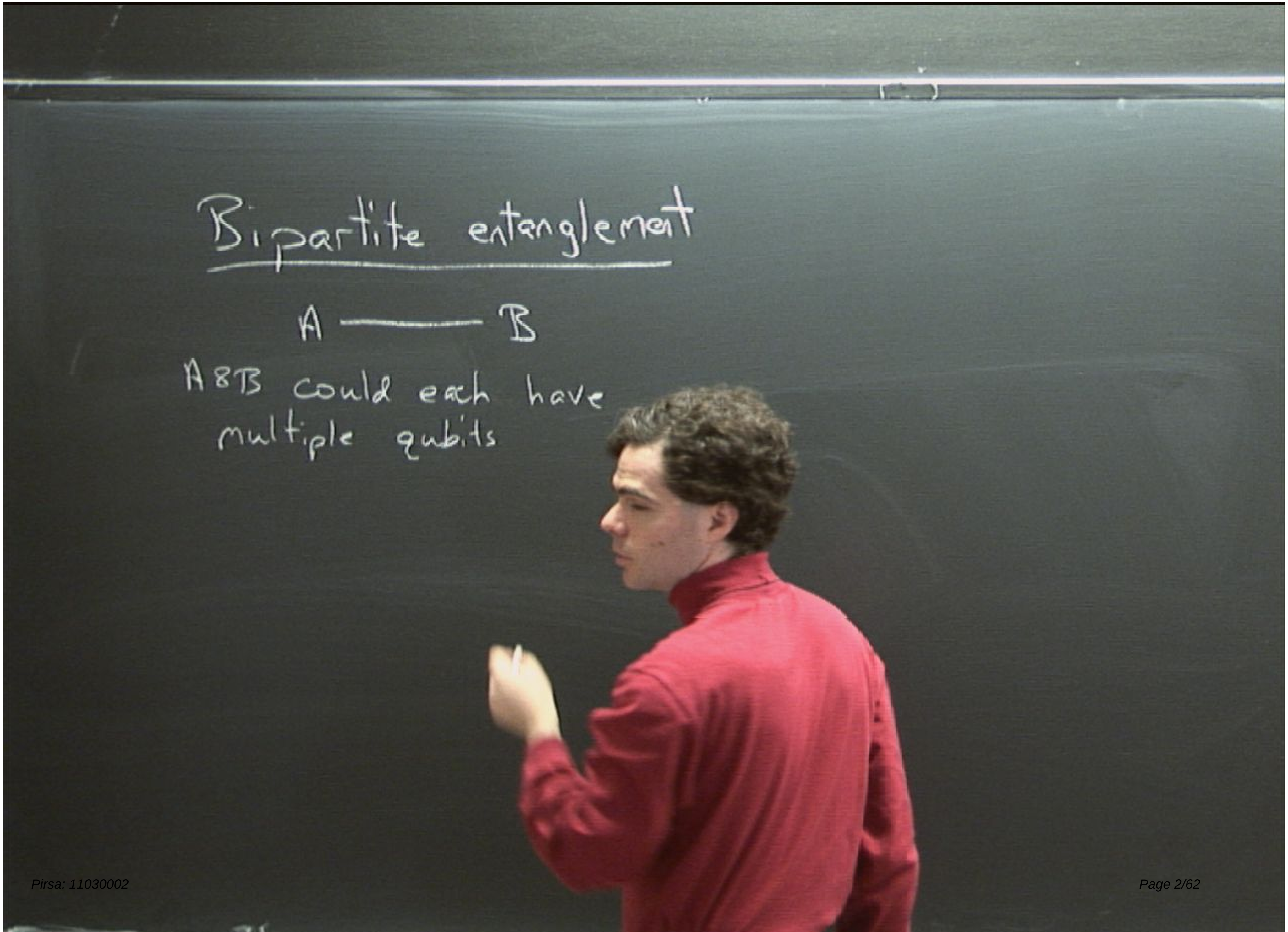
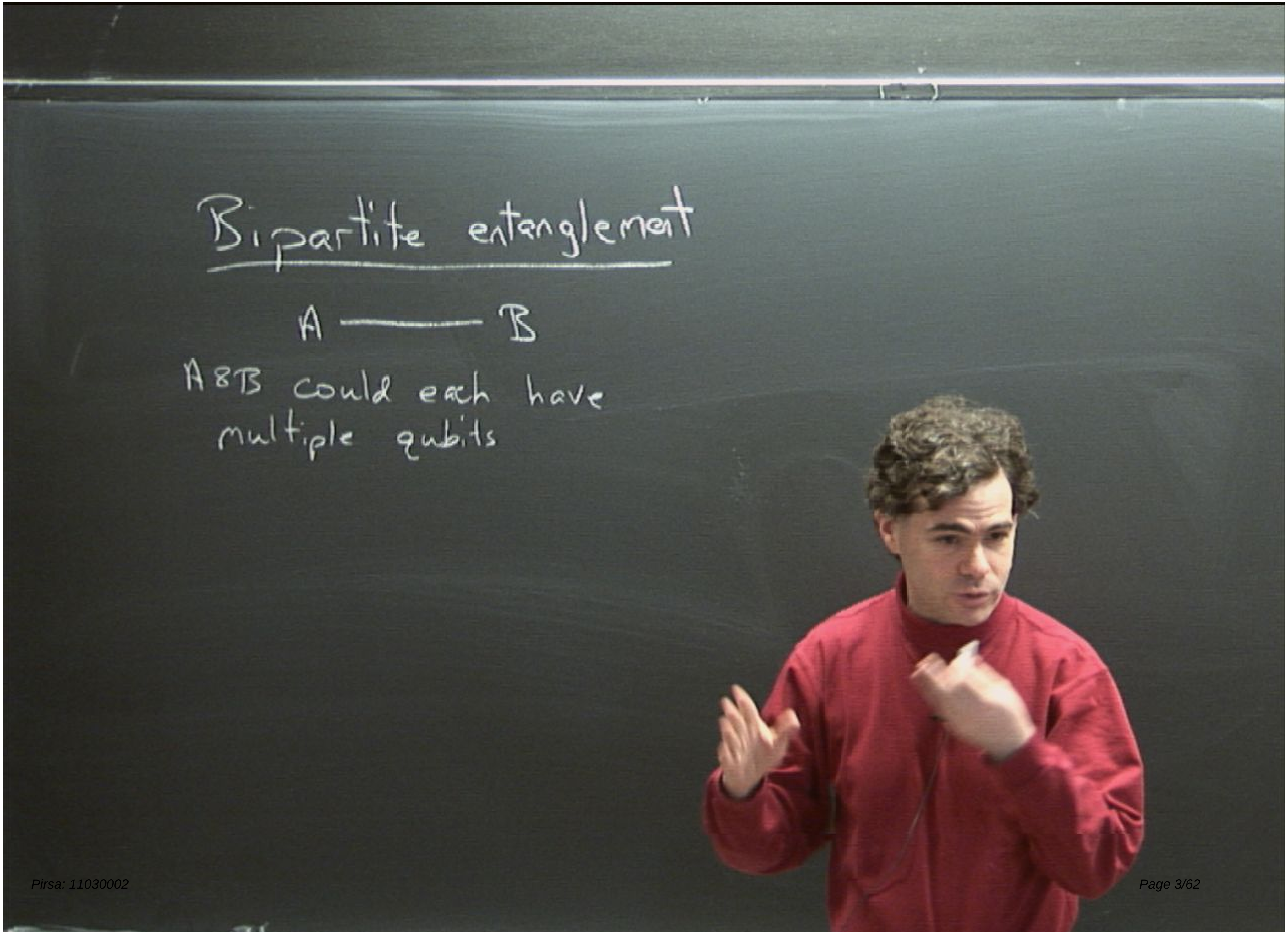
Title: Quantum Information Review - Lecture 13

Date: Mar 02, 2011 09:00 AM

URL: http://pirsa.org/11030002

Abstract:


Bipartite entanglement
A —— B
A & B could each have
multiple qubits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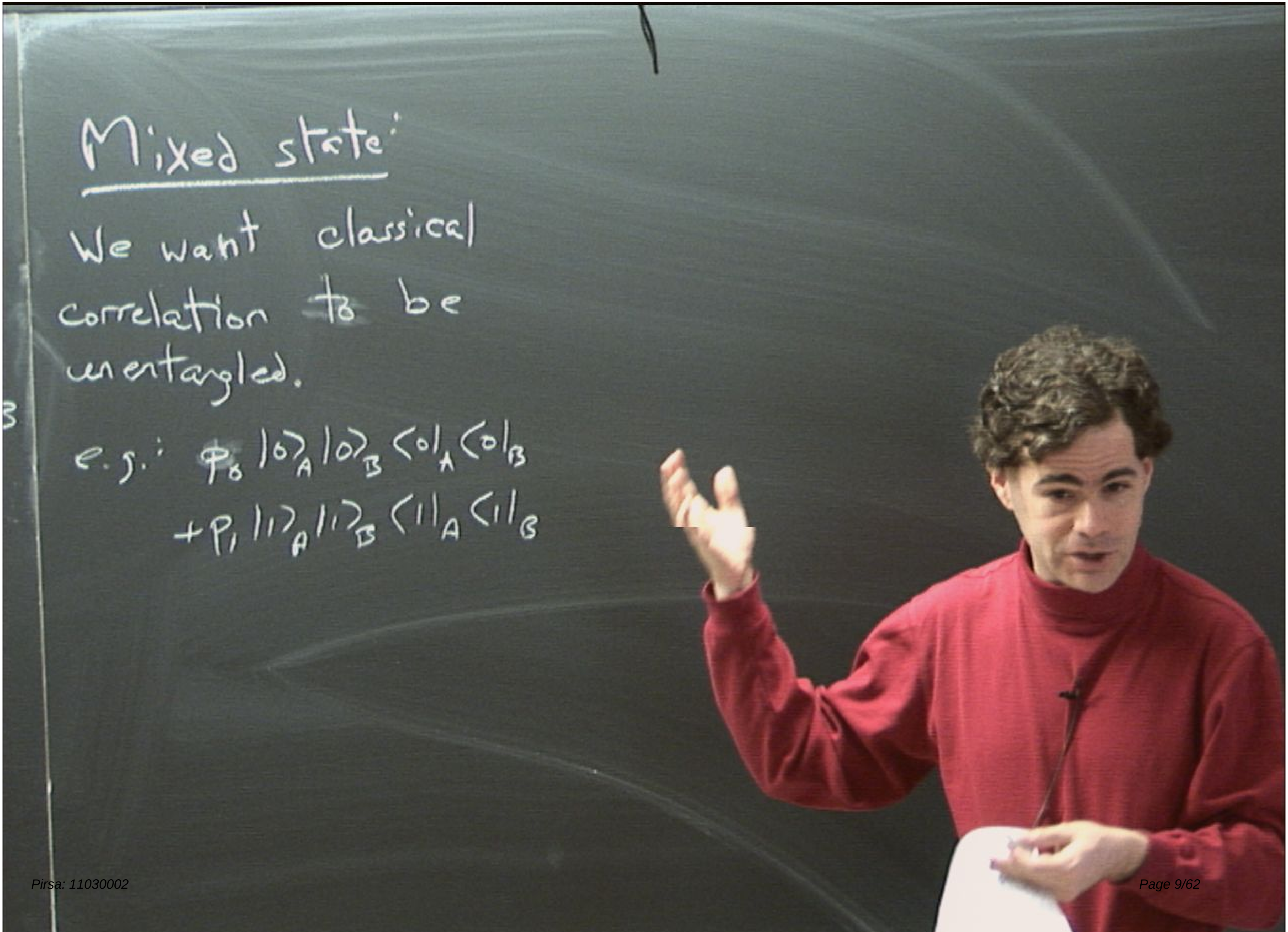
Mixed state:
We want classical correlation to be unentangled.
e.g.: $p_0 |0\rangle_A |0\rangle_B \langle 0|_A \langle 0|_B$
$+ p_1 |1\rangle_A |1\rangle_B \langle 1|_A \langle 1|_B$

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Def: A seperable mixed state ρ is one which can be written as a mixture

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not separable

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$\left(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\right)\left(\langle 0|_A \langle 0|_B + \langle 1|_A \langle 1|_B\right)$

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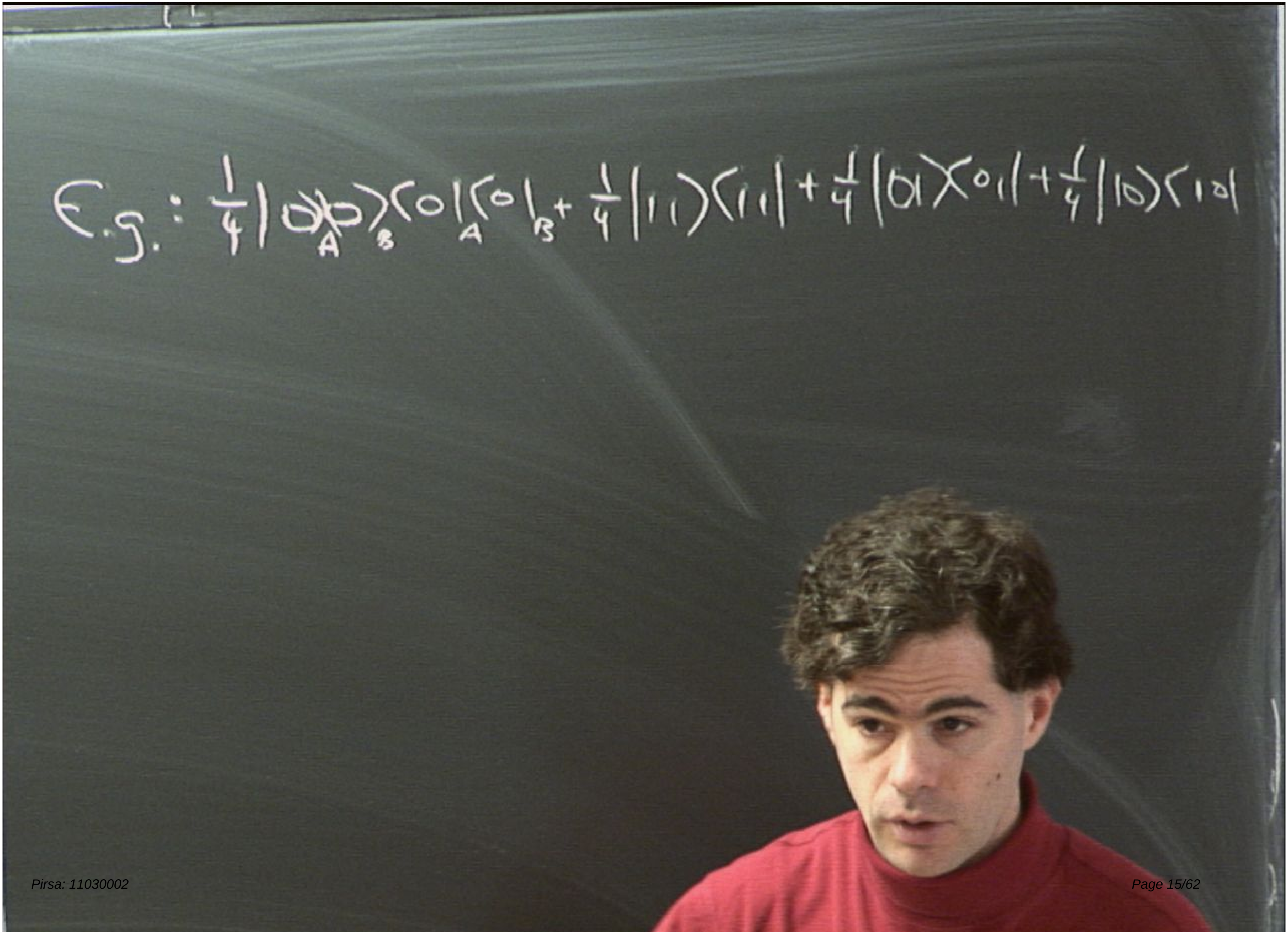
of tensor product states

E.g.: $\frac{1}{4}|00\rangle_{AB}\langle 0|_A \langle 0|_B + \frac{1}{4}|11\rangle\langle 11| + \frac{1}{4}|01\rangle\langle 01| + \frac{1}{4}|10\rangle\langle 10|$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E.g.: $\frac{1}{4}|0\rangle_A|0\rangle_B\langle 0|_A\langle 0|_B + \frac{1}{4}|11\rangle\langle 11| + \frac{1}{4}|01\rangle\langle 01| + \frac{1}{4}|10\rangle\langle 10|$

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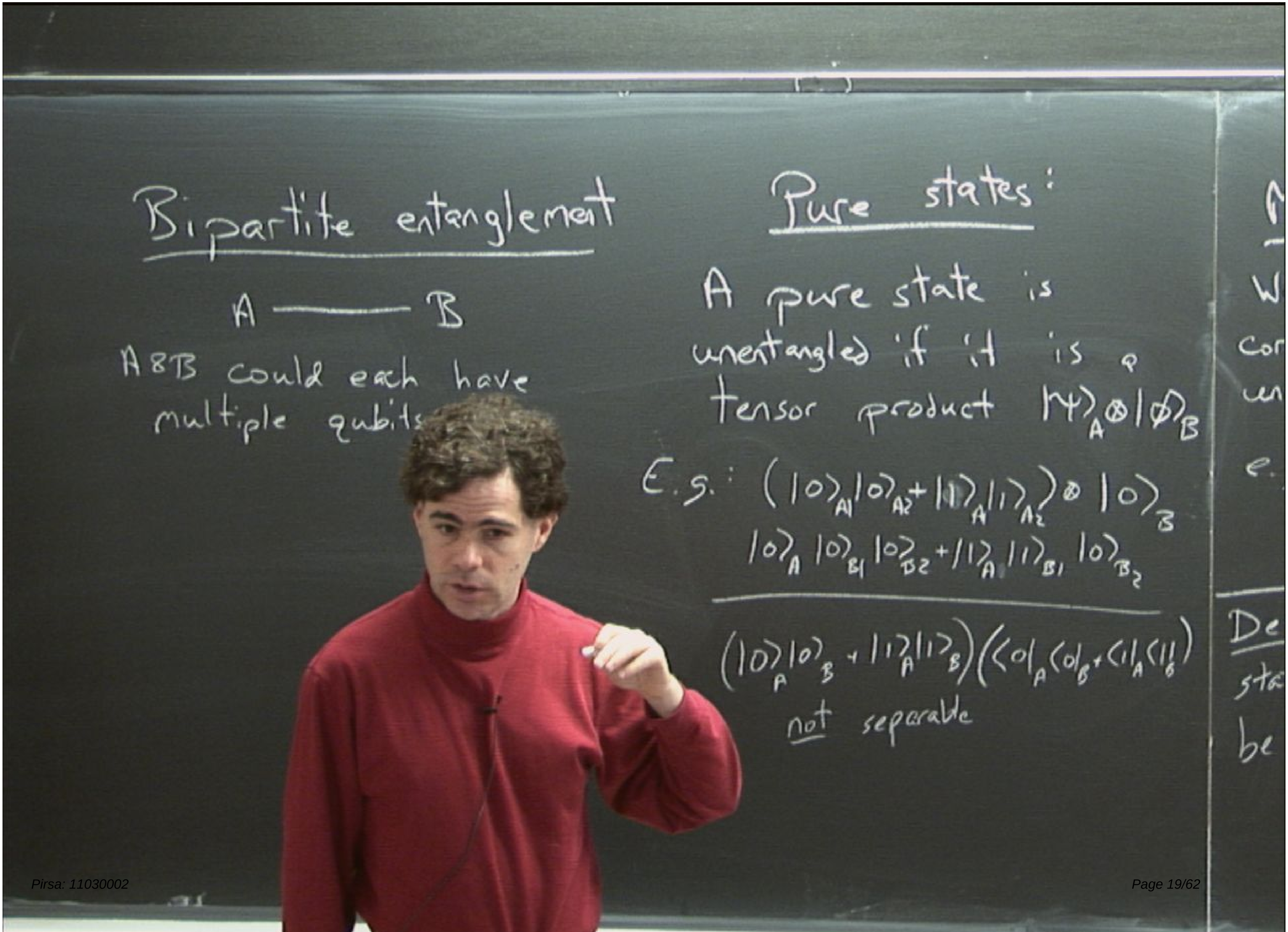
Question of whether a given state ρ is separable is NP-hard (as function of Hilbert space dimension)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(|0⟩_A |0⟩_B + |1⟩_A |1⟩_B)(⟨0|_A ⟨0|_B + ⟨1|_A ⟨1|_B)
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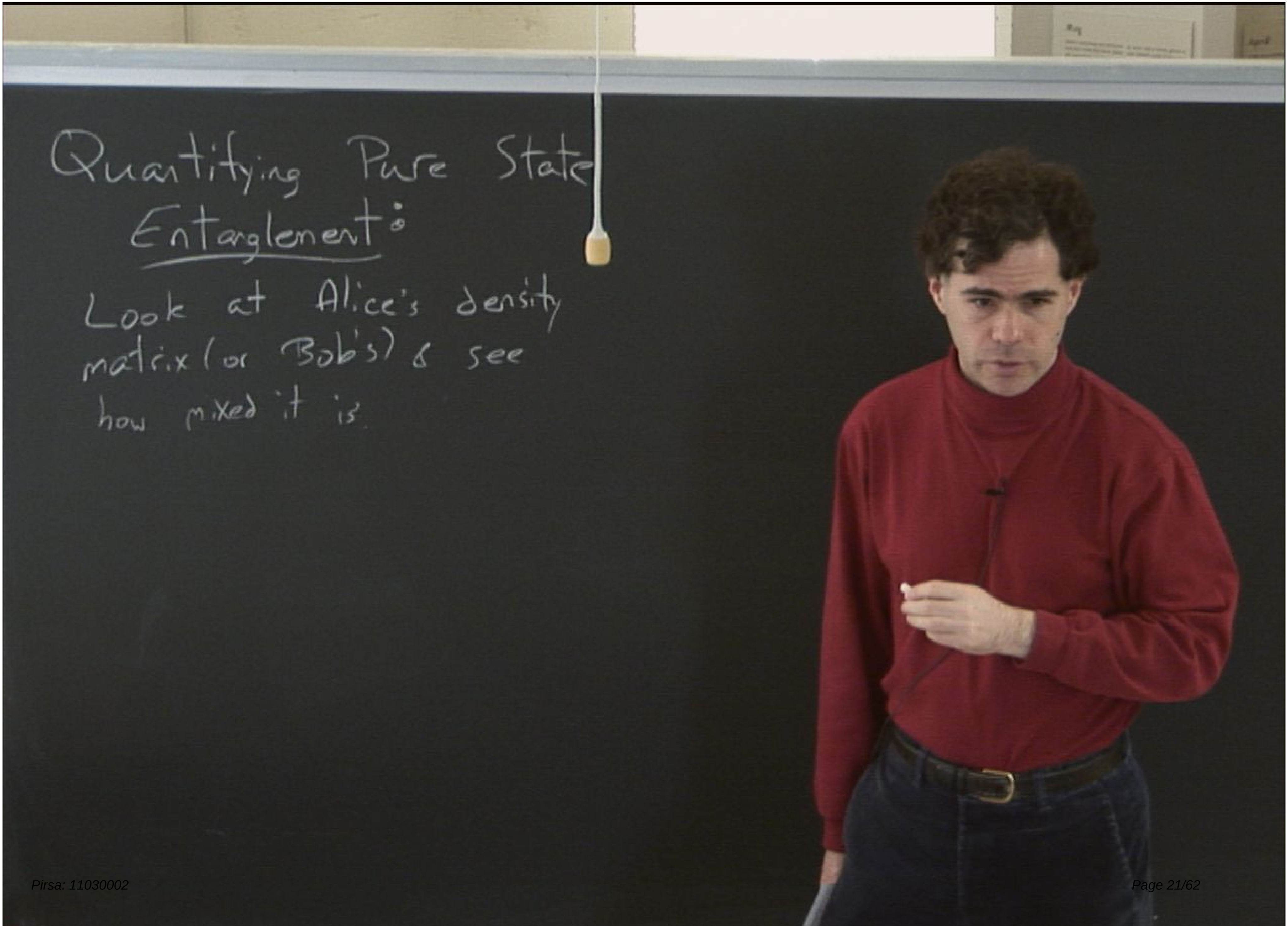
Quantifying Pure State
Entanglement:
Look at Alice's density
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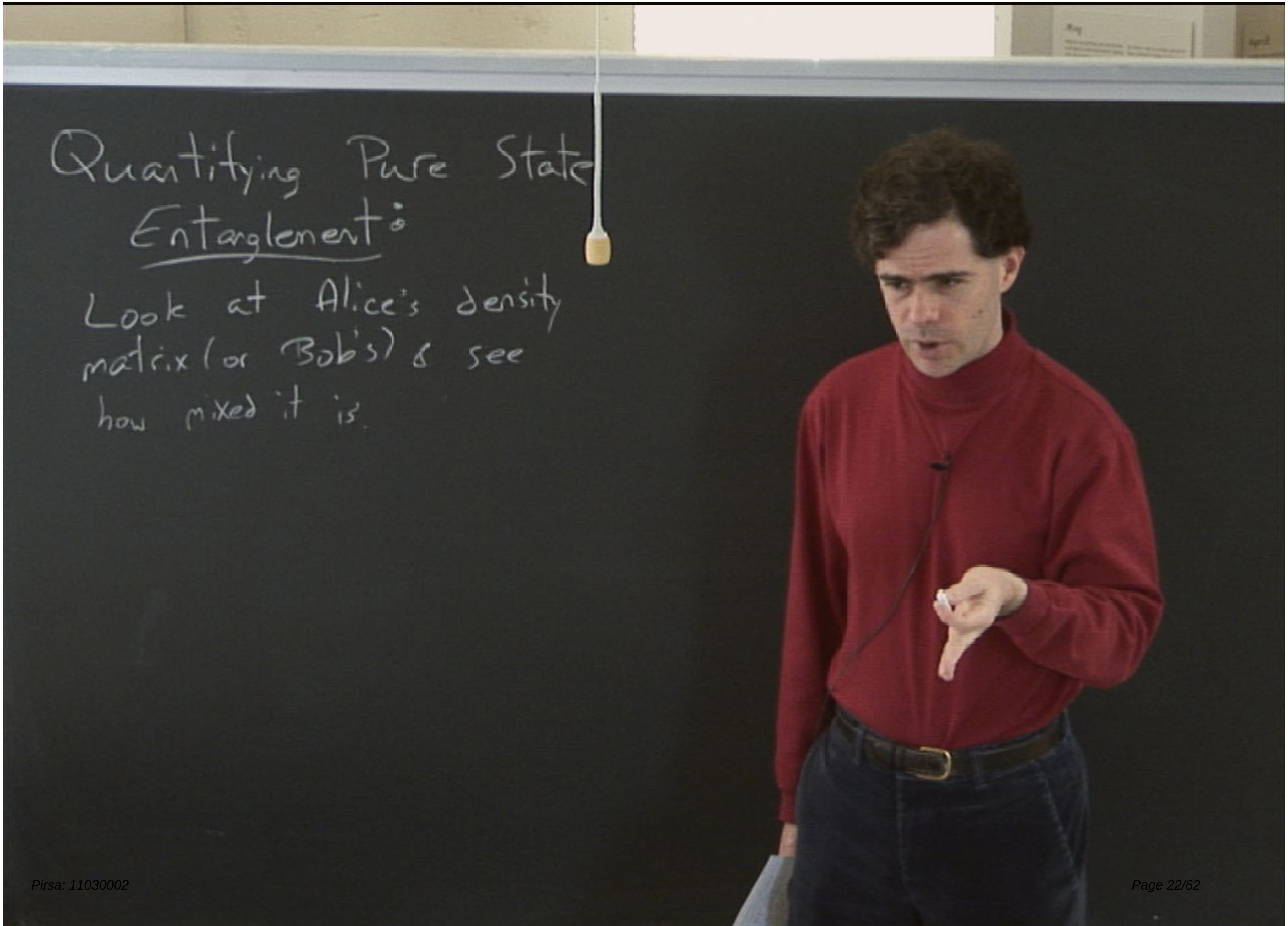
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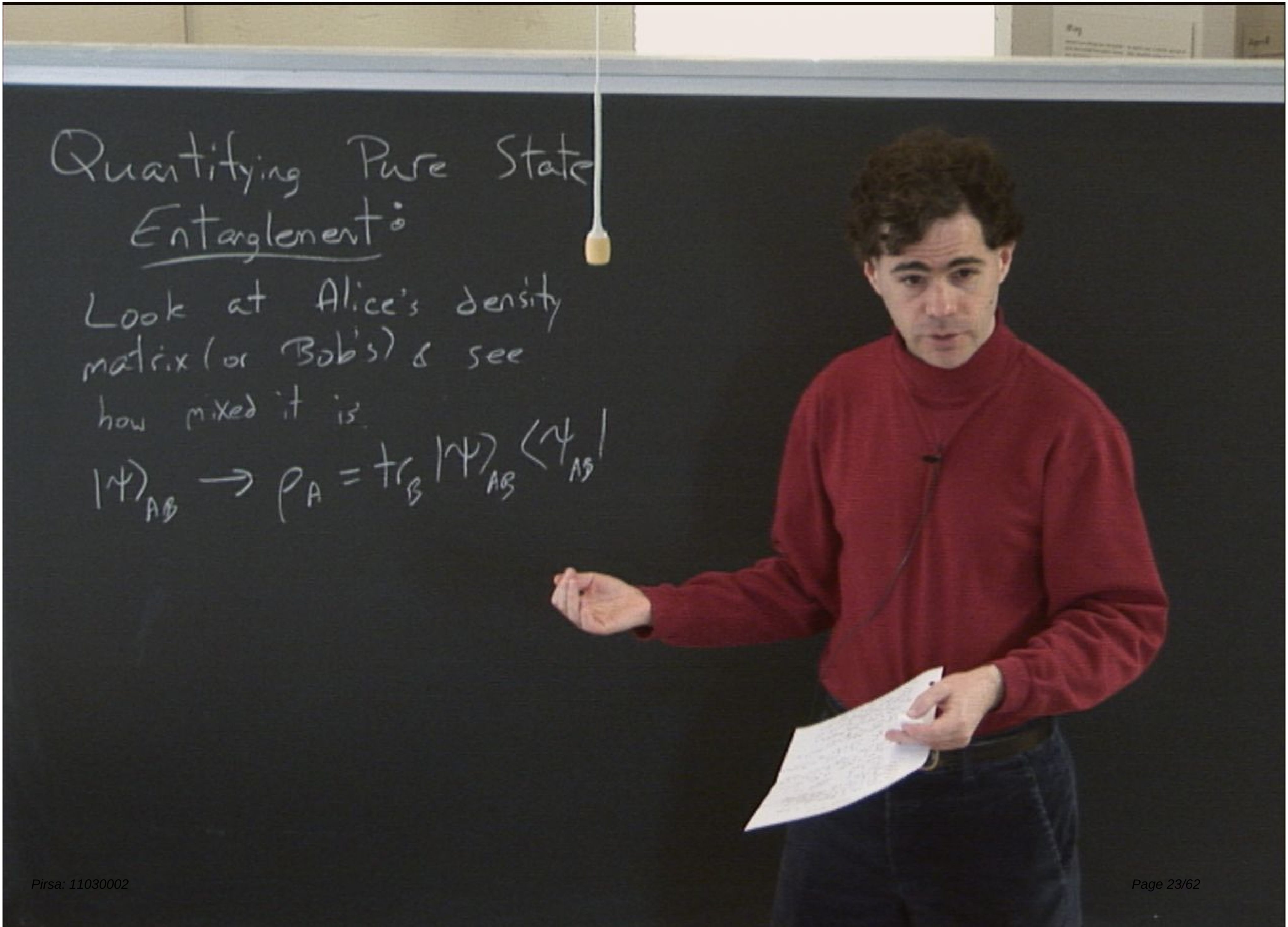
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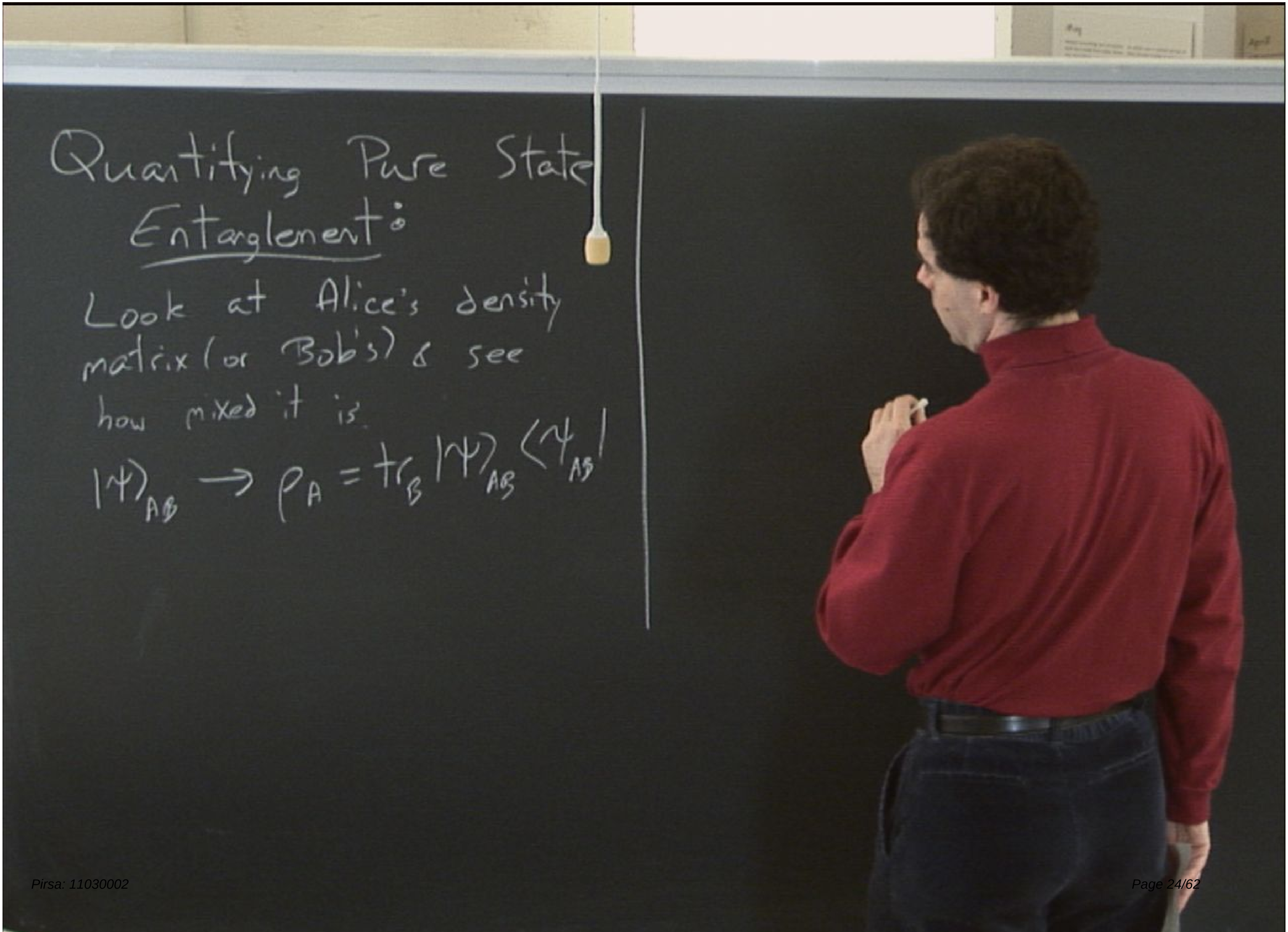
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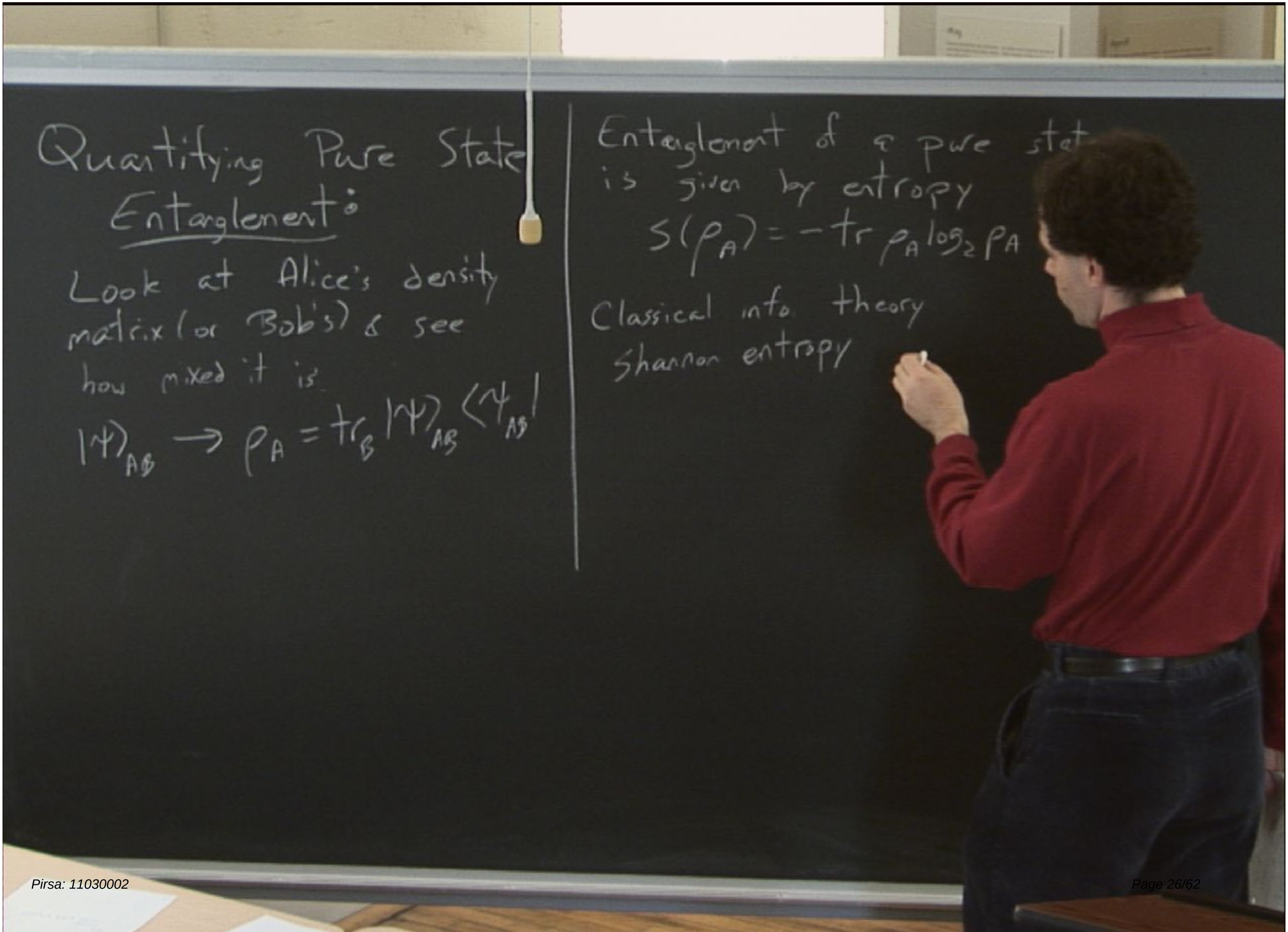
$$S(\rho_A) = -\mathrm{tr}\, \rho_A \log_2 \rho_A$$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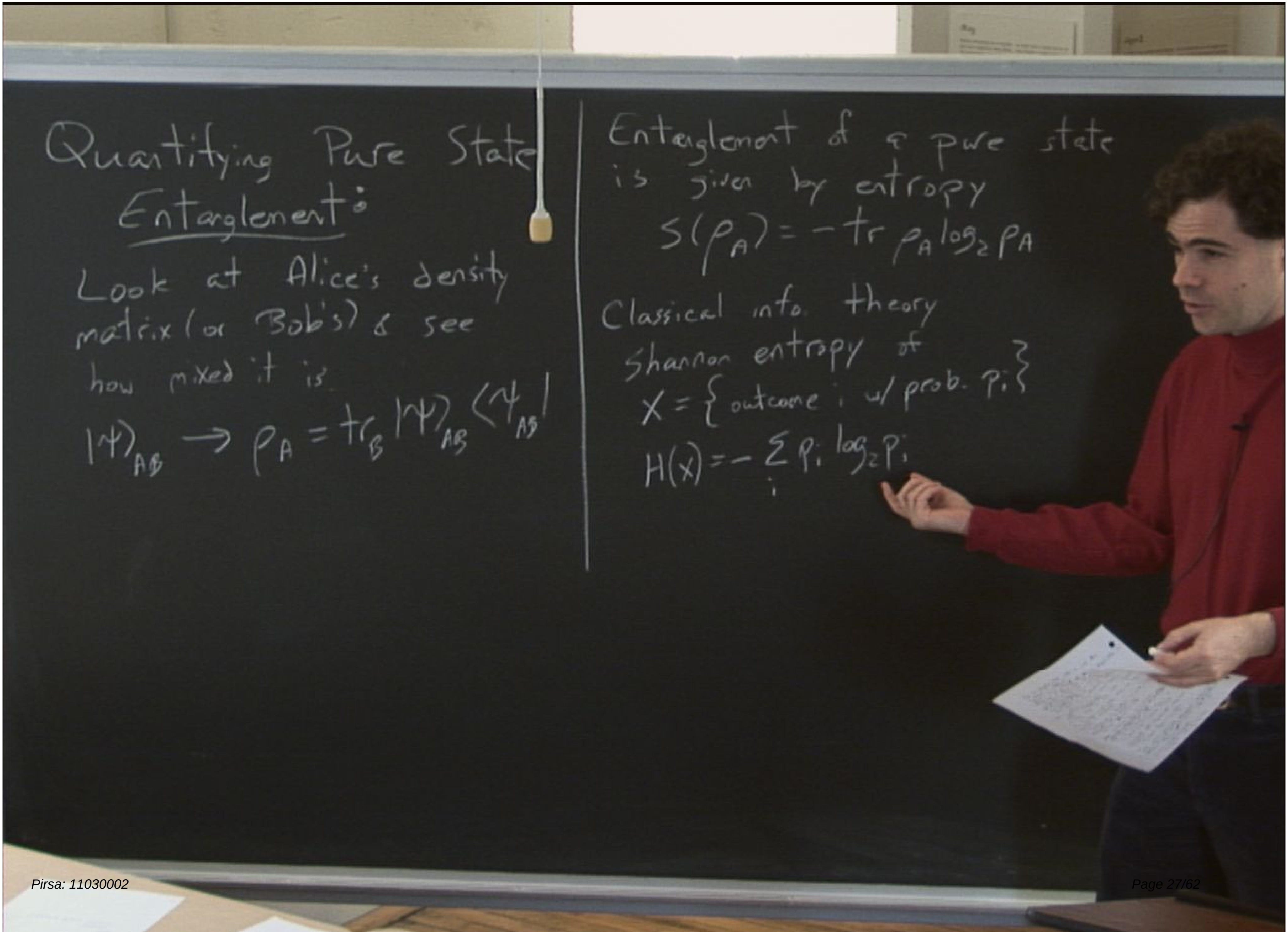
Classical info theory
Shannon entropy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$$S(A,B) \ge |S(A) - S(B)|$$

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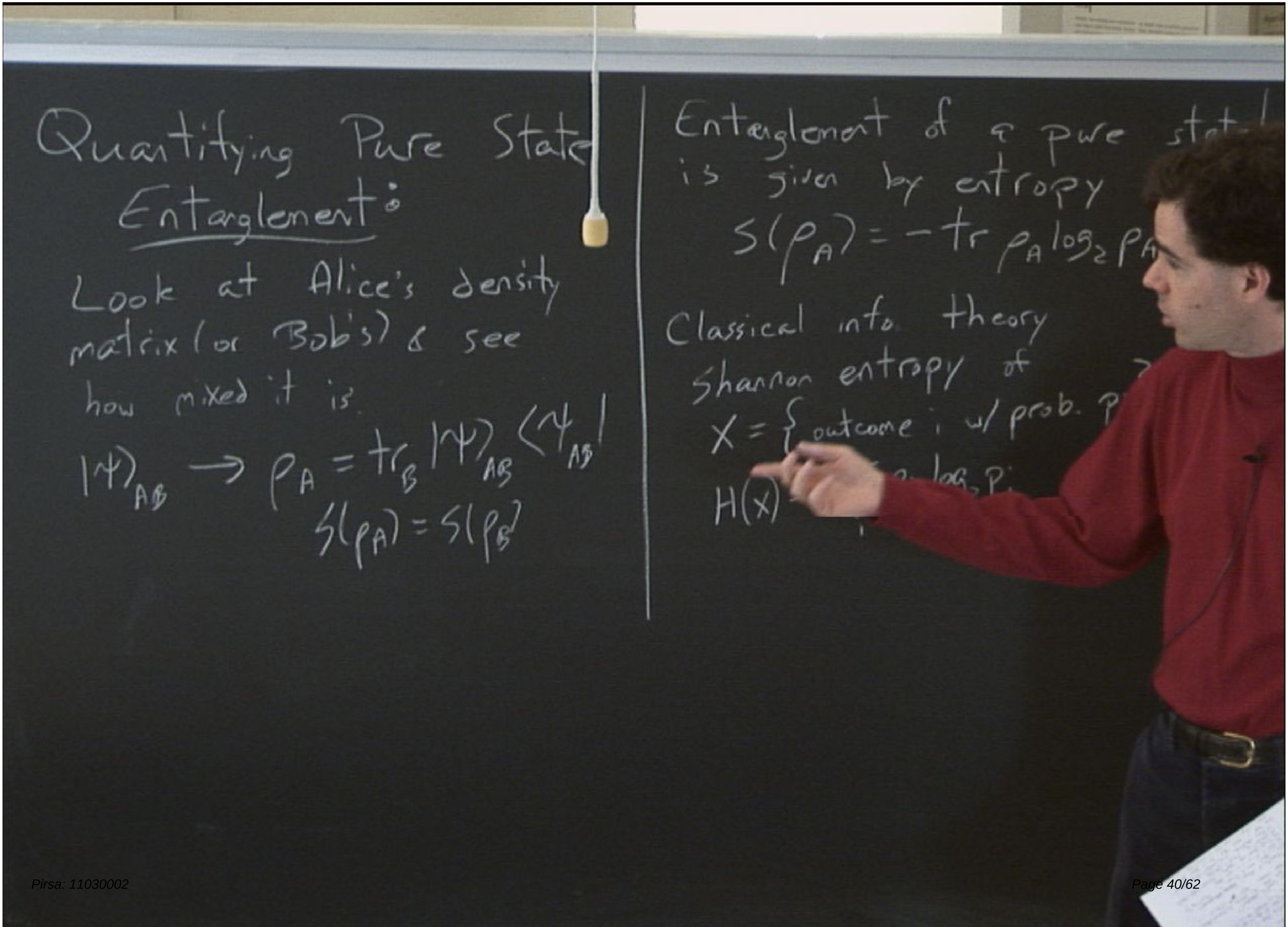
$$S(A,B,C) + S(B) \leq S(A,B) + S(B,C)$$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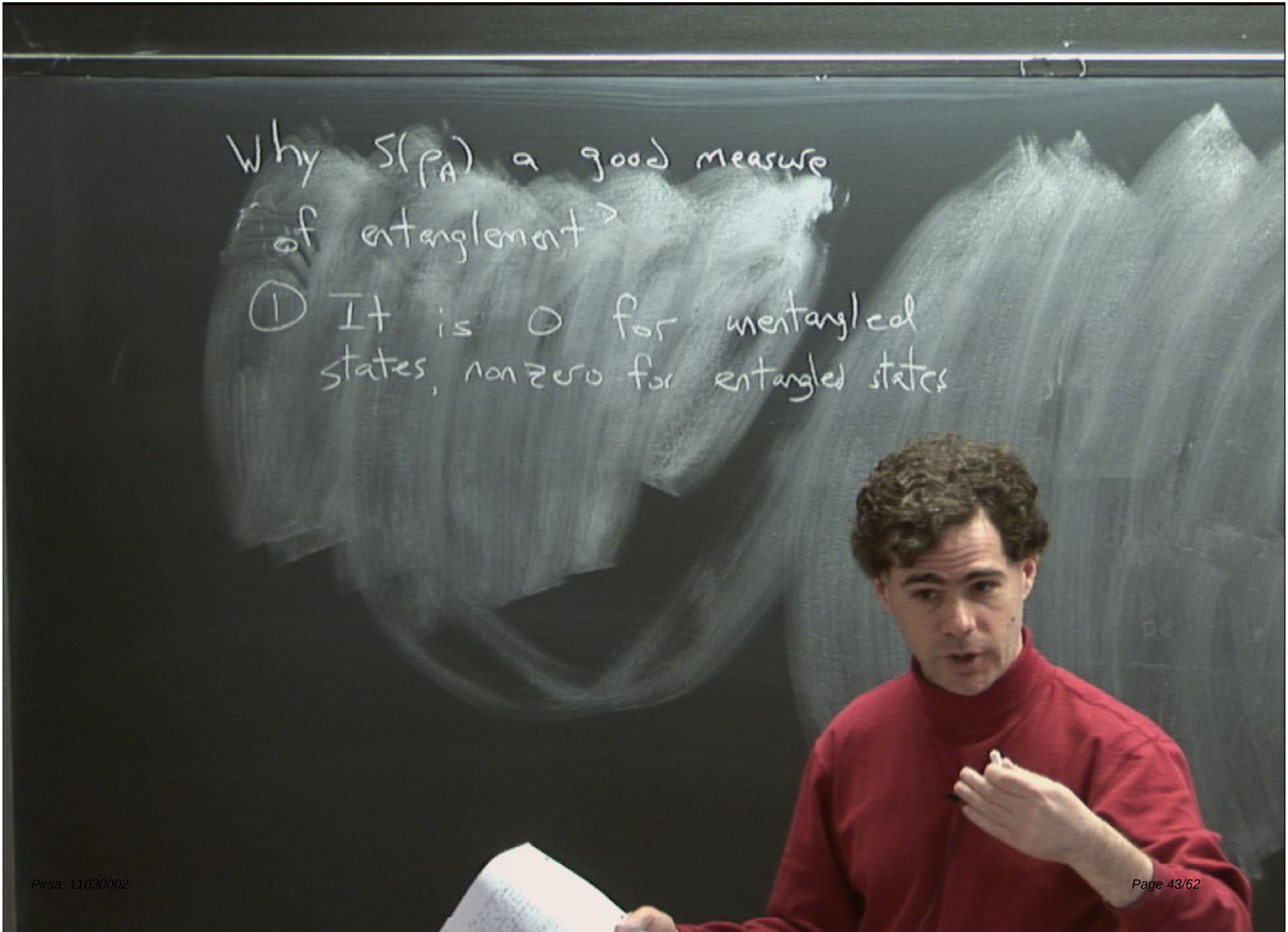
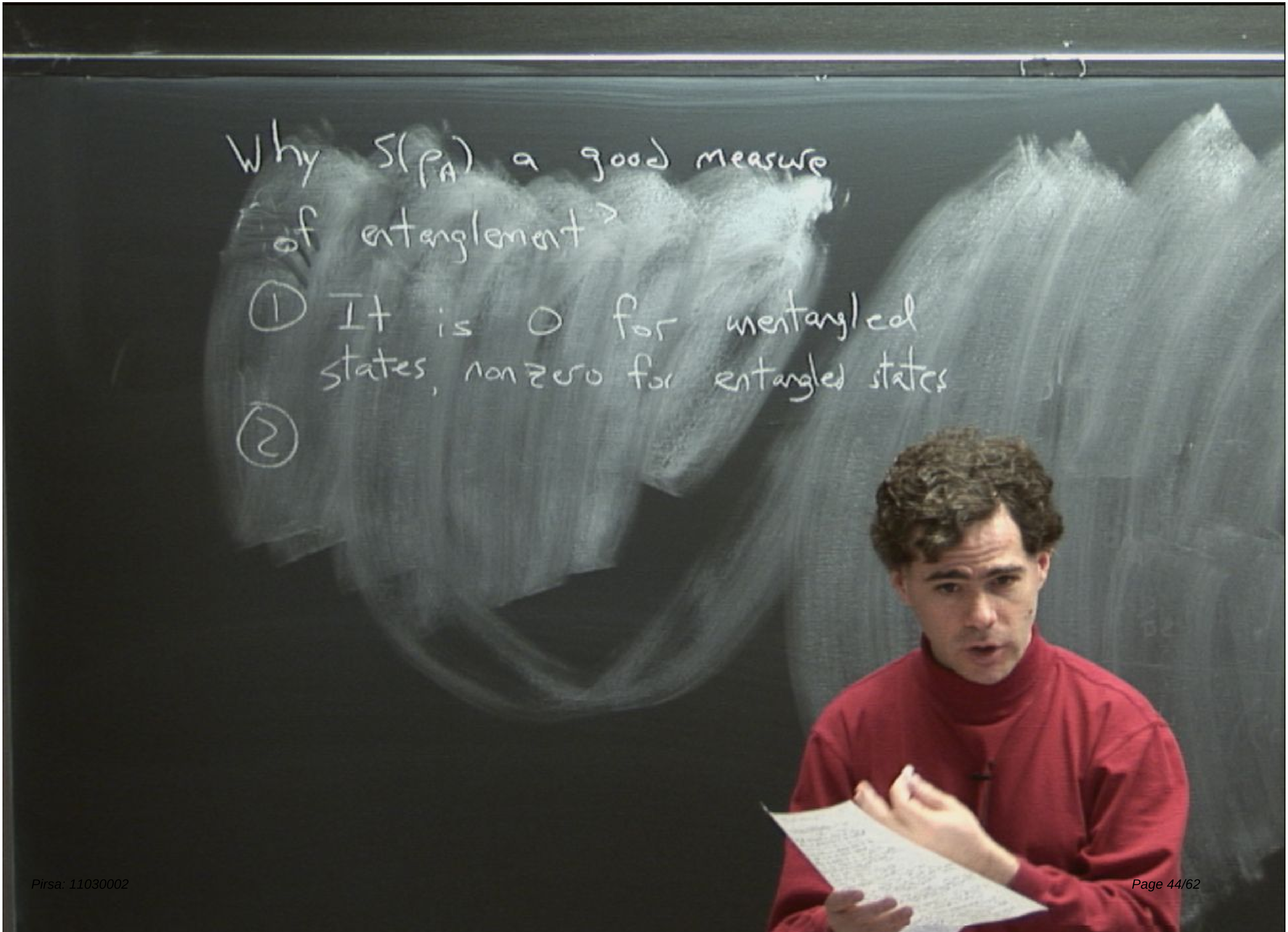

Why $S(\rho_A)$ a good measure
of entanglement?
① It is 0 for unentangled
states, nonzero for entangled states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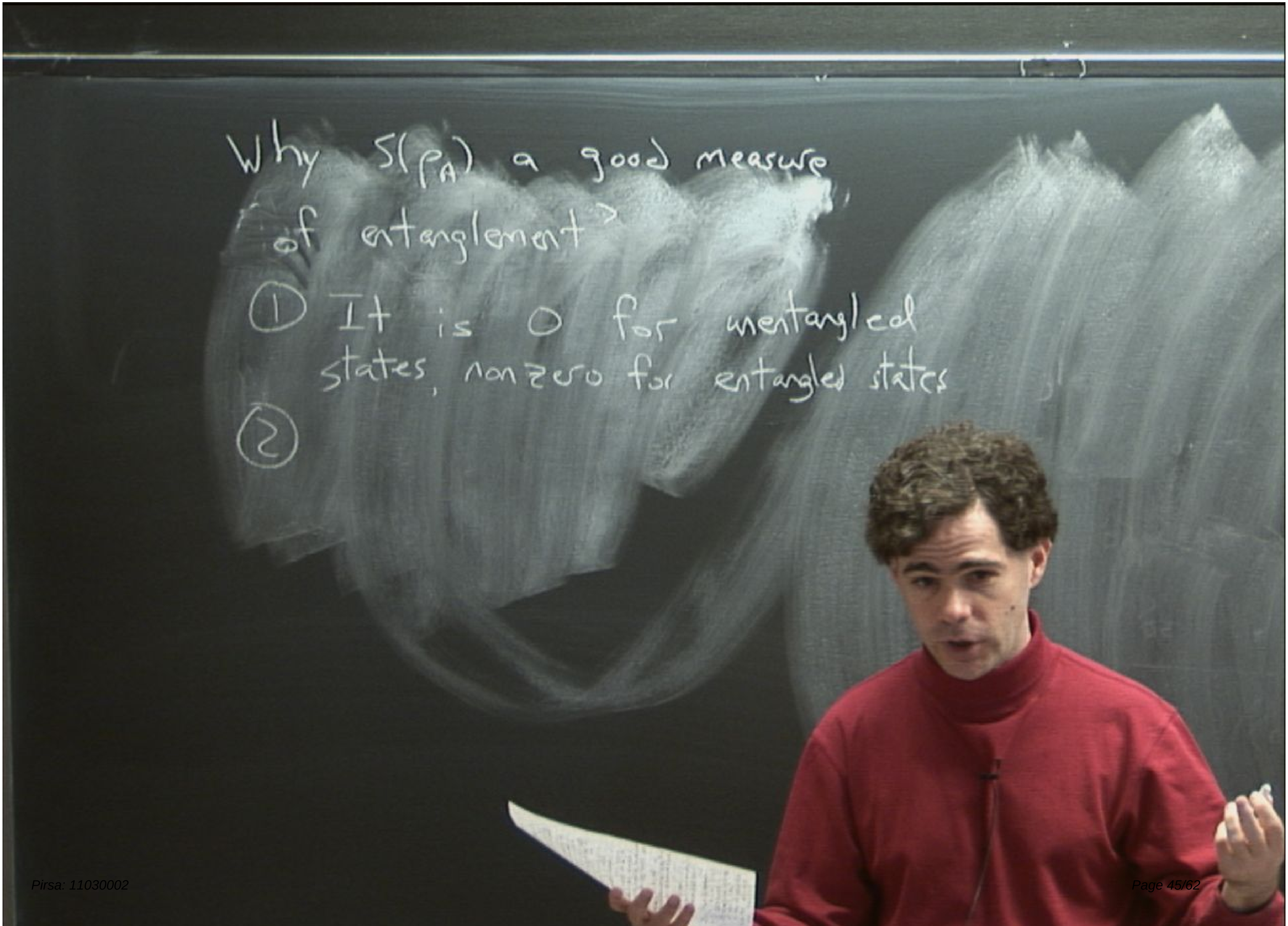
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Why $S(\rho_A)$ a good measure of entanglement?

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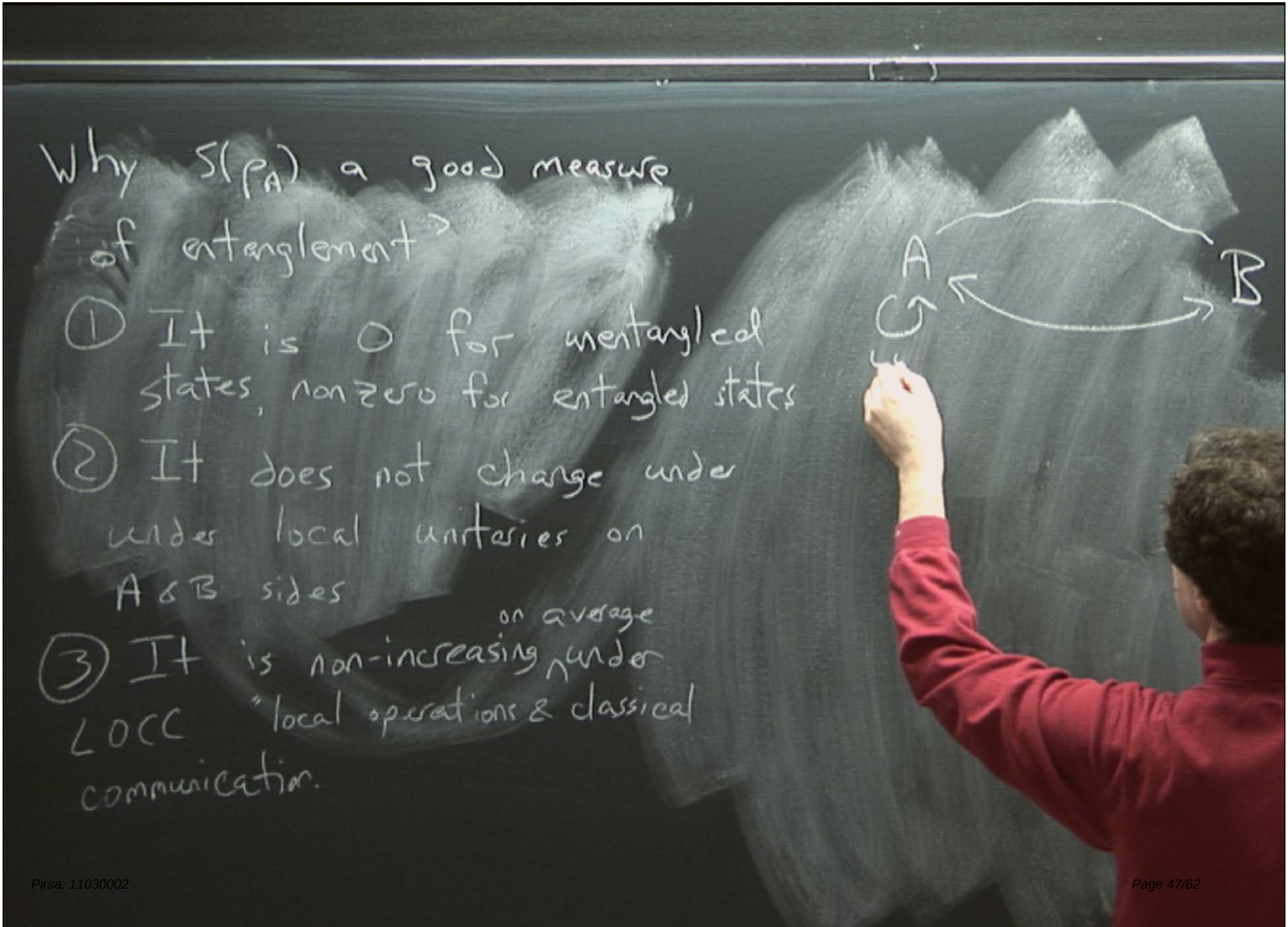
2. It does not change under under local unitaries on A & B sides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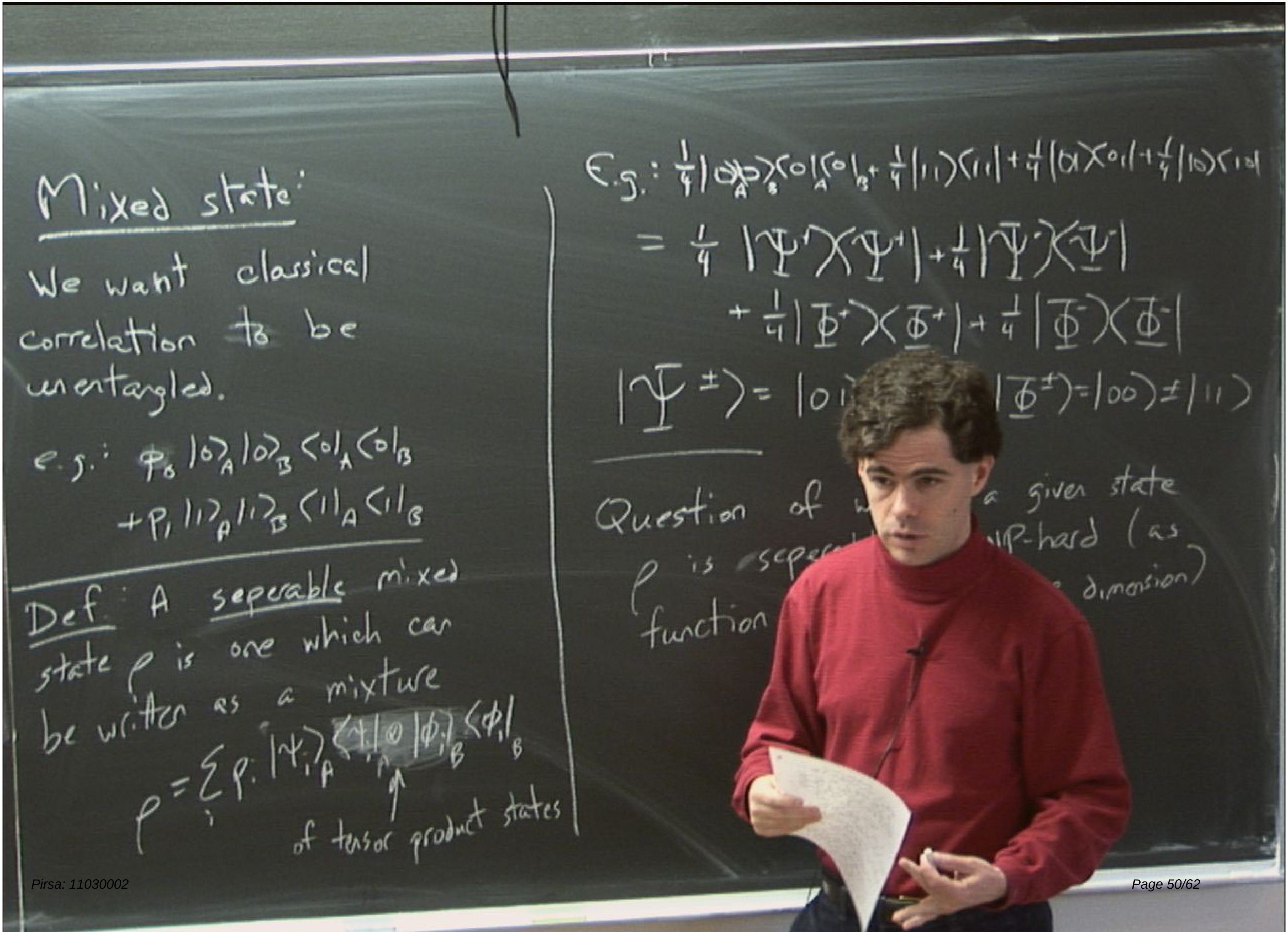
w/ LOCC, A & B can only create seperable states


Mixed state:
We want classical
correlation to be
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e.g.: $p_0 |0\rangle_A |0\rangle_B \langle 0|_A \langle 0|_B$
$+ p_1 |1\rangle_A |1\rangle_B \langle 1|_A \langle 1|_B$
Def: A seperable mixed
state ρ is one which can
be written as a mixture
$\rho = \sum_i p_i |\psi_i\rangle_A \langle \psi_i|_A \otimes |\phi_i\rangle_B \langle \phi_i|_B$
of tensor product states
E.g.: $\frac{1}{4}|00\rangle\langle 00| + \frac{1}{4}|11\rangle\langle 11| + \frac{1}{4}|01\rangle\langle 01| + \frac{1}{4}|10\rangle\langle 10|$
$= \frac{1}{4}|\Psi^+\rangle\langle\Psi^+| + \frac{1}{4}|\Psi^-\rangle\langle\Psi^-|$
$+ \frac{1}{4}|\Phi^+\rangle\langle\Phi^+| + \frac{1}{4}|\Phi^-\rangle\langle\Phi^-|$
$|\Phi^\pm\rangle = |00\rangle \pm |11\rangle$
Question of
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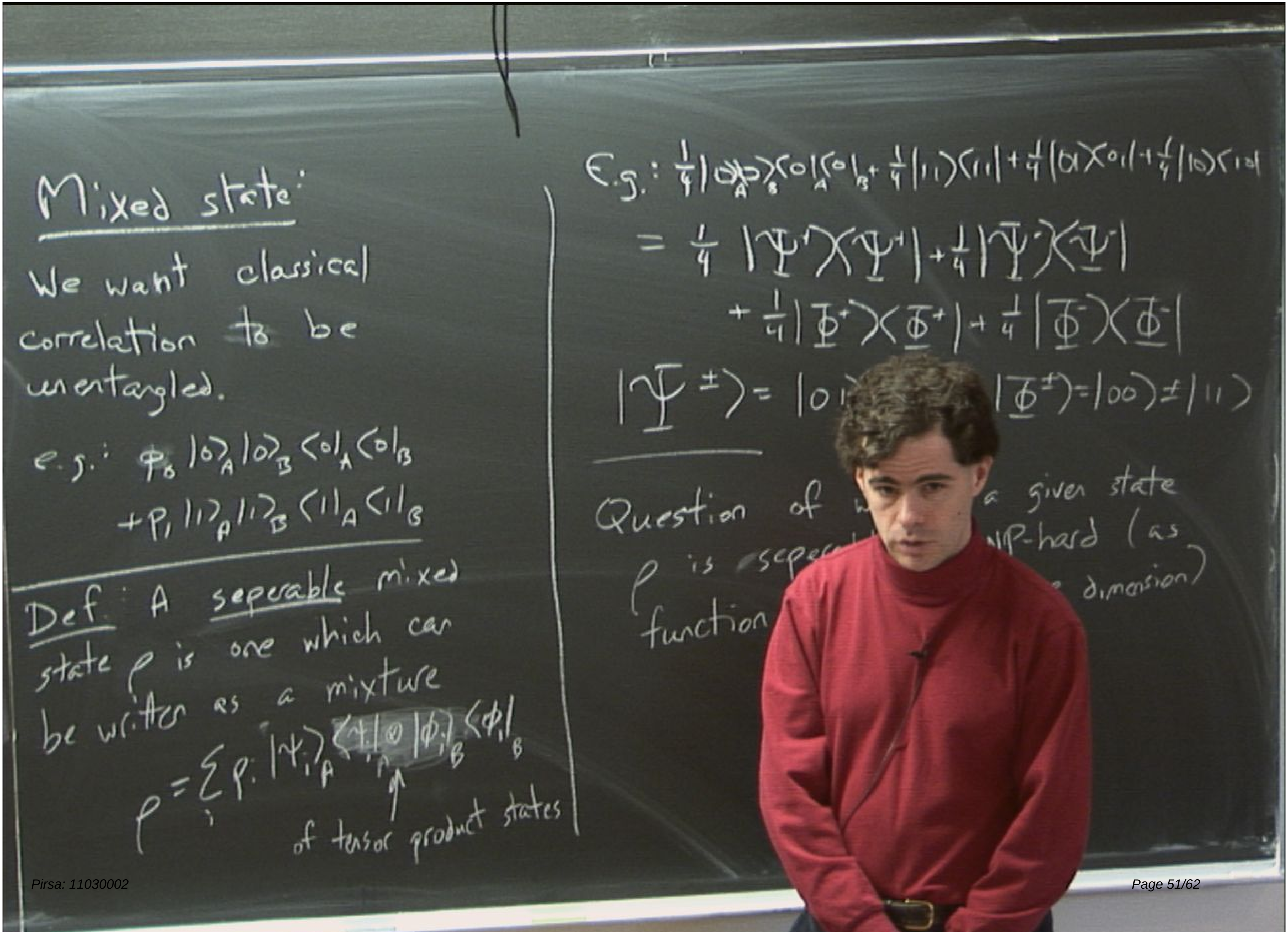
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Quantifying Pure State Entanglement:

Look at Alice's density matrix (or Bob's) & see how mixed it is.

$$|\Psi\rangle_{AB} \rightarrow \rho_A = tr_B |\Psi\rangle_{AB}\langle\Psi_{AB}|$$

$$S(\rho_A) = S(\rho_B)$$

Entanglement of a pure state is given by entropy

$$S(\rho_A) = -tr\, \rho_A \log_2 \rho_A$$

Classical info. theory

Shannon entropy of

$$X = \{\text{outcome } i \text{ w/ prob. } p_i\}$$

$$H(X) = -\sum_i p_i \log_2 p_i$$

c)

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<u>Thm</u>: (Properties of entropy)

a) On a Hilbert space of dimension D, $S(\rho) \le \log D$

b) $S(\rho) = 0 \Leftrightarrow \rho$ is pure

c) Triangle / Araki-Lieb inequality:

$$S(A,B) \ge |S(A) - S(B)|$$

d) Subadditivity $S(A,B) \le S(A) + S(B)$

e) Strong subadditivity

$$S(A,B,C) + S(B) \le S(A,B) + S(B,C)$$

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A ⟲ LO ⟷ classical communication ⟷ B ⟲ LO

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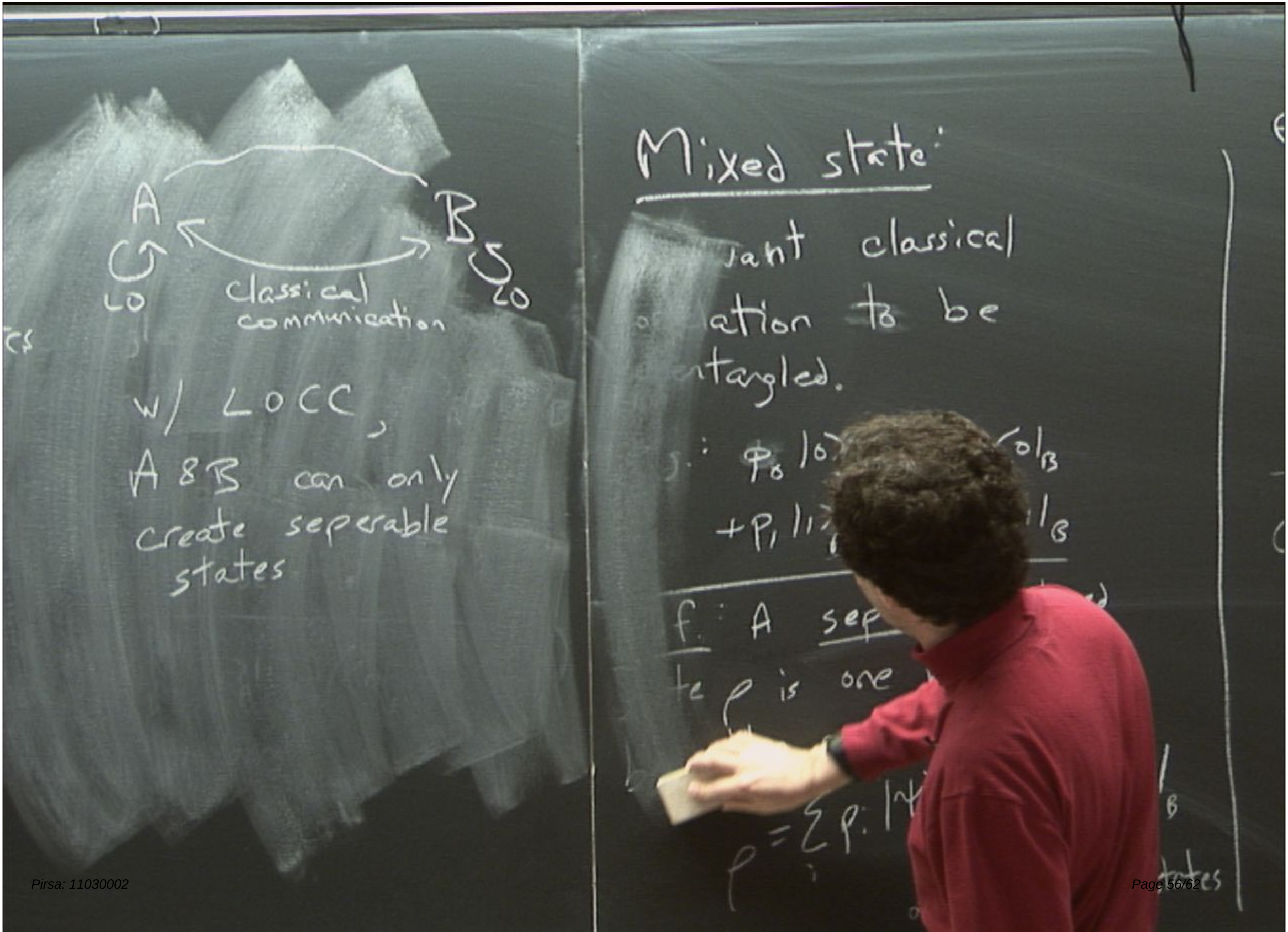
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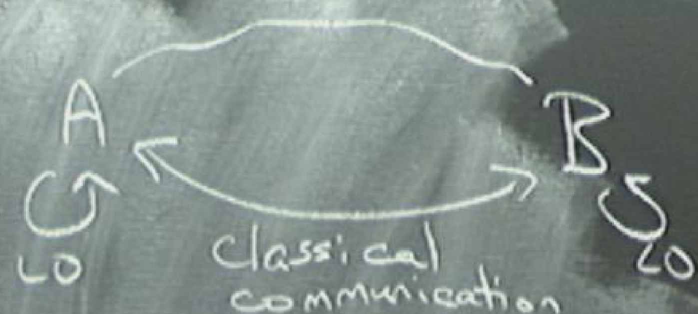
Mixed state:

Entanglement of formation:

minimal amount of entanglement need to create the state

$E_f(\rho_{AB}) =$



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where

$$\rho_{AB} = \sum_i p_i |\psi_i\rangle_{AB} \langle \psi_i|_{AB}$$

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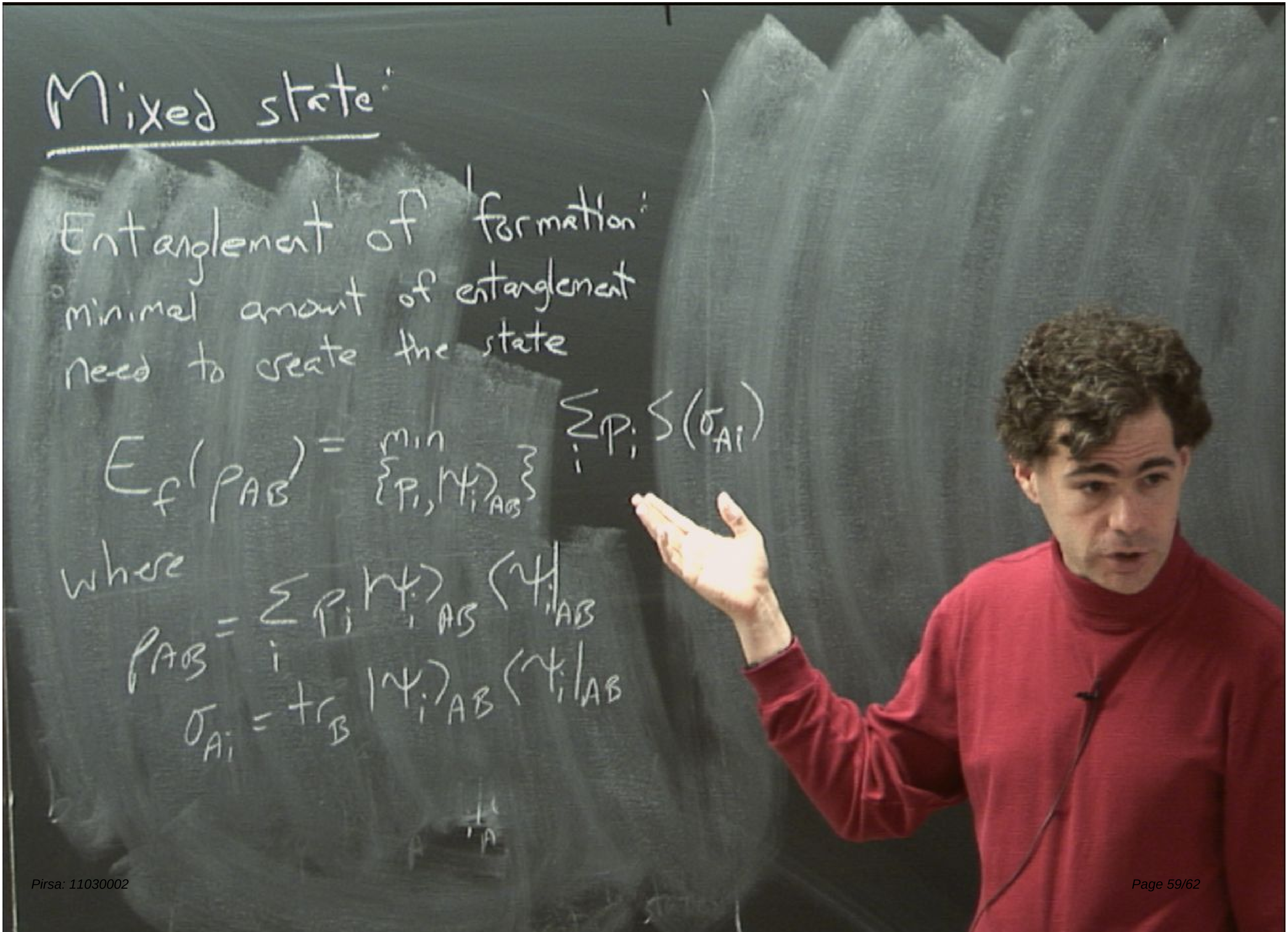
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Not additive:

Cases where

$$E_f(\rho_{AB} \otimes \sigma_{AB}) < E_f(\rho_{AB}) + E_f(\sigma_{AB})$$

<u>Entanglement cost</u>

Regularized entanglement of formation

$$E_c(\rho_{AB}) = \lim_{n \to \infty} \frac{1}{n} E_f(\rho_{AB}^{\otimes n})$$

$S(\sigma_{Ai})$

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Distillable entanglement: How much pure state entanglement can I get out?

Bound entanglement $E_D = 0$, $E_f > 0$

$E_D \le E_f$

$S(\sigma_{A_1})$

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$$E_f(\rho_{AB} \otimes \sigma_{AB}) \lneq E_f(\rho_{AB}) + E_f(\sigma_{AB})$$

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