Title: A Breathing mode for Compactifications

Date: Feb 25, 2011 02:30 PM

URL: http://pirsa.org/11020137

Abstract: Reducing a higher dimensional theory to a 4-dimensional effective theory results in a number of scalar fields describing, for instance, fluctuations of higher dimensional scalar fields (dilaton) or the volume of the compact space (volume modulus). But the fields in the effective theory must be constructed with care: artifacts from the higher dimensions, such as higher dimensional diffeomorphisms and constraint equations, can affect the identification of the degrees of freedom. The effective theory including these effects resembles in many ways cosmological perturbation theory. I will show how constraints and diffeomorphisms generically lead the dilaton and volume modulus to combine into a single degree of freedom in the effective theory, the " breathing mode". This has important implications for models of moduli stabilization and inflation with extra dimensions.

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Bret Underwood McGill University

A Breathing Mode for Compactifications

arXiv:1009.4200

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Hierarchy Problem

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Hierarchy Problem

Large (flat) Extra Dimensions

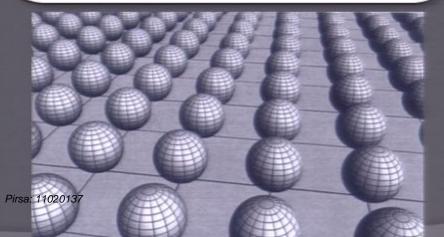
Fundamental scale of gravity is lower

$$M_p^2 = M_n^2 R^n$$

For n = 2, R < 0.1mm,

$$\Rightarrow M_n \sim (\text{few}) \times \text{TeV}$$

[ADD]



Hierarchy Problem

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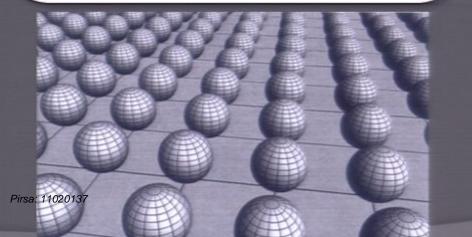
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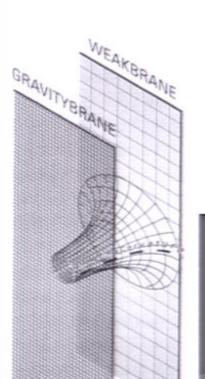
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[ADD]



Warped Extra Dimensions

$$ds^2 = e^{-A(y)} \eta_{\mu\nu} dx^{\mu} dx^{\nu} + dy^2$$



Fundamental scale of gravity depends on location in extra dimension

[Randall-Sundrum]

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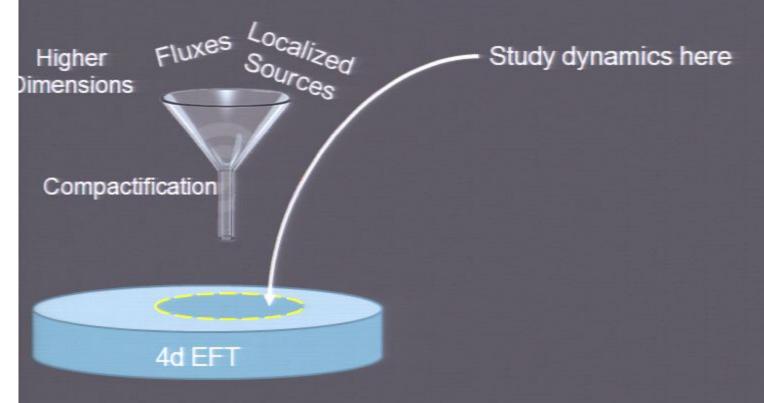
Fundamental scale of gravity depends on location in extra dimension

[Randall-Sundrum]

More generally, extra dimensions typically show up in high energy physics,

e.g. string theory...





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Higher Dimensions 1



Compactification



Study dynamics here

Example: Two *Universal* moduli

Volume Modulus

$$ds_D^2 = g_{\mu\nu}dx^{\mu}dx^{\nu} + \rho(x) \, \tilde{g}_{mn}dy^m dy^n$$

Dilaton (bulk scalar field) $\phi = \phi_0 + \delta \phi(x) \phi(y)$

$$\phi = \phi_0 + \frac{\delta \phi(x) \tilde{\phi}(y)}{\delta}$$

Dimensionally Reduced 4d EFT

$$\mathcal{L}_{eff} \sim rac{(\partial_{\mu}
ho)^2}{
ho^2} + rac{(\partial_{\mu}\delta\phi)^2}{\delta\phi^2} + V_{eff}(
ho,\delta\phi)$$

Example: Two Universal moduli Type IIA String Theory

$$V_{eff} = \left(\frac{A_{curvature}}{\rho} + \frac{A_{NSNS}}{\rho^3}\right)\tau^{-2} - (n_{O6} - n_{D6})A_6\tau^{-3} + \left(\sum_p \rho^{3-p}A_p^{RR}\right)\tau^{-4}$$

$$\left(\tau \equiv e^{-\delta\phi}\rho^{3/2}\right)$$

Can be used to study existence of dS vacua, inflation...

[Hertzberg et al; Silverstein; Underwood et al; Caviezel et al; Flauger et al; Hague et al;...]

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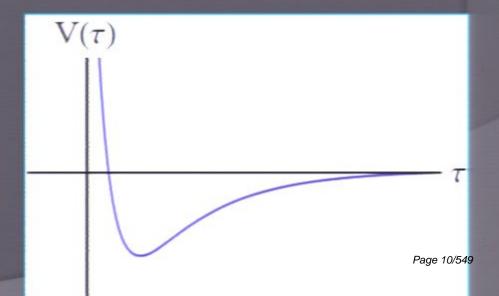
Positive Internal Curvature:



Pirsa: 11020137/e Internal Curvature:



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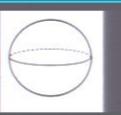
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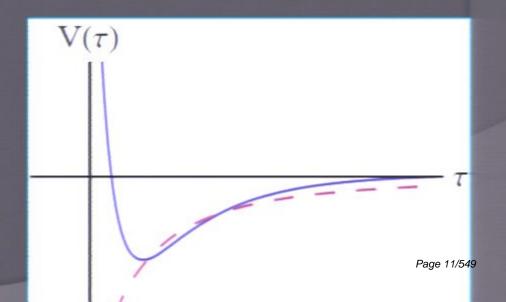
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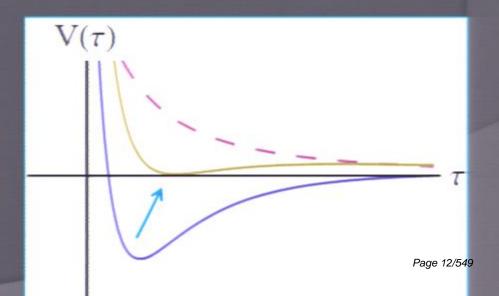
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Compactification

Study dynamics here

Example: Two *Universal* moduli

$$ds_{10}^2 = g_{\mu\nu}dx^{\mu}dx^{\nu} + \rho(x)\,\tilde{g}_{mn}dy^mdy^n$$



$$\phi = \phi_0 + \delta \phi(x) \tilde{\phi}(y)$$

Dimensionally Reduced 4d EFT

$$\mathcal{L}_{eff} = \frac{3}{4} \frac{(\partial_{\mu} \rho)^2}{\rho^2} + \frac{(\partial_{\mu} \delta \phi)^2}{\delta \phi^2} + V_{eff}(\rho, \delta \phi)$$

Are there features missed by the 4d EFT?

4d EFT

$$\partial_{\rho}V_{eff} = 0 \Leftrightarrow G_{mn} - T_{mn} = 0$$

$$\partial_{\delta\phi}V_{eff}=0\Leftrightarrow \text{ Dilaton EOM}=0$$

$$V_{eff}|_{min} = \Lambda \Leftrightarrow G_{\mu\nu} - T_{\mu\nu} = 0$$
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Fluxes Higher)imensions

Compactification



Study dynamics here

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Pirsa: 11/20137 nal Diffeomorphisms?

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Higher Fluxes Localized Sources

Compactification

More generally, want to study dynamics here – beyond validity of 4d Effective Theory

Intrinsically higher dimensional

Modulus
$$ds_{10}^2 = g_{\mu\nu}dx^{\mu}dx^{\nu} + \rho(x)g_{mn}dy^mdy^n$$

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<u>Dynamics in extra dimensions</u> Outline

- Cosmological Perturbation Theory (Review)
- Warped Perturbation Theory
- Example:
 - p-brane backgrounds
- Weakly warped limit ≠ unwarped limit
- Other examples of Warped Perturbation Theory
- Cosmological applications

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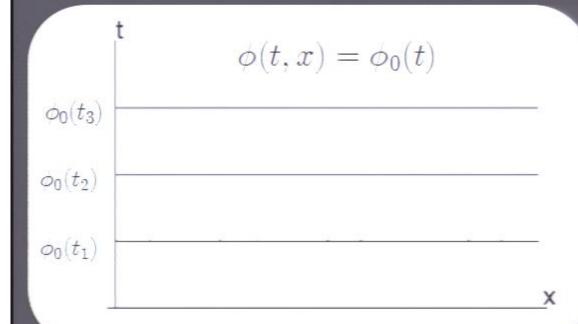
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4-dimensional FLRW spacetime with a homogeneous scalar field

$$ds^{2} = -dt^{2} + a(t)^{2} d\vec{x}^{2}, \quad \phi_{0}(t)$$

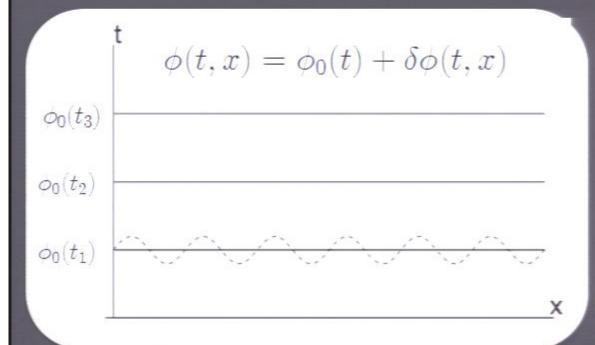


Homogeneous Mode: Constant cosmic time slices are constant in space.

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Homogeneous Mode:

Constant cosmic time slices are constant in space.

Fluctuation:

Constant cosmic time slices not constant on space.

But...distinction between fluctuation and background not a coordinate-independent statement: Under diffeo. $t \to t + \xi^0(t, x)$

$$\phi(t,x)
ightharpoonup \phi(t+\xi^0,x) = \phi_0(t) + \xi^0 \dot{\phi}_0 + \delta \phi = \phi_0(t)$$
 Page 19,

More generally, scalar perturbations about FLRW

$$\begin{split} ds^2 &= -(1+2\varphi(t,x))dt^2 + a^2(t)[(1-2\psi(t,x))\delta_{ij} + 2\partial_i\partial_j E(t,x)]dx^i dx^j \\ &\quad + a(t)\ \partial_i B(t,x) dt dx^i \\ \phi(t,x) &= \phi_0(t) + \delta\phi(t,x) \end{split} \qquad \qquad \text{5 scalar functions} \\ \phi(t,x) &= \phi_0(t) + \delta\phi(t,x) \qquad \qquad \{\varphi,\psi,E,B,\delta\phi\} \end{split}$$

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Transform non-trivially under diffeomorphisms
Can construct gauge-invariant scalar variables:

$$\Phi_B = \varphi - \frac{d}{dt} \left[a^2 (\dot{E} - B/a) \right]$$

$$\Psi_B = \psi \frac{\dot{a}}{a} a^2 (\dot{E} - B/a)$$

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3 gauge-invariant variables

Diffeomorphisms removed 2 degrees of freedom

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5 scalar functions $\{\varphi, \psi, E, B, \delta\phi\}$

3 gauge-invariant variables $\{\Phi_B, \Psi_B, \delta\Phi\}$

Gauge-invariant scalar variables must satisfy constraint equations arising from Einstein equations – non-dynamical equations.

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5-2-2=1 Single independent scalar degree of freedom

"Curvature Perturbation"

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"Curvature Perturbation"

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Now let's use the same reasoning for a theory with extra dimensions

$$ds_D^2 = e^{2A_0(y)} \hat{g}_{\mu\nu}(x) dx^\mu dx^\nu + e^{-2B_0(y)} \underbrace{\tilde{g}_{mn}} dy^m dy^n$$

$$\phi = \phi_0(y)$$
 Spacetime (p+1) Extra Dimensions (D-p-1)

(Some solution someone hands you)

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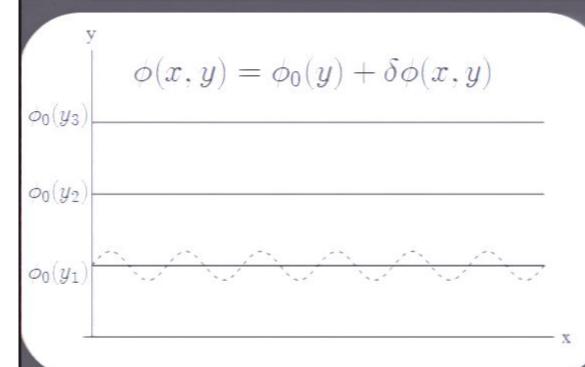
Generically, a solution will be "warped" – sources in extra dimensions always introduce a gravitational potential (warping)

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Fluctuations:

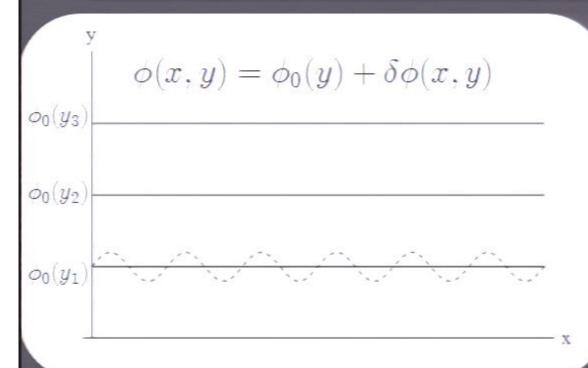
Constant slices along extra dimensions depend on spacetime.

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Fluctuations:

Constant slices along extra dimensions depend on spacetime.

But again, this is <u>not</u> a coordinate-independent statement.

$$y^m \to y^m + \xi^m(x,y)$$

More generally, scalar perturbations about background

$$\begin{split} ds_D^2 &= e^{2A_0(y)} \left[(1-2\psi(x,y)) \hat{g}_{\mu\nu} + 2\hat{\nabla}_{\mu}\partial_{\nu} E(x,y) \right] dx^{\mu} dx^{\nu} \\ &+ e^{2A_0(y)} \partial_{\mu} K_m(x,y) dx^{\mu} dy^m + e^{-2B_0(y)} (\tilde{g}_{mn}(y) + 2\varphi_{mn}(x,y)) dy^m dy^n ; \\ \phi &= \phi_0(y) + \delta\phi(x,y) \; . \end{split}$$

$$\begin{aligned} 2 + D + n(n+1)/2 \text{ scalar functions} \\ \{\psi, E, K_m, \varphi_{mn}, \delta\phi \} \end{aligned}$$

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Transform non-trivially under D-dim diffeomorphisms Can construct gauge-invariant scalar variables:

$$\begin{pmatrix} x^{\mu} \\ y^{m} \end{pmatrix} \rightarrow \begin{pmatrix} x^{\mu} + \xi^{\mu}(x, y) \\ y^{m} + \xi^{m}(x, y) \end{pmatrix}$$

$$\begin{split} &\Phi_{mn} = \varphi_{mn} + e^{2A_0}(\partial^p B_0)(K_p - \partial_p E)\tilde{g}_{mn} + \tilde{\nabla}_{(m} \left[e^{2A_0 + 2B_0}(\partial_n)E - K_n) \right]; \\ &\Psi = \psi + e^{2A_0}(\partial^p A_0)(K_p - \partial_p E); \\ &\delta\Phi = \delta\phi + e^{2A_0}(\partial^p \phi_0)(K_p - \partial_p E) \,. \end{split} \qquad \begin{array}{l} \text{Diffeomorphisms remove D} \\ \text{degrees of freedom} \end{split}$$

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Gauge-invariant scalar variables must satisfy constraint equations arising from Einstein equations – non-dynamical equations.

$$\begin{split} \delta G_{\mu\nu} - \kappa_D^2 \delta T_{\mu\nu} \big|_{\mu \neq \nu} &= \hat{\nabla}_{\mu} \partial_{\nu} \left[(p-1) \Psi - \Phi_p^{\tilde{p}} \right] = 0; \\ \delta G_{\mu m} - \kappa_D^2 \delta T_{\mu m} &= -\partial_{\mu} \partial_{m} \left[p \Psi + \Phi_p^{\tilde{p}} \right] + \partial_{\mu} \tilde{\nabla}_p \Phi_m^{\tilde{p}} + \partial_{\mu} \Phi_p^{\tilde{p}} \left[\partial_m A_0 + \partial_m B_0 \right] \\ &+ \partial_{\mu} \Phi_m^{\tilde{p}} \left[(p-1) \partial_p A_0 - (D-p-1) \partial_p B_0 \right] + \frac{1}{2} \partial_{\mu} \delta \Phi \partial_m \phi_0 = 0 \; . \end{split}$$

Constraint equations remove another D degrees of freedom.

Left with 1 + (D-p-1)(D-p+2)/2 independent scalar + metric d.o.f.

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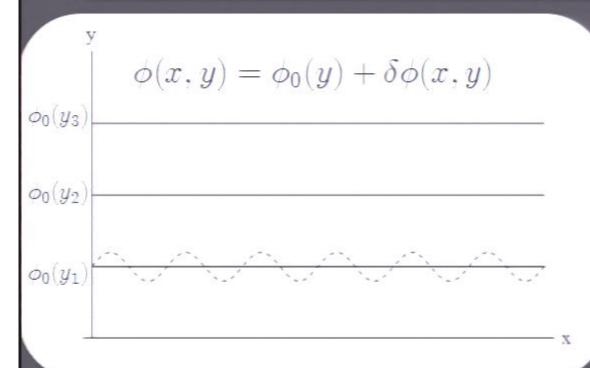
$$\begin{split} &\Phi_{mn} = \varphi_{mn} + e^{2A_0}(\partial^p B_0)(K_p - \partial_p E)\tilde{g}_{mn} + \tilde{\nabla}_{(m} \left[e^{2A_0 + 2B_0}(\partial_n E - K_n) \right]; \\ &\Psi = \psi + e^{2A_0}(\partial^p A_0)(K_p - \partial_p E); \\ &\delta\Phi = \delta\phi + e^{2A_0}(\partial^p \phi_0)(K_p - \partial_p E) \,. \end{split} \qquad \begin{array}{l} \text{Diffeomorphisms remove D} \\ \text{degrees of freedom} \end{split}$$

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Now let's use the same reasoning for a theory with extra dimensions

$$ds_D^2 = e^{2A_0(y)} \hat{g}_{\mu\nu}(x) dx^\mu dx^\nu + e^{-2B_0(y)} \tilde{g}_{mn} dy^m dy^n$$

$$\phi = \phi_0(y)$$
 Spacetime (p+1) Extra Dimensions (D-p-1)



Fluctuations:

Constant slices along extra dimensions depend on spacetime.

But again, this is <u>not</u> a coordinate-independent statement.

$$y^m \rightarrow y^m + \xi^m(x,y)$$

More generally, scalar perturbations about background

$$\begin{split} ds_D^2 &= e^{2A_0(y)} \left[(1-2\psi(\boldsymbol{x},\boldsymbol{y})) \hat{g}_{\mu\nu} + 2\hat{\nabla}_{\mu}\partial_{\nu}\boldsymbol{E}(\boldsymbol{x},\boldsymbol{y}) \right] dx^{\mu}dx^{\nu} \\ &+ e^{2A_0(y)} \partial_{\mu}\boldsymbol{K}_m(\boldsymbol{x},\boldsymbol{y}) dx^{\mu}dy^m + e^{-2B_0(y)} (\tilde{g}_{mn}(y) + 2\varphi_{mn}(\boldsymbol{x},\boldsymbol{y})) dy^m dy^n ; \\ \phi &= \phi_0(y) + \delta\phi(\boldsymbol{x},\boldsymbol{y}) \; . \end{split}$$

$$\begin{aligned} &2 + D + n(n+1)/2 \text{ scalar functions} \\ &\{\psi,E,K_m,\varphi_{mn},\delta\phi\} \end{aligned}$$

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More generally, scalar perturbations about background

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Gauge-invariant scalar variables must satisfy constraint equations arising from Einstein equations – non-dynamical equations.

$$\begin{split} \delta G_{\mu\nu} - \kappa_D^2 \delta T_{\mu\nu}\big|_{\mu\neq\nu} &= \hat{\nabla}_\mu \partial_\nu \left[(p-1)\Psi - \Phi_p^{\tilde{p}} \right] = 0; \\ \delta G_{\mu m} - \kappa_D^2 \delta T_{\mu m} &= -\partial_\mu \partial_m \left[p\Psi + \Phi_p^{\tilde{p}} \right] + \partial_\mu \tilde{\nabla}_p \Phi_m^{\tilde{p}} + \partial_\mu \Phi_p^{\tilde{p}} \left[\partial_m A_0 + \partial_m B_0 \right] \\ &+ \partial_\mu \Phi_m^{\tilde{p}} \left[(p-1)\partial_p A_0 - (D-p-1)\partial_p B_0 \right] + \frac{1}{2} \partial_\mu \delta \Phi \partial_m \phi_0 = 0 \; . \end{split}$$

Constraint equations remove another D degrees of freedom.

Left with 1 + (D-p-1)(D-p+2)/2 independent scalar + metric d.o.f.

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It is inconsistent to turn on only a dilaton fluctuation:

$$\delta G_{\mu m} - \kappa_D^2 \delta T_{\mu m} = \frac{1}{2} \partial_\mu \delta \phi(x,y) \partial_m \phi_0 = 0$$

Gauge-invariant dilaton fluctuation must combine with some metric fluctuation in order to create a consistent, gauge-invariant degree of freedom.

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Which fluctuation could this be?

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Back to the cosmological case:

Scalar field fluctuation $\phi(t,x)=\phi_0(t)+\delta\phi(t,x)$ mixes with the curvature perturbation (spatial volume perturbation)

$$g_{ij} = a^2(t)e^{2\zeta(t,x)}\delta_{ij}$$

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<u>Dynamics in extra dimensions</u> Outline

- Cosmological Perturbation Theory (Review)
- Warped Perturbation Theory
- Example:
 - p-brane backgrounds
- Weakly warped limit ≠ unwarped limit
- Other examples of Warped Perturbation Theory
- Cosmological applications

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D-dimensional gravity, dilaton, (p+2)-form, localized source

$$S = \frac{1}{2\kappa_D^2} \int d^D x \sqrt{g_D} \left[R_D - \frac{1}{2} (\partial \phi)^2 - \frac{e^{-\lambda \phi}}{2(p+2)!} F_{p+2}^2 \right] + S_{loc},$$

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p-brane solutions:

$$\begin{split} ds^2 &= e^{2A_0(y)} \hat{\eta}_{\mu\nu} dx^{\mu} dx^{\nu} + e^{-\left(\frac{p+1}{D-p-3}\right)A_0(y)} \tilde{g}_{mn}(y) dy^m dy^n; \\ \phi &= -\tilde{\lambda} A_0(y), \quad C_{p+1} = \pm e^{\tilde{a}A_0(y)} \hat{\epsilon}_{p+1} \end{split}$$

Harmonic Function:

$$\tilde{\nabla}^2 e^{-2\gamma A_0(y)} = \sum_n Q_n \delta^{(D-p-1)}(y - y_n) \,.$$

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Natural Shift-Invariance $e^{-2\gamma A_0(y)} \to e^{-2\gamma A_0(y)} + u$ implies an ansatz for dynamical "breathing" mode:

$$e^{-2\gamma A(x,y)} = e^{-2\gamma A_0(y)} + u(x)$$

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Almost...Need Weyl Rescaling and Compensator

$$\begin{split} ds_{D}^{2} &= e^{2A(y,u(x))}e^{2\Omega[u(x)]} \left[\hat{g}_{\mu\nu} + 2e^{(p-3)\Omega[u(x)]} (\hat{\nabla}_{\mu}\partial_{\nu}u(x))E(y) \right] dx^{\mu}dx^{\nu}, \\ &+ e^{-2\left(\frac{p+1}{D-p-3}\right)A(y,u(x))} \tilde{g}_{mn}(y)dy^{m}dy^{n} \\ &\phi(x,y) = -\tilde{\lambda}A(y,u(x))), \quad C_{p+1} = \pm e^{\tilde{a}A(y,u(x))} \hat{\epsilon}_{p+1} \\ &e^{-2\gamma A(x,y)} = e^{-2\gamma A_{0}(y)} + u(x) \end{split}$$

u(x) is a single (p+1)-dimensional degree of freedom

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$$\begin{split} ds_D^2 &= e^{2A(\boldsymbol{y},\boldsymbol{u}(\boldsymbol{x}))} e^{2\Omega[\boldsymbol{u}(\boldsymbol{x})]} \left[\hat{g}_{\mu\nu} + 2e^{(p-3)\Omega[\boldsymbol{u}(\boldsymbol{x})]} (\hat{\nabla}_{\mu}\partial_{\nu}\boldsymbol{u}(\boldsymbol{x}))\boldsymbol{E}(\boldsymbol{y}) \right] dx^{\mu}dx^{\nu}, \\ &+ e^{-2\left(\frac{p+1}{D-p-3}\right)A(\boldsymbol{y},\boldsymbol{u}(\boldsymbol{x}))} \tilde{g}_{mn}(\boldsymbol{y})dy^m dy^n \\ \hline \phi(\boldsymbol{x},\boldsymbol{y}) &= -\tilde{\lambda}A(\boldsymbol{y},\boldsymbol{u}(\boldsymbol{x})), \quad C_{p+1} = \pm e^{\tilde{a}A(\boldsymbol{y},\boldsymbol{u}(\boldsymbol{x}))} \hat{\epsilon}_{p+1} \end{split}$$

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Dilaton fluctuation

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$$\phi(x,y)=- ilde{\lambda}A(y,u(x)), \qquad C_{p+1}=\pm e^{ ilde{a}A(y,u(x))}\hat{\epsilon}_{p+1}$$
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Dilaton fluctuation + Volume modulus fluctuation

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$$ds_D^2 = e^{2\boldsymbol{A}(\boldsymbol{y},\boldsymbol{u}(\boldsymbol{x}))} e^{2\boldsymbol{\Omega}[\boldsymbol{u}(\boldsymbol{x})]} \left[\hat{g}_{\mu\nu} + 2e^{(p-3)\boldsymbol{\Omega}[\boldsymbol{u}(\boldsymbol{x})]} (\hat{\nabla}_{\mu}\partial_{\nu}\boldsymbol{u}(\boldsymbol{x}))\boldsymbol{E}(\boldsymbol{y}) \right] dx^{\mu}dx^{\nu},$$
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Dilaton fluctuation + Volume modulus fluctuation

4d degree of freedom of <u>dilaton fluctuation</u> also controls <u>volume</u> <u>modulus</u> degree of freedom – <u>warped breathing mode</u>

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Almost...Need Weyl Rescaling and Compensator

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$$\phi(x,y)=- ilde{\lambda}A(y,u(x)), \qquad C_{p+1}=\pm e^{ ilde{a}A(y,u(x))}\hat{\epsilon}_{p+1}$$
 $e^{-2\gamma A(x,y)}=e^{-2\gamma A_0(y)}+u(x)$

u(x) is a single (p+1)-dimensional degree of freedom

Dilaton fluctuation + Volume modulus fluctuation

4d degree of freedom of <u>dilaton fluctuation</u> also controls <u>volume</u> <u>modulus</u> degree of freedom – <u>warped breathing mode</u>

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$$\begin{split} ds_D^2 &= e^{2\boldsymbol{A}(\boldsymbol{y},\boldsymbol{u}(\boldsymbol{x}))} e^{2\boldsymbol{\Omega}[\boldsymbol{u}(\boldsymbol{x})]} \left[\hat{g}_{\mu\nu} + 2e^{(p-3)\boldsymbol{\Omega}[\boldsymbol{u}(\boldsymbol{x})]} (\hat{\nabla}_{\mu}\partial_{\nu}\boldsymbol{u}(\boldsymbol{x}))\boldsymbol{E}(\boldsymbol{y}) \right] dx^{\mu}dx^{\nu}, \\ &+ e^{-2\left(\frac{p+1}{D-p-3}\right)\boldsymbol{A}(\boldsymbol{y},\boldsymbol{u}(\boldsymbol{x}))} \tilde{g}_{mn}(\boldsymbol{y})dy^m dy^n \end{split}$$

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Dilaton fluctuation + Volume modulus fluctuation

4d degree of freedom of <u>dilaton fluctuation</u> also controls <u>volume</u> <u>modulus</u> degree of freedom – <u>warped breathing mode</u>

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Almost...Need Weyl Rescaling and Compensator

$$ds_D^2 = e^{2\mathbf{A}(\mathbf{y},\mathbf{u}(\mathbf{x}))} e^{2\mathbf{\Omega}[\mathbf{u}(\mathbf{x})]} \left[\hat{g}_{\mu\nu} + 2e^{(p-3)\Omega[\mathbf{u}(\mathbf{x})]} (\hat{\nabla}_{\mu}\partial_{\nu}\mathbf{u}(\mathbf{x}))\mathbf{E}(\mathbf{y}) \right] dx^{\mu}dx^{\nu}.$$
$$+ e^{-2\left(\frac{p+1}{D-p-3}\right)\mathbf{A}(\mathbf{y},\mathbf{u}(\mathbf{x}))} \tilde{g}_{mn}(\mathbf{y})dy^m dy^n$$

$$\phi(x,y) = -\tilde{\lambda}A(y,u(x)), \qquad C_{p+1} = \pm e^{\tilde{a}A(y,u(x))}\hat{\epsilon}_{p+1}$$

$$e^{-2\gamma A(x,y)} = e^{-2\gamma A_0(y)} + u(x)$$

u(x) is a single (p+1)-dimensional degree of freedom

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Almost...Need Weyl Rescaling and Compensator

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$$\phi(x,y)=- ilde{\lambda}A(y,u(x)), \qquad C_{p+1}=\pm e^{ ilde{a}A(y,u(x))}\hat{\epsilon}_{p+1}$$
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Effective Kinetic Term

Effective kinetic term for warped breathing mode comes from gravity and dilaton sectors:

$$\begin{split} S_{eff}^{kin} &= \int \sqrt{g_D} \left[R_D - \frac{1}{2} (\partial \phi)^2 \right] \\ &= \int \sqrt{\hat{g}} \left(\mathcal{G}_{uu}^{(g)} + \mathcal{G}_{uu}^{(o)} \right) \partial_{\mu} u(x) \partial^{\mu} u(x) \end{split}$$

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Gravity kinetic term gives the usual volume modulus kinetic term.

$$\mathcal{G}_{uu}^{(g)} = -\left(\frac{(p+1)^2(D-p-1)}{8\kappa_{p+1}^2(D-2)(p-1)}\right)\frac{1}{(u(x)+\tilde{V}_W^{(0)}/\tilde{V}_{D-p-1})^2}$$

Dilaton kinetic term is not as nice:

$$\mathcal{G}_{uu}^{(o)} = -\frac{1}{4\kappa_D^2} \int \sqrt{\tilde{g}} e^{(p-1)\Omega} \frac{\tilde{\lambda}^2}{4\gamma^2} e^{2\gamma A}$$

Pirsa: 11020137 Effective Theory is different than if had ignored these effects:

<u>Dynamics in extra dimensions</u> Outline

- Cosmological Perturbation Theory (Review)
- Warped Perturbation Theory
- Example:
 - p-brane backgrounds
- Weakly warped limit ≠ unwarped limit
- Other examples of Warped Perturbation Theory
- Cosmological applications

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Weakly Warped Limit

Take weakly warped (large volume) limit of background:

$$A_0(y) = \epsilon f(y), \quad \epsilon \propto (\text{Vol})^{-n}, \quad e^{2A(x,y)} \approx 1 - \frac{1}{\gamma} \mathbf{u}(\mathbf{x}) + 2\epsilon f(y)$$

$$\phi(x,y) = \phi_0(y) + \delta \phi(\mathbf{x}) \tilde{\phi}(y), \quad \phi_0(y) \approx -\tilde{\lambda} \epsilon f(y)$$

Can we treat dilaton u(x) and volume modulus $\delta \phi(x)$ as separate degrees of freedom in weakly warped limit?

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<u>Dynamics in extra dimensions</u> Outline

- Cosmological Perturbation Theory (Review)
- Warped Perturbation Theory
- Example:
 - p-brane backgrounds
- Weakly warped limit ≠ unwarped limit
- Other examples of Warped Perturbation Theory
- Cosmological applications

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Warped Breathing Mode (p-brane)

Almost...Need Weyl Rescaling and Compensator

$$\begin{split} ds_D^2 &= e^{2\boldsymbol{A}(\boldsymbol{y},\boldsymbol{u}(\boldsymbol{x}))} e^{2\boldsymbol{\Omega}[\boldsymbol{u}(\boldsymbol{x})]} \left[\hat{g}_{\mu\nu} + 2e^{(p-3)\boldsymbol{\Omega}[\boldsymbol{u}(\boldsymbol{x})]} (\hat{\nabla}_{\mu}\partial_{\nu}\boldsymbol{u}(\boldsymbol{x}))\boldsymbol{E}(\boldsymbol{y}) \right] dx^{\mu}dx^{\nu}, \\ &+ e^{-2\left(\frac{p+1}{D-p-3}\right)\boldsymbol{A}(\boldsymbol{y},\boldsymbol{u}(\boldsymbol{x}))} \tilde{g}_{mn}(\boldsymbol{y})dy^m dy^n \end{split}$$

$$\phi(x,y)=- ilde{\lambda}A(y,u(x)), \qquad C_{p+1}=\pm e^{ ilde{a}A(y,u(x))}\hat{\epsilon}_{p+1}$$
 $e^{-2\gamma A(x,y)}=e^{-2\gamma A_0(y)}+u(x)$

u(x) is a single (p+1)-dimensional degree of freedom

Dilaton fluctuation + Volume modulus fluctuation

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Warped Perturbation Theory

Gauge-invariant dilaton fluctuation must combine with some metric fluctuation in order to create a consistent, gauge-invariant degree of freedom.

Which fluctuation could this be?

Back to the cosmological case:

Scalar field fluctuation $\phi(t,x) = \phi_0(t) + \delta\phi(t,x)$ mixes with the curvature perturbation (spatial volume perturbation)

$$g_{ij} = a^2(t)e^{2\zeta(t,x)}\delta_{ij}$$

Similarly, expect *dilaton* fluctuation to combine with the *volume modulus* fluctuation of the metric:

$$g_{mn} = e^{-2B_0(y)} e^{2\beta\varphi(x)} \tilde{g}_{mn}(y)$$

(Need to generalize "volume modulus" to warped space)

Warped Perturbation Theory

Gauge-invariant scalar variables must satisfy constraint equations arising from Einstein equations – non-dynamical equations.

$$\begin{split} \delta G_{\mu\nu} - \kappa_D^2 \delta T_{\mu\nu} \big|_{\mu\neq\nu} &= \hat{\nabla}_{\mu} \partial_{\nu} \left[(p-1) \Psi - \Phi_p^{\tilde{p}} \right] = 0; \\ \delta G_{\mu m} - \kappa_D^2 \delta T_{\mu m} &= -\partial_{\mu} \partial_{m} \left[p \Psi + \Phi_p^{\tilde{p}} \right] + \partial_{\mu} \tilde{\nabla}_p \Phi_m^{\tilde{p}} + \partial_{\mu} \Phi_p^{\tilde{p}} \left[\partial_m A_0 + \partial_m B_0 \right] \\ &+ \partial_{\mu} \Phi_m^{\tilde{p}} \left[(p-1) \partial_p A_0 - (D-p-1) \partial_p B_0 \right] + \frac{1}{2} \partial_{\mu} \delta \Phi \partial_m \phi_0 = 0 \; . \end{split}$$

It is inconsistent to turn on only a dilaton fluctuation:

$$\delta G_{\mu m} - \kappa_D^2 \delta T_{\mu m} = \frac{1}{2} \partial_\mu \delta \phi(x, y) \partial_m \phi_0 = 0$$

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Warped Perturbation Theory

More generally, scalar perturbations about background

$$\begin{split} ds_D^2 &= e^{2A_0(y)} \left[(1-2\psi(\boldsymbol{x},\boldsymbol{y})) \hat{g}_{\mu\nu} + 2\hat{\nabla}_{\mu} \partial_{\nu} \boldsymbol{E}(\boldsymbol{x},\boldsymbol{y}) \right] dx^{\mu} dx^{\nu} \\ &+ e^{2A_0(y)} \partial_{\mu} \boldsymbol{K}_{m}(\boldsymbol{x},\boldsymbol{y}) dx^{\mu} dy^{m} + e^{-2B_0(y)} (\tilde{g}_{mn}(y) + 2\varphi_{mn}(\boldsymbol{x},\boldsymbol{y})) dy^{m} dy^{n} \\ \phi &= \phi_0(y) + \delta\phi(\boldsymbol{x},\boldsymbol{y}) \;. \end{split}$$

$$\begin{aligned} &2 + D + n(n+1)/2 \text{ scalar functions} \\ &\{\psi,E,K_m,\varphi_{mn},\delta\phi\} \end{aligned}$$

Transform non-trivially under D-dim diffeomorphisms Can construct gauge-invariant scalar variables:

$$\begin{pmatrix} x^{\mu} \\ y^{m} \end{pmatrix} \rightarrow \begin{pmatrix} x^{\mu} + \xi^{\mu}(x, y) \\ y^{m} + \xi^{m}(x, y) \end{pmatrix}$$

$$\begin{split} &\Phi_{mn}=\varphi_{mn}+e^{2A_0}(\partial^p B_0)(K_p-\partial_p E)\tilde{g}_{mn}+\tilde{\nabla}_{(m}\left[e^{2A_0+2B_0}(\partial_n)E-K_n)\right];\\ &\Psi=\psi+e^{2A_0}(\partial^p A_0)(K_p-\partial_p E);\\ &\Phi=\delta\phi+e^{2A_0}(\partial^p \phi_0)(K_p-\partial_p E)\,. \end{split} \qquad \begin{array}{l} \text{Diffeomorphisms remove D}\\ &\text{degrees of freedom} \end{split}$$

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Warped Breathing Mode (p-brane)

Almost...Need Weyl Rescaling and Compensator

$$\begin{split} ds_{D}^{2} &= e^{2A(y,u(x))}e^{2\Omega[u(x)]} \left[\hat{g}_{\mu\nu} + 2e^{(p-3)\Omega[u(x)]} (\hat{\nabla}_{\mu}\partial_{\nu}u(x))E(y) \right] dx^{\mu}dx^{\nu}, \\ &+ e^{-2\left(\frac{p+1}{D-p-3}\right)A(y,u(x))} \tilde{g}_{mn}(y)dy^{m}dy^{n} \\ &\phi(x,y) = -\tilde{\lambda}A(y,u(x))), \quad C_{p+1} = \pm e^{\tilde{a}A(y,u(x))} \hat{\epsilon}_{p+1} \\ &e^{-2\gamma A(x,y)} = e^{-2\gamma A_{0}(y)} + u(x) \end{split}$$

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Unwarped limit is singular limit — not smoothly connected to weakly warped limit!

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$$A_0(y) = \epsilon f(y), \quad \epsilon \propto (\text{Vol})^{-n}, \quad e^{2A(x,y)} \approx 1 - \frac{1}{\gamma} \mathbf{u}(\mathbf{x}) + 2\epsilon f(y)$$

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Unwarped limit is singular limit — not smoothly connected to weakly warped limit!

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Thus, axions are not gauge-invariant d.o.f.

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Similarly with open string degrees of freedom (D-brane positions):

$$Z_{\perp}^{i}(x) = Z_{\perp,0}^{i} + \delta Z_{\perp}^{i}(x), \quad Z_{\perp}^{i} \longrightarrow Z_{\perp,0}^{i} + \delta Z_{\perp}^{i}(x) + \xi^{i}(x) = Z_{\perp,0}^{i}$$

The combination of the dilaton and the volume modulus into the warped breathing mode is just one example of a larger class of such combinations:

e.g. when background fluxes are turned on: axions transform under diffeomorphisms

$$F_3 = F_3^{(0)} + db(x) \wedge \omega_2(y)$$

 $F_3 \to F_3 + d\left(\xi \cdot F_3^{(0)}\right) \to F_3^{(0)}$

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<u>Dynamics in extra dimensions</u> Outline

- Cosmological Perturbation Theory (Review)
- Warped Perturbation Theory
- Example:
 - p-brane backgrounds
- Weakly warped limit ≠ unwarped limit
- Other examples of Warped Perturbation Theory
- Cosmological applications

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$$Z_{\perp}^{i}(x) = Z_{\perp,0}^{i} + \delta Z_{\perp}^{i}(x), \quad Z_{\perp}^{i} \longrightarrow Z_{\perp,0}^{i} + \delta Z_{\perp}^{i}(x) + \xi^{i}(x) = Z_{\perp,0}^{i}$$

The combination of the dilaton and the volume modulus into the warped breathing mode is just one example of a larger class of such combinations:

e.g. when background fluxes are turned on: axions transform under diffeomorphisms

$$F_3 = F_3^{(0)} + db(x) \wedge \omega_2(y)$$

 $F_3 \to F_3 + d\left(\xi \cdot F_3^{(0)}\right) \to F_3^{(0)}$

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<u>Dynamics in extra dimensions</u> Outline

- Cosmological Perturbation Theory (Review)
- Warped Perturbation Theory
- Example:
 - p-brane backgrounds
- Weakly warped limit ≠ unwarped limit
- Other examples of Warped Perturbation Theory
- Cosmological applications

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Flat (unwarped) compactification, negatively curved internal space

$$ds_D^2 = e^{-n\psi(x)}g_{\mu\nu}dx^\mu dx^\nu + e^{2\psi(x)}\tilde{g}_{mn}dy^m dy^n$$

$$S_D = \int \sqrt{g_D} R_D \longrightarrow S_{eff,4} = \int \sqrt{g_4} \left[R_4 - \frac{n(n+2)}{2} (\partial \psi)^2 - V(\psi) \right]$$
$$V(\psi) = n(n-1)e^{-(n+2)\psi}$$

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acceleration

Accelerating universe has a time-dependent internal manifold.

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"Nice" cosmology with extra dimensions:

$$ds^{2} = e^{2A(y)} \left(-dt^{2} + a(t)^{2} d\vec{x}^{2} \right) + e^{-2A(y)} \tilde{g}_{mn}(y) dy^{m} dy^{n}$$

But ansatz required by warped perturbation theory is more intricate:

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Localized worldvolumes (branes) at different points in extra dimensional manifold can experience different cosmologies?







Are there features missed by the 4d EFT?

$$-G_{\mu m} - T_{\mu m} = 0$$
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- Internal Diffeomorphisms?

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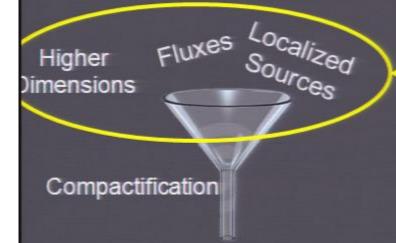


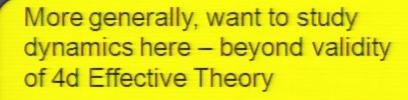
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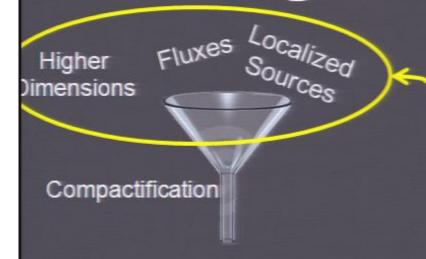
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4d EFT

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More generally, want to study dynamics here – beyond validity of 4d Effective Theory

Intrinsically higher dimensional

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- Internal Diffeomorphisms?



- Studying perturbations in warped spaces very similar to cosmological perturbation theory
- 2 "Universal" fluctuations: dilaton & volume modulus
 Combine into single fluctuation <u>Breathing Mode</u>
- Cosmology is inhomogeneous in extra dims:
 Different cosmologies at different points

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