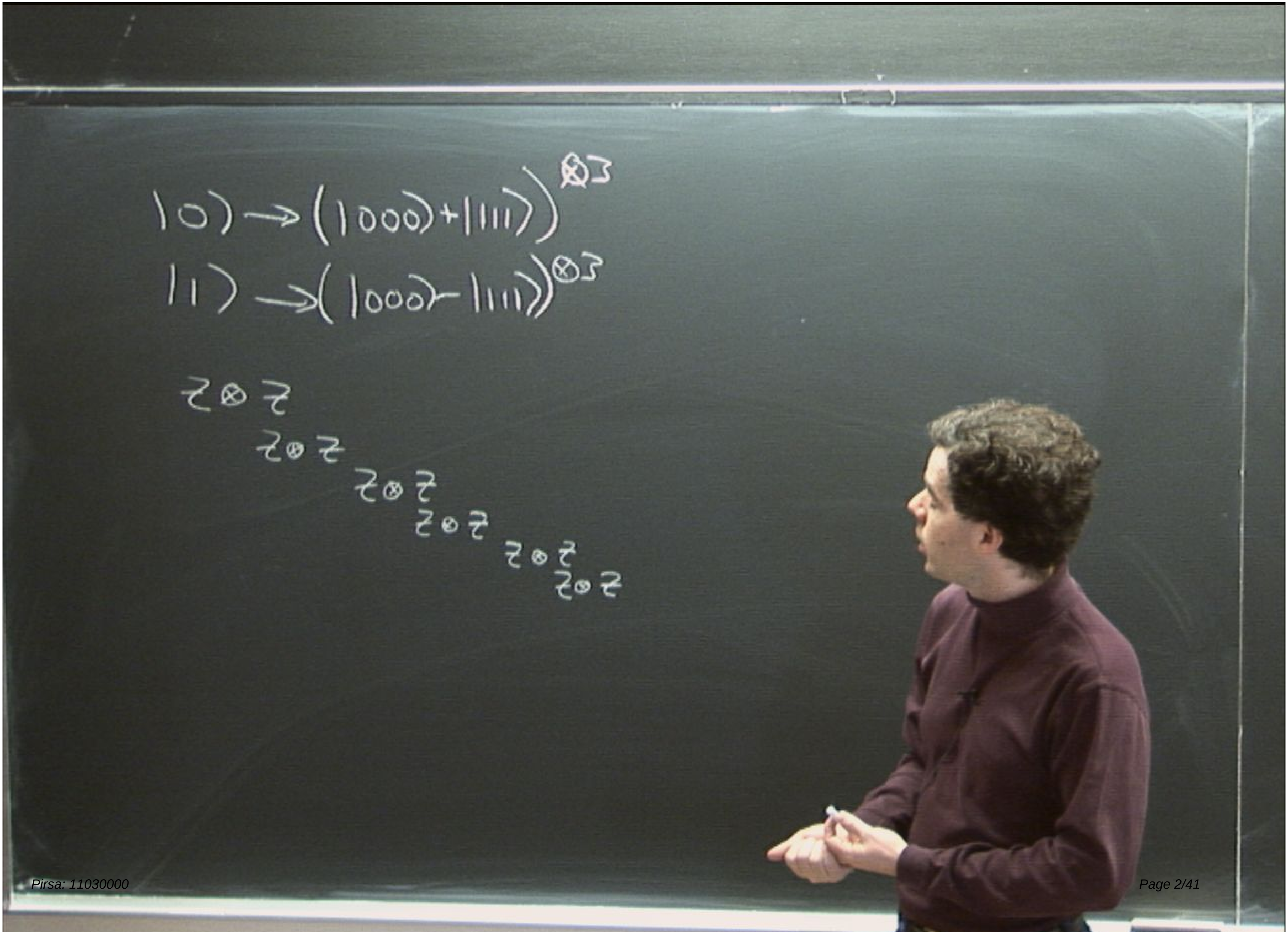
Title: Quantum Information Review - Lecture 11

Date: Mar 01, 2011 09:00 AM

URL: http://pirsa.org/11030000

Abstract:


$|0\rangle \rightarrow (|000\rangle + |111\rangle)^{\otimes 3}$
$|1\rangle \rightarrow (|000\rangle - |111\rangle)^{\otimes 3}$
$Z \otimes Z$
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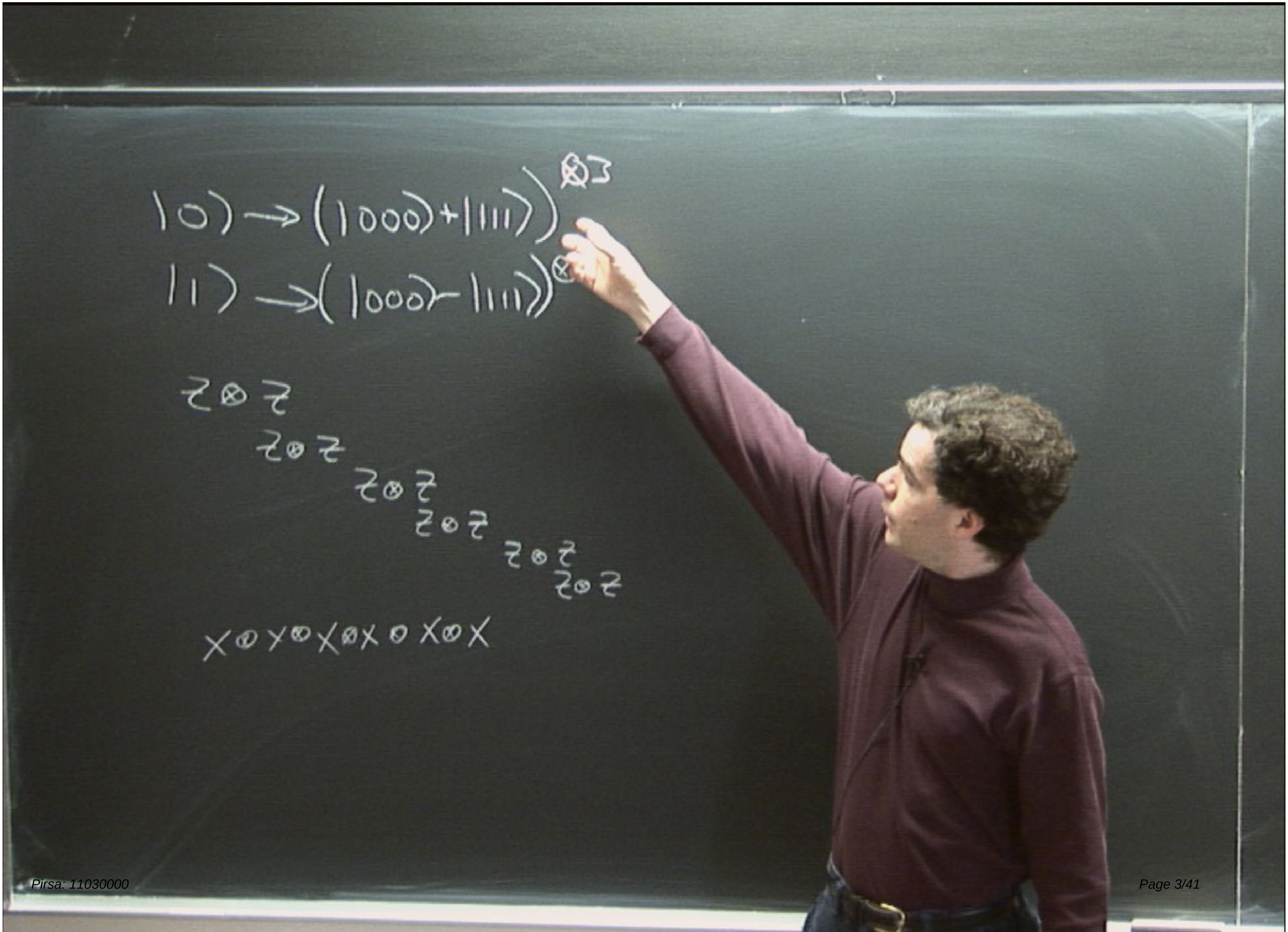

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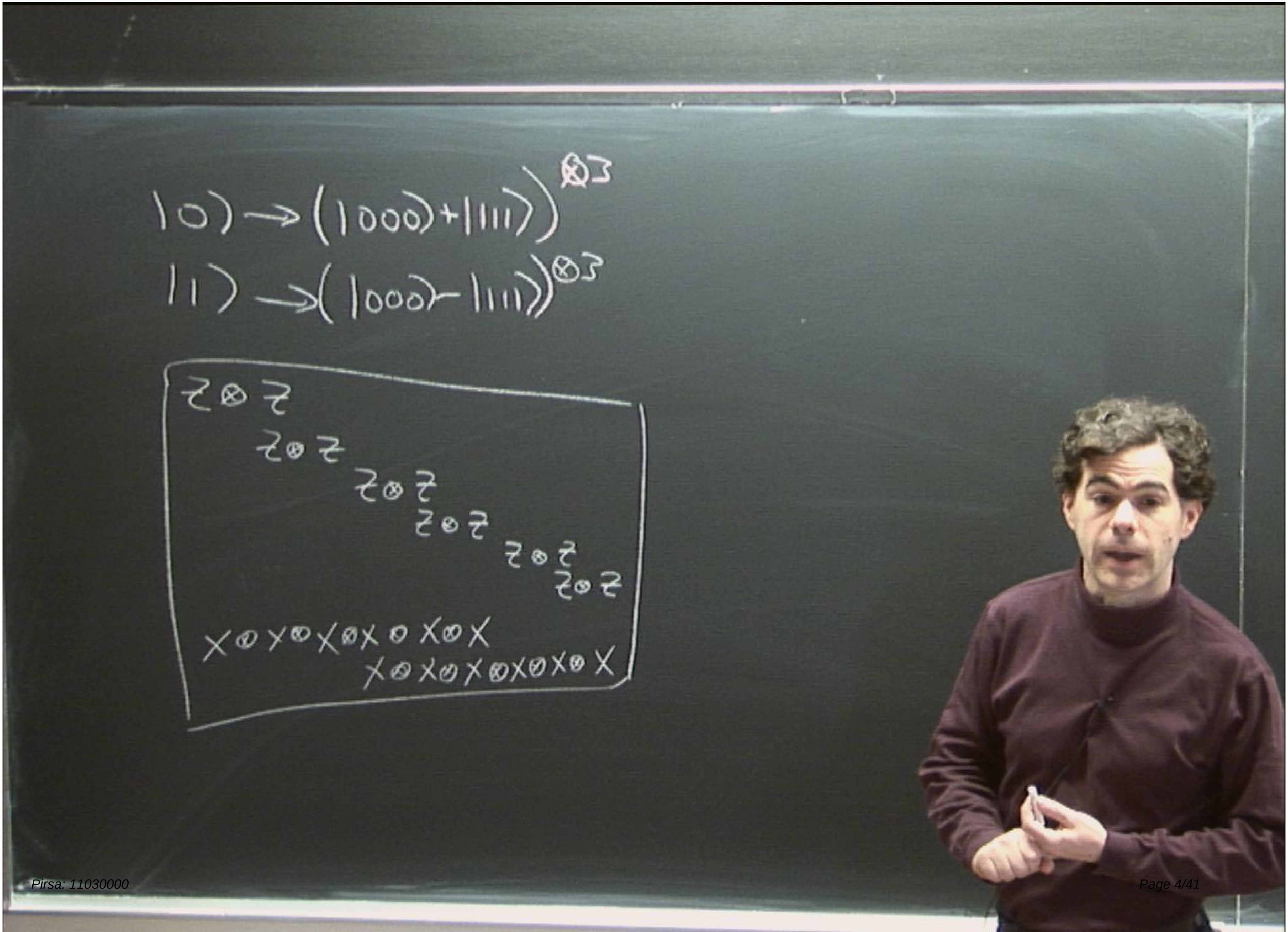
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Prop.: If S has r generators, on n physical qubits, then

$$\dim T(S) = 2^{n-r}$$

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E changes $+1$ eigenvector to -1 eigenvector

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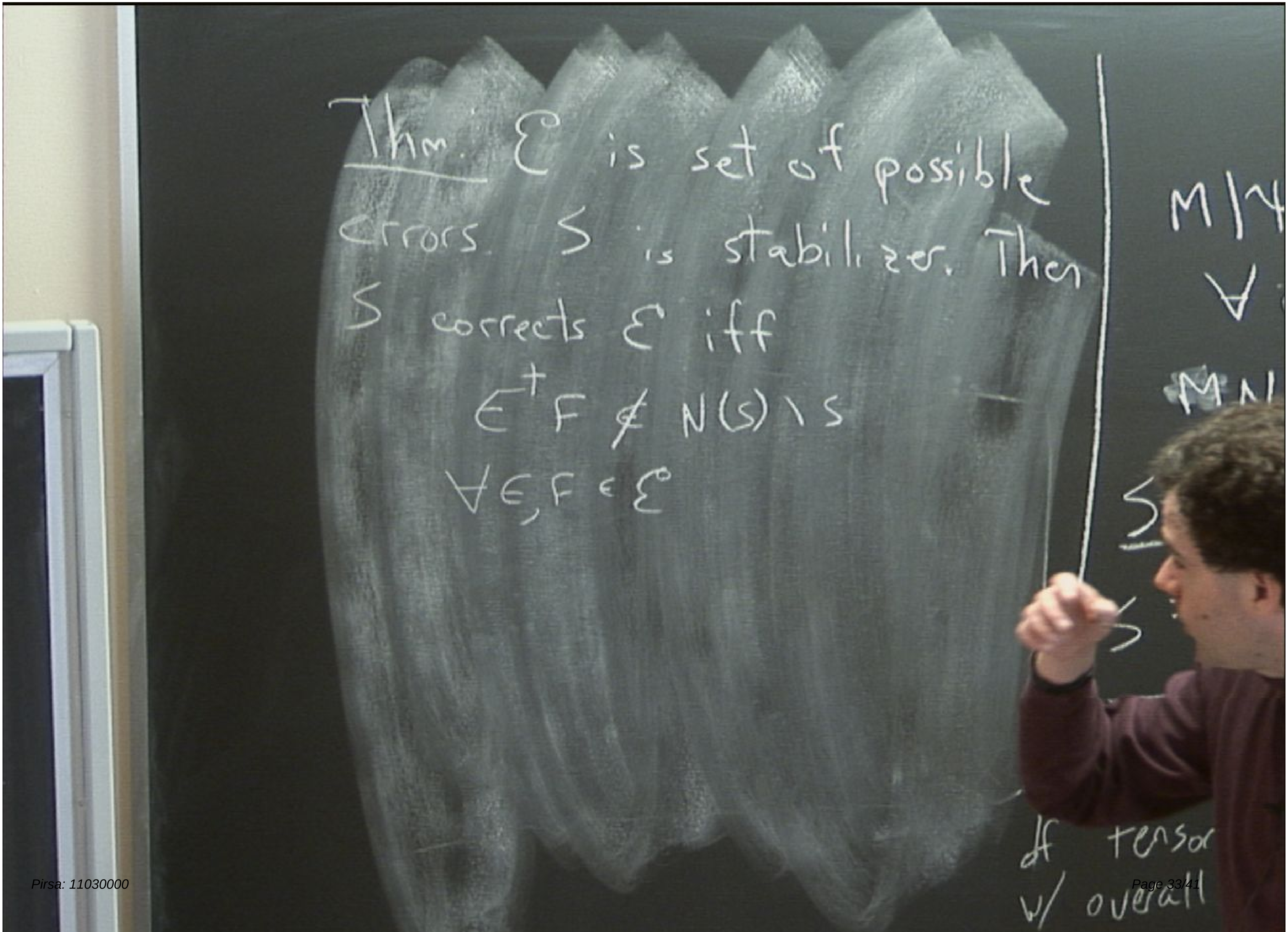
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$\forall$

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$\Rightarrow$ can be corrected the same way

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$[[5,1,3]]$

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$I \otimes X \otimes Z \otimes Z \otimes X$

$X \otimes I \otimes X \otimes Z \otimes Z$

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Def: Let

group,

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Prop: I

On