Title: Introduction to the de Broglie-Bohm Theory - Lecture 1

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Abstract:

PILOT-WAVE THEORY

Introduction to the ~~de Broglie-Bohm theory~~

Lecture 1: non-relativistic case and relativistic wave-equations

Samuel Colin

Griffith UNIVERSITY
Centre for Quantum Dynamics
Brisbane, Australia

22 February 2011

BOHMIAN MECHANICS

Introduction to ~~the de Broglie-Bohm theory~~

Lecture 1: non-relativistic case and relativistic wave-equations

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CAUSAL INTERPRETATION

Introduction to the ~~de Broglie-Bohm theory~~

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THEORIES OF DEBROGLIE AND BOHM

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. . . 84 years ago: a new theory of motion for particles

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de Broglie, Bohm and Bell

L. de Broglie (1897–1987).

David J. Bohm (1917–1992).

John S. Bell (1928–1990).

A possible quantum theory without observers: the de Broglie-Bohm theory

- Same predictions as nonrelativistic quantum mechanics.
- No distinction systems-observers.
- Deterministic.
- About particles in motion.

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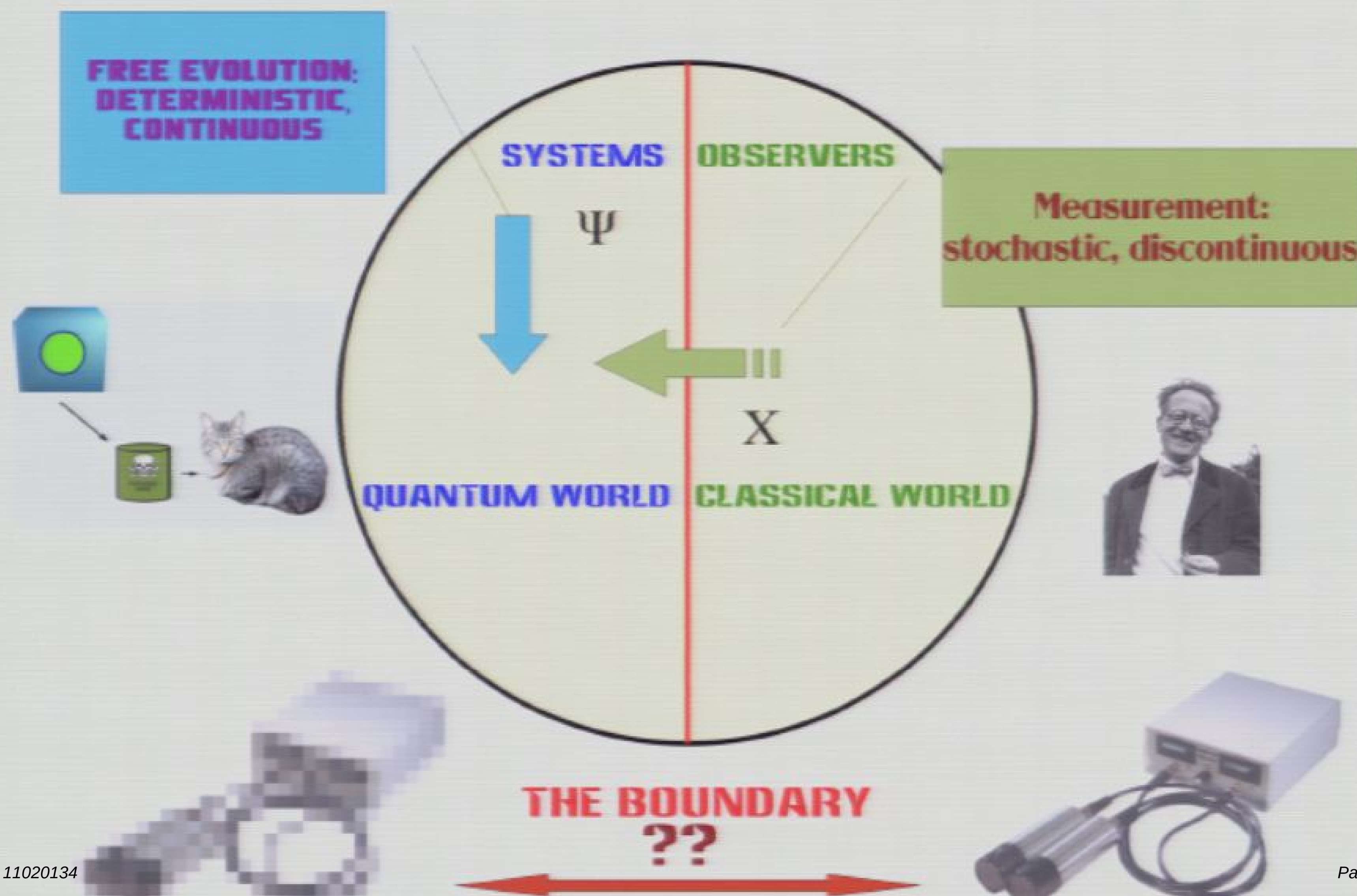
Standard quantum theory with observers



Other quantum theories without observers

- collapse models (Pearle, Ghirardi-Rimini-Weber, Diósi)
- nonlinear modifications of quantum mechanics (de Broglie, Bialynicki-Birula & Mycielski, Kibble, Penrose, Weinberg)
- many-world interpretation (Everett)

Books. . . and more to come.

Outline of the mini-course

- Lecture 1: the de Broglie-Bohm pilot-wave theory (inc. relativistic wave-equations)
- Lecture 2: pilot-wave models for quantum field theory
- Lecture 3: quantum non-equilibrium

Outline of this lecture

- The non-relativistic de Broglie-Bohm theory
  - The 1-particle dBB theory
  - The many-particle dBB theory and the measurement
  - Spin
  - Symmetries: Galilean invariance, time-reversal, identical particles.
- Some issues:
  - Justification of quantum equilibrium
  - Meaning of the wave-function
  - Surrealistic trajectories
  - Non-uniqueness of the guidance equation
  - Non-locality and Lorentz invariance
- Relativistic wave equations:
  - The Klein-Gordon equation
  - The Dirac equation

The de Broglie-Bohm theory for 1 particle (Bell's presentation of it)

- Complete description for one particle: $(\psi(t,\vec{x}),\vec{x}(t))$
- Schrödinger equation:

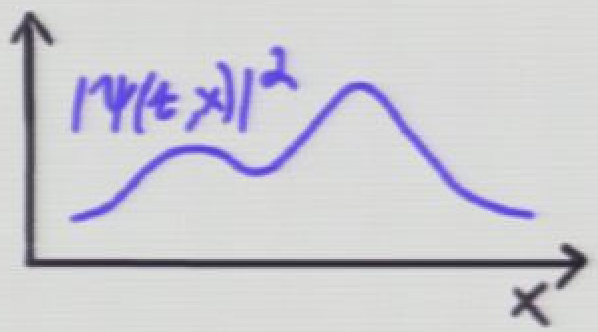
$$i\hbar\frac{\partial\psi(t,\vec{x})}{\partial t} = \left(-\hbar^2\frac{\Delta}{2m} + V(\vec{x})\right)\psi(t,\vec{x}) \tag{1}$$

- Guidance equation:

$$\vec{v}(t) = \left.\frac{\vec{j}(t,\vec{x})}{|\psi(t,\vec{x})|^2}\right|_{\vec{x}=\vec{x}(t)} = \frac{\hbar}{m}\mathfrak{Im}\left(\frac{\vec{\nabla}\psi(t,\vec{x})}{\psi(t,\vec{x})}\right)\Big|_{\vec{x}=\vec{x}(t)} = \left.\frac{\vec{\nabla}S(t,\vec{x})}{m}\right|_{\vec{x}=\vec{x}(t)} \tag{2}$$

where $\partial_t|\psi(t,\vec{x})|^2 + \vec{\nabla}\cdot\vec{j}(t,\vec{x}) = 0$.

Ensemble



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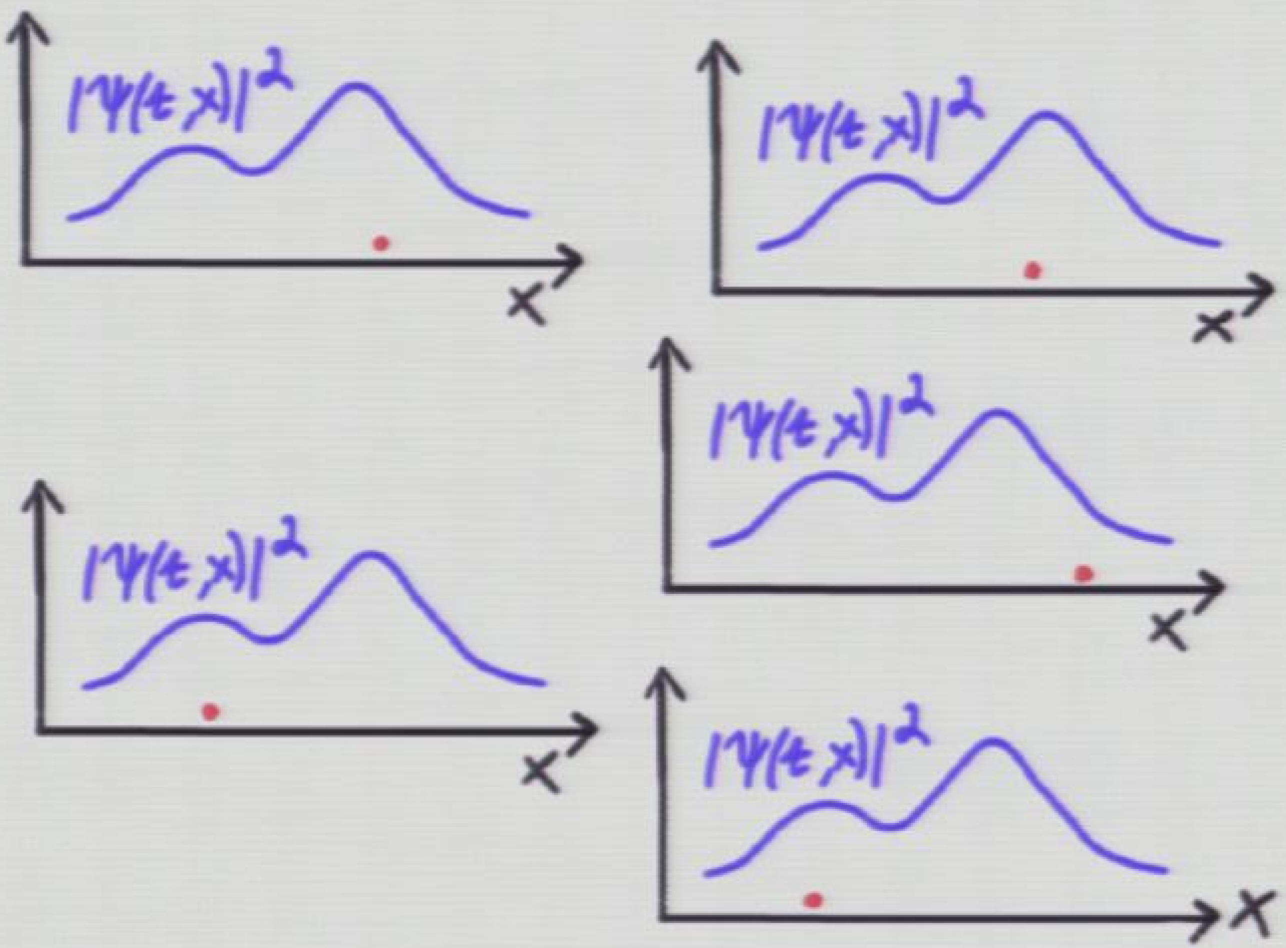
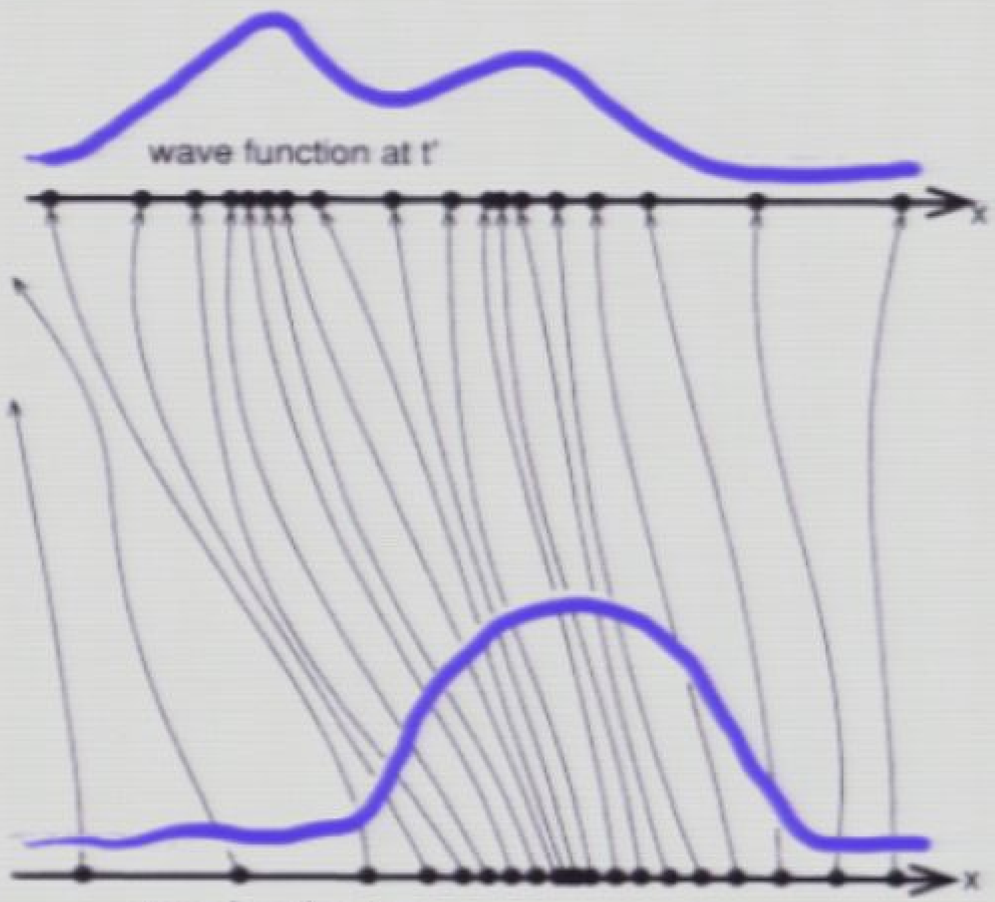


The distribution of positions $\rho(t,x)$ and $|\psi(t,x)|^2$ are two different things.

Equivariance I



Equivariance II

- $\rho(t,\vec{x}) = |\psi(t,\vec{x})|^2$ if $\rho(t_0,\vec{x}) = |\psi(t_0,\vec{x})|^2$
-
-

Equivariance II

- $$\rho(t_0,\vec{x}) = |\psi(t_0,\vec{x})|^2 \rightarrow \partial_t|\psi(t,\vec{x})|^2 = \partial_t\rho(t,\vec{x})$$
-
-

Equivariance II

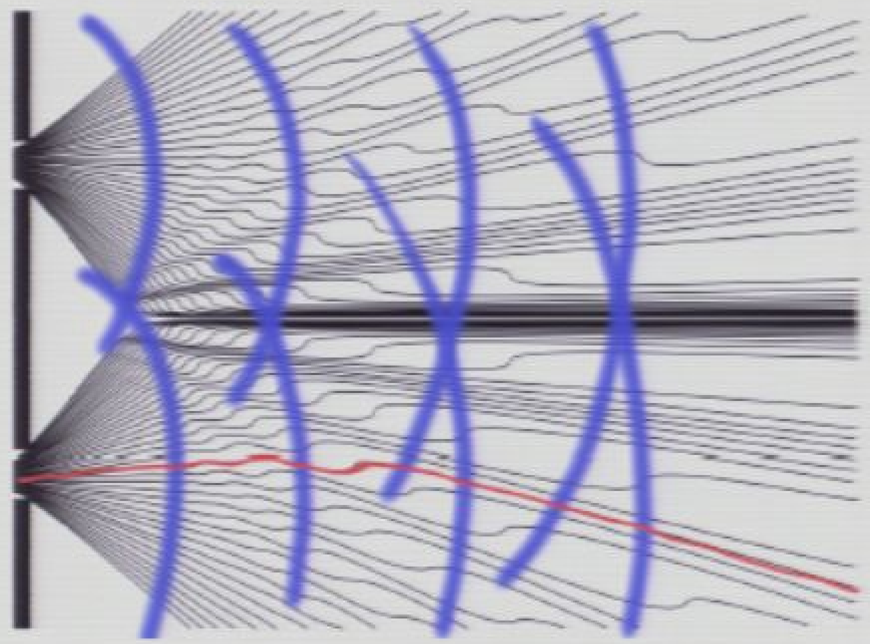
- $$\rho(t_0,\vec{x}) = |\psi(t_0,\vec{x})|^2 \to \partial_t|\psi(t,\vec{x})|^2 = \partial_t\rho(t,\vec{x})$$
- Consequence of

$$\partial_t|\psi|^2 + \vec{\nabla}\cdot\vec{j} = 0$$

$$\partial_t\rho + \vec{\nabla}\cdot(\rho\vec{v}) = 0 \quad \text{with } \vec{v} = \vec{j}/|\psi|^2 .$$

- $\rho(t,\vec{x}) = |\psi(t,\vec{x})|^2$: quantum equilibrium distribution

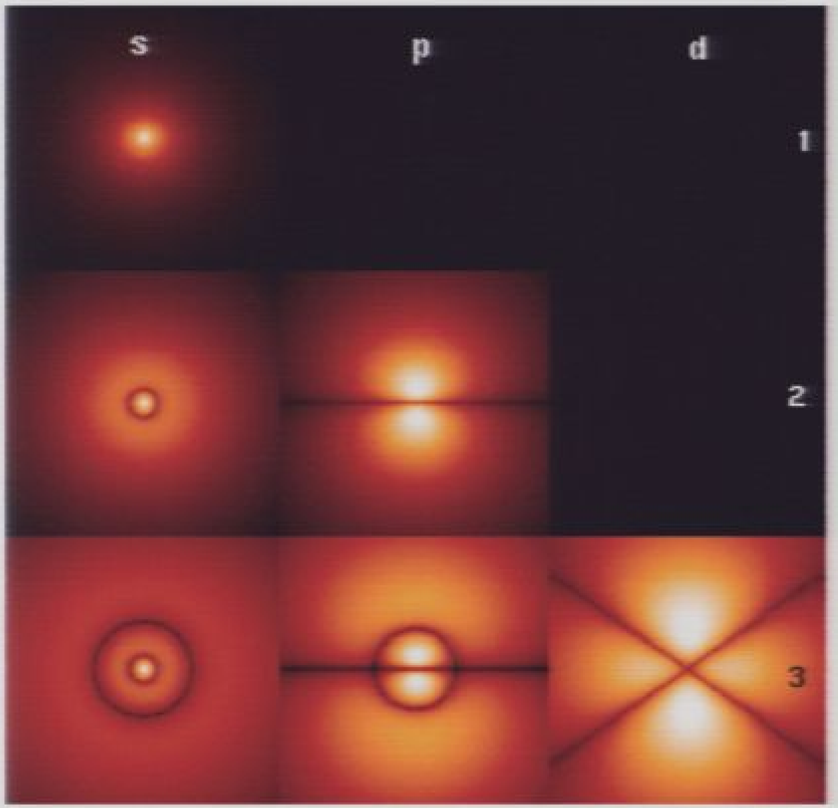
The 'mystery' of the 2-slit experiment



(Picture: after Gernot Bauer, after Chris Dewdney)

- Particle goes through one slot only, wave through both.
- Classical right after the slits.
- Highly non-classical when the waves interfere.
- Do not cross (but they can cross at different times of course - spin-term).
- Symmetry (reflexion with respect to the plane $x = 0$).

Example 2: the Hydrogen atom
$m = 0$ energy eigenstates



No motion for the eigenstates states with $m = 0$.

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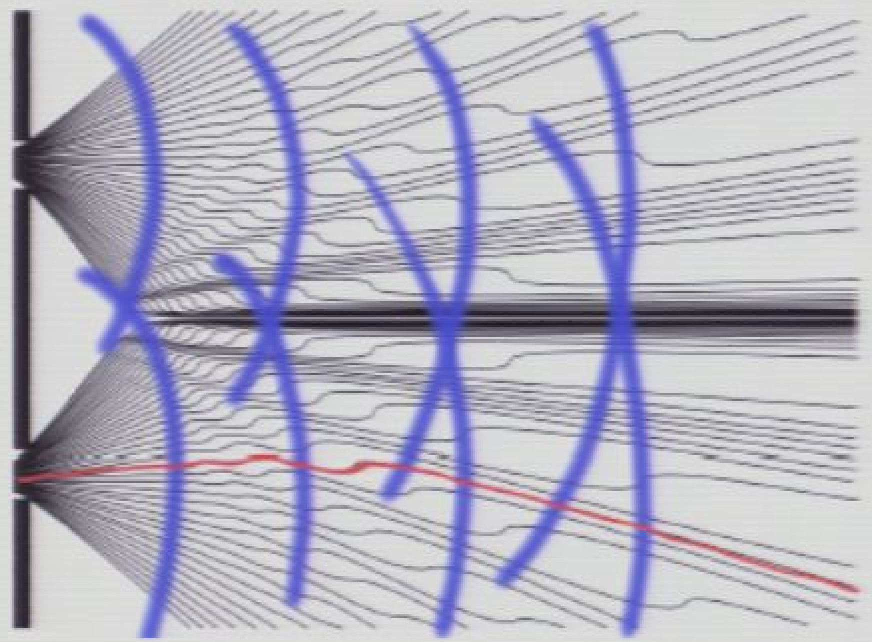

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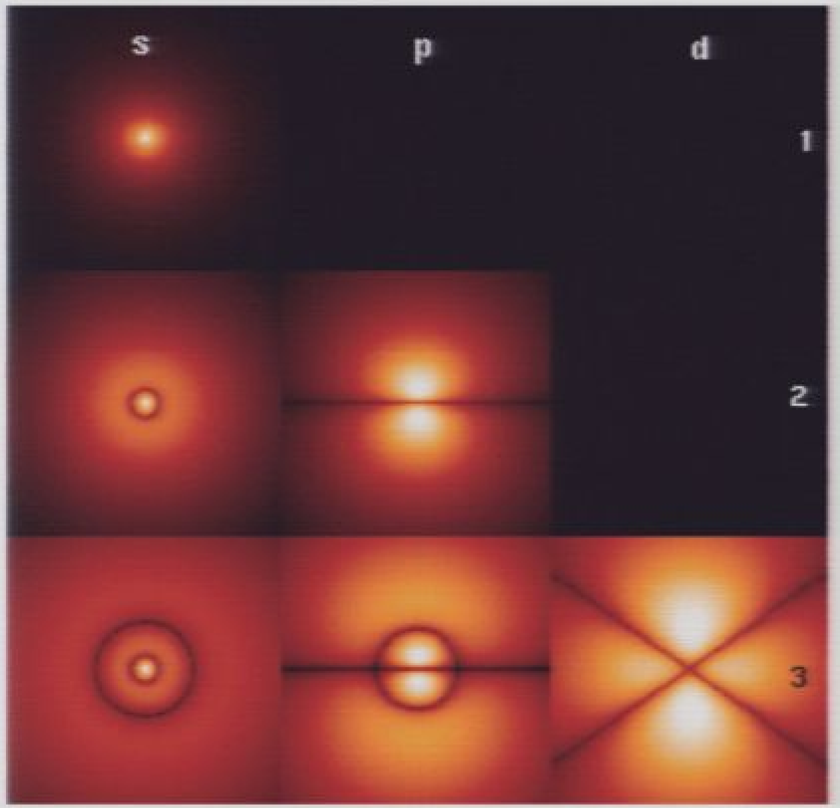

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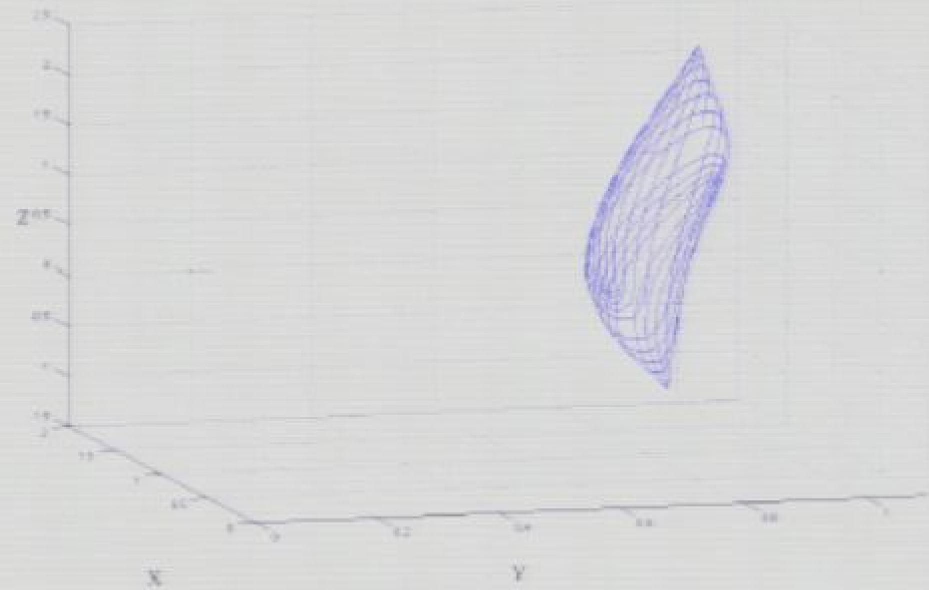
Example 2: the Hydrogen atom II

- Superpositions of eigenstates with $m = 0$: motion in an azimuthal plane.



- Spin-term. Simulations of trajectories (Colijn and Vrscay)[1].

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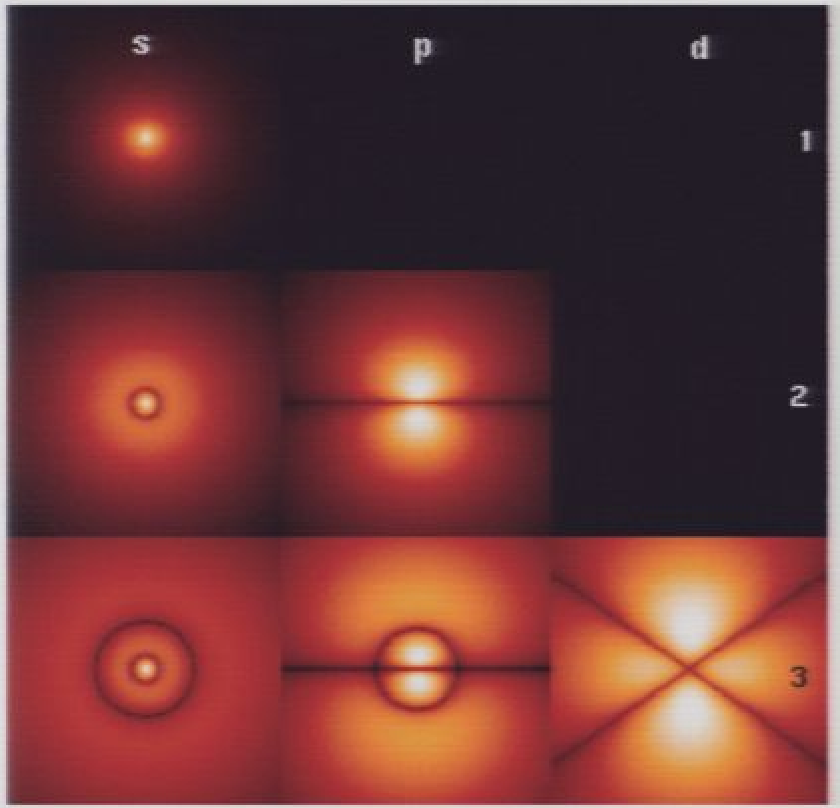

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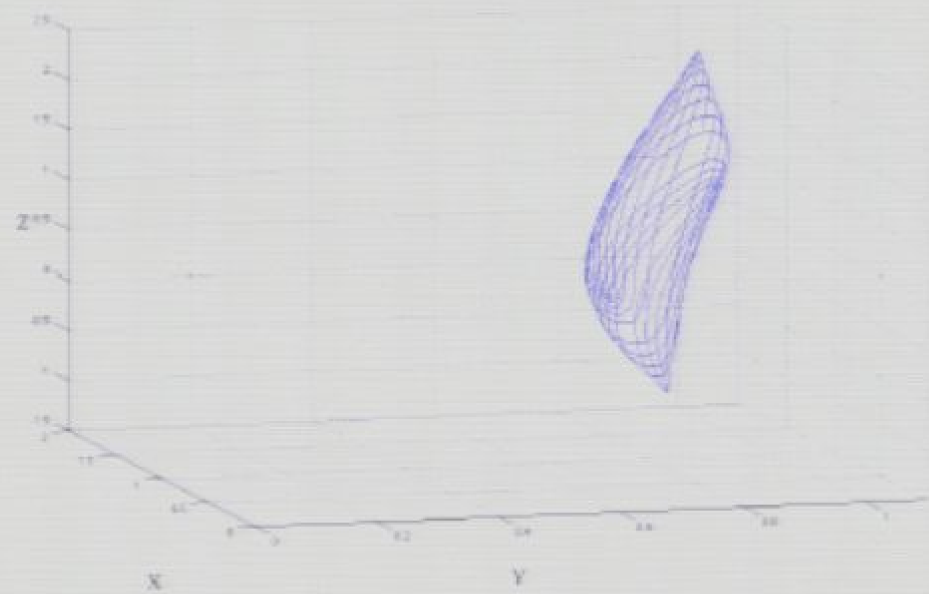

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Bohm's 1952 formulation I [2]

- Polar decomposition: $\psi = Re^{i\frac{S}{\hbar}}$
- The Schrödinger equation becomes:
  - Continuity equation:

$$\frac{\partial R^2}{\partial t} + \vec{\nabla} \cdot \left(R^2 \frac{\vec{\nabla} S}{m} \right) = 0 \,. \tag{3}$$

  - Hamilton-Jacobi equation:

$$\frac{\partial S}{\partial t} + \frac{\vec{\nabla} S \cdot \vec{\nabla} S}{2m} + V(\vec{x}) - \frac{\hbar^2}{2m} \frac{\Delta R}{R} = 0 \,. \tag{4}$$

- Interpretation of $\vec{v} = \frac{\vec{\nabla} S}{m}$.

Bohm's 1952 formulation II

Bohm goes one step further and adopts a second-order formulation:

- Newton-like equation of motion:

$$\vec{a} = -\vec{\nabla}(V + QP) \ . \tag{5}$$

- Constraint on the velocities:

$$\vec{v} = \frac{\vec{\nabla} S}{m} \ . \tag{6}$$

- Assumption on the initial distribution of particle positions over an ensemble:

$$\rho(t, \vec{x}) = |\psi(t, \vec{x})|^2 \ . \tag{7}$$

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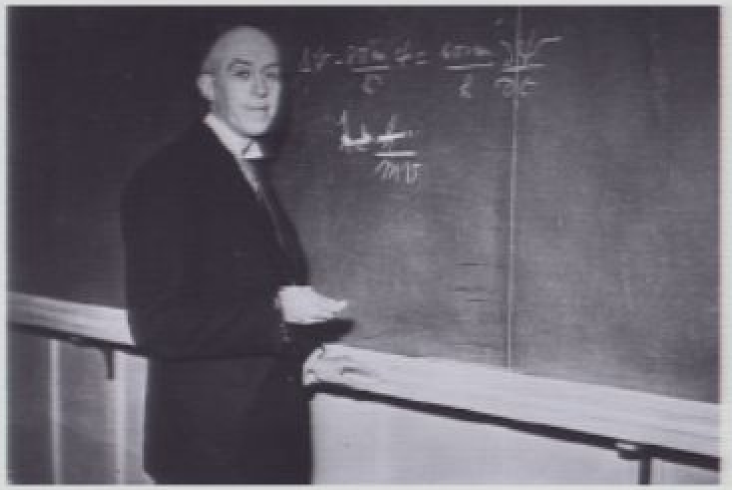
de Broglie's double-solution program

de Broglie back on tracks: the double-solution revival [3].



- Purely wave interpretation.
- Any solution:

$$\psi = Re^{iS/\hbar} + ue^{iS/\hbar} \quad (8)$$

(phase-matching)
(u moving singularity whose center moves according to $\vec{v} = \vec{\nabla} S/m$)

- Non-linear Schrödinger equation.
- Link with general relativity (Jean-Pierre Vigier).

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The many-particle de Broglie-Bohm theory

- Description of a system: $(\psi(t,\vec{X}),\vec{X}(t))$. $\vec{X}\in\mathbb{R}^{3n}$
- Laws of motion: the wave-function always evolves according to the Schrödinger equation

$$i\hbar\frac{\partial\psi(t,\vec{X})}{\partial t}=\left(-\hbar^2\sum_j\frac{\Delta_j}{2m_j}+V(t,\vec{X})\right)\psi(t,\vec{X})\,, \qquad (9)$$

whereas the positions are guided by the wave-function: the velocity of particle k is given by

$$\vec{v}_k(t)=\left.\frac{\vec{j}_k(t,\vec{X})}{|\psi(t,\vec{X})|^2}\right|_{\vec{X}=\vec{X}(t)}=\left.\frac{1}{m_k}\vec{\nabla}_k S(t,\vec{X})\right|_{\vec{X}=\vec{X}(t)}\,, \qquad (10)$$

where $\vec{J}=(\vec{j}_1,\ldots,\vec{j}_n)$ is the standard current associated to $|\psi(t,\vec{X})|^2$ through a continuity equation and $\psi=Re^{iS/\hbar}$.

$(\psi(t_0, x), x(t_0))$

$(\phi(t_0, y), y(t_0))$

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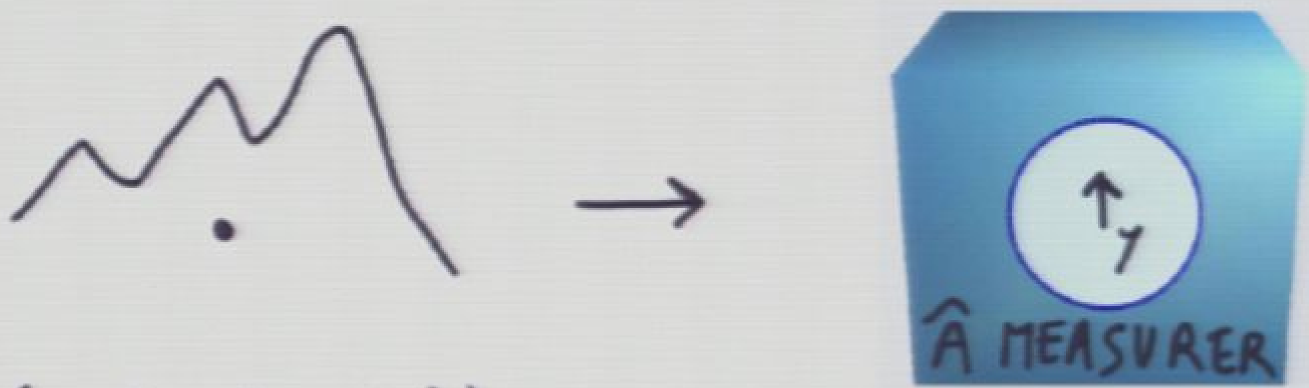
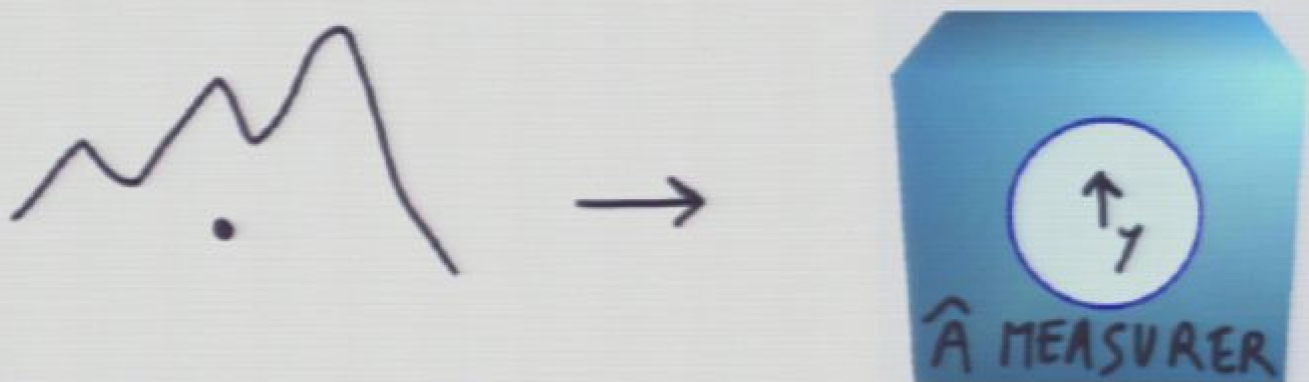
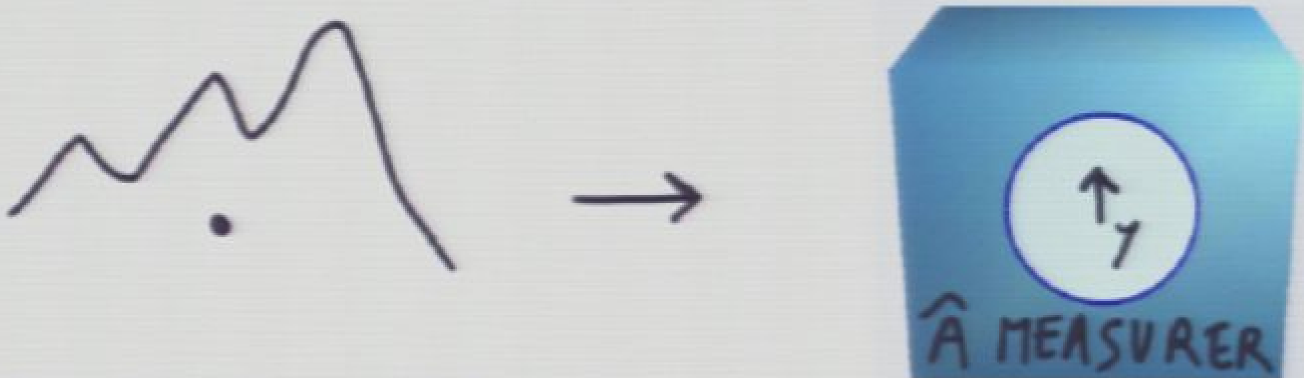


$(\psi(t_0, x), x(t_0))$ $(\phi(t_0, y), y(t_0))$



$(\psi(t_0,x), x(t_0))$ $\qquad$ $(\phi(t_0,y), y(t_0))$

$$\Psi(t_0,x,y) = \psi(t_0,x)\otimes\phi(t_0,y)$$
$$= \left(\sum_n c_n(t_0)\, A_n(x)\right)\otimes\phi(t_0,y)$$



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$$\hat{H}_I = g\,\hat{A}\,\hat{p}_y$$

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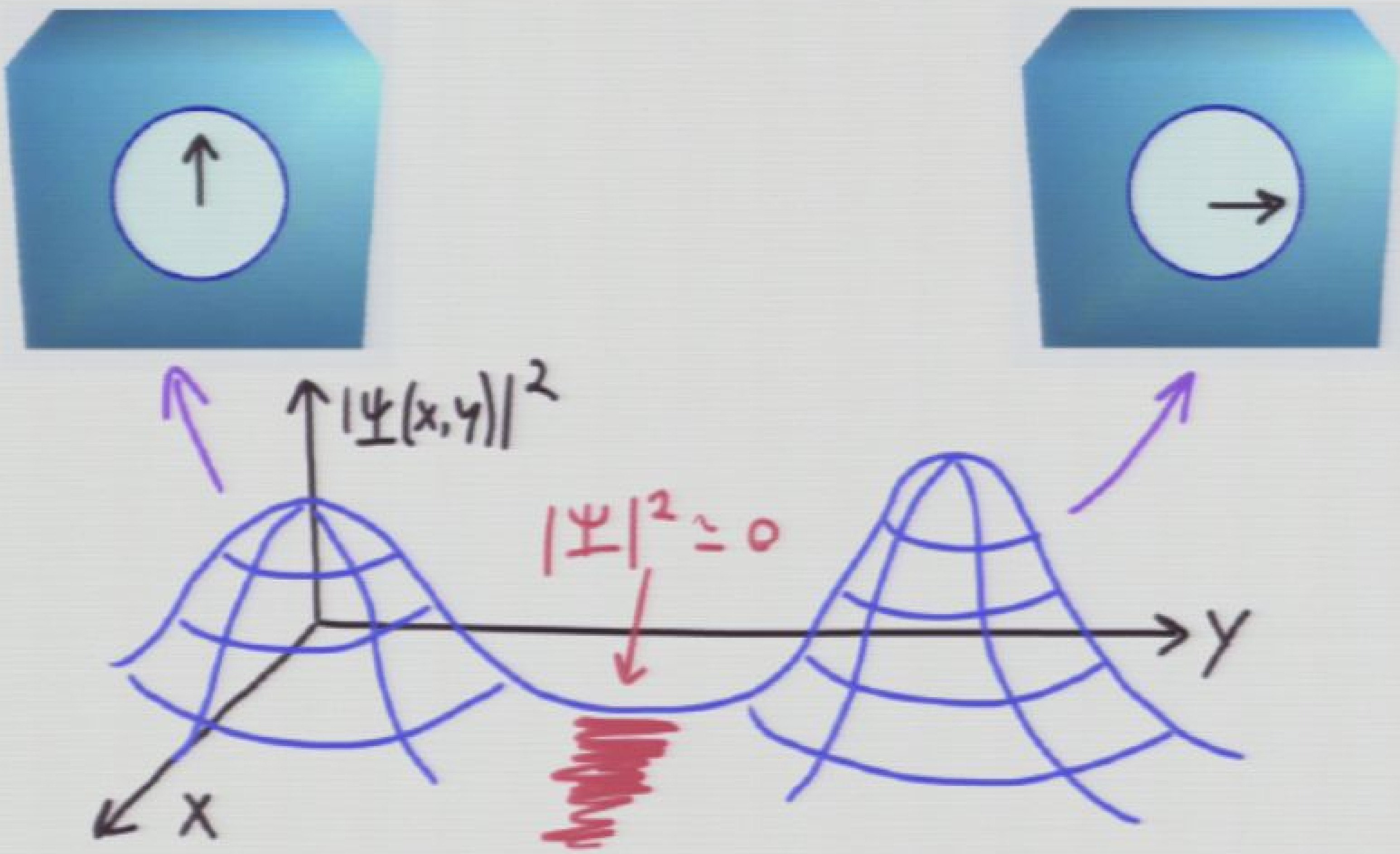
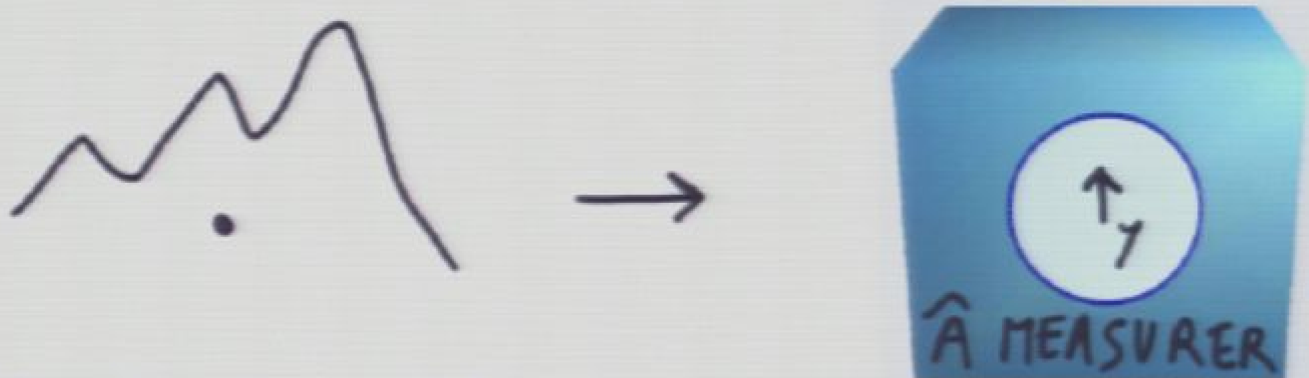


Separation of the branches in configuration space.



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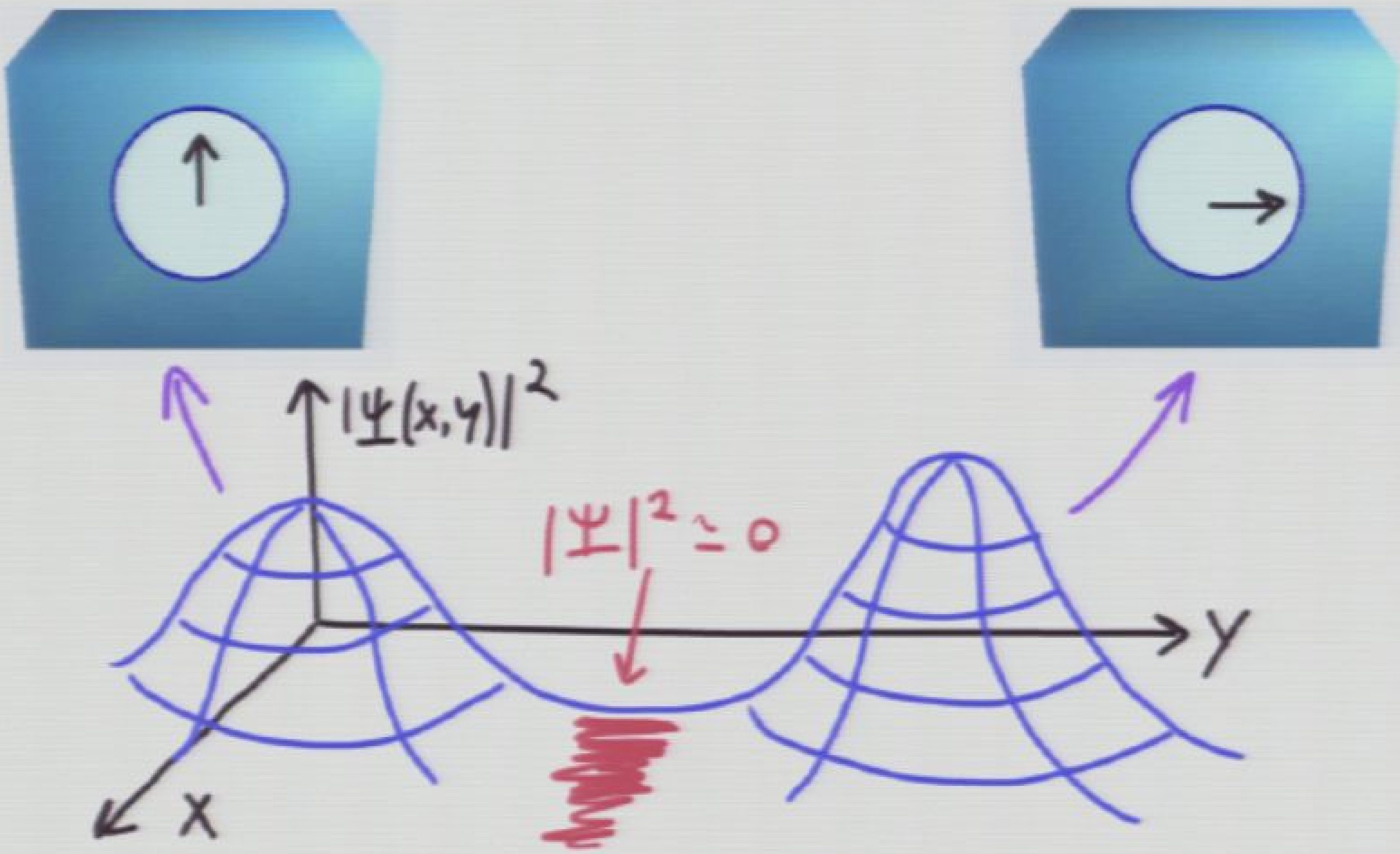
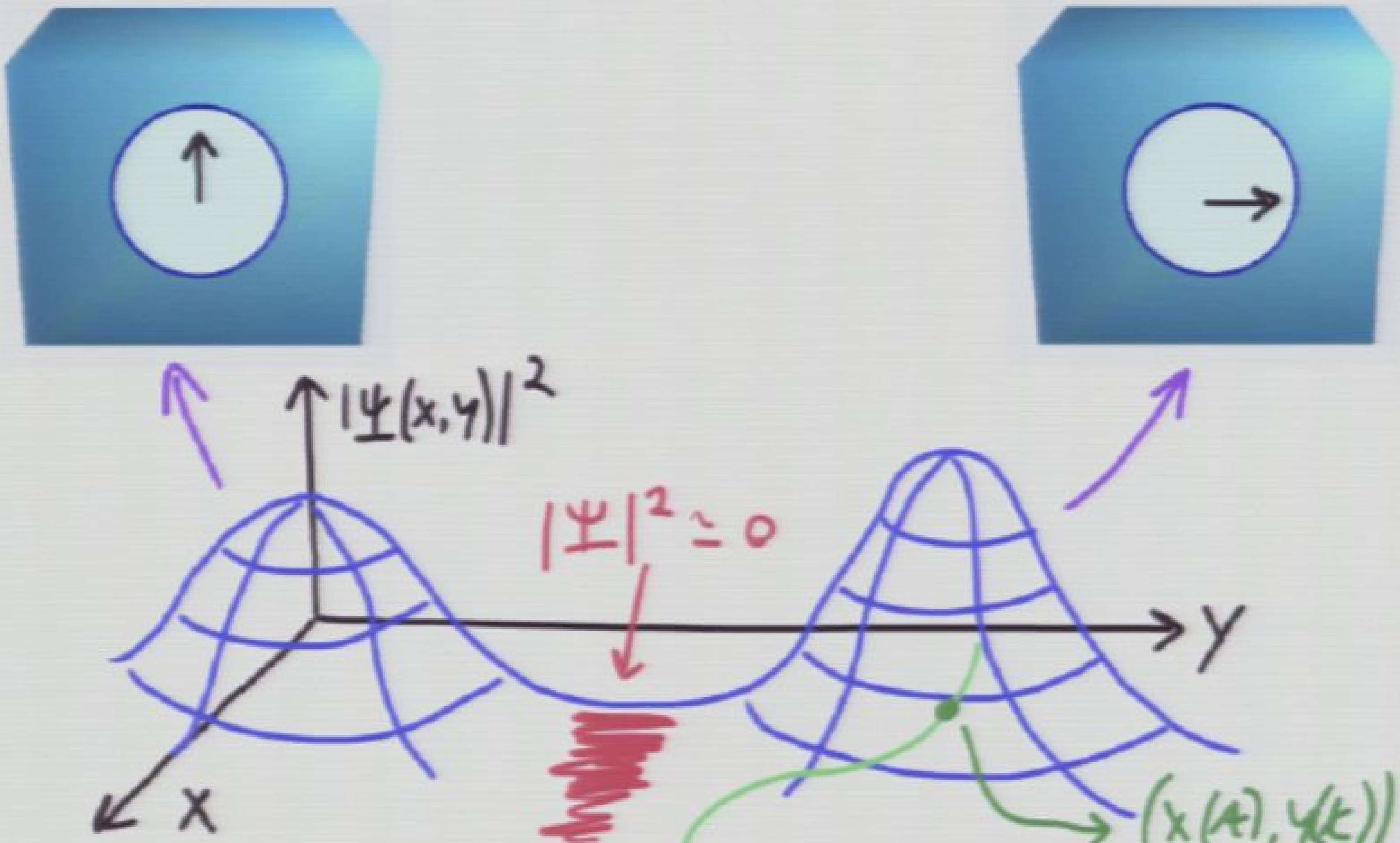


Separation of the branches in configuration space.



Effective collapse, effective wave-function, empty waves.
Statistics are reproduced provided that $\rho(t_0, \vec{x}, \vec{Y}) = |\psi(t_0, \vec{x}, \vec{Y})|^2$.

Spin $\frac{1}{2}$ particle and spin measurement

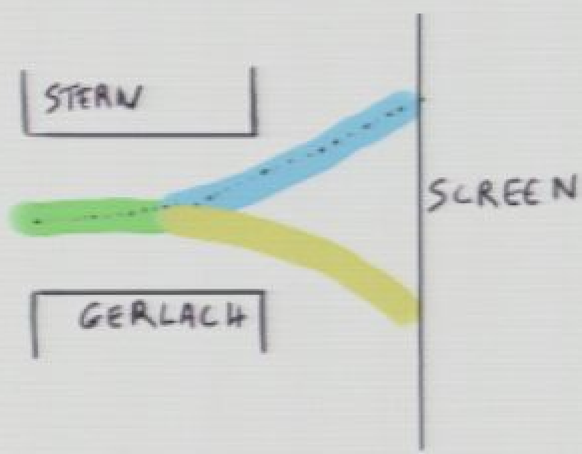
Spin-$\frac{1}{2}$ particle in magnetic field:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\Psi - \mu\vec{\sigma}\cdot\vec{B}\Psi \text{ with } \Psi = \begin{pmatrix}\Psi_1\\ \Psi_2\end{pmatrix} . \qquad (11)$$

Guidance equation:

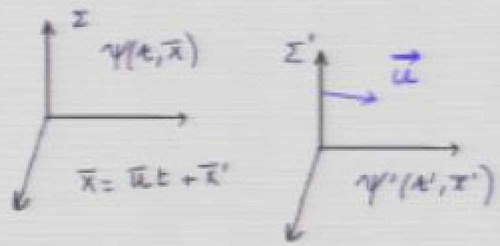
$$\vec{v}(t) = \frac{\hbar}{m}\mathfrak{Im}\left(\frac{\Psi^\dagger\vec{\nabla}\Psi}{\Psi^\dagger\Psi}\right)\Big|_{\vec{x}=\vec{X}(t)} \qquad (12)$$

Spin-measurement:



Spin is a property of the wave-function.

Symmetries: Galilean invariance



$$\psi'(t,\vec{x}') = e^{i(\frac{1}{2} m\, u^2\, t - m\vec{u}\cdot\vec{x})}\psi(t,\vec{x}) \quad (13)$$

- Invariance at the level of the wave-function:

$$i\hbar\frac{\partial\psi(t,\vec{x})}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi(t,\vec{x}) + V(\vec{x})\psi(t,\vec{x}) \quad (14)$$

- Invariance at the level of the guidance equation.

$$\vec{v}' = \frac{\vec{\nabla}' S'(t,\vec{x}')}{m} \iff \vec{v} = \frac{\vec{\nabla} S(t,\vec{x})}{m} \quad (15)$$

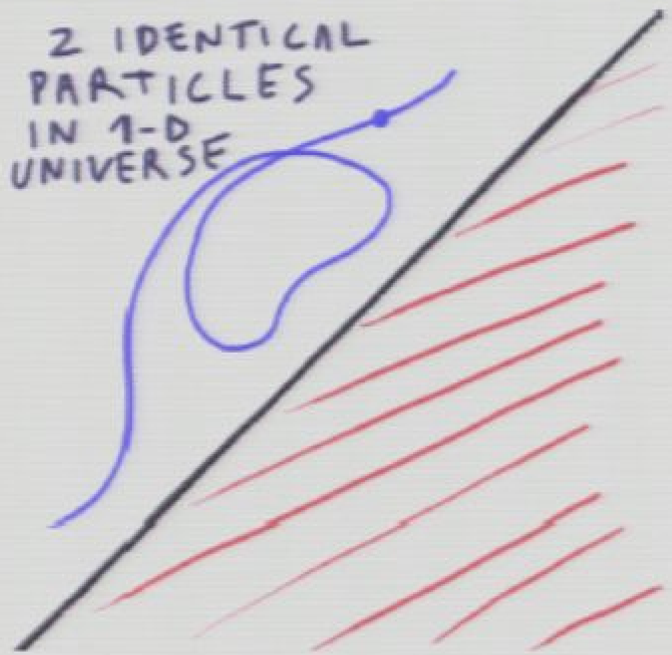
- Valentini: Galilean invariance is a fictitious symmetry [4]

Symmetries: time-reversal

- Standard QM: If $\psi(t, \vec{x})$ is solution, then $\psi^*(-t, \vec{x})$ is solution too.
- dBB theory: If $(\psi(t, \vec{x}), \vec{x}(t))$ is solution, then $(\psi^*(-t, \vec{x}), \vec{x}(-t))$ is solution too.

Symmetries: identical particles

- The wave-function is symmetric (or anti-symmetric) under a permutation of the particle labels.
- The trajectories do not depend on the choice of labels for the particles either.
- Reduced configuration space (Goldstein, Taylor, Tumulka, Zanghì [5]).



Justification of quantum equilibrium?

- $\rho(t_0, \vec{x}) = |\psi(t_0, \vec{x})|^2$ is an assumption (Bricmont [6]).
- In a typical universe, quantum equilibrium holds at the level of subsystems (Bell, Dürr, Goldstein and Zanghì [7]).
- Dynamical origin of quantum probabilities: quantum non-equilibrium distributions relax to quantum equilibrium (Valentini, Bohm).

[6] J. Bricmont, *Bayes, Boltzmann and Bohm: probabilities in physics* (2001)

We live in a ‘typical’ universe

- One Bohmian universe: $(\Psi_u(t, \vec{X}_u), \vec{X}_u(t))$
- Equilibrium at the level of subsystems. How is the subsystem defined?

$$\vec{X}_u = (\vec{x}_s, \vec{X}_e) \to \psi(t, \vec{x}_s) = \Psi_u(\vec{x}_s, \vec{X}_e(t)) \tag{16}$$

- Example:

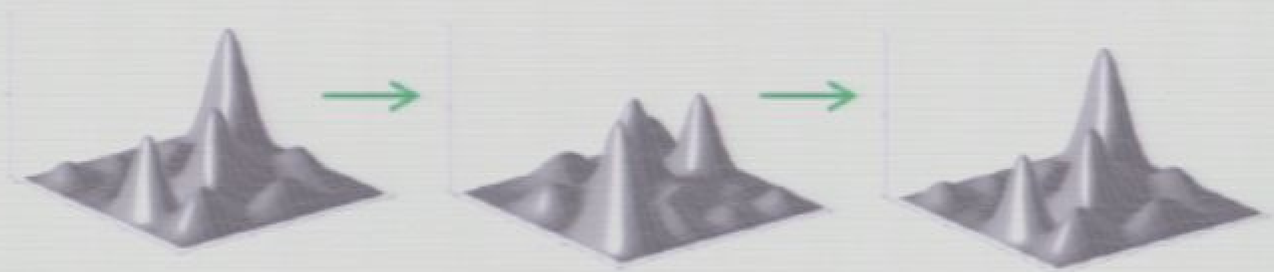
$$\Psi_u(\vec{x}_1, \ldots, \vec{x}_N) = \psi(\vec{x}_1) \otimes \psi(\vec{x}_2) \otimes \ldots \otimes \psi(\vec{x}_N)\ . \tag{17}$$

- ‘Most’ universal configurations lead to QE at the subsystem level. Most understood with respect to $|\Psi_u|^2$ measure.
- Typical configuration for the universe (pick $\vec{X}_u(t)$) lead to QE at the subsystem level.
- Why is $|\Psi_u|^2$ a good measure? Why not the Lebesgue measure? Because $|\Psi_u|^2$ is equivariant (stationarity). Is it the only equivariant measure (Goldstein-Struyve [8]).

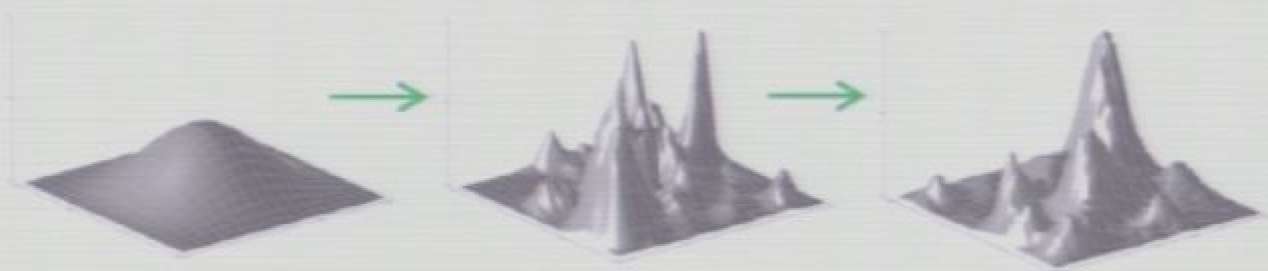
Dynamical origin of quantum probabilitities

Relaxation simulations: [9]

Time evolution of quantum equilibrium



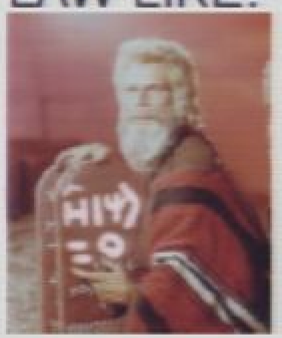
Non-equilibrium relaxes to equilibrium



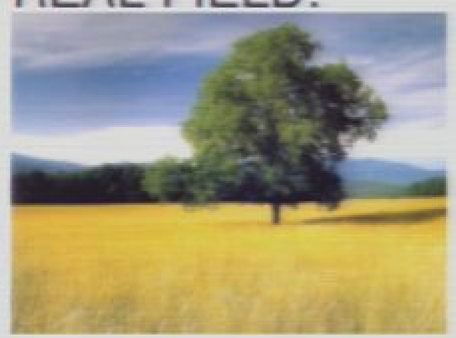
Status of the wave-function

No one can understand this theory until he is willing to think of ψ as a real objective field rather than just a 'probability amplitude'. Even though it propagates not in 3-space but in 3N-space.[10]

LAW-LIKE:



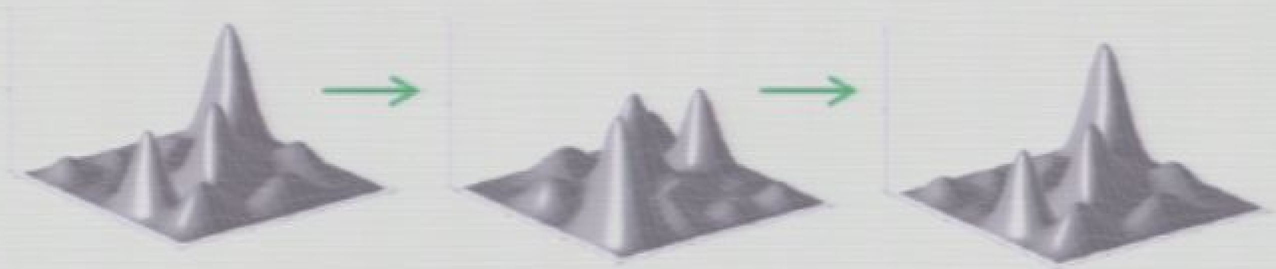
REAL FIELD:



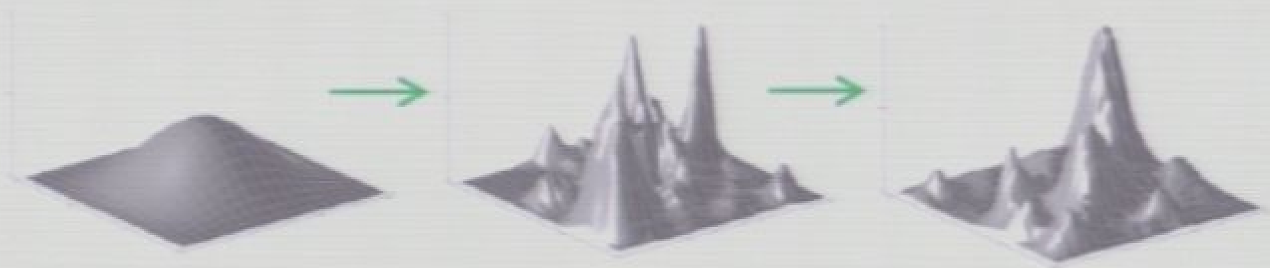
Dynamical origin of quantum probabilitities

Relaxation simulations: [9]

Time evolution of quantum equilibrium



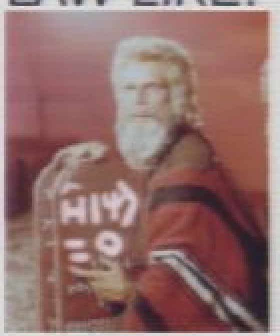
Non-equilibrium relaxes to equilibrium



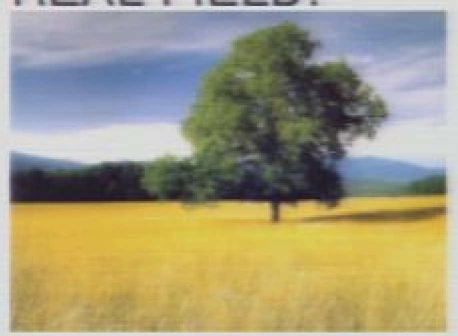
Status of the wave-function

No one can understand this theory until he is willing to think of ψ as a real objective field rather than just a 'probability amplitude'. Even though it propagates not in 3-space but in 3N-space.[10]

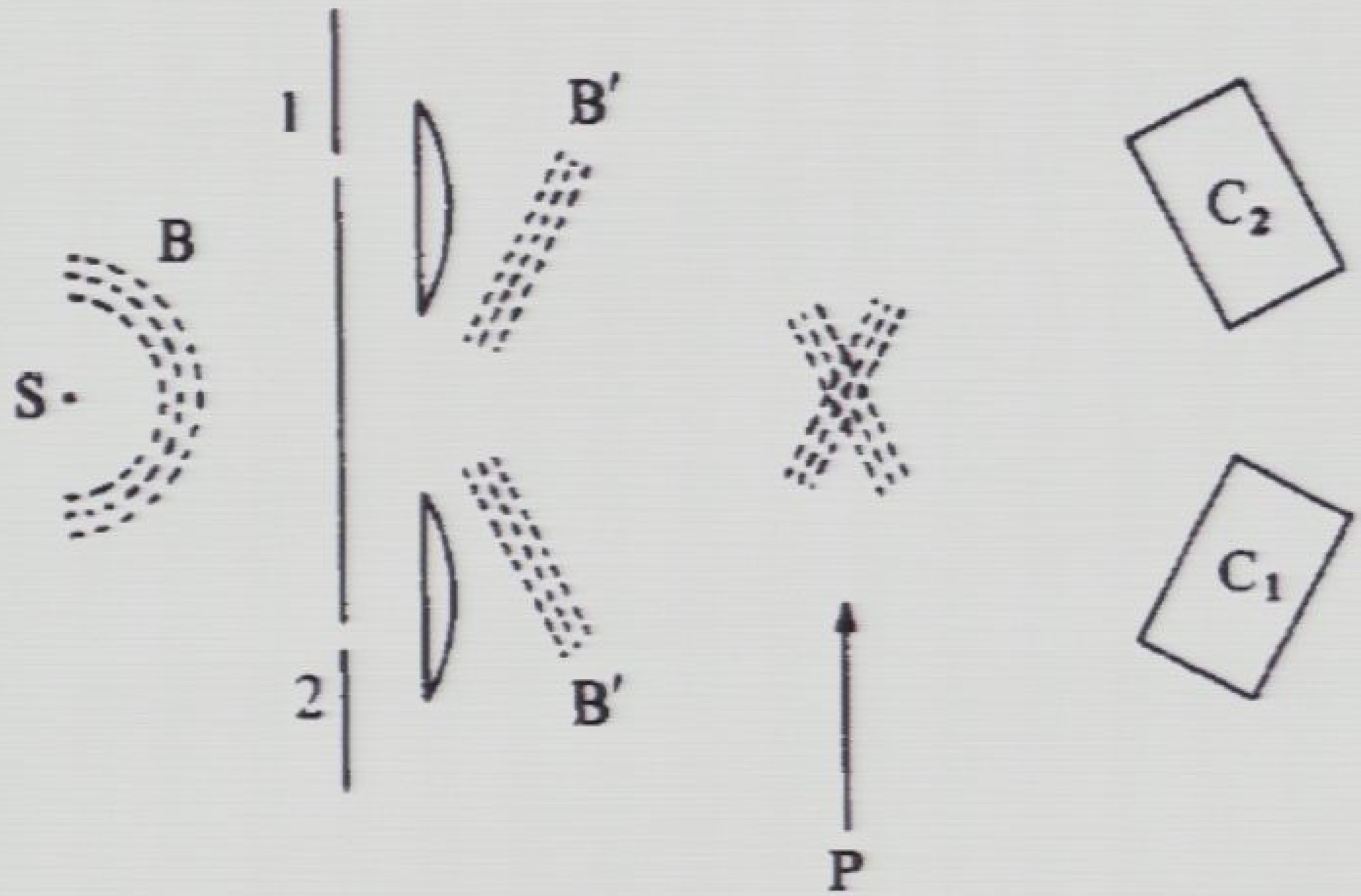
LAW-LIKE:



REAL FIELD:

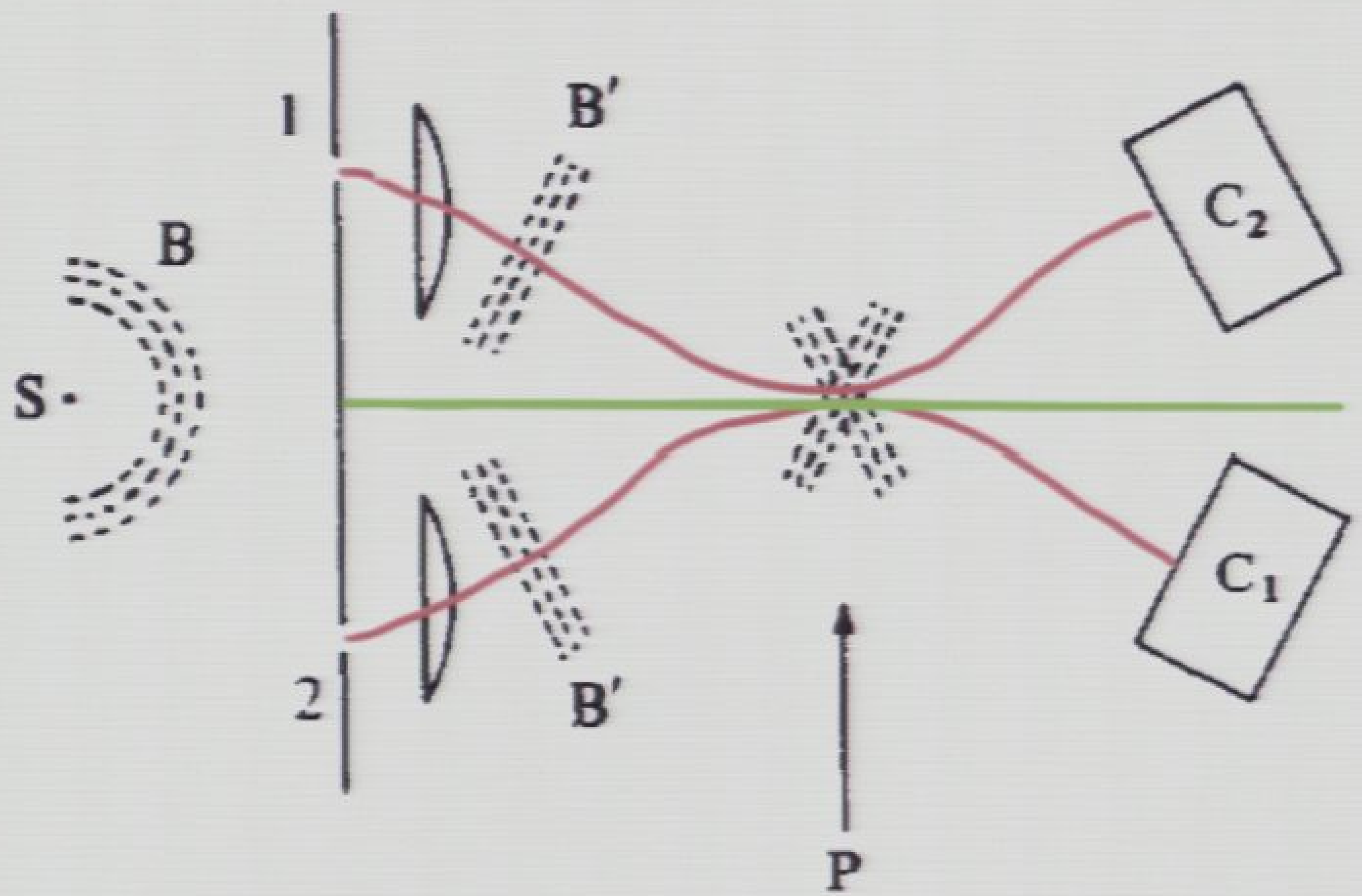


Surrealistic trajectories: the Wheeler delayed-choice experiment [11] [12]



[11] J. S. Bell, de Broglie Bohm, delayed choice double-slit experiment, and density matrix.

Surrealistic trajectories: the Wheeler delayed-choice experiment (dBB)



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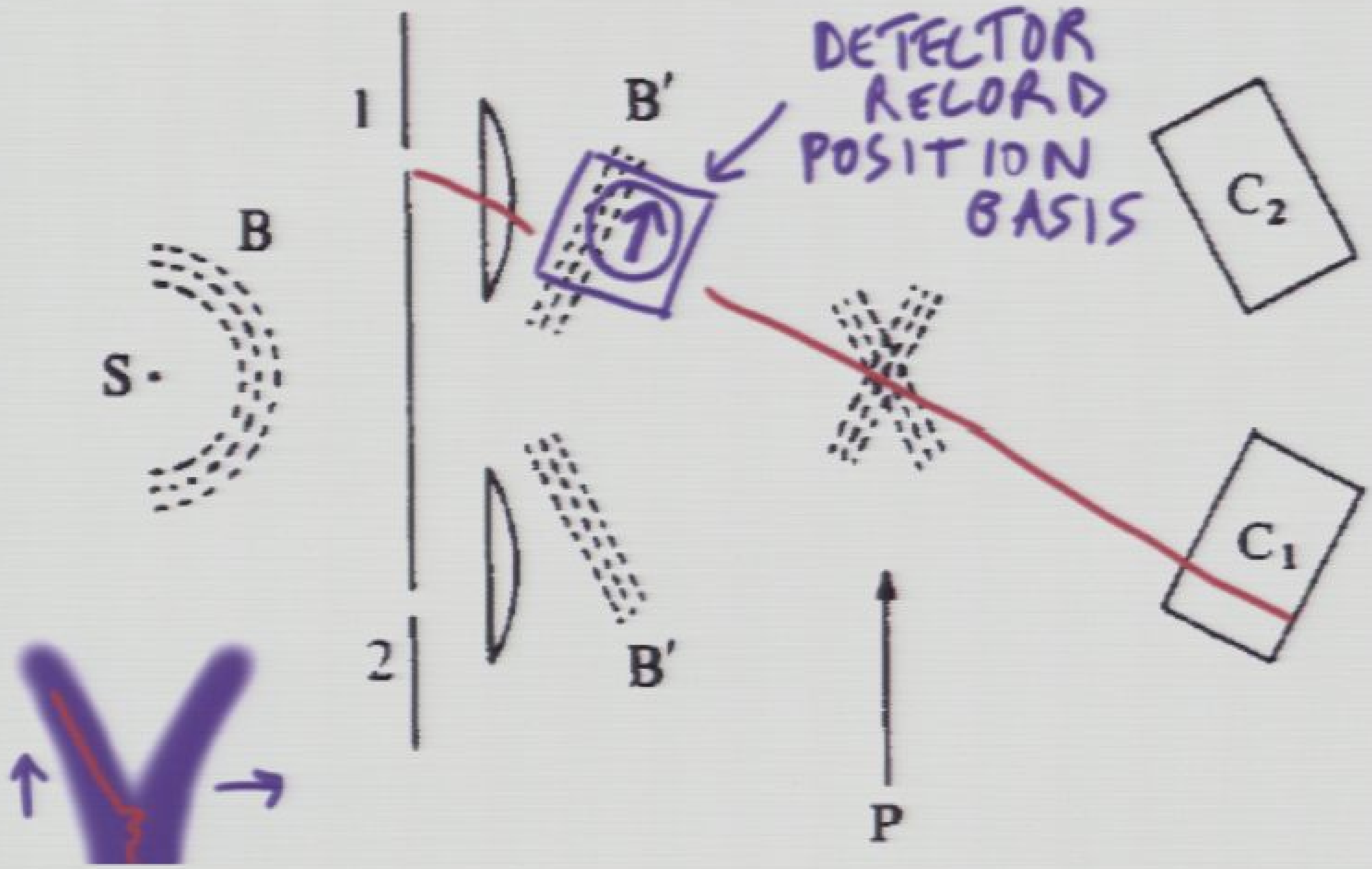
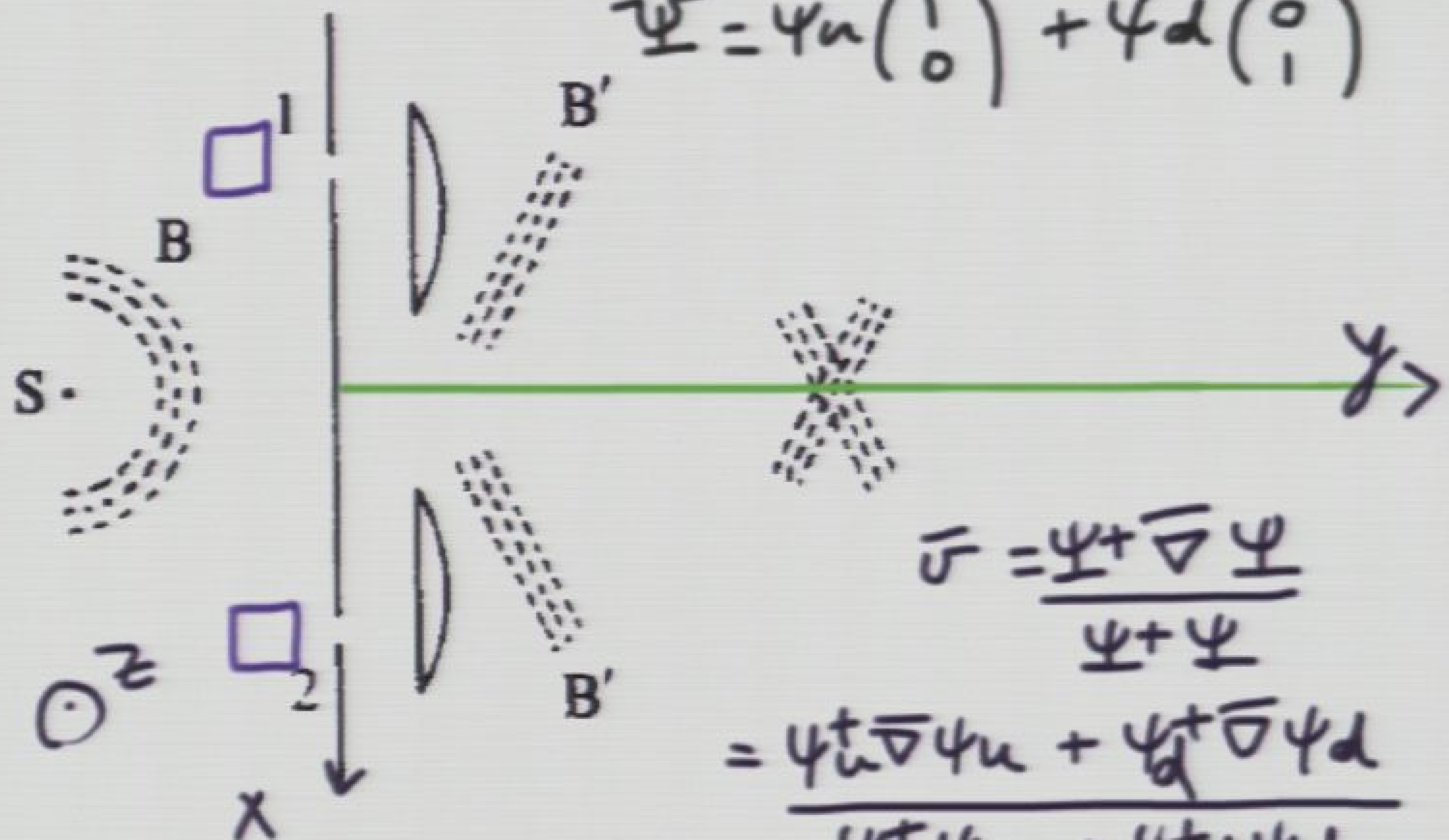


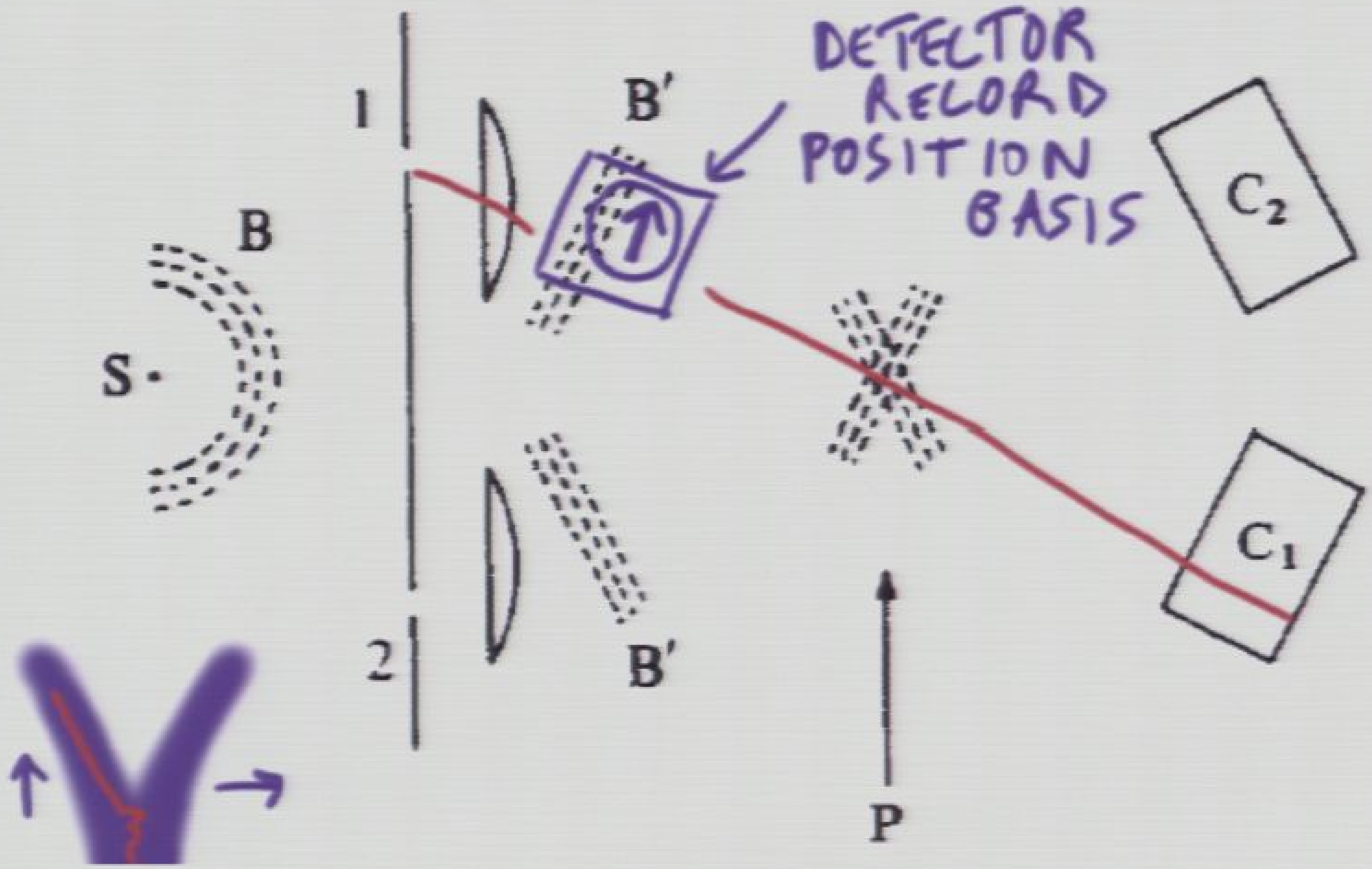
Surrealistic trajectories [13]

$$\Psi = \psi_u \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \psi_d \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



$$\bar{v} = \frac{\Psi^{+} \bar{\nabla} \Psi}{\Psi^{+} \Psi} = \frac{\psi_u^{+} \bar{\nabla} \psi_u + \psi_d^{+} \bar{\nabla} \psi_d}{\psi_u^{+} \psi_u + \psi_d^{+} \psi_d}$$

Surrealistic trajectories: the Wheeler delayed-choice experiment (dBB)



Non-uniqueness of the guidance equation: back to equivariance I

- $\rho(t,\vec{x}) = |\psi(t,\vec{x})|^2$ if $\rho(t_0,\vec{x}) = |\psi(t_0,\vec{x})|^2$.
- Consequence of

$$\vec{v} = \frac{\vec{j}}{|\psi|^2} \quad \text{where} \quad \partial_t|\psi|^2 + \vec{\nabla}\cdot\vec{j} = 0 \qquad (18)$$

- $\rho(t,\vec{x}) = |\psi(t,\vec{x})|^2$: quantum equilibrium distribution

Non-uniqueness of the guidance equation: back to equivariance II

- $\rho(t,\vec{x}) = |\psi(t,\vec{x})|^2$ if $\rho(t_0,\vec{x}) = |\psi(t_0,\vec{x})|^2$.
- Consequence of

$$\vec{v} = \frac{\vec{j} + \vec{\nabla} \times \vec{a}}{|\psi|^2} \quad \text{where} \quad \partial_t|\psi|^2 + \vec{\nabla} \cdot (\vec{j} + \vec{\nabla} \times \vec{a}) = 0 \qquad (19)$$

- $\rho(t,\vec{x}) = |\psi(t,\vec{x})|^2$: quantum equilibrium distribution

Non-uniqueness of the guidance equation

- ∞ number of de Broglie-Bohm-like models for which $|\psi(t,\vec{x})|^2$ is equivariant (E. Deotto and G.C. Ghirardi[14]):

$$\vec{v}' = \vec{v} + \frac{\vec{\nabla} \times \vec{a}}{|\psi(t,\vec{x})|^2} \tag{20}$$

  (even after imposing several constraints like Galilean invariance and global solution).
- The Pauli equation as the NR limit of the Dirac equation (Hestenes, D. Bohm and B. Hiley). New term: spin term.
- Arguments for the uniqueness of the guidance equation (Dürr, Goldstein & Zanghì, P. Holland, H. Wiseman)

Non-uniqueness of the guidance equation: arguments for the uniqueness of the guidance equation

- Dürr-Goldstein-Zanghì: simplicity.
- P. Holland [15]: in the relativistic case, we have the Dirac current $j^\mu = \bar{\psi}\gamma^\mu\psi$ satisfying $\partial_\mu j^\mu = 0$. Non-uniqueness becomes $j^\mu + a^\mu$ with $\partial_\mu a^\mu = 0$ and $a_0 = 0$ (in all inertial frames).
- H. Wiseman [16]: Operational definition of the velocity through weak measurements

$$\vec{v}(t,\vec{x}) = \lim_{dt\to 0} \frac{1}{dt} E[\hat{\vec{x}}_s(t+dt) - \hat{\vec{x}}_w(t) | \vec{x}_s(t+dt) = \vec{x}] \qquad (21)$$

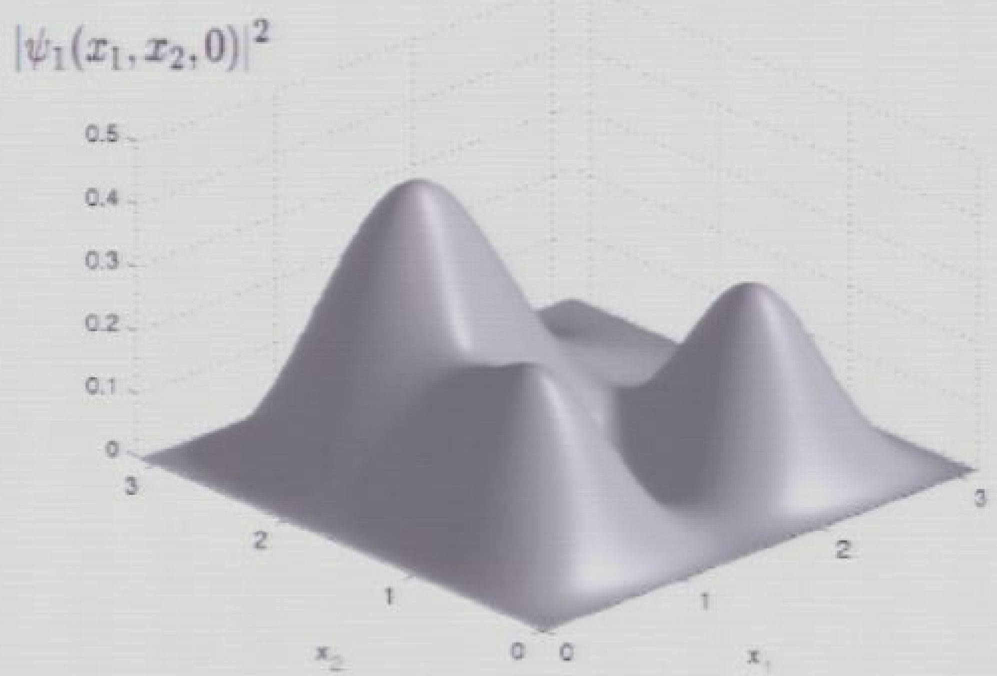
coincides with the dBB velocity. Implemented in the lab (S. Kocsis et al.).

[15] P. Holland, *Phys. Rev. A* **60**, 4316–4330 (1999).

Alternative guidance equations [17]

Particle in a 2-D Box $[0,\pi]x[0,\pi]$

$$\psi(t,x_1,x_2)=\frac{1}{\pi}\sum_{m=1}^{2}\sum_{n=1}^{2}\sin(mx_1)\sin(nx_2)e^{i(\theta_{mn}-\frac{m^2+n^2}{2}t)} \quad (22)$$



$$v_k'=v_k+\mu\frac{\epsilon_{kl}\partial_l f^{\psi}}{|\psi|^2} \quad (23)$$

The Klein-Gordon equation

- K-G equation:

$$(\Box + \mu^2)\phi(t,\vec{x}) = 0 \; . \tag{25}$$

- Continuity equation:

$$j^\mu = i\phi^*\partial^\mu\phi - i\partial^\mu\phi^*\phi \tag{26}$$

- The density

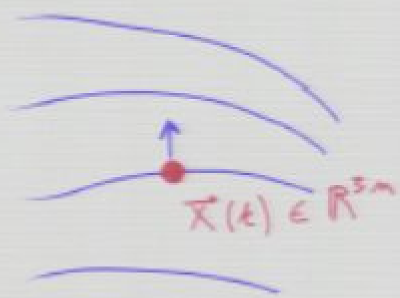
$$j^0 = i\phi^*\partial_t\phi - i\partial_t\phi^*\phi \tag{27}$$

can be negative, even for a superposition of positive-energy solutions.

- Various proposals to try to make sense of it (Holland, Dewdney-Horton, Ghose, Nikolic, Tumulka).

Non-locality and Lorentz invariance

- The many-particle de Broglie-Bohm theory is grounded in configuration space:



$$\vec{V}(t) = \left.\frac{\vec{J}(t,\vec{X})}{\rho_\Psi(t,\vec{X})}\right|_{\vec{X}=\vec{X}(t)} . \tag{24}$$

- The velocity of one particle, at time t depends on the positions of all the other particles at t, no matter how far they are separated.
- Distinction between Lorentz covariance and non-locality.
- Foliation in a Lorentz-covariant way [18].
- Serious Lorentz covariance, if any, will probably come from retro-causal models (Cramer, Price).

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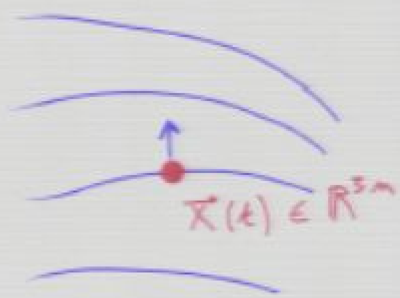

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