Title: Walking Beyond the Standard Model

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Abstract: There are good reasons to think that our understanding of particle physics is incomplete. The effective field theory describing the particles that we know about breaks down at the TeV scale, and new effective degrees of freedom must enter. In this talk I will discuss the role that strong dynamics might play in this new physics, focusing on the ways in which approximately scale-invariant dynamics could explain puzzling features of our low-energy Lagrangian. I will also describe recent theoretical and numerical results aimed at constraining the range of behavior that can occur in 4D conformal field theories.

Walking Beyond the Standard Model

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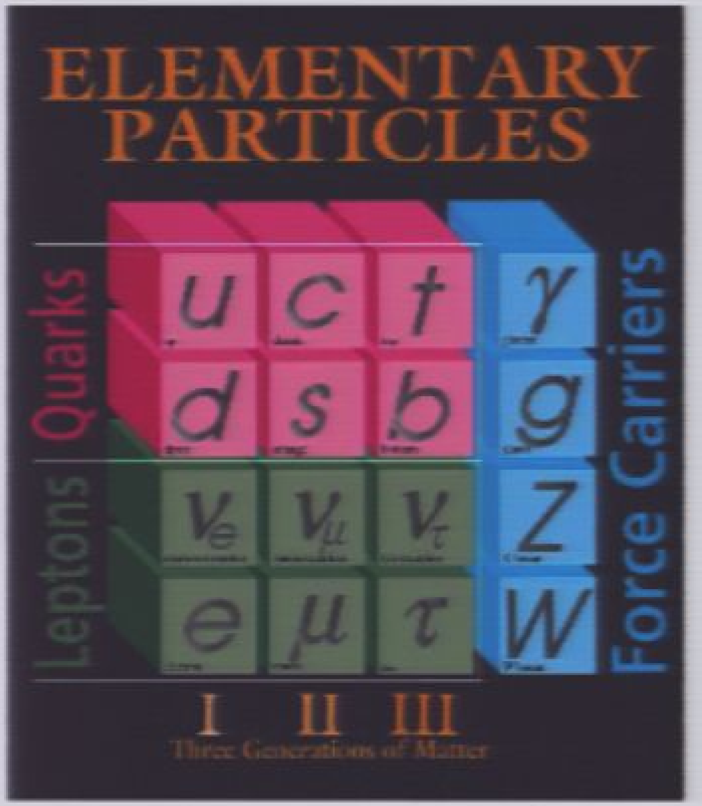
February 18, 2011

Perimeter Institute for Theoretical Physics

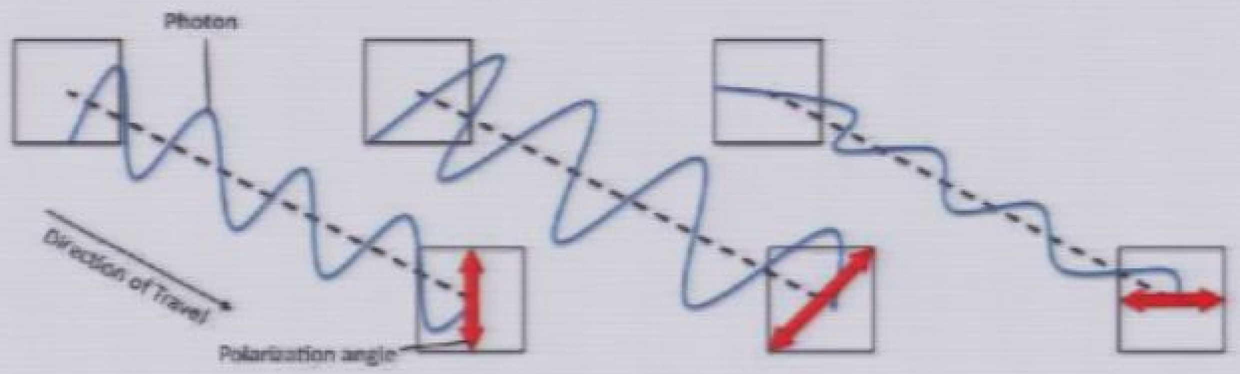
The Known

- The universe is well-described by local, Lorentz-invariant quantum field theory
- 3 generations of matter fields (spin 1/2)
  - Quarks
  - Leptons
- 3 non-gravitational forces (spin 1)
  - Electromagnetic
  - Strong
  - Weak
- Gravity (spin 2)



Electromagnetic and Strong Forces

- Massless spin-1 particles (γ, g) have 2 degrees of freedom



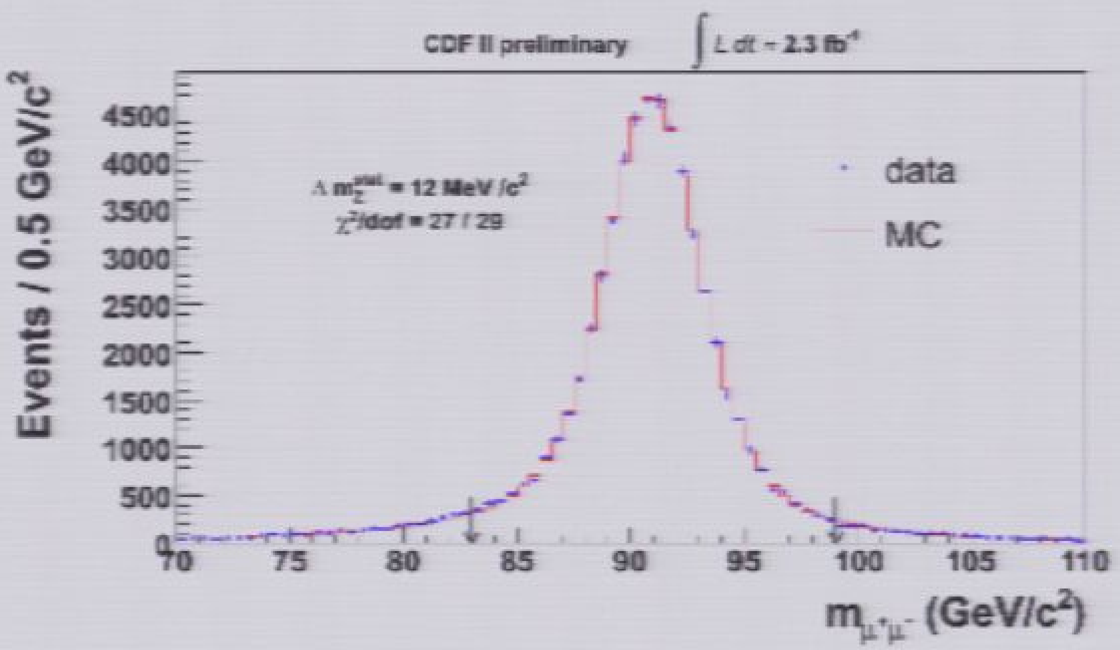
- However, Lorentz vectors A^μ have too many indices!
- Add gauge redundancies to keep Lorentz-invariance, locality

$$A^\mu \equiv A^\mu + \partial^\mu \alpha$$

  - Electromagnetic: $U(1)_{EM}$ gauge group
  - Strong: $SU(3)_C$ gauge group

Weak Force

- Massive spin-1 particles $(W_\pm, Z)$ have 3 degrees of freedom



- Interactions fit the pattern $SU(2)_W \times U(1)_Y \rightarrow U(1)_{EM}$

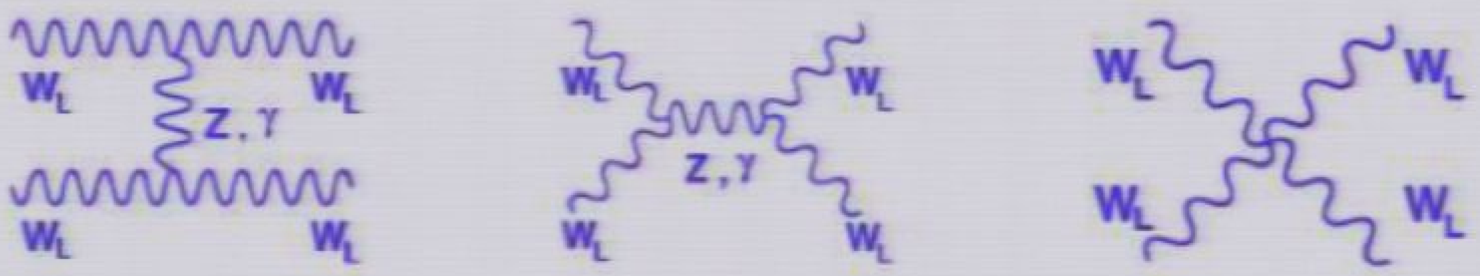
Electroweak symmetry breaking

- *Minimal* option:
  - Add 3 new fields $\pi^a(x)$, grouped into an object $\Sigma(x)$ that transforms simply under $SU(2)_W \times U(1)_Y$

$$\Sigma(x) = e^{i\pi^a(x)\sigma^a/v}$$

$$\mathcal{L} = v^2 \mathrm{Tr}\left[D_\mu \Sigma^\dagger D^\mu \Sigma\right] + \ldots$$

- The resulting effective field theory is remarkably successful at low energies, but becomes strongly-coupled around ~ 1 TeV



$$\mathcal{A}_{\mathrm{tree\ level}} \sim \mathbf{E}^2_{\mathrm{c.m.}}/\mathbf{M}^2_{\mathbf{W}}$$

TeV Scale Physics???

Many proposals for what it may look like:

- Higgs boson(s)
- Strong dynamics (e.g., technicolor)
- Flat or warped extra dimensions
- Supersymmetry
- ...

...and basically any combination of the above ideas that you can think of!

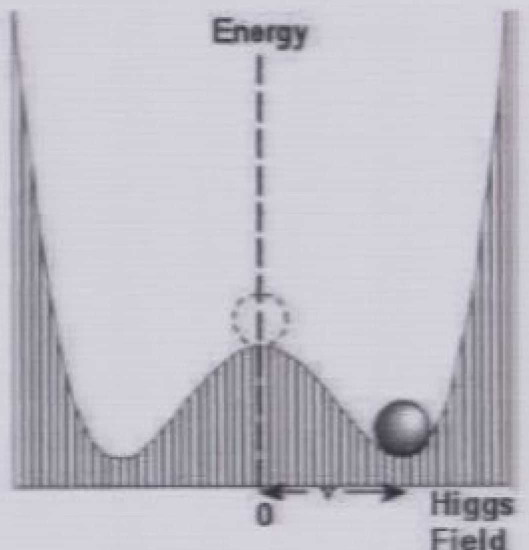
Higgs Boson

- Simplest UV completion: scalar doublet $H(x)$

$$H(x) = \Sigma(x) \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

- Adds one *new* degree of freedom: the Higgs Boson

$$\begin{aligned} V_{Higgs} &= -m^2(H^\dagger H) + \lambda(H^\dagger H)^2 \\ &\to v \sim \frac{m}{\sqrt{\lambda}} \sim 246\,\mathrm{GeV} \end{aligned}$$



Higgs Boson

The Good:

- Unitarizes WW scattering → Cutoff can become large $\sim M_{Pl}$

The Bad:

- Dimension 2 operator $H^\dagger H$ → "Hierarchy Problem"
  - Why is $m \sim \mathrm{TeV} \sim 10^{-15} M_{Pl}$?
- Fundamental scalar: All observed so far are composite

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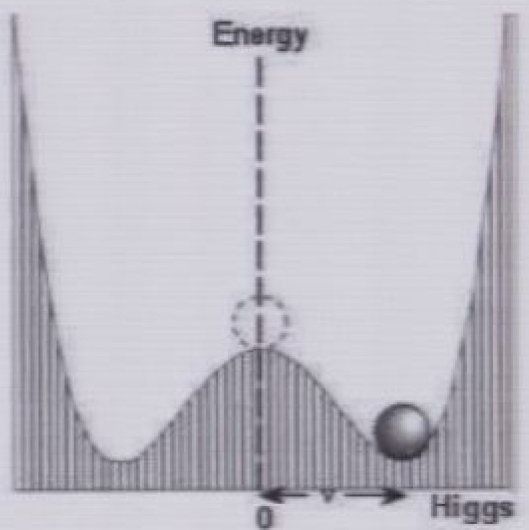
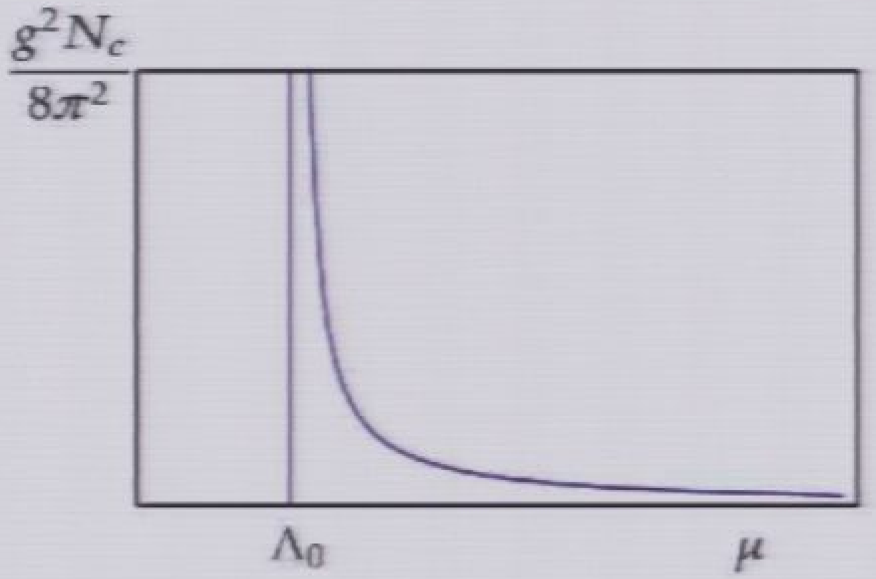


Strong Dynamics / Technicolor

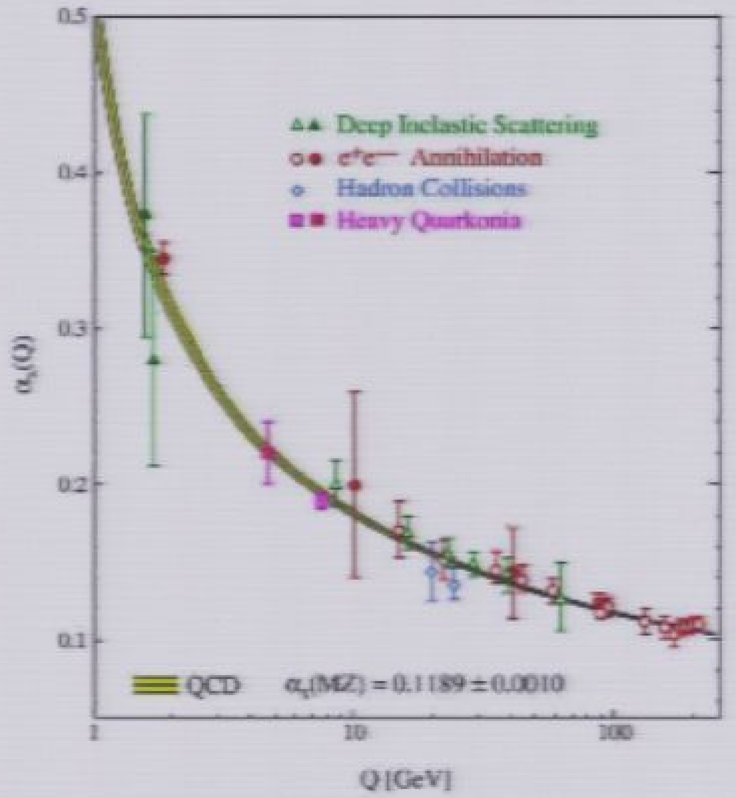


- An attractive idea is that the electroweak scale is *dynamically* generated rather than a fundamental parameter

$$\Lambda_{EW} \sim \Lambda_{UV} e^{-g_c^2/g_{UV}^2}$$

- Simple picture: Higgs operator is composite, $H \sim \overline{\psi}\psi$
  - ψ charged under new gauge group
  - $\langle \overline{\psi}\psi \rangle$ breaks $SU(2)_W \times U(1)_Y \to U(1)_{EM}$

Strong Dynamics / Technicolor



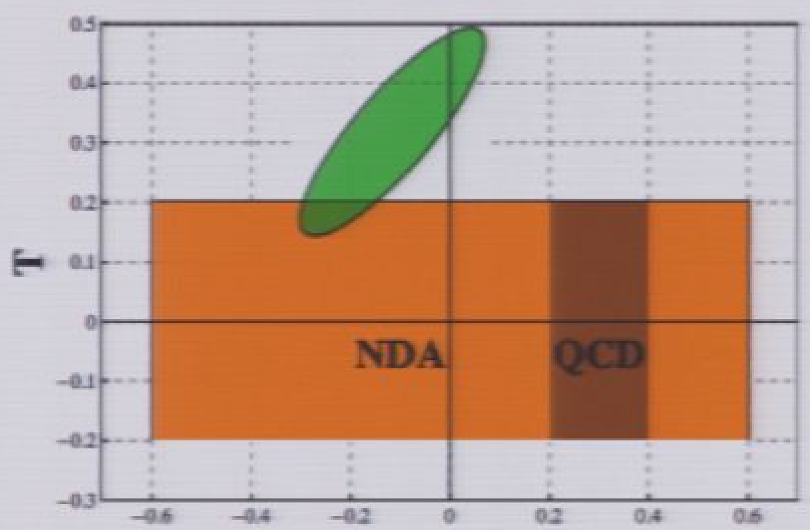
- Note: This *already* happens in QCD
  - $\Lambda_{QCD} \sim 200$ MeV dynamically generated
  - $\langle \bar{q}q \rangle = 4\pi f_\pi^3$ breaks electroweak symmetry, but at wrong scale!

Strong Dynamics / Technicolor: Challenges

- Top quark mass comes from operator suppressed by $\sim 1/\Lambda_F^2$, FCNC problems unless $\Lambda_F \gtrsim 10^3 - 10^4$ TeV

$$\frac{1}{\Lambda_F^2}(\overline{\psi}\psi)(t_L t_R) + \frac{1}{\Lambda_F^2}(\overline{q}_L^i \gamma^\mu q_L^j)^2 + \dots$$

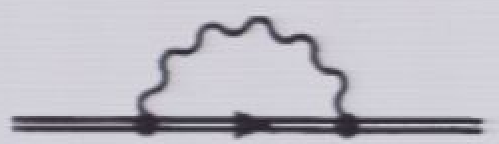
- Precision Electroweak Parameters
  - S Parameter: $SU(2)_W$ and $U(1)_Y$ kinetic mixing
  - T Parameter: relation between W and Z masses



A Solution: Anomalous Dimensions?

- The Bad: Flavor problems because $H = \bar{\psi}\psi$ has $\Delta_H = 3$
- The Good: Solves hierarchy problem because $\Delta_{H^\dagger H} = 6 > 4$

BUT....quantum effects can give operators *anomalous dimensions*



- Large effects need strong dynamics over range of energies...

$\rightarrow$ Scale-invariant or conformal dynamics!

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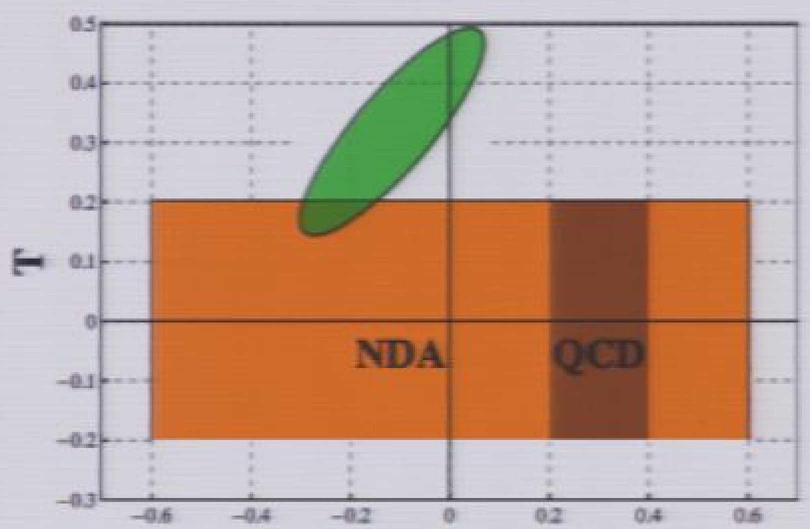

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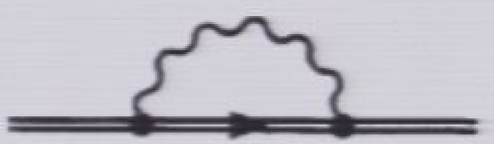

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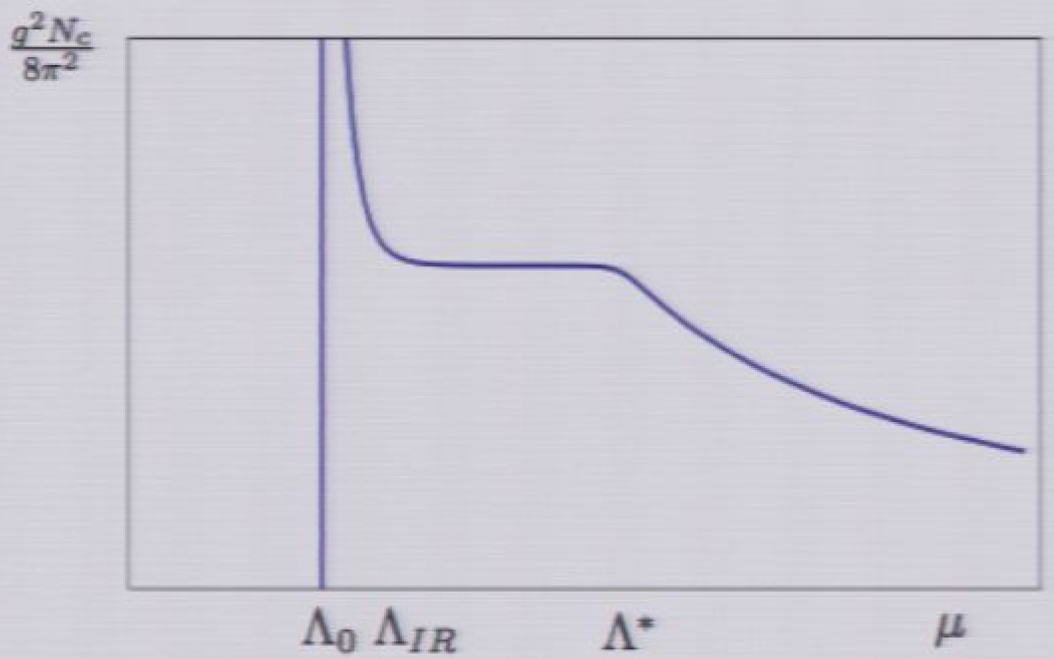
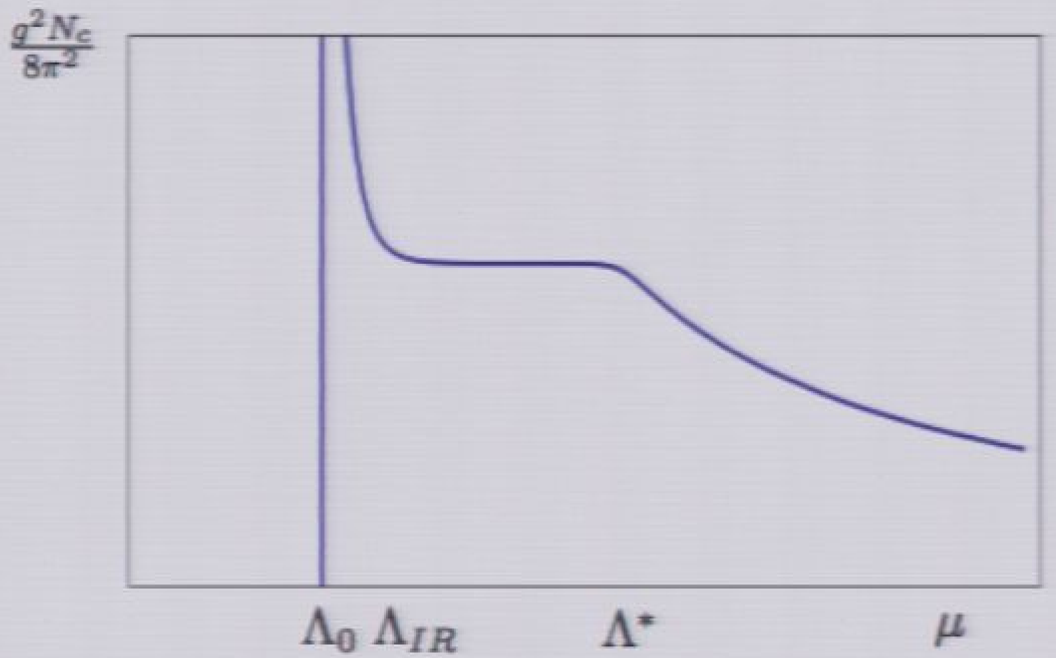
Walking / Conformal Technicolor



- Walking Technicolor: $\Delta_H \sim 2$ [Holdom '81; ...]
- Conformal Technicolor: $\Delta_H \sim 1$ [Luty, Okui '04]

$$\frac{1}{\Lambda_F^2}(\overline{\psi}\psi)(t_L t_R) \to \left(\frac{\Lambda^*}{\Lambda_{IR}}\right)^{3-\Delta_H} \frac{1}{\Lambda_F^2}(\overline{\psi}\psi)(t_L t_R)$$

Walking / Conformal Technicolor



"Ideal Scenario":

- $\Delta_H \sim 1.2 - 1.5$ would allow $\Lambda_F \sim \Lambda^* \sim 10^3 - 10^4$ TeV
- $\Delta_{H^\dagger H} > 4$ would solve hierarchy problem

But it is unknown if there exists a CFT with these properties

Other CFT Applications

- Large anomalous dimensions $\rightarrow$ power-law RG running

$$\mathcal{L}_{CFT} + \mathcal{O} \rightarrow \mathcal{L}_{CFT} + \left(\frac{\mu}{\Lambda^*}\right)^{\gamma_\mathcal{O}} \mathcal{O}$$

- This suppression (or enhancement) creates hierarchies!

- Many other possible new physics applications:
  - Flavor Hierarchies [Georgi, Nelson, Manohar '83; ...]
  - Conformal Sequestering [Luty, Sundrum '01; ...]
  - $\mu/B\mu$ Problem [Roy, Schmaltz '07; Murayama, Nomura, DP '07]
  - η Problem in Inflation [Baumann, Green '10]
  - ...

Example: Flavor Hierarchies

FERMIONS matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2

Flavor		Mass GeV/c^2	Electric charge
ν_e	electron neutrino	$<1\times10^{-8}$	0
e	electron	0.000511	–1
ν_μ	muon neutrino	<0.0002	0
μ	muon	0.106	–1
ν_τ	tau neutrino	<0.02	0
τ	tau	1.7771	–1

Quarks spin = 1/2

Flavor		Approx. Mass GeV/c^2	Electric charge
u	up	0.003	2/3
d	down	0.006	–1/3
c	charm	1.3	2/3
s	strange	0.1	–1/3
t	top	175	2/3
b	bottom	4.3	–1/3

- Fermion masses come from interactions with the Higgs:

$$\mathcal{L} = y^{ij} Q_i U_j H + \ldots$$

- The matrices y^{ij} have a bizarre hierarchical structure!

Example: Flavor Hierarchies

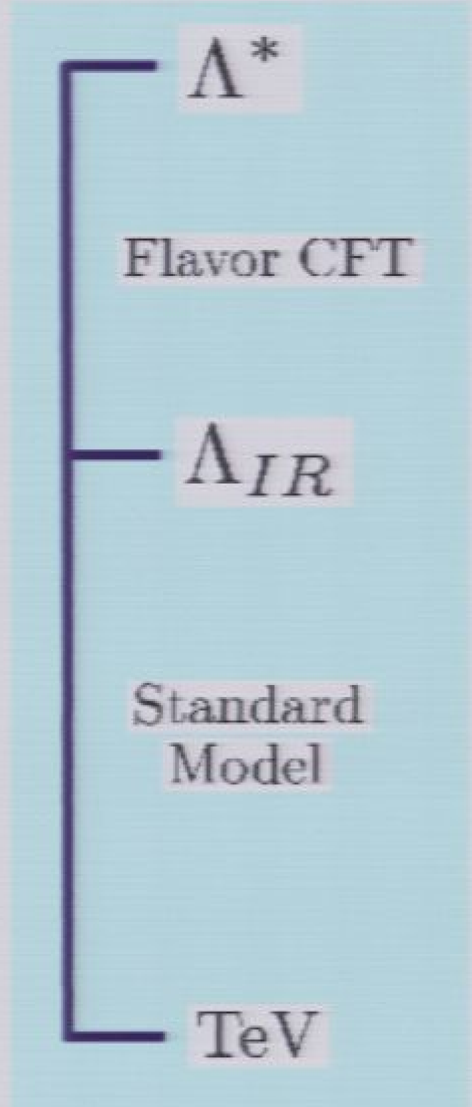
- Structure *explained* if generations have different anomalous dimensions

$$\mathcal{L}_{int} = Q_1 \mathcal{O}_1 + Q_2 \mathcal{O}_2 + \ldots$$

- These interactions lead to *power-law* suppressions of Yukawa couplings

$$y^{ij} Q_i U_j H \to \left(\frac{\mu}{\Lambda^*}\right)^{\gamma_{Q_i} + \gamma_{U_j}} y^{ij} Q_i U_j H$$

- Running stops at scale $\mu \sim \Lambda_{IR}$ when states in CFT decouple



Example: Flavor Hierarchies

- One can construct completely explicit supersymmetric models realizing this idea [Nelson, Strassler '00; DP, Simmons-Duffin '09]
- Dimensions of matter fields can even be *calculated* using a technique called a-maximization [Intriligator, Wecht '03]
  - Possible because matter fields are "chiral" , $\Delta_Q = \frac{3}{2} R_Q$
  - Reduces problem to one of maximizing polynomials!
- Hoped that these models can also solve *SUSY flavor problem*
  - Suppress flavor-violation in soft masses $\sim \frac{1}{M_{Pl}^2} Q_i^\dagger Q_j X^\dagger X$
  - Effects depend on $\Delta_{Q_i^\dagger Q_j}$, hard to calculate

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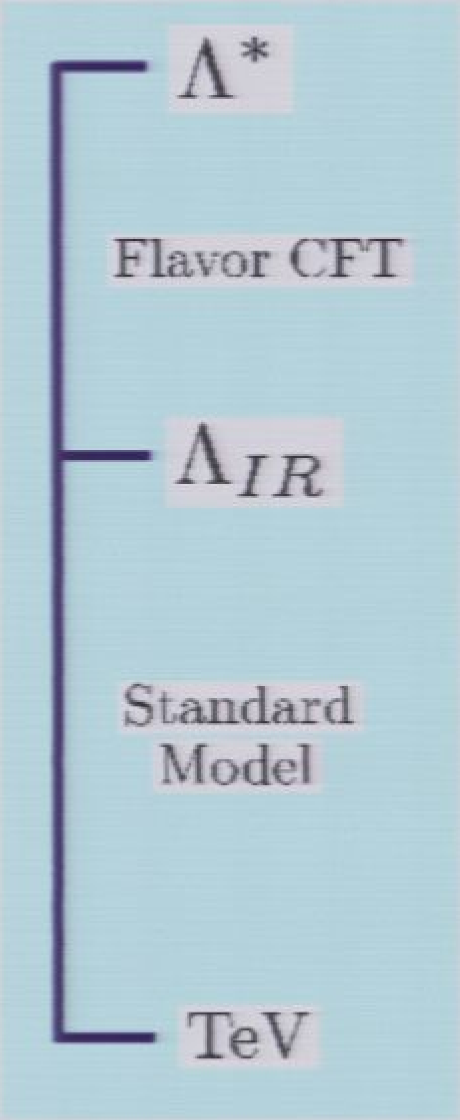

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Spectrum of Operator Dimensions

- It is an *important* problem to better understand the spectrum of operator dimensions in 4D CFTs
- Concrete questions in our examples
  - Conformal Technicolor: Is $\Delta_H \sim 1$ and $\Delta_{H^\dagger H} > 4$ possible?
  - Flavor Models: Can CFT dynamics solve SUSY flavor problem?
- Recent Progress: There are general *bounds* on CFT operator dimensions that may help answer these questions!
  - Consequences of crossing symmetry + unitarity

[Rattazzi, Rychkov, Tonni, Vichi '08; Rychkov, Vichi '09; Caracciolo, Rychkov '09; DP, Simmons-Duffin '10; Rattazzi, Rychkov, Vichi '10]

Conformal Block Decomposition

Basic Idea:

- 4-point functions can be expanded in *conformal blocks*

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{1}{x_{12}^{2d} x_{34}^{2d}} \sum_{\mathcal{O}\in\phi\times\phi} \lambda_{\mathcal{O}}^2\, g_{\Delta,l}(u,v)$$

- Comes from inserting the *operator product expansion*

$$\phi(x)\phi(0) = \sum_{\mathcal{O}\in\phi\times\phi} \lambda_{\mathcal{O}} C_I(x,\partial)\, \mathcal{O}^I(0)$$

- $g_{\Delta,l}(u,v)$ eigenfunctions of Casimir of conformal group
  - Known in terms of hypergeometric functions [Dolan, Osborn '00]

Crossing Symmetry

Basic Idea:

- $\langle\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle$ symmetric under permutations of x_i
- Switching $x_1 \leftrightarrow x_3$ (after OPE) gives the "crossing relation":

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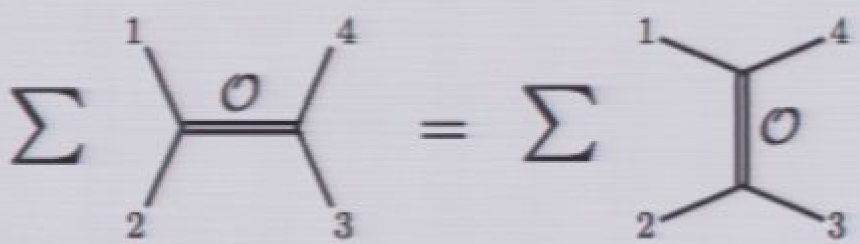
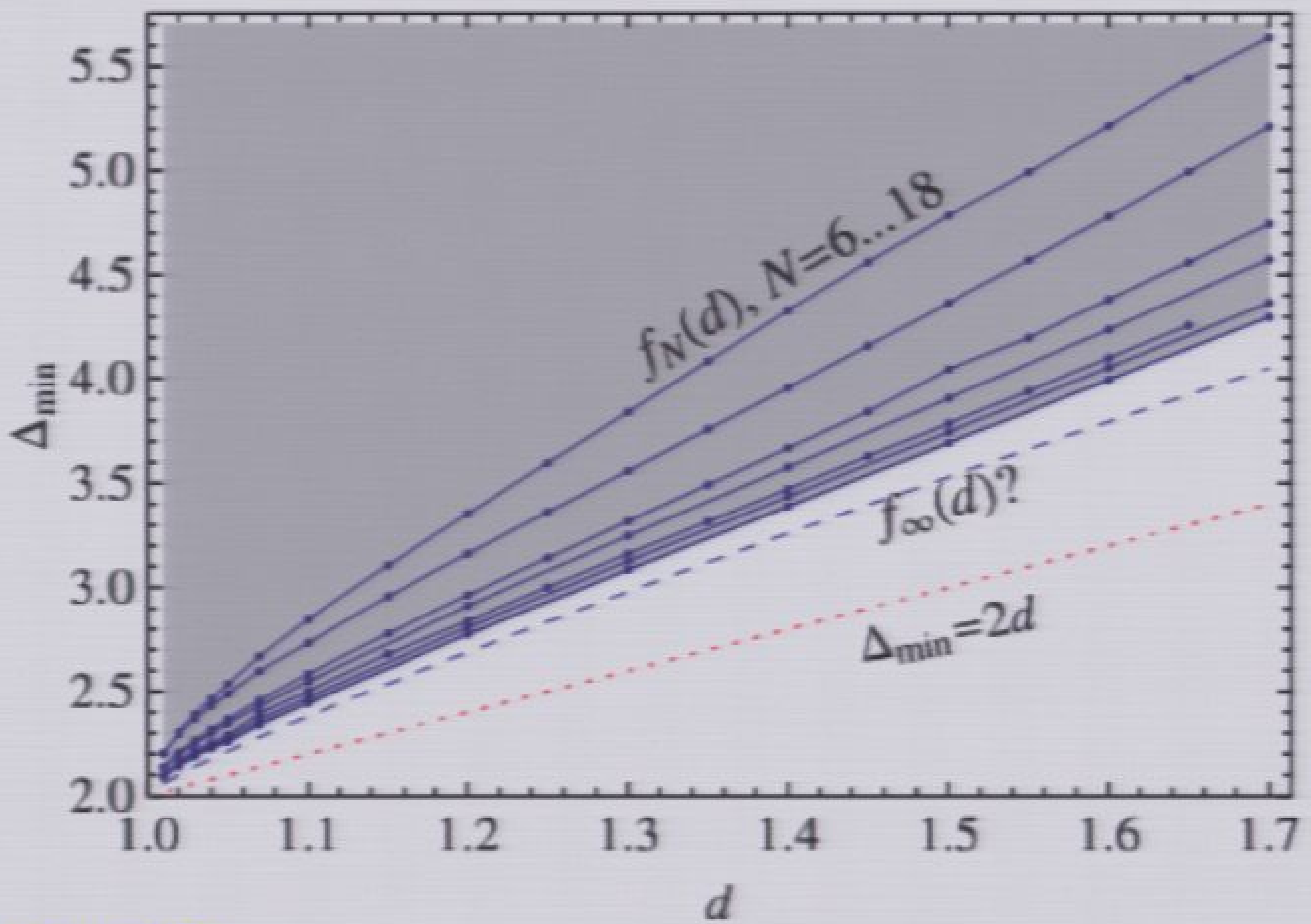


- Bounds follow from numerical studies of this constraint...

Upper Bound on Dimension of ϕ^2 in CFT



[Rychkov, Vichi '09]

- ϕ real scalar primary of dimension d
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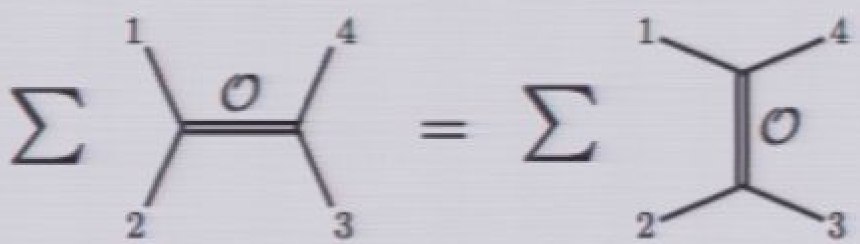

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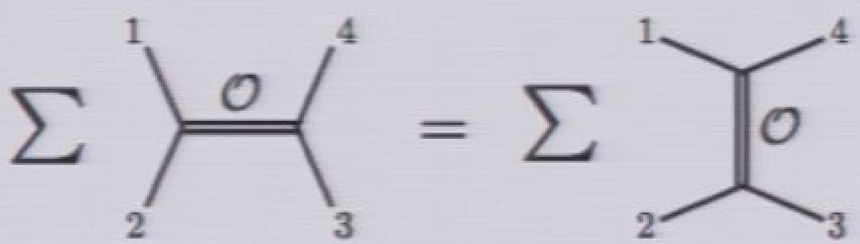

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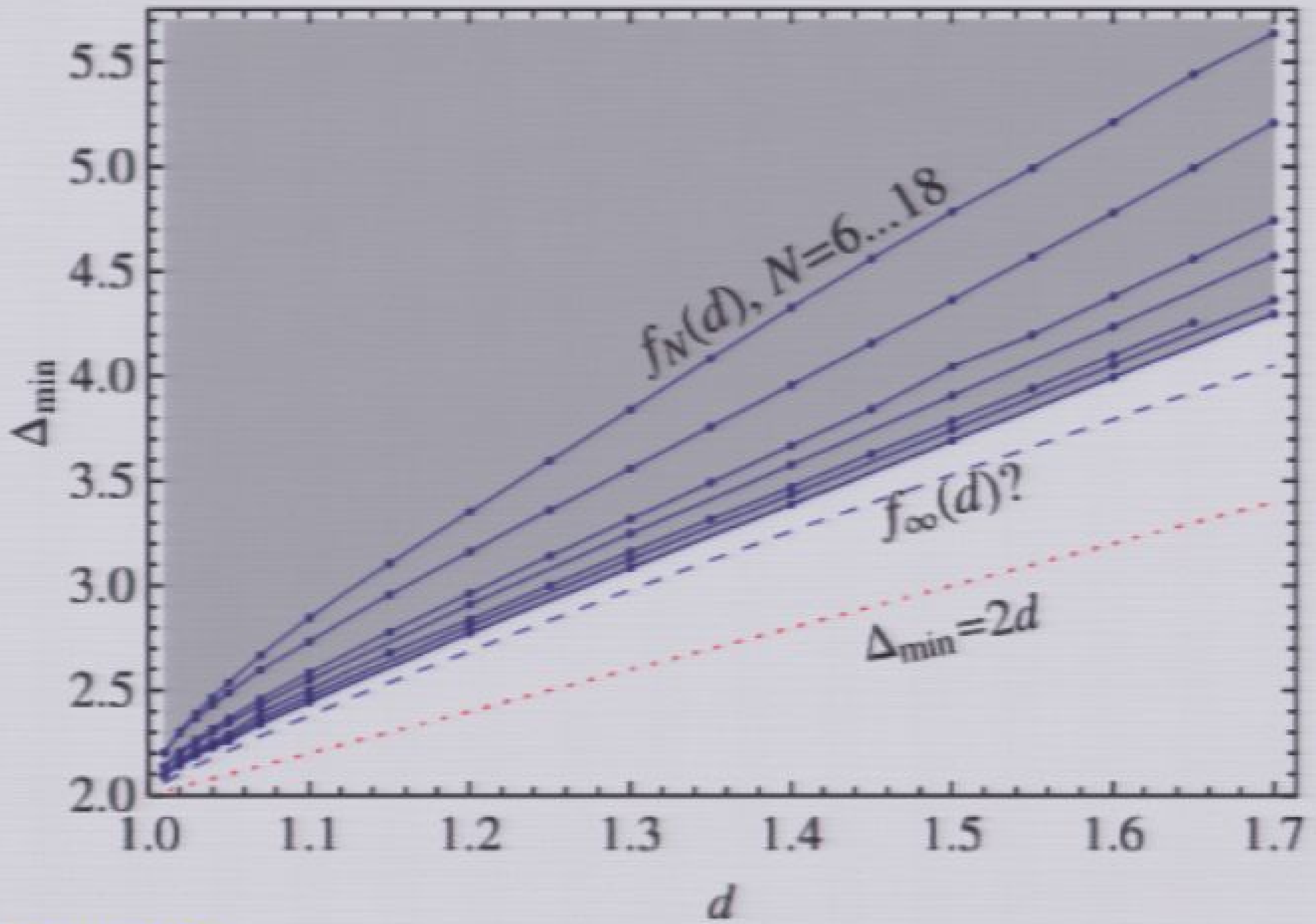

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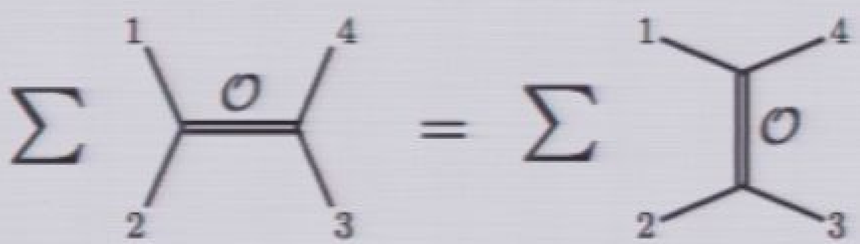

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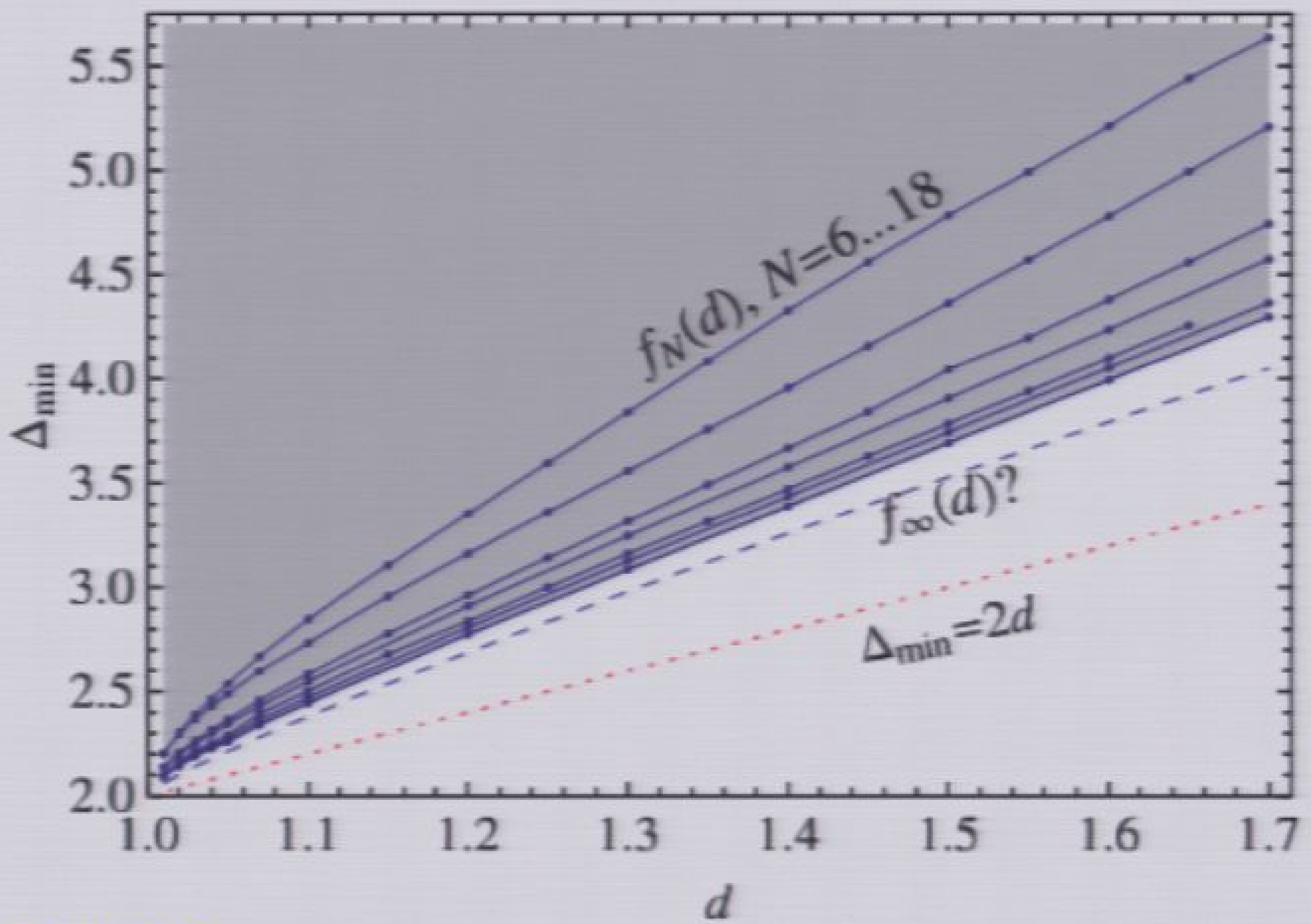

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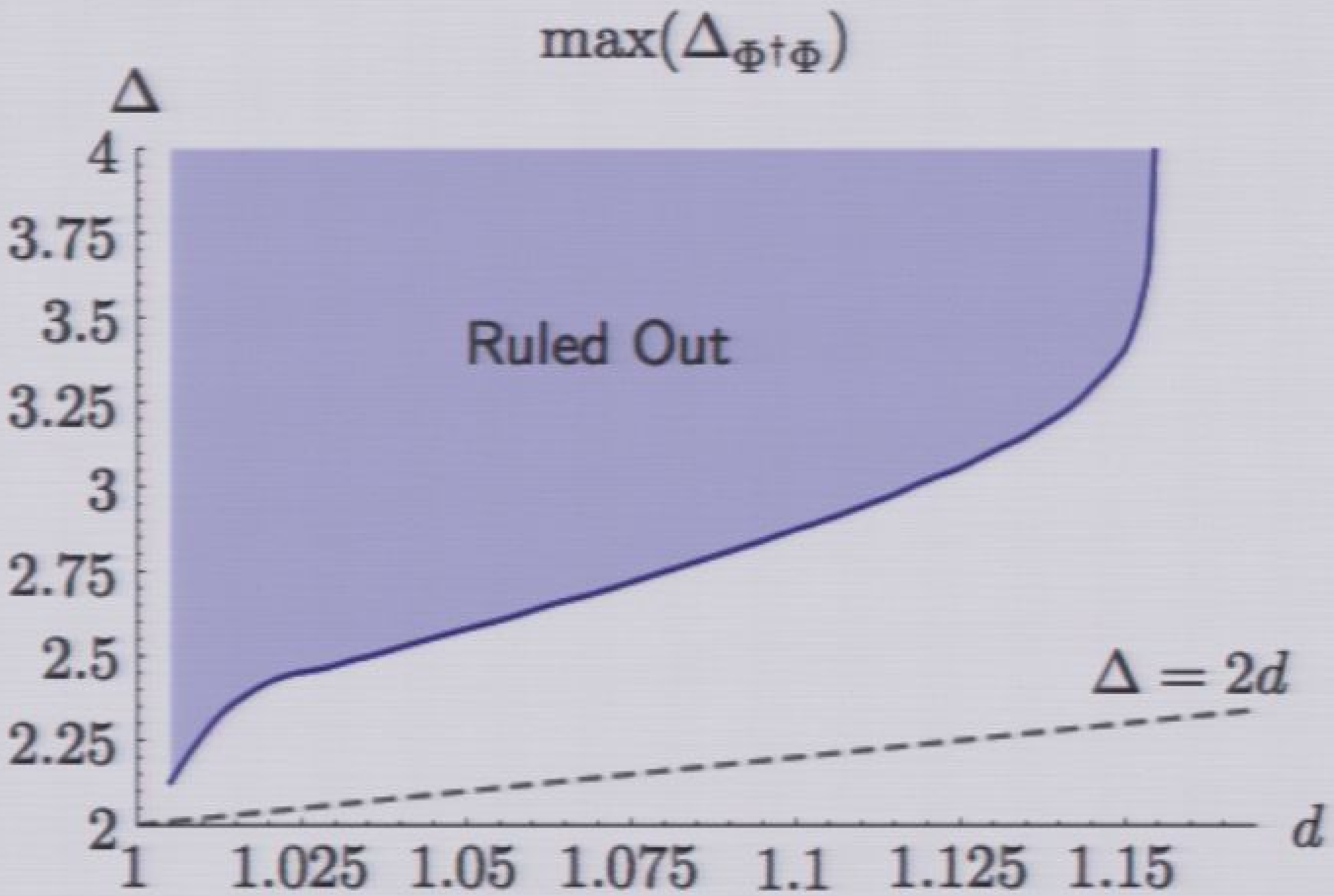
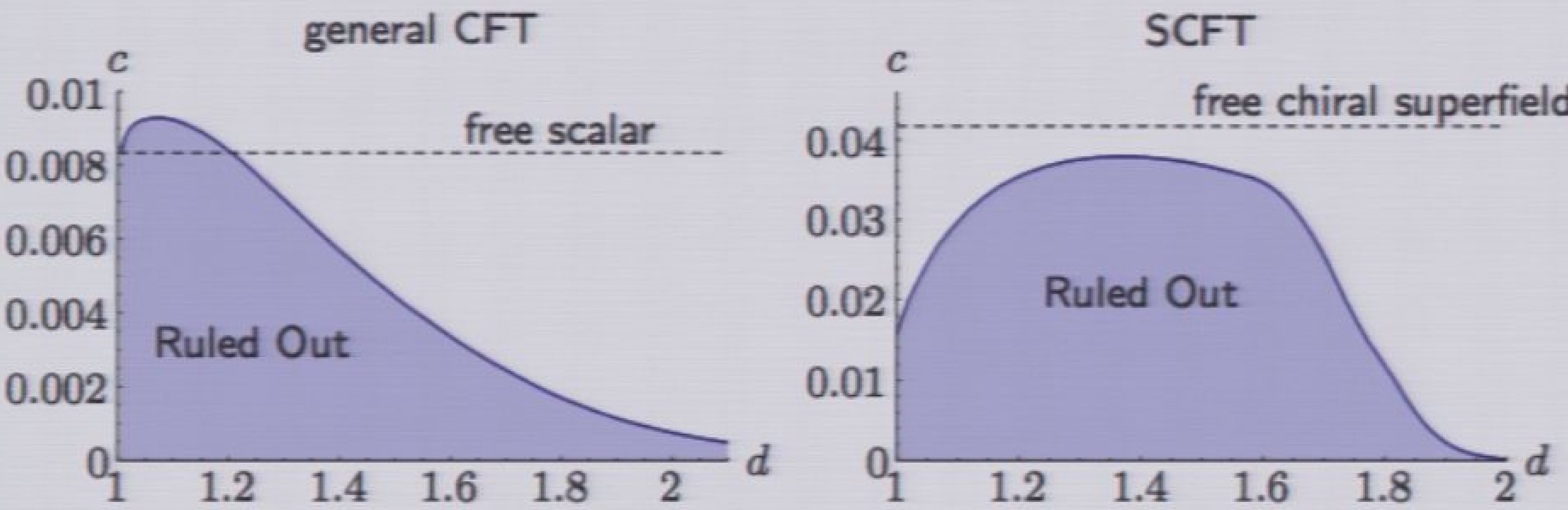
Upper Bound on Dimension of $\Phi^\dagger\Phi$ in SCFT



[DP, Simmons-Duffin '10]

- Φ chiral superconformal primary of dimension d

Lower Bound on Central Charge



[DP, Simmons-Duffin '10]

- Bound on coefficient $\langle TT \rangle \propto c$
- Lowest-dimension scalar in theory has dimension d
- (In dual AdS$_5$, $c \sim L^3 M_P^3$: suggests fundamental limit to strength of quantum gravity!)

Outlook

- Results are *close to* placing real constraints on BSM scenarios
  - Technicolor: still trying to disentangle $H^\dagger H$ from $H^\dagger \sigma^a H$
  - SCFT Flavor models: bound on $\Phi^\dagger \Phi$ still relatively weak

- Work to improve numerical algorithms and theoretical understanding is ongoing...

- Complementary approaches to learning about CFTs:
  - Study theories with lots of supersymmetry like $\mathcal{N} = 4$ SYM
  - Study theories numerically on a lattice
  - Study weakly-coupled CFTs in perturbation theory
  - Study large N theories and AdS/CFT
    - Effective field theories in AdS → "Effective CFTs" [Fitzpatrick, Katz, DP, Simmons-Duffin, '10]

Summary

- Approximately scale-invariant dynamics could play an important role in Beyond the Standard Model physics
- Spectrum of operator dimensions controls most interesting effects, needs to be better understood...
- Much progress is being made on several different fronts, but there's plenty of work still to be done!

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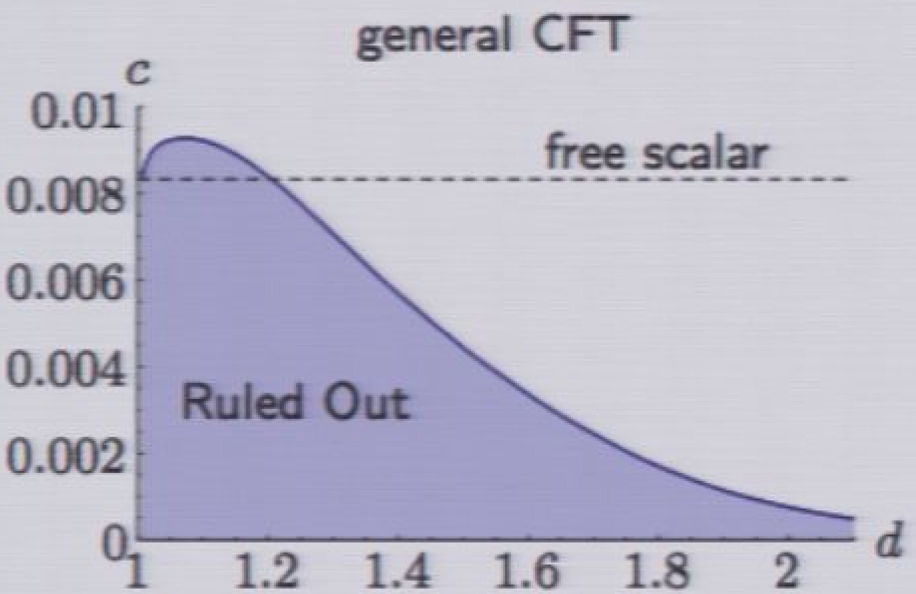

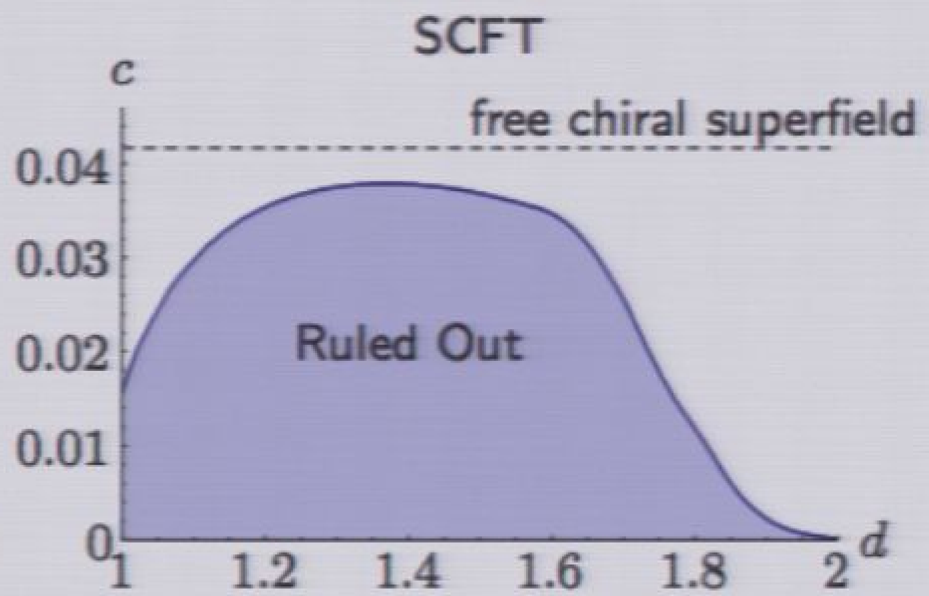

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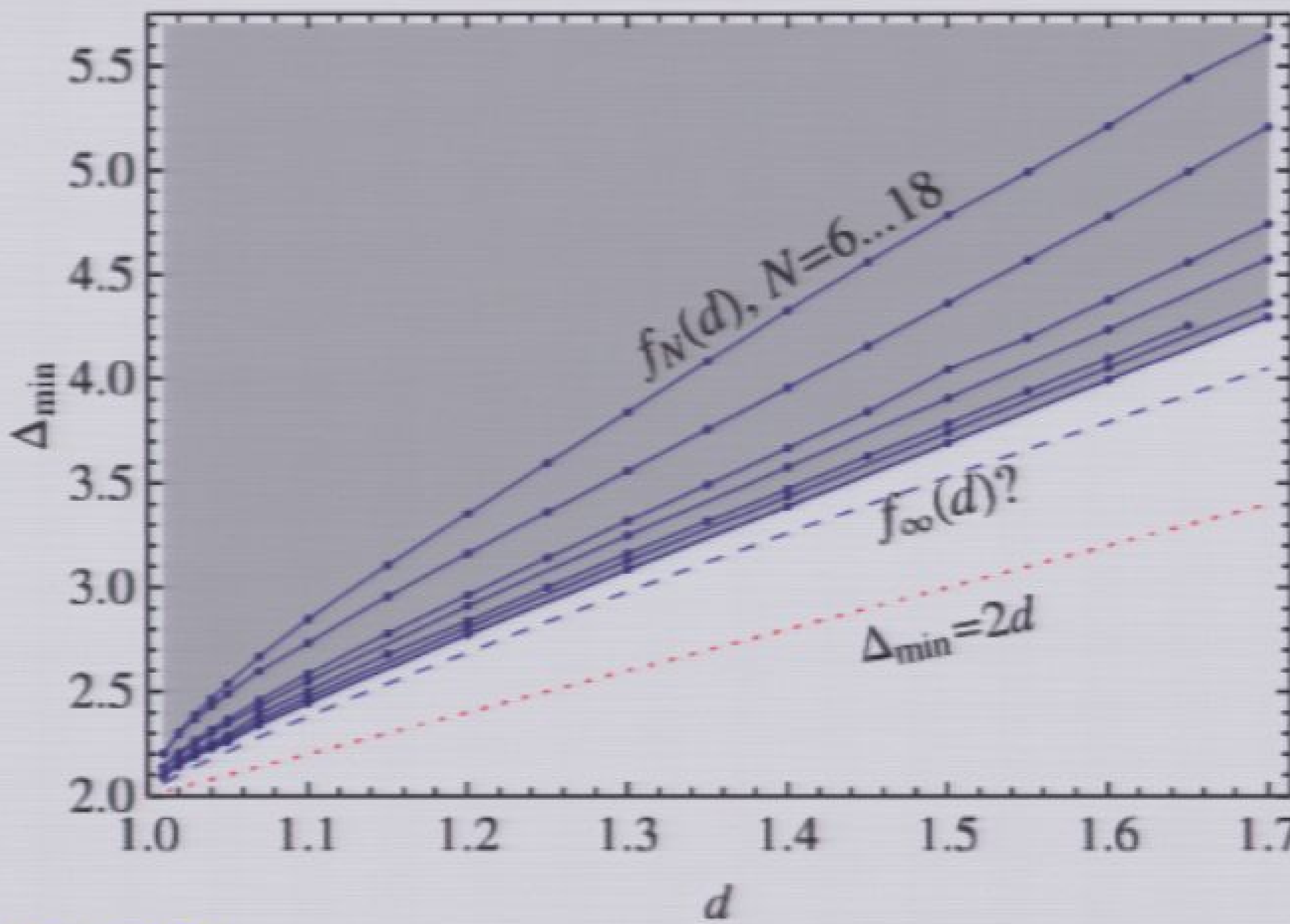
- Lowest-dimension scalar in theory has dimension d
- (In dual AdS$_5$, $c \sim L^3 M_P^3$: suggests fundamental limit to strength of quantum gravity!)

Upper Bound on Dimension of ϕ^2 in CFT



[Rychkov, Vichi '09]

- ϕ real scalar primary of dimension d
- ϕ^2 lowest dimension scalar appearing in $\phi \times \phi$ OPE

Spectrum of Operator Dimensions

- It is an *important* problem to better understand the spectrum of operator dimensions in 4D CFTs
- Concrete questions in our examples
  - Conformal Technicolor: Is $\Delta_H \sim 1$ and $\Delta_{H^\dagger H} > 4$ possible?
  - Flavor Models: Can CFT dynamics solve SUSY flavor problem?
- Recent Progress: There are general *bounds* on CFT operator dimensions that may help answer these questions!
  - Consequences of crossing symmetry + unitarity

[Rattazzi, Rychkov, Tonni, Vichi '08; Rychkov, Vichi '09; Caracciolo, Rychkov '09; DP, Simmons-Duffin '10; Rattazzi, Rychkov, Vichi '10]

Example: Flavor Hierarchies

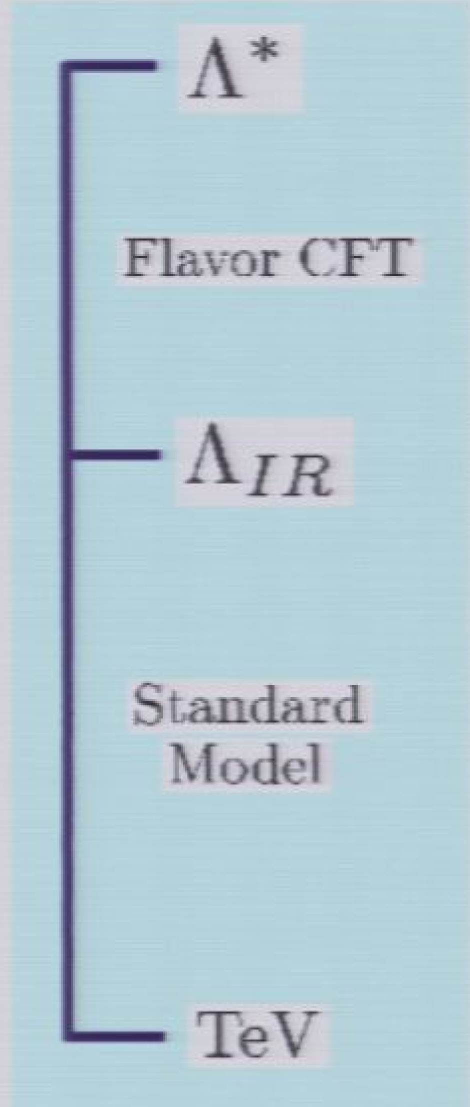
- Structure *explained* if generations have different anomalous dimensions

$$\mathcal{L}_{int} = Q_1 \mathcal{O}_1 + Q_2 \mathcal{O}_2 + \ldots$$

- These interactions lead to *power-law* suppressions of Yukawa couplings

$$y^{ij} Q_i U_j H \to \left(\frac{\mu}{\Lambda^*}\right)^{\gamma_{Q_i}+\gamma_{U_j}} y^{ij} Q_i U_j H$$

- Running stops at scale $\mu \sim \Lambda_{IR}$ when states in CFT decouple



Other CFT Applications

- Large anomalous dimensions → power-law RG running

$$\mathcal{L}_{CFT} + \mathcal{O} \to \mathcal{L}_{CFT} + \left(\frac{\mu}{\Lambda^*}\right)^{\gamma_{\mathcal{O}}} \mathcal{O}$$

- This suppression (or enhancement) creates hierarchies!

- Many other possible new physics applications:
  - Flavor Hierarchies [Georgi, Nelson, Manohar '83; ...]
  - Conformal Sequestering [Luty, Sundrum '01; ...]
  - $\mu/B\mu$ Problem [Roy, Schmaltz '07; Murayama, Nomura, DP '07]
  - η Problem in Inflation [Baumann, Green '10]
  - ...