Title: The instability of 5 dimensional black strings

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Abstract:

Outline

- Motivation : why study general relativity in higher dimensional spacetimes?
- Black strings in 5D
  - the Gregory-Laflamme instability
  - conjectures on the end-state
- A new numerical exploration of unstable black strings (work with Luis Lehner)
  - formulation
  - results
- Conclusions

Motivation: why study higher dimensional gravity?

- If string theory is providing the correct path to a consistent theory of nature valid at Planck scales, the universe is fundamentally higher dimensional
- Even if string theory is not correct, there has recently been a lot of work using the holographic dual correspondences of string theory (AdS/CFT in particular) to describe many aspects of conventional non-gravitational 4D physical processes in terms of higher dimensional gravity
  - superconductors, superfluidity, quark-gluon plasmas, etc.
  - interestingly, the gravitational dual to all the processes studied to date involves *black holes*
- Much interesting geometry in higher dimensional Ricci-flat Lorentzian manifolds, in particular the zoo of "black objects" – black spheres, rings, strings, saturns, drops, …

Higher dimensional black holes

- Higher dimensional black holes have many properties in common with their 4D counterparts, e.g.
  - can be defined using global (event horizons) or local (isolated horizons) properties of the spacetime
  - contain geometric singularities
  - quasi-stationary processes are governed by the usual laws of black hole mechanics (constant surface gravity, area can only increase, change in mass can be related to change in area/angular momenta/charges)
  - a couple of studies have shown the usual link between gravitational collapse and black hole formation, together with critical phenomena at threshold
  - Hawking radiate at the semi-classical level
- However, a few properties are in general drastically different, including
  - *no uniqueness* of stationary solutions
  - many black objects are *unstable* to perturbations

Black Strings

- Black strings are a particularly simple class of higher dimensional black hole solutions
  - in N spacetime dimensions, the metric is *4D Schwarzschild X (N-4)D Euclidean flatspace*; e.g. for N=5, in Schwarzschild coordinates

$$ds^2 = -(1-2m/r)dt^2 + \frac{1}{(1-2m/r)}dr^2 + r^2 d\Omega^2 + dw^2$$

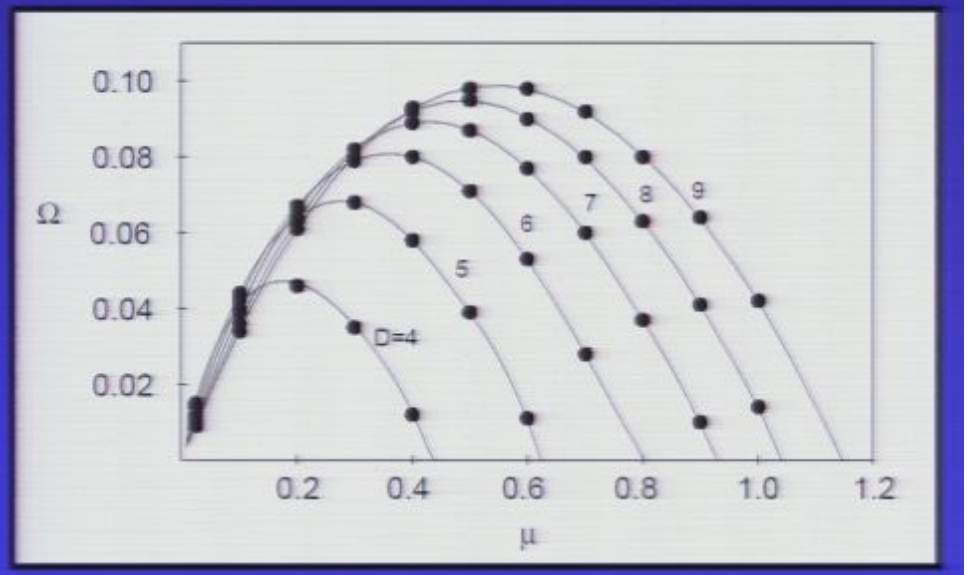
  - here m is interpreted as mass per unit length; a segment of length $\Delta w = L$ of the spacetime has asymptotic mass $M = mL$

Gregory-Laflamme instability

- Gregory and Laflamme *[PRL 70 (1993)]* first showed that black strings (and p-branes) are linearly unstable to long-wavelength perturbations

$$g = g_0 + \delta g \cdot e^{\Omega t/m + i\mu w/m}$$

  - Image from R. Gregory and R. Laflamme, Nucl.Phys.B428 (1994)



  - the D=4 curve corresponds to the 5D black string, and the critical wavelength above which modes are unstable is

$$\lambda_c \approx 14.3m$$

End-state of the instability?

- Based on the way the linear mode perturbed the horizon, and an entropic argument:
  - above a similar critical wavelength L_c the total *area/length* of a sequence of 5D hyper-spherical black holes, each a distance L_c apart, is *greater* than a 5D black string with the *same* total mass/length ($M=mL$):

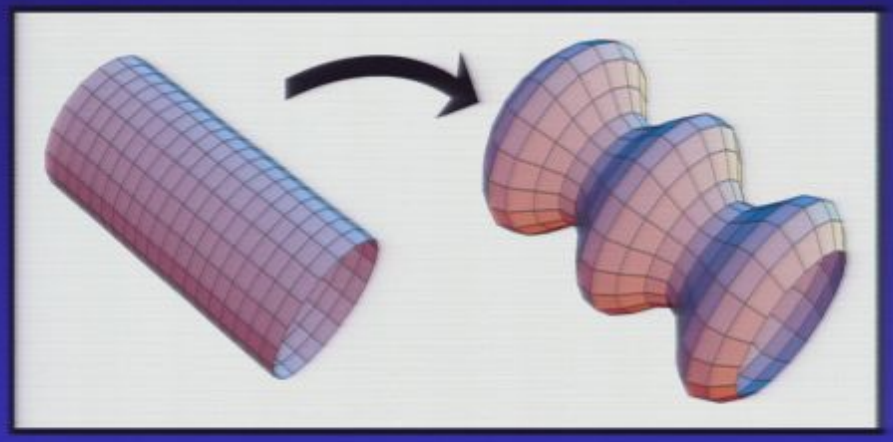


Image from:
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$$\frac{A_{BH}}{A_{BS}} = \sqrt{\frac{8}{27\pi}\frac{L}{m}} \Rightarrow L_c = \frac{27\pi}{8} m \approx 10.6m$$

they argued the black string would "pinch-off" into a sequence of spherical black holes

  - this cannot happen without the appearance of a naked singularity (the "no-bifurcation" theorems still hold in 5D) → a *generic* example of cosmic censorship violation in higher dimensional gravity

End-state of the instability?

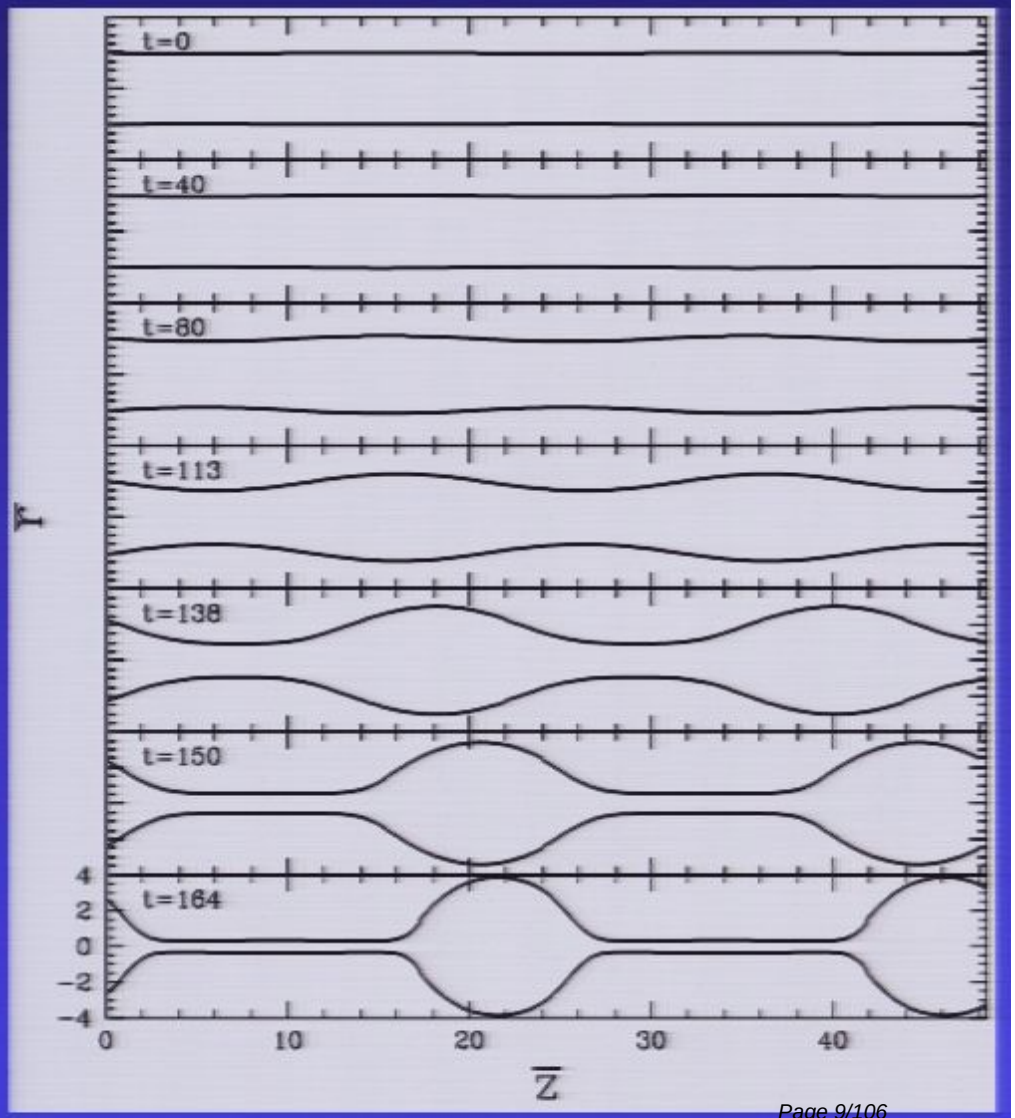
- However, Horowitz and Maeda *[PRL 87, 131301 (2001)]* proved that black string horizons cannot shrink to zero cross-sectional radius in *finite affine time* of the generators of the horizon
  - based on this, they conjectured the end-state would be a new, static, non-uniform solution with the same topology as the black string
  - this spurred a search for such solutions; a couple were found *[S. S. Gubser, CQG. 19, 4825 (2002), T. Wiseman, CQG. 20, 1137 (2003), E. Sorkin, PRD74:104027 (2006)]*; however, these solutions have less entropy (area) than the uniform black string, so could not be the end-state of the GL instability

End-state of the instability?

- The first (numerical) non-linear study was carried out by Choptuik et al. *[PRD 68, 044001 (2003)]*
  - simulation "crashed" before a conclusive statement about the end-state could be made
  - results more consistent with the GL pinch-off conjecture
  - affine time grows exponentially fast relative to asymptotic time [Garfinkle et al., PRD 71 (2005), Marolf PRD 71 (2005)]

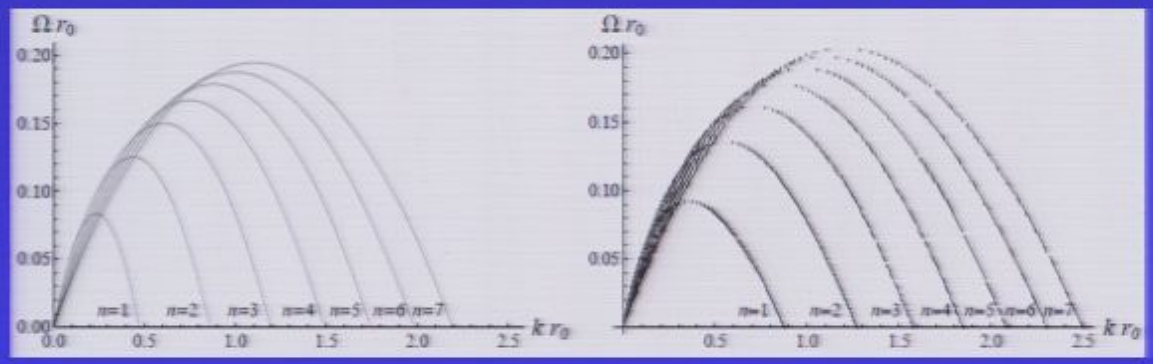
$$\ln \lambda \propto t/m \propto t/r_s$$

so not necessarily inconsistent with Horowitz & Maeda's theorem

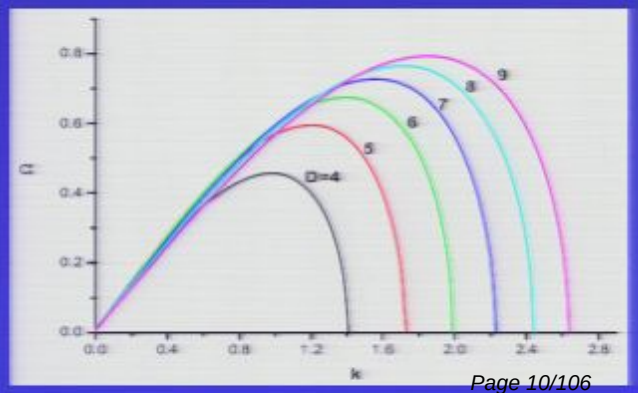


End-state of the instability?

- Further (anecdotal) evidence in favor of the pinch-off scenario has gathered in the form of various correspondences between equations governing viscous hydrodynamics and horizon dynamics
  - the membrane paradigm *[Thorne, Price, Macdonald, Eds. (1986)]* shows that the dynamics of a "stretched horizon" is governed by the Navier-Stokes equations for a relativistic fluid with very low shear-viscosity $\eta = 1/16\pi$.
  - more recently developed frameworks *[Bhattacharyya et al., JHEP 02 (2008), R. Emparan et al. JHEP 03 (2010)]* established similar relationships; *[J. Camps et al., arxiv:1003.3636 (2010)]* *(left figures)* used the "black folds" approach to re-derive the Gregory-Laflamme spectrum of modes to leading order
  - Cardoso and Dias *[PRL 96 (2006)]* *(right figure)* showed that the spectrum of unstable modes of a cylindrical flow of fluid with surface tension, subject to the Rayleigh-Plateau instability, was quantitatively similar to that of black strings



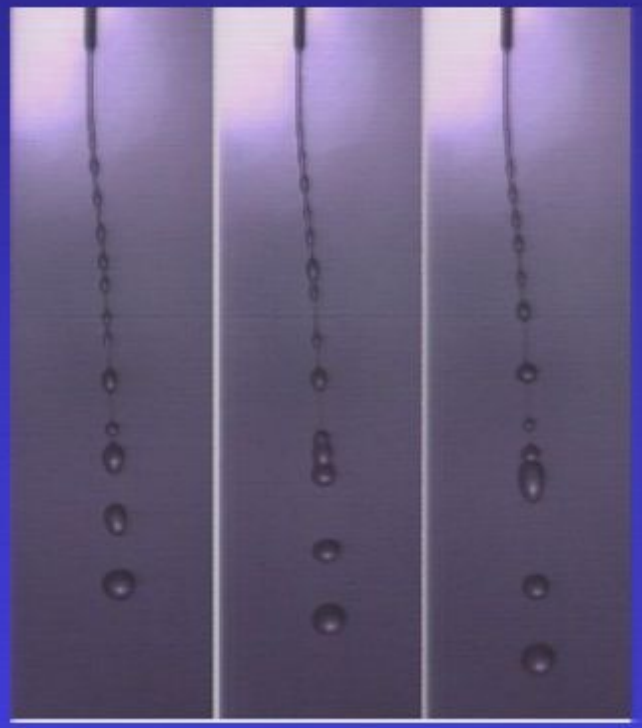
unstable sound waves in effective black string fluid (left) compared to GL modes (right)



Rayleigh-Plateau analogue

End-state of the instability?

- The reason why this could be considered evidence for pinch-off is that unstable fluid streams generically break up
  - For the Rayleigh-Plateau instability surface area is also the key explaining why one would expect a long-wavelength instability leading to pinch-off : above a critical length a sequence of spherical droplets has lower energy (due to surface tension) than a cylinder with the same volume/length
    - other analogues *[Cardoso and Gualtieri, CQG 23 (2006); Unruh and Wald, unpublished]* do not include surface tension, but the conclusion is the same
- The caveat with the fluid analogues is just that –they're analogues– and existent Einstein/horizon-hydrodynamic relationships is they're perturbative
  - thus, both end-state possibilities remain, and one needs to solve the full field equations to discover the answer



Numerical formalism

- We numerically solve the vacuum Einstein field equations

$$R_{\alpha\beta} = 0$$

for the (5-dimensional) spacetime metric

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$$

imposing harmonic coordinates

$$\nabla^\alpha \nabla_\alpha x^\mu \equiv \frac{1}{\sqrt{-g}} \partial_\alpha \left(\sqrt{-g} g^{\alpha\mu} \right) = 0$$

and using the harmonic formulation of the field equations

$$g^{\gamma\delta} g_{\alpha\beta,\gamma\delta} + 2 g^{\gamma\delta}{}_{,(\alpha} g_{\beta)\delta,\gamma} + 2\Gamma^\gamma_{\delta\beta}\Gamma^\delta_{\gamma\alpha} = 0$$

$$\Gamma^\delta_{\alpha\beta} \equiv \frac{1}{2} g^{\delta\varepsilon} \left(g_{\alpha\varepsilon,\beta} + g_{\beta\varepsilon,\alpha} - g_{\alpha\beta,\varepsilon} \right)$$

Constraint damping

- Free evolution of the "plain" harmonic equations typically suffers from exponential growth of the constraint conditions

$$C^{\mu} \equiv \nabla^{\alpha}\nabla_{\alpha}x^{\mu} = 0$$

- The (apparent) cure, as suggested by C. Gundlach et al (*[C. Gundlach, J. M. Martin-Garcia, G. Calabrese, I. Hinder, gr-qc/0504114]* based on earlier work by Brodbeck et al *[J. Math. Phys. 40, 909 (1999)]*) is to modify the Einstein equations in harmonic form as follows:

$$g^{\alpha\beta}g_{\mu\nu,\alpha\beta} + \ldots + \kappa\left(n_{\mu}C_{\nu} + n_{\nu}C_{\mu} - (1+\rho)g_{\mu\nu}n^{\alpha}C_{\alpha}\right) = 0$$

$n_u = -\alpha\partial_u t$ is a unit timelike vector normal to $t=const.$ hypersurfaces, with proper time measured by an observer moving along n_u given by the *lapse function* α, and κ,ρ are a constant parameters ($\kappa>=0; -1<\rho<=0$)

  - any solution to the field equations must have $C^u=0$, so we are adding "nothing" to them; however with a proper choice of parameters growth of truncation-error sourced violations of the constraints can be suppressed

  - all previous codes had $\rho=0$; we found we needed a non-zero ρ to suppress a zero-wavelength mode in the (periodic) extra dimension

Numerical formalism

- To make the simulations computationally feasible we restrict to spherically symmetry within the $w = constant$ sub-manifold

- Earlier attempts using coordinates adapted to this symmetry in conjunction with the harmonic scheme were not successful (Sorkin & Choptuik *[GRG 42 (2010)]* also found achieving long-term stable numerical evolution is more complicated in such coordinates), therefore we adopt *Cartesian* like coordinates; i.e. asymptotically the metric is

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 + dw^2 + O\left(\frac{1}{r}\right)$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

and demand that *these* coordinates are harmonic

Numerical formalism

- For efficient evolution we employ a variant of the "cartoon" method *[M. Alcubierre et al. Int.J.Mod D10 (2001); FP, CQG 22 (2005)]*, whereby we only discretize a 2+1 dimensional slice *(y=z=0)* of the spacetime
  - off-slice *(y & z)* derivatives of the metric in the field equations are replaced with in-slice *(x)* derivatives by using the Killing vectors of the chosen SO(3) symmetry

$$\xi_1^\alpha = x\left(\frac{\partial}{\partial y}\right)^\alpha - y\left(\frac{\partial}{\partial x}\right)^\alpha ;\ \xi_2^\alpha = y\left(\frac{\partial}{\partial z}\right)^\alpha - z\left(\frac{\partial}{\partial y}\right)^\alpha ;\ \xi_3^\alpha = z\left(\frac{\partial}{\partial x}\right)^\alpha - x\left(\frac{\partial}{\partial z}\right)^\alpha$$

For example, solving

$$L_{\xi_3} g_{\alpha\beta} = 0$$

for the z-gradient of the metric gives

$$g_{\alpha\beta,z} = \frac{1}{x}\left[z\, g_{\alpha\beta,x} - \delta^z_\alpha g_{\beta x} + \delta^x_\alpha g_{\beta z} - \delta^z_\beta g_{\alpha x} + \delta^x_\beta g_{\alpha z}\right]$$

1-Slide overview of the code & setup

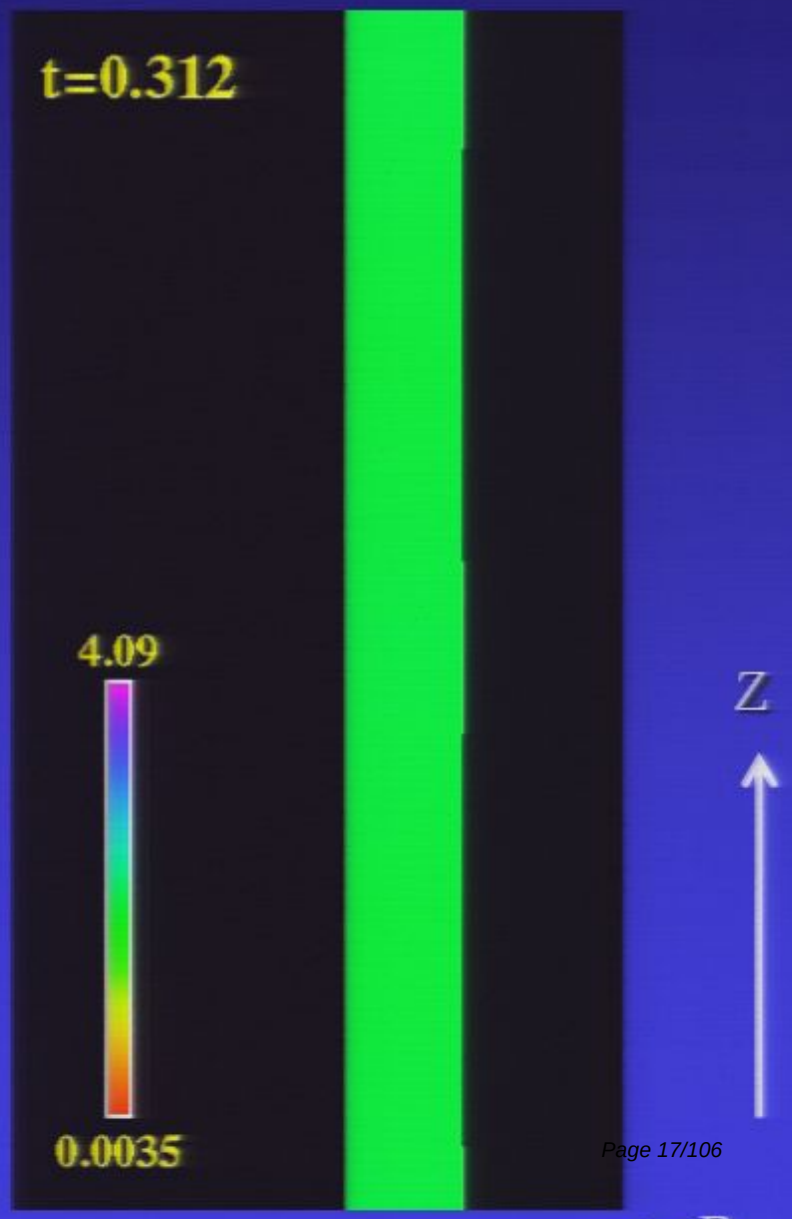
- 4th order finite difference discretization, Runge-Kutta time integration
- Berger and Oliger style adaptive mesh refinement (AMR) & parallel evolution as implemented with the PAMR/AMRD software libraries
- Using initial data as constructed in *Choptuik et al. [PRD 68, 044001 (2003)]*, describing a black string perturbed by a small gravitational wave
- Periodic in the string (w) direction
- Focusing on a single unstable case, with $L=20m$, i.e. $L\sim 1.4L_c$. (so with periodicity, the Gregory Laflamme analysis says this will have a *single* unstable mode, and close to the maximum growth rate)
- Choose spatial domain $r=[0..320m]$
- At outer boundary impose *Dirichlet* conditions, with the metric fixed to that of the initial data
  - not physically correct, hence we placed the outer boundary sufficiently far away from the horizon to be out of causal contact with it over the length of the simulation (t~230m)
- Excision (i.e. no boundary conditions) used at the inner boundary, which is dynamically adjusted to be some fractional distance within the apparent horizon (found via a flow-method) of the black string

Results : Apparent Horizon Embedding Diagram

- map the geometric 1D shape of each $t=x=y=constant$ slice of the apparent horizon to a flat (R,Z) Euclidean space; i.e. in parametric form

$$(R,Z)=\left(R(\xi),Z(\xi)\right)$$

- $R(\xi)$ is the areal radius of that point on the horizon, and $Z(\xi)$ is defined so that the proper length of the curve in the flat space is identical to that of the corresponding curve in the physical geometry
- the movie shows this curve spun around $R=0$ to form a surface for visual aid
- color is mapped to R
- note that time is "slowing down"



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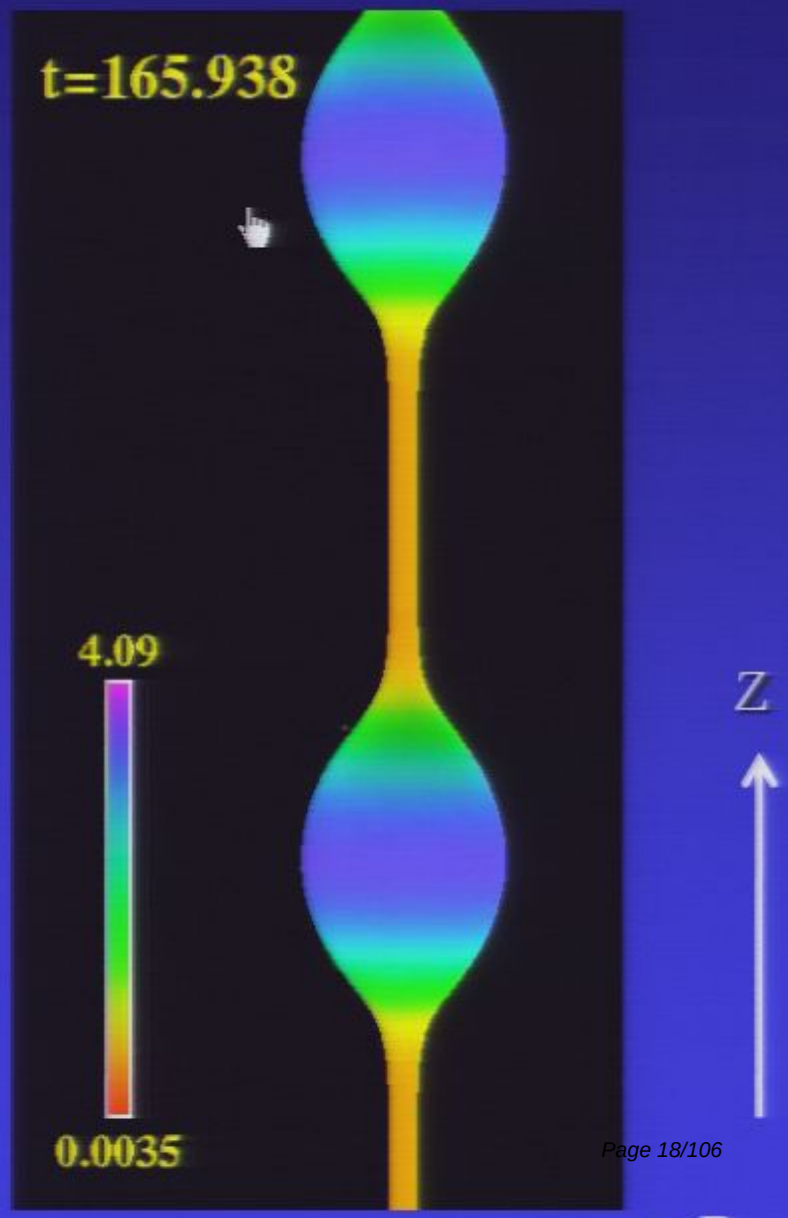

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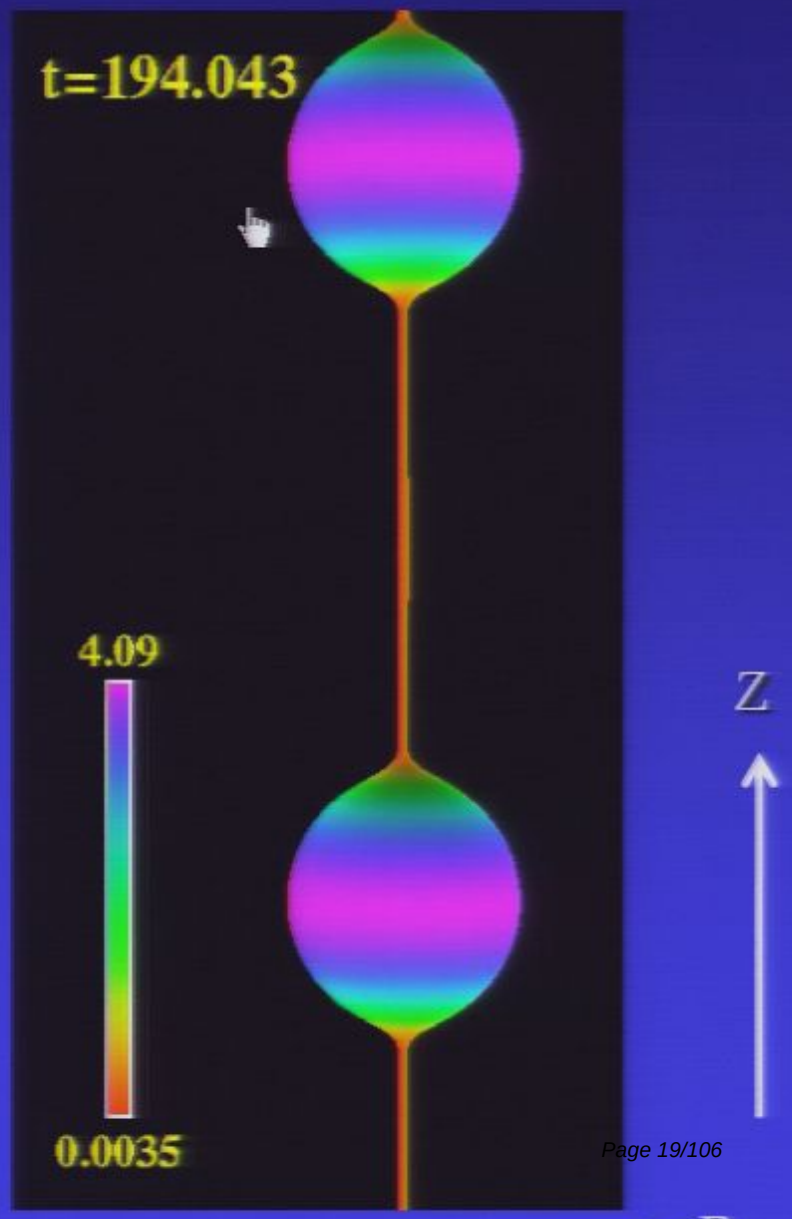

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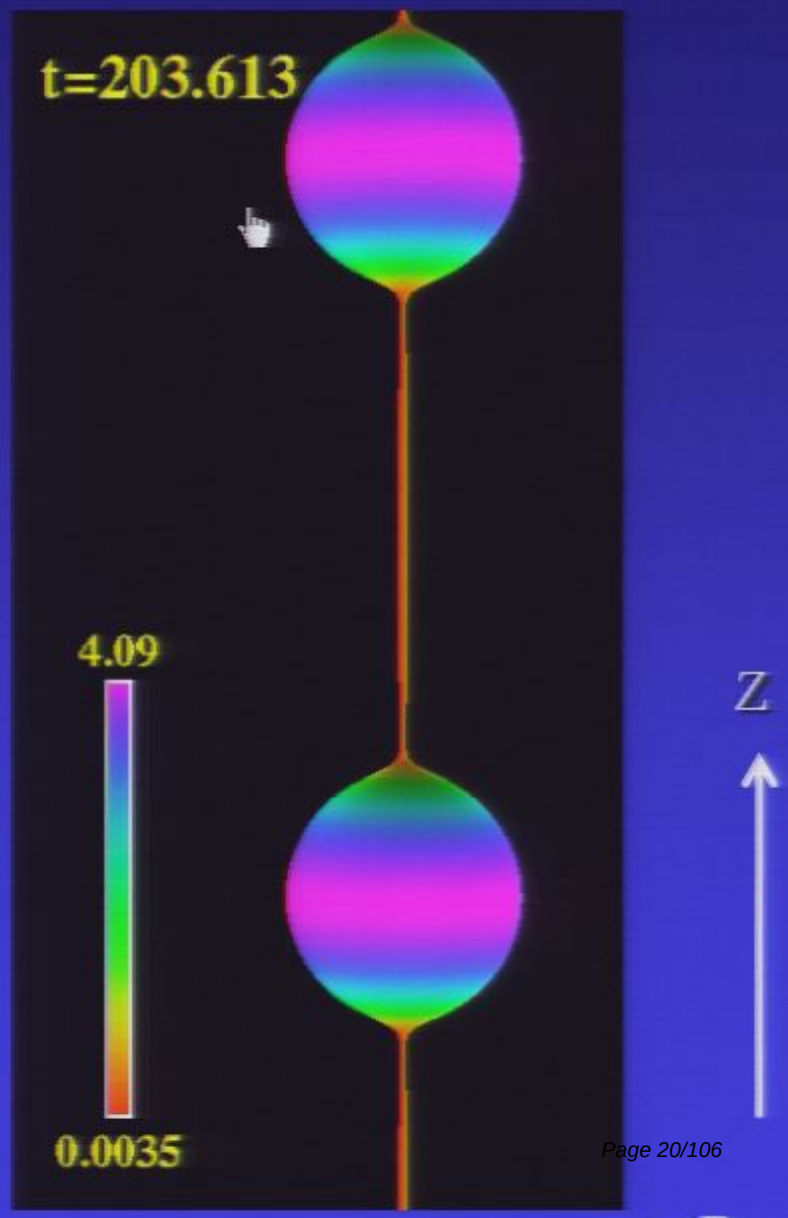

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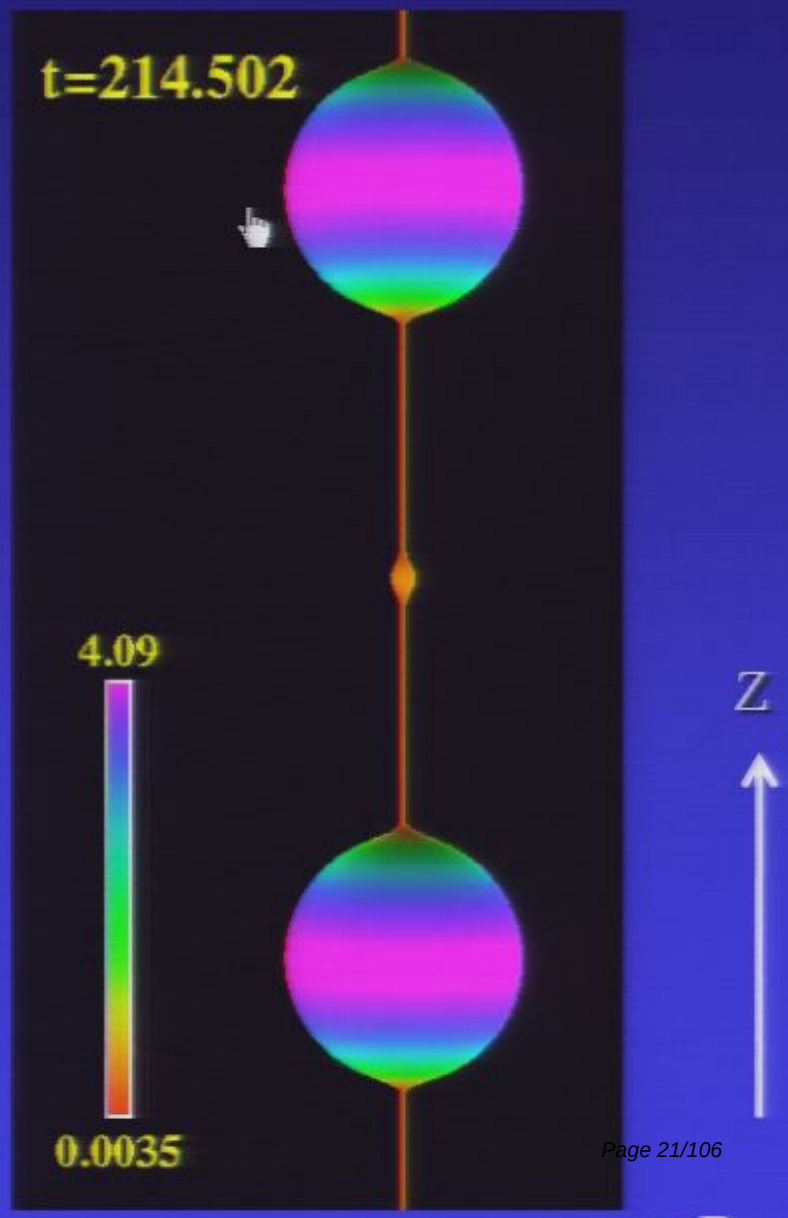

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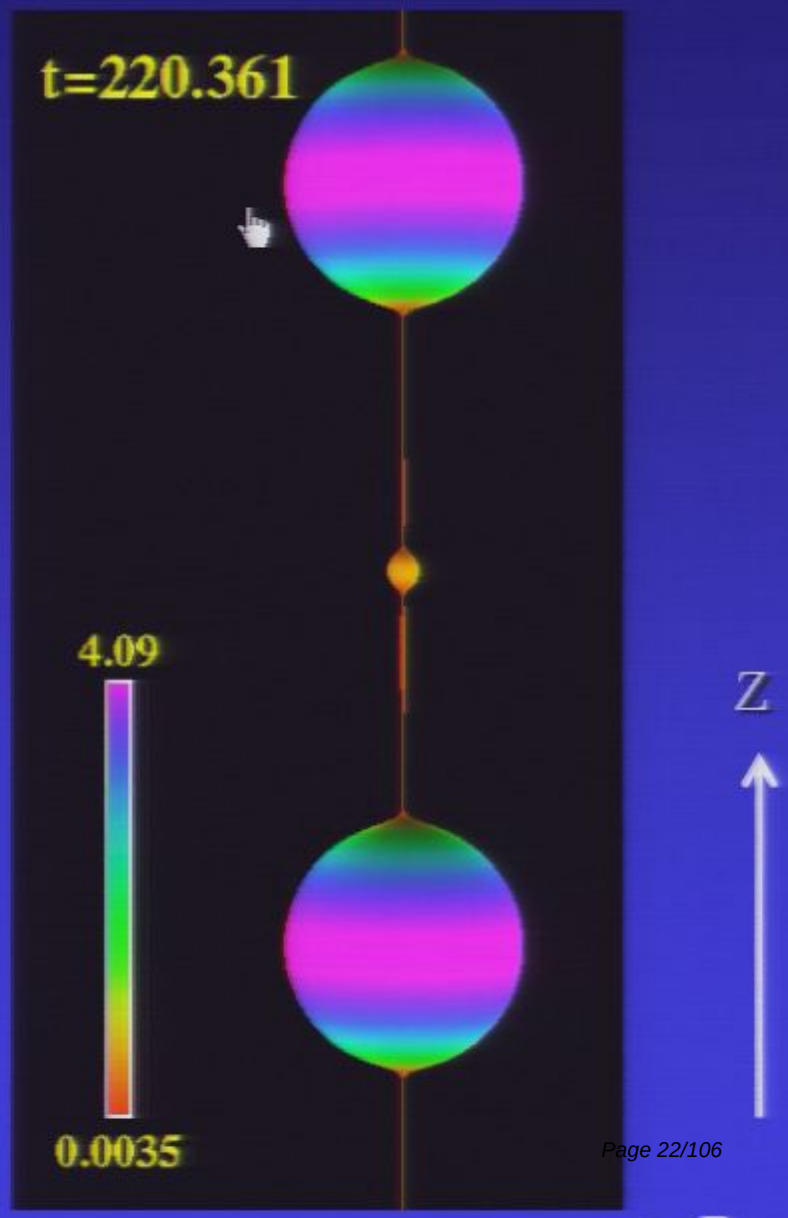

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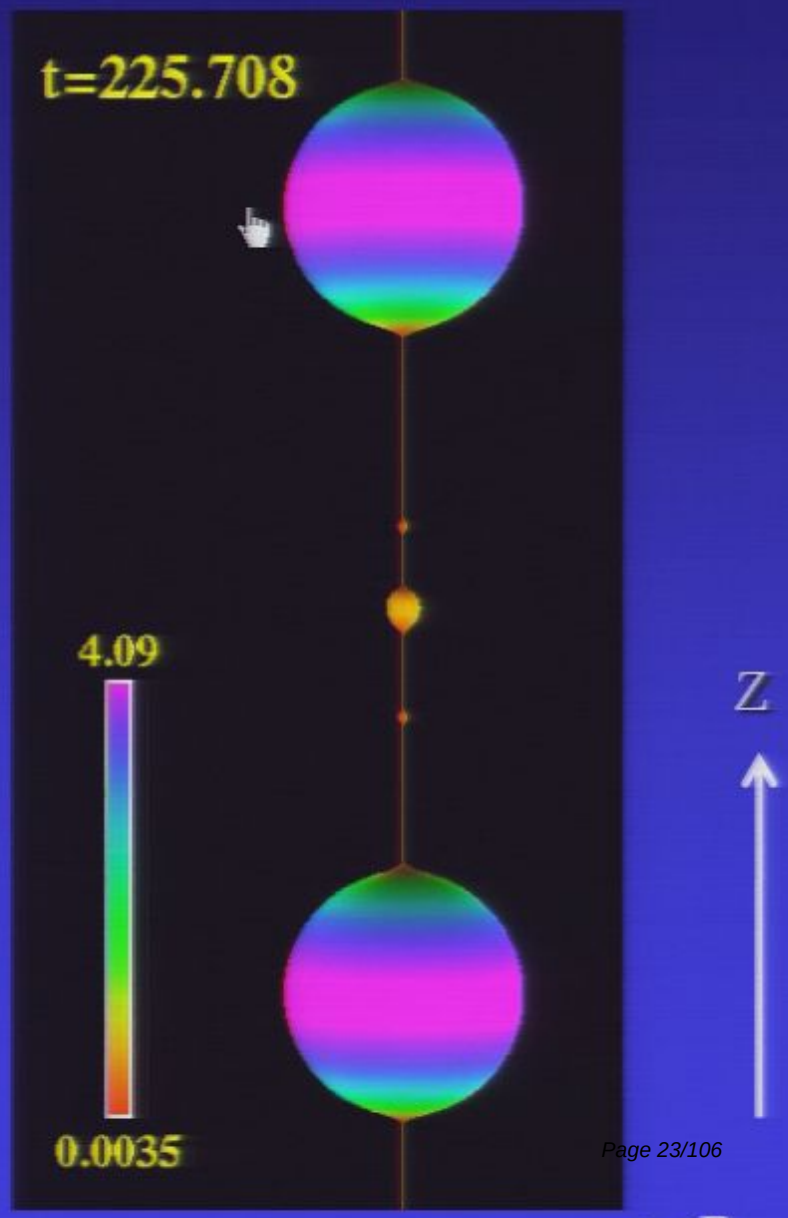

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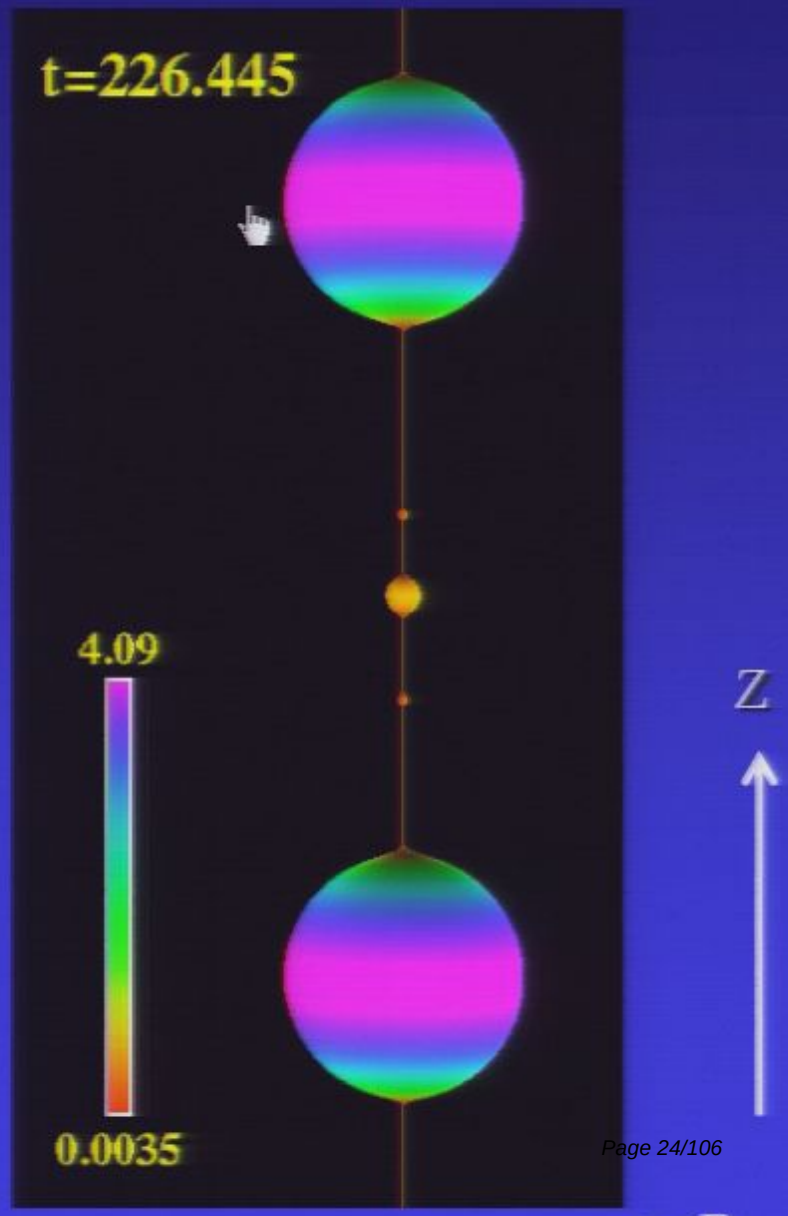

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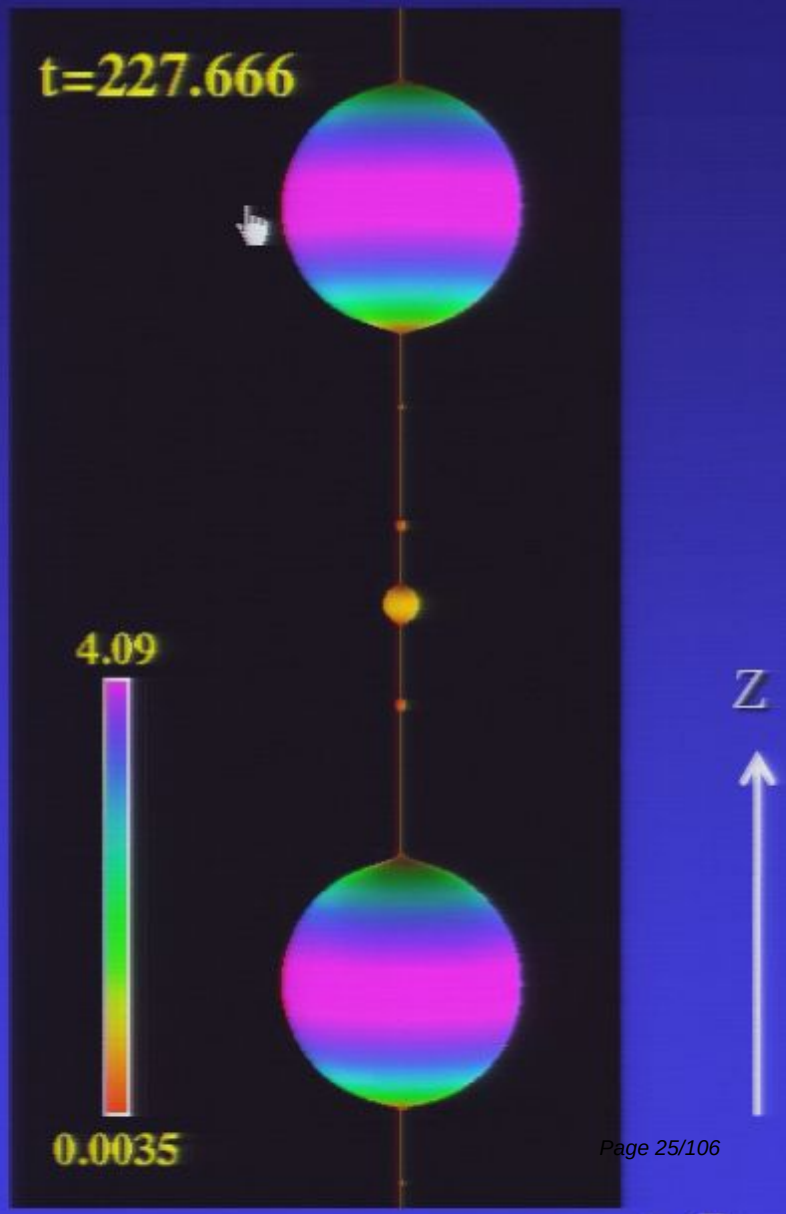

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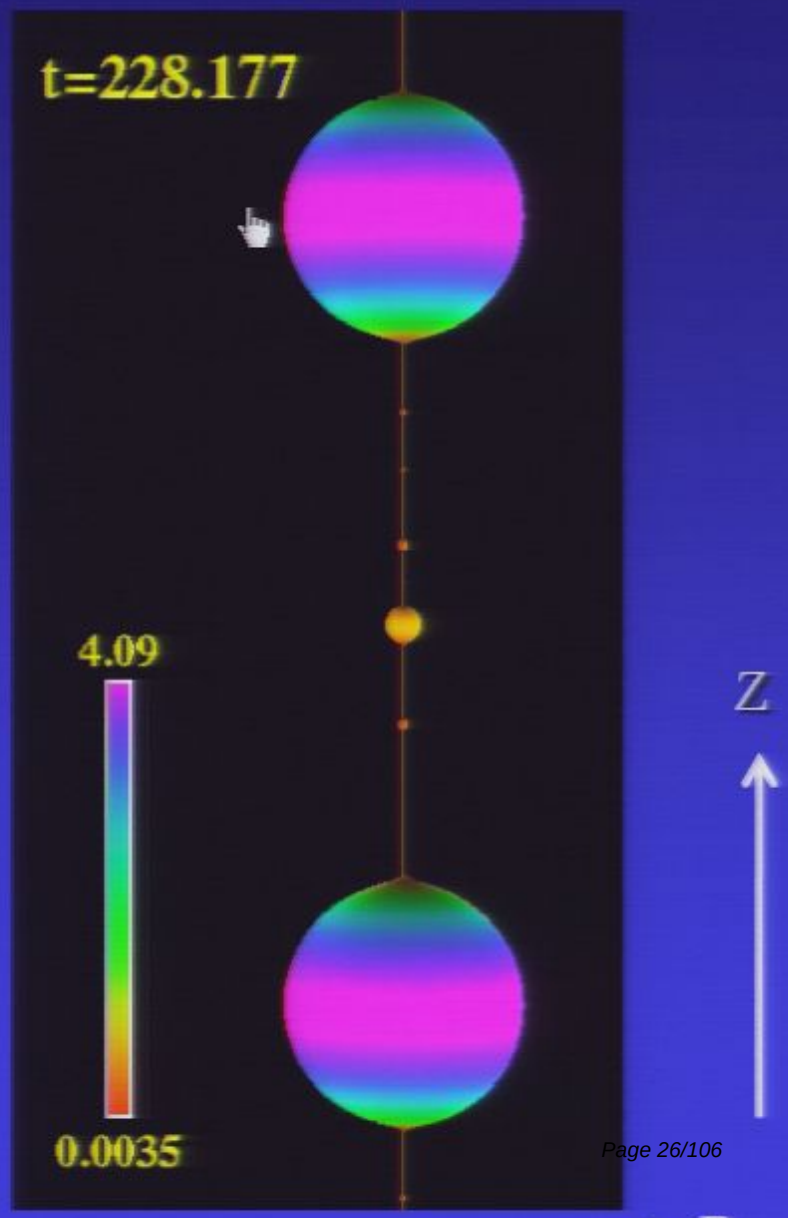

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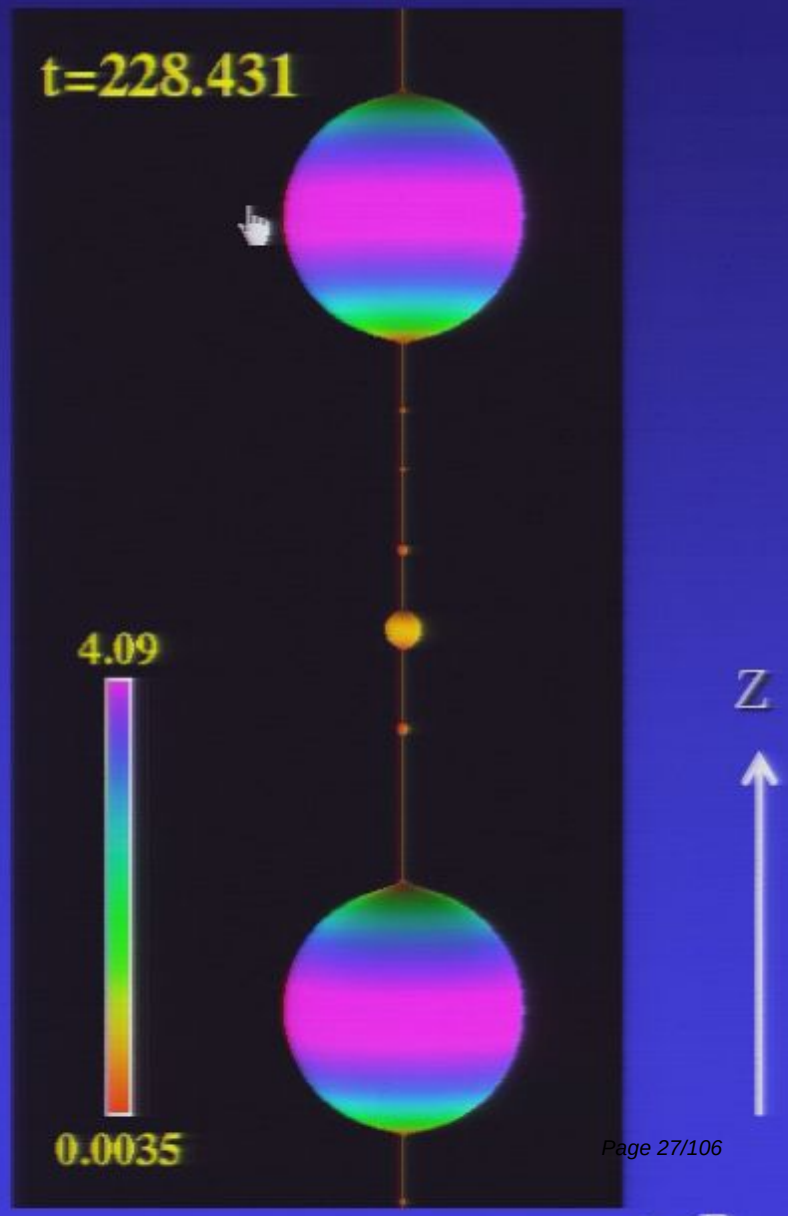

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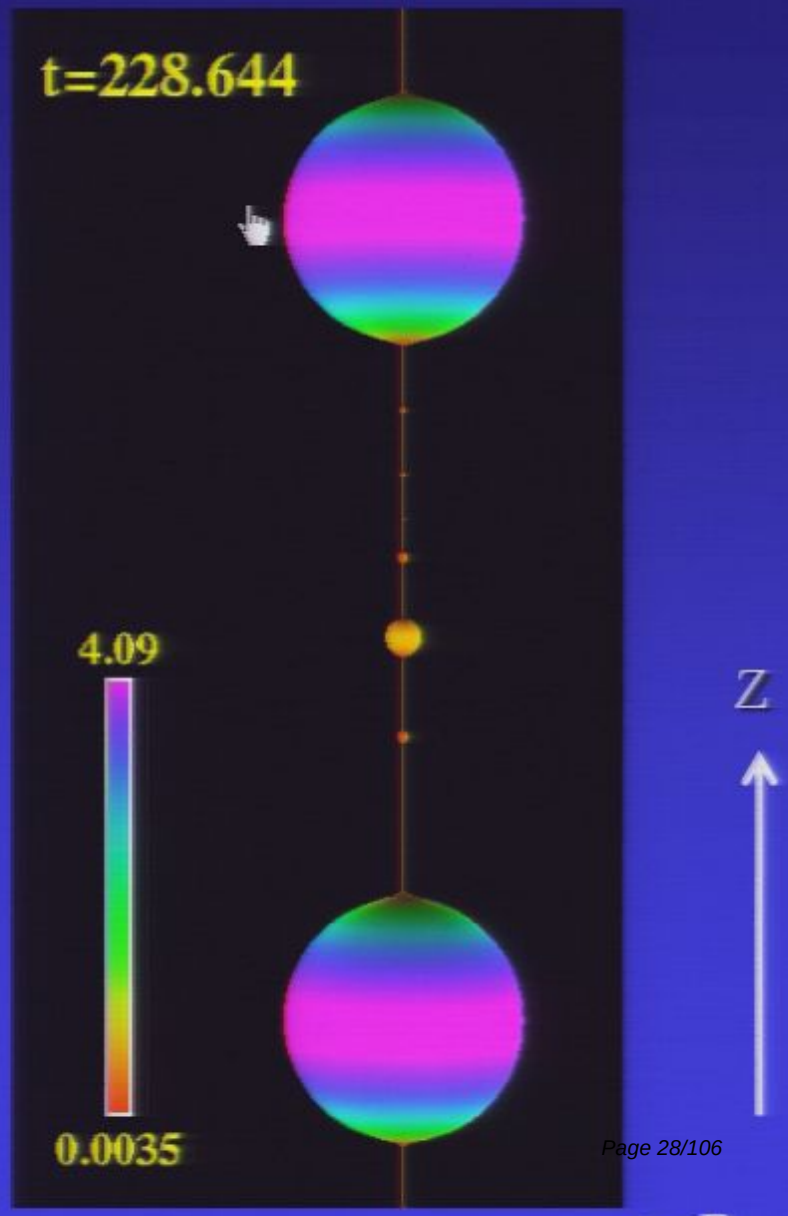

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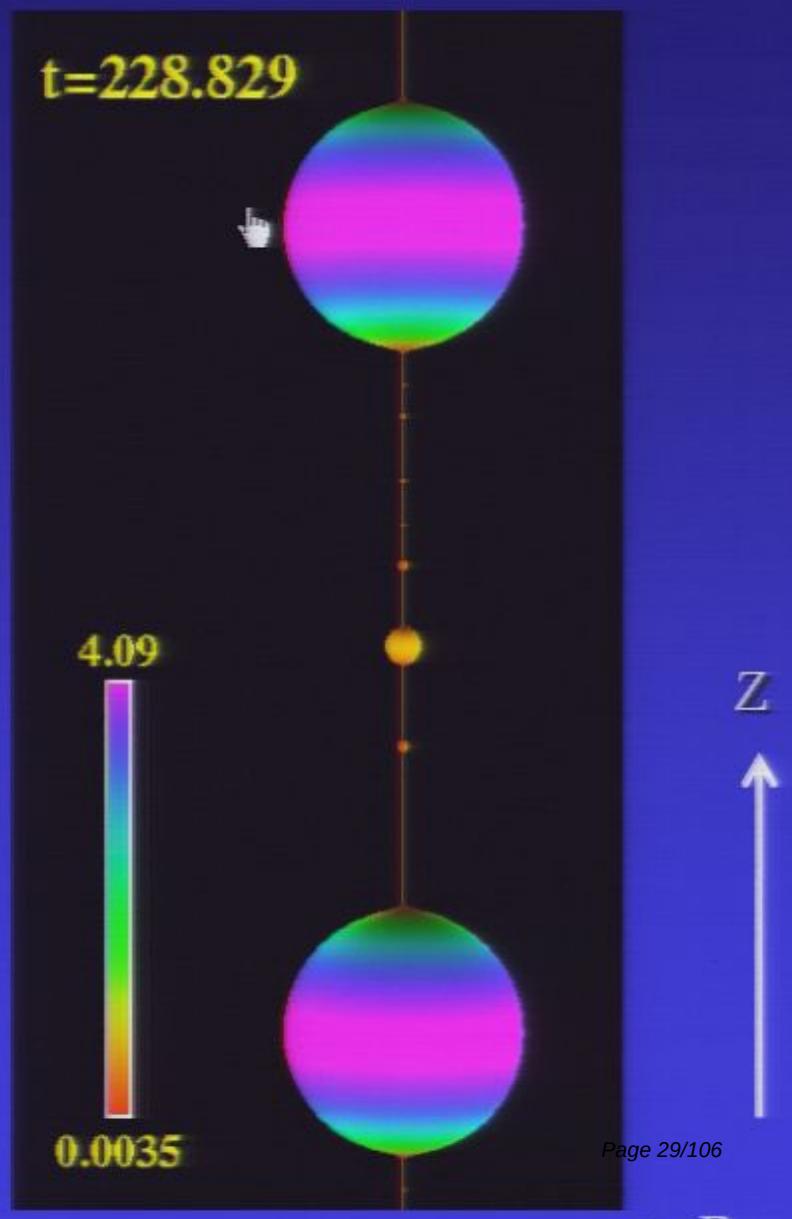

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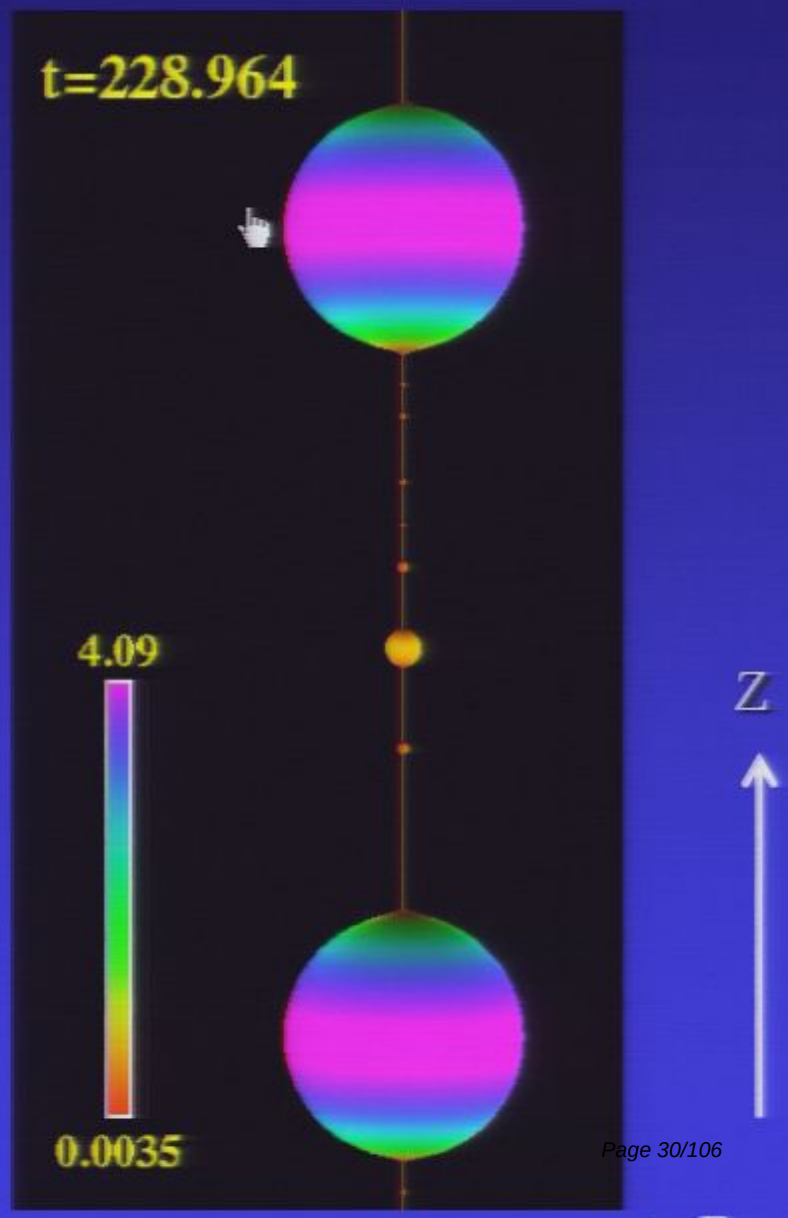

AH Embedding diagram: same data, zooming in

- map the geometric 1D shape of each $t=x=y=constant$ slice of the apparent horizon to a flat (R,Z) Euclidean space; i.e. in parametric form

$$(R,Z)=\left(R(\xi),Z(\xi)\right)$$

- $R(\xi)$ is the areal radius of that point on the horizon, and $Z(\xi)$ is defined so that the proper length of the curve in the flat space is identical to that of the corresponding curve in the physical geometry
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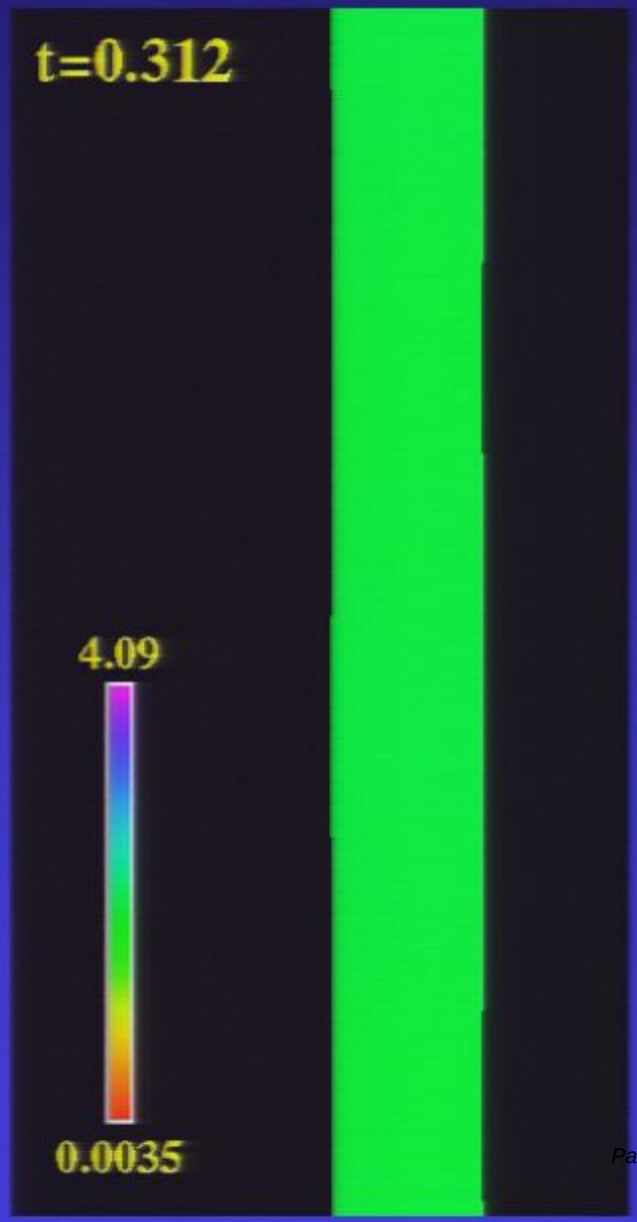

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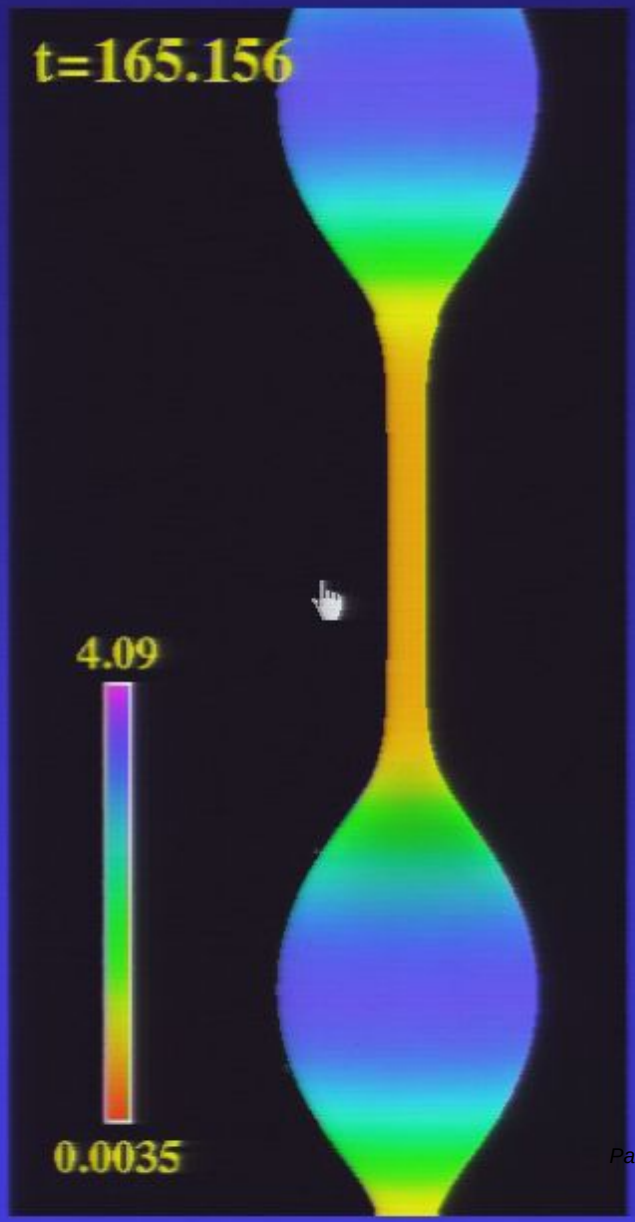

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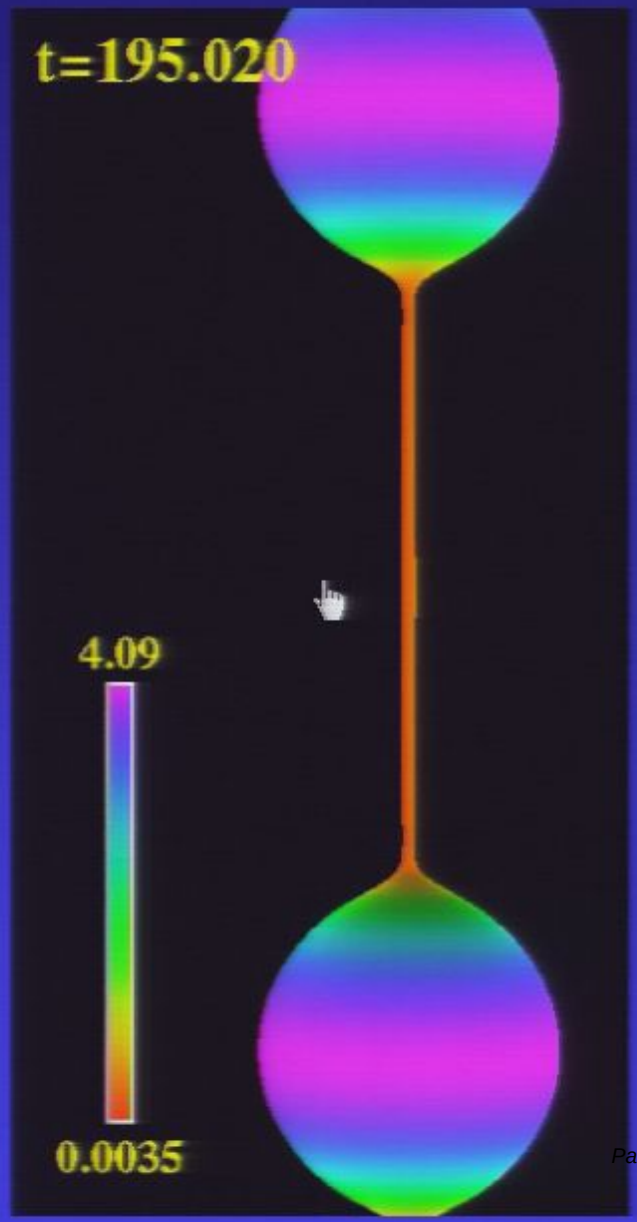

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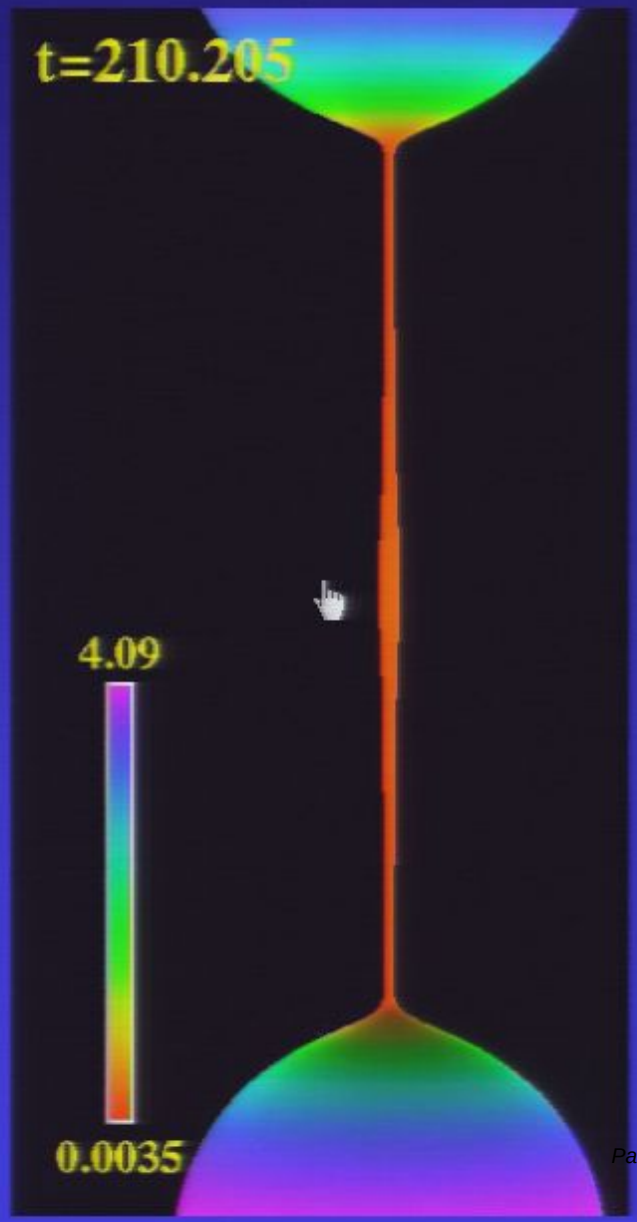

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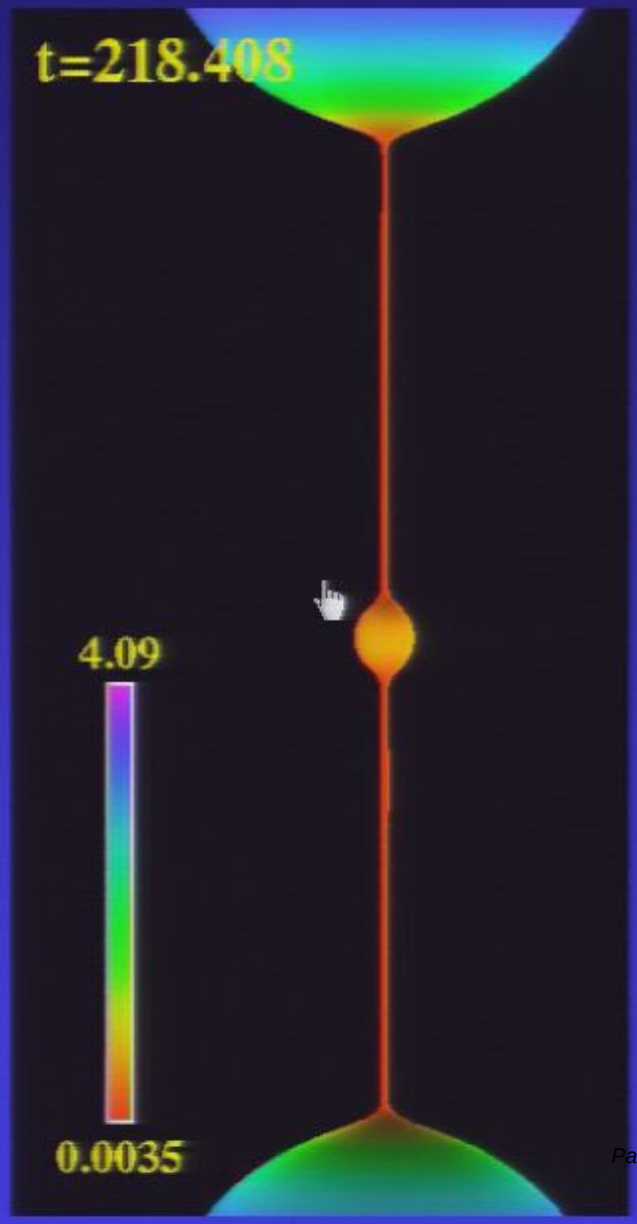

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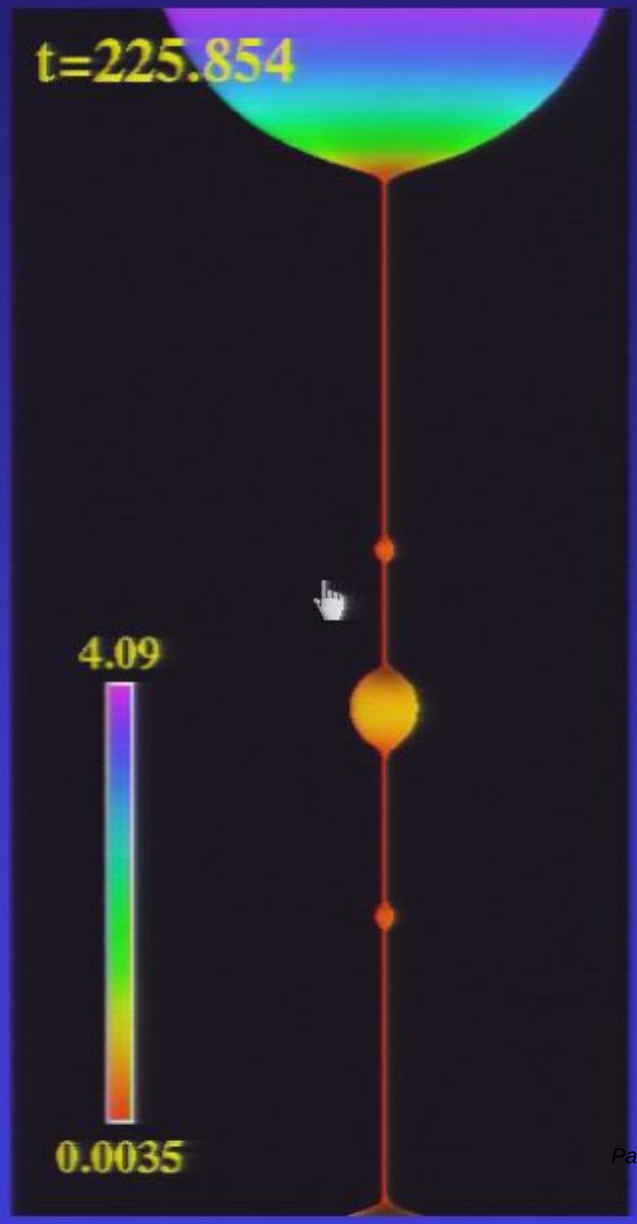

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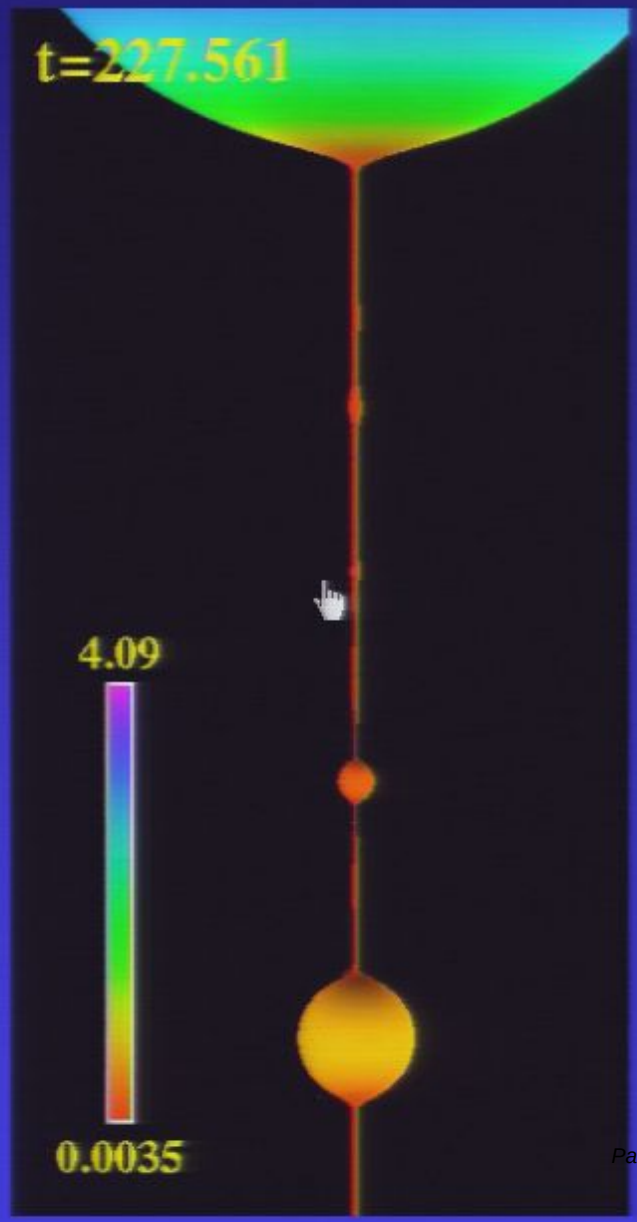

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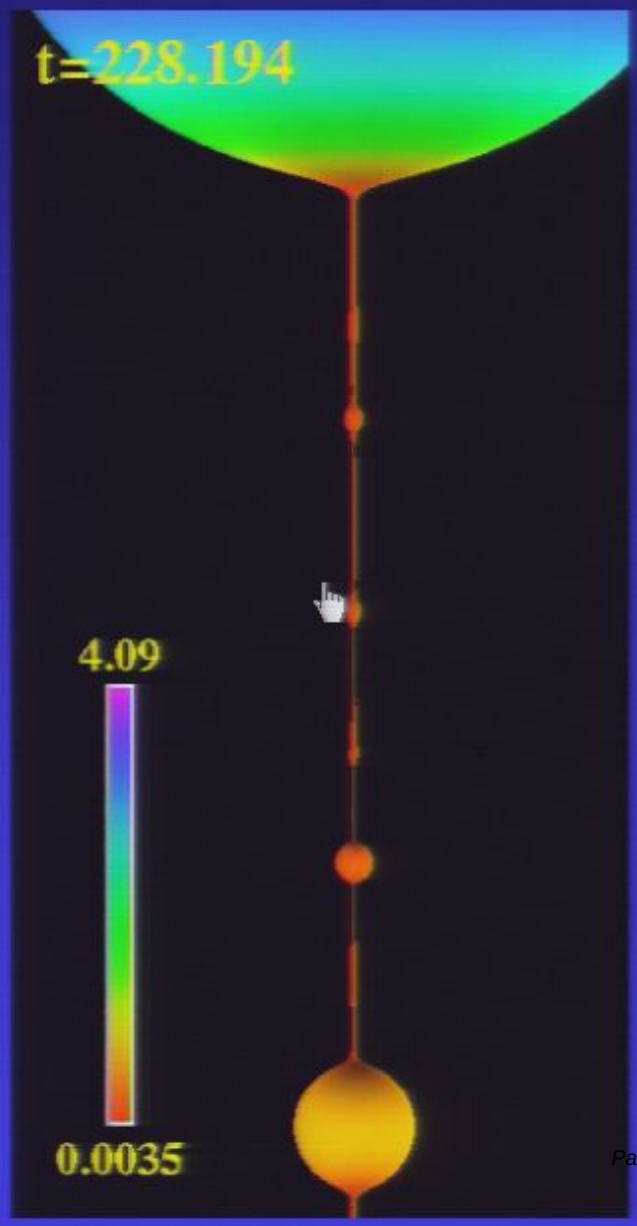

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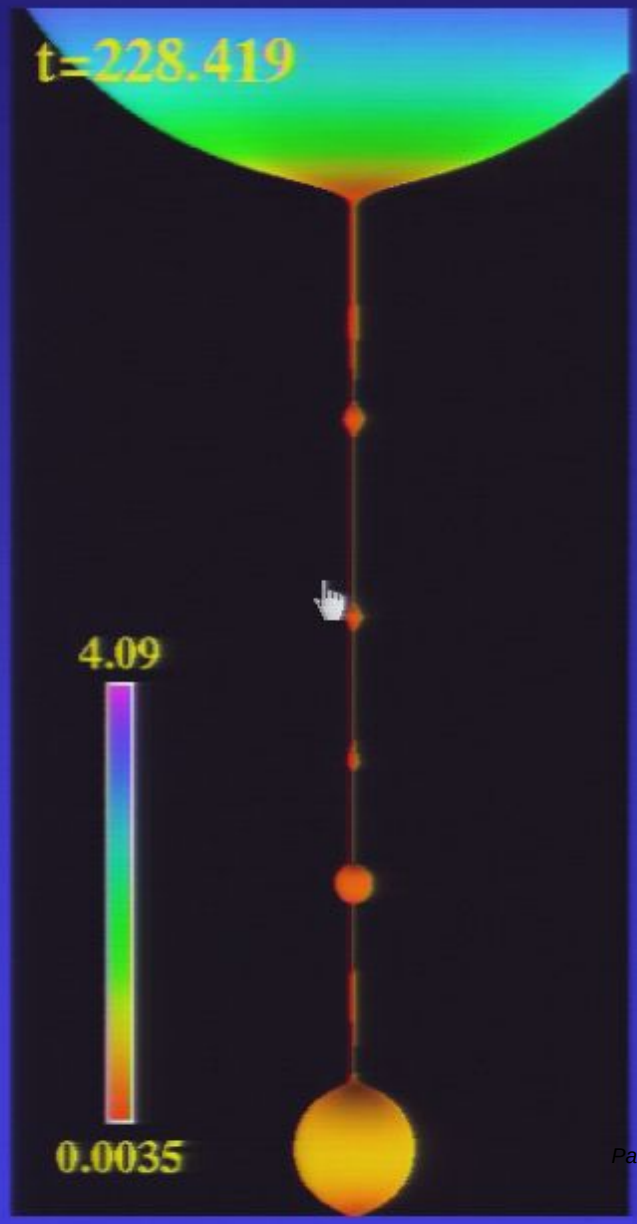

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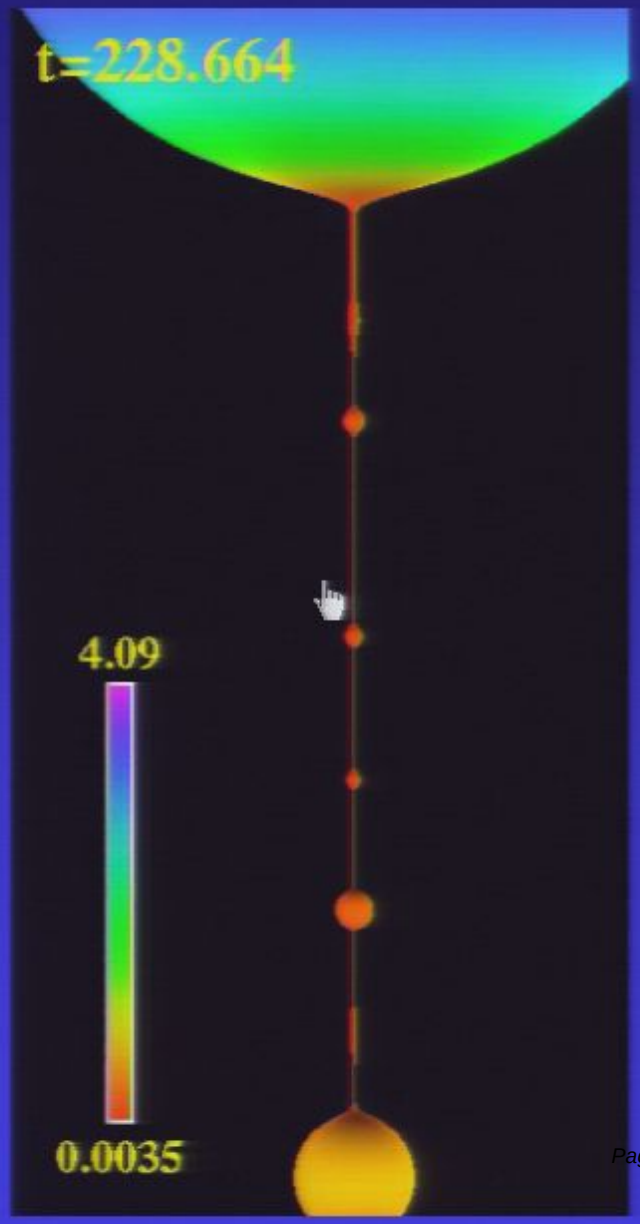

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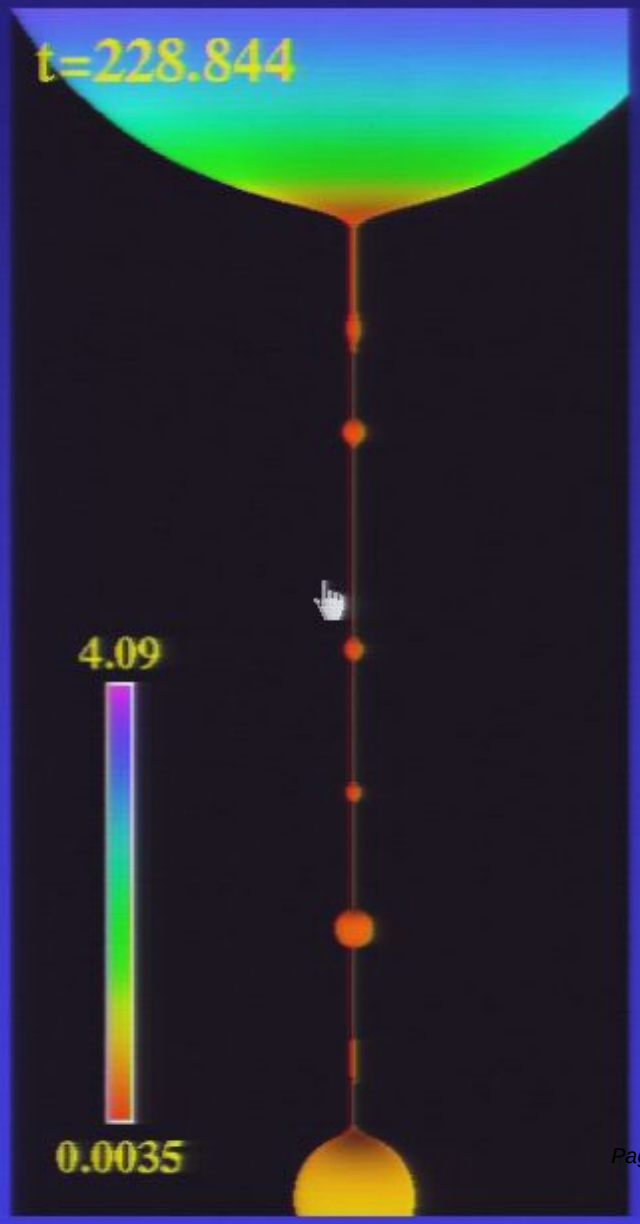

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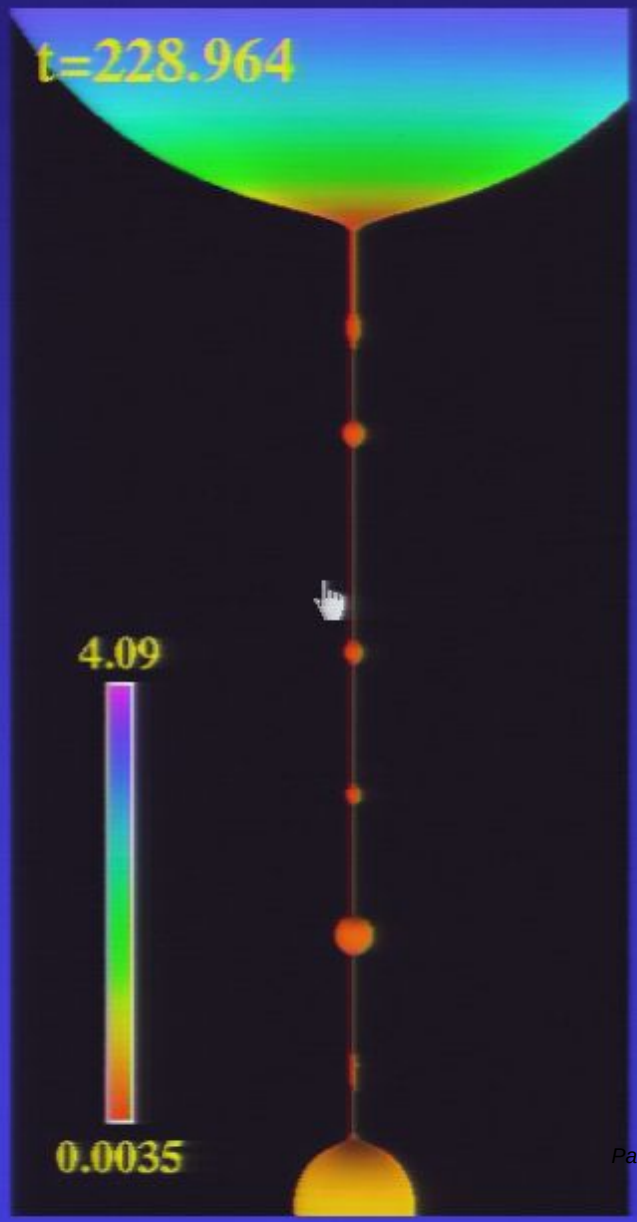
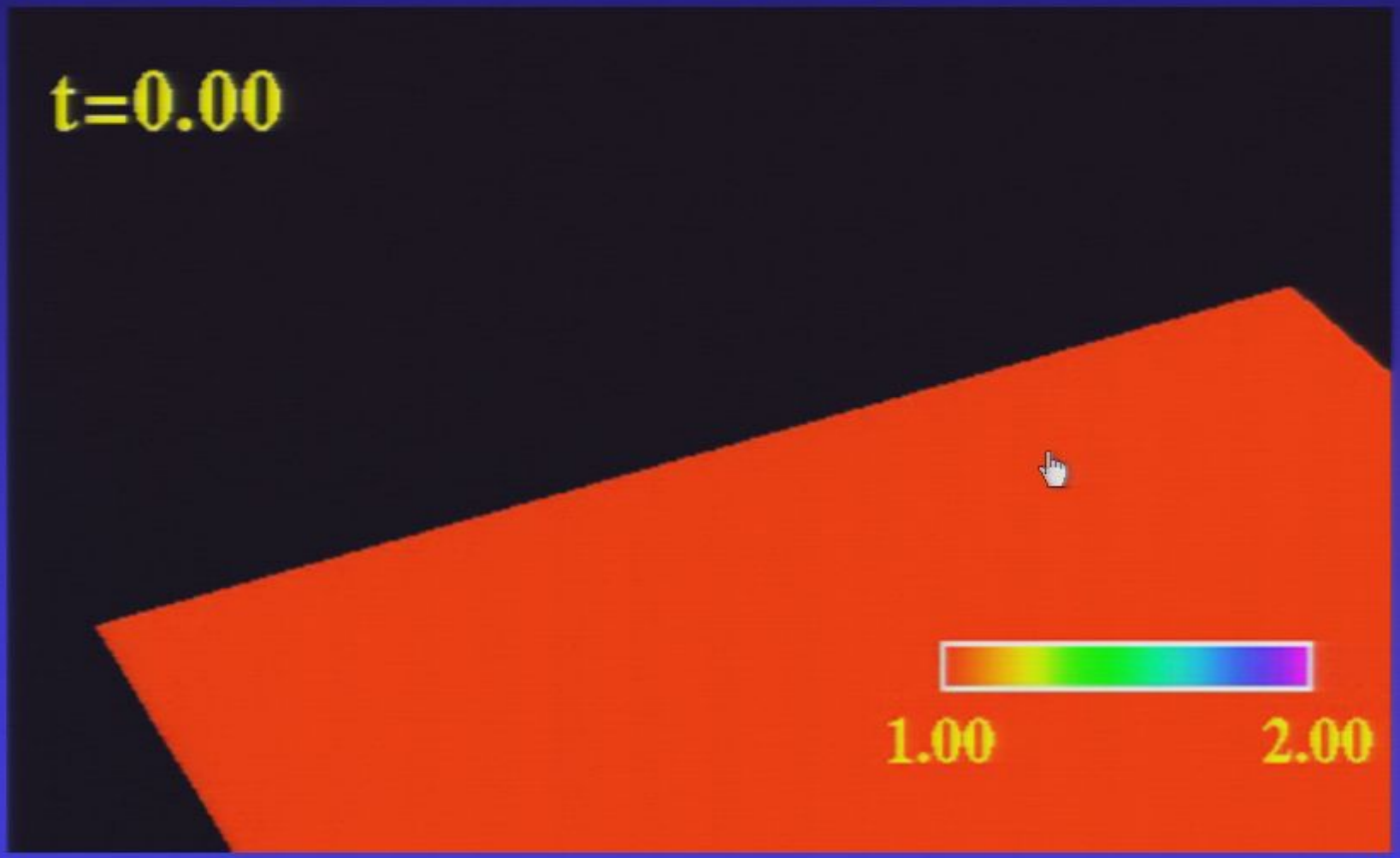


Metric Evolution

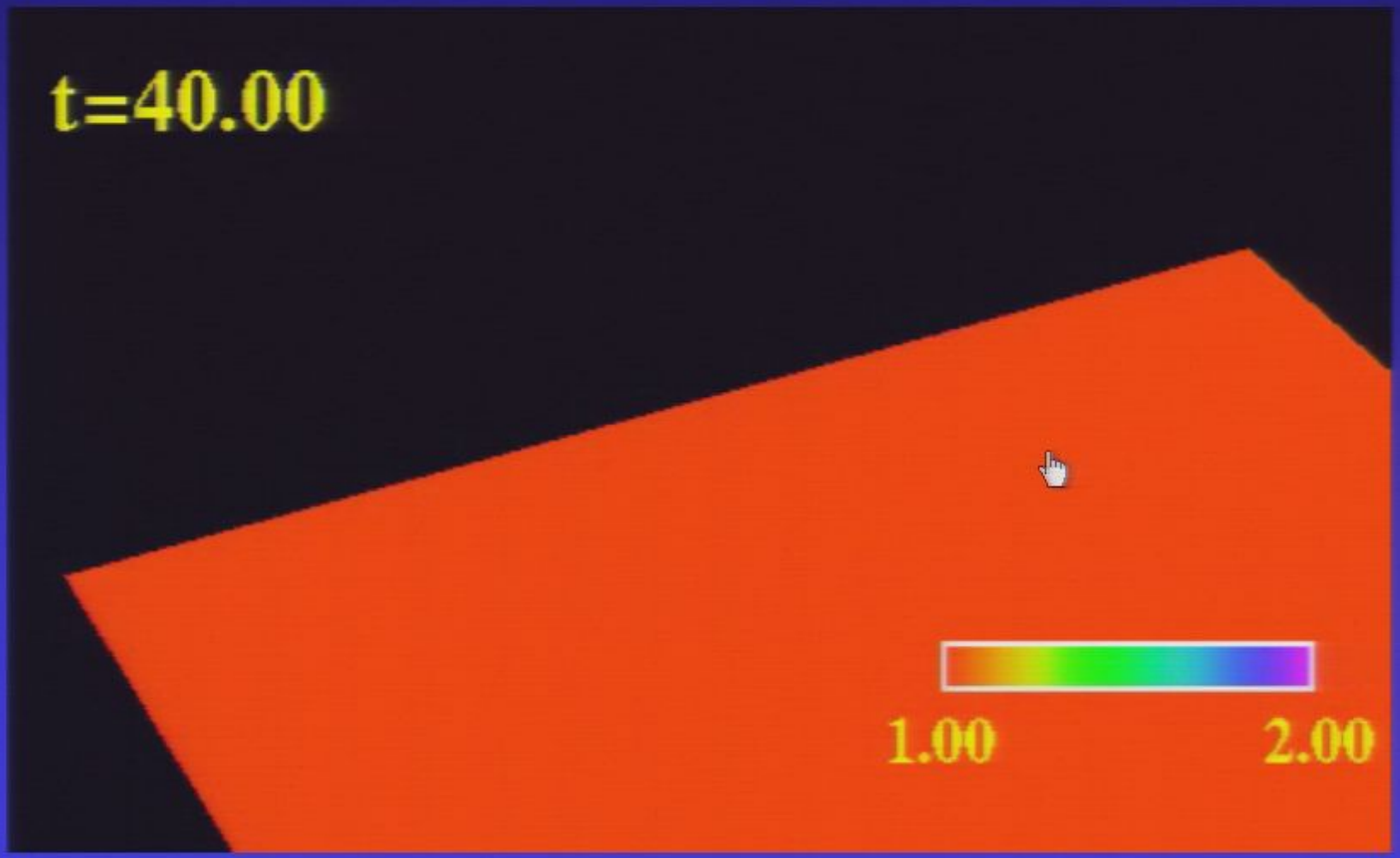


w

x

g_{ww}

Metric Evolution



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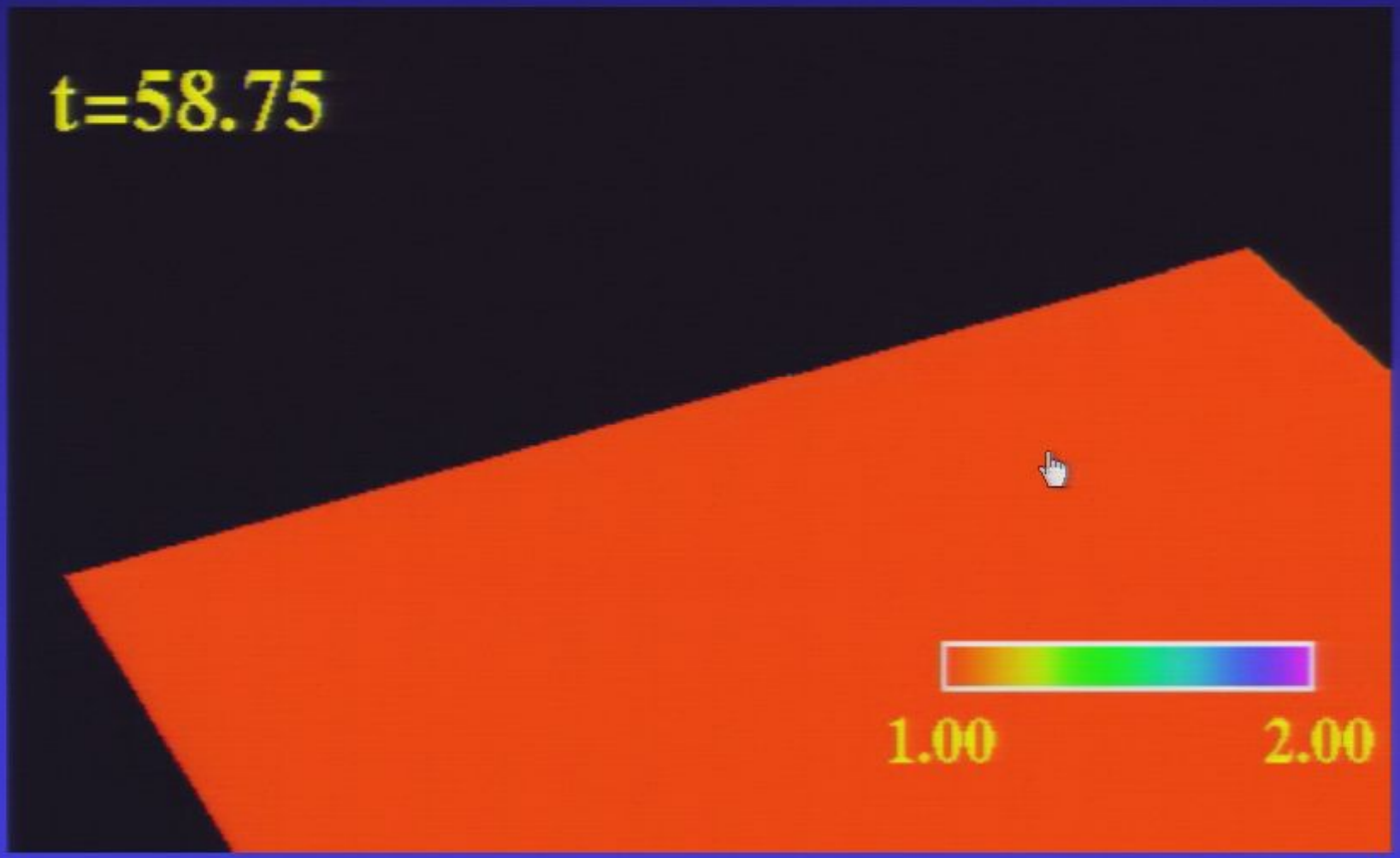

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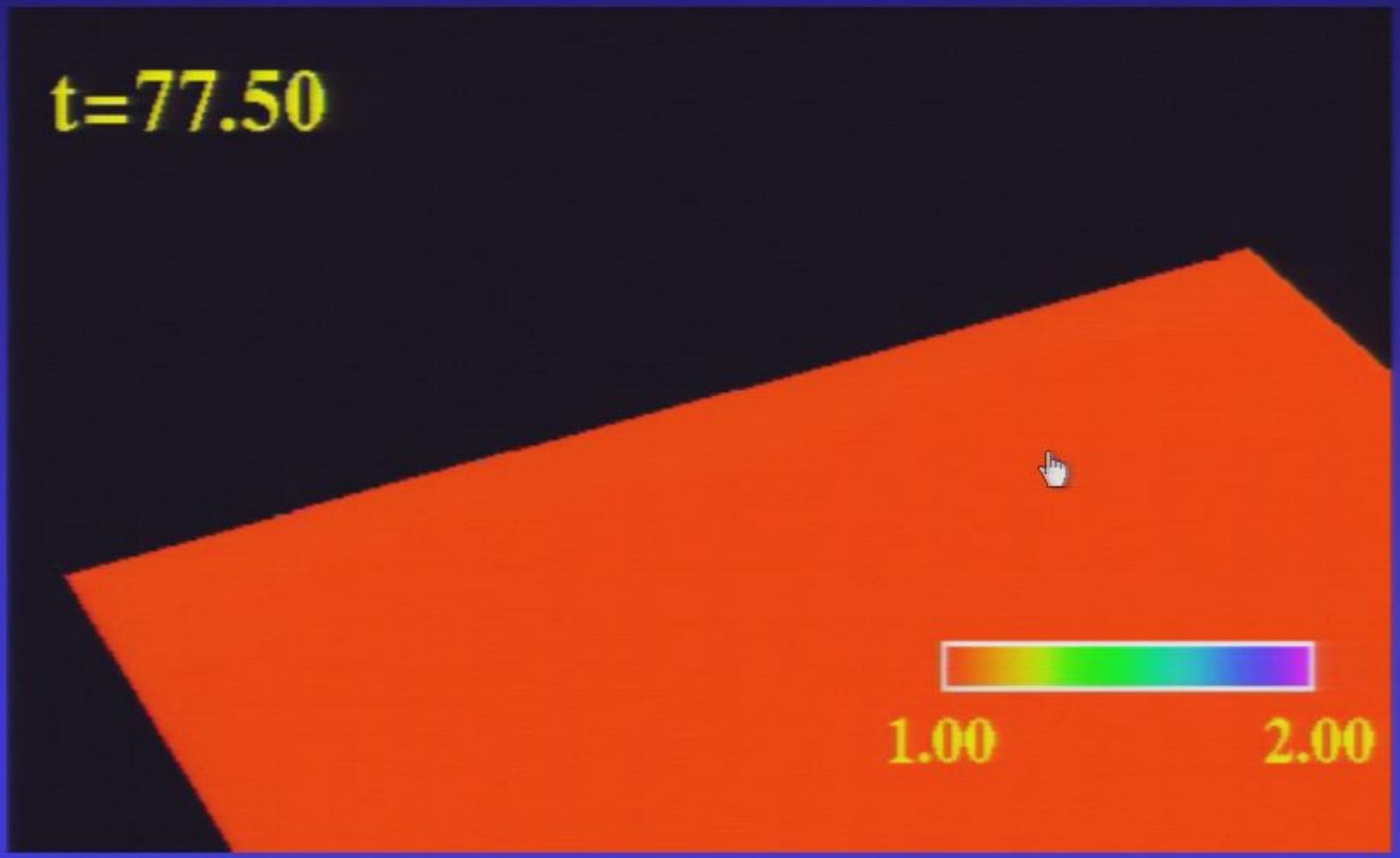

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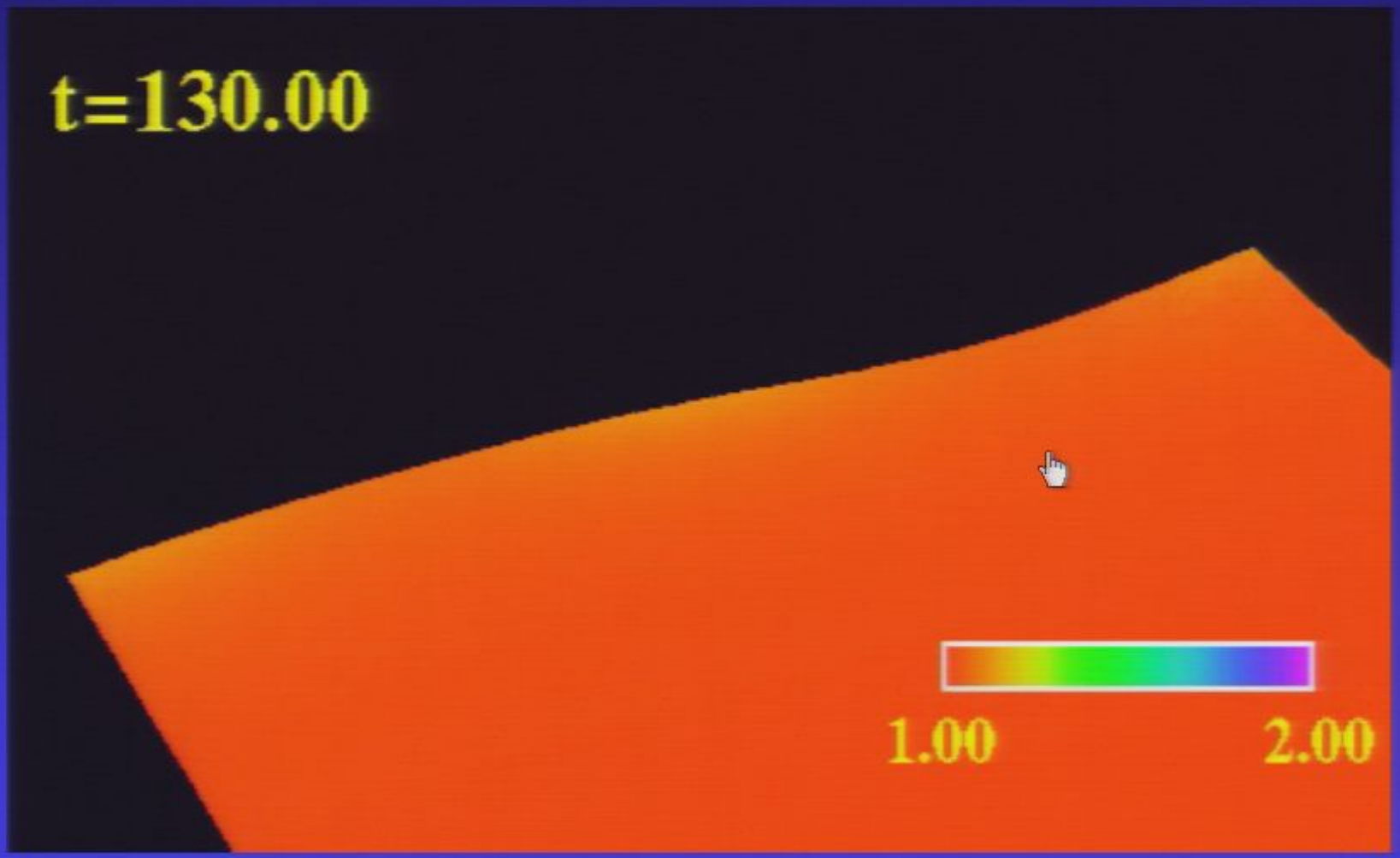

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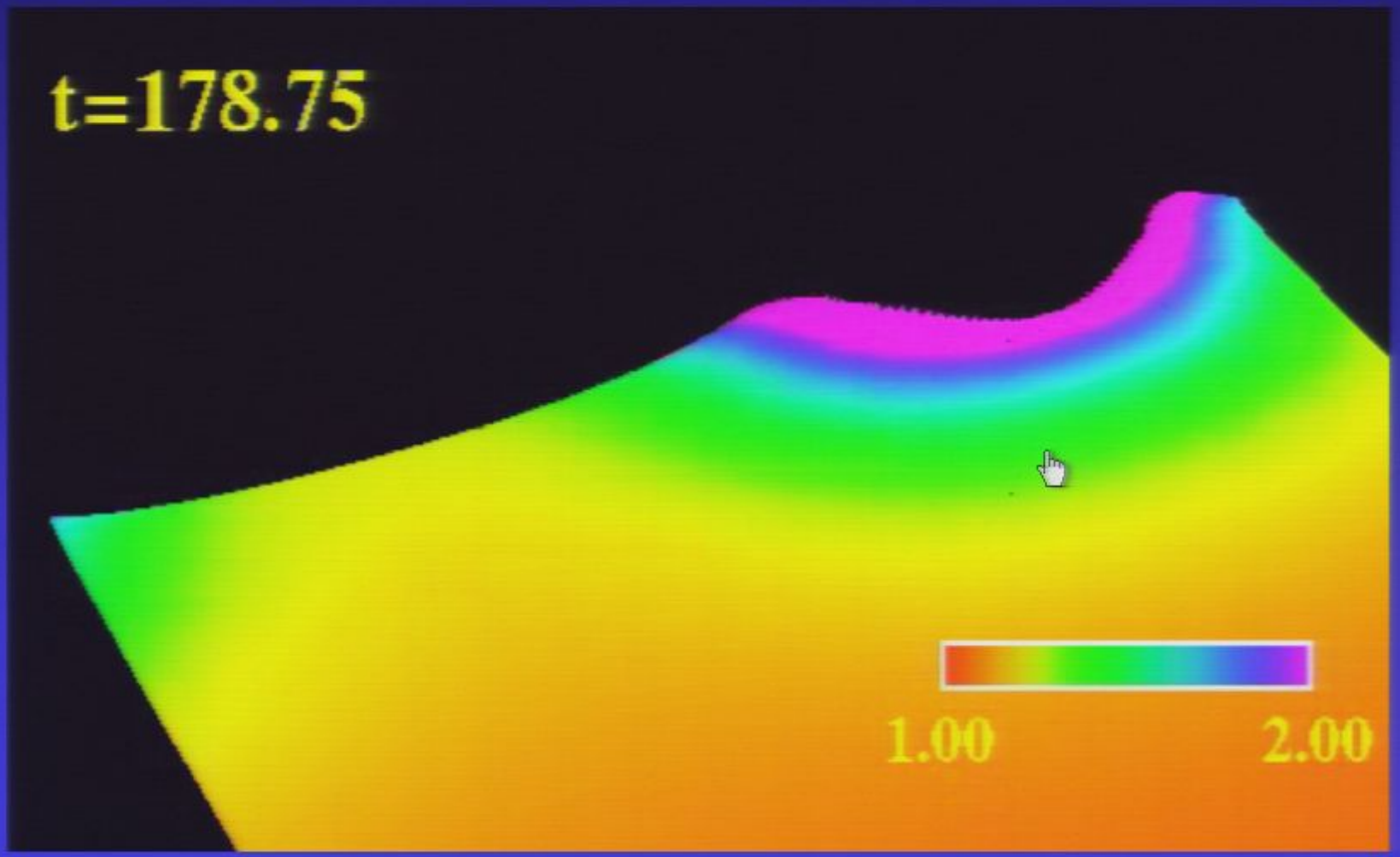

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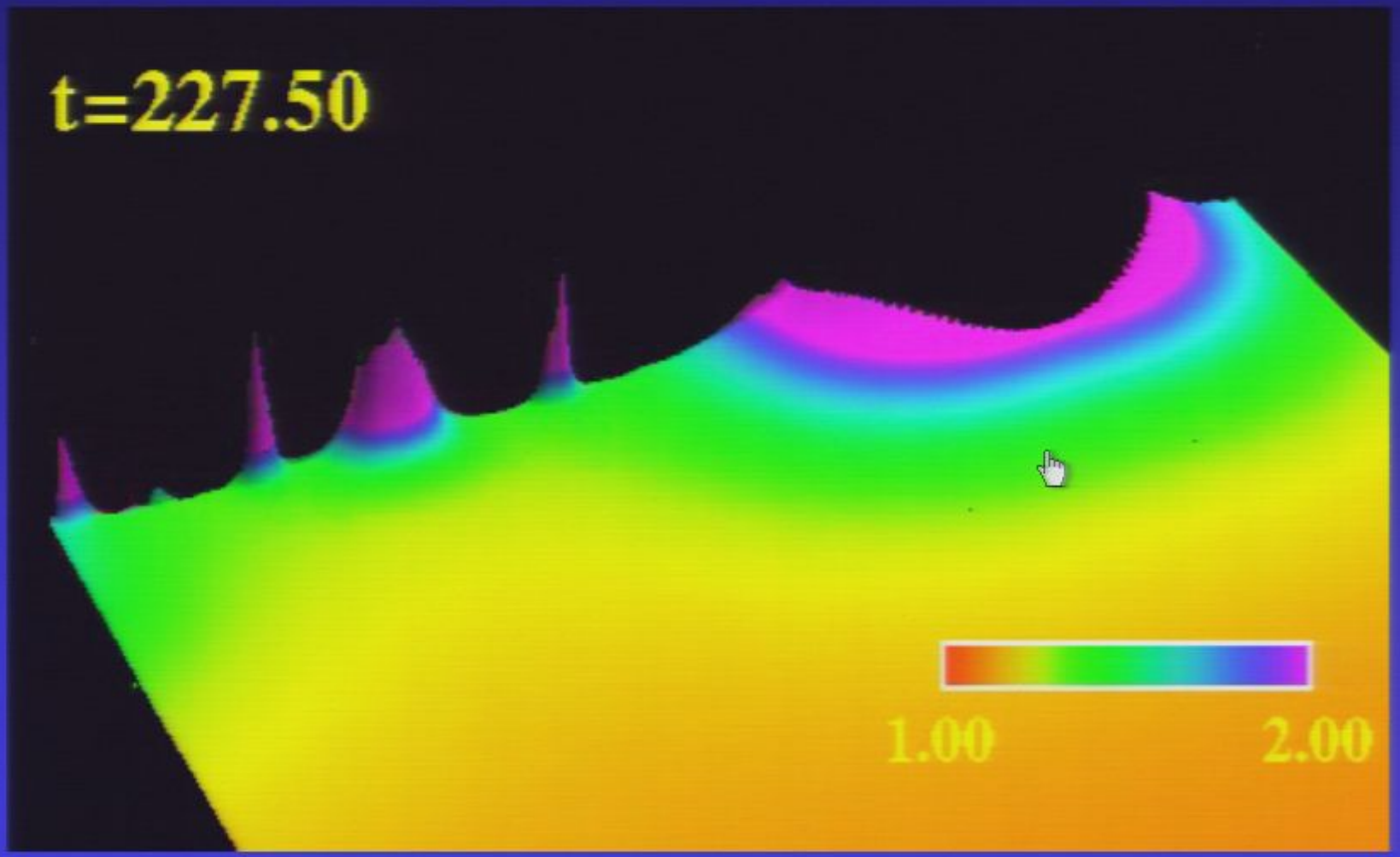

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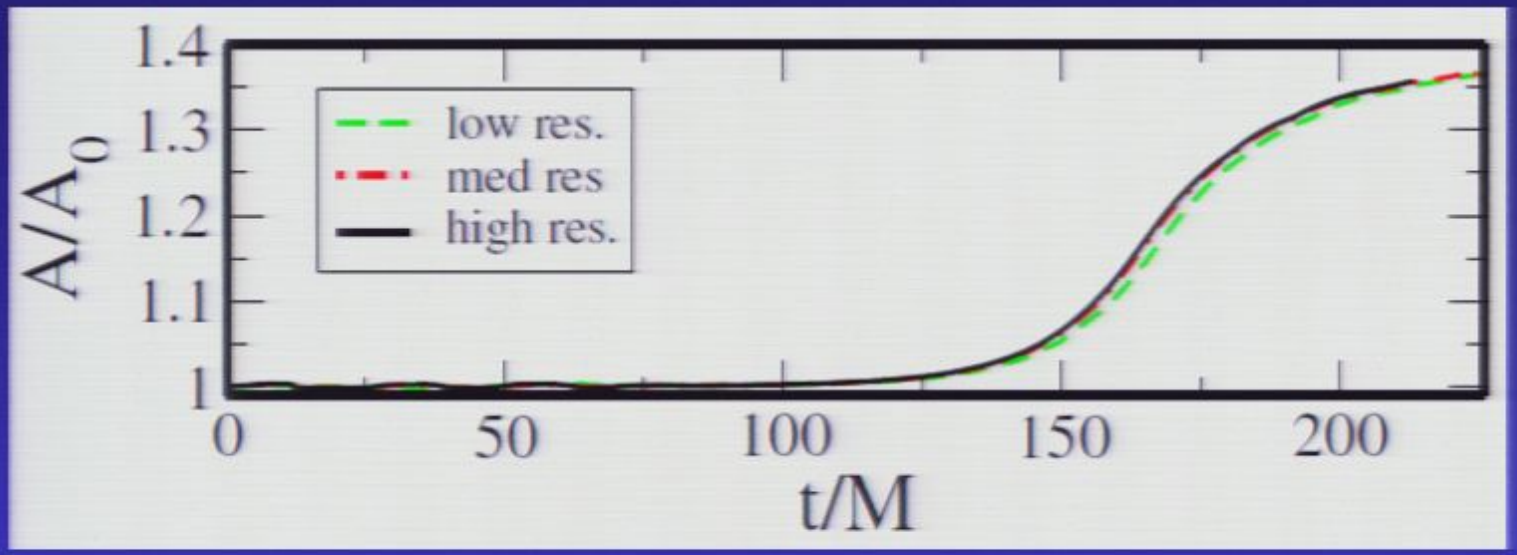
x

g_{ww}

Apparent Horizon Area



- Resolutions chosen via specification of the maximum estimated truncation error τ, from τ_0, $\tau_0/8$ to $\tau_0/64$ ("low" to "high")
- For this configuration, ignoring the (small amount of) energy from the initial perturbation, a sequence of spherical black holes (one per period) with the same energy as the initial string has an area

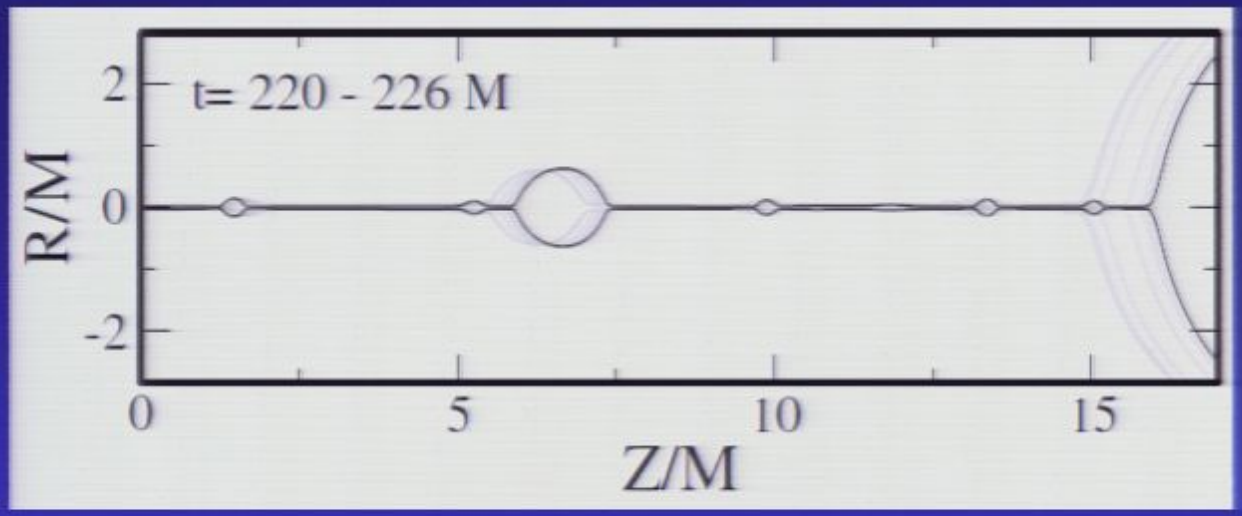
$$A_{BH}/A_{BS} = 1.374$$

The lowest resolution simulation, which has run the longest in physical time, has reached a value

$$A/A_0\,(t = t_{end}) \approx 1.369 \pm 0.005$$

- This is consistent with the argument that the dynamics of the instability is such as

Apparent Horizon Dynamics



- At late times the horizon certainly *looks* like it can be described as a sequence of spherical black holes connected by string segments; to quantify this a bit, we evaluate the following curvature invariants on the horizon:

$$I = R_{abcd}R^{abcd};\ J = R_{abcd}R^{cdef}R_{ef}{}^{ab}$$

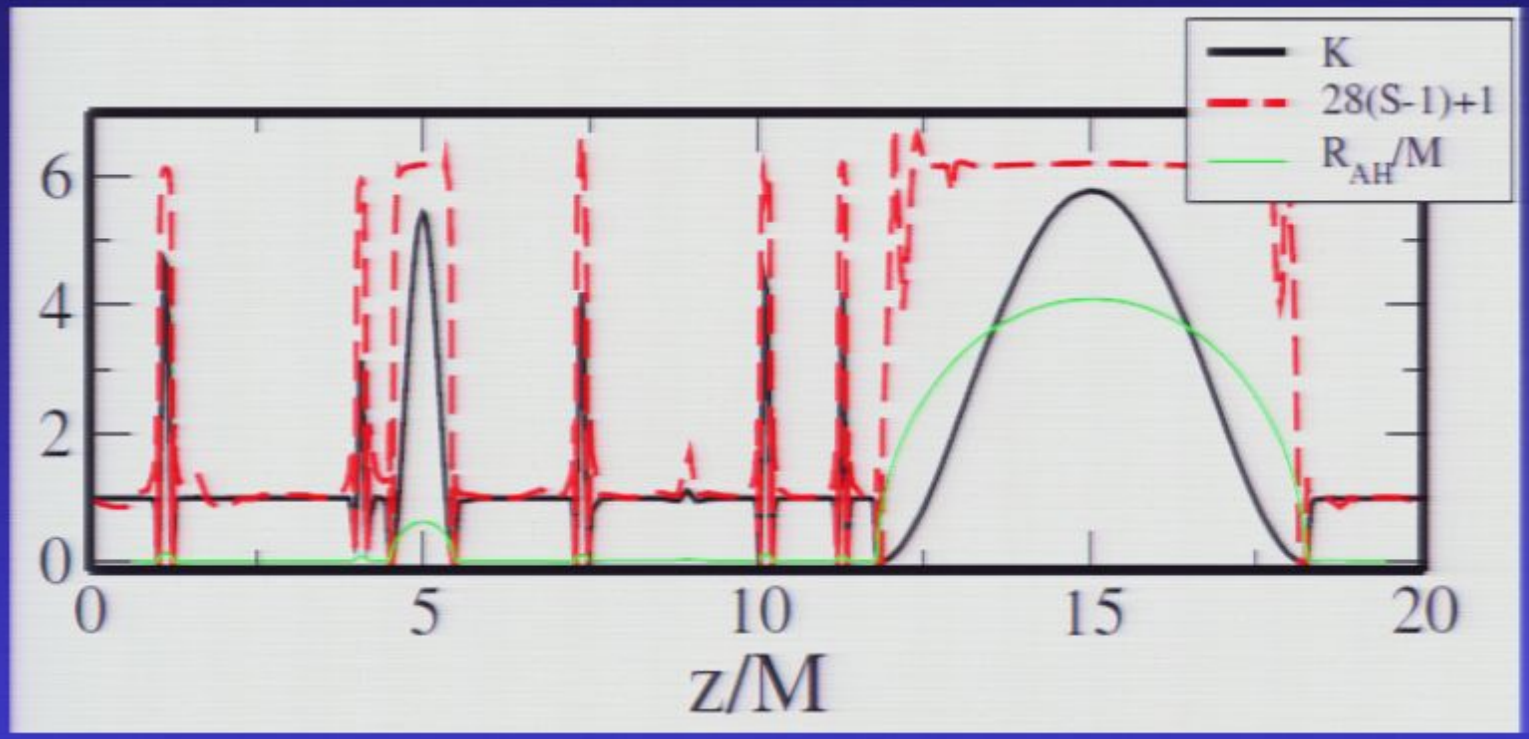
and construct the following dimensionless scalars

$$K = IR_{AH}^{4}/12;\ S = 12J^{2}I^{-3}$$

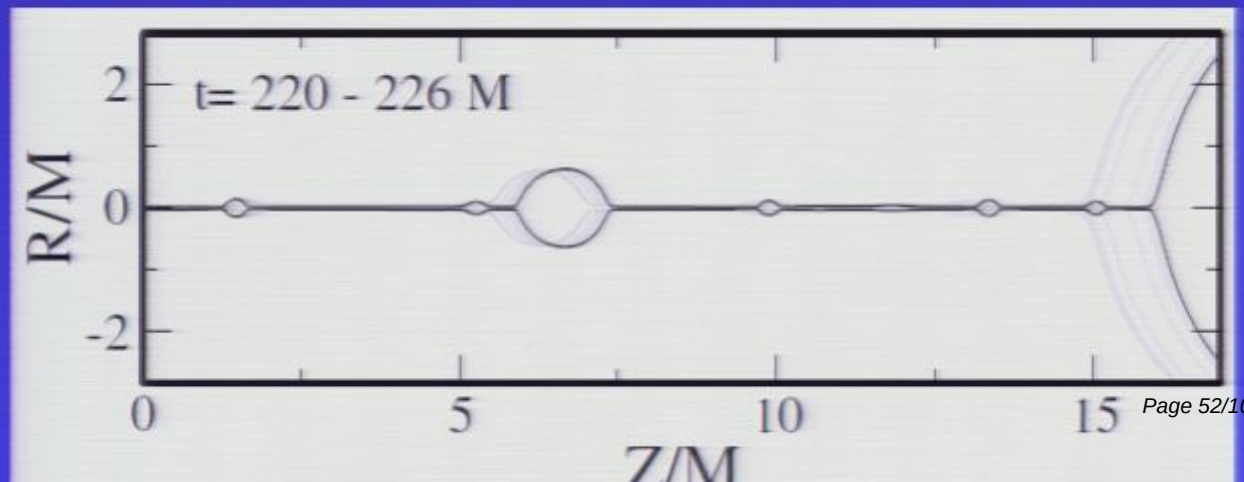
which evaluate to the following for the exact black sphere/black string solutions

$$K_{BH} = 6;\ 27(S_{BH} - 1) + 1 = 6 \qquad K_{BS} = 1;\ 27(S_{BS} - 1) + 1 = 1$$

Apparent Horizon Dynamics



Invariants above evaluated on the apparent horizon at the last time step of the (medium resolution) simulation depicted to right



Properties of satellites and string-segments

- Therefore, the spheres-connected-by-string-segments interpretation seems reasonable. With that interpretation, and that evolution proceeds through a sequence of unstable epochs, we extract the following properties from the horizon:

Gen.	t_i/M	n_s	$R_{s,i}/M$	$R_{AH,f}/M$	$L_{s,i}/R_{s,i}$
1	118.1 ± 0.5	1	2.00	$4.09 \pm 0.5\%$	10.0
2	203.1 ± 0.5	1	$0.148 \pm 1\%$	$0.63 \pm 2\%$	$105 \pm 1\%$
3	223 ± 2	> 1	$0.05 \pm 20\%$	$0.1 - 0.2$	$\approx 10^2$
4	≈ 227	$> 1(?)$	≈ 0.02	?	$\approx 10^2$

Gen: generation number

t_i: time of initial satellite formation (defined to be time when the areal radius has grown to 1.5 times that of the surrounding string-segment)

n_s: number of satellites that form

$R_{s,i}$: radius of local string segment

$R_{AH,f}$: radius of satellites by the time the simulation was stopped

L_{si}/R_{si}: Ratio of length to radius of local string-segment (recall GL critical ratio ~ 7.2)

Numerical formalism

- For efficient evolution we employ a variant of the "cartoon" method *[M. Alcubierre et al. Int.J.Mod D10 (2001); FP, CQG 22 (2005)]*, whereby we only discretize a 2+1 dimensional slice *(y=z=0)* of the spacetime
  - off-slice *(y & z)* derivatives of the metric in the field equations are replaced with in-slice *(x)* derivatives by using the Killing vectors of the chosen SO(3) symmetry

$$\xi_1^\alpha = x\left(\frac{\partial}{\partial y}\right)^\alpha - y\left(\frac{\partial}{\partial x}\right)^\alpha ;\ \xi_2^\alpha = y\left(\frac{\partial}{\partial z}\right)^\alpha - z\left(\frac{\partial}{\partial y}\right)^\alpha ;\ \xi_3^\alpha = z\left(\frac{\partial}{\partial x}\right)^\alpha - x\left(\frac{\partial}{\partial z}\right)^\alpha$$

For example, solving

$$L_{\xi_3} g_{\alpha\beta} = 0$$

for the z-gradient of the metric gives

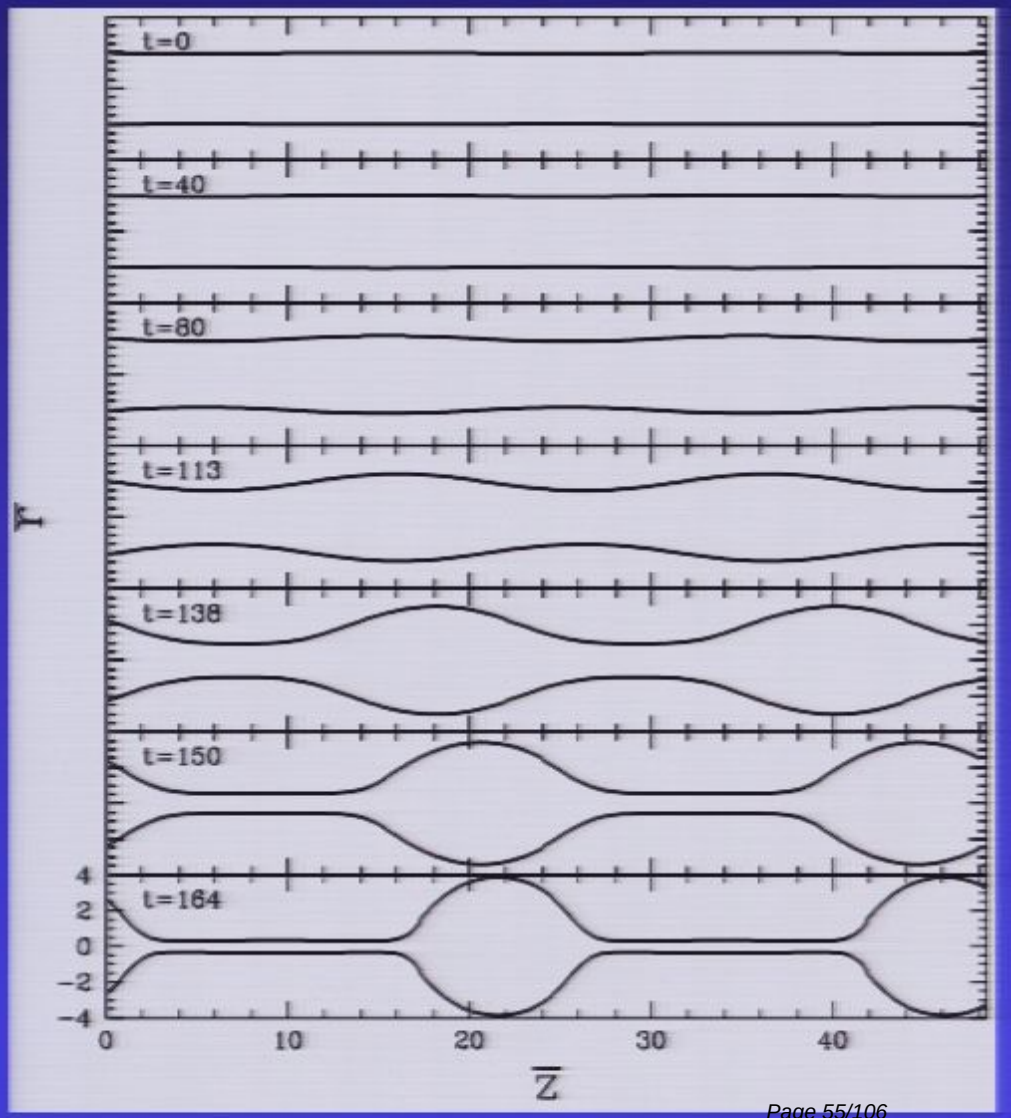
$$g_{\alpha\beta,z} = \frac{1}{x}\left[z\, g_{\alpha\beta,x} - \delta^z_\alpha g_{\beta x} + \delta^x_\alpha g_{\beta z} - \delta^z_\beta g_{\alpha x} + \delta^x_\beta g_{\alpha z}\right]$$

End-state of the instability?

- The first (numerical) non-linear study was carried out by Choptuik et al. *[PRD 68, 044001 (2003)]*
  - simulation "crashed" before a conclusive statement about the end-state could be made
  - results more consistent with the GL pinch-off conjecture
  - affine time grows exponentially fast relative to asymptotic time [Garfinkle et al., PRD 71 (2005), Marolf PRD 71 (2005)]

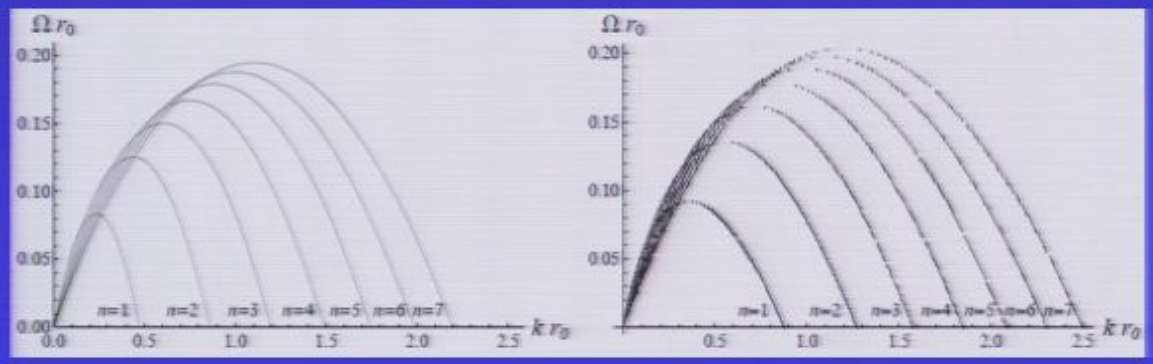
$$\ln \lambda \propto t / m \propto t / r_s$$

so not necessarily inconsistent with Horowitz & Maeda's theorem

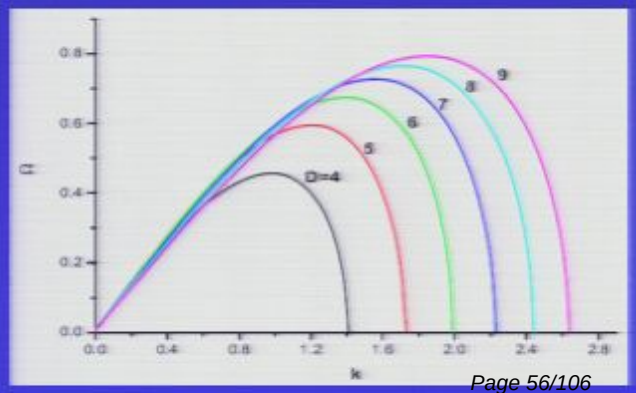


End-state of the instability?

- Further (anecdotal) evidence in favor of the pinch-off scenario has gathered in the form of various correspondences between equations governing viscous hydrodynamics and horizon dynamics
  - the membrane paradigm *[Thorne, Price, Macdonald, Eds. (1986)]* shows that the dynamics of a "stretched horizon" is governed by the Navier-Stokes equations for a relativistic fluid with very low shear-viscosity $\eta = 1/16\pi$.
  - more recently developed frameworks *[Bhattacharyya et al., JHEP 02 (2008), R. Emparan et al. JHEP 03 (2010)]* established similar relationships; *[J. Camps et al., arxiv:1003.3636 (2010)]* *(left figures)* used the "black folds" approach to re-derive the Gregory-Laflamme spectrum of modes to leading order
  - Cardoso and Dias *[PRL 96 (2006)]* *(right figure)* showed that the spectrum of unstable modes of a cylindrical flow of fluid with surface tension, subject to the Rayleigh-Plateau instability, was quantitatively similar to that of black strings



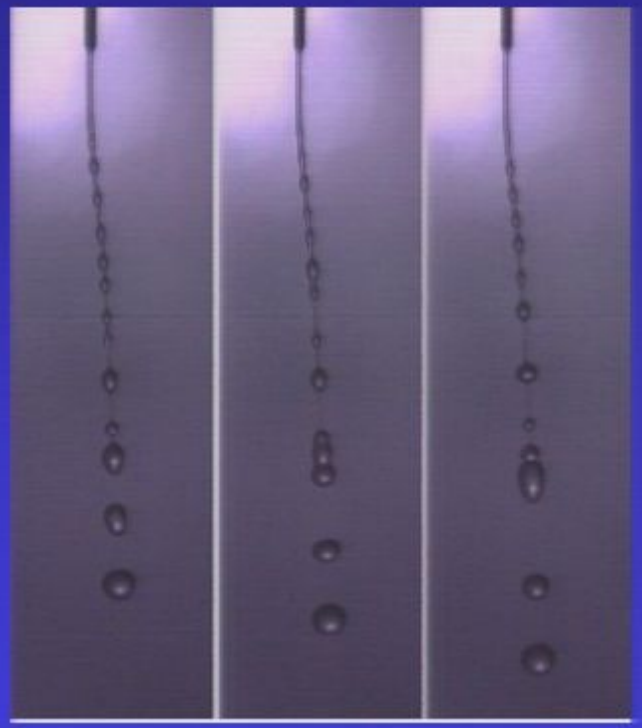
unstable sound waves in effective black string fluid (left) compared to GL modes (right)



Rayleigh-Plateau analogue

End-state of the instability?

- The reason why this could be considered evidence for pinch-off is that unstable fluid streams generically break up
  - For the Rayleigh-Plateau instability surface area is also the key explaining why one would expect a long-wavelength instability leading to pinch-off : above a critical length a sequence of spherical droplets has lower energy (due to surface tension) than a cylinder with the same volume/length
    - other analogues *[Cardoso and Gualtieri, CQG 23 (2006); Unruh and Wald, unpublished]* do not include surface tension, but the conclusion is the same
- The caveat with the fluid analogues is just that –they're analogues– and existent Einstein/horizon-hydrodynamic relationships is they're perturbative
  - thus, both end-state possibilities remain, and one needs to solve the full field equations to discover the answer



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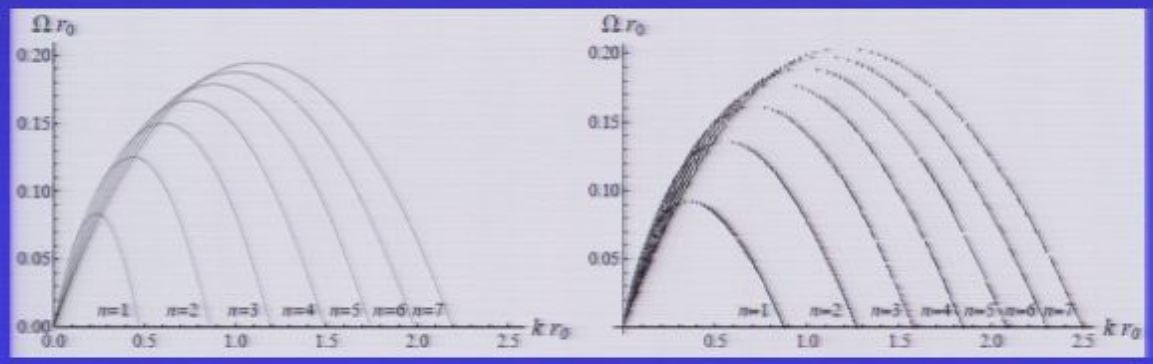

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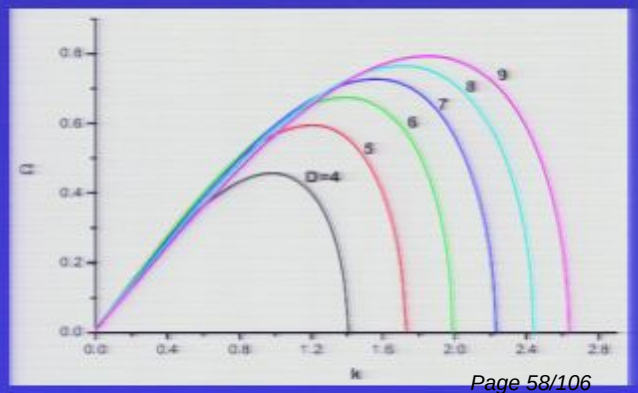
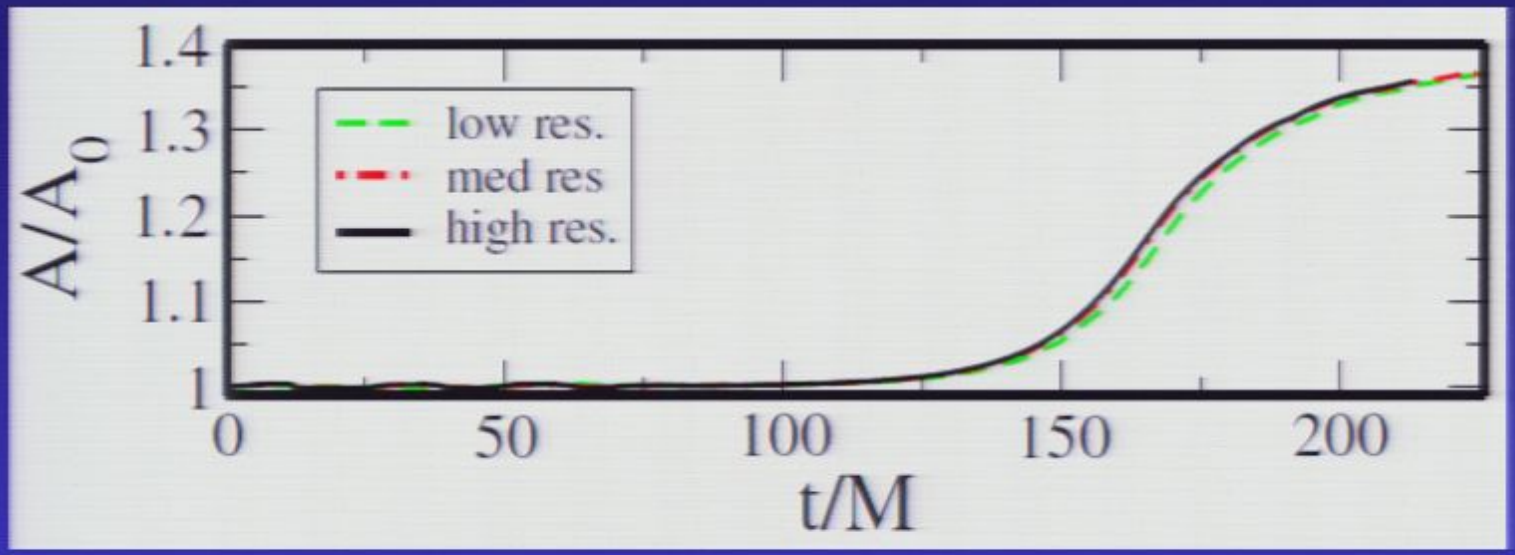


Rayleigh-Plateau analogue

Apparent Horizon Area



- Resolutions chosen via specification of the maximum estimated truncation error τ, from τ_0, $\tau_0/8$ to $\tau_0/64$ ("low" to "high")
- For this configuration, ignoring the (small amount of) energy from the initial perturbation, a sequence of spherical black holes (one per period) with the same energy as the initial string has an area

$$A_{BH}/A_{BS} = 1.374$$

The lowest resolution simulation, which has run the longest in physical time, has reached a value

$$A/A_0\,(t = t_{end}) \approx 1.369 \pm 0.005$$

- This is consistent with the argument that the dynamics of the instability is such as

Properties of satellites and string-segments

- Therefore, the spheres-connected-by-string-segments interpretation seems reasonable. With that interpretation, and that evolution proceeds through a sequence of unstable epochs, we extract the following properties from the horizon:

Gen.	t_i/M	n_s	$R_{s,i}/M$	$R_{AH,f}/M$	$L_{s,i}/R_{s,i}$
1	118.1 ± 0.5	1	2.00	$4.09 \pm 0.5\%$	10.0
2	203.1 ± 0.5	1	$0.148 \pm 1\%$	$0.63 \pm 2\%$	$105 \pm 1\%$
3	223 ± 2	> 1	$0.05 \pm 20\%$	$0.1 - 0.2$	$\approx 10^2$
4	≈ 227	$> 1(?)$	≈ 0.02	?	$\approx 10^2$

Gen: generation number

t_i: time of initial satellite formation (defined to be time when the areal radius has grown to 1.5 times that of the surrounding string-segment)

n_s: number of satellites that form

$R_{s,i}$: radius of local string segment

$R_{AH,f}$: radius of satellites by the time the simulation was stopped

L_{si}/R_{si}: Ratio of length to radius of local string-segment (recall GL critical ratio ~ 7.2)

Properties of satellites and string-segments

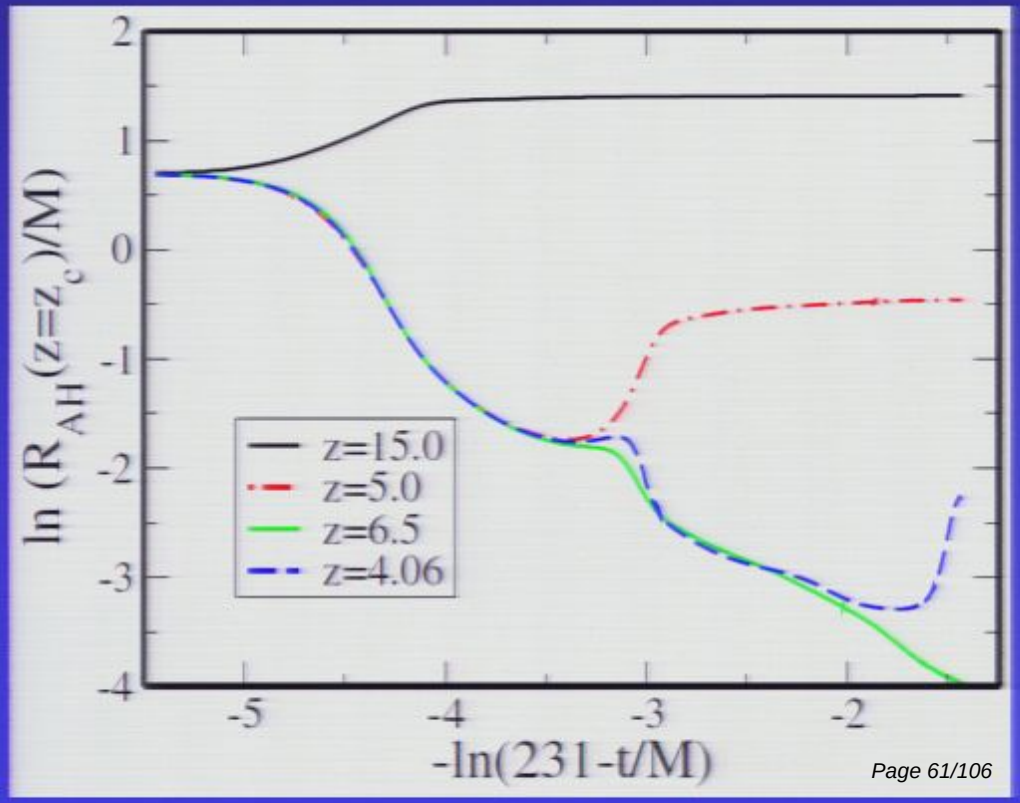
- The dynamics of the apparent horizon also suggests that the instability unfolds in a self-similar manner; if so, transforming to logarithmic coordinates in space and time should reveal this more clearly

- The following shows $R_{AH}(t,w=const.)$ at points (roughly coinciding) with the eventual maxima of satellites, and one representative point that is still string-like near the end of the simulation

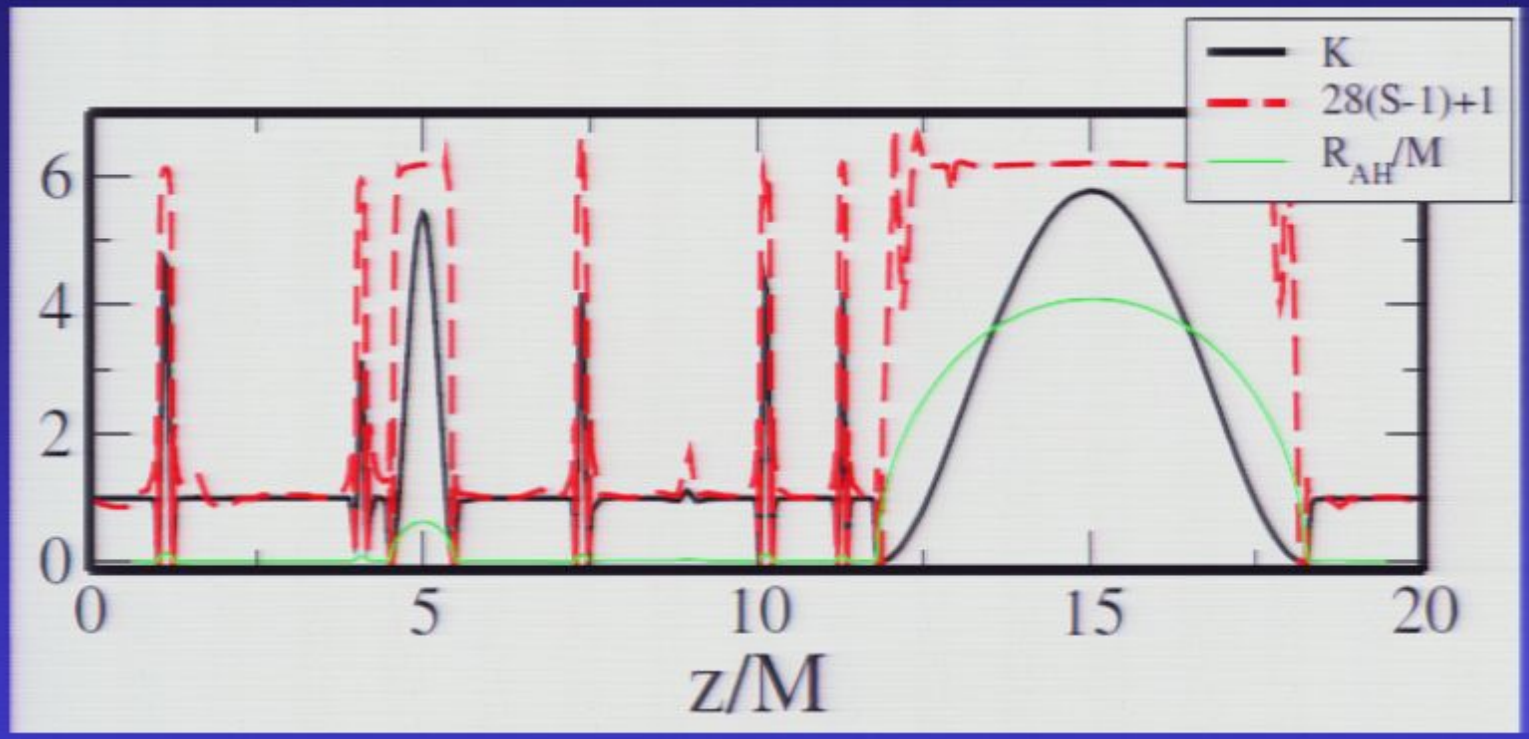
- Guess at "pinch-off time" by assuming the time scale for each later generation is a constant fraction X of the preceding one, with the exception of the first generation, whose time scale is controlled by the initial data:

$$\Delta T \sim T_0 + \sum_{i=0}^{\infty} T_1 X^i = T_0 + \frac{T_1}{1-X}$$

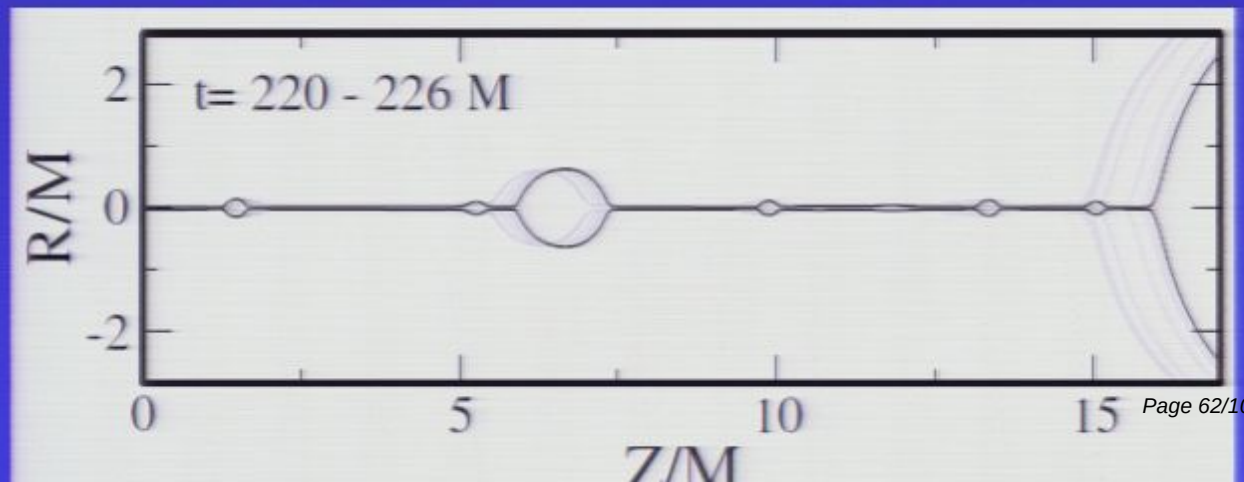
from the data in the table, we get $\Delta T \sim 231M$



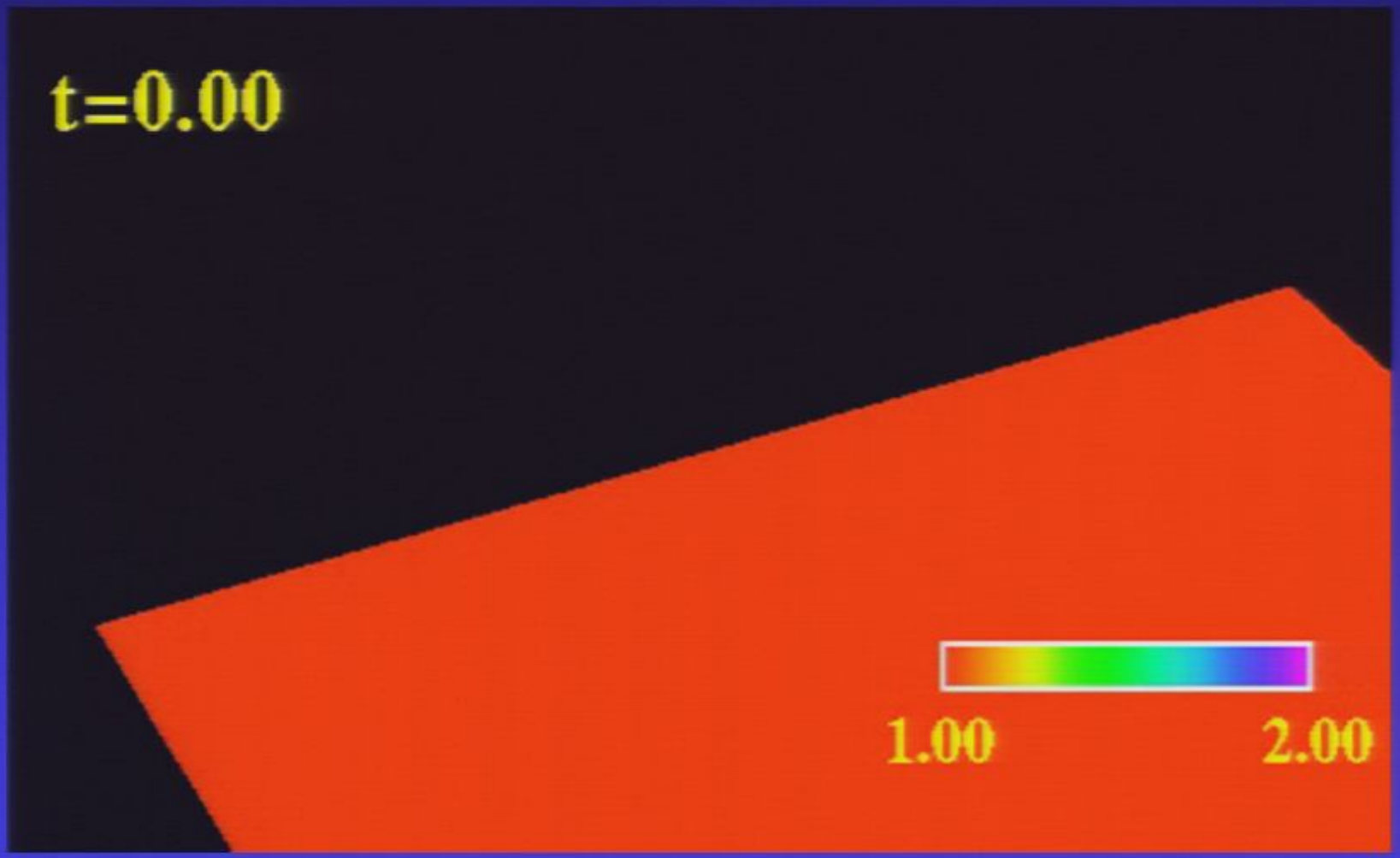
Apparent Horizon Dynamics



Invariants above evaluated on the apparent horizon at the last time step of the (medium resolution) simulation depicted to right



Metric Evolution



w

x

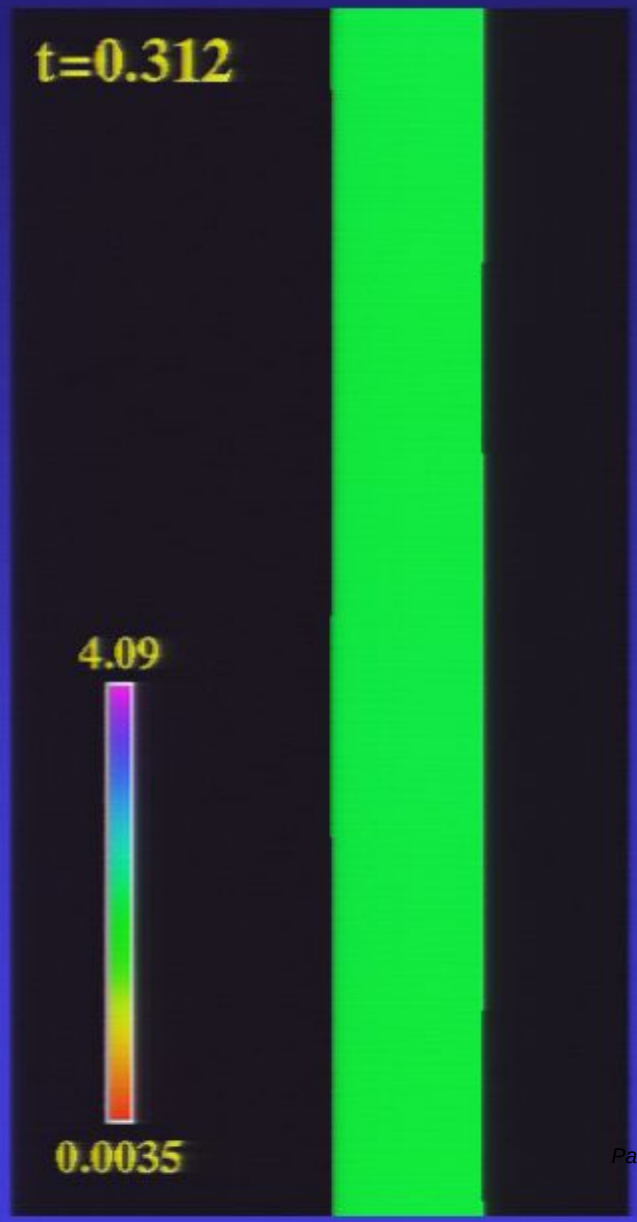
g_{ww}

AH Embedding diagram: same data, zooming in

- map the geometric 1D shape of each $t=x=y=constant$ slice of the apparent horizon to a flat (R,Z) Euclidean space; i.e. in parametric form

$$(R,Z)=\left(R(\xi),Z(\xi)\right)$$

- $R(\xi)$ is the areal radius of that point on the horizon, and $Z(\xi)$ is defined so that the proper length of the curve in the flat space is identical to that of the corresponding curve in the physical geometry
- the movie shows this curve spun around $R=0$ to form a surface for visual aid
- color is mapped to R
- note that time is "slowing down"



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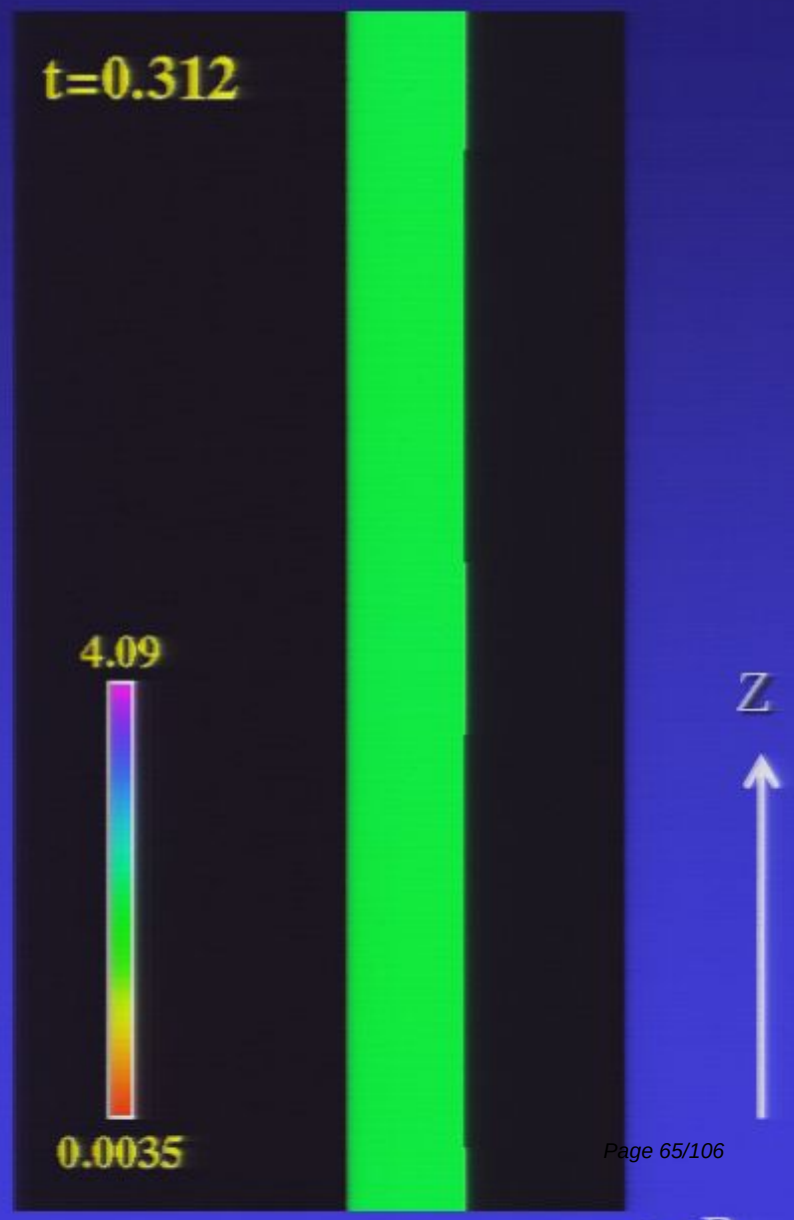

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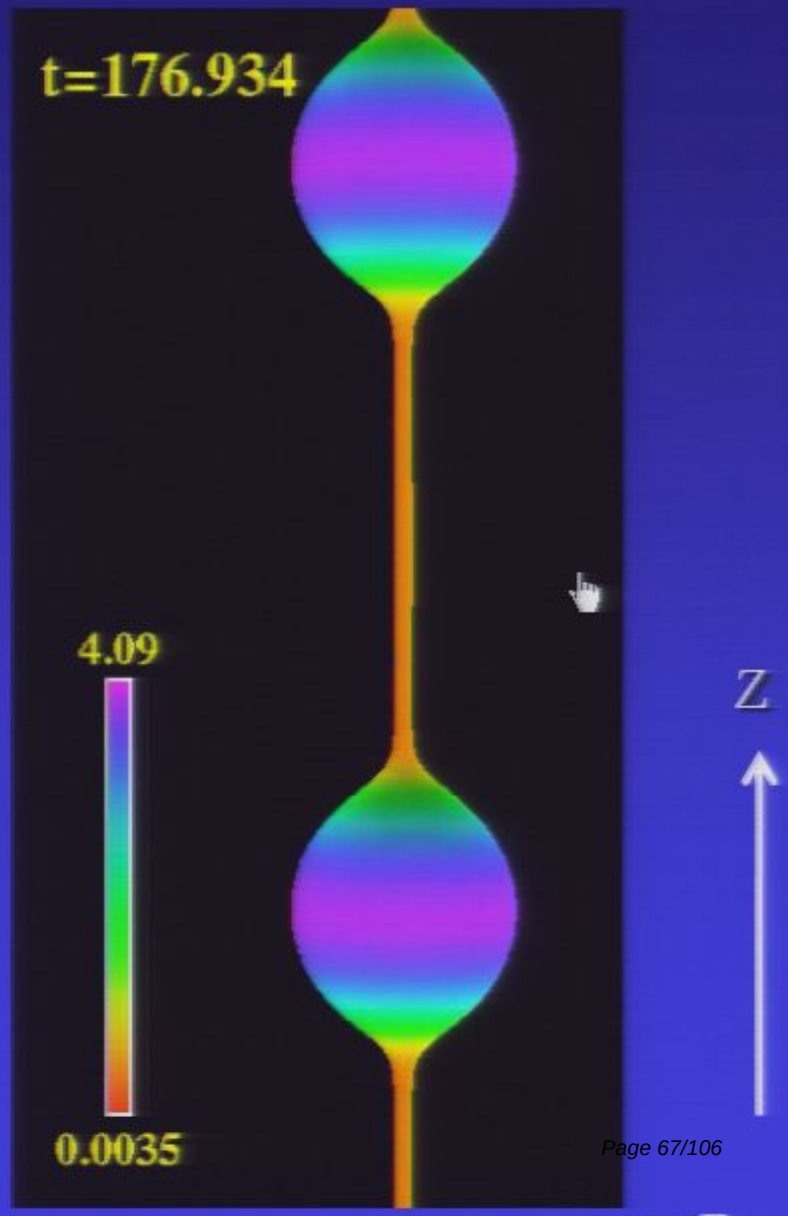

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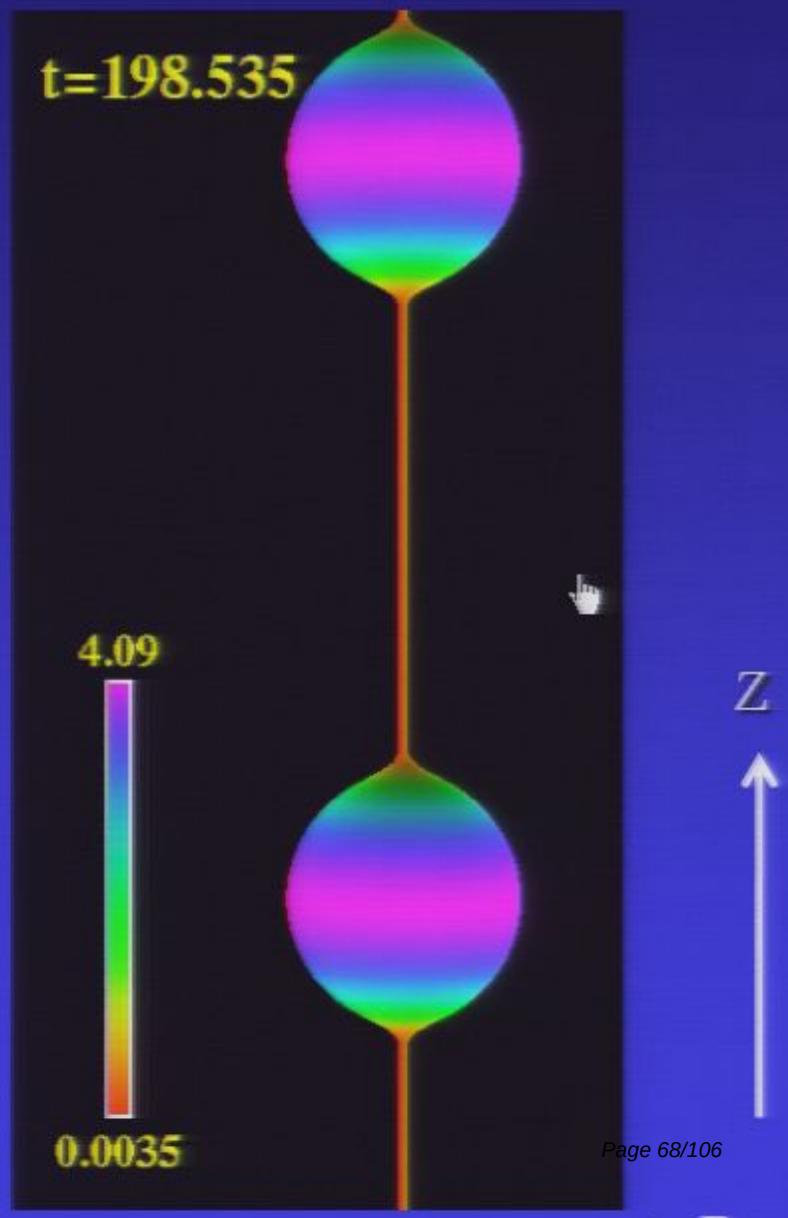

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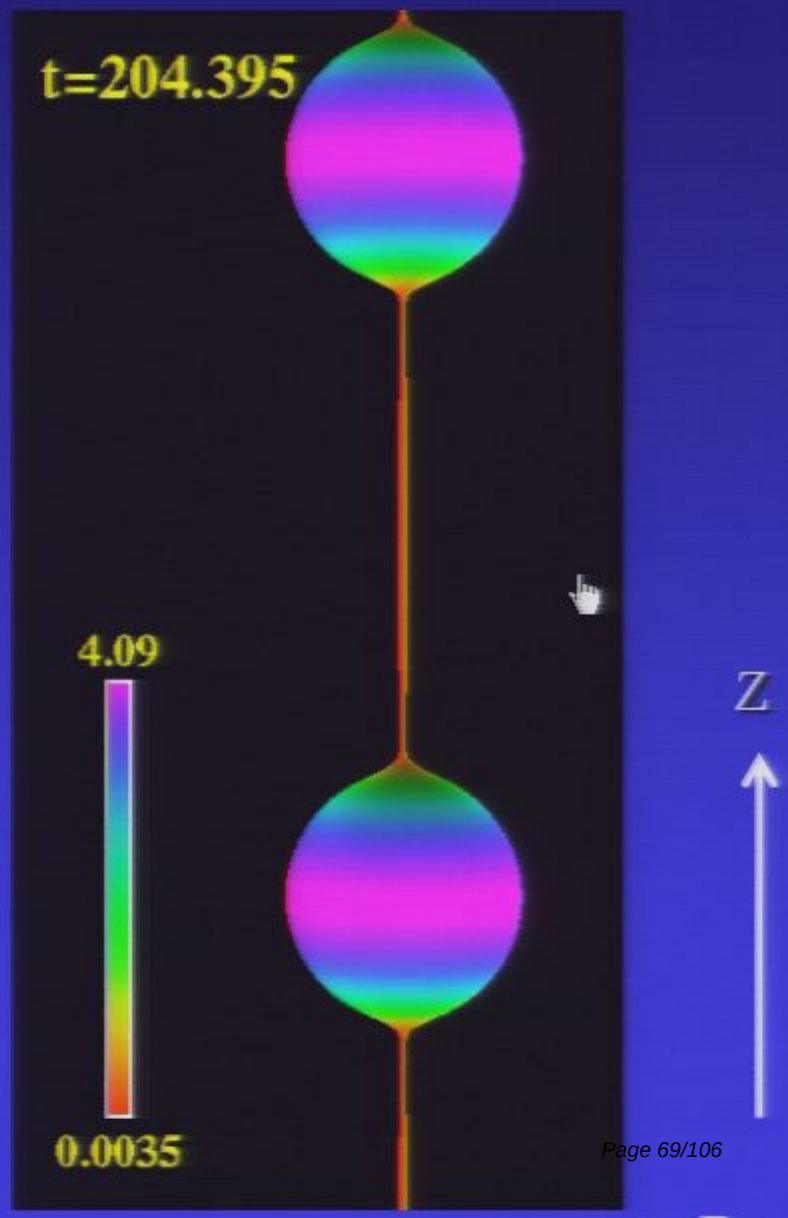

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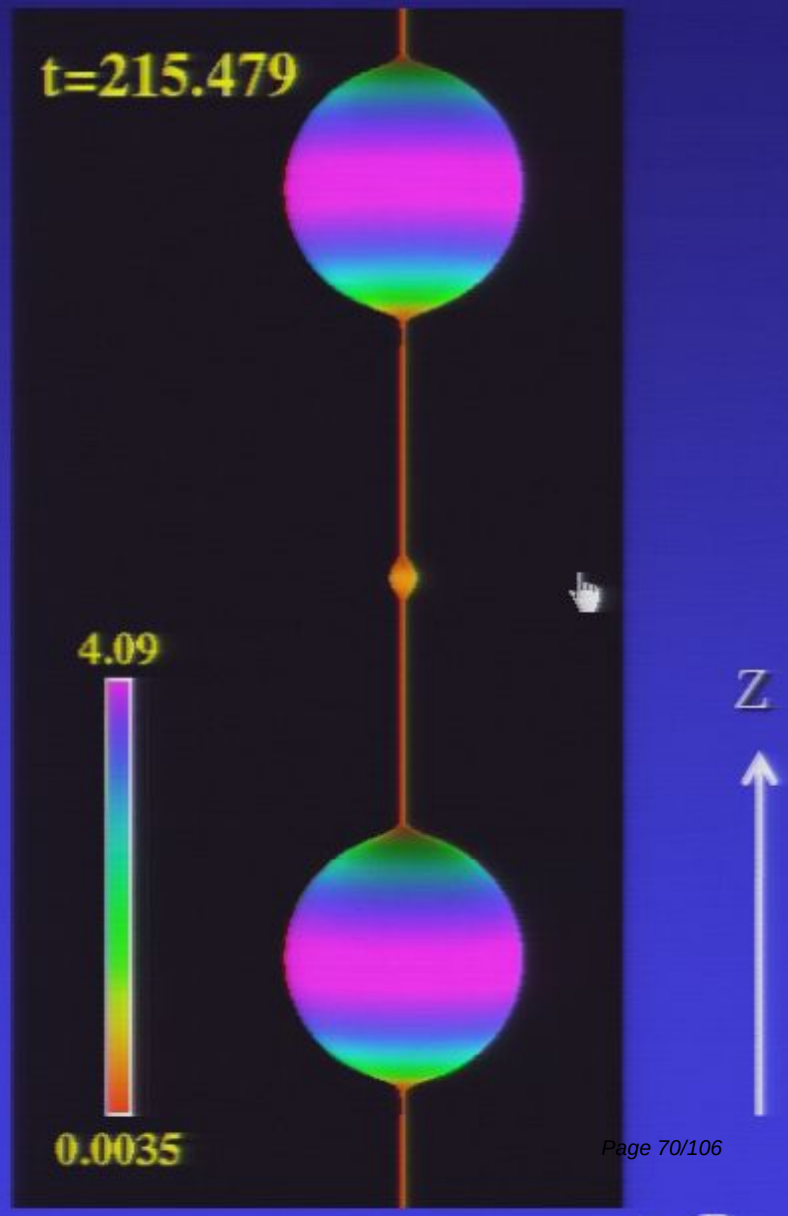

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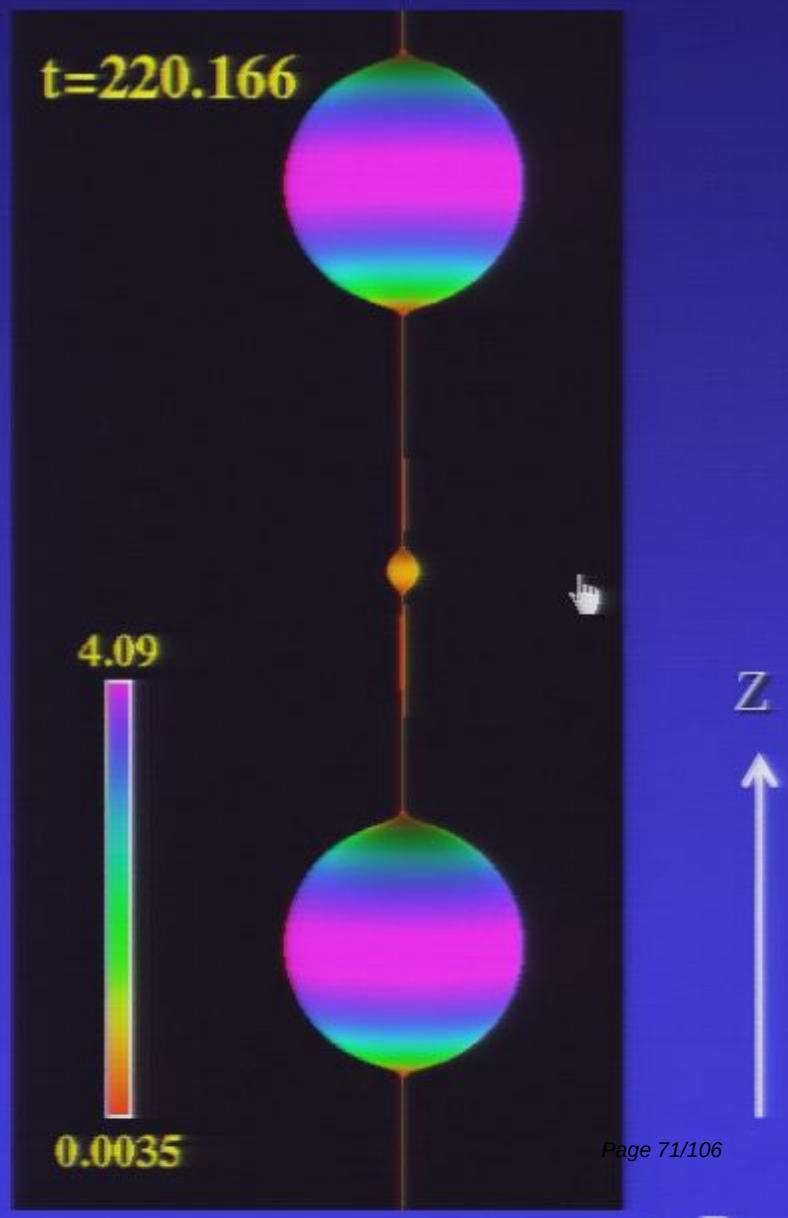

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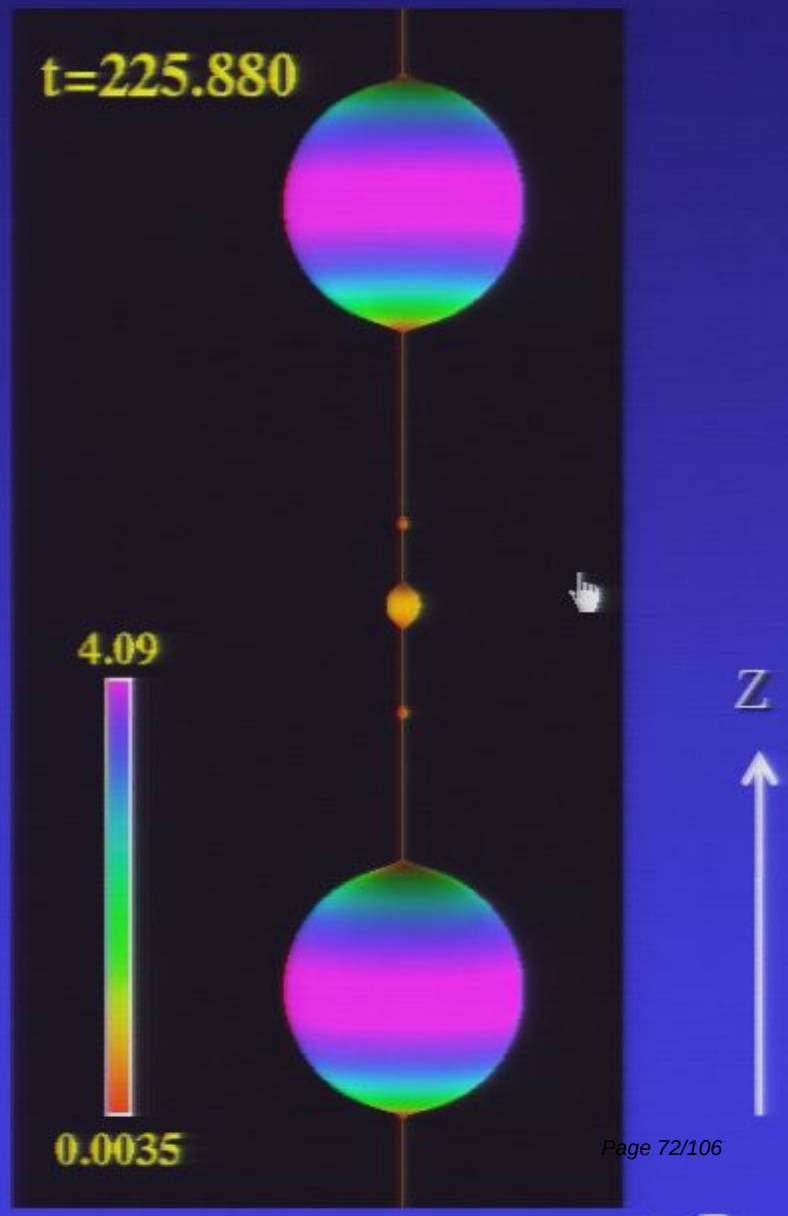

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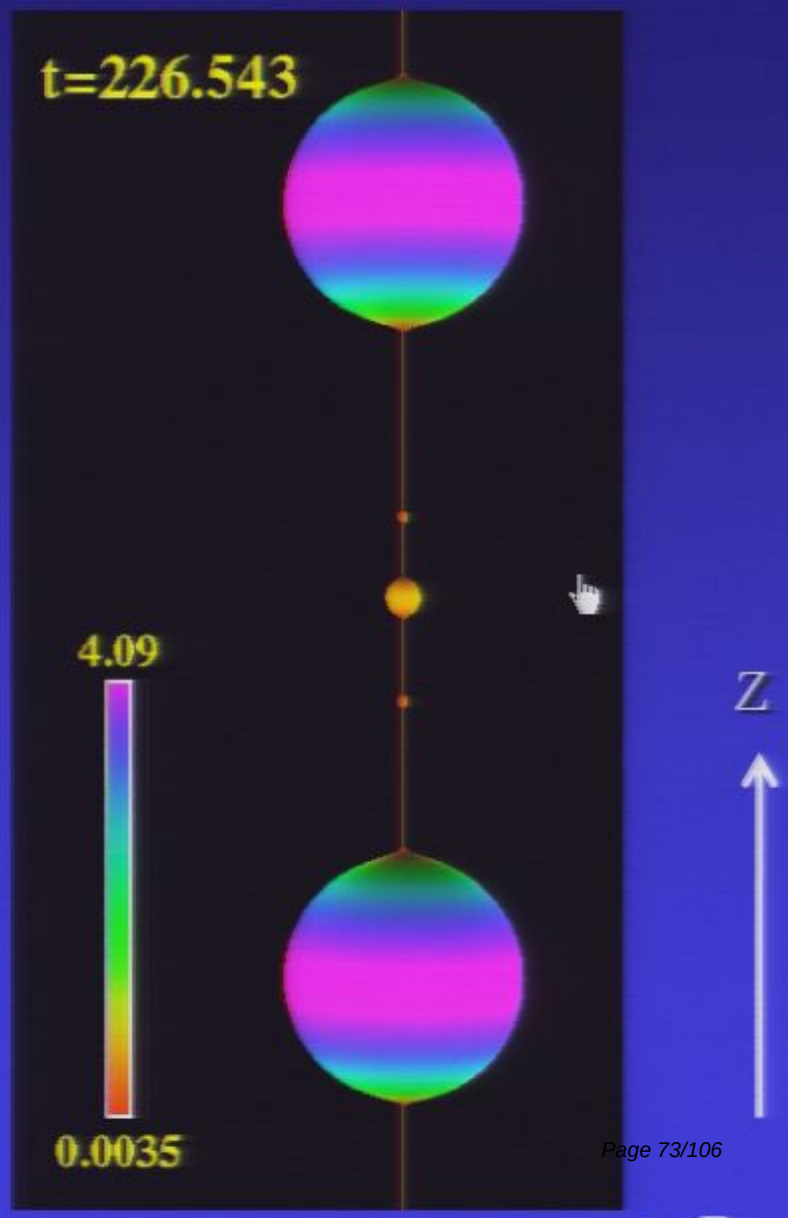

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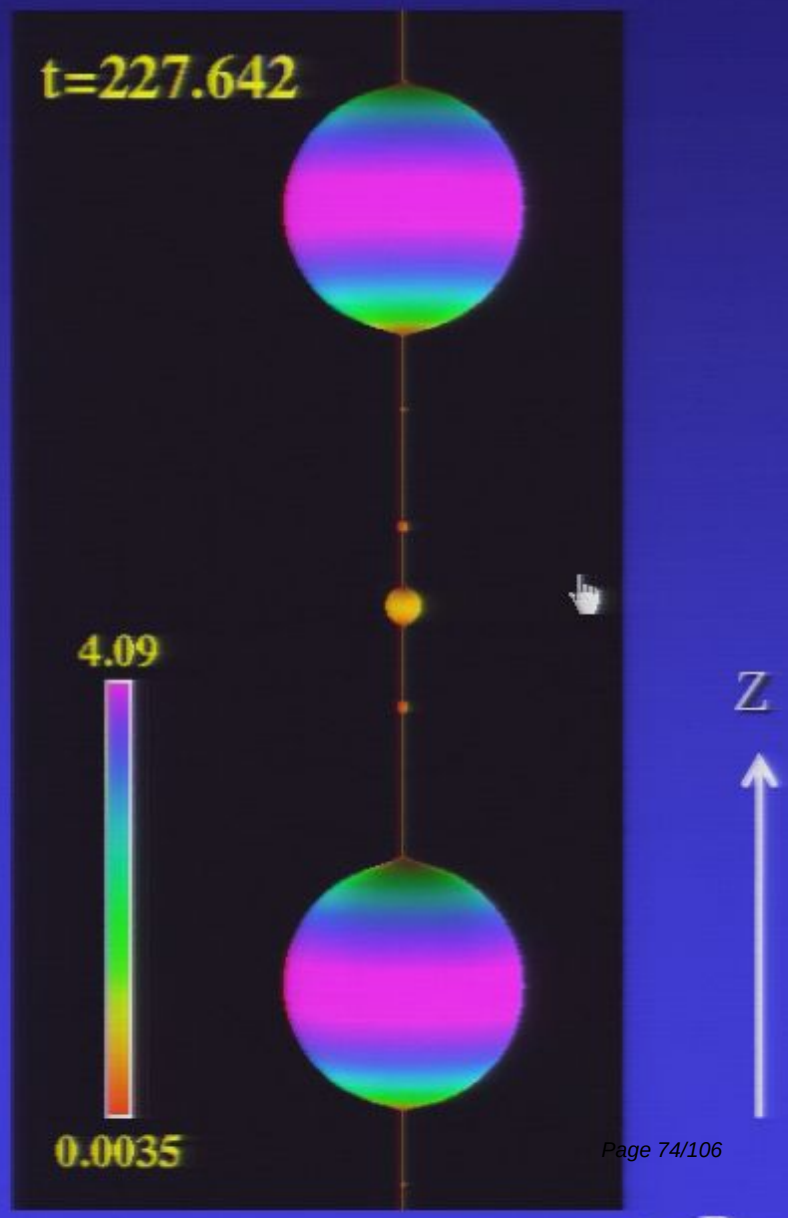

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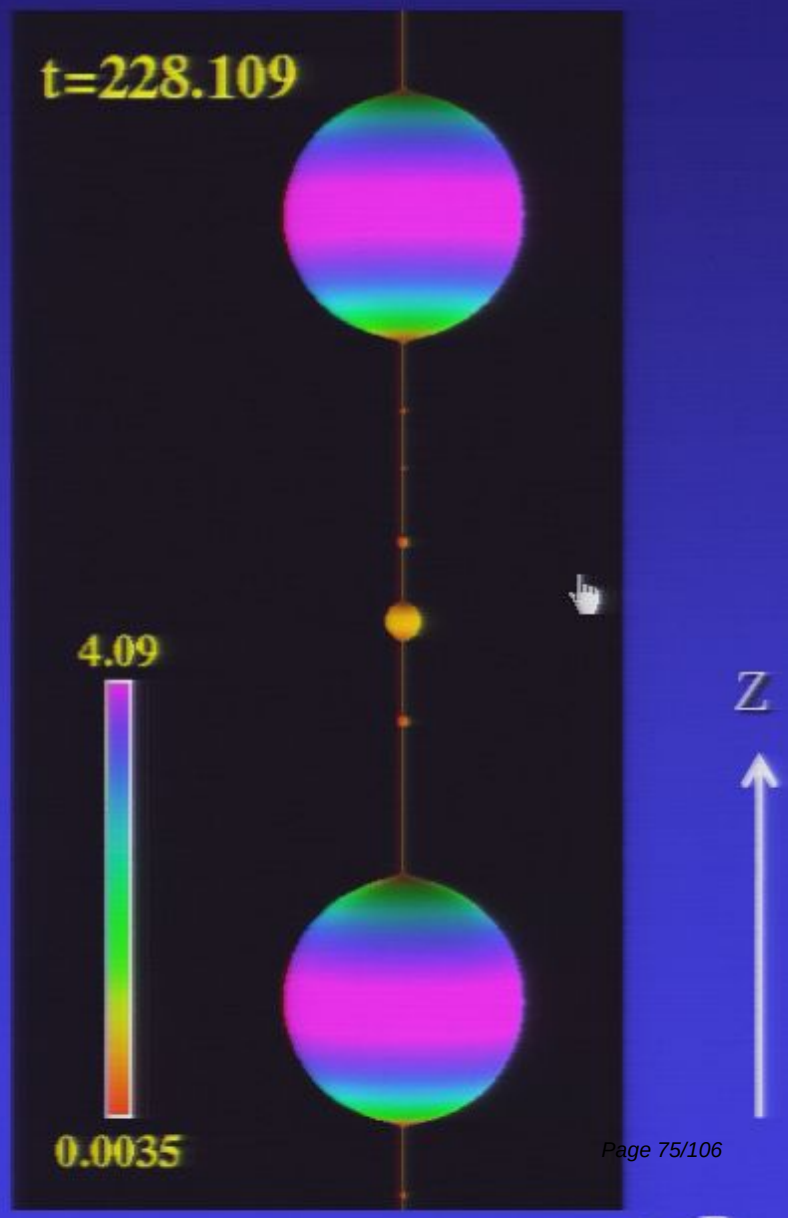

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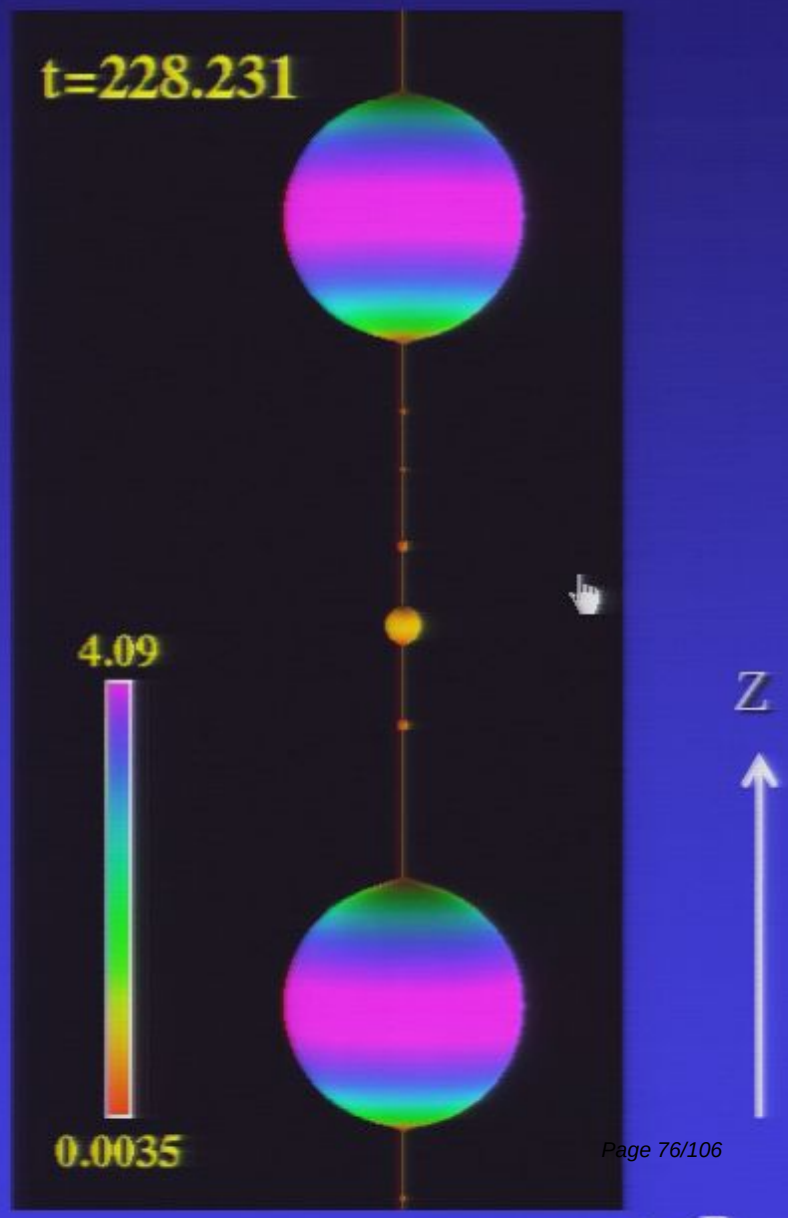

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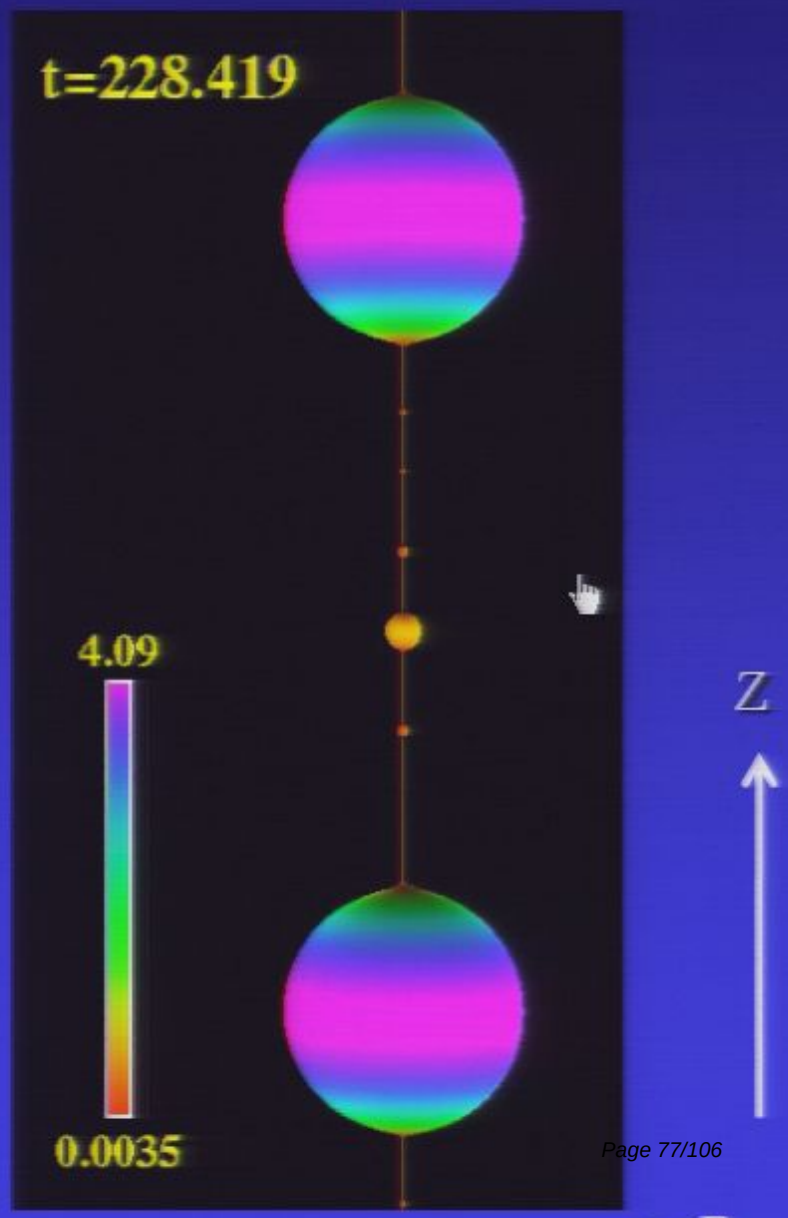

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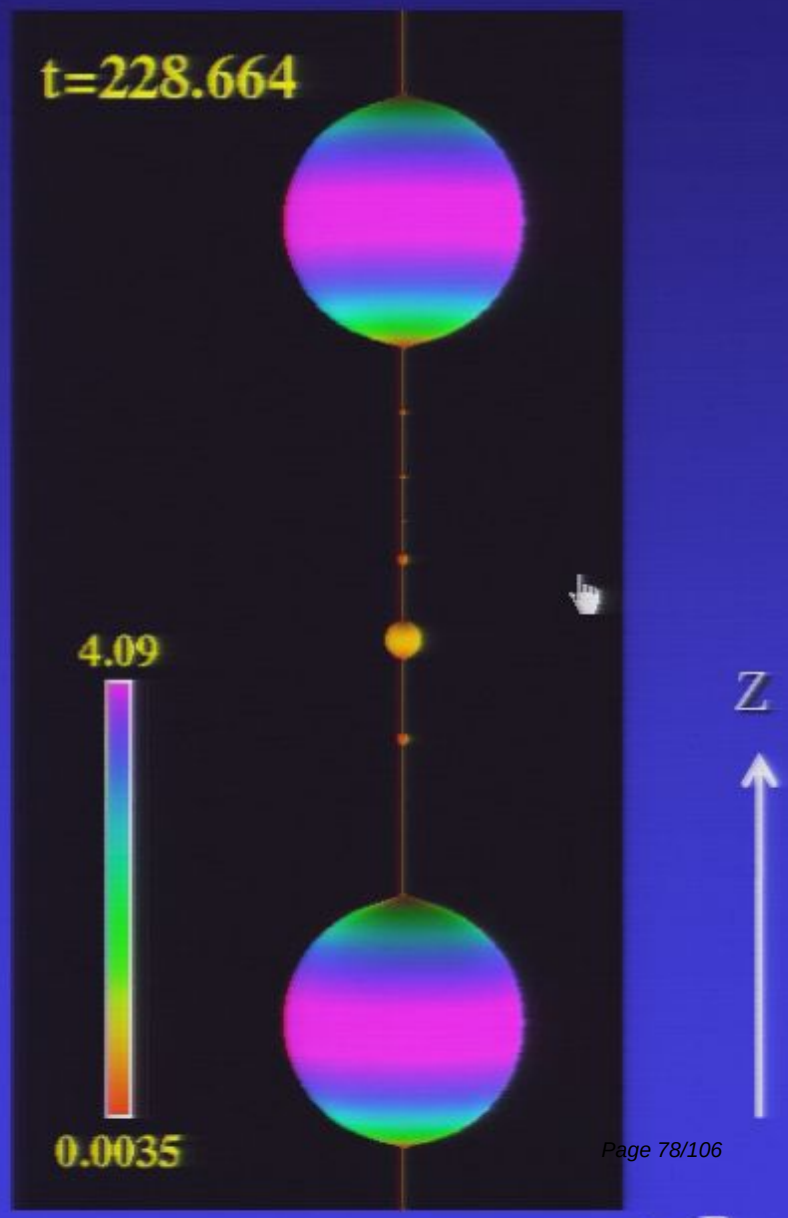

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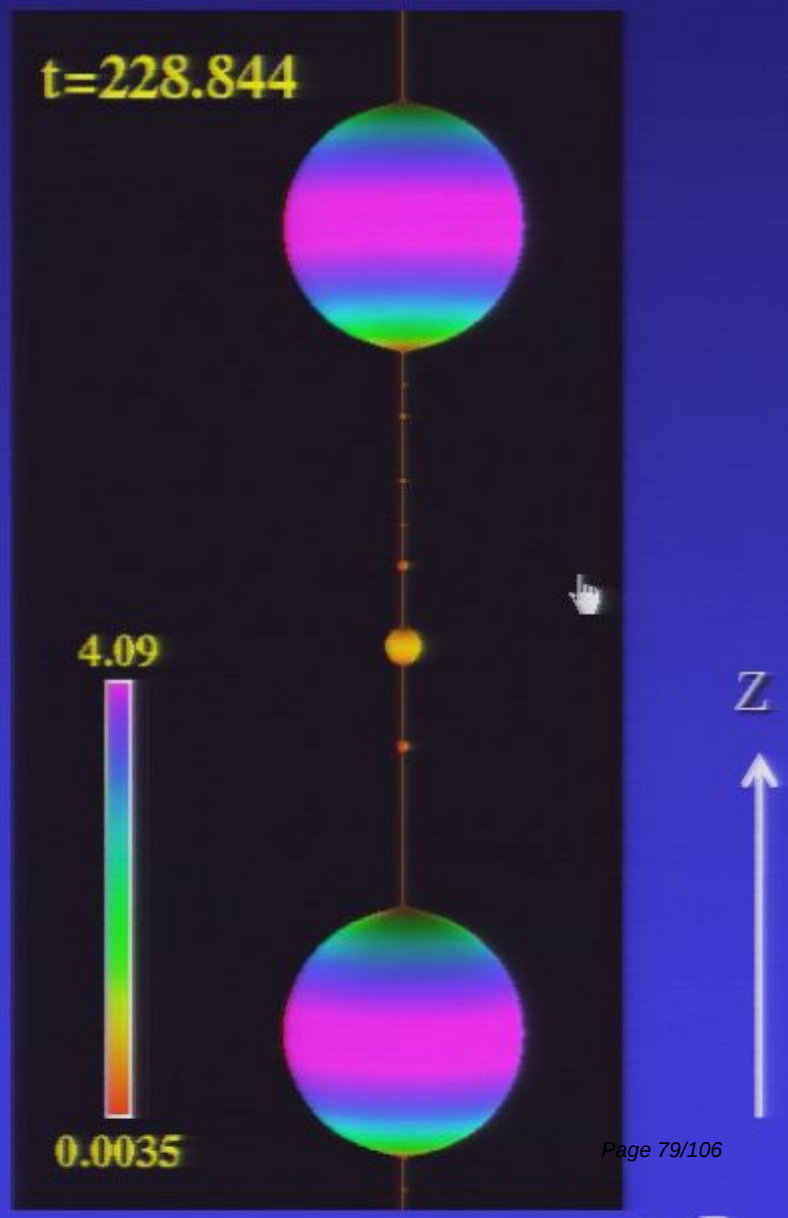

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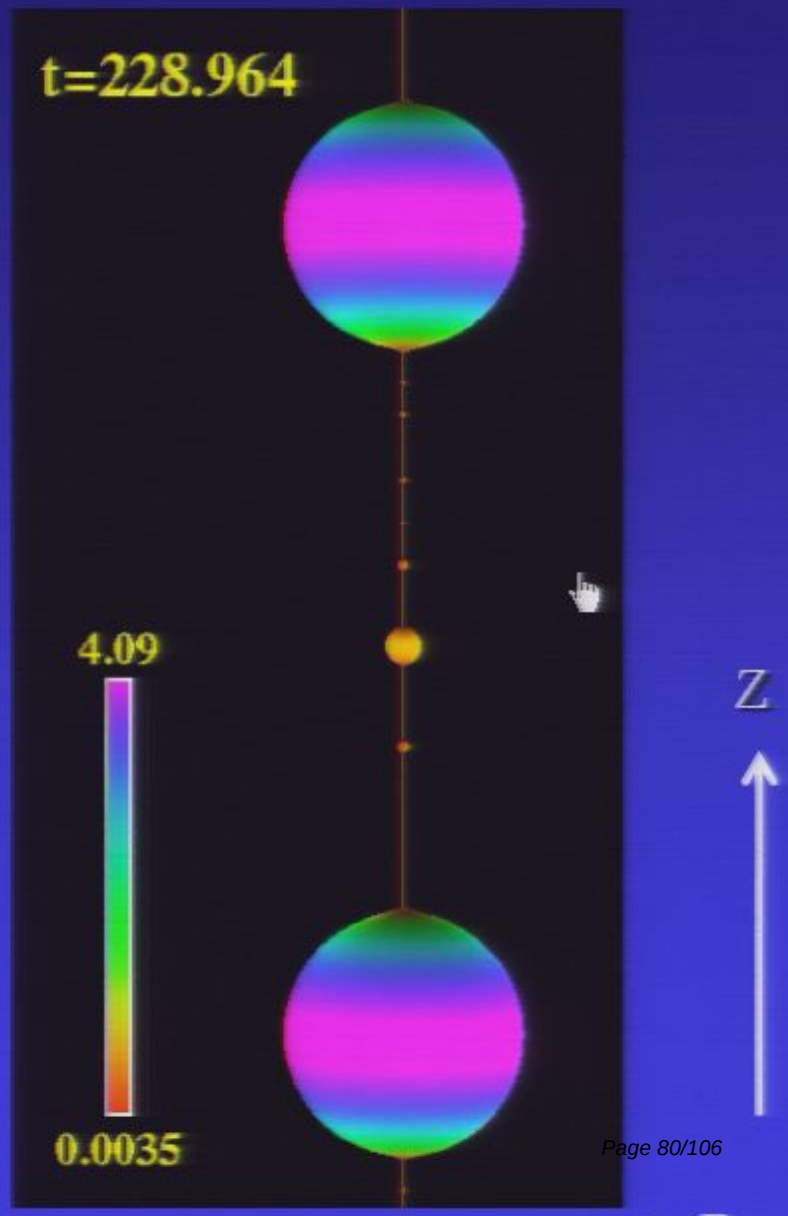

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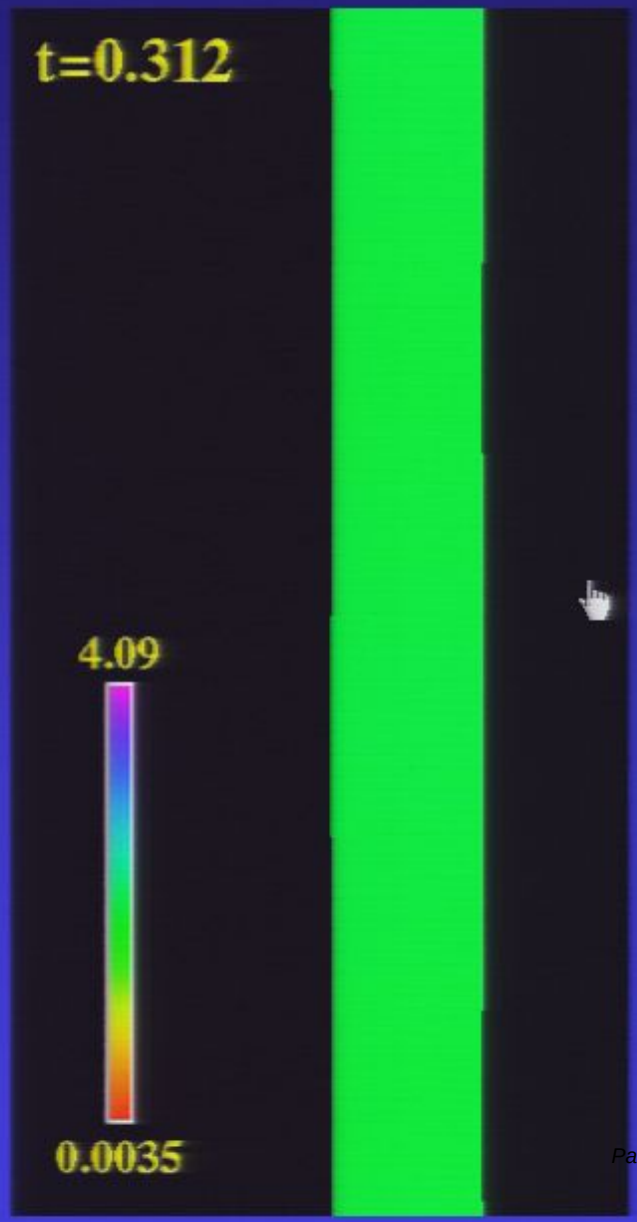

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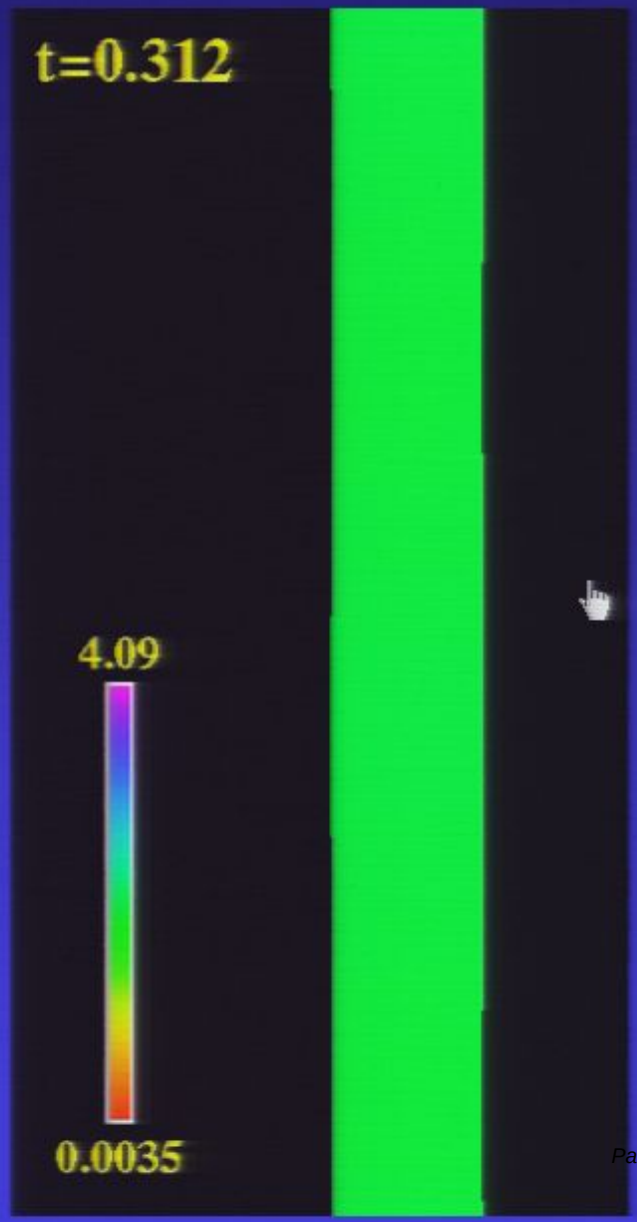

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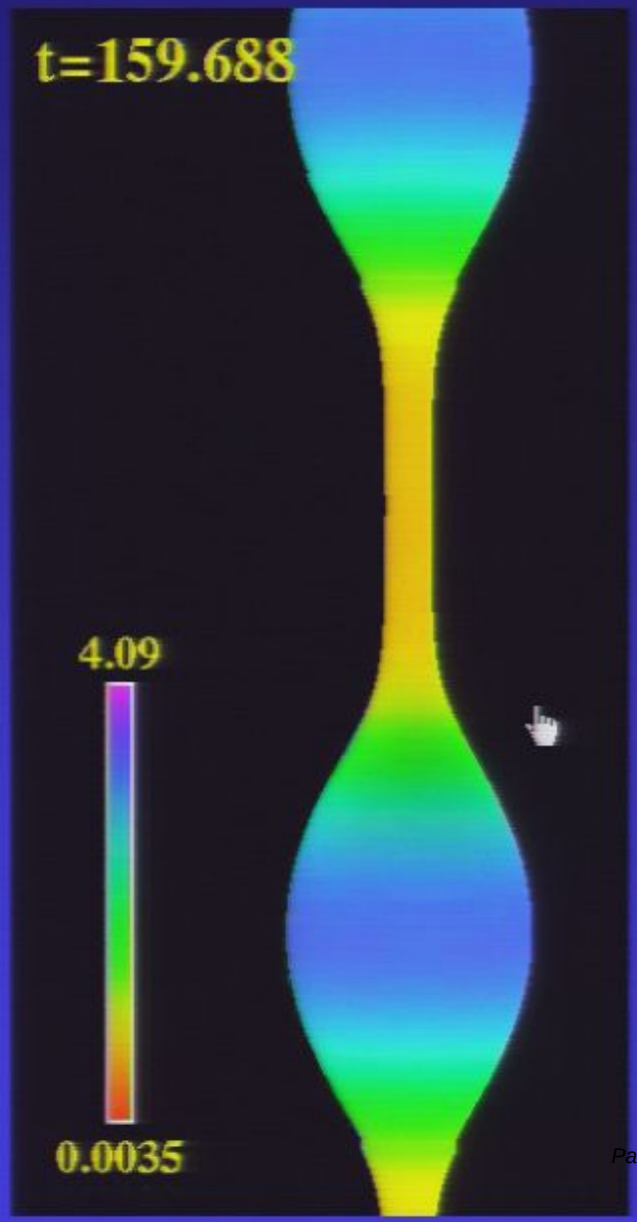

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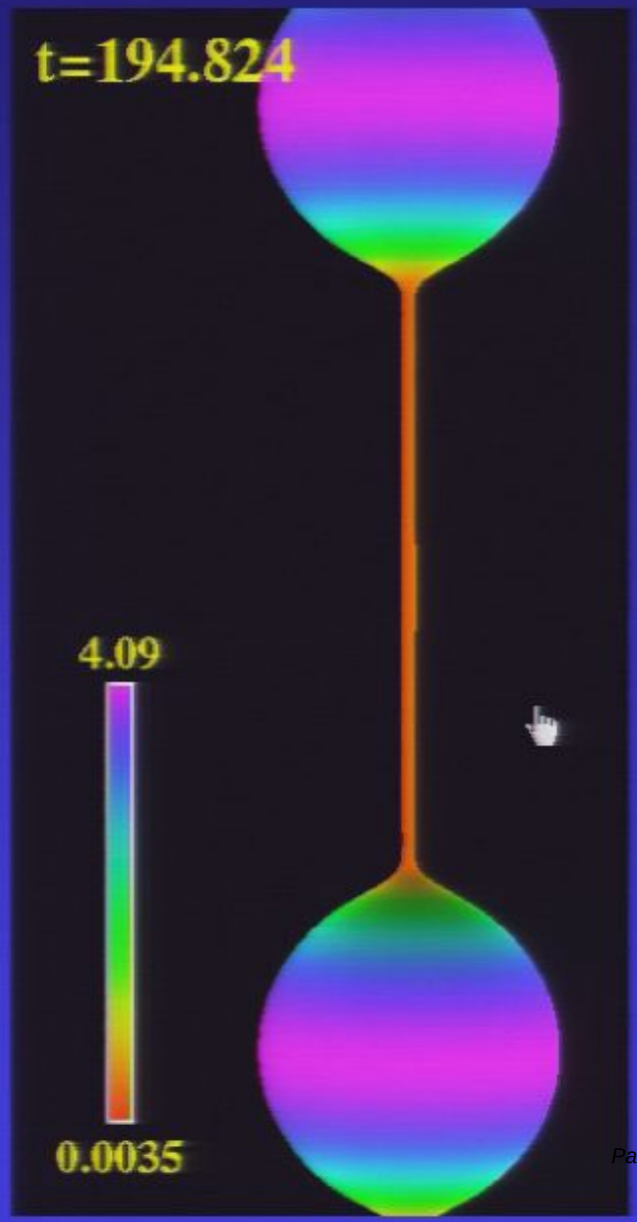

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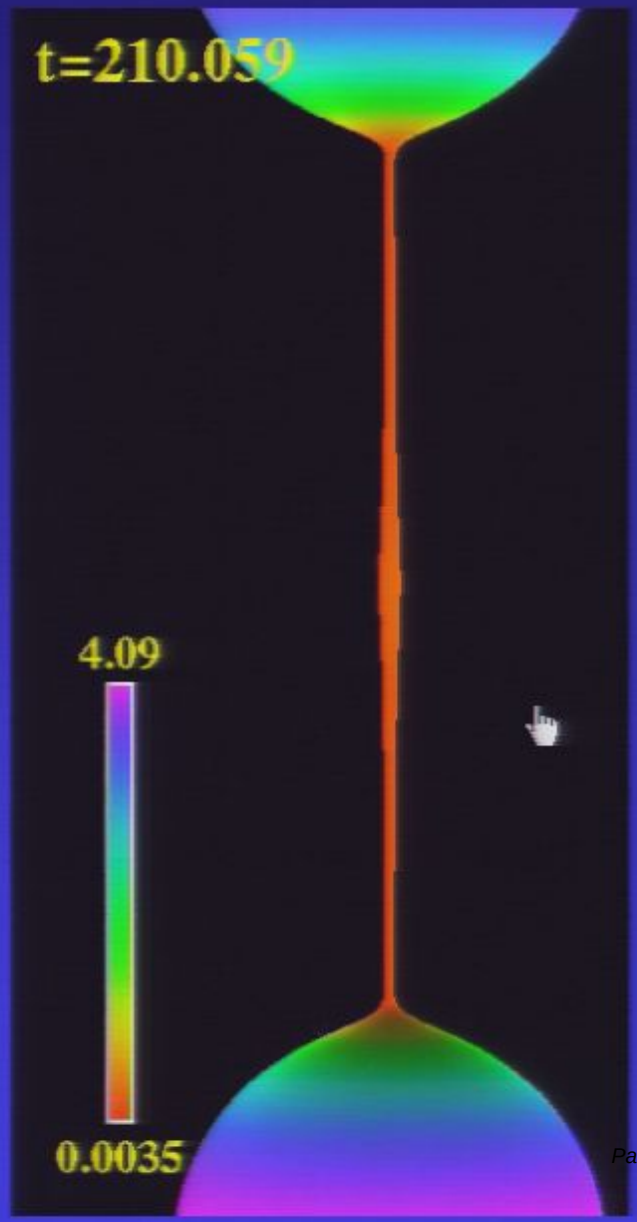

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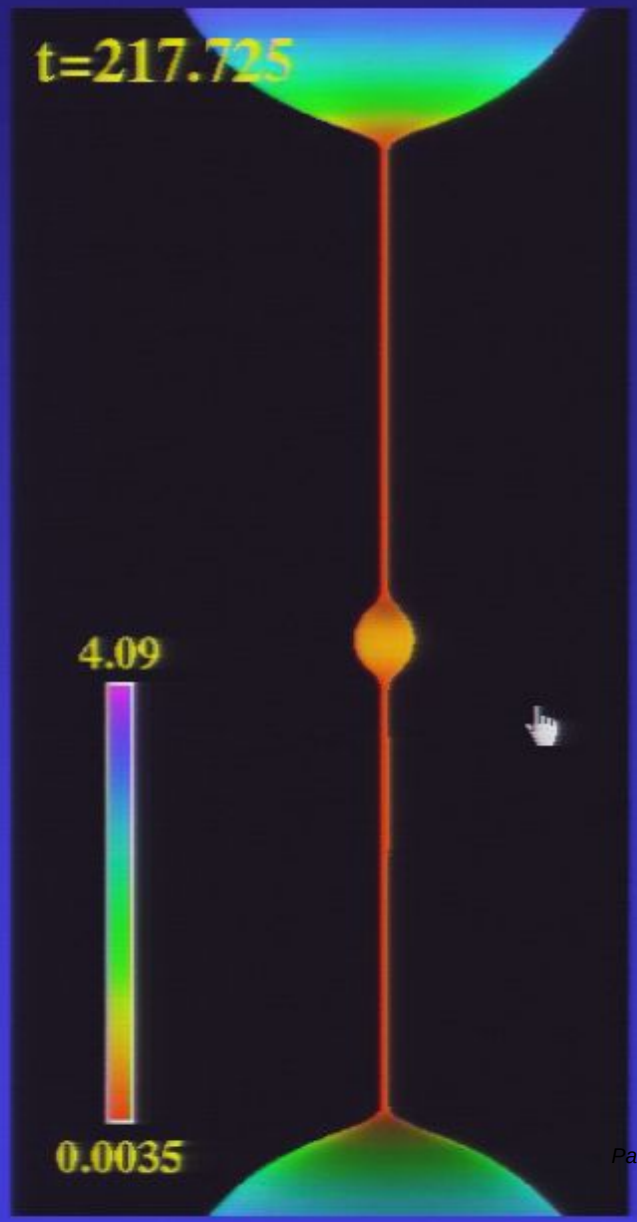

AH Embedding diagram: same data, zooming in

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$$(R,Z)=\left(R(\xi),Z(\xi)\right)$$

- $R(\xi)$ is the areal radius of that point on the horizon, and $Z(\xi)$ is defined so that the proper length of the curve in the flat space is identical to that of the corresponding curve in the physical geometry
- the movie shows this curve spun around $R=0$ to form a surface for visual aid
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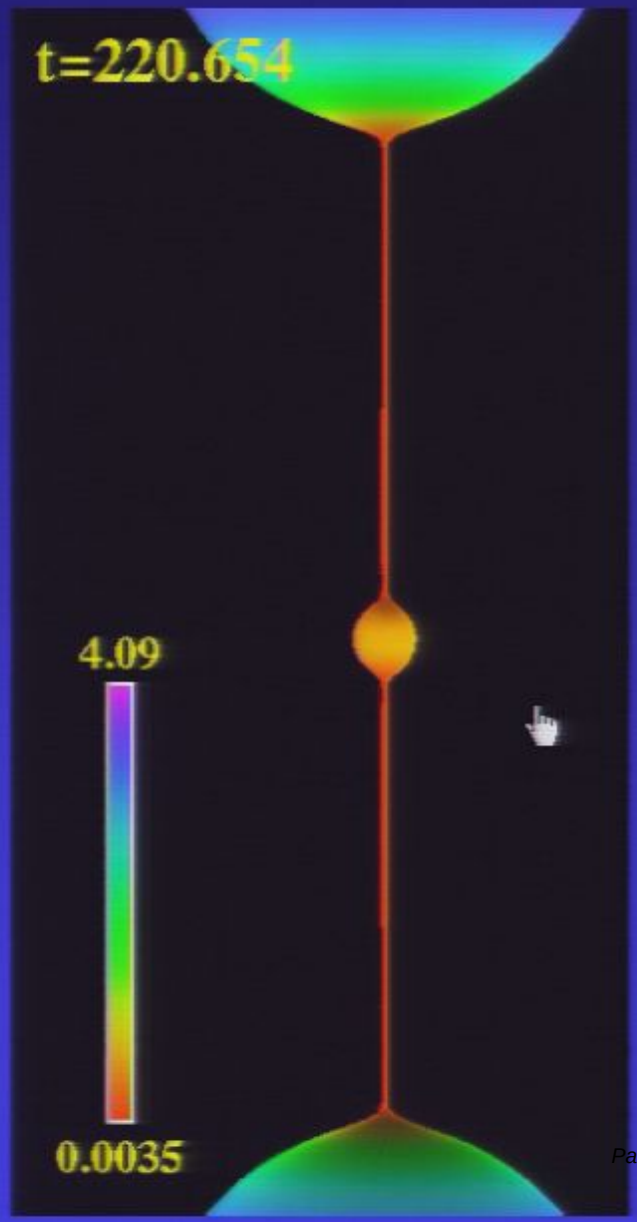

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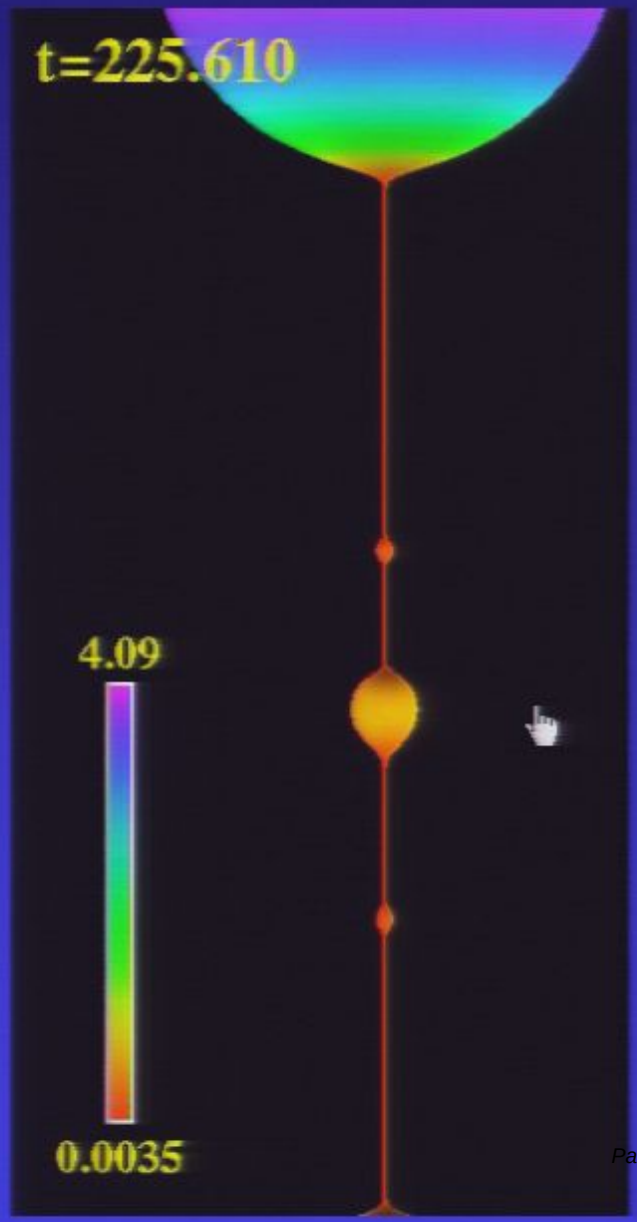

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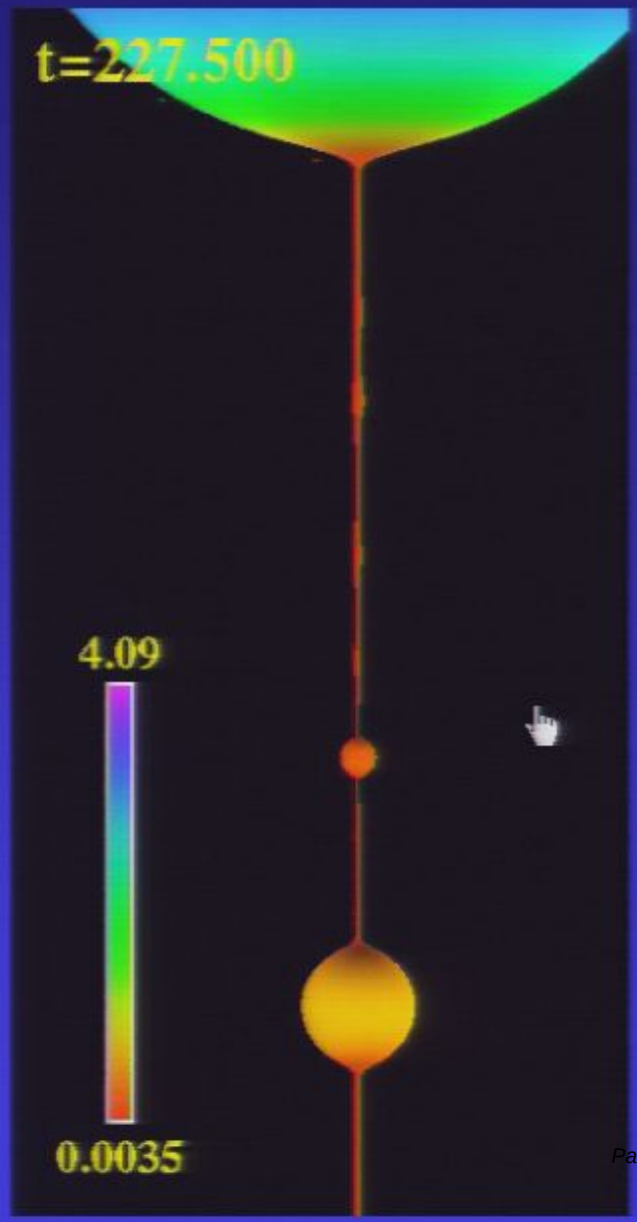

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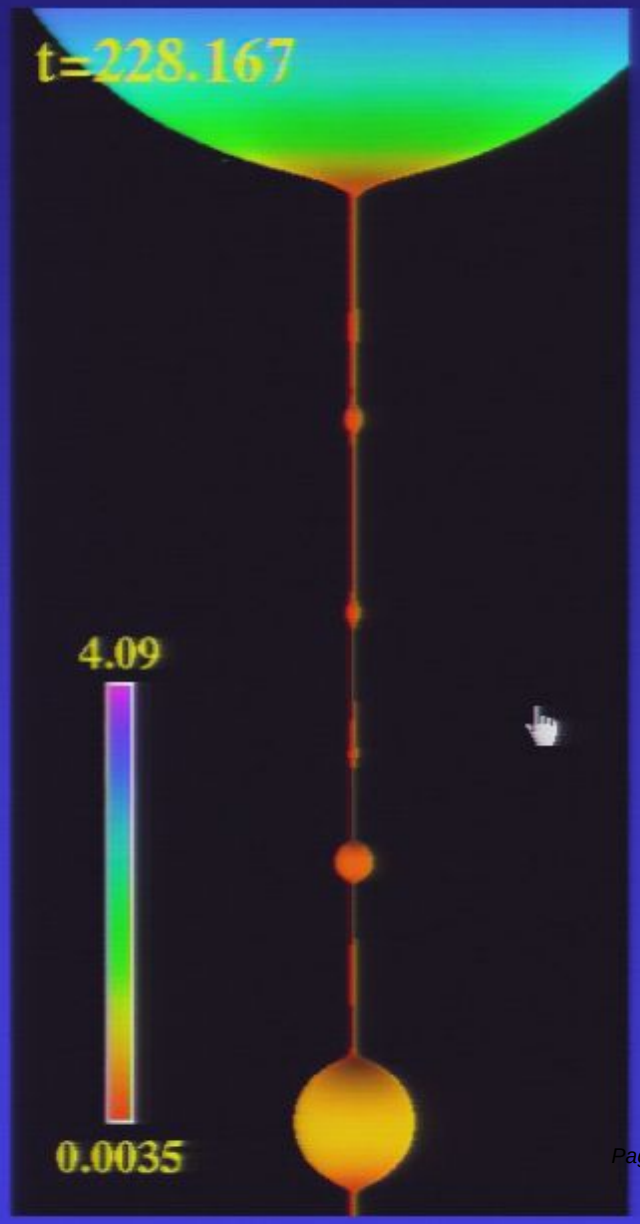

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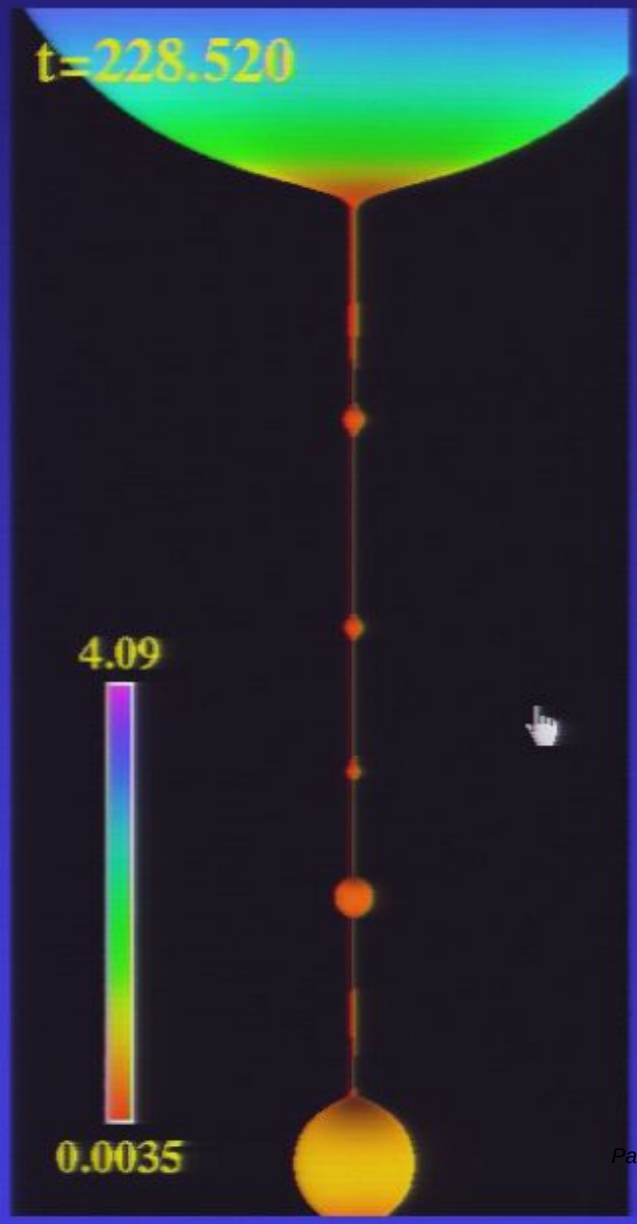

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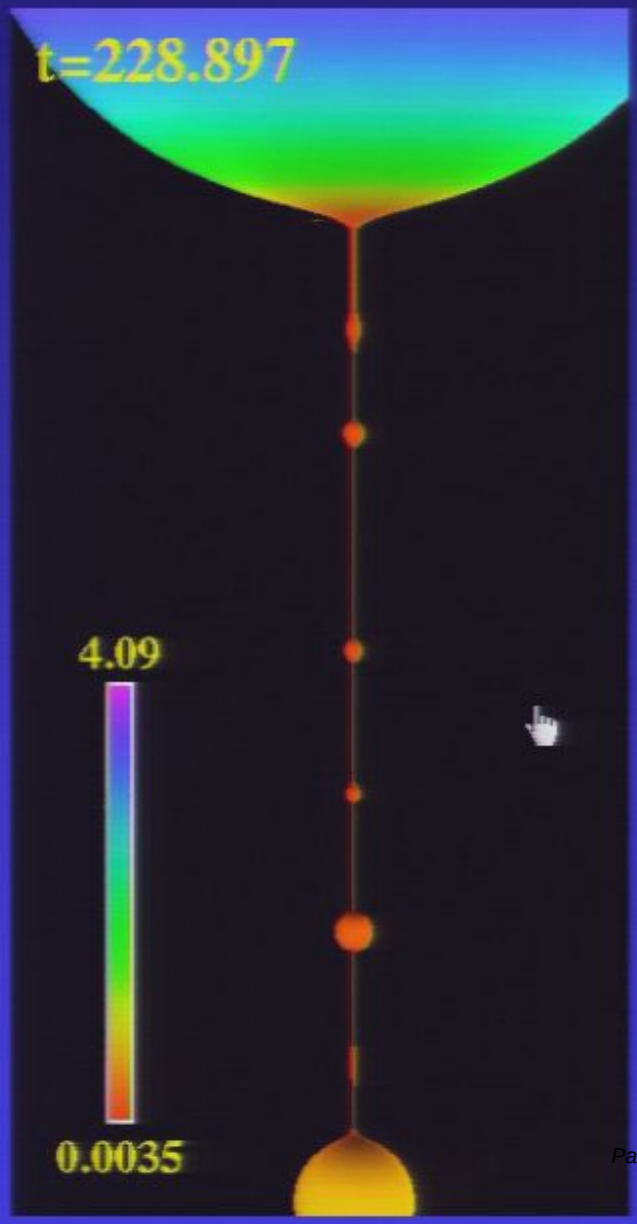

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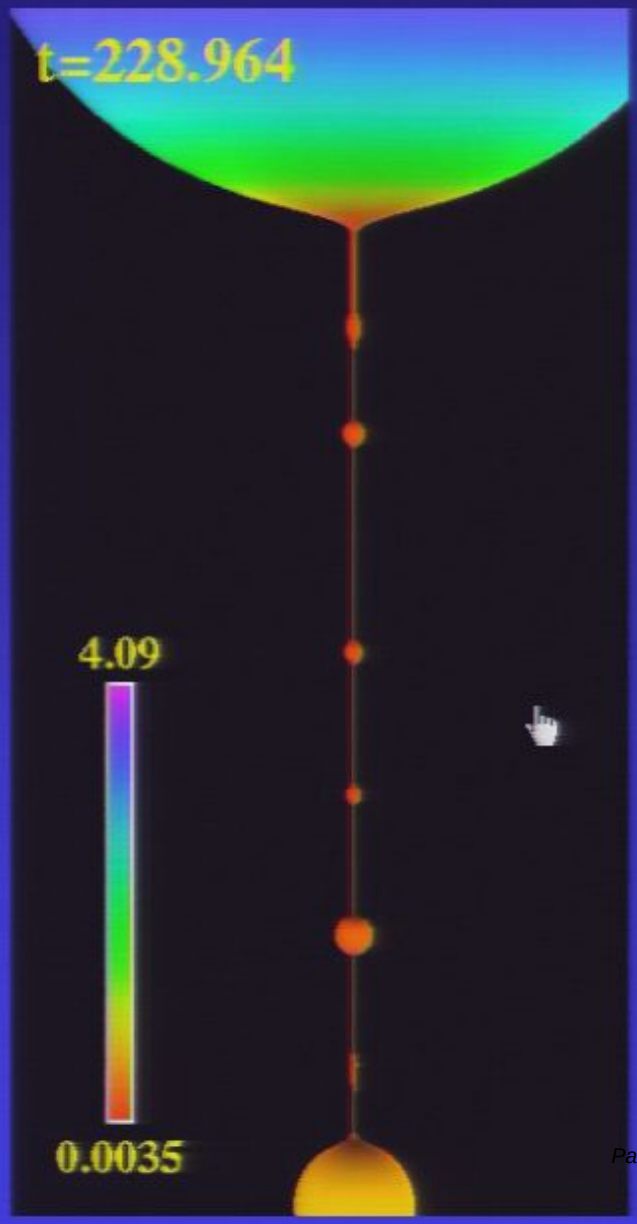
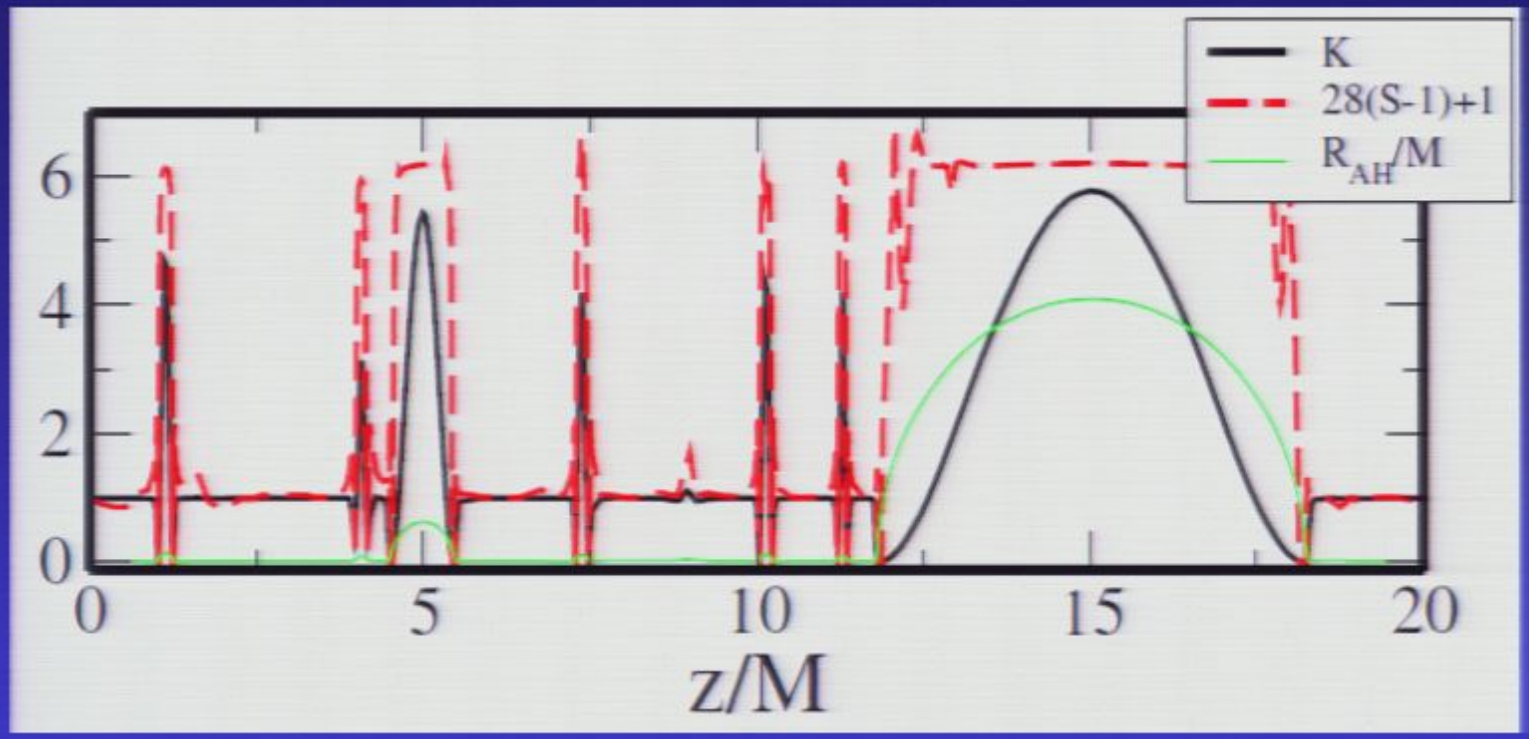


Apparent Horizon Dynamics



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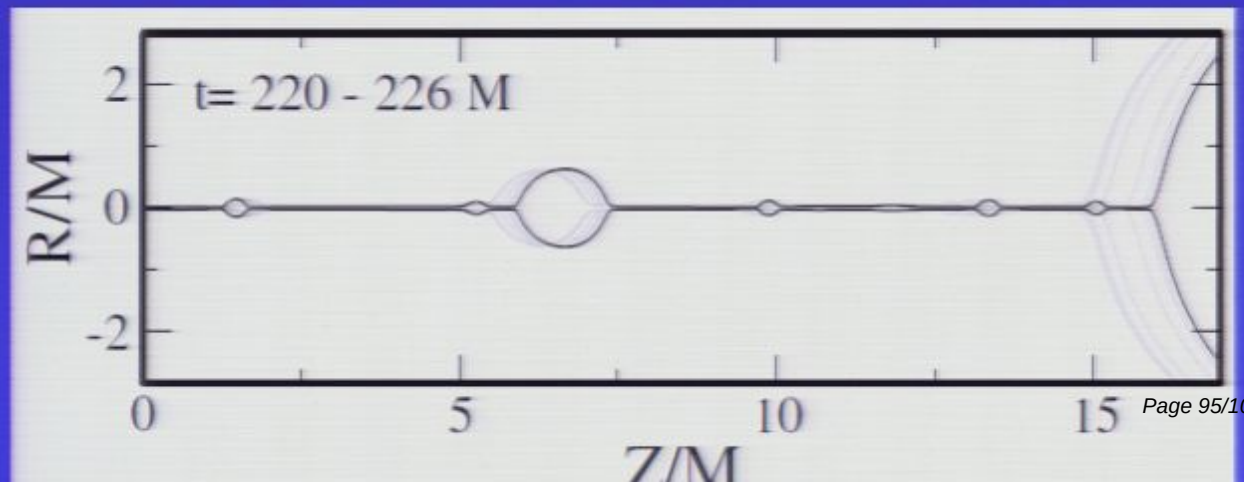

Properties of satellites and string-segments

- Therefore, the spheres-connected-by-string-segments interpretation seems reasonable. With that interpretation, and that evolution proceeds through a sequence of unstable epochs, we extract the following properties from the horizon:

Gen.	t_i/M	n_s	$R_{s,i}/M$	$R_{AH,f}/M$	$L_{s,i}/R_{s,i}$
1	118.1 ± 0.5	1	2.00	$4.09 \pm 0.5\%$	10.0
2	203.1 ± 0.5	1	$0.148 \pm 1\%$	$0.63 \pm 2\%$	$105 \pm 1\%$
3	223 ± 2	> 1	$0.05 \pm 20\%$	$0.1 - 0.2$	$\approx 10^2$
4	≈ 227	$> 1(?)$	≈ 0.02	?	$\approx 10^2$

Gen: generation number

t_i: time of initial satellite formation (defined to be time when the areal radius has grown to 1.5 times that of the surrounding string-segment)

n_s: number of satellites that form

$R_{s,i}$: radius of local string segment

$R_{AH,f}$: radius of satellites by the time the simulation was stopped

L_{si}/R_{si}: Ratio of length to radius of local string-segment (recall GL critical ratio ~ 7.2)

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- The dynamics of the apparent horizon also suggests that the instability unfolds in a self-similar manner; if so, transforming to logarithmic coordinates in space and time should reveal this more clearly

- The following shows $R_{AH}(t,w=const.)$ at points (roughly coinciding) with the eventual maxima of satellites, and one representative point that is still string-like near the end of the simulation

- Guess at "pinch-off time" by assuming the time scale for each later generation is a constant fraction X of the preceding one, with the exception of the first generation, whose time scale is controlled by the initial data:

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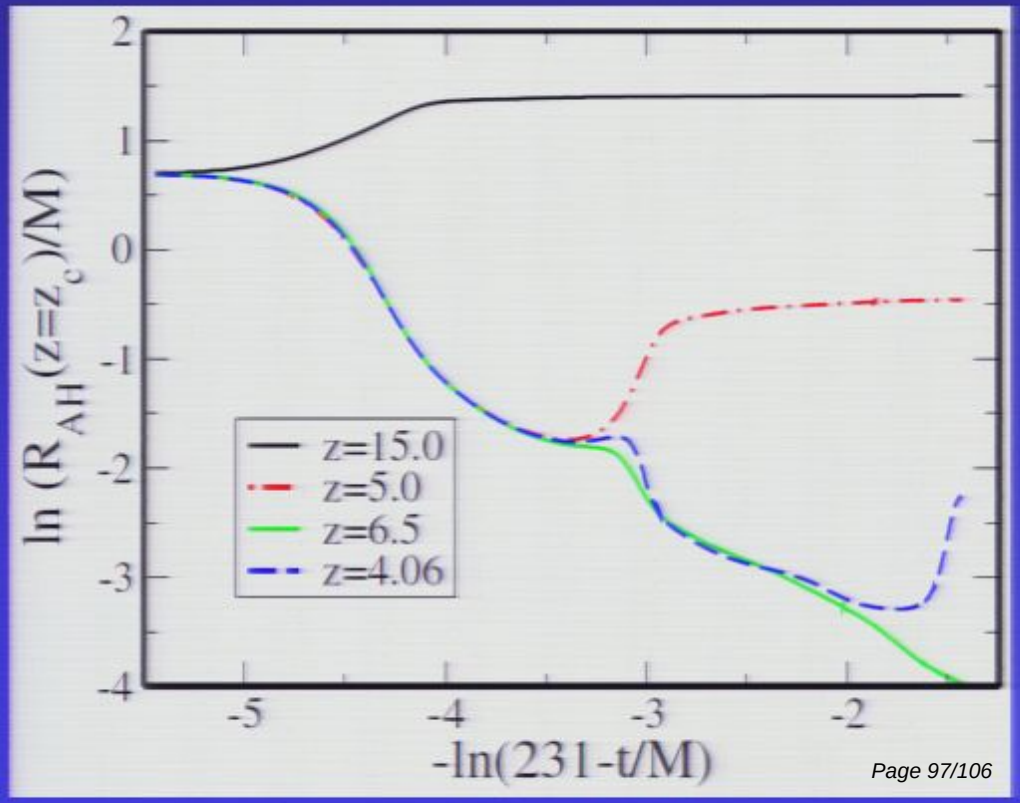
from the data in the table, we get $\Delta T \sim 231M$



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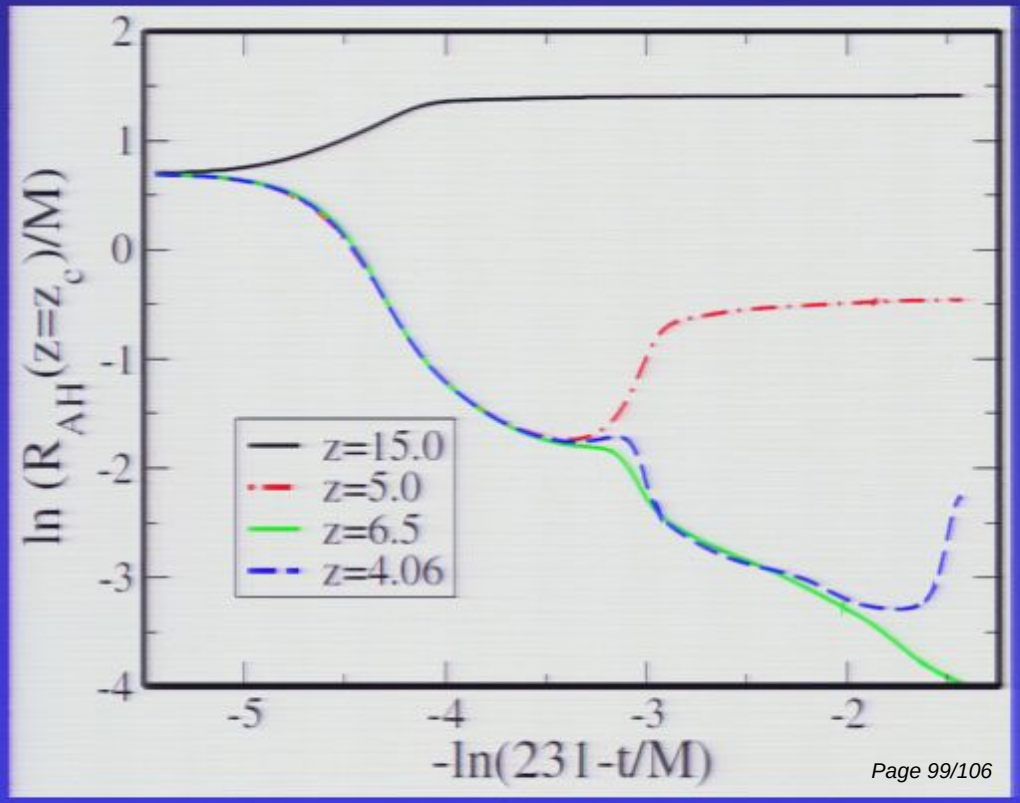


Properties of satellites and string-segments

- In a fluid with tension, the shrinking neck region exhibits a scaling solution of the form *[Eggers, PRL 71 (1993); Miyamoto, JHEP 1010 (2010)]*

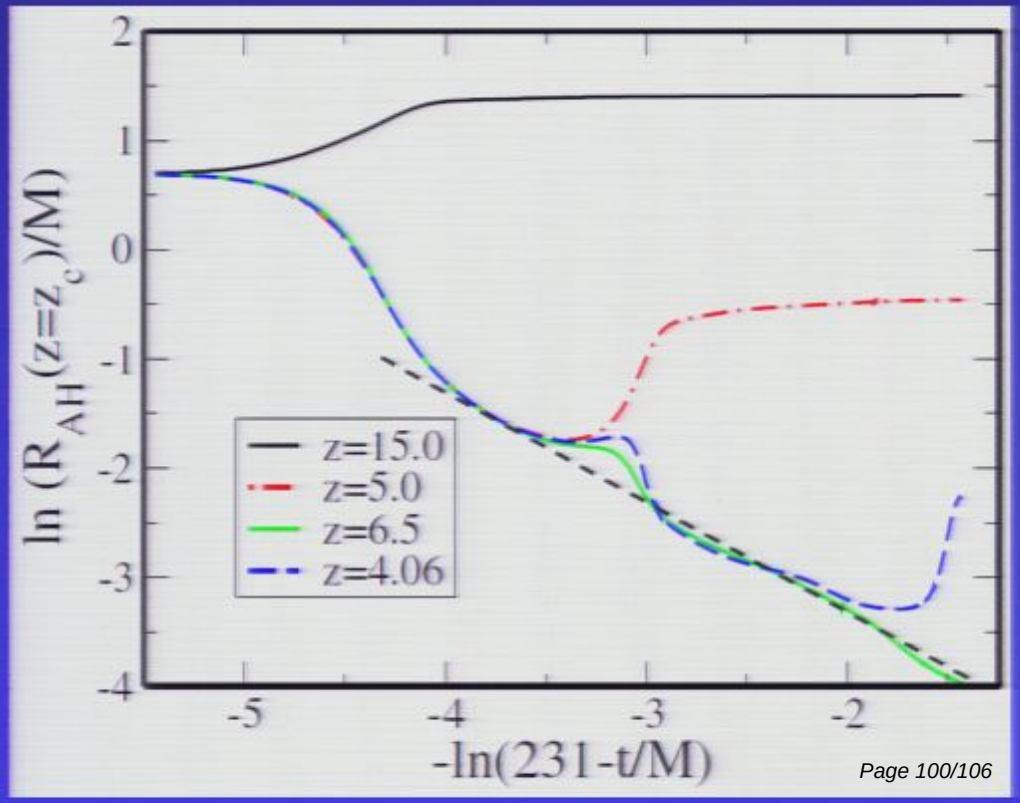
$$r \propto (t_0 - t)$$

or in logarithmic cooridantes

$$\frac{d \ln r}{d(-\ln(t_0 - t))} = -1$$

where t_0 is the pinch-off time

- The dashed line overlayed on the figure has slope ~-1
  - on average seems behavior of thinning string segment seems consistent with a self-similar pinch-off



Conclusions I

- New numerical solutions of the Gregory-Laflamme instability of 5 Dimensional black strings are revealing a rich dynamics
  - the original conjecture that the end state will be a sequence of spherical black holes seems correct, though this end-state is seemingly reached through a self-similar cascade, resulting (classically) in an infinite number of black holes per unit length, with arbitrarily small sizes
  - In the Rayleigh-Plateau hydrodynamic analogue, a self-similar cascade can occur
    - the lower the viscosity of the fluid, the more generations of self-similar behavior are observed before break-up; it is unknown at present whether below some critical viscosity the behavior continues indefinitely
    - Interestingly, the membrane paradigm suggests black holes have much lower viscosity that any "real-world" fluid

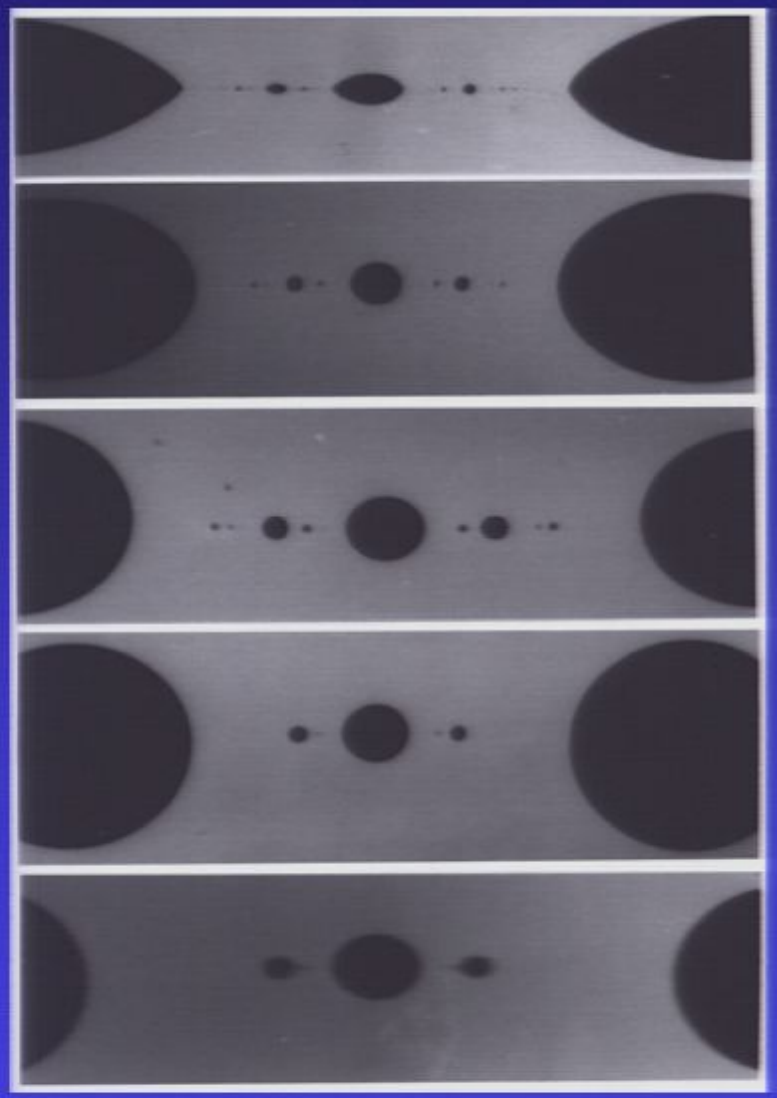


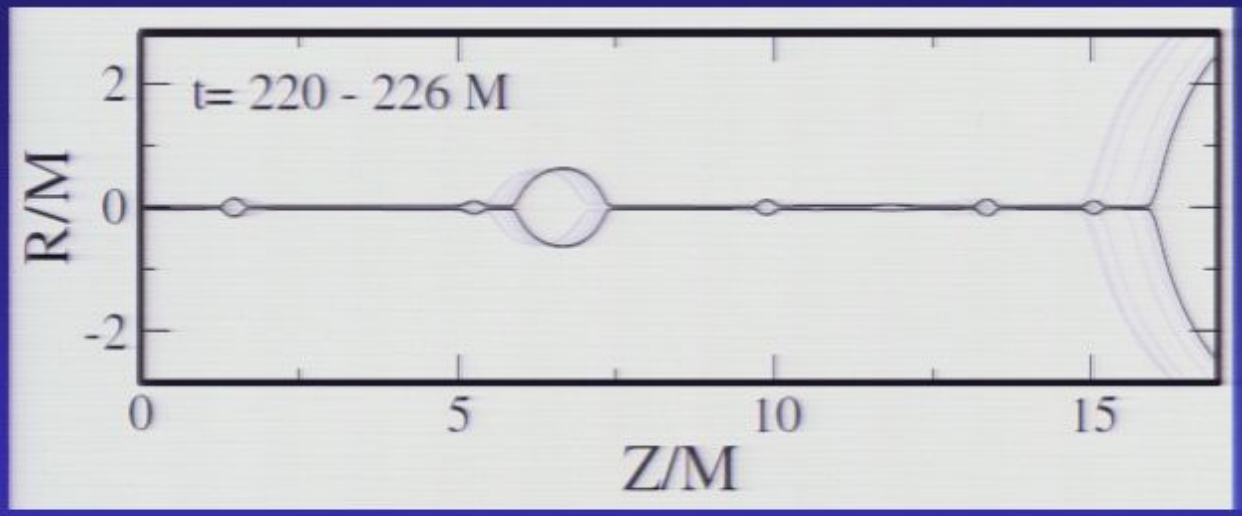
Image from Review article by Eggers *[Rev.Mod.Phys 59 (1997)]*, from work of Tjahjadi, Stone, and Ottino *[Fluid*

Conclusions II

- extrapolating from the first few generations, pinch-off will be reached in finite asymptotic time, at which time (classically) infinite geometric curvature will be revealed to the exterior universe
- this is then an example in 5D Einstein gravity (and presumably other dimensions where black holes exhibit similar instabilities) where generic violation of cosmic censorship occurs
- the "true" end-state will thus require some theory of quantum gravity to extend spacetime beyond the pinch-off

- Future work
  - presumably, as suggested in earlier simulations, the affine time along the generators of the horizon is diverging for consistency with the Horowitz-Maeda theorem; this should be confirmed
  - explore parameter space in the 5D case, in particular the initial spectrum of unstable modes excited
  - explore beyond 5D, in particular to investigate the work by Sorkin *[PRL.93 (2004)]* suggesting that for dimensions higher than 13, new non-uniform strings rather than bifurcation may result
  - investigate other classes of black hole instabilities
    - Shibata & Yoshino *[PRD 81 (2010)]* have begun such an investigation for rapidly spinning Myers-Perry black holes
  - see how far the hydrodynamic analogy can be extended; in particular see if the large body of work on break-up scenarios in the Navier-Stokes equations can be applied to the Einstein field equations (or vice-versa)

Apparent Horizon Dynamics



- At late times the horizon certainly *looks* like it can be described as a sequence of spherical black holes connected by string segments; to quantify this a bit, we evaluate the following curvature invariants on the horizon:

$$I = R_{abcd}R^{abcd};\ J = R_{abcd}R^{cdef}R_{ef}{}^{ab}$$

and construct the following dimensionless scalars

$$K = IR_{AH}^{4}/12;\ S = 12J^{2}I^{-3}$$

which evaluate to the following for the exact black sphere/black string solutions

$$K_{BH} = 6;\ 27(S_{BH}-1)+1 = 6 \qquad K_{BS} = 1;\ 27(S_{BS}-1)+1 = 1$$

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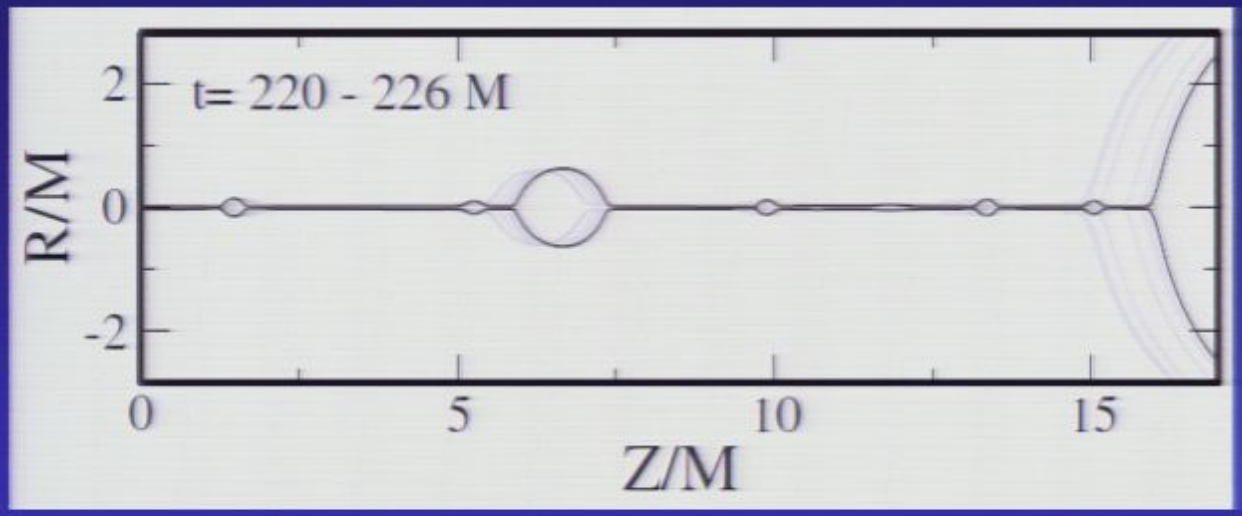

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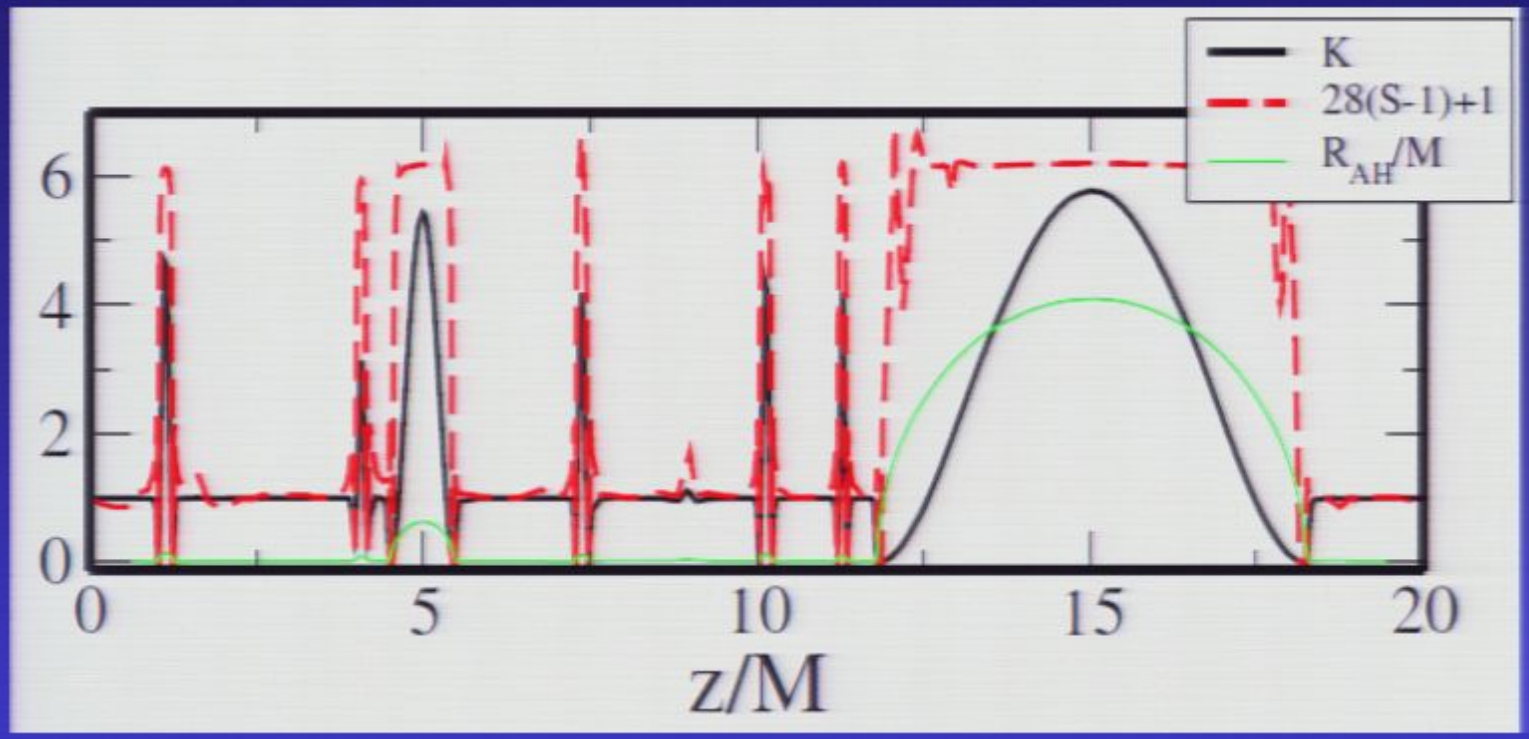

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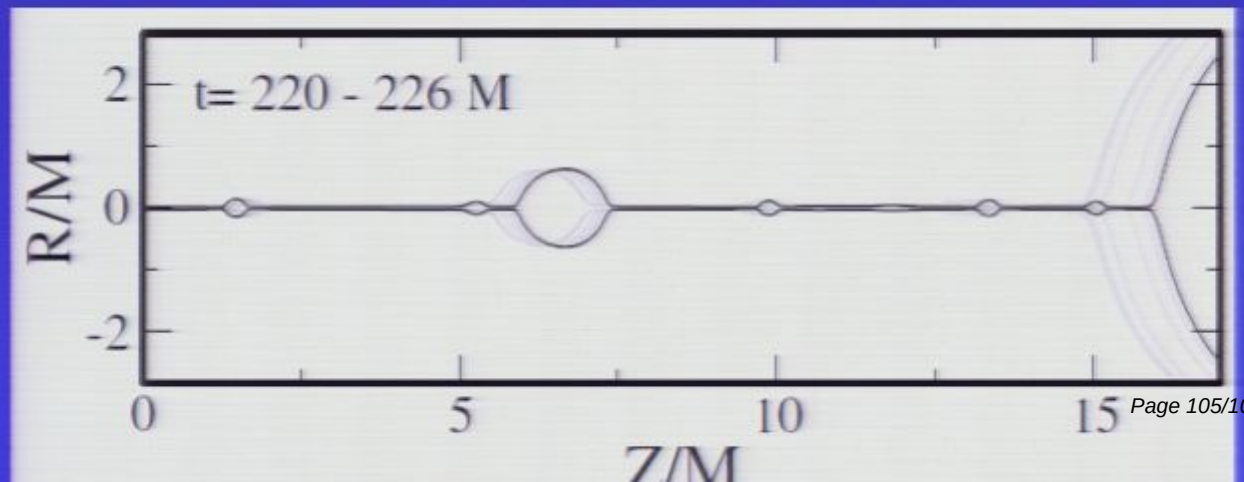

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