Title: On the instability of general relativistic wormholes

Date: Feb 17, 2011 01:00 PM

URL: http://pirsa.org/11020084

Abstract: Assuming exotic matter, several models representing static, spherically symmetric wormhole solutions of Einstein's field equations have been considered in the literature. We examine the dynamical stability of such wormholes in one of the simplest model, in which the matter is described by a massless ghost scalar field, and prove that all solutions are unstable with respect to linear fluctuations and possess precisely one unstable, exponentially in time growing mode. Numerical simulations of the nonlinear field equations suggest that these wormholes either expand or collapse and form a black hole. The stability problem for alternative models including electrically charged wormholes is also discussed.

On the instability of wormholes supported by ghost scalar fields

Perimeter Institute, Waterloo, Canada
February 17, 2011

Olivier Sarbach

Instituto de Física y Matemáticas

Universidad Michoacana de San Nicolás de Hidalgo

Morelia, México

Plan of the talk

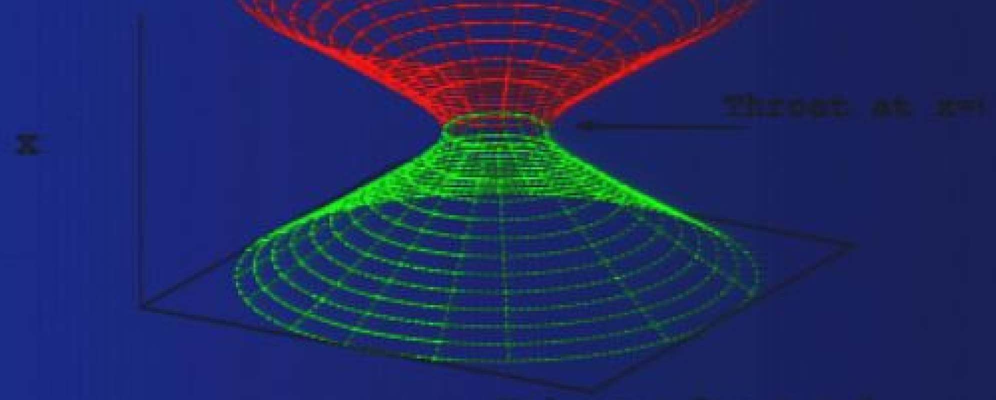
- Introduction
- A simple wormhole model
- Linear fluctuations and instability
- Nonlinear evolution
- Charging the wormholes
- Wormholes with exotic dust
- Conclusions

Collaborators at IFM in Morelia:
José A. González, Francisco S. Guzmán, Thomas Zannias

Introduction

Wormholes: Asymptotically flat solutions of Einstein's field equations with spacetime topology $\mathbb{R}_t \times \Sigma$, where Σ is a three-dimensional Riemannian manifold with nontrivial topology.

Example: $\Sigma = \mathbb{R} \times S^2$ (two asymptotic ends):

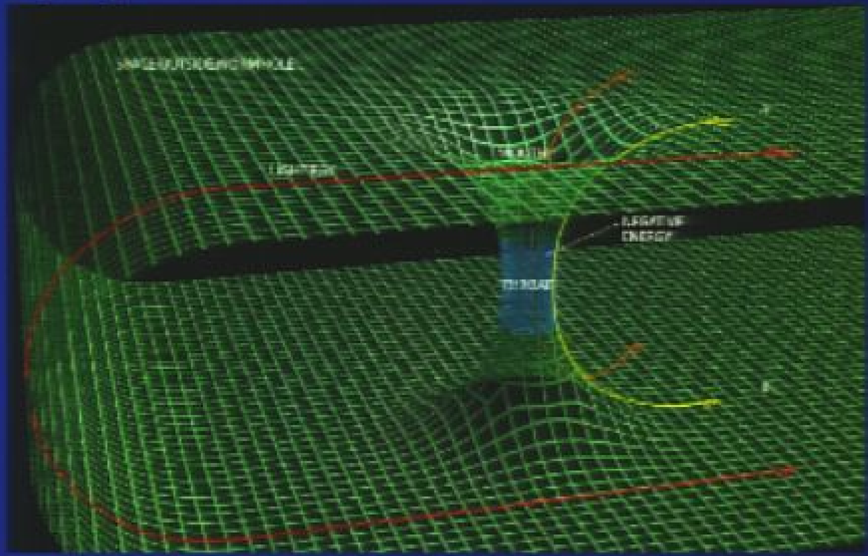


Traversability: Causal contact between the two universes.

Introduction

Wormholes: Asymptotically flat solutions of Einstein's field equations with spacetime topology $\mathbb{R}_t \times \Sigma$, where Σ is a three-dimensional Riemannian manifold with nontrivial topology.

Examples: One asymptotic end:



Interstellar travel, time machines

Morris, Thorne, Yurtsever, PRL **61**, 1446 (1988)

Introduction

Topological censorship theorem

Friedman, Schleich, Witt, PRL, 71, 1486 (1993)

"In a globally hyperbolic, asymptotically flat spacetime satisfying Einstein's field equations and the (averaged) null energy condition, every causal curve from $\mathcal{J}^-$ to $\mathcal{J}^+$ is continuously deformable to $\mathcal{J}$."

Null energy condition:

$T_{\mu\nu}k^\mu k^\nu \geq 0$ for all null vectors k^μ.

Therefore, in order to construct a wormhole, it is necessary to violate the null energy condition. In particular, it is necessary to violate the weak energy condition which states that the energy density is nonnegative *for all* timelike observers.

Introduction

- The existence of wormholes in GR requires exotic matter.
- Cosmological observations (dark energy models)
- Quantum effects
  see, for instance, Flanagan and Wald, PRD **54**, 6233 (1996).
- For the following, we assume the existence of exotic matter and analyze the classical dynamical stability of wormholes.
- For this, we fix some theory where stationary wormholes exist, and ask whether or not they are stable with respect to small perturbations of the fields in this theory.

A simple model

A simple model for exotic matter: A ghost scalar field

$$R_{\mu\nu} = \kappa \, \nabla_\mu \Phi \cdot \nabla_\nu \Phi,$$
$$g^{\mu\nu} \nabla_\mu \nabla_\nu \Phi = 0,$$

- Φ: A real scalar field
- $R_{\mu\nu}$: Ricci tensor associated to the spacetime metric $g_{\mu\nu}$.
- ∇_μ: Covariant derivative associated to $g_{\mu\nu}$.
- $\kappa = -8\pi G$: negative coupling constant.

A simple model

A simple model for exotic matter: A ghost scalar field

$$R_{\mu\nu} = \kappa \, \nabla_\mu \Phi \cdot \nabla_\nu \Phi,$$
$$g^{\mu\nu} \nabla_\mu \nabla_\nu \Phi = 0,$$

- Φ: A real scalar field
- $R_{\mu\nu}$: Ricci tensor associated to the spacetime metric $g_{\mu\nu}$.
- ∇_μ: Covariant derivative associated to $g_{\mu\nu}$.
- $\kappa = -8\pi G$: negative coupling constant.

Violation of the null energy condition:

$R_{\mu\nu} k^\mu k^\nu = \kappa (k^\mu \nabla_\mu \Phi)^2 < 0$ if $k^\mu \nabla_\mu \Phi \neq 0$ for a null vector k^μ.

A simple model

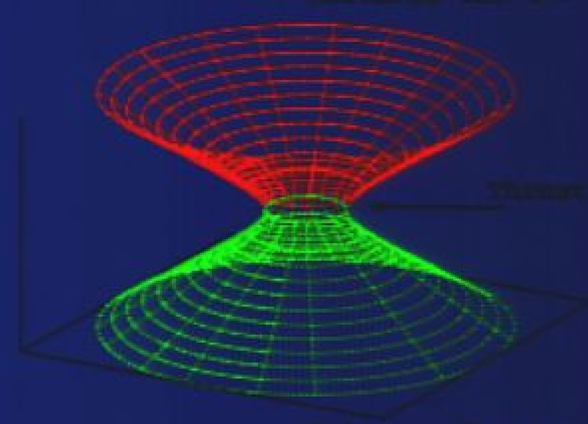
Static, spherically symmetric solutions

Ansatz:

$$ds^2 = -e^{-2a}dt^2 + e^{2a}dx^2 + e^{2c}\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right),$$

where a, c y Φ depend on the coordinate $x \in (-\infty, \infty)$ only.

- $r = e^c$: geometric radius
- Asymptotically flat, if $e^a \to 1$ and $r^2/x^2 \to 1$, $\Phi_x \to 0$ for $x \to \infty$ or $x \to -\infty$.
- Regular and traversable if a, c and Φ are regular for all x.



A simple model

A simple model for exotic matter: A ghost scalar field

$$R_{\mu\nu} = \kappa \nabla_\mu \Phi \cdot \nabla_\nu \Phi,$$
$$g^{\mu\nu} \nabla_\mu \nabla_\nu \Phi = 0,$$

- Φ: A real scalar field
- $R_{\mu\nu}$: Ricci tensor associated to the spacetime metric $g_{\mu\nu}$.
- ∇_μ: Covariant derivative associated to $g_{\mu\nu}$.
- $\kappa = -8\pi G$: negative coupling constant.

Violation of the null energy condition:

$R_{\mu\nu} k^\mu k^\nu = \kappa (k^\mu \nabla_\mu \Phi)^2 < 0$ if $k^\mu \nabla_\mu \Phi \neq 0$ for a null vector k^μ.

A simple model

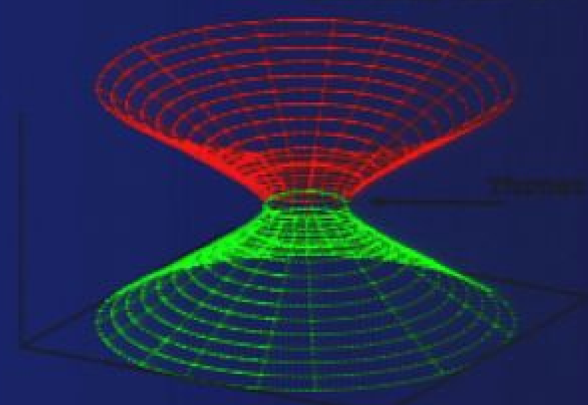
Static, spherically symmetric solutions

Ansatz:

$$ds^2 = -e^{-2a}dt^2 + e^{2a}dx^2 + e^{2c}\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right),$$

where a, c y Φ depend on the coordinate $x \in (-\infty, \infty)$ only.

- $r = e^c$: geometric radius
- Asymptotically flat, if $e^a \to 1$ and $r^2/x^2 \to 1$, $\Phi_x \to 0$ for $x \to \infty$ or $x \to -\infty$.
- Regular and traversable if a, c and Φ are regular for all x.



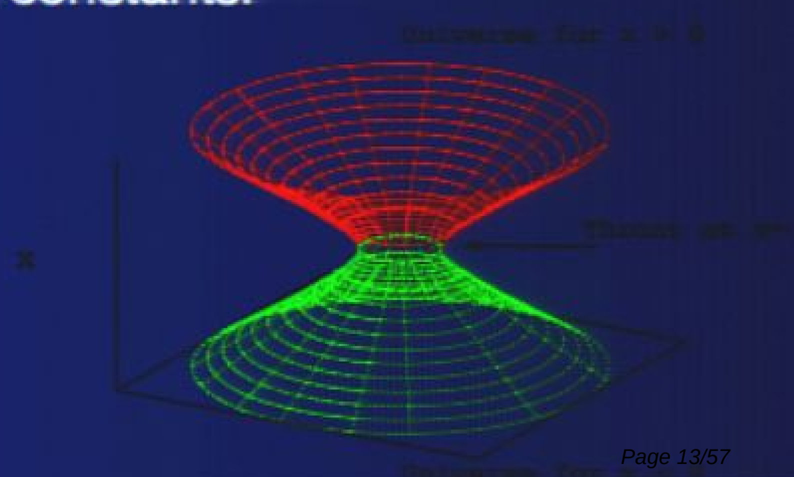
A simple model

Solutions (H.G. Ellis 1973, K.A. Bronnikov 1973)

$$ds^2 = -e^{2\gamma_1 \arctan(x/b)+2\gamma_0} dt^2$$
$$+e^{-2\gamma_1 \arctan(x/b)-2\gamma_0} \left[dx^2 + (x^2+b^2)\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right)\right],$$
$$\Phi(x) = \sqrt{\frac{2(1+\gamma_1^2)}{-\kappa}} \arctan\left(\frac{x}{b}\right),$$

where γ_1, γ_0 and $b > 0$ are integration constants.

- Scale freedom:
  $t \mapsto e^{-\Omega} t$, $x \mapsto e^{\Omega} x$, $b \mapsto e^{\Omega} b$,
  $\gamma_0 \mapsto \gamma_0 + \Omega$, $\gamma_1 \mapsto \gamma_1$.
- Invariants: $B := b e^{-\gamma_0}$, γ_1.



A simple model

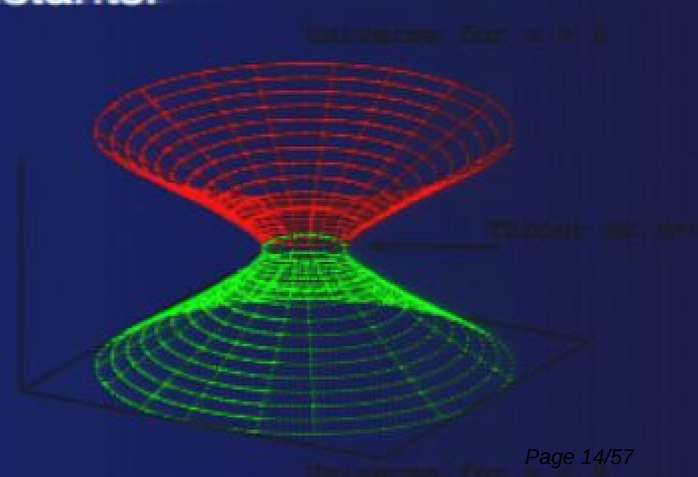
Solutions (H.G. Ellis 1973, K.A. Bronnikov 1973)

$$ds^2 = -e^{2\gamma_1 \arctan(x/b) + 2\gamma_0} dt^2$$
$$+ e^{-2\gamma_1 \arctan(x/b) - 2\gamma_0} \left[dx^2 + (x^2 + b^2)\left(d\vartheta^2 + \sin^2\vartheta \, d\varphi^2\right)\right],$$

$$\Phi(x) = \sqrt{\frac{2(1+\gamma_1^2)}{-\kappa}} \arctan\left(\frac{x}{b}\right),$$

where γ_1, γ_0 and $b > 0$ are integration constants.

- $B = be^{-\gamma_0}$.
- ADM mass $m_\infty = \gamma_1 B e^{-\gamma_1 \pi/2}$,
  ADM mass $m_{-\infty} = -\gamma_1 B e^{\gamma_1 \pi/2}$.
- Areal radius of the throat
  $r_{throat} = B\sqrt{1+\gamma_1^2}\, e^{-\gamma_1 \arctan(\gamma_1)}$.



Linear stability

- C. Armendáriz-Picón, PRD **65**, 104010 (2002):
  These wormhole solutions are *linearly stable.*
  (Analytic study for the massless case $\gamma_1 = 0$ and conjecture for the massive case)
- H. Shinkai and S.A. Hayward, PRD **66**, 044005 (2002):
  Numerical evolution of an initial nonlinear perturbation of the massless solutions indicates that the wormhole is *unstable*.
- So who is right?

Linear fluctuations

Consider small, time-dependent perturbations of the form

$$\Phi(\lambda) = \Phi + \lambda\delta\Phi + \mathcal{O}(\lambda^2), \qquad \Phi = \Phi_{static}, \quad \delta\Phi := \frac{d}{d\lambda}\Phi(\lambda)\Big|_{\lambda=0},$$

and similarly for the metric coefficients d, a and c in

$$ds^2 = -e^{2d}dt^2 + e^{2a}dx^2 + e^{2c}\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right).$$

Linear fluctuations

Consider small, time-dependent perturbations of the form

$$\Phi(\lambda) = \Phi + \lambda\delta\Phi + \mathcal{O}(\lambda^2), \qquad \Phi = \Phi_{static}, \quad \delta\Phi := \frac{d}{d\lambda}\Phi(\lambda)\Big|_{\lambda=0},$$

and similarly for the metric coefficients d, a and c in

$$ds^2 = -e^{2d}dt^2 + e^{2a}dx^2 + e^{2c}\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right).$$

Infinitesimal coordinate transformation $t \mapsto t + \xi^t$, $x \mapsto x + \xi^x$,

$$\delta a \mapsto \delta a + e^{-a}(e^a\xi^x)_x\,, \qquad \delta c \mapsto \delta c + \xi^x c_x\,, \qquad \delta\Phi \mapsto \delta\Phi + \xi^x\Phi_x\,.$$

Gauge invariant quantities: $A := \delta a - e^{-a}\left(e^a\frac{\delta\Phi}{\Phi_x}\right)_x$, $C := \delta c - c_x\frac{\delta\Phi}{\Phi_x}$.

Linear fluctuations

The linearized field equations yield the following pulsation equation for the gauge-invariant quantity $\Psi := e^{2a-c}C/c_x$:

$$\Psi_{tt} - e^{-2a}(e^{-2a}\Psi_x)_x + U(x)\Psi = 0,$$

where $a = -\gamma_1 \arctan(x)$ and

$$U(x) = \frac{e^{4\gamma_1 \arctan(x)}}{1+x^2}\left[1 - \frac{(x-\gamma_1)^2}{1+x^2} + \frac{2(1+\gamma_1^2)}{(x-\gamma_1)^2}\right].$$

- Notice that the potential *diverges* at the throat $x = \gamma_1$.
- Problem: Ψ is not well-defined at the throat, where $c_x = 0$.

Linear fluctuations

Example: reflection symmetric case $\gamma_1 = 0$:

- $$\Psi_{tt} - \Psi_{xx} + U(x)\Psi = 0,$$

  with the positive potential $U(x) = \frac{1}{(1+x^2)^2} + \frac{2}{x^2(1+x^2)}$.
- The potential is positive, so one could be led to the conclusion that the wormhole is linearly stable (Armendáriz-Picón, 2002).
- On the other hand, the singularity at the throat $x = 0$ requires Ψ to decay to zero sufficiently rapidly as $x \to 0$ (mirror).
- Clearly, this condition is artificial.
- Perturbations with compact support cannot grow in time as long as the support does not contain the throat $x = 0$.

Existence of an unstable mode

Transformation to a regular equation

- Consider the Hamiltonian operator $H := -\partial^2 + U(x)$, $\partial := e^{-2a}\partial_x$.
- Zero mode (from the family of static solutions):
  $\Psi_0 = \frac{\sqrt{1+x^2}\, e^{-\gamma_1 \arctan(x)}}{x-\gamma_1} \left(1 + \gamma_1 \frac{1+\gamma_1 x}{1+\gamma_1^2} \arctan(x)\right)$.
- Ψ_0 diverges at the throat $x = \gamma_1$; however $1/\Psi_0$ is regular.
- Define the intertwining operators

$$\mathcal{A} := \partial - \frac{\partial\Psi_0}{\Psi_0}, \qquad \mathcal{A}^\dagger := -\partial - \frac{\partial\Psi_0}{\Psi_0}.$$

  Then, $H = \mathcal{A}^\dagger\mathcal{A}$.
- The quantity $\chi := \mathcal{A}\Psi$ satisfies the new equation $\chi_{tt} + \mathcal{A}\mathcal{A}^\dagger\chi = 0$, where $\mathcal{A}\mathcal{A}^\dagger = -\partial^2 + W(x)$. The new potential W is everywhere regular and decays like $1/x^2$ asymptotically.

Existence of an unstable mode

- $\chi_{tt} + \mathcal{A}\mathcal{A}^\dagger\chi = 0$, $\mathcal{A} = \partial - \frac{\partial\Psi_0}{\Psi_0}$, $\mathcal{A}^\dagger = -\partial - \frac{\partial\Psi_0}{\Psi_0}$.
- The operator $\mathcal{A}\mathcal{A}^\dagger$ possesses the zero mode

$$1/\Psi_0 = \frac{(x-\gamma_1)e^{\gamma_1 \arctan(x)}}{\sqrt{1+x^2}} \frac{1}{1+\gamma_1 \frac{1+\gamma_1 x}{1+\gamma_1^2} \arctan(x)}.$$

- For $\gamma_1 \neq 0$ this mode decays like $1/x$ for $|x| \to \infty$, hence it represents a normalizable state (eigenfunction).
- Furthermore, $1/\Psi_0$ has exactly one node (namely at the throat $x = \gamma_1$), hence it represents the first excited state.
- Therefore, the ground state has *negative* energy $E_0 = -\beta^2 < 0$, and the corresponding eigenfunctions $\chi_\beta(x)$ gives rise to an exponentially growing mode of the form $\chi(t,x) = e^{\beta t}\chi_\beta(x)$.

Existence of an unstable mode

- For $\gamma_1 = 0$ it is possible to prove that the ground state has negative energy using the Rayleigh-Ritz variational principle,

$$E_0 = \inf_{\|\chi\|=1} (\chi, \mathcal{A}\mathcal{A}^\dagger \chi).$$

- The ground state gives rise to an *exponentially in time growing solution* for the gauge-invariants A and C which is everywhere regular in space.
- Conclusion 1: All static, spherically symmetric wormholes in the theory considered are *linearly unstable*.

Existence of an unstable mode

- $\chi_{tt} + \mathcal{A}\mathcal{A}^\dagger \chi = 0$, $\mathcal{A} = \partial - \frac{\partial \Psi_0}{\Psi_0}$, $\mathcal{A}^\dagger = -\partial - \frac{\partial \Psi_0}{\Psi_0}$.
- The operator $\mathcal{A}\mathcal{A}^\dagger$ possesses the zero mode

$$1/\Psi_0 = \frac{(x-\gamma_1)e^{\gamma_1 \arctan(x)}}{\sqrt{1+x^2}} \frac{1}{1+\gamma_1 \frac{1+\gamma_1 x}{1+\gamma_1^2} \arctan(x)}.$$

- For $\gamma_1 \neq 0$ this mode decays like $1/x$ for $|x| \to \infty$, hence it represents a normalizable state (eigenfunction).
- Furthermore, $1/\Psi_0$ has exactly one node (namely at the throat $x = \gamma_1$), hence it represents the first excited state.
- Therefore, the ground state has *negative* energy $E_0 = -\beta^2 < 0$, and the corresponding eigenfunctions $\chi_\beta(x)$ gives rise to an exponentially growing mode of the form $\chi(t,x) = e^{\beta t}\chi_\beta(x)$.

Existence of an unstable mode

- For $\gamma_1 = 0$ it is possible to prove that the ground state has negative energy using the Rayleigh-Ritz variational principle,

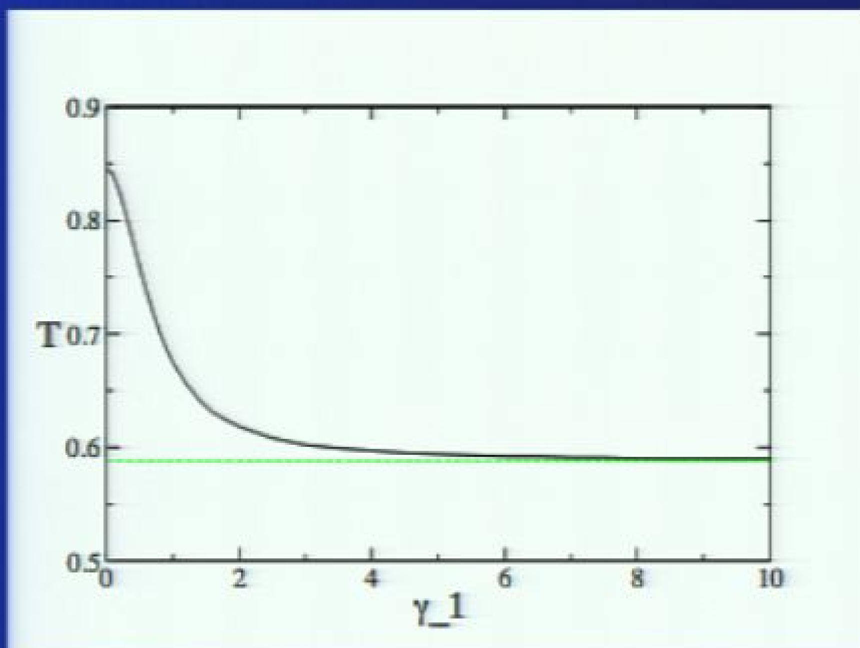
$$E_0 = \inf_{\|\chi\|=1} (\chi, \mathcal{A}\mathcal{A}^\dagger \chi).$$

- The ground state gives rise to an *exponentially in time growing solution* for the gauge-invariants A and C which is everywhere regular in space.
- Conclusion 1: All static, spherically symmetric wormholes in the theory considered are *linearly unstable*.

Time scale

- Unstable mode: solution proportional to $e^{\beta t}$, $\beta = \sqrt{-E_0}$.
- In terms of proper time at the throat,

$$\tau_{unstable} = \frac{e^{2\gamma_1 \arctan(\gamma_1)}}{\sqrt{1+\gamma_1^2}} \frac{r_{throat}}{\beta}.$$



A numerical shooting procedure gives the following values for $T = \tau_{unstable}/r_{throat}$

(The green line is a theoretical prediction for $\gamma_1 = \infty$.)

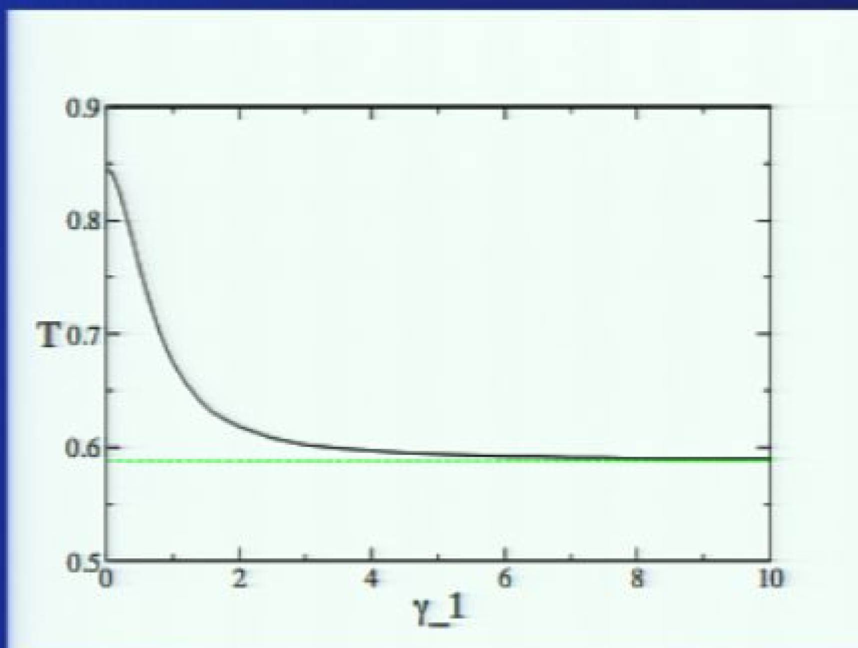
Time scale

- Conclusion 2: The time scale is of the order of r_{throat}/c.
- So the wormholes are "very" unstable.
- Example: $r_{throat} = 1km$, $\tau_{unstable} \simeq 5\mu s$.

Time scale

- Unstable mode: solution proportional to $e^{\beta t}$, $\beta = \sqrt{-E_0}$.
- In terms of proper time at the throat,

$$\tau_{unstable} = \frac{e^{2\gamma_1 \arctan(\gamma_1)}}{\sqrt{1+\gamma_1^2}} \frac{r_{throat}}{\beta} .$$



A numerical shooting procedure gives the following values for $T = \tau_{unstable}/r_{throat}$

(The green line is a theoretical prediction for $\gamma_1 = \infty$.)

Time scale

- Conclusion 2: The time scale is of the order of r_{throat}/c.
- So the wormholes are “very” unstable.
- Example: $r_{throat} = 1km$, $\tau_{unstable} \simeq 5\mu s$.

Nonlinear evolution

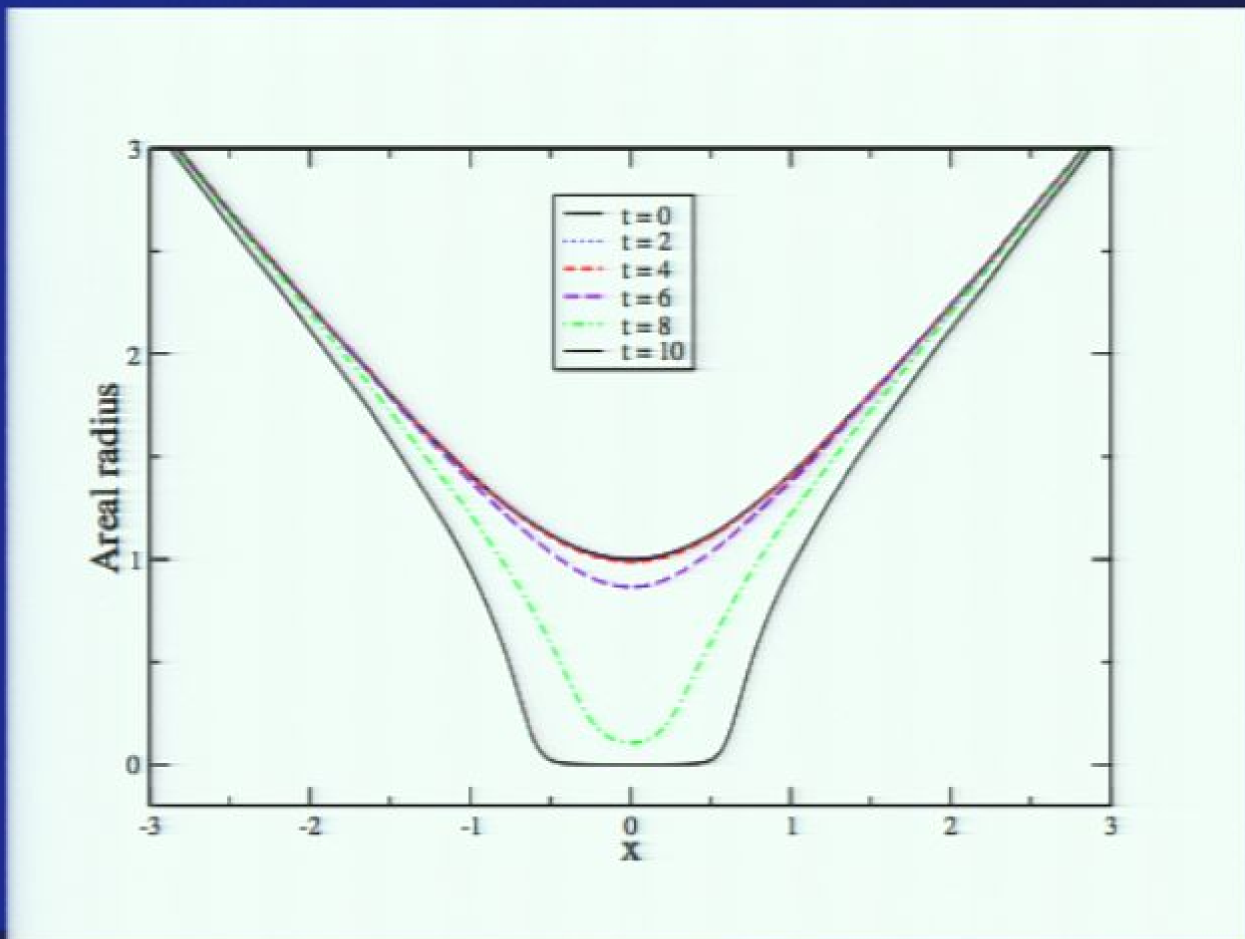
So the static, spherical symmetric wormholes supported by a massless ghost scalar field are linearly unstable.
What do they decay to?

Nonlinear evolution

- Numerical integration of the nonlinear field equations in spherical symmetry (system of $1+1$ wave equations with constraints).
- Constraint-satisfying initial data representing a static wormhole plus a small Gaussian perturbation.
- We check consistency with the time scale predicted by perturbation theory.
- Two cases: 1) The throat collapses, 2) The throat expands.

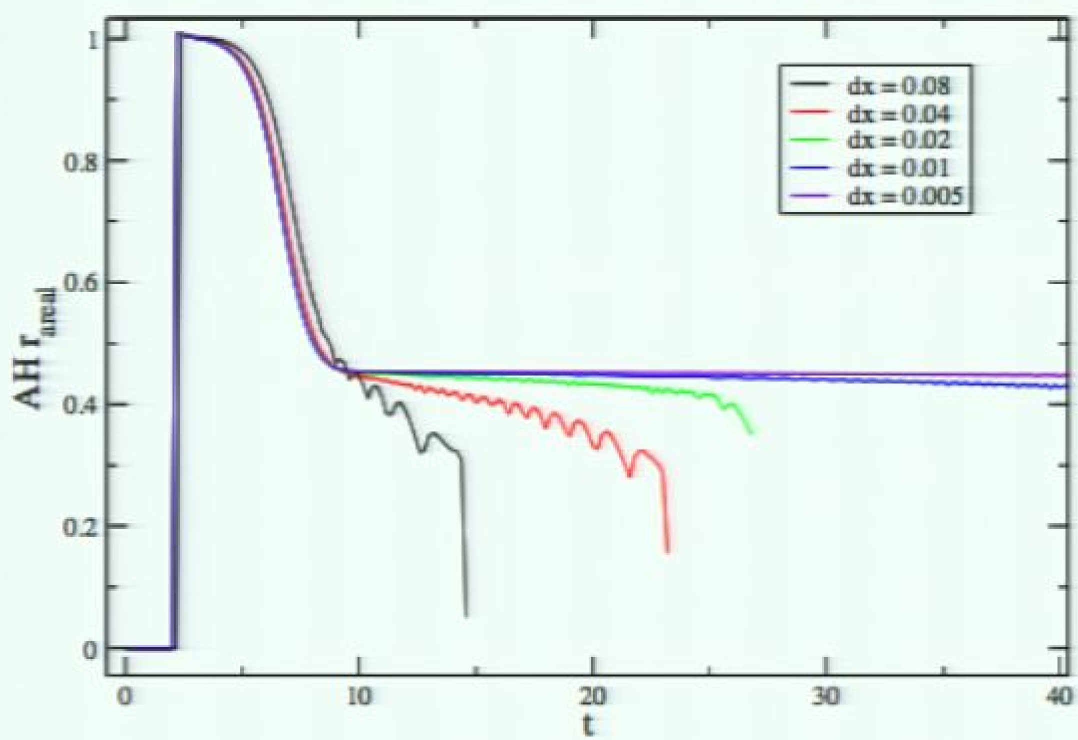
The collapsing case

The areal radius of the throat collapses.



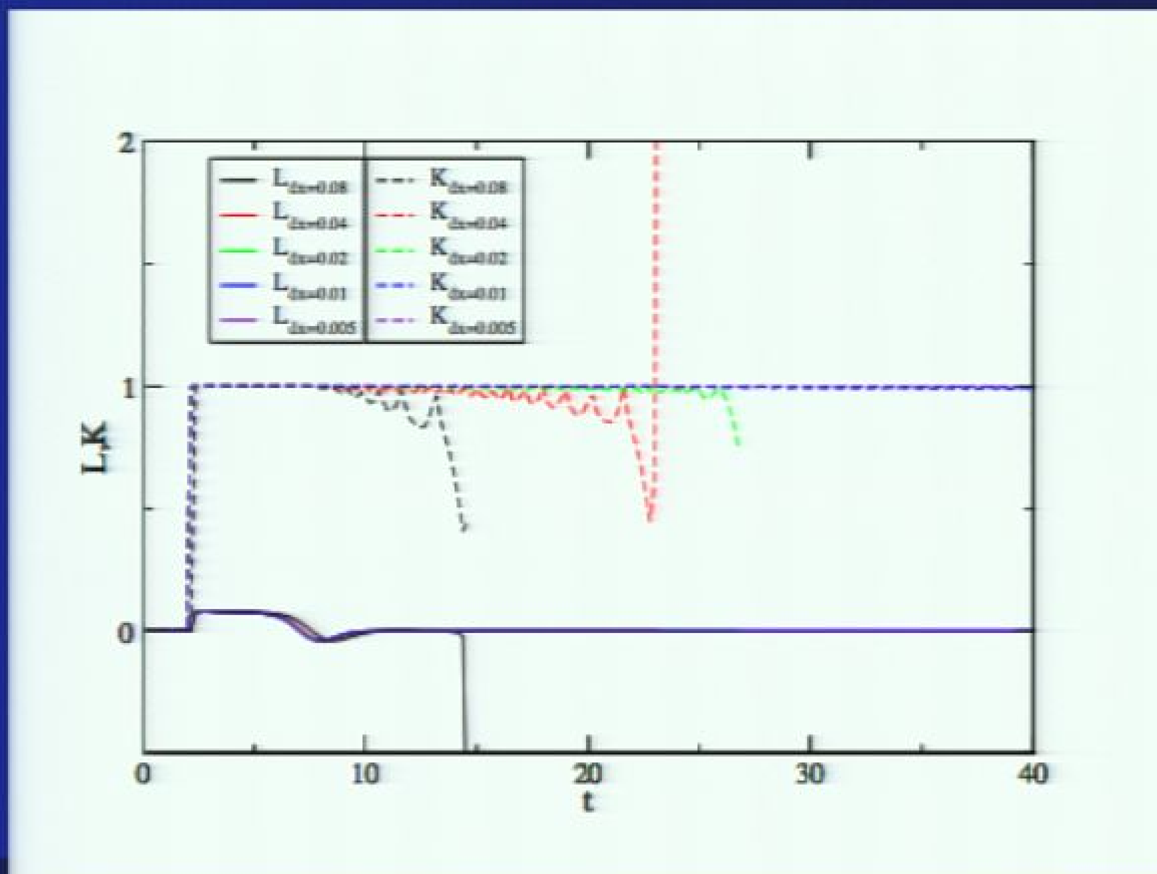
The collapsing case

We observe the formation of an *apparent horizon*.



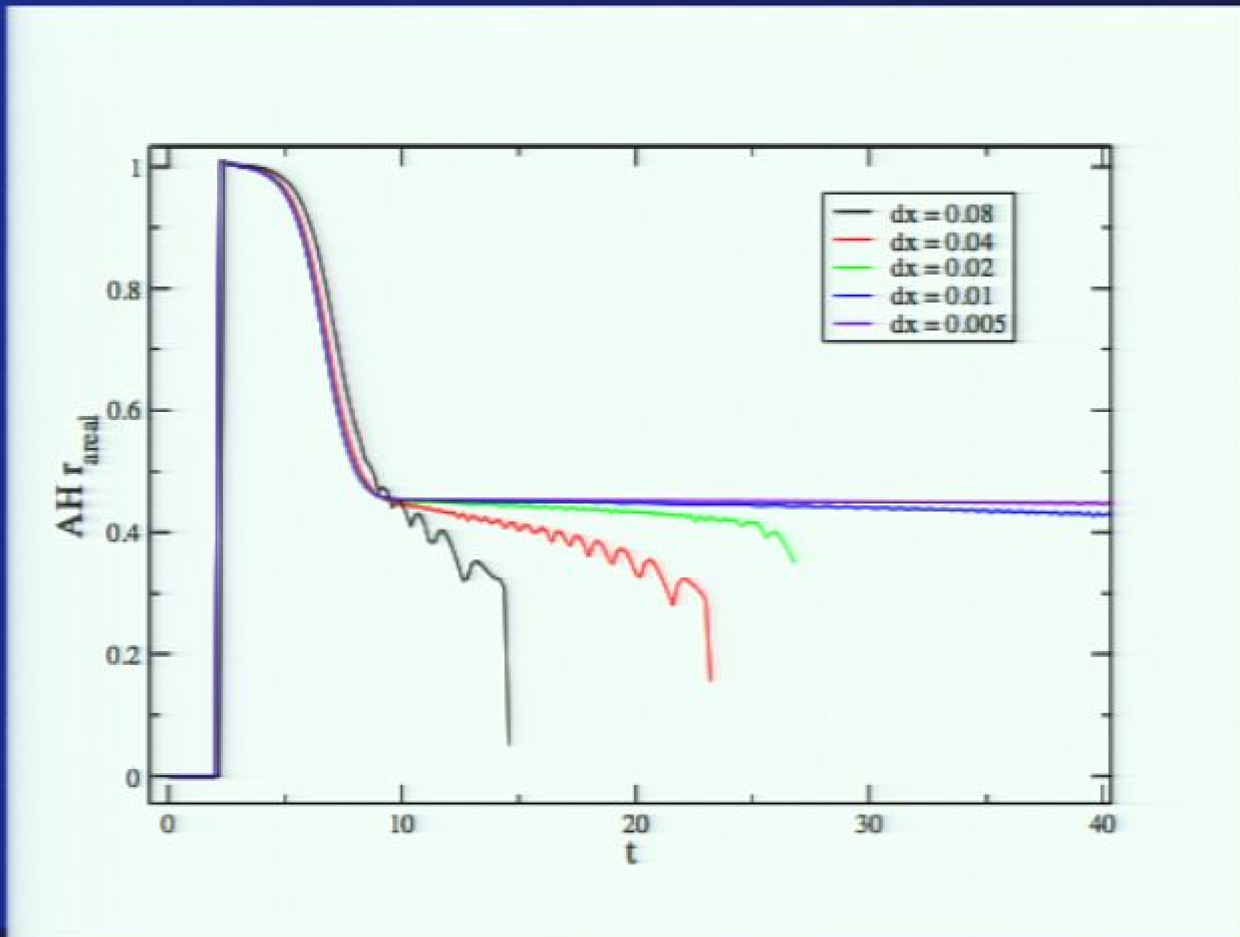
The collapsing case

Geometric invariants $L := g^{\mu\nu}\nabla_\mu\Phi\cdot\nabla_\nu\Phi$, $K := 1 - g^{\mu\nu}\nabla_\mu r\cdot\nabla_\nu r$, at the apparent horizon converge to the Schwarzschild values.



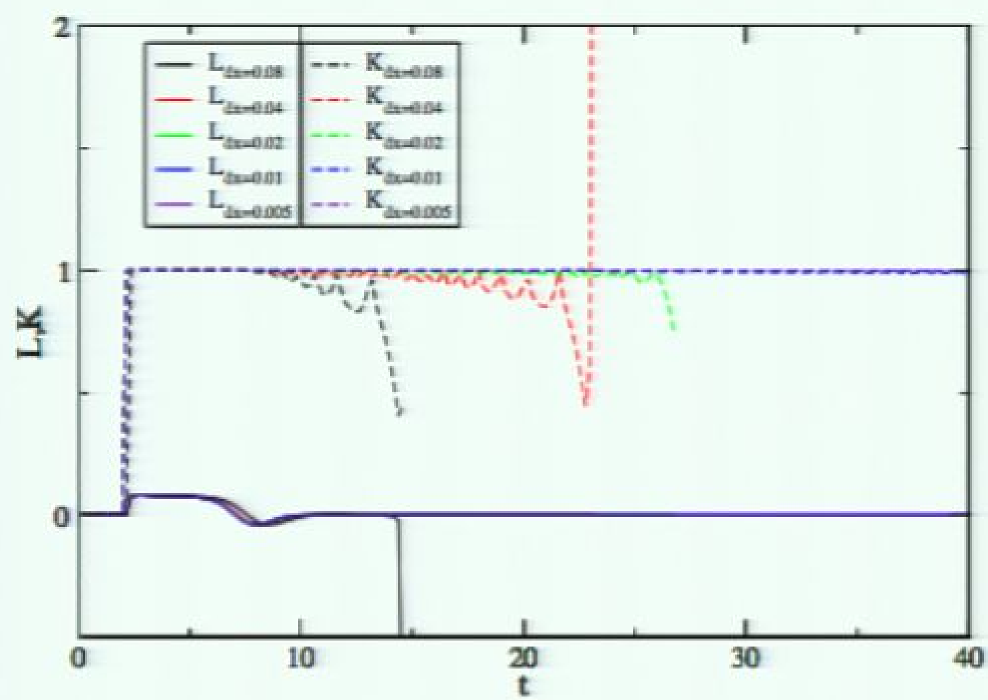
The collapsing case

We observe the formation of an *apparent horizon*.



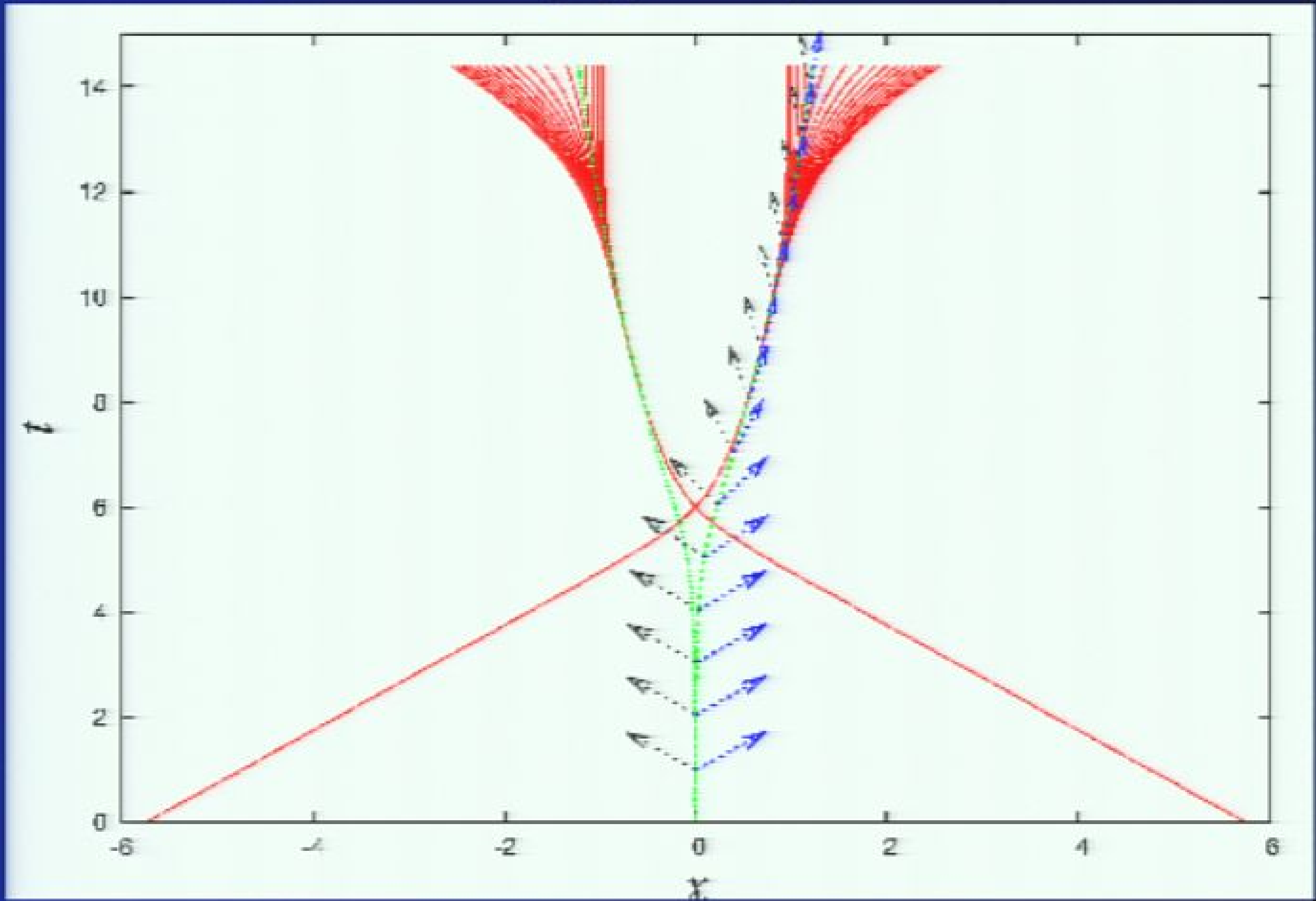
The collapsing case

Geometric invariants $L := g^{\mu\nu}\nabla_\mu \Phi \cdot \nabla_\nu \Phi$, $K := 1 - g^{\mu\nu}\nabla_\mu r \cdot \nabla_\nu r$, at the apparent horizon converge to the Schwarzschild values.



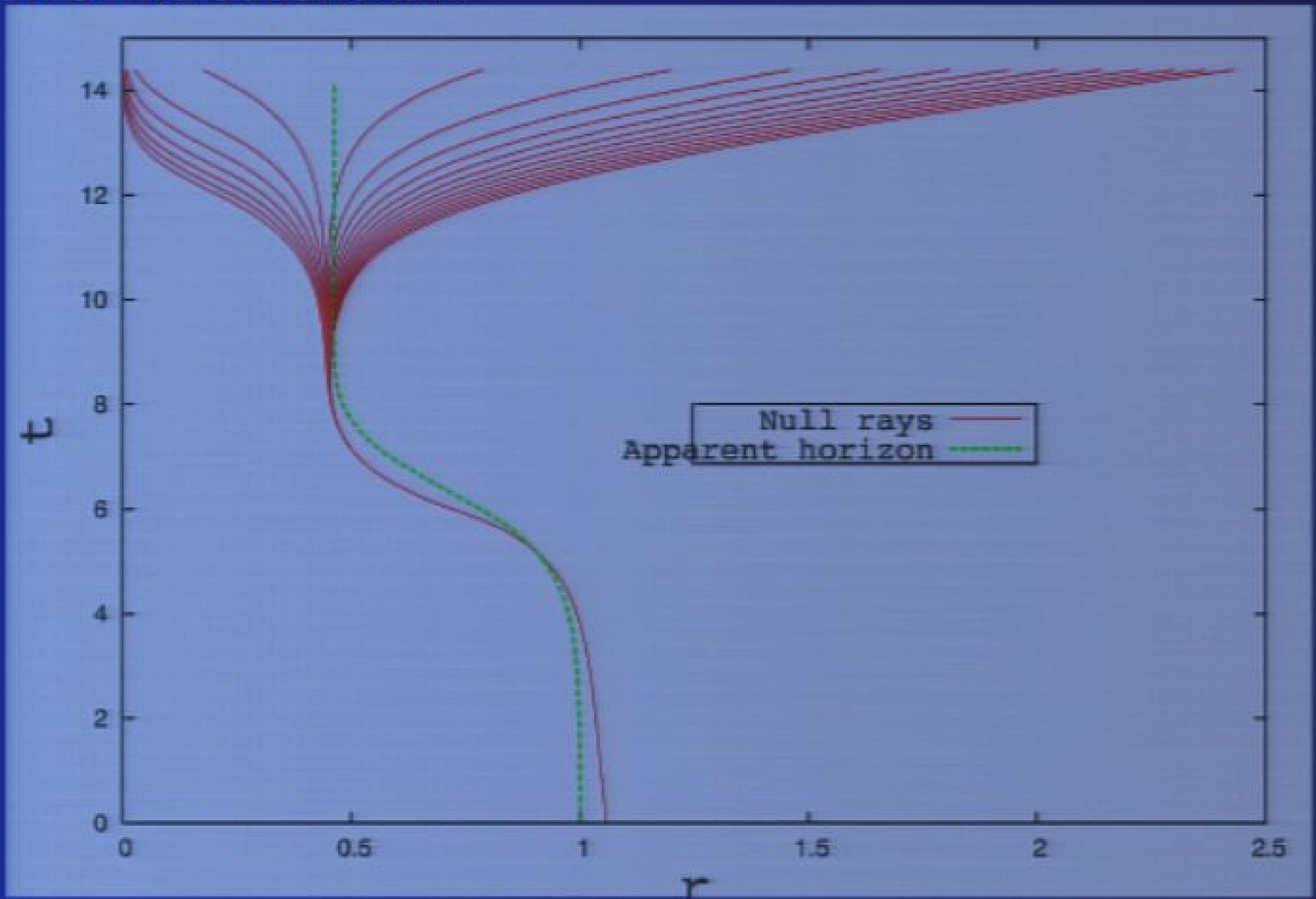
The collapsing case

Apparent horizons and converging null rays.



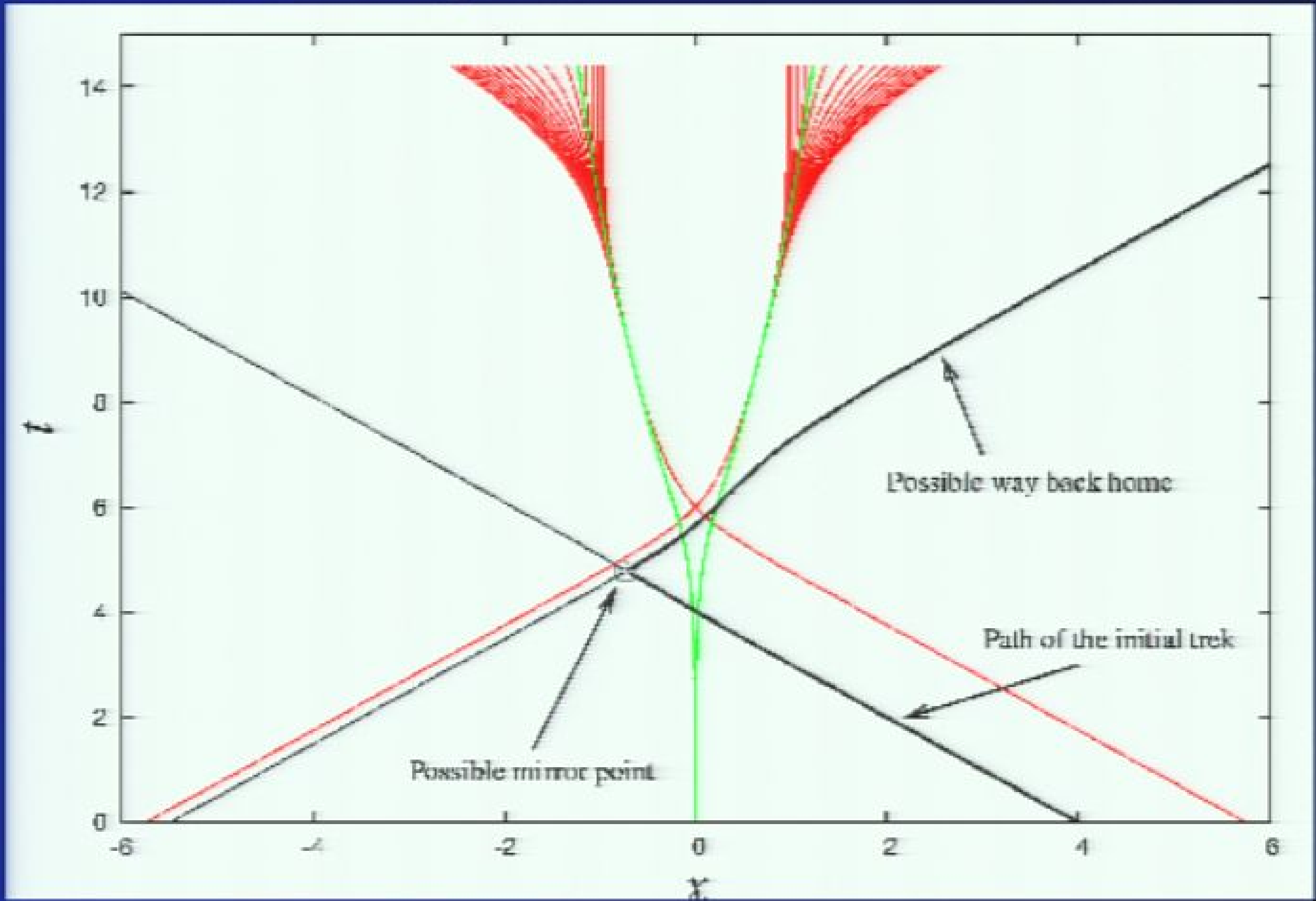
The collapsing case

Violation of the second law.



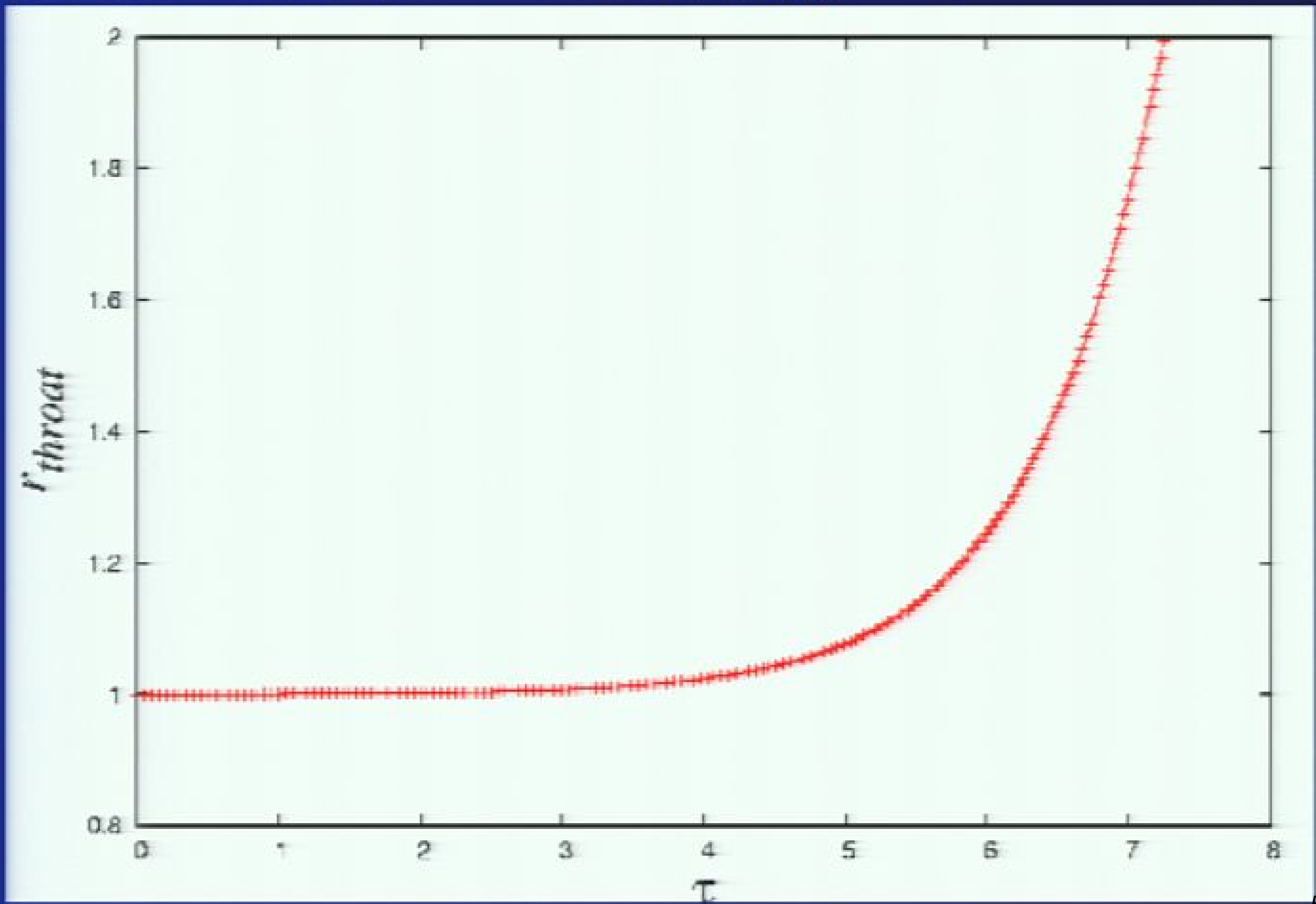
The collapsing case

How to throw a boomerang into a wormhole...



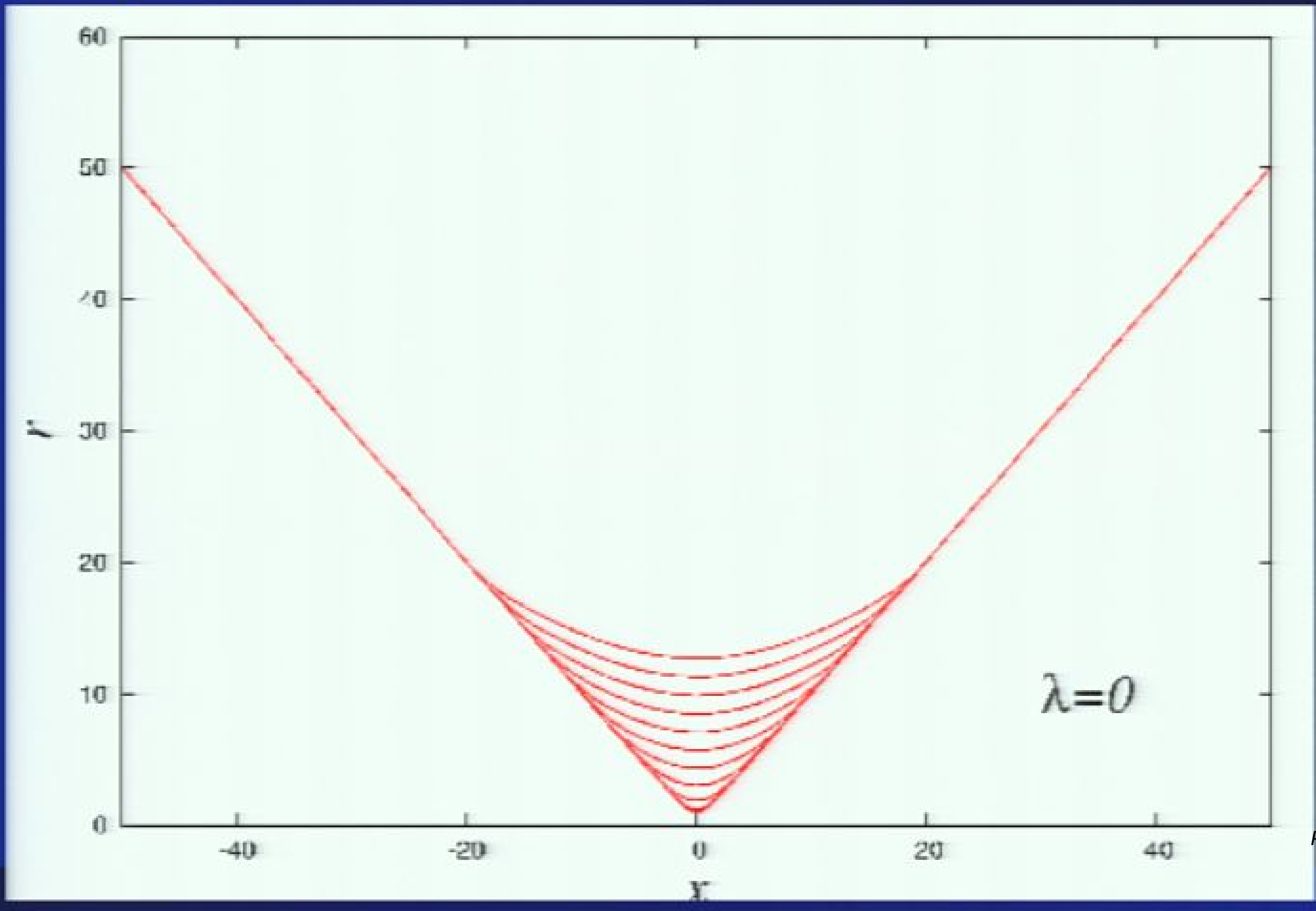
The expanding case

Wormhole seems to expand forever ($r_{throat} = Ae^{(\tau-\tau_0)/0.85}$).



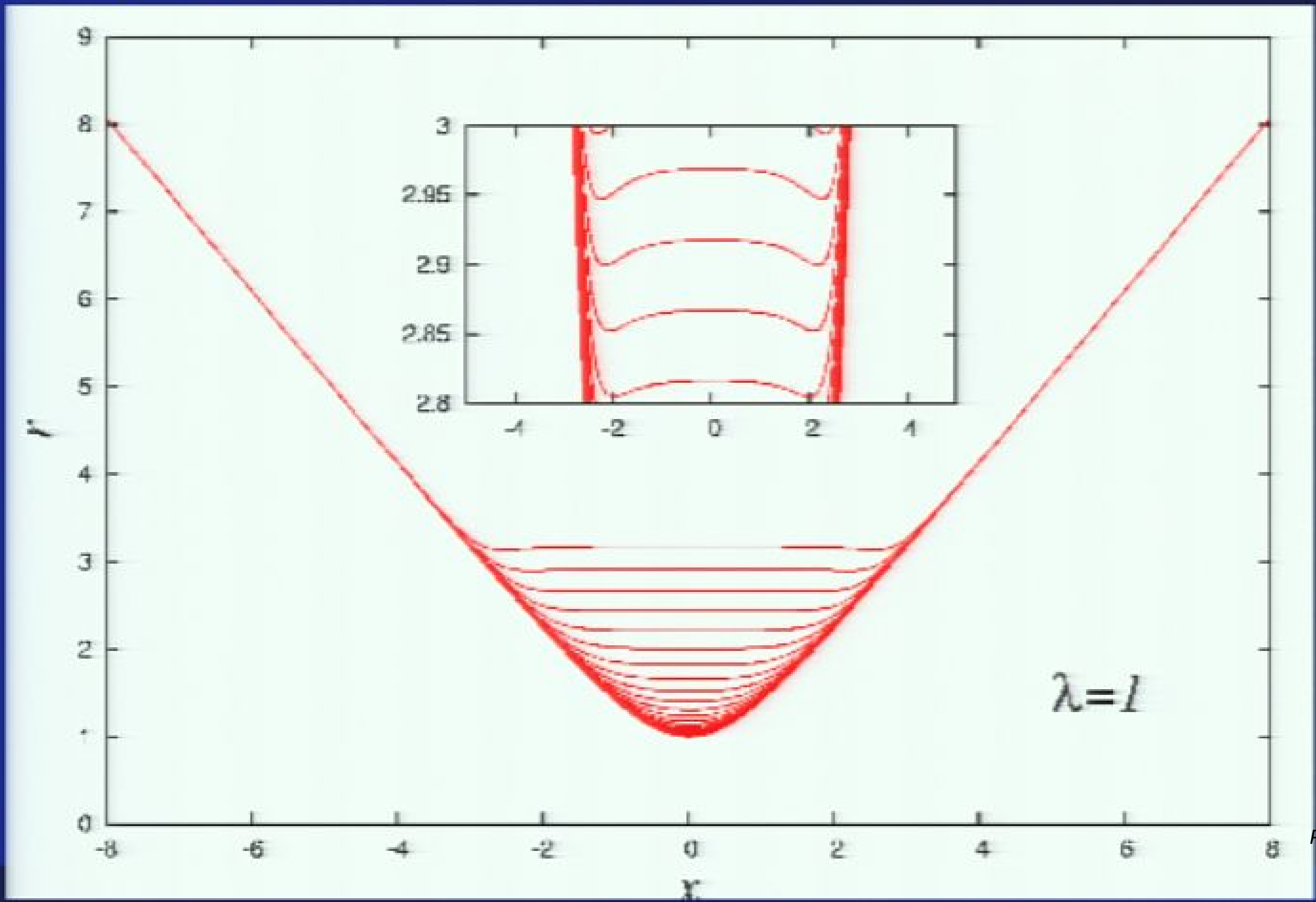
The expanding case

However, there are some subtleties on how to define the throat in an invariant way...



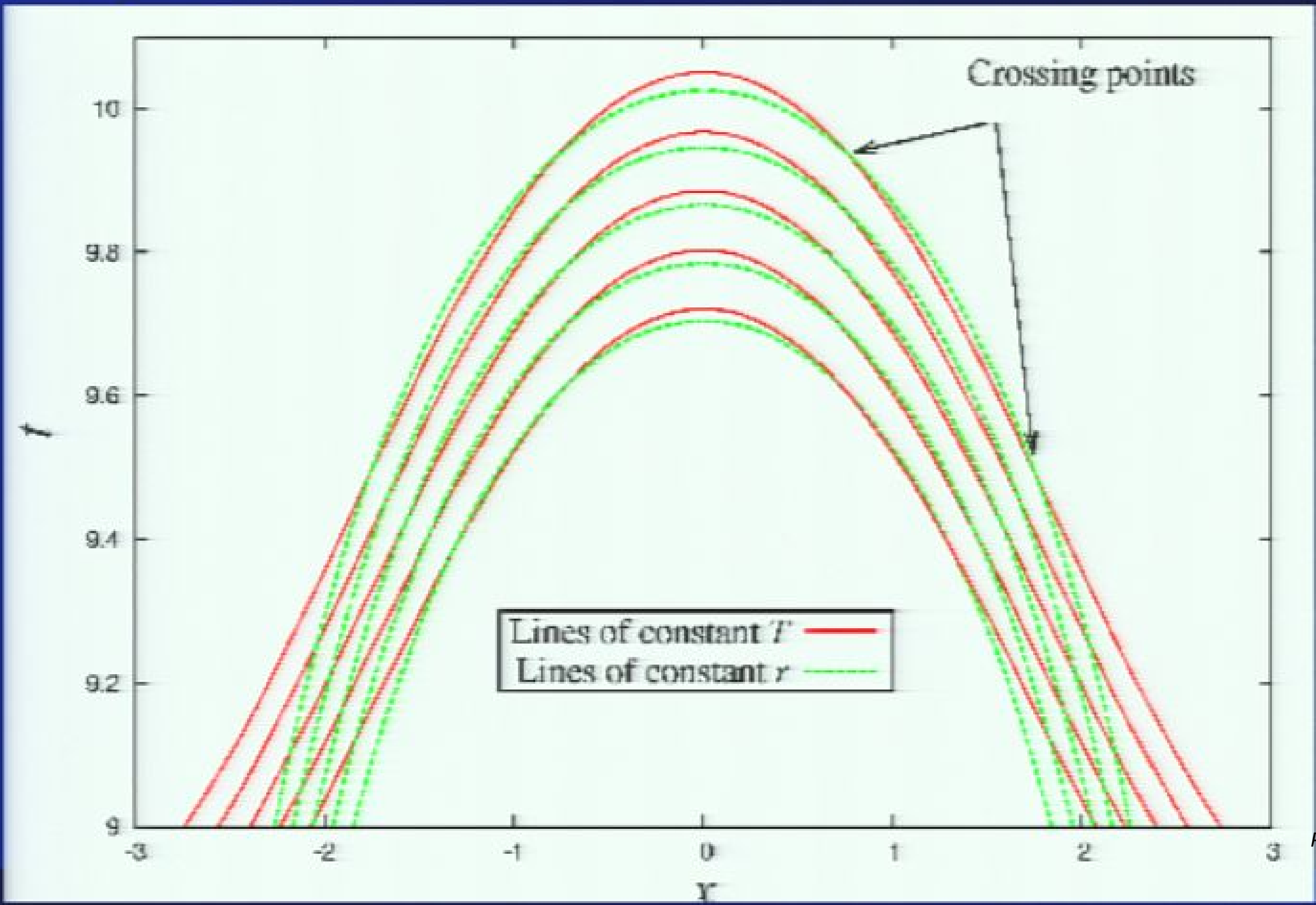
The expanding case

However, there are some subtleties on how to define the throat in an invariant way...



The expanding case

However, there are some subtleties on how to define the throat in an invariant way...

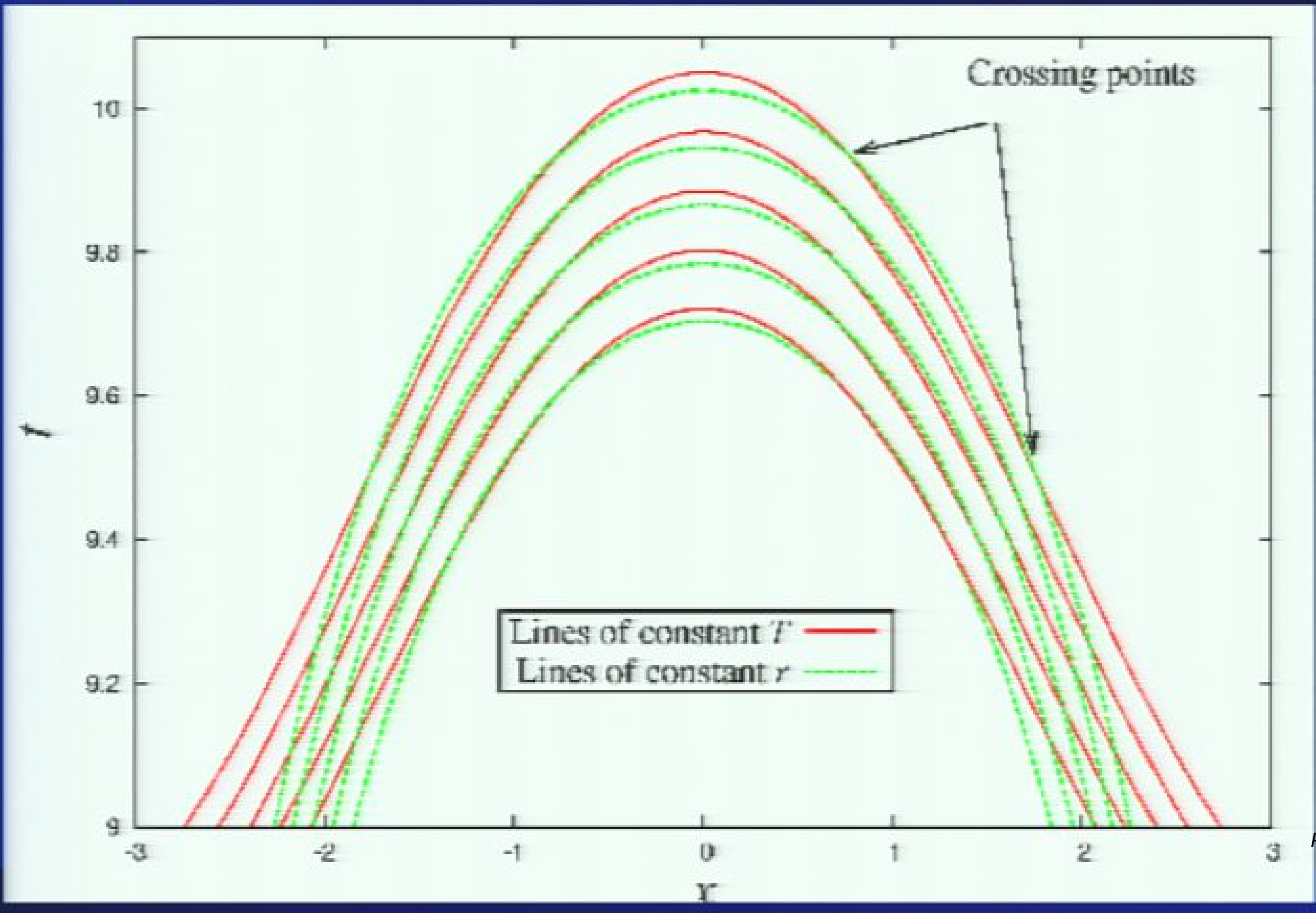


Charging the wormholes

- Is there a way to stabilize the wormholes?
- Adding angular momenta (Matos, Nuñez, Sushkov...)
- Since the resulting spacetime is not spherically symmetric anymore, the stability analysis is expected to be considerably more difficult in this case.
- Alternative: Keep spherical symmetry but add charge!

The expanding case

However, there are some subtleties on how to define the throat in an invariant way...



Charging the wormholes

- Is there a way to stabilize the wormholes?
- Adding angular momenta (Matos, Nuñez, Sushkov...)
- Since the resulting spacetime is not spherically symmetric anymore, the stability analysis is expected to be considerably more difficult in this case.
- Alternative: Keep spherical symmetry but add charge!

Charging the wormholes

- Hence, we consider, aside from the ghost scalar field, a Maxwell field. In spherically symmetry, this gives rise to a "Coulomb"-kind of field.
- Four-parameter family of static, spherically symmetric wormhole solutions:

$$\Phi = \sqrt{-2\kappa^{-1}(1+\Lambda^2)}\, y,$$
$$F = \frac{Q_e}{b} e^{2d} dt \wedge dy + Q_m\, d\vartheta \wedge \sin\vartheta\, d\varphi,$$
$$ds^2 = -e^{2d} dt^2 + e^{-2d}\left[dx^2 + (x^2+b^2)\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right)\right],$$

where $y = \arctan(x/b)$ and $e^{2d} = \left[\cosh(\Lambda y) - \gamma_1 \frac{\sinh(\Lambda y)}{\Lambda}\right]^{-2}$ with $\Lambda = \sqrt{\gamma_1^2 - \kappa_0 (Q_e^2 + Q_m^2) e^{2\gamma_0}/(2b^2)}$.

Charging the wormholes

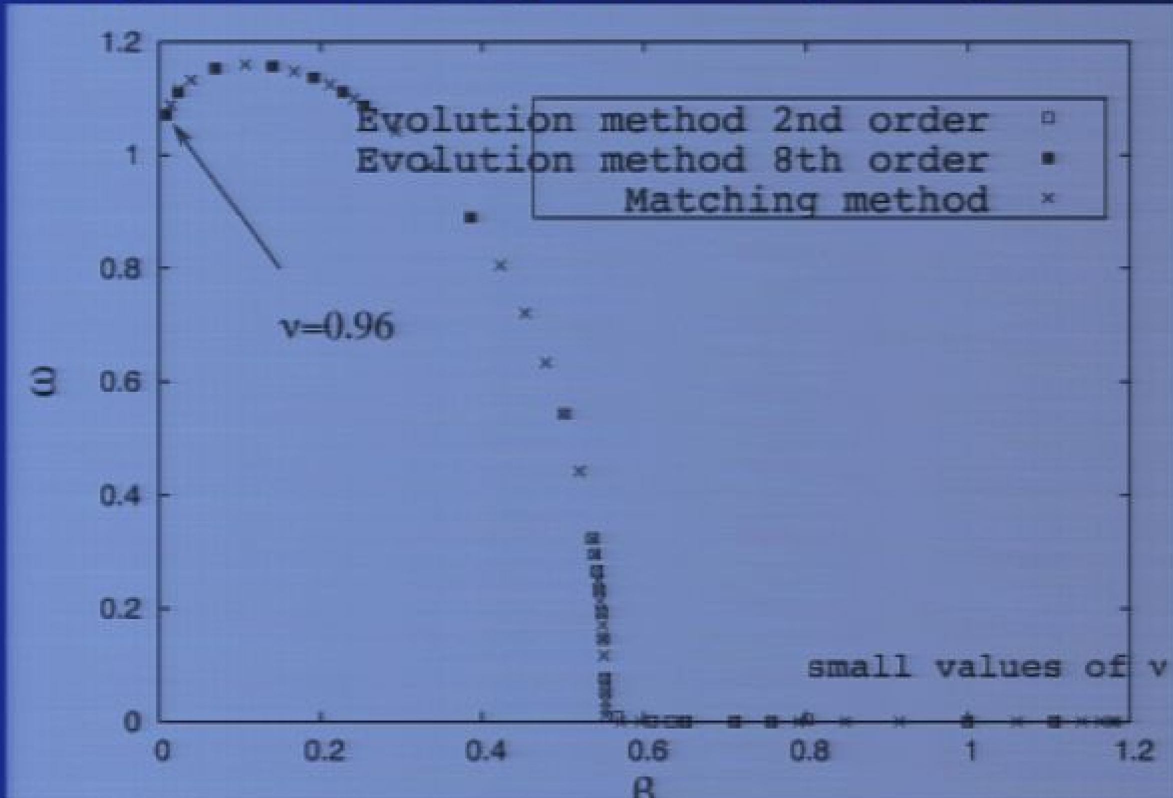
- Hence, we consider, aside from the ghost scalar field, a Maxwell field. In spherically symmetry, this gives rise to a "Coulomb"-kind of field.
- Four-parameter family of static, spherically symmetric wormhole solutions:

$$\Phi = \sqrt{-2\kappa^{-1}(1+\Lambda^2)}\, y,$$
$$F = \frac{Q_e}{b} e^{2d} dt \wedge dy + Q_m\, d\vartheta \wedge \sin\vartheta\, d\varphi,$$
$$ds^2 = -e^{2d} dt^2 + e^{-2d}\left[dx^2 + (x^2+b^2)\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right)\right],$$

where $y = \arctan(x/b)$ and $e^{2d} = \left[\cosh(\Lambda y) - \gamma_1 \frac{\sinh(\Lambda y)}{\Lambda}\right]^{-2}$ with $\Lambda = \sqrt{\gamma_1^2 - \kappa_0(Q_e^2 + Q_m^2)e^{2\gamma_0}/(2b^2)}$.

Charging the wormholes

- All solutions with $\Lambda > 0$ are linearly unstable.
- The subfamily $\gamma_1 = 0$, $\Lambda = i\nu$ has an interesting behavior: For ν large enough we numerically find a *complex* eigenvalue $\beta + i\omega$.



Wormholes with exotic dust

- Spherically symmetric metric, coupled to dust with negative density and a magnetic charge Q.
  Doroshkevich, Hansen, Novikov, Shatskii, Astr. Rep. **53**, 1079 (2009)
- Field equations reduce to a one-dimensional mechanical system for each mass shell x,

$$\frac{1}{2}\dot{r}^2 + V(r,x) = E(x), \qquad V(r,x) = -\frac{\mu(x)}{r} + \frac{Q^2}{2r^2},$$

  where $\mu(x)$ is the mass function for the dust.
- Potential well with minimum at $r = Q^2/\mu(x)$, where each shell x is in equilibrium.
- Stability?

Wormholes with exotic dust

- Unstable due to the formation of shell-crossing singularities!
- When perturbed, each shell undergoes periodic motion with period

$$T(x) = \frac{\pi}{\sqrt{2}} \frac{\mu(x)}{|E(x)|^{3/2}}.$$

- Since the period is slightly different for neighboring shells, the shells will cross eventually.
- The function $\nu(\tau, x) := r_x(\tau, x)/r_x(0, x)$, measuring the norm of the normal geodesic deviation vector field ∂_x undergoes wild oscillations. For fixed x and $n = 1, 2, 3, ...,$

$$\nu(\tau_1 + nT) = \nu(\tau_1) = const \qquad \text{at turning points}$$

$$\nu(nT) = 1 - const \times n \qquad \text{at the minimum.}$$

Conclusions

- The (theoretical) existence of wormholes in Einstein's theory of gravity is due to an equilibrium between the attractive gravitational force and the repulsion of an exotic matter field.

Conclusions

- The (theoretical) existence of wormholes in Einstein's theory of gravity is due to an equilibrium between the attractive gravitational force and the repulsion of an exotic matter field.
- For the simple model of a massless ghost scalar field, this equilibrium is *unstable*, at least for solutions which are static and spherically symmetric.

Conclusions

- The (theoretical) existence of wormholes in Einstein's theory of gravity is due to an equilibrium between the attractive gravitational force and the repulsion of an exotic matter field.
- For the simple model of a massless ghost scalar field, this equilibrium is *unstable*, at least for solutions which are static and spherically symmetric.
- Furthermore, the timescale associated to this instability is very small (compared to the areal radius of the throat divided by c).

Conclusions

- The (theoretical) existence of wormholes in Einstein's theory of gravity is due to an equilibrium between the attractive gravitational force and the repulsion of an exotic matter field.
- For the simple model of a massless ghost scalar field, this equilibrium is *unstable*, at least for solutions which are static and spherically symmetric.
- Furthermore, the timescale associated to this instability is very small (compared to the areal radius of the throat divided by c).
- Numerical simulations suggest that the wormhole either expands forever or collapses to a Schwarzschild black hole.

Conclusions

- The (theoretical) existence of wormholes in Einstein's theory of gravity is due to an equilibrium between the attractive gravitational force and the repulsion of an exotic matter field.
- For the simple model of a massless ghost scalar field, this equilibrium is *unstable*, at least for solutions which are static and spherically symmetric.
- Furthermore, the timescale associated to this instability is very small (compared to the areal radius of the throat divided by c).
- Numerical simulations suggest that the wormhole either expands forever or collapses to a Schwarzschild black hole.
- Addition of a charge does not seem to stabilize the wormholes

Conclusions

- The (theoretical) existence of wormholes in Einstein's theory of gravity is due to an equilibrium between the attractive gravitational force and the repulsion of an exotic matter field.
- For the simple model of a massless ghost scalar field, this equilibrium is *unstable*, at least for solutions which are static and spherically symmetric.
- Furthermore, the timescale associated to this instability is very small (compared to the areal radius of the throat divided by c).
- Numerical simulations suggest that the wormhole either expands forever or collapses to a Schwarzschild black hole.
- Addition of a charge does not seem to stabilize the wormholes
- "Bad news" for interstellar travel agencies...

Conclusions

- The (theoretical) existence of wormholes in Einstein's theory of gravity is due to an equilibrium between the attractive gravitational force and the repulsion of an exotic matter field.
- For the simple model of a massless ghost scalar field, this equilibrium is *unstable*, at least for solutions which are static and spherically symmetric.
- Furthermore, the timescale associated to this instability is very small (compared to the areal radius of the throat divided by *c*).
- Numerical simulations suggest that the wormhole either expands forever or collapses to a Schwarzschild black hole.
- Addition of a charge does not seem to stabilize the wormholes
- "Bad news" for interstellar travel agencies...