Title: Quantum correlations in applications and foundations

Date: Feb 07, 2011 04:00 PM

URL: http://pirsa.org/11020126

Abstract: Quantum key distribution (QKD) is an application of quantum theory as its security relies on quantum foundations, at the same time there is development in the information-theoretic point of view to quantum theory. The security is related to impossible quantum performance, for instance, neither perfect quantum cloning nor perfect quantum state discrimination are possible. In this talk, I would like to discuss issues relevant to practical and fundamental sides of QKD: i) toward characterization of quantum correlations from which a secret key can be distilled, ii) determination whether quantum states shared by two honest parties in distance are entangled or separable, and iii) limitations on quantum performance by fundamental principles in quantum theory.

Quantum correlations in applications and foundations

Joonwoo Bae

Korea Institute for Advanced Study (KIAS), Seoul, Korea

Summary of QKD protocols

$$\rho_{AB}$$

Alice

$$P_{AB}(a,b\,|\,x,y) = tr[Q_a^{(x)} \otimes Q_b^{(y)} \rho_{AB}]$$

Bob

x

↓

BOX

↓

a

y

↓

BOX

↓

b

Alice

ρ_{AB}

$$P_{AB}(a,b \mid x,y) = tr[Q_a^{(x)} \otimes Q_b^{(y)} \rho_{AB}]$$

Bob

x

$\downarrow$

BOX

$\downarrow$

a

Entanglement means security?

y

$\downarrow$

BOX

$\downarrow$

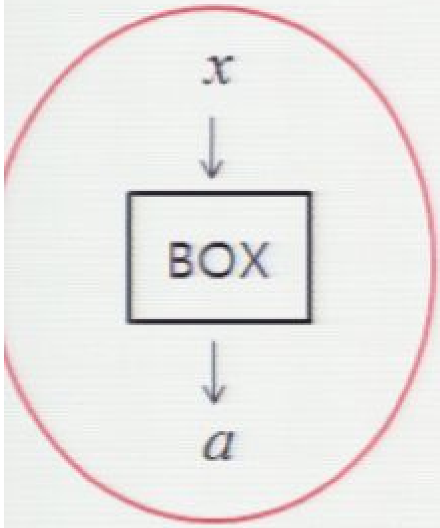
b

Alice

$$\rho_{AB}$$

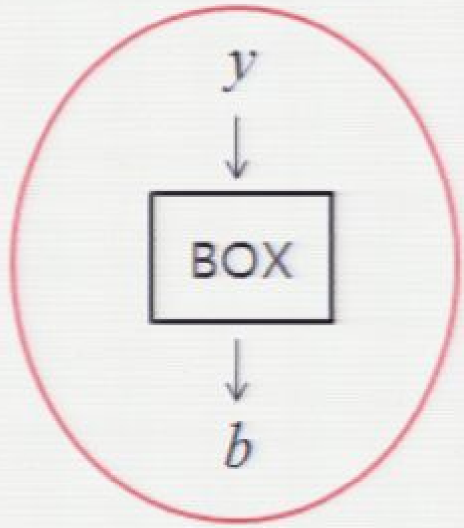
$$P_{AB}(a,b\,|\,x,y) = tr[Q_a^{(x)} \otimes Q_b^{(y)} \rho_{AB}]$$

Bob



Entanglement means security?

Fundamental limitations on local operations? E.g. quantum cloning and quantum state estimation.



Thanks for the attention !

Quantum correlations in applications and foundations

Joonwoo Bae

Korea Institute for Advanced Study (KIAS), Seoul, Korea

Summary of QKD protocols

Alice

$$\rho_{AB}$$

$$P_{AB}(a,b \mid x,y) = tr[Q_a^{(x)} \otimes Q_b^{(y)} \rho_{AB}]$$

Bob

x → BOX → a

y → BOX → b

ρ_{AB}

Alice

$$P_{AB}(a,b \mid x,y) = tr[Q_a^{(x)} \otimes Q_b^{(y)} \rho_{AB}]$$

Bob

x → BOX → a

y → BOX → b

Entanglement means security?

Fundamental limitations on local operations? E.g. quantum cloning and quantum state estimation.

Alice

ρ_{AB}

Bob

$$P_{AB}(a,b \mid x,y) = tr[Q_a^{(x)} \otimes Q_b^{(y)} \rho_{AB}]$$

x ↓ BOX ↓ a

y ↓ BOX ↓ b

Entanglement means security?

Fundamental limitations on local operations? E.g. quantum cloning and quantum state estimation.

Separable states are useless !

Outline of the talk

Quatnum vs. Classical correlations

- -Multipartite cases
- -Bipartite cases

Relations among fundamental no-go theorems

- -No-signaling
- -No perfect quantum cloning
- -No perfect quantum state estimation

Fundamental theorem in entanglement detection

- -Conjecture : Approximations to positive maps are entanglement-breaking
- -An approximate partial transpose and application to entanglement detection

Outline of the talk

Quatnum vs. Classical correlations

-Multipartite cases
-Bipartite cases

Relations among fundamental no-go theorems

-No-signaling
-No perfect quantum cloning
-No perfect quantum state estimation

Fundamental theorem in entanglement detection

-Conjecture : Approximations to positive maps are entanglement-breaking
-An approximate partial transpose and application to entanglement detection

PART I

Relations among fundamental no-go theorems

-No-signaling
-No perfect quantum cloning
-No perfect quantum state estimation

Antonio Acín, *ICFO – Institut de Ciències Fotòniques, Mediterranean Technology Park Castelldefels (Barcelona), Spain, ICREA-Institució Catalana de Recerca i Estudis Avançats, Barcelona, Spain*

Won-Young Hwang, *Department of Physics Education, Chonnam National University, Gwangju, Korea*

Yeong-Duk Han, *Department of Game Contents, Woosuk University, Wanju, Korea*

KIAS
KOREA INSTITUTE FOR ADVANCED STUDY

ICFO
Institut de Ciències Fotòniques

No-go theorems in quantum theory

Quantum states

i) Cannot be perfectly cloned : **No-cloning**

ii) Cannot be perfectly estimated : **No-perfect state estimation**

iii) Cannot be sent faster than light : **No-signaling**

Outline of the talk

Quatnum vs. Classical correlations

- -Multipartite cases
- -Bipartite cases

Relations among fundamental no-go theorems

- -No-signaling
- -No perfect quantum cloning
- -No perfect quantum state estimation

Fundamental theorem in entanglement detection

- -Conjecture : Approximations to positive maps are entanglement-breaking
- -An approximate partial transpose and application to entanglement detection

PART I

Relations among fundamental no-go theorems

-No-signaling
-No perfect quantum cloning
-No perfect quantum state estimation

Antonio Acín, *ICFO – Institut de Ciències Fotòniques, Mediterranean Technology Park Castelldefels (Barcelona), Spain, ICREA-Institució Catalana de Recerca i Estudis Avançats, Barcelona, Spain*

Won-Young Hwang, *Department of Physics Education, Chonnam National University, Gwangju, Korea*

Yeong-Duk Han, *Department of Game Contents, Woosuk University, Wanju, Korea*

KIAS
KOREA INSTITUTE FOR ADVANCED STUDY

ICFO
Institut de Ciències Fotòniques

No-go theorems in quantum theory

Quantum states

i) Cannot be perfectly cloned : **No-cloning**

ii) Cannot be perfectly estimated : **No-perfect state estimation**

iii) Cannot be sent faster than light : **No-signaling**

No-go theorems in quantum theory

Cannot be perfectly cloned : **No-cloning**

$$|\psi\rangle|B\rangle \Longrightarrow |\psi\rangle|\psi\rangle$$

$$|?\rangle|B\rangle \not\Longrightarrow |?\rangle|?\rangle$$

No-go theorems in quantum theory

No-perfect discrimination:

Nonorthogonal quantum states cannot be perfectly discriminated.

The no-signaling principle: **Information cannot be sent faster than light.**

Correlations exsiting in quantum systems $P_{AB}(a,b|x,y) = Tr[Q_a^{(x)} \otimes Q_b^{(y)} \rho_{AB}]$

ρ_{AB}

Non-signalling probabilities

Alice

$$\sum_y P_{AB}(a,b \mid x,y) = \sum_{y'} P_{AB}(a,b \mid x,y') = P_A(a \mid x)$$

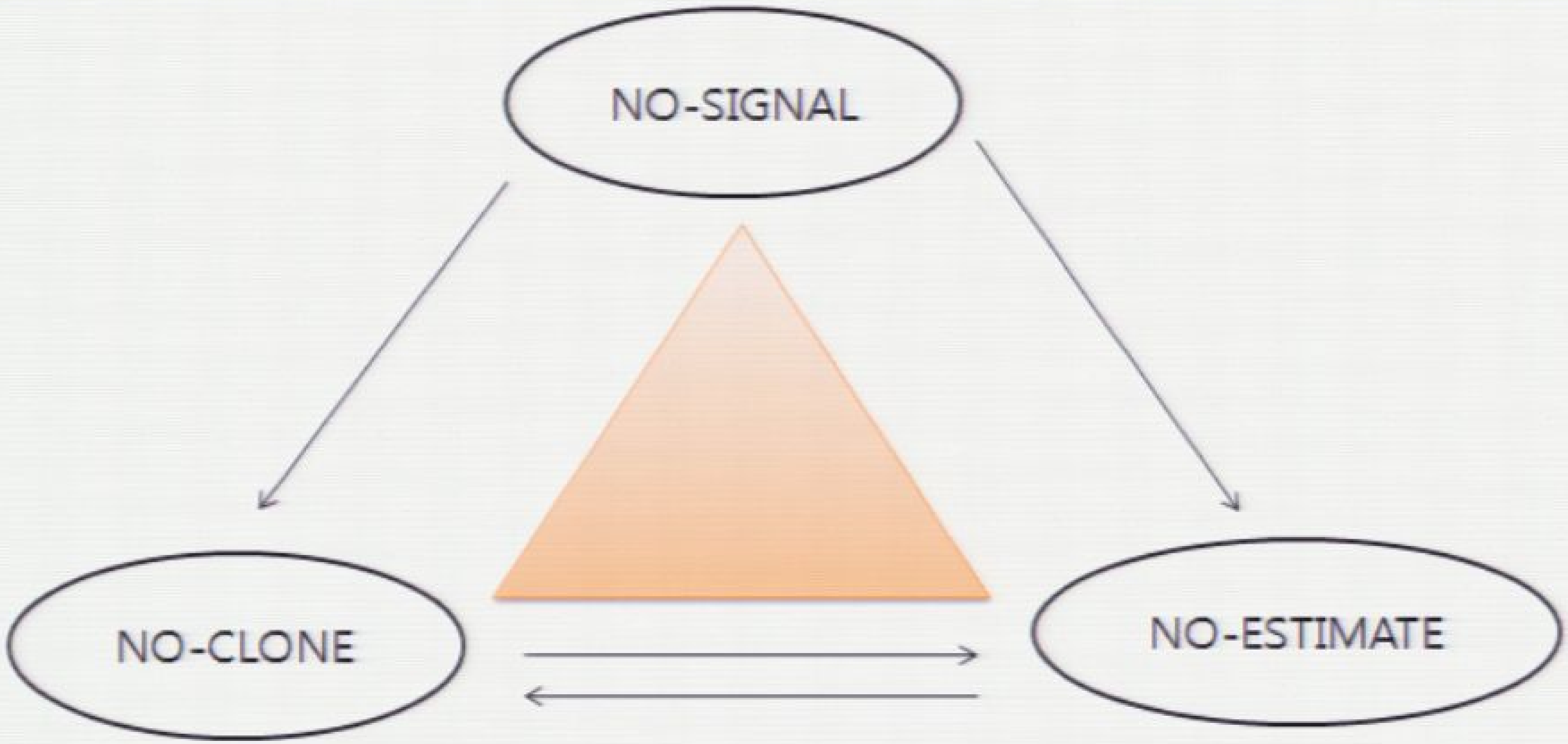
Bob

$Q^{(x)} \to$ BOX $\to a$

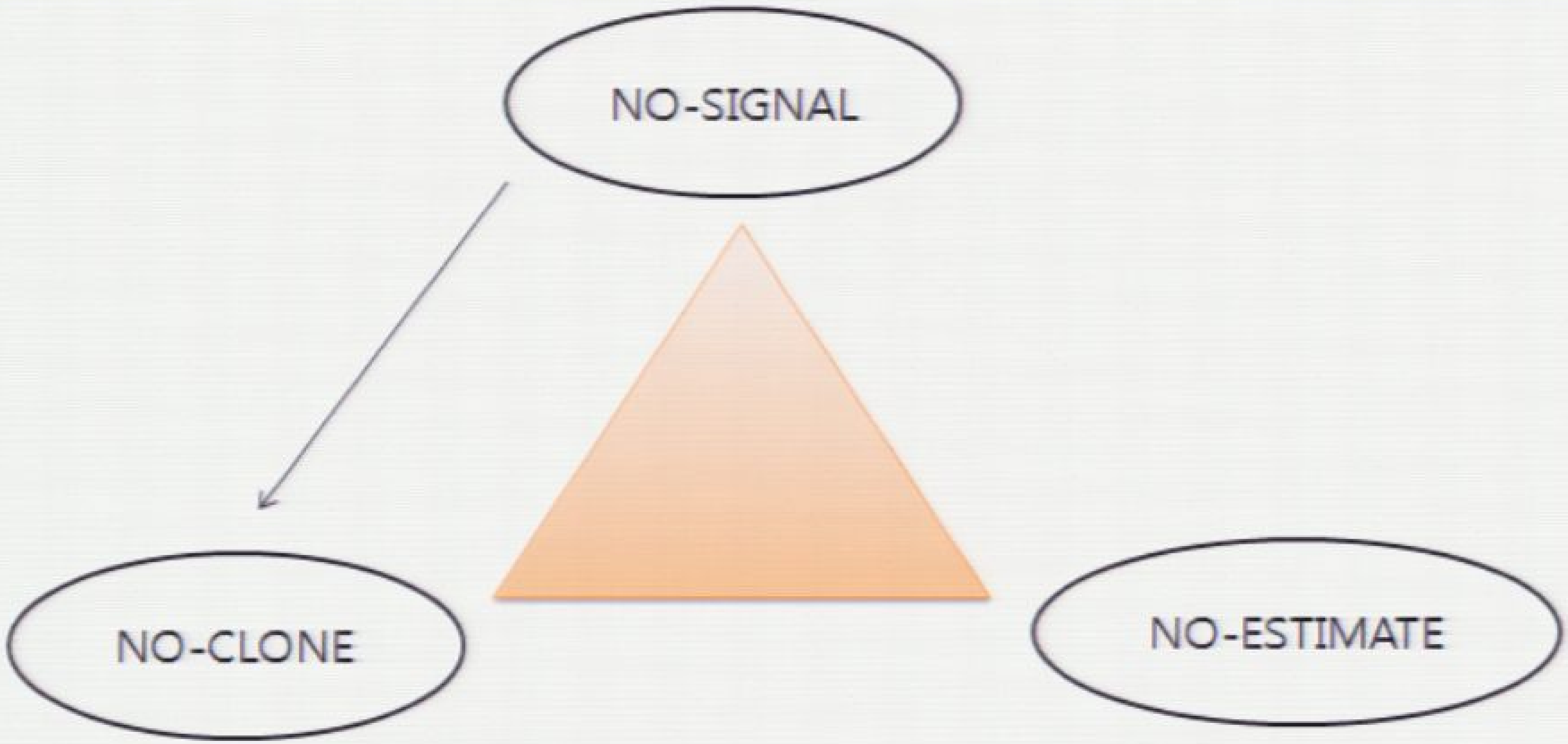
$$\begin{aligned}\sum_y P_{AB}(a,b|x,y) &= \sum_y tr[Q_a^{(x)} \otimes Q_b^{(y)} \rho_{AB}] \\ &= tr[Q_a^{(x)} \otimes \sum_y Q_b^{(y)} \rho_{AB}] \\ &= tr[Q_a^{(x)} \otimes \sum_{y'} Q_b^{(y')} \rho_{AB}] \\ &= \sum_{y'} P_{AB}(a,b|x,y') = P_A(a|x)\end{aligned}$$

$Q^{(y)} \to$ BOX $\to b$

Interrelations between no-go theorems

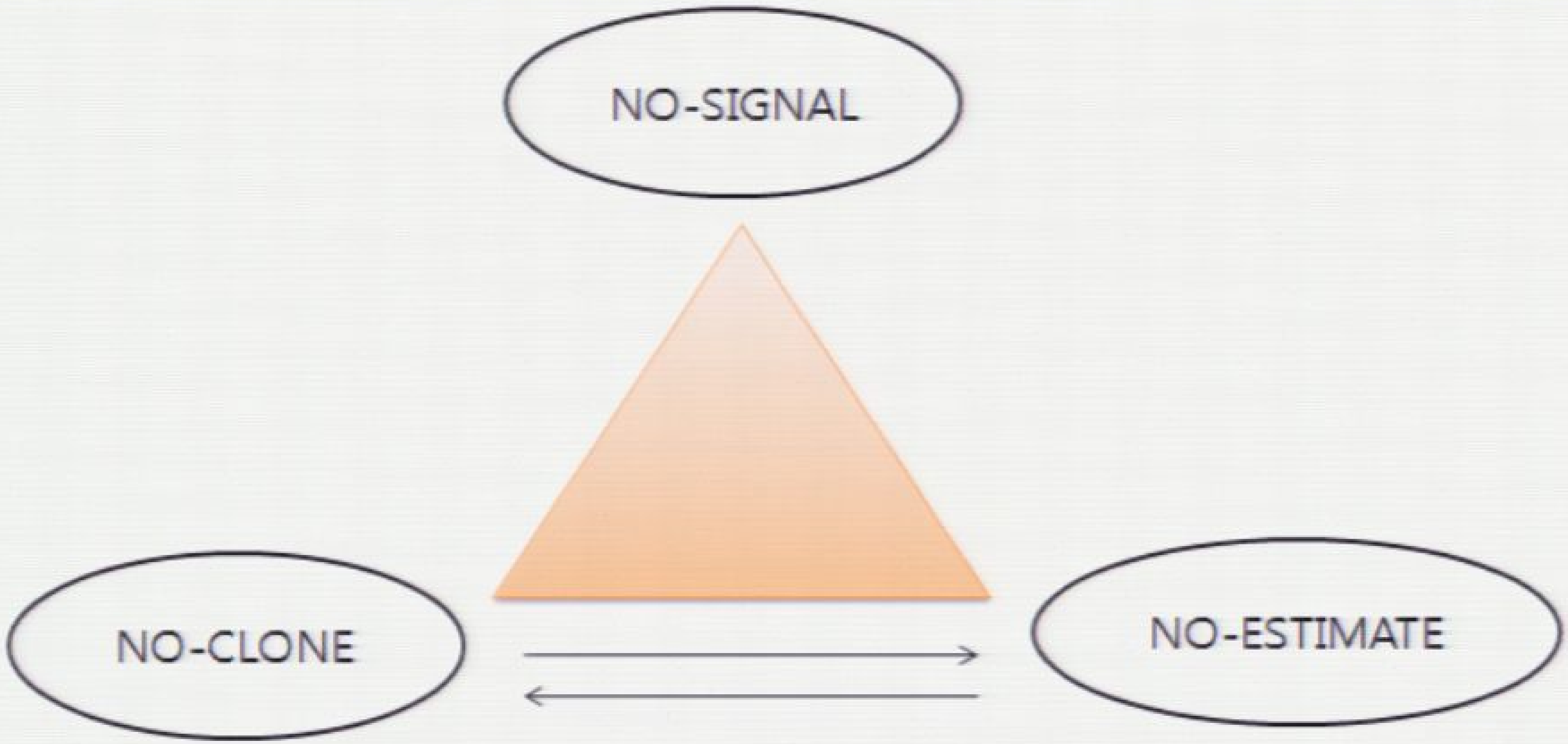


Interrelations between no-go theorems



N. Gisin, PLA 242 1 (1998), D. Bruss, G. M. D'Ariano, C. Macchiavello, and M. F. Sacchi, PRA 62 62302 (2000)

Interrelations between no-go theorems



i) Universal case : N Gisin S Massar PRL 79 2153 (1997),
R Werner PRA 58 1827 (1999)
ii) Phase-covariant case : D Bruss *et al PRA 62 12302 (2000)*

$m \to n$ cloning operation

Quantum cloning ε

$$F_Q = \langle \psi | \rho_Q | \psi \rangle = \frac{nm + n + m}{n(m+2)}$$

$|\psi\rangle = a|0\rangle + b|1\rangle$

$$F_Q \geq F_M$$

ρ_Q

$$F_M = \langle \psi | \rho_M | \psi \rangle = \frac{m+1}{m+2}$$

ρ_M

State estimation M

S Massar S Popescu PRL 74 1259

D Bruss A Ekert C Macchiavello PRL 81 2598 (1998)
M Keyl R Werner J Math Phys 40 3283 (1999)

i) Universal case : N Gisin S Massar PRL 79 2153 (1997),
R Werner PRA 58 1827 (1999)
ii) Phase-covariant case : D Bruss *et al PRA 62 12302 (2000)*

$m \rightarrow n$ cloning operation

Quantum cloning $\mathcal{E}$

$$F_Q = \langle \psi | \rho_Q | \psi \rangle = \frac{nm + n + m}{n(m+2)} \xrightarrow[n \to \infty]{} \frac{m+1}{m+2}$$

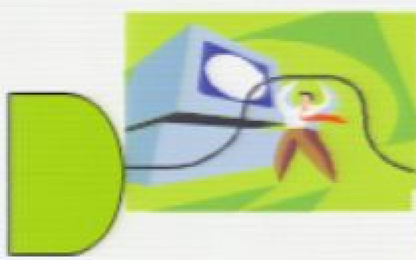
ρ_Q

$$|\psi\rangle = a|0\rangle + b|1\rangle \qquad F_Q \geq F_M$$

$$F_M = \langle \psi | \rho_M | \psi \rangle = \frac{m+1}{m+2}$$

ρ_M

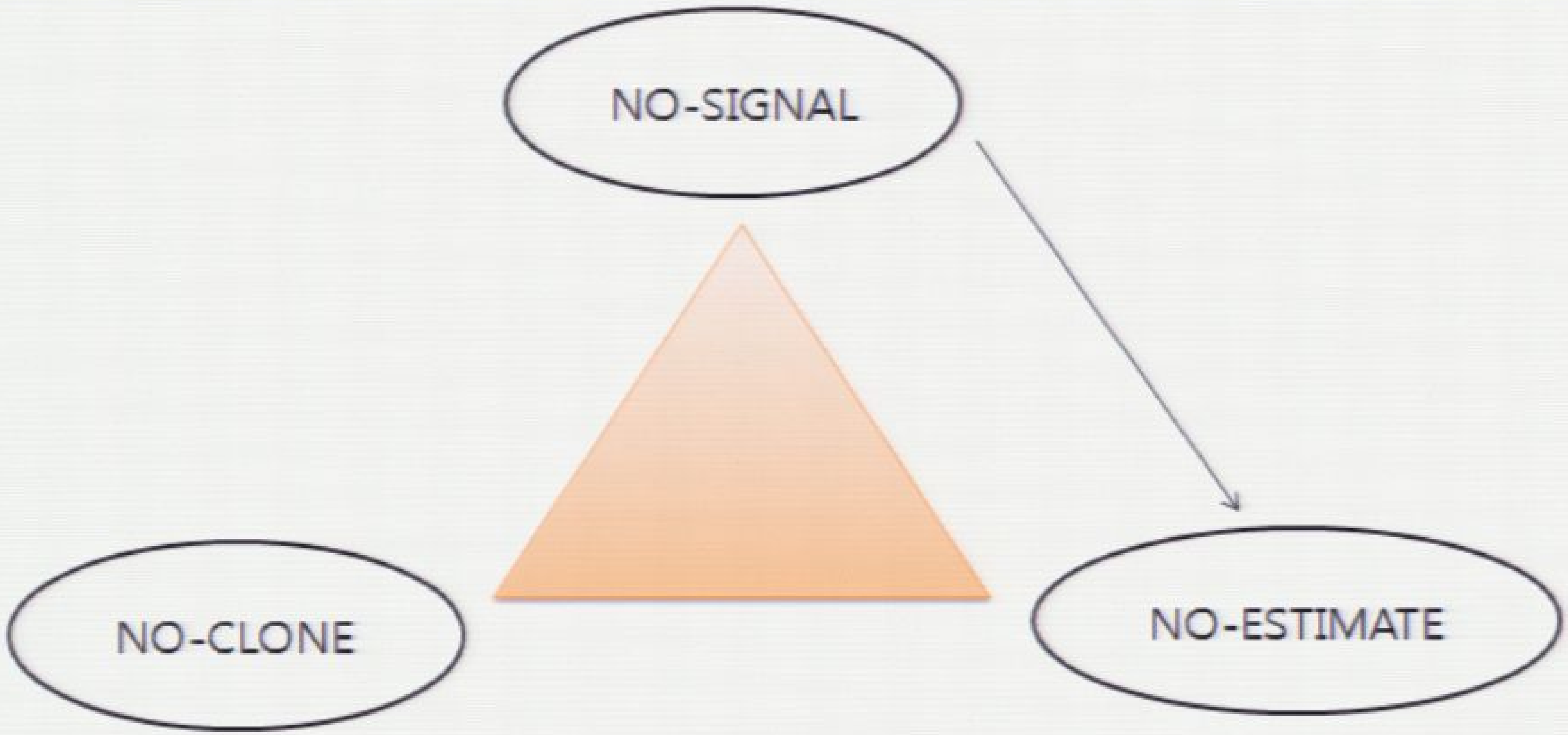
State estimation M



S Massar S Popescu PRL 74 1259

D Bruss A Ekert C Macchiavello PRL 81 2598 (1998)
M Keyl R Werner J Math Phys 40 3283 (1999)

Interrelations between no-go theorems



No-signaling principle can determine optimal quanutm state estimation (qubits)

Y.-D. Han, J. Bae, W.-Y. Hwang, PRA (2010

No-signaling principle can determine optimal quantum state discrimination

W.-Y. Hwang, PRA (2005)

J Bae et al, PRA (2008)

W.-Y. Hwang, J Bae JMP (2010)

i) Universal case : N Gisin S Massar PRL 79 2153 (1997),
R Werner PRA 58 1827 (1999)
ii) Phase-covariant case : D Bruss *et al PRA 62 12302 (2000)*

$m \to n$ cloning operation

Quantum cloning ε

$$F_Q = \langle \psi | \rho_Q | \psi \rangle = \frac{nm + n + m}{n(m+2)} \xrightarrow[n \to \infty]{} \frac{m+1}{m+2}$$

ρ_Q

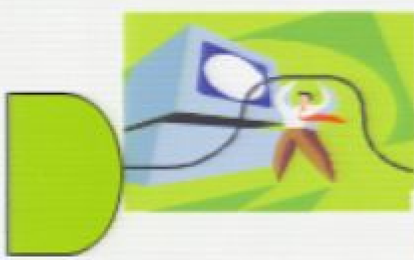
$$|\psi\rangle = a|0\rangle + b|1\rangle$$

$$F_Q \geq F_M$$

$$F_M = \langle \psi | \rho_M | \psi \rangle = \frac{m+1}{m+2}$$

ρ_M

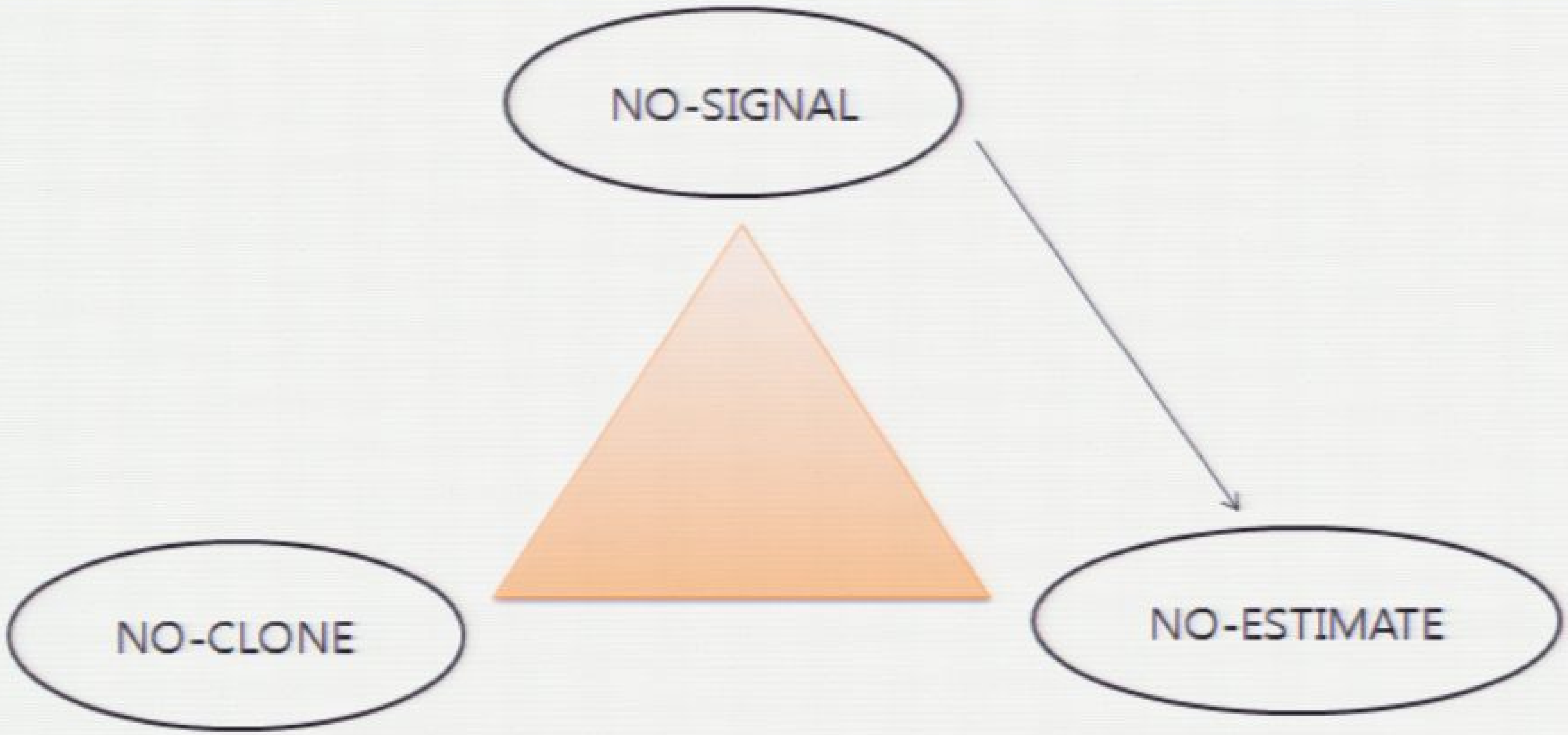
State estimation M



S Massar S Popescu PRL 74 1259

D Bruss A Ekert C Macchiavello PRL 81 2598 (1998)
M Keyl R Werner J Math Phys 40 3283 (1999)

Interrelations between no-go theorems



No-signaling principle can determine optimal quanutm state estimation (qubits)

Y.-D. Han, J. Bae, W.-Y. Hwang, PRA (2010

No-signaling principle can determine optimal quantum state discrimination

W.-Y. Hwang, PRA (2005)

J Bae et al, PRA (2008)

W.-Y. Hwang, J Bae JMP (2010)

Identical ensembles can be in different decompositions

Indistinguishability of quantum states

(Mixed) Quantum states can have infintely many decomposition.

Example. (A single qubit state)

$$\rho = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| = \frac{1}{2}|+\rangle\langle +| + \frac{1}{2}|-\rangle\langle -| = \ldots$$

Identical ensembles can be in different decompositions

Indistinguishability of quantum states

$$|\phi^+\rangle_{AB} = (|00\rangle + |11\rangle)/\sqrt{2}$$

$$P_{AB}(a,b\,|\,x,y) = tr[Q_a^{(x)} \otimes Q_b^{(y)} \rho_{AB}]$$

Alice **Bob**

$\sigma^{(z)} = \{|0_z\rangle\langle 0_z|, |1_z\rangle\langle 1_z|\}$ $\{|0_z\rangle\langle 0_z|, |1_z\rangle\langle 1_z|\}$

$|0_z\rangle$ $|0_z\rangle$

$|1_z\rangle$ $|1_z\rangle$

$\sigma^{(x)} = \{|0_x\rangle\langle 0_x|, |1_x\rangle\langle 1_x|\}$ $\{|0_z\rangle\langle 0_z|, |1_z\rangle\langle 1_z|\}$

$|0_x\rangle$ $|0_x\rangle = (|0_z\rangle + |1_z\rangle)/\sqrt{2}$

$|1_x\rangle$ $|1_x\rangle = (|0_z\rangle - |1_z\rangle)/\sqrt{2}$

Identical ensembles can be in different decompositions

Indistinguishability of quantum states

$$|\phi^+\rangle_{AB} = (|00\rangle + |11\rangle)/\sqrt{2}$$

$$P_{AB}(a,b\,|\,x,y) = tr[Q_a^{(x)} \otimes Q_b^{(y)} \rho_{AB}]$$

Alice **Bob**

$$\{|0_z\rangle\langle 0_z|, |1_z\rangle\langle 1_z|\}$$

$$\rho_B = p_0 tr_A[|0\rangle\langle 0| \otimes I |\phi^+\rangle\langle\phi^+|] + p_1 tr_A[|1\rangle\langle 1| \otimes I |\phi^+\rangle\langle\phi^+|]$$

$$|0_z\rangle$$

$$= \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| = \frac{I}{2}$$

$$|1_z\rangle$$

$$= p_0 tr_A[|+\rangle\langle +| \otimes I |\phi^+\rangle\langle\phi^+|] + p_1 tr_A[|-\rangle\langle -| \otimes I |\phi^+\rangle\langle\phi^+|]$$

$$\{|0_z\rangle\langle 0_z|, |1_z\rangle\langle 1_z|\}$$

$$|0_x\rangle = (|0_z\rangle + |1_z\rangle)/\sqrt{2}$$

$$|1_x\rangle = (|0_z\rangle - |1_z\rangle)/\sqrt{2}$$

Identical ensembles can be in different decompositions

Indistinguishability of quantum states

$$|\phi^+\rangle_{AB} = (|00\rangle + |11\rangle)/\sqrt{2}$$

$$P_{AB}(a,b\,|\,x,y) = tr[Q_a^{(x)} \otimes Q_b^{(y)} \rho_{AB}]$$

Alice

Bob

$\sigma^{(z)} = \{|0_z\rangle\langle 0_z|, |1_z\rangle\langle 1_z|\}$ $\{|0_z\rangle\langle 0_z|, |1_z\rangle\langle 1_z|\}$

$|0_z\rangle$ $|0_z\rangle$

$|1_z\rangle$ $|1_z\rangle$

$\sigma^{(x)} = \{|0_x\rangle\langle 0_x|, |1_x\rangle\langle 1_x|\}$ $\{|0_z\rangle\langle 0_z|, |1_z\rangle\langle 1_z|\}$

$|0_x\rangle$ $|0_x\rangle = (|0_z\rangle + |1_z\rangle)/\sqrt{2}$

$|1_x\rangle$ $|1_x\rangle = (|0_z\rangle - |1_z\rangle)/\sqrt{2}$

Identical ensembles can be in different decompositions

Indistinguishability of quantum states

$$\left|\phi^+\right\rangle_{AB} = (|00\rangle + |11\rangle)/\sqrt{2}$$

$$P_{AB}(a,b\,|\,x,y) = tr[Q_a^{(x)} \otimes Q_b^{(y)} \rho_{AB}]$$

Alice **Bob**

$$\{|0_z\rangle\langle 0_z|, |1_z\rangle\langle 1_z|\}$$

$$\rho_B = p_0 tr_A[|0\rangle\langle 0| \otimes I |\phi^+\rangle\langle\phi^+|] + p_1 tr_A[|1\rangle\langle 1| \otimes I |\phi^+\rangle\langle\phi^+|]$$

$$|0_z\rangle$$

$$= \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| = \frac{I}{2}$$

$$|1_z\rangle$$

$$= p_0 tr_A[|+\rangle\langle +| \otimes I |\phi^+\rangle\langle\phi^+|] + p_1 tr_A[|-\rangle\langle -| \otimes I |\phi^+\rangle\langle\phi^+|]$$

$$\{|0_z\rangle\langle 0_z|, |1_z\rangle\langle 1_z|\}$$

$$|0_x\rangle = (|0_z\rangle + |1_z\rangle)/\sqrt{2}$$

$$|1_x\rangle = (|0_z\rangle - |1_z\rangle)/\sqrt{2}$$

Identical ensembles can be in different decompositions

Any decomposition in the Bob's ensemble can be prepared by Alice

$$P_{AB}(a,b\,|\,x,y) = tr[Q_a^{(x)} \otimes Q_b^{(y)} \rho_{AB}]$$

Alice **Bob**

$M = \{M_0, M_1\}$ $\quad M_0 + M_1 = I$ $\quad \rho_B = p_1\rho_1 + (1-p_1)\sigma_1$

M_0 $\quad \rho_1 \propto Tr_A[M_0 \otimes I |\phi^+\rangle\langle\phi^+|]$

M_1 $\quad \sigma_1 \propto Tr_A[M_1 \otimes I |\phi^+\rangle\langle\phi^+|]$

$N = \{N_0, N_1\}$ $\quad N_0 + N_1 = I$ $\quad \rho_B = p_2\rho_2 + (1-p_2)\sigma_2$

N_0 $\quad \rho_2 \propto Tr_A[N_0 \otimes I |\phi^+\rangle\langle\phi^+|]$

N_1 $\quad \sigma_2 \propto Tr_A[N_1 \otimes I |\phi^+\rangle\langle\phi^+|]$

No-signaling constraint in quantum state discrimination

How the no-signaling principle is applied?

$$P_{AB}(a,b|x,y) = tr[Q_a^{(x)} \otimes Q_b^{(y)} \rho_{AB}]$$

Alice **Bob**

$M = \{M_0, M_1\}$ $\rho_B{}^M = p_1\rho_1 + (1-p_1)\sigma_1$

$N = \{N_0, N_1\}$ $\rho_B{}^N = p_2\rho_2 + (1-p_2)\sigma_2$

Equal and indisntguishable ensemble

Suppose $p_1 P(\rho_1|\rho_1) + p_2 P(\rho_2|\rho_2) > 1$

$\longrightarrow$ $p_j P(\rho_j|\rho_j) > 1/2$ Alice's measurement decision can be found by Bob
Contradiction to the no-signaling principle !

$\longrightarrow$ $p_1 P(\rho_1|\rho_1) + p_2 P(\rho_2|\rho_2) \leq 1$

$$q_1 = \frac{p_1}{p_1 + p_2}$$

$$P_{guess} = q_1 P(\rho_1|\rho_1) + q_2 P(\rho_2|\rho_2) \leq \frac{1}{p_1 + p_2}$$

$$q_2 = \frac{p_2}{p_1 + p_2}$$

No-signaling constraint in quantum state discrimination

How the no-signaling principle is applied?

Alice

$$P_{AB}(a,b\,|\,x,y) = tr[Q_a^{(x)} \otimes Q_b^{(y)} \rho_{AB}]$$

Bob

$M = \{M_0, M_1\}$ $\rho_B{}^M = p_1\rho_1 + (1-p_1)\sigma_1$

$N = \{N_0, N_1\}$ $\rho_B{}^N = p_2\rho_2 + (1-p_2)\sigma_2$

} Equal and indisntguishable ensemble

$$\longrightarrow \quad P_{guess} = q_1 P(\rho_1\,|\,\rho_1) + q_2 P(\rho_2\,|\,\rho_2) \le \frac{1}{p_1 + p_2}$$

$$\rho_B{}^M = \rho_B{}^N$$

$$\longrightarrow \quad P_{guess} \le \frac{1}{2}(1 + \|\, q_1\rho_1 - q_2\rho_2 \,\|)$$

Helstrom bound

No-perfect state estimation/discrimination :

Non-orthogonal states cannot be perfectly discriminated.

$$\rho_1 \qquad \rho_2 \qquad tr[\rho_1\rho_2] \neq 0$$

Discrimination process:
One has to first build measurement, described by Positive-Operator-Valued-Measure M_1, M_2

$$p(i \mid j) = tr[M_i \rho_j]$$

Note that $M_1 + M_2 = I$ since $p(1 \mid j) + p(2 \mid j) = 1$

Those measurements are optimized such that the success probability is maximized

$$\begin{aligned} P_{guess} &= \max_{M_1, M_2} q_1 p(1 \mid 1) + q_2 p(2 \mid 2) \\ &= \max_{M_1, M_2} q_1 tr[\rho_1 M_1] + q_2 tr[\rho_2 M_2] \end{aligned}$$

The solution is known as Helstrom bound $P_{guess} = \frac{1}{2}(1 + \| q_1\rho_1 - q_2\rho_2 \|)$

No-signaling constraint in quantum state discrimination

How the no-signaling principle is applied?

Alice

$$P_{AB}(a,b \mid x,y) = tr[Q_a^{(x)} \otimes Q_b^{(y)} \rho_{AB}]$$

Bob

$$M = \{M_0, M_1\} \qquad \rho_B^{\ M} = p_1\rho_1 + (1-p_1)\sigma_1$$

$$N = \{N_0, N_1\} \qquad \rho_B^{\ N} = p_2\rho_2 + (1-p_2)\sigma_2$$

Equal and indisntguishable ensemble

$$\longrightarrow \qquad P_{guess} = q_1 P(\rho_1 \mid \rho_1) + q_2 P(\rho_2 \mid \rho_2) \le \frac{1}{p_1 + p_2}$$

$$\rho_B^{\ M} = \rho_B^{\ N}$$

$$\longrightarrow \qquad P_{guess} \le \frac{1}{2}(1 + \| q_1\rho_1 - q_2\rho_2 \|)$$

Helstrom bound

No-perfect state estimation/discrimination :

Non-orthogonal states cannot be perfectly discriminated.

$$\rho_1 \qquad \rho_2 \qquad tr[\rho_1\rho_2] \neq 0$$

Discrimination process:
One has to first build measurement, described by Positive-Operator-Valued-Measure M_1, M_2

$$p(i\,|\,j) = tr[M_i\rho_j]$$

Note that $M_1 + M_2 = I$ since $p(1\,|\,j) + p(2\,|\,j) = 1$

Those measurements are optimized such that the success probability is maximized

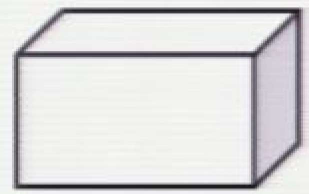
$$P_{guess} = \max_{M_1,M_2} q_1 p(1\,|\,1) + q_2 p(2\,|\,2)$$
$$= \max_{M_1,M_2} q_1 tr[\rho_1 M_1] + q_2 tr[\rho_2 M_2]$$

The solution is known as Helstrom bound $P_{guess} = \frac{1}{2}(1 + \|\, q_1\rho_1 - q_2\rho_2 \,\|)$

No-signaling constraint and quantum state discrimination

Summary in application of the no-signaling principle in quantum state discrimination

Problem. $(q_j, \rho_j) \quad j = 1, \ldots, N$



$\longrightarrow \quad \rho_j \quad q_j$

1. Find equal ensembles

$$\rho_B^{(1)} = p_1\rho_1 + (1-p_1)\sigma_1$$
$$\rho_B^{(2)} = p_2\rho_2 + (1-p_2)\sigma_2$$
$$\vdots$$

such that $q_j = \dfrac{p_j}{p_1 + p_2 + \ldots + p_N}$

2. Applying the no-signaling, $\sum_j p_j P(\rho_j \mid \rho_j) \leq 1$

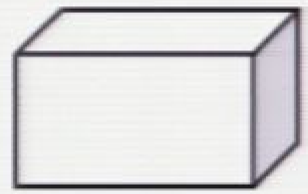
3. A closed form, $P_{guess} = \sum_j q_j P(\rho_j \mid \rho_j) \leq \dfrac{1}{p_1 + p_2 + \ldots + p_N}$

4. Compute $\dfrac{1}{p_1 + p_2 + \ldots + p_N}$ from the equal ensemble condition

No-signaling constraint and quantum state discrimination

Summary in application of the no-signaling principle in quantum state discrimination

Problem. $(q_j, \rho_j) \quad j = 1, \ldots, N$



$\longrightarrow \quad \rho_j \quad q_j$

1. Find equal ensembles

$$\rho_B^{(1)} = p_1 \rho_1 + (1 - p_1)\sigma_1$$
$$\rho_B^{(2)} = p_2 \rho_2 + (1 - p_2)\sigma_2$$
$$\vdots$$

such that $q_j = \dfrac{p_j}{p_1 + p_2 + \ldots + p_N}$

$tr[M_j \sigma_j] = 0$

2. Applying the no-signaling, $\sum_j p_j P(\rho_j | \rho_j) \leq 1$

3. A closed form, $P_{guess} = \sum_j q_j P(\rho_j | \rho_j) \leq \dfrac{1}{p_1 + p_2 + \ldots + p_N}$

4. Compute $\dfrac{1}{p_1 + p_2 + \ldots + p_N}$ from the equal ensemble condition

Quantum state discrimination in semidefinite programming

Discrimination process:

One has to first build measurement, described by Positive-Operator-Valued-Measure M_j

$$p(i|j) = tr[M_i \rho_j]$$

Note that $M_1 + \ldots + M_N = I$ since $p(1|j) + \ldots + p(N|j) = 1$

Those measurements are optimized such that $P_{guess} = \max_{M_j} \sum_j q_j p(j|j)$

Max $\quad f(M_j) = \sum_j q_j tr[M_j \rho_j]$

(Primal)

Subject to $\quad M_j \geq 0$

$$\sum_j M_j = I$$

Quantum state discrimination in semidefinite programming

(Primal)

Max $$f(M_j) = \sum_j q_j tr[M_j \rho_j]$$

Subject to $$M_j \geq 0$$

$$\sum_j M_j = I$$

Lagrangian $$L(M_j, \sigma_j, K) = f(M_j) - \sum_j tr[M_j \sigma_j] + tr[K(\sum_j M_j - I)]$$

(Dual)

Min $$tr[K]$$

Subject to $$K \geq q_j \rho_j$$

Quantum state discrimination in semidefinite programming

Lagrangian $$L(M_j, \sigma_j, K) = f(M_j) - \sum_j tr[M_j \sigma_j] + tr[K(\sum_j M_j - I)]$$

KKT condition $$K = q_j \rho_j + \sigma_j \quad \text{for all} \quad j = 1, \ldots, N$$

$$tr[M_j \sigma_j] = 0$$

Constraints in the primal and dual problems

Actually, they mean...

i) Identical ensembles $$K = q_1 \rho_1 + \sigma_1 = \ldots = q_N \rho_N + \sigma_N$$

ii) Optimal measurement is orthogonal to dual variables

$$tr[M_j \sigma_j] = 0$$

Quantum state discrimination in semidefinite programming

Lagrangian $L(M_j,\sigma_j,K)=f(M_j)-\sum_j tr[M_j\sigma_j]+tr[K(\sum_j M_j-I)]$

KKT condition $K=q_j\rho_j+\sigma_j$ for all $j=1,\ldots,N$

$tr[M_j\sigma_j]=0$

Constraints in the primal and dual

Actually, they mean...

i) Identical ensembles $K=q_1\rho_1+\sigma_1=\ldots=q_N$

ii) Optimal measurement is orthogonal to dual variabl

$tr[M_j\sigma_j]=0$

V3 Internet Security
스마트 업데이트 실행
새로운 엔진 또는 패치 파일을 확인하기 위해 스마트 업데이트가 실행되었습니다.
스마트 업데이트 실행 결과:
인터넷에 연결할 수 없습니다. 연결 상태를 확인하십시오.
[1/1]
확인
같은 알림 창 다시 띄우지 않기 알림 설정

Quantum state discrimination in semidefinite programming

Lagrangian $L(M_j, \sigma_j, K) = f(M_j) - \sum_j tr[M_j \sigma_j] + tr[K(\sum_j M_j - I)]$

KKT condition $K = q_j \rho_j + \sigma_j$ for all $j = 1, \ldots, N$

$tr[M_j \sigma_j] = 0$

Constraints in the primal and dual problems

Actually, they mean...

i) Identical ensembles $K = q_1 \rho_1 + \sigma_1 = \ldots = q_N \rho_N + \sigma_N$

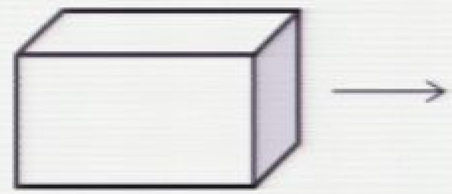
ii) Optimal measurement is orthogonal to dual variables

$$tr[M_j \sigma_j] = 0$$

No-signaling constraint and quantum state discrimination

Summary in application of the no-signaling principle in quantum state discrimination

Problem. $(q_j, \rho_j) \quad j = 1, \ldots, N$



$\rho_j \quad q_j$

1. Find equal ensembles

$$\rho_B^{(1)} = p_1\rho_1 + (1-p_1)\sigma_1$$
$$\rho_B^{(2)} = p_2\rho_2 + (1-p_2)\sigma_2$$
$$\vdots$$

such that $q_j = \dfrac{p_j}{p_1 + p_2 + \ldots + p_N}$

$$tr[M_j \sigma_j] = 0$$

2. Applying the no-signaling, $\sum_j p_j P(\rho_j \mid \rho_j) \le 1$

3. A closed form, $P_{guess} = \sum_j q_j P(\rho_j \mid \rho_j) \le \dfrac{1}{p_1 + p_2 + \ldots + p_N}$

4. Compute $\dfrac{1}{p_1 + p_2 + \ldots + p_N}$ from the equal ensemble condition

Quantum state discrimination in semidefinite programming

(Dual)

Min $tr[K]$

Subject to $K \geq q_j \rho_j$

Solving KKT conditions $\overline{K} = \frac{q_j}{tr[K]} \rho_j + \frac{1}{tr[K]} \sigma_j$

Remind $p_j = \frac{q_j}{tr[K]}$ $\qquad q_j = \frac{p_j}{p_1 + p_2 + \ldots + p_N}$

Solution $tr[K] = \frac{1}{p_1 + p_2 + \ldots + p_N}$

Quantum state discrimination in semidefinite programming

Lagrangian $$L(M_j, \sigma_j, K) = f(M_j) - \sum_j tr[M_j \sigma_j] + tr[K(\sum_j M_j - I)]$$

KKT condition
$$K = q_j \rho_j + \sigma_j \qquad \text{for all} \quad j = 1, \ldots, N$$
$$tr[M_j \sigma_j] = 0$$

Constraints in the primal and dual problems

Actually, they mean...

i) Identical ensembles $$K = q_1 \rho_1 + \sigma_1 = \ldots = q_N \rho_N + \sigma_N$$

ii) Optimal measurement is orthogonal to dual variables

$$tr[M_j \sigma_j] = 0$$

Quantum state discrimination in semidefinite programming

(Dual)

Min $tr[K]$

Subject to $K \geq q_j \rho_j$

Solving KKT conditions $\overline{K} = \frac{q_j}{tr[K]} \rho_j + \frac{1}{tr[K]} \sigma_j$

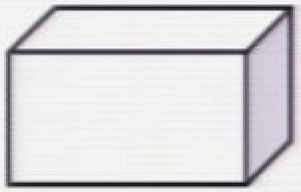
Remind $p_j = \frac{q_j}{tr[K]}$ $\quad q_j = \frac{p_j}{p_1 + p_2 + \ldots + p_N}$

Solution $tr[K] = \frac{1}{p_1 + p_2 + \ldots + p_N}$

No-signaling constraint and quantum state discrimination

Summary in application of the no-signaling principle in quantum state discrimination

Problem. $(q_j, \rho_j) \quad j = 1, \dots, N$



$\longrightarrow \quad \rho_j \quad q_j$

1. Find equal ensembles

$$\rho_B^{(1)} = p_1\rho_1 + (1 - p_1)\sigma_1$$
$$\rho_B^{(2)} = p_2\rho_2 + (1 - p_2)\sigma_2$$
$$\vdots$$

such that $q_j = \dfrac{p_j}{p_1 + p_2 + \dots + p_N}$

$tr[M_j \sigma_j] = 0$

2. Applying the no-signaling, $\sum_j p_j P(\rho_j | \rho_j) \le 1$

3. A closed form, $P_{guess} = \sum_j q_j P(\rho_j | \rho_j) \le \dfrac{1}{p_1 + p_2 + \dots + p_N}$

4. Compute $\dfrac{1}{p_1 + p_2 + \dots + p_N}$ from the equal ensemble condition

Quantum state discrimination in semidefinite programming

(Dual)

Min $tr[K]$

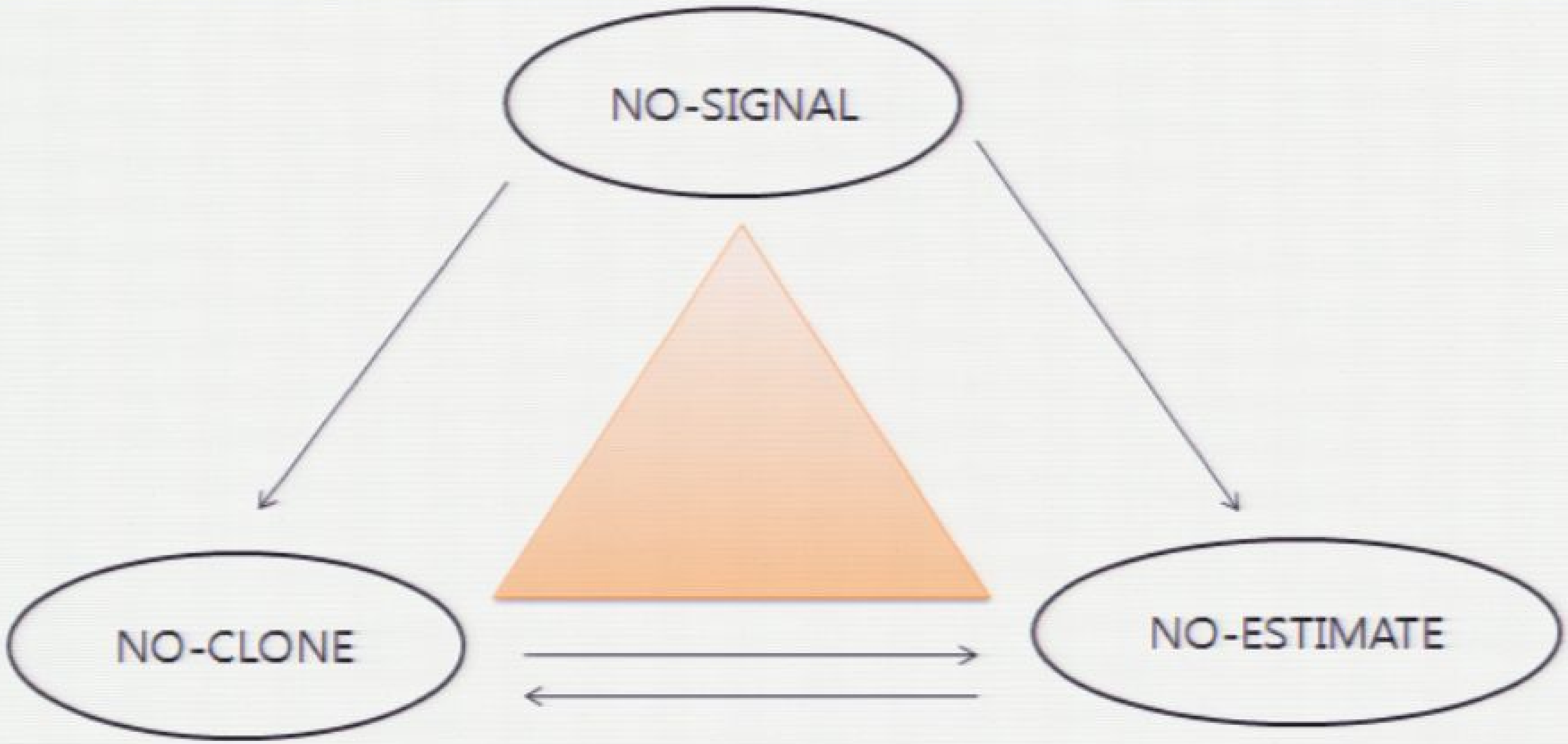
Subject to $K \geq q_j \rho_j$

Solving KKT conditions $\overline{K} = \frac{q_j}{tr[K]} \rho_j + \frac{1}{tr[K]} \sigma_j$

Remind $p_j = \frac{q_j}{tr[K]}$ $q_j = \frac{p_j}{p_1 + p_2 + \ldots + p_N}$

Solution $tr[K] = \frac{1}{p_1 + p_2 + \ldots + p_N}$

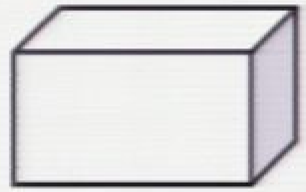
Interrelations between no-go theorems



No-signaling constraint and quantum state discrimination

Summary in application of the no-signaling principle in quantum state discrimination

Problem. $(q_j, \rho_j) \quad j = 1, \ldots, N$



$\longrightarrow \quad \rho_j \quad q_j$

1. Find equal ensembles

$$\rho_B^{(1)} = p_1 \rho_1 + (1 - p_1)\sigma_1$$
$$\rho_B^{(2)} = p_2 \rho_2 + (1 - p_2)\sigma_2$$
$$\vdots$$

such that $q_j = \dfrac{p_j}{p_1 + p_2 + \ldots + p_N}$

$$tr[M_j \sigma_j] = 0$$

2. Applying the no-signaling, $\sum_j p_j P(\rho_j \mid \rho_j) \leq 1$

3. A closed form, $P_{guess} = \sum_j q_j P(\rho_j \mid \rho_j) \leq \dfrac{1}{p_1 + p_2 + \ldots + p_N}$

4. Compute $\dfrac{1}{p_1 + p_2 + \ldots + p_N}$ from the equal ensemble condition

Quantum state discrimination in semidefinite programming

(Dual)

Min $tr[K]$

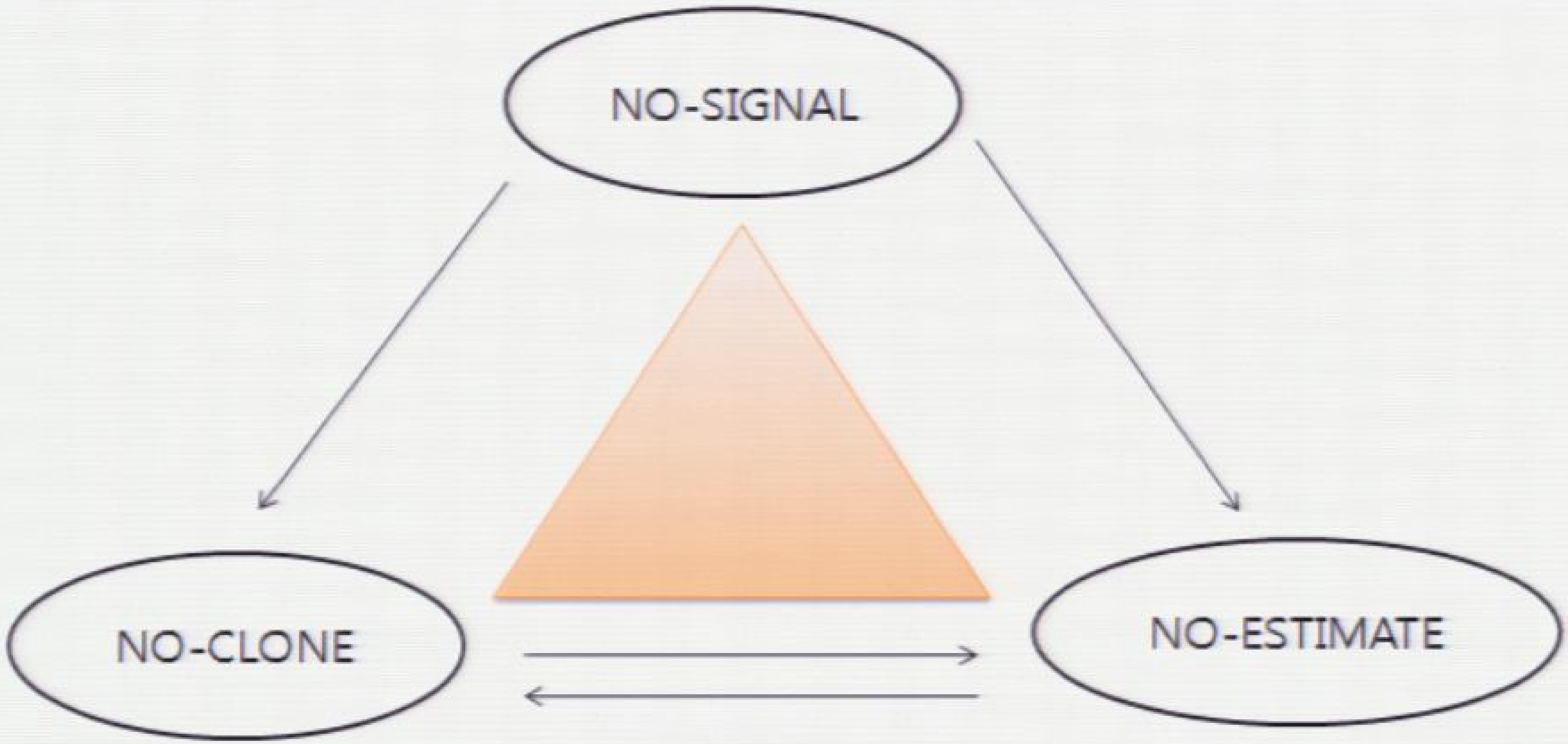
Subject to $K \geq q_j \rho_j$

Solving KKT conditions $\overline{K} = \frac{q_j}{tr[K]} \rho_j + \frac{1}{tr[K]} \sigma_j$

Remind $p_j = \frac{q_j}{tr[K]}$ $q_j = \frac{p_j}{p_1 + p_2 + \ldots + p_N}$

Solution $tr[K] = \frac{1}{p_1 + p_2 + \ldots + p_N}$

Interrelations between no-go theorems



PART II

Fundamental theorem in entanglement detection

-Conjecture : Approximations to positive maps are entanglement-breaking

Jarosław Korbicz *ICFO – Institut de Ciències Fotòniques, Mediterranean Technology Park Castelldefels (Barcelona), Spain, Technical University of Gdansk, Gdansk, Poland, and National Quantum Information Center of Gdansk, Sopot, Poland*

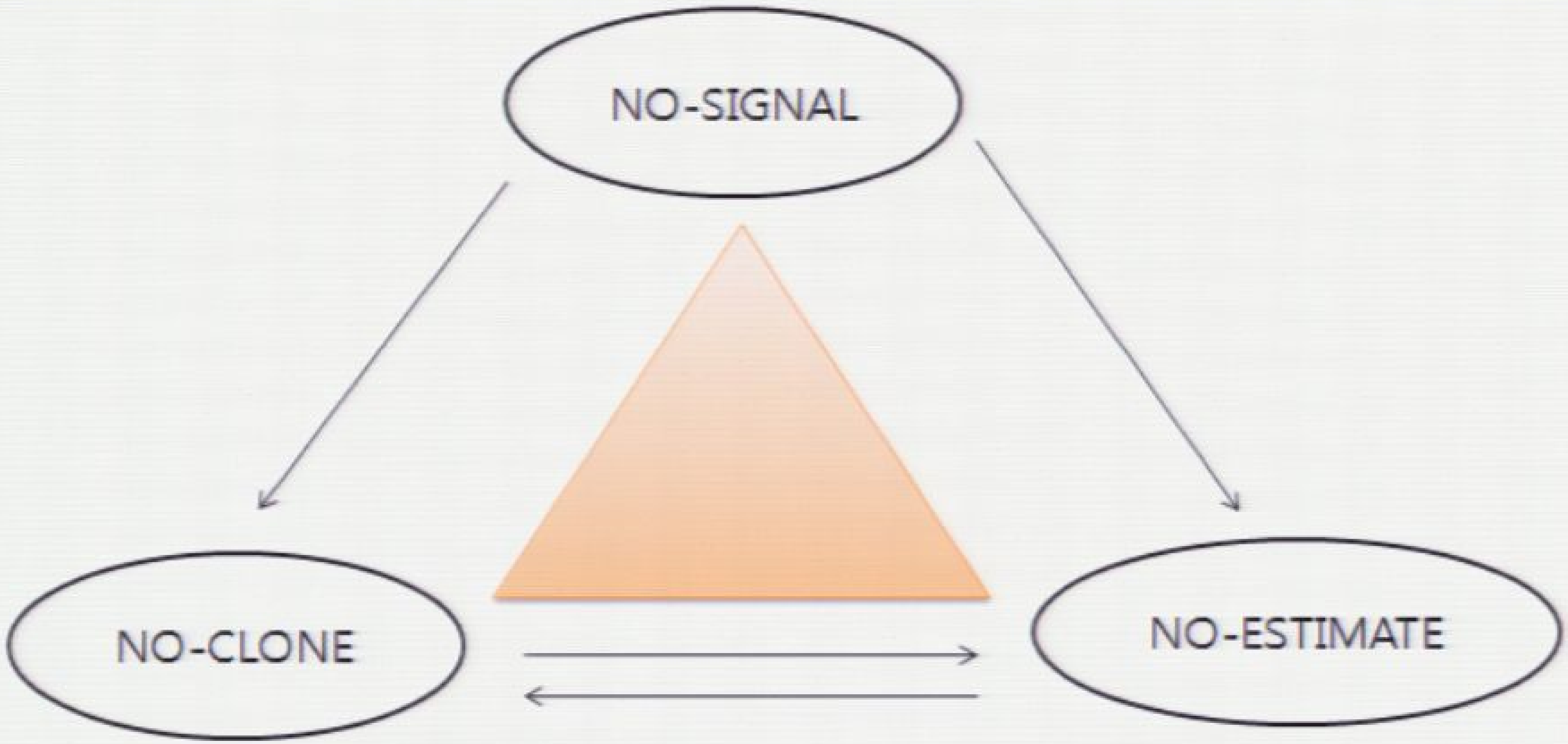
Mafalda Almeida *Center for Quantum Technologies, Singapore*

Remigiusz Augusiak *ICFO – Institut de Ciències Fotòniques, Mediterranean Technology Park Castelldefels (Barcelona), Spain*

Maciej Lewenstein *ICFO – Institut de Ciències Fotòniques, Mediterranean Technology Park Castelldefels (Barcelona), Spain, ICREA-Institució Catalana de Recerca i Estudis Avançats, Barcelona, Spain*

Antonio Acín, *ICFO – Institut de Ciències Fotòniques, Mediterranean Technology Park Castelldefels (Barcelona), Spain, ICREA-Institució Catalana de Recerca i Estudis Avançats, Barcelona, Spain*

Interrelations between no-go theorems



PART II

Fundamental theorem in entanglement detection

-Conjecture : Approximations to positive maps are entanglement-breaking

Jaroslaw Korbicz *ICFO – Institut de Ciències Fotòniques, Mediterranean Technology Park Castelldefels (Barcelona), Spain, Technical University of Gdansk, Gdansk, Poland, and National Quantum Information Center of Gdansk, Sopot, Poland*

Mafalda Almeida *Center for Quantum Technologies, Singapore*

Remigiusz Augusiak *ICFO – Institut de Ciències Fotòniques, Mediterranean Technology Park Castelldefels (Barcelona), Spain*

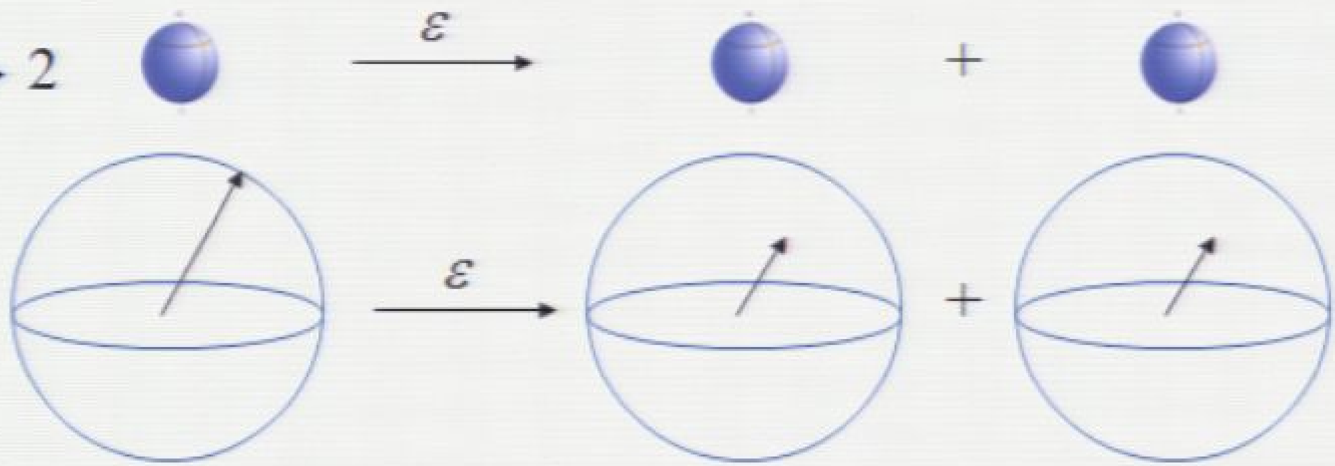
Maciej Lewenstein *ICFO – Institut de Ciències Fotòniques, Mediterranean Technology Park Castelldefels (Barcelona), Spain, ICREA-Institució Catalana de Recerca i Estudis Avançats, Barcelona, Spain*

Antonio Acín, *ICFO – Institut de Ciències Fotòniques, Mediterranean Technology Park Castelldefels (Barcelona), Spain, ICREA-Institució Catalana de Recerca i Estudis Avançats, Barcelona, Spain*

Contents

Quantum Cloning

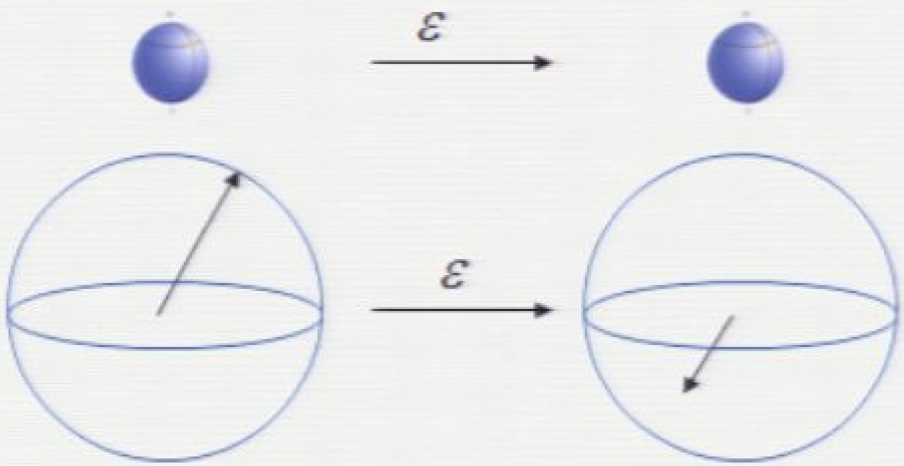
$1 \rightarrow 2$



$$F_Q = \langle \psi | \rho_Q | \psi \rangle = \frac{5}{6}$$

$$F_M = \langle \psi | \rho_M | \psi \rangle = \frac{2}{3}$$

Universal NOT operation



$$F_Q = \langle \psi | \rho_Q | \psi \rangle = \frac{2}{3}$$

$$F_M = \langle \psi | \rho_M | \psi \rangle = \frac{2}{3}$$

H. Bechmann-Pasquinucci, N. Gisin, arXiv:quant-ph/9807041

$m \to n$ (approximate) universal NOT operation

$+ \quad + \quad + \cdots$

UNOT ε

$$F_Q = \langle \psi | \rho_Q | \psi \rangle = \frac{m+1}{m+2}$$

$\times m$

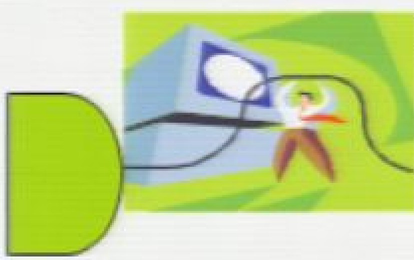
$$F_Q = F_M$$

ρ_Q

$$F_M = \langle \psi | \rho_M | \psi \rangle = \frac{m+1}{m+2}$$

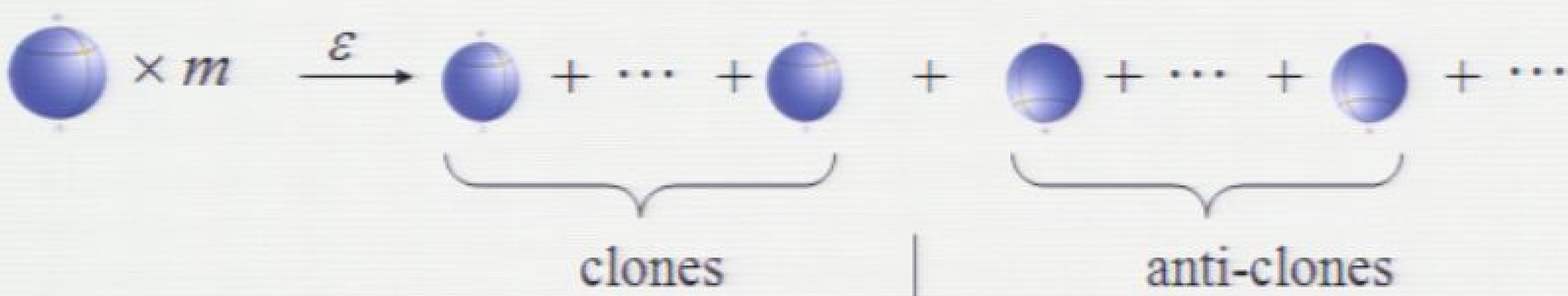
ρ_M

State estimation M



$+ \quad + \quad + \cdots$

Quantum Cloning	Universal NOT operation
Impossible	Impossible
$\exists$ Advantage by quantum approximation	$\nexists$ Advantage by quantum approximation
$F_Q \geq F_M$	$F_Q = F_M$



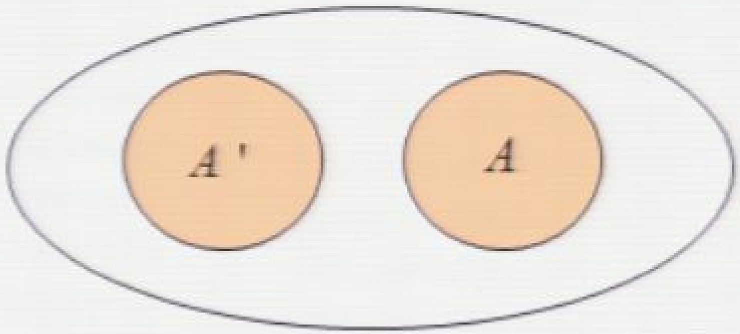
WHY ? Unitarity, linearity, …	WHY ? Completely positive

H. Bechmann-Pasquinucci, N. Gisin, arXiv:quant-ph/9807041
S. van Enk Phys. Rev. Lett. **95**, 010502 (2005)

Quantum operations

i) Positive (P) $\rho_A \geq 0$

$$\Lambda : \rho_A \rightarrow \Lambda[\rho_A] \geq 0$$



ii) Completely positive(CP) $\rho_{A'A} \geq 0$

$$I \otimes \Lambda : \rho_{A'A} \rightarrow I \otimes \Lambda[\rho_{A'A}]$$

$$I \otimes \Lambda[\rho_{A'A}] \geq 0$$

Lemma. For separable states, P immediately implies CP.

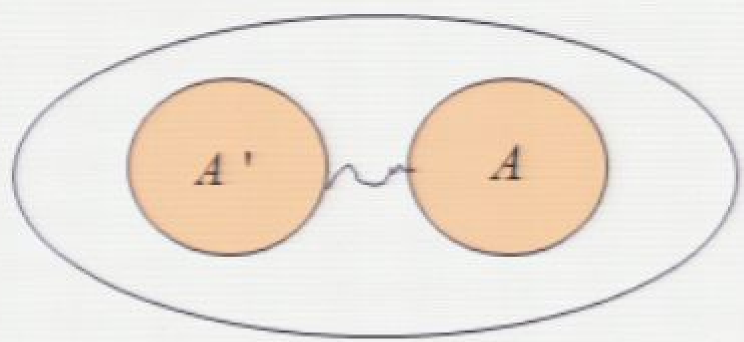
Proof. $\rho_{A'A} = \sum_j p_j \rho^j_{A'} \otimes \rho^j_A$

$$I \otimes \Lambda[\rho_{A'A}] = \sum_j p_j \rho^j_{A'} \otimes \Lambda[\rho^j_A] \geq 0$$

Non-CP positive map does not preserve positivity for certain entangled states

$$I \otimes \Lambda : \rho_{A'A} \rightarrow I \otimes \Lambda[\rho_{A'A}]$$

$$I \otimes \Lambda[\rho_{A'A}] \not\geq 0$$



Example. Tranpose / Partial transpose

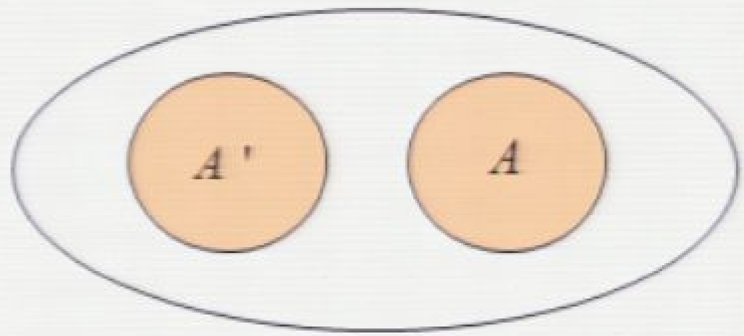
$$T : |i\rangle\langle j| \rightarrow |j\rangle\langle i|$$

$I \otimes T[|\psi\rangle\langle\psi|] \not\geq 0 \quad \forall |\psi\rangle$ entangled states

Quantum operations

i) Positive (P) $\rho_A \geq 0$

$$\Lambda : \rho_A \rightarrow \Lambda[\rho_A] \geq 0$$



ii) Completely positive(CP) $\rho_{A'A} \geq 0$

$$I \otimes \Lambda : \rho_{A'A} \rightarrow I \otimes \Lambda[\rho_{A'A}]$$

$$I \otimes \Lambda[\rho_{A'A}] \geq 0$$

Lemma. For separable states, P immediately implies CP.

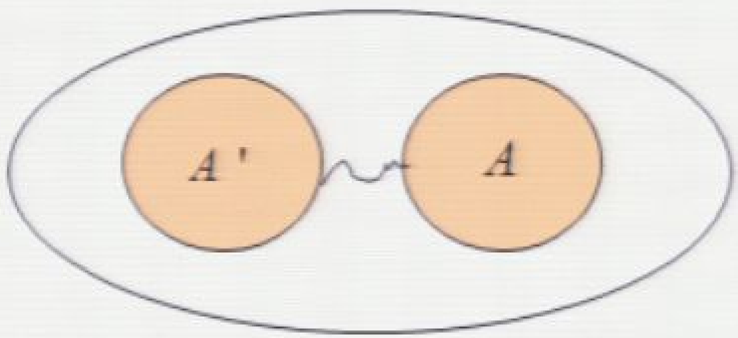
Proof. $\rho_{A'A} = \sum_j p_j \rho^j_{A'} \otimes \rho^j_A$

$$I \otimes \Lambda[\rho_{A'A}] = \sum_j p_j \rho^j_{A'} \otimes \Lambda[\rho^j_A] \geq 0$$

Non-CP positive map does not preserve positivity for certain entangled states

$$I \otimes \Lambda : \rho_{A'A} \rightarrow I \otimes \Lambda[\rho_{A'A}]$$

$$I \otimes \Lambda[\rho_{A'A}] \ngeq 0$$



Example. Tranpose / Partial transpose

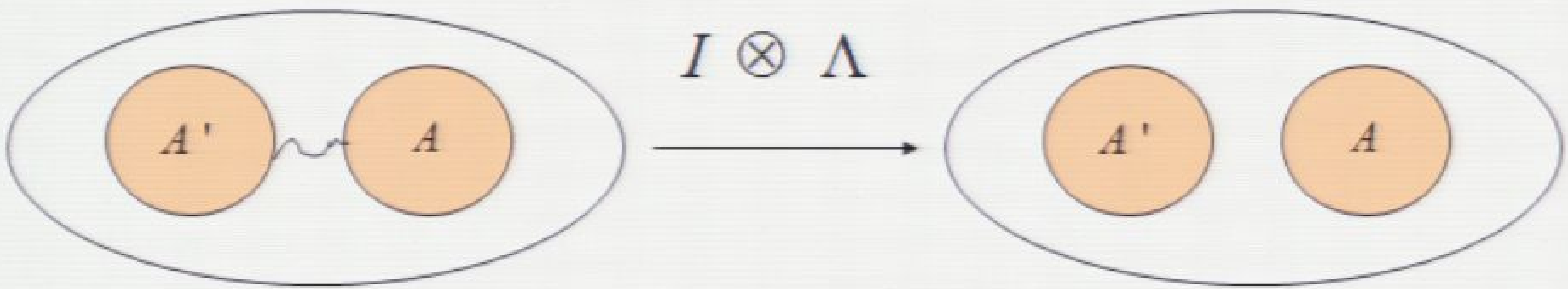
$$T : |i\rangle\langle j| \rightarrow |j\rangle\langle i|$$

$$I \otimes T[|\psi\rangle\langle\psi|] \ngeq 0 \quad \forall |\psi\rangle$$ entangled states

Entanglement-breaking operation

Λ is entanglement-breaking if and only if

$$I \otimes \Lambda : \rho_{ent} \rightarrow I \otimes \Lambda[\rho_{ent}] = \rho_{sep}$$



Theorem Entanglement breaking operation is of the following form

$$\varepsilon(\rho) = \sum_j tr(M_j \rho) \rho_j$$

⇒ Measurement plus State Preparation
(State Estimation)

Quantum Cloning

Impossible

$\exists$ Advantage by quantum approximation

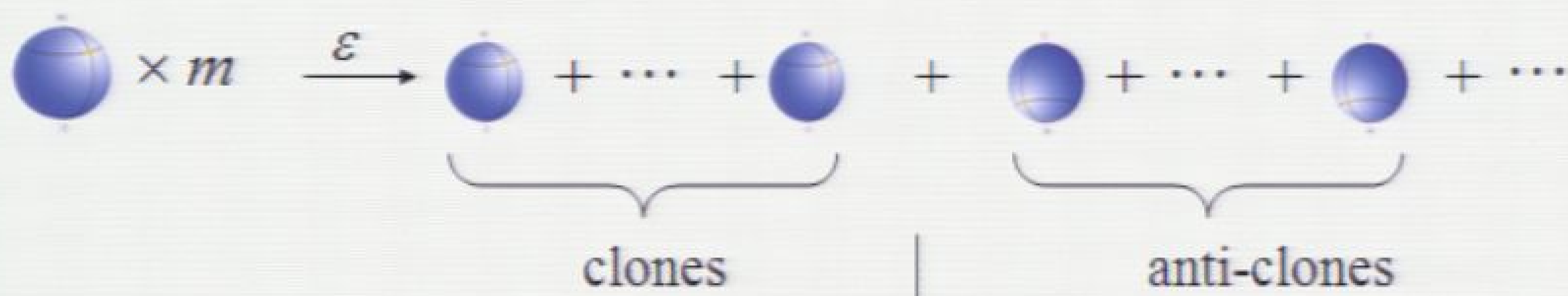
$$F_Q \geq F_M$$

Universal NOT operation

Impossible

$\nexists$ Advantage by quantum approximation

$$F_Q = F_M$$



WHY ? Unitarity, linearity, …

WHY ? Completely positivity

H. Bechmann-Pasquinucci, N. Gisin, arXiv:quant-ph/9807041

S. van Enk Phys. Rev. Lett. **95**, 010502 (2005)

Possible Interpretation to UNOT operation

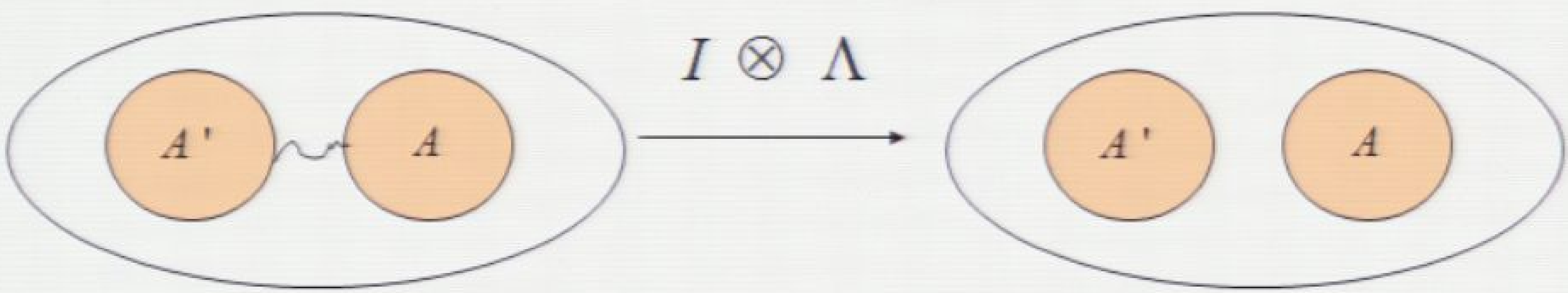
Fact. UNOT is not physical since it is not completely positive

i) UNOT is completely positive (physical) if no entangled state exists

i)' UNOT is not non-physical for separable states

A quantum operation that approximates UNOT can be understood as a map that removes existing quantum correlations

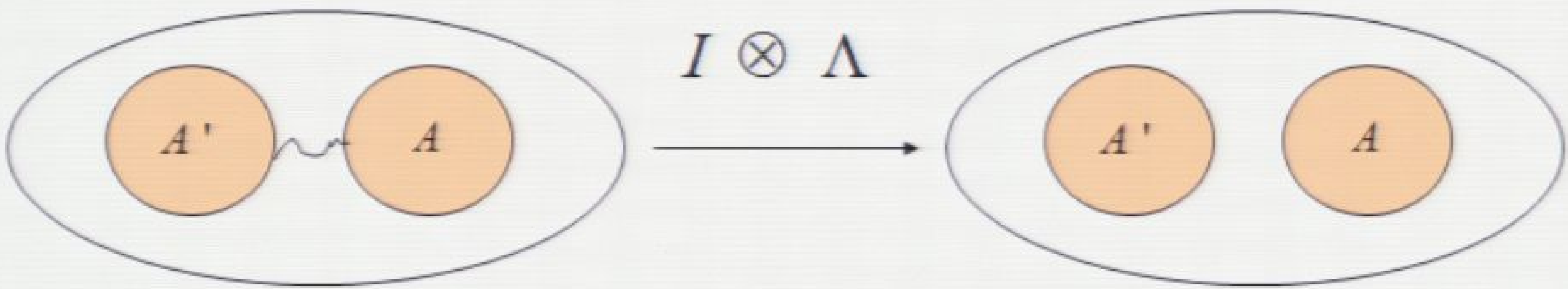
$$I \otimes \Lambda : \rho_{ent} \rightarrow I \otimes \Lambda[\rho_{ent}] = \rho_{sep}$$



Entanglement-breaking operation

Λ is entanglement-breaking if and only if

$$I \otimes \Lambda : \rho_{ent} \rightarrow I \otimes \Lambda[\rho_{ent}] = \rho_{sep}$$



Theorem Entanglement breaking operation is of the following form

$$\varepsilon(\rho) = \sum_j tr(M_j \rho)\rho_j$$

⇨ Measurement plus State Preparation
(State Estimation)

Quantum Cloning

Impossible

∃ Advantage by quantum approximation

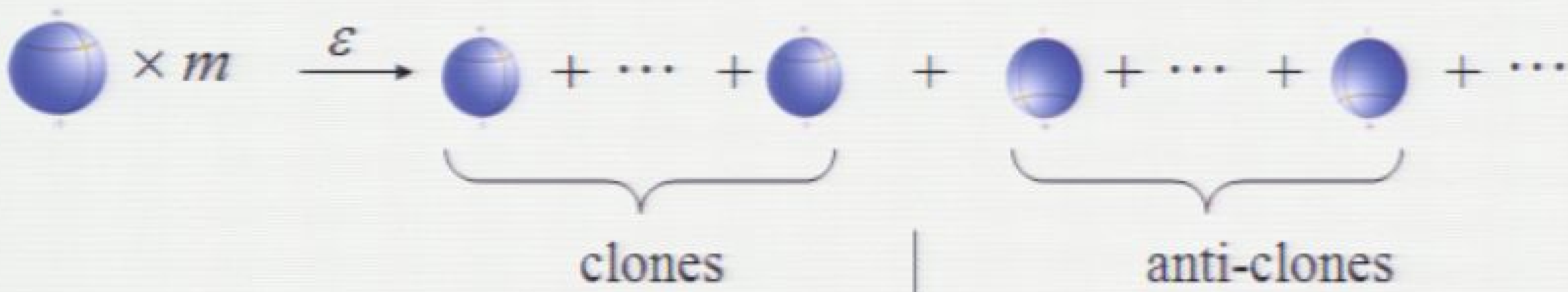
$$F_Q \geq F_M$$

Universal NOT operation

Impossible

∄ Advantage by quantum approximation

$$F_Q = F_M$$



WHY ? Unitarity, linearity, …

WHY ? Completely positivity

H. Bechmann-Pasquinucci, N. Gisin, arXiv:quant-ph/9807041

S. van Enk Phys. Rev. Lett. **95**, 010502 (2005)

Possible Interpretation to UNOT operation

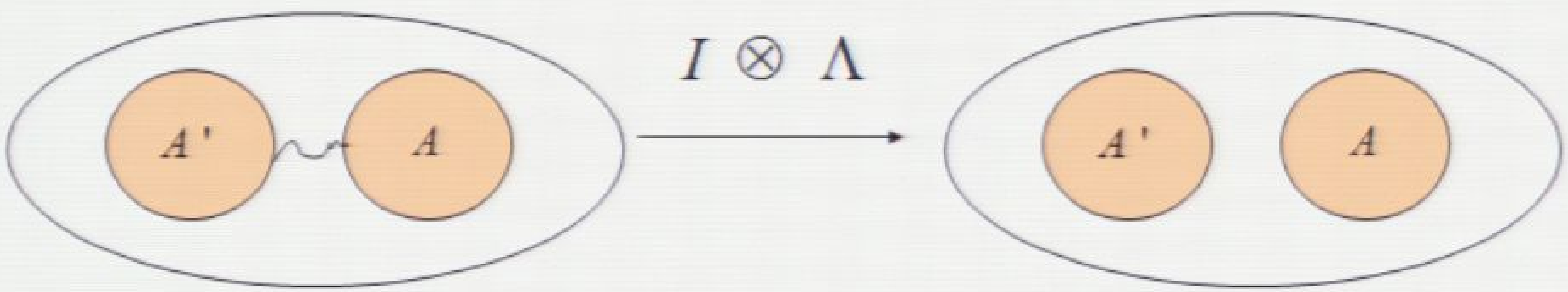
Fact. UNOT is not physical since it is not completely positive

i) UNOT is completely positive (physical) if no entangled state exists

i)' UNOT is not non-physical for separable states

A quantum operation that approximates UNOT can be understood as a map that removes existing quantum correlations

$$I \otimes \Lambda : \rho_{ent} \rightarrow I \otimes \Lambda[\rho_{ent}] = \rho_{sep}$$



Quantum Cloning

Impossible

∃ Advantage by quantum approximation

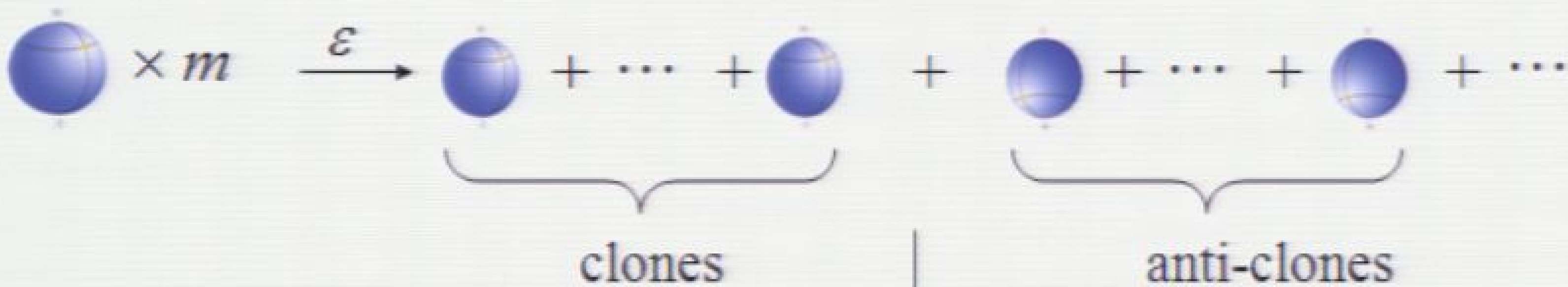
$$F_Q \geq F_M$$

Universal NOT operation

Impossible

∄ Advantage by quantum approximation

$$F_Q = F_M$$



WHY ? Unitarity, linearity, …

WHY ? Completely positivity

H. Bechmann-Pasquinucci, N. Gisin, arXiv:quant-ph/9807041

S. van Enk Phys. Rev. Lett. **95**, 010502 (2005)

Possible Interpretation to UNOT operation

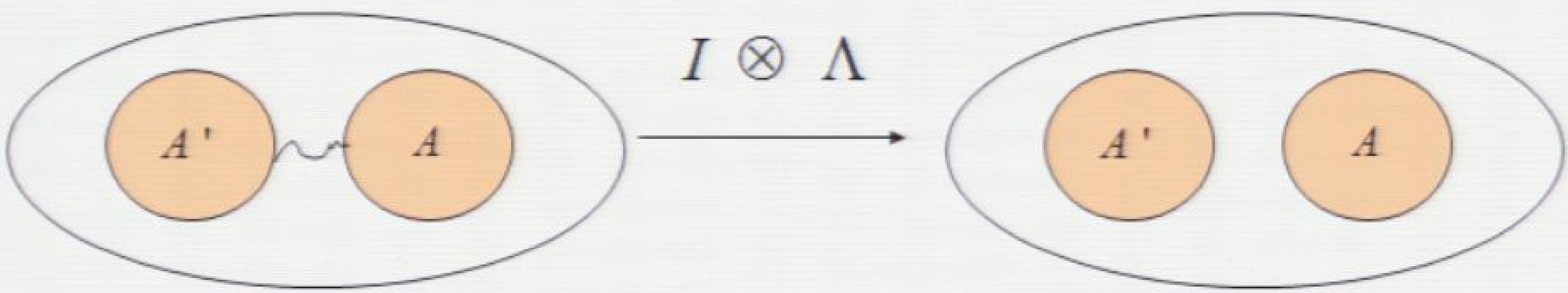
Fact. UNOT is not physical since it is not completely positive

i) UNOT is completely positive (physical) if no entangled state exists

i)' UNOT is not non-physical for separable states

A quantum operation that approximates UNOT can be understood as a map that removes existing quantum correlations

$$I \otimes \Lambda : \rho_{ent} \rightarrow I \otimes \Lambda[\rho_{ent}] = \rho_{sep}$$



Quantum Cloning | Universal NOT operation

Quantum Cloning

Impossible

$\exists$ Advantage by quantum approximation

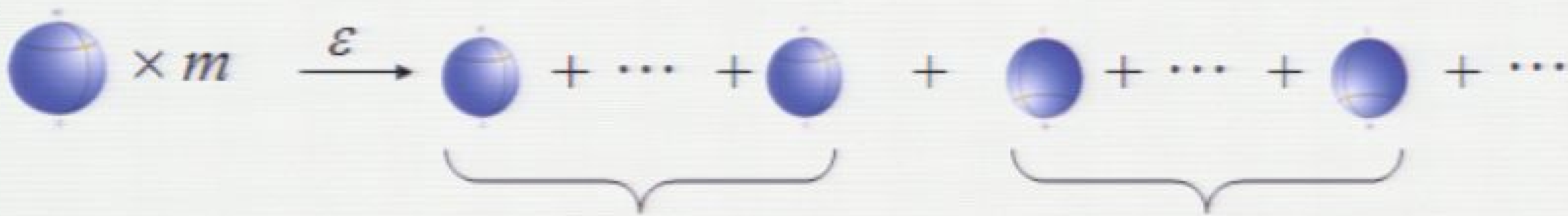
$$F_Q \geq F_M$$

Universal NOT operation

Impossible

$\nexists$ Advantage by quantum approximation

$$F_Q = F_M$$



clones

anti-clones

WHY ? Unitarity, linearity, …

WHY ? Completely positivity

H. Bechmann-Pasquinucci, N. Gisin, arXiv:quant-ph/9807041

S. van Enk Phys. Rev. Lett. **95**, 010502 (2005)

Contents

Optimal positive maps are those maps that correspond to optimal entanglement witness

$$tr[\rho_{ent} W] < 0 \quad \exists \rho_{ent}$$

$$tr[\rho_{sep} W] \geq 0 \quad \forall \rho_{sep}$$

$$W_{A'A} = (I \otimes \Lambda)(\phi^+) \not\geq 0$$

$$\phi^+ = \frac{1}{\sqrt{d}} \sum_j |jj\rangle$$

Jamiolkowski isomorphism $\quad \Lambda[\rho] = d_A tr_{A'}[W(\rho^T \otimes I)]$

Optimal entanglement witness

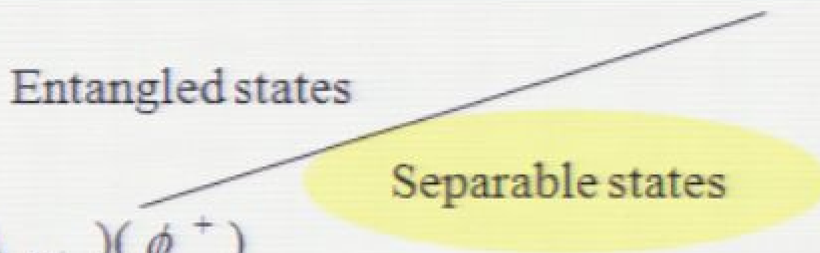
$$D_W = \{\rho : tr[\rho W] < 0\}$$

W_2 is finer than W_1 if $D_{W_1} \subseteq D_{W_2}$

Def. W is optimal if there exists no other witness which is finer than W

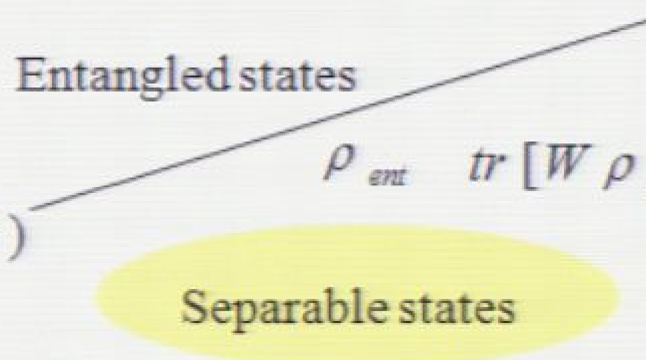
The Conjecture

> Any physical approximations to optimal positive maps correspond to entanglement breaking maps.
> Equivalently structural physical approximations to optimal entanglement witnesses are given by separable states



$$W_{opt} = (I \otimes \Lambda_{opt})(\phi^{+})$$

Why do we restrict to optimal positive maps?



If W is not optimal

$D_W \subseteq D_{W_{opt}}$

$$W = (I \otimes \Lambda)(\phi^{+})$$

ρ_{ent} $tr[W \rho_{ent}] > 0$

is not non-physical for those states in $D_{W_{opt}} - D_W$

Optimal positive maps are those maps that correspond to optimal entanglement witness

$tr[\rho_{ent} W] < 0 \quad \exists \rho_{ent}$

$tr[\rho_{sep} W] \geq 0 \quad \forall \rho_{sep}$

$$W_{A'A} = (I \otimes \Lambda)(\phi^{+}) \ngeq 0$$

$$\phi^{+} = \frac{1}{\sqrt{d}} \sum_j |jj\rangle$$

Jamiolkowski isomorphism $\quad \Lambda[\rho] = d_{A'} tr_{A'}[W(\rho^{T} \otimes I)]$

Optimal entanglement witness

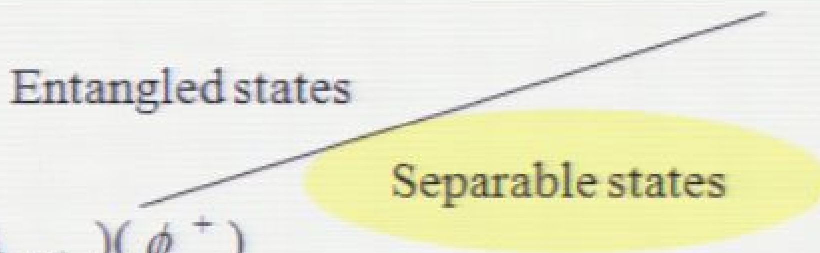
$$D_W = \{\rho : tr[\rho W] < 0\}$$

W_2 is finer than W_1 if $D_{W_1} \subseteq D_{W_2}$

Def. W is optimal if there exists no other witness which is finer than W

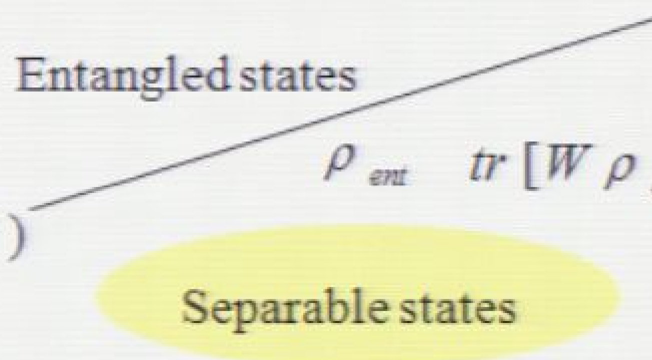
The Conjecture

Any physical approximations to optimal positive maps correspond to entanglement breaking maps.
Equivalently structural physical approximations to optimal entanglement witnesses are given by separable states



$$W_{opt} = (I \otimes \Lambda_{opt})(\phi^{+})$$

Why do we restrict to optimal positive maps?



If W is not optimal

$\rho_{ent} \quad tr[W \rho_{ent}] > 0$

$D_W \subseteq D_{W_{opt}}$

$$W = (I \otimes \Lambda)(\phi^{+})$$

is not non-physical for those states in $D_{W_{opt}} - D_W$

Structural Physical Approximation (SPA) to Positive Maps

$$\Lambda \geq 0 \quad I \otimes \Lambda \ngeq 0$$

$$\overline{\Lambda} = p^* \frac{\mathrm{I}}{d_A} + (1 - p^*)\,\Lambda, \quad \overline{\Lambda}[\rho] = p^* \frac{\mathrm{I}}{d_A} tr[\rho] + (1 - p^*)\,\Lambda[\rho]$$

$$p^* = \min \; p \qquad \text{such that} \quad \mathrm{I} \otimes \overline{\Lambda} \geq 0$$

$$I \otimes \overline{\Lambda}[\rho] = p^* \frac{\mathrm{I} \otimes \mathrm{I}}{d_A^2} + (1 - p^*)\, I \otimes \Lambda[\rho]$$

P Horodecki, A Ekert, PRL 89 127902 (2002)
P Horodecki, quant-ph/0111036

The Conjecture

The general form

Any physical approximations to optimal positive maps correspond to entanglement breaking maps.

In SPA

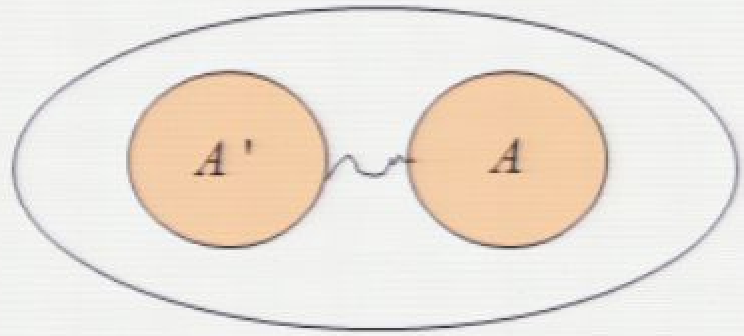
Structural physical approximations to optimal positive maps correspond to entanglement breaking maps.

Contents

Non-CP positive map can detect ALL entangled states

$\forall \rho_{A'A}$ entangled states

$\exists \Lambda$ such that $I \otimes \Lambda[\rho_{A'A}] \not\geq 0$



Example. Tranpose / Partial transpose

$$T : |i\rangle\langle j| \rightarrow |j\rangle\langle i|$$

$$I \otimes T[|\psi_{ent}\rangle\langle\psi_{ent}|] \not\geq 0$$

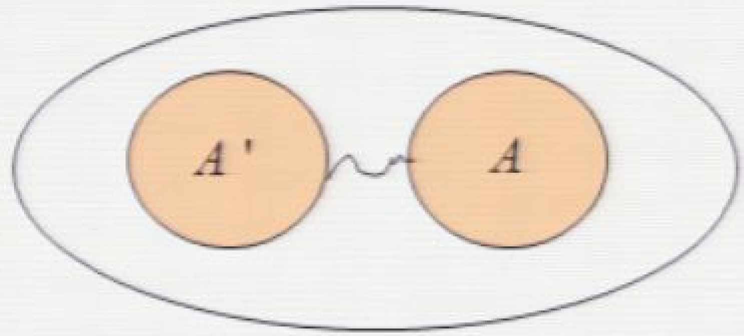
$\rho \in 2 \otimes 2$ or $2 \otimes 3$

ρ is entangled if and only if $I \otimes T[\rho] \not\geq 0$

Non-CP positive map can detect ALL entangled states

$\forall \rho_{A'A}$ entangled states

$\exists \Lambda$ such that $I \otimes \Lambda[\rho_{A'A}] \not\geq 0$



Example. Tranpose / Partial transpose

$$T : |i\rangle\langle j| \rightarrow |j\rangle\langle i|$$

$$I \otimes T[|\psi_{ent}\rangle\langle\psi_{ent}|] \not\geq 0$$

$\rho \in 2 \otimes 2$ or $2 \otimes 3$

ρ is entangled if and only if $I \otimes T[\rho] \not\geq 0$

Any Entangled States Are Something in Quantum Information Processing

i) Teleportation, non-locality (Bell Inq. violation), quantum computation

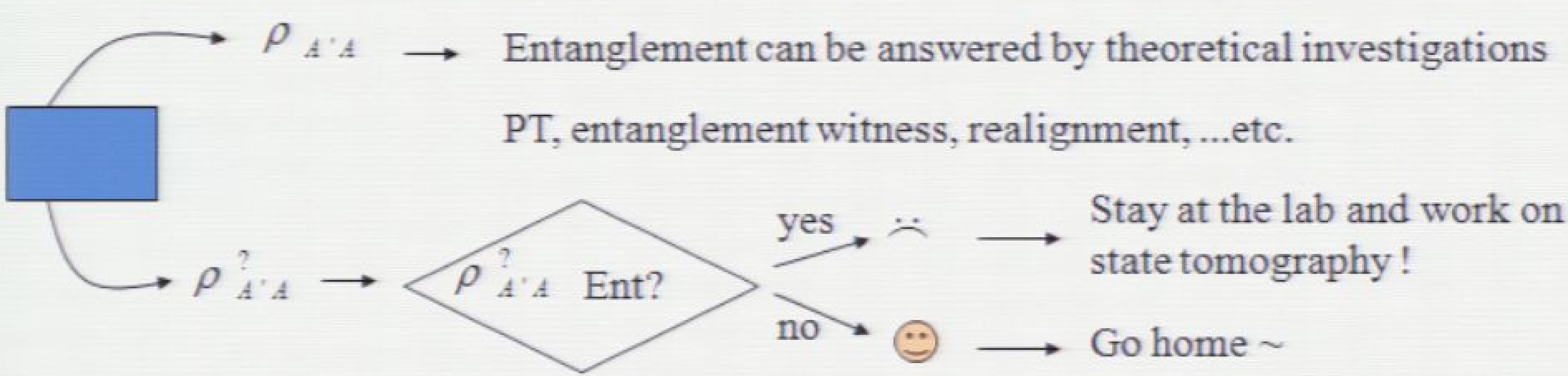
ii) Key distillation

iii) Activating entanglement distillation, and quantum channel capacity

iv) Useful in general

In practical quantum information processing (experiment),

When copies of some quantum state are given, it should be answered first if the state is entangled or not, before identifying what the state is (state tomography).

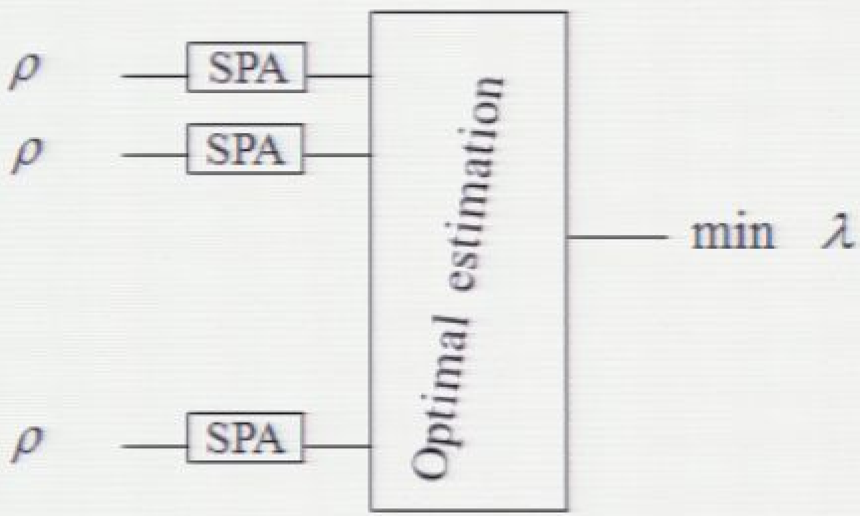


Ll. Masanes, PRL 91 150501 (2006)
Ll. Masanes, Y.-C. Liang, A. C. Doherty, PRL 100 090403 (2008)

Experimental detection of entanglement

Example. Partial transpose $I \otimes T$

$$\overline{I \otimes T} = p\frac{\mathrm{I} \otimes \mathrm{I}}{d^2} + (1-p) I \otimes T \qquad p \geq \frac{d^3}{d^3+1}$$



ρ is entangled if $\min \lambda < \dfrac{d}{d^3+1}$

P Horodecki, A Ekert, PRL 89 127902 (2002)
M Keyl, R Werner, PRA 64 052311 (2001)

Contents

Sketch of Proofs $\Lambda \geq 0 \quad I \otimes \Lambda \ngeq 0$

i) Make SPA

$$\overline{\Lambda} = p^* \frac{\mathrm{I}}{d_A} + (1 - p^*)\,\Lambda, \qquad p^* = \min \; p \qquad \text{such that} \quad \mathrm{I} \otimes \overline{\Lambda} \geq 0$$

$$I \otimes \overline{\Lambda}[\rho] = p^* \frac{\mathrm{I} \otimes \mathrm{I}}{d_A . d_A} + (1 - p^*)\, I \otimes \Lambda[\rho]$$

ii) Is $\overline{\Lambda}$ entanglement-breaking? $\mathrm{I} \otimes \overline{\Lambda}[\rho_{ent}] = \rho_{sep} \quad \forall \rho_{ent}$

Theorem. $\overline{\Lambda}$ is entanglement breaking if and only if

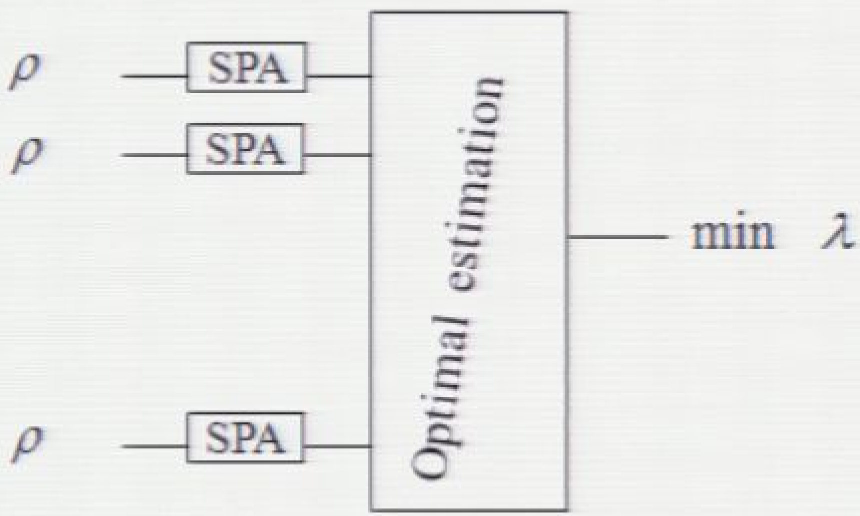
$\mathrm{I} \otimes \overline{\Lambda}[\phi^+]$ is separable. $\phi^+ = \frac{1}{\sqrt{d}} \sum_j |jj\rangle$

ii)' Is $\mathrm{I} \otimes \overline{\Lambda}[\phi^+]$ separable?

Experimental detection of entanglement

Example. Partial transpose $I \otimes T$

$$\overline{I \otimes T} = p \frac{\mathrm{I} \otimes \mathrm{I}}{d^2} + (1-p) I \otimes T \qquad p \geq \frac{d^3}{d^3+1}$$



ρ is entangled if $\min \lambda < \dfrac{d}{d^3+1}$

P Horodecki, A Ekert, PRL 89 127902 (2002)
M Keyl, R Werner, PRA 64 052311 (2001)

Sketch of Proofs $\Lambda \geq 0 \quad I \otimes \Lambda \ngeq 0$

i) Make SPA

$$\overline{\Lambda} = p^* \frac{\mathrm{I}}{d_A} + (1 - p^*)\,\Lambda, \qquad p^* = \min \; p \qquad \text{such that} \quad \mathrm{I} \otimes \overline{\Lambda} \geq 0$$

$$I \otimes \overline{\Lambda}[\rho] = p^* \frac{\mathrm{I} \otimes \mathrm{I}}{d_A \cdot d_A} + (1 - p^*)\, I \otimes \Lambda[\rho]$$

ii) Is $\overline{\Lambda}$ entanglement-breaking? $\mathrm{I} \otimes \overline{\Lambda}[\rho_{ent}] = \rho_{sep} \quad \forall \rho_{ent}$

Theorem. $\overline{\Lambda}$ is entanglement breaking if and only if

$\mathrm{I} \otimes \overline{\Lambda}[\phi^+]$ is separable. $\phi^+ = \frac{1}{\sqrt{d}} \sum_j |jj\rangle$

ii)' Is $\mathrm{I} \otimes \overline{\Lambda}[\phi^+]$ separable?

Contents

Sketch of Proofs $\Lambda \geq 0 \quad I \otimes \Lambda \ngeq 0$

i) Make SPA

$$\overline{\Lambda} = p^* \frac{\mathrm{I}}{d_A} + (1 - p^*)\,\Lambda, \qquad p^* = \min\ p \quad \text{such that} \quad \mathrm{I} \otimes \overline{\Lambda} \geq 0$$

$$I \otimes \overline{\Lambda}[\rho] = p^* \frac{\mathrm{I} \otimes \mathrm{I}}{d_A \cdot d_A} + (1 - p^*)\, I \otimes \Lambda[\rho]$$

ii) Is $\overline{\Lambda}$ entanglement-breaking? $\mathrm{I} \otimes \overline{\Lambda}[\rho_{ent}] = \rho_{sep} \quad \forall \rho_{ent}$

Theorem. $\overline{\Lambda}$ is entanglement breaking if and only if

$\mathrm{I} \otimes \overline{\Lambda}[\phi^+]$ is separable. $\phi^+ = \frac{1}{\sqrt{d}} \sum_j |jj\rangle$

ii)' Is $\mathrm{I} \otimes \overline{\Lambda}[\phi^+]$ separable?

Evidence to the conjecture: Transpose

i) Tranpose $T : |i\rangle\langle j| \rightarrow |j\rangle\langle i|$

$$I \otimes \overline{T} \geq 0$$

$$I \otimes \overline{T} = p^* \frac{\mathrm{I} \otimes \mathrm{I}}{d^2} + (1 - p^*)\, I \otimes T \qquad p^* = \frac{d}{d+1}$$

ii) Is $\mathrm{I} \otimes \overline{T}[\phi^+]$ separable?

$$I \otimes \overline{T}[\phi^+] = p^* \frac{\mathrm{I} \otimes \mathrm{I}}{d^2} + (1 - p^*)\, F$$

$$= a(p^*)\, A + s(p^*)\, S \qquad \text{Werner states}$$

$I \otimes \overline{T}[\phi^+]$ is separable if and only if $p^* \geq \dfrac{d}{d+1}$

Conclusion: SPA of the transpose map is entanglement breaking

Application: SPA of the transpose map can be measurement and state preparation

Evidence to the conjecture: Partial Transpose

ii) Partial Tranpose $I \otimes \overline{T}$

$$(I \otimes I)_{B'B} \otimes (\overline{I \otimes \overline{\overline{T}}})_{A'A} \geq 0 \qquad p^* = \frac{d^3}{d^3+1}$$

ii) Is $(I \otimes I)_{B'B} \otimes (\overline{I \otimes \overline{\overline{T}}})_{A'A}$ separable in the A'A / B'B?

$(I \otimes I)_{B'B} \otimes (\overline{I \otimes \overline{\overline{T}}})_{A'A}$ is separable if and only if $p^* \geq \dfrac{d^3}{d^3+1}$

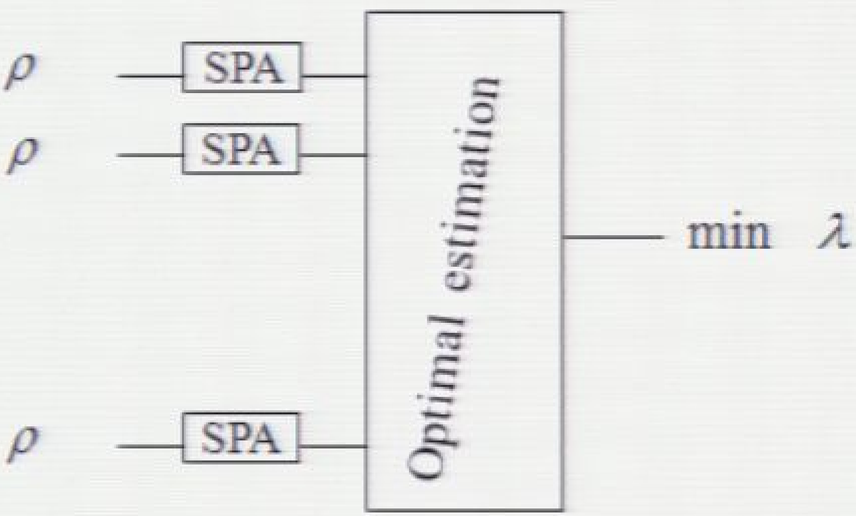
Conclusion: SPA of the partial transpose map is entanglement breaking

Application: SPA of the transpose map can be measurement and state preparation

Experimental detection of entanglement

Example. Partial transpose $I \otimes T$

$$\overline{I \otimes T} = p\,\frac{\mathrm{I} \otimes \mathrm{I}}{d^2} + (1-p)\,I \otimes T \qquad p \geq \frac{d^3}{d^3+1}$$



ρ is entangled if $\min \lambda < \dfrac{d}{d^3+1}$

P Horodecki, A Ekert, PRL 89 127902 (2002)
M Keyl, R Werner, PRA 64 052311 (2001)

Evidence to the conjecture: Partial Transpose

ii) Partial Tranpose $I \otimes \overline{T}$

$$(I \otimes I)_{B'B} \otimes (\overline{I \otimes \overline{T}})_{A'A} \geq 0 \qquad p^* = \frac{d^3}{d^3+1}$$

ii) Is $(I \otimes I)_{B'B} \otimes (\overline{I \otimes \overline{T}})_{A'A}$ separable in the A'A / B'B?

$(I \otimes I)_{B'B} \otimes (\overline{I \otimes \overline{T}})_{A'A}$ is separable if and only if $p^* \geq \dfrac{d^3}{d^3+1}$

Conclusion: SPA of the partial transpose map is entanglement breaking

Application: SPA of the transpose map can be measurement and state preparation

Evidence to the conjecture: Partial Transpose

ii) Partial Tranpose $I \otimes \overline{T}$

$$(I \otimes I)_{B'B} \otimes (\overline{I \otimes \overline{\overline{T}}})_{A'A} \geq 0 \qquad p^* = \frac{d^3}{d^3 + 1}$$

ii) Is $(I \otimes I)_{B'B} \otimes (\overline{I \otimes \overline{\overline{T}}})_{A'A}$ separable in the A'A / B'B?

$(I \otimes I)_{B'B} \otimes (\overline{I \otimes \overline{\overline{T}}})_{A'A}$ is separable if and only if $p^* \geq \dfrac{d^3}{d^3 + 1}$

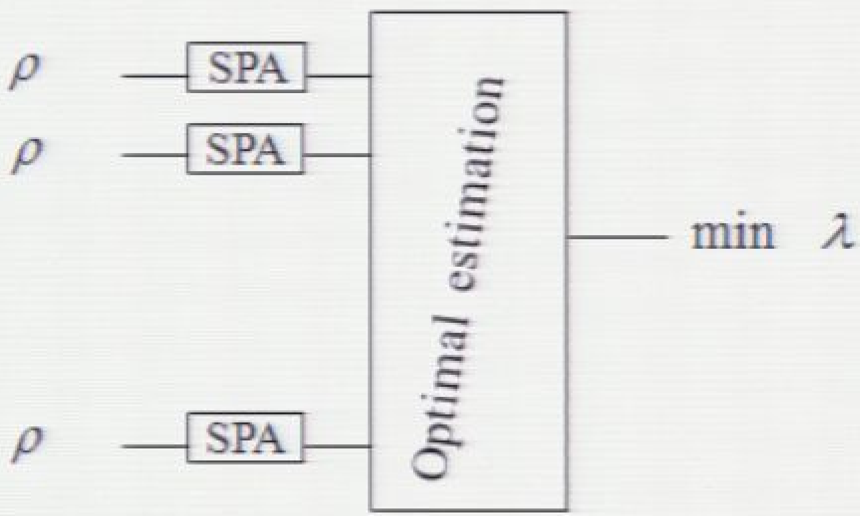
Conclusion: SPA of the partial transpose map is entanglement breaking

Application: SPA of the transpose map can be measurement and state preparation

Experimental detection of entanglement

Example. Partial transpose $I \otimes T$

$$\overline{I \otimes T} = p\,\frac{\mathrm{I} \otimes \mathrm{I}}{d^2} + (1-p)\,I \otimes T \qquad p \geq \frac{d^3}{d^3+1}$$



ρ is entangled if $\min \lambda < \frac{d}{d^3+1}$

P Horodecki, A Ekert, PRL 89 127902 (2002)
M Keyl, R Werner, PRA 64 052311 (2001)

ii) Partial Tranpose $I \otimes \overline{T}$

$$(I \otimes I)_{B'B} \otimes (\overline{I \otimes \overline{T}})_{A'A} \geq 0 \qquad p^* = \frac{d^3}{d^3+1}$$

ii) Is $(I \otimes I)_{B'B} \otimes (\overline{I \otimes \overline{T}})_{A'A}$ separable in the A'A / B'B?

$(I \otimes I)_{B'B} \otimes (\overline{I \otimes \overline{T}})_{A'A}$ is separable if and only if $p^* \geq \dfrac{d^3}{d^3+1}$

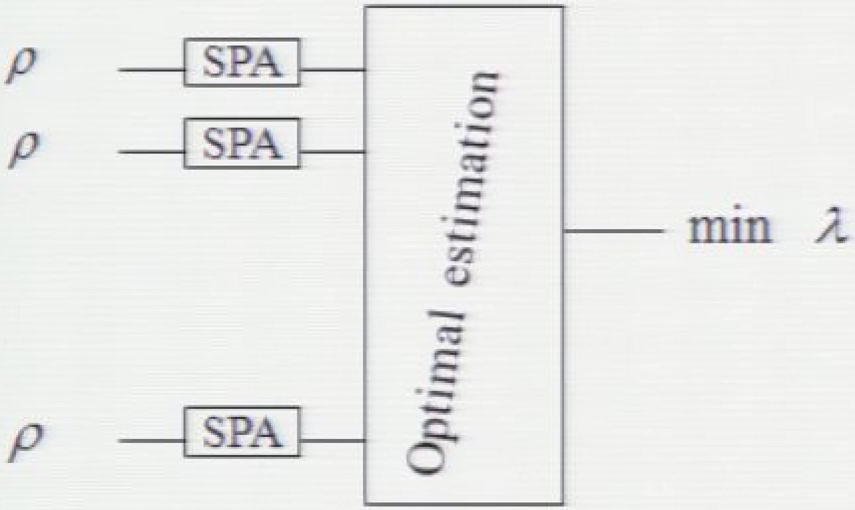
Conclusion: SPA of the partial transpose map is entanglement breaking

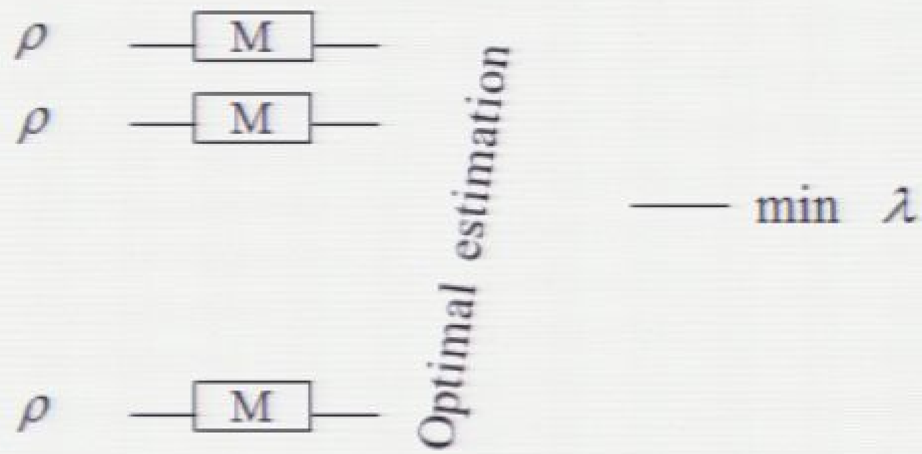
Application: SPA of the transpose map can be measurement and state preparation

Experimental detection of entanglement

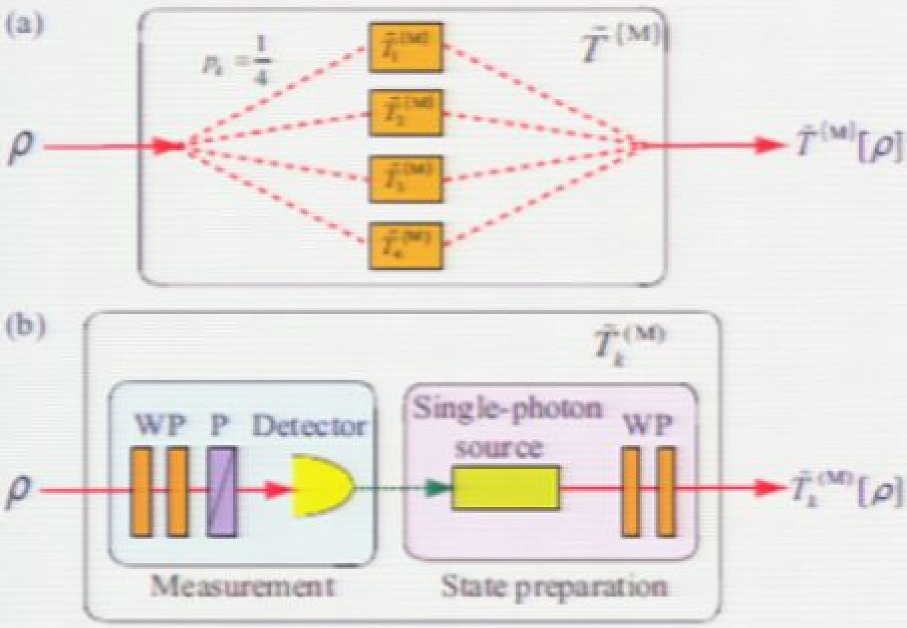
Example. Partial transpose $I \otimes T$

$$\overline{I \otimes T} = p \frac{I \otimes I}{d^2} + (1 - p) I \otimes T \qquad p \geq \frac{d^3}{d^3 + 1}$$



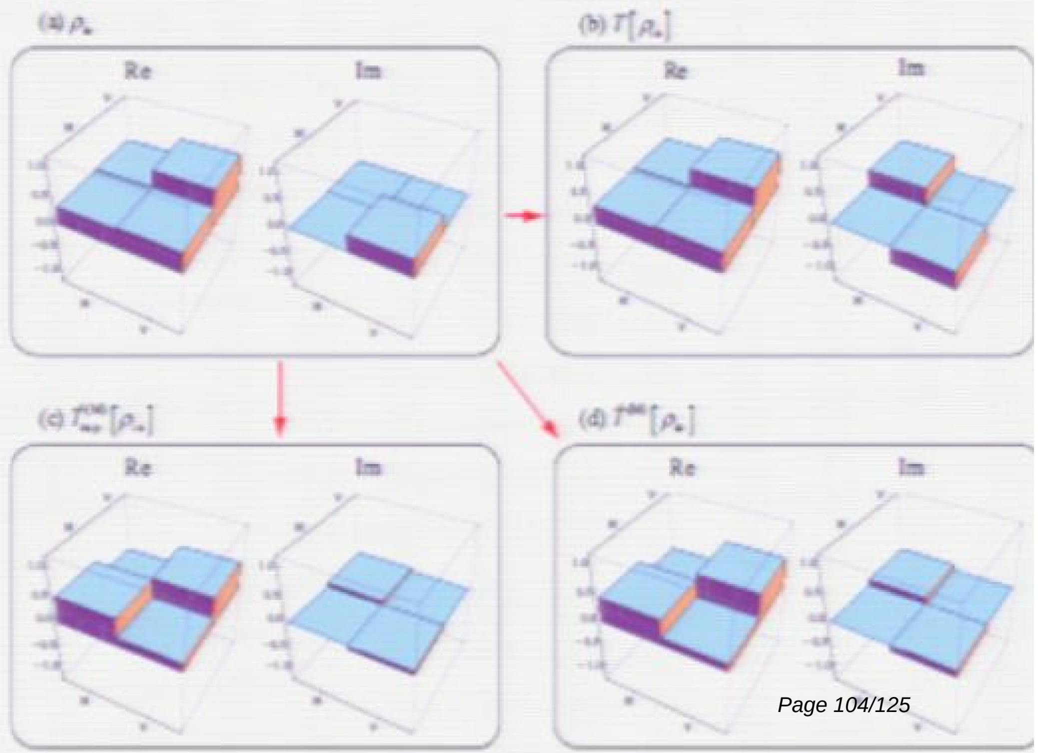


SPA of Partial Transpose can be implemented by Single-copy level measurement



$$\overline{T} = \frac{1}{3}T + \frac{2}{3}I$$

$$F = 0.99$$



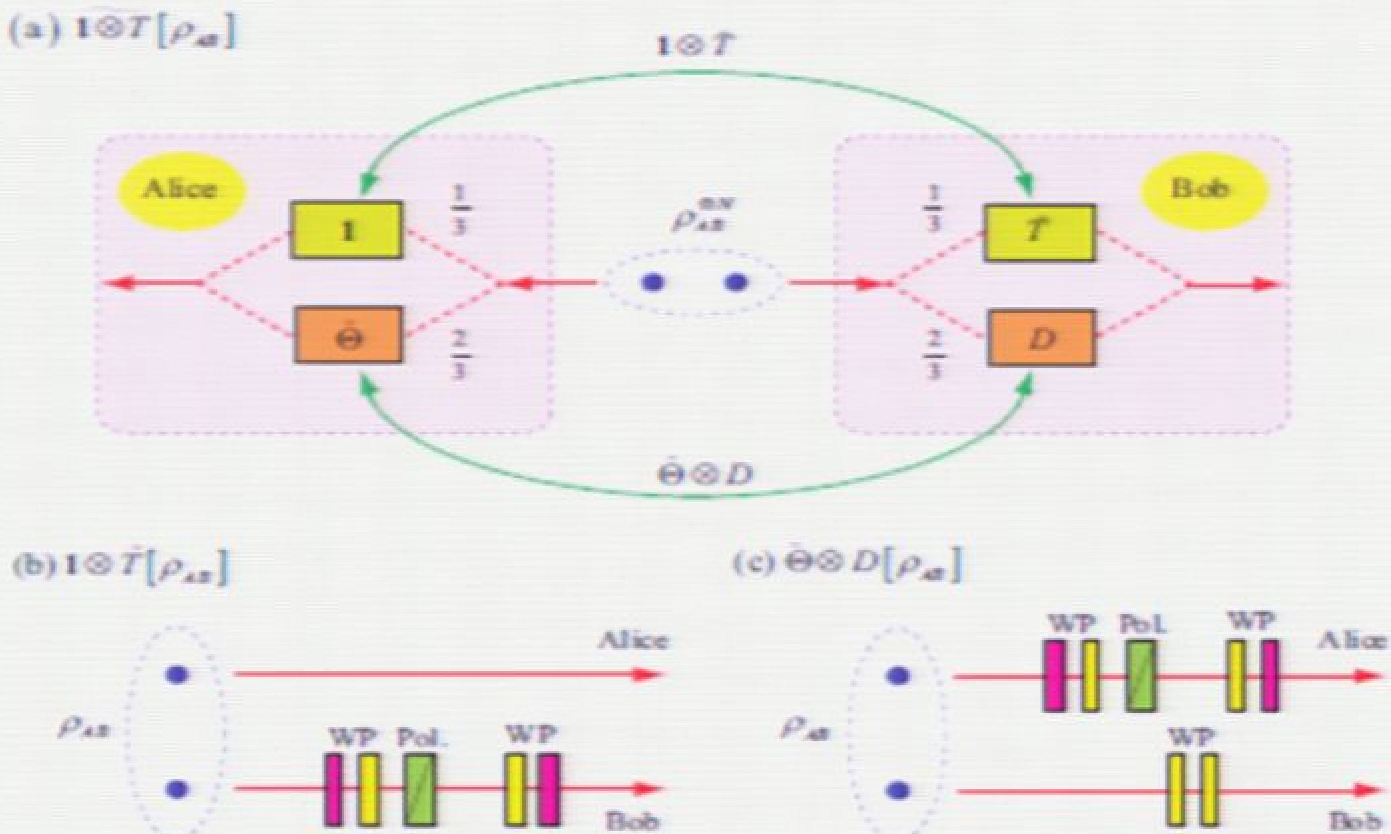
$$\tilde{T}[\sigma] = \sum_{k=1}^{4} \mathrm{tr}\left[\frac{1}{2}|v_k^*\rangle\langle v_k^*|\,\sigma\right]|v_k\rangle\langle v_k|.$$

$$|v_1\rangle \propto |0\rangle + \frac{ie^{i\pi 2/3}}{i+e^{-i\pi 2/3}}|1\rangle,\quad |v_2\rangle \propto |0\rangle - \frac{ie^{i\pi 2/3}}{i-e^{-i\pi 2/3}}|1\rangle,$$

$$|v_3\rangle \propto |0\rangle + \frac{ie^{i\pi 2/3}}{i-e^{-i\pi 2/3}}|1\rangle,\quad |v_4\rangle \propto |0\rangle - \frac{ie^{i\pi 2/3}}{i+e^{-i\pi 2/3}}|1\rangle.$$

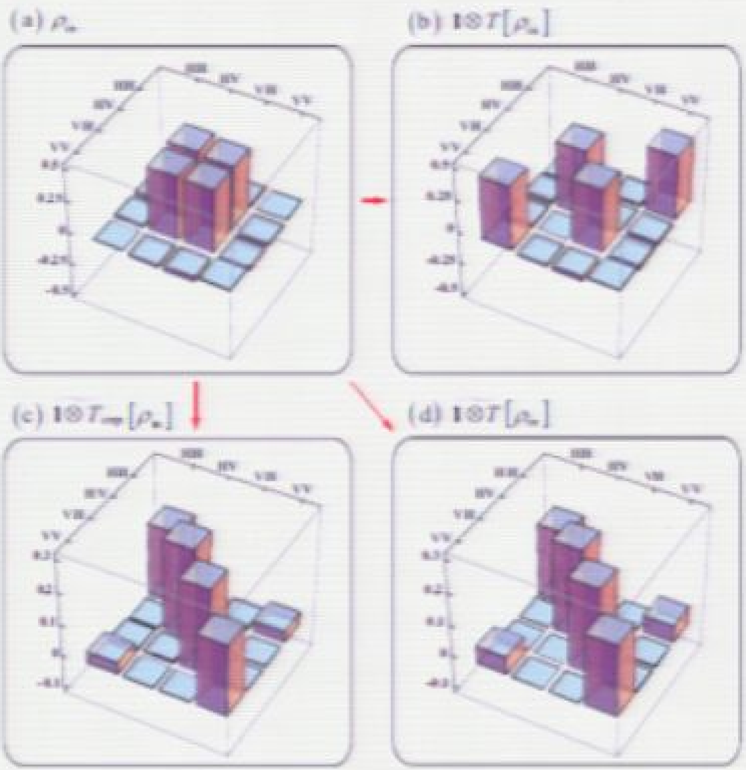
$$\overline{I \otimes T} = \frac{1}{9} I \otimes T + \frac{2}{9} \mathrm{I} \otimes \mathrm{I} = \frac{1}{3} I \otimes \overline{T} + \frac{2}{3} \overline{\Theta} \otimes D$$

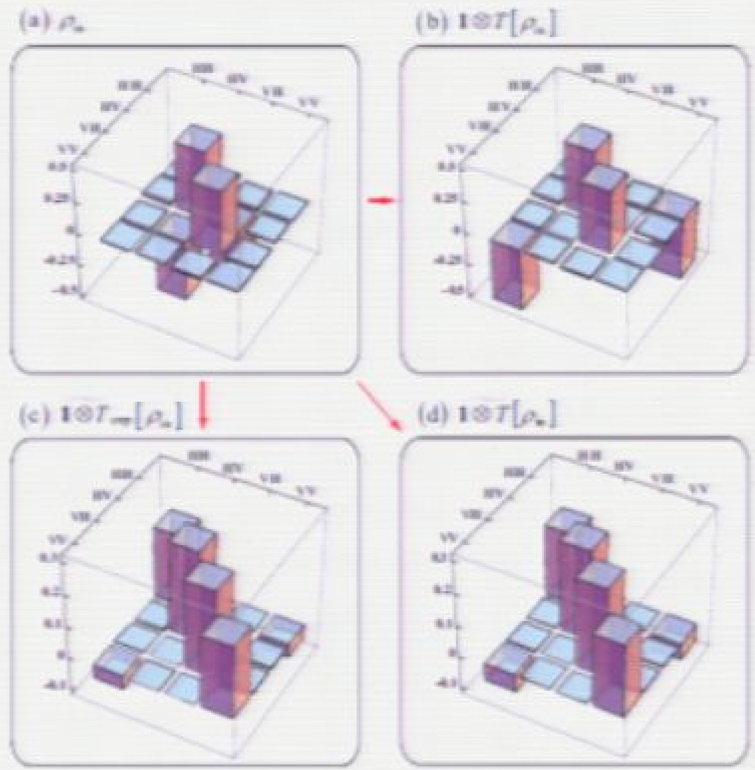
$$\overline{T} = \sigma_y \overline{\Theta} \sigma_y$$



C. M. Alves at al. PRA 68 032306 (2003)

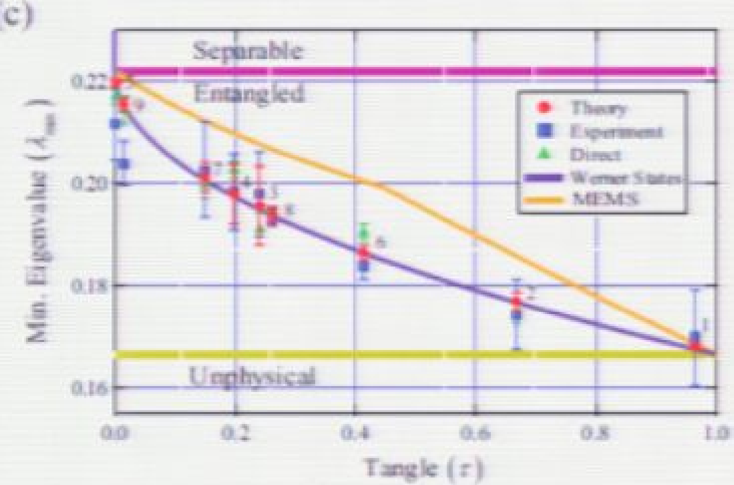
$|\psi^{+}\rangle$ $|\psi^{-}\rangle$



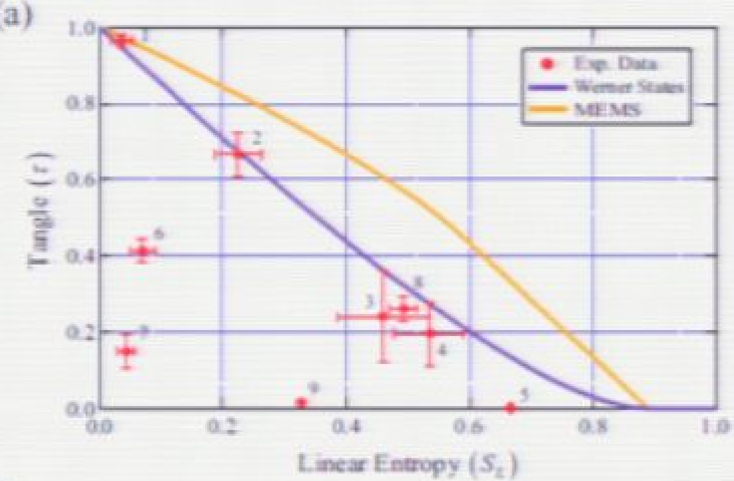


$F = 0.99$ $F = 0.99$

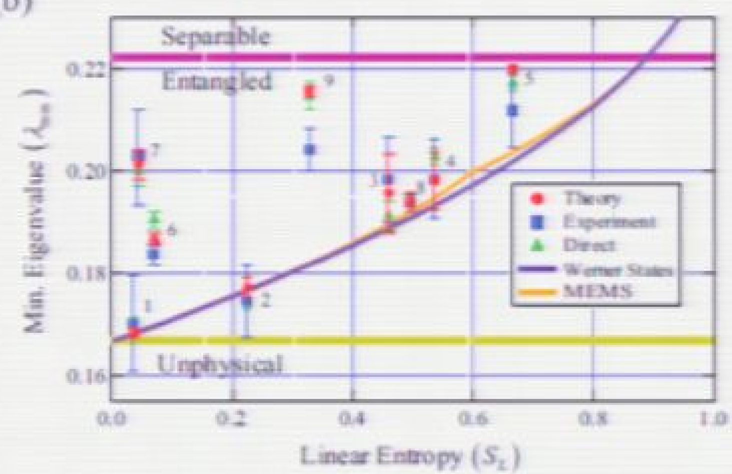
(c)



(a)



(b)



The following quantum states are considered in the experiment: $\rho(p,\alpha) = (1-p)\left|\psi\right\rangle\left\langle\psi\right| + p\left|\psi^{\perp}\right\rangle\left\langle\psi^{\perp}\right|$ where $|\psi\rangle = \alpha|01\rangle - \sqrt{1-|\alpha|^2}|10\rangle$. We generated 9 different quantum states in the following, where the number (p,α) are identified by quantum state tomography.

(p,α) $(0, 2^{-1/2})$ $(0.12, 2^{-1/2})$ $(0.25, 2^{-1/2})$ $(0.3, 2^{-1/2})$ $(0.51, 2^{-1/2})$ $(0, 0.92)$ $(0, 0.97)$ $(0.37, 0.86)$ $(0.42, 0.92)$

Evidence to the conjecture: Reduction map M Horodecki, P Horodecki, PRA 59 4206 (1999)

$$\Lambda_R[\rho] = \frac{1}{d-1}(tr[\rho]I - \rho)$$

$\overline{\Lambda}_R$ is entanglement breaking

Evidences to the conjecture: Non-decomposable maps

i) Choi's map $3 \otimes 3$ M D Choi, J. Operator Theory, 4 271 (1980)

$$\Lambda_C[\rho] = \frac{1}{2}(-\rho + \sum_{j=0}^{2} \rho_{jj}(2|j\rangle\langle j| + |j-1\rangle\langle j-1|))$$

$\overline{\Lambda}_C$ is entanglement breaking

ii) Breuer-Hall Map H.P. Breuer, PRL 97 080501 (2006), W. Hall, JPA 39 14119 (2006)

$$\Lambda_{BH}[\rho] = \frac{1}{d-2}(I tr[\rho] - \rho - U\rho^T U^+) \qquad U^T = -U$$

$\overline{\Lambda}_{BH}$ is entanglement breaking

Conclusion: SPA of the those maps are entanglement breaking

Application: SPA of the those maps can be measurement and state preparation

Evidence to the conjecture: UPB map C Bennett D DiVicenzo T Mor P Shor J Smolin B Terhal, PRL 82 5385 (199

UPB state $I \otimes \Lambda[\phi^+] = \frac{1}{d^2 - n}(\mathrm{I} \otimes \mathrm{I} - \sum_{j=1}^{n} |v_j\rangle\langle v_j|)$

UPB map $I \otimes \Lambda_{UPB}[\phi^+] = \frac{1}{n - \varepsilon d^2}(\sum_{j=1}^{n} |v_j\rangle\langle v_j| - \varepsilon \mathrm{I} \otimes \mathrm{I})$

SPAed UPB map

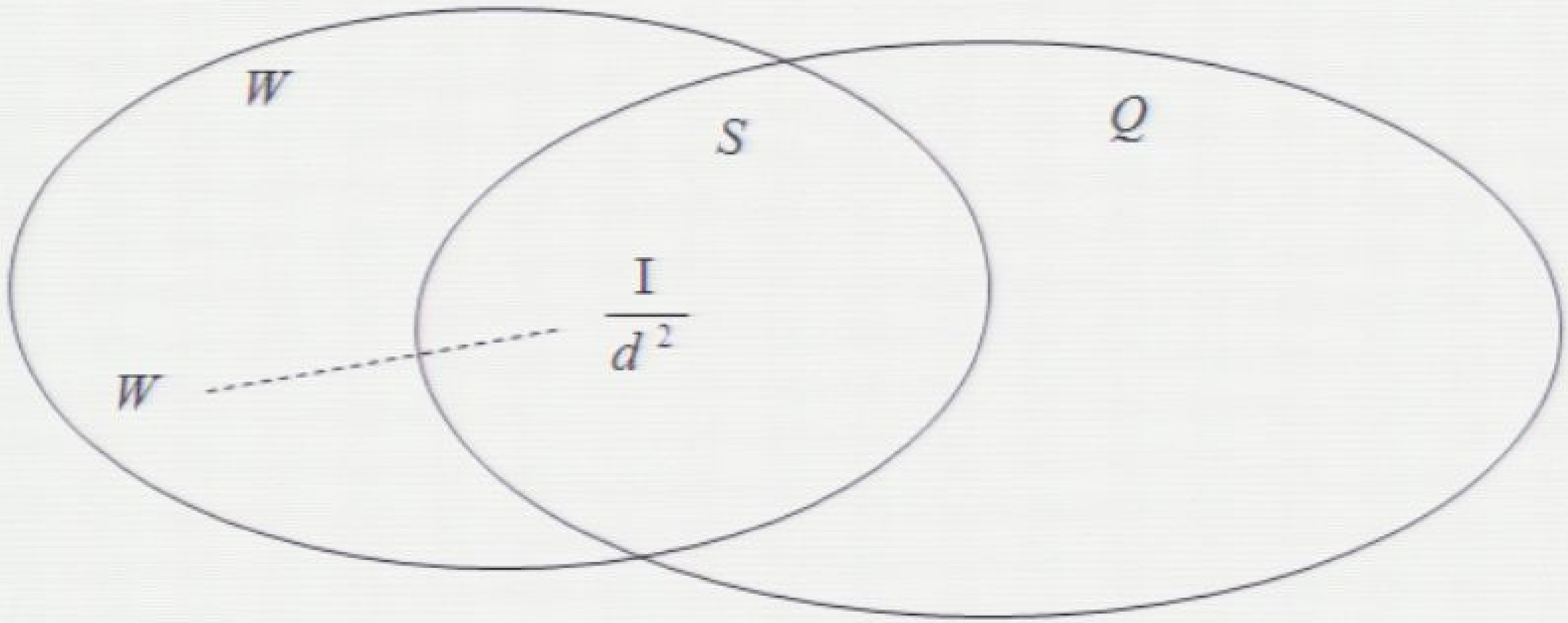
$$I \otimes \overline{\Lambda}_{UPB}[\phi^+] = p\frac{\mathrm{I} \otimes \mathrm{I}}{d^2} + (1-p)\frac{1}{n - \varepsilon d^2}(\sum_{j=1}^{n} |v_j\rangle\langle v_j| - \varepsilon \mathrm{I} \otimes \mathrm{I})$$

$$= \frac{1}{n - \varepsilon d^2}(\frac{np - \varepsilon d^2}{d^2}\mathrm{I} \otimes \mathrm{I} + (1-p)\sum_{j=1}^{n} |v_j\rangle\langle v_j|) \geq 0$$

Conclusion: SPA of the UPB map is entanglement breaking

Application: SPA of the UPB map can be measurement and state preparation

Implications to the geometry of the set of entangled and separable states



If the conjecture is true, it would mean that optimal positive maps (witnesses) enter the area of physical states, when adding white noise, via the area of separable states.

Conclusion

Quatnum vs. Classical correlations

- -Multipartite cases
- -Bipartite cases

Relations among fundamental no-go theorems

- -No-signaling
- -No perfect quantum cloning
- -No perfect quantum state estimation

Fundamental theorem in entanglement detection

- -Conjecture : Approximations to positive maps are entanglement-breaking
- -An approximate partial transpose and application to entanglement detection

Conclusion

Quatnum vs. Classical correlations

-Multipartite cases

-Bipartite cases

Relations among fundamental no-go theorems

-No-signaling

-No perfect quantum cloning

-No perfect quantum state estimation

Fundamental theorem in entanglement detection

-Conjecture : Approximations to positive maps are entanglement-breaking

-An approximate partial transpose and application to entanglement detection

Thanks for the attention !

Thanks for the attention !

Thanks for the attention !

Thanks for the attention !

Thanks for the attention !

Thanks for the attention !

Thanks for the attention !

Thanks for the attention !

Thanks for the attention !

Thanks for the attention !

Thanks for the attention !

Thanks for the attention !

Conclusion

Quatnum vs. Classical correlations

-Multipartite cases

-Bipartite cases

Relations among fundamental no-go theorems

-No-signaling

-No perfect quantum cloning

-No perfect quantum state estimation

Fundamental theorem in entanglement detection

-Conjecture : Approximations to positive maps are entanglement-breaking

-An approximate partial transpose and application to entanglement detection