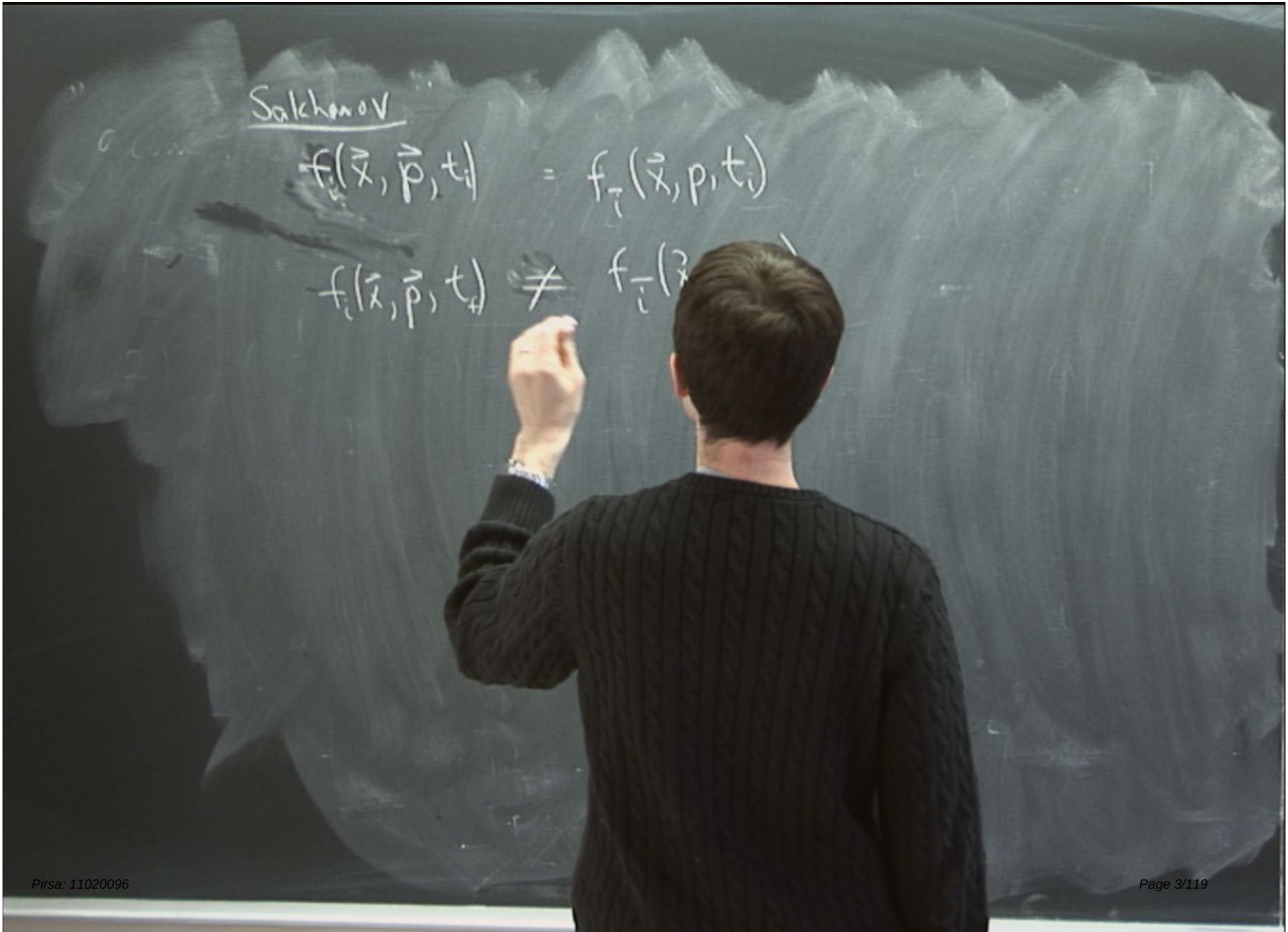
Title: Cosmology Review - Lecture 13

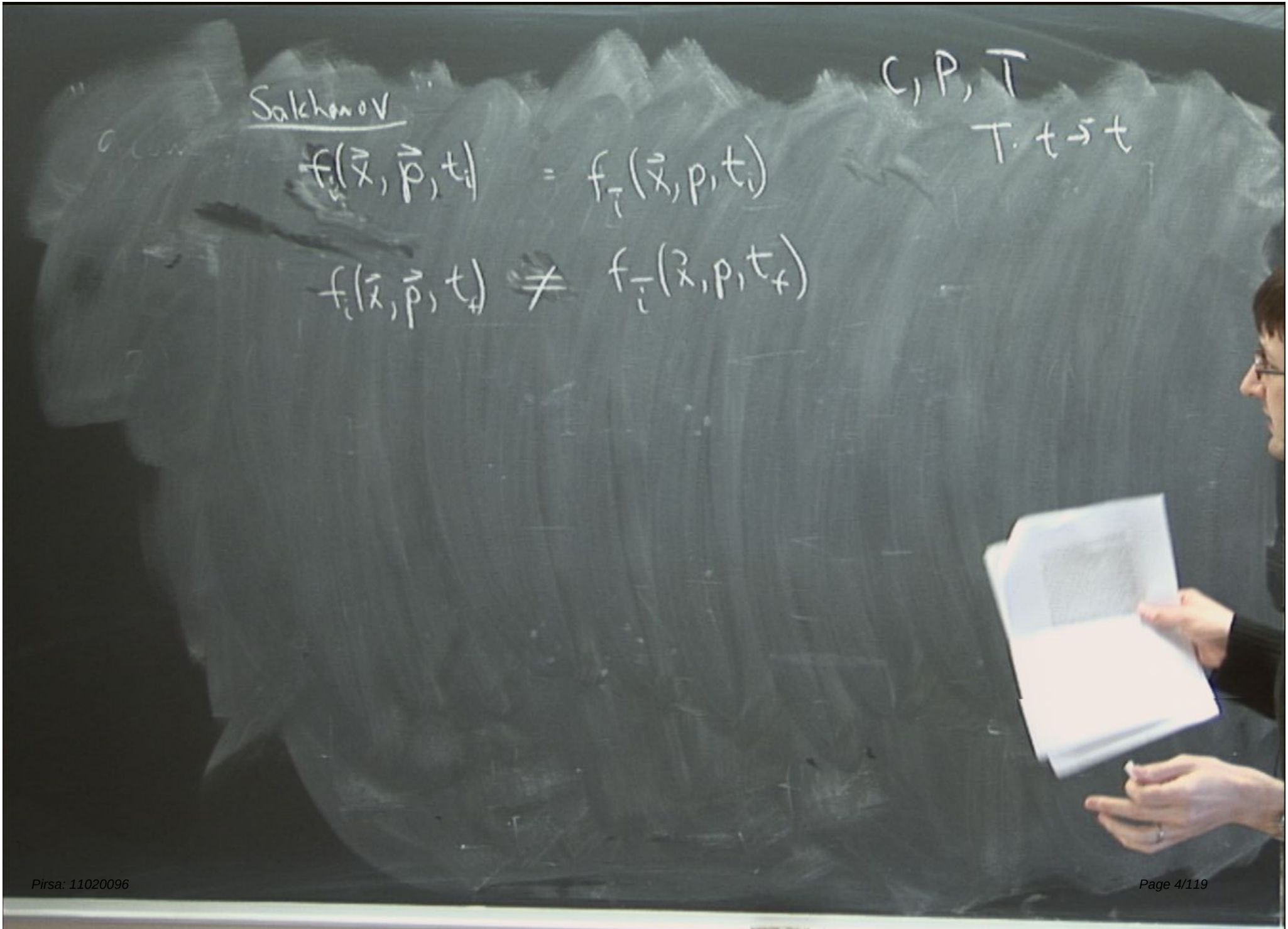
Date: Feb 09, 2011 11:30 AM

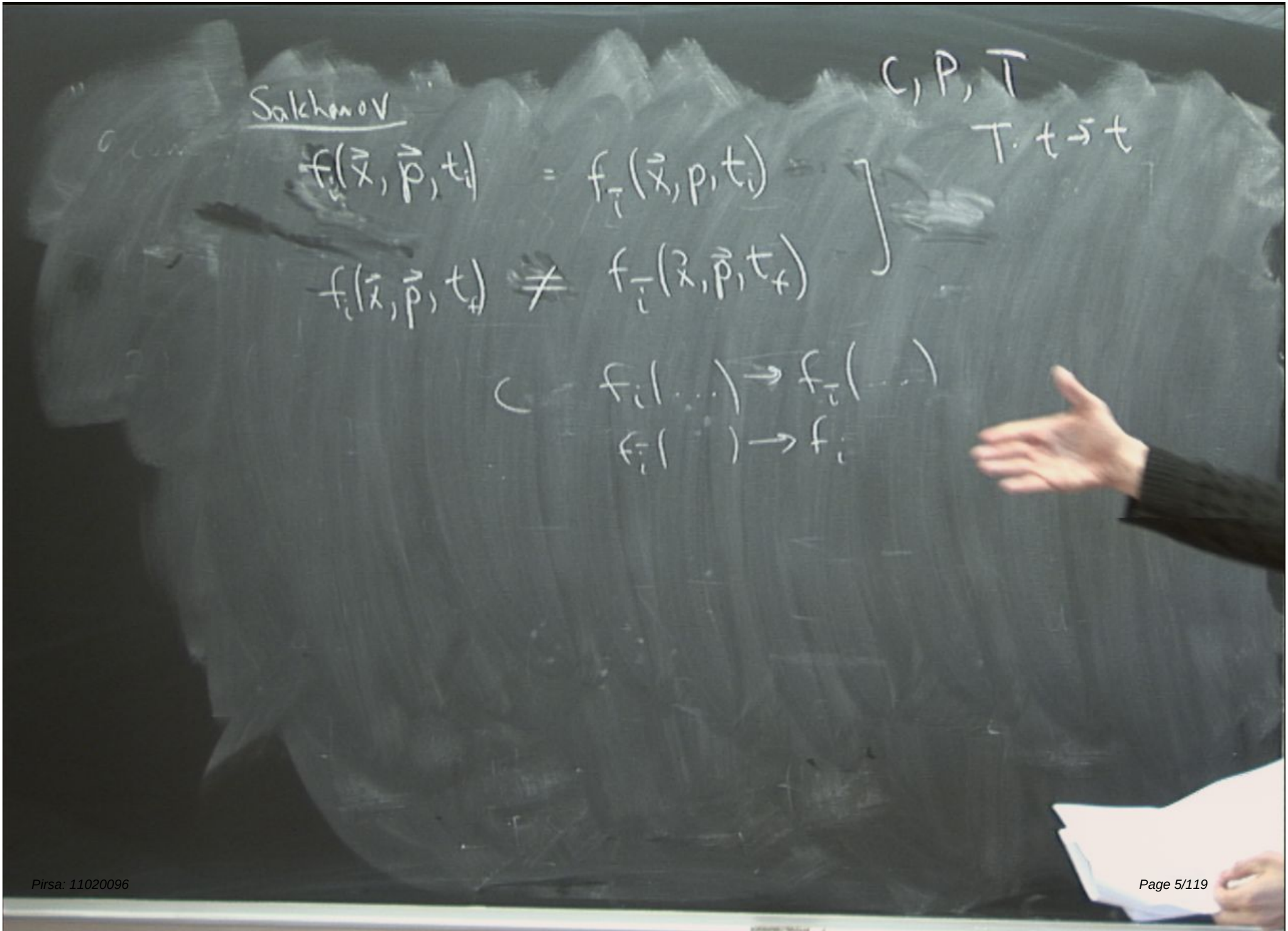
URL: http://pirsa.org/11020096

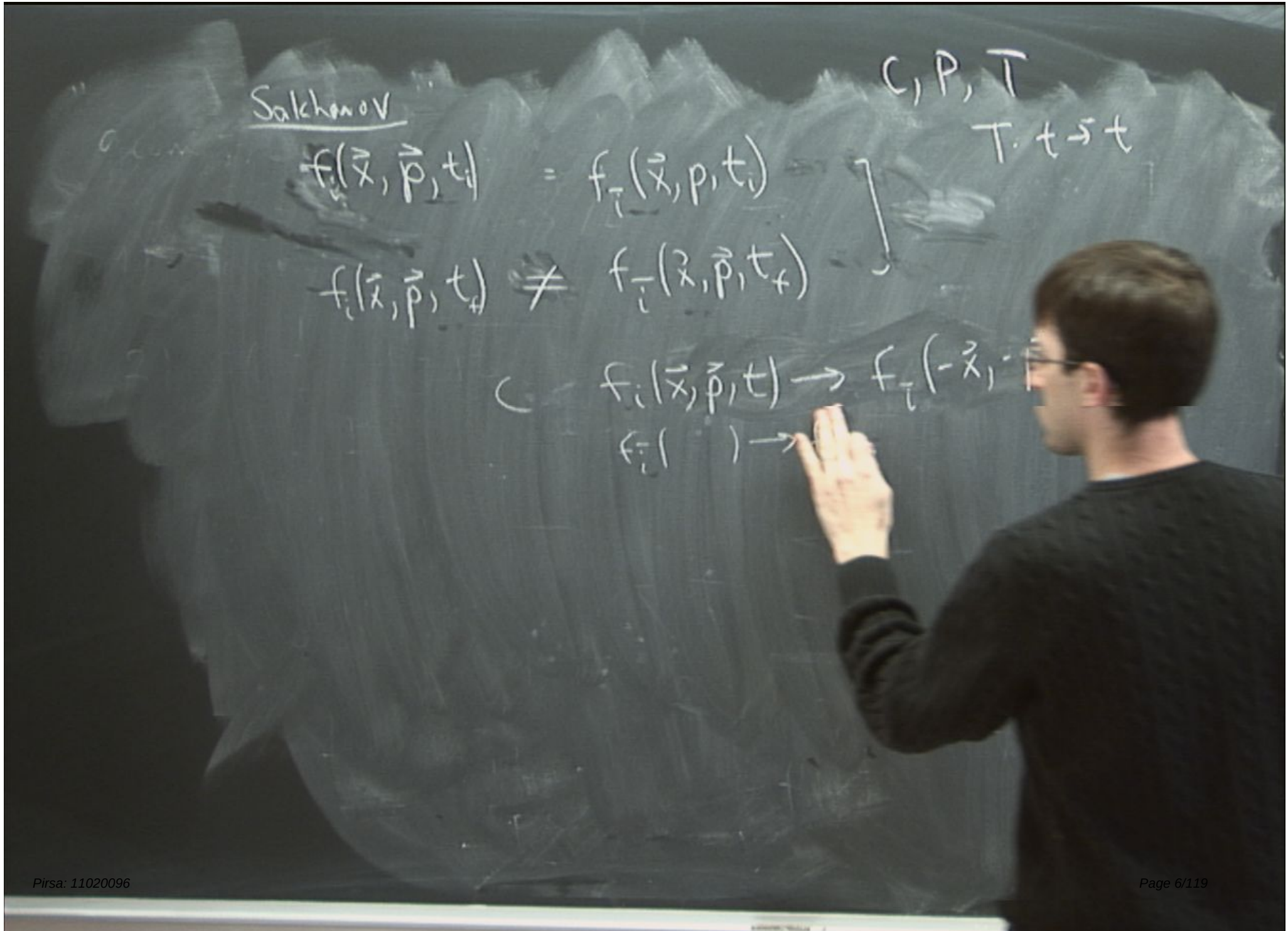
Abstract:

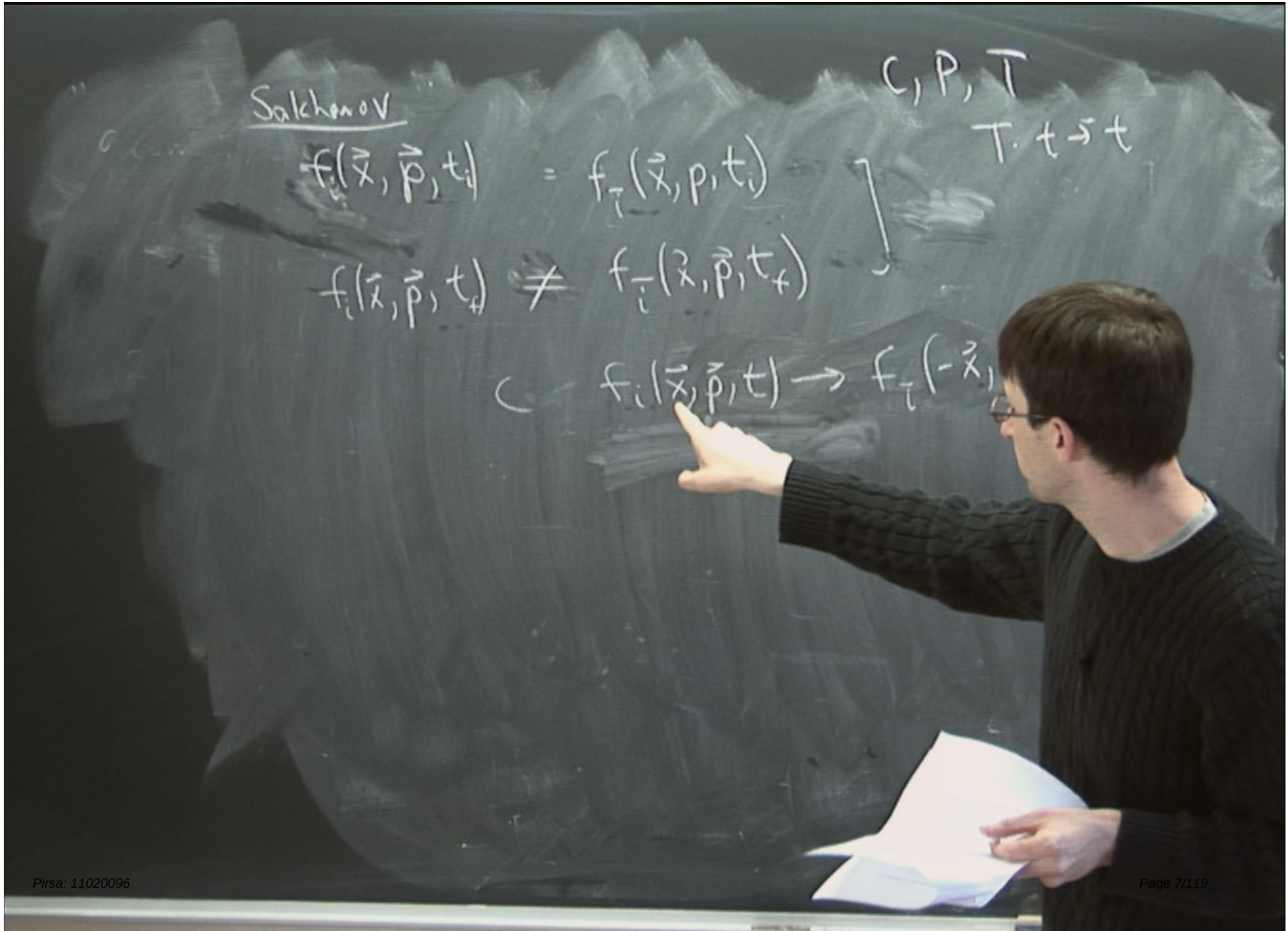
Salchanov

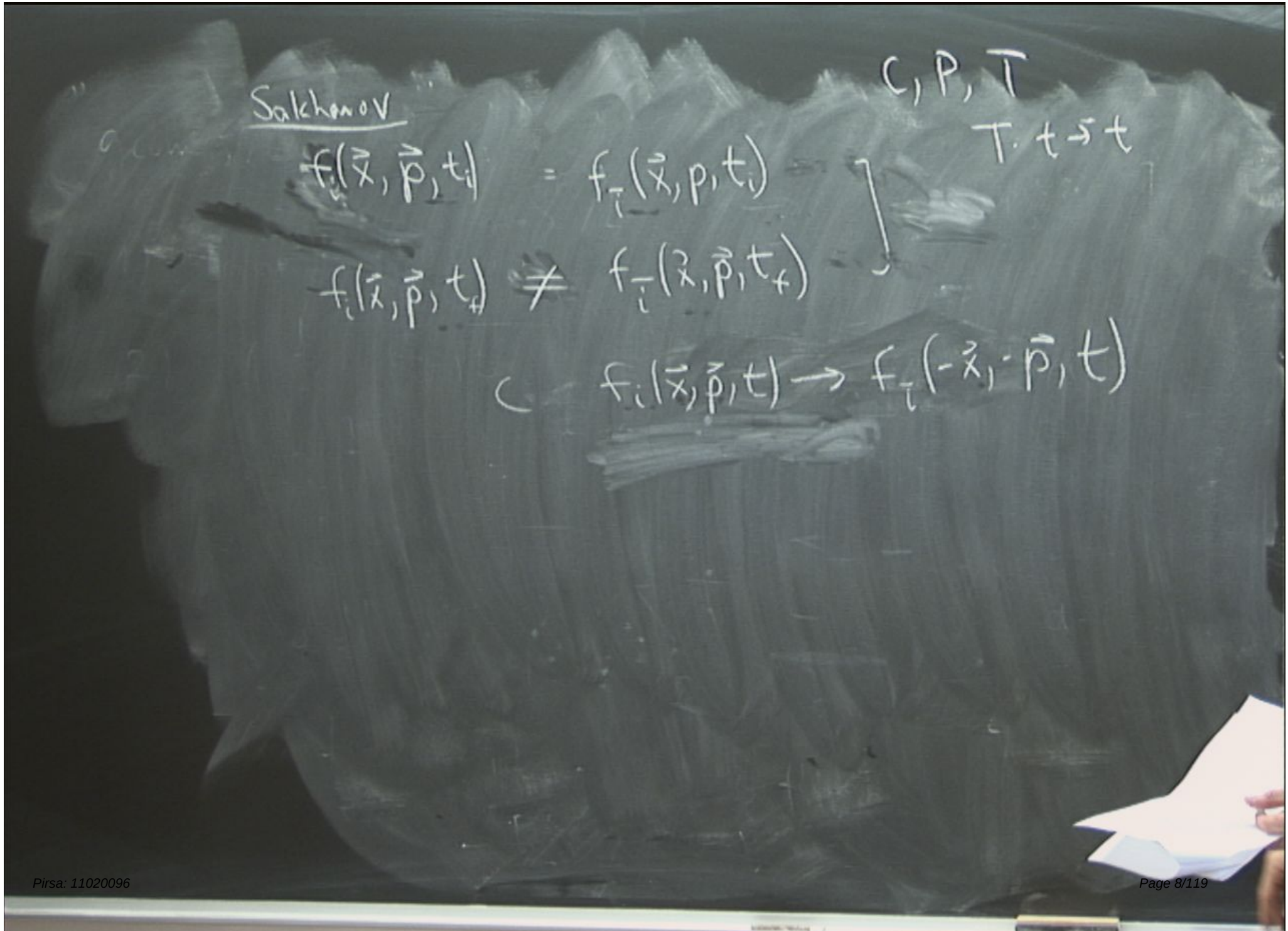

Sakharov
$f_i(\vec{x}, \vec{p}, t_i) = f_{\bar{i}}(\vec{x}, p, t_i)$
$f_i(\vec{x}, \vec{p}, t_f) \neq f_{\bar{i}}(\vec{x}$

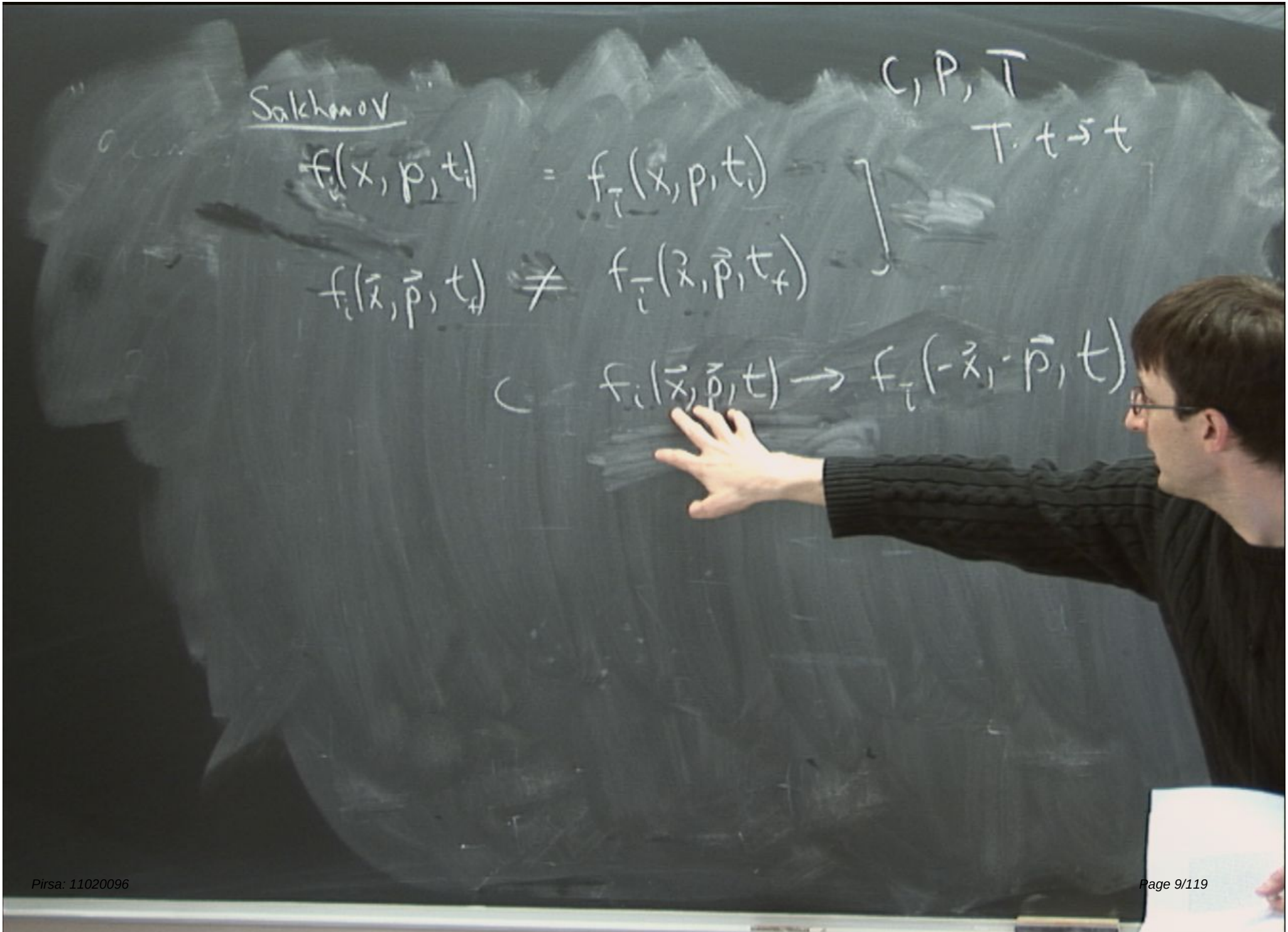

Sakharov
$f_i(\vec{x}, \vec{p}, t_i) = f_{\bar{i}}(\vec{x}, p, t_i)$
$f_i(\vec{x}, \vec{p}, t_f) \neq f_{\bar{i}}(\vec{x}, p, t_f)$
C, P, T
T: t → -t


Saltchanov
C, P, T
T: t → t
$f_i(\vec{x}, \vec{p}, t_i) = f_{\bar{i}}(\vec{x}, p, t_i)$
$f_i(\vec{x}, \vec{p}, t_f) \neq f_{\bar{i}}(\vec{x}, \vec{p}, t_f)$
$f_i(\ldots) \to f_{\bar{i}}(\ldots)$
$f_{\bar{i}}(\ \ ) \to f_i$


C, P, T
T: t → -t
Solchonov
$f_i(\vec{x}, \vec{p}, t_i) = f_{-i}(\vec{x}, p, t_i)$
$f_i(\vec{x}, \vec{p}, t_f) \neq f_{-i}(\vec{x}, \vec{p}, t_f)$
$f_i(\vec{x}, \vec{p}, t) \rightarrow f_{-i}(-\vec{x},$
$f_{-i}(\quad) \rightarrow$


Solchanov
C, P, T
T: t → t
$f_i(\vec{x}, \vec{p}, t_i) = f_{\bar{i}}(\vec{x}, p, t_i)$
$f_i(\vec{x}, \vec{p}, t_f) \neq f_{\bar{i}}(\vec{x}, \vec{p}, t_f)$
$f_i(\vec{x}, \vec{p}, t) \rightarrow f_{\bar{i}}(-\vec{x},$


C, P, T
Sakharov
$f_i(\vec{x}, \vec{p}, t_i) = f_{\bar{i}}(\vec{x}, p, t_i)$
$T: t \to -t$
$f_i(\vec{x}, \vec{p}, t_f) \neq f_{\bar{i}}(\vec{x}, \vec{p}, t_f)$
$f_i(\vec{x}, \vec{p}, t) \to f_{\bar{i}}(-\vec{x}, -\vec{p}, t)$


Solchanov
C, P, T
T: t → -t
$f_i(x, p, t_i) = f_f(x, p, t_f)$
$f_i(\vec{x}, \vec{p}, t_i) \neq f_f(\vec{x}, \vec{p}, t_f)$
$f_i(\vec{x}, \vec{p}, t) \rightarrow f_f(-\vec{x}, -\vec{p}, t)$

Sakharov

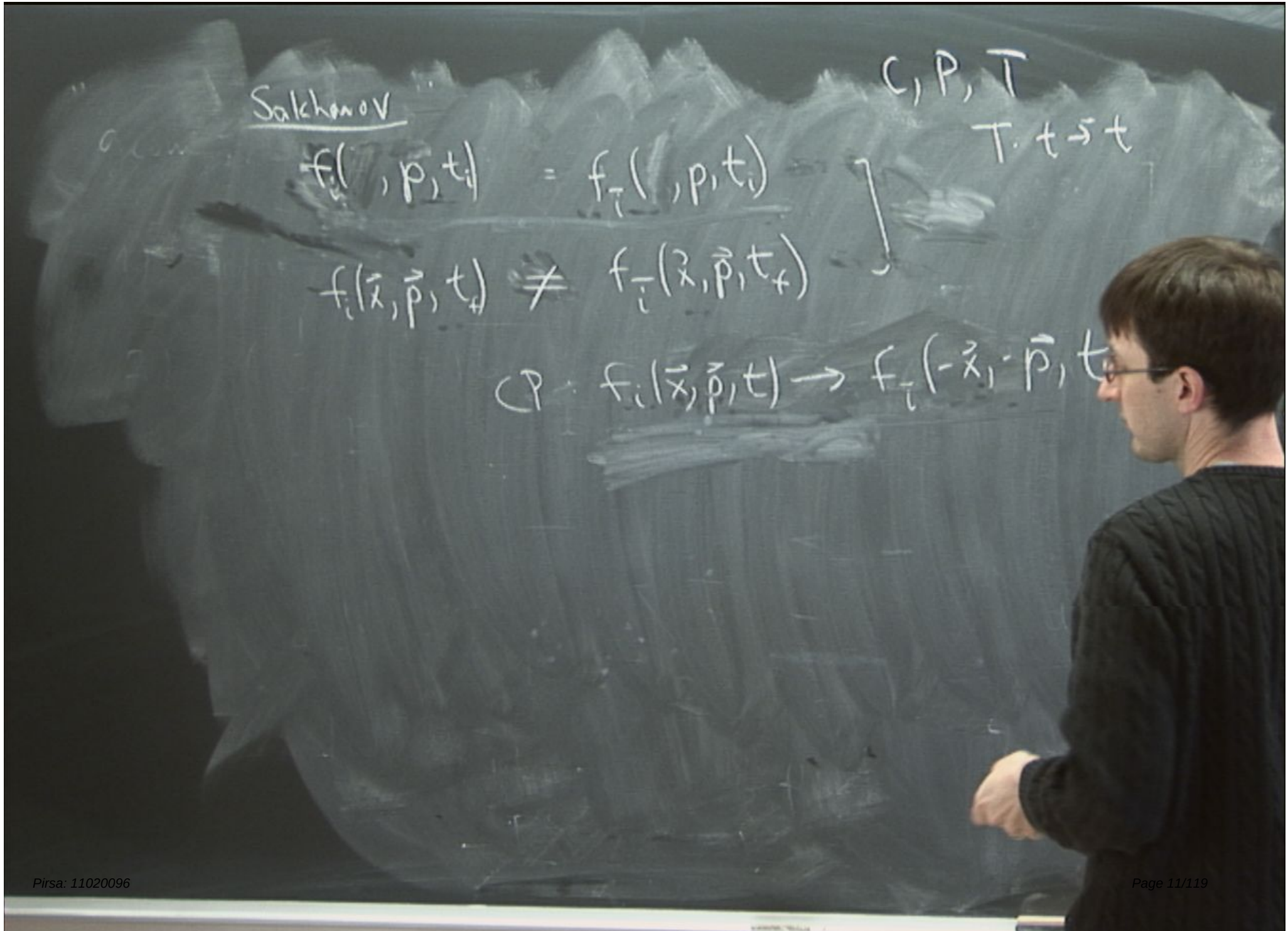
C, P, T

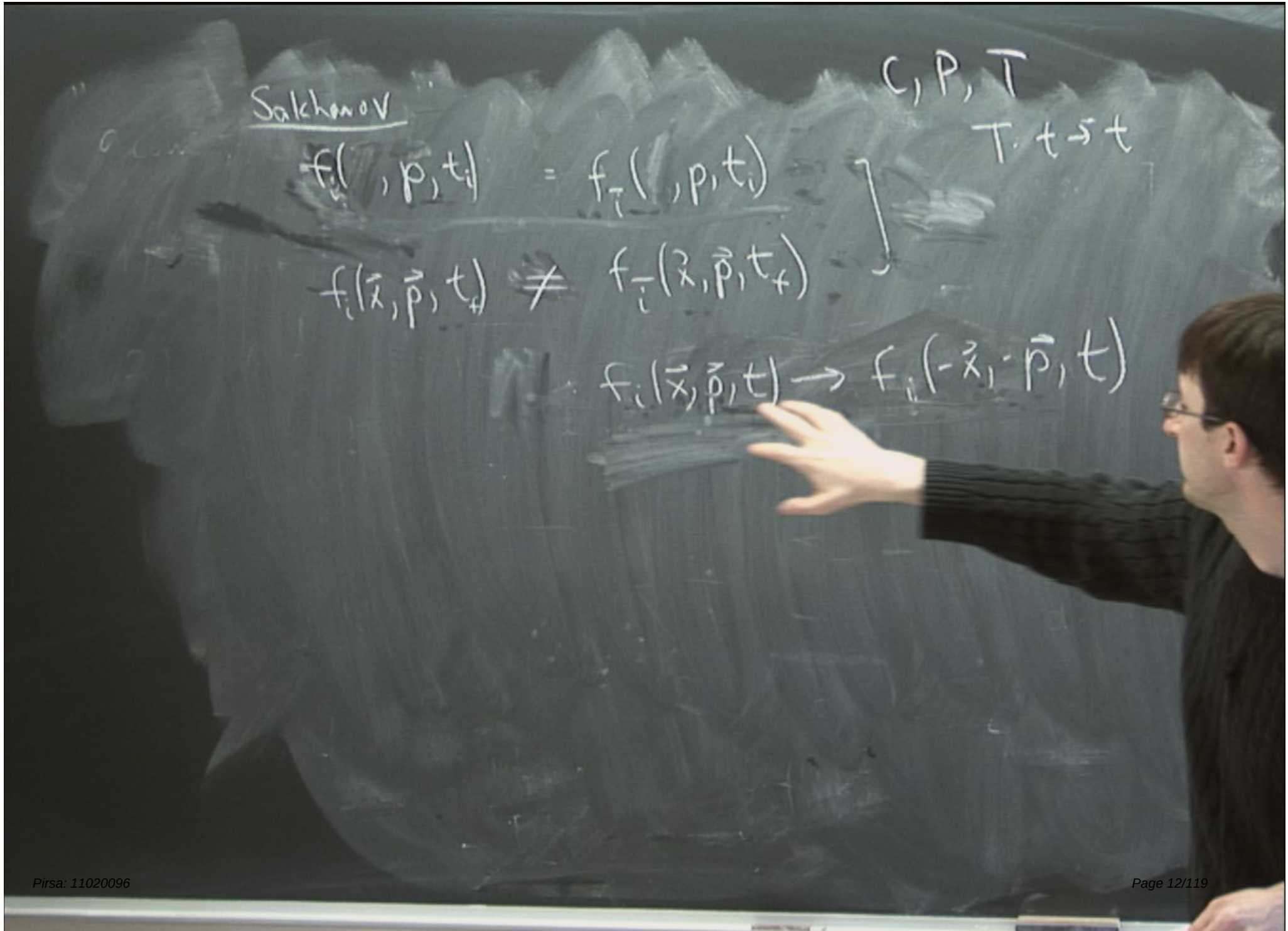
$T: t \to t$

$$f_i(x, p, t_i) = f_{\bar{i}}(x, p, t_i)$$

$$f_i(\vec{x}, \vec{p}, t_f) \neq f_{\bar{i}}(\vec{x}, \vec{p}, t_f)$$

$$C \quad f_i(\vec{x}, \vec{p}, t) \to f_{\bar{i}}(-\vec{x}, -\vec{p}, t)$$


Sakharov
C, P, T
T: t → t
f_i(, p, t_i) = f_ī(, p, t_i)
f_i(x⃗, p⃗, t_f) ≠ f_ī(x⃗, p⃗, t_f)
CP: f_i(x⃗, p⃗, t) → f_ī(-x⃗, -p⃗, t


Sakharov
C, P, T
T: t → t
f_i(, p, t_i) = f_ī(, p, t_i)
f_i(x⃗, p⃗, t_f) ≠ f_ī(x⃗, p⃗, t_f)
f_i(x⃗, p⃗, t) → f_i(-x⃗, -p⃗, t)

Sakharov

C, P, T

$T: t \to t$

$$f_i(\ , p, t_i) = f_{\bar{i}}(\ , p, t_i)$$

$$f_i(\vec{x}, \vec{p}, t_f) \neq f_{\bar{i}}(\vec{x}, \vec{p}, t_f)$$

$$f_i(\vec{x}, \vec{p}, t) \to f_i(-\vec{x}, -\vec{p}, t)$$

Sakharov

C, P, T

T: $t \to -t$

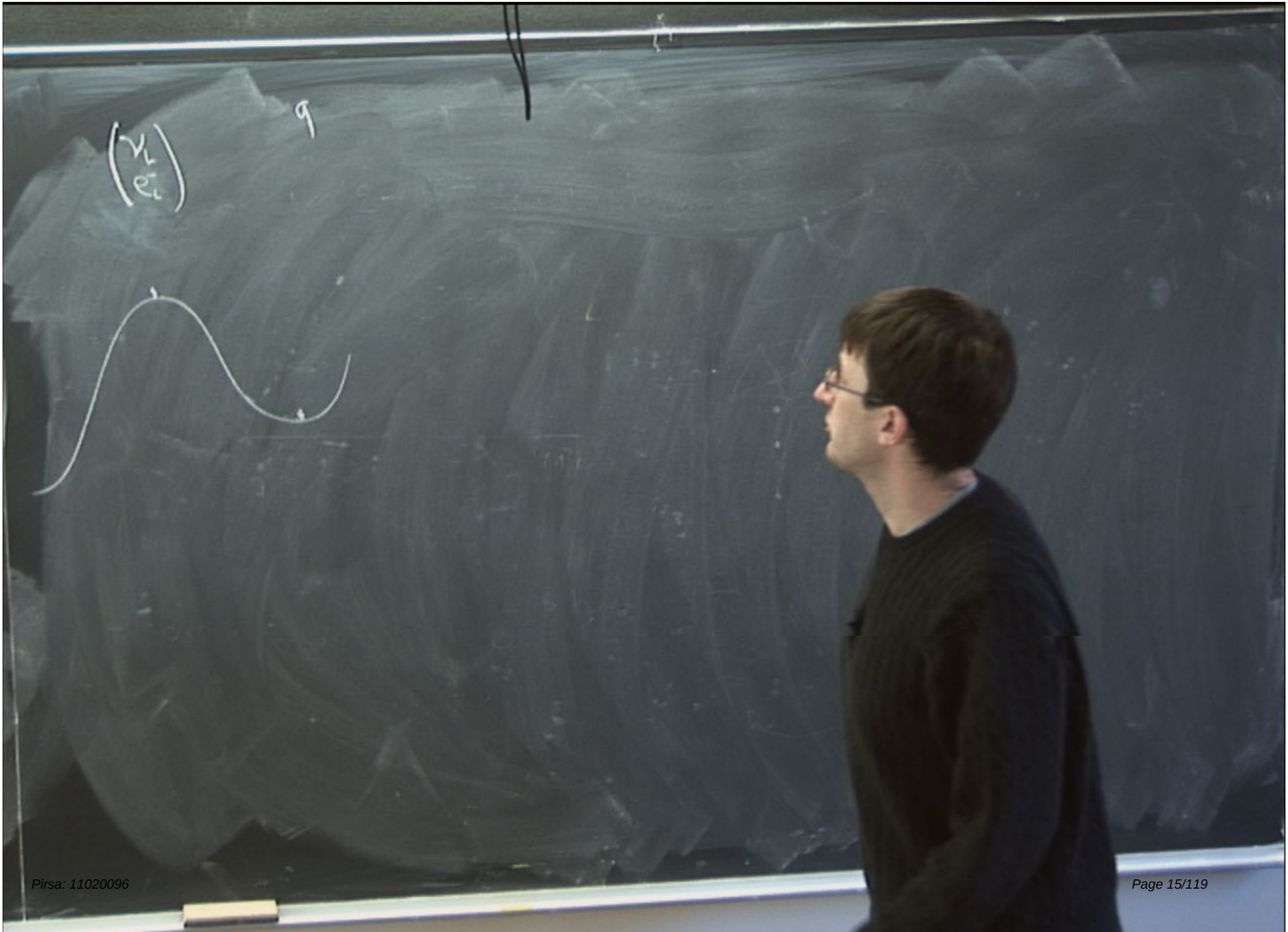
C, CP

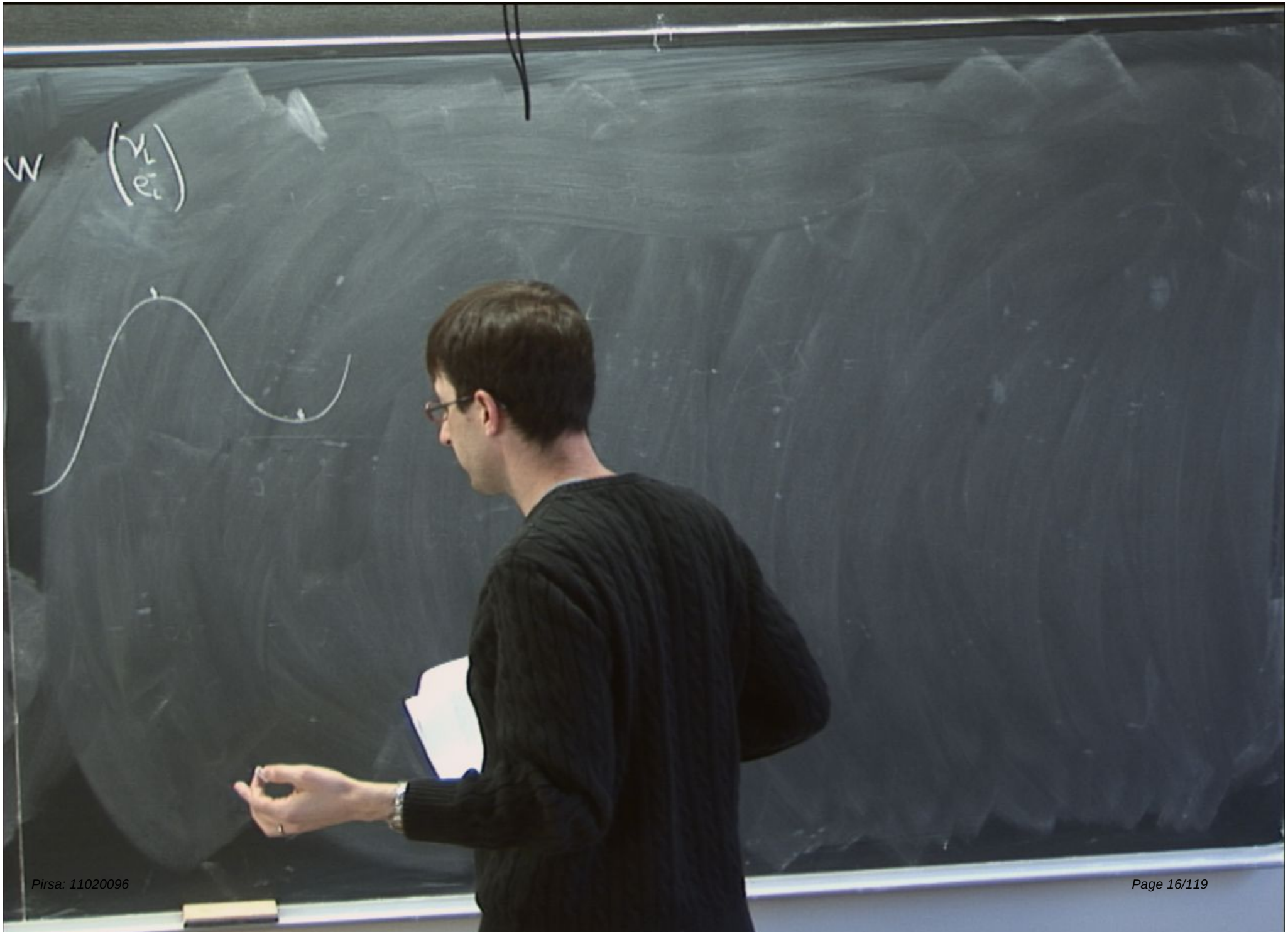
$f_i(\vec{x}, \vec{p}, t_i) = f_{\bar{i}}(\quad, \vec{p}, t_i)$

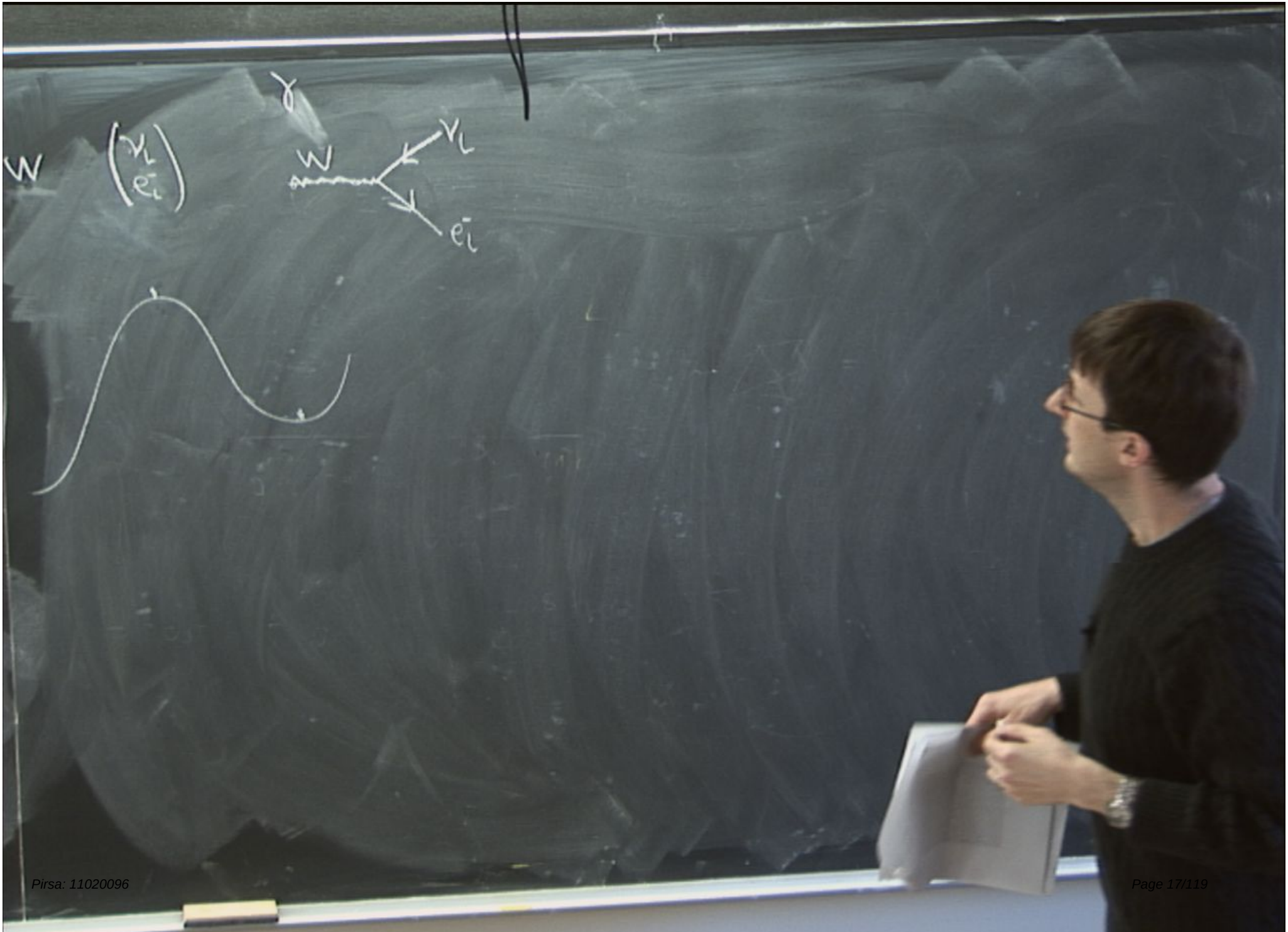
$f_i(\vec{x}, \vec{p}, t_f) \neq f_{\bar{i}}(\vec{x}, \vec{p}, t_f)$

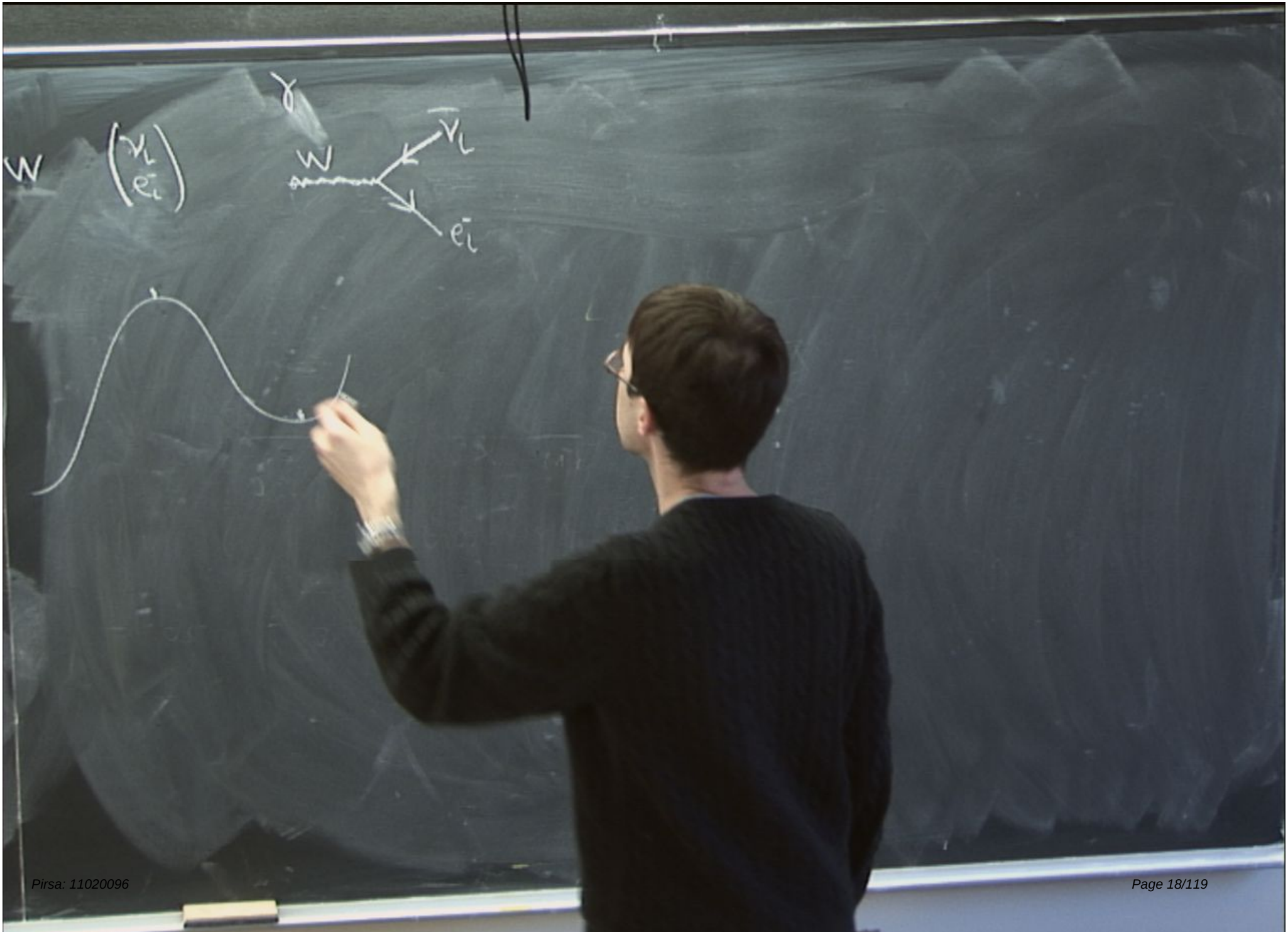
$f_i(\vec{x}, \vec{p}, t) \to f_{\bar{i}}(-\vec{x}, \vec{p}, t)$

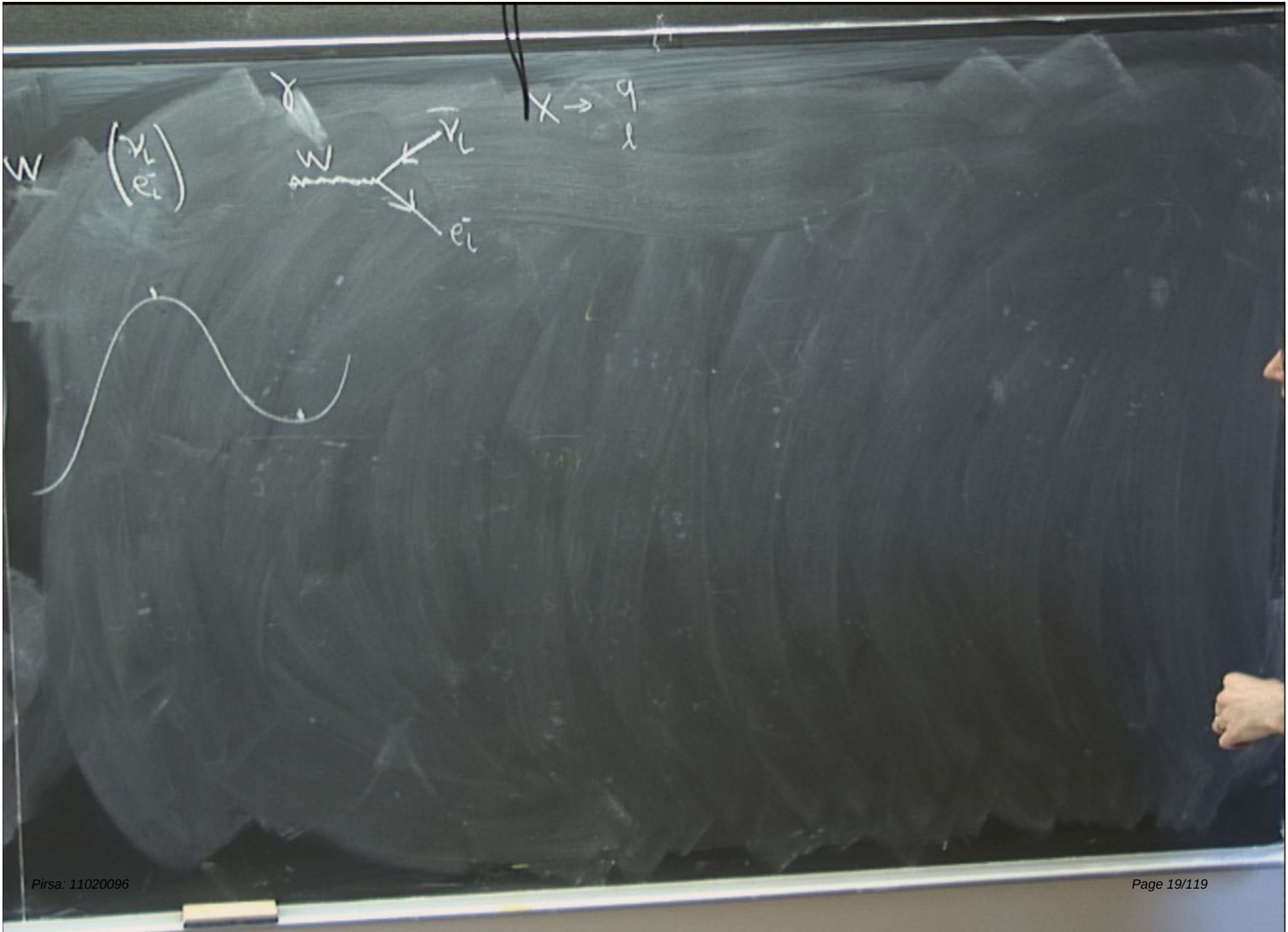
$B_1 \rightleftharpoons B_2$

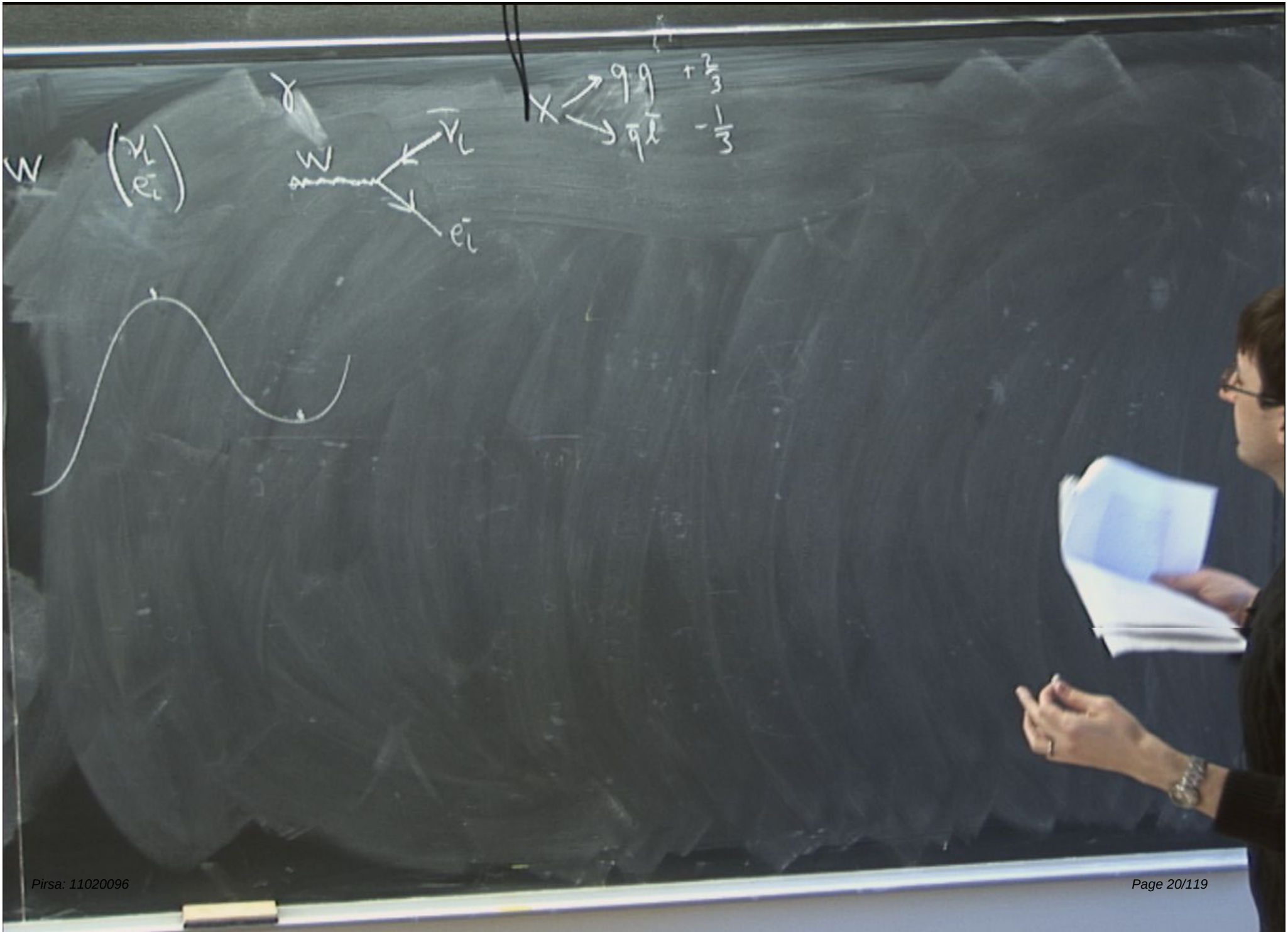

$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$
q

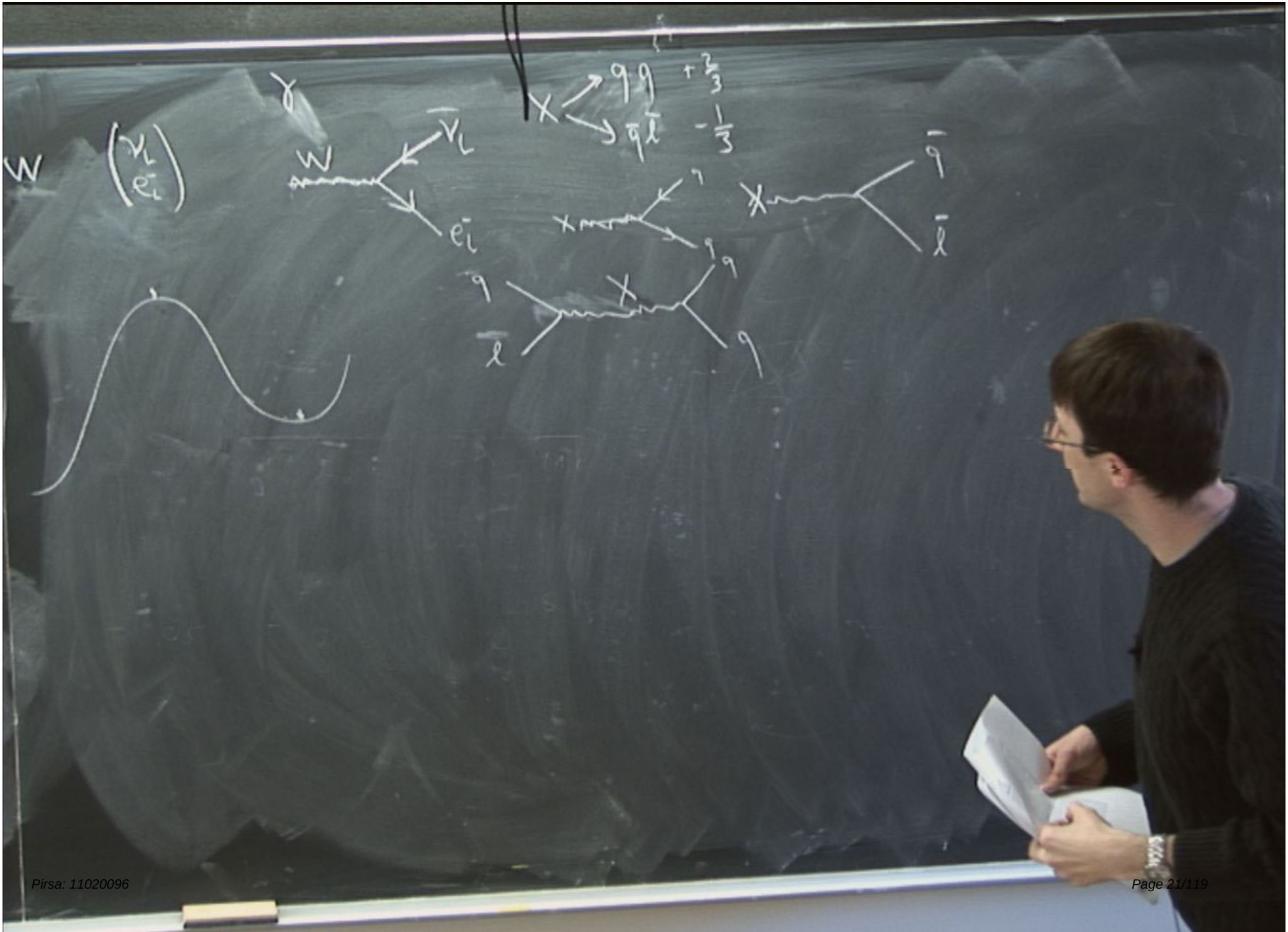

W

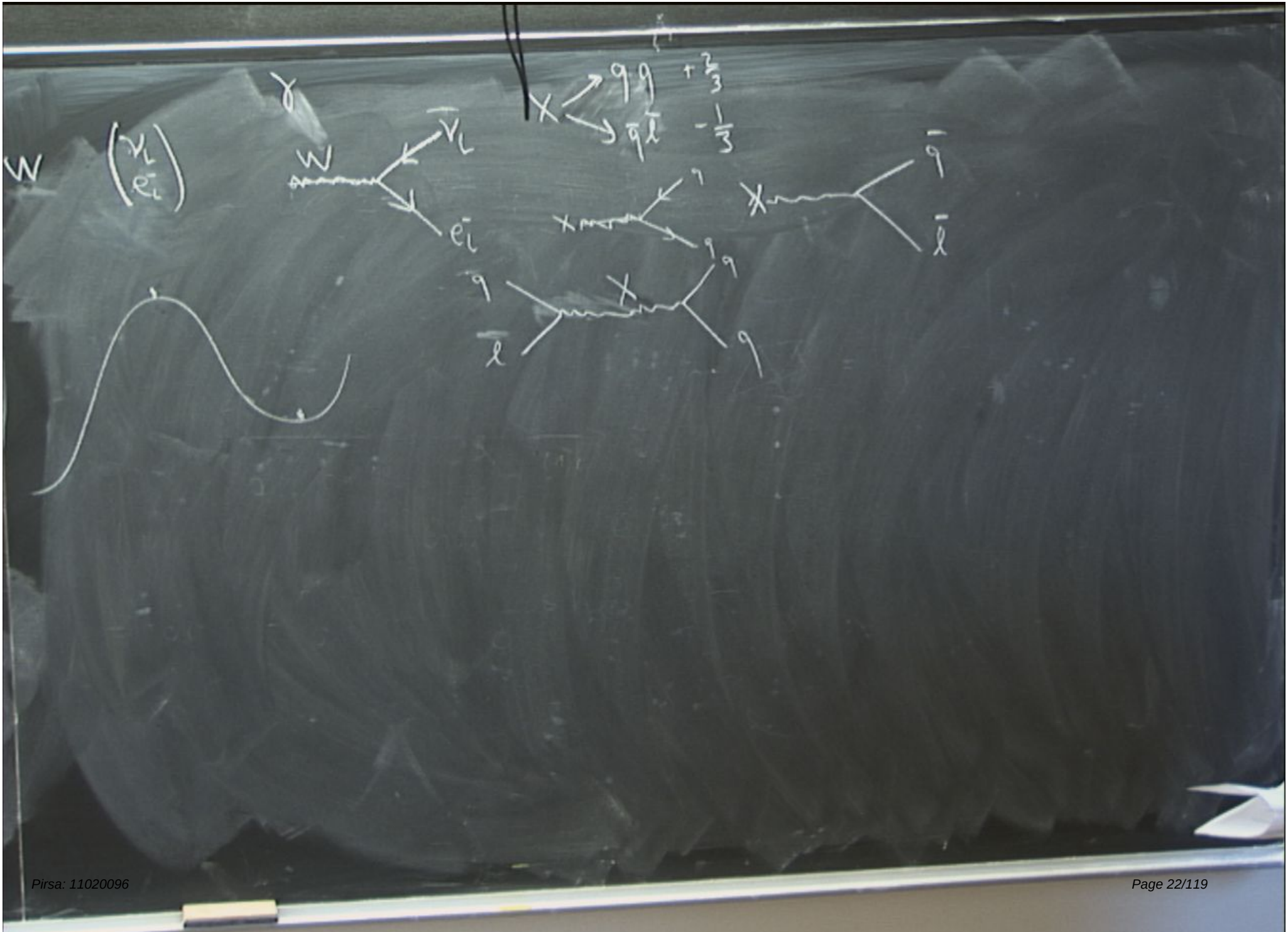

γ
W
$\begin{pmatrix} \nu_l \\ e_l^- \end{pmatrix}$
W
ν_l
e_l^-

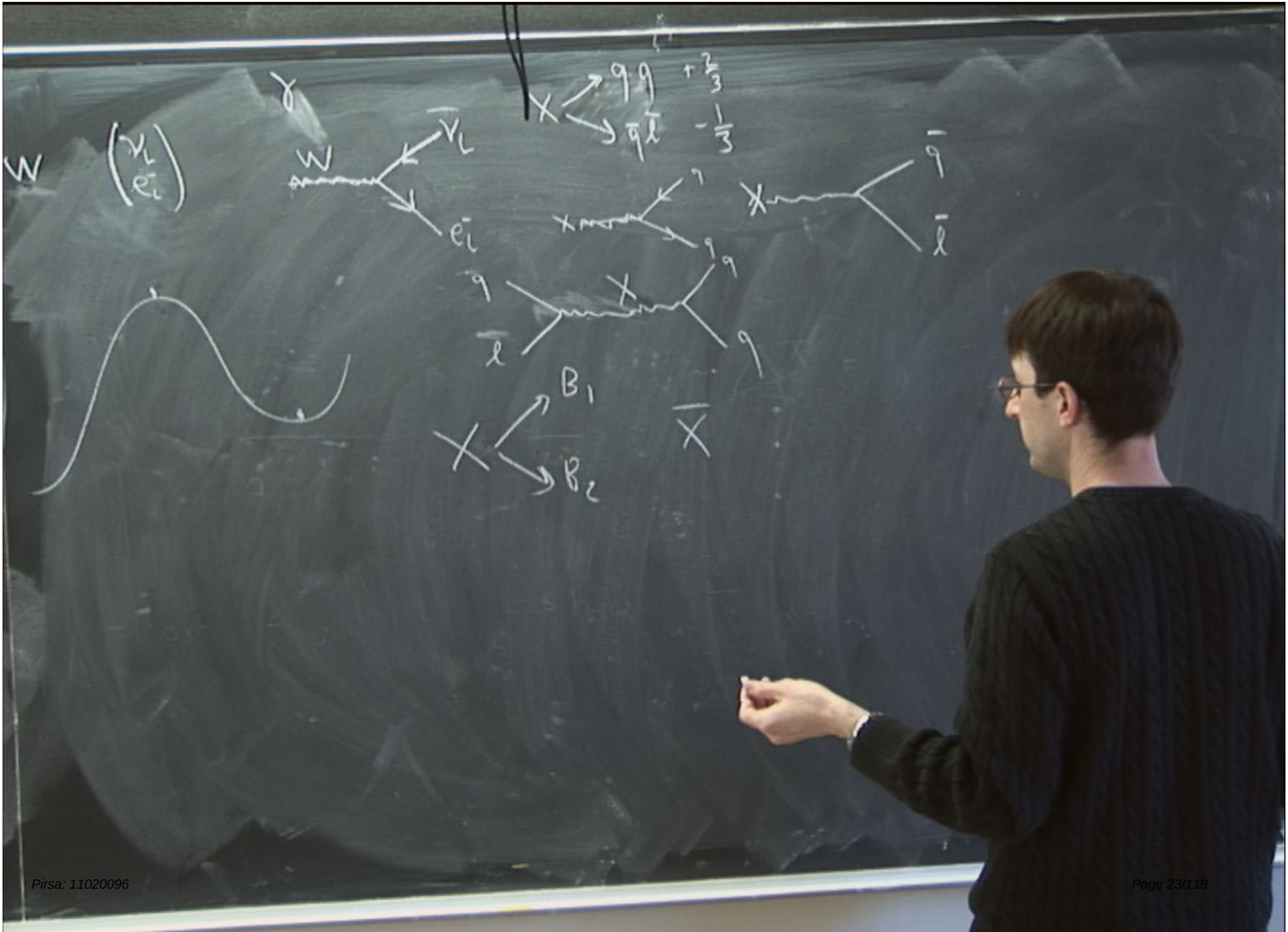

W
$\begin{pmatrix} \nu_l \\ e_i^- \end{pmatrix}$
γ
W
$\bar{\nu}_l$
e_i^-

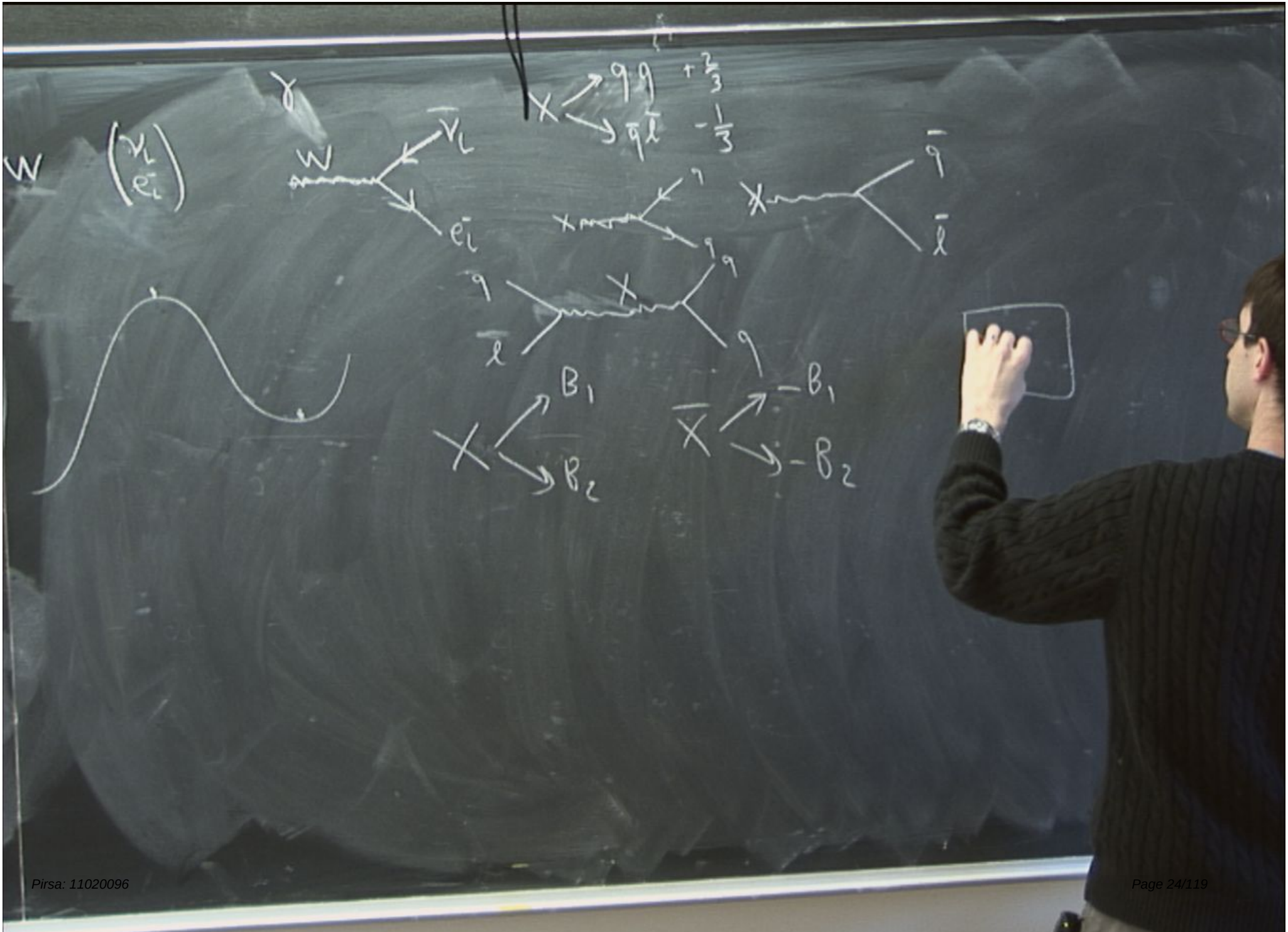

W
W
X → q
l

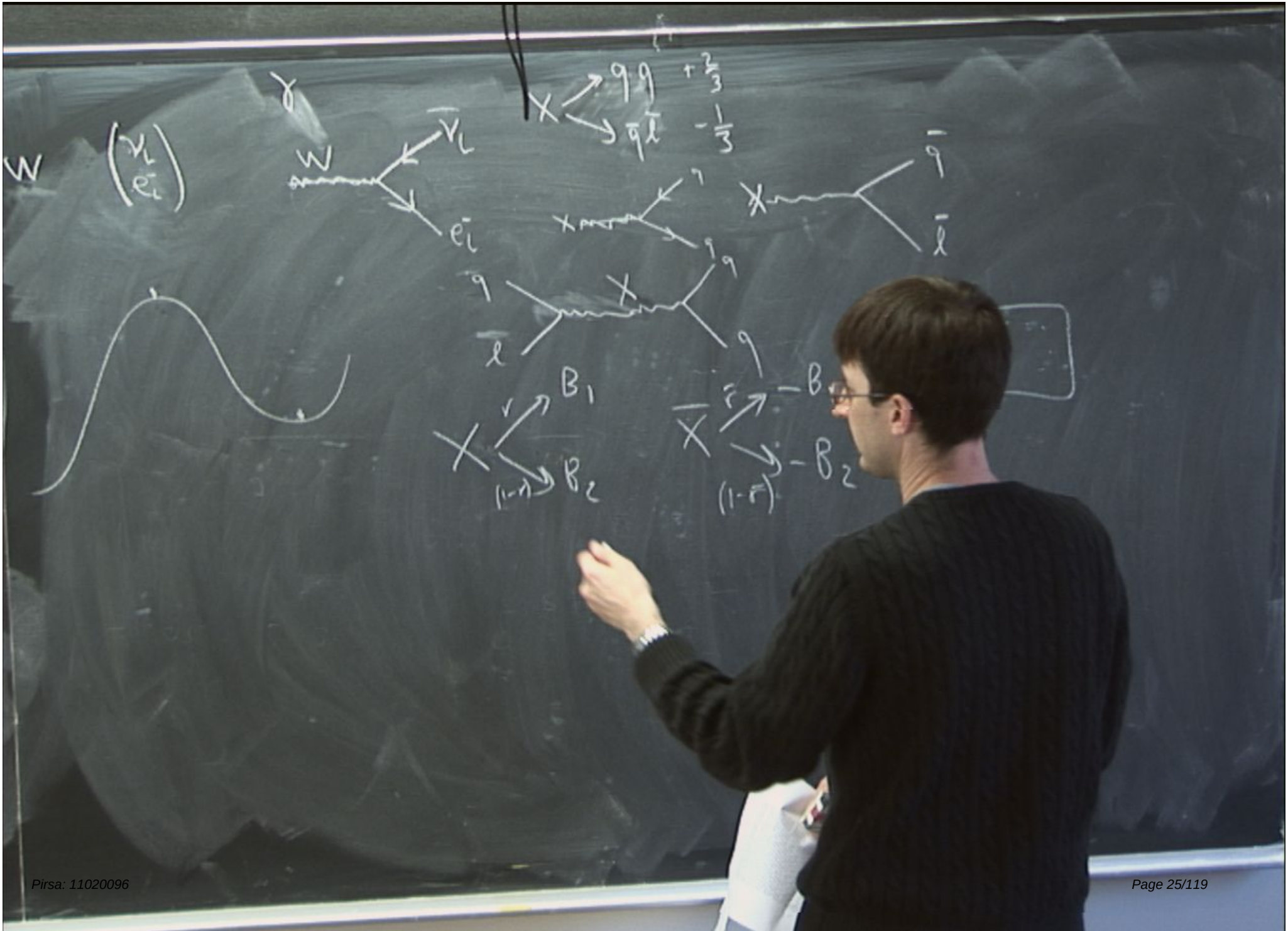

W
$\begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix}$
γ
W
$\bar{\nu}_L$
e_L^-
X
$q\bar{q}$ $+\frac{2}{3}$
$q\bar{q}$ $-\frac{1}{3}$

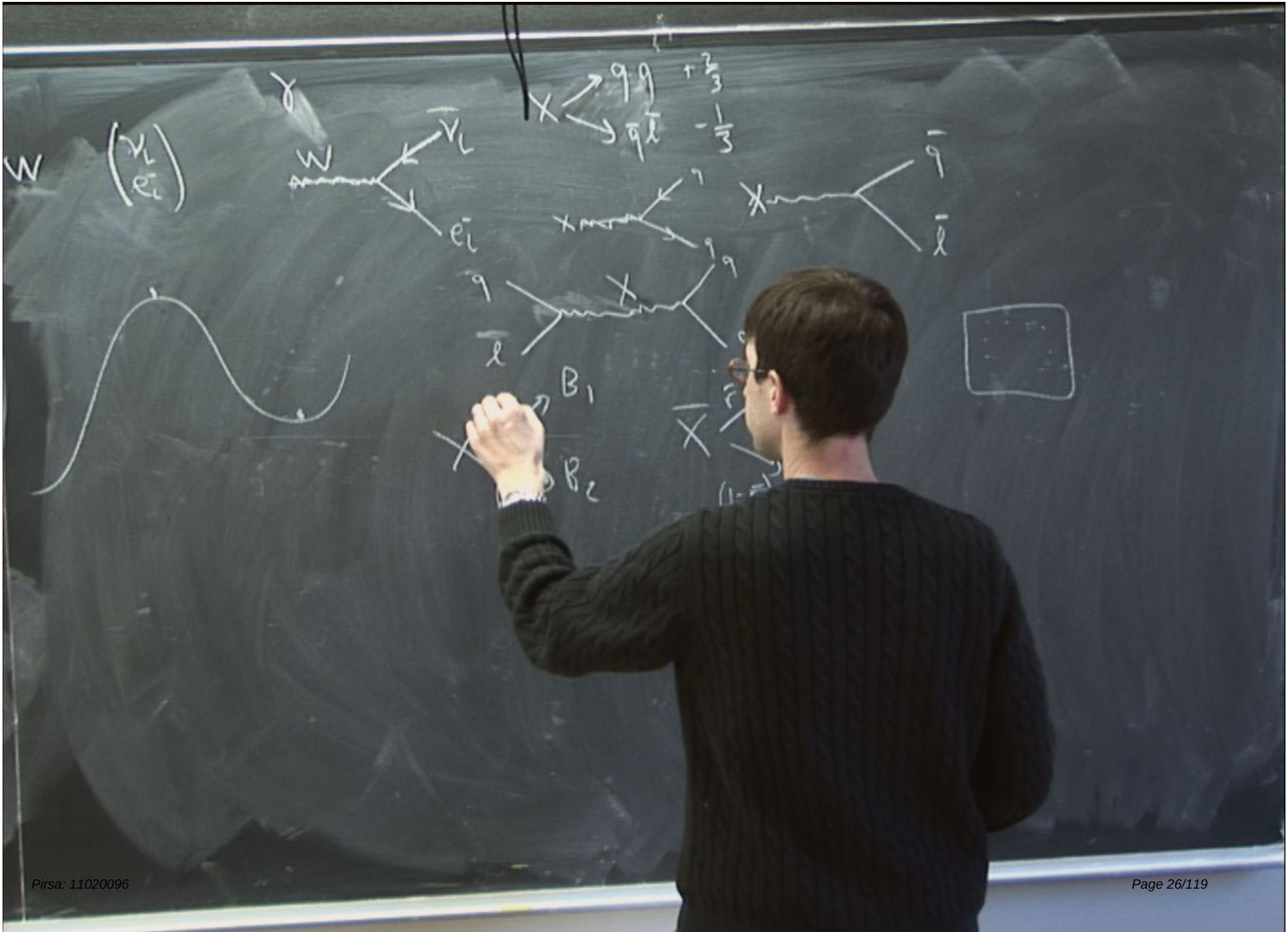

W
$\begin{pmatrix} \nu_l \\ e_l^- \end{pmatrix}$
γ
W
$\bar{\nu}_l$
e_l^-
X
$q\bar{q}$ $+\frac{2}{3}$
$q\bar{l}$ $-\frac{1}{3}$
$\bar{q}$
$\bar{l}$
$\bar{l}$
q
q
q

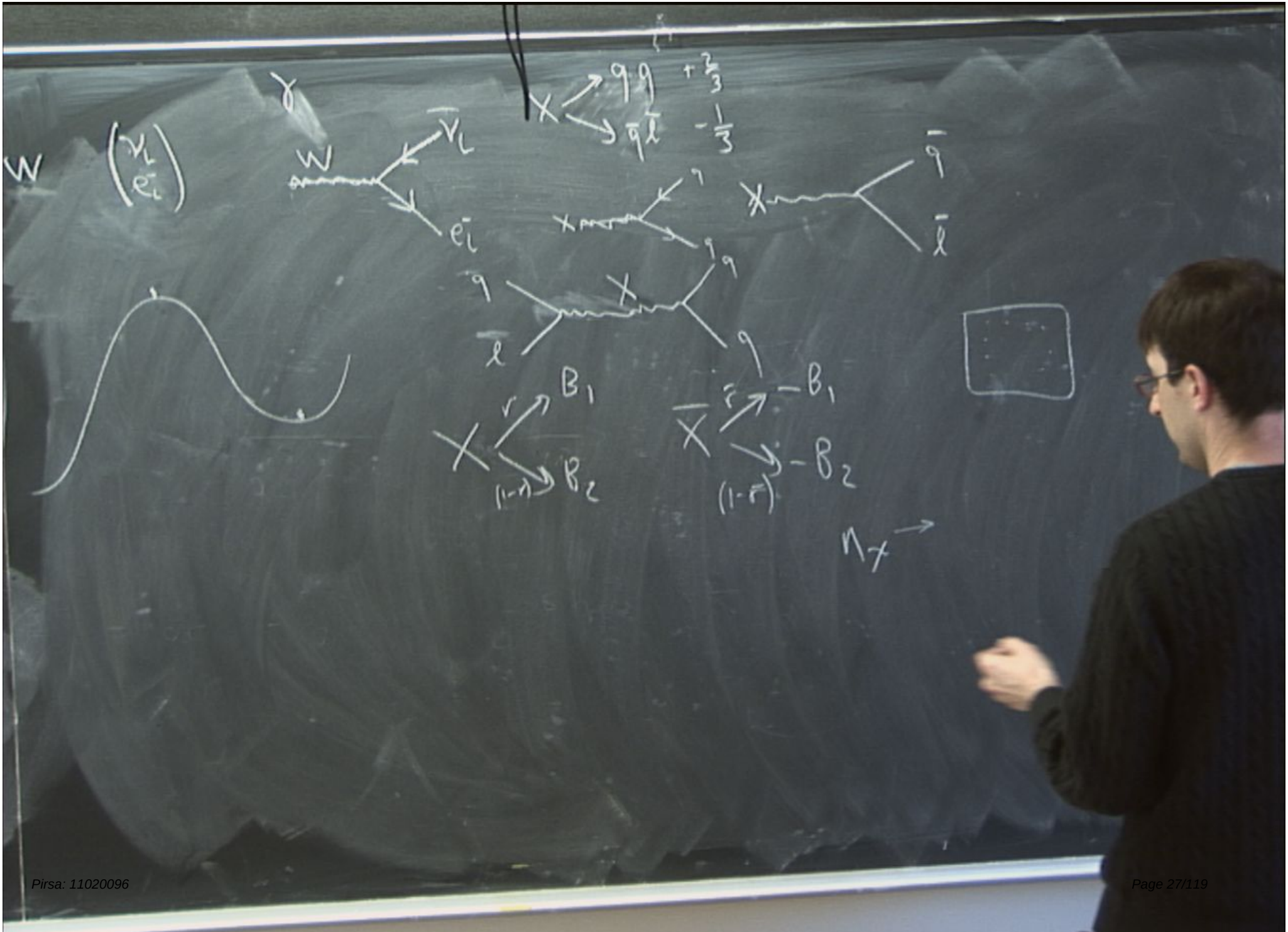

W
$\begin{pmatrix} \nu_L \\ e_i^- \end{pmatrix}$
γ
W
$\bar{\nu}_L$
e_i^-
X
$q\bar{q}$ $+\frac{2}{3}$
$q\bar{l}$ $-\frac{1}{3}$
q
X
$\bar{q}$
$\bar{l}$
q
$\bar{l}$
X
q
q

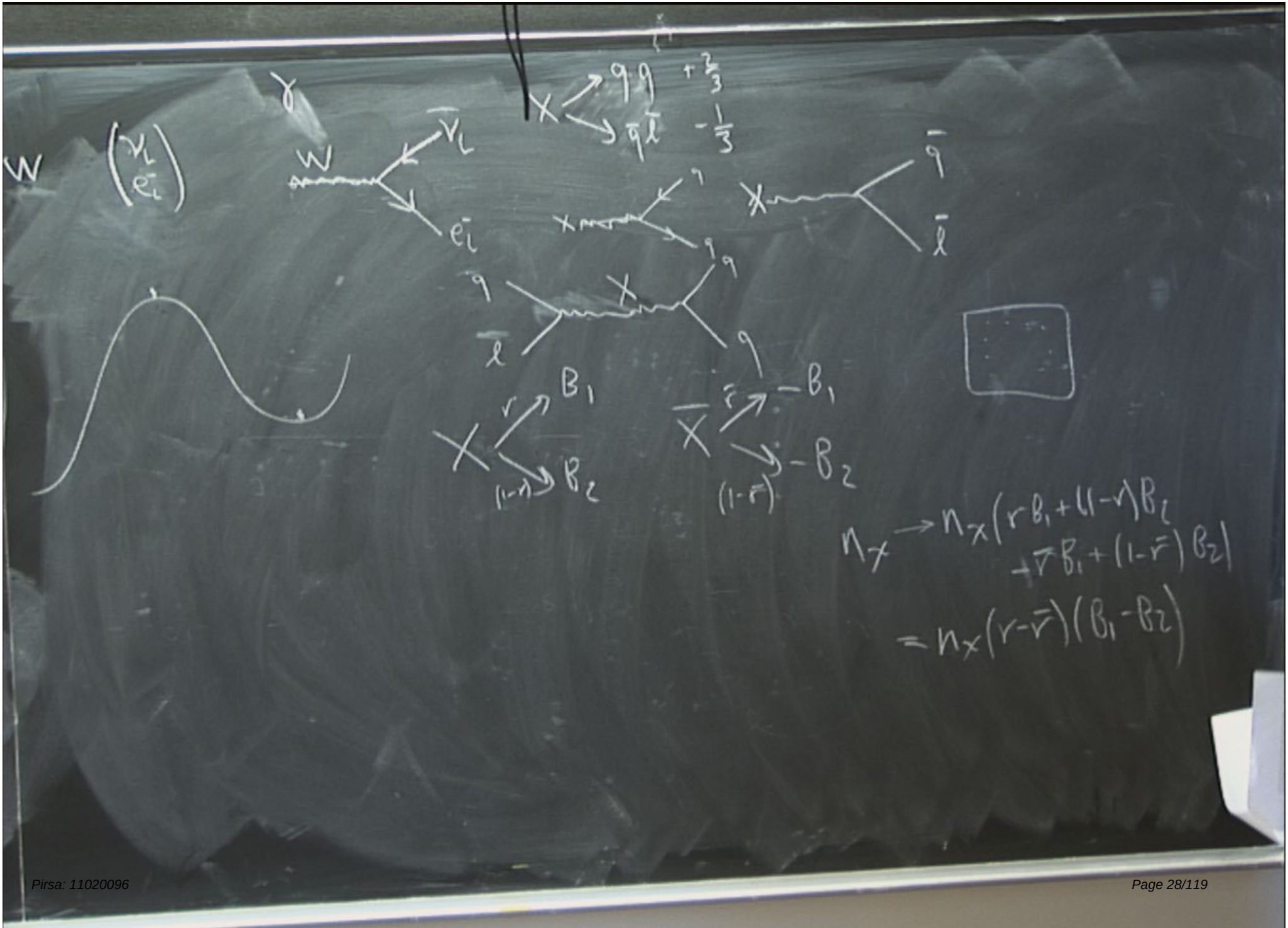

W
$\begin{pmatrix} \nu_l \\ e_i \end{pmatrix}$
γ
W
$\bar{\nu}_l$
e_i
X
$q\bar{q}$ $+\frac{2}{3}$
$q\bar{l}$ $-\frac{1}{3}$
$\bar{q}$
$\bar{l}$
q
$\bar{l}$
B_1
B_2
$\bar{X}$

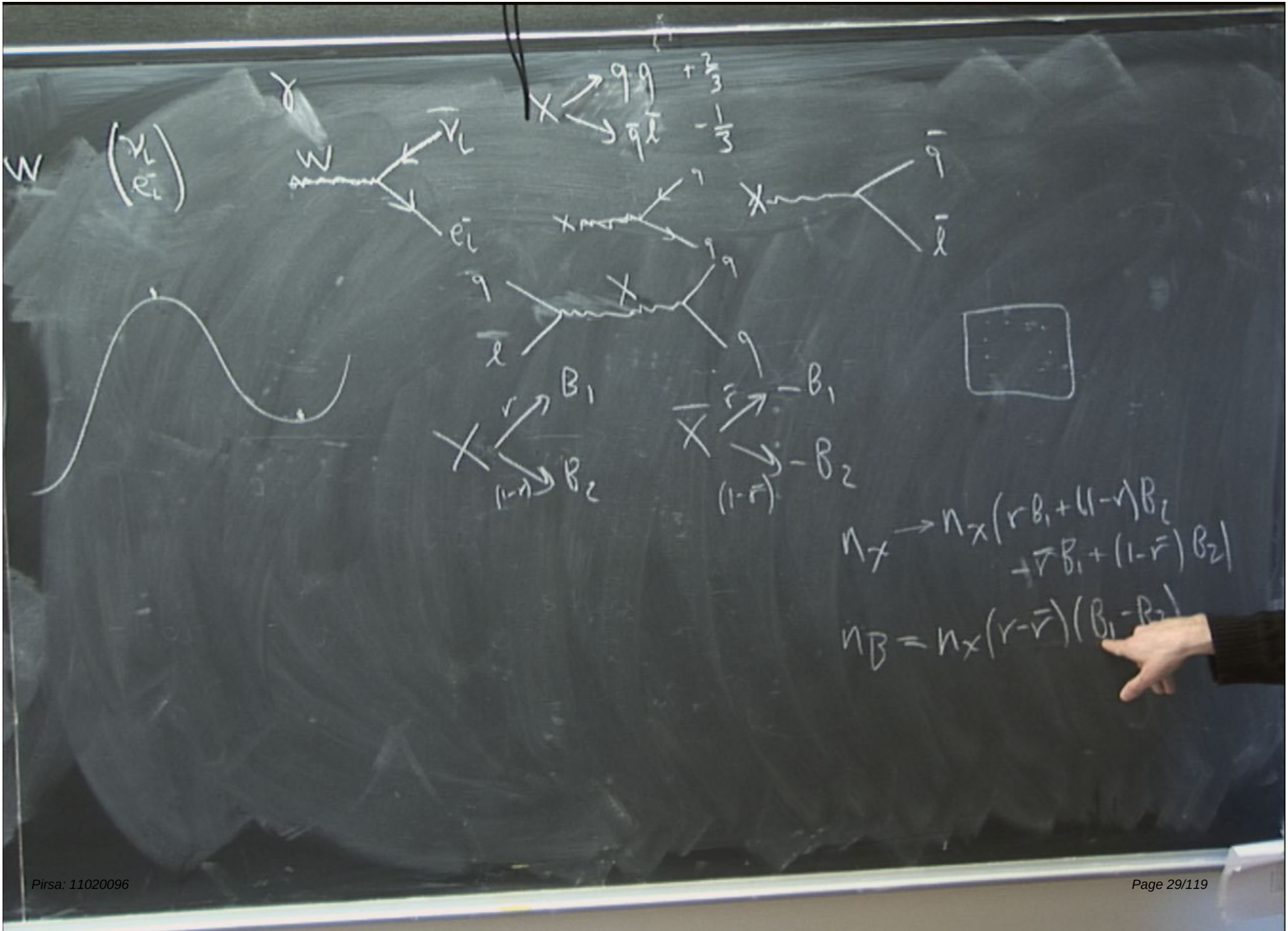

W
$\begin{pmatrix} \nu_l \\ e_i^- \end{pmatrix}$
γ
W
$\bar{\nu}_l$
e_i^-
X
$q\bar{q}$ $+\frac{2}{3}$
$q\bar{l}$ $-\frac{1}{3}$
γ
$q\bar{q}$
X
$\bar{q}$
$\bar{l}$
q
$\bar{l}$
X
q
B_1
X
B_2
$\bar{B}_1$
$\bar{X}$
$\bar{B}_2$

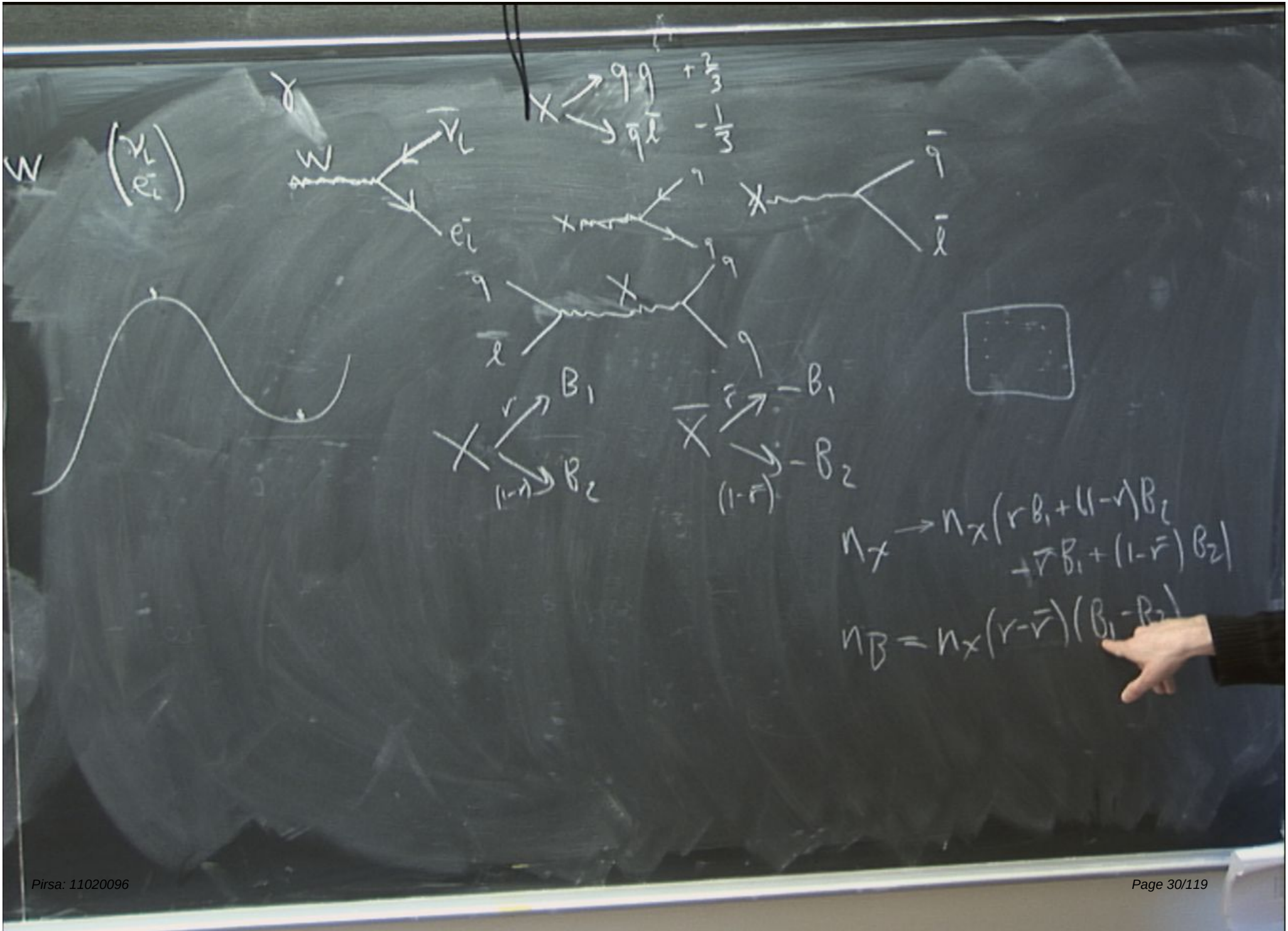

W
γ
W
X
q q
+2/3
q̄ l̄
-1/3
B1
B2
(1-r)
-B2
(1-r̄)

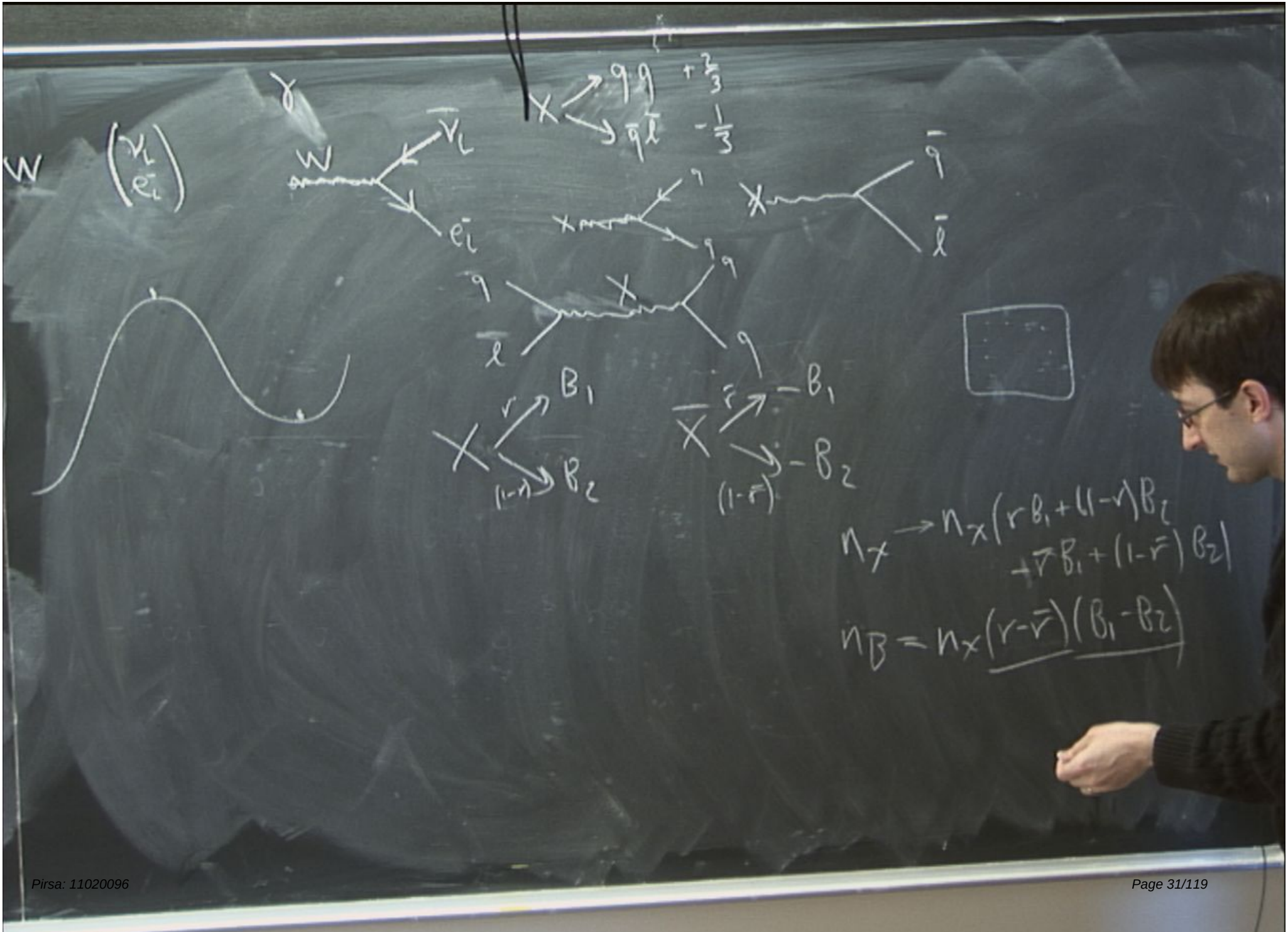

W
γ
W
X
B1
B2

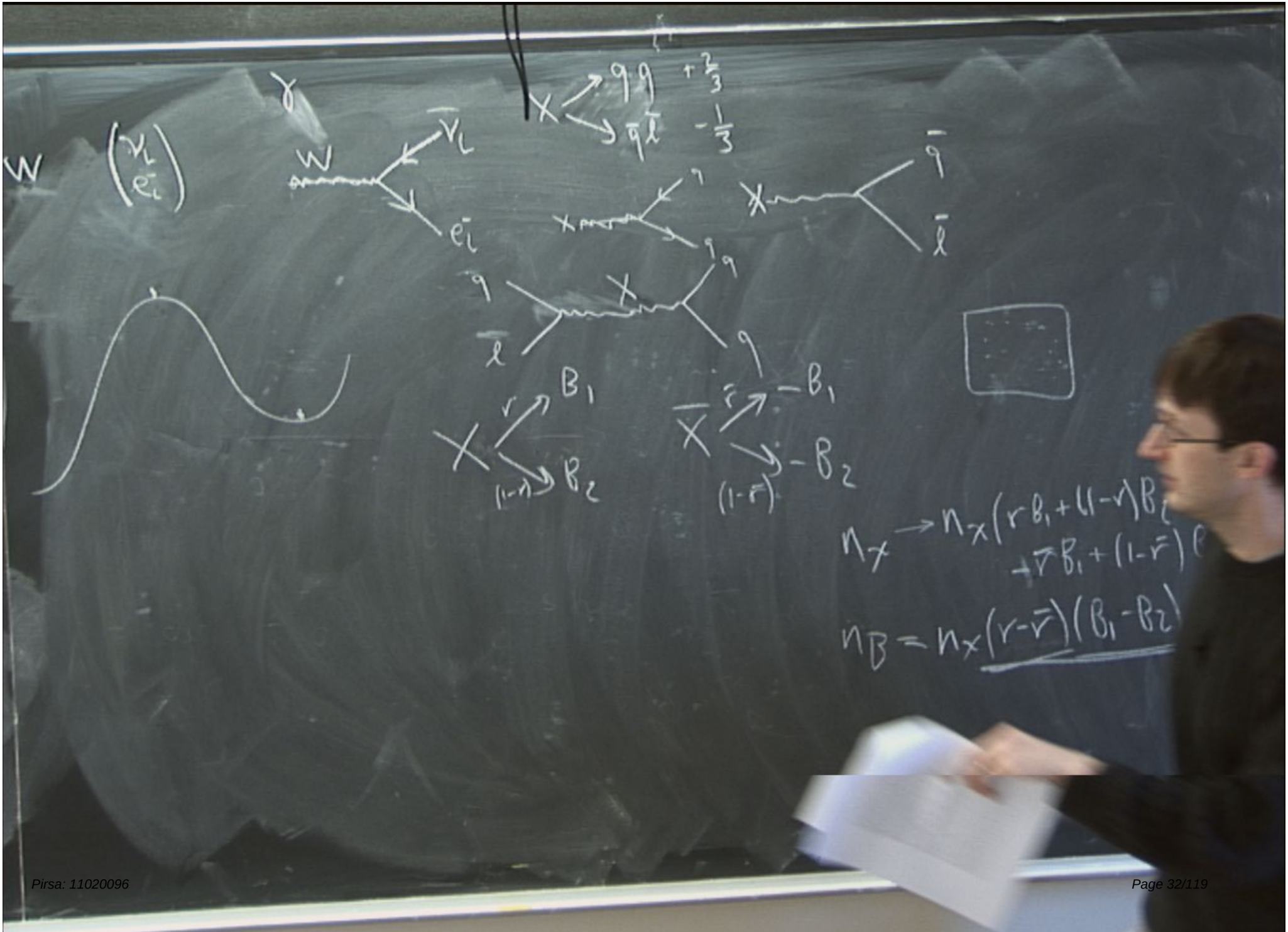

W
$\begin{pmatrix}\nu_L \\ e_L^-\end{pmatrix}$
W
$\bar{\nu}_L$
e_L^-
γ
X
$q\bar{q}$ $+\frac{2}{3}$
$q\bar{l}$ $-\frac{1}{3}$
$\bar{q}$
$\bar{l}$
B_1
B_2
$(1-r)$
n_X

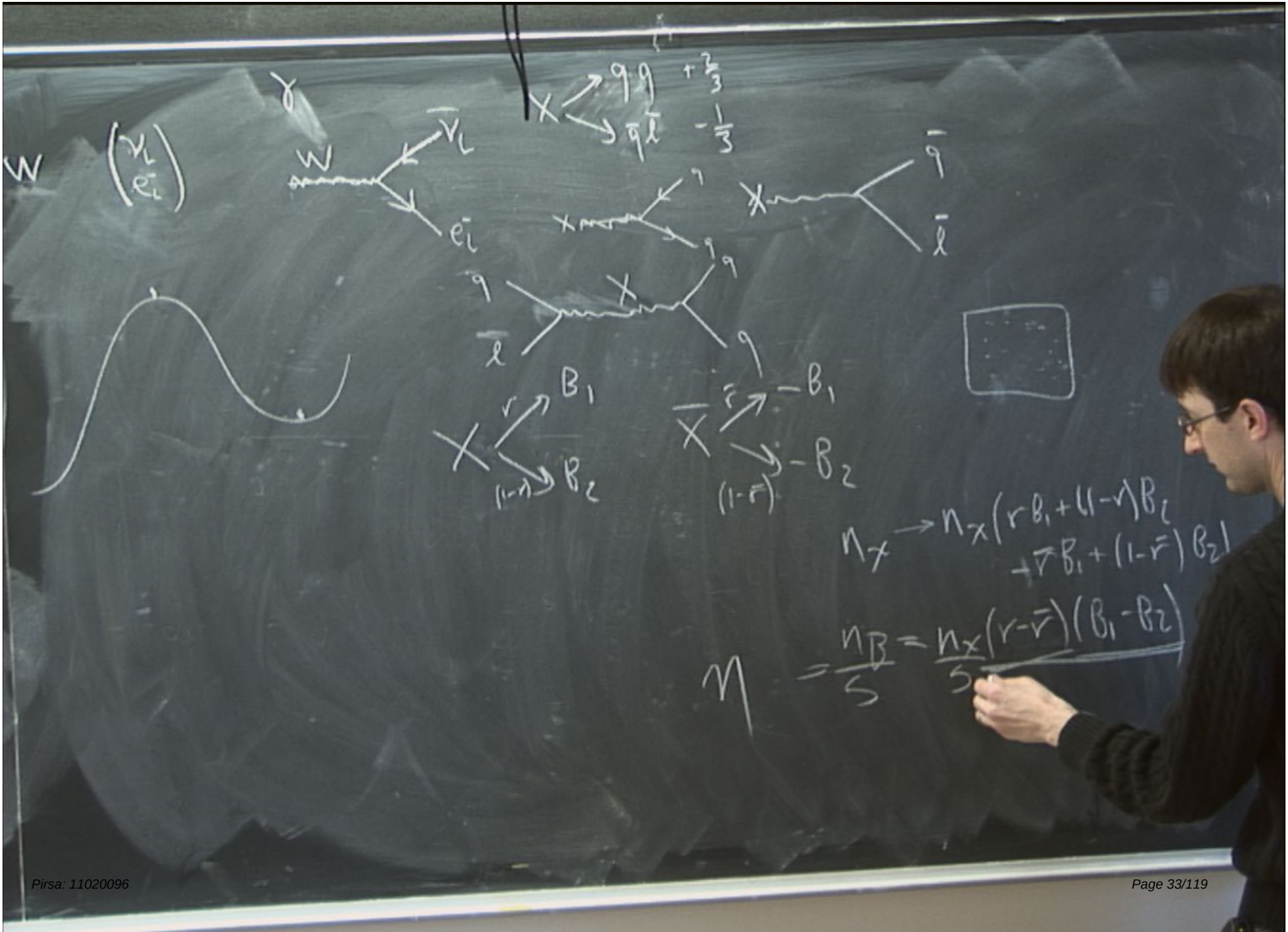

W
γ
X → qq +2/3
X → q̄l̄ −1/3
X → q q
X → q̄ l̄
X → q q
X → q q
X → r B1
X → (1−r) B2
X̄ → r̄ −B1
X̄ → (1−r̄) −B2
nX → nX(rB1 + (1−r)B2 + r̄B1 + (1−r̄)B2)
= nX(r−r̄)(B1−B2)

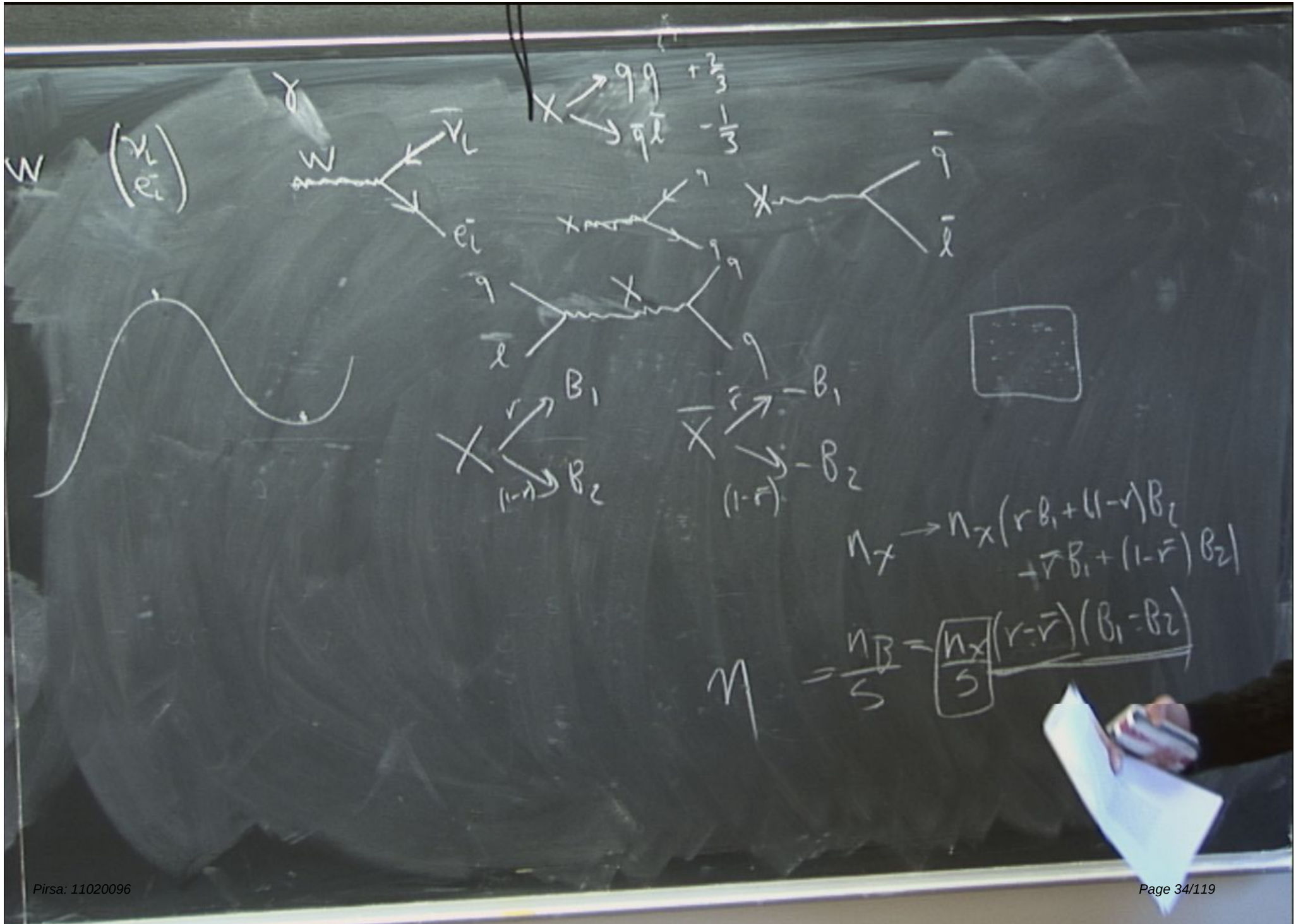

W
$\begin{pmatrix}\nu_l\\ e_l^-\end{pmatrix}$
γ
W
$\bar{\nu}_l$
e_l^-
X
$q\bar{q}$ $+\frac{2}{3}$
$q\bar{l}$ $-\frac{1}{3}$
$\bar{q}$
$\bar{l}$
q
$\bar{l}$
B_1
B_2
$(1-r)$
$-B_1$
$-B_2$
$(1-\bar{r})$
$n_X \to n_X(r B_1 + (1-r) B_2 + \bar{r} B_1 + (1-\bar{r}) B_2)$
$n_B = n_X(r-\bar{r})(B_1 - B_2)$

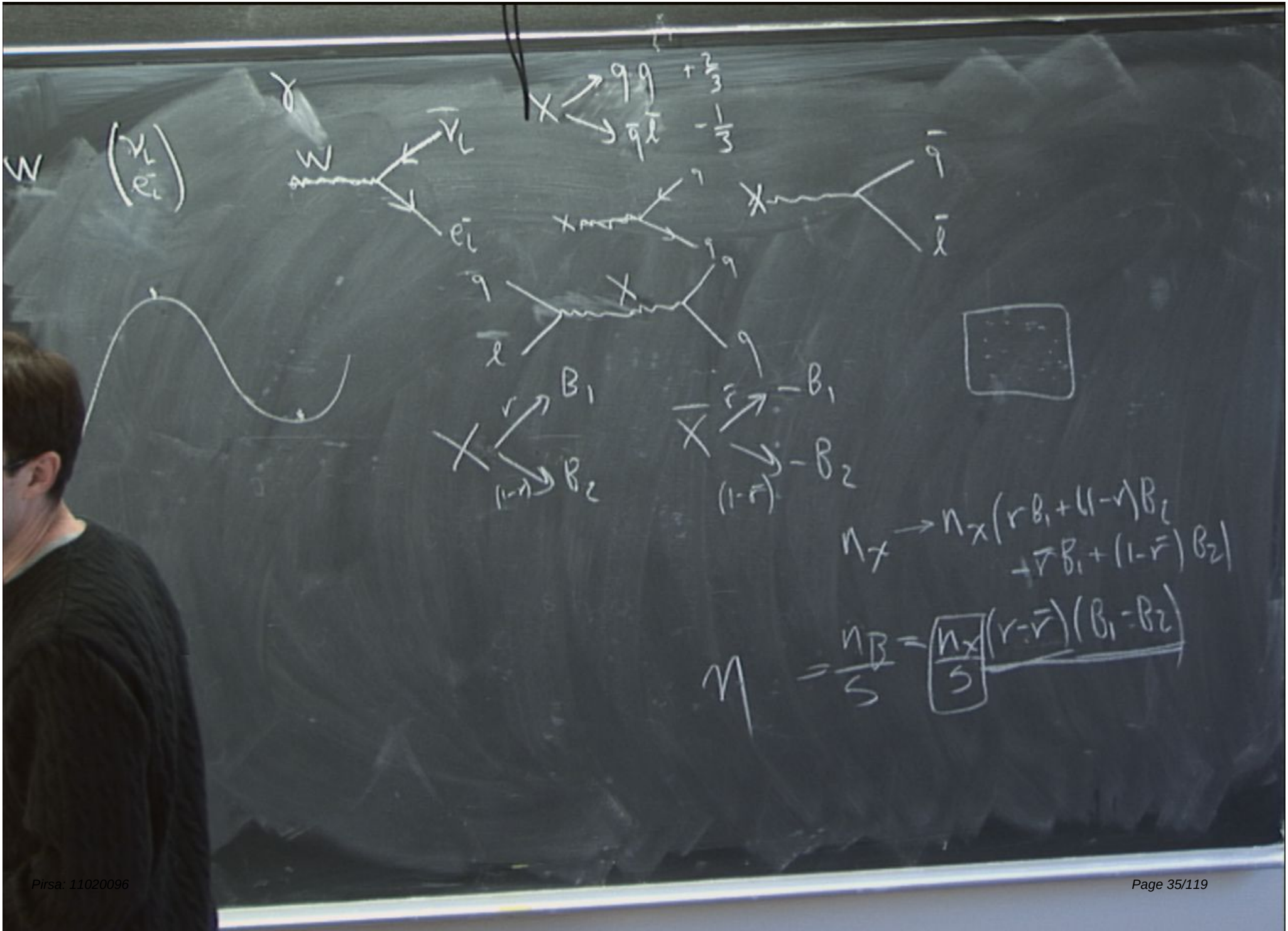

W
$\begin{pmatrix}\nu_l\\ e_l^-\end{pmatrix}$
γ
W
$\bar{\nu}_l$
e_l^-
X
$q\bar{q}$ $+\frac{2}{3}$
$q\bar{l}$ $-\frac{1}{3}$
$\bar{q}$
$\bar{l}$
B_1
B_2
$(1-r)$
$-B_1$
$-B_2$
$(1-\bar{r})$
$n_X \to n_X(r B_1 + (1-r) B_2$
$+ \bar{r} B_1 + (1-\bar{r}) B_2|$
$n_B = n_X(r-\bar{r})(B_1 - B_2)$

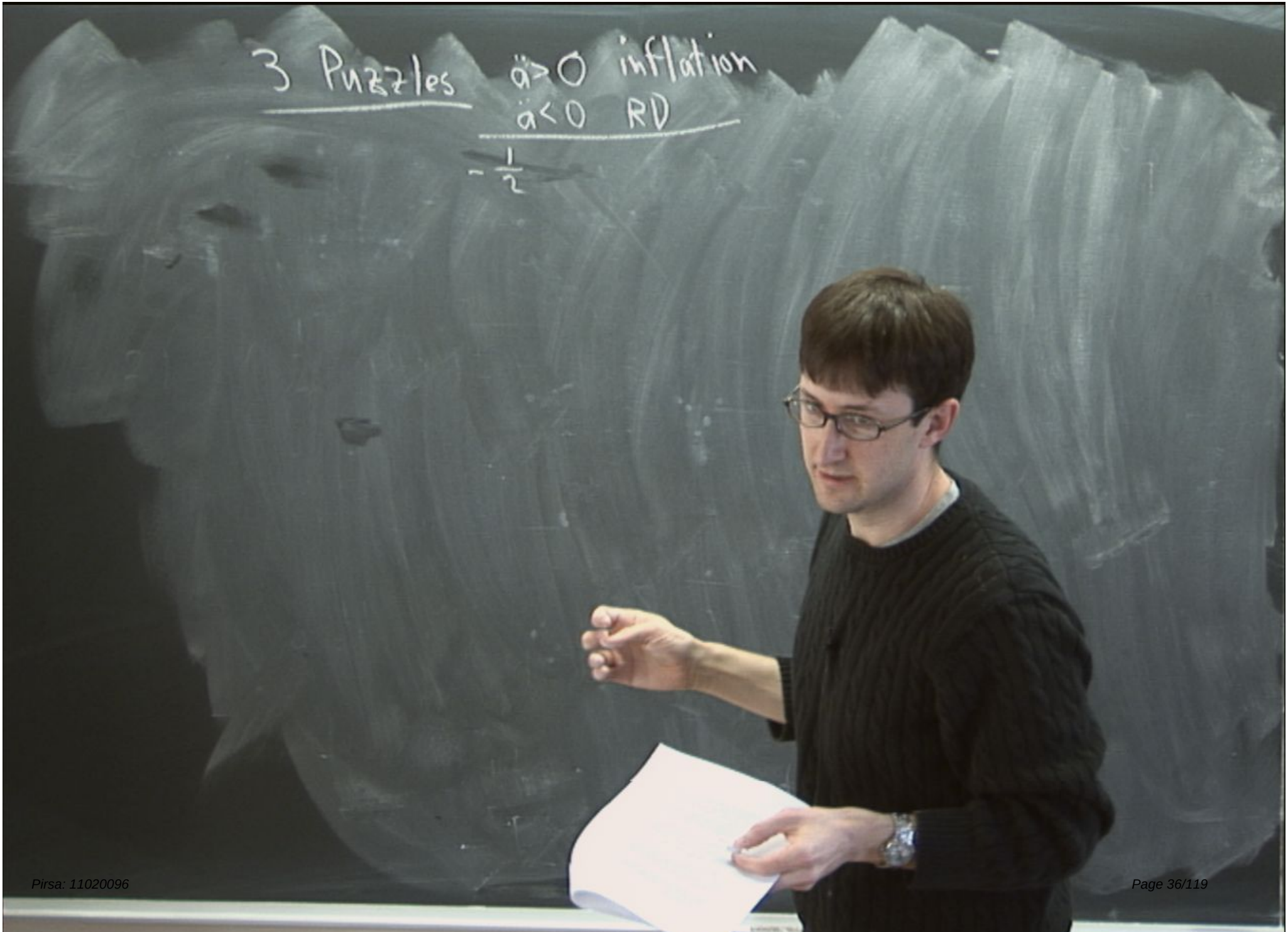

W
$\begin{pmatrix}\nu_L \\ e_L^-\end{pmatrix}$
γ
W
$\bar{\nu}_L$
e_L^-
X
$q\bar{q}$ $+\frac{2}{3}$
$q\bar{l}$ $-\frac{1}{3}$
$\bar{q}$
$\bar{l}$
B_1
B_2
$(1-r)$
$\bar{r}$
$n_X \rightarrow n_X(r B_1 + (1-r) B_2 + \bar{r} B_1 + (1-\bar{r}) B_2)$
$n_B = n_X (r-\bar{r})(B_1 - B_2)$

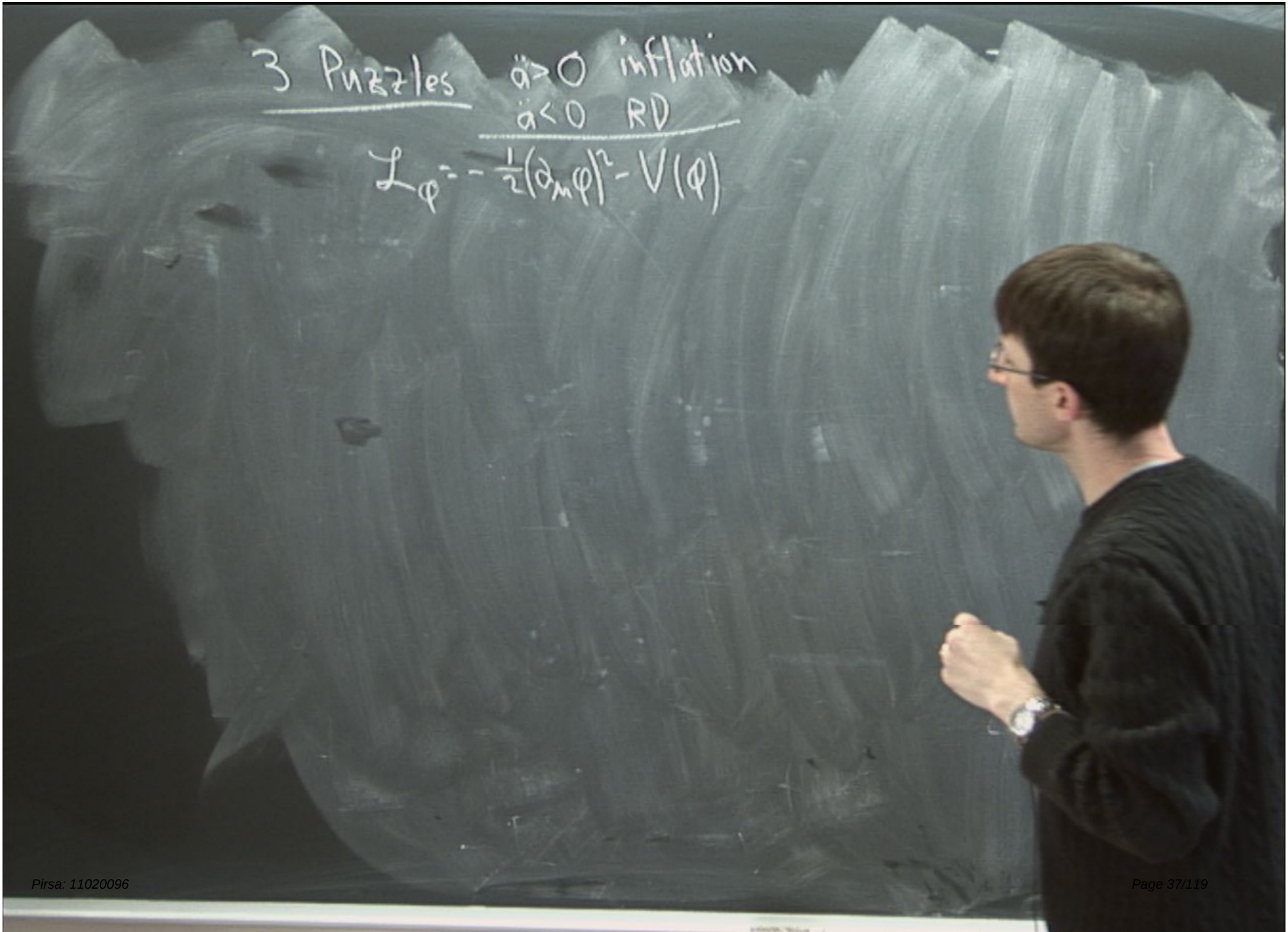

W
γ
qq +2/3
q̄l̄ −1/3
B1
B2
−B1
−B2
(1−r)
(1−r̄)
nχ → nχ(rB1 + (1−r)B2
+ r̄B1 + (1−r̄)
nB = nχ(r−r̄)(B1−B2)


W
γ
X → qq +2/3
X → q̄l̄ −1/3
X → B1
X → B2
X̄ → −B1
X̄ → −B2
n_X → n_X(rB1 + (1−r)B2 + r̄B1 + (1−r̄)B2)
η = n_B/s = n_X(r−r̄)(B1−B2)/s


W
$\begin{pmatrix}\nu_i\\ e_i\end{pmatrix}$
γ
W
$\bar{\nu}_i$
e_i
X
$q\bar{q}$ $+\frac{2}{3}$
$q\bar{l}$ $-\frac{1}{3}$
$\bar{q}$
$\bar{l}$
B_1
B_2
$(1-r)$
$-B_1$
$-B_2$
$(1-\bar{r})$
$n_X \to n_X(r B_1 + (1-r) B_2 + \bar{r} B_1 + (1-\bar{r}) B_2)$
$\eta = \frac{n_B}{s} = \frac{n_X}{s}(r-\bar{r})(B_1 - B_2)$


W
γ
X → qq +2/3
X → q̄l̄ −1/3
X → r B1
X → (1−r) B2
X̄ → r̄ −B1
X̄ → (1−r̄) −B2
nX → nX(rB1 + (1−r)B2 + r̄B1 + (1−r̄)B2)
η = nB/s = nX/s (r−r̄)(B1−B2)


3 Puzzles
ä>0 inflation
ä<0 RD
-1/2


3 Puzzles
$\ddot{a}>0$ inflation
$\ddot{a}<0$ RD
$\mathcal{L}_\varphi = -\frac{1}{2}(\partial_\mu\varphi)^2 - V(\varphi)$

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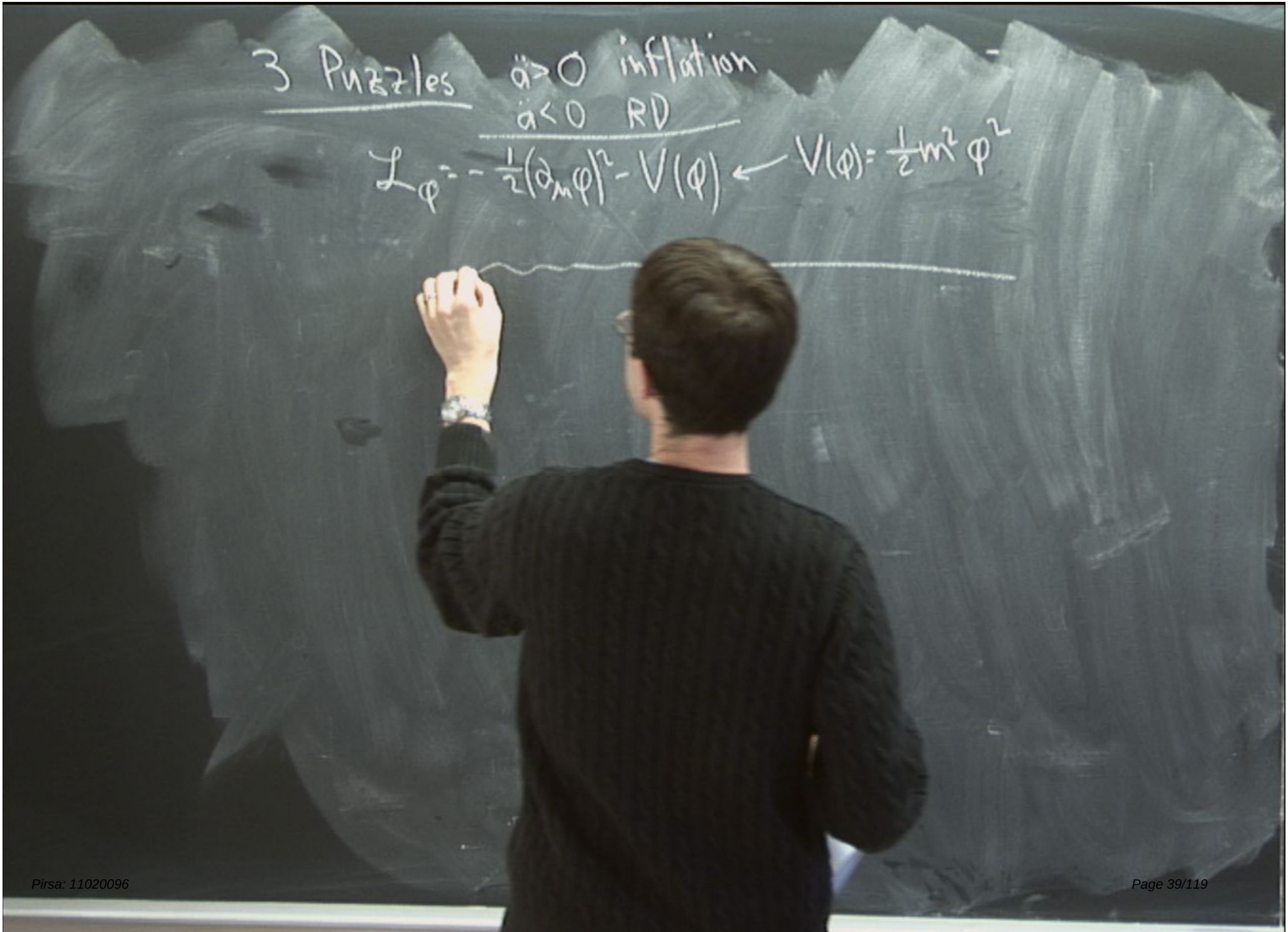
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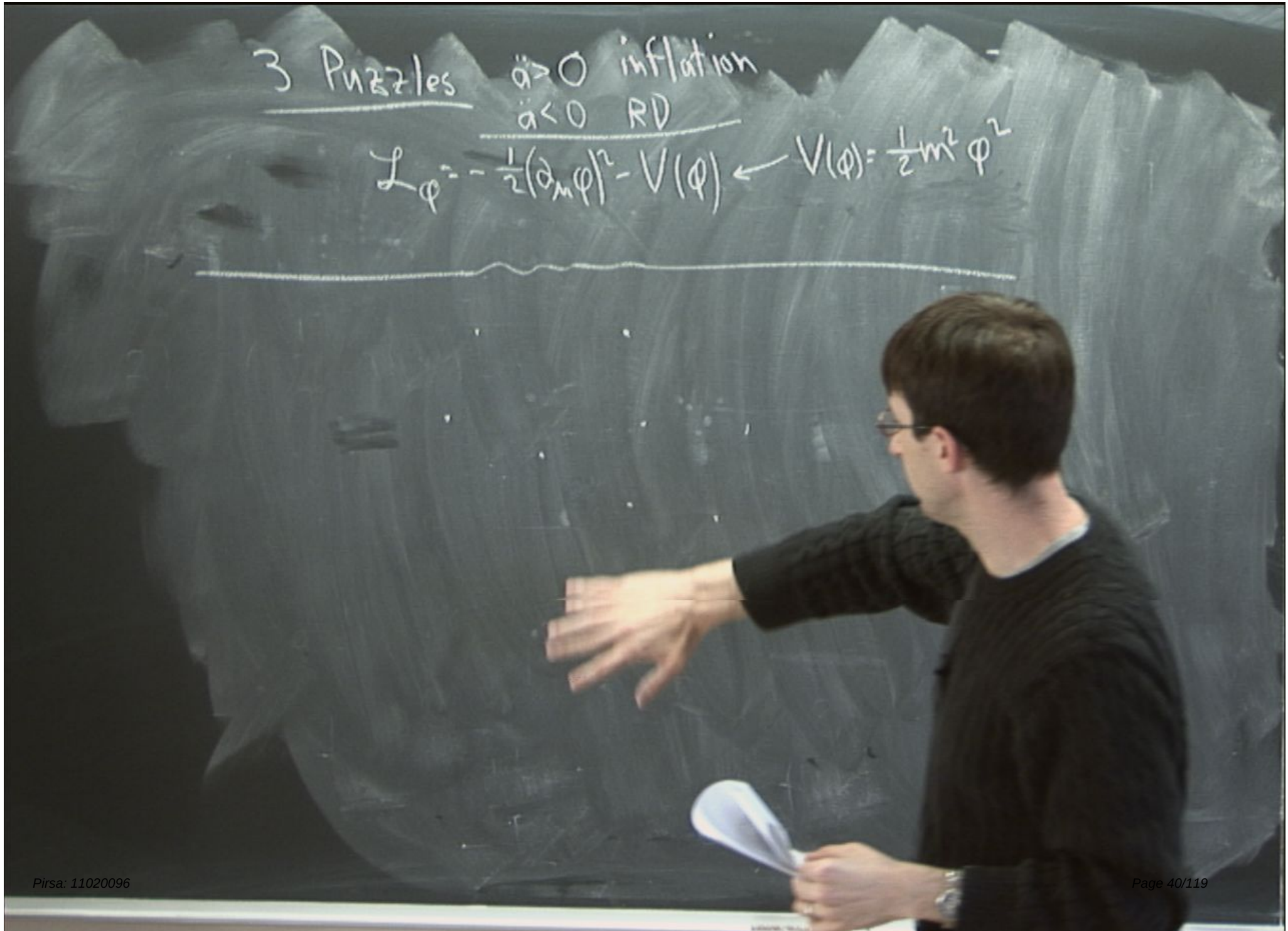
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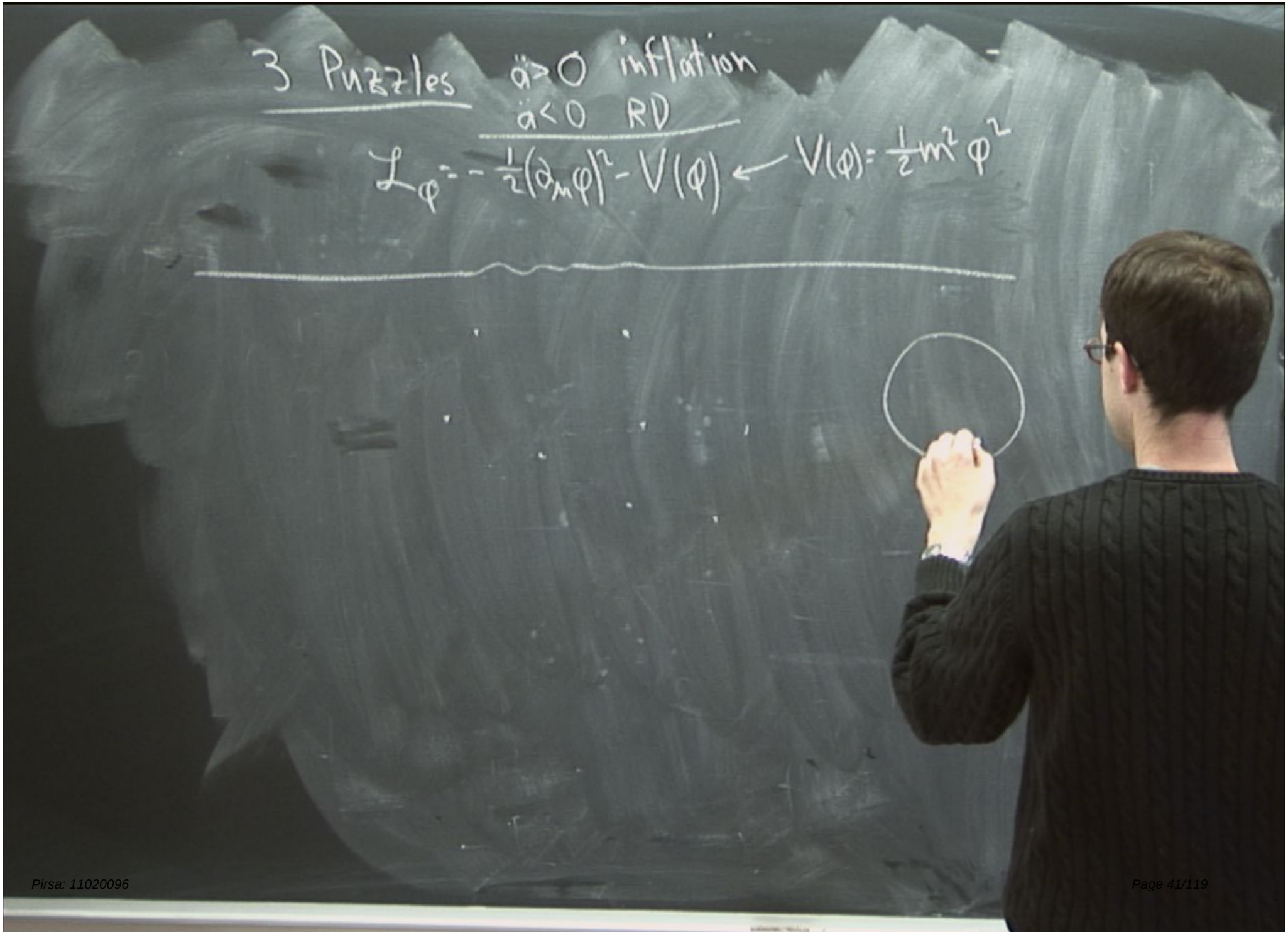
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$c = \frac{L}{T}$

$K = +1$

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$H(t) = \frac{\dot{a}(t)}{a(t)}$

$\frac{1}{H(t)} =$

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$c = \frac{L}{T}$

$\frac{1}{H(t)} =$

$V(\phi) = \frac{1}{2} m^2 \phi^2$

$ds^2 = -dt^2 + a^2 [$

$c = \frac{\lambda}{T}$

$K = +1$

t

$H(t) = \frac{\dot{a}(t)}{a(t)}$

$\frac{1}{H(t)} =$

$V(\phi) = \frac{1}{2} m^2 \phi^2$

$ds^2 = -dt^2 + a^2 [dx^2 + \dots]$

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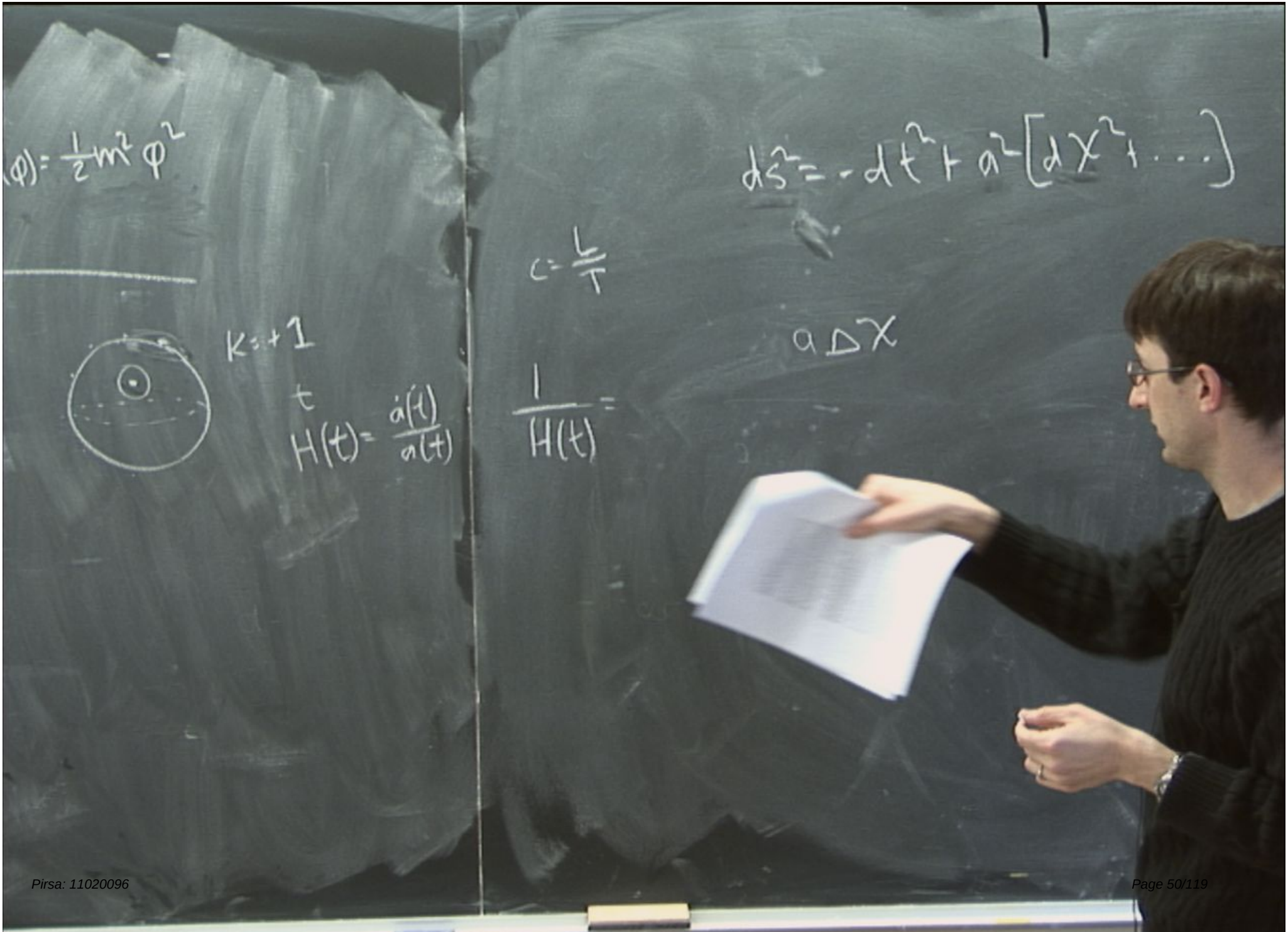
$\phi) = \frac{1}{2}m^2\phi^2$
$K = +1$
$H(t) = \frac{\dot{a}(t)}{a(t)}$
$c = \frac{L}{T}$
$\frac{1}{H(t)} =$
$ds^2 = -dt^2 + a^2[dx^2 + \ldots]$
$a\Delta x$

$(\phi) = \frac{1}{2} m^2 \phi^2$

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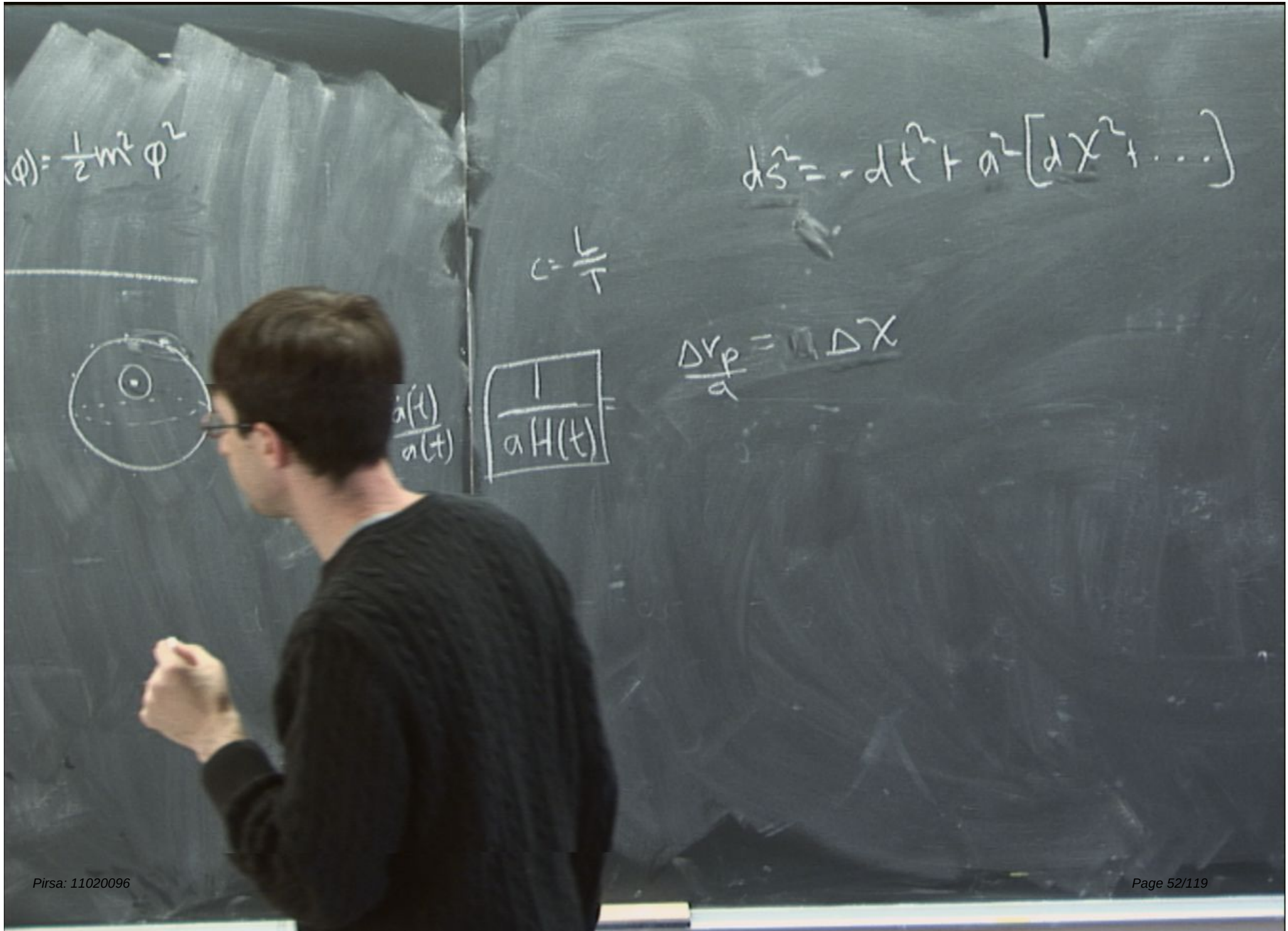
$H(t) = \frac{\dot{a}(t)}{a(t)}$

$\frac{1}{H(t)} =$

$\Delta r_p = a \Delta x$


$\frac{1}{2}m^2\varphi^2$
$ds^2 = -dt^2 + a^2[dx^2 + \dots]$
$c = \frac{L}{T}$
$\frac{\Delta r_p}{a} = H \Delta x$
$\frac{\dot{a}(t)}{a(t)}$
$\frac{1}{aH(t)}$

3 Puzzles $\ddot{a}>0$ inflation

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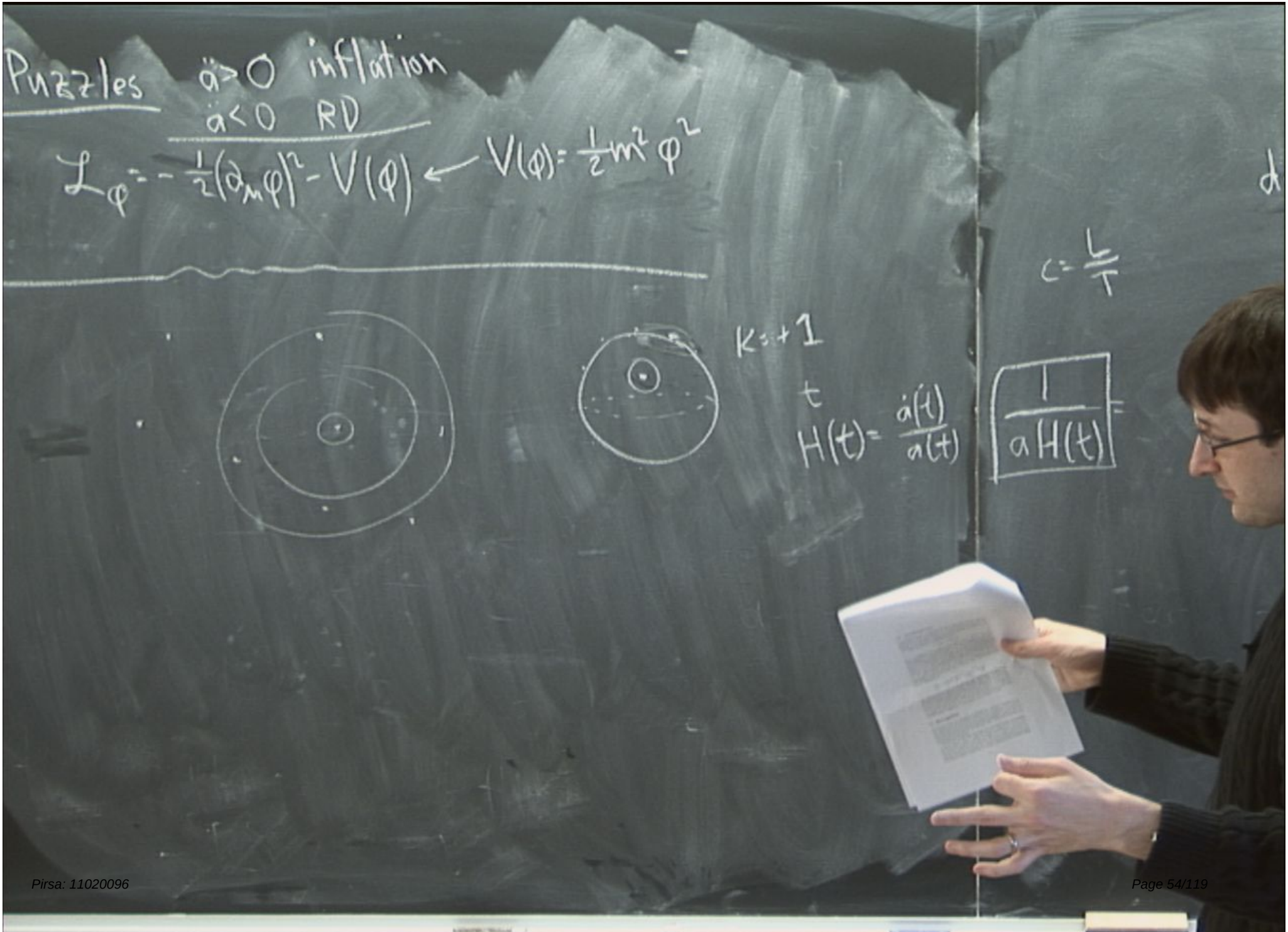
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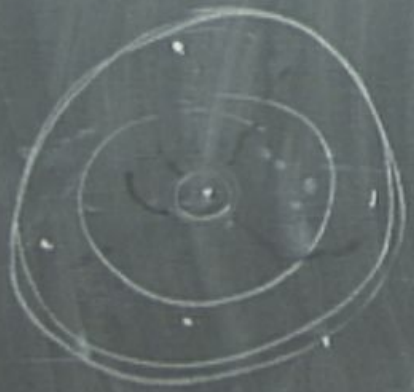

$K = +1$

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$c = \frac{L}{T}$

$\frac{1}{aH(t)} =$

$\frac{\Delta r_p}{a}$

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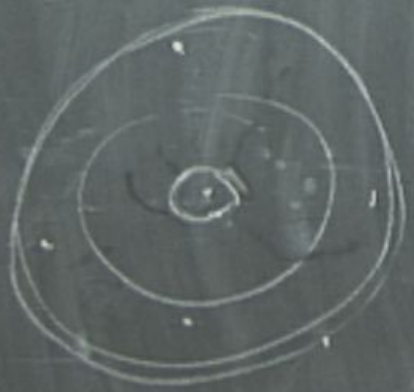

$K = +1$

$H(t) = \frac{\dot{a}(t)}{a(t)}$ t

$c = \frac{L}{T}$

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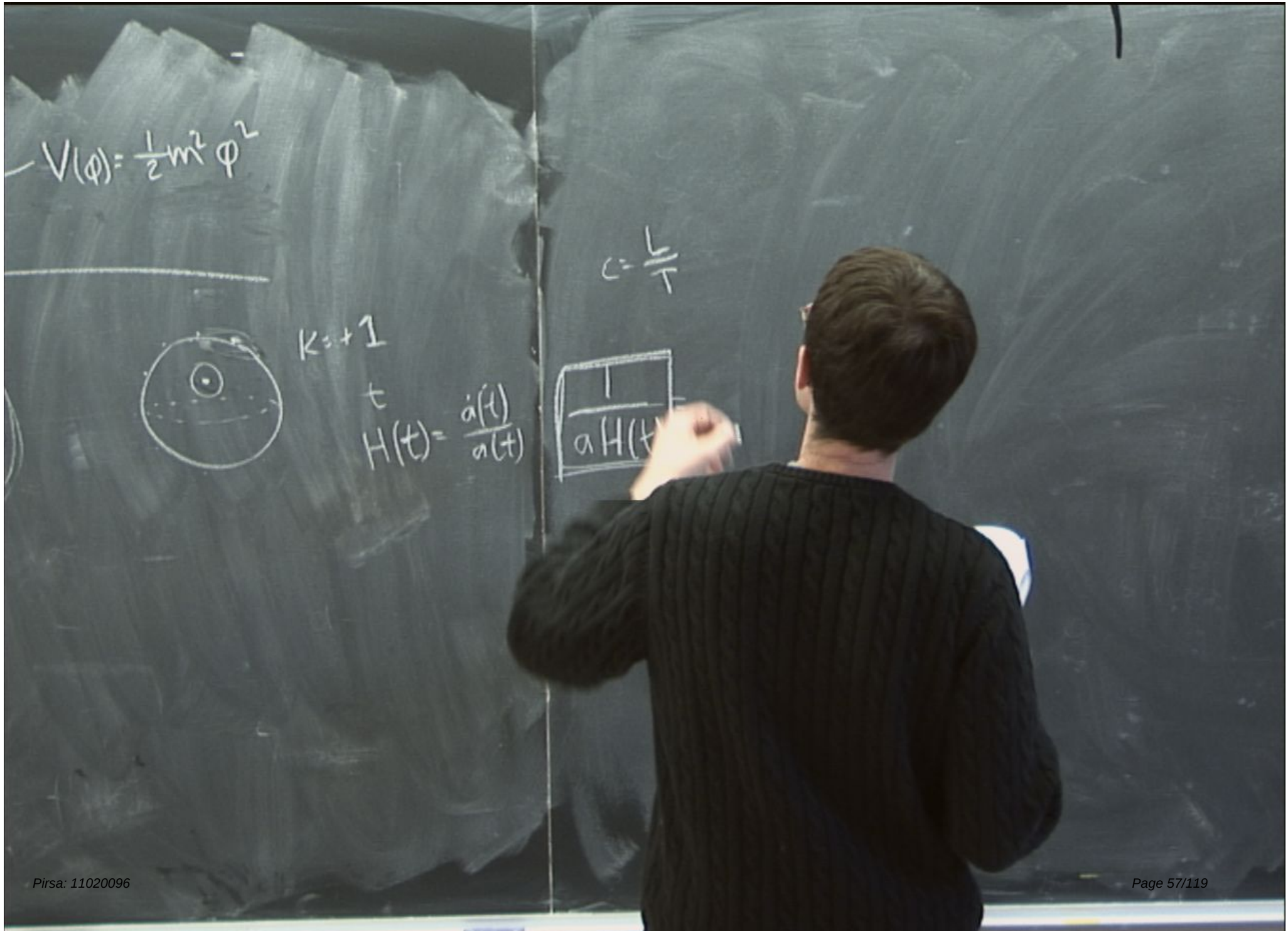
$V(\phi) = \frac{1}{2} m^2 \phi^2$
$K = +1$
$H(t) = \frac{\dot{a}(t)}{a(t)}$
$c = \frac{L}{T}$
$a H(t)$

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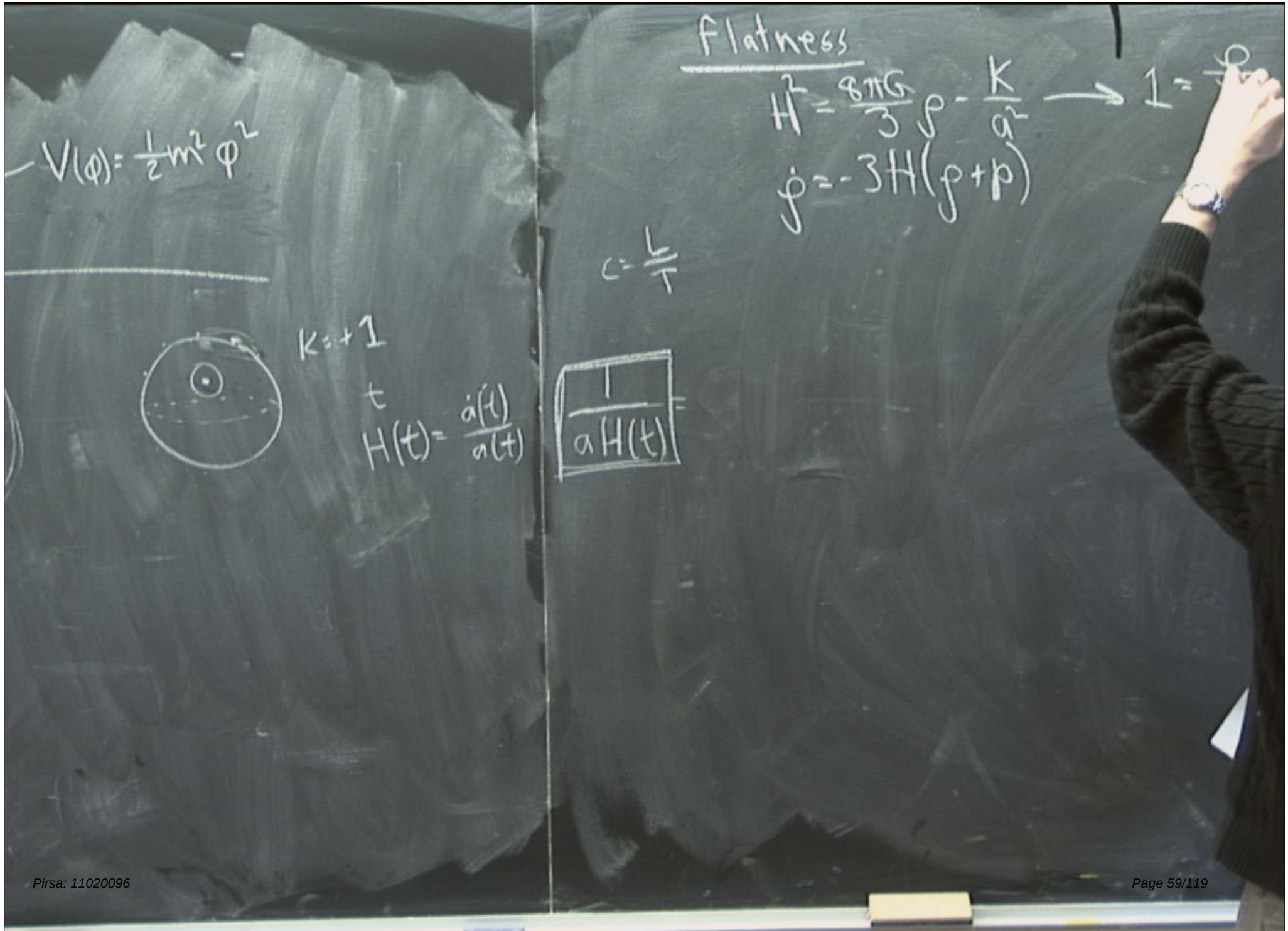
$\frac{1}{a H(t)} =$

Flatness

$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$

$\dot{\rho} = -3H(\rho + p)$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$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$
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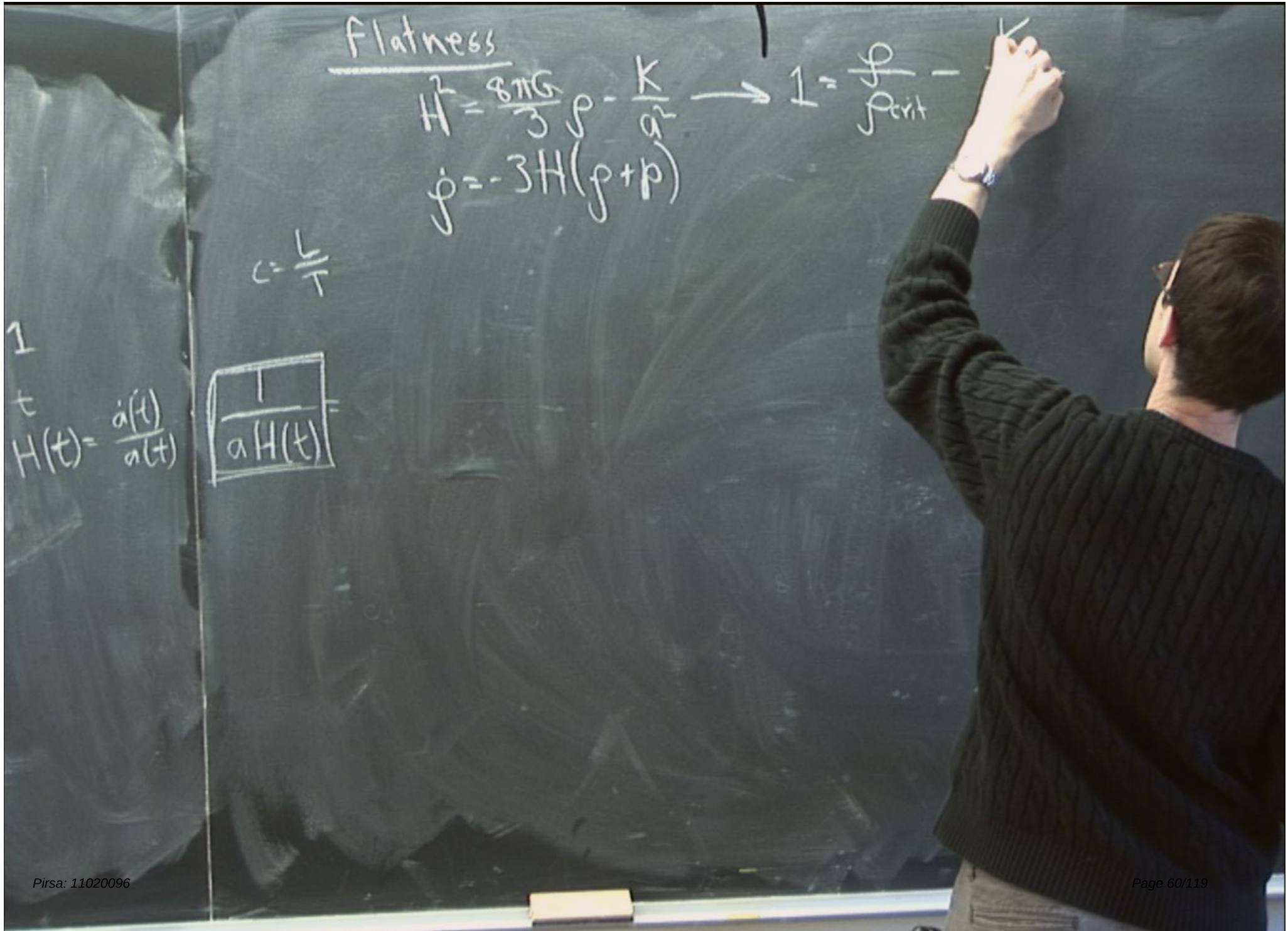
$\frac{1}{aH(t)}$


Flatness
$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$
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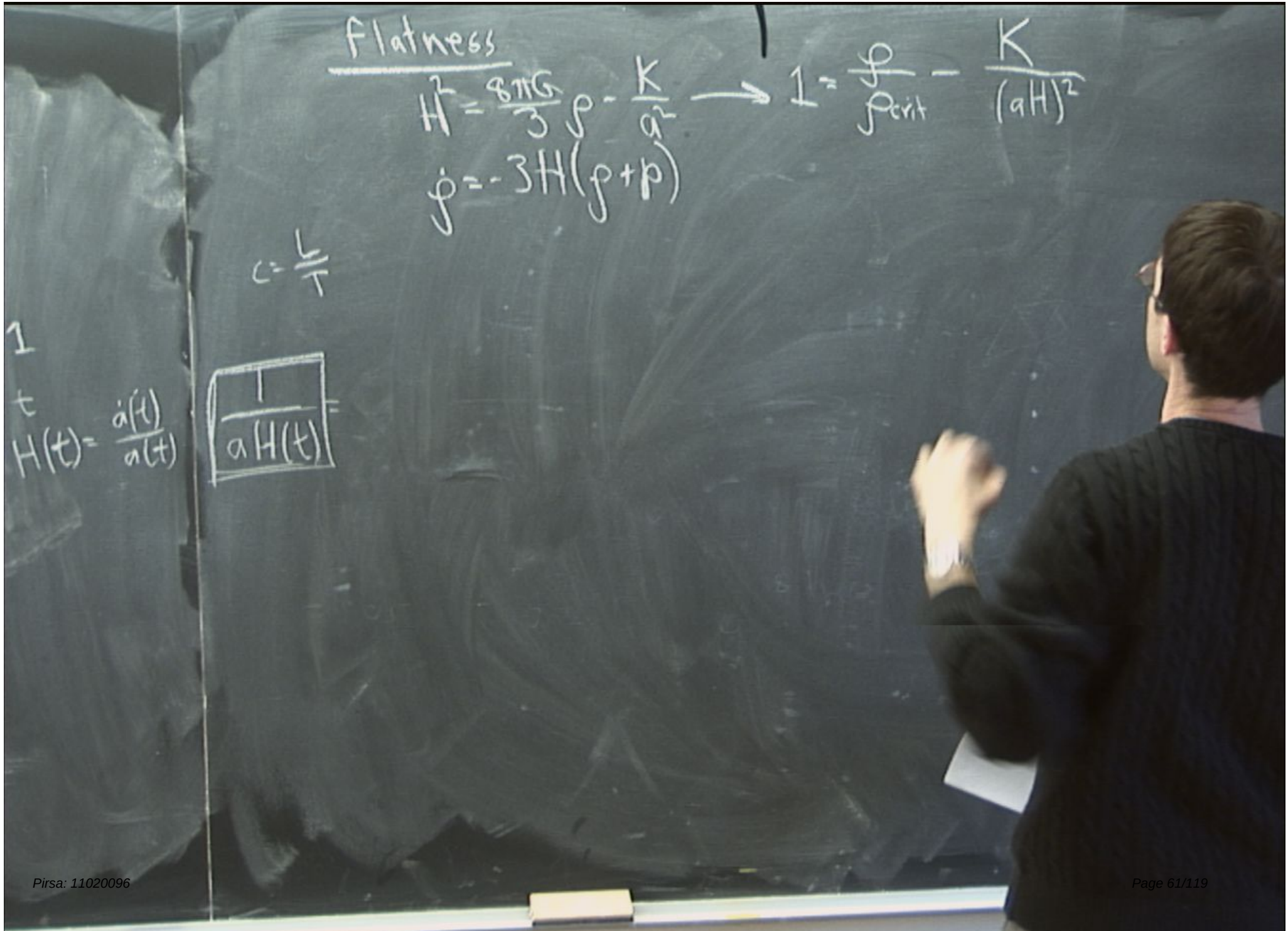
$1 = \frac{\rho}{\rho_{crit}} -$
$c = \frac{L}{T}$
1
t
$H(t) = \frac{\dot{a}(t)}{a(t)}$
$aH(t)$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$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} \longrightarrow 1 = \frac{\rho}{\rho_{crit}} - \frac{K}{(aH)^2} \qquad \rho_{crit} = \frac{3H^2}{8\pi G}$$

$$\dot{\rho} = -3H(\rho + p)$$

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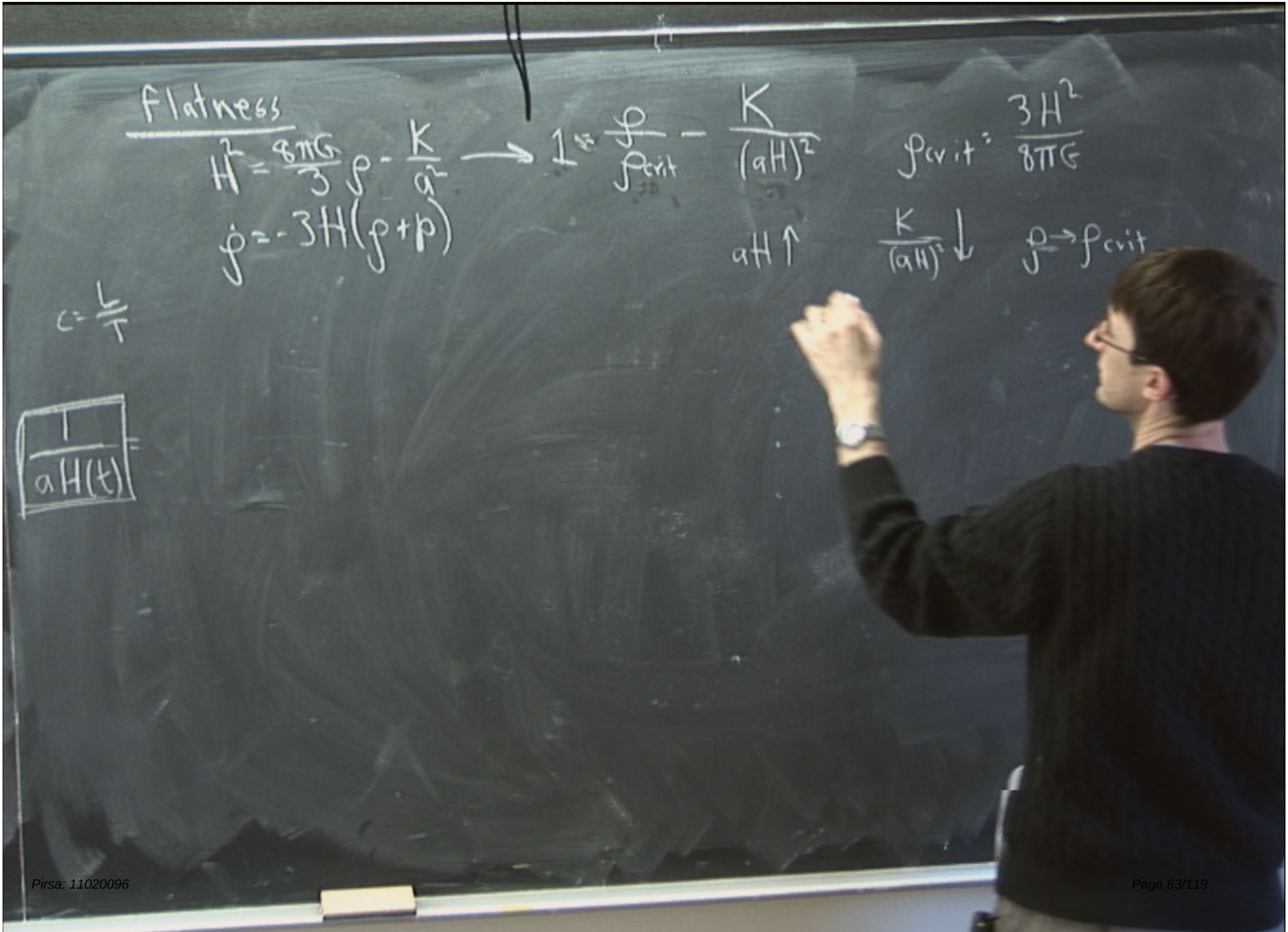
$\frac{1}{aH(t)}$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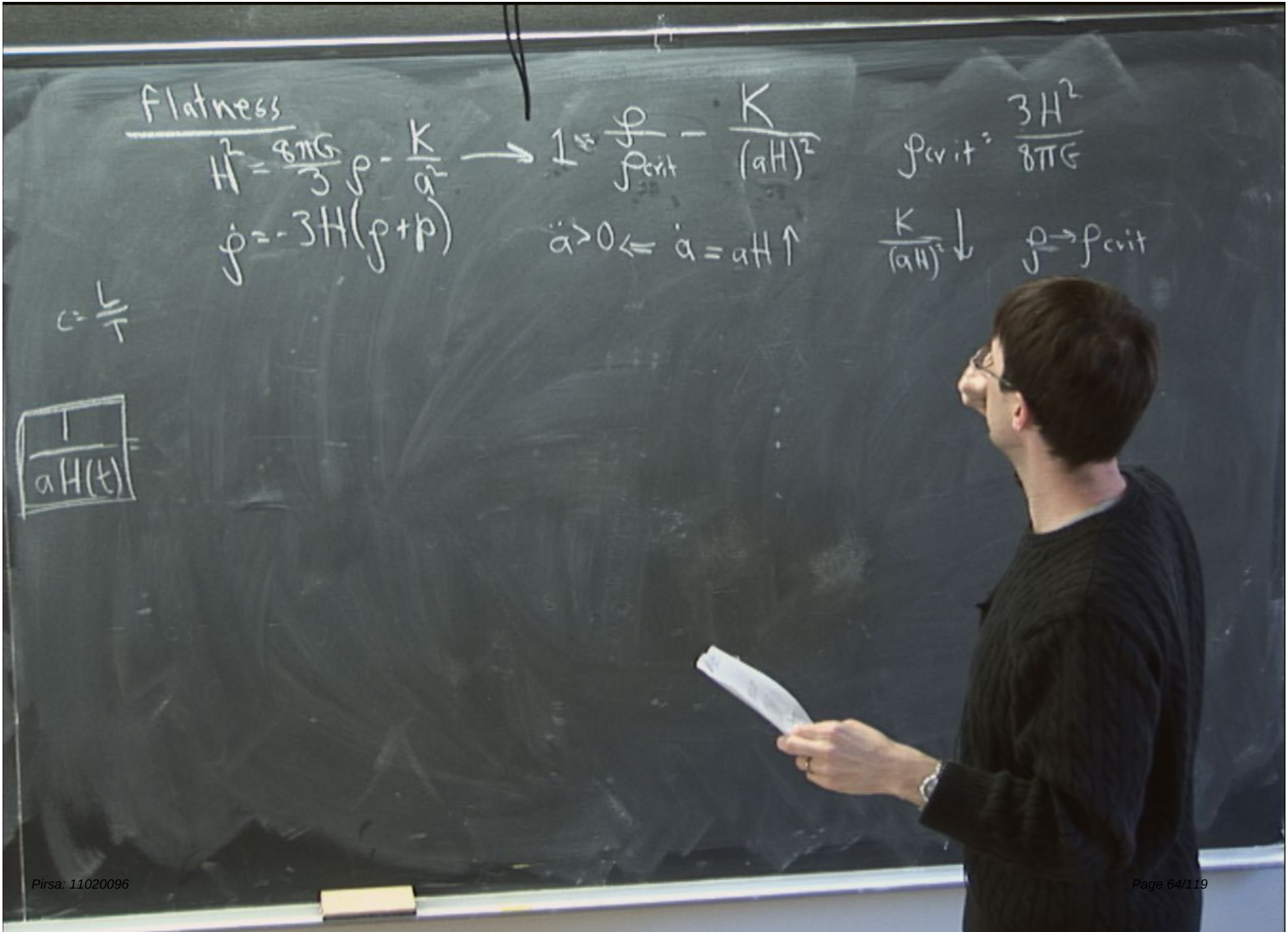
$aH \uparrow$
$\frac{K}{(aH)^2} \downarrow$
$\rho \to \rho_{crit}$
$c = \frac{L}{T}$
$\frac{1}{aH(t)}$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$\ddot{a} > 0 \Leftarrow \dot{a} = aH \uparrow$
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$\frac{K}{(aH)^2} \downarrow$
$\rho \to \rho_{crit}$
$c = \frac{L}{T}$
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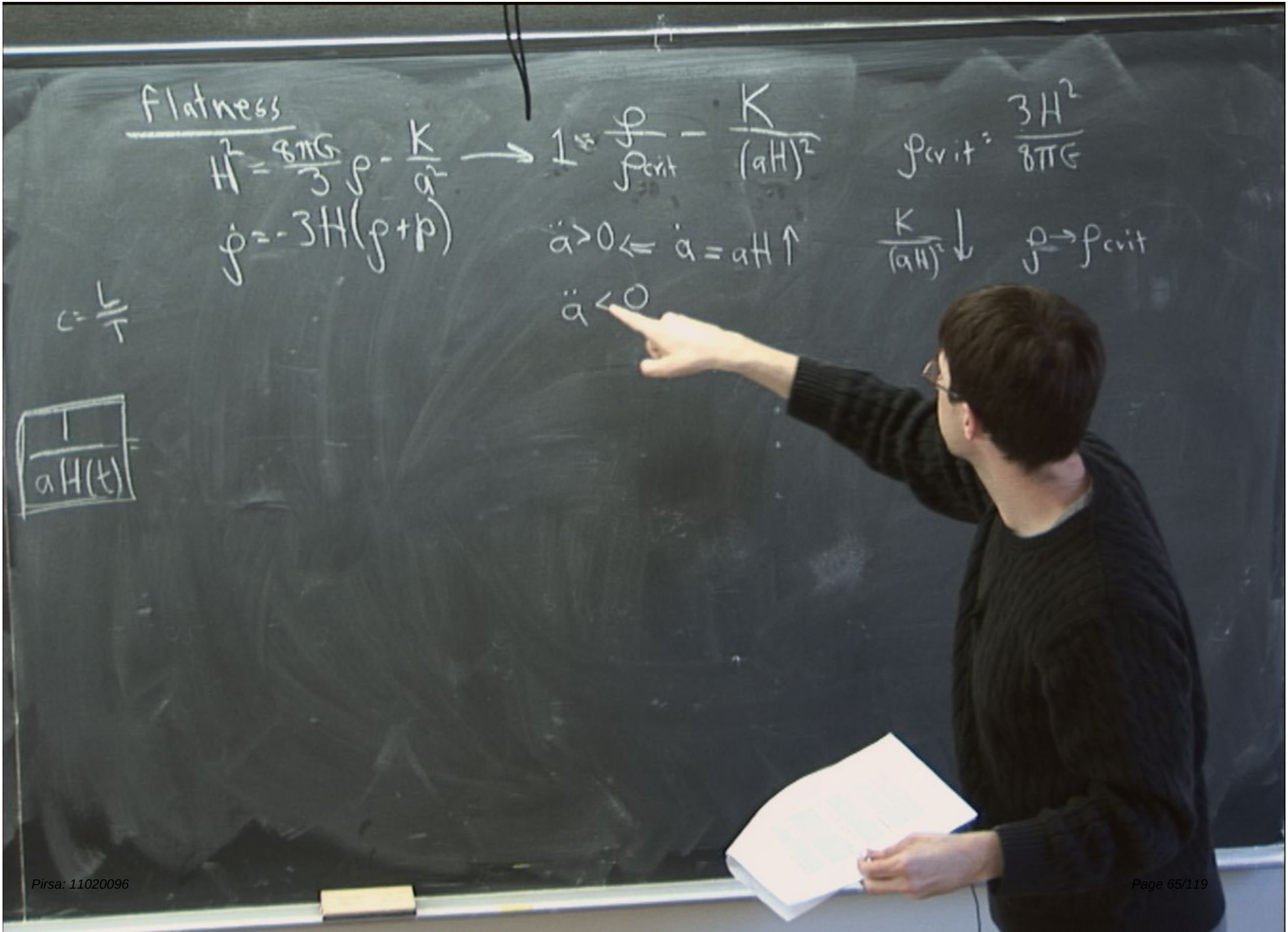
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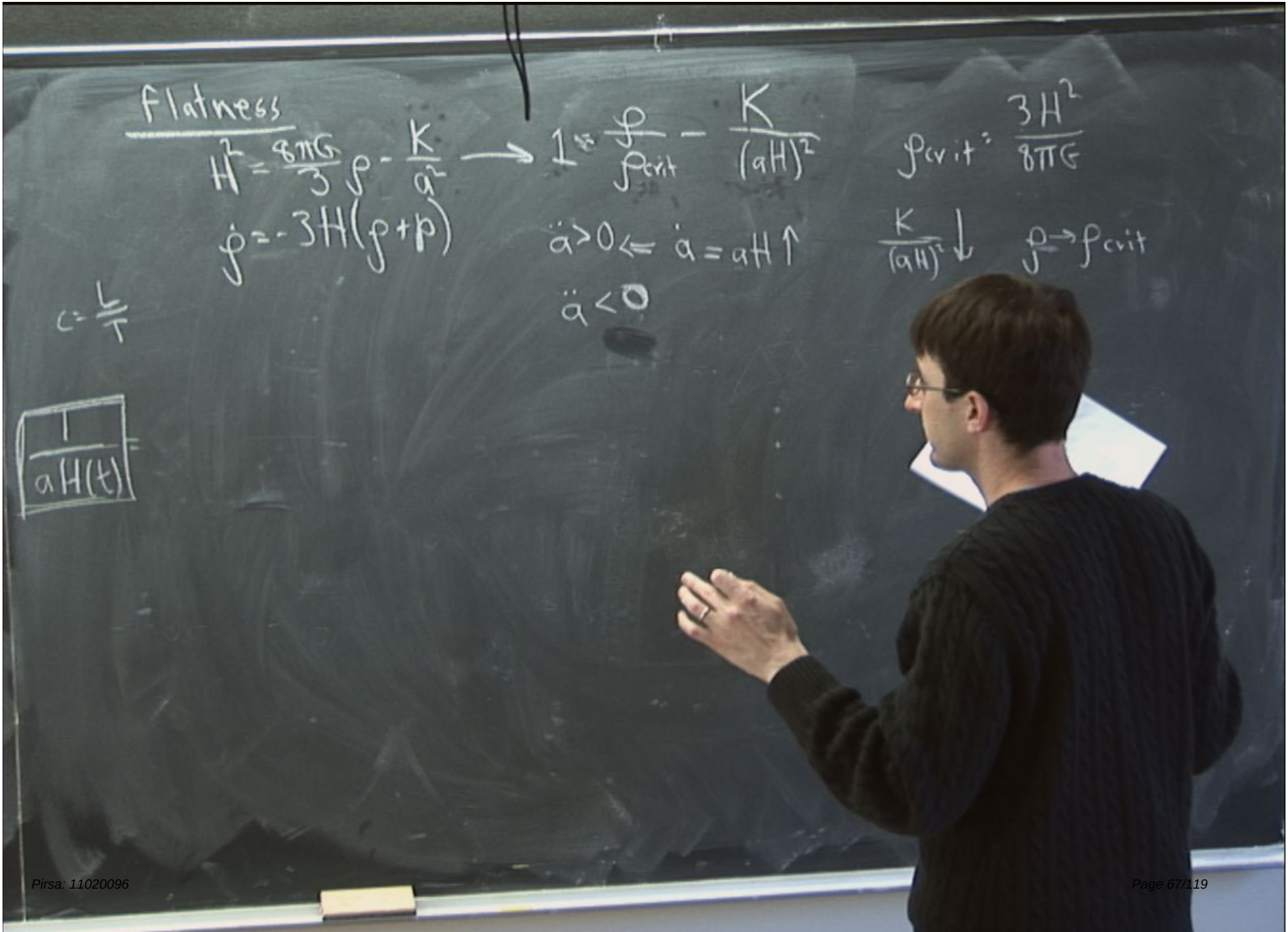
$c = \frac{L}{T}$

$$\boxed{\frac{1}{aH(t)}}$$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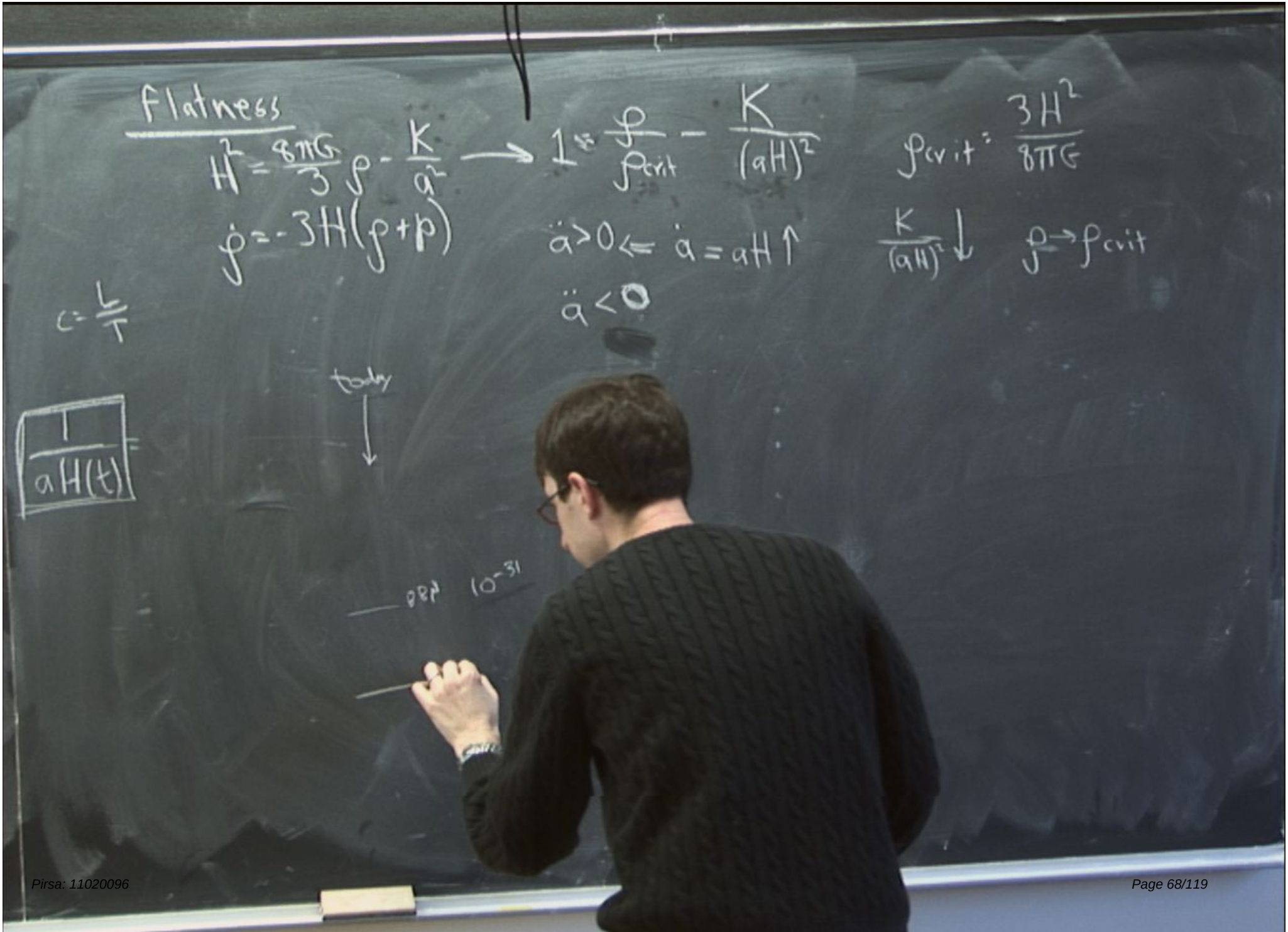
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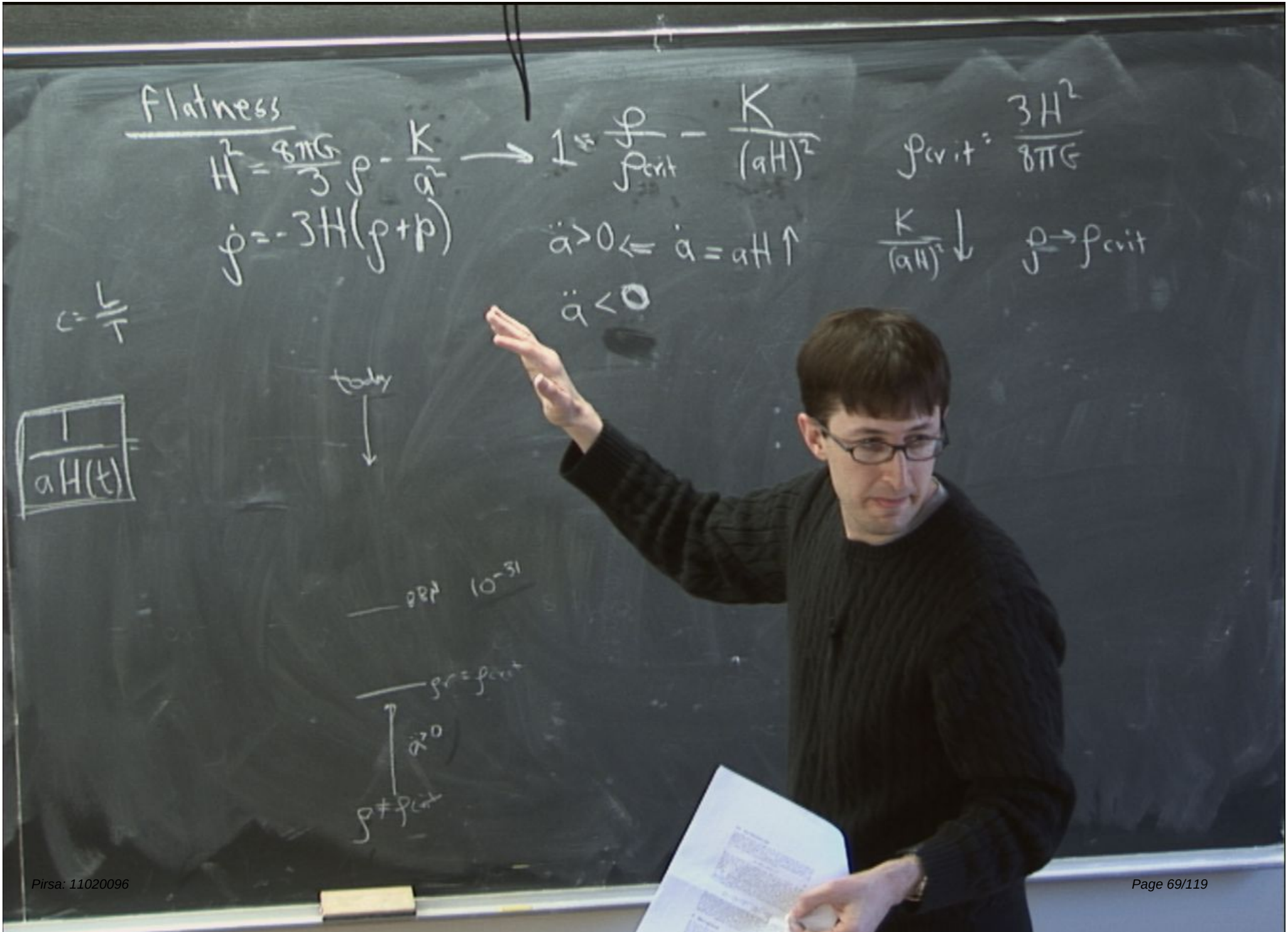
$\ddot{a} > 0 \Leftarrow \dot{a} = aH \uparrow$
$\frac{K}{(aH)^2} \downarrow$
$\rho \to \rho_{crit}$
$\ddot{a} < 0$
$c = \frac{L}{T}$
today
$\frac{1}{aH(t)}$
BBN 10^{-31}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$\ddot{a} > 0 \Leftarrow \dot{a} = aH \uparrow$
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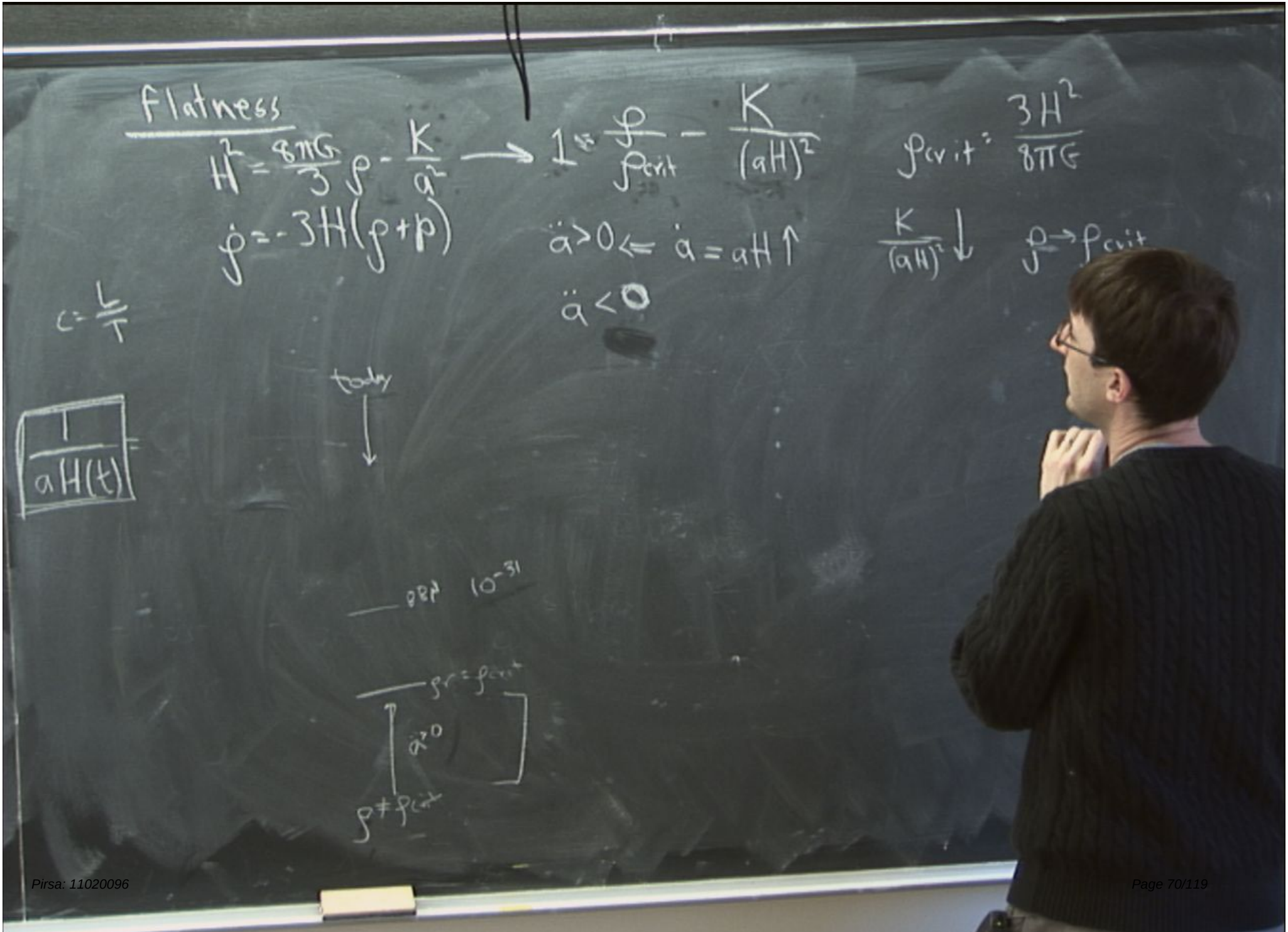
$\rho \to \rho_{crit}$
$\ddot{a} < 0$
$c = \frac{L}{T}$
today
$\frac{1}{aH(t)}$
10^{-31}
$\rho_r = \rho_{crit}$
$\ddot{a} > 0$
$\rho \neq \rho_{crit}$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today
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10^{-31}
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$\ddot{a} > 0$
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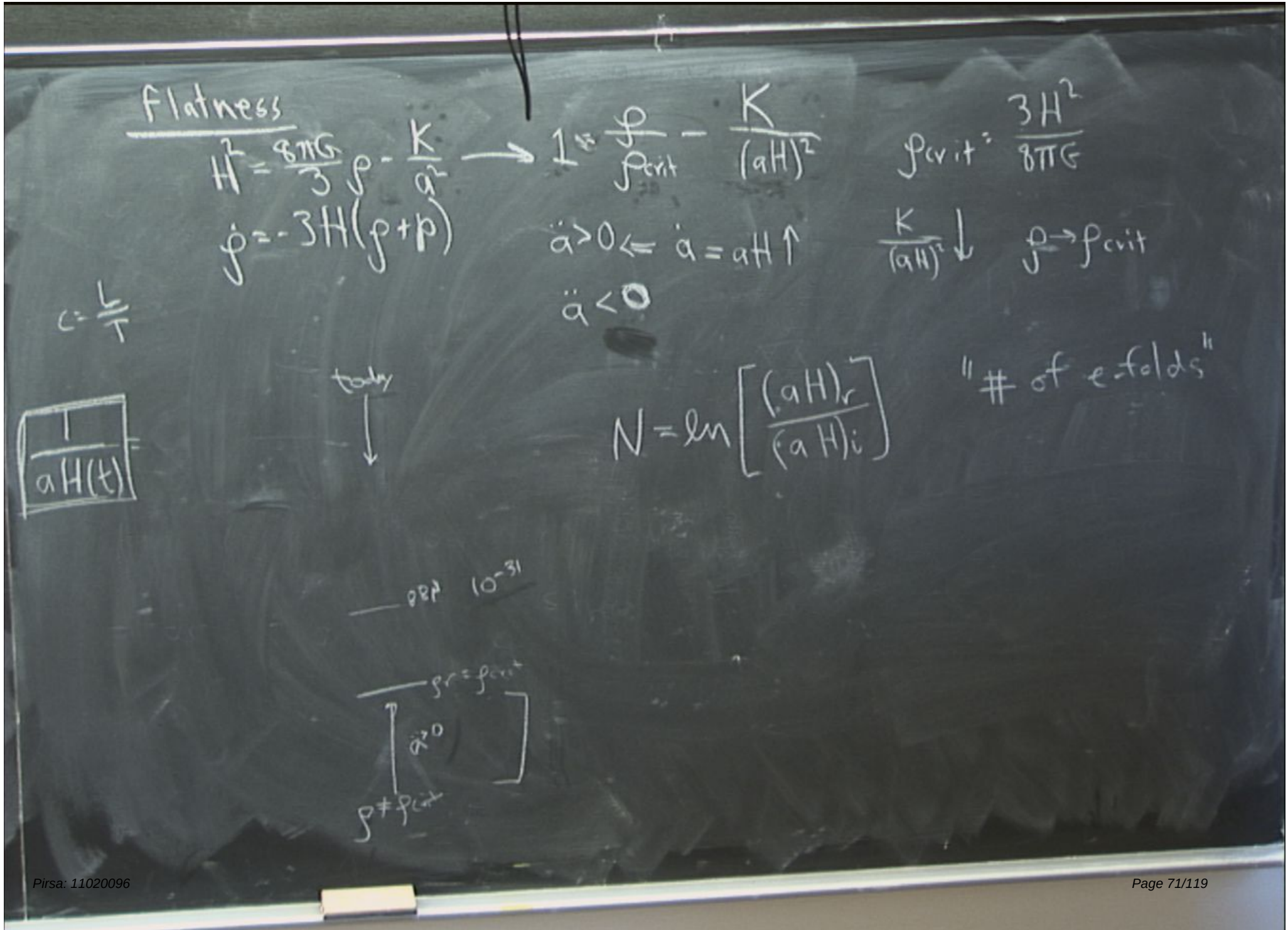

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$$\dot{\rho} = -3H(\rho + p) \qquad \ddot{a} > 0 \Leftarrow \dot{a} = aH \uparrow \qquad \frac{K}{(aH)^2} \downarrow \qquad \rho \to \rho_{crit}$$

$$\ddot{a} < 0$$

$$c = \frac{L}{T}$$

$$\boxed{\frac{1}{aH(t)}} =$$

today $\downarrow$

$$N = \ln\left[\frac{(aH)_r}{(aH)_i}\right]$$

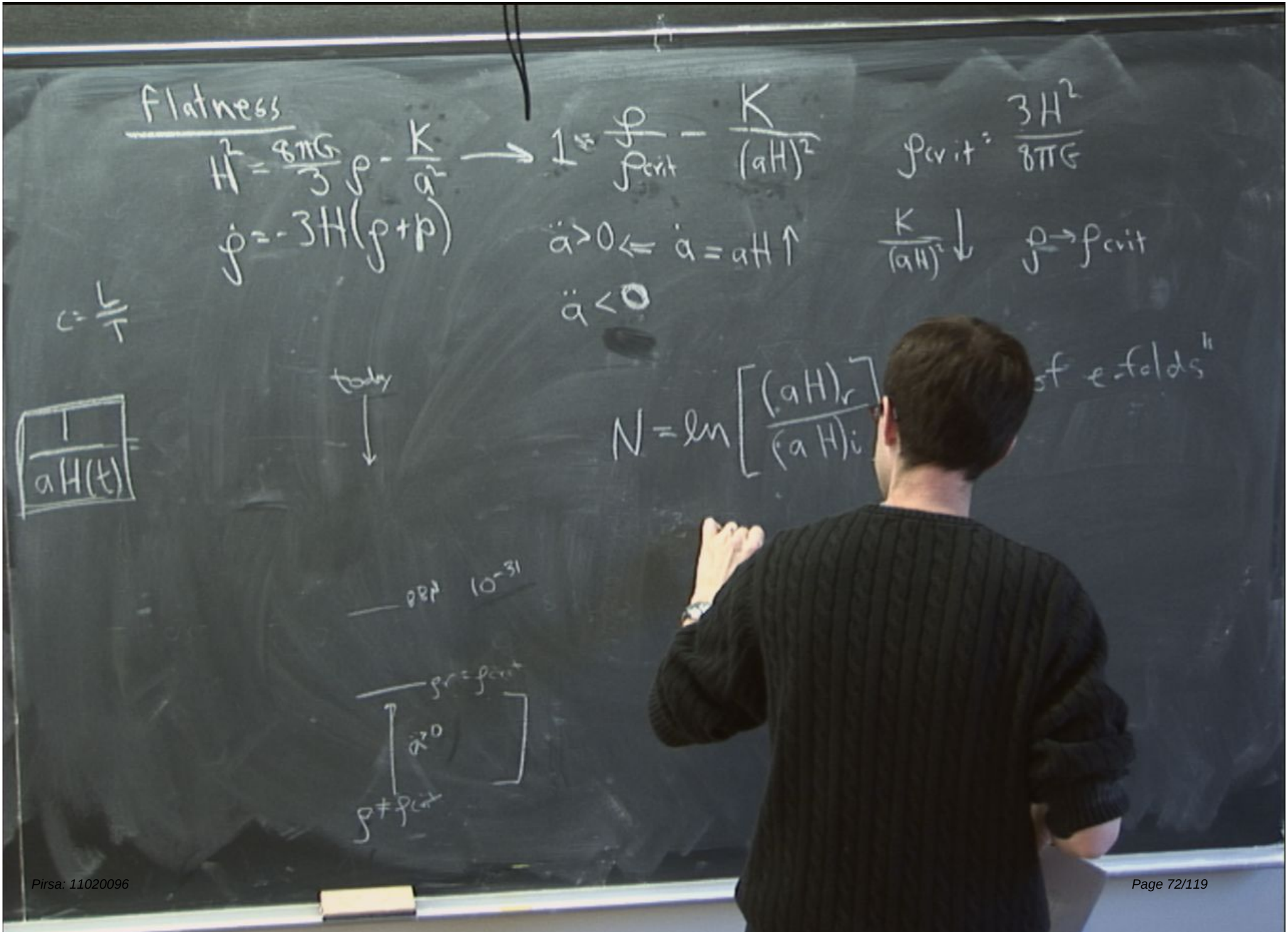
"# of e-folds"

— BBN 10^{-31}

— $\rho_r = \rho_{crit}$

$\ddot{a} > 0$

$\rho \neq \rho_{crit}$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$N = \ln\left[\frac{(aH)_r}{(aH)_i}\right]$
of e-folds"
10^{-31}
$\rho_r = \rho_{crit}$
$\ddot{a} > 0$
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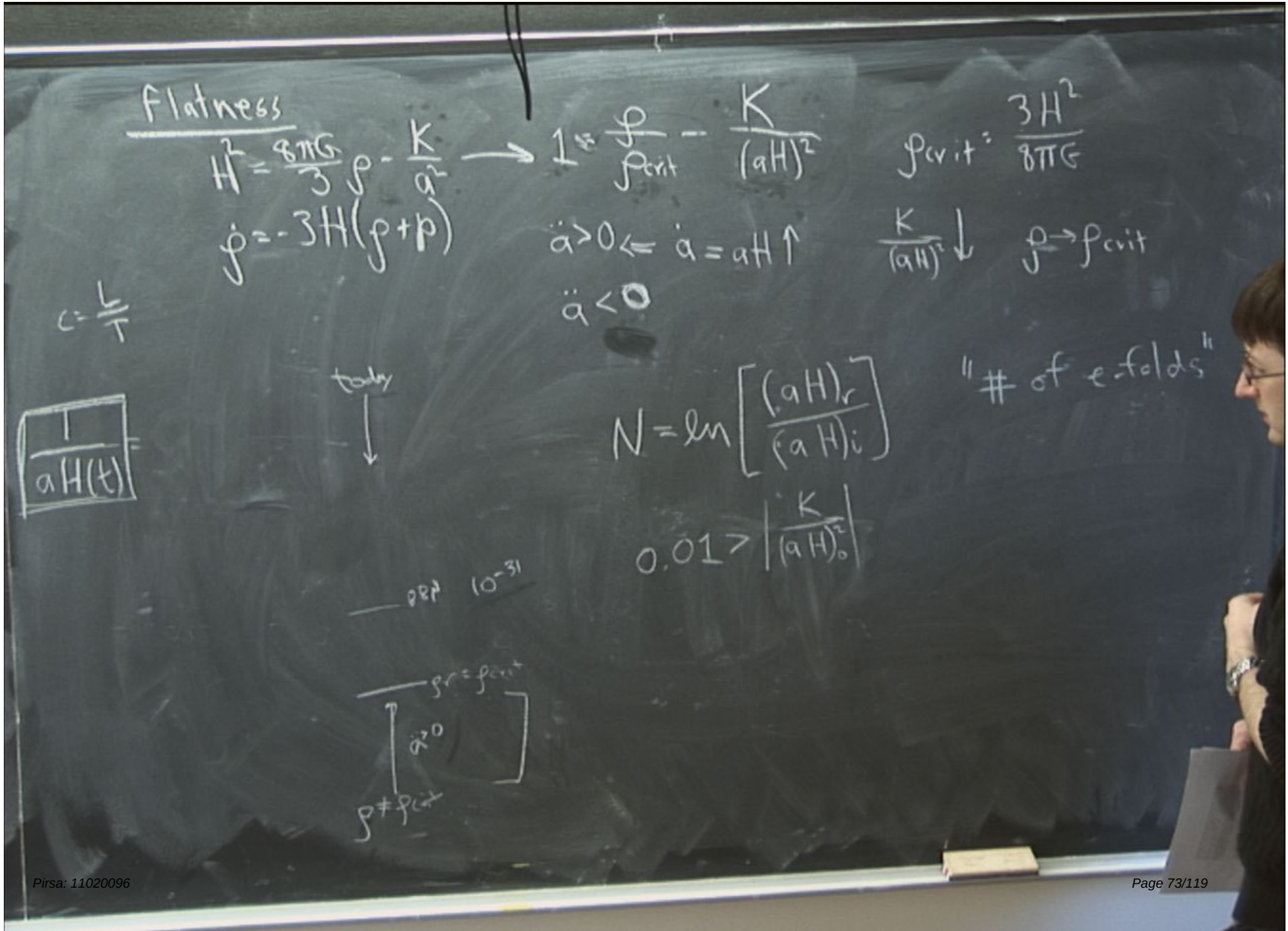
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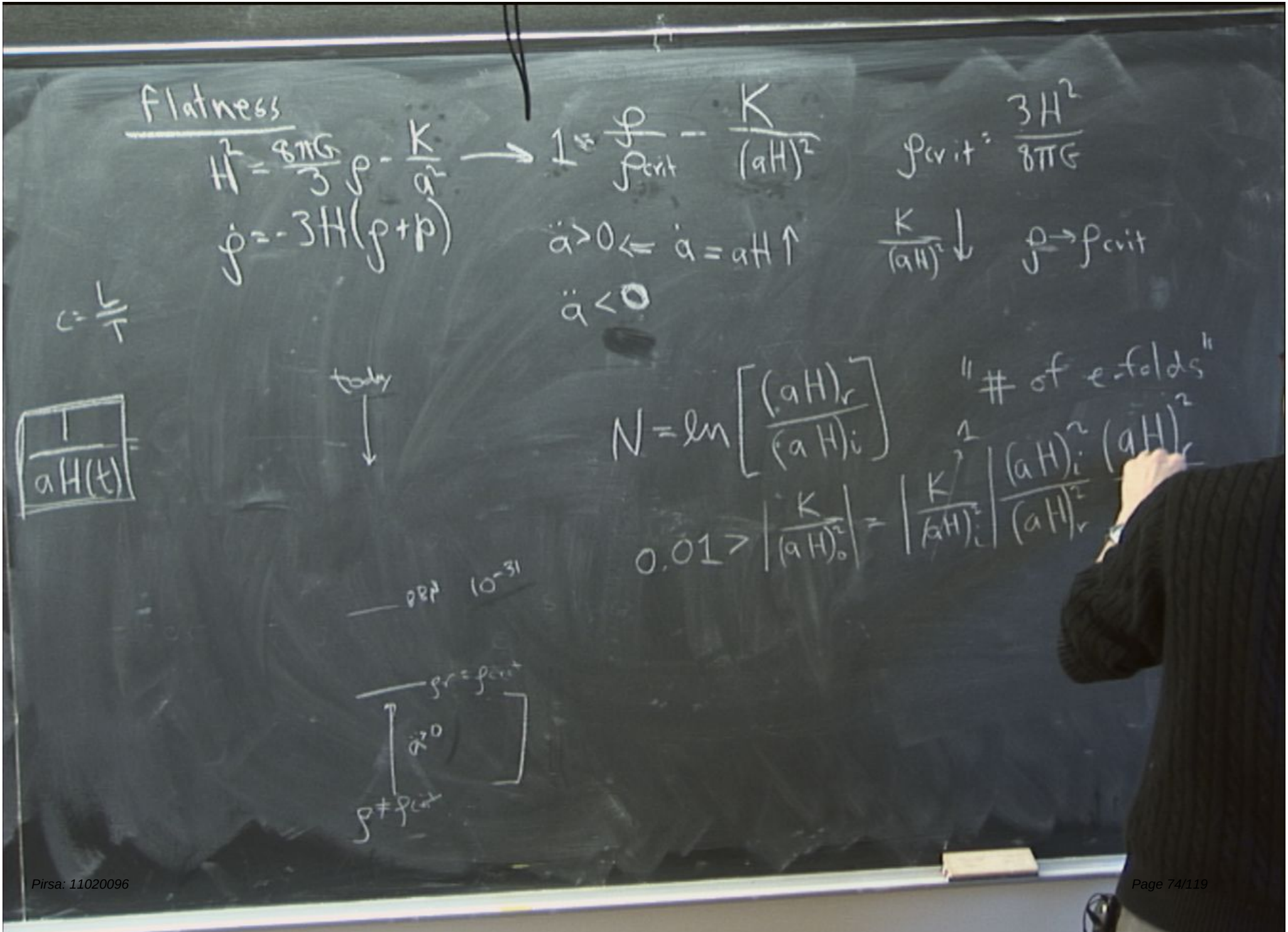
$\frac{K}{(aH)^2} \downarrow$ $\rho \to \rho_{crit}$
$\ddot{a} < 0$
$c = \frac{L}{T}$
today
$\frac{1}{aH(t)}$
$N = \ln\left[\frac{(aH)_f}{(aH)_i}\right]$
"# of e-folds"
$0.01 > \left|\frac{K}{(aH)_0^2}\right|$
BBN 10^{-31}
$\rho_f = \rho_{crit}$
$\ddot{a} > 0$
$\rho \neq \rho_{crit}$


Flatness
$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$
$\dot{\rho} = -3H(\rho + p)$
$1 = \frac{\rho}{\rho_{crit}} - \frac{K}{(aH)^2}$
$\rho_{crit} = \frac{3H^2}{8\pi G}$
$\ddot{a} > 0 \Leftarrow \dot{a} = aH \uparrow$
$\ddot{a} < 0$
$\frac{K}{(aH)^2} \downarrow \quad \rho \to \rho_{crit}$
$c = \frac{L}{T}$
today
$\frac{1}{aH(t)}$
$N = \ln\left[\frac{(aH)_f}{(aH)_i}\right]$
"# of e-folds"
$0.01 > \left|\frac{K}{(aH)_0^2}\right| = \left|\frac{K}{(aH)_i^2}\right| \frac{(aH)_i^2}{(aH)_f^2}$
10^{-31}
$\rho_f = \rho_{crit}$
$\ddot{a} > 0$
$\rho \neq \rho_{crit}$

Flatness

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} \longrightarrow 1 = \frac{\rho}{\rho_{crit}} - \frac{K}{(aH)^2}$$

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$$\dot{\rho} = -3H(\rho + p)$$

$\ddot{a} > 0 \Leftarrow \dot{a} = aH \uparrow$

$\ddot{a} < 0$

$\frac{K}{(aH)^2} \downarrow \quad \rho \to \rho_{crit}$

$c = \frac{L}{T}$

today

$$\frac{1}{aH(t)}$$

$$N = \ln\left[\frac{(aH)_f}{(aH)_i}\right]$$

"# of e-folds"

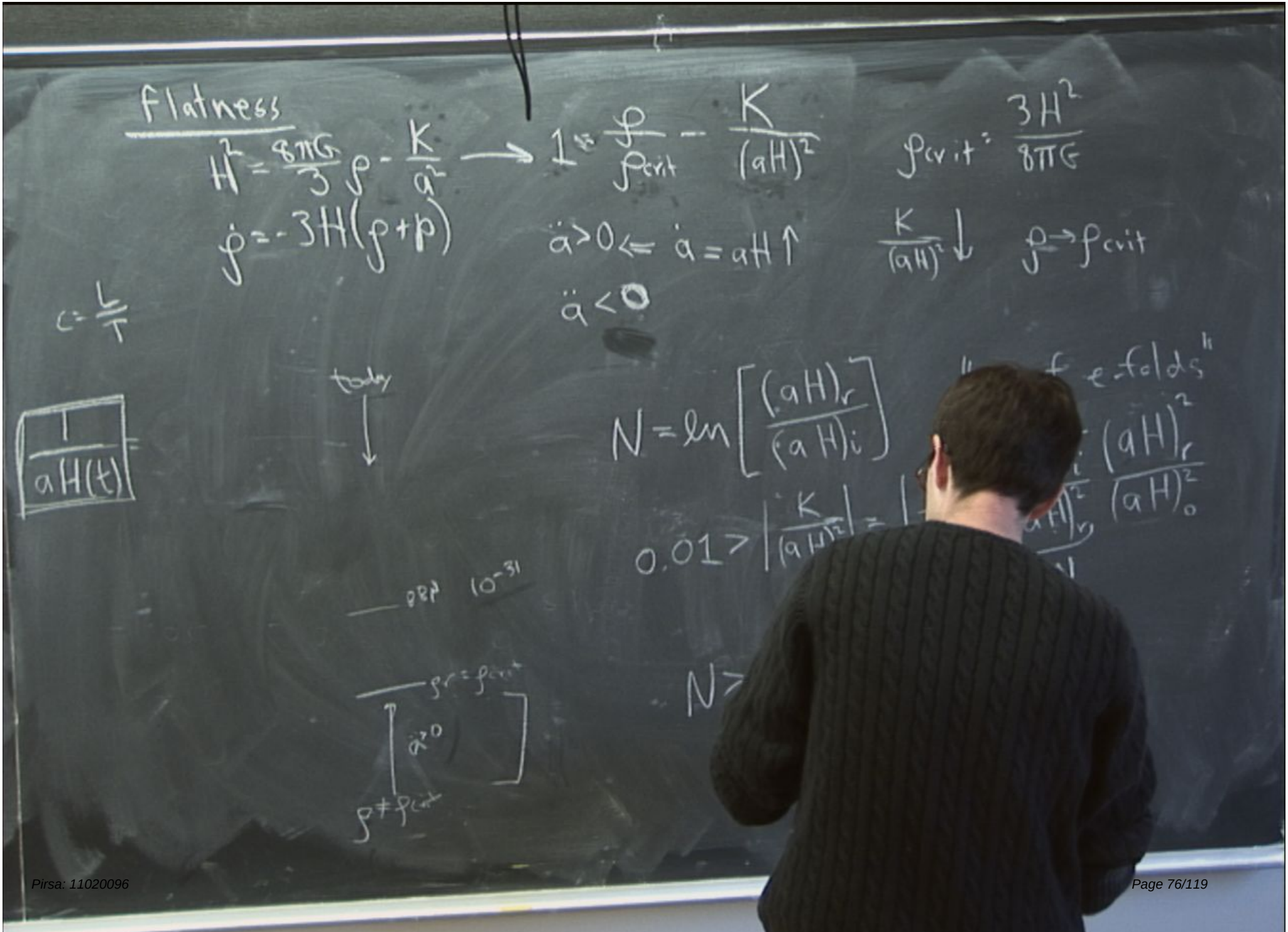
$$0.01 > \left|\frac{K}{(aH)_0^2}\right| = \left|\frac{K}{(aH)_i^2}\right|^{-1} \frac{(aH)_i^2}{(aH)_f^2} \frac{(aH)_f^2}{(aH)_0^2}$$

BBN 10^{-31}

$\rho_f = \rho_{crit}$

$\ddot{a} > 0$

$\rho \neq \rho_{crit}$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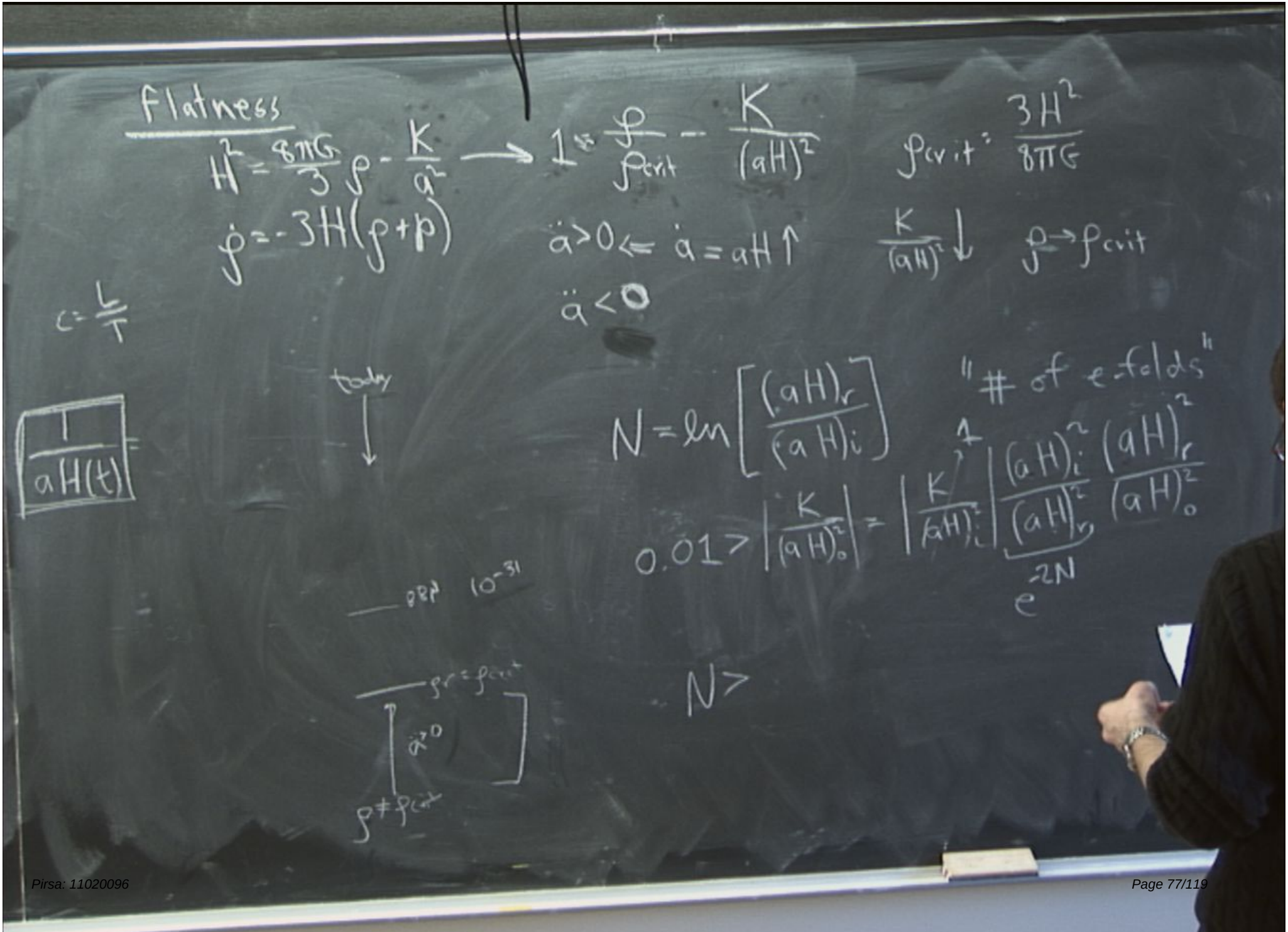
$c = \frac{L}{T}$
$\frac{1}{aH(t)}$
today
$N = \ln\left[\frac{(aH)_f}{(aH)_i}\right]$
$0.01 > \left|\frac{K}{(aH)^2}\right| =$
$\frac{(aH)_f^2}{(aH)_0^2}$
10^{-31}
$\rho_r = \rho_{crit}$
$\ddot{a} > 0$
$\rho \neq \rho_{crit}$
$N >$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$c = \frac{L}{T}$
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$\frac{1}{aH(t)}$
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$N = \ln\left[\frac{(aH)_f}{(aH)_i}\right]$
$0.01 > \left|\frac{K}{(aH)_0^2}\right| = \left|\frac{K}{(aH)_i^2}\right| \underbrace{\frac{(aH)_i^2}{(aH)_f^2}}_{e^{-2N}} \frac{(aH)_f^2}{(aH)_0^2}$
BBN 10^{-31}
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Flatness

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} \longrightarrow 1 = \frac{\rho}{\rho_{crit}} - \frac{K}{(aH)^2}$$

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$c = \frac{L}{T}$

today

$$\boxed{\frac{1}{aH(t)}}$$

$$N = \ln\left[\frac{(aH)_f}{(aH)_i}\right]$$

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$$0.01 > \left|\frac{K}{(aH)_0^2}\right| = \left|\frac{K}{(aH)_i^2}\right| \underbrace{\frac{(aH)_i^2}{(aH)_f^2}}_{e^{-2N}} \frac{(aH)_f^2}{(aH)_0^2}$$

BBN 10^{-31}

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$\frac{K}{(aH)^2} \downarrow$ $\rho \to \rho_{crit}$

$c = \frac{L}{T}$

$\left[\frac{1}{aH(t)}\right] =$

today

$$N = \ln\left[\frac{(aH)_f}{(aH)_i}\right]$$

"# of e-folds"

$$0.01 > \left|\frac{K}{(aH)_0^2}\right| = \left|\frac{K}{(aH)_i^2}\right| \underbrace{\frac{(aH)_i^2}{(aH)_f^2}}_{e^{-2N}} \frac{(aH)_f^2}{(aH)_0^2}$$

BBN 10^{-31}

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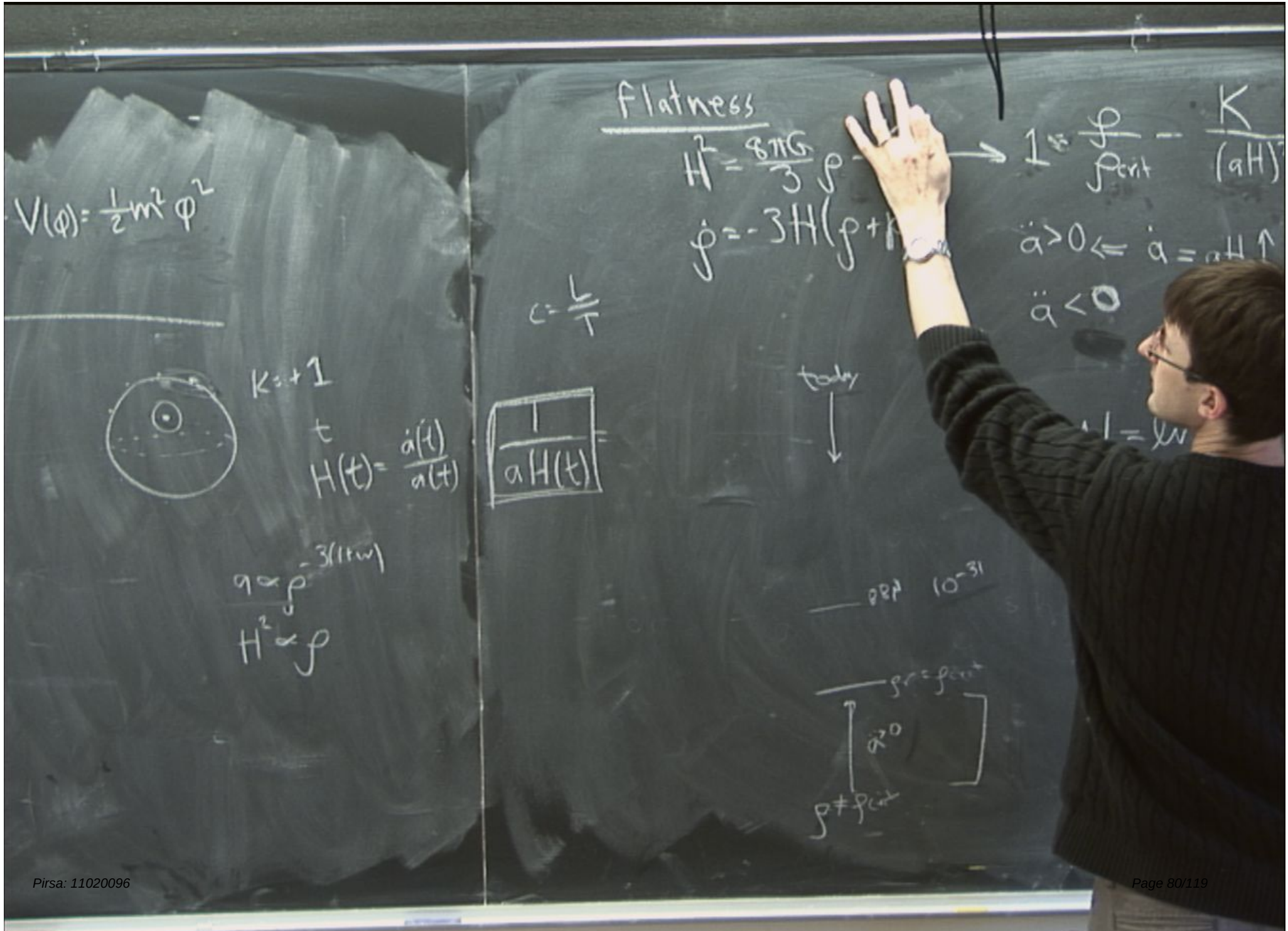
Flatness
$V(\phi) = \frac{1}{2} m^2 \phi^2$
$H^2 = \frac{8\pi G}{3} \rho$
$\dot{\rho} = -3H(\rho + $
$1 = \frac{\rho}{\rho_{crit}} - \frac{K}{(aH)^2}$
$\ddot{a} > 0 \Leftarrow \dot{a} = aH \uparrow$
$\ddot{a} < 0$
$c = \frac{L}{T}$
$K = +1$
$H(t) = \frac{\dot{a}(t)}{a(t)}$
$\frac{1}{aH(t)}$
today
$a \propto \rho^{-3(1+w)}$
$H^2 \propto \rho$
10^{-31}
$\rho \neq \rho_{crit}$

$V(\phi) = \frac{1}{2} m^2 \phi^2$

$K = +1$

t

$H(t) = \frac{\dot{a}(t)}{a(t)}$

$\frac{1}{aH(t)} =$

$a \propto \rho^{-3(1+w)}$

$H^2 \propto \rho$

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today

Flatness

$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$ → $1 = \frac{\rho}{\rho_{crit}} - \frac{K}{(aH)^2}$

$\dot{\rho} = -3H(\rho + p)$

$\ddot{a} > 0 \Leftarrow \dot{a} = aH \uparrow$

$\ddot{a} < 0$

$N = \ln\left[\frac{(a$

$(a$

$0.01 >$

BBN 10^{-31}

$\rho_c = \rho_{crit}$

$\ddot{a} > 0$

$\rho \neq \rho_{crit}$

N

Flatness

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} \longrightarrow 1 = \frac{\rho}{\rho_{crit}} - \frac{K}{(aH)^2} \qquad \rho_{crit} = \frac{3H^2}{8\pi G}$$

$$\dot{\rho} = -3H(\rho + p) \qquad \ddot{a} > 0 \Leftarrow \dot{a} = aH \uparrow \qquad \frac{K}{(aH)^2} \downarrow \qquad \rho \to \rho_{crit}$$

$$\ddot{a} < 0$$

$$c = \frac{L}{T}$$

$$\boxed{\frac{1}{aH(t)}} =$$

today

$$N = \ln\left[\frac{(aH)_r}{(aH)_i}\right] \qquad \text{"\# of e-folds"}$$

$$0.01 > \left|\frac{K}{(aH)_0^2}\right| = \left|\frac{K}{(aH)_i^2}\right| \underbrace{\frac{(aH)_i^2}{(aH)_r^2}}_{e^{-2N}} \frac{(aH)_r^2}{(aH)_0^2}$$

$$N > \ln\left[\frac{(aH)_r}{(aH)_0}\right] = 30 + \ln\frac{\rho^{1/4}}{\text{GeV}}$$

— BBN 10^{-31}

— $\rho_r = \rho_{crit}$

$\ddot{a} > 0$

$\rho \neq \rho_{crit}$

Flatness

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} \longrightarrow 1 = \frac{\rho}{\rho_{crit}} - \frac{K}{(aH)^2} \qquad \rho_{crit} = \frac{3H^2}{8\pi G}$$

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today ↓

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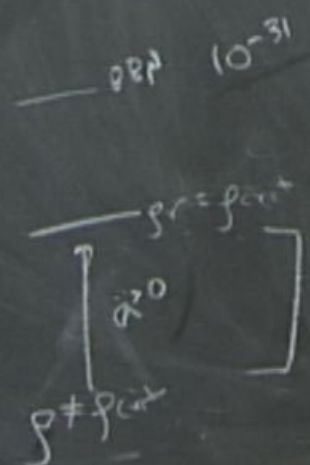

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$$\rho_{crit} = \frac{3H^2}{8\pi G} \qquad \rho = g_{30}\frac{\pi^2}{30}T^4$$

$$\dot{\rho} = -3H(\rho + p)$$

$\ddot{a} > 0 \Leftarrow \dot{a} = aH \uparrow$

$\ddot{a} < 0$

$\frac{K}{(aH)^2} \downarrow \quad \rho \to \rho_{crit}$

$c = \frac{L}{T}$

today

$\frac{1}{aH(t)}$

"# of e-folds"

$$N = \ln\left[\frac{(aH)_f}{(aH)_i}\right]$$

$$0.01 > \left|\frac{K}{(aH)_0^2}\right| = \left|\frac{K}{(aH)_i^2}\right| \underbrace{\frac{(aH)_i^2}{(aH)_f^2}}_{e^{-2N}} \frac{(aH)_f^2}{(aH)_0^2}$$

BBN 10^{-31}

$\rho_f = \rho_{crit}$

$\ddot{a} > 0$

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$$\ddot{a} > 0 \Leftarrow \dot{a} = aH \uparrow \qquad \ddot{a} < 0$$

$$\frac{K}{(aH)^2} \downarrow \quad \rho \to \rho_{crit}$$

$c = \frac{L}{T}$

$\frac{1}{aH(t)} =$

today

BBN: $N > 25$

Planck $N > 70$

BBN 10^{-31}

$\rho_r = \rho_{crit}$

$\ddot{a} > 0$

$\rho \neq \rho_{crit}$

"# of e-folds"

$$N = \ln\left[\frac{(aH)_f}{(aH)_i}\right]$$

$$0.01 > \left|\frac{K}{(aH)_0^2}\right| = \left|\frac{K}{(aH)_i^2}\right| \underbrace{\frac{(aH)_i^2}{(aH)_f^2}}_{e^{-2N}} \frac{(aH)_f^2}{(aH)_0^2}$$

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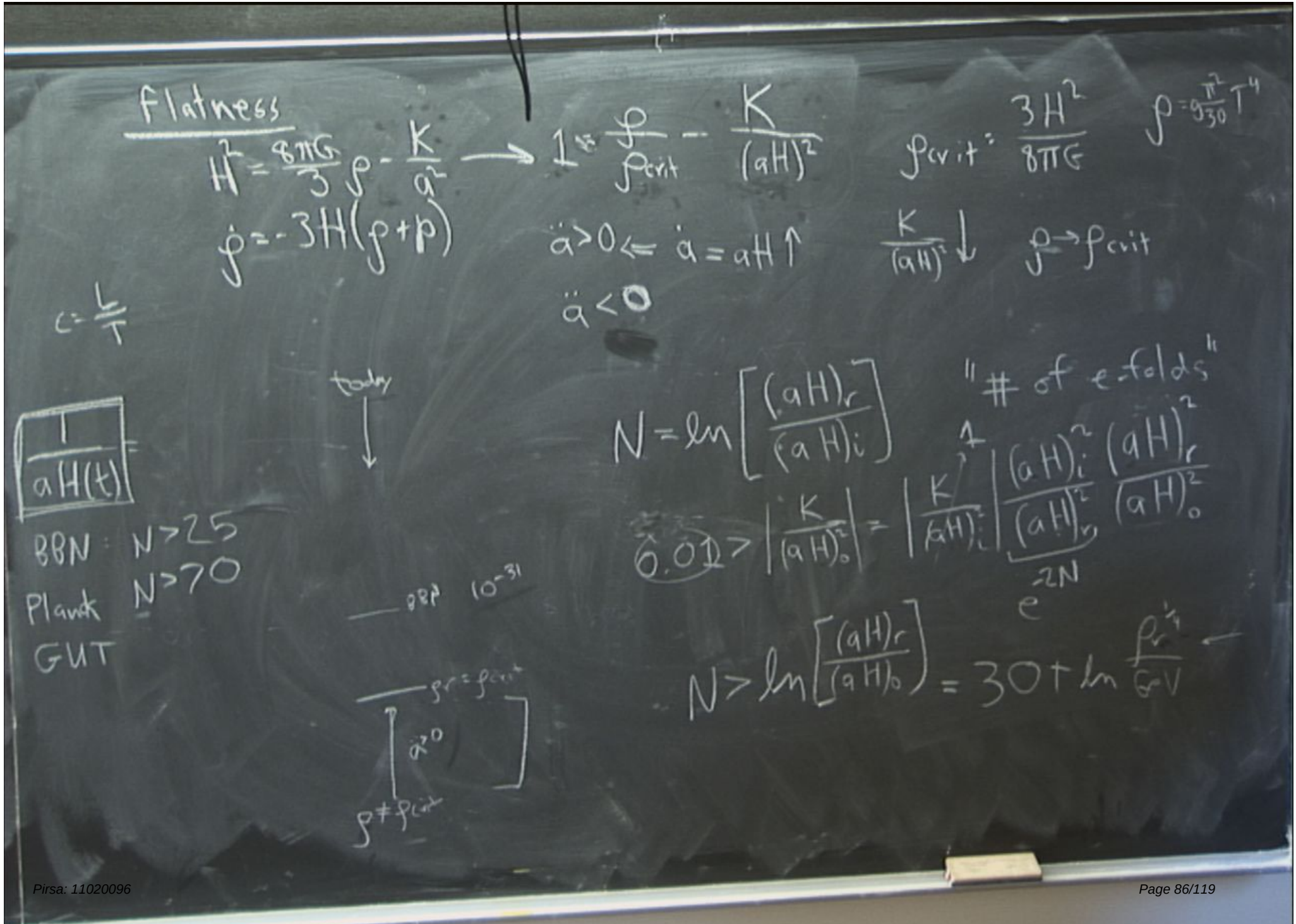

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$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} \longrightarrow 1 = \frac{\rho}{\rho_{crit}} - \frac{K}{(aH)^2}$$

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$$\dot{\rho} = -3H(\rho + p)$$

$$\ddot{a} > 0 \Leftarrow \dot{a} = aH \uparrow \qquad \frac{K}{(aH)^2} \downarrow \qquad \rho \to \rho_{crit}$$

$$\ddot{a} < 0$$

$$c = \frac{L}{T}$$

today

$$\boxed{\frac{1}{aH(t)}} =$$

BBN : $N > 25$

Planck $N > 70$

GUT

BBN 10^{-31}

$\rho_r = \rho_{crit}$

$\ddot{a} > 0$

$\rho \neq \rho_{crit}$

$$N = \ln\left[\frac{(aH)_r}{(aH)_i}\right]$$

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$$0.01 > \left|\frac{K}{(aH)_0^2}\right| = \left|\frac{K}{(aH)_i^2}\right| \underbrace{\frac{(aH)_i^2}{(aH)_r^2}}_{e^{-2N}} \frac{(aH)_r^2}{(aH)_0^2}$$

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3 Puzzles $\ddot{a}>0$ inflation
$\ddot{a}<0$ RD

$\mathcal{L}_\varphi = -\frac{1}{2}(\partial_\mu\varphi)^2 - V(\varphi)$ ← $V(\varphi) = \frac{1}{2}m^2\varphi^2$

aH

$K = +1$

t

$H(t) = \frac{\dot{a}(t)}{a(t)}$

$\rho \propto \rho^{-3(1+w)}$

$H^2 \propto \rho$

3 Puzzles

$\ddot{a} > 0$ inflation

$\ddot{a} < 0$ RD

$\mathcal{L}_\varphi = -\frac{1}{2}(\partial_\mu \varphi)^2 - V(\varphi)$ ← $V(\varphi) = \frac{1}{2} m^2 \varphi^2$

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t

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3 Puzzles

$\ddot{a} > 0$ inflation

$\ddot{a} < 0$ RD

$$\mathcal{L}_\varphi = -\frac{1}{2}(\partial_\mu \varphi)^2 - V(\varphi) \leftarrow V(\varphi) = \frac{1}{2} m^2 \varphi^2$$

aH

$\frac{H'}{a} \ll 1$

$\frac{1}{aH} \ll 1$

$K = +1$

t

$H(t) = \frac{\dot{a}(t)}{a(t)}$

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aH

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$K = +1$

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3 Puzzles

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$\mathcal{L}_\varphi = -\frac{1}{2}(\partial_\mu \varphi)^2 - V(\varphi)$ ← $V(\varphi) = \frac{1}{2} m^2 \varphi^2$

aH

$\frac{H^{-1}}{a} \ll 1$ $\quad \frac{1}{aH} \ll 1$

$K = +1$

t

$H(t) = \frac{\dot{a}(t)}{a(t)}$

$a \propto \rho^{-3(1+w)}$

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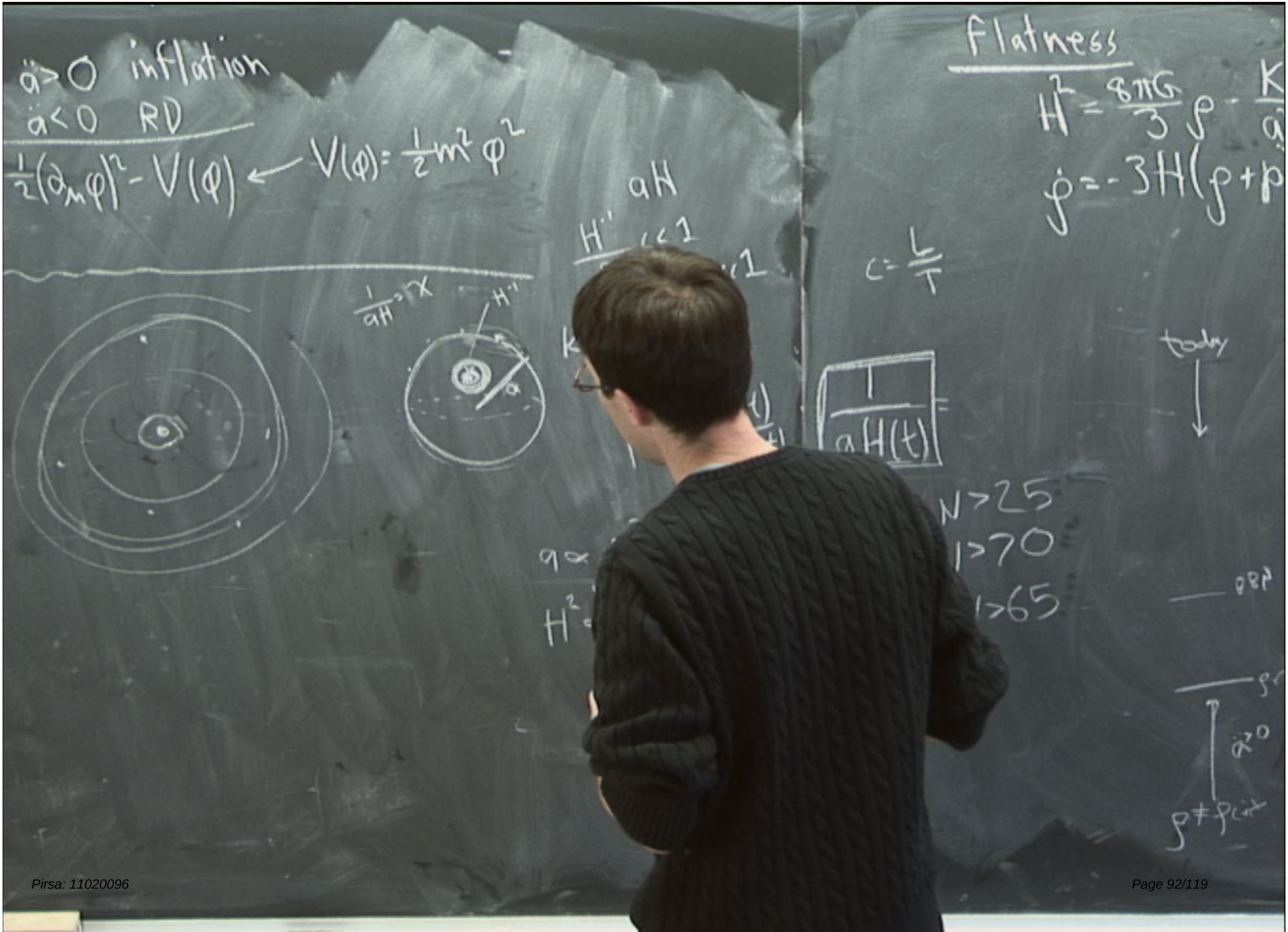
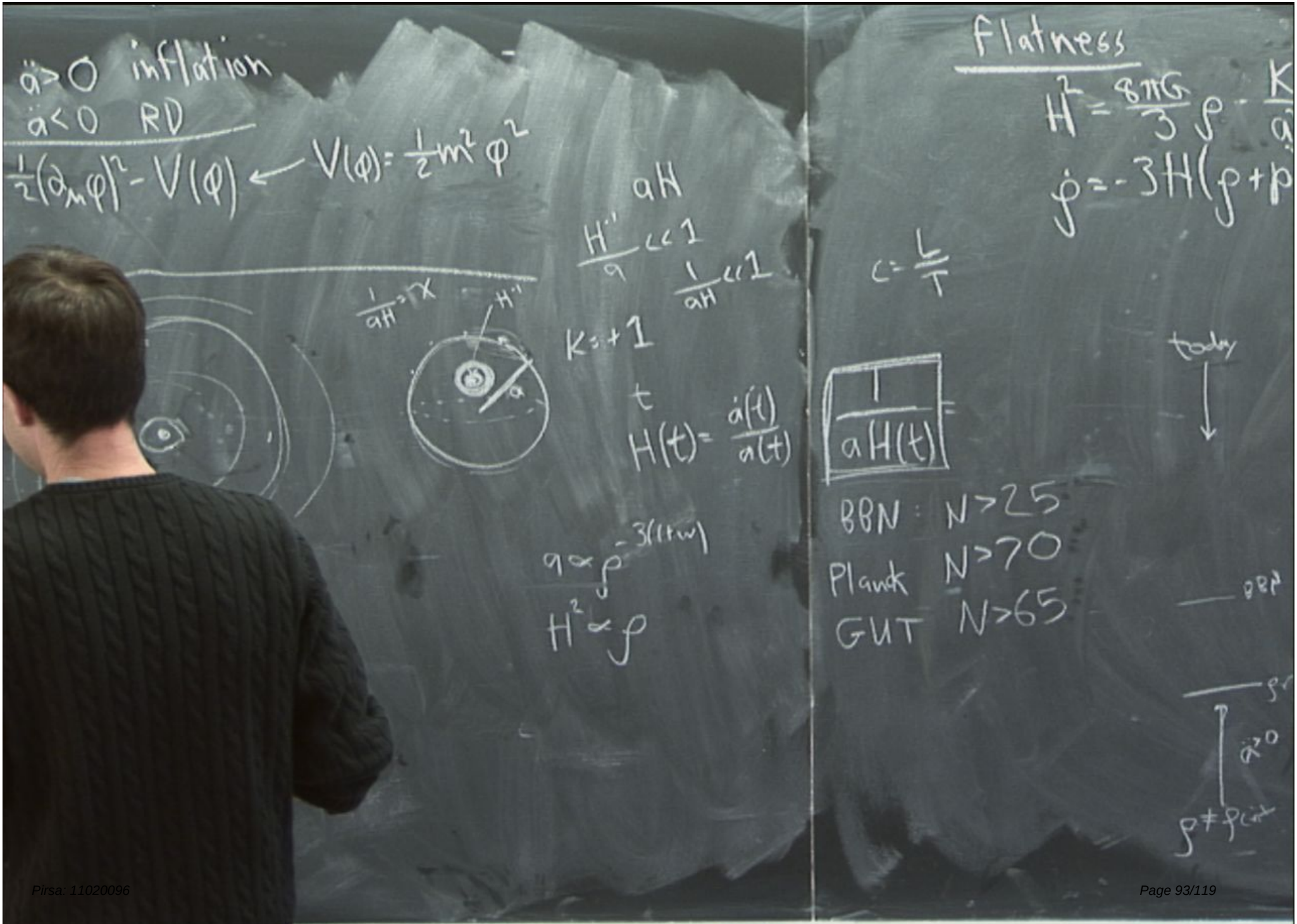

ä>0 inflation
ä<0 RD
½(∂μφ)² - V(φ) ← V(φ) = ½m²φ²
aH
Flatness
H² = 8πG/3 ρ - K/a²
ρ̇ = -3H(ρ+p
c = L/T
today
1/aH(t)
N>25
N>70
N>65
ρ≠ρcrit


ä>0 inflation
ä<0 RD
$\frac{1}{2}(\partial_\mu\varphi)^2 - V(\varphi)$ ← $V(\varphi) = \frac{1}{2}m^2\varphi^2$
$\frac{1}{aH} = r_H$
H^{-1}
a
aH
$\frac{H'}{a} \ll 1$
$\frac{1}{aH} \ll 1$
K = +1
t
$H(t) = \frac{\dot{a}(t)}{a(t)}$
$a \propto \rho^{-3(1+w)}$
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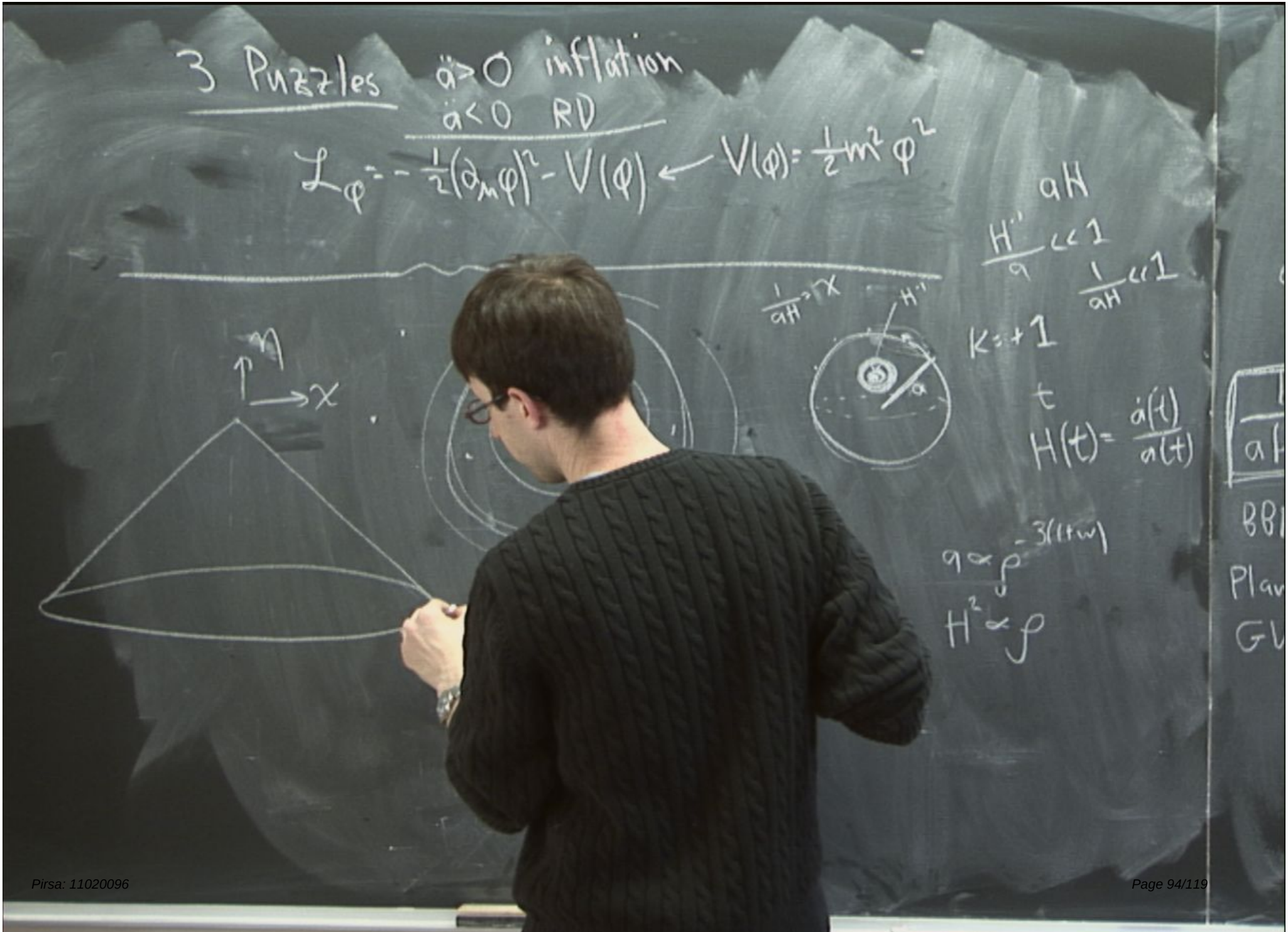
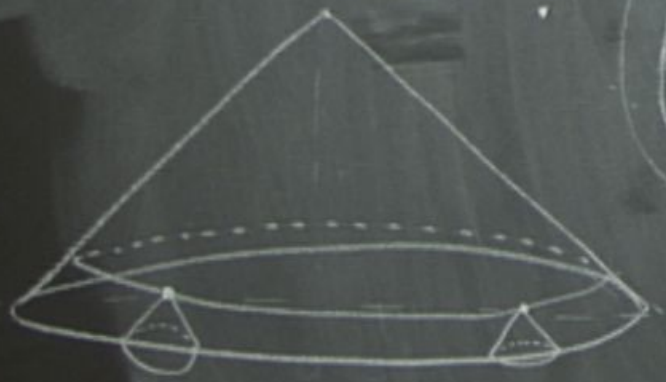
Flatness
$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$
$\dot{\rho} = -3H(\rho + p$
$c = \frac{L}{T}$
1
aH(t)
today
BBN : N>25
Planck N>70
GUT N>65
BBN
$\ddot{a}>0$
$\rho \neq \rho_{cst}$


3 Puzzles
ä>0 inflation
ä<0 RD
$\mathcal{L}_\varphi = -\frac{1}{2}(\partial_\mu\varphi)^2 - V(\varphi)$ ← $V(\varphi) = \frac{1}{2}m^2\varphi^2$
aH
$\frac{H'}{a} \ll 1$
$\frac{1}{aH} \ll 1$
$\frac{1}{aH} \to \infty$
H'
K = +1
t
$H(t) = \frac{\dot{a}(t)}{a(t)}$
$\rho \propto \rho^{-3(1+w)}$
$H^2 \propto \rho$
BB
Pla
GU

3 Puzzles $\ddot{a}>0$ inflation
$\ddot{a}<0$ RD

$\mathcal{L}_\varphi = -\frac{1}{2}(\partial_\mu\varphi)^2 - V(\varphi)$ ← $V(\varphi) = \frac{1}{2}m^2\varphi^2$



$\frac{aH}{a^2} \ll 1$

$\frac{1}{aH} \ll 1$

$K = +1$

$H(t) = \frac{\dot{a}(t)}{a(t)}$

$\rho \propto \rho^{-3(1+w)}$

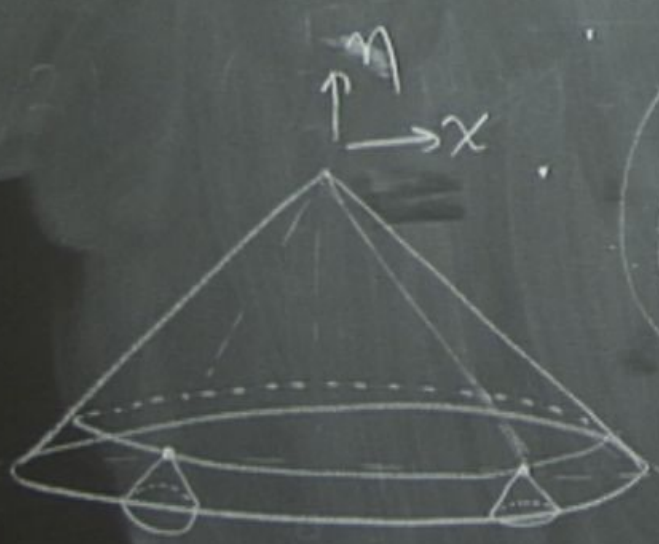
$H^2 \propto \rho$

3 Puzzles

$\ddot{a} > 0$ inflation

$\ddot{a} < 0$ RD

$\mathcal{L}_\varphi = -\frac{1}{2}(\partial_\mu \varphi)^2 - V(\varphi) \leftarrow V(\varphi) = \frac{1}{2} m^2 \varphi^2$



aH

$\frac{H'}{a} \ll 1$

$\frac{1}{aH} \ll 1$

$K = +1$

t

$H(t) = \frac{\dot{a}(t)}{a(t)}$

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BB

Pla

GU

3 Puzzles

$\ddot{a} > 0$ inflation

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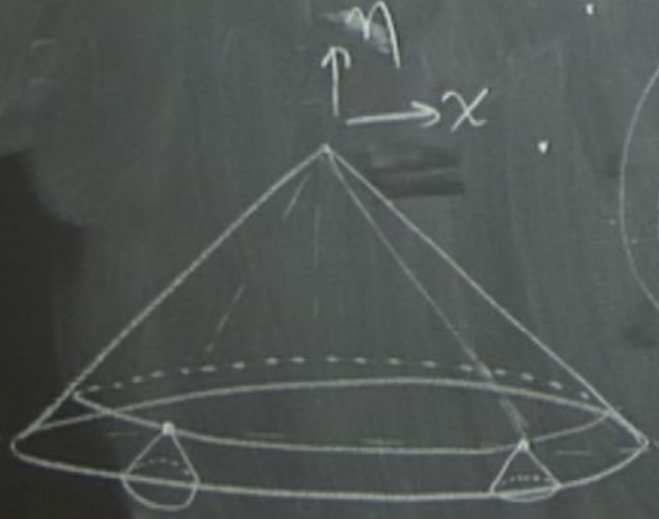
$\mathcal{L}_\varphi = -\frac{1}{2}(\partial_\mu \varphi)^2 - V(\varphi)$ ← $V(\varphi) = \frac{1}{2} m^2 \varphi^2$

aH

$\frac{H'}{a} \ll 1$

$\frac{1}{aH} \ll 1$



$K = +1$

t

$H(t) = \frac{\dot{a}(t)}{a(t)}$

$\rho \propto \rho^{-3(1+w)}$

$H^2 \propto \rho$

BB

Pla

GL

Flatness

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} \longrightarrow$$

$$\dot{\rho} = -3H(\rho + p)$$

$$ds^2 = -dt^2 + a^2[dx + \dots]$$

$$= a^2[-d\eta^2 + dx^2 + \dots]$$

$$d\eta = -dx$$

$c = \frac{L}{T}$

today

$\frac{1}{aH(t)}$

BBN: $N > 25$

Planck $N > 70$

GUT $N > 65$

BBN 10^{-31}

$\rho_r = \rho_{crit}$

$\ddot{a} > 0$

$\rho \neq \rho_{crit}$

Flatness

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} \longrightarrow$$

$$\dot{\rho} = -3H(\rho + p)$$

$$ds^2 = -dt^2 + a^2[dx^2 + \dots]$$

$$= a^2[-d\eta^2 + dx^2 + \dots]$$

$$d\eta = -dx$$

$$c = \frac{L}{T}$$

$$x = \eta_0 - \eta_{BB} = \int_{t_{BB}}^{t_0} \frac{dt}{a} = \int \frac{dt}{da}\frac{1}{a}\,da$$

today

$$\frac{1}{aH(t)}$$

BBN: $N > 25$

Planck $N > 70$

GUT $N > 65$

BBN 10^{-31}

$\rho_r = \rho_{crit}$

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$c = \frac{L}{T}$

today

$\frac{1}{aH(t)}$ =

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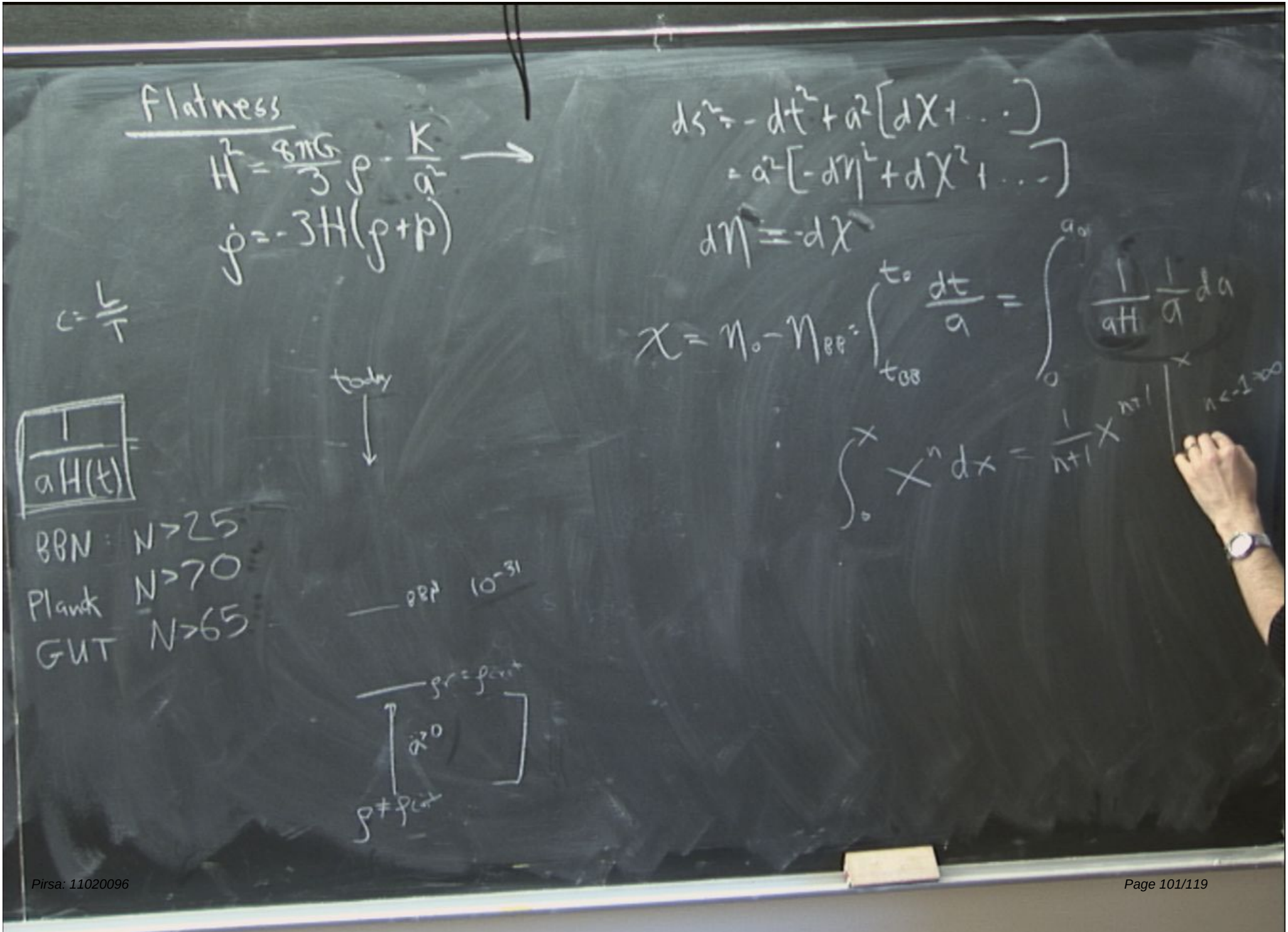
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today
$\frac{1}{aH(t)}$
$\int_0^x x^n dx = \frac{1}{n+1}x^{n+1}$
$n < -1$
BBN: $N > 25$
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$$\int_0^x x^n dx = \frac{1}{n+1}x^{n+1}\Big|_0^x \quad \begin{matrix} n<-1 \to \infty \\ n<-1 \to 0 \end{matrix}$$

$c = \frac{L}{T}$

today

$\frac{1}{aH(t)}$

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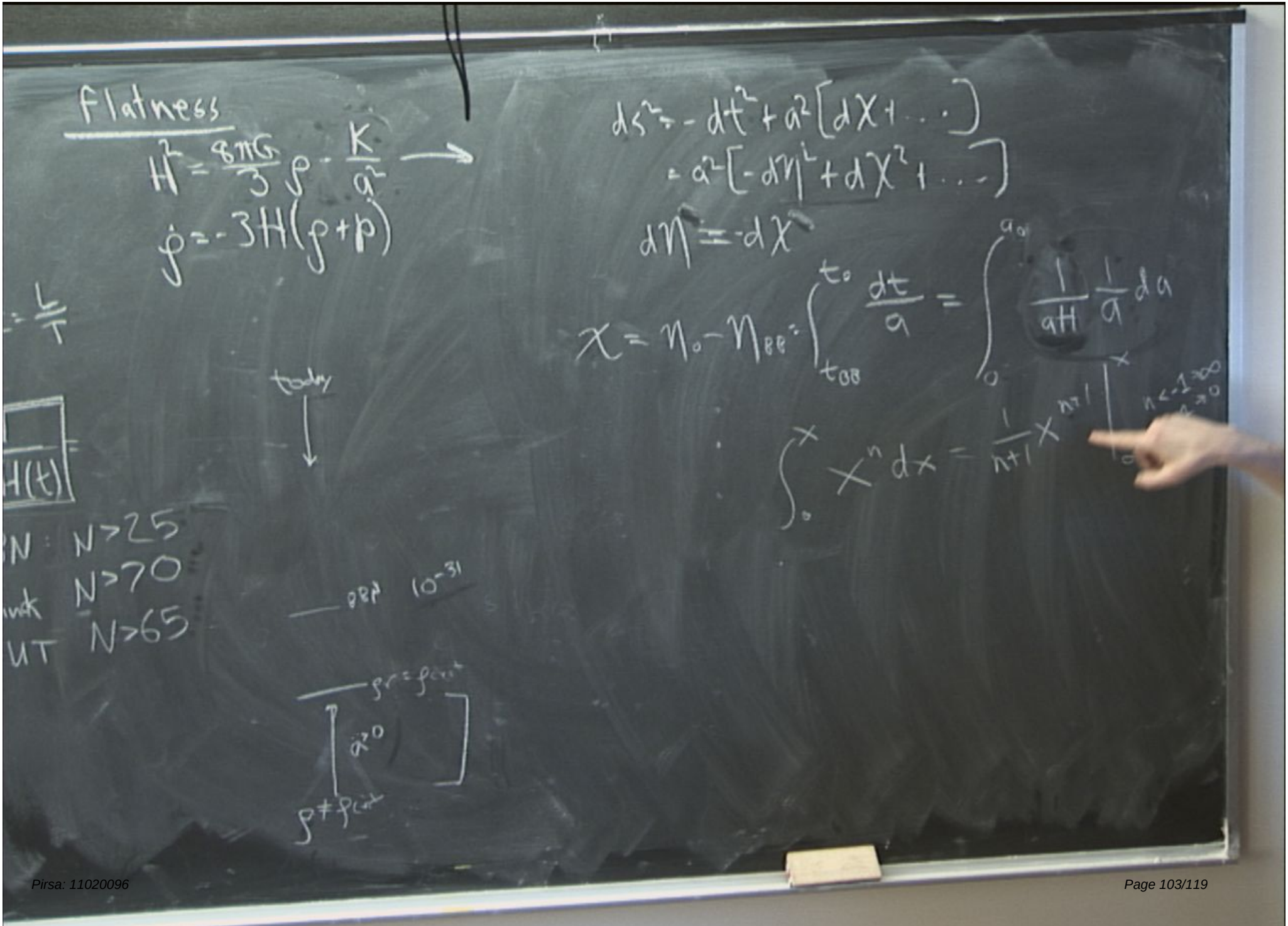
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today
$H(t)$
N > 25
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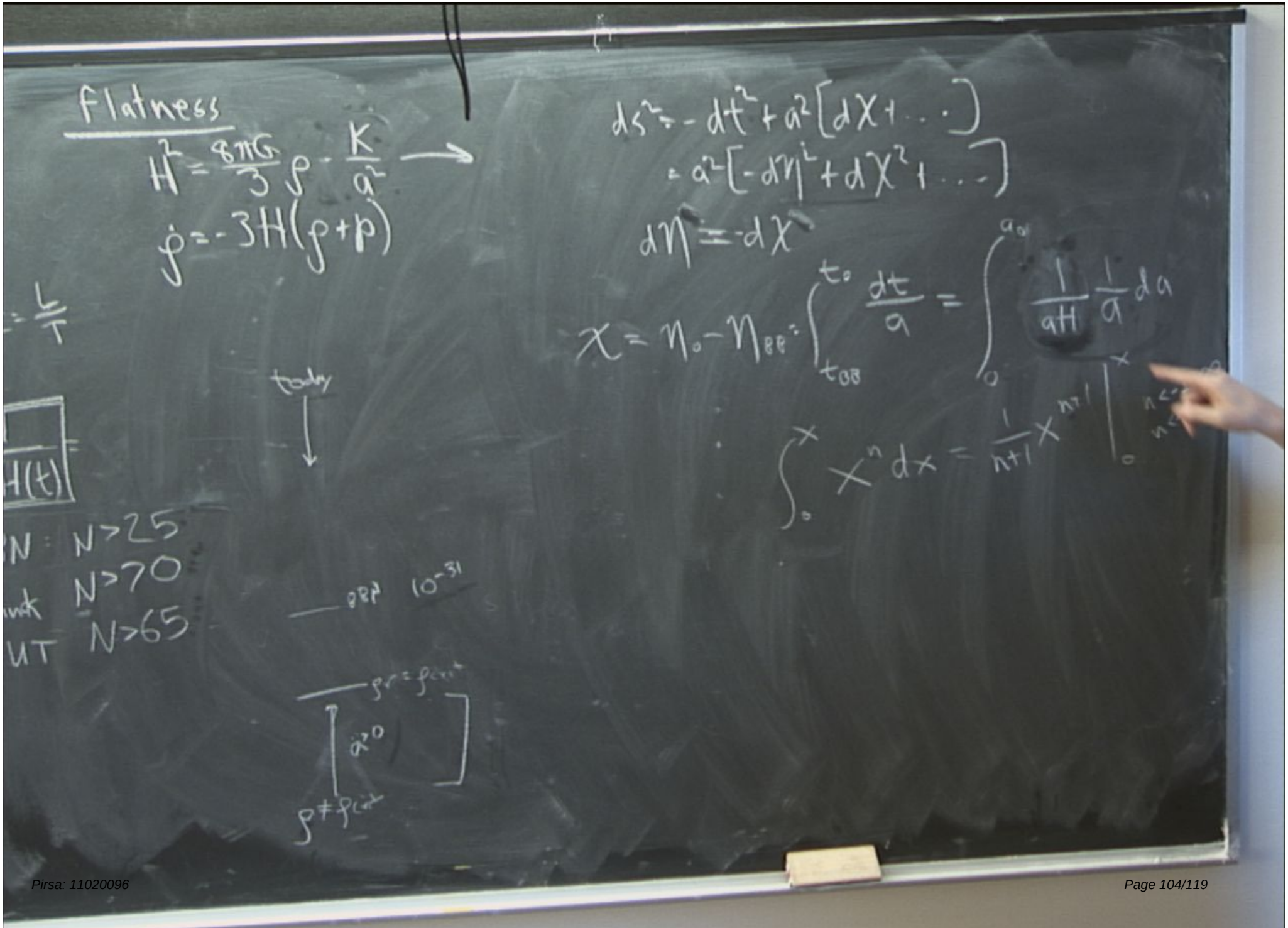
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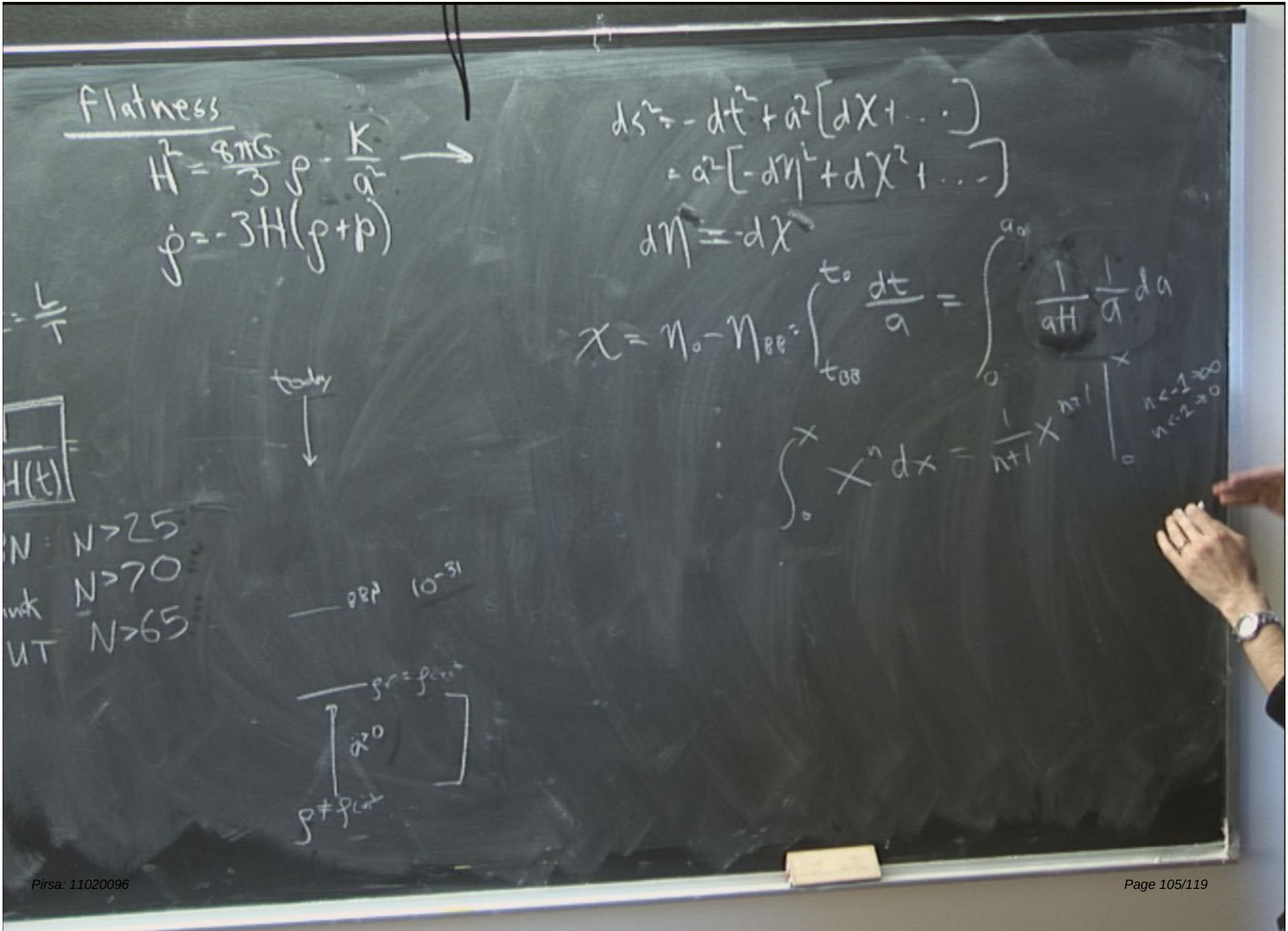
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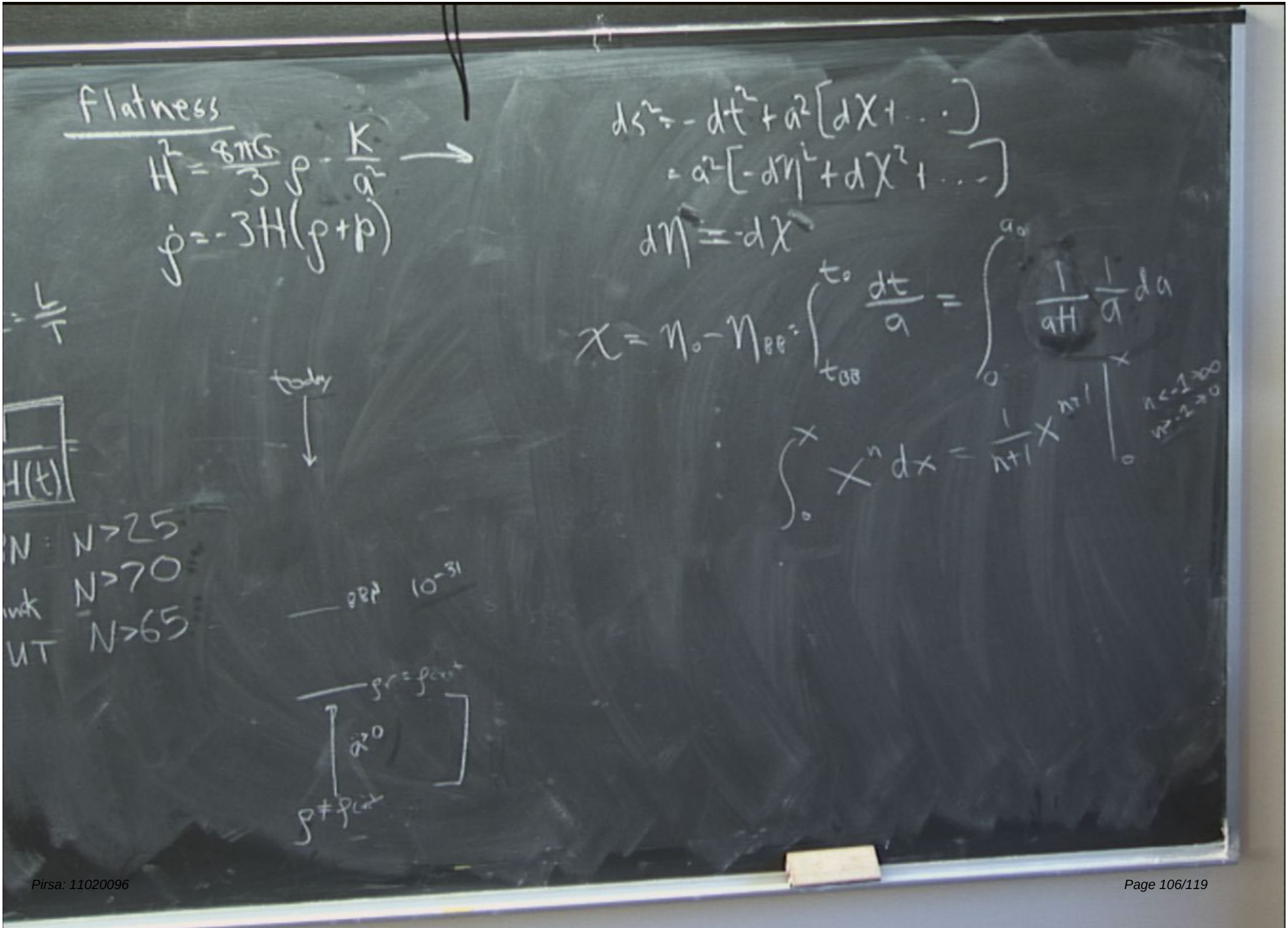
$\int_0^x x^n dx = \frac{1}{n+1}x^{n+1}\Big|_0^x$
$n<-1 \to \infty$
$n<-2 \to 0$
today
BBN 10^{-31}
$\rho_r = \rho_{crit}$
$\dot{a}>0$
$\rho \neq \rho_{crit}$
H(t)
N: N>25
N>70
UT N>65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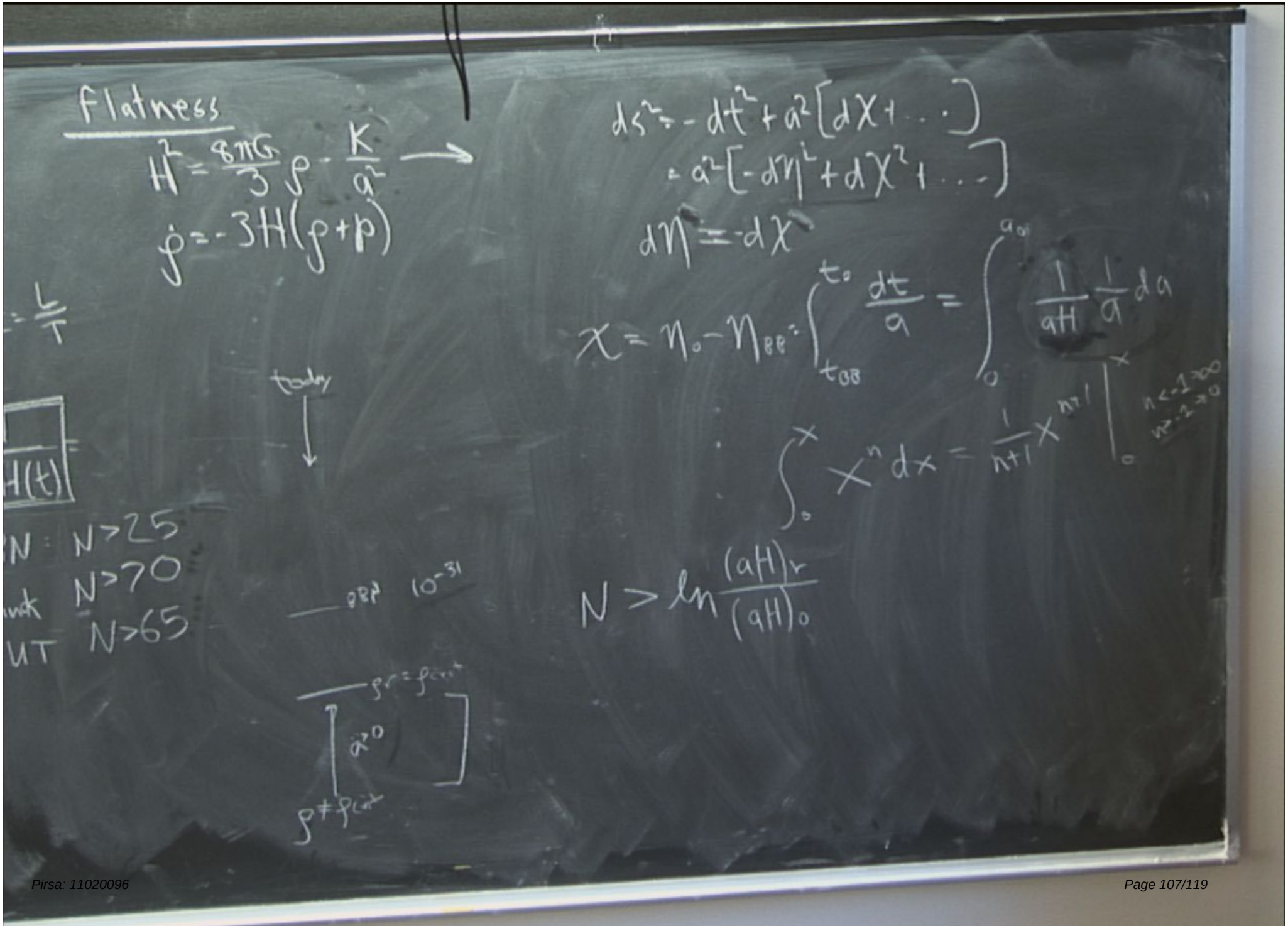

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$$\int_0^x x^n\,dx = \frac{1}{n+1}x^{n+1}\Big|_0^x \qquad n<-1 \to \infty,\ n>-1 \to 0$$

$$N > \ln\frac{(aH)_r}{(aH)_0}$$

$= \frac{L}{T}$

today

$\boxed{\frac{1}{H(t)}}$

N : $N>25$

nk $N>70$

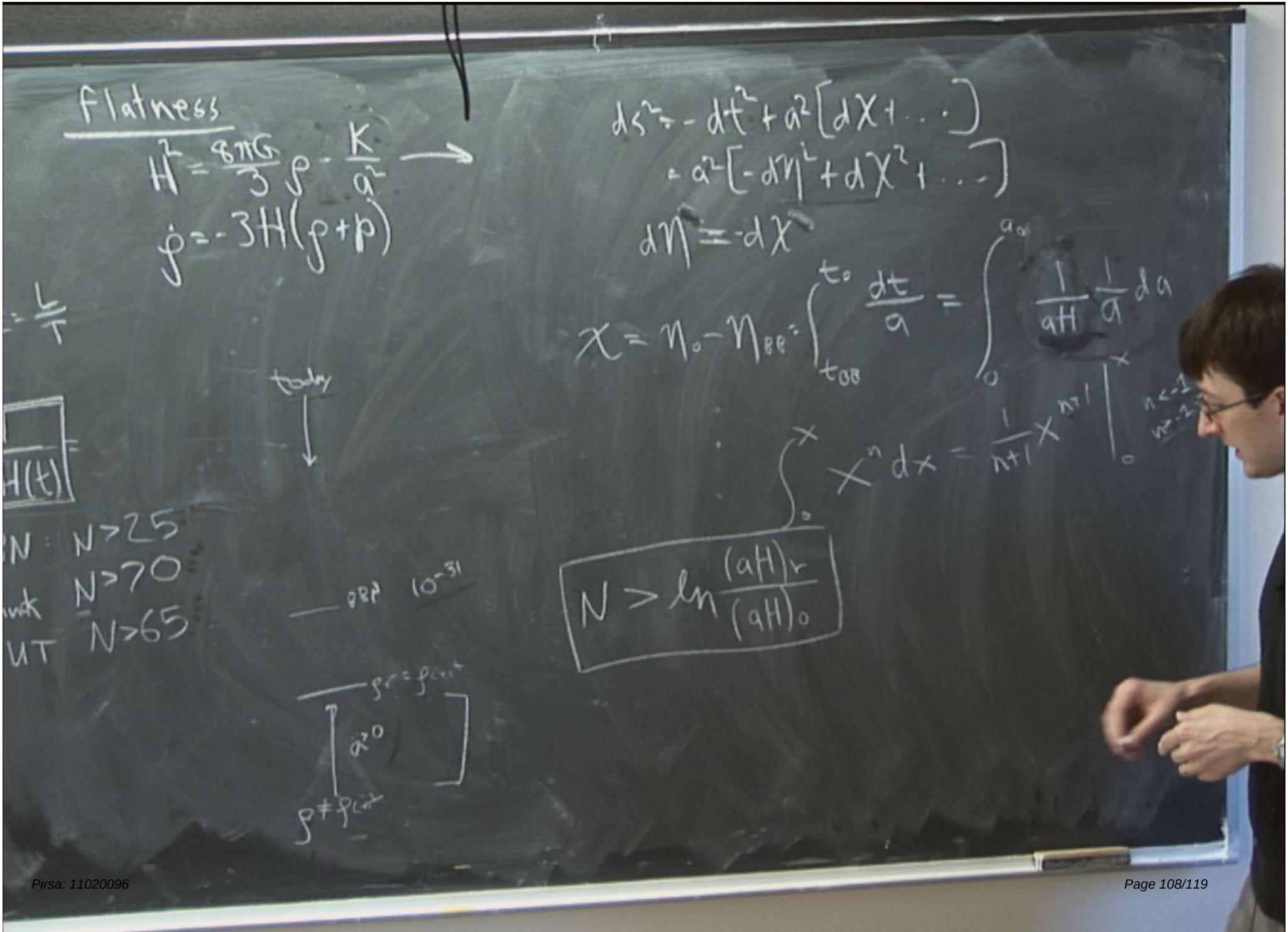
UT $N>65$

BBN 10^{-31}

$\rho_r = \rho_{crit}$

$a>0$

$\rho \neq \rho_{crit}$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today
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10^{-31}
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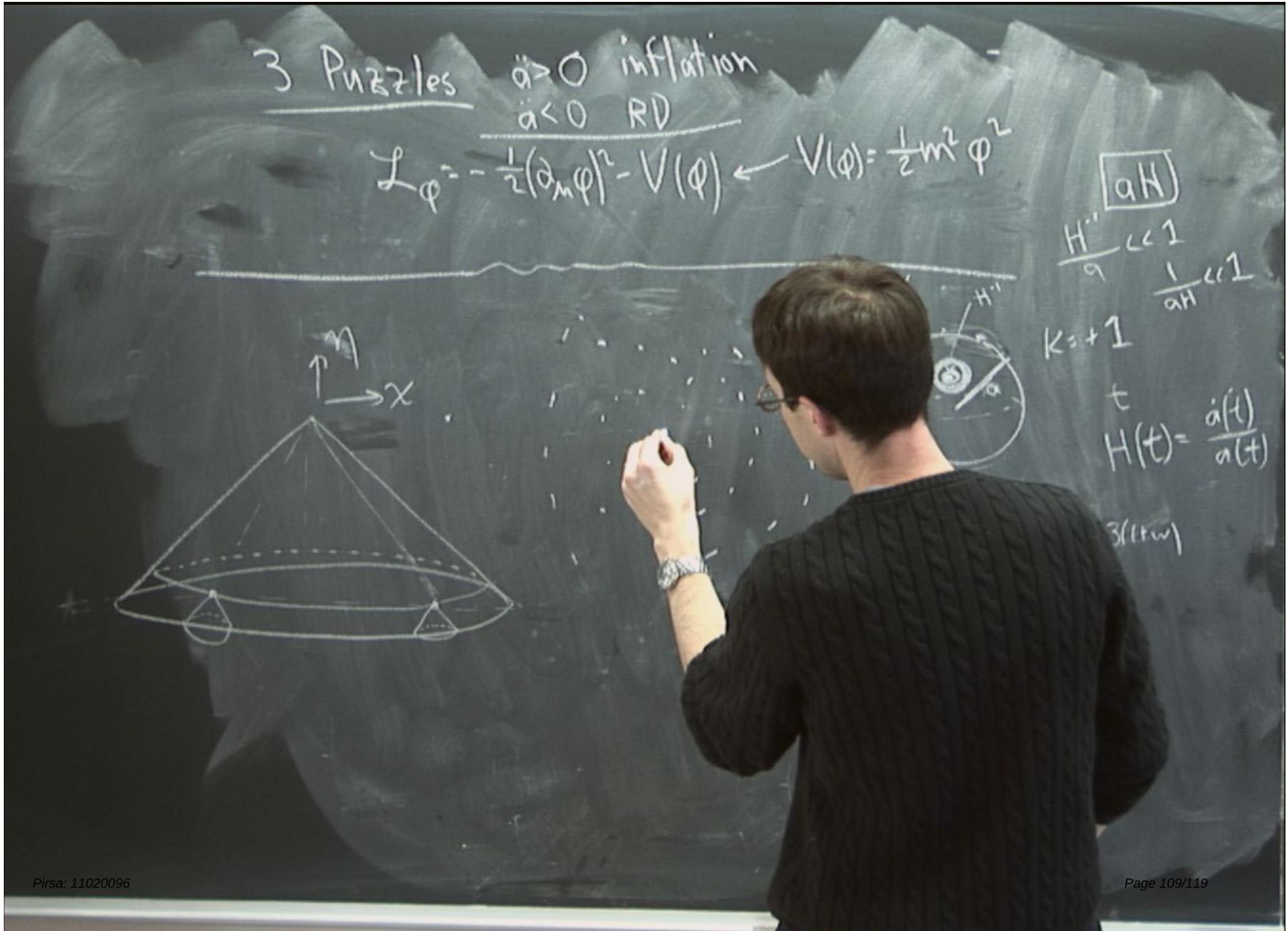
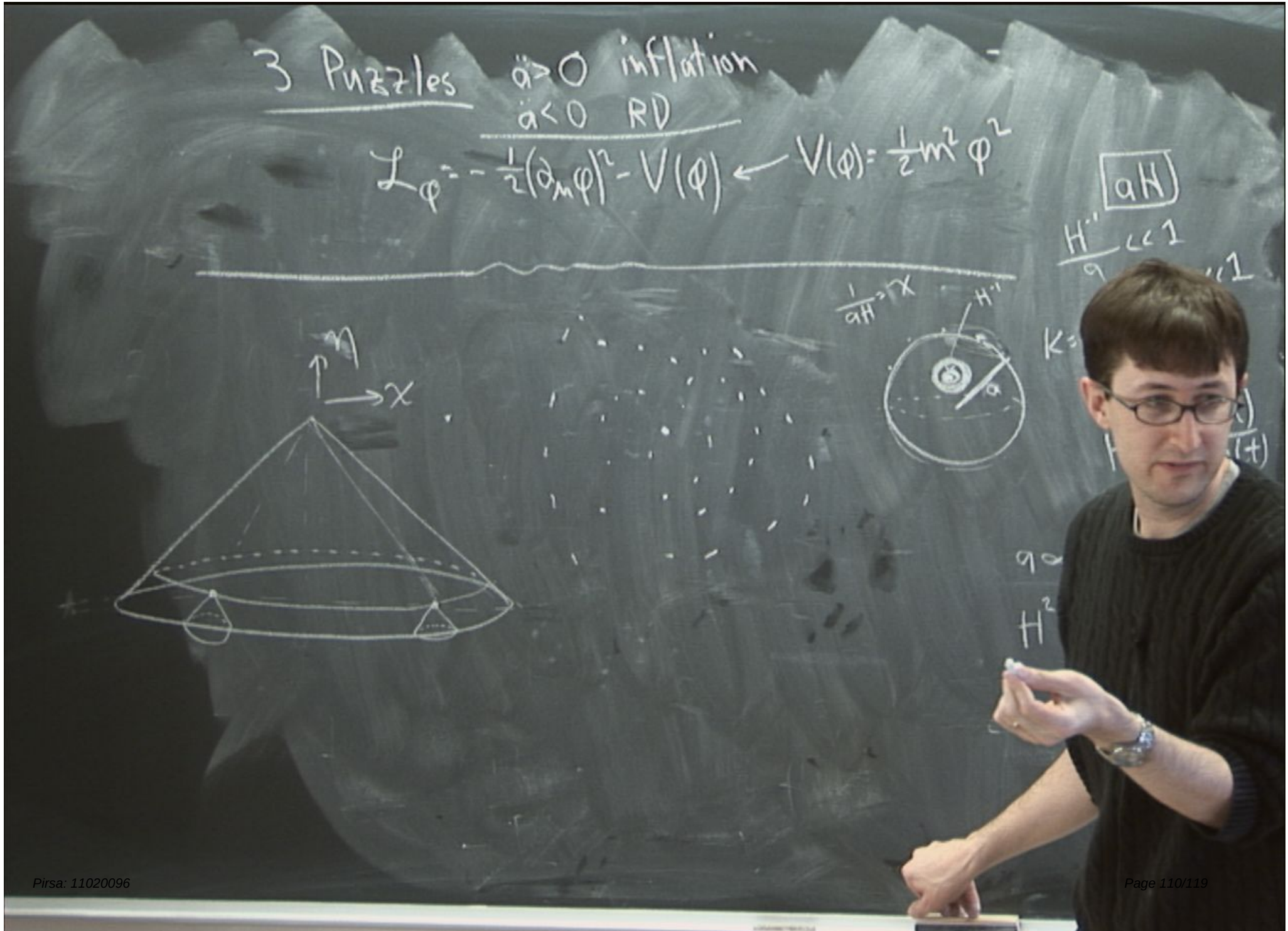

3 Puzzles
$\ddot{a}>0$ inflation
$\ddot{a}<0$ RD
$\mathcal{L}_\varphi = -\frac{1}{2}(\partial_\mu\varphi)^2 - V(\varphi)$ ← $V(\varphi) = \frac{1}{2}m^2\varphi^2$
aH
$\frac{H'}{a} \ll 1$
$\frac{1}{aH} \ll 1$
H'
K = +1
t
$H(t) = \frac{\dot{a}(t)}{a(t)}$
y
x
a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ä>0 inflation
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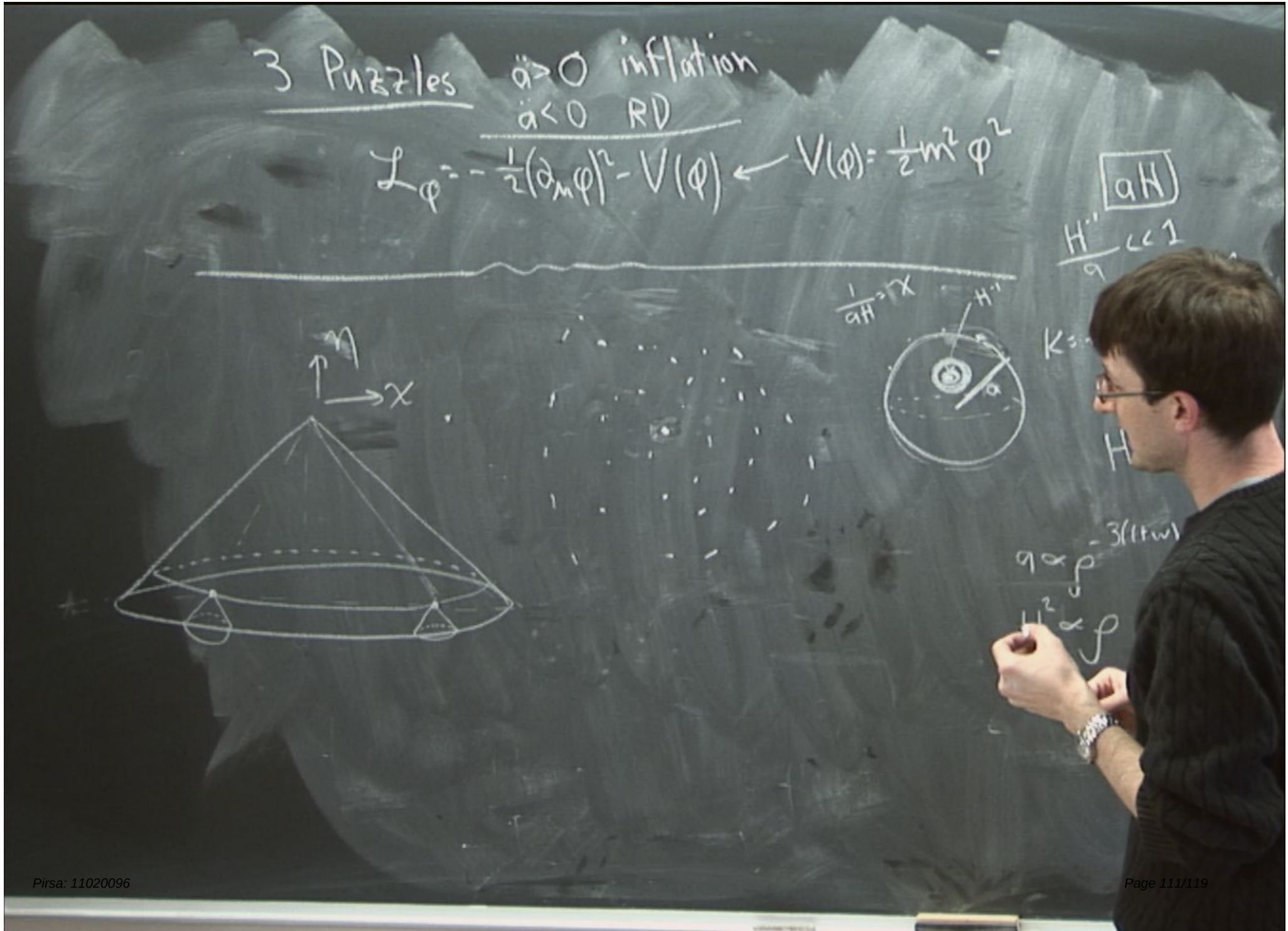
$\frac{H^{-1}}{a} \ll 1$
$\frac{1}{aH} = \chi$
H^{-1}
η
χ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3 Puzzles

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$\frac{H'}{a} \ll 1$

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$K = +1$

t

$$H(t) = \frac{\dot{a}(t)}{a(t)}$$

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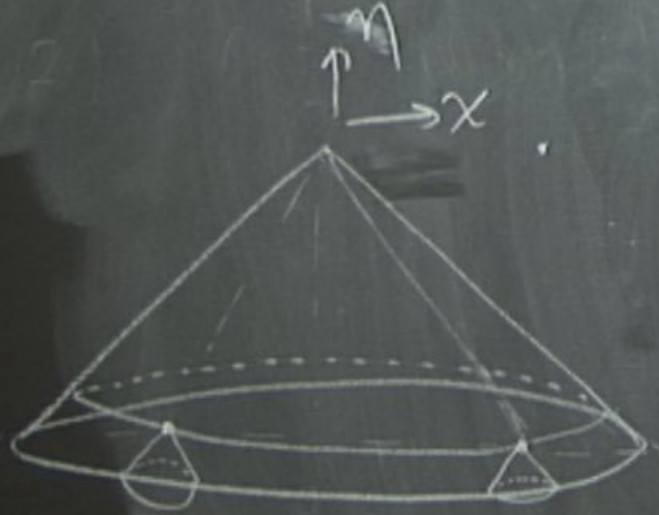
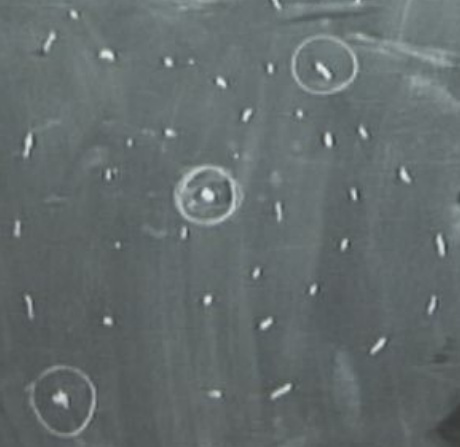
aH

$\frac{H'}{a} \ll 1$ $\quad \frac{1}{aH} \ll 1$

$(aH)^{-1} = \chi$ $\quad \frac{1}{aH} = \chi$ $\quad H^{-1}$





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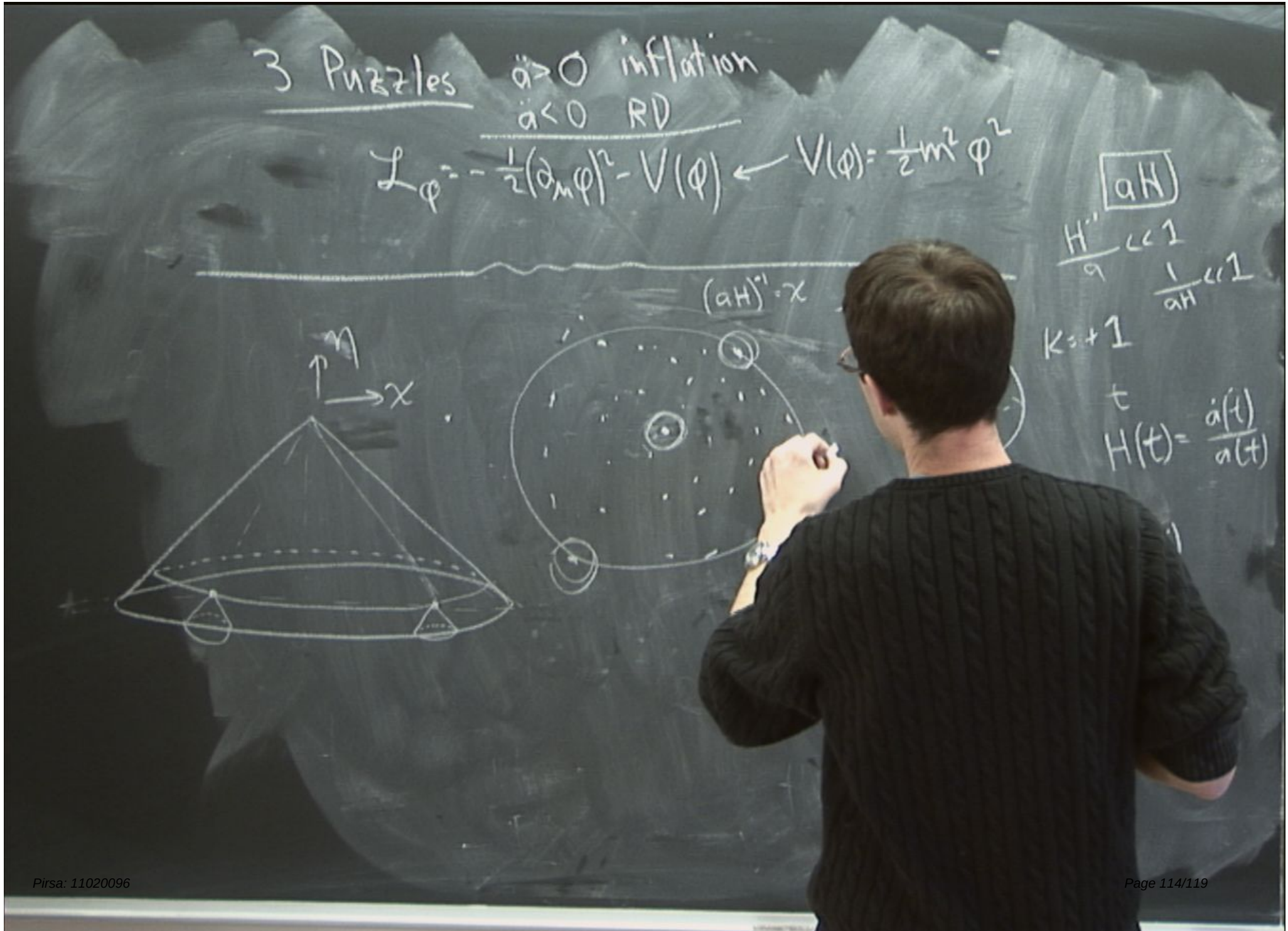
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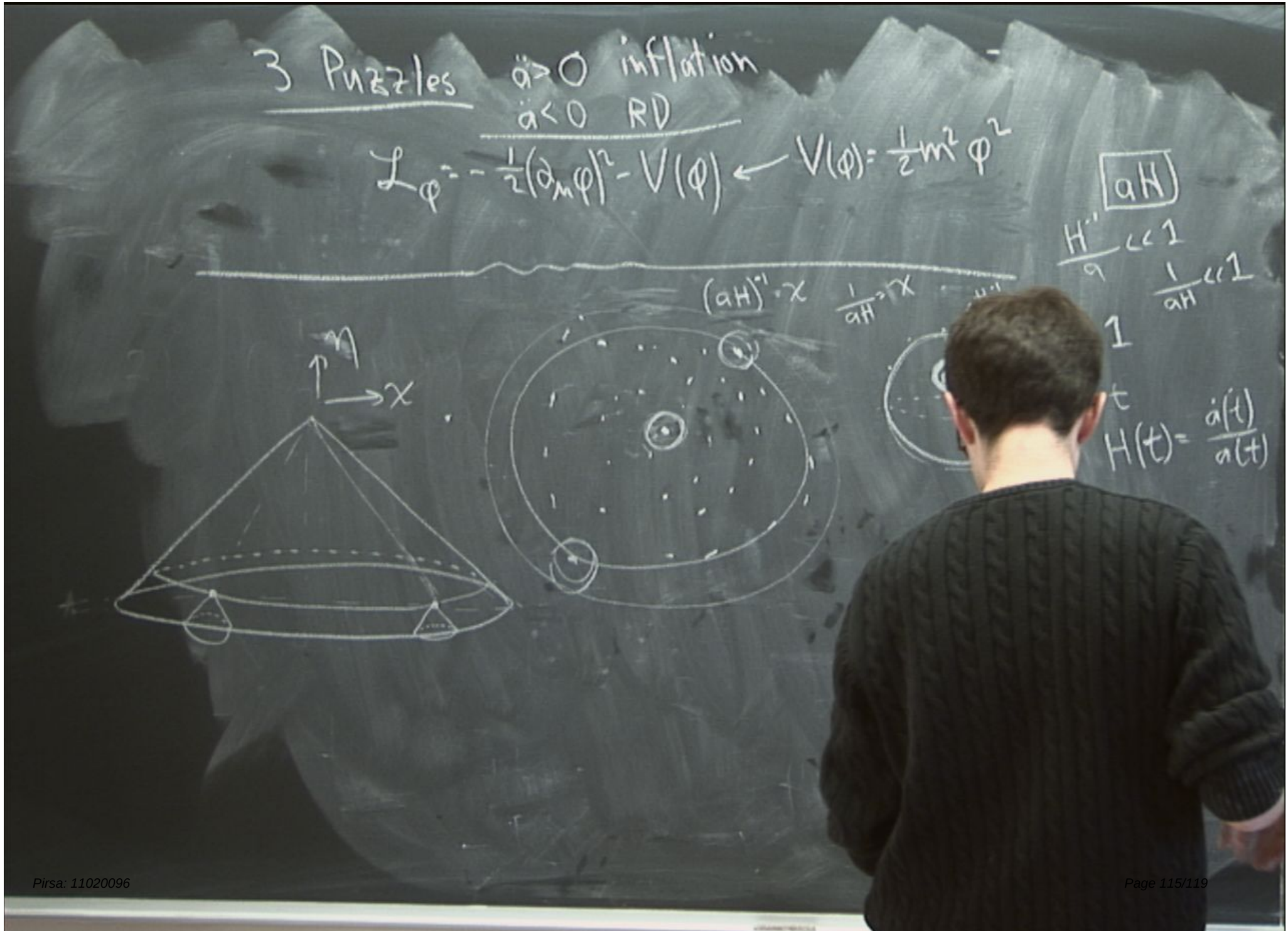
η
χ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$(aH)^{-1} \cdot \chi$
$\frac{1}{aH} \cdot \chi$
η
χ
1
t
$H(t) = \frac{\dot{a}(t)}{a(t)}$

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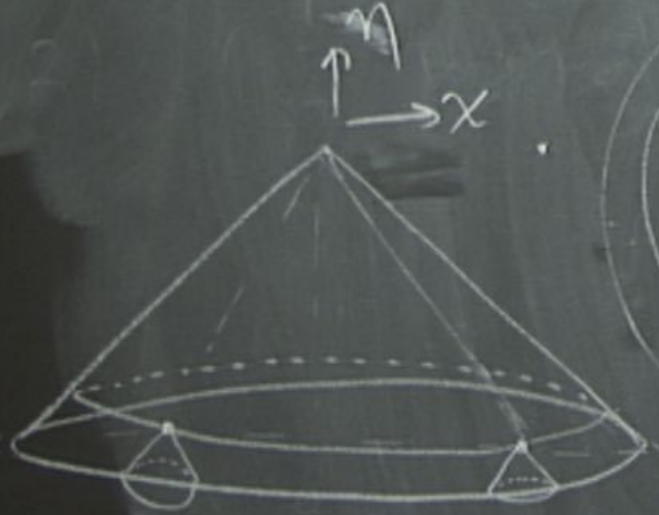
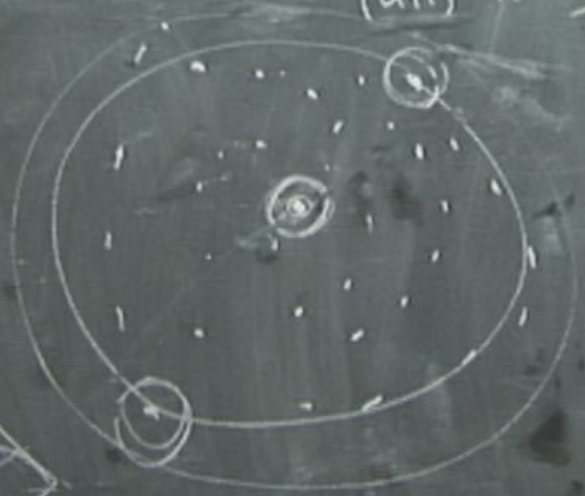
H^{-1}





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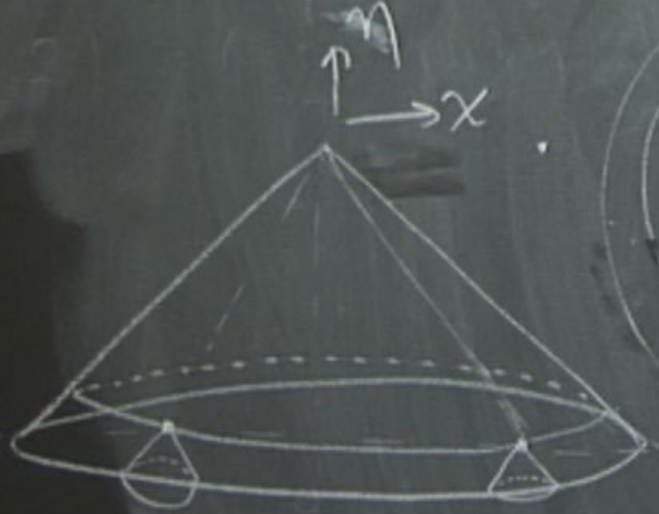

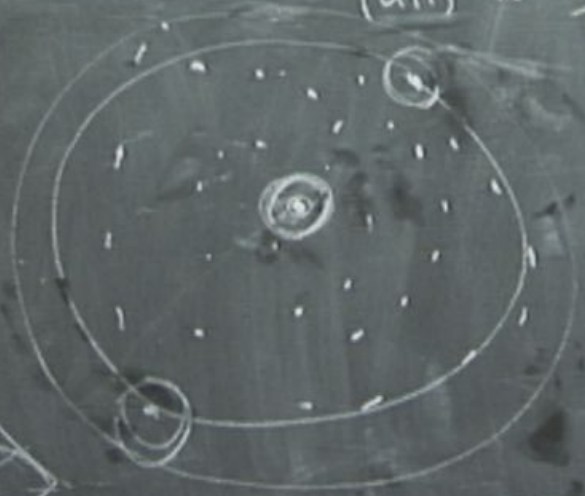

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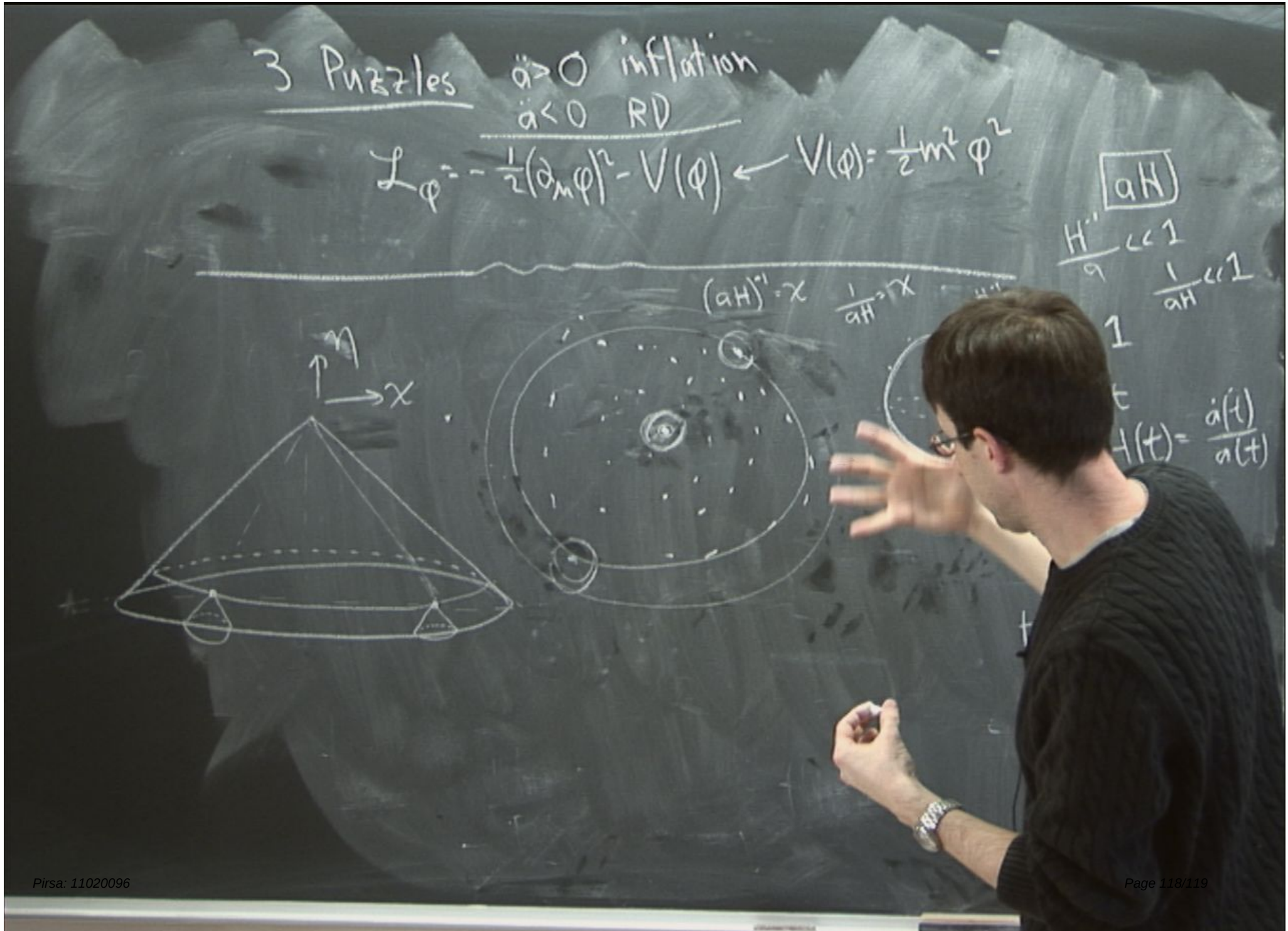
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η
x

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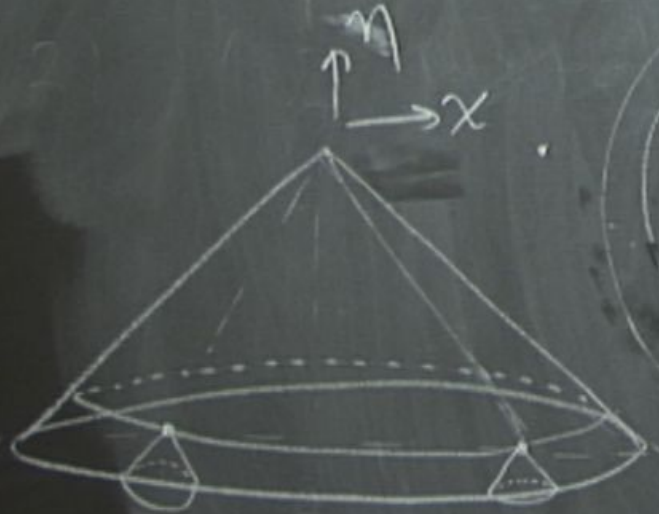

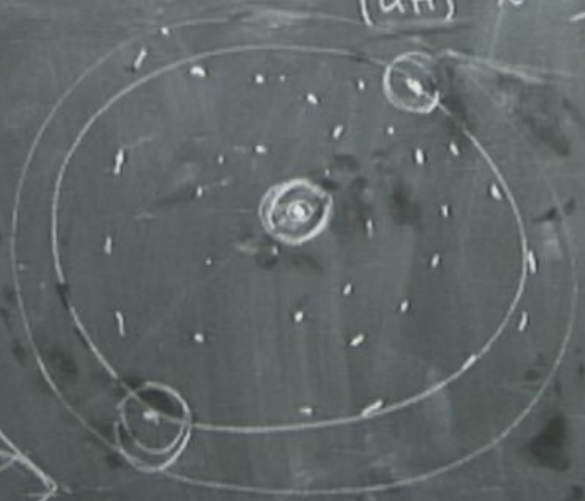

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