Title: Detecting Majorana Modes via Non-local Two Particle Interferometry

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Abstract: We consider one dimensional devices supporting a pair of Majorana bound states at their ends We firstly show [1] that edge Majorana bound modes allow for processes with an actual transfer of electronic material between well-separated points and provide an explicit computation of the tunnelling amplitude for this process. We then show [2] that these devices can produce remarkable Hanbury-Brown Twiss like interference effects between well separated Dirac fermions of pertinent energies: we find indeed that, at these energies, the simultaneous scattering of two incoming electrons or two incoming holes from the Majorana bound states leads exclusively to an electron-hole final state. This "anti-bunching" in electron-hole internal pseudospin space can be detected through a measure of current-current correlations. Finally, we show [2] that, by scattering appropriate spin polarized electrons from the Majorana bound states, one can engineer a non-local entangler of electronic spins useful for quantum information applications. [1] G. W. Semenoff and P. Sodano: J. Phys. B: At. Mol. Opt. Phys. 40, 1479 (2007); [2] S. Bose and P. Sodano: ÃÂƒÂ¢ÃÂ‚ÂÂ€ÃÂ‚ÂÂœNon-local Handbury- Brown Twiss Interferometry & Entanglement Generation from Majorana

Detecting Majorana Modes via Non-Local Two Particle Interferometry

Pasquale Sodano

- Sougato Bose, P.S., arXiv: 1010.0709
- G. W. Semenoff, P.S., J. Phys. B 40, 1479 (2007) arXiv: cond-mat/065147

Outline

- Dirac vs Majorana Fermions
- Midgap States
  - Soliton – Fermion interaction
  - Kitaev wire
- Non-local effects
- Two Particle Handbury – Brown Twiss interferometer (non-local) for Dirac electrons
- Non-local Entangler of fermion spin

Dirac vs Majorana Fermions

Dirac fermions

$$i\frac{\partial}{\partial t}\Psi(\vec{x},t) = H_0\Psi(\vec{x},t)$$

$$\left\{\Psi(\vec{x},t),\Psi^\dagger(\vec{y},t)\right\} = \delta(\vec{x}-\vec{y})$$

$$H = \int dx : \Psi^\dagger(x,t)H_0\Psi(x,t) :$$

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = [\Psi(x,t),H]$$

Dirac vs Majorana Fermions

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$$H_0\psi_E(x) = E\psi_E(x)$$

Orthonormality conditions:

$$\int d\vec{x}\,\psi^\dagger_E(\vec{x})\psi_{E'}(\vec{x}) = \delta_{EE'} \quad \sum_E \psi_E(\vec{x})\psi^\dagger_E(\vec{y}) = \delta(\vec{x}-\vec{y})$$

Field operator expansion:

$$\Psi(\vec{x},t) = \sum_{E>0}\psi_E(\vec{x})e^{-iEt/\hbar}a_E + \sum_{E<0}\psi_E(\vec{x})e^{-iEt/\hbar}b^\dagger_{-E}$$

with:

$$\left\{a_E, a^\dagger_{E'}\right\} = \delta_{EE'}\ , \ \left\{b_{-E}, b^\dagger_{-E}\right\} = \delta_{EE'}$$

$$a_E|0\rangle = 0 = b_{-E}|0\rangle$$

Dirac vs Majorana Fermions

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Majorana fermions

Reality condition: $\psi_{-E}(x) = \Gamma \psi_E^*(x)$

Majorana field operator:

$$\Phi(\vec{x},t) = \sum_{E>0} \left(\psi_E(\vec{x}) e^{-iEt/\hbar} a_E + \Gamma \psi_E^*(\vec{x}) e^{iEt/\hbar} a_E^\dagger \right)$$

$$a_E|0\rangle = 0 \qquad \forall E > 0$$

$$\Phi(\vec{x},t) = \Gamma \Phi^\dagger(\vec{x},t)$$

$$\left\{ \Phi(\vec{x},t), \Phi^\dagger(\vec{y},t) \right\} = \delta(\vec{x} - \vec{y})$$

From Dirac to Majorana:

$$\Phi_1(\vec{x},t) = \frac{1}{\sqrt{2}} \left(\Psi(\vec{x},t) + \Gamma \Psi^\dagger(\vec{x},t) \right)$$

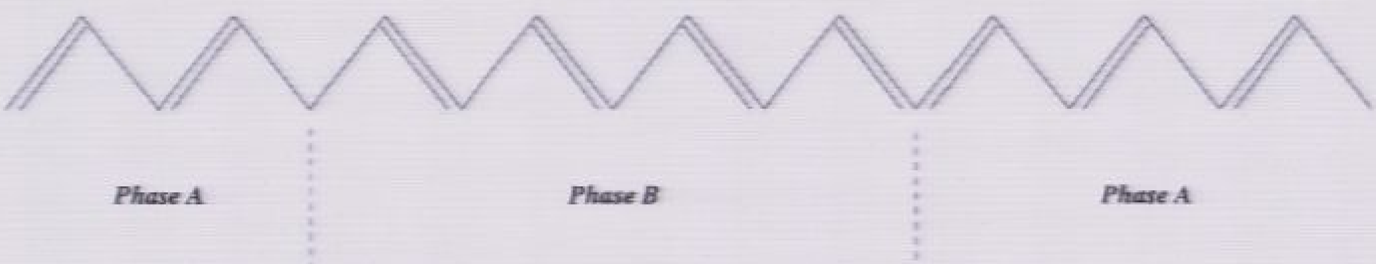
$$\Phi_2(\vec{x},t) = \frac{1}{i\sqrt{2}} \left(\Psi(\vec{x},t) - \Gamma \Psi^\dagger(\vec{x},t) \right)$$

Fractionally charged soliton

R.Jackiw, C.Rebbi, Phys.Rev.D13,3398 (1976)
W.P.Su, J.R.Schrieffer, A.Heeger, Phys.Rev.Lett.42,1698(1979)
R.Jackiw, J.R.Schreiffer, Nucl.Phys.B190[FS3],253(1981).

Fermion interacting with a topological solition can have quantum states with fractional charge $q = \left(n + \frac{1}{2}\right) e$ (also can be non-($\frac{1}{2}$)integer in systems with less symmetry)

Example in 1+1-dimensions which models polyacetylene



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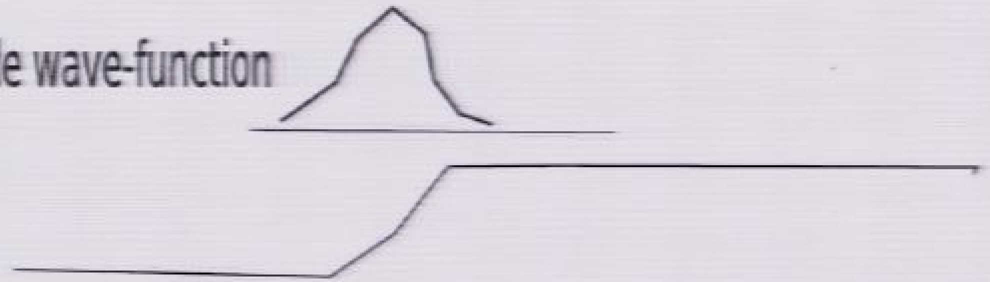
zero mode wave-function



soliton

Scalar field with soliton profile: $\phi(x) = \mu \tanh \mu x$

Dirac Eqn. in soliton background: $(i\gamma \cdot \partial - \phi(x))\, \psi(x,t) = 0$

$\psi(x,t) = e^{iEt}\psi_E(x)$

Hamiltonian $\left(i\vec{\alpha}\cdot\vec{\nabla} + \beta\phi(x)\right)\psi_E(x) = E\psi_E(x)$

$$\begin{bmatrix} 0 & -\partial_x + \phi(x) \\ \partial_x + \phi(x) & 0 \end{bmatrix} \begin{bmatrix} u_E(x) \\ v_E(x) \end{bmatrix} = E \begin{bmatrix} u_E(x) \\ v_E(x) \end{bmatrix} \; , \; \begin{bmatrix} u_{-E} \\ v_{-E} \end{bmatrix} = \begin{bmatrix} u_E \\ -v_E \end{bmatrix}$$

$\exists$ zero frequency $E=0$ mode. $u_0(x) \sim e^{-\int_0^x \phi(x')dx'}$, $v_0 = 0$

$$\psi(x,t) = \psi_0(x)a + \sum_{E>0}\left[e^{iEt}\psi_E(x)a_E + e^{-iEt}\psi_{-E}(x)b_E^\dagger\right]$$

3

- Charge conjugation symmetry

$$\psi(x,t) \rightarrow C\psi^{\dagger}(c,t) \ \ , \ \ \psi_E(x) = C\psi^{*}_{-E}(x) \ \ , \ \ C = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$Q \rightarrow CQC^{-1} = -Q$$

For every state $|q>$ with $Q|q>= q|q>, \exists \ |-q>= C|q>$ with $Q|-q>= -q|-q>$

- charges are quantized: if $\exists \ |q>$ with $Q|q>= q|q>$ then $\exists \ |q+e>$ with $Q|q+e>= (q+e)|Q+e>$

Then $q-(-q) = 2q = e \cdot$integer and **either**

- $q = n \cdot e \quad n \in \mathcal{Z}$

  or

- $q = \left(n + \frac{1}{2}\right) \cdot e \quad n \in \mathcal{Z}$

[4]

$$\psi(x,t) = \psi_0(x)a + \sum_{E>0}\left[e^{iEt}\psi_E(x)a_E + e^{-iEt}\psi_{-E}(x)b_E^\dagger\right]$$

Zero mode creation/annihlation operator

$$\{a,a^\dagger\} = 1 \ , \ \{a,a_E\} = 0 = \{a,b_E\} \ , \ \{a_E,a_{E'}^\dagger\} = \delta_{EE'} \ \text{ etc.}$$

Zero mode has two-dimensional representation $\{|\downarrow>,|\uparrow>\}$

$$a^\dagger|\downarrow> = |\uparrow> \ , \ a^\dagger|\uparrow> = 0 \ , \ a|\uparrow> = |\downarrow> \ , \ a|\downarrow> = 0$$

$a_E|\uparrow> = a_E|\downarrow> = 0 = b_E|\uparrow> = b_E|\downarrow>$

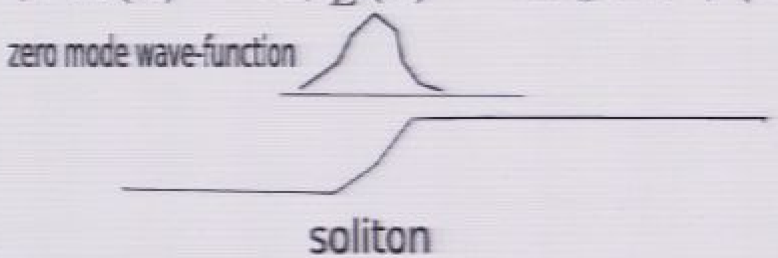
$a_{E_1}^\dagger ... b_{E_1'}^\dagger ...|\uparrow> \ , \ a_{E_1}^\dagger ... b_{E_1'}^\dagger ...|\downarrow>$ have energy $E_1 + ... + E_1'' + ...$

$$Q/e = \int dx \psi^\dagger(x,t)\psi(x,t) - \text{const} = a^\dagger a - \frac{1}{2} + \sum_E \left(a_E^\dagger a_E - b_E^\dagger b_E\right)$$

$$Q|\downarrow> = -\frac{e}{2}|\downarrow> \ , \ Q|\uparrow> = \frac{e}{2}|\uparrow> \ , \ Q = \left(n + \frac{1}{2}\right)e$$

Majorana fermion: identify particles and anti-particles [5]

$\psi_{-E}(x) = C\psi^*_E(x) \rightarrow$ impose $\psi(x,t) = C\psi^*(x,t)$.



$$\psi(x,t) = \psi_0(x)a + \sum_{E>0}\left[e^{iEt}\psi_E(x)a_E + e^{-iEt}\psi_{-E}(x)a^\dagger_E\right]$$

$$a = a^\dagger \ , \ a^2 = \frac{1}{2} \ , \ \{a, a_E\} = 0 \ , \ \{a_E, a^\dagger_{E'}\} = \delta_{EE'}$$

Zero mode has 2 inequiv. 1-dim.reps.

$$a = (\pm)\frac{1}{\sqrt{2}}(-1)^{\sum a^\dagger_E a_E} \ , \ a_E|0>= 0 \ , \ a|0>= (\pm)\frac{1}{\sqrt{2}}|0>$$

States: $a^\dagger_{E_1}...|0>$ energy $E_1 + ...$ **but** $<0|\psi(x,t)|0>= (\pm)\frac{1}{\sqrt{2}}\psi_0(x)$

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Majorana fermions do not have conserved fermion number: [6]
($Q = e\int \psi^\dagger\psi = 0$)

They do have a discrete symmetry, $(-1)^F\psi(x,t) + \psi(x,t)(-1)^F = 0$
(**fermion parity**: conservation of fermion number mod 2)

$$[(-1)^F, \psi^\dagger M\psi] = 0$$

However, since $a|0> = (\pm)\frac{1}{\sqrt{2}}|0>$, $|0>$ not an eigenstate of $(-1)^F$

Fixed by using 2-dim. rep of algebra

$$a^2 = \frac{1}{2}\ ,\ b^2 = \frac{1}{2}\ ,\ \{a,b\} = 0\ ,\ \alpha = a + ib\ ,\ \alpha^\dagger = a - ib$$

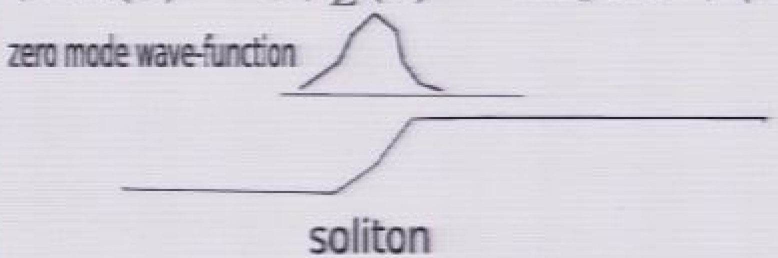
$\{\alpha, \alpha^\dagger\} = 1$, $\alpha|-> = 0$, $\alpha^\dagger|-> = |+>$, $\alpha^\dagger|+> = 0$, $\alpha^\dagger|+> = |->$

Fermion parity: $(-1)^F|-> = -|->$, $(-1)^F|+> = |+>$

Soliton-antisoliton system $-->$

Majorana fermion: identify particles and anti-particles [5]

$\psi_{-E}(x) = C\psi_E^*(x) \to$ impose $\psi(x,t) = C\psi^*(x,t)$.



$$\psi(x,t) = \psi_0(x)a + \sum_{E>0}\left[e^{iEt}\psi_E(x)a_E + e^{-iEt}\psi_{-E}(x)a_E^\dagger\right]$$

$$a = a^\dagger \ , \ a^2 = \frac{1}{2} \ , \ \{a, a_E\} = 0 \ , \ \{a_E, a_{E'}^\dagger\} = \delta_{EE'}$$

Zero mode has 2 inequiv. 1-dim.reps.

$$a = (\pm)\frac{1}{\sqrt{2}}(-1)^{\sum a_E^\dagger a_E} \ , \ a_E|0> = 0 \ , \ a|0> = (\pm)\frac{1}{\sqrt{2}}|0>$$

States: $a_{E_1}^\dagger ...|0>$ energy $E_1 + ...$ **but** $<0|\psi(x,t)|0> = (\pm)\frac{1}{\sqrt{2}}\psi_0(x)$

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Fixed by using 2-dim. rep of algebra

$$a^2 = \frac{1}{2}\,,\; b^2 = \frac{1}{2}\,,\; \{a,b\} = 0\,,\; \alpha = a + ib\,,\; \alpha^\dagger = a - ib$$

$\{\alpha, \alpha^\dagger\} = 1\,,\; \alpha|-> = 0\,,\; \alpha^\dagger|-> = |+>\;,\; \alpha^\dagger|+> = 0\,,\; \alpha^\dagger|+> = |->$

Fermion parity: $(-1)^F|-> = -|->\;,\; (-1)^F|+> = |+>$

Soliton-antisoliton system $-->$

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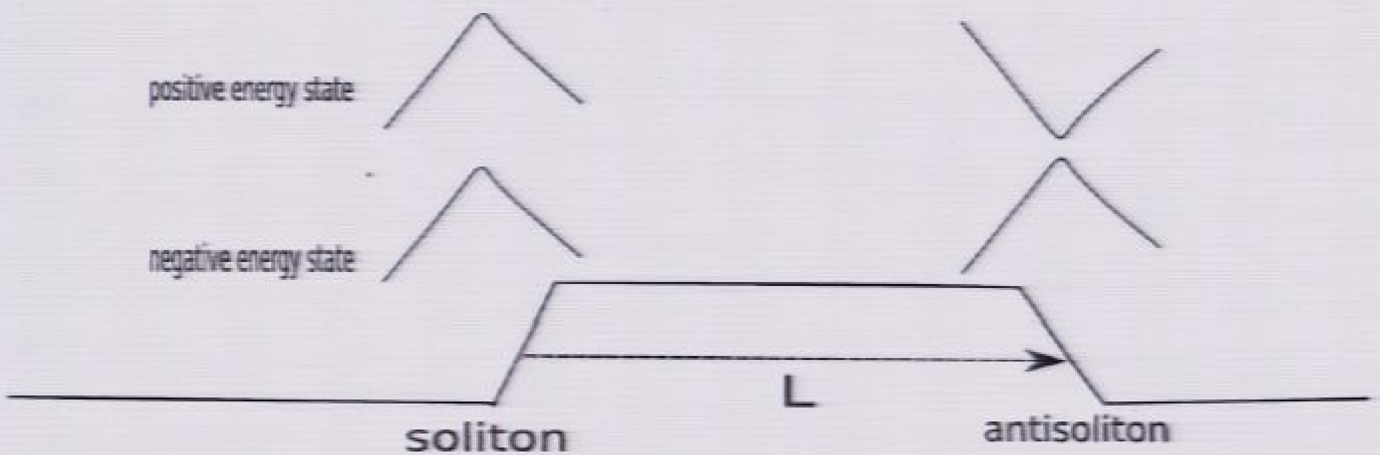
[7]

Fractional charge in a two-soliton system

R.Jackiw, A.Kerman, I.Klebanov, G.Semenoff, Nucl.Phys.B225 [FS9], 233 (1983)

R.Jackiw J.R.Schrieffer, cond-mat/0012370

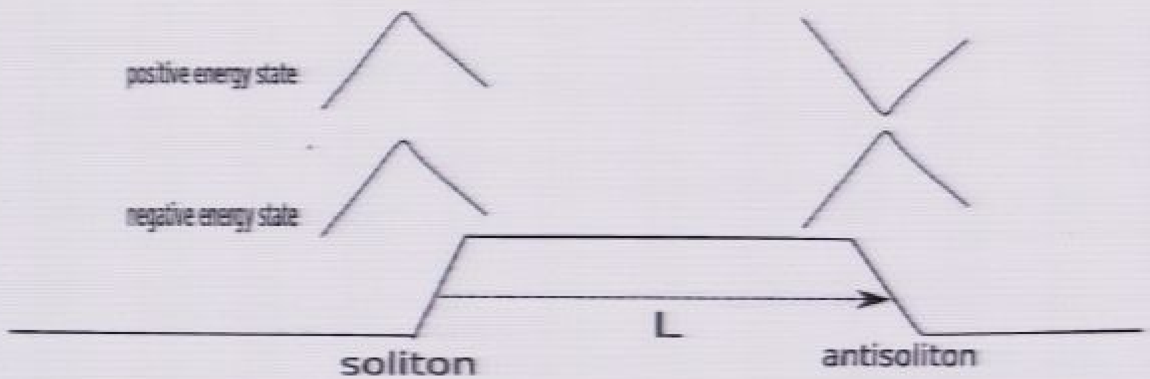


Two bound states with $\omega = \pm\epsilon \sim e^{-L}$ $\quad L \to \infty$

$$\psi = \psi_+(x)e^{i\epsilon t}\alpha + \psi_-(x)e^{-i\epsilon t}\beta^\dagger + \sum_{E>0}\left[e^{iEt}\psi_E(x)\alpha_E + e^{-iEt}\psi_{-E}(x)\beta_E^\dagger\right]$$

$$= \left[\frac{\psi_+e^{i\epsilon t} + \psi_-e^{-i\epsilon t}}{\sqrt{2}}\right]\frac{\alpha+\beta^\dagger}{\sqrt{2}} + \left[\frac{\psi_+e^{i\epsilon t} - \psi_-e^{-i\epsilon t}}{-\sqrt{2}i}\right]\frac{\alpha-\beta^\dagger}{\sqrt{2}i} + \ldots$$

[8]



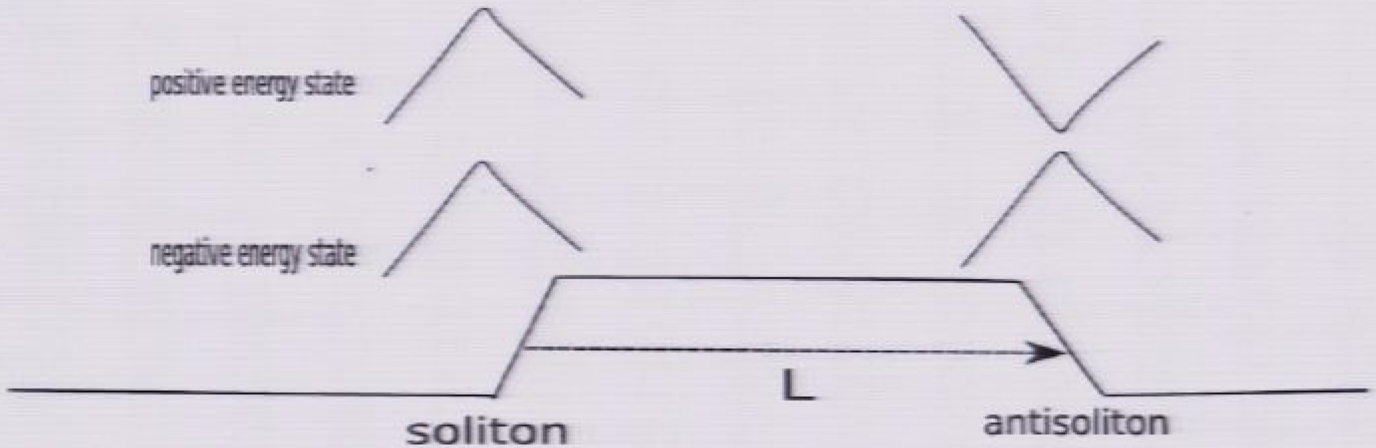
$$\psi = \left[\frac{\psi_+ e^{i\epsilon t} + \psi_- e^{-i\epsilon t}}{\sqrt{2}}\right]\frac{\alpha+\beta^\dagger}{\sqrt{2}} + \left[\frac{\psi_+ e^{i\epsilon t} - \psi_- e^{-i\epsilon t}}{-\sqrt{2}i}\right]\frac{\alpha-\beta^\dagger}{\sqrt{2}i} + \ldots$$

$$\sim \begin{bmatrix} u_0 \\ 0 \end{bmatrix} a + \begin{bmatrix} 0 \\ v_0 \end{bmatrix} b + \ldots$$

$$a = \frac{\alpha+\beta^\dagger}{\sqrt{2}}\ ,\ a^\dagger = \frac{\alpha^\dagger+\beta}{\sqrt{2}}\ ,\ \{a, a^\dagger\} = 1$$

$$b = \frac{\alpha-\beta^\dagger}{\sqrt{2}i}\ ,\ b^\dagger = \frac{\alpha^\dagger-\beta}{-\sqrt{2}i}\ ,\ \{b, b^\dagger\} = 1$$

[9]



$$a = \frac{\alpha + \beta^\dagger}{\sqrt{2}} \ , \ a^\dagger = \frac{\alpha^\dagger + \beta}{\sqrt{2}} \ , \ \{a, a^\dagger\} = 1$$

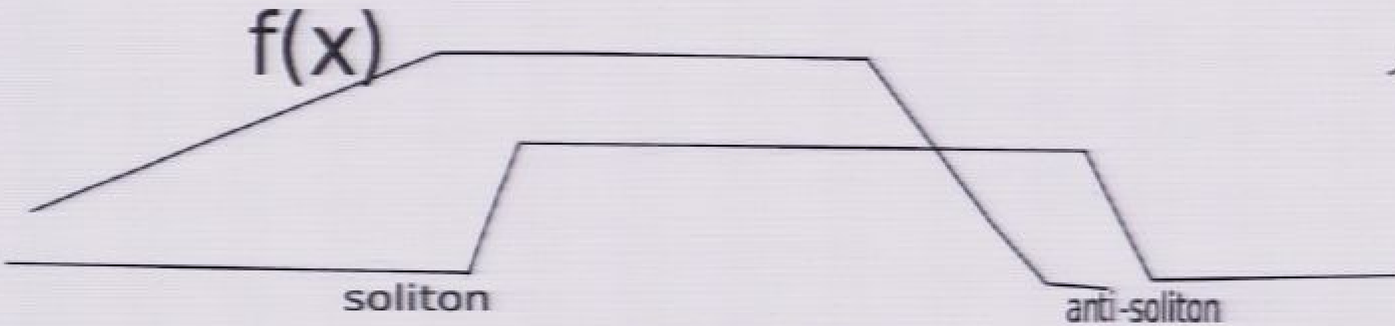
$$b = \frac{\alpha - \beta^\dagger}{\sqrt{2}i} \ , \ b^\dagger = \frac{\alpha^\dagger - \beta}{-\sqrt{2}i} \ , \ \{b, b^\dagger\} = 1$$

has a four-dimensional representation

$$a|\downarrow> = 0 \ , \ a^\dagger|\downarrow> = |\uparrow> \ , \ a|\uparrow> = |\downarrow> \ , \ a^\dagger|\uparrow> = 0$$

$$b|\downarrow> = 0 \ , \ b^\dagger|\downarrow> = |\uparrow> \ , \ b|\uparrow> = |\downarrow> \ , \ b^\dagger|\uparrow> = 0$$

10



States are direct products $\begin{matrix} |\uparrow\uparrow> & |\uparrow\downarrow> \\ |\downarrow\uparrow> & |\downarrow\downarrow> \end{matrix}$ with fractional charge.

$$Q = \lim_{f\to 1}\lim_{L\to\infty} e\int dx f(x)\psi^\dagger(x,t)\psi(x,t) \ , \quad \begin{matrix} Q|\uparrow\uparrow> = \frac{e}{2}|\uparrow\uparrow> \\ Q|\downarrow\uparrow> = -\frac{e}{2}|\downarrow\uparrow> \end{matrix}$$

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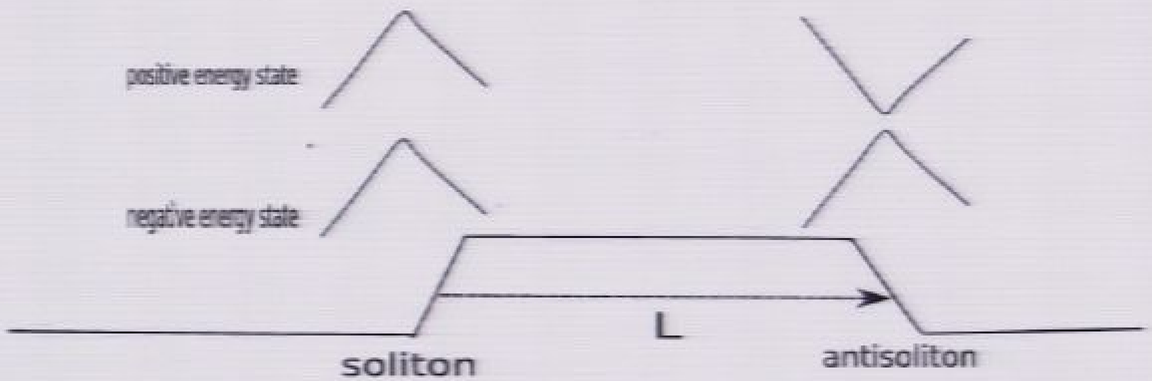
With Majorana fermions

[11]



$$\psi = \psi_+(x)e^{i\epsilon t}\alpha + \psi_-(x)e^{-i\epsilon t}\alpha^\dagger + \sum_{E>0}\left[e^{iEt}\psi_E(x)\alpha_E + e^{-iEt}\psi_{-E}(x)\alpha_E^\dagger\right]$$

$$= \frac{\psi_+e^{i\epsilon t}+\psi_-e^{-i\epsilon t}}{\sqrt{2}}\frac{\alpha+\alpha^\dagger}{\sqrt{2}} + \frac{\psi_+e^{i\epsilon t}-\psi_-e^{-i\epsilon t}}{-\sqrt{2}i}\frac{\alpha-\alpha^\dagger}{\sqrt{2}i} + \ldots$$

$$\alpha|->=0\ ,\ \alpha^\dagger|->=|+>\ ,\ \alpha^\dagger|+>=0\ ,\ \alpha|+>=|->$$

'local variables': $a = \frac{\alpha+\alpha^\dagger}{\sqrt{2}} = a^\dagger\ ,\ a^2 = \frac{1}{2}\ ;\ b = \frac{\alpha-\alpha^\dagger}{\sqrt{2}i} = b^\dagger\ ,\ b^2 = \frac{1}{2}$

Superpositions of $|+>, |->$ violate $(-1)^F$

classical switch off$= |->$ **on**$= |+>$ (stretched states)

Position of switch measured by $iab|+>=+|+>,\ iab|->=-|->$

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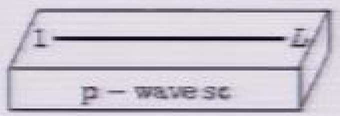
quantum wire + $p_x + ip_y$-superconductor

[12]

A.Kitaev cond-mat/0010440



$$\left\{\psi_k, \psi^\dagger_{k'}\right\} = \delta_{kk'}$$

$$H = \sum_{k=1}^{L} \left\{ \frac{t}{2}\left[\psi^\dagger_{k+1}\psi_k + \psi^\dagger_k\psi_{k+1}\right] + \mu\psi^\dagger_k\psi_k + \frac{\Delta}{2}\psi^\dagger_{k+1}\psi^\dagger_k + \frac{\Delta^*}{2}\psi_k\psi_{k+1} \right\}$$

$t=$ amplitude for electron hopping between neighboring sites

$\Delta=$ amplitude for electrons leave wire as cooper pair

p-wave condensate $\sim <\psi_\sigma \vec{r} \times \vec{\nabla}\psi_\sigma>$, consider one spin state

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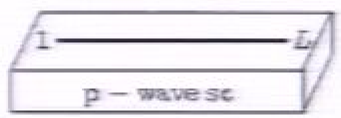

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p-wave condensate $\sim<\psi_\sigma \vec{r}\times\vec{\nabla}\psi_\sigma>$, consider one spin state

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[13]

Choose phases so that Δ, t are real, positive. $t > \Delta > \mu$

Real and imaginary parts of electron $\psi_k(t) = b_k(t) + ic_k(t)$

Schrödinger equation:

$$i\frac{d}{dt}\begin{bmatrix} b_k \\ c_k \end{bmatrix} = \begin{bmatrix} 0 & \mathcal{D} \\ \mathcal{D}^\dagger & 0 \end{bmatrix}\begin{bmatrix} b_k \\ c_k \end{bmatrix} \;, \quad \begin{bmatrix} b_0 \\ c_0 \end{bmatrix} = 0 = \begin{bmatrix} b_{L+1} \\ c_{L+1} \end{bmatrix}$$

$$\mathcal{D}b_k = i\left\{\frac{t}{2}(b_{k+1} + b_{k-1}) + \mu b_k + \frac{\Delta}{2}(b_{k+1} - b_{k-1})\right\}$$

$$\mathcal{D}^\dagger c_k = -i\left\{\frac{t}{2}(c_{k+1} + c_{k-1}) + \mu c_k - \frac{\Delta}{2}(c_{k+1} - c_{k-1})\right\}$$

1-1 mapping of positive to negative energy levels $\begin{bmatrix} b \\ c \end{bmatrix} \to \begin{bmatrix} b \\ -c \end{bmatrix}$

# of zero modes mod2 is a topological invariant

Can be explicitly solved for the half-line: $(L \to \infty)$ $\quad c_k^0 = 0$

$$b_k^0 = 2\sqrt{\frac{t^2\sin^2\phi + \Delta^2\cos^2\phi}{(t^2-\Delta^2)\sin^2\phi}}\left(\frac{t-\Delta}{t+\Delta}\right)^k \sin k\phi \,, \; \phi = \cos^{-1}\frac{\mu}{\sqrt{t^2-\Delta^2}}$$

14

All other modes are in the continuum with

$$E(p) = \sqrt{(t\cos p + \mu)^2 + \Delta^2 \sin^2 p}$$

$$\psi_k(t) = b_k^0 \beta + \int_0^\infty dp \left((b_k(p) + i c_k(p)) \beta_p e^{iE(p)t} + (b_k^*(p) - i c_k^*(p)) e^{-iE(p)t} \gamma_p^\dagger \right)$$

$$\beta = \beta^\dagger \ . \ \beta^2 = \frac{1}{2} \ , \ \{\beta, \beta_p\} = 0 \ , \ \{\beta, \gamma_p\} = 0$$

[15]

- When L is finite, there are two bound states with energies $\pm\epsilon \sim e^{-L}$.
- b_k^0 is localized near $k=1$ and, up to exponentially small corrections, is the same as in the half-line. $c_k^0 = b_{L+1-k}^0$.
- The negative energy bound state has wave-function $(b_k^0 + ic_k^0)e^{-i\epsilon t}$ and the positive energy state is $(b_k^0 - ic_k^0)e^{i\epsilon t}$.

$$\psi_k(t) = (b_k^0 + ib_{L+1-k}^0)e^{i\epsilon t}\beta + (b_k^0 - ib_{L+1-k}^0)e^{-i\epsilon t}\beta^\dagger + \ldots$$

$$= b_k^0\, a \; + \; ib_{L+1-k}^0\, b \; + \ldots$$

$$a = \frac{\beta+\beta^\dagger}{2}\ ,\ a = a^\dagger\ ,\ a^2 = \frac{1}{2}\ ,\ b = \frac{\beta-\beta^\dagger}{2i}\ ,\ b = b^\dagger\ ,\ b^2 = \frac{1}{2}$$

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(16)

$$a = \frac{\beta + \beta^\dagger}{2}\ ,\ a = a^\dagger\ ,\ a^2 = \frac{1}{2}\ ,\ b = \frac{\beta - \beta^\dagger}{2i}\ ,\ b = b^\dagger\ ,\ b^2 = \frac{1}{2}$$

stretched states

$$\beta|- >= 0\ ,\ \beta^\dagger|- >= |+>\ ,\ \beta|+ >= |->\ ,\ \beta|- >= 0$$

$$|\uparrow\rangle = \frac{1}{\sqrt{2}}(|+\rangle + i|-\rangle)$$

$$|\uparrow\rangle = \frac{1}{\sqrt{2}}(|+\rangle - i|-\rangle)$$

$$(-1)^F|\uparrow\rangle = |\uparrow\rangle \qquad (-1)^F|\downarrow\rangle = |\downarrow\rangle$$

What is the quantum amplitude for the transition - after time T – of this state to one with the electron locate at L ?

$$\mathcal{A}_{1L} = \frac{\langle\uparrow|a_L e^{iHt} a_1^\dagger|\uparrow\rangle}{\mathcal{N}} = 2|c_1^0|^2 + p = 4\frac{\Delta}{t}\frac{t^2+\mu^2}{(t+\Delta)^2} \sim \frac{1}{2}$$

$$\mathcal{N} = |a_L^\dagger|\uparrow\rangle|\,|a_1^\dagger|\uparrow\rangle|$$

14

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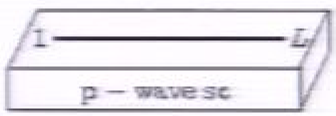
$$\beta = \beta^\dagger \ . \ \beta^2 = \frac{1}{2} \ , \ \{\beta, \beta_p\} = 0 \ , \ \{\beta, \gamma_p\} = 0$$

quantum wire + $p_x + ip_y$-superconductor

[12]

A.Kitaev cond-mat/0010440



$$\left\{\psi_k, \psi_{k'}^\dagger\right\} = \delta_{kk'}$$

$$H = \sum_{k=1}^{L} \left\{ \frac{t}{2} \left[\psi_{k+1}^\dagger \psi_k + \psi_k^\dagger \psi_{k+1} \right] + \mu \psi_k^\dagger \psi_k + \frac{\Delta}{2} \psi_{k+1}^\dagger \psi_k^\dagger + \frac{\Delta^*}{2} \psi_k \psi_{k+1} \right\}$$

t= amplitude for electron hopping between neighboring sites

Δ= amplitude for electrons leave wire as cooper pair

p-wave condensate $\sim < \psi_\sigma \vec{r} \times \vec{\nabla} \psi_\sigma >$, consider one spin state

Choose phases so that Δ, t are real, positive. $t > \Delta > \mu$ [13]

Real and imaginary parts of electron $\psi_k(t) = b_k(t) + ic_k(t)$

Schrödinger equation:

$$i\frac{d}{dt}\begin{bmatrix} b_k \\ c_k \end{bmatrix} = \begin{bmatrix} 0 & \mathcal{D} \\ \mathcal{D}^\dagger & 0 \end{bmatrix}\begin{bmatrix} b_k \\ c_k \end{bmatrix} \quad , \quad \begin{bmatrix} b_0 \\ c_0 \end{bmatrix} = 0 = \begin{bmatrix} b_{L+1} \\ c_{L+1} \end{bmatrix}$$

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1-1 mapping of positive to negative energy levels $\begin{bmatrix} b \\ c \end{bmatrix} \to \begin{bmatrix} b \\ -c \end{bmatrix}$

# of zero modes mod2 is a topological invariant

Can be explicitly solved for the half-line: $(L \to \infty)$ $\quad c_k^0 = 0$

$$b_k^0 = 2\sqrt{\frac{t^2\sin^2\phi + \Delta^2\cos^2\phi}{(t^2 - \Delta^2)\sin^2\phi}}\left(\frac{t-\Delta}{t+\Delta}\right)^k \sin k\phi\,, \ \phi = \cos^{-1}\frac{\mu}{\sqrt{t^2 - \Delta^2}}$$

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16

$$a = \frac{\beta + \beta^\dagger}{2} \ , \ a = a^\dagger \ , \ a^2 = \frac{1}{2} \ , \ b = \frac{\beta - \beta^\dagger}{2i} \ , \ b = b^\dagger \ , \ b^2 = \frac{1}{2}$$

stretched states

$$\beta|- >= 0 \ , \ \beta^\dagger|- >= |+ > \ , \ \beta|+ >= |- > \ , \ \beta|- >= 0$$

[15]

- When L is finite, there are two bound states with energies $\pm\epsilon \sim e^{-L}$.
- b^0_k is localized near $k = 1$ and, up to exponentially small corrections, is the same as in the half-line. $c^0_k = b^0_{L+1-k}$.
- The negative energy bound state has wave-function $(b^0_k + ic^0_k)e^{-i\epsilon t}$ and the positive energy state is $(b^0_k - ic^0_k)e^{i\epsilon t}$.

$$\psi_k(t) = (b^0_k + ib^0_{L+1-k})e^{i\epsilon t}\beta + (b^0_k - ib^0_{L+1-k})e^{-i\epsilon t}\beta^\dagger + \ldots$$

$$= b^0_k \; a \; + \; ib^0_{L+1-k} \; b \; + \ldots$$

$$a = \frac{\beta + \beta^\dagger}{2} \; , \; a = a^\dagger \; , \; a^2 = \frac{1}{2} \; , \; b = \frac{\beta - \beta^\dagger}{2i} \; , \; b = b^\dagger \; , \; b^2 = \frac{1}{2}$$

All other modes are in the continuum with

$$E(p) = \sqrt{(t\cos p + \mu)^2 + \Delta^2 \sin^2 p}$$

$$\psi_k(t) = b_k^0 \beta + \int_0^\infty dp \left((b_k(p) + ic_k(p)) \beta_p e^{iE(p)t} + (b_k^*(p) - ic_k^*(p)) e^{-iE(p)t} \gamma_p^\dagger \right)$$

$$\beta = \beta^\dagger \ , \ \beta^2 = \frac{1}{2} \ , \ \{\beta, \beta_p\} = 0 \ , \ \{\beta, \gamma_p\} = 0$$

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$$|\uparrow\rangle = \frac{1}{\sqrt{2}}(|+\rangle + i|-\rangle)$$

$$|\uparrow\rangle = \frac{1}{\sqrt{2}}(|+\rangle - i|-\rangle)$$

$$(-1)^F|\uparrow\rangle = |\uparrow\rangle \qquad (-1)^F|\downarrow\rangle = |\downarrow\rangle$$

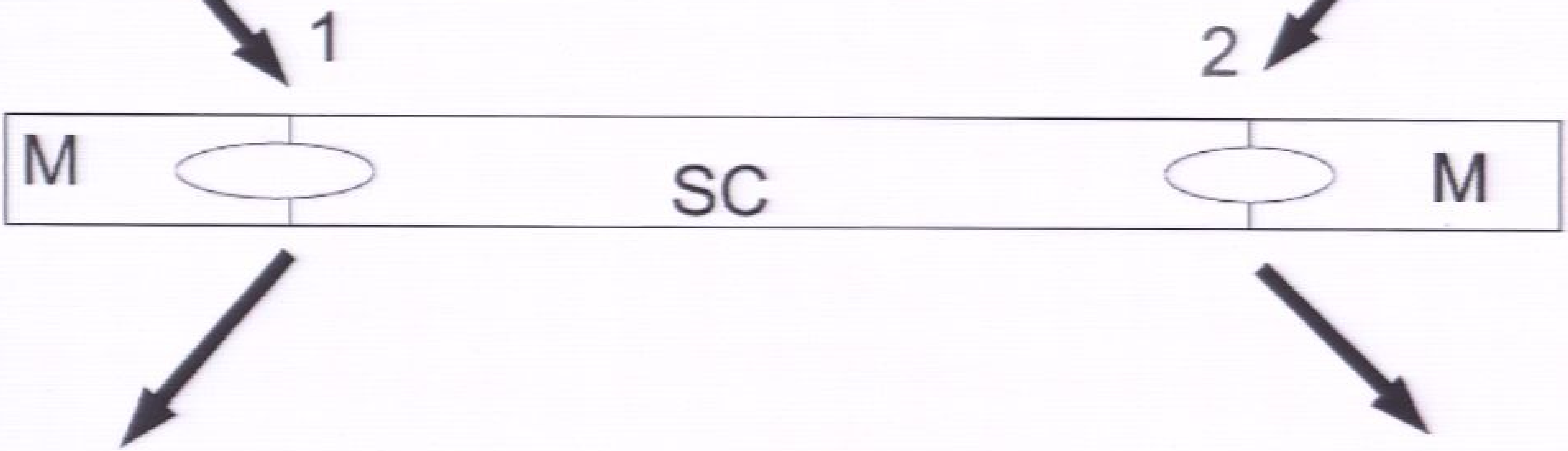
What is the quantum amplitude for the transition - after time T – of this state to one with the electron locate at L ?

$$\mathcal{A}_{1L} = \frac{\langle\uparrow|a_L e^{iHt} a_1^\dagger|\uparrow\rangle}{\mathcal{N}} = 2|c_1^0|^2 + p = 4\frac{\Delta}{t}\frac{t^2+\mu^2}{(t+\Delta)^2} \sim \frac{1}{2}$$

$$\mathcal{N} = |a_L^\dagger|\uparrow\rangle|\,|a_1^\dagger|\uparrow\rangle|$$

Weakly coupled Majorana Bound States

S. Bose - P.S. arXiv 1010.0709 1a



$$H_M = iE_M \gamma_1 \gamma_2$$

$$\gamma_j \gamma_k + \gamma_k \gamma_j = 2\delta_{ij}$$

e.g., Kitaev, Fu-Kane, Beenakker et al, Refael et.al.

$$S(E) = 1 + 2\pi i W^\dagger (H_M - E - i\pi W W^\dagger)^{-1} W$$

(2a)

$E \approx E_M$ (a *different* regime than the crossed Andreev reflection of Beenakker et al)

$$\{|e_1\rangle, |e_2\rangle, |h_1\rangle, |h_2\rangle\}$$

$$S = \frac{1}{2}\begin{pmatrix} 1 & -i & -1 & -i \\ i & 1 & i & -1 \\ -1 & -i & 1 & -i \\ i & -1 & i & 1 \end{pmatrix}$$

Four-port beam splitter, but ports are both spatial & in isospin space

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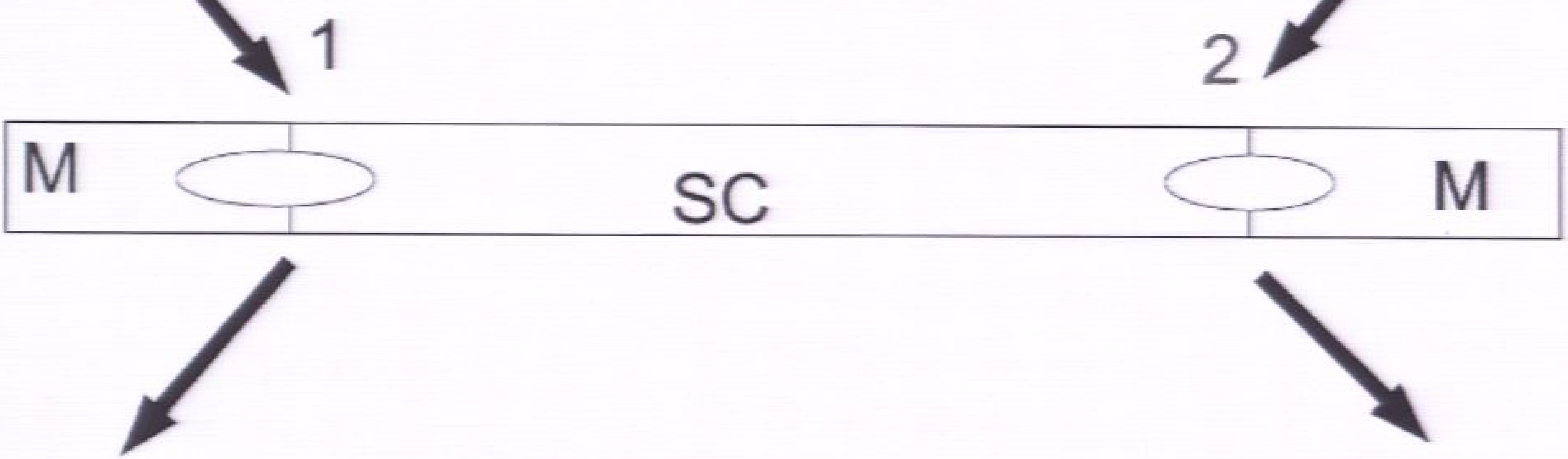

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Bose & Sodano, arXiv:1010.0709 (oct, 2010)

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$$c_1^\dagger|0\rangle \xrightarrow{\text{MBS}} \frac{1}{2}(c_1^\dagger + ic_2^\dagger - d_1^\dagger + id_2^\dagger)$$

$$c_1^\dagger c_2^\dagger|0\rangle \xrightarrow{MBS} \frac{1}{2}(c_1^\dagger + ic_2^\dagger - d_1^\dagger$$

$$+id_2^\dagger)\ \frac{1}{2}(-ic_1^\dagger + c_2^\dagger - id_1^\dagger - d_2^\dagger)|0\rangle$$

$$= \frac{1}{2}(ic_1^\dagger d_1^\dagger - c_1^\dagger d_2^\dagger + c_2^\dagger d_1^\dagger + ic_2^\dagger d_2^\dagger)|0\rangle.$$

$$d_j^\dagger(E) = c_j(-E)$$

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$$\begin{aligned} c_1^\dagger c_2^\dagger|0\rangle \xrightarrow{MBS} & \frac{1}{2}(c_1^\dagger + ic_2^\dagger - d_1^\dagger \\ & +id_2^\dagger) \; \frac{1}{2}(-ic_1^\dagger + c_2^\dagger - id_1^\dagger - d_2^\dagger)|0\rangle \\ = & \frac{1}{2}(ic_1^\dagger d_1^\dagger - c_1^\dagger d_2^\dagger + c_2^\dagger d_1^\dagger + ic_2^\dagger d_2^\dagger)|0\rangle. \end{aligned}$$

$$d_j^\dagger(E) = c_j(-E)$$

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Scattering matrix for deviations

$$S_{\delta E} = \frac{i\Gamma}{\delta E + i\Gamma} S + \frac{\delta E}{\delta E + i\Gamma} I$$

Current noise correlations

$$P_{ij} = \frac{e^2}{h\nu} \frac{\Gamma^2}{\{(\delta E)^2 + \Gamma^2\}^2} \{(\delta E)^2 - \Gamma^2\} \delta_{\epsilon_1, \epsilon_2}$$

Conventional anti-bunching

Anti-bunching in iso-spin space

Refael, Oreg, van Oppen:

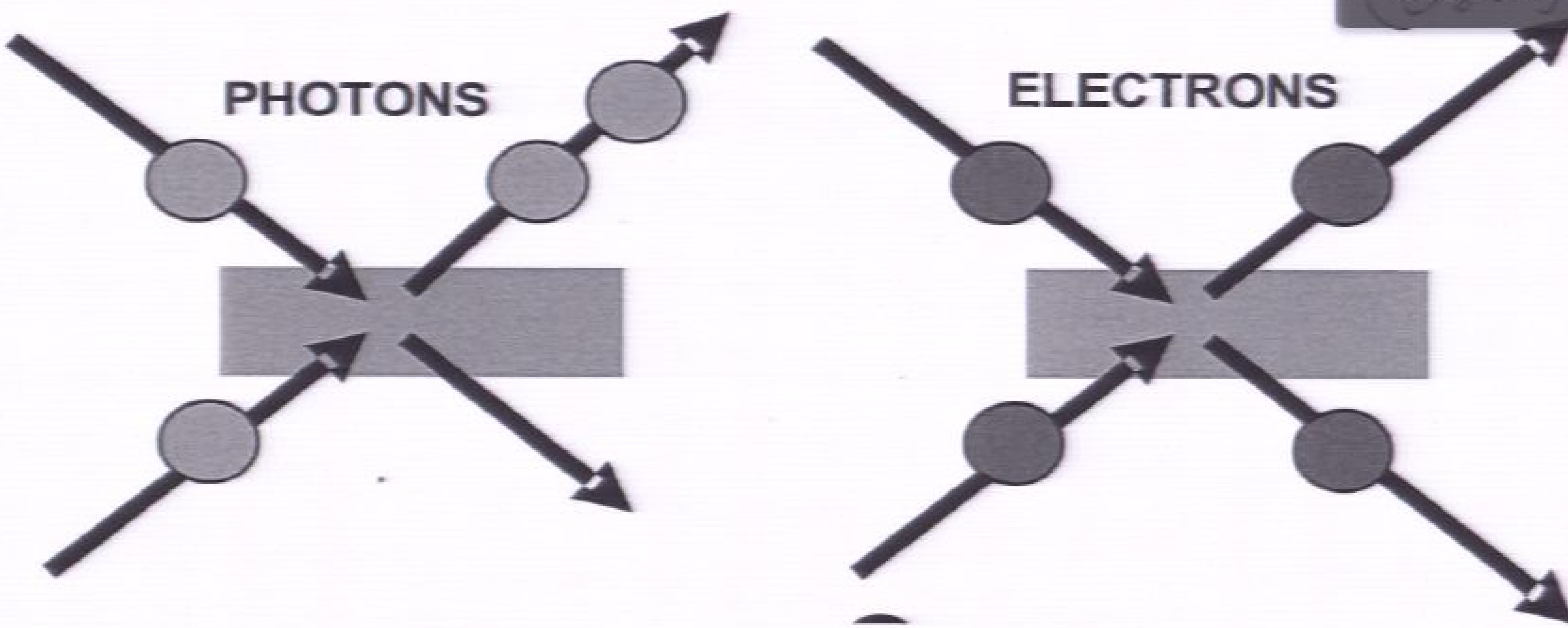
$$\gamma_1 = \frac{1}{\sqrt{2}}(c_{1,\uparrow} - ic_{1,\downarrow} + ic^\dagger_{1,\downarrow} + c^\dagger_{1,\downarrow})$$

$$\gamma_2 = \frac{1}{\sqrt{2}}(c_{1,\uparrow} + ic_{1,\downarrow} - ic^\dagger_{1,\downarrow} + c^\dagger_{1,\downarrow}),$$

$$c^\dagger_{1,\uparrow}c^\dagger_{2,\uparrow}|0\rangle \xrightarrow{MBS} \frac{1}{4}(c^\dagger_{1,\uparrow}c^\dagger_{2,\uparrow} - c^\dagger_{1,\downarrow}c^\dagger_{2,\downarrow} + 2c^\dagger_{1,\uparrow}c^\dagger_{2,\downarrow} + \quad ...)|0\rangle,$$

$$|\xi\rangle_{12} = \frac{1}{\sqrt{6}}(|\uparrow\rangle_1|\uparrow\rangle_2 - |\downarrow\rangle_1|\downarrow\rangle_2 + 2|\uparrow\rangle_1|\downarrow\rangle_2)$$

Bose & Sodano, arXiv:1010.0709 (oct, 2010)



To test quantum indistinguishability you need to bring identical particles together at a beam splitter

Spatial bunching and anti-bunching has been tested.

Conclusions:

- Relevant non local effects of Majorana bound modes

A) Handbury - Brown Twiss interferometer between well separated Dirac fermions

B) Non-local entangler of electronic spin

A) – B) can be observed in various settings

enable us to detect presence of Majoranas