Title: Detecting extra dimensions with gravitational waves

Date: Feb 02, 2011 02:00 PM

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Abstract: Cosmic strings, generic in brane inflationary models, may be detected by the current generation of gravitational wave detectors. An important source of gravitational wave emission is from isolated events on the string called cusps and kinks. I first review cosmic strings, discussing their effective action and motion, and showing how cusps and kinks arise dynamically. I then show how allowing for the motion of the strings in extra dimensions gives a potentially significant reduction in signal strength, and comment on current LIGO bounds.

Durham University

DETECTING EXTRA DIMENSIONS WITH GRAVITATIONAL WAVES

"CAPTAIN...IT'S A COSMIC STRING FRAGMENT!"

RUTH GREGORY

2/2/11

SARAH CHADBURN, GHAZAL GESHNIZJANI, EIMEAR O'CALLAGHAN, AND IVONNE ZAVALA

SETTING

Inflation is a successful paradigm for explaining the origin of perturbations and solving other issues with the Big Bang description of the universe.
It is generic, powerful and widely accepted.
It lacks a robust theoretical foundation. (Big industry.)

String theory is a successful mechanism for unifying gravity with particle interactions.
It is powerful and widely accepted.
It lacks practical applications. (Big industry.)

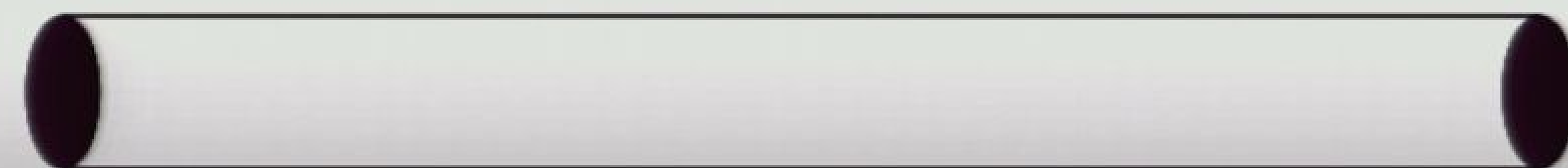
The two come together in BRANE INFLATION, which might leave observational traces in COSMIC STRINGS: This is the subject of this talk.

OUTLINE

- INTRODUCTION AND MOTIVATION
- COSMIC STRINGS AND SUPERSTRINGS
- GRAVITATIONAL WAVES
- CUSPS AND KINKS WITH EXTRA DIMENSIONS
- PROSPECTS FOR DETECTION

KALUZA AND KLEIN

FIRST SUGGESTED UNIFYING GRAVITY AND ELECTROMAGNETISM 80 YEARS AGO, BY ADDING EXTRA DIMENSION TO SPACETIME - A SMALL CIRCLE, WHICH GEOMETRICALLY REPRESENTED THE U(1) GAUGE GROUP OF ELECTROMAGNETISM IN A VERY DIRECT FASHION.



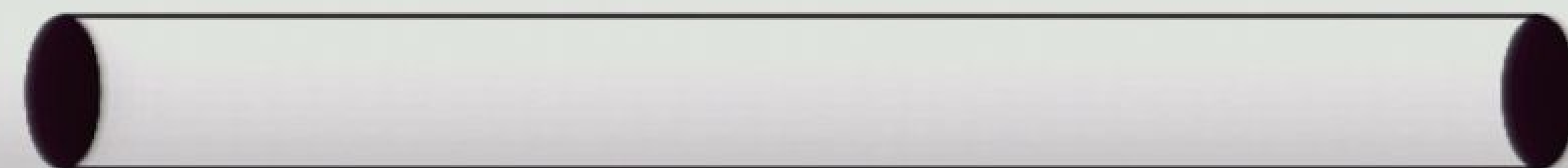
$$G^{(5)} \longrightarrow G^{(4)} + \Phi + A_{(em)}$$

Outline

- Introduction and Motivation
- Cosmic strings and superstrings
- Gravitational waves
- Cusps and Kinks with Extra dimensions
- Prospects for detection

Kaluza and Klein

First suggested unifying
Gravity and Electromagnetism 80 years ago, by adding
extra dimension to spacetime - a small circle, which
geometrically represented the U(1) gauge group of
electromagnetism in a very direct fashion.

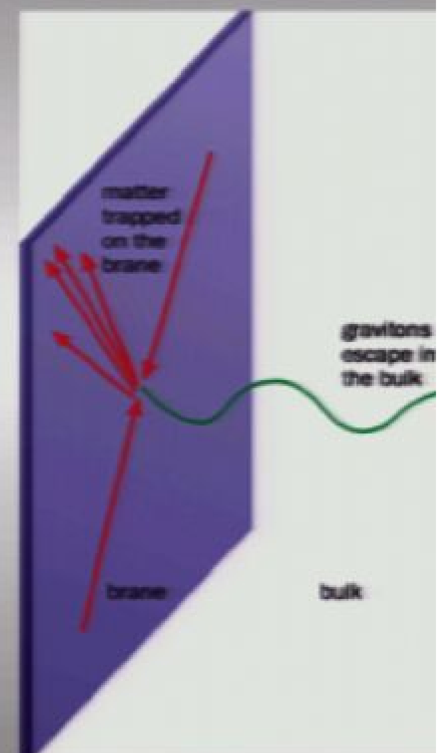


$$G^{(5)} \longrightarrow \underbrace{G^{(4)} + \Phi}_{\text{Scalar-tensor gravity}} + \underbrace{A_{(em)}}_{\text{Electromagnetism}}$$

BRANEWORLDS

…provide an alternative way of dealing with extr
dimensions…..

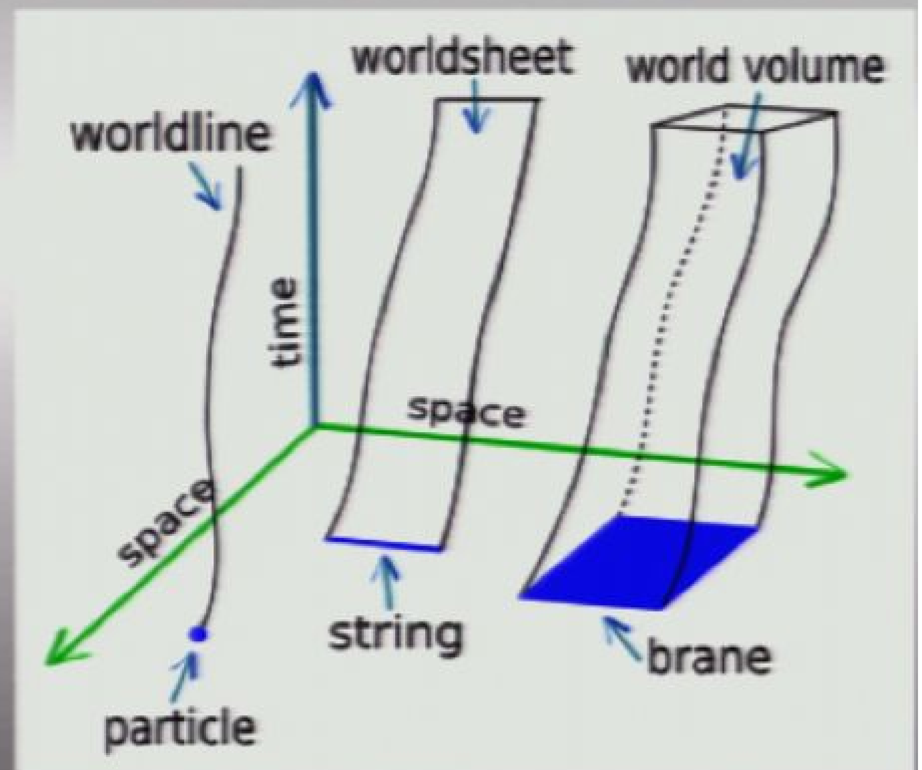
A braneworld is a slice through spacetime on which we live. We cannot (easily) see the extra dimensions perpendicular to our slice, all of our standard physics is confined.



String theory is full of branes, extended objects charged under RR or NSNS fields present in the low energy effective theory.

Brane?

We use the term "brane" to mean an extended object: 0D is a particle, 1D a string, 2D a membrane, so more D is a p-brane!



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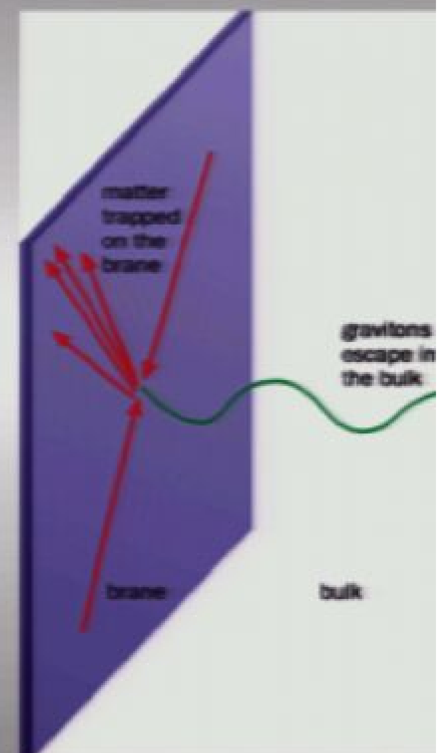

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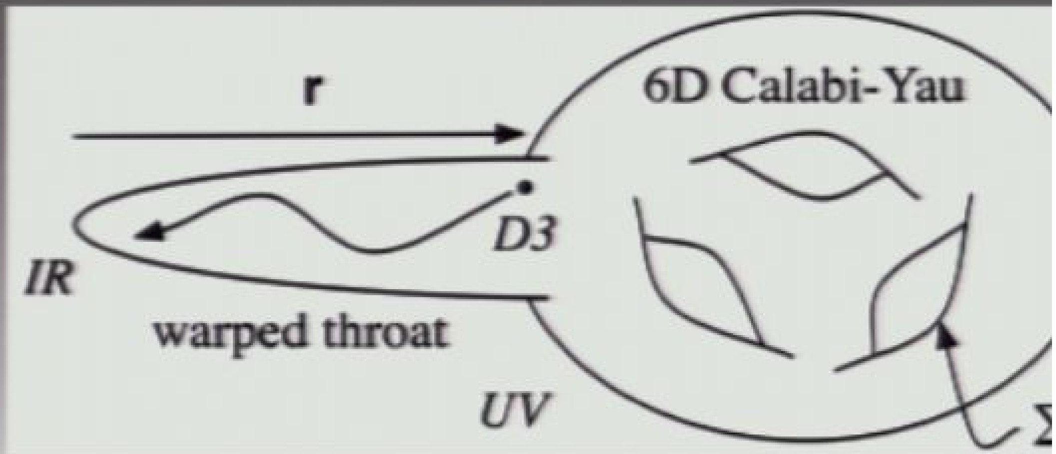
Brane Inflation

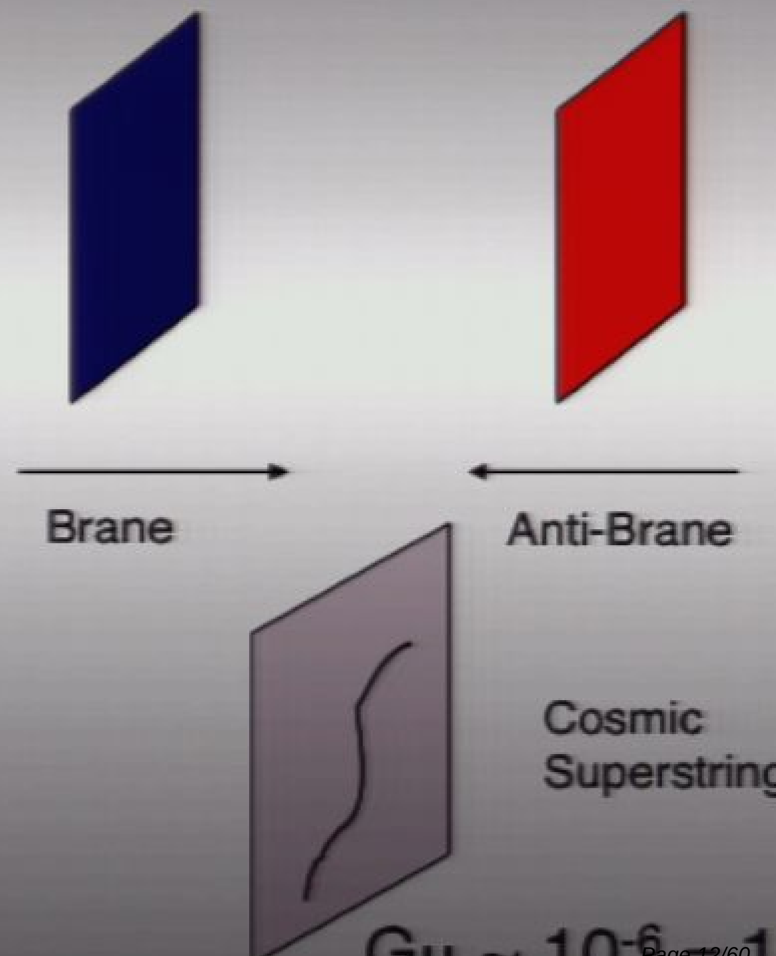


In brane inflation, we imagine an (anti-)brane moving on the internal extra dimensions and extended along the noncompact directions.

The higher dimensional information is encoded in a scalar describing the brane position: **The Inflaton.**

Inflation ends when the anti-brane meets the brane and annihilates.

Brane Inflation – the movie!



$G\mu \sim 10^{-6} - 10^{-12}$

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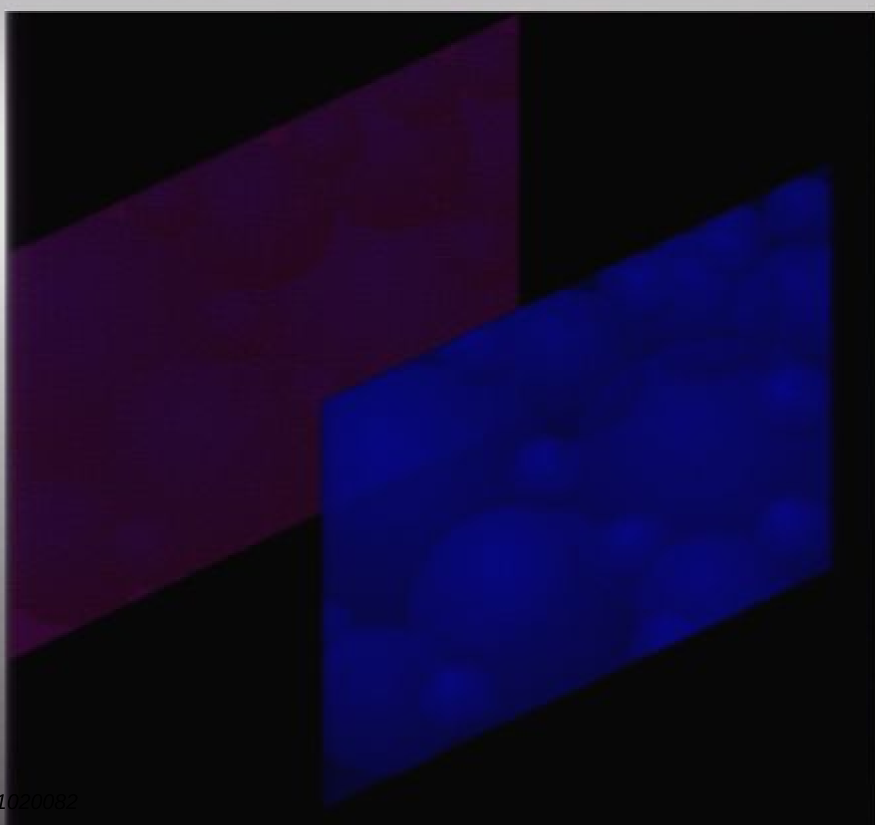

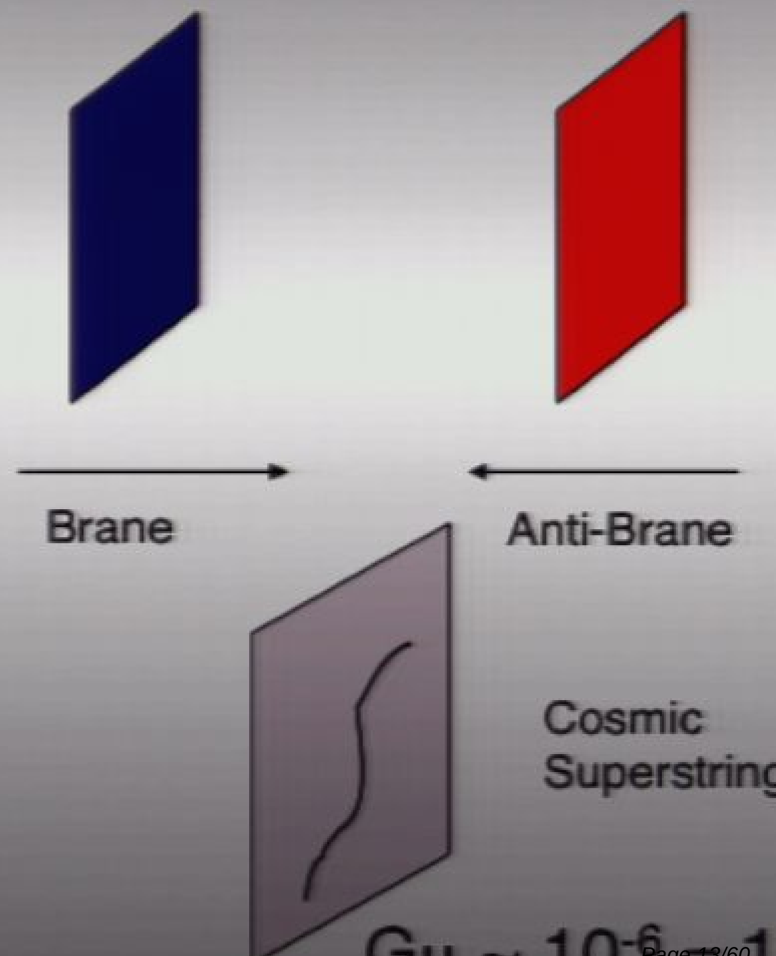

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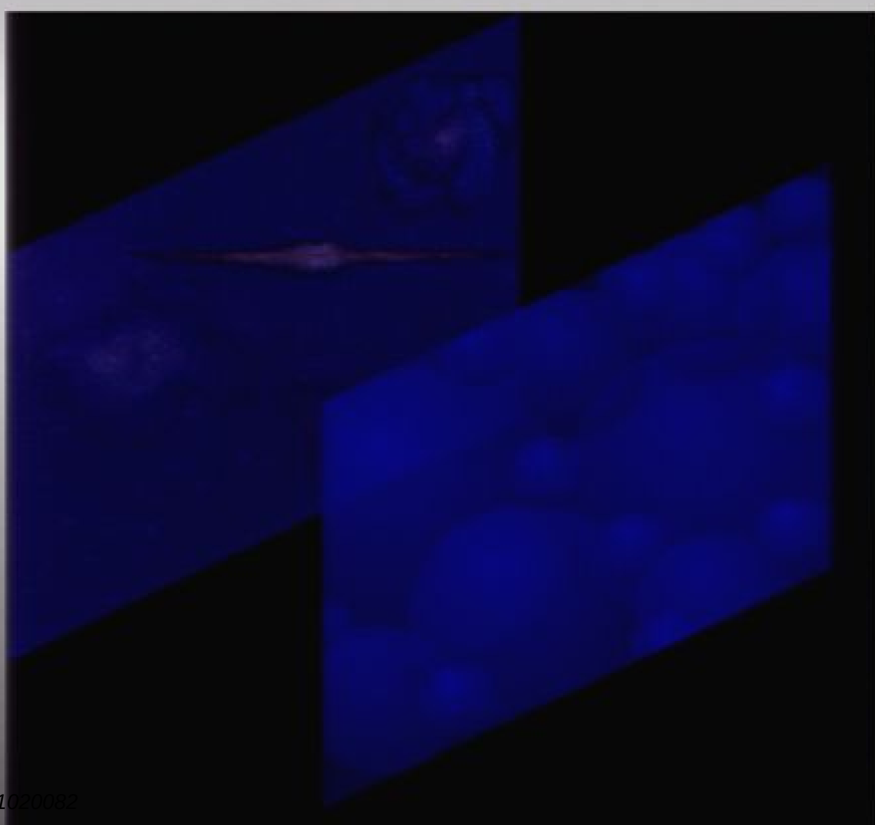
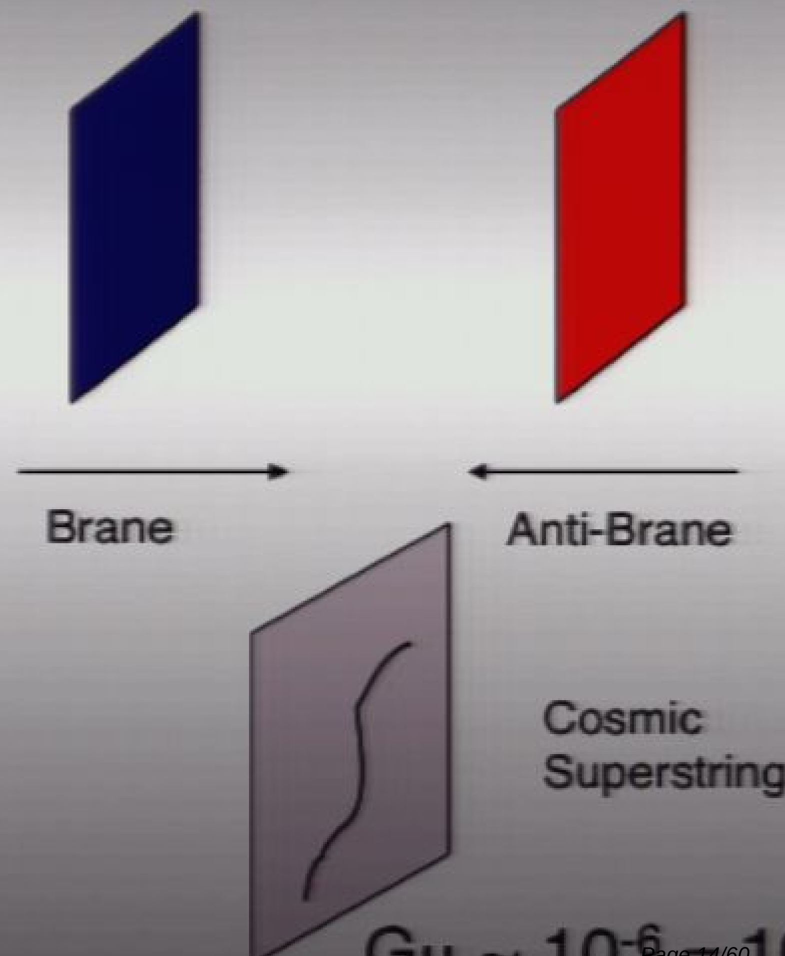
BRANE INFLATION – THE MOVIE!





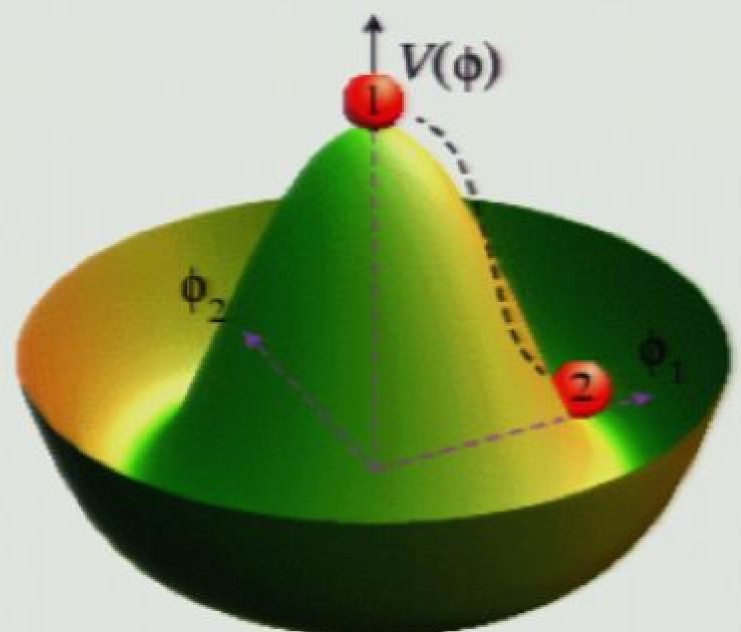
$G\mu \sim 10^{-6} - 10^{-12}$

WHAT IS A COSMIC STRING? 1

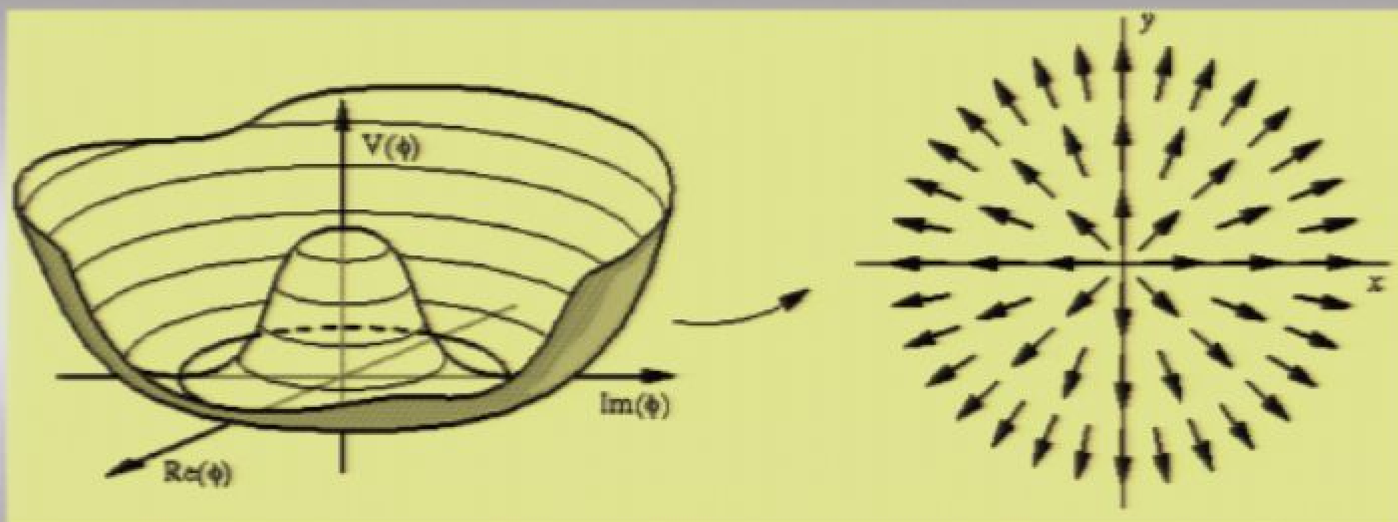
A cosmic string is a defect in the quantum vacuum of our Universe. It is a 1D cousin of the 2D domain wall. Usually described by using the Abelian Higgs as a toy model.

$$|D\phi|^2 - \tfrac{1}{4}F_{\mu\nu}^2 - V(\phi)$$

The potential V is U(1) symmetric, but has a mexican hat shape, so the vacuum is nontrivial: a circle.



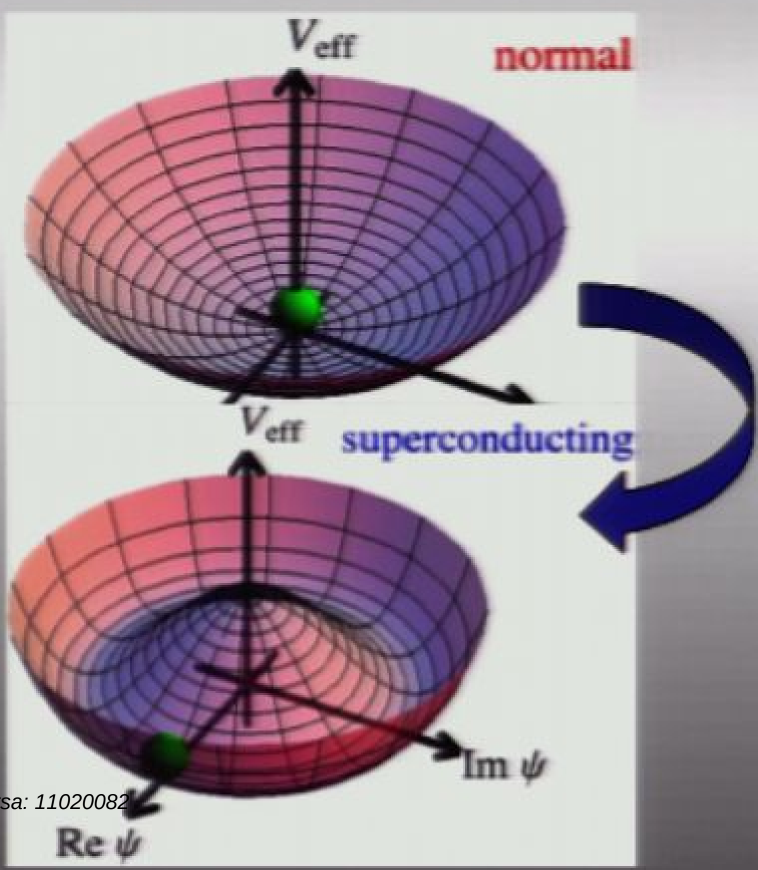
WHAT IS A COSMIC STRING? 2

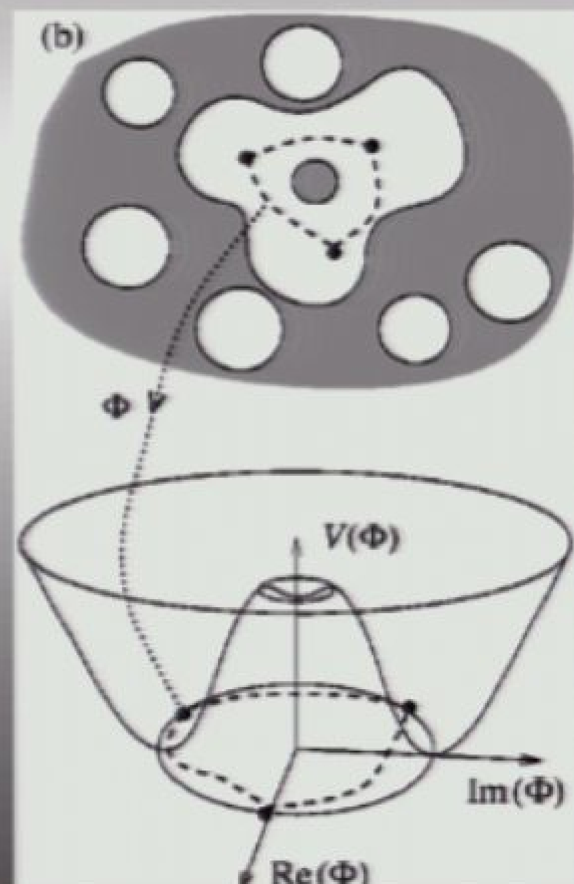


The phase of the Higgs winds around as we circle the origin. Shrinking our loop in space eventually lifts us out of the vacuum and gives a linear energy density.

HOW DO THEY FORM?

At high T, V is symmetric, but at lower T a "superconducting" phase forms, symmetry is spontaneously broken and the vector bosons become massive.





But we can fall into different vacua at differen places in space Generates the winding needed for the string / vortex.

Properties of a cosmic string

- Strings protected by topology – to unwind the string costs infinite energy.
- Strings either infinite, or closed loops (or end on black holes!)
- Width and mass (p.u.l.) set by massive gauge and scalar bosons

GUT String: $W \sim 10^{-27}$ cm
$\mu \sim 10^{21}$ Kg m^{-1}

Or, mass of

100M

=

HOW DO THEY EVOLVE?

Motion of the string is governed by its *effective action*, obtained by integrating out over the detailed core structure. This gives the Nambu action:

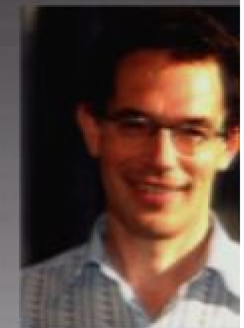
$$S = -\mu \int d\tau d\sigma \sqrt{\gamma}$$

AREA OF WORLDSHEET

Essentially the same action as string theory. The loops have left moving and right moving waves.

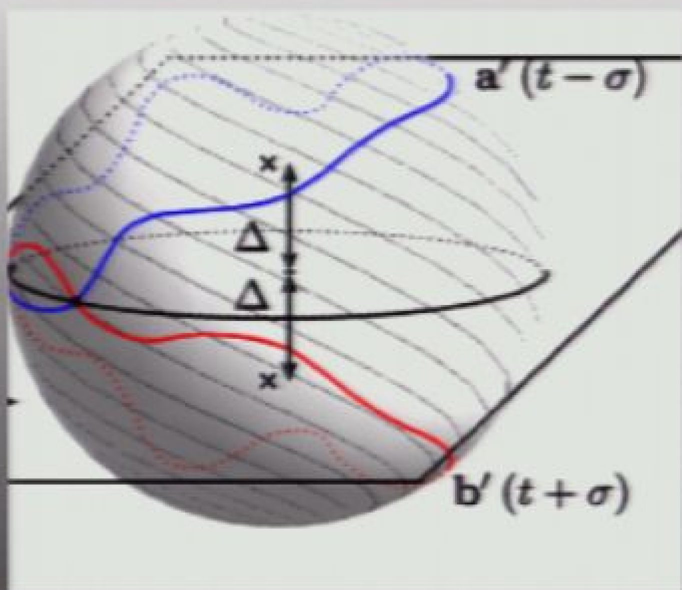
$$(\partial^2_\tau - \partial^2_\sigma)\, X^\mu = 0 \qquad X^\mu = (t, \tfrac{1}{2}[\underline{a}(t-\sigma) + \underline{b}(t+\sigma)])$$

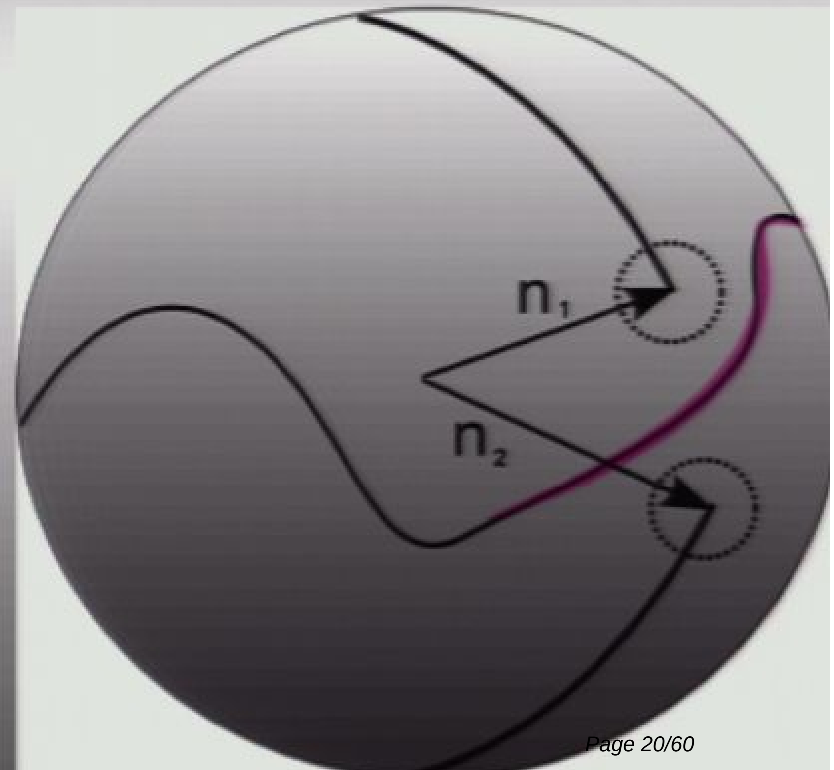
Kibble-Turok Sphere



Gauge constraints: $\dot{X}^2 = X'^2, \quad \dot{X} \cdot X' = 0$

$$\Rightarrow \quad (\underline{a}')^2 = (\underline{b}')^2 = 1$$





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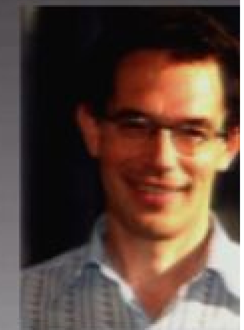

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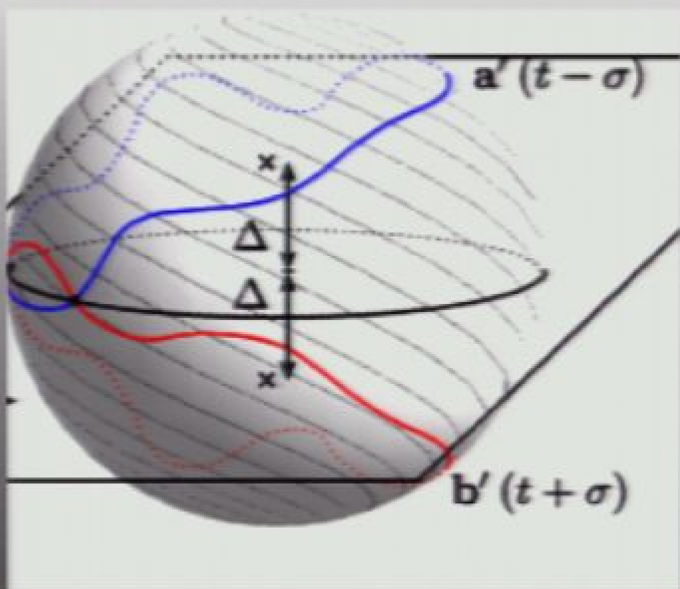
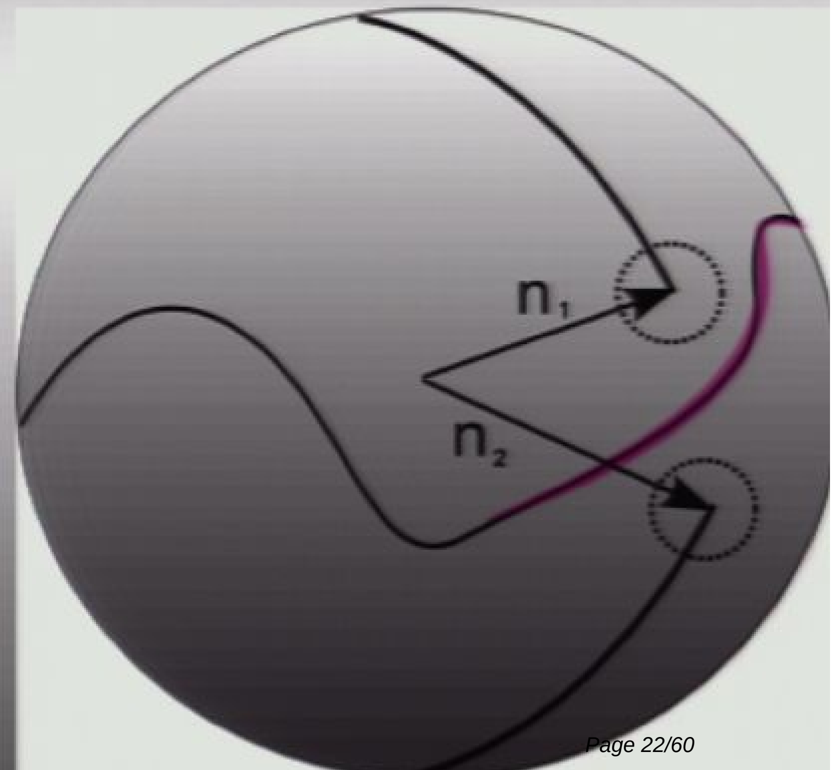




Cusps and Kinks

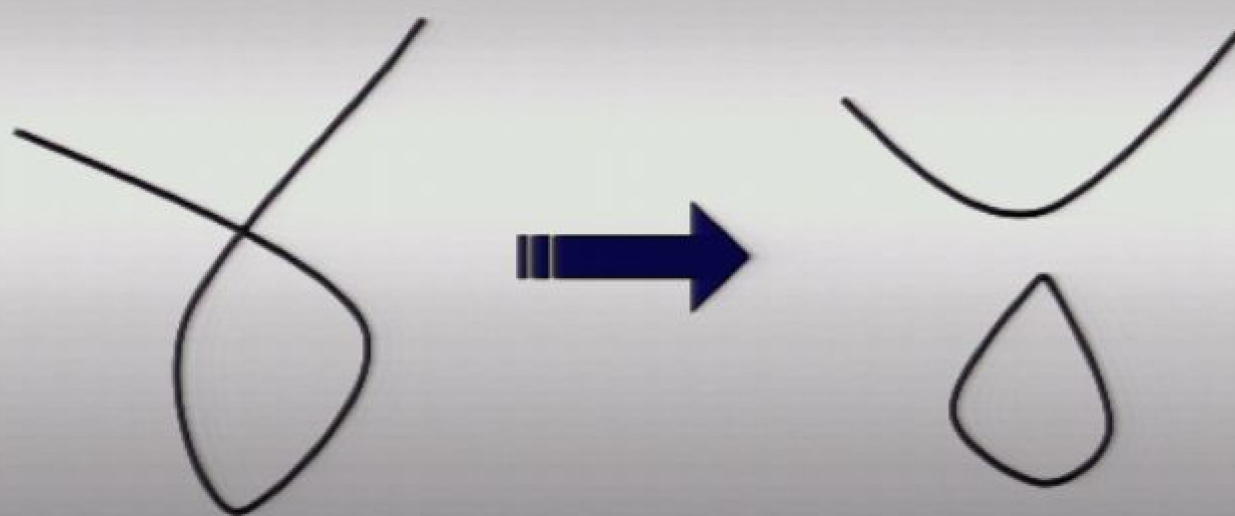
The string can have a sharp profile for two reasons:

• KINKS: occur when a string self-intersects and cuts off; the loop or string has a kink in it. [One of **a′** or **b′** discontinuous]

• CUSPS: occur when the left and right moving waves constructively interfere to allow the string to (instantaneously) move at the speed of light. [**a′** and **b′** align]

INTERCOMMUTATION

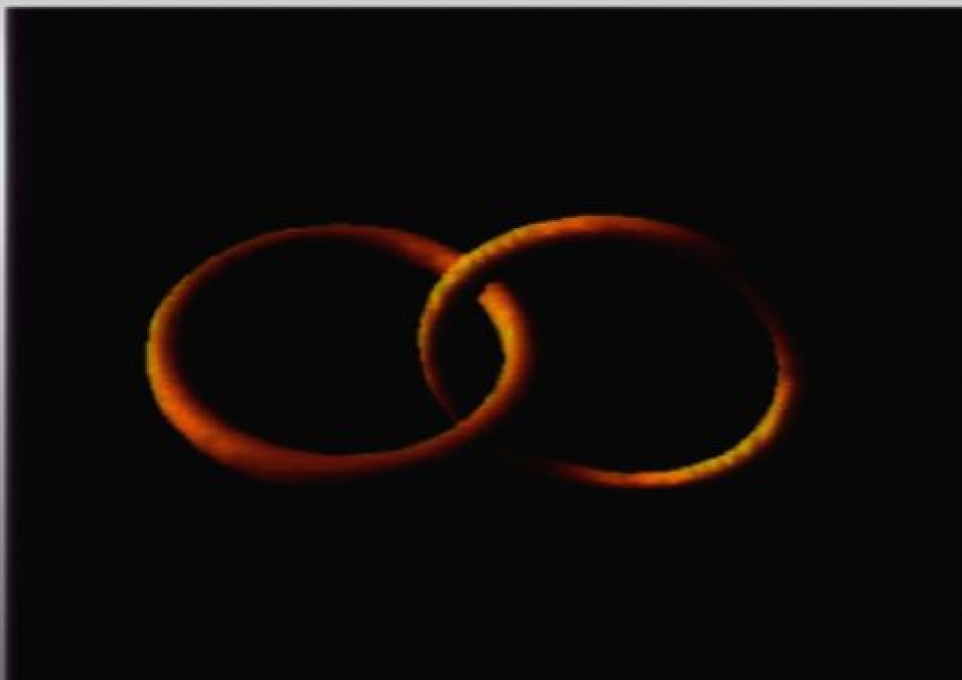
The other key feature of string motion is that strings tend to intercommute, or break off when they self-intersect:



This is related to a standard "90° scattering" property of solitons.

LOOPS AND GRAVITY

For loops, we can use the Nambu action and linearized gravity.



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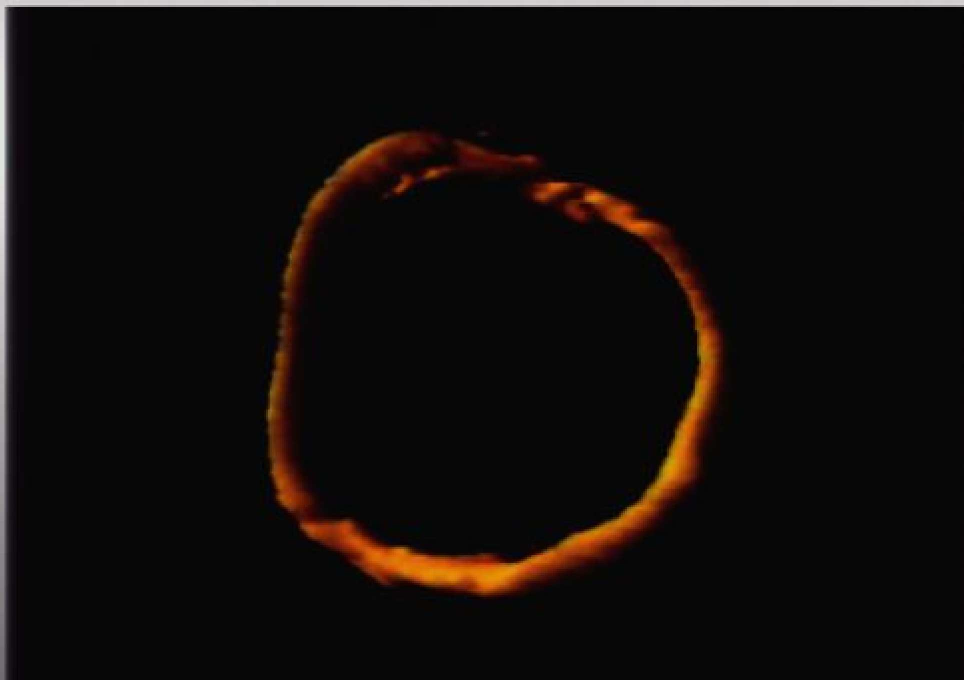

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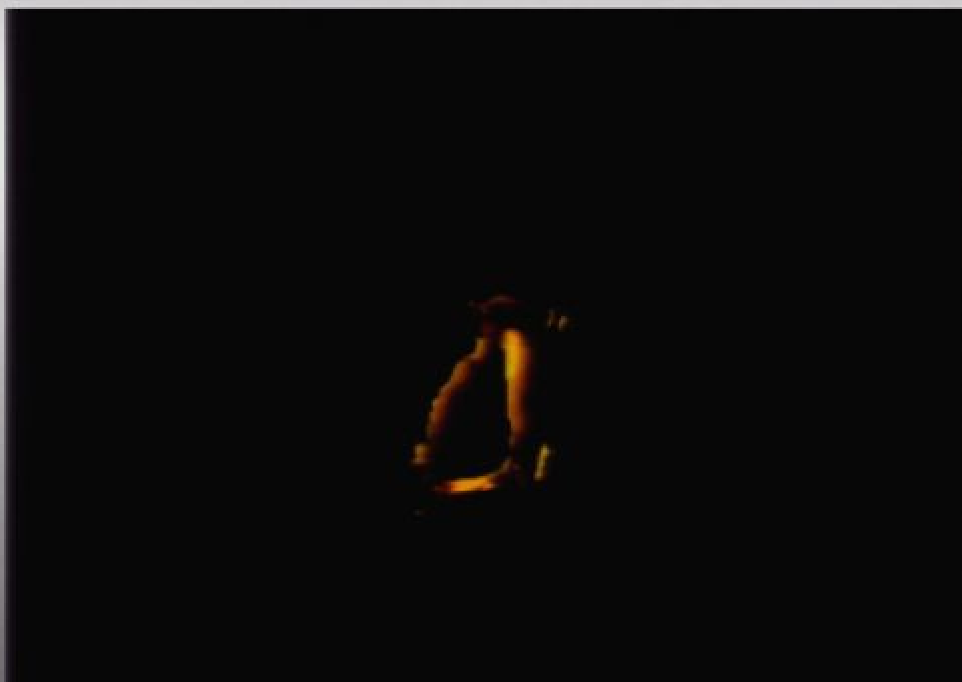

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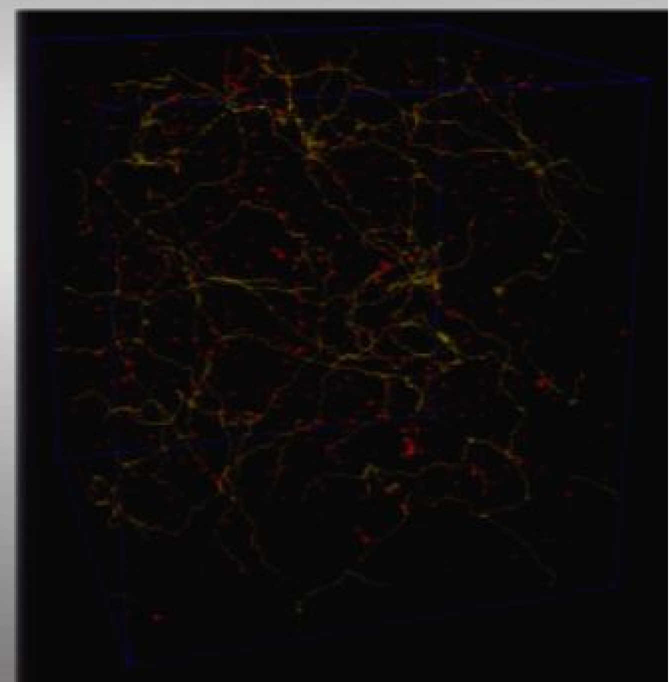
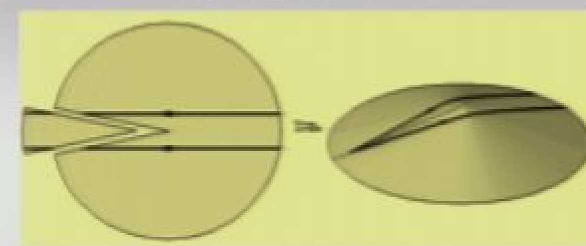
Network

These properties allow a computation of the evolution of the initial configuration of strings in the background of an expanding universe. The network is found to scale. Roughly, the length of a typical loop is proportional to cosmological time, and the density inversely proportional to T^3.



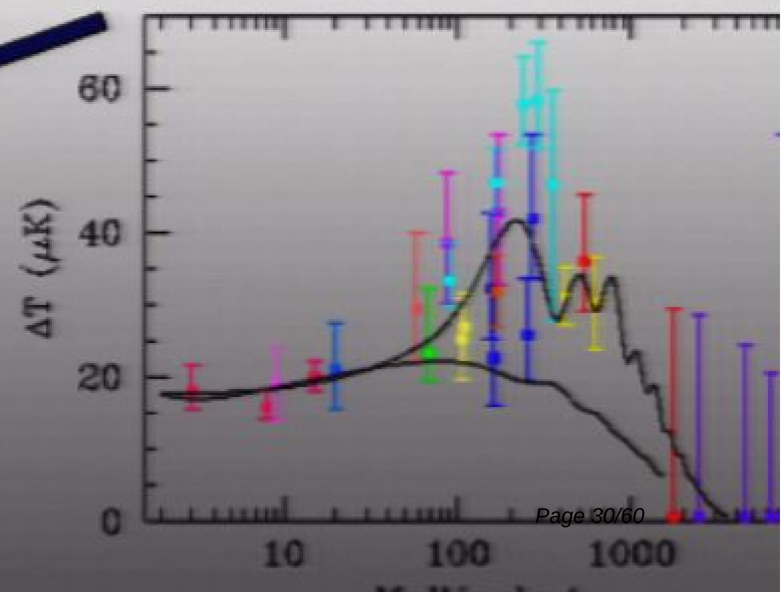
Detecting Strings?

Strings are detected via gravity, rather than particle physics:



- Gravitational lensing

  Some candidates, no detection



- Gravitational perturbations

  CMB: $G\mu < 10^{-7}$, Pulsar: $G\mu < 10^{-6}$

- Gravity wave background

  No positive detection

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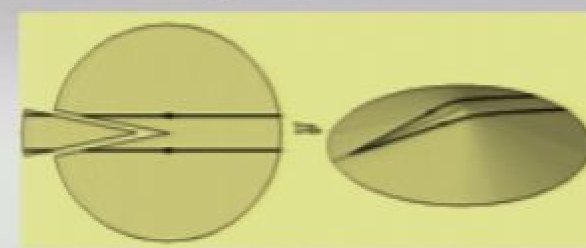

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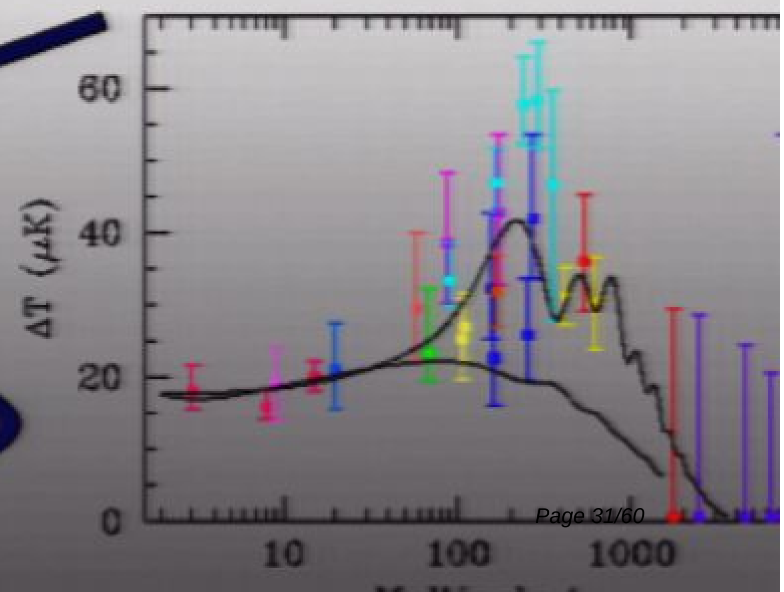

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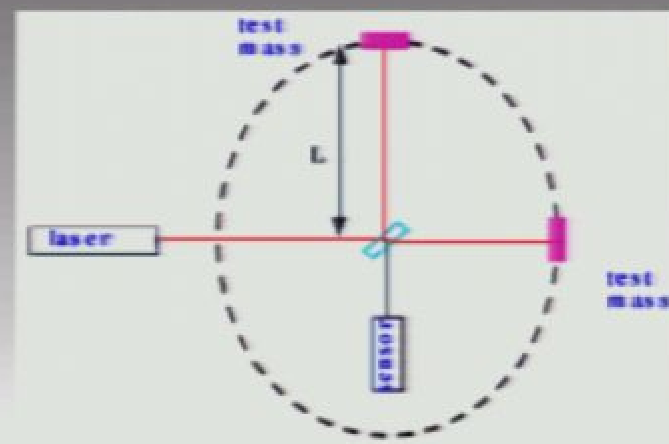
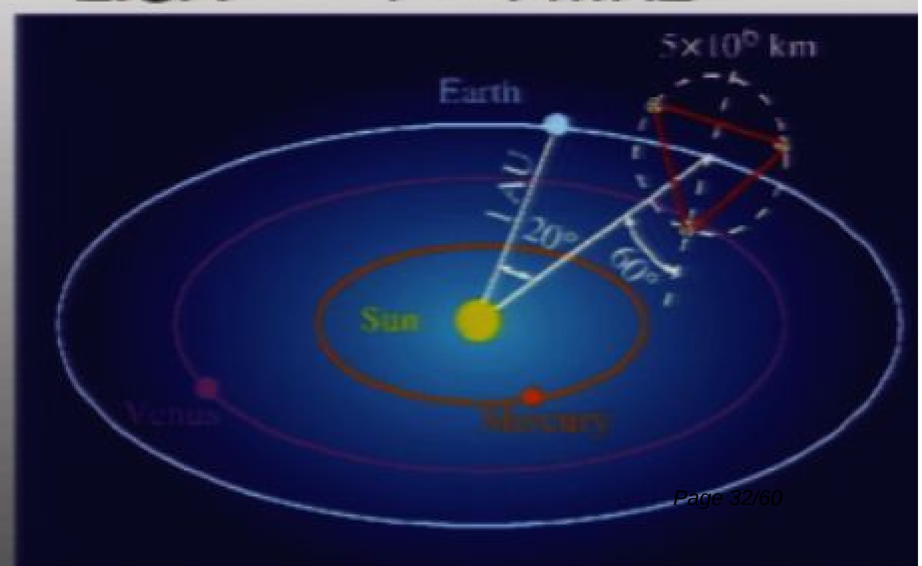
Gravitational wave observations

A gravitational wave is a distortion of spacetime:



LIGO f ~ 150 Hz

LISA f ~ 4 mHz



LINEARIZED GRAVITY

In the far field the metric is well approximated by:

$$\bar{h}_{\mu\nu} \approx \frac{4G}{r} \sum_{\omega} e^{-i\omega(t-r)} T_{\mu\nu}(\underline{k},\omega)$$

For a cusp or kink, the Fourier integral is dominated when the momentum aligns with the cusp vector. This gives a characteristic high frequency power law tail, giving a distinct signal.

$$I_{\pm}{}^{\mu} = \int d\sigma_{\pm}\left(l^{\mu} + \sigma_{\pm}\ddot{X}_{\pm}{}^{\mu}\right)\exp[-\tfrac{2\pi i m}{L} l_{\mu} X_{\pm}{}^{\mu}] \approx \frac{\ddot{X}_{\pm}{}^{\mu}}{|\ddot{X}_{\pm}|^{4/3}}\left(\frac{12}{f}\right)^{2/3}$$

Damour and Vilenkin

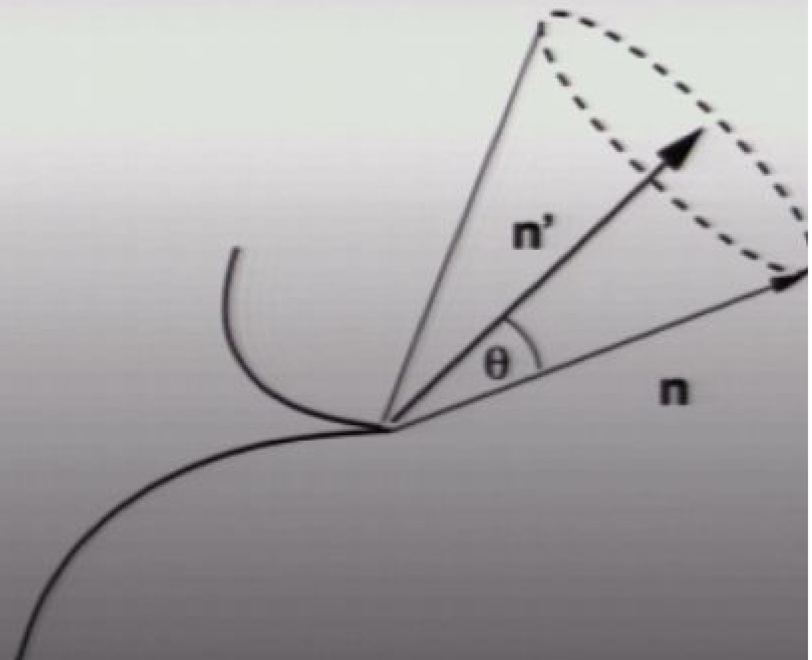
Cusp Signal

The left and right moving modes each contribute to $h_{\mu\nu}$ so when these align, a strong signal is produced – THE CUSP

- Cusp beams out gravity waves in a tight cone around the cusp vector
- Opening angle defined by saddle point of $I_{\pm}$

$$h^{cusp}(f,\theta) \approx \frac{G\mu L^{2/3}}{r|f|^{1/3}} H[\theta_m - \theta]$$

$$\left(\frac{2}{Lf}\right)^{1/3}$$



KINK SIGNAL

The discontinuity dominates the integral for the left mover, but the right mover still has the saddle around its wave vector, signal a combination of these.

- Kink beams out gravity waves along the cts wave-vector, localised transverse – a FAN
- Opening angle defined by saddle point of I_+

$$h^{cusp}(f,\theta) \approx \frac{G\mu L^{1/3}}{r|f|^{2/3}} H[\theta_m - \theta]$$

$$\theta_m = \left(\frac{2}{Lf}\right)^{1/3}$$

COSMOLOGICAL SIGNAL

The expanding universe modifies the signal from an individual cusp/kink

$$f \rightarrow (1+z)f \qquad\qquad r \rightarrow a_0 r = (1+z)D_A(z)$$

Then must estimate the total signal from the network

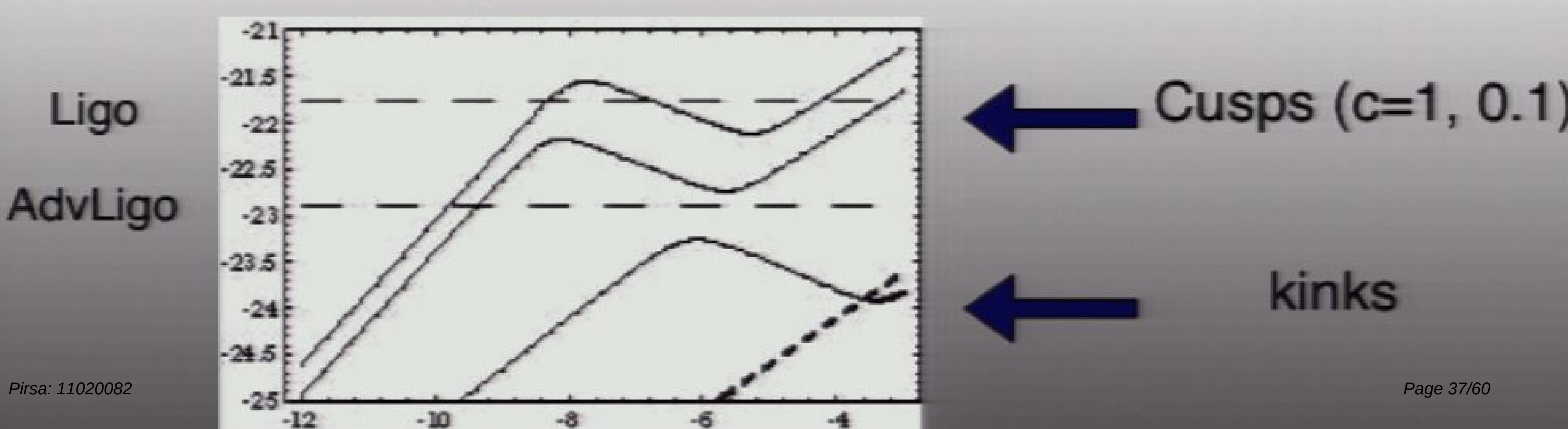
$$\frac{Cn_L}{PT_L} \approx \frac{2C}{P\alpha^2 t^4}$$

$$d\dot{N} \approx \frac{\nu(z)}{(1+z)} \frac{\pi\theta_m^2(z)D_A(z)^2}{(1+z)H(z)} dz$$

C – number cusps/kinks per loop

DV Ligo result

Damour and Vilenkin use analytic approximations to variables, and take a desired event rate of 1 per year. Signal dominated by maximal redshift, use this to obtain amplitude at desired fiducial frequency.



Cosmological Signal

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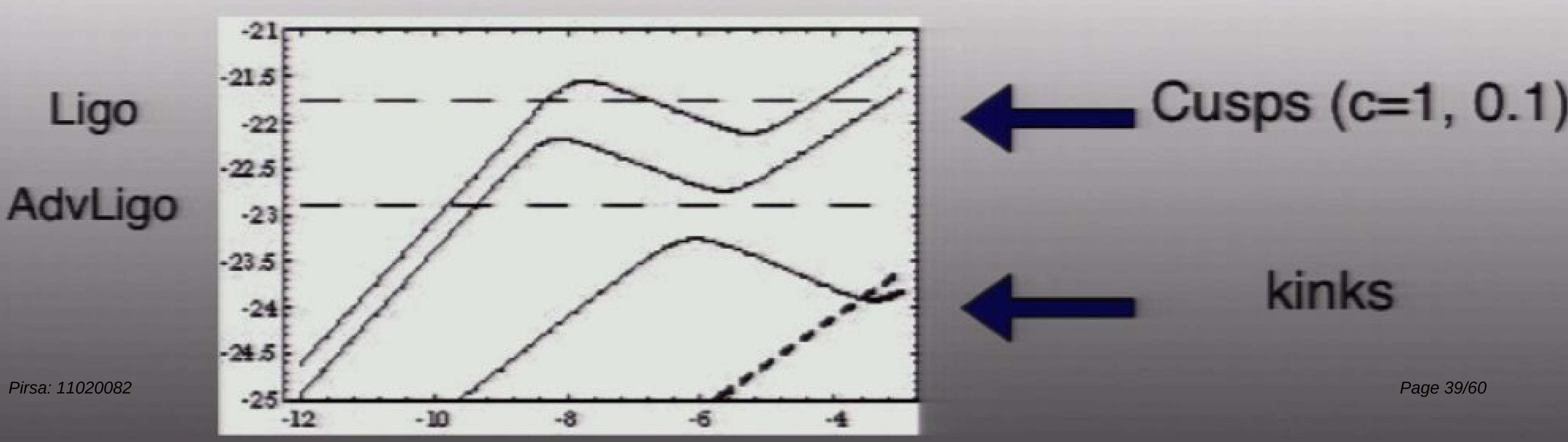
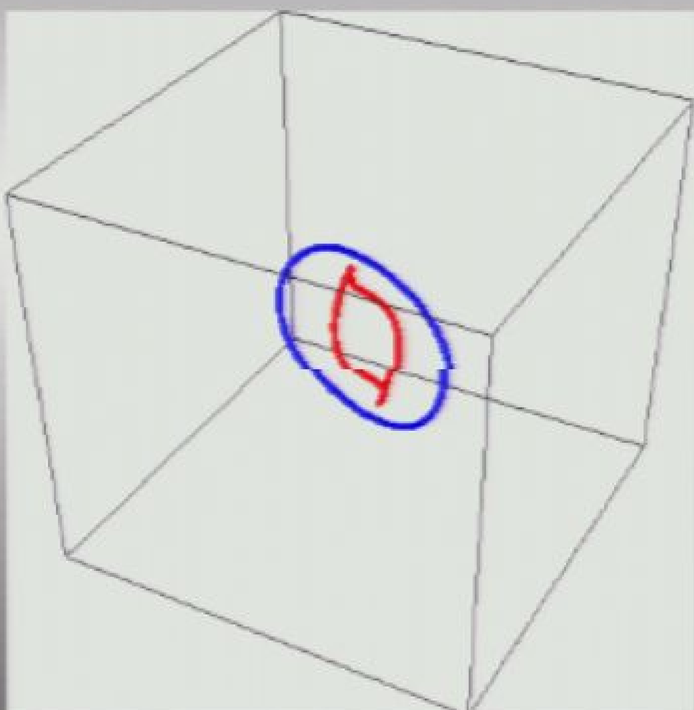


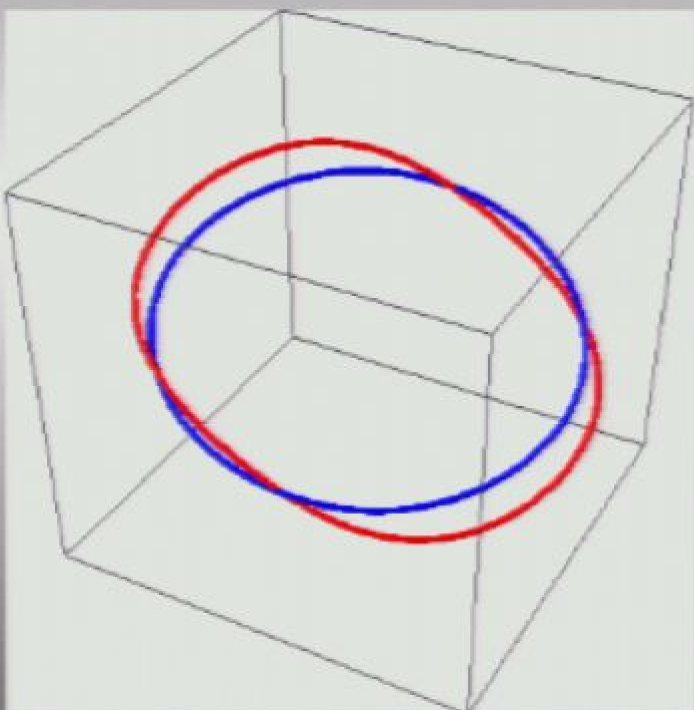
The effect of extra dims?



Extra dimensions reduce the probability of intercommutation, but also the kinematics of strings will be different (more freedom in solns to Nambu eqns). This reduces the probability of cusp formation in particular and hence the gravity wave background.

E.o'C, SC, GG, RG, IZ 1003.4395, 1005.3220 [hep-th]

THE EFFECT OF EXTRA DIMS?



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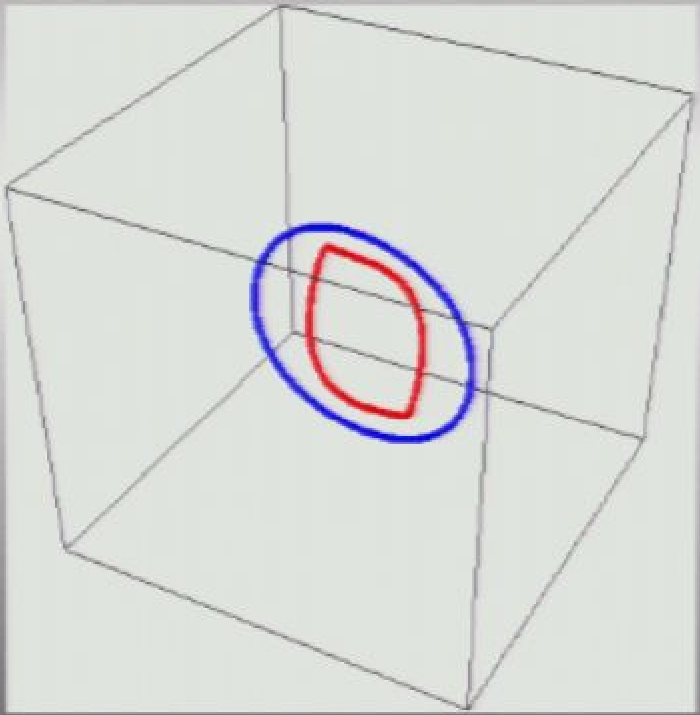

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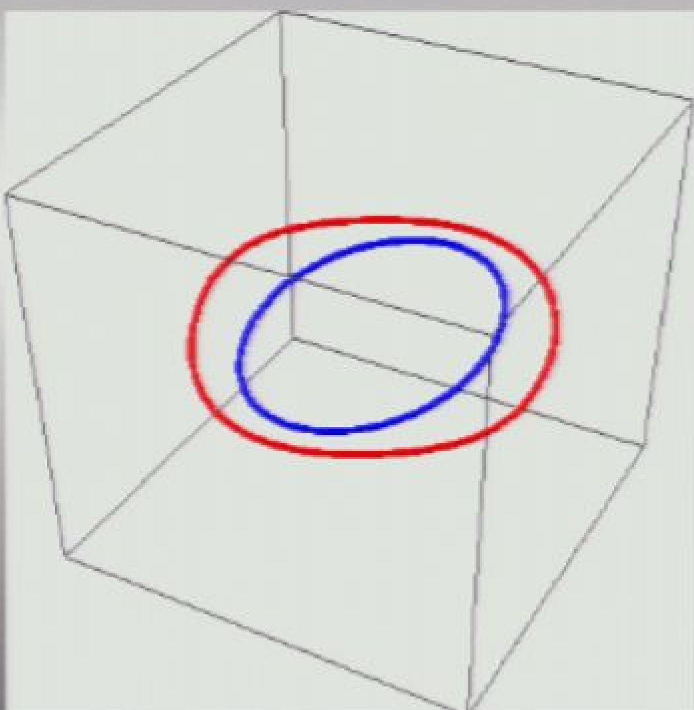

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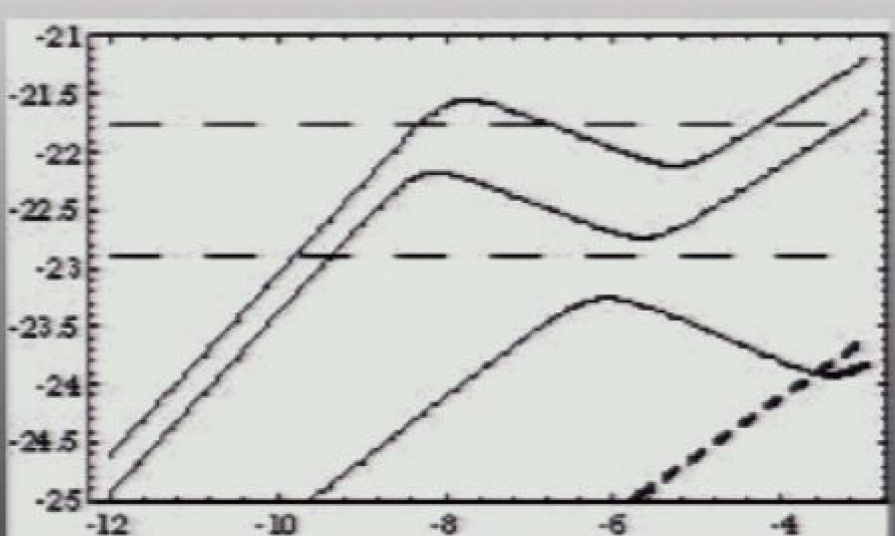
Ligo

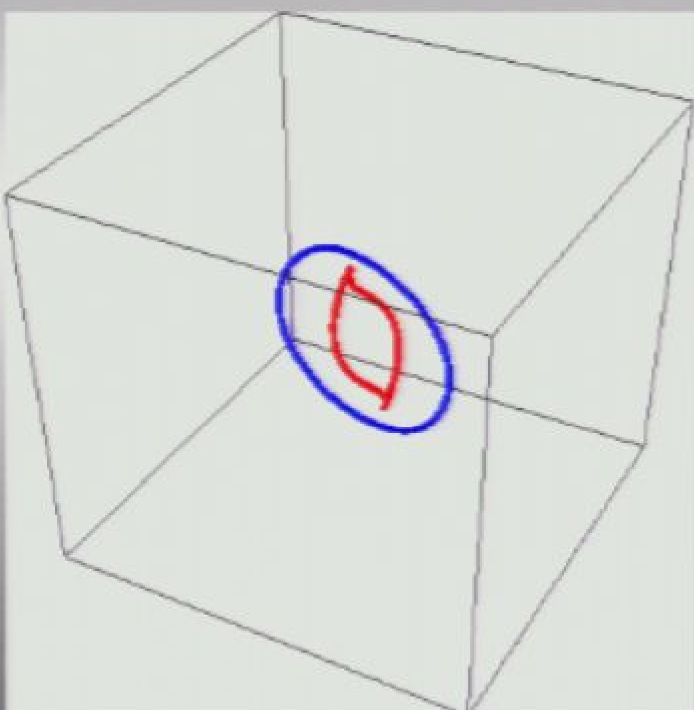
AdvLigo



Cusps (c=1, 0.1)

kinks

THE EFFECT OF EXTRA DIMS?

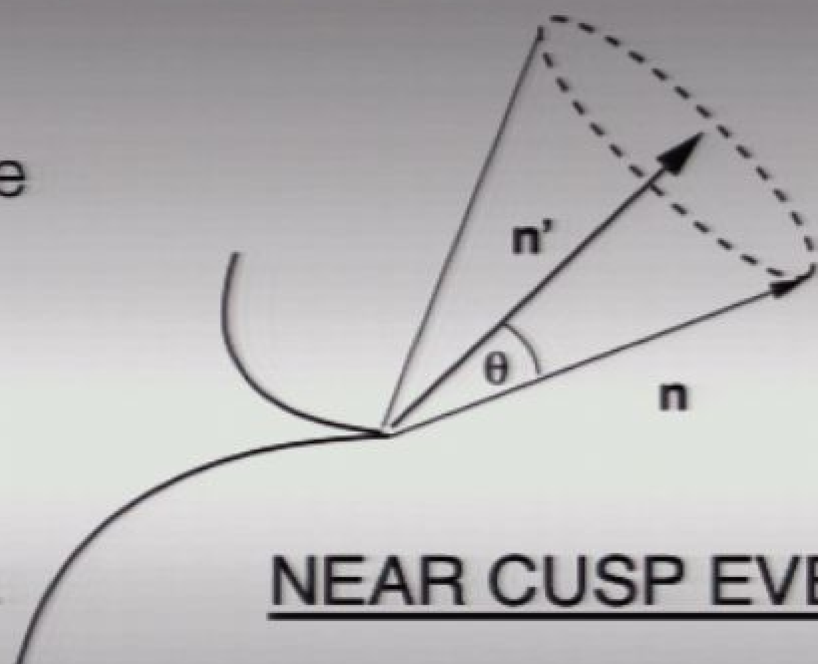


Extra dimensions reduce the probability of intercommutation, but also the kinematics of strings will be different (more freedom in solns to Nambu eqns). This reduces the probability of cusp formation in particular and hence the gravity wave background.

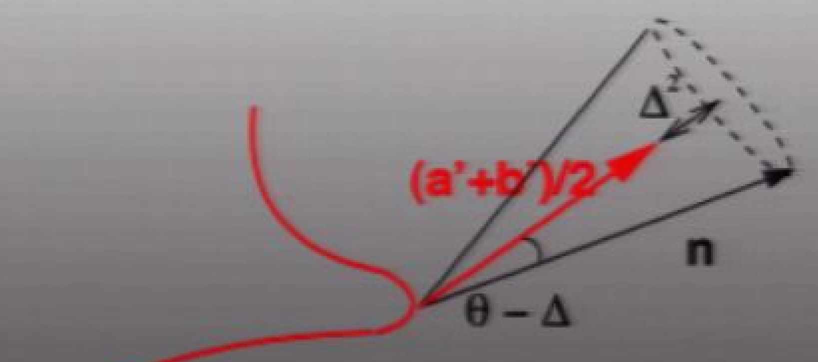
E.o'C, SC, GG, RG, IZ 1003.4395, 1005.3220 [hep-th]

Cusp Rounding

The first clear effect is that the wave vectors of left & right movers no longer need be null.
This rounds off the cusp, narrowing the beaming cone.



NEAR CUSP EVENT



$$\left|\underline{a}' - \underline{b}'\right| = 2\Delta << 1$$

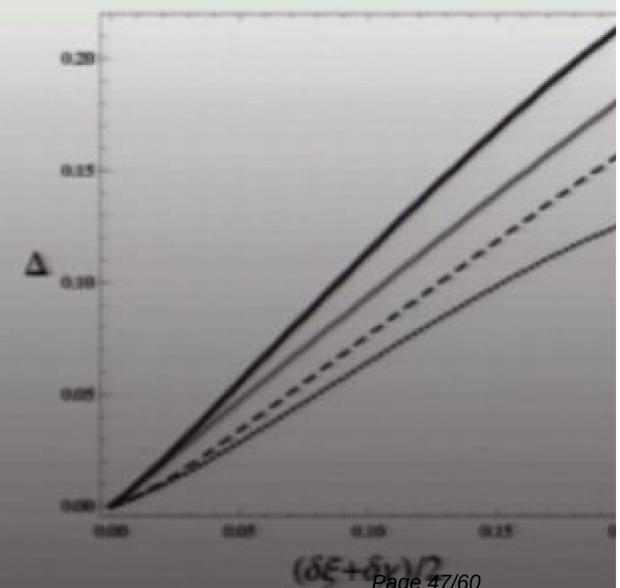
$$\theta_m \rightarrow \theta_m - \Delta$$

Measure reduction

In 3d, cusps are generic, but in higher dimensions the Kibble-Turok sphere allows lines not to cross – have to estimate probability of near cusp event.

$$C \rightarrow C(\Delta)$$

From cusp constraint, co-dimensionality of solns with exact cusps is *n*. Test trajectories give



$$\mathcal{N}(\Delta) \approx \Delta^n \qquad C(\Delta) \approx n\Delta^{n-1}$$

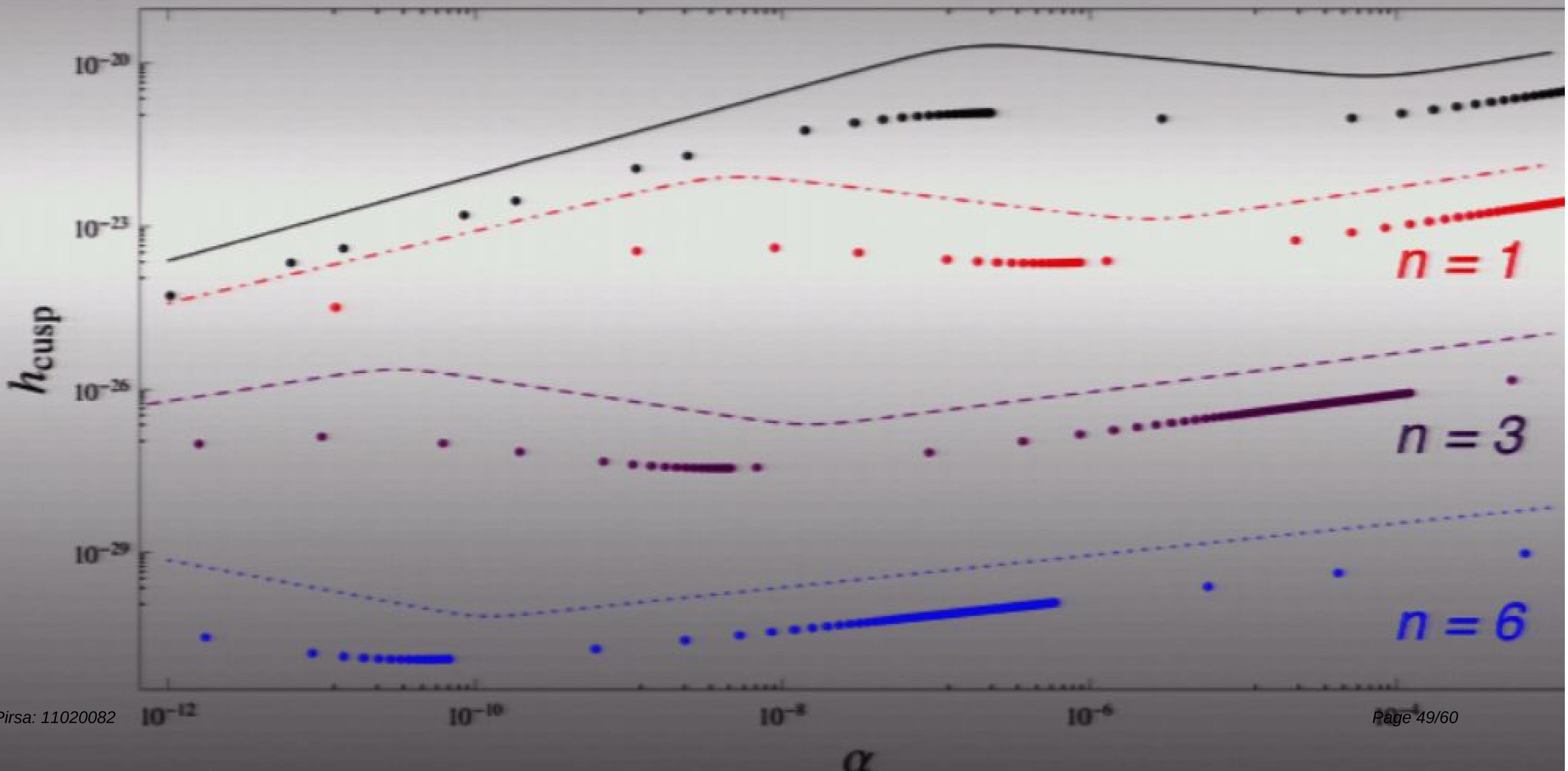
TOTAL EFFECT:

General network has range of NCE's, up to critical Δ where cone closes off. Must integrate cusp event rate over this range:

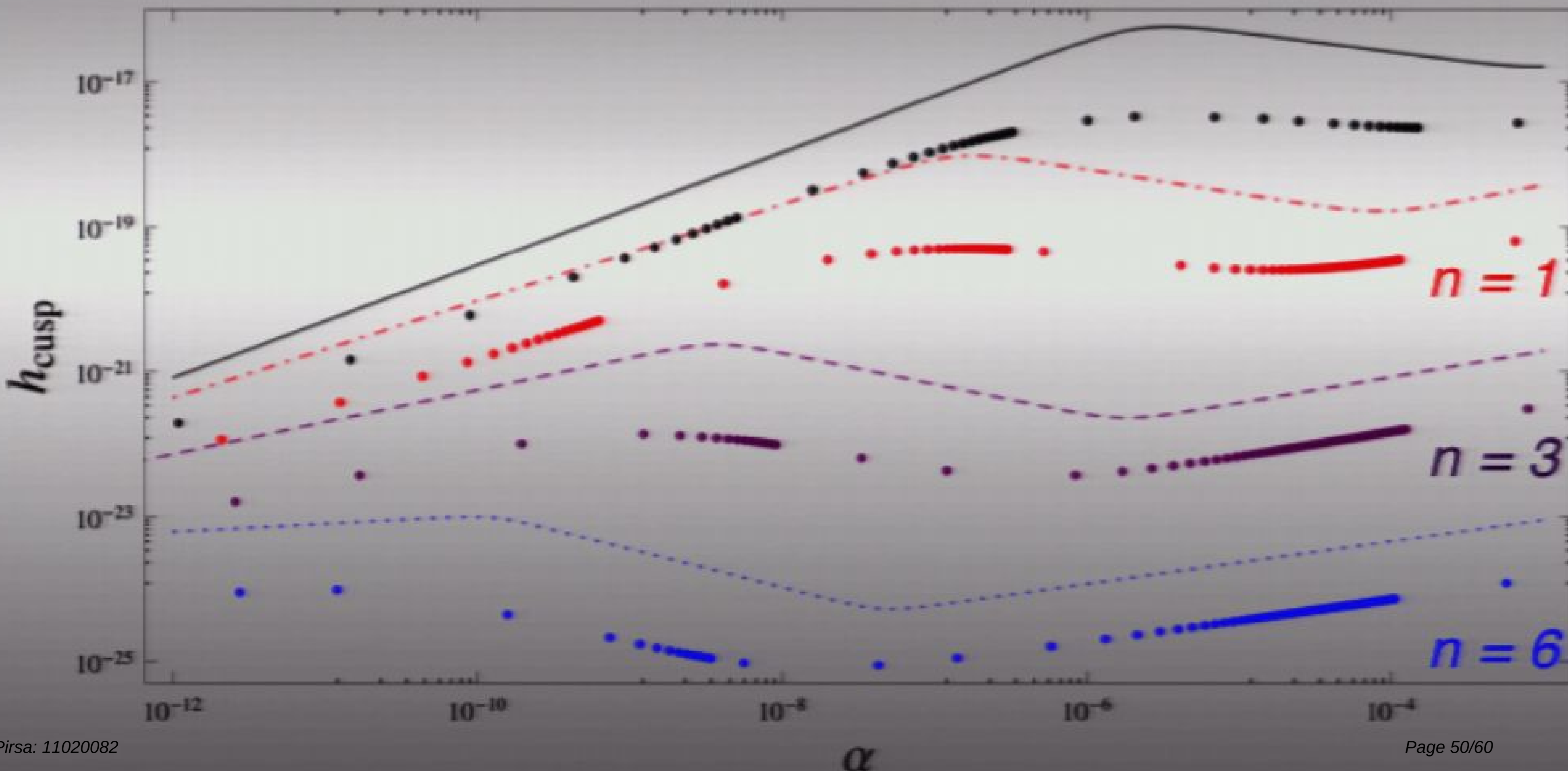
$$\frac{d\dot{N}_{NCE}}{dz} \approx \frac{2\theta_m^{\ n+2}(z)}{(n+1)(n+2)} \frac{n_L(z)}{PT_L(z)} \frac{\pi D_A(z)^2}{(1+z)^2 H(z)}$$

Can then use interpolating functions, as DV, or calculate $D_A(z)$ and $H(z)$ exactly for ΛCDM.

LIGO (150 Hz)



LISA (3.9 mHz)



h_{cusp}
10^{-17}
10^{-19}
10^{-21}
10^{-23}
10^{-25}
10^{-12}
10^{-10}
10^{-8}
10^{-6}
10^{-4}
α
$n = 1$
$n = 3$
$n = 6$

CAVEATS

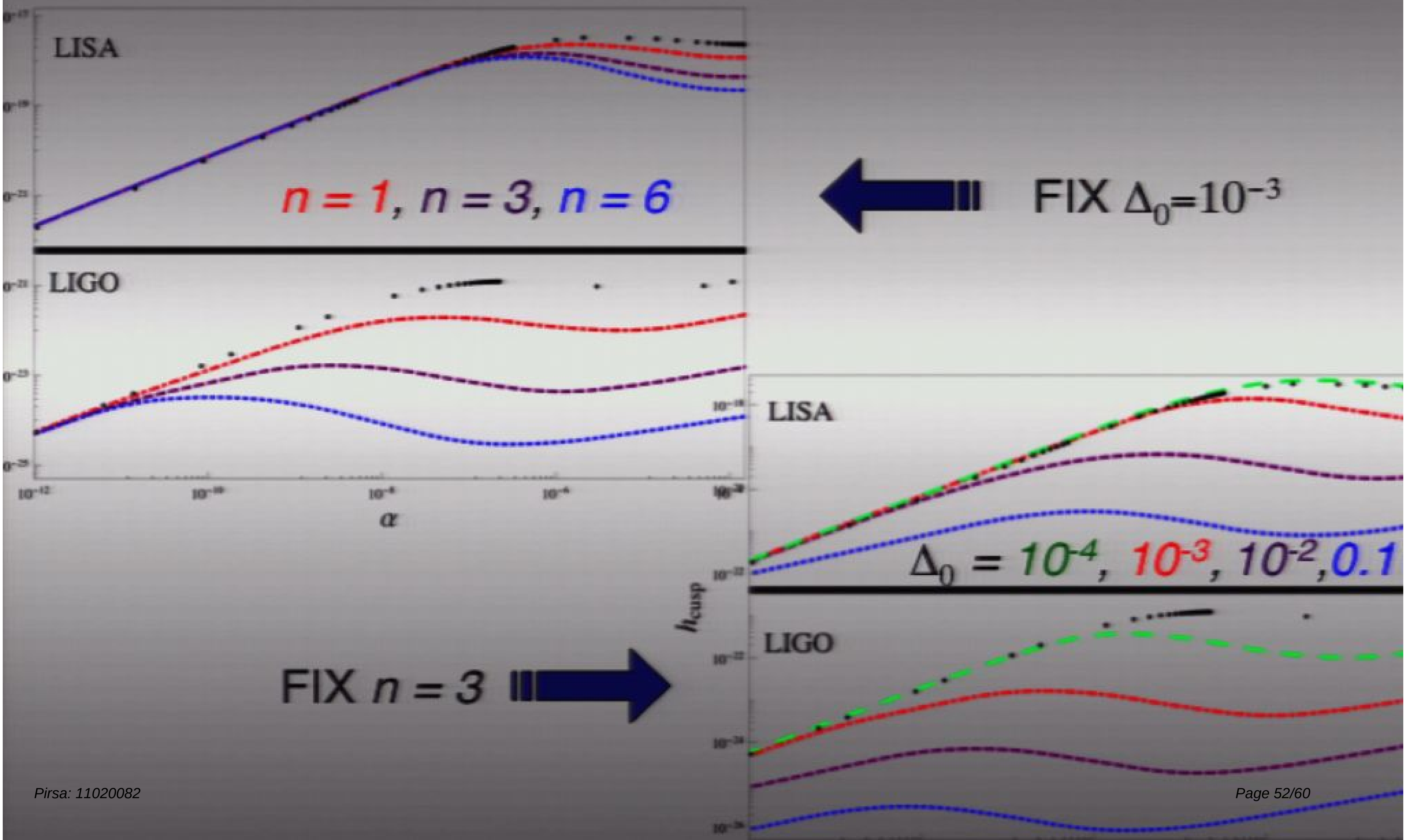
All tests with Nambu strings back up this picture, but strings have finite width. This will give more self-intersection as size of extra dims decreases, and possibly restrict parameter space. Model this empirically by changing measure on solution space to peak around 3d.

$$\Delta \in [0, \Delta_0] \quad \Rightarrow \quad C(\Delta) = \frac{n\Delta^{n-1}}{\Delta_0^{\,n}}$$

On integration gives new rate dependence on θ, Δ_0

$$\frac{2\theta_m^{\,n+2}}{(n+1)(n+2)} \rightarrow \frac{2\theta_m(z)^{n+2}}{(n+1)(n+2)}\frac{H[\Delta_0-\theta_m]}{\Delta_0^{\,n}} + \left(\theta_m^{\,2}(z) - \frac{2n\Delta_0\theta_m(z)}{(n+1)} + \frac{n\Delta_0^{\,2}}{(n+2)}\right) H[\theta_m - \Delta_0]$$

NEW MEASURE

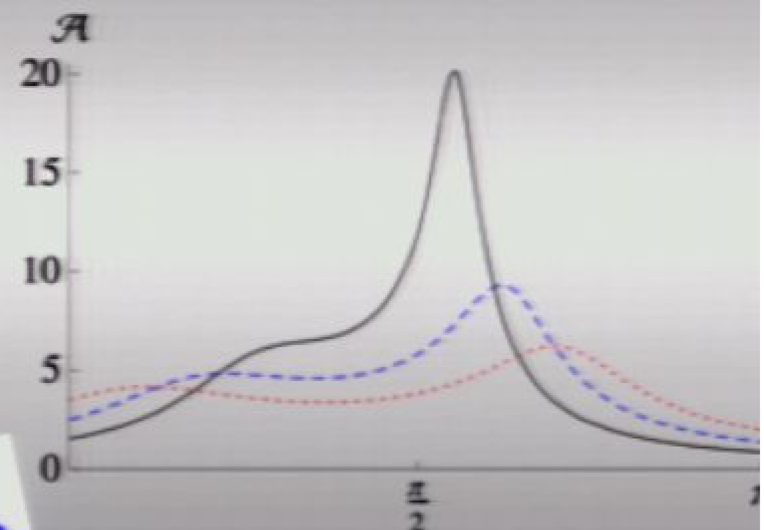

LISA
LIGO
$n = 1$, $n = 3$, $n = 6$
FIX $\Delta_0 = 10^{-3}$
α
LISA
LIGO
h_{cusp}
$\Delta_0 = 10^{-4}$, 10^{-3}, 10^{-2}, 0.1
FIX $n = 3$

KINK WAVEFORM

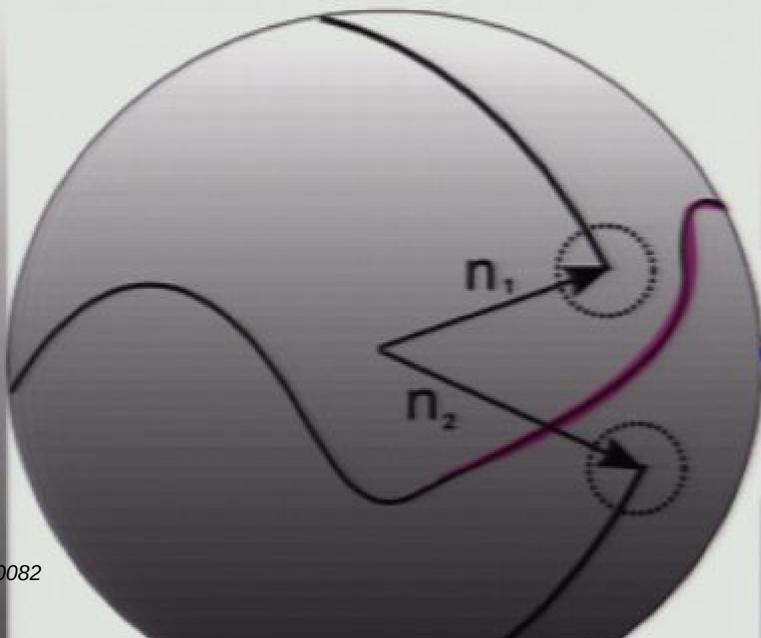
E.o'C & RG 1010.3942 [hep

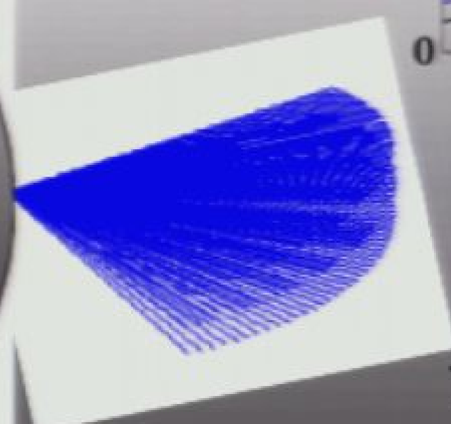
For the kink, the right mover has a saddle as with the cusp, but the discontinuity contributes from its end points:

$$I_{-} \propto \frac{v^{\mu}}{f} \qquad v^{\mu} = \frac{n_1^{\mu}}{\hat{k}\cdot n_1} - \frac{n_2^{\mu}}{\hat{k}\cdot n_2}$$



Kink amplitude for rang
of discontinuity angles

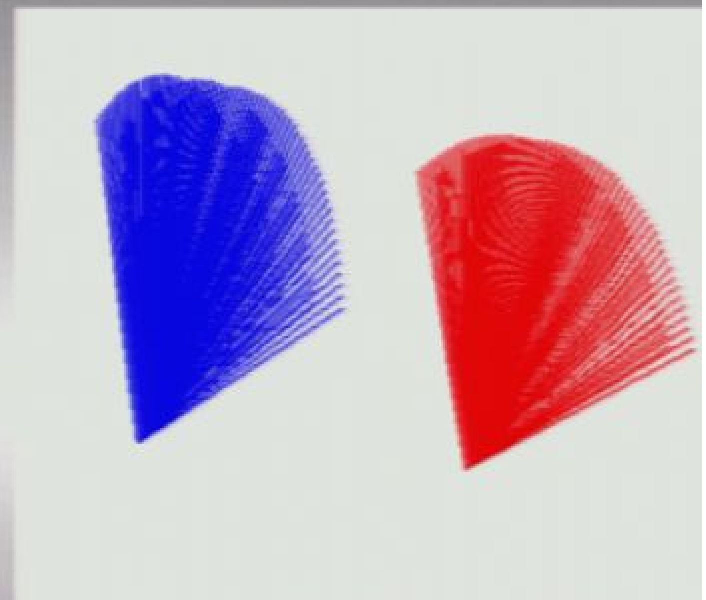




fan

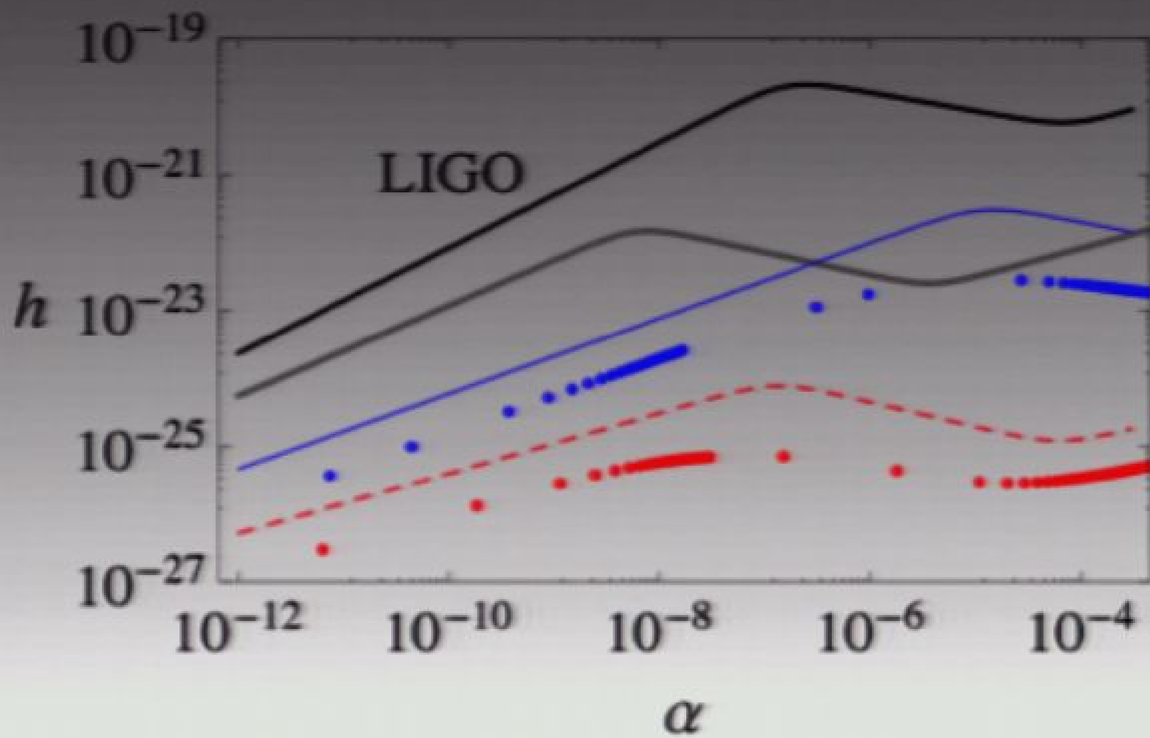
KINKS:

For the kink, the only factor from extra dimensions is the thinning of the fan to $(\theta-\delta)$ which must be integrated over:

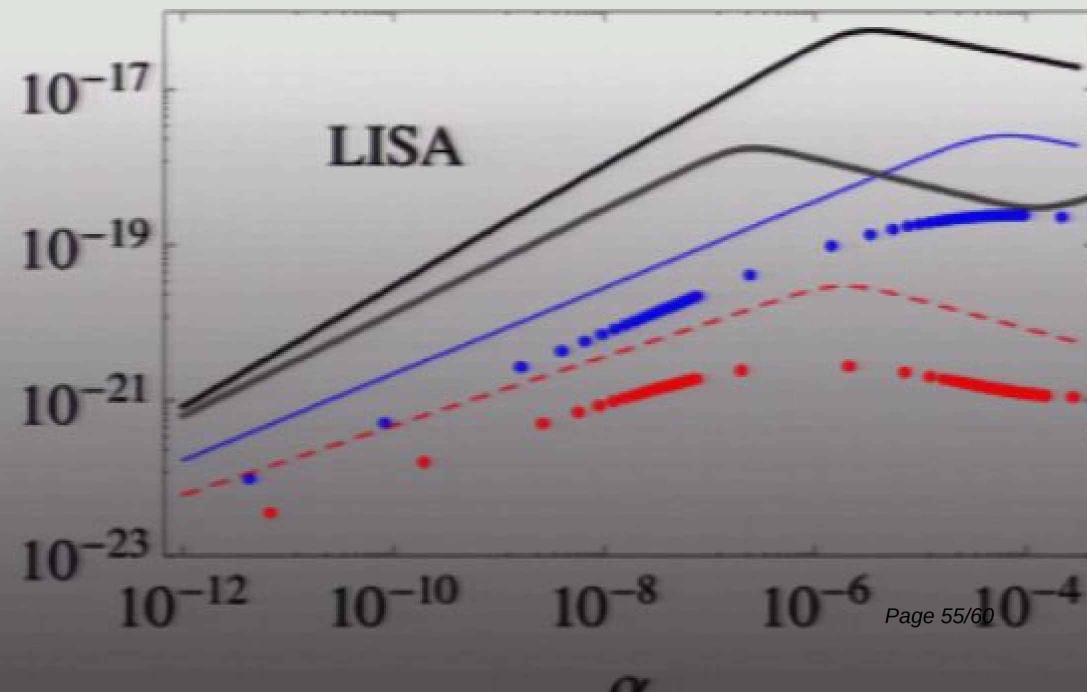


$$\frac{d\dot{N}_K}{dz} \approx K \frac{n_L(z)\theta_m{}^2(z)}{PT_L(z)} \frac{\pi D_A(z)^2}{(1+z)^2 H(z)}$$

NB: This is *independent* of the number of extra dimensions



Plots show amplitudes are suppressed from D[...] result, but not more tha[...] cusp with n=3

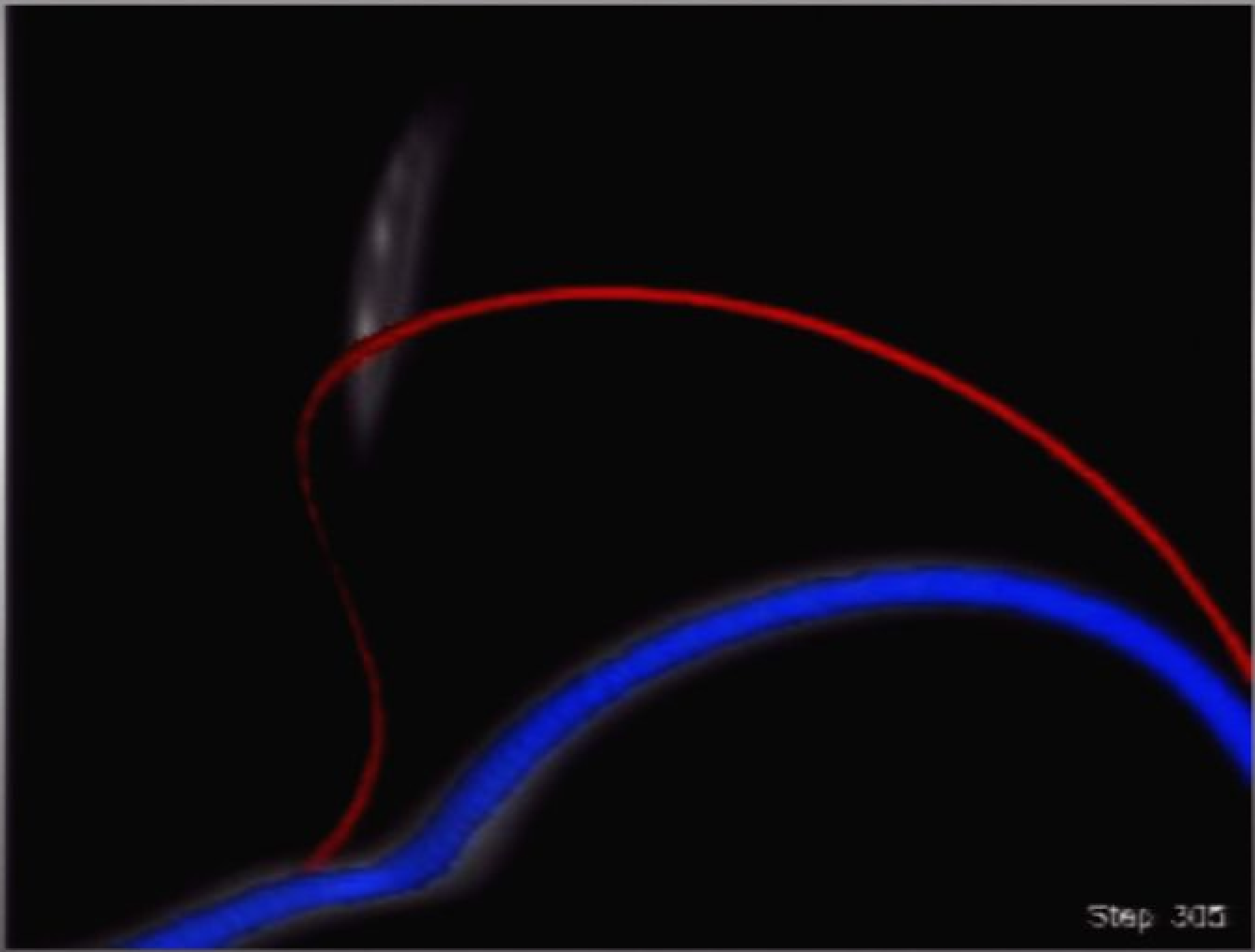


If kinks proliferate by more than 6 orders of magnitude, they will be easily detectable.

SUMMARY

- Kinematics of extra dimensions can have a strong effect on the gravitational wave signal
- Cusp signal is particularly sensitive to number of extra dimensions
- But there are caveats, better understanding needed: Warped extra dimensions? Effect of cosmological expansion?
- Lower frequency bands have less signal differentiation
- Kinks offer better bet for detection

Finite width effects



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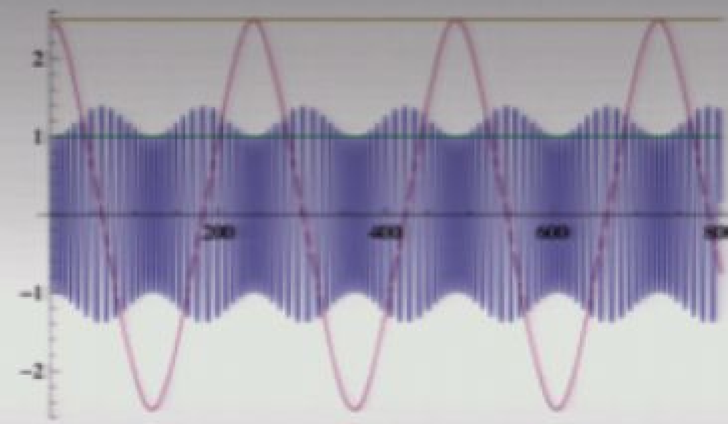
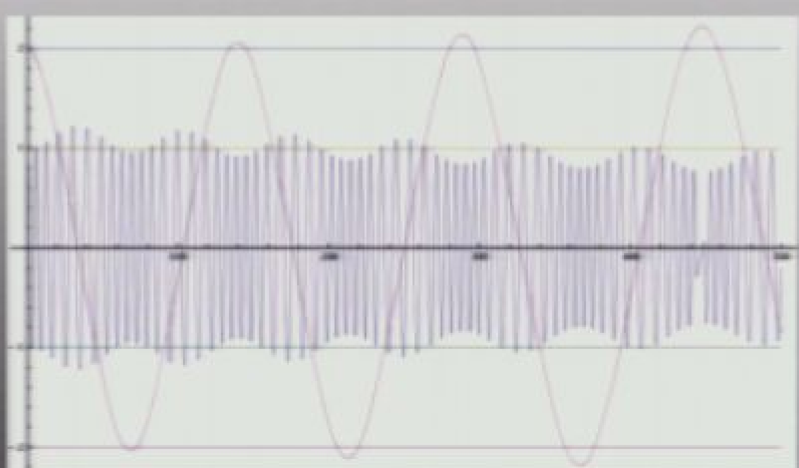
Warped Extra Dimensions

Energy flows elastically between the compact and noncompact dims.





If motion damped in our noncompact dims, the amplitude increases in the internal space.

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