Title: Constraints on dark matter annihilation in Milky Way halo from spherical harmonics of Fermi gamma-rays

Date: Feb 04, 2011 02:30 PM

URL: http://pirsa.org/11020115

Abstract: Gamma-ray production by dark matter annihilation is one of the most universal indirect dark matter signals. In order to avoid intensive astrophysical background, one can study the gamma-rays away from the Galactic plane. The problems is that the dark matter annihilation signal at high latitudes is smooth and most probably subdominant to Galactic and extragalactic fluxes. I will discuss the use of spherical harmonics decomposition as a tool to distinguish a large scale small amplitude dark matter signal from astrophysical fluxes. The sensitivity of this method for currently available Fermi data is similar to the signal from thermal WIMP dark matter annihilation into W+W- or b-bbar.

Outline

1. Motivation

   DM annihilation at high latitudes is a large scale small amplitude signal: ``Fourier transform" is a tool of choice for such signals

2. Illustration of the idea: Fourier transform of a signa
   on an interval

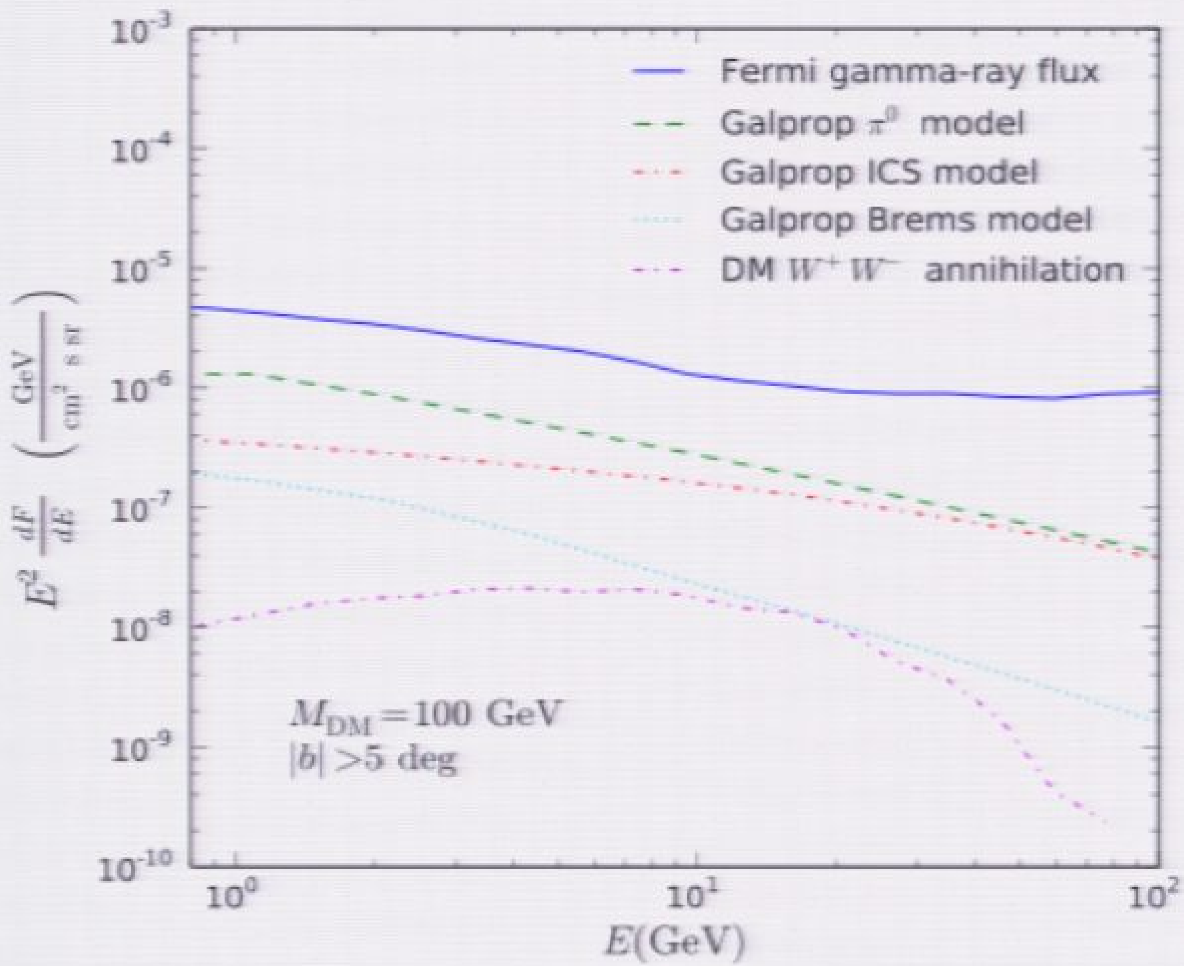
3. Fitting the large scale structure in the space of spherical harmonics

Sources of gamma-ray flux

1. Extragalactic sources:
   1. Point sources
   2. Isotropic flux (very faint point sources)
2. Galactic sources:
   1. Point sources
   2. Pion production
   3. Inverse Compton scattering
   4. Bremsstrahlung
3. Exotic sources:
   1. New astrophysics, e.g., past AGN activity
   2. Dark matter annihilation or decay

Sources of gamma-ray flux

(Galactic plane and point sources are masked)



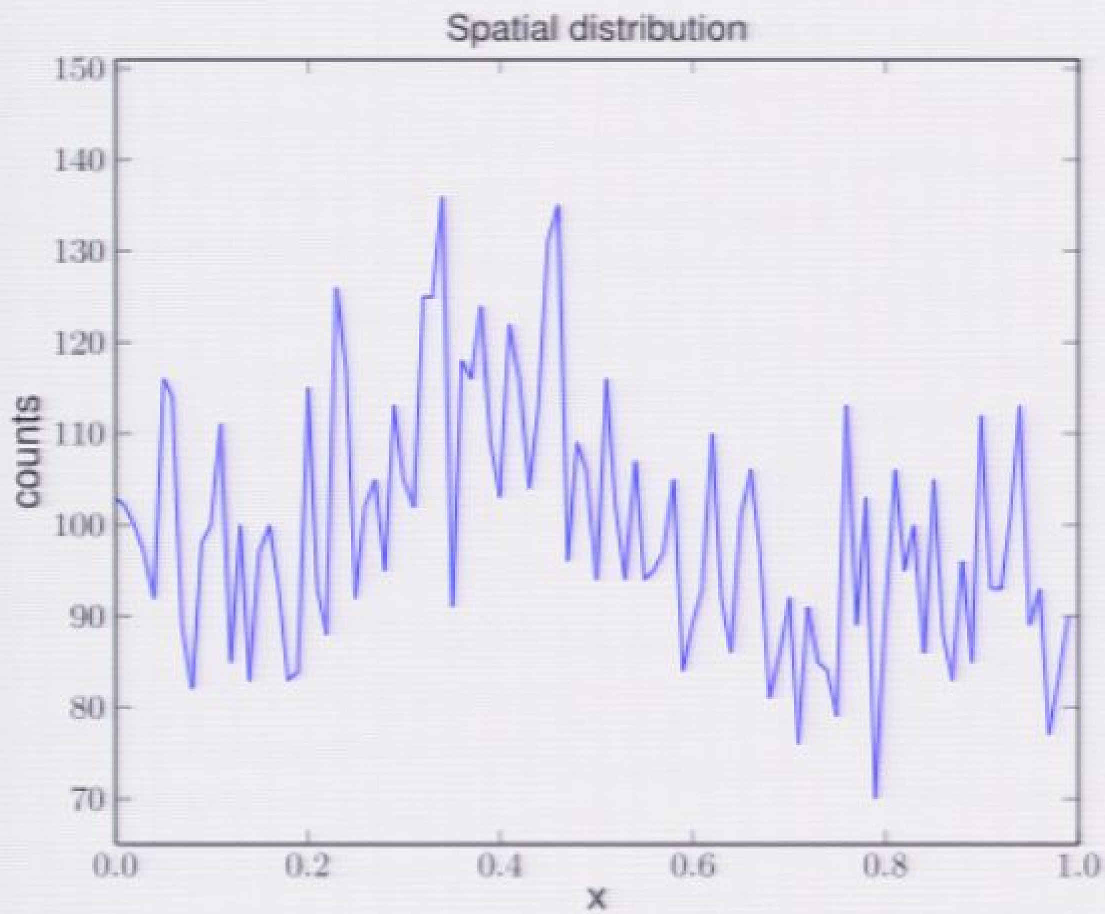
Expected DM annihilat
signal is about 1% from
the total gamma-ray flu

DM mass and annihila
cross section:

$M_{\rm DM} = 100\ {\rm GeV}$
$\sigma v = 3 \times 10^{-26}\ {\rm cm}^3/$

DM distribution:
NFW profile

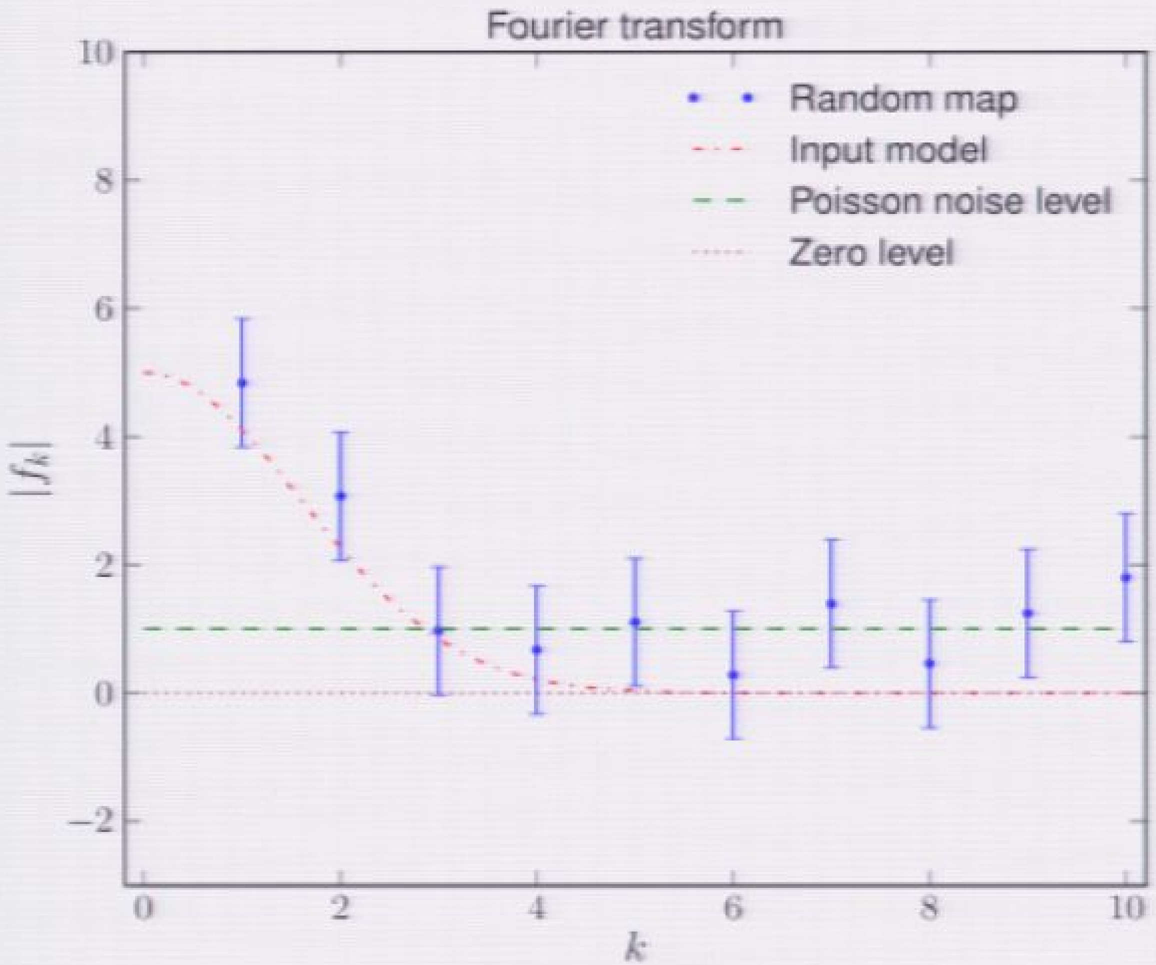
One dimensional random map with a small signal



This map has a significant homogeneous component and a small Gaussian distribution of points.

There is a clear access arou x = 0.4, but how significant it and what is its amplitude

Number of events: $N = 10^4$
Number of bins: $N_{\rm bins} = 10^2$



$$f_k = \frac{1}{\sqrt{N}} \int_0^1 dx f(x) e^{2\pi i}$$

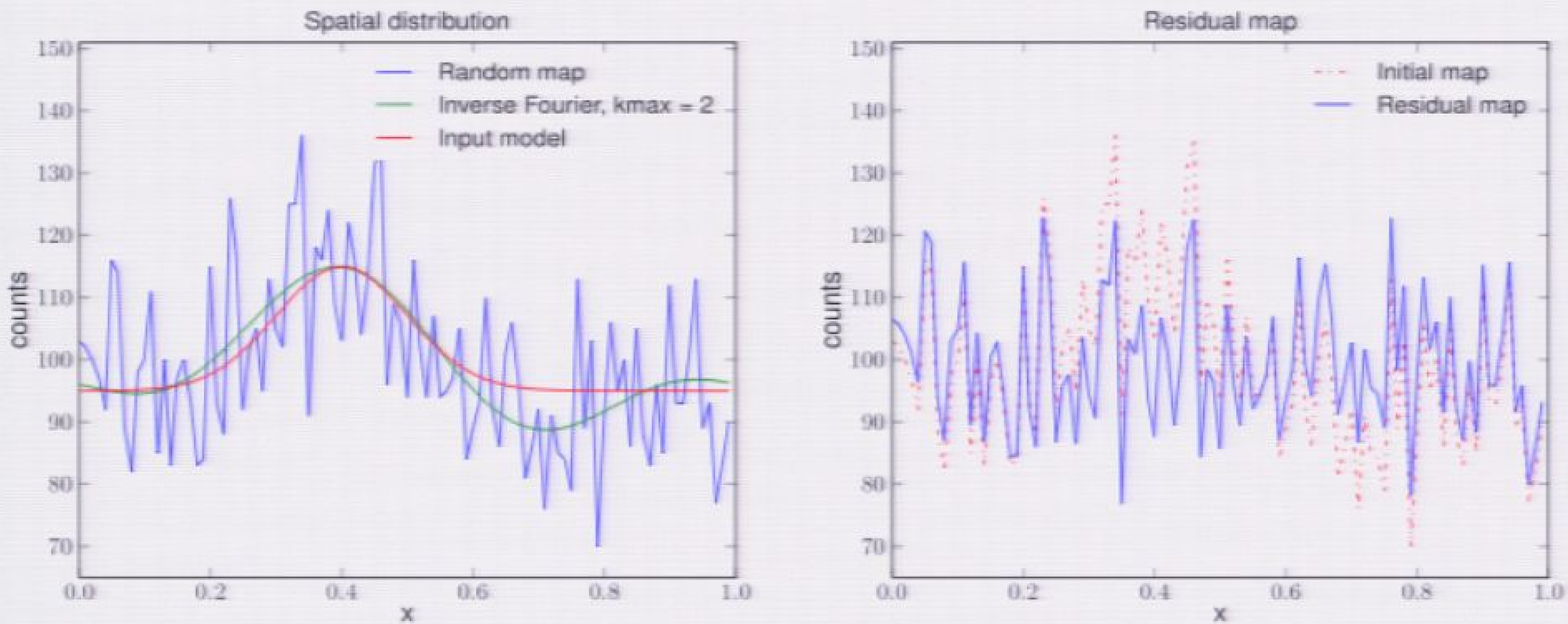
The normalization is chosen such that for th Poisson noise $\langle |f_k|^2 \rangle =$

The Poisson noise level also gives an estimate c uncertainty of f_k's

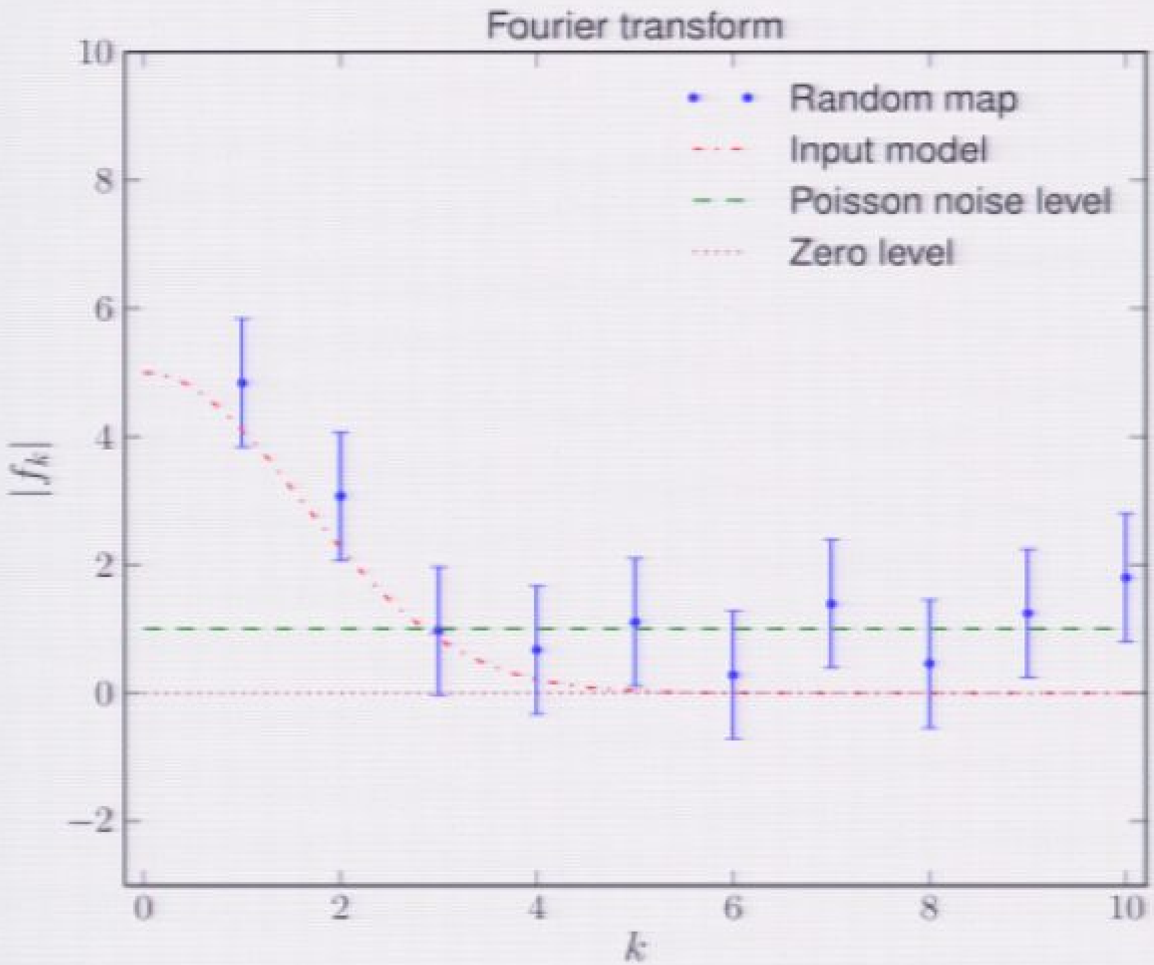
The significance of f_1 is about 5 sigma, the significance of f_2 is about 3 sigma.

The non-homogeneous contribution is $\sqrt{N} f_0 = 500$ which is 5% in this case

Inverse Fourier transform and the residual map



The inverse Fourier transform (green line)
of the first non-trivial harmonics does an OK job
in approximating the form of the excess (red line)



$$f_k = \frac{1}{\sqrt{N}} \int_0^1 dx f(x) e^{2\pi i}$$

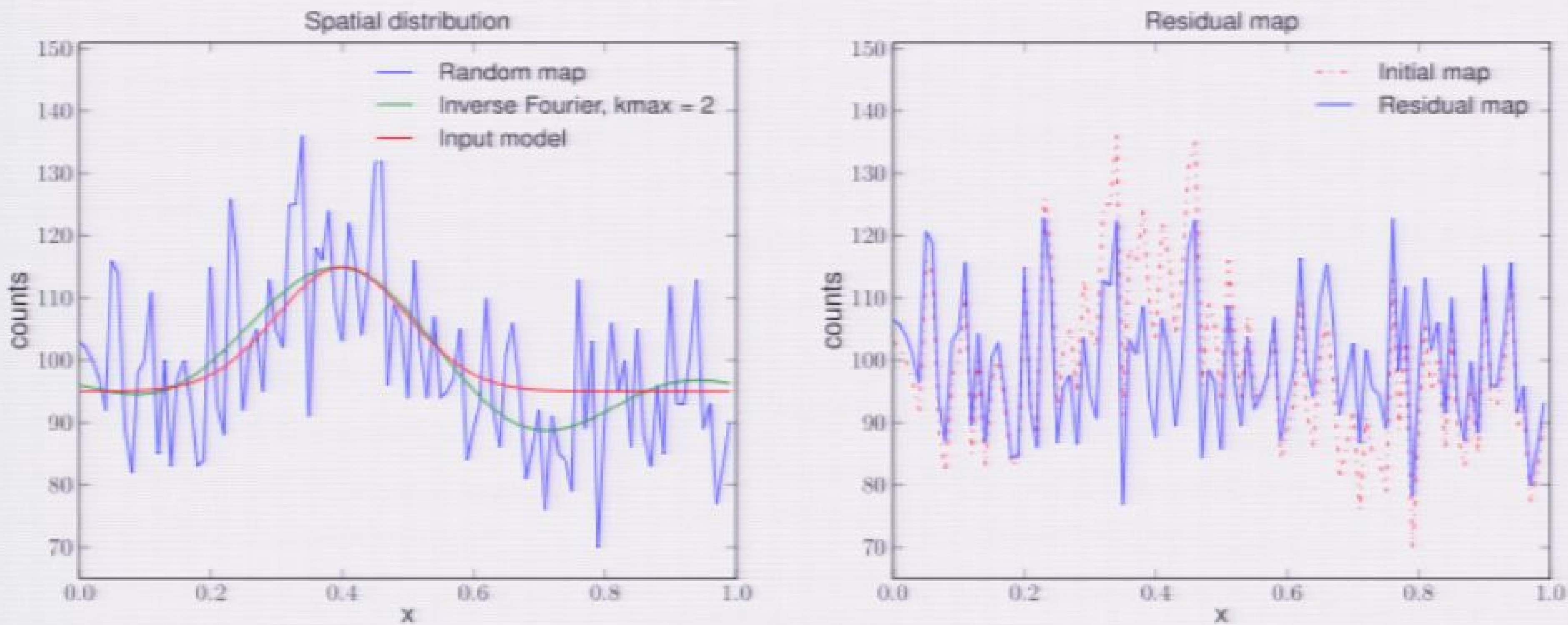
The normalization is chosen such that for th Poisson noise $\langle |f_k|^2 \rangle =$

The Poisson noise level also gives an estimate o uncertainty of f_k's

The significance of f_1 is about 5 sigma, the significance of f_2 is about 3 sigma.

The non-homogeneous contribution is $\sqrt{N} f_0 = 500$ which is 5% in this case

Inverse Fourier transform and the residual map



The inverse Fourier transform (green line)
of the first non-trivial harmonics does an OK job
in approximating the form of the excess (red line)

Fitting in Fourier space and in coordinate space

Fourier space:

1. Find Fourier transform of the data and of the templates.
2. Determine the standard deviation of the Fourier modes (given by the Poisson noise level).
3. Find the maximum likelihood model by minimizing the chi^2:

$$\chi^2(\alpha) = \sum_k \frac{|f_k^{\rm data} - f_k^{\rm model}(\alpha)|^2}{\sigma_k^2}$$

Coordinate space:

1. Determine the model in terms of the expected numbers of photons in every pixel $x_p(\alpha)$.
2. The likelihood is determined as a product over pixels of Poisson probabilities:

$$L(\alpha) = \prod_p \frac{x_p(\alpha)^{n_p}}{n_p!} e^{-x_p(\alpha)}$$

Comparison of fitting a small gaussian excess on top of the homogeneous distribution

Quantity	Input	Fourier	Coordinate
Fraction	0.05	0.06 $\pm$0.01	0.06 $\pm$0.01
Position	0.40	0.38 $\pm$0.01	0.38 $\pm$0.01
Dispersion	0.10	0.10 $\pm$0.02	0.10 $\pm$0.01

The results are practically the same but in Fourier space I used only 4 Fourier modes instead of 100 pixels in coordinate space fitting

Fitting in Fourier space and in coordinate space

Fourier space:

1. Find Fourier transform of the data and of the templates.
2. Determine the standard deviation of the Fourier modes (given by the Poisson noise level).
3. Find the maximum likelihood model by minimizing the chi^2:

$$\chi^2(\alpha) = \sum_k \frac{|f_k^{\text{data}} - f_k^{\text{model}}(\alpha)|^2}{\sigma_k^2}$$

Coordinate space:

1. Determine the model in terms of the expected numbers of photons in every pixel $x_p(\alpha)$.
2. The likelihood is determined as a product over pixels of Poisson probabilities:

$$L(\alpha) = \prod \frac{x_p(\alpha)^{n_p}}{n_p!} e^{-x_p(\alpha)}$$

Comparison of fitting a small gaussian excess on top of the homogeneous distribution

Quantity	Input	Fourier	Coordinate
Fraction	0.05	0.06 ± 0.01	0.06 ± 0.01
Position	0.40	0.38 ± 0.01	0.38 ± 0.01
Dispersion	0.10	0.10 ± 0.02	0.10 ± 0.01

The results are practically the same but in Fourier space I used only 4 Fourier modes instead of 100 pixels in coordinate space fitting

Fourier space fitting	Coordinate space fitting
1. Linear fitting procedure	Non-linear likelihood function
2. Chi^2 doesn't depend on pixel size (for small k)	Absolute likelihood depends on pixel size
3. Stable wrt negative expected numbers of photons	Poisson probability for negative expectation is not defined

DM annihilation via spherical harmonics of gamma-ray

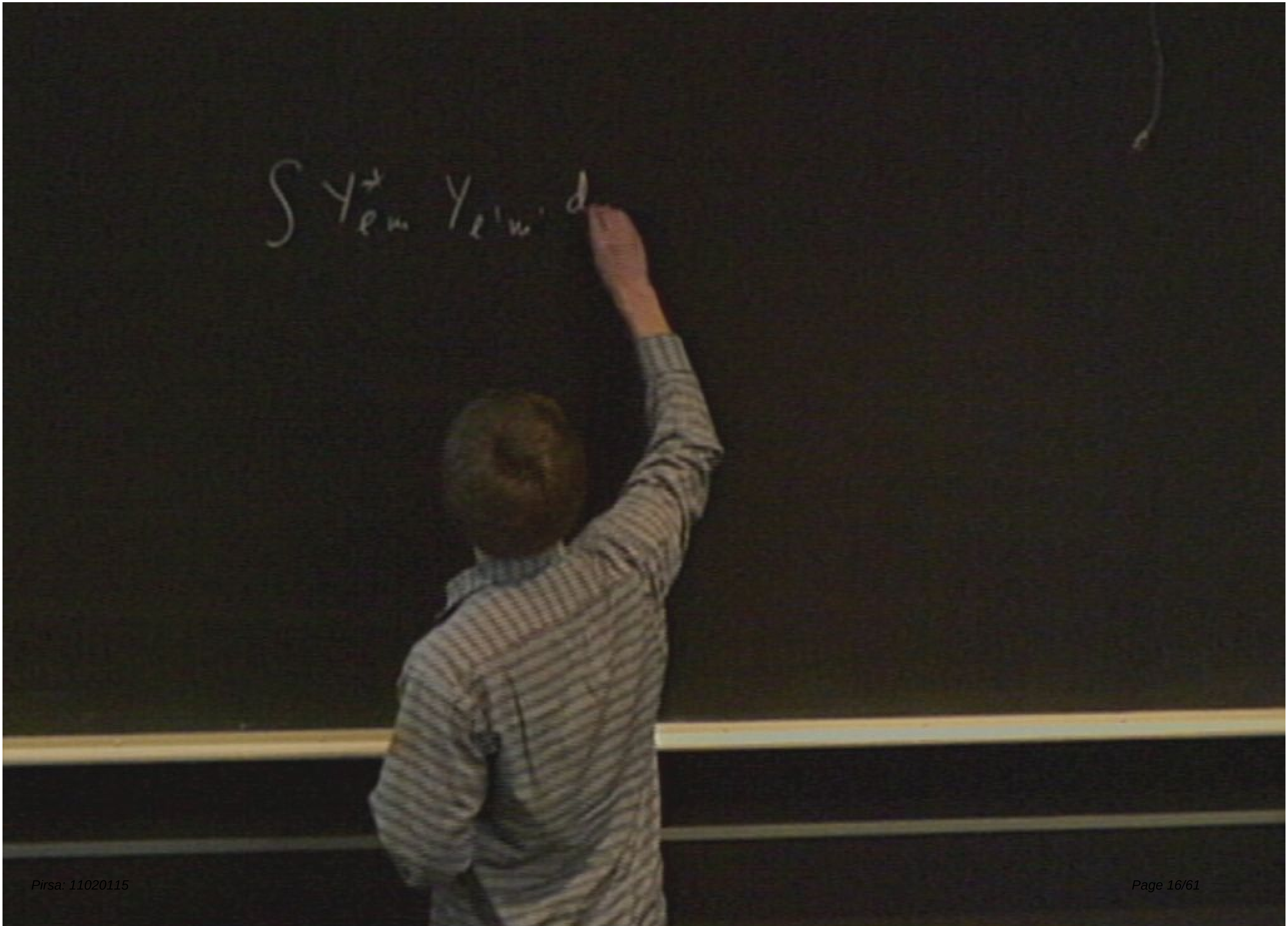
The goal is to find a small amplitude large scale gamma-ray signal due to annihilation of dark matter in the main halo of our Galaxy using the decomposition in spherical harmonics.

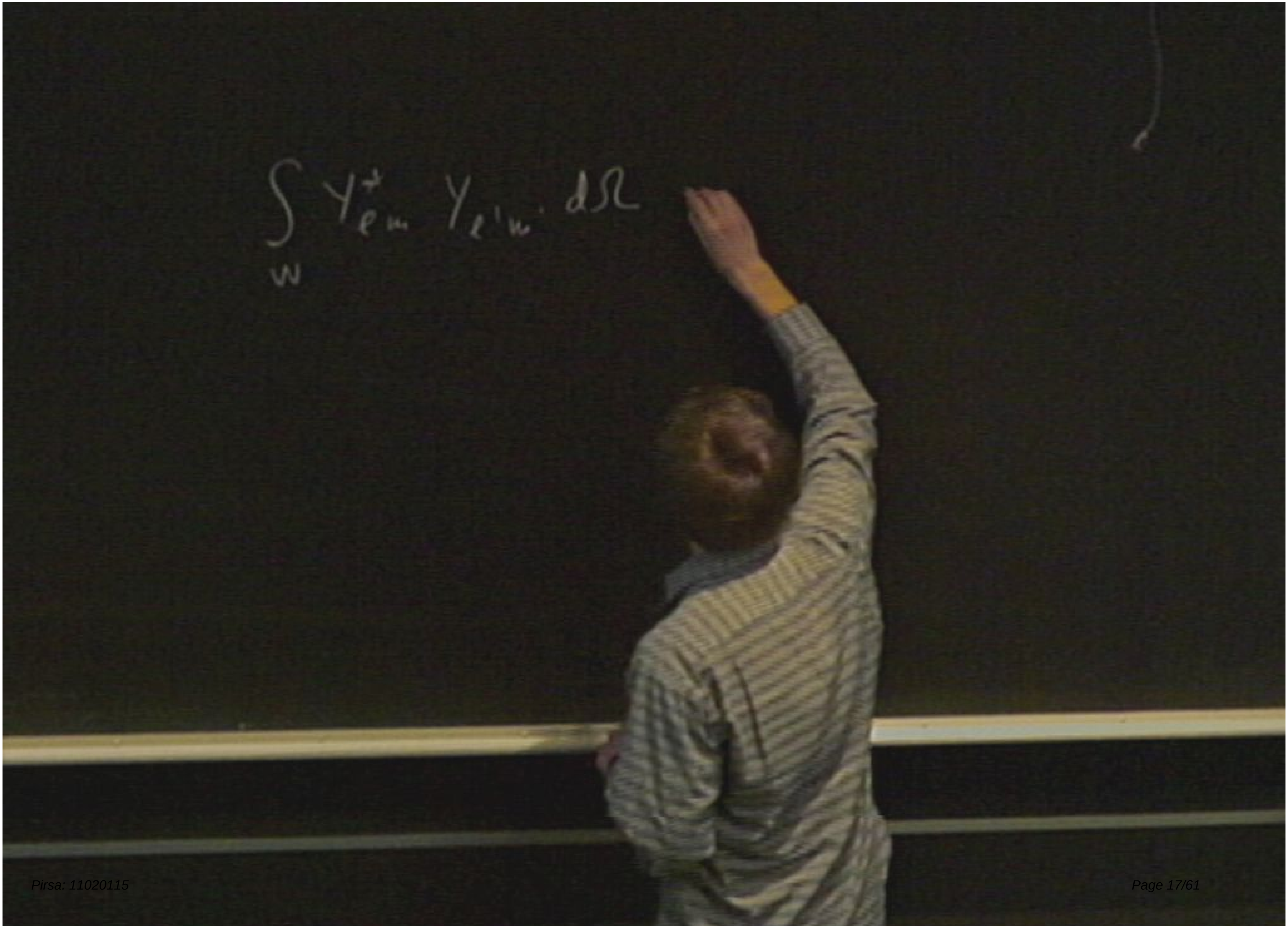
The algorithm has the following steps:

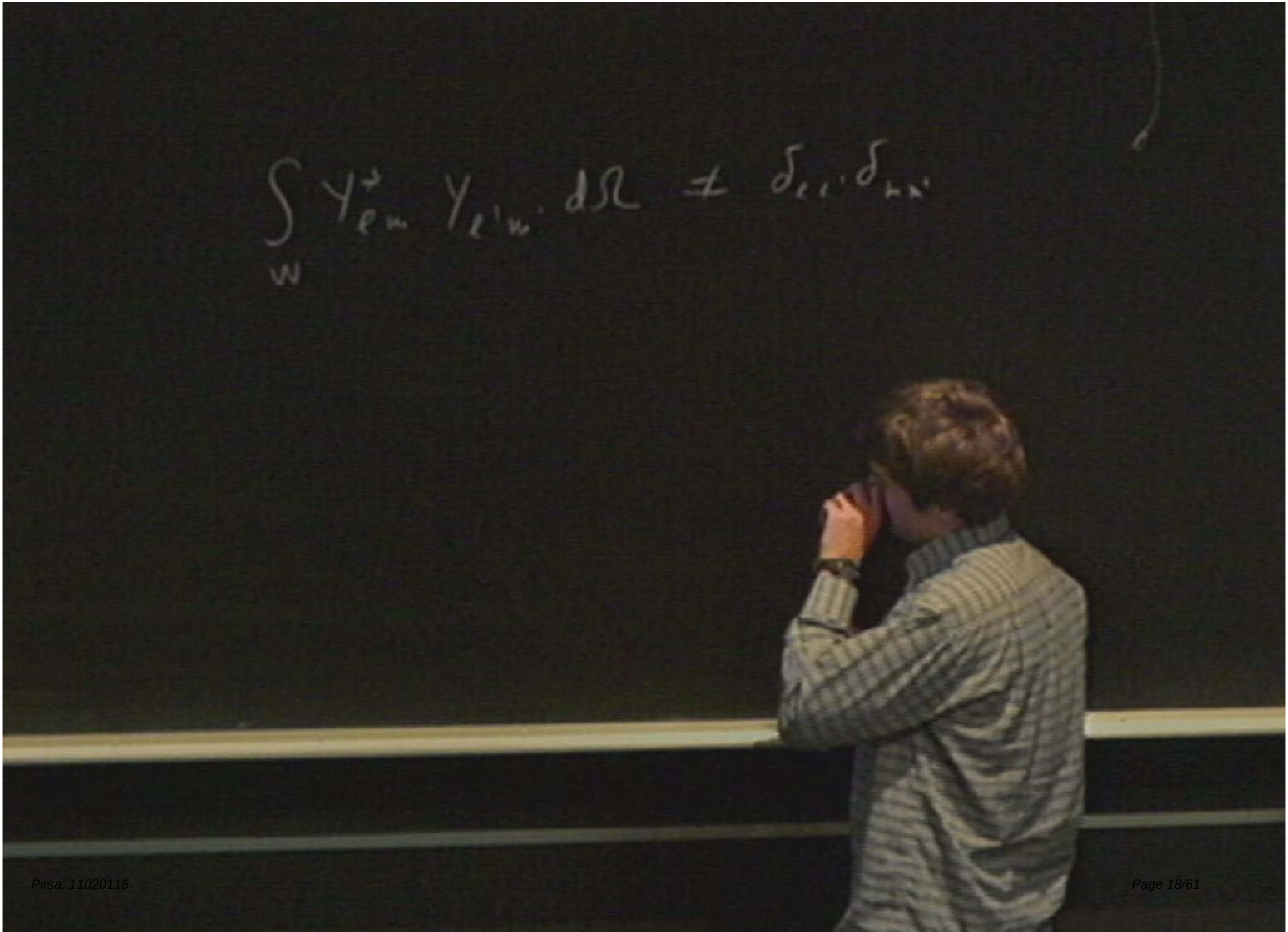
1. Mask the Galactic plane and point sources
2. Choose the astrophysics template(s)
3. Decompose the data and the templates in a_{lm}'s
4. Calculate the variance of a_{lm}'s
5. The χ^2 is defined as

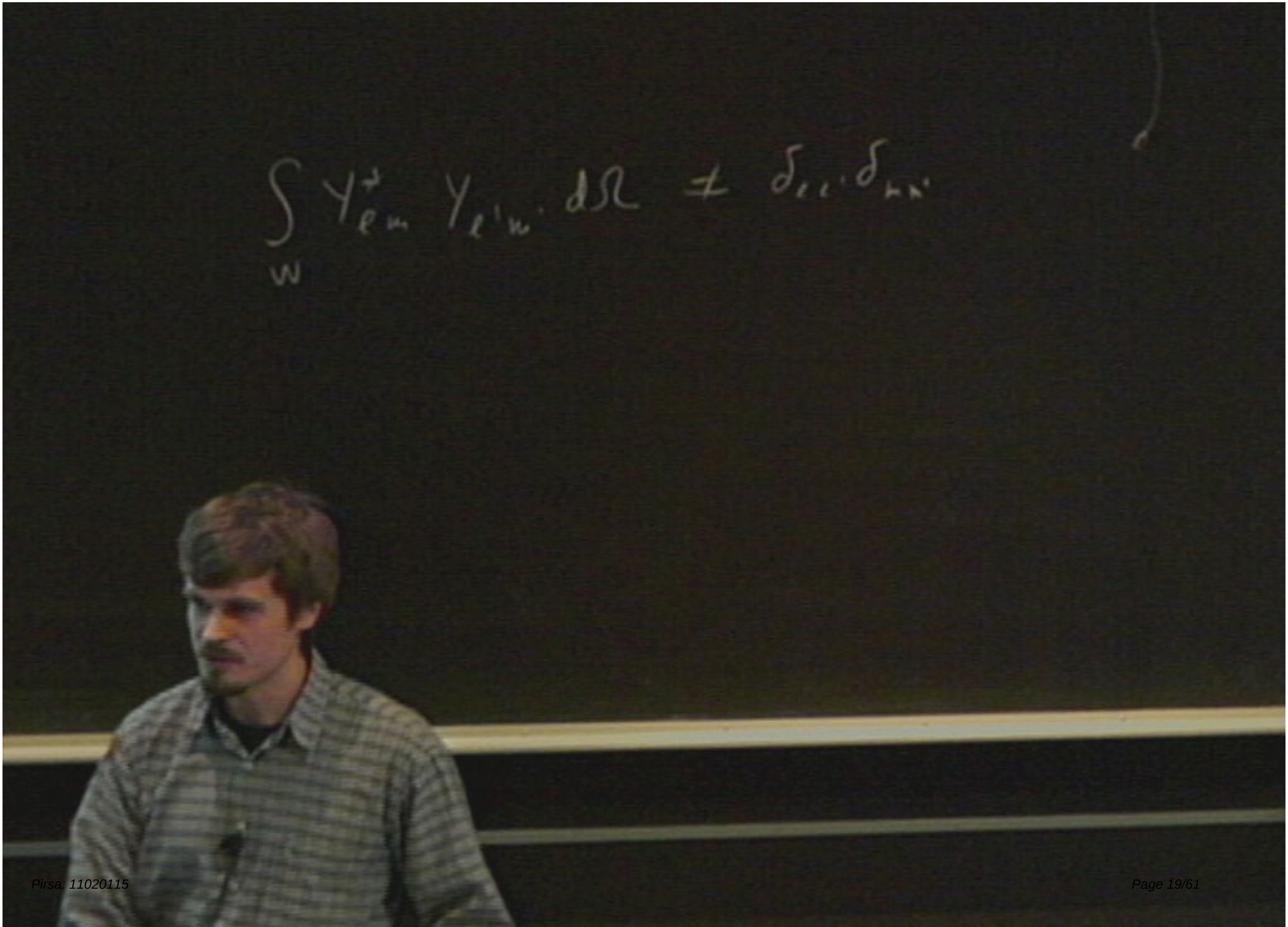
$$\chi^2 = \sum_{lm} \frac{(a_{lm}^{\mathrm{data}} - a_{lm}^{\mathrm{model}})^2}{\sigma^2(a_{lm})}$$

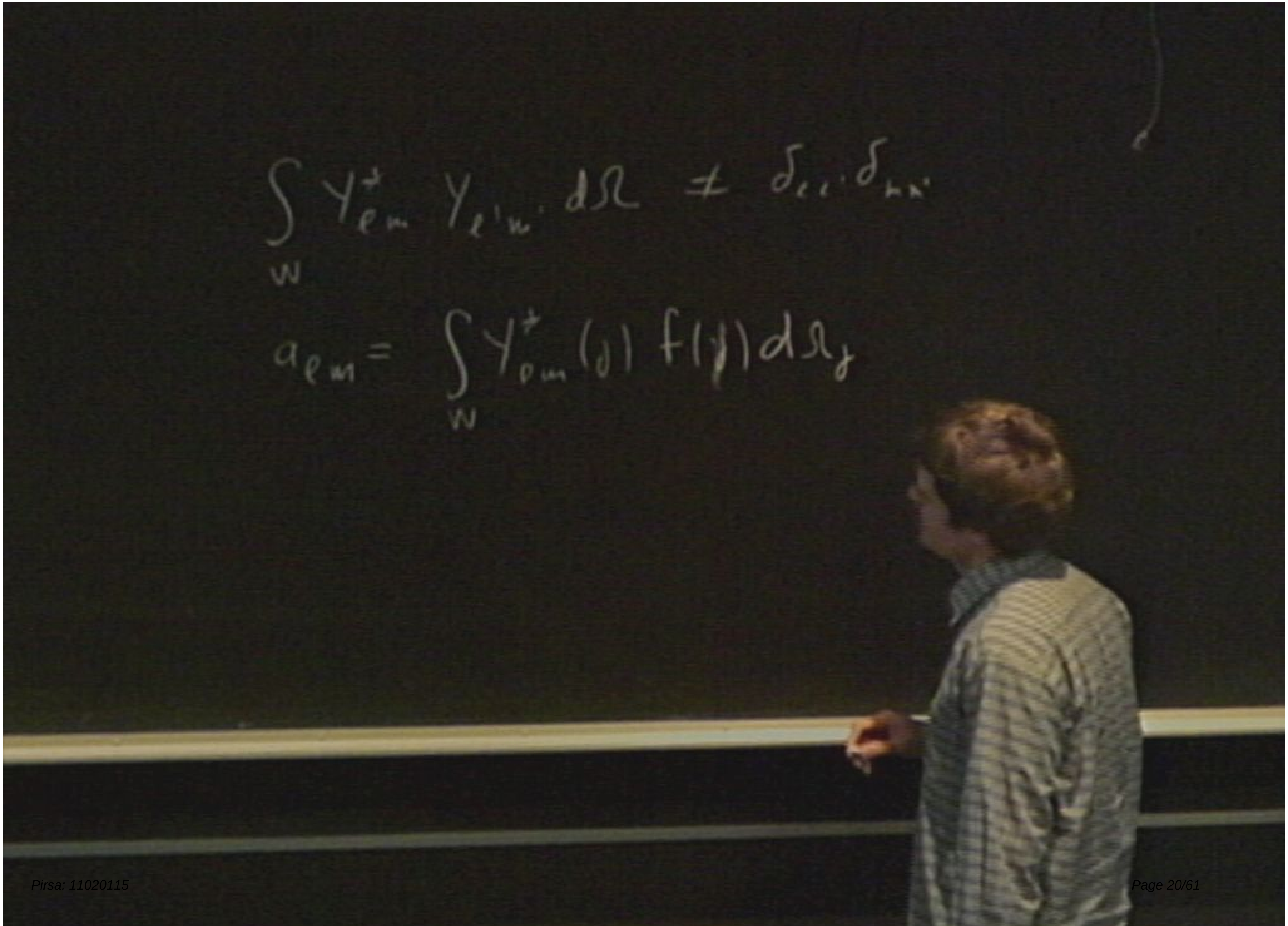
where a_{lm}^{model} is a linear sum of an isotropic component (mostly extragalactic), Galactic astrophysics, and any extra templates.


$\int Y^*_{\ell m} Y_{\ell' m'} d$

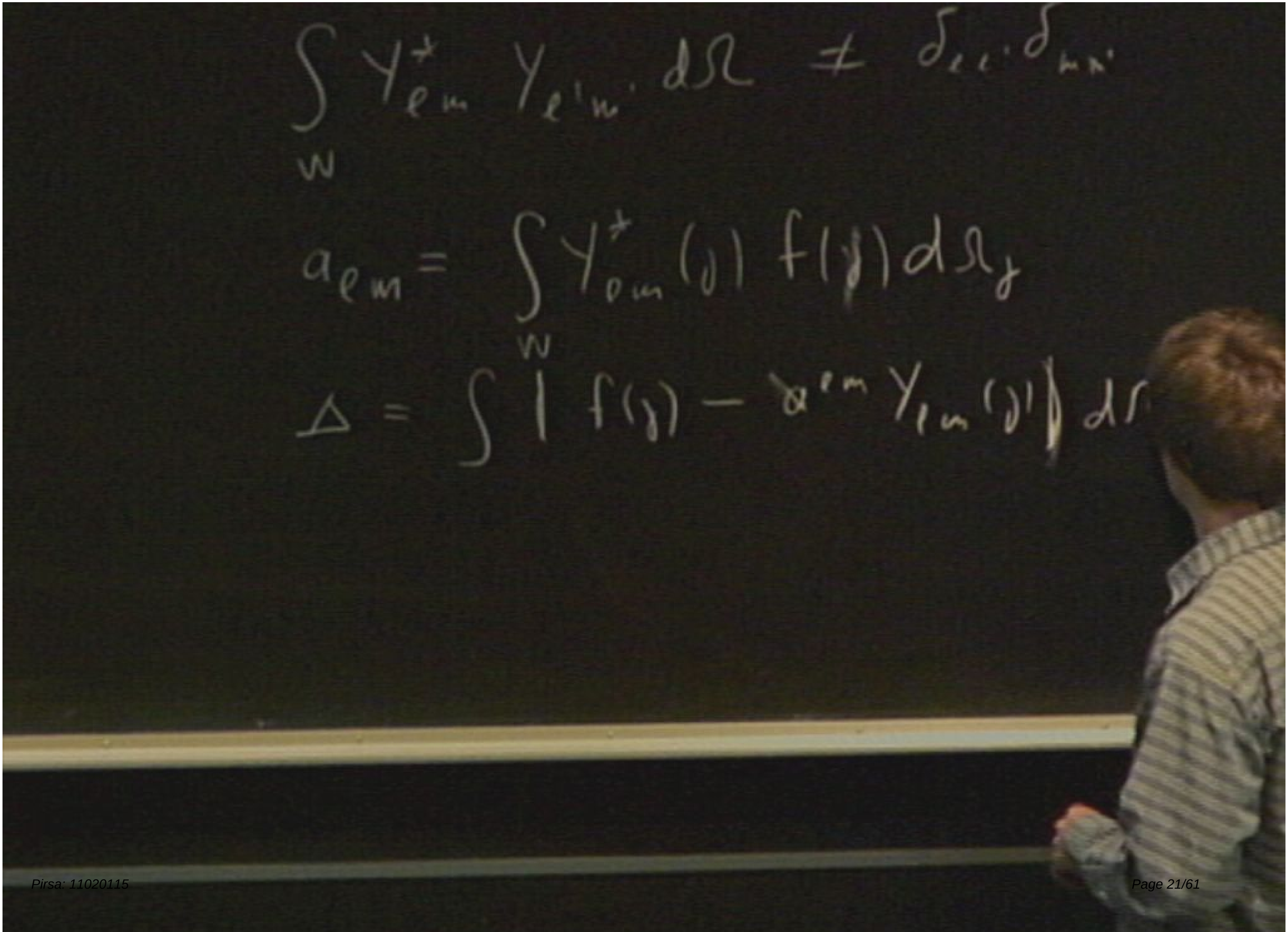

$\int_W Y^*_{\ell m} Y_{\ell' m'} \, d\Omega$

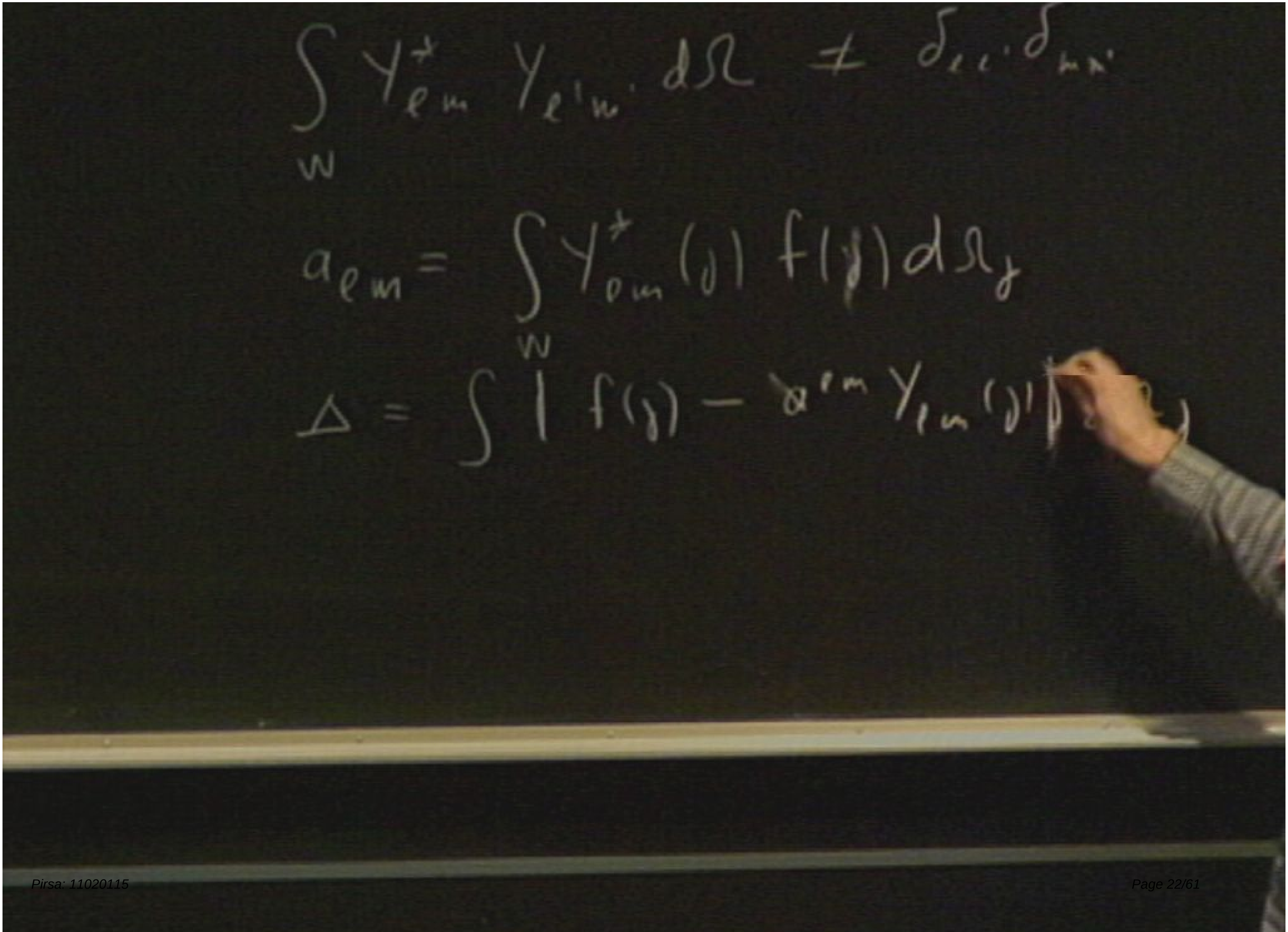

$\int_W Y^*_{\ell m} Y_{\ell' m'} \, d\Omega \neq \delta_{\ell\ell'}\delta_{mm'}$

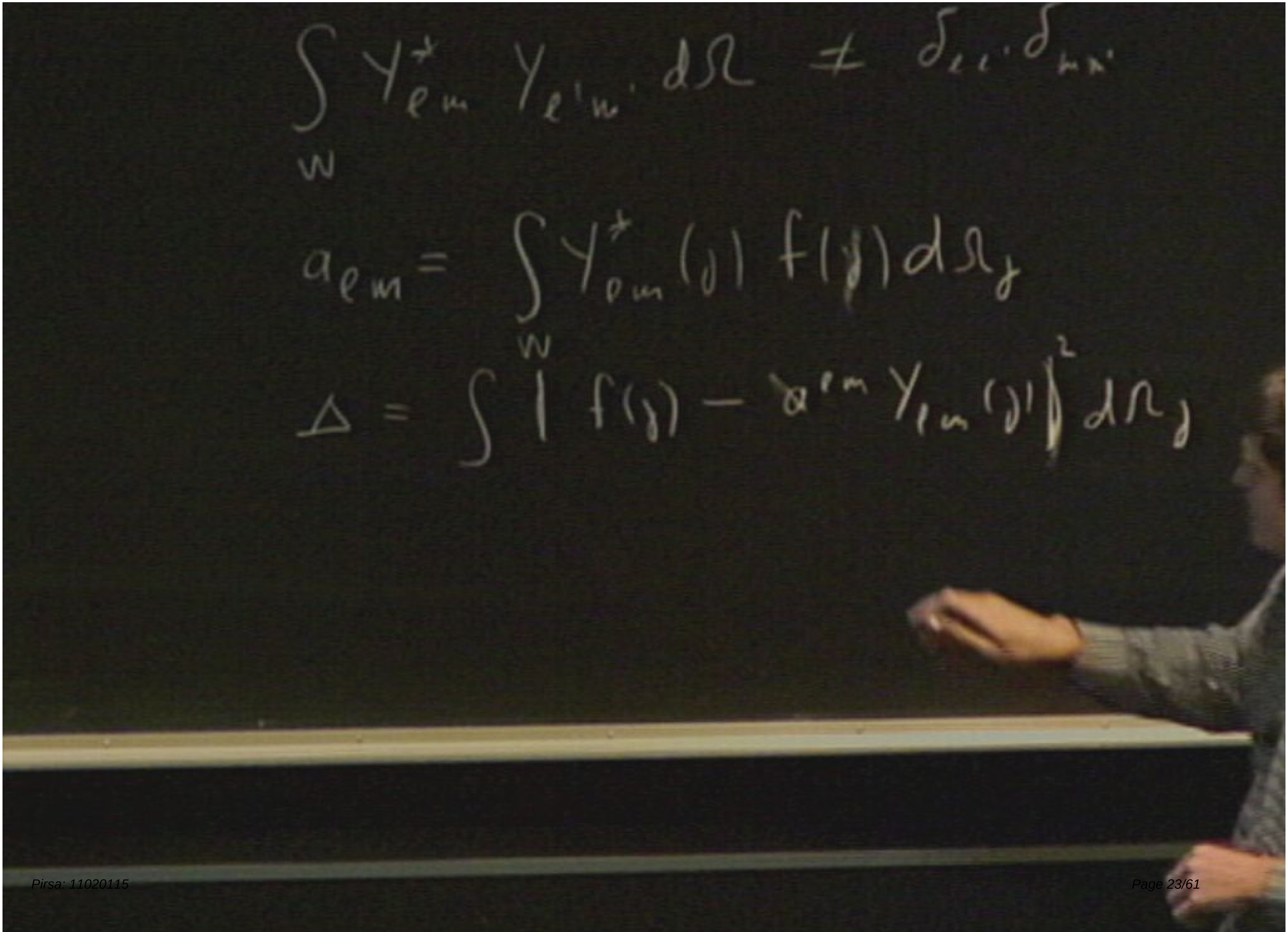

$\int_W Y^*_{\ell m} Y_{\ell' m'} \, d\Omega \neq \delta_{\ell\ell'} \delta_{mm'}$

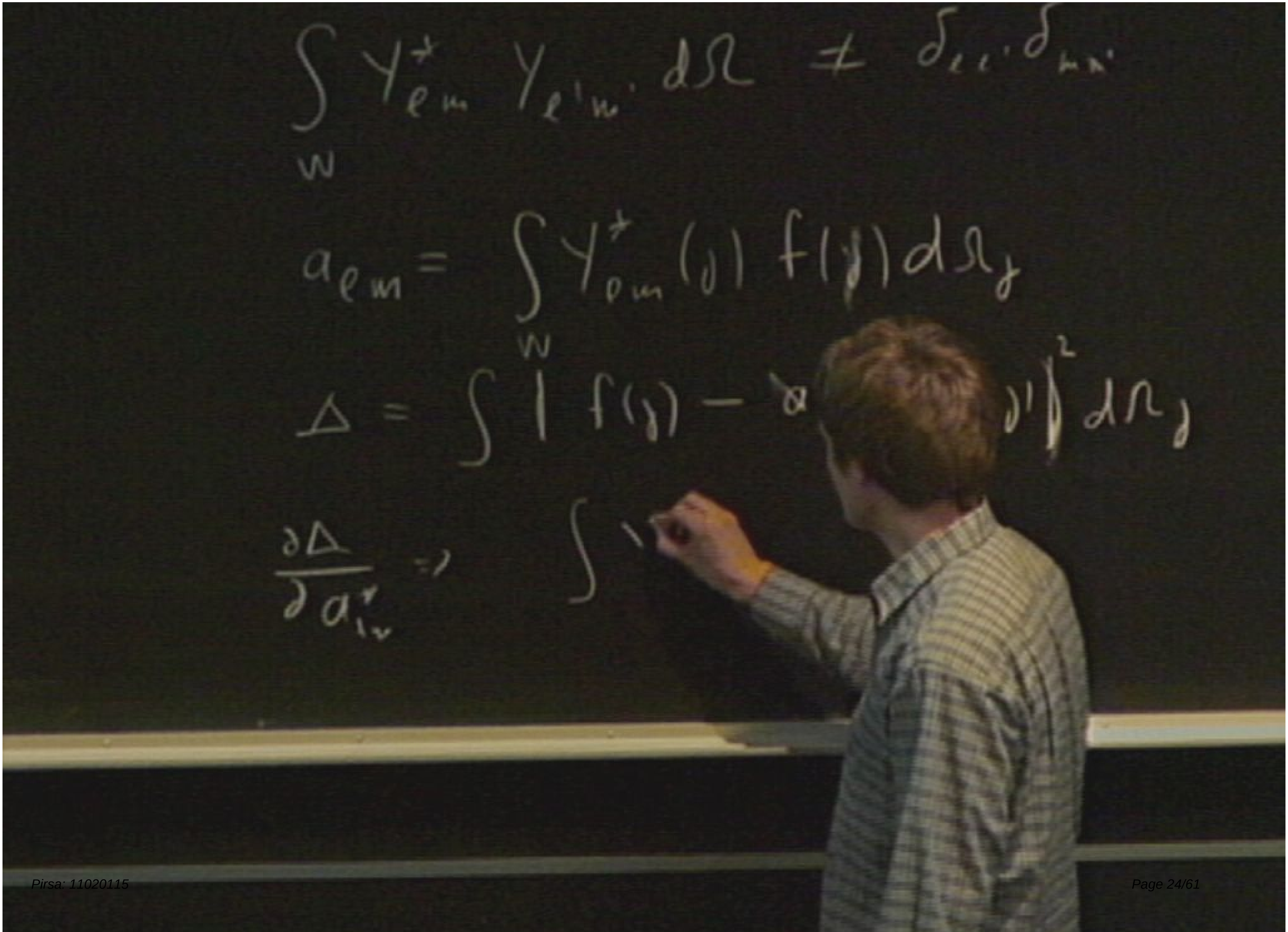


$\int_W Y^*_{\ell m} Y_{\ell' m'} \, d\Omega \neq \delta_{\ell \ell'} \delta_{m m'}$
$a_{\ell m} = \int_W Y^*_{\ell m}(\gamma) f(\gamma) \, d\Omega_\gamma$


∫_W Y*_{lm} Y_{l'm'} dΩ ≠ δ_{ll'} δ_{mm'}
a_{lm} = ∫_W Y*_{lm}(γ) f(γ) dΩ_γ
Δ = ∫ | f(γ) − a^{lm} Y_{lm}(γ) | dΩ


$\int_W Y^*_{\ell m} Y_{\ell' m'} \, d\Omega \neq \delta_{\ell\ell'} \delta_{mm'}$
$a_{\ell m} = \int_W Y^*_{\ell m}(\gamma) f(\gamma) \, d\Omega_\gamma$
$\Delta = \int \left| f(\gamma) - a^{\ell m} Y_{\ell m}(\gamma) \right|$


$\int_W Y^*_{\ell m} Y_{\ell' m'} \, d\Omega \neq \delta_{\ell \ell'} \delta_{m m'}$
$a_{\ell m} = \int_W Y^*_{\ell m}(\gamma) f(\gamma) \, d\Omega_\gamma$
$\Delta = \int | f(\gamma) - a^{\ell m} Y_{\ell m}(\gamma) |^2 \, d\Omega_\gamma$



$\int_W Y^*_{\ell m} Y_{\ell' m'} d\Omega \neq \delta_{\ell\ell'}\delta_{mm'}$
$a_{\ell m} = \int_W Y^*_{\ell m}(\gamma) f(\gamma) d\Omega_\gamma$

$$\int_W Y^*_{\ell m} Y_{\ell' m'} \, d\Omega \neq \delta_{\ell \ell'} \delta_{m m'}$$

$$a_{\ell m} = \int_W Y^*_{\ell m}(\gamma) \, f(\gamma) \, d\Omega_\gamma$$

$$\Delta = \int \left| f(\gamma) - a^{\ell m} Y_{\ell m}(\gamma) \right|^2 d\Omega_\gamma$$

$$\frac{\partial \Delta}{\partial a^*_{\ell m}} \Rightarrow \quad \int Y^*_{\ell m} Y_{\ell' m'} \, d\Omega \; a^{\ell' m'}$$

$$\int_W Y^*_{\ell m} Y_{\ell' m'} \, d\Omega \neq \delta_{\ell\ell'}\delta_{mm'}$$

$$a_{\ell m} = \int_W Y^*_{\ell m}(\gamma) \, f(\gamma) \, d\Omega_\gamma$$

$$\Delta = \int \left| f(\gamma) - a^{\ell m} Y_{\ell m}(\gamma) \right|^2 d\Omega_\gamma$$

$$\frac{\partial \Delta}{\partial a^*_{\ell m}} \Rightarrow \sum_{\ell' m'} \int Y^*_{\ell m} Y_{\ell' m'} \, d\Omega \; a^{\ell' m'} =$$

$$\int_W Y^*_{\ell m} Y_{\ell' m'} \, d\Omega \neq \delta_{\ell\ell'}\delta_{mm'}$$

$$a_{\ell m} = \int_W Y^*_{\ell m}(\gamma) \, f(\gamma) \, d\Omega_\gamma$$

$$\Delta = \int \left| f(\gamma) - a^{\ell m} Y_{\ell m}(\gamma') \right|^2 d\Omega_\gamma$$

$$\frac{\partial \Delta}{\partial a^*_{\ell m}} \Rightarrow \sum_{\ell' m'} \int Y^*_{\ell m} Y_{\ell' m'} \, d\Omega \; a^{\ell' m'} = a_{\ell m}$$

$$\int_W Y^*_{\ell m} Y_{\ell' m'} \, d\Omega \neq \delta_{\ell \ell'} \delta_{m m'}$$

$$a_{\ell m} = \int_W Y^*_{\ell m}(\gamma) \, f(\gamma) \, d\Omega_\gamma$$

$$\Delta = \int \left| f(\gamma) - a_{\ell m} Y_{\ell m}(\gamma) \right|^2 d\Omega_\gamma$$

$$\frac{\partial \Delta}{\partial a^*_{\ell m}} \Rightarrow \sum_{\ell' m'} \underbrace{\int Y^*_{\ell m} Y_{\ell' m'} \, d\Omega}_{g_{\ell m \ell' m'}} \cdots {}_{m'} = a_{\ell m}$$

$$\int_W Y^*_{\ell m} Y_{\ell' m'} \, d\Omega \neq \delta_{\ell\ell'} \delta_{mm'}$$

$$a_{\ell m} = \int_W Y^*_{\ell m}(\gamma) f(\gamma) \, d\Omega_\gamma$$

$$\Delta = \int \left| f(\gamma) - a^{\ell m} Y_{\ell m}(\gamma) \right|^2 d\Omega_\gamma$$

$$\frac{\partial \Delta}{\partial a^*_{\ell m}} \Rightarrow \sum_{\ell' m'} \underbrace{\int Y^*_{\ell m} Y_{\ell' m'} \, d\Omega}_{g_{\ell m \, \ell' m'}} \, a^{\ell' m'} = a_{\ell m}$$

$\neq \delta_{ll'} \delta_{mm'}$

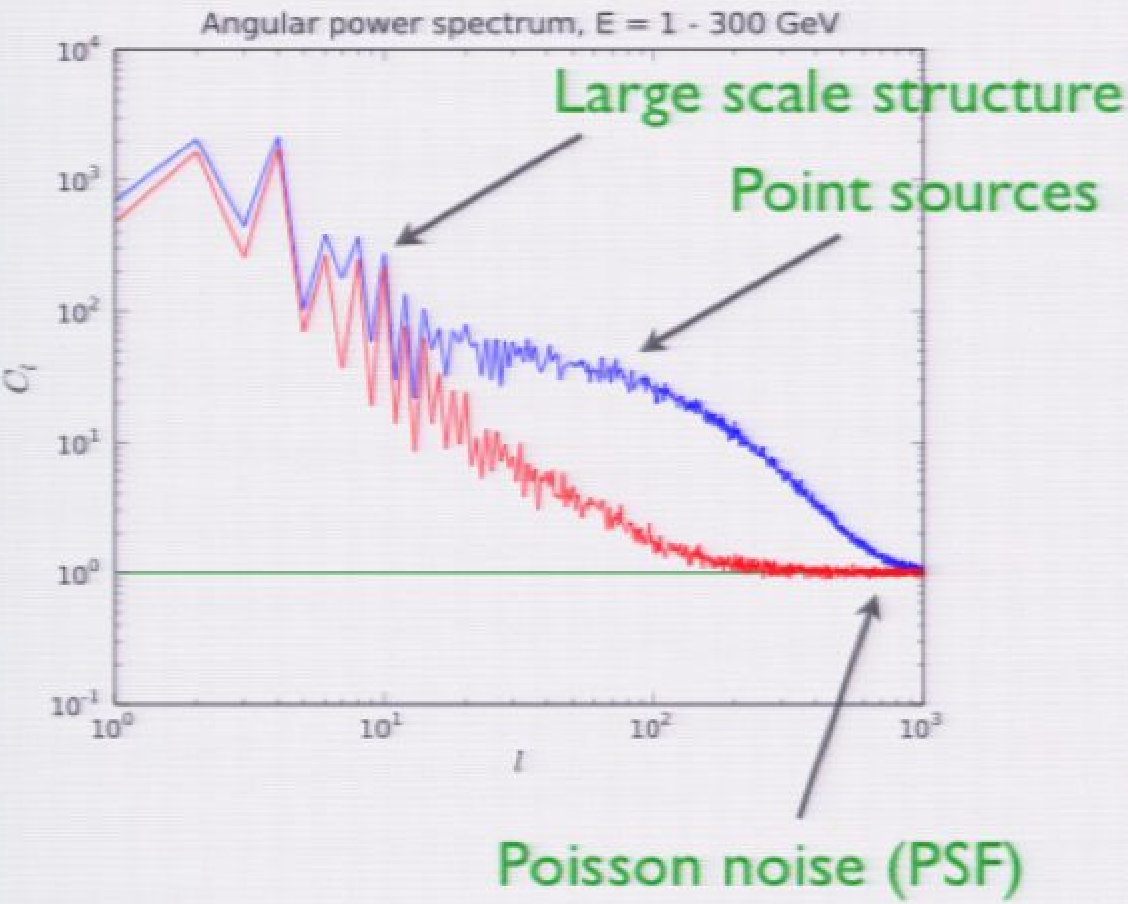
$\tilde{f}(\gamma) = \sum_{l'm'} a^{l'm'} Y_{l'm'}(\gamma)$

$f(\gamma) \, d\Omega_\gamma$

$a^{lm} Y_{lm}(\gamma) |^2 \, d\Omega_\gamma$

$l'm' d\Omega \; a^{l'm'} = a_{lm}$

Angular power spectrum of Fermi data above 1 GeV



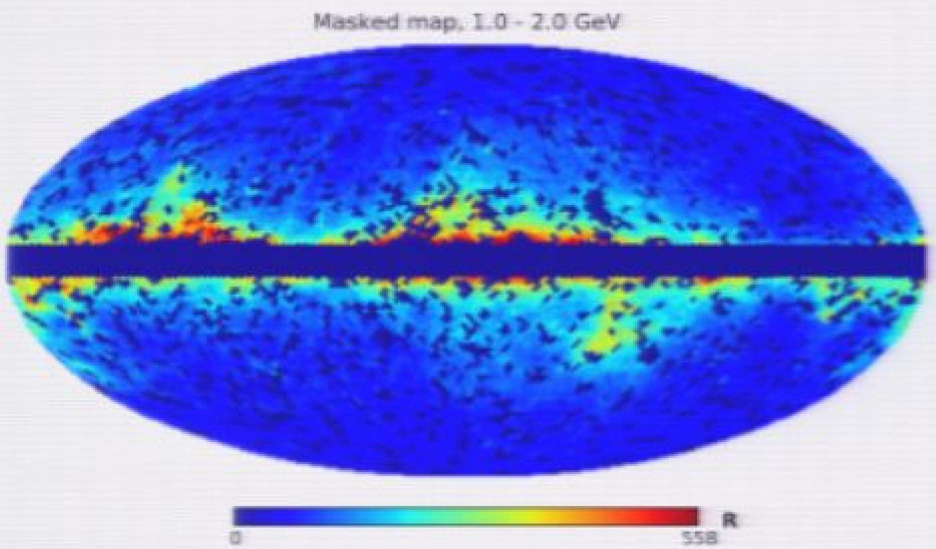
Blue curve - all Fermi data at |b| > 5 deg

Red curve - gamma-ray point sources are masked

Point sources increase the level of Poisson noise.
In order to study the large scale distributions for $\ell \gtrsim 10$ one needs to mask the point sources.

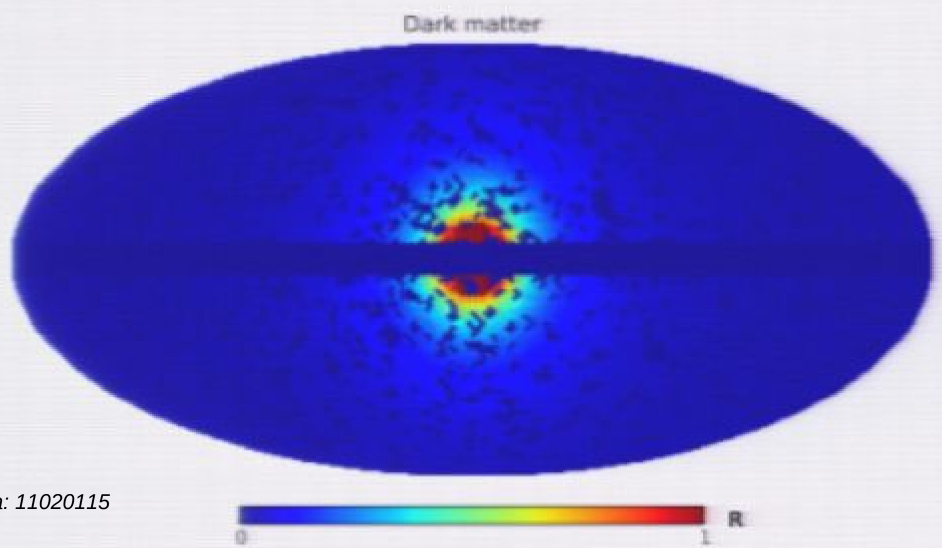
In the analysis we will use $\ell_{\max} = 15$

Templates



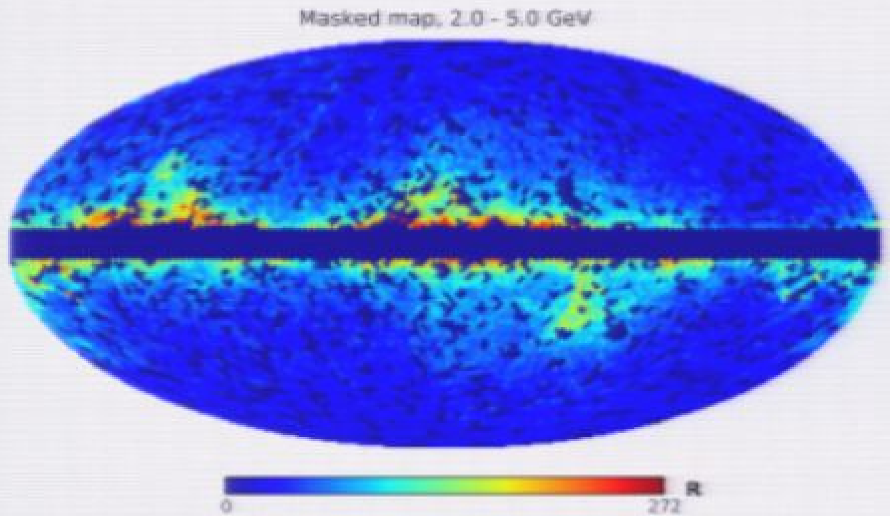
Masked Fermi data in 1-2 GeV energy bin.

Photon counts in pixels with pixel size about 2 deg

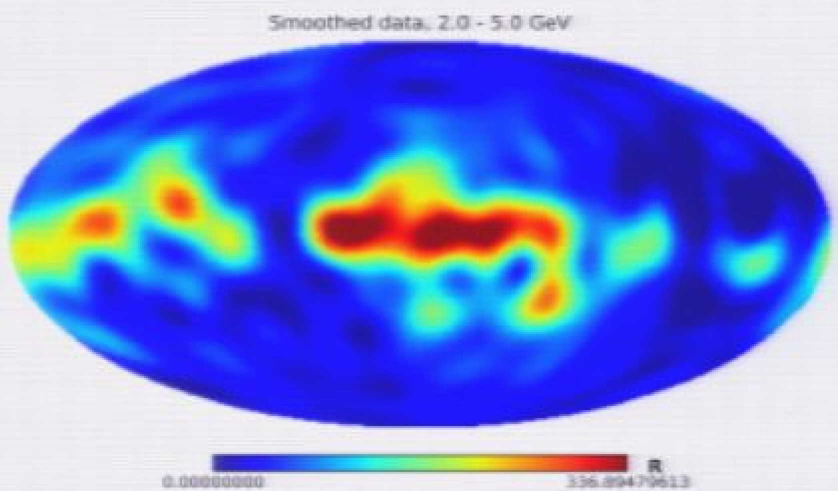


Dark matter annihilation in NF profile (arbitrary units)

Data

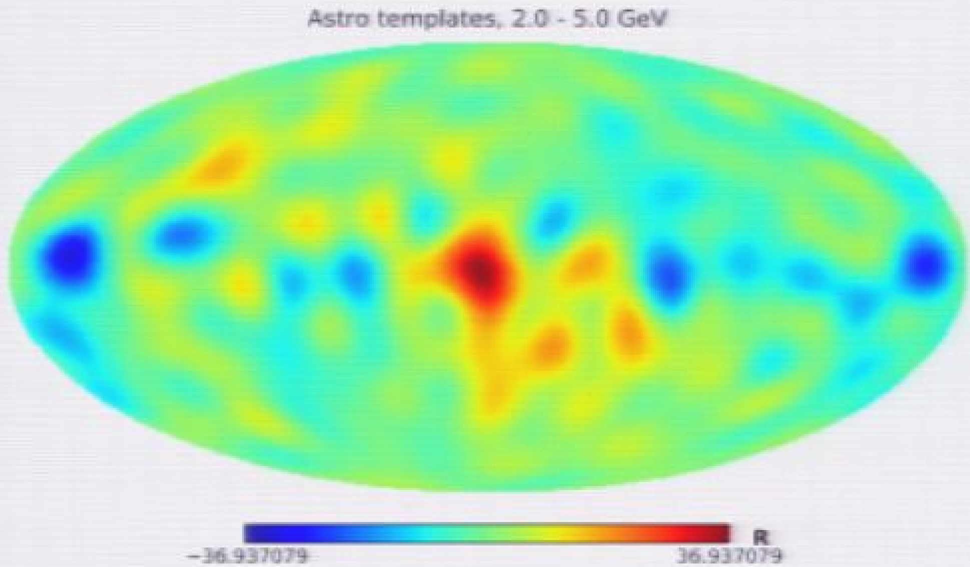


Consider the Fermi data in 2-5 GeV energy bin and make the spherical harmonics decomposition



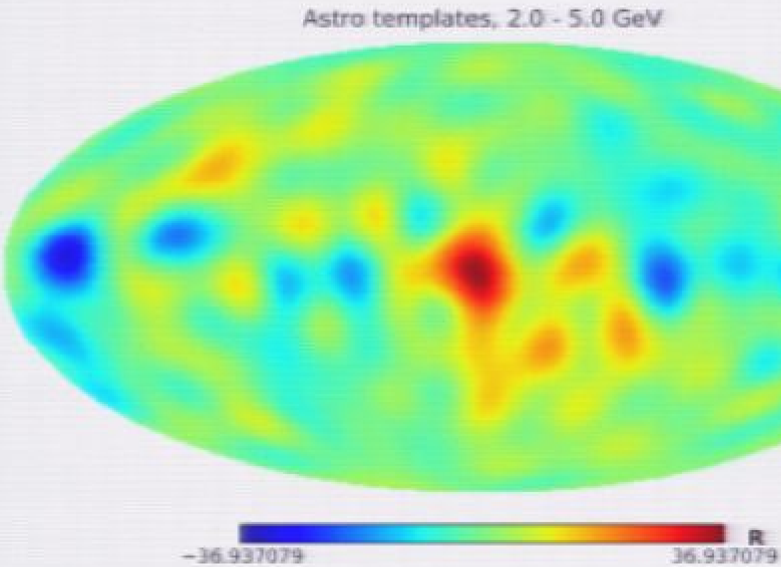
Inverse transform of the first 1 spherical harmonics

2 - 5 GeV astro templates

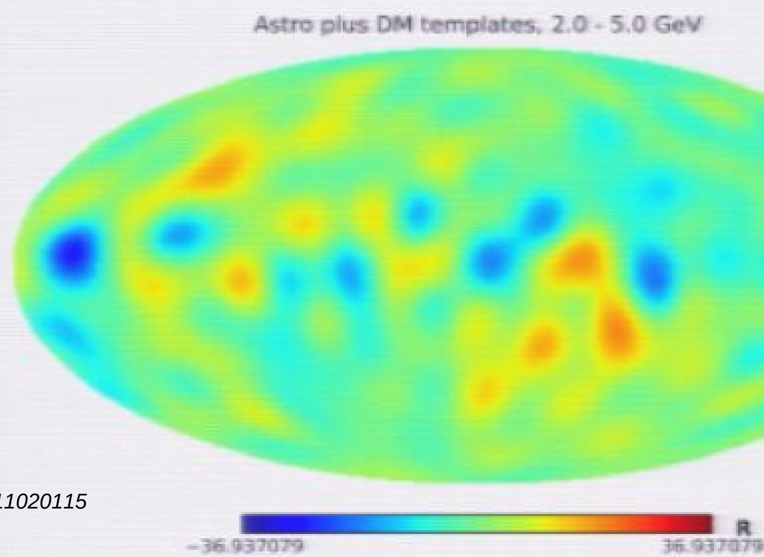


Inverse transform of the residual for low E bin and isotropic templates

2 - 5 GeV astro+DM templates

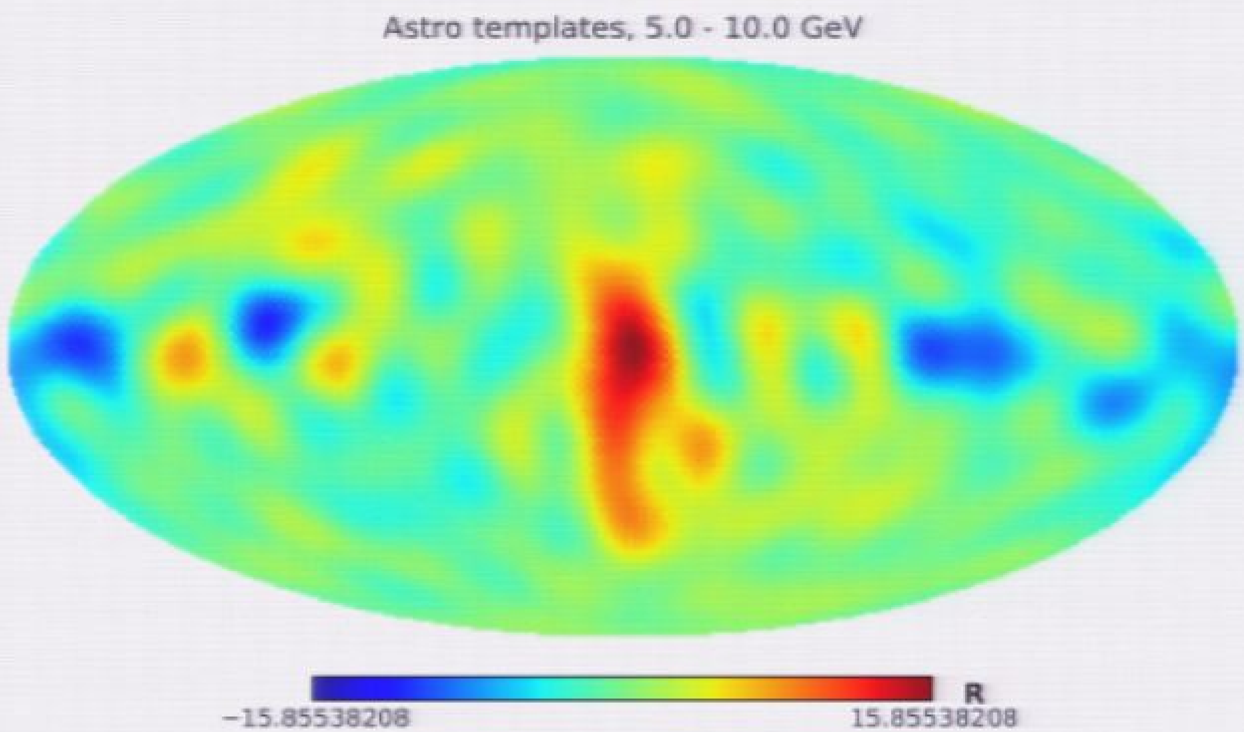


Inverse transform of the residual for low E bin and isotropic templates

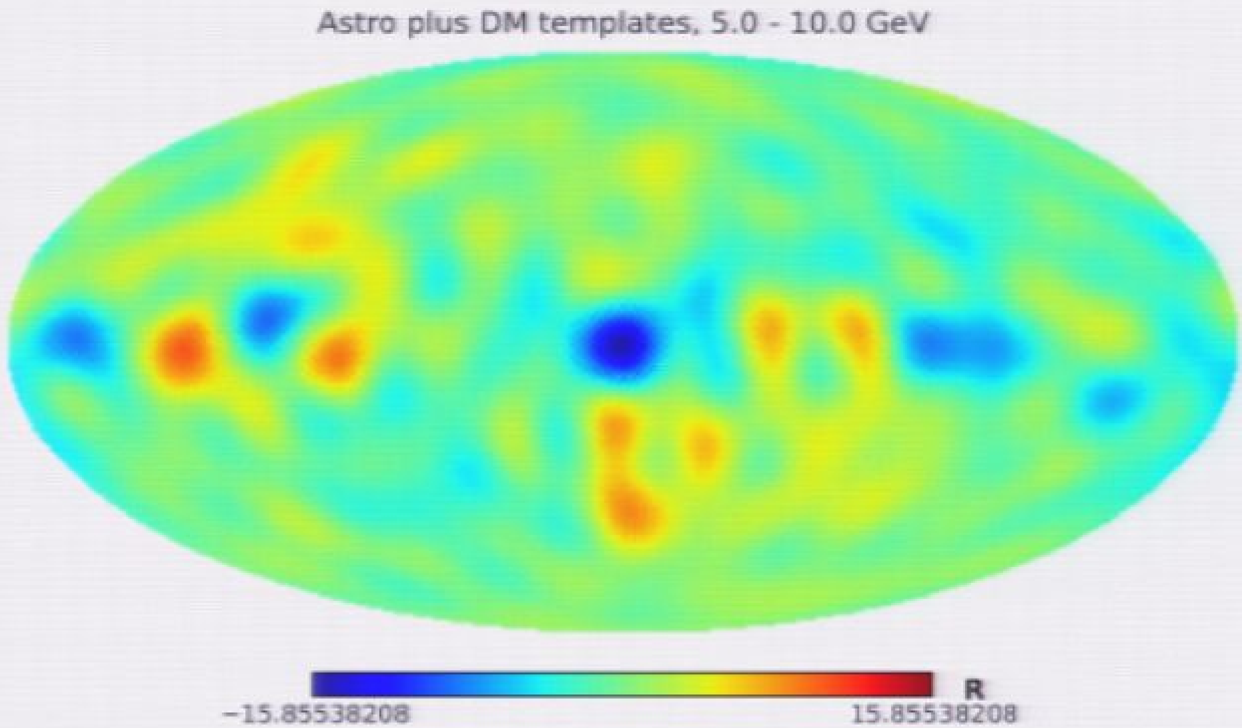


Inverse transform with DM template added

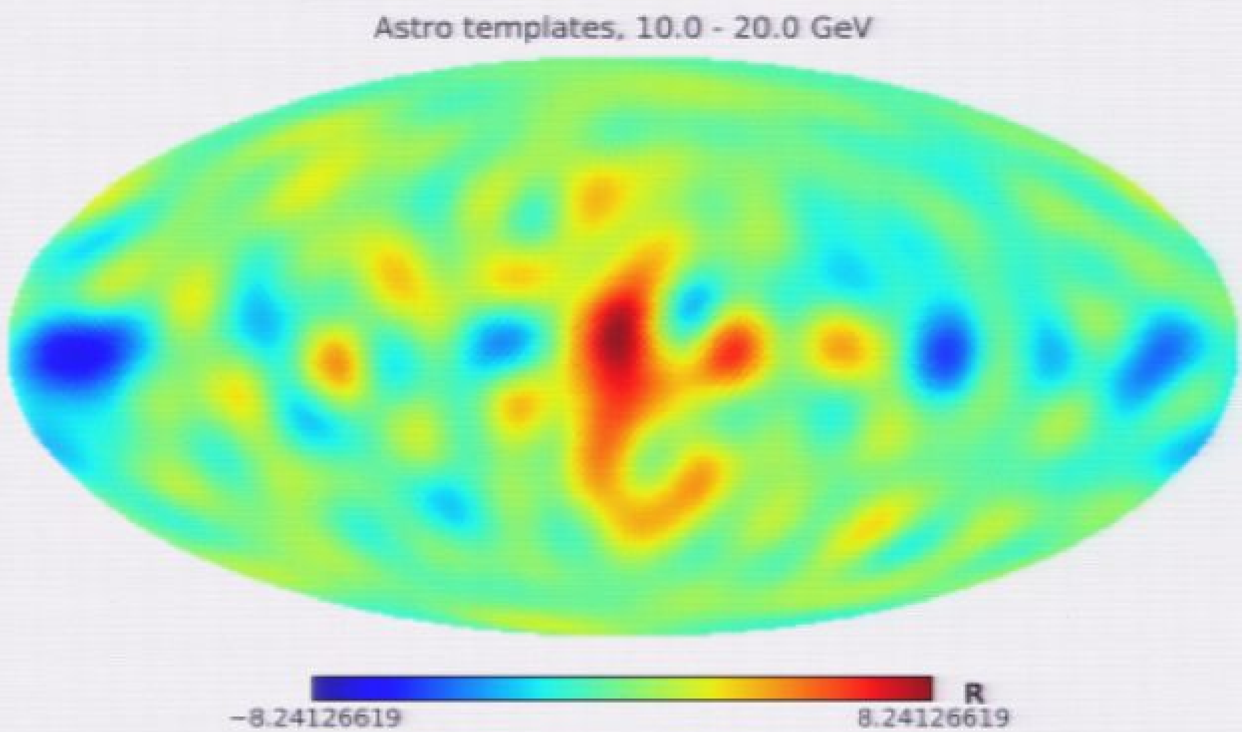
5 - 10 GeV astro templates



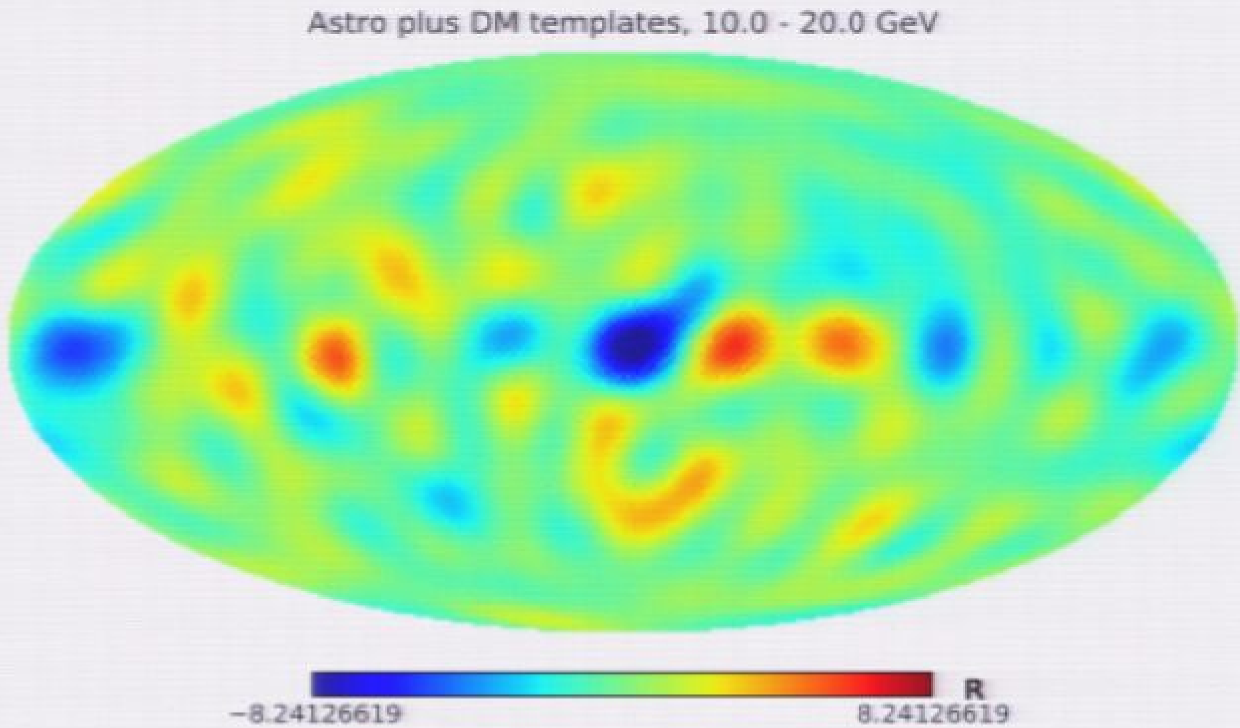
5 - 10 GeV astro+DM templates



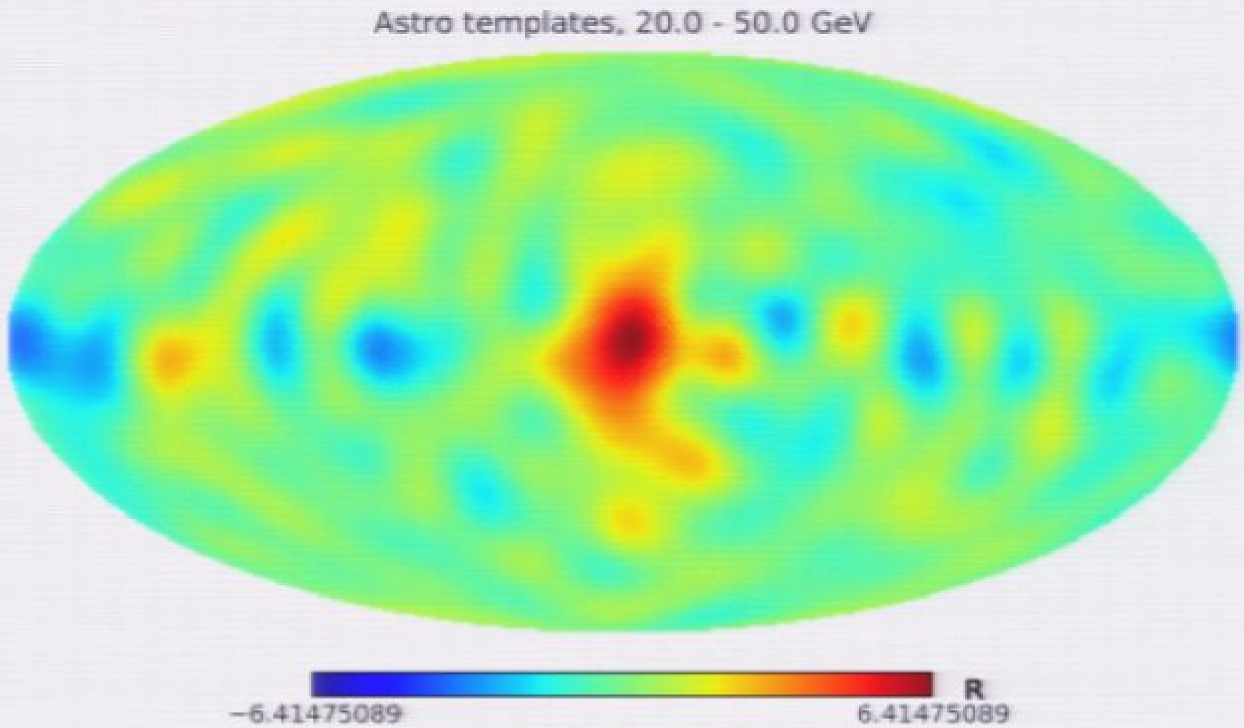
10 - 20 GeV astro templates



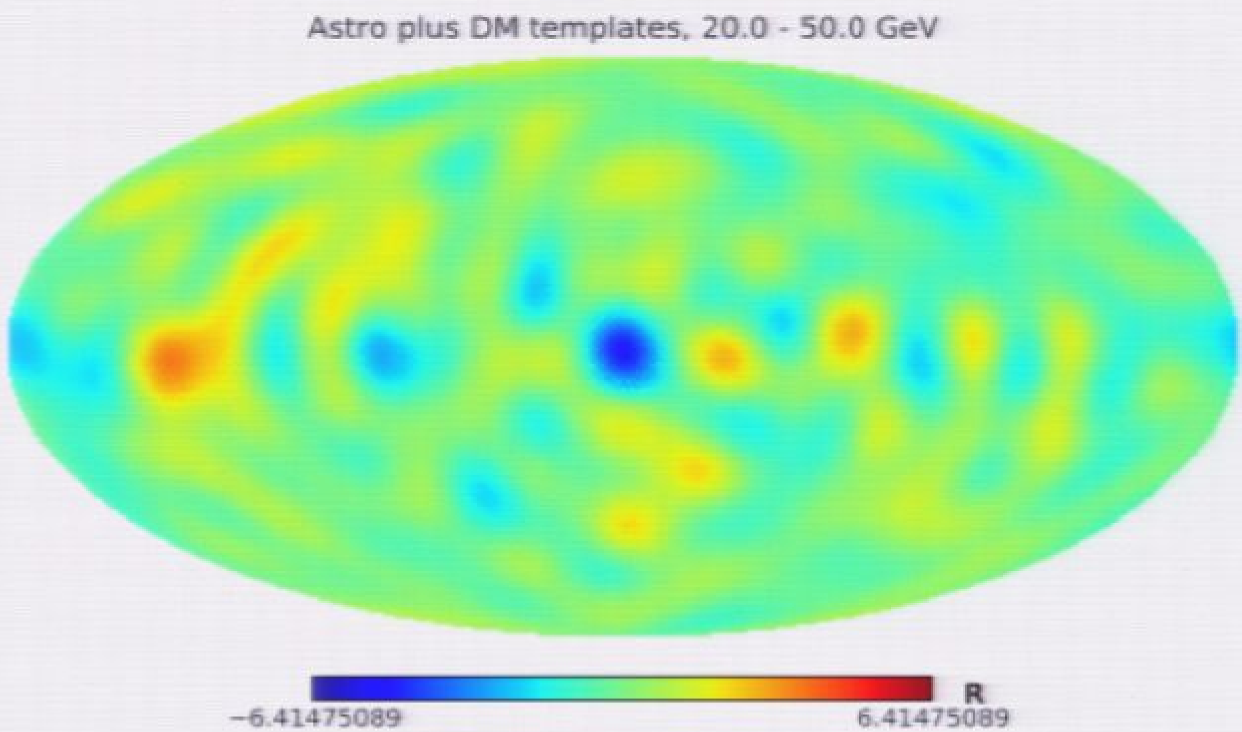
10 - 20 GeV astro+DM templates



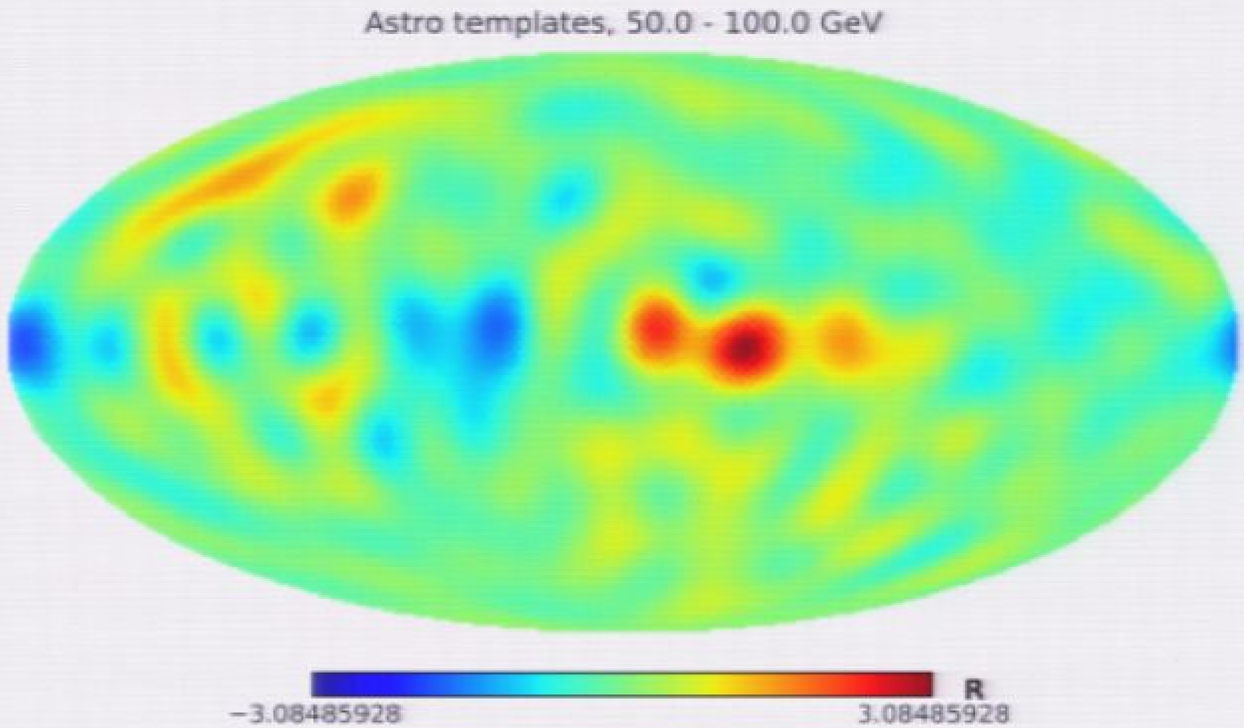
20 - 50 GeV astro templates



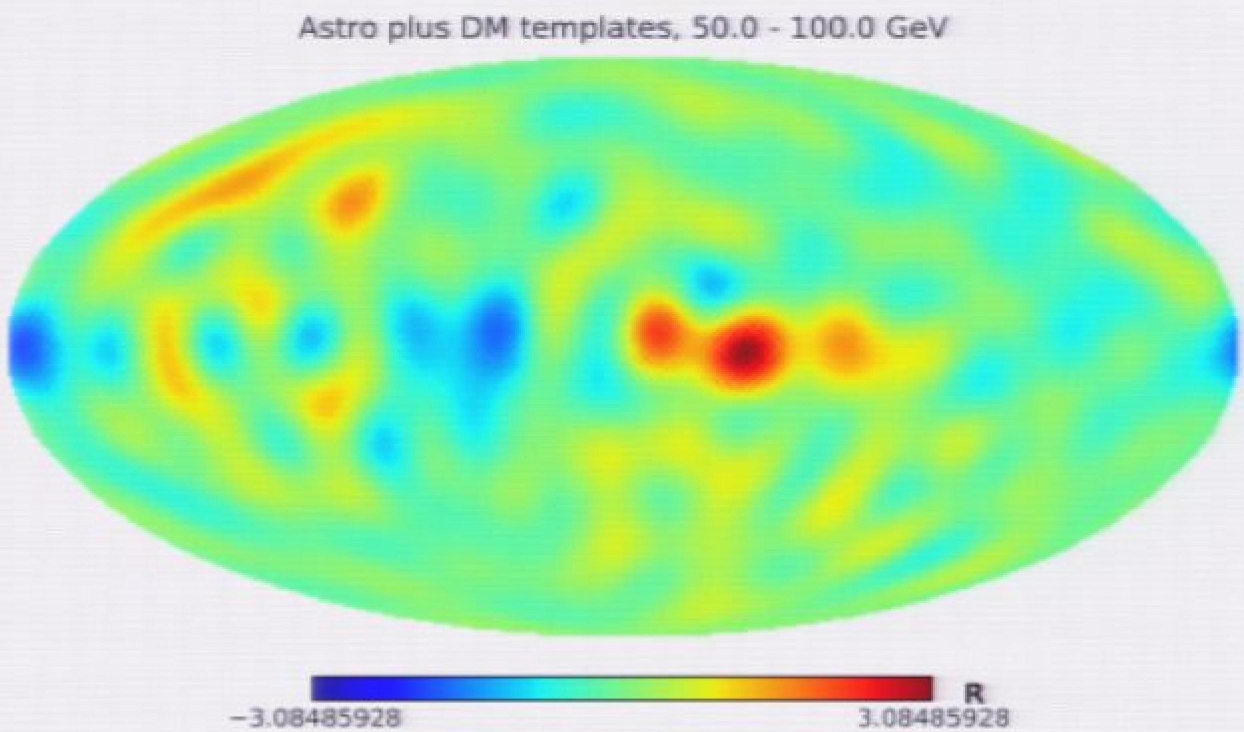
20 - 50 GeV astro+DM templates



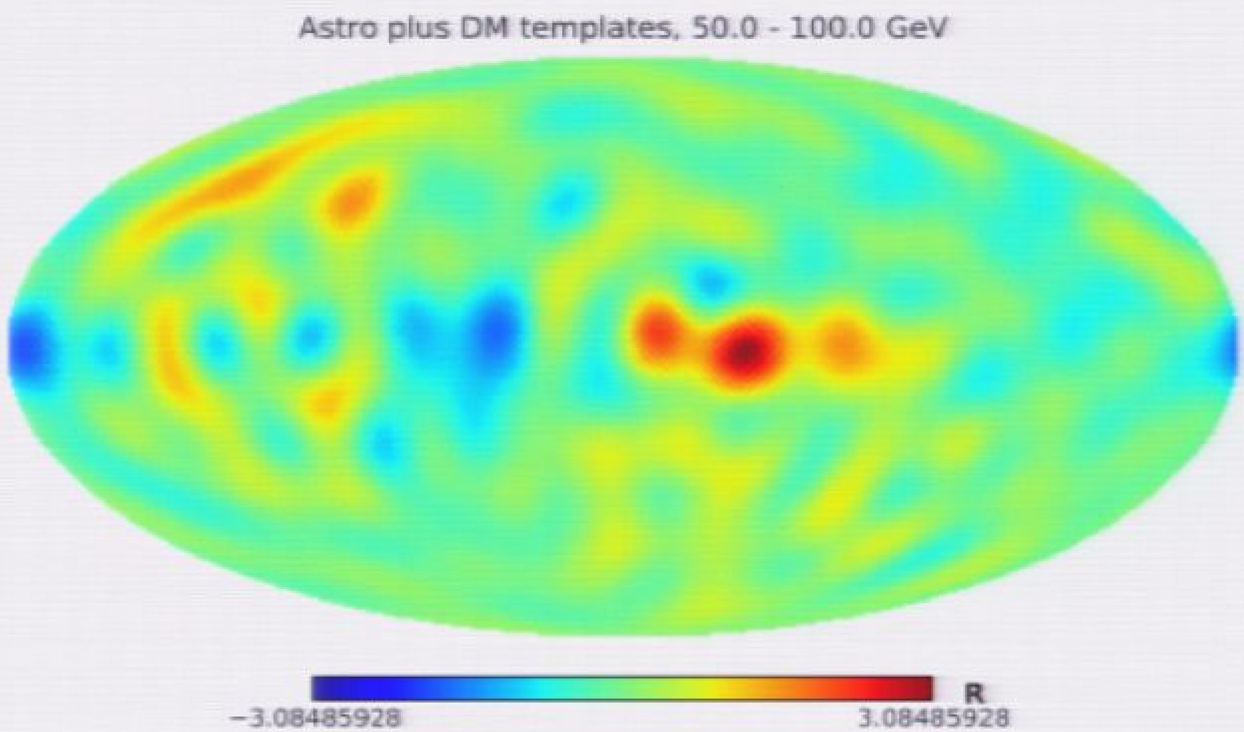
50 - 100 GeV astro templates



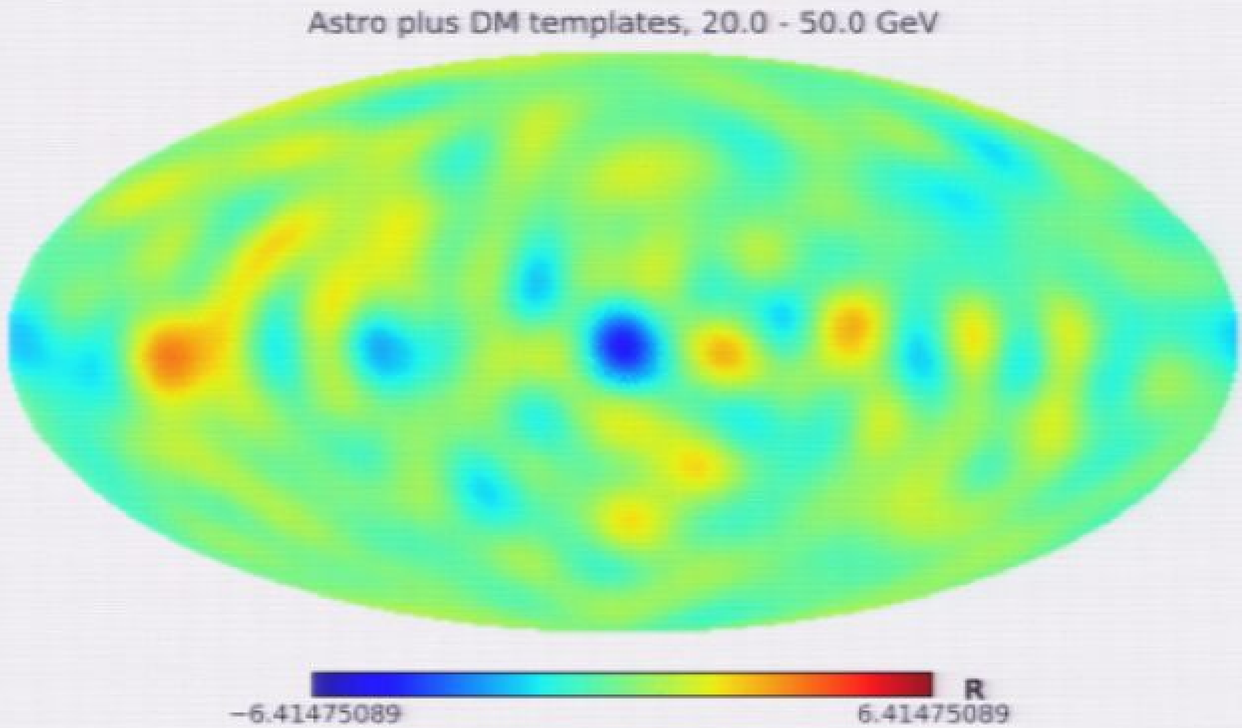
50 - 100 GeV astro+DM templates



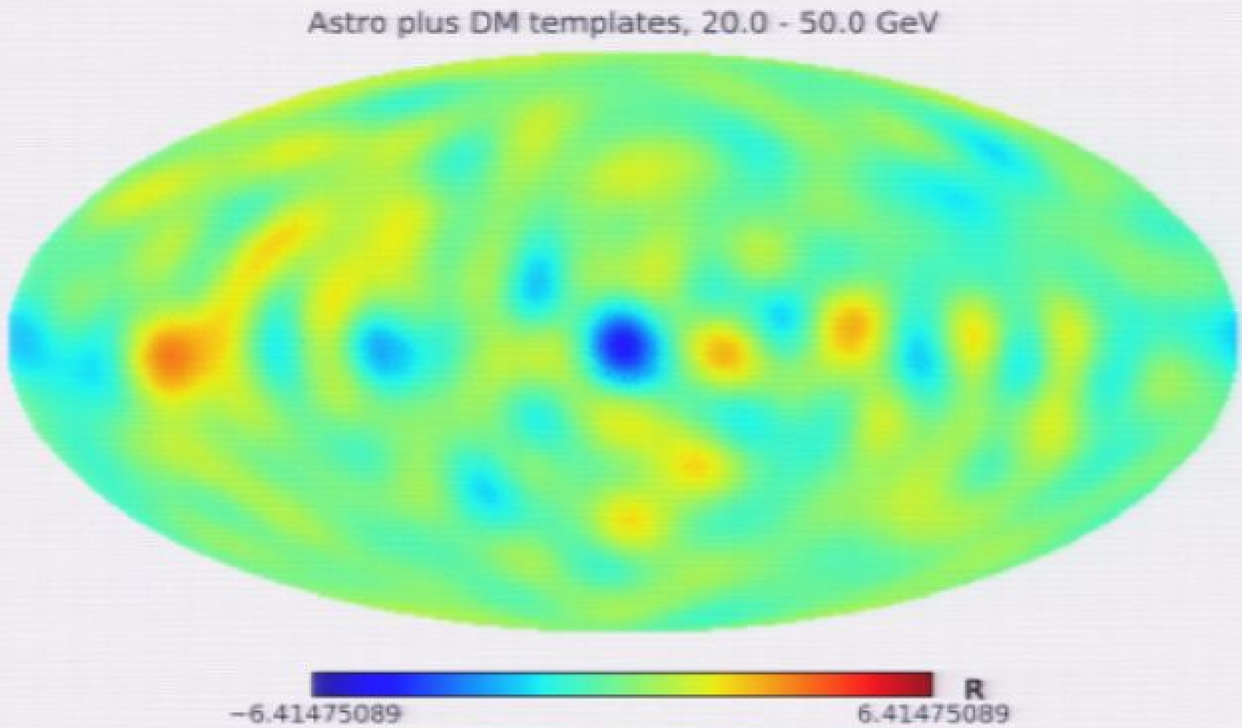
50 - 100 GeV astro+DM templates



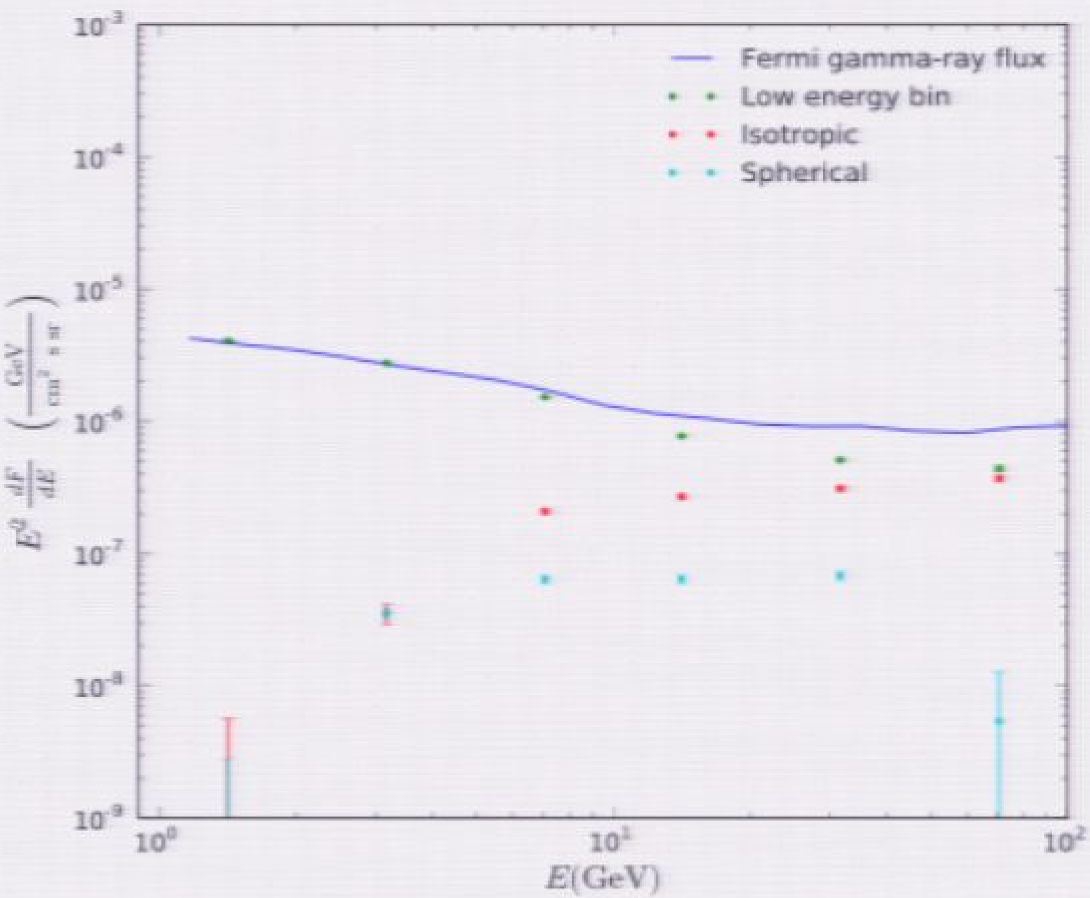
20 - 50 GeV astro+DM templates



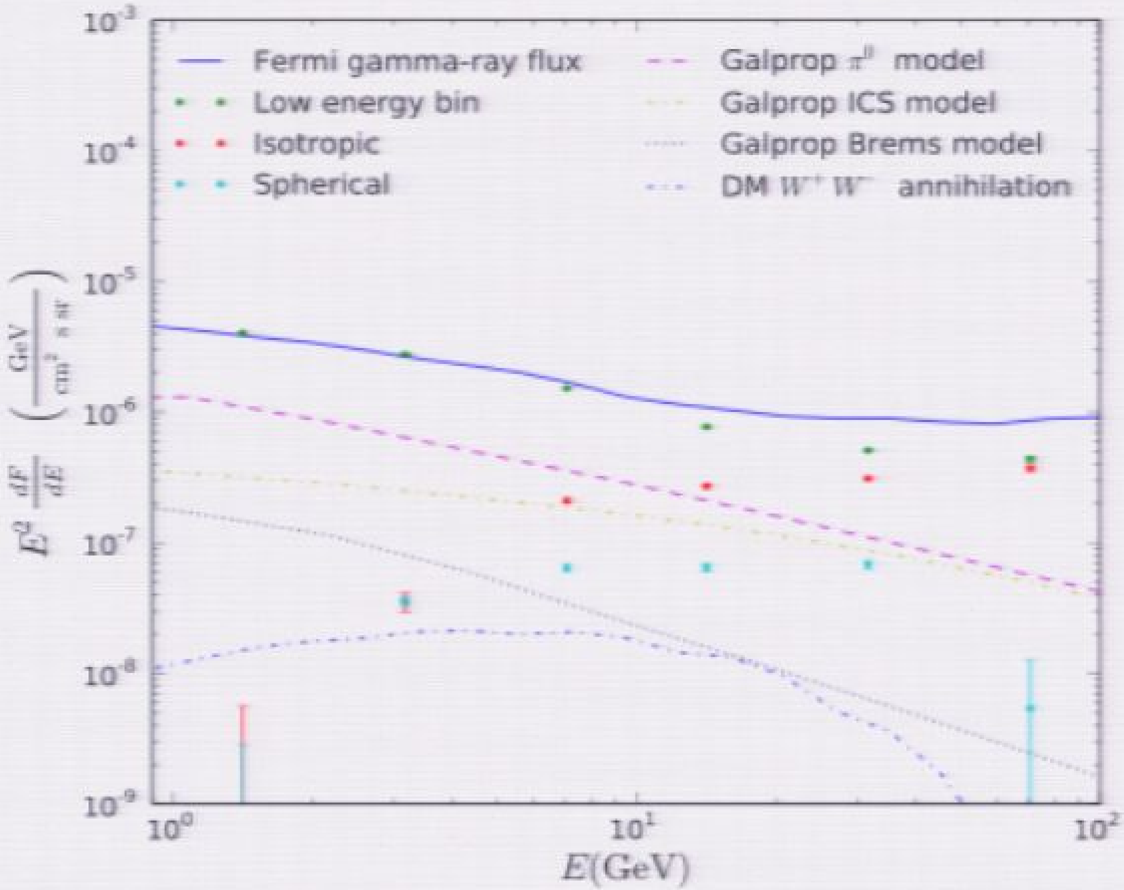
20 - 50 GeV astro+DM templates



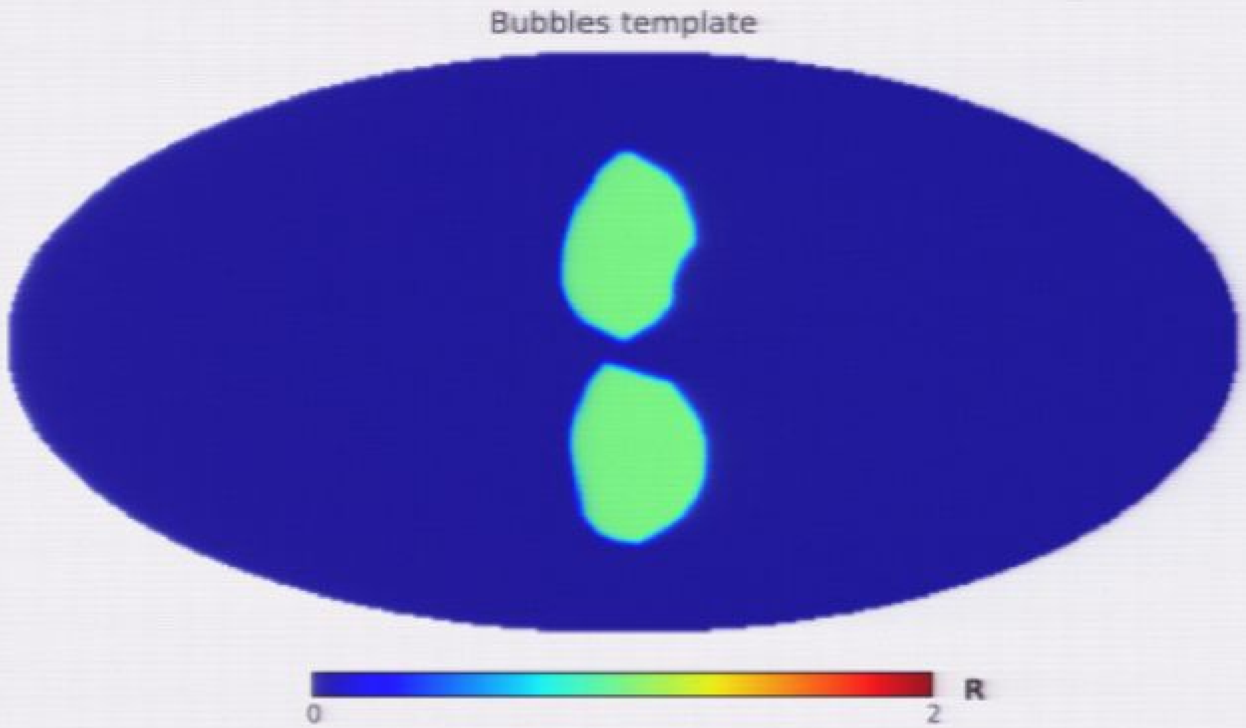
Fluxes for astro and DM templates



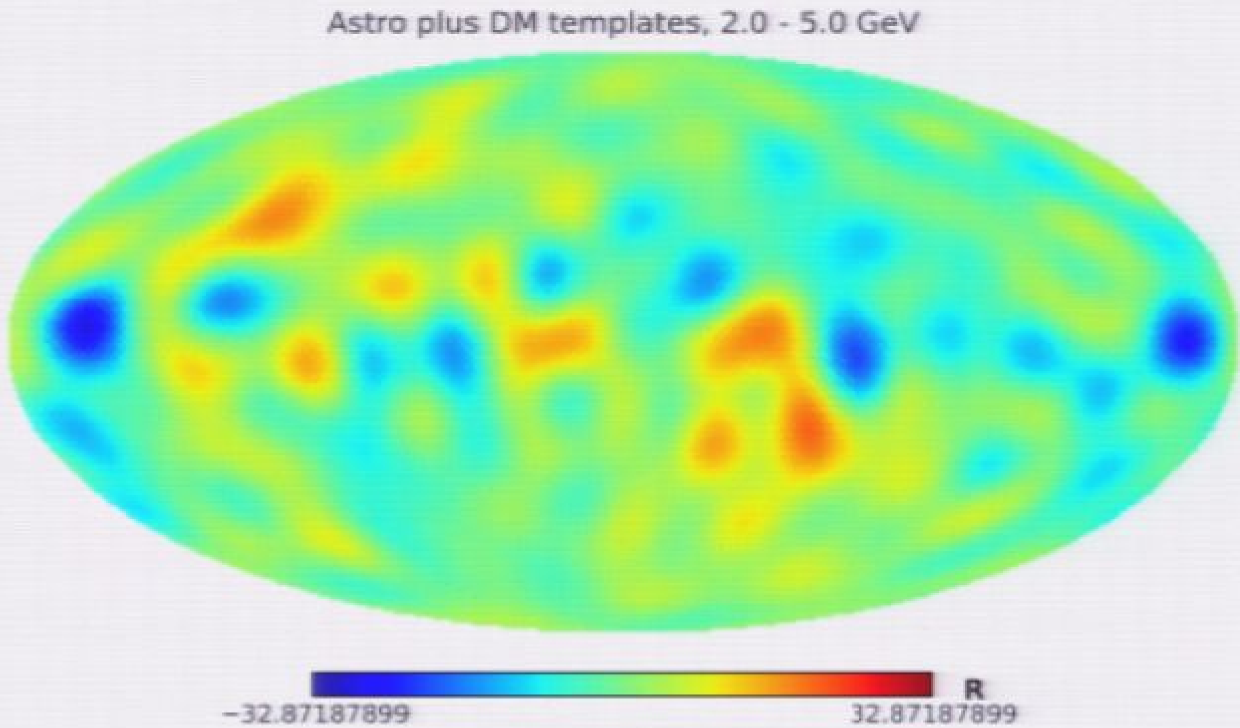
Comparison with Galprop and DM annihilation



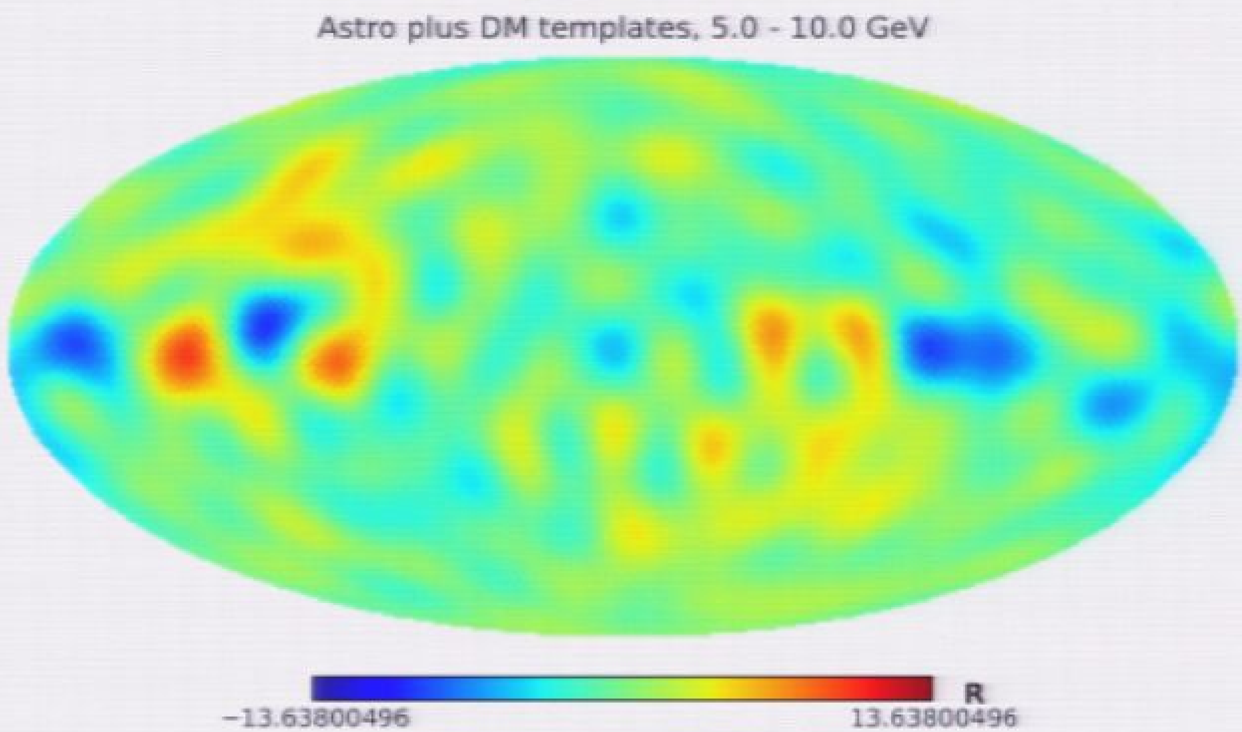
Add bubbles template (Su, Slatyer, Finkbeiner 2010)



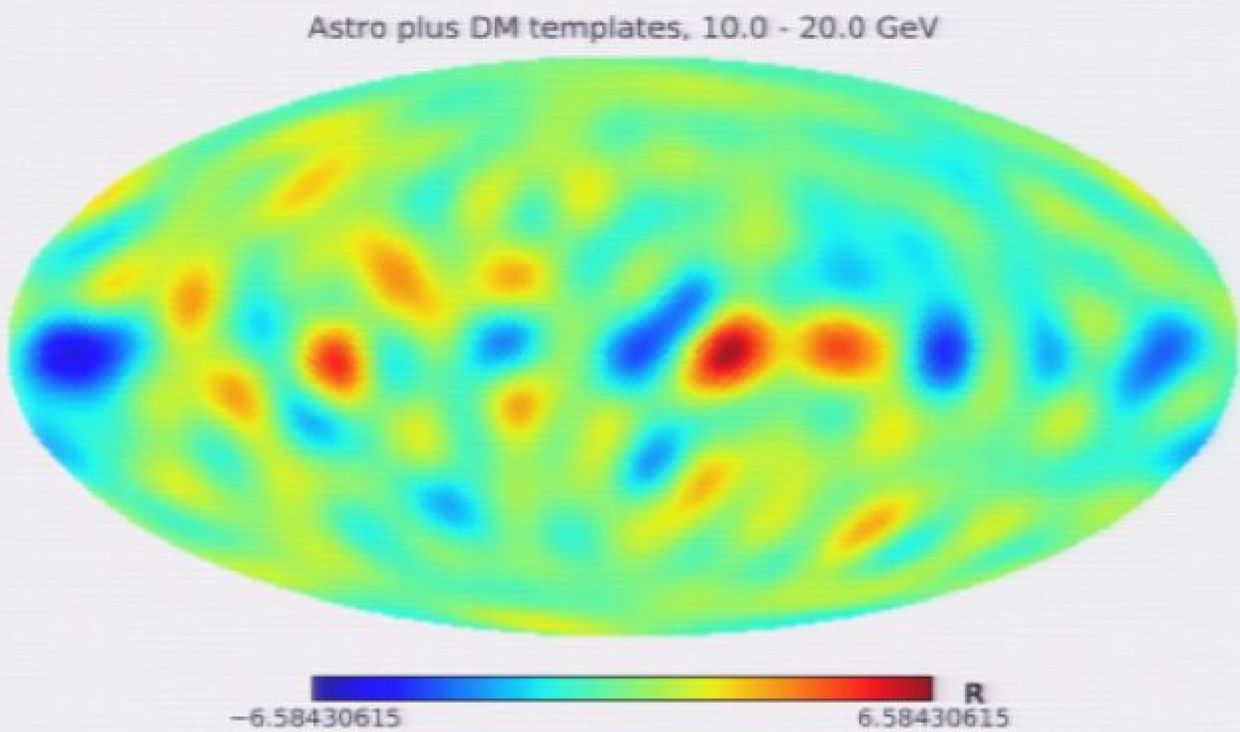
2 - 5 GeV astro+bubbles+DM templates



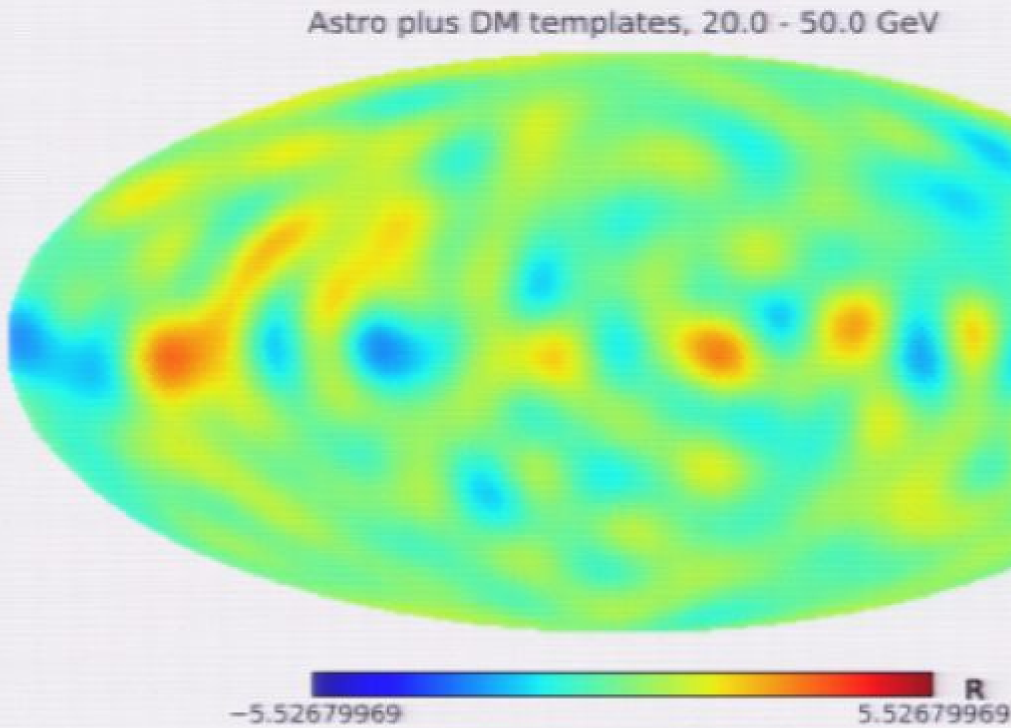
5 - 10 GeV astro+bubbles+DM templates



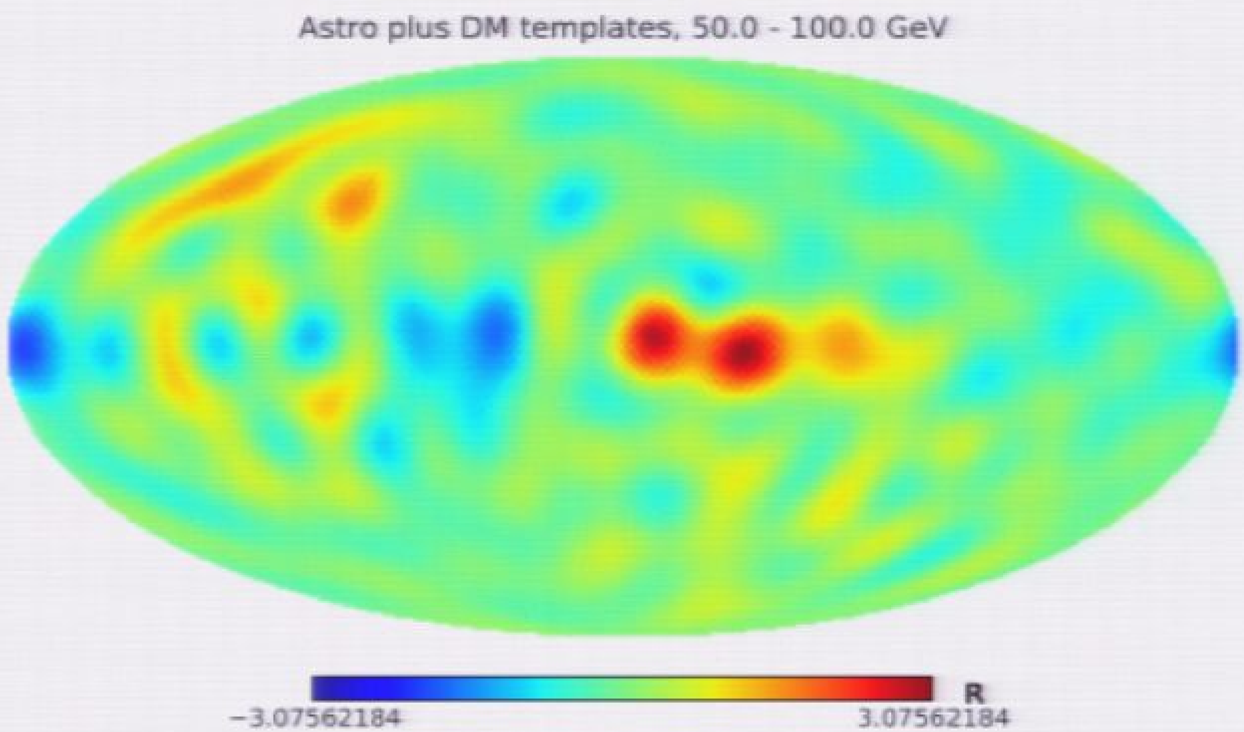
10 - 20 GeV astro+bubbles+DM templates



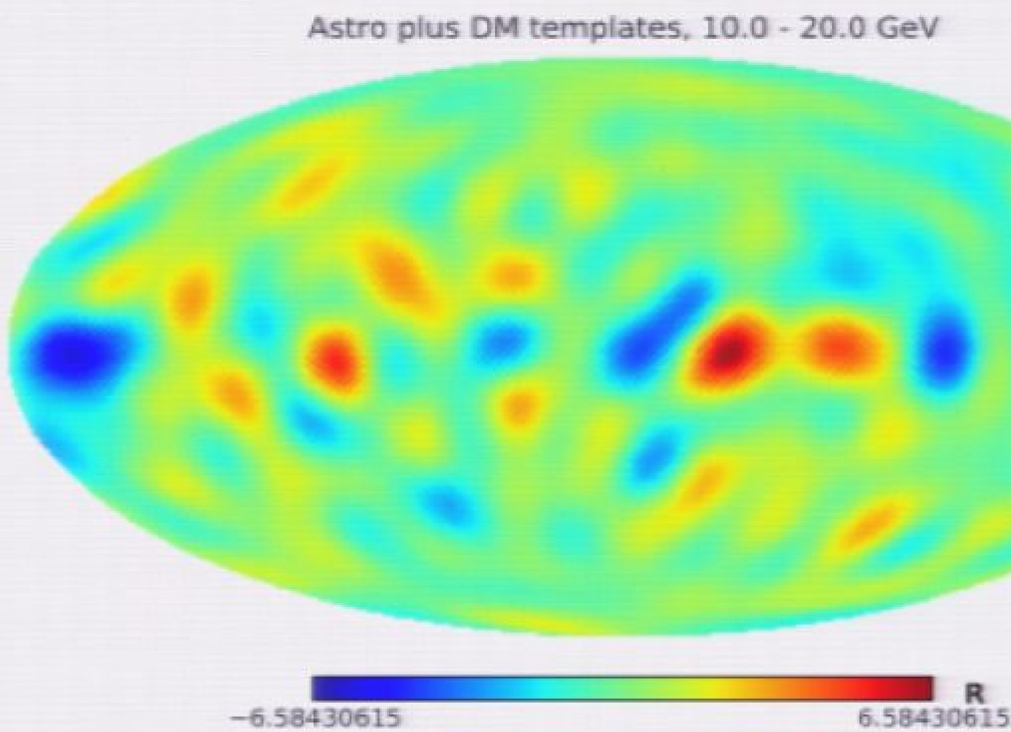
20 - 50 GeV astro+bubbles+DM templates



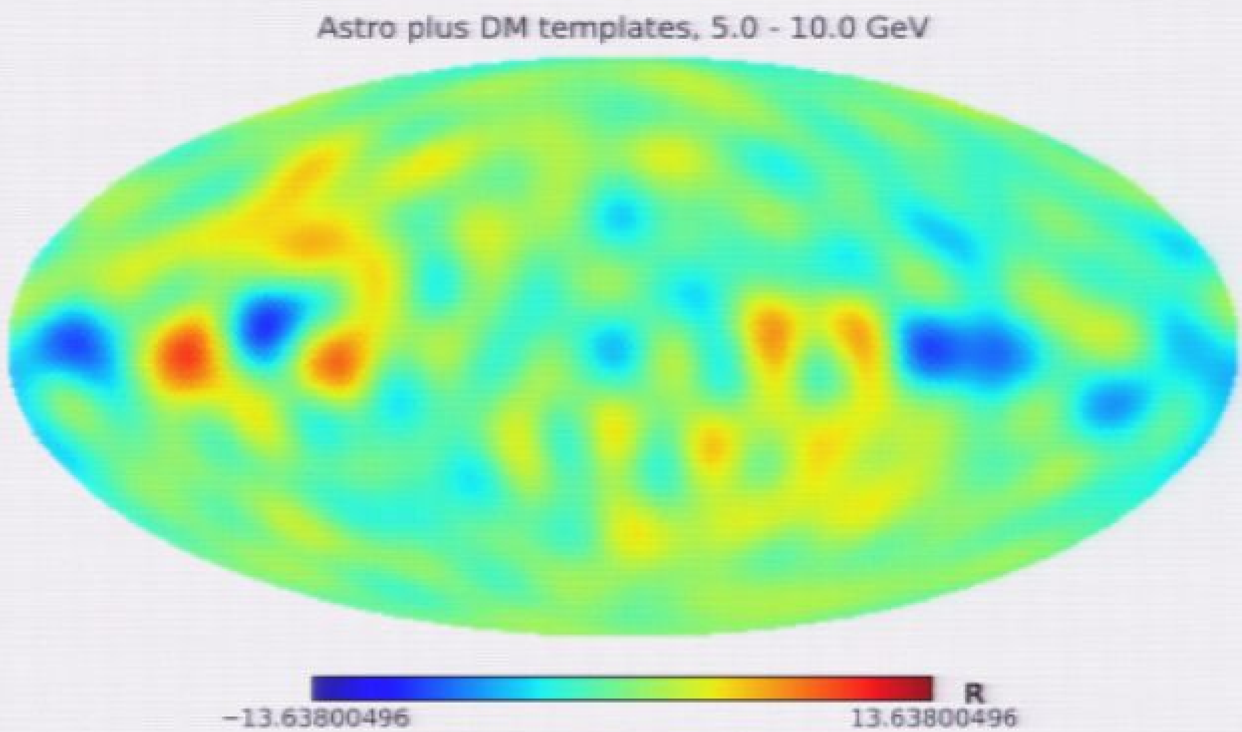
50 - 100 GeV astro+bubbles+DM templates



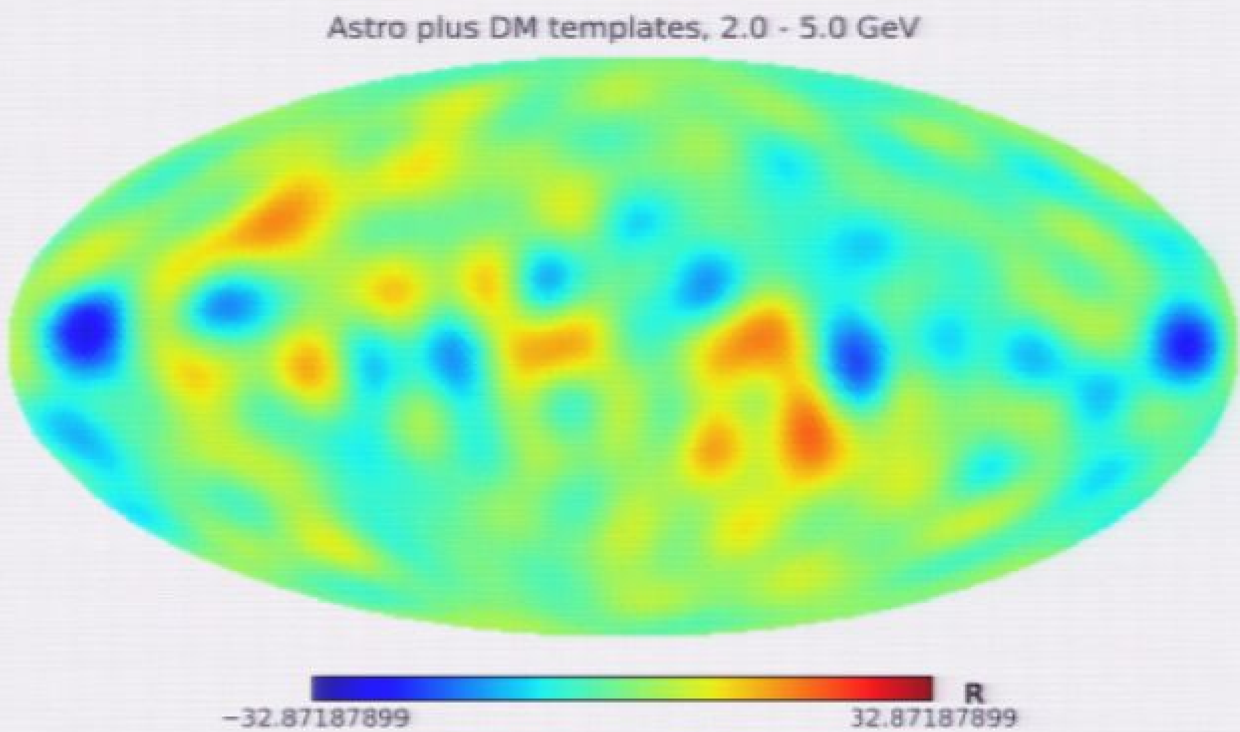
10 - 20 GeV astro+bubbles+DM templates



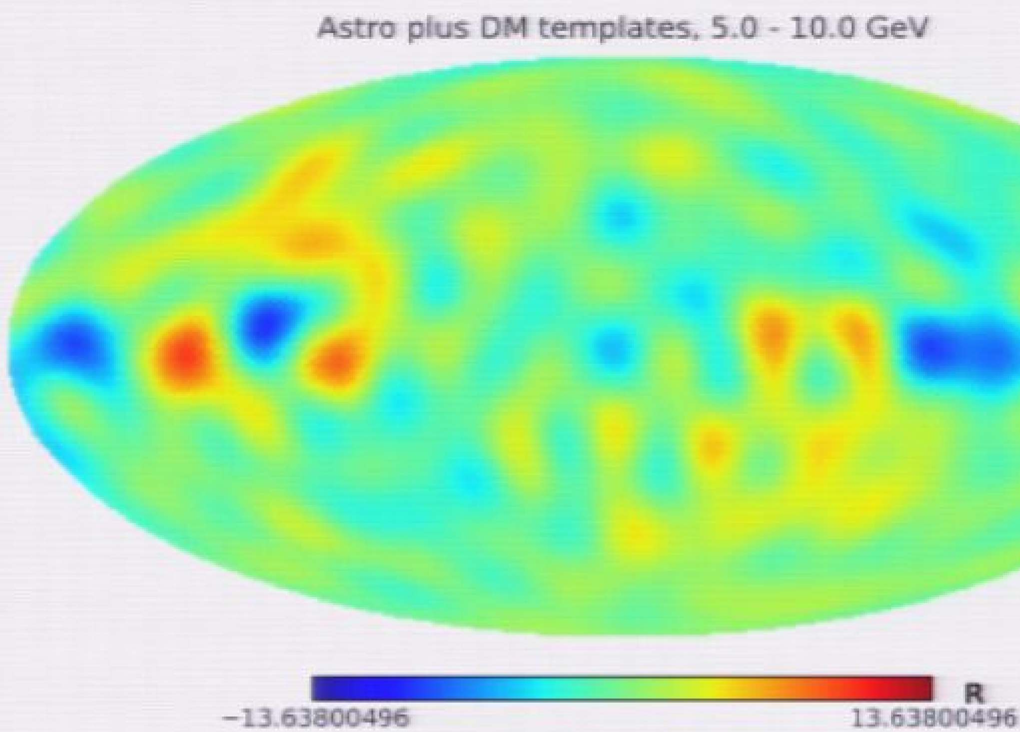
5 - 10 GeV astro+bubbles+DM templates



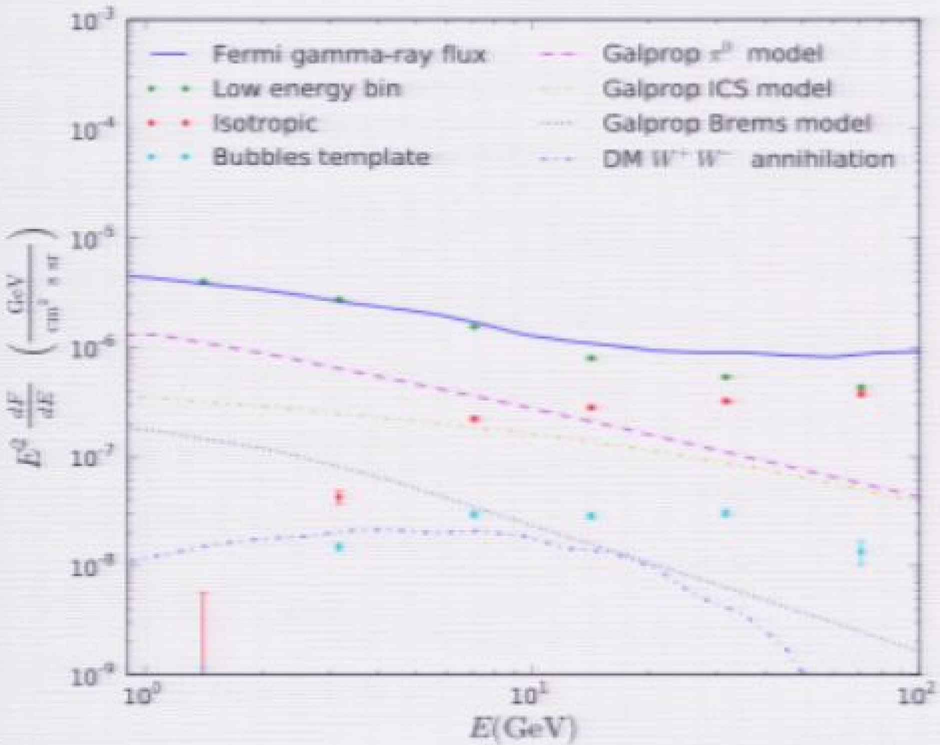
2 - 5 GeV astro+bubbles+DM templates



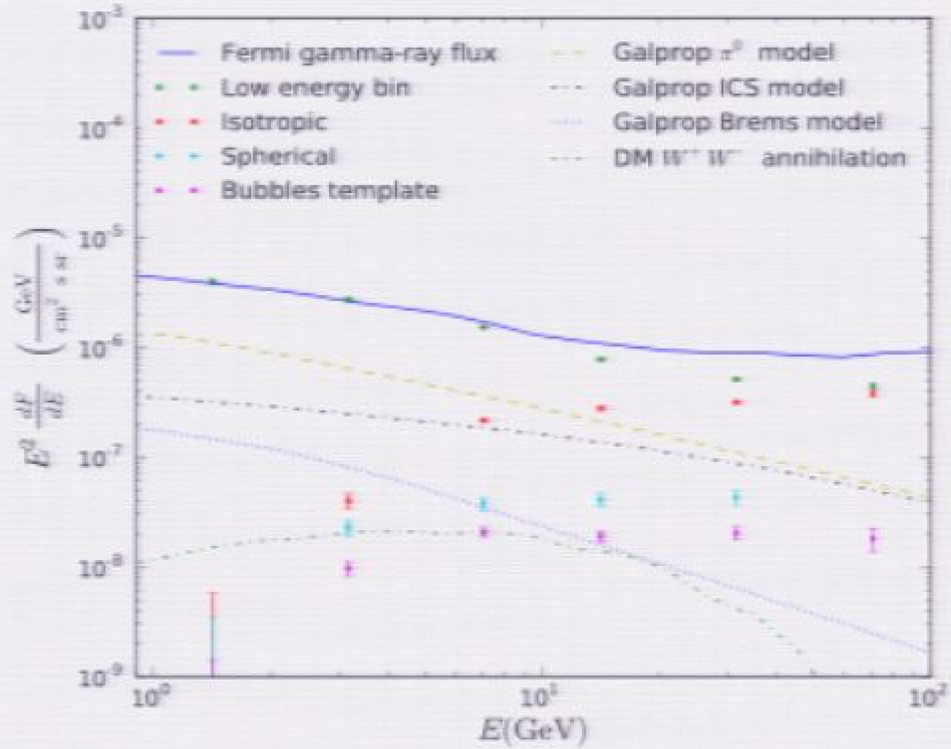
5 - 10 GeV astro+bubbles+DM templates



Astro + bubbles



Astro + DM + bubbles



Adding the bubble template reduces the power in the spherical template.
The spherical template contribution remains significant and a little closer to the expected DM annihilation signal

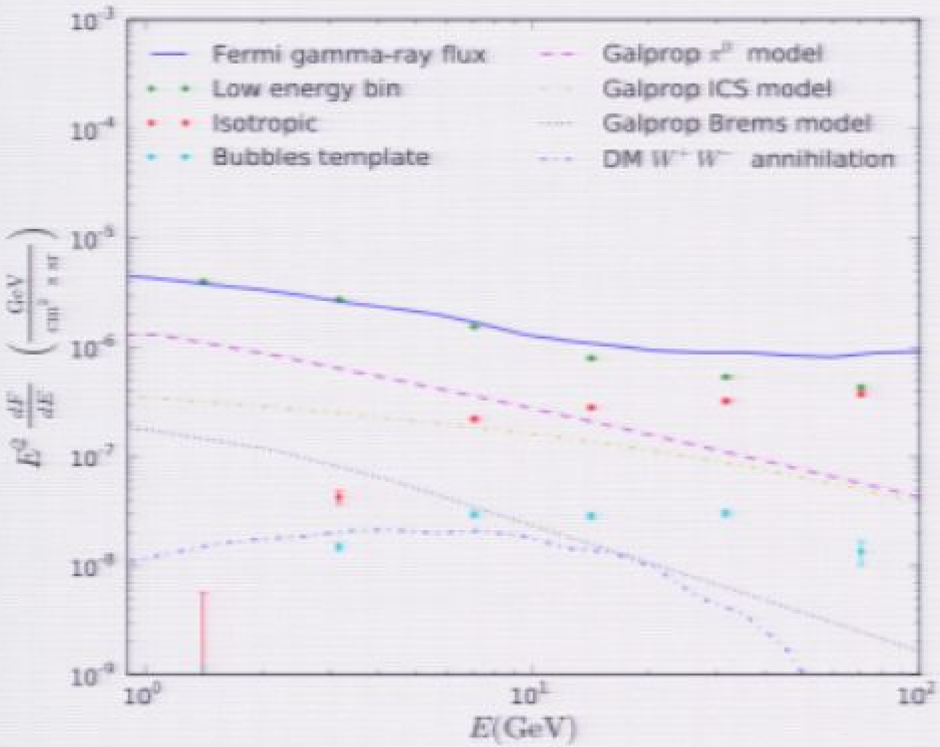
Conclusions

1. Spherical harmonics are a natural choice for fitting the large scale structures in gamma-rays. It is a linear procedure, that avoids many difficulties associated with coordinate space fitting (e.g., upgrading / degrading of templates to fit to the same pixelation etc.)

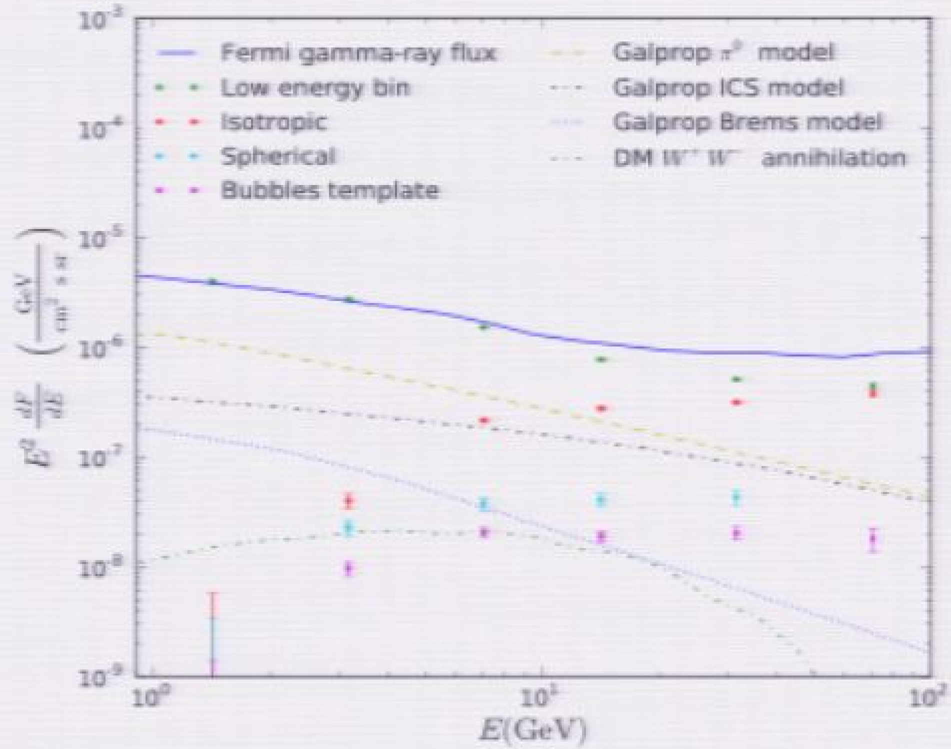
2. The calculation of χ^2 and the fitting procedure are simple. The data is conveniently separated into low ℓ modes (relevant for the fitting) and high ℓ modes (dominated by the noise).

3. We observe that adding an extra spherical template significantly reduces the χ^2. Adding the bubbles template reduces χ^2 but doesn't eliminate the DM-like spherical contribution.

Astro + bubbles



Astro + DM + bubbles



Adding the bubble template reduces the power in the spherical template.
The spherical template contribution remains significant and a little closer to the expected DM annihilation signal