Title: Quantum Information Review - Lecture 12

Date: Mar 01, 2011 10:10 AM

URL: http://pirsa.org/11030001

Abstract:

Fault Tolerance

- Fault tolerant circuits are designed to avoid error propagation.

Transversal gates

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7-qubit code is particularly good.

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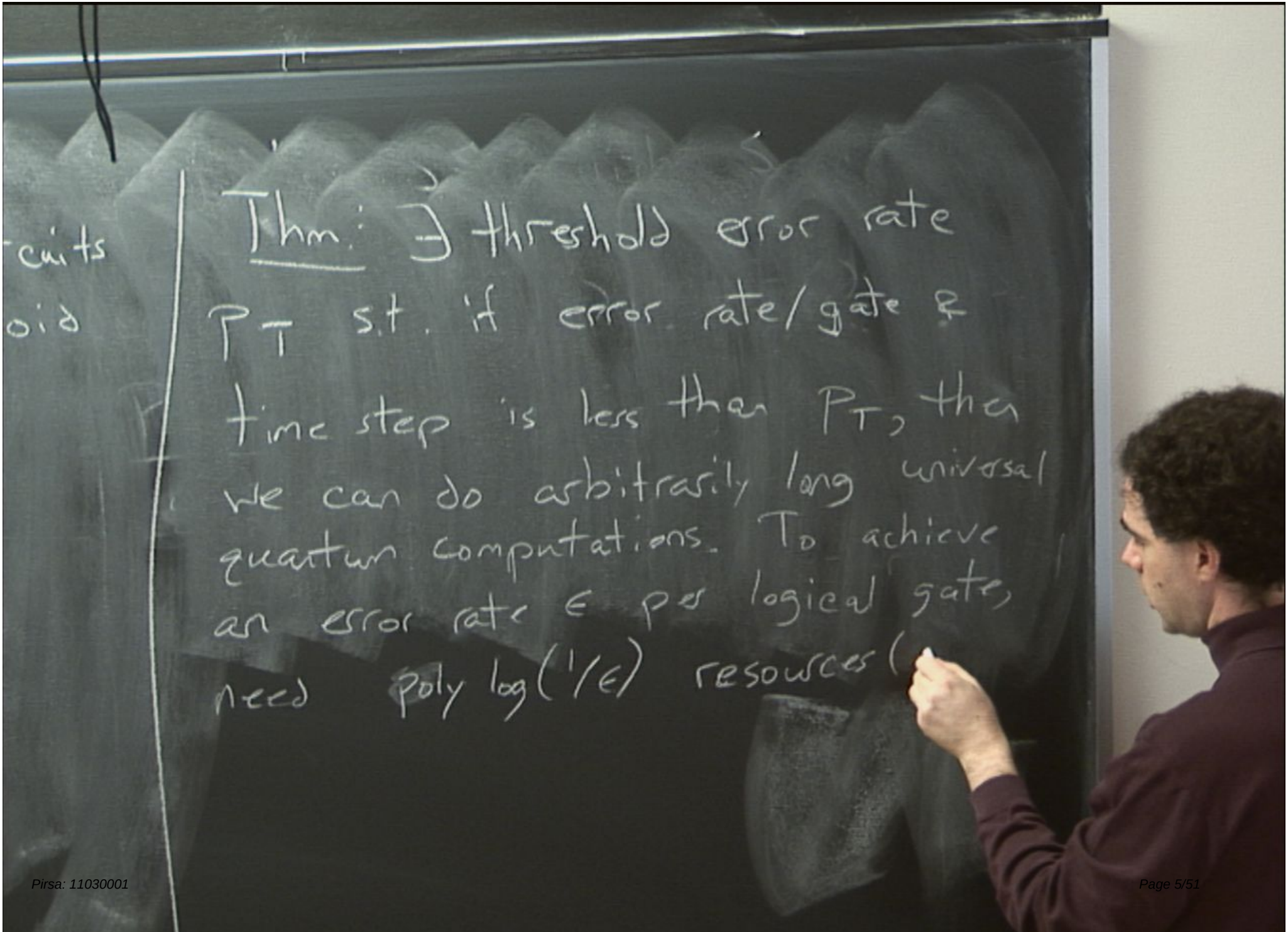
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Thm: ∃ threshold error rate
p_T s.t. if error rate/gate &
time step is less than p_T, then
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Threshold ~ 5%

2D ~ 0.7%

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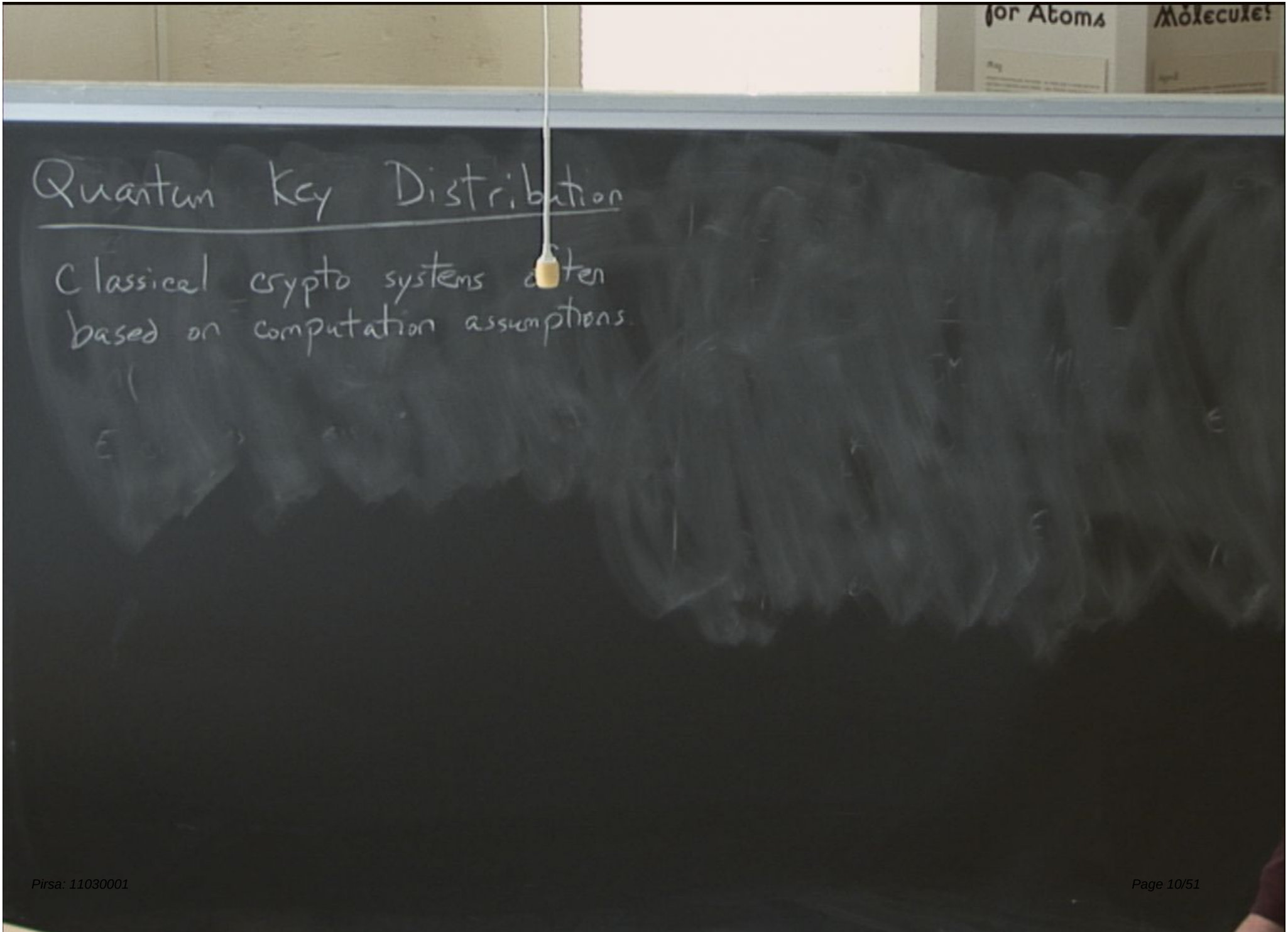
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Molecules
Quantum Key Distribution
Classical crypto systems often
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Encrypt m (n bits long): $m \oplus k$
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Quantum Key Distribution

Classical crypto systems often based on computational assumptions.

One-time pad: Alice & Bob share
key k: n random bits
Encrypt m (n bits long): $m \oplus k = e$ ← Completely random to Eve
(bitwise XOR)
Decrypt: $e \oplus k = m \oplus k \oplus k = m$

Key Distribution

crypto systems often computational assumptions.

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(bitwise XOR)
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Completely random to Eve

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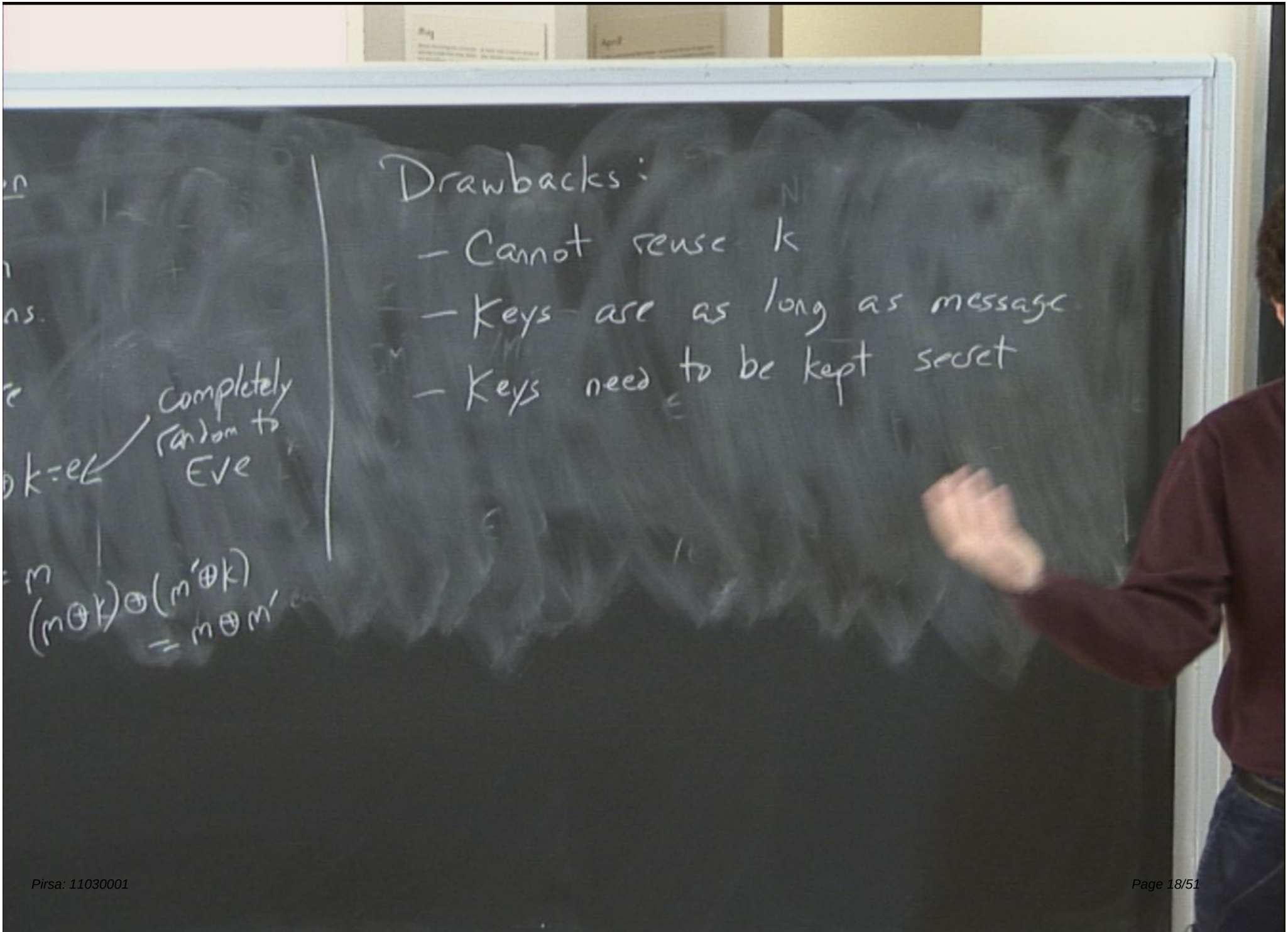
Drawbacks:
- Cannot reuse k
- Keys are as long as message
- Keys need to be kept secret
Completely random to Eve
(m⊕k)⊕(m'⊕k)
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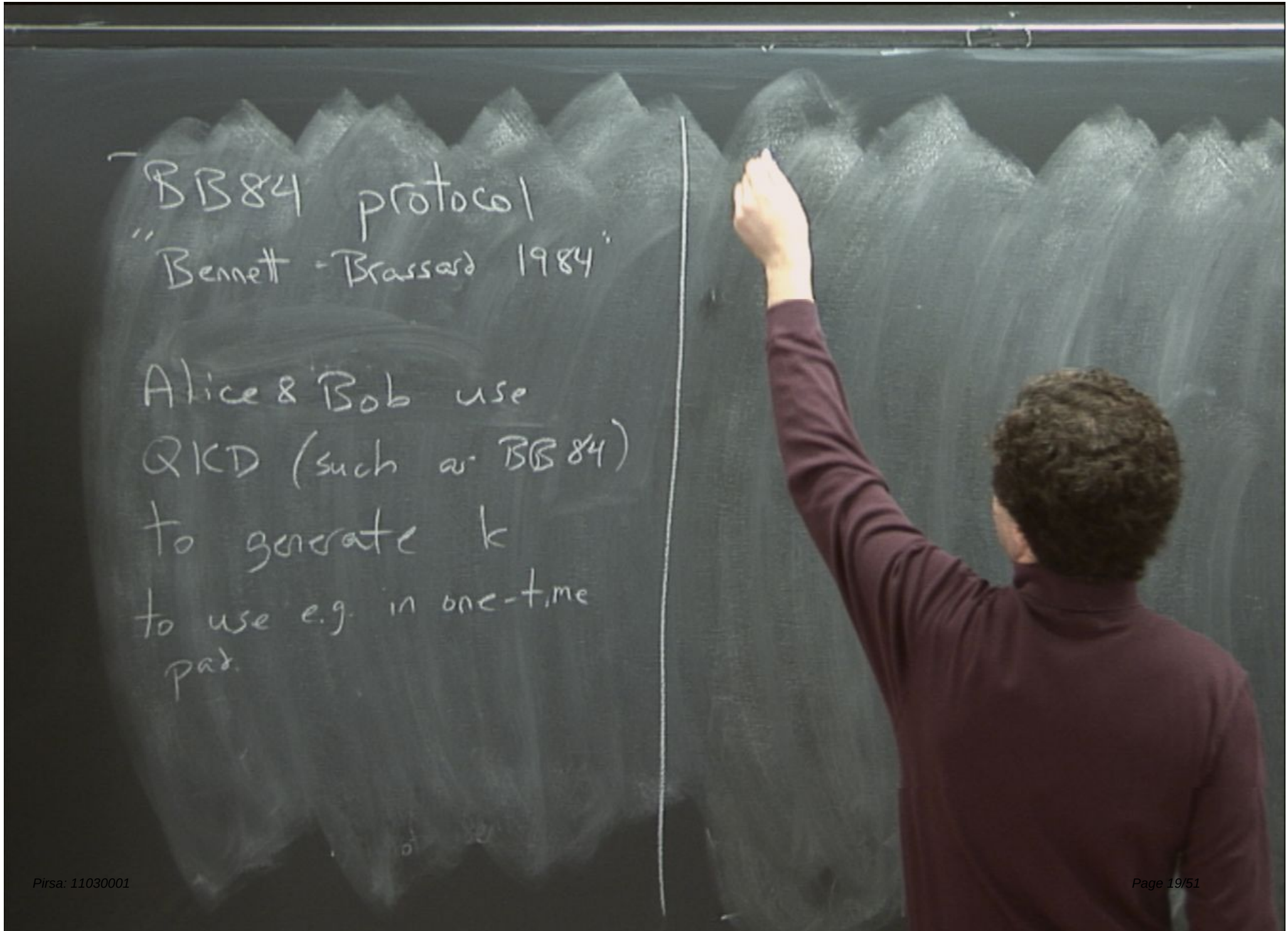
BB84 protocol
"Bennett - Brassard 1984"
Alice & Bob use
QKD (such as BB84)
to generate k
to use e.g. in one-time
pad.

1. Alice generate random bit pairs (q_i, r_i).

q_i	r_i
0	0
0	1
1	0
1	1

2. q_i specifies basis
   r_i specifies state

1984

e

BB 84)

-time

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0	1	$\|1\rangle$
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Tr

$|1\rangle$

~~$|0\rangle$~~

$|0\rangle$ —

7-q

part

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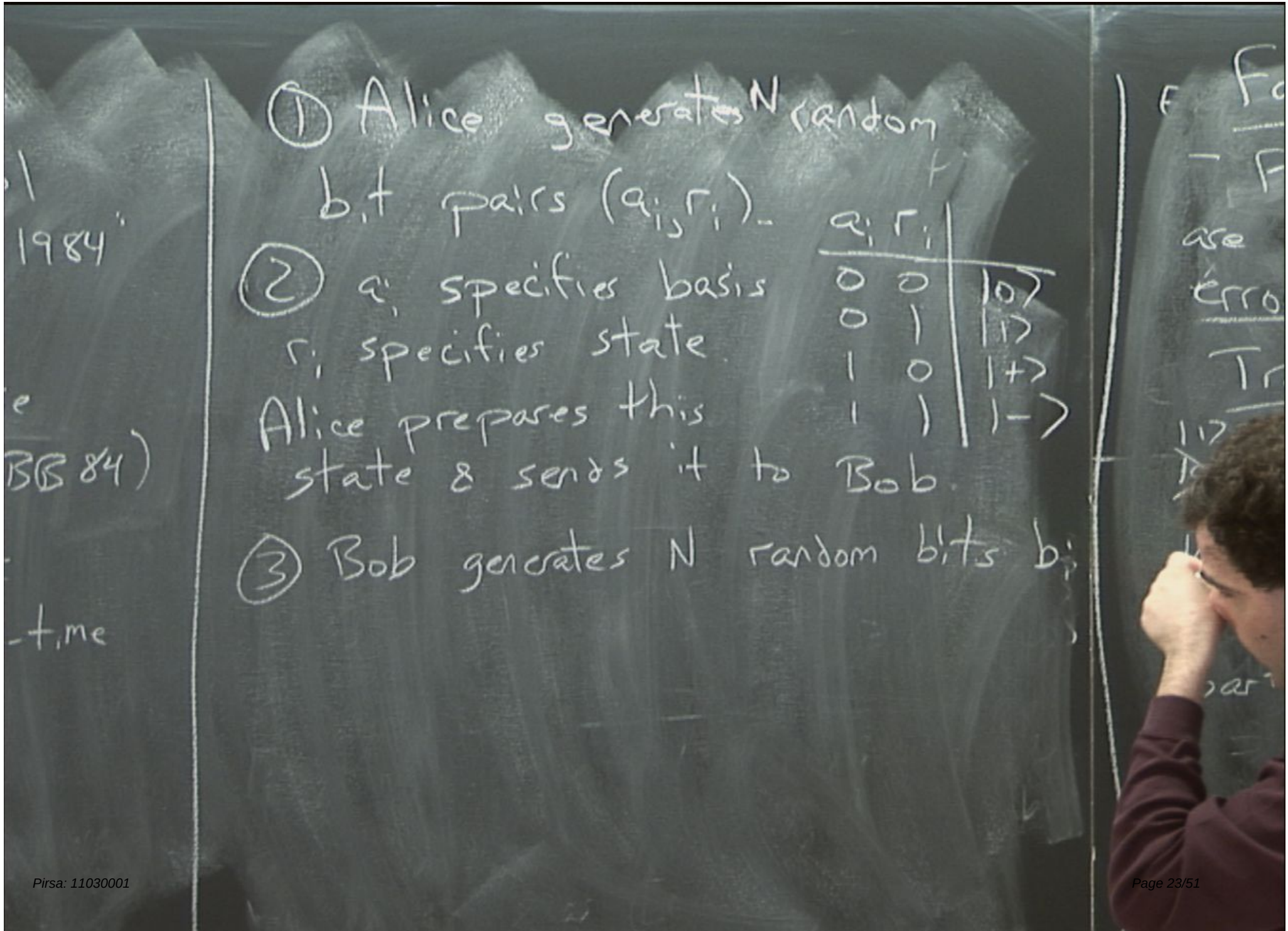
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0 0 |0>
0 1 |1>
1 0 |+>
1 1 |->
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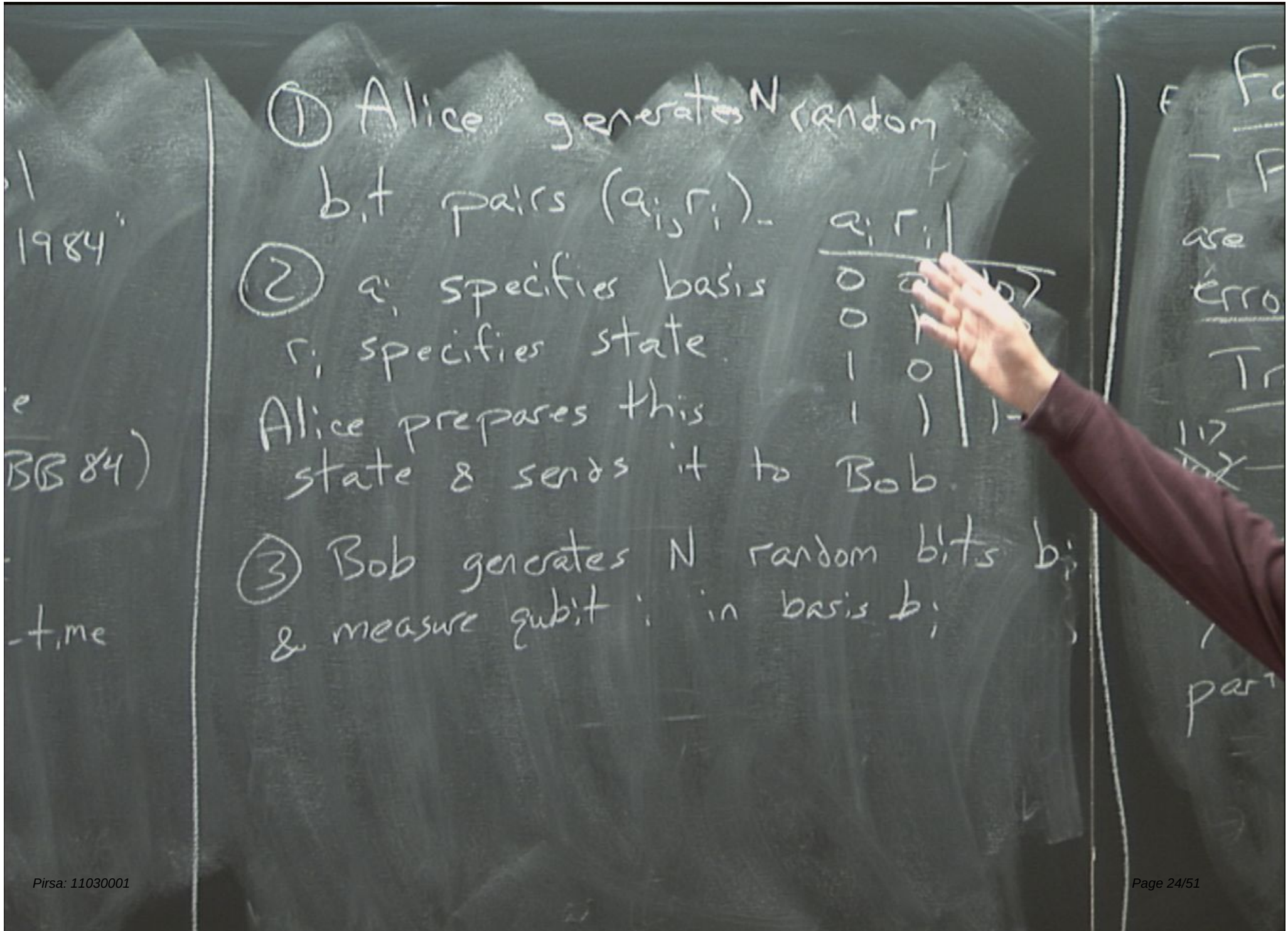
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0 0
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③ Bob generates N random bits b_i & measure qubit i in basis b_i, getting result s_i

④ Alice & Bob announce publicly a_i & b_i, $\forall i$
If $a_i = b_i$, keep bit i
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|0⟩
|0⟩ —
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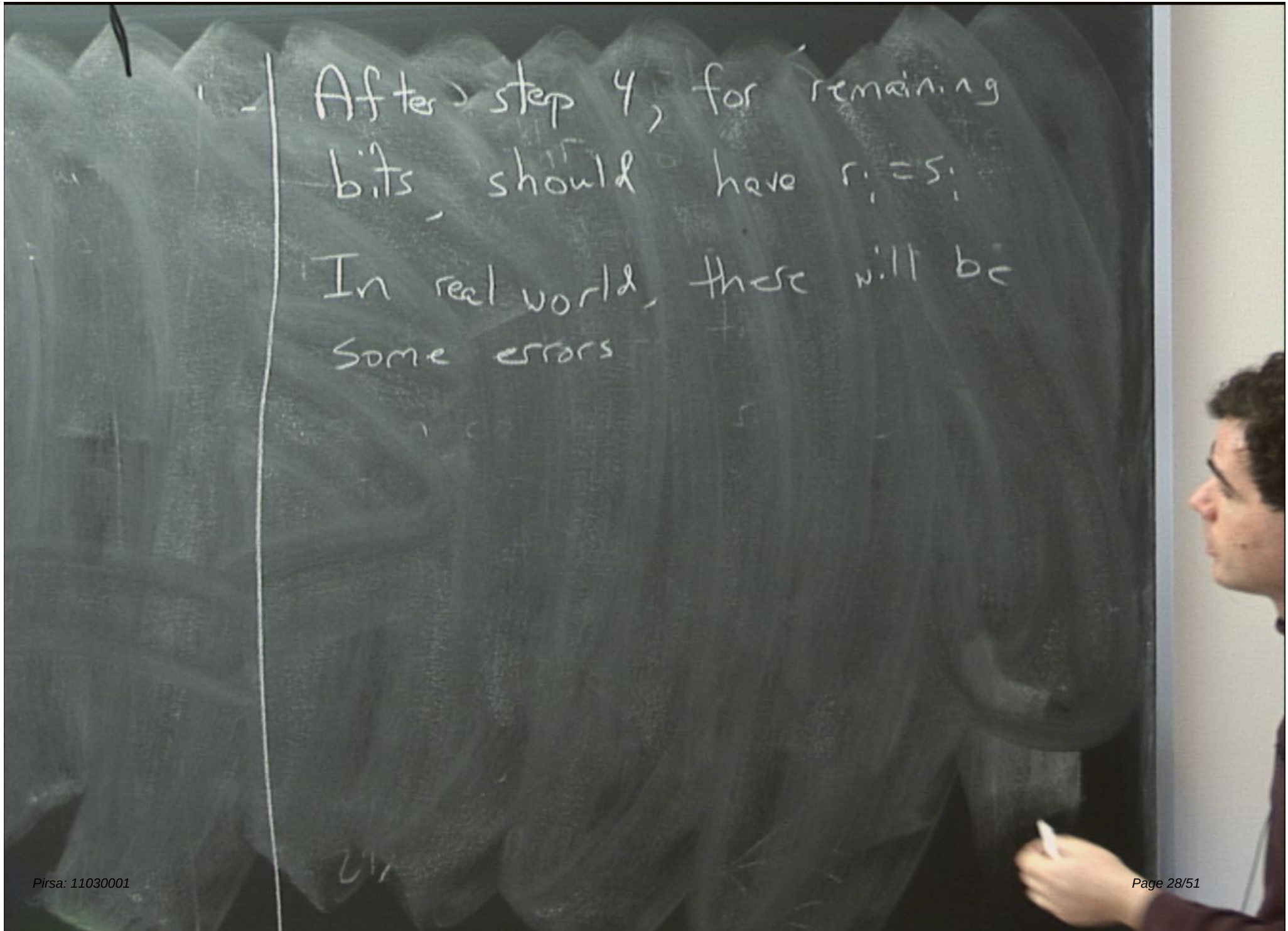
After step 4, for remaining bits should have $r_i = s_i$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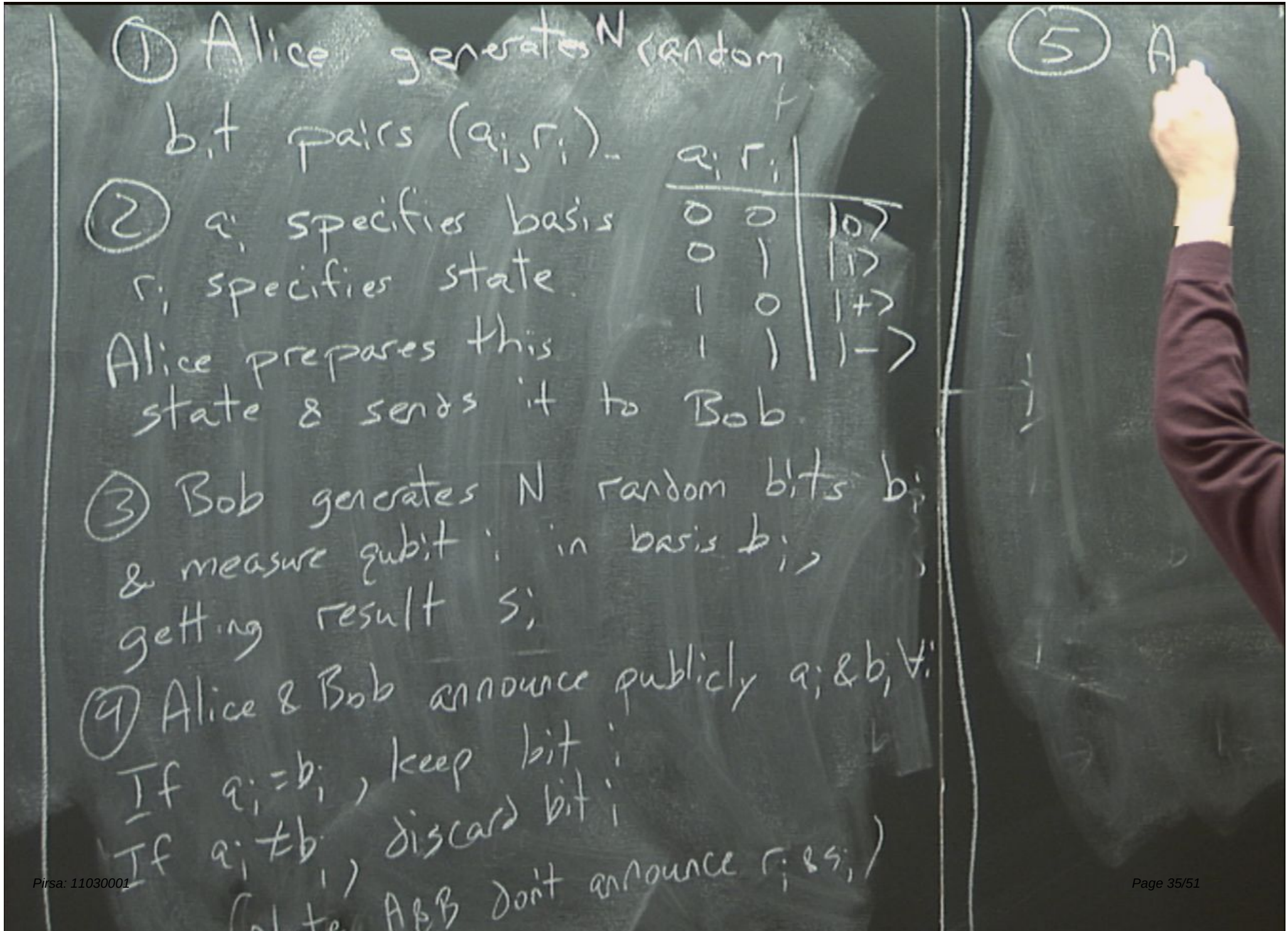
$\Rightarrow$ 25% bit error rate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③ Bob generates N random bits b_i & measure qubit i in basis b_i, getting result s_i
④ Alice & Bob announce publicly a_i & b_i, $\forall i$
If $a_i = b_i$, keep bit i
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Alice & Bob don't announce r_i & s_i)
⑤ A

(5) A&B choose some subset of remaining bits & announce #'s for those bits.

After > step
bits, shou

In real wor
Some error
Eavesdropper
- In fact, E
- e.g. Alice s
know a; - s
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⑥ Do error correction by revealing some parities of key bits (must discard some key bits).

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|1>
|+>
|->

s b

; & b, ∀

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(7) Privacy amplification:

After step
bits, shou

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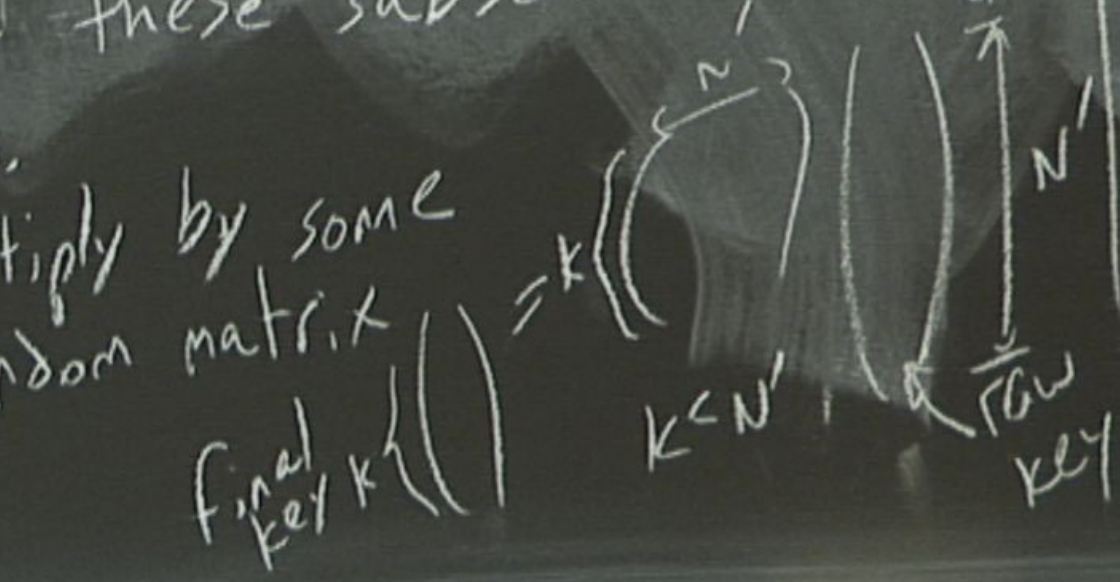
⑦ Privacy amplification: A & B choose random subsets & use the parities of these subsets as final key bits.

i.e. multiply by some random matrix



dom

r_i

0	0	$\|0\rangle$
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Bob.

m bits b_i

is $b_i\rangle$

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$$\text{final key } k\left\{\left(\;\right)\right. = k\left\{\left(\;\overset{N'}{\;}\;\right)\right.\left(\;\right)\uparrow N' \quad \text{raw key}$$

$k < N'$

After step
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BB84 protocol

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Alice & Bob use QKD (such as BB84) to generate k to use e.g. in one-time pad.

Uses quantum channel

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Alice prepares this state & sends it to Bo

a_i
0
0
1
1

(3) Bob generates N random & measure qubit i in basis getting result s_i

(4) Alice & Bob announce publicly
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- Uses classical channels

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  - Can be public

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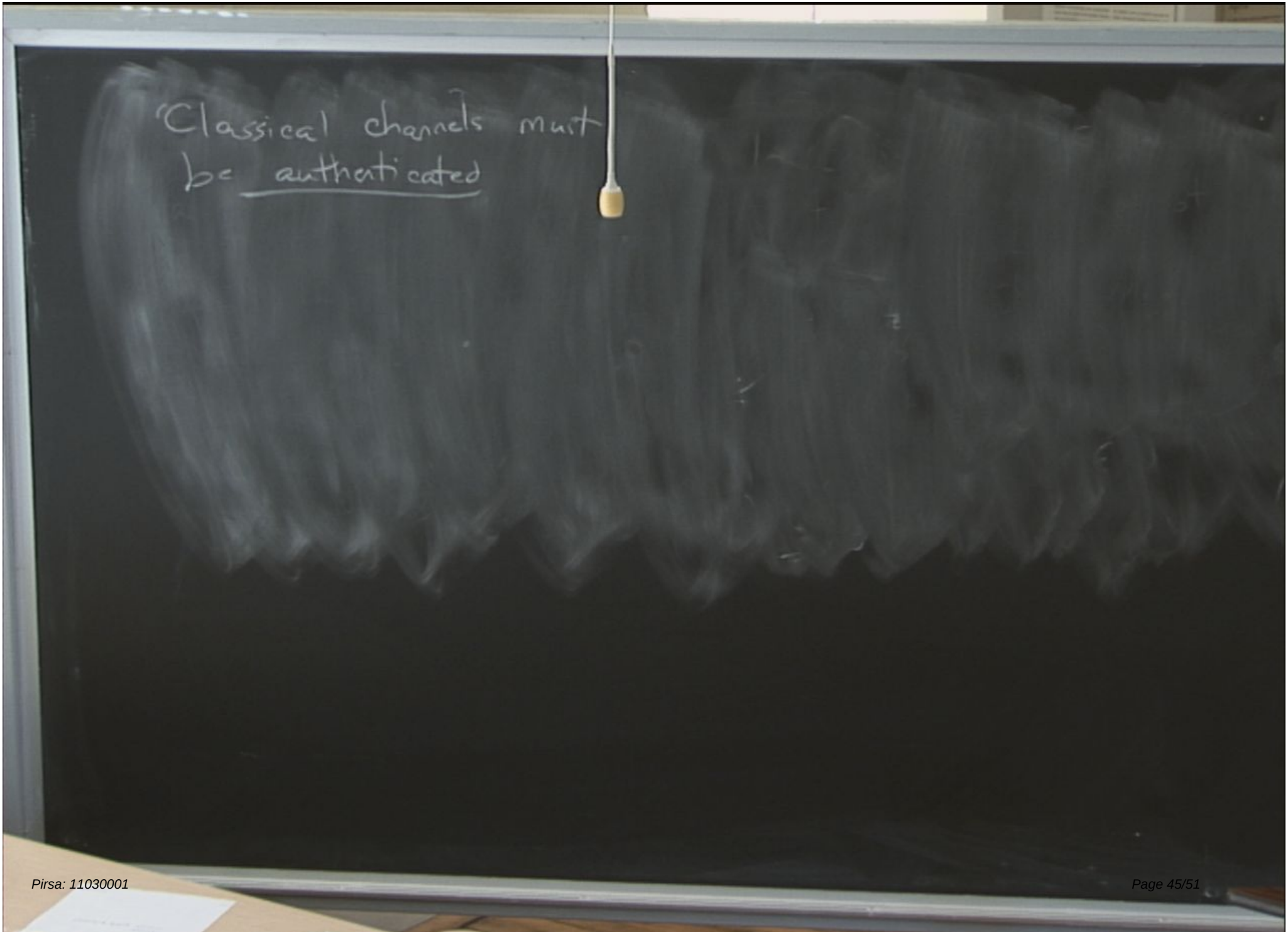
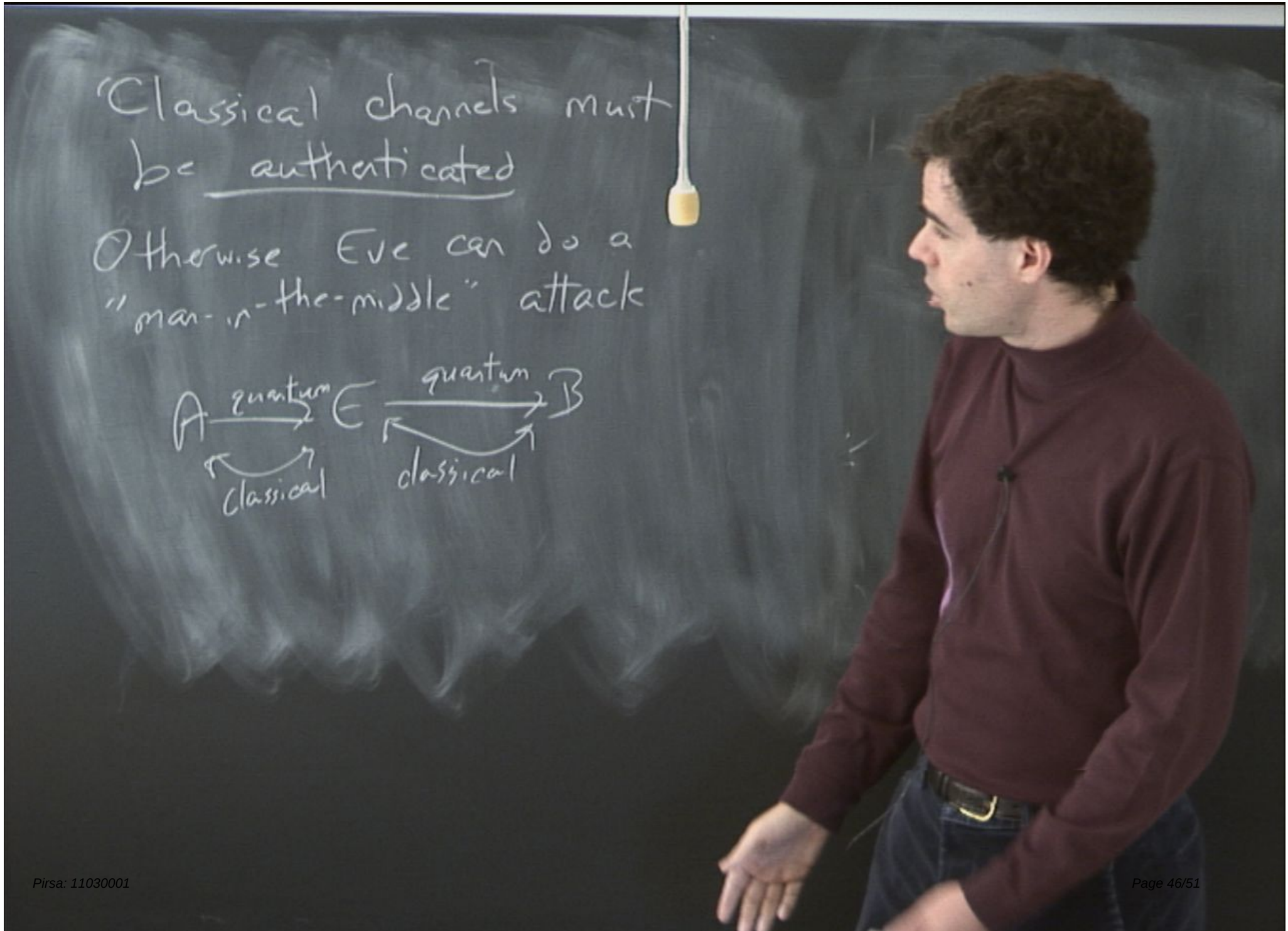

Classical channels must
be authenticated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Otherwise Eve can do a "man-in-the-middle" attack
A
quantum
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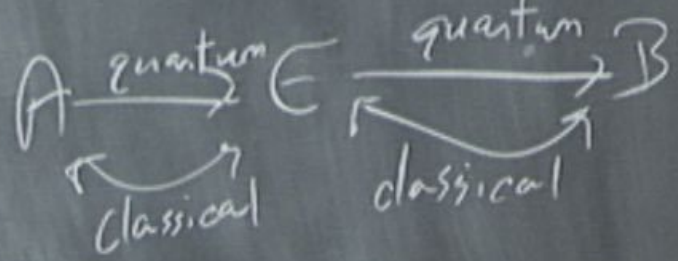

Classical channels must be authenticated → Can use secret key

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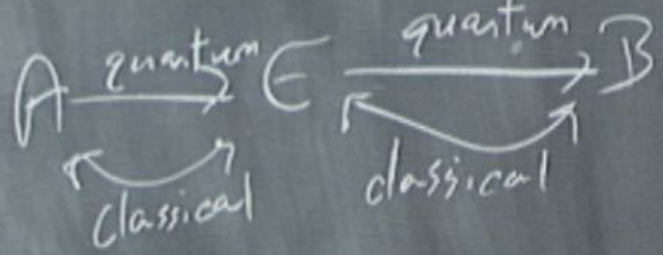

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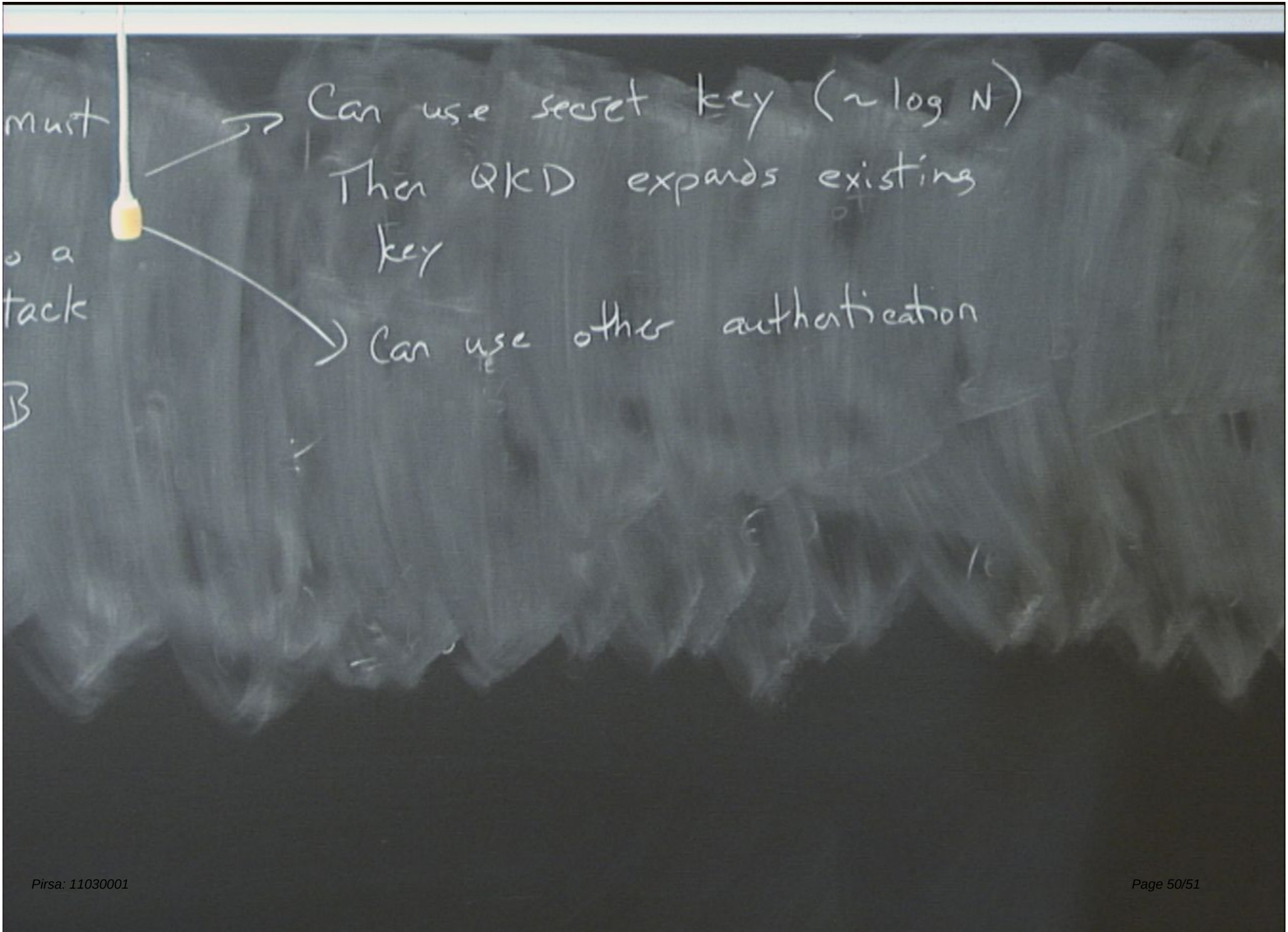
→ Can use secret key ($\sim \log N$)
Then QKD expands existing key


must
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Can use secret key (~ log N)
Then QKD expands existing
key
Can use other authentication

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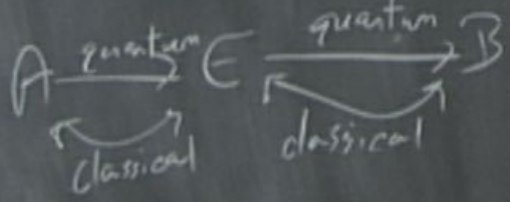

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