Title: A Breathing mode for Compactifications

Date: Feb 25, 2011 02:30 PM

URL: http://pirsa.org/11020137

Abstract: Reducing a higher dimensional theory to a 4-dimensional effective theory results in a number of scalar fields describing, for instance, fluctuations of higher dimensional scalar fields (dilaton) or the volume of the compact space (volume modulus). But the fields in the effective theory must be constructed with care: artifacts from the higher dimensions, such as higher dimensional diffeomorphisms and constraint equations, can affect the identification of the degrees of freedom. The effective theory including these effects resembles in many ways cosmological perturbation theory. I will show how constraints and diffeomorphisms generically lead the dilaton and volume modulus to combine into a single degree of freedom in the effective theory, the "breathing mode". This has important implications for models of moduli stabilization and inflation with extra dimensions.

Bret Underwood
McGill University

A Breathing Mode for Compactifications

arXiv:1009.4200

Extra Dimensions: Why?

Hierarchy Problem

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Hierarchy Problem

Large (flat) Extra Dimensions

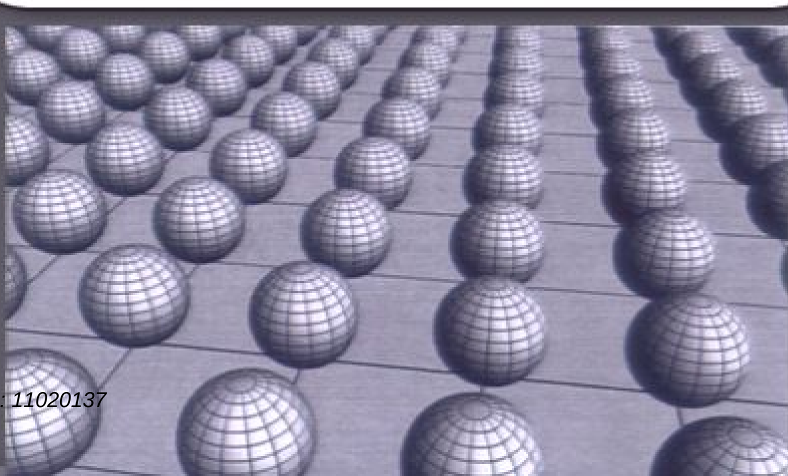
Fundamental scale of gravity is lower

$$M_p^2 = M_n^2 R^n$$

For $n = 2, R < 0.1\text{mm},$

$\Rightarrow M_n \sim (\text{few}) \times \text{TeV}$

[ADD]



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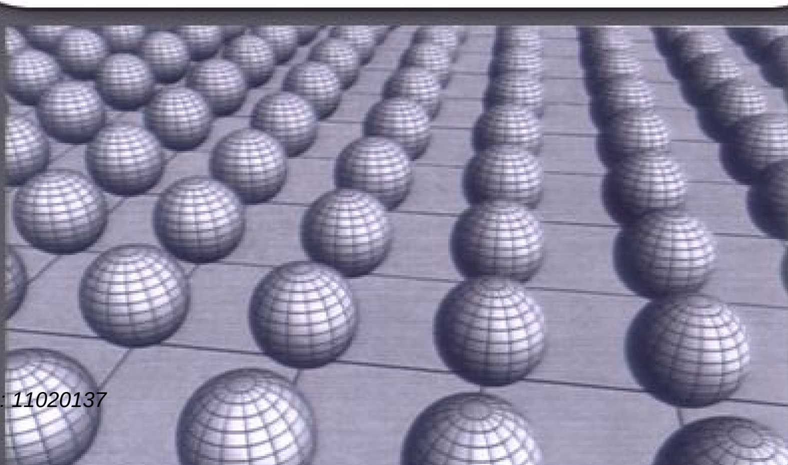
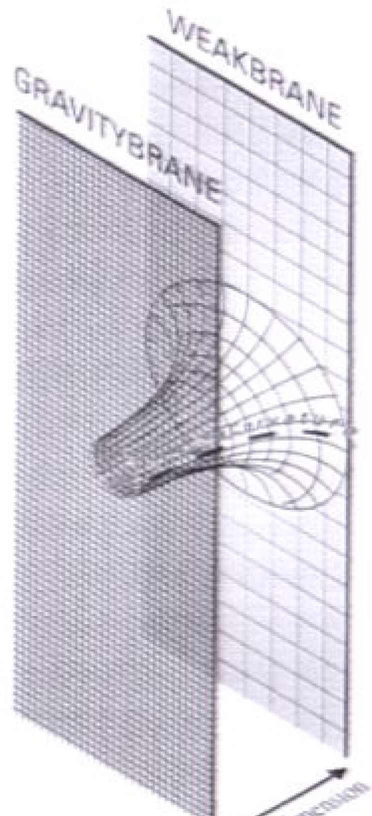


Warped Extra Dimensions

$$ds^2 = e^{-A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$$

Fundamental scale of gravity depends on location in extra dimension

[Randall-Sundrum]



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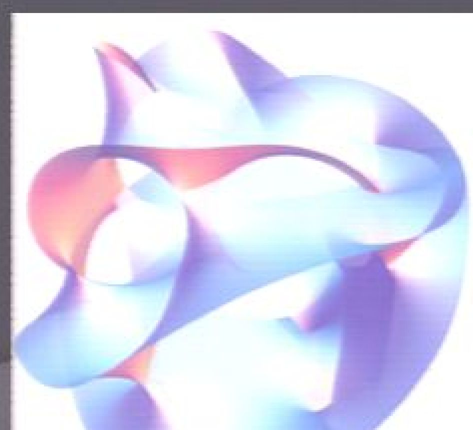
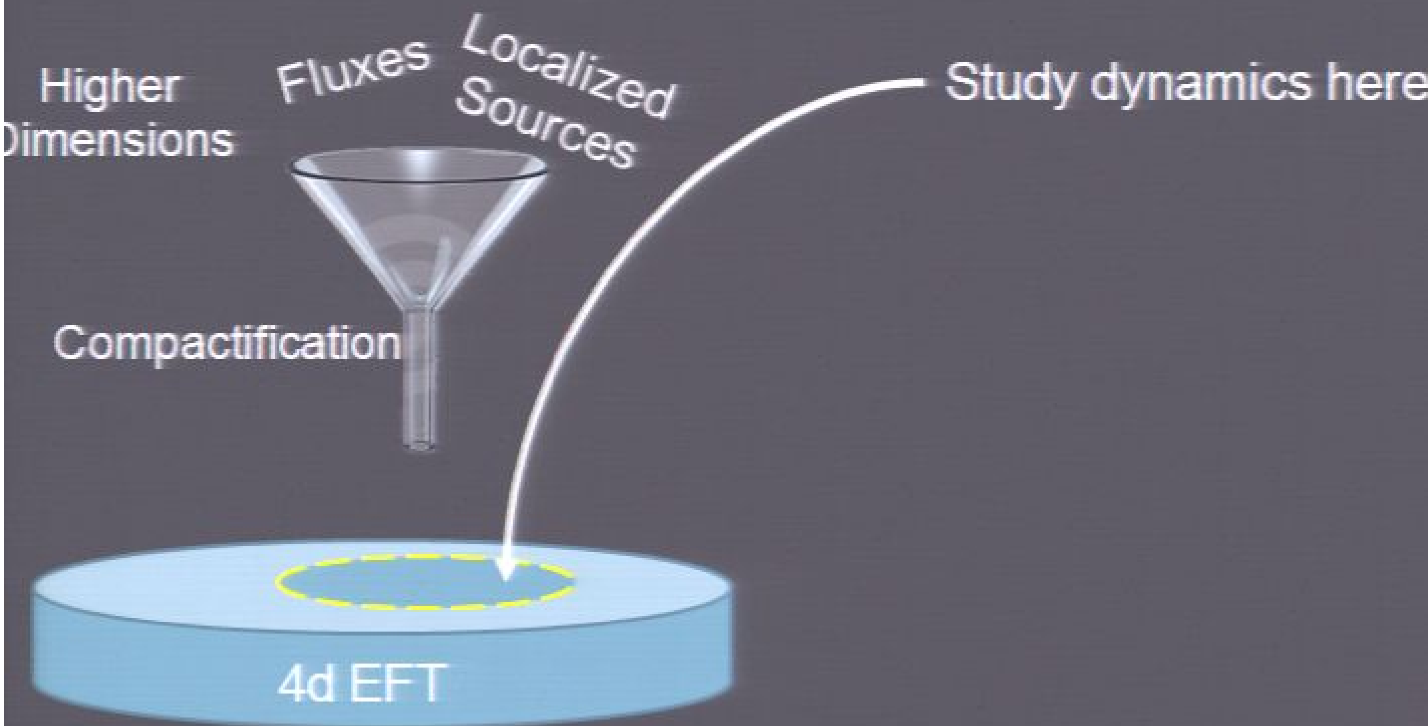
More generally, extra dimensions typically show up in high energy physics,

e.g. string theory…



Describing Extra Dimensions



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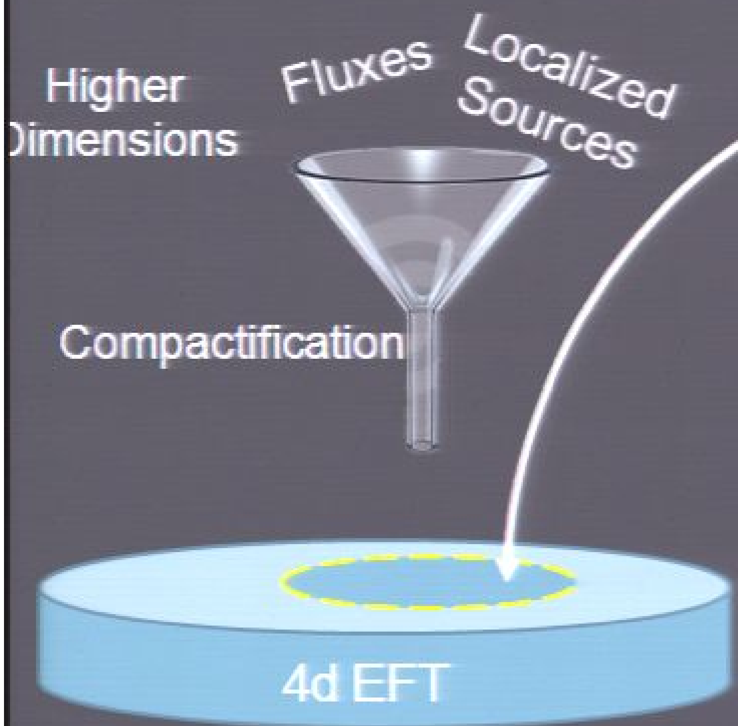

Study dynamics here

Example: Two *Universal* moduli

Volume Modulus $ds_D^2 = g_{\mu\nu} dx^\mu dx^\nu + \rho(x)\, \tilde{g}_{mn} dy^m dy^n$

Dilaton (bulk scalar field) $\phi = \phi_0 + \delta\phi(x)\tilde{\phi}(y)$

Dimensionally Reduced 4d EFT

$$\mathcal{L}_{eff} \sim \frac{(\partial_\mu \rho)^2}{\rho^2} + \frac{(\partial_\mu \delta\phi)^2}{\delta\phi^2} + V_{eff}(\rho, \delta\phi)$$

Describing Extra Dimensions

Example: Two *Universal* moduli **Type IIA String Theory**

$$V_{eff} = \left(\frac{A_{curvature}}{\rho} + \frac{A_{NSNS}}{\rho^3}\right)\tau^{-2} - (n_{O6} - n_{D6})A_6\tau^{-3} + \left(\sum_p \rho^{3-p} A_p^{RR}\right)\tau^{-4}$$

$$\left(\tau \equiv e^{-\delta o}\rho^{3/2}\right)$$

Can be used to study existence of dS vacua, inflation…

[Hertzberg et al; Silverstein; Underwood et al; Caviezel et al; Flauger et al; Haque et al;…]

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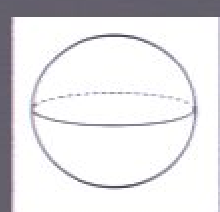
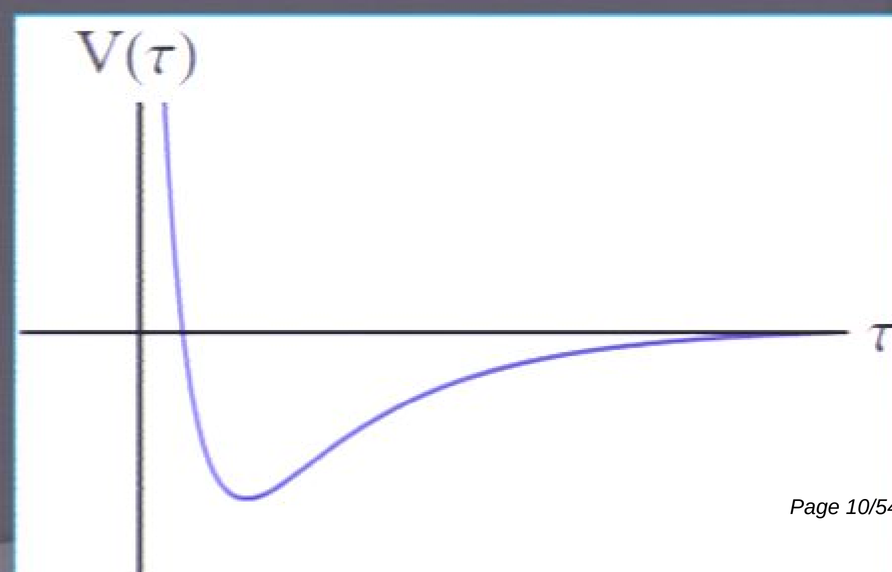
Zero Internal Curvature:

Positive Internal Curvature:



Negative Internal Curvature:



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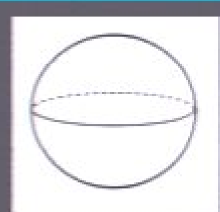

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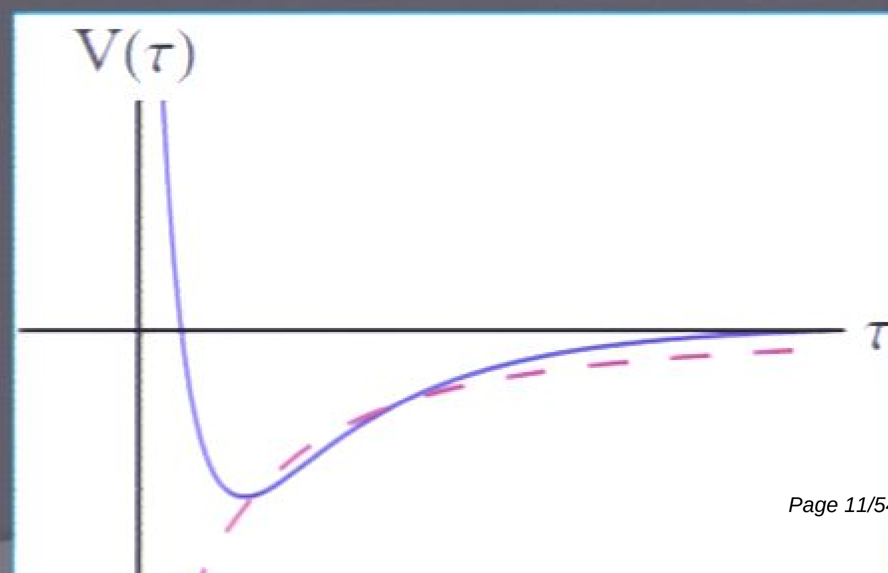

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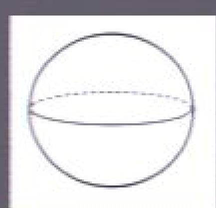

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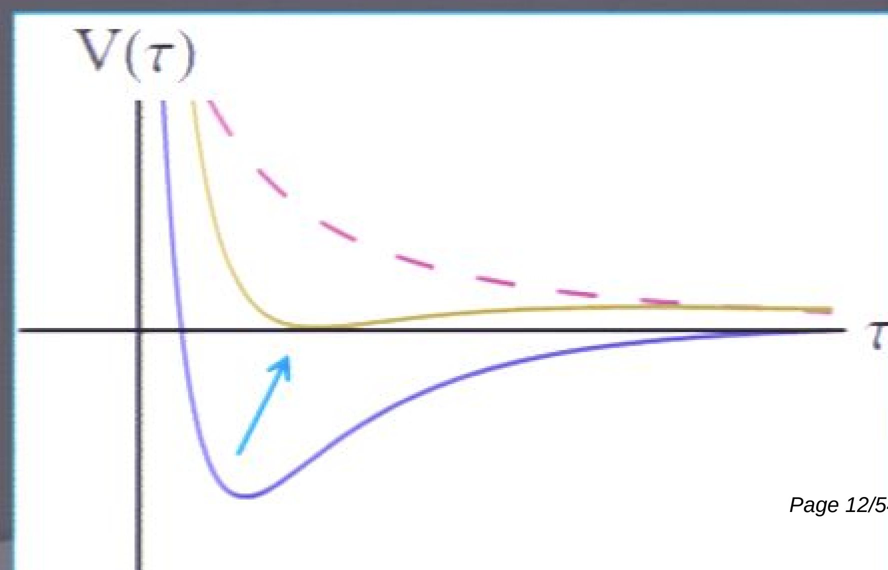

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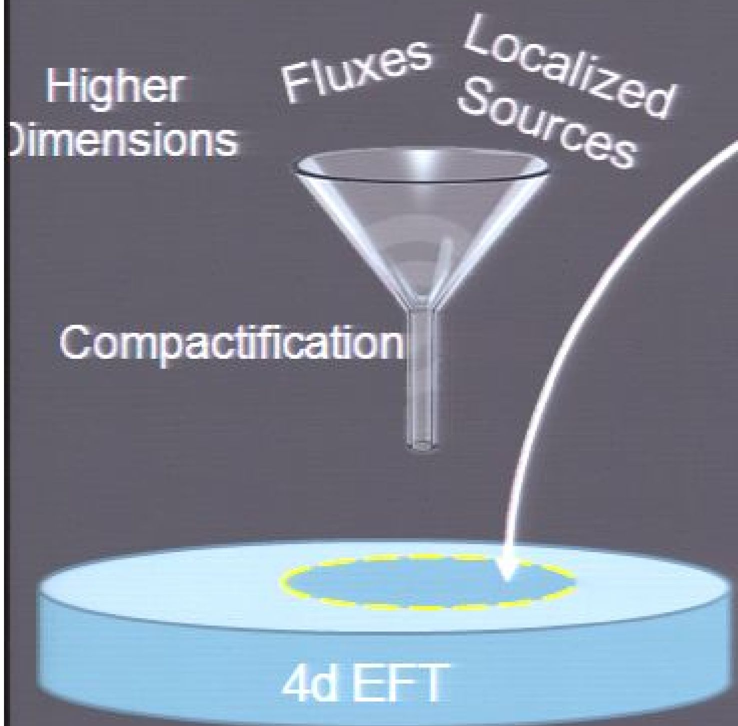

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Dimensionally Reduced 4d EFT

$$\mathcal{L}_{eff} = \frac{3}{4}\frac{(\partial_\mu \rho)^2}{\rho^2} + \frac{(\partial_\mu \delta\phi)^2}{\delta\phi^2} + V_{eff}(\rho, \delta\phi)$$

Are there features missed by the 4d EFT?

$$\partial_\rho V_{eff} = 0 \Leftrightarrow G_{mn} - T_{mn} = 0$$

$$\partial_{\delta\phi} V_{eff} = 0 \Leftrightarrow \text{Dilaton EOM} = 0$$

$$V_{eff}|_{min} = \Lambda \Leftrightarrow G_{\mu\nu} - T_{\mu\nu} = 0$$

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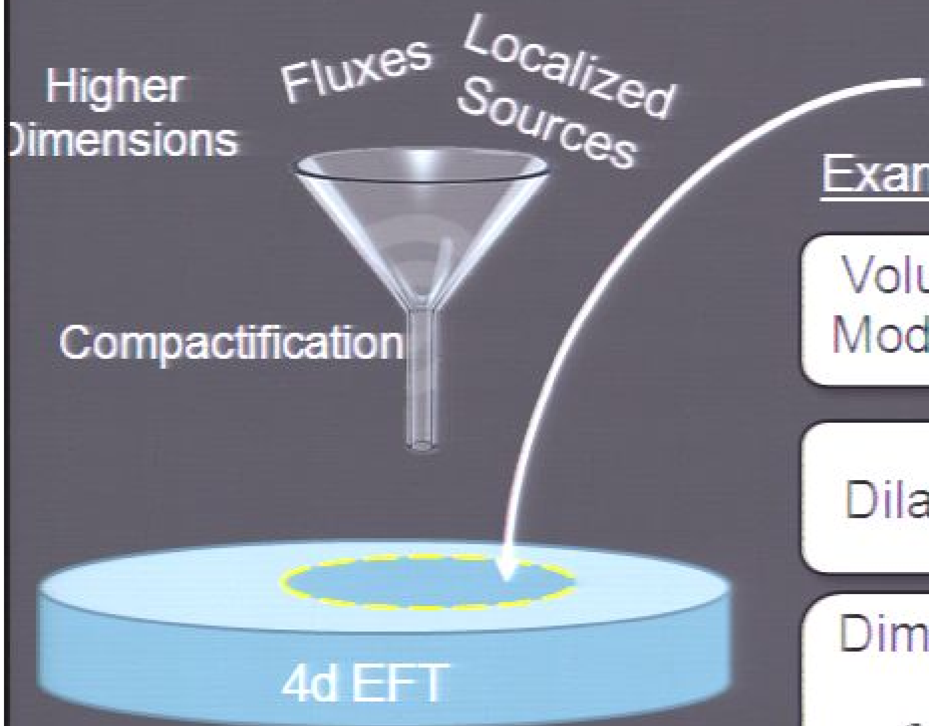

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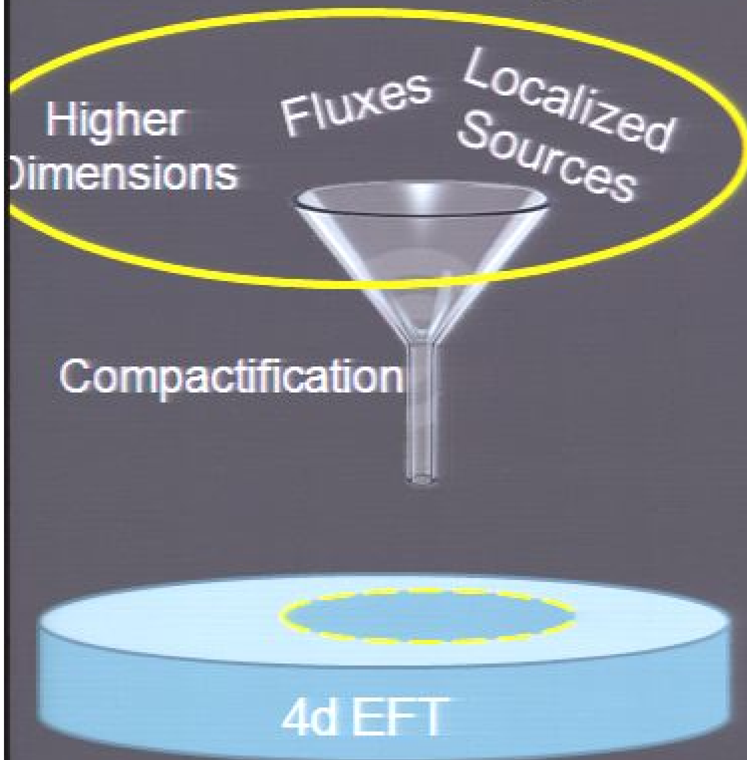

More generally, want to study dynamics here – beyond validity of 4d Effective Theory

Intrinsically higher dimensional

Ex...

Volume Modulus $ds^2_{10} = g_{\mu\nu} dx^\mu dx^\nu + \rho(x)\, \tilde{g}_{mn} dy^m dy^n$

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Dynamics in extra dimensions

Outline

- Cosmological Perturbation Theory (Review)
- Warped Perturbation Theory
- Example:
  - p-brane backgrounds
- Weakly warped limit $\neq$ unwarped limit
- Other examples of Warped Perturbation Theory
- Cosmological applications

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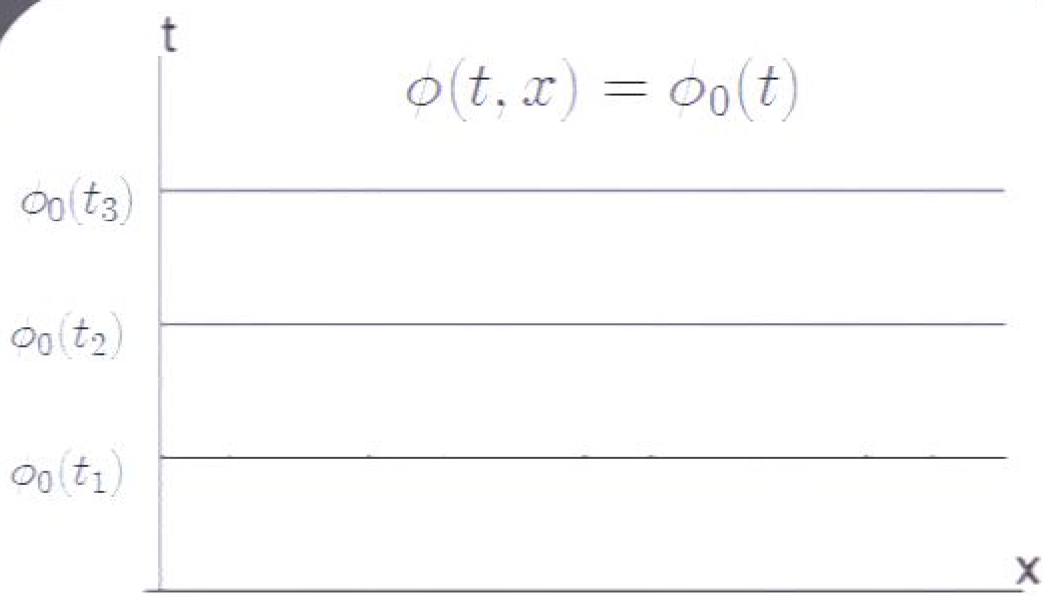
Cosmological Perturbation Theory

4-dimensional FLRW spacetime with a homogeneous scalar field

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2, \quad \phi_0(t)$$

$$\phi(t, x) = \phi_0(t)$$



Homogeneous Mode:
Constant cosmic time slices are constant in space.

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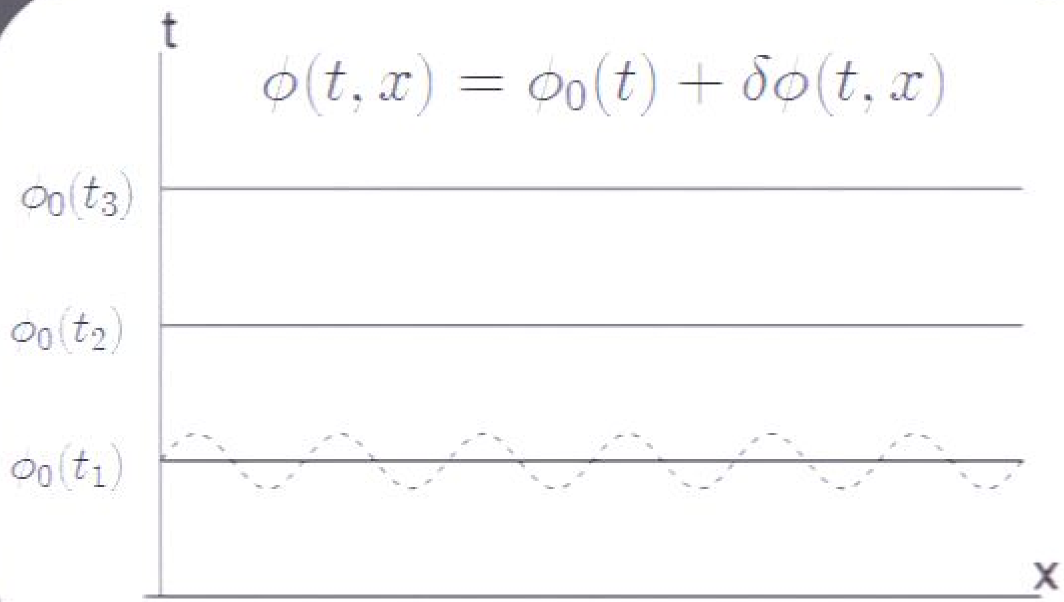

Homogeneous Mode:
Constant cosmic time slices are constant in space.

Fluctuation:
Constant cosmic time slices not constant on space.

But…distinction between fluctuation and background not a coordinate-independent statement: Under diffeo. $t \to t + \xi^0(t,x)$

$$\phi(t,x) \to \phi(t+\xi^0,x) = \phi_0(t) + \xi^0\dot{\phi}_0 + \delta\phi = \phi_0(t)$$

Cosmological Perturbation Theory

More generally, scalar perturbations about FLRW

$$ds^2 = -(1+2\varphi(t,x))dt^2 + a^2(t)[(1-2\psi(t,x))\delta_{ij} + 2\partial_i\partial_j E(t,x)]dx^i dx^j$$

$$+a(t)\ \partial_i B(t,x) dt dx^i$$

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5 scalar functions

$$\{\varphi, \psi, E, B, \delta\phi\}$$

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Transform non-trivially under diffeomorphisms
Can construct gauge-invariant scalar variables:

$$\Phi_B = \varphi - \frac{d}{dt}\left[a^2(\dot{E} - B/a)\right]$$

$$\Psi_B = \psi \frac{\dot{a}}{a} a^2(\dot{E} - B/a)$$

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3 gauge-invariant variables

Diffeomorphisms removed 2 degrees of freedom

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Mixture of scalar field and metric fluctuations

Cosmological Perturbation Theory

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$$\delta G_{00} - 8\pi G \delta T_{00} = 3H\left(\dot{\Psi}_B + H\Phi_B\right) + \nabla^2\Psi_B + 4\pi G\delta\rho = 0$$

$$\delta G_{0i} - 8\pi G \delta T_{0i} = 2\partial_i\left[\dot{\Psi}_B + \frac{\dot{a}}{a}\Phi_B - 4\pi G\dot{\phi}_0\,\delta\Phi\right] = 0$$

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"Curvature Perturbation"

Mixture of scalar field and metric fluctuations

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Warped Perturbation Theory

Now let's use the same reasoning for a theory with extra dimensions

$$ds_D^2 = e^{2A_0(y)}\hat{g}_{\mu\nu}(x)dx^\mu dx^\nu + e^{-2B_0(y)}\tilde{g}_{mn}dy^m dy^n$$

$$\phi = \phi_0(y)$$

Spacetime (p+1)

Extra Dimensions (D-p-1)

(Some solution someone hands you)

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Spacetime (p+1)

Extra Dimensions (D-p-1)

(Some solution someone hands you)

Generically, a solution will be "warped" – sources in extra dimensions always introduce a gravitational potential (warping)

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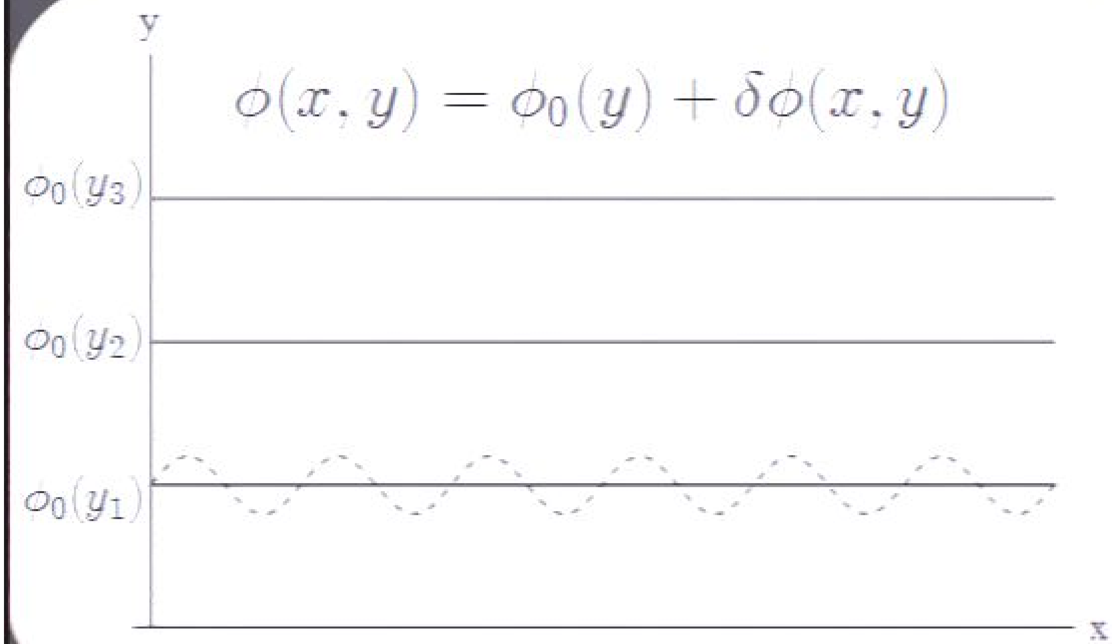
Spacetime (p+1) — Extra Dimensions (D-p-1)

$$\phi(x,y) = \phi_0(y) + \delta\phi(x,y)$$



Fluctuations:
Constant slices along extra dimensions depend on spacetime .

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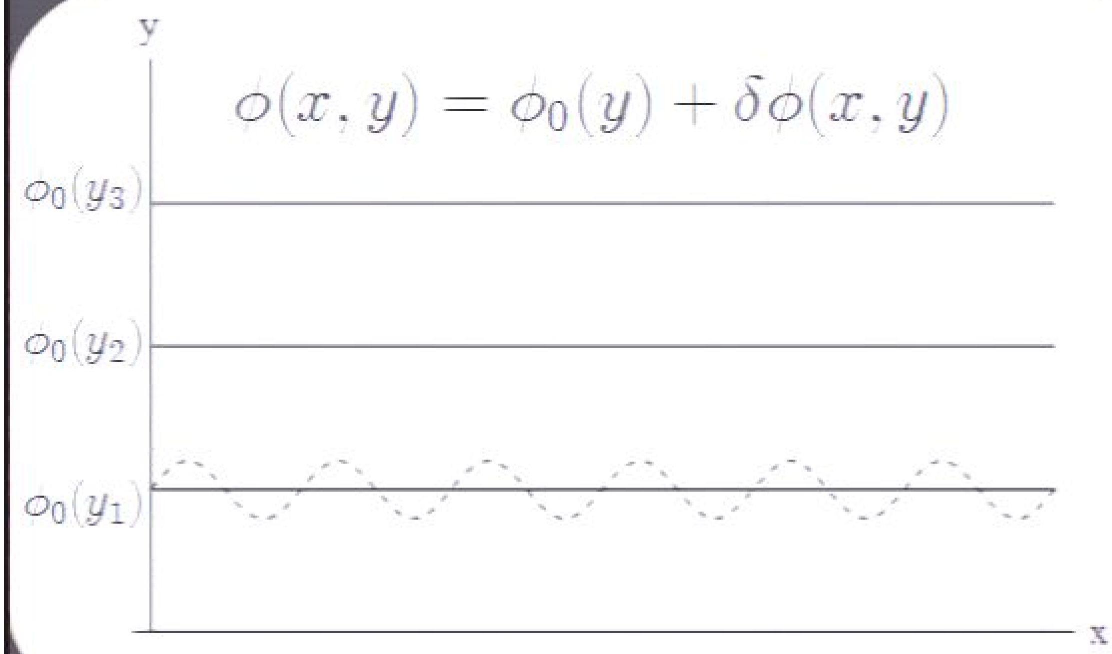
Spacetime (p+1) **Extra Dimensions (D-p-1)**



Fluctuations:
Constant slices along extra dimensions depend on spacetime .

But again, this is <u>not</u> a coordinate-independent statement.

$$y^m \rightarrow y^m + \xi^m(x, y)$$

$$\phi(x, y) \rightarrow \phi(x, y + \xi) = \phi_0(y) + \xi^m \partial_m \phi_0 + \delta\phi = \phi_0(y)$$

Warped Perturbation Theory

More generally, scalar perturbations about background

$$ds_D^2 = e^{2A_0(y)}\left[(1-2\psi(x,y))\hat{g}_{\mu\nu} + 2\hat{\nabla}_\mu\partial_\nu E(x,y)\right] dx^\mu dx^\nu$$

$$+e^{2A_0(y)}\partial_\mu K_m(x,y)dx^\mu dy^m + e^{-2B_0(y)}(\tilde{g}_{mn}(y) + 2\varphi_{mn}(x,y))dy^m dy^n;$$

$$\phi = \phi_0(y) + \delta\phi(x,y)\,.$$

2+D+n(n+1)/2 scalar functions

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Transform non-trivially under D-dim diffeomorphisms
Can construct gauge-invariant scalar variables:

$$\begin{pmatrix} x^\mu \\ y^m \end{pmatrix} \rightarrow \begin{pmatrix} x^\mu + \xi^\mu(x,y) \\ y^m + \xi^m(x,y) \end{pmatrix}$$

$$\Phi_{mn} = \varphi_{mn} + e^{2A_0}(\partial^p B_0)(K_p - \partial_p E)\tilde{g}_{mn} + \tilde{\nabla}_{(m}\left[e^{2A_0+2B_0}(\partial_{n)}E - K_{n)})\right];$$

$$\Psi = \psi + e^{2A_0}(\partial^p A_0)(K_p - \partial_p E);$$

$$\delta\Phi = \delta\phi + e^{2A_0}(\partial^p\phi_0)(K_p - \partial_p E)\,.$$

Diffeomorphisms remove *D* degrees of freedom

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$$\begin{pmatrix} x^\mu \\ y^m \end{pmatrix} \to \begin{pmatrix} x^\mu + \xi^\mu(x,y) \\ y^m + \xi^m(x,y) \end{pmatrix}$$

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Diffeomorphisms remove *D* degrees of freedom

Mixture of dilaton and metric fluctuations

Warped Perturbation Theory

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Constraint equations remove another *D* degrees of freedom.

Left with *1 + (D-p-1)(D-p+2)/2* independent scalar + metric d.o.f.

Warped Perturbation Theory

More generally, scalar perturbations about background

$$ds_D^2 = e^{2A_0(y)}\left[(1-2\psi(x,y))\hat{g}_{\mu\nu} + 2\hat{\nabla}_\mu\partial_\nu E(x,y)\right]dx^\mu dx^\nu$$

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$2+D+n(n+1)/2$ scalar functions

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Diffeomorphisms remove D degrees of freedom

Warped Perturbation Theory

Now let's use the same reasoning for a theory with extra dimensions

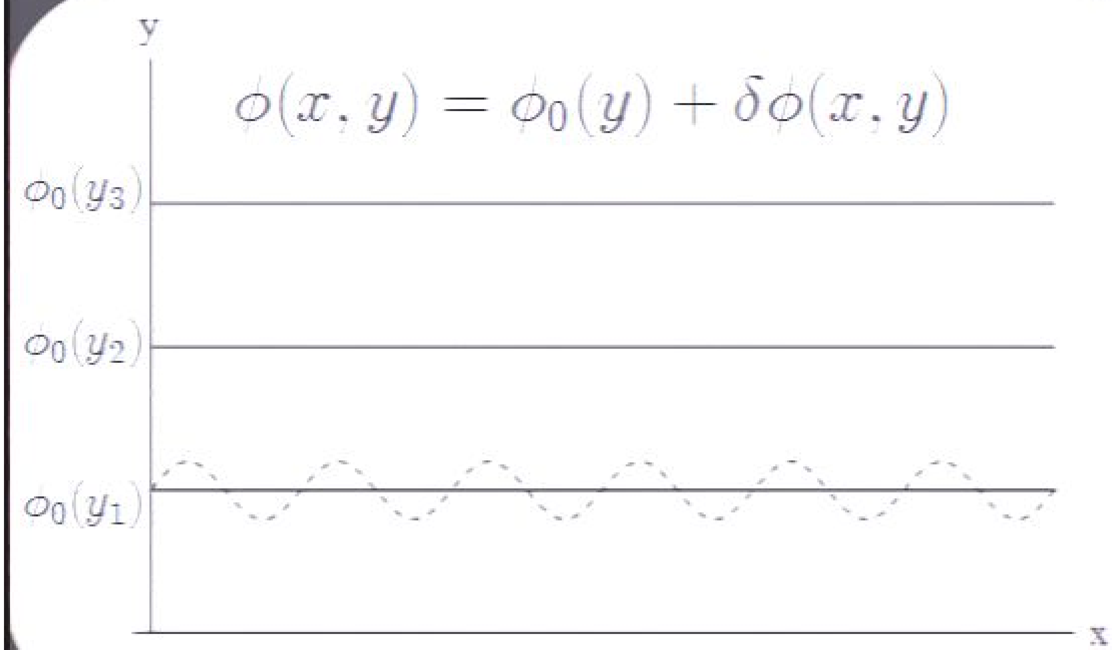
$$ds_D^2 = e^{2A_0(y)}\hat{g}_{\mu\nu}(x)dx^\mu dx^\nu + e^{-2B_0(y)}\tilde{g}_{mn}dy^m dy^n$$

$$\phi = \phi_0(y)$$

Spacetime (p+1)

Extra Dimensions (D-p-1)

$$\phi(x,y) = \phi_0(y) + \delta\phi(x,y)$$



Fluctuations:
Constant slices along extra dimensions depend on spacetime .

But again, this is <u>not</u> a coordinate-independent statement.

$$y^m \rightarrow y^m + \xi^m(x,y)$$

$$\phi(x,y) \rightarrow \phi(x,y+\xi) = \phi_0(y) + \xi^m\partial_m\phi_0 + \delta\phi = \phi_0(y)$$

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It is inconsistent to turn on *only* a dilaton fluctuation:

$$\delta G_{\mu m} - \kappa_D^2 \delta T_{\mu m} = \frac{1}{2}\partial_\mu \delta\phi(x,y) \partial_m \phi_0 = 0$$

Gauge-invariant dilaton fluctuation must combine with some metric fluctuation in order to create a consistent, gauge-invariant degree of freedom.

Which fluctuation could this be?

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$$g_{mn} = e^{-2B_0(y)} e^{2\beta\varphi(x)} \tilde{g}_{mn}(y)$$

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Dynamics in extra dimensions

Outline

- Cosmological Perturbation Theory (Review)
- Warped Perturbation Theory
- Example:
  - p-brane backgrounds
- Weakly warped limit ≠ unwarped limit
- Other examples of Warped Perturbation Theory
- Cosmological applications

Warped Breathing Mode (p-brane)

D-dimensional gravity, dilaton, (p+2)-form, localized source

$$S = \frac{1}{2\kappa_D^2}\int d^D x\sqrt{g_D}\left[R_D - \frac{1}{2}(\partial\phi)^2 - \frac{e^{-\lambda\phi}}{2(p+2)!}F_{p+2}^2\right] + S_{loc}\,,$$

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p-brane solutions:

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Harmonic Function:

$$\tilde{\nabla}^2 e^{-2\gamma A_0(y)} = \sum_n Q_n \delta^{(D-p-1)}(y-y_n)\,.$$

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Natural Shift-Invariance $e^{-2\gamma A_0(y)} \rightarrow e^{-2\gamma A_0(y)} + u$ implies an ansatz for dynamical "breathing" mode:

$$e^{-2\gamma A(x.y)} = e^{-2\gamma A_0(y)} + u(x)$$

Warped Breathing Mode (p-brane)

Almost…Need Weyl Rescaling and Compensator

$$ds_D^2 = e^{2A(y,u(x))} e^{2\Omega[u(x)]} \left[\hat{g}_{\mu\nu} + 2e^{(p-3)\Omega[u(x)]} (\hat{\nabla}_\mu \partial_\nu u(x)) E(y) \right] dx^\mu dx^\nu .$$

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4d degree of freedom of dilaton fluctuation also controls volume modulus degree of freedom – *warped breathing mode*

Volume modulus and dilaton fluctuations are not independent degrees of freedom

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4d degree of freedom of dilaton fluctuation also controls volume modulus degree of freedom – *warped breathing mode*

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4d degree of freedom of dilaton fluctuation also controls volume modulus degree of freedom – *warped breathing mode*

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4d degree of freedom of dilaton fluctuation also controls volume modulus degree of freedom – *warped breathing mode*

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Effective kinetic term for warped breathing mode comes from gravity and dilaton sectors:

$$S_{eff}^{kin} = \int \sqrt{g_D} \left[R_D - \frac{1}{2}(\partial\phi)^2 \right]$$
$$= \int \sqrt{\hat{g}} \left(\mathcal{G}_{uu}^{(g)} + \mathcal{G}_{uu}^{(\phi)} \right) \partial_\mu u(x) \partial^\mu u(x)$$

Gravity kinetic term gives the usual volume modulus kinetic term.

$$\mathcal{G}_{uu}^{(g)} = -\left(\frac{(p+1)^2(D-p-1)}{8\kappa_{p+1}^2(D-2)(p-1)} \right) \frac{1}{(u(x) + \tilde{V}_W^{(0)}/\tilde{V}_{D-p-1})^2}$$

Dilaton kinetic term is not as nice:

$$\mathcal{G}_{uu}^{(\phi)} = -\frac{1}{4\kappa_D^2} \int \sqrt{\tilde{g}} e^{(p-1)\Omega} \frac{\tilde{\lambda}^2}{4\gamma^2} e^{2\gamma A}$$

Effective Theory is different than if had ignored these effects:

Dynamics in extra dimensions

Outline

- Cosmological Perturbation Theory (Review)
- Warped Perturbation Theory
- Example:
  - p-brane backgrounds
- Weakly warped limit ≠ unwarped limit
- Other examples of Warped Perturbation Theory
- Cosmological applications

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Take *weakly warped (large volume)* limit of background:

$$A_0(y) = \epsilon f(y), \quad \epsilon \propto (\mathrm{Vol})^{-n}, \quad e^{2A(x,y)} \approx 1 - \frac{1}{\gamma} u(x) + 2\epsilon f(y)$$

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Dynamics in extra dimensions

Outline

- Cosmological Perturbation Theory (Review)
- Warped Perturbation Theory
- Example:
  - p-brane backgrounds
- Weakly warped limit ≠ unwarped limit
- Other examples of Warped Perturbation Theory
- Cosmological applications

Warped Breathing Mode (p-brane)

Almost…Need Weyl Rescaling and Compensator

$$ds_D^2 = e^{2A(y,u(x))} e^{2\Omega[u(x)]} \left[\hat{g}_{\mu\nu} + 2e^{(p-3)\Omega[u(x)]} (\hat{\nabla}_\mu \partial_\nu u(x)) E(y)\right] dx^\mu dx^\nu .$$

$$+ e^{-2\left(\frac{p+1}{D-p-3}\right)A(y,u(x))} \tilde{g}_{mn}(y) dy^m dy^n$$

$$\phi(x,y) = -\tilde{\lambda} A(y,u(x)), \qquad C_{p+1} = \pm e^{\tilde{a}A(y,u(x))} \hat{\epsilon}_{p+1}$$

$$e^{-2\gamma A(x,y)} = e^{-2\gamma A_0(y)} + u(x)$$

$u(x)$ is a single (p+1)-dimensional degree of freedom

Dilaton fluctuation + Volume modulus fluctuation

Warped Perturbation Theory

Gauge-invariant dilaton fluctuation must combine with some metric fluctuation in order to create a consistent, gauge-invariant degree of freedom.

Which fluctuation could this be?

Back to the cosmological case:

Scalar field fluctuation $\phi(t,x) = \phi_0(t) + \delta\phi(t,x)$ mixes with the curvature perturbation (spatial volume perturbation)

$$g_{ij} = a^2(t) e^{2\zeta(t,x)} \delta_{ij}$$

Similarly, expect *dilaton* fluctuation to combine with the *volume modulus* fluctuation of the metric:

$$g_{mn} = e^{-2B_0(y)} e^{2\beta\varphi(x)} \tilde{g}_{mn}(y)$$

(Need to generalize "volume modulus" to warped space)

Already seen in Randall-Sundrum context

Warped Perturbation Theory

Gauge-invariant scalar variables must satisfy constraint equations arising from Einstein equations – non-dynamical equations.

$$\delta G_{\mu\nu} - \kappa_D^2 \delta T_{\mu\nu}\Big|_{\mu\neq\nu} = \hat{\nabla}_\mu \partial_\nu \left[(p-1)\Psi - \Phi_p^{\tilde{p}}\right] = 0;$$

$$\delta G_{\mu m} - \kappa_D^2 \delta T_{\mu m} = -\partial_\mu \partial_m \left[p\Psi + \Phi_p^{\tilde{p}}\right] + \partial_\mu \tilde{\nabla}_p \Phi_m^{\tilde{p}} + \partial_\mu \Phi_p^{\tilde{p}} \left[\partial_m A_0 + \partial_m B_0\right]$$
$$+ \partial_\mu \Phi_m^{\tilde{p}} \left[(p-1)\partial_p A_0 - (D-p-1)\partial_p B_0\right] + \frac{1}{2}\partial_\mu \delta\Phi \partial_m \phi_0 = 0\,.$$

It is inconsistent to turn on *only* a dilaton fluctuation:

$$\delta G_{\mu m} - \kappa_D^2 \delta T_{\mu m} = \frac{1}{2}\partial_\mu \delta\phi(x,y)\partial_m \phi_0 = 0$$

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Warped Perturbation Theory

More generally, scalar perturbations about background

$$ds_D^2 = e^{2A_0(y)}\left[(1-2\psi(x,y))\hat{g}_{\mu\nu} + 2\hat{\nabla}_\mu\partial_\nu E(x,y)\right] dx^\mu dx^\nu$$

$$+e^{2A_0(y)}\partial_\mu K_m(x,y)dx^\mu dy^m + e^{-2B_0(y)}(\tilde{g}_{mn}(y)+2\varphi_{mn}(x,y))dy^m dy^n;$$

$$\phi = \phi_0(y) + \delta\phi(x,y).$$

$2+D+n(n+1)/2$ scalar functions

$$\{\psi, E, K_m, \varphi_{mn}, \delta\phi\}$$

Transform non-trivially under D-dim diffeomorphisms
Can construct gauge-invariant scalar variables:

$$\begin{pmatrix} x^\mu \\ y^m \end{pmatrix} \to \begin{pmatrix} x^\mu + \xi^\mu(x,y) \\ y^m + \xi^m(x,y) \end{pmatrix}$$

$$\Phi_{mn} = \varphi_{mn} + e^{2A_0}(\partial^p B_0)(K_p - \partial_p E)\tilde{g}_{mn} + \tilde{\nabla}_{(m}\left[e^{2A_0+2B_0}(\partial_{n)}E - K_{n)})\right];$$

$$\Psi = \psi + e^{2A_0}(\partial^p A_0)(K_p - \partial_p E);$$

$$\delta\Phi = \delta\phi + e^{2A_0}(\partial^p\phi_0)(K_p - \partial_p E).$$

Diffeomorphisms remove *D* degrees of freedom

Mixture of dilaton and metric fluctuations

Warped Breathing Mode (p-brane)

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Take *weakly warped (large volume)* limit of background:

$$A_0(y) = \epsilon f(y), \quad \epsilon \propto (\mathrm{Vol})^{-n}, \quad e^{2A(x,y)} \approx 1 - \frac{1}{\gamma} u(x) + 2\epsilon f(y)$$

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Take *weakly warped (large volume)* limit of background:

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$$A_0(y) = \epsilon f(y), \quad \epsilon \propto (\mathrm{Vol})^{-n}, \quad e^{2A(x,y)} \approx 1 - \frac{1}{\gamma} u(x) + 2\epsilon f(y)$$

$$\phi(x,y) = \phi_0(y) + \delta\phi(x)\tilde{\phi}(y), \quad \phi_0(y) \approx -\tilde{\lambda}\epsilon f(y)$$

Can we treat dilaton u(x) and volume modulus δϕ(x) as separate degrees of freedom in weakly warped limit?

$$G_{\mu m} - T_{\mu m} = 0 \sim (\partial_\mu u(x))(\partial_m A_0(y)) + (\partial_\mu \delta\phi(x))(\partial_m \phi_0(y))$$

Weakly warped limit: $\epsilon \neq 0$ Dilaton and volume modulus combine into a single degree of freedom $u(x) = \delta\phi(x)$

Unwarped limit: $\epsilon = 0$ Dilaton and volume modulus decouple, become separate degrees of freedom $u(x) \neq \delta\phi(x)$

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Take *weakly warped (large volume)* limit of background:

$$A_0(y) = \epsilon f(y), \quad \epsilon \propto (\mathrm{Vol})^{-n}, \quad e^{2A(x,y)} \approx 1 - \frac{1}{\gamma} u(x) + 2\epsilon f(y)$$

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Dynamics in extra dimensions

Outline

- Cosmological Perturbation Theory (Review)
- Warped Perturbation Theory
- Example:
  - p-brane backgrounds
- Weakly warped limit ≠ unwarped limit
- Other examples of Warped Perturbation Theory
- **Cosmological applications**

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Other Examples

The combination of the dilaton and the volume modulus into the warped breathing mode is just one example of a larger class of such combinations:

e.g. when background fluxes are turned on:

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Dynamics in extra dimensions

Outline

- Cosmological Perturbation Theory (Review)
- Warped Perturbation Theory
- Example:
  - p-brane backgrounds
- Weakly warped limit ≠ unwarped limit
- Other examples of Warped Perturbation Theory
- **Cosmological applications**

Cosmology with Extra Dimensions

Flat (unwarped) compactification, negatively curved internal space

$$ds_D^2 = e^{-n\psi(x)} g_{\mu\nu} dx^\mu dx^\nu + e^{2\psi(x)} \tilde{g}_{mn} dy^m dy^n$$

$$S_D = \int \sqrt{g_D}\, R_D \longrightarrow S_{eff.4} = \int \sqrt{g_4} \left[R_4 - \frac{n(n+2)}{2} (\partial\psi)^2 - V(\psi) \right]$$

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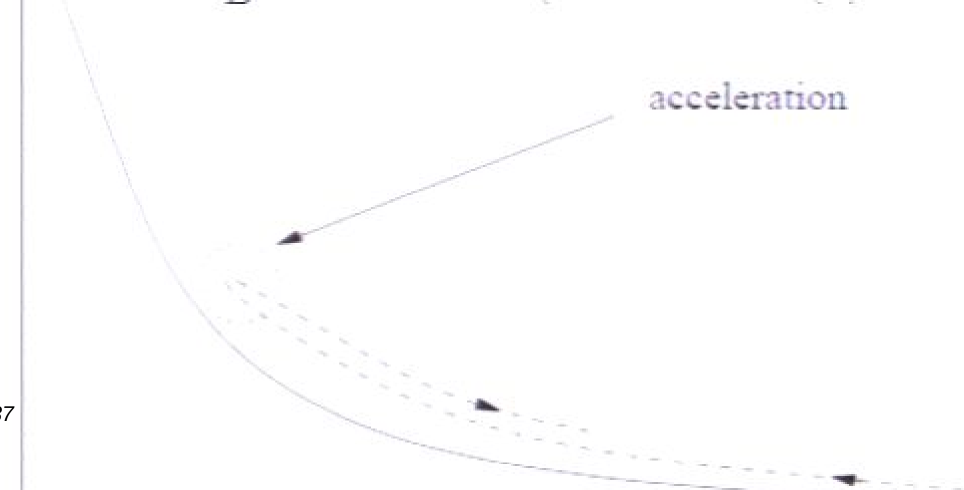

Accelerating universe has a time-dependent internal manifold.

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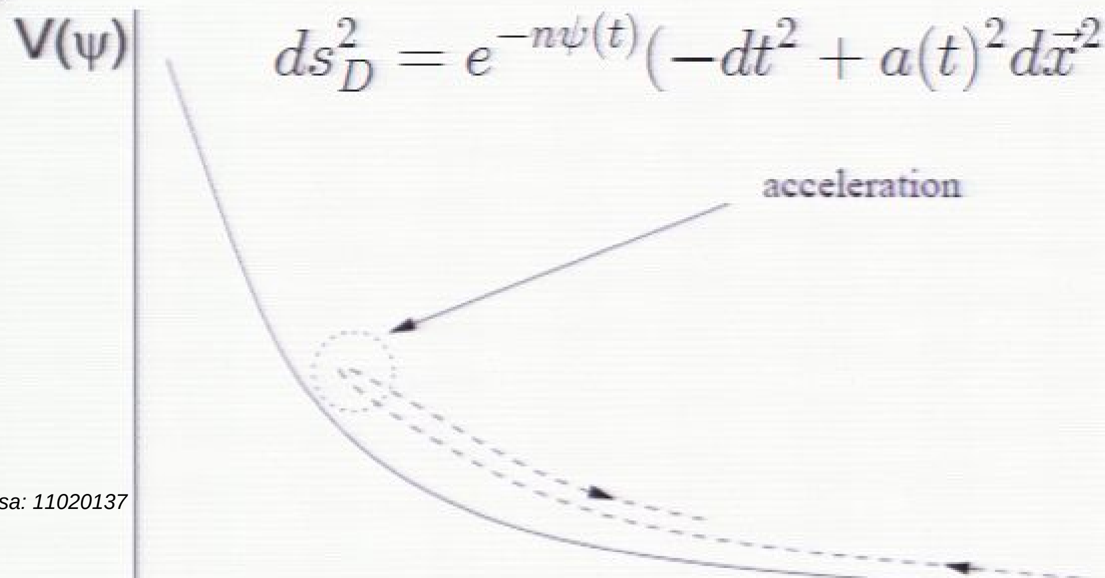

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$$ds^2 = e^{2A(y)}\left(-dt^2 + a(t)^2 d\vec{x}^2\right) + e^{-2A(y)}\tilde{g}_{mn}(y)dy^m dy^n$$

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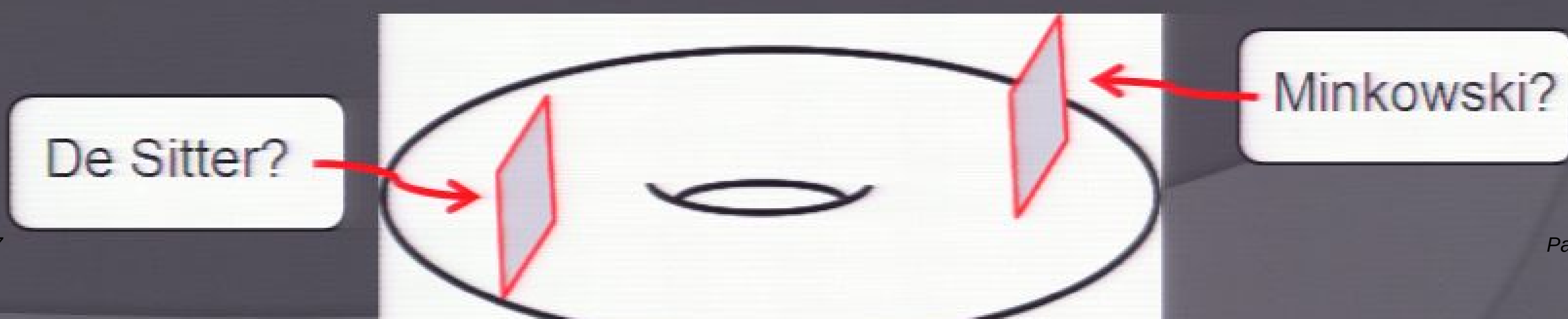
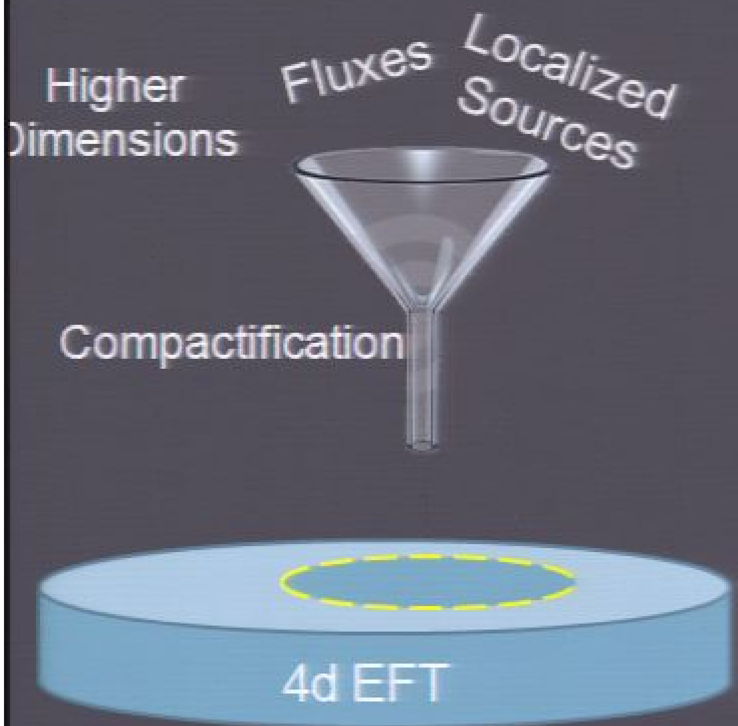
Localized worldvolumes (branes) at different points in extra dimensional manifold can experience different cosmologies?



Describing Extra Dimensions



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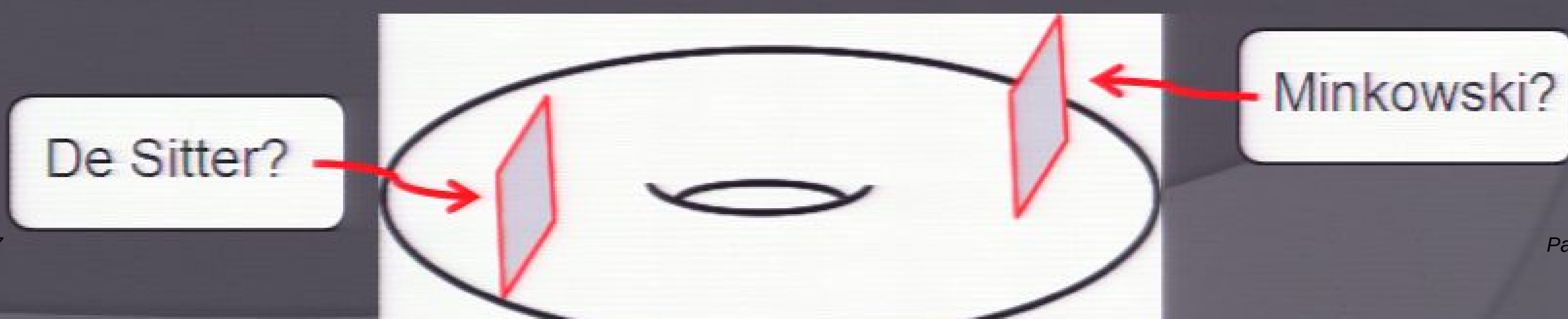

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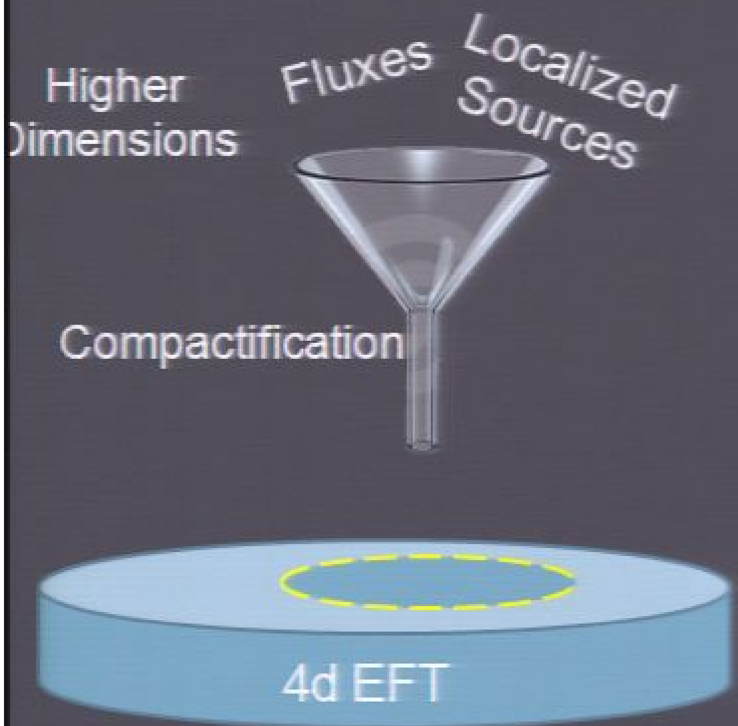

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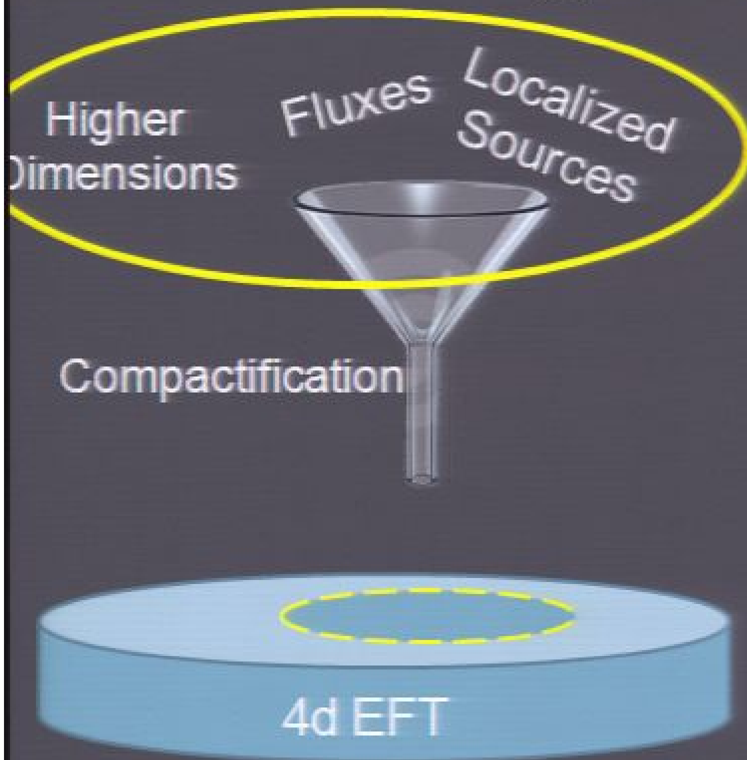

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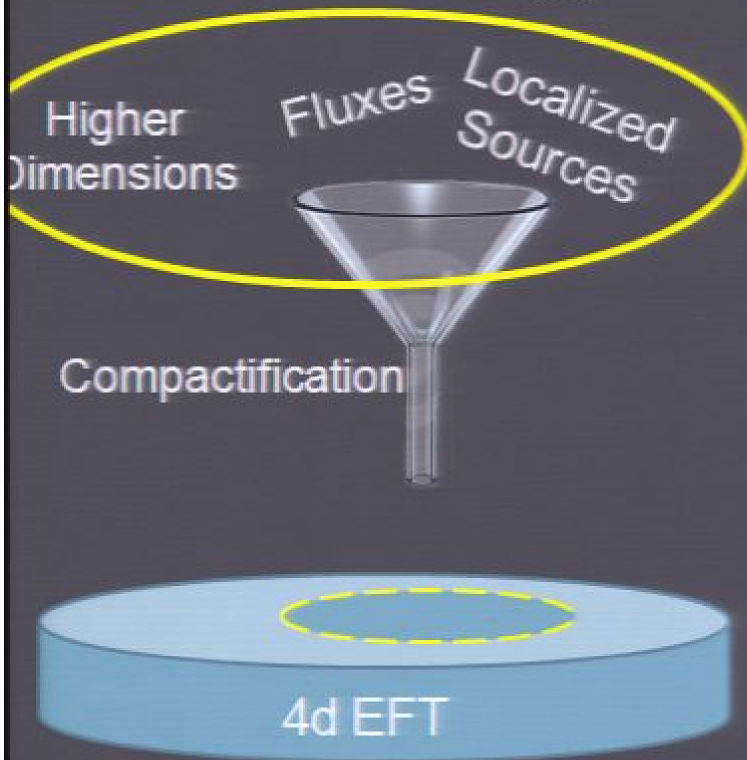

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- Studying perturbations in warped spaces very similar to cosmological perturbation theory
- 2 "Universal" fluctuations: dilaton & volume modulus
  Combine into single fluctuation – Breathing Mode
- Cosmology is inhomogeneous in extra dims:
  Different cosmologies at different points

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