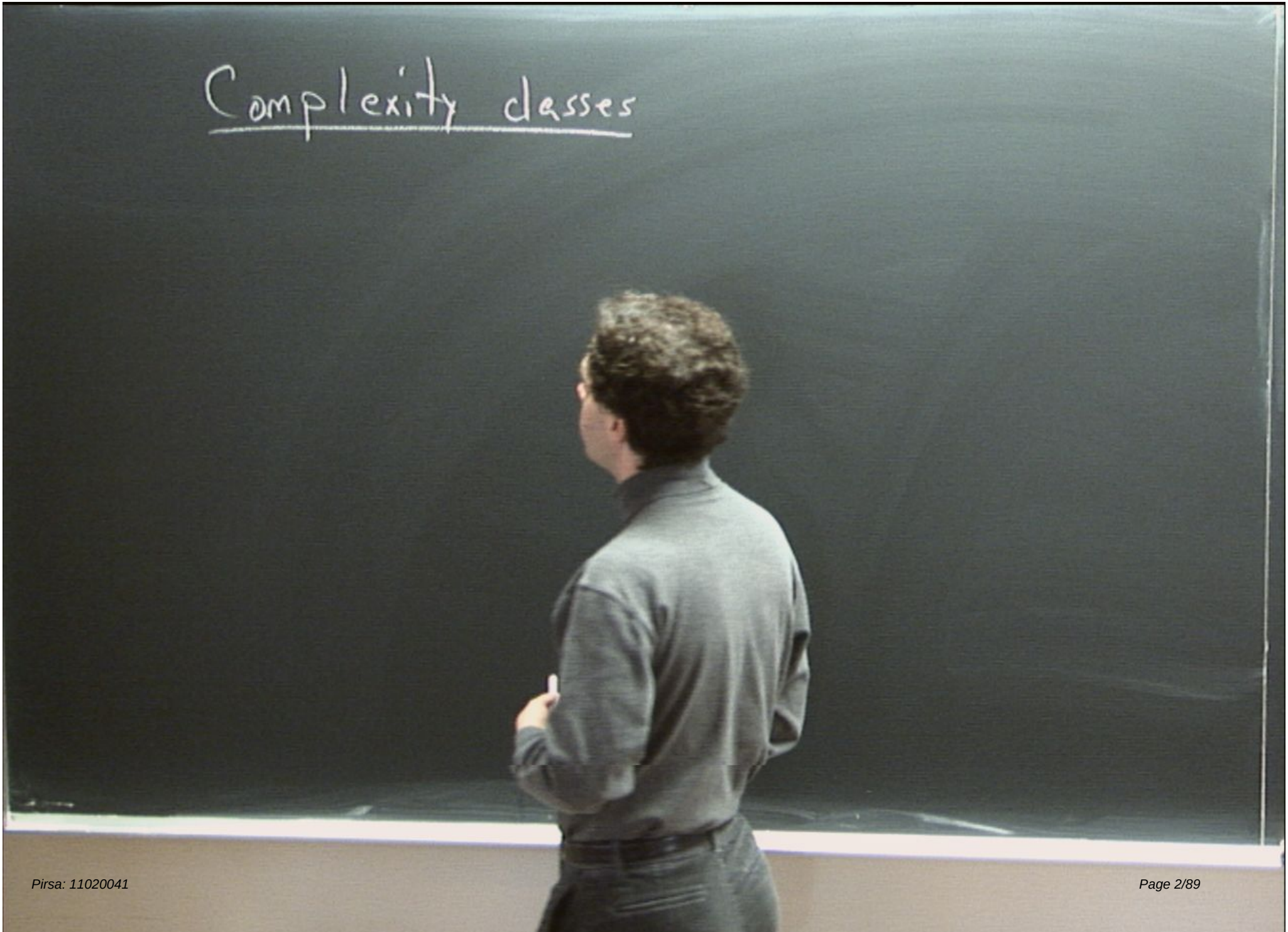
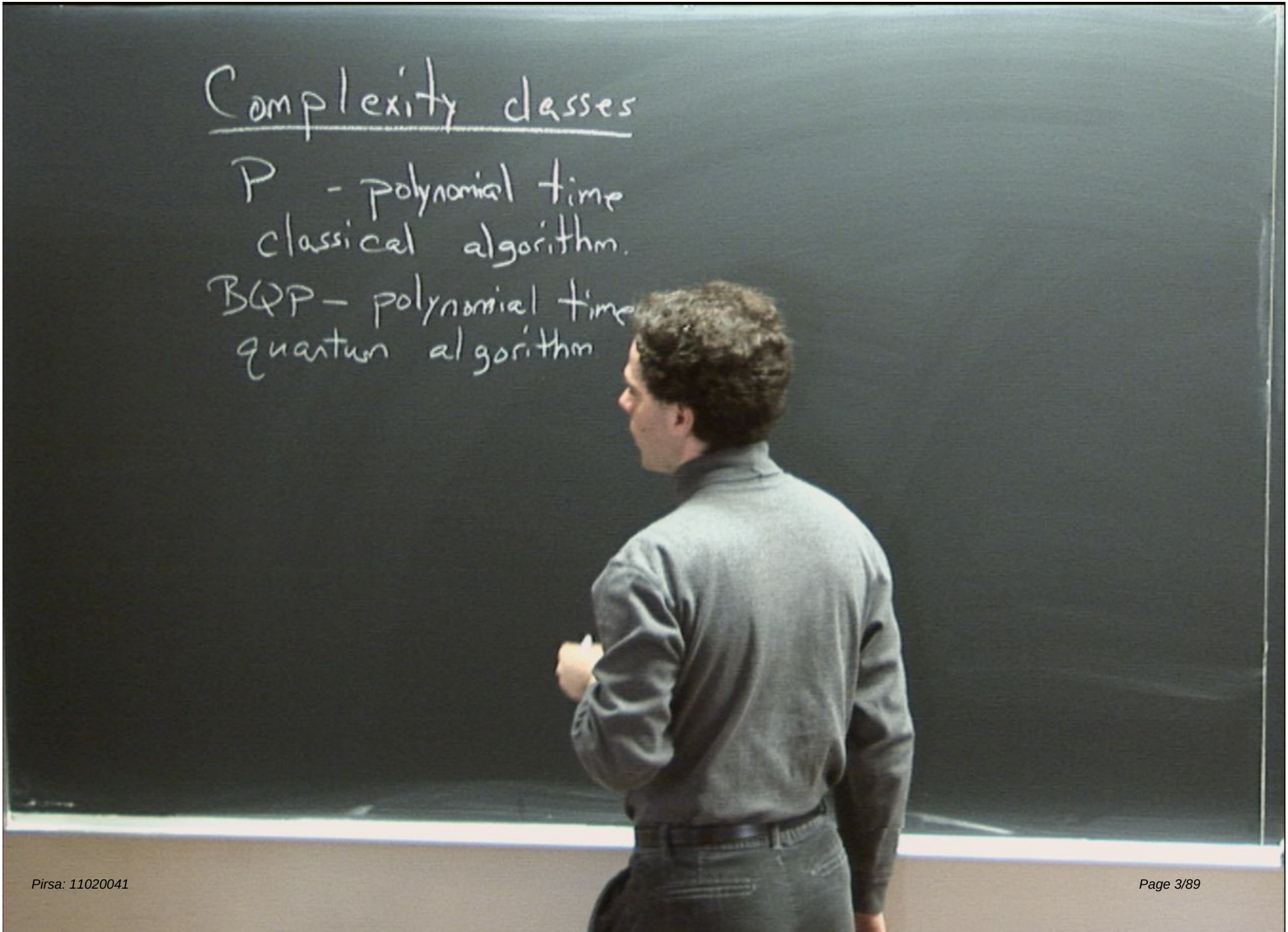
Title: Quantum Information Review - Lecture 6

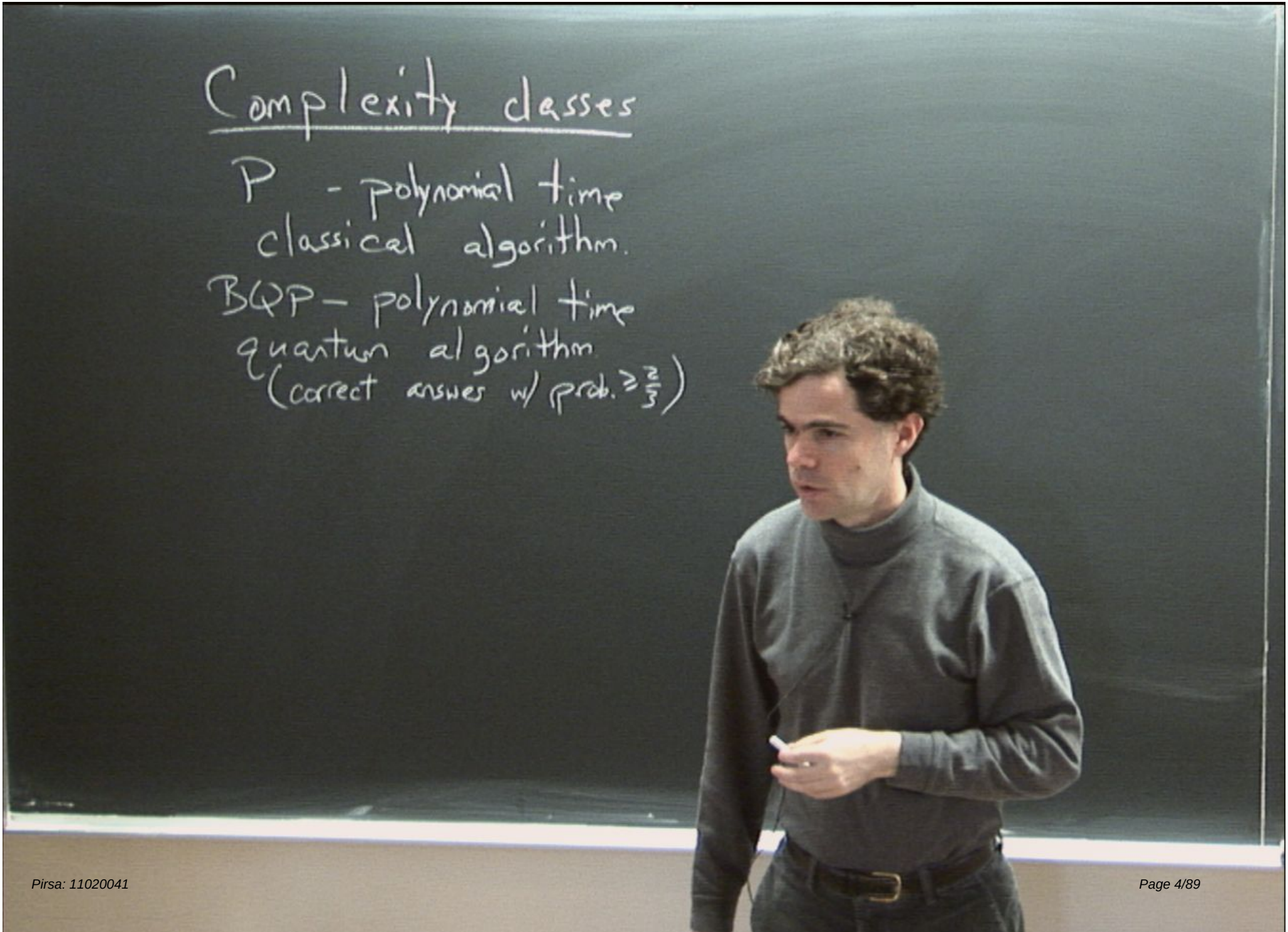
Date: Feb 22, 2011 09:00 AM

URL: http://pirsa.org/11020041

Abstract:


Complexity classes


Complexity classes
P - polynomial time classical algorithm.
BQP - polynomial time quantum algorithm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P - polynomial time classical algorithm.
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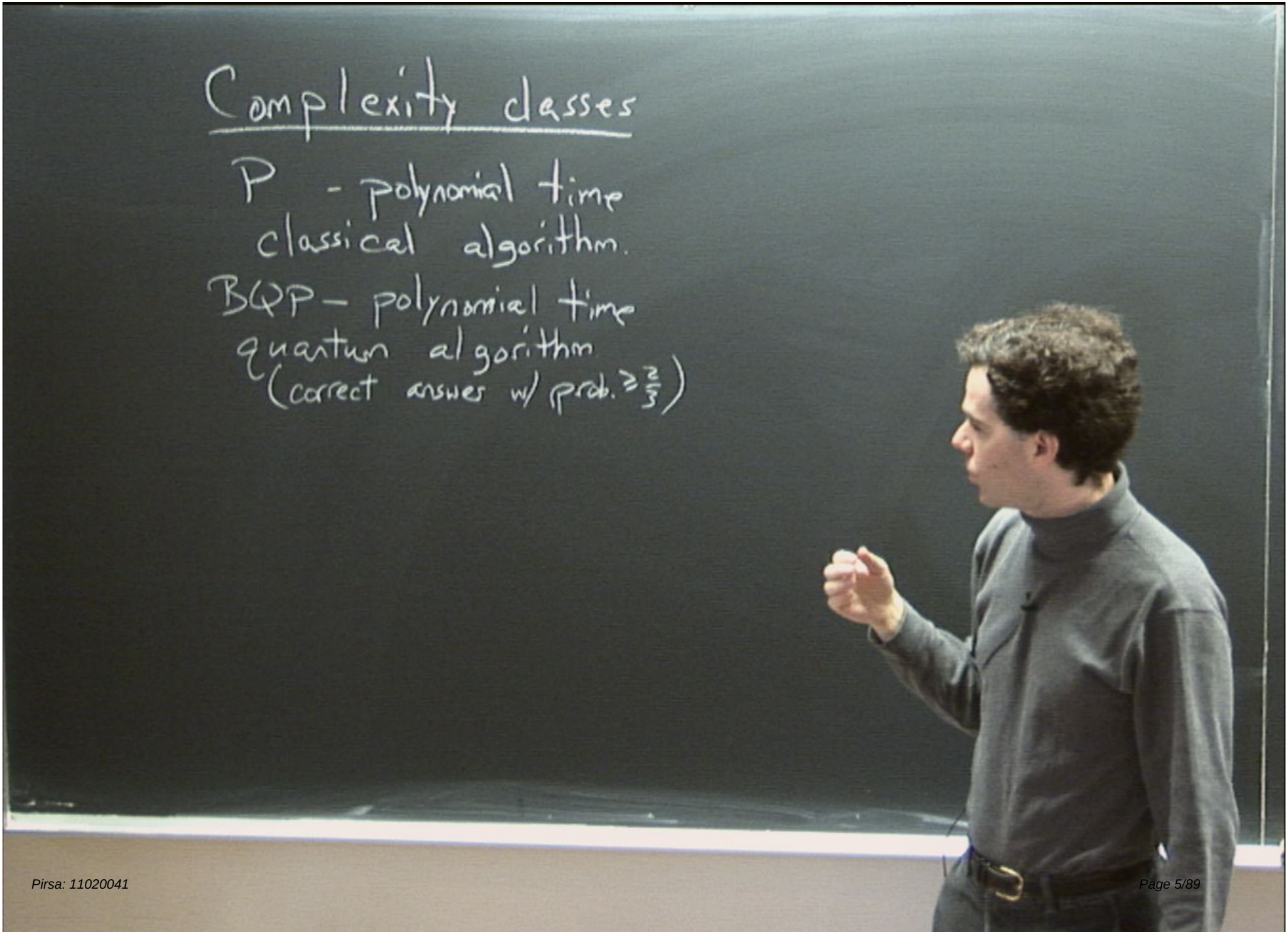
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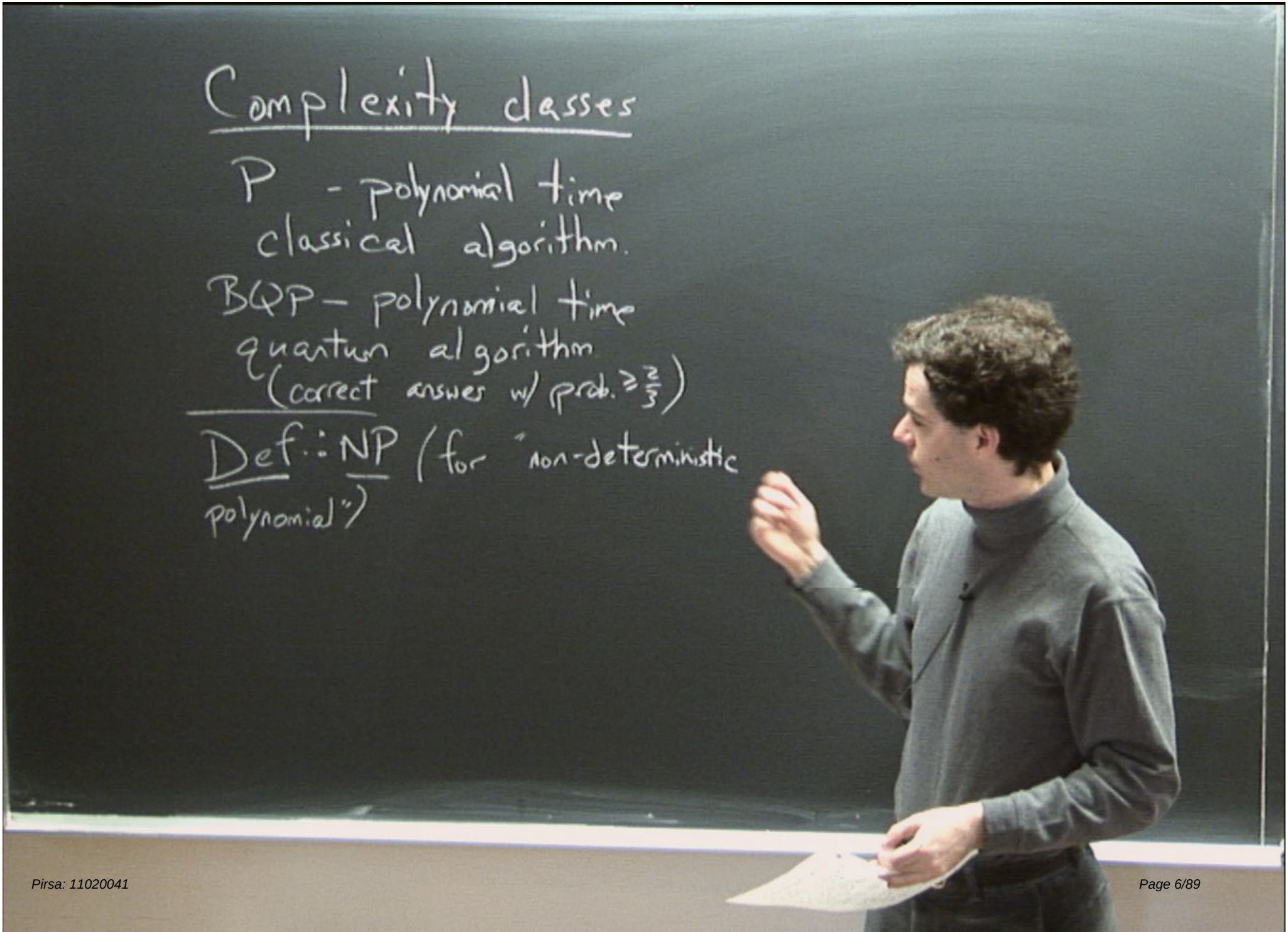
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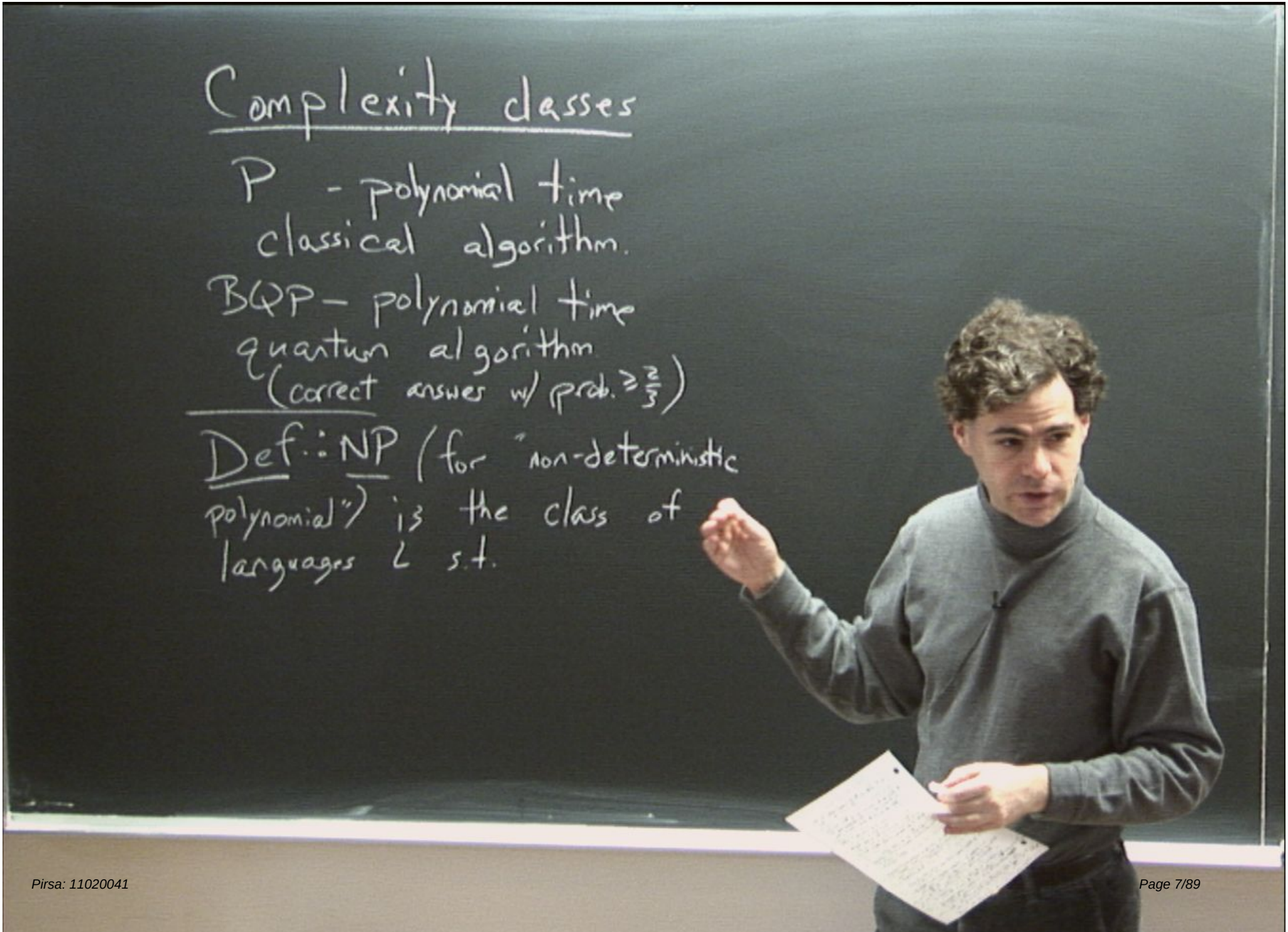
Def: NP (for "non-deterministic polynomial")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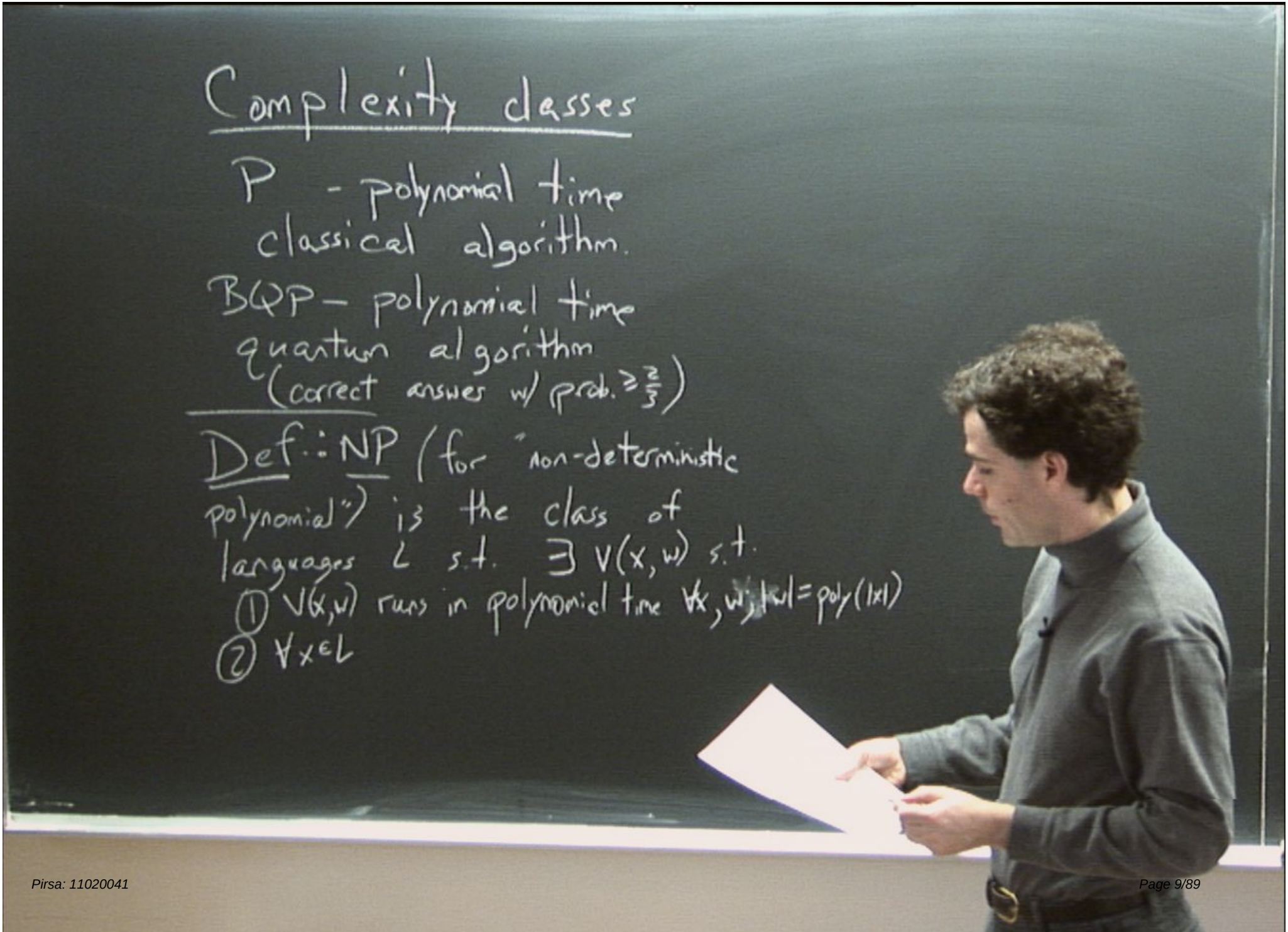
Def.: NP (for "non-deterministic polynomial") is the class of languages L s.t. $\exists\, V(x, w)$ s.t.

① $V(x,w)$ runs in polynomial time $\forall x, w$; $|w| = \text{poly}(|x|)$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② ∀x∈L

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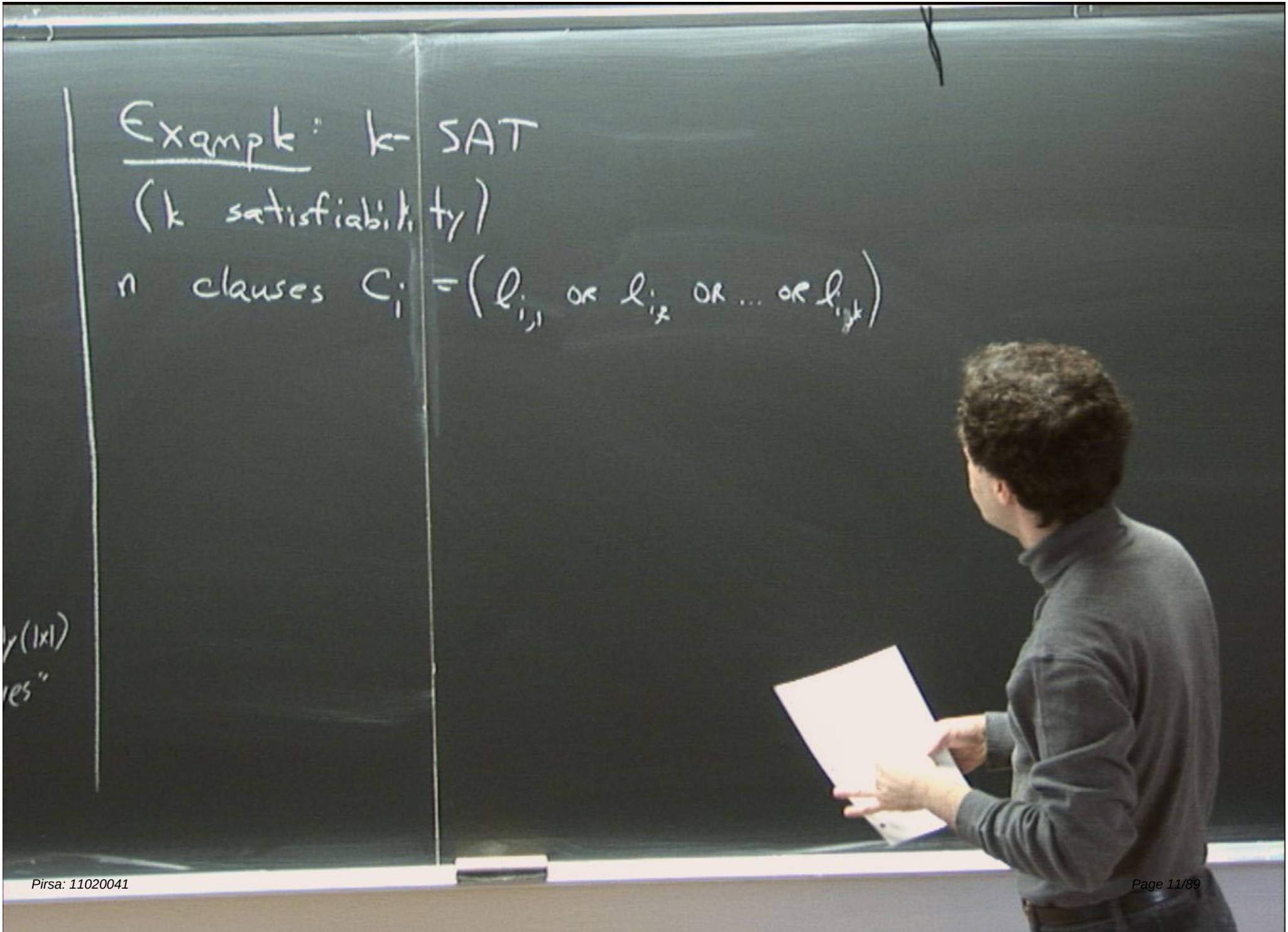
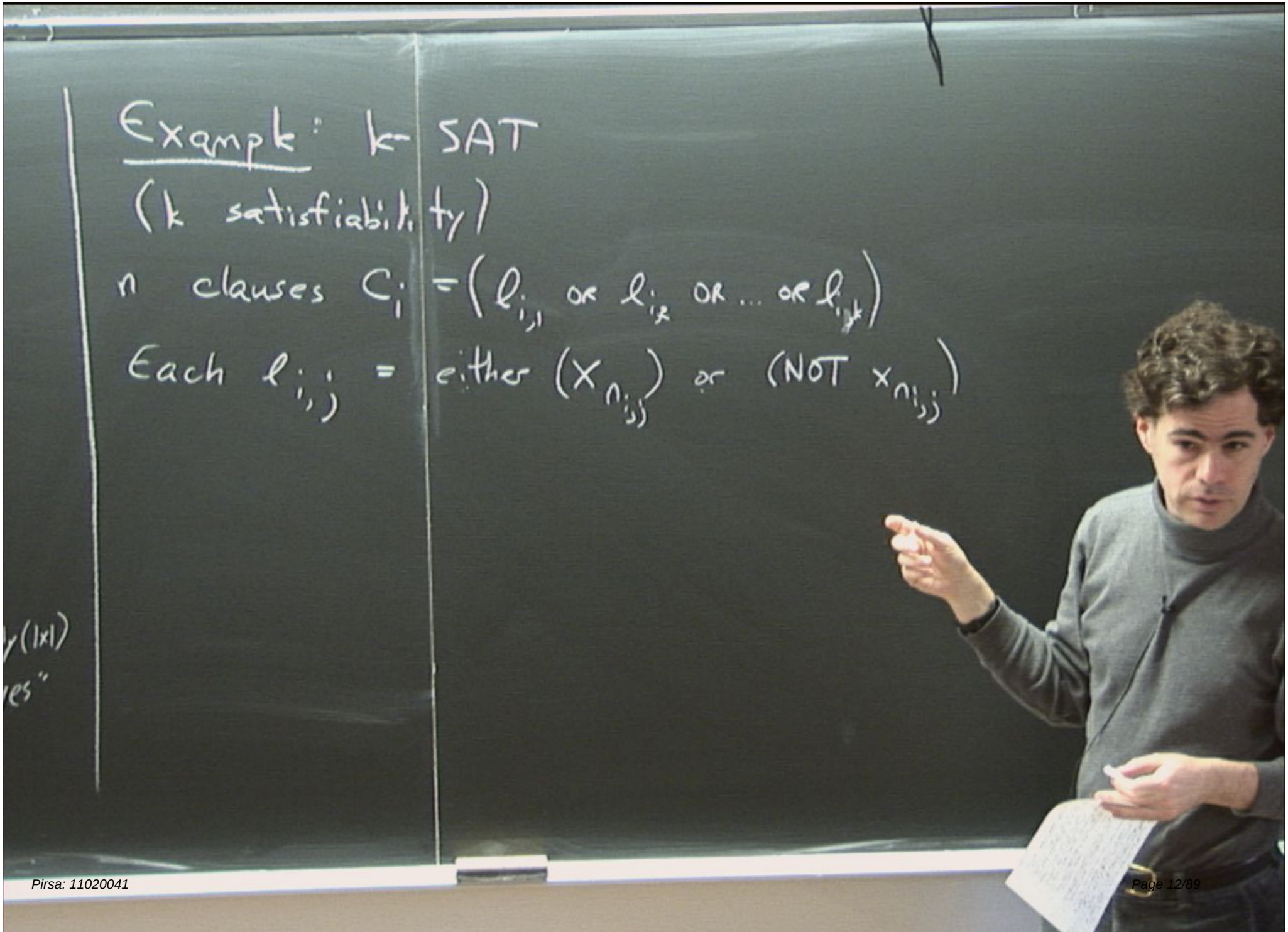
Def.: NP (for "non-deterministic polynomial") is the class of languages L s.t. $\exists\, V(x,w)$ s.t.

1. $V(x,w)$ runs in polynomial time $\forall x, w$; $|w| = \text{poly}(|x|)$
2. $\forall x \in L$, $\exists w_x$ s.t. $|w_x| = \text{poly}(|x|)$, $V(x,w_x) = \text{"yes"}$
3. $\forall x \notin L$, $\forall w\ (|w| = \text{poly}(|x|))$, $V(x,w) = \text{"no"}$

w_x is the witness for a yes instance x.


Example: k-SAT
(k satisfiability)
n clauses $C_i = (\ell_{i,1}$ OR $\ell_{i,2}$ OR ... OR $\ell_{i,k})$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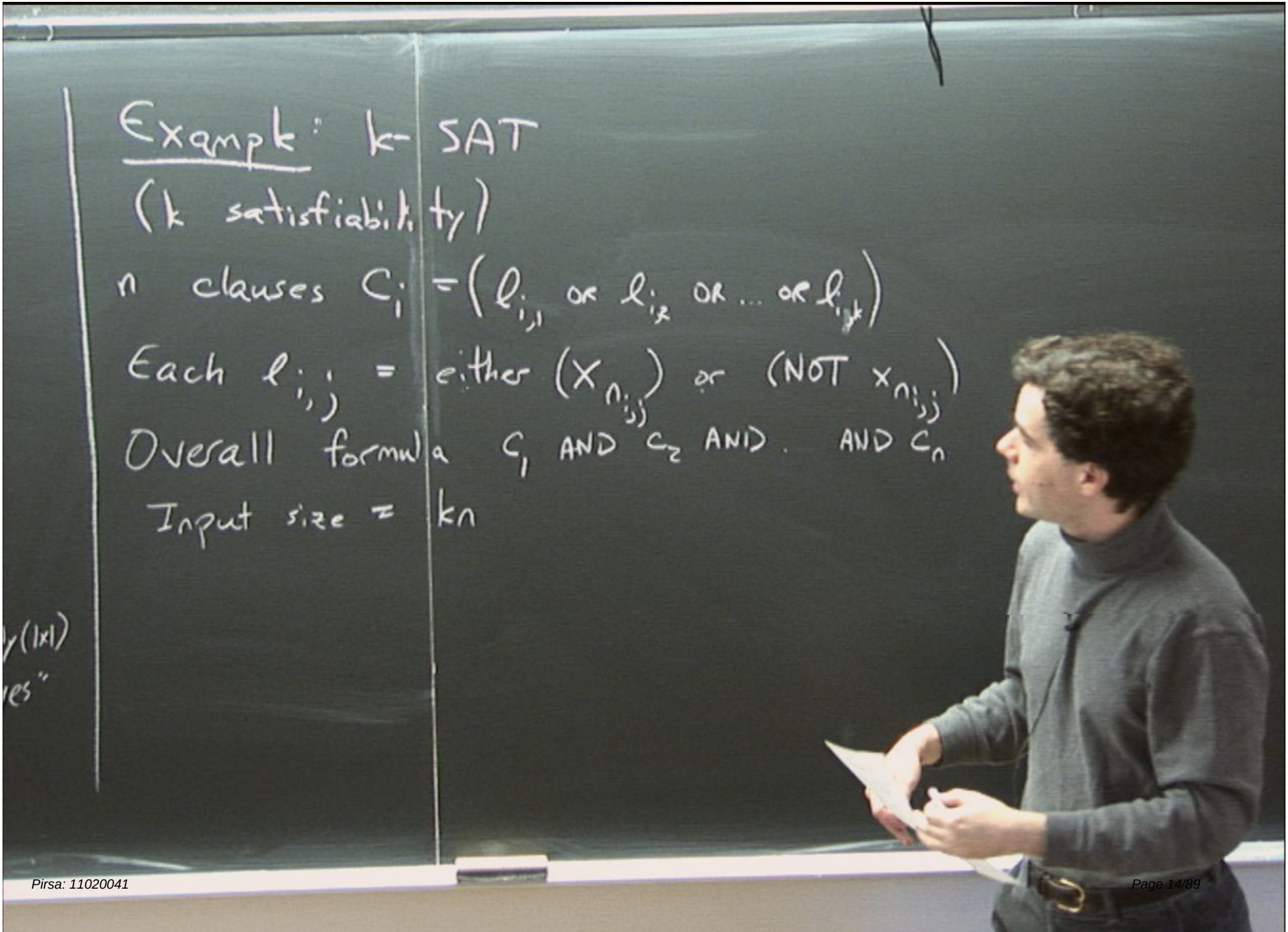
Overall formula C_1 AND C_2 AND . AND C_n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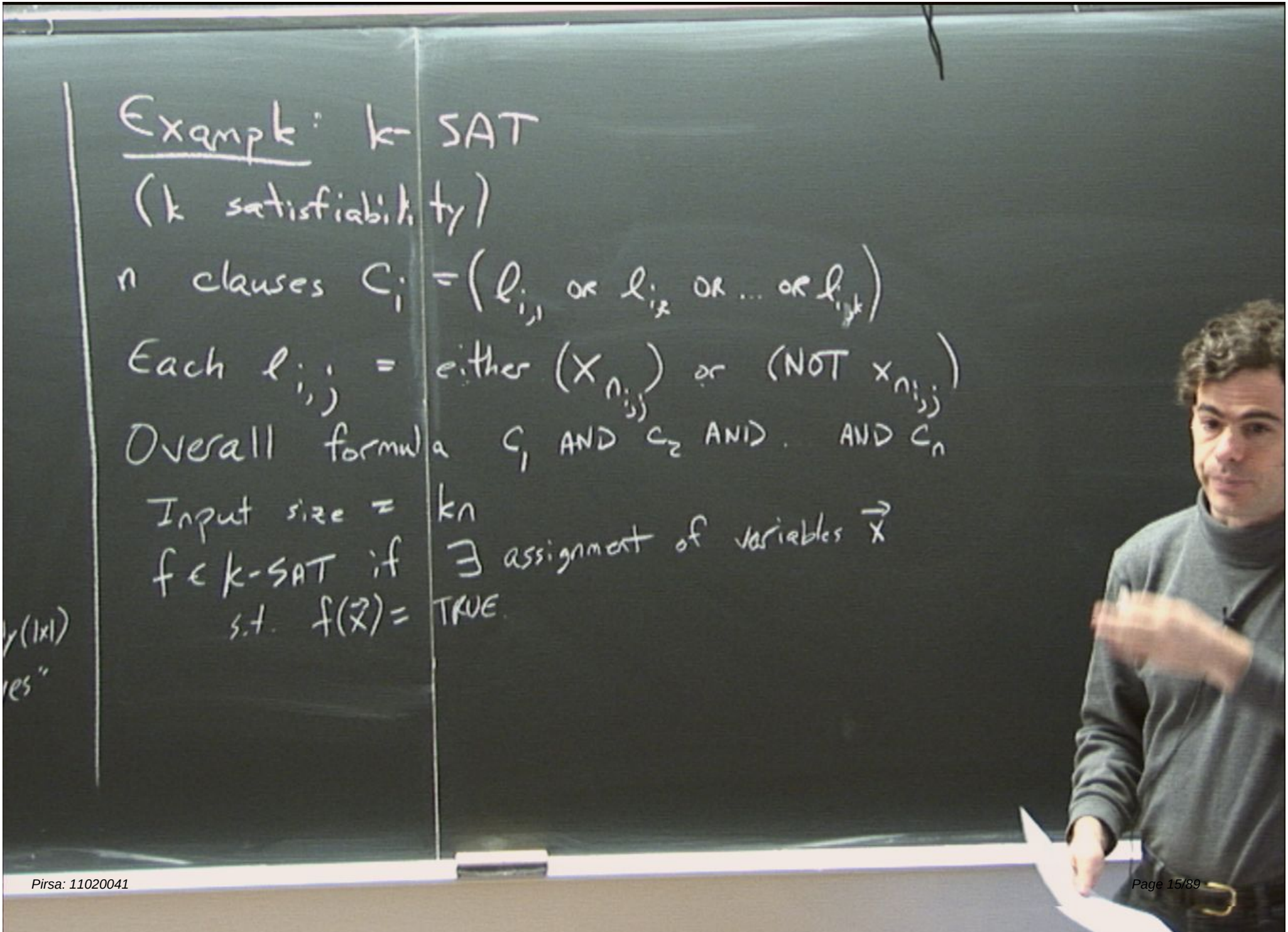
Input size = kn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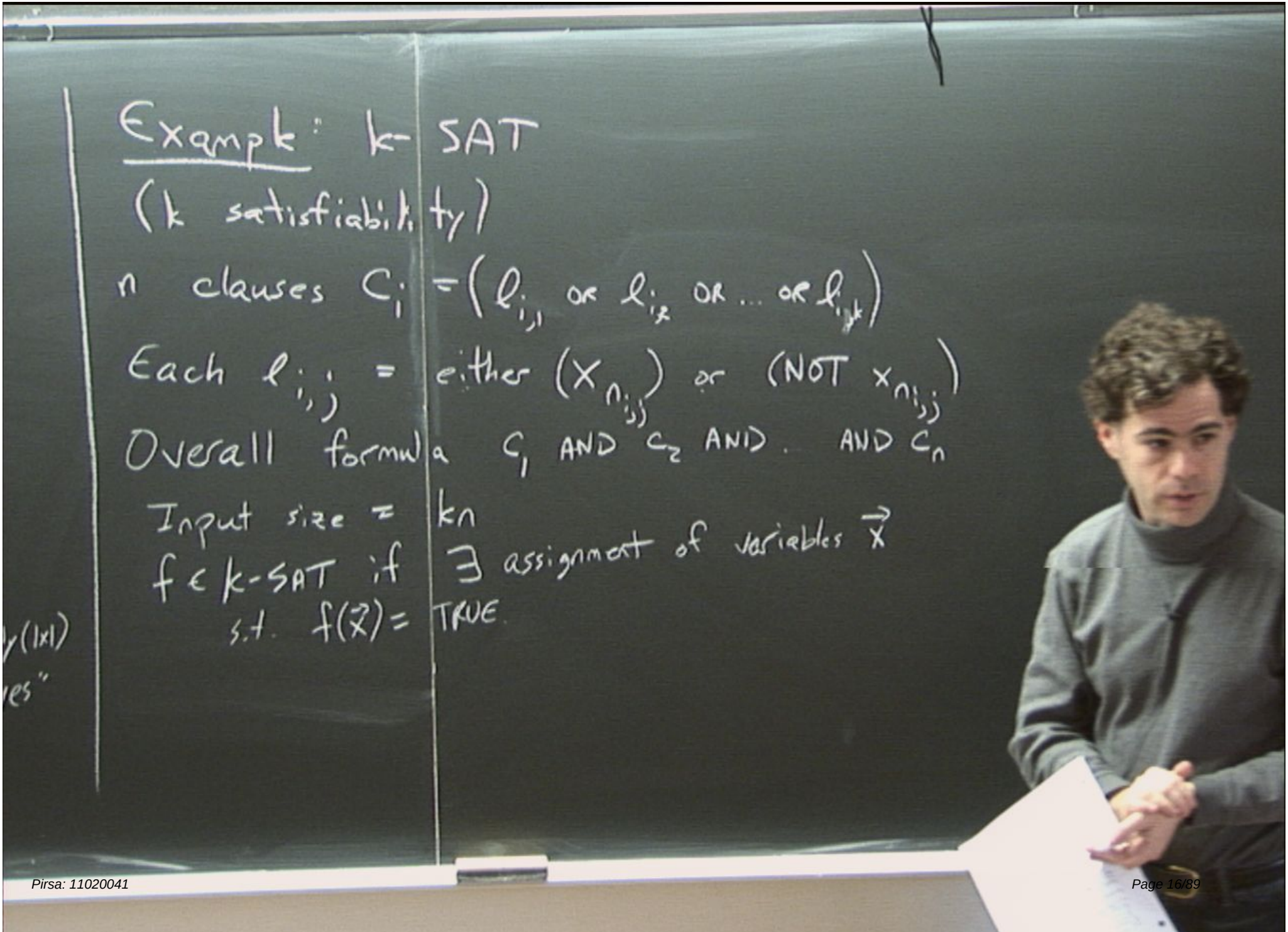
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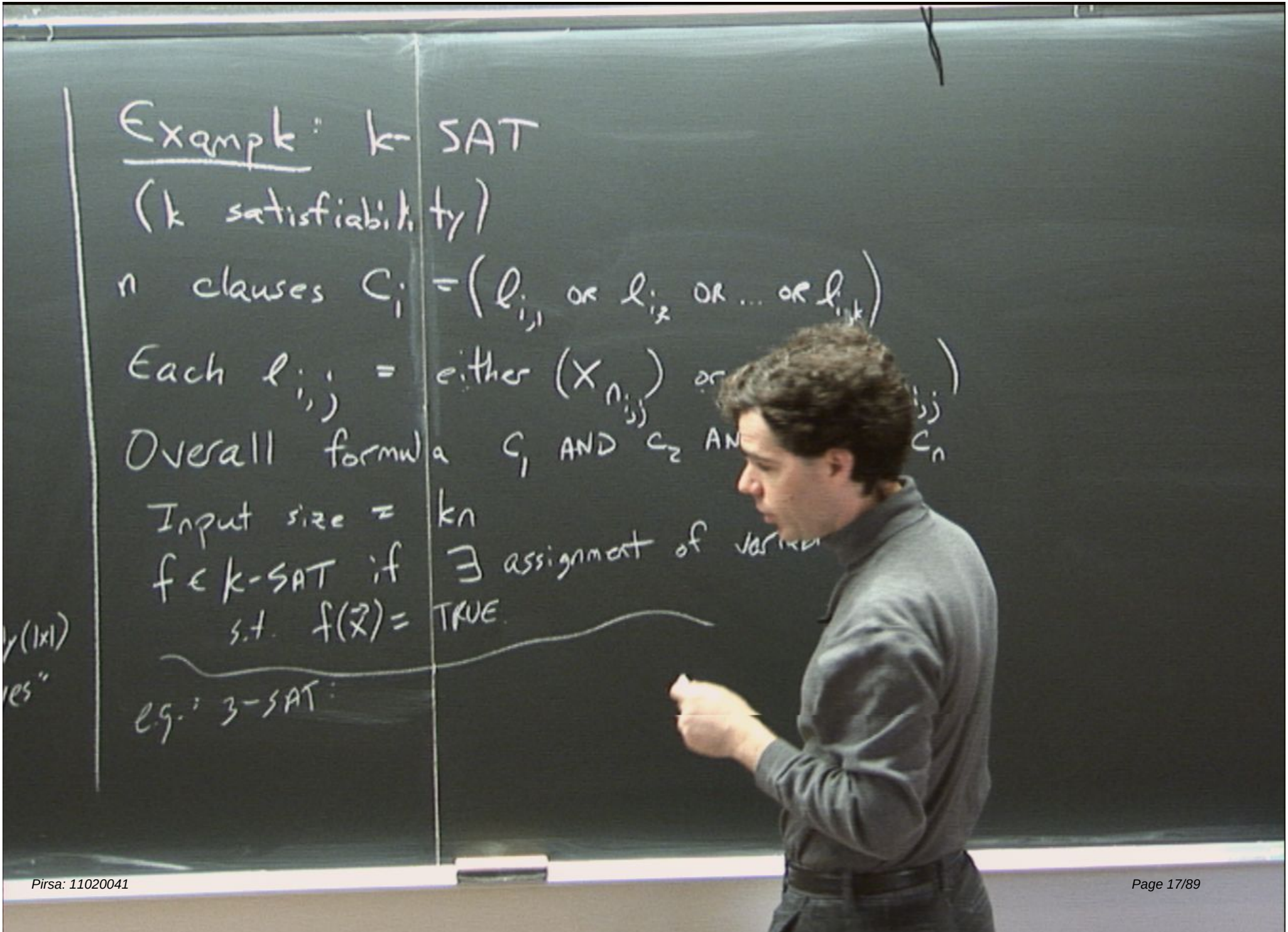
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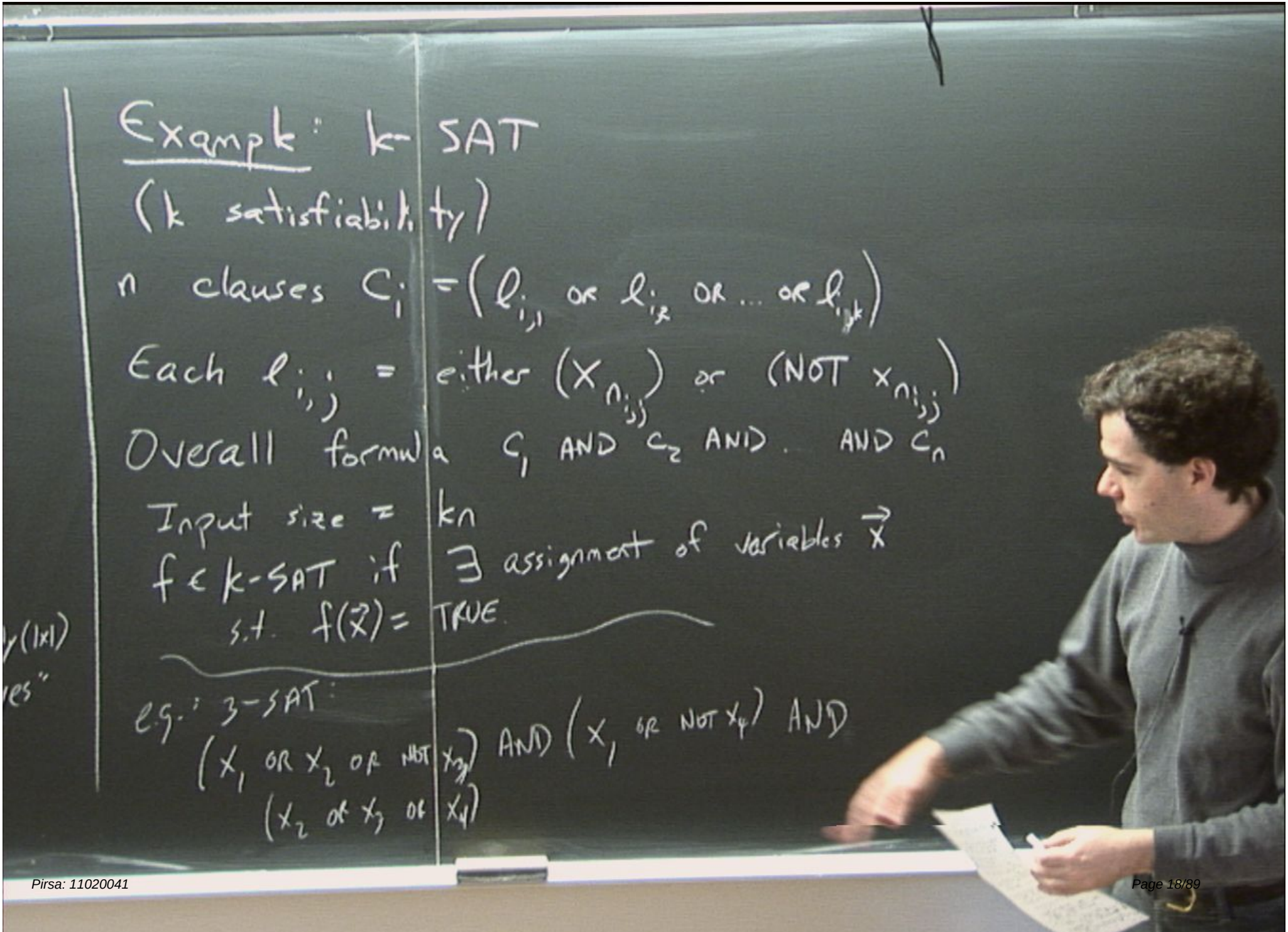
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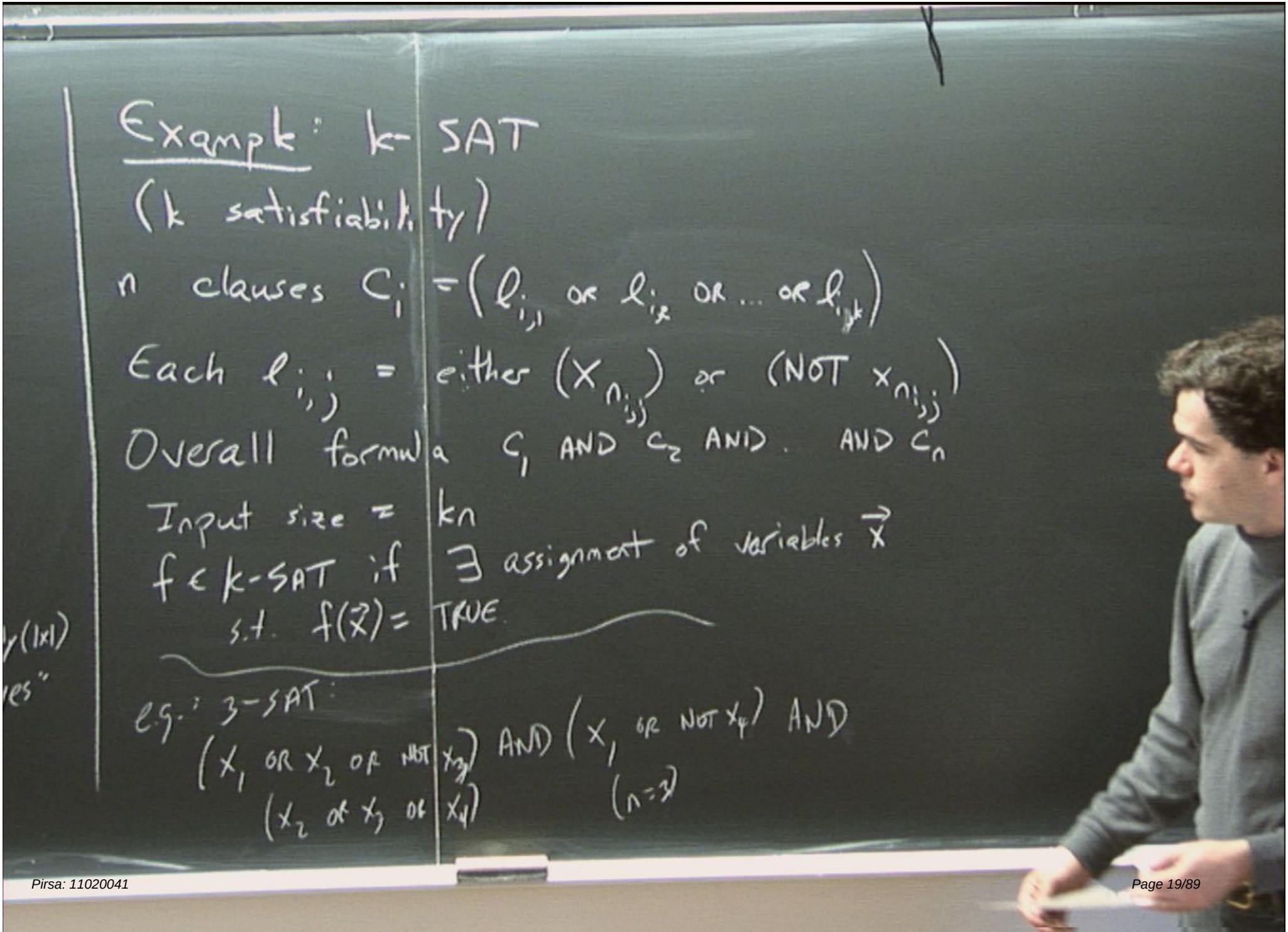
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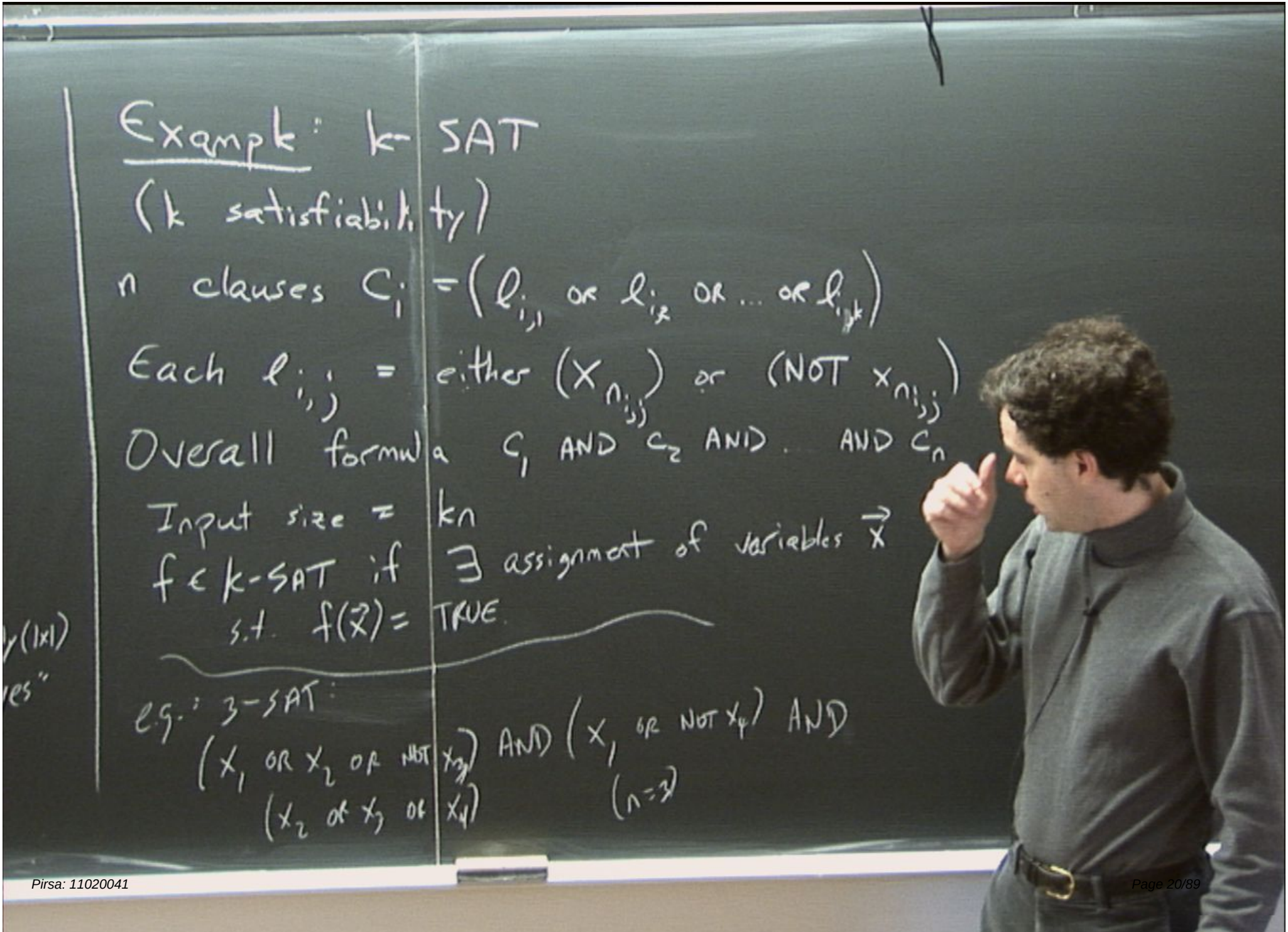
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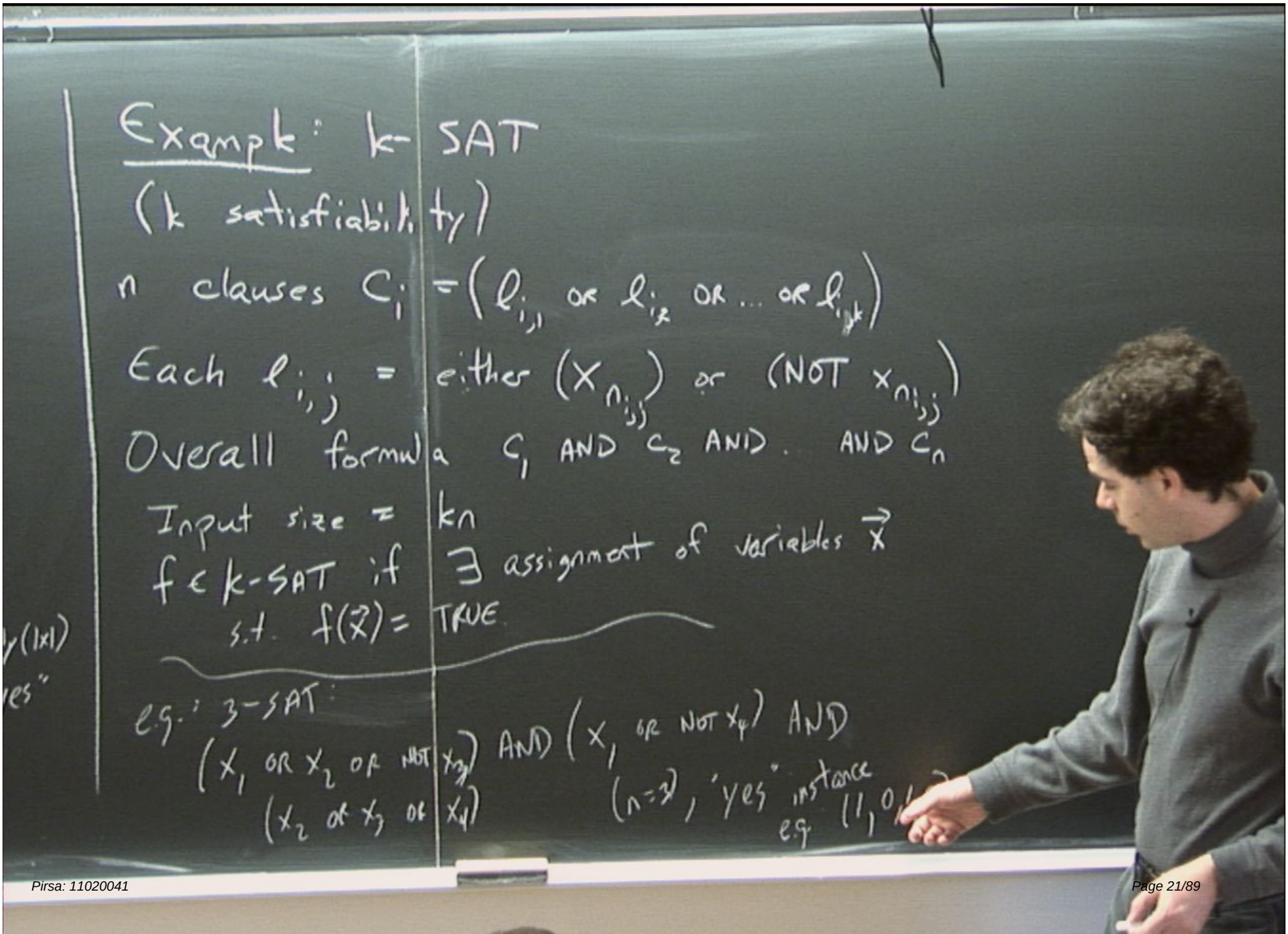
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e.g.: 3-SAT

$(x_1$ OR x_2 OR NOT $x_3)$ AND $(x_1$ OR NOT $x_4)$ AND

$(x_2$ OR x_3 OR $x_4)$

$(n=3)$, "yes" instance

e.g. $(1,0,1,0)$

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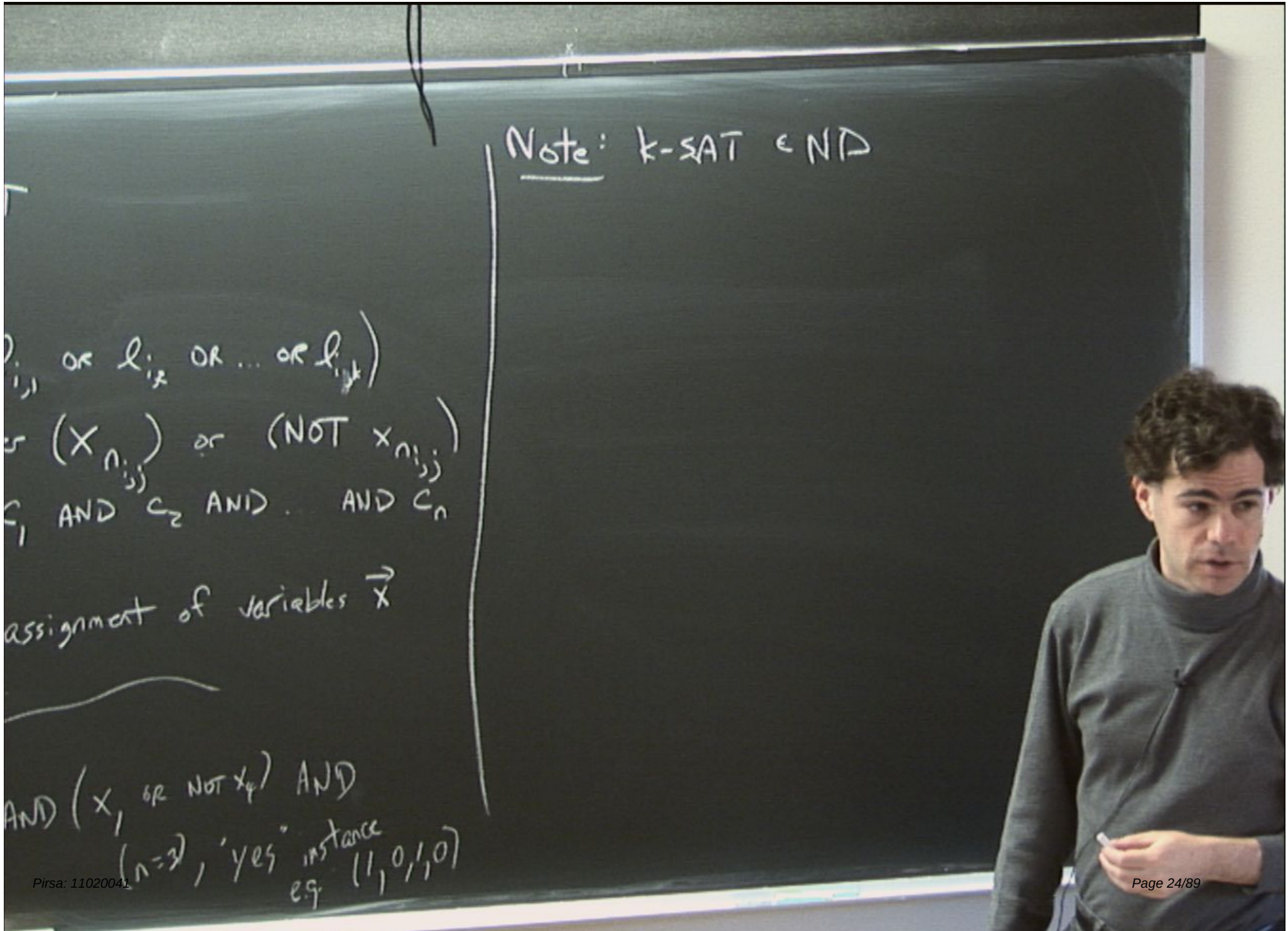
Note: k-SAT ∈ NP
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$(x_{n_{i,j}})$ or (NOT $x_{n_{i,j}}$)
C_1 AND C_2 AND . AND C_n
assignment of variables $\vec{x}$
AND (x_1 OR NOT x_4) AND
(n=3, 'yes' instance
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Note: k-SAT $\in$ NP

Witness is $\vec{x}$ that makes $f(\vec{x}) = 1$

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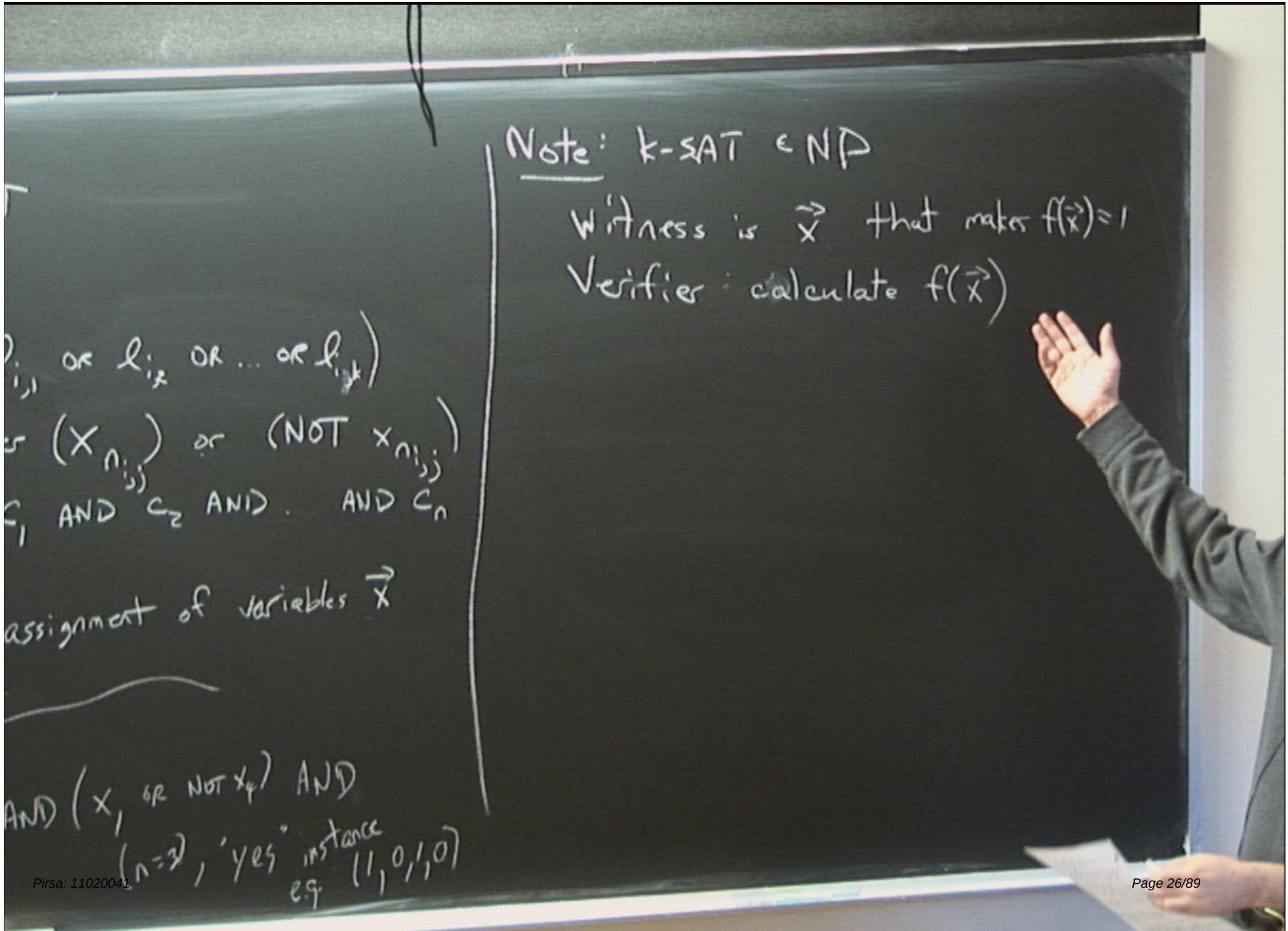
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Witness is $\vec{x}$ that makes $f(\vec{x})=1$
Verifier: calculate $f(\vec{x})$
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e.g. $(1,0,1,0)$

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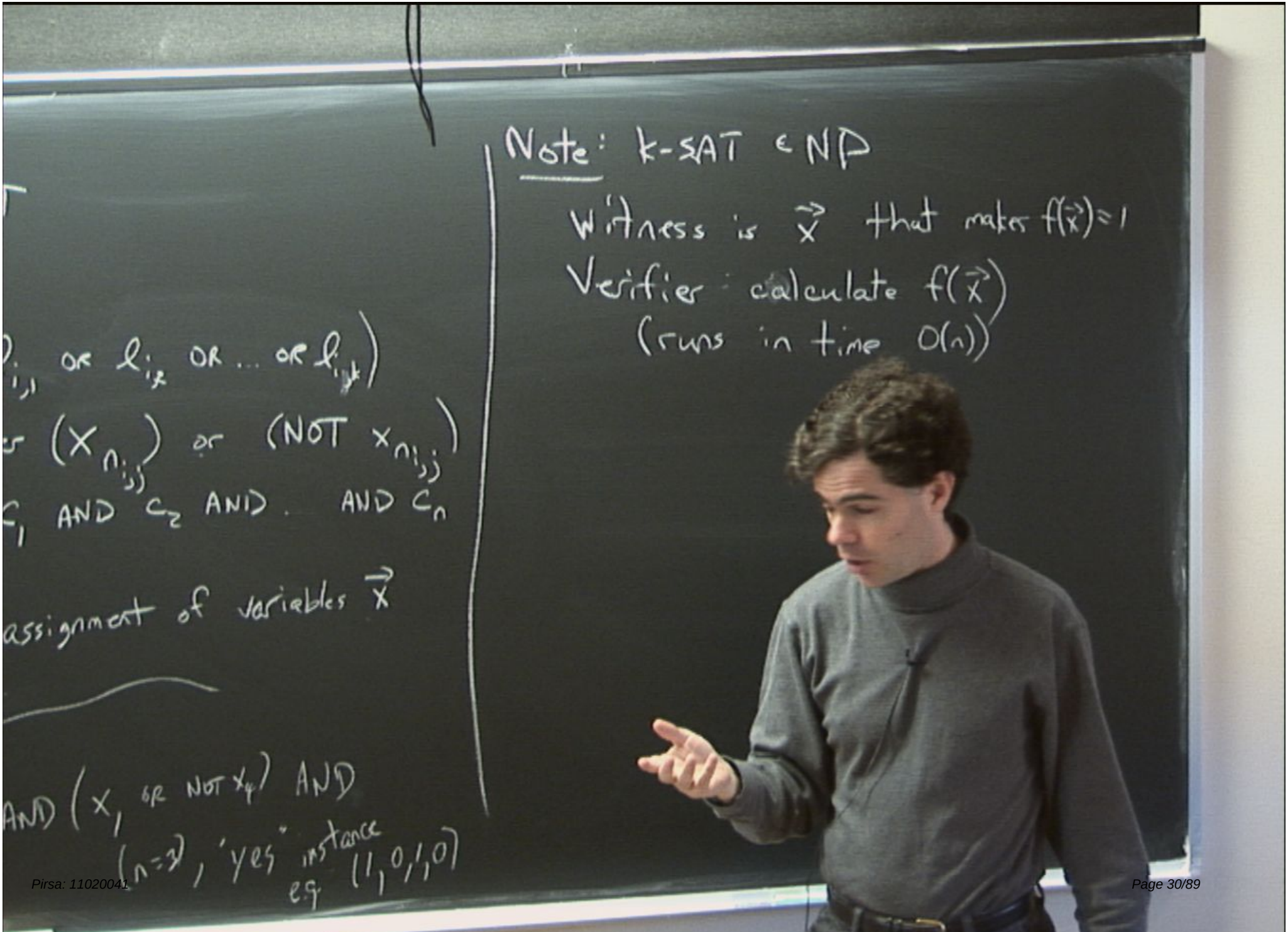
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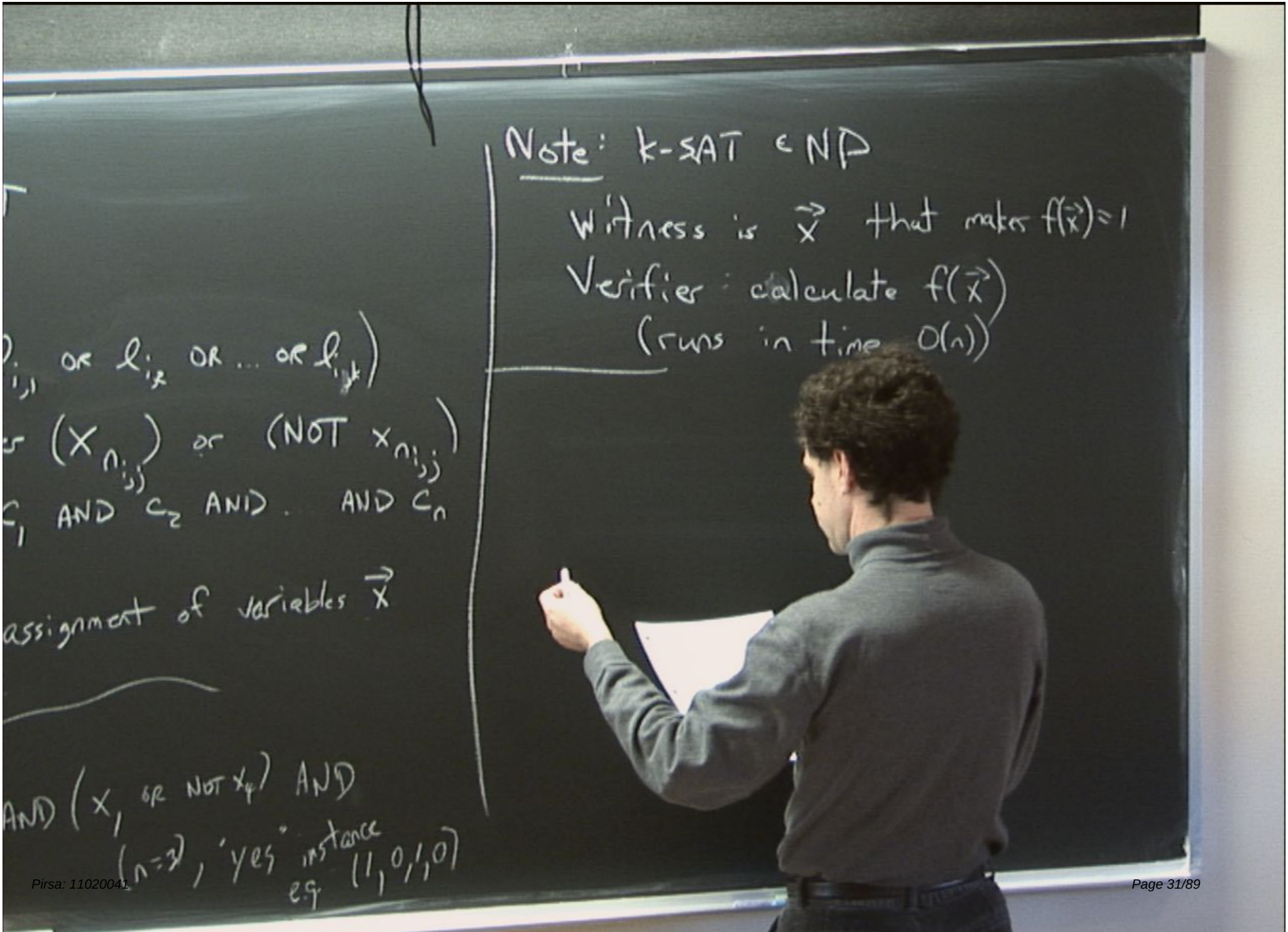
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Def: Language L **reduces** to language M if $\exists$ poly-time $f: \{0,1\}^* \to \{0,1\}^*$

s.t. $x \in L \Leftrightarrow f(x) \in M$

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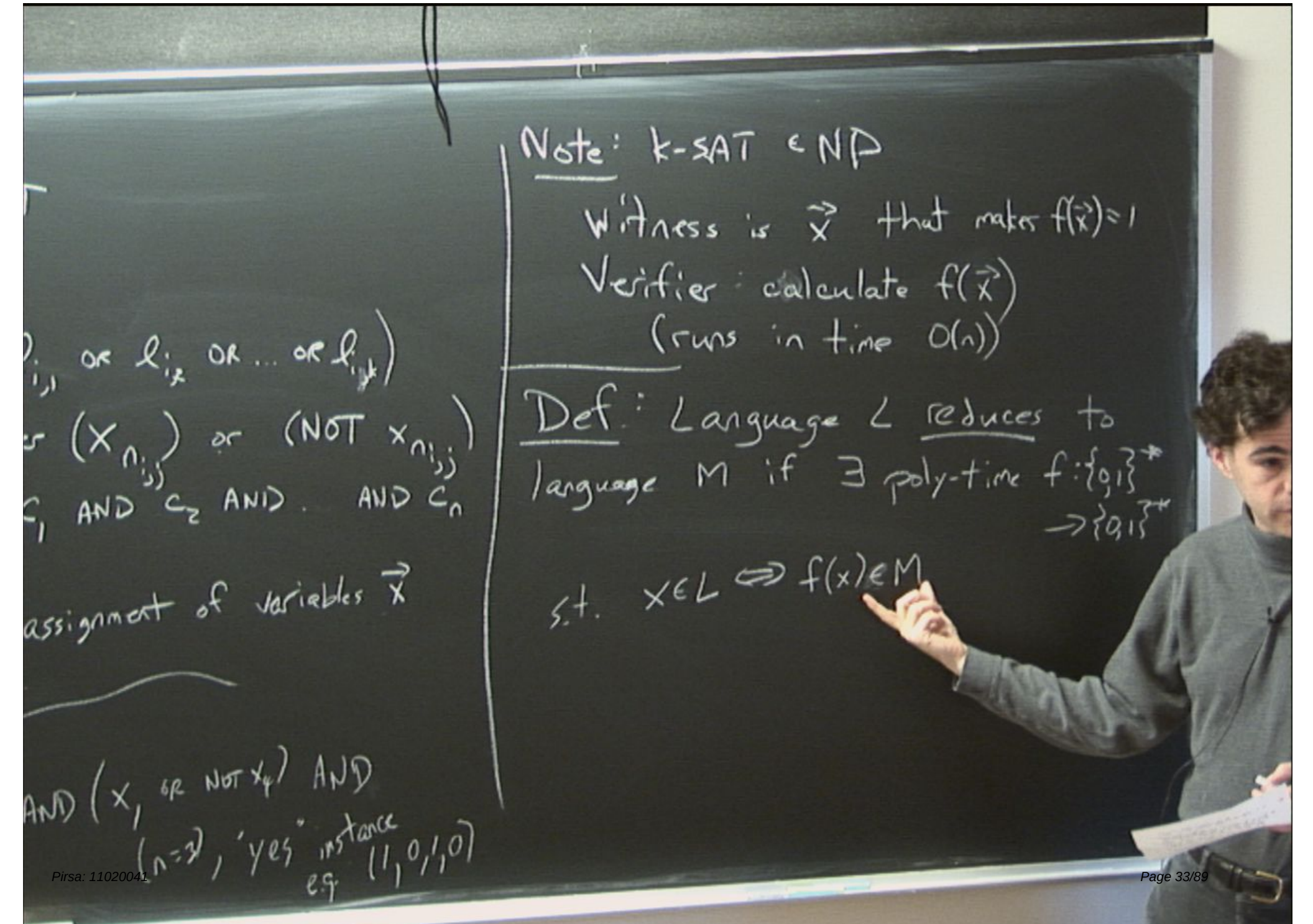

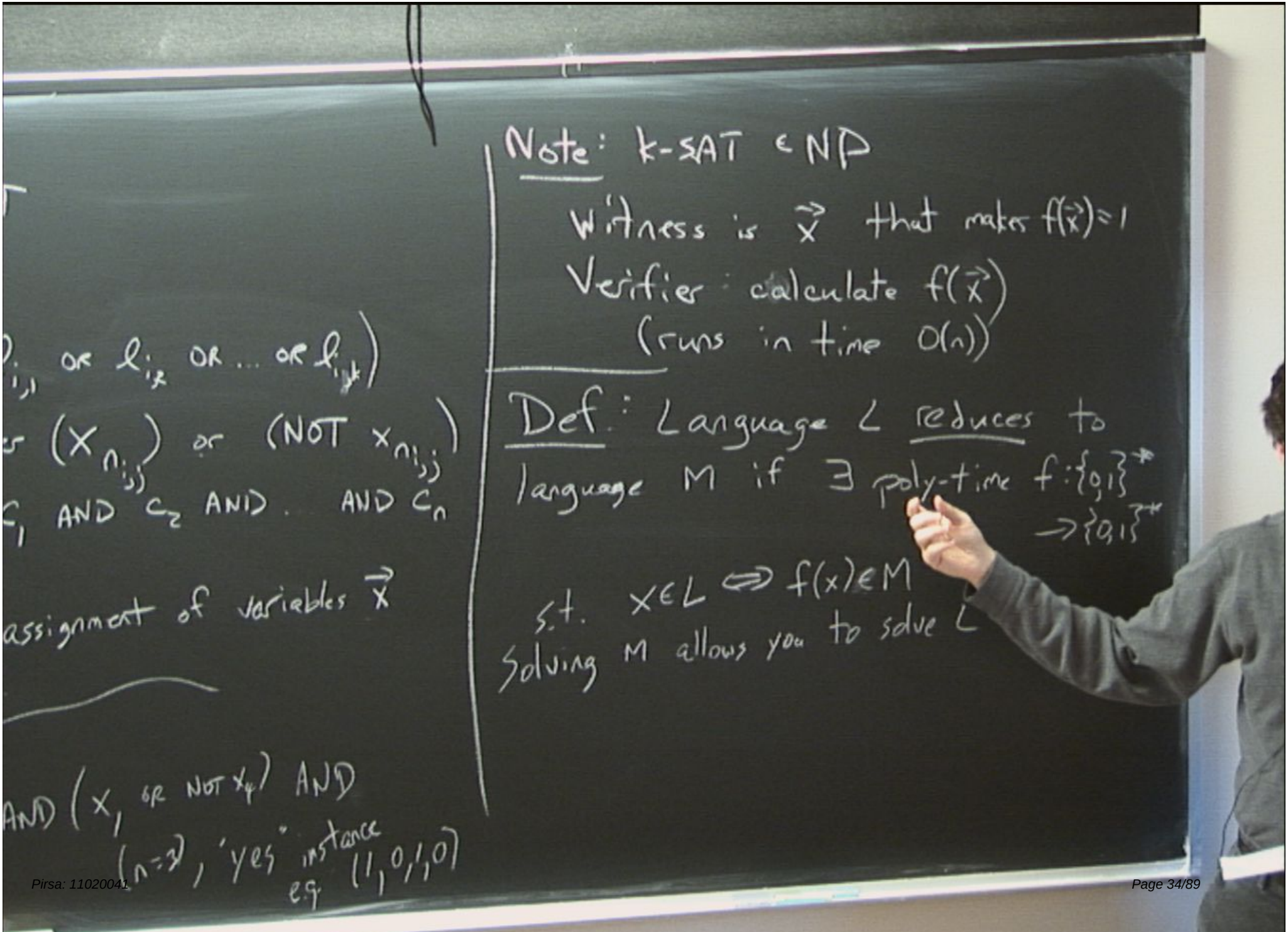
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Solving M allows you to solve L
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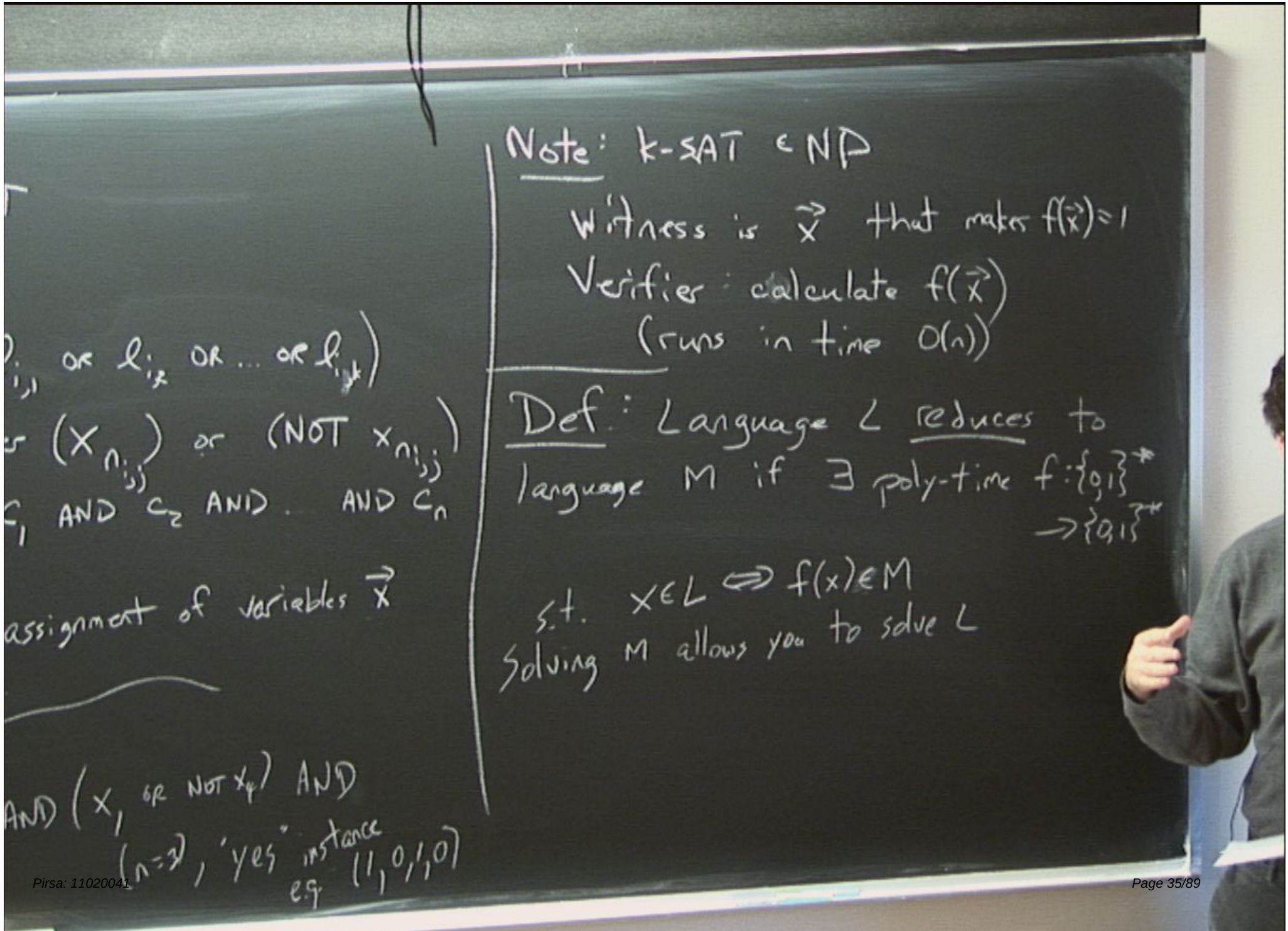
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or $(x_{n_{i,j}})$ or $(\text{NOT } x_{n_{i,j}})$

C_1 AND C_2 AND . AND C_n

assignment of variables $\vec{x}$

AND $(x_1$ OR NOT $x_4)$ AND

$(n=3$, "yes" instance
e.g. $(1,0,1,0)$

Note: k-SAT $\in$ NP

Witness is $\vec{x}$ that makes $f(\vec{x})=1$

Verifier: calculate $f(\vec{x})$
(runs in time $O(n)$)

Def: Language L **reduces** to language M if $\exists$ poly-time $f: \{0,1\}^* \to \{0,1\}^*$

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Solving M allows you to solve L

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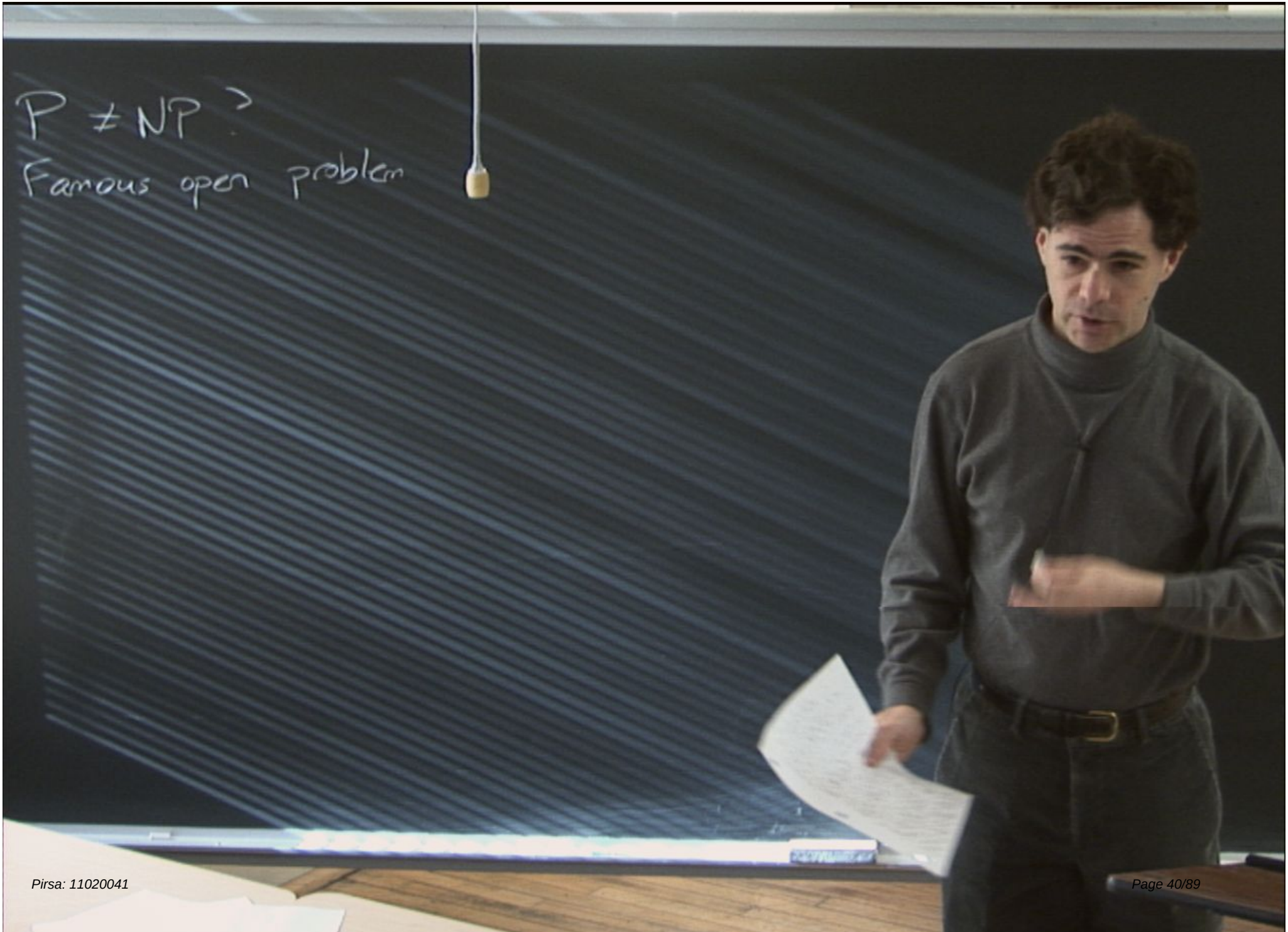
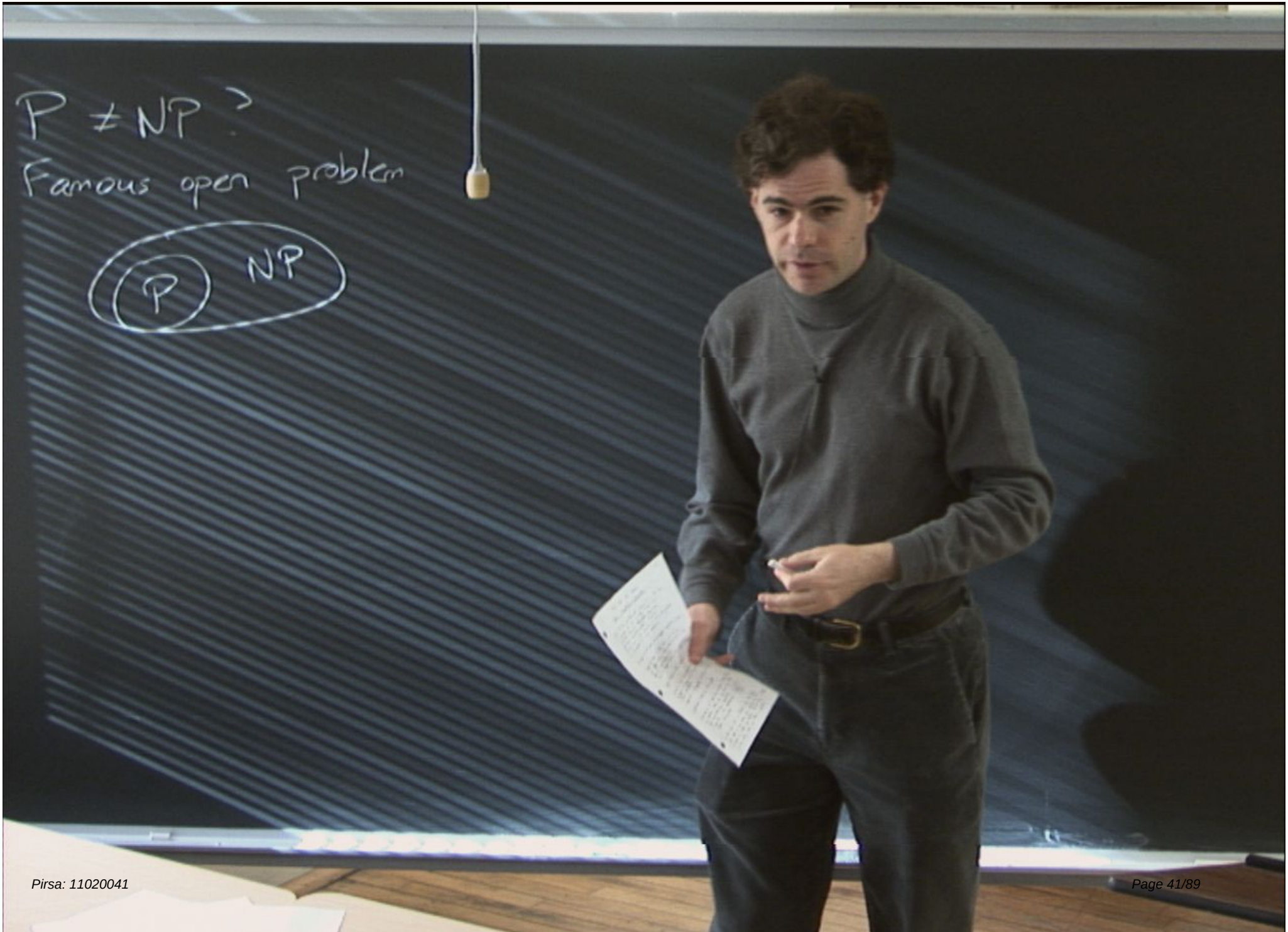
P ≠ NP ?


P ≠ NP ?
Famous open problem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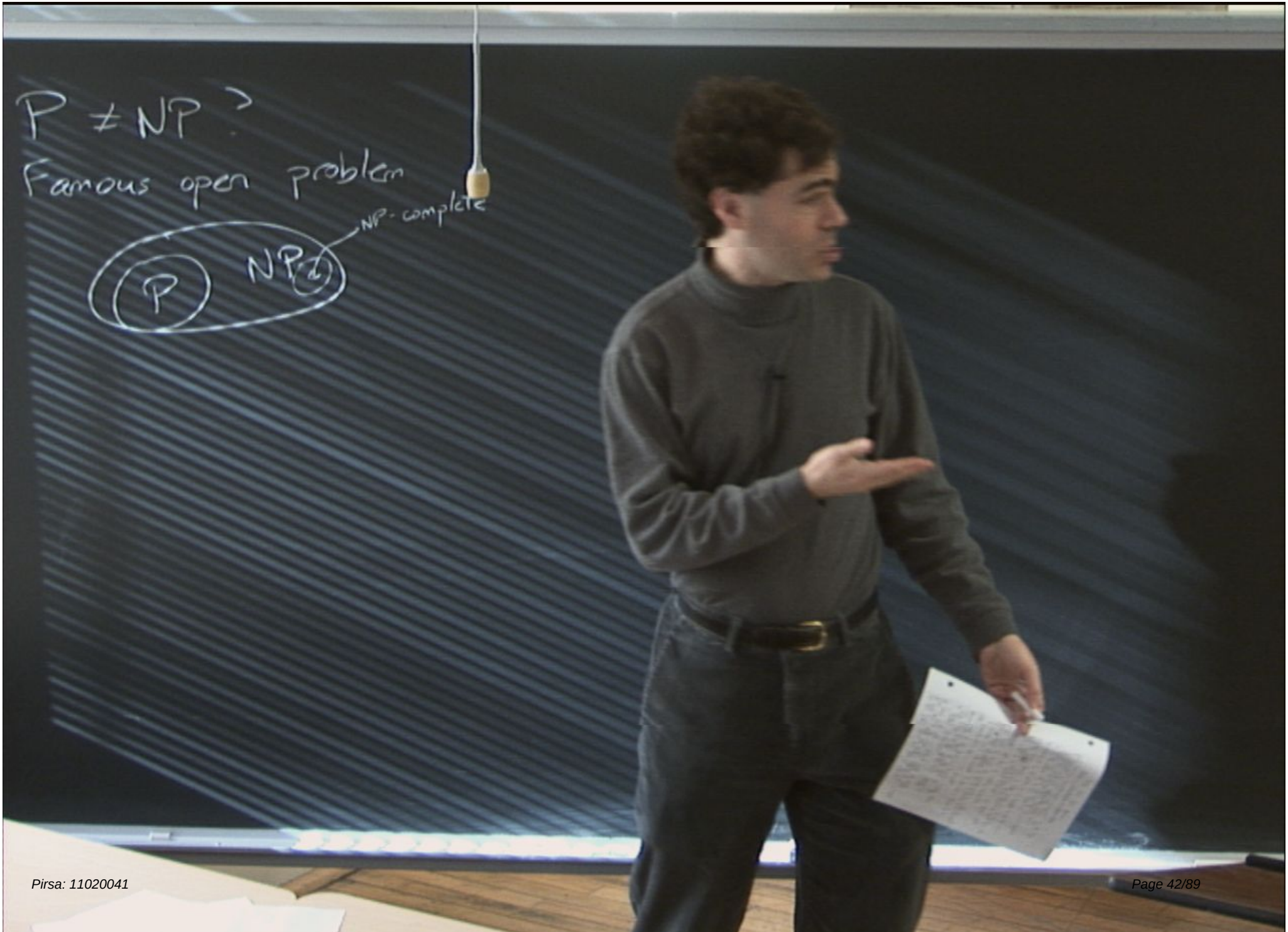
Famous open problem
P
NP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NP - complete
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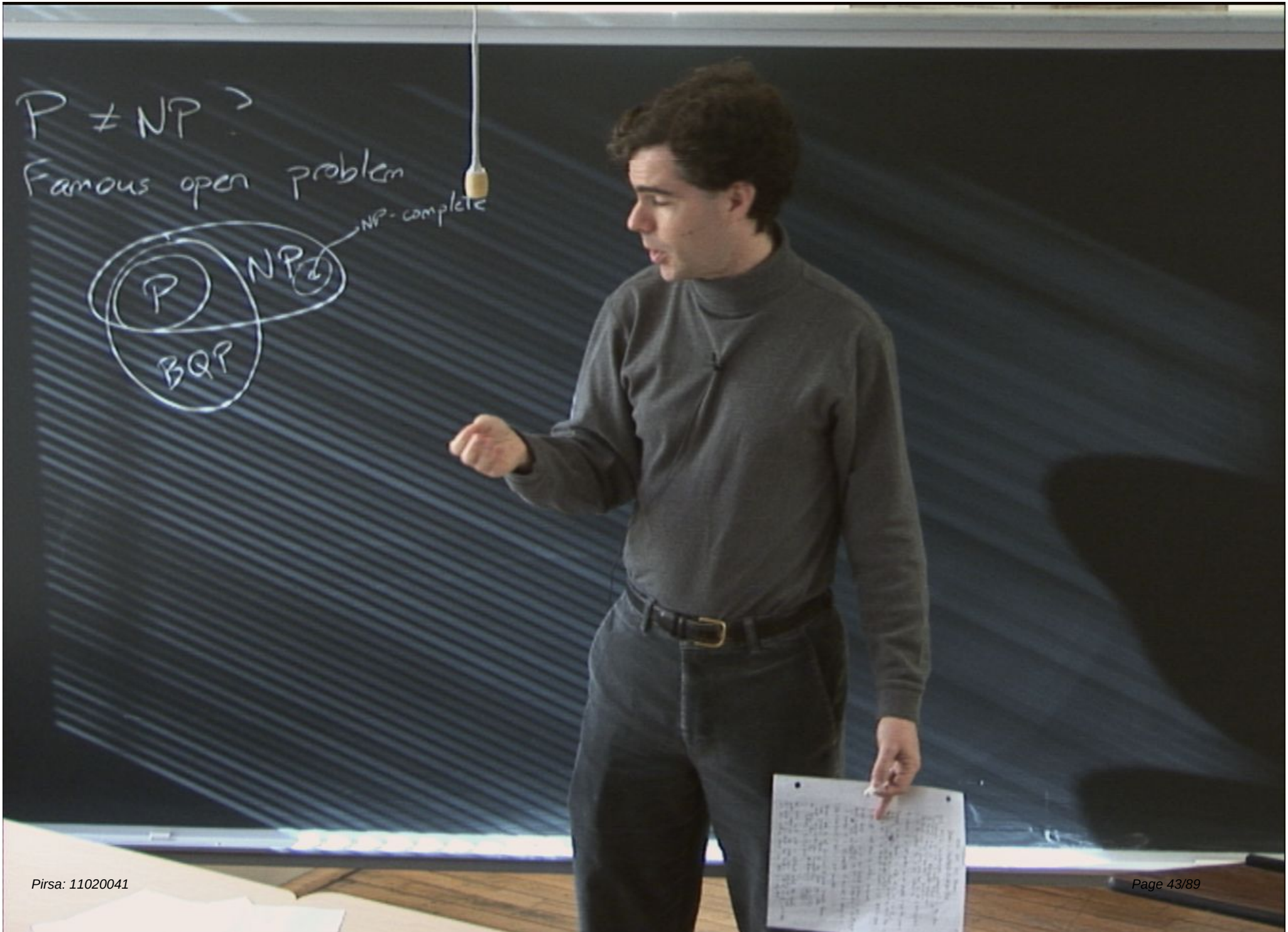
NP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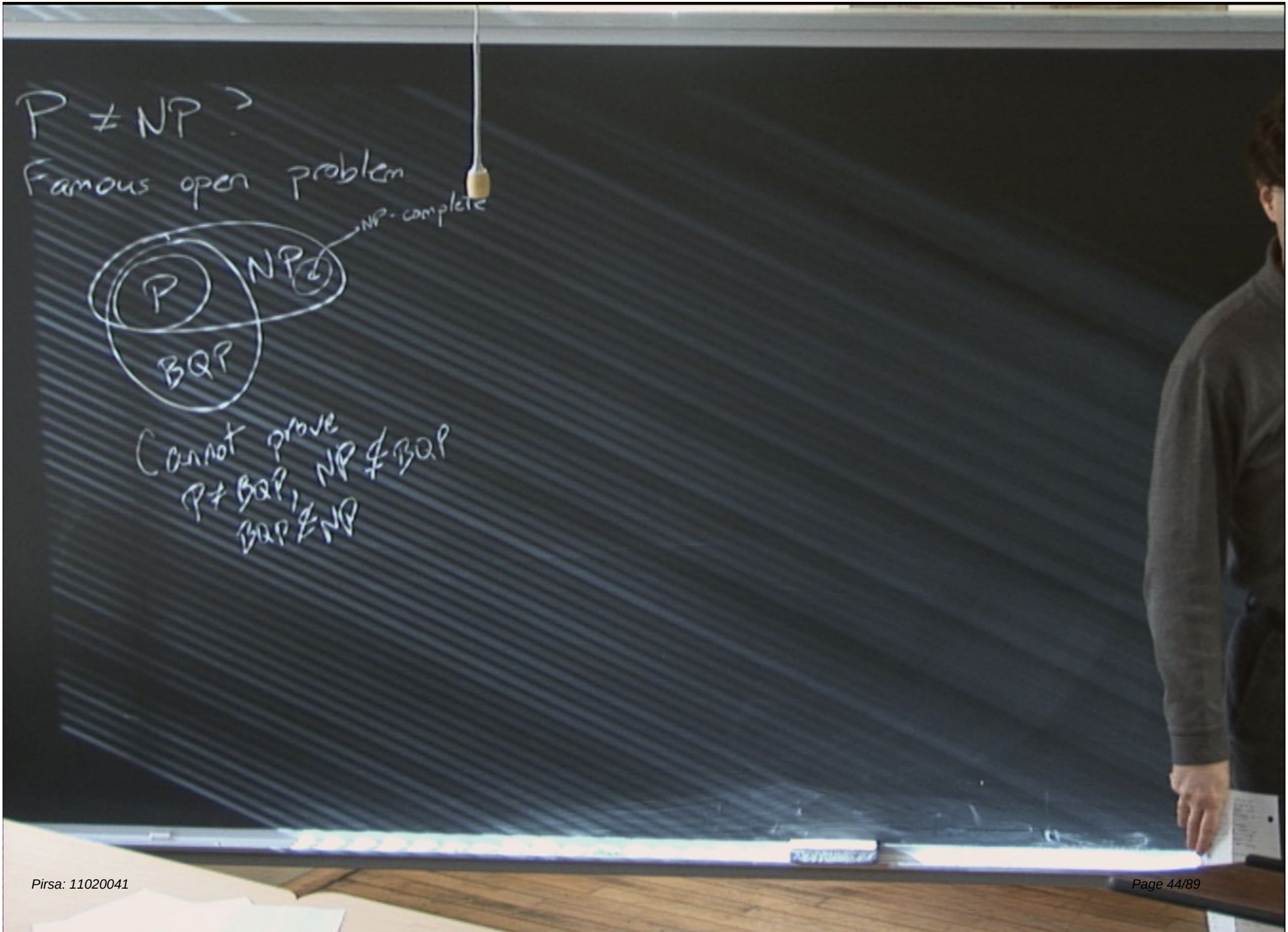
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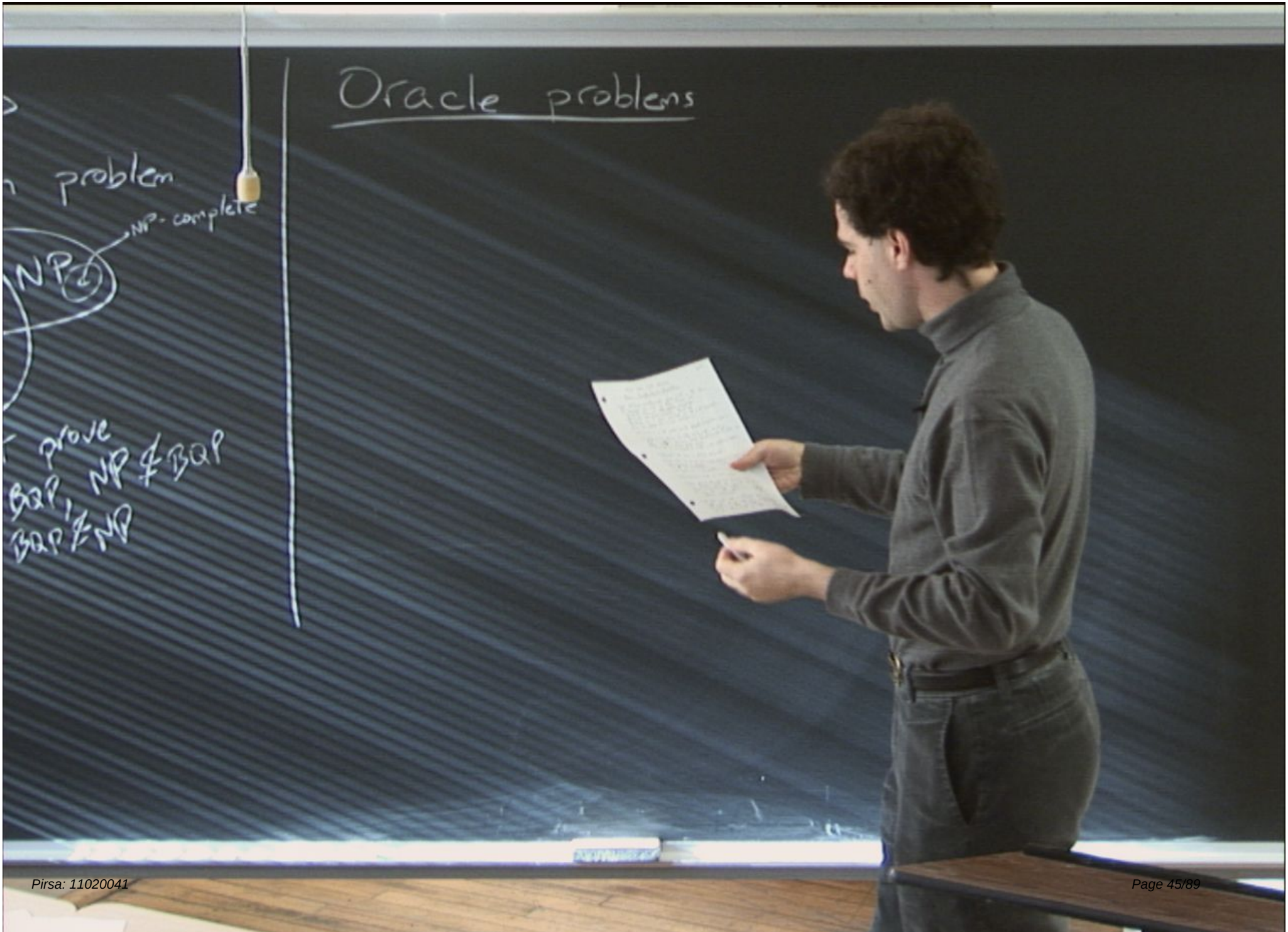
NP-complete
NP
P
BQP
Cannot prove
NP ⊄ BQP
P ⊄ BQP, BQP ⊄ NP


Oracle problems
problem
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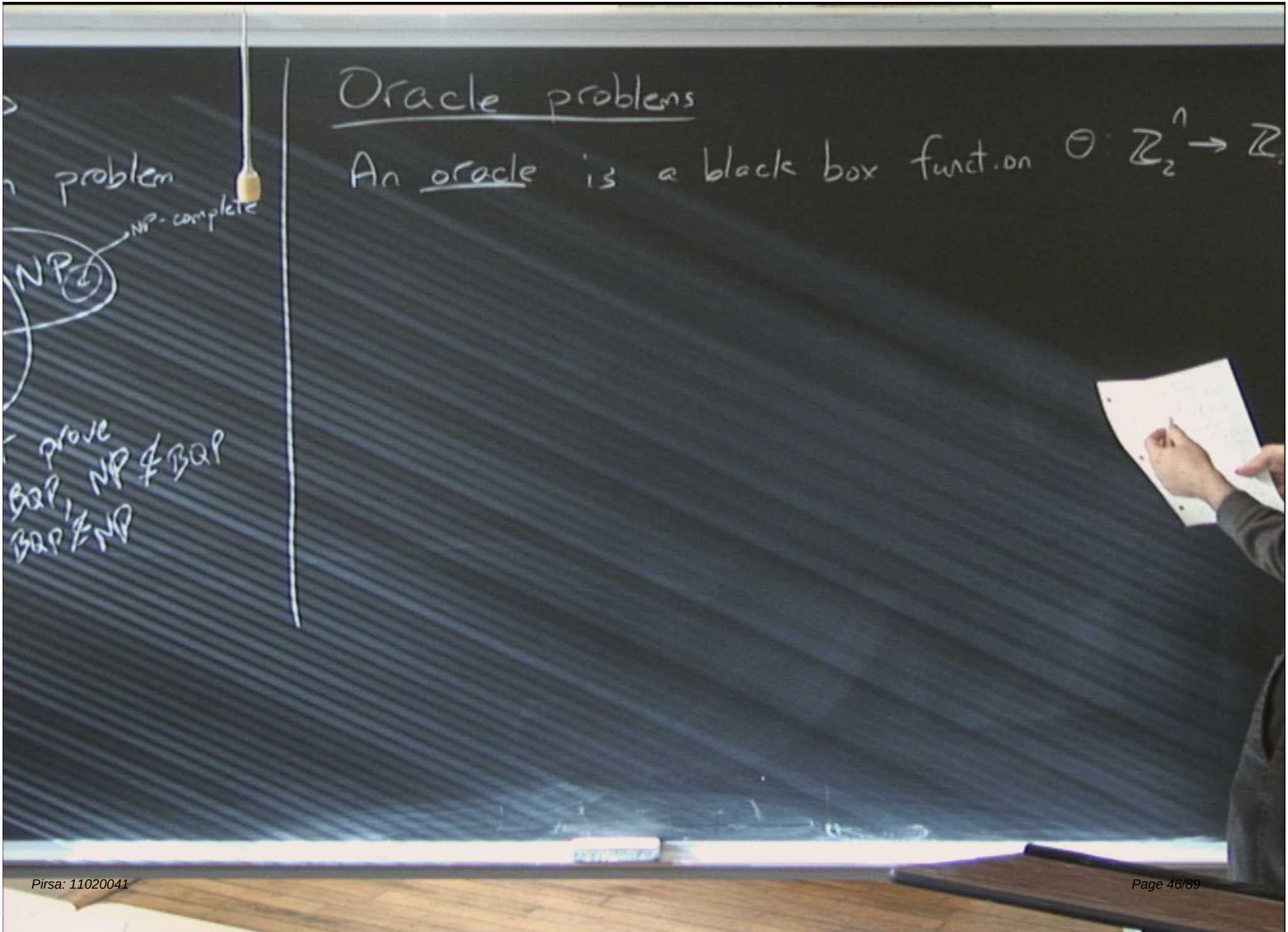
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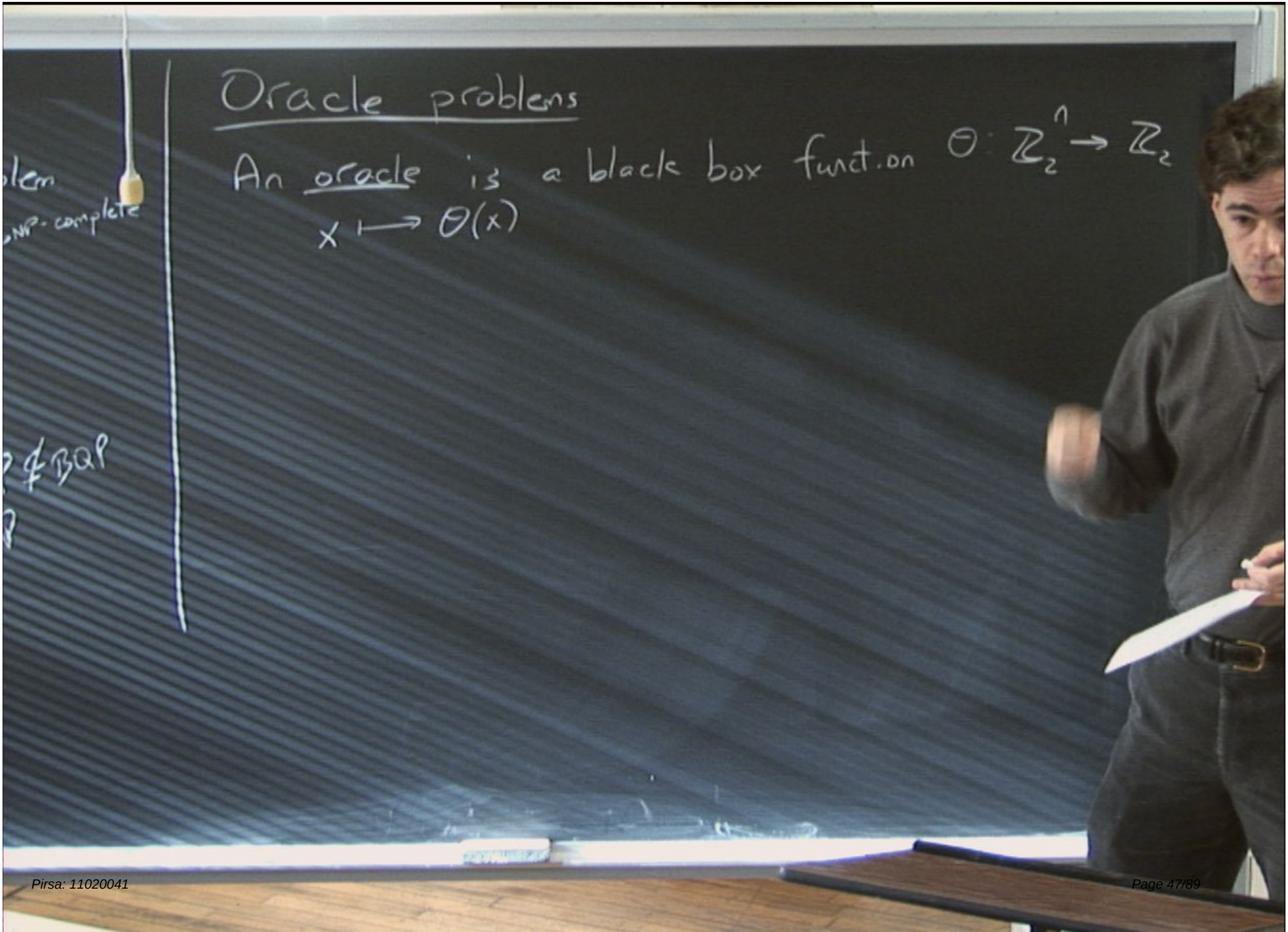
An oracle is a black box function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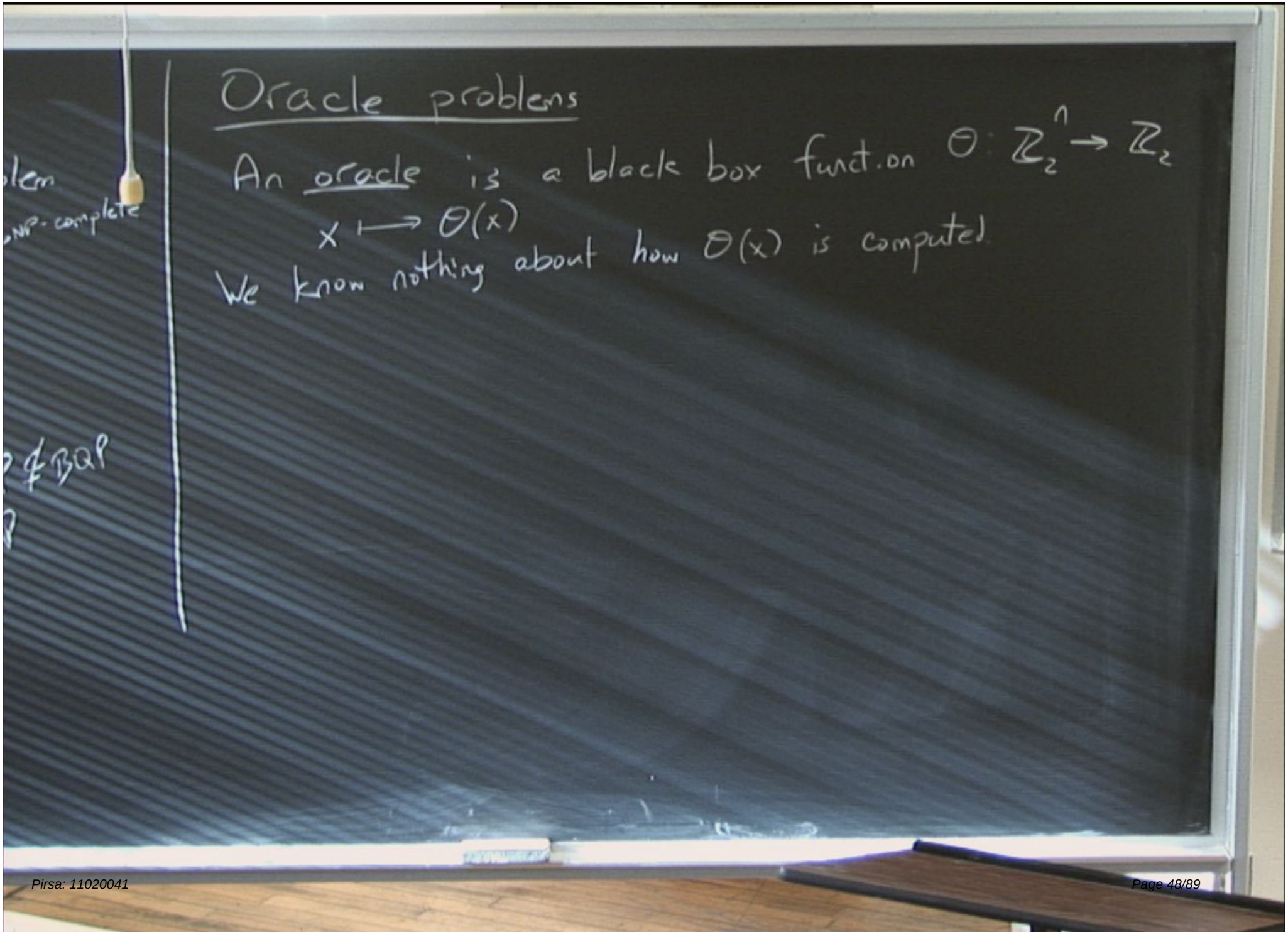
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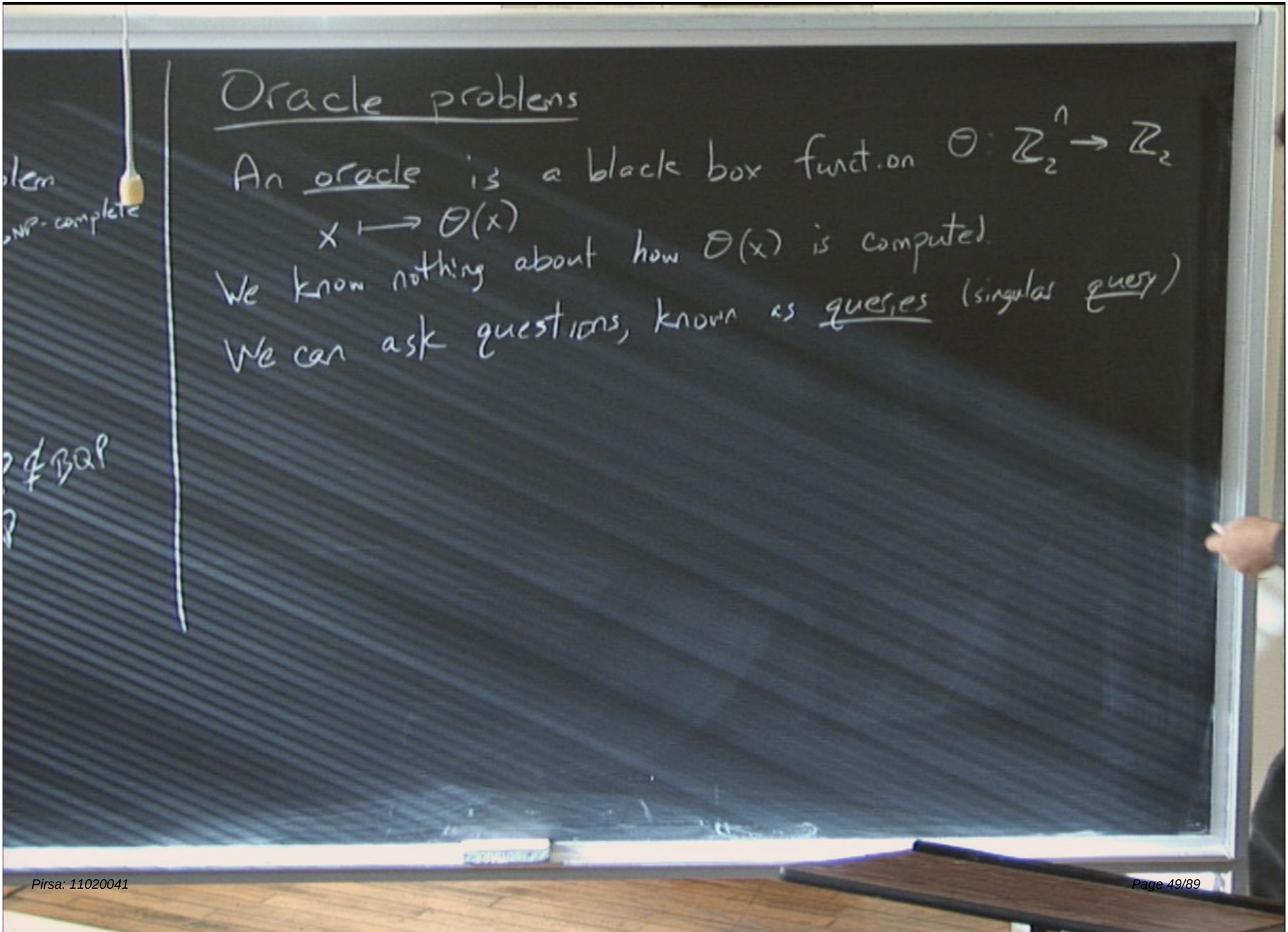
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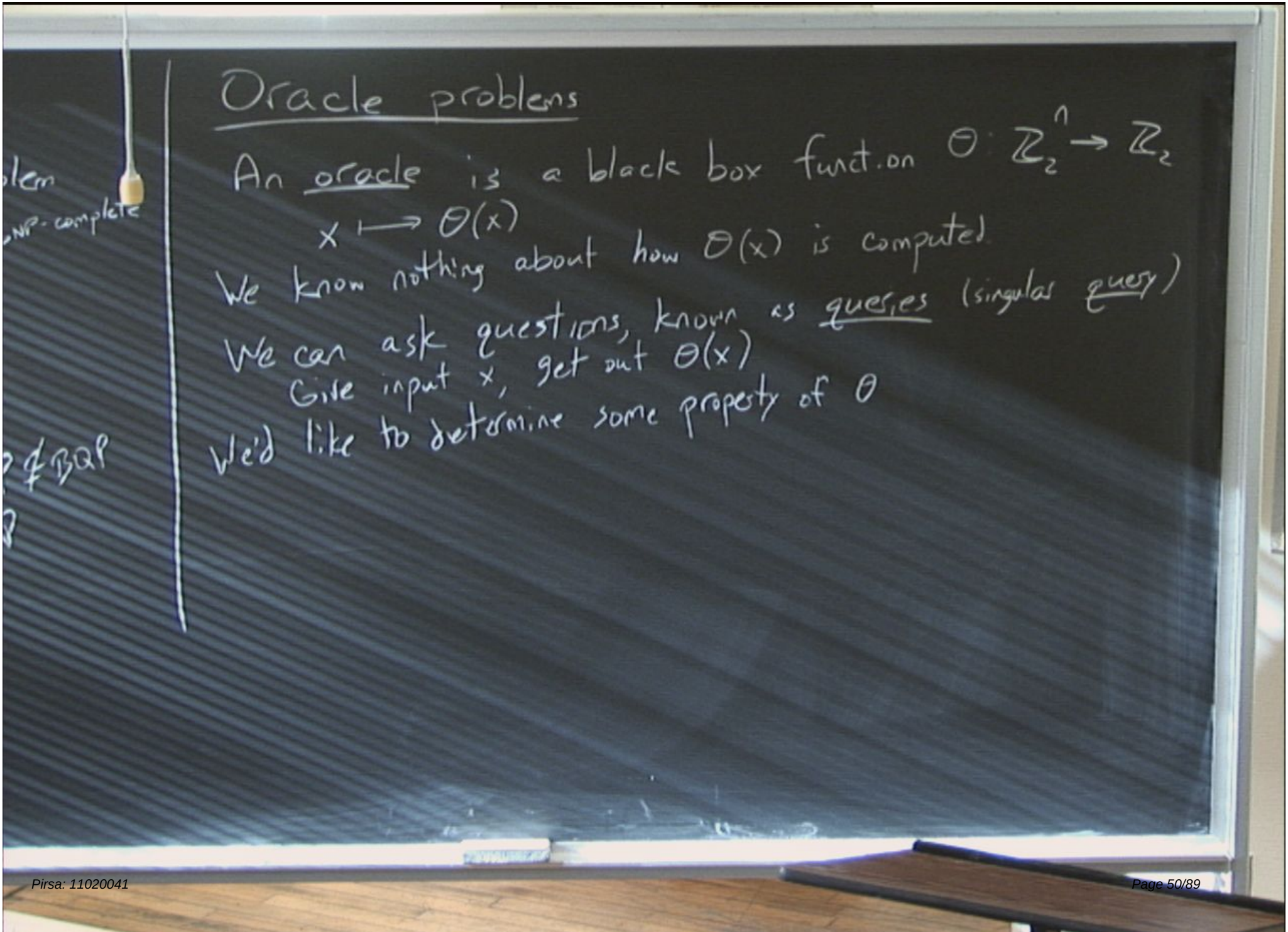
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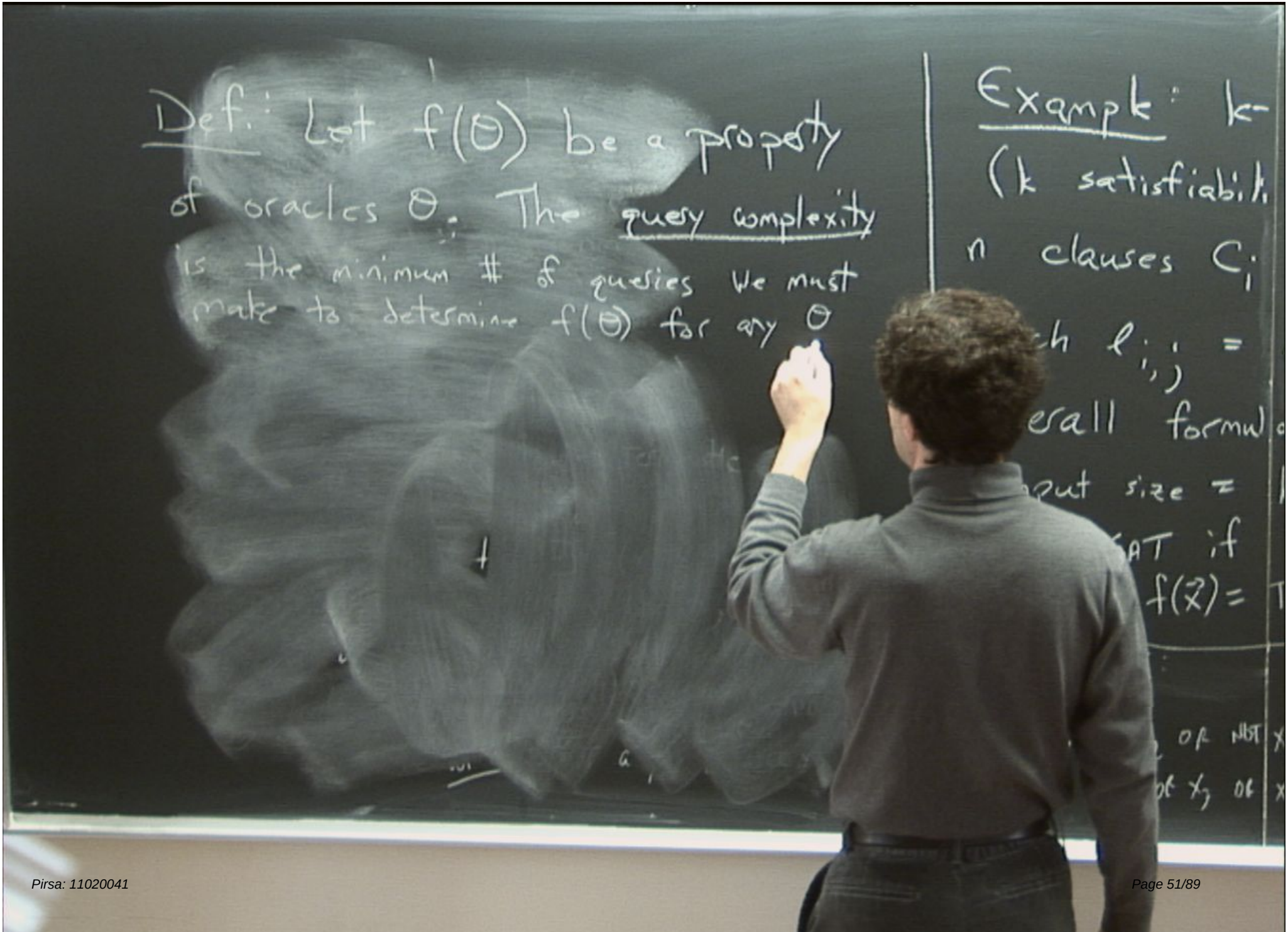
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Example: k-
(k satisfiabi
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put size =
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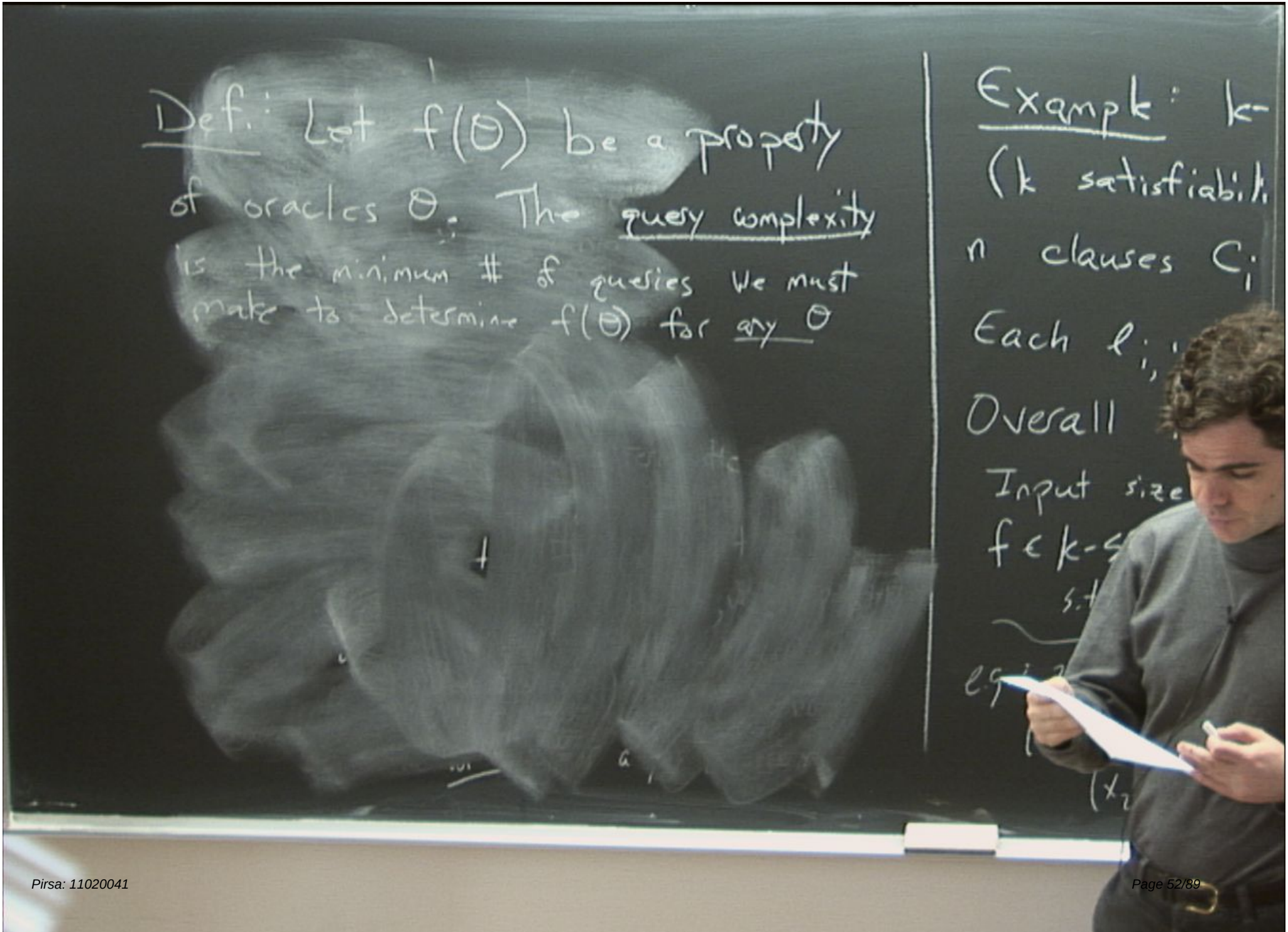
f(x) =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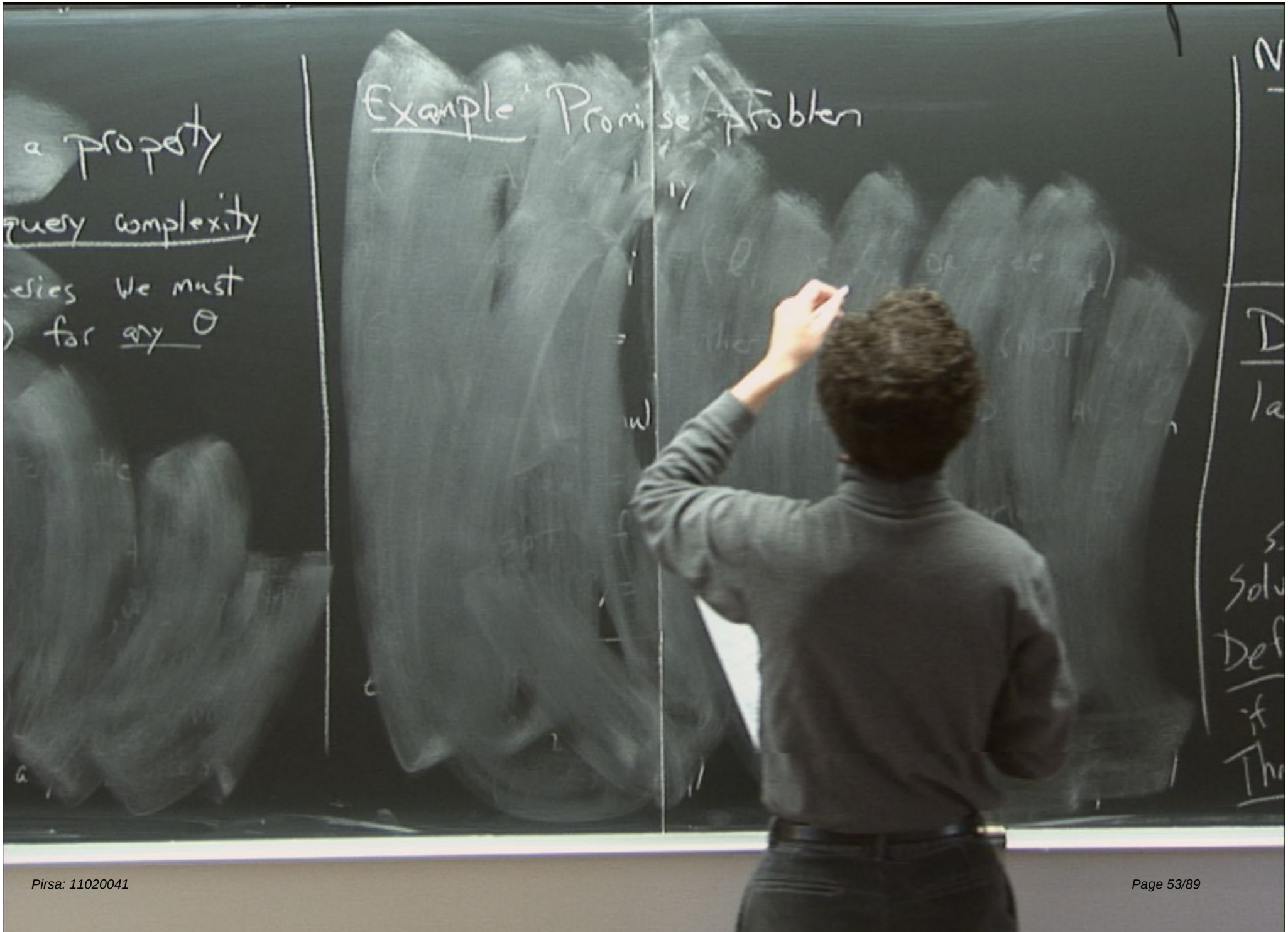
a property
query complexity
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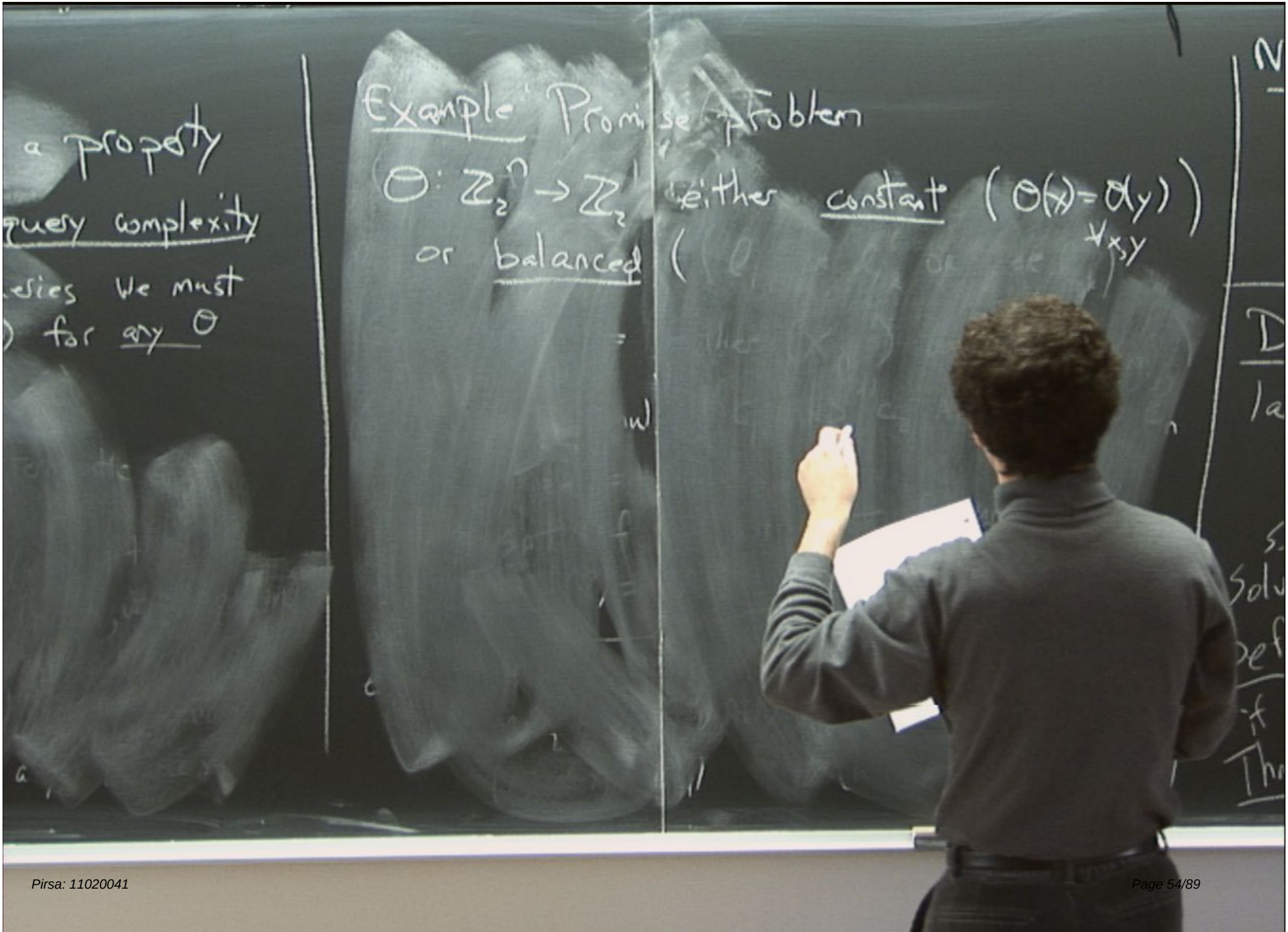
Example Promise Problem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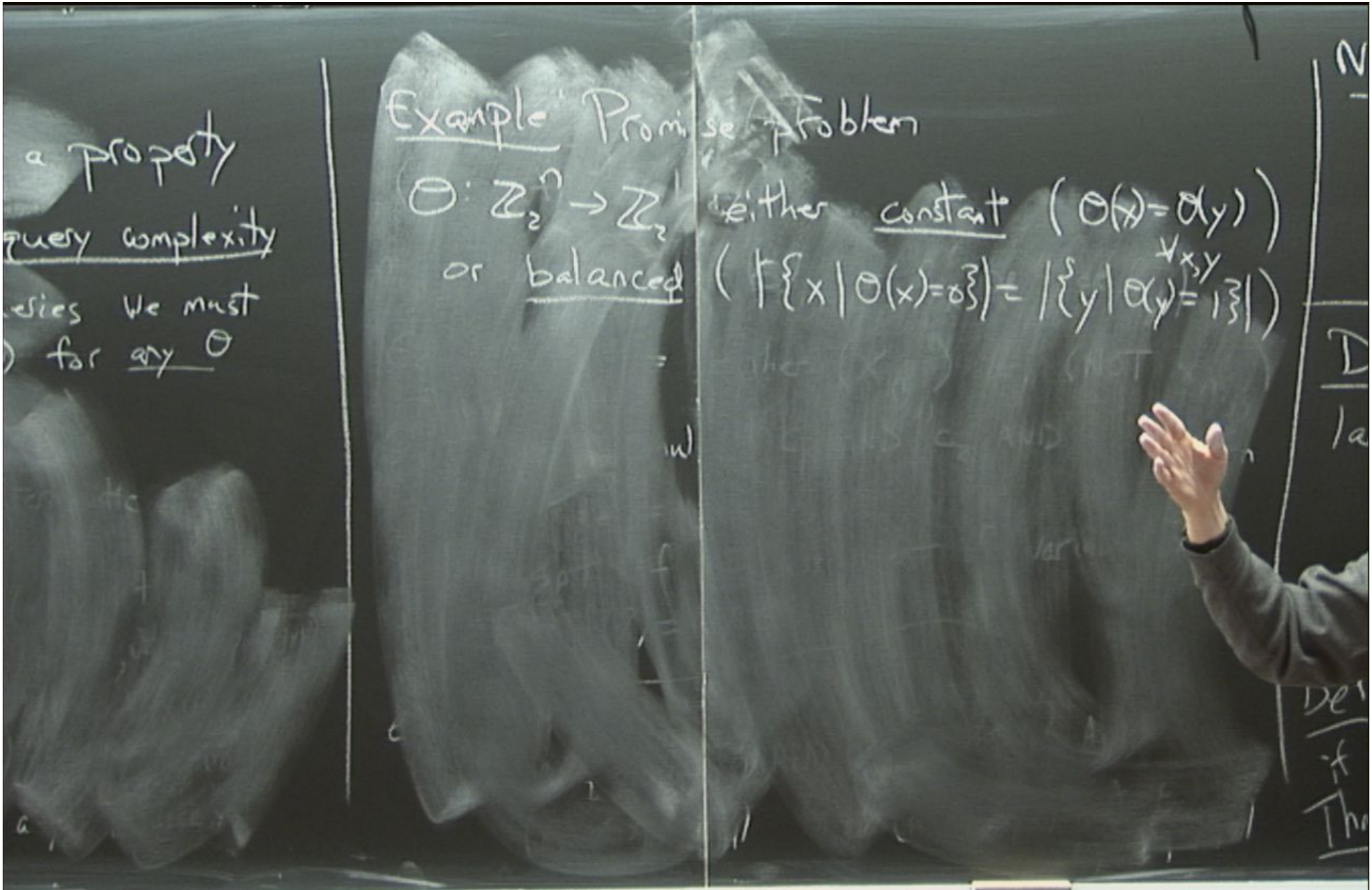
either constant
or balanced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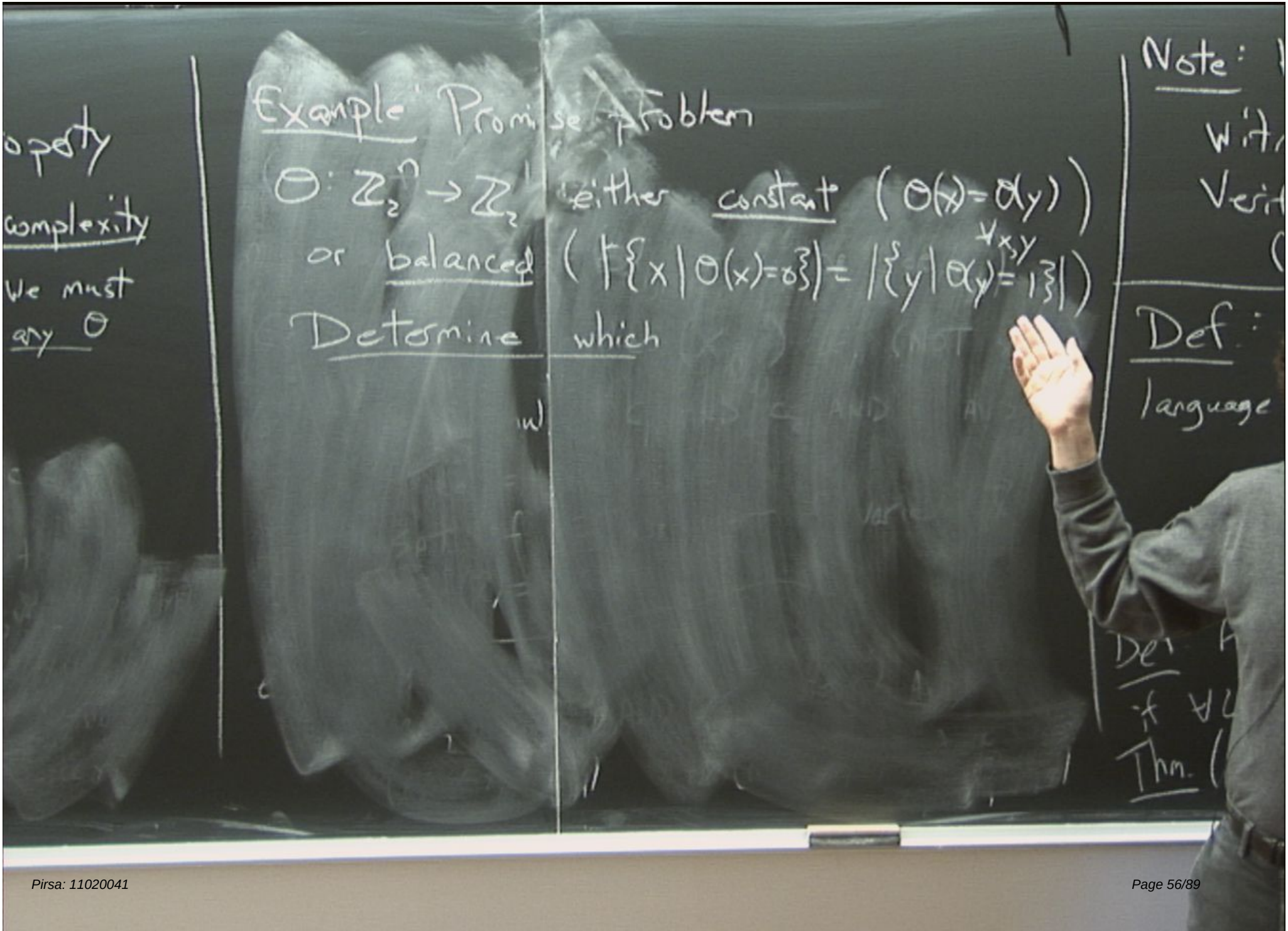
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Determine which
Note:
Def:
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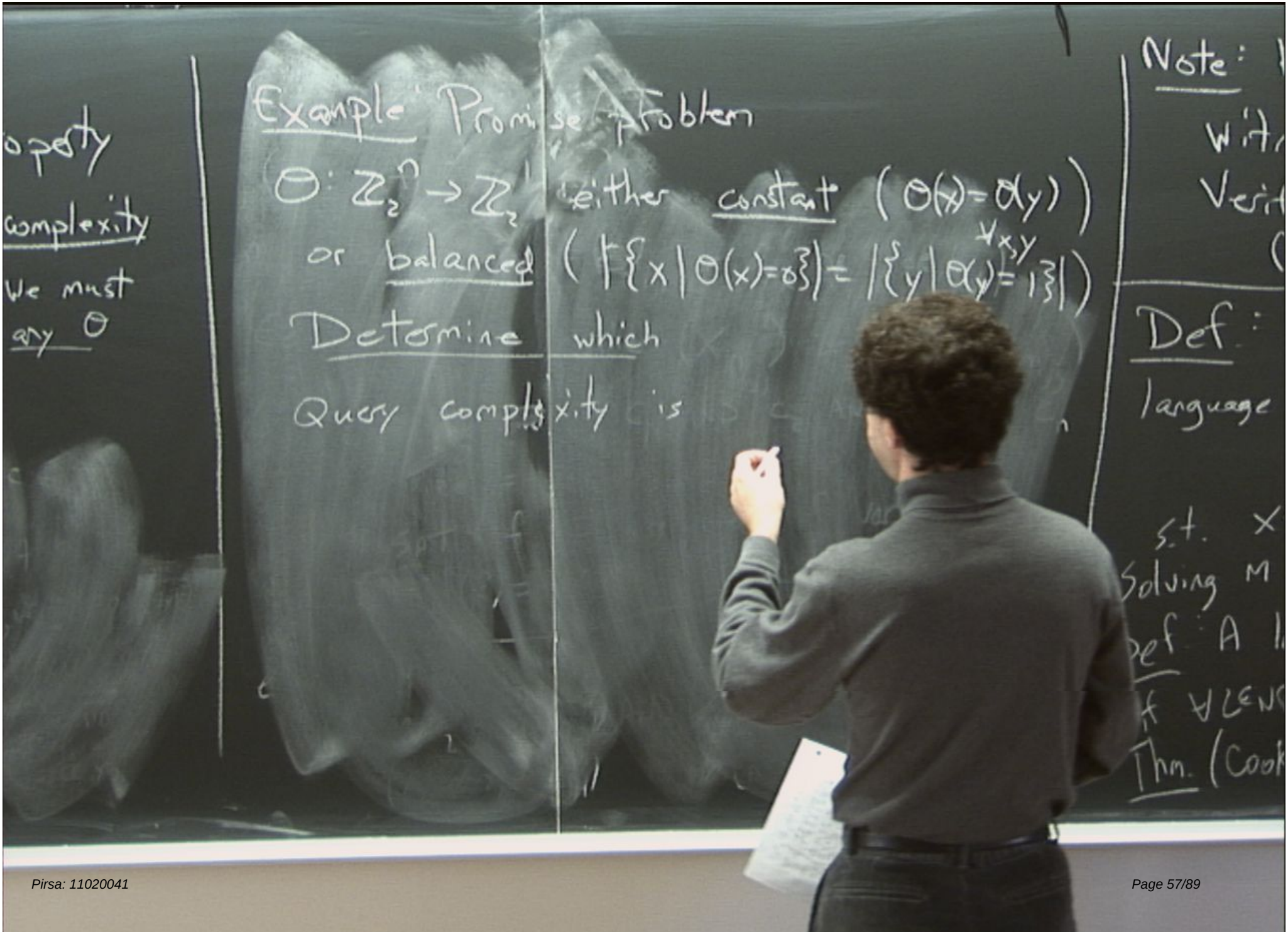
Thm.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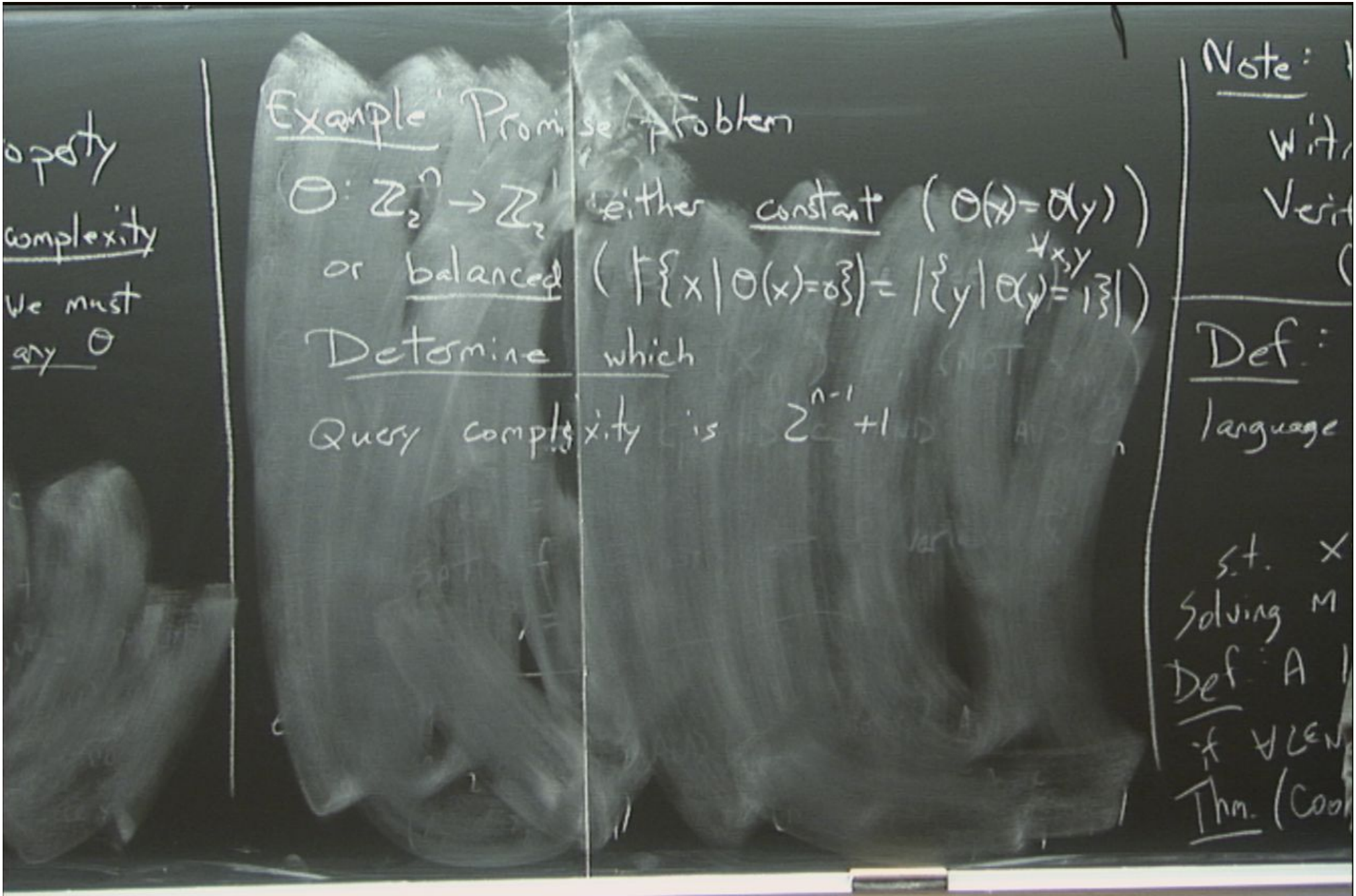
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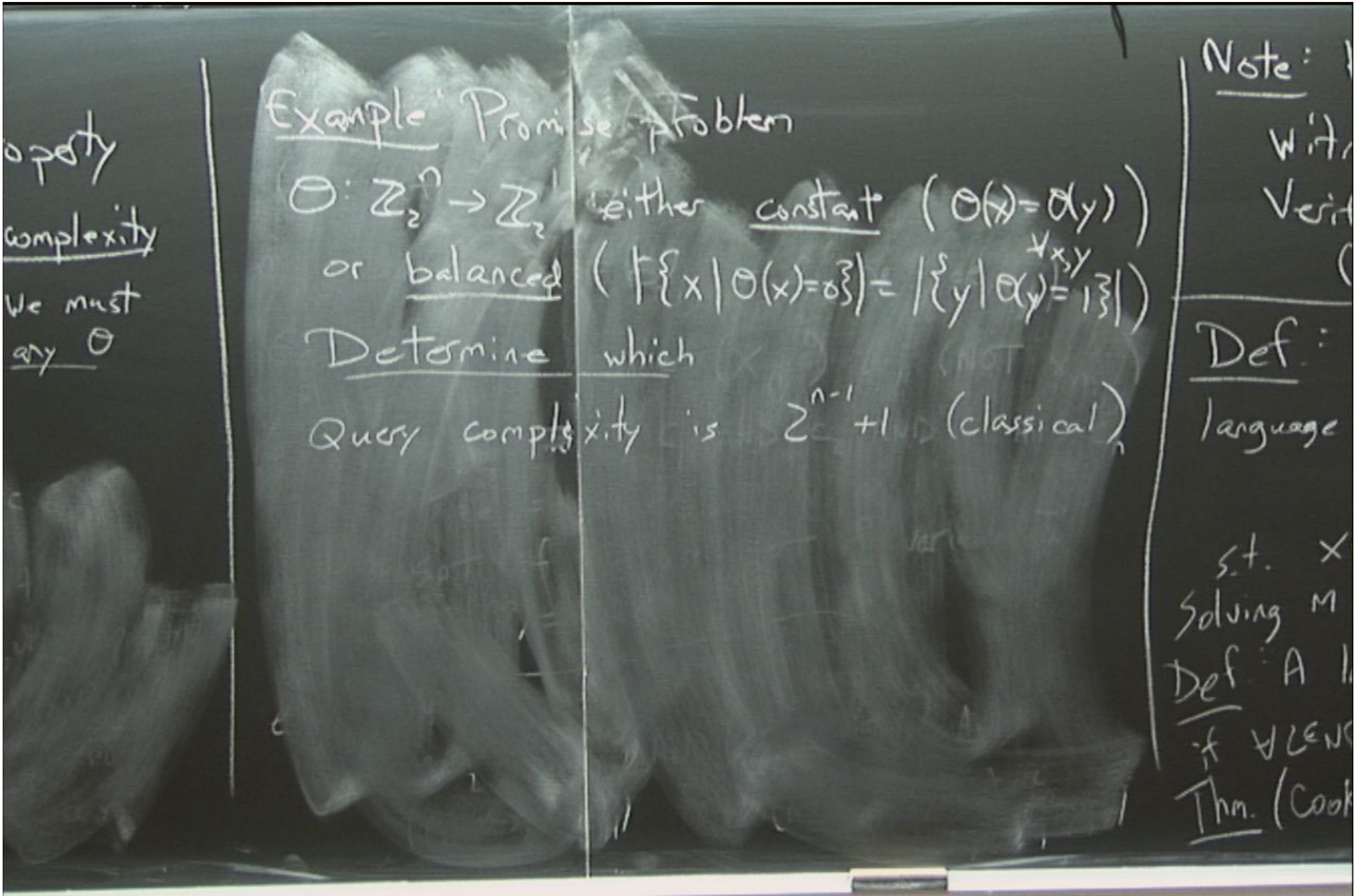
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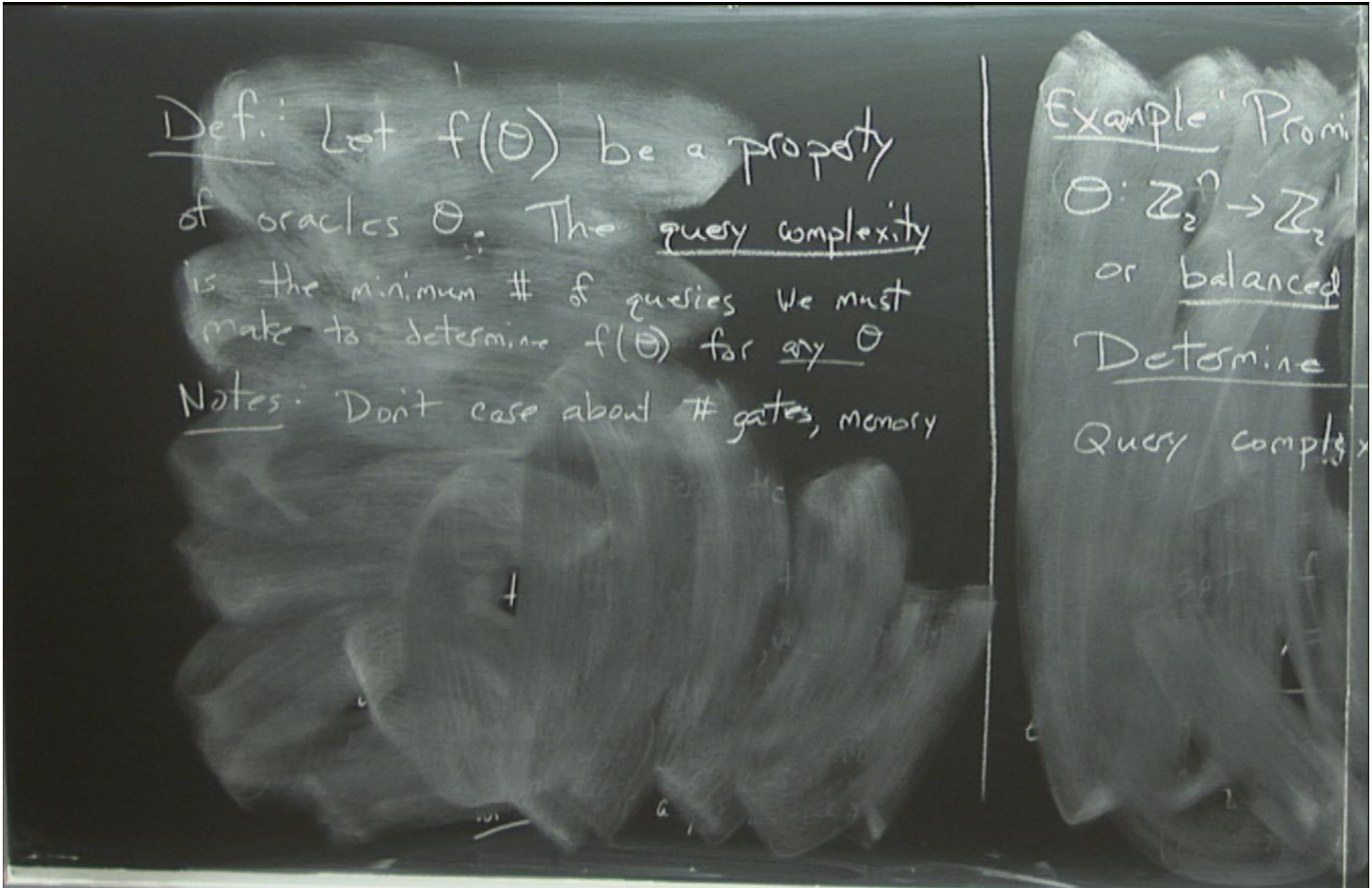
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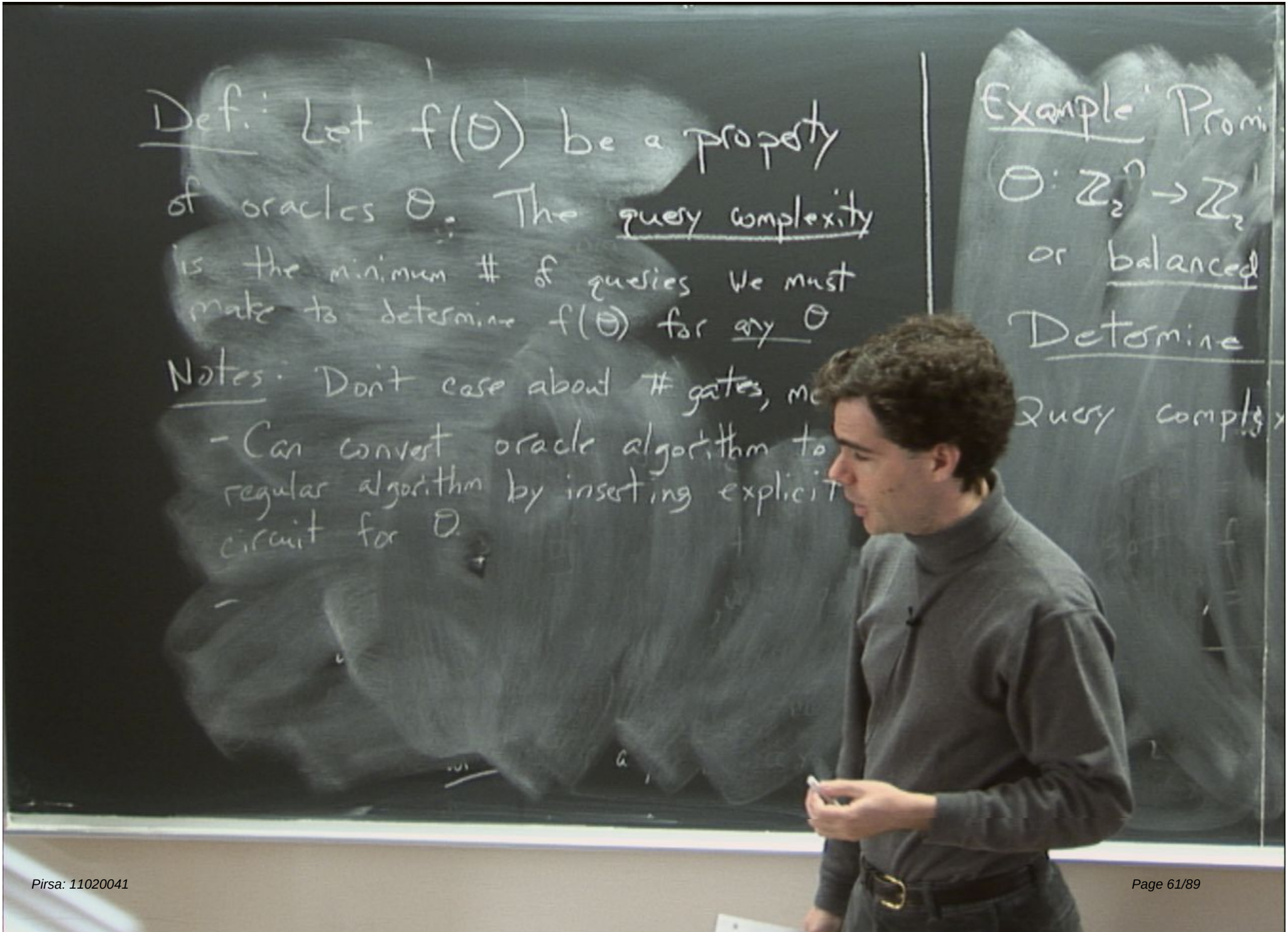
Notes: Don't care about # gates, memory
Example: Promi
Θ: Z2^n → Z2
or balanced
Determine
Query complex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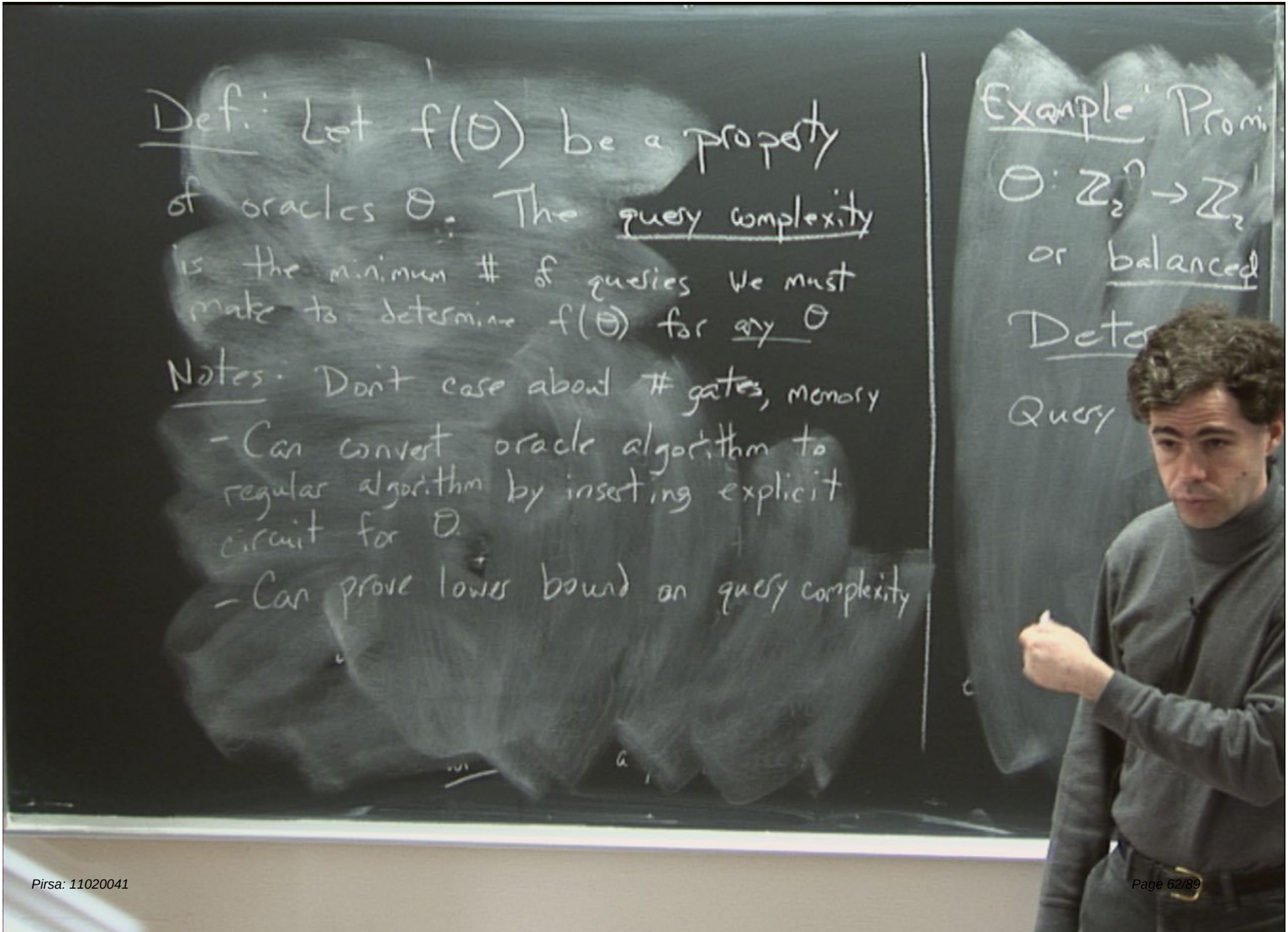
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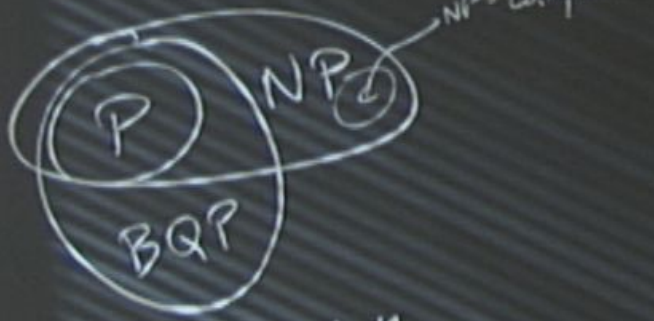
$P \neq NP$?

Famous open problem



Cannot prove
$P \neq BQP$, $NP \not\subseteq BQP$
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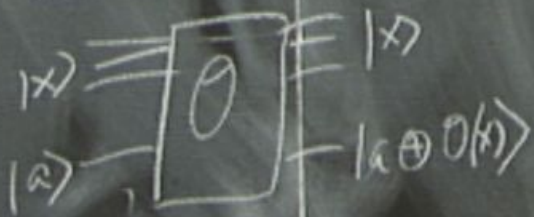
Query complexity is $2^{n-1}+1$ (classical)

For quantum oracle, make oracle reversible:



Note: k-SA

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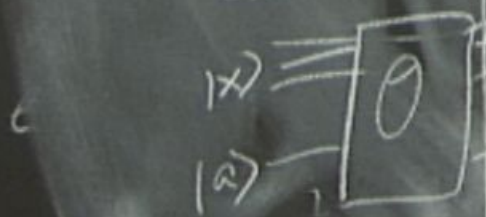

Superpositions work by linearity

$(|x\rangle + |x'\rangle)|0\rangle \to |x\rangle|O(x)\rangle + |x'\rangle|O(x')\rangle$

Note: k-SA

Witness

Verifier

(runs

Def: Lan

language M

s.t. $x \in L$

Solving M allow

Def: A langua

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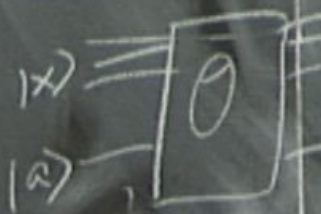

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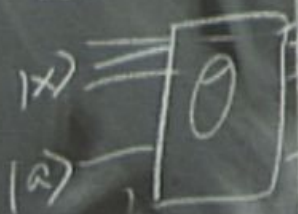

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Quantum query complexity is ≤ 2

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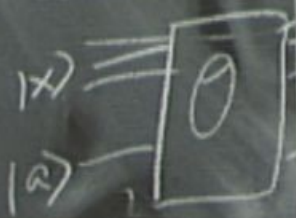

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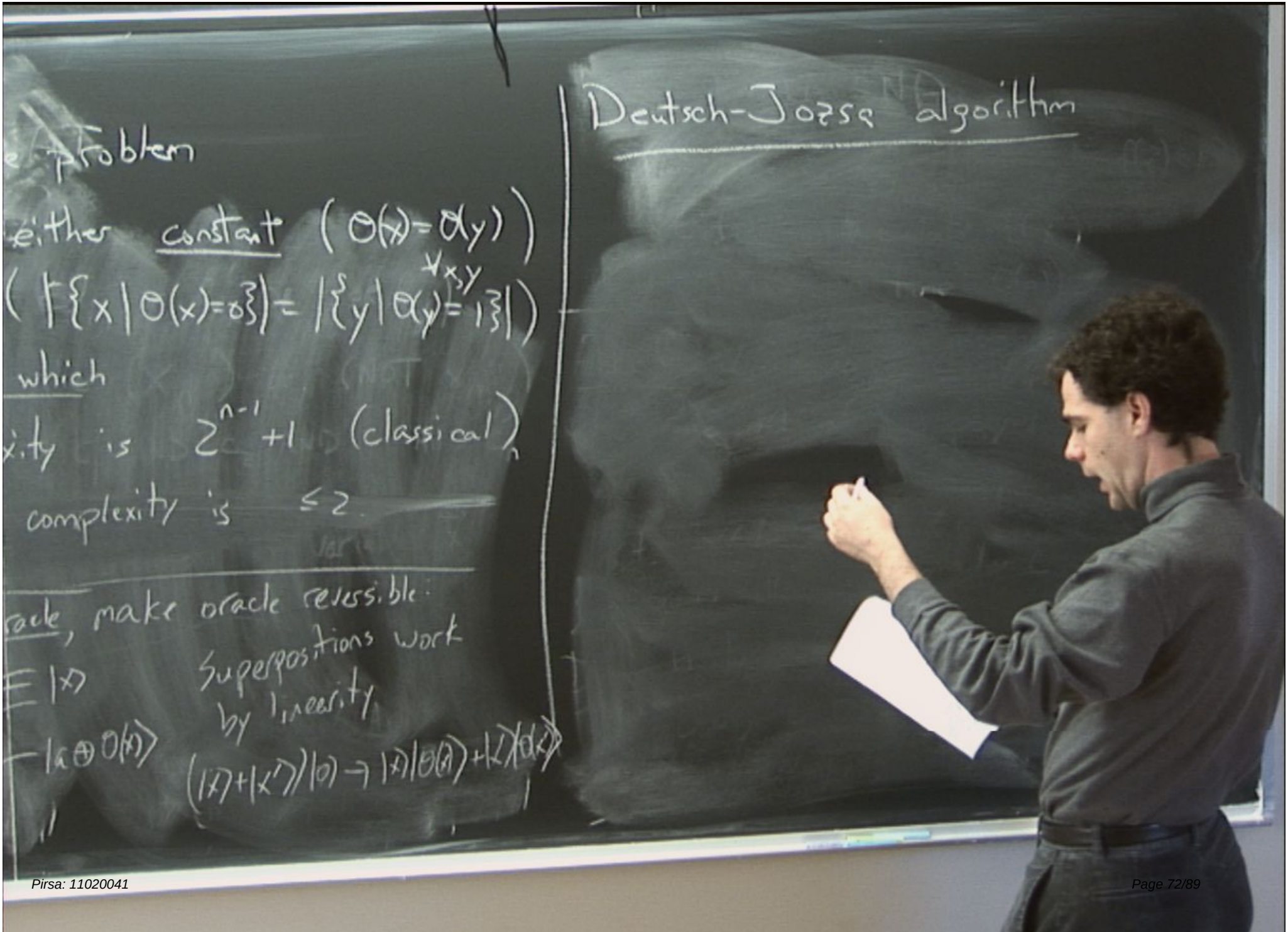
Deutsch


Deutsch-Josza algorithm
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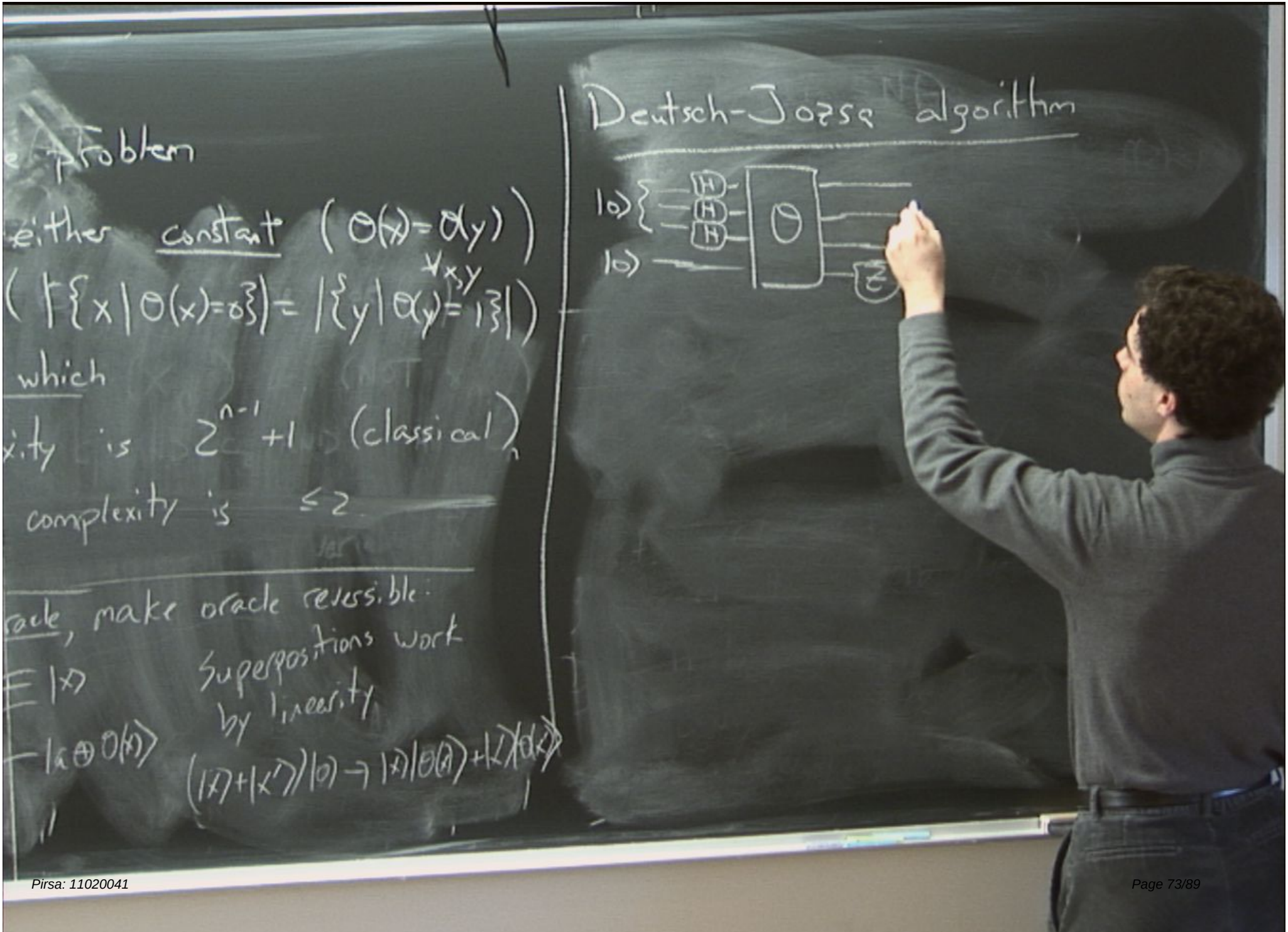
$|a \oplus f(x)\rangle$
$(|x\rangle+|x'\rangle)|0\rangle \rightarrow |x\rangle|f(x)\rangle+|x'\rangle|f(x')\rangle$


Problem
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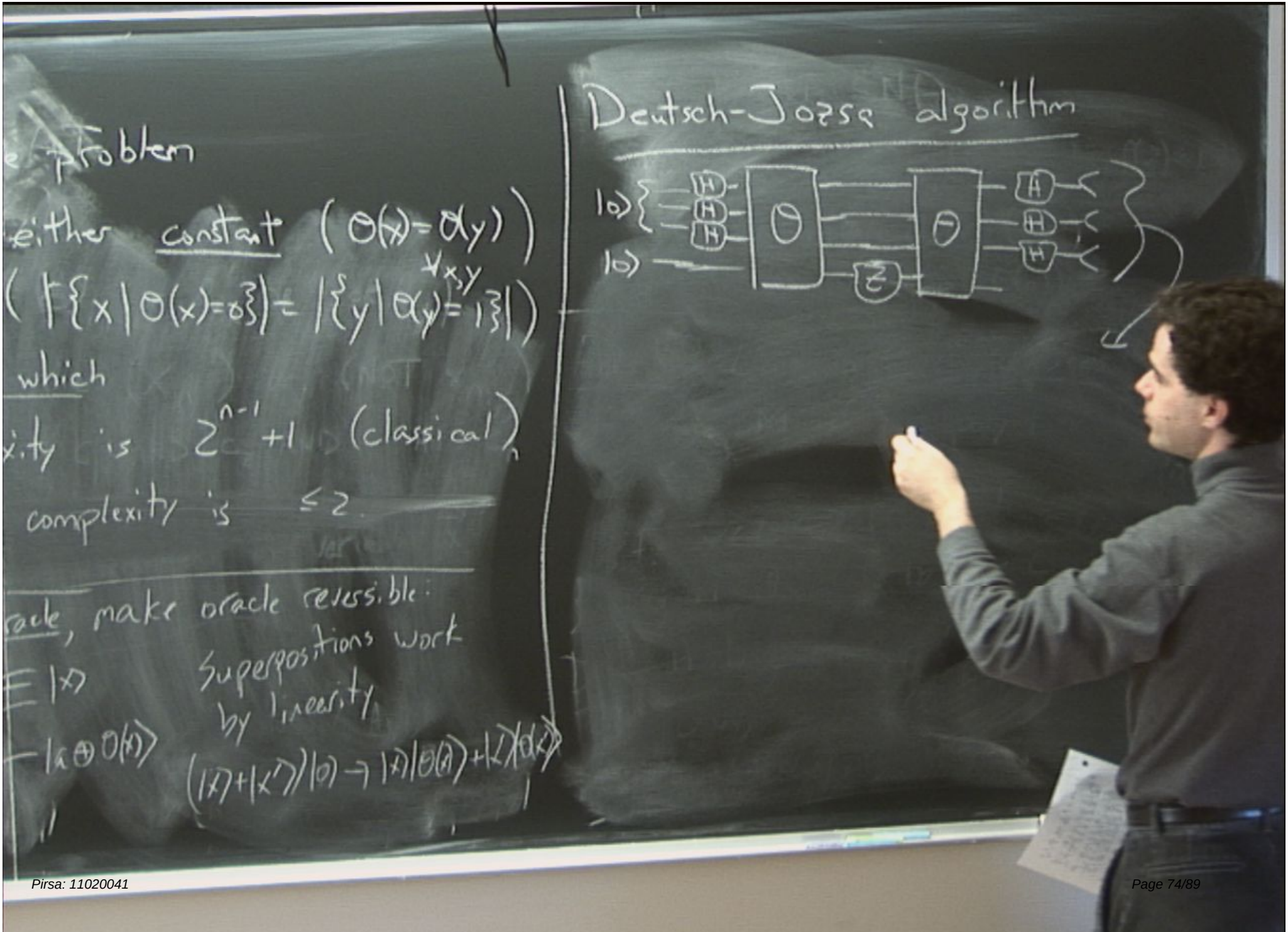
|x⟩
|a⊕O(x)⟩
(|x⟩+|x'⟩)|0⟩ → |x⟩|O(x)⟩+|x'⟩|O(x')⟩
Deutsch-Josza algorithm
|0⟩
|0⟩
H
H
H
O
H


Deutsch-Jozsa algorithm
problem
either constant
which
complexity is
complexity is
oracle, make oracle reversible:
Superpositions work
by linearity

e problem

either constant $(f(x)=f(y)\ \forall x,y)$

$(|\{x \mid f(x)=0\}| = |\{y \mid f(y)=1\}|)$

which

xity is $2^{n-1}+1$ (classical),

complexity is ≤ 2

racle, make oracle reversible:

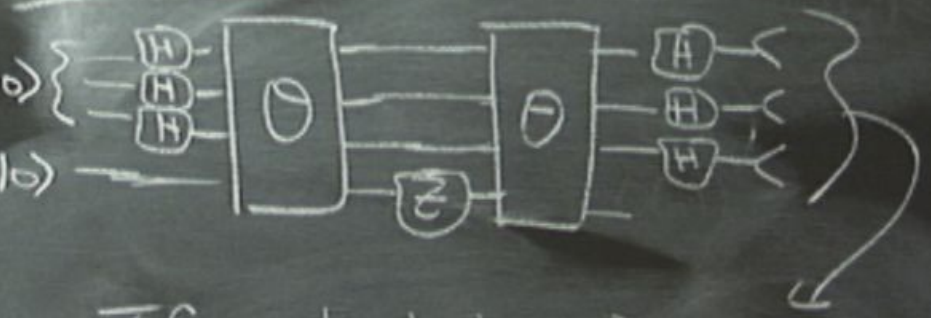
$|x\rangle$

$|a \oplus f(x)\rangle$

Superpositions work by linearity

$(|x\rangle+|x'\rangle)|0\rangle \to |x\rangle|f(x)\rangle+|x'\rangle|f(x')\rangle$

Deutsch-Jozsa algorithm



If output = |00⟩ ⟶

oblem

constant $(\forall_{x,y}\ f(x)=f(y))$

$|\{x \mid f(x)=0\}| = |\{y \mid f(y)=1\}|)$

is $2^{n-1}+1$ (classical)

lexity is ≤ 2

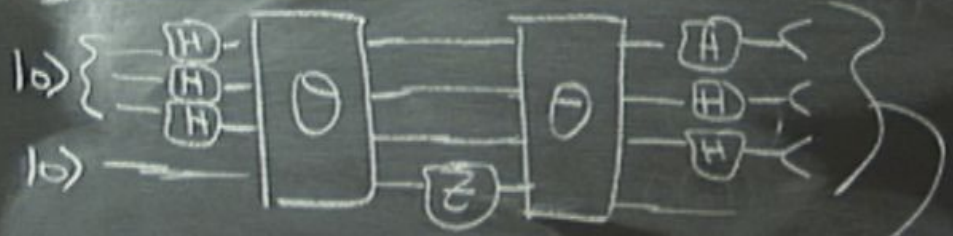
make oracle reversible:

Superpositions work by linearity

$|x\rangle$ $f(x)$

$(|x\rangle+|x'\rangle)|0\rangle \to |x\rangle|f(x)\rangle+|x'\rangle|f(x')\rangle$

Deutsch-Jozsa algorithm



If output = |00...0⟩ ⇒ constant

output ≠ |00...0⟩ ⇒ balanced.

oblem

constant $(\Theta(x)=\Theta(y))$

$\forall x,y$

$|\{x|\Theta(x)=0\}| = |\{y|\Theta(y)=1\}|)$

is $2^{n-1}+1$ (classical)

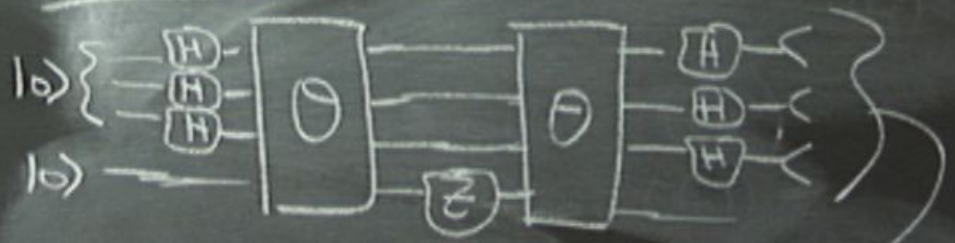
lexity is ≤ 2

make oracle reversible:

Superpositions work by linearity

$O(|x\rangle)$ $(|x\rangle+|x'\rangle)|0\rangle \rightarrow |x\rangle|\Theta(x)\rangle+|x'\rangle|\Theta(x')\rangle$

Deutsch-Jozsa algorithm



If output = $|00\ldots0\rangle$ → constant

output ≠ $|00\ldots0\rangle$ → balanced.

e problem

either constant $(O(x)=O(y)\ \forall x,y)$

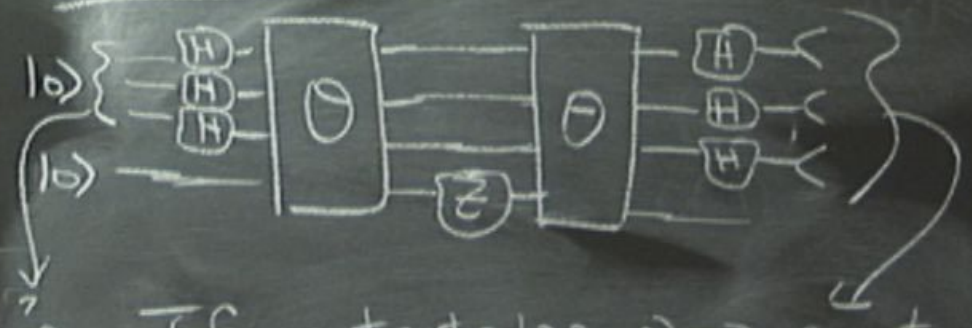
$\left(|\{x \mid O(x)=0\}| = |\{y \mid O(y)=1\}|\right)$

which

$\ldots$ity is $2^{n-1}+1$ (classical),

complexity is ≤ 2

Deutsch-Jozsa algorithm



If output = |00⟩ → constant
output ≠ |00⟩ → balanced.

scale, make oracle reversible:

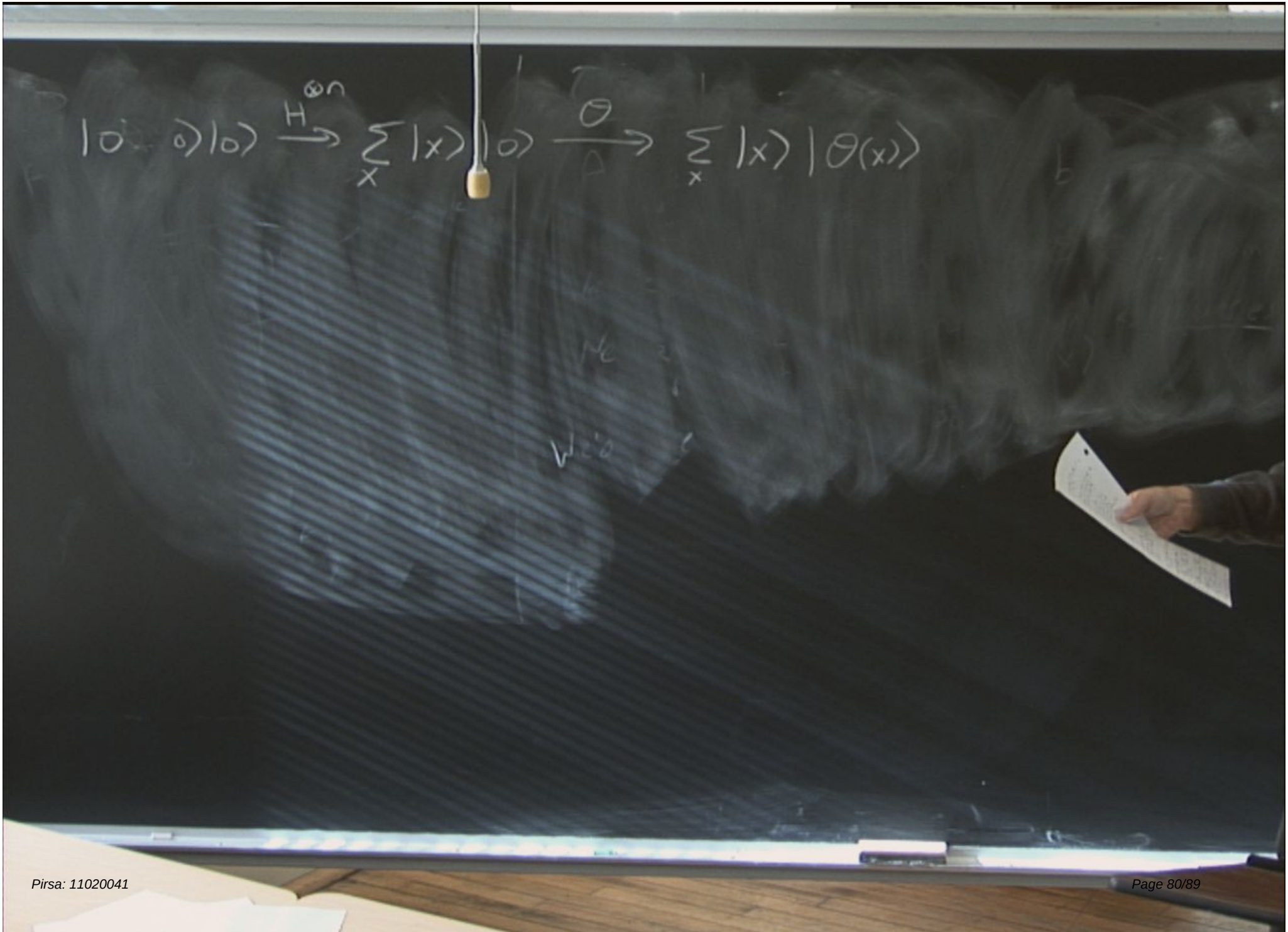
$|x\rangle$

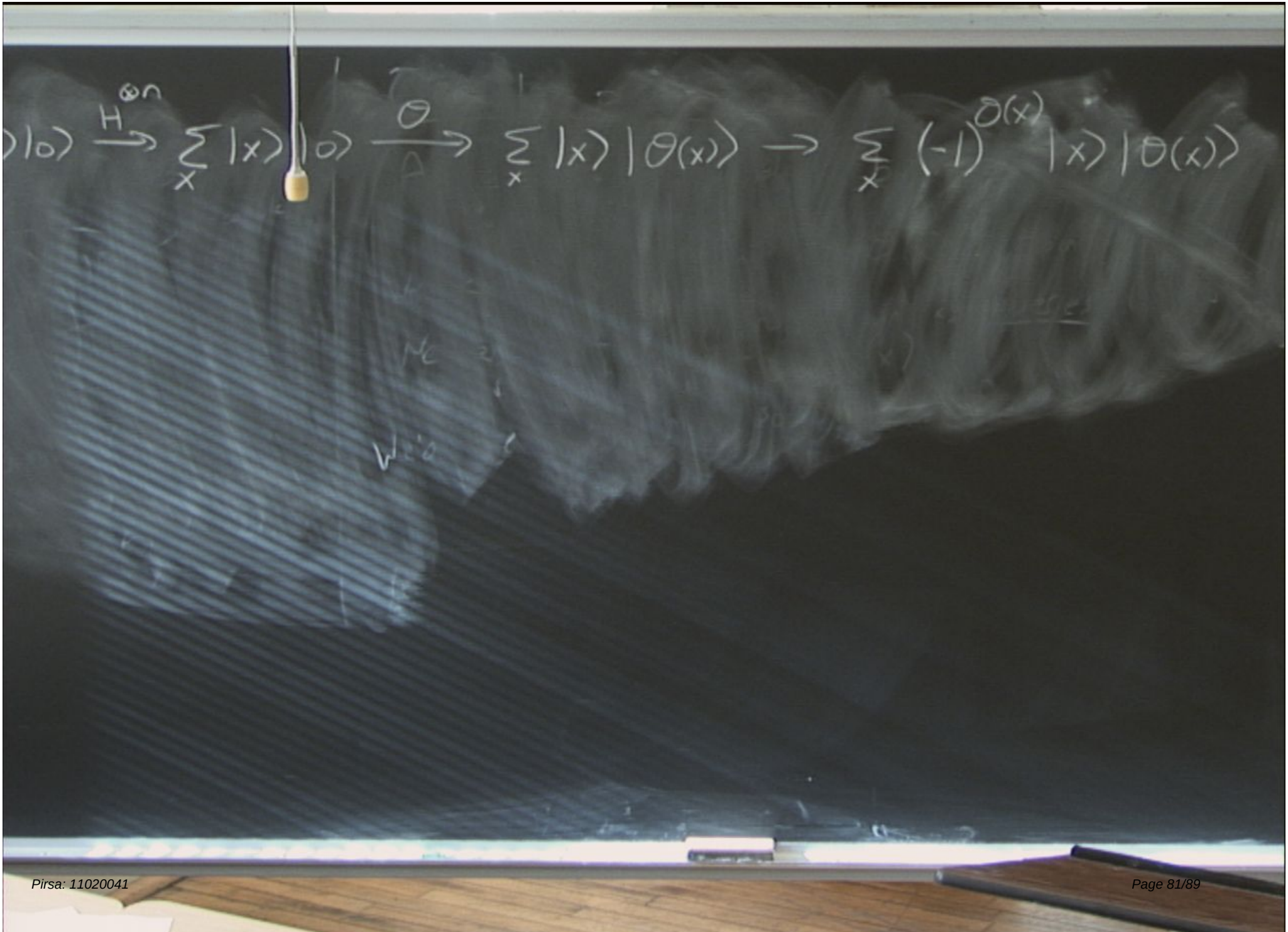
$|a \oplus O(x)\rangle$

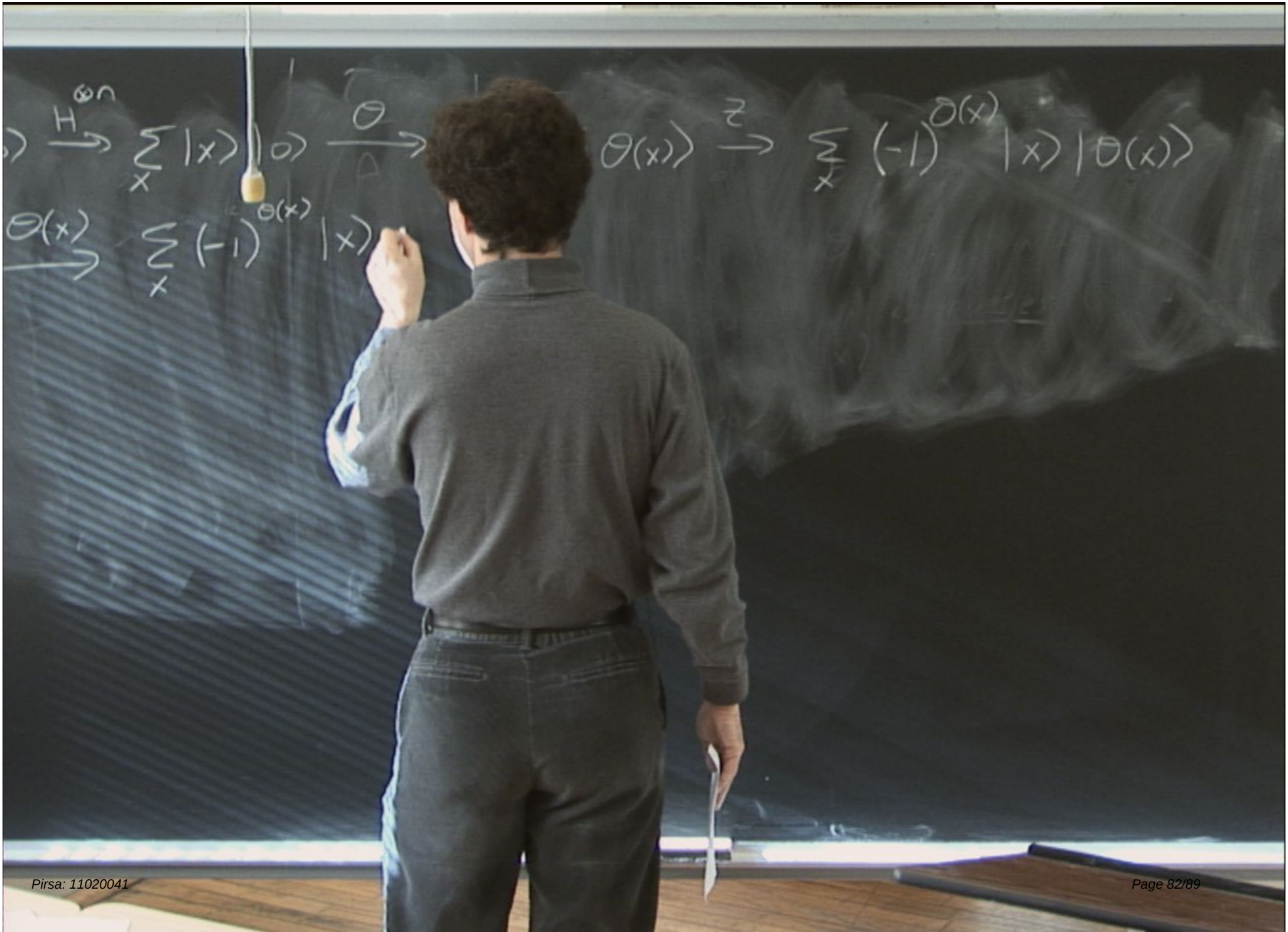
Superpositions work by linearity

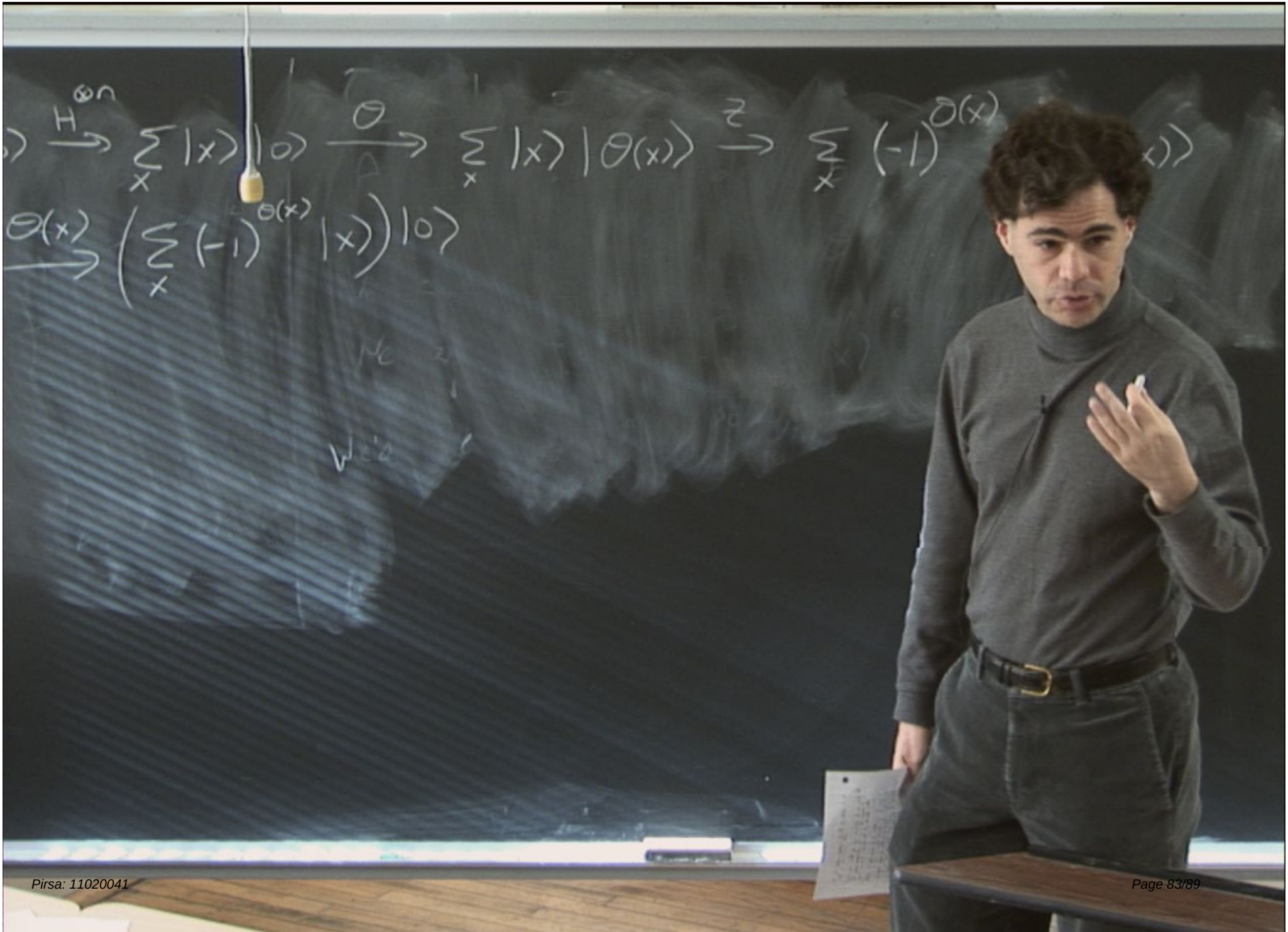
$(|x\rangle+|x'\rangle)|0\rangle \rightarrow |x\rangle|O(x)\rangle+|x'\rangle|O(x')\rangle$

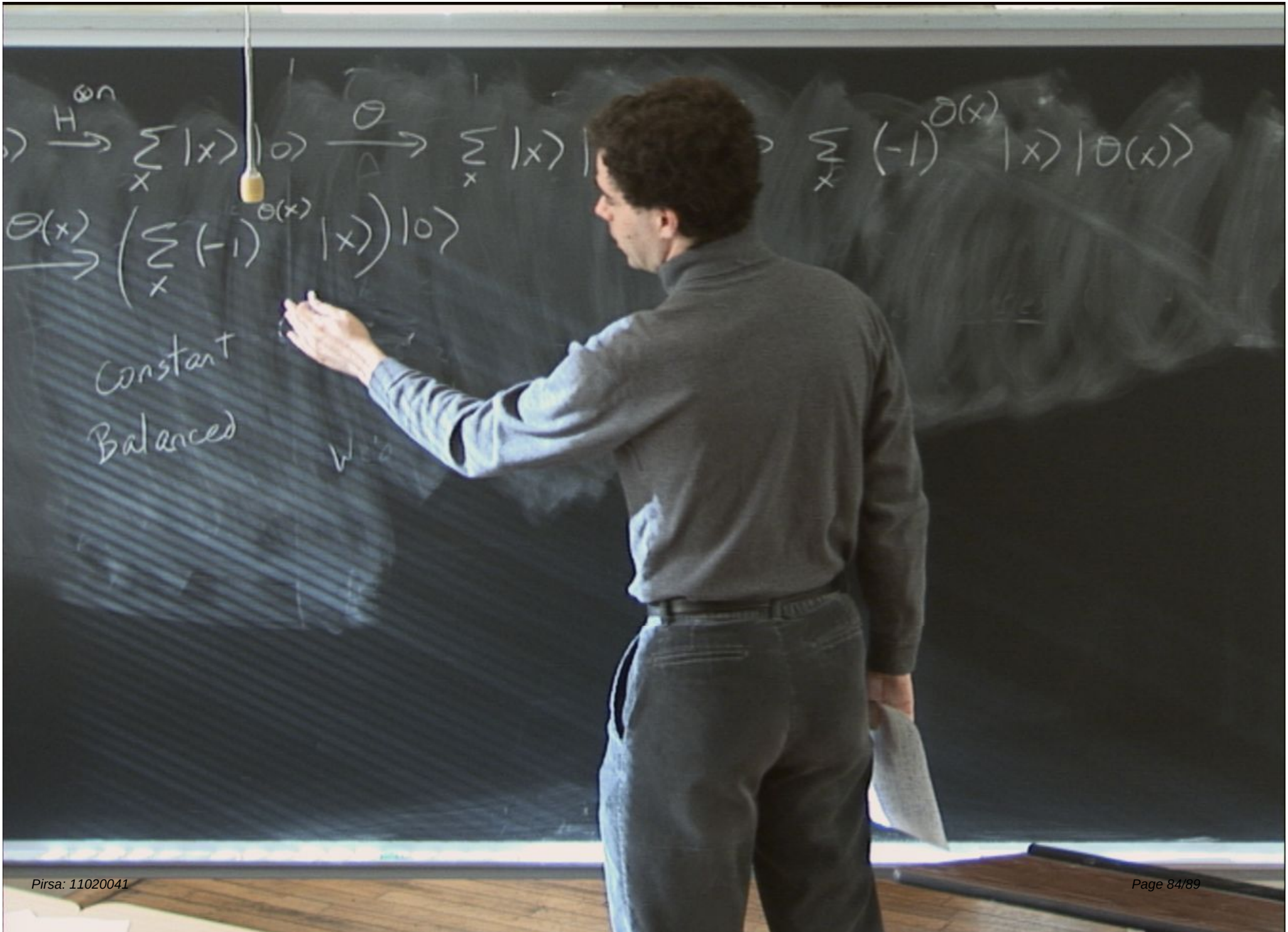

$|0\ldots 0\rangle|0\rangle \xrightarrow{H^{\otimes n}} \sum_x |x\rangle|0\rangle \xrightarrow{O}$

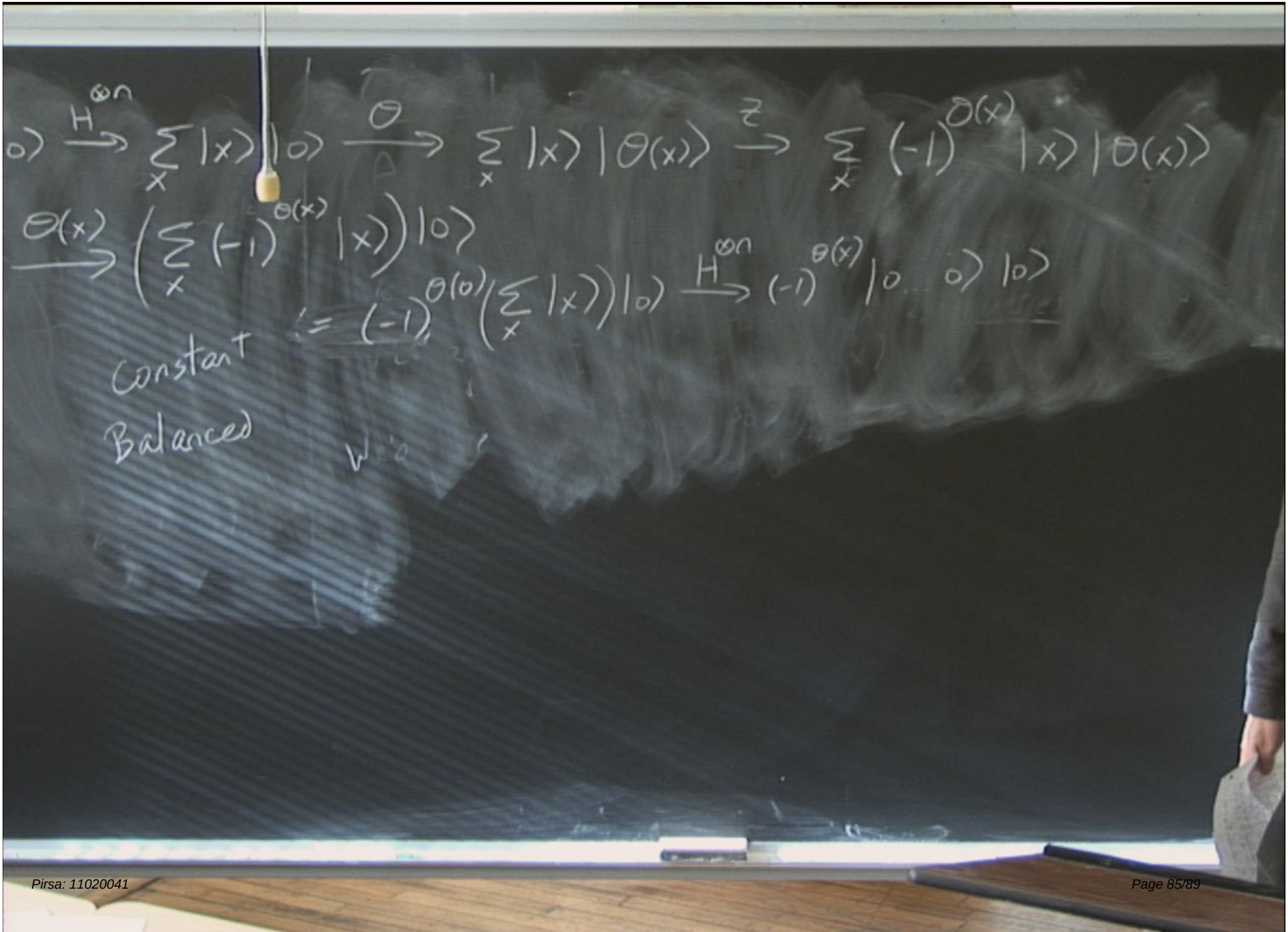

$|0\ldots 0\rangle|0\rangle \xrightarrow{H^{\otimes n}} \sum_x |x\rangle|0\rangle \xrightarrow{\theta} \sum_x |x\rangle|\theta(x)\rangle$

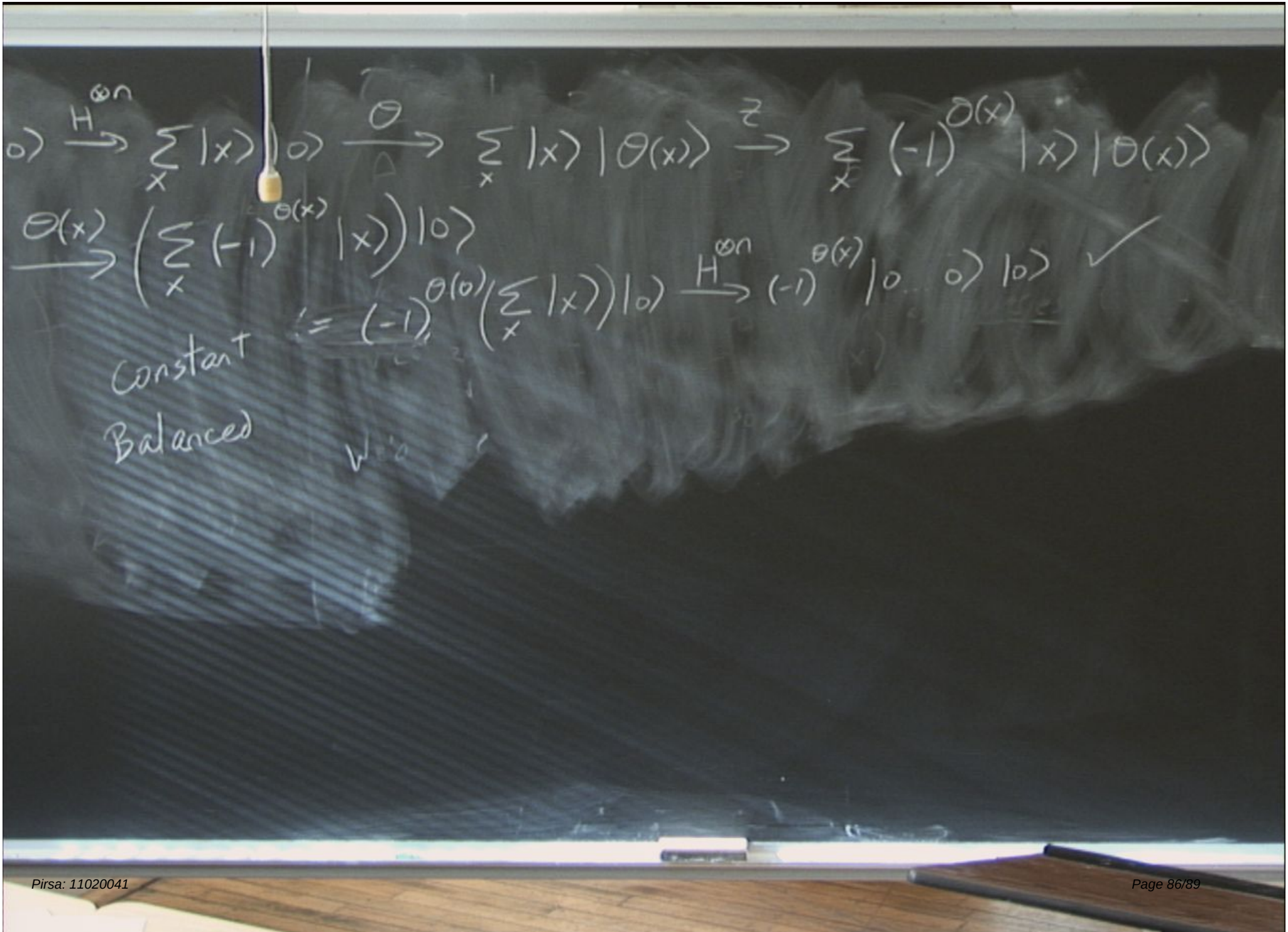

$|0\rangle \xrightarrow{H^{\otimes n}} \sum_x |x\rangle|0\rangle \xrightarrow{\Theta} \sum_x |x\rangle|\theta(x)\rangle \rightarrow \sum_x (-1)^{\theta(x)} |x\rangle|\theta(x)\rangle$

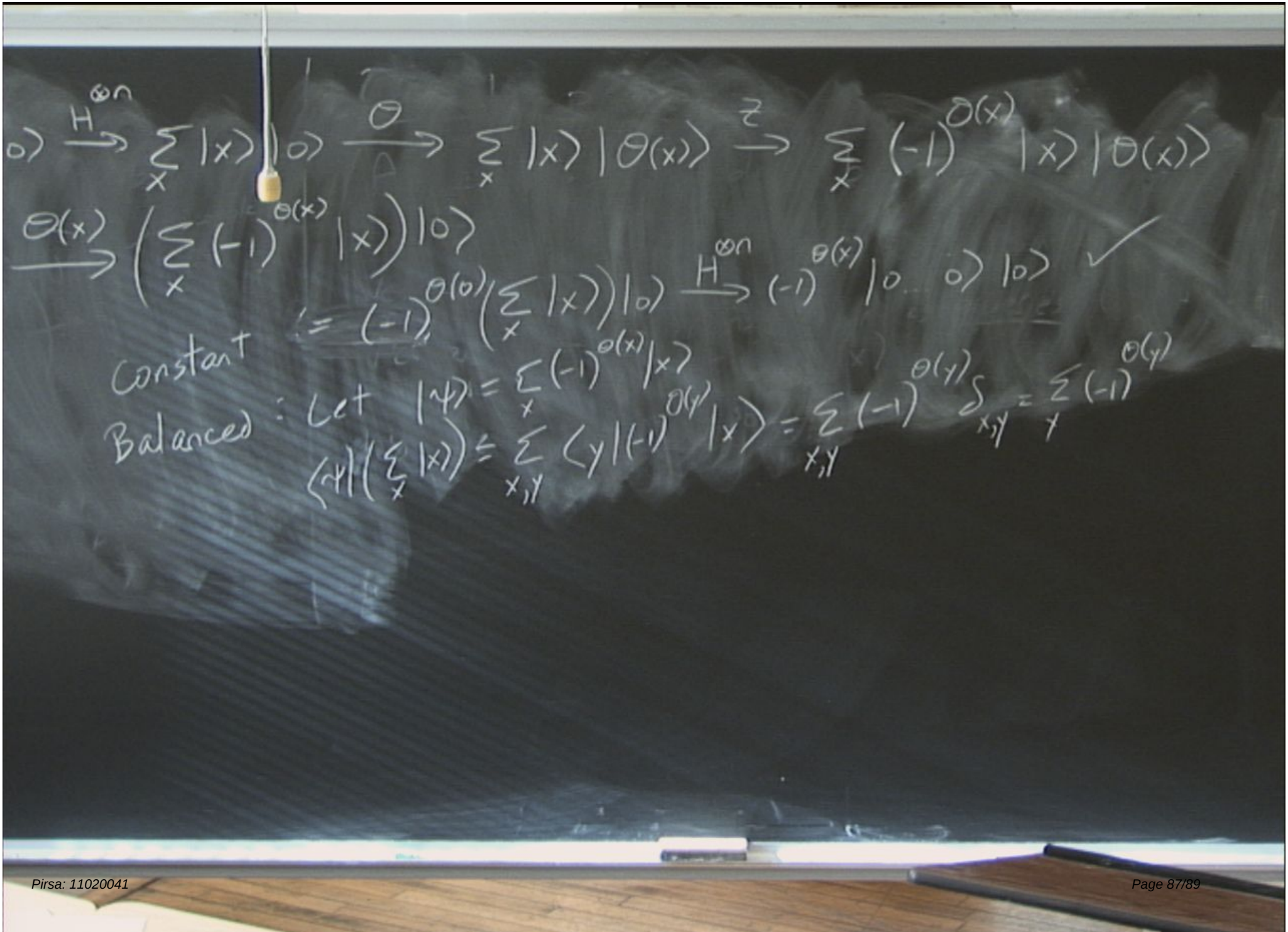

H^⊗n
Σ_x |x⟩|0⟩
O
|θ(x)⟩
Z
Σ_x (-1)^θ(x) |x⟩|θ(x)⟩
θ(x)
Σ_x (-1)^θ(x) |x⟩

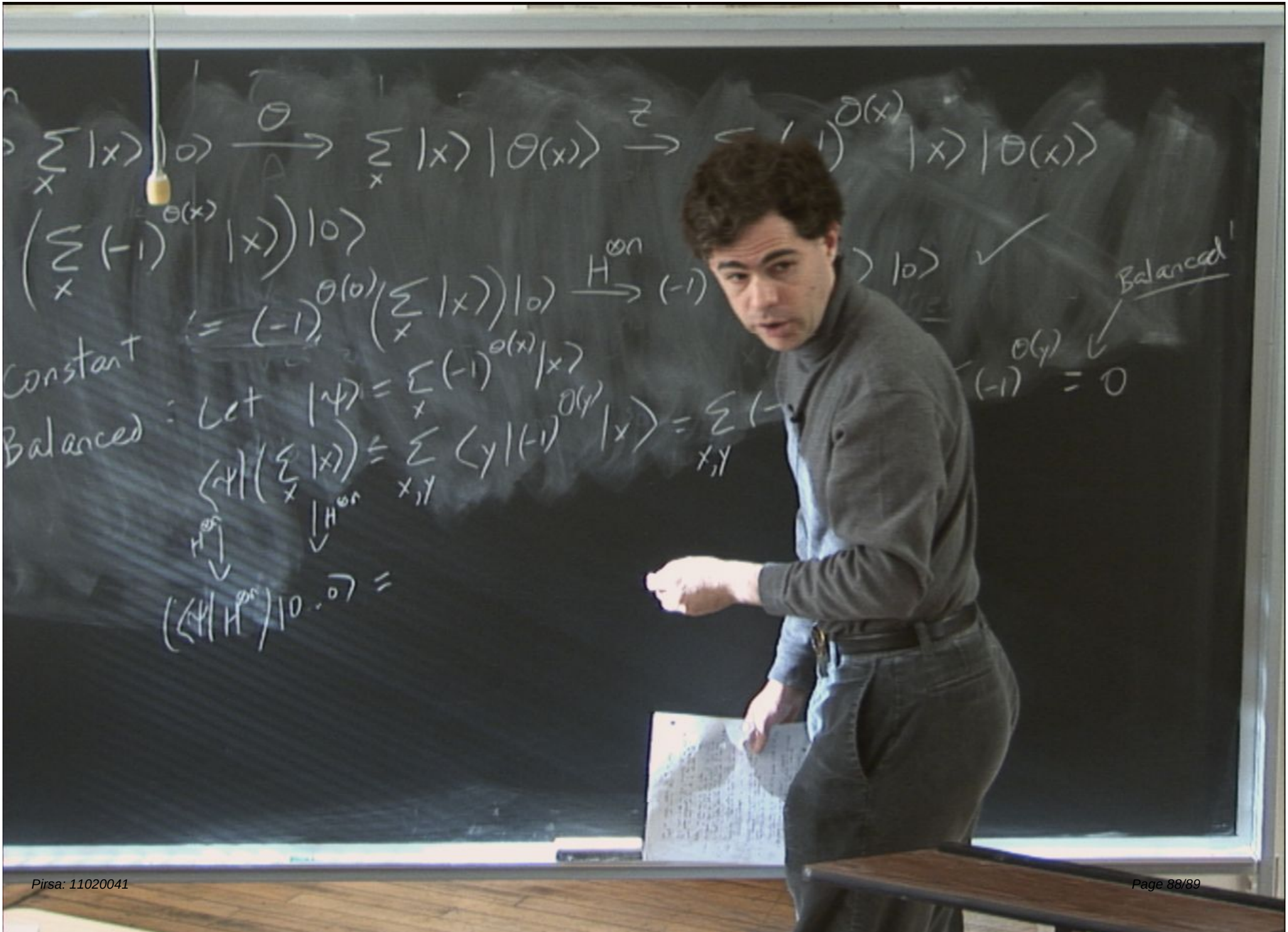

$\xrightarrow{H^{\otimes n}} \sum_x |x\rangle|0\rangle \xrightarrow{\theta} \sum_x |x\rangle|\theta(x)\rangle \xrightarrow{z} \sum_x (-1)^{\theta(x)}$
$\xrightarrow{\theta(x)} \left(\sum_x (-1)^{\theta(x)}|x\rangle\right)|0\rangle$

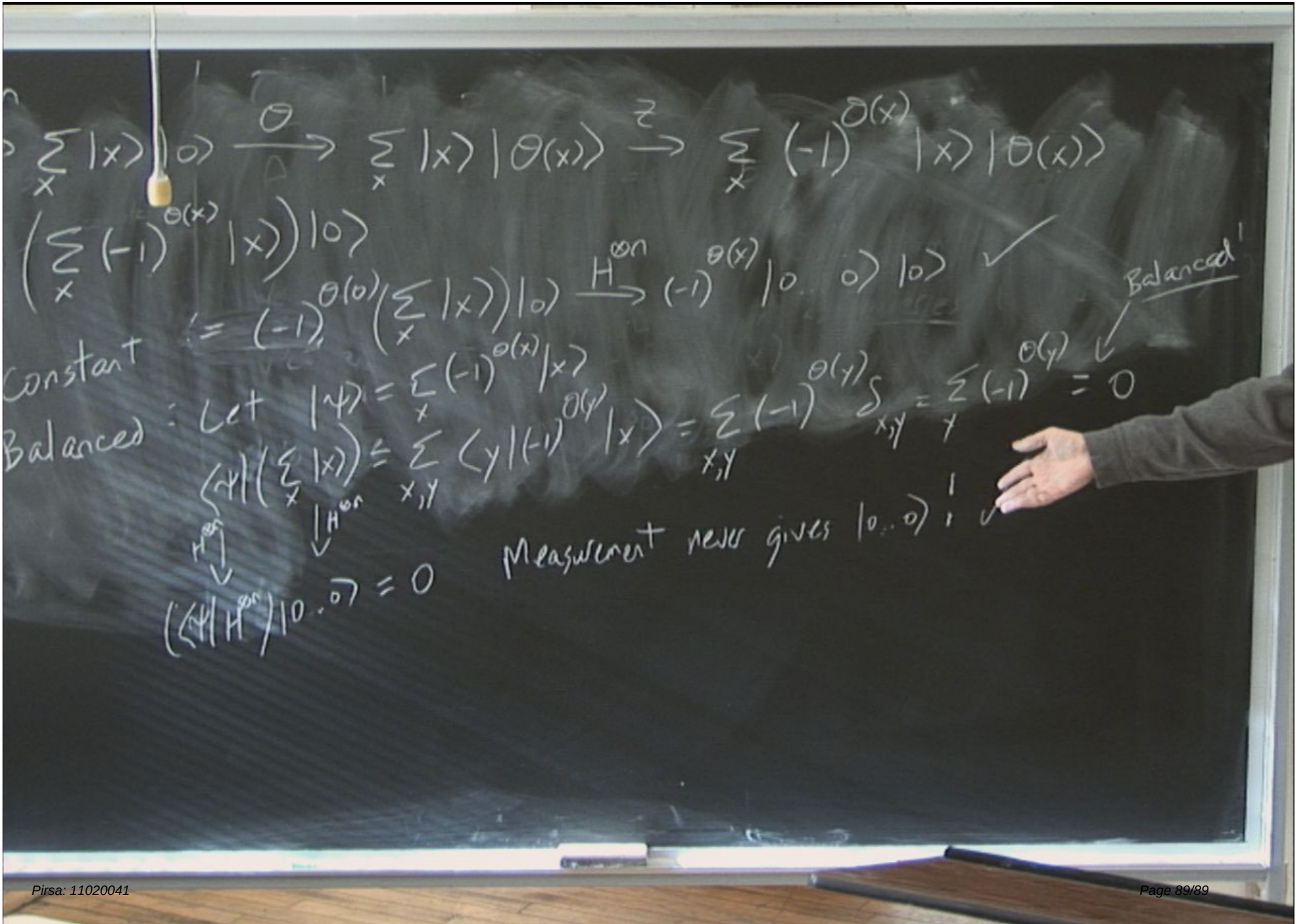

Constant
Balanced


$|0\rangle \xrightarrow{H^{\otimes n}} \sum_x |x\rangle|0\rangle \xrightarrow{\theta} \sum_x |x\rangle|\theta(x)\rangle \xrightarrow{Z} \sum_x (-1)^{\theta(x)}|x\rangle|\theta(x)\rangle$
$\xrightarrow{\theta(x)} \left(\sum_x (-1)^{\theta(x)}|x\rangle\right)|0\rangle$
$= (-1)^{\theta(0)}\left(\sum_x |x\rangle\right)|0\rangle \xrightarrow{H^{\otimes n}} (-1)^{\theta(x)}|0\ldots0\rangle|0\rangle$
Constant
Balanced


$|0\rangle \xrightarrow{H^{\otimes n}} \sum_x |x\rangle |0\rangle \xrightarrow{\theta} \sum_x |x\rangle |\theta(x)\rangle \xrightarrow{Z} \sum_x (-1)^{\theta(x)} |x\rangle |\theta(x)\rangle$
$\xrightarrow{\theta(x)} \left(\sum_x (-1)^{\theta(x)} |x\rangle\right) |0\rangle$
$= (-1)^{\theta(0)} \left(\sum_x |x\rangle\right) |0\rangle \xrightarrow{H^{\otimes n}} (-1)^{\theta(x)} |0 \ldots 0\rangle |0\rangle$ ✓
Constant
Balanced


$|0\rangle \xrightarrow{H^{\otimes n}} \sum_x |x\rangle|0\rangle \xrightarrow{\theta} \sum_x |x\rangle|\theta(x)\rangle \xrightarrow{Z} \sum_x (-1)^{\theta(x)}|x\rangle|\theta(x)\rangle$
$\xrightarrow{\theta(x)} \left(\sum_x (-1)^{\theta(x)}|x\rangle\right)|0\rangle$
$= (-1)^{\theta(0)}\left(\sum_x |x\rangle\right)|0\rangle \xrightarrow{H^{\otimes n}} (-1)^{\theta(x)}|0\ldots 0\rangle|0\rangle$ ✓
Constant
Balanced: Let $|\psi\rangle = \sum_x (-1)^{\theta(x)}|x\rangle$
$\langle\psi|\left(\sum_x |x\rangle\right) = \sum_{x,y}\langle y|(-1)^{\theta(y)}|x\rangle = \sum_{x,y}(-1)^{\theta(y)}\delta_{x,y} = \sum_y (-1)^{\theta(y)}$


Constant
Balanced
Balanced!


$\sum_x |x\rangle|0\rangle \xrightarrow{\Theta} \sum_x |x\rangle|\Theta(x)\rangle \xrightarrow{Z} \sum_x (-1)^{\Theta(x)}|x\rangle|\Theta(x)\rangle$
$\left(\sum_x (-1)^{\Theta(x)}|x\rangle\right)|0\rangle$
$= (-1)^{\Theta(0)}\left(\sum_x |x\rangle\right)|0\rangle \xrightarrow{H^{\otimes n}} (-1)^{\Theta(x)}|0\ldots0\rangle|0\rangle$ ✓
Balanced!
Constant
Balanced: Let $|\psi\rangle = \sum_x (-1)^{\Theta(x)}|x\rangle$
$\langle\psi|\left(\sum_x |x\rangle\right) = \sum_{x,y}\langle y|(-1)^{\Theta(y)}|x\rangle = \sum_{x,y}(-1)^{\Theta(y)}\delta_{x,y} = \sum_y (-1)^{\Theta(y)} = 0$
$H^{\otimes n}$ $H^{\otimes n}$
$(\langle\psi|H^{\otimes n})|0\ldots0\rangle = 0$
Measurement never gives $|0\ldots0\rangle$; !