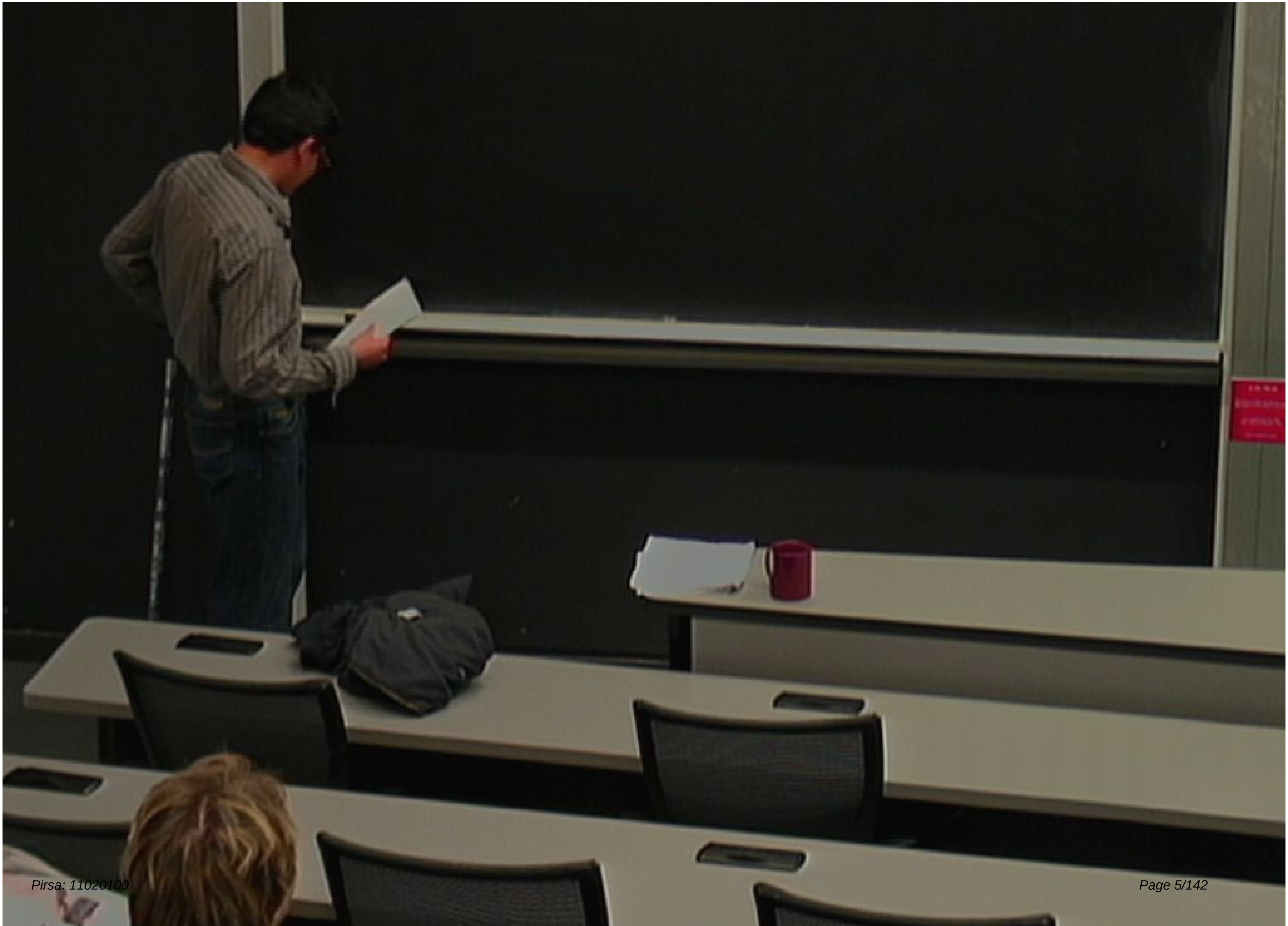
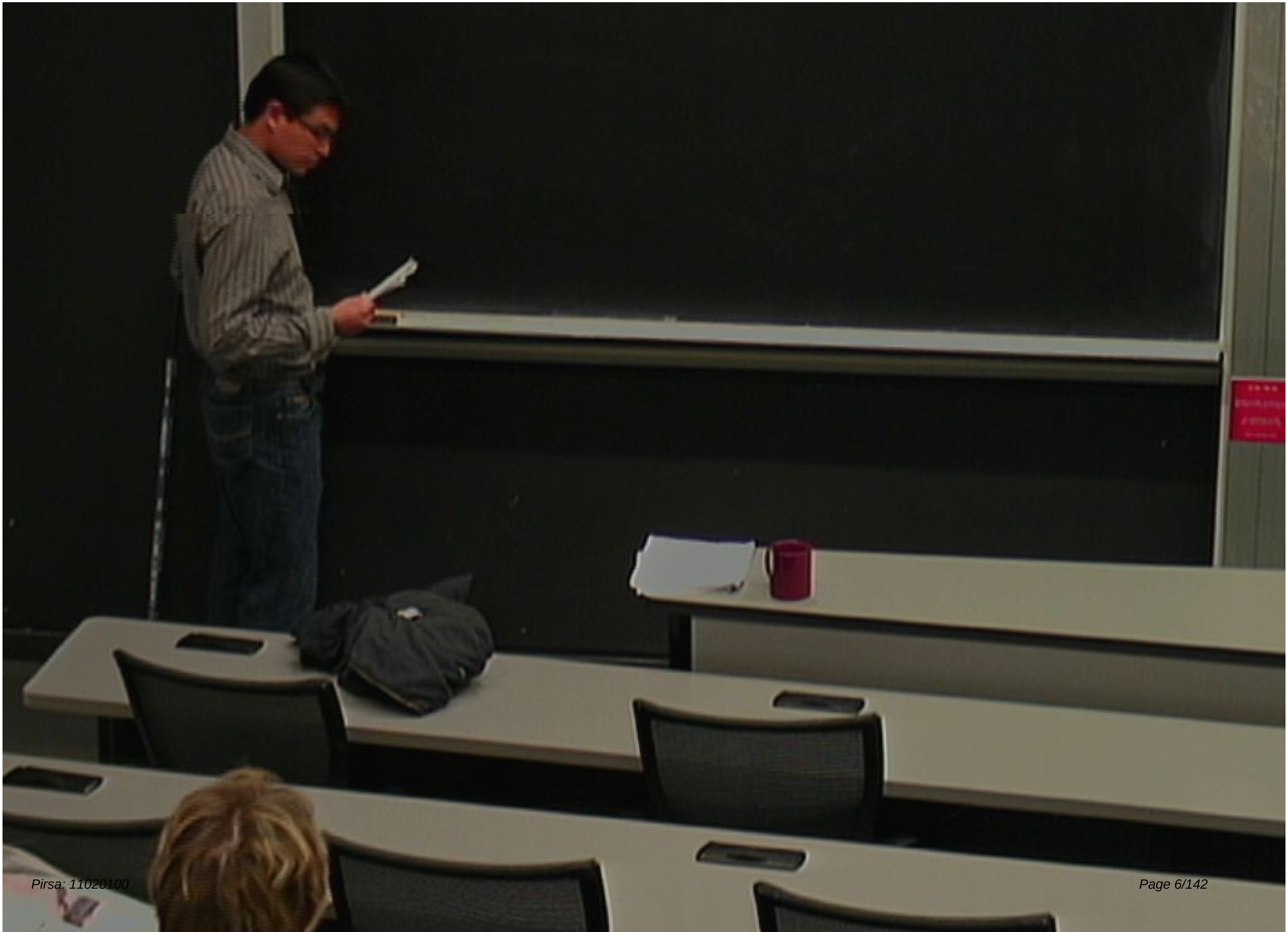
Title: N=2 S-dualities in A_N theories

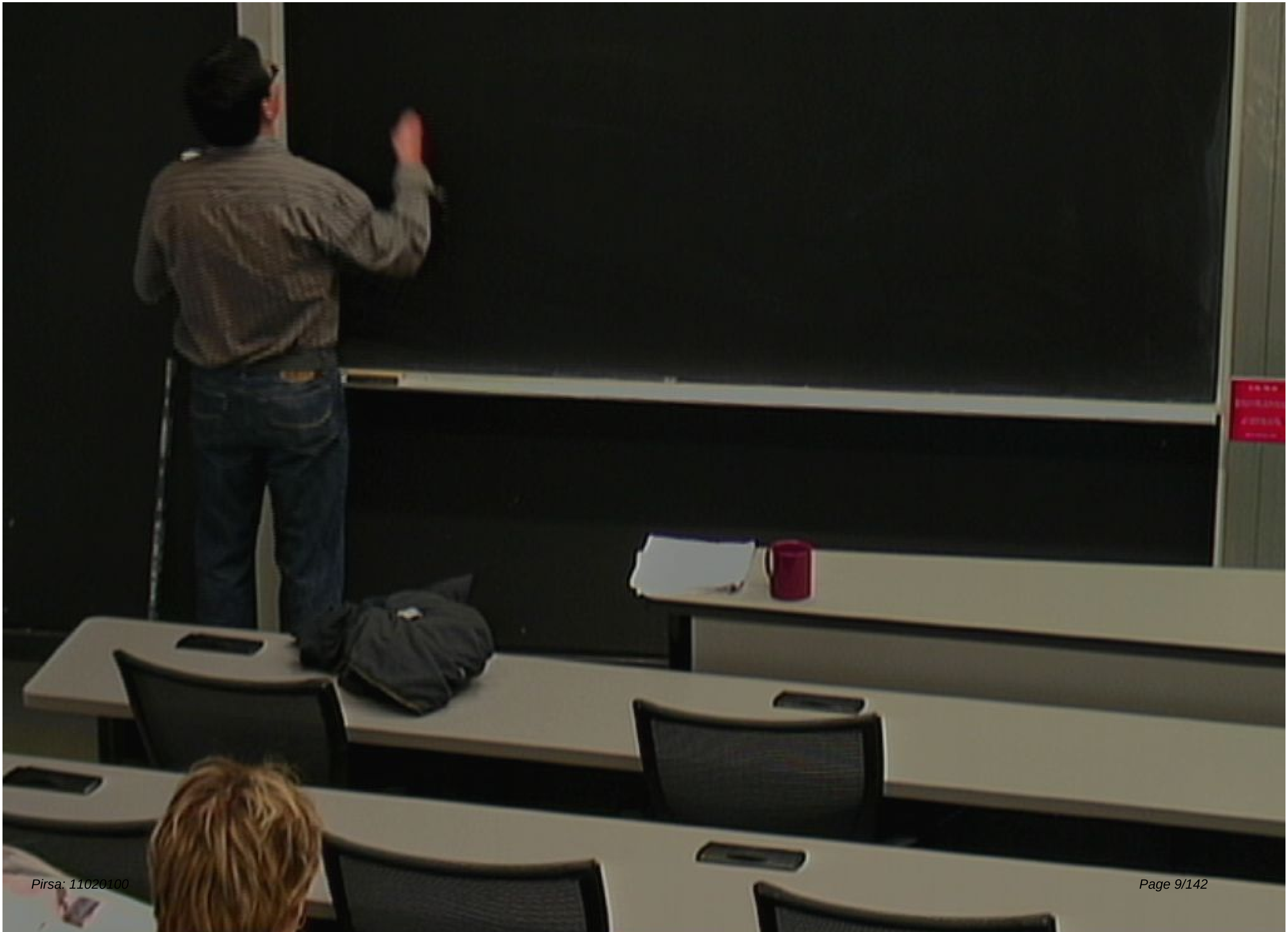
Date: Feb 01, 2011 11:00 AM

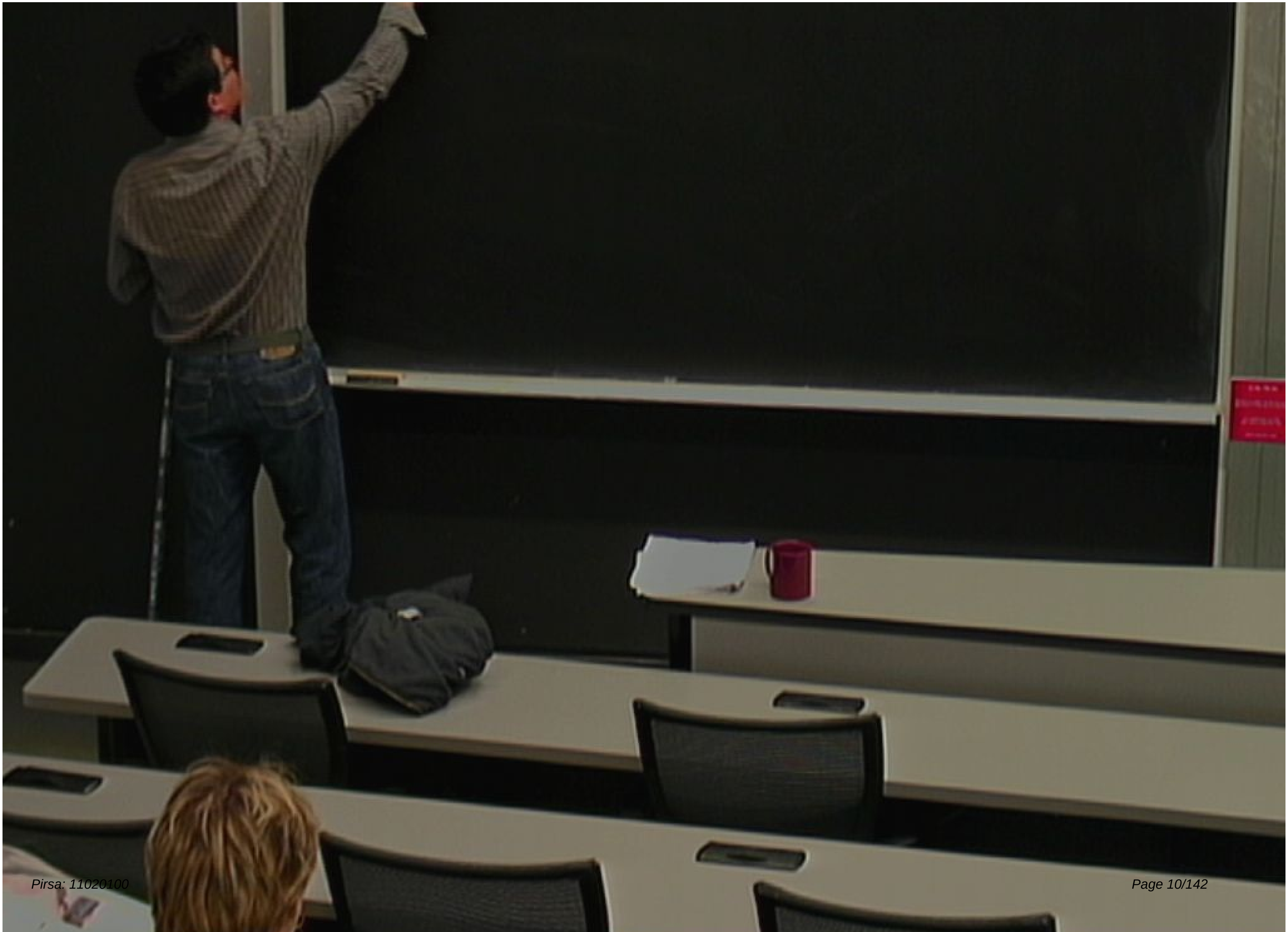
URL: http://pirsa.org/11020100

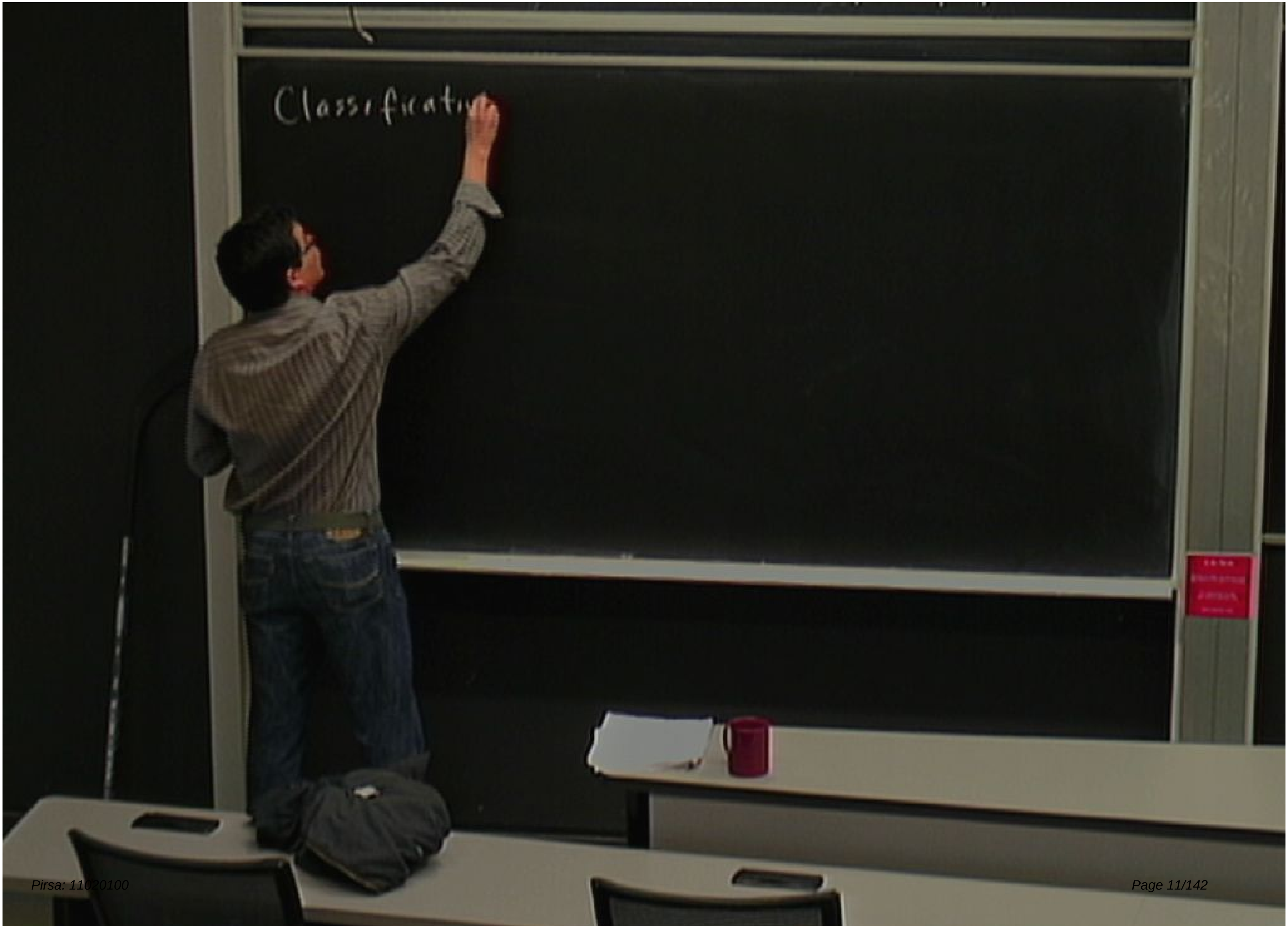
Abstract: We describe a classification of 4d N=2 superconformal theories obtained from the compactification of 6d N=(2,0) A_N theories on punctured Riemann surfaces. We show the basic building blocks to construct any such theory and its various S-dual frames. A host of new S-dualities and interacting (non-Lagrangian) superconformal theories are unconvered. We also compute various properties of these interacting superconformal theories.







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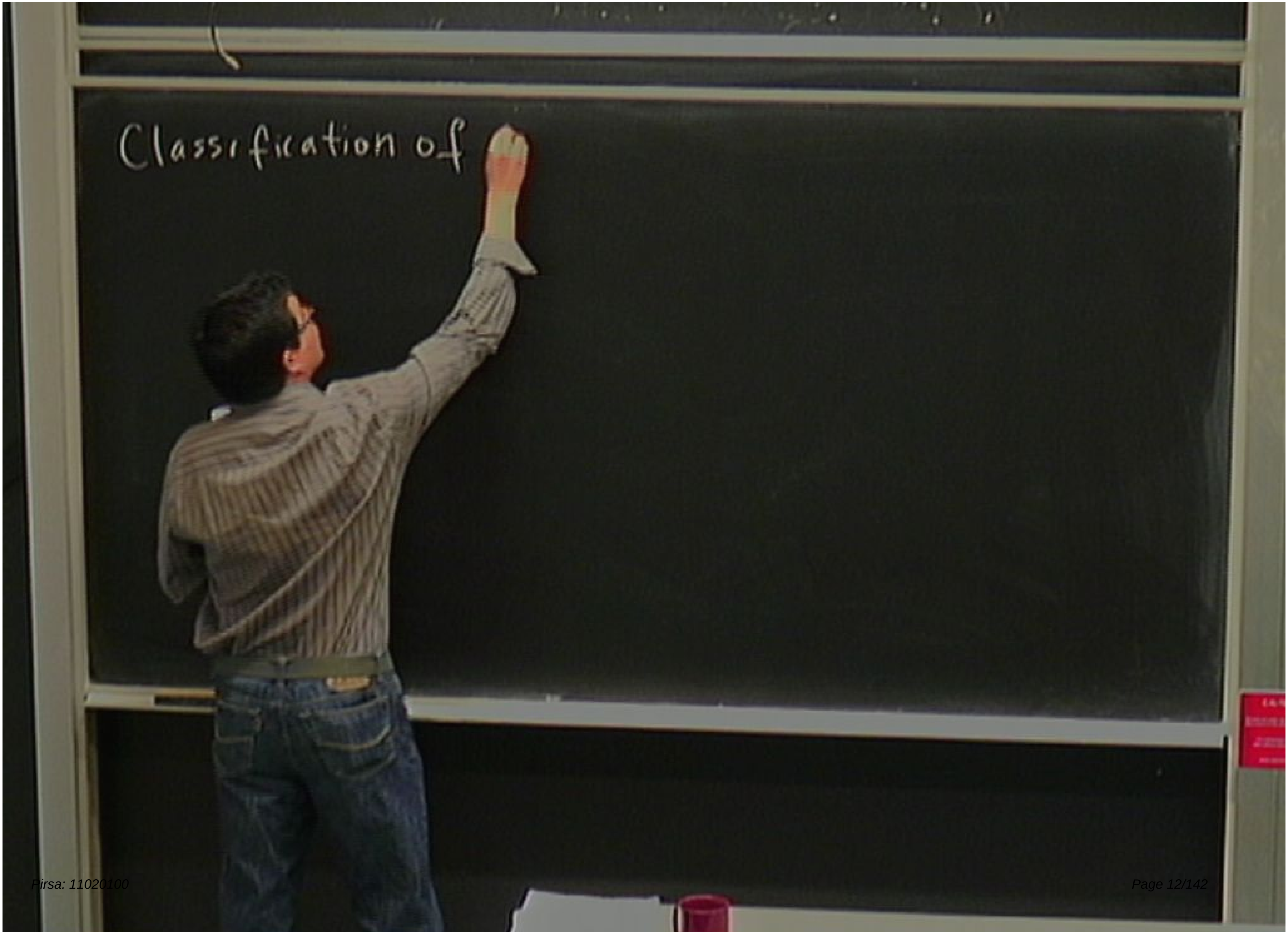
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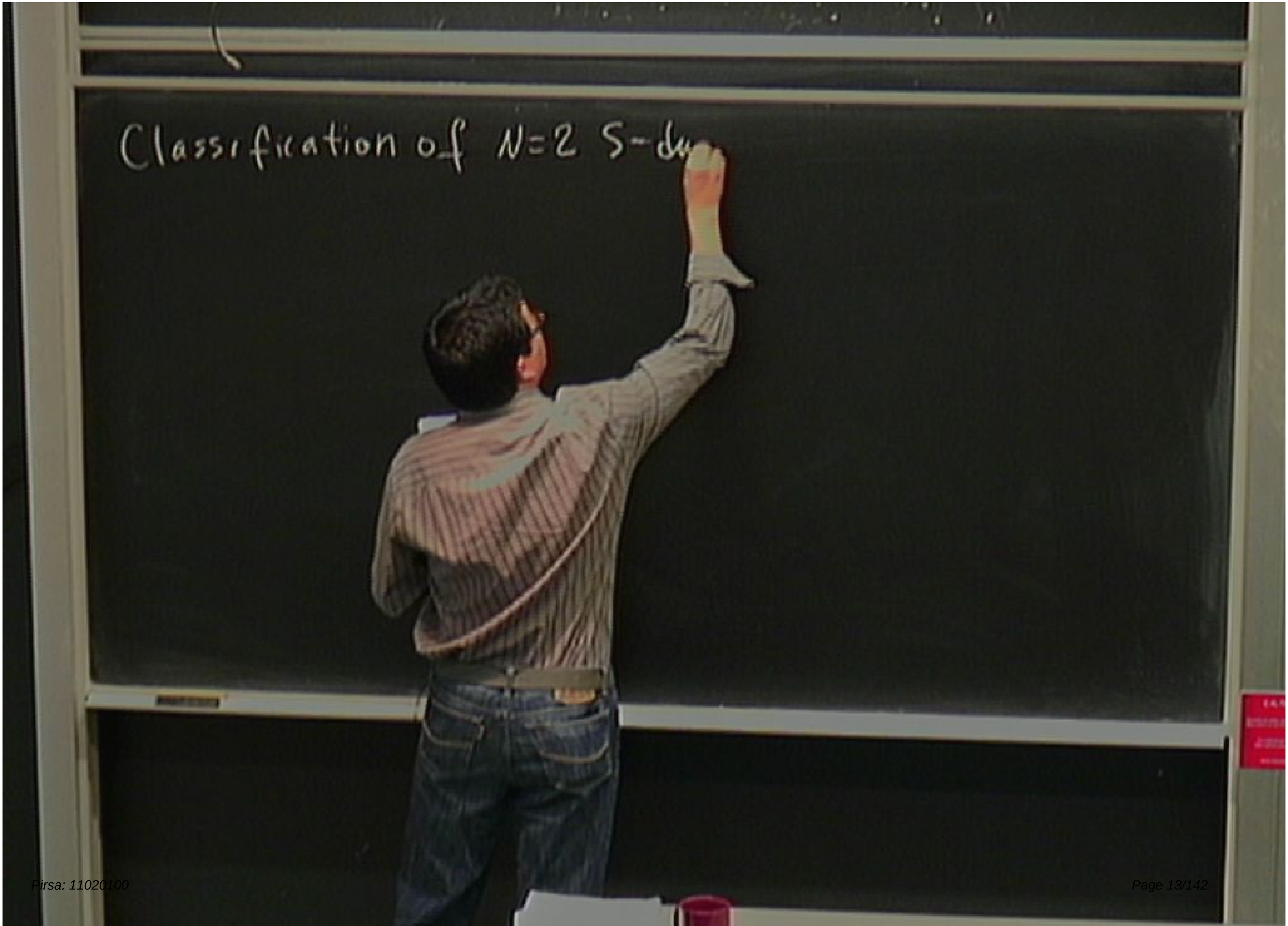
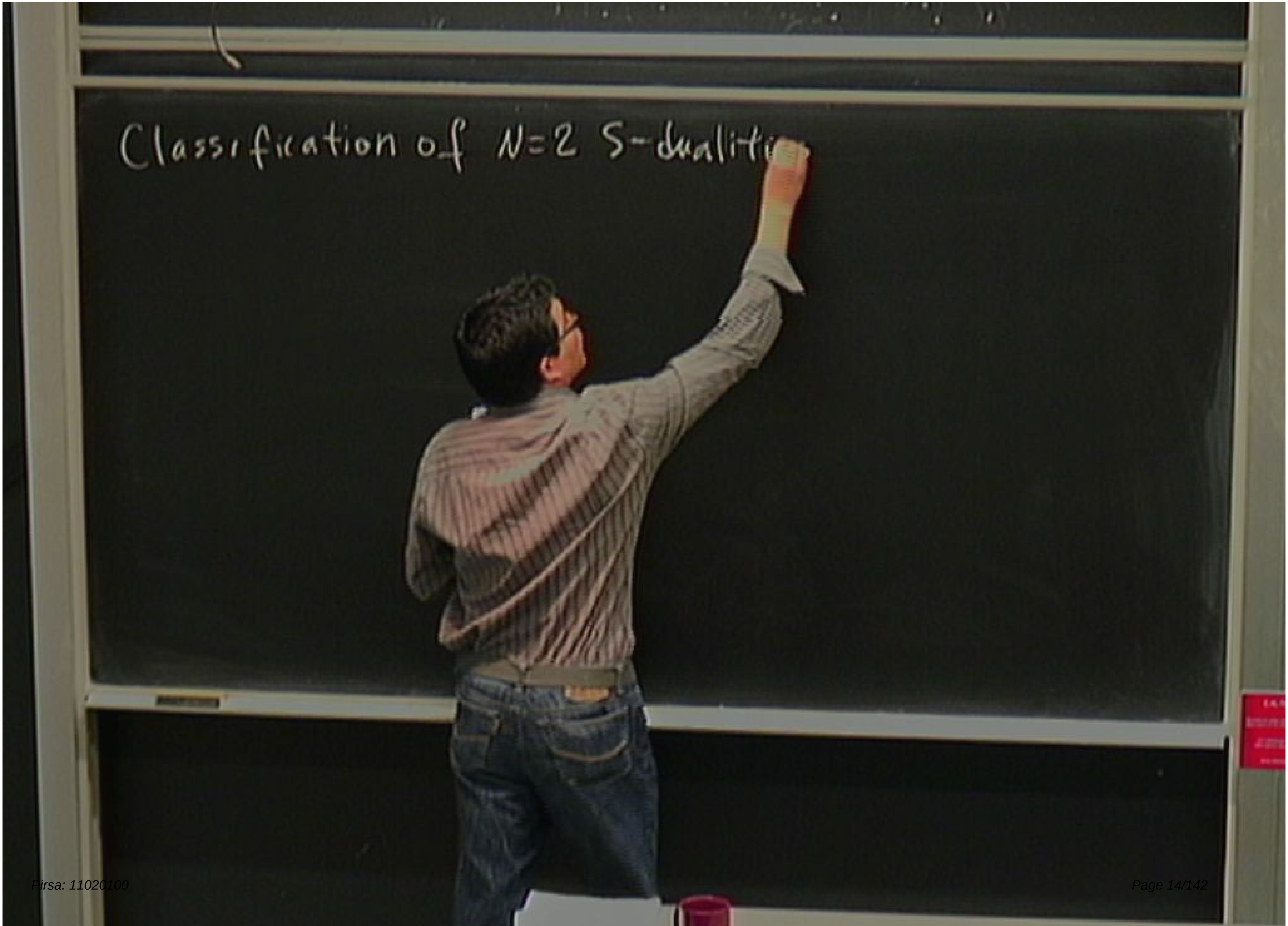

Classification of N=2 S-du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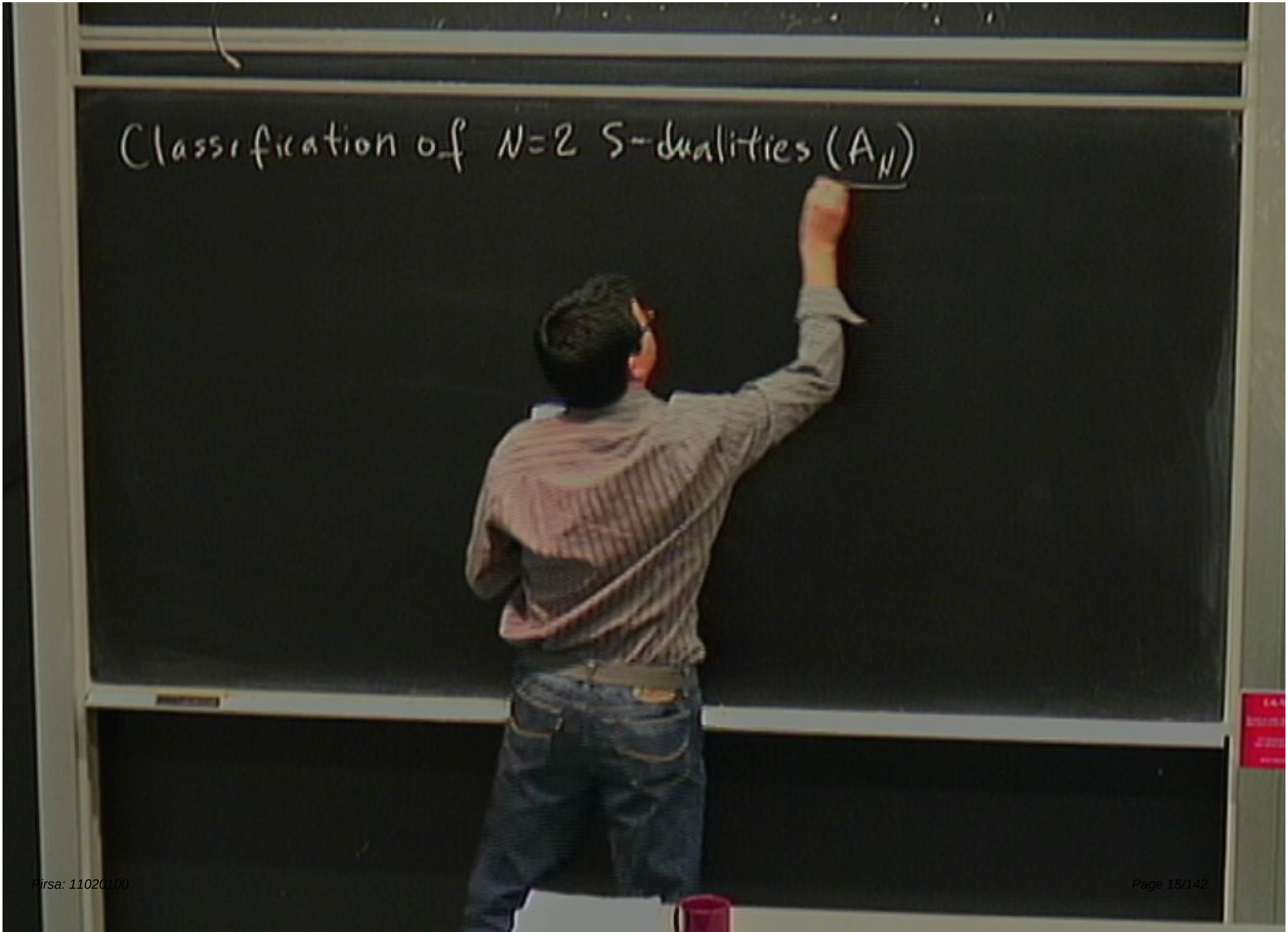
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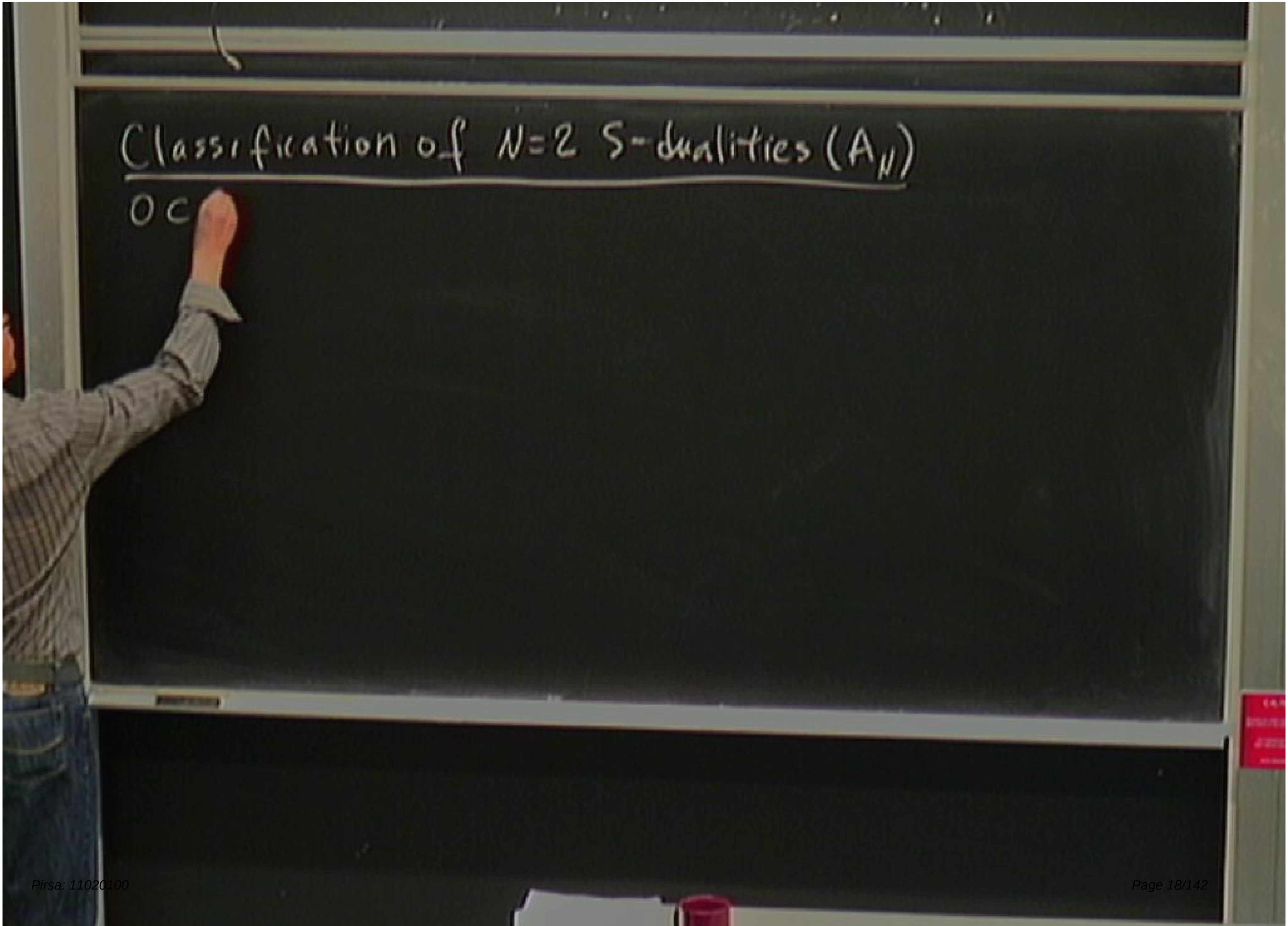
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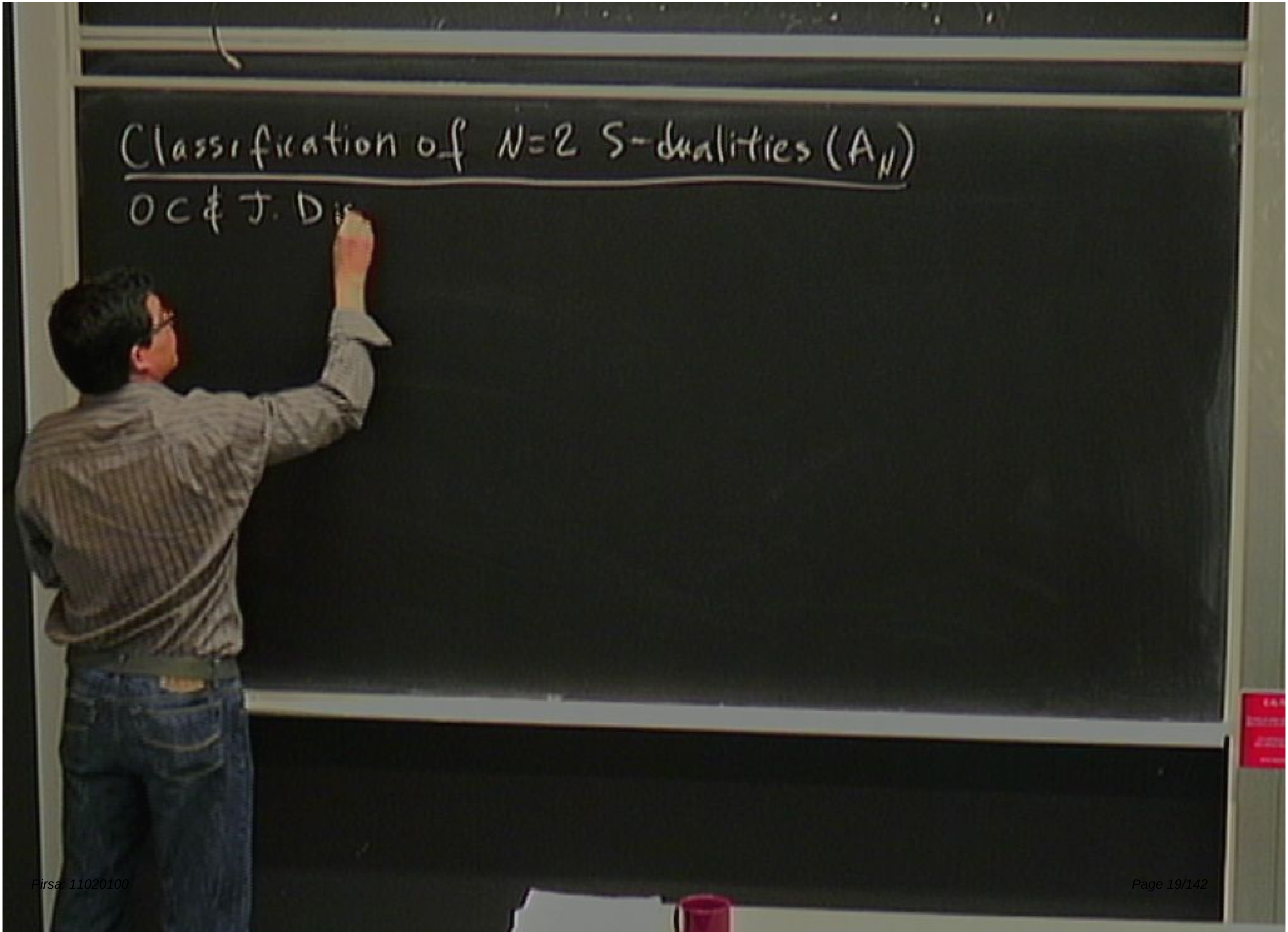
Classification of N=2 S-dualities (A_N)
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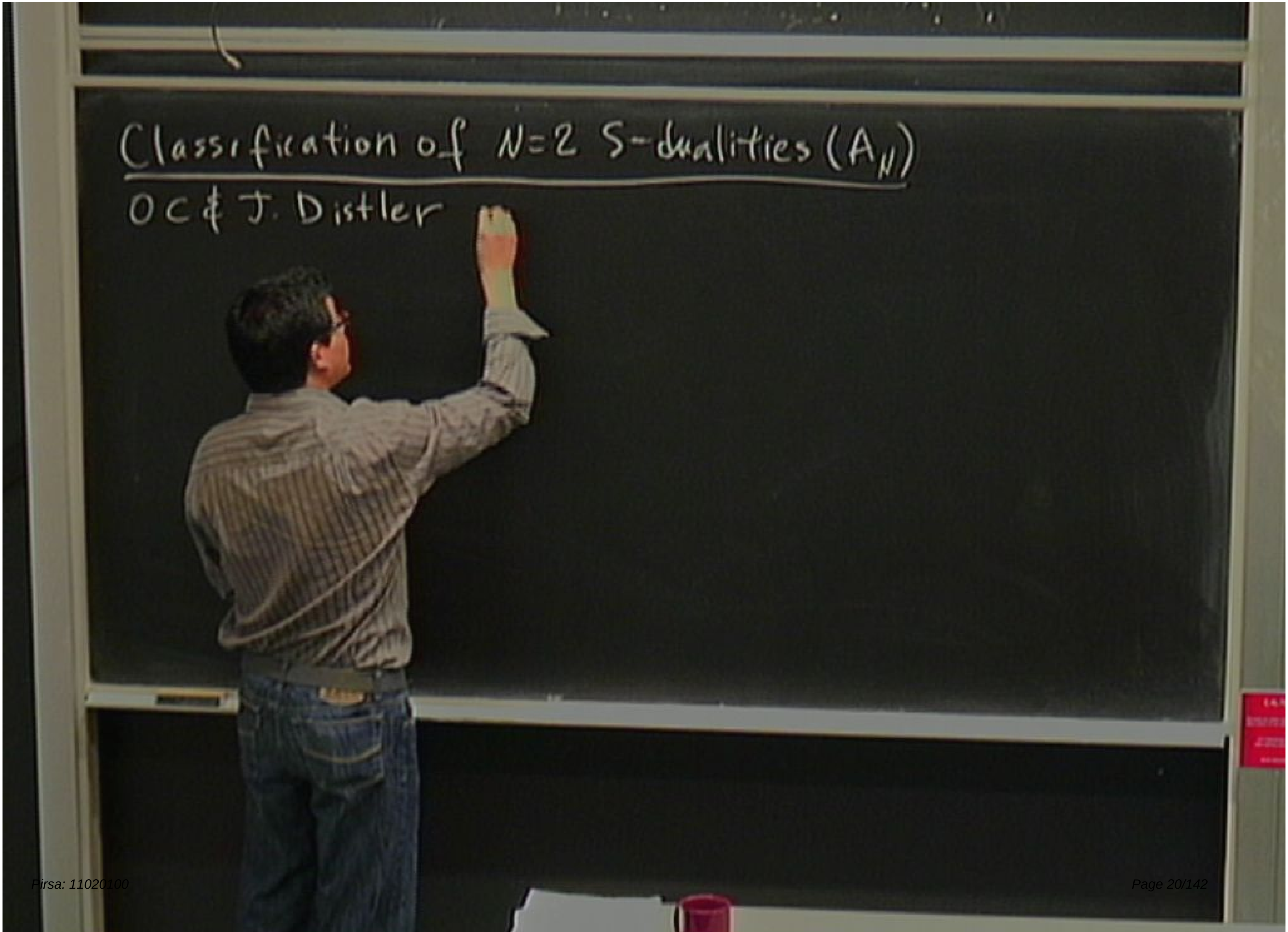
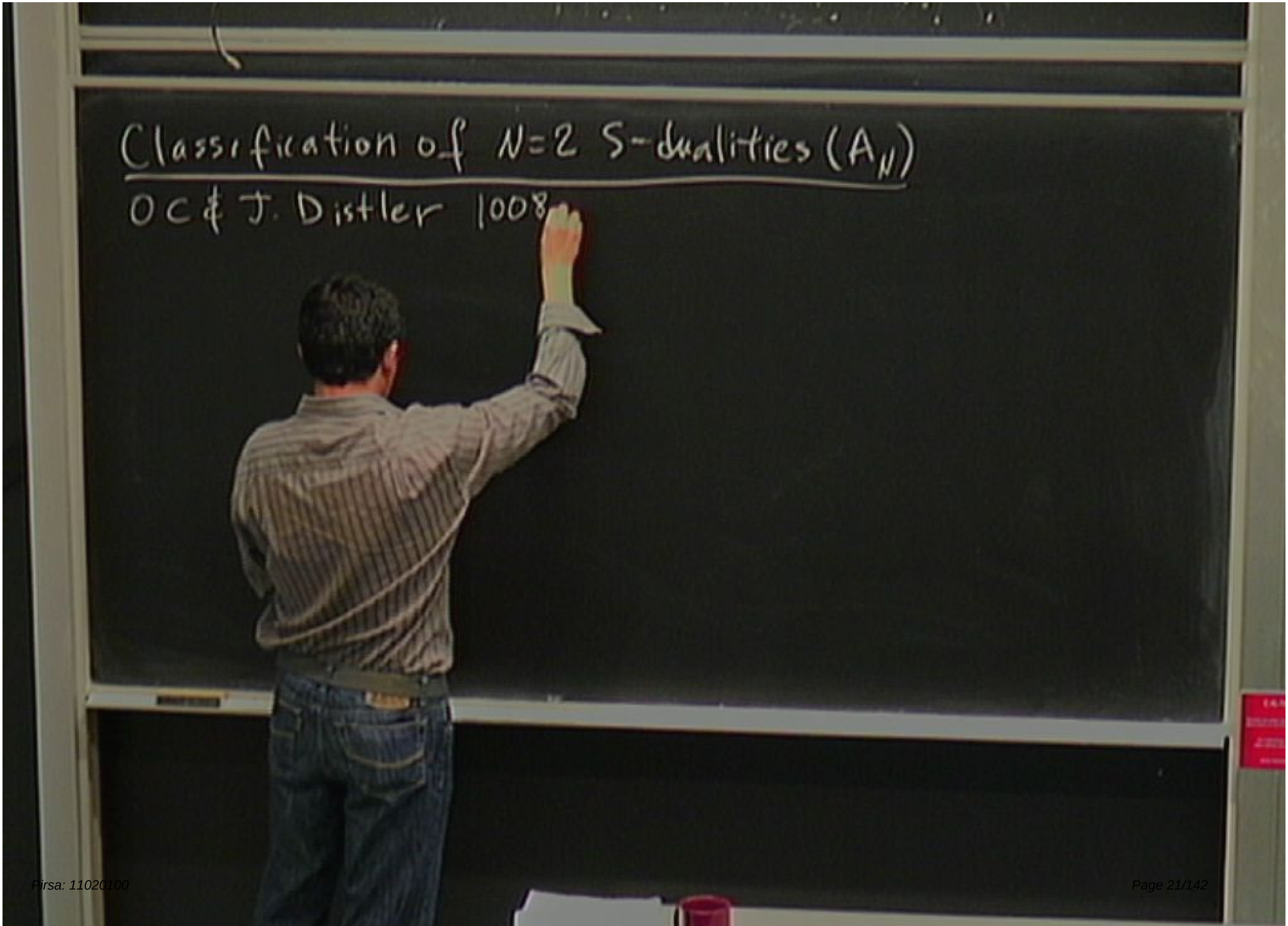
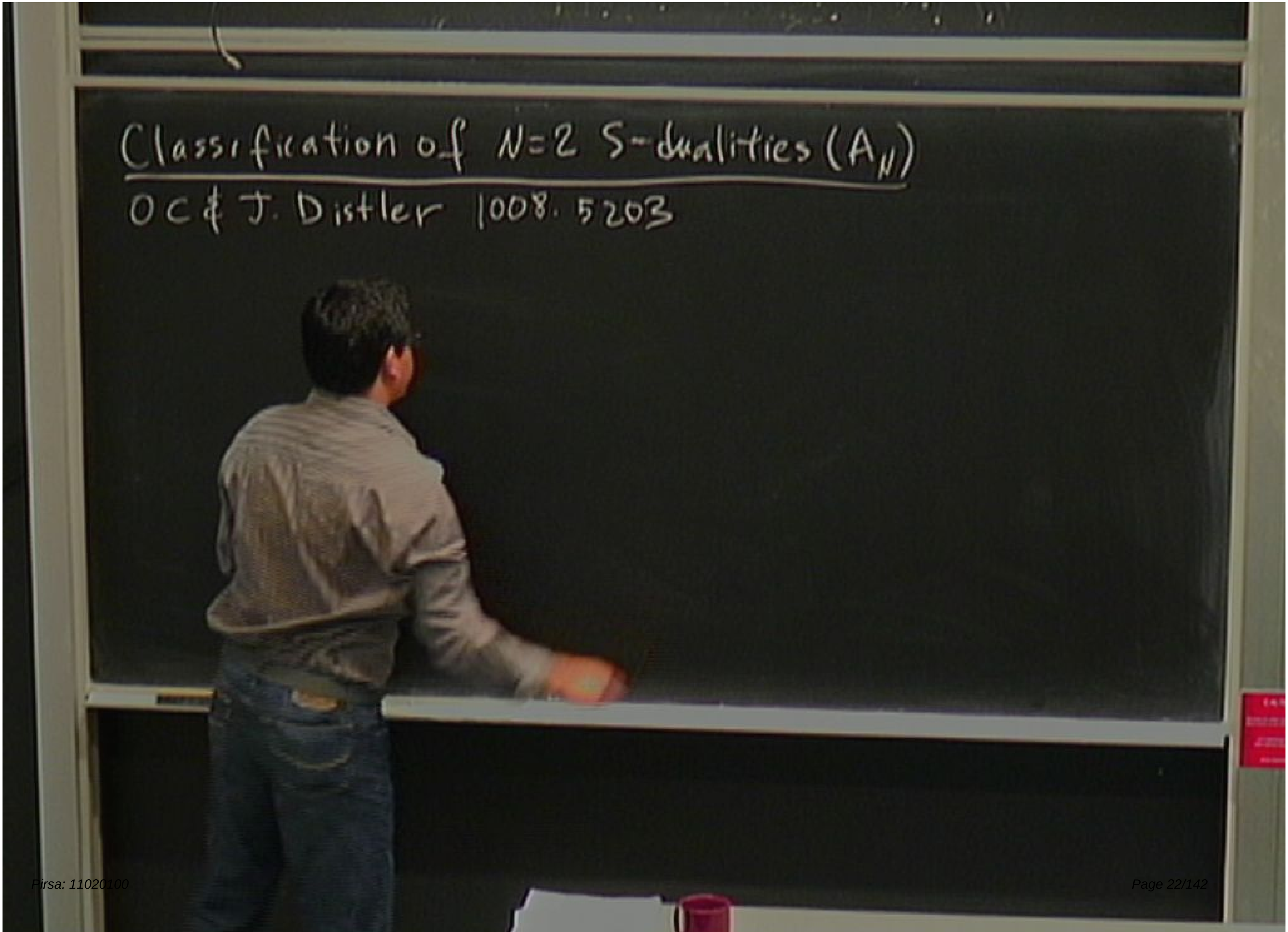

Classification of N=2 S-dualities (A_N)
OC & J. Distler


Classification of $\mathcal{N}=2$ S-dualities (A_N)
OC & J. Distler 1008


Classification of N=2 S-dualities (A_N)
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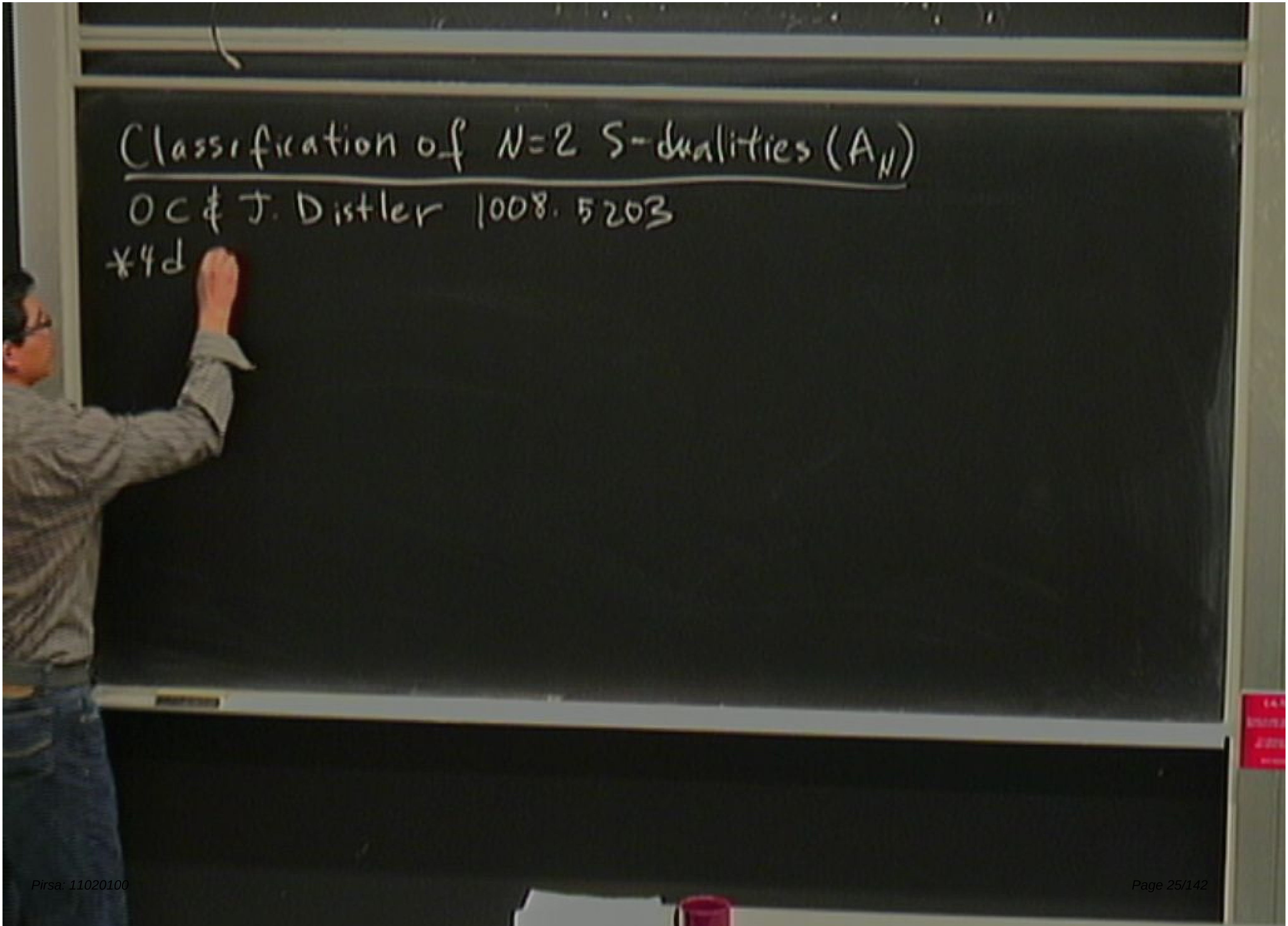
Classification of N=2 S-dualities (A_N)
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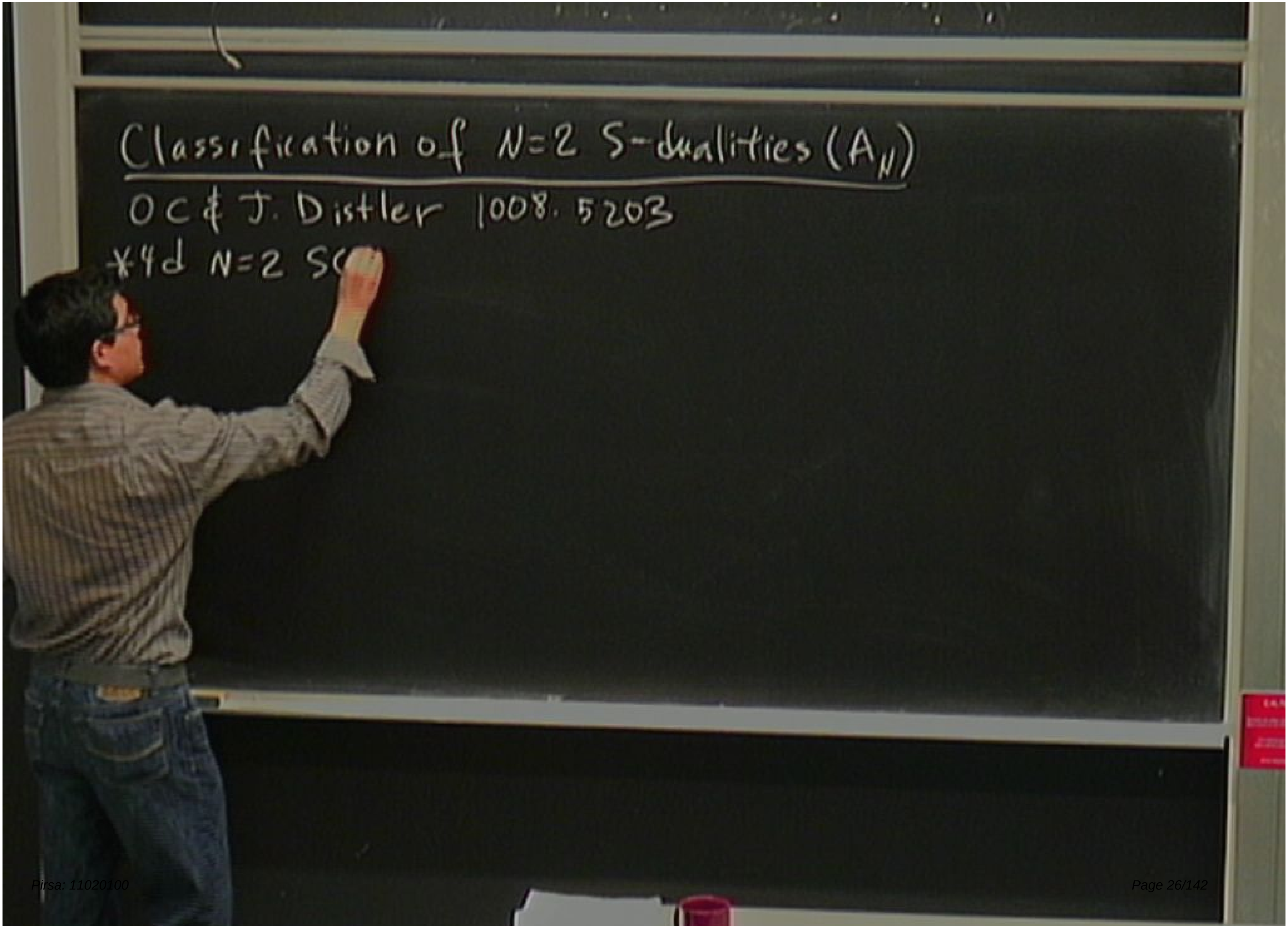
*4d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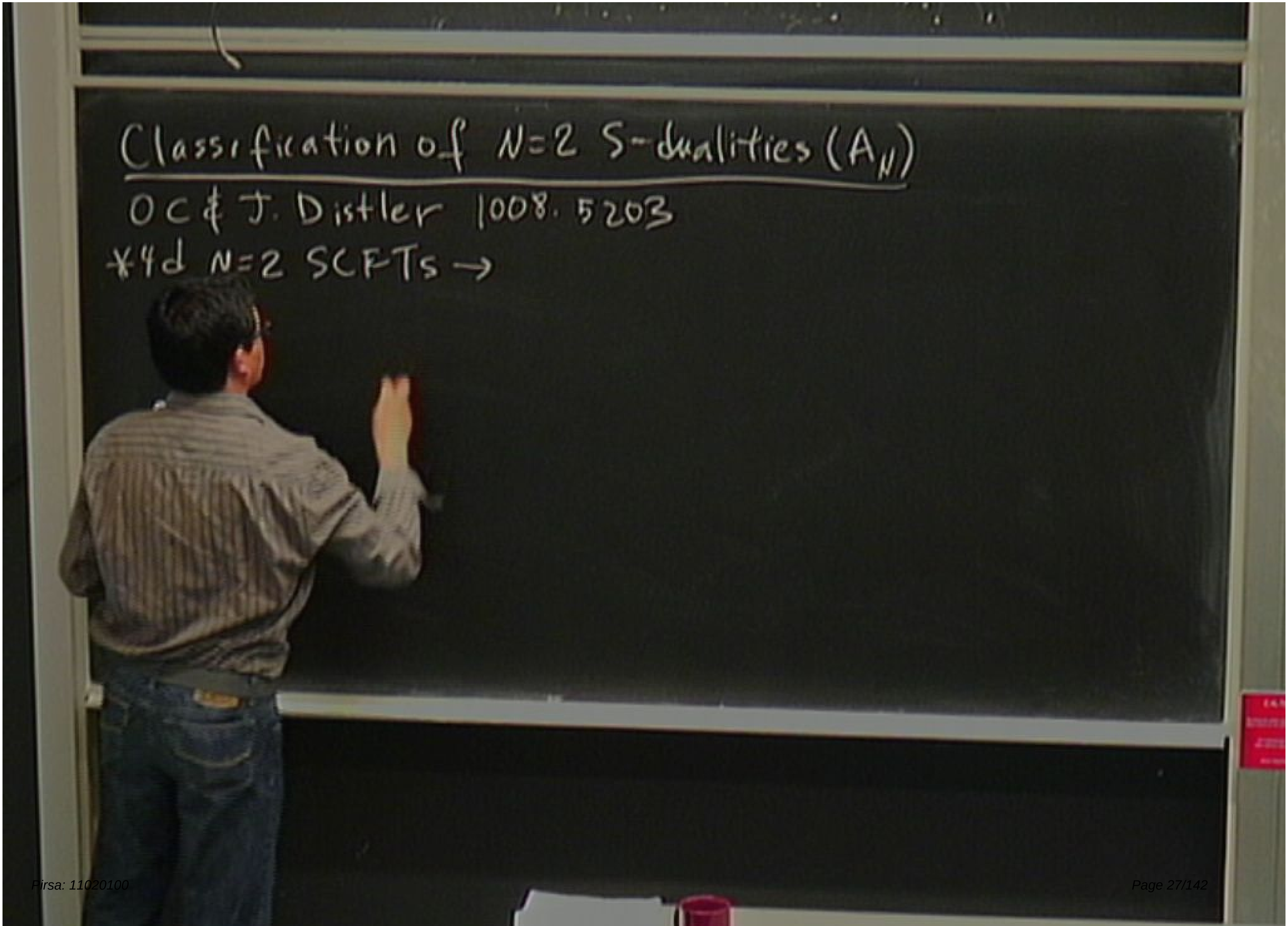
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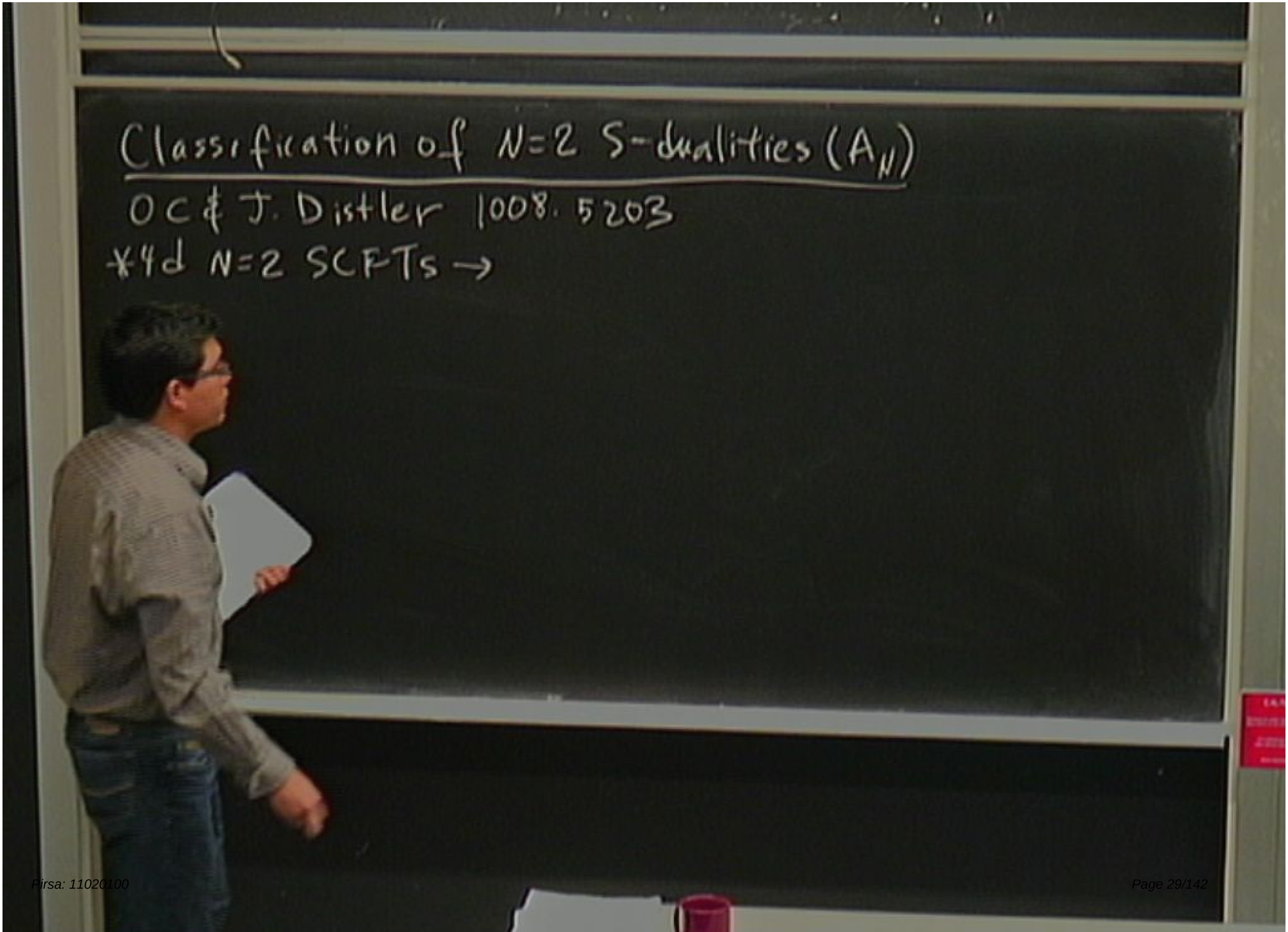
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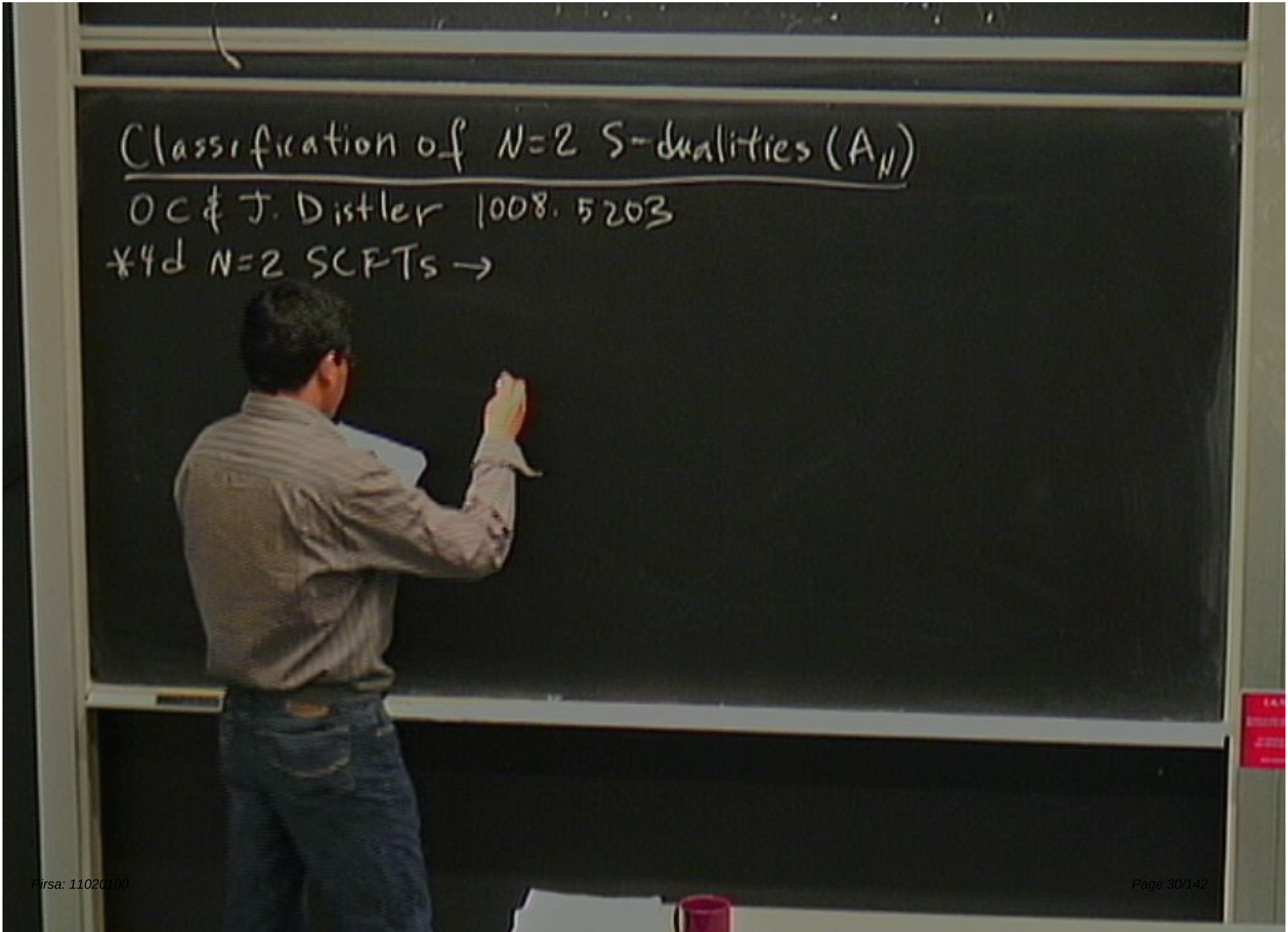

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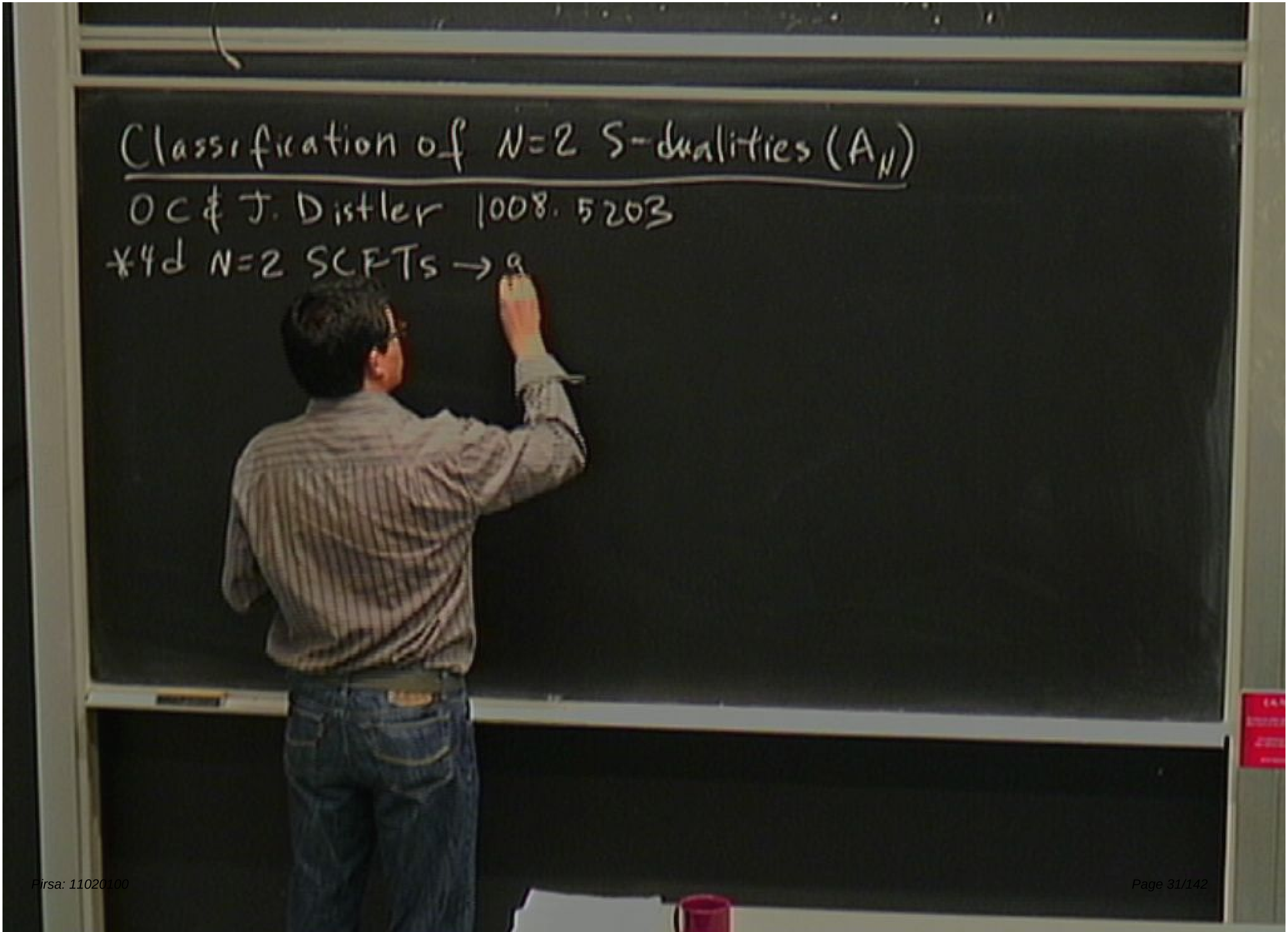
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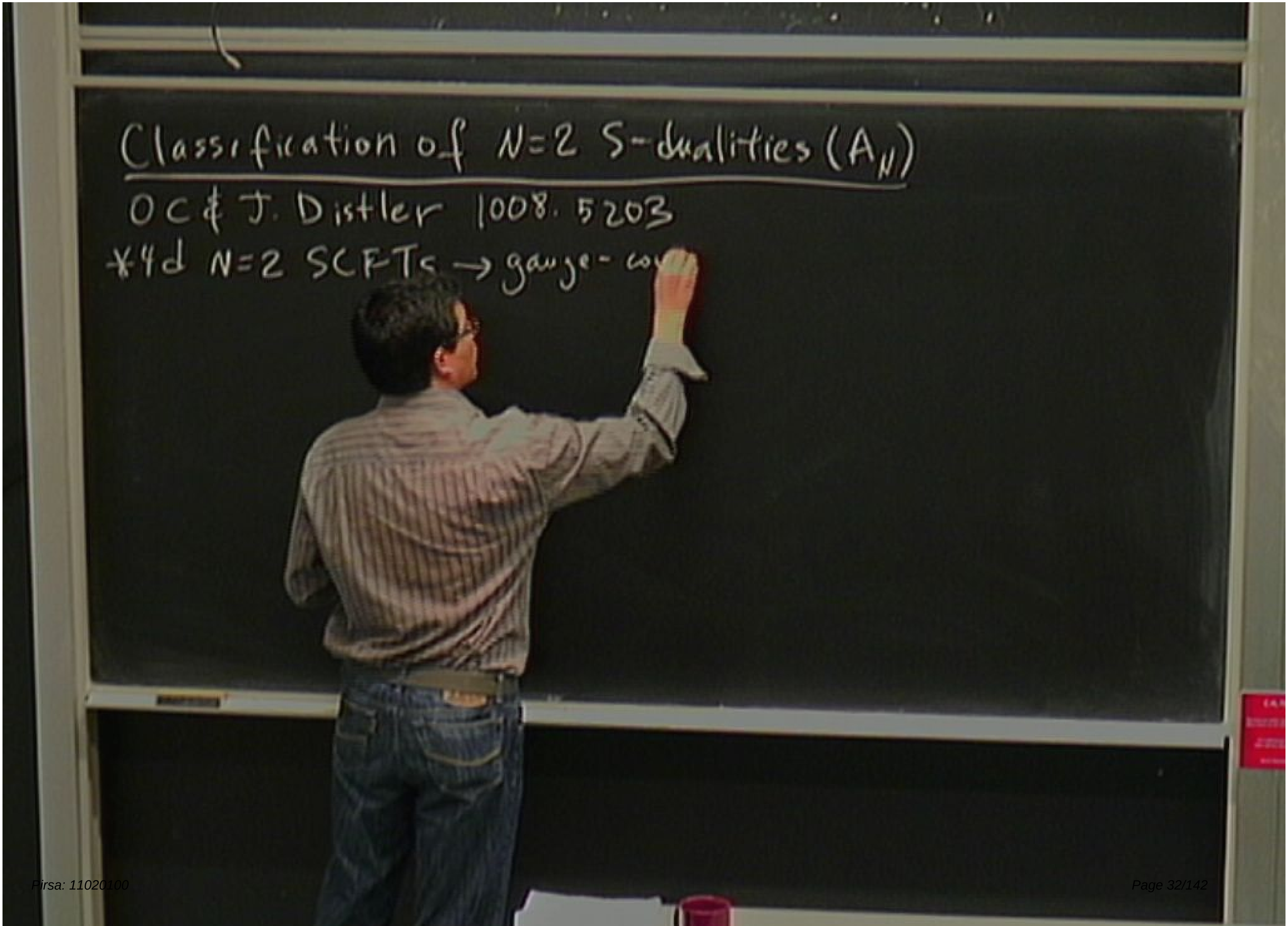
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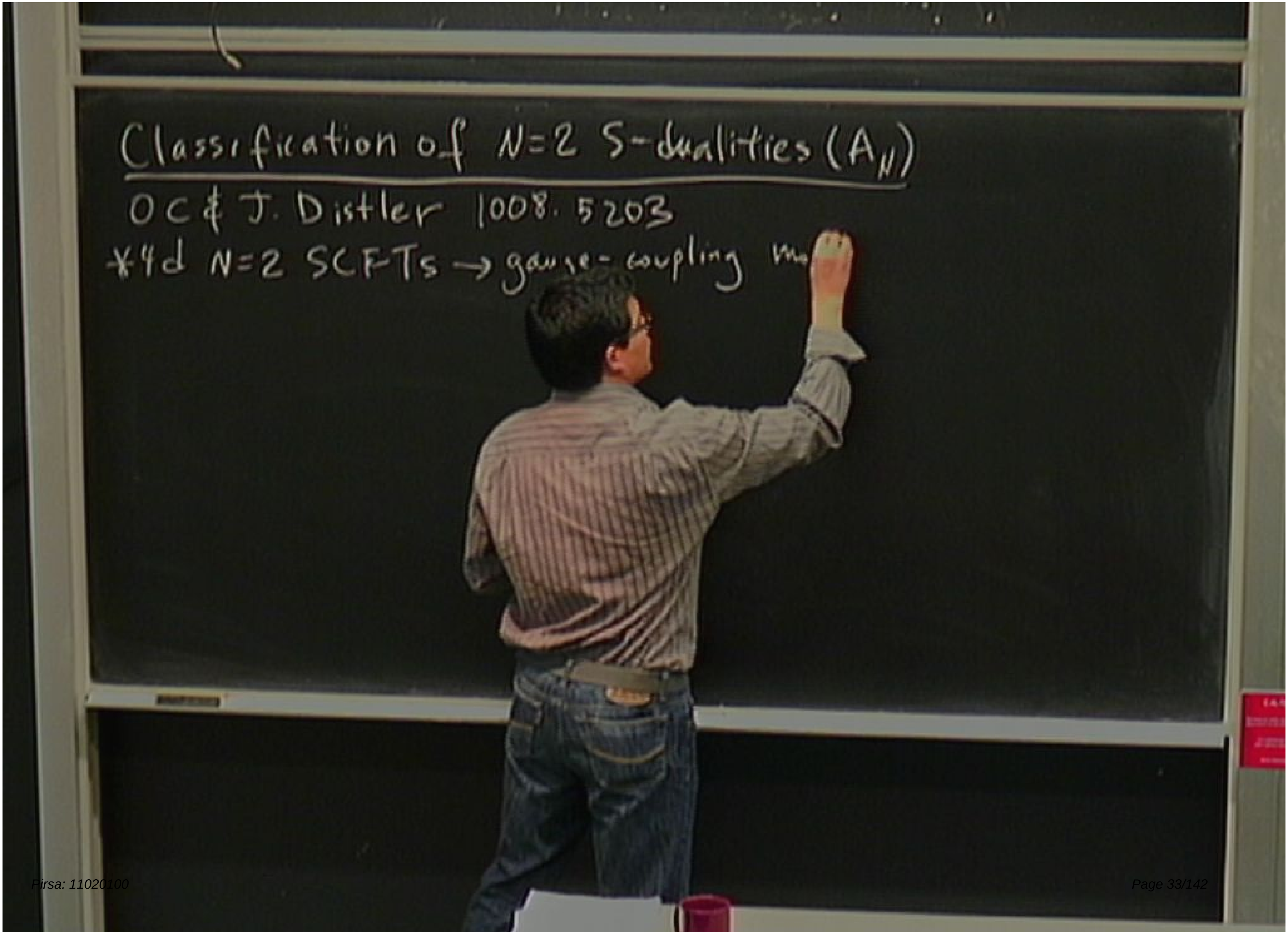
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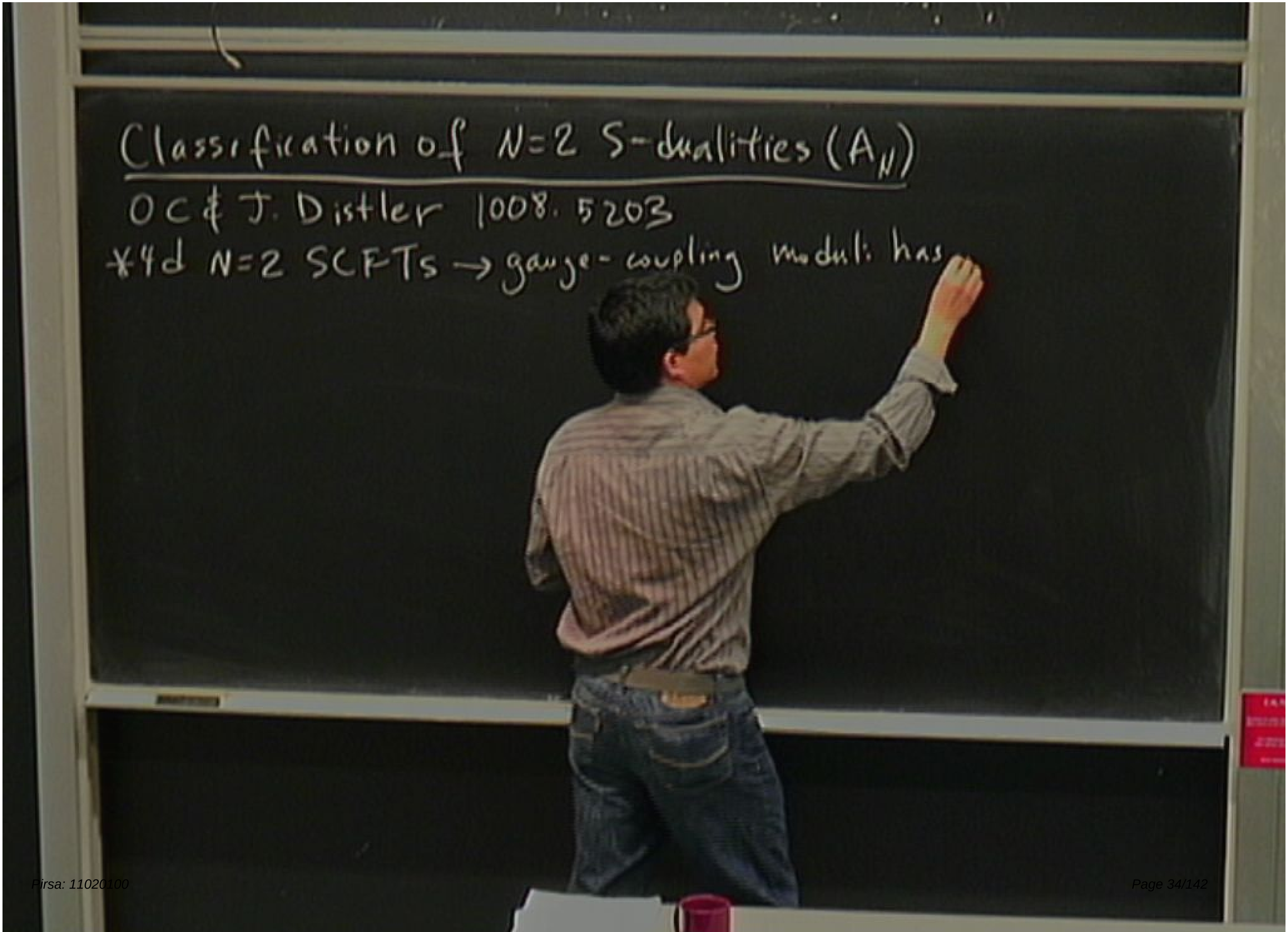
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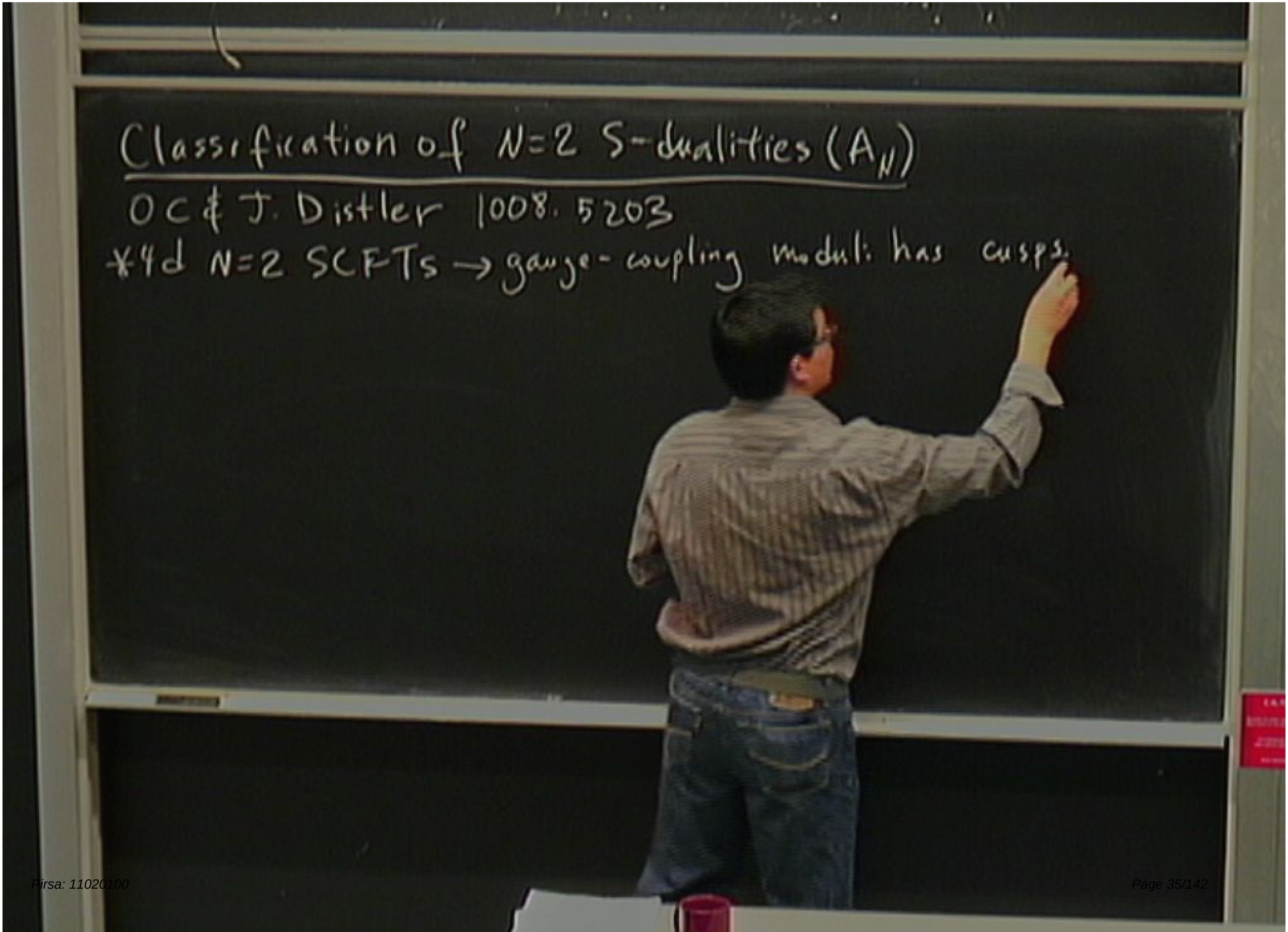
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* 4d N=2 SCFTs → gauge-coupling moduli has cusps.

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Classification of $\mathcal{N}=2$ S-dualities (A_N)

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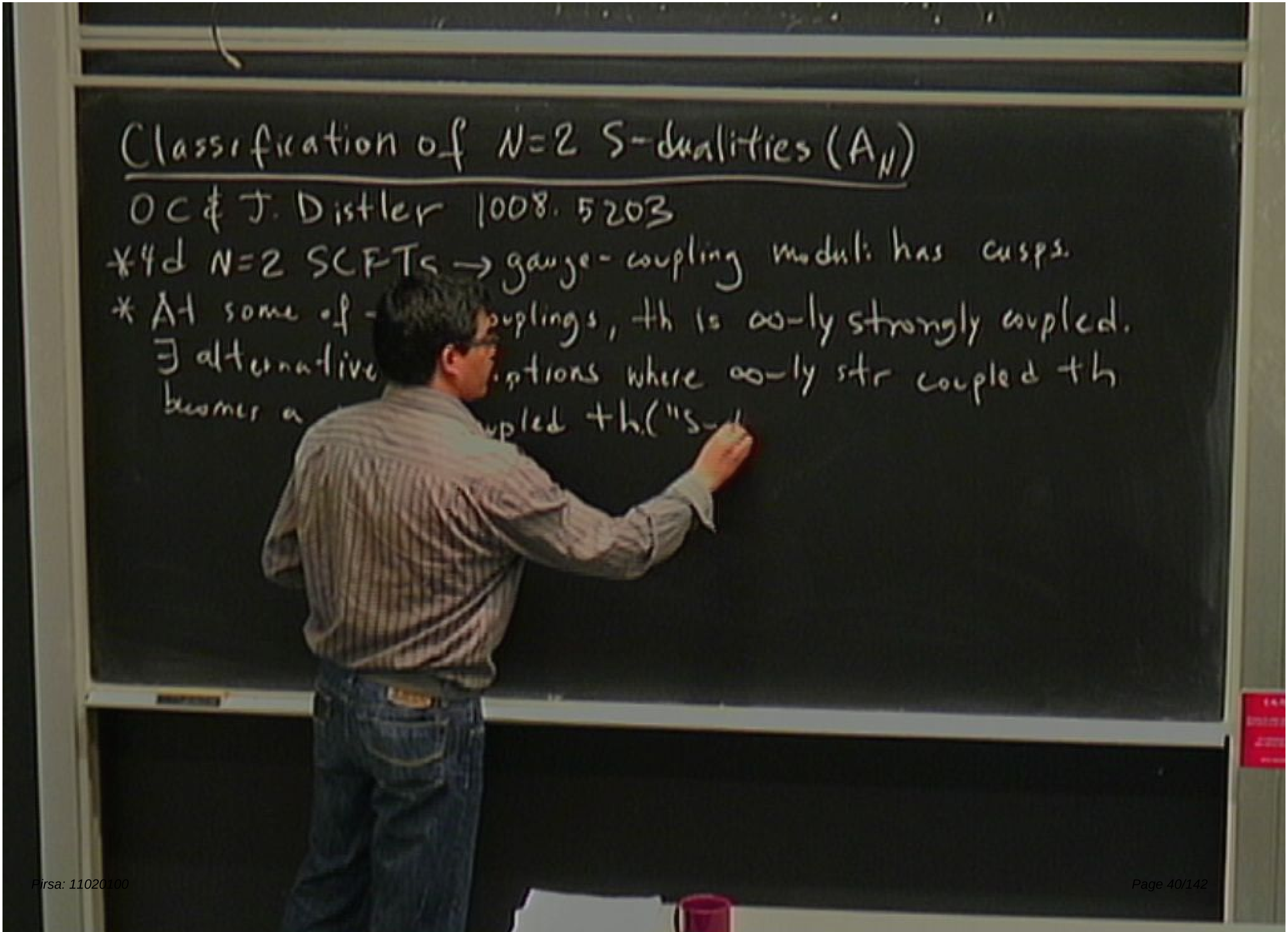
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Classification o[illegible]ualities (A_N)

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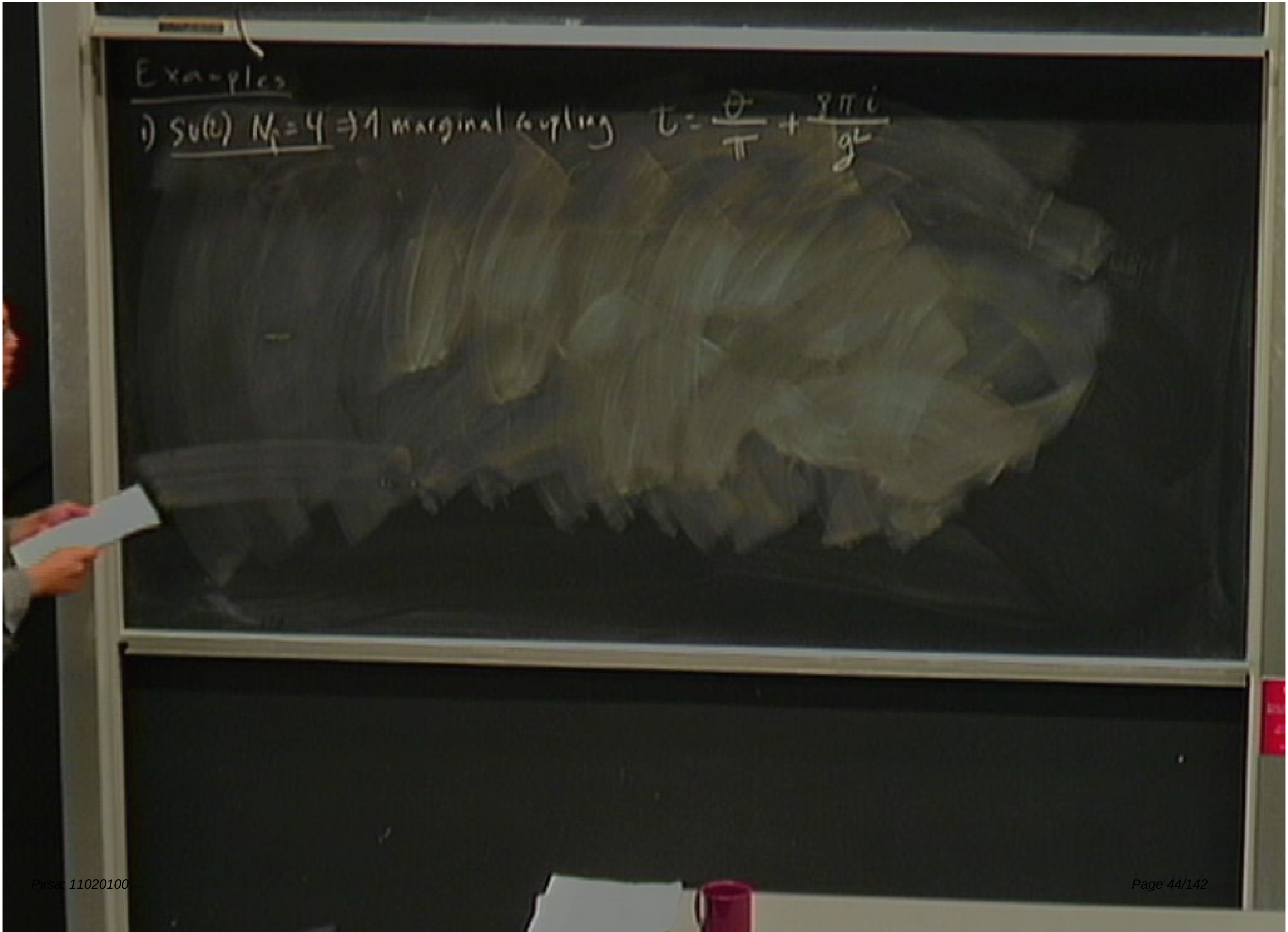
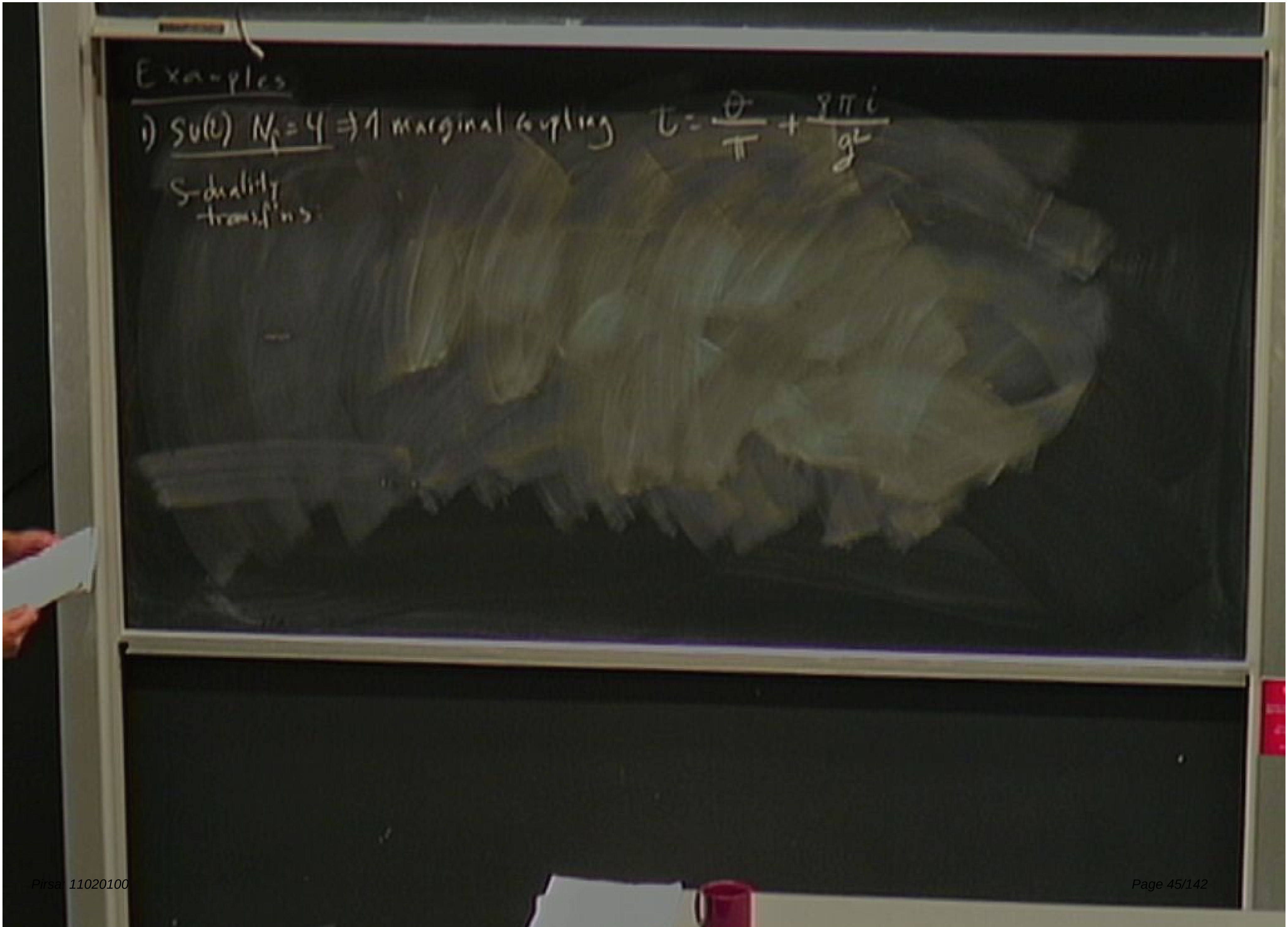
Examples

1) $SU(2)$ N_f

Examples

1) $SU(2)$ $N_f = 4$ $\Rightarrow$ 1 marginal coupling


Examples
1) SU(2) N_f = 4 ⇒ 1 marginal coupling τ = θ/π + 8πi/g²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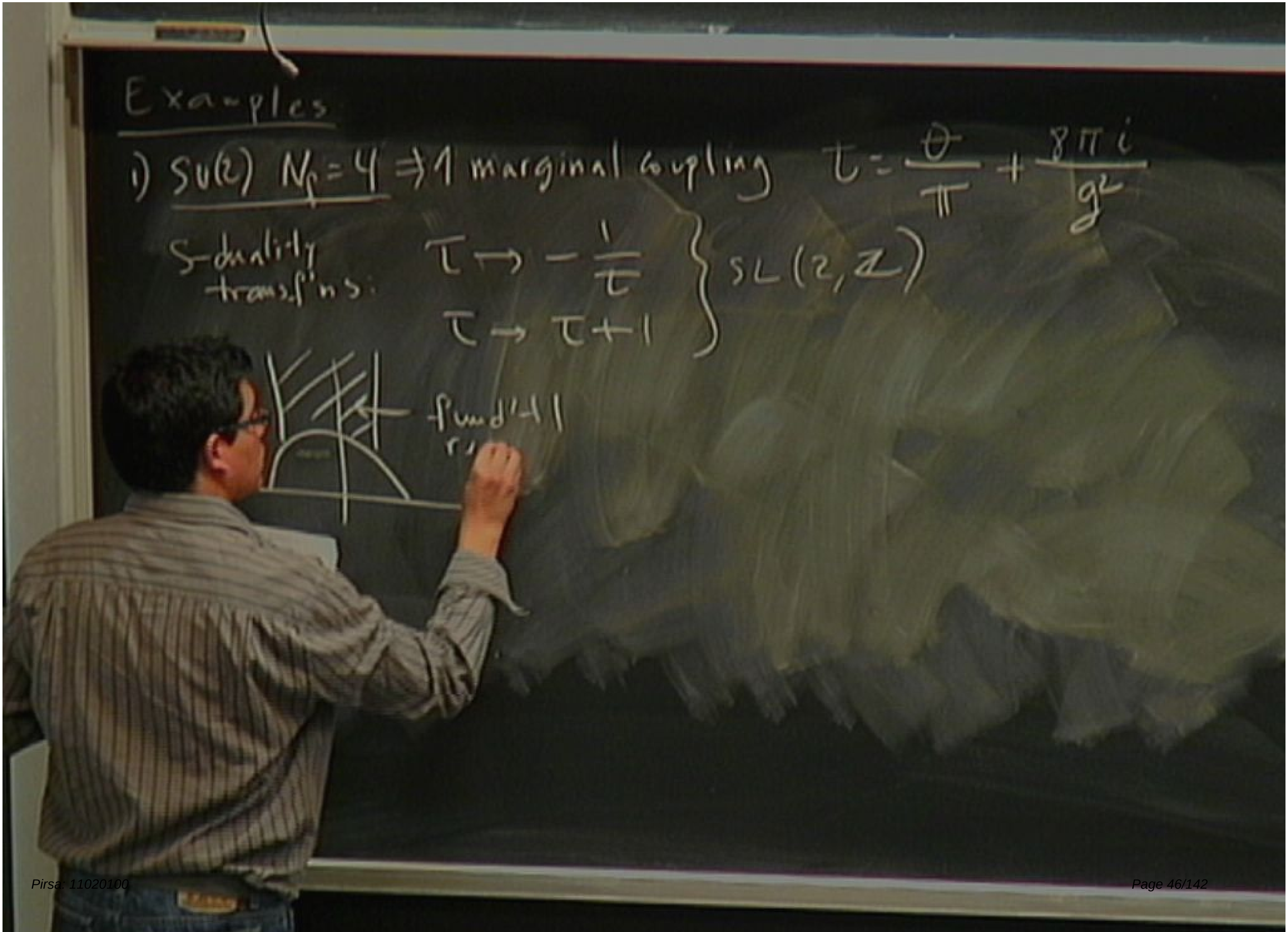
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1) SU(2) $N_f = 4$ ⇒ 1 marginal coupling $\tau = \frac{\theta}{\pi} + \frac{8\pi i}{g^2}$
S-duality transf'ns: $\tau \mapsto -\frac{1}{\tau}$, $\tau \to \tau + 1$ $\}$ $SL(2,\mathbb{Z})$
fund'l

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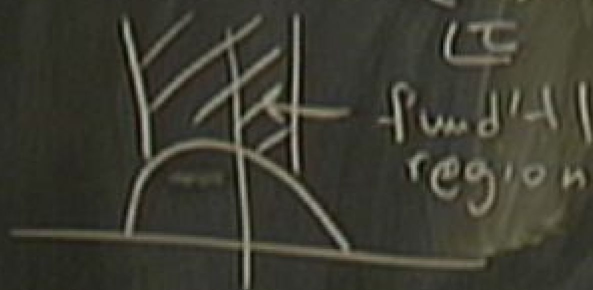
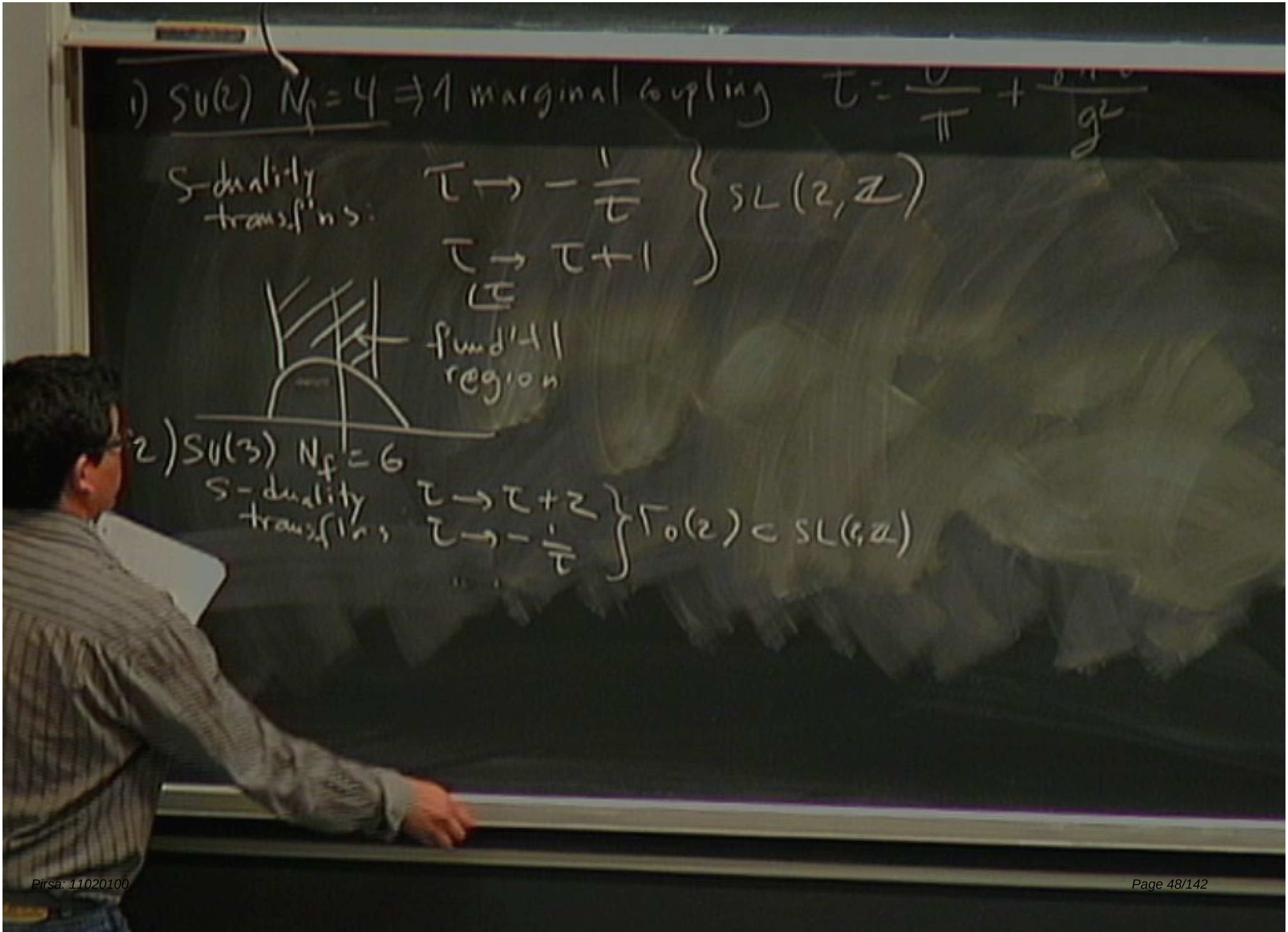
S-duality transf'ns: $\left.\begin{array}{l} \tau \to -\frac{1}{\tau} \\ \tau \to \tau + 1 \end{array}\right\} SL(2,\mathbb{Z})$

$\mathbb{C}\tau$





1) SU(2) N_f = 4 ⇒ 1 marginal coupling τ = θ/π + …/g²
S-duality transf'ns: τ → −1/τ, τ → τ+1 } SL(2,Z)
fund'tl region
2) SU(3) N_f = 6
S-duality transf'ns τ → τ+2, τ → −1/τ } Γ₀(2) ⊂ SL(2,Z)

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$\Im\tau$

fund'l region

2) SU(3) $N_f = 6$

S-duality transf'ns $\left.\begin{array}{l} \tau \to \tau + 2 \\ \tau \to -\frac{1}{\tau} \end{array}\right\} \Gamma_0(2) \subset SL(2,\mathbb{Z})$

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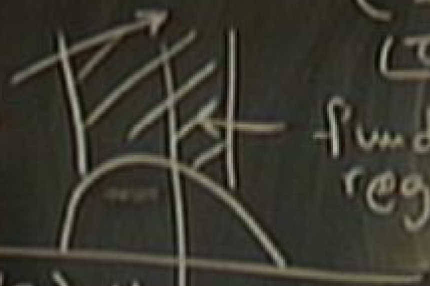
$\llcorner \tau$

weakly coupled



fund'tl region

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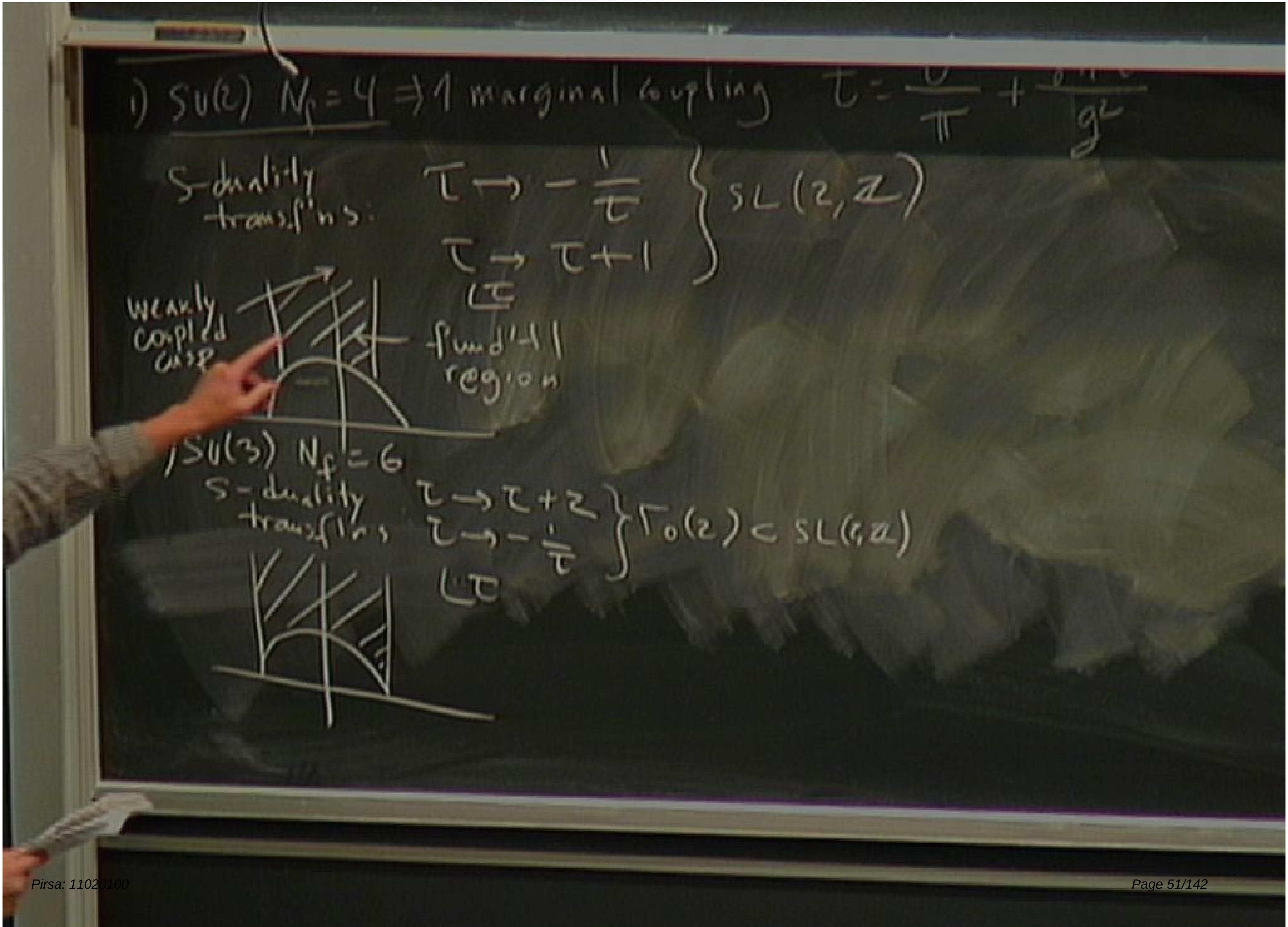
1) SU(2) Nf = 4 ⇒ 1 marginal coupling
S-duality transf'ns:
τ → − 1/τ
τ → τ + 1
SL(2, Z)
weakly coupled cusp
fund'l region
SU(3) Nf = 6
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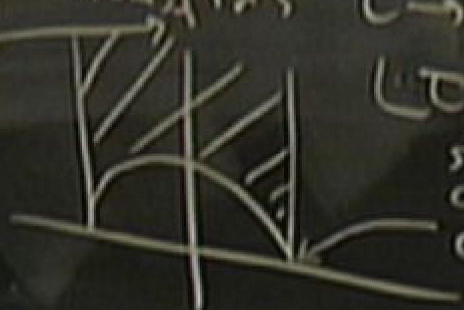
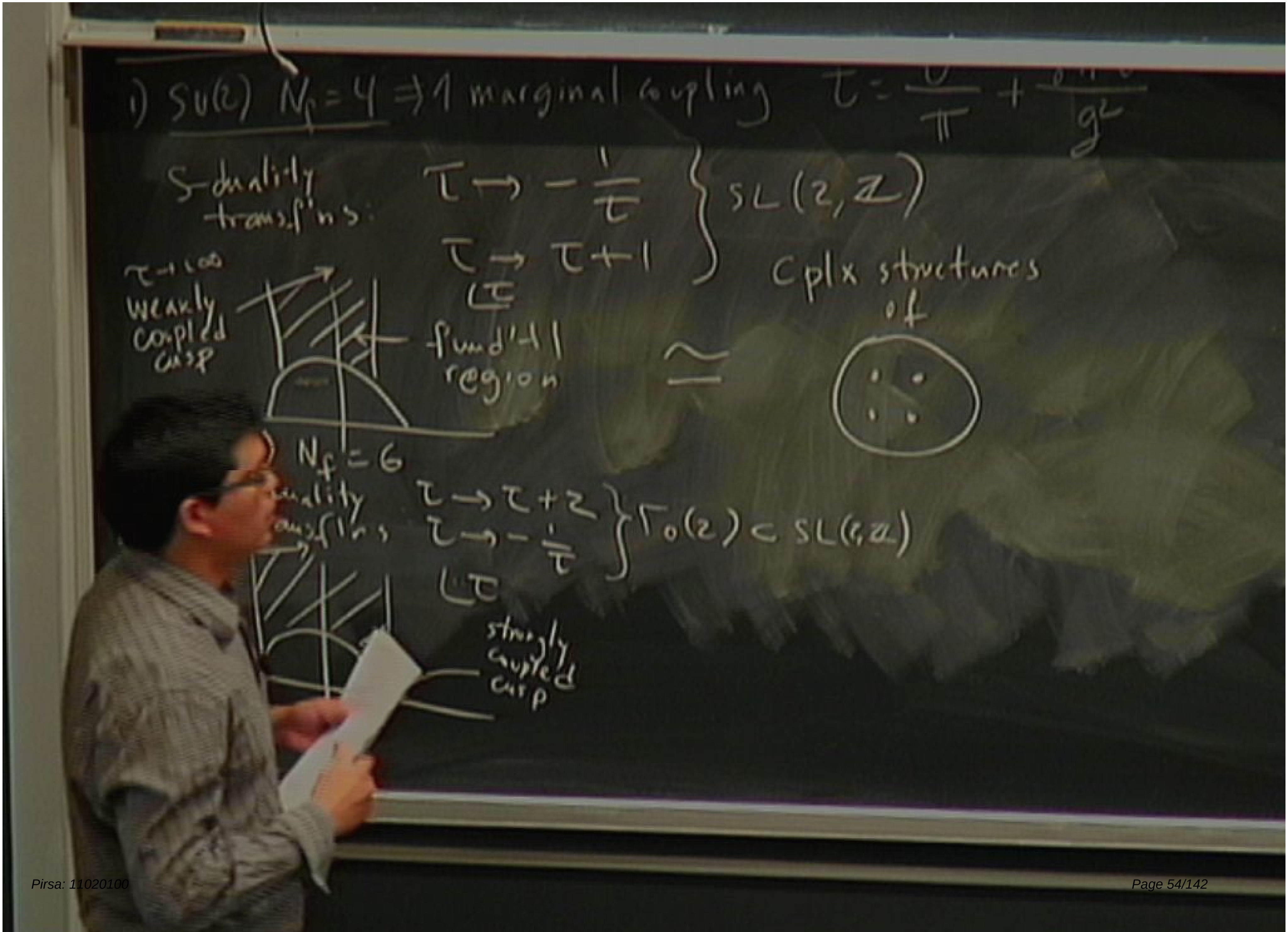


strongly coupled cusp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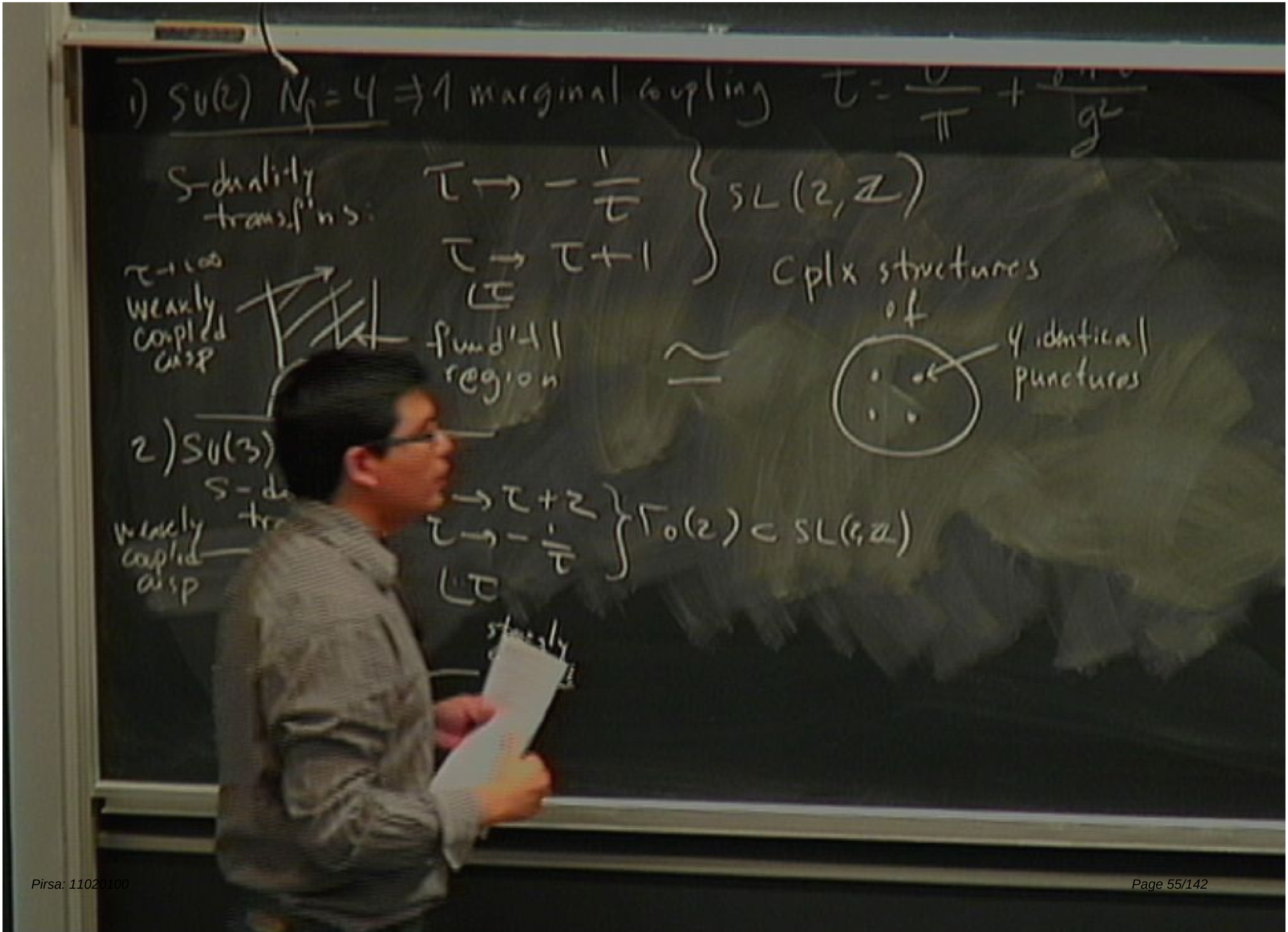
1) SU(2) N_f = 4 ∃ 1 marginal coupling
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SL(2,Z)
Cplx structures of
4 identical punctures
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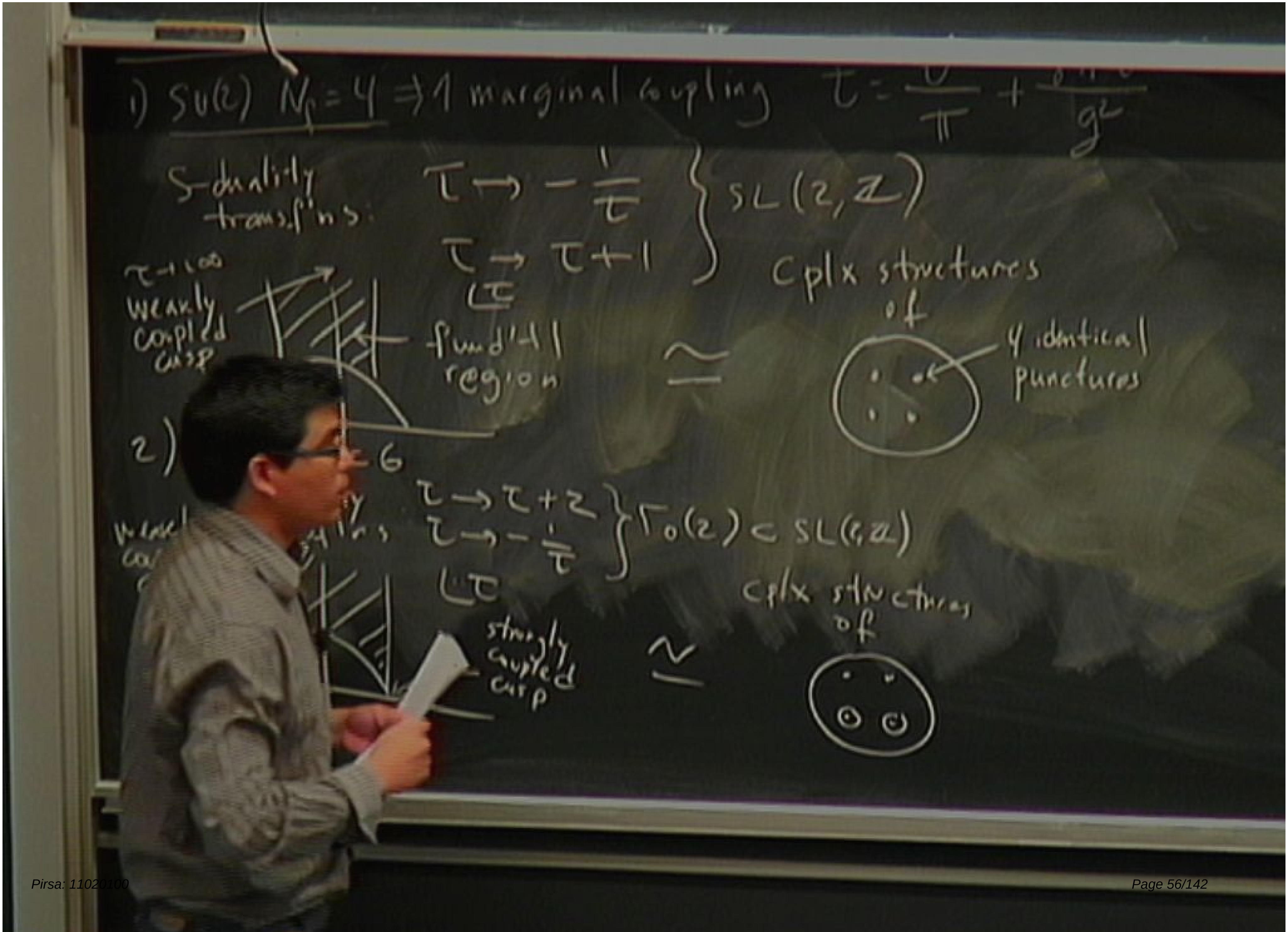
fund'tl region
2) SU(3)
τ → τ+2
Γ_0(2) ⊂ SL(2,Z)
weakly coupled cusp



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$\subset \tau$

$\tau \to i\infty$ weakly coupled cusp

fund'tl region $\simeq$ Cplx structures of

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2) SU(3) $N_f = 6$

S-duality transf'ns

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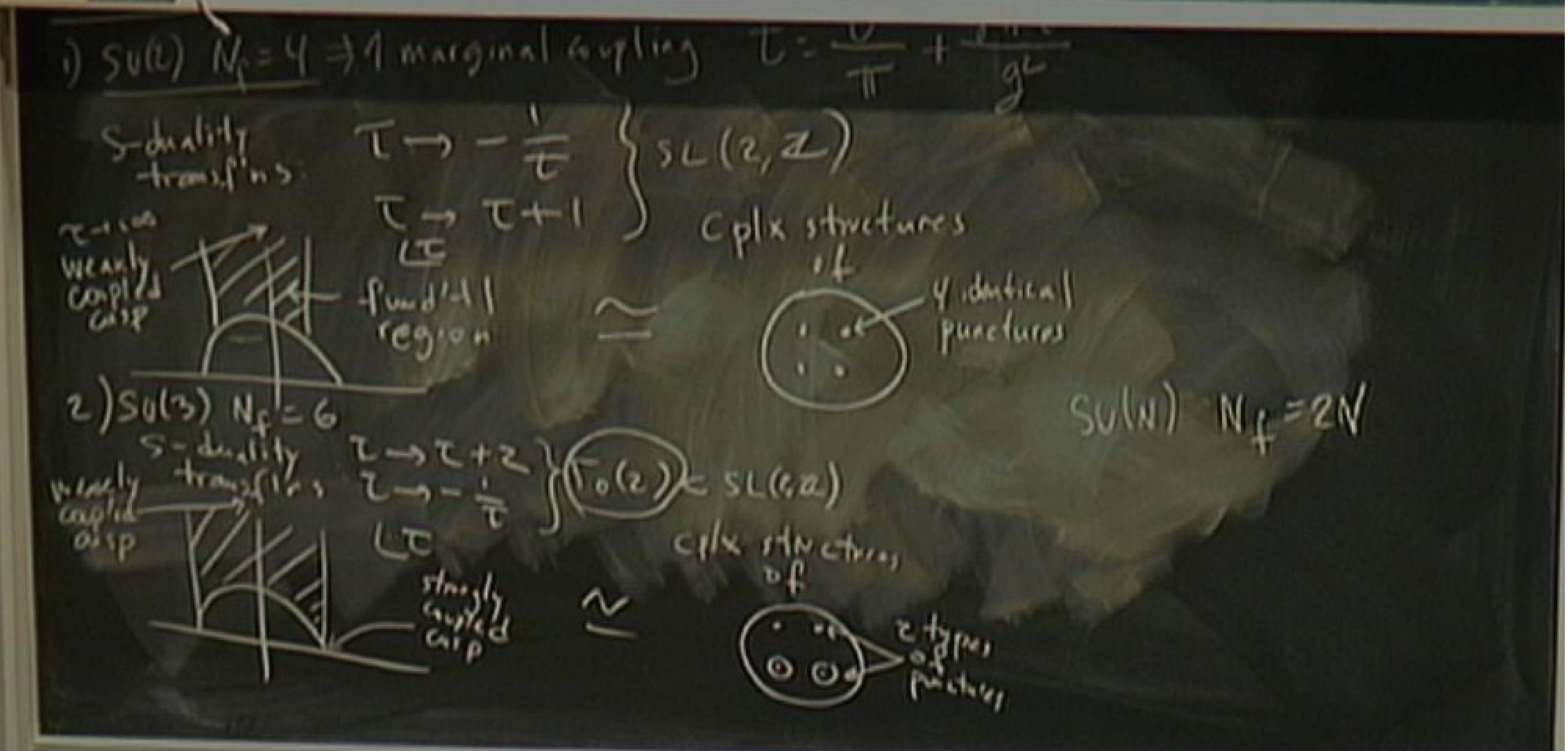
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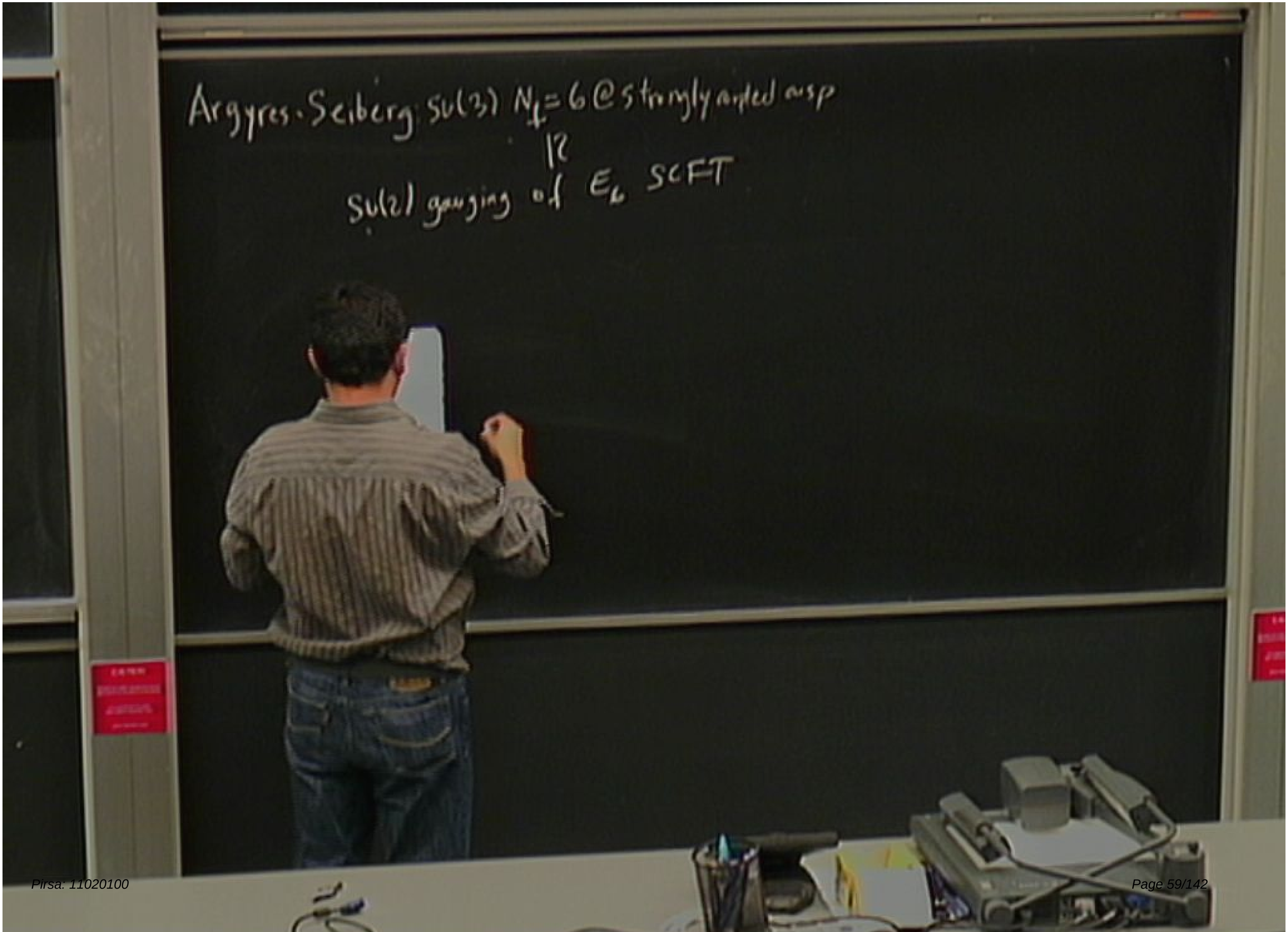
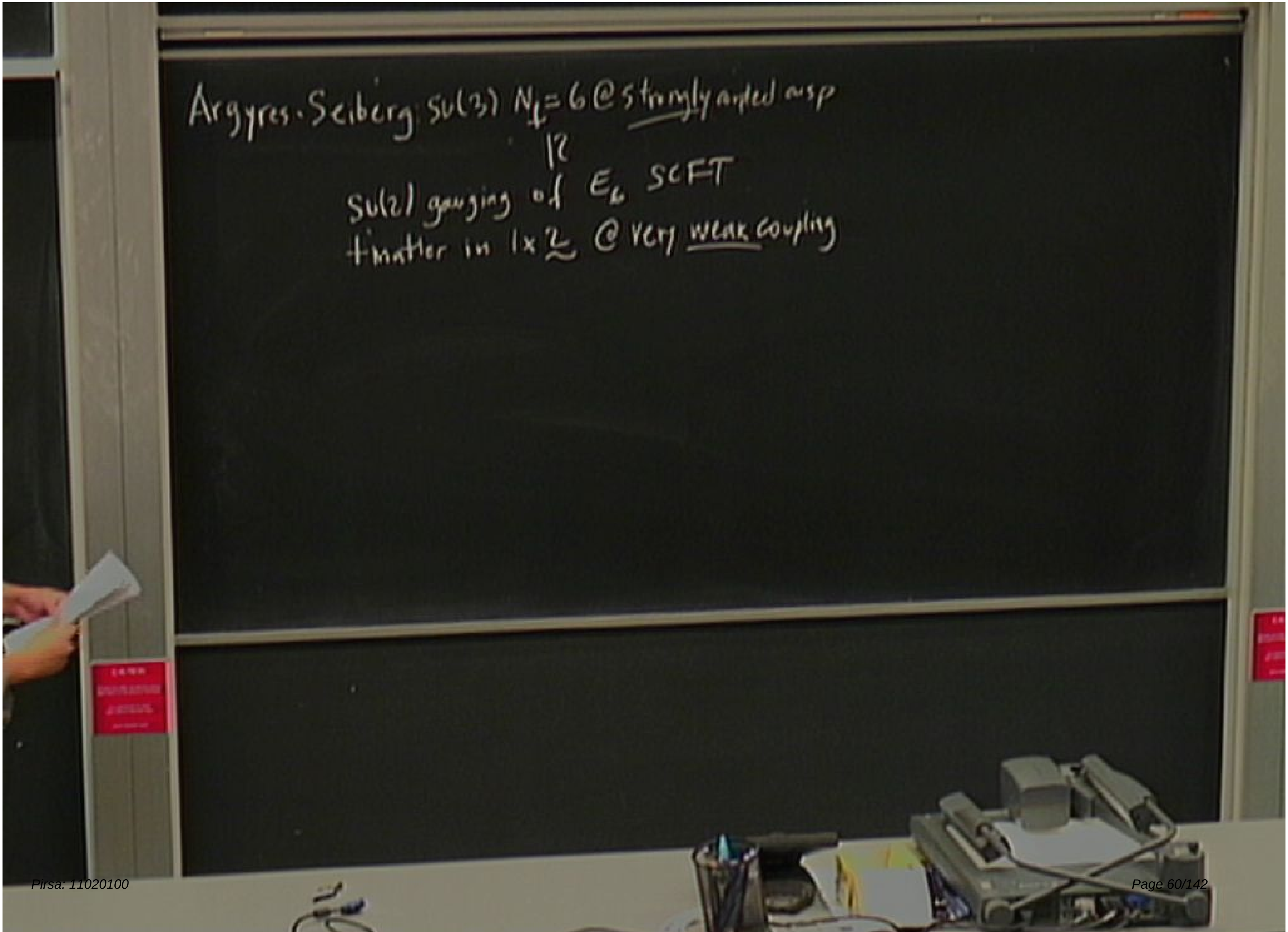
SU(N) N_f = 2N


Argyres-Seiberg: SU(3) $N_f = 6$ @ strongly coupled cusp
≅
SU(2) gauging of E_6 SCFT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$\cong$
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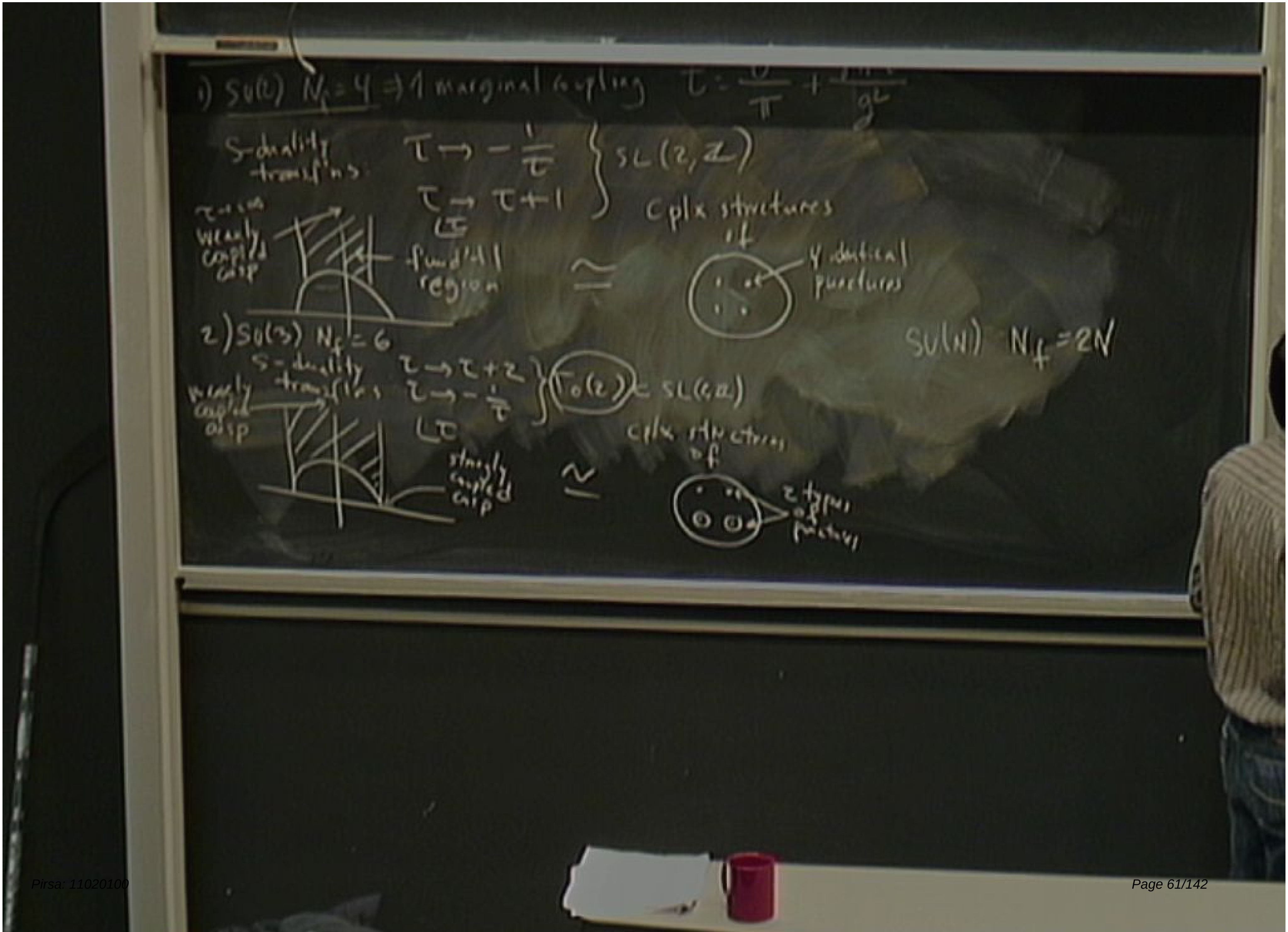
+ matter in $1 \times \underset{\sim}{2}$ @ very weak coupling



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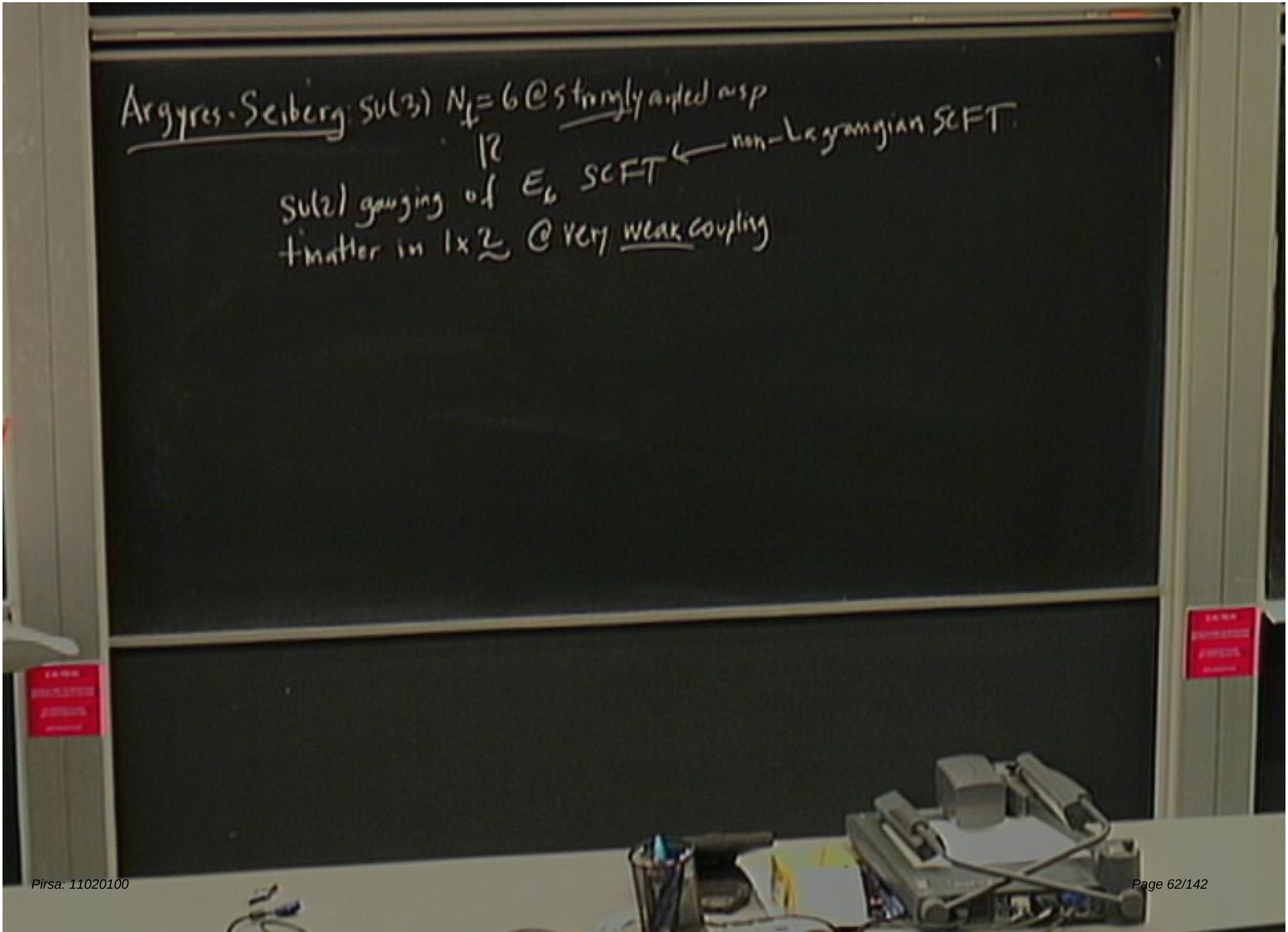
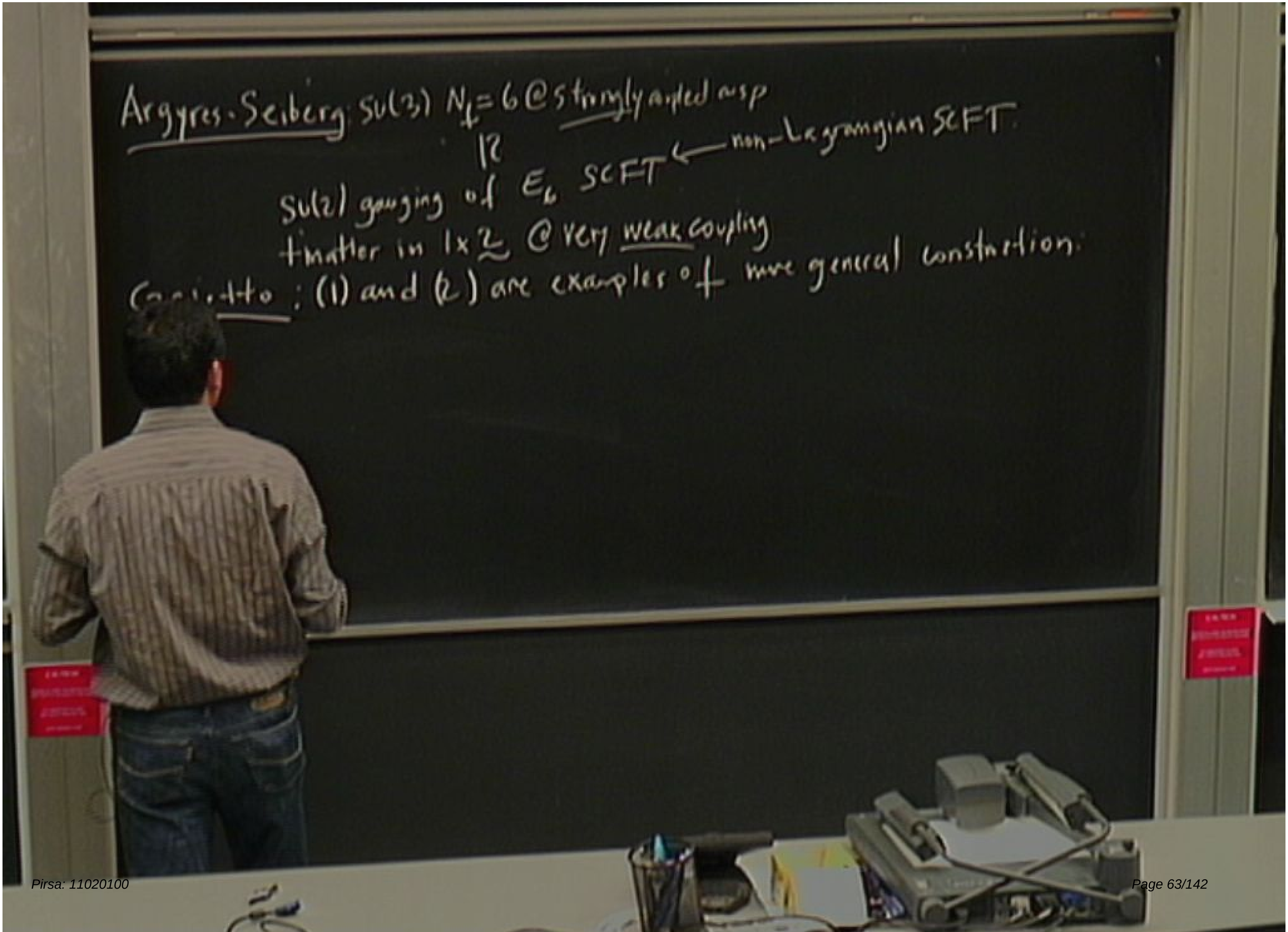
2 types of punctures
SU(N) N_f = 2N


Argyres-Seiberg: SU(3) $N_f = 6$ @ strongly coupled cusp
≅
SU(2) gauging of E_6 SCFT ← non-Lagrangian SCFT
+ matter in $1 \times \underset{\sim}{2}$ @ very weak coupling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$\cong$
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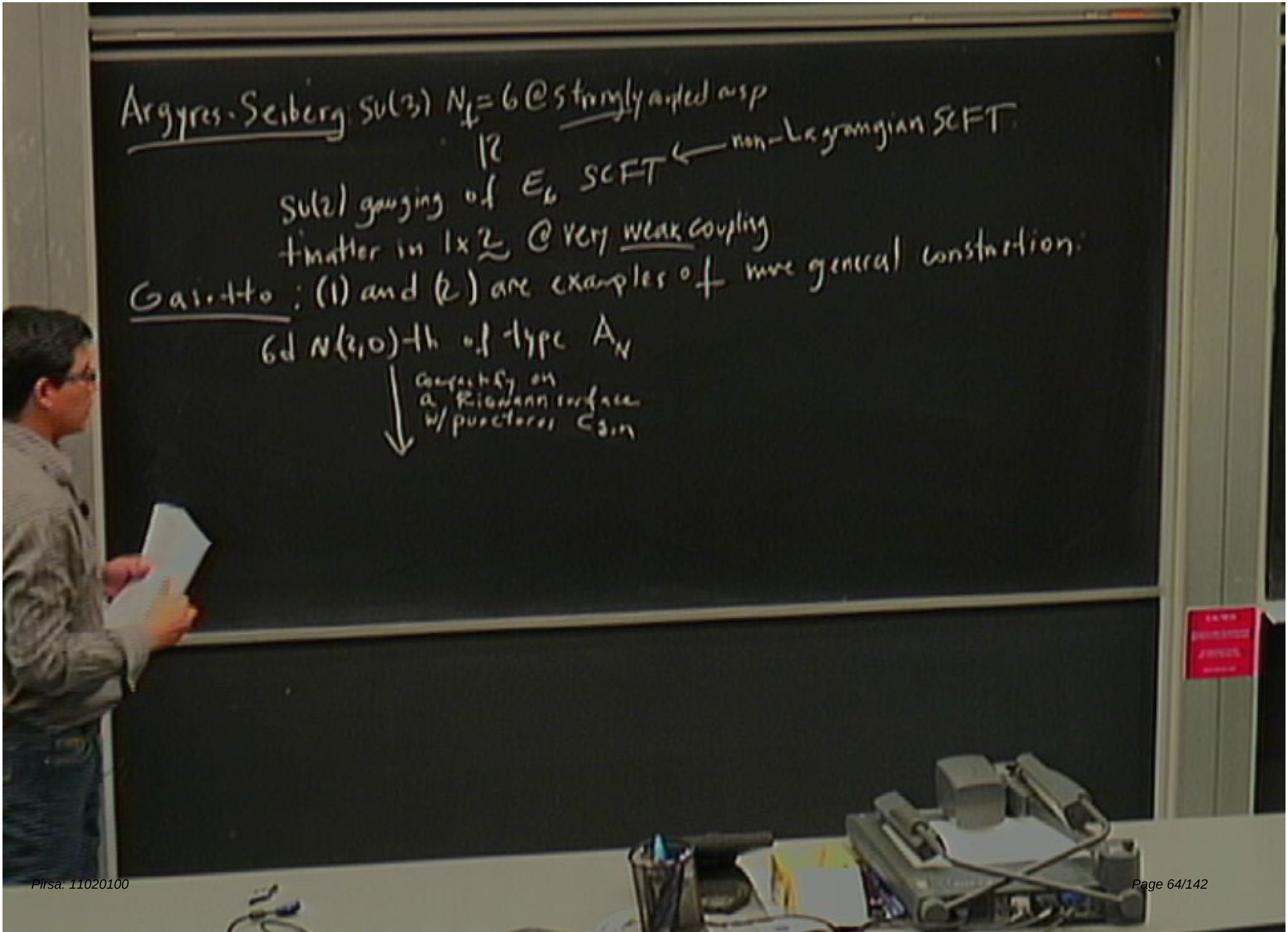
Gaiotto: (1) and (2) are examples of more general construction.


Argyres-Seiberg: SU(3) N_f = 6 @ strongly coupled cusp
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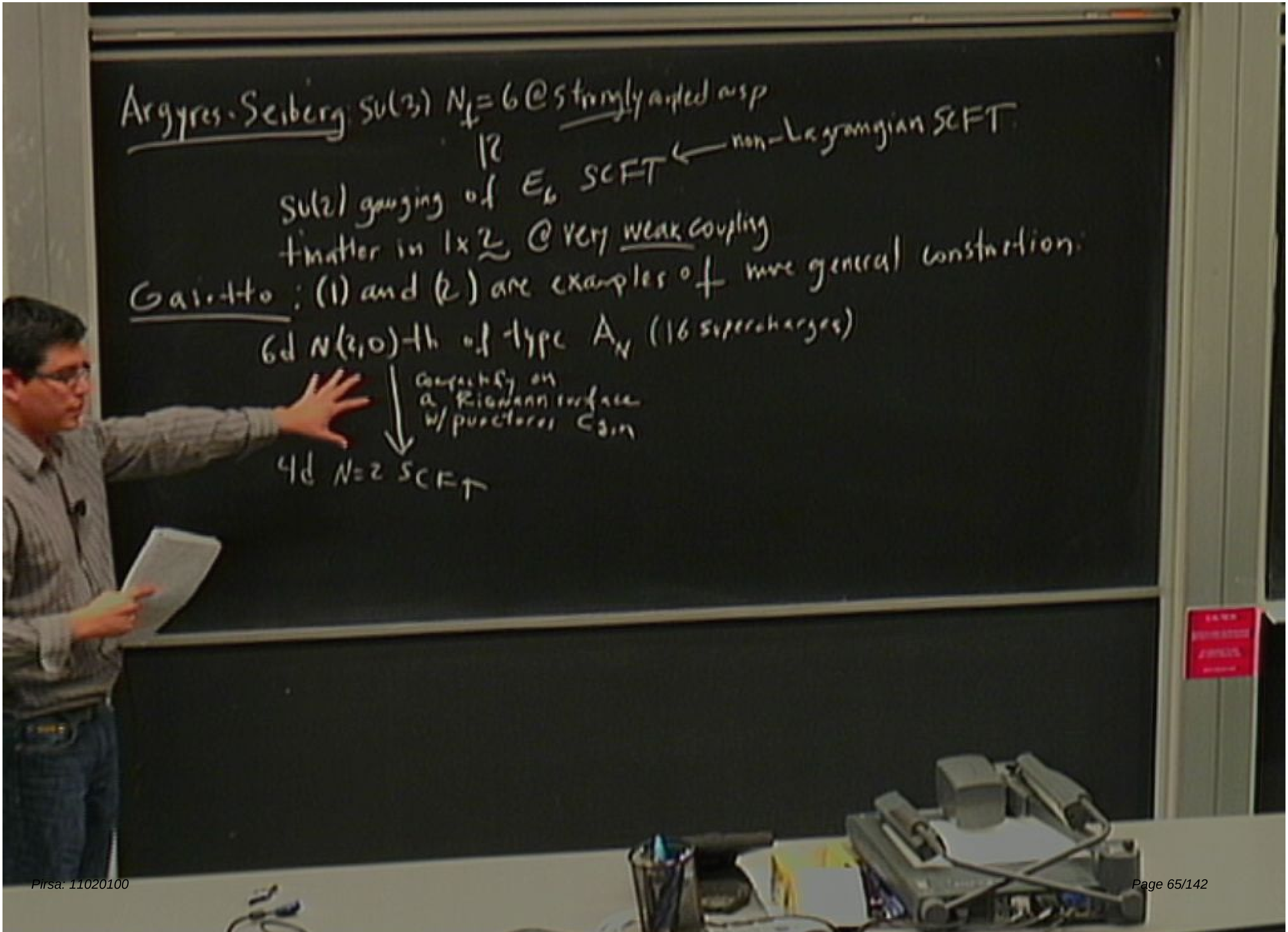
Gaiotto: (1) and (2) are examples of more general construction:
6d N(2,0)-th of type A_N
compactify on
a Riemann surface
w/ punctures C_g,n


Argyres-Seiberg: SU(3) N_f=6 @ strongly coupled cusp
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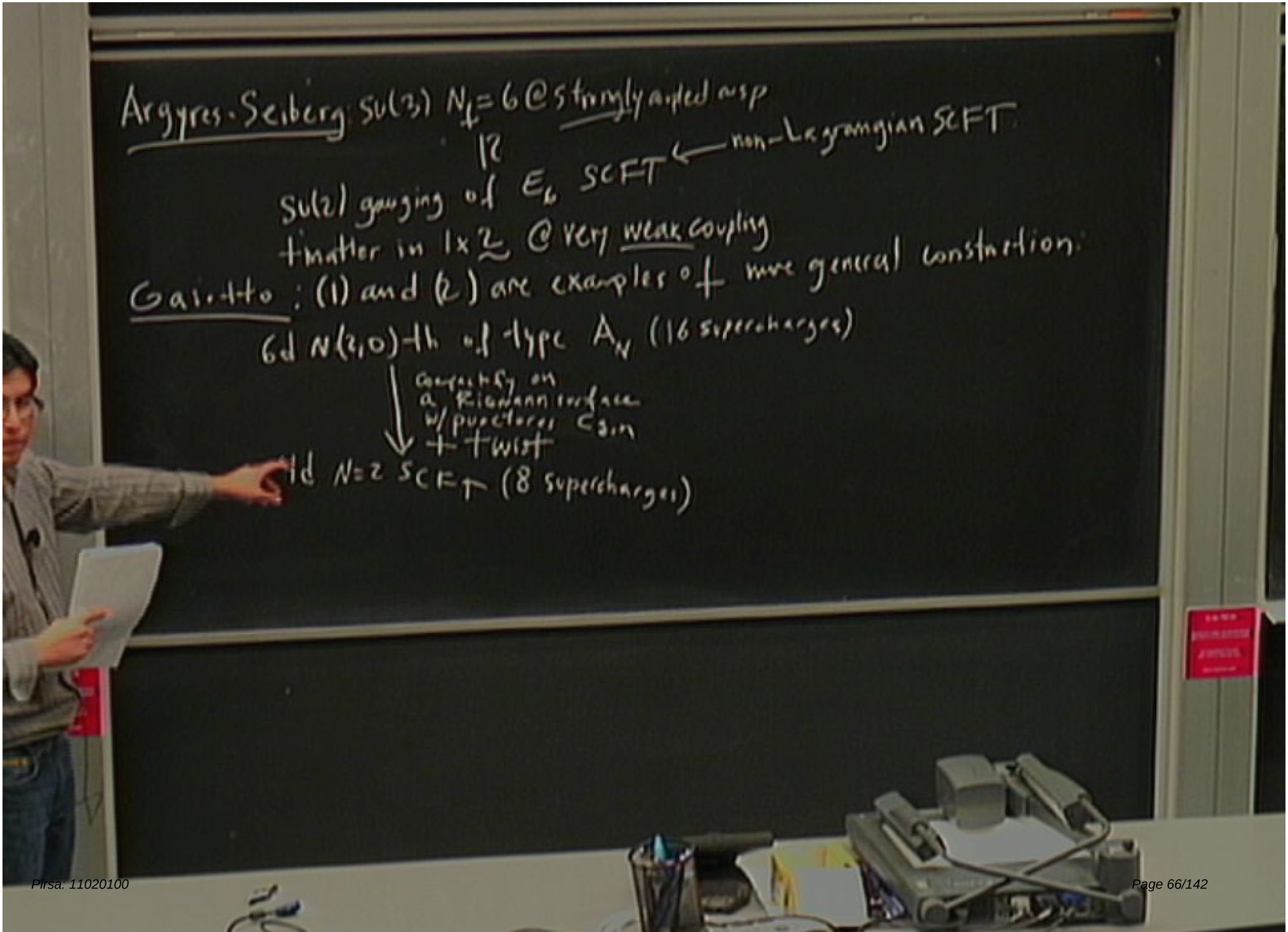
Gaiotto: (1) and (2) are examples of more general construction:
6d N(2,0) th of type A_N (16 supercharges)
compactify on a Riemann surface w/punctures C_g,n
4d N=2 SCFT


Argyres-Seiberg: SU(3) N_f=6 @ strongly coupled cusp
≅
SU(2) gauging of E_6 SCFT ← non-Lagrangian SCFT
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compactify on a Riemann surface w/ punctures C_{g,n}
+ twist
4d N=2 SCFT (8 supercharges)

Argyres-Seiberg: $SU(3)$ $N_f = 6$ @ strongly coupled pt
$\wr\wr$
$SU(2)$ gauging of E_6 SCFT ⟵ non-Lagrangian SCFT
+ matter in $1 \times \underset{\sim}{2}$ @ very weak coupling

Gaiotto: (1) and (2) are examples of more general construction:

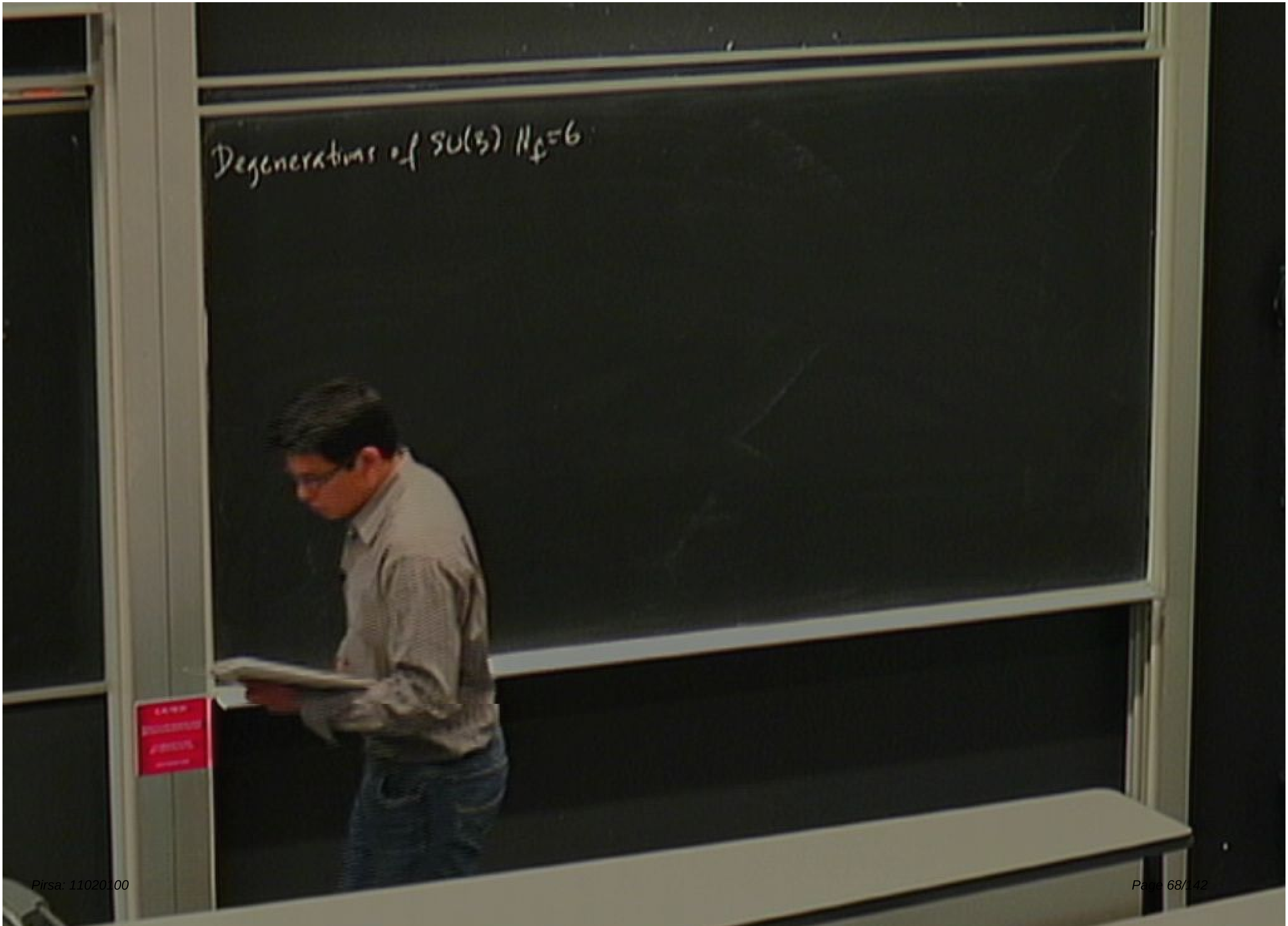
6d $\mathcal{N}(2,0)$ th of type A_N (16 supercharges)

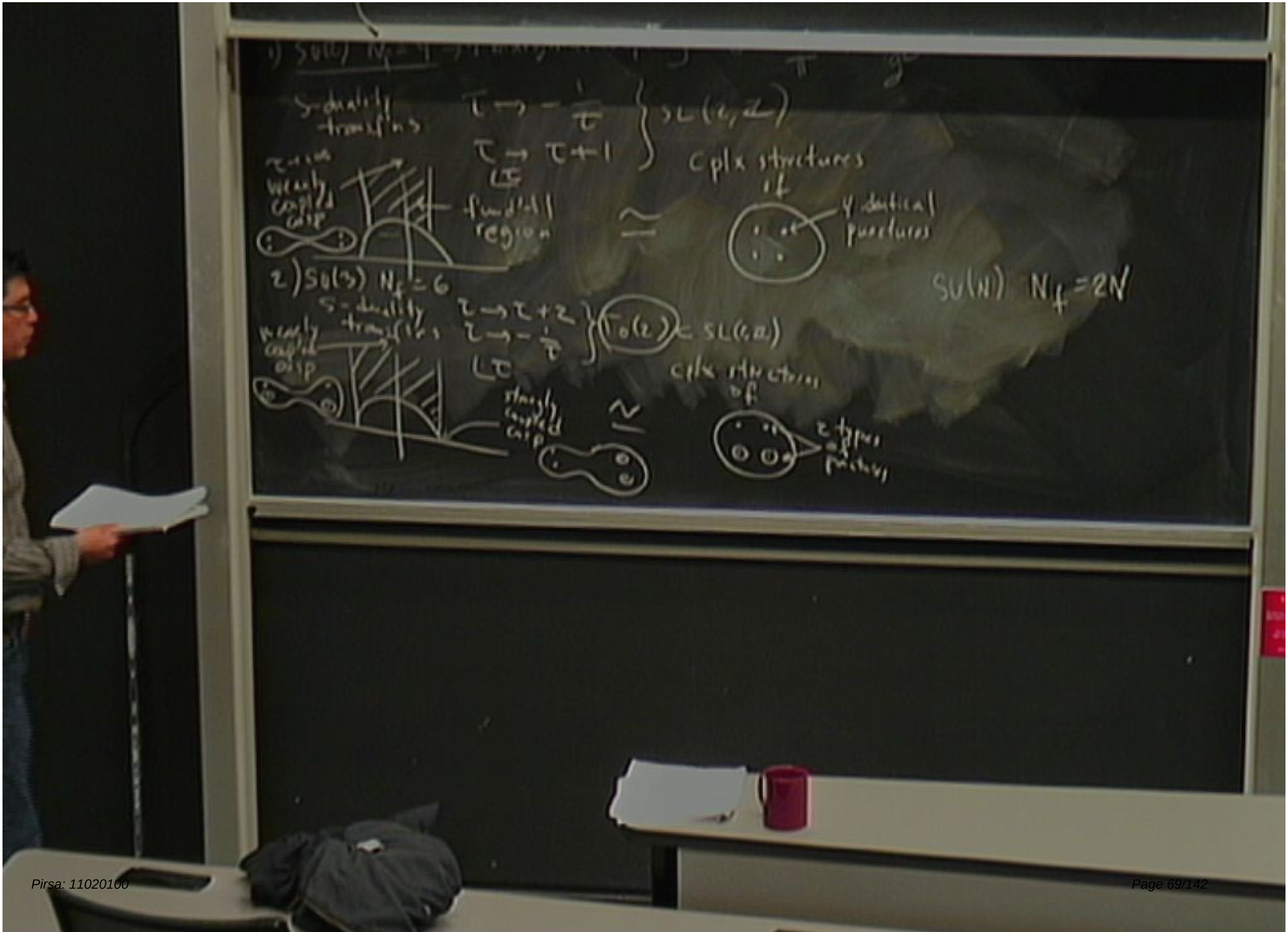
$\Big\downarrow$ compactify on a Riemann surface w/ punctures $C_{g,n}$ + twist

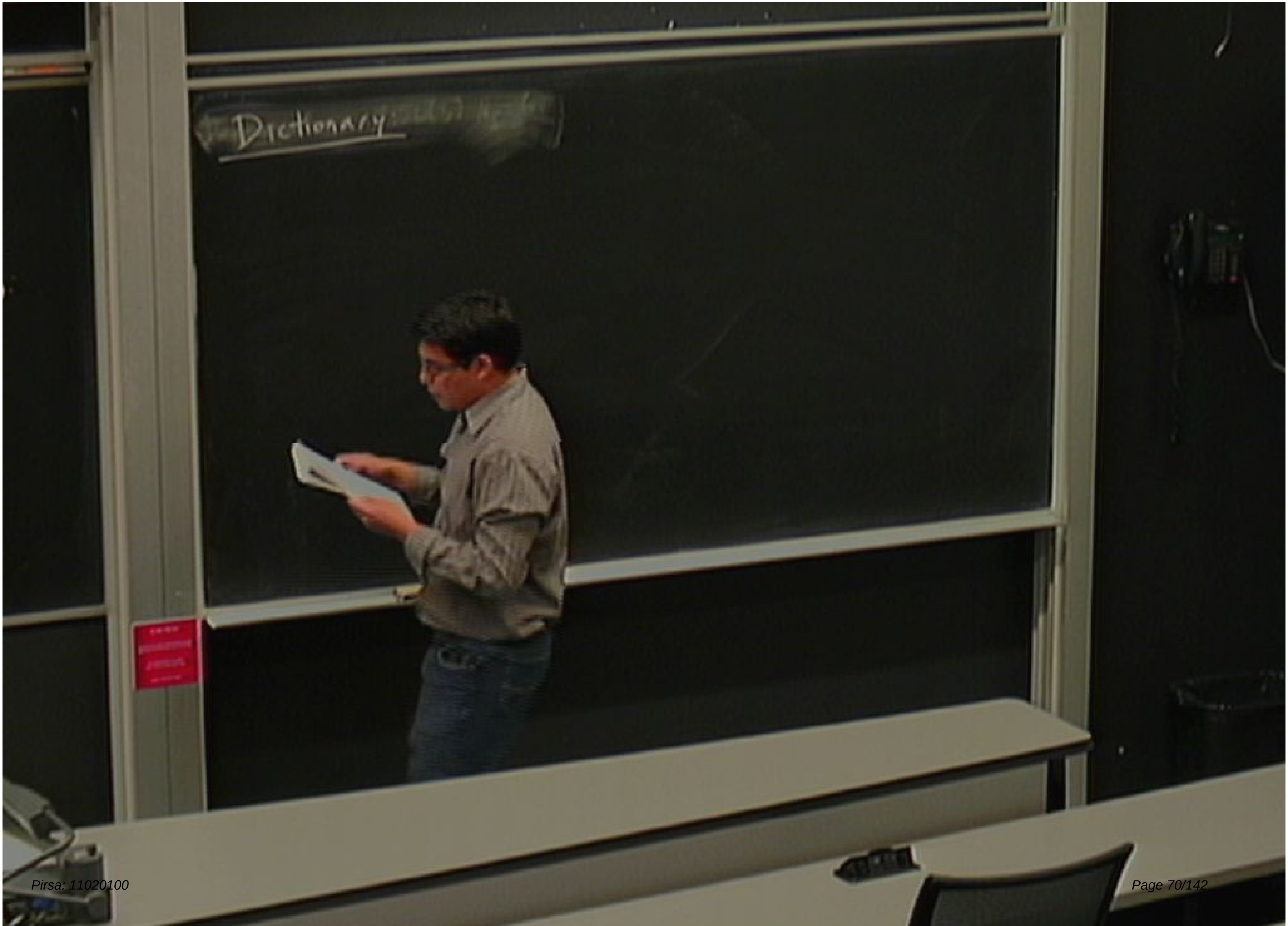
4d $\mathcal{N}=2$ SCFT (8 supercharges)

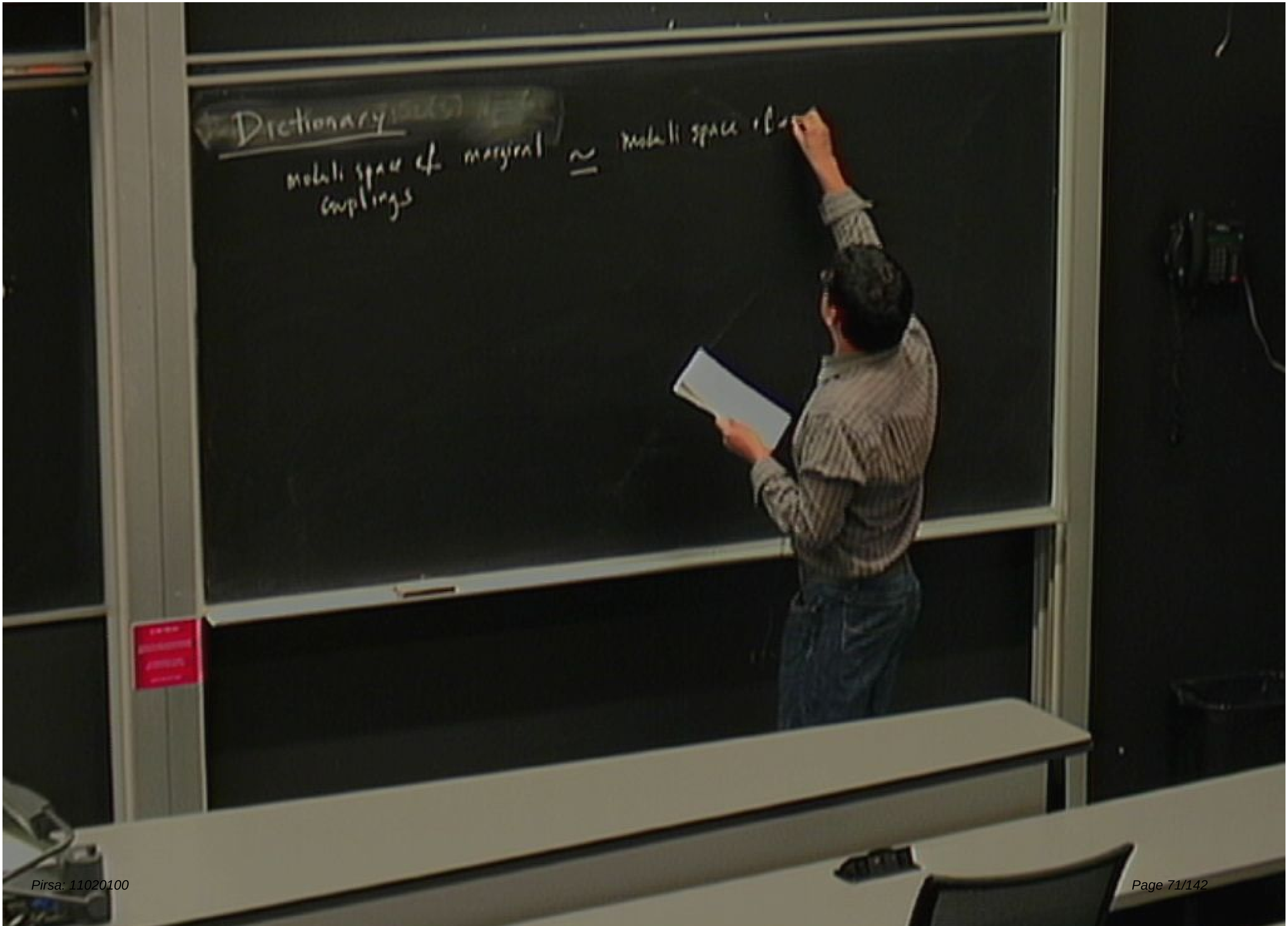
Ex. $SU(2)$ $N_f = 4 \to A_1$ th on

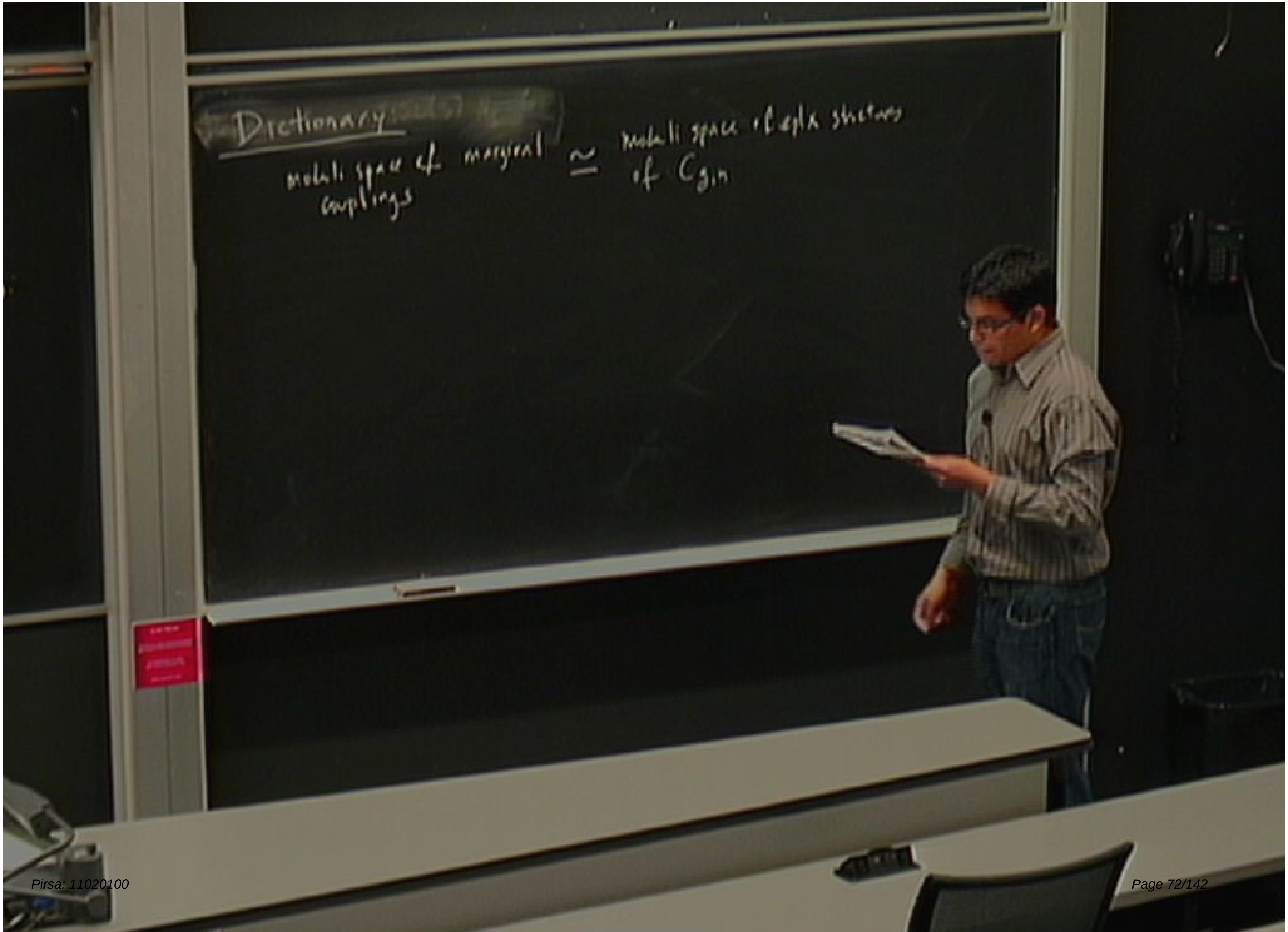
$SU(3)$ $N_f = 6 \to A_2$ th on

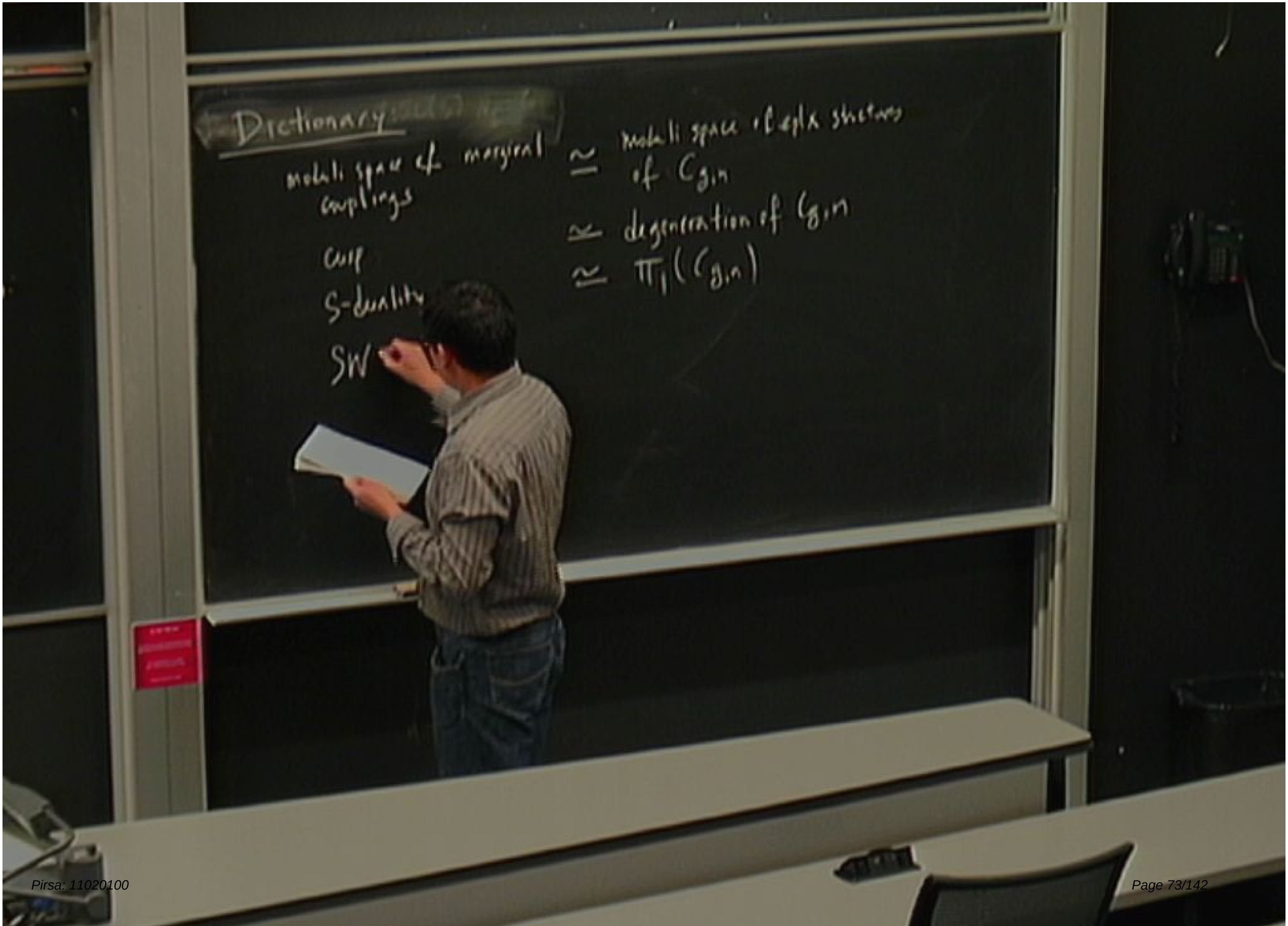

Degenerations of SU(3) $N_f=6$


S-duality transf'ns
τ → -1/τ
τ → τ+1
SL(2,Z)
cplx structures of
4 identical punctures
weakly coupled cusp
fund'l region
2) SU(3) N_f = 6
S-duality transf'ns
τ → τ+2
τ → -1/τ
Γ_0(2) ⊂ SL(2,Z)
cplx structures of
strongly coupled cusp
2 types of punctures
SU(N) N_f = 2N

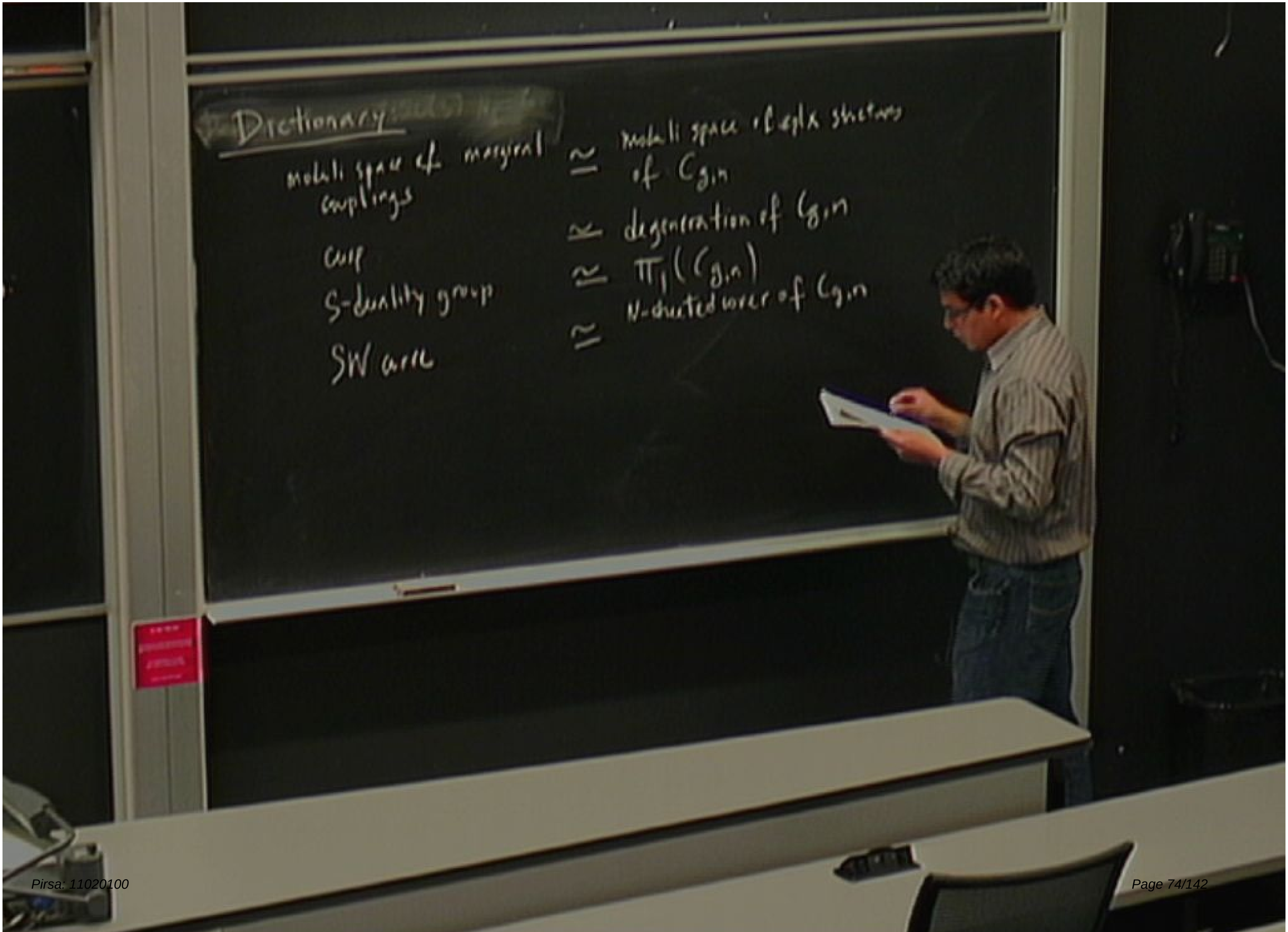

Dictionary:


Dictionary
moduli space of marginal couplings ≃ moduli space

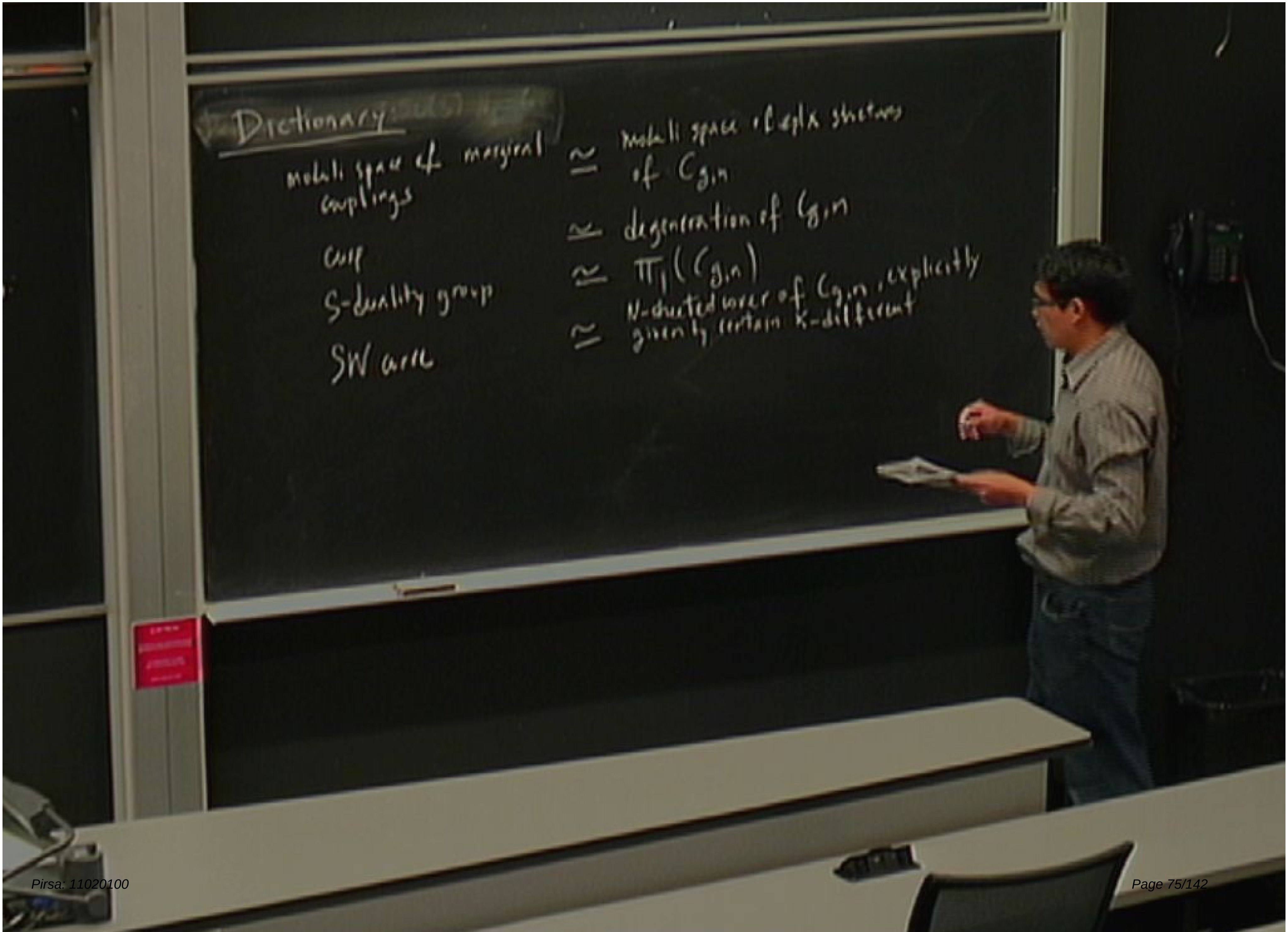

Dictionary
moduli space of marginal couplings ≃ moduli space of cplx structures of $C_{g,n}$

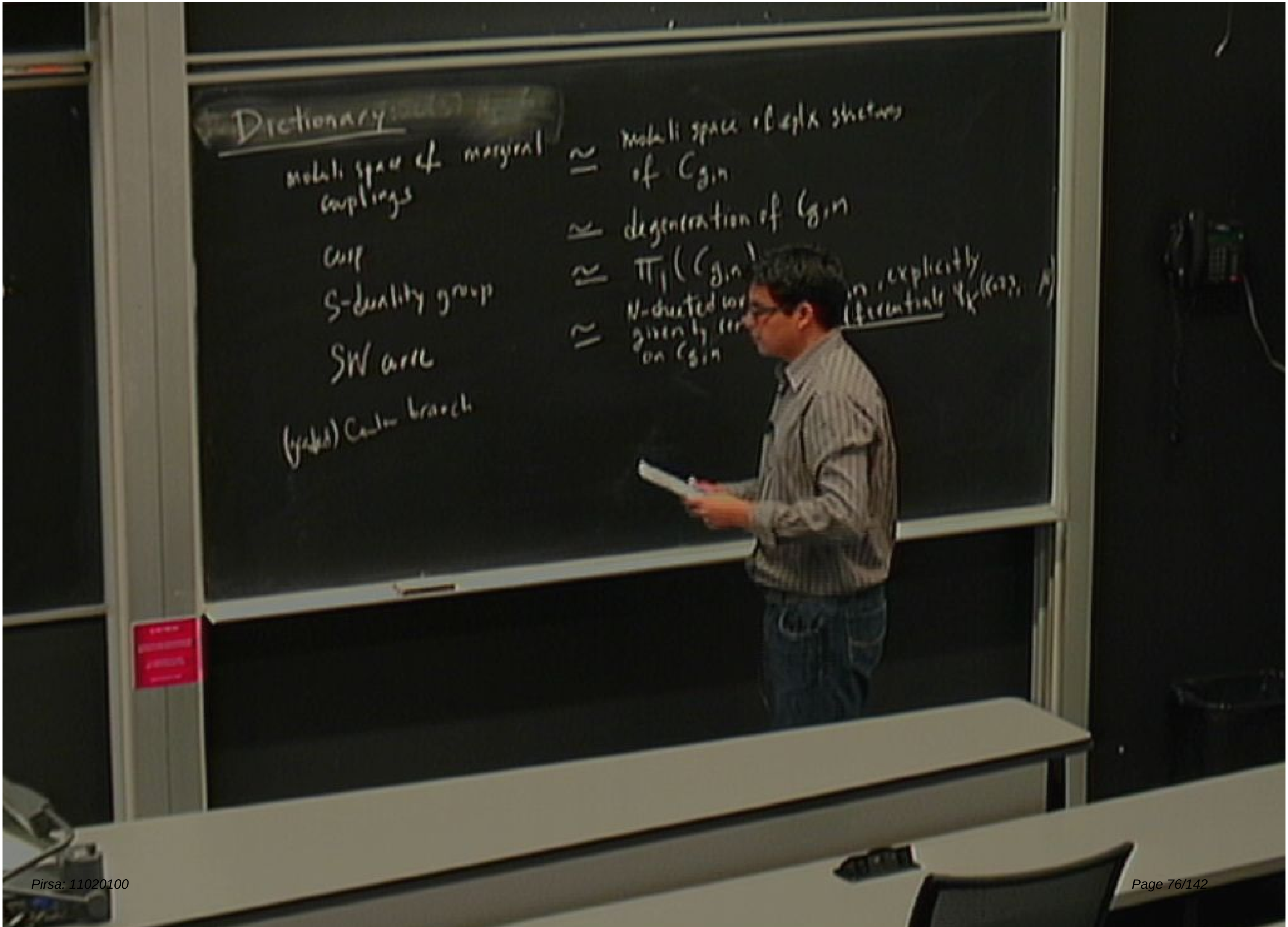


Dictionary
moduli space of marginal couplings
≃ moduli space of cplx structures of $C_{g,n}$
cusp
≃ degeneration of $C_{g,n}$
S-duality
≃ $\pi_1(C_{g,n})$
SW



Dictionary
moduli space of marginal couplings
≃ moduli space of cplx structures of $C_{g,n}$
cusp
≃ degeneration of $C_{g,n}$
S-duality group
≃ $\pi_1(C_{g,n})$
SW curve
≃ N-sheeted cover of $C_{g,n}$


Dictionary
moduli space of marginal couplings
≃ moduli space of cplx structures of $C_{g,n}$
≃ degeneration of $C_{g,n}$
cusp
S-duality group
≃ $\pi_1(C_{g,n})$
SW curve
≃ N-sheeted cover of $C_{g,n}$, explicitly given by certain k-different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moduli space of marginal couplings
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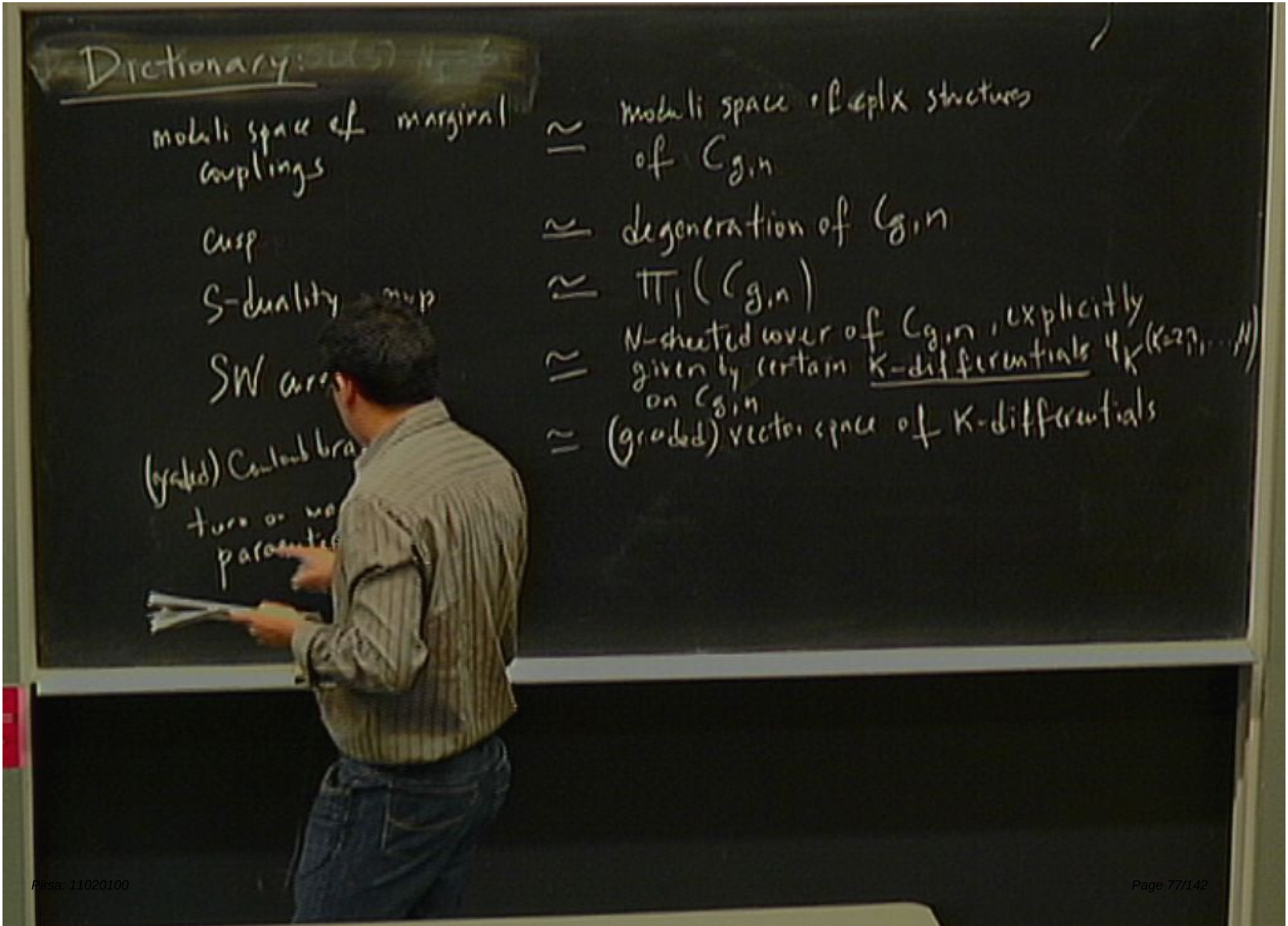
SW curve
explicitly
differentials
on C_{g,n}
(graded) Coulomb branch


Dictionary:
moduli space of marginal couplings
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cusp
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S-duality group
≃ $\pi_1(C_{g,n})$
SW
≃ N-sheeted cover of $C_{g,n}$, explicitly given by certain K-differentials φ_K $(K=2,\ldots,N)$ on $C_{g,n}$
(graded) Coulomb bra
≃ (graded) vector space of K-differentials
turn on
paramet

Dictionary:

th @ conformal point:

- moduli space of marginal couplings $\simeq$ moduli space of cplx structures of $C_{g,n}$
- cusp $\simeq$ degeneration of $C_{g,n}$
- S-duality group $\simeq$ $\pi_1(C_{g,n})$

th @ generic pt in Coulomb br…:

- …rve $\simeq$ N-sheeted cover of $C_{g,n}$, explicitly given by certain K-differentials φ_K $(K=2,\dots,N)$ on $C_{g,n}$
- …anch $\simeq$ (graded) vector space of K-differentials
- (graded) … $\simeq$ assign labels to punctures
- tu… $\simeq$
- t…

Dictionary:

th @ conformal point:

- moduli space of marginal couplings $\simeq$ moduli space of cplx structures of $C_{g,n}$
- cusp $\simeq$ degeneration of $C_{g,n}$ $\quad \varphi_K \sim (dz)^K$
- S-duality group $\simeq$ $\pi_1(C_{g,n})$

th @ generic pt in Coulomb branch:

- SW curve $\simeq$ N-sheeted cover of $C_{g,n}$, explicitly given by certain K-differentials $\varphi_K(K=2,\dots,N)$ on $C_{g,n}$
- (graded) Coulomb branch $\simeq$ (graded) vector space of K-differentials
- turn on mass parameters $\simeq$ assign labels to punctures

Dictionary:

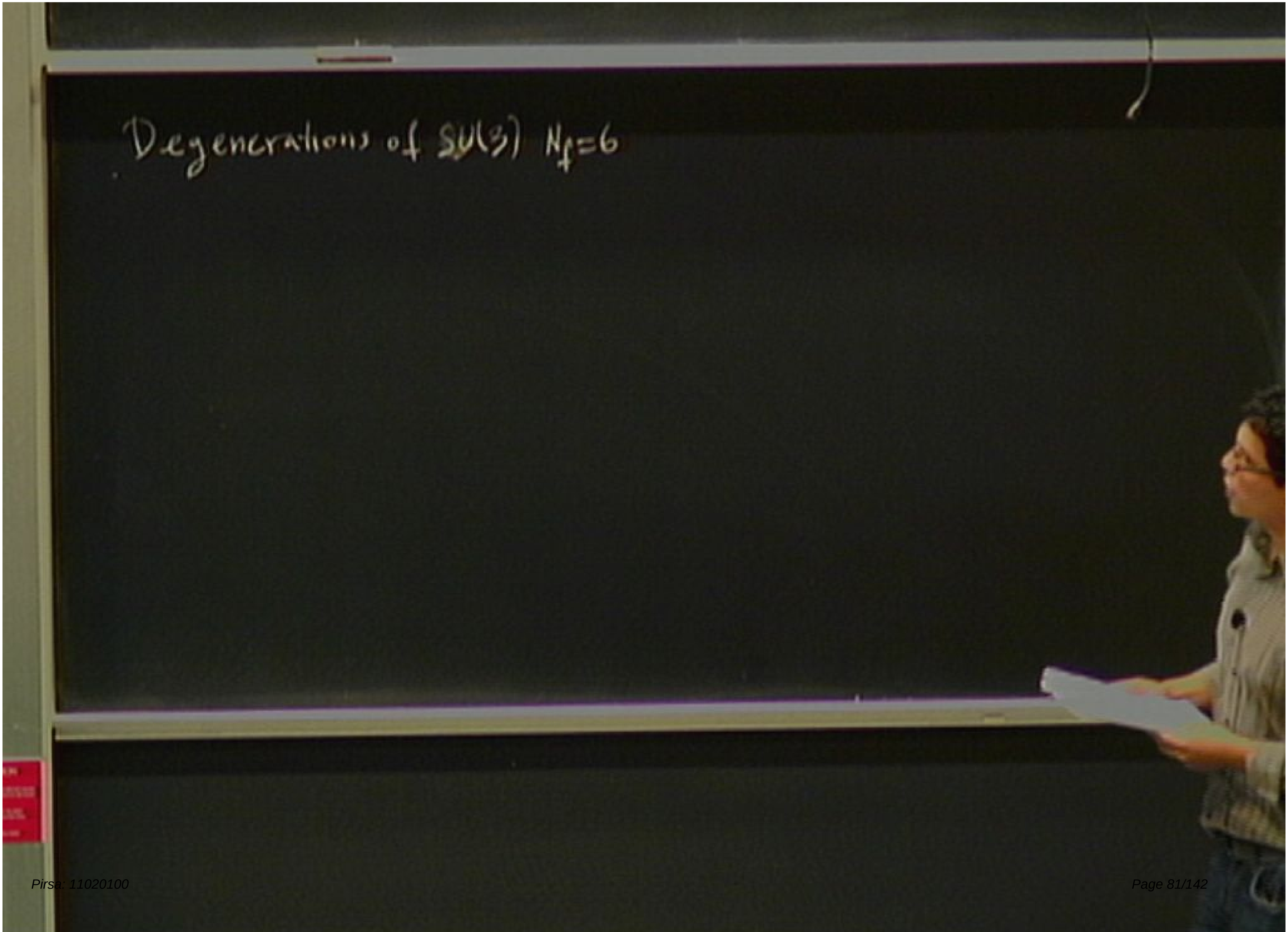
th @ conformal point:

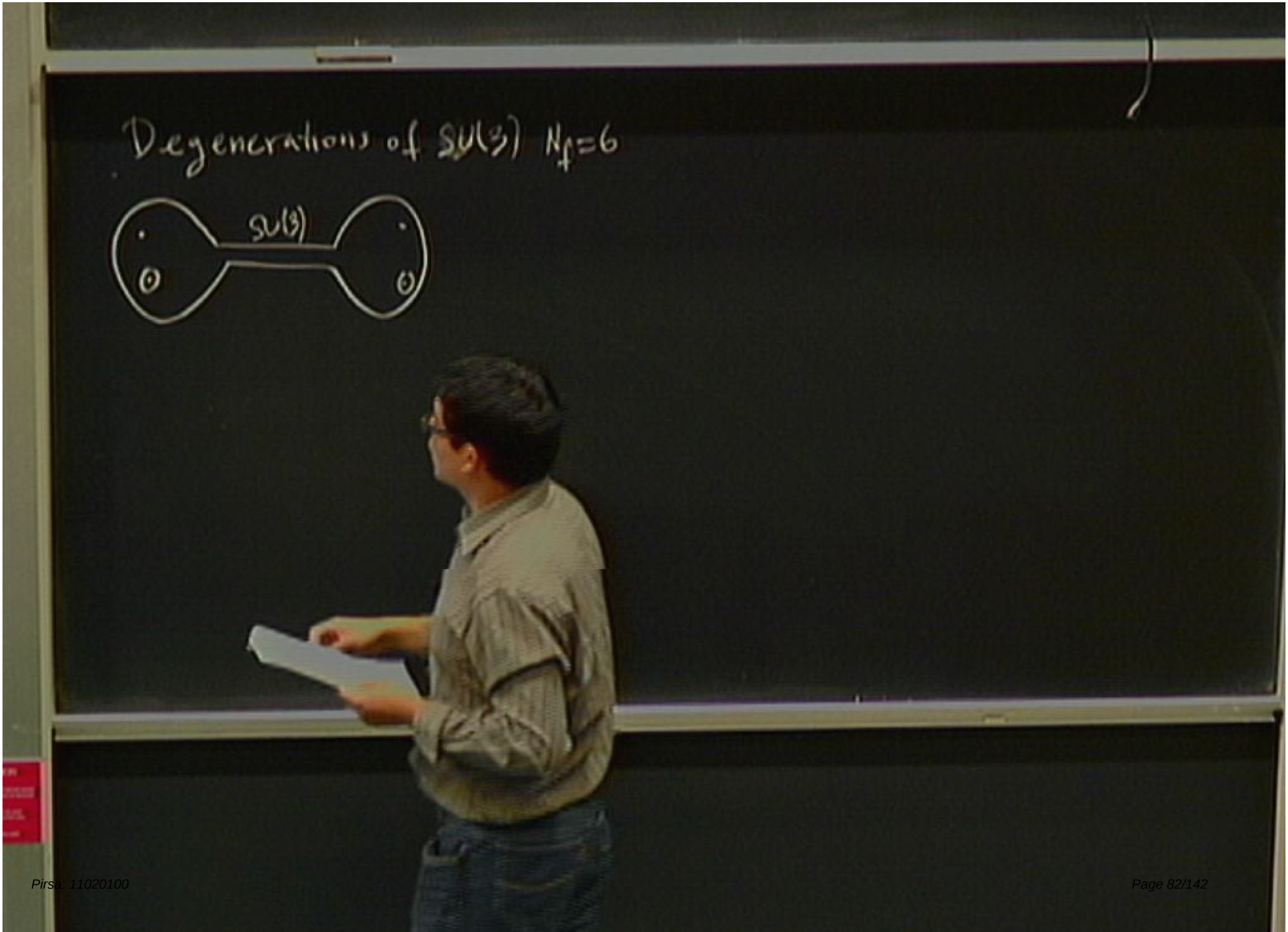
- moduli space of marginal couplings $\simeq$ [space] of cplx structures of $C_{g,n}$ $\quad \mathrm{Tr}\,\Phi^K$
- cusp $\simeq$ degeneration of $C_{g,n}$ $\quad \varphi_K \sim (dz)^K \sim (\text{mass})^K$
- S-duality group $\simeq$ $\pi_1(\mathcal{C}_{g,n})$

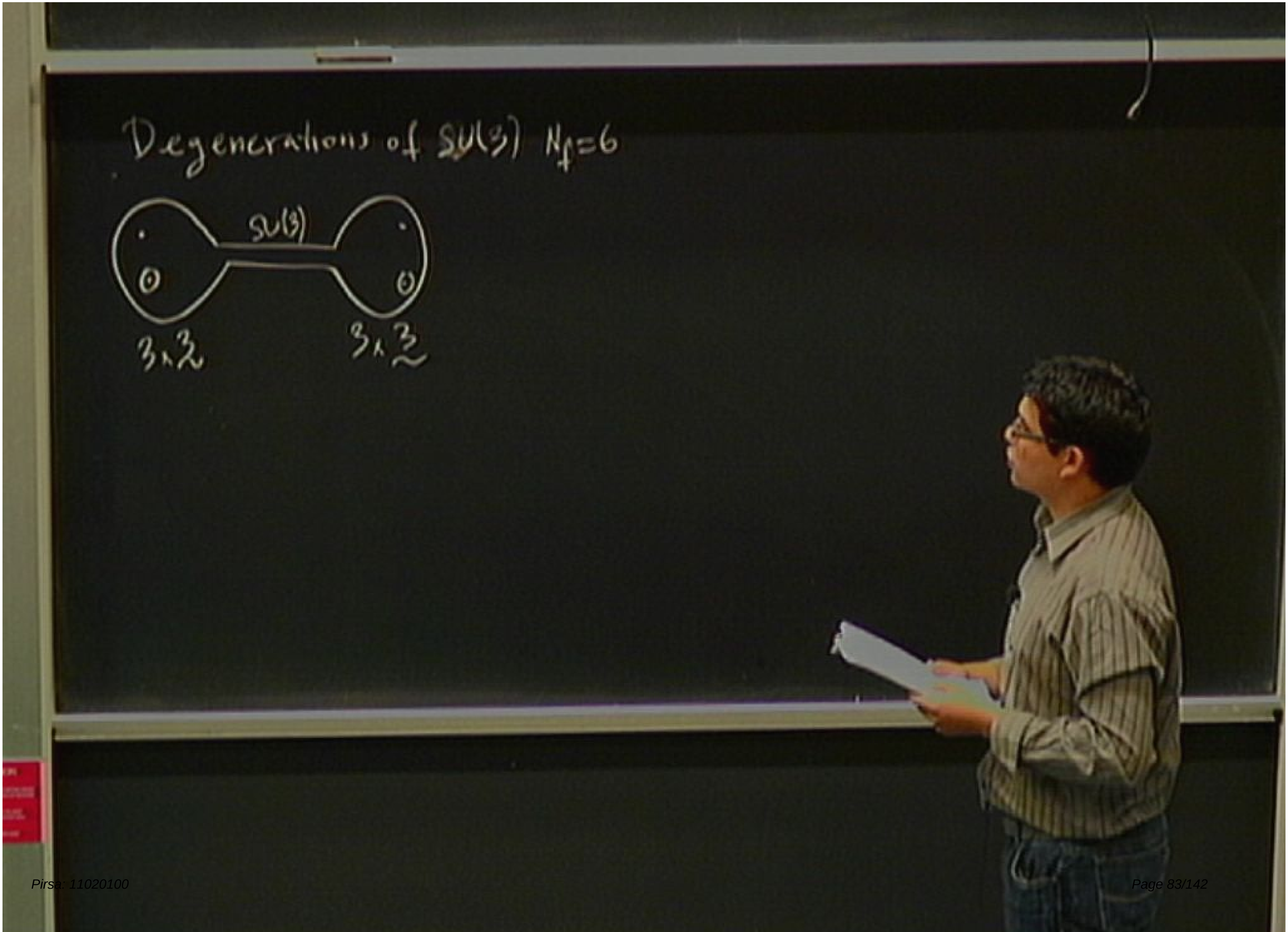
th @ generic pt in Coulomb branch:

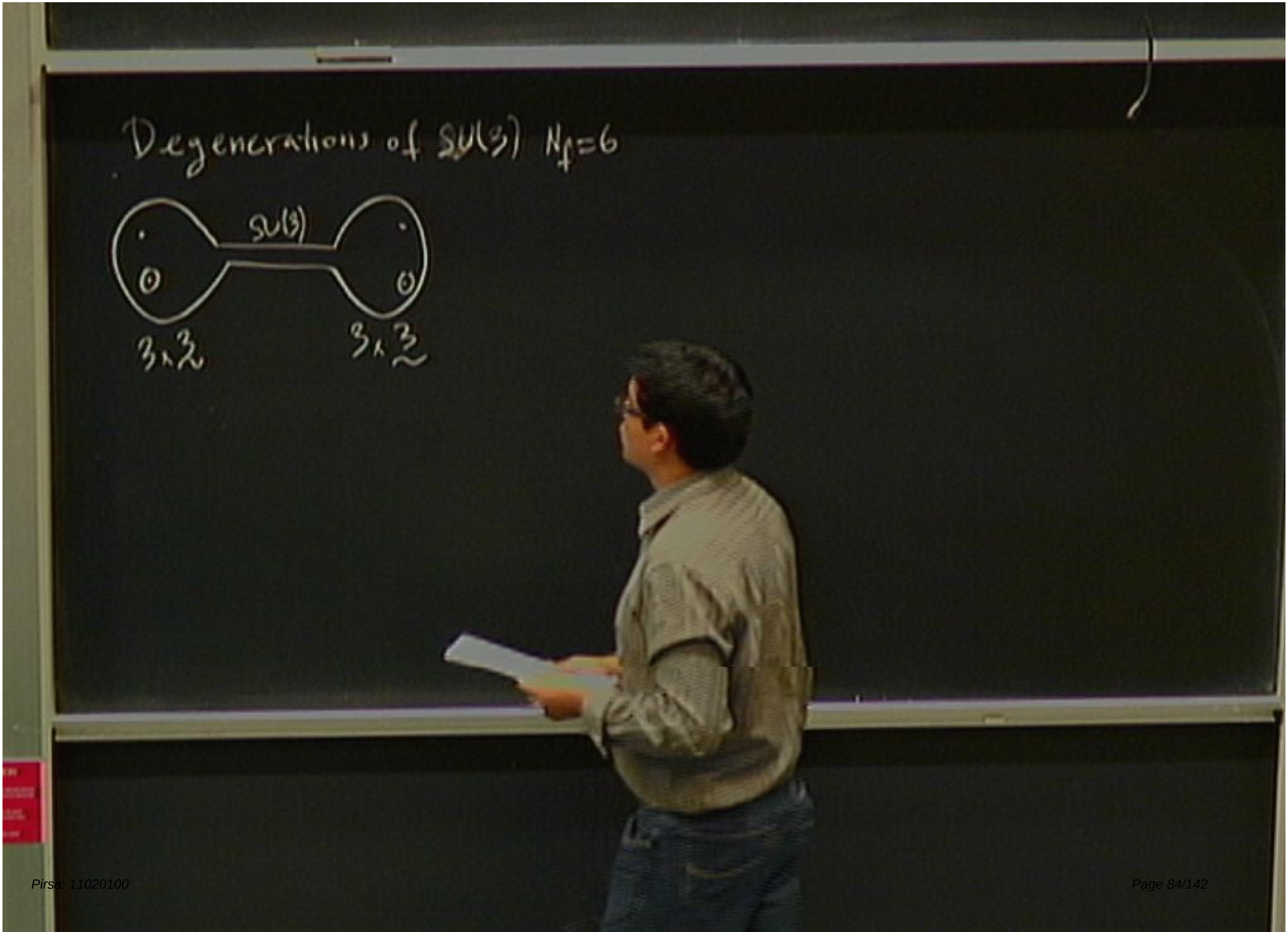
- SW curve $\simeq$ N-sheeted cover of $C_{g,n}$, explicitly given by certain $\underline{K\text{-differentials}}$ φ_K $(K=2,\ldots,N)$ on $C_{g,n}$
- (graded) Coulomb branch $\simeq$ (graded) vector space of K-differentials
- turn on mass parameters $\simeq$ assign labels to punctures $\quad SU(n)$: $2,3,4,\ldots,N$

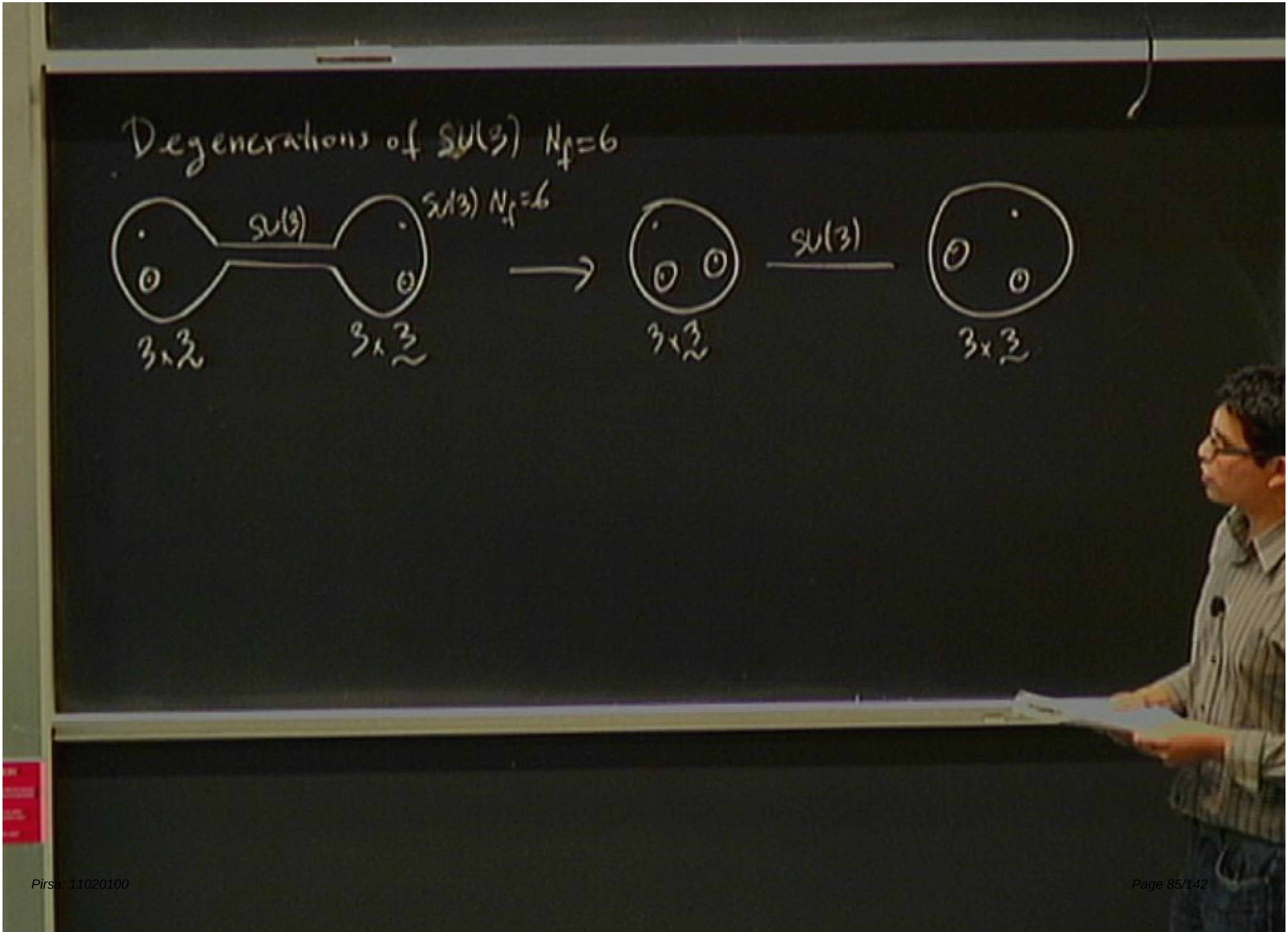
Deg

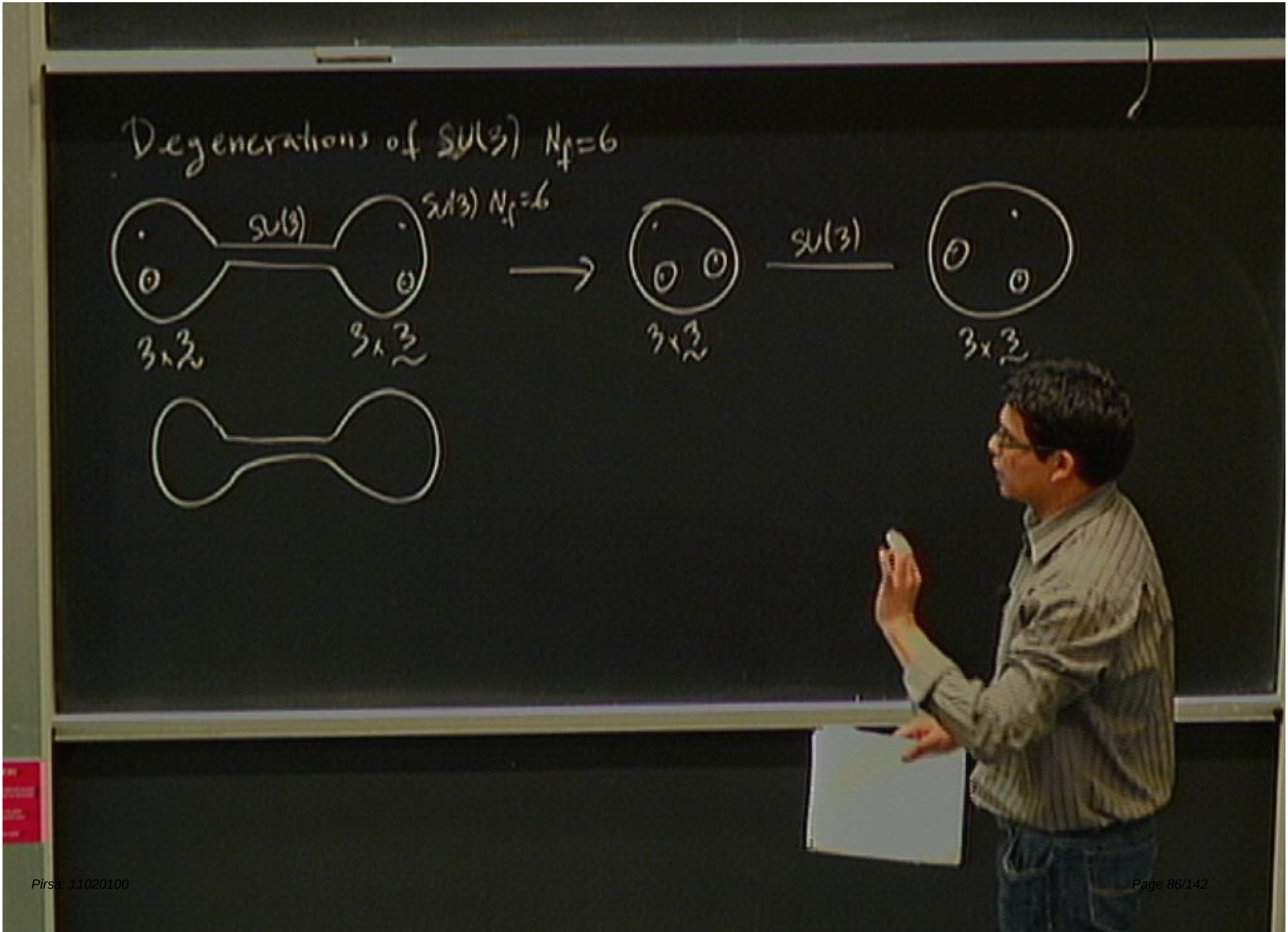

Degenerations of SU(3) $N_f=6$

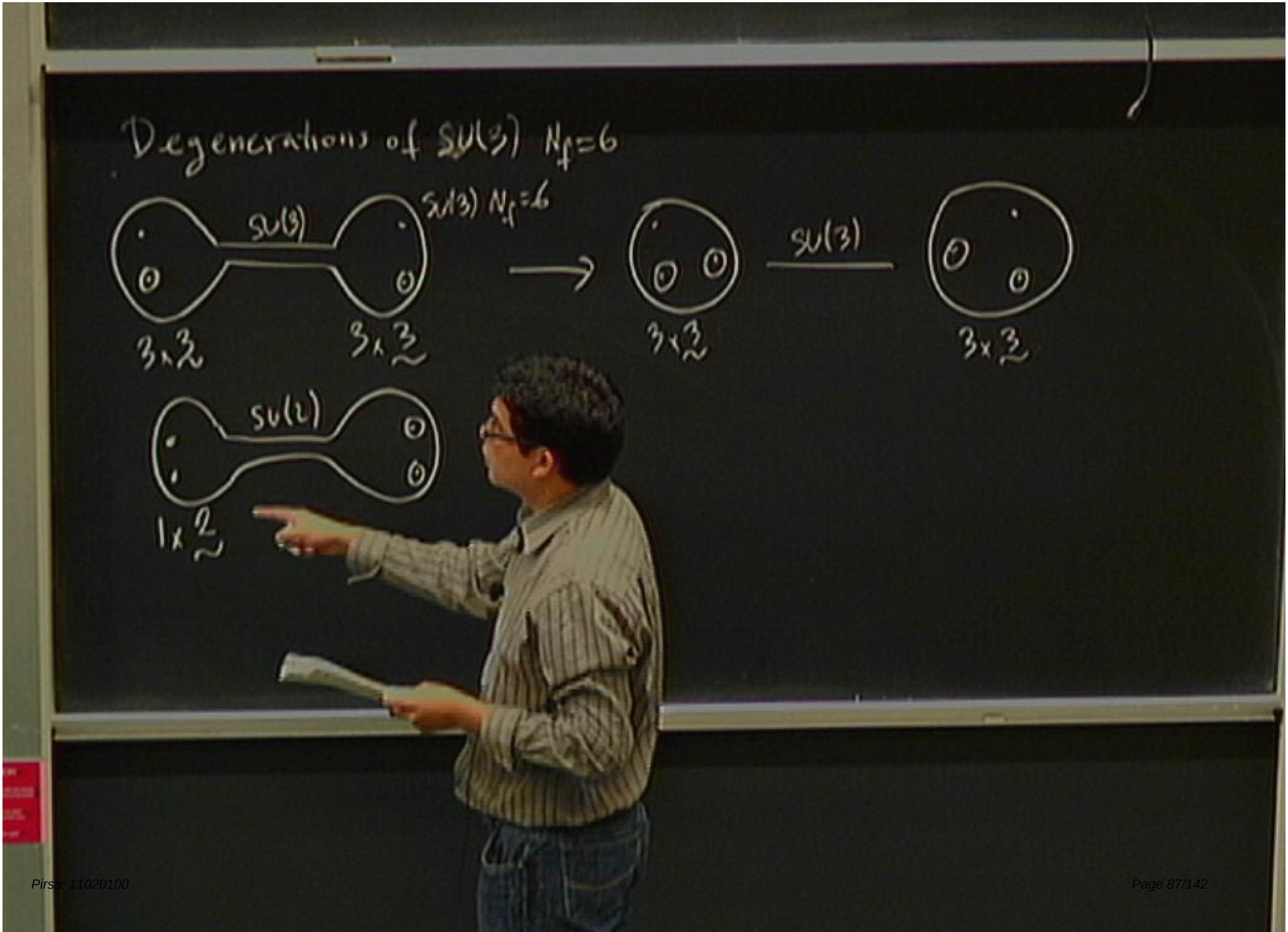

Degenerations of SU(3) $N_f=6$
SU(3)

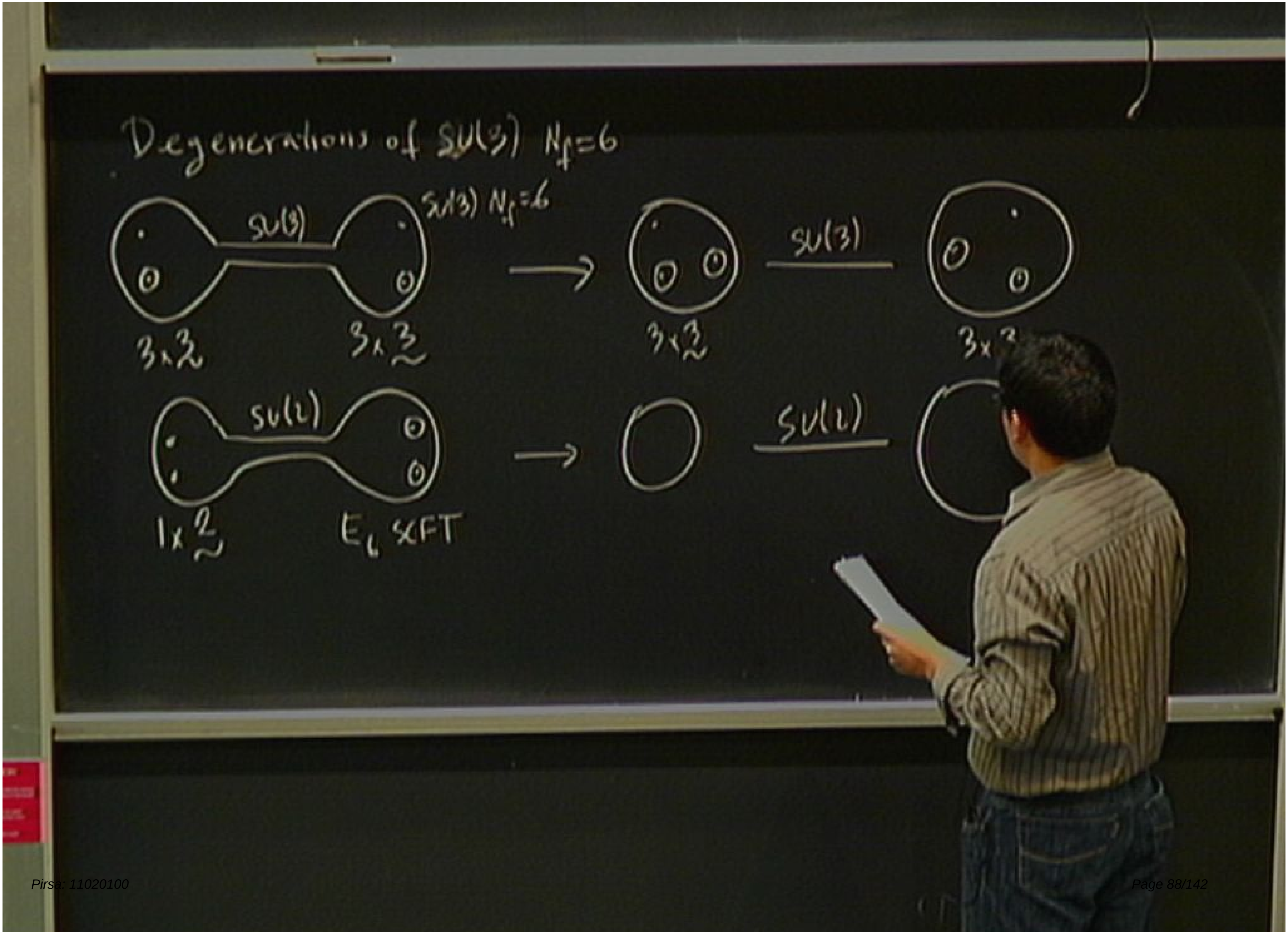

Degenerations of SU(3) $N_f=6$
SU(3)
3×2
3×2

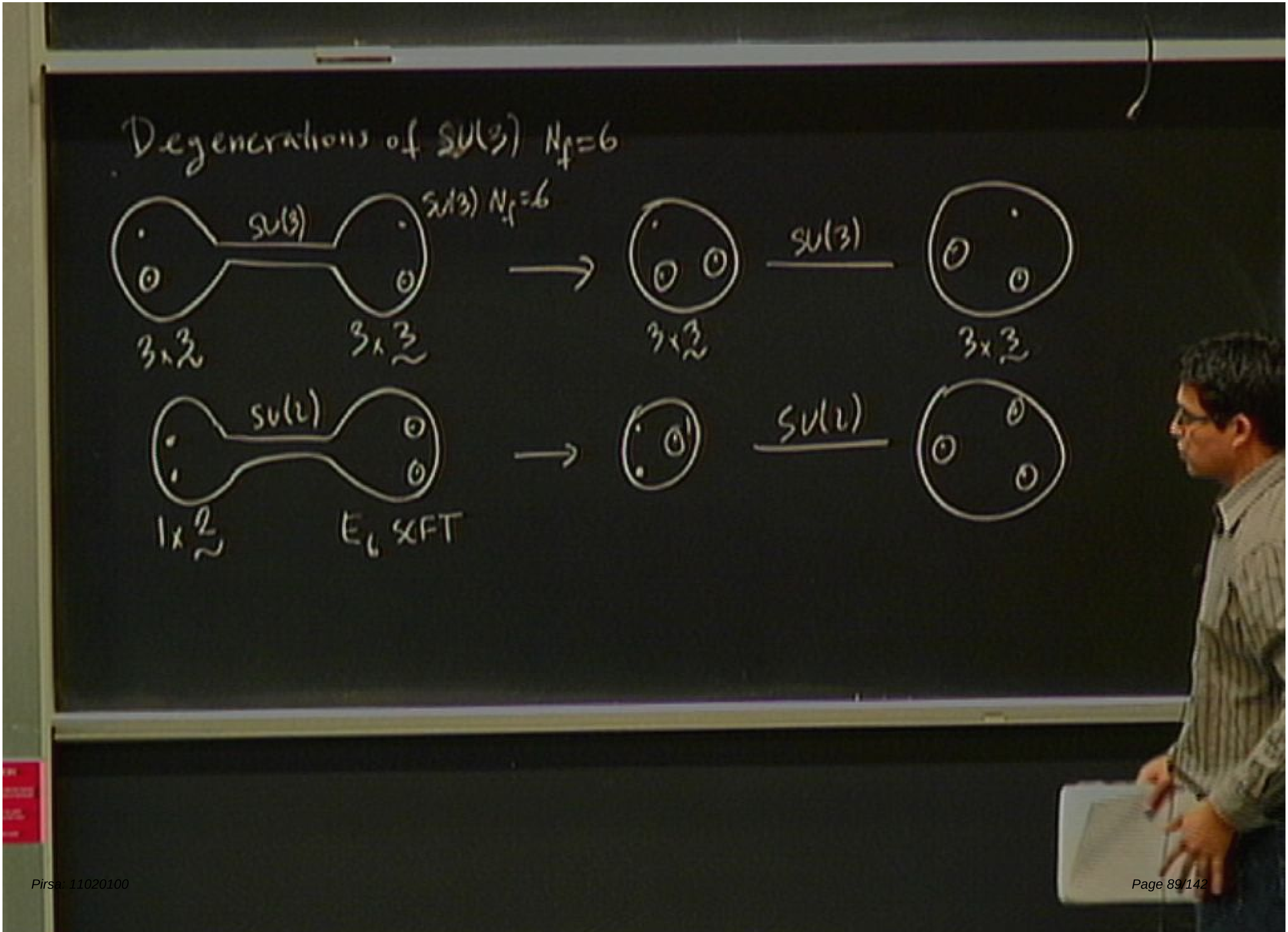

Degenerations of SU(3) $N_f=6$
SU(3)
3×2
3×2

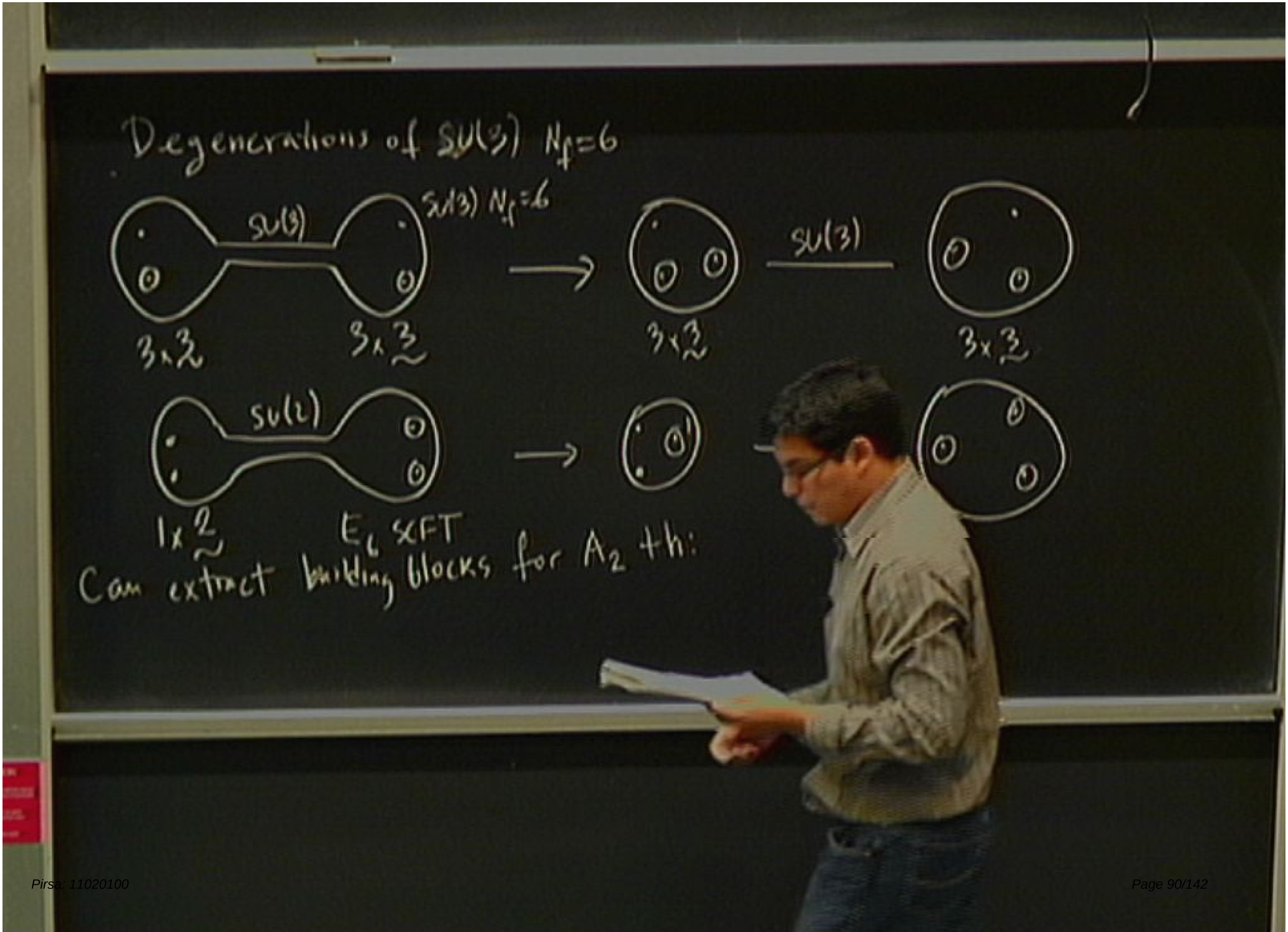

Degenerations of SU(3) N_f=6
SU(3) N_f=6
SU(3)
3x3
3x3
SU(3)
3x3
3x3

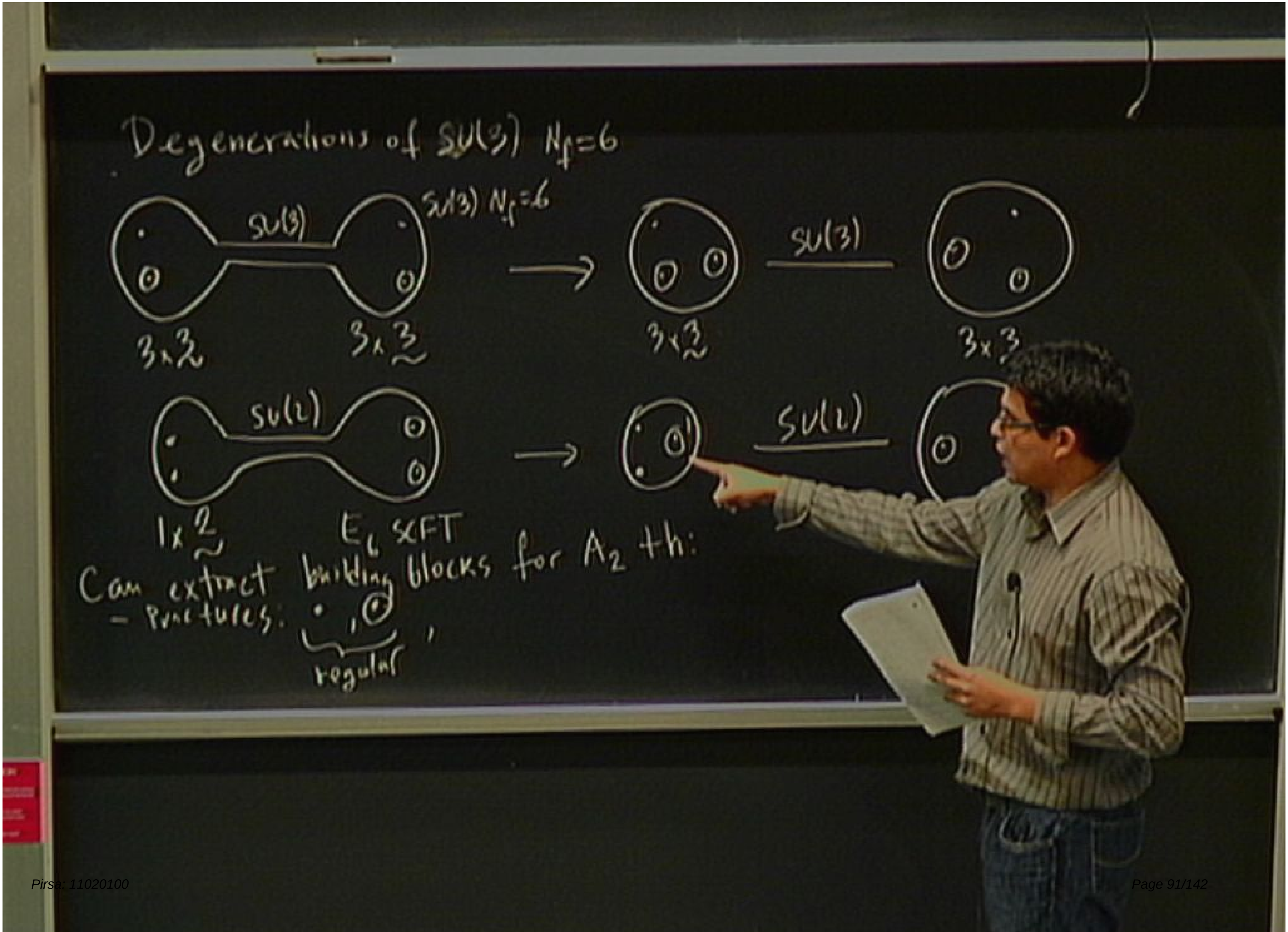

Degenerations of SU(3) N_f=6
SU(3) N_f=6
SU(3)
3×3
3×3
SU(3)
3×3
3×3

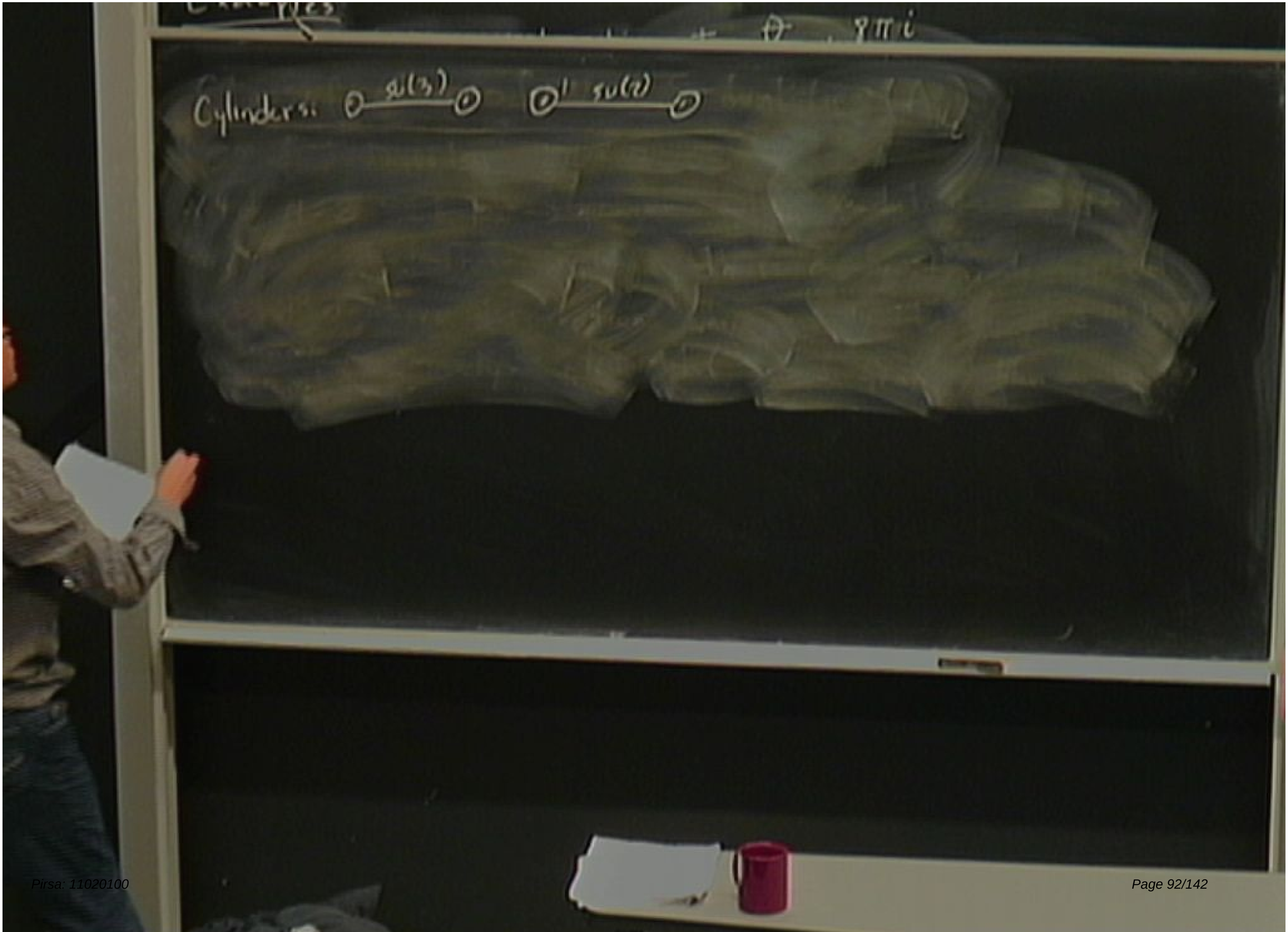

Degenerations of SU(3) Nf=6
SU(3)
SU(3) Nf=6
3x2
3x2
SU(3)
3x2
3x2
SU(2)
1x2

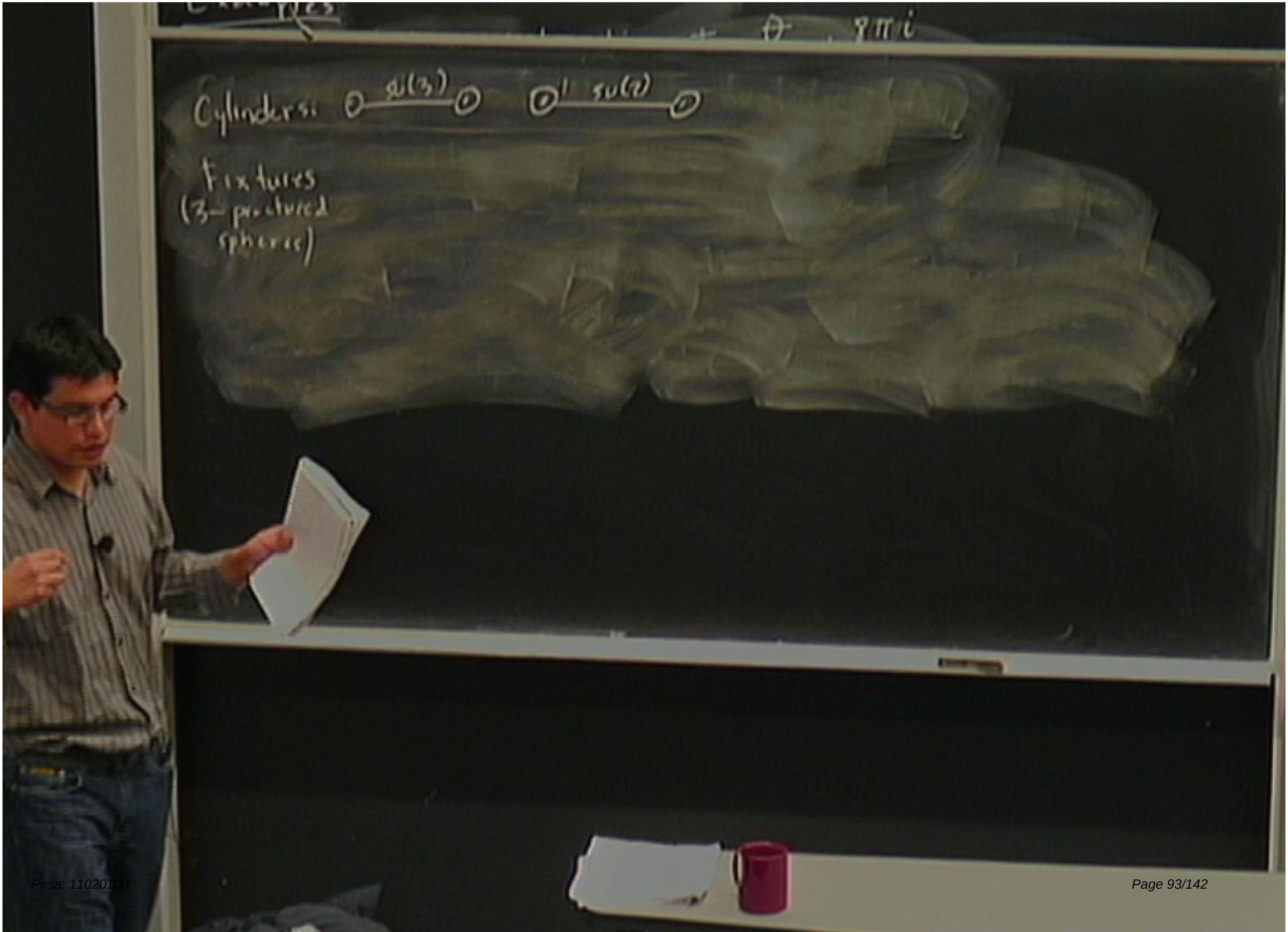

Degenerations of SU(3) N_f=6
SU(3) N_f=6
SU(3)
3x3
3x3
SU(3)
3x3
SU(2)
SU(2)
1x2
E_6 SCFT

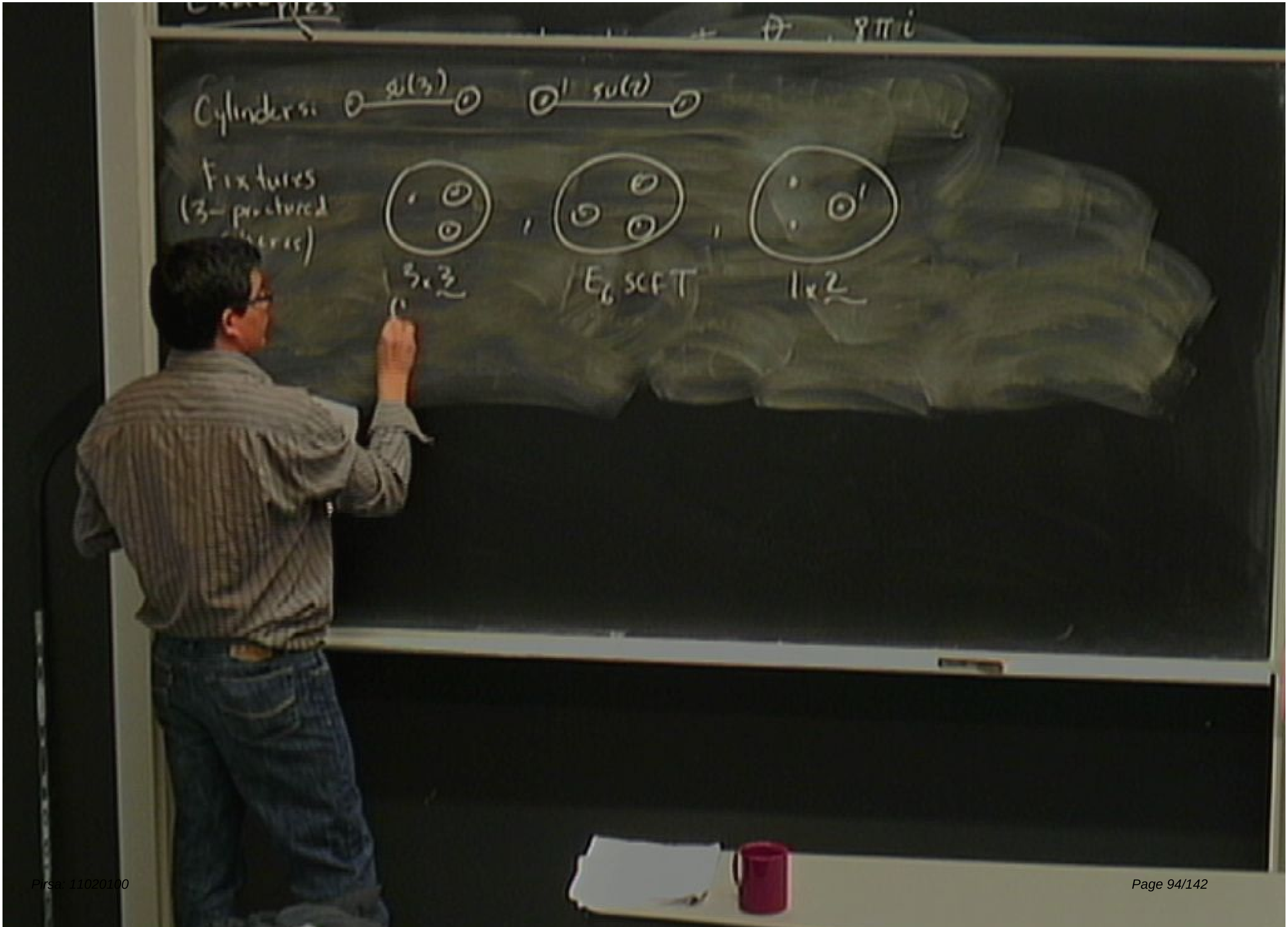

Degenerations of SU(3) N_f=6
SU(3) N_f=6
SU(3)
3×3
3×3
→
3×3
SU(3)
3×3
SU(2)
1×2
E_6 SCFT
→
SU(2)

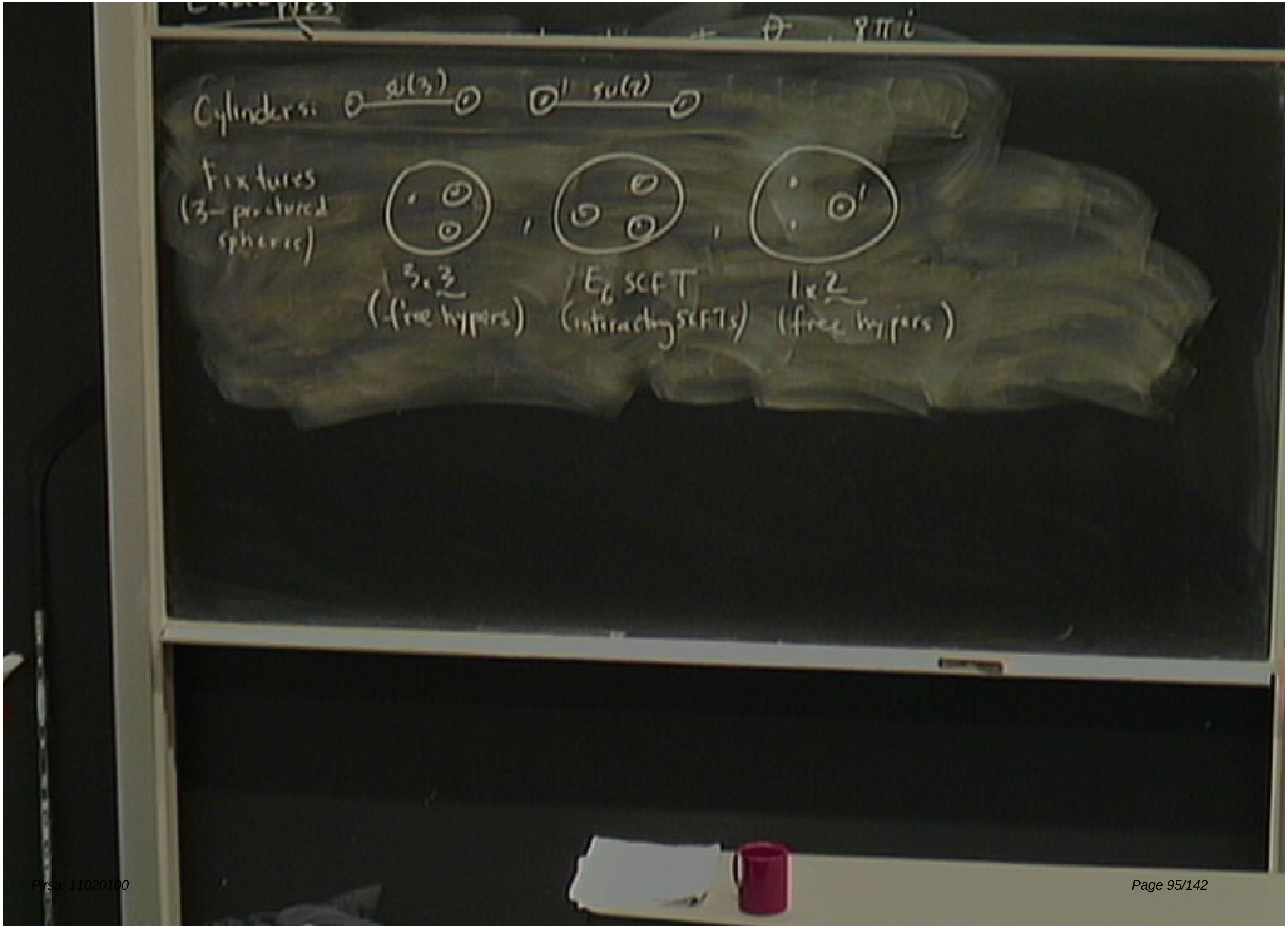

Degenerations of SU(3) Nf=6
SU(3) Nf=6
SU(3)
SU(3)
3x3
3x3
3x3
3x3
SU(2)
1x2
E6 SCFT
Can extract building blocks for A2 th:

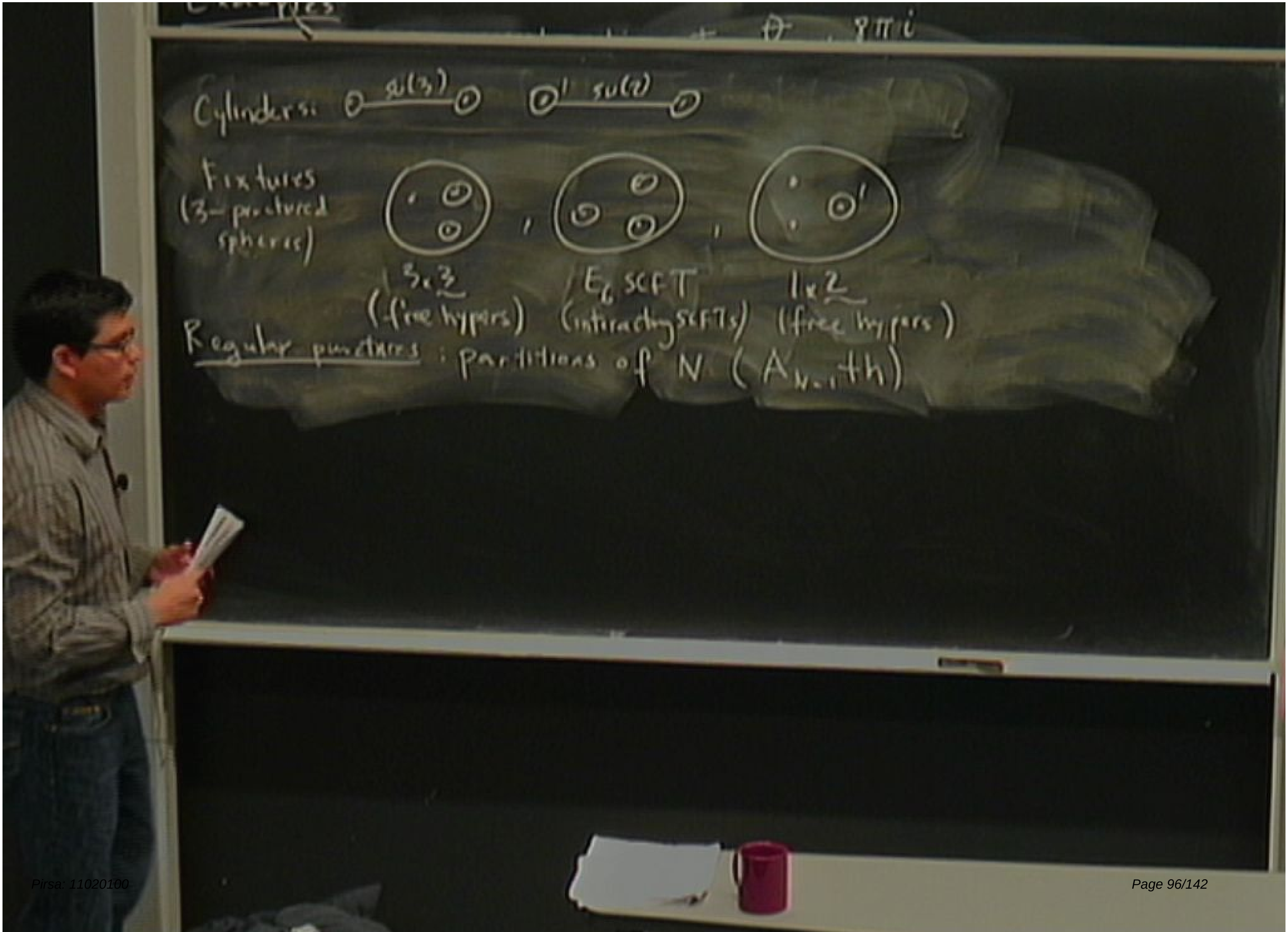

Degenerations of SU(3) N_f=6
SU(3) N_f=6
SU(3)
3x3
3x3
SU(3)
3x3
3x3
SU(2)
SU(2)
1x2
E_6 SCFT
Can extract building blocks for A_2 th:
- Punctures: • , ⊙ ,
regular

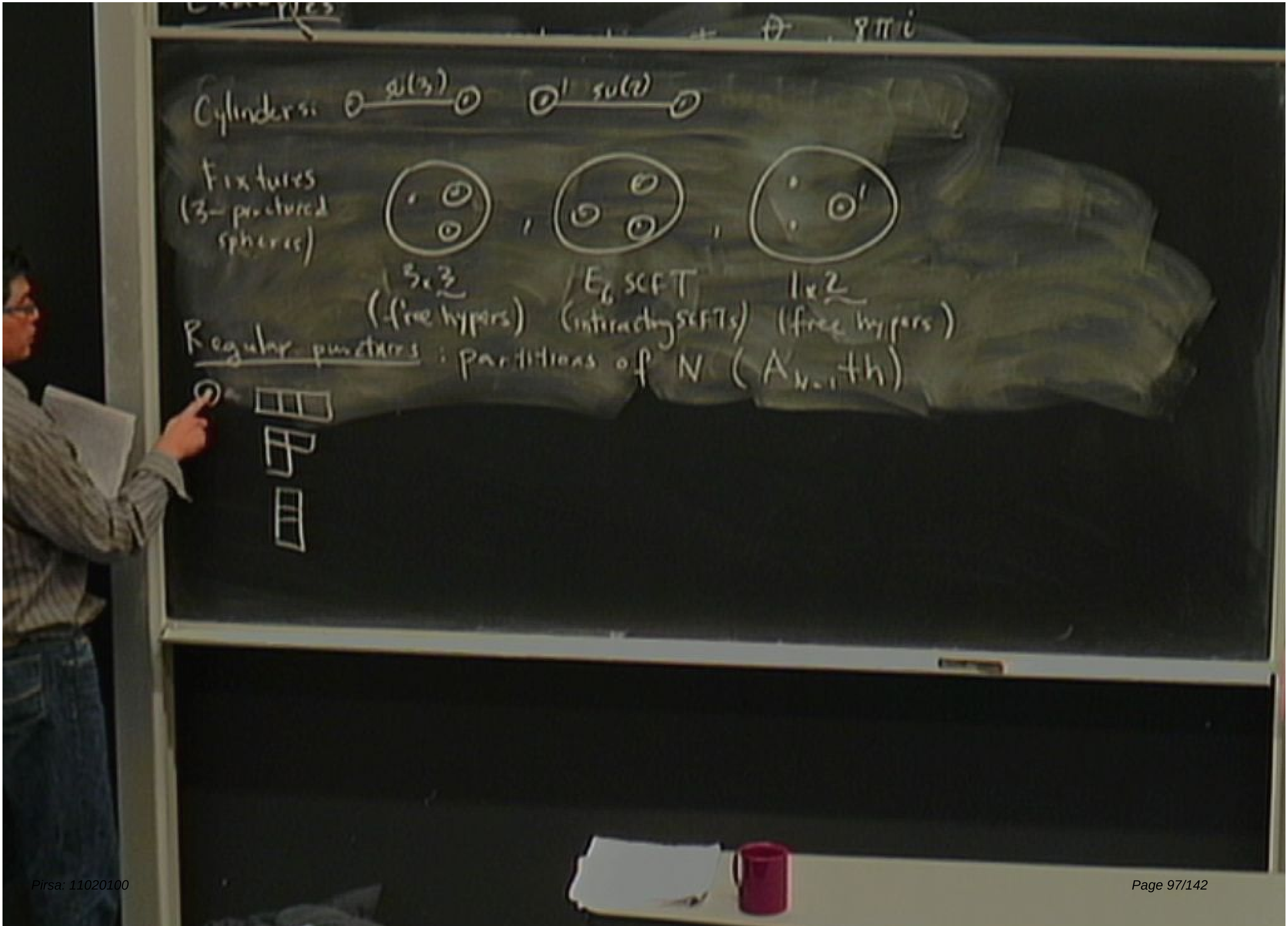

Cylinders:


Cylinders:
su(2)
Fixtures
(3-punctured spheres)

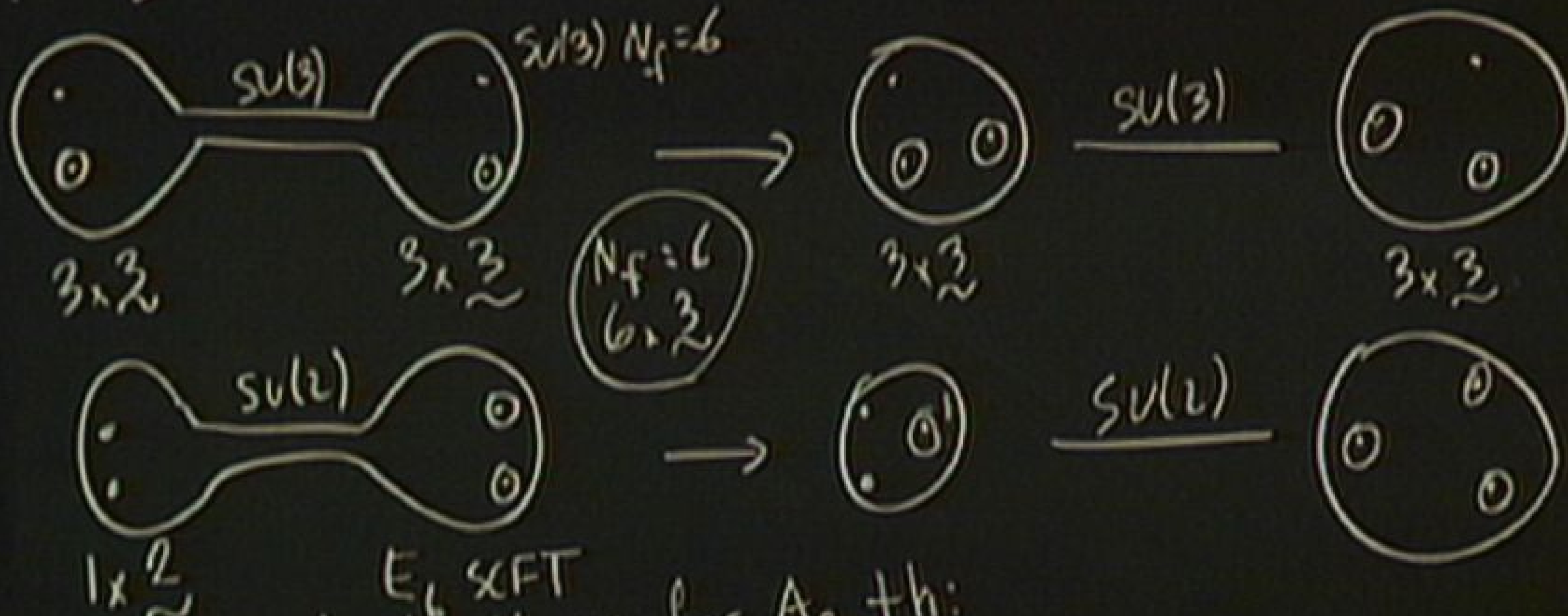

Cylinders:
su(3)
su(2)
Fixtures
(3-punctured spheres)
3 x 3
E6 SCFT
1 x 2


Cylinders:
su(3)
su(2)
Fixtures
(3-punctured
spheres)
3×3
(free hypers)
E6 SCFT
(interacting SCFTs)
1×2
(free hypers)


Cylinders: su(3) su(2)
Fixtures (3-punctured spheres)
3×3 (free hypers)
E6 SCFT (interacting SCFTs)
1×2 (free hypers)
Regular punctures: partitions of N (A_{N-1} th)

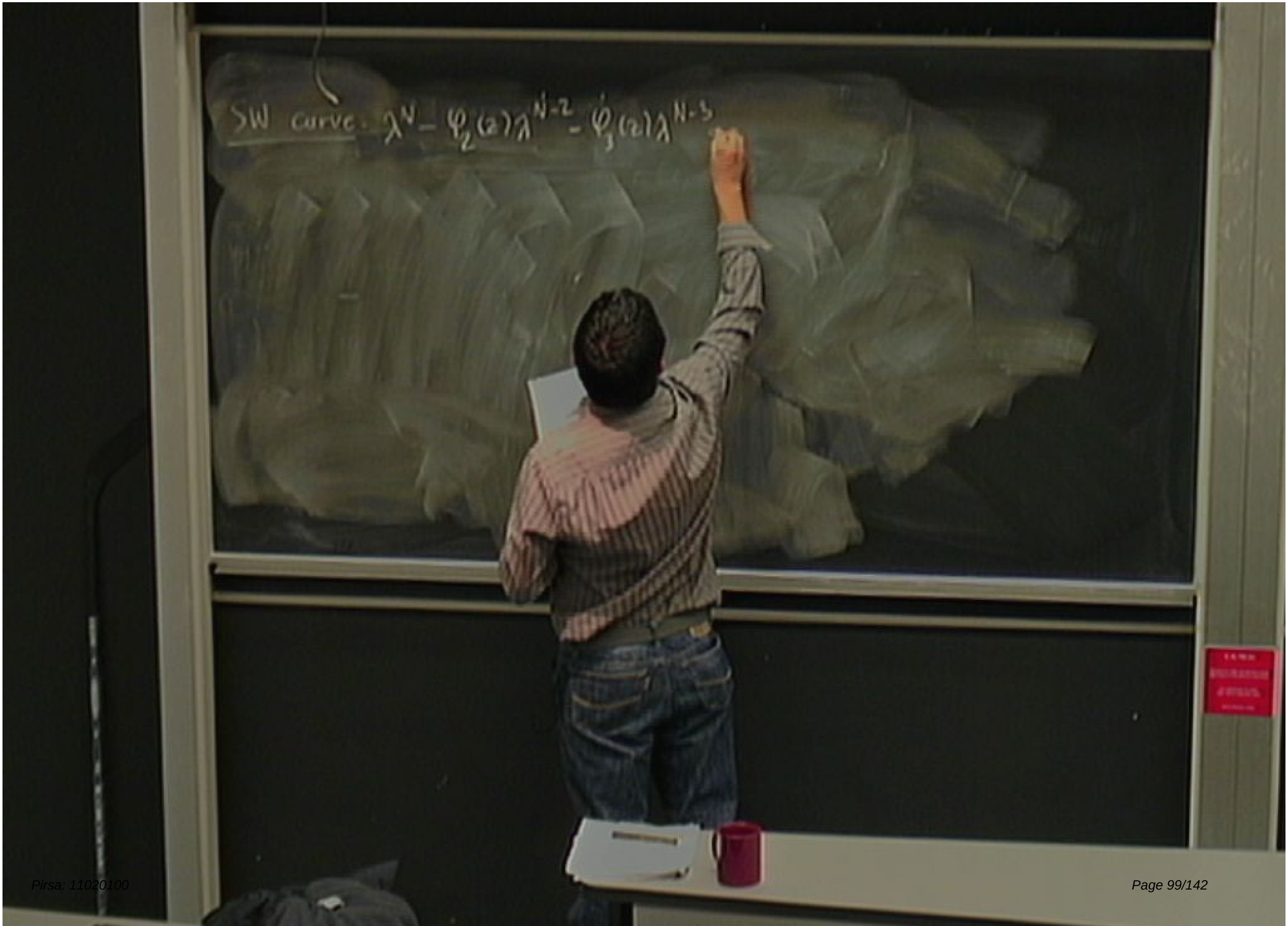

Cylinders:
su(3)
su(2)
Fixtures
(3-punctured spheres)
3 x 3
(free hypers)
E6 SCFT
(interacting SCFTs)
1 x 2
(free hypers)
Regular punctures : partitions of N (A N-1 th)

Degenerations of SU(3) $N_f=6$



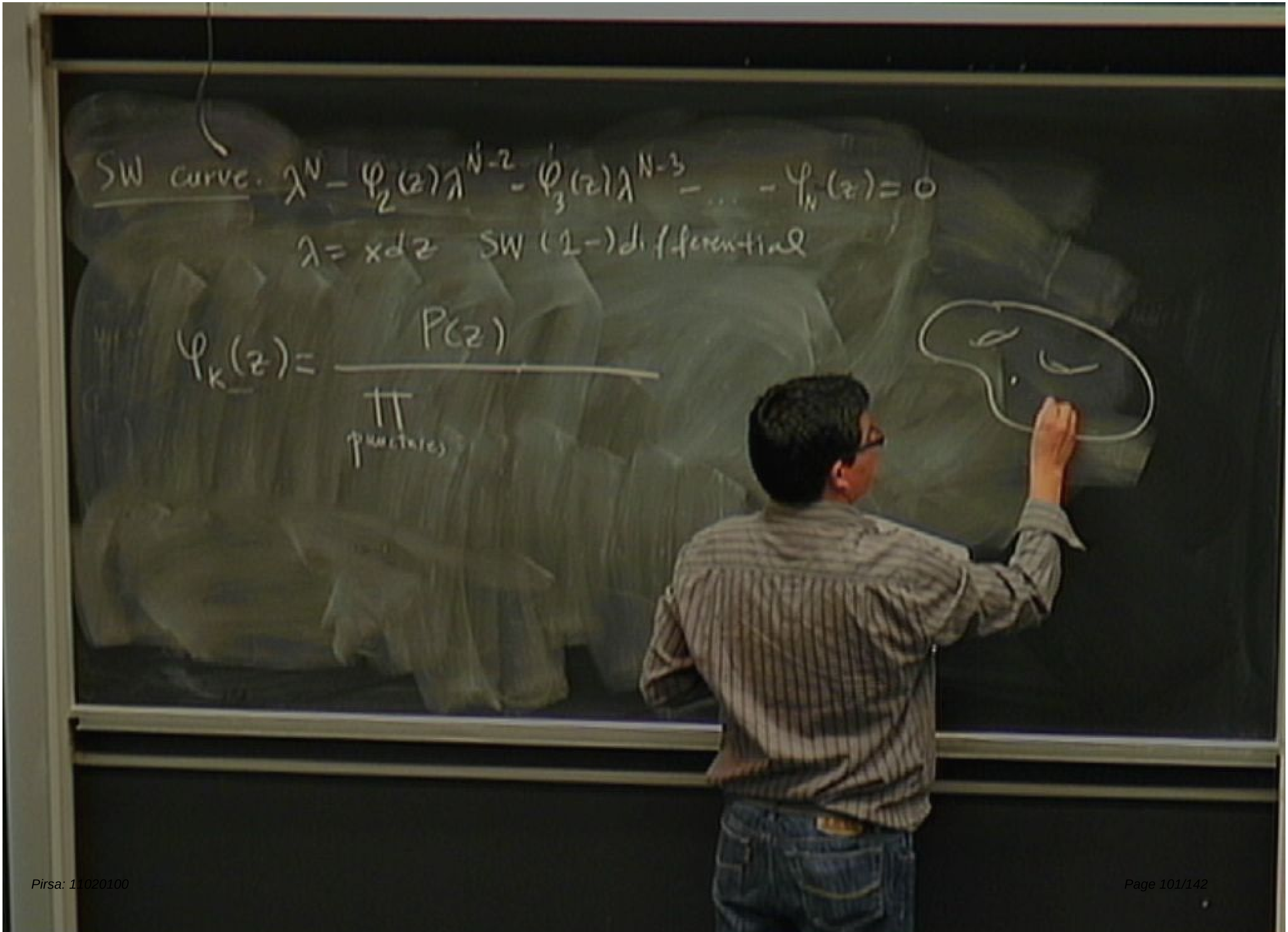
Can extract building blocks for A_2 th:

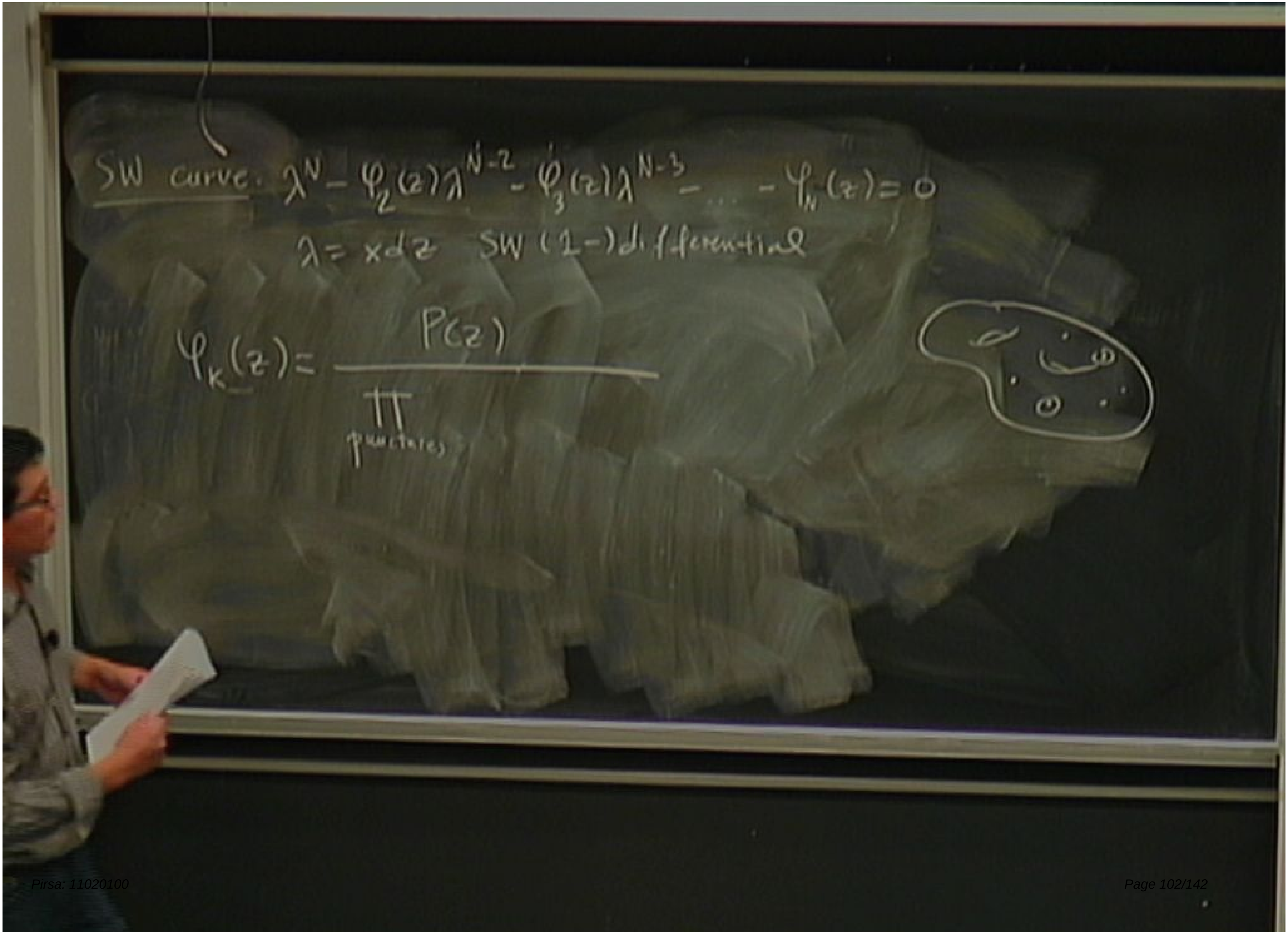
- Punctures: $\underbrace{\bullet\,,\ \odot}_{\text{regular}}$, $\odot'$ ← irreg puncture


SW curve: $\lambda^N - \varphi_2(z)\lambda^{N-2} - \varphi_3(z)\lambda^{N-3}$

SW curve: $\lambda^N - \varphi_2(z)\lambda^{N-2} - \varphi_3(z)\lambda^{N-3} - \dots - \varphi_N(z) = 0$

$\lambda = x\,dz$ SW (1-) differential


SW curve. $\lambda^N - \varphi_2(z)\lambda^{N-2} - \varphi_3(z)\lambda^{N-3} - \dots - \varphi_N(z) = 0$
$\lambda = x\,dz$ SW (1-)differential
$\varphi_k(z) = \dfrac{P(z)}{\prod_{\text{punctures}}}$


SW curve: λ^N − φ_2(z) λ^{N-2} − φ_3(z) λ^{N-3} − ... − φ_N(z) = 0
λ = x dz SW (1-) differential
φ_k(z) = P(z) / ∏ punctures

SW curve: $\lambda^N - \varphi_2(z)\lambda^{N-2} - \varphi_3(z)\lambda^{N-3} - \dots - \varphi_N(z) = 0$

$\lambda = x\,dz$ SW (1-)differential

$$\varphi_k(z) = \frac{P(z)}{\prod_{a \in \text{punctures}} (z - z_a)^{p_a^k}} (dz)^k$$

$k = 2, 3, \dots, N$

SW curve: $\lambda^N - \varphi_2(z)\lambda^{N-2} - \varphi_3(z)\lambda^{N-3} - \dots - \varphi_N(z) = 0$

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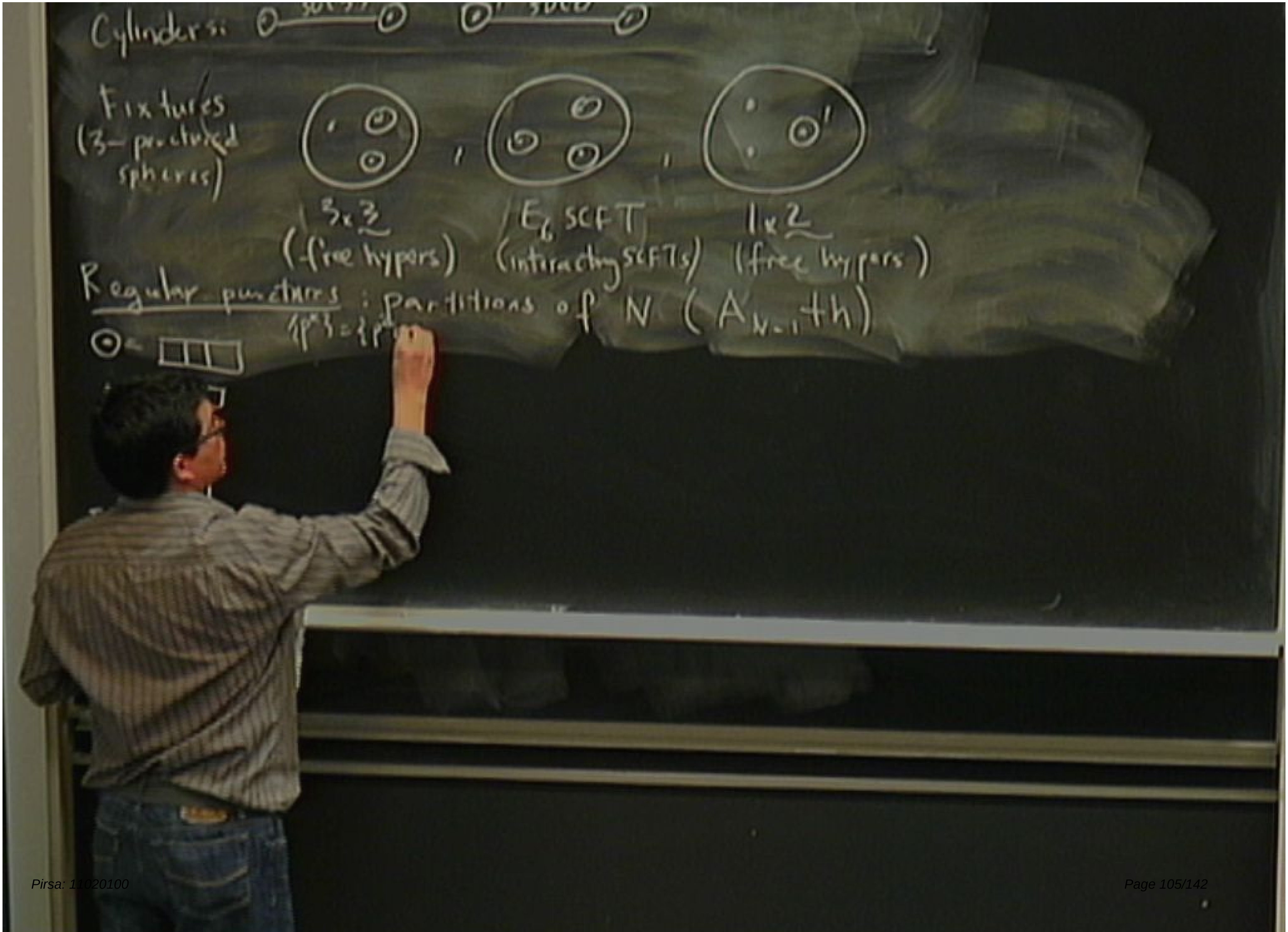
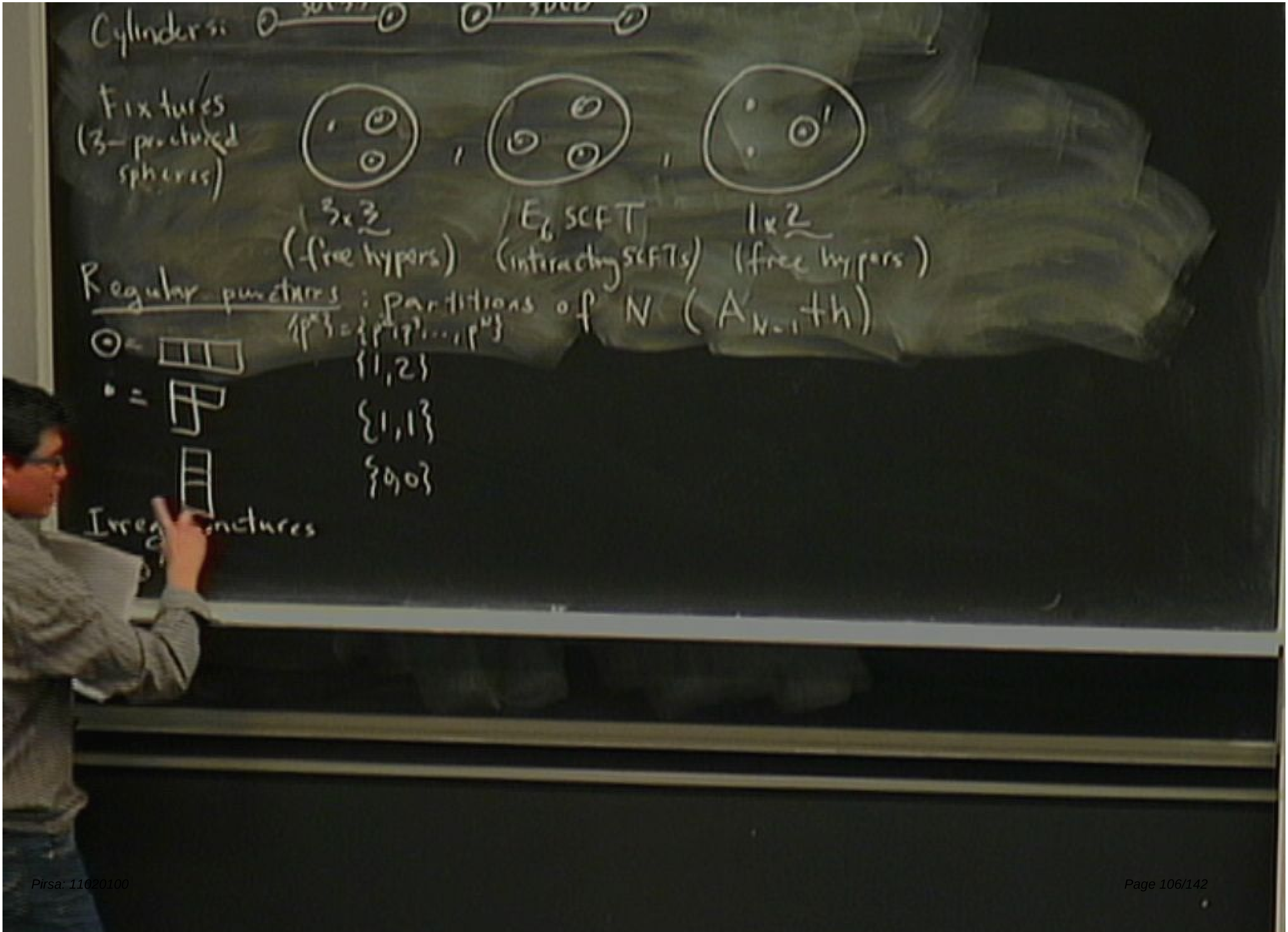
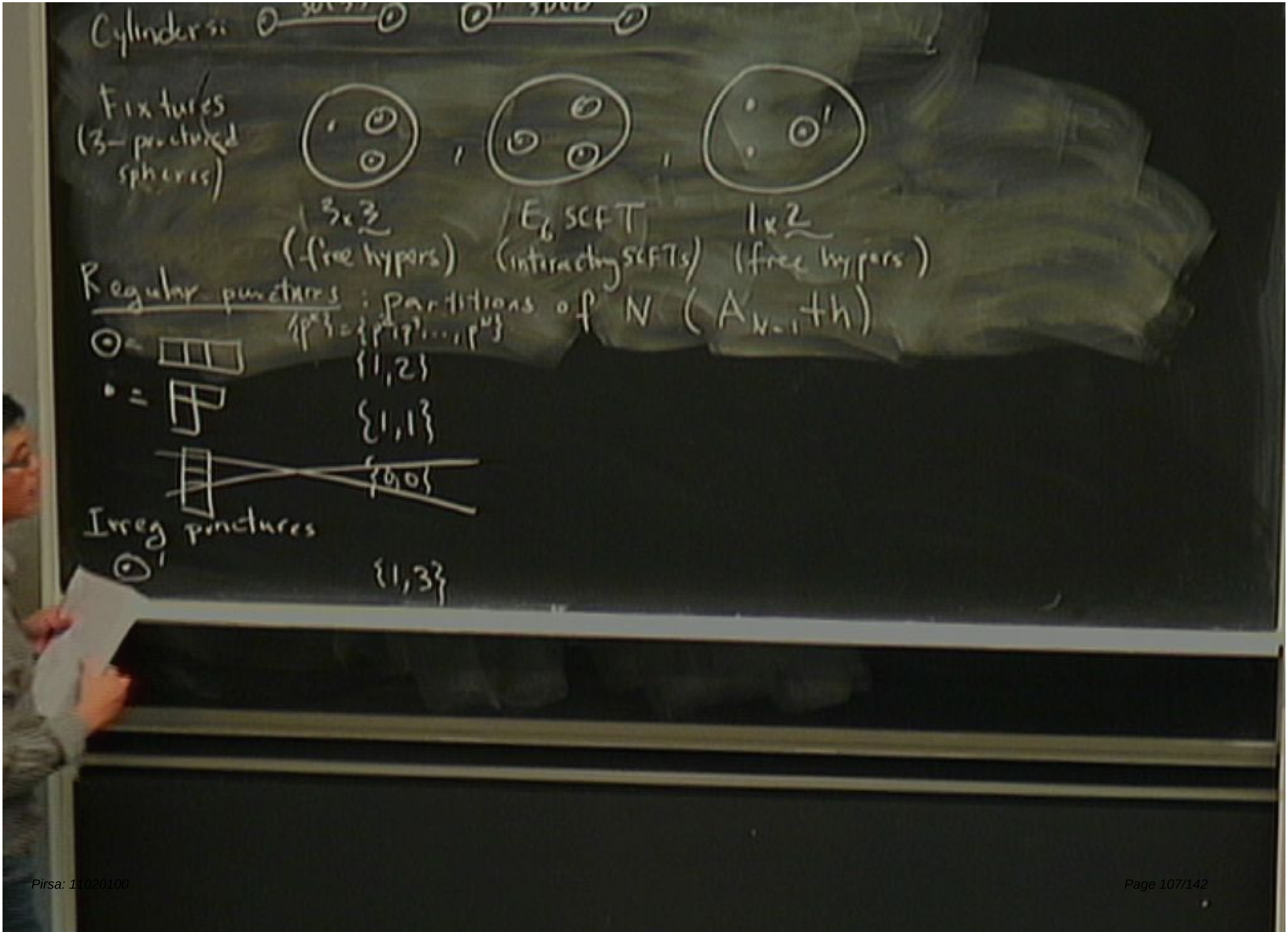

Cylinders:
Fixtures
(3-punctured
spheres)
3 x 3
(free hypers)
E6 SCFT
(interacting SCFTs)
1 x 2
(free hypers)
Regular punctures : partitions of N (A N-1 th)


Cylinders:
Fixtures
(3-punctured
spheres)
3 x 3
(free hypers)
E SCFT
(interacting SCFTs)
1 x 2
(free hypers)
Regular punctures : partitions of N (A th)
{1,2}
{1,1}
{0,0}
nctures


Cylinders:
Fixtures
(3-punctured spheres)
3 x 2
(free hypers)
E6 SCFT
(interacting SCFTs)
1 x 2
(free hypers)
Regular punctures : partitions of N (A N-1 th)
{1,2}
{1,1}
Irreg punctures
{1,3}

Irreg punctures

$\odot'$ $\{1,3\}$

SW curve. $\lambda^N - \varphi_2(z)\lambda^{N-2} - \varphi_3(z)\lambda^{N-3} - \dots - \varphi_N(z) = 0$

$\lambda = x\,dz$ SW (1-)differential

$$\varphi_K(z) = \frac{P(z)}{\prod_{a \in \text{punctures}} (z - z_a)^{p_a^K}} (dz)^K$$

$K = 2, 3, \dots, N$

Irreg punctures

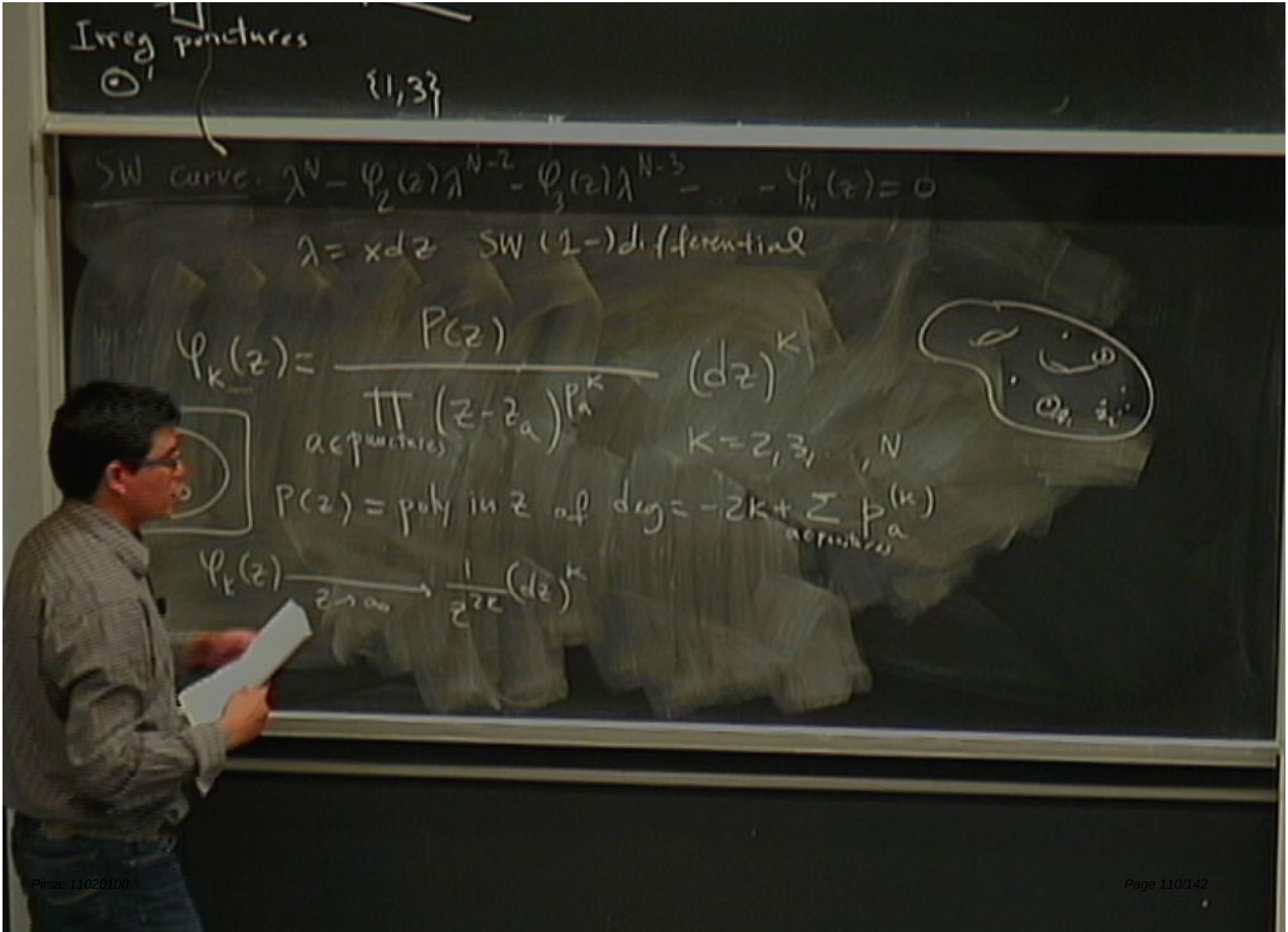
$\{1,3\}$

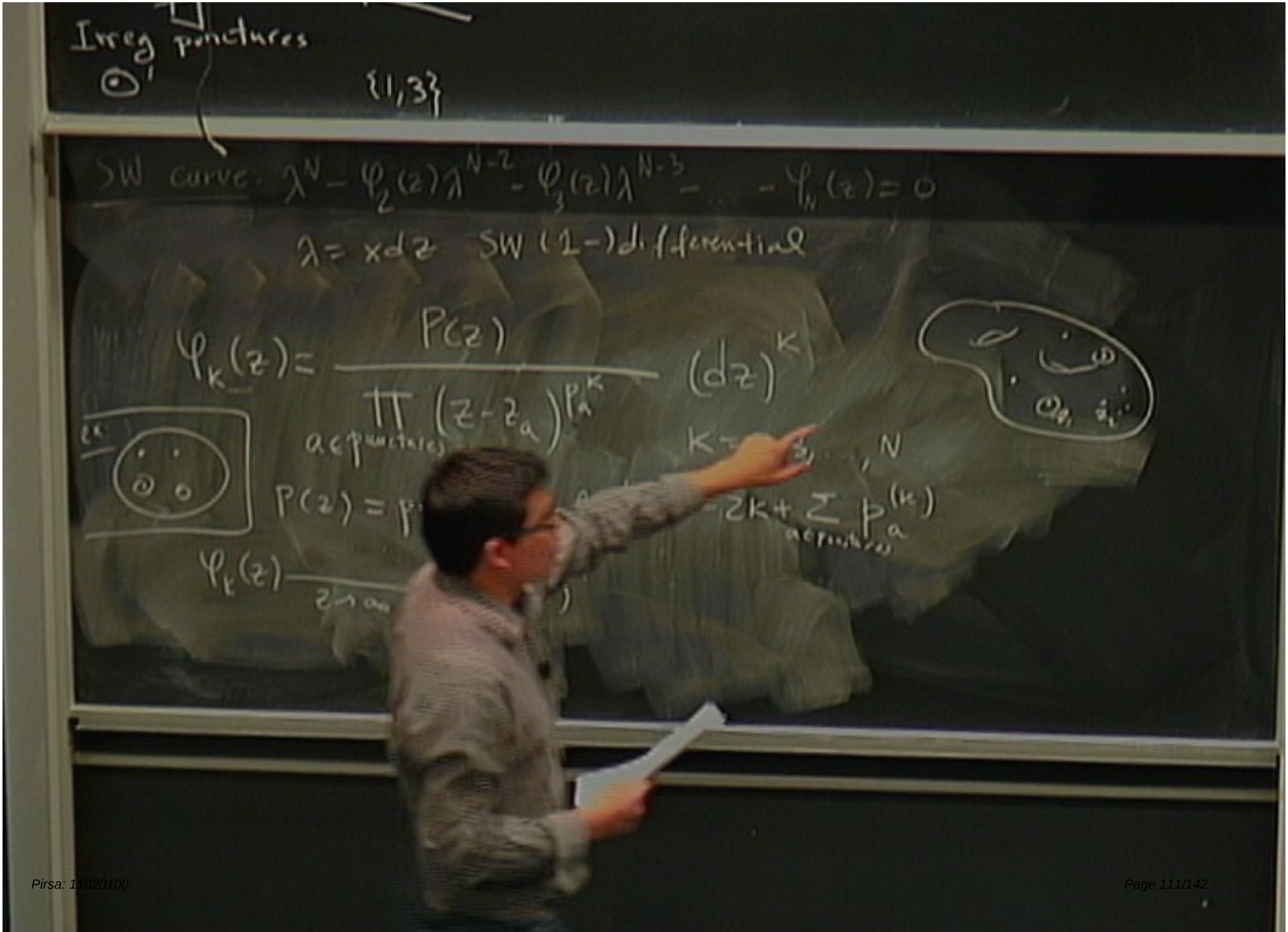
SW curve: $\lambda^N - \varphi_2(z)\lambda^{N-2} - \varphi_3(z)\lambda^{N-3} - \dots - \varphi_N(z) = 0$

$\lambda = x\,dz$ SW (1-)differential

$$\varphi_k(z) = \frac{P(z)}{\prod_{a \in \text{punctures}} (z - z_a)^{p_a^k}} (dz)^k \qquad k = 2, 3, \dots, N$$

$P(z)$ = poly in z of $\deg = -2k + \sum_{a \in \text{punctures}} p_a^{(k)}$


Irreg punctures
{1,3}
SW curve: $\lambda^N - \varphi_2(z)\lambda^{N-2} - \varphi_3(z)\lambda^{N-3} - \cdots - \varphi_N(z) = 0$
$\lambda = x\,dz$ SW (1-)differential
$\varphi_K(z) = \frac{P(z)}{\prod_{a \in \text{punctures}} (z - z_a)^{p_a^K}} (dz)^K$
$K = 2, 3, \ldots, N$
$P(z)$ = poly in z of deg $= -2K + \sum_{a \in \text{punctures}} p_a^{(K)}$
$\varphi_K(z) \xrightarrow[z \to \infty]{} \frac{1}{z^{2K}} (dz)^K$


Irreg punctures
{1,3}
SW curve: λ^N − φ_2(z)λ^{N−2} − φ_3(z)λ^{N−3} − … − φ_N(z) = 0
λ = x dz SW (1-)differential
φ_k(z) = P(z) / ∏_{a∈punctures} (z − z_a)^{p_a^k} (dz)^k
P(z) =
φ_k(z)

Irreg punctures

$\{1,3\}$

SW curve: $\lambda^N - \varphi_2(z)\lambda^{N-2} - \varphi_3(z)\lambda^{N-3} - \dots - \varphi_N(z) = 0$

$\lambda = x\,dz$ SW (1-)differential

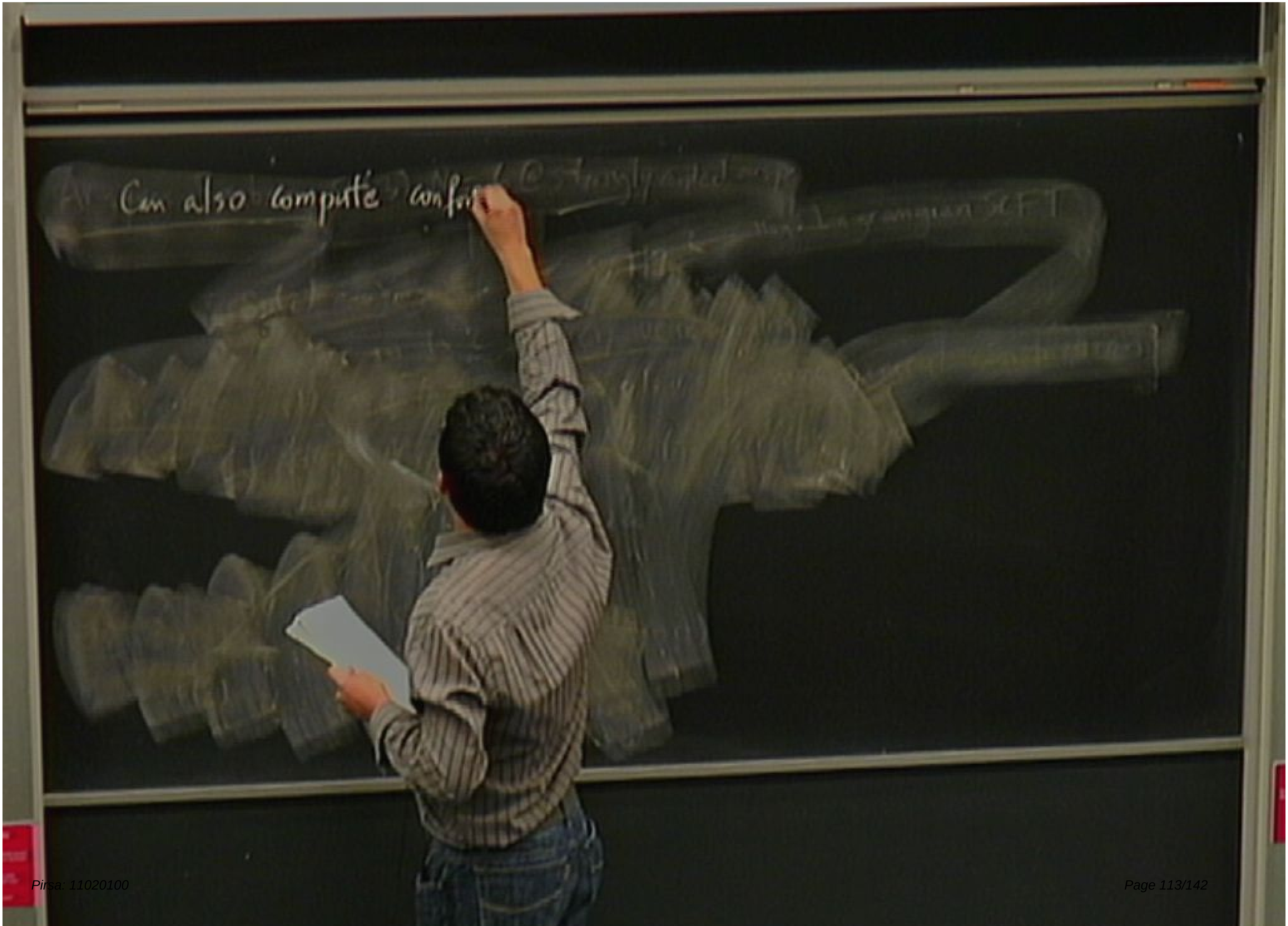
$$\varphi_k(z) = \frac{P(z)}{\prod_{a\in \text{punctures}} (z-z_a)^{p_a^{(k)}}} (dz)^k \qquad k = 2,3,\dots,N$$

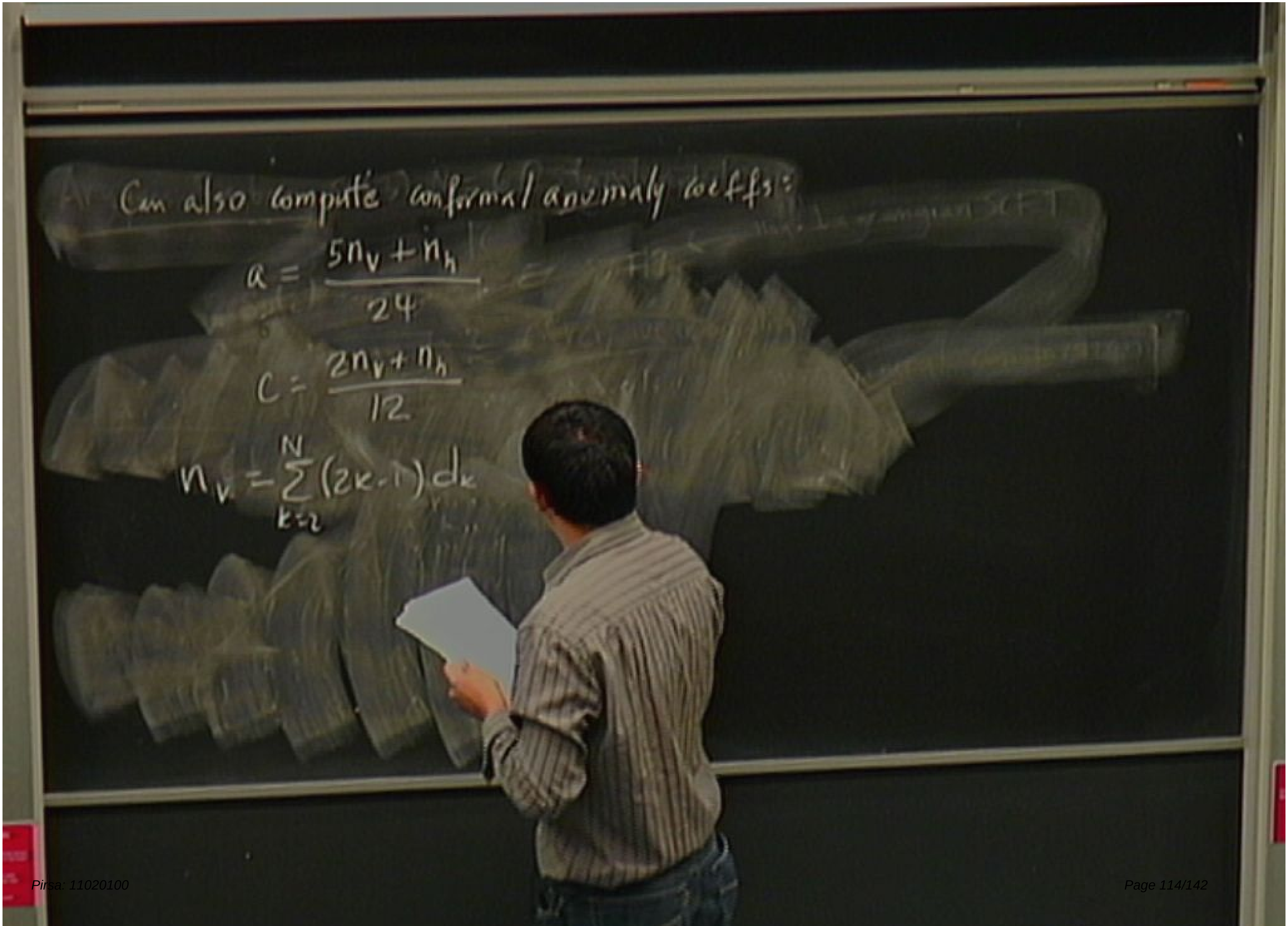
ex:

$P(z) = \text{poly in } z \text{ of } \deg = -2k + \sum_{a\in\text{punctures}} p_a^{(k)}$

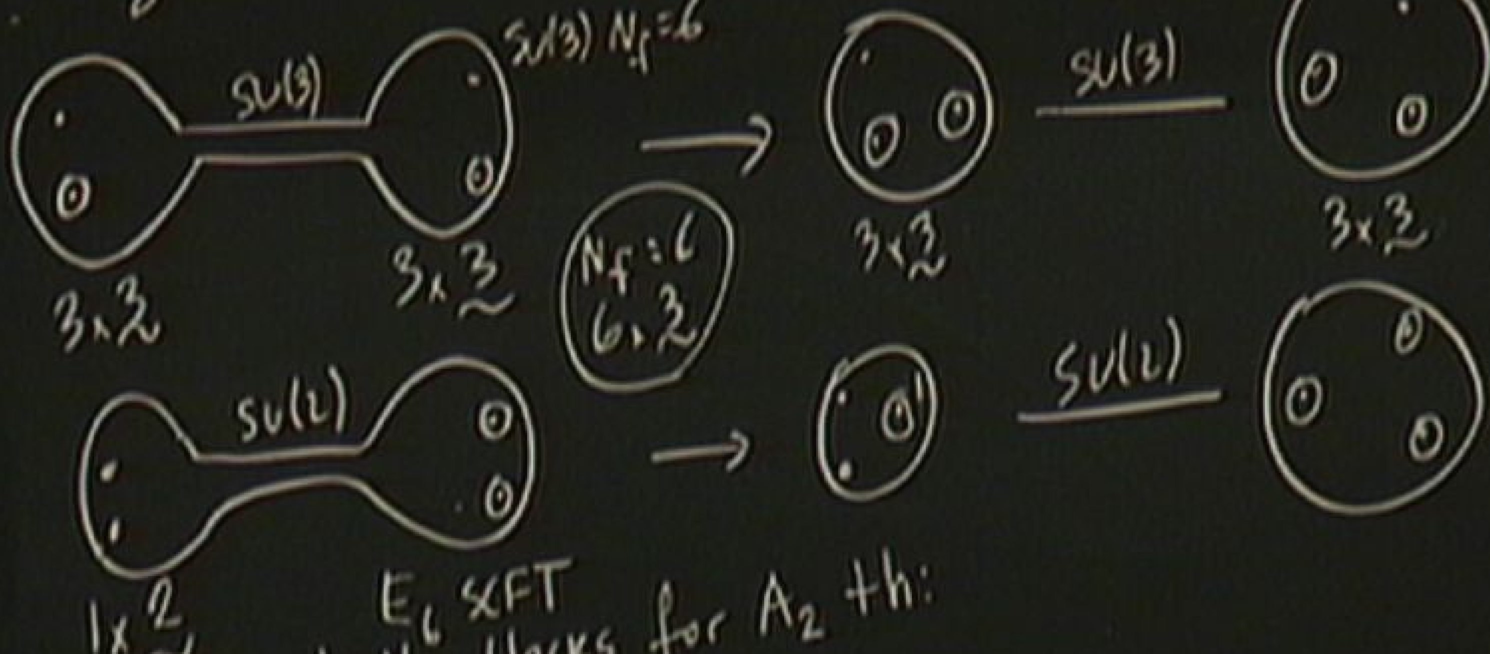
$(1-g)$

$$\varphi_k(z) \xrightarrow[z\to\infty]{} \frac{1}{z^{2k}}(dz)^k$$


Can also compute

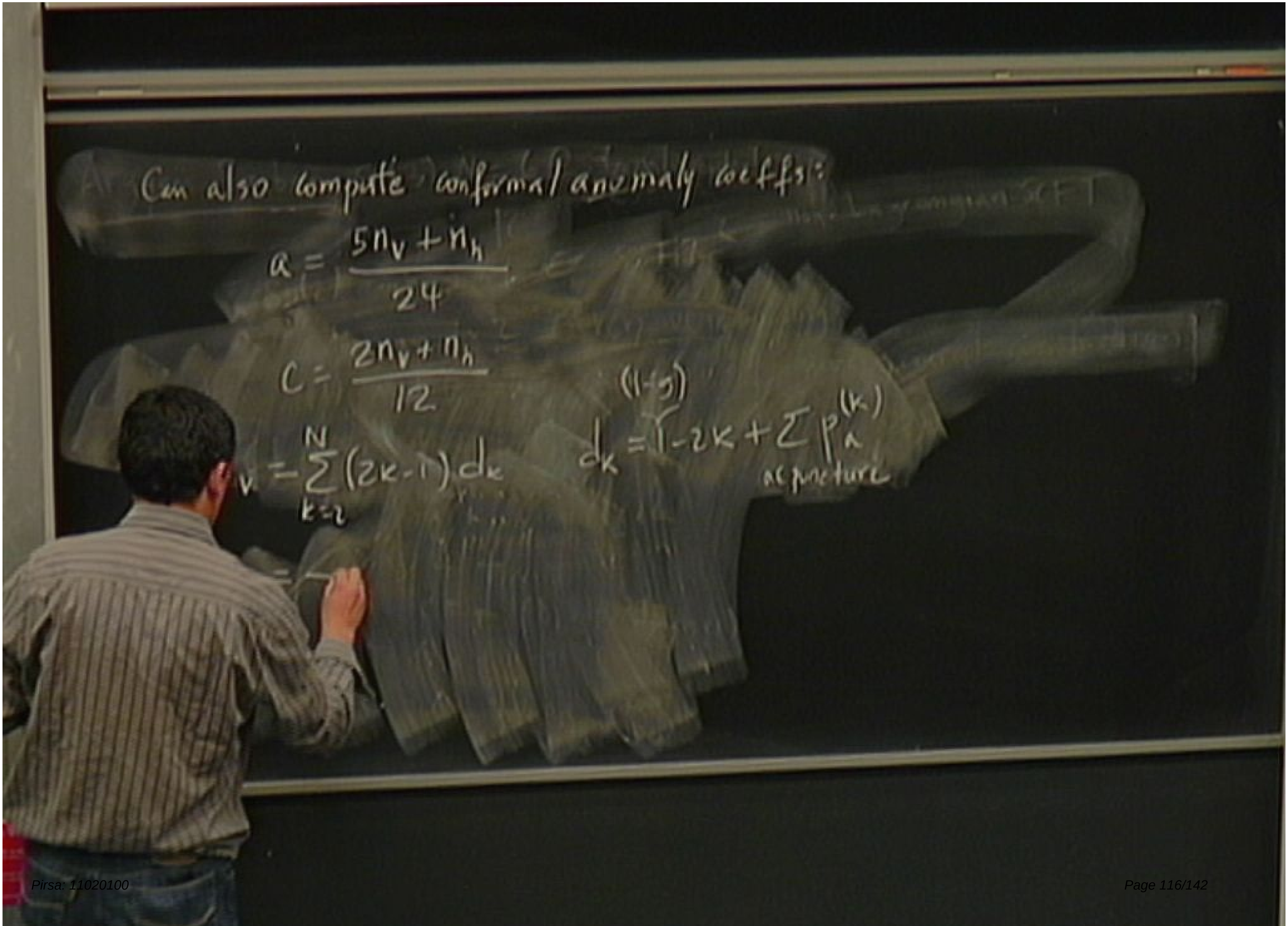

Can also compute conformal anomaly coeffs:
$a = \frac{5n_v + n_h}{24}$
$c = \frac{2n_v + n_h}{12}$
$n_v = \sum_{k=2}^{N} (2k-1) d_k$

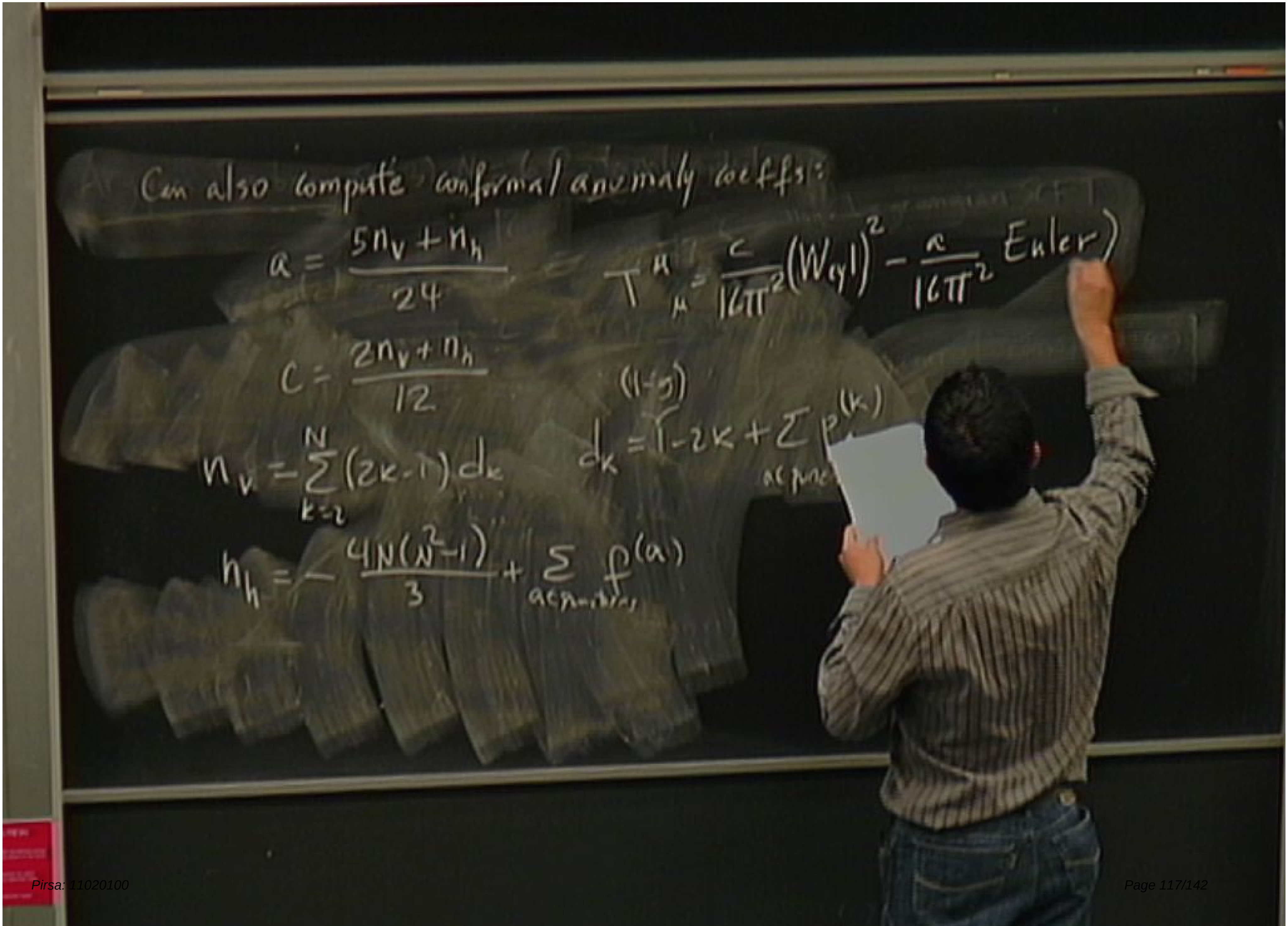
Degenerations of SU(3) $N_f=6$



Can extract building blocks for A_2 th:

– punctures: $\underbrace{\bullet\,,\ \odot}_{\text{regular}}\,,\ \odot'$ ← irreg puncture


Can also compute conformal anomaly coeffs:
$a = \frac{5n_v + n_h}{24}$
$c = \frac{2n_v + n_h}{12}$
$v = \sum_{k=2}^{N} (2k-1) d_k$
$d_k = \underbrace{1}_{(1-g)} - 2k + \sum_{a \in \text{puncture}} p_a^{(k)}$


Can also compute conformal anomaly coeffs:
$a = \frac{5n_v + n_h}{24}$
$T^\mu{}_\mu = \frac{c}{16\pi^2}(\mathrm{Weyl})^2 - \frac{a}{16\pi^2}\,\mathrm{Euler})$
$c = \frac{2n_v + n_h}{12}$
$(1-g)$
$n_v = \sum_{k=2}^{N}(2k-1)d_k$
$d_k = 1 - 2k + \sum p^{(k)}$
$n_h = -\frac{4N(N^2-1)}{3} + \sum f^{(a)}$

Can also compute conformal anomaly coeffs:

$$a = \frac{5n_v + n_h}{24} \qquad T^\mu{}_\mu = \frac{c}{16\pi^2}(\text{Weyl})^2 - \frac{a}{16\pi^2}(\text{Euler})$$

$$c = \frac{2n_v + n_h}{12}$$

$$n_v = \sum_{k=2}^{N} (2k-1)\, d_k \qquad d_k = \overbrace{1-2k}^{(1-g)} + \sum_{a \in \text{puncture}} p_a^{(k)}$$

$$n_h = -\frac{4N(N^2-1)}{3} + \sum_{a \in \text{punctures}} f^{(a)}$$

Can also compute conformal anomaly coeffs:

$$a = \frac{5n_v + n_h}{24} \qquad T^\mu{}_\mu = \frac{c}{16\pi^2}(\text{Weyl})^2 - \frac{a}{16\pi^2}(\text{Euler})$$

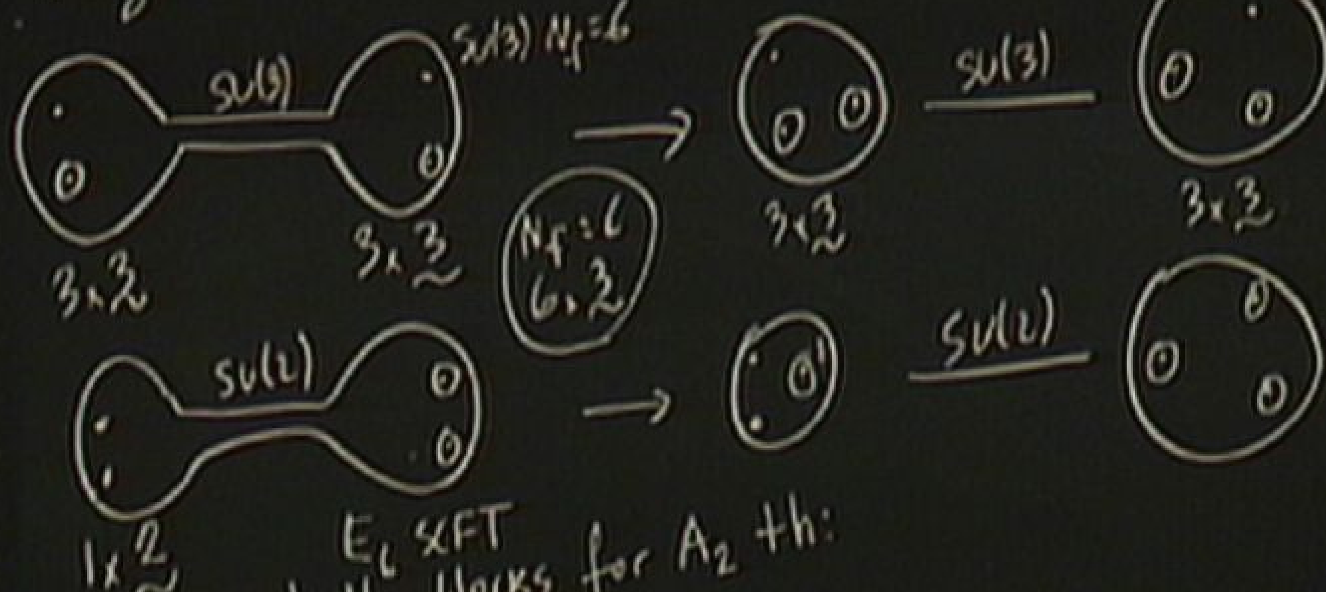
$$c = \frac{2n_v + n_h}{12}$$

$$n_v = -\sum_{k=2}^{N} (2k-1)\, d_k \qquad d_k = (1-g)(1-2k) + \sum_{a \in \text{puncture}} p_k^{(a)}$$

$$n_h = -\frac{4N(N^2-1)}{3} + \sum_{a \in \text{punctures}} f^{(a)}$$

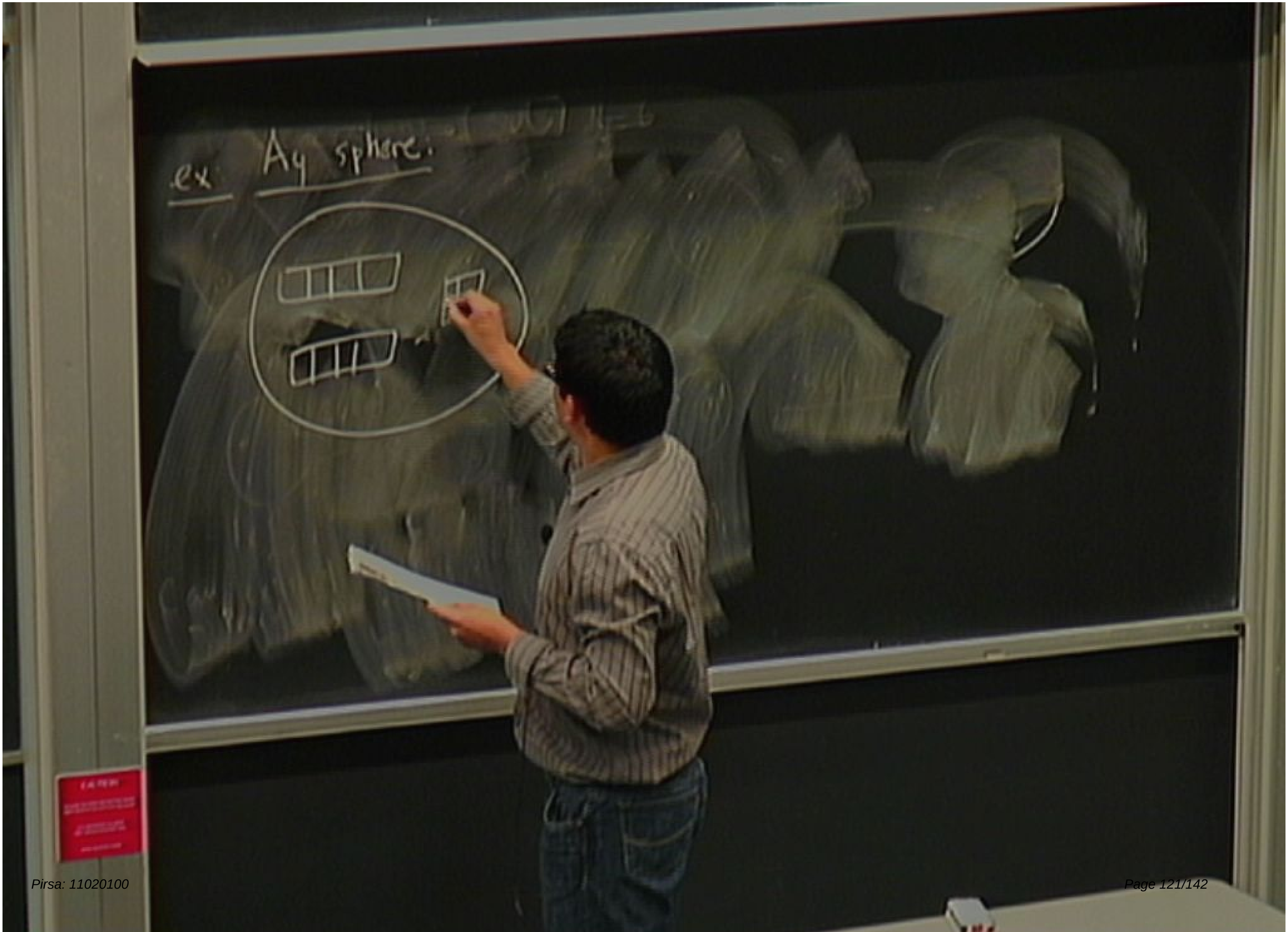
$$f^{(a)} = \frac{1}{2}\left(-N + \sum \ell_r^2\right) + \sum (2k-1)\, p_k$$

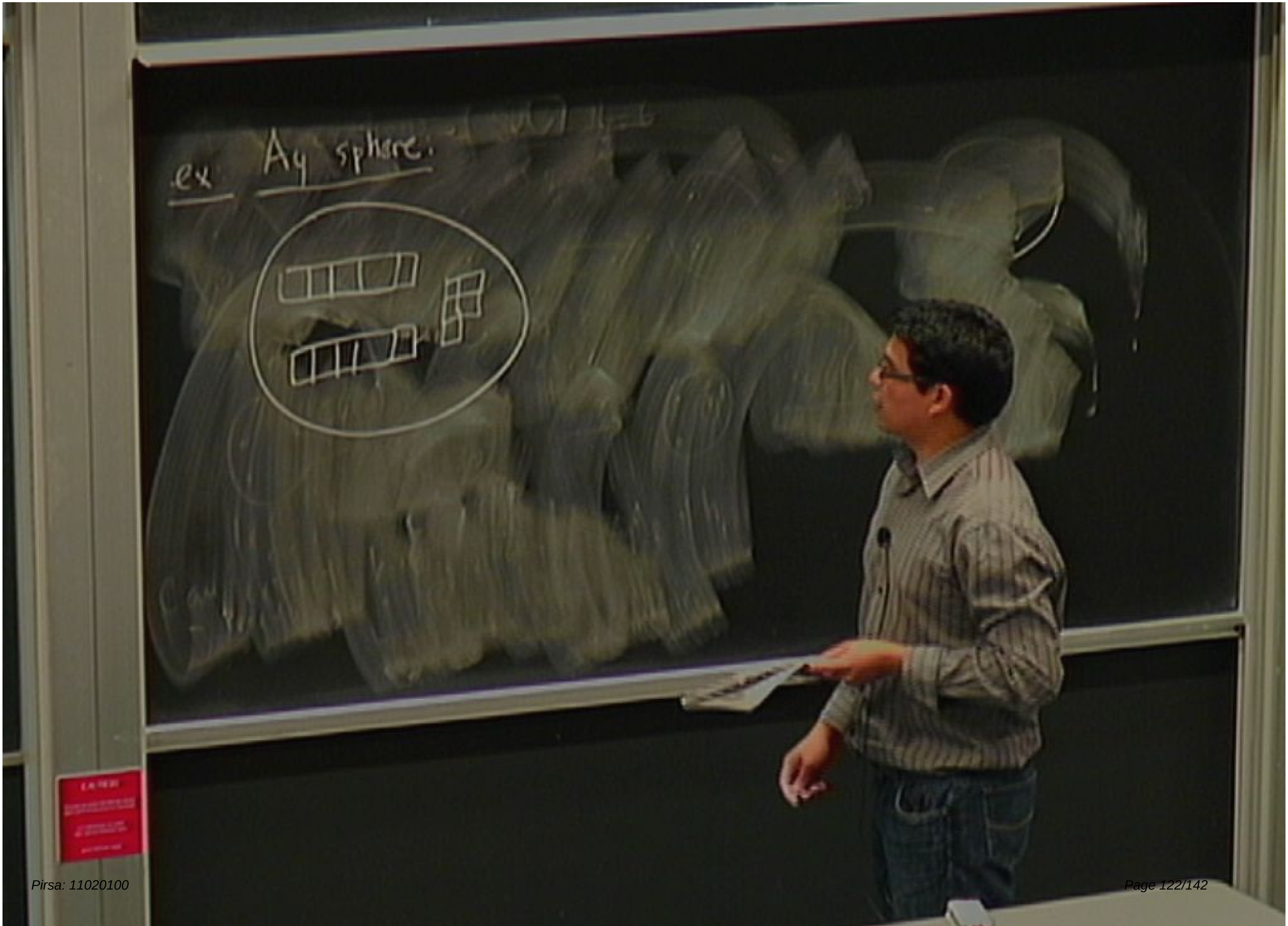
Degenerations of $SU(3)$ $N_f=6$

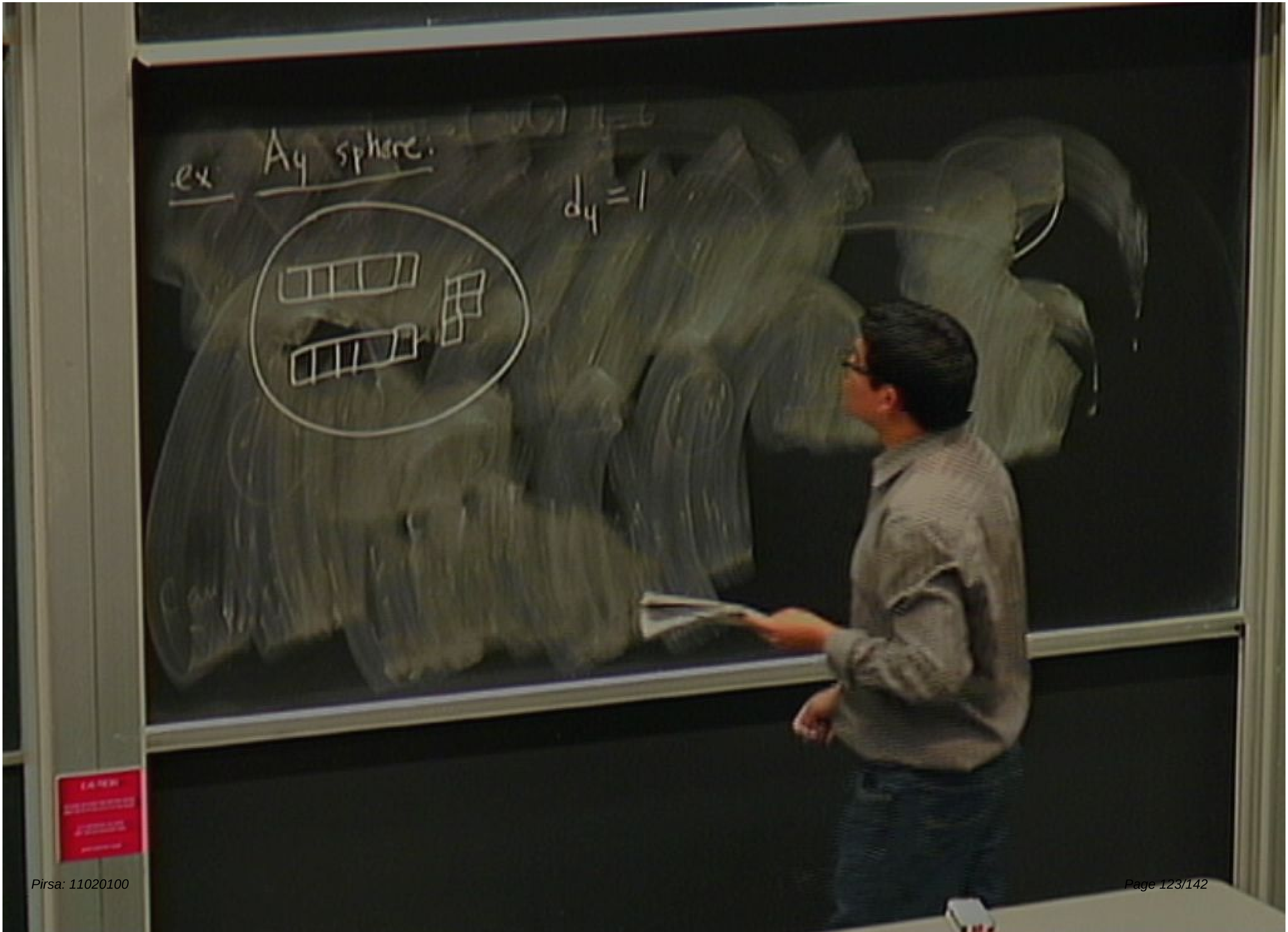


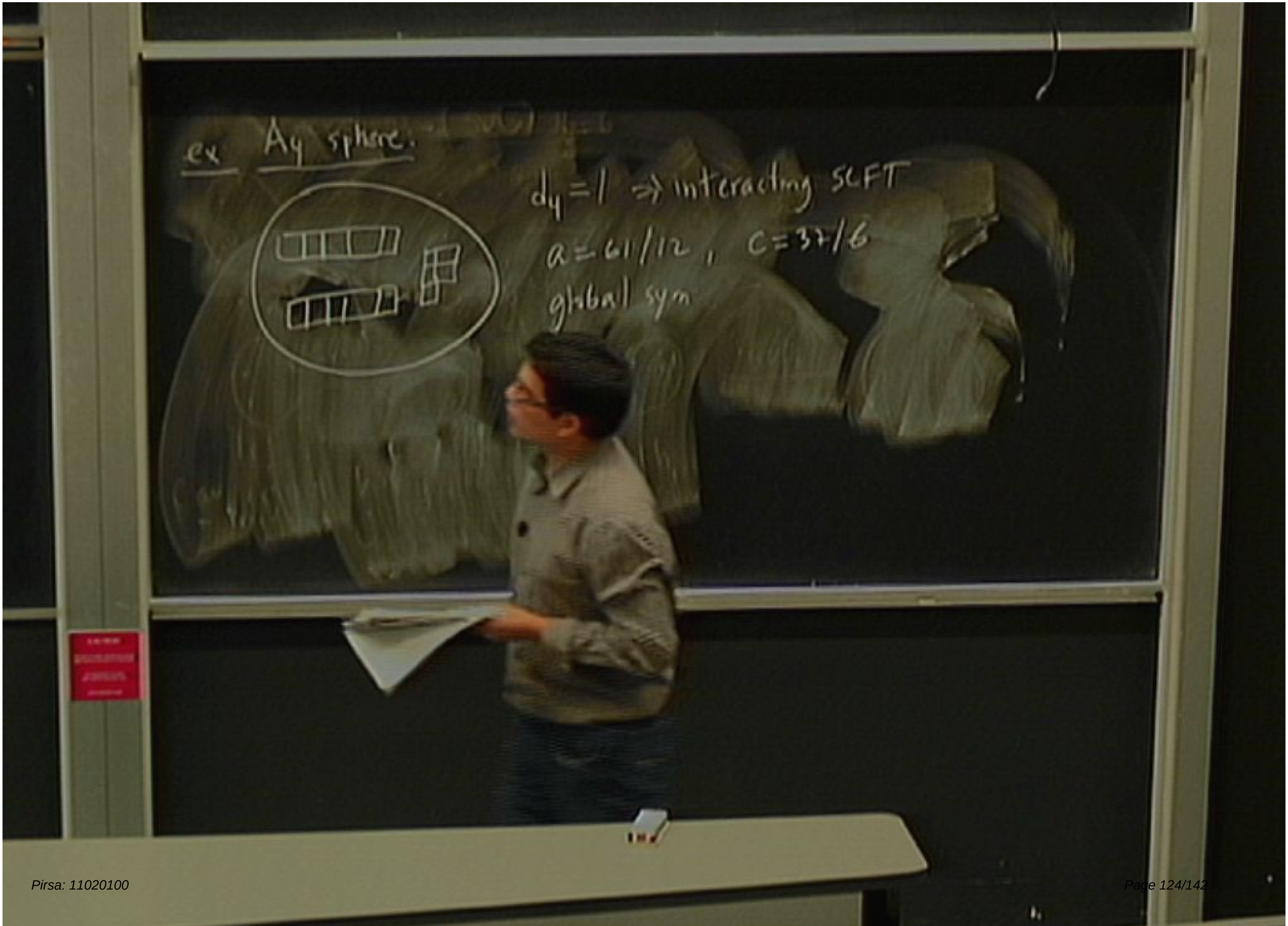
Can extract building blocks for A_2 th:

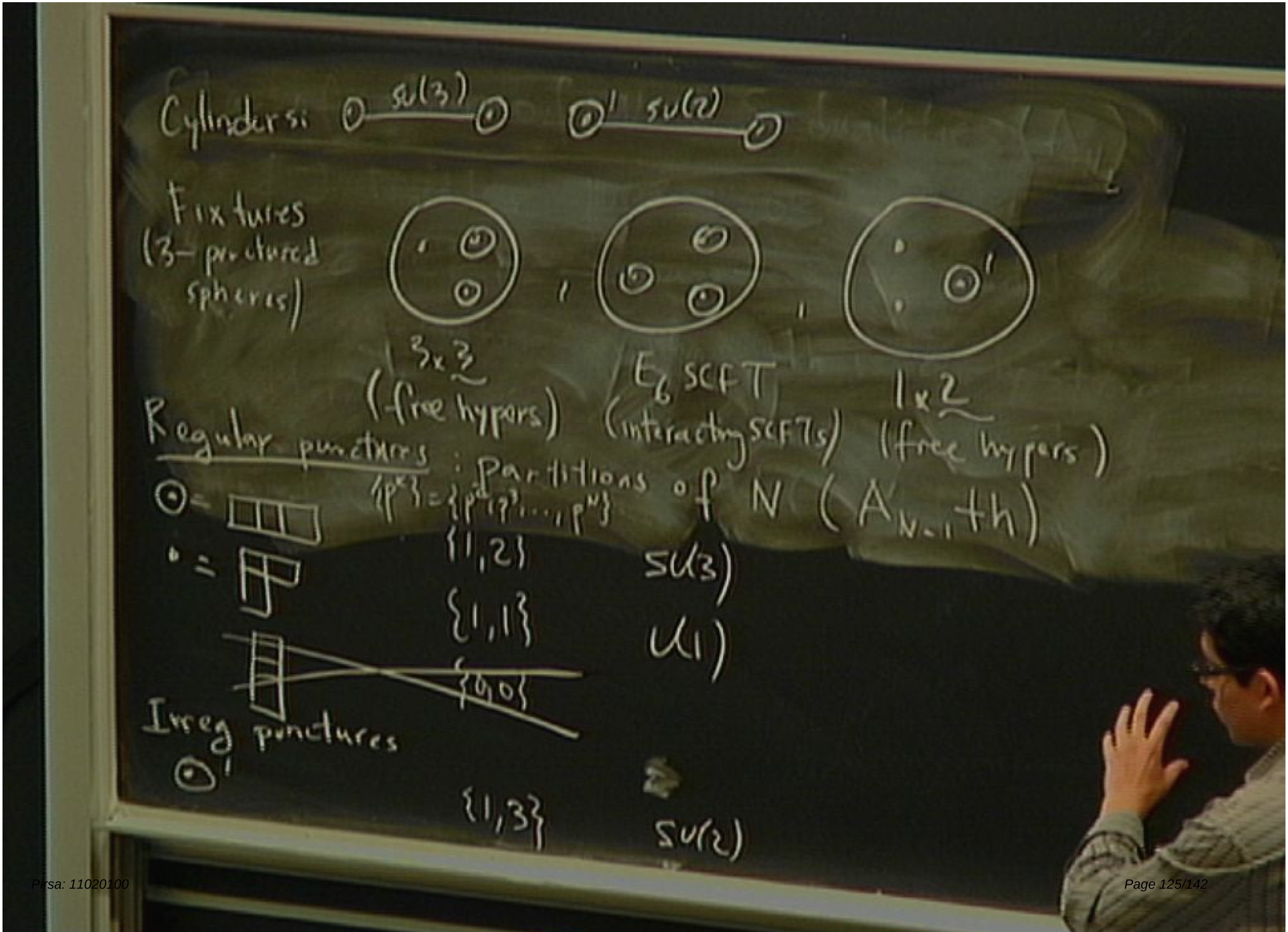
- punctures: $\underbrace{\bullet\ ,\ \odot}_{\text{regular}}$, $\odot$ ← irreg puncture


ex
A4 sphere:


ex
sphere.


ex A4 sphere:
d4 = 1

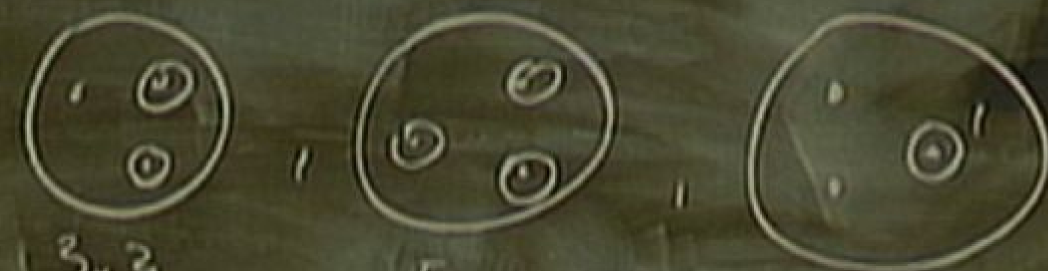

ex
A4 sphere:
d4 = 1 ⇒ interacting SCFT
a = 61/12, c = 37/6
global sym



Cylinders: su(3) su(2)
Fixtures (3-punctured spheres)
3 x 3 (free hypers)
E6 SCFT (interacting SCFTs)
1 x 2 (free hypers)
Regular punctures : Partitions of N (A N-1 th)
{p^k} = {p^1, p^2, ..., p^N}
{1,2} SU(3)
{1,1} U(1)
{0,0}
Irreg punctures
{1,3} SU(2)

Cylinders: $\odot \overset{SU(3)}{—} \odot$ $\quad \odot' \overset{SU(2)}{—} \odot$

Fixtures
(3-punctured spheres)



$\underset{\sim}{3} \times \underset{\sim}{3}$ (free hypers) , E_6 SCFT (interacting SCFTs) , $1 \times \underset{\sim}{2}$ (free hypers)

<u>Regular punctures</u> : Partitions of N (A_{N-1} th)

$\{p^k\} = \{p^1, p^2, \ldots, p^N\}$ $\quad$ <u>global sym</u>

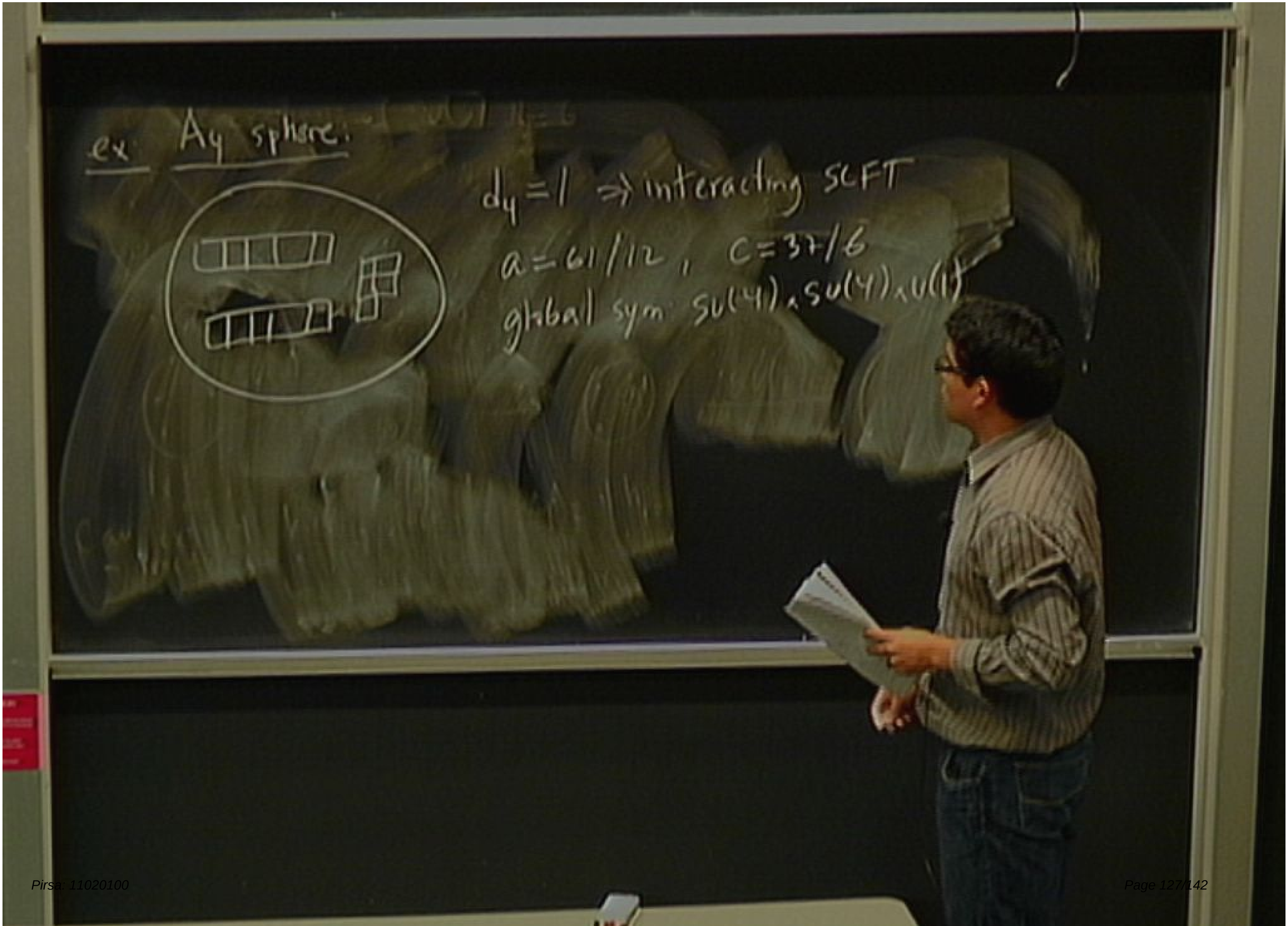
$\odot =$ $\quad \{1,2\}$ $\quad SU(3)$

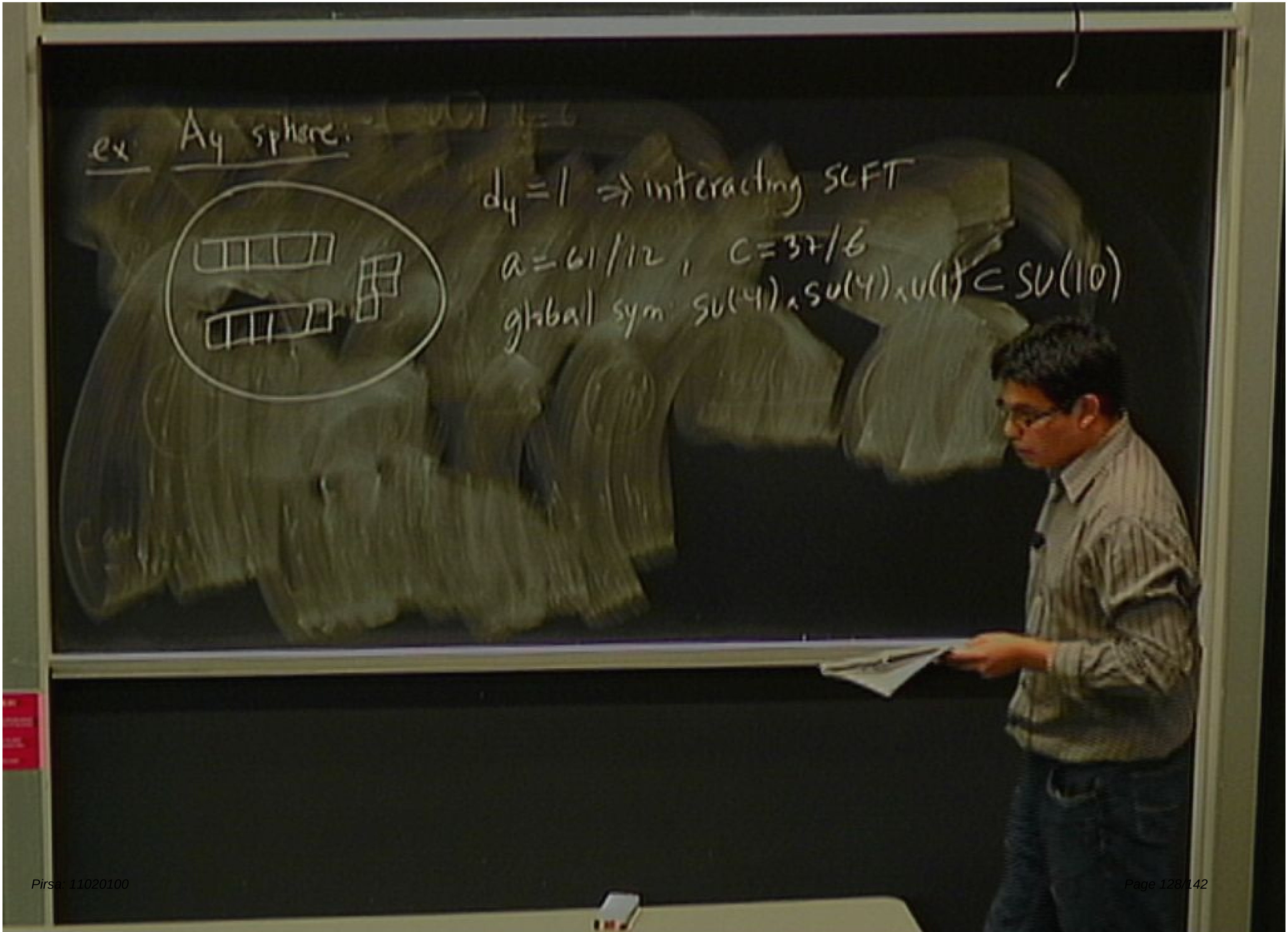
$\bullet =$ $\quad \{1,1\}$ $\quad U(1)$

~~{0,0}~~

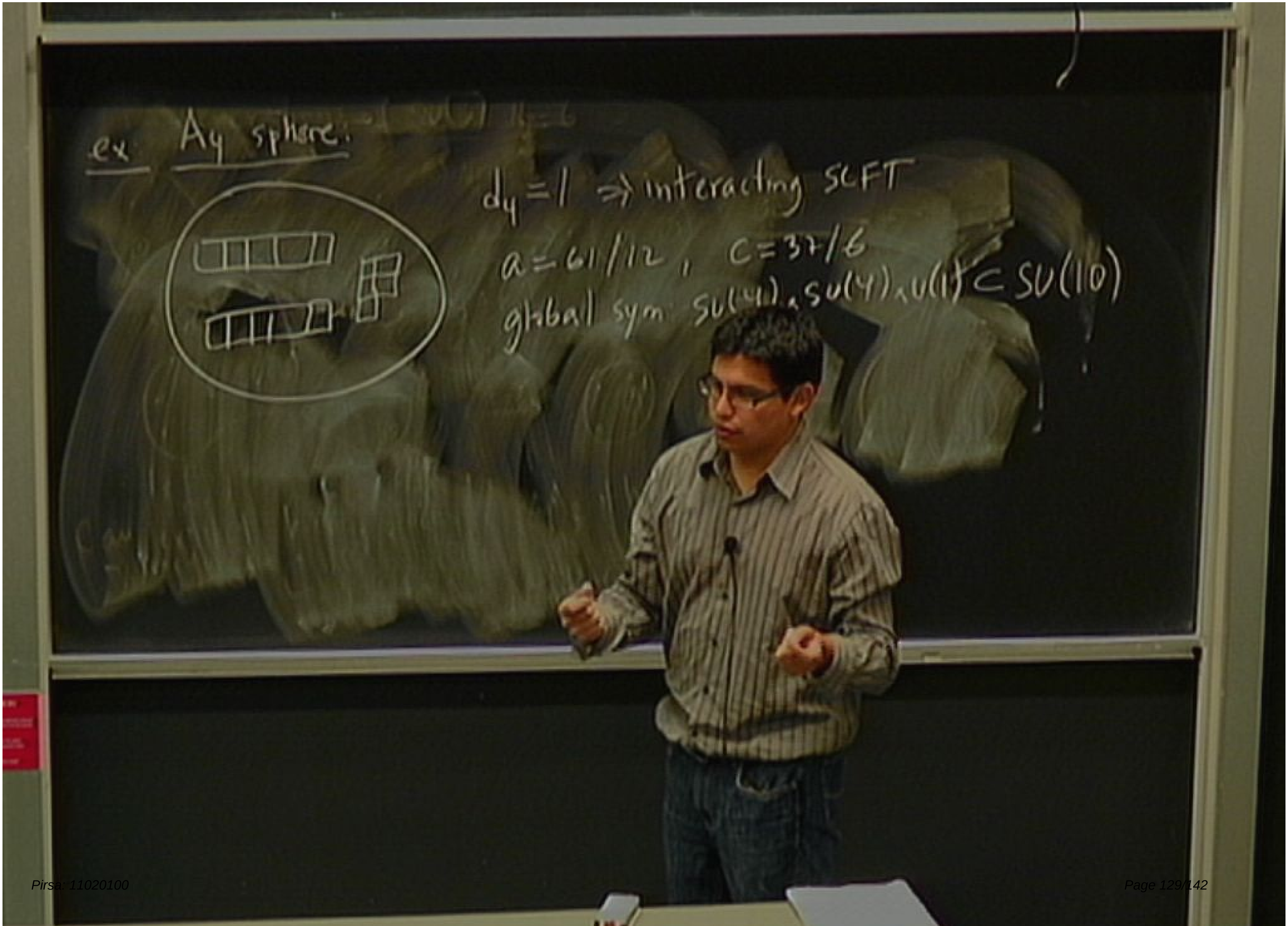
Irreg punctures

$\odot'$ $\quad \{1,3\}$ $\quad SU(2)$


ex A4 sphere:
d4 = 1 ⇒ interacting SCFT
a = 61/12 , c = 37/6
global sym SU(4)×SU(4)×U(1)



ex A4 sphere:
d4 = 1 ⇒ interacting SCFT
a = 61/12 , c = 37/6
global sym SU(4) x SU(4) x U(1) ⊂ SU(10)


ex A4 sphere:
d4 = 1 ⇒ interacting SCFT
a = 61/12 , c = 37/6
global sym SU(4)×SU(4)×U(1) ⊂ SU(10)

Can also compute conformal anomaly coeffs:

$$a = \frac{5n_v + n_h}{24} \qquad T^{\mu}{}_{\mu} = \frac{c}{16\pi^2}(\text{Weyl})^2 - \frac{a}{16\pi^2}(\text{Euler})$$

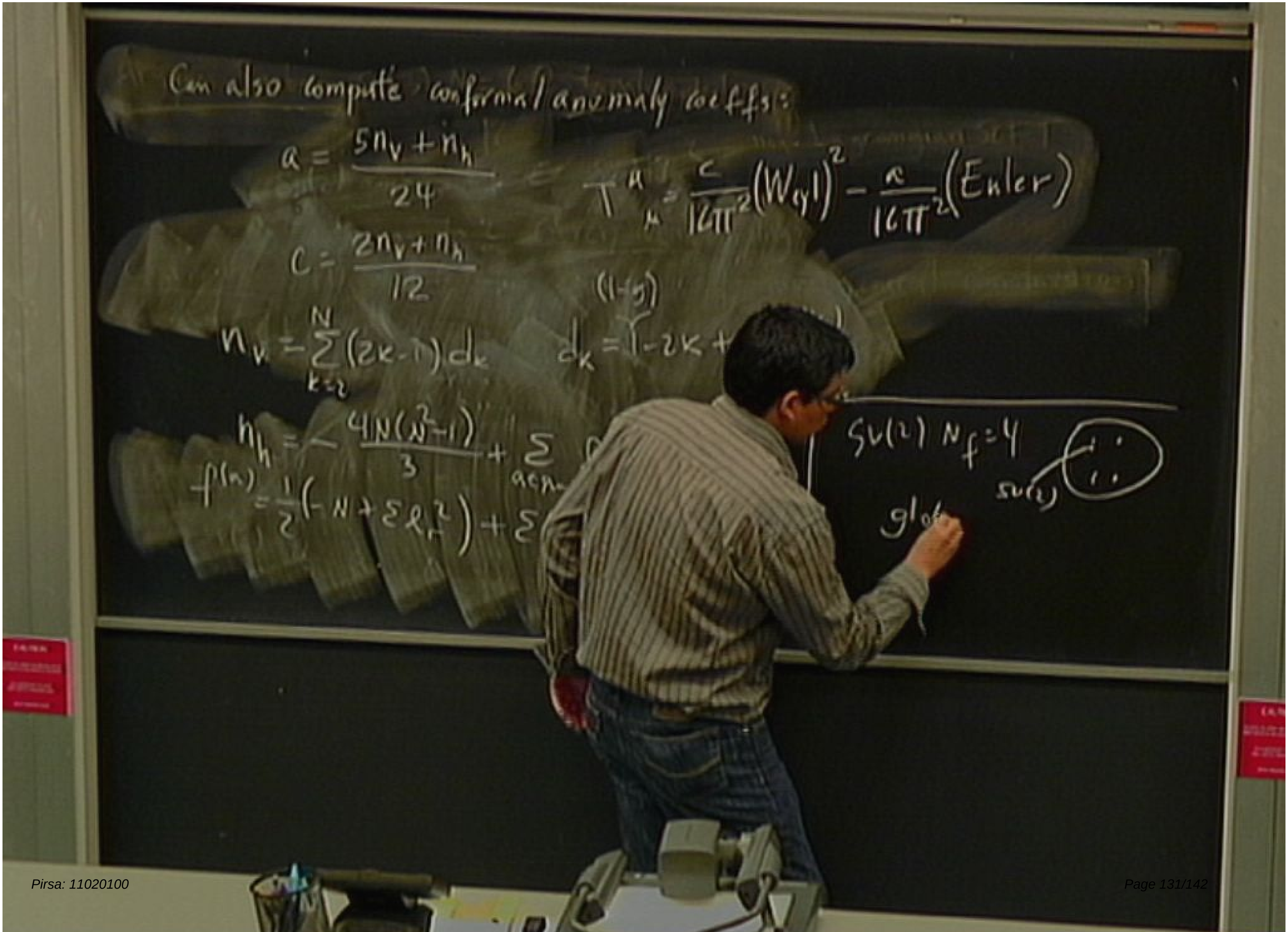
$$c = \frac{2n_v + n_h}{12}$$

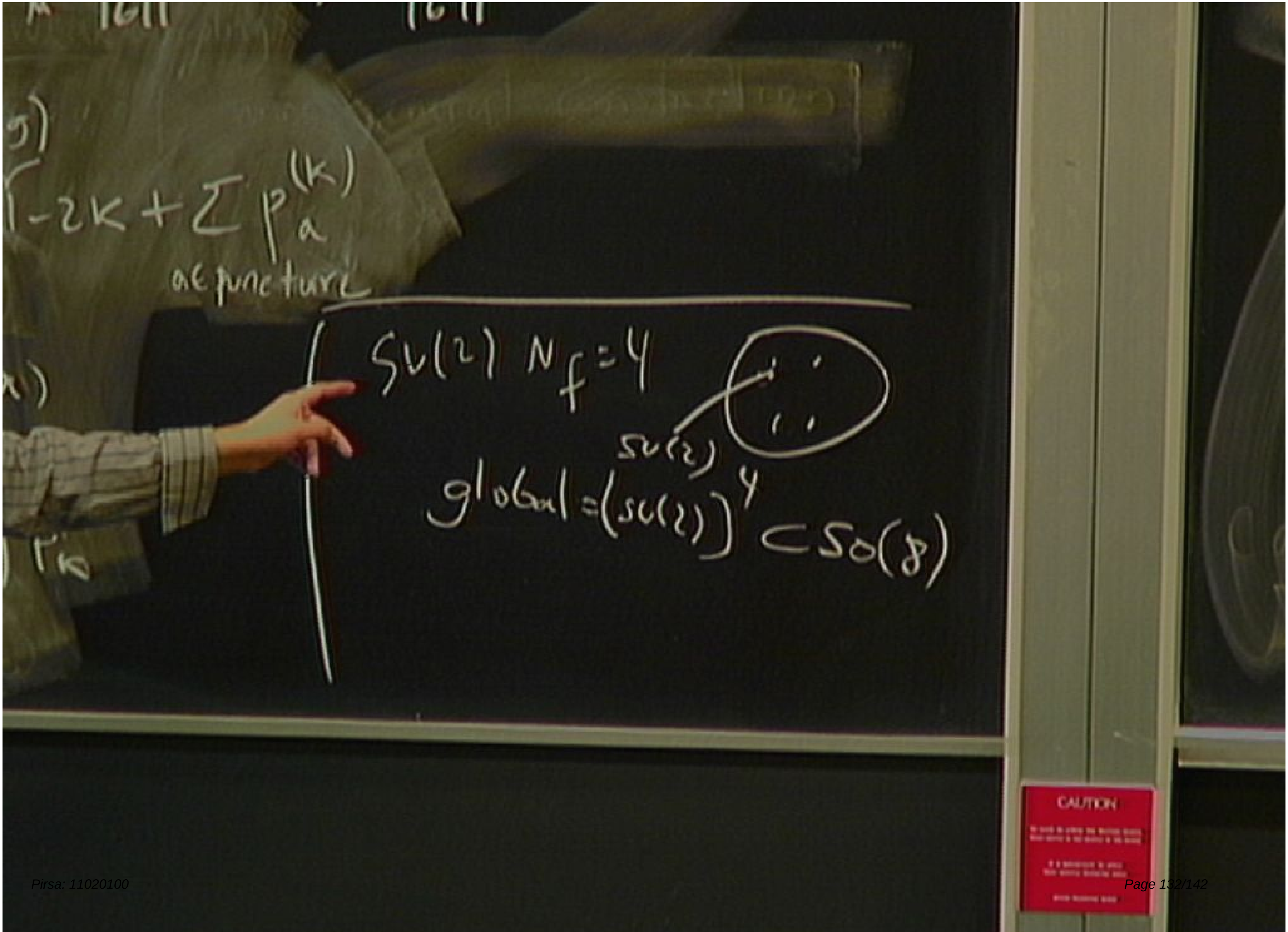
$$n_v = \sum_{k=2}^{N} (2k-1)\, d_k \qquad d_k = \overbrace{1}^{(1-g)} - 2k + \sum_{a \in \text{puncture}} p_a^{(k)}$$

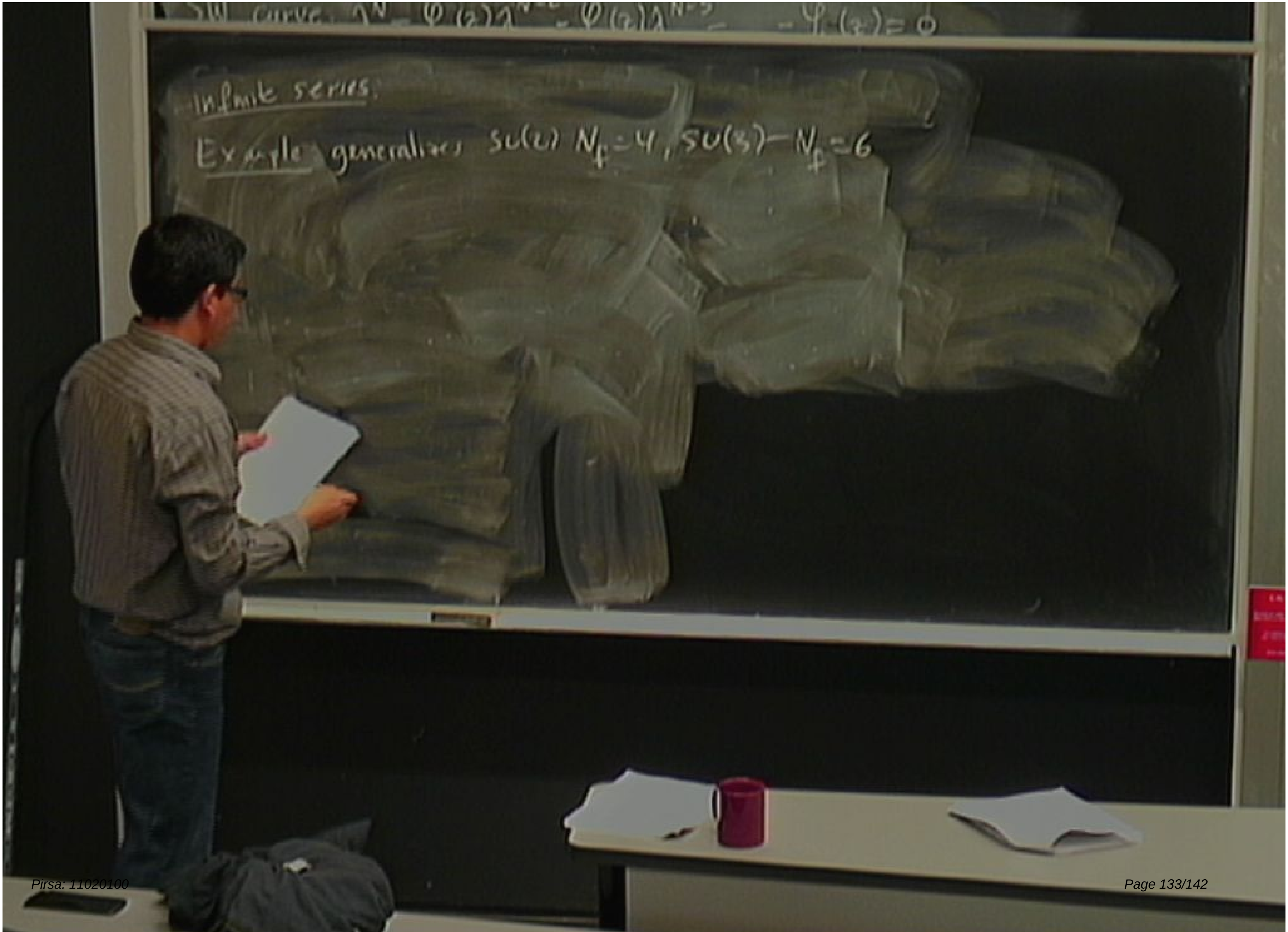
$$n_h = -\frac{4N(N^2-1)}{3} + \sum_{a \in \text{punctures}} f^{(a)}$$

$$f^{(a)} = \frac{1}{2}\left(-N + \sum \ell_r^2\right) + \sum (2k-1)\, p_k$$

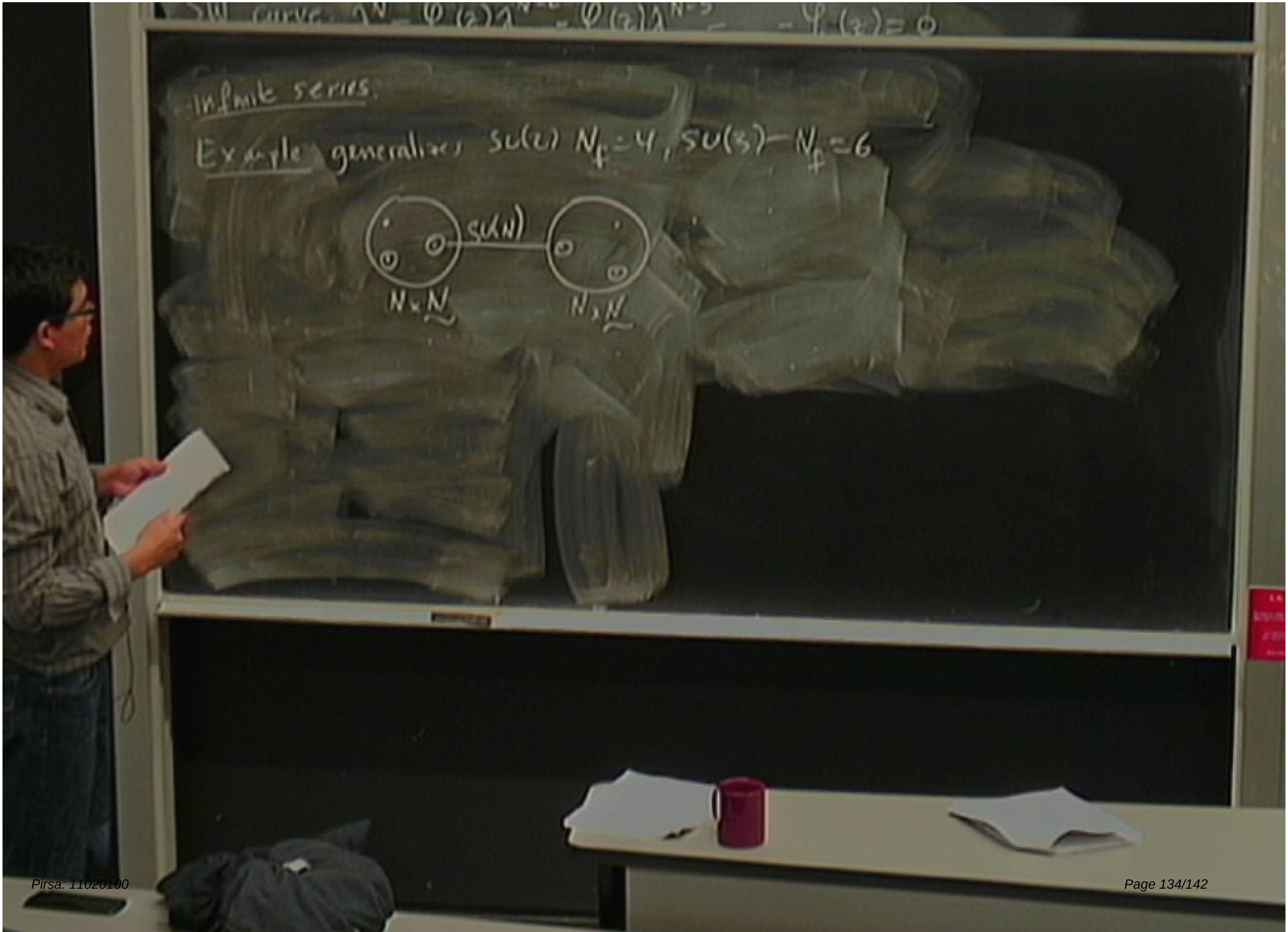
$SU(2)\ N_f =$

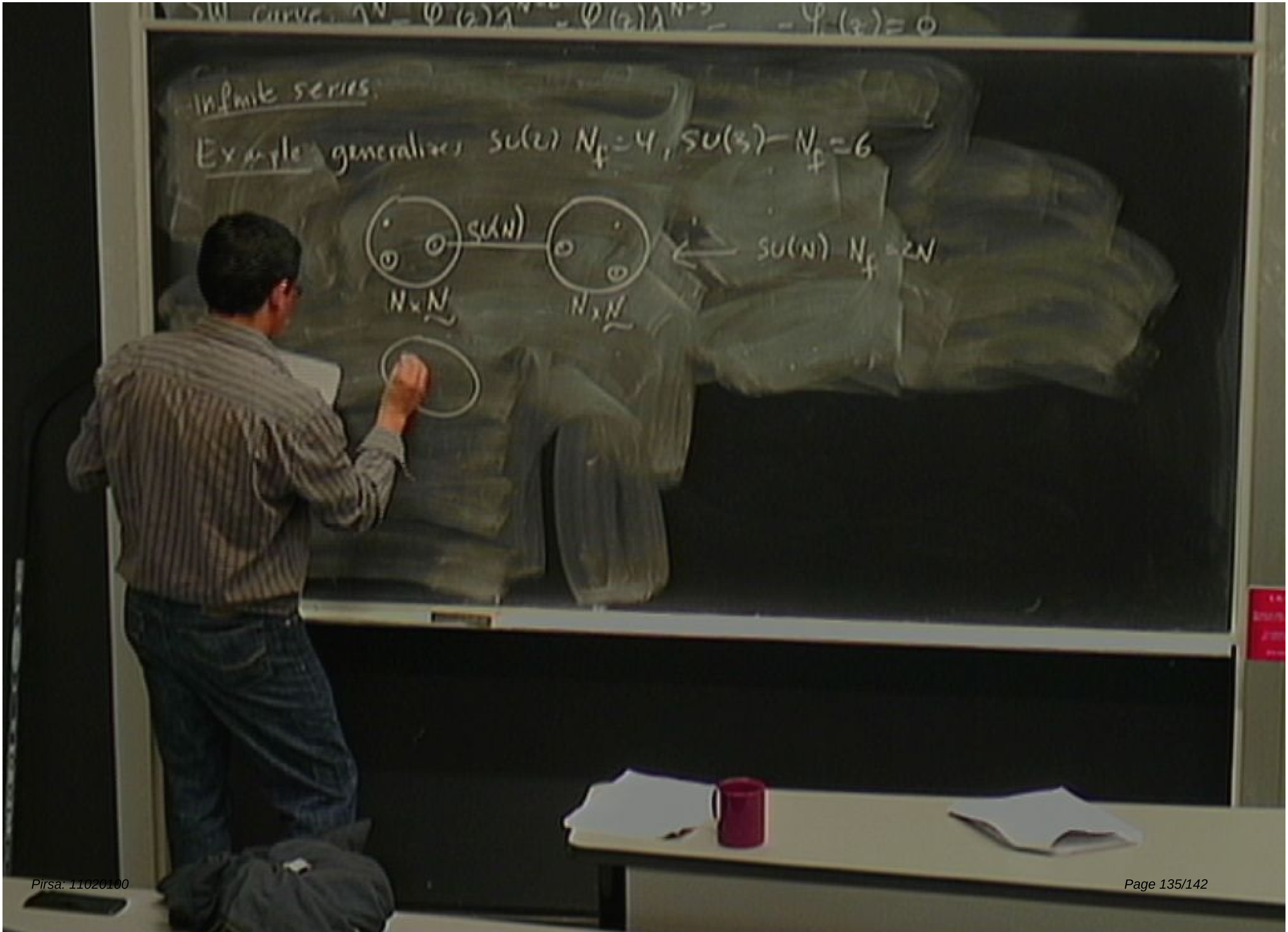

Can also compute conformal anomaly coeffs:
a = (5n_v + n_h)/24
T^μ_μ = c/16π^2 (Weyl)^2 - a/16π^2 (Euler)
c = (2n_v + n_h)/12
n_v = Σ_{k=2}^{N} (2k-1) d_k
d_k = 1-2k +
n_h = 4N(N^2-1)/3 + Σ
f(n) = 1/2(-N + Σ ℓ_r^2) + Σ
SU(2) N_f = 4
SU(2)
glob

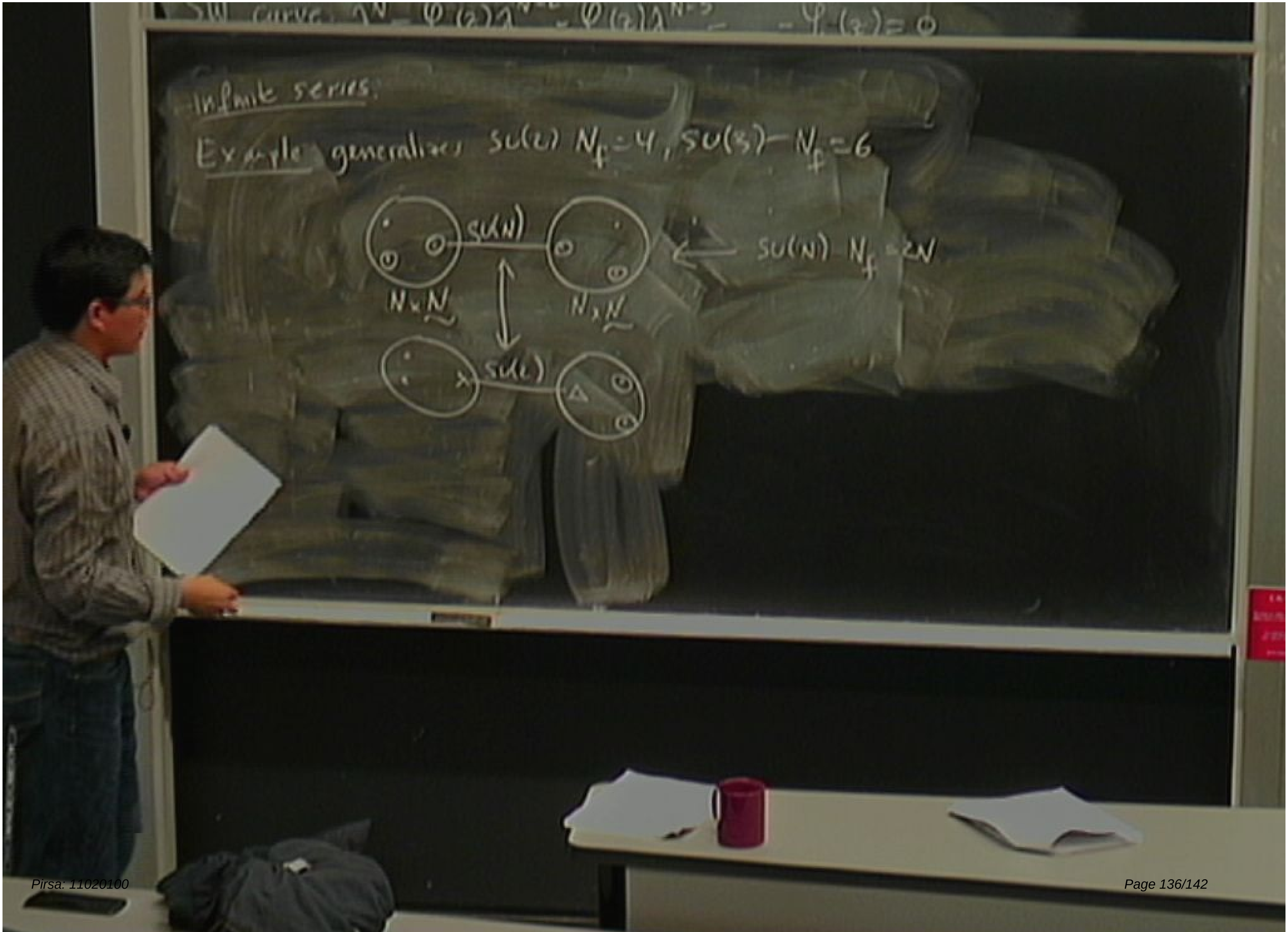

-2k + Σ p_a^(k)
a ∈ puncture
SU(2) N_f = 4
SU(2)
global = (SU(2))^4 ⊂ SO(8)

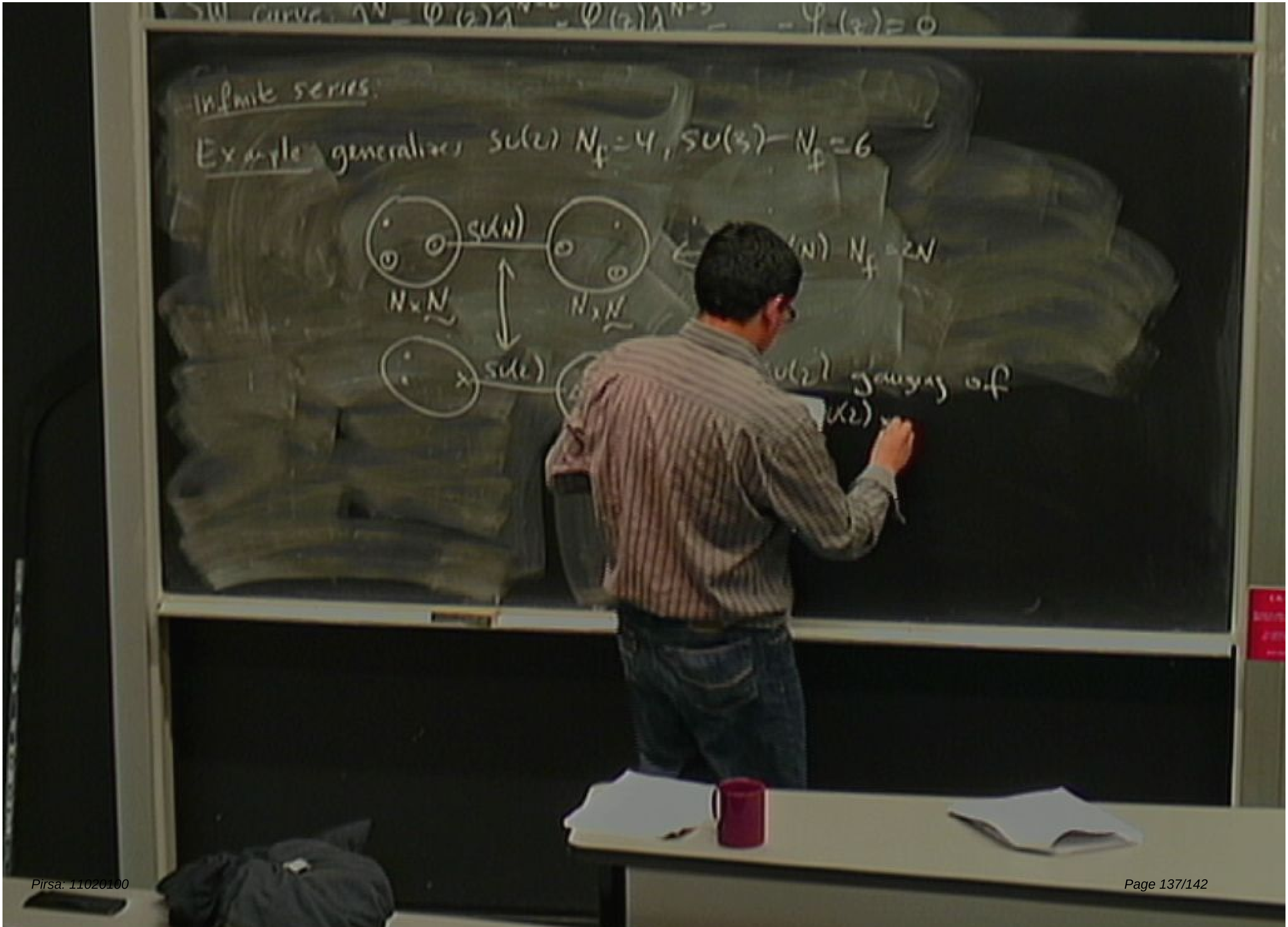


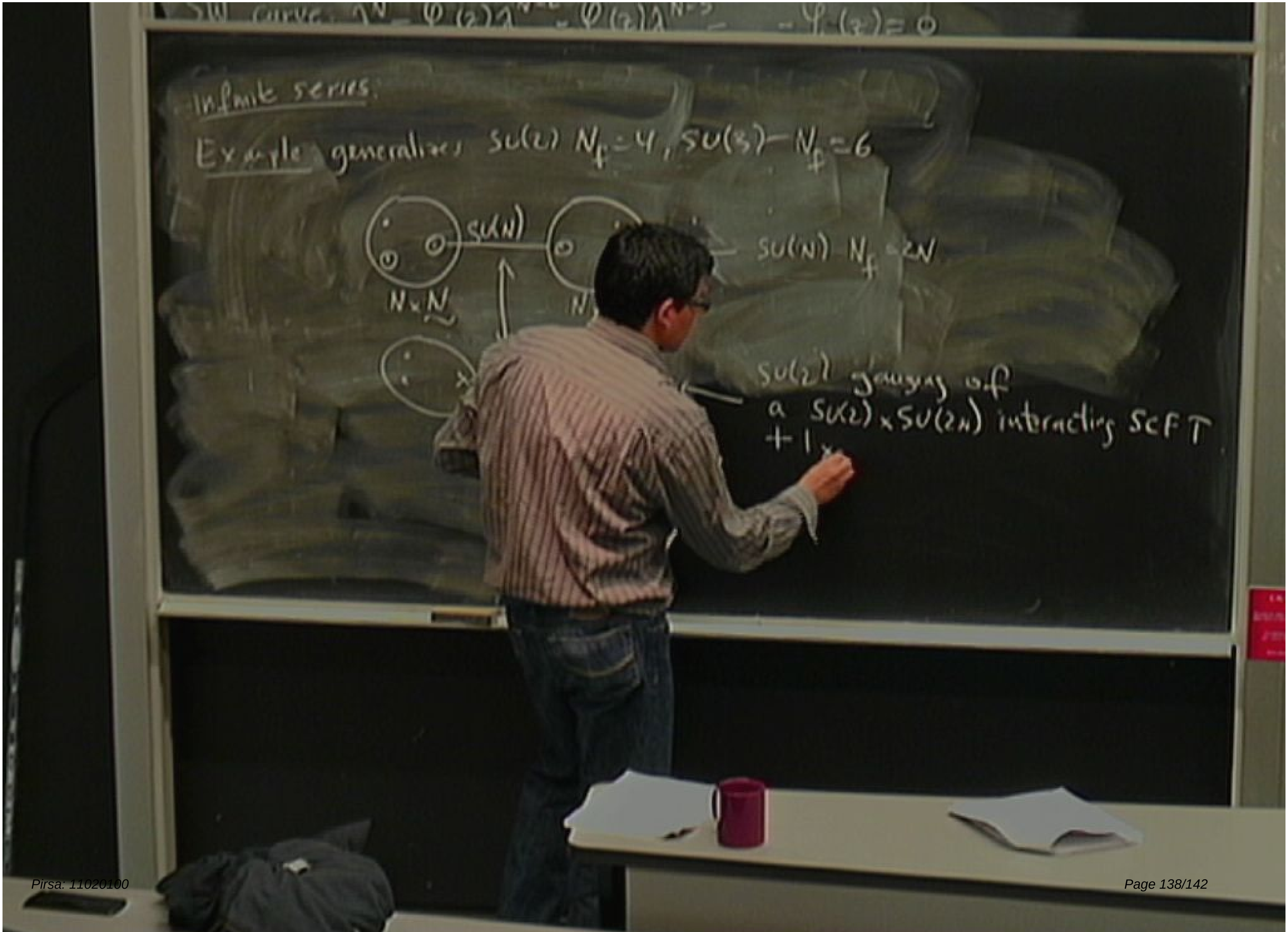
Infinit series.
Example generalizes SU(2) N_f = 4, SU(3) N_f = 6

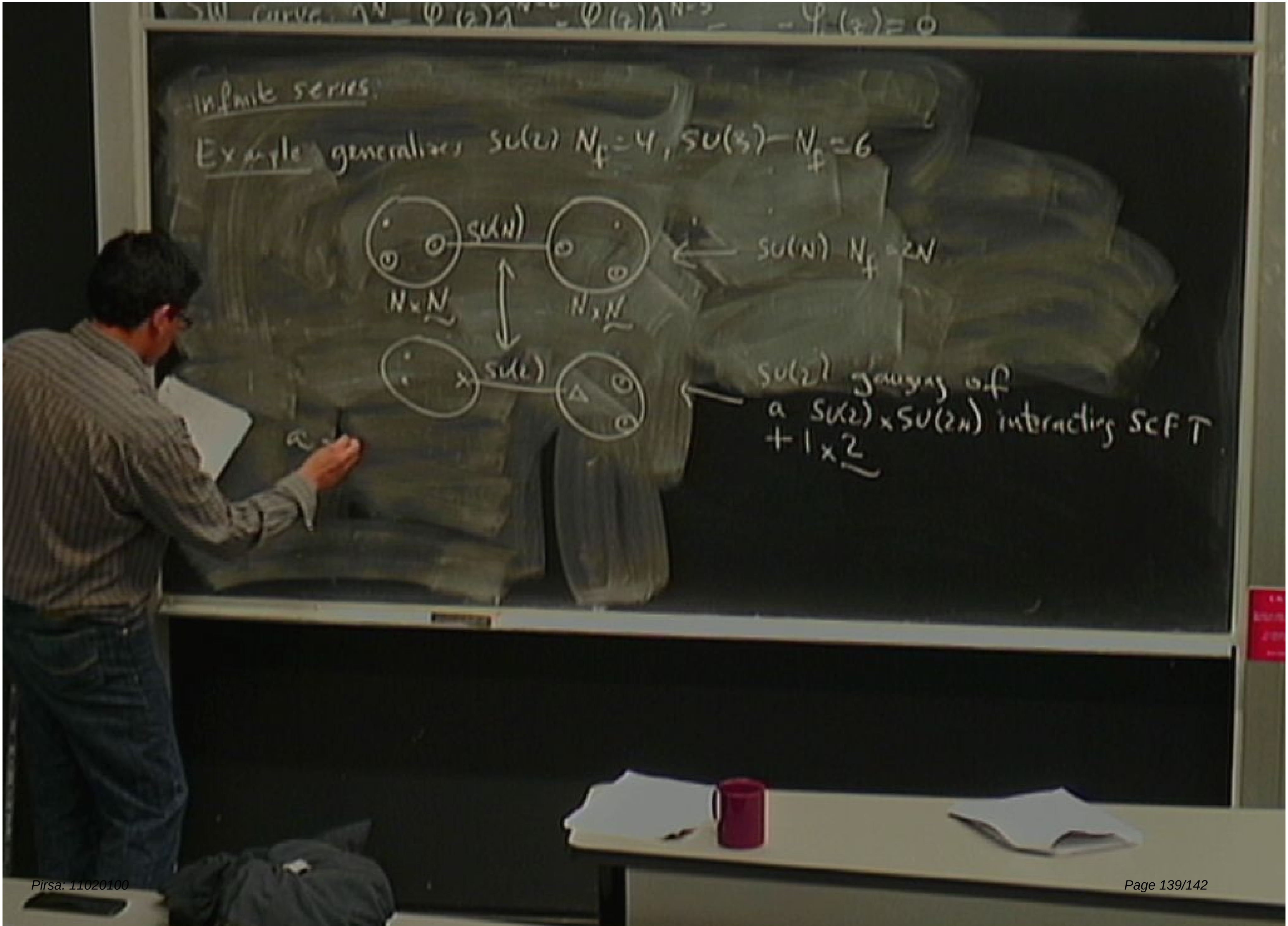

Infinite series.
Example generalizes SU(2) N_f=4, SU(3) N_f=6
SU(N)
N x N
N x N

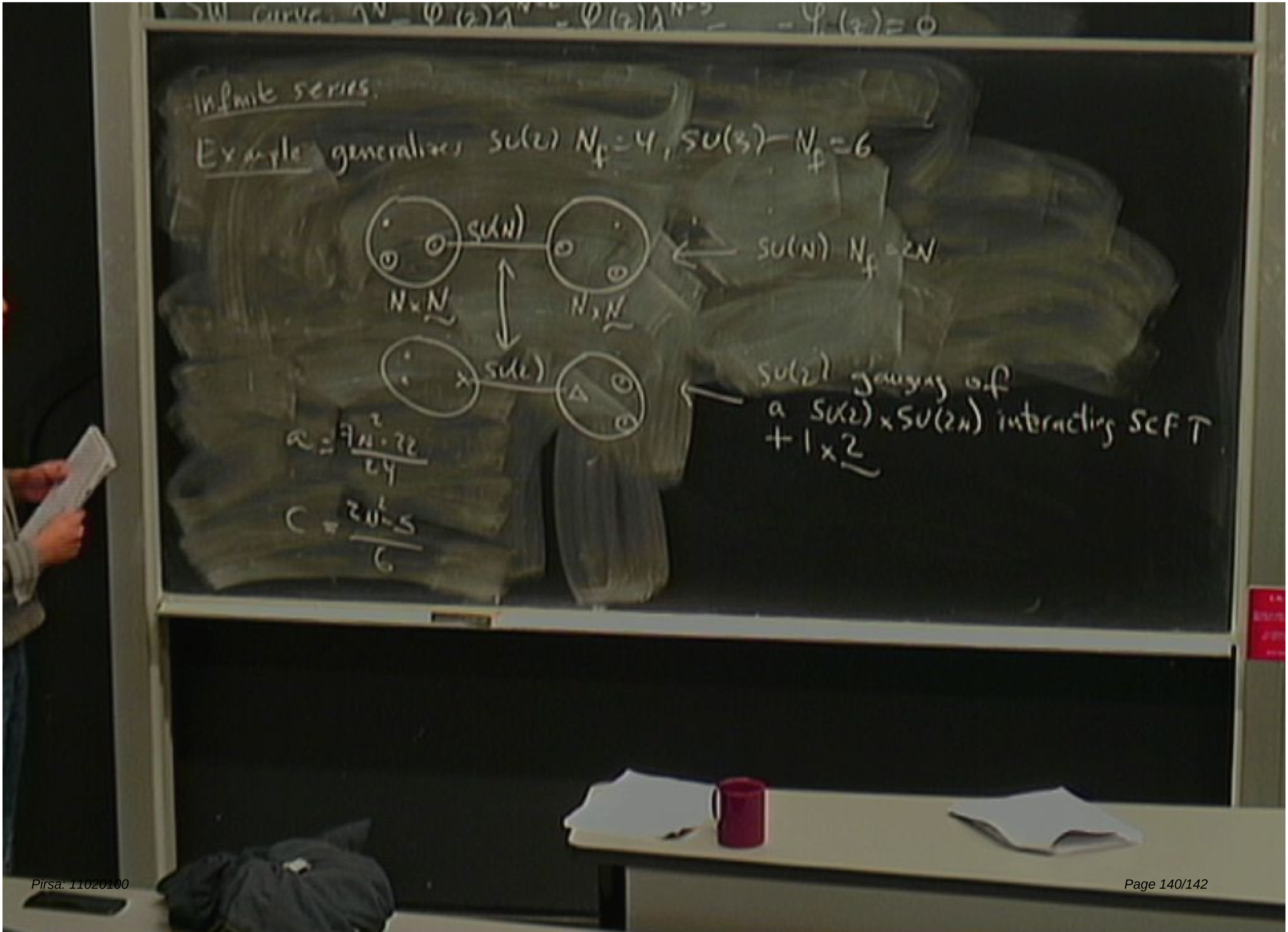

Infinite series
Example generalizes SU(2) Nf = 4, SU(3) Nf = 6
SU(N)
N x N
N x N
SU(N) Nf = 2N

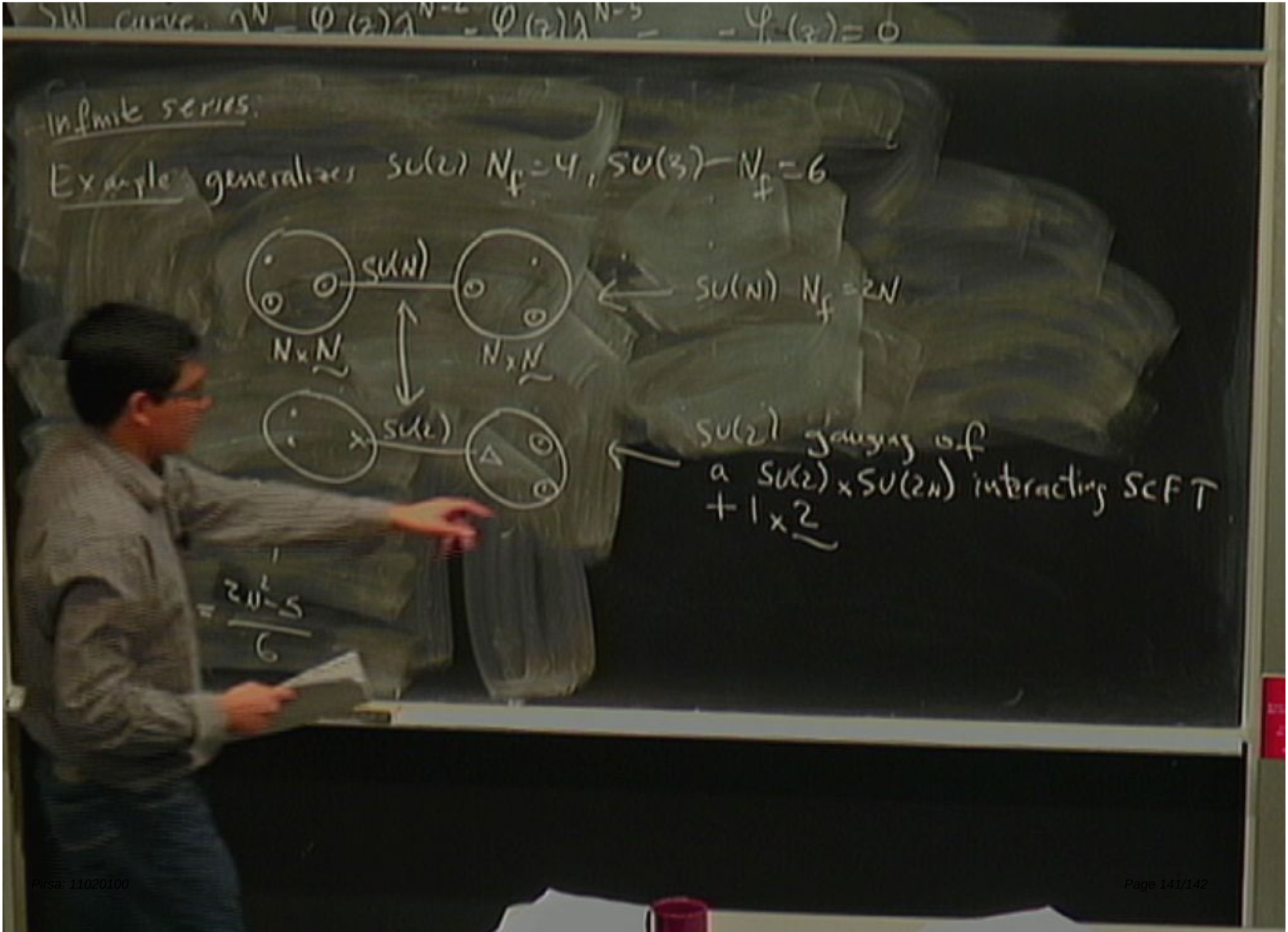

Infinite series
Example generalizes SU(2) N_f = 4, SU(3) N_f = 6
SU(N)
SU(N) N_f = 2N
N x N
N x N
SU(2)

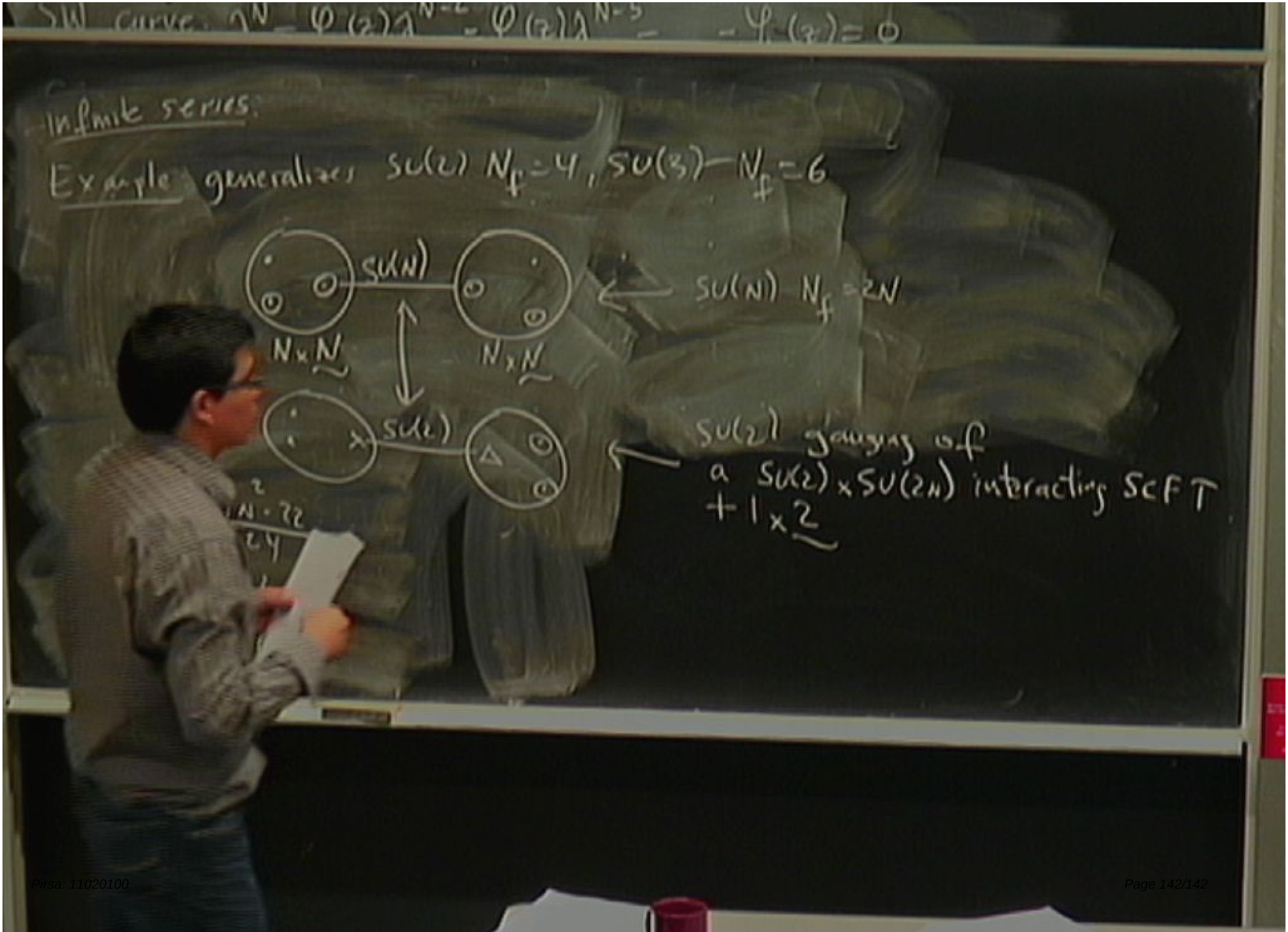

Infinite series:
Example generalizes SU(2) N_f=4, SU(3) N_f=6
SU(N)
N×N
N×N
SU(2)
gauging of


Infinite series
Example generalizes SU(2) Nf=4, SU(3) Nf=6
SU(N)
N×N
SU(N) Nf=2N
SU(2) gauging of
a SU(2)×SU(2N) interacting SCFT
+ 1 ×


Infinite series:
Example generalizes SU(2) Nf=4, SU(3) Nf=6
SU(N)
N x N
N x N
SU(N) Nf = 2N
SU(2)
SU(2) gauging of
a SU(2) x SU(2N) interacting SCFT
+ 1 x 2


Infinite series.
Example generalizes SU(2) N_f=4, SU(3) N_f=6
SU(N)
N×N
SU(N) N_f=2N
SU(2)
SU(2) gauging of
a SU(2)×SU(2N) interacting SCFT
+1×2


SW curve
Infinite series
Example generalizes SU(2) Nf=4, SU(3) Nf=6
SU(N)
N x N
N x N
SU(N) Nf=2N
SU(2)
SU(2) gauging of
a SU(2) x SU(2N) interacting SCFT
+ 1 x 2


Infinite series:
Example generalizes SU(2) N_f = 4, SU(3) N_f = 6
SU(N)
N x N
N x N
SU(N) N_f = 2N
SU(2)
SU(2) gauging of
a SU(2) x SU(2N) interacting SCFT
+ 1 x 2