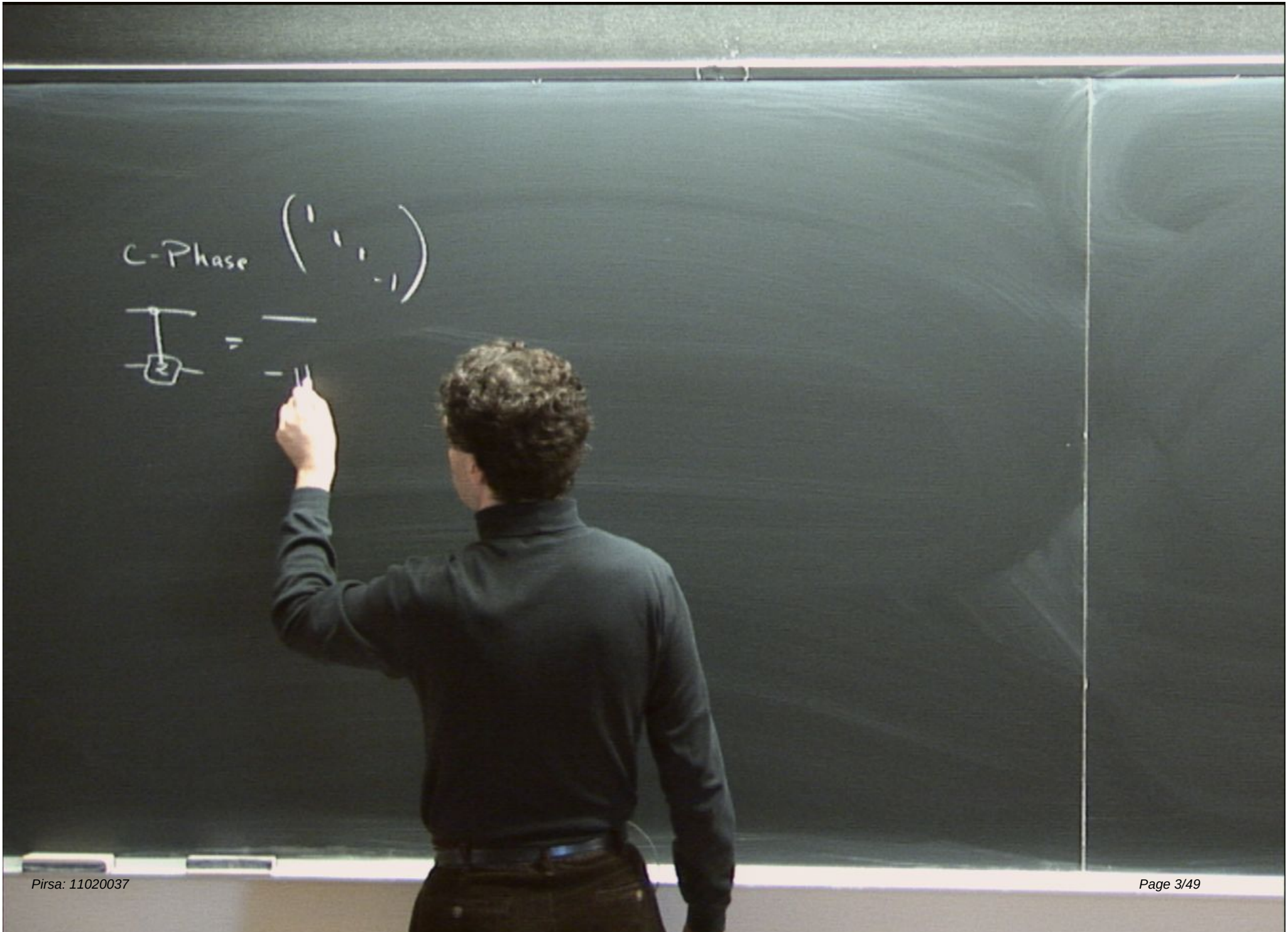
Title: Quantum Information Review - Lecture 5

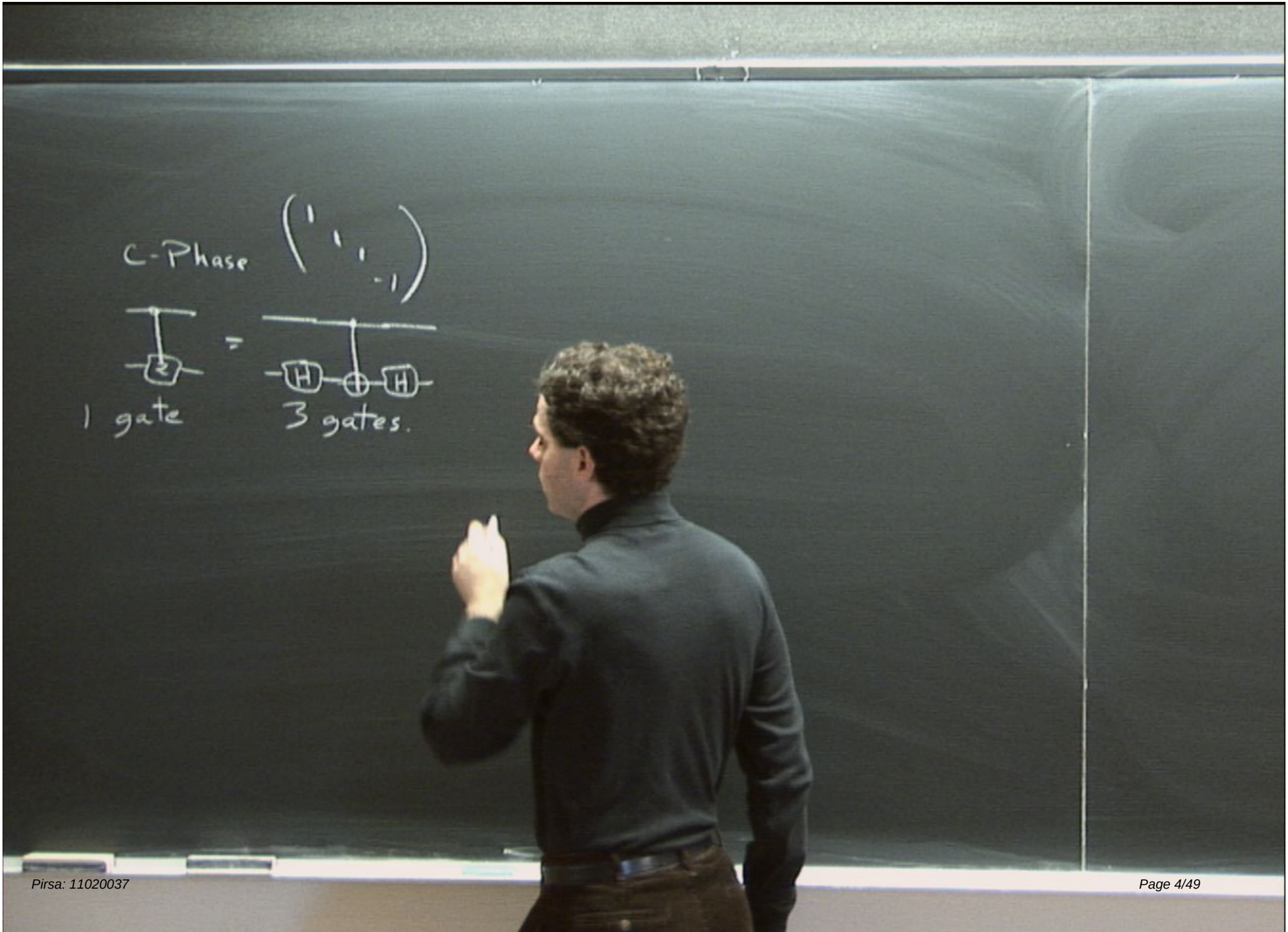
Date: Feb 18, 2011 09:00 AM

URL: http://pirsa.org/11020037

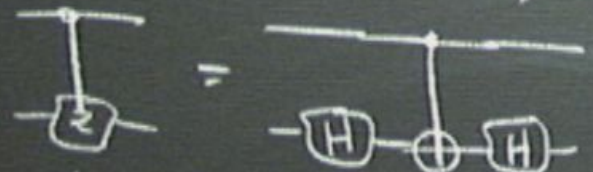
Abstract:


PERIMETER
INSTITUTE FOR THEORETICAL PHYSICS


C-Phase
(1 1 1 -1)

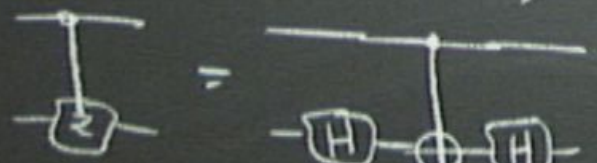

C-Phase
1 gate
3 gates.

C-Phase $\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}$



1 gate 3 gates.

C-Phase $(1, 1, 1, -1)$



1 gate = 3 gates.

Def.: $\{0,1\}^*$ is the set of bit strings of any length.

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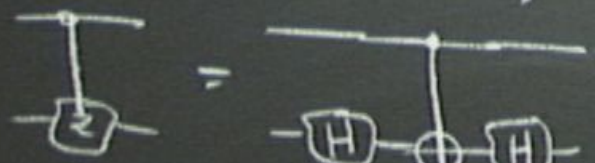

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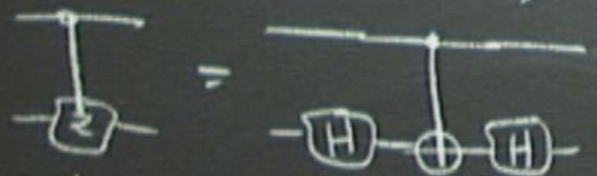

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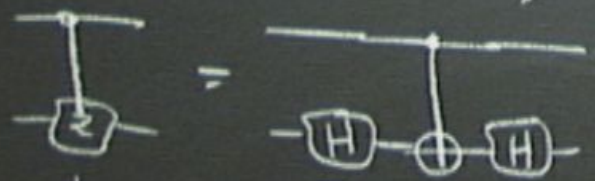

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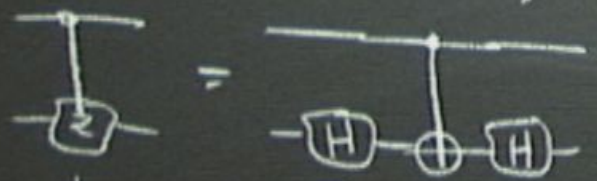

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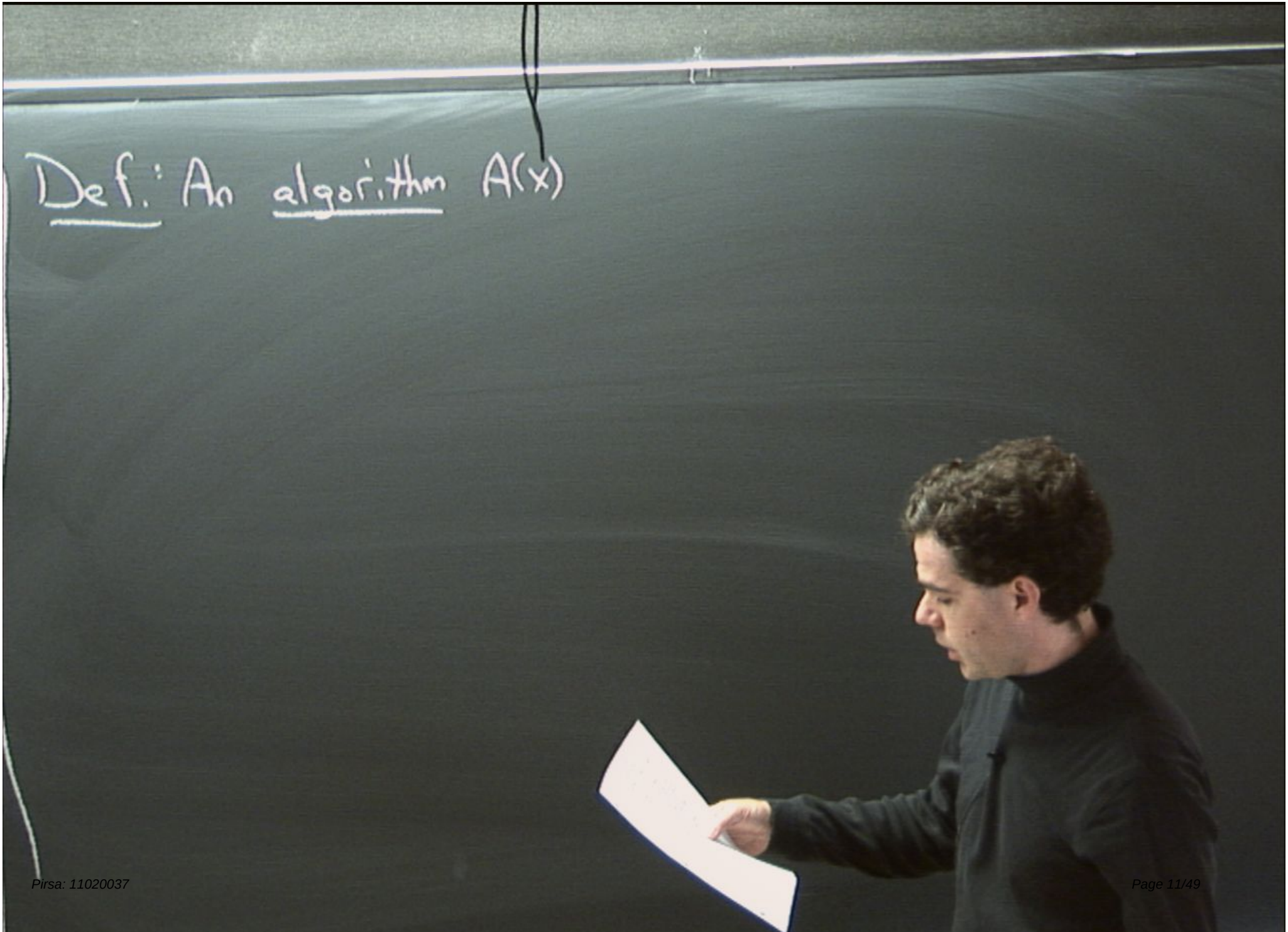
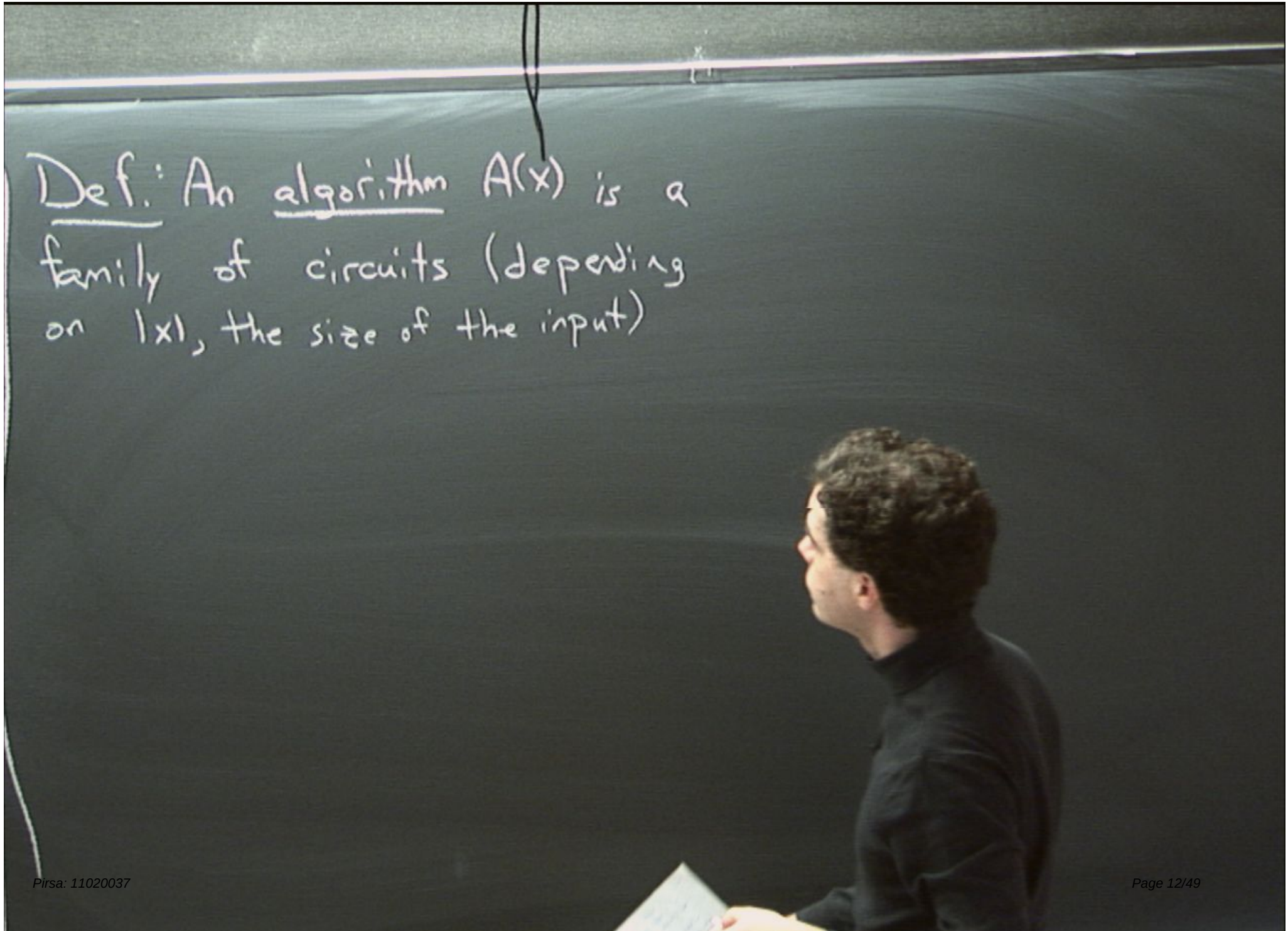

Def.: An algorithm A(x)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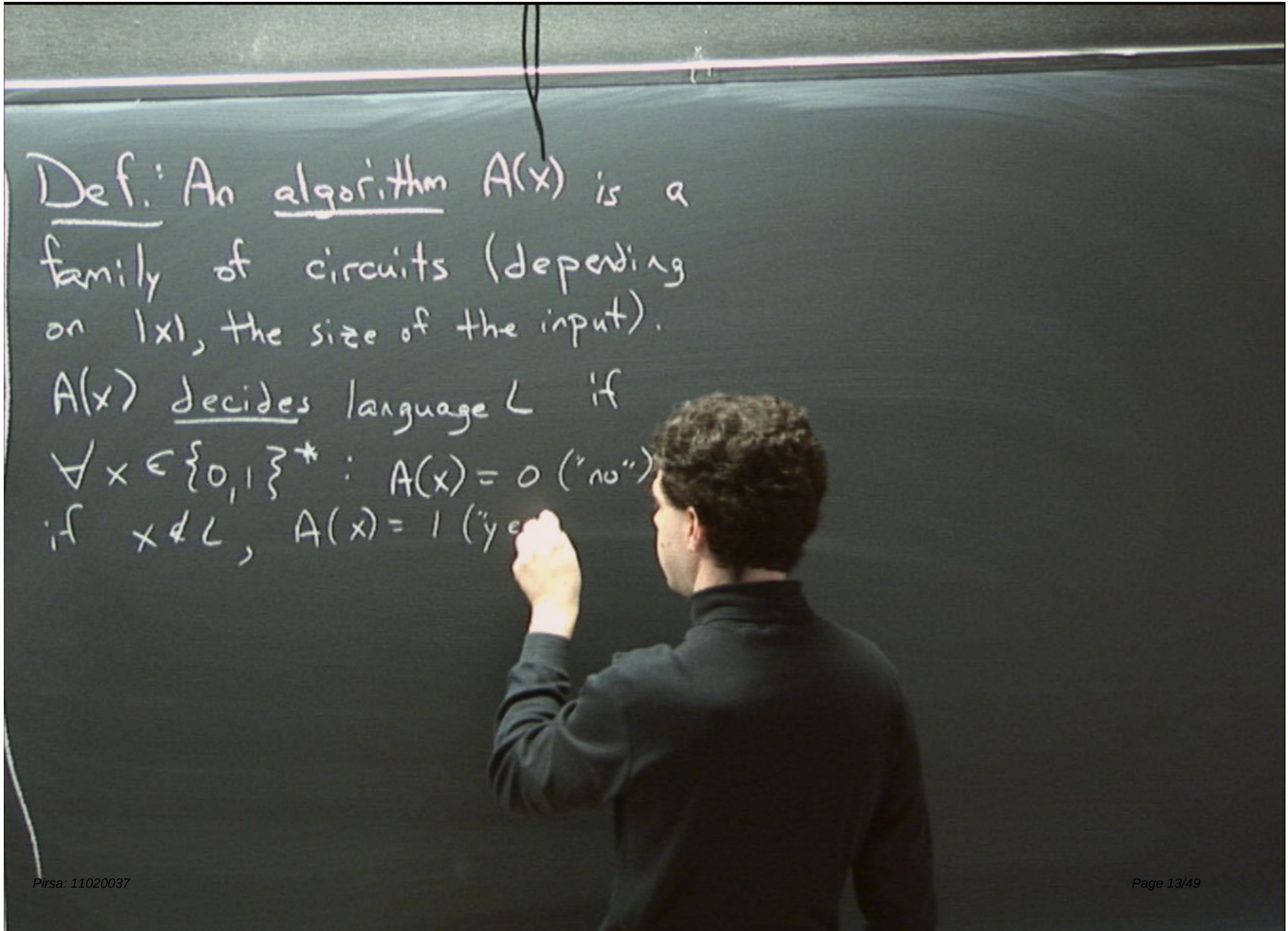
Def.: An algorithm A(x) is a
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A(x) decides language L if
∀ x ∈ {0,1}* : A(x) = 0 ("no")
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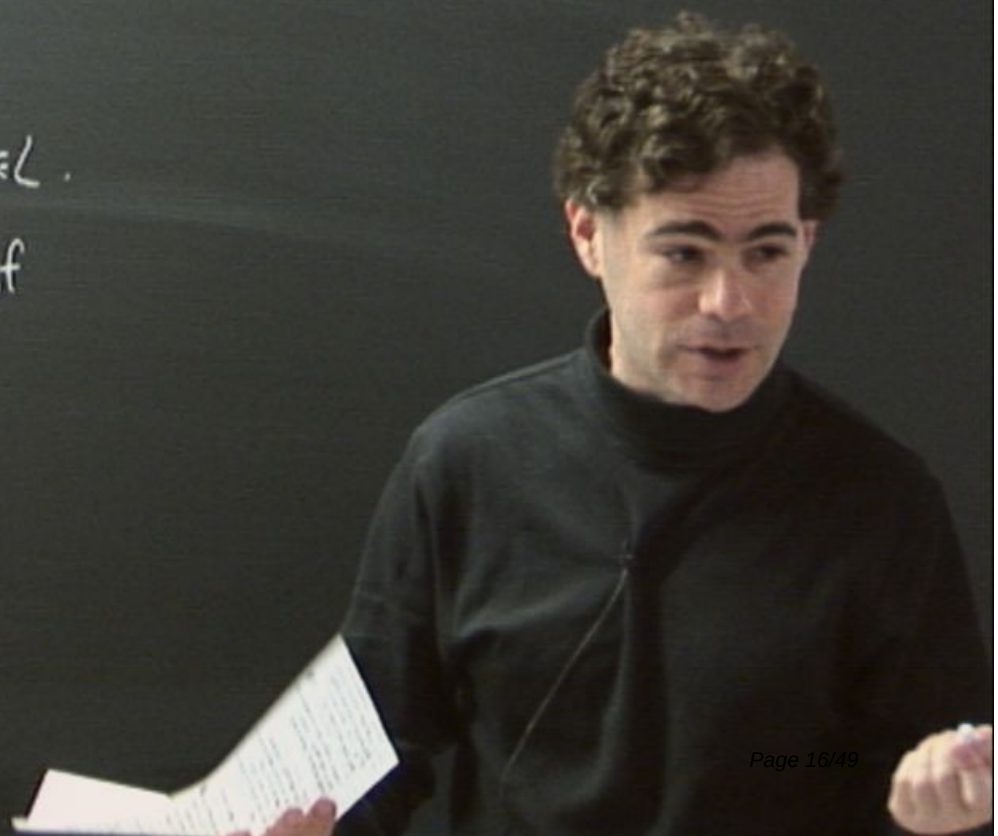

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possible inputs.

Language L is set of "yes" answers to a yes/no question (decision problem).

E.g.: Primality testing $L = \{p \in \mathbb{Z}^+ \mid p \text{ prime}\}$

Algorithm to decide should say if p is prime or composite.

if $x \notin L$

The input

L.

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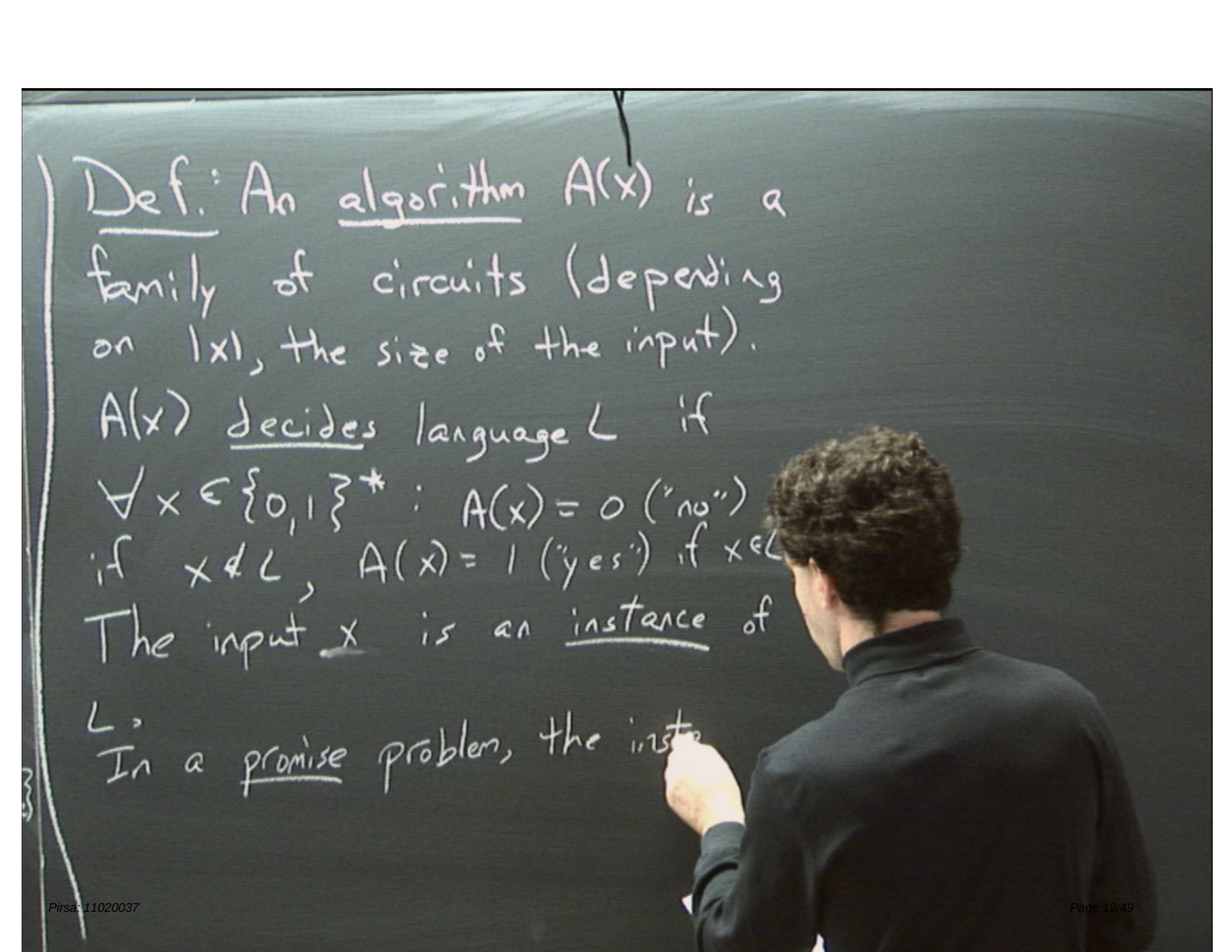

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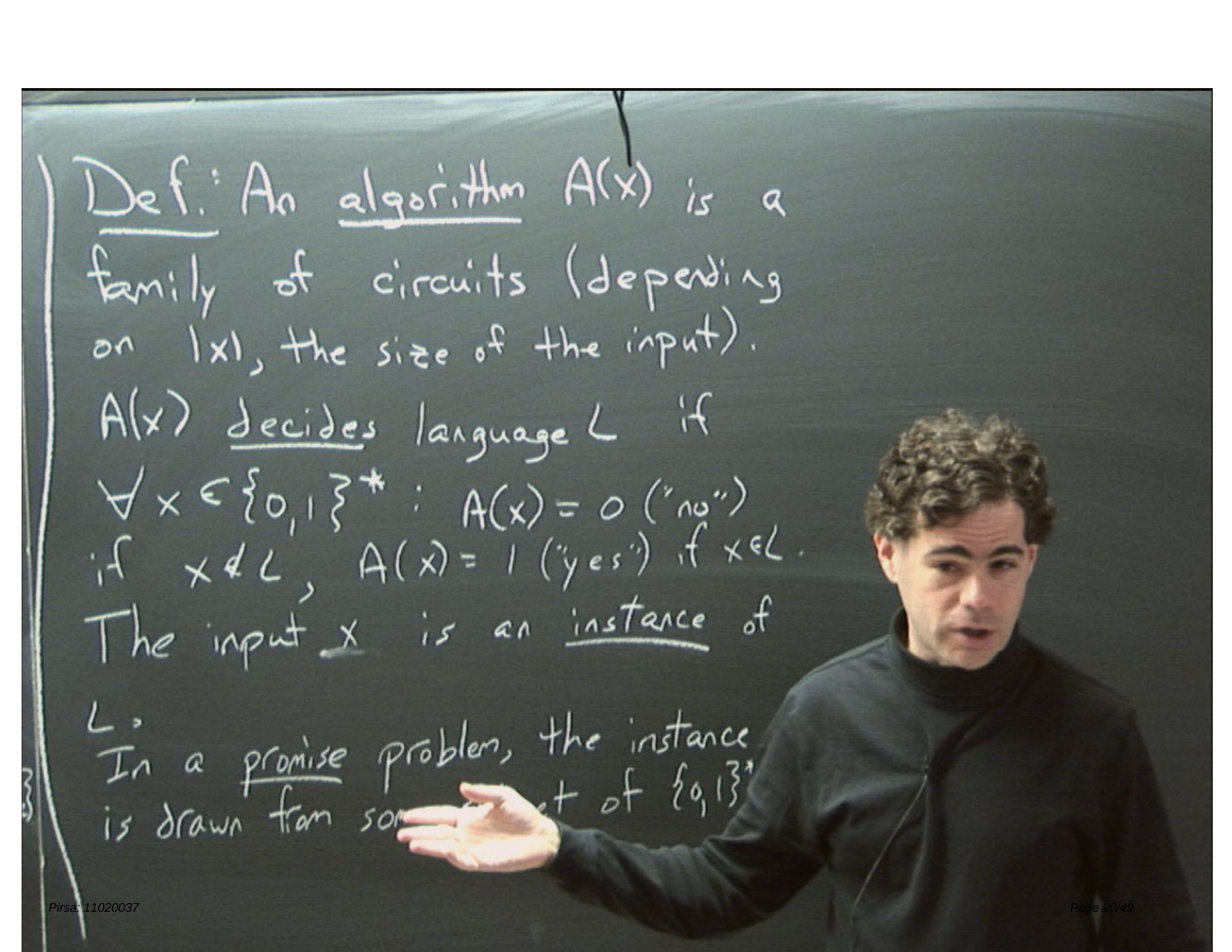

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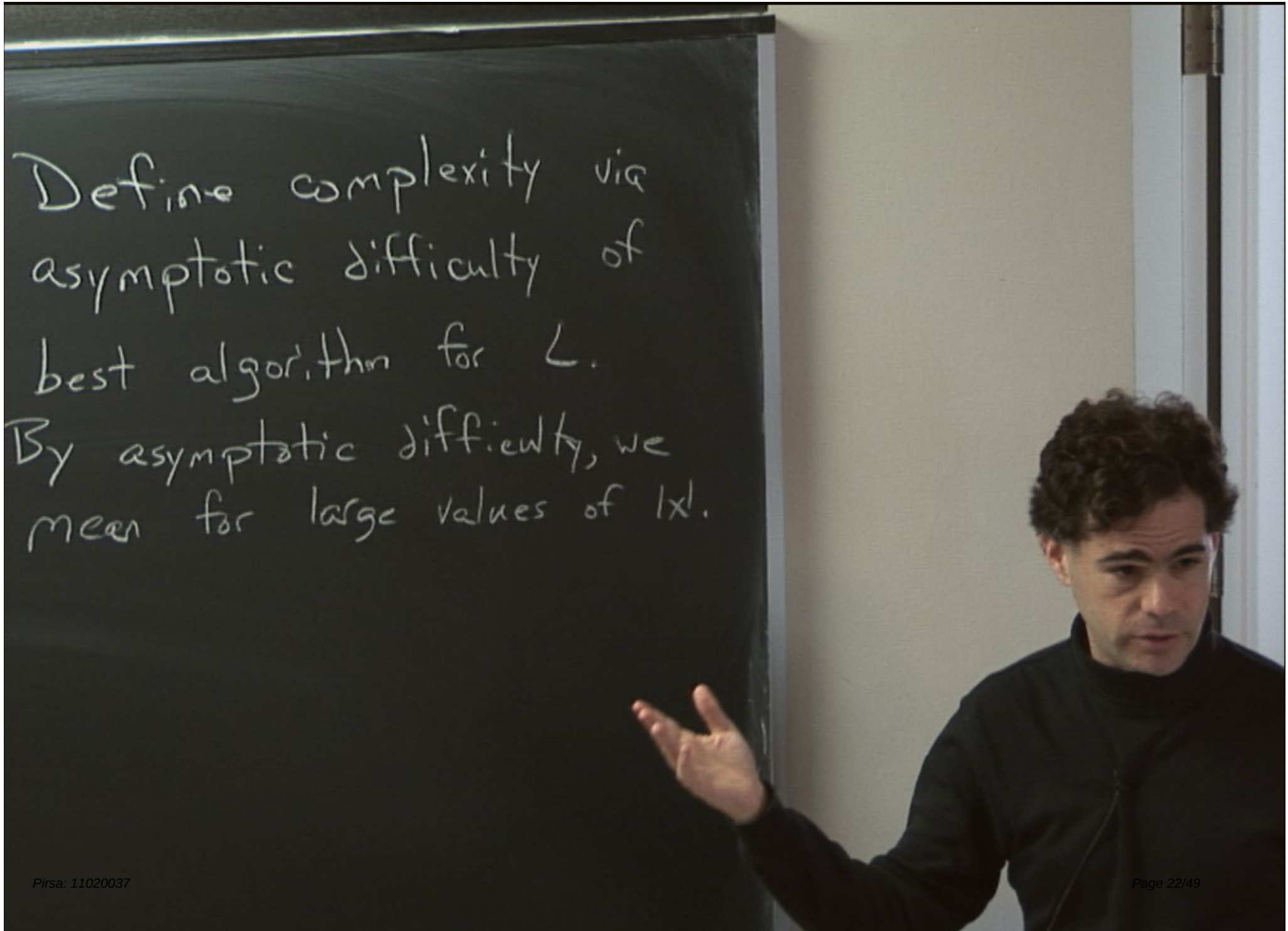
Define complexity via asymptotic difficulty of best algorithm for L.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By asymptotic difficulty, we
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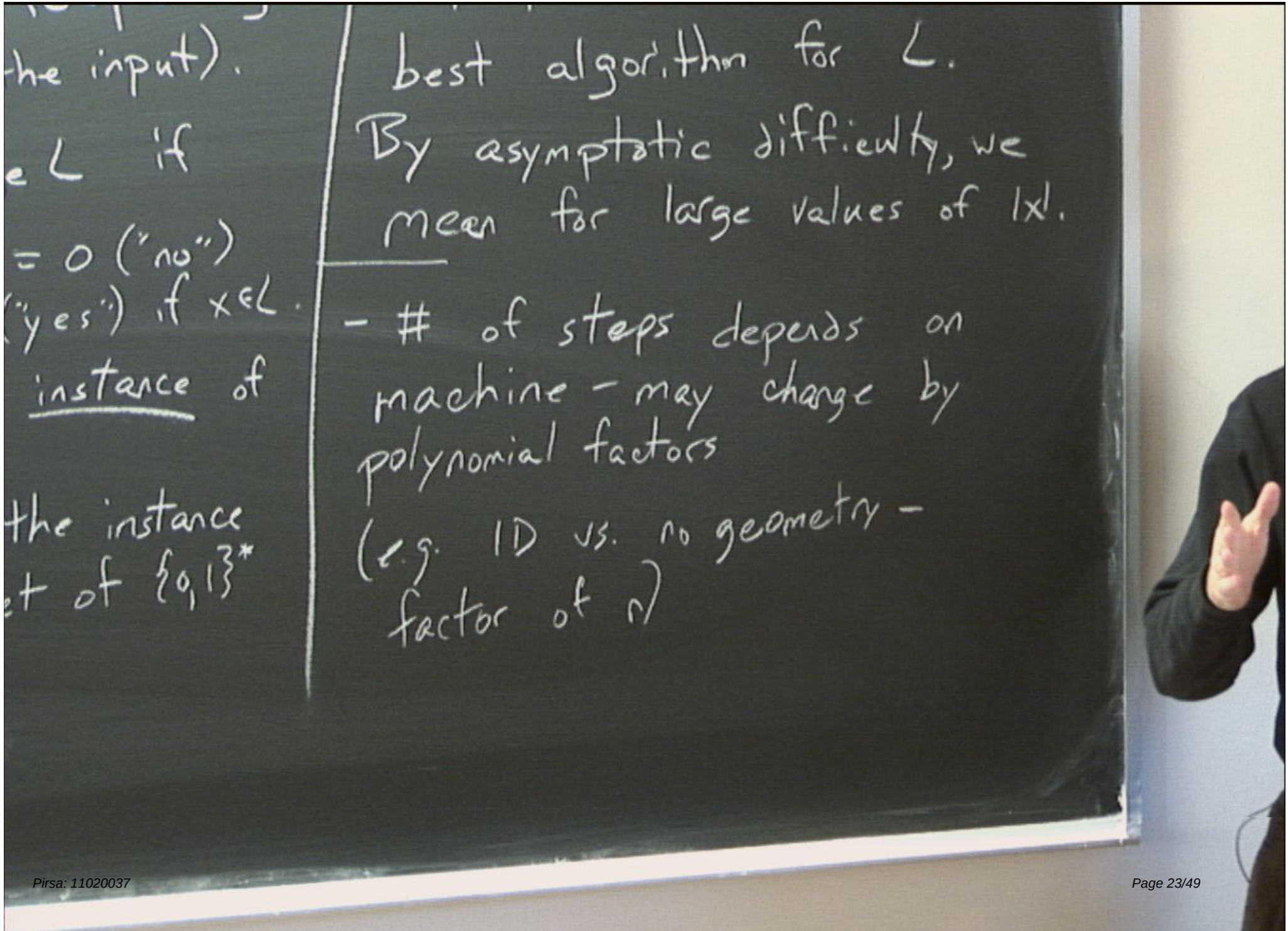
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(e.g. 1D vs. no geometry -
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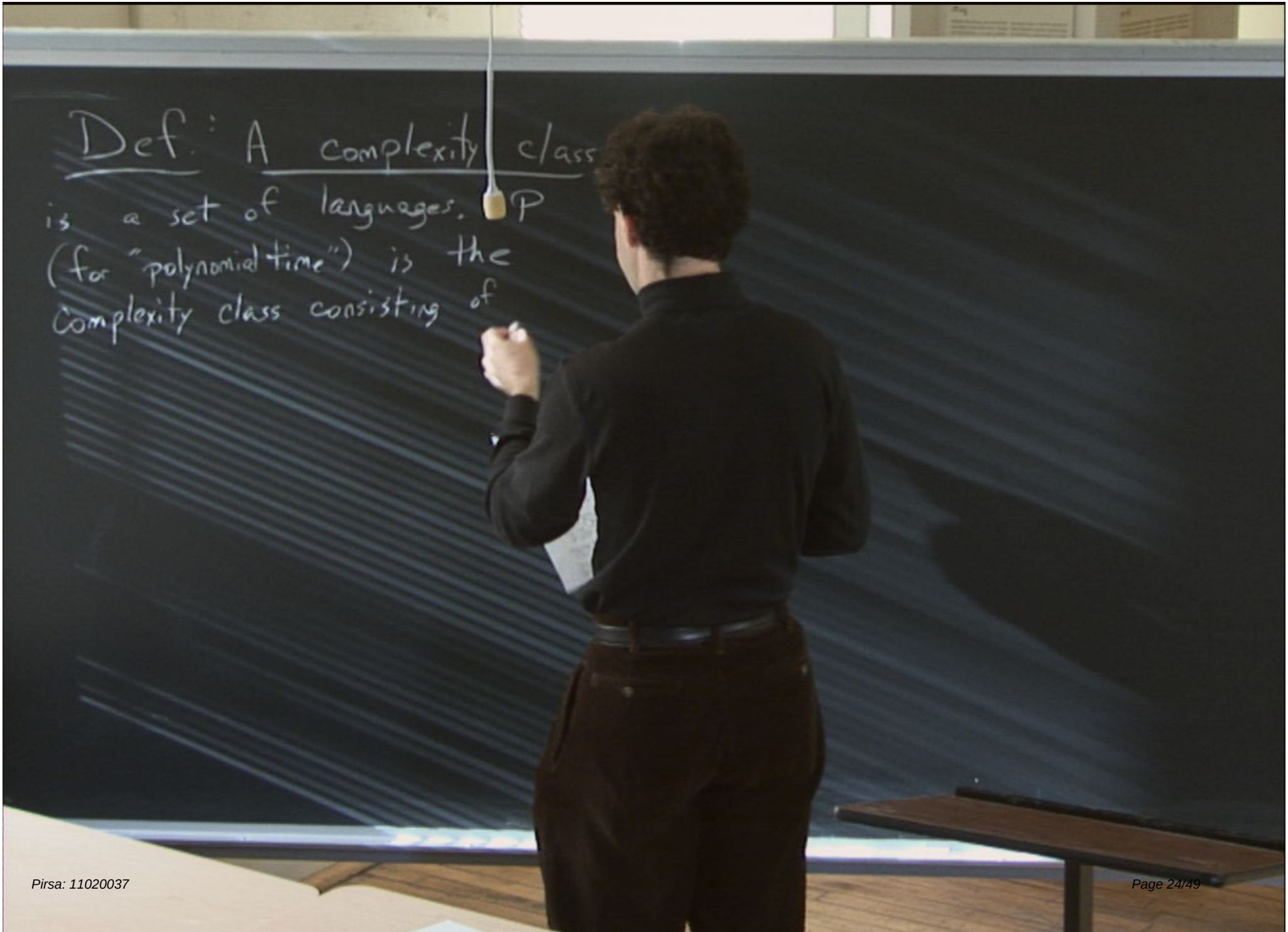
Def: A complexity class
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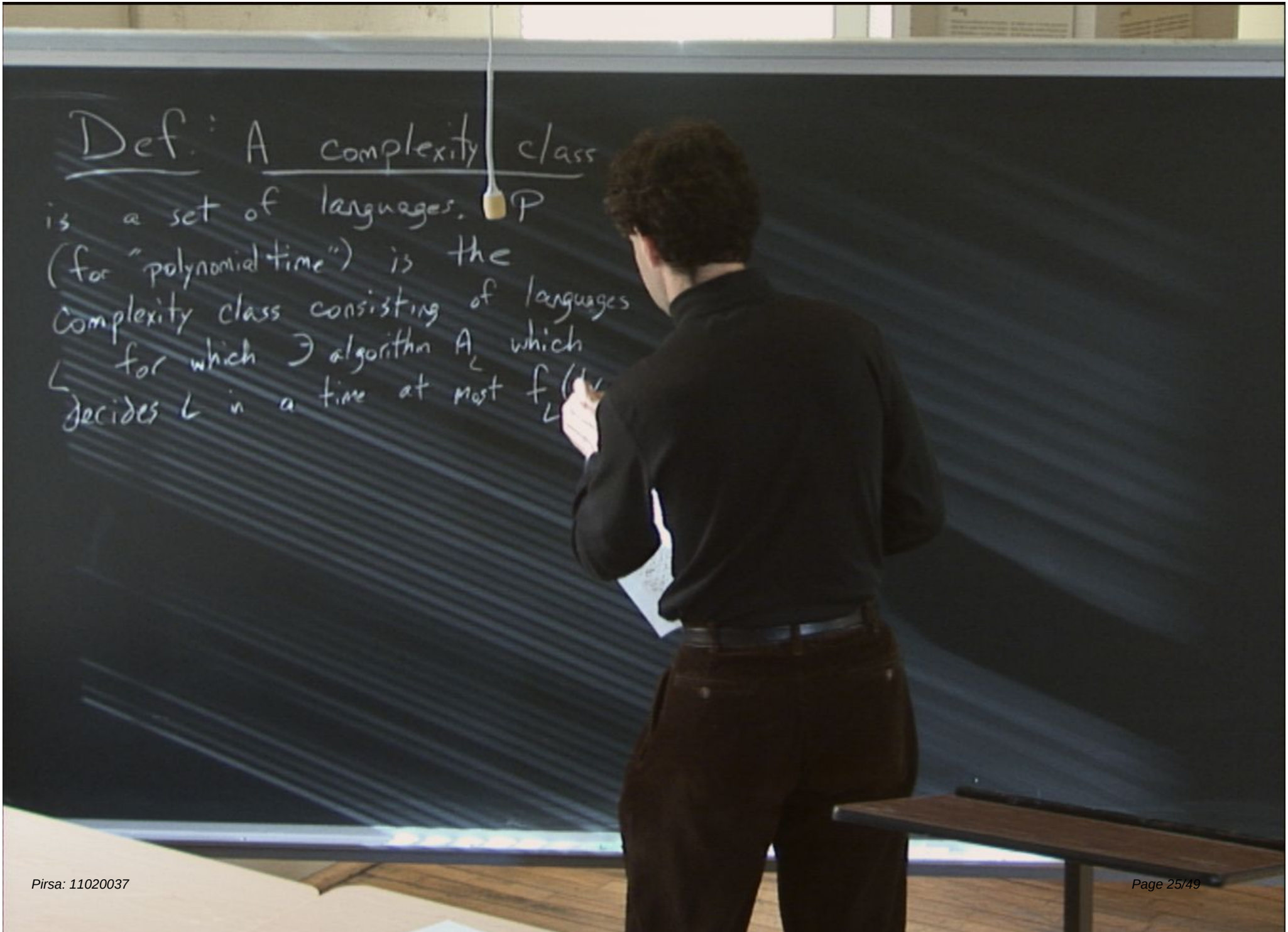
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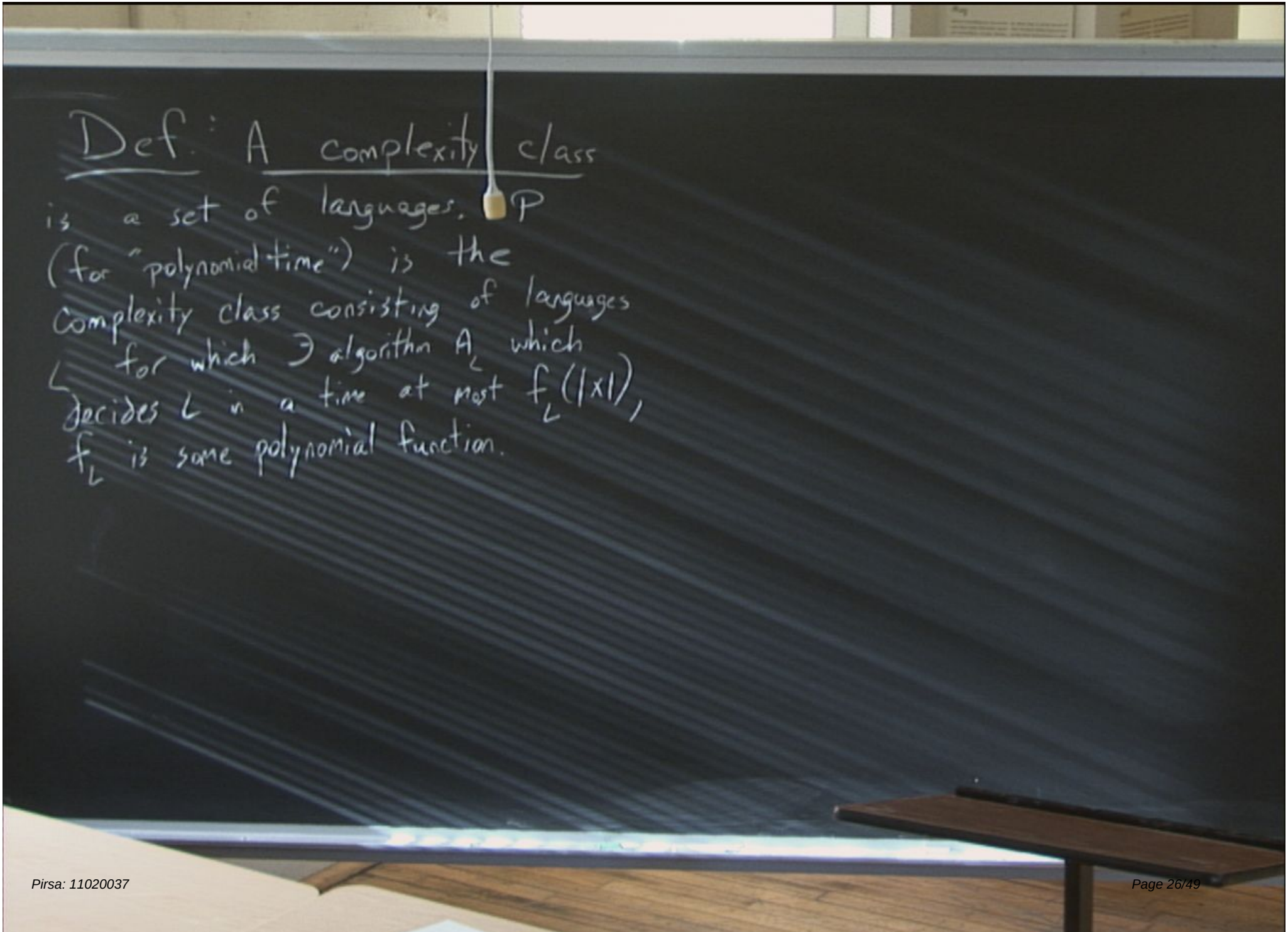
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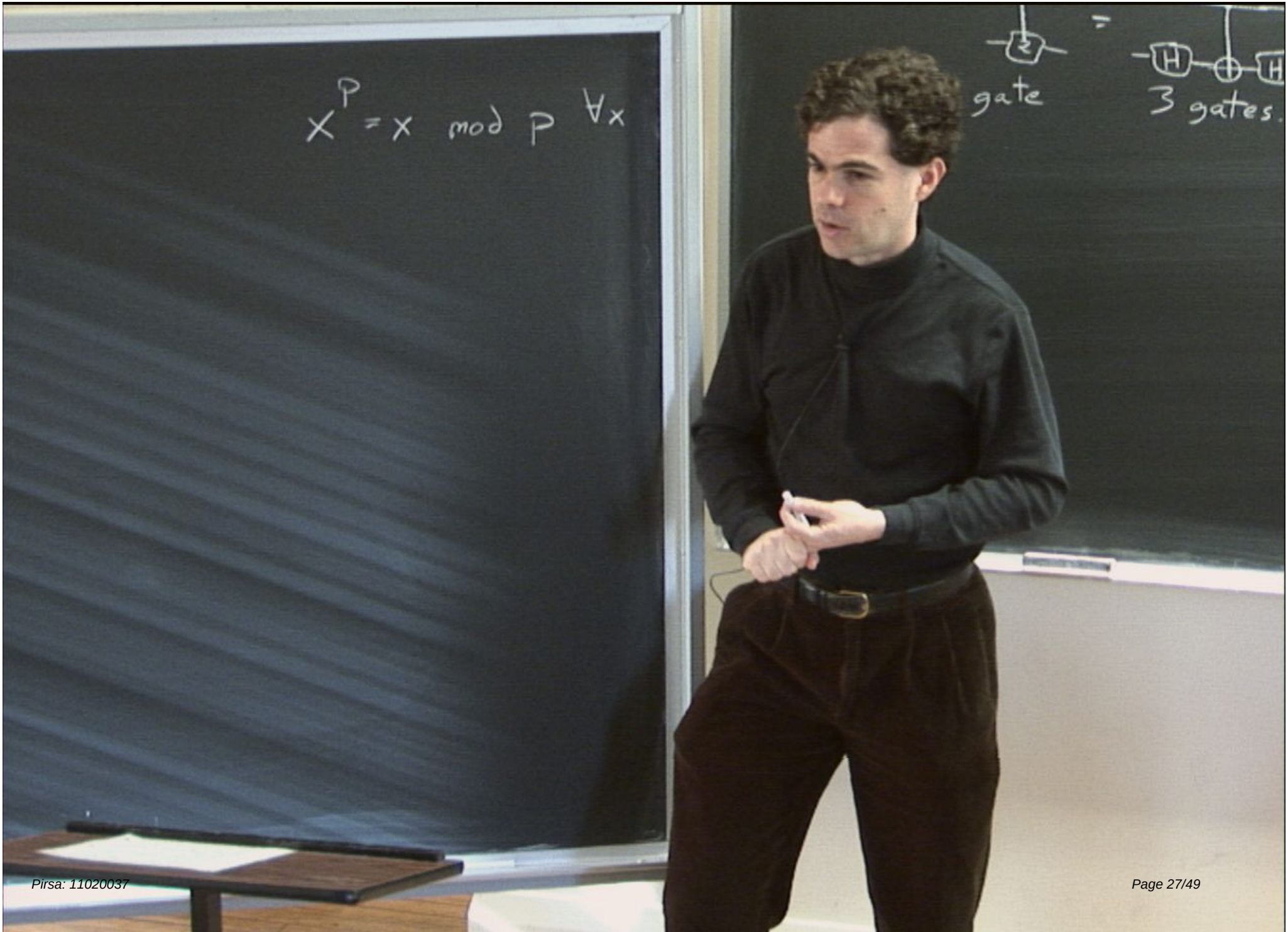
$x^p = x \mod p \quad \forall x$
gate
3 gates.

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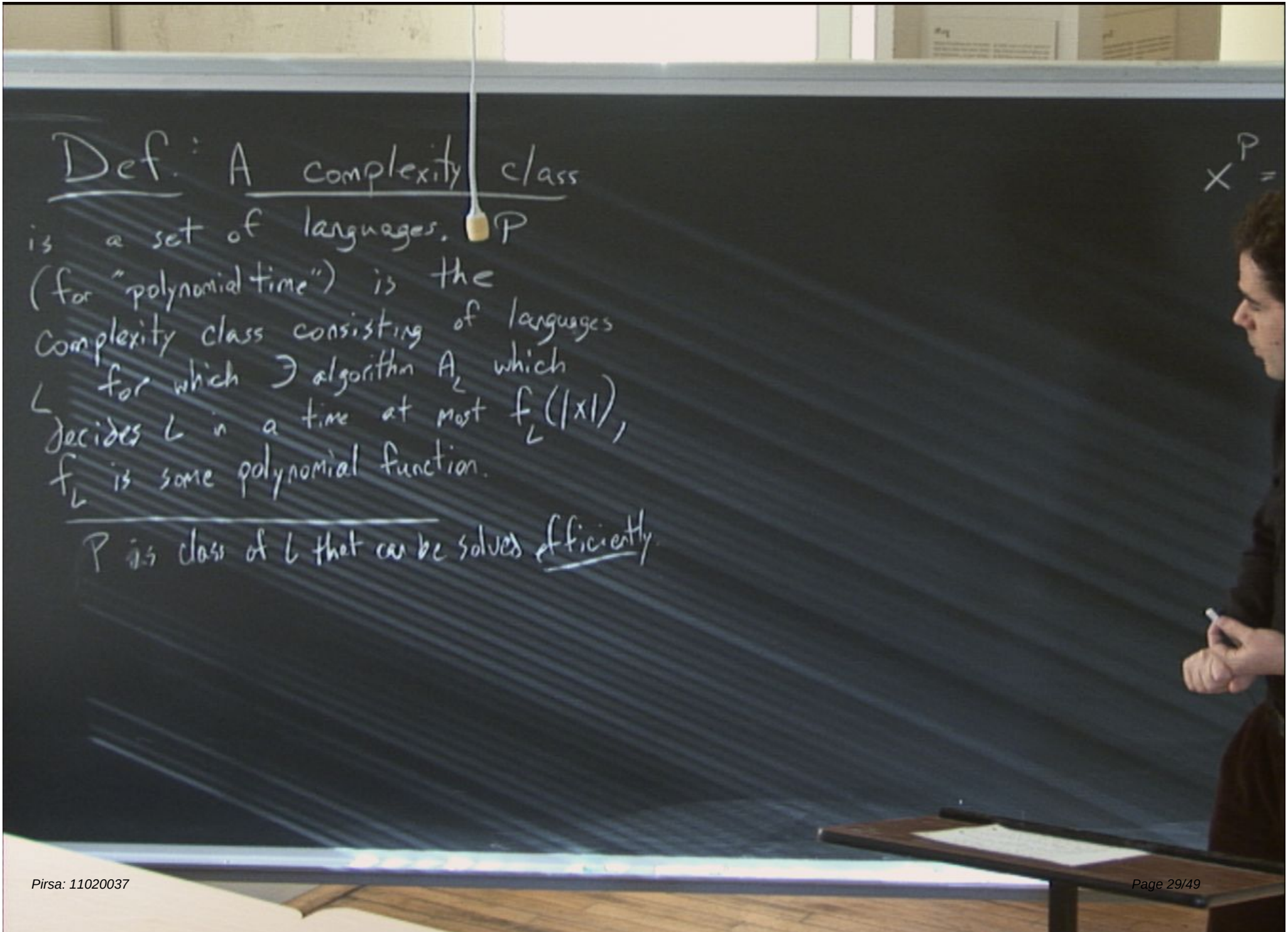
$A(x)$ must work even for worst case.

best algorithm for L.

By asymptotic difficulty, we mean for large values of $|x|$.

- # of steps depends on machine - may change by polynomial factors

(e.g. 1D vs. no geometry - factor of n)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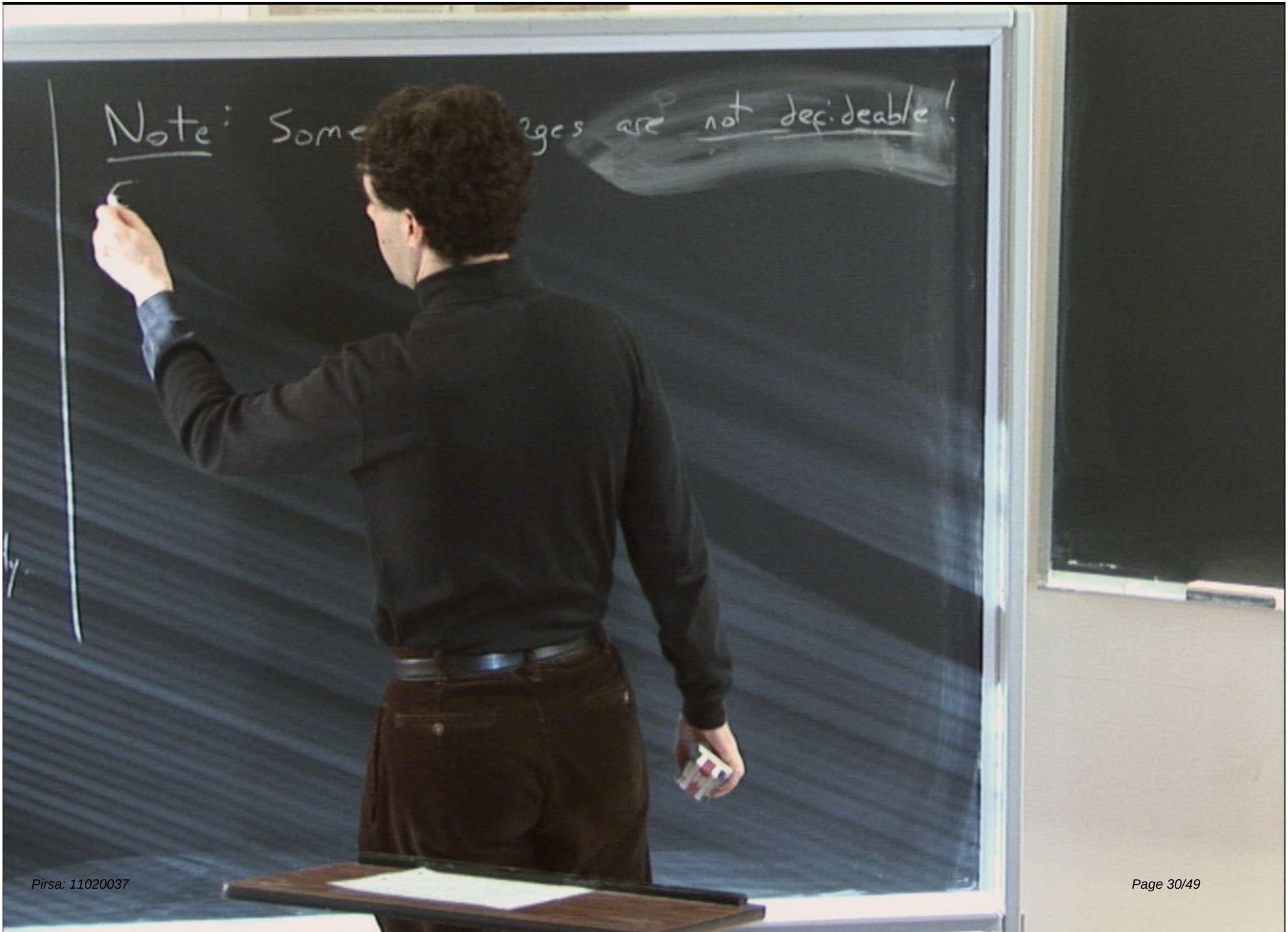
P is class of L that can be solved efficiently.
$X^P =$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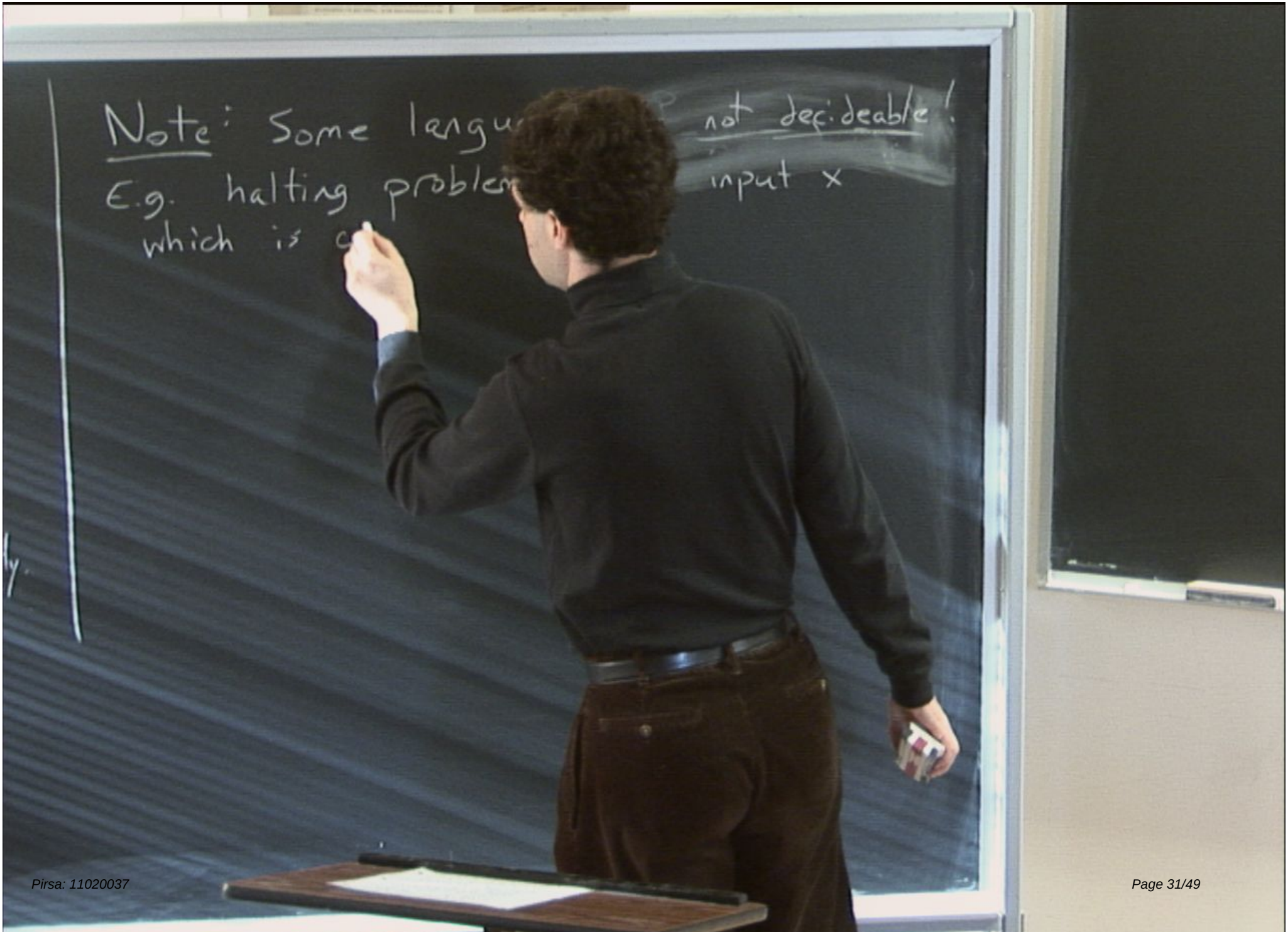
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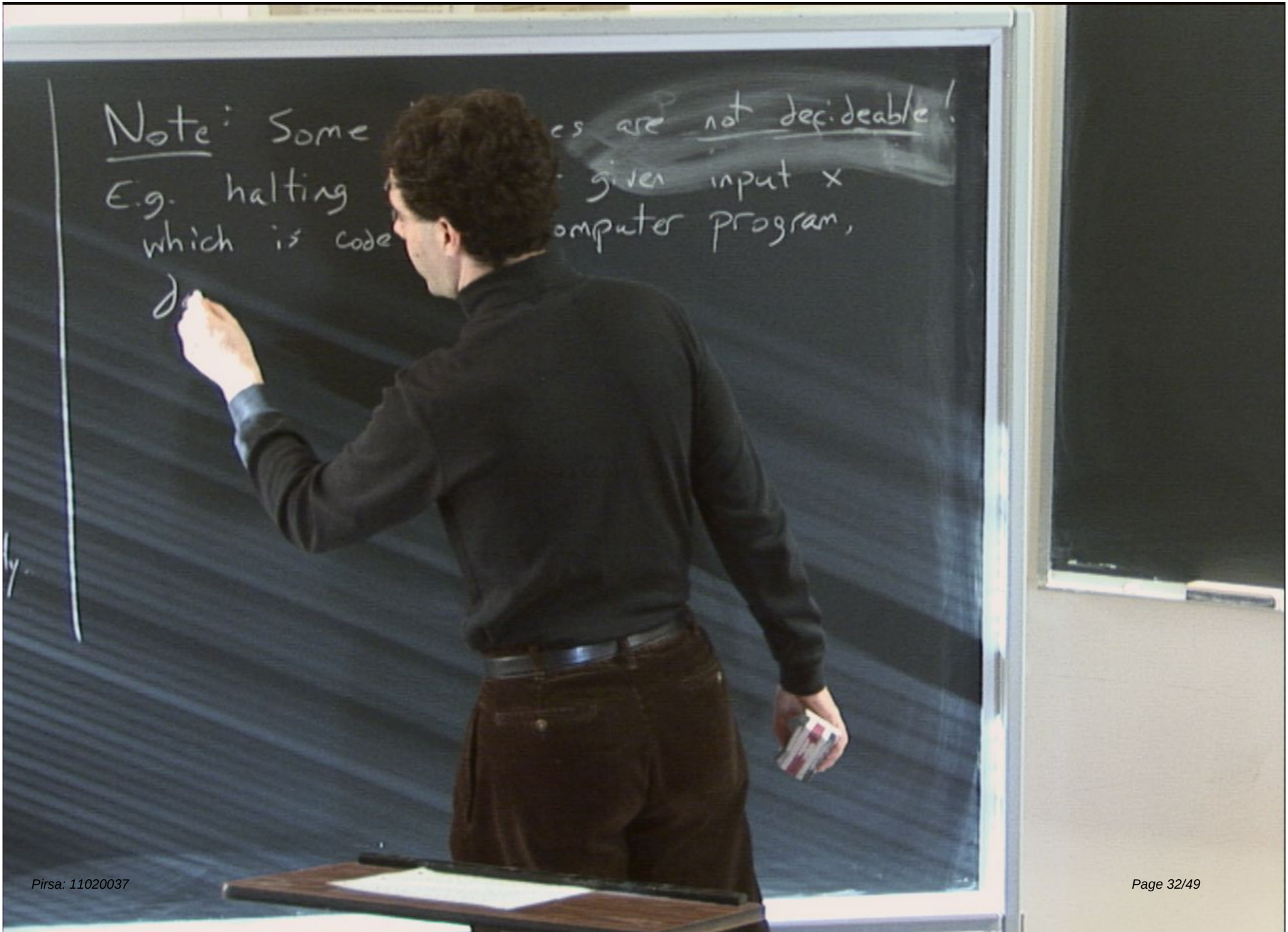
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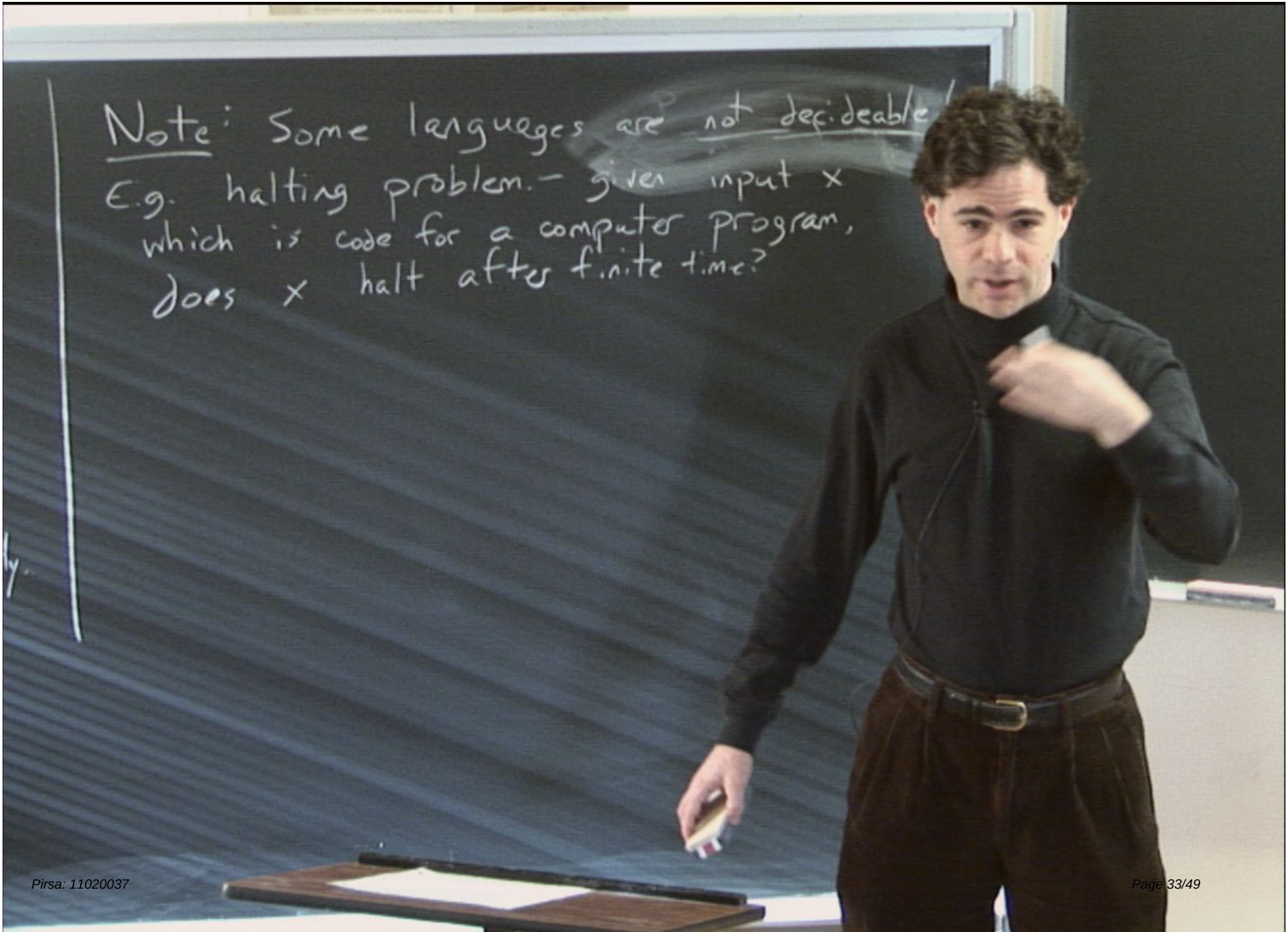
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E.g. halting problem. – given input x
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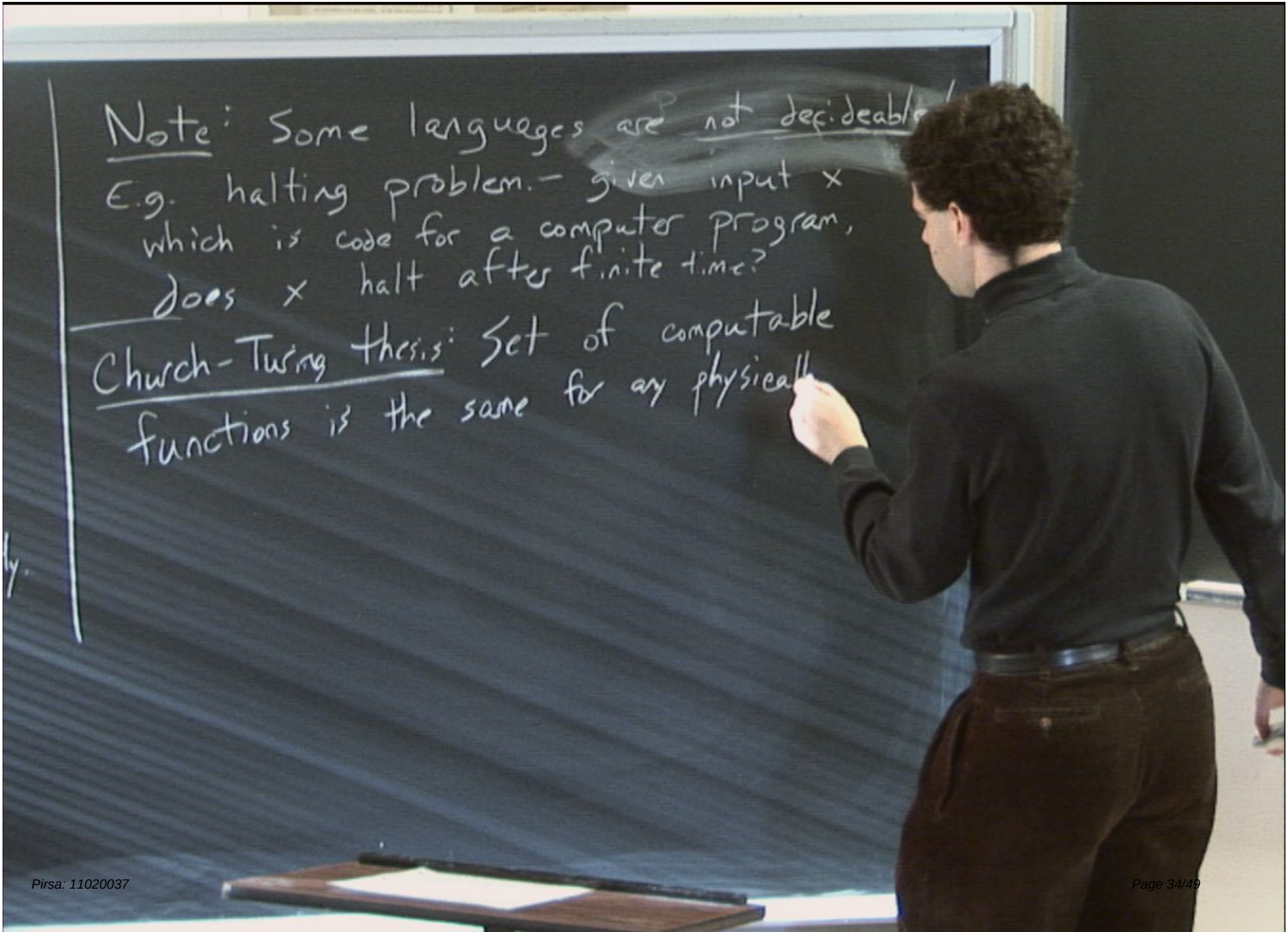
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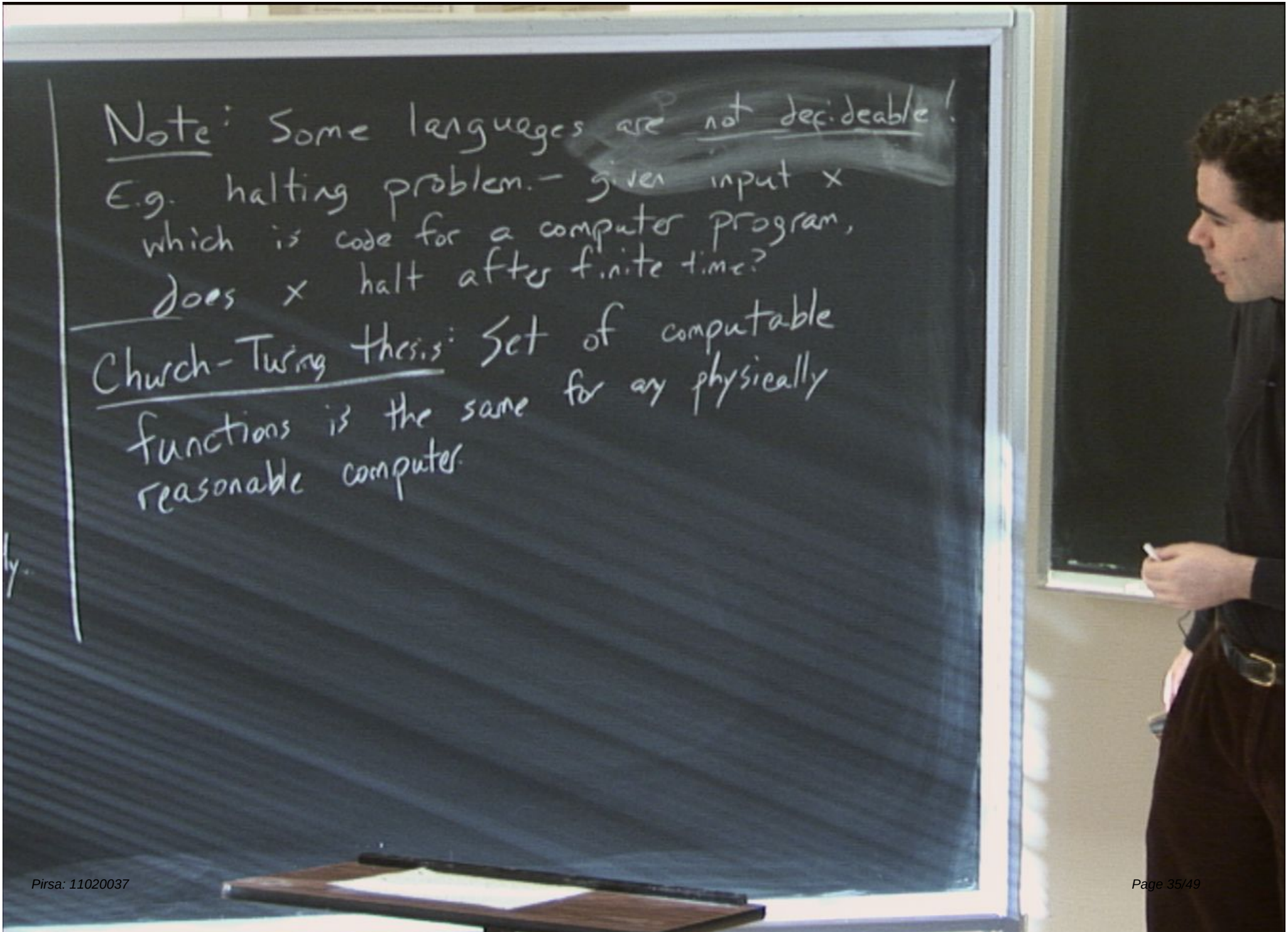
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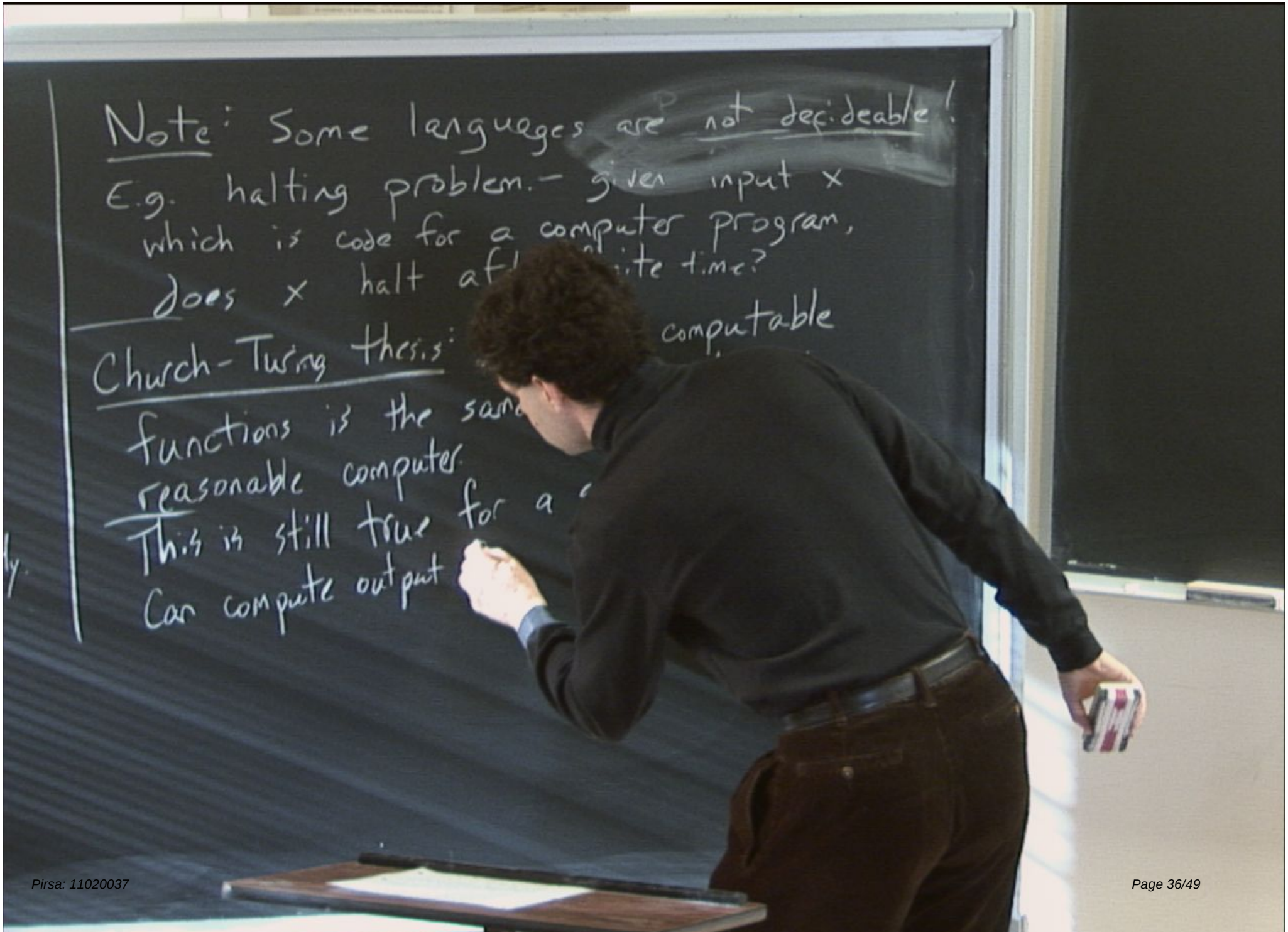
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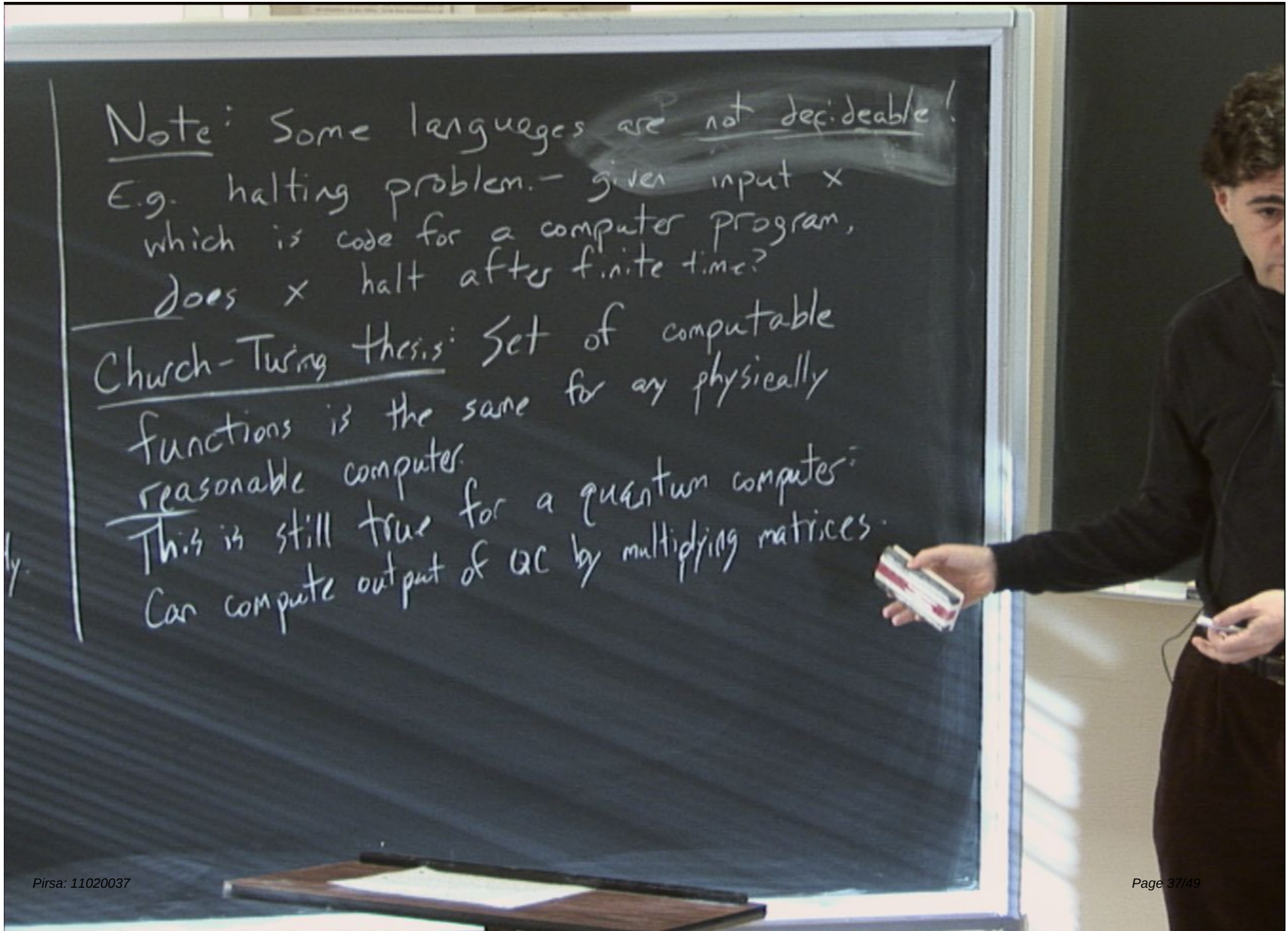
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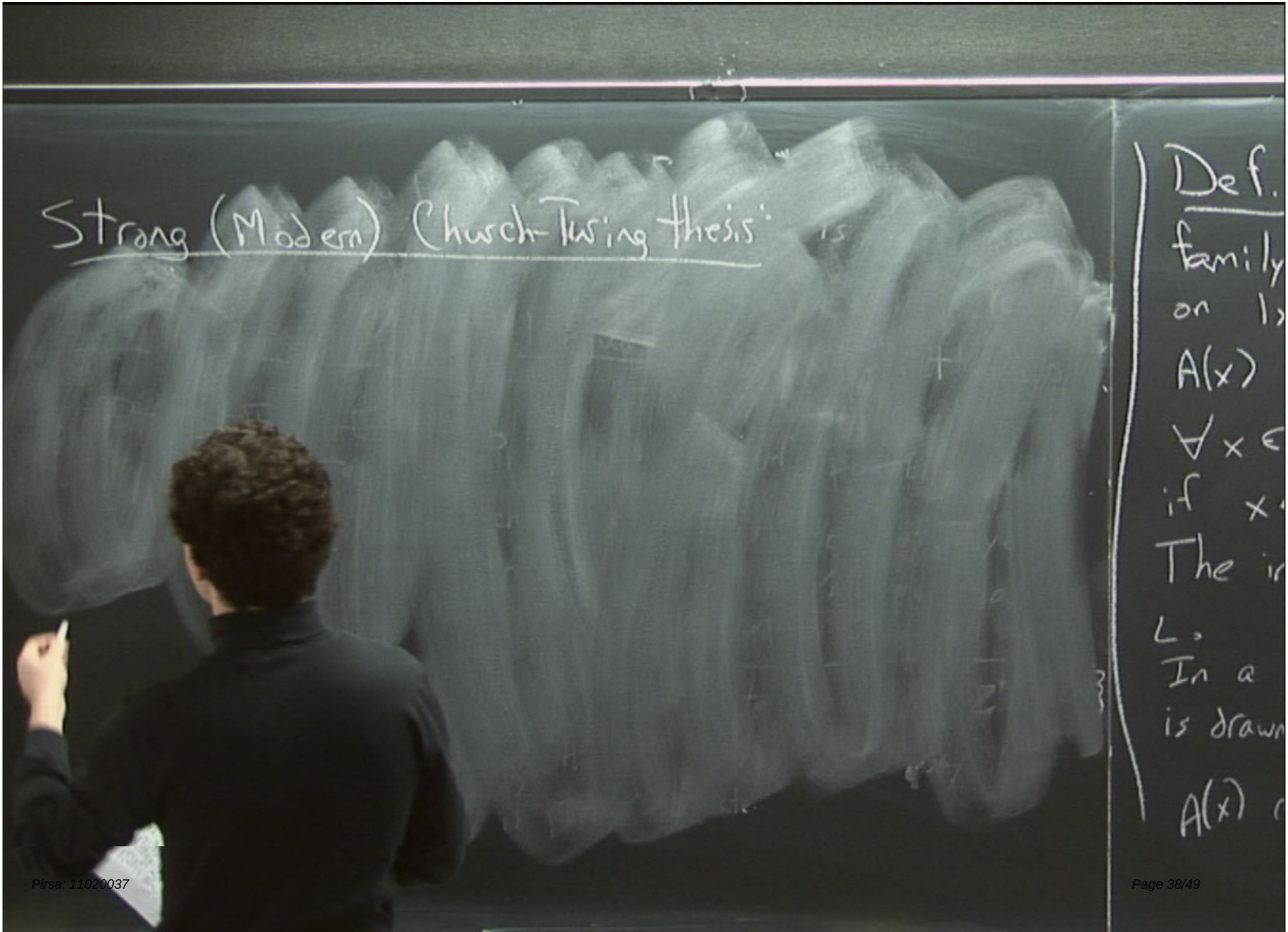
This is still true for a quantum computer:
Can compute output of QC by multiplying matrices.


Strong (Modern) Church-Turing thesis:
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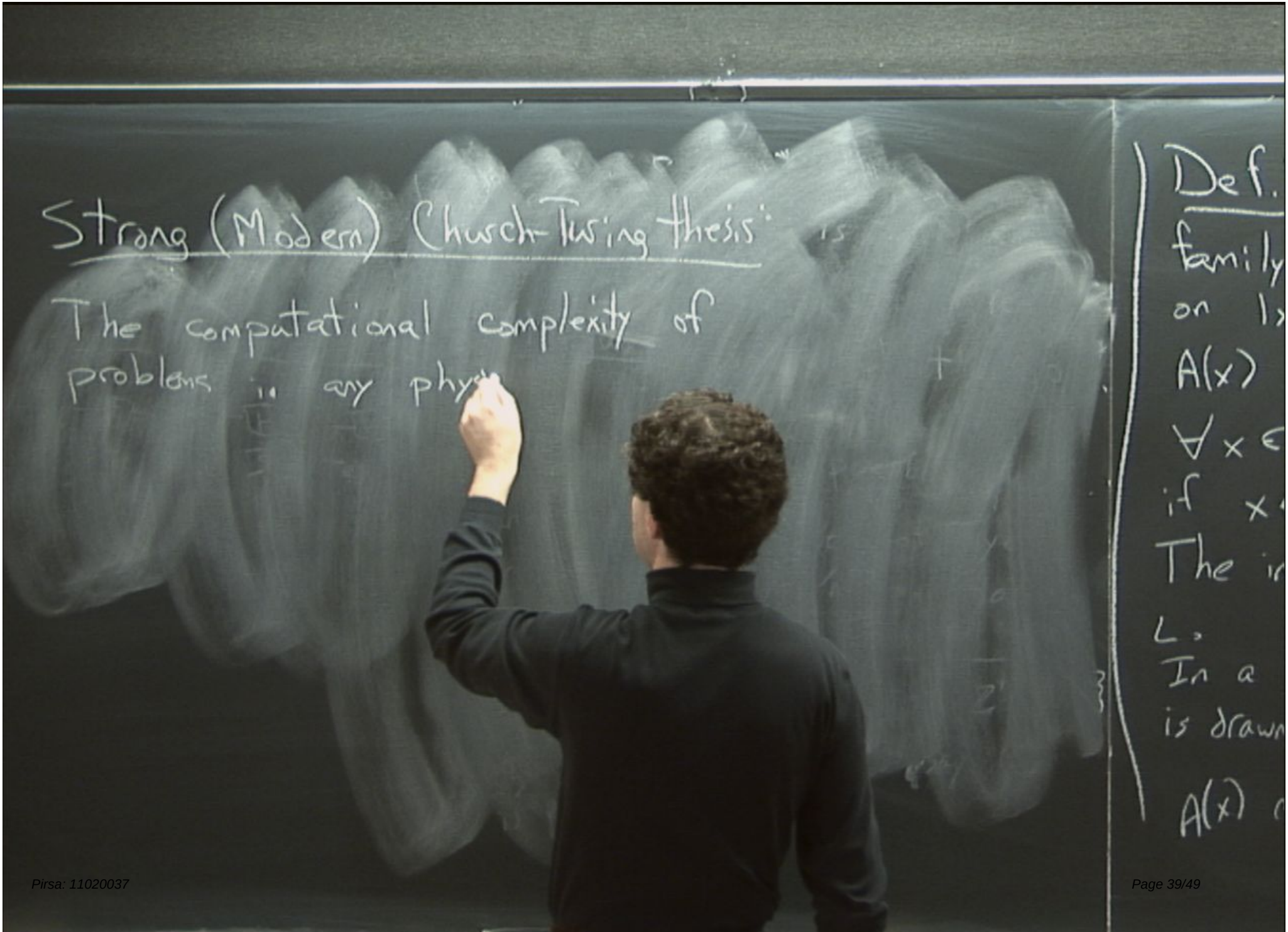
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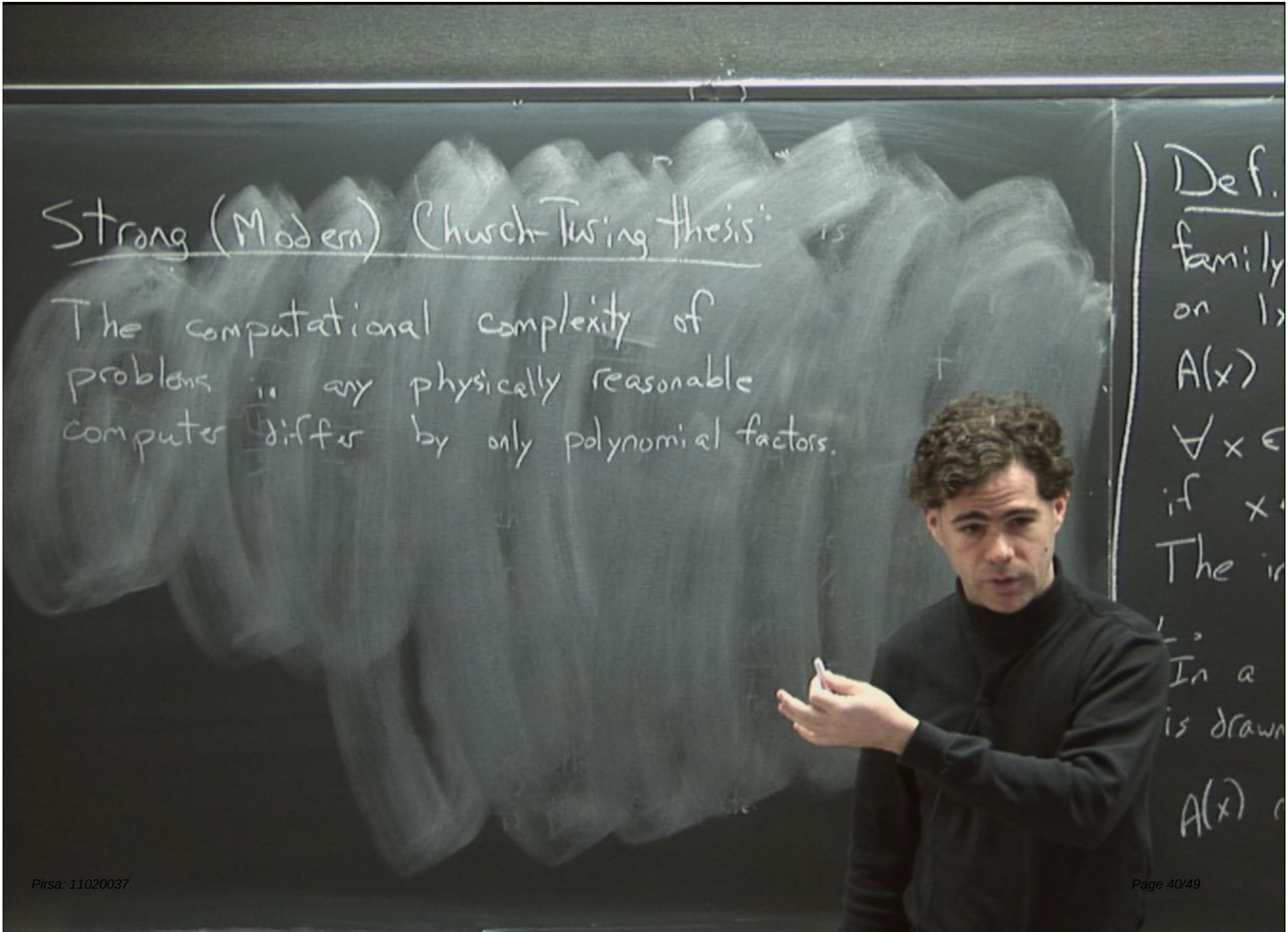
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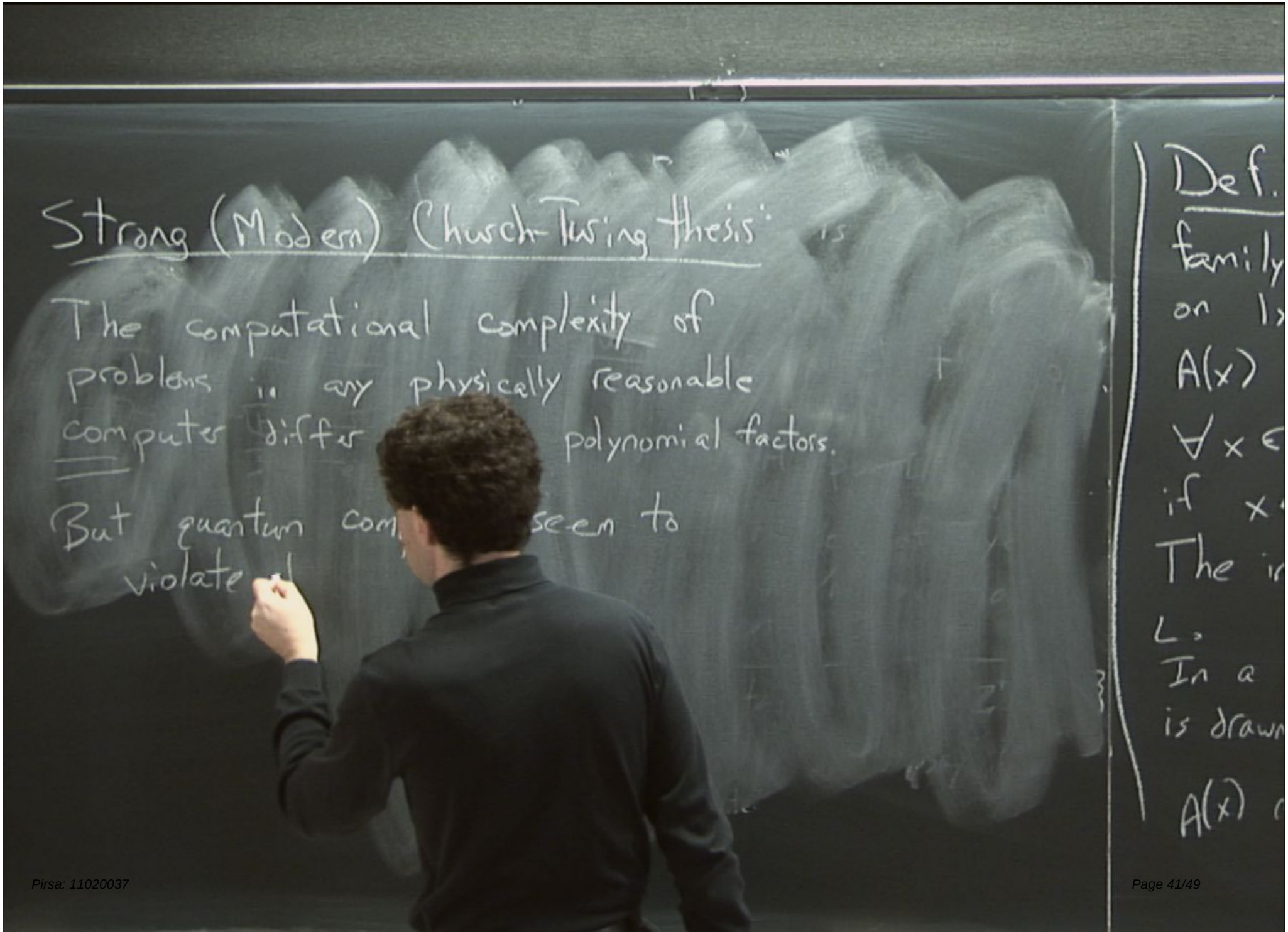
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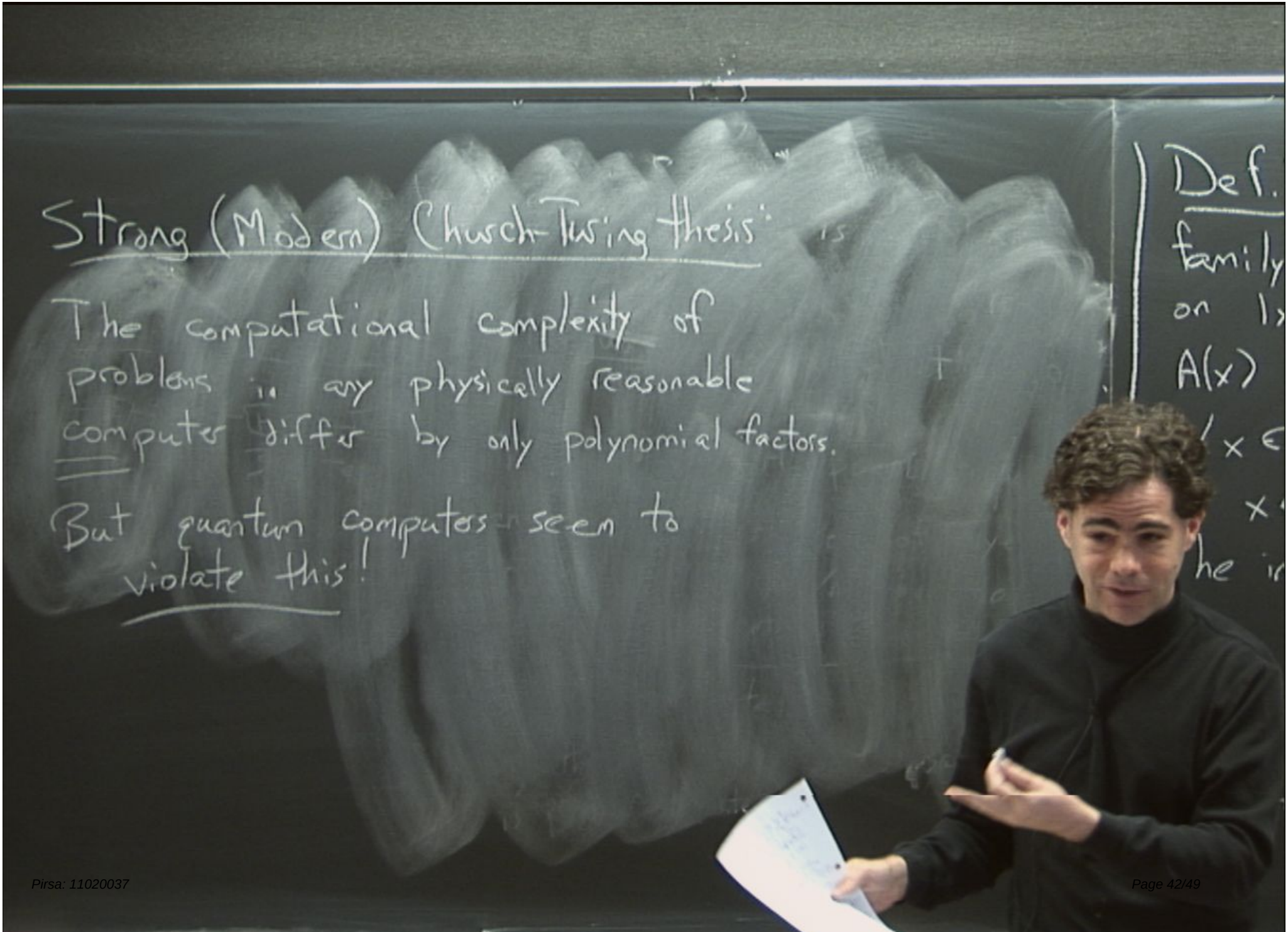
Strong (Modern) Church-Turing thesis:
The computational complexity of problems in any physically reasonable computer differ by only polynomial factors.
But quantum computers seem to violate this!
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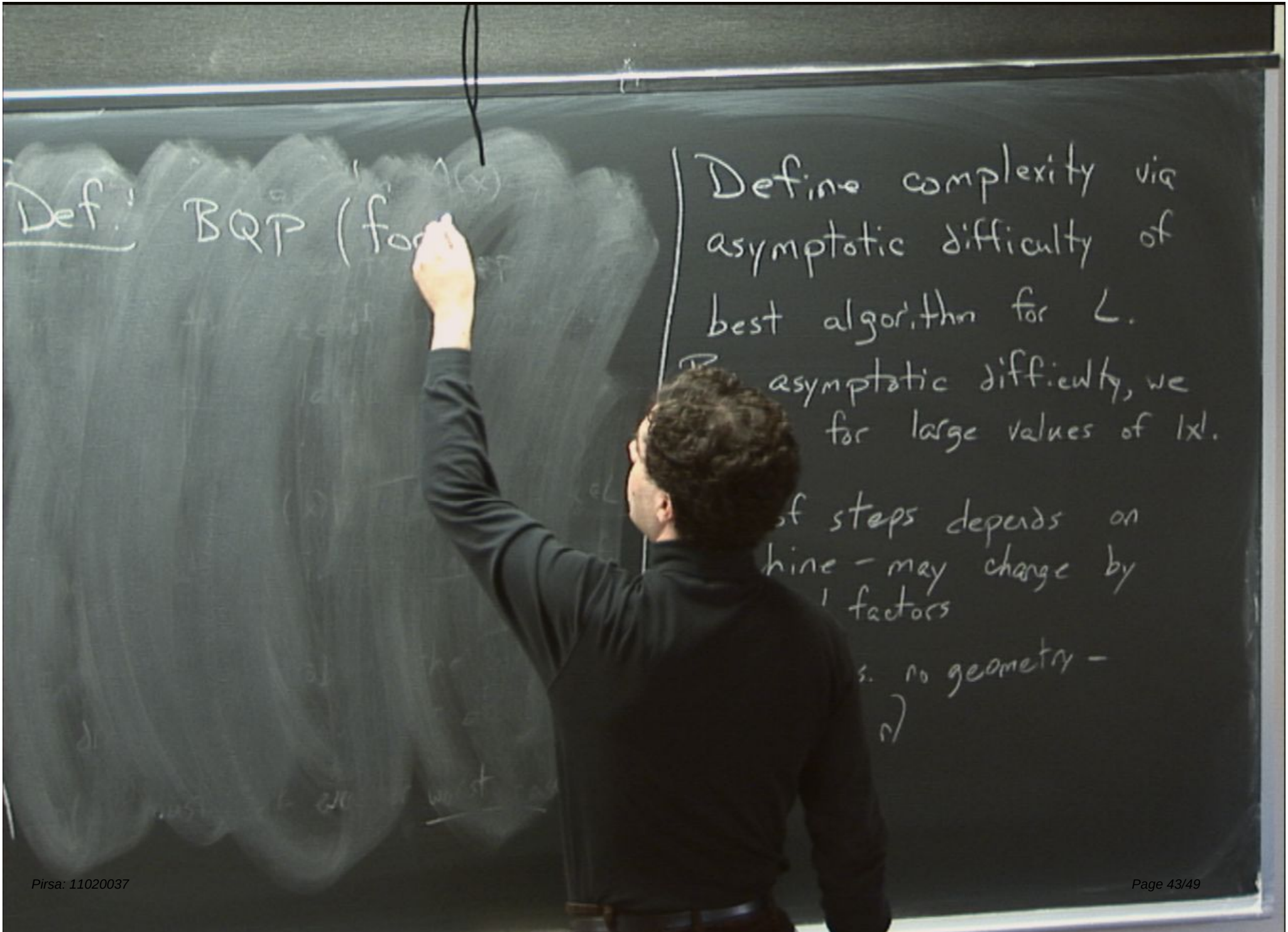
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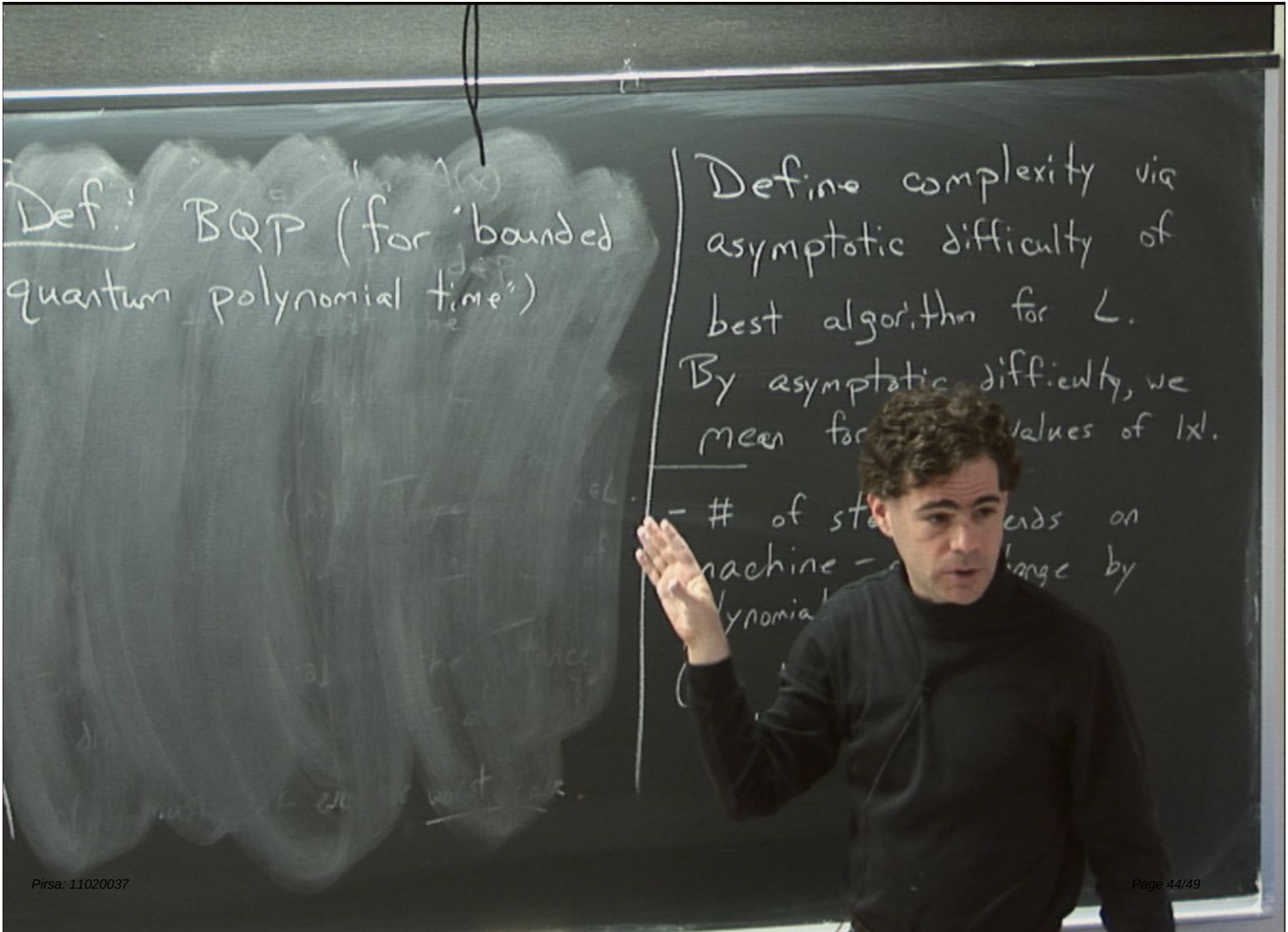
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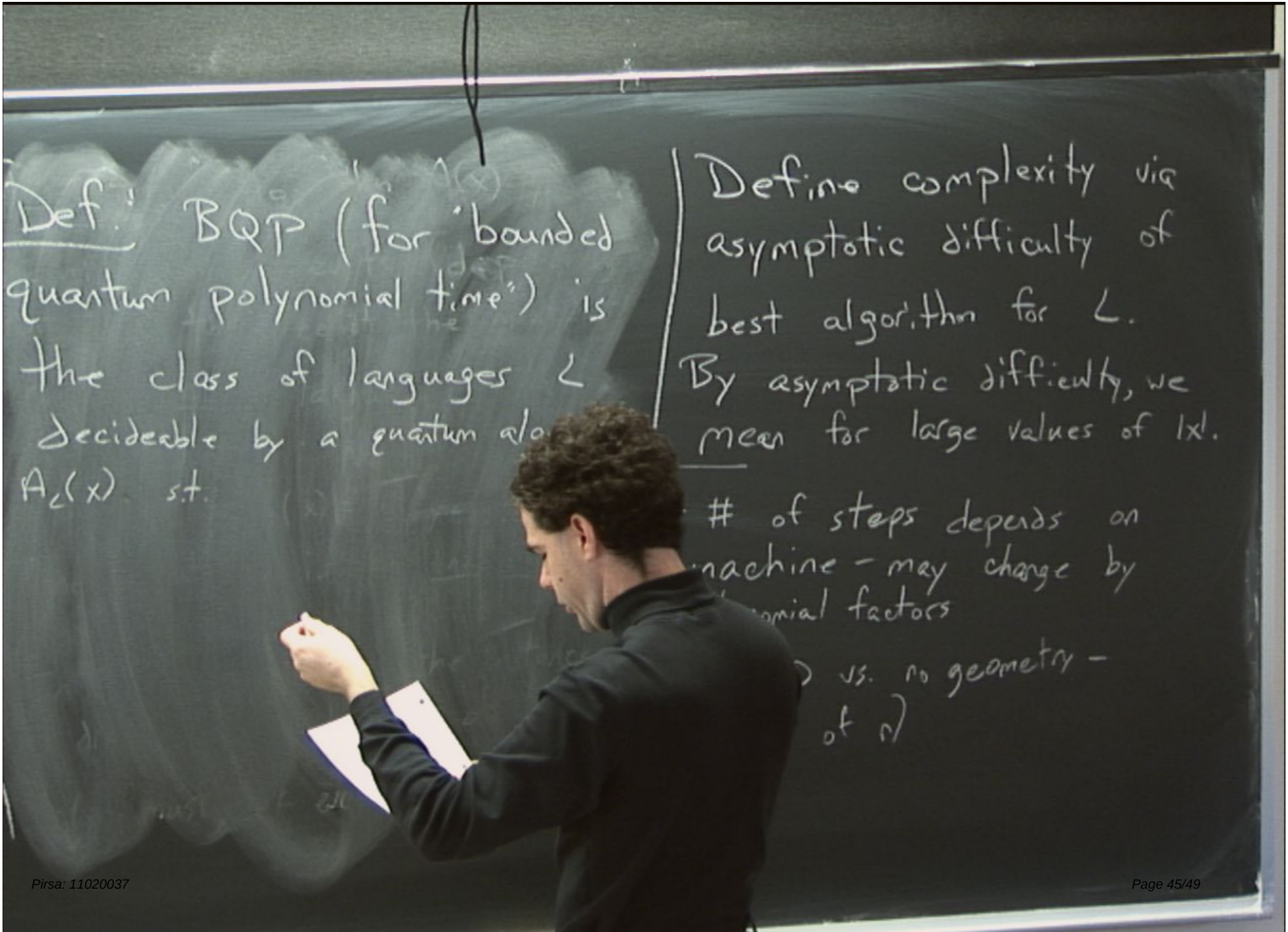
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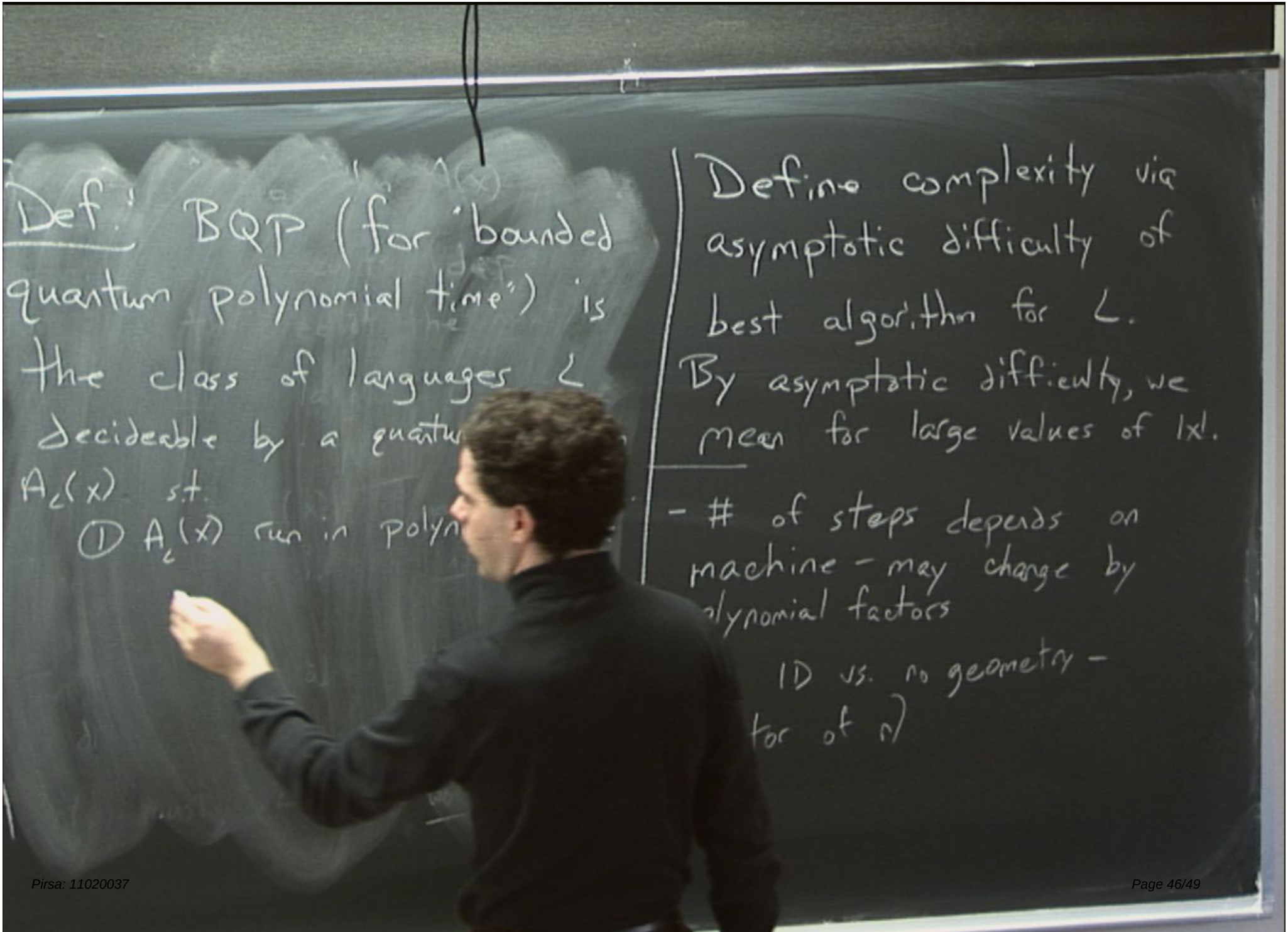
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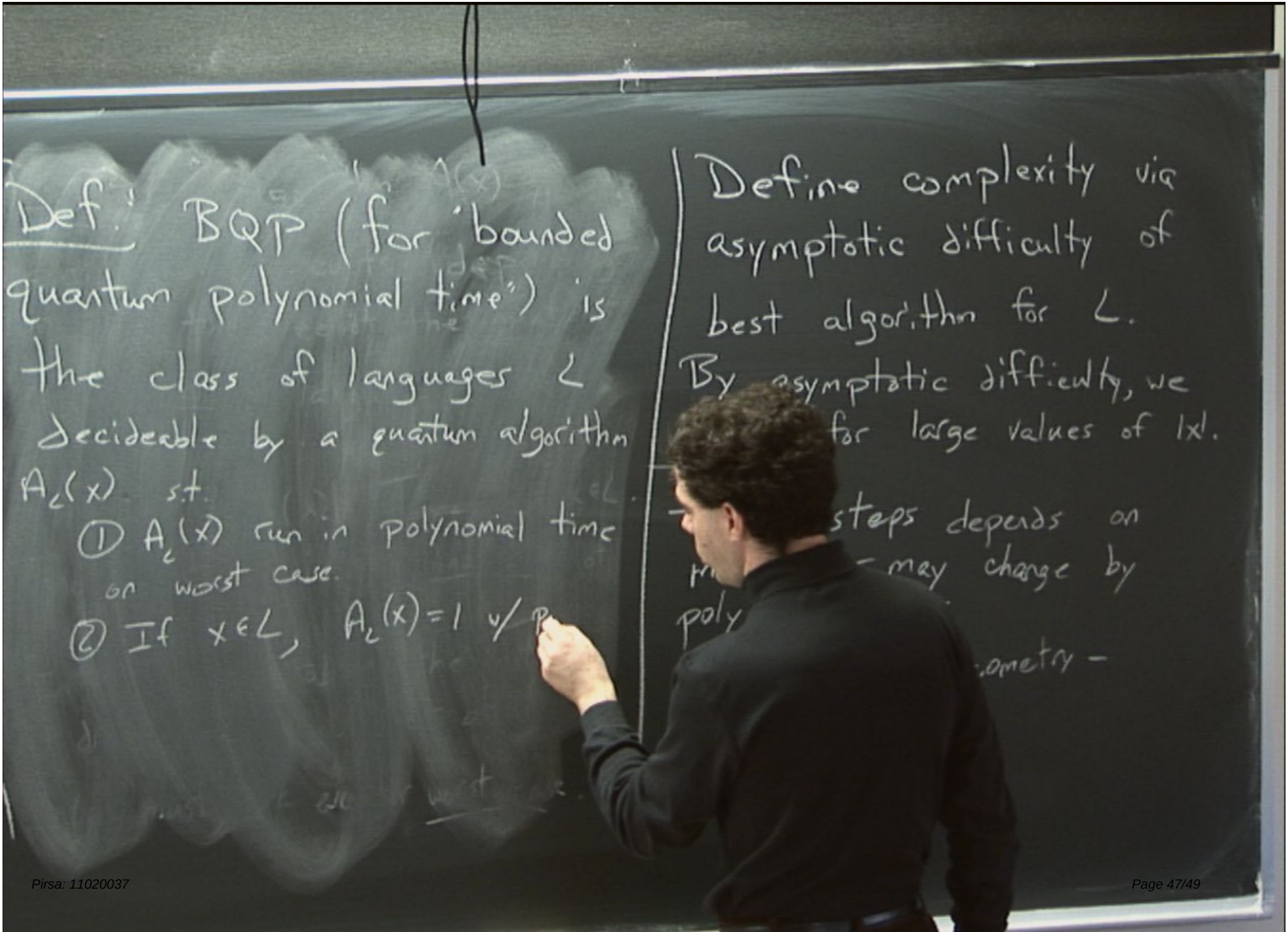
1D vs. no geometry -
tor of n)


Def: BQP (for 'bounded quantum polynomial time') is the class of languages L decideable by a quantum algorithm $A_L(x)$ s.t.
① $A_L(x)$ run in polynomial time on worst case.
② If $x \in L$, $A_L(x) = 1$ w/
Define complexity via asymptotic difficulty of best algorithm for L.
By asymptotic difficulty, we for large values of |x|.
steps depends on
may change by
poly
ometry -

Def: BQP (for "bounded quantum polynomial time") is the class of languages L decidable by a quantum algorithm $A_L(x)$ s.t.

1. $A_L(x)$ run in polynomial time on worst case.
2. If $x \in L$, $A_L(x) = 1$ w/ prob. $\geq \frac{2}{3}$
3. If $x \notin L$, $A_L(x) = 0$ w/ prob. $\geq \frac{2}{3}$

($A_L(x)$ outputs a random variable w/ this value)

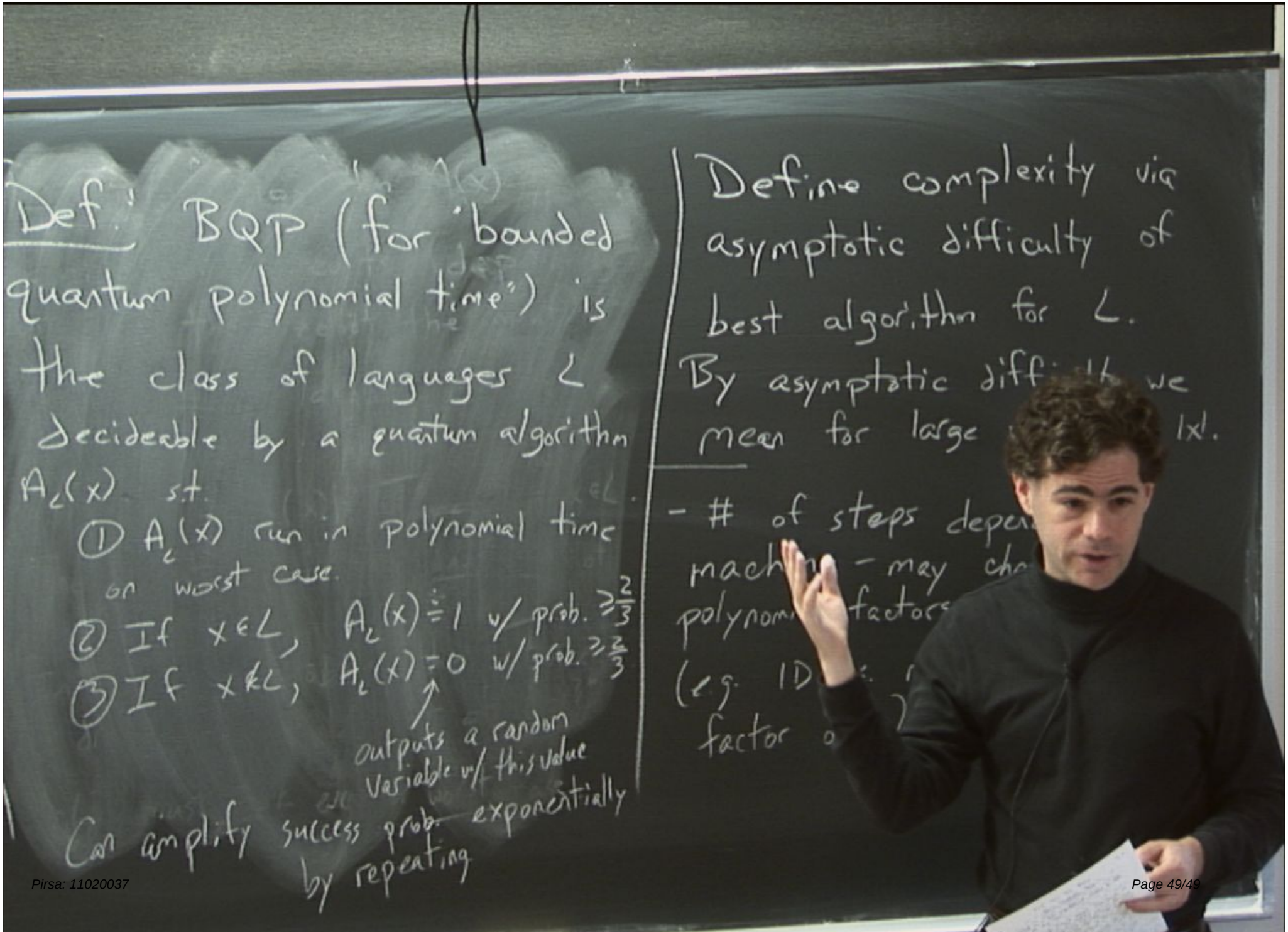
Can amplify success prob. exponentially by repeating

Define complexity via asymptotic difficulty of best algorithm for L.

By asymptotic difficulty, we mean for lar[illegible] of $|x|$.

- # of steps [illegible] on machine - may [illegible] by polynomial fact[illegible]

(e.g. ID vs. [illegible] factor of n)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