Title: Quantum Mechanics with Extended Probabilities

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Abstract: We present a new formulation of quantum mechanics for closed systems like the universe using an extension of familiar probability theory that incorporates negative probabilities. Probabilities must be positive for alternative histories that are the basis of settleable bets. However, quantum mechanics describes alternative histories are not the basis for settleable bets as in the two-slit experiment. These alternatives can be assigned extended probabilities that are sometimes negative. We will compare this with the decoherent (consistent) histories formulation of quantum theory. The prospects for using this formulation as a starting point for testable alternatives to quantum theory or further generalizations of it will be briefly discussed.

Quantum Mechanics with Extended Probabilities

Jim Hartle, UCSB

Perimeter Institute, March 7, 2011

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Probabilities as Instructions for Betting

De Finetti

Asserting that the probability of an event A is p means that, if a bookie offers a payoff S if A occurs, you will put up pS and consider it a fair bet. (S can be negative.)

All the rules of probability follow from the requirement that it not be possible for the bookie to pick payoffs such that you always lose. (A Dutch book.)

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Why Probabilities are Positive

You bet on events A and $\bar{A}$ (not A)
with probabilities p_A and $p_{\bar{A}}$
and stakes S_A and $S_{\bar{A}}$.
Your gains G_A and $G_{\bar{A}}$ are:

$$G_A = S_A - p_A S_A - p_{\bar{A}} S_{\bar{A}}$$

$$G_{\bar{A}} = S_{\bar{A}} - p_{\bar{A}} S_{\bar{A}} - p_A S_A$$

Suppose $p_A < 0$.

The bookie picks $S_A < 0,\ S_{\bar{A}} = 0$.

$$G_A = (1 - p_A) S_A$$

$$G_{\bar{A}} = -p_A S_A$$

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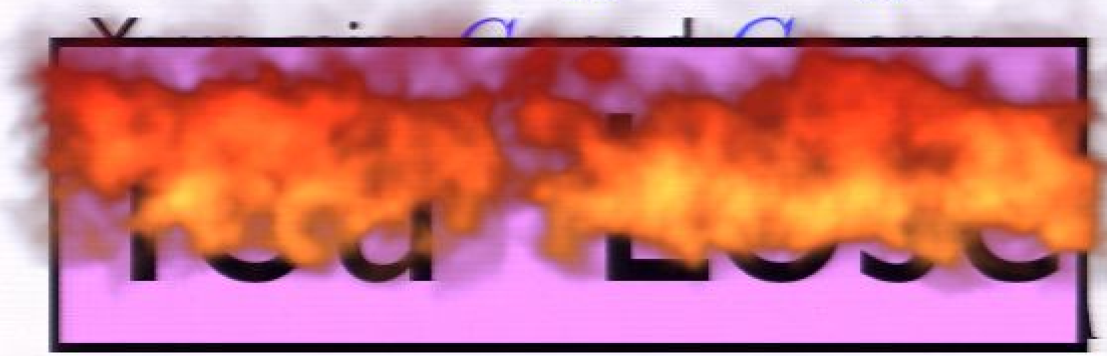

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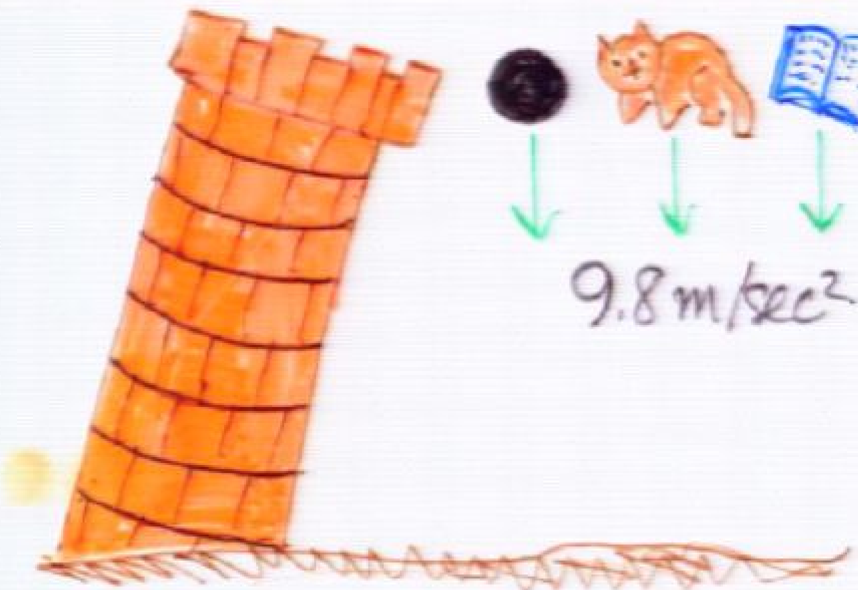

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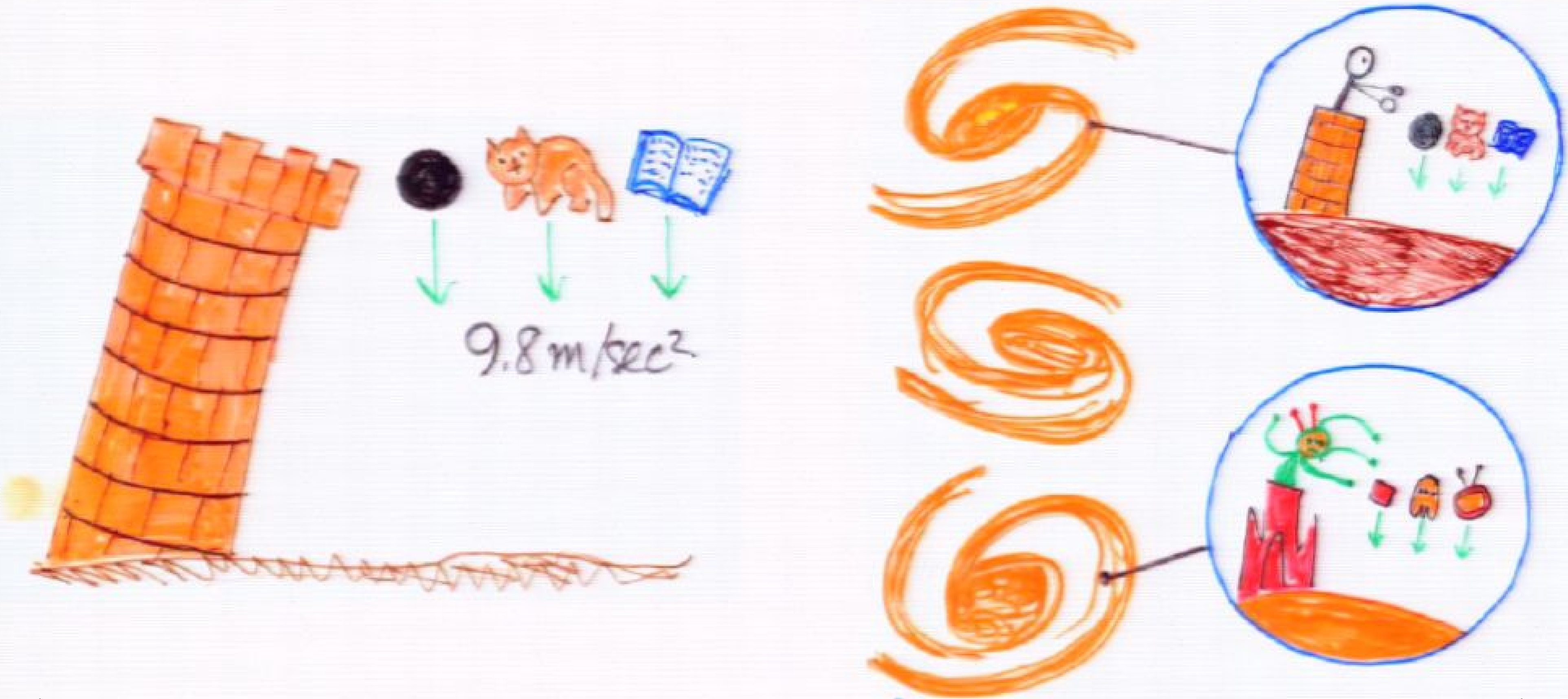
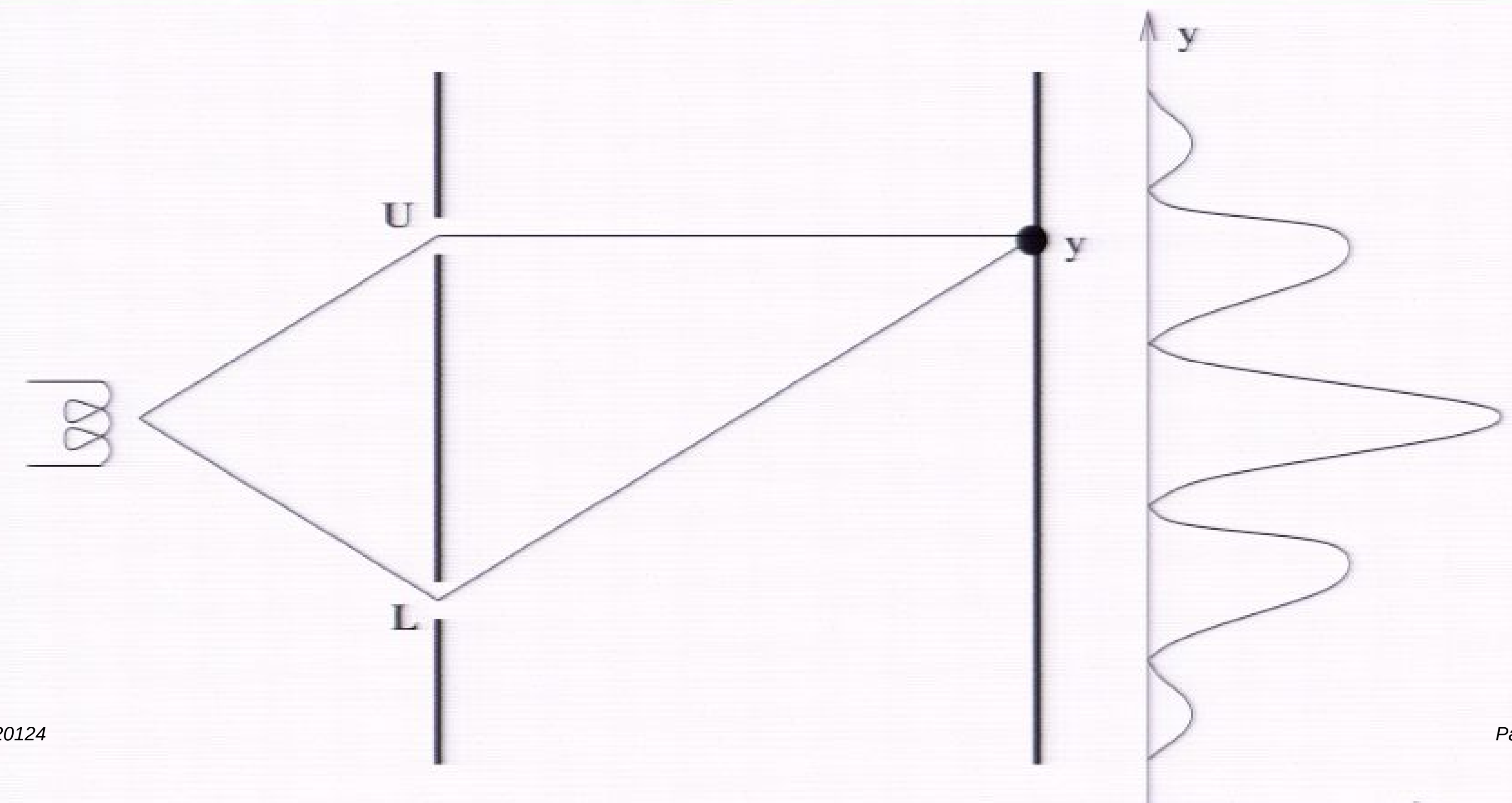
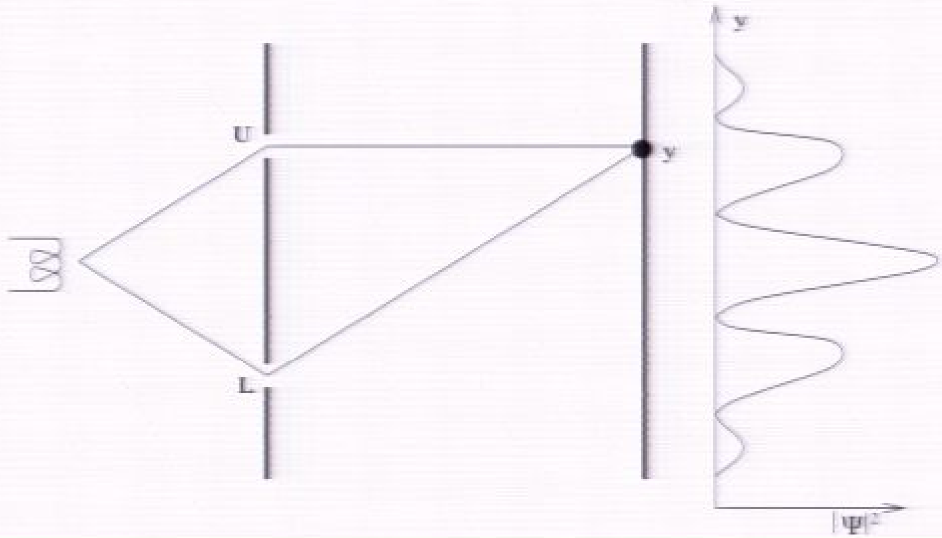


The de Finetti derivation of the rules of probability assumes that all alternatives described by the theory are the basis of a settleable bet.

Quantum mechanics describes sets of alternative histories that are not the basis of settleable bets.



Non-Settleable Alternatives Present a Choice



Assign probabilities only to alternatives that are settleable in principle (a decoherence condition).

Extend the notion of probability and assign extended probabilities (not necessarily postive) to all alternatives.

Extended Probabilities

Extended probabilities satisfy the usual rules except they are not necessarily positive or less than one for non-settleable bets.

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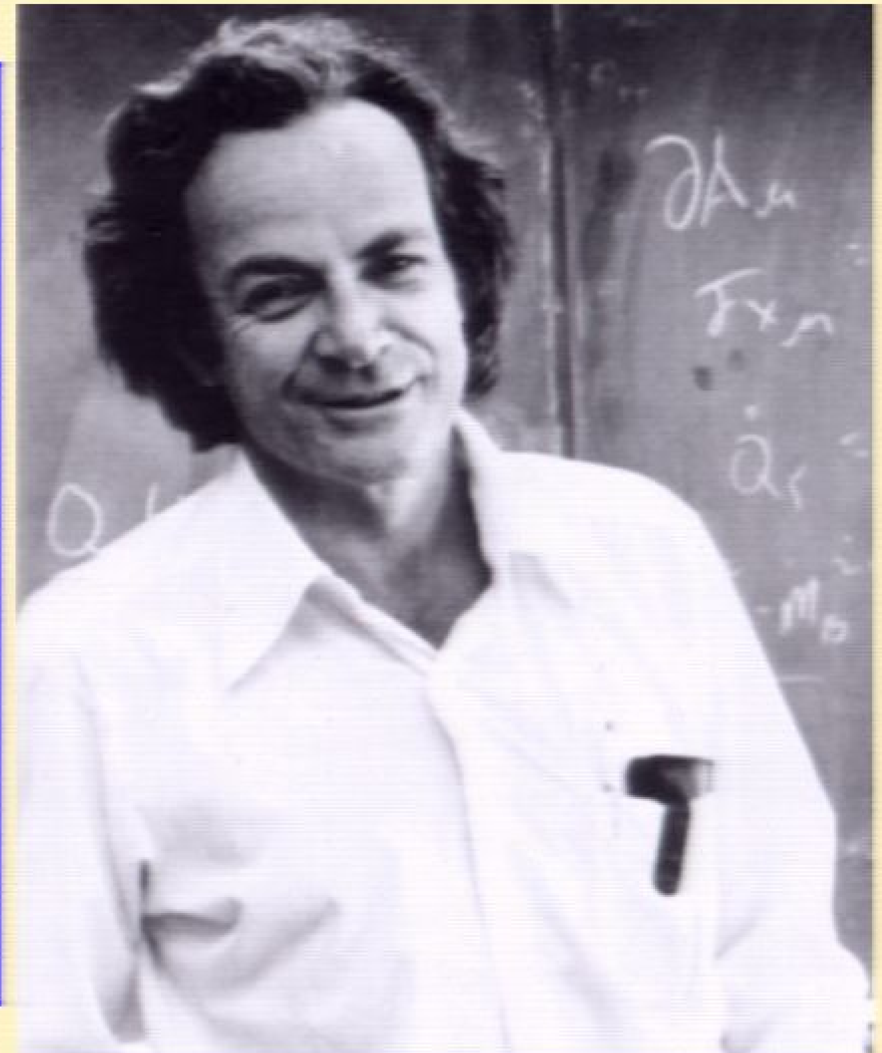
A negative probability or one bigger than one for an alternative is an instruction not to bet --- it can't be settled.

Wigner Distribution

$$w(x,p) = \frac{1}{2\pi}\int d\xi \; \psi^*(x+\frac{\xi}{2}) \; e^{ip\xi/\hbar} \; \psi(x-\frac{\xi}{2})$$

- Not generally positive.
- x and p are not simultaneously measurable and do not correspond to a settleable bet.
- Coarse graining x and/or p leads to positive probabilities and settleable bets.

If a physical theory for calculating probabilities yields a negative probability ... we need not conclude that the theory is incorrect. ... [A] possibility is that the situation for which the probability appears to be negative is not one that can be verified.



A Model Universe in a Box

The most general objective of any quantum theory are the (extended) probabilities for the members of sets of coarse-grained alternative histories of the closed system.

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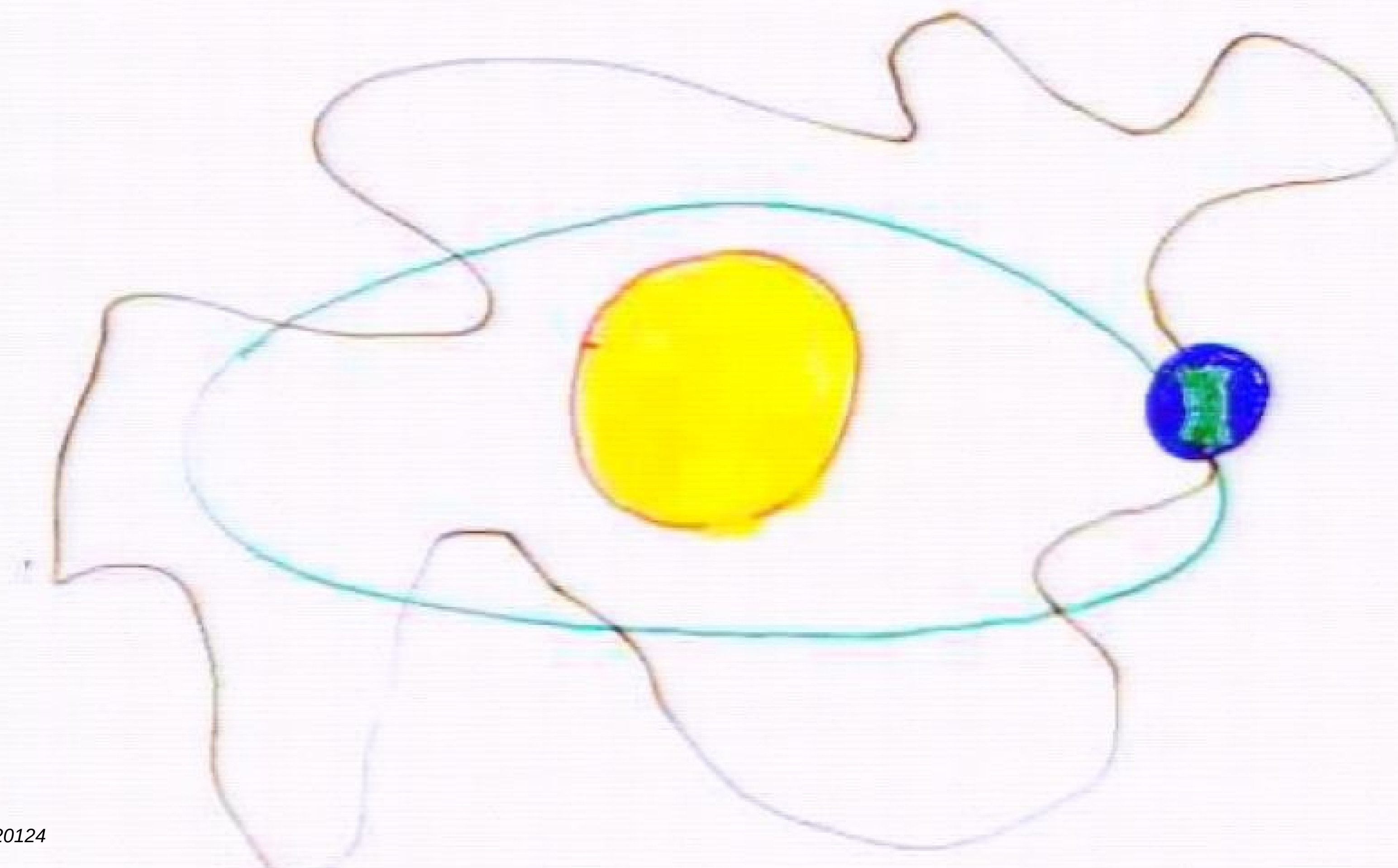
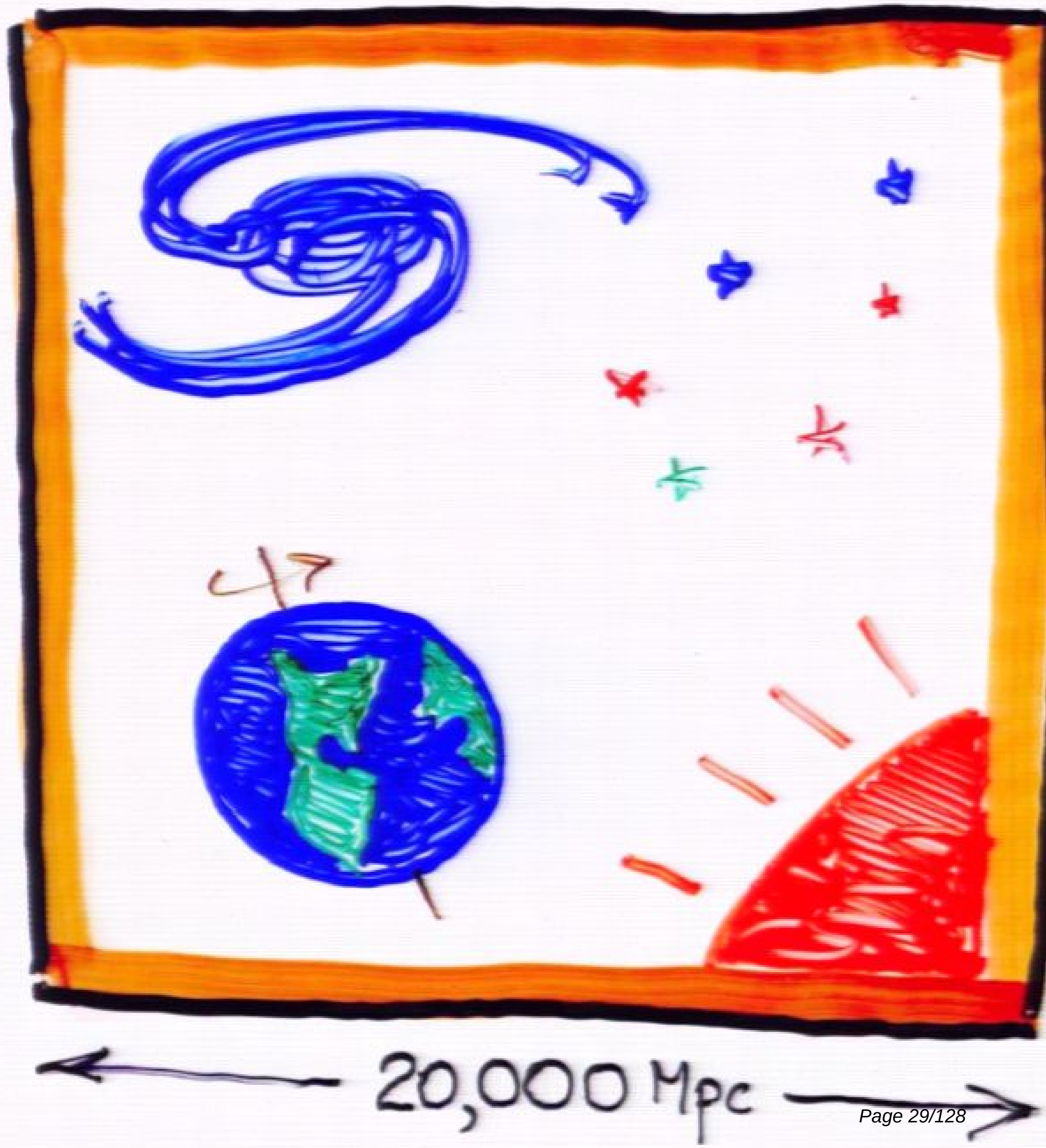


Theoretical Inputs

$$H$$

$$|\Psi\rangle$$



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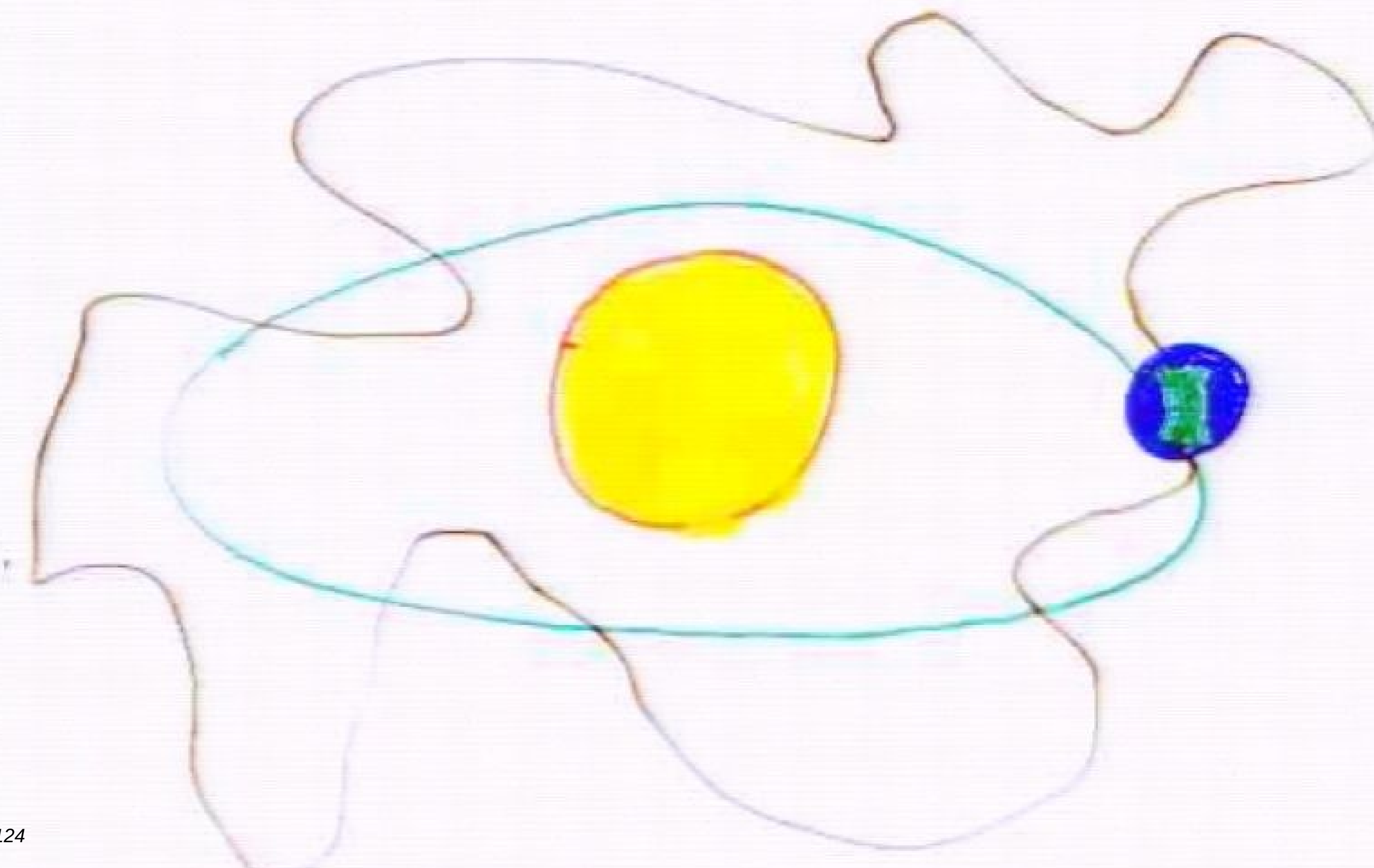

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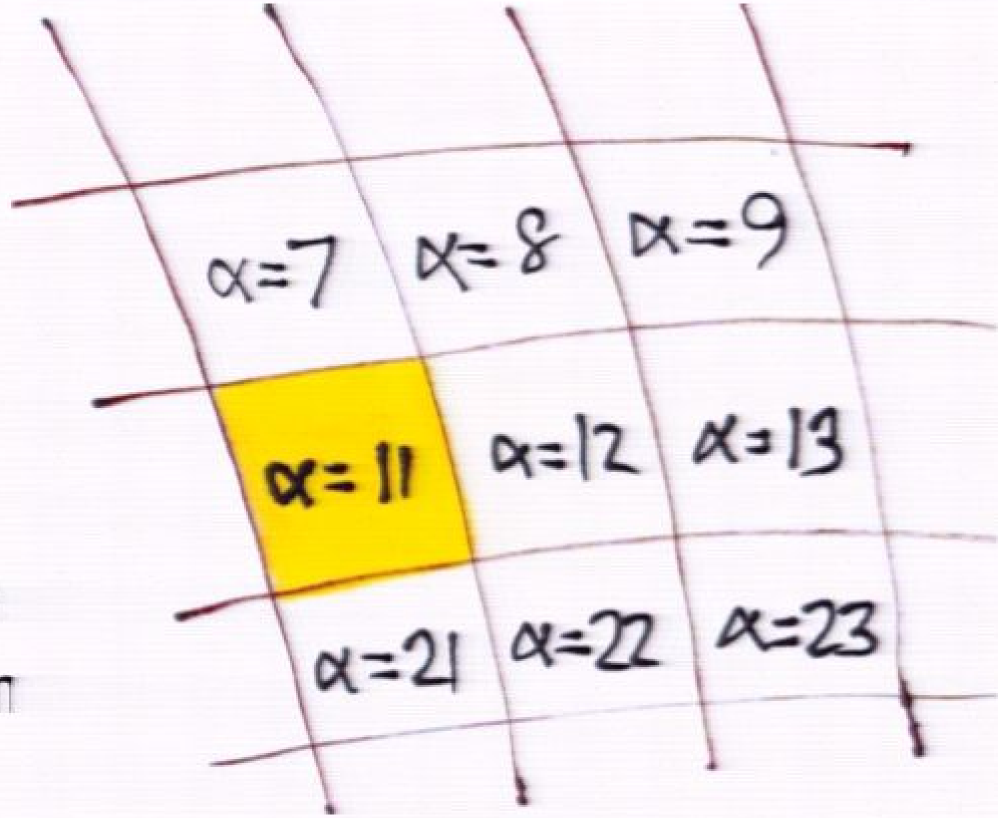
Two things necessary to specify an extended probability quantum theory (EP)

- The description of sets of fine- and coarse-grained alternative histories.
- The prescription for assigning extended probabilities for each member of these sets.

Alternatives at a Moment of Time



Yes/no alternatives at a moment of time are represented by exhaustive sets of exclusive projection operators $\{P_\alpha(t)\}$

$$P_\alpha(t)P_\beta(t) = \delta_{\alpha\beta}P_\alpha(t) \qquad \sum_\alpha P_\alpha(t) = I$$

$$P_\alpha(t) = e^{iHt/\hbar}P_\alpha(0)e^{-iHt/\hbar}$$

One-dimensional P's are fine-grained, others are coarse-grained.

Histories

- Sets of histories are specified by a sequence of sets of alternatives at a series of times $t_1, t_2, \cdots t_n$

$$\{P^1_{\alpha_1}(t_1)\}, \{P^2_{\alpha_2}(t_2)\}, \cdots, \{P^n_{\alpha_n}(t_n)\}$$

- An individual history c_α is a particular sequence of alternatives: $\alpha = (\alpha_1, \alpha_2, \cdots \alpha_n)$ represented by the chain

$$C_\alpha = P^n_{\alpha_n}(t_n) \cdots P^1_{\alpha_1}(t_1)$$

Fine-grained and Coarse-grained.

$$C_\alpha = P^n_{\alpha_n}(t_n)\cdots P^1_{\alpha_1}(t_1)$$

- Fine grained histories a have one-dimensional P's at each and every time (like Feynman paths)
- Coarse graining is partitioning a set of histories $\{c_\alpha\}$ into bigger classes $\{\bar{c}_{\bar{\alpha}}\}$.

$$\bar{c}_{\bar{\alpha}} = \cup_{\alpha\in\bar{\alpha}}\, c_\alpha \qquad\qquad \bar{C}_{\bar{\alpha}} = \sum_{\alpha\in\bar{\alpha}} C_\alpha$$

Extended Probabilities

- Assumed Input: $H \quad |\Psi\rangle$
- Histories: $C_\alpha = P^n_{\alpha_n}(t_n)\cdots P^1_{\alpha_1}(t_1)$

$$p(\alpha) = \mathrm{Re}\langle\Psi|C_\alpha|\Psi\rangle$$

- Not generally positive.
- Agrees with usual QM for single time histories

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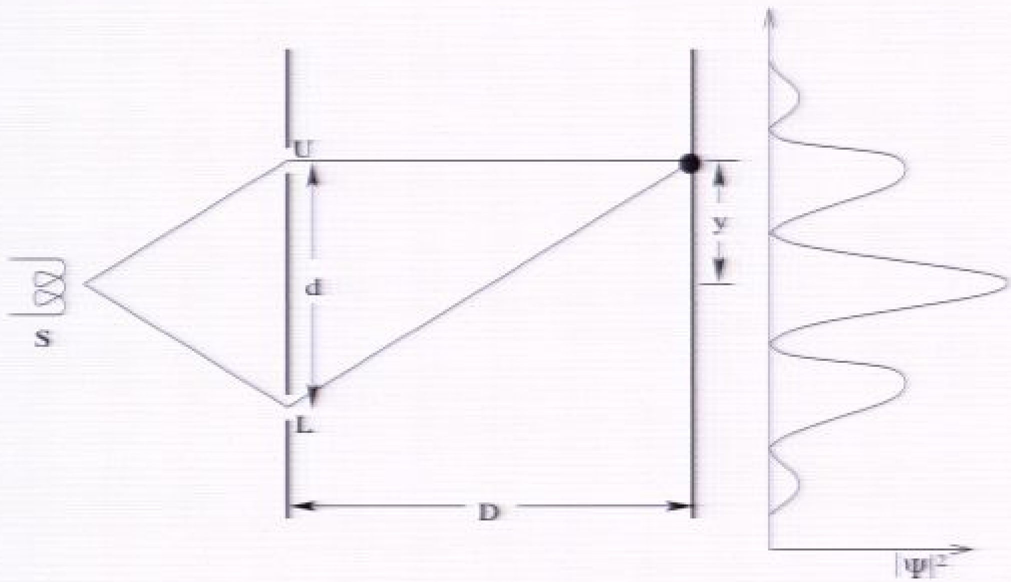
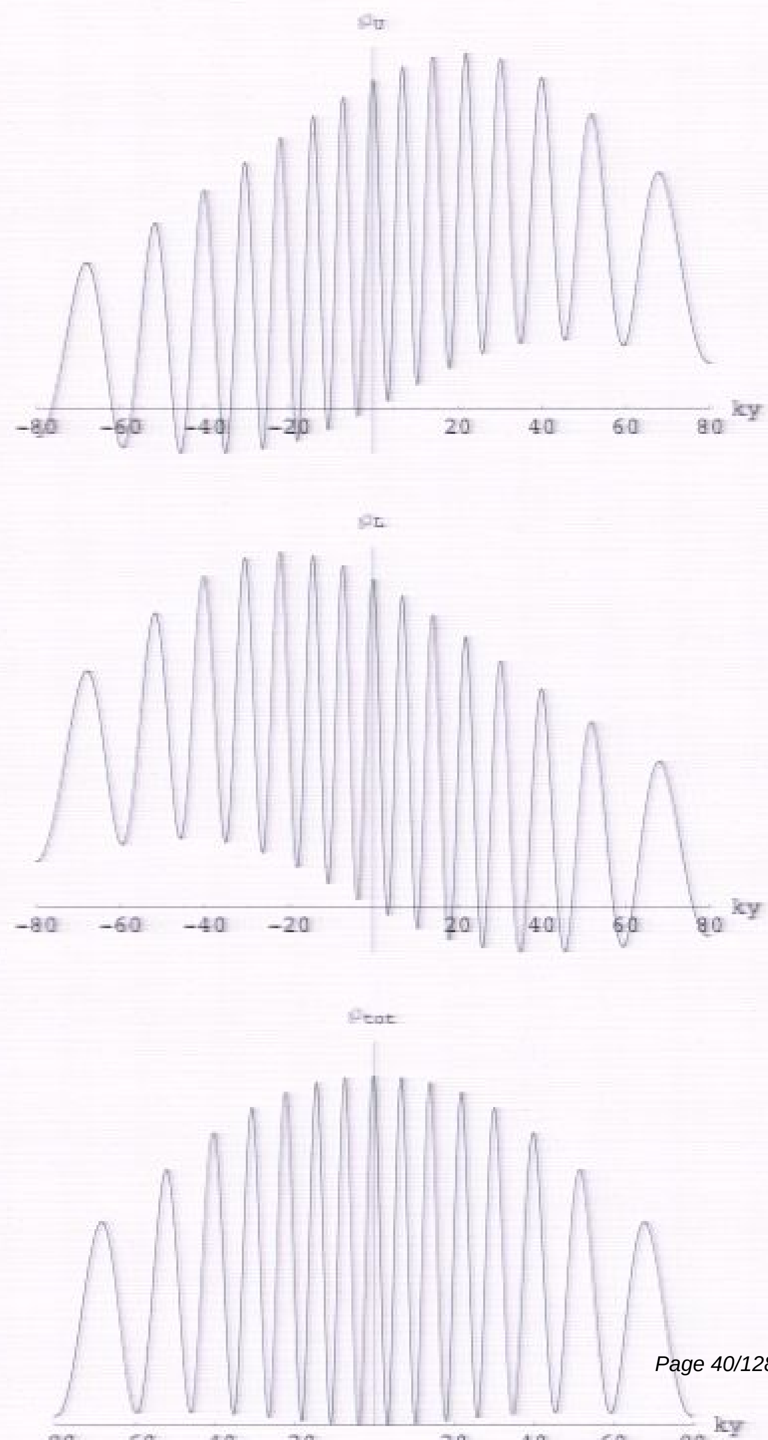
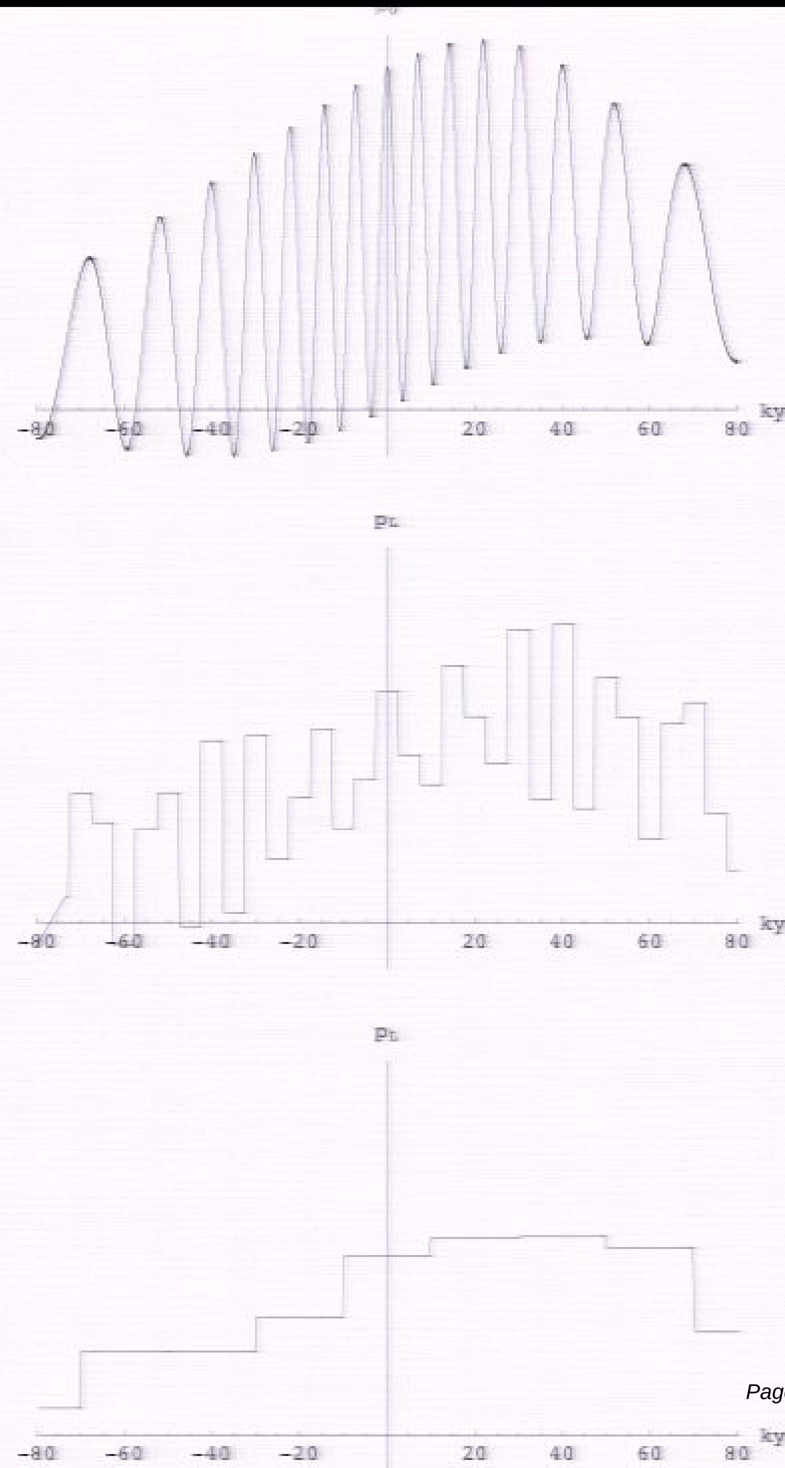
Extended Probabilities for the Two-Slit Expt.





Coarse-Graining and Positive Probabilities



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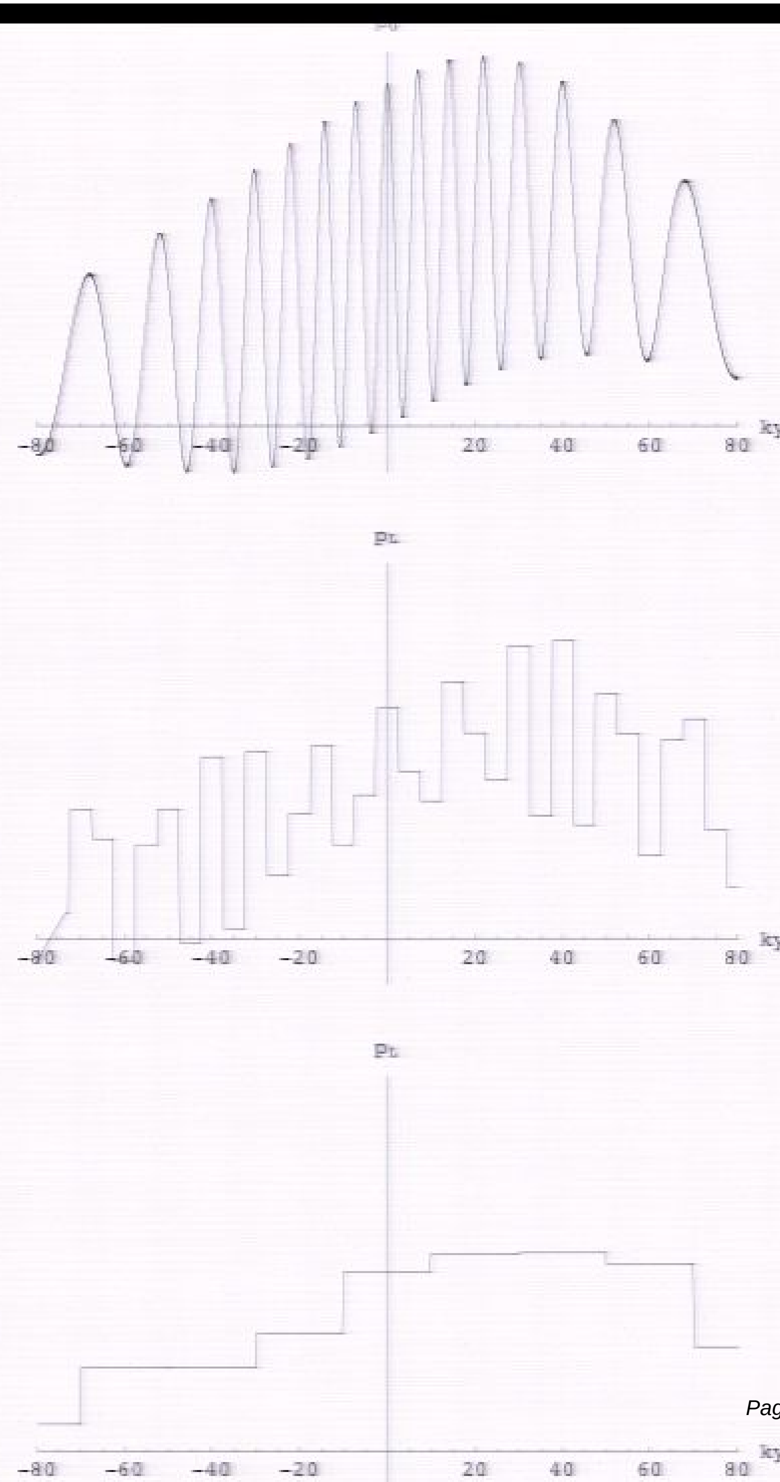

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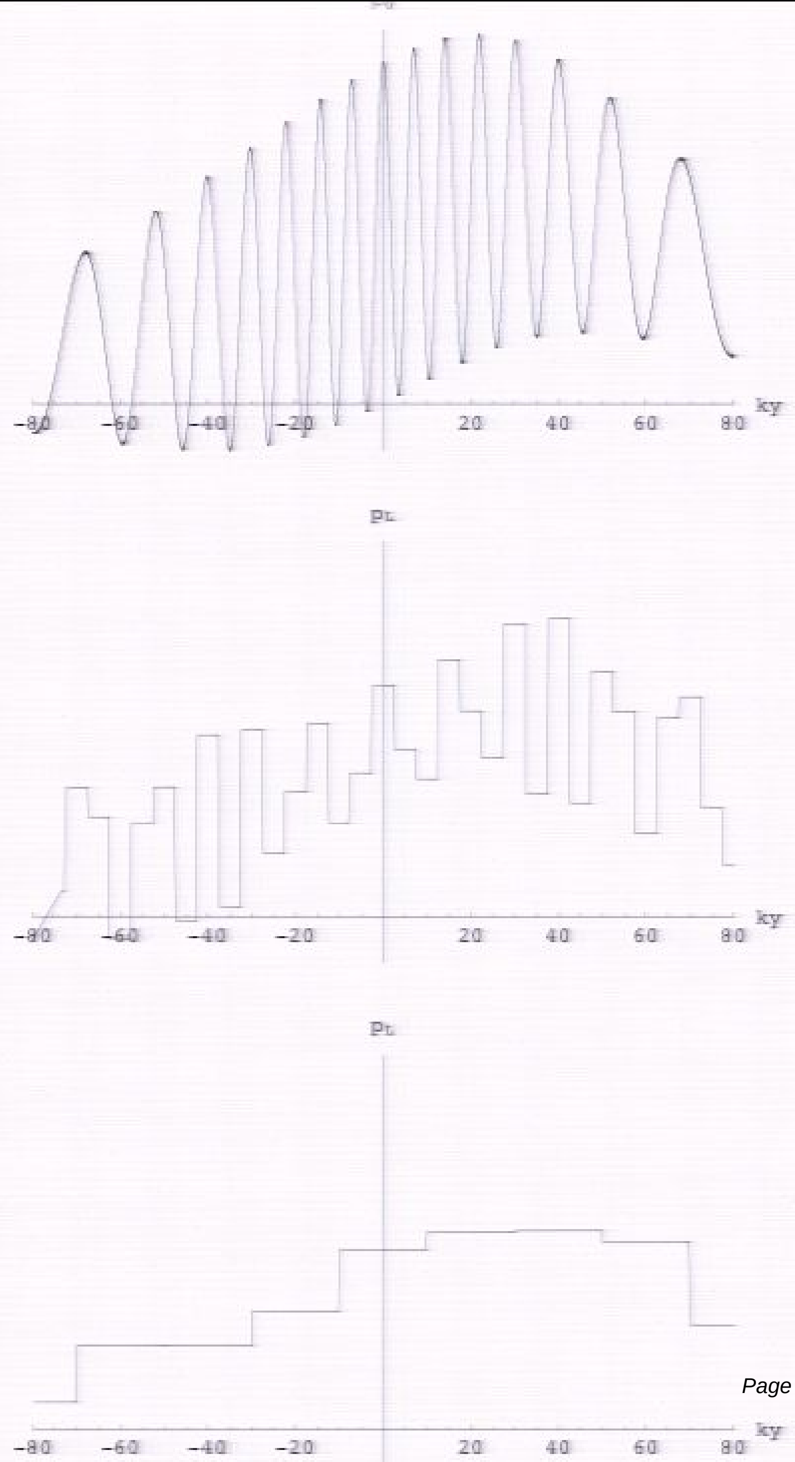

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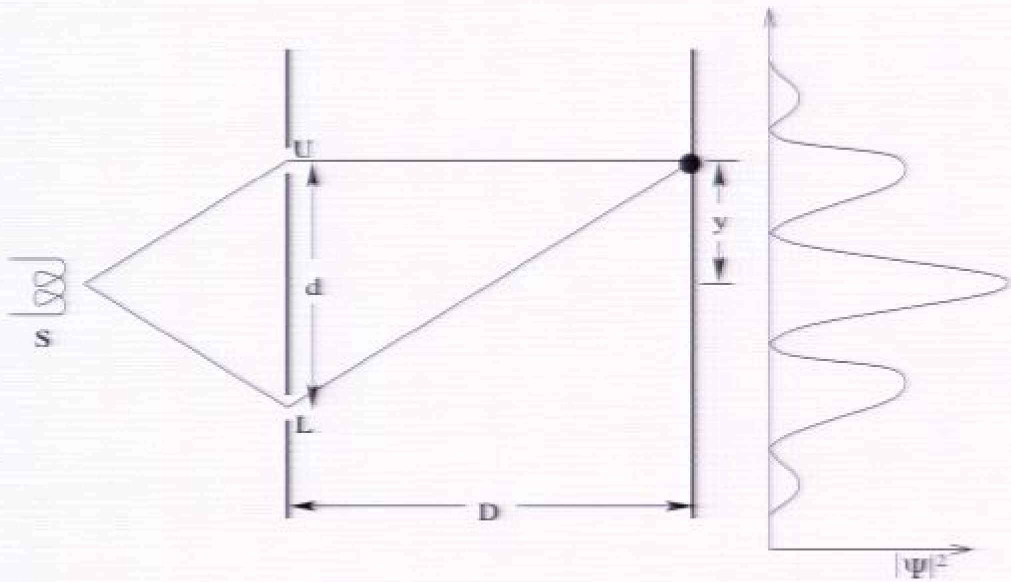

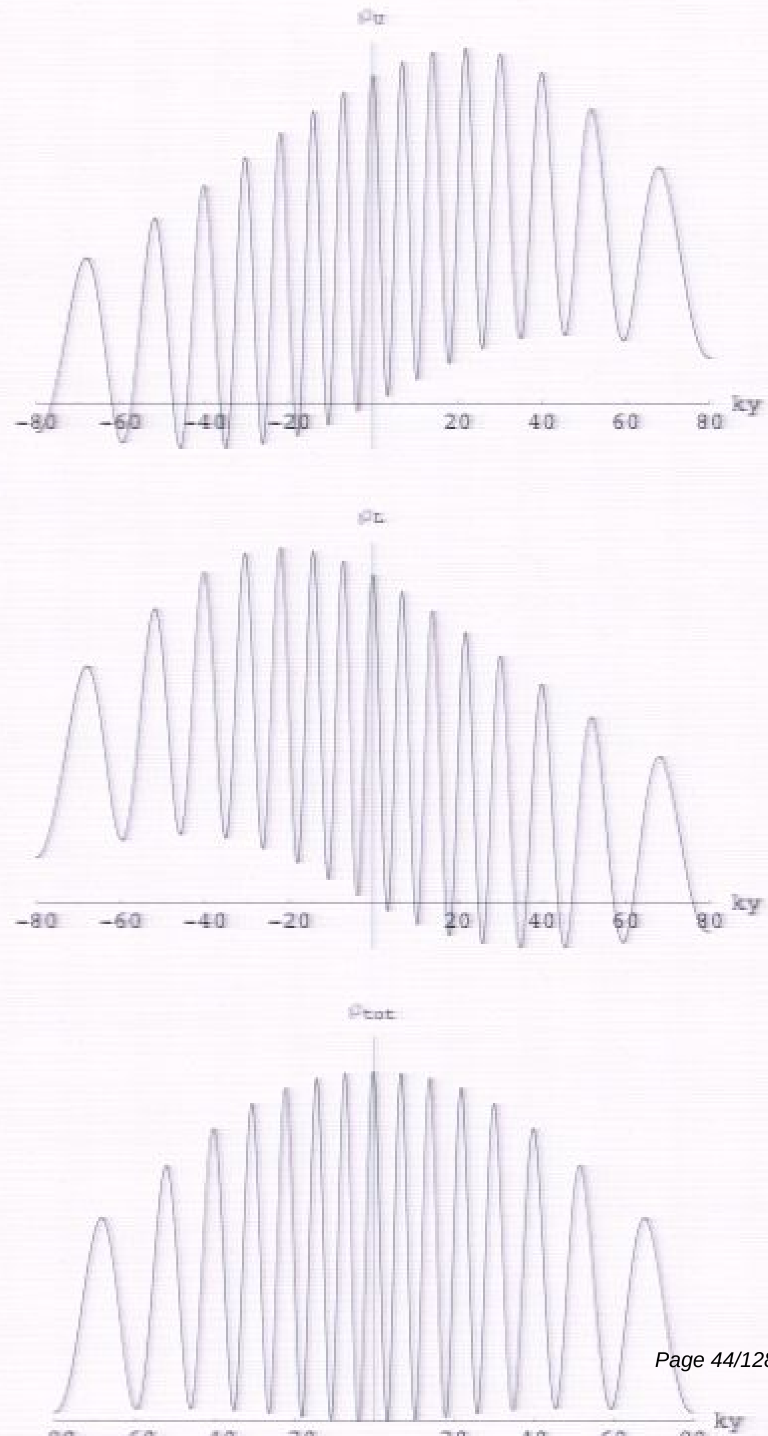

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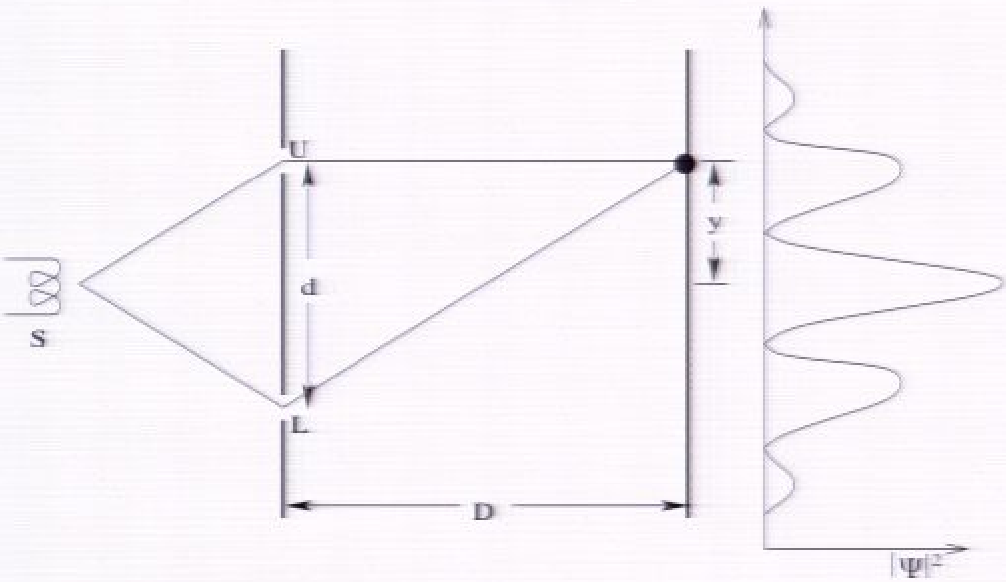
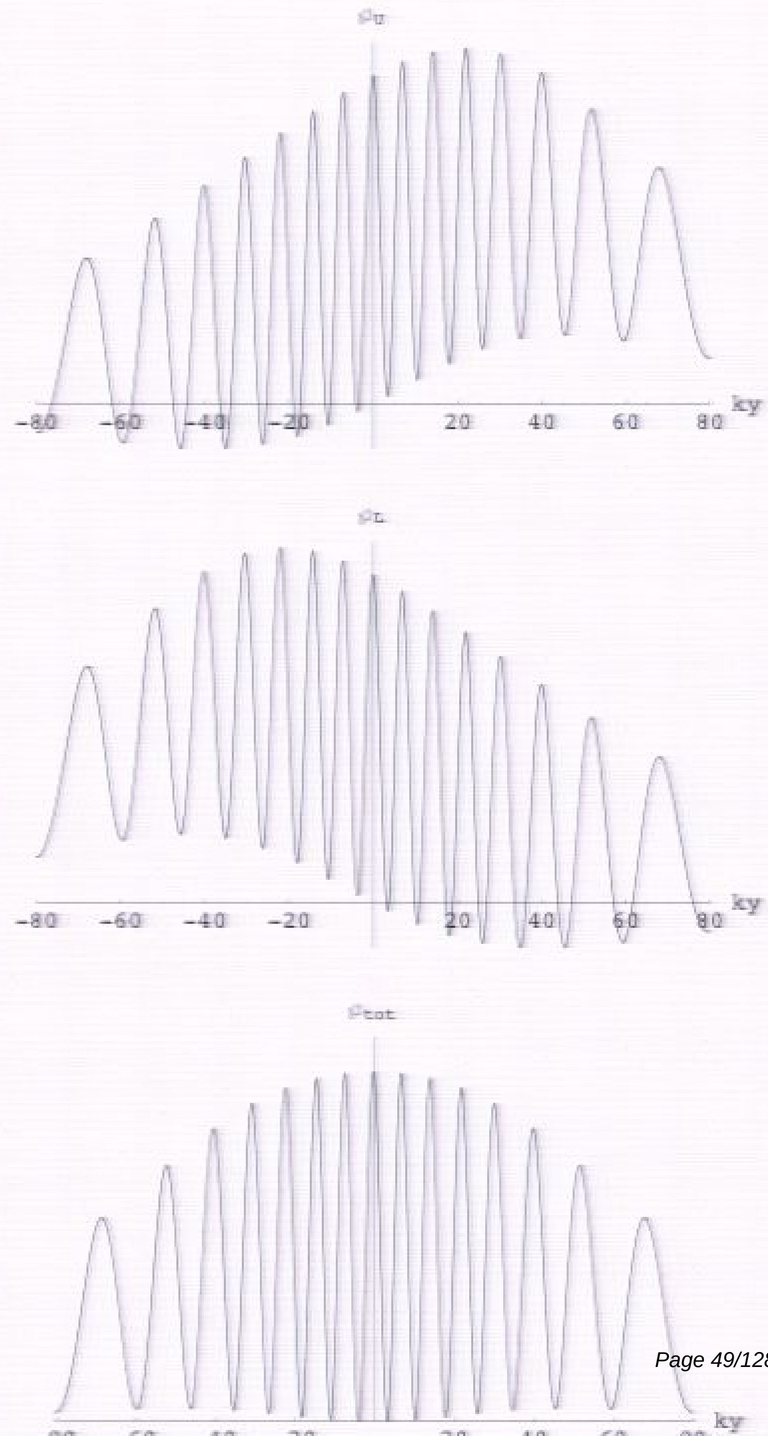
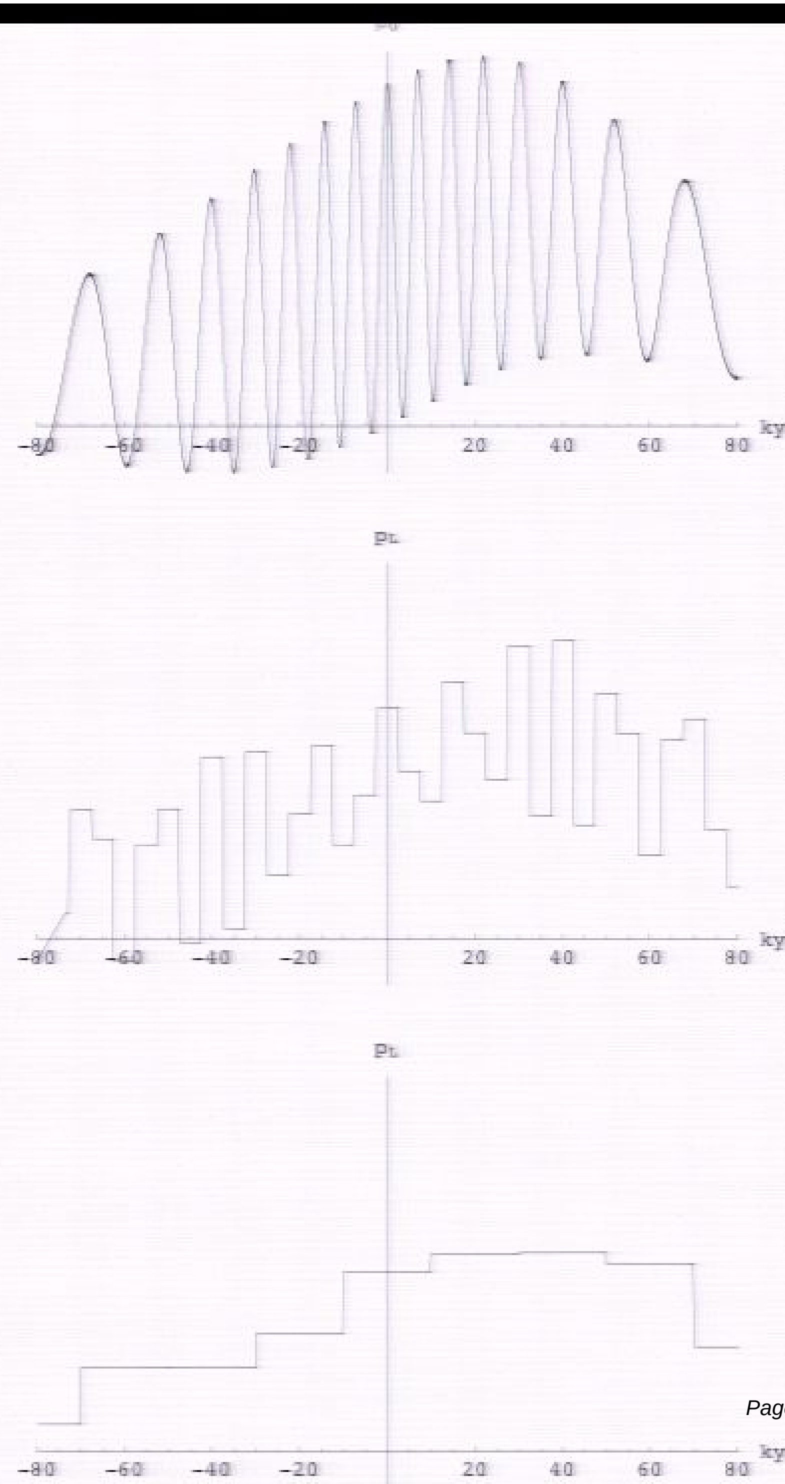




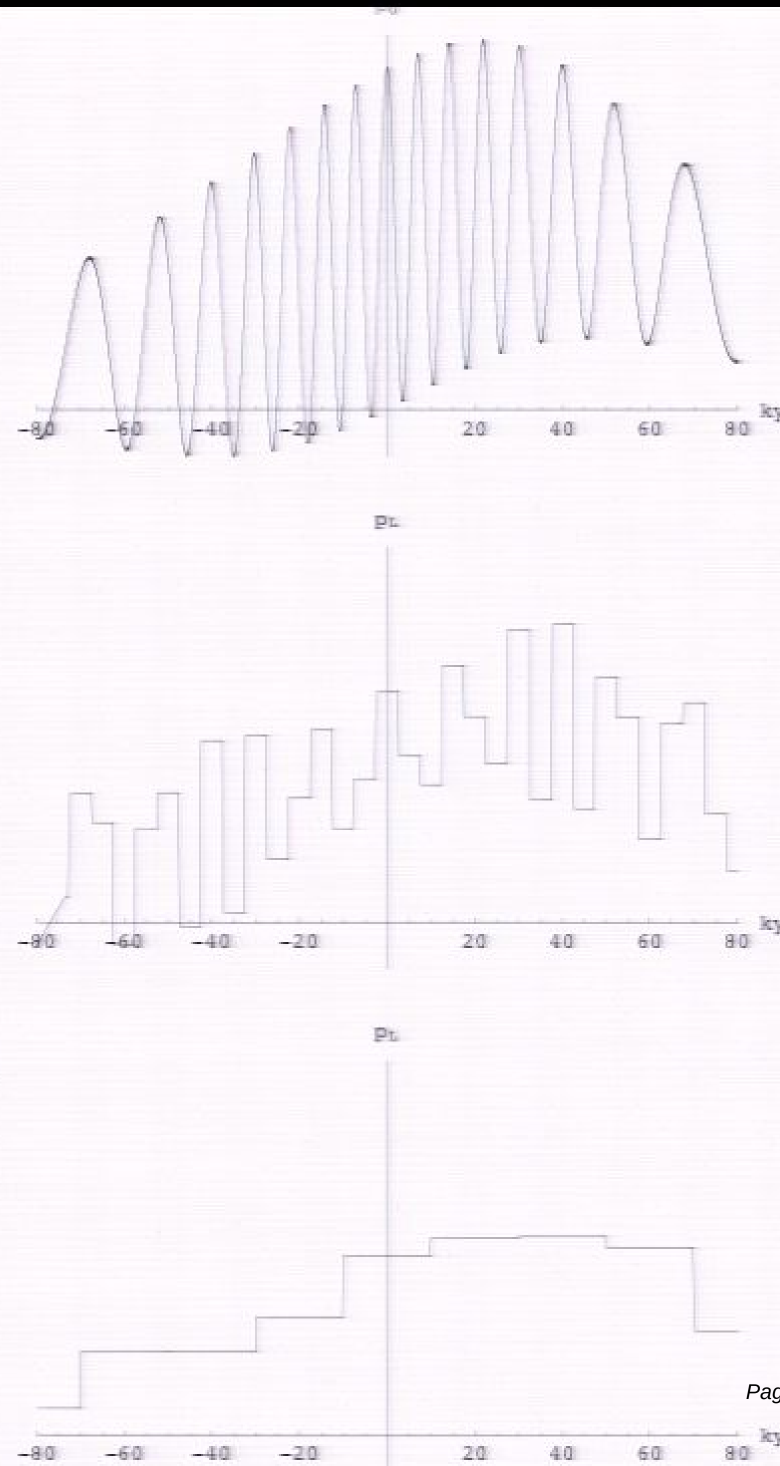
Coarse-Graining and Positive Probabilities



Coarse-Graining and Positive Probabilities

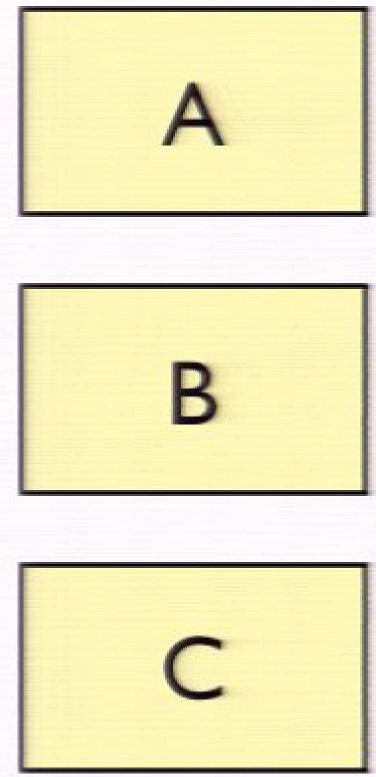
Appropriate coarse-graining can make probabilities positive.

How negative p is is a measure of how much coarse graining is needed.



Aharonov
Vaidman

Three-Box Example



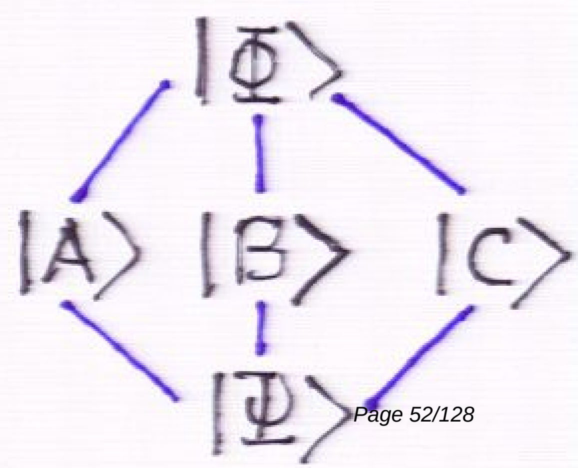
A single particle that can be in one of three boxes A, B, C in states |A>, |B>, |C>. H=0.

Initial State:

$$|\Psi\rangle \equiv \frac{1}{\sqrt{3}}\left(|A\rangle + |B\rangle + |C\rangle\right)$$

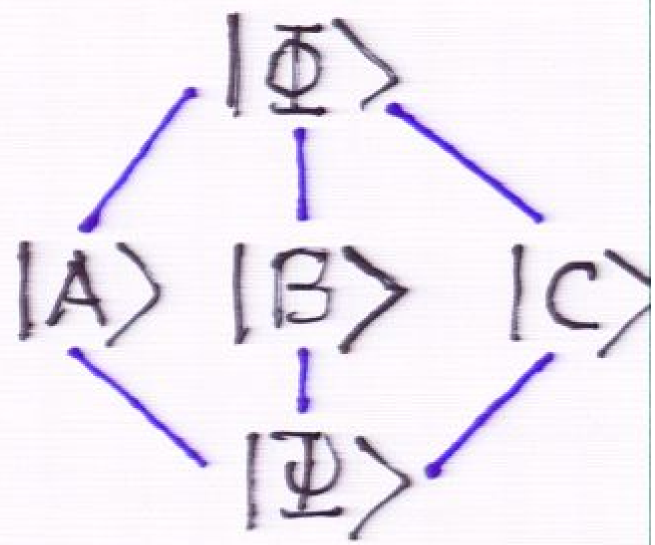
Final State:

$$|\Phi\rangle \equiv \frac{1}{\sqrt{3}}\left(|A\rangle + |B\rangle - |C\rangle\right)$$



Given that the particle is in the final state | Φ >, what is the probability that it was in box A, B, or C at an intermediate time?

Three Box --- Histories



Projections: $P_A = |A\rangle\langle A|, \quad P_{\bar{A}} = I - P_A$

Histories: $P_\Phi P_A, \quad P_\Phi P_B, \quad P_\Phi P_C.$

Extended Probabilities: $p(\Phi, A) = \mathrm{Re}\langle\Psi|P_\Phi P_A|\Psi\rangle$

Result: $p(A|\Phi) = 1, \quad p(B|\Phi) = 1, \quad p(C|\Phi) = -1$

$$|\Psi\rangle \equiv \frac{1}{\sqrt{3}}\left(|A\rangle + |B\rangle + |C\rangle\right) \qquad |\Phi\rangle \equiv \frac{1}{\sqrt{3}}\left(|A\rangle + |B\rangle - |C\rangle\right)$$

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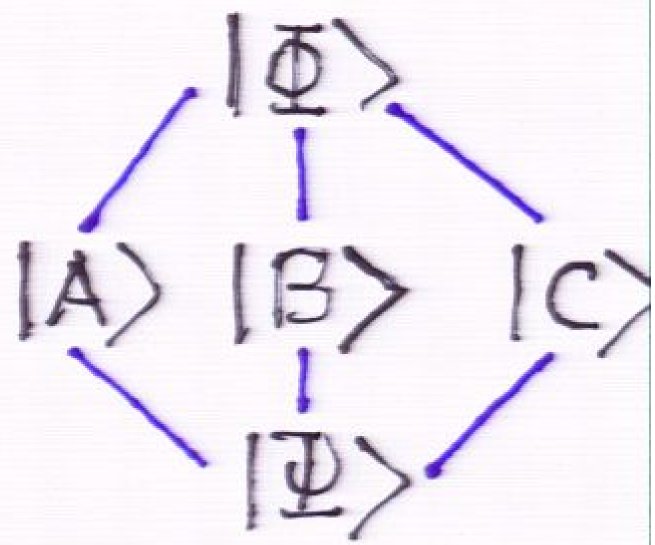

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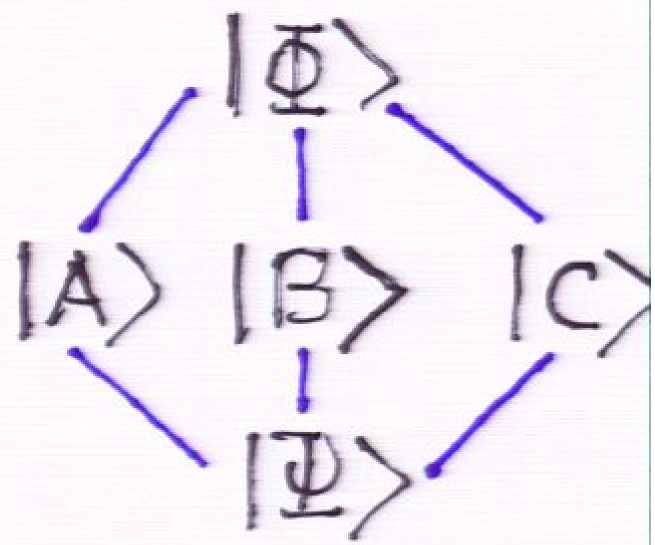

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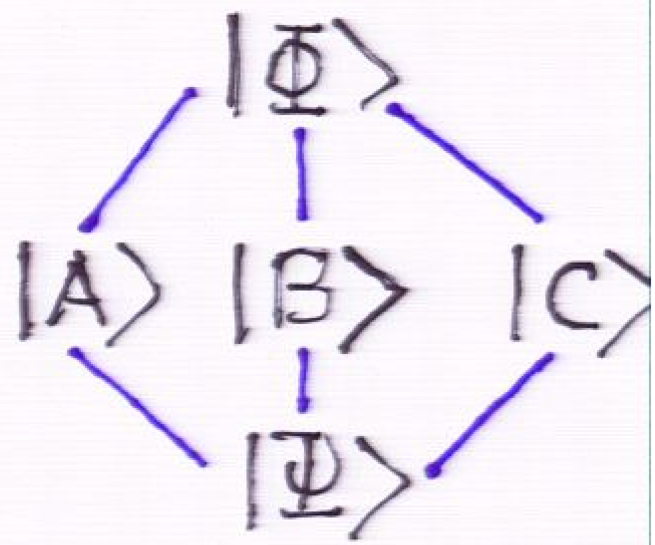

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No settleable bet is possible on whether the particle is in box A, B, or C at the intermediate time.

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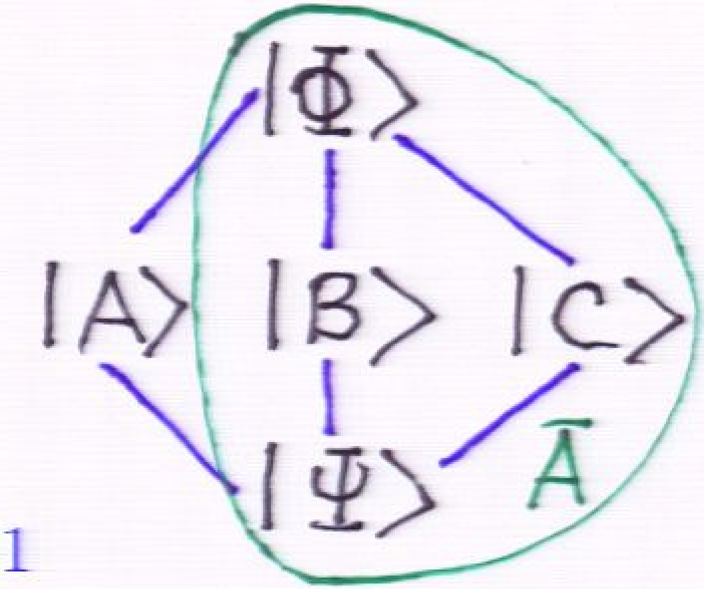
Coarse Graining



Coarse grained question:
Is the particle in box A or not at the intermediate time?

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$$p(A|\Phi) = 1, \qquad p(\bar{A}|\Phi) = 0,$$

$$p(B|\Phi) = 1, \qquad p(\bar{B}|\Phi) = 0,$$

$$p(C|\Phi) = -1, \qquad p(\bar{C}|\Phi) = 2.$$

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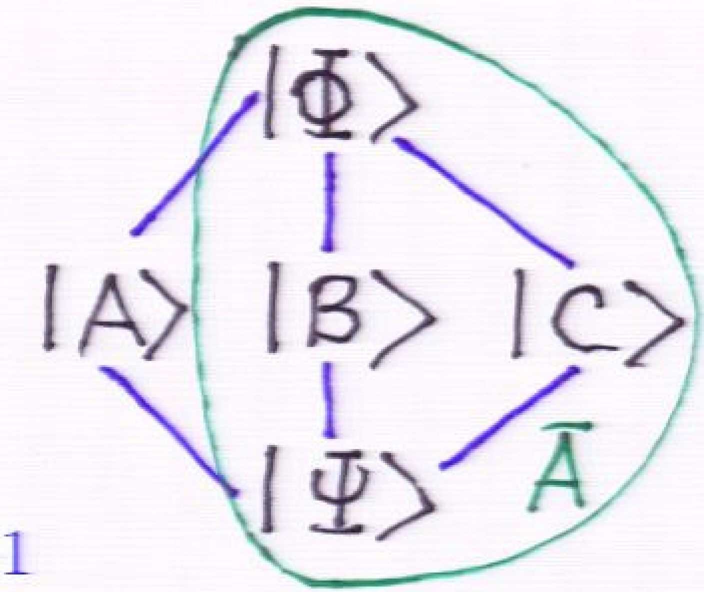

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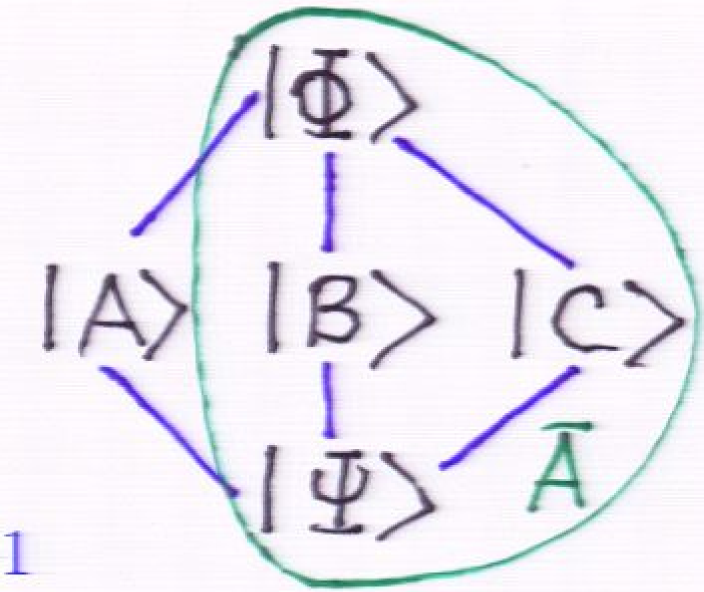

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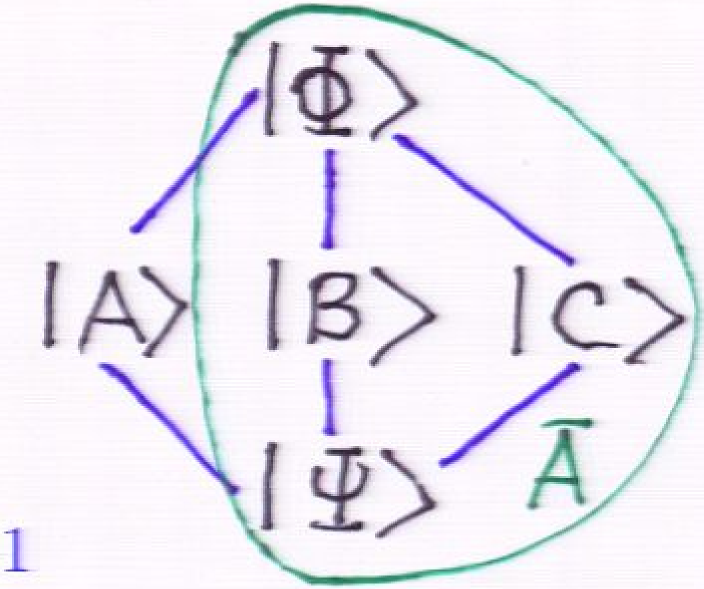

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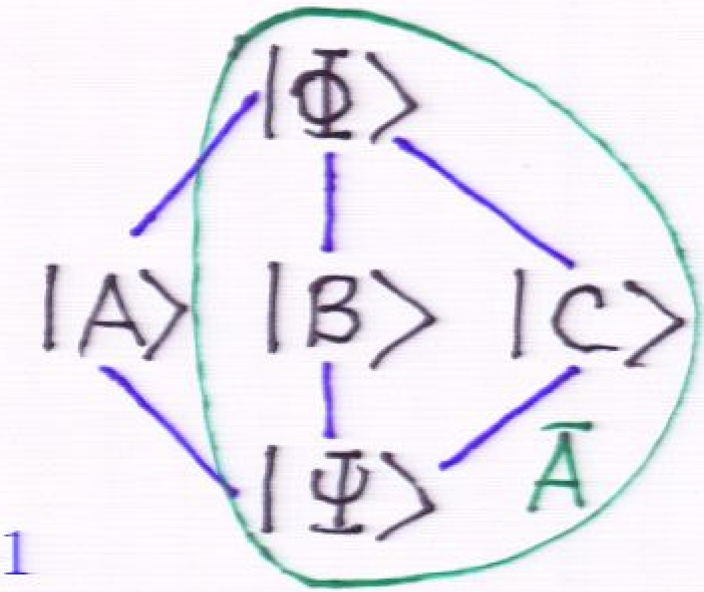

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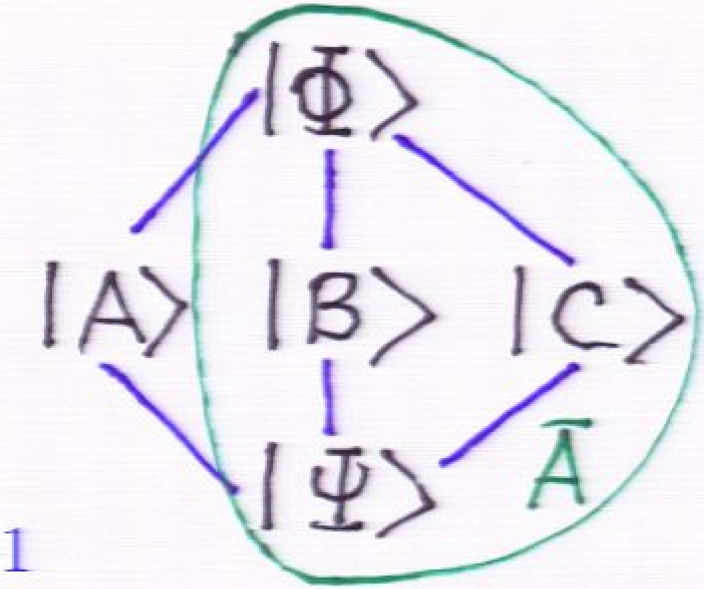

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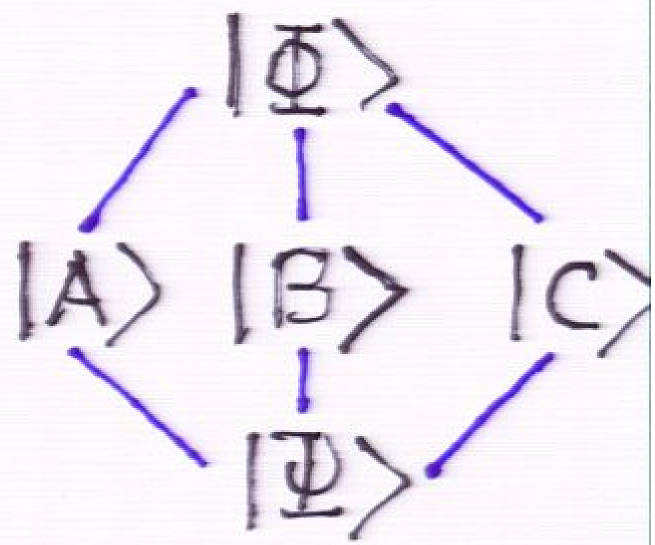

Projections: $P_A = |A\rangle\langle A|, \quad P_{\bar{A}} = I - P_A$

Histories: $P_\Phi P_A, \quad P_\Phi P_B, \quad P_\Phi P_C.$

Extended Probabilities: $p(\Phi, A) = \mathrm{Re}\langle\Psi|P_\Phi P_A|\Psi\rangle$

Result: $p(A|\Phi) = 1, \quad p(B|\Phi) = 1, \quad p(C|\Phi) = -1$

$$|\Psi\rangle \equiv \frac{1}{\sqrt{3}}\left(|A\rangle + |B\rangle + |C\rangle\right) \qquad |\Phi\rangle \equiv \frac{1}{\sqrt{3}}\left(|A\rangle + |B\rangle - |C\rangle\right)$$

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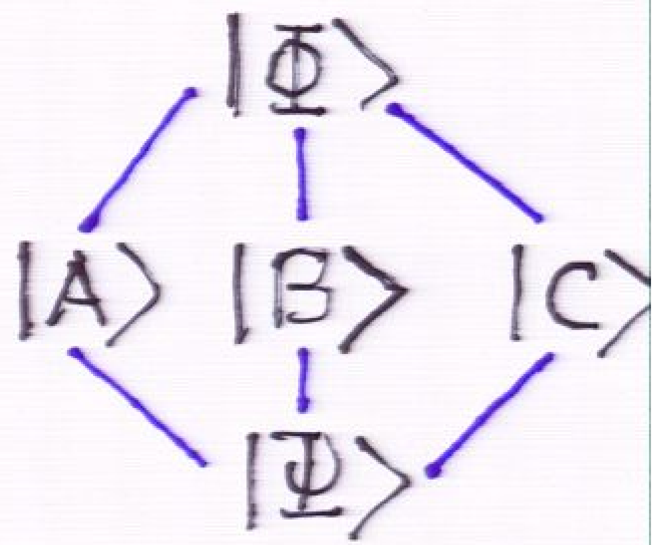

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No settleable bet is possible on whether the particle is in box A, B, or C at the intermediate time.

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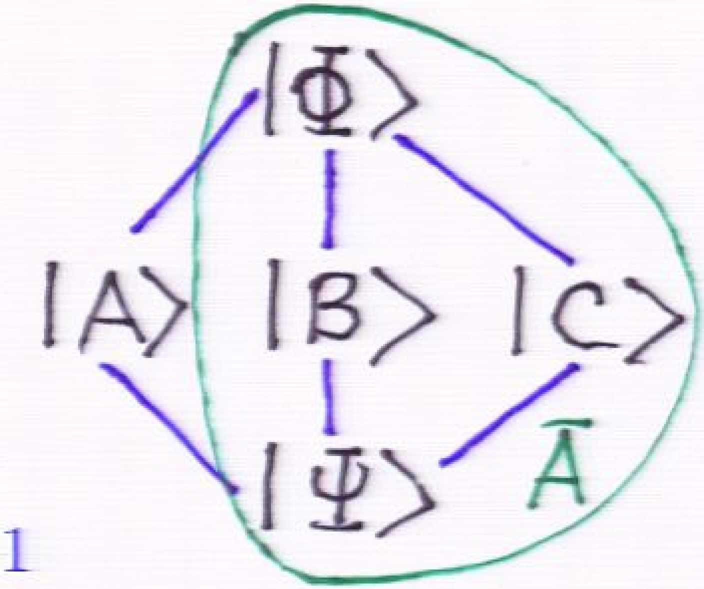

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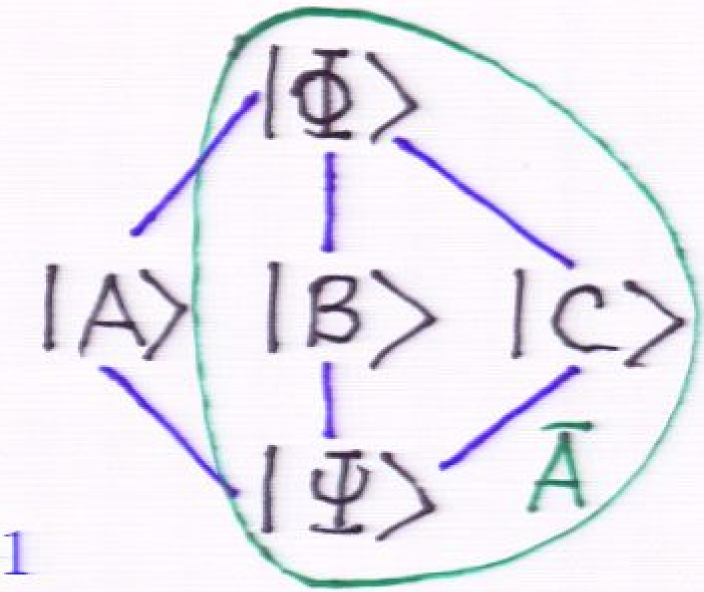

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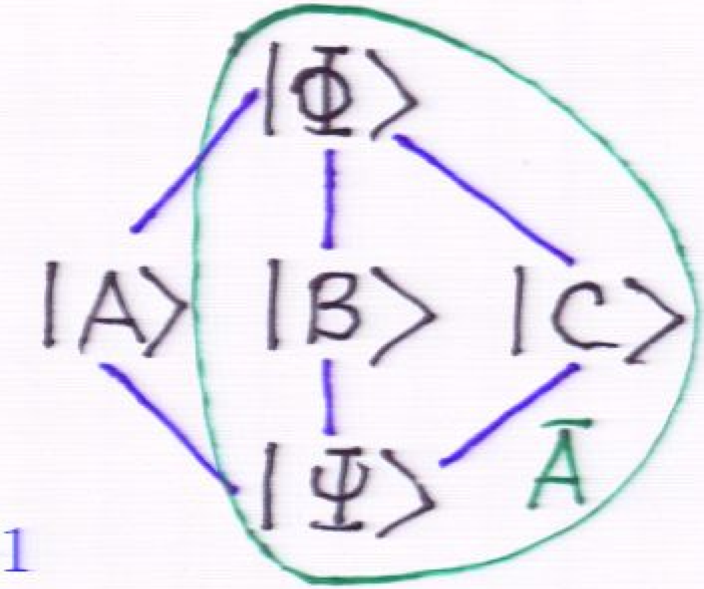

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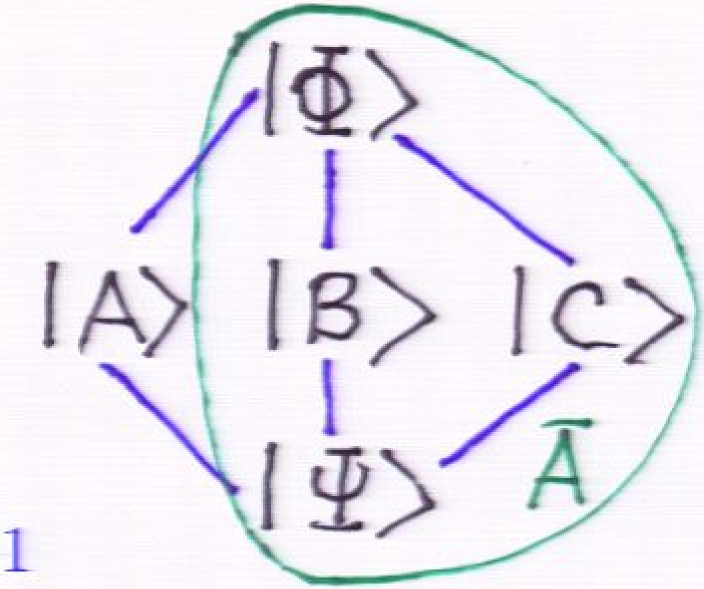

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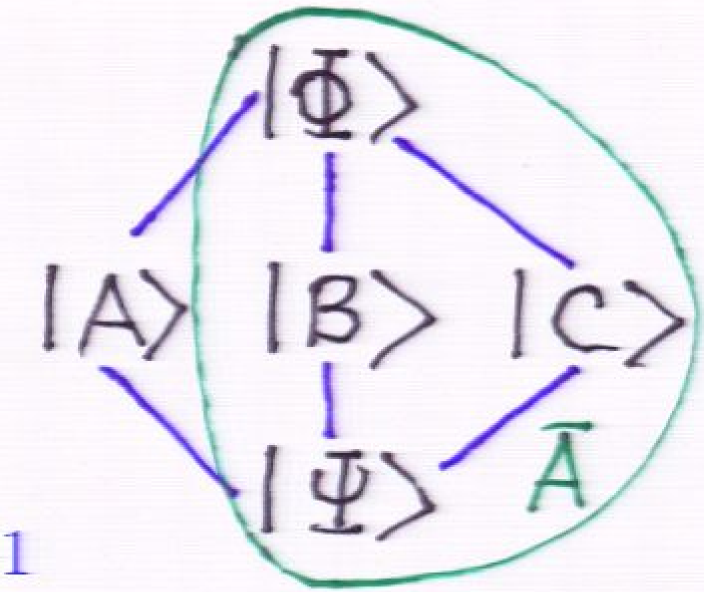

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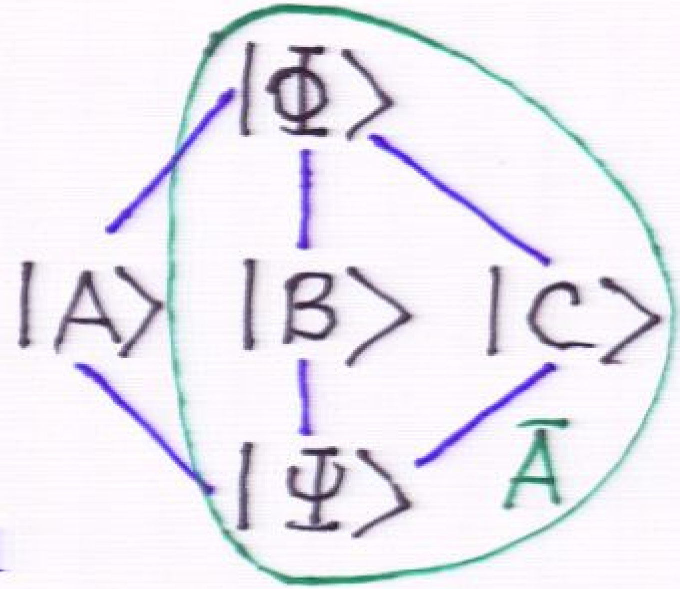

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Recorded Histories are the basis for Settleable Bets

$$R_\alpha C_\beta|\Psi\rangle = \delta_{\alpha\beta} C_\beta|\Psi\rangle.$$

Summing over β gives:

$$R_\alpha|\Psi\rangle = C_\alpha|\Psi\rangle$$

thus,

$$p(\alpha) \equiv \mathrm{Re}\langle\Psi|C_\alpha|\Psi\rangle = \langle\Psi|R_\alpha|\Psi\rangle \geq 0.$$

Extended probabilities are positive for recorded histories and bets on them are settleable.

Recorded Histories are Decoherent

$$R_\alpha|\Psi\rangle = C_\alpha|\Psi\rangle \equiv |\Psi_\alpha\rangle$$

$|\Psi_\alpha\rangle$ is the branch state vector for the history α.

$$\langle\Psi_\alpha|\Psi_\beta\rangle = \langle\Psi|R_\alpha R_\beta|\Psi\rangle = \delta_{\alpha\beta}||C_\alpha|\Psi\rangle||^2$$

Recorded sets are decoherent sets (and vice versa).
Summing over β we get:

$$\langle\Psi|R_\alpha|\Psi\rangle = \langle\Psi|C_\alpha|\Psi\rangle = ||C_\alpha\Psi\rangle||^2$$

This is the usual probability rule. Recorded (decoherent) sets have positive probabilities

$$p(\alpha) = \text{Re}\langle\Psi|C_\alpha|\Psi\rangle = ||C_\alpha|\Psi\rangle||^2$$

Recorded Histories are Decoherent

Decoherent sets have positive probabilities and bets on them can be settled by their records.

$|\Psi_\alpha\rangle$ is ... y α.

$$\langle\Psi_\alpha ... |\Psi\rangle||^2$$

Recorde... ersa).

Summing ...

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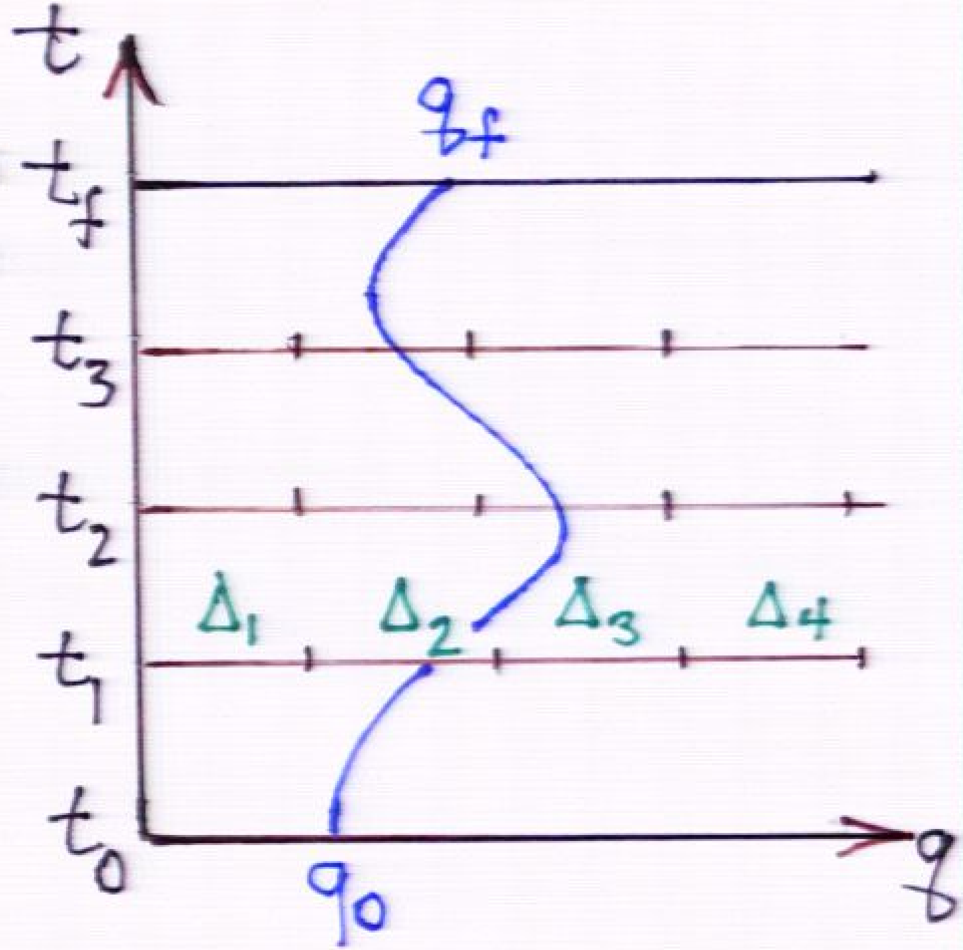
Comparison of EP and DH

	Decoherent Histories	Extended Probabilities
Probs/ Exprobs	Assigned to decoherent sets	Assigned to all sets
Probs/ Exprobs for fine grained histories	No	Yes
Probability Sum Rules	Satisfied to the accuracy of decoherence	Satisfied exactly
Records correlated with Recorded	Approximately	Approximately
Settleable bets	Decoherent Sets	Recorded Sets

Sum-Over-Histories

- Fine-grained histories are paths q(t) in configuration space. (Like field histories.)
- Coarse-grainings are partitions of the set of fine-grained histories into classes c_α .



- Translation identity:

$$\langle\Psi|P^n_{\alpha_n}(t_n)\cdots P^1_{\alpha_1}(t_1)|\Psi\rangle = \int_{\alpha_n\cdots\alpha_1}\delta q(t)\ \hat{\Psi}(q_f,t_f)\exp\{iS[q(t)]/\hbar\}\hat{\Psi}(q_0,t_0)$$

- SOH formula for extended probabilities

$$p(\alpha) = \mathrm{Re}\langle\Psi|\int \delta q\exp\{iS[q(t)]/\hbar\}|\Psi\rangle$$

The Fundamental Distribution

- A set of alternative fine grained histories {q(t)}.
- A fundamental distribution w[q(t)] for the extended probabilities of the fine-grained histories.

$$w[q(t)] \equiv \mathrm{Re}\left[\hat{\Psi}^*(q_f, t_f)\exp\{i\mathcal{S}[q(t)]/\hbar\}\hat{\Psi}(q_0, t_0)\right] .$$

- Coarse-grained histories as partitions of the fine-grained set into classes c_α .
- Extended probabilities for coarse-grained histories as sums over the ex. probabilities for fine-grained ones

$$p(\alpha) = \int_{c_\alpha} \delta q \; w[q(t)]$$

Quantum Mechanics can be thought of as a classical stochastic theory with extended probabilities in a preferred set of variables.

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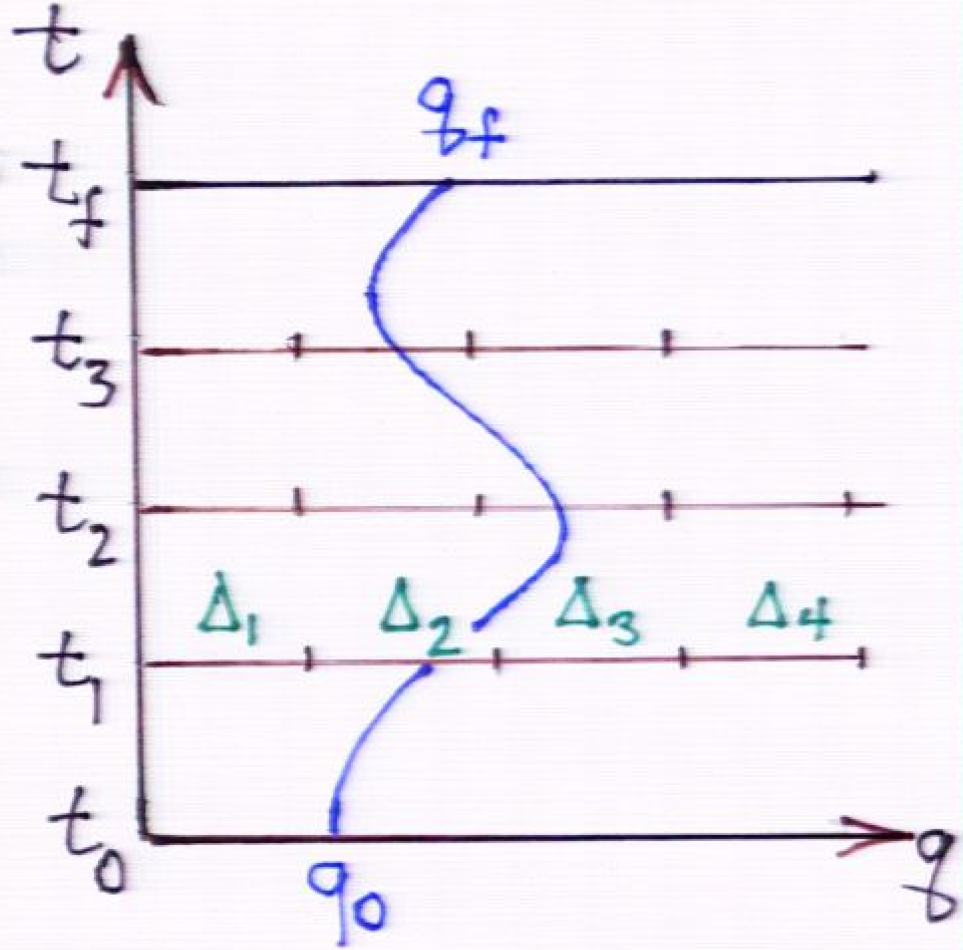

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That's hidden variables but not in conflict with the Bell inequalities because they require positive probabilities in their derivation.

Classical Stat. Mech. and Quantum Ensemble

	Classical Statistical Mechanics	Quantum Mechanics
real fine-grained ine-grained history	a path in phase space $z(t)$ obeying equation of motion	a path in configuration space, $q(t)$ between t_0 and t_f
ensemble	alternative phase space paths	alternative configuration space paths between t_0 and t_f
betting instructions	with probabilities	with extended probabilities
state	distribution on phase space $\rho(z_0, t_0)$	wave function $\hat{\Psi}(q_0, t_0)$
fundamental distribution	$w[z(t)] \equiv \int dz_0 \delta[z(t) - z_t(z_0)] \rho(z_0)$	$w[q(t)] \equiv \mathrm{Re}\left[\hat{\Psi}^*(q_f, t_f) \exp\{i\mathcal{S}[q(t)]/\hbar\} \hat{\Psi}(q_0, t_0)\right]$
coarse graining	partitions of the ensemble into classes c_α (coarse-grained histories)	partitions of the ensemble into classes c_α (coarse-grained histories)
probabilities for coarse-grained	$p(\alpha) = \int_{c_\alpha} \delta z w[z(t)]$	$p(\alpha) = \int_{c_\alpha} \delta q \; w[q(t)]$

Reality

Quantum Reality in Decoherent Histories

- Fine grained sets are not decoherent.
- Mutually incompatible sets for which there is no common fine graining that is also decoherent.
- One can say that one history in each set is real (or all the histories in all sets are real).
- Restricting to SOH variables doesn't help.

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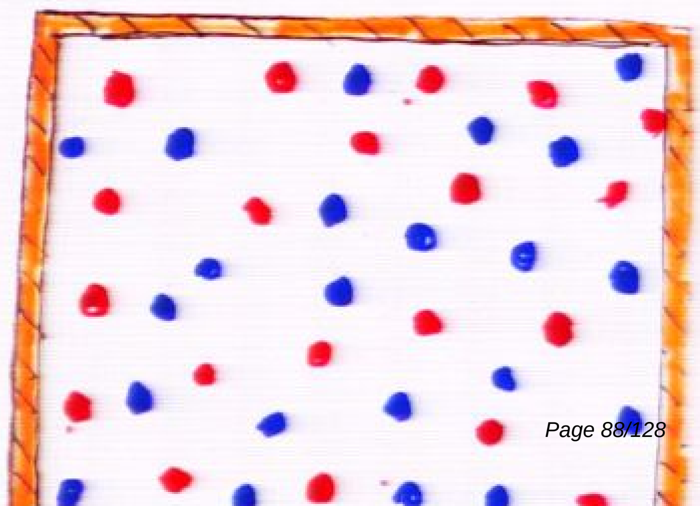

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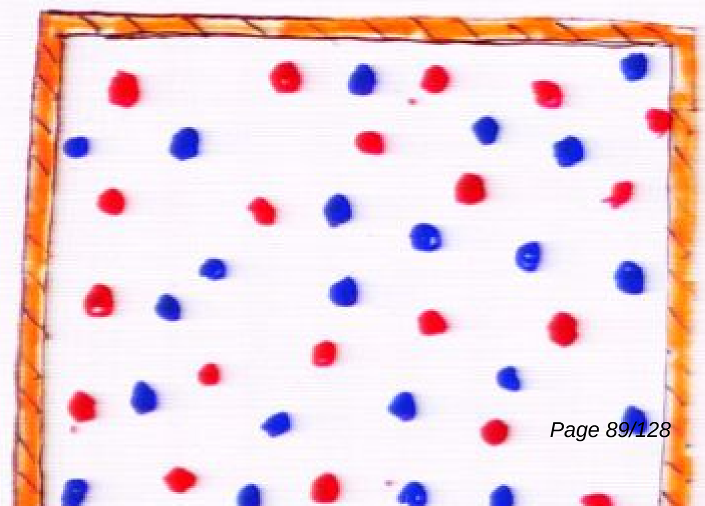
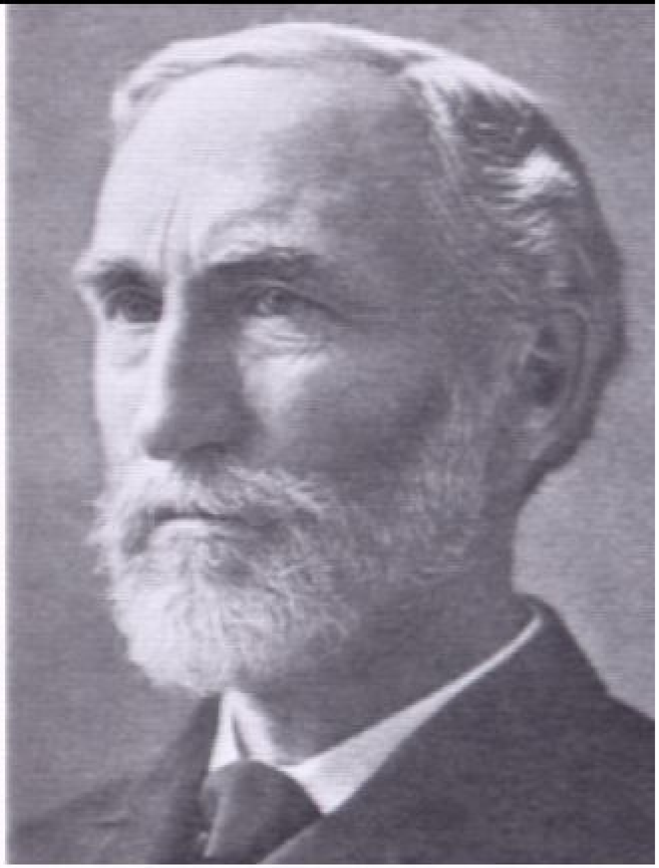


In DH reality is relative to the set of alternatives.

Gibbs' Ensemble Method

Coarse-grained regularities can be described by conceptually embedding the real history in an ensemble of alternative fine-grained histories and assigning probabilities to them so the coarse-grained regularities have high probability.



- Then we can use these probabilities to bet on coarse grained regularities.
- The probabilities are a measure of our ignorance of the fine-grained history.

Classical Gas of N Particles

- The gas has one real fine-grained history z(t) in 6N dimensional phase space.
- Newton's laws imply regularities of this fine-grained history but it is not possible to measure, store, retrieve, or compute the information in them.
- Accessible regularities are highly coarse grained such as those summarized by the Navier-Stokes equation (eg the approach to equilibrium.)
- If we knew the real history we could deduce the coarse-grained regularities but we don't.

QM as an Ensemble Theory

- There is one real fine-grained history q(t).
- The fine-grained regularities (if any) are inaccessible to measurement.
- To exhibit the coarse-grained regularities implied by the real q(t) we conceptually embed it in an ensemble of other histories with extended probabilities

$$w[q(t)] \equiv \mathrm{Re}\left[\hat{\Psi}^*(q_f, t_f)\exp\{i\mathcal{S}[q(t)]/\hbar\}\hat{\Psi}(q_0, t_0)\right]$$

- We can use the extended probabilities to bet on the coarse grained regularities when the bets are settleable (recorded).

Extended Probabilities as Measures of Ignorance

- Extended probabilities are measures of ignorance of the real fine-grained history.
- Usual probabilities implicitly assume that in principle we could find out what the real history is.
- QM requires a new level of ignorance to represent the physical situation that the real fine-grained history cannot be determined by any means.
- It is also impossible to determine the real history of the 10^{80} particles in the classical universe or even write down their description.

Advantages of this Reality

- It is close to the classical idea of reality.
- It provides a unified perspective on coarse-graining. A unique fine-grained history is realized but is inaccessible to discovery by experiment or observation.
- It allows the use of ordinary language without qualification by the set of histories referred to. E.g. the coarse grained history which happened is the one containing the real fine-grained history.
- It allows for an interpretation of extended probabilities as measures of ignorance.

A Starting Point for Generalization

``It is striking that so far it has not been possible to find a logically consistent theory that is close to quantum mechanics other than quantum mechanics itself."

But we can investigate fundamental distributions that are small deformations of

$$w[q(t)] \equiv \mathrm{Re}\left[\hat{\Psi}^*(q_f, t_f)\exp\{i\mathcal{S}[q(t)]/\hbar\}\hat{\Psi}(q_0, t_0)\right]$$

and use them to parametrize experimental tests.

Emergent Features of QM

States at a moment of time,
Unitary evolution,
Hilbert Space,
Linearity,
Principle of Superposition

are features emergent from the particular form of the FD

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But, in quantum gravity we don't expect states evolving through spacelike surfaces because there is no fixed spacetime, and in cosmology there is only one state.

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Here it is!

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The Main Points Again

- Quantum mechanics can be formulated as a theory of sets of alternative histories in which each history is assigned an extended probability giving instructions for betting.
- Records of histories can be used to settle bets. Recorded sets have positive probabilities and are decoherent. DH and EP are equivalent for recorded sets.
- In a sum-over-histories formulation, quantum mechanics is a classical stochastic history with a fundamental distribution giving extended probabilities for fine-grained histories. One history is real.
- This formulation can be the basis for generalizations of and alternatives to quantum theory.

Don’t be afraid
of negative probabilities.

Don't be afraid
of negative probabilities.

They give a new
perspective on quantum theory
that may be useful.

Ψ

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quant-ph/0401108

forthcoming with
M. Gell-Mann

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Unitary evolution,
Hilbert Space,
Linearity,
Principle of Superposition

are features emergent from the particular form of the FD

$$w[q(t)] \equiv \mathrm{Re}\left[\hat{\Psi}^*(q_f, t_f)\exp\{i\mathcal{S}[q(t)]/\hbar\}\hat{\Psi}(q_0, t_0)\right]$$

We cannot expect them in any generalization.

But, in quantum gravity we don't expect states evolving through spacelike surfaces because there is no fixed spacetime, and in cosmology there is only one state.

A fundamental formulation of quantum mechanics should be based on:

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We cannot expect them in any generalization.

A fundamental formulation of quantum mechanics should be based on:

A set of fine-grained histories in a preferred set of variables,

In which one history is real.

Outstanding Questions

An information theory for extended probabilities.

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Classical Gas of N Particles

- The gas has one real fine-grained history z(t) in 6N dimensional phase space.
- Newton's laws imply regularities of this fine-grained history but it is not possible to measure, store, retrieve, or compute the information in them.
- Accessible regularities are highly coarse grained such as those summarized by the Navier-Stokes equation (eg the approach to equilibrium.)
- If we knew the real history we could deduce the coarse-grained regularities but we don't.

QM as an Ensemble Theory

- There is one real fine-grained history q(t).
- The fine-grained regularities (if any) are inaccessible to measurement.
- To exhibit the coarse-grained regularities implied by the real q(t) we conceptually embed it in an ensemble of other histories with extended probabilities

$$w[q(t)] \equiv \mathrm{Re}\left[\hat{\Psi}^*(q_f, t_f)\exp\{i\mathcal{S}[q(t)]/\hbar\}\hat{\Psi}(q_0, t_0)\right]$$

- We can use the extended probabilities to bet on the coarse grained regularities when the bets are settleable (recorded).

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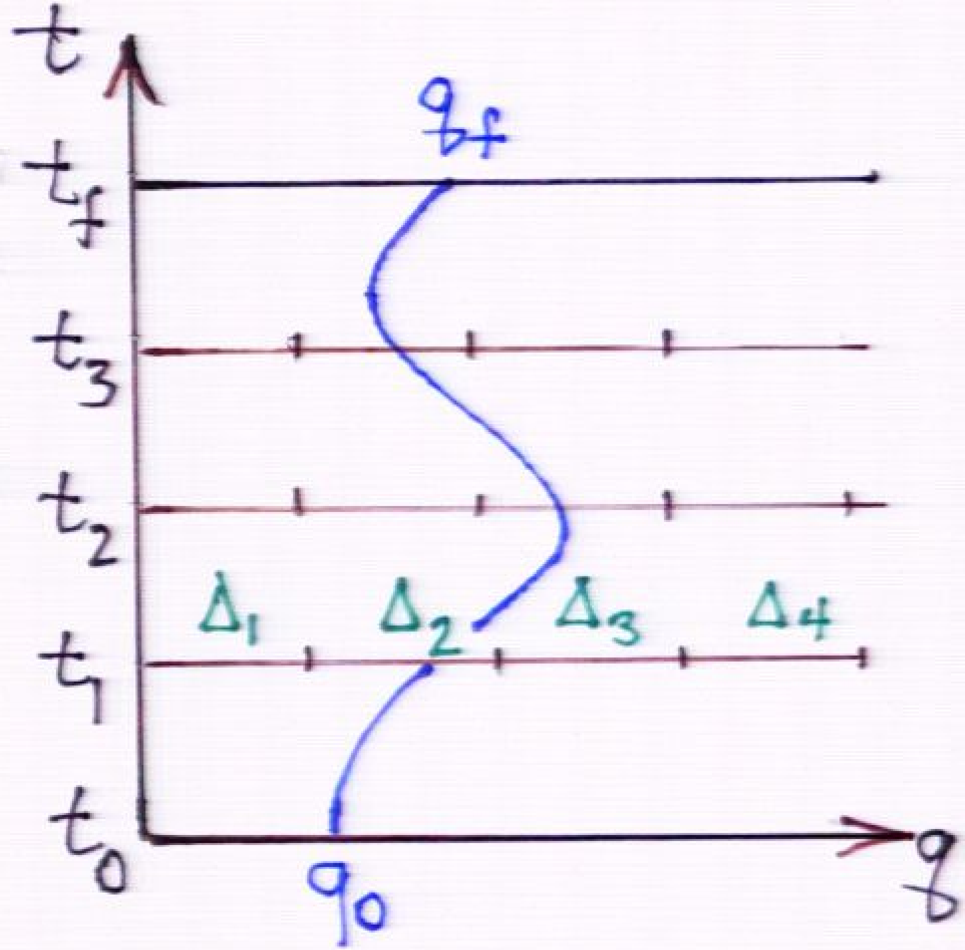
Reality

Sum-Over-Histories

- Fine-grained histories are paths q(t) in configuration space. (Like field histories.)
- Coarse-grainings are partitions of the set of fine-grained histories into classes c_α .



- Translation identity:

$$\langle\Psi|P^n_{\alpha_n}(t_n)\cdots P^1_{\alpha_1}(t_1)|\Psi\rangle = \int_{\alpha_n\cdots\alpha_1} \delta q(t)\ \hat{\Psi}(q_f,t_f)\exp\{iS[q(t)]/\hbar\}\hat{\Psi}(q_0,t_0)$$

- SOH formula for extended probabilities

$$p(\alpha) = \mathrm{Re}\langle\Psi|\int \delta q\exp\{iS[q(t)]/\hbar\}|\Psi\rangle$$

Recorded Histories are Decoherent

$$R_\alpha|\Psi\rangle = C_\alpha|\Psi\rangle \equiv |\Psi_\alpha\rangle$$

$|\Psi_\alpha\rangle$ is the branch state vector for the history α.

$$\langle\Psi_\alpha|\Psi_\beta\rangle = \langle\Psi|R_\alpha R_\beta|\Psi\rangle = \delta_{\alpha\beta}||C_\alpha|\Psi\rangle||^2$$

Recorded sets are decoherent sets (and vice versa).

Summing over β we get:

$$\langle\Psi|R_\alpha|\Psi\rangle = \langle\Psi|C_\alpha|\Psi\rangle = ||C_\alpha\Psi\rangle||^2$$

This is the usual probability rule. Recorded (decoherent) sets have positive probabilities

$$p(\alpha) = \text{Re}\langle\Psi|C_\alpha|\Psi\rangle = ||C_\alpha|\Psi\rangle||^2$$

Records

- No bet is complete without a prescription of how to settle it.
- A record of the outcome is one way of settling a bet.
- A set of histories is strongly recorded if there are an exhaustive set of exclusive projections such that

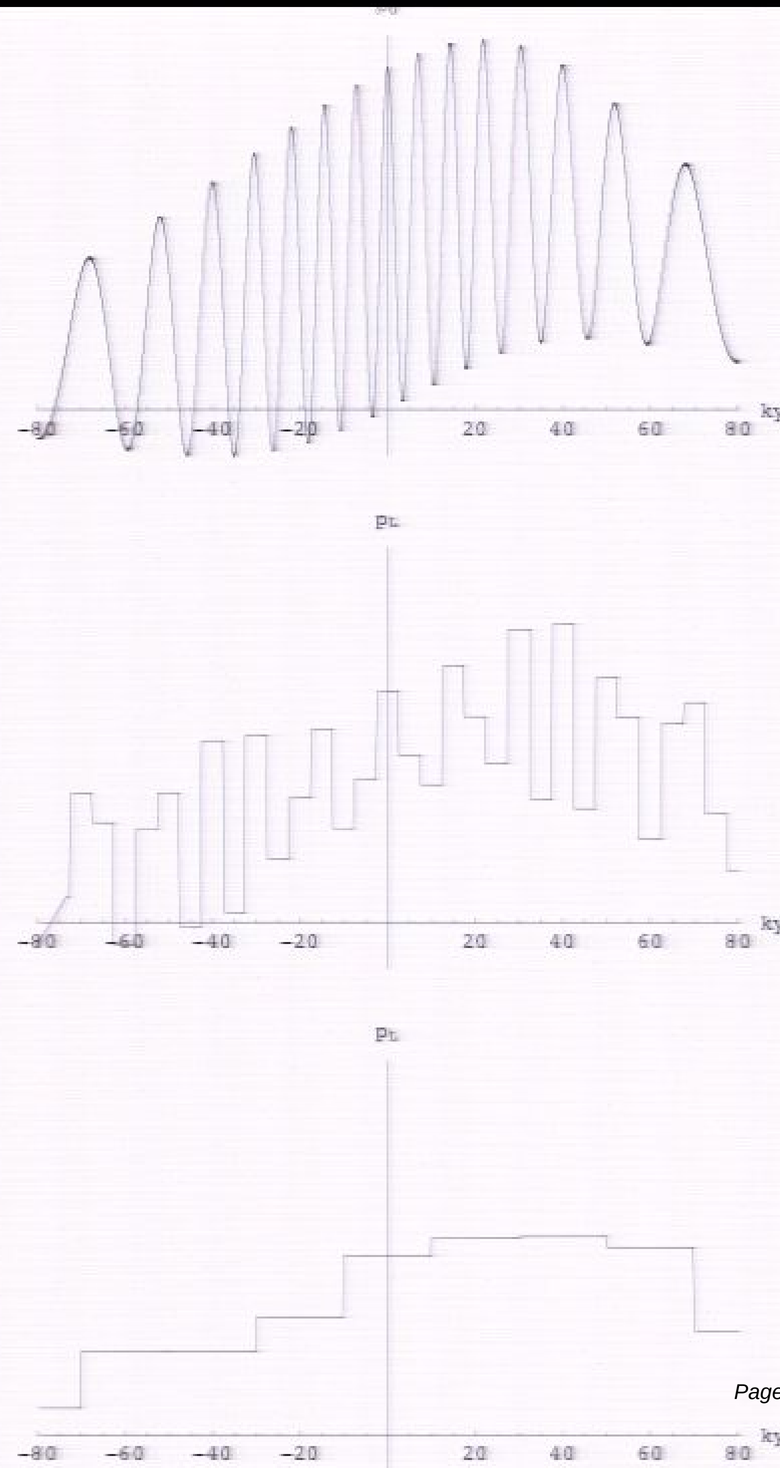
$$R_\alpha C_\beta |\Psi\rangle = \delta_{\alpha\beta} C_\beta |\Psi\rangle .$$

- Then there is a correlation between histories and their records

$$p(\alpha, \beta) = \delta_{\alpha\beta} p(\beta)$$

- Individual bettors may require more than just correlation e.g. the NY Times.

Coarse-Graining and Positive Probabilities



Fine-grained and Coarse-grained.

$$C_\alpha = P^n_{\alpha_n}(t_n)\cdots P^1_{\alpha_1}(t_1)$$

- Fine grained histories a have one-dimensional P's at each and every time (like Feynman paths)
- Coarse graining is partitioning a set of histories $\{c_\alpha\}$ into bigger classes $\{\bar{c}_{\bar{\alpha}}\}$.

$$\bar{c}_{\bar{\alpha}} = \cup_{\alpha\in\bar{\alpha}}\ c_\alpha \qquad\qquad \bar{C}_{\bar{\alpha}} = \sum_{\alpha\in\bar{\alpha}} C_\alpha$$

Extended Probabilities

- Assumed Input: $H \quad |\Psi\rangle$
- Histories: $C_\alpha = P^n_{\alpha_n}(t_n)\cdots P^1_{\alpha_1}(t_1)$

$$p(\alpha) = \mathrm{Re}\langle\Psi|C_\alpha|\Psi\rangle$$

- Not generally positive.
- Agrees with usual QM for single time histories

$$p(\alpha) = \langle\Psi|P_\alpha|\Psi\rangle$$

- Sum rules satisfied exactly:

$$\bar{C}_{\bar{\alpha}} = \sum C_\alpha \qquad p(\bar{\alpha}) = \sum_{\alpha\in\bar{\alpha}} p(\alpha) \qquad \sum p(\alpha) = 1$$