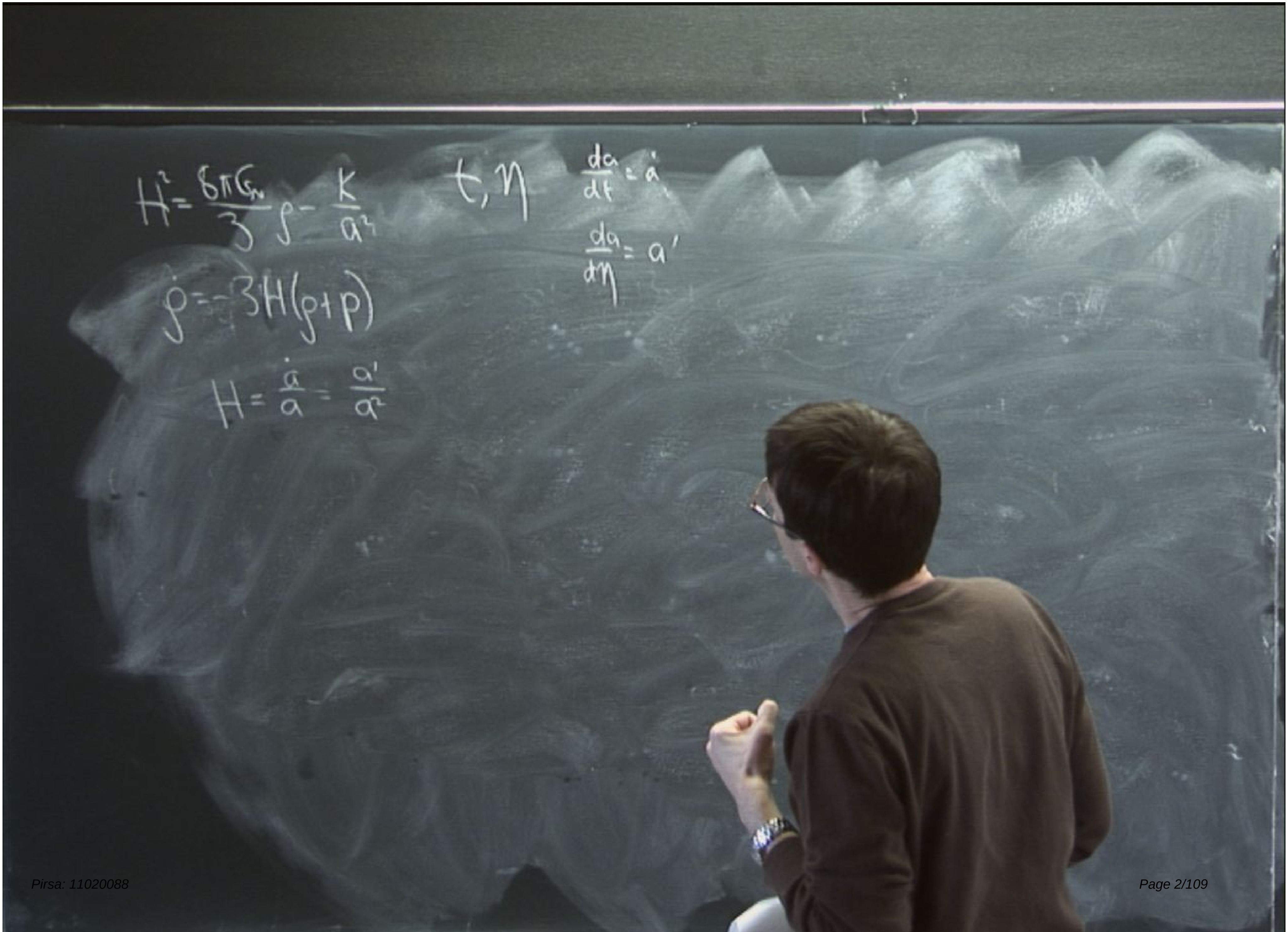
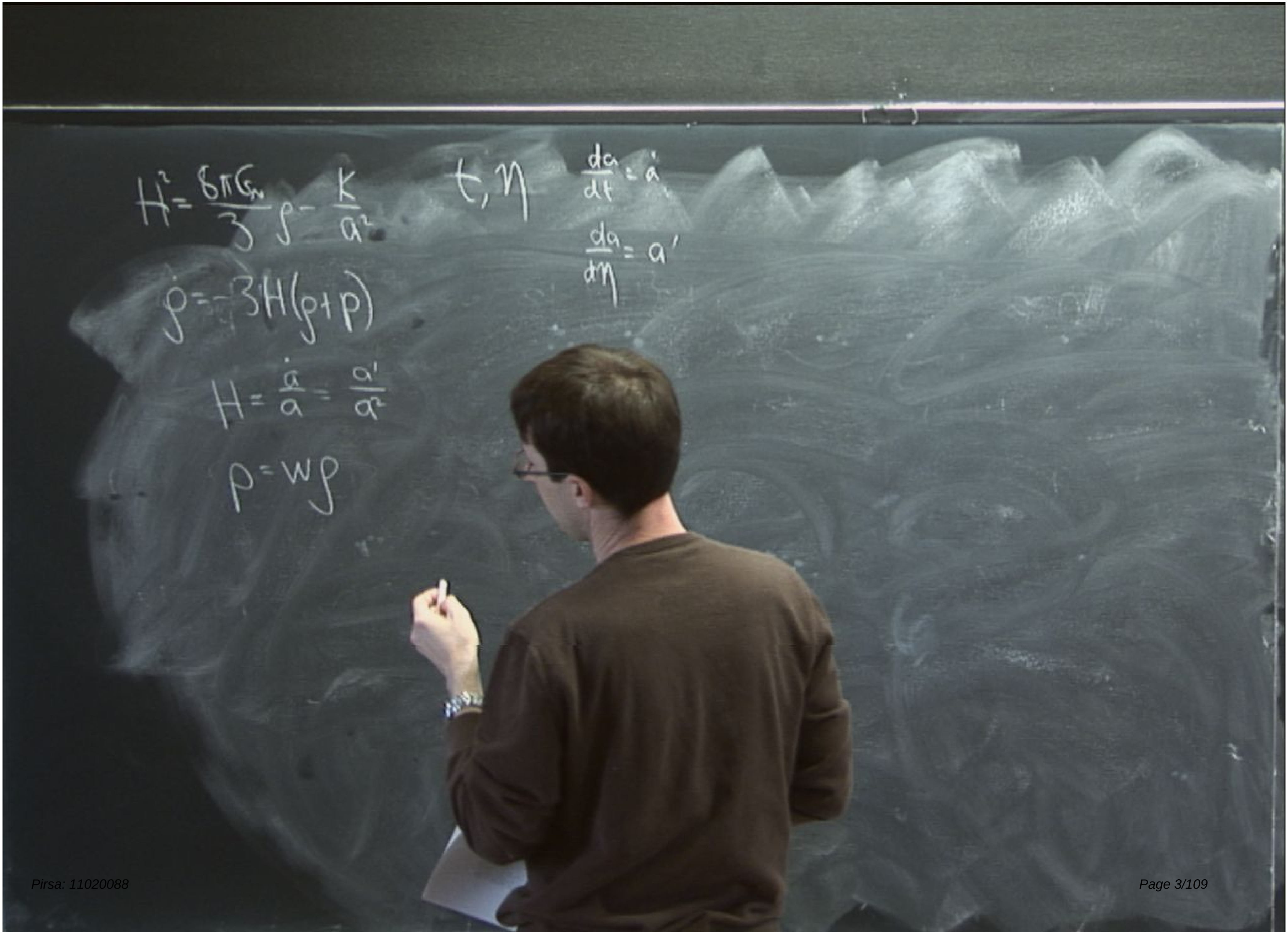
Title: Cosmology Review - Lecture 7

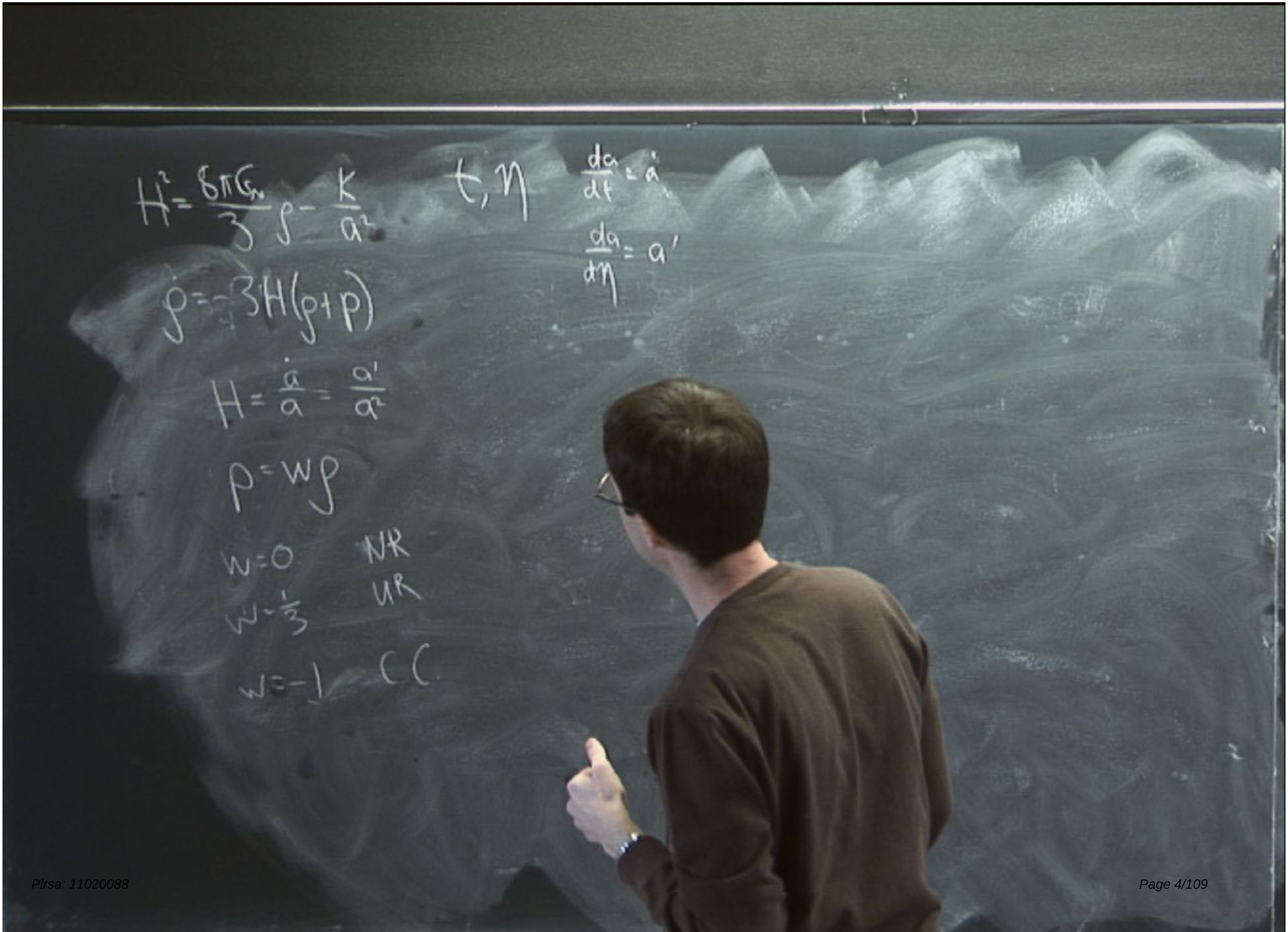
Date: Feb 01, 2011 11:30 AM

URL: http://pirsa.org/11020088

Abstract:


$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$
t, η
$\frac{da}{dt} = \dot{a}$
$\frac{da}{d\eta} = a'$
$\dot{\rho} = -3H(\rho + p)$
$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$


$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$
t, η
$\frac{da}{dt} = \dot{a}$
$\frac{da}{d\eta} = a'$
$\dot{\rho} = -3H(\rho + p)$
$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$
$p = w\rho$


$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$
t, η
$\frac{da}{dt} = \dot{a}$
$\frac{da}{d\eta} = a'$
$\dot{\rho} = -3H(\rho + p)$
$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$
$p = w\rho$
$w=0$ NR
$w=\frac{1}{3}$ UR
$w=-1$ CC

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$$

t, η

$$\frac{da}{dt} = \dot{a}$$

$$\frac{da}{d\eta} = a'$$

$$H^2 = \frac{16\pi G_N \rho - 2\Lambda}{n(n-1)} - \frac{K}{a^2}$$

$$\dot{\rho} = -3H(\rho + p)$$

$$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$$

$$p = w\rho$$

$w = 0$ NR

$w = \frac{1}{3}$ UR

$w = -1$ CC $\leftarrow$ vacuum

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$$

t, η

$$\frac{da}{dt} = \dot{a}$$

$$\frac{da}{d\eta} = a'$$

$$H^2 = \frac{16\pi G_N \rho + 2\Lambda}{n(n-1)} - \frac{K}{a^2}$$

$$= \frac{16\pi G_N}{n(n-1)}\left(\rho + \rho\right.$$

$$\dot{\rho} = -3H(\rho + p)$$

$$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$$

$$p = w\rho$$

$w = 0$ NR

$w = \frac{1}{3}$ UR

$w = -1$ CC $\leftarrow$ vacuum

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$$

t, η

$$\frac{da}{dt} = \dot{a}$$

$$\frac{da}{d\eta} = a'$$

$$H^2 = \frac{16\pi G_N \rho + 2\Lambda}{n(n-1)} - \frac{K}{a^2}$$

$$= \frac{16\pi G_N}{n(n-1)}\left(\rho + \rho\right.$$

$$\dot{\rho} = -3H(\rho + p)$$

$$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$$

$$p = w\rho$$

$w = 0$ NR

$w = \frac{1}{3}$ UR

$w = -1$ CC ← vacuum

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$$

t, η

$$\frac{da}{dt} = \dot{a}$$

$$\frac{da}{d\eta} = a'$$

$$H^2 = \frac{16\pi G_N \rho + 2\Lambda}{n(n-1)} - \frac{K}{a^2}$$

$$= \frac{16\pi G_N}{n(n-1)}$$

$$\dot{\rho} = -3H(\rho + p)$$

$$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$$

$$p = w\rho$$

$w = 0$ NR

$w = \frac{1}{3}$ UR

$w = -1$ CC $\leftarrow$ vacuum

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$$

t, η

$$\frac{da}{dt} = \dot{a}$$

$$\frac{da}{d\eta} = a'$$

$$H^2 = \frac{16\pi G_N \rho + 2\Lambda}{n(n-1)} - \frac{K}{a^2}$$

$$= \frac{16\pi G_N}{n(n-1)}\tilde{\rho}$$

$$\dot{\rho} = -3H(\rho + p)$$

$$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$$

$$p = w\rho$$

$w = 0$ NR

$w = \frac{1}{3}$ UR

$w = -1$ CC $\leftarrow$ vacuum

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$$

t, η

$$\frac{da}{dt} = \dot{a}$$

$$\frac{da}{d\eta} = a'$$

$$H^2 = \frac{16\pi G_N \rho + 2\Lambda}{n(n-1)} - \frac{K}{a^2}$$

$$= \frac{16\pi G_N}{n(n-1)}\tilde{\rho}$$

$\rho +$

$$\dot{\rho} = -3H(\rho + p)$$

$$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$$

$$p = w\rho$$

$w = 0$ NR

$w = \frac{1}{3}$ UR

$w = -1$ CC ← vacuum

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$$

t, η

$$\frac{da}{dt} = \dot{a}$$

$$\frac{da}{d\eta} = a'$$

$$H^2 = \frac{16\pi G_N \rho + 2\Lambda}{n(n-1)} - \frac{K}{a^2}$$

$$= \frac{16\pi G_N}{n(n-1)}\bar{\rho}$$

$$\rho + \frac{\Lambda}{8\pi G_N}$$

$$\dot{\rho} = -3H(\rho + p)$$

$$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$$

$$p = w\rho$$

$w = 0$ NR

$w = \frac{1}{3}$ UR

$w = -1$ CC ← vacuum

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$$

t, η

$$\frac{da}{dt} = \dot{a}$$

$$\frac{da}{d\eta} = a'$$

$$H^2 = \frac{16\pi G_N \rho + 2\Lambda}{n(n-1)} - \frac{K}{a^2}$$

$$= \frac{16\pi G_N}{n(n-1)}\bar{\rho}$$

$$\rho + \frac{\Lambda}{8\pi G}$$

$$\dot{\rho} = -3H(\rho + p)$$

$$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$$

$$p = w\rho$$

$w = 0$ NR

$w = \frac{1}{3}$ UR

$w = -1$ CC ← vacuum

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$$

t, η

$$\frac{da}{dt} = \dot{a}$$

$$\frac{da}{d\eta} = a'$$

$$H^2 = \frac{16\pi G_N \rho + 2\Lambda}{n(n-1)} - \frac{K}{a^2}$$

$$= \frac{16\pi G_N}{n(n-1)}\tilde{\rho}_{\prime\prime}$$

$$\rho + \frac{\Lambda}{8\pi G_N}$$

$$\dot{\rho} = -3H(\rho + p)$$

$$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$$

$$p = w\rho$$

$w = 0$ NR

$w = \frac{1}{3}$ UR

$w = -1$ CC ← vacuum

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$$

t, η

$$\frac{da}{dt} = \dot{a}$$

$$\frac{da}{d\eta} = a'$$

$$H^2 = \frac{16\pi G_N \rho + 2\Lambda}{n(n-1)} - \frac{K}{a^2}$$

$$= \frac{16\pi G_N}{n(n-1)} \bar{\rho}_{\shortparallel}$$

$$\rho + \frac{\Lambda}{8\pi G_N}$$

$$\rho + \rho_{vac}$$

$$\dot{\rho} = -3H(\rho + p)$$

$$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$$

$$p = w\rho$$

$w = 0$ NR

$w = \frac{1}{3}$ UR

$w = -1$ CC $\leftarrow$ vacuum

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$$

t, η

$$\frac{da}{dt} = \dot{a}$$

$$\frac{da}{d\eta} = a'$$

$$H^2 = \frac{16\pi G_N \rho + 2\Lambda}{n(n-1)} - \frac{K}{a^2}$$

$$= \frac{16\pi G_N}{n(n-1)} \bar{\rho}$$

$$\bar{\rho} = \rho + \frac{\Lambda}{8\pi G_N}$$

$$\rho + \rho_{vac}$$

$$\dot{\rho} = -3H(\rho + p)$$

$$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$$

$$p = w\rho$$

$w = 0$ NR

$w = \frac{1}{3}$ UR

$w = -1$ CC $\leftarrow$ vacuum

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$$

t, η

$$\frac{da}{dt} = \dot{a}$$

$$\frac{da}{d\eta} = a'$$

$$H^2 = \frac{16\pi G_N \rho + 2\Lambda}{n(n-1)} - \frac{K}{a^2}$$

$$= \frac{16\pi G_N}{n(n-1)}\tilde{\rho}''$$

$$\dot{\rho} = -3H(\rho + p)$$

$$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - V(\phi)$$

$$\rho + \frac{\Lambda}{8\pi}$$

$$\rho + \rho_{va}$$

$$p = w\rho$$

$w = 0$ NR

$w = \frac{1}{3}$ UR

$w = -1$ CC ← vacuum

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$$

t, η

$$\frac{da}{dt} = \dot{a}$$

$$\frac{da}{d\eta} = a'$$

$$H^2 = \frac{16\pi G_N \rho + 2\Lambda}{n(n-1)} - \frac{K}{a^2}$$

$$= \frac{16\pi G_N}{n(n-1)}\tilde{\rho}_{''}$$

$$\dot{\rho} = -3H(\rho + p)$$

$$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - V(\varphi)$$

$$p = w\rho$$

$$T_{\mu\nu} = \ldots$$

$w = 0$ NR

$w = \frac{1}{3}$ UR

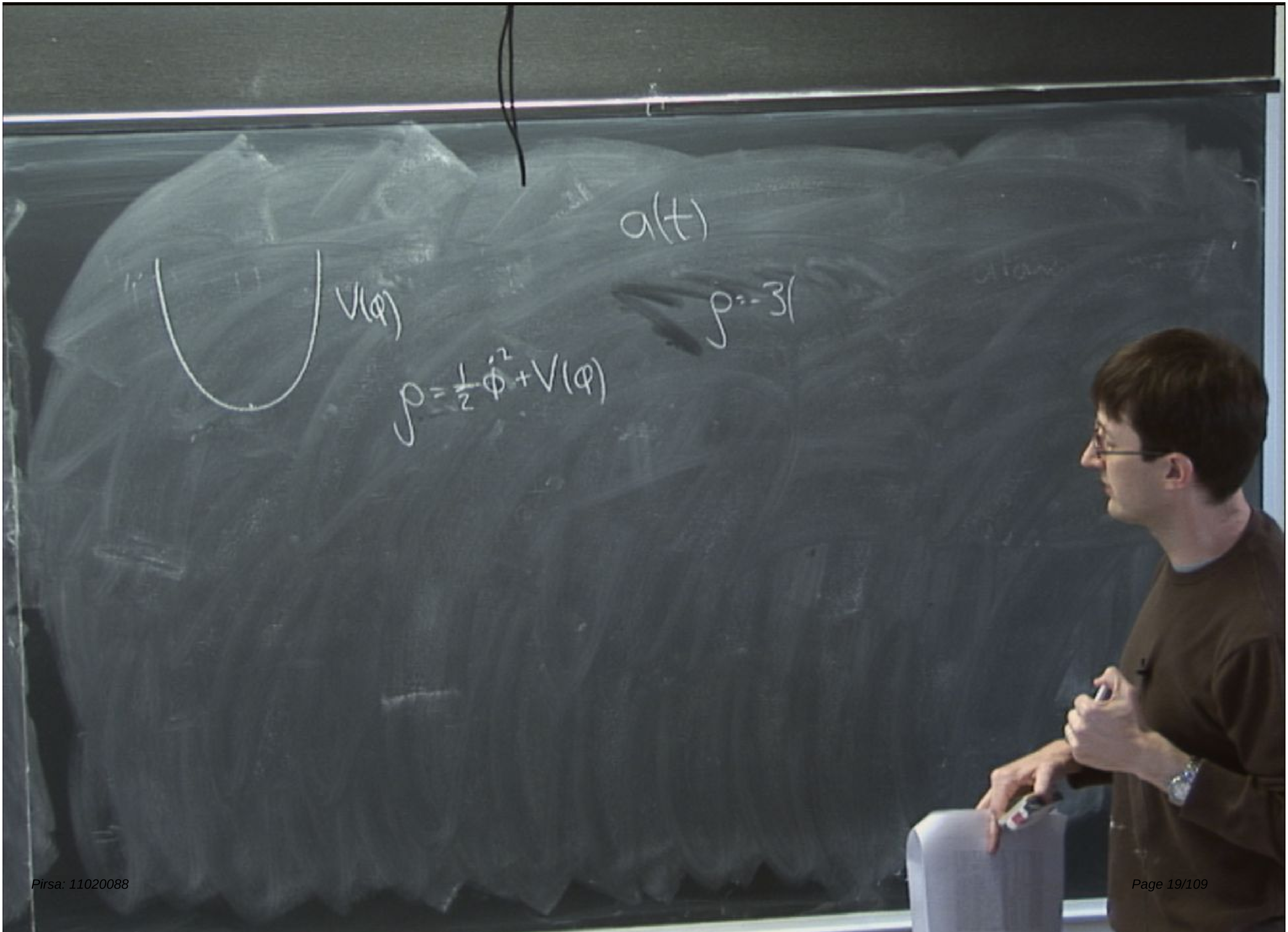
$w = -1$ CC ← vacuum

$a(t)$

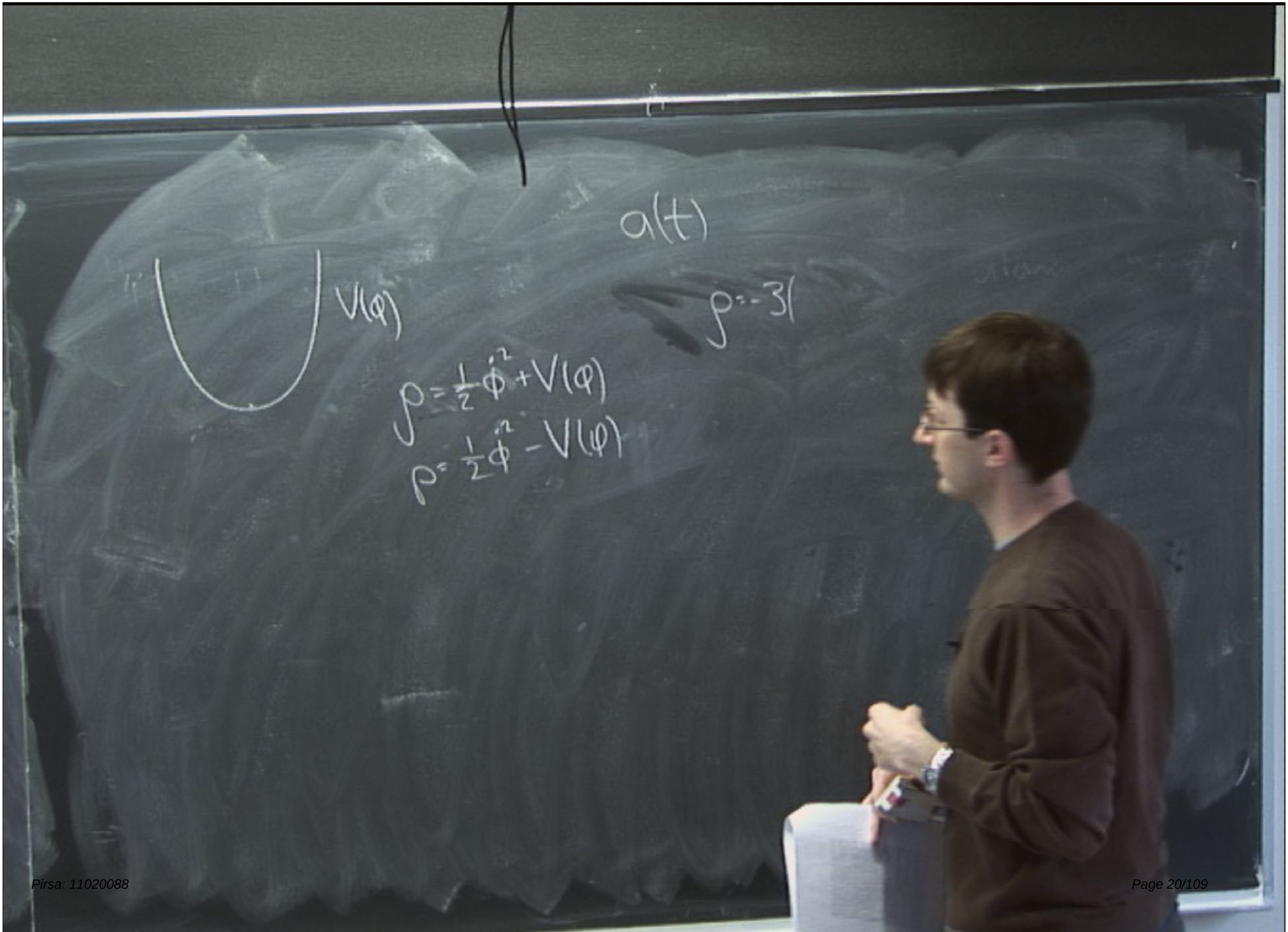
$V(\varphi)$

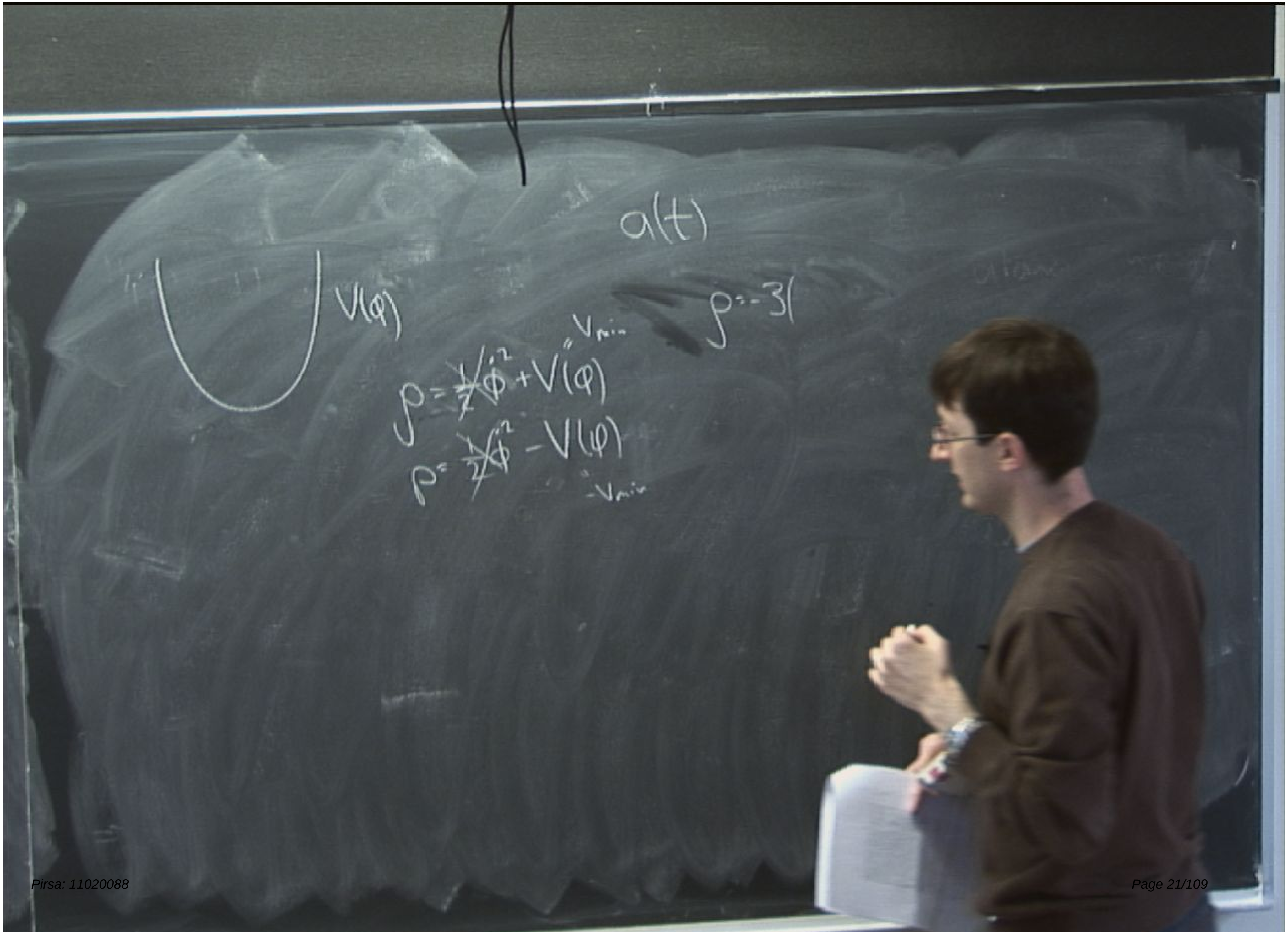
$\rho = -3($

ρ



a(t)
V(φ)
ρ = 1/2 φ̇² + V(φ)
ρ = -3(


a(t)
V(φ)
ρ = ½ φ̇² + V(φ)
p = ½ φ̇² − V(φ)
p = −3(

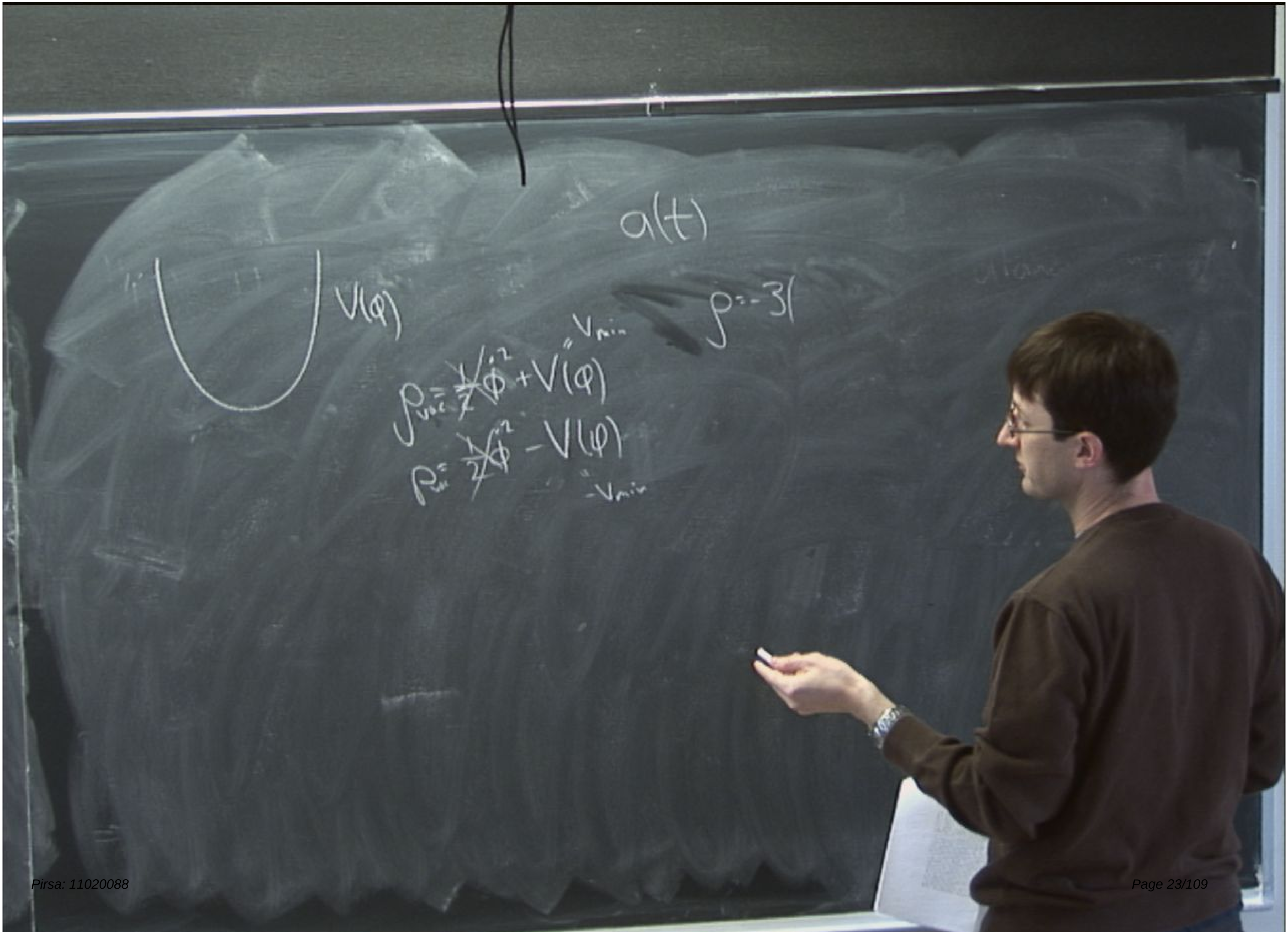

a(t)
V(φ)
ρ = ½φ̇² + V(φ)
V_min
p = ½φ̇² − V(φ)
−V_min

$a(t)$

$\rho = -3($

$\rho = \cancel{\tfrac{1}{2}}\dot{\phi}^2 + \underbrace{V(\phi)}_{} = V_{min}$

$p = \cancel{\tfrac{1}{2}}\dot{\phi}^2 - V(\phi) = -V_{min}$


V(φ)
a(t)
p=-3(
ρ_vac = ½ φ̇² + V(φ) = V_min
p_vac = ½ φ̇² − V(φ) = −V_min

$a(t)$

$V(\phi)$

$$p = -3($$

$$\rho_{vac} = \cancel{\tfrac{1}{2}\dot{\phi}^2} + V(\phi) \overset{\shortparallel}{} V_{min}$$

$$p_{vac} = \cancel{\tfrac{1}{2}\dot{\phi}^2} - V(\phi) \overset{\shortparallel}{} -V_{min}$$

$$p_{vac} = -\rho_{vac}$$

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2}$$

t, η

$$\frac{da}{dt} = \dot{a}$$

$$\frac{da}{d\eta} = a'$$

$$H^2 = \frac{16\pi G_N \rho + 2\Lambda}{n(n-1)} - \frac{k}{a^2}$$

$$= \frac{16\pi G_N}{n(n-1)}\tilde{\rho}_{11}$$

$$\dot{\rho} = -3H(\rho + p)$$

$$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)$$

$$p = w\rho$$

$$T_{\mu\nu} = -V_{min}\, g_{\mu\nu}$$

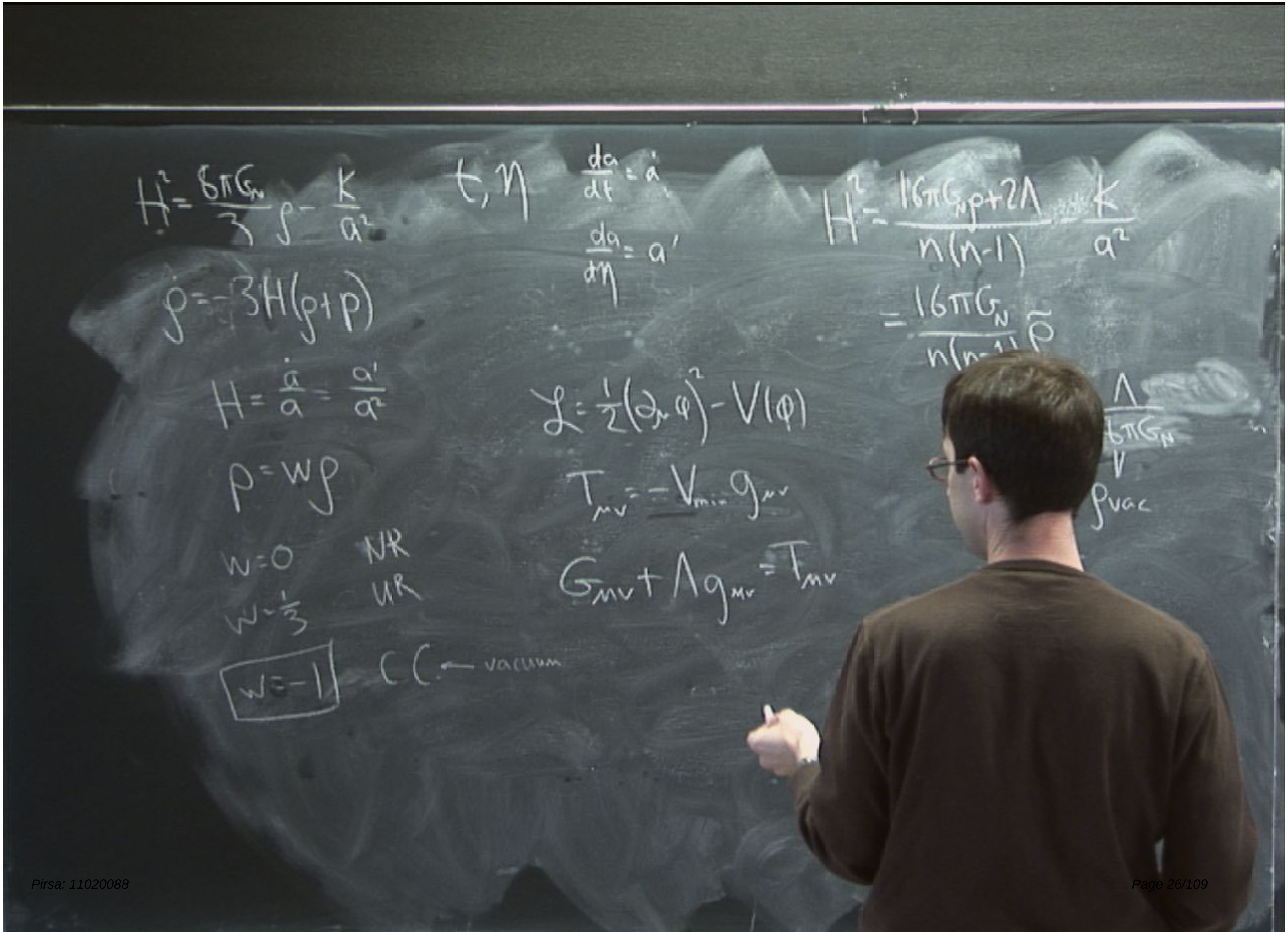
$w = 0$ NR

$w = \frac{1}{3}$ UR

$\boxed{w = -1}$ CC ← vacuum

$\rho +$

$\rho + \rho$


$H^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2}$
t, η
$\frac{da}{dt} = \dot{a}$
$\frac{da}{d\eta} = a'$
$\dot{\rho} = -3H(\rho + p)$
$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$
$p = w\rho$
$w = 0$ NR
$w = \frac{1}{3}$ UR
$\boxed{w = -1}$ CC ← vacuum
$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)$
$T_{\mu\nu} = -V_{min}\, g_{\mu\nu}$
$G_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$
$H^2 = \frac{16\pi G_N \rho + 2\Lambda}{n(n-1)} - \frac{k}{a^2}$
$= \frac{16\pi G_N}{n(n-1)}\tilde{\rho}$
$\frac{\Lambda}{8\pi G_N}$
ρ_{vac}

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$$

t, η

$$\frac{da}{dt} = \dot{a}$$

$$\frac{da}{d\eta} = a'$$

$$\dot{\rho} = -3H(\rho + p)$$

$$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$$

$$p = w\rho$$

$w = 0$ NR

$w = \frac{1}{3}$ UR

$\boxed{w = -1}$ CC ← vacuum

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)$$

$$T_{\mu\nu} = -V_{min}\, g_{\mu\nu}$$

$$G_{\mu\nu} = T_{\mu\nu} - \Lambda g_{\mu\nu}$$

$$H^2 = \frac{16\pi G_N \rho + 2\Lambda}{n(n-1)} - \frac{K}{a^2}$$

$$= \frac{16\pi G_N}{n(n-1)}\tilde{\rho}$$

$$\rho + \frac{\Lambda}{8\pi G_N}$$

$$\rho + \rho_{vac}$$

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$$

t, η

$$\frac{da}{dt} = \dot{a}$$

$$\frac{da}{d\eta} = a'$$

$$H^2 = \frac{16\pi G_N \rho + 2\Lambda}{n(n-1)} - \frac{K}{a^2}$$

$$= \frac{16\pi G_N}{n(n-1)}\tilde{\rho}_{''}$$

$$\dot{\rho} = -3H(\rho + p)$$

$$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)$$

$$\rho + \frac{\Lambda}{8\pi G_N}$$

$$p = w\rho$$

$$T_{\mu\nu} = -V_{min}\, g_{\mu\nu}$$

$$\rho + \rho_{vac}$$

$w = 0$ NR

$w = \frac{1}{3}$ UR

$$G_{\mu\nu} = T_{\mu\nu} - \Lambda g_{\mu\nu}$$

$\boxed{w = -1}$ CC ← vacuum

$$= \tilde{T}_{\mu\nu}$$

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2}$$

t, η

$$\frac{da}{dt} = \dot{a}$$

$$\frac{da}{d\eta} = a'$$

$$\dot{\rho} = -3H(\rho + p)$$

$$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$$

$$p = w\rho$$

$w = 0$ NR

$w = \frac{1}{3}$ UR

$\boxed{w = -1}$ CC ← vacuum

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)$$

$$T_{\mu\nu} = -V_{min}\, g_{\mu\nu}$$

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu} - \Lambda g_{\mu\nu}$$

$$= \tilde{T}_{\mu\nu}$$

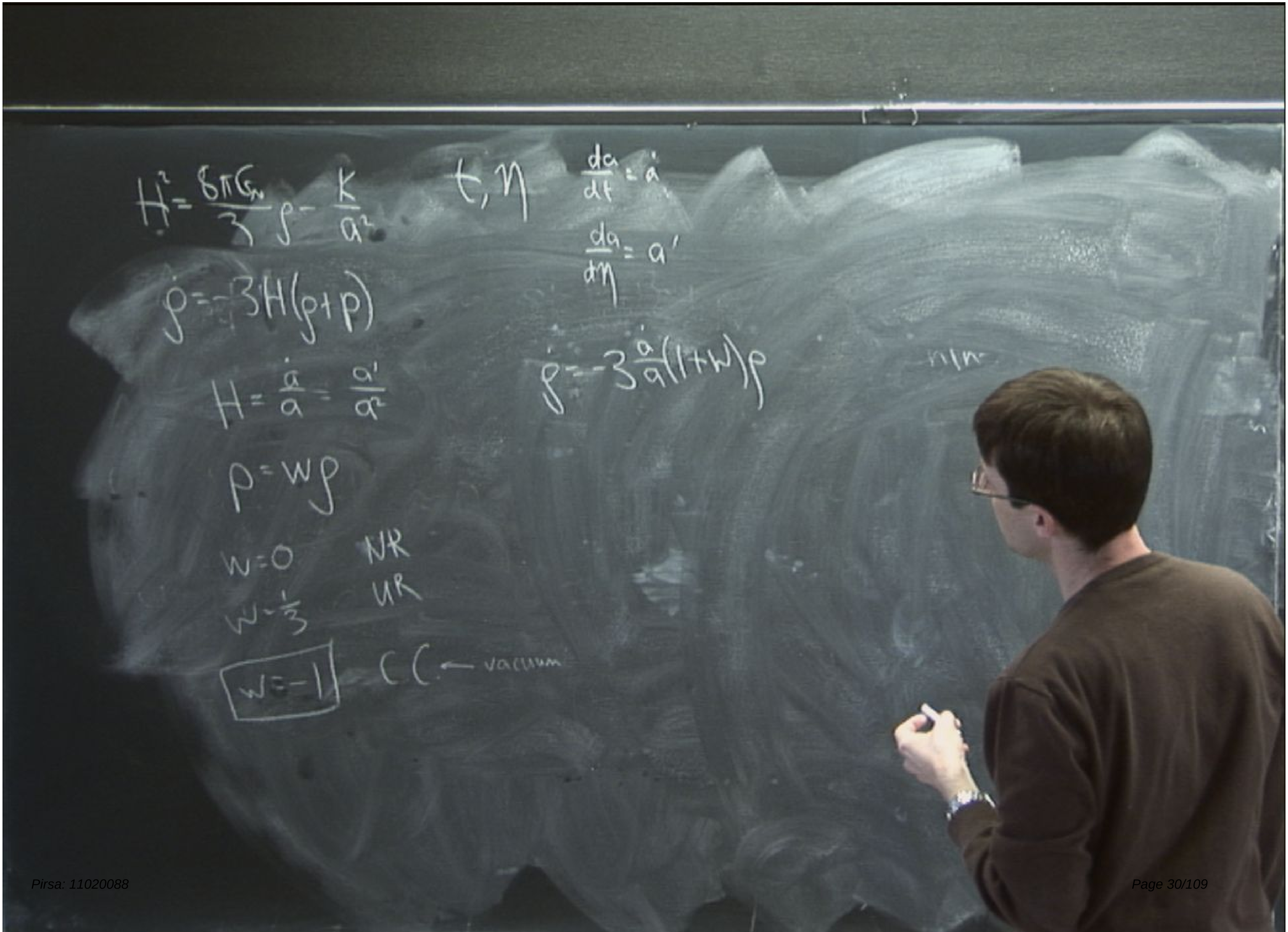
$$H^2 = \frac{16\pi G_N \rho + 2\Lambda}{n(n-1)} - \frac{k}{a^2}$$

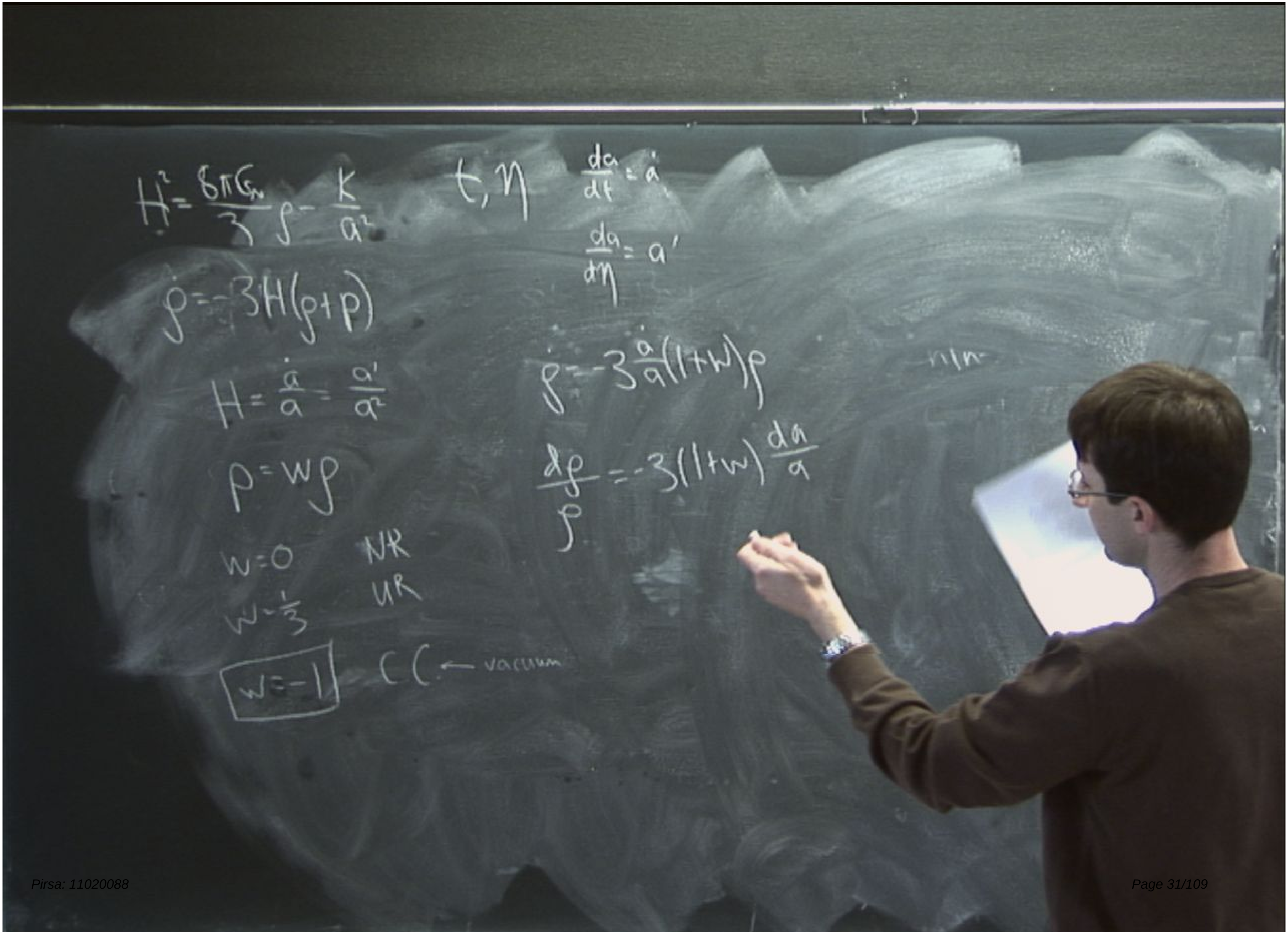
$$= \frac{16\pi G_N}{n(n-1)}\tilde{\rho}$$

$$\rho + \frac{\Lambda}{8\pi G_N}$$

$$\rho + \rho_{vac}$$

$$\rho_{vac} = \frac{\Lambda}{8\pi G_N}$$


$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$
t, η
$\frac{da}{dt} = \dot{a}$
$\frac{da}{d\eta} = a'$
$\dot{\rho} = -3H(\rho + p)$
$\dot{\rho} = -3\frac{\dot{a}}{a}(1+w)\rho$
$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$
$p = w\rho$
$w = 0$ NR
$w = \frac{1}{3}$ UR
$\boxed{w = -1}$ CC ← vacuum


$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$
t, η
$\frac{da}{dt} = \dot{a}$
$\frac{da}{d\eta} = a'$
$\dot{\rho} = -3H(\rho + p)$
$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$
$p = w\rho$
$\dot{\rho} = -3\frac{\dot{a}}{a}(1+w)\rho$
$\frac{d\rho}{\rho} = -3(1+w)\frac{da}{a}$
w=0 NR
w=1/3 UR
$\boxed{w=-1}$ CC ← vacuum

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$$

t, η

$$\frac{da}{dt} = \dot{a}$$

$$\frac{da}{d\eta} = a'$$

$$\dot{\rho} = -3H(\rho + p)$$

$$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$$

$$p = w\rho$$

$w = 0$ NR

$w = \frac{1}{3}$ UR

$\boxed{w = -1}$ CC ← vacuum

$$\dot{\rho} = -3\frac{\dot{a}}{a}(1+w)\rho$$

$$\frac{d\rho}{\rho} = -3(1+w)\frac{da}{a}$$

$$\rho = \rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)}$$

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2}$$

t, η

$$\frac{da}{dt} = \dot{a}$$

$$\frac{da}{d\eta} = a'$$

$$\dot{\rho} = -3H(\rho + p)$$

$$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$$

$$p = w\rho$$

$w = 0$ NR

$w = \frac{1}{3}$ UR

$\boxed{w = -1}$ CC ← vacuum

$$\dot{\rho} = -3\frac{\dot{a}}{a}(1+w)\rho$$

$$\frac{d\rho}{\rho} = -3(1+w)\frac{da}{a}$$

$$\rho = \rho_0 \left(\frac{a}{a_0}\right)^{-3(1+w)}$$

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$$

t, η

$$\frac{da}{dt} = \dot{a}$$

$$\frac{da}{d\eta} = a'$$

$$\dot{\rho} = -3H(\rho + p)$$

$$H = \frac{\dot{a}}{a} = \frac{a'}{a^2}$$

$$p = w\rho$$

$w = 0$ NR

$w = \frac{1}{3}$ UR

$\boxed{w = -1}$ CC ← vacuum

$$\dot{\rho} = -3\frac{\dot{a}}{a}(1+w)\rho$$

$$\frac{d\rho}{\rho} = -3(1+w)\frac{da}{a}$$

$$\underbrace{\rho = \rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)}}$$

$$H^2 = \frac{8\pi G_N}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$$

$K = 0,\ K = +1,\ K = -1$

t, η

$$\frac{da}{dt} = \dot{a}$$

$$\frac{da}{d\eta} = a'$$

$$\dot{\rho} = -3\frac{\dot{a}}{a}(1+w)\rho$$

$$\frac{d\rho}{\rho} = -3(1+w)\frac{da}{a}$$

$$\rho = \rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)}$$

— vacuum

$$H^2 = \frac{8\pi G_N}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$$

$K=0,\ K=+1,\ K=-1$

$$H^2 = \frac{8\pi G}{3}\rho_0\left(\frac{a}{a}\right)^{-\cancel{3}(1+w)}$$

t, η $\frac{da}{dt} = \dot{a}$

$\frac{da}{d\eta} = a'$

$$H^2 = \frac{8\pi G_N}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$$

$K=0,\ K=+1,\ K=-1$

$$\dot{\rho} = -3\frac{\dot{a}}{a}(1+w)\rho$$

$$H^2 = \frac{8\pi G}{3}\rho_0\left(\frac{a}{a}\right)^{-3(1+w)}$$

$\rho > 0$

$$\frac{d\rho}{\rho} = -3(1+w)\frac{da}{a}$$

$$\frac{\dot{a}}{a} = \pm\sqrt{\quad\left(\frac{a}{a_0}\right)^{-3(1+w)}}$$

$$\rho = \rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)}$$

— vacuum

t, η

$$\frac{da}{dt} = \dot{a}$$

$$\frac{da}{d\eta} = a'$$

$$\dot{\rho} = -3\frac{\dot{a}}{a}(1+w)\rho$$

$$\frac{d\rho}{\rho} = -3(1+w)\frac{da}{a}$$

$$\rho = \rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)}$$

— vacuum

$$H^2 = \frac{8\pi G_N}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$$

$K=0,\ K=+1,\ K=-1$

$$H^2 = \frac{8\pi G}{3}\rho_0\left(\frac{a}{a}\right)^{-3(1+w)}$$

$\rho > 0$

$$\frac{\dot{a}}{a} = \pm\sqrt{}\left(\frac{a}{a_0}\right)^{-\frac{3(1+w)}{2}}$$

$$a(t) = a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$$

t, η

$$\frac{da}{dt} = \dot{a}$$

$$\frac{da}{d\eta} = a'$$

$$H^2 = \frac{8\pi G_N}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$$

$K=0,\ K=+1,\ K=-1$

$$\dot{\rho} = -3\frac{\dot{a}}{a}(1+w)\rho$$

$$H^2 = \frac{8\pi G}{3}\rho_0\left(\frac{a}{a}\right)^{-3(1+w)}$$

$\rho > 0$

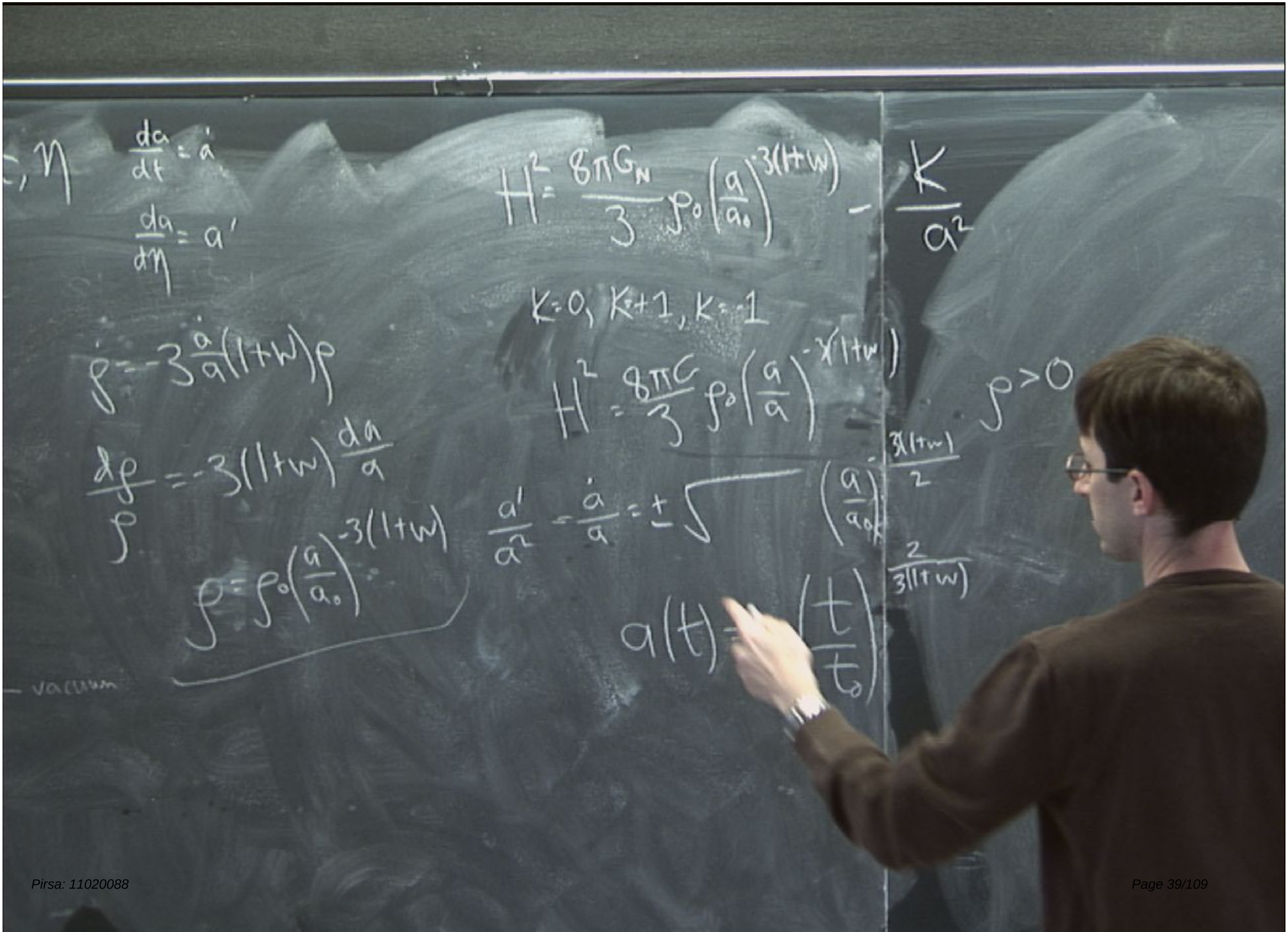
$$\frac{d\rho}{\rho} = -3(1+w)\frac{da}{a}$$

$$\frac{\dot{a}}{a} = \pm\sqrt{}\left(\frac{a}{a_0}\right)^{-\frac{3(1+w)}{2}}$$

$$\boxed{\rho = \rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)}}$$

$$a(t) = a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$$

— vacuum



t, η
$\frac{da}{dt} = \dot{a}$
$\frac{da}{d\eta} = a'$
$\dot{\rho} = -3\frac{\dot{a}}{a}(1+w)\rho$
$\frac{d\rho}{\rho} = -3(1+w)\frac{da}{a}$
$\rho = \rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)}$
— vacuum
$H^2 = \frac{8\pi G_N}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$
$K=0,\ K=+1,\ K=-1$
$H^2 = \frac{8\pi G}{3}\rho_0\left(\frac{a}{a}\right)^{-3(1+w)}$
$\rho > 0$
$\frac{a'}{a^2} = \frac{\dot{a}}{a} = \pm\sqrt{\quad}\left(\frac{a}{a_0}\right)^{-\frac{3(1+w)}{2}}$
$a(t) = \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$

$= \dot{a}$

$= a'$

$3\frac{\dot{a}}{a}(1+w)\rho$

$= -3(1+w)\frac{da}{a}$

$$\boxed{\rho = \rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)}}$$

$$H^2 = \frac{8\pi G_N}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$$

$K=0,\ K=+1,\ K=-1$

$$H^2 = \frac{8\pi G}{3}\rho_0\left(\frac{a}{a}\right)^{-3(1+w)}$$

$$\frac{a'}{a^2} = \frac{\dot{a}}{a} = \pm\sqrt{}\left(\frac{a}{a_0}\right)^{-\frac{3(1+w)}{2}}$$

$$a(t) = a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}} = a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{2}{1+3w}}$$

$\rho > 0$

$\rho_{vac} = \cancel{\frac{1}{2}}\dot{\phi}^2$

$\rho_{\cdot} = \cancel{\frac{1}{2}}\dot{\phi}^2$

$$H^2 = \frac{8\pi G_N}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$$

$K=0,\ K=+1,\ K=-1$

$(1+w)\rho$

$$H^2 = \frac{8\pi G}{3}\rho_0\left(\frac{a}{a}\right)^{-3(1+w)} \qquad \rho > 0$$

$(1+w)\frac{da}{a}$

$$\frac{a'}{a^2} = \frac{\dot{a}}{a} = \pm\sqrt{\quad}\left(\frac{a}{a_0}\right)^{-\frac{3(1+w)}{2}}$$

$\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)}$

$$a(t) = a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}} = a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{2}{1+3w}}$$

$\rho_{vac} = \cancel{\frac{1}{2}}\dot{\phi}^2 + V(\phi)$

$p_{vac} = \cancel{\frac{1}{2}}\dot{\phi}^2 - V($

$0 < a\ \infty$

$\infty > a > 0$

$$H^2 = \frac{8\pi G_N}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$$

$K=0,\ K=+1,\ K=-1$

$+w)\rho$

$$H^2 = \frac{8\pi G}{3}\rho_0\left(\frac{a}{a}\right)^{-3(1+w)} \qquad \rho > 0$$

$(1+w)\frac{da}{a}$

$\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)}$

$$\frac{a'}{a^2} = \frac{\dot a}{a} = \pm\sqrt{\quad}\left(\frac{a}{a_0}\right)^{-\frac{3(1+w)}{2}}$$

$$a(t) = a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}} = a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{2}{1+3w}}$$

$$\rho_{vac} = \cancel{\tfrac{1}{2}\dot\phi^2} + V(\phi$$

$$p_{vac} = \cancel{\tfrac{1}{2}\dot\phi^2} - V($$

big $\eta \to 0$

$0 < a\, \infty$

$\infty > a > 0$

$$H^2 = \frac{8\pi G_N}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$$

$K=0,\ K=+1,\ K=-1$

$(1+w)\rho$

$$H^2 = \frac{8\pi G}{3}\rho_0\left(\frac{a}{a}\right)^{-3(1+w)}$$

$\rho > 0$

$(1+w)\frac{da}{a}$

$$\frac{a'}{a^2} = \frac{\dot a}{a} = \pm\sqrt{\quad}\left(\frac{a}{a_0}\right)^{-\frac{3(1+w)}{2}}$$

$\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)}$

$$a(t) = a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}} = a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{2}{1+3w}}$$

$\rho_{vac} = \cancel{\tfrac{1}{2}\dot\phi^2} + V(\phi)$

$p_{vac} = \cancel{\tfrac{1}{2}\dot\phi^2} - V(\phi)$

$0 < a\ \infty$

$\infty > a > 0$

$$H^2 = \frac{8\pi G_N}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$$

$K=0,\ K=+1,\ K=-1$

$(1+w)\rho$

$$H^2 = \frac{8\pi G}{3}\rho_0\left(\frac{a}{a}\right)^{-3(1+w)} \qquad \rho > 0$$

$(1+w)\frac{da}{a}$

$$\frac{a'}{a^2} = \frac{\dot{a}}{a} = \pm\sqrt{}\left(\frac{a}{a_0}\right)^{-\frac{3(1+w)}{2}}$$

$\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)}$

$$a(t) = a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}} = a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{2}{1+3w}}$$

$\rho_{vac} = \frac{1}{2}\dot{\phi}^2 + V(\phi)$

$p = \frac{1}{2}\dot{\phi}^2 - V($

$0 < a\ \infty$

$\infty > a > 0$

$$H^2 = \frac{8\pi G_N}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$$

$K=0,\ K=+1,\ K=-1$

$$H^2 = \frac{8\pi G}{3}\rho_0\left(\frac{a}{a}\right)^{-3(1+w)} \quad \rho > 0$$

$$\frac{a'}{a^2} = \frac{\dot a}{a} = \pm\sqrt{\quad}\left(\frac{a}{a_0}\right)^{-\frac{3(1+w)}{2}}$$

$$a(t) = a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}} = a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{2}{1+3w}}$$

Big

Bang

$0 < a\ \infty$

$\infty > a > 0 \leftarrow$ Big

$a(t)$

$\rho_{vac} = \cancel{\tfrac{1}{2}\dot\phi^2} + V(\phi) \quad {}^{= V_{min}}$

$\rho_{vac} = \cancel{\tfrac{1}{2}\dot\phi^2} - V(\phi) \quad {}^{= -V_{min}}$

$\rho = -3($

$\rho_{vac} =$

$$H^2 = \frac{8\pi G_N}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$$

$K=0,\ K=+1,\ K=-1$

$$H^2 = \frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} \qquad \rho>0$$

$$\frac{a'}{a^2} = \frac{\dot a}{a} = \pm\sqrt{}\left(\frac{a}{a_0}\right)^{-\frac{3(1+w)}{2}}$$

$$a(t) = a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}} = a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{2}{1+3w}}$$

Big Bang $\downarrow$ $0 < a\ \infty$

$\infty > a > 0 \leftarrow$ Big Crunch

$a(t)$

$$\rho_{vac} = \cancel{\tfrac{1}{2}\dot\phi^2} + V(\phi) \quad V_{min}$$

$$p_{vac} = \cancel{\tfrac{1}{2}\dot\phi^2} - V(\phi) \quad -V_{min}$$

$\rho = -3($

$\rho_{vac} =$

$$H^2 = \frac{8\pi G_N}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$$

$$\frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} > \frac{1}{a^2}$$

$K=0,\ K=+1,\ K=-1$

$$H^2 = \frac{8\pi G}{3}\rho_0\left(\frac{a}{a}\right)^{-3(1+w)}$$

$\rho > 0$

$$\frac{a'}{a^2} = \frac{\dot a}{a} = \pm\sqrt{}\left(\frac{a}{a_0}\right)^{-\frac{3(1+w)}{2}}$$

$$a(t) = a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}} = a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{2}{1+3w}}$$

Big Bang

$0 < a\ \infty$

$\infty > a > 0 \leftarrow$ Big Crunch

$$H^2 = \frac{8\pi G_N}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$$

$K=0,\ K=+1,\ K=-1$

$$H^2 = \frac{8\pi G}{3}\rho_0\left(\frac{a}{a}\right)^{-3(1+w)} \qquad \rho > 0$$

$$\frac{a'}{a^2} = \frac{\dot a}{a} = \pm\sqrt{}\left(\frac{a}{a_0}\right)^{-\frac{3(1+w)}{2}}$$

$$a(t) = a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}} = a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{2}{1+3w}}$$

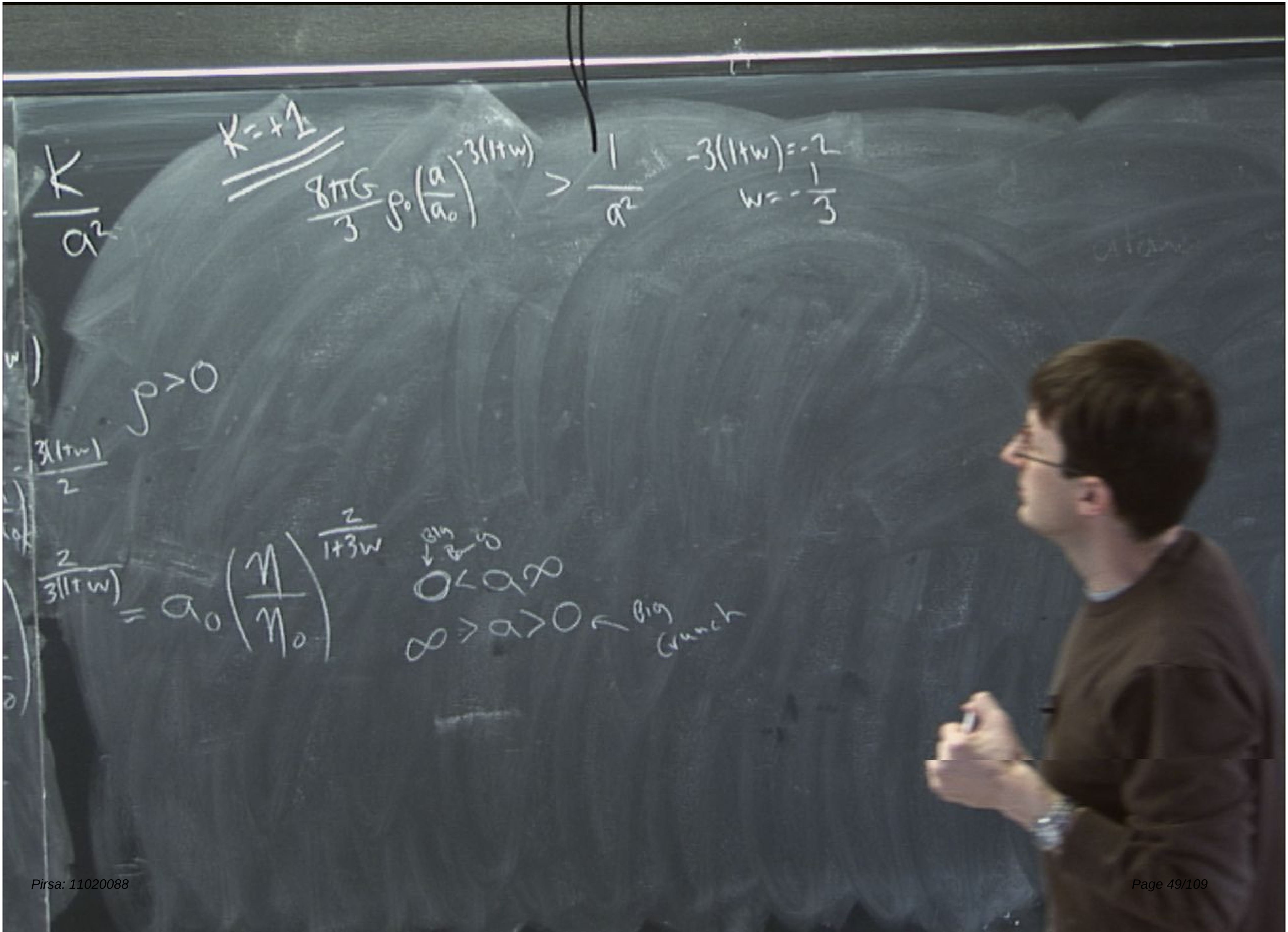
Big Bang $\downarrow$

$0 < a\ \infty$

$\infty > a > 0 \leftarrow$ Big Crunch

$\underline{\underline{K=+1}}$

$$\frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} > \frac{1}{a^2}$$


K = +1
K / a²
(8πG/3) ρ₀ (a/a₀)^(-3(1+w)) > 1/a²
-3(1+w) = -2
w = -1/3
ρ > 0
3(1+w)/2
2/3(1+w)
= a₀ (η/η₀)^(2/(1+3w))
Big Bang
0 < a ∞
∞ > a > 0 ← Big Crunch

$\frac{K}{a^2}$

$K=+1$

$\frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} > \frac{1}{a^2}$

$-3(1+w) = -2$

$w = -\frac{1}{3}$

$w > -\frac{1}{3}$

$\rho > 0$

$\frac{3(1+w)}{2}$

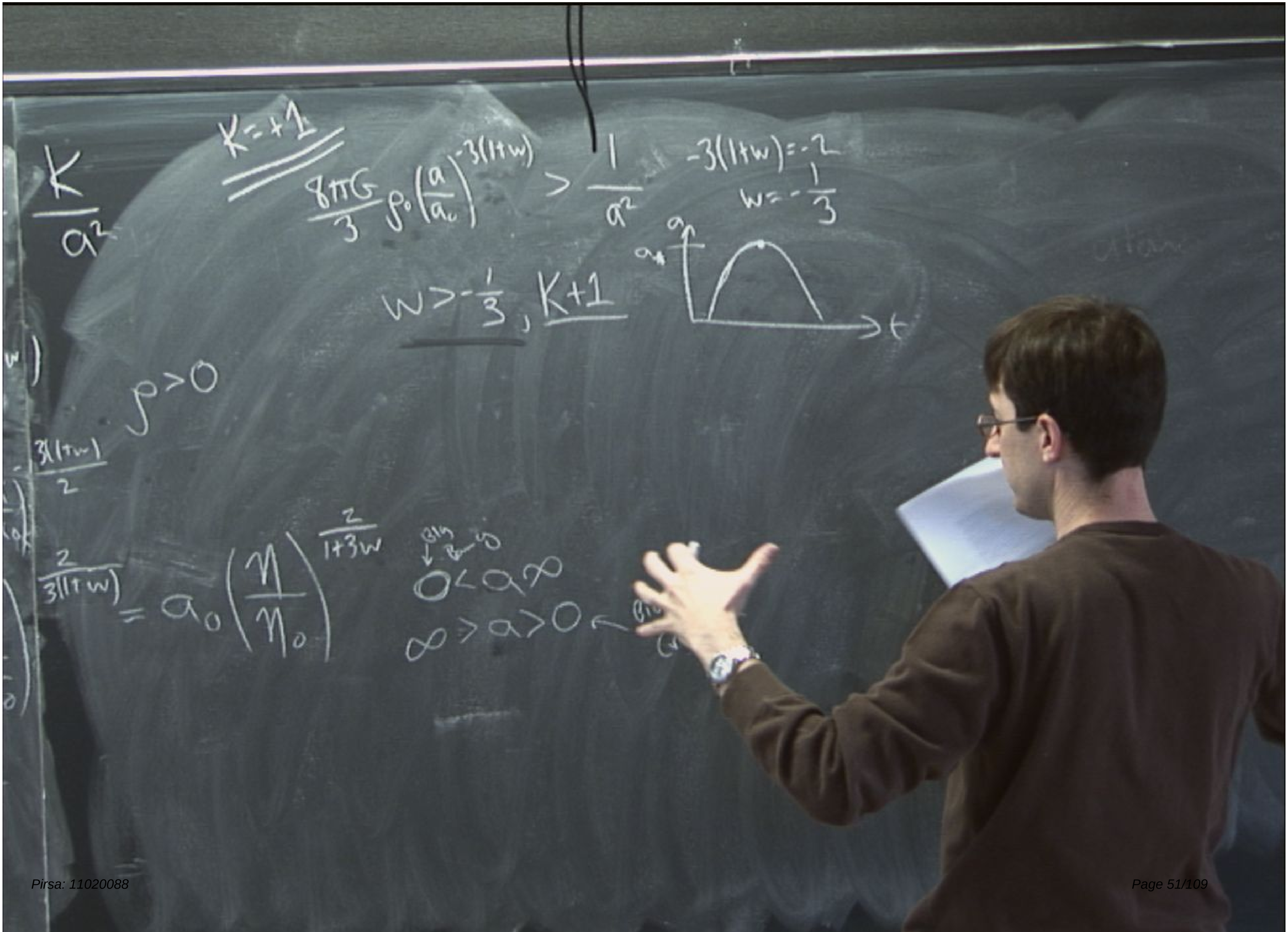
$\frac{2}{3(1+w)}$

$= a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{2}{1+3w}}$

Big Bang

$0 < a \infty$

$\infty > a > 0 \leftarrow$ Big Crunch


K = +1
K / a^2
8πG/3 ρ0 (a/a0)^-3(1+w) > 1/a^2
-3(1+w) = -2
w = -1/3
w > -1/3 , K+1
a
a*
t
ρ > 0
3(1+w)/2
2/3(1+w)
= a0 (η/η0)^(2/1+3w)
Big Bang
0 < a ∞
∞ > a > 0

$\frac{K}{a^2}$

$K = +1$

$$\frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} > \frac{1}{a^2}$$

$-3(1+w) = -2$

$w = -\frac{1}{3}$

$w > -\frac{1}{3}$, $K+1$

$\rho > 0$

$\frac{3(1+w)}{2}$

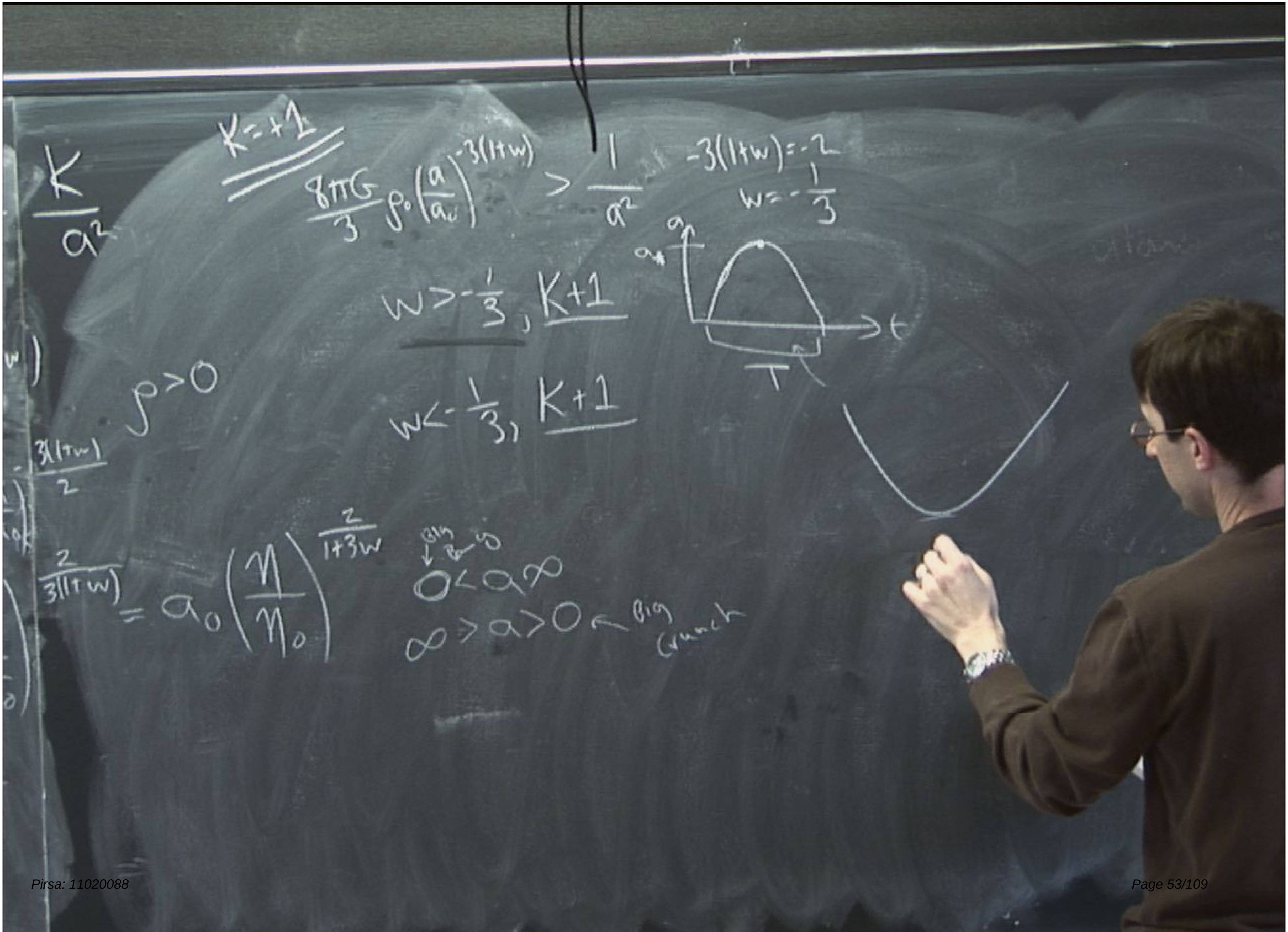
$\frac{2}{3(1+w)}$

$$= a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{2}{1+3w}}$$

Big Bang

$0 < a\infty$

$\infty > a > 0$ ← Big Crunch


K=+1
K / a^2
8πG/3 ρ0 (a/a0)^-3(1+w) > 1/a^2
-3(1+w) = -2
w = -1/3
w > -1/3, K+1
w < -1/3, K+1
ρ > 0
a
a*
t
T
= a0 (η/η0)^(2/(1+3w))
Big Bang
0 < a ∞
∞ > a > 0 ← Big Crunch

$\frac{K}{a^2}$

$K=+1$

$$\frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} > \frac{1}{a^2}$$

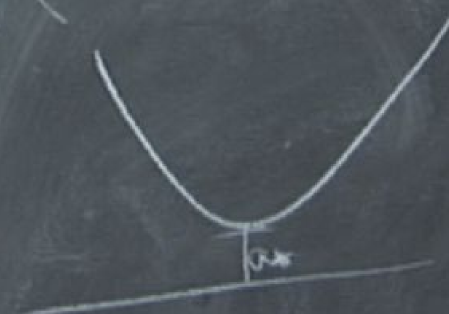
$-3(1+w)=-2$

$w=-\frac{1}{3}$

$w>-\frac{1}{3}$, $\underline{K+1}$

$\rho>0$

$w<-\frac{1}{3}$, $\underline{K+1}$



$\frac{3(1+w)}{2}$

$\frac{2}{3(1+w)}$ $= a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{2}{1+3w}}$

Big Bang

$0<a\infty$

$\infty>a>0$ ← Big Crunch

$K = +1$

$$\frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} > \frac{1}{a^2}$$

$-3(1+w) = -2$

$w = -\frac{1}{3}$

$\rho > 0$

$w > -\frac{1}{3}$, $K+1$

$w < -\frac{1}{3}$, $K+1$

$K = -1$

$$a = a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{2}{1+3w}}$$

Big Bang

$0 < a\infty$

$\infty > a > 0 \leftarrow$ Big Crunch

$$H^2 = \frac{8\pi G_N}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$$

$K=0,\ K=+1,\ K=-1$

$$H^2 = \frac{8\pi G}{3}\rho_0\left(\frac{a}{a}\right)^{-3(1+w)}$$

$\rho > 0$

$1+w)$

$$\frac{a'}{a^2} = \frac{\dot a}{a} = \pm\sqrt{\ }\left(\frac{a}{a_0}\right)^{-\frac{3(1+w)}{2}}$$

$$a(t) = a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}} = a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{2}{1+3w}}$$

$\underline{\underline{K=+1}}$

$$\frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} > \frac{1}{a^2}$$

$-3(1+w)=$

$w=-$

$w > -\frac{1}{3}$, $\underline{K+1}$

$w < -\frac{1}{3}$, $\underline{K+1}$

Big Bang $\to 0$

$0 < a\,\infty$

$\infty > a > 0 \leftarrow$ Big Crunch

$$H^2 = \frac{8\pi G_N}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$$

$K=0,\ K=+1,\ K=-1$

$$H^2 = \frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} \qquad \rho>0$$

$$H = \frac{\dot a}{a} = \pm\sqrt{}\left(\frac{a}{a_0}\right)^{-\frac{3(1+w)}{2}}$$

$$a(t) = a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}} = a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{2}{1+3w}}$$

$K=+1$

$$\frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} > \frac{1}{a^2}$$

$-3(1+w) = -2$

$$\boxed{w = -\frac{1}{3}}$$

$w > -\frac{1}{3},\ K+1$

$w < -\frac{1}{3},\ K+1$

Big Bang

$0 < a\ \infty$

$\infty > a > 0 \leftarrow$ Big Crunch

$K=-$

$K = +1$

$\frac{8\pi G}{3} \rho_0 \left(\frac{a}{a_0}\right)^{-3(1+w)} >$

$a(\eta) = a_0 A^\alpha S_K^\alpha\left(\frac{\eta}{\alpha}\right)$

$w > -\frac{1}{3}, K = 1$

$\rho > 0$

$w < -\frac{1}{3}, K = -1$

$= a_0 \left(\frac{\eta}{\eta_0}\right)^{\frac{2}{1+3w}}$

Big Bang

$0 < a\infty$

$\infty > a > 0 \leftarrow$ Big Crunch

$K=+1$

$\frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} >$

$w > -\frac{1}{3}, \ K=+1$

$w < -\frac{1}{3}, \ K=-1$

$a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{2}{1+3w}}$

Big Bang

$0 < a \infty$

$\infty > a > 0 \leftarrow$ Big Crunch

$a(\eta) = a_0 A^\alpha S_K^\alpha\left(\frac{\eta}{\alpha}\right)$

$S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \sinh\chi & K=-1 \\ \chi & K=0 \end{cases}$

$r \to \chi$

$\underline{\underline{K=+1}}$

$$\frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} >$$

$$w > -\frac{1}{3}, \quad \underline{K=+1}$$

$$w < -\frac{1}{3}, \quad \underline{K=-1}$$

0

$$\boxed{a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{2}{1+3w}}}$$

Big Bang

$0 < a\ \infty$

$\infty > a > 0 \leftarrow$ Big Crunch

$$a(\eta) = a_0 A^{\alpha} S_K^{\alpha}\left(\frac{\eta}{\alpha}\right)$$

$$S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \sinh\chi & K=-1 \\ \chi & K=0 \end{cases}$$

$r \to \chi$

$\underline{\underline{K=+1}}$

$$\frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} >$$

$w > -\frac{1}{3}$, $\underline{K=+1}$

$w < -\frac{1}{3}$, $\underline{K=-1}$

$$a(\eta) = a_0 A^\alpha S_K^\alpha\left(\frac{\eta}{\alpha}\right)$$

$$\alpha = \frac{2}{1+3w}$$

$$S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \sinh\chi & K=-1 \\ \chi & K=0 \end{cases}$$

$r \to \chi$

0

$$\boxed{a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{2}{1+3w}}}$$

Big Bang

$0 < a\infty$

$\infty > a > 0 \leftarrow$ Big Crunch

$K=+1$

$$\frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} >$$

$w > -\frac{1}{3}$, $K=+1$

$w < -\frac{1}{3}$, $K=-1$

$$a(\eta) = a_0 A^\alpha S_K^\alpha\left(\frac{\eta}{\alpha}\right)$$

$$\alpha = \frac{2}{1+3w}$$

$$S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \sinh\chi & K=-1 \\ \chi & K=0 \end{cases}$$

$r \to \chi$

0

$$\boxed{a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{2}{1+3w}}}$$

Big Bang

$0 < a\ \infty$

$\infty > a > 0 \leftarrow$ Big Crunch

$K = +1$

$\frac{8\pi G}{3} \rho_0 \left(\frac{a}{a_0}\right)^{-3(1+w)} >$

$a(\eta) = a_0 A^{\alpha} S_K^{\alpha}\left(\frac{\eta}{\alpha}\right)$

$w > -\frac{1}{3}, \quad K = 1$

$\alpha = \frac{2}{1+3w}$

$S_K(\chi) = \begin{cases} \sin\chi & K = +1 \\ \sinh\chi & K = -1 \\ \chi & K = 0 \end{cases}$

0

$w < -\frac{1}{3}, \quad K = -1$

$r \to \chi$

$a = 0$

$\boxed{\ldots_0 \left(\frac{\eta}{\eta_0}\right)^{\frac{2}{1+3w}}}$

Big Bang

$0 < a \infty$

$\infty > a > 0 \leftarrow$ Big Crunch

$K=+1$

$$\frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} >$$

$$a(\eta) = a_0 A^{\alpha} S_K^{\alpha}\left(\frac{\eta}{\alpha}\right)$$

$w > -\frac{1}{3}$, $K=1$

$$\alpha = \frac{2}{1+3w}$$

$$S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \sinh\chi & K=-1 \\ \chi & K=0 \end{cases}$$

0

$w < -\frac{1}{3}$, $K=-1$

$r \to \chi$

$r^2 d\varphi^2$

$a=0$

$$\boxed{a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{2}{1+3w}}}$$

Big Bang

$0 < a \infty$

$\infty > a > 0 \leftarrow$ Big Crunch

$K=+1$

$\frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} >$

$w>-\frac{1}{3}$, $K=1$

O

$w<-\frac{1}{3}$, $K=-1$

$\left(\frac{\eta}{\eta_0}\right)^{\frac{2}{1+3w}}$

Big Bang

$0<a\infty$

$\infty>a>0 \leftarrow$ Big Crunch

$a(\eta) = a_0 A^{\alpha} S_K^{\alpha}\left(\frac{\eta}{\alpha}\right)$

$\alpha = \frac{2}{1+3w}$

$$S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \sinh\chi & K=-1 \\ \chi & K=0 \end{cases}$$

$r \to \chi$

$r^2 d\varphi^2$

$\boxed{a=0}$

$\underline{\underline{K=+1}}$

$\frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} >$

$w > -\frac{1}{3}$, $\underline{K=+1}$

$w < -\frac{1}{3}$, $\underline{K=-1}$

$$a(\eta) = a_0 A^{\alpha} S_K^{\alpha}\left(\frac{\eta}{\alpha}\right)$$

$$\alpha = \frac{2}{1+3w}$$

$$S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \sinh\chi & K=-1 \\ \chi & K=0 \end{cases}$$

$$R = \frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2}$$

$r \to \chi$

$$\boxed{a=0}$$

$$\boxed{a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{2}{1+3w}}}$$

Big Bang

$0 < a\ \infty$

$\infty > a > 0 \leftarrow$ Big Crunch

$$a(\eta) = a_0 A\, S_K\left(\frac{\eta}{\alpha}\right)$$

$$\alpha = \frac{2}{1+3w}$$

$$\underbrace{S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \sinh\chi & K=-1 \\ \chi & K=0 \end{cases}}$$

$$R = \frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2}$$

$$r \to \chi$$

$$\boxed{a = 0}$$

R

Big Crunch

$)^{-3(1+w)}$ >

$-\frac{1}{3}$, $\underline{K=+1}$

$-\frac{1}{3}$, $\underline{K=-1}$

$a(\eta) = a_0 A^\alpha S_K^\alpha\left(\frac{\eta}{\alpha}\right)$

$\alpha = \frac{2}{1+3w}$

$$\underbrace{S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \sinh\chi & K=-1 \\ \chi & K=0 \end{cases}}$$

$r \to \chi$

$R = \frac{\ddot{a}}{a} \left(\frac{\dot{a}}{a}\right)^2 - \frac{K}{a^2}$

$\boxed{a=0}$

R

Big

$< a \infty$

$> a > 0 \leftarrow$ Big Crunch

Penrose & Hawking

$$a(\eta) = a_0 A^\alpha S_K^\alpha\left(\frac{\eta}{\alpha}\right)$$

$$\alpha = \frac{2}{1+3w}$$

$$S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \sinh\chi & K=-1 \\ \chi & K=0 \end{cases}$$

$r \to \chi$

$$R = \frac{\ddot{a}}{a}\left(\frac{\dot{a}}{a}\right)^2 - \frac{K}{a^2}$$

$$\boxed{a=0}$$

R

$)^{-3(1+w)}$ >

$-\frac{1}{3}$, $\underline{K=+1}$

$-\frac{1}{3}$, $\underline{K=-1}$

$< a\infty$

$> a > 0 \leftarrow$ Big Crunch

$)^{-3(1+w)}$ $>$

Penrose + Hawking

$$a(\eta) = a_0 A^\alpha S_K^\alpha\left(\frac{\eta}{\alpha}\right)$$

$-\frac{1}{3}$, $K=+1$

$$\alpha = \frac{2}{1+3w}$$

$$S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \sinh\chi & K=-1 \\ \chi & K=0 \end{cases}$$

$-\frac{1}{3}$, $K=-1$

$$r \to \chi$$

$$R = \frac{\ddot{a}}{a}\left(\frac{\dot{a}}{a}\right)^2 - \frac{K}{a^2}$$

$$\boxed{a=0}$$

R

Bang

$< a \infty$

$> a > 0 \leftarrow$ Big Crunch

Penrose & Hawking

$-3(1+w)$

$>$

$$a(\eta) = a_0 A^\alpha S_K^\alpha\left(\frac{\eta}{\alpha}\right)$$

$-\frac{1}{3},\ \underline{K=+1}$

$\alpha = \frac{2}{1+3w}$

$$S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \sinh\chi & K=-1 \\ \chi & K=0 \end{cases}$$

$\frac{1}{3},\ \underline{K=-1}$

$r \to \chi$

$R = \frac{\ddot a}{a} \left(\frac{\dot a}{a}\right)^2 - \frac{K}{a^2}$

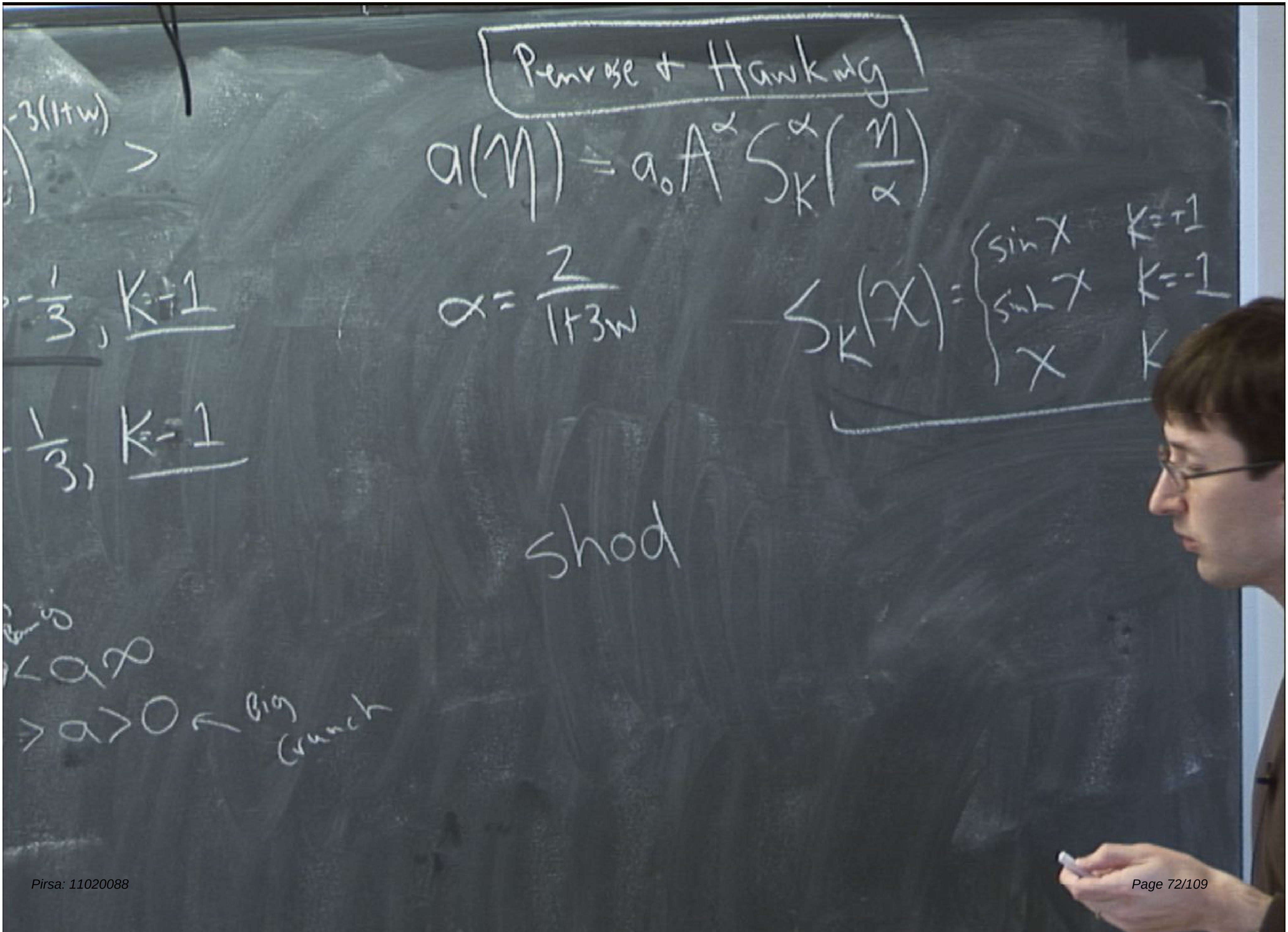
$\boxed{a=0}$

R

Bang

$<a\infty$

$> a > 0 \leftarrow$ Big Crunch


Penrose & Hawking
$a(\eta) = a_0 A^\alpha S_K^\alpha\left(\frac{\eta}{\alpha}\right)$
$\alpha = \frac{2}{1+3w}$
$S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \sinh\chi & K=-1 \\ \chi & K \end{cases}$
shod
$-3(1+w)$
$-\frac{1}{3}$, $K=+1$
$\frac{1}{3}$, $K=-1$
$> a > 0 \leftarrow$ Big Crunch

$)^{-3(1+w)}$ >

$-\frac{1}{3}$, $\underline{K=+1}$

$\frac{1}{3}$, $\underline{K=-1}$

Bang

$< a\infty$

$> a > 0 \leftarrow$ Big Crunch

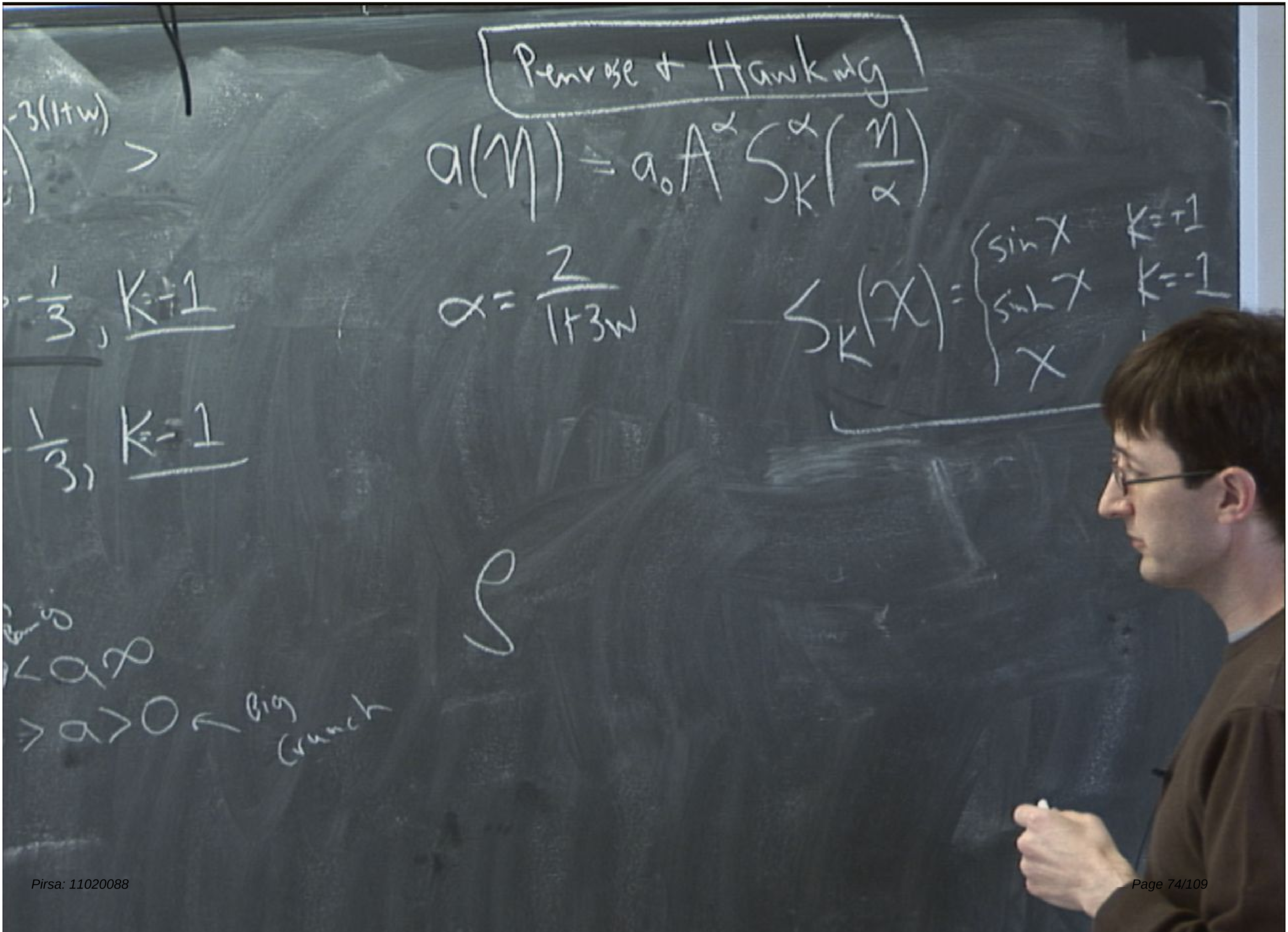
Penrose + Hawking

$$a(\eta) = a_0 A^{\alpha} S_K^{\alpha}\left(\frac{\eta}{\alpha}\right)$$

$$\alpha = \frac{2}{1+3w}$$

$$S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \sinh\chi & K=-1 \\ \chi & K=0 \end{cases}$$

shod


Penrose & Hawking
a(η) = a₀ A^α S_K^α(η/α)
α = 2/(1+3w)
S_K(χ) = { sin χ K=+1; sinh χ K=-1; χ
K=+1
K=-1
Big Crunch

$)^{-3(1+w)}$ >

$-\frac{1}{3}$, $K=+1$

$\frac{1}{3}$, $K=-1$

$< a\infty$

$> a > 0 \leftarrow$ Big Crunch

Penrose & Hawking

$$a(\eta) = a_0 A^\alpha S_K^\alpha\left(\frac{\eta}{\alpha}\right)$$

$$\alpha = \frac{2}{1+3w}$$

$$S_K(x) = \begin{cases} \sin x & K=+1 \\ \sinh x & K=-1 \\ x & K=0 \end{cases}$$

$$\rho = \rho_m\left(\frac{a}{a}\right)^{-3} + \rho_r$$

$$\boxed{\text{Penrose \& Hawking}}$$

$${}^{-3(1+w)}$$

$$>$$

$$a(\eta) = a_0 A^\alpha S_K^\alpha\left(\frac{\eta}{\alpha}\right)$$

$$-\frac{1}{3},\ \underline{K=1}$$

$$\alpha = \frac{2}{1+3w}$$

$$S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \sinh\chi & K=-1 \\ \chi & K=0 \end{cases}$$

$$\frac{1}{3},\ \underline{K=-1}$$

$$\rho = \rho_m\left(\frac{a}{a}\right)^{-3} + \rho_r\left(\frac{a}{a_0}\right)^{-4}$$

$$< a\ \infty$$

$$> a > 0 \leftarrow \text{Big Crunch}$$

$$H^2 = \frac{8\pi G_N}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$$

$$= \frac{8\pi G_N}{3}\left[\rho_m\left(\frac{a}{a_0}\right)^{-3} + \rho_r\left(\frac{a}{a_0}\right)^{-4}\right] - \frac{K}{a^2}$$

$\underline{\underline{K=+1}}$

$$\frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} >$$

$w > -\frac{1}{3}$, $\underline{K=1}$

$w < -$

$0 < a <$

$\infty > a >$

$+w)\rho$

$1+w)\frac{da}{a}$

$\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)}$

$$H^2 = \frac{8\pi G_N}{3} \rho_0 \left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$$

$$= \frac{8\pi G_N}{3}\left[\rho_m \left(\frac{a}{a_0}\right)^{-3} + \rho_r \left(\frac{a}{a_0}\right)^{-4}\right] - \frac{K}{a^2}$$

$(1+w)\rho$

$(1+w)\frac{da}{a}$

$\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)}$

$\underline{\underline{K=+1}}$

$\frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} >$

$\underline{w > -\frac{1}{3}}, \ \underline{K=1}$

$w < -\frac{1}{3}, \ \underline{K=-1}$

Big Bang

$0 < a \infty$

$\infty > a > 0 \leftarrow$ Big Crunch

$$H^2 = \frac{8\pi G_N}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$$

$$= \frac{8\pi G_N}{3}\left[\rho_m\left(\frac{a}{a_0}\right)^{-3} + \rho_r\left(\frac{a}{a_0}\right)^{-4}\right] - \frac{K}{a^2}$$

$(1+w)\rho$

$(1+w)\frac{da}{a}$

$\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)}$

$K=+1$

$\frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} >$

$w > -\frac{1}{3}$, $K=1$

$w < -\frac{1}{3}$, $K=-1$

Big Bang

$0 < a\ \infty$

$\infty > a > 0 \leftarrow$ Big Crunch

$$H^2 = \frac{8\pi G_N}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$$

$$= \frac{8\pi G_N}{3}\left[\rho_m\left(\frac{a}{a_0}\right)^{-3} + \rho_r\left(\frac{a}{a_0}\right)^{-4}\right] - \underbrace{\frac{K}{a^2}}$$

$+\rho_{int}$

$\uparrow$ flatness

$K = +1$

$$\frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} >$$

$w > -\frac{1}{3}$, $K = 1$

$w < -\frac{1}{3}$, $K = -1$

Big Bang

$0 < a \, \infty$

$\infty > a > 0$

$+w)$

$$H^2 = \frac{8\pi G_N}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$$

$$= \frac{8\pi G_N}{3}\left[\rho_m\left(\frac{a}{a_0}\right)^{-3} + \rho_r\left(\frac{a}{a_0}\right)^{-4} + \rho_{inf}\left(\frac{a}{a_0}\right)^{-3(1+w)}\right] - \frac{K}{a^2}$$

$\uparrow$ flatness

$K = +1$

$$\frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} >$$

$w > -\frac{1}{3}$, $K = +1$

$w < -\frac{1}{3}$, K

Big Bang

$0 < a \infty$

$\infty > a >$

$$H^2 = \frac{8\pi G_N}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$$

$K=+1$

$$\frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} >$$

$$= \frac{8\pi G_N}{3}\left[\rho_m\left(\frac{a}{a_0}\right)^{-3} + \rho_r\left(\frac{a}{a_0}\right)^{-4}\right] - \underbrace{\frac{K}{a^2}}$$

$$+\rho_{inf}\left(\frac{a}{a_0}\right)^{-3(1+w)}$$

$w > -\frac{1}{3}$, $K=+1$

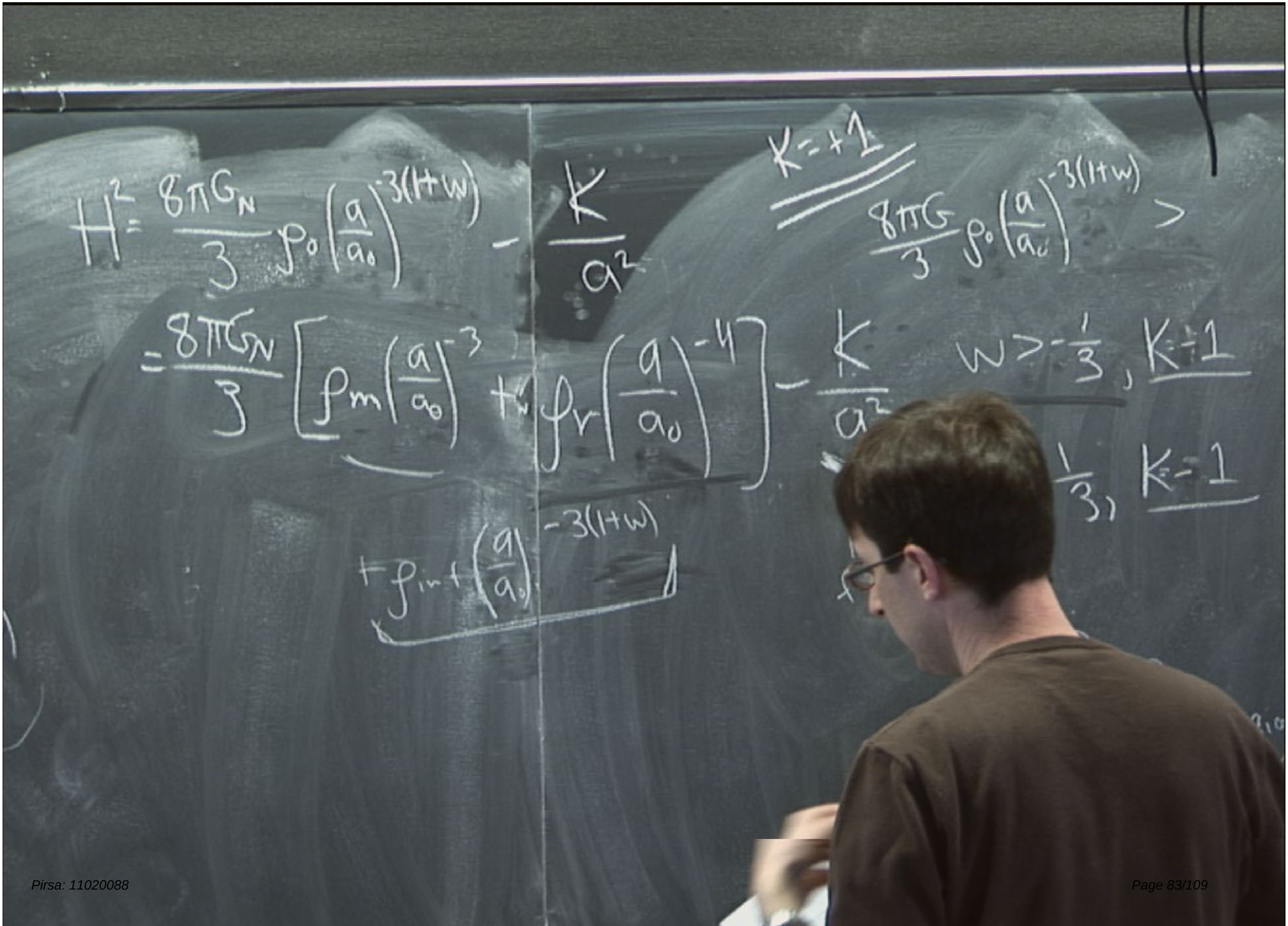
$w < -\frac{1}{3}$, $K=-1$

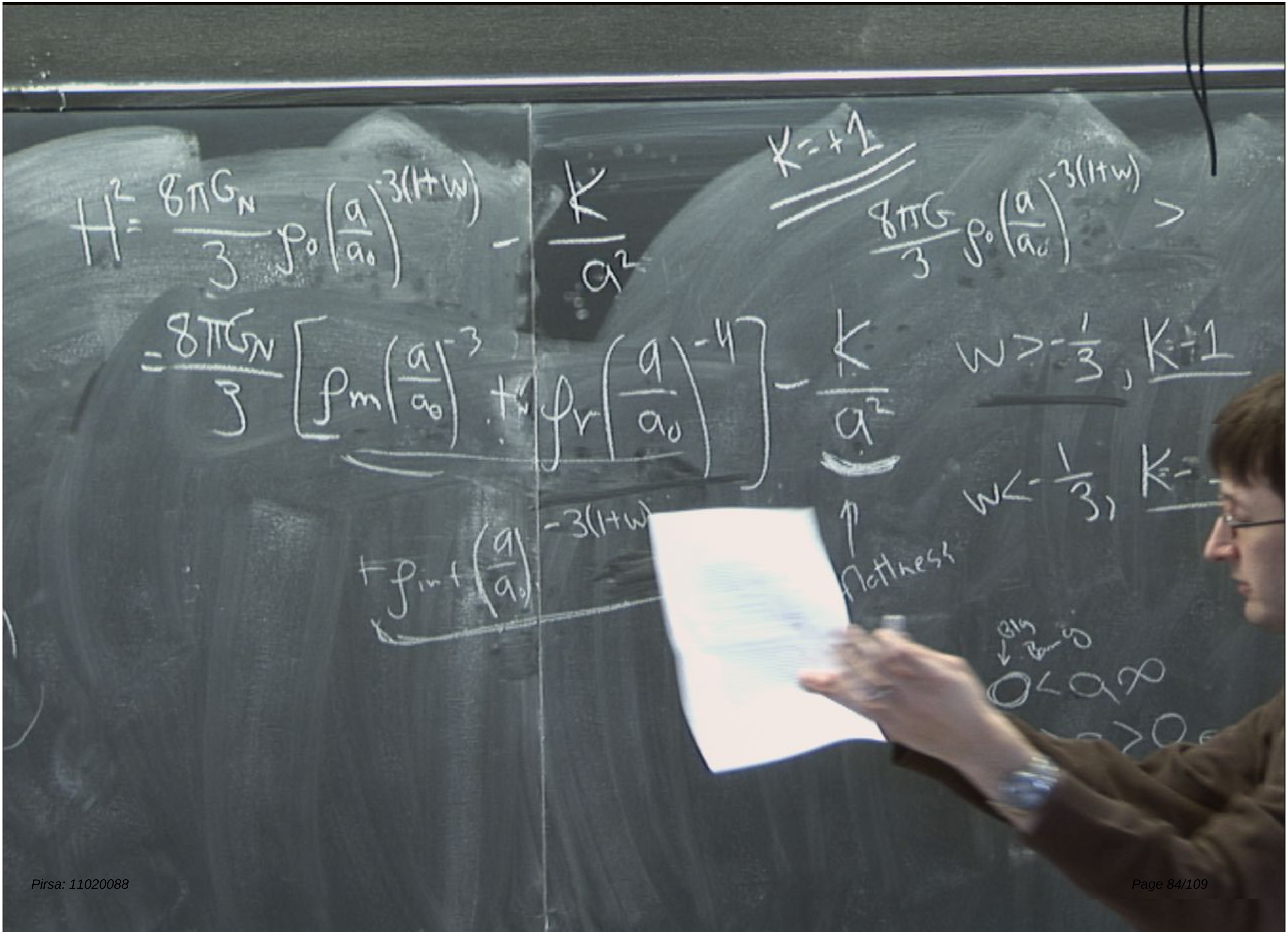
flatness

Big Bang

$0 < a \infty$

$\infty > a > 0$


$K = +1$
$H^2 = \frac{8\pi G_N}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$
$\frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} >$
$= \frac{8\pi G_N}{3}\left[\rho_m\left(\frac{a}{a_0}\right)^{-3} + \rho_r\left(\frac{a}{a_0}\right)^{-4} + \rho_{inf}\left(\frac{a}{a_0}\right)^{-3(1+w)}\right] - \frac{K}{a^2}$
$w > -\frac{1}{3}$, $K = +1$
$\frac{1}{3}$, $K = -1$


$H^2 = \frac{8\pi G_N}{3} \rho_0 \left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$
$= \frac{8\pi G_N}{3} \left[\rho_m \left(\frac{a}{a_0}\right)^{-3} + \rho_r \left(\frac{a}{a_0}\right)^{-4} \right] - \frac{K}{a^2}$
$+ \rho_{inf} \left(\frac{a}{a_0}\right)^{-3(1+w)}$
flatness
$K = +1$
$\frac{8\pi G}{3} \rho_0 \left(\frac{a}{a_0}\right)^{-3(1+w)} >$
$w > -\frac{1}{3}, K = +1$
$w < -\frac{1}{3}, K = -$
Big Bang
$0 < a \infty$

$$H^2 = \frac{8\pi G_N}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$$

$$= \frac{8\pi G_N}{3}\left[\rho_m\left(\frac{a}{a_0}\right)^{-3} + \rho_r\left(\frac{a}{a_0}\right)^{-4} + \rho_{inf}\left(\frac{a}{a_0}\right)^{-3(1+w)}\right] - \underbrace{\frac{K}{a^2}}_{}$$

↑ flatness

$K = +1$

$$\frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} >$$

$w > -\frac{1}{3}$, $K = +1$

$w < -\frac{1}{3}$, $K = -1$

Big Bang

$0 < a \infty$

$\infty > a > 0 \leftarrow$ Big

$$H^2 = \frac{8\pi G_N}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$$

$$= \frac{8\pi G_N}{3}\left[\rho_m\left(\frac{a}{a_0}\right)^{-3} + \rho_r\left(\frac{a}{a_0}\right)^{-4} + \rho_{vac}\left(\frac{a}{a}\right) + \rho_{inf}\left(\frac{a}{a_0}\right)^{-3(1+w)}\right]$$

flatness

$\underline{\underline{K=+1}}$

$$\frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} >$$

$3,\ \underline{K=+1}$

$3,\ K$

$$H^2 = \frac{8\pi G_N}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$$

$$K = +1$$

$$\frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)} >$$

$$= \frac{8\pi G_N}{3}\left[\rho_m\left(\frac{a}{a_0}\right)^{-3} + \rho_r\left(\frac{a}{a_0}\right)^{-4} + \rho_{vac}\left(\frac{a}{a}\right)^{0} + \rho_{inf}\left(\frac{a}{a_0}\right)^{-3(1+w)}\right]$$

↑ flatness

$K = -1$

$w < -\frac{1}{3}$, $K = -1$

Big Bang

$0 < a\,\infty$

$\infty > a > 0 \leftarrow$ Big

$$H^2 = \frac{8\pi G_N}{3} \rho_0 \left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$$

$K = +1$

$$\frac{8\pi G}{3} \rho_0 \left(\frac{a}{a_0}\right)^{-3(1+w)} >$$

$$= \frac{8\pi G_N}{3}\left[\rho_m \left(\frac{a}{a_0}\right)^{-3} + \rho_r \left(\frac{a}{a_0}\right)^{-4} + \rho_{vac} \left(\frac{a}{a}\right)^{0}\right]$$

$, K = +1$

$w < -\frac{1}{3}, K = -1$

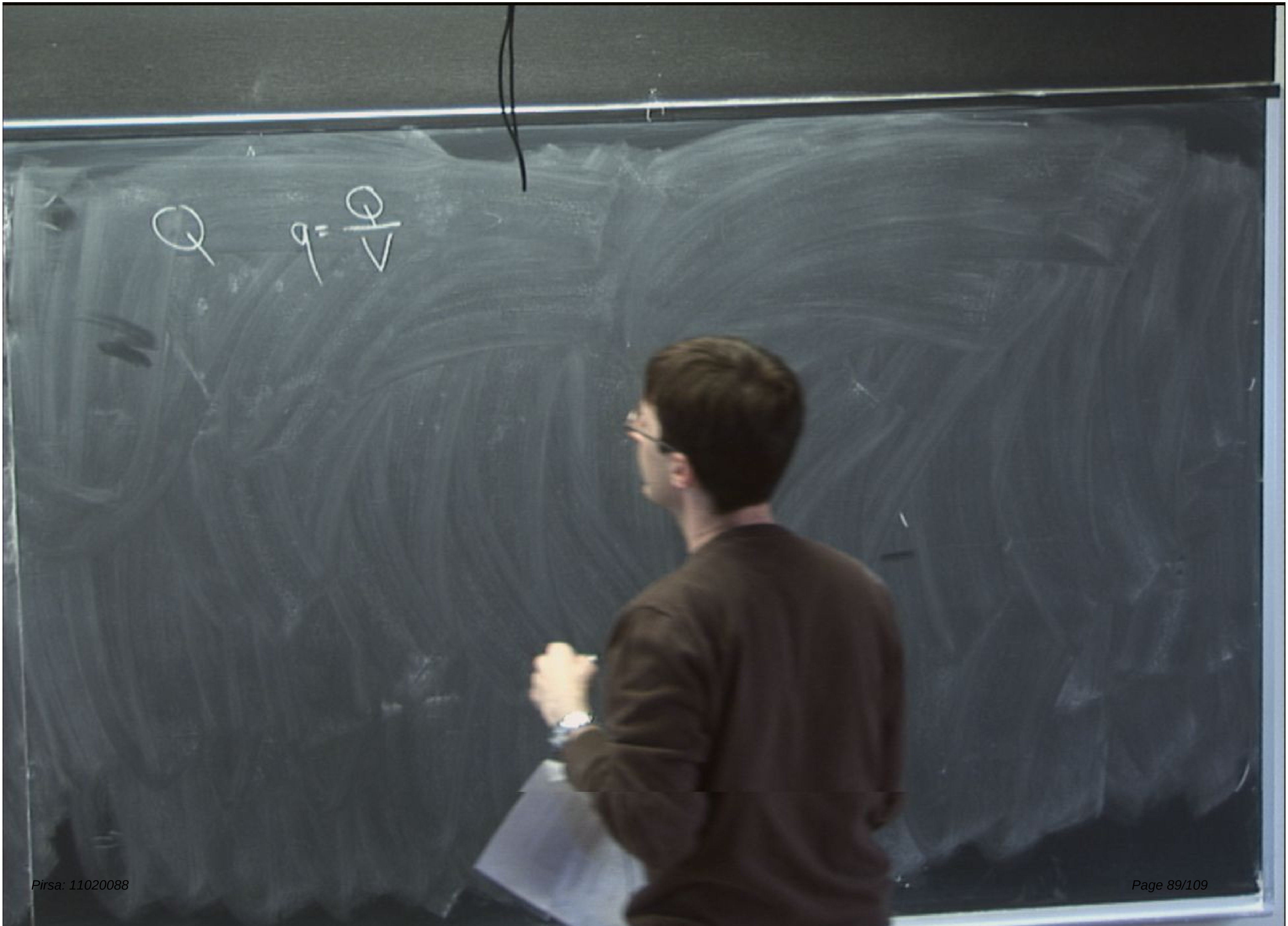
$$+ \rho_{inf} \left(\frac{a}{a_0}\right)^{-3(1+w)}$$

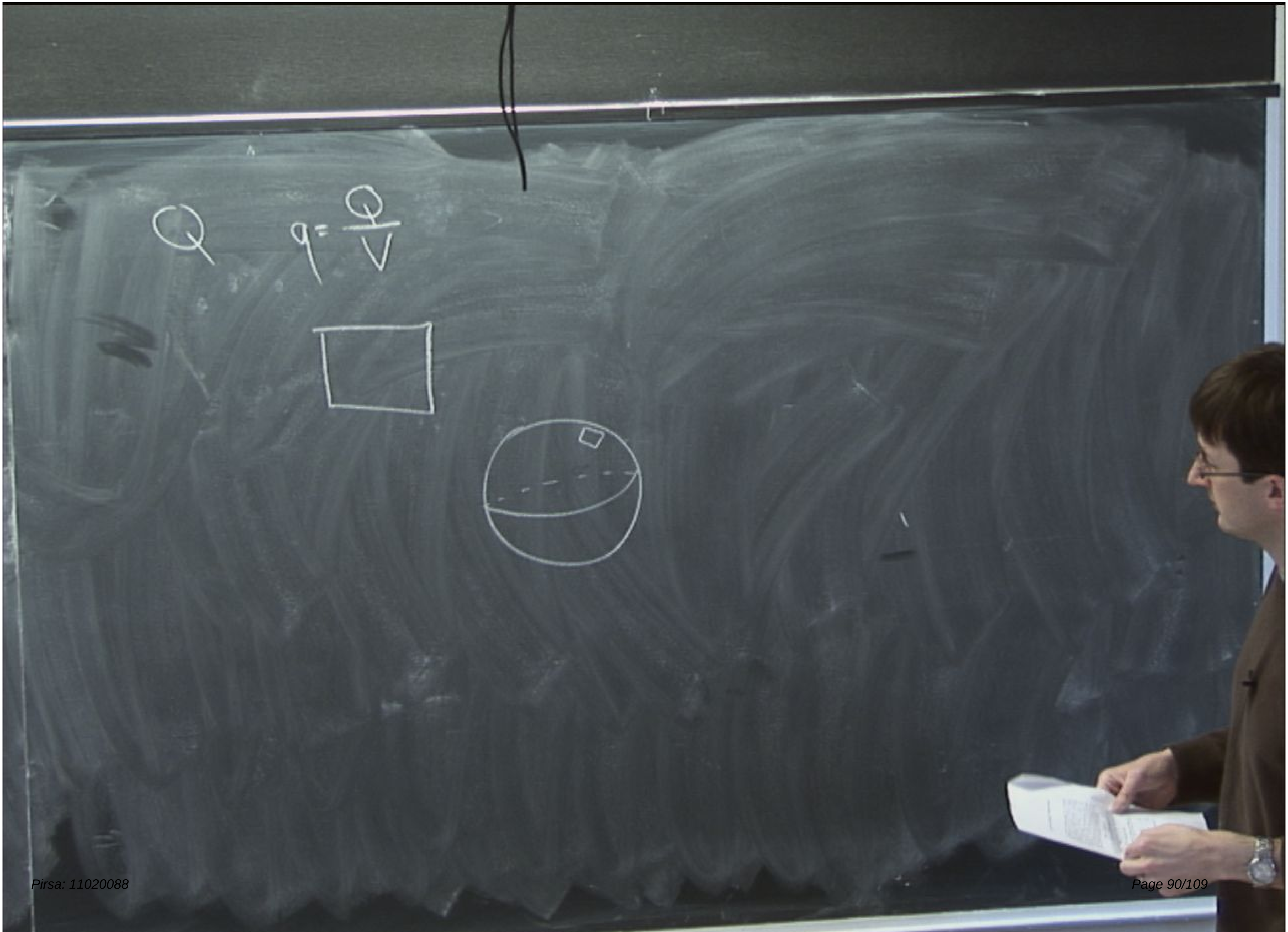
flatness

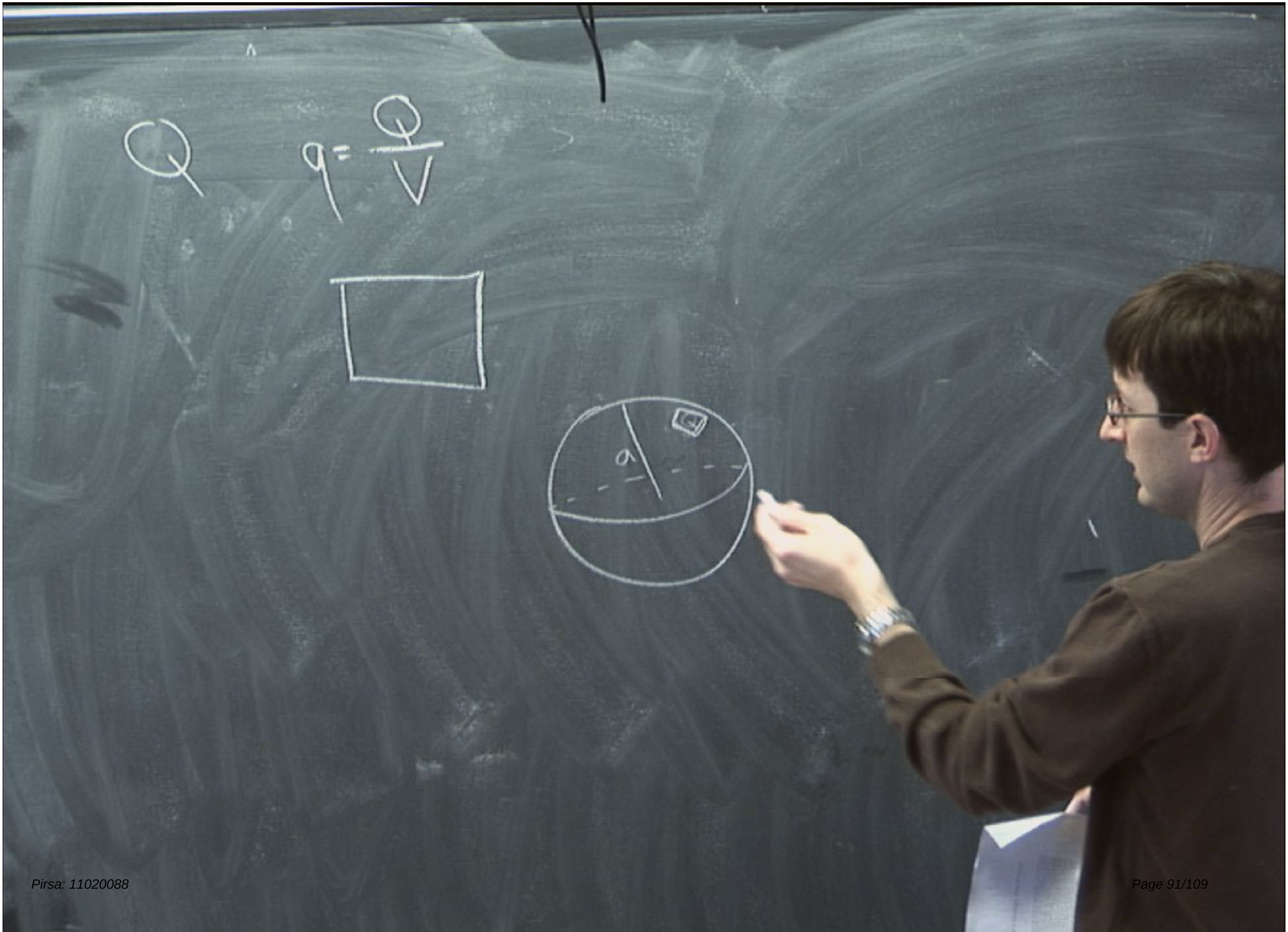
Big Bang

$0 < a \infty$

$\infty > a > 0 \leftarrow$ Big

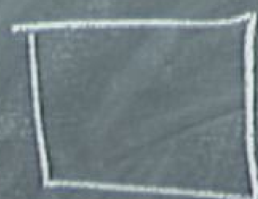

Q
q = Q/V


Q
q = Q/V


q = Q/V
a

Q

$q = \frac{Q}{V}$



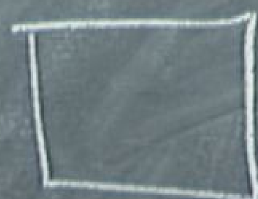
a^3

Q

$$q = \frac{Q}{V} \propto \frac{1}{a^3}$$

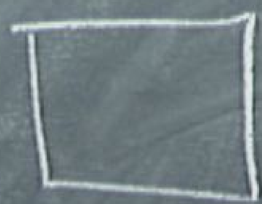
a^3

$Q \qquad q = \frac{Q}{V} \propto \frac{1}{a^3} \qquad n \propto \frac{1}{a^3}$



a^3

$Q \qquad q = \frac{Q}{V} \propto \frac{1}{a^3} \qquad n \propto \frac{1}{a^3} \qquad \rho \propto \frac{E}{a^3}$



a^3

$Q \qquad q = \frac{Q}{V} \propto \frac{1}{a^3} \qquad n \propto \frac{1}{a^3} \qquad \rho \propto \frac{E}{a^3} \qquad E^2 = m^2 + \vec{p}^{\,2}$

$\rho \propto \frac{1}{a}$

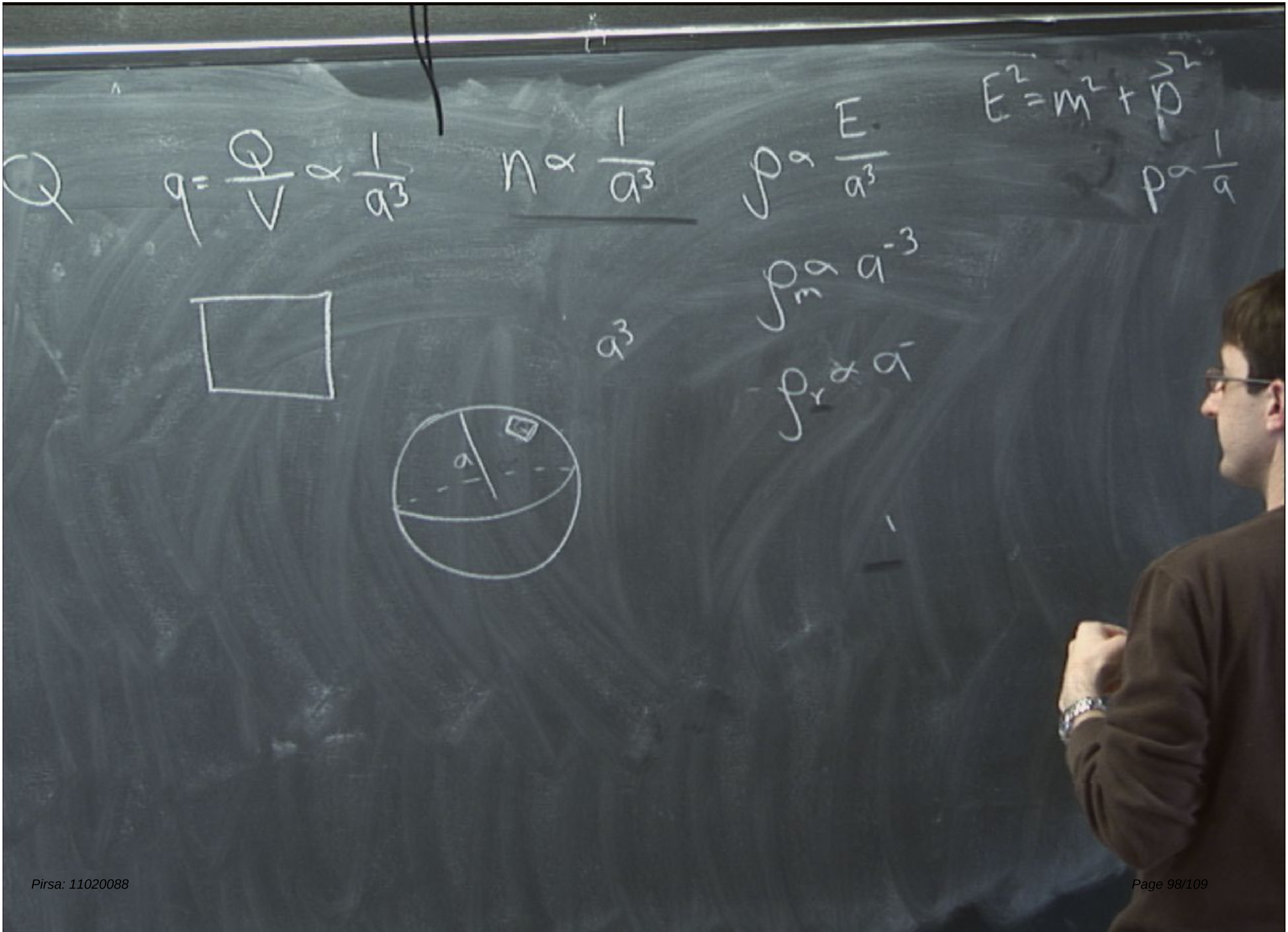
a^3

Q $\quad q = \frac{Q}{V} \propto \frac{1}{a^3}$ $\quad n \propto \frac{1}{a^3}$ $\quad \rho \propto \frac{E}{a^3}$ $\quad E^2 = m^2 + \vec{p}^{\,2}$

$\rho \propto \frac{1}{a}$

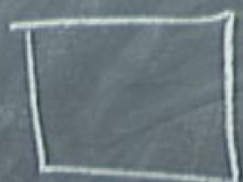
$\rho \propto a^{-3}$

a^3


Q
$q = \frac{Q}{V} \propto \frac{1}{a^3}$
$n \propto \frac{1}{a^3}$
$\rho \propto \frac{E}{a^3}$
$E^2 = m^2 + \vec{p}^2$
$\rho \propto \frac{1}{a}$
$\rho_m \propto a^{-3}$
a^3
$\rho_r \propto a^{-}$
a

$Q \quad q = \frac{Q}{V} \propto \frac{1}{a^3} \qquad n \propto \frac{1}{a^3} \qquad \rho \propto \frac{E}{a^3} \qquad E^2 = m^2 + \vec{p}^{\,2}$

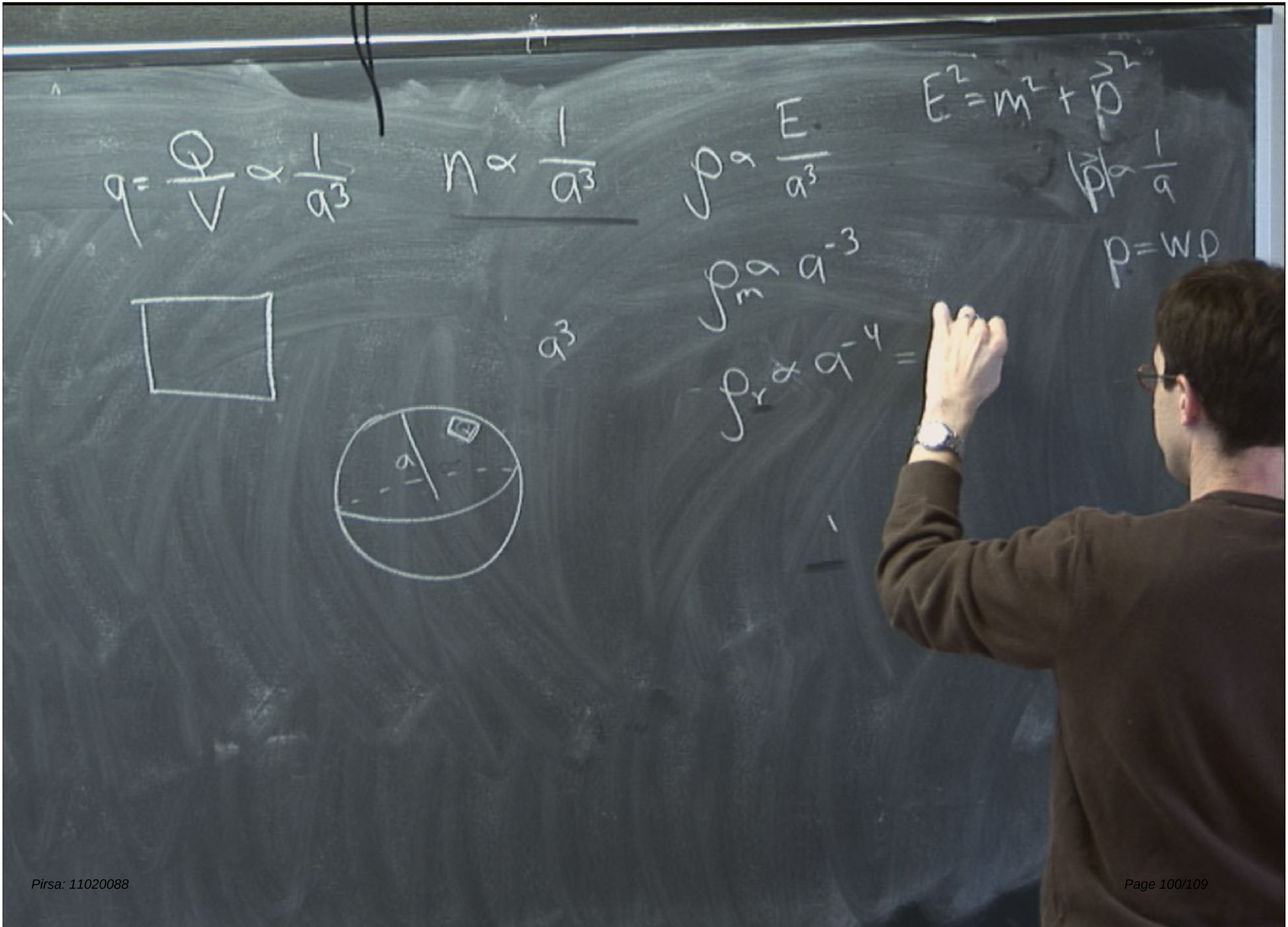
$\rho \propto \frac{1}{a}$



$\rho_m \propto a^{-3}$

a^3

$\rho_r \propto a^{-4}$


$q = \frac{Q}{V} \propto \frac{1}{a^3}$
$n \propto \frac{1}{a^3}$
$\rho \propto \frac{E}{a^3}$
$E^2 = m^2 + \vec{p}^2$
$|\vec{p}| \propto \frac{1}{a}$
$p = w\rho$
$\rho_m \propto a^{-3}$
a^3
$\rho_r \propto a^{-4} =$
a

$q = \frac{Q}{V} \propto \frac{1}{a^3}$

$n \propto \frac{1}{a^3}$

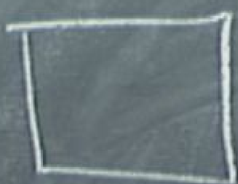
$\rho \propto \frac{E}{a^3}$

$E^2 = m^2 + \vec{p}^{\,2}$

$|\vec{p}| \propto \frac{1}{a}$

$\rho_m \propto a^{-3}$

$p = w\rho$



a^3

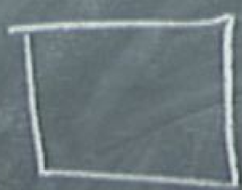
$\rho_r \propto a^{-4} = a^{-3(1+w)}$

$$q = \frac{Q}{V} \propto \frac{1}{a^3} \qquad n \propto \frac{1}{a^3} \qquad \rho \propto \frac{E}{a^3} \qquad E^2 = m^2 + \vec{p}^{\,2}$$

$$|\vec{p}| \propto \frac{1}{a}$$

$$p = w\rho$$

$$\rho_m \propto a^{-3}$$



a^3

$$\rho_r \propto a^{-4} = a^{-3(1+w)}$$

$q = \frac{Q}{V} \propto \frac{1}{a^3}$

$n \propto \frac{1}{a^3}$

$\rho \propto \frac{E}{a^3}$ $\boxed{w=0}$

$E^2 = m^2 + \vec{p}^{\,2}$

$|\vec{p}| \propto \frac{1}{a}$

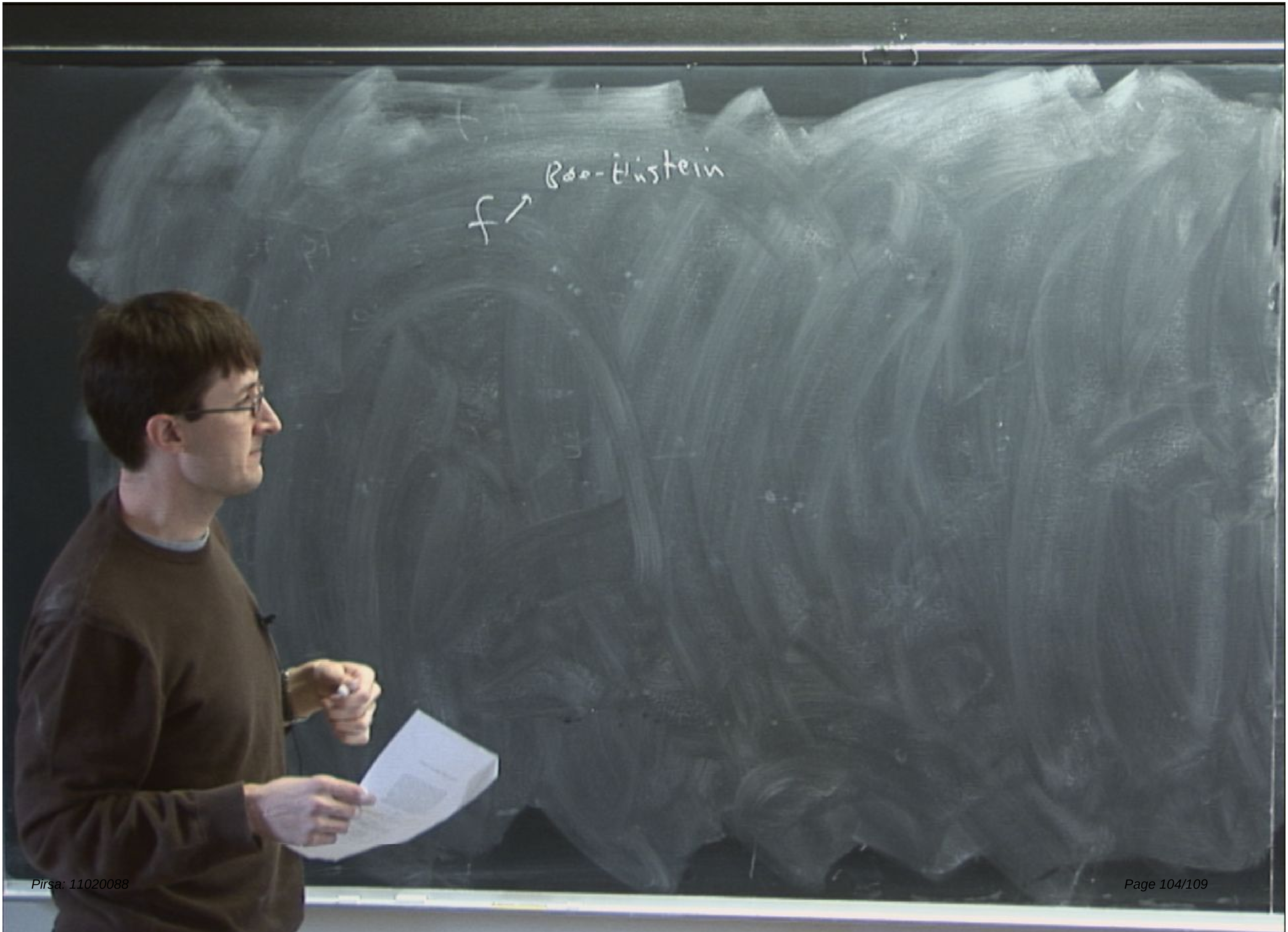
$\rho_m \propto a^{-3} = a^{-3(1+w)}$

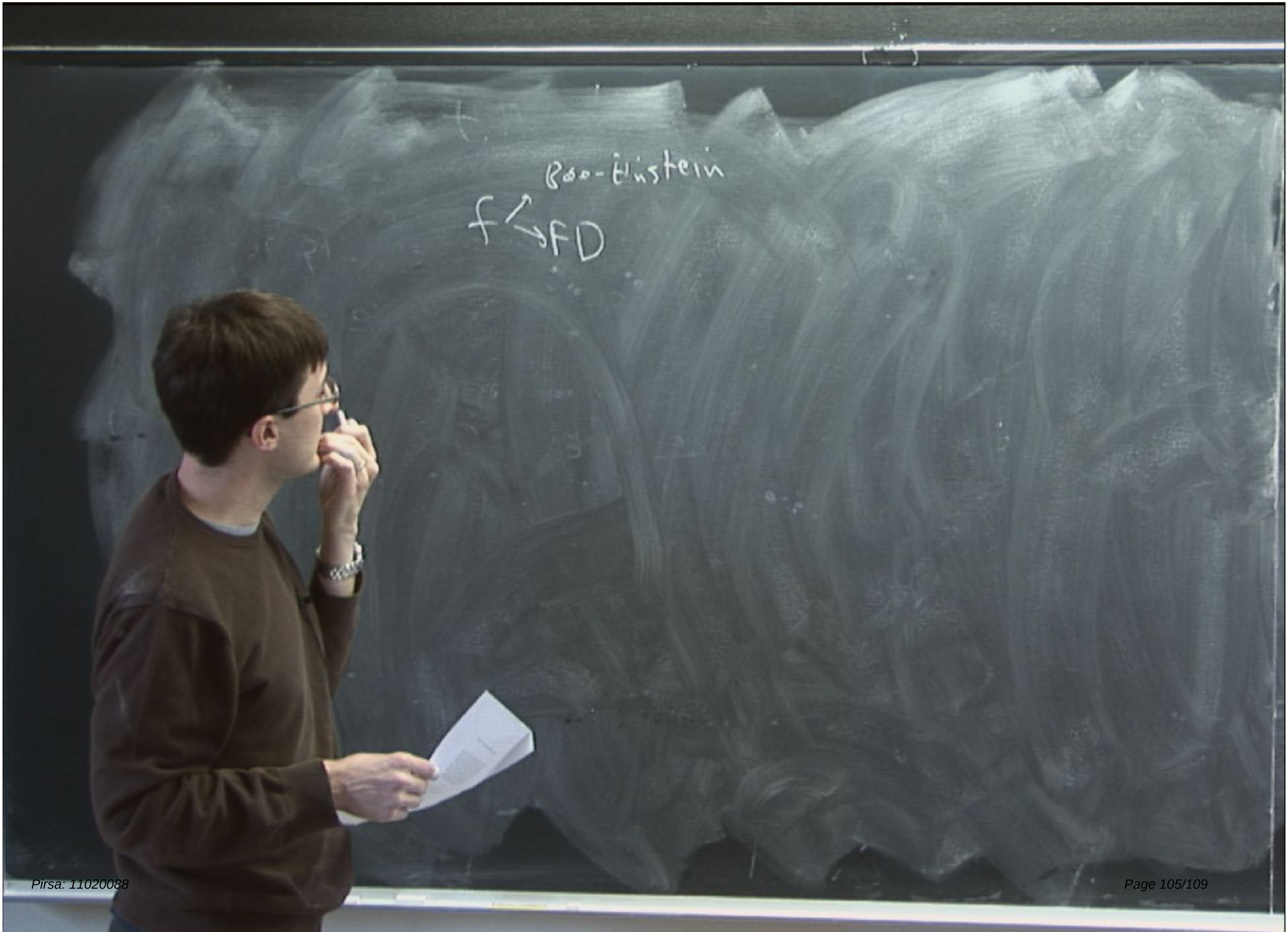
$p = w\rho$

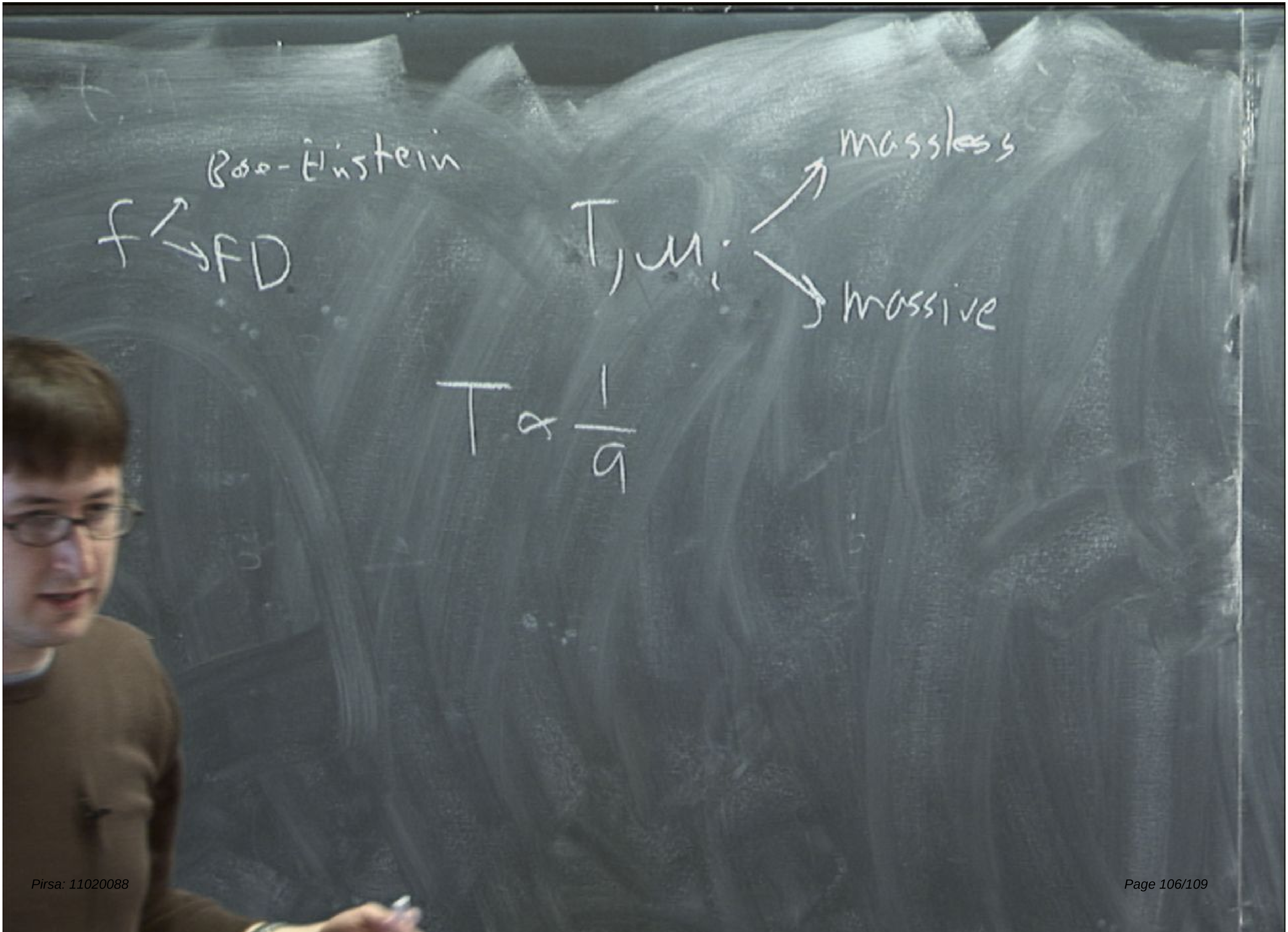
a^3

$\rho_r \propto a^{-4} = a^{-3(1+w)}$

$\boxed{w=\frac{1}{3}}$


Bose-Einstein
f


Bose-Einstein
f
FD


Bose-Einstein
f
FD
T, μi
massless
massive
T ∝ 1/a

Bose-Einstein

f → FD

T, μ_i → massless

→ massive

$$T \propto \frac{1}{a}$$

Bose-Einstein

$f \nearrow \searrow FD$

T, m_i : massless / massive

$$T \propto \frac{1}{a}$$

Bose-Einstein

$f \nearrow$ / $\searrow$ FD

T, m_i : $\nearrow$ massless / $\searrow$ massive

$$T \propto \frac{1}{a}$$

$$p \propto \frac{1}{a}$$