Title: Free energy of ABJM theory

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Abstract: ABJM theory is a world-volume theory for an arbitrary number of M2-branes. One of the unique features of ABJM theory is its characteristic scaling behaviour, exhibited for example by the free energy and correlation functions of chiral primary operators. In more detail, ABJM theory has a holographic dual where thermodynamics at strong coupling is determined by a system of black M2-branes. The zero-coupling (black-body radiation) free energy disagrees with the strong coupling result. Even the scaling in the 't Hooft coupling is different (strongly suppressed at strong coupling). It is therefore important to check that the weak and strong coupling results converge as loop corrections are taken into account. The leading order computation indeed confirms that the first correction goes in the right direction.


WANTED
WORLD-VOLUME THEORY FOR M2-BRANES
REWARD

History Highlights

wanted: field theory for M2-branes

Schwarz '04: no-go for U(N)

BL '06: three-algebra

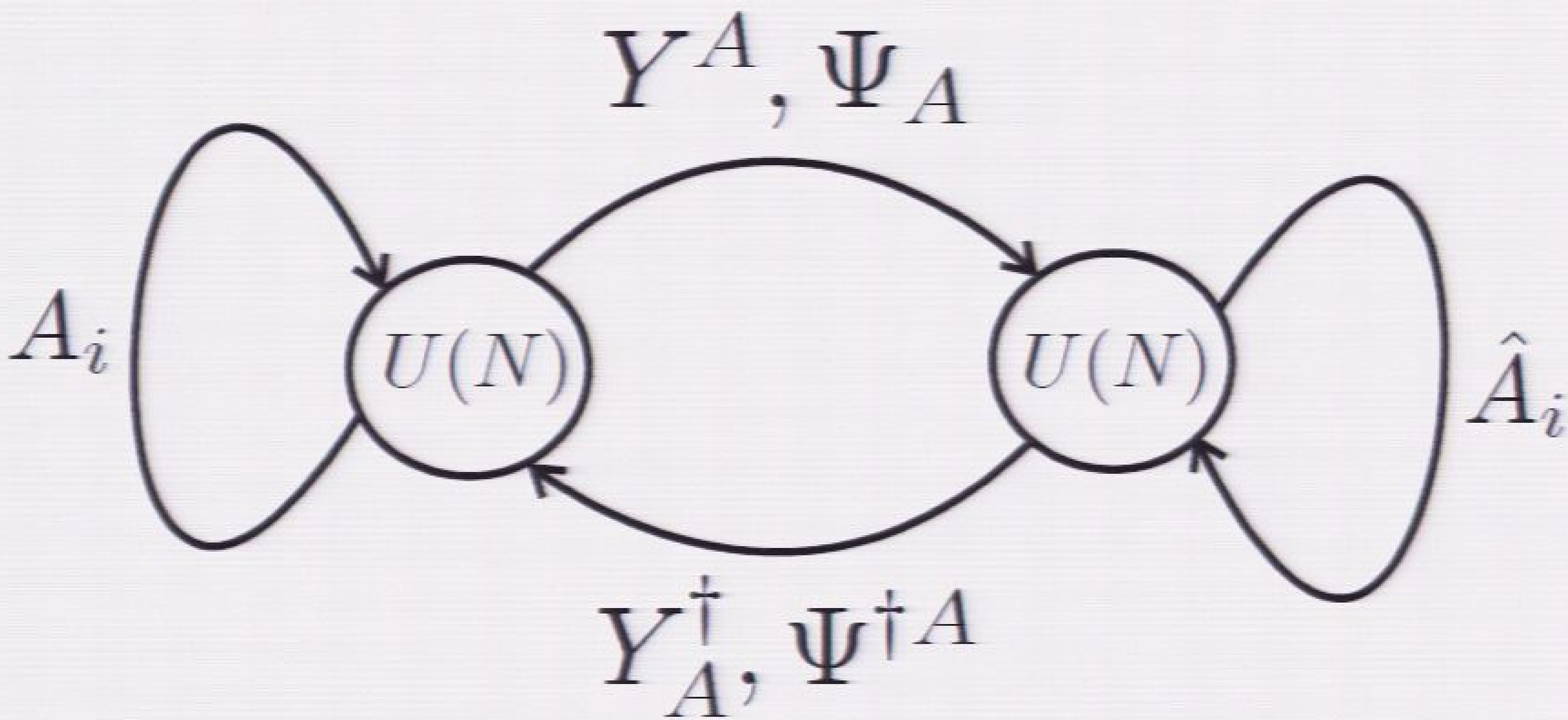
BLG '07 SUSY algebra closes

only valid for two M2-branes

ABJM '08: U(N) x U(N) Chern-Simons theory

[Gustavsson Sept '07; Bagger, Lambert Nov ´06, Nov '07, Dec '0
Aharony, Bergman, Jafferis, Maldacena June '08]

ABJM Theory



ABJM Theory

- $\mathcal{N} = 2$ superspace formulation:

$$\boxed{\mathcal{S} = \mathcal{S}_{\rm CS} + \mathcal{S}_{\rm mat} + \mathcal{S}_{\rm pot}}$$

- gauge and matter fields: $\mathcal{V}, \hat{\mathcal{V}}, \mathcal{Z}^A, \mathcal{W}_A (A = 1, 2)$

bifundamental → $\mathcal{Z} = Z + \theta\zeta + \theta^2 F$

conjugate rep. → $\mathcal{W} = W + \theta\omega + \theta^2 G$

ABJM Theory

- change the superpotential: (equal for N=2)

SU(2) x SU(2) global

$$W_{\mathrm{ABJM}} = \epsilon_{AC}\epsilon^{BD}\,\mathrm{tr}\,\mathcal{Z}^A\mathcal{W}_B\mathcal{Z}^C\mathcal{W}_D$$

$$W_{\mathrm{BLG}} = \epsilon_{ABCD}\,\mathrm{tr}\,\tilde{\mathcal{Z}}^A\tilde{\mathcal{Z}}^{\ddagger B}\tilde{\mathcal{Z}}^C\tilde{\mathcal{Z}}^{\ddagger D}$$ (BKKS 2008)

SU(4) global

$$Z^{\ddagger A} = X^{\dagger A} + iX^{\dagger A+4}$$

only N=2

ABJM and AdS/CFT

AdS/CFT

The ABJM model is a U(N) x U(N) gauge theory. It describes an **arbitrary number of M2-branes** on $\mathbb{C}^4/\mathbf{Z}_k$. Its gravity dual is M-theory on $AdS_4 \times S^7/\mathbf{Z}_k$.

[Aharony, Bergman, Jafferis, Maldacena 2008]

AdS/CFT

$$\lambda = \frac{N}{k} = \frac{R^4}{l_s^4}$$

k

low energy field theory on
N M2-branes at $\mathbb{C}^4/\mathbf{Z}_k$
(k=1: IR limit of field theory on D2's)

$k = N$

IIA theory on $AdS_4 \times \mathbb{C}\mathbf{P}^3$

$k = N^{1/5}$

M-theory on $AdS_4 \times S^7/\mathbf{Z}_k$

Interpretation

- analyze the vacuum state V=0 (moduli space).
- potential vanishes for diagonal Z^A , W_A .
- naive expectation: moduli space is $\mathbb{C}^{4N}$.
- but: there are residual gauge transformations.

$$U(N) \times U(N) \rightarrow U(1)^N \times U(1)^N \times S_N$$

Conclusion: The moduli space is $\left(\mathbb{C}^4/\mathbf{Z}_k\right)^N/S_N$.
Interpretation: N M2-branes on a space with a $\mathbf{Z}_k$ singularity ($y^A \rightarrow e^{2\pi i/k} y^A$ for A=1,2,3,4).

SUSY: Gravity Side

- 8 transverse coordinates: $y^A \in \mathbb{C}^4$
- ABJM model, $\mathbb{Z}_k$ orbifold:

$$y^A \to e^{2\pi i/k} y^A$$
$$\Psi \to e^{2\pi i(s_1+s_2+s_3+s_4)/k} \Psi$$

- need two spins up, two spins down: $\boxed{\mathcal{N}=6}$
- for k=1,2: even $\boxed{\mathcal{N}=8}$
- but no more, due to the chirality condition

$$s_1 + s_2 + s_3 + s_4 \in 2\mathbb{Z}$$

SUSY: Field Theory Side

- Generic: Only $U(1)_R$ symmetry, i.e. $\mathcal{N}=2$
- ABJM: The orbifold action preserves an SU(4):

$$Y^A = \{Z^A, W^{\dagger A}\}$$

R-charges are (+1,+1,-1,-1).

- Claim: The action is **invariant under this SU(4)**, which mixes R-charges in more general ways.
- So the R-symmetry is **enhanced** to SU(4), i.e. $\mathcal{N}=6$ SUSY.

ABJM: Bosonic Potential

ABJM made the SU(4) R-symmetry **manifest in the bosonic potential** by a field redefinition.

$$Y^A = \{Z^A, W^{\dagger A}\}$$

$$Y^\dagger_A = \{Z^\dagger_A, W_A\}$$

$$V^{\text{bos}} = \text{tr}\Big[Y^A Y^\dagger_A Y^B Y^\dagger_B Y^C Y^\dagger_C + Y^\dagger_A Y^A Y^\dagger_B Y^B Y^\dagger_C Y^C + 4 Y^A Y^\dagger_B Y^C Y^\dagger_A Y^B Y^\dagger_C - 6 Y^A Y^\dagger_B Y^B Y^\dagger_A Y^C Y^\dagger_C\Big]$$

ABJM: Fermionic Potential

Benna, Klebanov, Klose, M.S. 2008: we show how to make the SU(4) R-symmetry manifest also in the fermionic potential, concluding the proof of $\mathcal{N} = 6$ SUSY in ABJM.

$$\psi_A = \{\epsilon_{AB}\zeta^B e^{-i\pi/4}, -\epsilon_{AB}\omega^{\dagger B} e^{i\pi/4}\}$$

$$V^{\text{ferm}} = \frac{iL}{4}\text{tr}\Big[Y_A^\dagger Y^A \psi^{B\dagger}\psi_B - Y^A Y_A^\dagger \psi_B \psi^{B\dagger} + 2Y^A Y_B^\dagger \psi_A \psi^{B\dagger} - 2Y_A^\dagger Y^B \psi^{A\dagger}\psi_B$$

$$- \epsilon^{ABCD} Y_A^\dagger \psi_B Y_C^\dagger \psi_D + \epsilon_{ABCD} Y^A \psi^{B\dagger} Y^C \psi^{D\dagger}\Big]$$

Schwarz (July '08): finds the explicit supercharges

SU(4)-invariant Action:

$$S = \frac{k}{2\pi}\int d^3x\, L$$

$$\begin{aligned} L = \epsilon^{ijk}\mathrm{tr}[&- \frac{i}{2}A_i\partial_j A_k + \tfrac{1}{3}A_i A_j A_k \\ &- (A \leftrightarrow \hat{A})] \\ &+ \mathrm{tr}(D_i Y_A)^\dagger D^i Y^A \\ &+ i\mathrm{tr}\psi^{\dagger A}\gamma^\mu D_\mu \psi_A \\ &+ V^{\mathrm{bos}}(Y) + V^{\mathrm{ferm}}(Y,\Psi)] \end{aligned}$$

- Right symmetries: $\mathrm{OSp}(6|4)$ and parity-invariance

Free energy

Unique Features of ABJM:

- world-volume theory for M2-branes
  (matching symmetries, matching moduli space)
- condensed matter systems
  (conformal fixed points of 3D Chern-Simons systems)
- study the landscape
  (4D string backgrounds with negative cosm. const.)
- monopole operators
  (needed for enhancement to $\mathcal{N} = 8$)
- spin chains and integrability
  (intregrability of ABJM)
- scaling properties
  (unique scaling $\sim \lambda^{-1/2}$)

Thermodynamics and AdS/CFT

- Finite temperature dual to black holes in AdS

In $\mathcal{N} = 4$ super-Yang Mills:

- free energy consistent with smooth interpolation
- weak coupling: next-to-leading order correction non-analytical in the coupling $(\lambda^{3/2})$.

In $\mathcal{N} = 6$ ABJM theory:

- free energy has been computed at strong coupling
- similar behaviour at weak coupling?

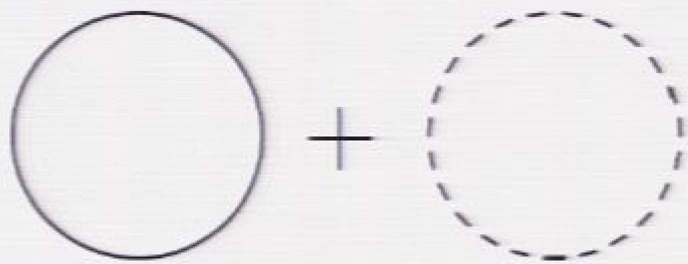
[Gubser, Klebanov, Peet,Tseytlin 1996, 1998]

[Fotopoulos, Taylor 1998; Vazquez-Mozo; Kim, Rey; Nieto, Tytgat 1999]

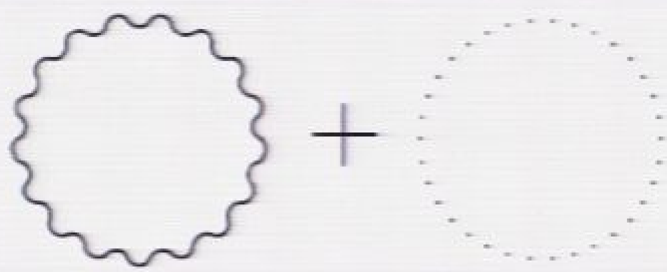
Prescription to Compute F

- Write down all vacuum diagrams.
- Sum them up.
- Can we do this order by order?

F=0 in Flat Theory. Ex: 1-loop:

$\Delta F =$  $= N^2 \left(8\tfrac{1}{2}A - 8\tfrac{1}{2}A\right) = 0$

supersymmetry

$\Delta F =$  $= 0$

non-propagating

$$A = \int \frac{d^3p}{(2\pi)^3} \log(p^2)$$

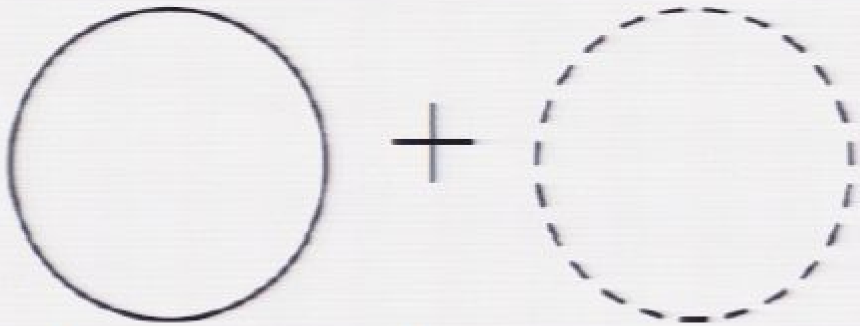
F(T) on $\mathbb{R}^2 \times S^1$:

- Compactify the time direction.
- Write down all vacuum diagrams.
- Sum them up, using appropriate boundary conditions:

$$Y^A(x^1, x^2, x^3 + L) = +Y^A(x^1, x^2, x^3),$$
$$\psi_A(x^1, x^2, x^3 + L) = -\psi_A(x^1, x^2, x^3),$$
$$A_i(x^1, x^2, x^3 + L) = +A_i(x^1, x^2, x^3),$$
$$c(x^1, x^2, x^3 + L) = +c(x^1, x^2, x^3).$$

- Breaks supersymmetry and conformal invariance.

1-loop Order:

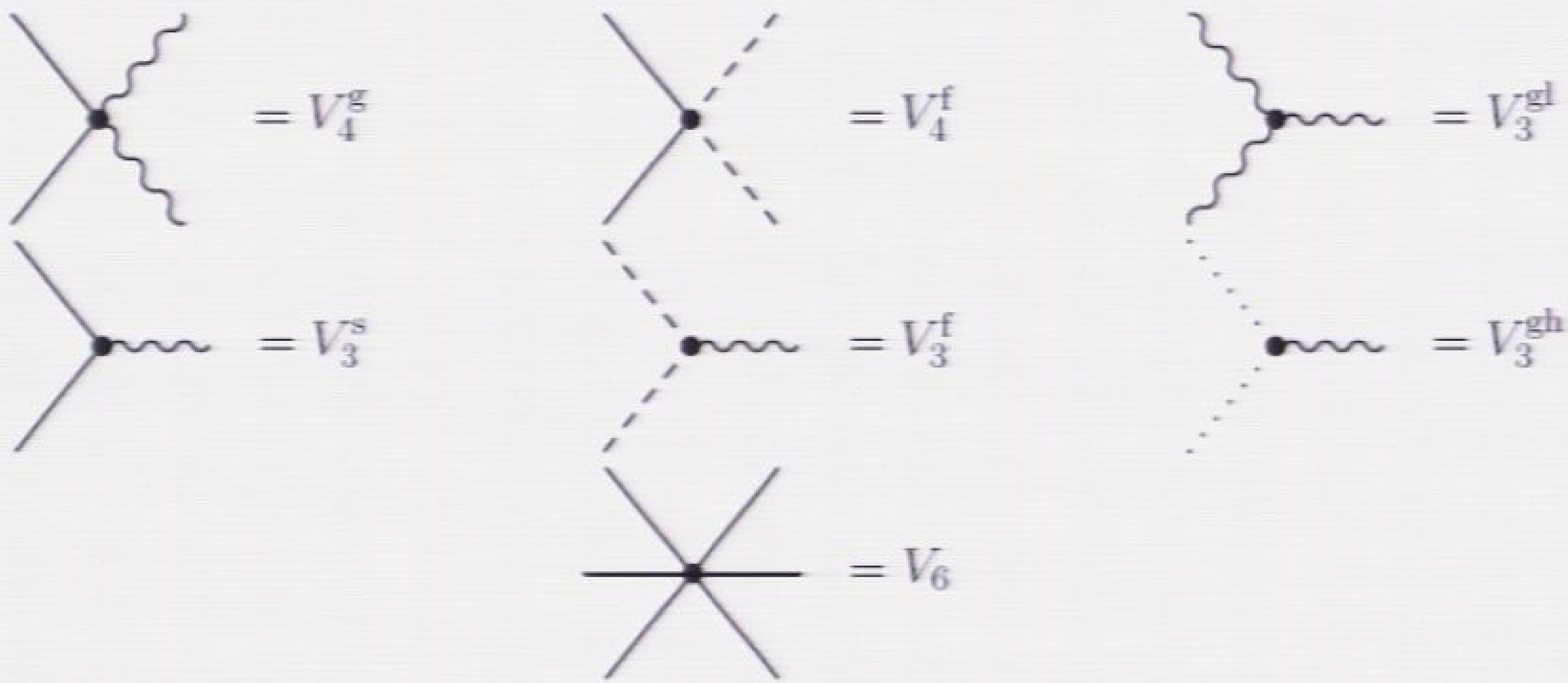


$$F = N^2 \left(8\tfrac{1}{2}A_0 - 8\tfrac{1}{2}A_{1/2}\right) = -N^2 T^3 \frac{7\zeta(3)}{\pi}$$

$$A_\nu = \int \frac{d^2p}{(2\pi)^2} T \sum_{n=\in\mathbb{Z}+\nu} \log(\vec{p}^2 + \omega_n^2)$$

$$\omega_n = 2\pi T n$$

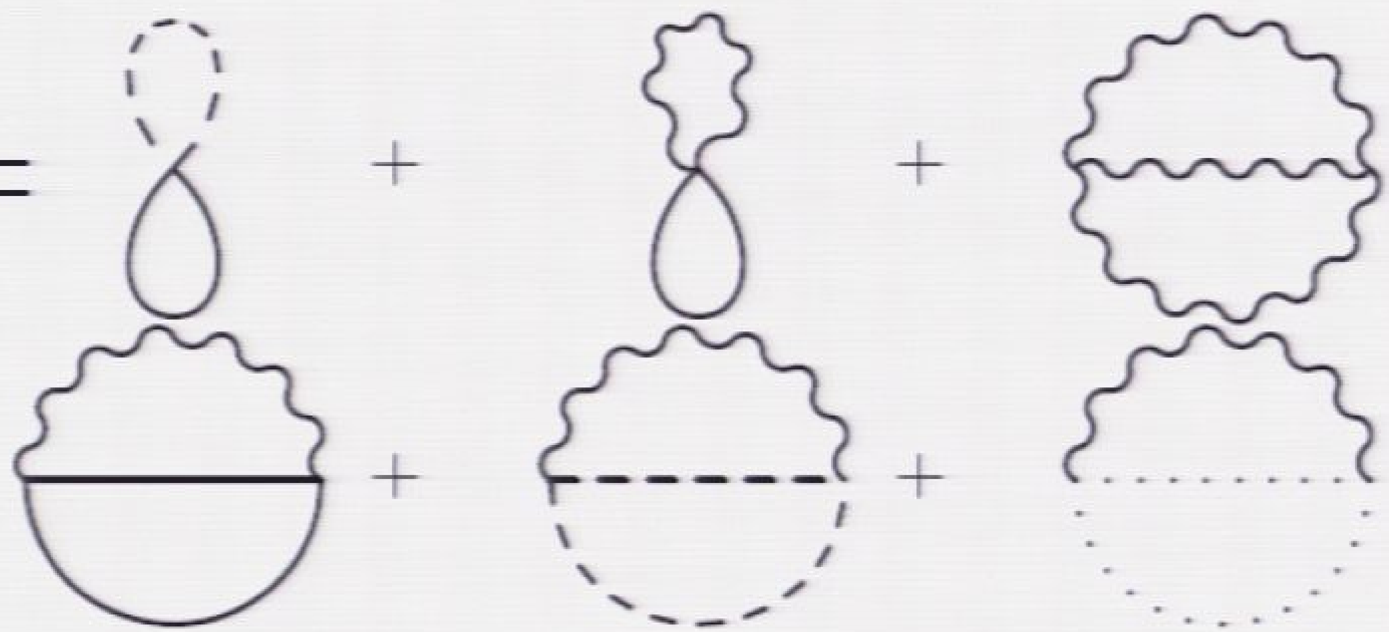
Higher Loops: Interactions



- 2-loop order: type (4), (33)
- 3-loop order: type (6), (44), (433), (3333)

2-loop Order: No $\mathcal{O}(\lambda)$ Correction

$F =$



$= 0$

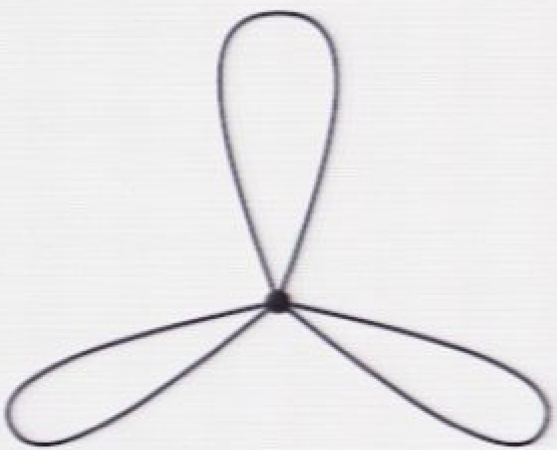
Example Diagram:

$= 0$

$$< A_n(\vec{p}) A_{-n}(-\vec{p}) >= -\frac{2\pi}{kT}\epsilon^{ijk}\frac{p_k}{\vec{p}^2+\omega_n^2}$$

$$V_4^{\mathrm{g}} = \mathrm{tr}[Y_A^\dagger A^i A^i Y^A]$$

3-Loop Problem: IR Divergences



$$\sim \left(\int \frac{d^2p}{(2\pi)^2} T \sum_{n\in\mathbb{Z}} \frac{1}{\vec{p}^2 + \omega_n^2} \right)^3$$

$$< Y_n(\vec{p})\, Y^{\dagger}_{-n}(-\vec{p}) >= \frac{2\pi}{kT} \frac{1}{\vec{p}^2 + \omega_n^2}$$

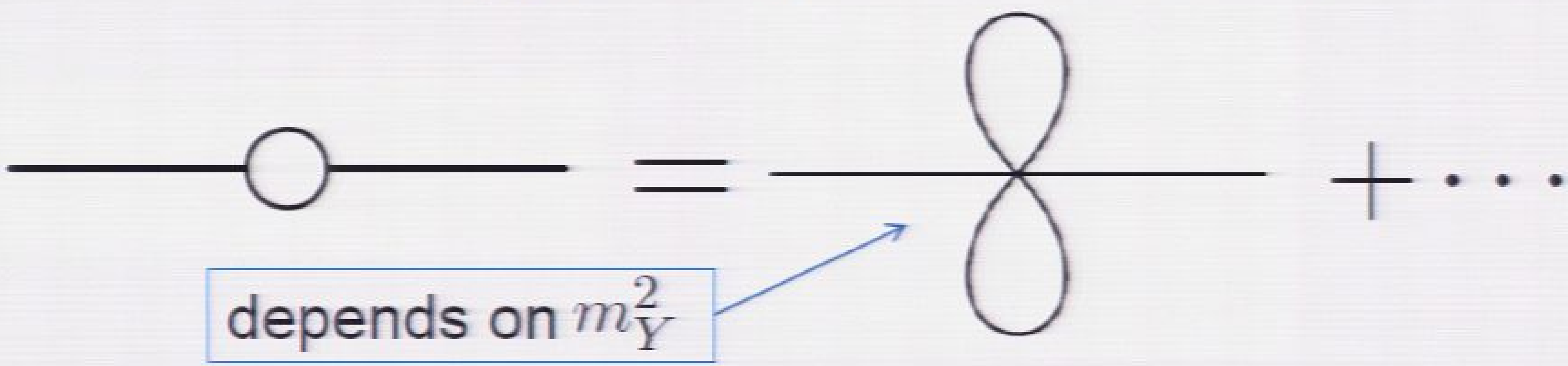
$$\omega_n = 2\pi T n$$

problem: zero modes

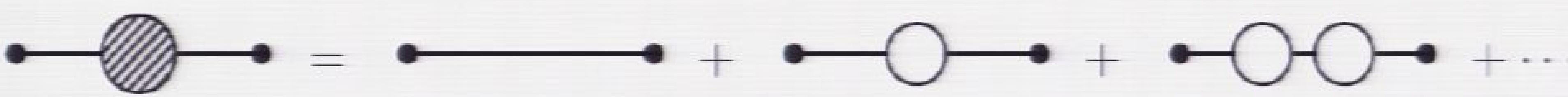
Thermal mass

Regularization: Thermal Mass

- Scalar self-energy (1PI diagrams in static limit):



- Sum them up to find the scalar thermal mass m_Y^2:



1-loop:

$$\sim q_i \left(\gamma^i\right)_\alpha{}^\alpha = 0$$

$$\sim \epsilon^{ijk}\delta_{ij} = 0$$

$$\sim \epsilon^{ijk} q_i q_j = 0$$

Use static limit (no external momentum) to regularize the zero modes (resummation choice)

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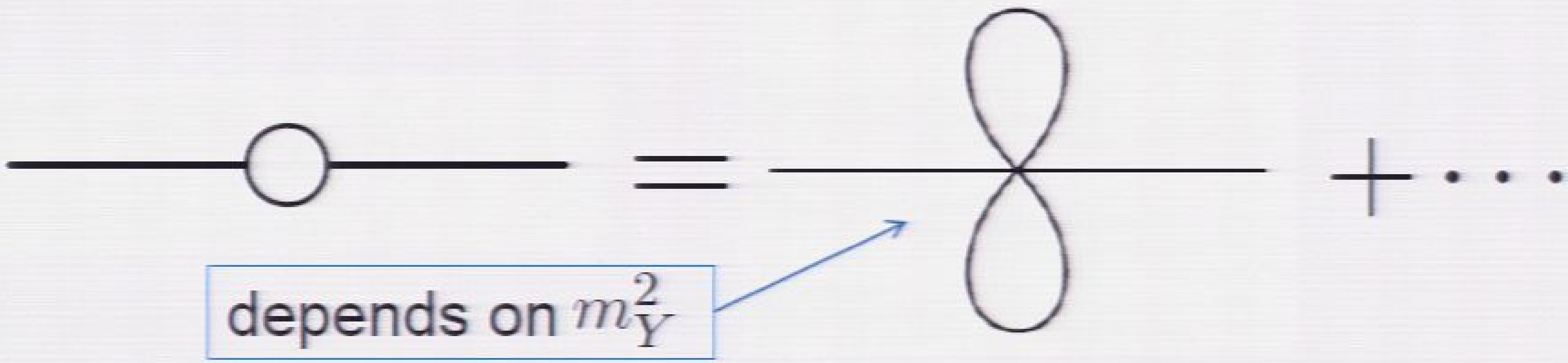

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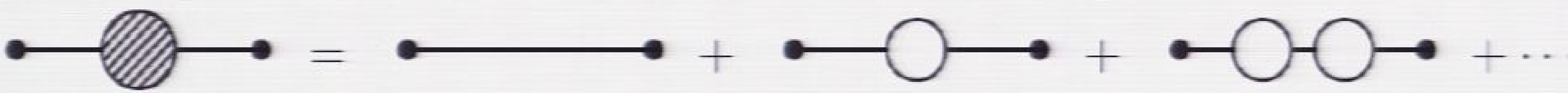

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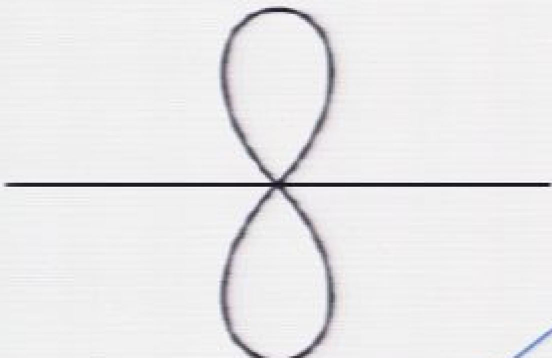
2-loop Example Diagram:



$$= -42 \cdot 2\pi k T \lambda^2 \left(\int \frac{d^2p}{(2\pi)^2} T \sum_{n\in\mathbb{Z}} \frac{1}{\vec{p}^2 + \omega_n^2 + m_Y^2} \right)^2$$

one vertex

two loops

$$V^{\text{bos}} = A^{BDF}_{ACE} Y^A Y^\dagger_B Y^C Y^\dagger_D Y^E Y^\dagger_F$$

$$A^{BDF}_{ACE} = -\tfrac{1}{3}\delta^B_A\delta^D_C\delta^F_E - \tfrac{1}{3}\delta^F_A\delta^B_C\delta^D_E - \tfrac{4}{3}\delta^D_A\delta^B_E\delta^F_C + 2\delta^D_A\delta^B_C\delta^F_E$$

Gauge Choice:

- Lorentz gauge $\partial_1 A^1 + \partial_2 A^2 + \partial_3 A^3 = 0$:

$$< A_n(\vec{p}) A_{-n}(-\vec{p}) >= -\frac{2\pi}{kT}\epsilon^{ijk}\frac{p_k}{\vec{p}^2 + \omega_n^2}$$

- Coulomb gauge $\partial_1 A^1 + \partial_2 A^2 = 0$:

$$< A_n(\vec{p}) A_{-n}(-\vec{p}) >= -\frac{2\pi}{kT}(\epsilon^{ij1}p_1 + \epsilon^{ij2}p_2)\frac{1}{\vec{p}^2}$$

- Lorentz gauge: simpler combinatorics
- Coulomb gauge: simpler loop integrals

Thermal Mass - Result:

$$= \frac{2\pi}{kT}\frac{1}{\vec{p}^2+\omega_n^2+m_Y^2}$$

$$m_Y^2(\lambda) = (2\pi T)^2\mu^2(\lambda)$$

$$\mu^2(\lambda) = \frac{118}{3(2\pi)^2}\lambda^2\log(\mu)^2 + \mathcal{O}(\lambda^2\log(\lambda))$$

$$\omega_n = 2\pi T n$$

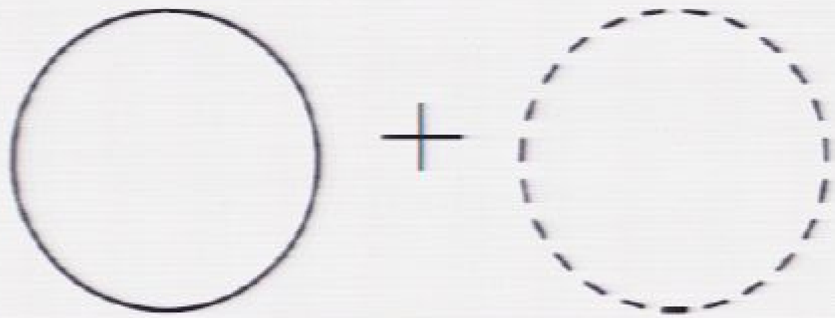
both sides depend on m_Y^2

[self-consistent treatment: D’Hoker 198

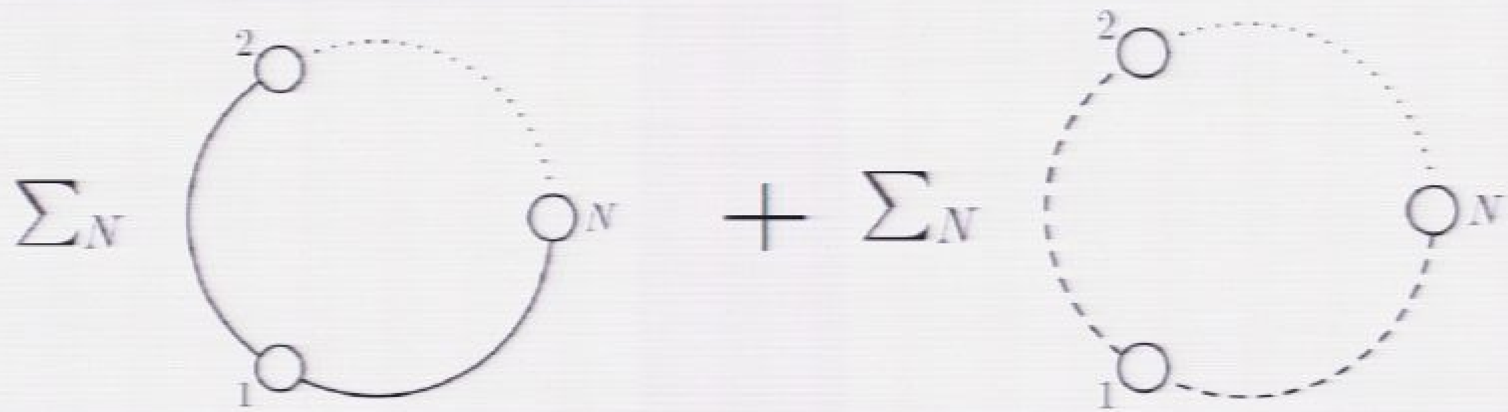
Reorganized Expansion

Redo 1-loop Free Energy

- Before:



- After (resummation i.e. massive propagators):



- Reorganized perturbation theory: all orders in λ.

In Formulas:

$$F = N^2\left(8\tfrac{1}{2}A_0(m_Y^2) - 8\tfrac{1}{2}A_{1/2}\right) = -N^2T^3 f(\lambda)$$

$$A_\nu(m) = \int \frac{d^2p}{(2\pi)^2} T \sum_{n=\in\mathbb{Z}+\nu} \log(\vec{p}^2 + \omega_n^2 + m^2)$$

$$< Y_n(\vec{p})\, Y^\dagger_{-n}(-\vec{p}) >= \frac{2\pi}{kT}\,\frac{1}{\vec{p}^2 + \omega_n^2 + m_Y^2}$$

Coupling Dependence

- answer: $F = -N^2 T^3 f(\lambda)$

$$f(\lambda) = \left[\frac{7\zeta(3)}{\pi} + \frac{m_Y^2(\lambda)}{\pi T^2} \log\left(\frac{m_Y^2(\lambda)}{T^2}\right) + \mathcal{O}(\lambda^2 \log(\lambda)^2) \right]$$

- approximate answer:

$$f_{\text{approx}}(\lambda) = \left[\frac{7\zeta(3)}{\pi} + \frac{236}{3\pi} \lambda^2 \log(\lambda)^3 + \mathcal{O}(\lambda^2 \log(\lambda)^2) \right]$$

similar behaviour observed by Gaiotto, Yin (2007) in other abelian 3D CS-matter theories

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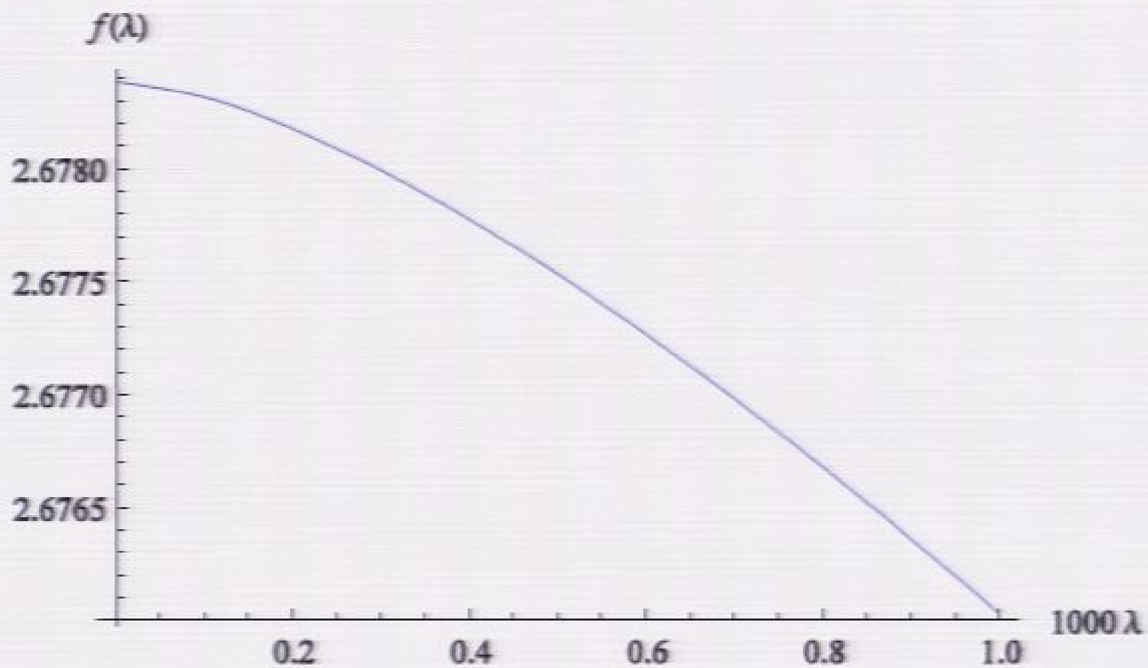
Dependence on the Coupling



- indicates smooth matching to strong coupling result:

$$f_s(\lambda) = \left[\frac{2^{7/2}}{9} \pi^2 \frac{1}{\sqrt{\lambda}} + \cdots \right]$$

Differences SYM – ABJM:

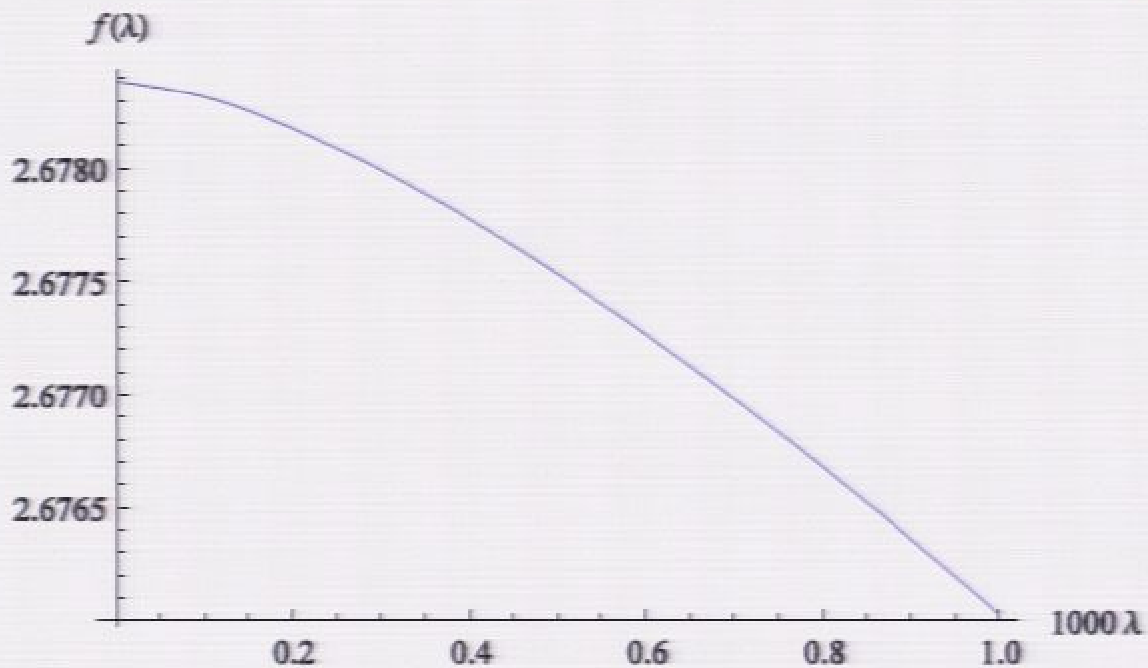
In $\mathcal{N} = 4$ super-Yang Mills:

- weak coupling: next-to-leading order correction non-analytical in the coupling $(\lambda^{3/2})$
- 1-loop diagrams defining thermal mass are regular

In $\mathcal{N} = 6$ ABJM theory:

- already first non-vanishing correction is non-analytical in the coupling $(\lambda^2 \log(\lambda)^3)$.
- self-consistent treatment necessary

Dependence on the Coupling



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Summary of free energy

- We have computed the first non-vanishing quantum correction to the free energy in ABJM theory on $\mathbb{R}^2 \times S^1$.
- The correction is non-analytic in λ, and is approximately of order $\lambda^2 \log(\lambda)^3$.
- The result is consistent with the existence of a smooth interpolation to the strong coupling region.

Chiral Primary Operators

Motivation

IR divergences destroy perturbation theory, so the free energy calculation is only valid when $-\log(\lambda) \gg 1$. Are there other objects which exhibit the same scaling $\sim \lambda^{-1/2}$ at strong coupling?

Yes, various objects:

- $Z(S^3, \mathcal{N}=6)$ [Drukker, Marino, Putrov 2010]
- $Z(S^3, \mathcal{N}=3)$ [Herzog, Klebanov, Pufu, Tesileanu 2010]
- $Z(\mathbb{R}^2 \times S^1) = F$ [M.S.]
- correlation functions of chiral primary operators

CPO's in super Yang-Mills

- Lee, Minwalla, Rangamani, Seiberg 1998: conjectured that n-point functions of CPO's constant in λ
- D'Hoker, Freedman, Skiba 1998: loop corrections
- Howe, West 1996: superconformal invariants
- Intriligator, Skiba 1998-1999:
  - showed that they are invariant under "bonus symmetry" $U(1)_Y$
  - conjectured that **more** correlation functions are invariant under this $U(1)_Y$
  - showed that this implies non-renormalization
- Eden, Howe, West 1999: proof of $U(1)_Y$ (valid for supergravity on $AdS_5 \times S^5$)

Full Statement:

2- and 3-point functions of gauge-invariant operators corresponding to KK multiplets in AdS supergravity are not renormalized

Chiral Primary Operators in ABJM Theory

Chiral Primary Operators:

- **4**-multiplet of SU(4) in ABJM:

$$Y^A = (Z^1, Z^2, W^{\dagger 1}, W^{\dagger 2})$$

- chiral primary operators (CPO's) in ABJM:

$$\mathcal{O}^I(x) = M^{I,(\bar{A}_1,\cdots,\bar{A}_L)}_{(A_1,\cdots,A_L)} \mathrm{tr}\left(Y^{A_1} Y^{\dagger}_{\bar{A}_1} \cdots Y^{A_L} Y^{\dagger}_{\bar{A}_L}\right)(x)$$

M symmetric in upper and lower indices and no traces.

Correlators

Consider length one, L=1 (maximizes wrapping effects).

- 2-point function:

$$g(x,y) = \langle \mathrm{tr}\left[Y^A(x)Y^\dagger_{\bar{A}}(x)\right]\mathrm{tr}\left[Y^B(y)Y^\dagger_{\bar{B}}(y)\right]\rangle$$

- 3-point function:

$$h(x,y,w) = \langle \mathrm{tr}\left[Y^A(x)Y^\dagger_{\bar{A}}(x)\right]\mathrm{tr}\left[Y^B(y)Y^\dagger_{\bar{B}}(y)\right]$$
$$\mathrm{tr}\left[Y^C(w)Y^\dagger_{\bar{C}}(w)\right]\rangle$$

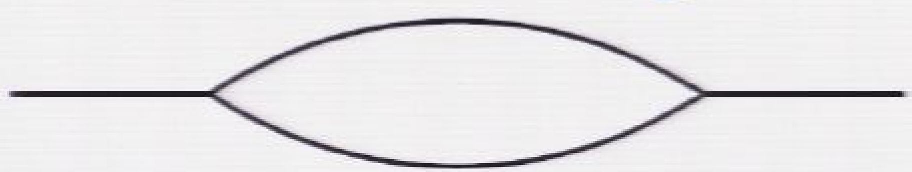
Tree-level: 2-pt

Free and planar field theory:

$$g_0(x,y) = \left(\frac{\lambda}{2}\right)^2 \frac{1}{|x-y|^2}(\delta^A_{\bar{B}}\delta^B_{\bar{A}})$$

$$\tilde{g}_0(p) = \frac{\pi^2}{2}\lambda^2 \frac{1}{\sqrt{p^2}}(\delta^A_{\bar{B}}\delta^B_{\bar{A}})$$

1-loop

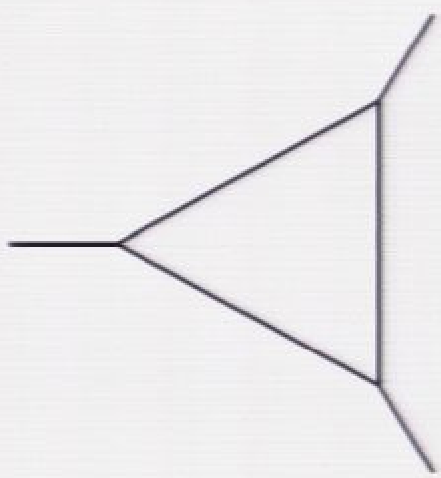


Tree-level: 3-pt

Free and planar field theory:

$$h_0(x,y,w) = \left(\delta^{A}_{\bar{B}}\delta^{B}_{\bar{C}}\delta^{C}_{\bar{A}} + \delta^{A}_{\bar{C}}\delta^{C}_{\bar{B}}\delta^{B}_{\bar{A}}\right)\frac{1}{N}\left(\frac{\lambda}{2}\right)^3 \frac{1}{|x-y||x-w||y-w|}$$

$$\tilde{h}_0(p_1,p_2) = \left(\delta^{A}_{\bar{B}}\delta^{B}_{\bar{C}}\delta^{C}_{\bar{A}} + \delta^{A}_{\bar{C}}\delta^{C}_{\bar{B}}\delta^{B}_{\bar{A}}\right)\frac{1}{N}\pi^3\lambda^3 \frac{1}{\sqrt{p_1^2 p_2^2 (p_1+p_2)^2}}$$



Loop corrections

Quantum Corrections

Chiral Primary Operators:

- dimension protected
- but normalization **not** protected

$$\tilde{g}(p) = \frac{\pi^2}{2}(1+c)\lambda^2 \frac{1}{\sqrt{p^2}}(\delta^A_{\bar{B}}\delta^B_{\bar{A}})$$

$$\tilde{h}(p_1,p_2) = \pi^3(1+d)\frac{\lambda^3}{N}\frac{1}{\sqrt{p_1^2 p_2^2 (p_1+p_2)^2}}(\delta^A_{\bar{B}}\delta^B_{\bar{C}}\delta^C_{\bar{A}} + \delta^A_{\bar{C}}\delta^C_{\bar{B}}\delta^B_{\bar{A}})$$

Contributions:

- 1-loop order: type (4), (33)
- 2-loop order: type (6), (44), (433), (3333)

Vertex Types

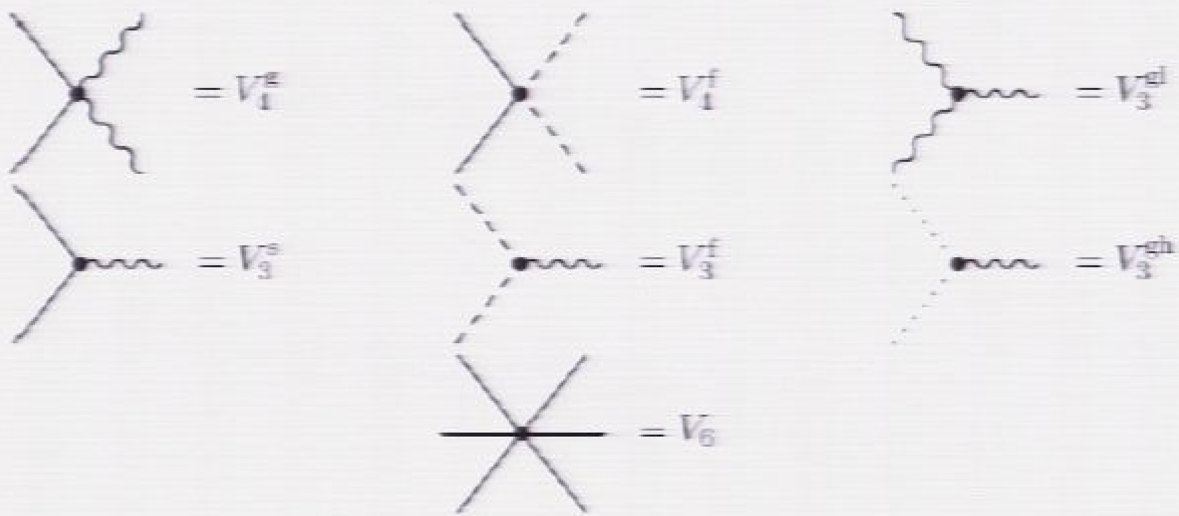
2 loops

$$c = c_1 + c_4 + c_{33} + c_6 + c_{44} + c_{433} + c_{3333}$$

$$d = d_1 + d_4 + d_{33} + d_6 + d_{44} + d_{433} + d_{3333}$$

one-leg contributions

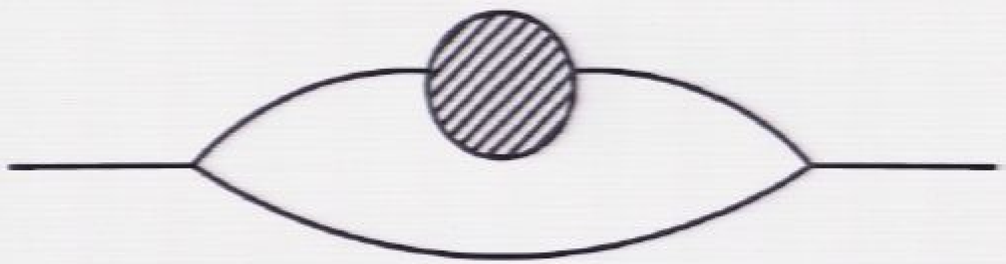
no 1-loop corrections

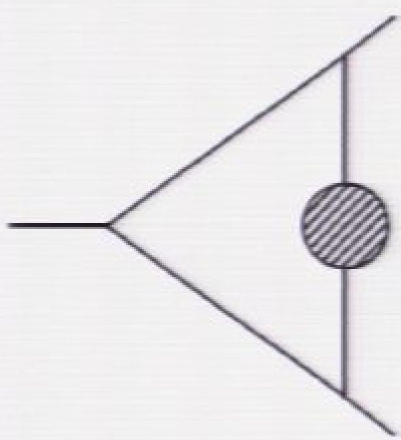


Single Leg Corrections

Compute c_1 and d_1:

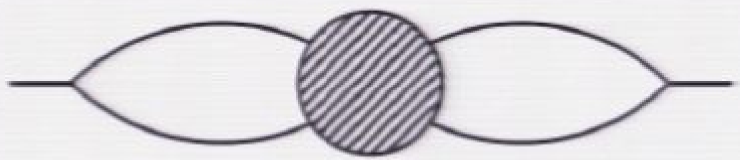
- given by scalar self-energy
- extract single leg corrections

$c_1 =$ 

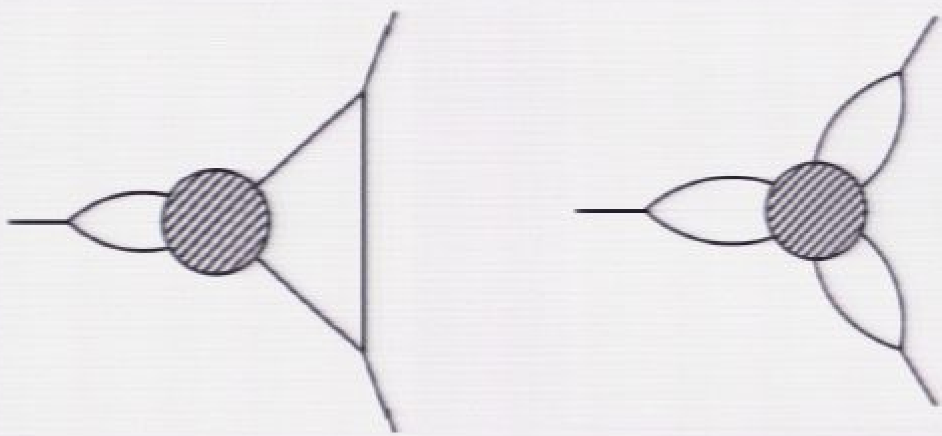
$d_1 =$ 

Multiple Leg Corrections

- 2-pt fcn: 2-leg corrections:



- 3-pt fcn: 2- and 3-leg corrections:



Combinatorics and Loops

$$c_i = \int \frac{d^3 q_1}{(2\pi)^3} \int \frac{d^3 q_2}{(2\pi)^3} \int \frac{d^3 q_3}{(2\pi)^3} C_i$$

$$i \in \{(6), (44), (433), (3333)\}$$

lower-case: after integration

(similar for d_i)

Upper-case: before integration: combinatorics determined by diagrams satisfying the following conditions:

- planarity (large N)
- connectedness (correlation functions)
- no tadpoles (cancelled by counterterms)
- no selfcontractions (vanishes in dimensional regularization)
- existence (realizable in ABJM theory)

Expectations

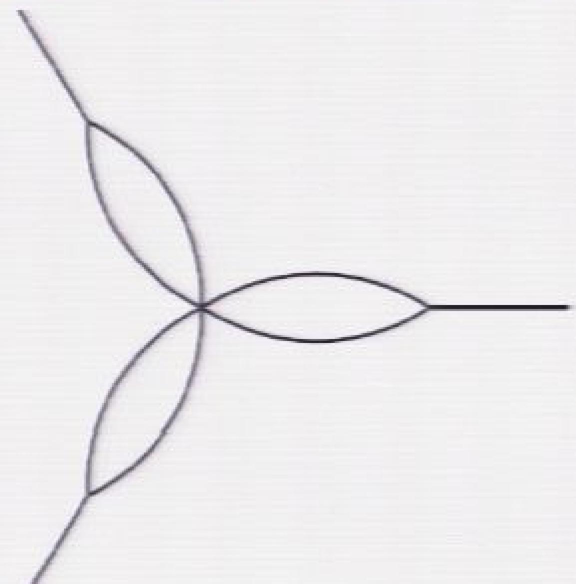
- more rewarding than free energy computation in terms of range of validity
- however, both similar and unique challenges
- free energy: no contribution from (3333)-vertices
- combinatorics: composite operators
- CPO's: divergent piece cancels
- must work at three loops and extract finite piece

Example: (6)-vertex

- contributes only to the 3-pt fcn
- before integration:

$$D_6 = -2(2\pi)^5 \Delta(q_1)\Delta(q_2)\Delta(q_3)$$
$$\Delta(p_{\text{in}1}+q_1)\Delta(p_{\text{in}2}+q_2)\Delta(p_{\text{in}1}+p_{\text{in}2}+q_3)$$

$$\Delta(q) = \frac{1}{q^2}$$

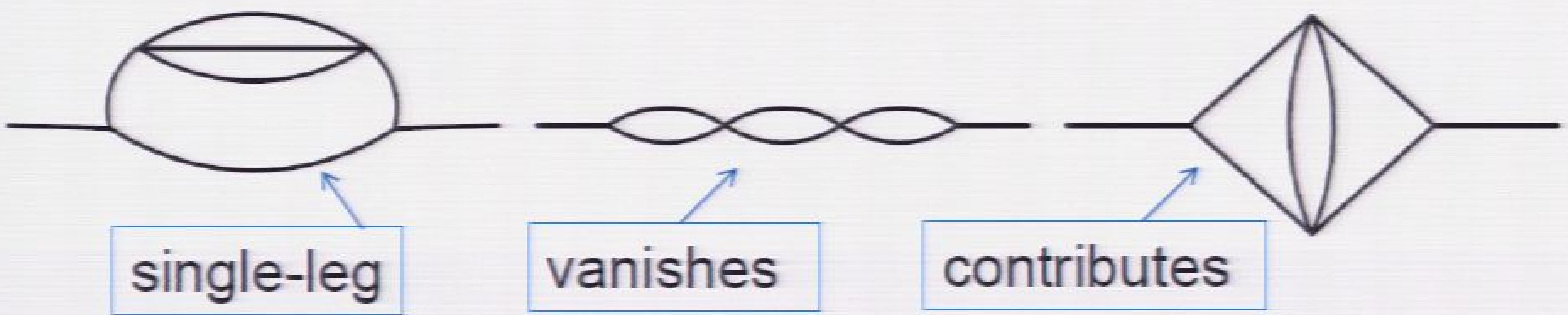


- after integration:

$$\frac{\pi^3}{\lambda^2} d_6 = -\frac{\pi^5}{8}$$

Example: (44)-vertices

- consider corrections to 2-pt fcn
- diagrams allowed by conditions:



$$C_{44} = 12(2\pi)^4 (q_2 \cdot q_3)\Delta(q_1)\Delta(q_2)\Delta(q_3)$$
$$\Delta(p+q_1)\Delta(q_1+q_2+q_3)\Delta(p+q_1+q_2+q_3)$$

$$\frac{\pi^2}{2\lambda^2} c_{44} = +\frac{3\pi^2}{8\epsilon} - \frac{3\pi^2}{8}\left(-2+3\gamma-\log(256\pi^3)\right)$$

$$d = 3 - 2\epsilon$$

Scaling Properties

Normalization

- Suppose:

$$\langle \mathcal{O}\mathcal{O}\rangle \sim 1 + c\lambda^2$$

$$\langle \mathcal{O}\mathcal{O}\mathcal{O}\rangle \sim 1 + d\lambda^2$$

where $\mathcal{O}$ is a general chiral primary operator.

- Rescale: $\mathcal{O} \to (1 + e\lambda^2)\mathcal{O}$ with $e = d - c$.
- Gives a common normalization:

$$\langle \mathcal{O}\mathcal{O}\rangle \sim 1 + (3c - 2d)\lambda^2$$

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$$\langle \mathcal{O}\mathcal{O}\mathcal{O}\rangle \sim 1 + (3c - 2d)\lambda^2$$

- Known behaviour at strong coupling:

$$\langle \mathcal{O}\mathcal{O}\rangle \sim \lambda^{-1/2}$$

$$\langle \mathcal{O}\mathcal{O}\mathcal{O}\rangle \sim \lambda^{-1/2}$$

- Expectation:

$$3c - 2d < 0$$

Partial Result

If a smooth interpolation to the strong coupling behaviour exists, then loop corrections to n-point functions of chiral primary operators should satisfy

$$3c - 2d < 0$$

- Partial result:

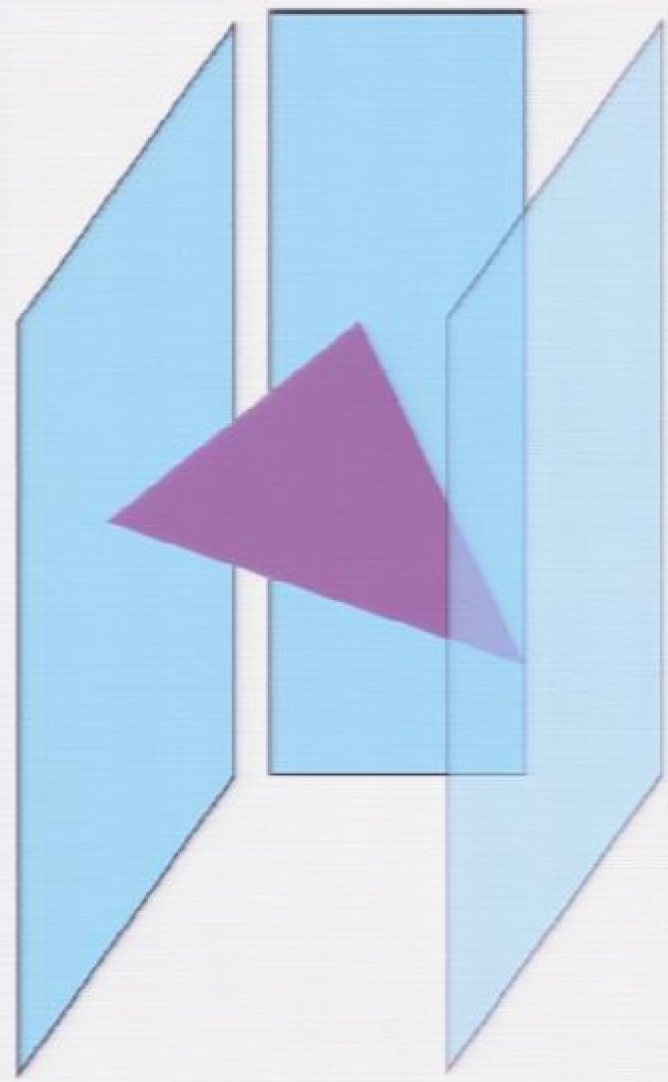
$$c_p = c_1 + c_6 + c_{44} + c_{433} = -57.4698$$
$$d_p = d_1 + d_6 = -97.5758$$
$$\Rightarrow 3c_p - 2d_p = +22.7422$$

- Known: $C_{3333}, D_{44}, D_{433}$

Summary

Summary

- We have been interested in the degrees of freedom of ABJM theory and their dependence on the coupling λ.
- We have computed the first non-vanishing quantum correction in the planar limit both to the free energy on $\mathbb{R}^2 \times S^1$ and 3-point fcn's of length one CPO's (in progress).
- Indicates the existence of a smooth interpolation to strong coupling.



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