Title: Resonant Tunneling in Superfluid Helium-3

Date: Feb 04, 2011 09:30 AM

URL: http://pirsa.org/11020127

Abstract: The resonant tunneling phenomenon is well understood in quantum mechanics. I argue why a similar phenomenon must be present in quantum field theory. Using the functional Schr\"odinger method I show how resonant tunneling through multiple barriers takes place in quantum field theory with a single scalar field. I also show how this phenomenon in scalar quantum field theory can lead to an exponential enhancement of the single-barrier tunneling rate. My analysis is carried out in the thin-wall approximation. I discuss a possible explanation of the fast nucleation of the B phase of superfluid Helium-3 as an application.

Resonant Tunneling in Superfluid Helium-3

Dan Wohns

Cornell University

arXiv:0910.1088 and work in progress with S.-H. Henry Tye

February 4, 2011

Motivation

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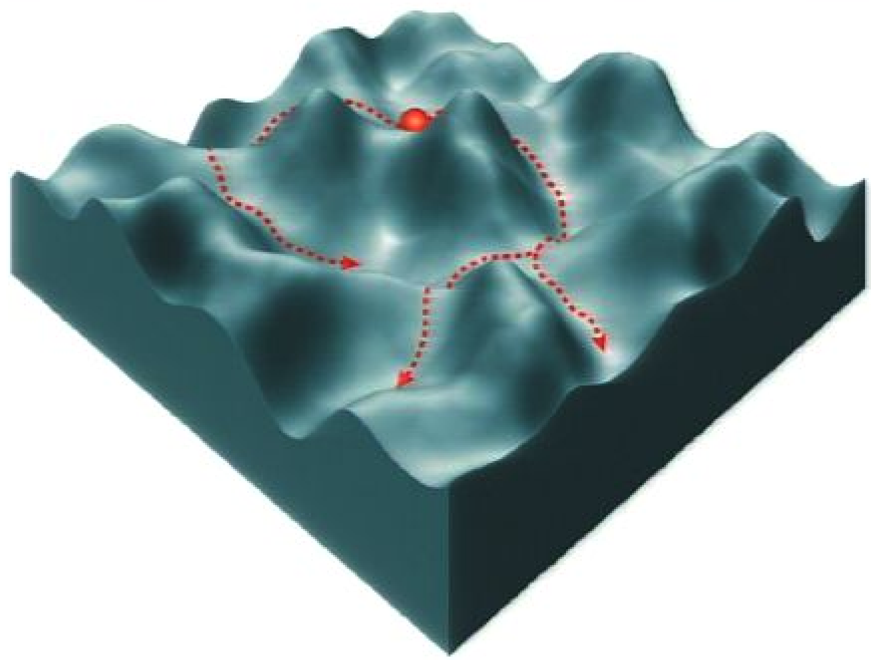
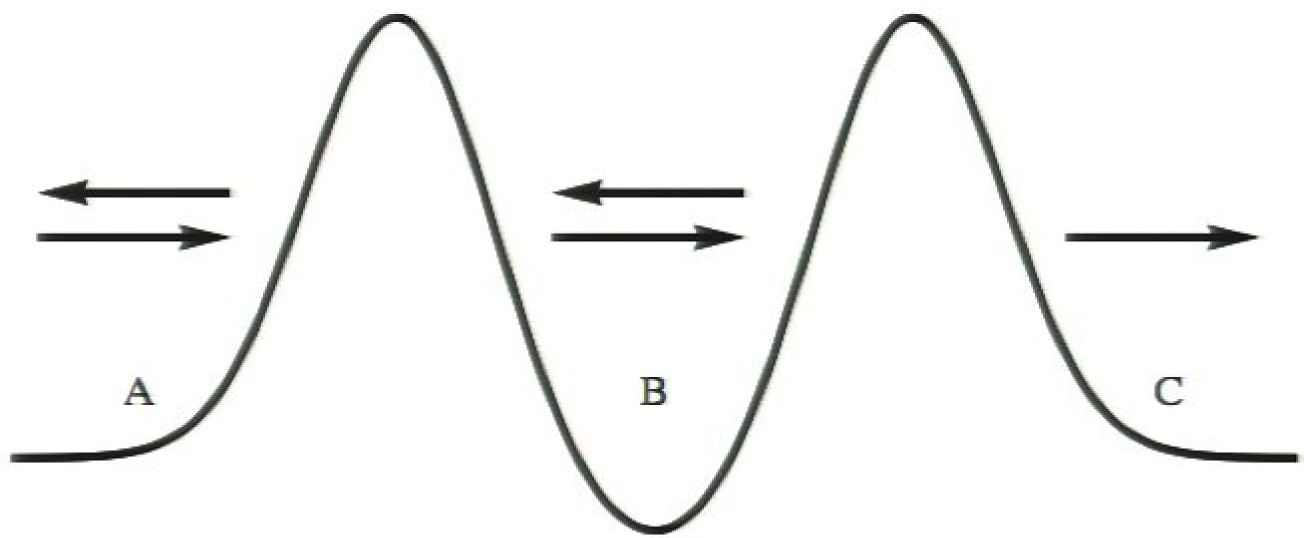
- Relevant to study of potentials with many minima
- Generic existence of resonance effects
- Superfluid Helium-3 may exhibit resonant tunneling

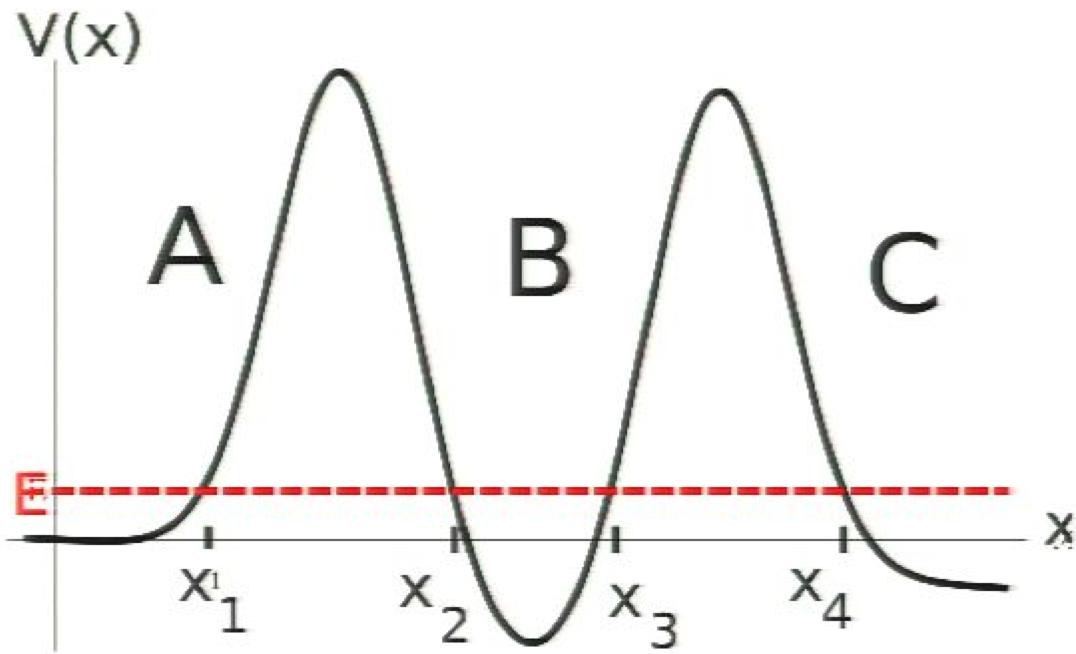


General Argument



- Tunneling rate for single-barrier tunneling is $\Gamma_{A \to B} = Ae^{-S}$
- Tunneling probability for single-barrier tunneling is $T_{A \to B} = Ke^{-S}$

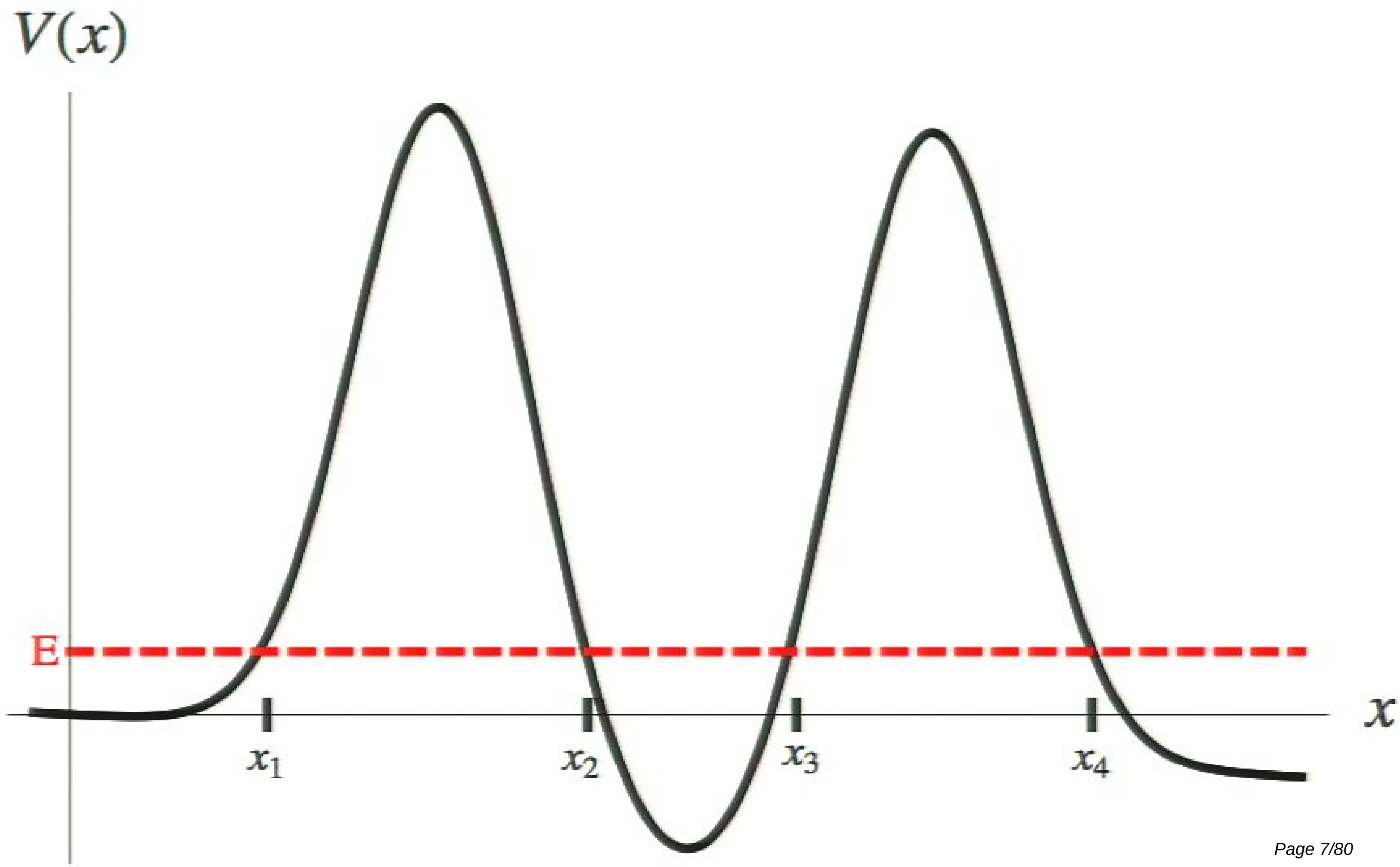
General Argument



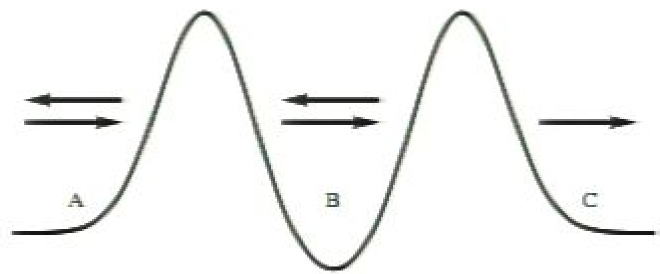
- Naive WKB analysis $T_{A\to C} \approx T_{A\to B} T_{B\to C}$ is incorrect
- Instead $t_{A\to C} = t_{A\to B} + t_{B\to C}$ or equivalently $T_{A\to C} = \frac{T_{A\to B} T_{B\to C}}{T_{B\to C}+T_{B\to C}}$.
- Enhancement due to resonant tunneling effect.

Outline

Potential



WKB Approximation



- Expand a general wavefunction $\Psi(x) = e^{if(x)/\hbar}$ in powers of $\hbar$
- $\psi_{L,R}(x) \approx \frac{1}{\sqrt{k(x)}} \exp\left(\pm i \int dx k(x) \right)$ in classically allowed region, where $k(x) = \sqrt{\frac{2m}{\hbar^2}(E - V(x))}$
- $\psi_{\pm}(x) \approx \frac{1}{\sqrt{\kappa(x)}} \exp\left(\pm \int dx \kappa(x) \right)$ in the classically forbidden region, where $\kappa(x) = \sqrt{\frac{2m}{\hbar^2}(V(x) - E)}$

Matching Conditions

- Complete solution $\psi(x) = \alpha_L \psi_L(x) + \alpha_R \psi_R(x)$ in vacuum A
- $\psi(x) = \alpha_+ \psi_+(x) + \alpha_- \psi_-(x)$ in the classically forbidden region
- $\psi(x) = \beta_L \psi_L(x) + \beta_R \psi_R(x)$ in vacuum B
- $\begin{pmatrix} \alpha_R \\ \alpha_L \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \Theta + \Theta^{-1} & i(\Theta - \Theta^{-1}) \\ -i(\Theta - \Theta^{-1}) & \Theta + \Theta^{-1} \end{pmatrix} \begin{pmatrix} \beta_R \\ \beta_L \end{pmatrix}$
- $\Theta \simeq 2 \exp\left(\frac{1}{\hbar} \int_{x_1}^{x_2} dx \sqrt{2m(V(x) - E)} \right)$
- Tunneling probability $T_{A \to B} = |\frac{\beta_R}{\alpha_R}|^2 = 4 \left(\Theta + \frac{1}{\Theta}\right)^{-2} \simeq \frac{4}{\Theta^2}$

Double-Barrier Tunneling

- Same method of analysis
- $T_{A\to C} = 4\left(\left(\Theta\Phi + \frac{1}{\Theta\Phi}\right)^2 \cos^2 W + \left(\frac{\Theta}{\Phi} + \frac{\Phi}{\Theta}\right)^2 \sin^2 W\right)^{-1}$
- $\Phi \simeq 2\exp\left(\frac{1}{\hbar}\int_{x_3}^{x_4} dx \sqrt{2m\left(V(x) - E\right)}\right)$
- $W = \frac{1}{\hbar}\int_{x_2}^{x_3} dx \sqrt{2m(E - V(x))}$

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- If B has zero width $T_{A\to C} \simeq 4\Theta^{-2}\Phi^{-2} = T_{A\to B}T_{B\to C}/4$

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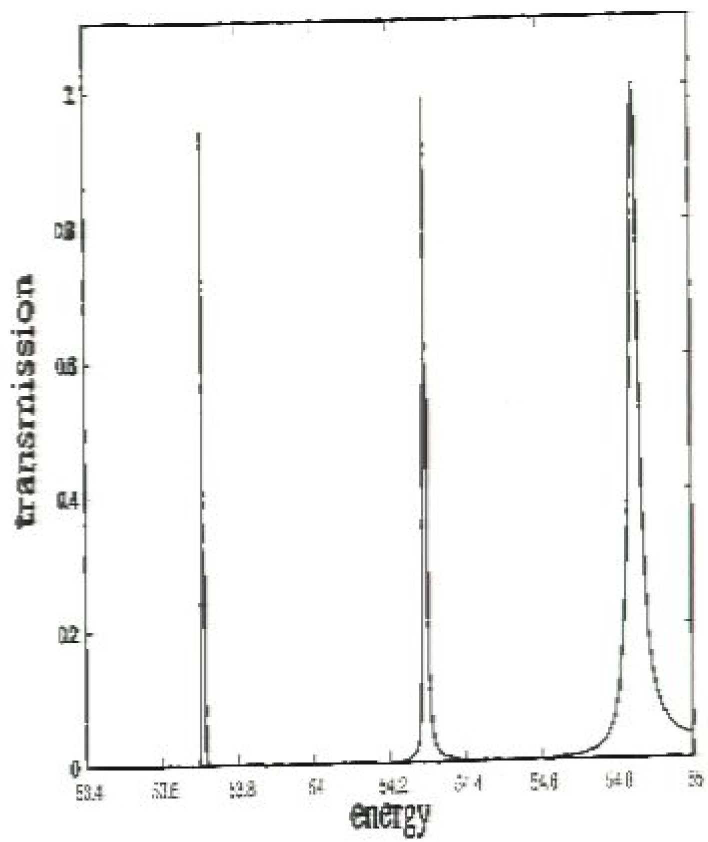
- If $W = (n_B + 1/2)\pi$ then $T_{A\to C} = \frac{4}{(\Theta/\Phi + \Phi/\Theta)^2}$

Tunneling Probability versus Energy



[1] Copeland, Padilla, Saffin

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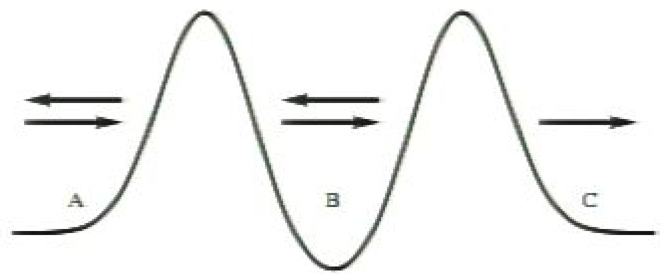

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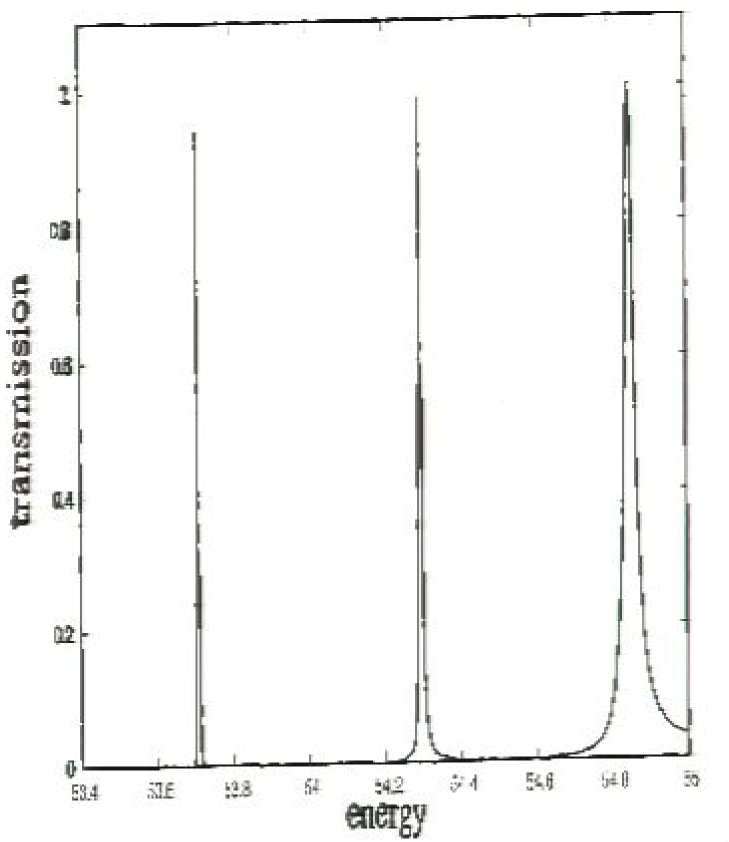

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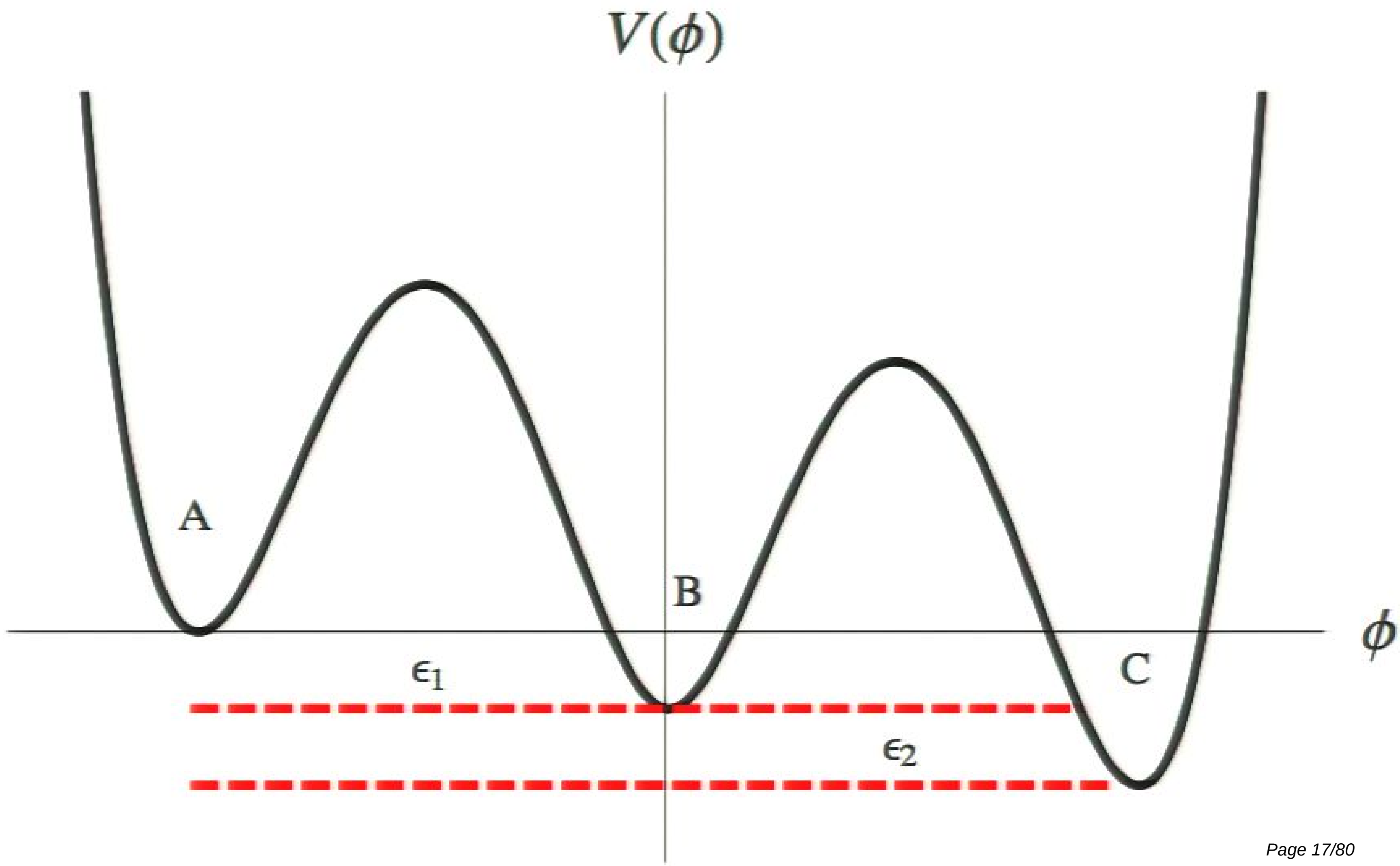
Potential



Thin-Wall Approximation

- Tunneling rate per unit volume [2] is $\Gamma/V = A\exp(-S_E/\hbar)$

[2] Coleman

Thin-Wall Approximation

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- Consider $V(\phi) = \frac{1}{4}g(\phi^2 - c^2)^2 - B(\phi + c)$
- Euclidean EOM is $\left(\frac{\partial^2}{\partial\tau^2} + \nabla^2\right)\phi = V'(\phi)$
- Assume O(4) symmetry
- Solution to Euclidean EOM is
  $\phi_{DW}(\tau, x, R) = -c\tanh\left(\frac{\mu}{2}(r - R)\right)$
- Inverse thickness of domain wall $\mu = \sqrt{2gc^2}$

[2] Coleman

Euclidean Action

- $S_E = \int d\tau d^3x \left[\frac{1}{2}\left(\frac{\partial\phi}{\partial\tau}\right)^2 + \frac{1}{2}(\nabla\phi)^2 + V(\phi)\right]$
- $S_E = -\frac{1}{2}\pi^2 R^4 \epsilon + 2\pi^2 R^3 S_1$
- Domain-wall tension is $S_1 = \int_{-c}^{c} d\phi \sqrt{2V(\phi)} = \frac{2}{3}\mu c$

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- $\frac{dS_E}{dR} = 0$ implies $\mathcal{E} = -\frac{4}{3}\pi R^3 \epsilon + 4\pi R^2 S_1 = 0$
- $R = \lambda_c \equiv 3S_1/\epsilon$
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- Bubble grows if $d\mathcal{E}/dR < 0$ or $R > 2\lambda_c/3$

Functional Schrödinger Method

Basic Idea

Infinite-dimensional QFT $\rightarrow$ one-dimensional QM problem

- In semiclassical limit, the vacuum tunneling rate is dominated by a discrete set of classical paths [3] [4] [5]
- Equivalent to Euclidean instanton method for single-barrier tunneling
- Easily generalizes to multiple-barrier tunneling

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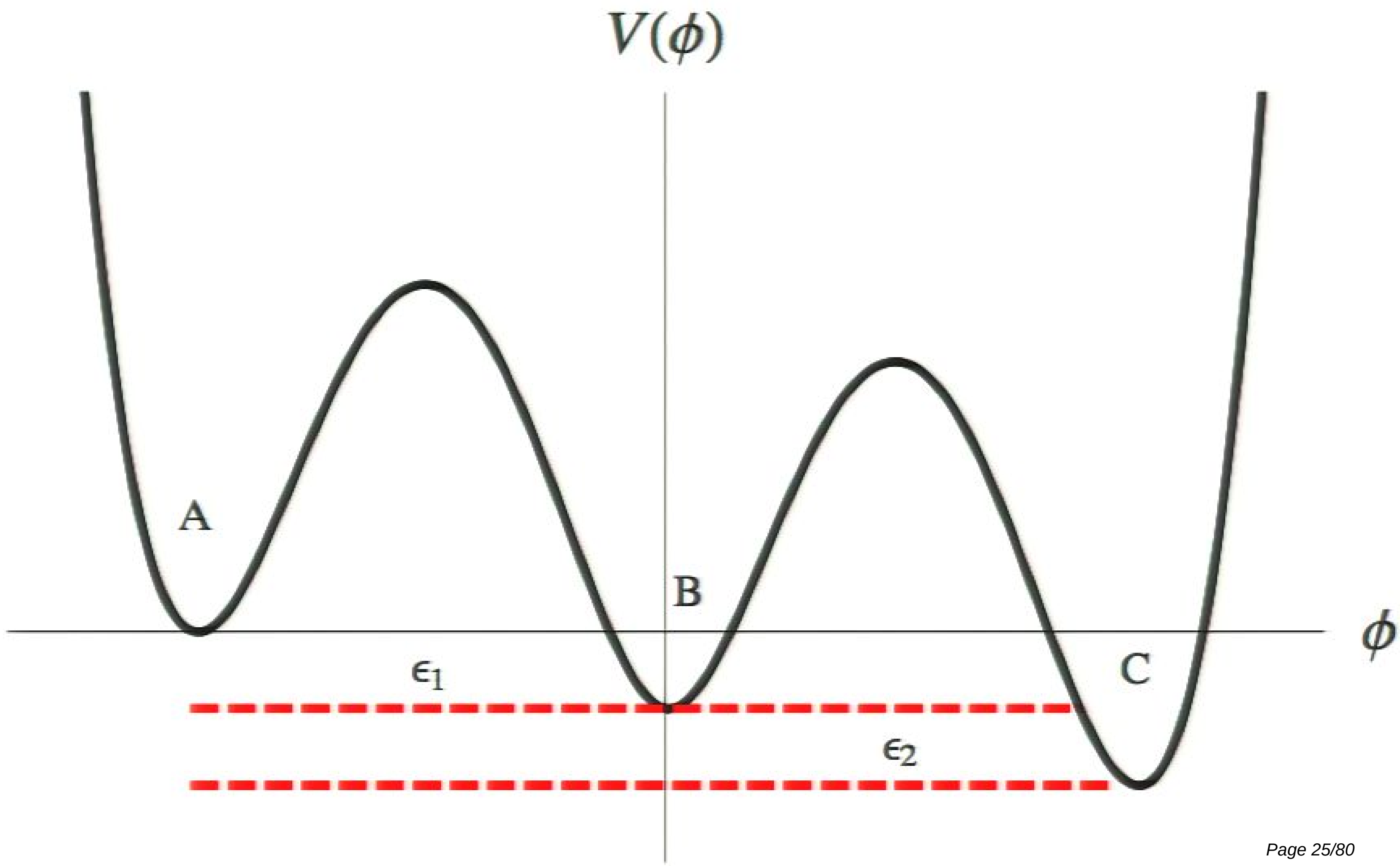
[3] Bender, Banks, Wu
[4] Gervais, Sakita
[5] Bitar, Chang

Functional Schrödinger Equation

- $H = \int d^3x \left(\frac{\dot{\phi}^2}{2} + \frac{1}{2}(\nabla\phi)^2 + V(\phi) \right)$
- Quantize using $[\dot{\phi}(x), \phi(x')] = i\hbar\delta^3(x - x')$
- $H = \int d^3x \left(-\frac{\hbar^2}{2} \left(\frac{\delta}{\delta\phi(x)} \right)^2 + \frac{1}{2}(\nabla\phi)^2 + V(\phi) \right)$

Potential



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- Make ansatz $\Psi(\phi) = A \exp(-\frac{i}{\hbar} S(\phi))$
- $H\Psi(\phi(x)) = E\Psi(\phi(x))$

Semiclassical Expansion

- $S(\phi) = S_{(0)}(\phi) + \hbar S_{(1)}(\phi) + \ldots$
- $\int d^3x \left[\frac{1}{2} \left(\frac{\delta S_{(0)}(\phi)}{\delta \phi} \right)^2 + \frac{1}{2} (\nabla \phi)^2 + V(\phi) \right] = E$
- $\int d^3x \left[-i \frac{\delta^2 S_{(0)}(\phi)}{\delta \phi^2} + 2 \frac{\delta S_{(0)}(\phi)}{\delta \phi} \frac{\delta S_{(1)}(\phi)}{\delta \phi} \right] = 0$
- Ignore higher-order terms

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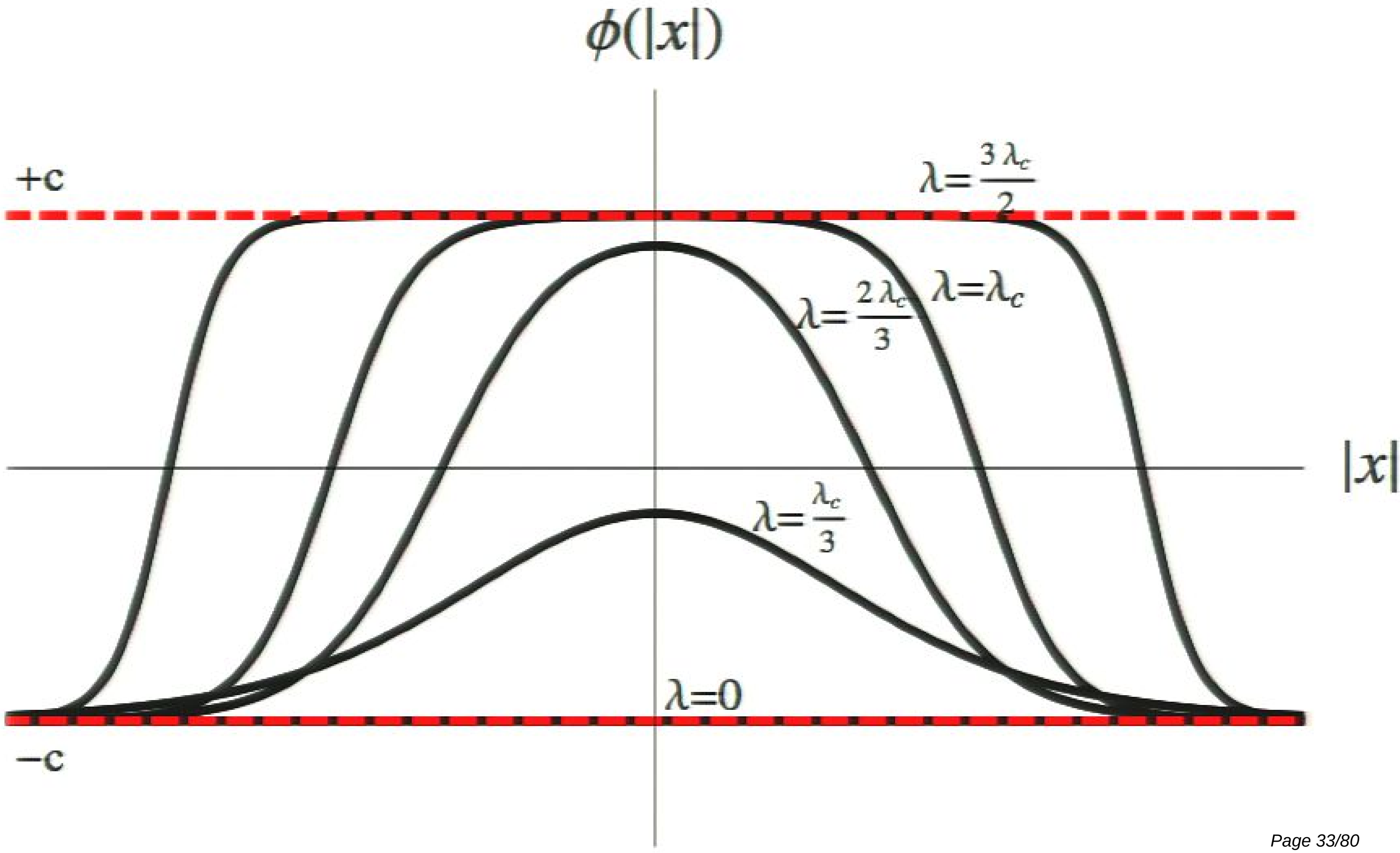
Goal

Determine value of $S_{(0)}$ that gives dominant contribution to the tunneling probability

Most Probable Escape Path

- $\phi_0(x, \lambda)$
- Trajectory in the configuration space of $\phi(x)$ parameterized by λ which gives dominant contribution to tunneling probability
- Variation of $S_{(0)}$ vanishes perpendicular to the MPEP
- Variation of $S_{(0)}$ is nonvanishing along the MPEP
- MPEP satisfies the same Euler-Lagrange equation as instanton in Euclidean instanton method

Most Probable Escape Path



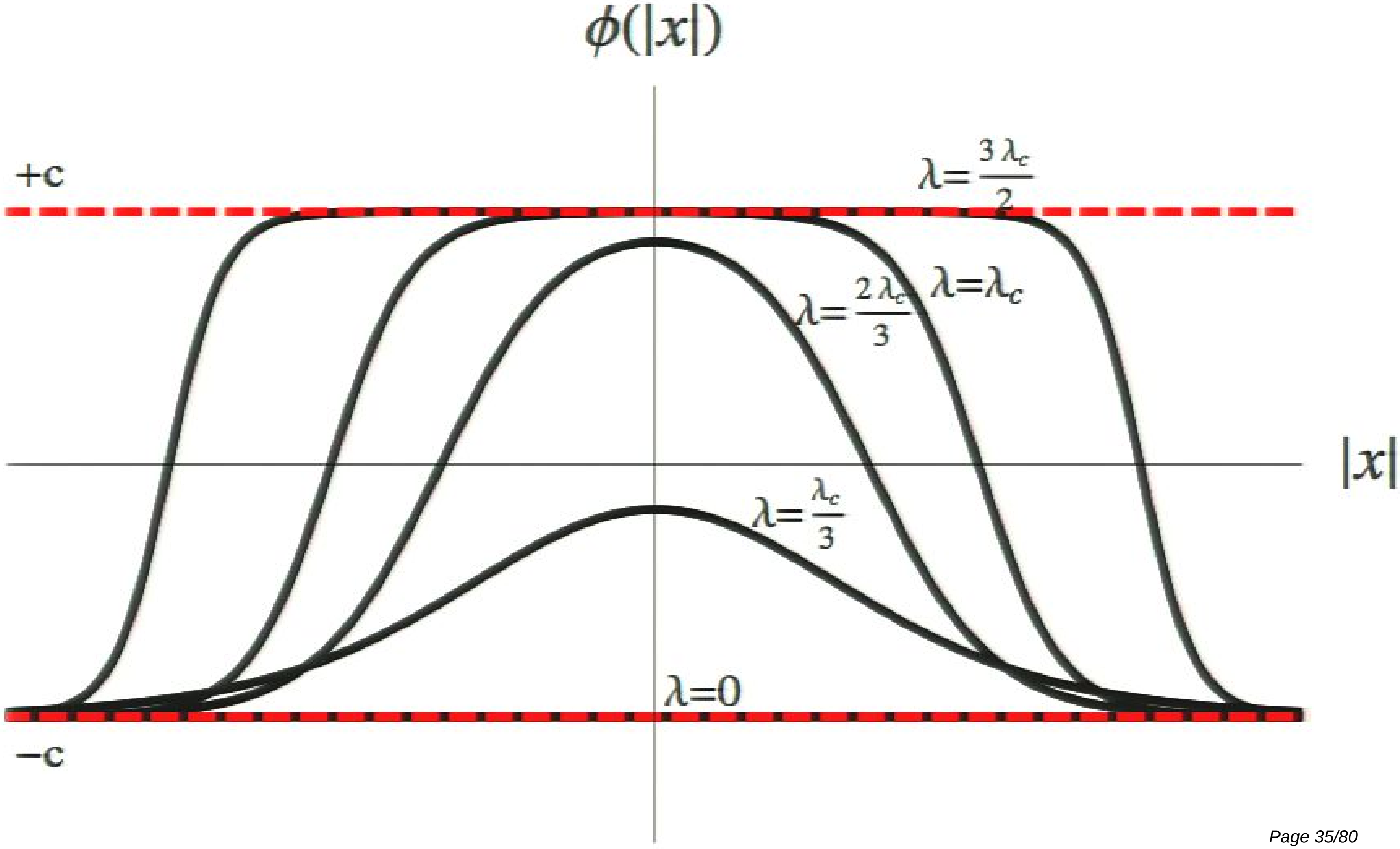
Determining $S_{(0)}$

- Effective tunneling potential
$U(\lambda) = U(\phi(x,\lambda)) = \int d^3x \left(\frac{1}{2}(\nabla\phi(x,\lambda))^2 + V(\phi(x,\lambda))\right)$
- Path length $(ds)^2 = \int d^3x (d\phi(x))^2 =$
$(d\lambda)^2 \int d^3x \left(\frac{\partial\phi(x,\lambda)}{\partial\lambda}\right)^2 = (d\lambda)^2 m(\phi(x,\lambda))$

Zeroth-Order Solution (Classically Forbidden Region)

$$S_{(0)} = i\int_0^s ds' \sqrt{2[U(\phi(x,s')) - E]} =$$
$$i\int_{\lambda_1}^{\lambda_2} d\lambda \left(\frac{ds}{d\lambda}\right)\sqrt{2[U(\phi(x,\lambda)) - E]}$$

Most Probable Escape Path



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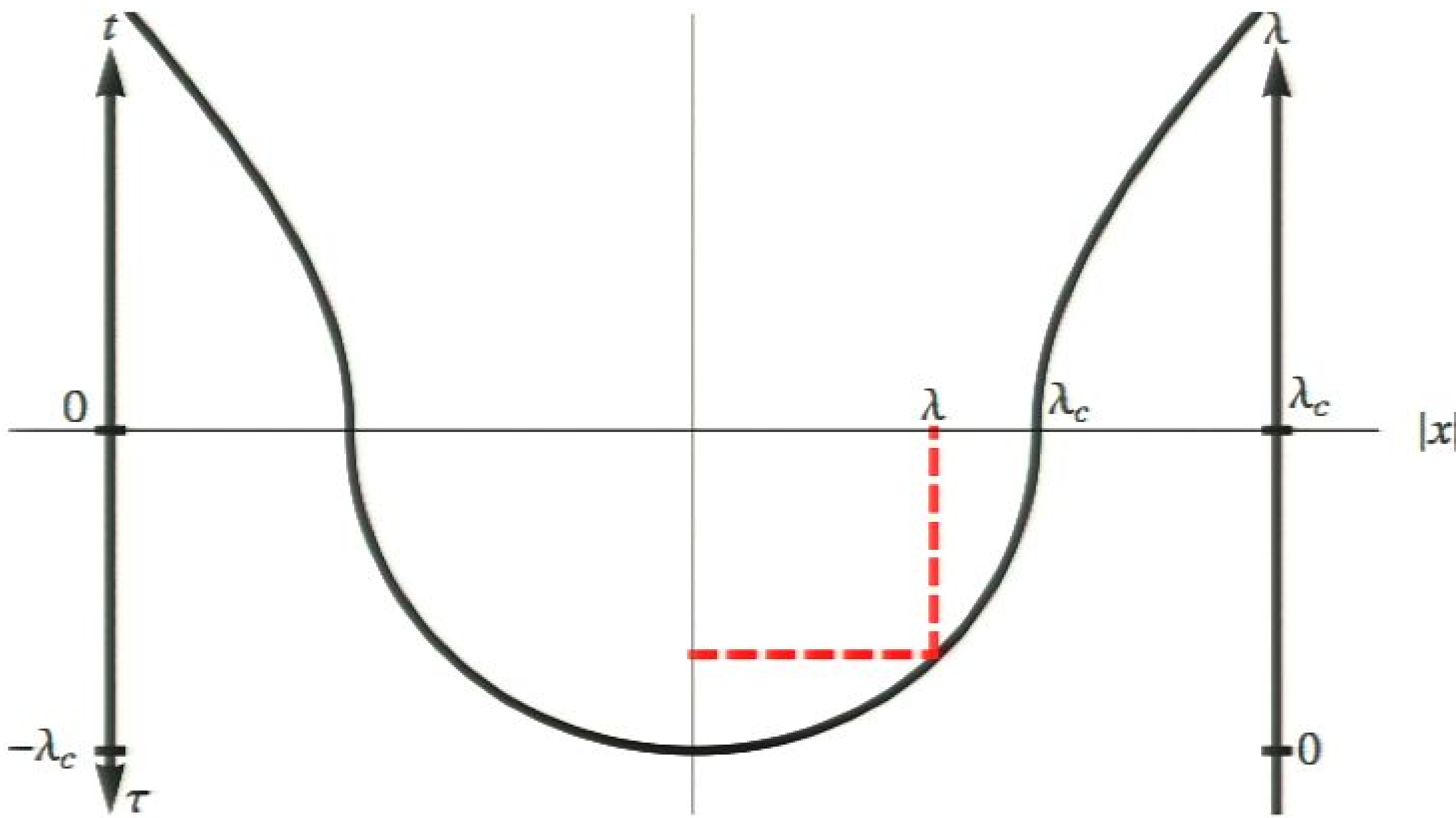
Parameterization of MPEP



- Solution parameterized by $\lambda = \sqrt{\lambda_c^2 - \tau^2}$

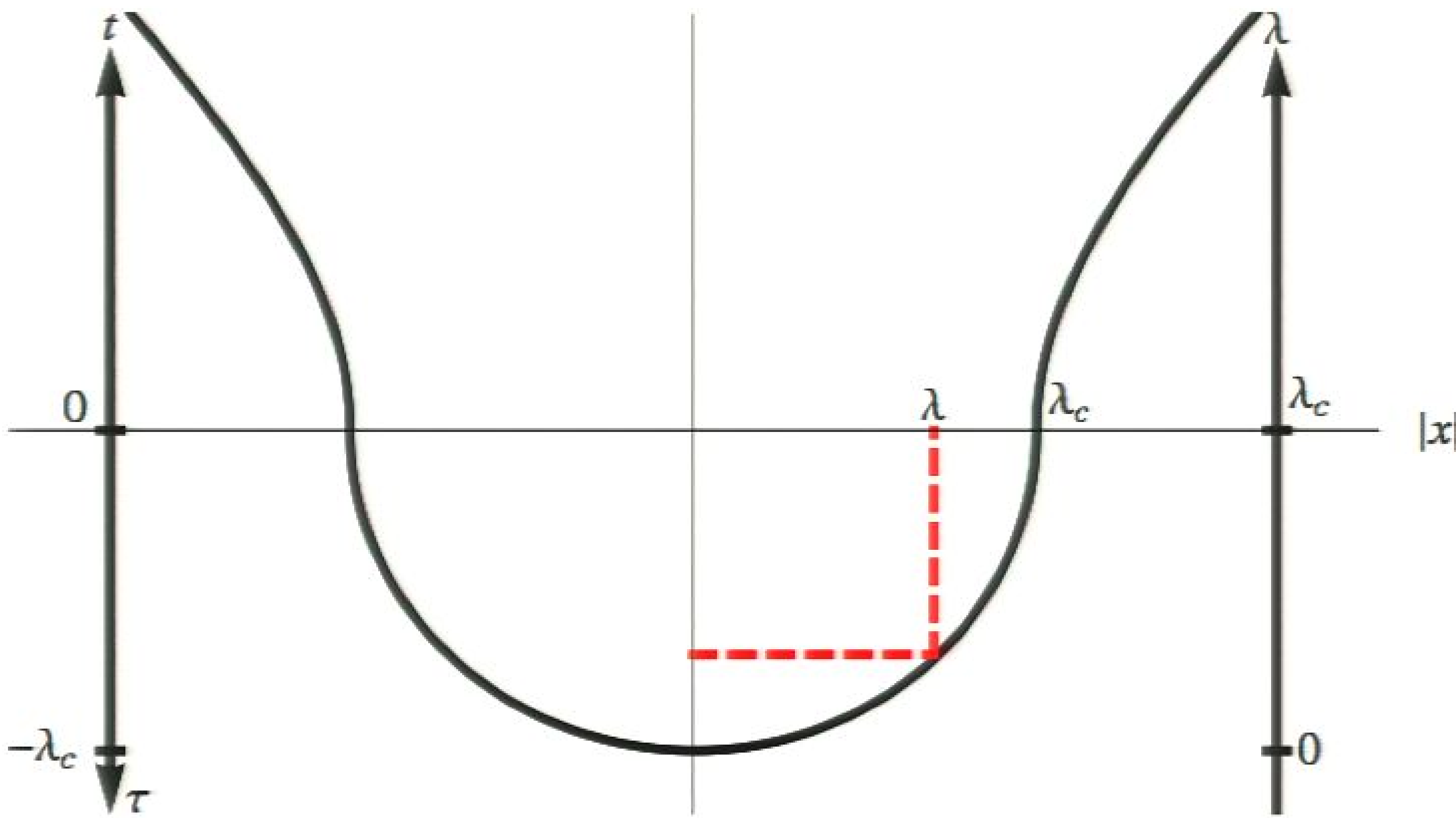
WKB Wavefunction

WKB Wavefunction

$$\Psi(\phi(x,\lambda)) = Ae^{iS_0/\hbar} = A\exp\left(-\frac{1}{\hbar}\left[\int_0^{\lambda_c} d\lambda\sqrt{2m(\lambda)[U(\lambda)-E]}\right]\right)$$

- Position-dependent mass $m(\lambda) \equiv \int d^3x \left(\frac{\partial\phi_0(x,\lambda)}{\partial\lambda}\right)^2$
- Effective tunneling potential $U(\phi(x,\lambda)) = \int d^3x \left(\frac{1}{2}(\nabla\phi(x,\lambda))^2 + V(\phi(x,\lambda))\right)$
- Now have one-dimensional QM problem

Parameterization of MPEP



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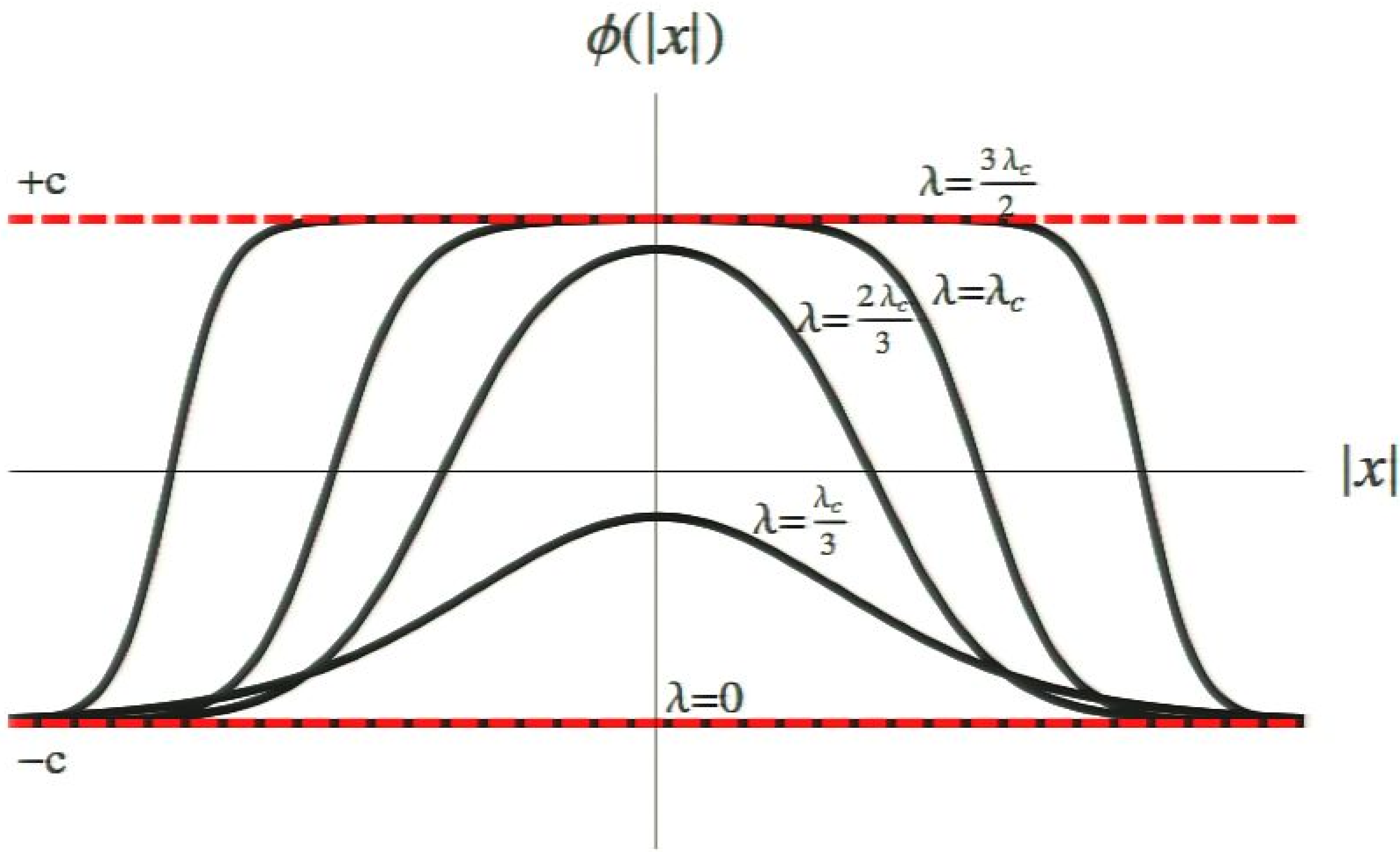

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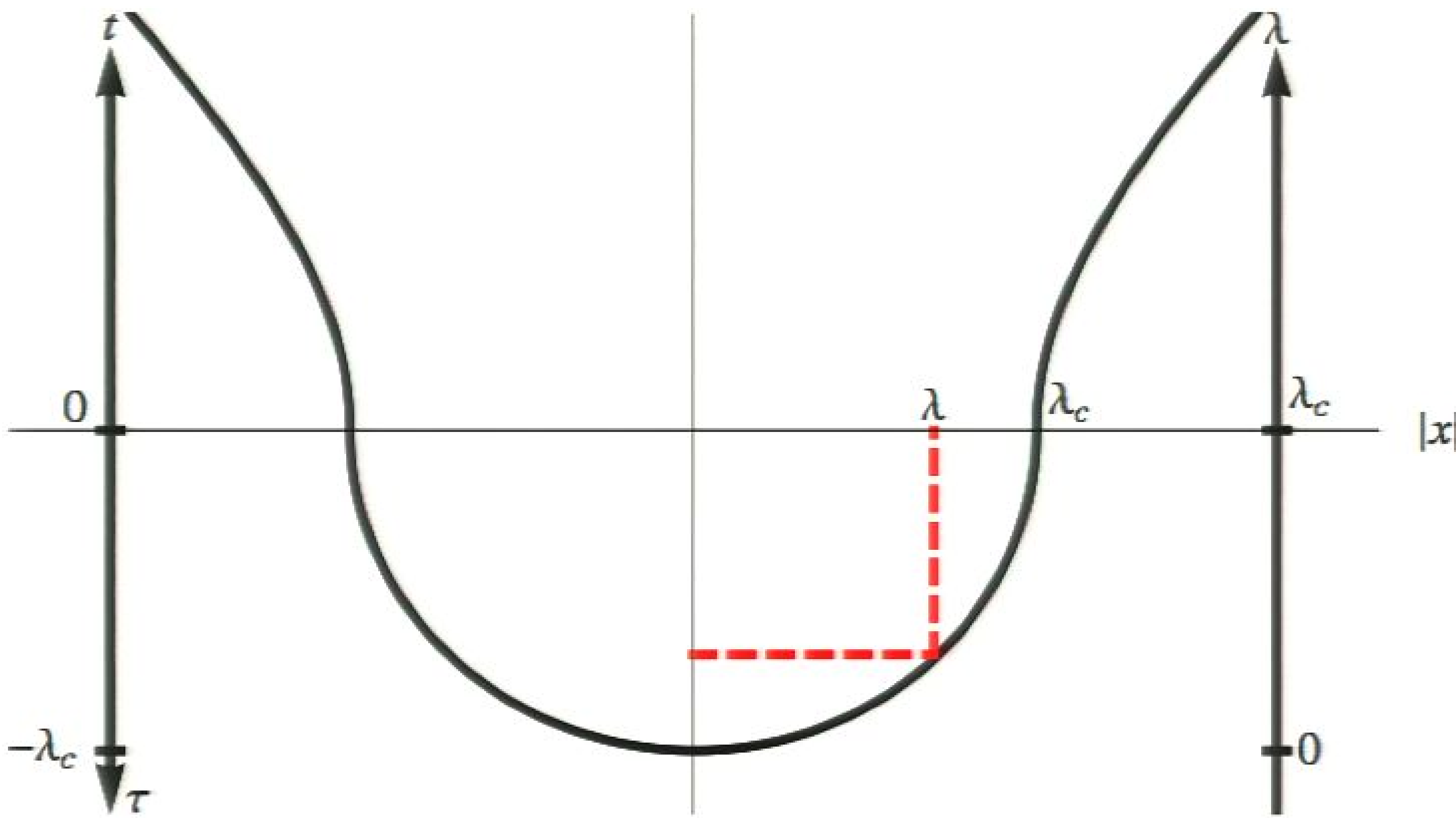

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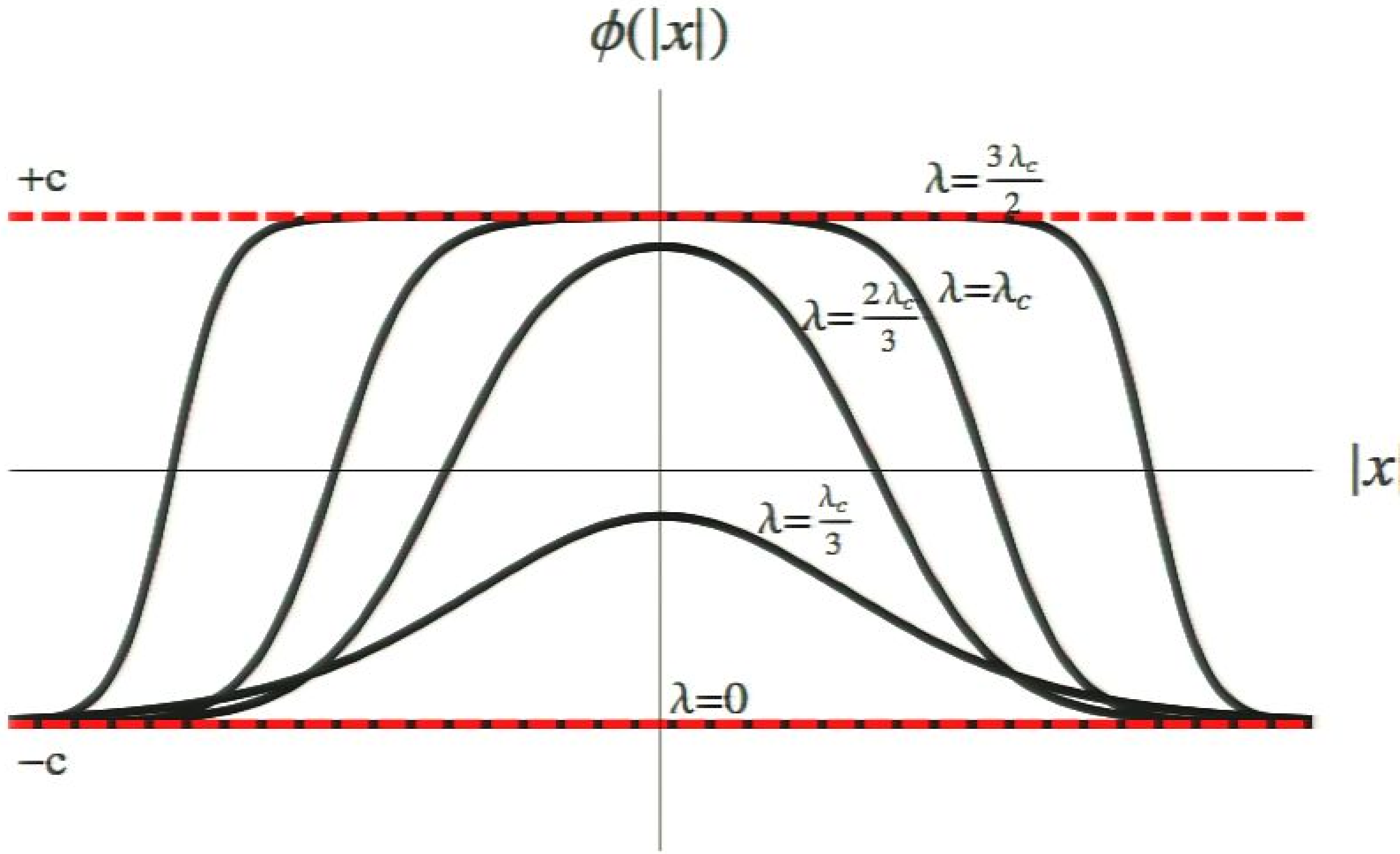

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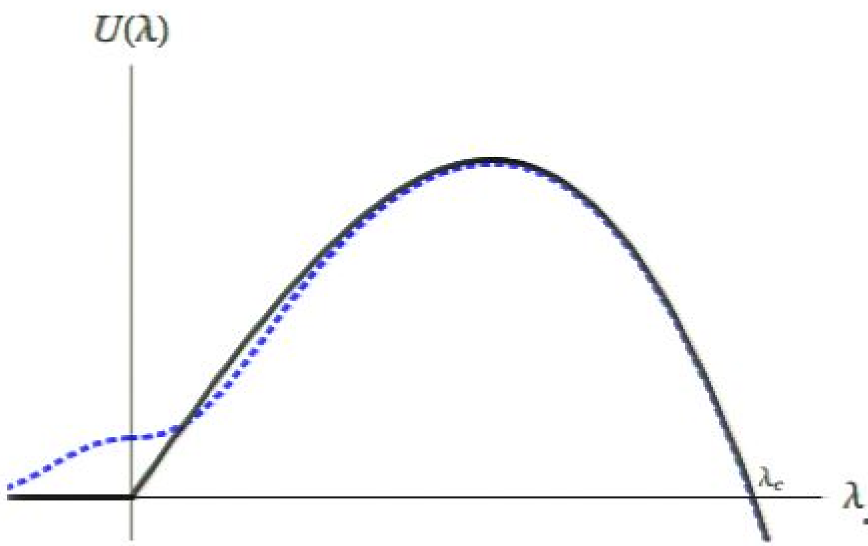
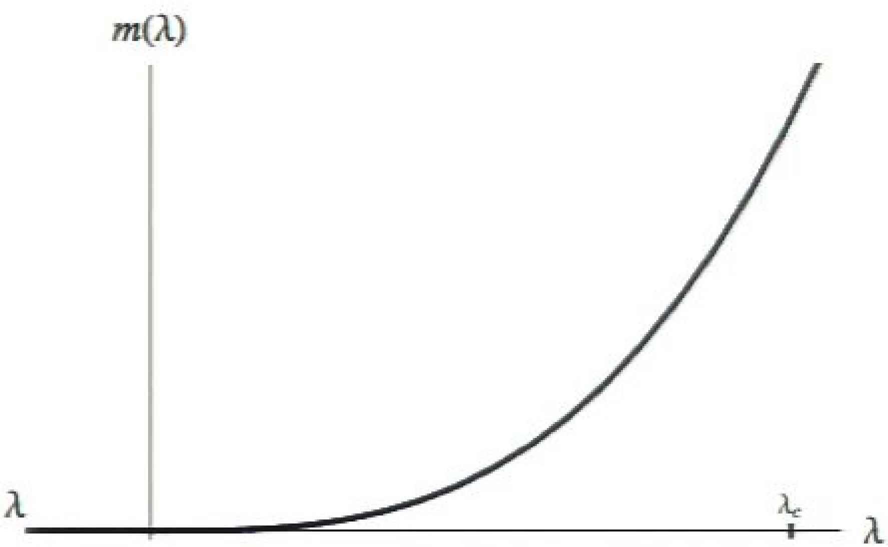
Equivalence of FSM with EIM



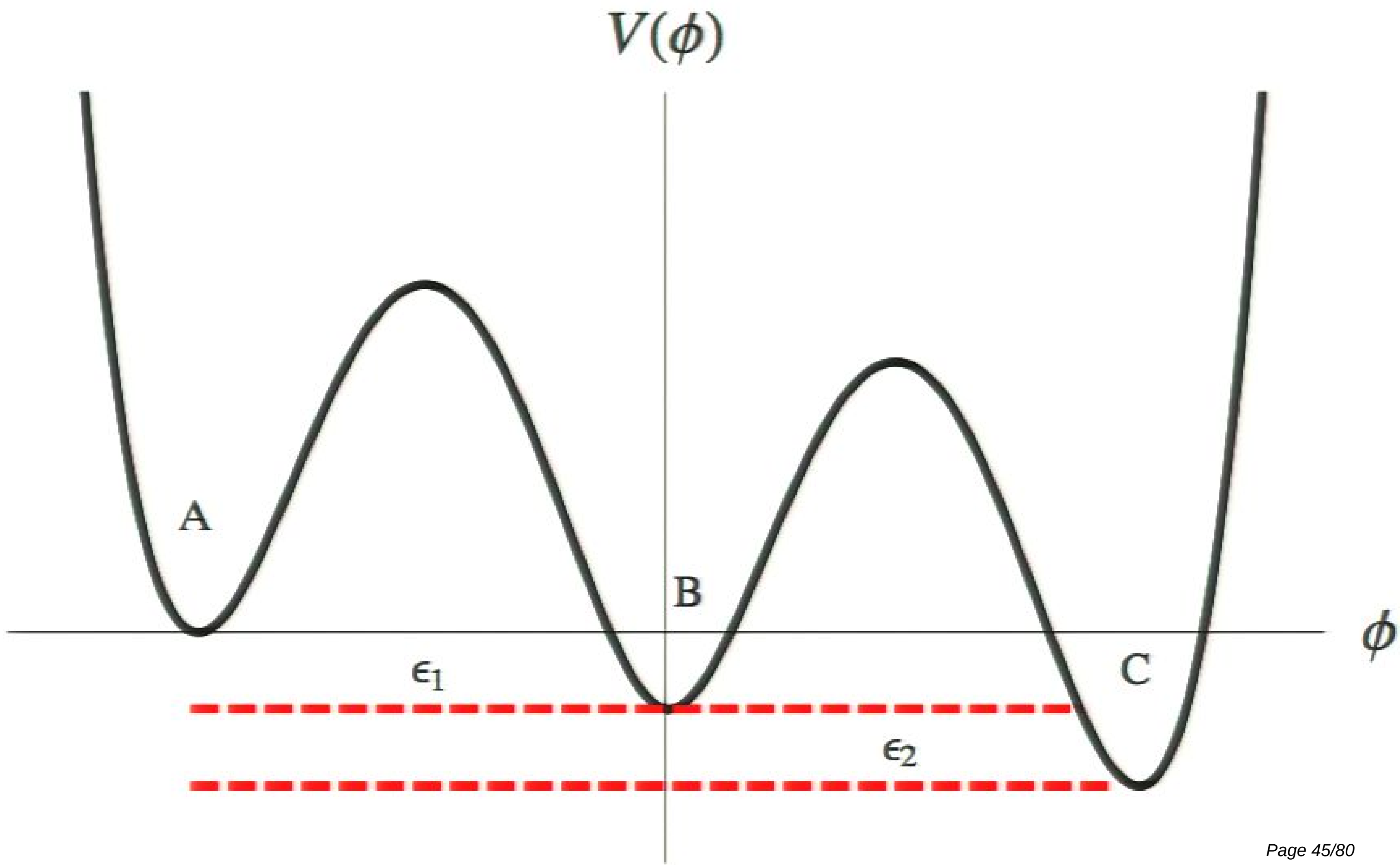


- Integrate spatially to get $U(\lambda) \approx \frac{2\pi S_1}{\lambda_c}\lambda(\lambda_c^2 - \lambda^2)$
- Position dependent mass $m(\lambda) \approx 4\pi S_1 \frac{\lambda^3}{\lambda_c}$
- Amplitude is $\exp\left(-\frac{27\pi^2}{4}\frac{S_1^4}{\epsilon^3}\right) = \exp\left(-S_E/2\right)$

Advantages of FSM

- Same arguments lead to $S_{(0)}(\phi(x,\lambda)) = \int d\lambda \sqrt{2m((\phi(x,\lambda))[E - U((\phi(x,\lambda))]}$ and Lorentzian EOM in classically allowed regions
- If we choose $\lambda = \sqrt{\lambda_c^2 + t^2}$ as parameter, MPEP takes form $\phi_0(x,\lambda) \approx -c\tanh\left(\frac{\mu}{2}(|x| - \lambda)\frac{\lambda}{\lambda_c}\right) = -c\tanh\left(\frac{\mu}{2}\frac{(|x|-\lambda)}{\sqrt{1-\dot{\lambda}^2}}\right)$
- Single real parameter describes entire system

Potential



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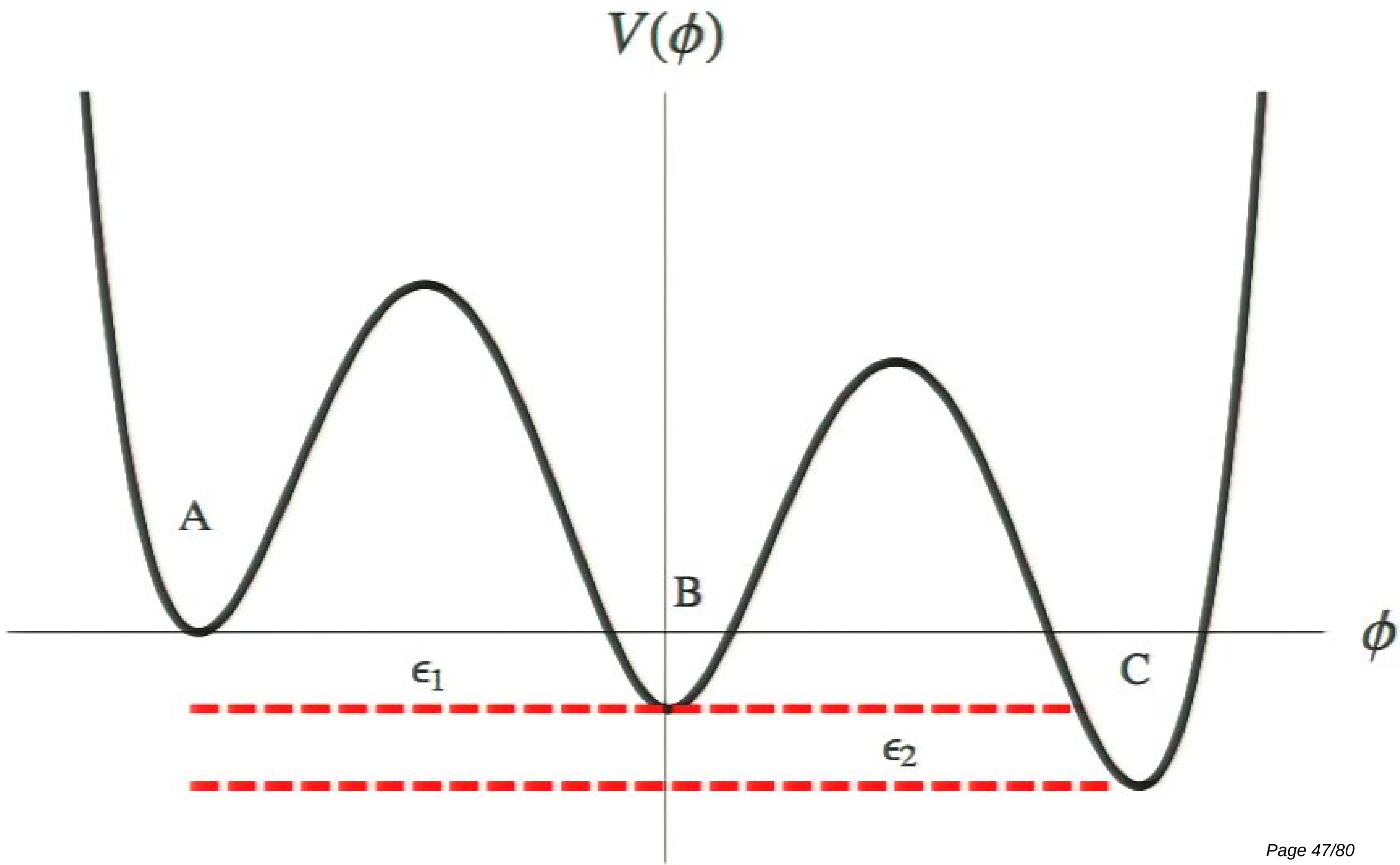
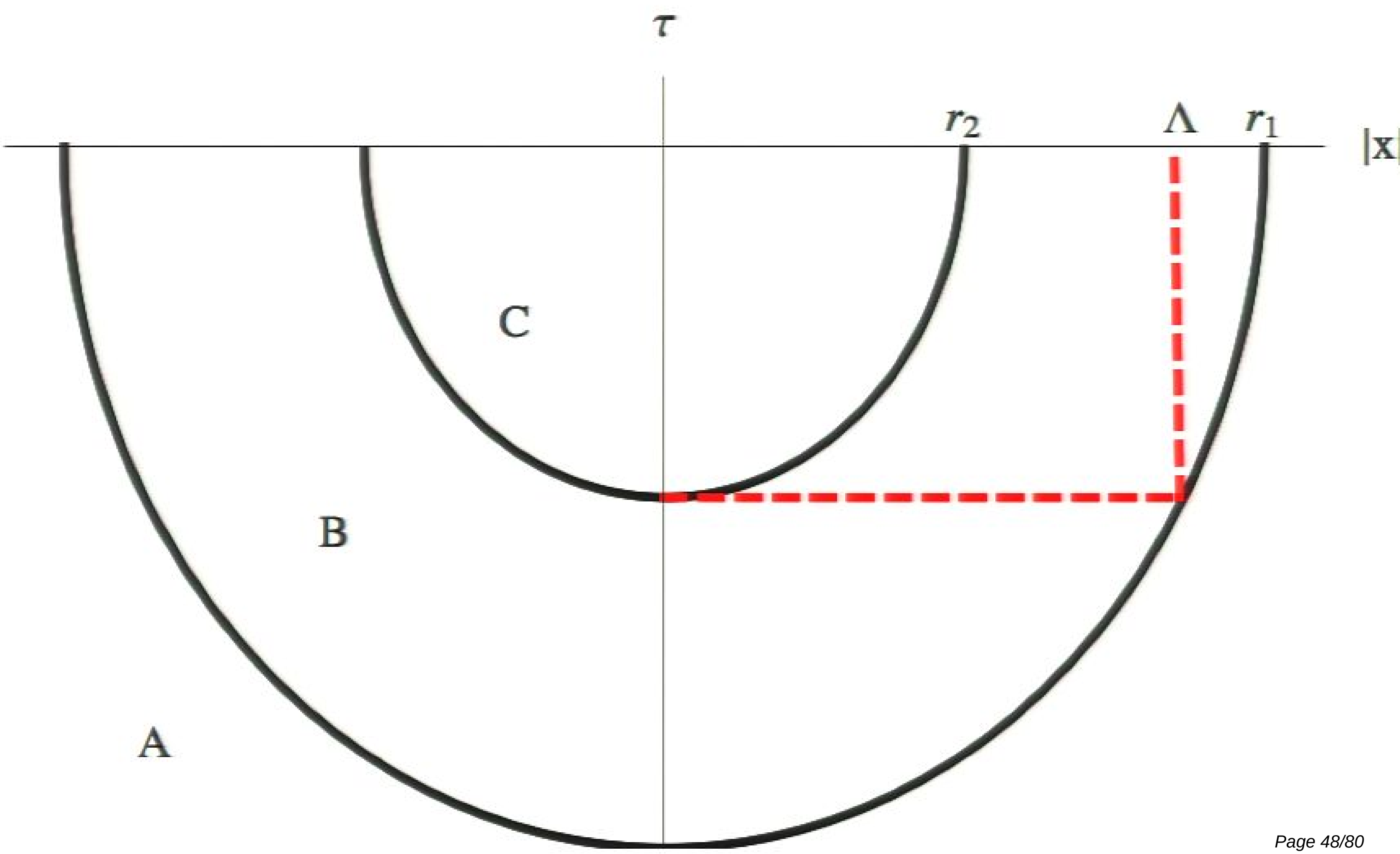


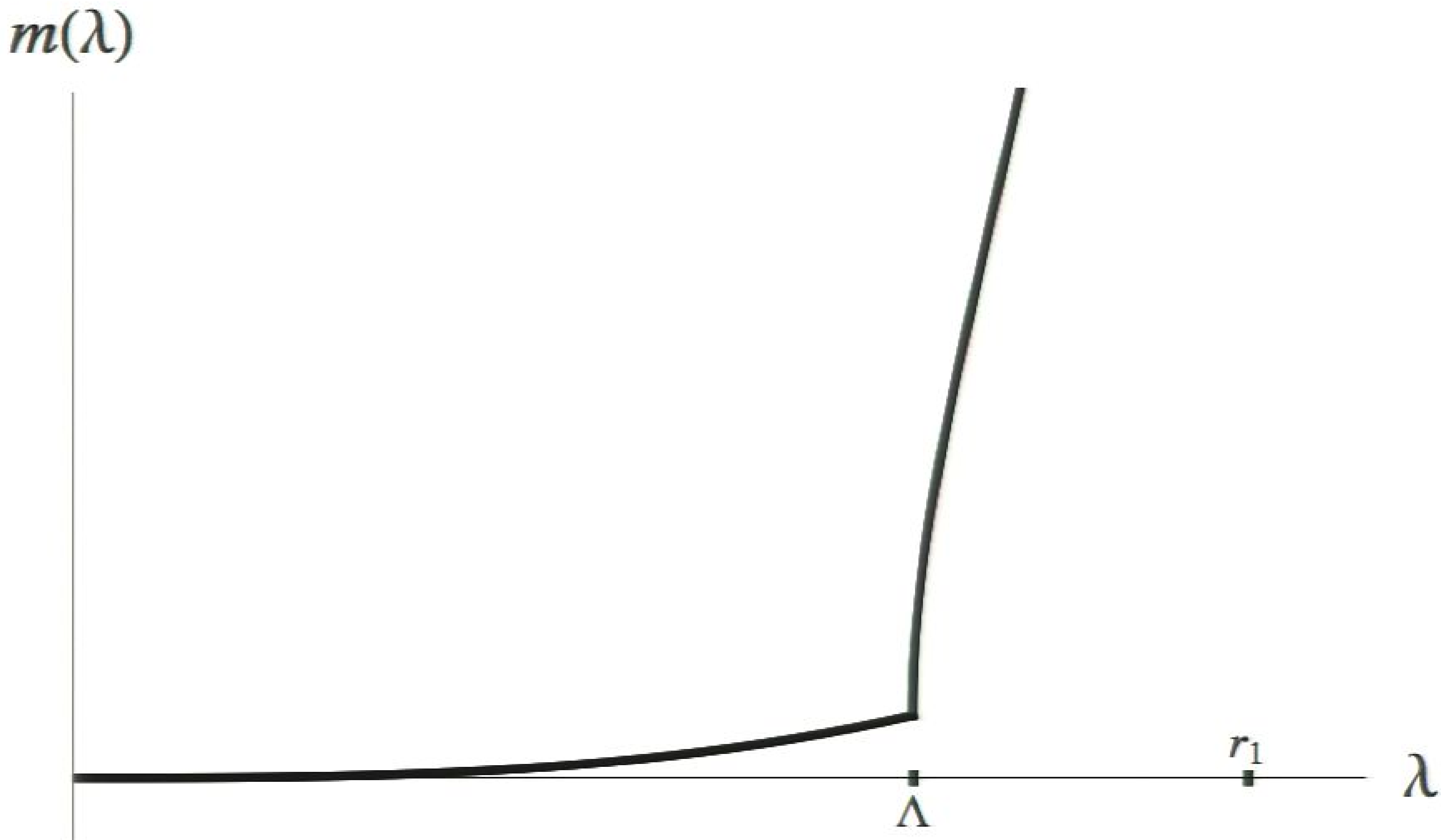
MPEP



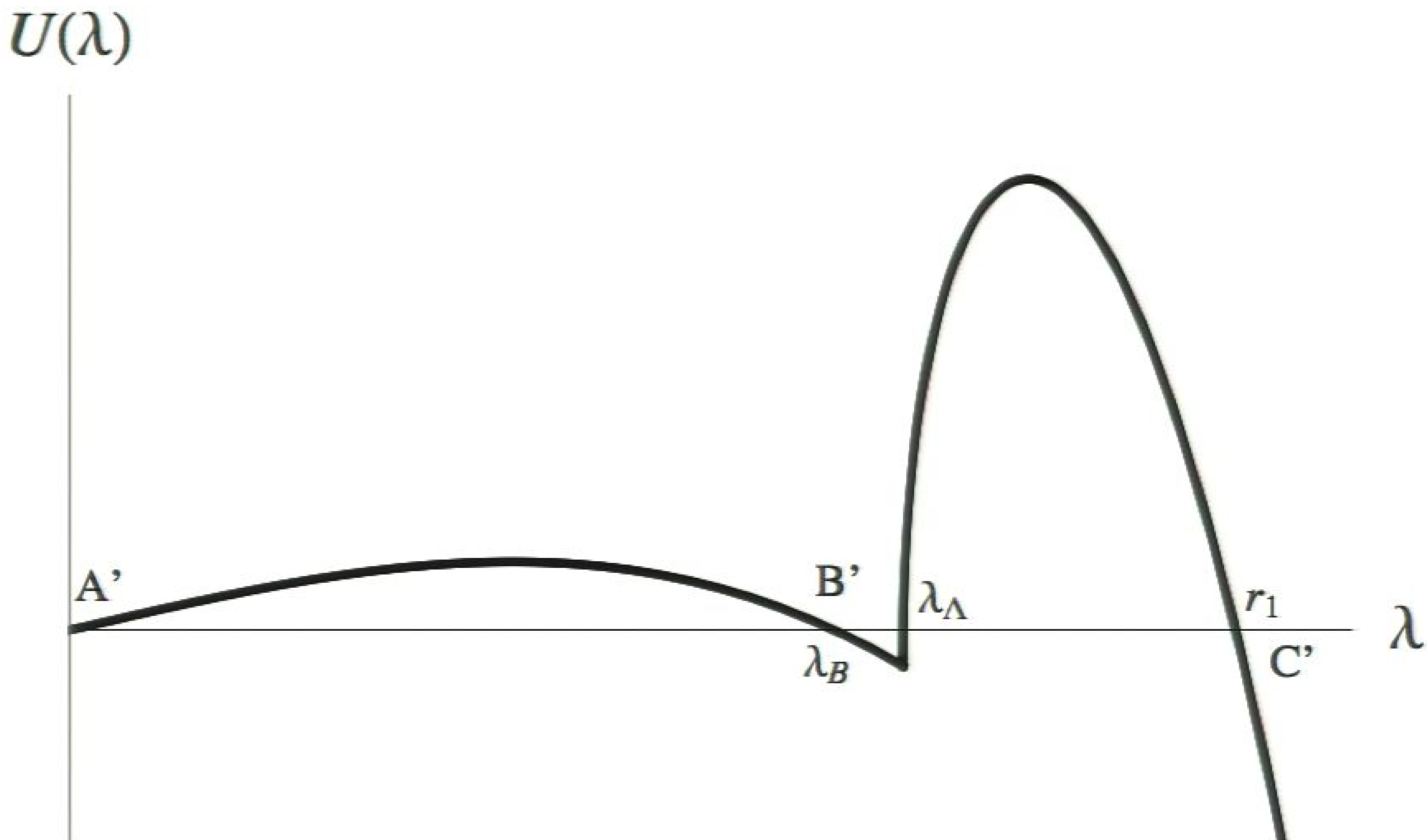
Resonant Tunneling in QFT

- $T_{A\to C} = 4\left(\left(\Theta\Phi + \frac{1}{\Theta\Phi}\right)^2 \cos^2 W + \left(\frac{\Theta}{\Phi} + \frac{\Phi}{\Theta}\right)^2 \sin^2 W\right)^{-1}$
- $W = \int_{\lambda_2}^{\lambda_3} d\lambda \sqrt{2m(\lambda)(-U(\lambda))}$
- $W = \frac{S_1^{(1)}\lambda_\Lambda}{\lambda_B}\sqrt{\lambda_\Lambda^2 - \lambda_B^2} - S_1^{(1)}\lambda_B \log\left[\frac{\lambda_\Lambda + \sqrt{\lambda_\Lambda^2 - \lambda_B^2}}{\lambda_B}\right]$
- Resonance condition is $W = (n + \frac{1}{2})\pi$

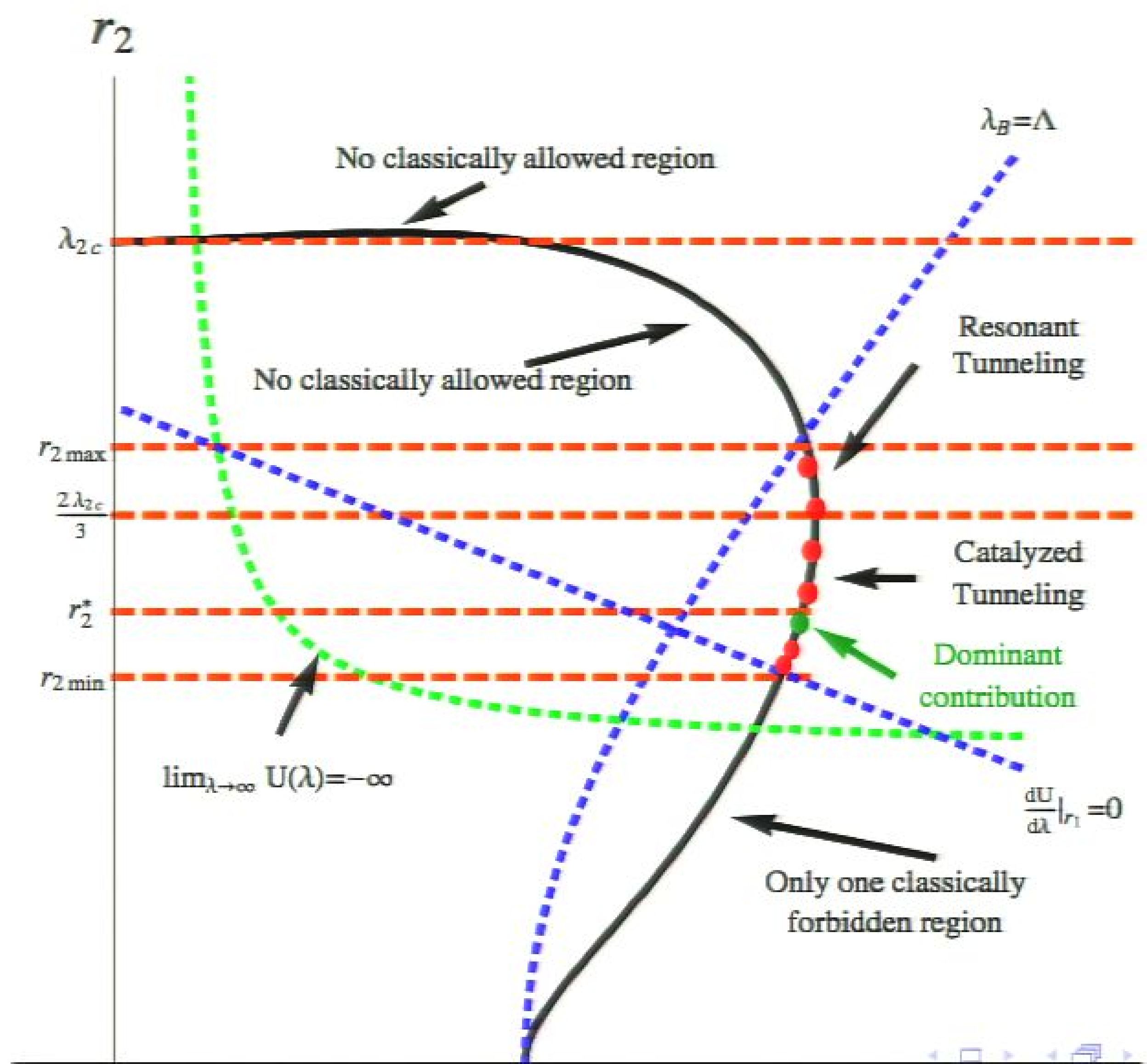
Position-Dependent Mass



Effective Tunneling Potential



Consistency Conditions



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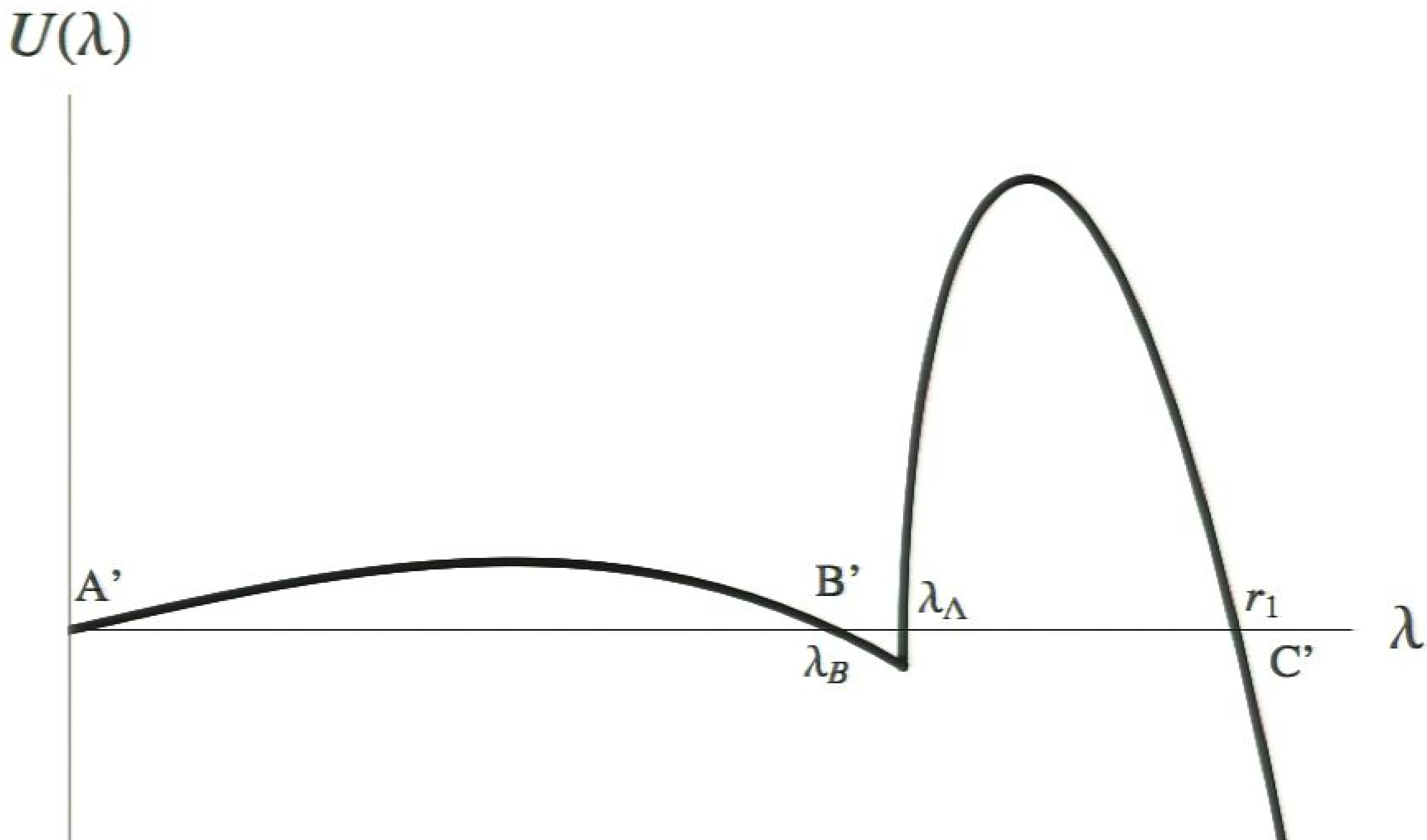
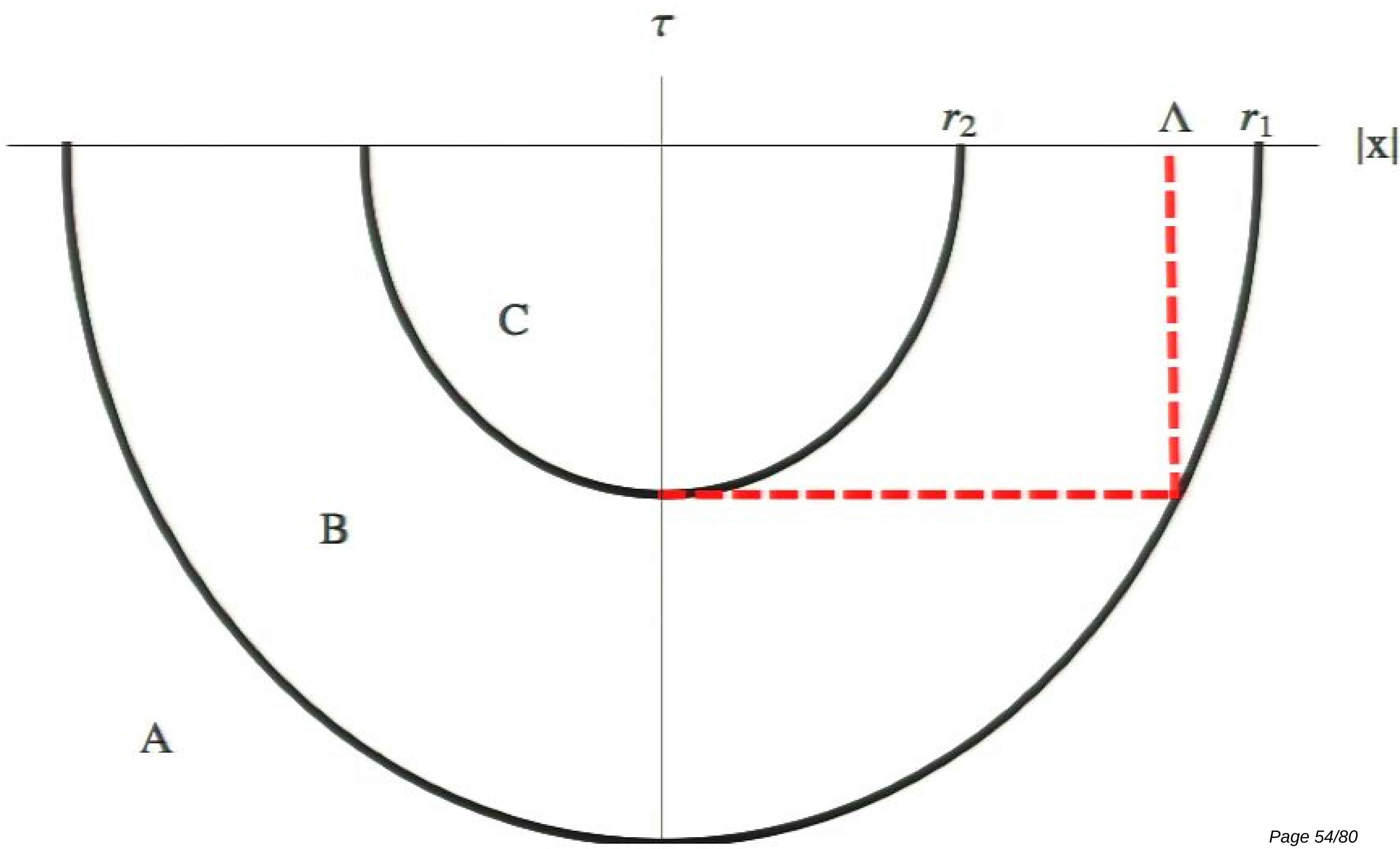


MPEP



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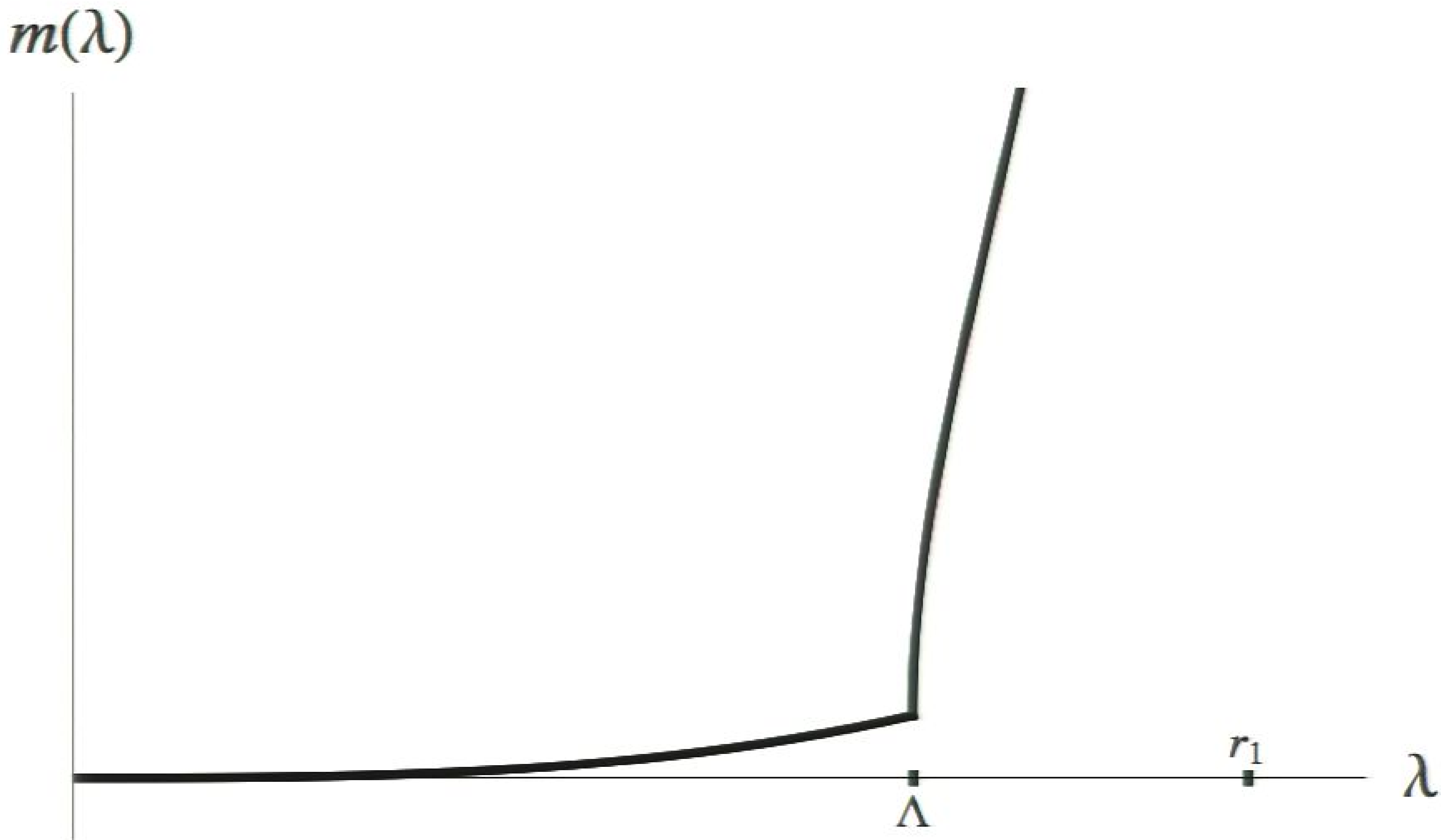

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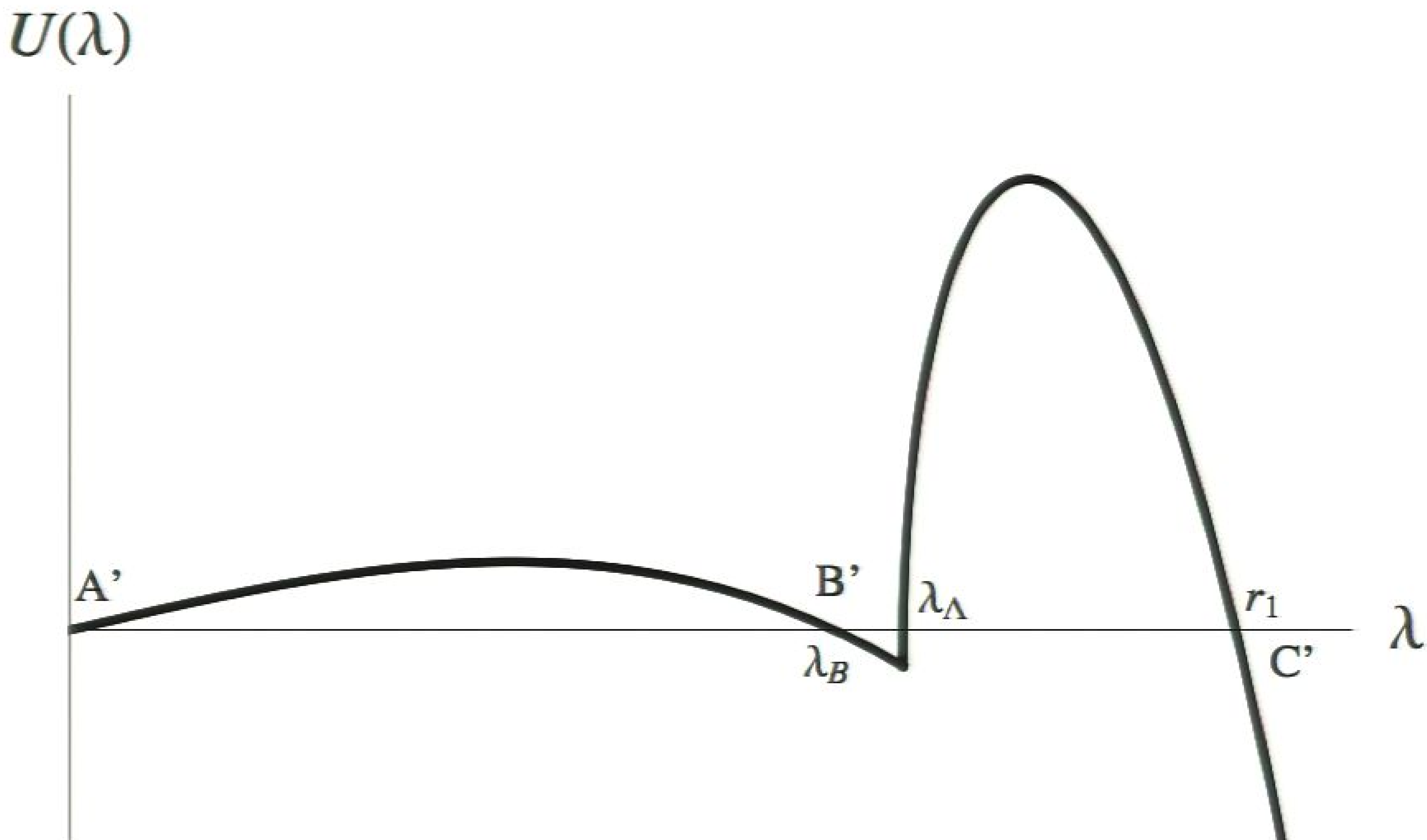

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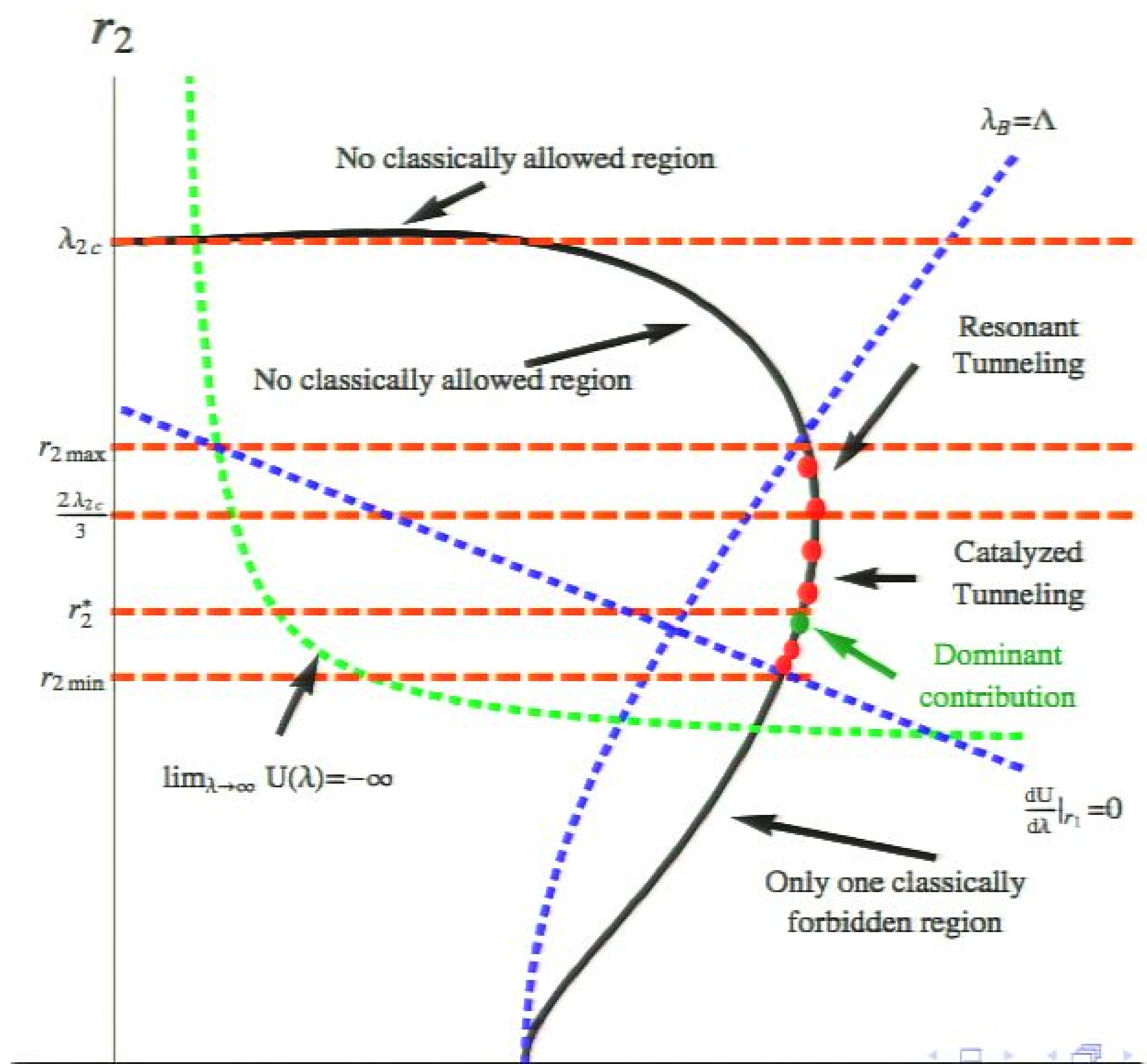

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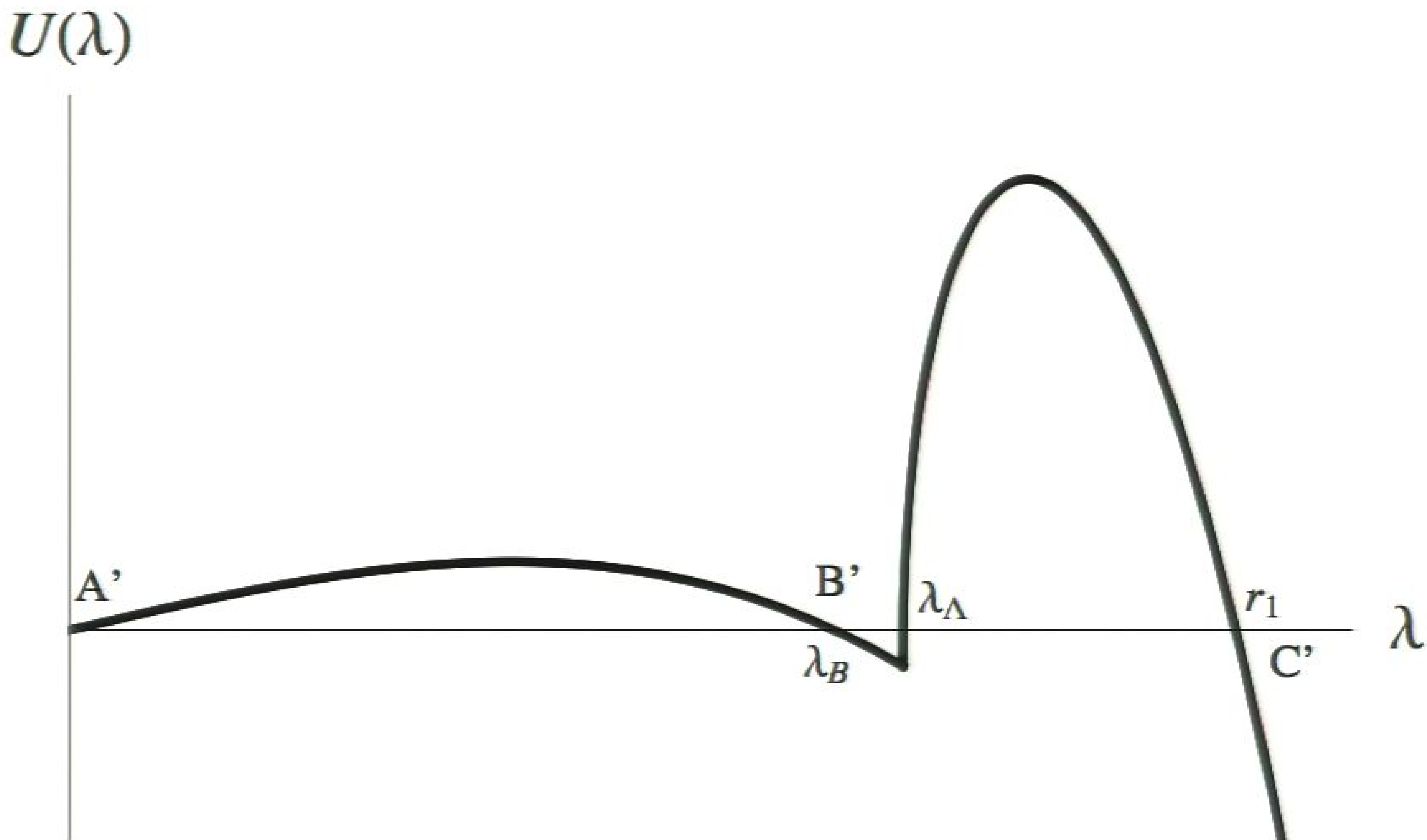

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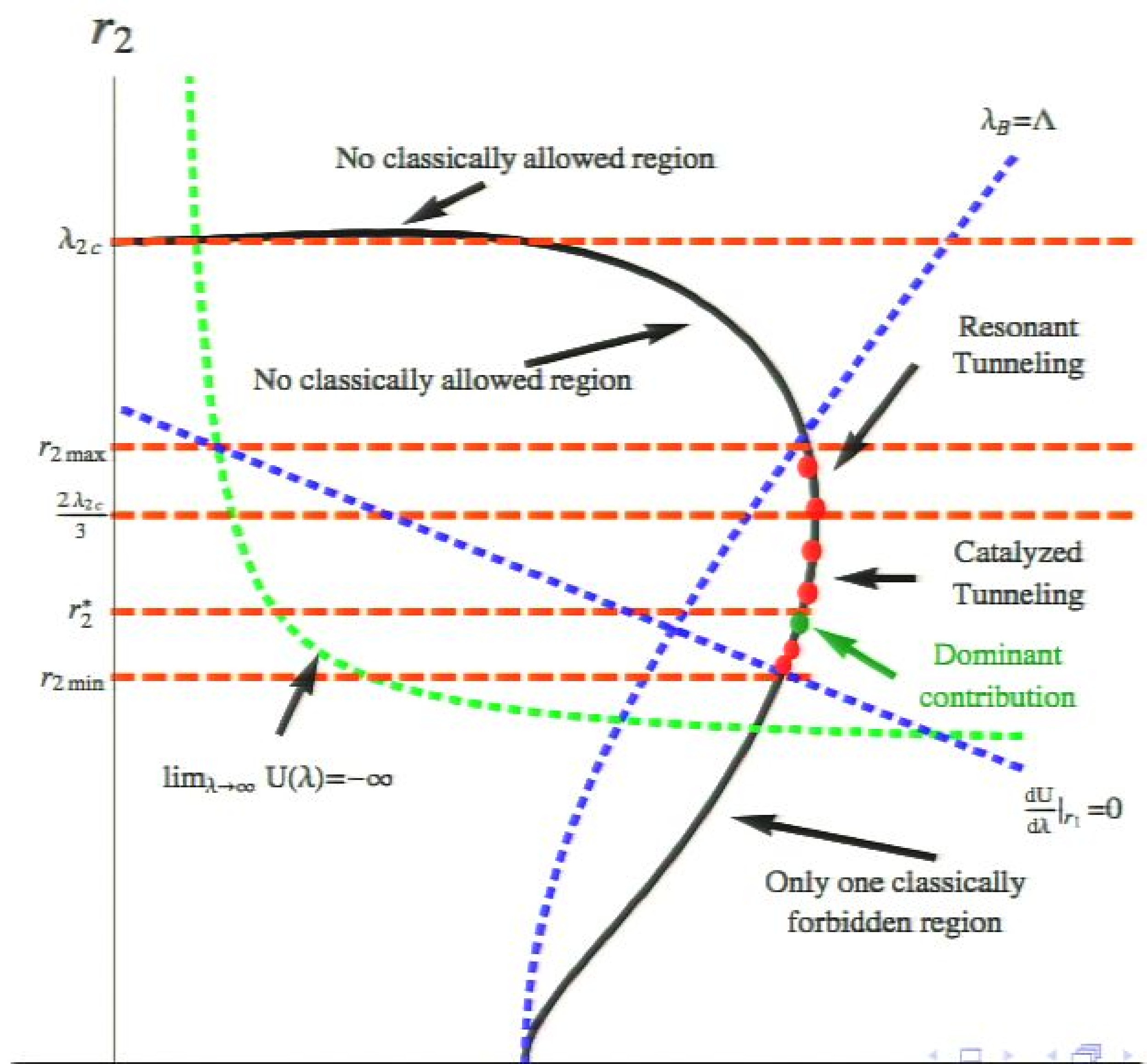

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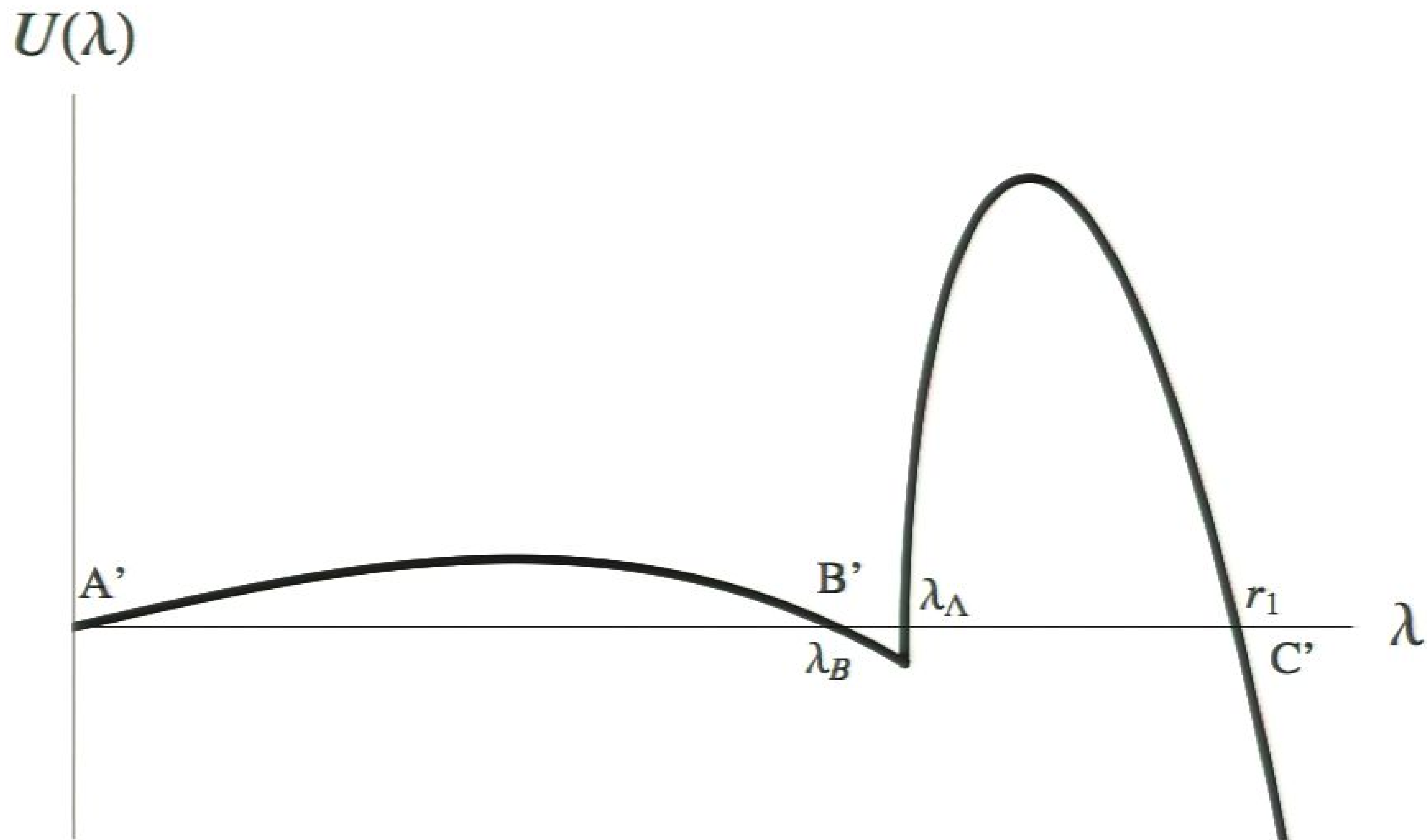

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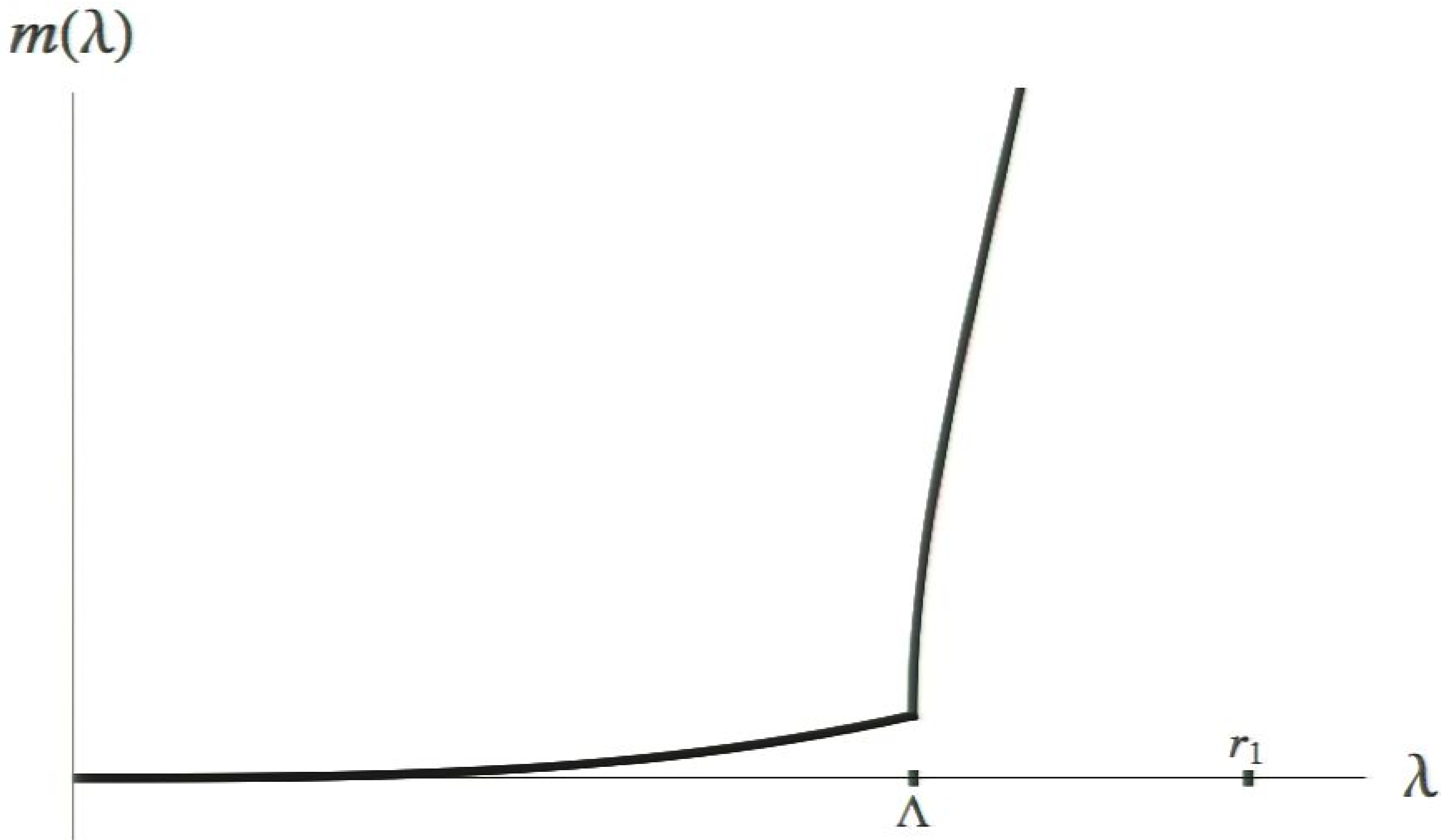

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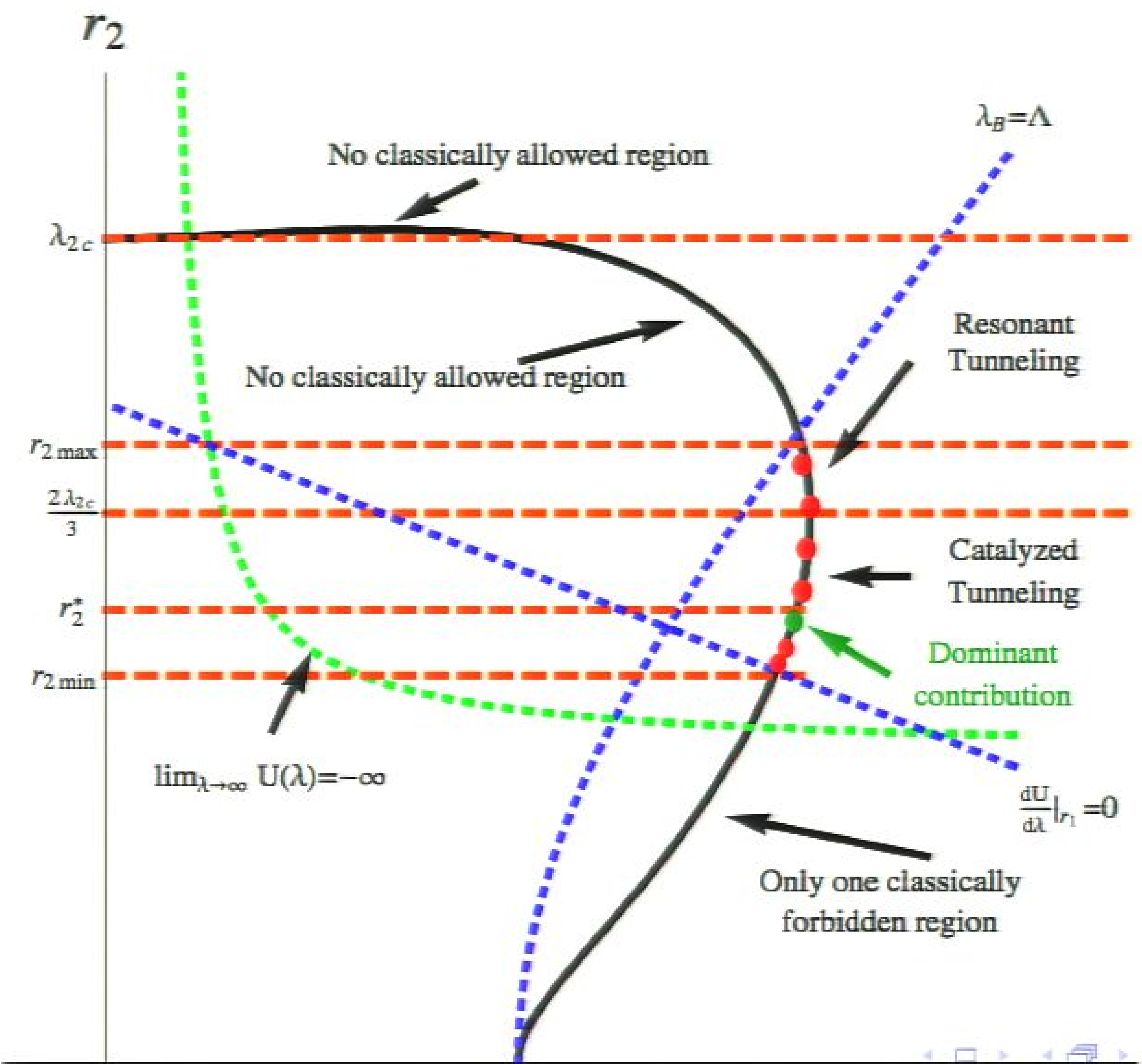

Resonant Tunneling or Catalytic Tunneling

Resonant Tunneling

If the inside bubble is large enough $\lambda_{2c} > r_2 > 2\lambda_{2c}/3$ tunneling from A to C will complete.

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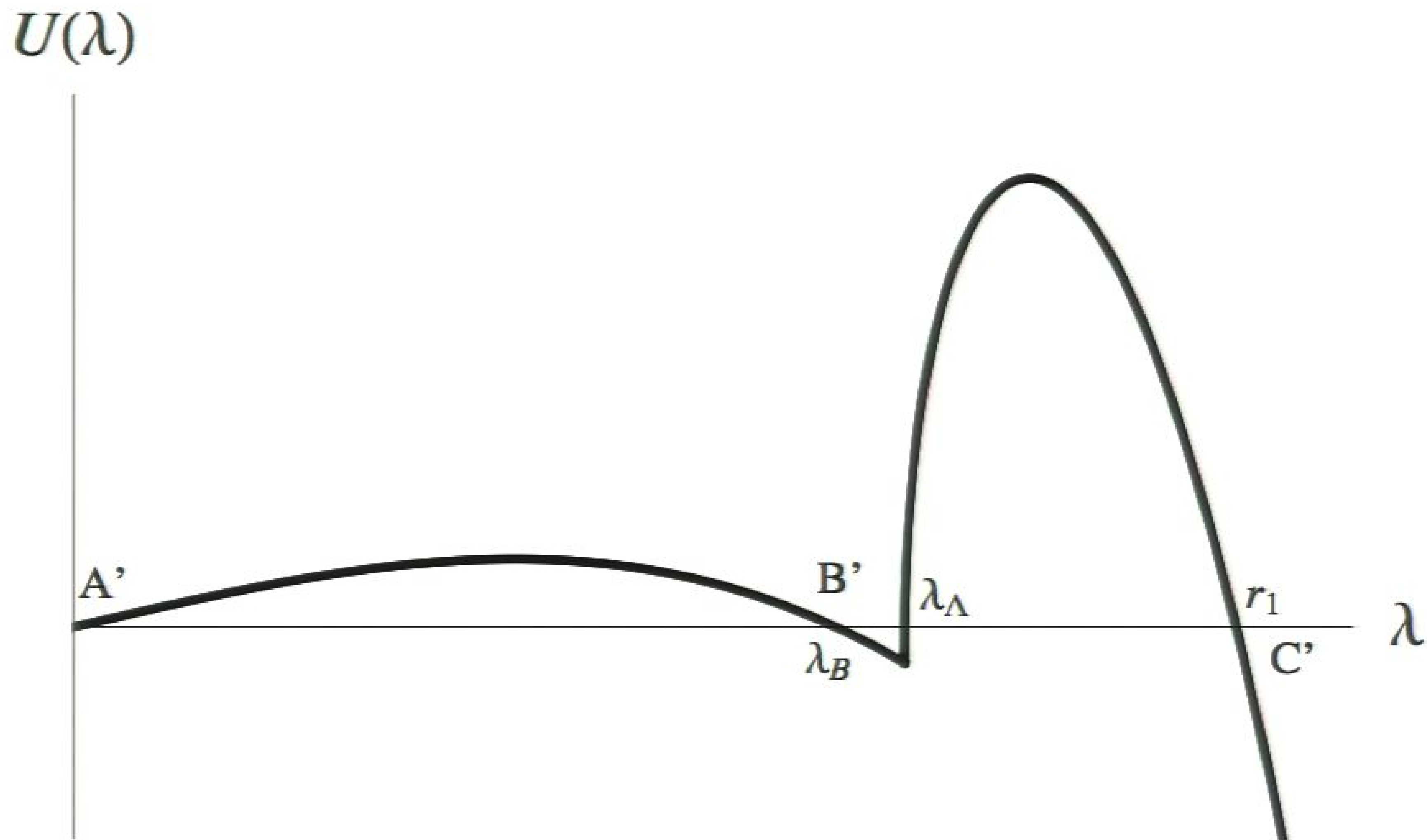

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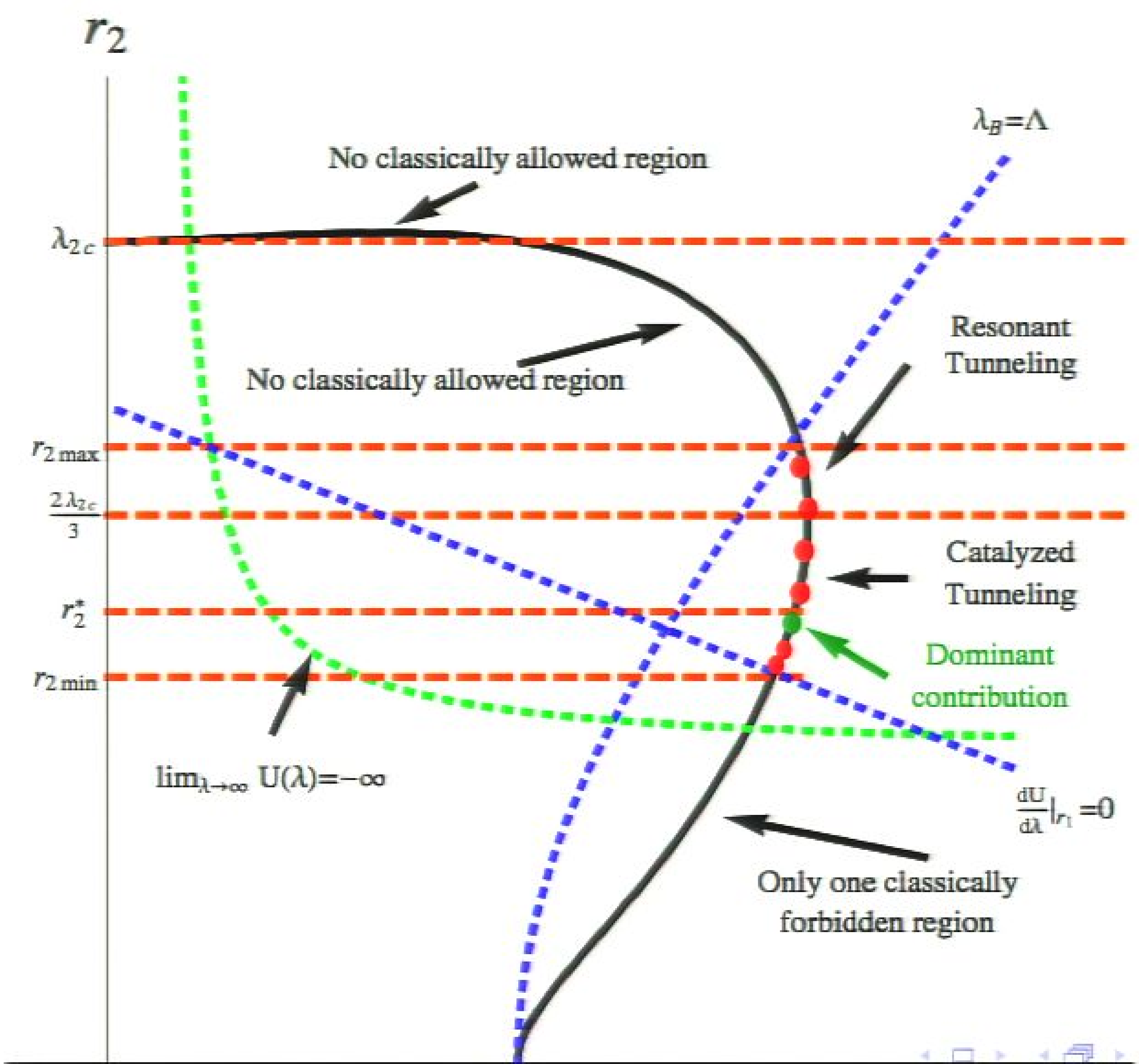

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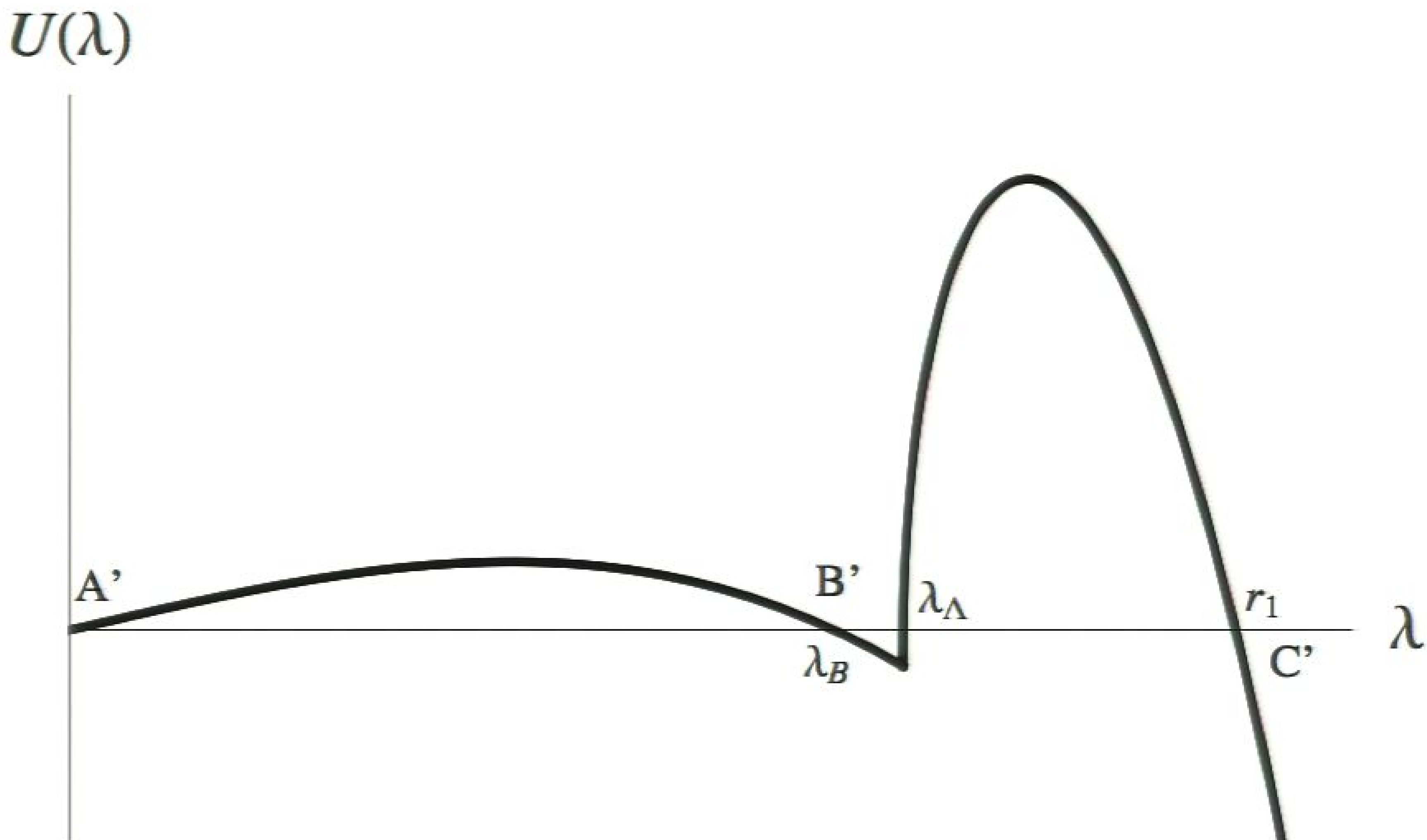

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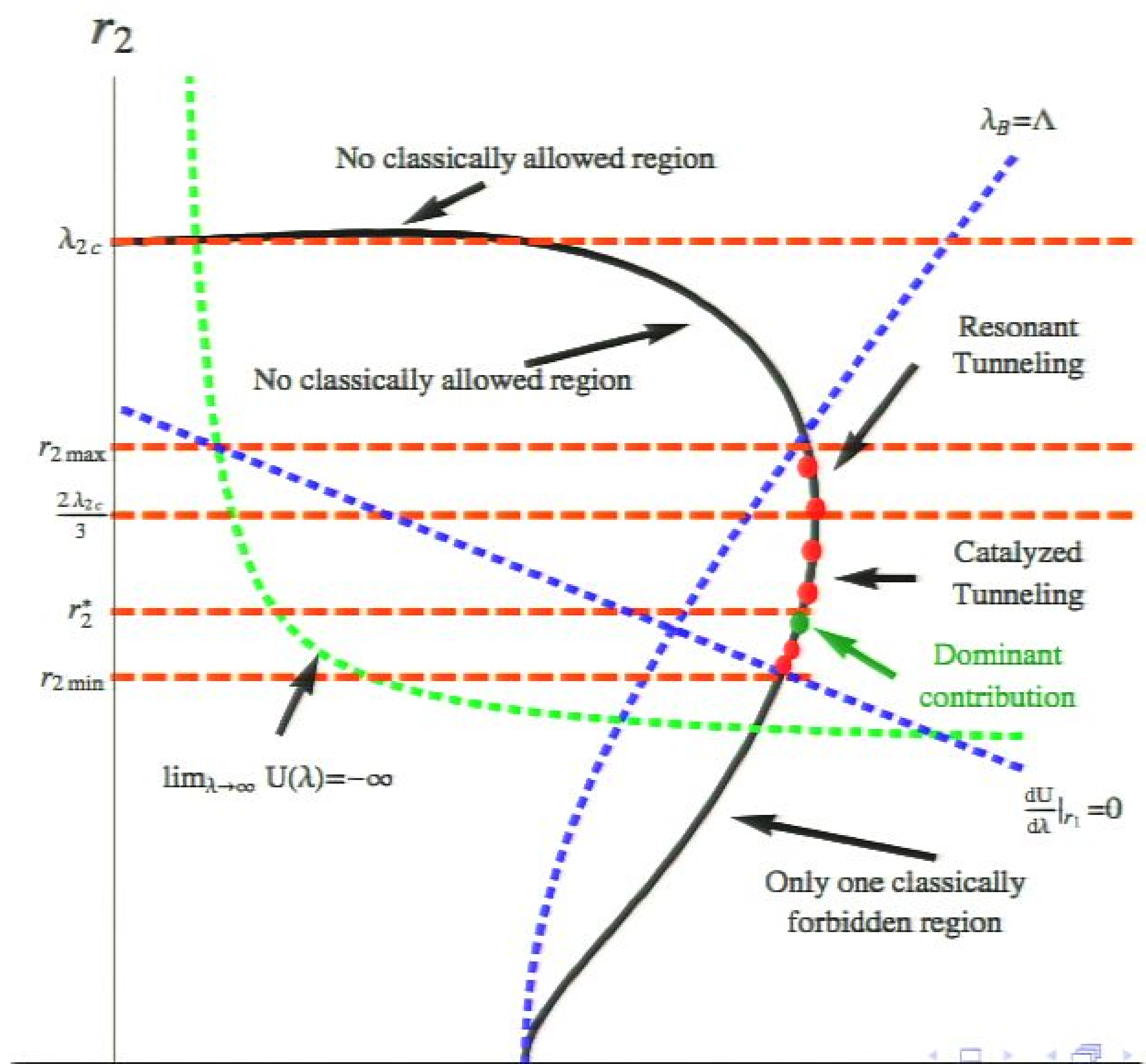

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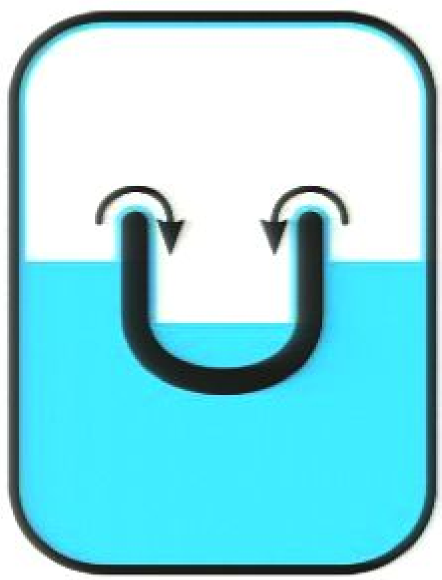
Catalytic Tunneling

- If the inside bubble is too small $0 < r_2 < 2\lambda_{2c}/3$, inside bubble will collapse
- Effect will be tunneling from A to B
- Tunneling rate is exponentially enhanced by presence of C

Superfluid Helium-3

- Bose condensate of Cooper pairs
- Many phases believed to exist
- With a small magnetic field, the A phase is stable near the superfluid transition temperature
- B phase is stable at zero temperature



First-Order Phase Transition from A to B

- A to B transition typically occurs in minutes or hours
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- A to B transition typically occurs in minutes or hours
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- Transition time due to thermal fluctuation is $\sim 10^{1,470,000}$ years
- Transition time due to quantum fluctuations is $\sim 10^{20,000}$ years

“Baked Alaska”

- Proposal that B-phase nucleation is caused by cosmic rays
- Incoming particle dumps a large amount of heat into a small volume
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- Proposal that B-phase nucleation is caused by cosmic rays
- Incoming particle dumps a large amount of heat into a small volume
- Heat transport causes center to cool sufficiently for B to nucleate
- Experiments found no correlations between cosmic rays and nucleation

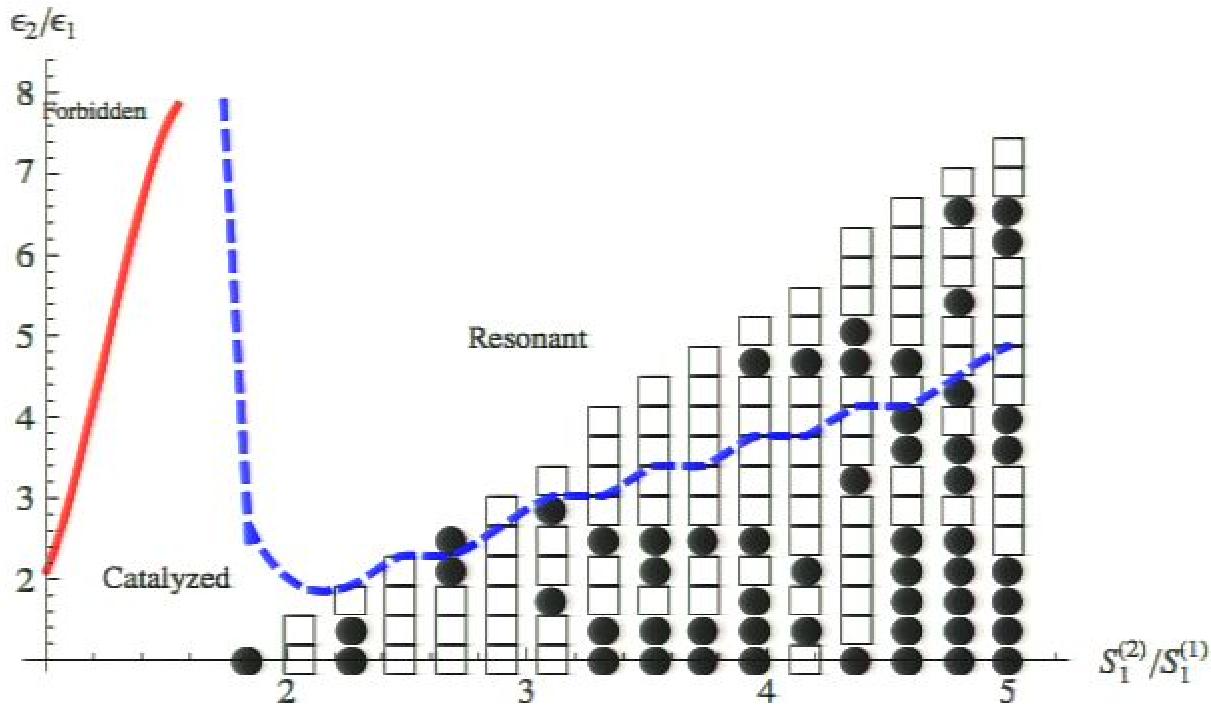
Boundary Effects

- Near the boundary, the symmetry of the A phase is broken
- Orientation vector must be normal to the boundary
- Effectively causes the A phase to become two distinct phases near the boundary
- All samples to date consist of multiple domains of each of these "subphases"
- In the bulk this distinction disappears

Resonant Tunneling?

- Free energy difference between these "subphases" is small and spatially varying
- Cooling may provide tuning needed to satisfy resonance conditions
- Nucleation occurs near boundary

Tunneling Rate



- $\epsilon_1 = 0.25$ and $S_1^{(2)} = 1$ are fixed (so $S_E^{A\to B} \sim 4 \cdot 10^3$)
- Squares $100 < -S_E^{A\to B} / \ln T < 1000$
- Circles $1000 < -S_E^{A\to B} / \ln T$

Conclusions and Future Directions

- Resonant tunneling may explain the fast nucleation of the B phase of Helium-3
- More detailed analysis of the domain wall tension and free energy densities in progress
- May be tested experimentally in the near future
- Interesting to study the more general case, especially including gravity

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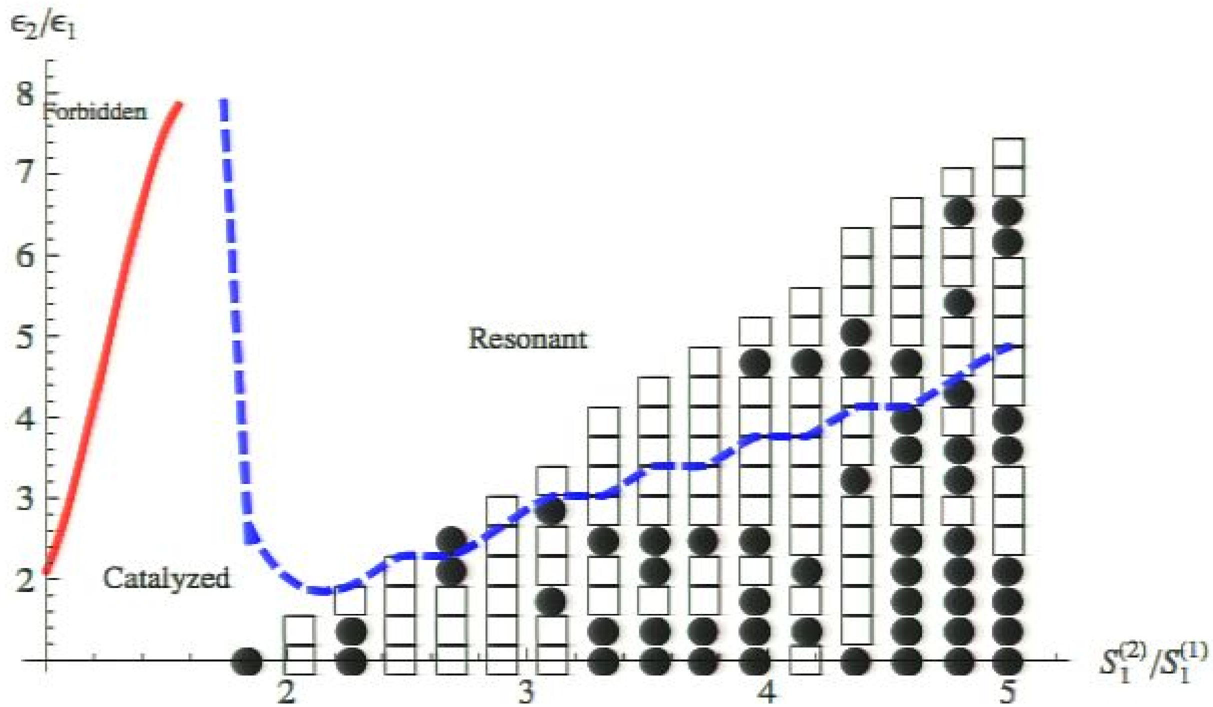

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