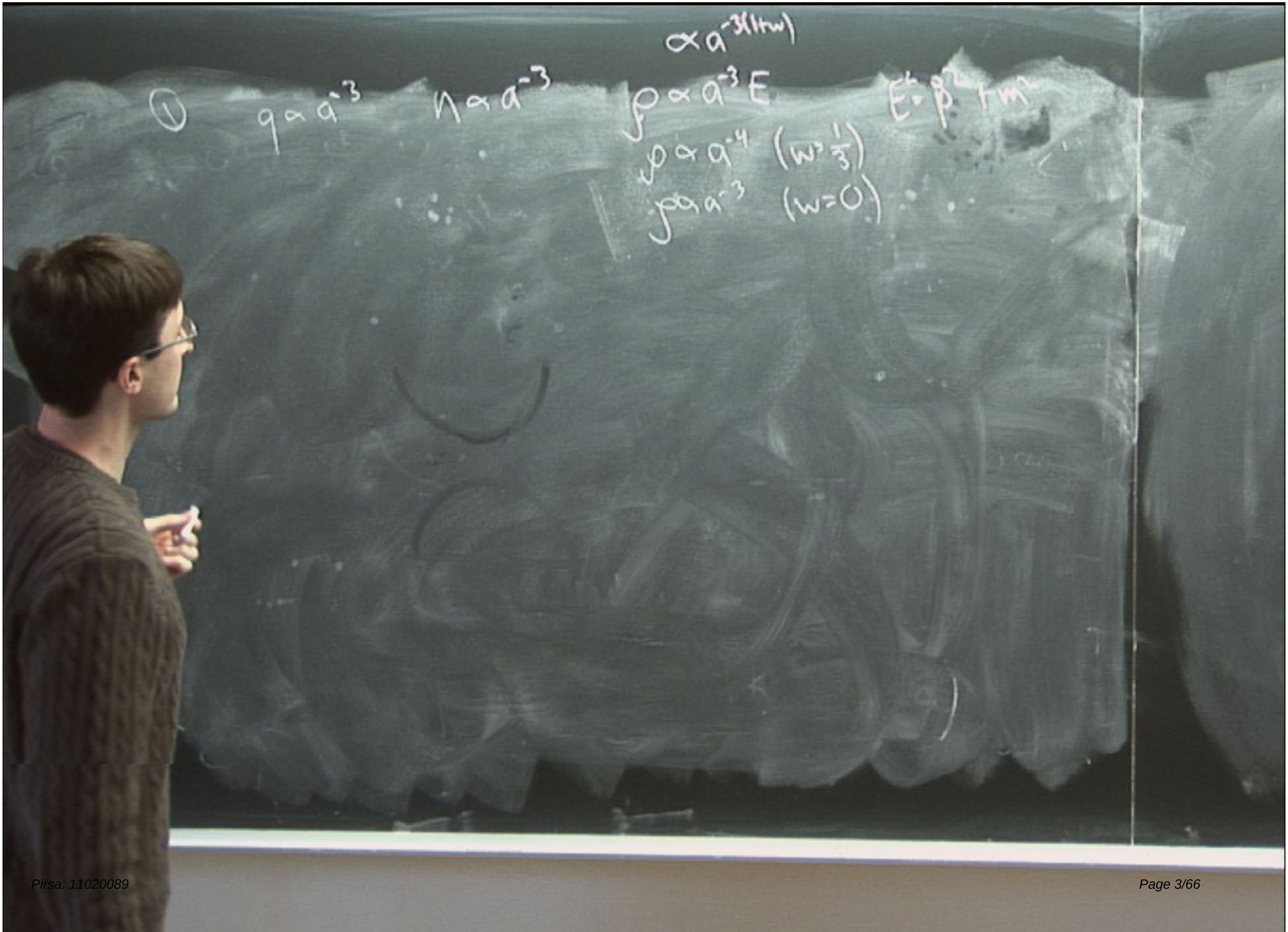
Title: Cosmology Review - Lecture 8

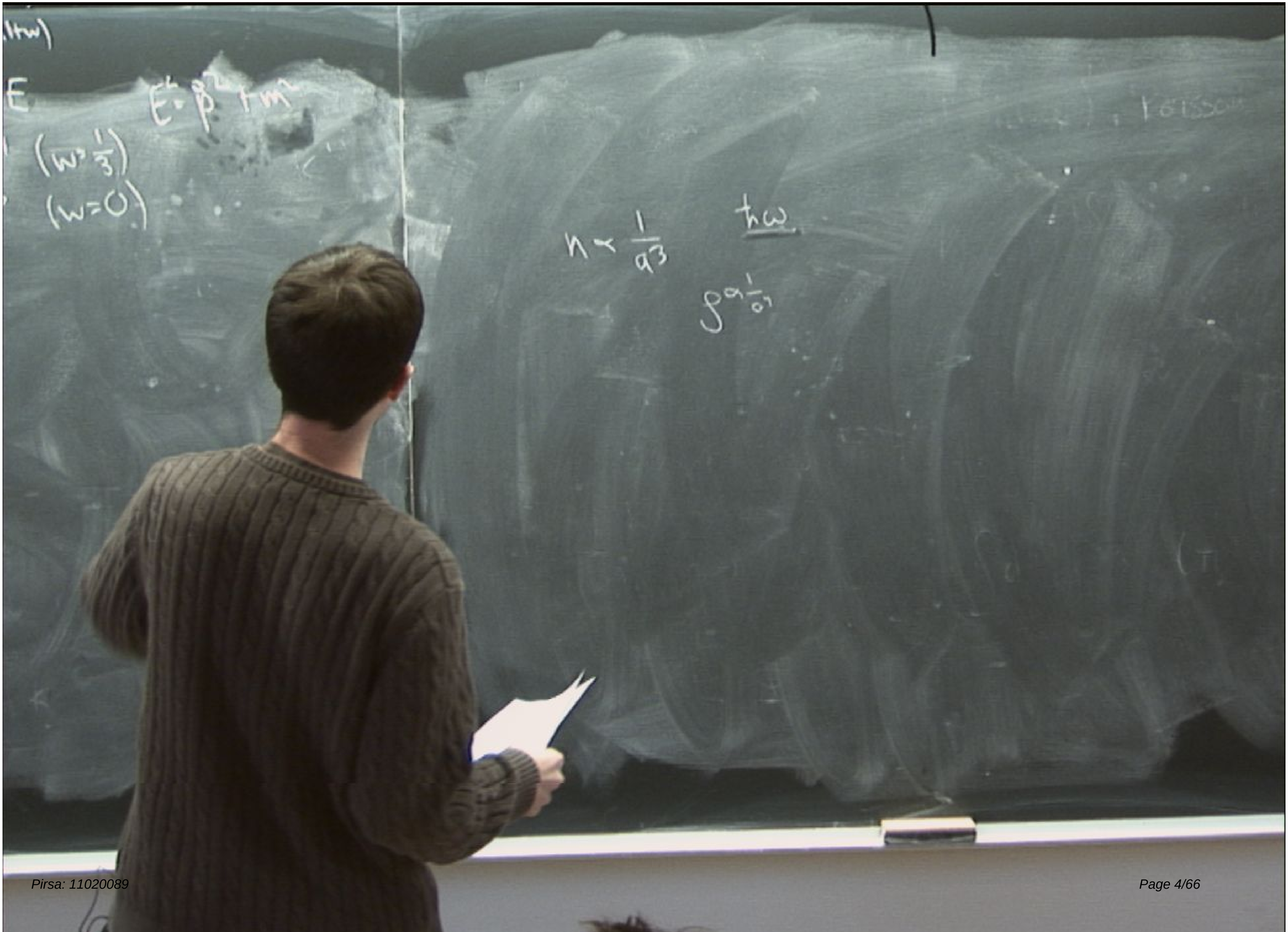
Date: Feb 02, 2011 11:30 AM

URL: http://pirsa.org/11020089

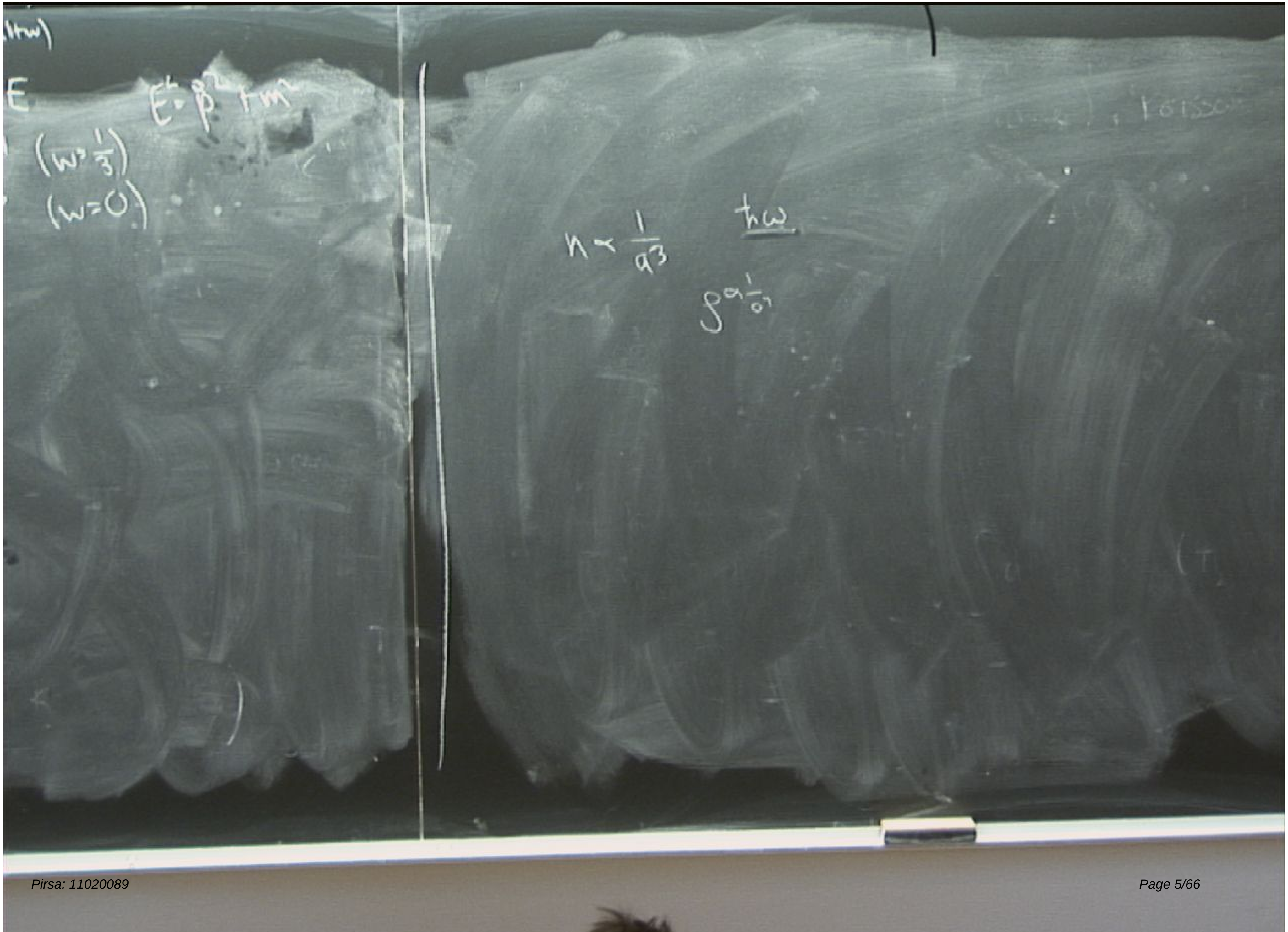
Abstract:

1 $q \propto a^{-3}$ $n \propto a^{-3}$ $\rho \propto a^{-3} E$ $E^2 = p^2 + m^2$

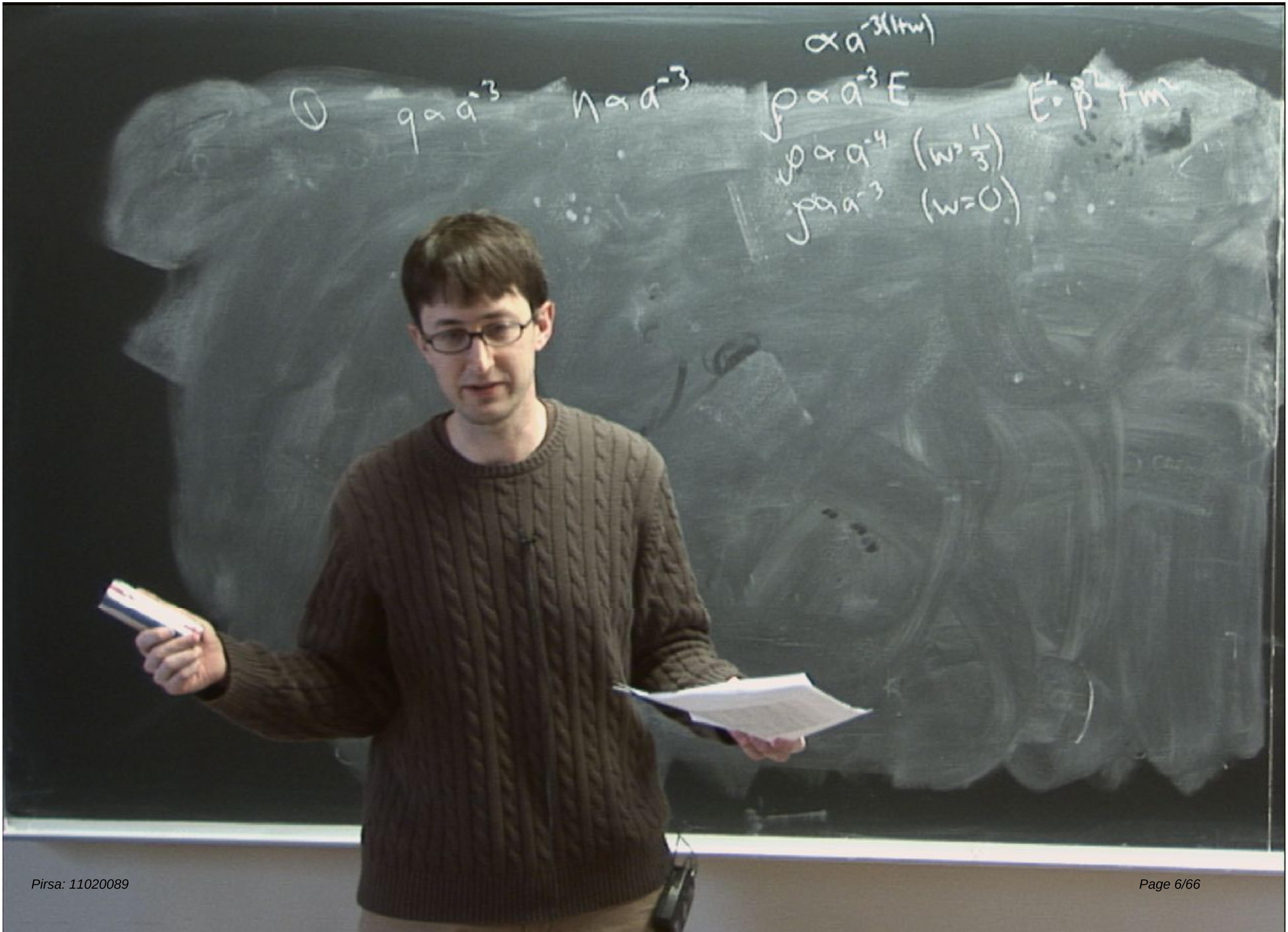

① q ∝ a^-3
n ∝ a^-3
ρ ∝ a^-3 E ∝ a^-3(1+w)
E^2 = p^2 + m^2
ρ ∝ a^-4 (w = 1/3)
ρ ∝ a^-3 (w = 0)



$(w = \frac{1}{3})$
$(w = 0)$
$n \propto \frac{1}{a^3}$
$\hbar\omega$



E
E² = p² + m²
(w = 1/3)
(w = 0)
n ∝ 1/a³
ħω
ρ ∝ 1/a


① q ∝ a^-3
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E² = p² + m²
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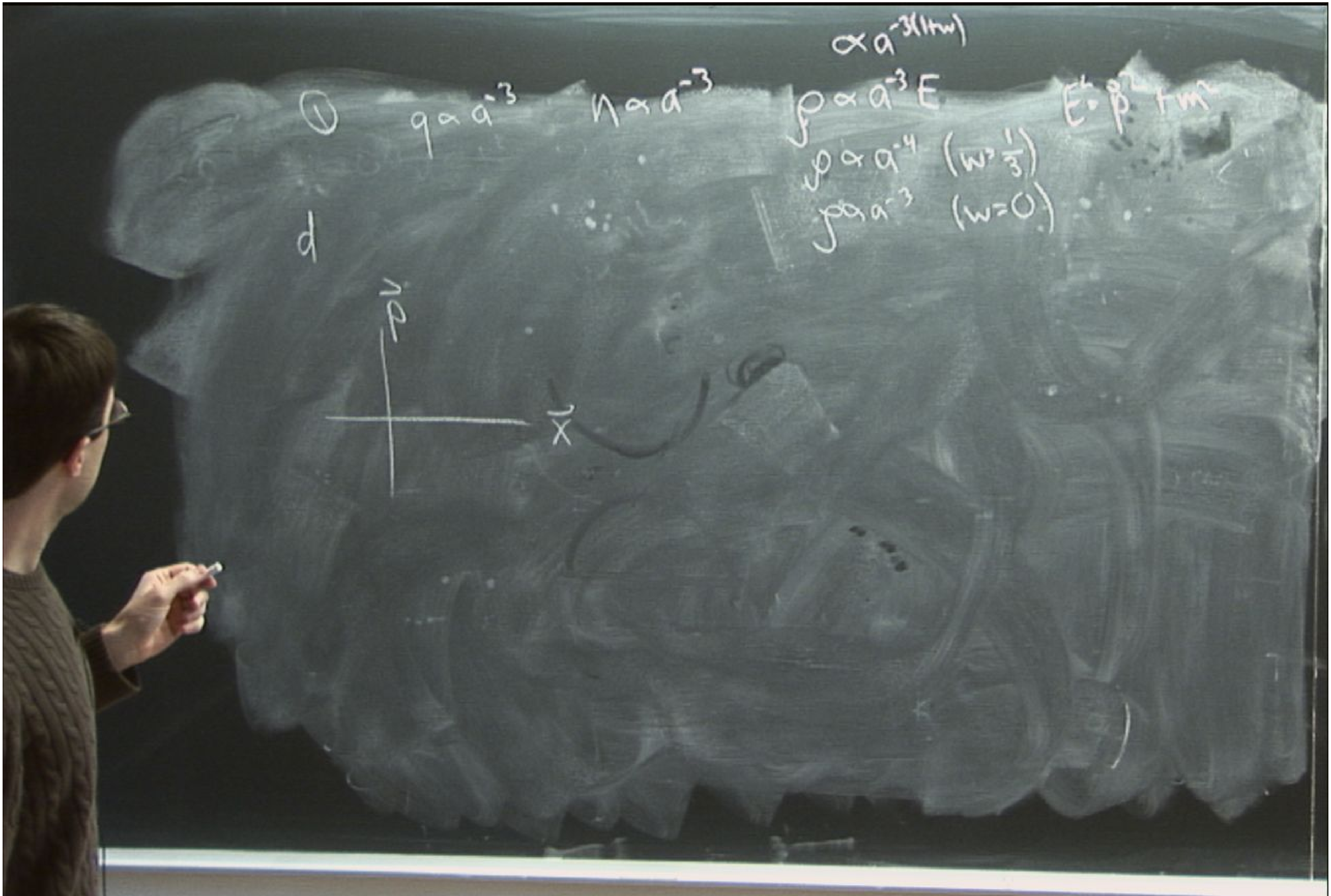
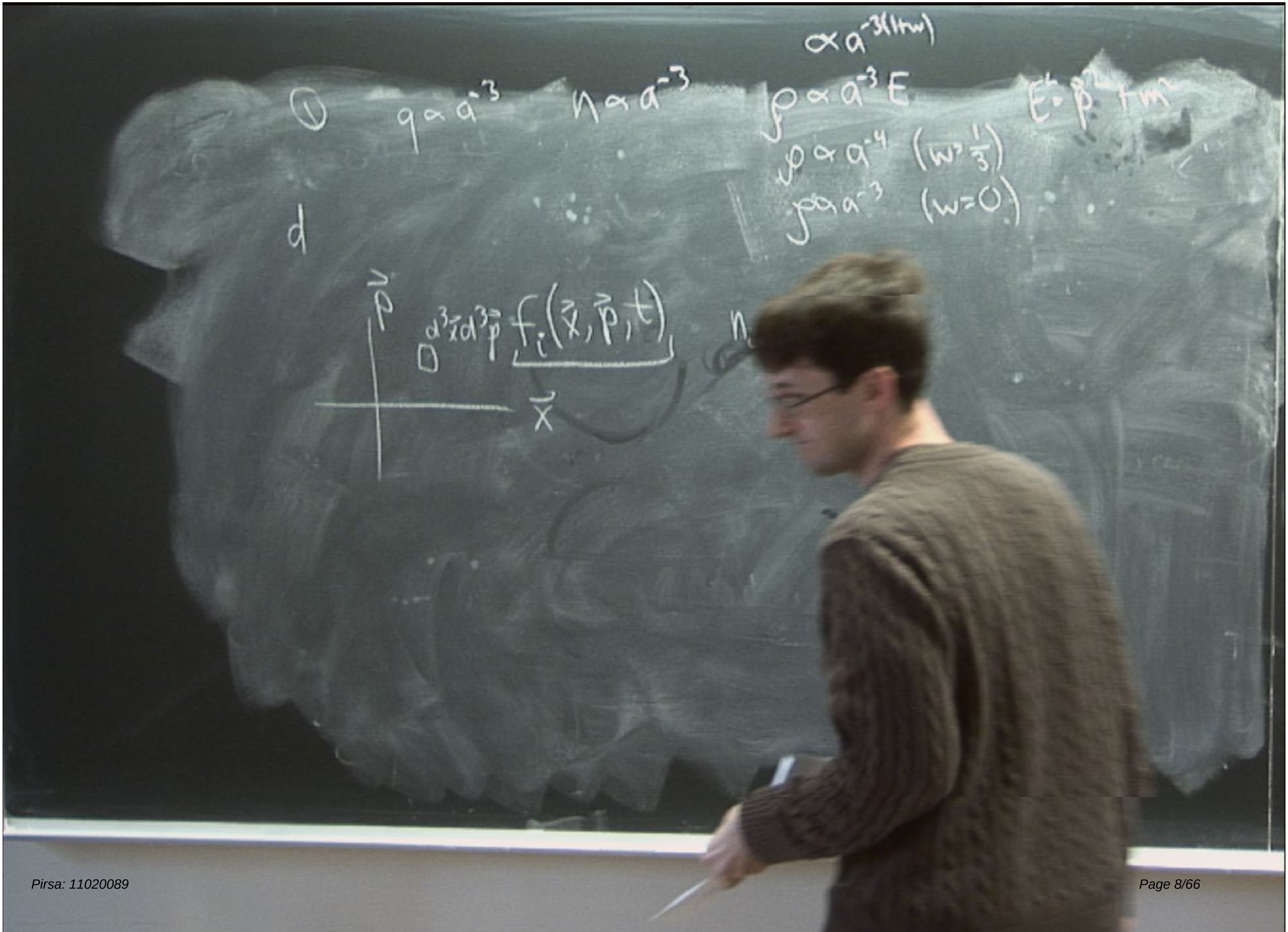

$\propto a^{-3(1+w)}$
① $q \propto a^{-3}$
$n \propto a^{-3}$
$\rho \propto a^{-3} E$
$E^2 = p^2 + m^2$
$\rho \propto a^{-4}$ $(w = \frac{1}{3})$
$\rho \propto a^{-3}$ $(w = 0)$
d
$\vec{p}$
$\vec{x}$


$\propto a^{-3(1+w)}$
① $q \propto a^{-3}$
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$E^2 = \vec{p}^2 + m^2$
$\rho \propto a^{-4}$ $(w = \frac{1}{3})$
$\rho \propto a^{-3}$ $(w = 0)$
d
$\vec{p}$
$d^3\vec{x}\, d^3\vec{p}\; f_i(\vec{x}, \vec{p}, t)$
$\vec{x}$

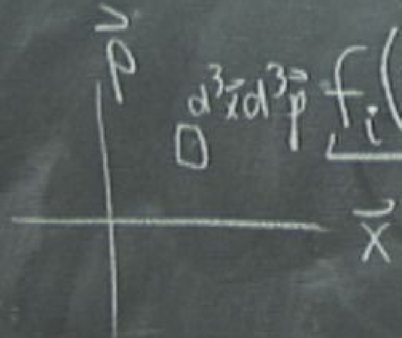
$\propto a^{-3(1+w)}$

① $q \propto a^{-3}$ $\quad n \propto a^{-3}$ $\quad \rho \propto a^{-3} E$ $\quad E^2 = \vec{p}^2 + m^2$

$\rho \propto a^{-4}$ $\quad (w = \frac{1}{3})$

$\rho \propto a^{-3}$ $\quad (w = 0)$

d



$d^3\vec{x}\, d^3\vec{p}\, f_i(\vec{x}, \vec{p}, t)$

$$n_i(\vec{x}, t) = \int d^3\vec{p}\, f_i(\vec{x}, \vec{p}, t)$$

$$\rho_i(\vec{x}, t) = \int d^3\vec{p}\, f_i(\vec{x}, \vec{p}, t)\, E(\vec{p}$$

① $q \propto a^{-3}$ $\quad n \propto a^{-3}$ $\quad \rho \propto a^{-3} E \propto a^{-3(1+w)}$ $\quad E^2 = p^2 + m^2$

$\rho \propto a^{-4} \quad (w = \frac{1}{3})$

$\rho \propto a^{-3} \quad (w = 0)$

d

$\vec{p}$ $\quad d^3\vec{x}\, d^3\vec{p}\; \underbrace{f_i(\vec{x}, \vec{p}, t)}$ $\quad \vec{x}$

$h^3 = (2\pi\hbar)^3$

$n_i(\vec{x}, t) = \int d^3\vec{p}\; f_i(\vec{x}, \vec{p}, t)$

$\rho_i(\vec{x}, t) = \int d^3\vec{p}\; f_i(\vec{x}, \vec{p}, t)\; E(\vec{p}, \vec{x}, t)$

$n \propto$

$\propto a^{-3(1+w)}$

① $q \propto a^{-3}$ $\quad n \propto a^{-3}$ $\quad \rho \propto a^{-3} E$ $\quad E^2 = \vec{p}^2 + m^2$

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d

$\vec{p}$

$d^3\vec{x}\, d^3\vec{p}\ \underline{f_i(\vec{x}, \vec{p}, t)}$

$\vec{x}$

$h^3 = (2\pi\hbar)^3$

$n_i(\vec{x}, t) = \int d^3\vec{p}\, f_i(\vec{x}, \vec{p}, t)$

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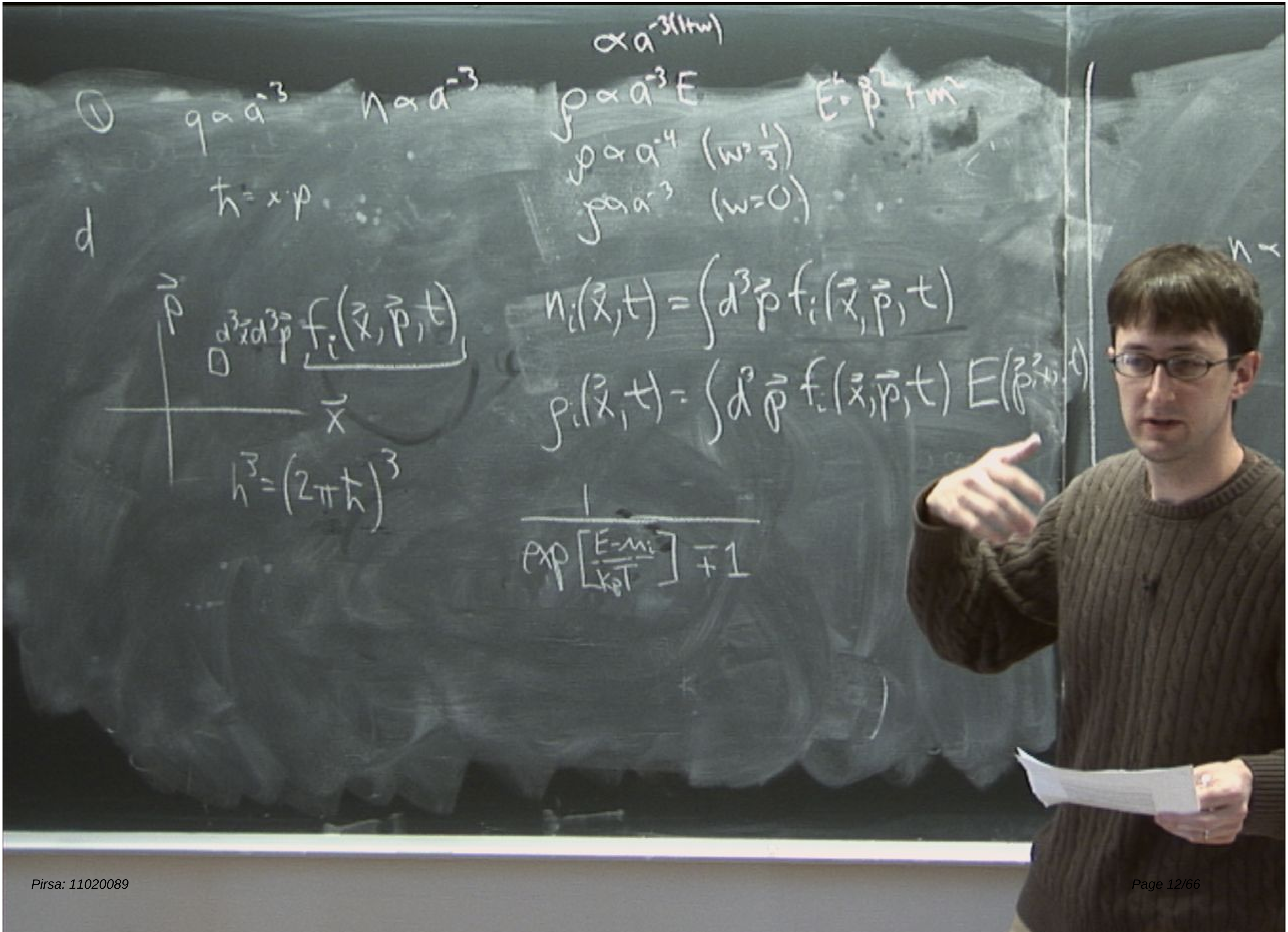
$n \propto$


∝ a^{-3(1+w)}
① q ∝ a^{-3} n ∝ a^{-3} ρ ∝ a^{-3} E E² = p² + m²
ρ ∝ a^{-4} (w = 1/3)
ħ = x·p ρ ∝ a^{-3} (w = 0)
d
n_i(x, t) = ∫ d³p f_i(x, p, t)
p d³x d³p f_i(x, p, t)
ρ_i(x, t) = ∫ d³p f_i(x, p, t) E(p², x, t)
x
h³ = (2πħ)³
1 / (exp[(E − μ_i)/(k_B T)] ∓ 1)

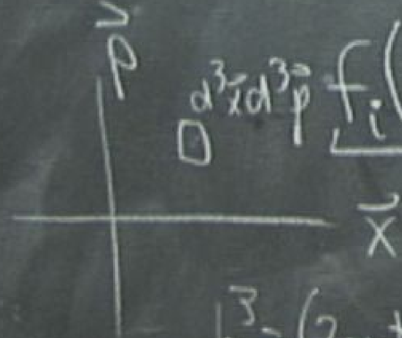
$\propto a^{-3(1+w)}$

① $q \propto a^{-3}$ $\quad n \propto a^{-3}$ $\quad \rho \propto a^{-3} E$ $\quad E^2 = \vec{p}^2 + m^2$

$\rho \propto a^{-4} \quad (w = \frac{1}{3})$

$\rho \propto a^{-3} \quad (w = 0)$

d $\quad \hbar = x \cdot p$



$d^3\vec{x}\, d^3\vec{p}\, \underbrace{f_i(\vec{x}, \vec{p}, t)}$

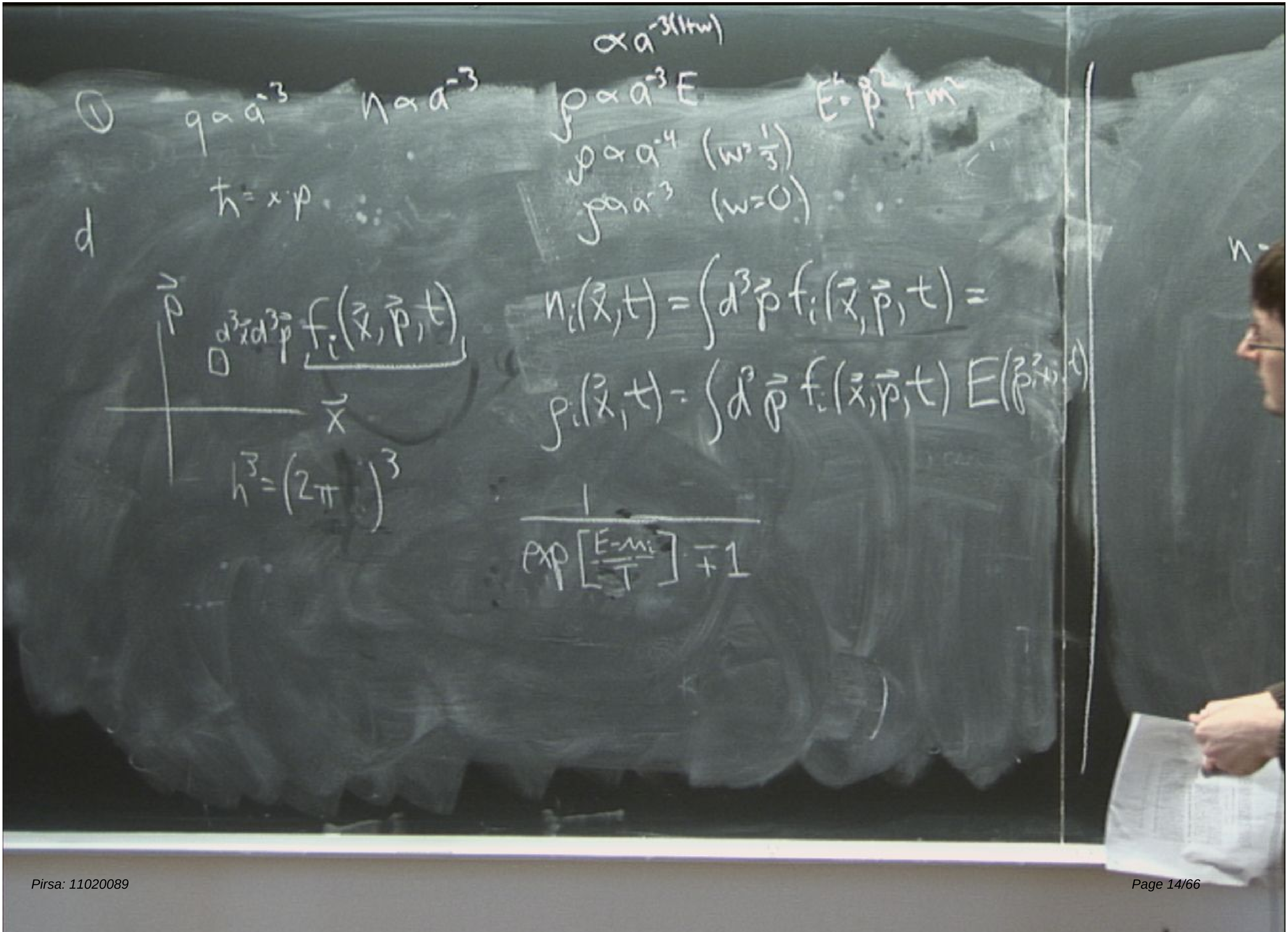
$h^3 = (2\pi\hbar)^3$

$$n_i(\vec{x}, t) = \int d^3\vec{p}\, f_i(\vec{x}, \vec{p}, t)$$

$$\rho_i(\vec{x}, t) = \int d^3\vec{p}\, f_i(\vec{x}, \vec{p}, t)\, E(\vec{p}^2, x_i, t)$$

$$\frac{1}{\exp\left[\frac{E - \mu_i}{T}\right] \mp 1}$$

$n \propto$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$\hbar = x \cdot p$
$\vec{p}$
$d^3\vec{x}\, d^3\vec{p}\, f_i(\vec{x}, \vec{p}, t)$
$\vec{x}$
$h^3 = (2\pi\ )^3$
$n_i(\vec{x}, t) = \int d^3\vec{p}\, f_i(\vec{x}, \vec{p}, t) =$
$\rho_i(\vec{x}, t) = \int d^3\vec{p}\, f_i(\vec{x}, \vec{p}, t)\, E(\vec{p}^2, \ldots)$
$\frac{1}{\exp\left[\frac{E - \mu_i}{T}\right] \mp 1}$

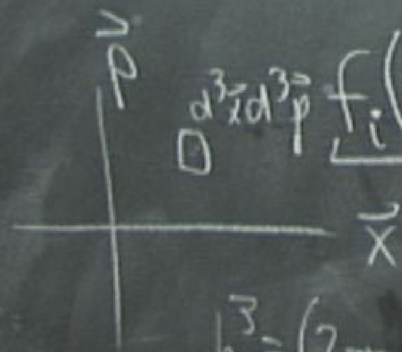
$\propto a^{-3(1+w)}$

① $q \propto a^{-3}$ $\quad n \propto a^{-3}$ $\quad \rho \propto a^{-3} E$ $\quad E^2 = \vec{p}^2 + m^2$

$\rho \propto a^{-4}$ $\left(w = \frac{1}{3}\right)$

$\hbar = x \cdot p$ $\quad \rho \propto a^{-3}$ $(w = 0)$ $\quad 4\pi \vec{p}^2 d\vec{p}$

d



$d^3\vec{x}\, d^3\vec{p}\ \underbrace{f_i(\vec{x}, \vec{p}, t)}$

$h^3 = (2\pi\hbar)^3$

$$n_i(\vec{x}, t) = \int d^3\vec{p}\, f_i(\vec{x}, \vec{p}, t) =$$

$$\rho_i(\vec{x}, t) = \int d^3\vec{p}\, f_i(\vec{x}, \vec{p}, t)\, E(\vec{p}^2 \ldots)$$

$$\frac{1}{\exp\left[\frac{E-\mu_i}{T}\right] \mp 1}$$

$E^2 = \vec{p}^2 + m^2$

$n \propto$

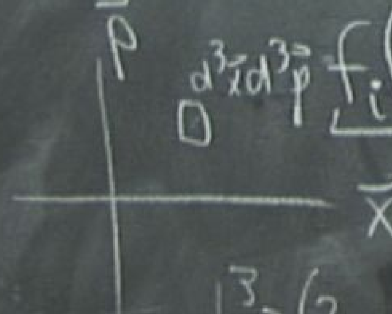
$\propto a^{-3(1+w)}$

① $q \propto a^{-3}$ $\quad n \propto a^{-3}$ $\quad \rho \propto a^{-3} E$ $\quad E^2 = \vec{p}^{\,2} + m^2$

$\rho \propto a^{-4}$ $\quad (w = \frac{1}{3})$

$\hbar = x \cdot p$ $\quad \rho \propto a^{-3}$ $\quad (w = 0)$ $\quad 4\pi p^2 dp$

d



$d^3\vec{x}\, d^3\vec{p}\, f_i(\vec{x}, \vec{p}, t)$

$h^3 = (2\pi \;\;)^3$

$$n_i(\vec{x}, t) = \int d^3\vec{p}\, f_i(\vec{x}, \vec{p}, t) =$$

$$\rho_i(\vec{x}, t) = \int d^3\vec{p}\, f_i(\vec{x}, \vec{p}, t)\, E(\vec{p}^{\,2}, t)$$

$$\frac{1}{\exp\left[\frac{E - \mu_i}{T}\right] \mp 1} \qquad E^2 = \vec{p}^{\,2} + m^2$$

$\propto a^{-3(1+w)}$

① $q \propto a^{-3}$ $\quad n \propto a^{-3}$ $\quad \rho \propto a^{-3} E$ $\quad E^2 = \vec{p}^2 + m^2$

$\rho \propto a^{-4}$ $\quad (w = \frac{1}{3})$

$\rho \propto a^{-3}$ $\quad (w = 0)$ $\quad 4\pi p^2 dp$

$\hbar = x \cdot p$

d

$\vec{p}$ $\quad d^3\vec{x}\, d^3\vec{p}\, \underbrace{f_i(\vec{x}, \vec{p}, t)}$ $\quad \vec{x}$

$h^3 = (2\pi\ )^3$

$$n_i(\vec{x}, t) = \int d^3\vec{p}\, f_i(\vec{x}, \vec{p}, t) =$$

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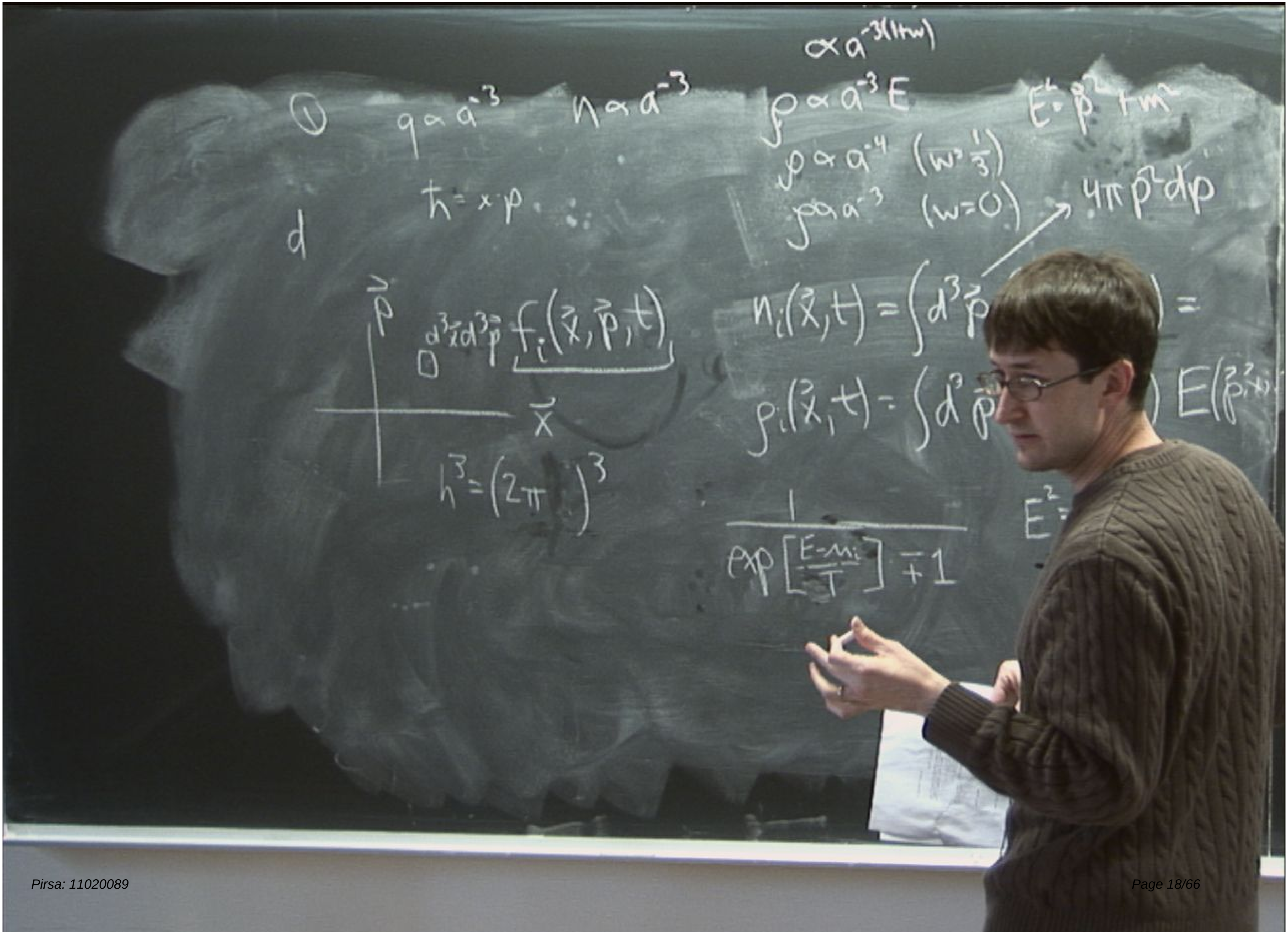
$$\frac{1}{\exp\left[\frac{E - \mu_i}{T}\right] \mp 1}$$

$$E^2 = \vec{p}^2 + m^2$$


∝ a^-3(1+w)
① q ∝ a^-3
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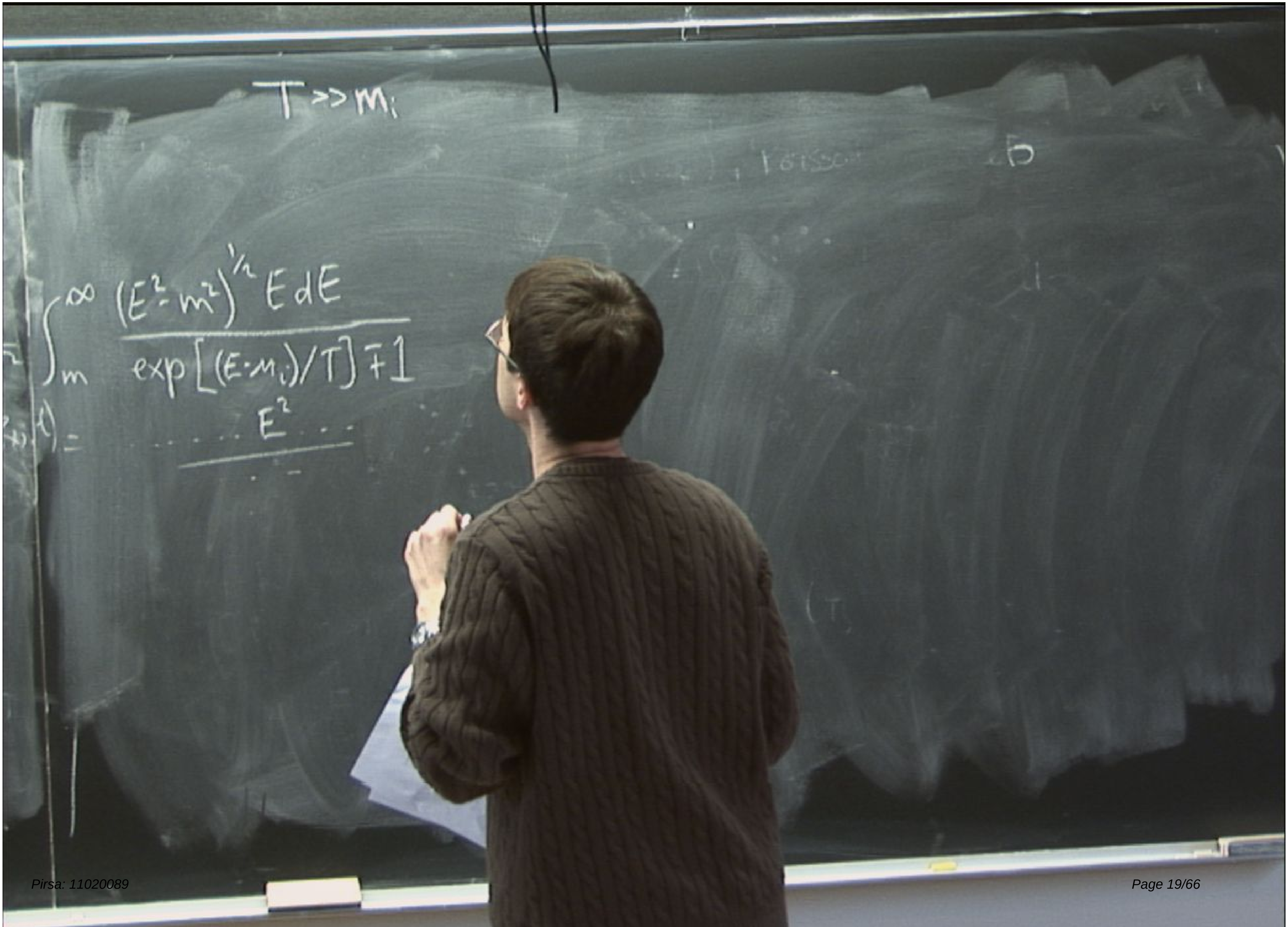
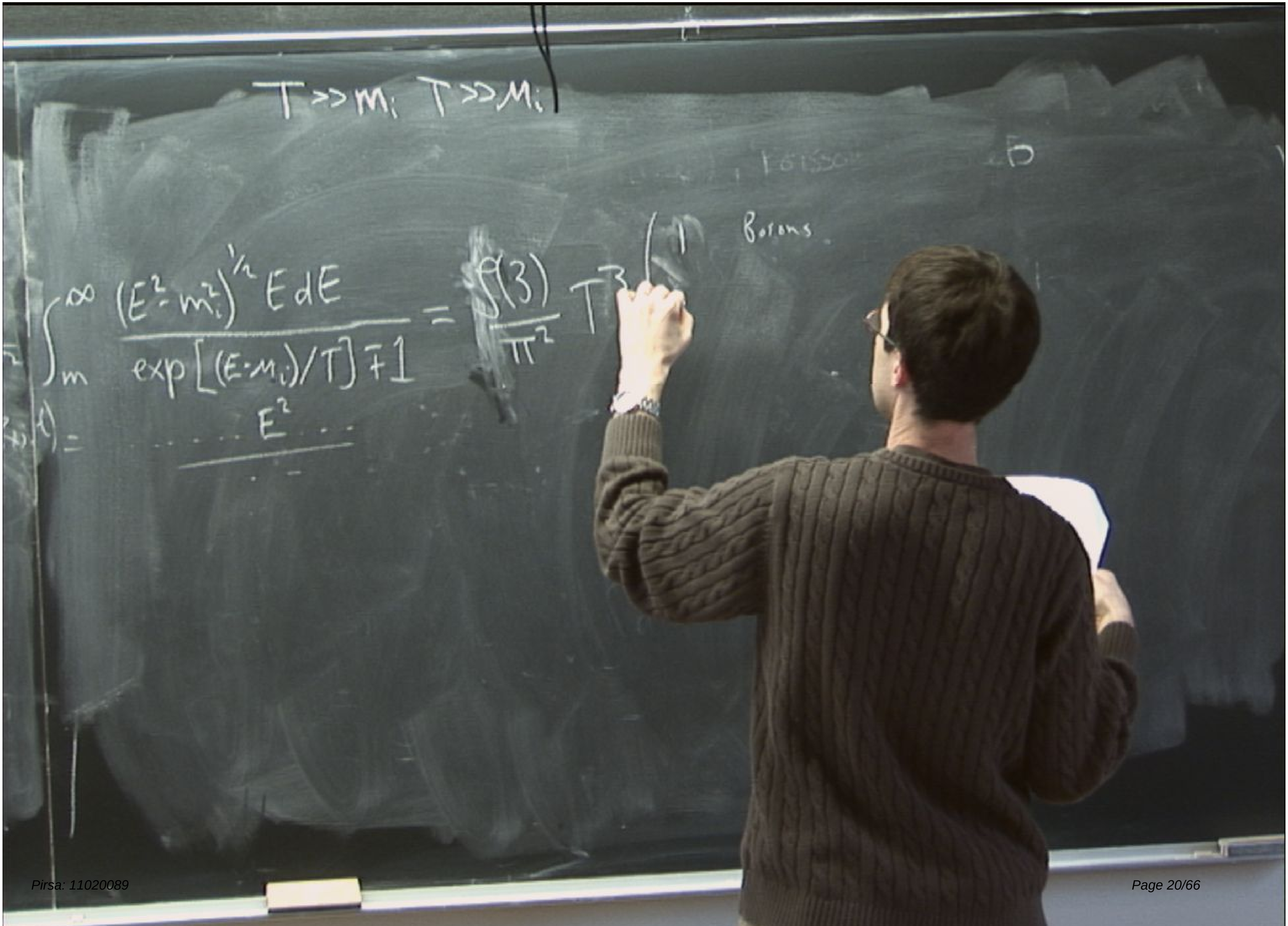
ρ ∝ a^-3 E
E² = p² + m²
ρ ∝ a^-4 (w = 1/3)
ρ ∝ a^-3 (w = 0)
4π p² dp
ħ = x·p
d
p
d³x d³p f_i(x, p, t)
x
h³ = (2π)³
n_i(x, t) = ∫d³p
ρ_i(x, t) = ∫d³p
E(p²,
1
exp[(E-μ_i)/T] ∓ 1
E² =


$T \gg m_i$
$\int_m^\infty \frac{(E^2 - m^2)^{1/2} E\,dE}{\exp[(E - m_i)/T] \mp 1}$
E^2


T>>m_i T>>M_i
Bosons
∫_m^∞ (E²-m_i²)^½ E dE / (exp[(E-m_i)/T] ∓ 1) = ζ(3)/π² T³
E²

$T \gg m_i \quad T \gg M_i$

$$\int_m^\infty \frac{(E^2 - m_i^2)^{1/2} E\, dE}{\exp[(E-m_i)/T] \mp 1} = \frac{\zeta(3)}{\pi^2} T^3 \begin{pmatrix} 1 & \text{Bosons} \\ \frac{3}{4} & \text{Fermions} \end{pmatrix}$$

$$\ldots \frac{E^2 \ldots}{\ldots} = \frac{\pi^2}{30} T^4 \begin{pmatrix} 1 & \text{Bosons} \\ \frac{7}{8} & \text{Fermions} \end{pmatrix}$$

$n \propto T^3$

$\rho \propto T^4$

$T \gg m_i \quad T \gg M_i$

$e^-, e^+, 3\nu, 3\bar{\nu}, \gamma$

$$\int_m^\infty \frac{(E^2 - m_i^2)^{1/2} E\, dE}{\exp[(E - m_i)/T] \mp 1} = \frac{\zeta(3)}{\pi^2} T^3 \begin{pmatrix} 1 & \text{Bosons} \\ \frac{3}{4} & \text{Fermions} \end{pmatrix}$$

$$\ldots \frac{E^2}{\ldots} = \frac{\pi^2}{30} T^4 \begin{pmatrix} 1 & \text{Bosons} \\ \frac{7}{8} & \text{Fermions} \end{pmatrix}$$

$n \propto g_{\text{eff}} T^3$

$\rho \propto g_{\text{eff}} T^4$

$T \gg m_i \quad T \gg M_i$

$e^-, e^+, 3\nu, 3\bar{\nu}, \gamma$

$$\int_m^\infty \frac{(E^2 - m_i^2)^{1/2} E\,dE}{\exp[(E - m_i)/T] \mp 1} = \frac{\zeta(3)}{\pi^2} T^3 \begin{pmatrix} 1 & \text{Bosons} \\ \frac{3}{4} & \text{Fermions} \end{pmatrix}$$

$$\frac{E^2}{\quad} = \frac{\pi^2}{30} T^4 \begin{pmatrix} 1 & \text{Bosons} \\ \frac{7}{8} & \text{Fermions} \end{pmatrix}$$

$n \propto g_{\text{eff}} T^3$

$\rho \propto g_{\text{eff}} T^4$

$T \gg m_i \quad T \gg m_i$

$e^-, e^+, 3\nu, 3\bar{\nu}, \gamma$

$$\int_m^\infty \frac{(E^2 - m_i^2)^{1/2} E\, dE}{\exp[(E - m_i)/T] \mp 1} = \frac{\zeta(3)}{\pi^2} T^3 \begin{pmatrix} 1 & \text{Bosons} \\ \frac{3}{4} & \text{Fermions} \end{pmatrix}$$

$$= \frac{E^2}{\quad} = \frac{\pi^2}{30} T^4 \begin{pmatrix} 1 & \text{Bosons} \\ \frac{7}{8} & \text{Fermions} \end{pmatrix}$$

$$\left.\begin{array}{l} n \propto g_{\text{eff}} T^3 \\ \rho \propto g_{\text{eff}} T^4 \end{array}\right\}$$

$k_B = \hbar = c = 1$

$T = m = \vec{p} = E$

$L = T = E^{-1}$

$T \gg m_i \quad T \gg M_i$

$e^-, e^+, 3\nu, 3\bar{\nu}, \gamma$

$$\int_m^\infty \frac{(E^2 - m_i^2)^{1/2} E\, dE}{\exp[(E-m_i)/T] \mp 1} = \frac{\zeta(3)}{\pi^2} T^3 \begin{pmatrix} 1 & \text{Bosons} \\ \frac{3}{4} & \text{Fermions} \end{pmatrix}$$

$$\ldots \frac{E^2}{\ldots} \ldots = \frac{\pi^2}{30} T^4 \begin{pmatrix} 1 & \text{Bosons} \\ \frac{7}{8} & \text{Fermions} \end{pmatrix}$$

$$\left.\begin{array}{l} n \propto g_{\text{eff}} T^3 \\ \rho \propto g_{\text{eff}} T^4 \end{array}\right\}$$

$k_B = \hbar = c = 1$

$T = m = \vec{p} = E$

$L = T = E^{-1}$

$T \gg m_i \quad T \gg M_i$

$e^-, e^+, 3\nu, 3\bar{\nu}, \gamma$

$$\int_0 \frac{(E^2 - m_i^2)^{1/2}\, E\, dE}{\exp[(E-\mu_i)/T] \mp 1} = \frac{\zeta(3)}{\pi^2} T^3 \begin{pmatrix} 1 & \text{Bosons} \\ \frac{3}{4} & \text{Fermions} \end{pmatrix}$$

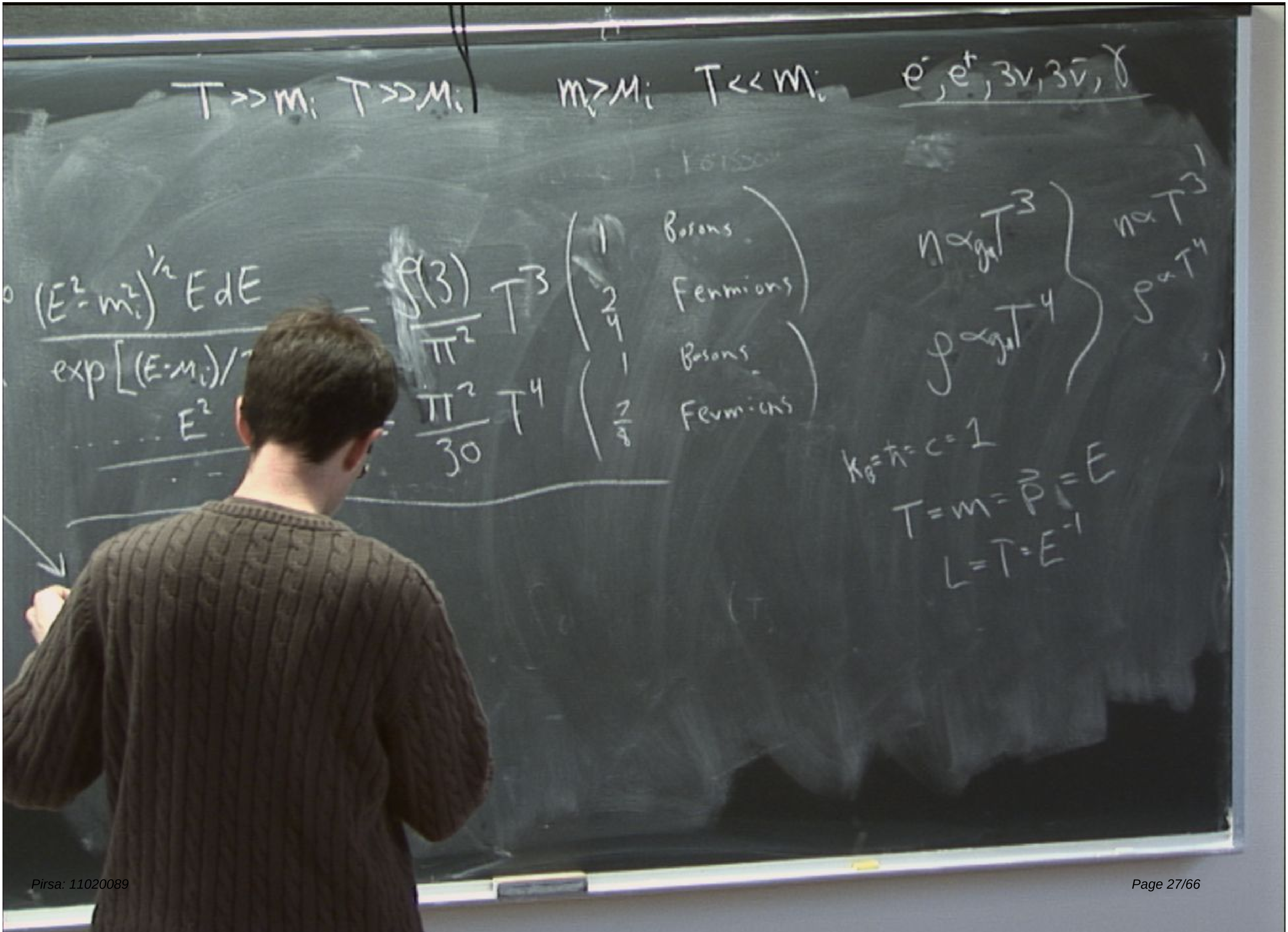
$$\frac{\cdots E^2 \cdots}{\cdots} = \frac{\pi^2}{30} T^4 \begin{pmatrix} 1 & \text{Bosons} \\ \frac{7}{8} & \text{Fermions} \end{pmatrix}$$

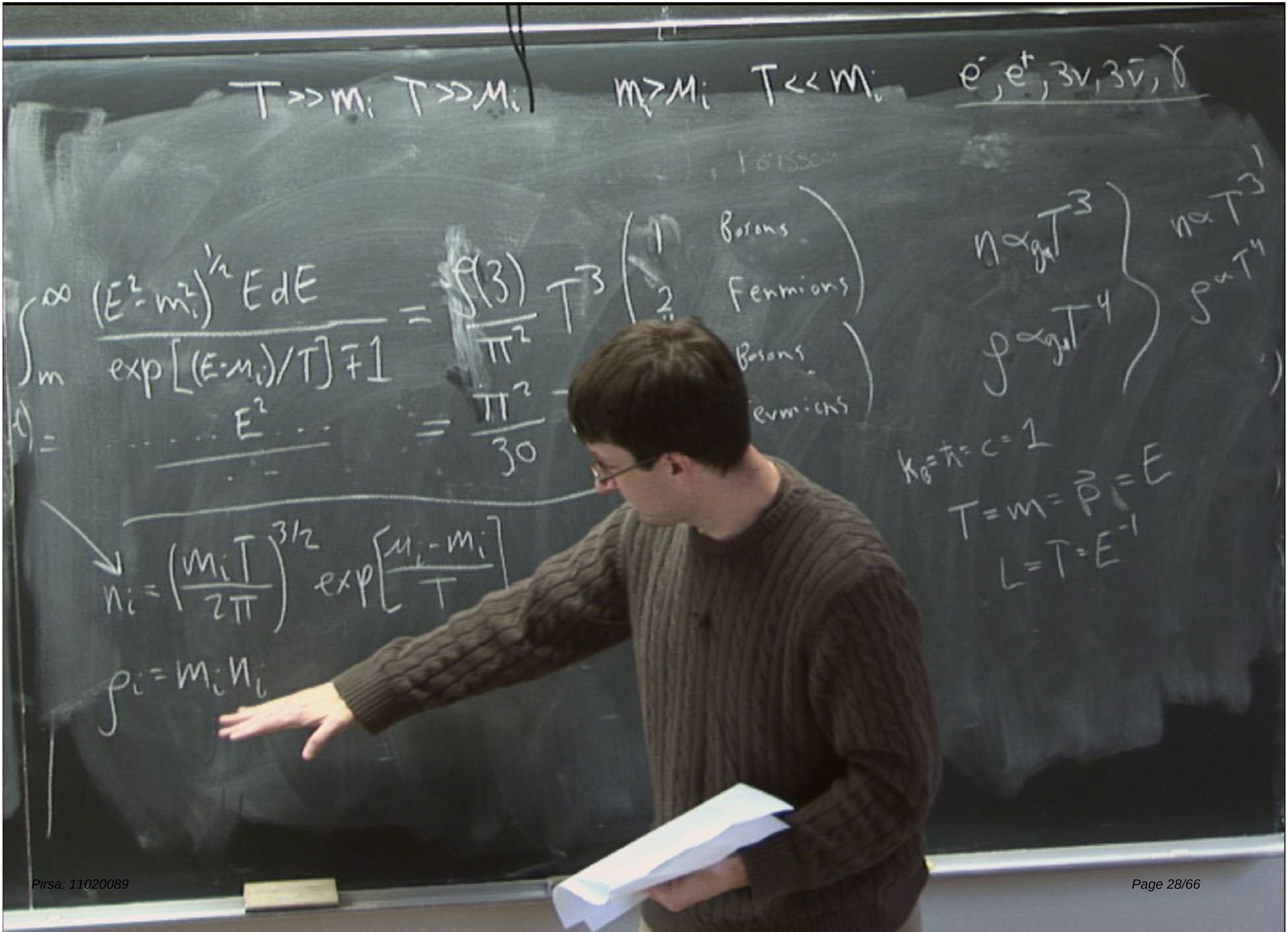
$$\left.\begin{matrix} n \propto g_* T^3 \\ \rho \propto g_* T^4 \end{matrix}\right\}$$

$k_B = \hbar = c = 1$

$T = m = p = E$

$L = T = E^{-1}$


T >> m_i T >> M_i
m_i > M_i T << M_i
e^-, e^+, 3ν, 3ν̄, γ
(E² - m_i²)^½ E dE
exp[(E - M_i)/
E²
ζ(3)/π² T³
π²/30 T⁴
1 Bosons
3/4 Fermions
1 Bosons
7/8 Fermions
n ∝ g_eff T³
ρ ∝ g_eff T⁴
n ∝ T³
ρ ∝ T⁴
k_B = ħ = c = 1
T = m = p = E
L = T = E⁻¹


$T \gg m_i \quad T \gg \mu_i$
$m_i > \mu_i \quad T \ll m_i$
$e^-, e^+, 3\nu, 3\bar{\nu}, \gamma$
$\int_m^\infty \frac{(E^2 - m_i^2)^{1/2} E\,dE}{\exp[(E-\mu_i)/T] \mp 1} = \frac{\zeta(3)}{\pi^2} T^3$
Bosons
Fermions
Bosons
Fermions
$= \frac{\pi^2}{30}$
$n \propto g_* T^3$
$\rho \propto g_* T^4$
$n \propto T^3$
$\rho \propto T^4$
$k_B = \hbar = c = 1$
$T = m = \vec{p} = E$
$L = T = E^{-1}$
$n_i = \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left[\frac{\mu_i - m_i}{T}\right]$
$\rho_i = m_i n_i$

$$\int_m^\infty \frac{(E^2-m_i^2)^{1/2}\, E\, dE}{\exp\left[(E-m_i)/T\right] \mp 1} = \frac{\zeta(3)}{\pi^2} T^3 \begin{pmatrix} 1 & \text{Bosons} \\ \frac{3}{4} & \text{Fermions} \end{pmatrix}$$

$$= \frac{\dots E^2 \dots}{\dots} = \frac{\pi^2}{30} T^4 \begin{pmatrix} 1 & \text{Bosons} \\ \frac{7}{8} & \text{Fermions} \end{pmatrix}$$

$$\left.\begin{matrix} n \propto g_{\rm eff} T^3 \\ \rho \propto g_{\rm eff} T^4 \end{matrix}\right\} \begin{matrix} n \propto T^3 \\ \rho \propto T^4 \end{matrix}$$

$$k_B = \hbar = c = 1$$

$$T = m = \vec{p} = E$$

$$L = T = E^{-1}$$

$$n_i = \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left[\frac{\mu_i - m_i}{T}\right]$$

$$\rho_i = m_i n_i$$

$$\rho \propto a^{-4}$$

$$\int_m^\infty \frac{(E^2-m_i^2)^{1/2} E\,dE}{\exp[(E-\mu_i)/T] \mp 1} = \frac{\zeta(3)}{\pi^2} T^3 \begin{pmatrix} 1 & \text{Bosons} \\ \frac{3}{4} & \text{Fermions} \end{pmatrix}$$

$$= \frac{E^2}{\quad} = \frac{\pi^2}{30} T^4 \begin{pmatrix} 1 & \text{Bosons} \\ \frac{7}{8} & \text{Fermions} \end{pmatrix}$$

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$$k_B = \hbar = c = 1$$

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$$L = T = E^{-1}$$

$$n_i = \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left[\frac{\mu_i - m_i}{T}\right]$$

$$\rho_i = m_i n_i$$

$\rho \propto a^{-4}$ $T \propto \frac{1}{a}$

$$\int_m^\infty \frac{(E^2 - m_i^2)^{1/2} E\,dE}{\exp[(E-\mu_i)/T] \mp 1} = \frac{\zeta(3)}{\pi^2} T^3 \begin{pmatrix} 1 & \text{Bosons} \\ \frac{3}{4} & \text{Fermions} \end{pmatrix}$$

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$k_B = \hbar = c = 1$

$T = m = \vec{p}_i = E$

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$$n_i = \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left[\frac{\mu_i - m_i}{T}\right]$$

$$\rho_i = m_i n_i$$

$\rho \propto a^{-4}$ $\boxed{T \propto \frac{1}{a}}$

$$\frac{(E^2 - m_i^2)^{1/2} E\,dE}{\exp[(E - \mu_i)/T] \mp 1} = \frac{\zeta(3)}{\pi^2} T^3 \begin{pmatrix} 1 & \text{Bosons} \\ \frac{3}{4} & \text{Fermions} \end{pmatrix}$$

$$\frac{E^2}{\ldots} = \frac{\pi^2}{30} T^4 \begin{pmatrix} 1 & \text{Bosons} \\ \frac{7}{8} & \text{Fermions} \end{pmatrix}$$

$$\left.\begin{array}{l} n \propto g_{*} T^3 \\ \rho \propto g_{*} T^4 \end{array}\right\} \quad \begin{array}{l} n \propto T^3 \\ \rho \propto T^4 \end{array}$$

$k_B = \hbar = c = 1$

$T = m = p = E$

$L = T = E^{-1}$

$$n_i = \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left[\frac{\mu_i - m_i}{T}\right]$$

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$$\left.\begin{matrix} n \propto g_{\text{eff}} T^3 \\ \rho \propto g_{\text{eff}} T^4 \end{matrix}\right\} \quad \begin{matrix} n \propto T^3 \\ \rho \propto T^4 \end{matrix}$$

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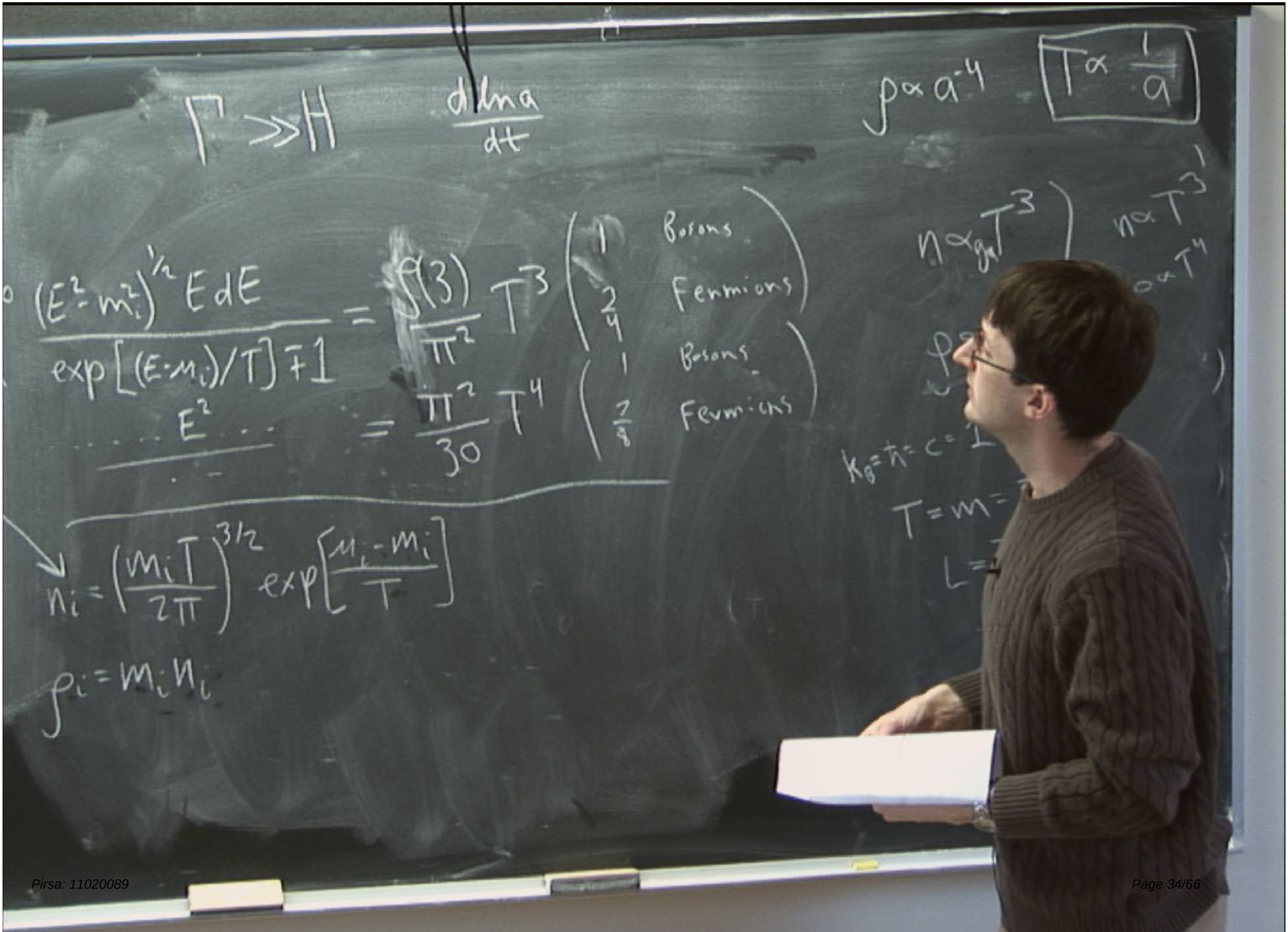
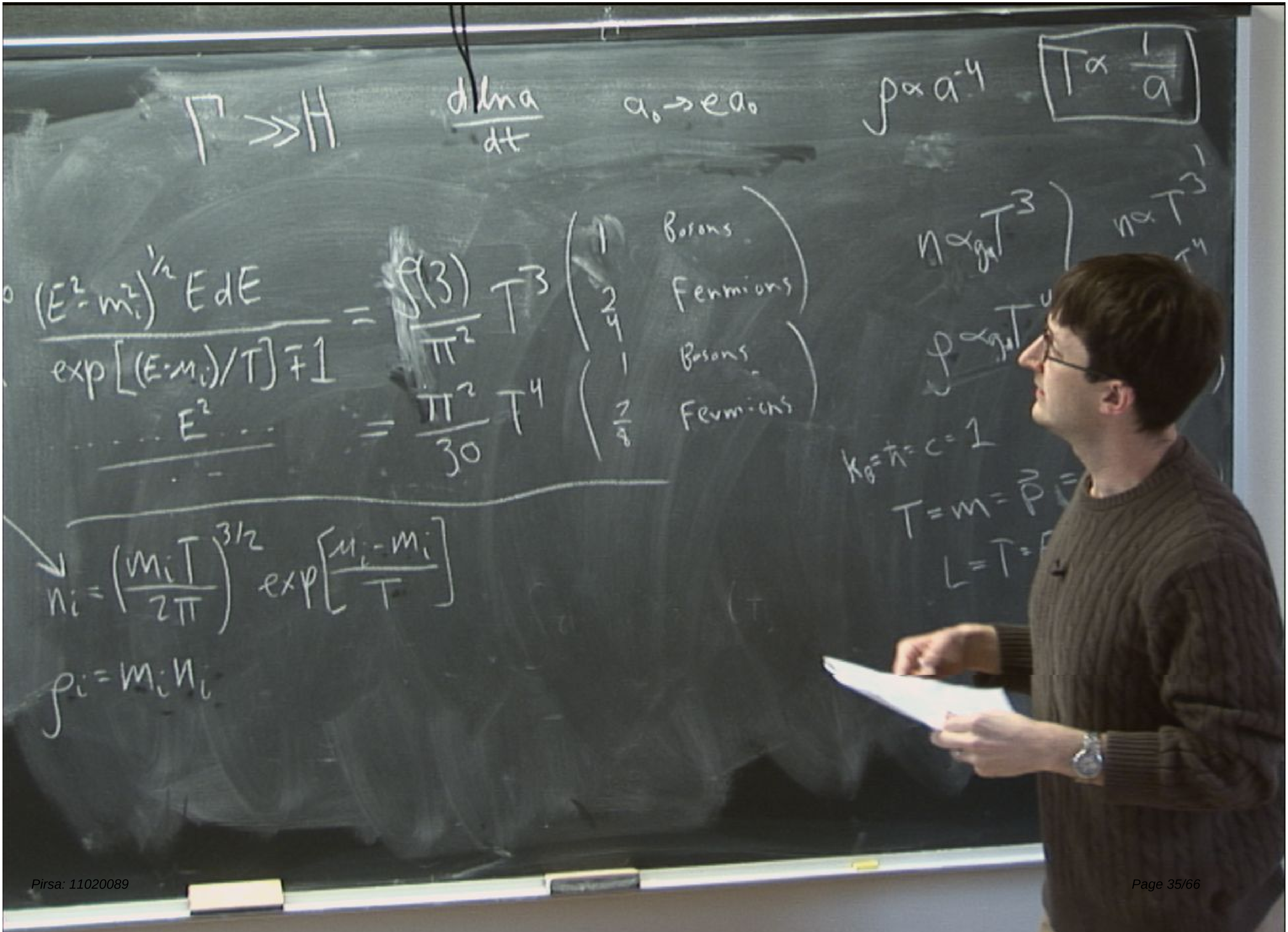


Γ >> H
d ln a / dt
ρ ∝ a^-4
T ∝ 1/a
Bosons
Fermions
Bosons
Fermions
n ∝ T^3
ζ(3)/π^2 T^3
π^2/30 T^4
exp[(E-m_i)/T] ∓ 1
n_i = (m_i T / 2π)^3/2 exp[(μ_i - m_i)/T]
ρ_i = m_i n_i
k_B = ħ = c = 1


Γ >> H
d ln a / dt
a₀ → e a₀
ρ ∝ a⁻⁴
T ∝ 1/a
Bosons
Fermions
Bosons
Fermions
n ∝ T³
k_B = ħ = c = 1
T = m =
L = T =
(E² - m²ᵢ)^½ E dE / exp[(E - mᵢ)/T] ∓ 1
= ζ(3)/π² T³
= π²/30 T⁴
nᵢ = (mᵢT/2π)^3/2 exp[(μᵢ - mᵢ)/T]
ρᵢ = mᵢ nᵢ

$\Gamma \gg H$ $\quad \frac{d\ln a}{dt}$ $\quad a_0 \to e a_0$ $\quad \rho \propto a^{-4}$ $\quad \boxed{T \propto \frac{1}{a}}$

$$\frac{(E^2 - m_i^2)^{1/2} E\,dE}{\exp[(E-\mu_i)/T] \mp 1} = \frac{\zeta(3)}{\pi^2} T^3 \begin{pmatrix} 1 & \text{Bosons} \\ \frac{3}{4} & \text{Fermions} \end{pmatrix}$$

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$$n \propto g_{\text{eff}} T^3$$

$$\rho \propto g_{\text{eff}} T^4$$

$$\rho \propto T$$

$$k_B = \hbar = c = 1$$

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$$\Gamma \gg H \qquad \frac{d\ln a}{dt} \qquad a_0 \to e\,a_0 \qquad \rho \propto a^{-4} \qquad \boxed{T \propto \frac{1}{a}}$$

$$\frac{(E^2 - m_i^2)^{1/2} E\,dE}{\exp[(E - \mu_i)/T] \mp 1} = \frac{\zeta(3)}{\pi^2} T^3 \begin{pmatrix} 1 & \text{Bosons} \\ \frac{3}{4} & \text{Fermions} \end{pmatrix}$$

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$T \gg m_\chi$

$T \lesssim m_\chi$

$\propto a^{-3(1+w)}$

$\rho \propto a^{-3} E \qquad E^2 = \vec{p}^2 + m^2$

$\rho \propto a^{-4} \quad (w = \frac{1}{3})$

$\rho \propto a^{-3} \quad (w = 0) \qquad 4\pi p^2 dp$

$n_i(\vec{x}, t) = \int d^3\vec{p}\, f_i(\vec{x}, \vec{p}, t) = \frac{1}{2\pi^2}$

$\rho_i(\vec{x}, t) = \int d^3\vec{p}\, f_i(\vec{x}, \vec{p}, t)\, E(\vec{p}^2 \ldots)$

$\frac{1}{\exp\left[\frac{E - \mu_i}{T}\right] \mp 1} \qquad E^2 = \vec{p}^2 + m_i^2$

$$\Gamma \gg H \qquad \frac{d\ln a}{dt} \qquad a_0 \to e a_0 \qquad \rho \propto a^{-4} \qquad \boxed{T \propto \frac{1}{a}}$$

$$\int_m^\infty \frac{(E^2-m_i^2)^{1/2} E\,dE}{\exp[(E-m_i)/T] \mp 1} = \frac{\zeta(3)}{\pi^2} T^3 \begin{pmatrix} 1 & \text{Bosons} \\ \frac{3}{4} & \text{Fermions} \end{pmatrix}$$

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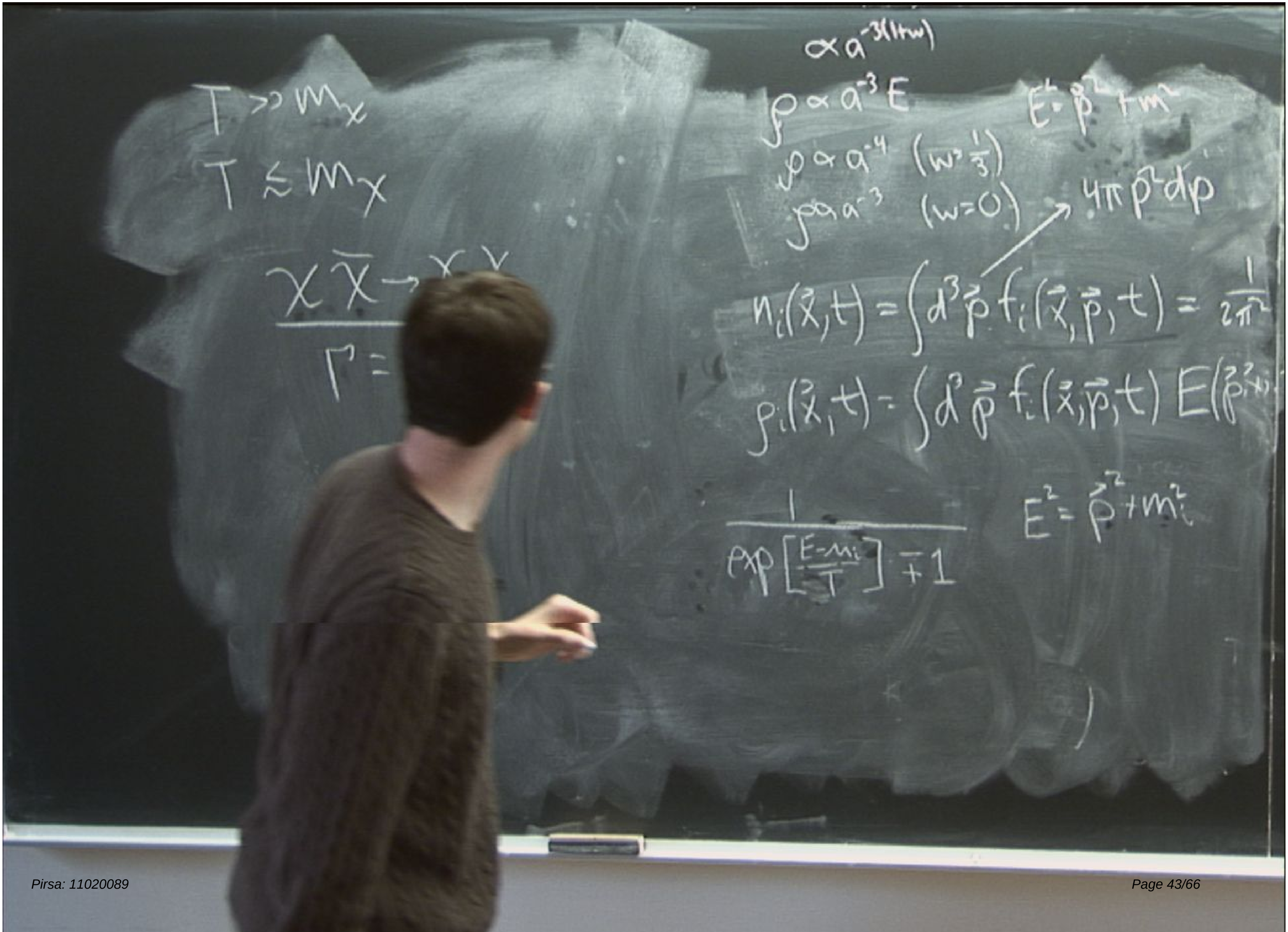
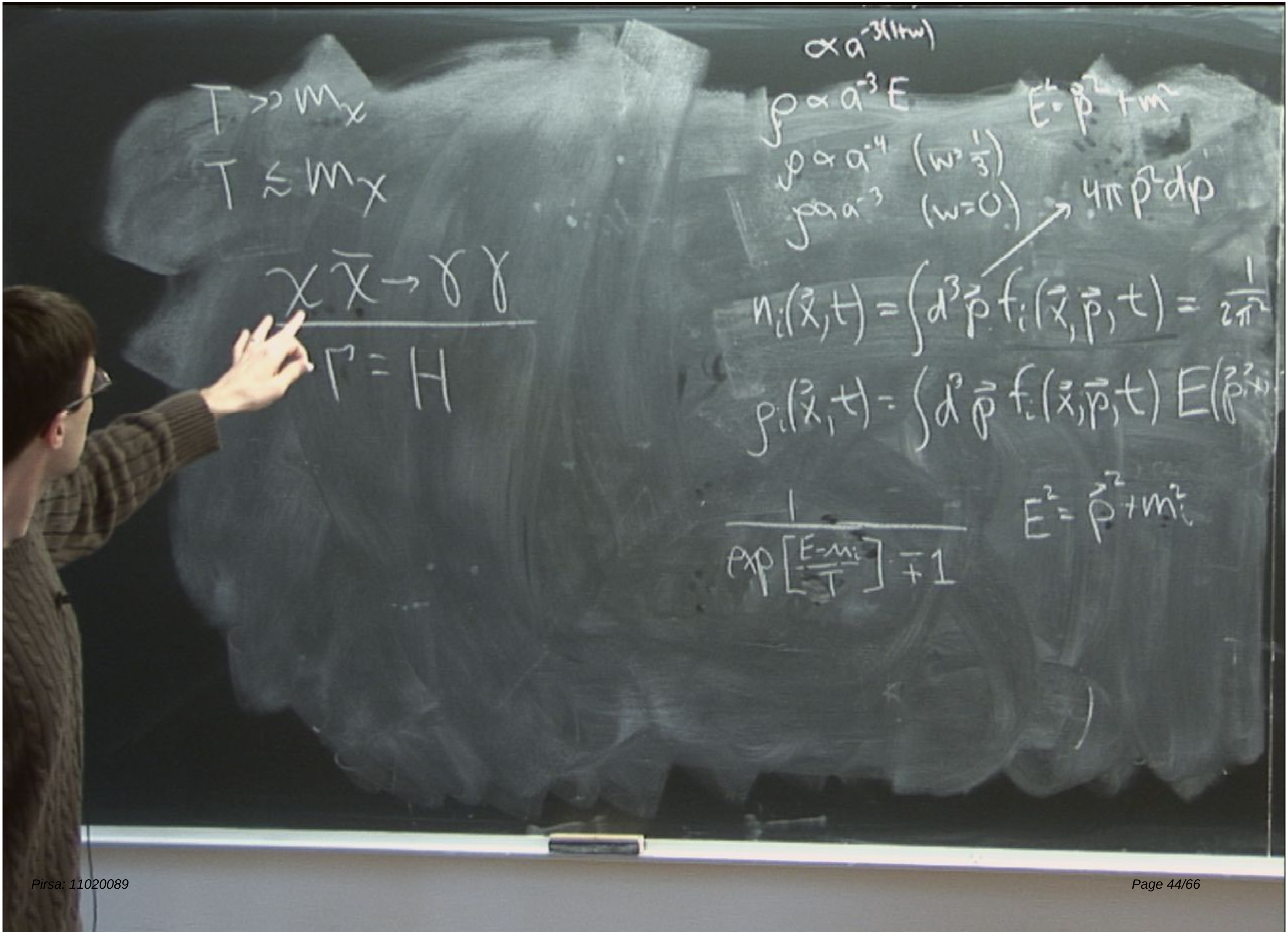

$\propto a^{-3(1+w)}$
$T \gg m_\chi$
$T \lesssim m_\chi$
$\chi\bar{\chi} \to$
$\Gamma =$
$\rho \propto a^{-3} E$
$\rho \propto a^{-4} \quad (w = \frac{1}{3})$
$\rho \propto a^{-3} \quad (w = 0)$
$E^2 = \vec{p}^2 + m^2$
$4\pi p^2 dp$
$n_i(\vec{x},t) = \int d^3\vec{p}\, f_i(\vec{x},\vec{p},t) = \frac{1}{2\pi^2}$
$\rho_i(\vec{x},t) = \int d^3\vec{p}\, f_i(\vec{x},\vec{p},t)\, E(\vec{p}^2$
$\frac{1}{\exp\left[\frac{E-\mu_i}{T}\right] \mp 1}$
$E^2 = \vec{p}^2 + m_i^2$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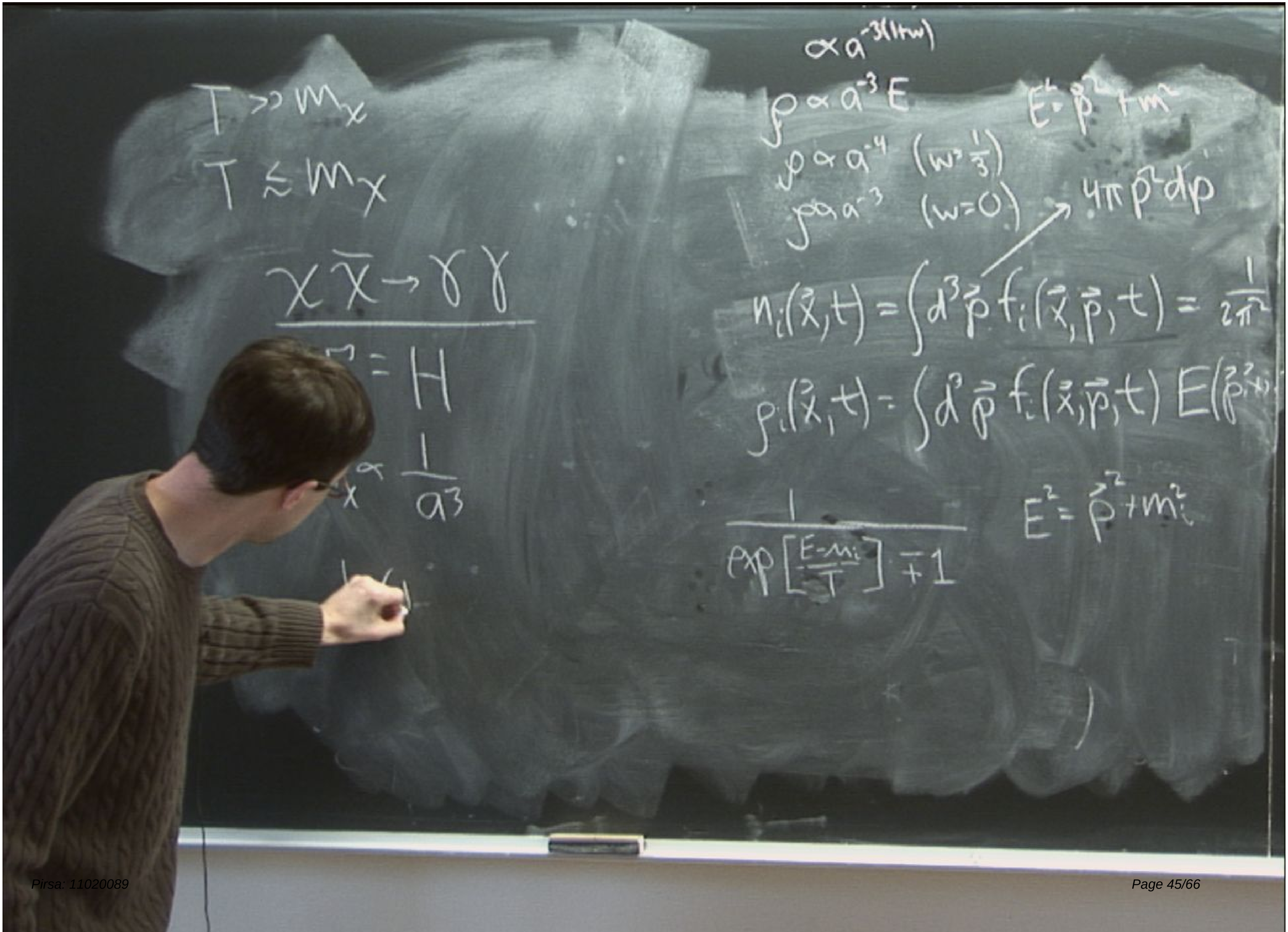
$E^2 = \vec{p}^2 + m_i^2$



$T \gg m_\chi$
$T \lesssim m_\chi$
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$\Gamma = H$
$n_\chi \propto \frac{1}{a^3}$
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WIMP

Miracle

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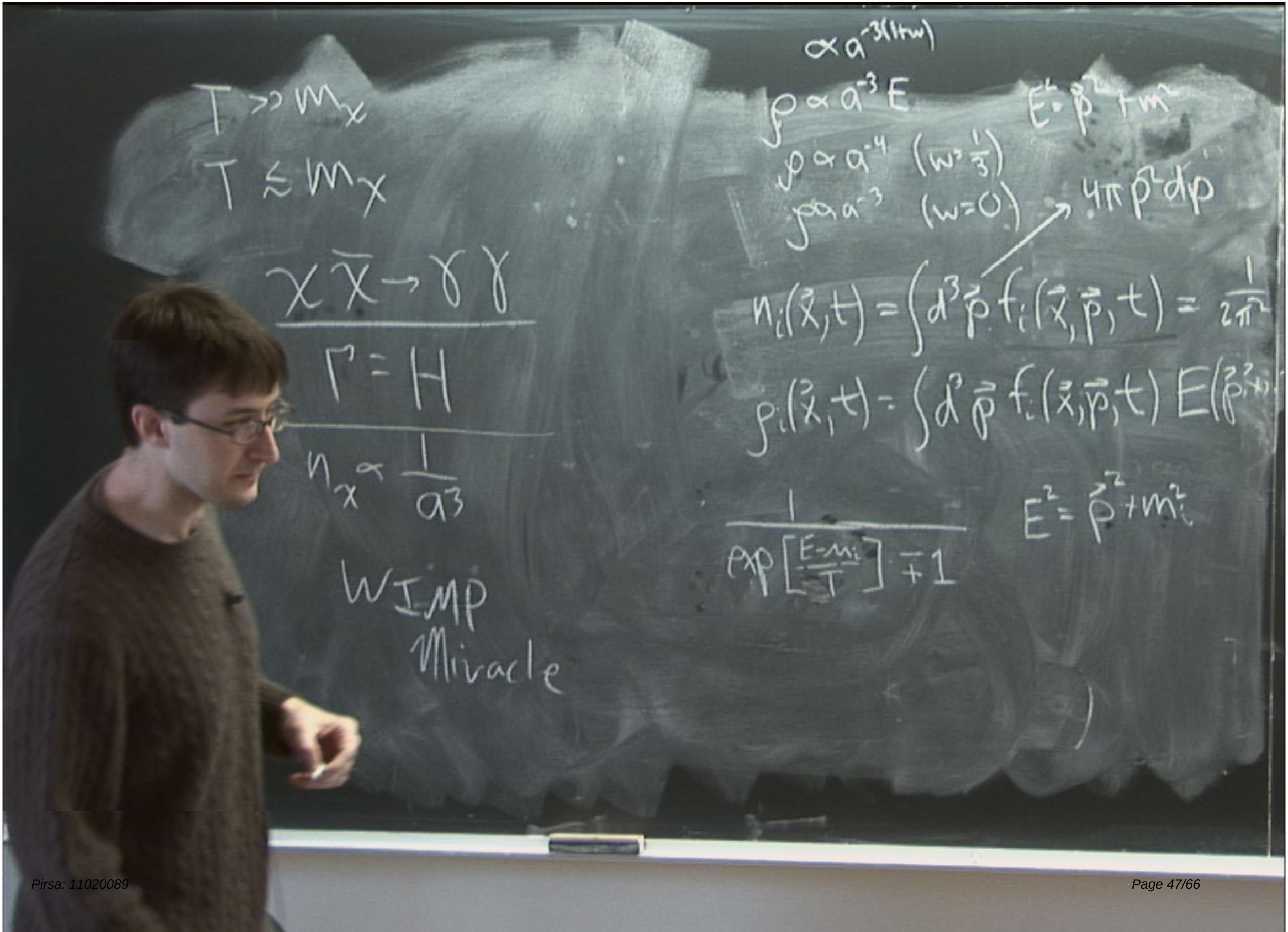

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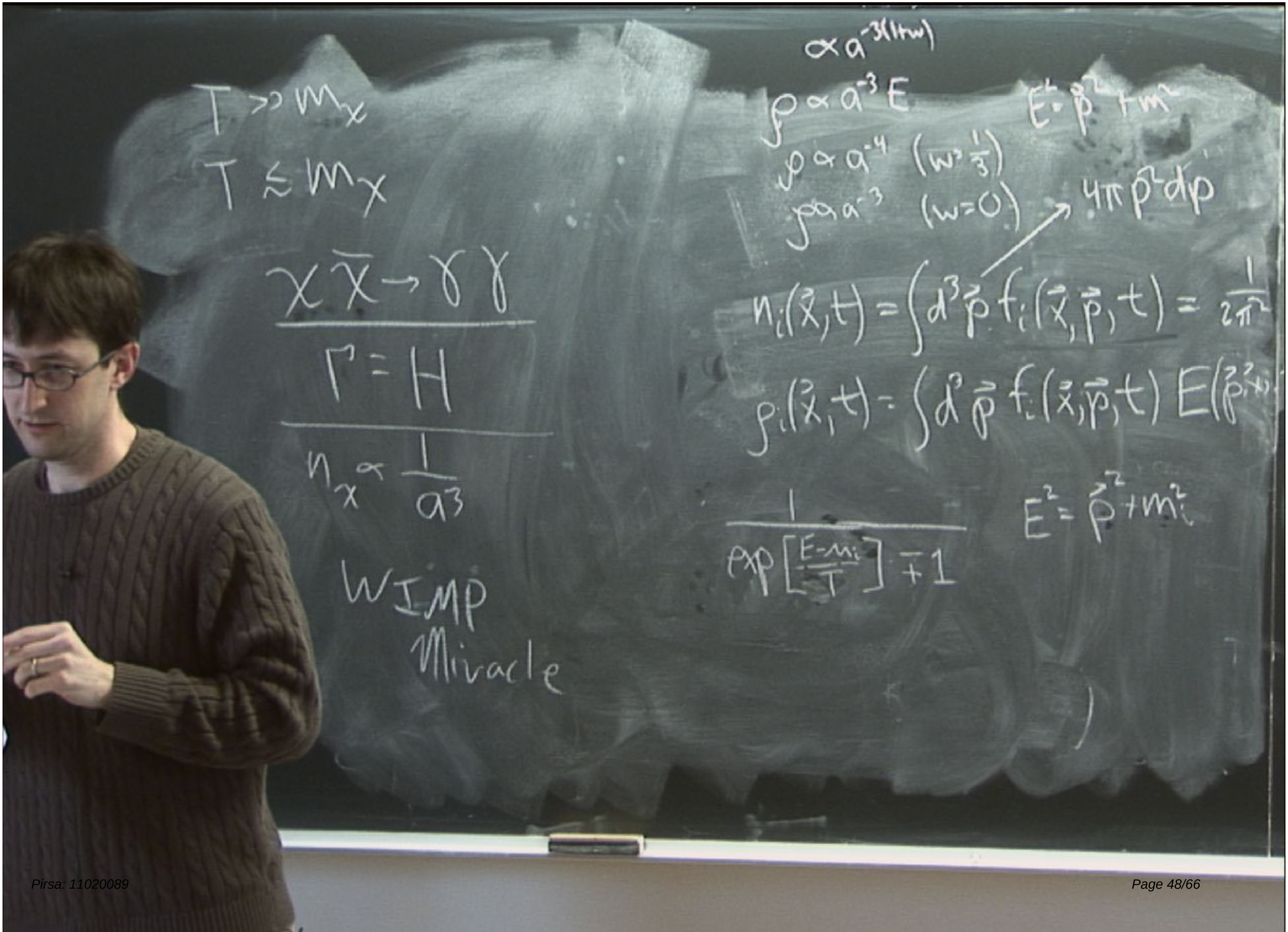
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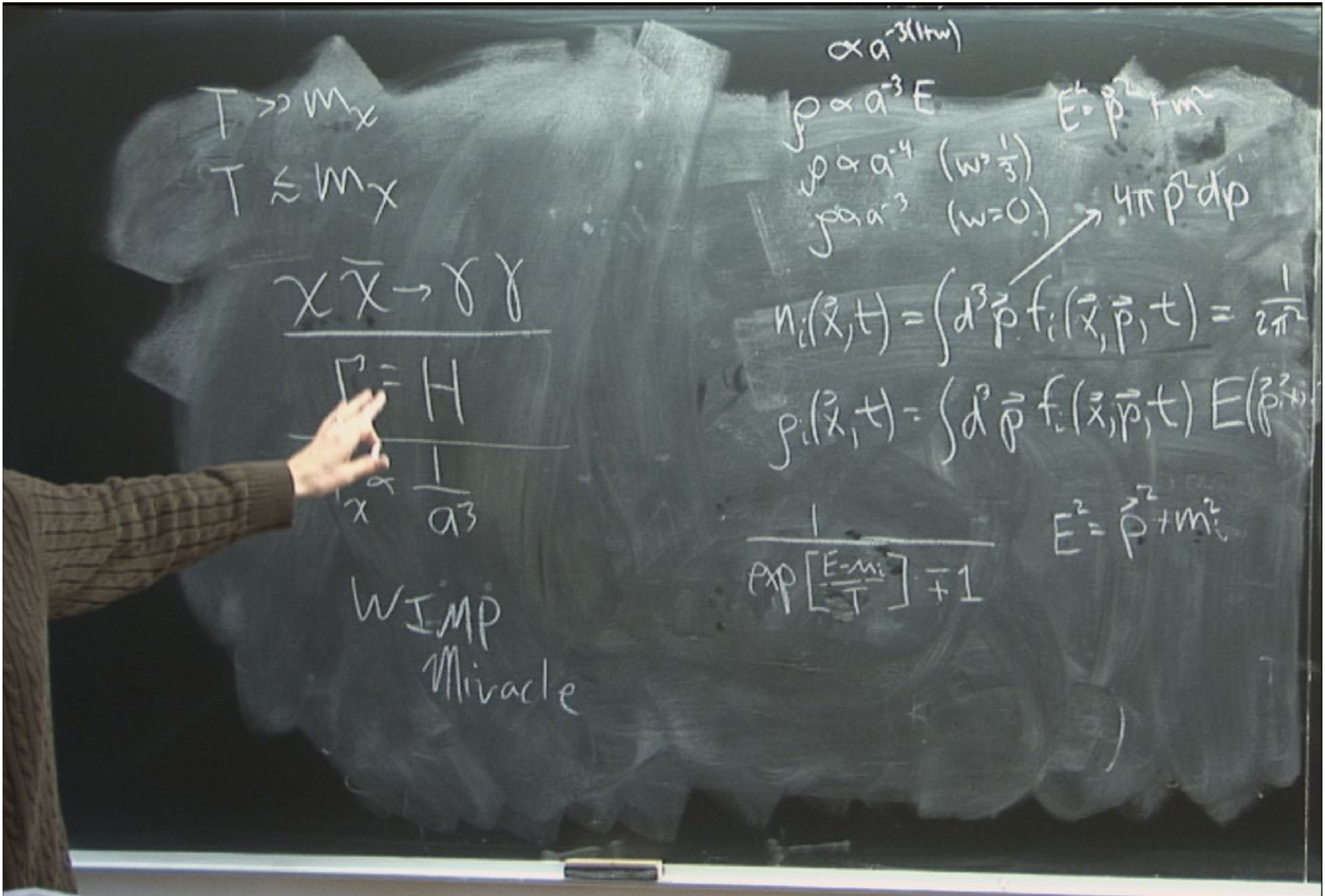
$T \gg m_\chi$
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Miracle

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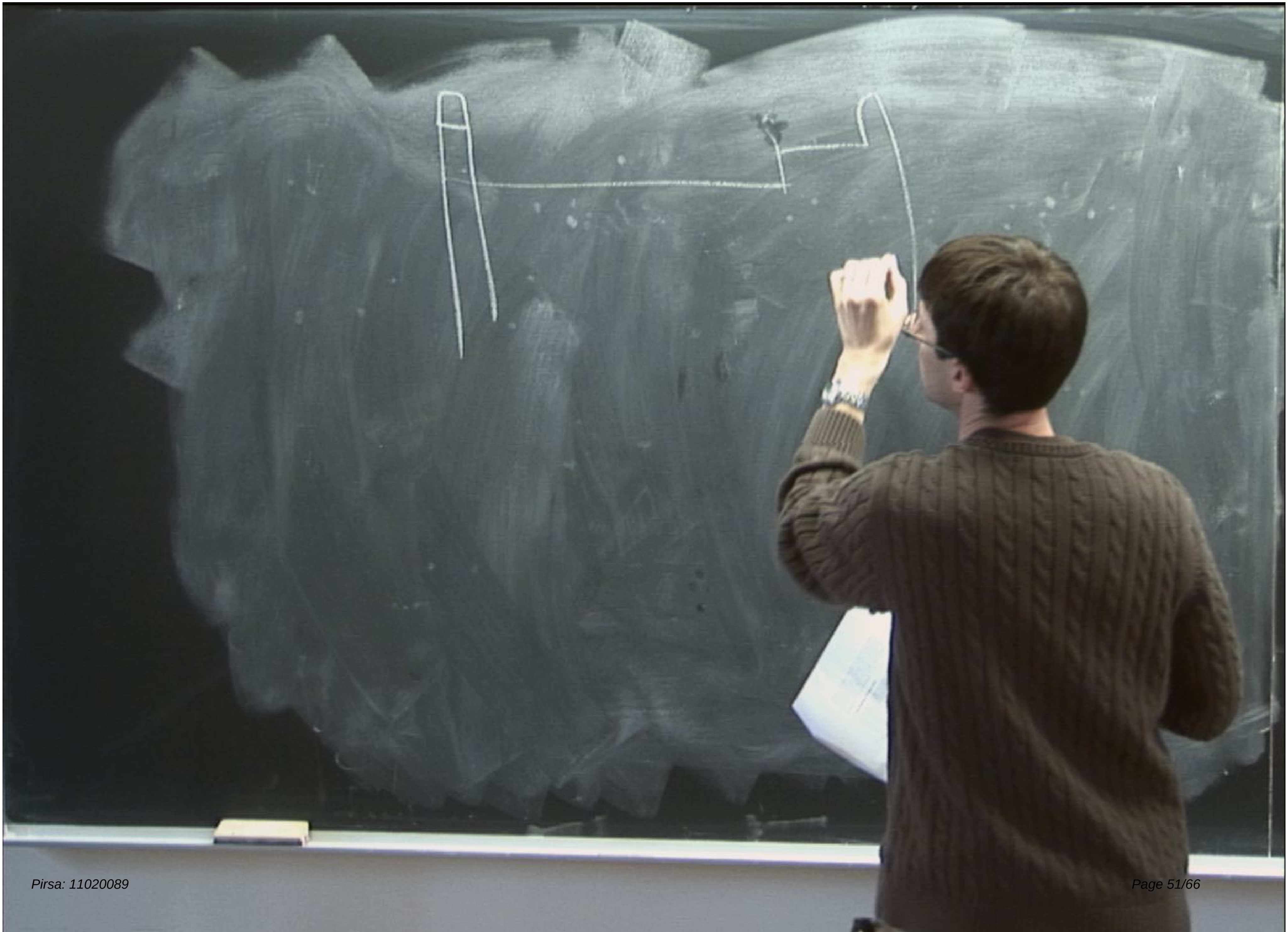
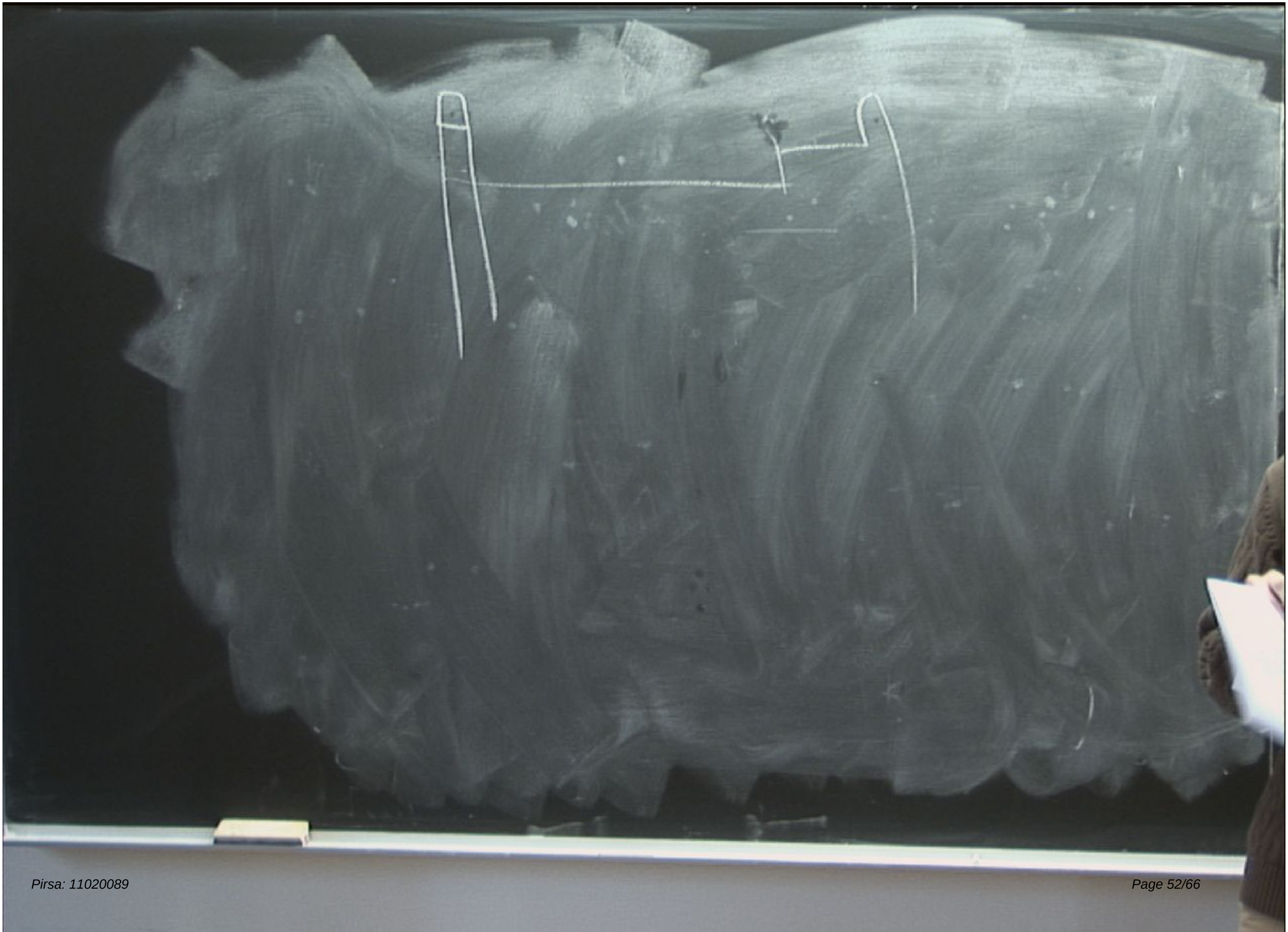
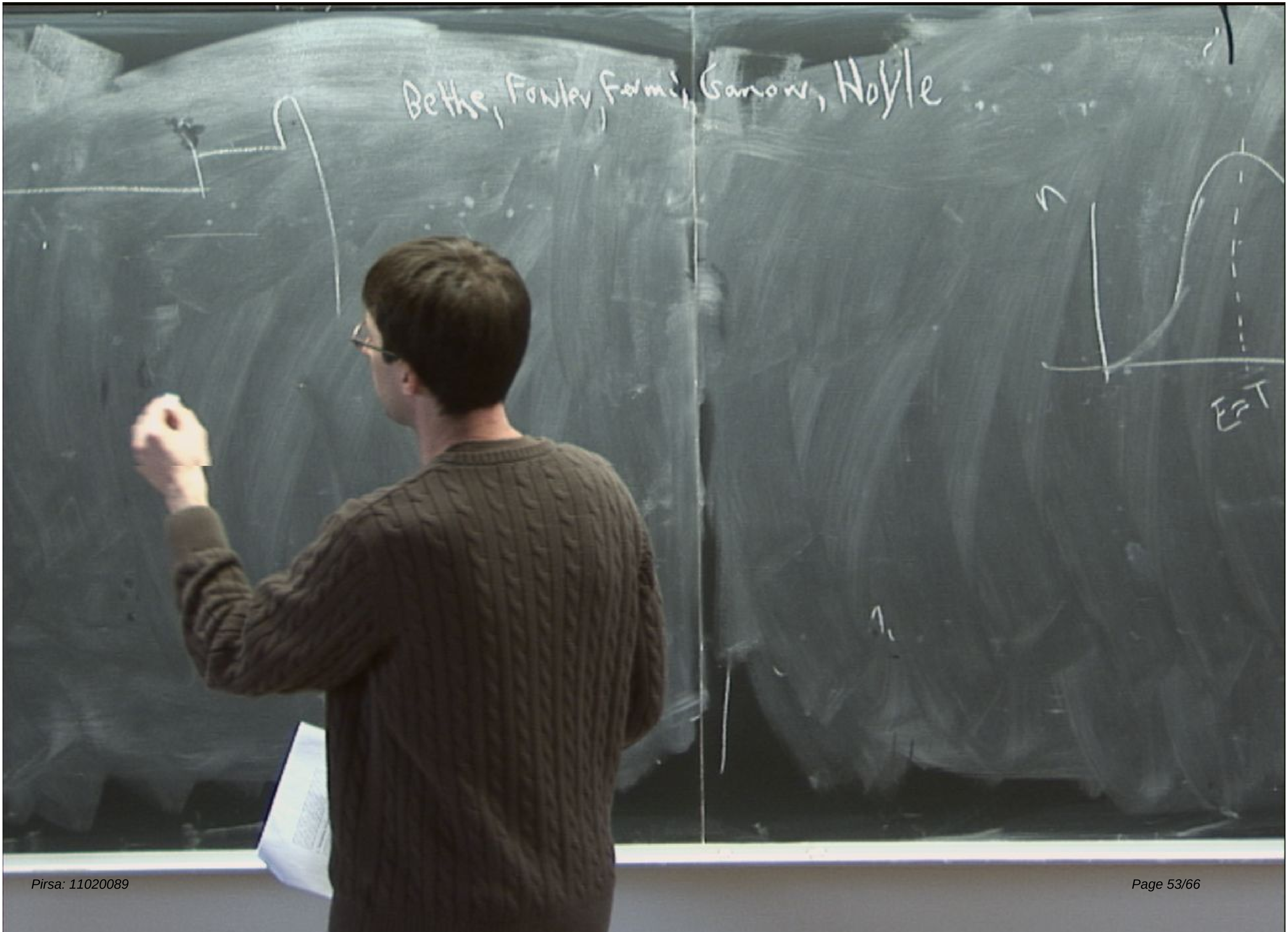
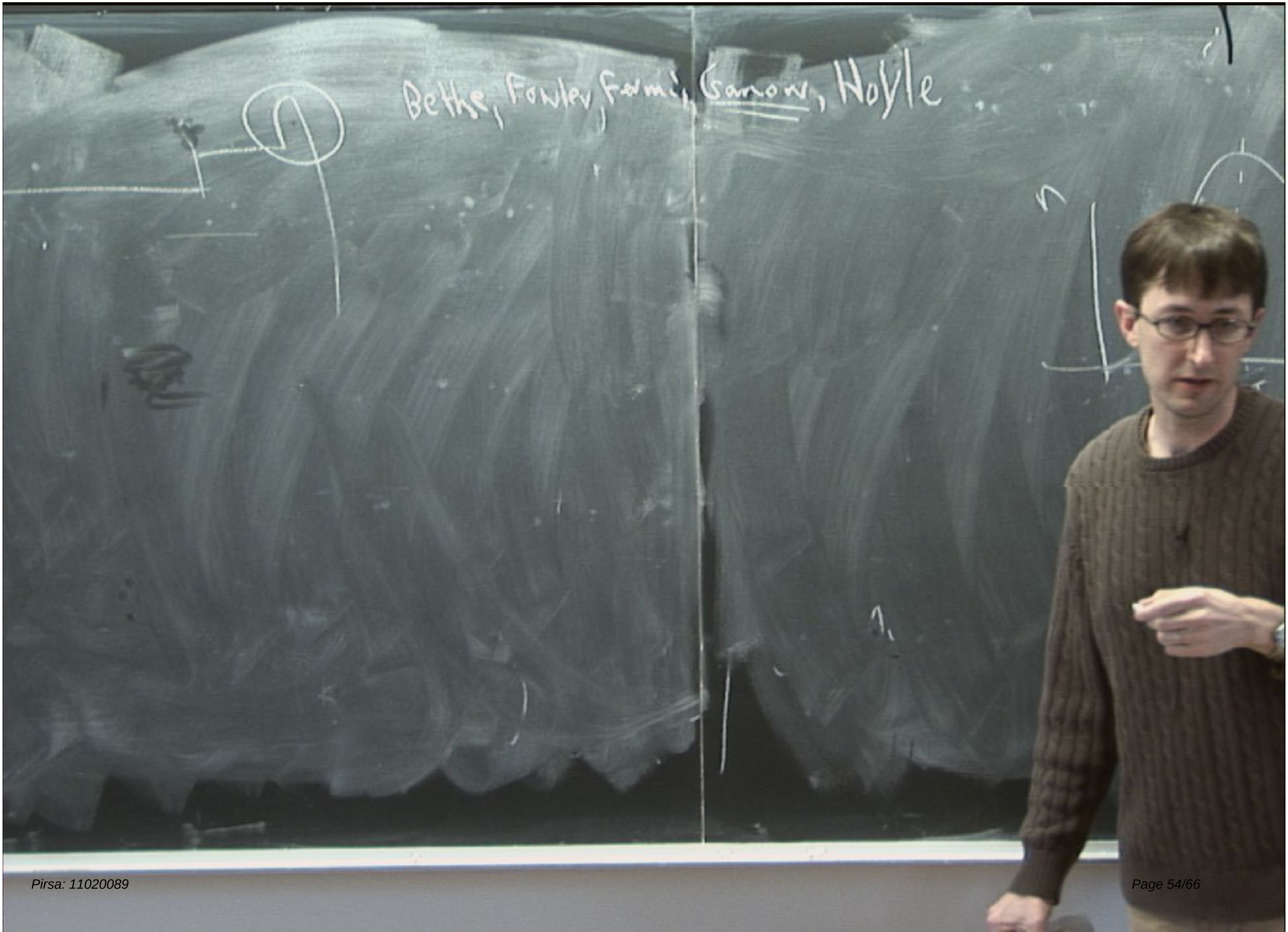





Bethe, Fowler, Fermi, Gamow, Hoyle
E≈T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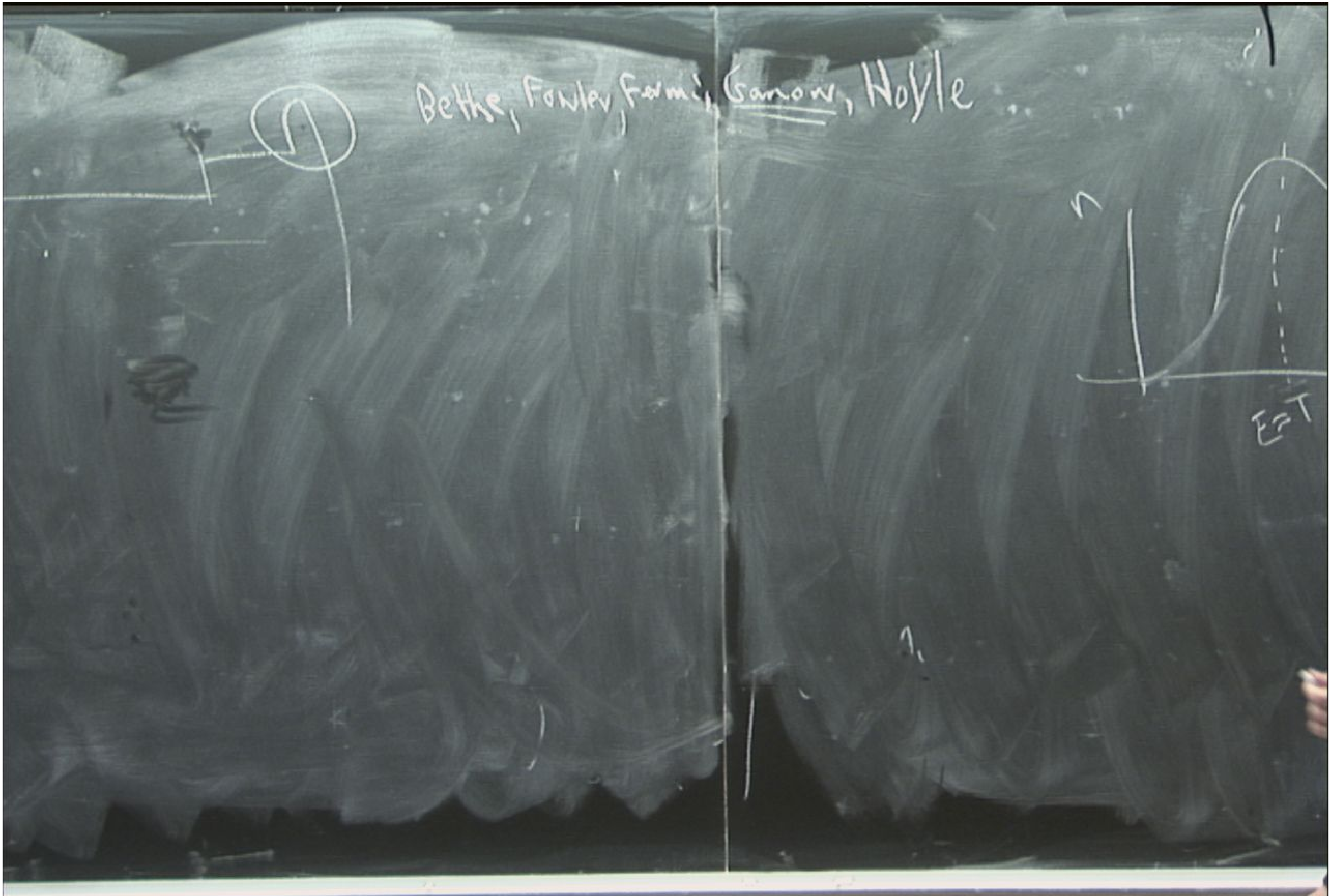
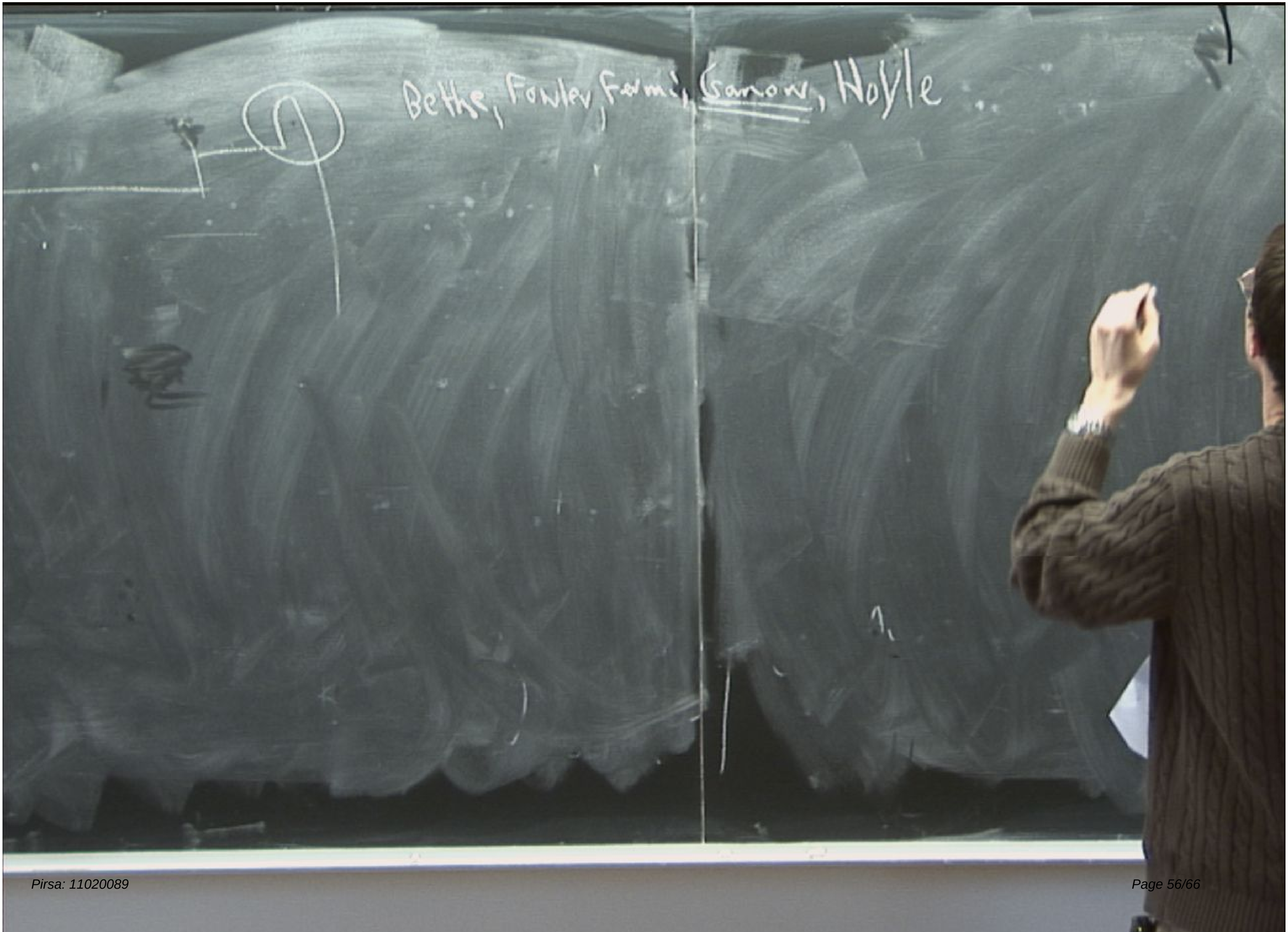

Bethe, Fowler, Fermi, Gamow, Hoyle
n
E≈T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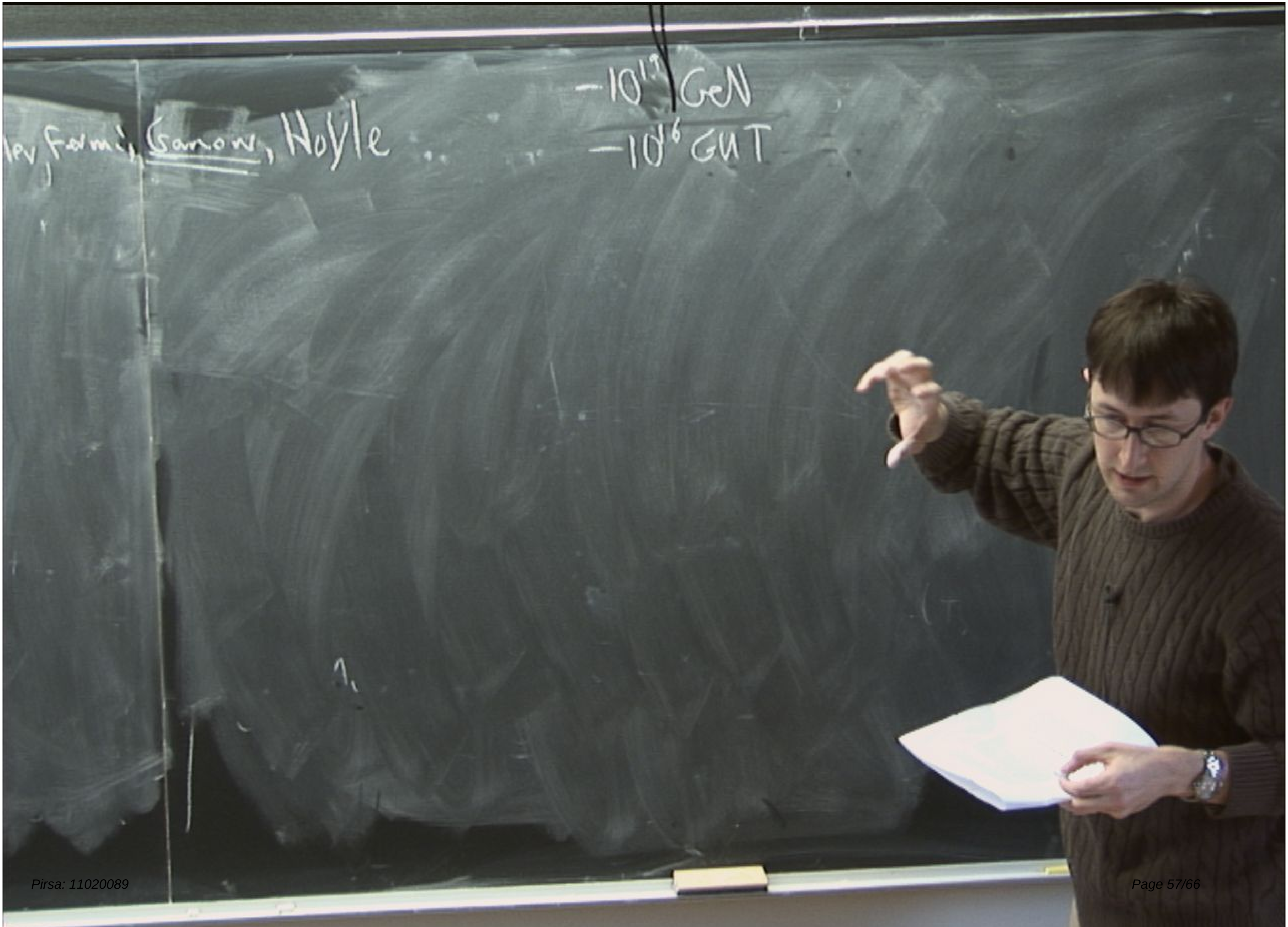
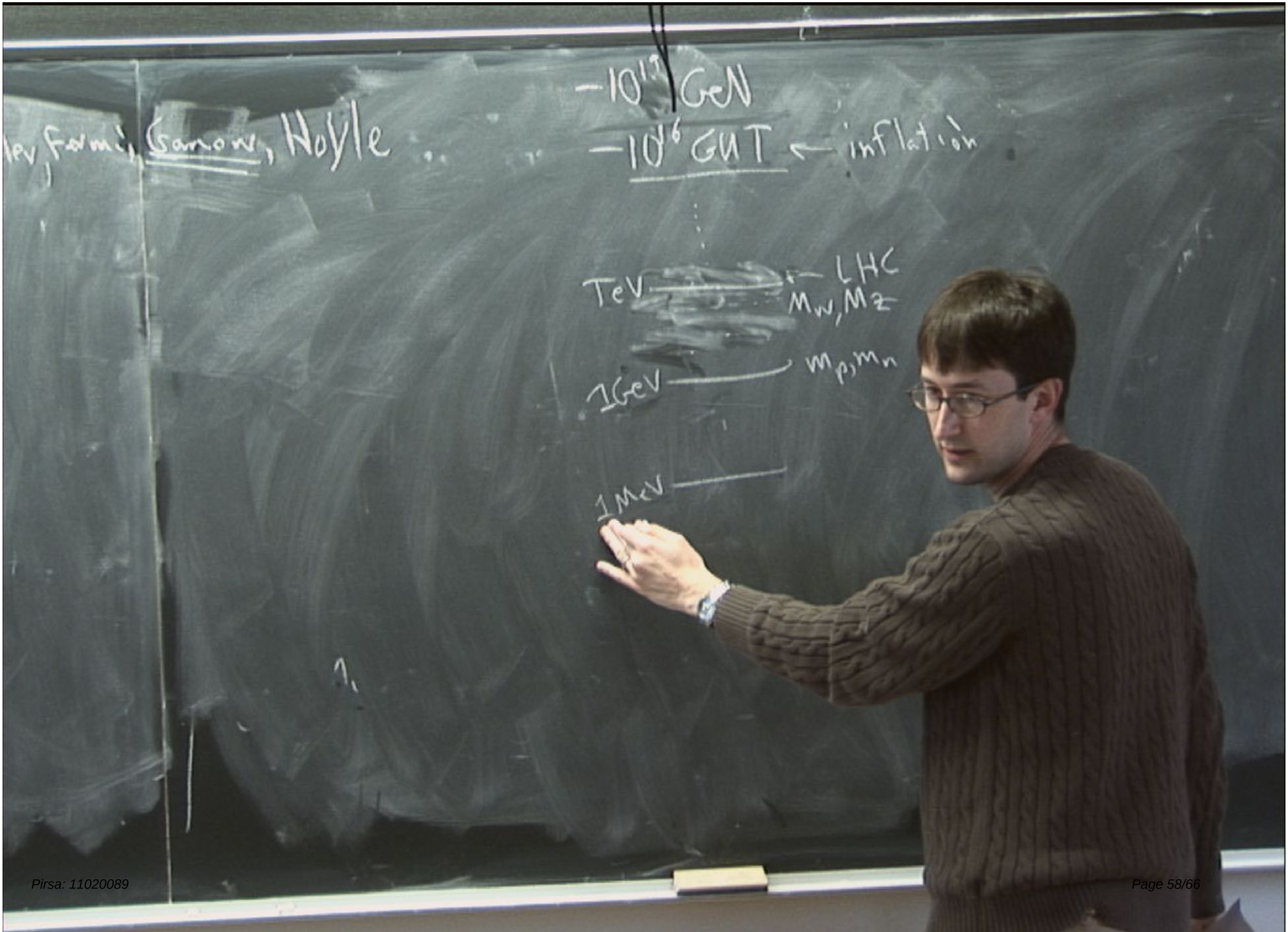

Fermi, Gamow, Hoyle
-10^{19} GeV
-10^{16} GUT


ev, Fermi, Gamow, Hoyle
10^{19} GeV
10^{16} GUT ← inflation
TeV
LHC
M_W, M_Z
1 GeV
m_p, m_n
1 MeV

lev, Fermi, Gamow, Hoyle

$- 10^{19}$ GeV

$- 10^{16}$ GUT ← inflation

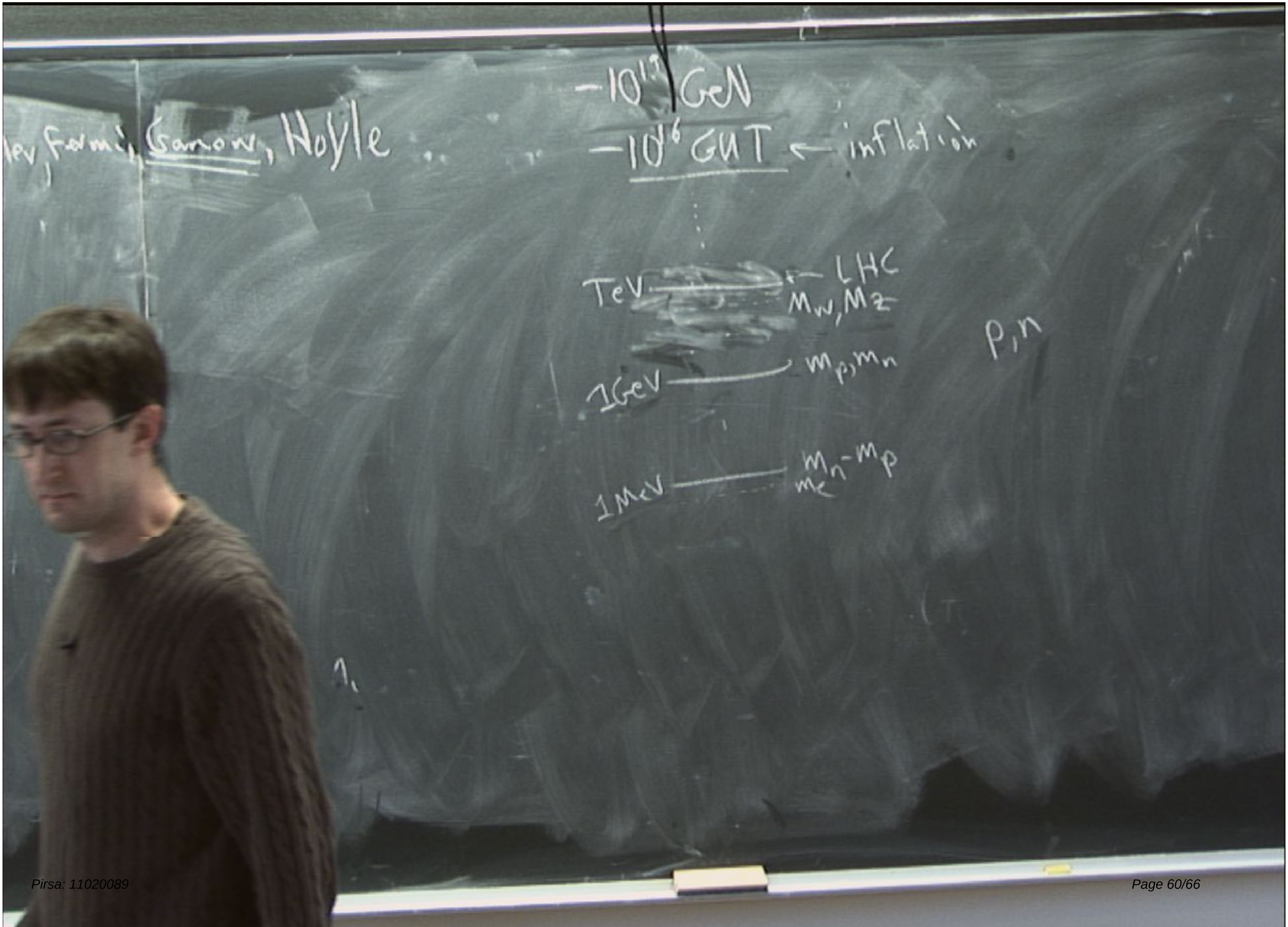
TeV ← LHC

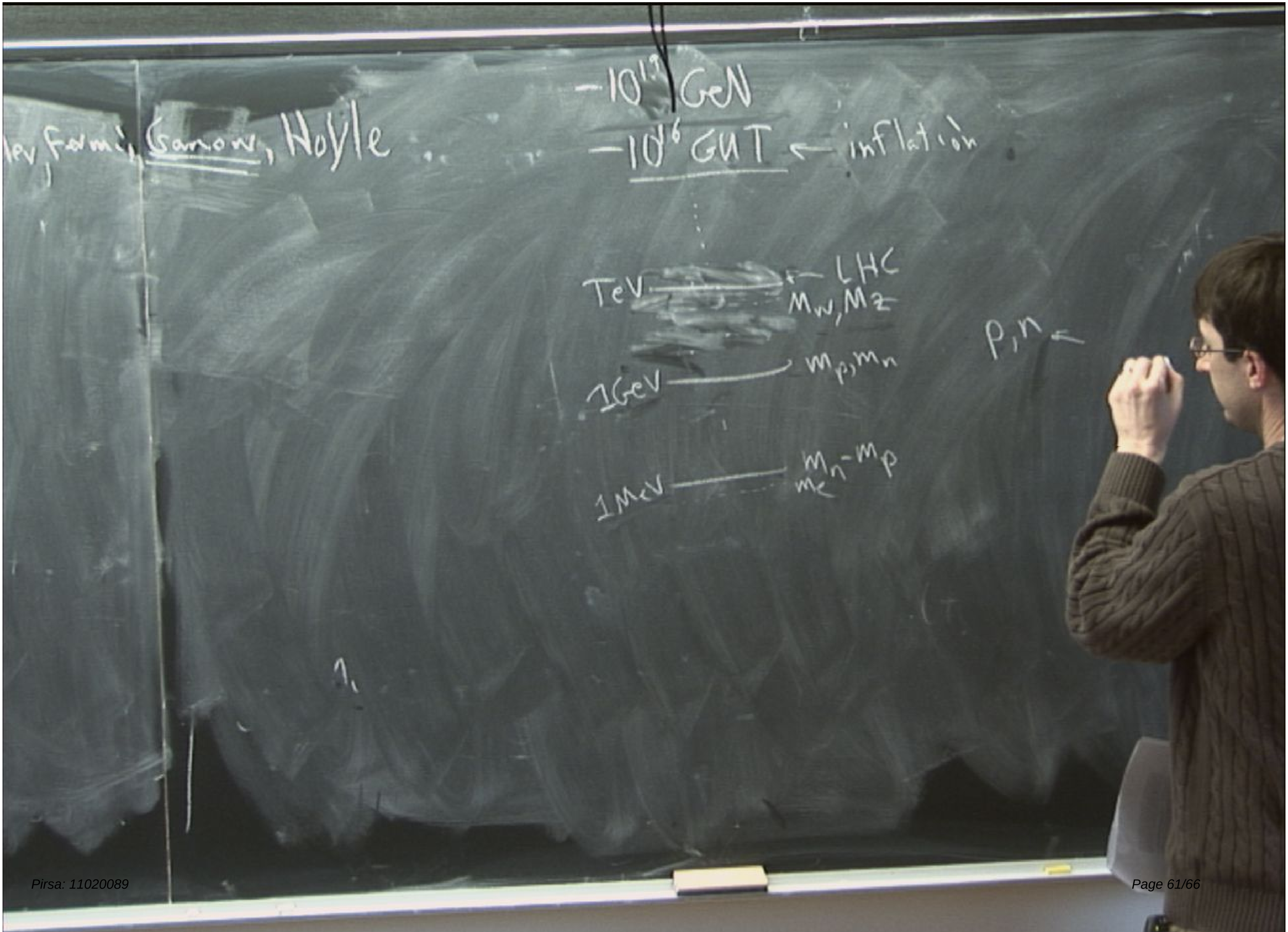
M_W, M_Z

1 GeV m_p, m_n

1 MeV $m_n - m_p$

m_e


Fermi, Gamow, Hoyle
GeV
10^16 GUT ← inflation
TeV
LHC
M_W, M_Z
p, n
m_p, m_n
1GeV
m_n - m_p
m_e
1MeV


lev, Fermi, Gamow, Hoyle
10^19 GeV
10^16 GUT ← inflation
TeV ← LHC
M_W, M_Z
p, n ←
1GeV m_p, m_n
1MeV m_n - m_p
m_e

lev, Fermi, Gamow, Hoyle ...

10^{19} GeV

10^{16} GUT ← inflation

TeV ← LHC

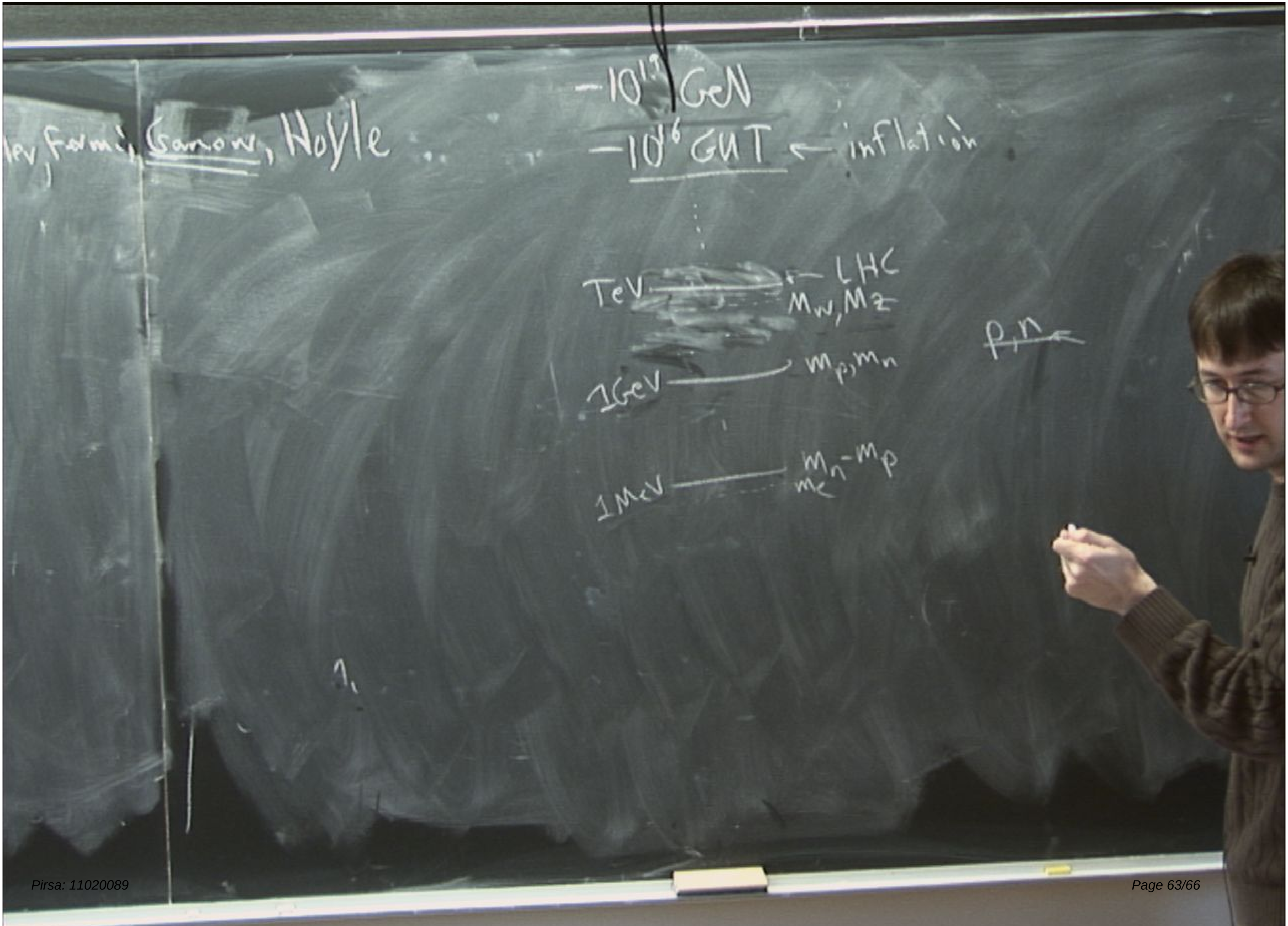
M_W, M_Z

1 GeV m_p, m_n

p, n ←

1 MeV $m_n - m_p$

m_e


Fermi, Gamow, Hoyle
10^19 GeV
10^16 GUT ← inflation
TeV ← LHC
M_W, M_Z
1 GeV
m_p, m_n
1 MeV
m_n - m_p
m_e
p, n

lev, Fermi, Gamow, Hoyle

— 10^{19} GeV

— 10^{16} GUT ← inflation

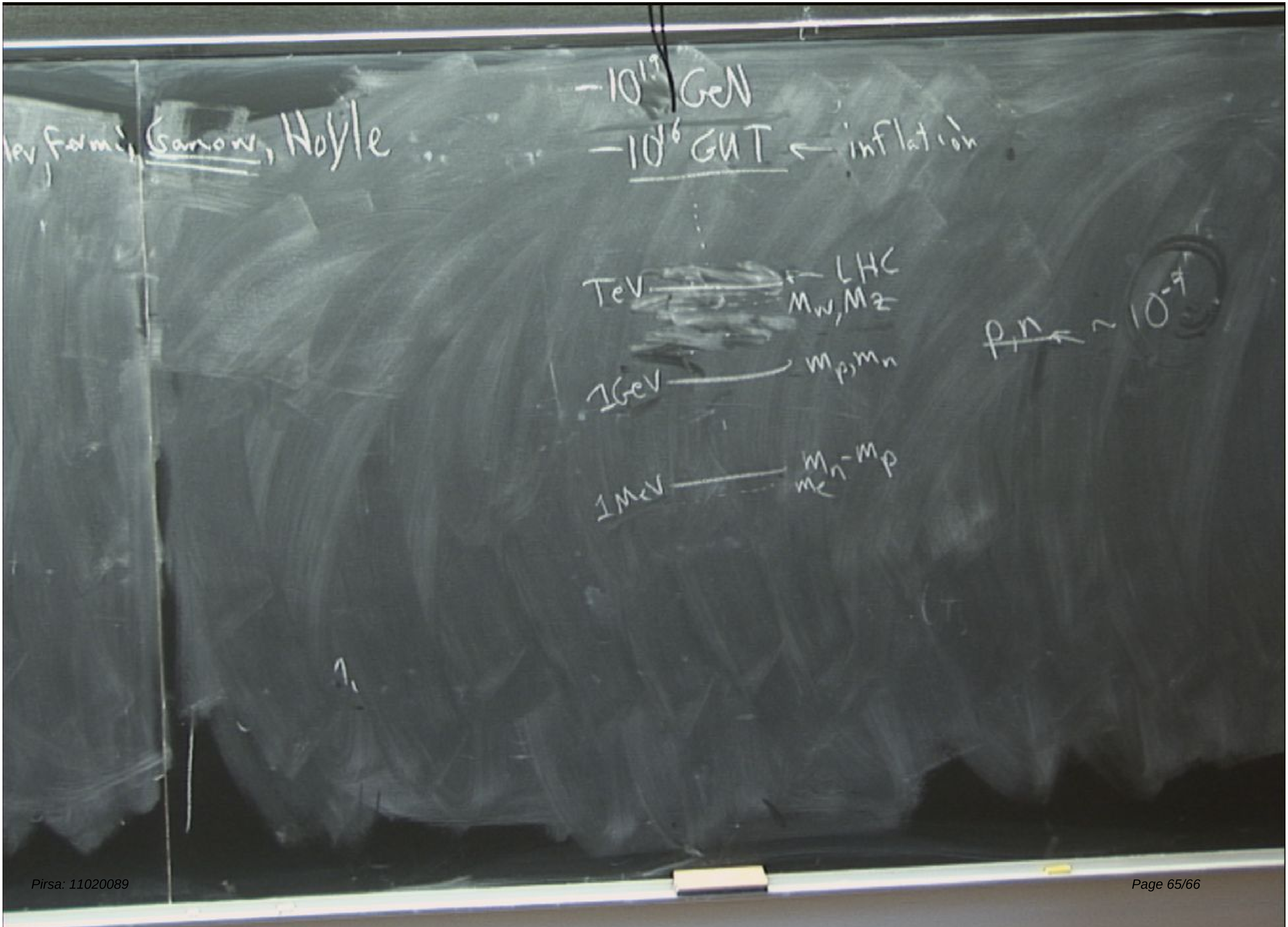
TeV ← LHC

M_W, M_Z

p, n ← $\sim 10^{-9}$

1GeV — m_p, m_n

1MeV — $m_n - m_p$, m_e


lev, Fermi, Gamow, Hoyle
-10^{19} GeV
-10^{16} GUT ← inflation
TeV ← LHC
M_W, M_Z
p, n ← $\sim 10^{-9}$
1GeV m_p, m_n
1MeV $m_n - m_p$
m_e

lev, Fermi, Gamow, Hoyle

— 10^{19} GeV

— 10^{16} GUT ← inflation

TeV — ← LHC

M_W, M_Z

1 GeV — m_p, m_n

1 MeV — $m_n - m_p$, m_e

p, n ← $\sim 10^{-9}$