Title: Tensor-net states: a new perspective on many-body quantum systems

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Abstract: Traditional condensed matter physics is based on two theories: symmetry breaking theory for phases and phase transitions, and Fermi liquid theory for metals. Mean-field theory is a powerful method to describe symmetry breaking phases and phase transitions by assuming the ground state wavefunctions for many-body systems can be approximately described by direct product states. The Fermi liquid theory is another powerful method to study electron systems by assuming that the ground state wavefunctions for the electrons can be approximately described by Slater determinants. From the encoding point of view, both methods only use a polynomial amount of information to approximately encode many-body ground state wavefunctions which contain an exponentially large amount of information. Moreover, another nice property of both approaches is that all the physical quantities (energy, correlation functions, etc.) can be efficiently calculated (polynomially hard). In this talk, I'll introduce a new class of states: (Grassmann-number) tensor-net states. These states only need polynomial amount of information to approximately encode many-body ground states. Many classes of states, such as Slater determinant states, projective states, string-net states and their generalizations, etc., are subclasses of (Grassmann-number) tensor-net states. However, calculating the physical quantities for these state can be exponentially hard in general. To solve this difficulty, we develop the Tensor-Entanglement Renormalization Group (TERG) method to efficiently calculate the physical quantities. We demonstrate our algorithm by studying several interesting boson/fermion models, including t-J model on a honeycomb lattice.

Tensor product states: a new perspective on strongly correlated systems

Zheng-cheng Gu (KITP)

Collaborators:

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Dr. Hong Yao(UC Berkeley)

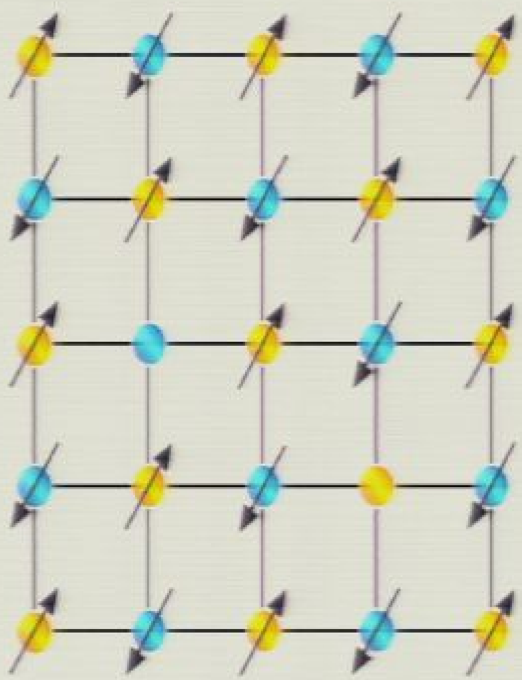
PI.Feb. 2011

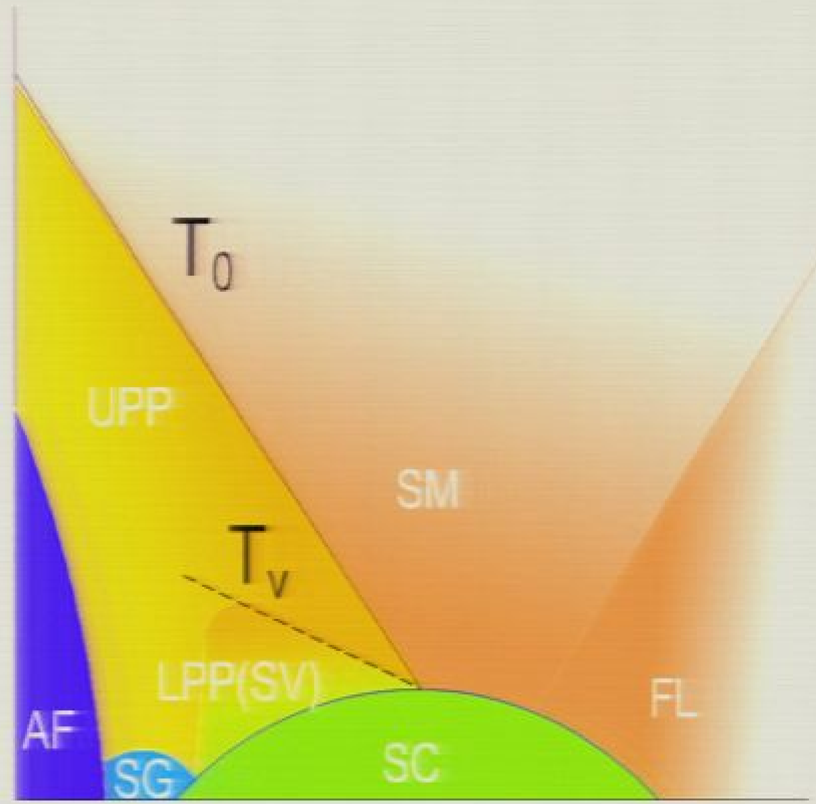
The ambitious goal

t-J model on square lattice(strong-coupling limit of Hubbard model)

$$H_{\text{t-J}} = -t\sum_{\langle ij\rangle\sigma}\tilde{c}^{\dagger}_{i\sigma}\tilde{c}_{j\sigma} + H.c. + J\sum_{\langle ij\rangle}(\hat{S}_i\hat{S}_j - \frac{1}{4}\hat{n}_i\hat{n}_j) - \mu\sum_i \hat{n}_i$$

(F. C. Zhang and T. M. Rice 1988)





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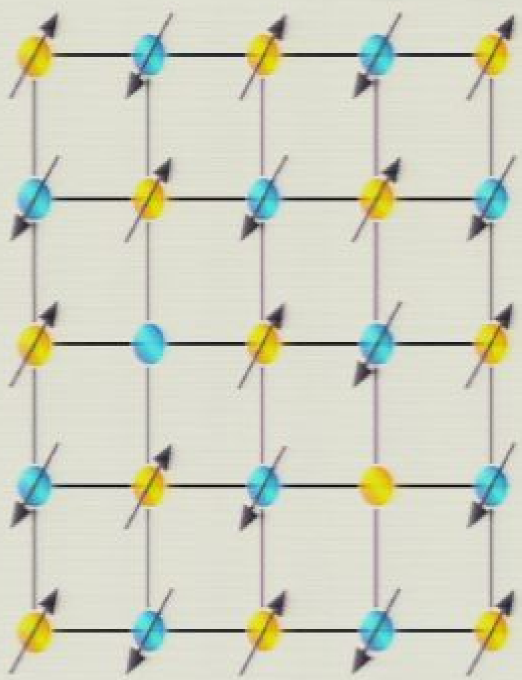

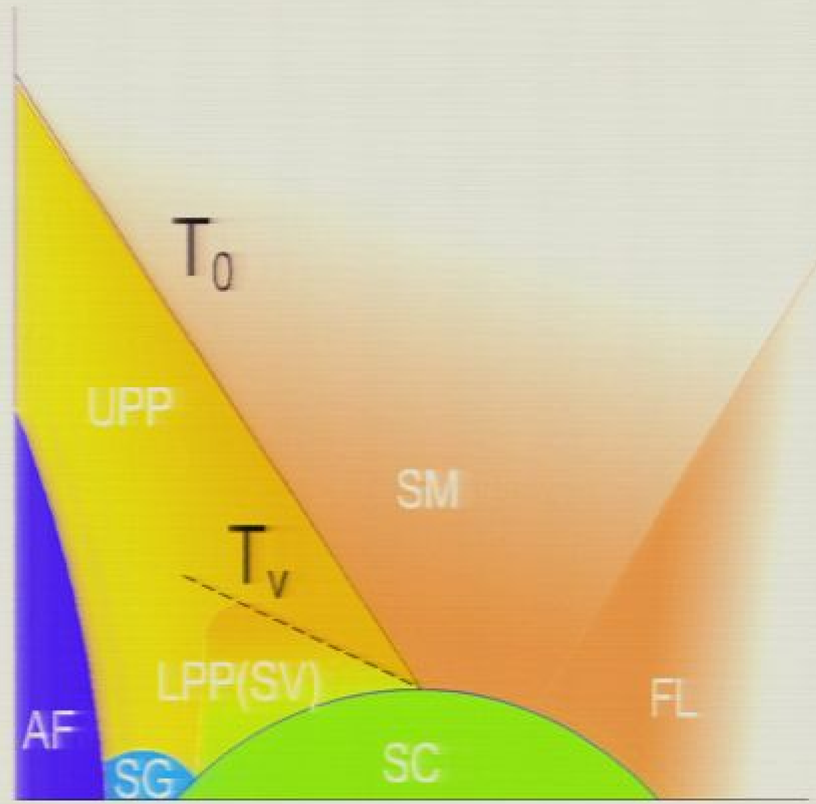

A small step today

t-J model on honeycomb lattice

- AF ordering at half-filling

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Outline

- Tensor Product States(TPS)

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Landau's paradigm of phases and phase transitions

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Symmetry breaking

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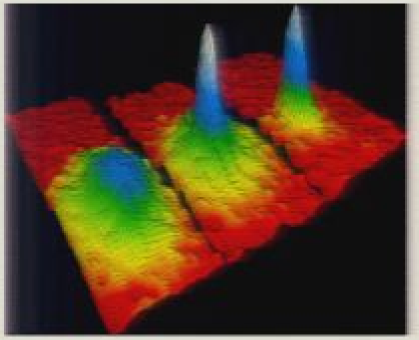
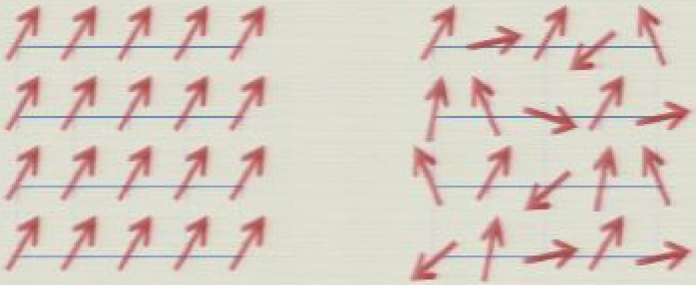


- Bose Einstein Condensation(BEC)
- Various of magnetic orders in spin systems



Landau's paradigm of phases and phase transitions

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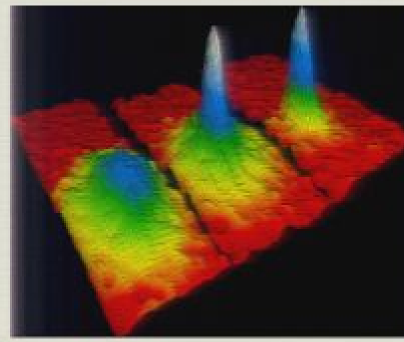

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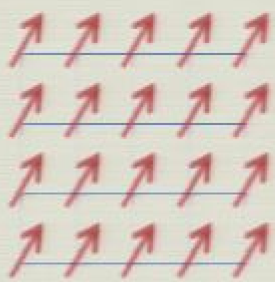
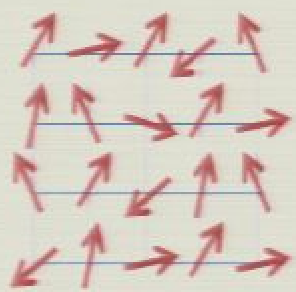
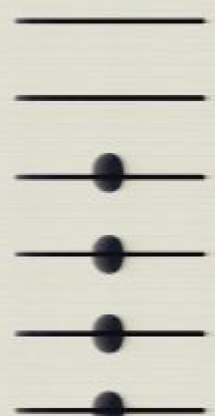




Fermi Liquid theory for electron systems.



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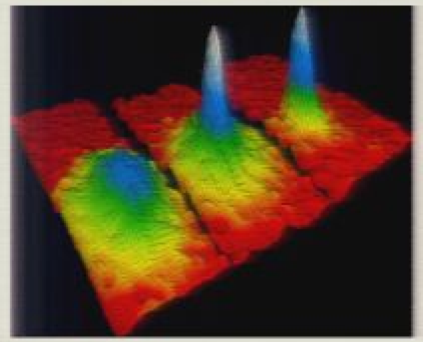

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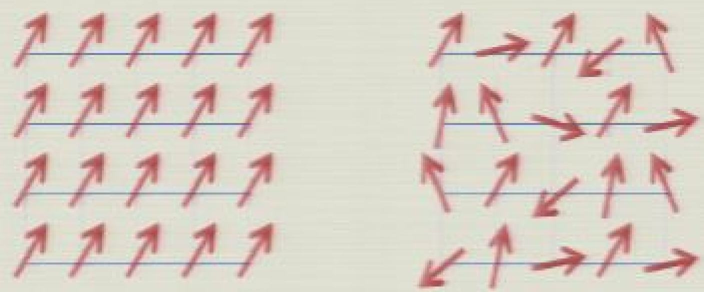

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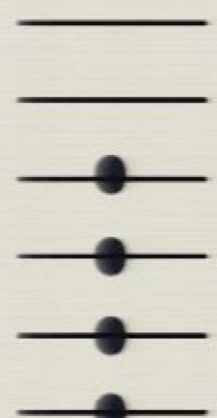

- Metal,Semiconductors,Band Insulators
- Integer Quantum Hall and Topological Insulators
- Superconductivity

Methods-trial wavefunctions

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Mean-field description for symmetry breaking phases and phase transitions:

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$$|\Psi_f\rangle = \exp\left(\frac{1}{2}\sum_{ij} u_{ij} c_j^{\dagger} c_i^{\dagger}\right)|0\rangle = \prod_m \left(1+\lambda_m c_{m+}^{\dagger} c_{m-}^{\dagger}\right)|0\rangle$$

$$\propto \prod_m \left(v_m c_{m+}^{\dagger} + u_m c_{m-}\right)\left(u_m c_{m+} - v_m c_{m-}^{\dagger}\right)|0\rangle \quad \text{with} \quad |u_m|^2+|v_m|^2=1;\ \frac{v_m}{u_m}=\lambda_m$$

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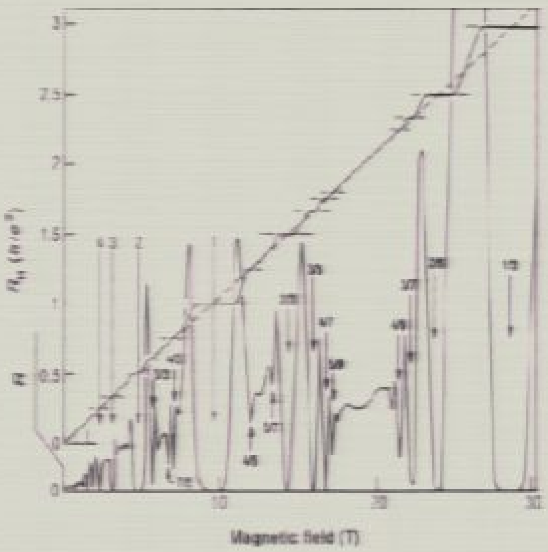

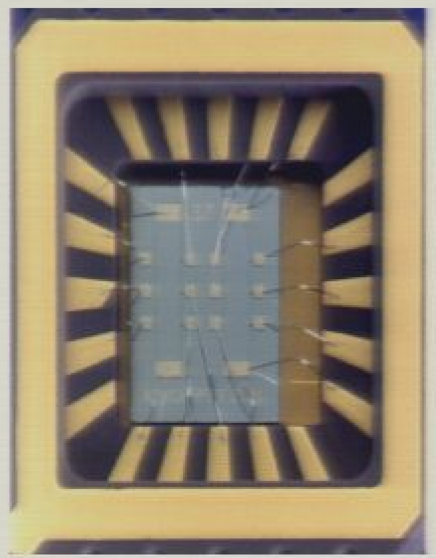

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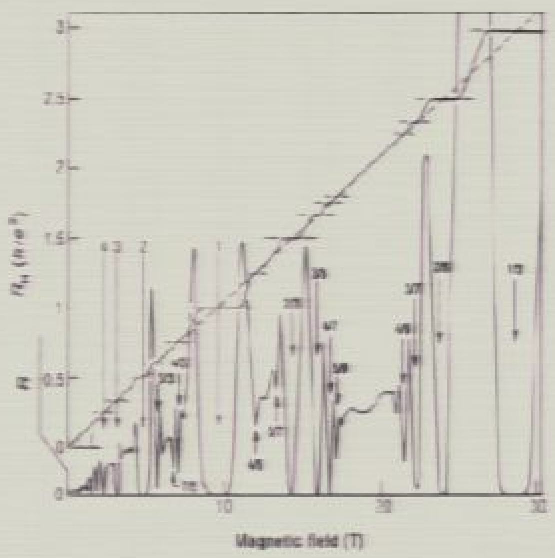

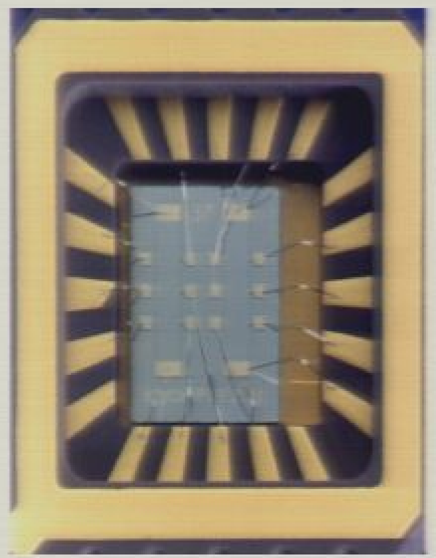

Spin liquid

- Frustrated magnets

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- Ground state degeneracies are robust against any local perturbations.
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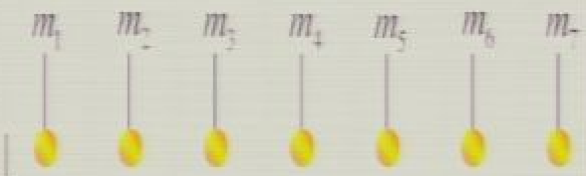
- Protected chiral edge states(chiral topological order).

Tensor Product States

Mean-field states: $\uparrow \longrightarrow u^{\uparrow} ; \quad \downarrow \longrightarrow u^{\downarrow}$



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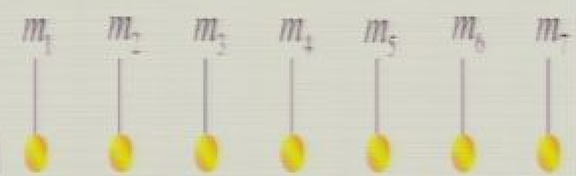

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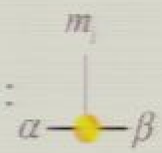
$A^{m_i}_{\alpha\beta}$:



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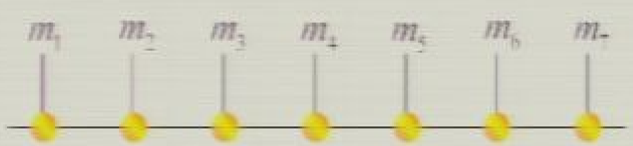
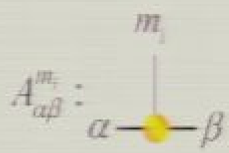
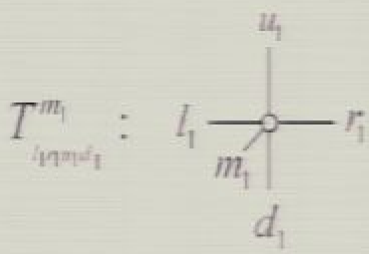
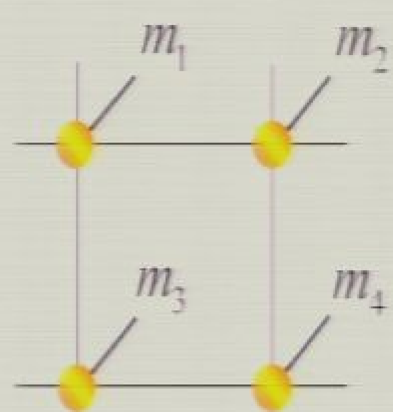




TPS: $\uparrow \longrightarrow T^{\uparrow}_{lrud}; \quad \downarrow \longrightarrow T^{\downarrow}_{lrud}$ (F. Verstraete and J. I. Cirac 2004)

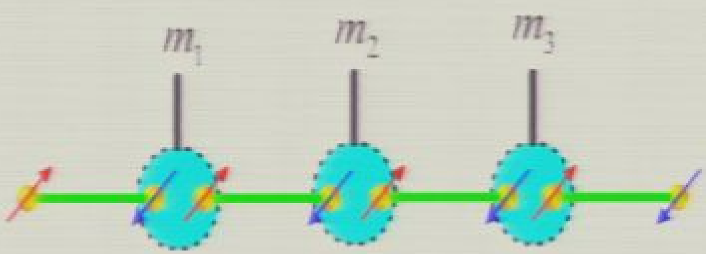




Examples:

1D AKLT state: $H = \sum_i P_2(\mathbf{S}_i + \mathbf{S}_{i+1})$ Ian Affleck, *etal*, PRL(1987)

$$= \sum_i \left[\tfrac{1}{2} \mathbf{S}_i \cdot \mathbf{S}_{i+1} + \tfrac{1}{6} (\mathbf{S}_i \cdot \mathbf{S}_{i+1})^2 + \tfrac{1}{3} \right].$$



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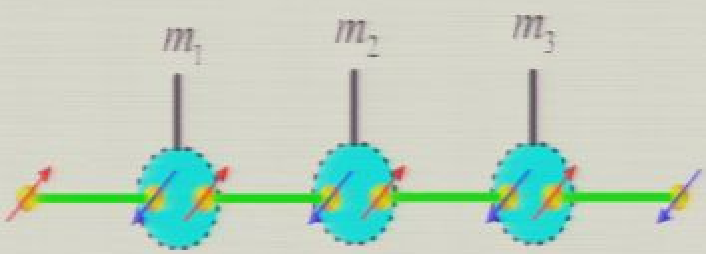

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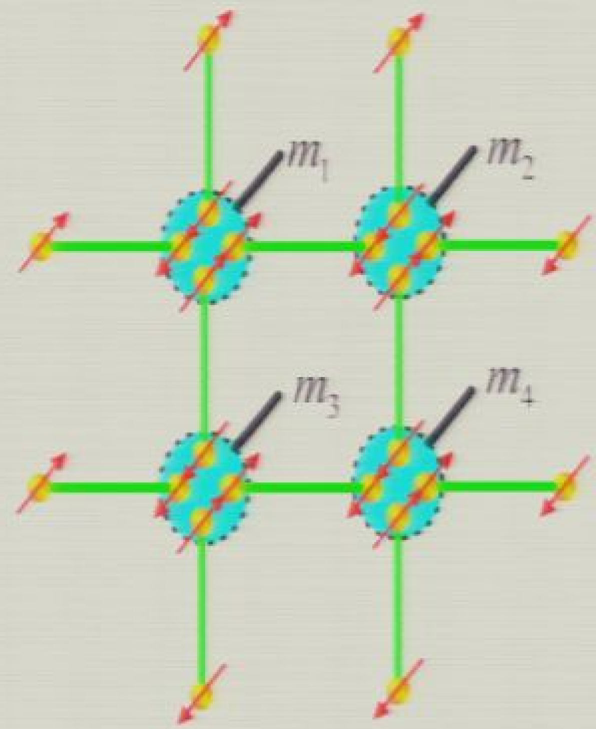

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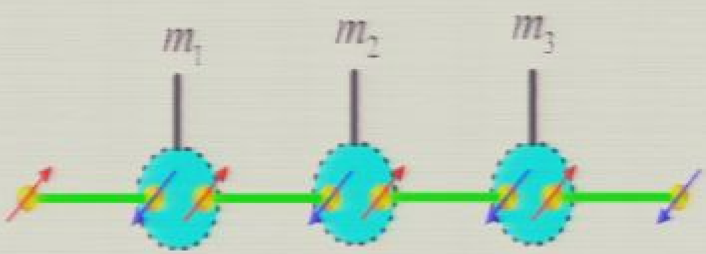

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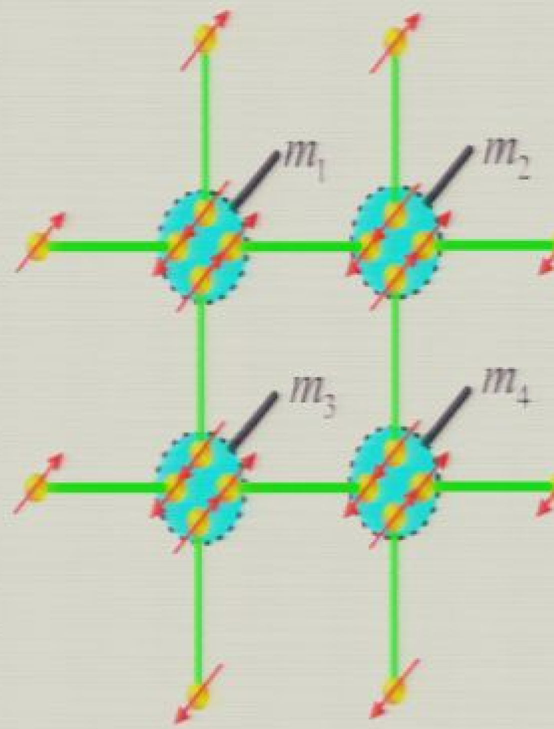

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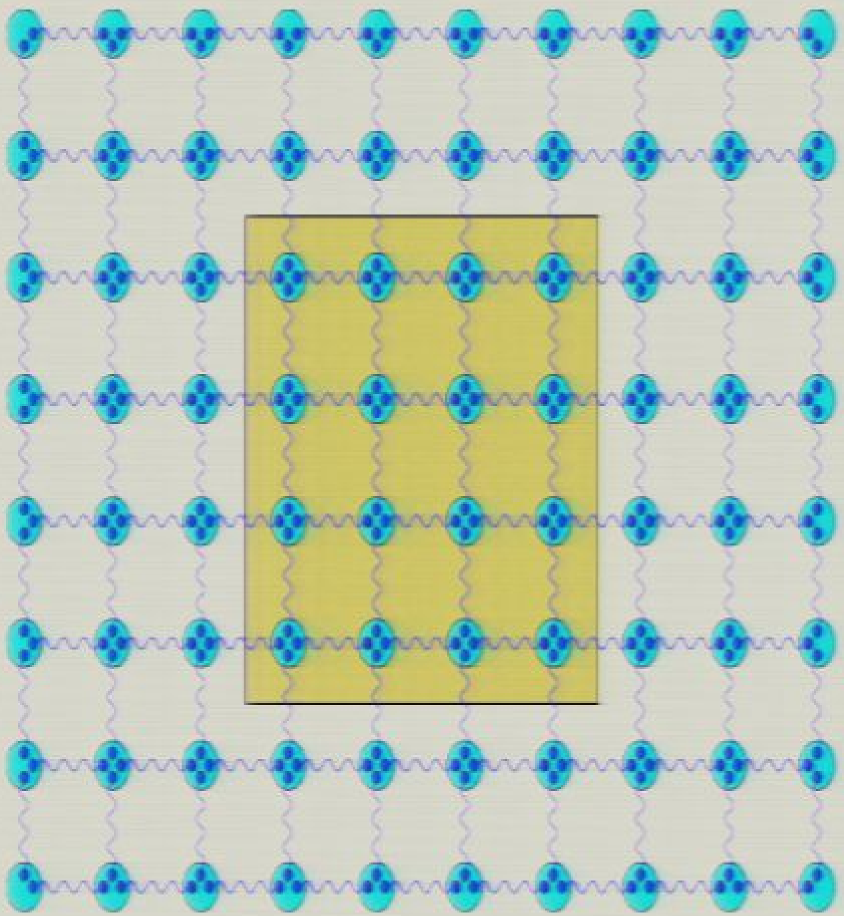
string-net states(all the non-chiral topological order)

(Z.C. Gu, *etal*., PRB, 2008, O. Buerschaper, *etal*., PRB, 2008)

Properties of TPS:

Informational perspective



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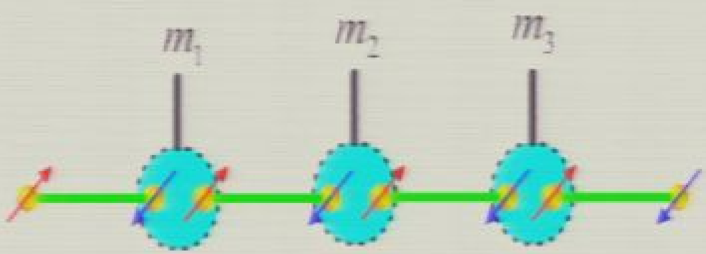

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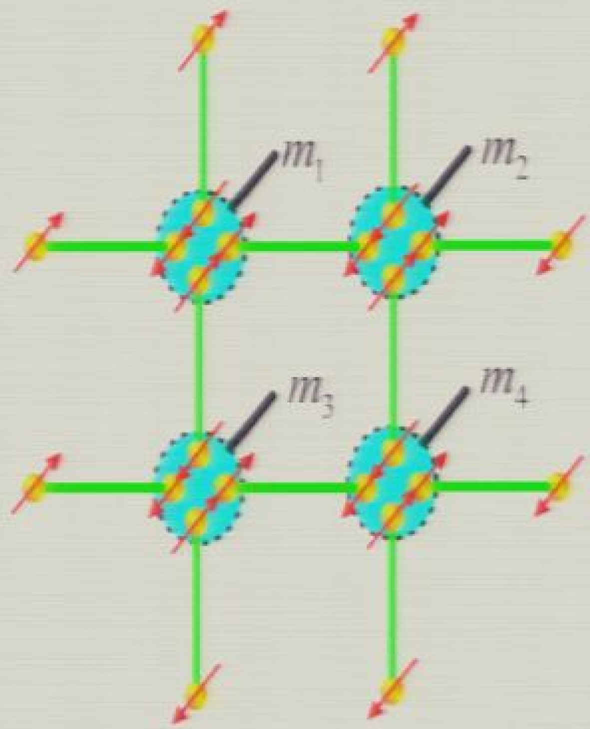

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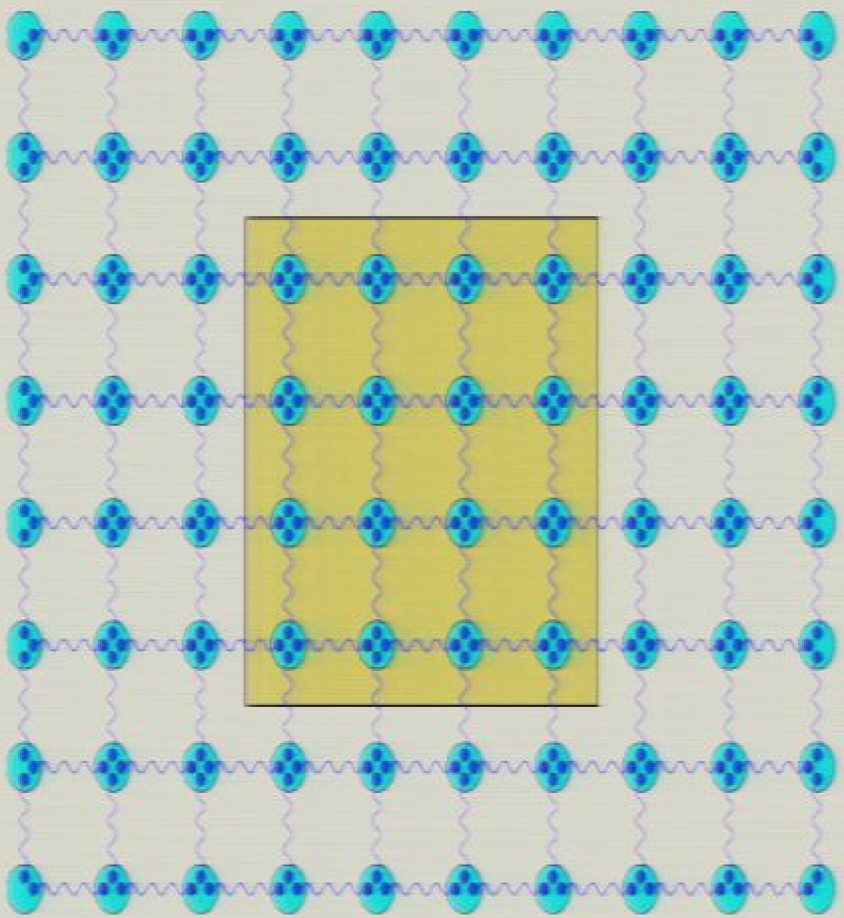

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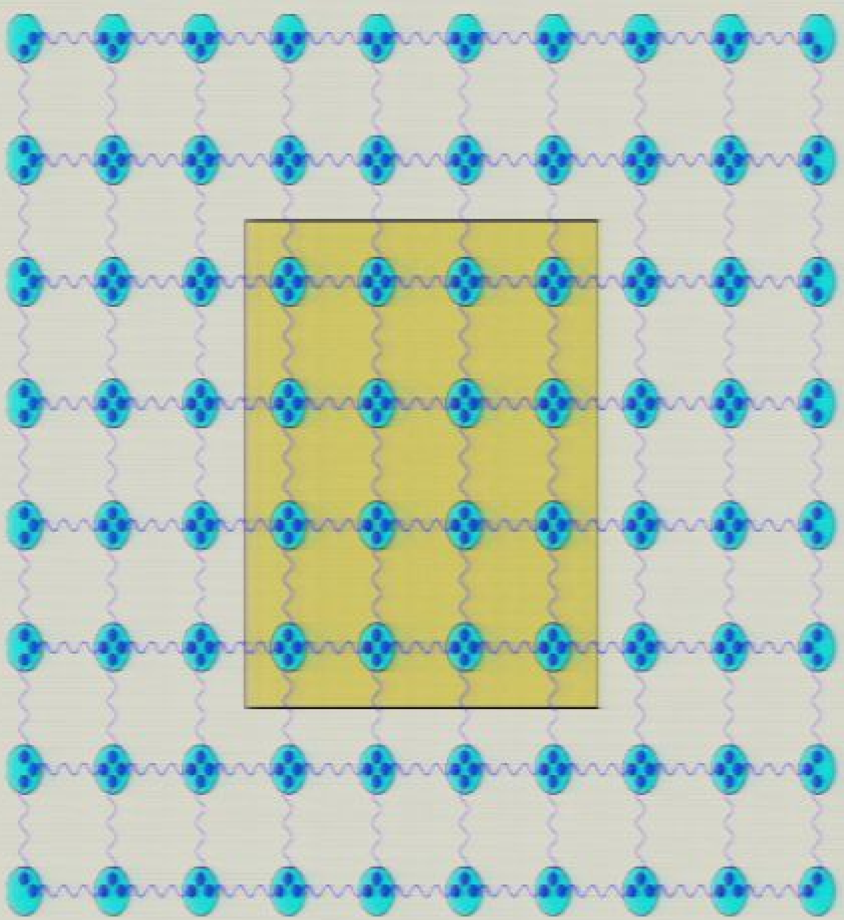

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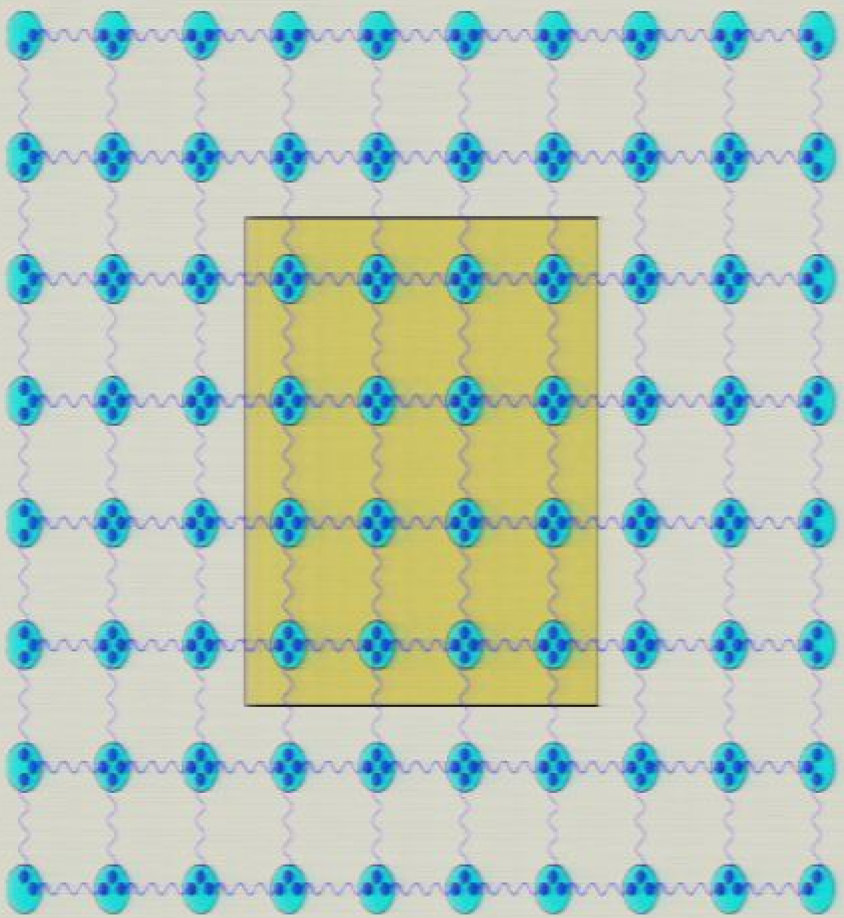

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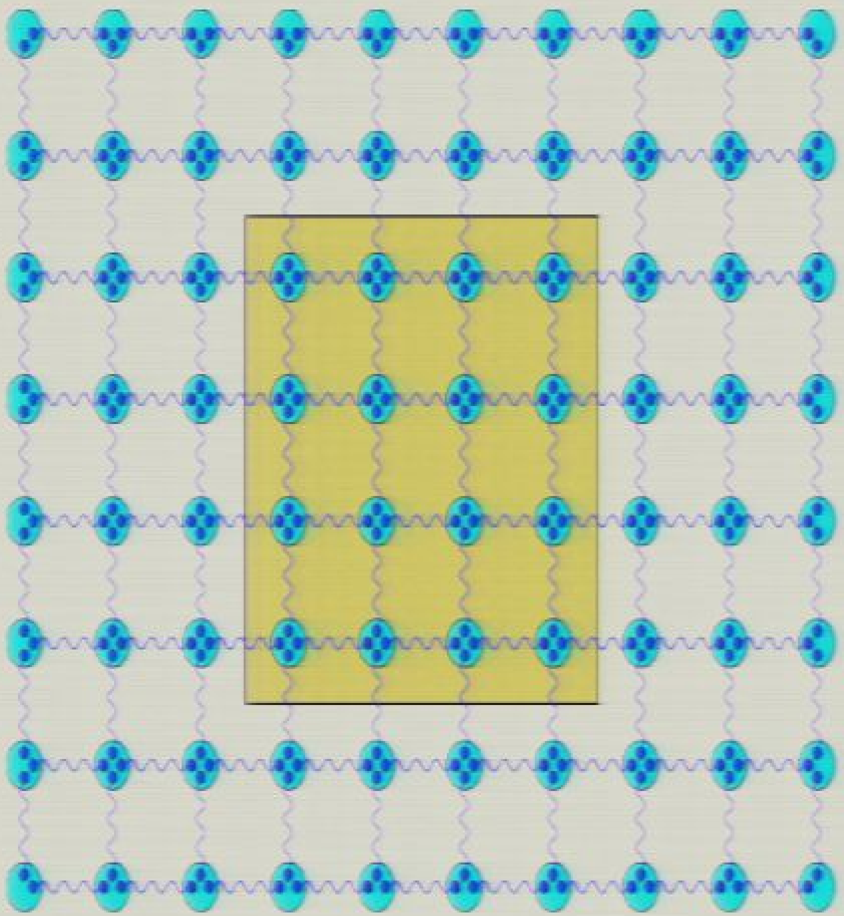
- Systems with topological order

$$S(\rho_L) = \alpha L - \gamma$$

- Tensor Product States(TPS)
- Tensor-Entanglement Renormalization Group
- Application in Spin systems
- Generalization to Fermion systems
- Application in t-J model on honeycomb lattice
- Summary and outlook

Properties of TPS:

Informational perspective



- Area law for entanglement entropy

$$S(\rho_L) = \alpha L$$

- Systems with topological order

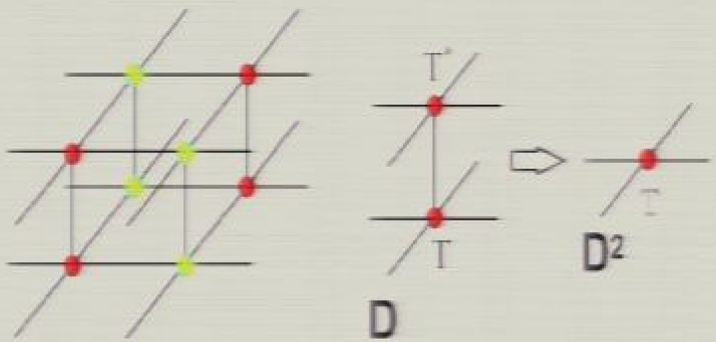
$$S(\rho_L) = \alpha L - \gamma$$

- Tensor Product States(TPS)
- Tensor-Entanglement Renormalization Group
- Application in Spin systems
- Generalization to Fermion systems
- Application in t-J model on honeycomb lattice
- Summary and outlook

Calculate physical quantities

Calculate physical quantities

calculate the norm



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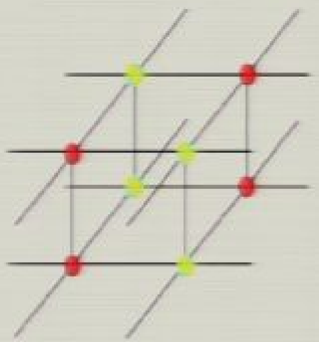

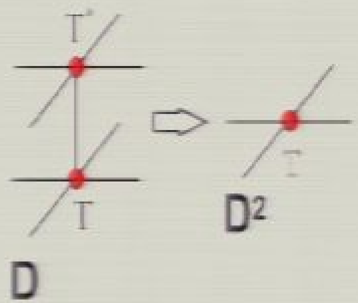

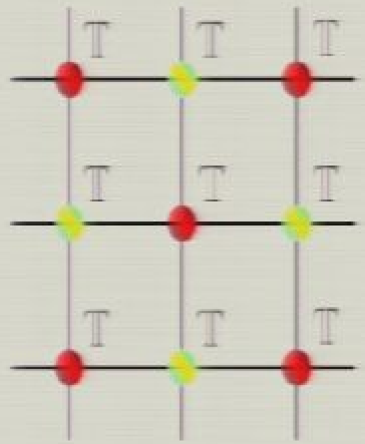

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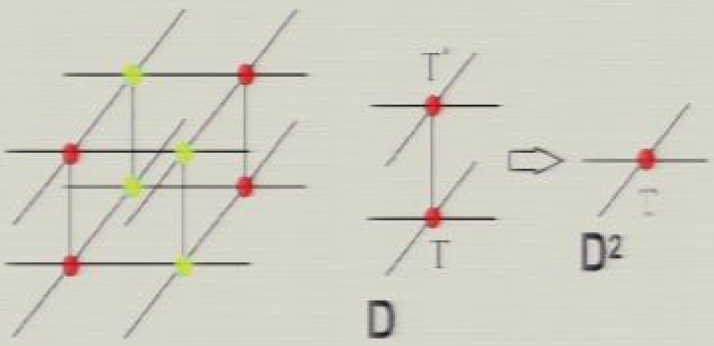
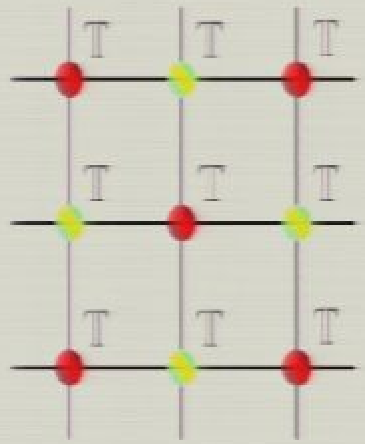
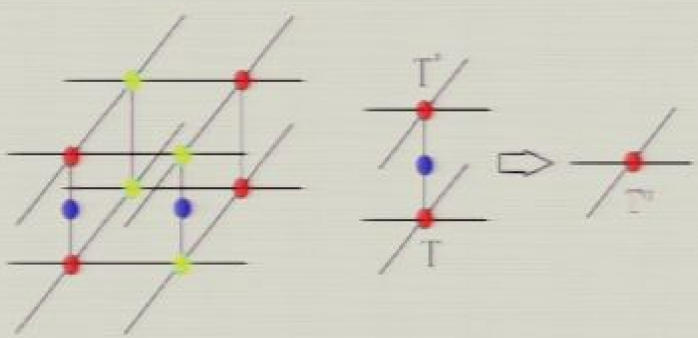




calculate the energy



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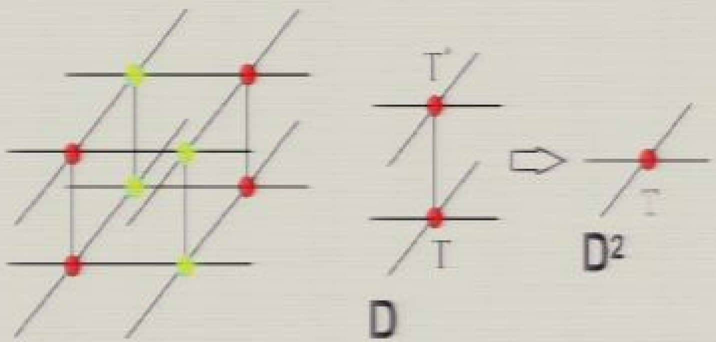

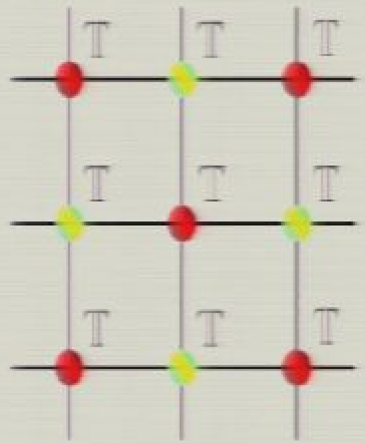

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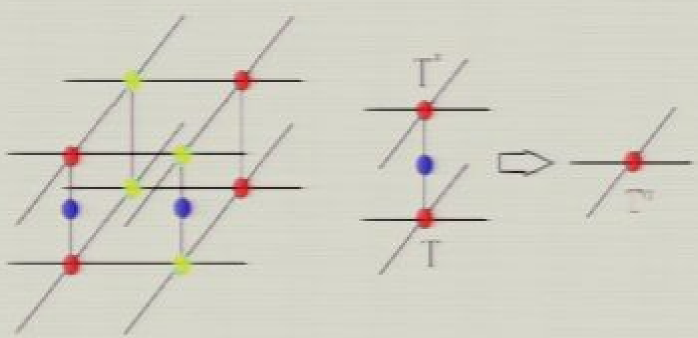

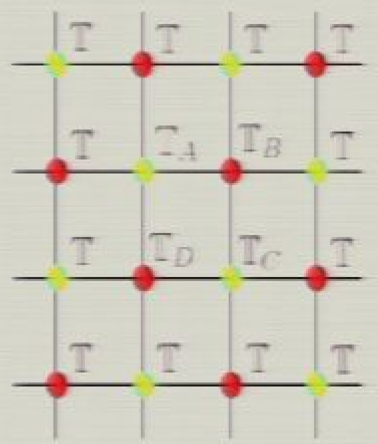

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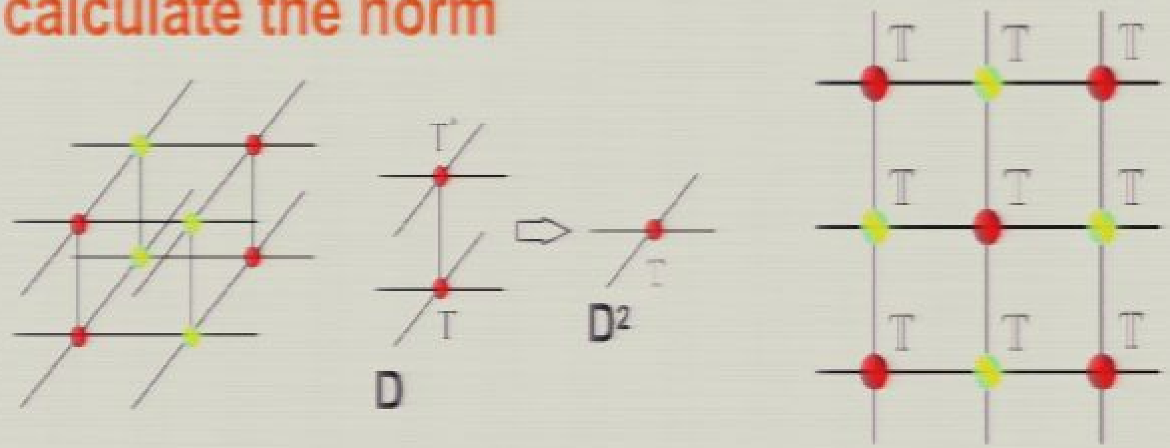

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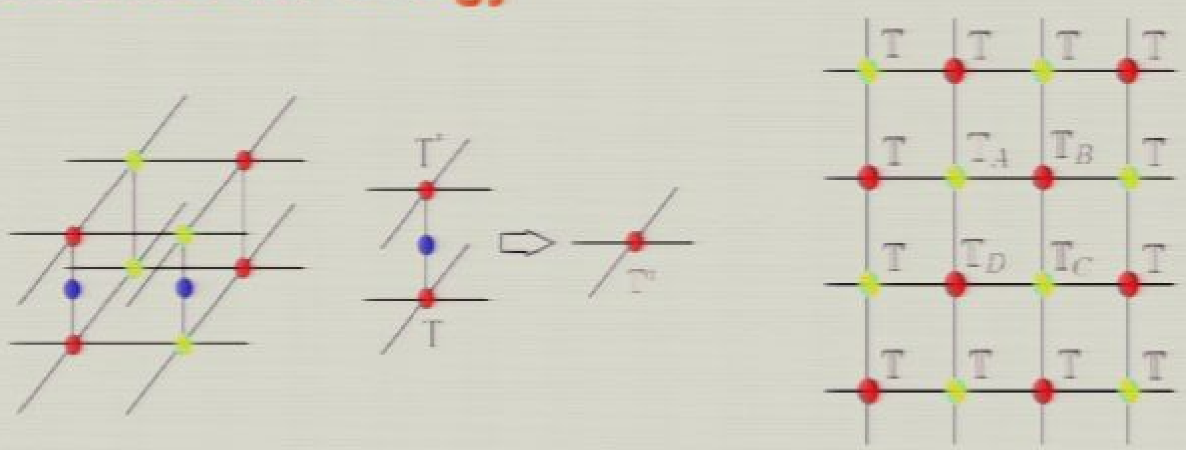
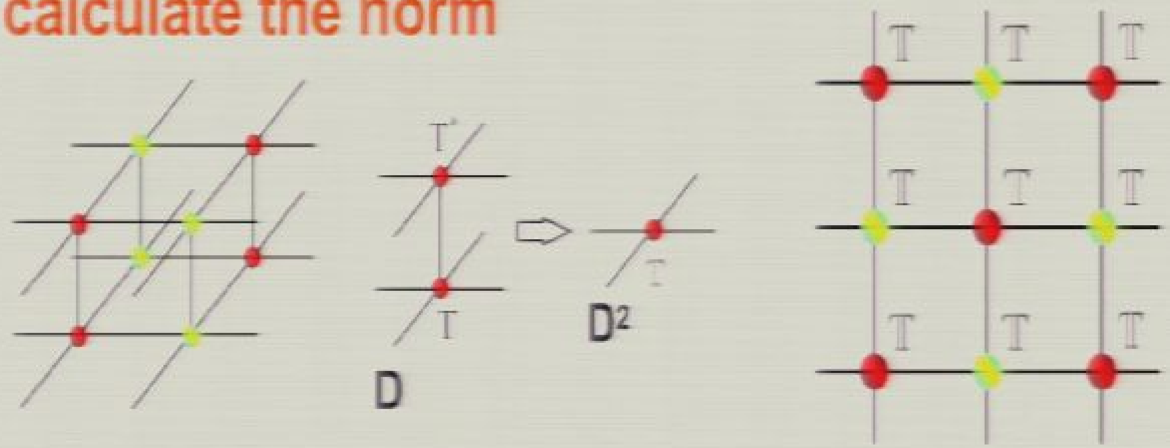


The difficulties

Calculate physical quantities

calculate the norm



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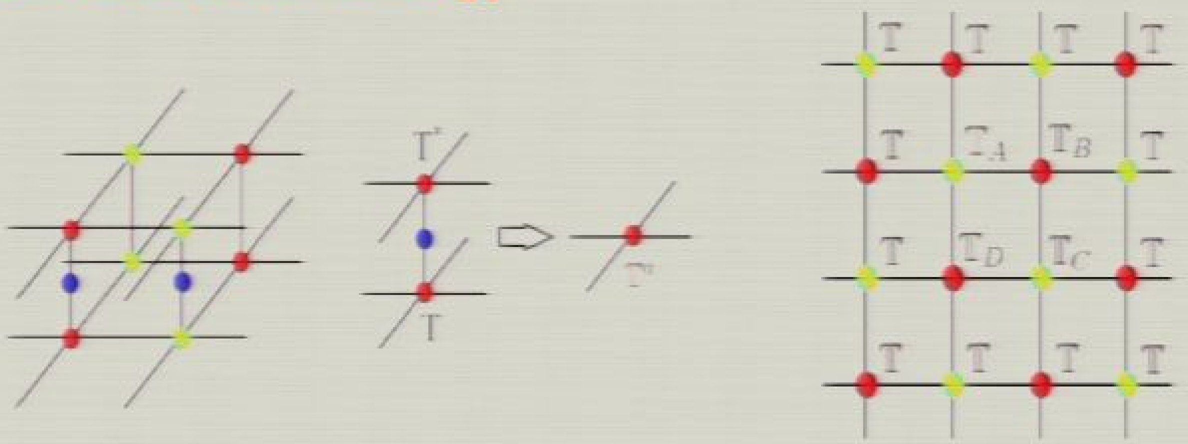
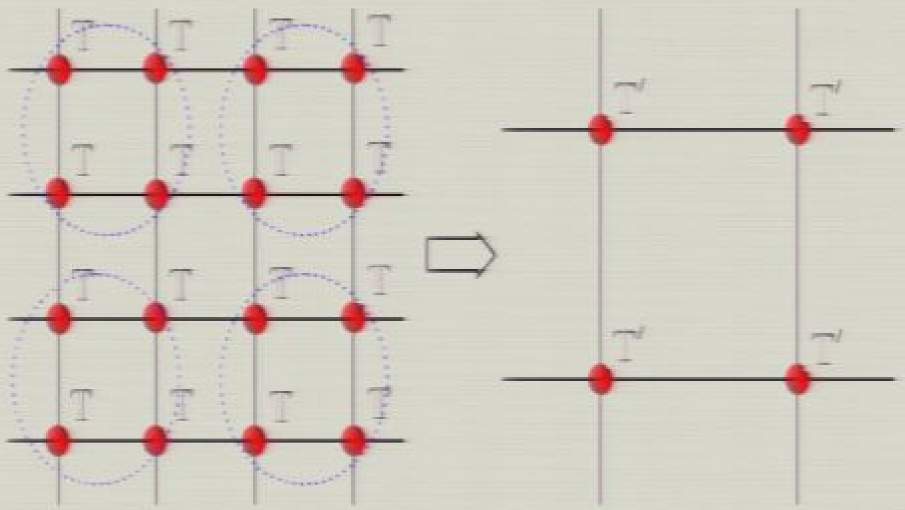


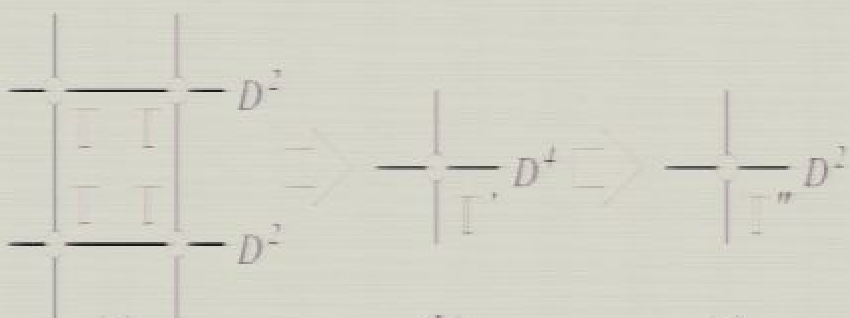
The difficulties

- Calculate the norm and energy for 2D tensor-net are

Tensor-Entanglement Renormalization Group algorithm

Basic idea

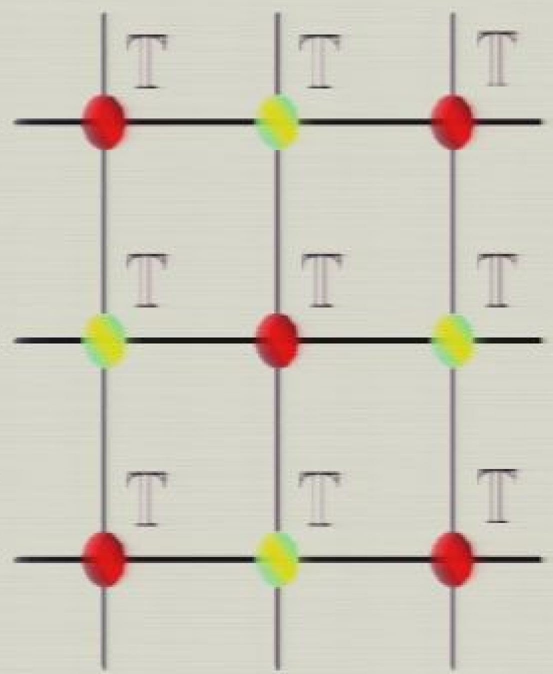




(a) (b) (c)

$$\mathrm{tTr}[\mathbb{T}\otimes\mathbb{T}\cdots] \simeq \mathrm{tTr}[\mathbb{T}''\otimes\mathbb{T}''\cdots]$$

Detail implementation:



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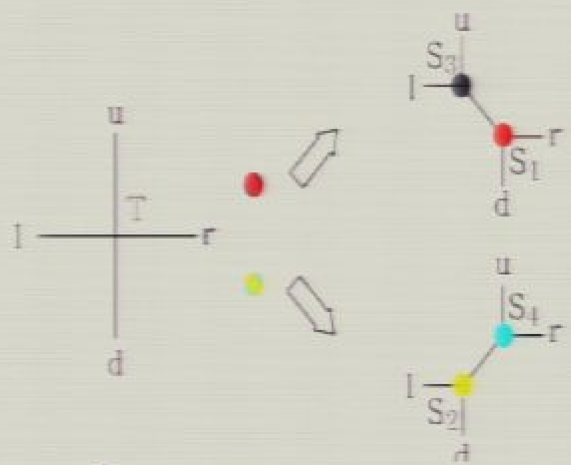
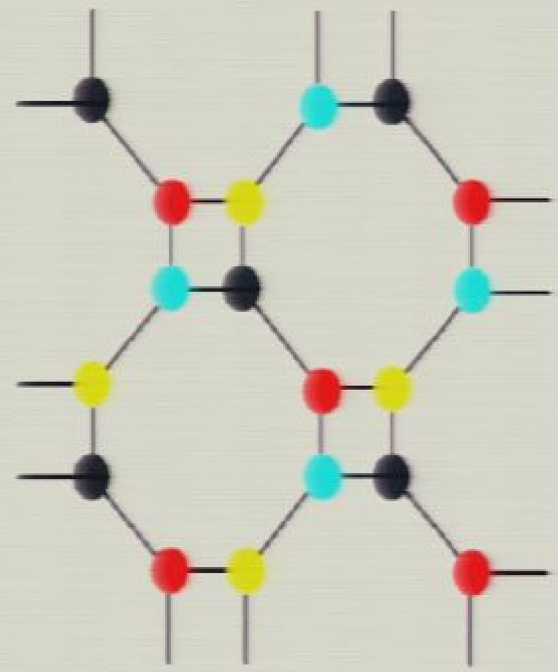


$$M^{\text{red}}_{rd:lu} = \mathbb{T}_{lrud} \quad M^{\text{red}} = USV^{\dagger}$$



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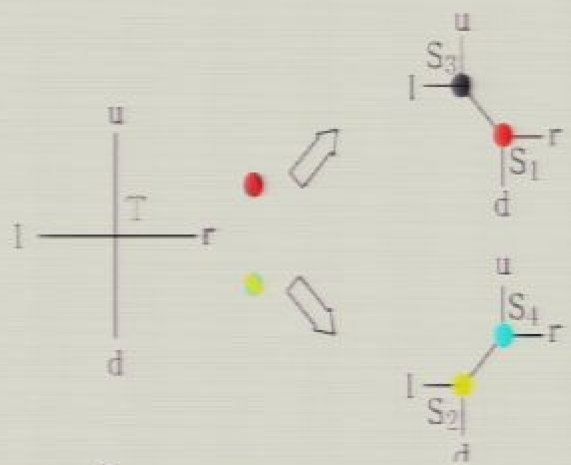

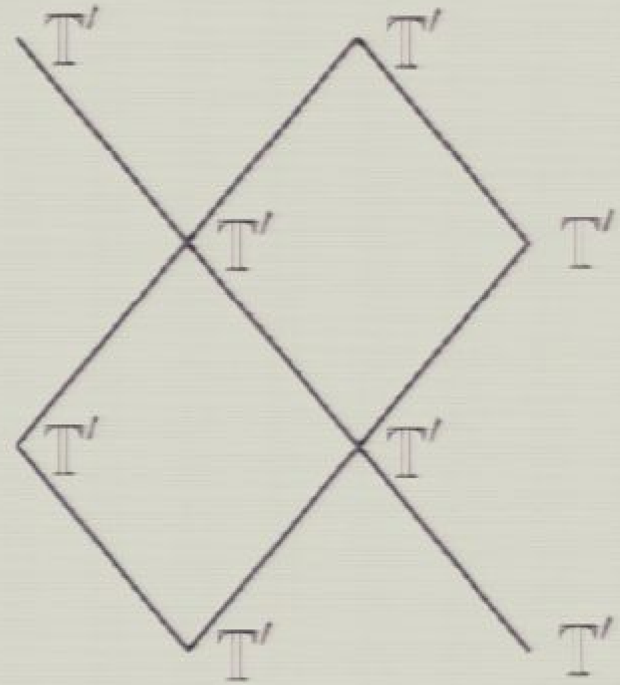

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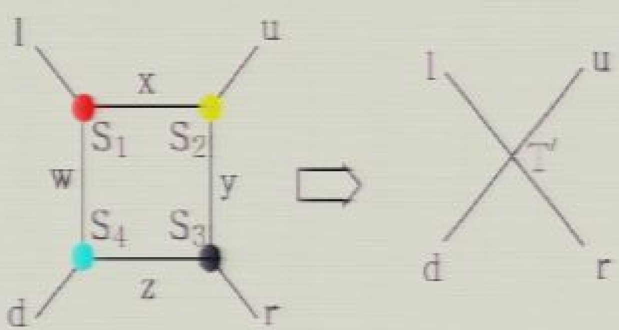

M. Levin, *etal*. 2007, Z. C. Gu, *etal*, 2008

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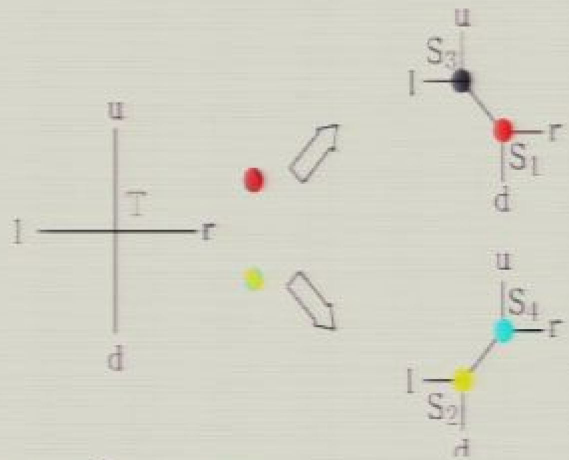

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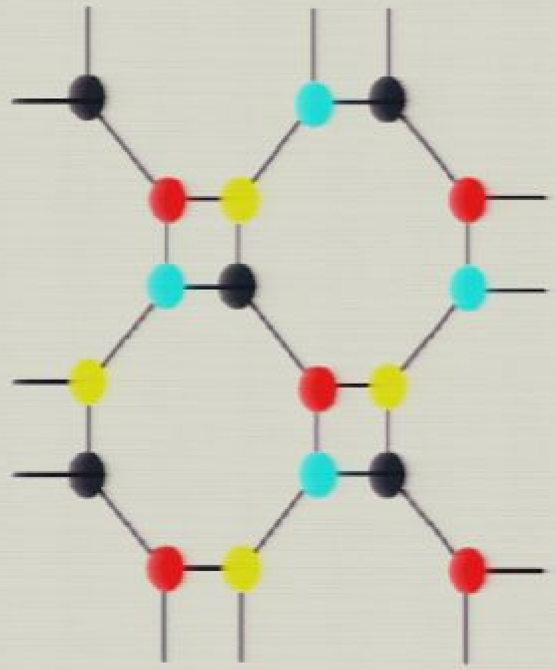

Tensor-Entanglement Renormalization Group algorithm

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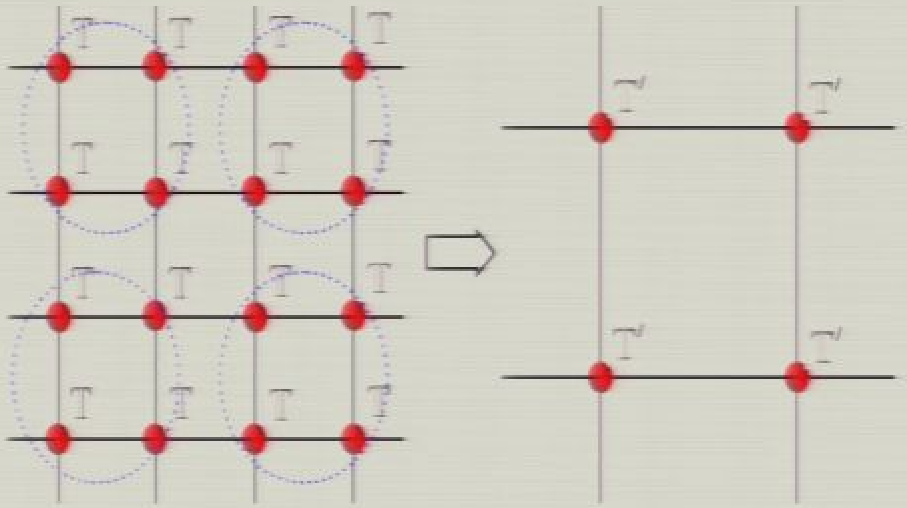

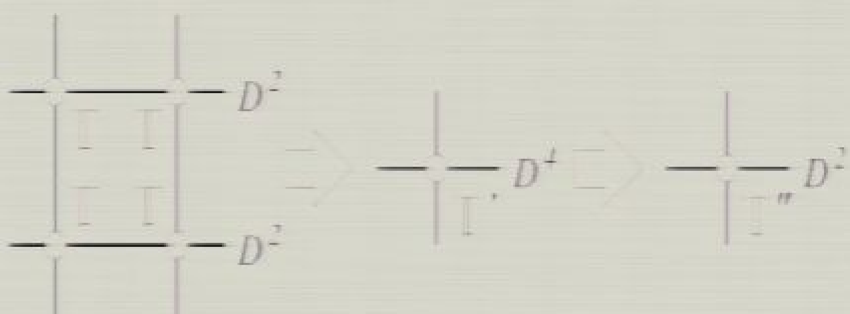

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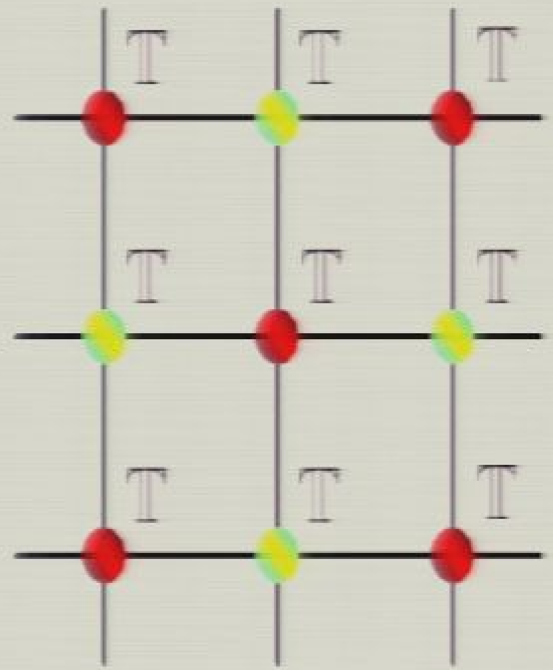

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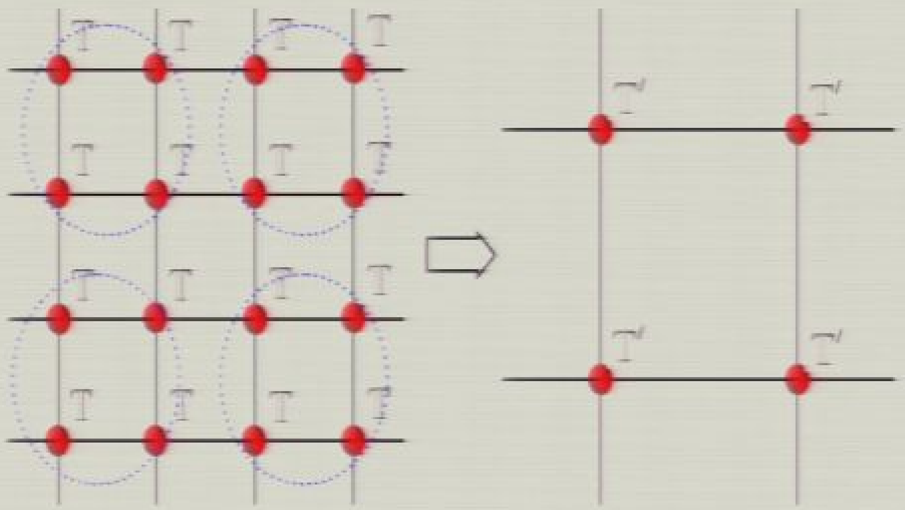

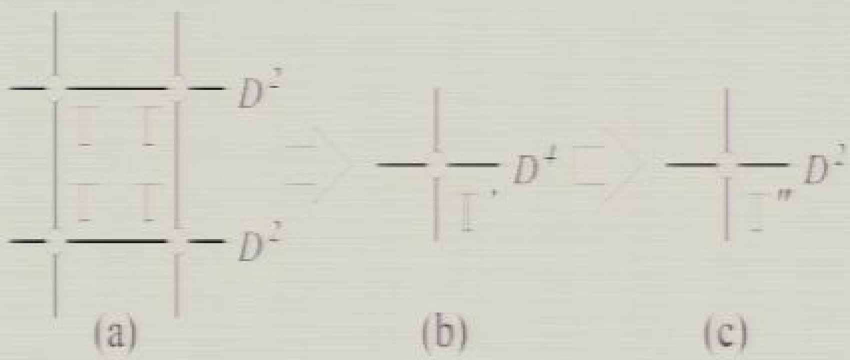

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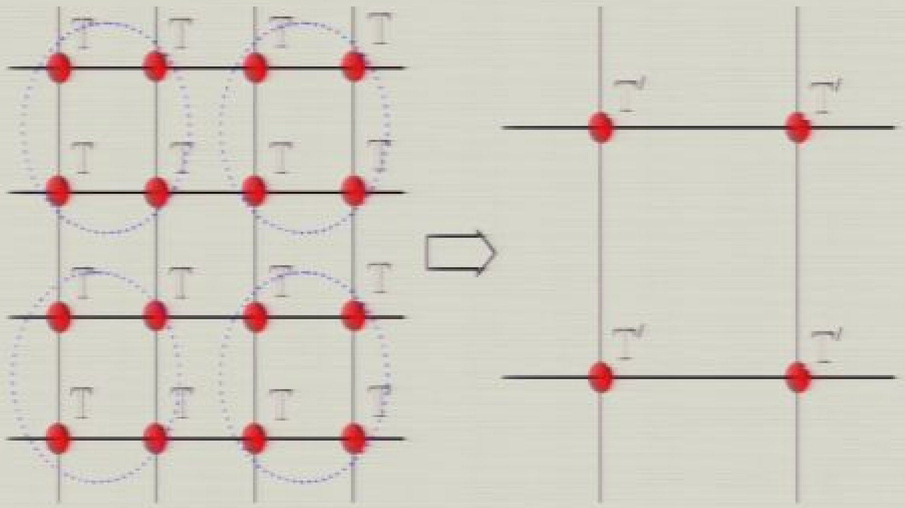

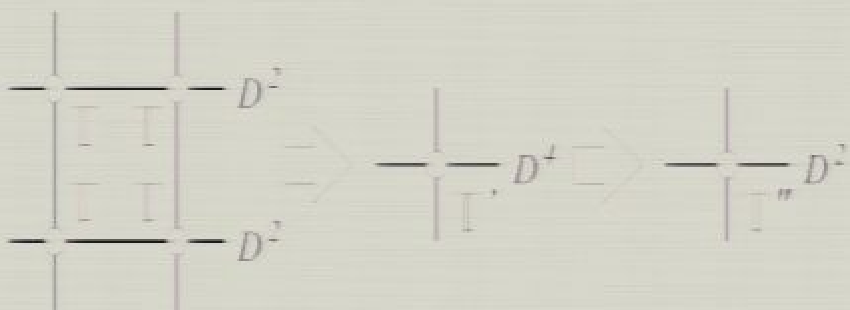

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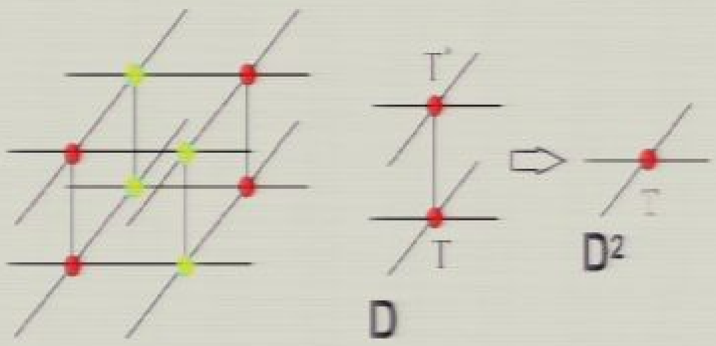
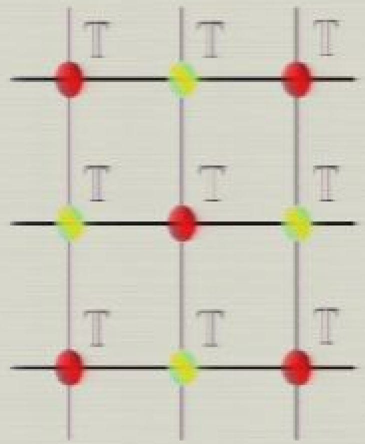
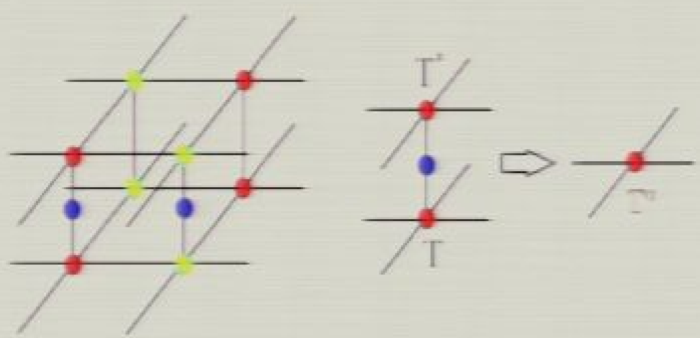
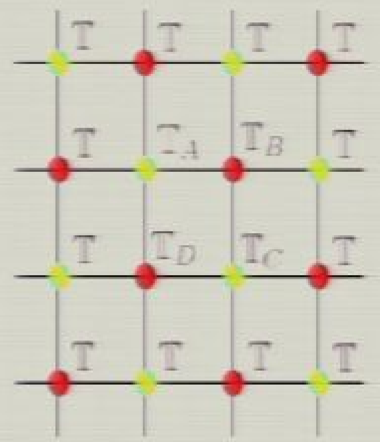
calculate the norm





calculate the energy





The difficulties

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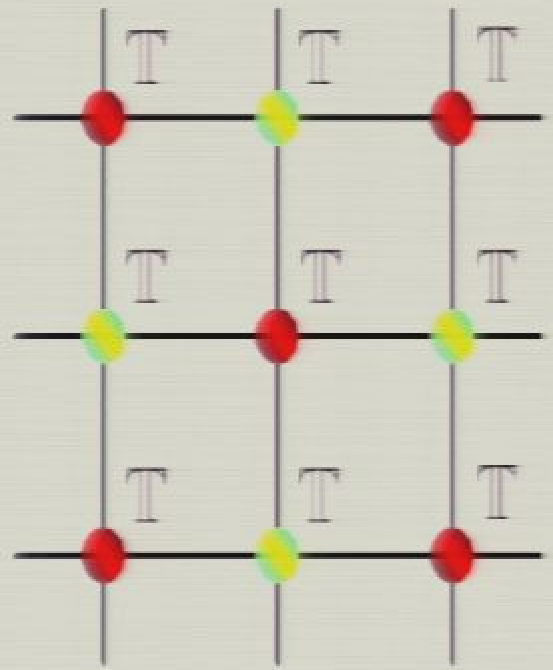

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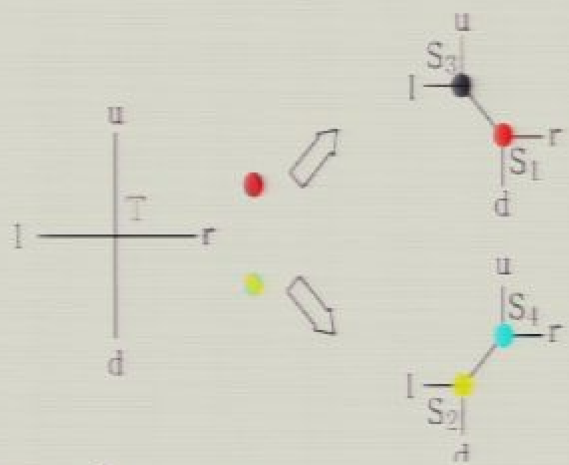

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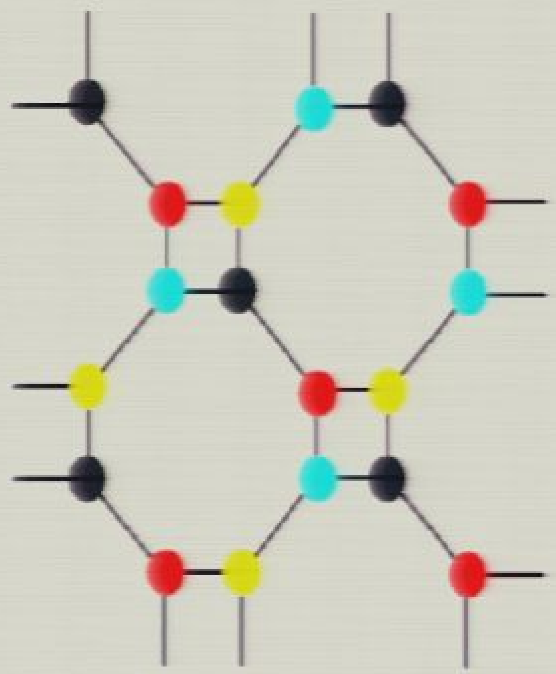

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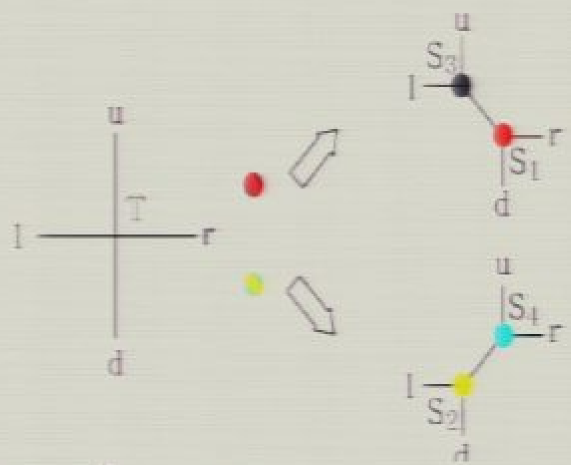

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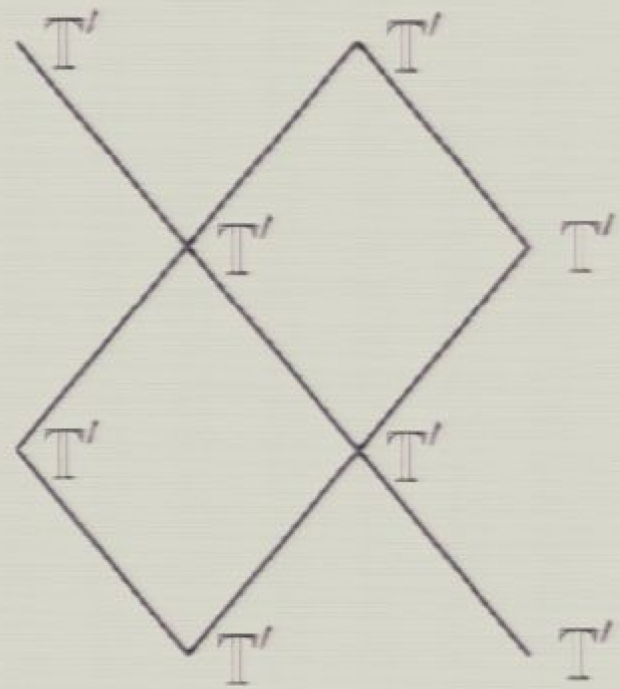

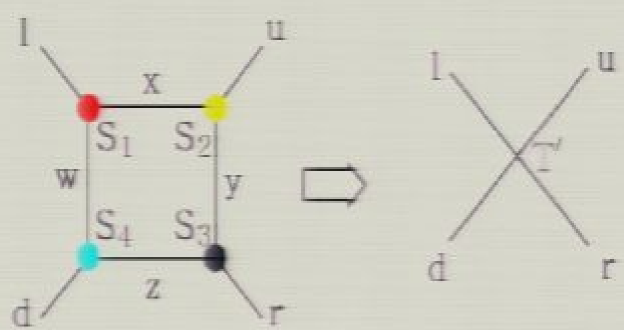

M. Levin, *etal*. 2007, Z. C. Gu, *etal*, 2008

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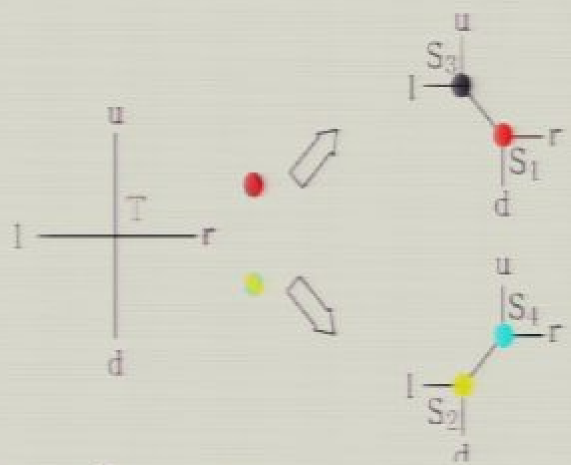

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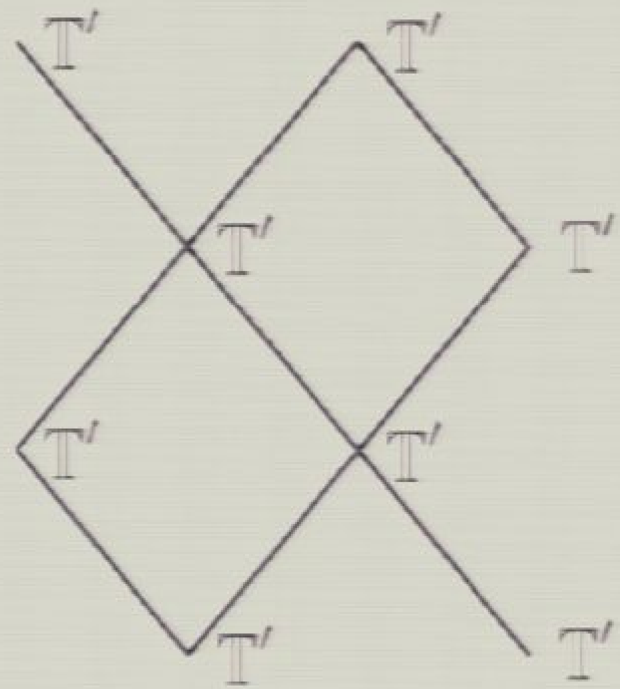

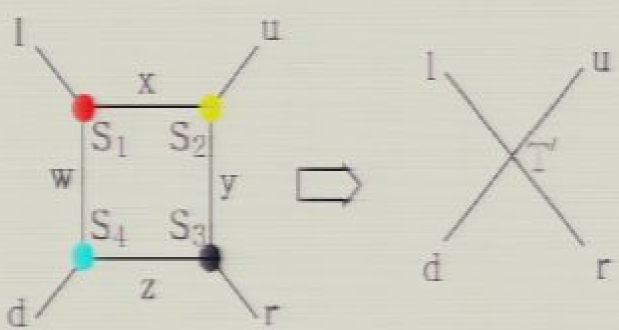

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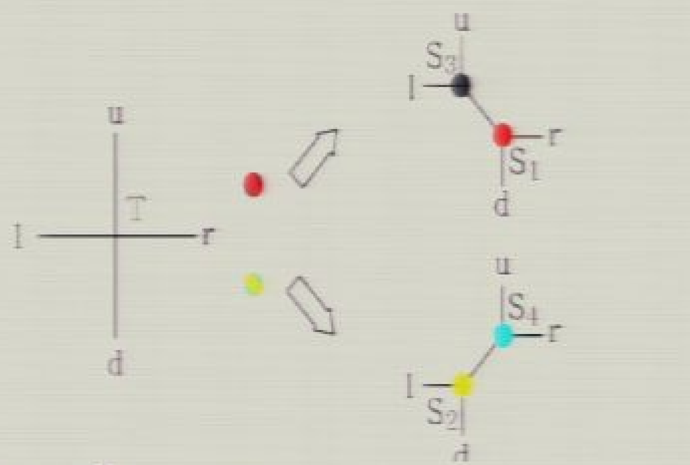
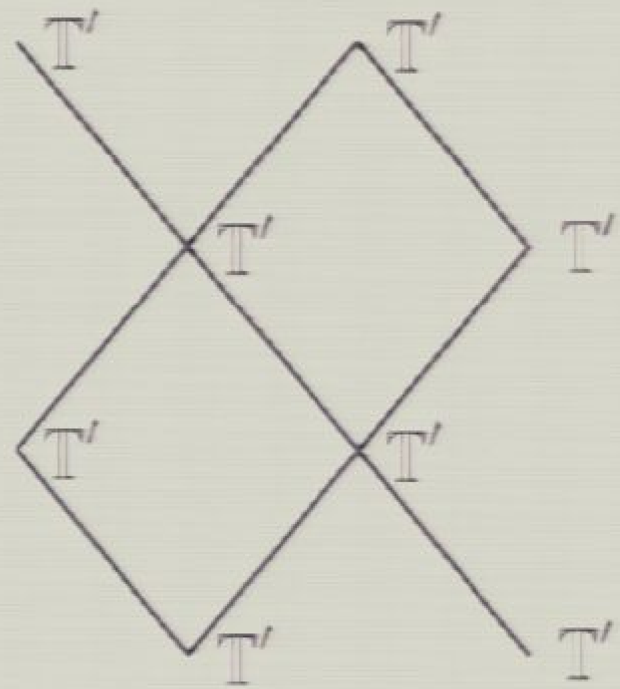
- All the tensors that represent string-net states are fixed point tensors.(States not faraway from fixed point have controlled errors)

Detail implementation:





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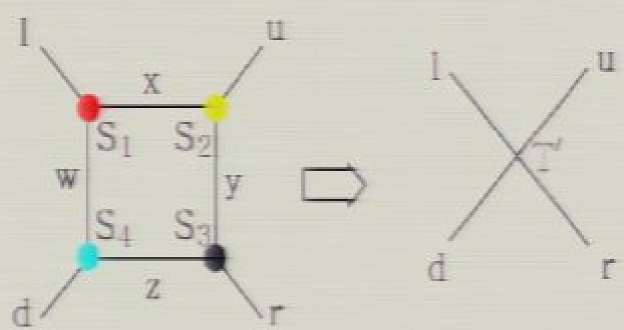

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- Recent development: SRG(Tao Xiang,*etal*, PRL, 2009),

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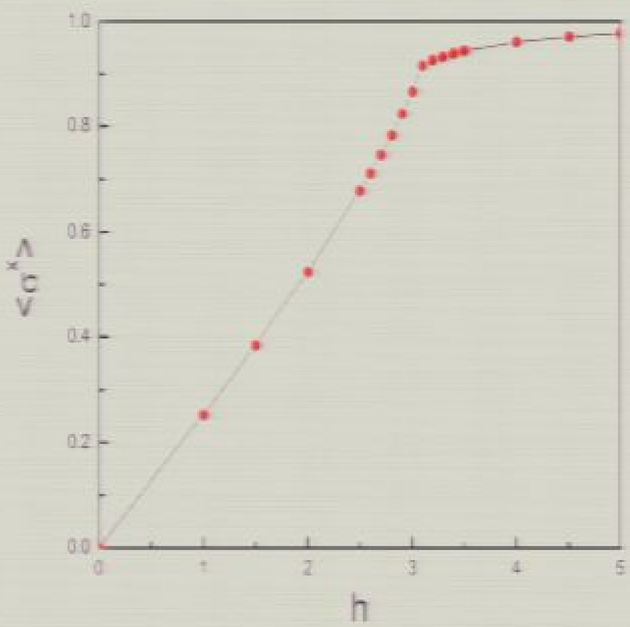
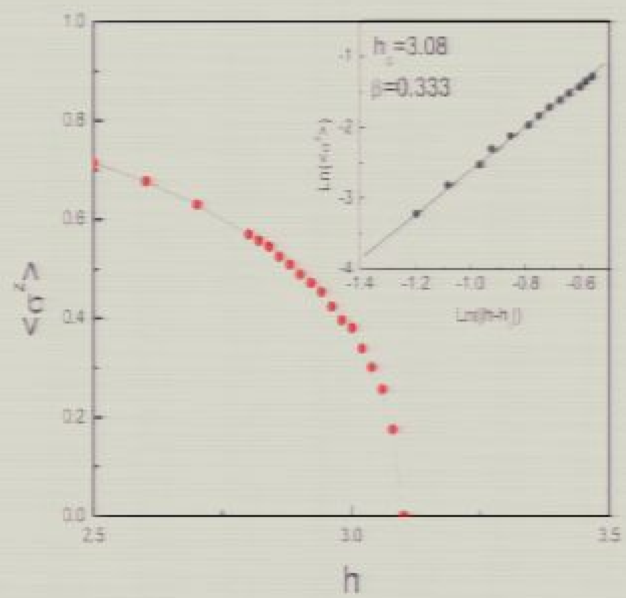
Example: symmetry breaking order

Transverse Ising model:

$$H = -\sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x$$

$D = 2 \quad D_{\rm cut} = 18$





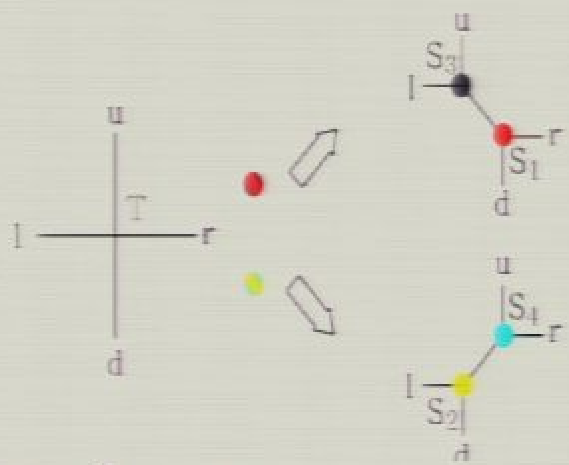
- System size: $N=2^{18}$

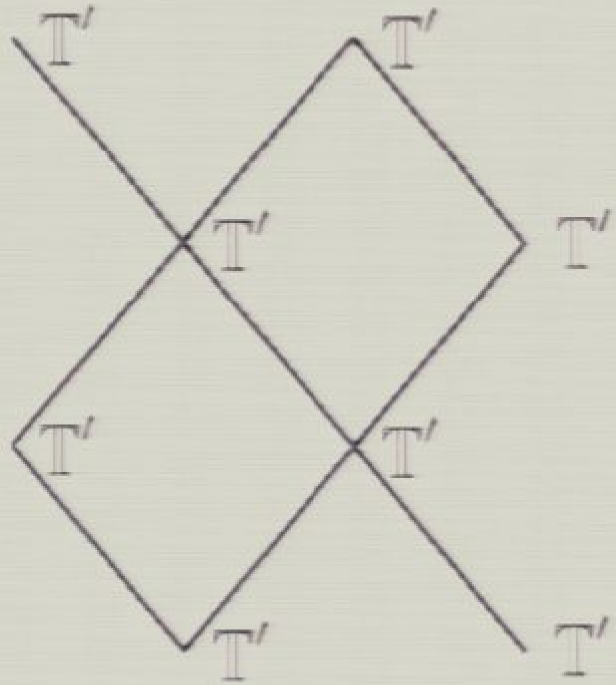
$\langle \sigma^z \rangle = A|h - h_c|^\beta$

$\beta^{QMC} \simeq 0.327 \quad h_c^{QMC} \simeq 3.044$

Much better than simple mean-field!

Detail implementation:





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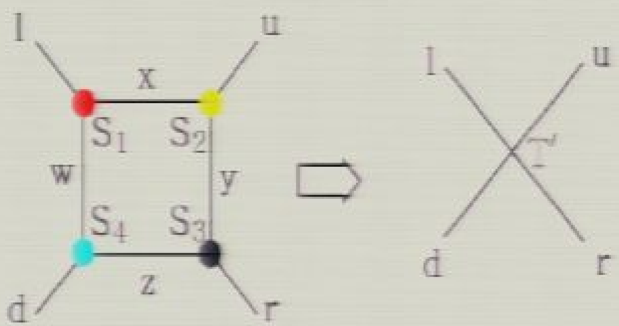

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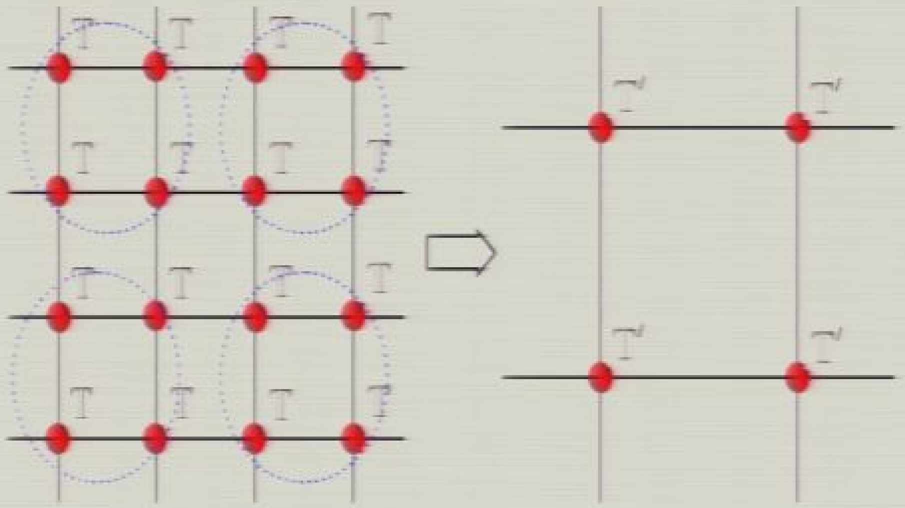

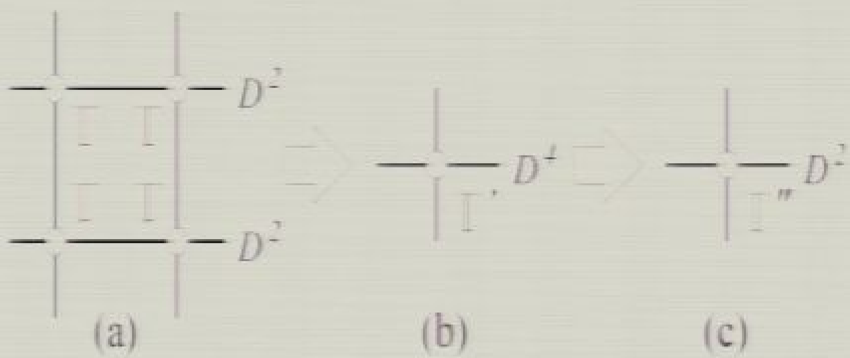

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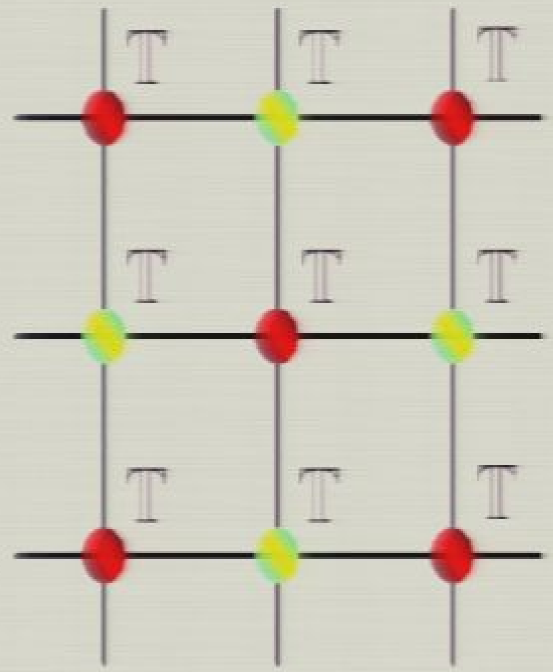
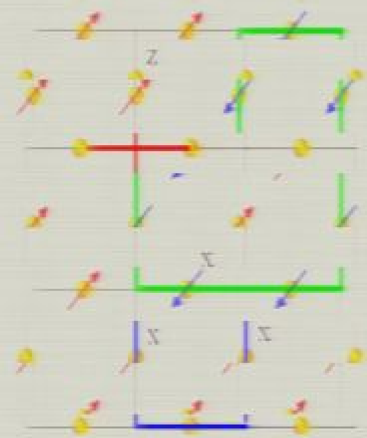


Example: topological order

Z_2 gauge model:

$$H = U\sum_{v}\left(1-\prod_{l\in v}\sigma_l^z\right) - t\sum_{p}\prod_{l\in p}\sigma_l^x$$

(Kitaev 2003, M. Levin and X.G. Wen 2005)



$|\uparrow\rangle$ — no string; $|\downarrow\rangle$ — one string

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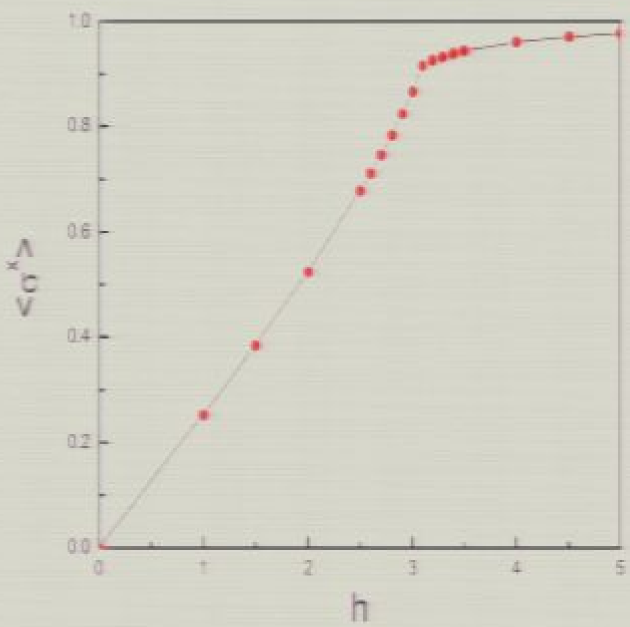

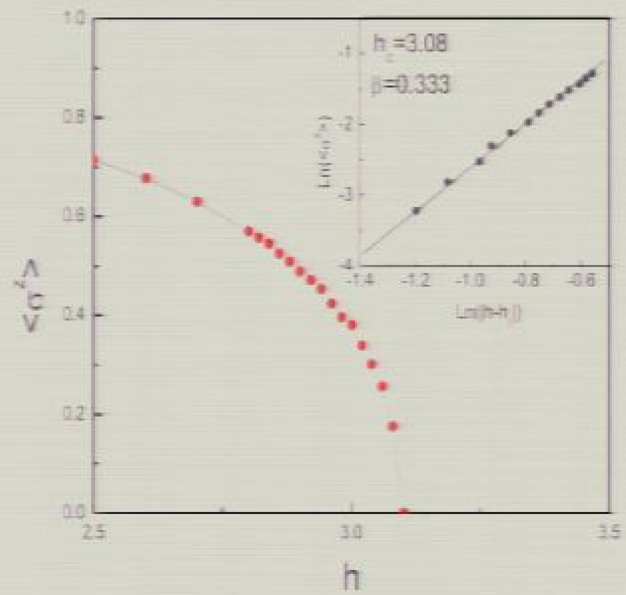

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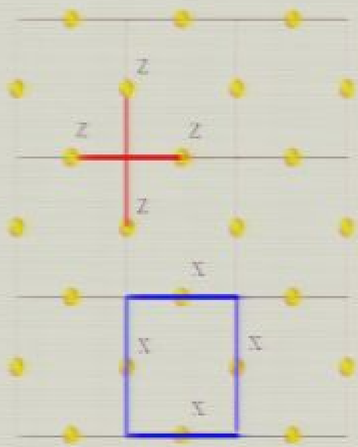

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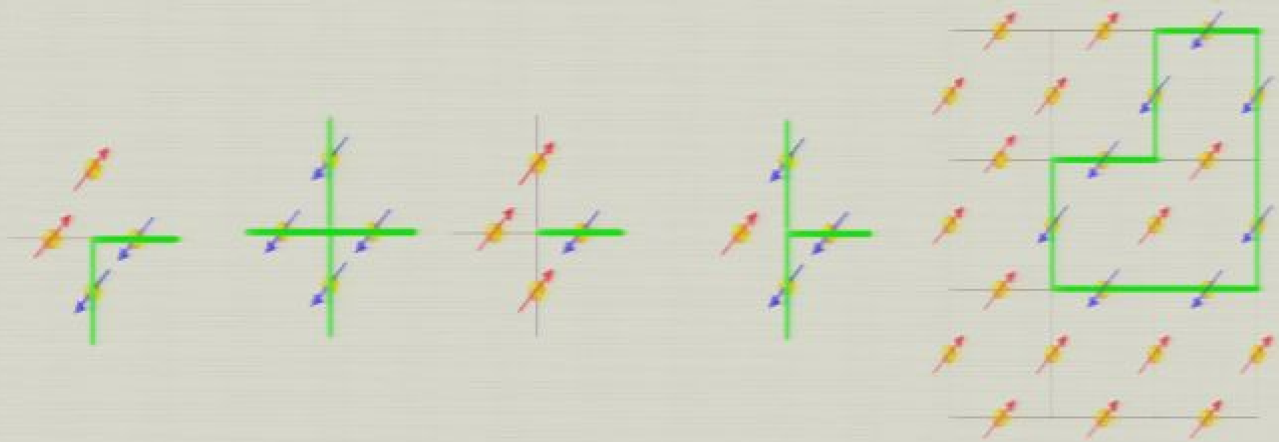

$|\uparrow\rangle$ — no string; $|\downarrow\rangle$ — one string

Example: topological order

Z_2 gauge model:

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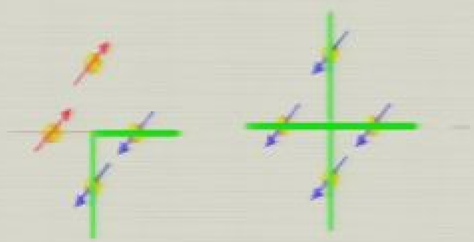

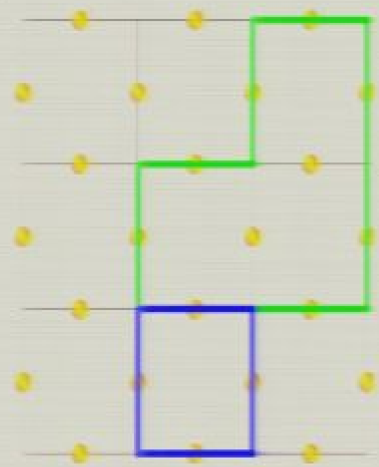

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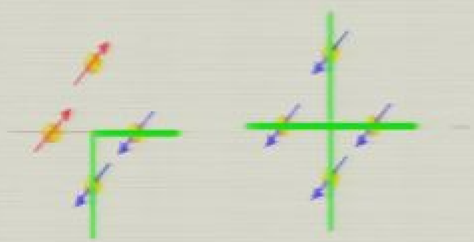

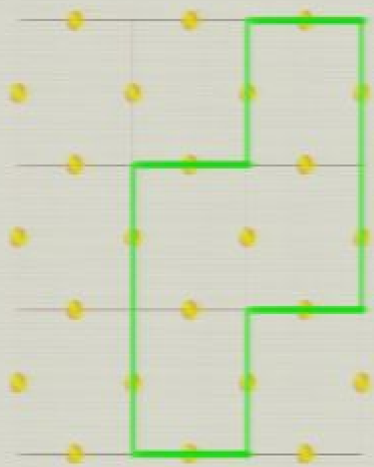

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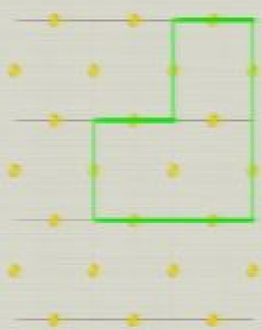
=



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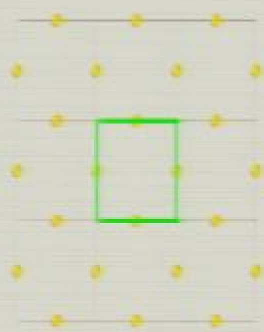

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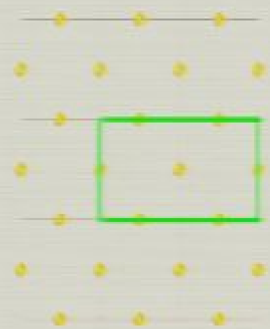

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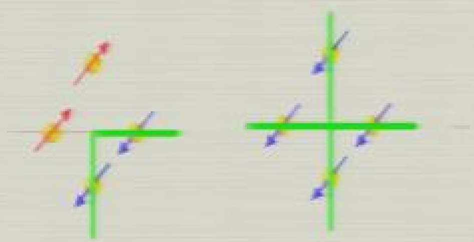

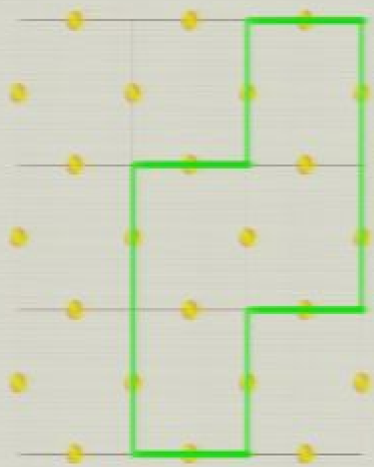

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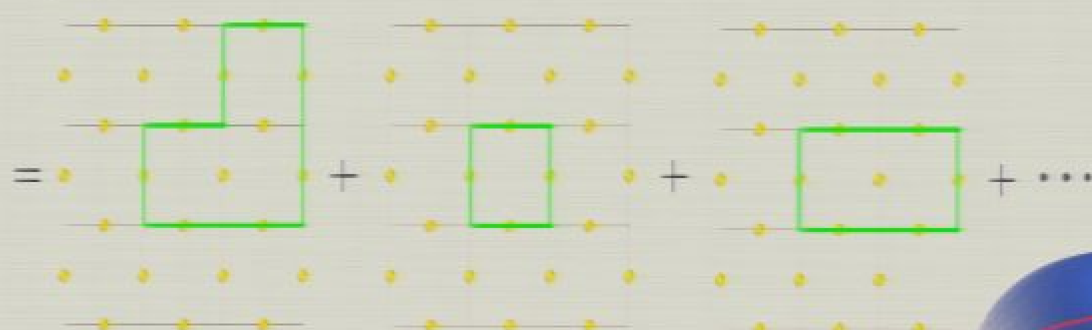

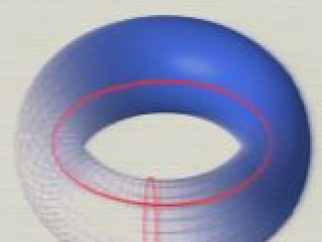

- Four fold ground state degeneracies

A generic model

Z_2 gauge model with string tension:

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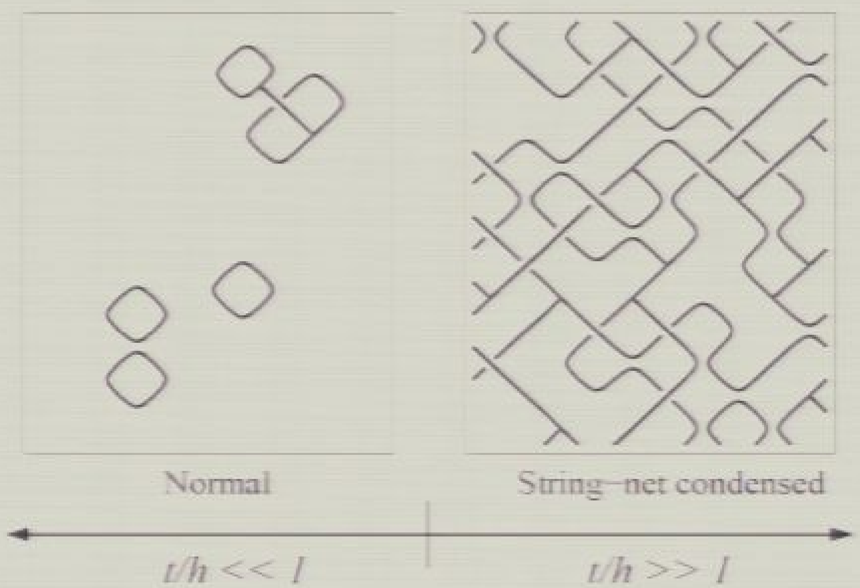
- Small-loop to large-loop phase transition



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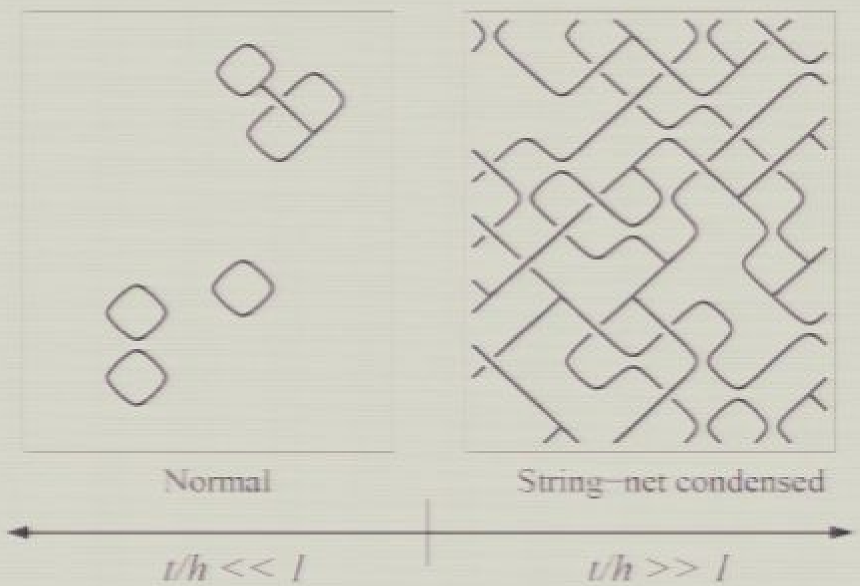

Do we have a mean-field solution?

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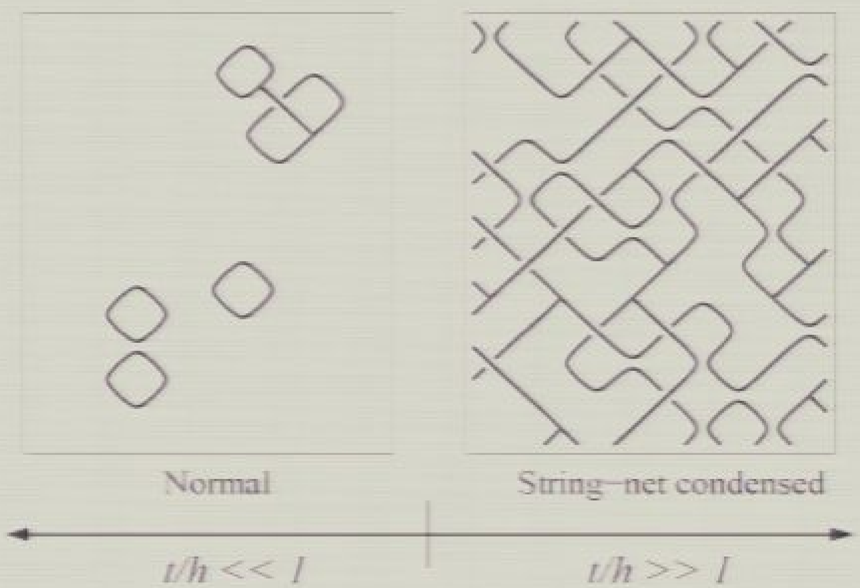


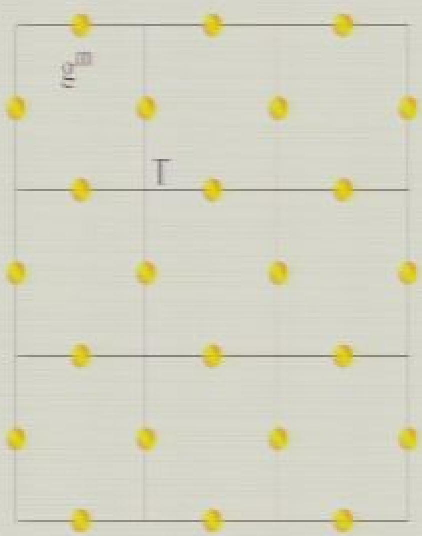
Do we have a mean-field solution? No

How to understand such kind of systems?

TPS representations for topologically ordered states

TPS representation for ground state of Z_2 gauge model

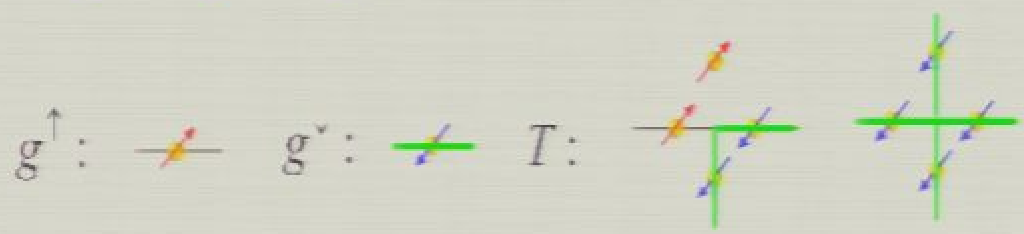
$$|\Psi_{Z_2}\rangle = \sum_{m_1, m_2, \ldots} \mathrm{tTr}[\otimes_v T \otimes_l g^{m_l}] |m_1, m_2, \ldots\rangle$$



$$T_{\alpha\beta\gamma\delta} = \begin{cases} 1 & \text{if} \quad \alpha+\beta+\gamma+\delta \quad \text{even} \\ 0 & \text{if} \quad \alpha+\beta+\gamma+\delta \quad \text{odd} \end{cases}$$

$$g^{\uparrow}_{00} = 1, \quad g^{\downarrow}_{11} = 1, \quad \text{others} = 0.$$

with internal indices like α running over $0, 1$



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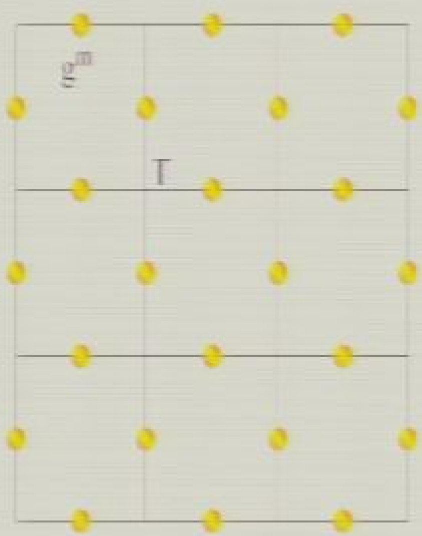

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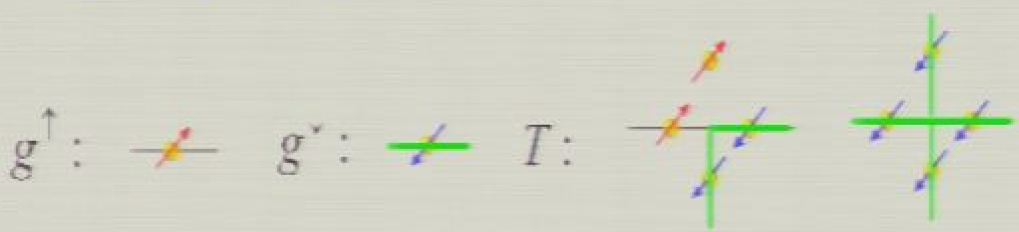


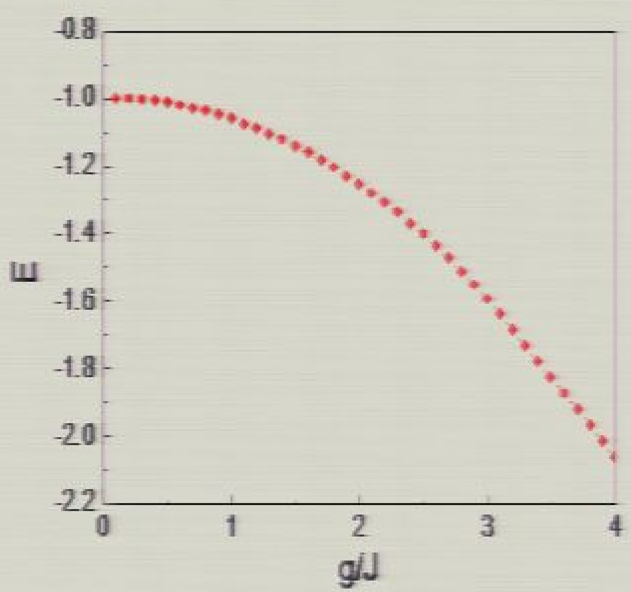
- It's easy to study local (Hamiltonian) perturbations of the system

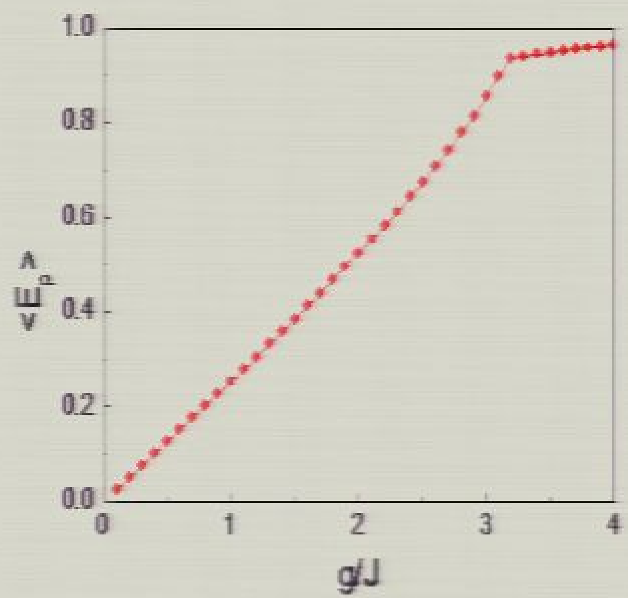
Phase diagram

Z_2 gauge model with string tension:

$$H = U\sum_{v}\left(1-\prod_{l\in v}\sigma_l^z\right) - g\sum_{p}\prod_{l\in p}\sigma_l^x - J\sum_{l}\sigma_l^z$$

- D=4, five parameters





- Second order transition at g/J=3.1, very close to QMC result with g/J=3.044

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Fermion

How to simulate fermion systems?

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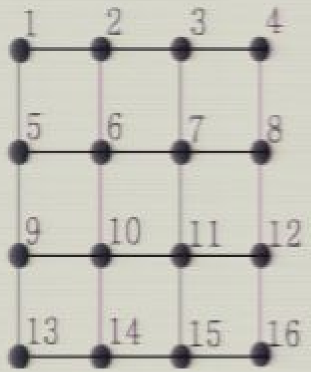
- Treat fermion systems as ordinary hardcore boson/spin systems.



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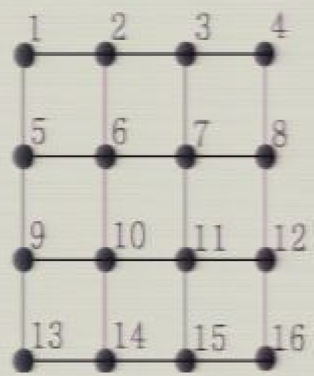

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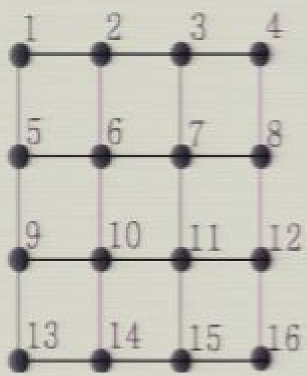

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- A naive wavefunction

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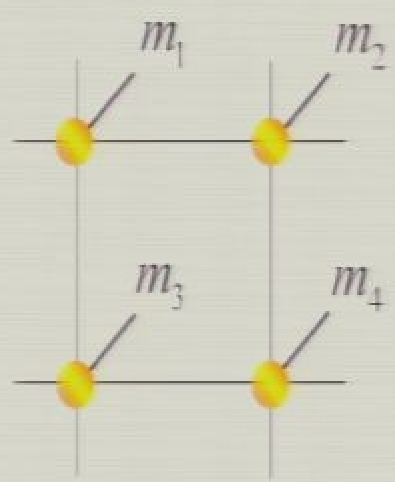

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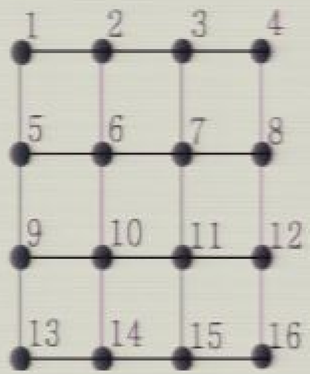

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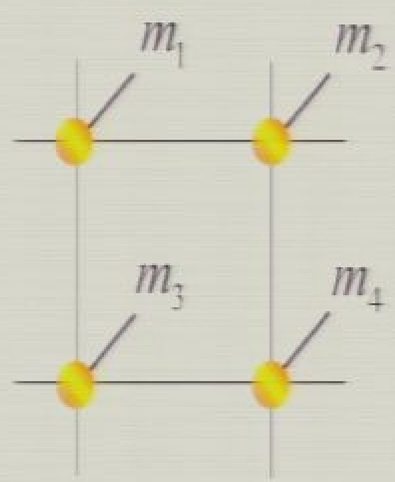

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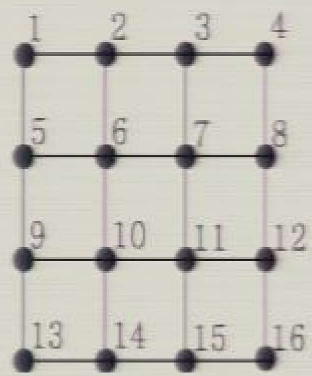

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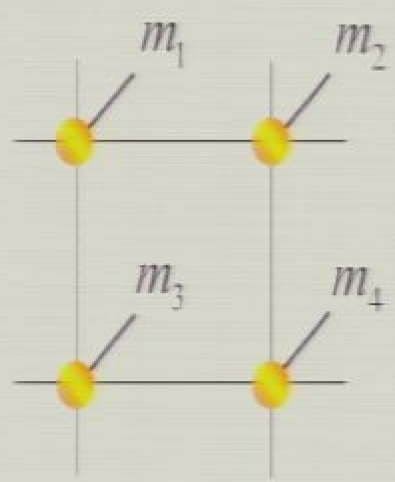

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Grassmann TPS (Z C Gu *etal*. 2010)

• A fermion wavefunction should give out the correct sign under different orderings.

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The magic of Grassmann algebra:

0,1

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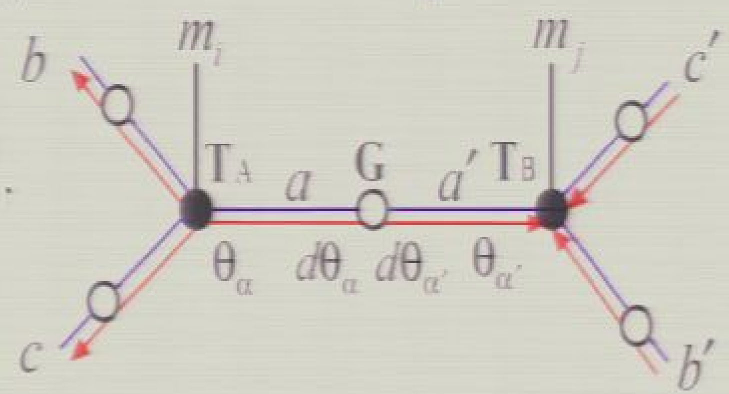

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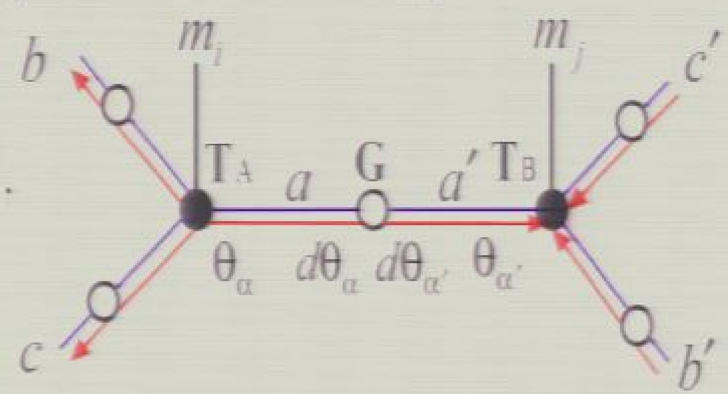


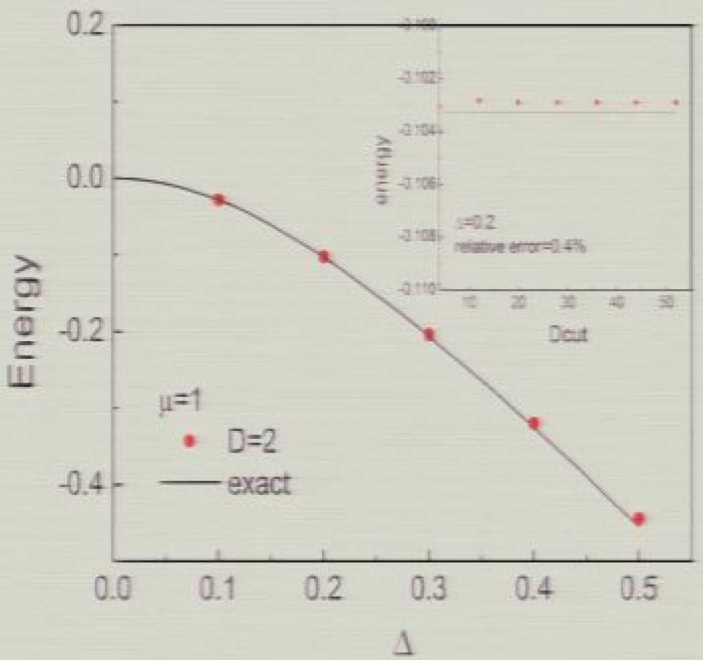
$$\Psi(\{m_i\},\{m_j\}) = \sum_{\{a\},\{a'\}} \int \prod_{\langle ij\rangle} \mathbf{G}_{aa'} \prod_{i\in A} \mathbf{T}_{A_{abc}}^{m_i} \prod_{j\in B} \mathbf{T}_{B_{a'b'c'}}^{m_j}$$

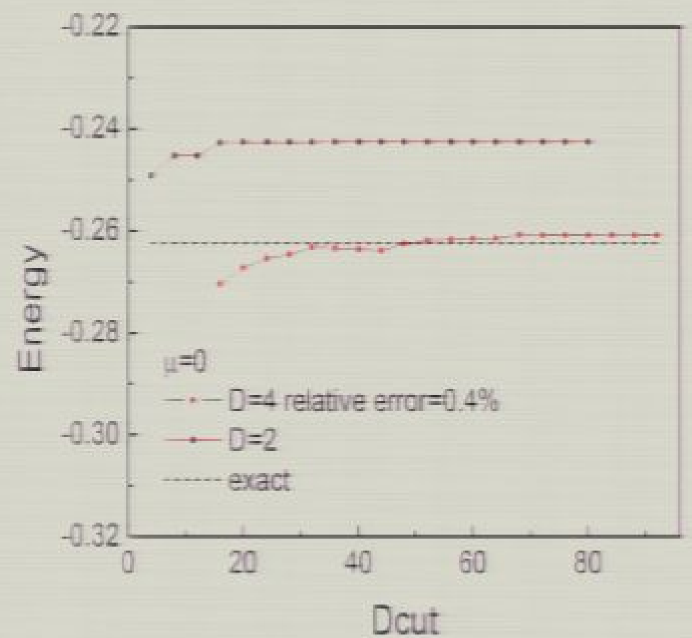
A free fermion model:

Free fermion model on honeycomb lattice:

$$H = -2\Delta \sum_{\langle i\in A j\in B\rangle} c_i^\dagger c_j^\dagger + H.c. + \mu \sum_i n_i \qquad N=2*3^6$$

- Imaginary time evolution is performed to find the ground state.





- The energy is correct even with extremely small D.
- Truncation error is larger for critical systems.

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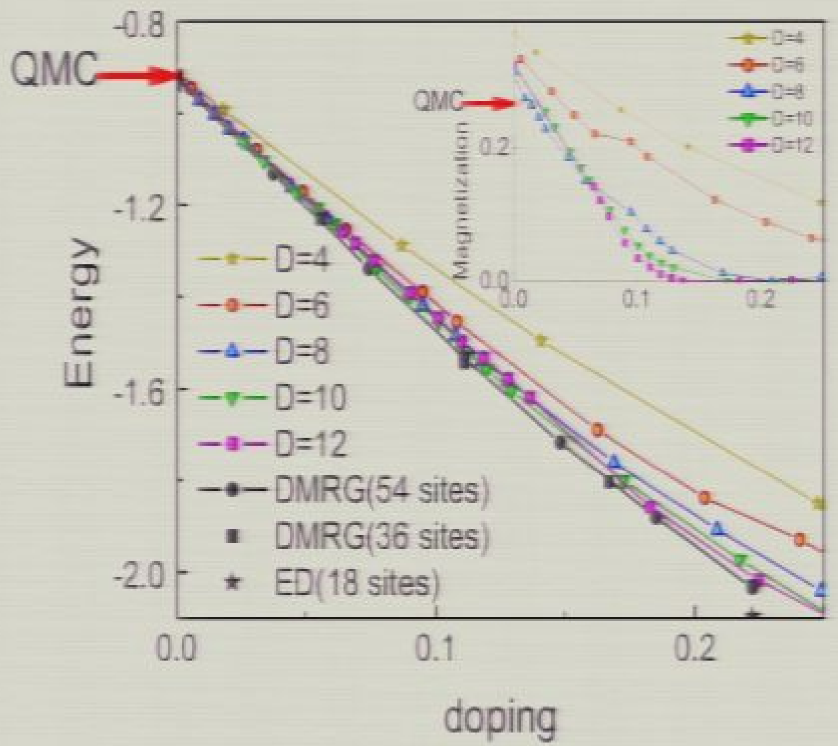
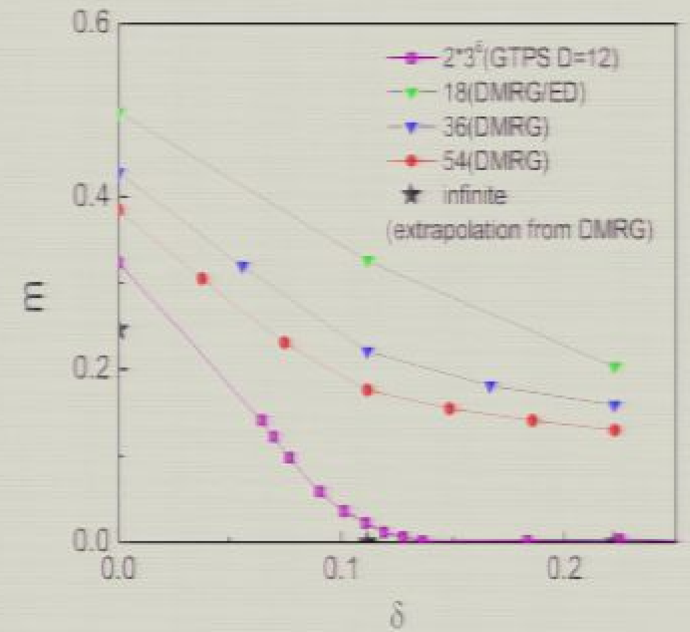
A more challenge model:

Honeycomb lattice t-J model (t=3J)

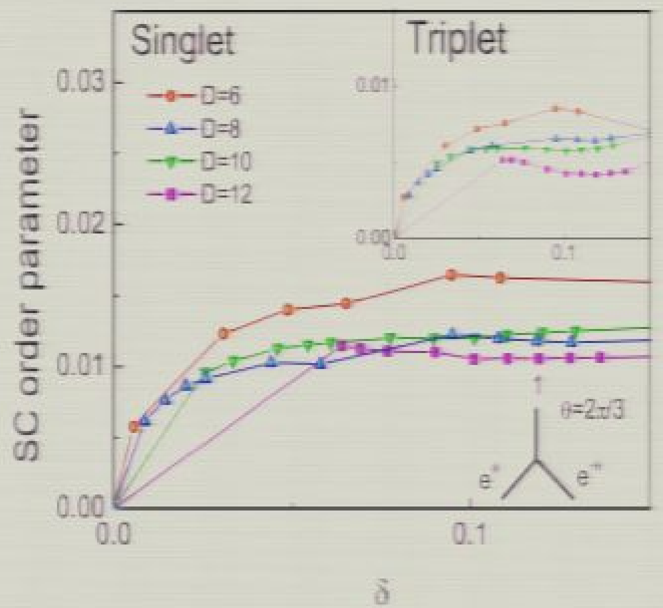
$$H_{t\text{-}J} = -t\sum_{\langle ij\rangle\sigma} \tilde{c}^\dagger_{i\sigma}\tilde{c}_{j\sigma} + H.c. + J\sum_{\langle ij\rangle}\left(\hat{S}_i\hat{S}_j - \frac{1}{4}\hat{n}_i\hat{n}_j\right) - \mu\sum_i \hat{n}_i \quad \tilde{c}_{i\sigma} = \hat{c}_{i\sigma}(1-\hat{c}^\dagger_{i\bar{\sigma}}\hat{c}_{i\bar{\sigma}})$$





- Energy agree with QMC at half filling, slightly larger magnetization.
- Energy is pretty good comparing with ED/DMRG for low doping.

Chiral superconductivity:



- A robust chiral SC phase is found over a large doping regime.
- Coexist with AF at low doping.
- With both singlet and triplet paring.
- Triplet d vector anti-parallel with Neel vector.

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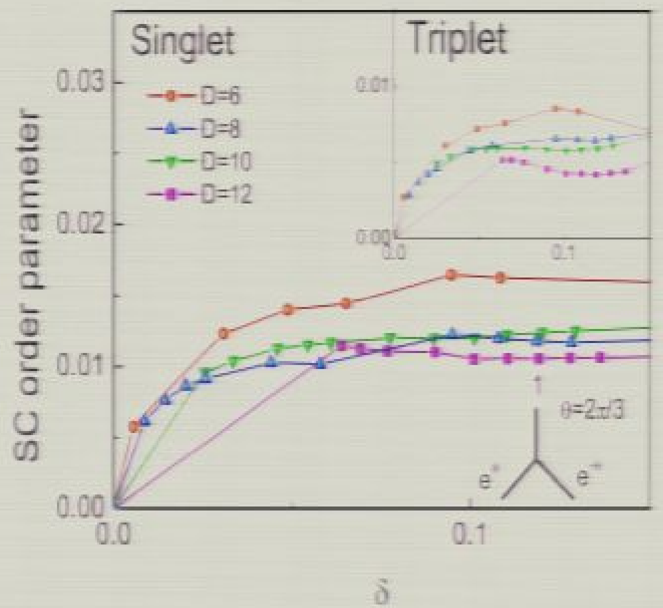

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Possible realizations: $InCu_{2/3}V_{1/3}O_3$.

- S=1/2 AF on honeycomb lattice.(Phys. Rev. B 78, 024420 (2008))
- Dope: chiral topological superconductor?

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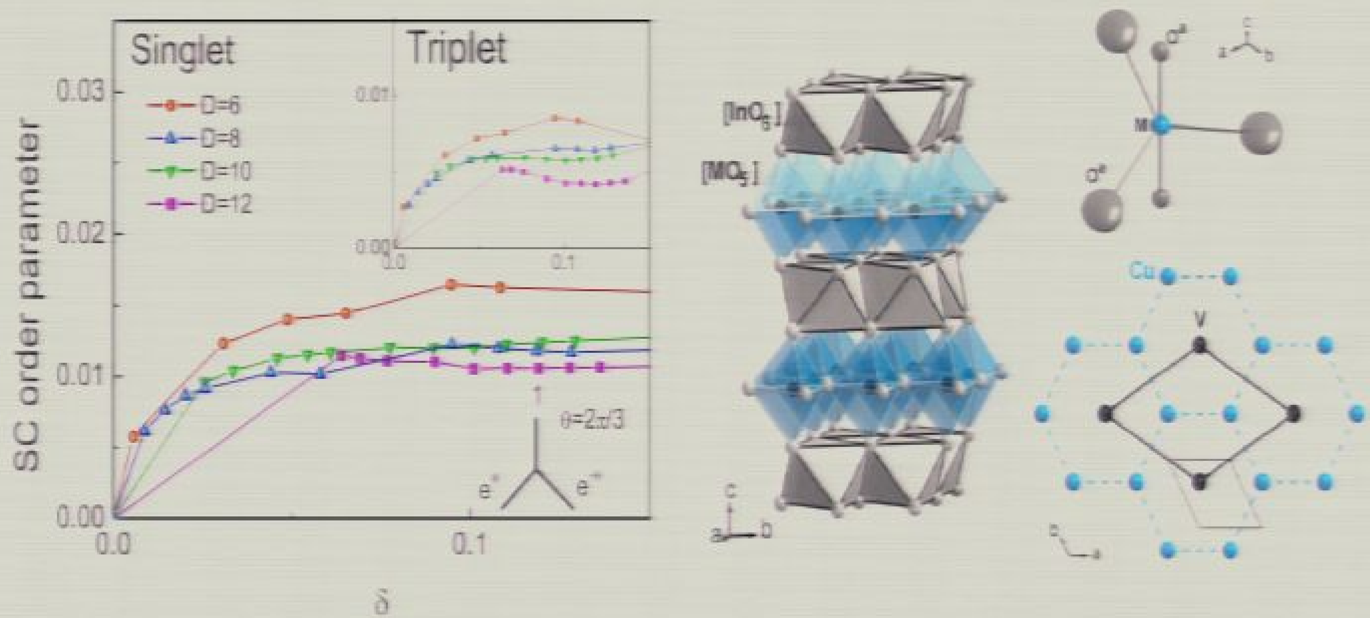

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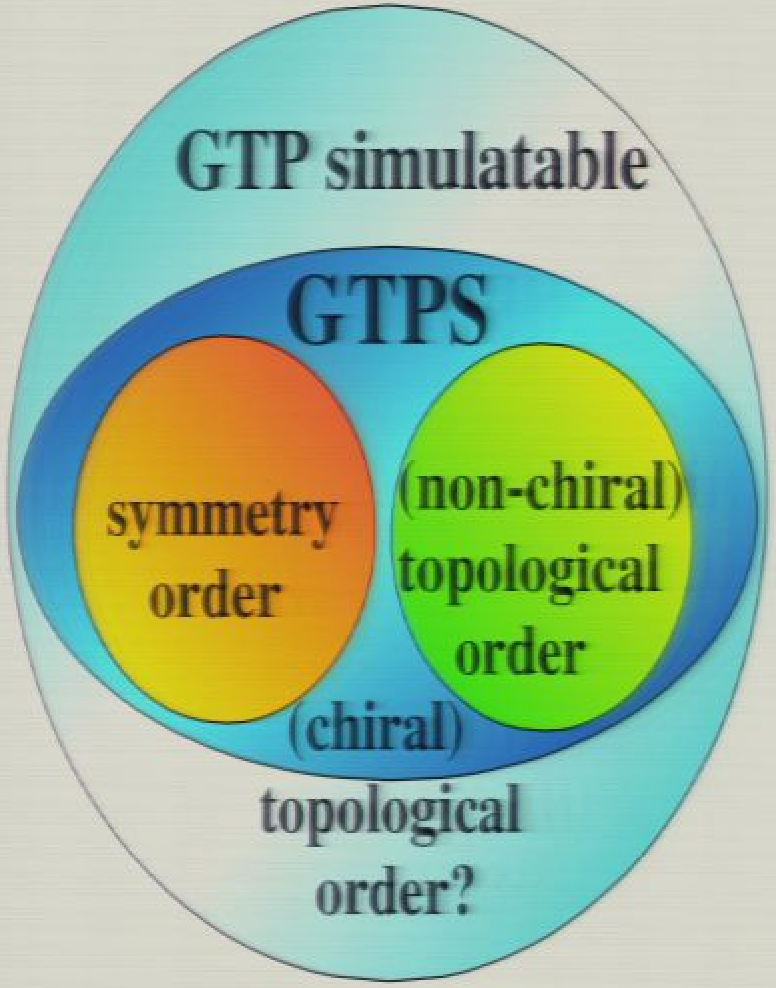
Summaries and future works



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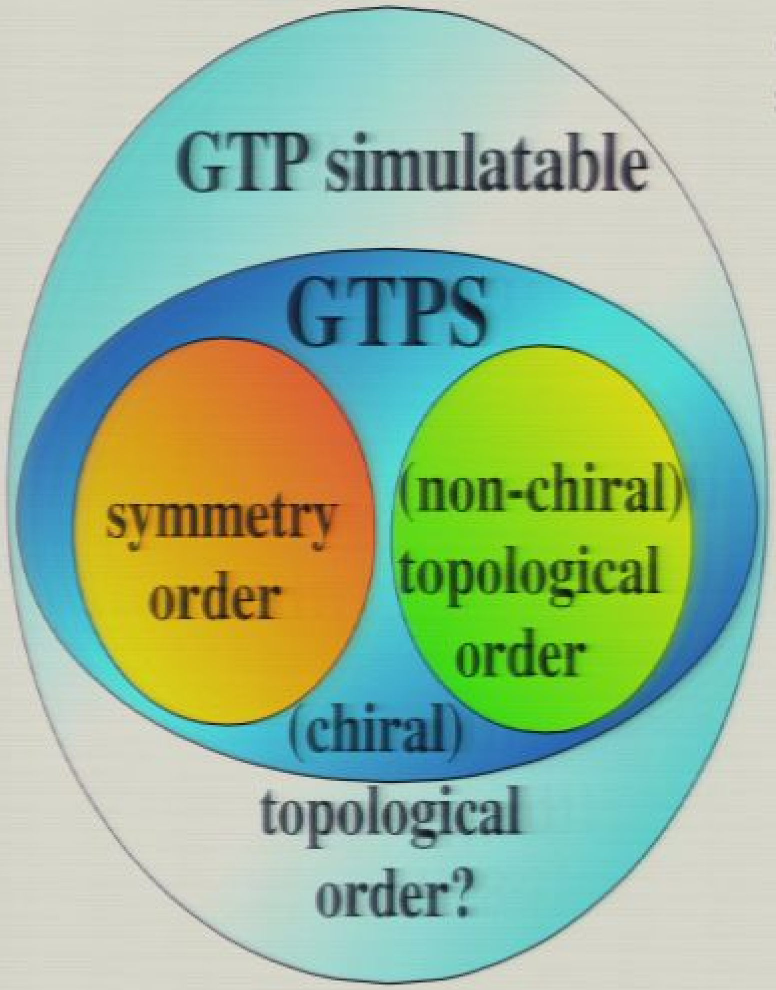

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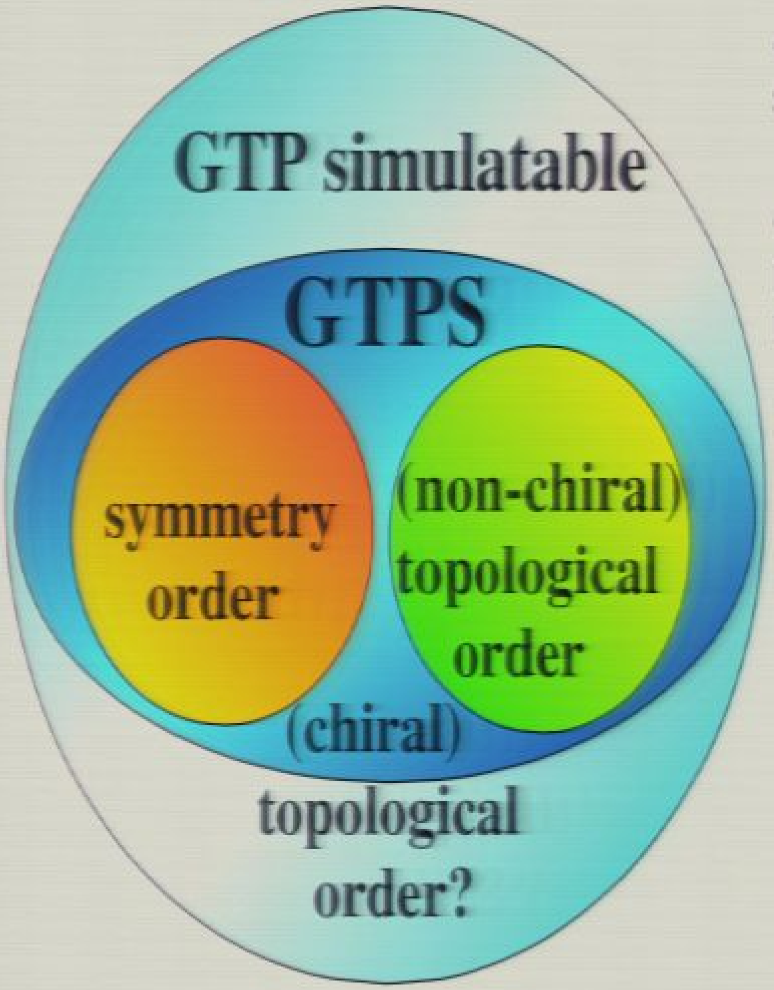
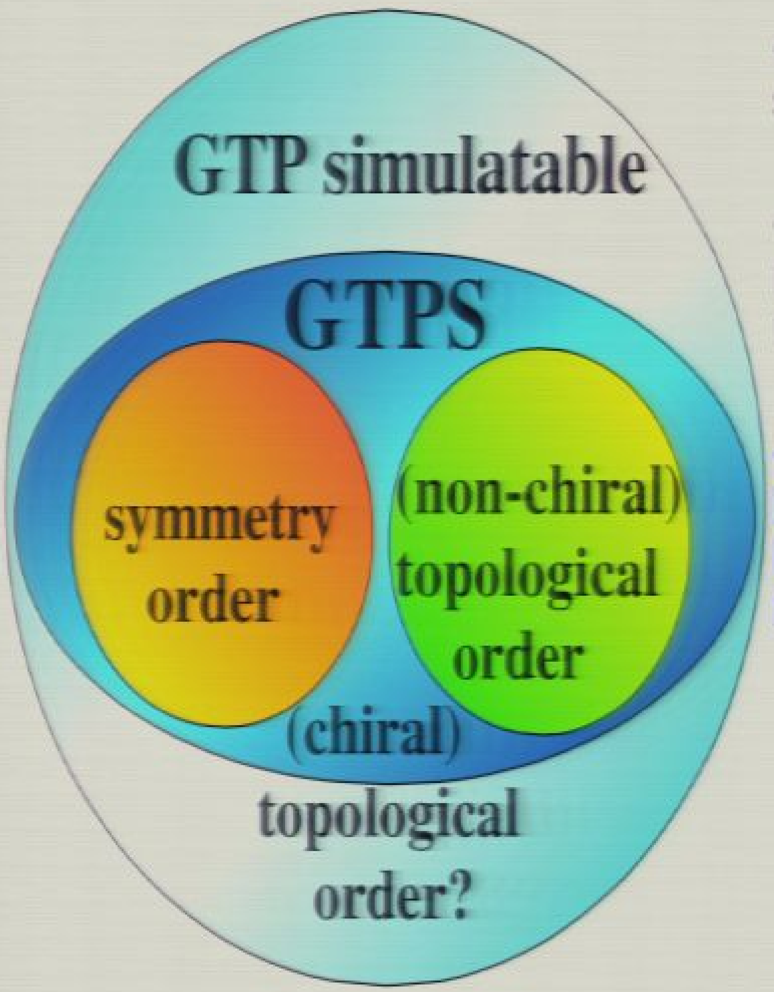


- The concept of RG is a key for efficient simulations.

- A chiral topological superconducting phase is predicted on the honeycomb lattice t-J model

Summaries and future works



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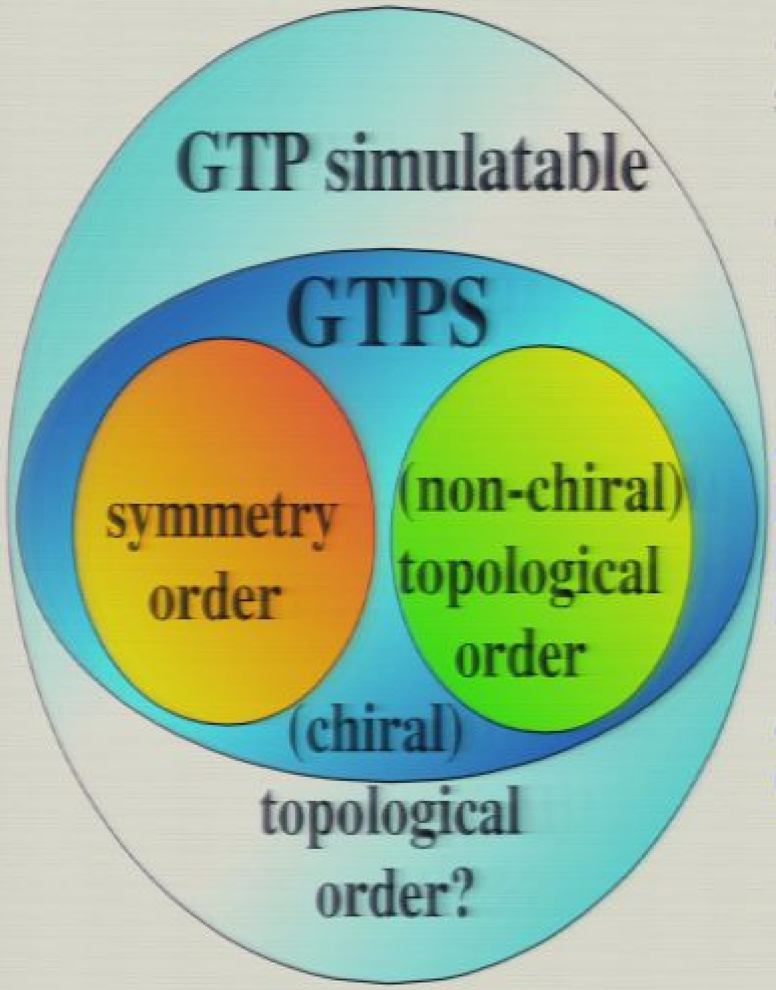

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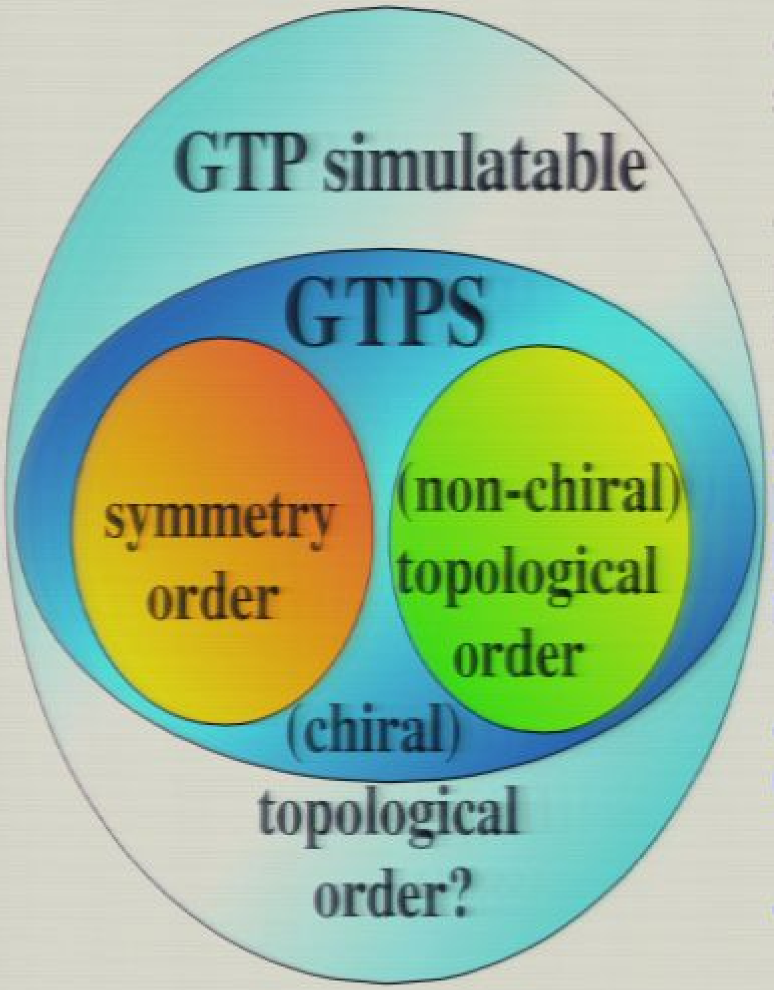

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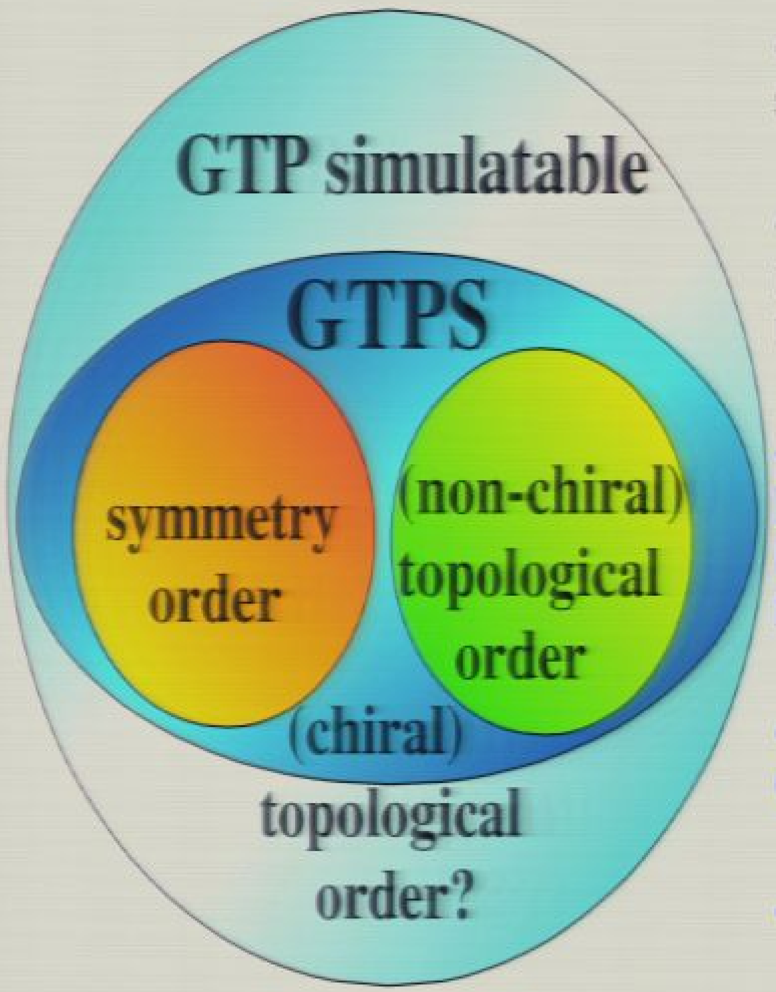

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