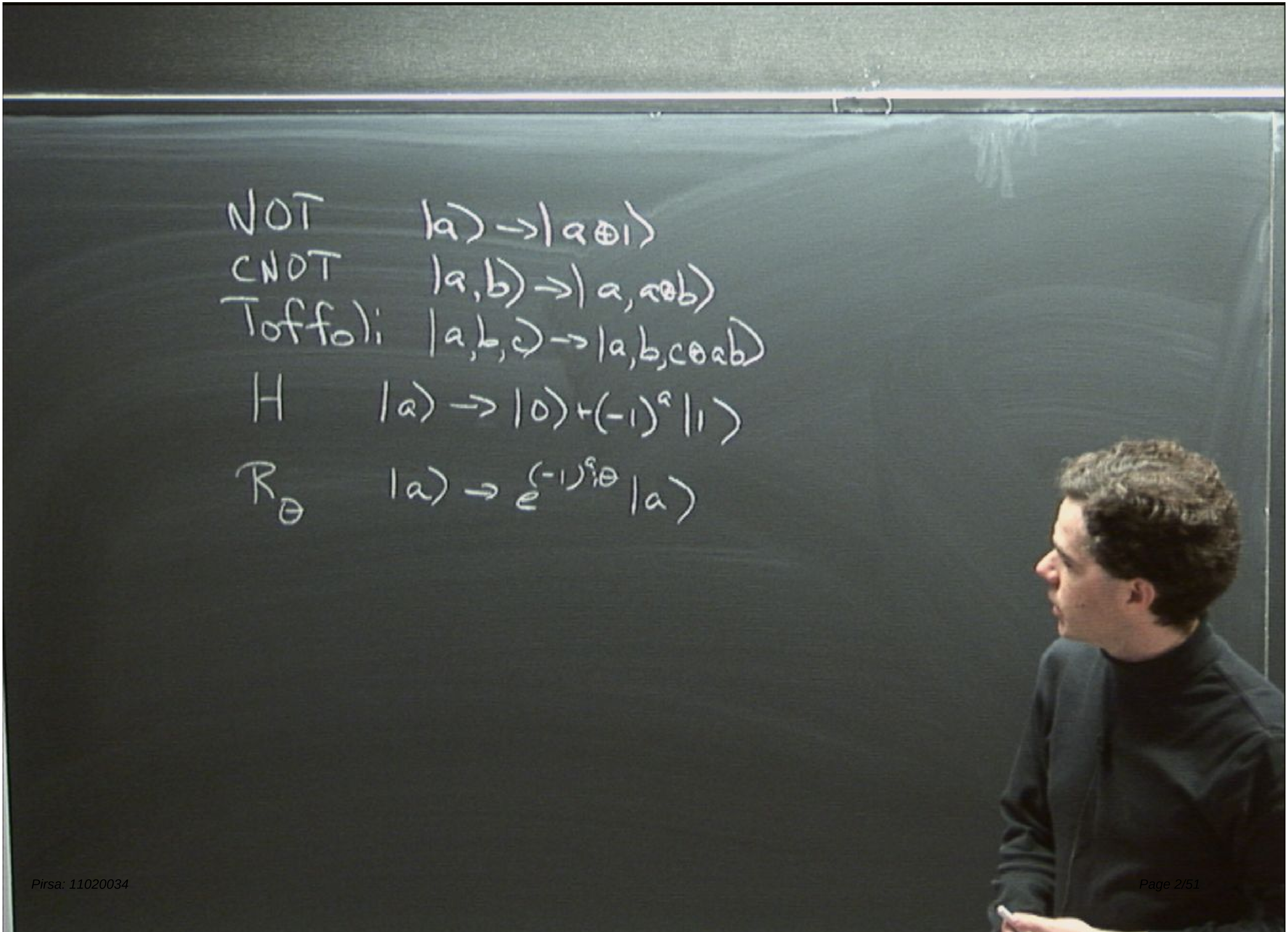
Title: Quantum Information Review - Lecture 2

Date: Feb 15, 2011 09:00 AM

URL: http://pirsa.org/11020034

Abstract:


NOT $|a\rangle \to |a \oplus 1\rangle$
CNOT $|a,b\rangle \to |a, a\oplus b\rangle$
Toffoli: $|a,b,c\rangle \to |a,b,c\oplus ab\rangle$
H $|a\rangle \to |0\rangle + (-1)^a |1\rangle$
R_θ $|a\rangle \to e^{(-1)^a i\theta} |a\rangle$

NOT $|a\rangle \to |a \oplus 1\rangle$

CNOT $|a,b\rangle \to |a, a \oplus b\rangle$

Toffoli: $|a,b,c\rangle \to |a,b,c \oplus ab\rangle$

H $|a\rangle \to |0\rangle + (-1)^a |1\rangle$

R_θ $|a\rangle \to e^{(-1)^a i\theta} |a\rangle$

We want to be able to create any unitary on n qubits $SU(2^n)$

(global phase doesn't matter)

NOT $|a\rangle \to |a \oplus 1\rangle$

CNOT $|a,b\rangle \to |a, a\oplus b\rangle$

Toffoli: $|a,b,c\rangle \to |a,b,c\oplus ab\rangle$

H $|a\rangle \to |0\rangle + (-1)^a |1\rangle$

R_θ $|a\rangle \to e^{(-1)^a i\theta} |a\rangle$

We want to be able to create <u>any</u> unitary on n qubits $SU(2^n)$

(global phase doesn't matter)

A <u>universal</u> set of gates is some set for which any other gate can be written as a

NOT $|a\rangle \to |a \oplus 1\rangle$

CNOT $|a,b\rangle \to |a, a\oplus b\rangle$

Toffoli: $|a,b,c\rangle \to |a,b,c \oplus ab\rangle$

H $|a\rangle \to |0\rangle + (-1)^a |1\rangle$

R_θ $|a\rangle \to e^{(-1)^a i\theta} |a\rangle$

We want to be able to create any unitary on n qubits $SU(2^n)$

(global phase doesn't matter)

A universal set of gates is some set for which any other gate can be written as a product of gates from the universal set.

sal set

es is some
which any
ate can
as a
of gates
the universal

Classically:

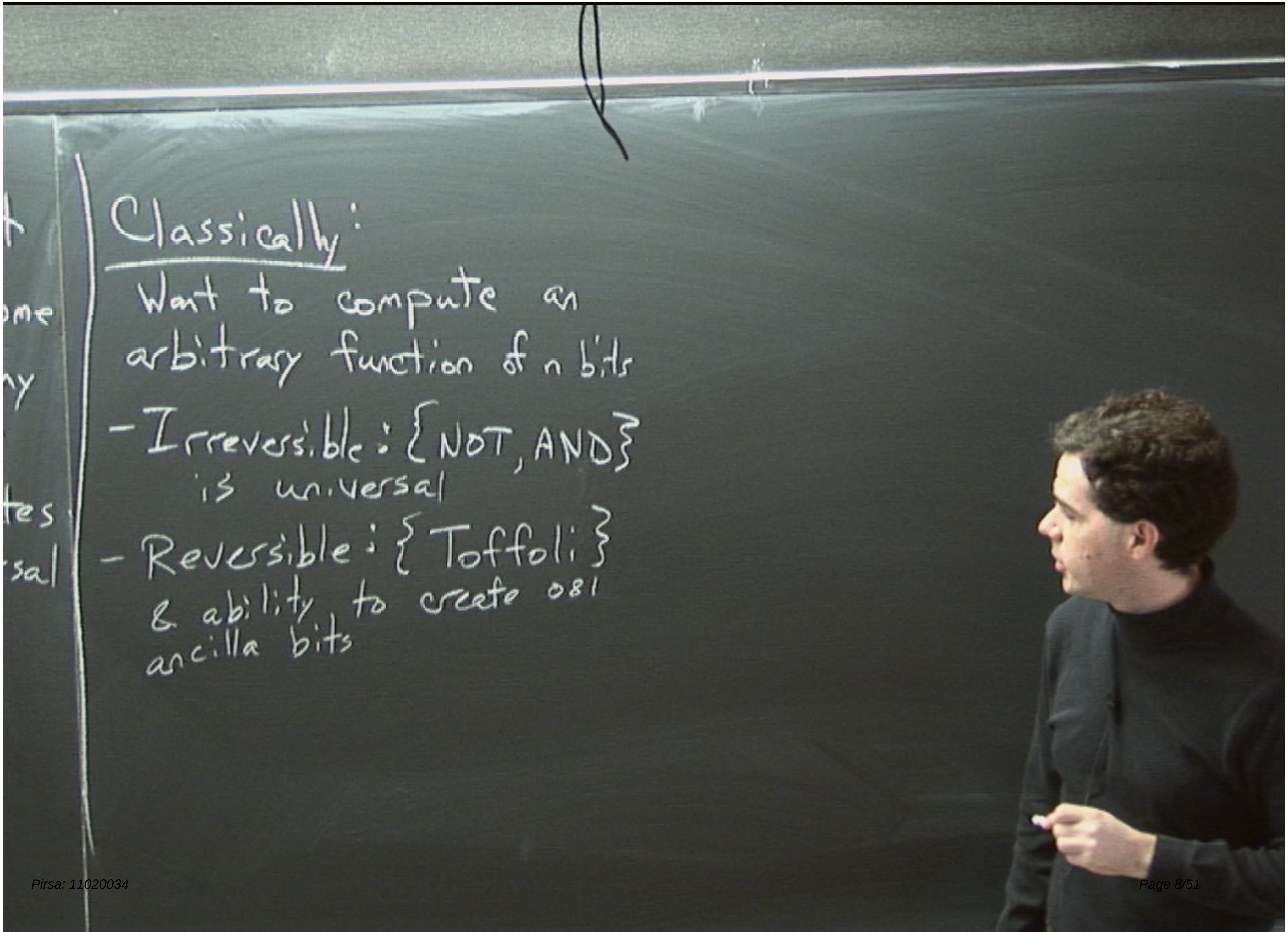
Want to compute an arbitrary function of n bits

- Irreversible: {NOT, AND}

Classically:

Want to compute an arbitrary function of n bits

- Irreversible: {NOT, AND} is universal
- Reversible: {Toffoli}


Classically:
Want to compute an arbitrary function of n bits
- Irreversible: {NOT, AND} is universal
- Reversible: {Toffoli} & ability to create 0&1 ancilla bits

assically:

nt to compute an
bitrary function of n bits

rreversible: {NOT, AND}
is universal

eversible: {Toffoli}
ability to create 0&1
cilla bits

Quantum:

$- \{ \text{CNOT}, \underbrace{\text{single-qubit unitaries}}_{U(2)} \}$

assically:

nt to compute an
bitrary function of n bits

rreversible: {NOT, AND}
is universal

eversible: {Toffoli}
ability to create 0&1
cilla bits

Quantum:

– $\{\text{CNOT}, \underbrace{\text{single-qubit unitaries}}_{U(2)}\}$
is universal

assically:
nt to compute an
bitrary function of n bits

rreversible: {NOT, AND}
is universal

eversible: {Toffoli}
ability to create 0&1
ncilla bits

Quantum:

- $\{\text{CNOT}, \underbrace{\text{single-qubit unitaries}}_{U(2)}\}$ is universal

- $\{\cancel{\text{CNOT}}, H, \text{Tof}, R_{\pi/4}\}$
- $\{\text{CNOT}, H, R_{\pi/8}\}$

assically:

nt to compute an
bitrary function of n bits

rreversible: {NOT, AND}
is universal

eversible: {Toffoli}
ability to create 0&1
ncilla bits

Quantum:

$- \{\text{CNOT}, \underbrace{\text{single-qubit unitaries}}_{U(2)}\}$
is universal

$- \{\cancel{\text{CNOT}}, H, \text{Tof}, R_{\pi/4}\}$

$- \{\text{CNOT}, H, R_{\pi/8}\}$

are universal but only in a weaker sense: Products of these gates are dense in $SU(2^n)$

assically:

nt to compute an
bitrary function of n bits

rreversible: {NOT, AND}
is universal

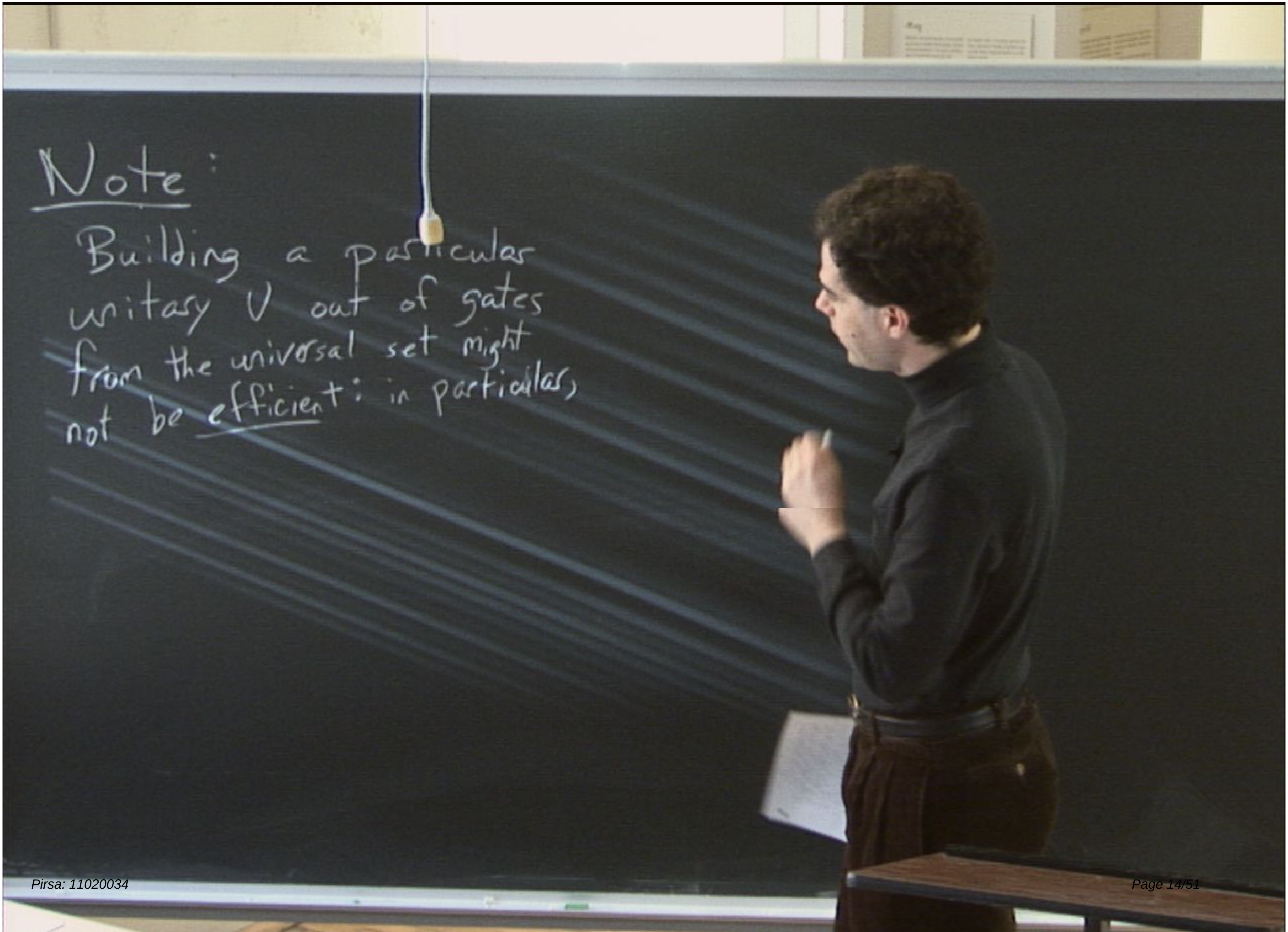
eversible: {Toffoli}
ability to create 0&1
ncilla bits

Quantum:

- {CNOT, $\underbrace{\text{single-qubit uniteries}}_{U(2)}$}
is universal

- {~~CNOT~~, H, Tof, $R_{\pi/4}$}
- {CNOT, H, $R_{\pi/8}$}

are universal but only in a weaker sense: Products of these gates are dense in $SU(2^n)$
(Can approximate unitaries to arbitrary precision)


Note:
Building a particular
unitary U out of gates
from the universal set might
not be efficient: in particular,

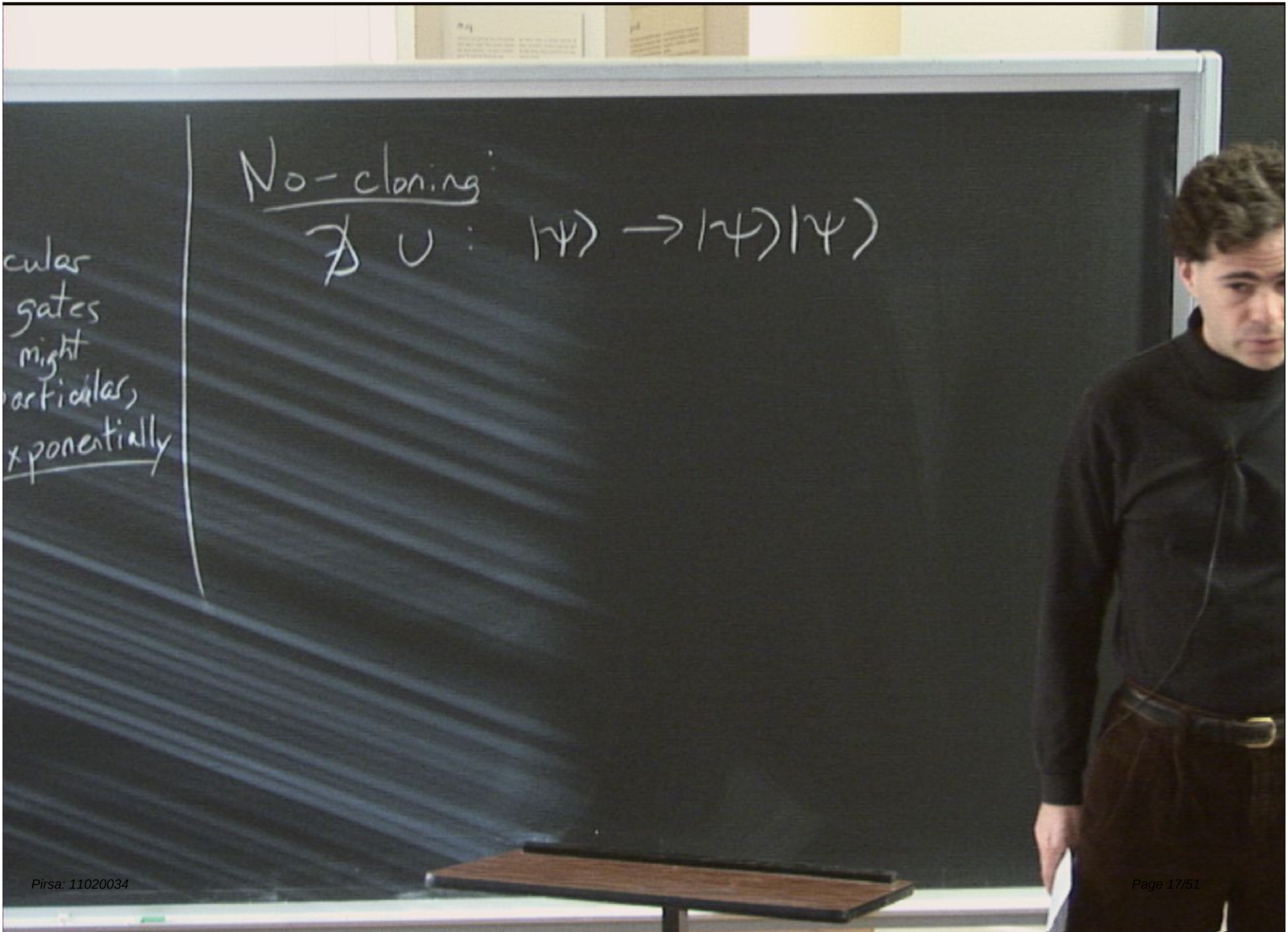
Note:

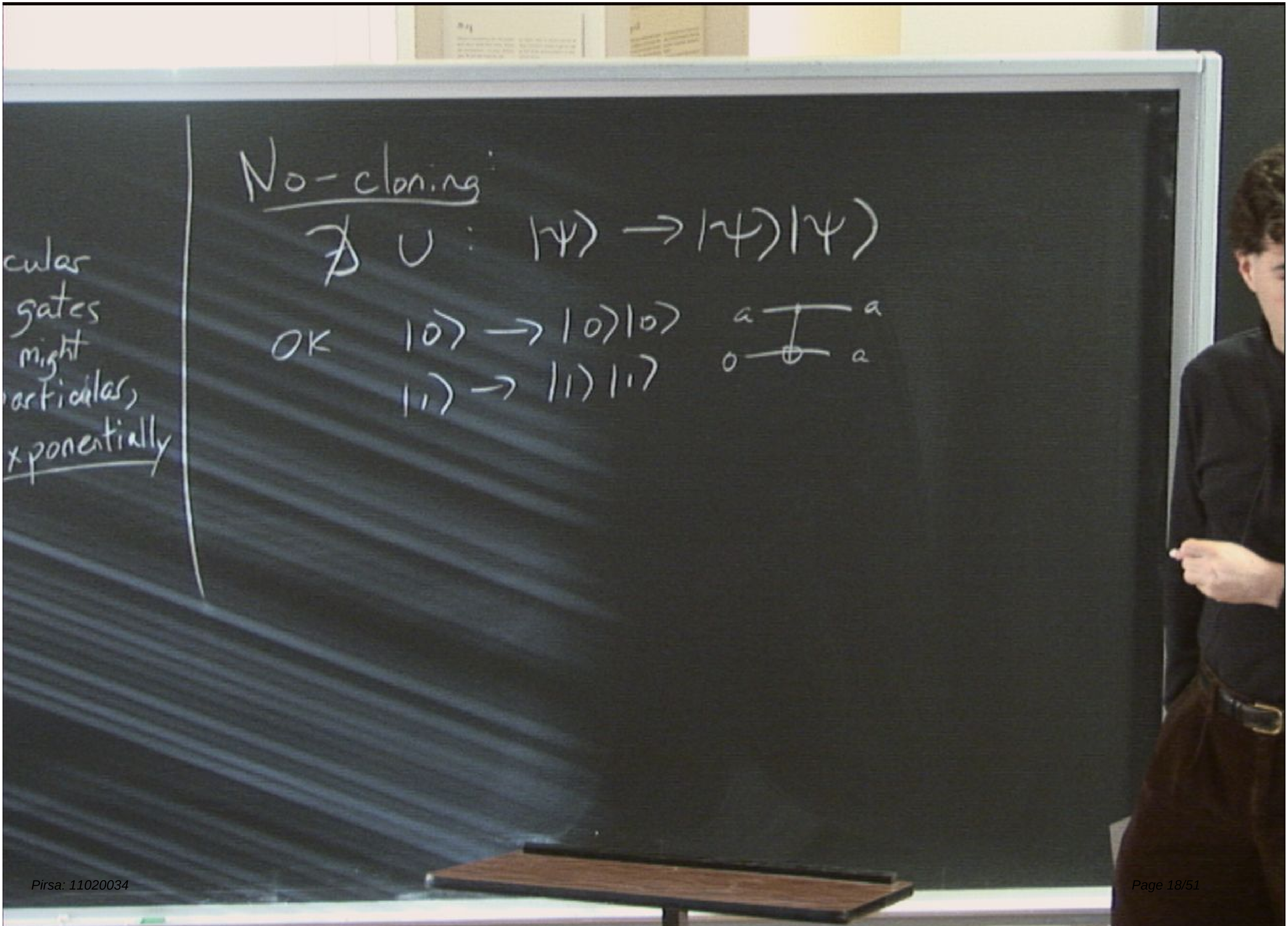
Building a particular unitary U out of gates from the universal set might not be efficient; in particular, some U will require exponentially many gates.

cular
gates
might
articular,
xponentially

No-cloning:

$$\nexists\, U : |\psi\rangle \rightarrow |\psi\rangle|\psi\rangle$$


No-cloning:
$\nexists\, U : |\psi\rangle \rightarrow |\psi\rangle|\psi\rangle$
cular
gates
might
articular,
xponentially


cular
gates
might
articular,
xponentially
No-cloning:
$\nexists\, U : |\psi\rangle \rightarrow |\psi\rangle|\psi\rangle$
OK $|0\rangle \rightarrow |0\rangle|0\rangle$
$|1\rangle \rightarrow |1\rangle|1\rangle$
a a
0 a

cular
gates
might
particular,
exponentially

No-cloning:

$$\nexists\, U : |\psi\rangle \rightarrow |\psi\rangle|\psi\rangle$$

OK: $\begin{cases} |0\rangle \rightarrow |0\rangle|0\rangle \\ |1\rangle \rightarrow |1\rangle|1\rangle \end{cases}$

Doesn't take $\alpha|0\rangle+\beta|1\rangle \rightarrow (\alpha|0\rangle+\beta|1\rangle)\otimes(\alpha|0\rangle+\beta|1\rangle)$

cular
gates
might
articular,
xponentially

No-cloning:

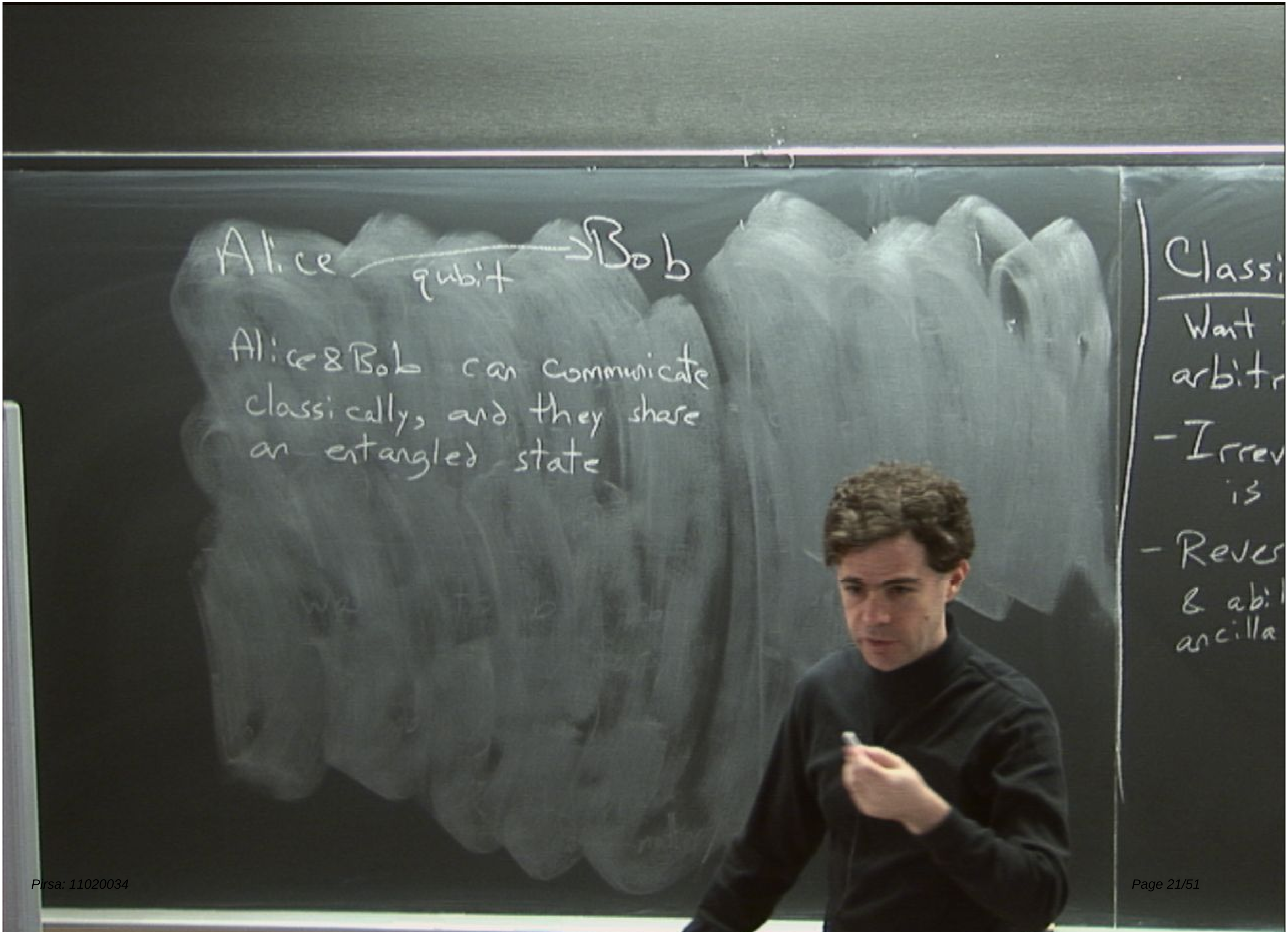
$\nexists\, U : |\psi\rangle \rightarrow |\psi\rangle|\psi\rangle$

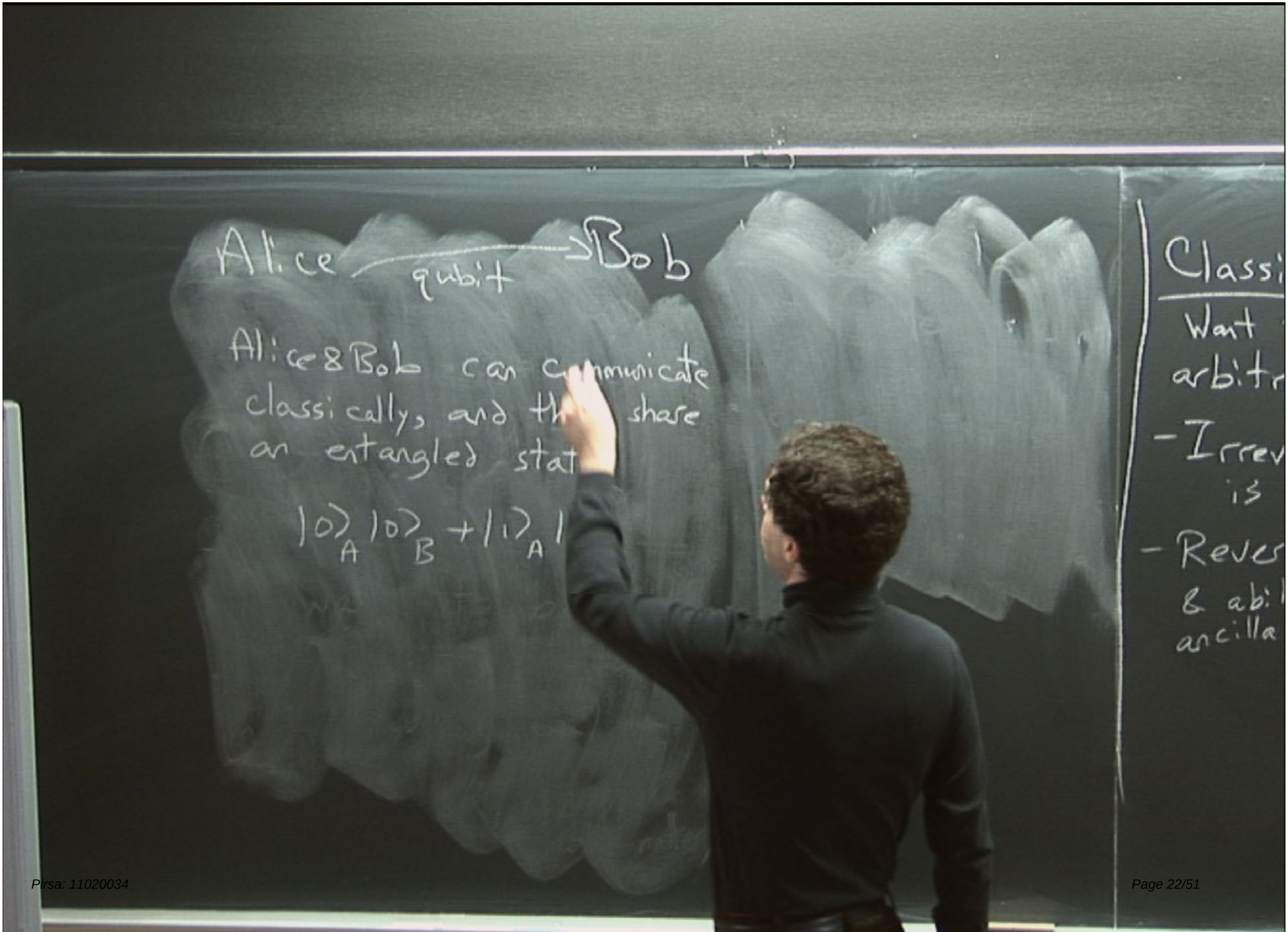
OK: $\begin{cases} |0\rangle \rightarrow |0\rangle|0\rangle \\ |1\rangle \rightarrow |1\rangle|1\rangle \end{cases}$

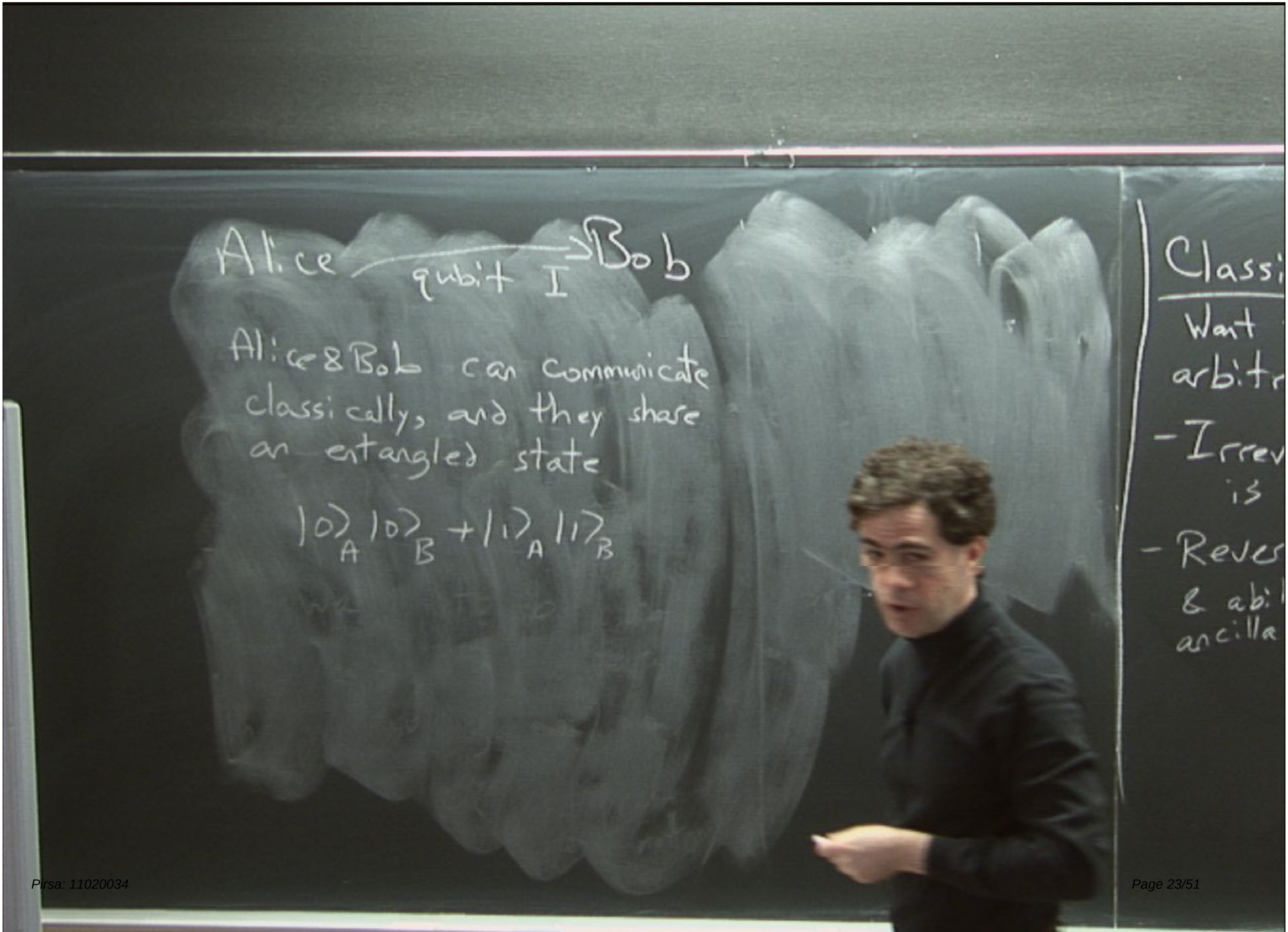
a — a
0 — a

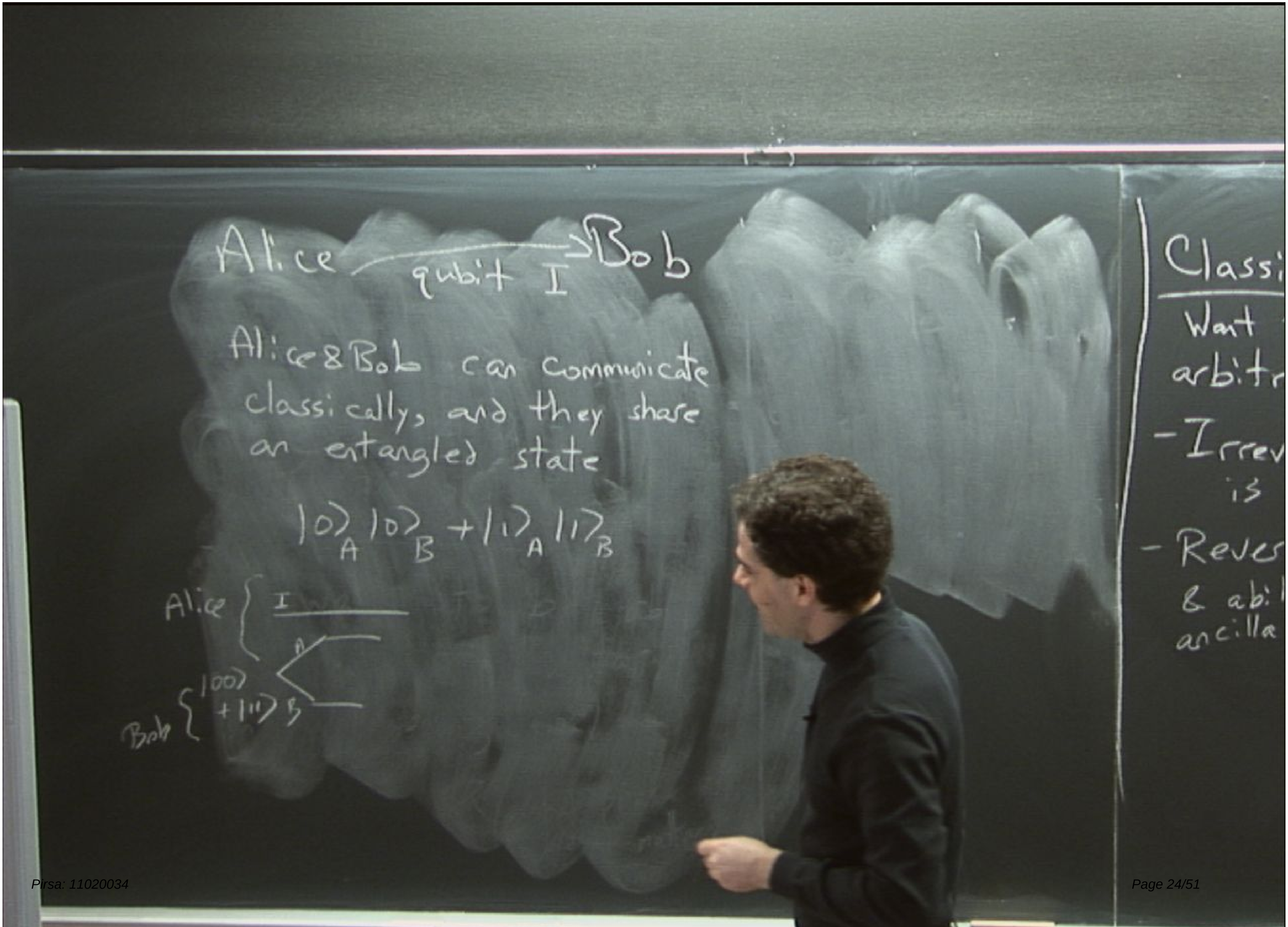
Doesn't take $\alpha|0\rangle+\beta|1\rangle \rightarrow (\alpha|0\rangle+\beta|1\rangle)\otimes(\alpha|0\rangle+\beta|1\rangle)$

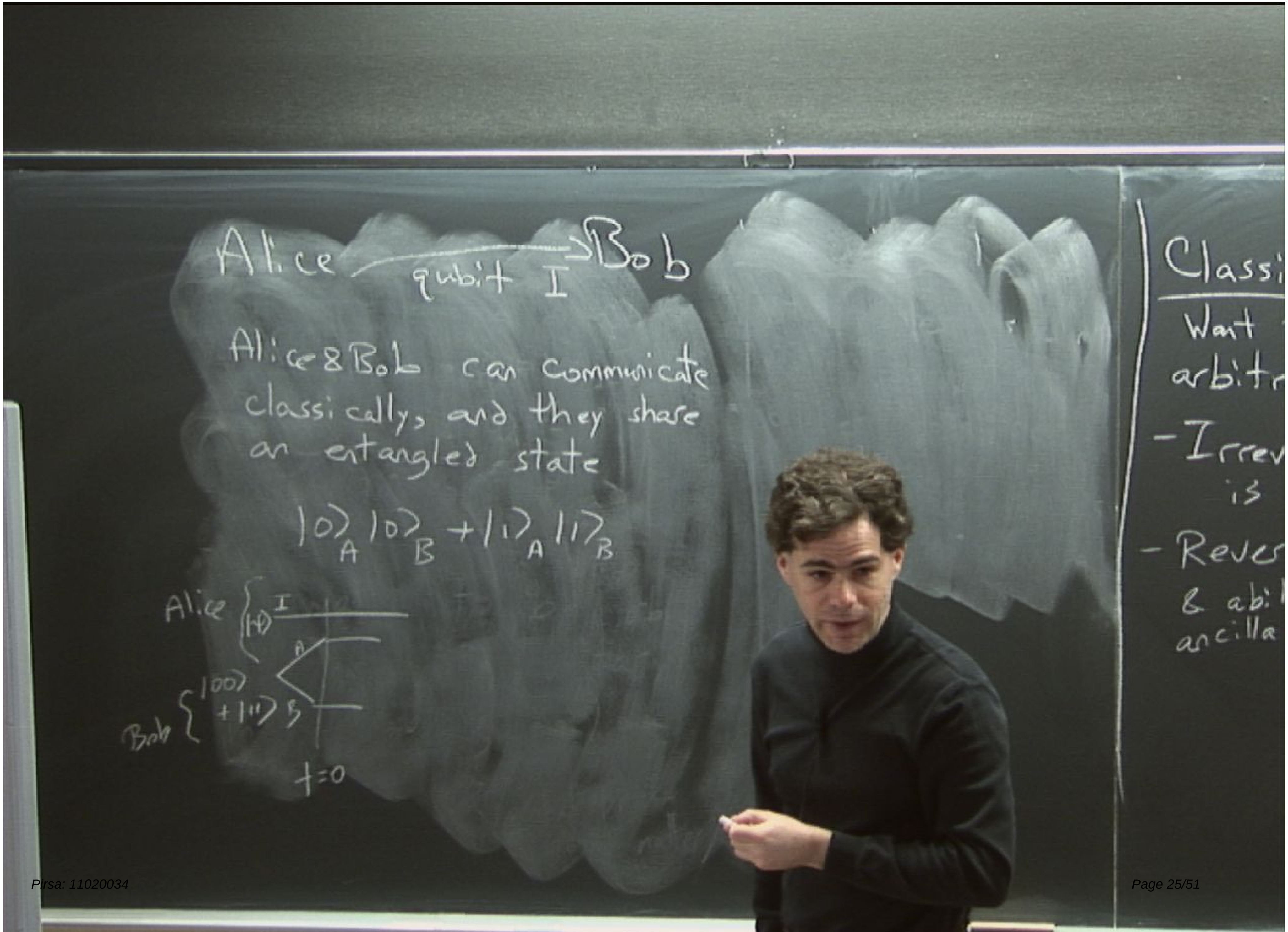
You get $\alpha|00\rangle+\beta|11\rangle$


Alice → Bob
qubit
Alice & Bob can communicate
classically, and they share
an entangled state


Alice qubit Bob
Alice & Bob can communicate
classically, and they share
an entangled state
$|0\rangle_A |0\rangle_B + |1\rangle_A$
Classi
Want
arbitr
- Irrev
is
- Rever
& abi
ancilla


Alice qubit I Bob
Alice & Bob can communicate classically, and they share an entangled state
$|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B$
Classi
Want
arbitr
- Irrev
is
- Rever
& abi
ancilla

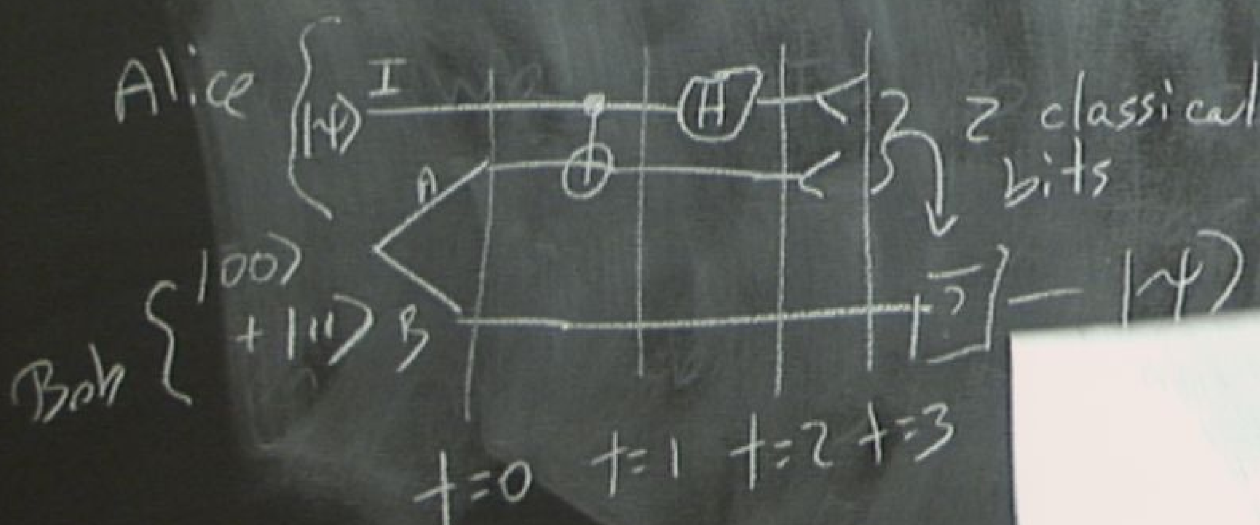

Alice → Bob
qubit I
Alice & Bob can communicate classically, and they share an entangled state
|0⟩A |0⟩B + |1⟩A |1⟩B
Alice
I
A
Bob
|00⟩ + |11⟩
B
Classi
Want
arbitr
- Irrev
is
- Rever
& abi
ancilla


Alice → Bob
qubit I
Alice & Bob can communicate classically, and they share an entangled state
$|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B$
Alice
Bob
t=0
Want

Alice $\xrightarrow[\text{qubit I}]{}$ Bob

Alice & Bob can communicate classically, and they share an entangled state

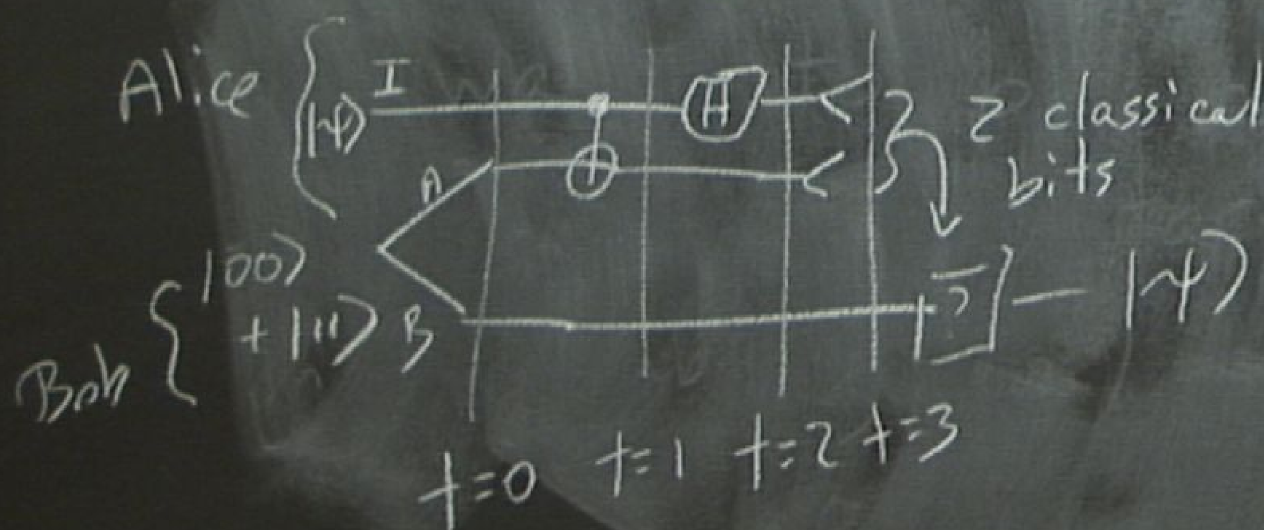
$$|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B$$



Alice $\xrightarrow[\text{qubit } I]{}$ Bob

Alice & Bob can communicate classically, and they share an entangled state

$$|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B$$



$$t=0: \quad |\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$(\alpha|0\rangle_I + \beta|1\rangle_I) \otimes (|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$$

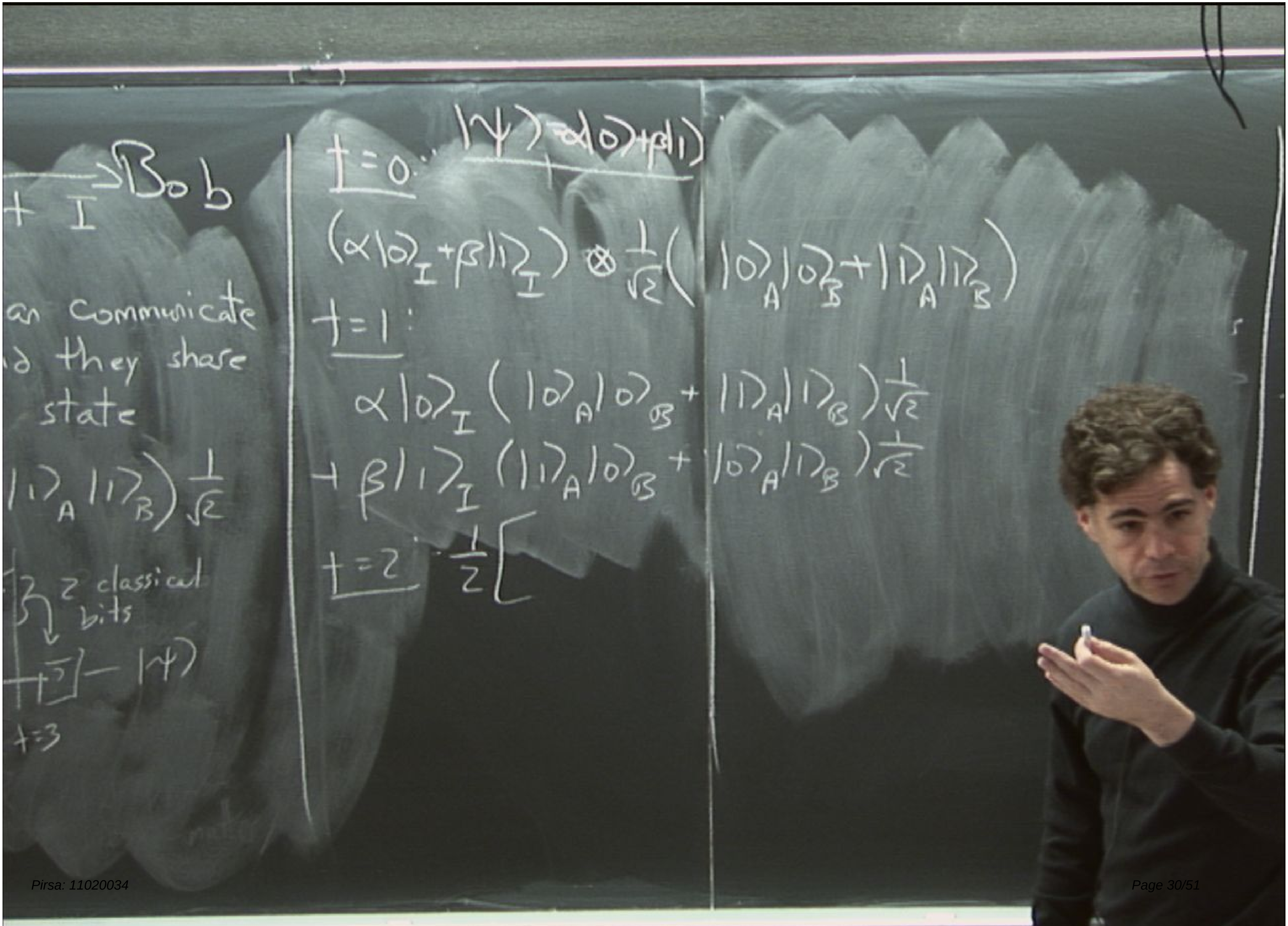
$\underline{t=0}$: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$$(\alpha|0\rangle_I + \beta|1\rangle_I) \otimes (|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$$

$\underline{t=1}$:

$$\alpha|0\rangle_I (|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$$

$$+ \beta|1\rangle_I (|1\rangle_A|0\rangle_B + |0\rangle_A|1\rangle_B)$$


→ Bob
+ I
an communicate
d they share
state
|1⟩_A |1⟩_B) 1/√2
2 classical bits
|ψ⟩
t=3
t = 0: |ψ⟩ = α|0⟩ + β|1⟩
(α|0⟩_I + β|1⟩_I) ⊗ 1/√2 (|0⟩_A|0⟩_B + |1⟩_A|1⟩_B)
t = 1:
α|0⟩_I (|0⟩_A|0⟩_B + |1⟩_A|1⟩_B) 1/√2
+ β|1⟩_I (|1⟩_A|0⟩_B + |0⟩_A|1⟩_B) 1/√2
t = 2: 1/2 [

$\to$ Bob

I

...an communicate

...d they share

... state

$|1\rangle_A |1\rangle_B)\frac{1}{\sqrt{2}}$

2 classical bits

$- |\psi\rangle$

$t=3$

$t=0$: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$$(\alpha|0\rangle_I + \beta|1\rangle_I) \otimes \frac{1}{\sqrt{2}}\left(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\right)$$

$t=1$:

$$\alpha|0\rangle_I\left(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\right)\frac{1}{\sqrt{2}}$$

$$+\beta|1\rangle_I\left(|1\rangle_A|0\rangle_B + |0\rangle_A|1\rangle_B\right)\frac{1}{\sqrt{2}}$$

$t=2$:

$$\frac{1}{2}\Big[\alpha|0\rangle_I|0\rangle_A|0\rangle_B + \alpha|1\rangle_I|0\rangle_A|0\rangle_B$$

$$+\alpha|0\rangle_I|1\rangle_A|1\rangle_B + \alpha|1\rangle_I|1\rangle_A|1\rangle_B +$$

$$+\beta|0\rangle_I|1\rangle_A|0\rangle_B - \beta|1\rangle_I|1\rangle_A|0\rangle_B$$

$\to$ Bob

$+\ I$

an communicate

d they share

state

$|1\rangle_A |1\rangle_B) \frac{1}{\sqrt{2}}$

$\}$ 2 classical bits

$-|\psi\rangle$

$t=3$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$\underline{t=0}$:

$$(\alpha|0\rangle_I + \beta|1\rangle_I) \otimes \frac{1}{\sqrt{2}}\left(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\right)$$

$\underline{t=1}$:

$$\alpha|0\rangle_I\left(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\right)\frac{1}{\sqrt{2}}$$

$$+\beta|1\rangle_I\left(|1\rangle_A|0\rangle_B + |0\rangle_A|1\rangle_B\right)\frac{1}{\sqrt{2}}$$

$\underline{t=2}$:

$$\frac{1}{2}\Big[\alpha|0\rangle_I|0\rangle_A|0\rangle_B + \alpha|1\rangle_I|0\rangle_A|0\rangle_B$$

$$+\alpha|0\rangle_I|1\rangle_A|1\rangle_B + \alpha|1\rangle_I|1\rangle_A|1\rangle_B +$$

$$+\beta|0\rangle_I|1\rangle_A|0\rangle_B - \beta|1\rangle_I|1\rangle_A|0\rangle_B +$$

$$+\beta|0\rangle_I|0\rangle_A|1\rangle_B - \beta|1\rangle_I|0\rangle_A|1\rangle_B\Big)$$

Bob

municate

share

$\frac{1}{\sqrt{2}}$

ssical

$|\psi\rangle$

$t=0$: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$$(\alpha|0\rangle_I + \beta|1\rangle_I) \otimes \frac{1}{\sqrt{2}}\left(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\right)$$

$t=1$:

$$\alpha|0\rangle_I\left(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\right)\frac{1}{\sqrt{2}}$$

$$+\beta|1\rangle_I\left(|1\rangle_A|0\rangle_B + |0\rangle_A|1\rangle_B\right)\frac{1}{\sqrt{2}}$$

$t=2$:

$$\frac{1}{2}\Big[\alpha|0\rangle_I|0\rangle_A|0\rangle_B + \alpha|1\rangle_I|0\rangle_A|0\rangle_B +$$

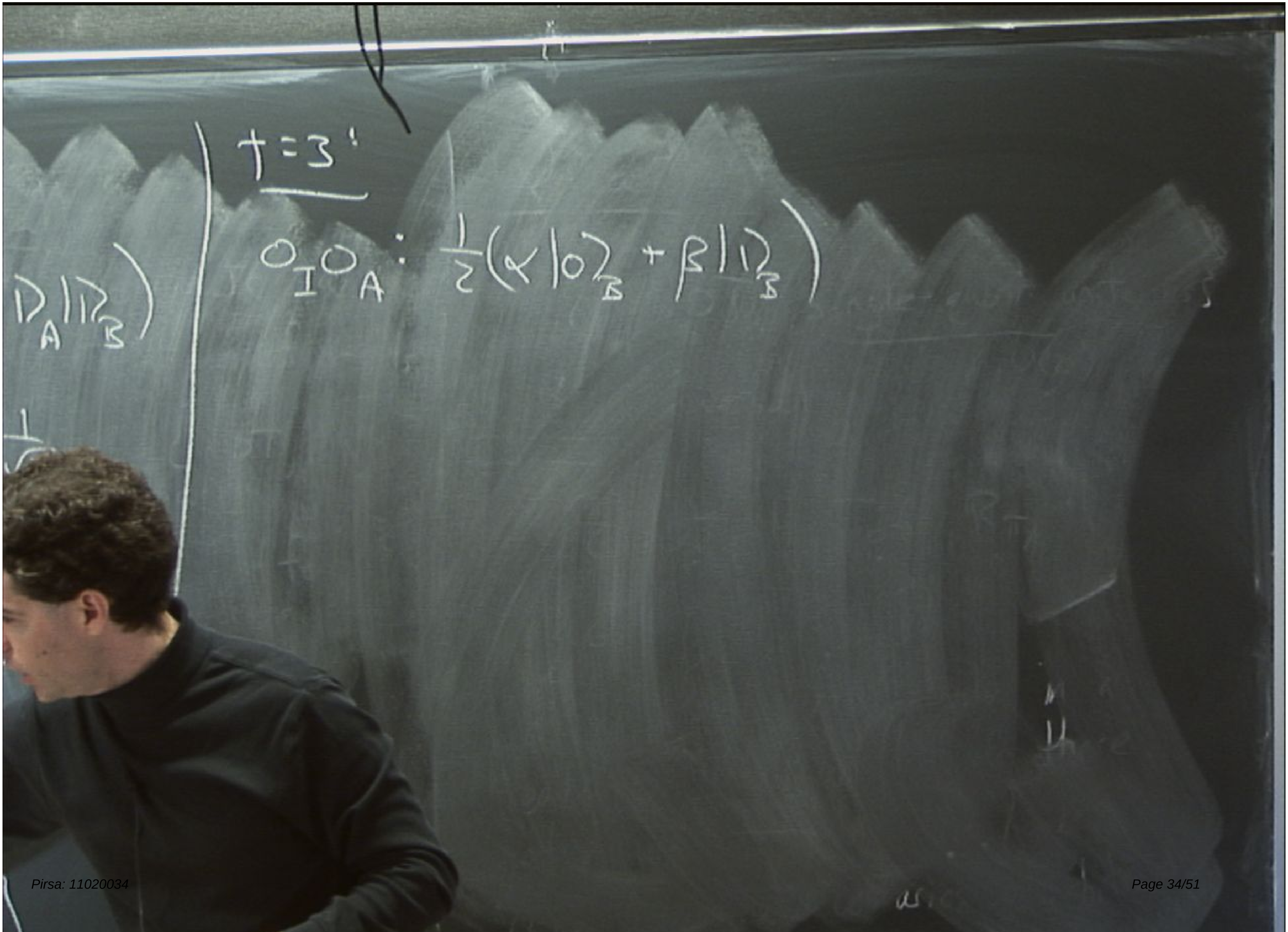
$$+\alpha|0\rangle_I|1\rangle_A|1\rangle_B + \alpha|1\rangle_I|1\rangle_A|1\rangle_B +$$

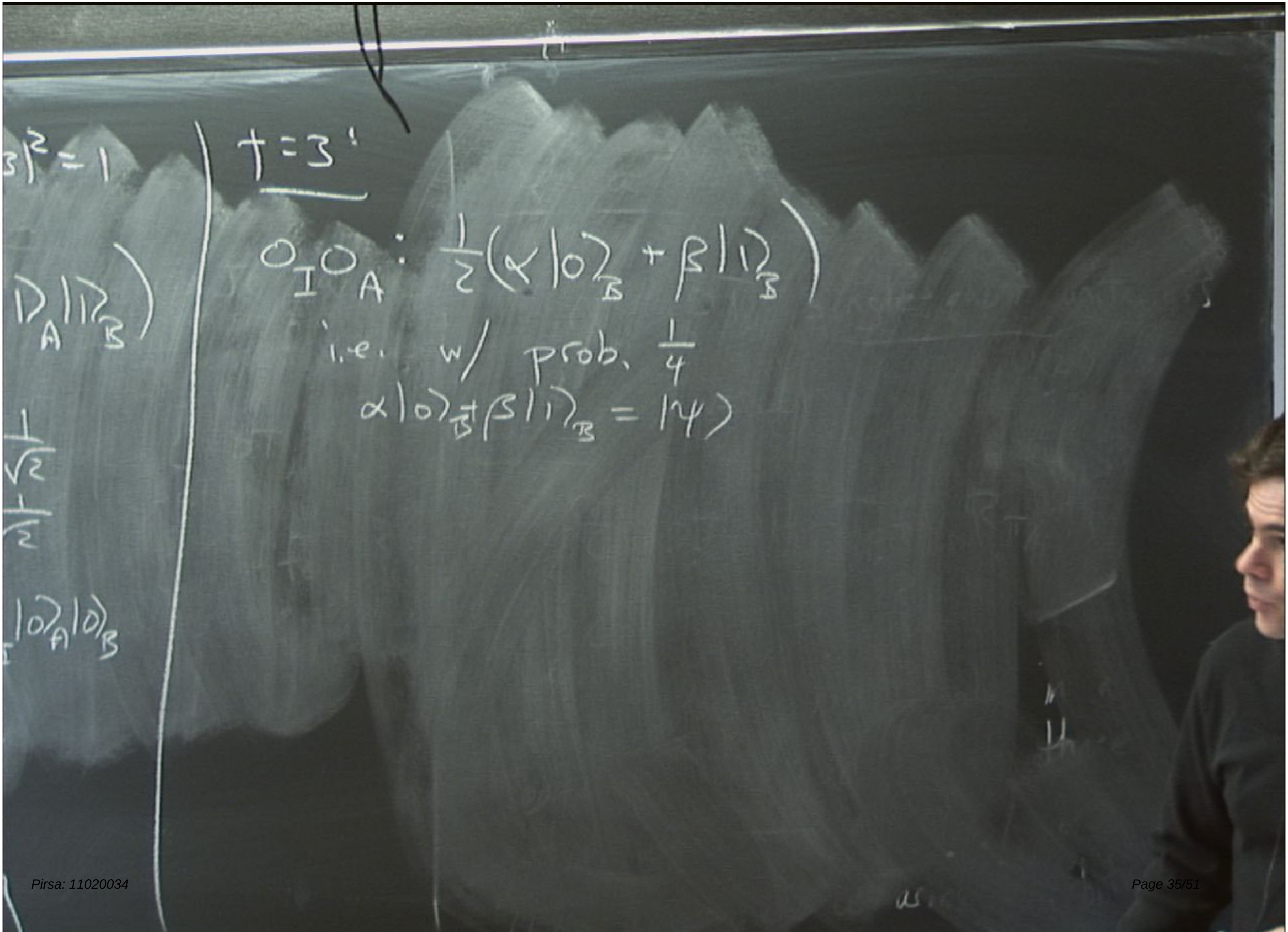
$$+\beta|0\rangle_I|1\rangle_A|0\rangle_B - \beta|1\rangle_I|1\rangle_A|0\rangle_B +$$

$$+\beta|0\rangle_I|0\rangle_A|1\rangle_B - \beta|1\rangle_I|0\rangle_A|1\rangle_B\Big]$$

$t=$

0


$t=3$:
$0_I 0_A : \frac{1}{2}(\alpha|0\rangle_B + \beta|1\rangle_B)$
$1\rangle_A|1\rangle_B)$


t = 3'
$O_I O_A : \frac{1}{2}(\alpha|0\rangle_B + \beta|1\rangle_B)$
i.e. w/ prob. $\frac{1}{4}$
$\alpha|0\rangle_B + \beta|1\rangle_B = |\psi\rangle$

$\beta|^2 = 1$

$1\rangle_A |1\rangle_B)$

$\frac{1}{\sqrt{2}}$

$\frac{1}{2}$

$|0\rangle_A |0\rangle_B$

$t = 3$!

$0_I 0_A : \frac{1}{2}(\alpha|0\rangle_B + \beta|1\rangle_B)$

i.e. w/ prob. $\frac{1}{4}$

$\alpha|0\rangle_B + \beta|1\rangle_B = |\psi\rangle$

$0_I 1_A : \alpha|1\rangle_B + \beta|0\rangle_B =$

t=3:

$0_I 0_A : \frac{1}{2}(\alpha|0\rangle_B + \beta|1\rangle_B)$

i.e. w/ prob. $\frac{1}{4}$

$\alpha|0\rangle_B + \beta|1\rangle_B = |\psi\rangle$

$0_I 1_A : \alpha|1\rangle_B + \beta|0\rangle_B = X|\psi\rangle$

$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$t=3$:

$0_I 0_A : \frac{1}{2}\left(\alpha|0\rangle_B + \beta|1\rangle_B\right)$

i.e. w/ prob. $\frac{1}{4}$

$\alpha|0\rangle_B + \beta|1\rangle_B = |\psi\rangle$

$0_I 1_A : \alpha|1\rangle_B + \beta|0\rangle_B = X|\psi\rangle$

w/ prob. $\frac{1}{4}$

$1_I 0_A : \alpha|0\rangle_B - \beta|1\rangle_B = Z|\psi\rangle$

$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$t=3$:

$0_I 0_A : \frac{1}{2}(\alpha|0\rangle_B + \beta|1\rangle_B)$

i.e. w/ prob. $\frac{1}{4}$

$\alpha|0\rangle_B + \beta|1\rangle_B = |\psi\rangle$

$0_I 1_A : \alpha|1\rangle_B + \beta|0\rangle_B = X|\psi\rangle$

w/ prob. $\frac{1}{4}$

$1_I 0_A : \alpha|0\rangle_B - \beta|1\rangle_B = Z|\psi\rangle$

$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$t=3$:

$0_I 0_A : \frac{1}{2}\left(\alpha|0\rangle_B + \beta|1\rangle_B\right)$

i.e. w/ prob. $\frac{1}{4}$

$\alpha|0\rangle_B + \beta|1\rangle_B = |\psi\rangle$

$0_I 1_A : \alpha|1\rangle_B + \beta|0\rangle_B = X|\psi\rangle$

w/ prob. $\frac{1}{4}$

$1_I 0_A : \alpha|0\rangle_B - \beta|1\rangle_B = Z|\psi\rangle$

w/ prob. $\frac{1}{4}$

$1_I 1_A : \alpha|1\rangle_B - \beta|0\rangle_B = -iY|\psi\rangle$

$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

t=3:

$0_I 0_A : \frac{1}{2}\left(\alpha|0\rangle_B + \beta|1\rangle_B\right)$

i.e. w/ prob. $\frac{1}{4}$

$\alpha|0\rangle_B + \beta|1\rangle_B = |\psi\rangle$

$0_I 1_A : \alpha|1\rangle_B + \beta|0\rangle_B = X|\psi\rangle$

w/ prob. $\frac{1}{4}$

$1_I 0_A : \alpha|0\rangle_B - \beta|1\rangle_B = Z|\psi\rangle$

w/ prob. $\frac{1}{4}$

$1_I 1_A : \alpha|1\rangle_B - \beta|0\rangle_B = -iY|\psi\rangle$

w/ prob. $\frac{1}{4}$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Bob

mmunicate

y share

$\rangle_B) \frac{1}{\sqrt{2}}$

ssical

$|\psi\rangle$

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad |\alpha|^2 + |\beta|^2 = 1$

$\underline{t=0}$:

$$(\alpha|0\rangle_I + \beta|1\rangle_I) \otimes \frac{1}{\sqrt{2}}\left(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\right)$$

$\underline{t=1}$:

$$\alpha|0\rangle_I\left(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\right)\frac{1}{\sqrt{2}}$$

$$+\beta|1\rangle_I\left(|1\rangle_A|0\rangle_B + |0\rangle_A|1\rangle_B\right)\frac{1}{\sqrt{2}}$$

$\underline{t=2}$:

$$\frac{1}{2}\Big[\alpha|0\rangle_I|0\rangle_A|0\rangle_B + \alpha|1\rangle_I|0\rangle_A|0\rangle_B$$

$$+\alpha|0\rangle_I|1\rangle_A|1\rangle_B + \alpha|1\rangle_I|1\rangle_A|1\rangle_B +$$

$$+\beta|0\rangle_I|1\rangle_A|0\rangle_B - \beta|1\rangle_I|1\rangle_A|0\rangle_B +$$

$$+\beta|0\rangle_I|0\rangle_A|1\rangle_B - \beta|1\rangle_I|0\rangle_A|1\rangle_B\Big)$$

$\underline{t=3}$:

$0_I 0_A$

i.e.

α

$0_I 1_A$

w/ F

$1_I 0_A$:

w

pr

Bob

...mmunicate
...y share

$\left.\right)_B\Big)\frac{1}{\sqrt{2}}$

...ssical

$|\psi\rangle$

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad |\alpha|^2 + |\beta|^2 = 1$

$t=0$:

$$(\alpha|0\rangle_I + \beta|1\rangle_I) \otimes \frac{1}{\sqrt{2}}\left(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\right)$$

$t=1$:

$$\alpha|0\rangle_I\left(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\right)\frac{1}{\sqrt{2}}$$

$$+\beta|1\rangle_I\left(|1\rangle_A|0\rangle_B + |0\rangle_A|1\rangle_B\right)\frac{1}{\sqrt{2}}$$

$t=2$:

$$\frac{1}{2}\Big[\alpha|0\rangle_I|0\rangle_A|0\rangle_B + \alpha|1\rangle_I|0\rangle_A|0\rangle_B$$

$$+\alpha|0\rangle_I|1\rangle_A|1\rangle_B + \alpha|1\rangle_I|1\rangle_A|1\rangle_B +$$

$$+\beta|0\rangle_I|1\rangle_A|0\rangle_B - \beta|1\rangle_I|1\rangle_A|0\rangle_B +$$

$$+\beta|0\rangle_I|0\rangle_A|1\rangle_B - \beta|1\rangle_I|0\rangle_A|1\rangle_B\Big]$$

$t=3$:

$0_I 0_A$

i.e.

α

$0_I 1_A$:

w/ p...

$1_I 0_A$:

w/ ...

$1_I 1_A$:

w/ prob...

$\Rightarrow$ Bob

communicate
they share
ate

$|1\rangle_B) \frac{1}{\sqrt{2}}$

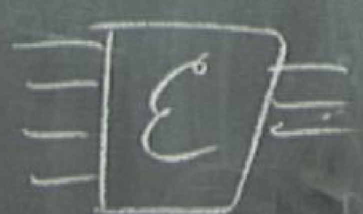
classical
bits

$\rangle - |\psi\rangle$

(If $\mathcal{E}$ is a completely positive, trace-preserving map,)

$t=3$:

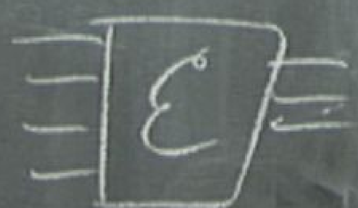
(If $\mathcal{E}$ is a completely positive, trace-preserving map, then $\mathcal{E}$ can be purified:)



$O_I O_A$:

i.e.

(If $\mathcal{E}$ is a completely positive, trace-preserving map, then $\mathcal{E}$ can be purified:



$t=3$:

$0_I 0_A$:

i.e.

$\alpha|0$

$0_I 1_A$: α

w/ pro

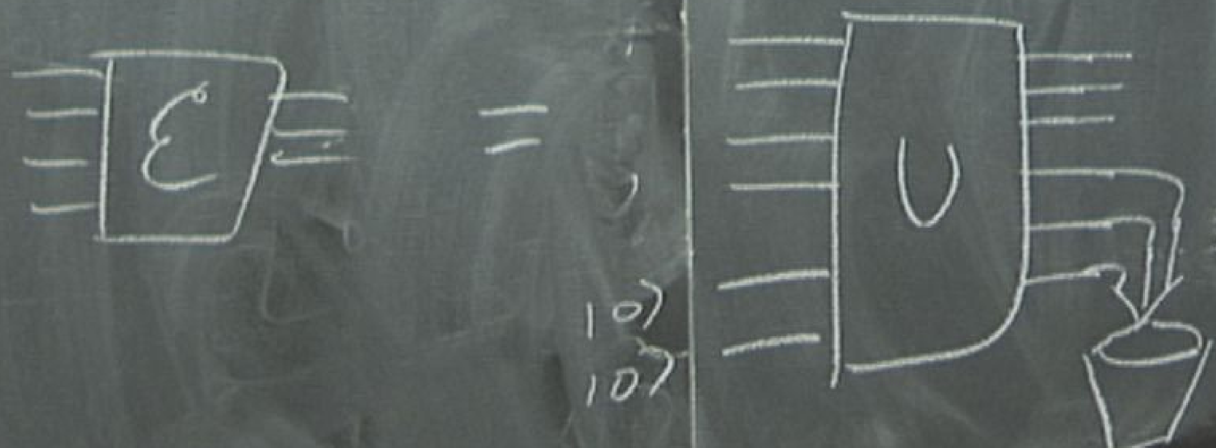
$1_I 0_A$: α

w/ pr

$1_I 1_A$: $\alpha|$

w/ pro

(If $\mathcal{E}$ is a completely positive, trace-preserving map, then $\mathcal{E}$ can be purified:



$t=3$:

$0_I 0_A$:

i.e.

$\alpha|0$

$0_I 1_A$: α

w/ pro

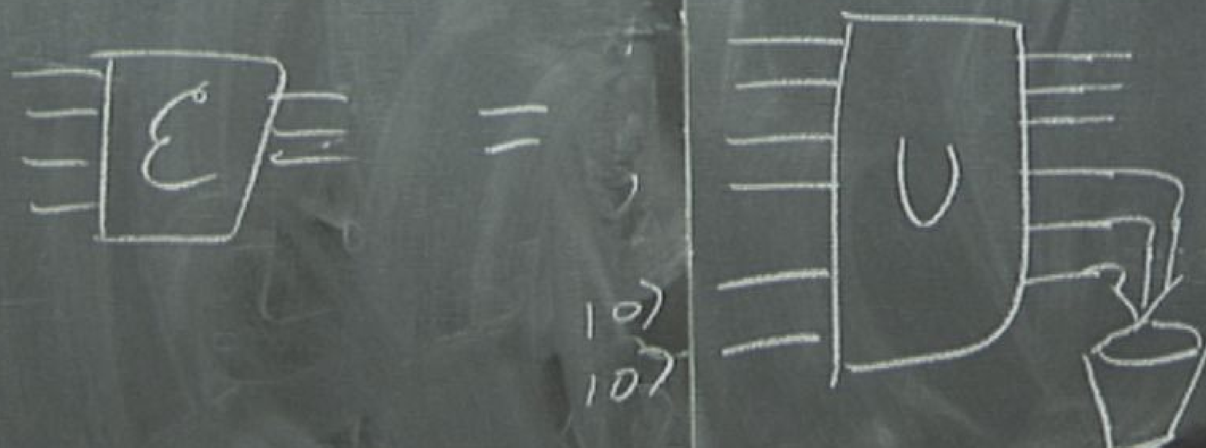
$1_I 0_A$: α

w/ pro

$1_I 1_A$: $\alpha|$

w/ prob.

(If $\mathcal{E}$ is a completely positive, trace-preserving map, then $\mathcal{E}$ can be purified:)



$t=3$:

$0_I 0_A$:

i.e.

$\alpha|0\rangle$

$0_I 1_A$: α

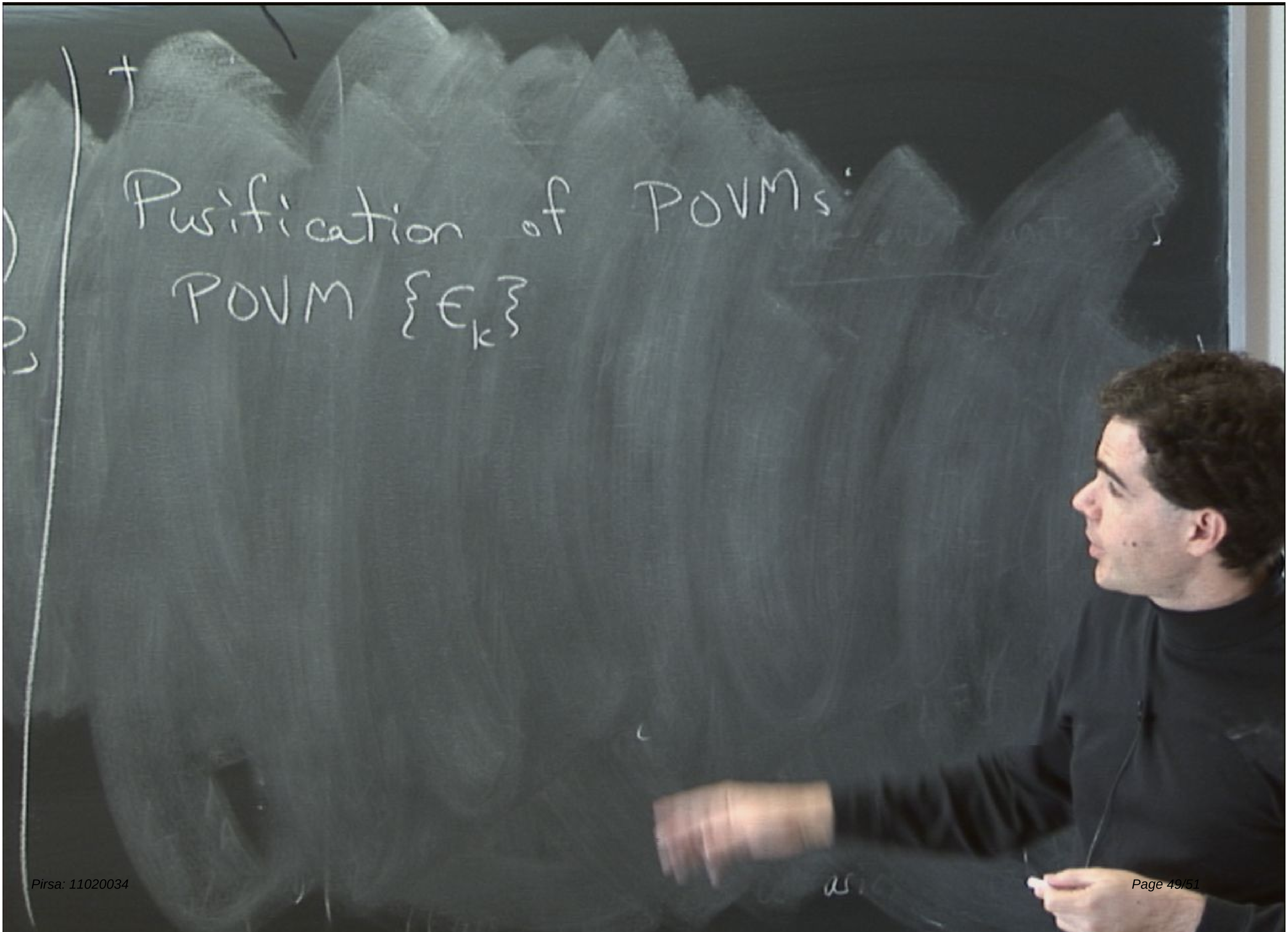
w/ prob

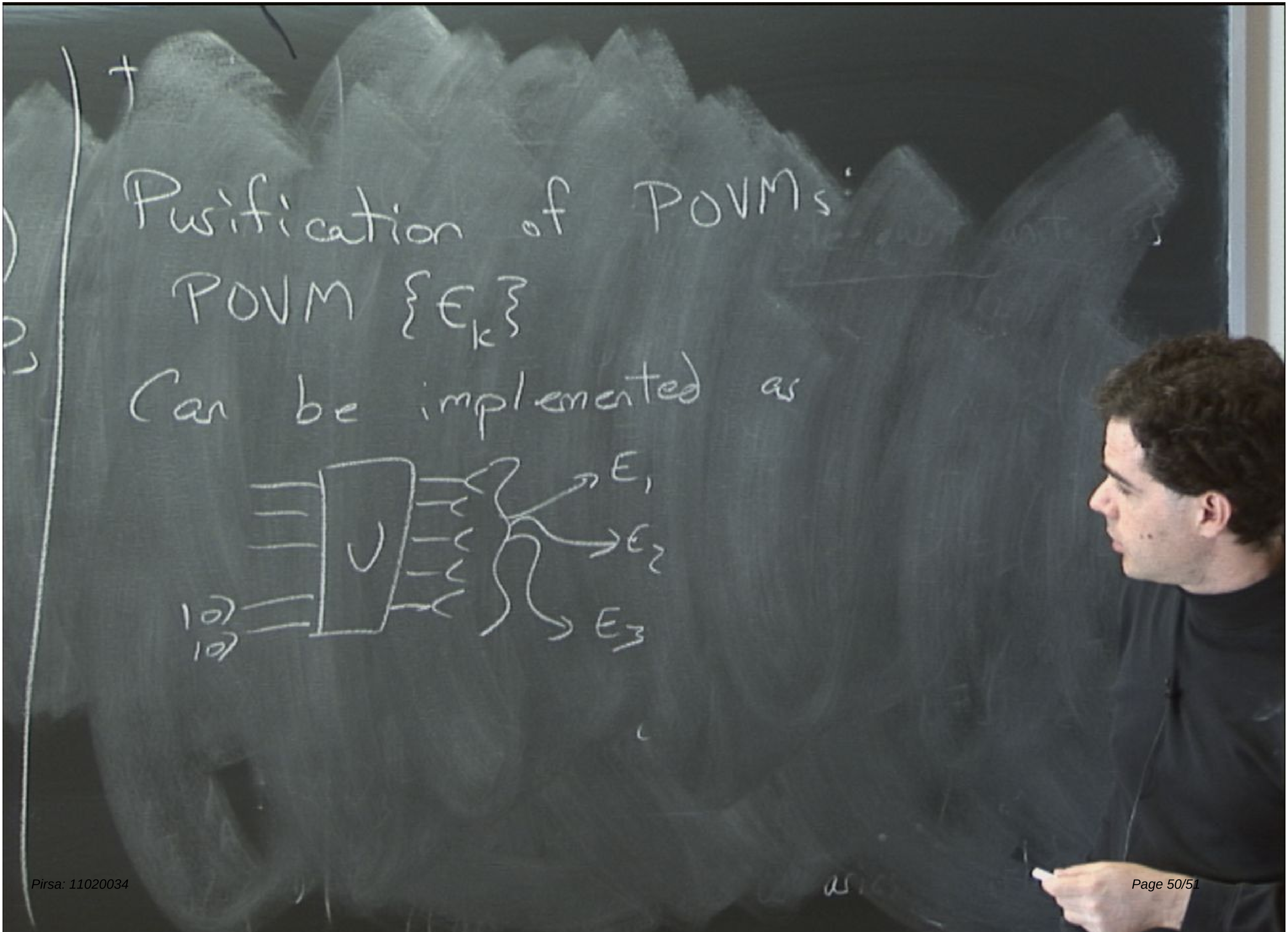
$1_I 0_A$: α

w/ prob

$1_I 1_A$: $\alpha|$

w/ prob.


Purification of POVMs:
POVM $\{E_k\}$


Purification of POVMs:
POVM $\{E_k\}$
Can be implemented as
U
$|0\rangle$
$|0\rangle$
E_1
E_2
E_3

Purification of POVMs:

POVM $\{E_k\}$

Can be implemented as