Title: The principle of relative locality

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Abstract: Several current experiments probe physics in the approximation in which Planck's constant and Newton's constant may be neglected, but, the Planck mass, is relevant. These include tests of the symmetry of the ground state of quantum gravity such as time delays in photons of different energies from gamma ray bursts. I will describe a new approach to quantum gravity phenomenology in this regime, developed with Giovanni Amelino-Camelia, Jersy Kowalski-Glikman and Laurent Freidel.

This approach is based on a deepening of the relativity principle, according to which the invariant arena for non-quantum physics is a phase space rather than spacetime. Descriptions of particles propagating and interacting in spacetimes are constructed by observers, but different observers, separated from each other by translations, construct different spacetime projections from the invariant phase space. Nonetheless, all observers agree that interactions are local in the spacetime coordinates constructed by observers local to them.

This framework, in which absolute locality is replaced by relative locality, results from deforming momentum space, just as the passage from absolute to relative simultaneity results from deforming the linear addition of velocities. Different aspects of momentum space geometry, such as its curvature, torsion and non-metricity, are reflected in different kinds of deformations of the energy-momentum conservation laws. These are in principle all measurable by appropriate experiments.

The Principle of Relative Locality

Lee Smolin
PI February 2011

with Giovanni Amelino-Camelia, Laurent Freidel, Jerzy Kowalski-Glikman

arXiv:1101.0931 _and papers in preparation.

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Many thanks to Sabine Hossenfelder and to R Schutzhold and Bill Unruh for raising the issue of non-locality in theories with deformed lorentz invariance.

Thanks also to Michele Arzano, Florian Girelli, Etera Livine, Seth Major, ...

Look around: do you see space?

Look around: do you see space?

"I don't see space...I see things"

-Diego Rivera

Look around: do you see space?

No, we see spacetime...

So, look around: do you see spacetime?

If fact we don't see spacetime, we see momentum space...

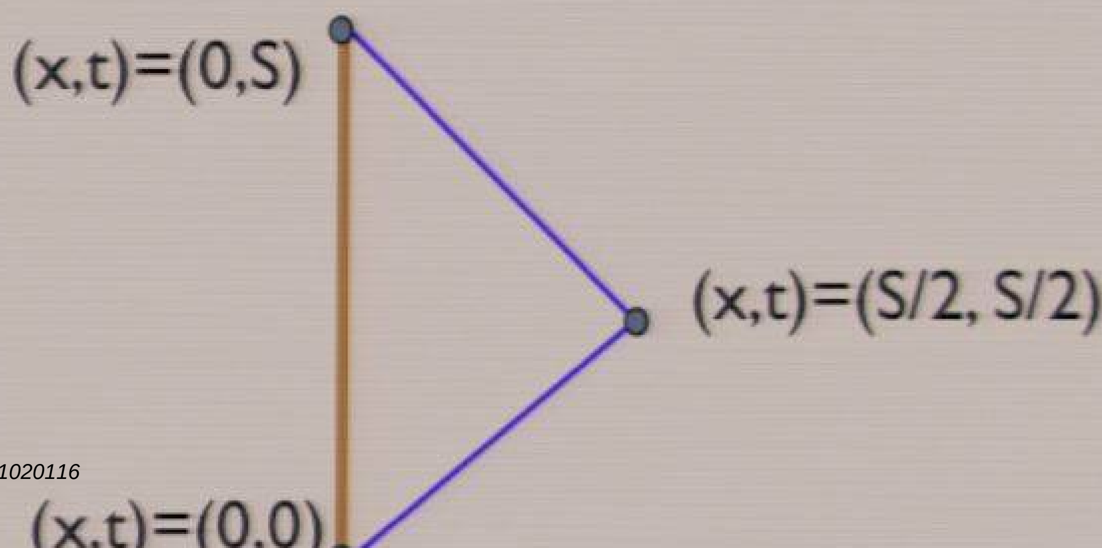
We see photons arriving with different momenta and energies at different angles.

So, look around: do you see spacetime?

If fact we don't see spacetime, we see momentum space...

We see photons arriving with different momenta and energies at different angles.

Spacetime is inferred. As Einstein taught us, distant spacetime coordinates are inferred from momentum space measurements.



But, do we all infer the same spacetime?
Do we infer the same spacetime at different energies?

In special relativity the answers are yes. Why?

- The conservation laws that generate transformations between observers are linear in momenta.

$$\mathcal{P}_c^{tot} = \sum_I p_c^I$$

- Total momentum generates translations:

$$\delta x_I^a = \{\delta x_I^a, b^c \mathcal{P}_c^{tot}\} = b^a$$

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- How much a worldline is translated, does not depend on how much momentum and energy it carries.
- Hence we all construct the same spacetime.
- If an interaction is local for one observer it is local for all observers

- How much the spacetime coordinates of a worldline are translated, will now depend on how much momentum and energy it carries.
- The description of events are different at different energies.
- If an interaction is local for one observer *it will not be inferred to be local for distant observers.*

- *For every interaction, observers local to it will infer it to be local.*

We call this the principle of relative locality.

There is a simple and coherent mathematical framework for it, based on the geometry of momentum space.

The classical Planck-mass regime

$$\begin{aligned} G_{Newton} &\rightarrow 0 \\ \hbar &\rightarrow 0 \\ m_p &= \sqrt{\frac{\hbar}{G_{Newton}}} \rightarrow \text{constant} \end{aligned}$$

e relativistic particle mechanics with invariant mass, m_p and velocity
:, characterizing non-linearities in momentum space.

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The experimental question: *what is the symmetry of the ground state of quantum spacetime?*

Possibilities:

- *Poincare invariance (as in classical GR)*
- *broken Poinare invariance (ie. a prefered frame)*
- *deformed or modified Poincare invariance*

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What is the scale of quantum gravity effects in this limit?

$$l_{Planck} = \sqrt{\hbar G_{Newton}} \rightarrow 0$$

Quantum gravity effects now show up at very large scales:

$$\Delta x \approx x \left(\frac{E}{m_p} \right)^p$$

p=1,2

The classical Planck-mass regime

$$
\begin{aligned}
G_{Newton} &\rightarrow 0 \\
\hbar &\rightarrow 0 \\
m_p &= \sqrt{\frac{\hbar}{G_{Newton}}} \rightarrow \text{constant}
\end{aligned}
$$

Experimental windows into this regime:

- Gamma Ray burst time of flight measurements at Fermi etc
- Tests of GZK cutoff at AUGER
- Birefringence of photons, ie polarized radio galaxies, Gamma rays etc.

These can be modeled by positing non-linearities in momentum space.

Since its launch Fermi, nee GLAST, has seen 8 GRB's with Gev scale photons.

Two have been the source of new bounds on M_{QG}.

$$v = c(1 \pm \frac{E}{M_{QG}})$$

$$\Delta T = T_{flight}\frac{\Delta E}{M_{QG}} = 1sec\frac{m_P}{M_{QG}}\frac{T_{flight}}{10^{10}\ years}\frac{\Delta E}{10\ Gev}$$

GRB	Redshift	Duration	$\text{counts}\vert_{\text{LAT}}$	E_{max}	t_i^{LAT}	t_f^{LAT}
080916C	4.35	Long	Strong	13 GeV	4.5 s	$>10^3$ s
081024B		Short		3 GeV	0.2 s	
090510	0.9	Short	Strong	>1 GeV	<1 s	$\geq$ 60 s
090328	0.7	Long		>1 GeV		$\approx$ 900 s
090323	4	Long	Strong	>1 GeV		$>10^3$ s
090217		Long			~1 s	$\approx$ 20 s
080825C		Long	Weak	0.6 GeV	3 s	>40 s
081215A			Weak	0.2 GeV		

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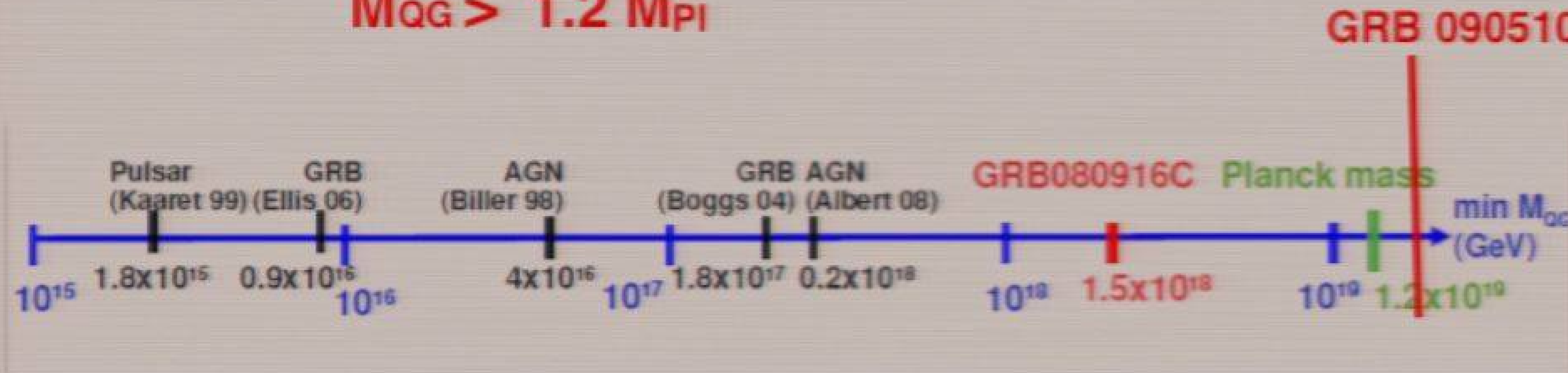
GRB 090510

redshift: .9
short burst
highest energy photon: 32 Gev comes .8 sec into the burst

$M_{QG} > 1.2\ M_{Pl}$



Can we use these experiments to put bounds on the geometry of momentum space?

Geometry of momentum space

Operational point of view: an observer is equipped with a calorimeter and a clock.

From her measurement she conclude that each isolated system possess 4 conserved quantities: Energy momentum space $\mathcal{P}$

She can realise two type of measurements:

One particle measurements: measurement of the mass and kinetic energy *determines the metric*

Multi particle measurements: sacttering processes, interactions, merging. *determines the connection.*

Geometry of momentum space

One postulate that single particle measurements determine the geometry of $\mathcal{P}$

$\mathcal{P}$ is a lorentzian metric manifold

The mass is interpreted as the timelike distance from the origin

$$D^2(p) \equiv D^2(p,0) = m^2.$$

The kinetic energy defines the geodesic spacelike distance between a particle p at rest and a particle p' of identical mass $D(p) = D(p') = m$

$$D^2(p,p') = -2mK.$$

from these measurements we can reconstruct the metric on $\mathcal{P}$

$$dk^2 = h^{ab}(k)dk_a dk_b$$

Geometry of momentum space

When we define operationally momentum space $\mathcal{P}$

we used one type of calorimeter,

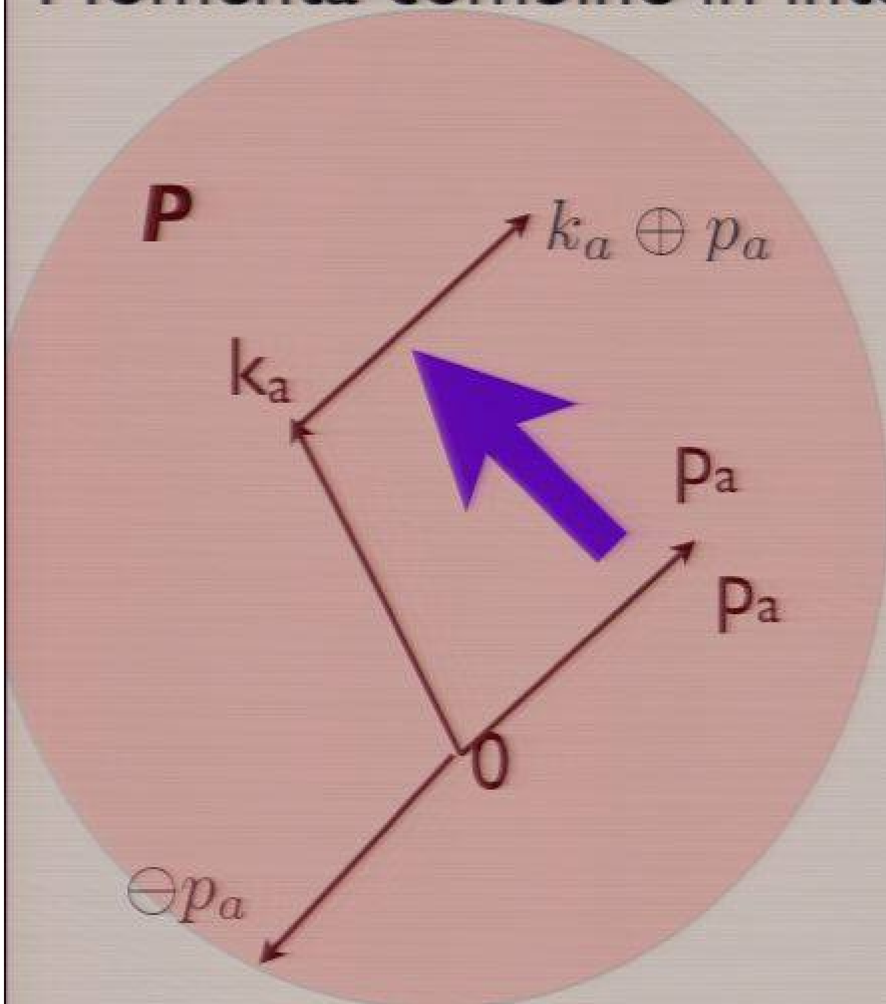
chosing another calorimeter will amount to a redefinition

$$p \to p' = \phi(p)$$

The theory has to be invariant under diffeomorphism on momentum space.

Geometry of momentum space

Momenta combine in interactions: we need a rule:



$$(k,q) \rightarrow k'_a = k_a \oplus q_a$$

This is a rule for combining geodesics on a curved manifold, so it defines a connection or parallel transport.

$$\begin{aligned} k_a \oplus dp_a &= k_a + U(k)_a^b dp_b \\ &= k_a + dp_a + \Gamma_a^{bc} k_b dp_c + \end{aligned}$$

Complicated process are built up:

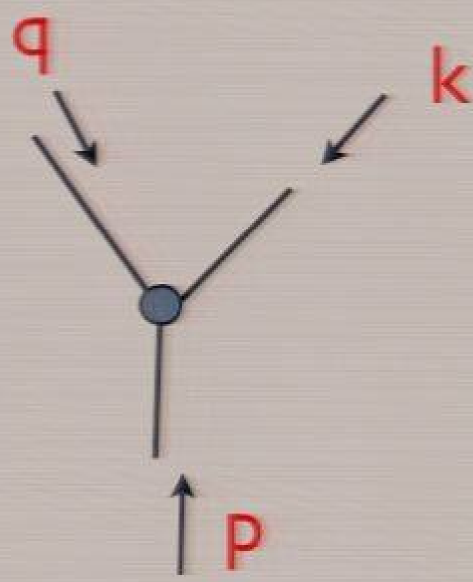
$$(k_a \oplus q_a) \oplus p_a$$

We assume neither commutativity nor associativity.

We do assume there is an inverse

$$p_a \rightarrow \ominus p_a, \quad (\ominus p_a) \oplus p_a = 0$$

The non-linear composition rule is used to define conservation laws at interactions.



$$\mathcal{K}(k,p,q)_a = (k_a \oplus p_a) \oplus q_a = 0$$

This requires choices when the composition rule is non-commutative or non-associative.

Geometry of momentum space

The composition rules defines an affine connection on $\mathcal{P}$

$$\frac{\partial}{\partial p_a}\frac{\partial}{\partial q_b}(p\oplus q)_c|_{q,p=o} = -\Gamma_c^{ab}(0)$$

transform as an affine connexion

Curvature measure non associativity

$$2\frac{\partial}{\partial p_{[a}}\frac{\partial}{\partial q_{b]}}\frac{\partial}{\partial k_c}((p\oplus q)\oplus k - p\oplus(q\oplus k))_d\,|_{q,p,k=o} = R^{abc}{}_d(0)$$

Three aspects of geometry, which can be measured:

$$p_a \oplus q_a = p_a + q_a + \Gamma_a^{bc} p_b q_c + ...$$

- Torsion: measures non-commutativity of interactions.

$$T_a^{bc} = \Gamma_a^{bc} - \Gamma_a^{cb}$$

- Curvature: measures non-associativity of interactions.

$$R^{abc}{}_d = \partial^a \Gamma_d^{bc} - \partial^b \Gamma_d^{ac} + \Gamma\Gamma$$

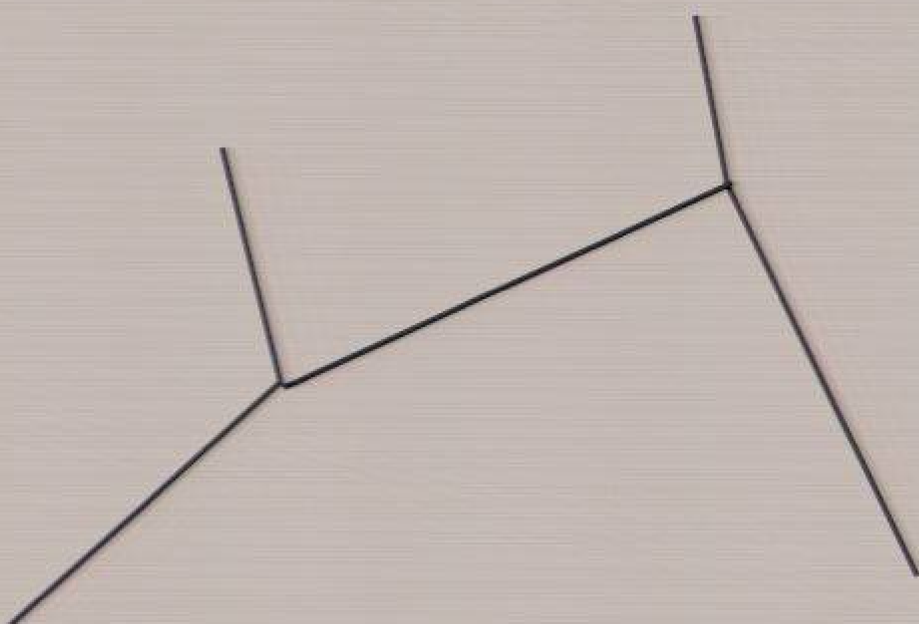
- Non-metricity: if the connection defined by interactions is not the metric connection defined from propagation.

$$N^{abc} = \nabla^a g^{bc}$$

Dynamics

- Spacetime emerges from dynamics on momentum space.
- In our limit, we study first classical particle dynamics
- Each process has an action principle

$$S^{process} = \sum_{trajectories, I} S_I^{free} + \sum_{interactions, \alpha} S_\alpha^{int}$$



The free relativistic particle:

• Canonical coordinates, x^a, and canonical momenta k_b

$$S_{free} = \int ds \left(x^a \dot{k}_a + \mathcal{N}\mathcal{C}(k) \right)$$

Energy-momentum relations expressed as a constraint:

$$\mathcal{C}(k) = -k_0^2 + \vec{k} \cdot \vec{k} + m^2 = 0$$

Canonical Poisson brackets:

$$\{x_I^a, k_b^J\} = \delta_b^a \delta_I^J$$

Equations of motion: *N*=lagrange multiplier

$$\begin{aligned} \dot{k}_a^J &= 0 \\ \dot{x}_J^a &= \mathcal{N}_J \frac{\delta \mathcal{C}^J}{\delta k_a^J} = \mathcal{N}_J p^a \\ \mathcal{C}^J(k) &= 0 \end{aligned}$$

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Equations of motion: *N*=lagrange multiplier

Notice that the free particle action makes no reference to a metric for spacetime. Spacetime geometry is inferred from the geometry of momentum space.

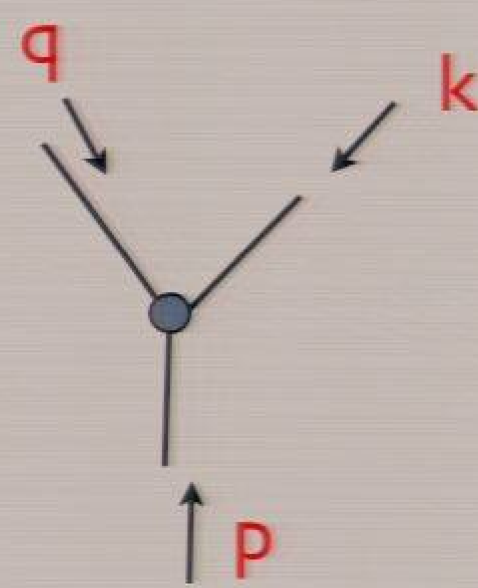
The interaction imposes a conservation law at each node

$$\boxed{S^{int} = \mathcal{K}(k(o))_a z^a}$$

$$\frac{\delta S^{int}}{\delta z^a} = \mathcal{K}_a = 0$$

$$\mathcal{K}(k,p,q)_a = (k_a \oplus p_a) \oplus q_a = 0$$

z^a is a lagrange multiplier that enforces the conservation law $K_a = 0$.



But, in turn, z^a become the point representing the interaction in spacetime.

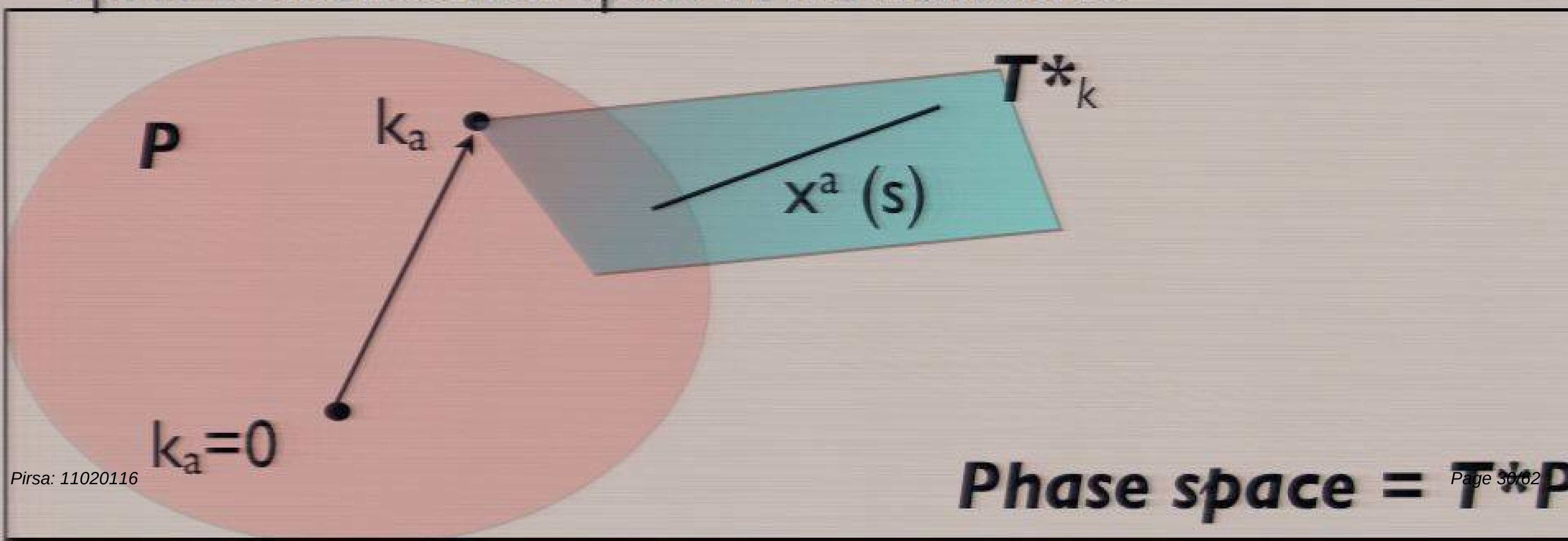
Two kinds of spacetime coordinates:

- Canonical coordinates, x^a_I, from the variation of the free action

$$S^I_{free} = \int ds \left(x^a_J \dot{k}^J_a + \mathcal{N}_J \mathcal{C}^J(k) \right)$$

$$\{x^a_I, k^J_b\} = \delta^a_b \delta^J_I \qquad \{x^a_I, x^b_J\} = 0$$

These are momentum dependent. They live in the cotangent space of momentum space at momentum k.



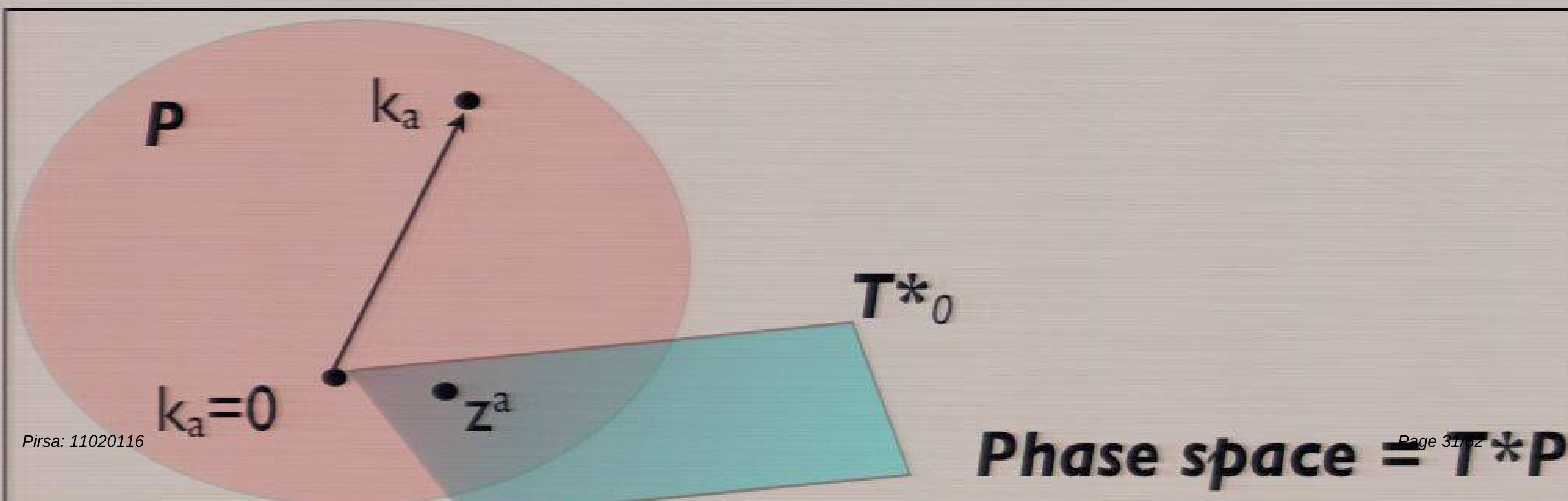
Two kinds of spacetime coordinates:

• Interaction coordinates, z^a, from the variation of the interaction

$$S^{int} = \mathcal{K}(k(o))_a z^a \qquad \frac{\delta S^{int}}{\delta z^a} = \mathcal{K}_a = 0$$

These are non-commutative. They live in the cotangent space of momentum space at momentum k=0.

$$\{z^a, z^b\} = T^{ab}_d z^d + R^{abc}{}_d p_c z^d + \cdots$$



Relating the two kinds of spacetime coordinates:

- Is a consequence of the equations of motion at the endpoints

$$\delta S = (\frac{\delta \mathcal{K}(k(o))_a}{\delta k^I_a(0)} z^a - x^a(0))\delta k_a(0)$$

The interaction point is related to the endpoint of the worldline by a parallel transport between the spaces where they live.

$$\boxed{x^a(0) = U(k)^a_b z^b,} \quad U(k)^a_b = \frac{\delta \mathcal{K}_b}{\delta k_a}$$

If the conservation K_a is linear, U=I and $x^a = z^a$.
Then the interaction is local.

When K_a is non-linear, the interaction is relatively local

ie x^a =0 when z^a =0.

Relating the two kinds of spacetime coordinates:

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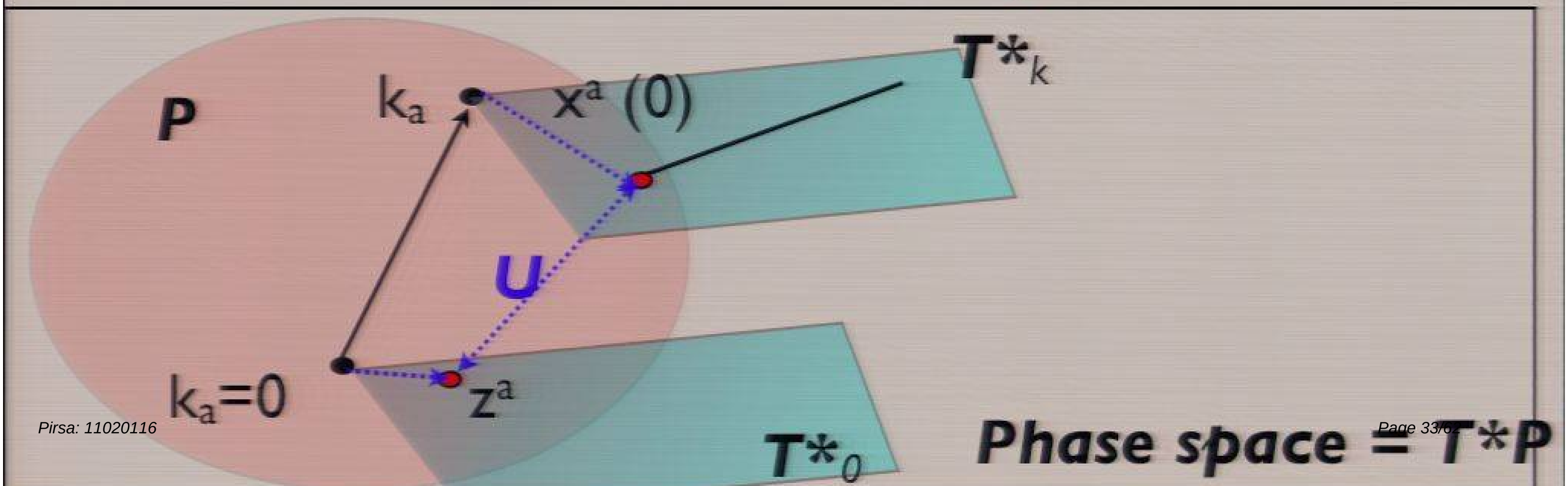
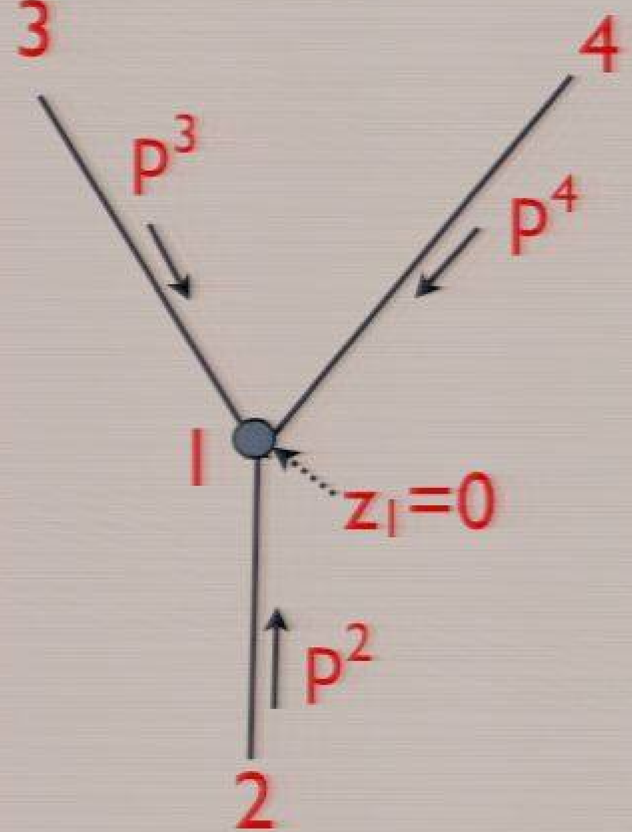
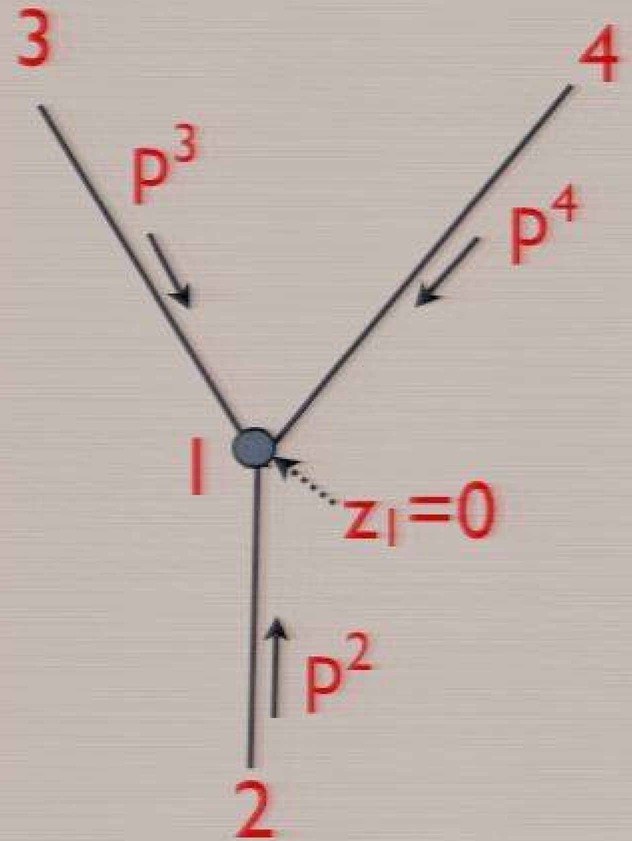


Vertices look local to local observers, for which $z^a = 0$



local observer

Vertices look non-local to distant observers

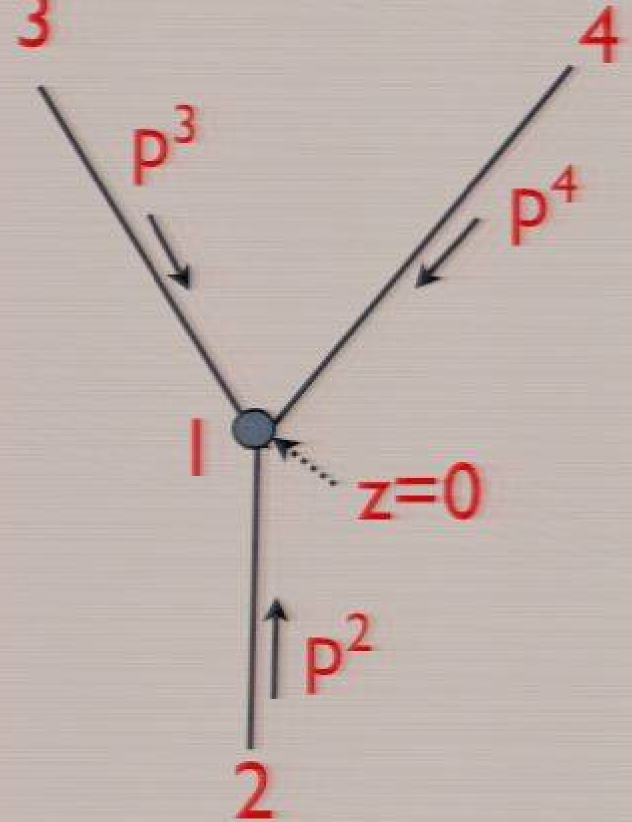


local observer

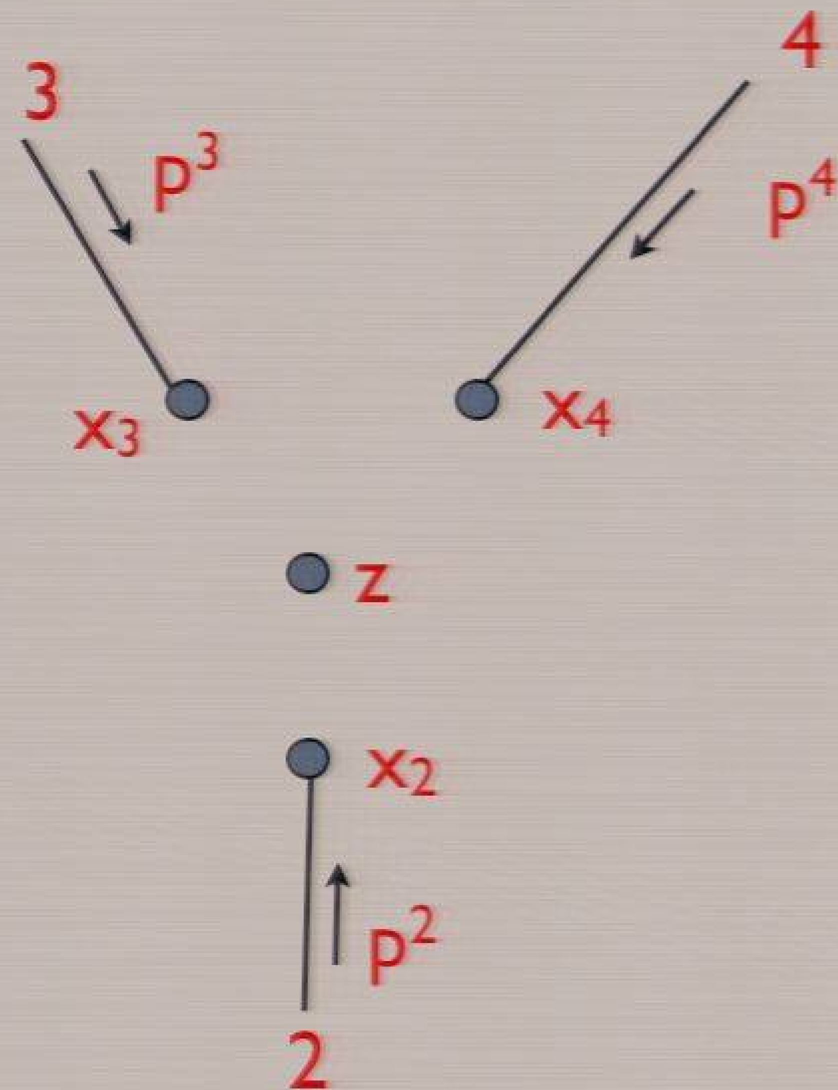
Translate the endpoints by

$$\delta x^a_I = \pm\{b^c \mathcal{K}_c, x^a_I\} = b^a + \Gamma^{ac}_b b^a p^I_c + \ldots$$

Vertices look non-local to distant observers



local observer

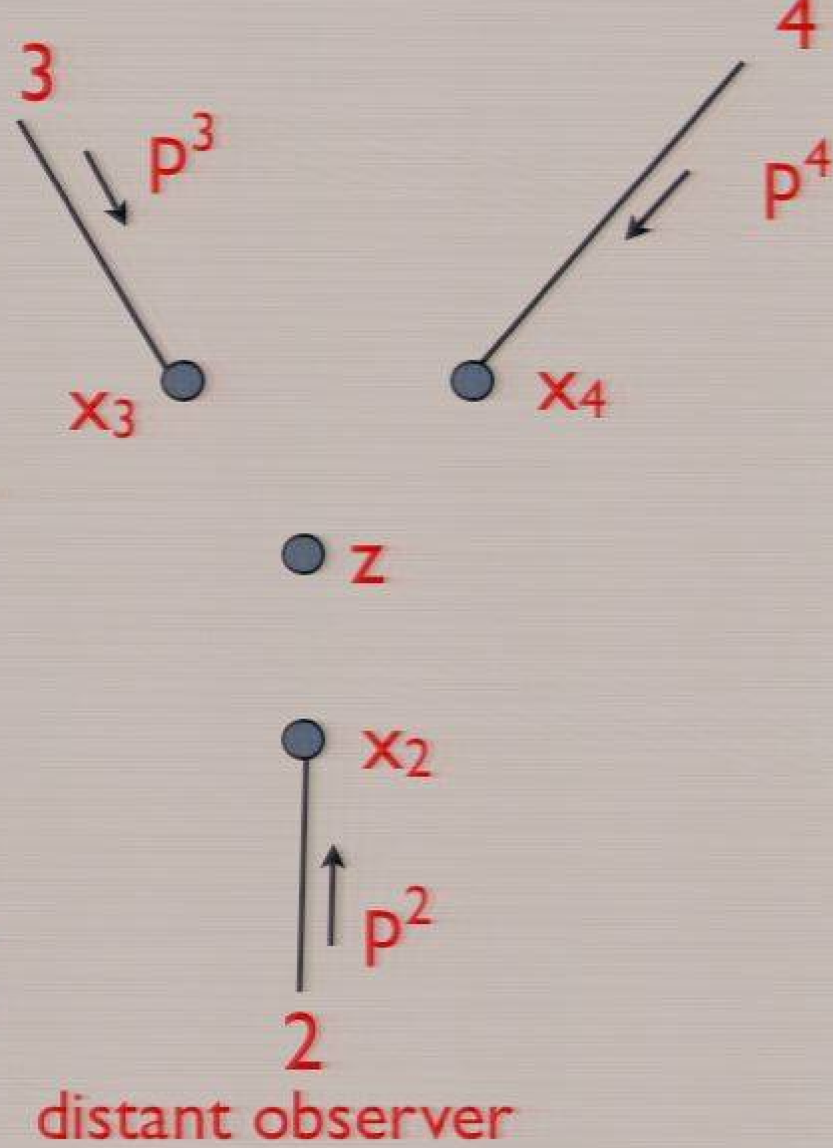


distant observer

$$\delta x_I^a = \pm\{b^c \mathcal{K}_c, x_I^a\} = b^a + \Gamma_b^{ac} b^a p_c^I + \ldots$$

$$x_I^a = z^b \frac{\partial \mathcal{K}_b}{\partial p_a^I}$$

$$= z^a + z^b \Gamma_b^{ac} p_c + \ldots$$



$$|x^a - z^a| \approx |z||\Gamma_b^{ac}||p| \approx |z|\frac{E}{M_{QG}}$$

Specializing the geometry

The correspondence principle:

Special relativity holds for momentum smaller than a mass scale M_{QG}

- Torsion and non-metricity $= O(M_{QG}^{-1})$
- Curvature $= O(M_{QG}^{-2})$

The dual equivalence principle:

The geometry of momentum space is universal.

Maximal symmetry:

Momentum space has as many symmetries as flat spacetime

ie it has a deSitter or AdS geometry with radius of curvature M_Q

Theorists propose but experiments decide.

The Gamma Ray Burst (GRB) problem

Long ago and far away there was a GRB.

Two photons were created simultaneously (according to a local observer there) but with very different energies.

Are they detected by the Fermi satellite simultaneously?

The Gamma Ray Burst (GRB) problem

Long ago and far away there was a GRB.

Two photons were created simultaneously (according to a local observer there) but with very different energies.

Are they detected by the Fermi satellite simultaneously?

Naive (wrong) argument: you can choose coordinates on curved momentum space so that the speed of light is energy dependent.

$$c(E) = \frac{dE}{dp} = c(1 - \frac{E}{M_{QG}} + ...)$$

Hence there is a time delay

$$\Delta T = T_{flight} \frac{\Delta E}{M_{QG}} = 1sec \frac{m_P}{M_{QG}} \frac{T_{flight}}{10^{10}\ years} \frac{\Delta E}{10\ Gev}$$

The problem with this: you can also choose coordinates on momentum space so the speed of light is a constant!

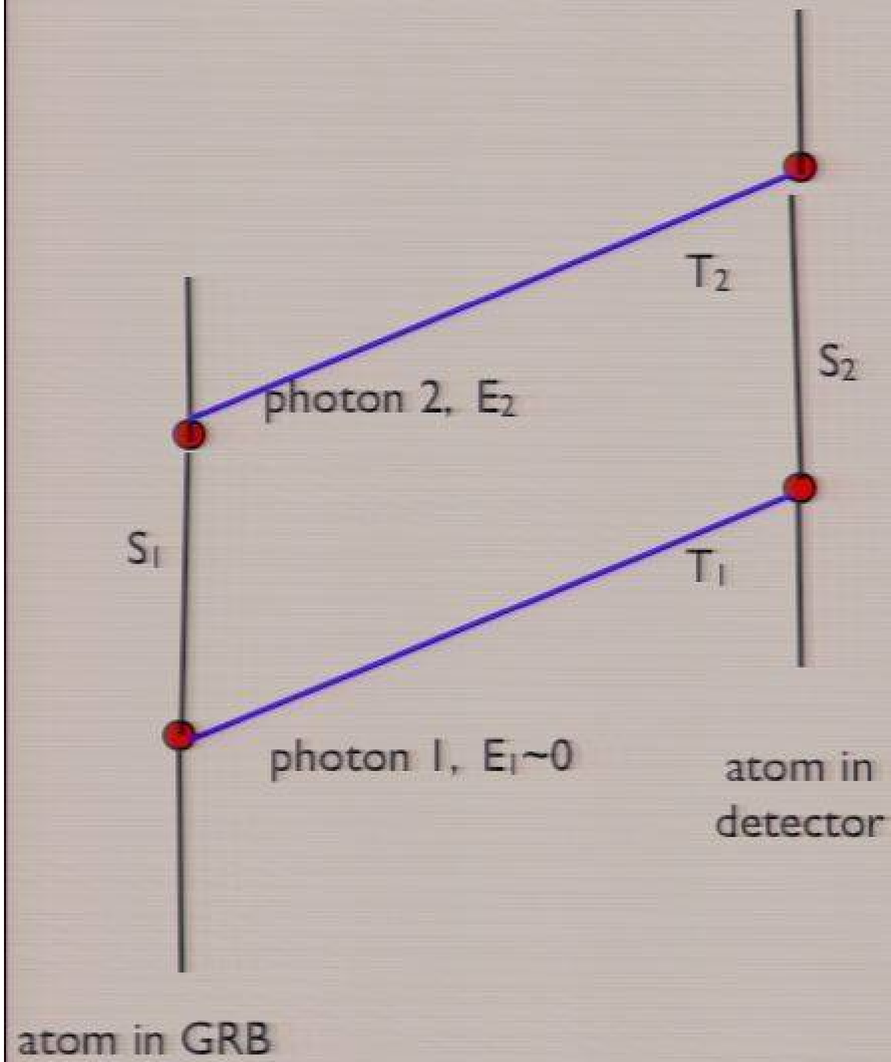
These are Riemann normal coordinates:

$$D(p) = \eta^{ab} p_a p_b$$

$$\partial^b g^{bc}|_{p=0} = 0 \rightarrow \Gamma = T + N$$

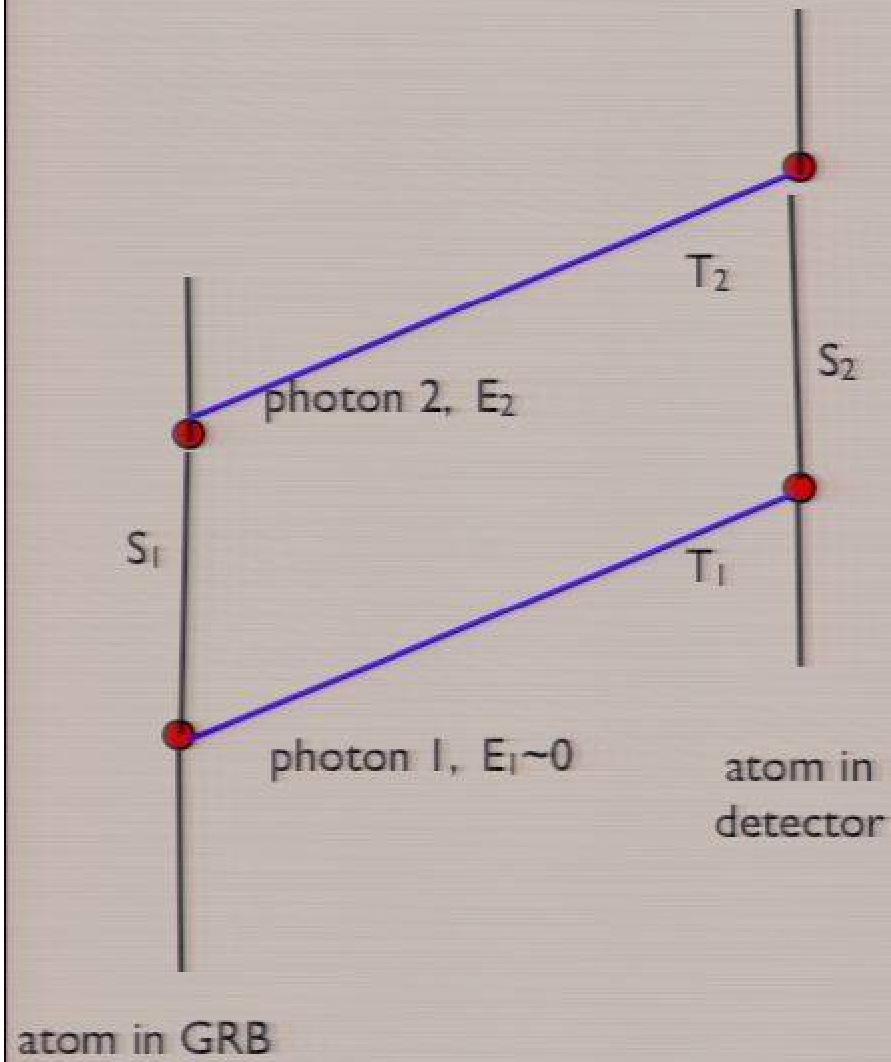
So is there no time delay??

To find out you have to compute the proper time between detections of the two photons.



Neglect all energies except E_2

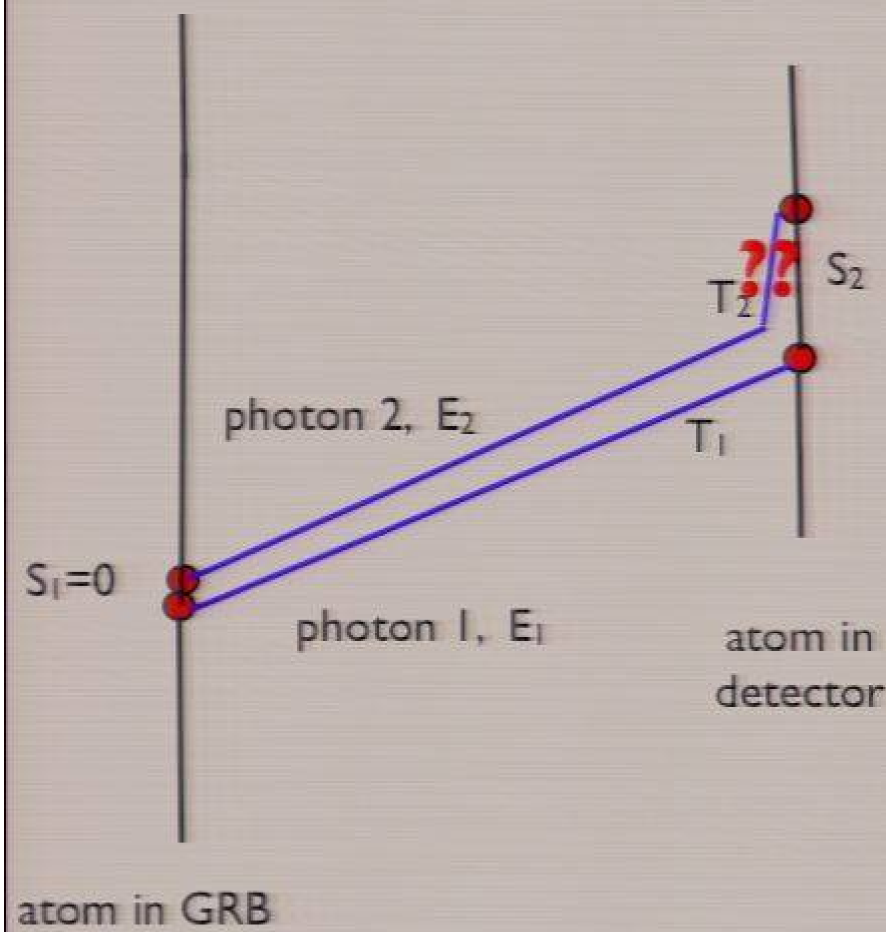
$T_1 \sim T_2 = T >> S_{1,2}$



Neglect all energies except E_2
$T_1 \sim T_2 = T >> S_{1,2}$

$$\boxed{\begin{aligned} S_2 - S_1 &= E_2\, T\, \Gamma^{++}_{-} \\ &= E_2\, T\, N^{++}_{-} \end{aligned}}$$

The leading order effect
is due to non-metricity.



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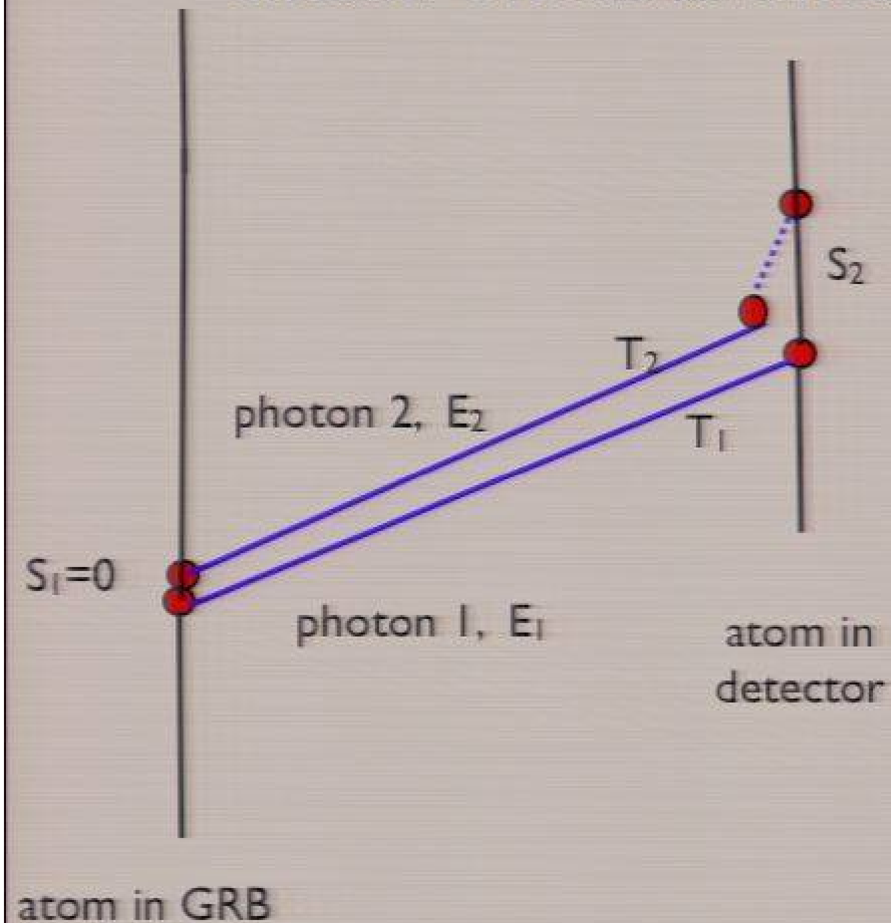
$$\boxed{\begin{aligned} S_2 - S_1 &= E_2\, T\, \Gamma^{++}_{-} \\ &= E_2\, T\, N^{++}_{-} \end{aligned}}$$

The leading order effect is due to non-metricity.

If emission is simultaneous in the GRB frame, so S_1=0, *there is still a time delay!*

The time delay is due to relative locality: observers see distant events as smeared out non-locally.

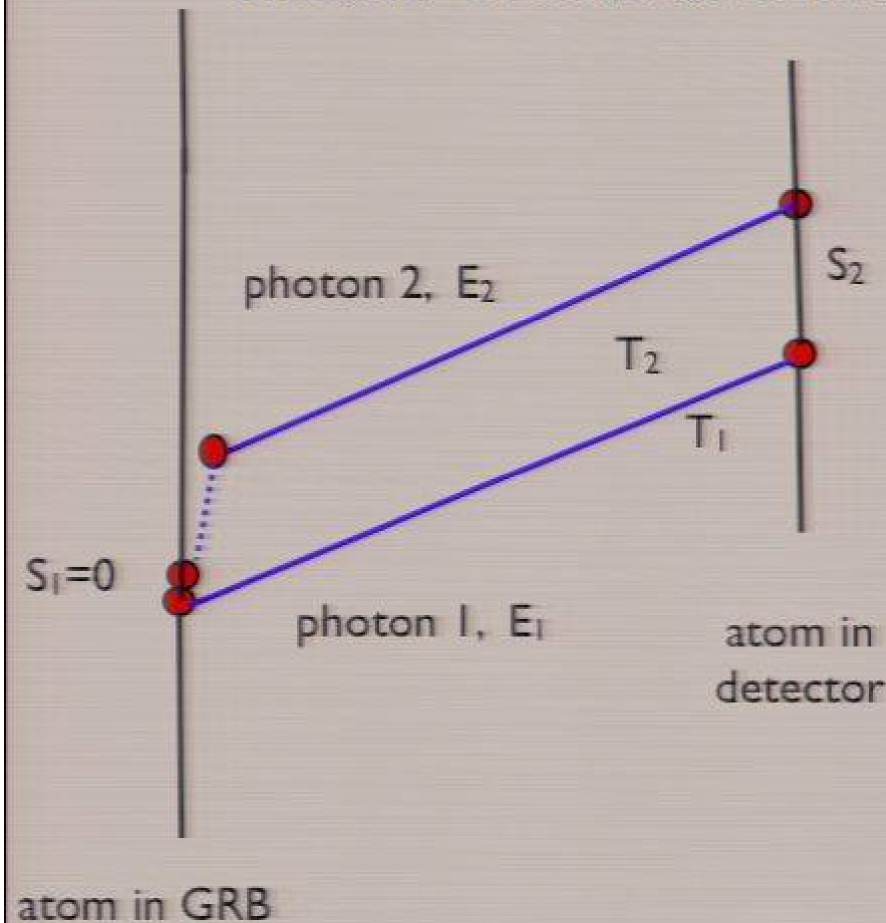


Observers at the GRB see the emission events to be local while the detection events are smeared out proportionally to distance and energy.

$$S_2 - S_1 = E_2\, T\, \Gamma^{++}_{-}$$

$$= E_2\, T\, N^{++}$$

The time delay is due to relative locality: observers see distant events as smeared out non-locally.



Observers at the detector see the detection events to be local while the emission events are smeared out proportionally to distance and energy.

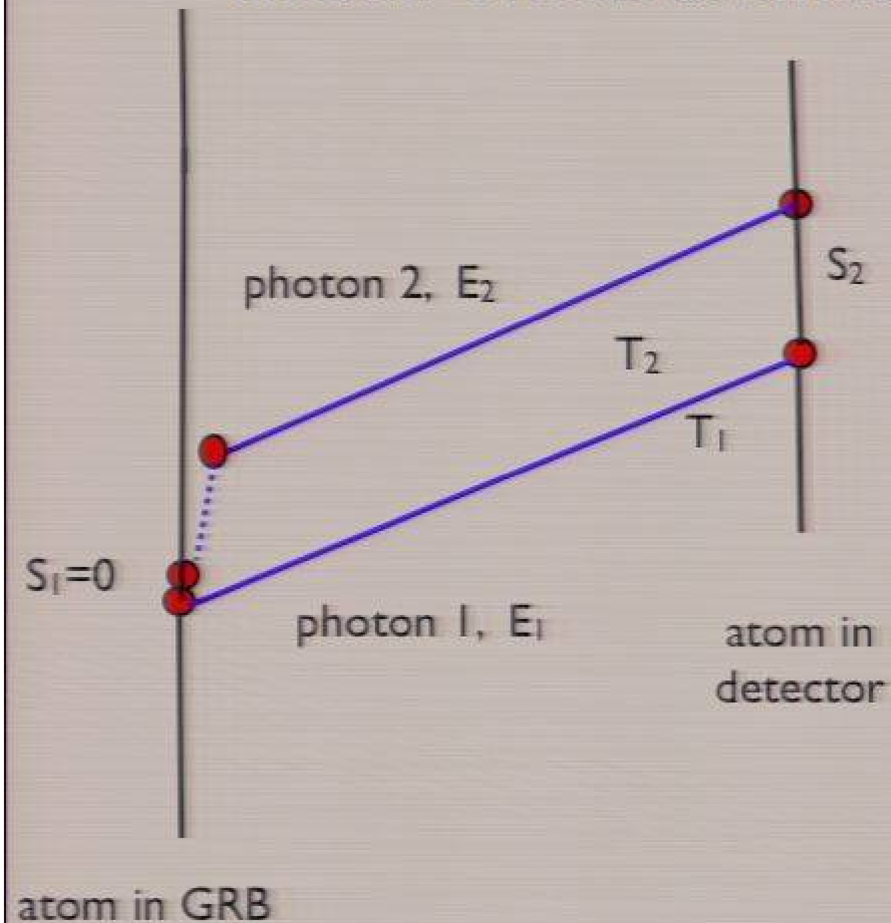
All observers agree that there is a time delay and agree on its value.

There will be higher order curvature terms.

$$S_2 - S_1 = E_2 \, T \, \Gamma^{++}_{-}$$

$$= E_2 \, T \, N^{++}$$

The time delay is due to relative locality: observers see distant events as smeared out non-locally.



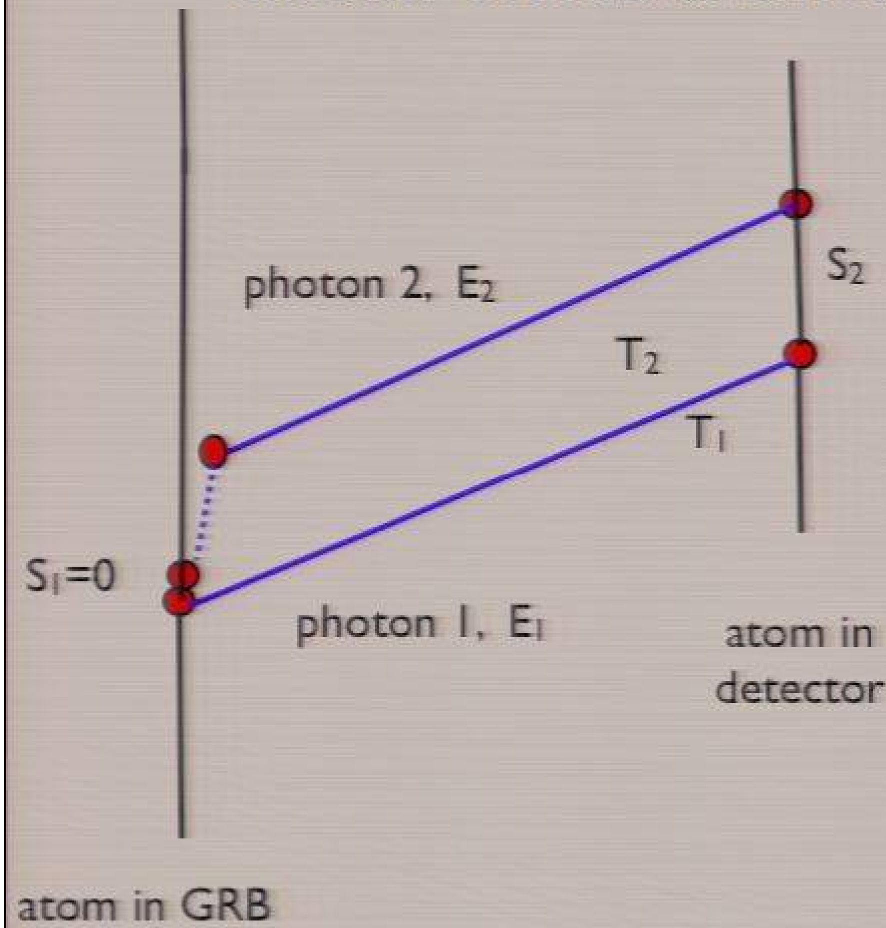
The Fermi event **GRB 090510** bounds the non-metricity tensor:

$$N^{++}_{-} < \frac{1}{1.2 M_{planck}}$$

$$S_2 - S_1 = E_2\, T\, \Gamma^{++}_{-}$$

$$= E_2\, T\, N^{++}$$

The time delay is due to relative locality: observers see distant events as smeared out non-locally.

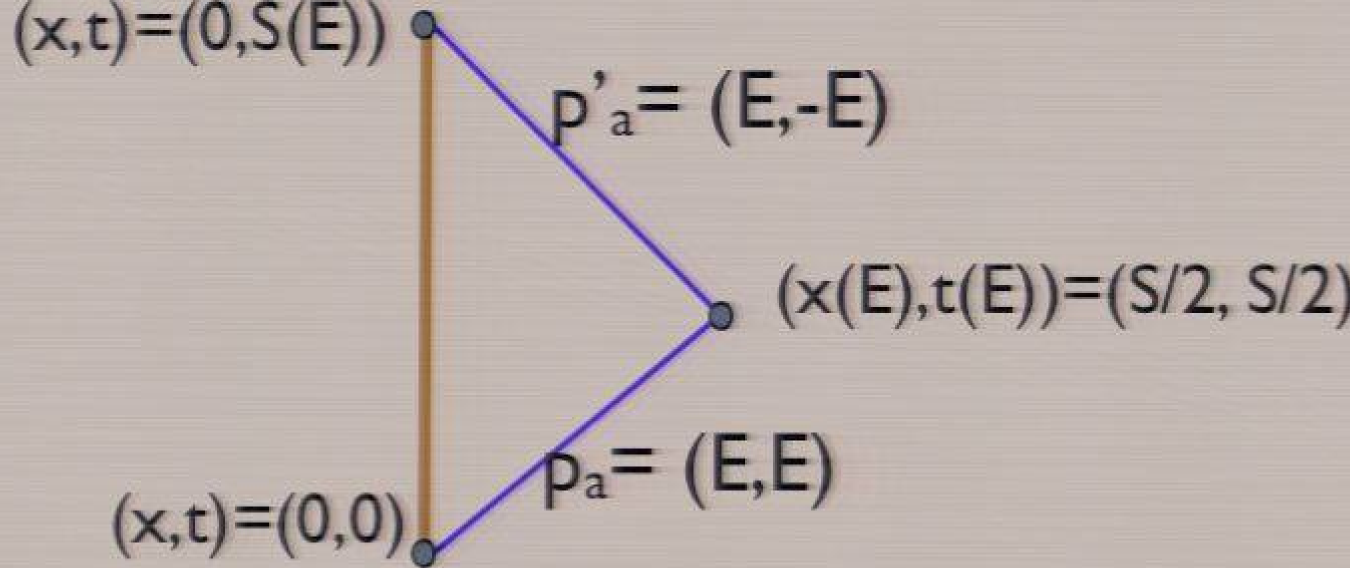


Observers at the detector see the detection events to be local while the emission events are smeared out proportionally to distance and energy.

Observers at the GRB see the emission events to be local while the detection events are smeared out proportionally to distance and energy.

This is paradoxical if you insist that both observers see events unfolding in an invariant spacetime. Once you understand spacetime is relative and phase space is invariant, the paradox

Einstein localization gives energy dependent spacetime coordinates, determined by curvature of momentum space.

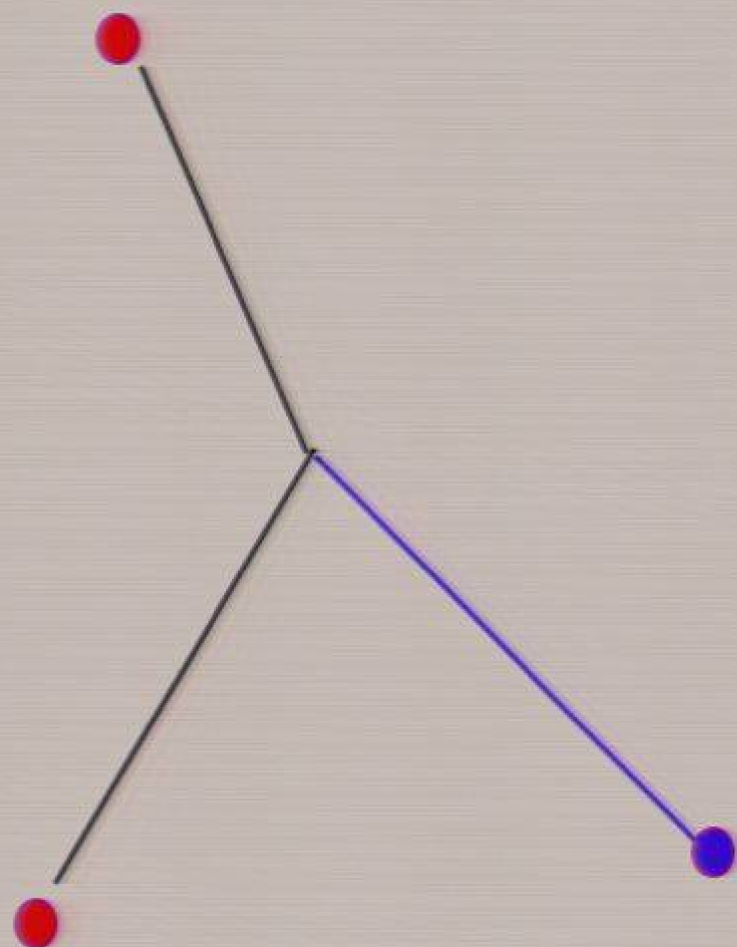


The soccer ball problem

The soccer ball problem

If elementary particles scatter like:

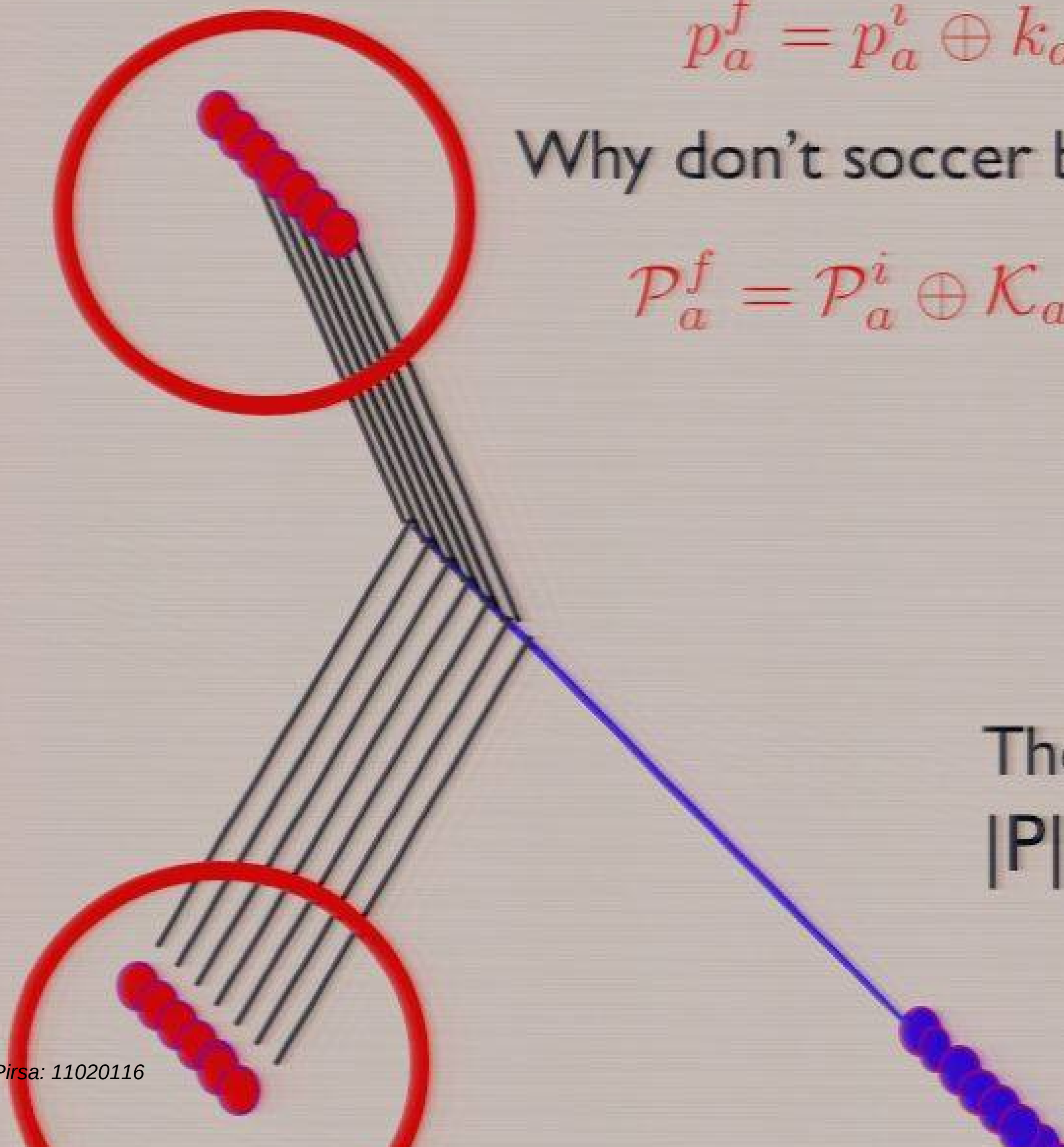
$$p^f_a = p^i_a \oplus k_a = p^i_a + k_a + \Gamma^{bc}_a p^i_b k_c + ...$$



The soccer ball problem

If elementary particles scatter like:

$$p^f_a = p^i_a \oplus k_a = p^i_a + k_a + \Gamma^{bc}_a p^i_b k_c + \ldots$$



Why don't soccer balls scatter like:

$$\mathcal{P}^f_a = \mathcal{P}^i_a \oplus \mathcal{K}_a = \mathcal{P}^i_a + \mathcal{K}_a + \Gamma^{bc}_a \mathcal{P}^i_b \mathcal{K}_c + \ldots$$

$$\begin{aligned} \mathcal{P}_a &= N p_a \\ \mathcal{K}_a &= N k_a \end{aligned}$$

They clearly don't because $|P| >> m_P$

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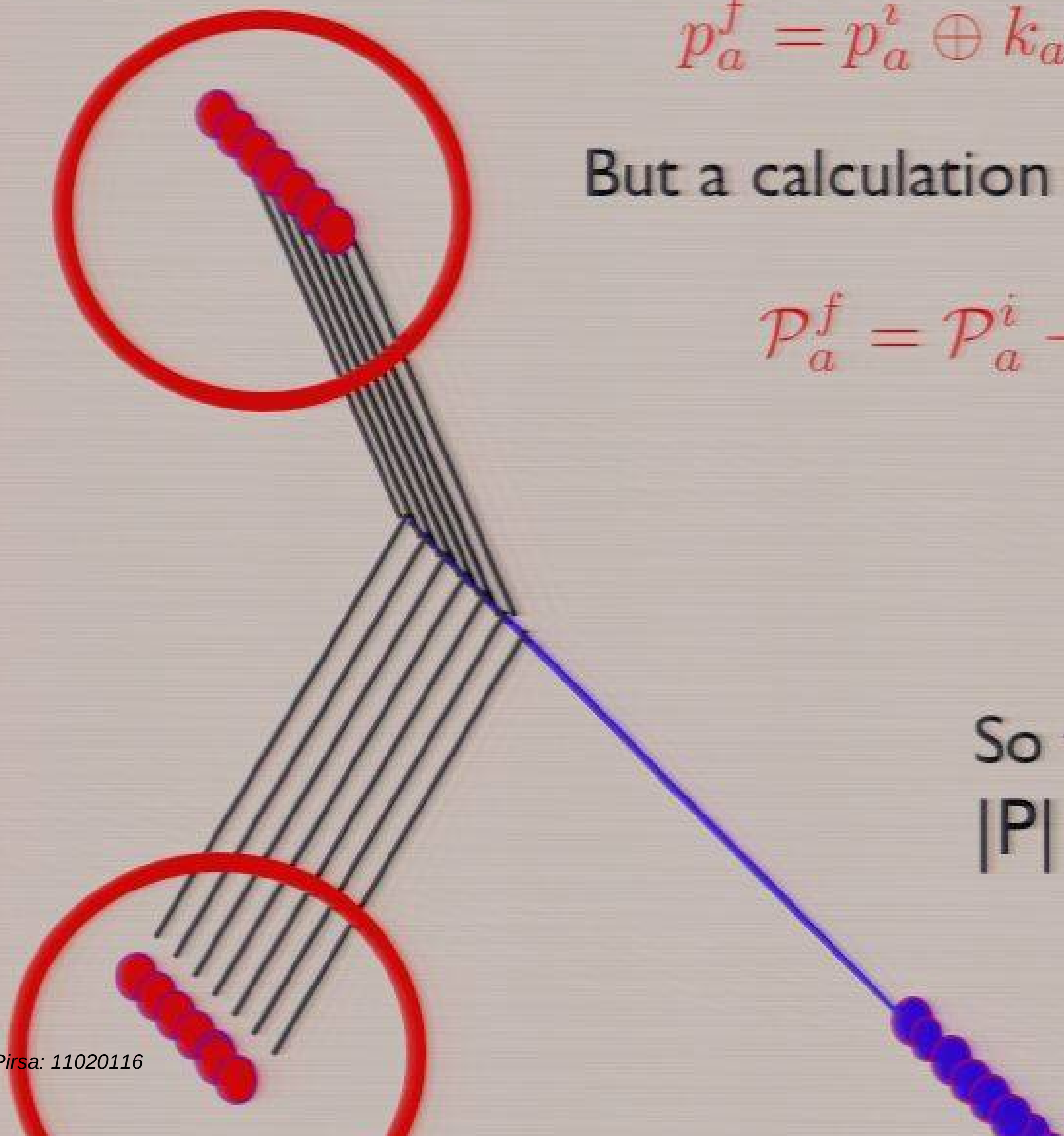


But a calculation shows that:

$$\mathcal{P}^f_a = \mathcal{P}^i_a + \mathcal{K}_a + \frac{1}{N}\Gamma^{bc}_a \mathcal{P}^i_b \mathcal{K}_c + ...$$

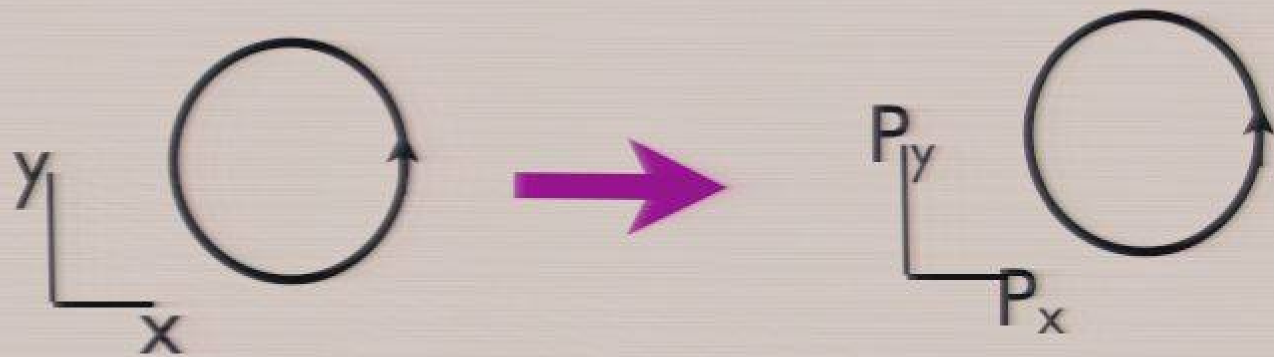
$$\mathcal{P}_a = N p_a$$

$$\mathcal{K}_a = N k_a$$

So there is no problem because $|P| << N\, m_P$

Other experimental windows:

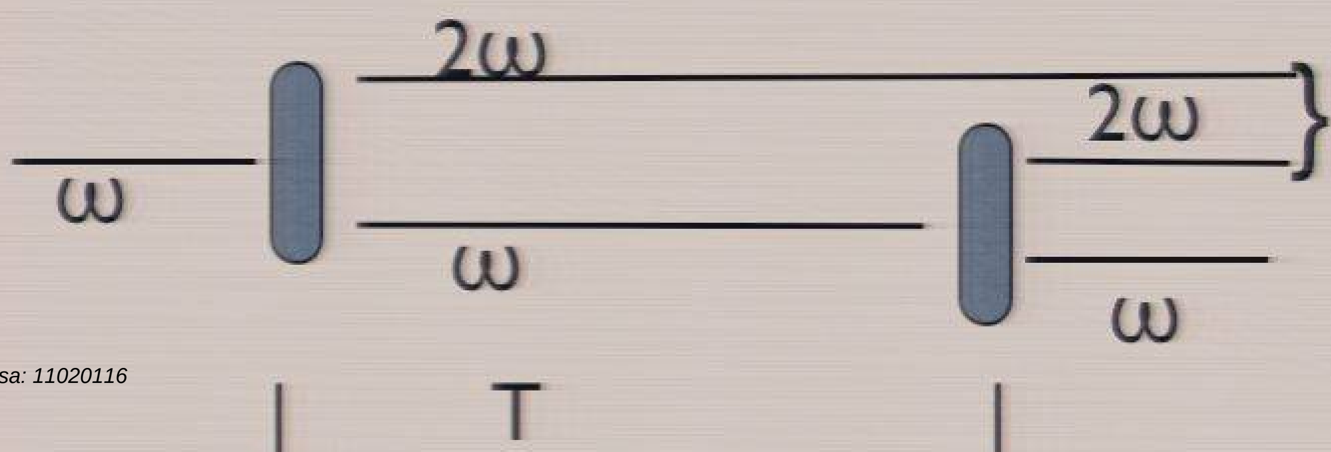
•Closed loops in momentum space create new effects from curvature of momentum space, analogous to Thomas precession.



rotation: $\Lambda \approx \frac{E^2}{m_p^2}\hat{A}$

Girelli, Livine

• Proper time is energy dependent: interferometry in phase space.



$\Delta\varphi \sim Th\omega^2/m_{QG}$

Speculative remark on the black hole information loss problem

Relative locality implies that there is an ambiguity in the localization of a particle at a time T in the future of length

$$\Delta x \approx T \left(\frac{p}{m_p} \right)^n$$

If the particle has momentum p, and fits into a black hole of mass M

$$|p| > \Delta p > \hbar / GM \quad \rightarrow \Delta x > \frac{t_p T}{GM} \left(\frac{m_p}{M} \right)^{n-1}$$

At T_{nl} the uncertainty in position larger than R_{Schw} :

$$T_{nl} = \frac{(GM)^{n+1}}{t_p^n}$$

The evaporation time is

$$T_e = \frac{(GM)^3}{t_p^2}$$

Hence, for n=1 $T_{rl} < T_e$. For n=2 $T_{rl} \sim T_e$.

For these cases, by the time the black hole evaporates the uncertainty in the location of a particle is at least as large as R_{Schw} . So we cannnot predict whether a bit of information is at that time inside or outside of the horizon.

Conclusions:

Physics takes place in Hilbert space.
There is an experimental regime, in which the arena is a phase space

$$
\begin{aligned}
G_{Newton} &\rightarrow 0 \\
\hbar &\rightarrow 0 \\
m_p &= \sqrt{\frac{\hbar}{G_{Newton}}} \rightarrow \text{constant}
\end{aligned}
$$

m_p can measure the geometry of momentum space, *P*.

- If momentum space is curved there is no invariant notion of spacetime.
- There is only an invariant phase space, $T^*(P)$

If so, spacetime is as misleading a concept as space is in special relativity.
$O(m_p)$ phenomena appear paradoxical if one attempts to describe them using a notion of invariant spacetime.

Conclusions:

These apparent paradoxes appear to be resolved by working consistently in the phase space.

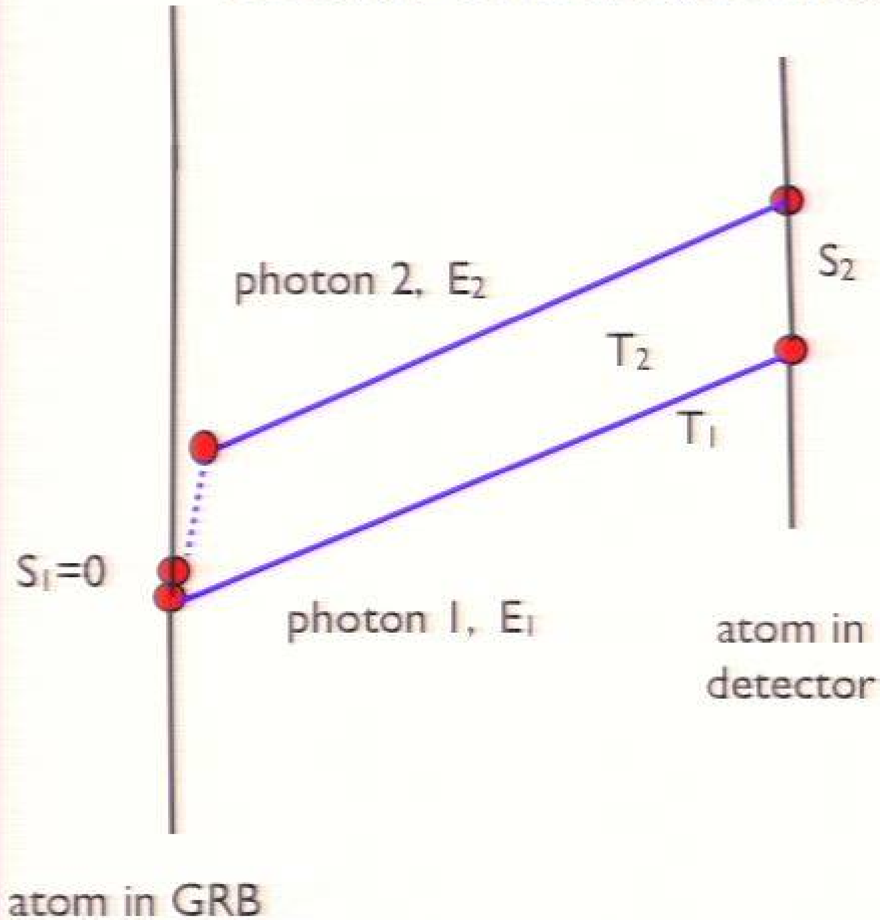
- No soccer ball problem
- Distant coordinate ambiguities not paradoxical, leads instead to consistent predictions of phenomena like GRB time delays.

Geometry of momentum space is measurable and characterizes an interesting regime of accessible quantum gravity phenomena.

- GRB time delays.
- Interferometry in phase space.
- Dual Thomas precession. *(Girelli-Livine precession.)*

And around the corner: turn on h and G

The time delay is due to relative locality: observers see distant events as smeared out non-locally.



Observers at the detector see the detection events to be local while the emission events are smeared out proportionally to distance and energy.

Observers at the GRB see the emission events to be local while the detection events are smeared out proportionally to distance and energy.

This is paradoxical if you insist that both observers see events unfolding in an invariant spacetime. Once you understand spacetime is relative and phase space is invariant, the paradox

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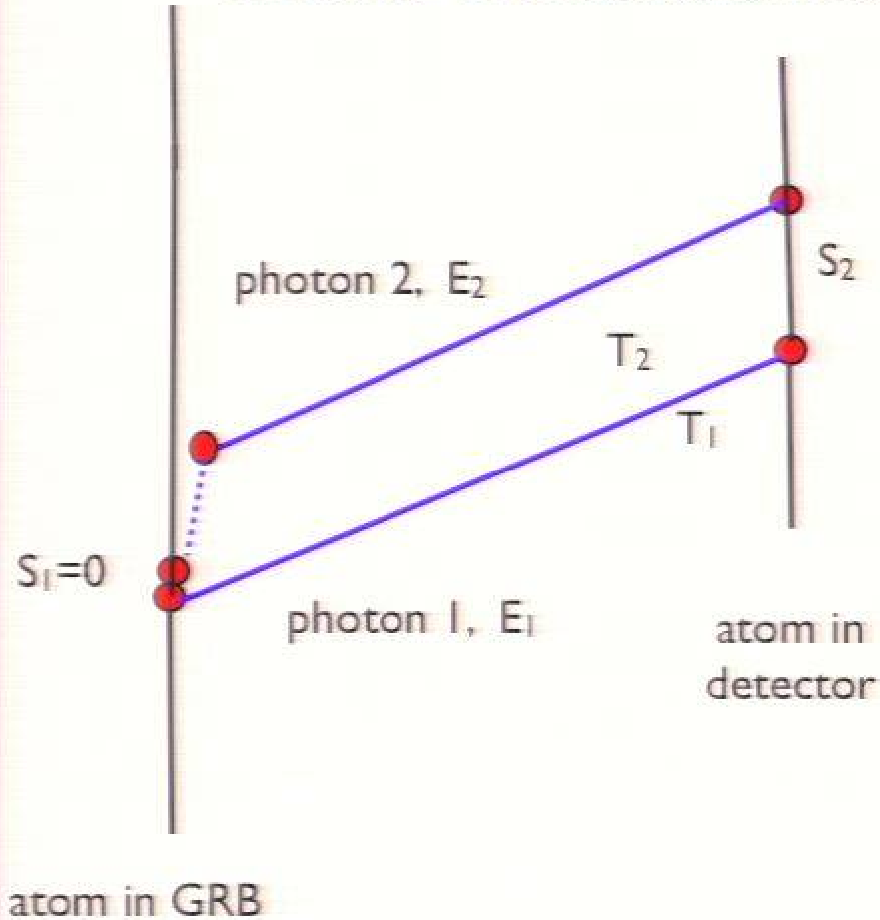

The Fermi event **GRB 090510** bounds the non-metricity tensor:

$$N_{-}^{++} < \frac{1}{1.2 M_{planck}}$$

$$S_2 - S_1 = E_2 \, T \, \Gamma_{-}^{++}$$

$$= E_2 \, T \, N^{++}$$

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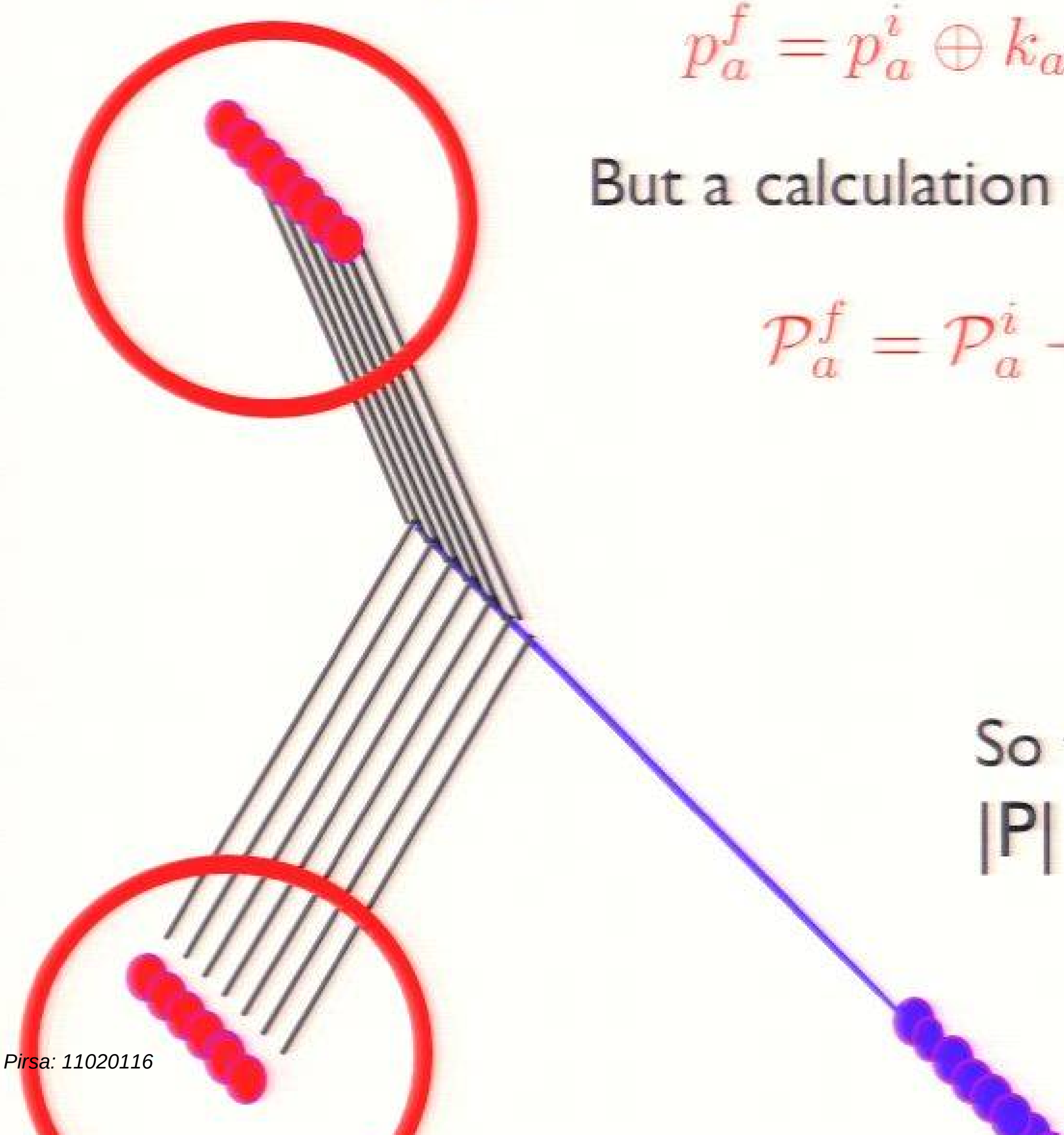

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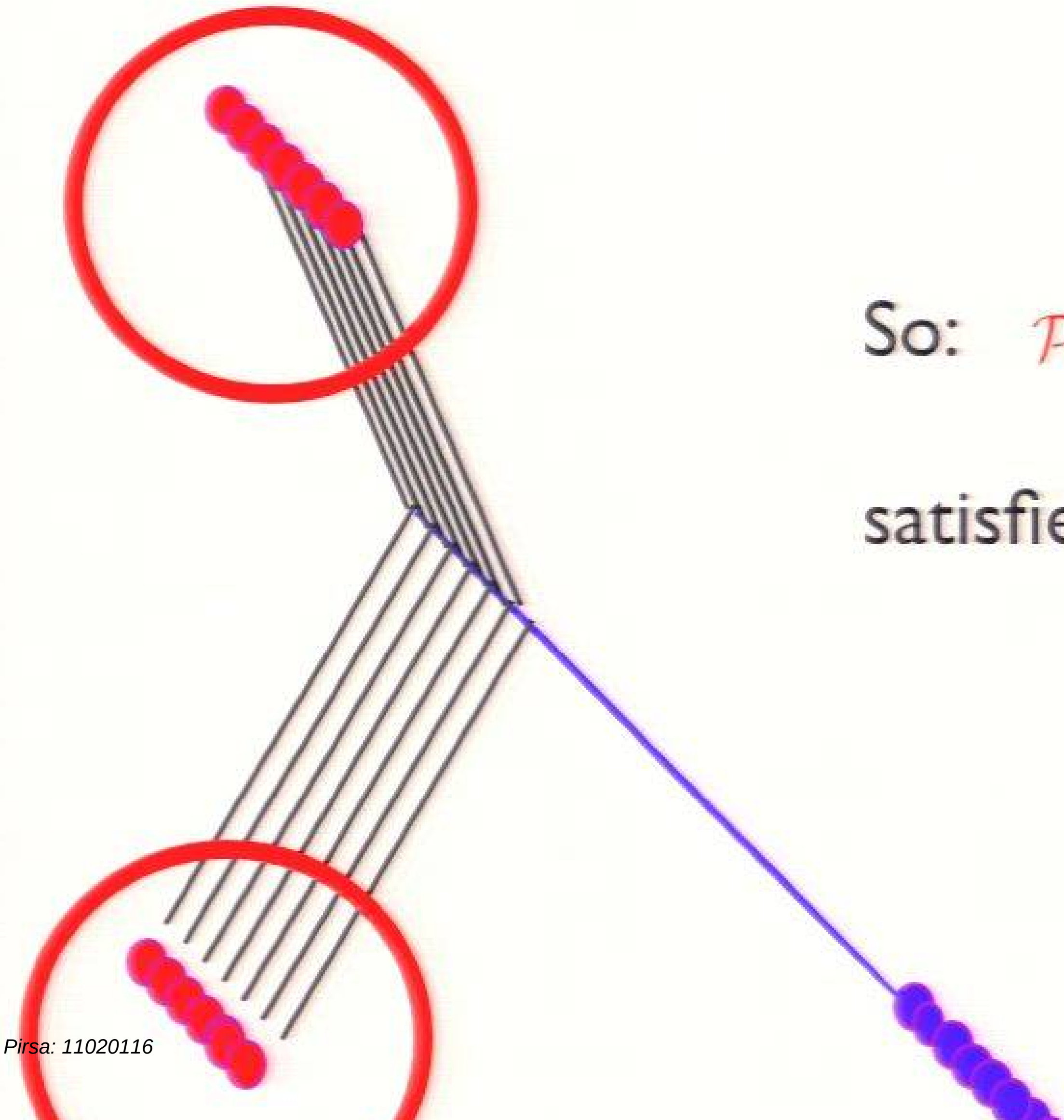
So there is no problem because
$|P| << N\ m_P$

The soccer ball problem: energy-momentum?

In Riemann normal coordinates:

$$D^2(p) = \eta^{ab} p_a p_b = m^2$$

$$p_a \oplus p_a = 2p_a$$



So: $\mathcal{P}_a = N p_a$

satisfies $\eta^{ab} \mathcal{P}_a \mathcal{P}_b = (Nm)^2$