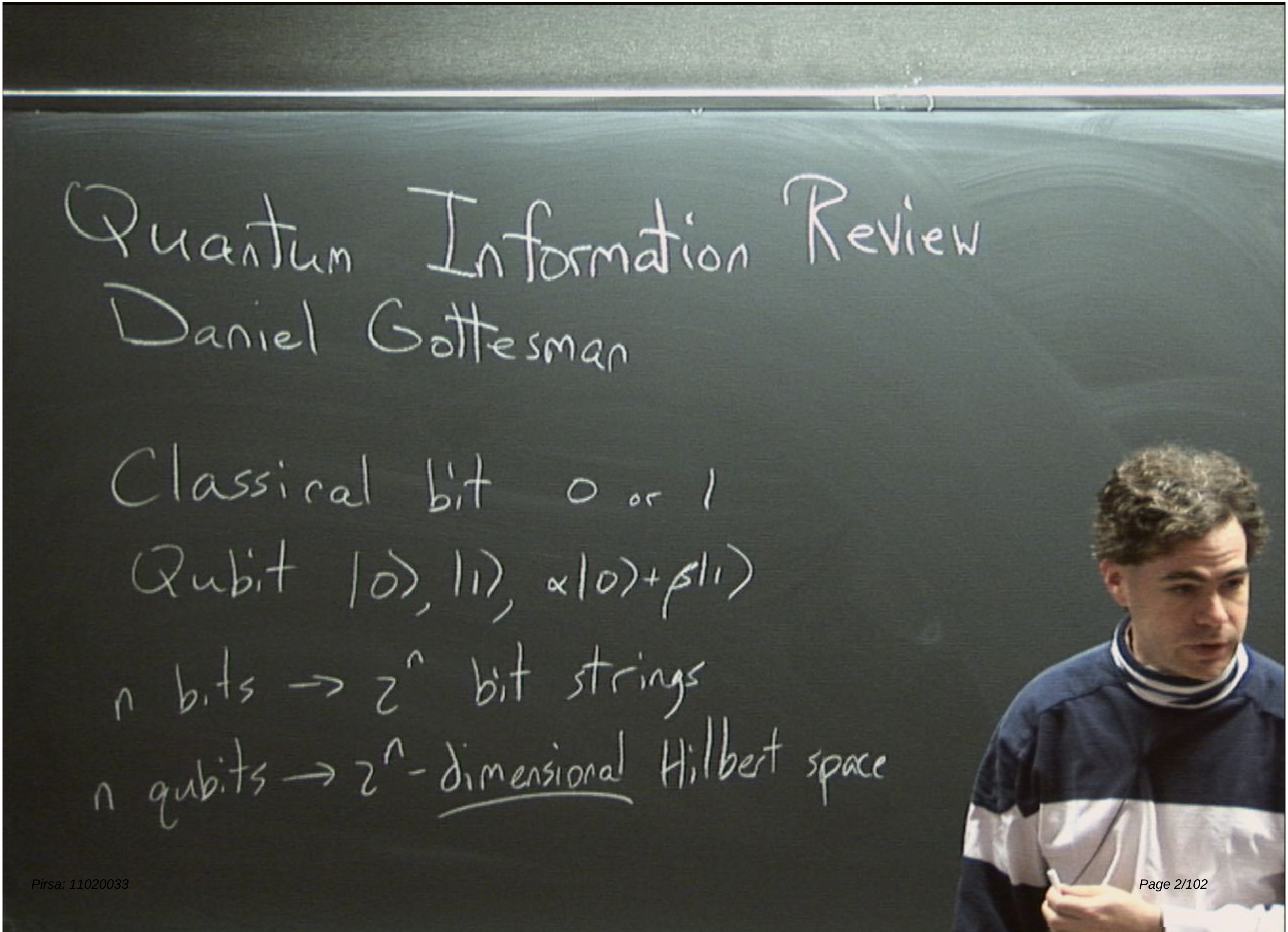
Title: Quantum Information Review - Lecture 1

Date: Feb 14, 2011 09:00 AM

URL: http://pirsa.org/11020033

Abstract:


Quantum Information Review
Daniel Gottesman
Classical bit 0 or 1
Qubit $|0\rangle$, $|1\rangle$, $\alpha|0\rangle+\beta|1\rangle$
n bits $\rightarrow$ 2^n bit strings
n qubits $\rightarrow$ 2^n-dimensional Hilbert space

Quantum Information Review

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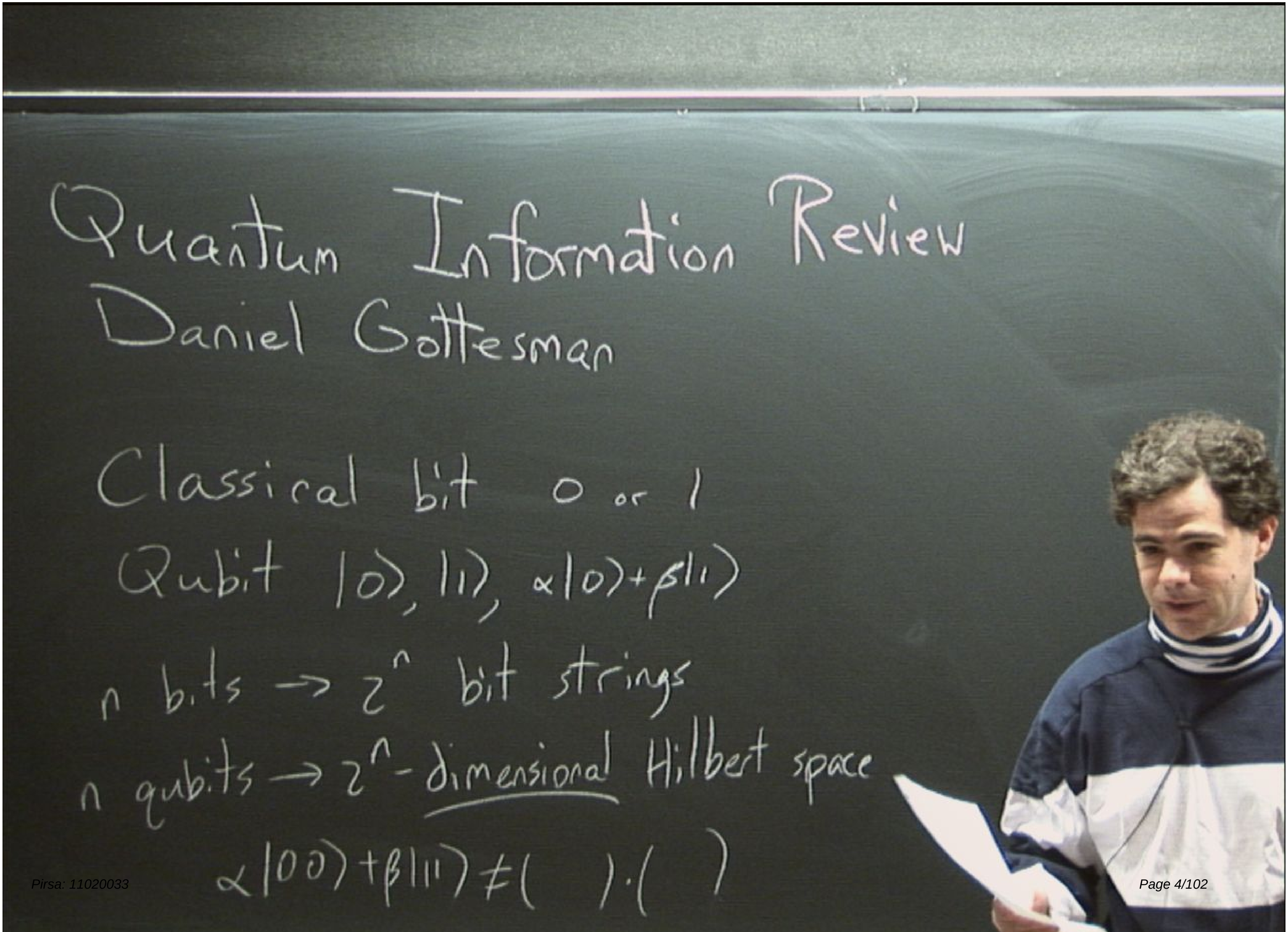
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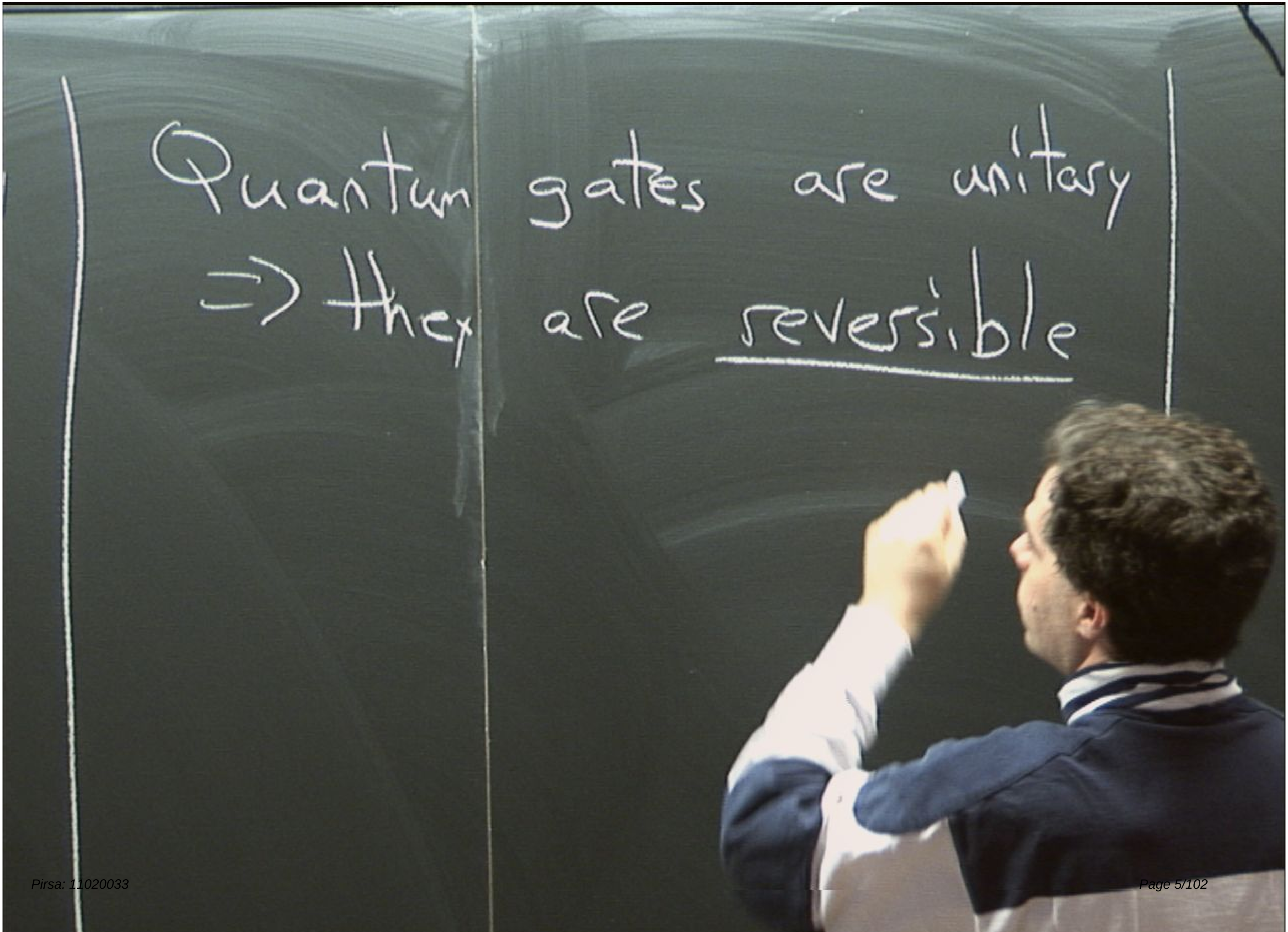
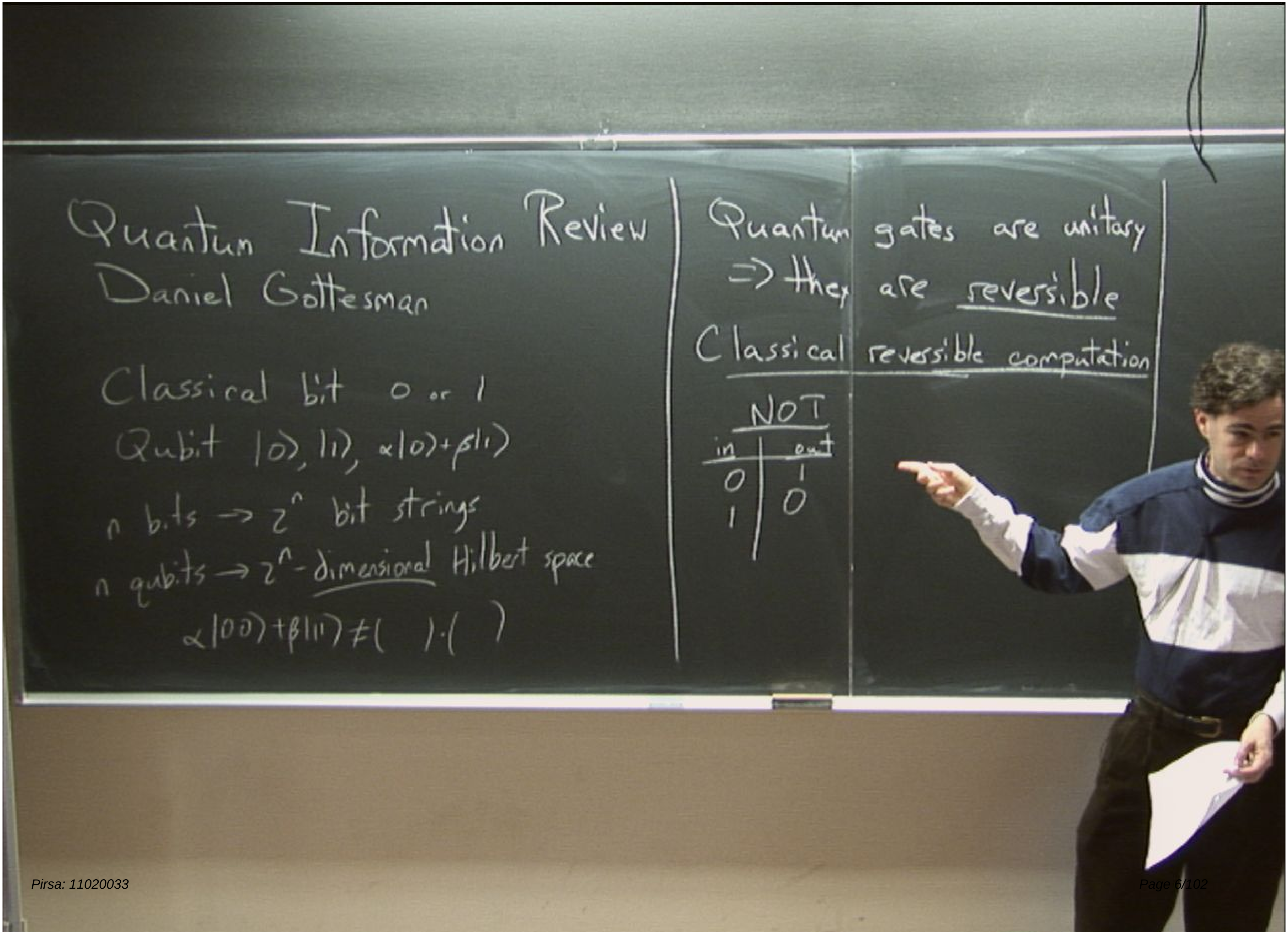
Qubit $|0\rangle, |1\rangle, \alpha|0\rangle+\beta|1\rangle$
n bits → 2^n bit strings
n qubits → 2^n-dimensional Hilbert space
$\alpha|00\rangle+\beta|11\rangle \neq (\quad)\cdot(\quad)$


Quantum gates are unitary
⇒ they are reversible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α|00⟩+β|11⟩ ≠ ( )·( )
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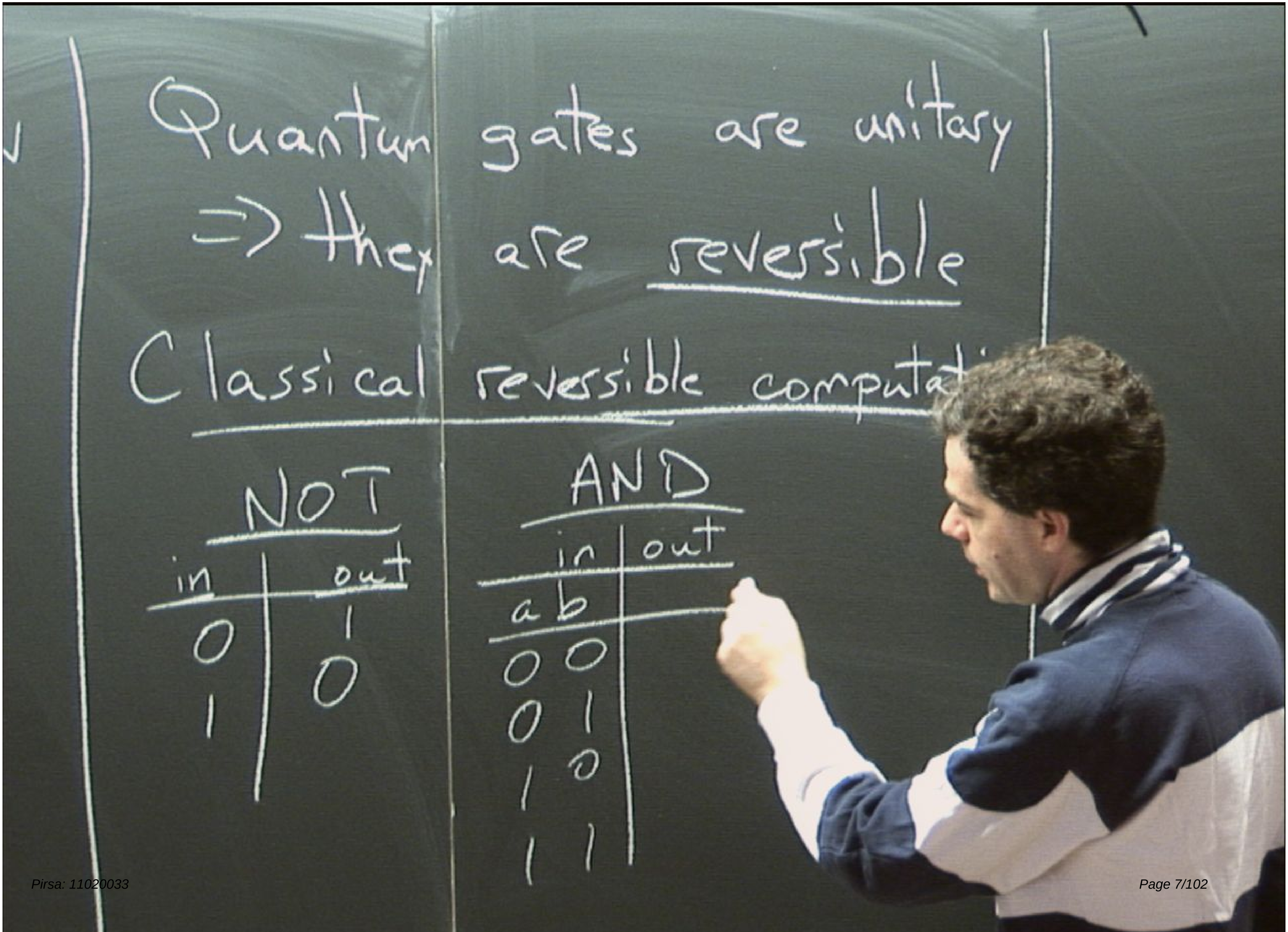
Classical reversible computation
NOT
in out
0 1
1 0



Quantum gates are unitary
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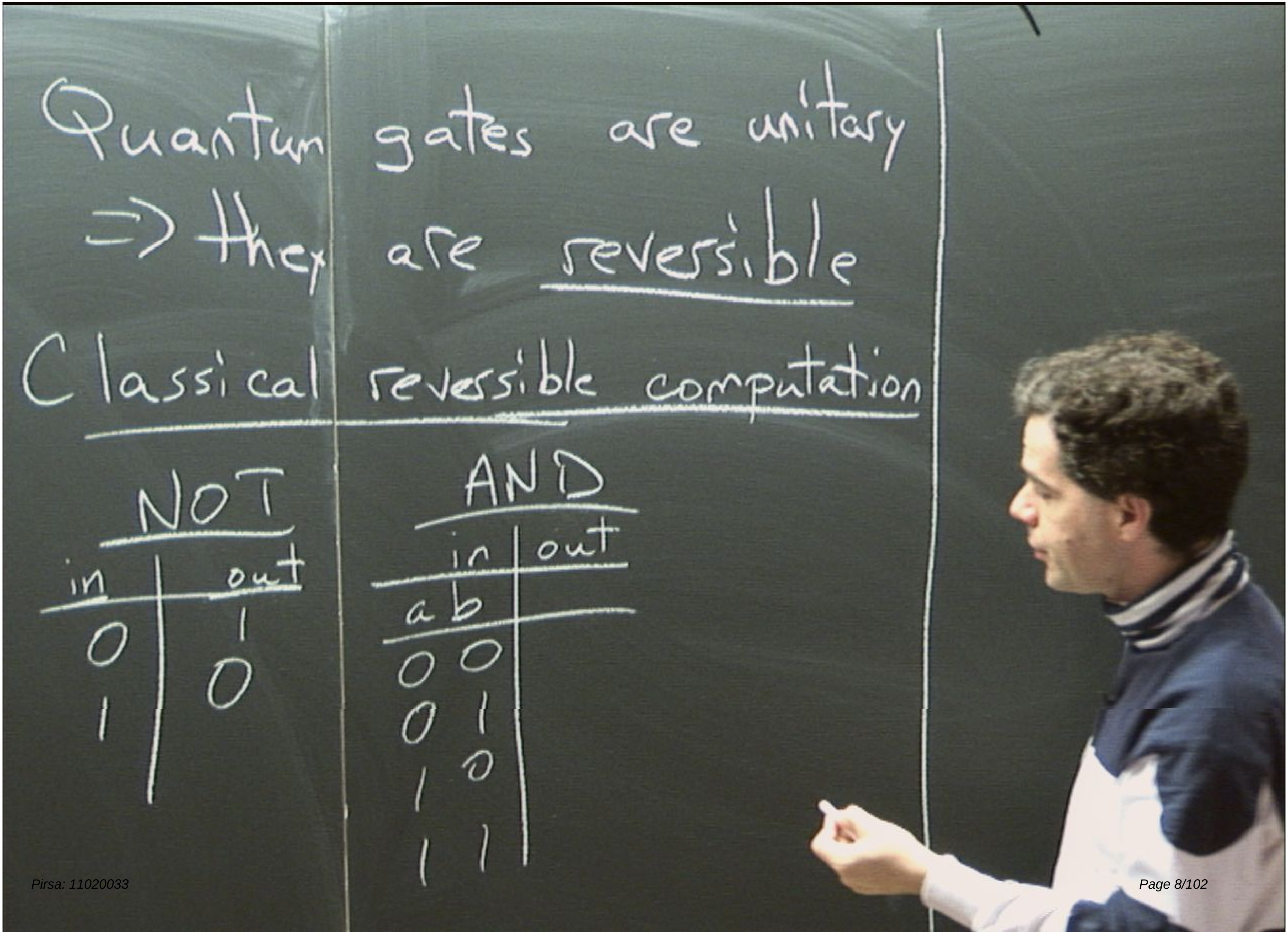
Classical reversible computat
NOT
in | out
0 | 1
1 | 0
AND
in | out
a b
0 0
0 1
1 0
1 1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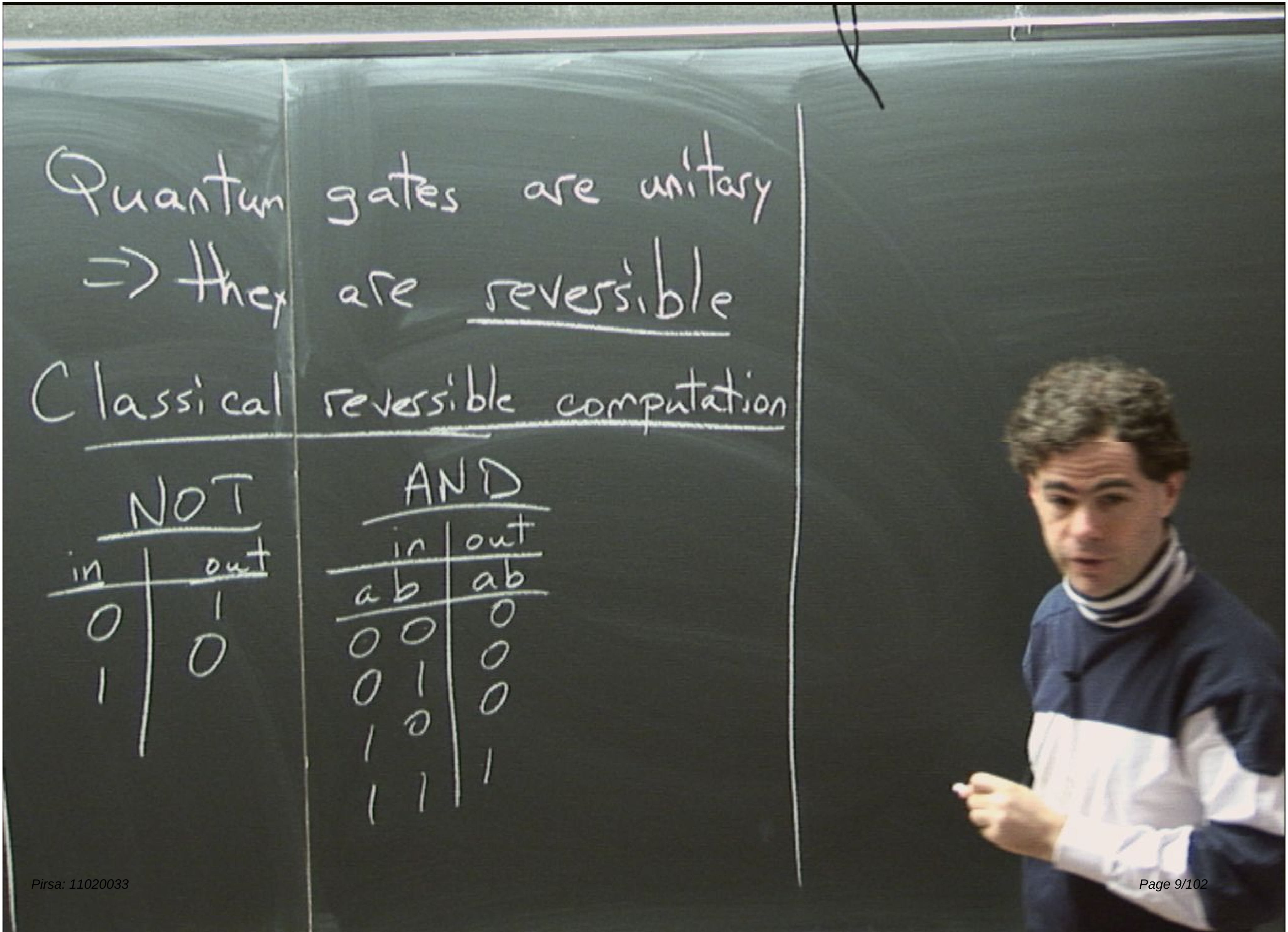
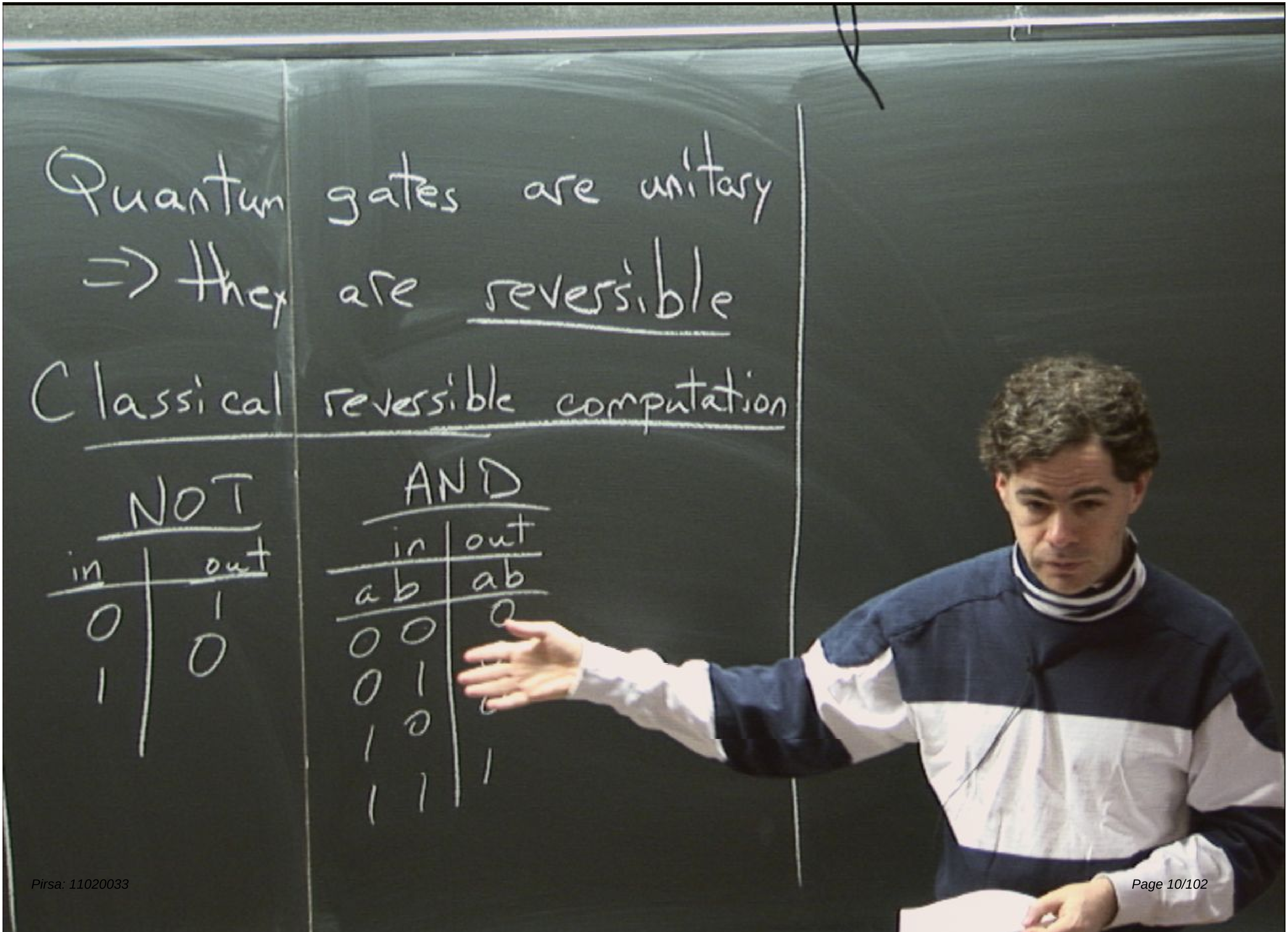

Quantum gates are unitary
=> they are reversible
Classical reversible computation
NOT
in | out
0 | 1
1 | 0
AND
in | out
a b | ab
0 0 | 0
0 1 | 0
1 0 | 0
1 1 | 1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0 0 | 0
0 1
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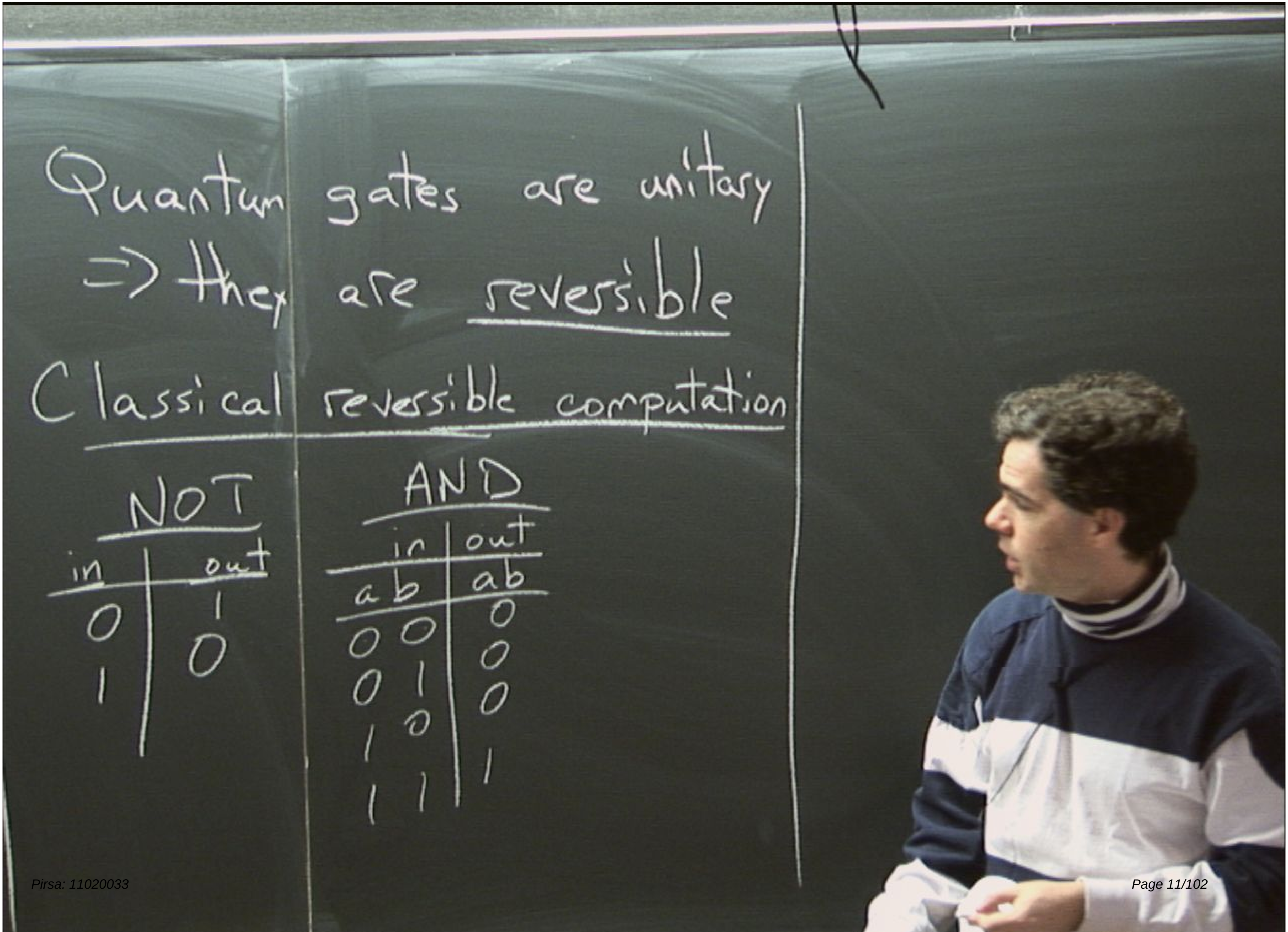
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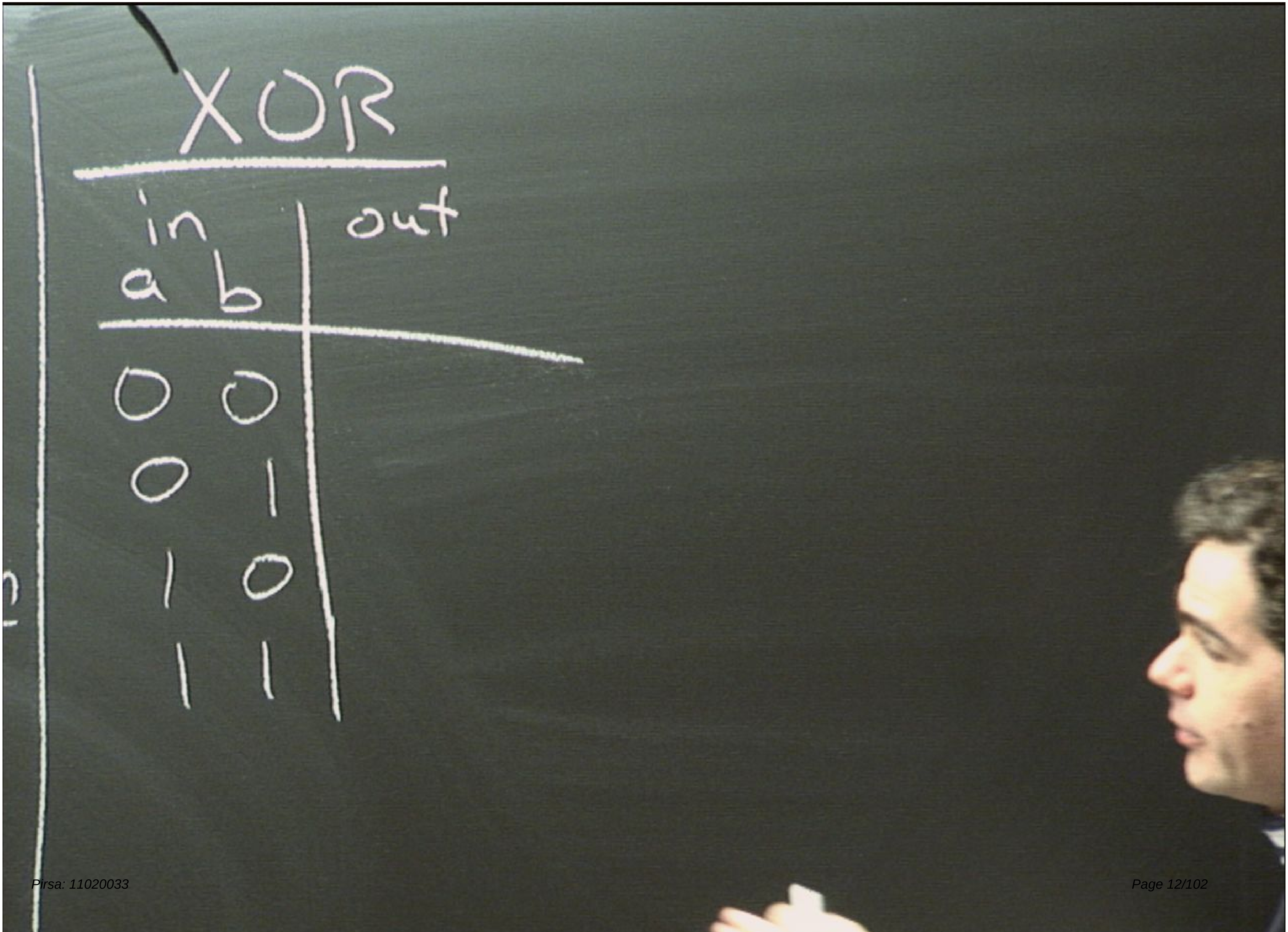
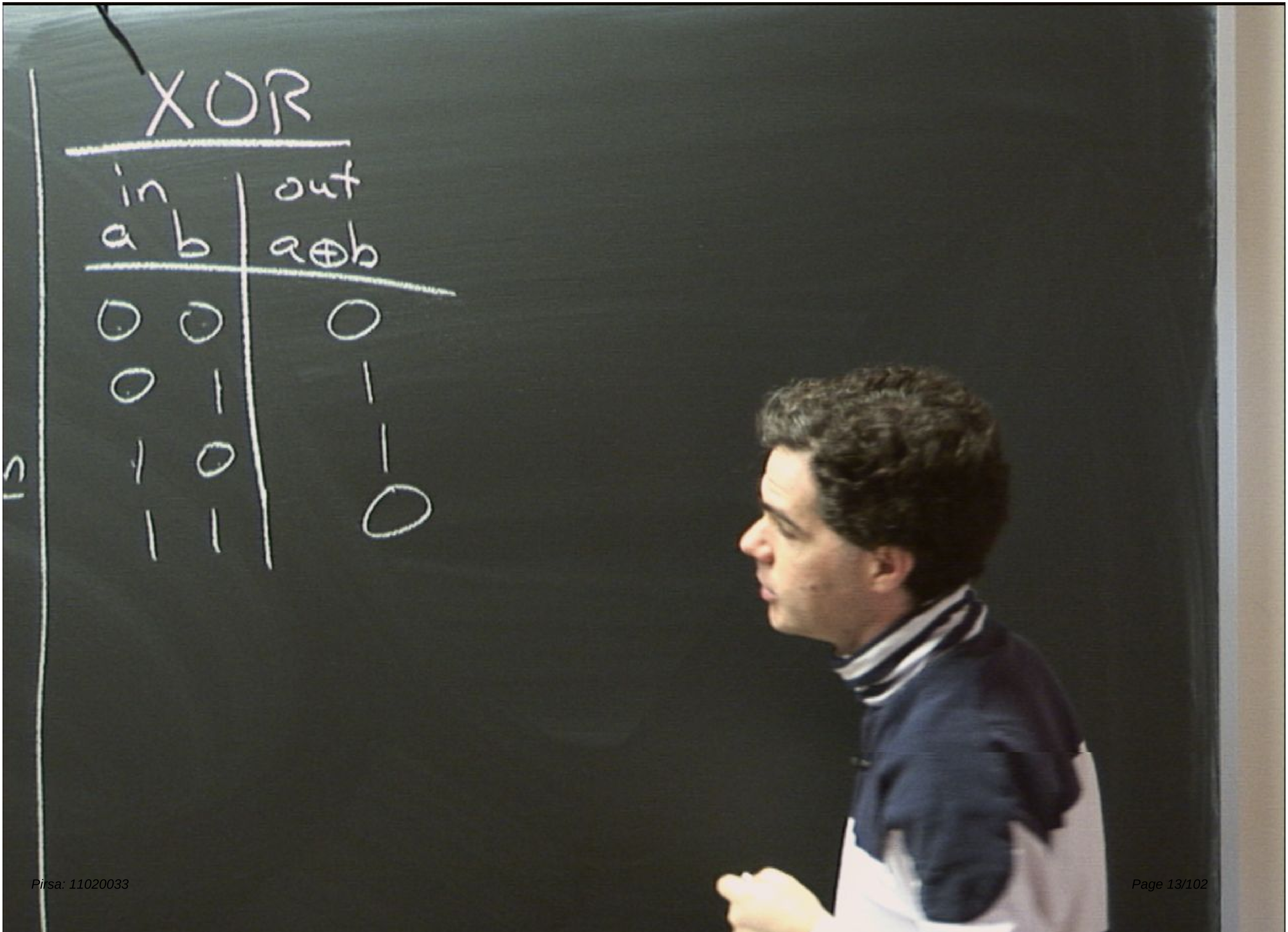
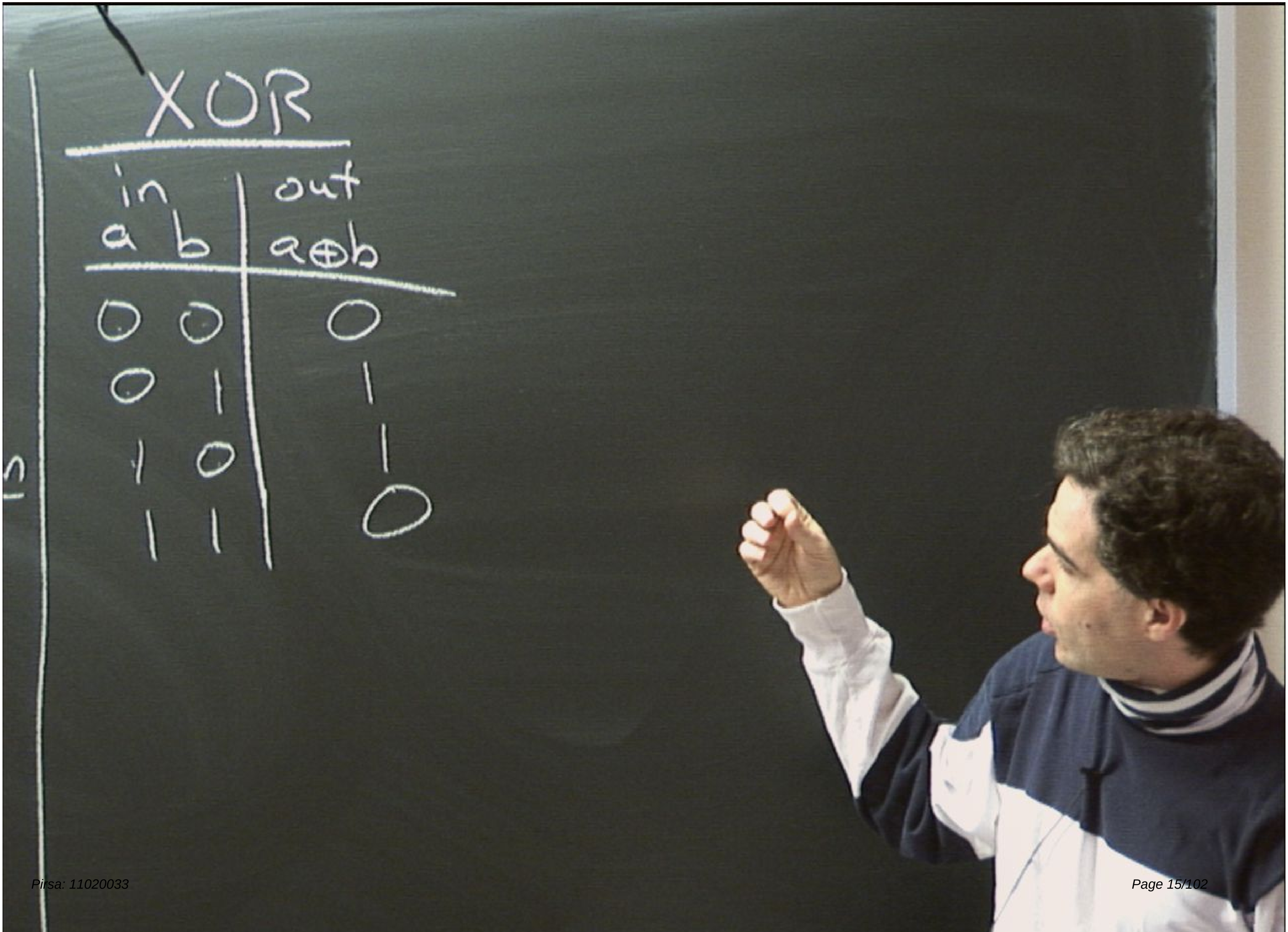
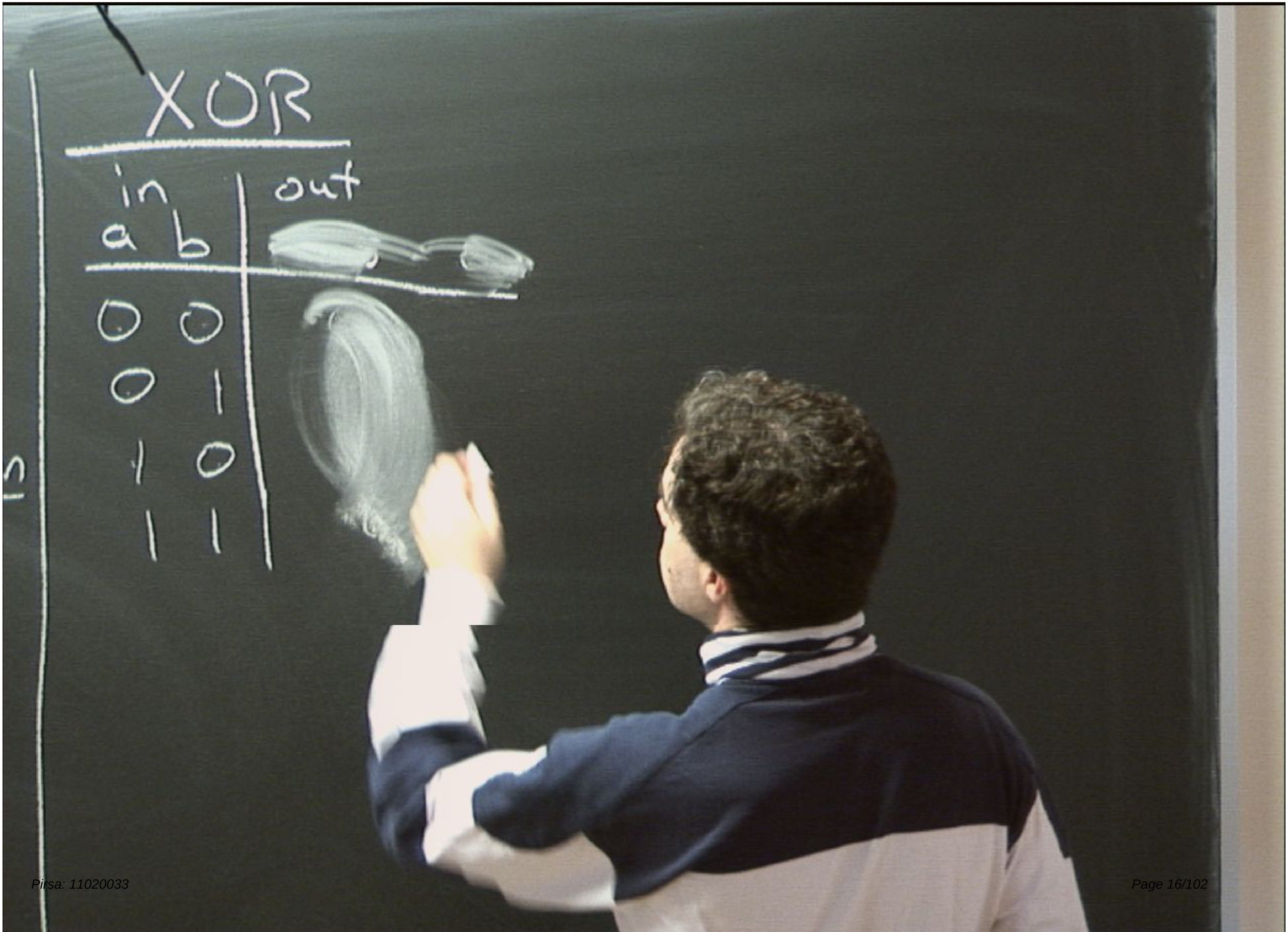
in | out
ab | ab
0 0 | 0
0 1 | 0
1 0 | 0
1 1 | 1


XOR
in
out
a b
0 0
0 1
1 0
1 1


XOR
in | out
a b | a⊕b
0 0 | 0
0 1 | 1
1 0 | 1
1 1 | 0


XOR
in | out
a b | a⊕b
0 0 | 0
0 1 | 1
1 0 | 1
1 1 | 0


XOR
in out
a b a⊕b
0 0 0
0 1 1
1 0 1
1 1 0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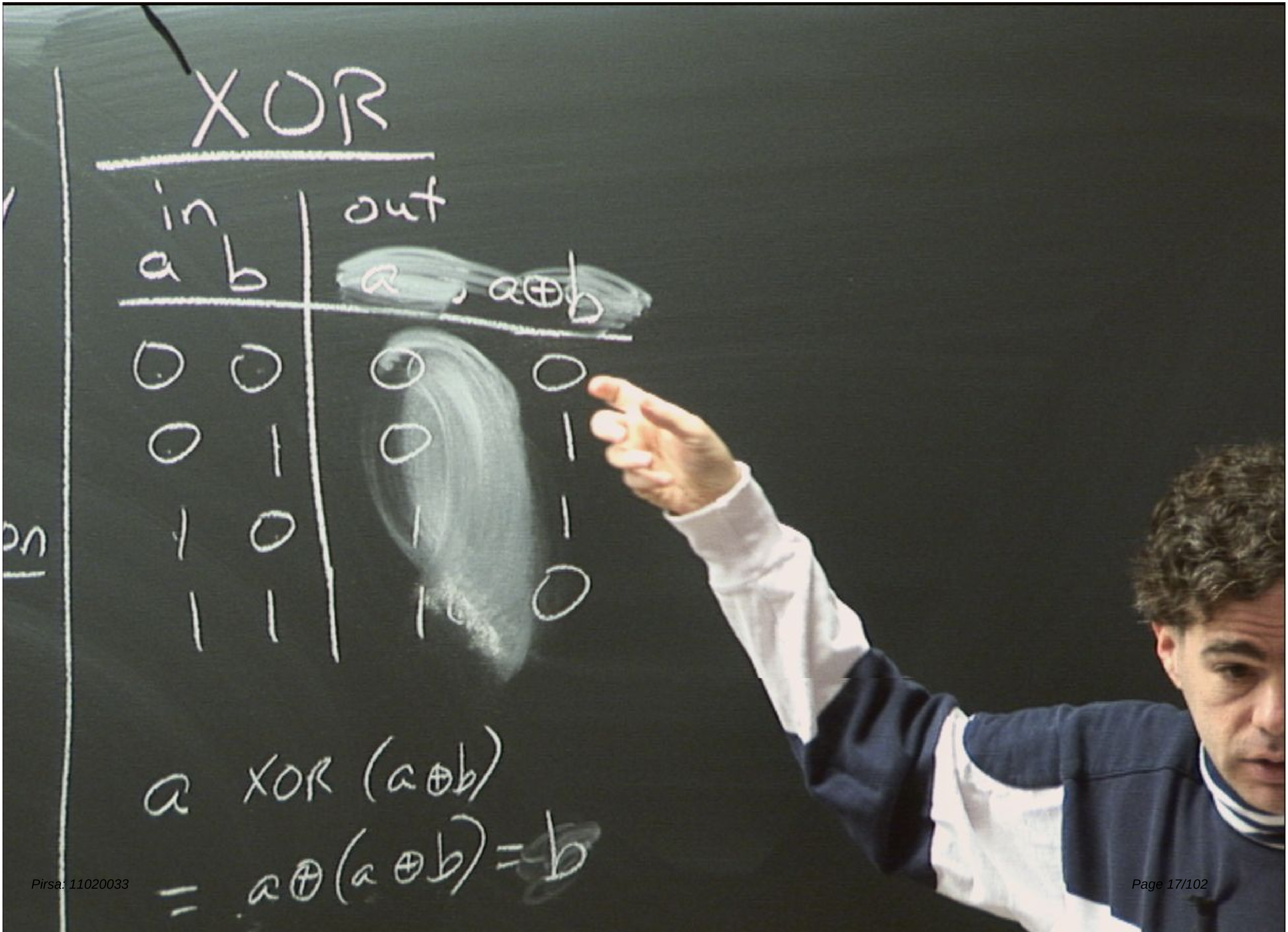
in
out
a b
0 0
0 1
1 0
1 1


XOR
in | out
a b | a a⊕b
0 0 | 0 0
0 1 | 0 1
1 0 | 1 1
1 1 | 1 0
a XOR (a⊕b)
= a⊕(a⊕b) = b

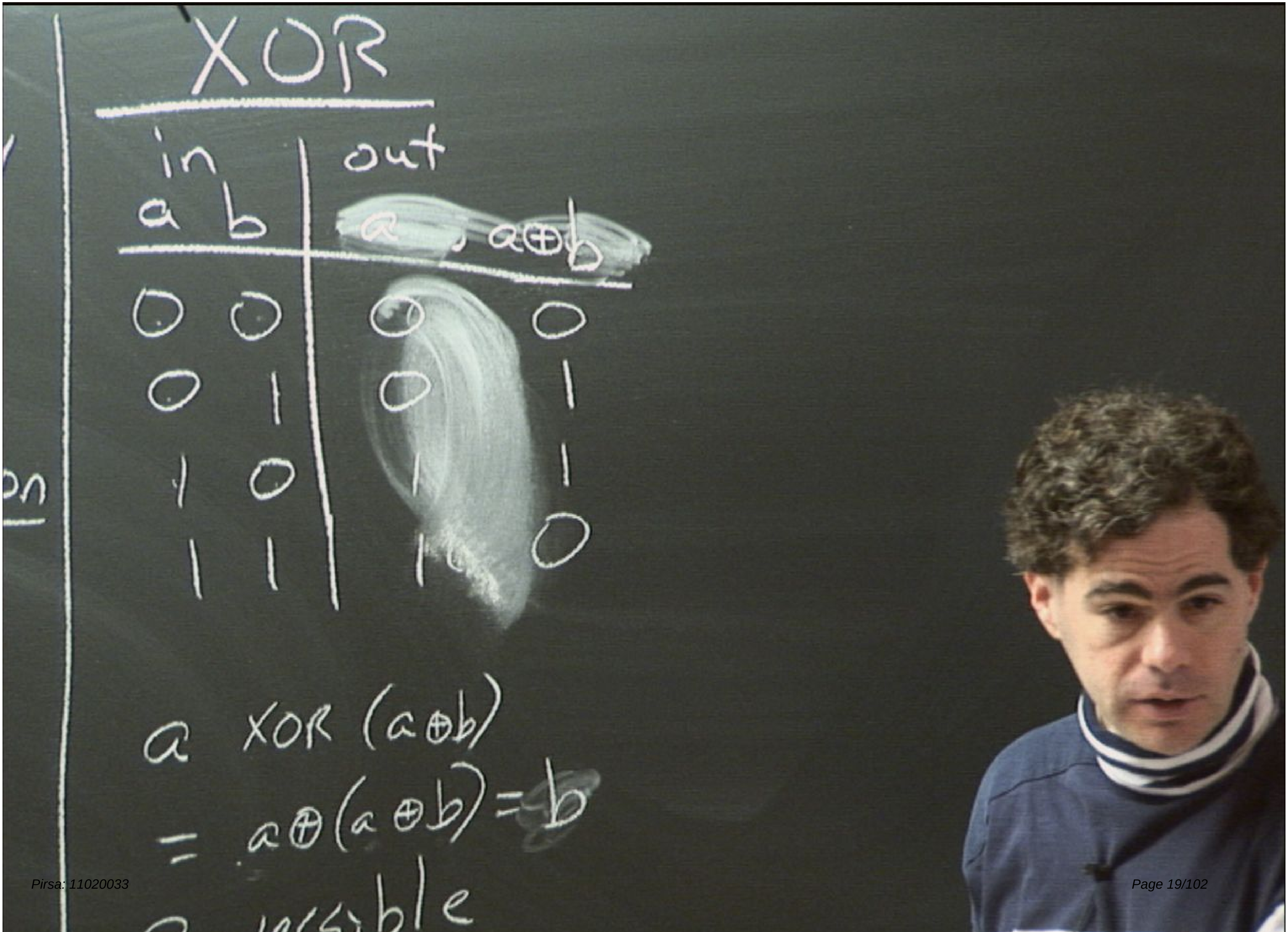
XOR

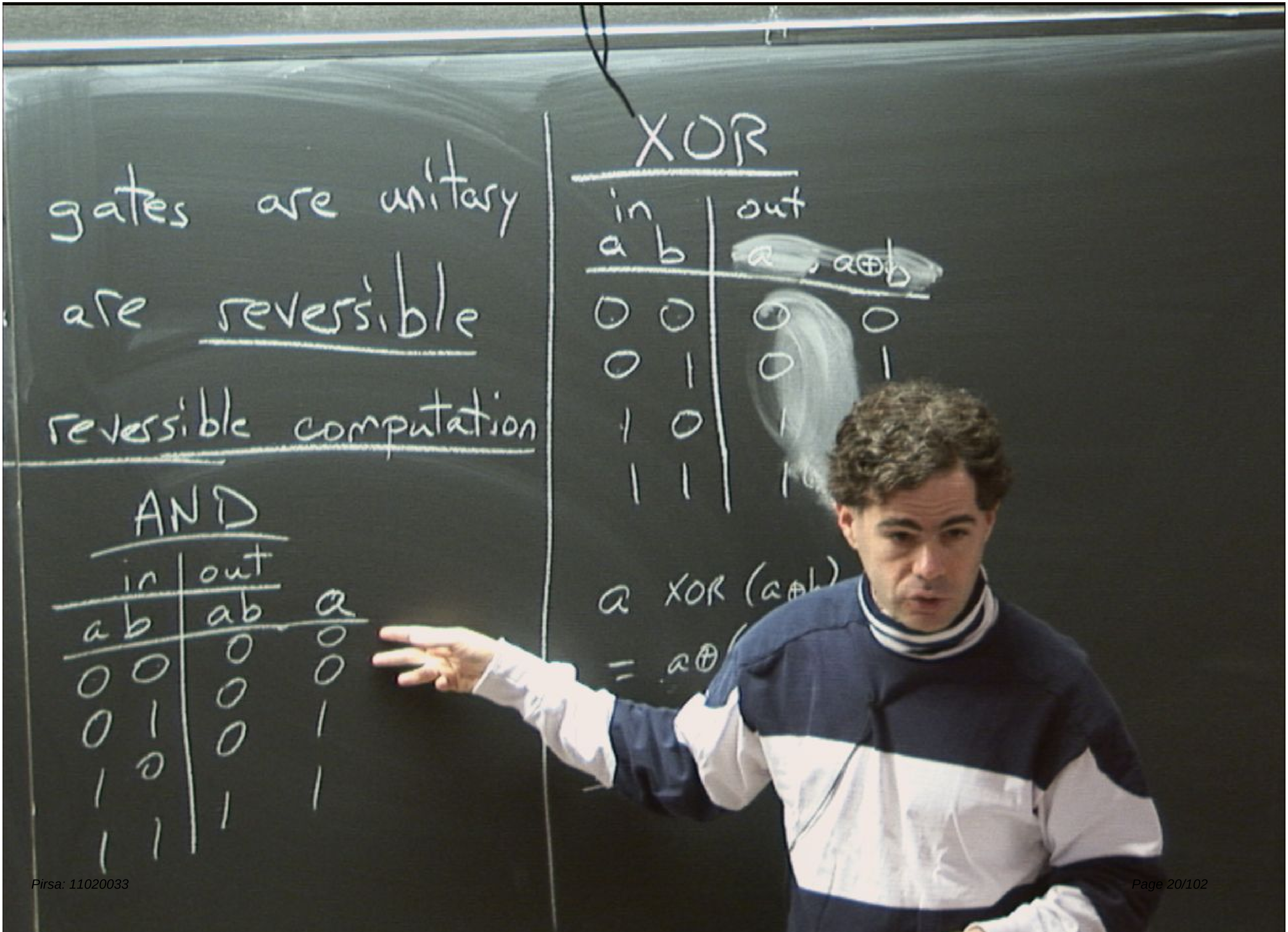
in		out	
a	b	a	a⊕b
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

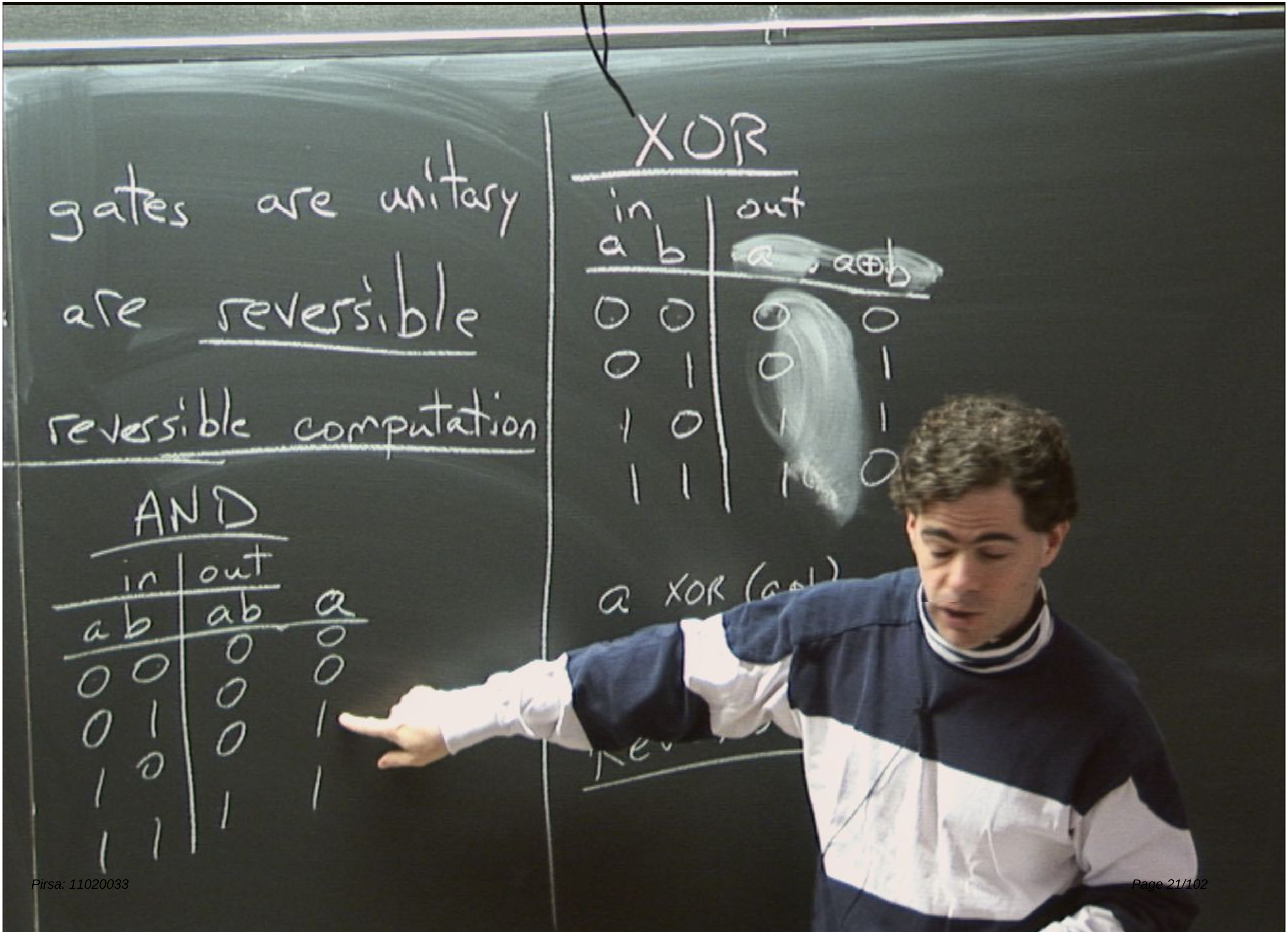
$a \text{ XOR } (a \oplus b)$

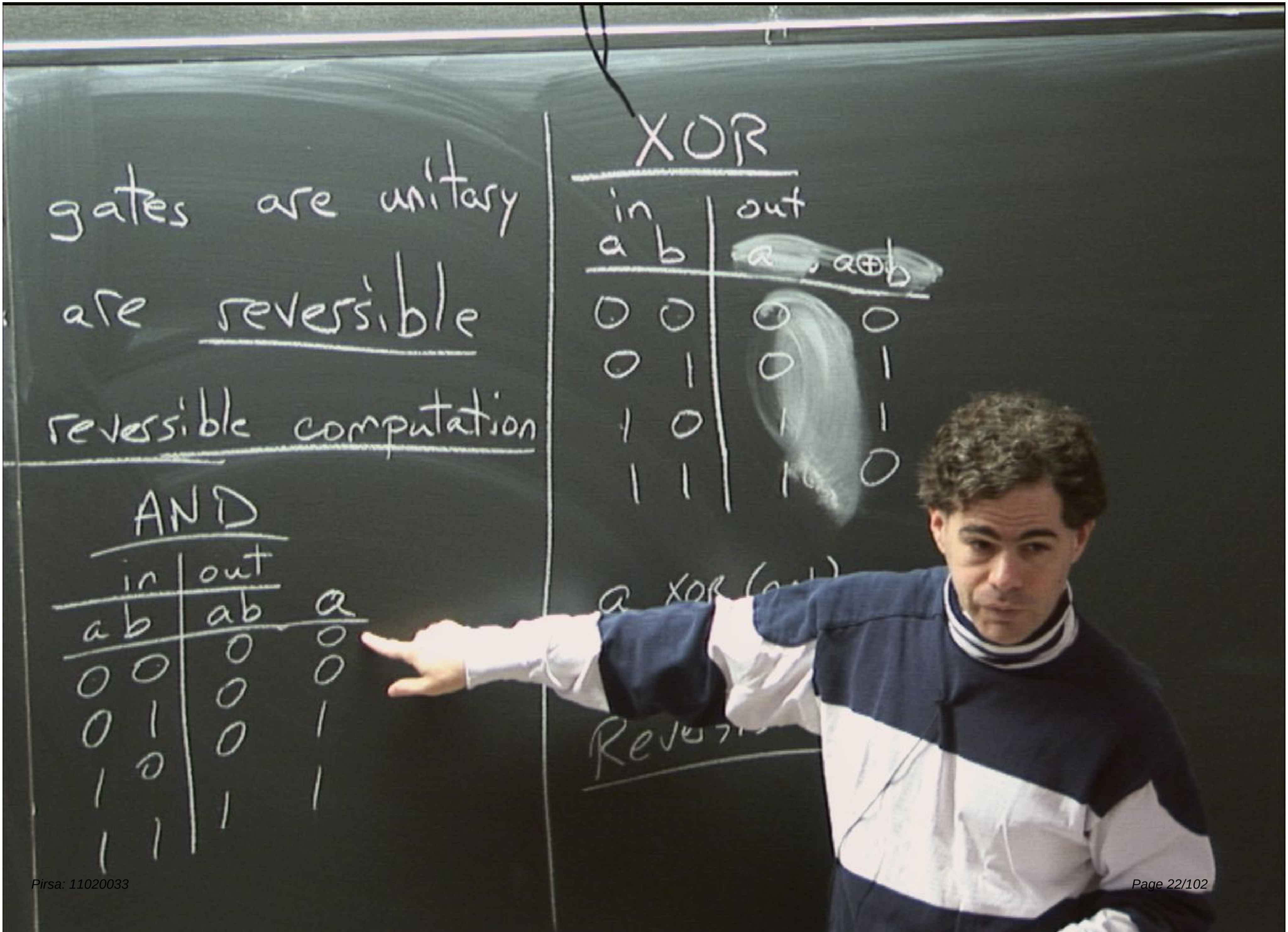
$= a \oplus (a \oplus b) = b$

reversible


XOR
in | out
a b | a , a⊕b
0 0 | 0 0
0 1 | 0 1
1 0 | 1 1
1 1 | 1 0
a XOR (a⊕b)
= a⊕(a⊕b) = b


gates are unitary
are reversible
reversible computation
AND
in out
a b ab a
0 0 0 0
0 1 0 1
1 0 0 1
1 1 1 1
XOR
in out
a b a a⊕b
0 0 0 0
0 1 0 1
1 0 1
1 1 1


gates are unitary
are reversible
reversible computation
AND
in out
ab ab a
00 0 0
01 0 1
10 0 1
11 1
XOR
in out
a b a , a⊕b
0 0 0 0
0 1 0 1
1 0 1 1
1 1 1 0
a XOR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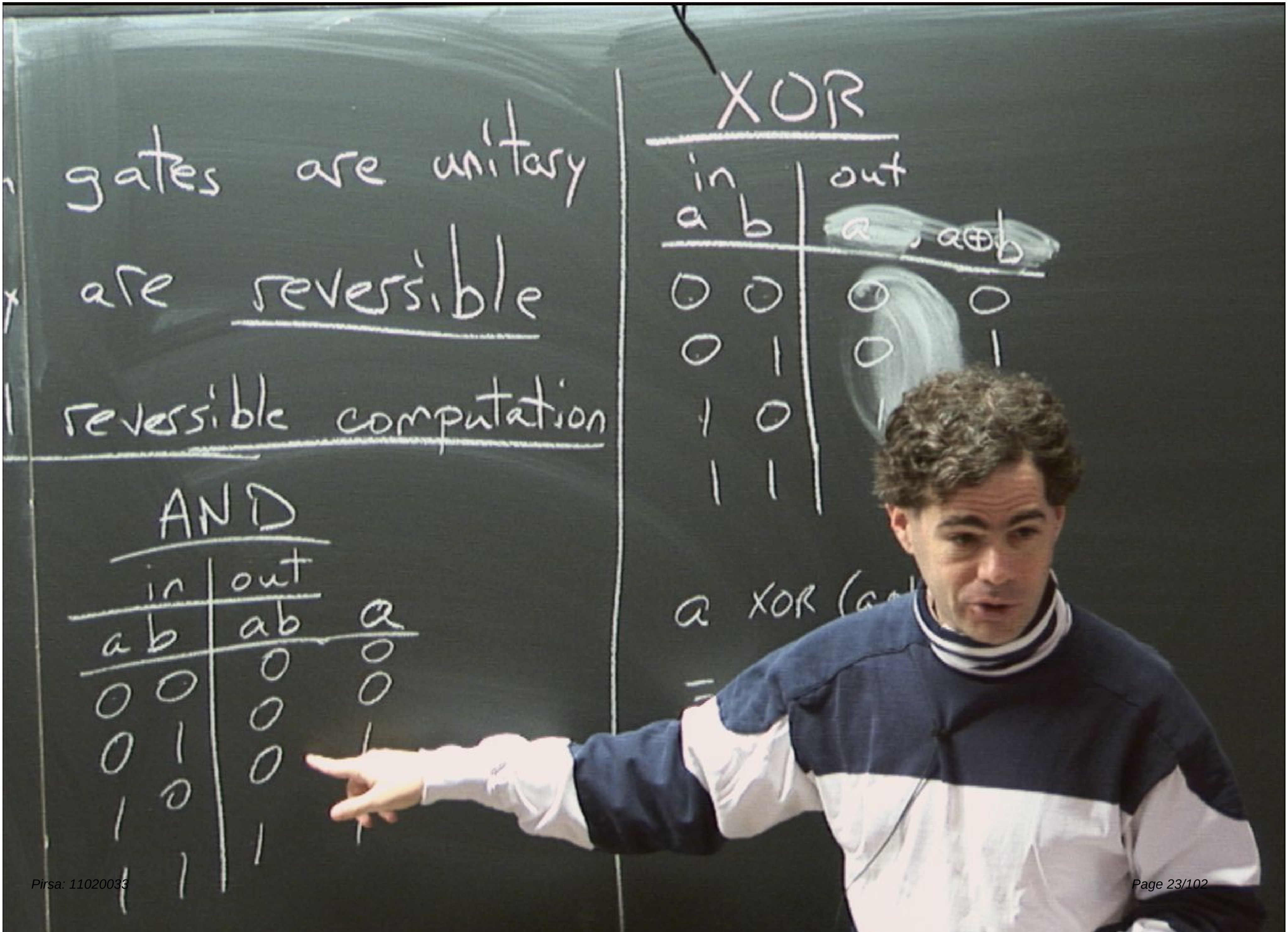
ab
a
XOR
in
out
a b
a , a⊕b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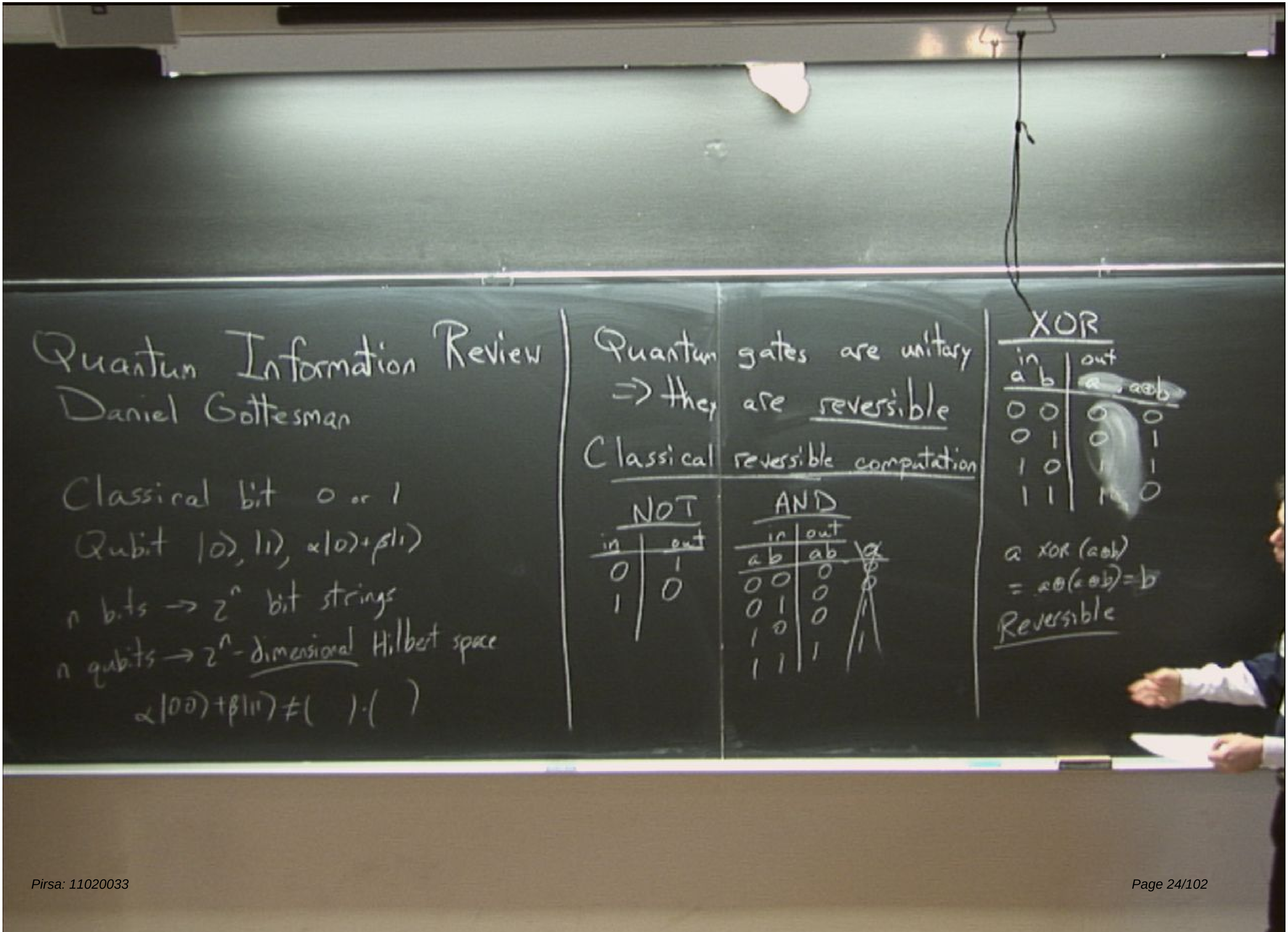
AND
in | out
a b | ab a
0 0 | 0 0
0 1 | 0 1
1 0 | 0 1
1 1 | 1
XOR
in | out
a b | a a⊕b
0 0 | 0 0
0 1 | 0 1
1 0 | 1
1 1
a XOR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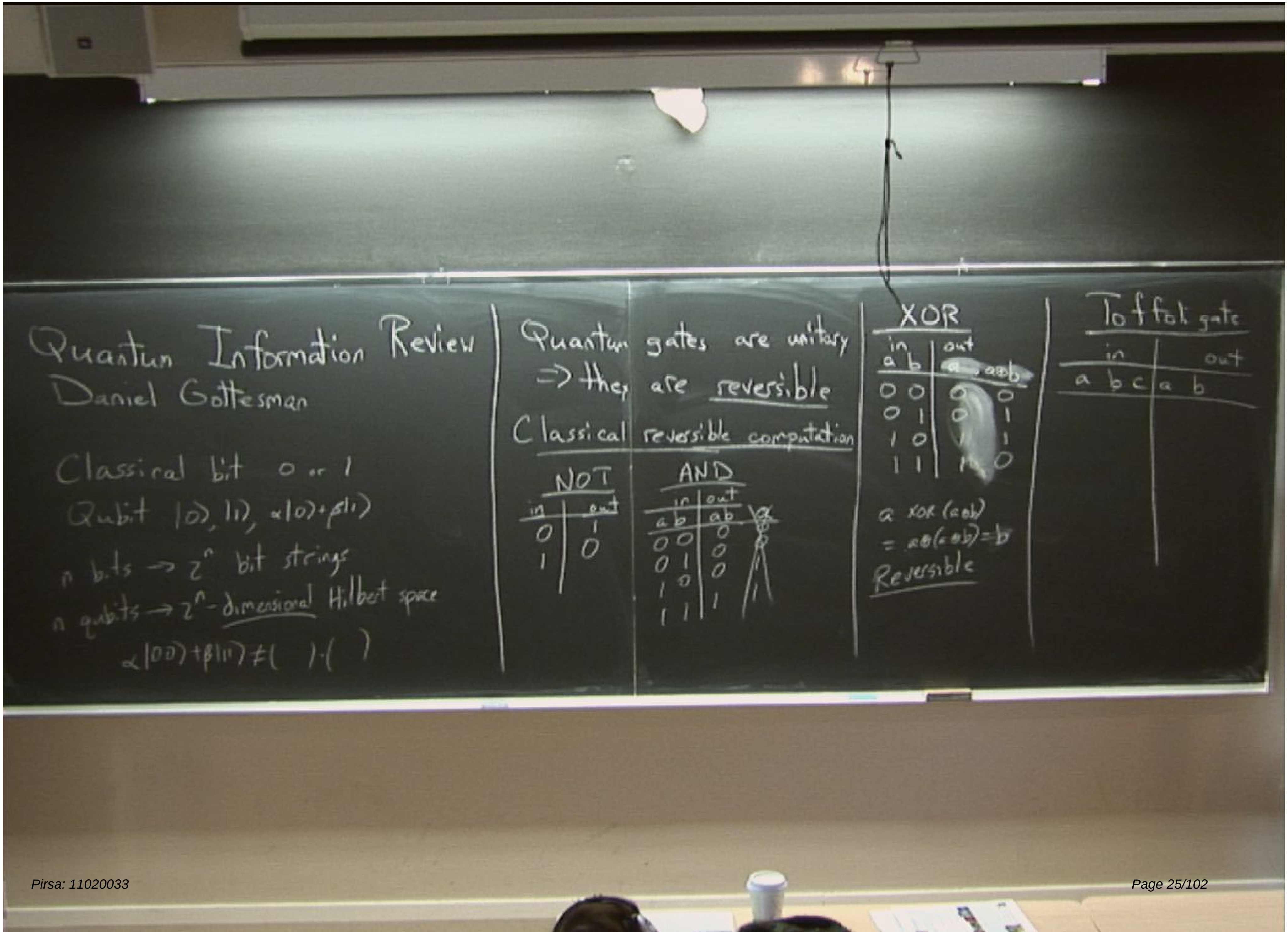
XOR
in | out
a b | a, a⊕b
0 0 | 0 0
0 1 | 0 1
1 0 | 1 1
1 1 | 1 0
a XOR (a⊕b)
= a⊕(a⊕b) = b
Reversible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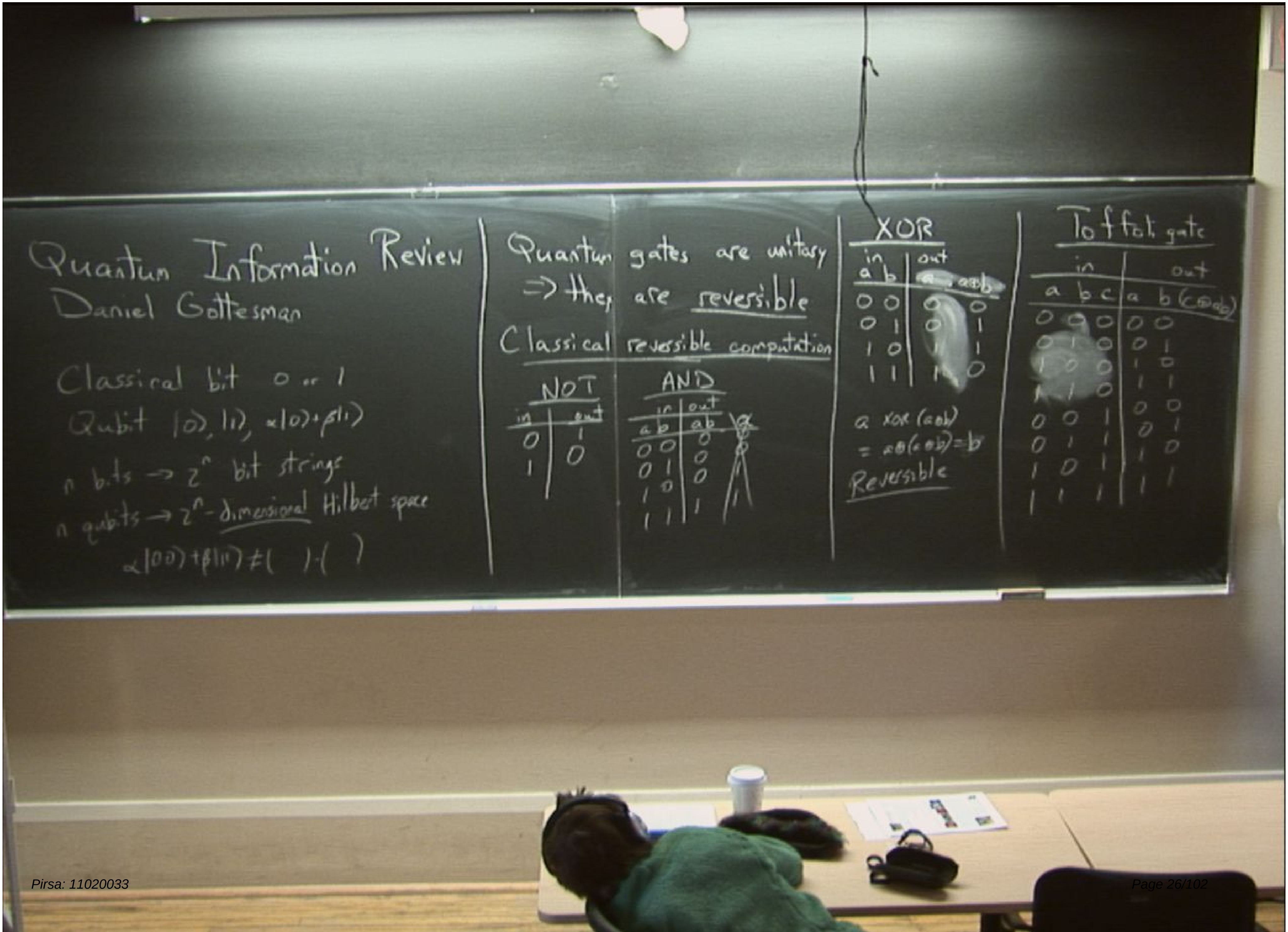
⇒ they are reversible
Classical reversible computation
NOT
in out
0 1
1 0
AND
in out
a b a b
0 0 0
0 1 0
1 0 0
1 1 1
XOR
in out
a b a⊕b
0 0 0
0 1 1
1 0 1
1 1 0
a XOR (a⊕b)
= a⊕(a⊕b) = b
Reversible
Toffoli gate
in out
a b c a b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in out
0 1
1 0
AND
in out
a b ab
0 0 0
0 1 0
1 0 0
1 1 1
XOR
in out
a b a⊕b
0 0 0
0 1 1
1 0 1
1 1 0
a XOR (a⊕b)
= a⊕(a⊕b) = b
Reversible
Toffoli gate
in out
a b c a b c⊕ab

unitary

...sible

...mputation

XOR

in a	in b	out a	out $a\oplus b$
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

a XOR $(a\oplus b)$

$= a\oplus(a\oplus b) = b$

Reversible

Toffoli gate

in a	in b	in c	out a	out b	out $(c\oplus ab)$
0	0	0	0	0	0
0	1	0	0	1	0
1	0	0	1	0	0
1	1	0	1	1	1
0	0	1	0	0	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	1	1	1	0

XOR

in a	in b	out a	out $a \oplus b$
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

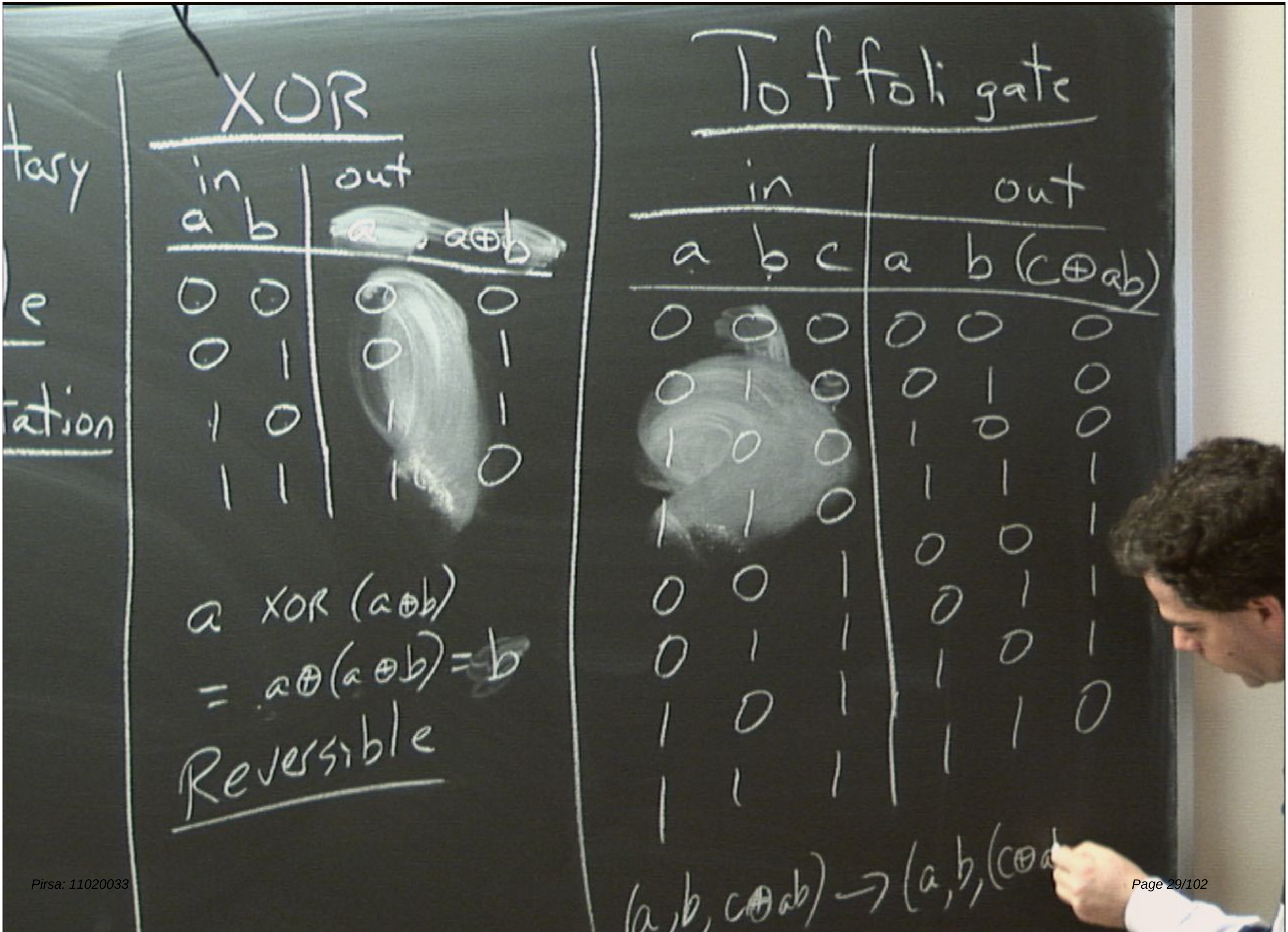
$a \text{ XOR } (a \oplus b)$

$= a \oplus (a \oplus b) = b$

Reversible

Toffoli gate

in a	in b	in c	out a	out b	out $(c \oplus ab)$
0	0	0	0	0	0
0	1	0	0	1	0
1	0	0	1	0	0
1	1	0	1	1	1
0	0	1	0	0	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	1	1	1	0


tary
e
tation
XOR
in out
a b a a⊕b
0 0 0 0
0 1 0 1
1 0 1 1
1 1 1 0
a XOR (a⊕b)
= a⊕(a⊕b)=b
Reversible
Toffoli gate
in out
a b c a b (c⊕ab)
0 0 0 0 0 0
0 1 0 0 1 0
1 0 0 1 0 0
1 1 0 1 1 1
0 0 1 0 0 1
0 1 1 0 1 1
1 0 1 1 0 1
1 1 1 1 1 0
(a,b,c⊕ab) → (a,b,(c⊕a

tary

e

tation

XOR

in a	in b	out a	out $a \oplus b$
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

$a \text{ XOR } (a \oplus b)$

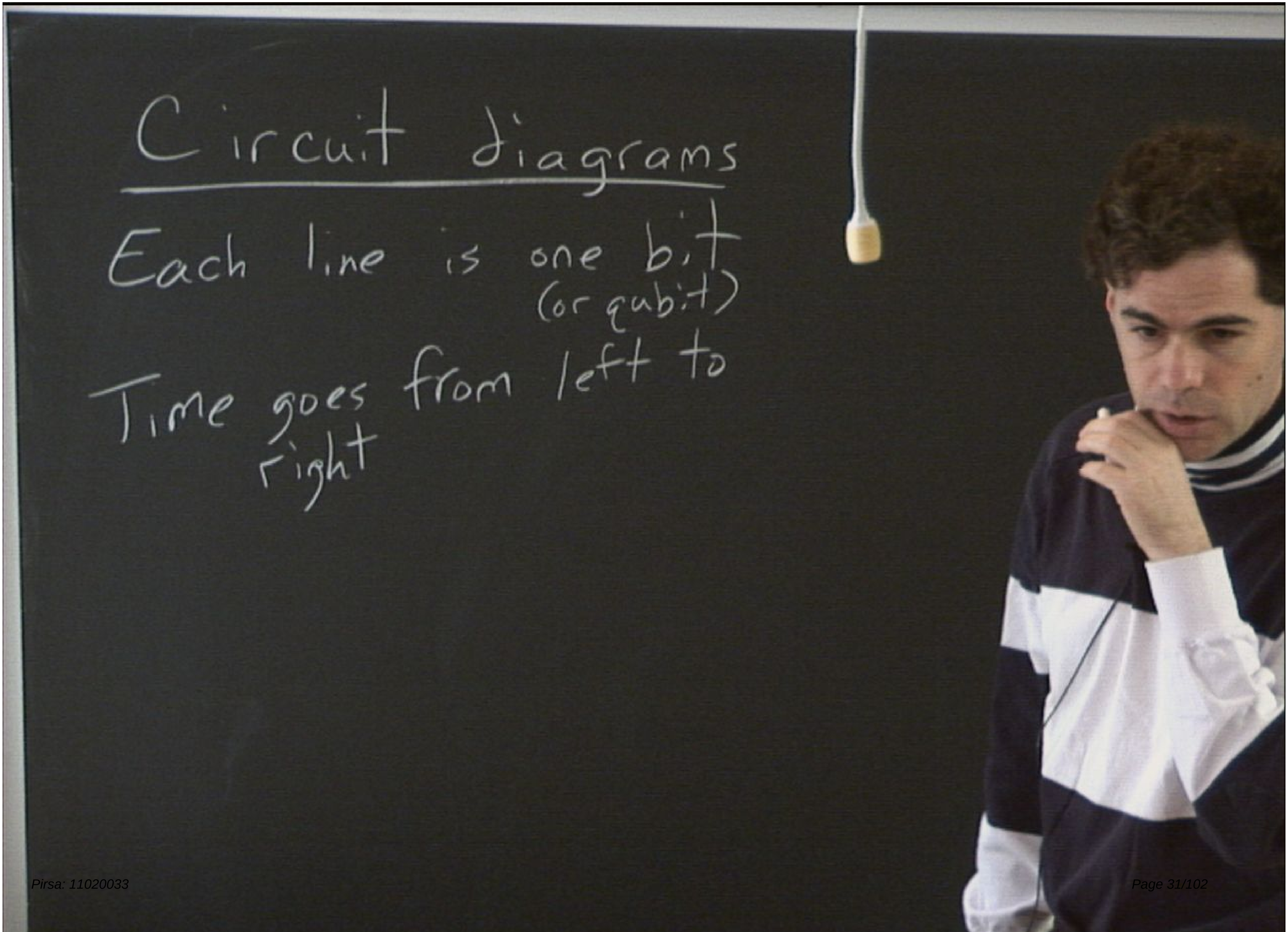
$= a \oplus (a \oplus b) = b$

Reversible

Toffoli gate

in a	in b	in c	out a	out b	out $(c \oplus ab)$
0	0	0	0	0	0
0	1	0	0	1	0
1	0	0	1	0	0
1	1	0	1	1	1
0	0	1	0	0	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	1	1	1	0

$(a, b, c \oplus ab) \rightarrow (a, b, (c \oplus ab) \oplus ab)$


Circuit diagrams
Each line is one bit
(or qubit)
Time goes from left to right

Circuit diagrams

Each line is one bit
(or qubit)

Time goes from left to
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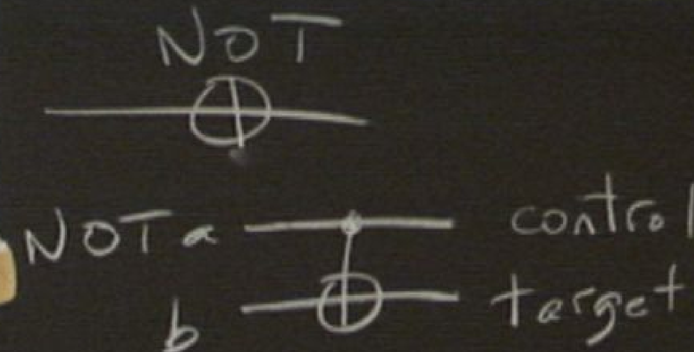

Circuit diagrams

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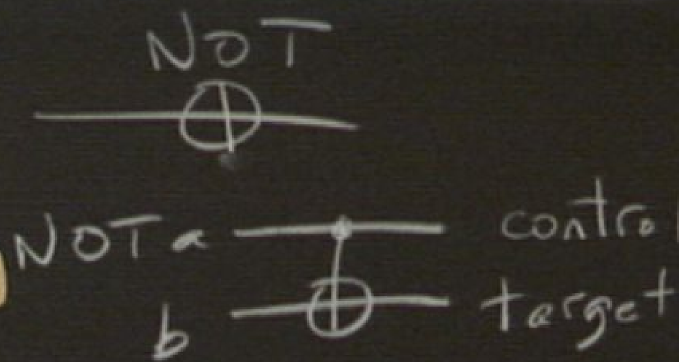
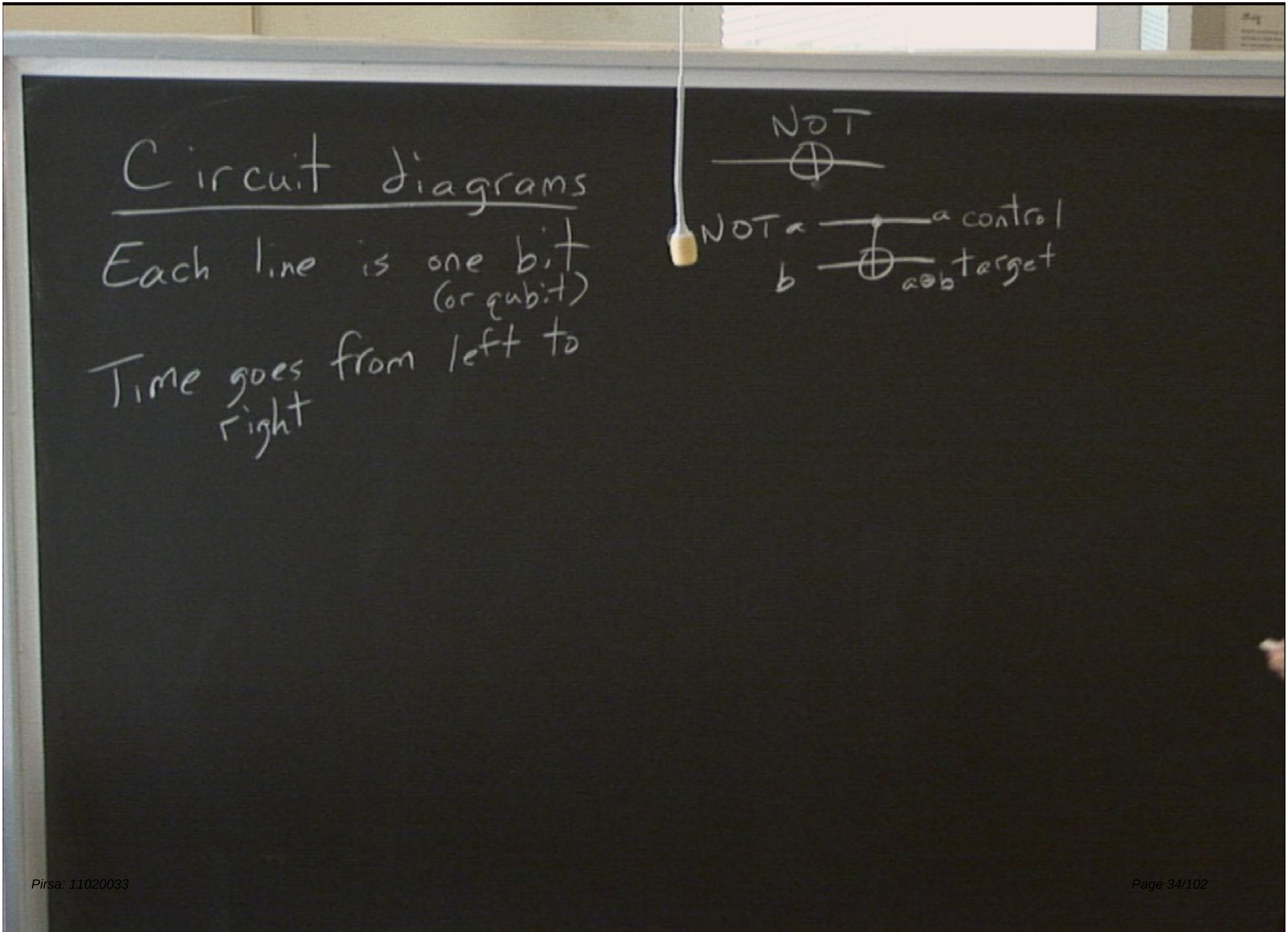
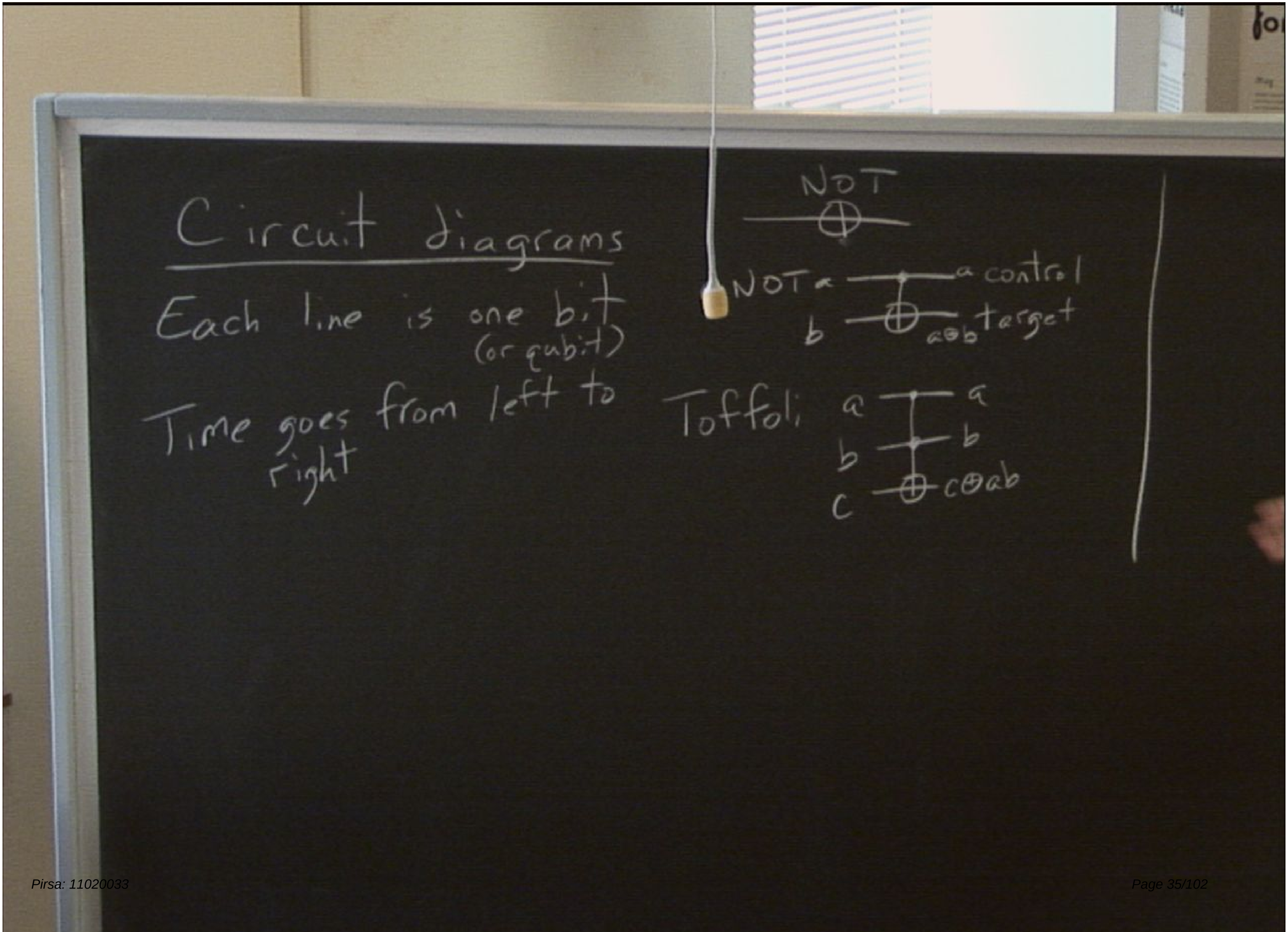
Time goes from left to right



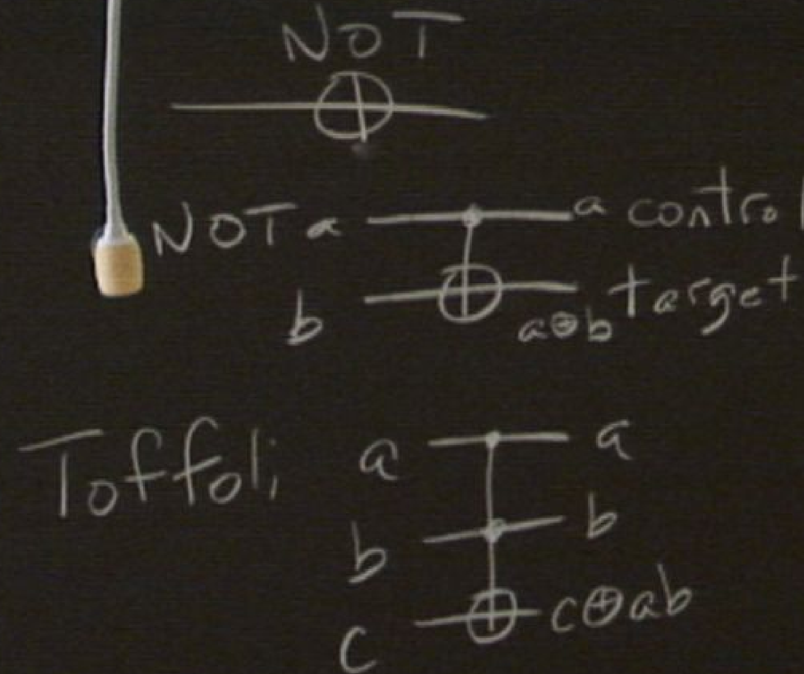

Circuit diagrams
Each line is one bit
(or qubit)
Time goes from left to right
NOT
C-NOT
a
a
control
b
a⊕b
target

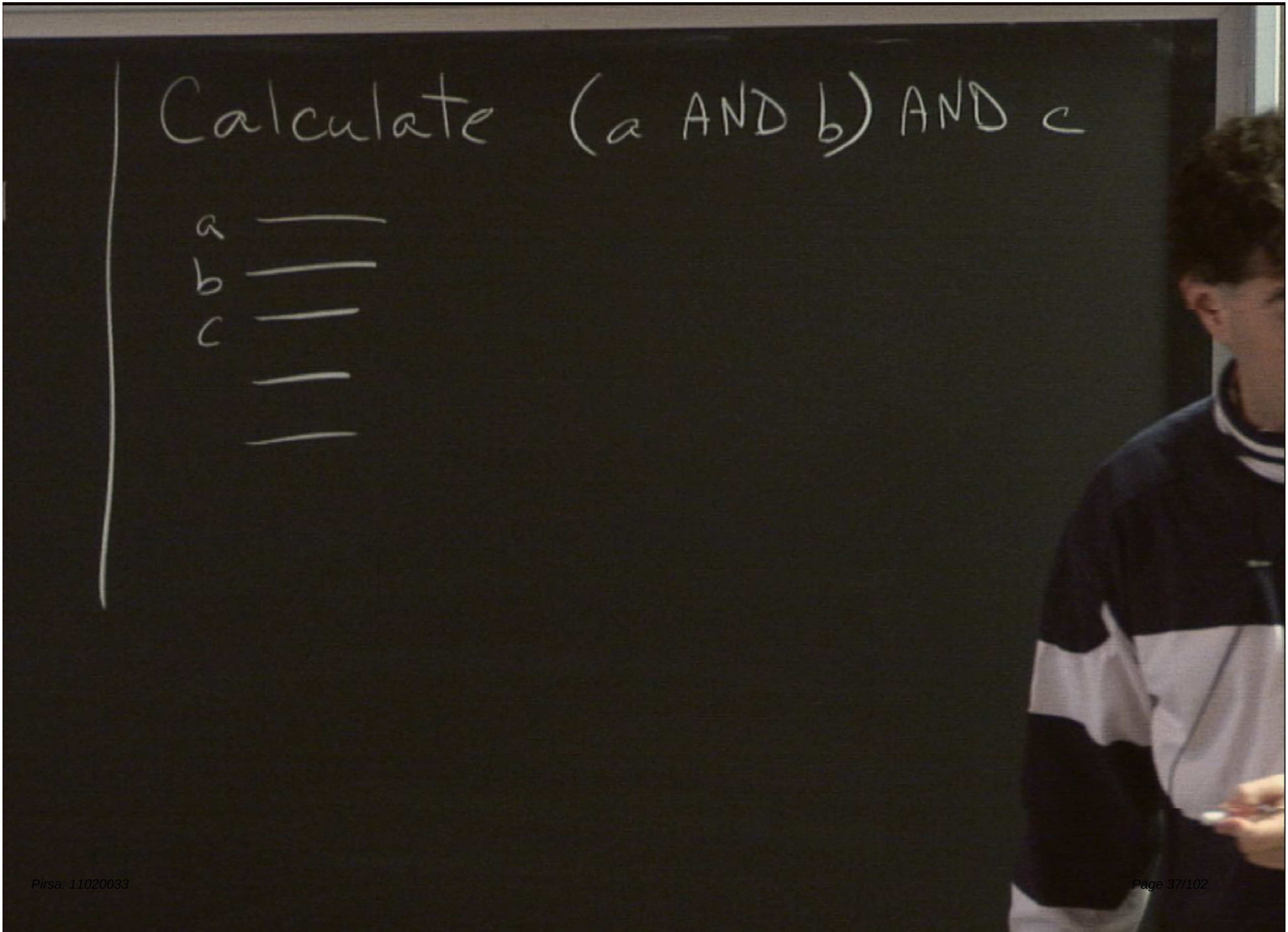

Circuit diagrams
Each line is one bit (or qubit)
Time goes from left to right
NOT
NOT a
b
a control
target
a⊕b
Toffoli
a
b
c
a
b
c⊕ab

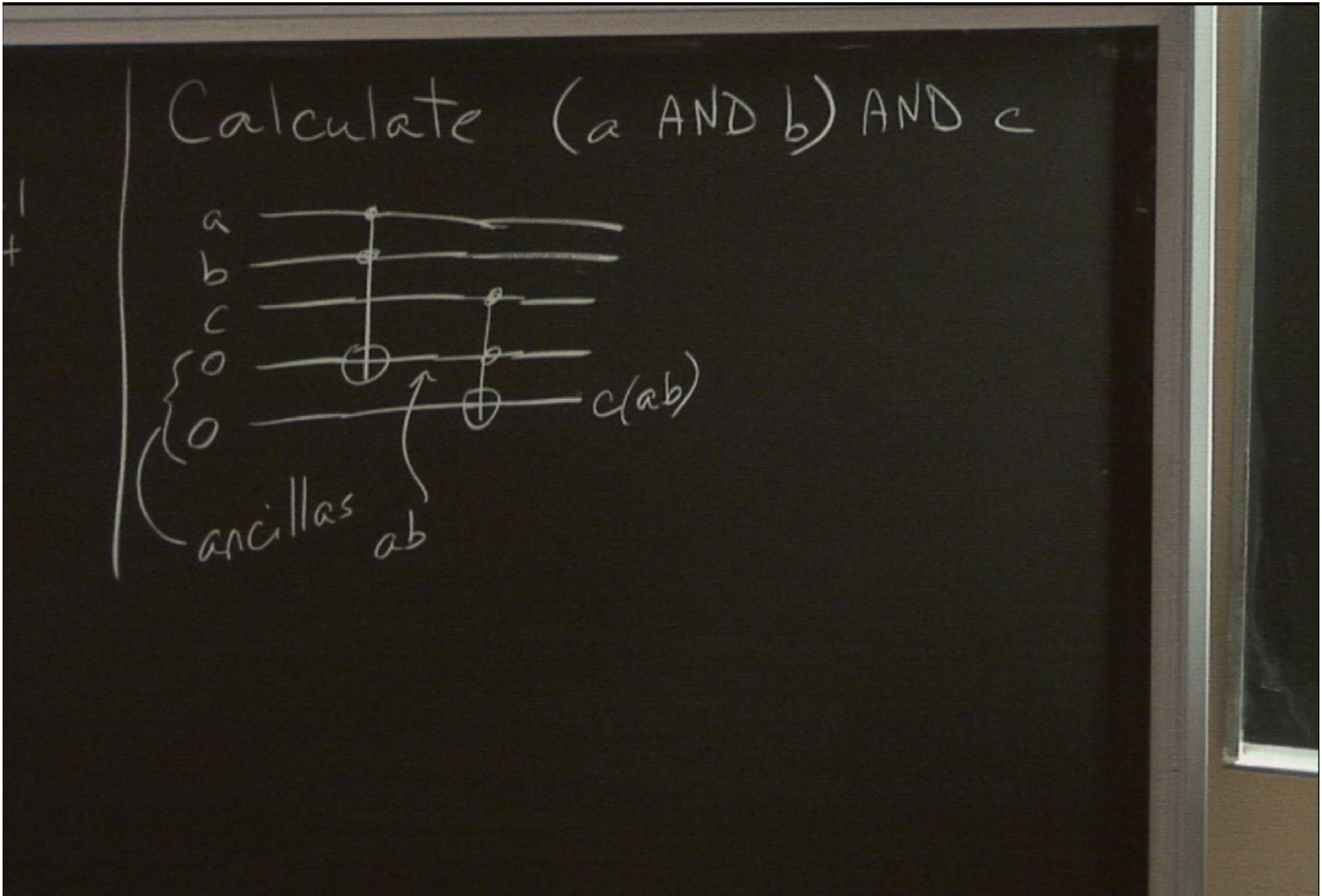
Circuit diagrams

Each line is one bit
(or qubit)

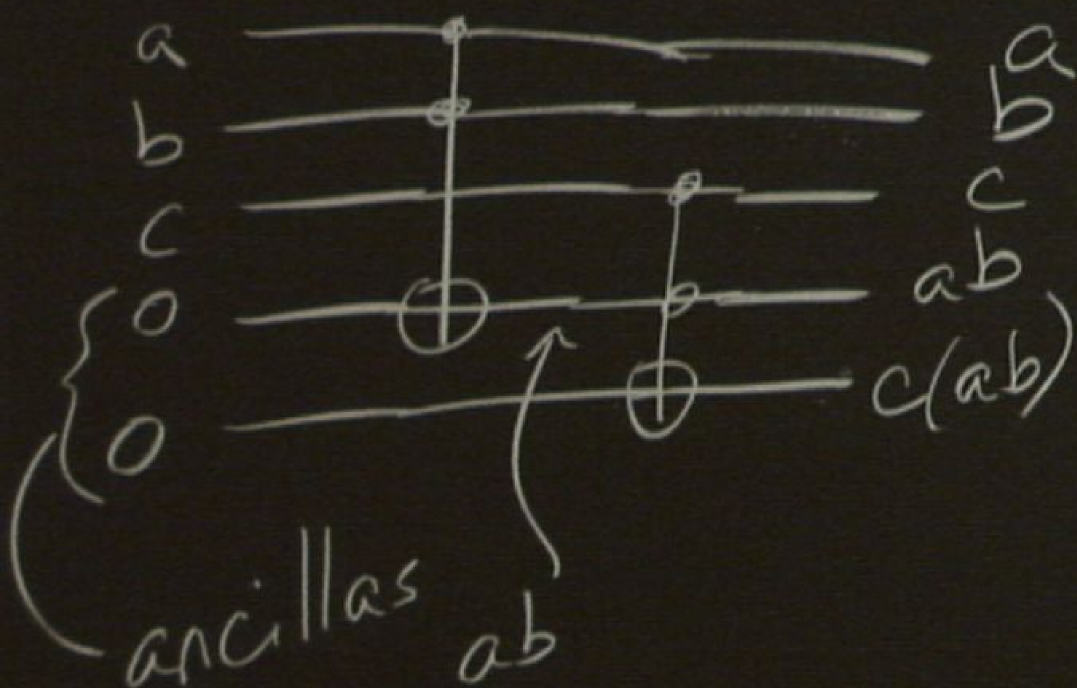
Time goes from left to right



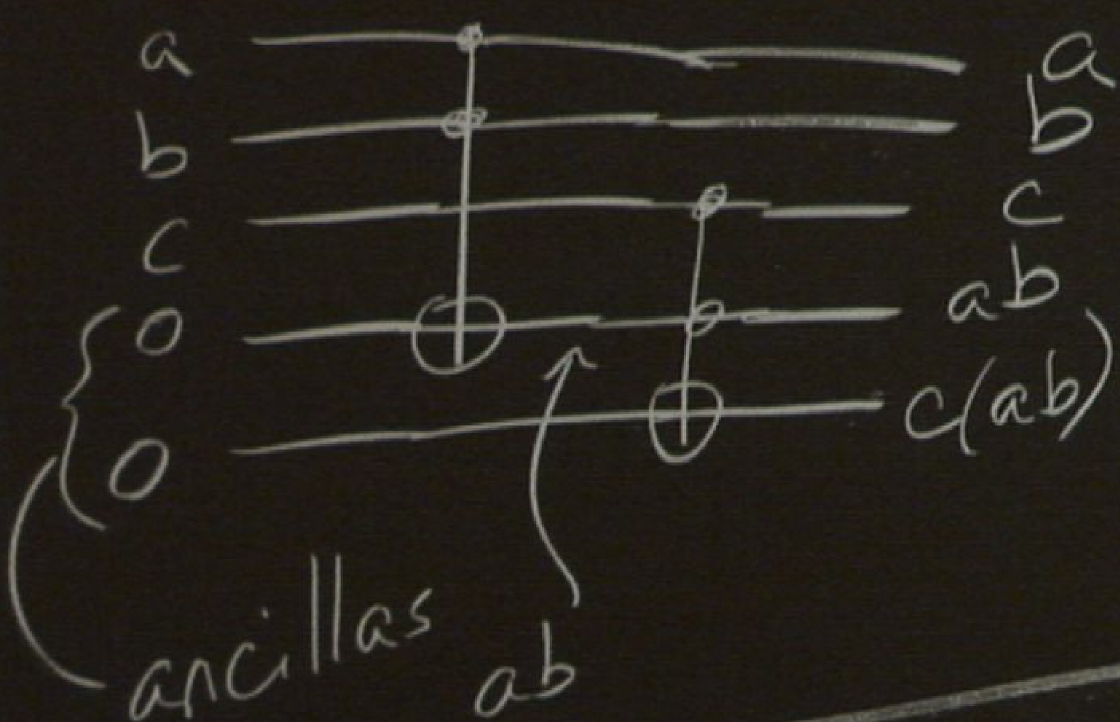

Calculate (a AND b) AND c
a
b
c


Calculate (a AND b) AND c
a
b
c
0
0
c(ab)
ancillas
ab

Calculate (a AND b) AND c

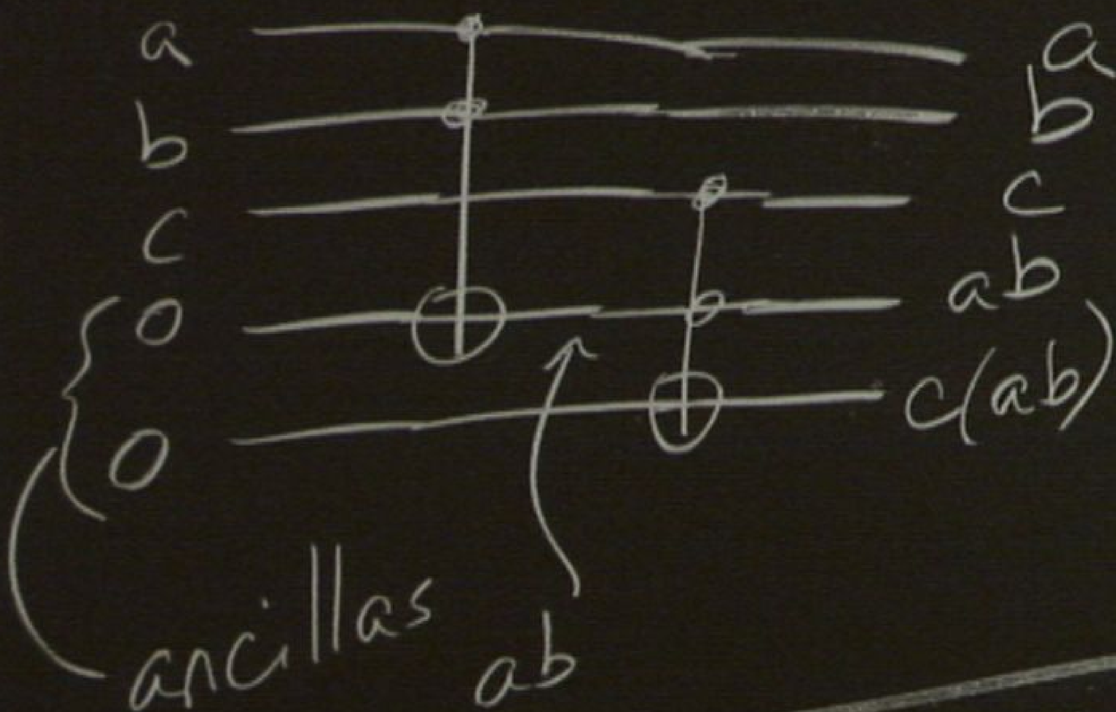


Calculate (a AND b) AND c



Classical function $(a_1, a_2, \ldots, a_n$

Calculate (a AND b) AND c



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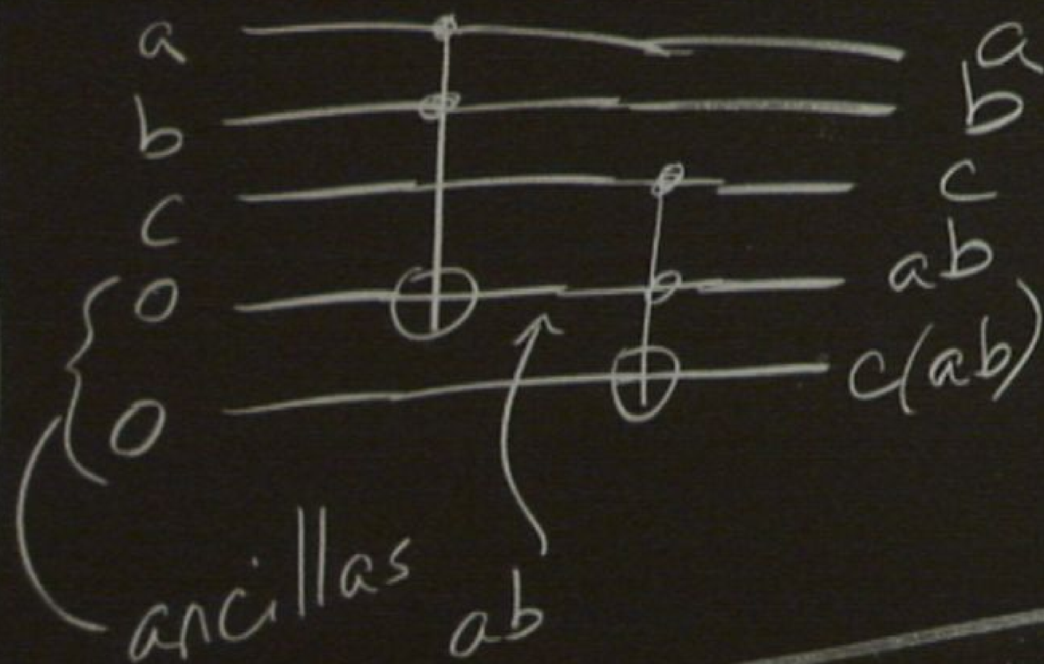
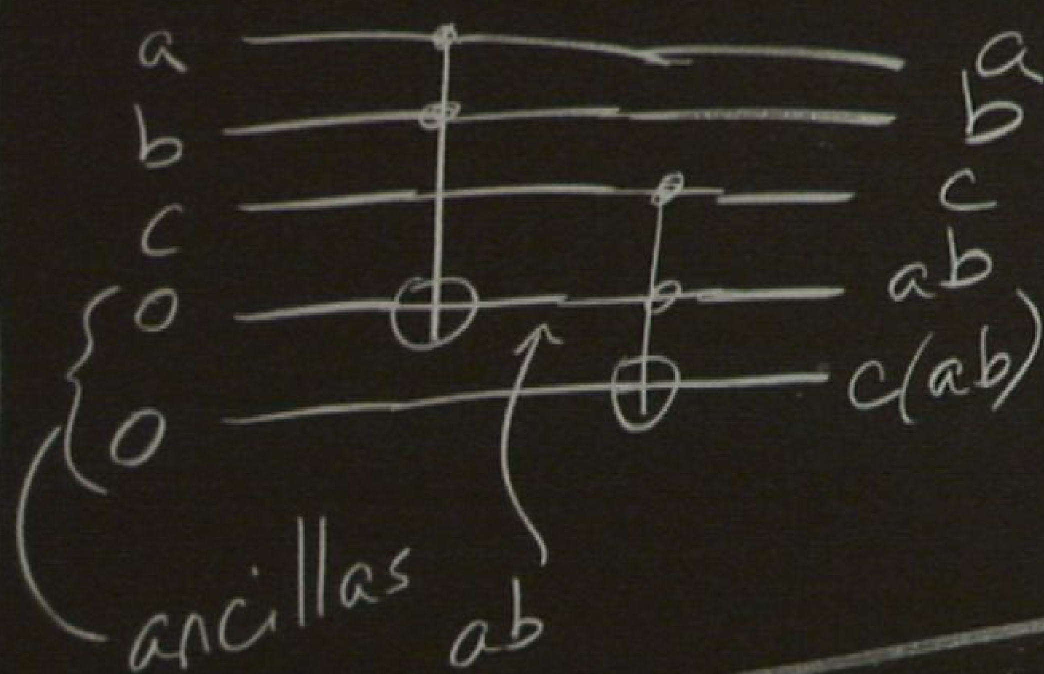
Calculate (a AND b) AND c



Classical function $(a_1, a_2, \dots, a_n) \to f(a_1, \dots, a_n)$

$\to (a_1, a_2, \dots, a_n, f(a_1, \dots, a_n))$

Calculate (a AND b) AND c



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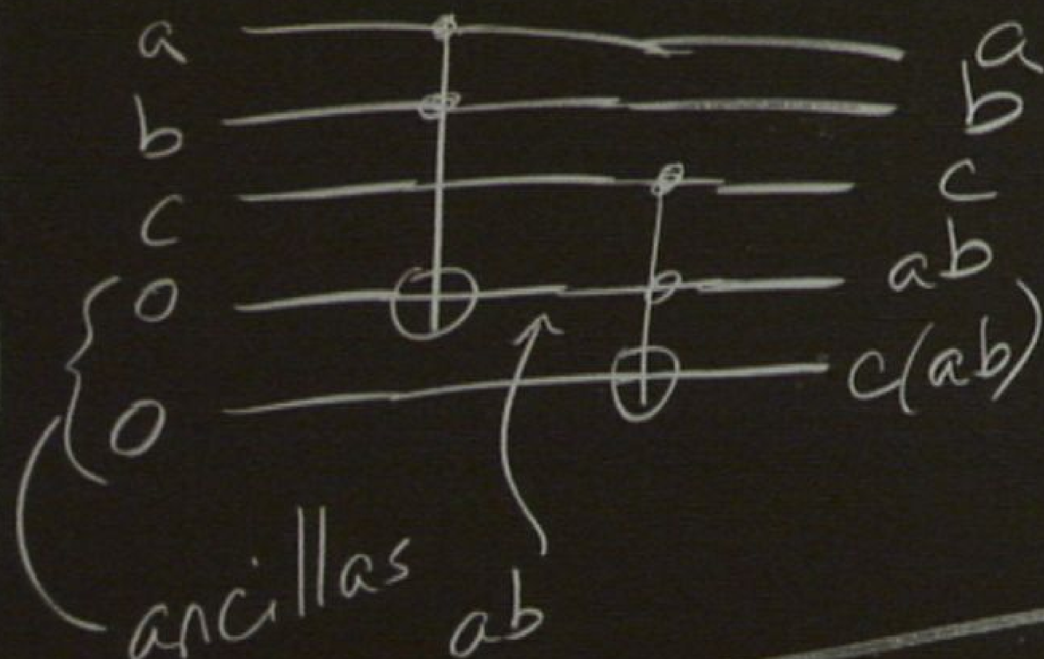
Calculate (a AND b) AND c



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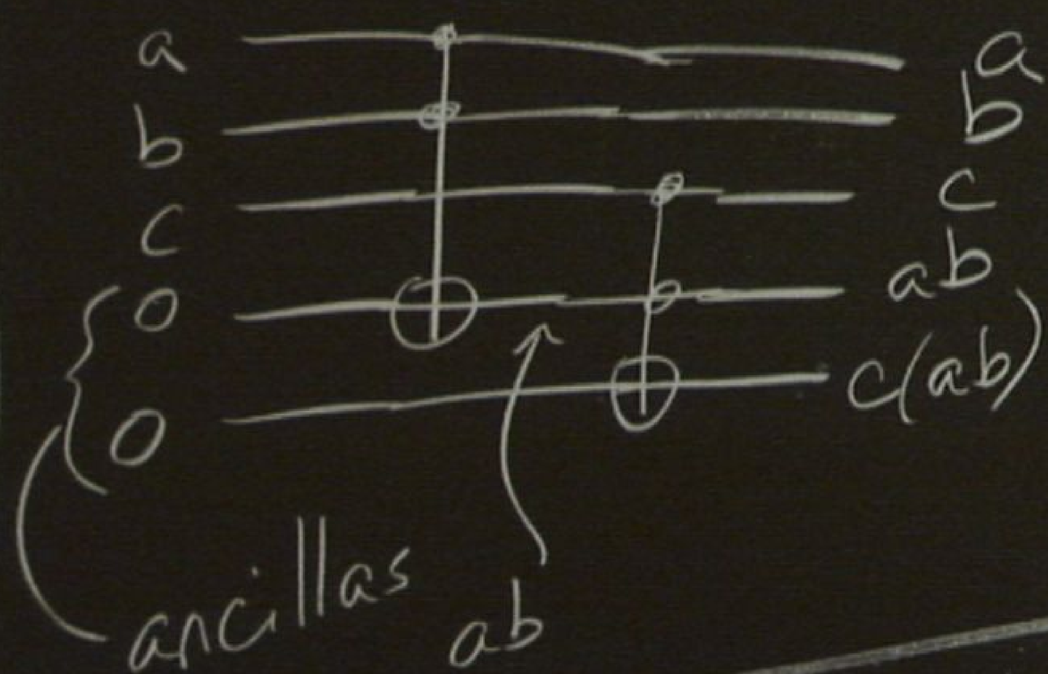

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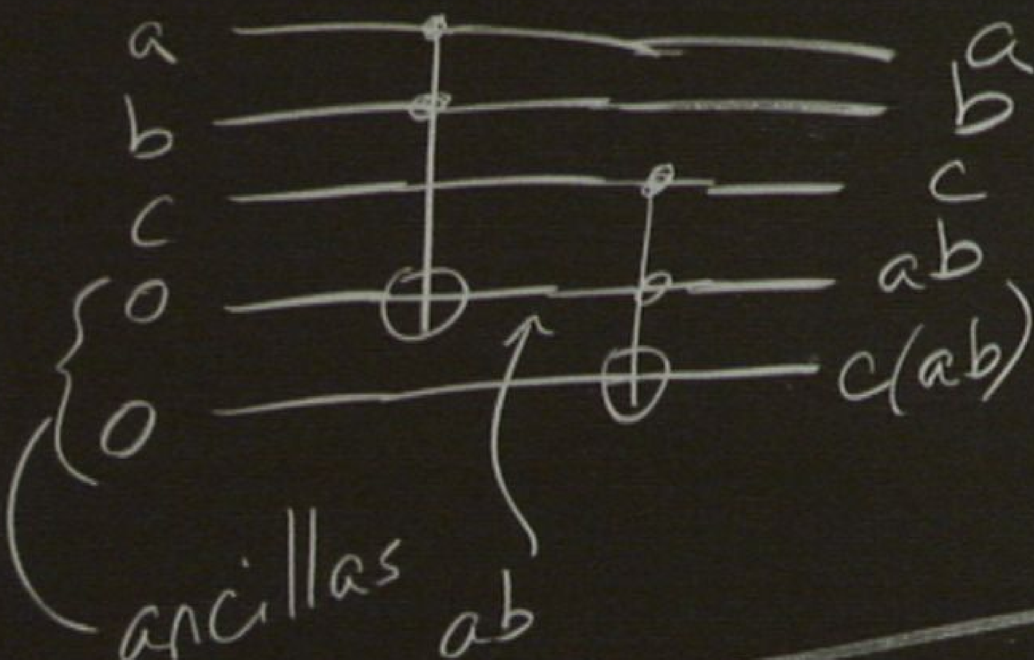
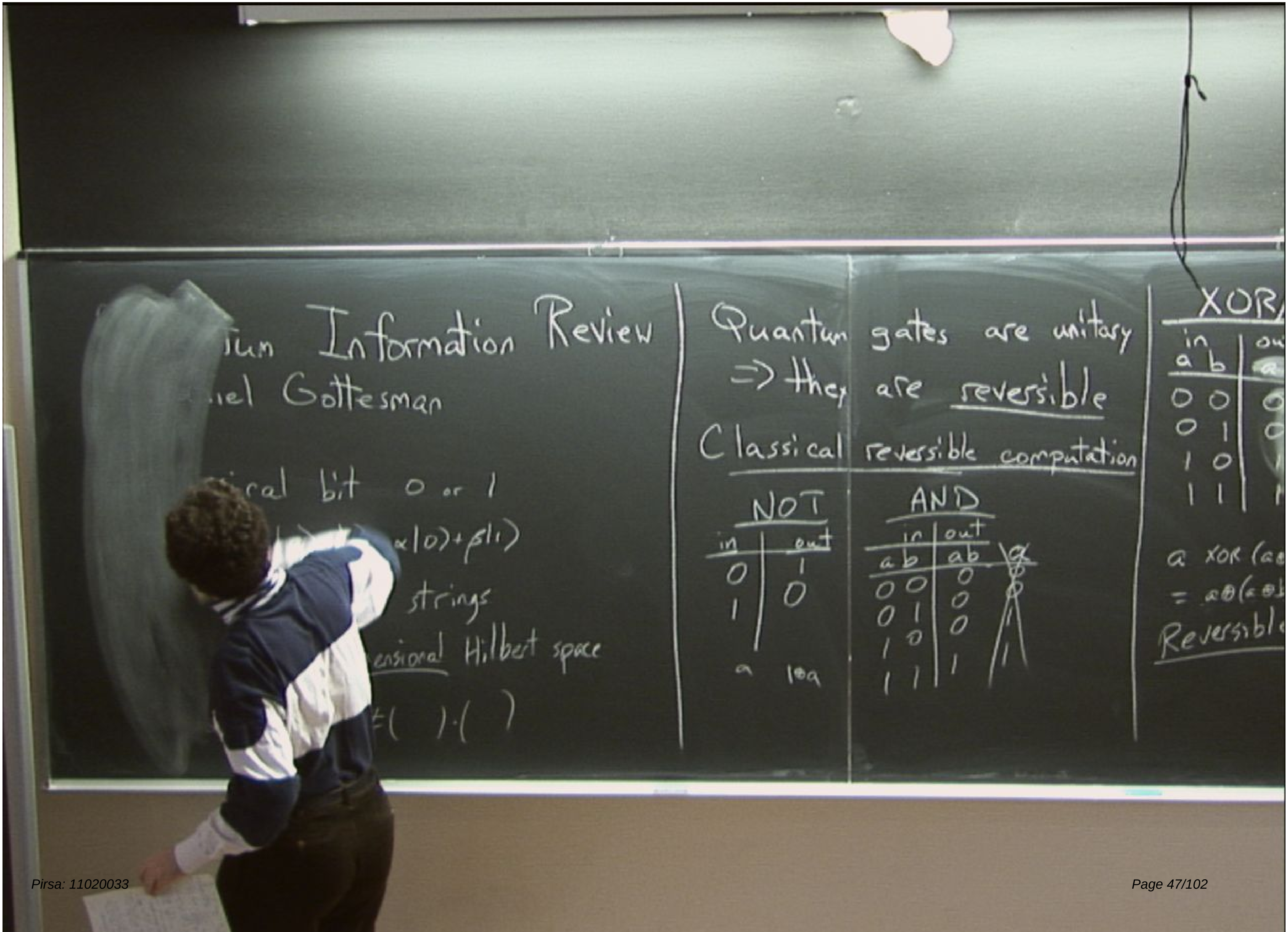


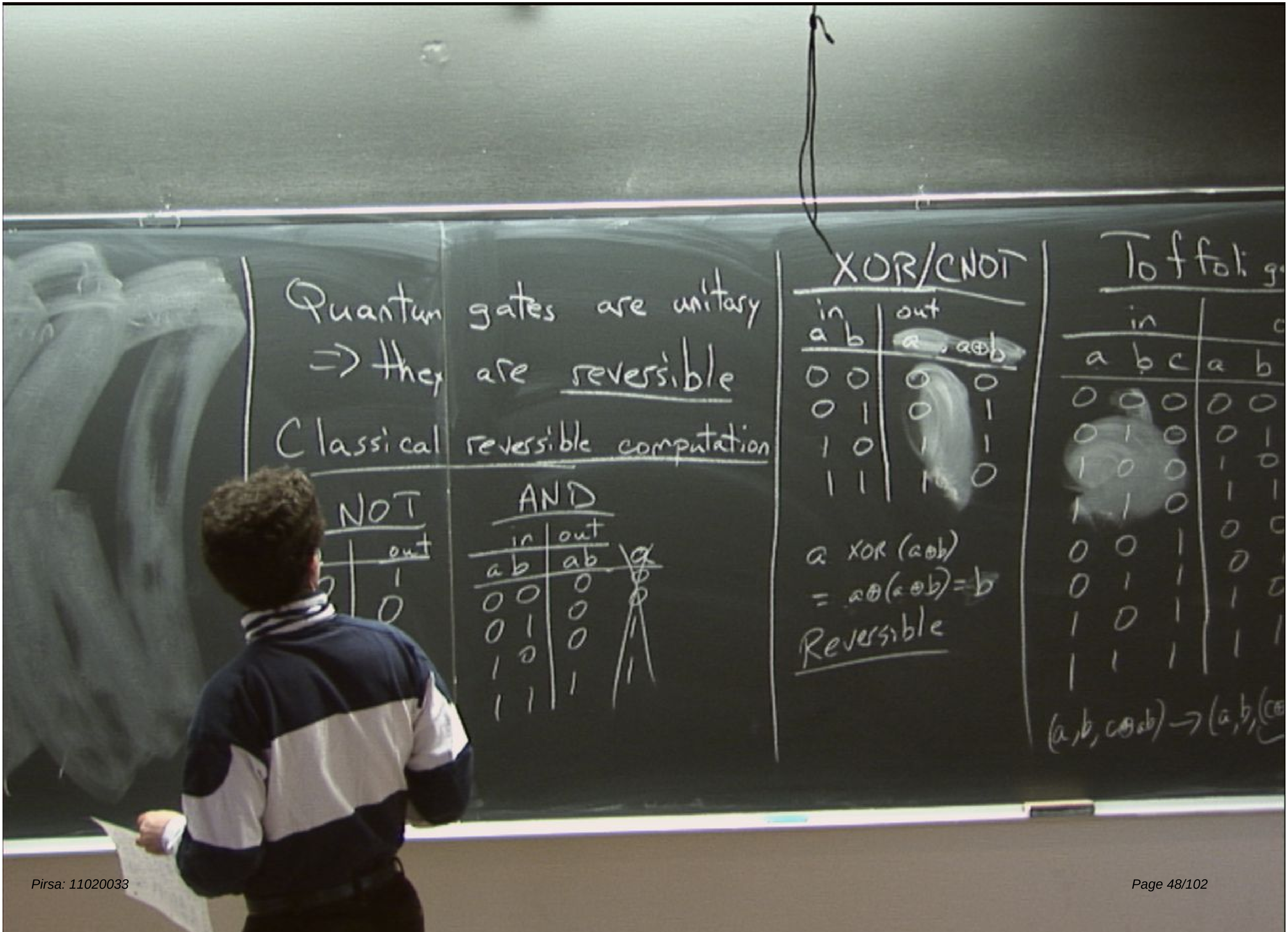
extra information

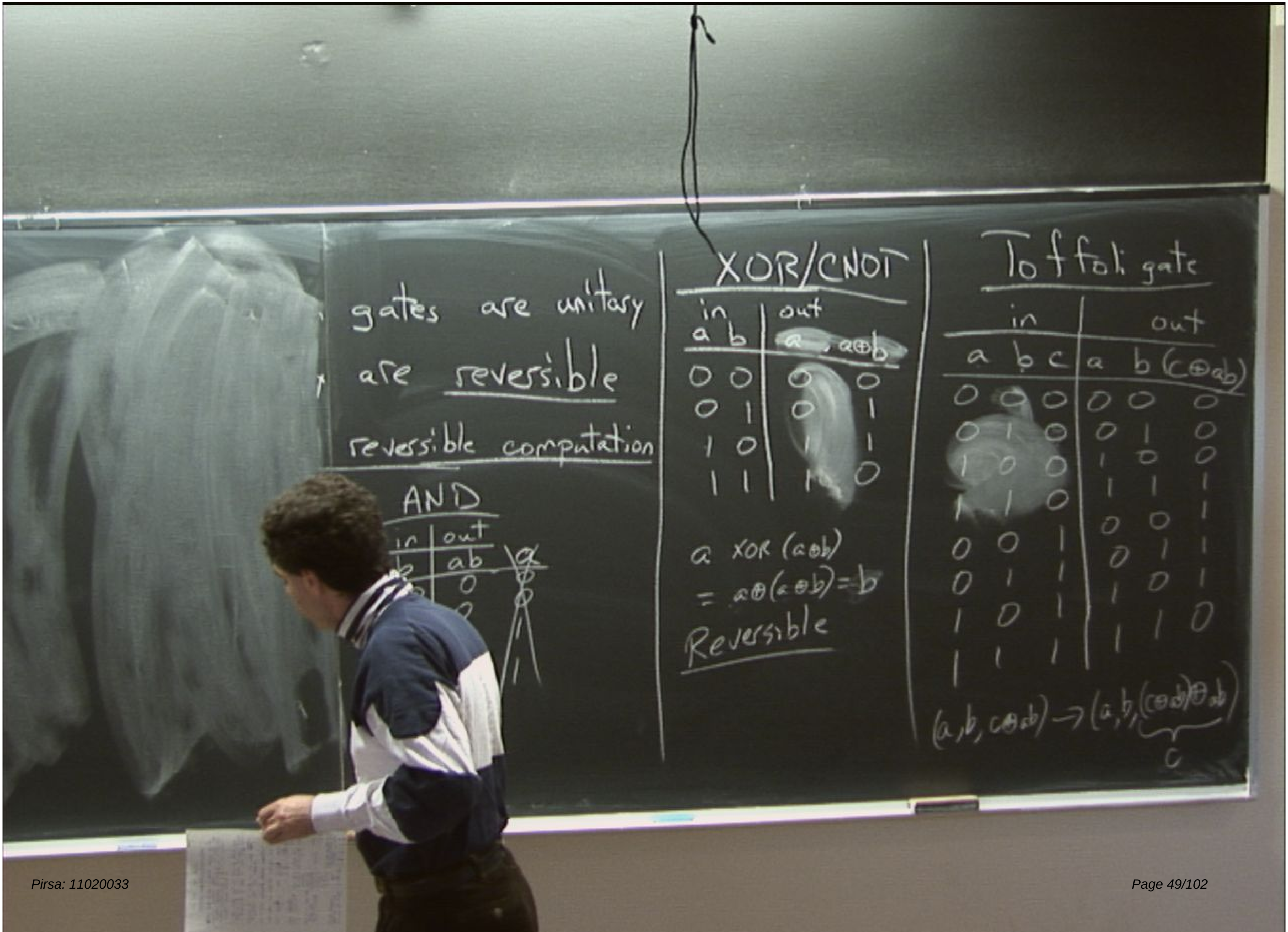
Classical function $(a_1, a_2, \dots, a_n) \to f(a_1, \dots, a_n)$

$$(a_1, \dots, a_n, c) \to (a_1, a_2, \dots, a_n, c \oplus f(a_1, \dots, a_n))$$

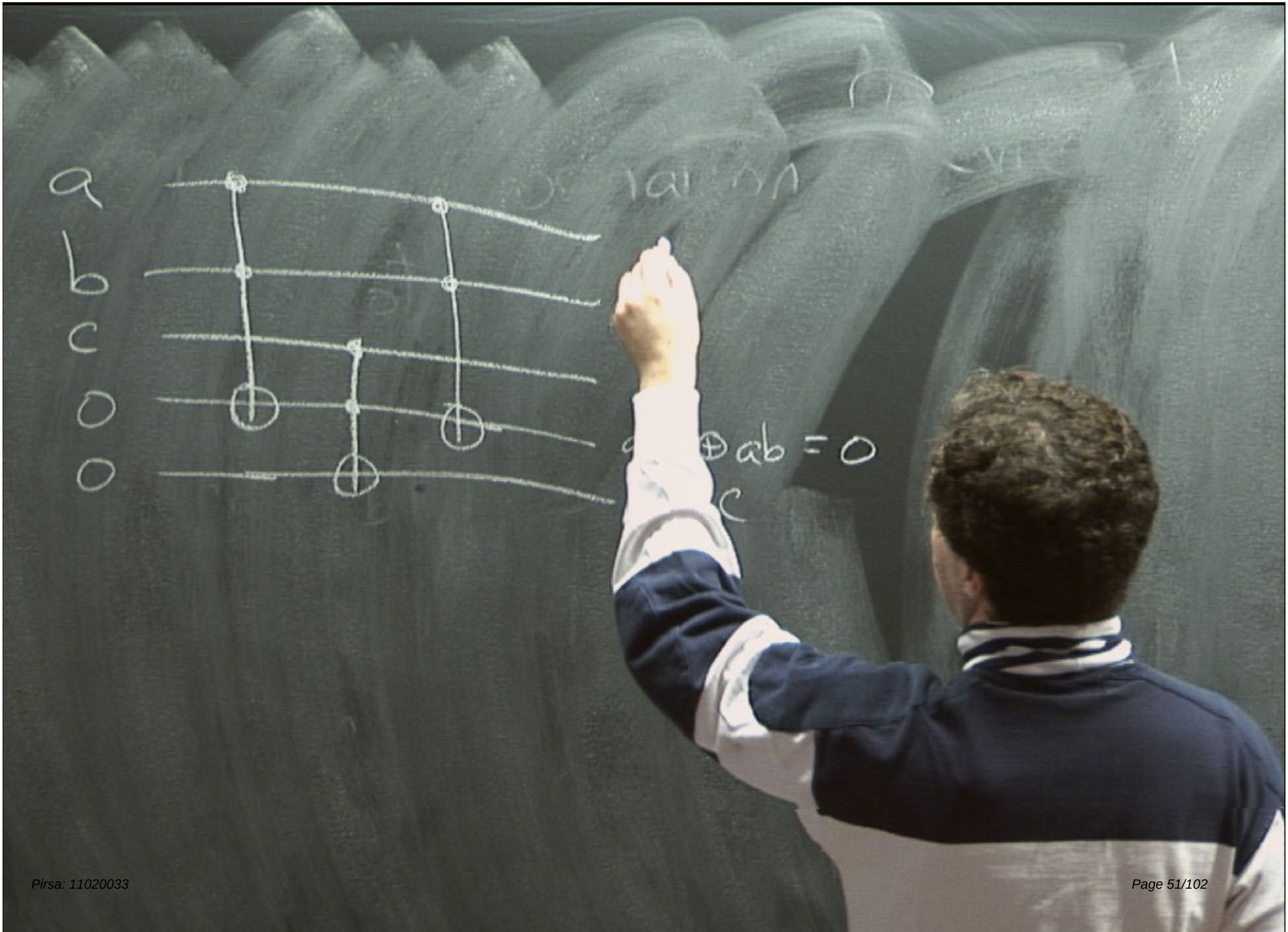
reversible!

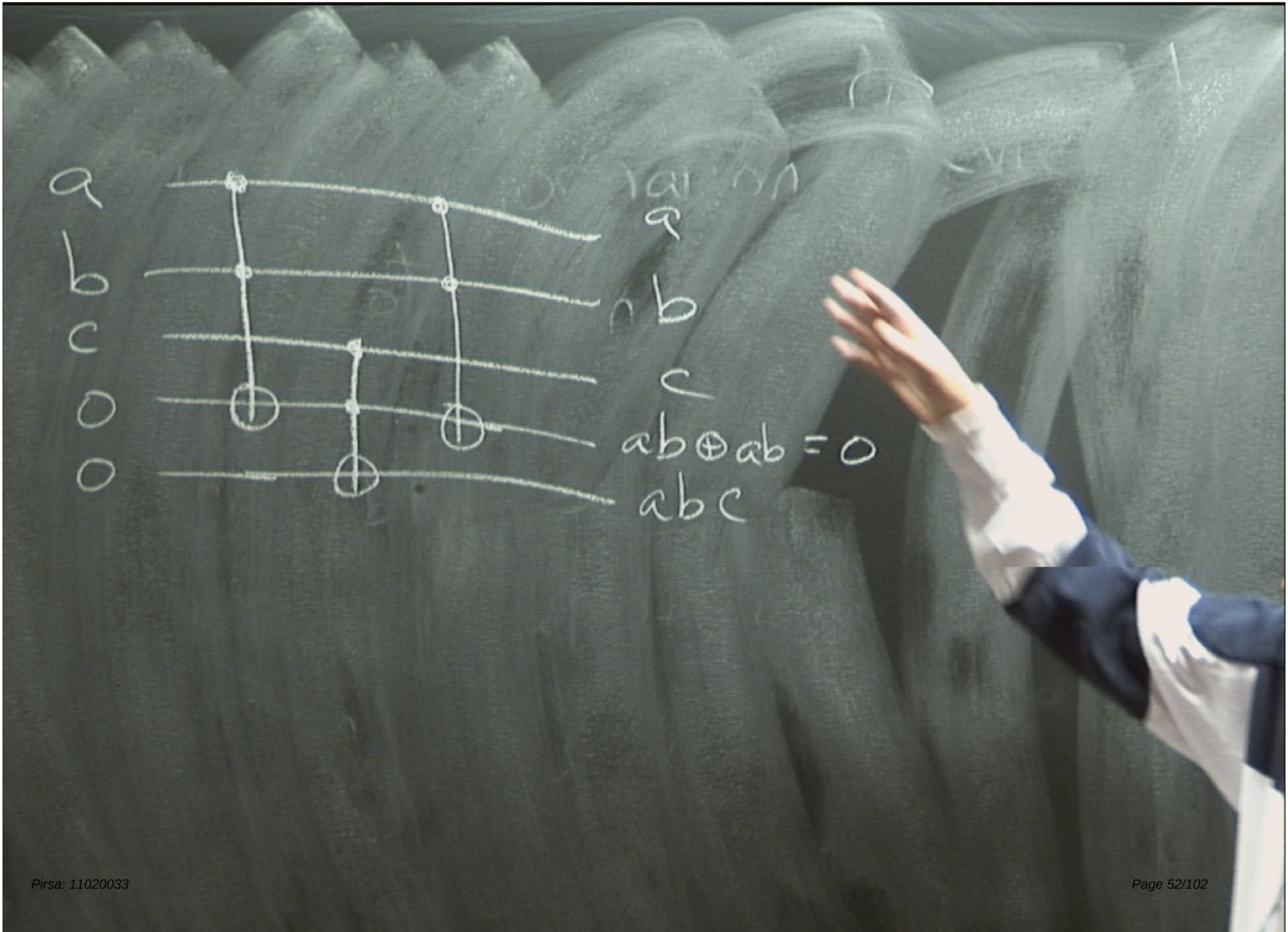

Information Review
Gottesman
bit 0 or 1
strings
Hilbert space
Quantum gates are unitary
⇒ they are reversible
Classical reversible computation
NOT
in out
0 1
1 0
AND
in out
ab ab
00 0
01 0
10 0
11 1
XOR
in
a b
00
01
10
11
a XOR
Reversibl

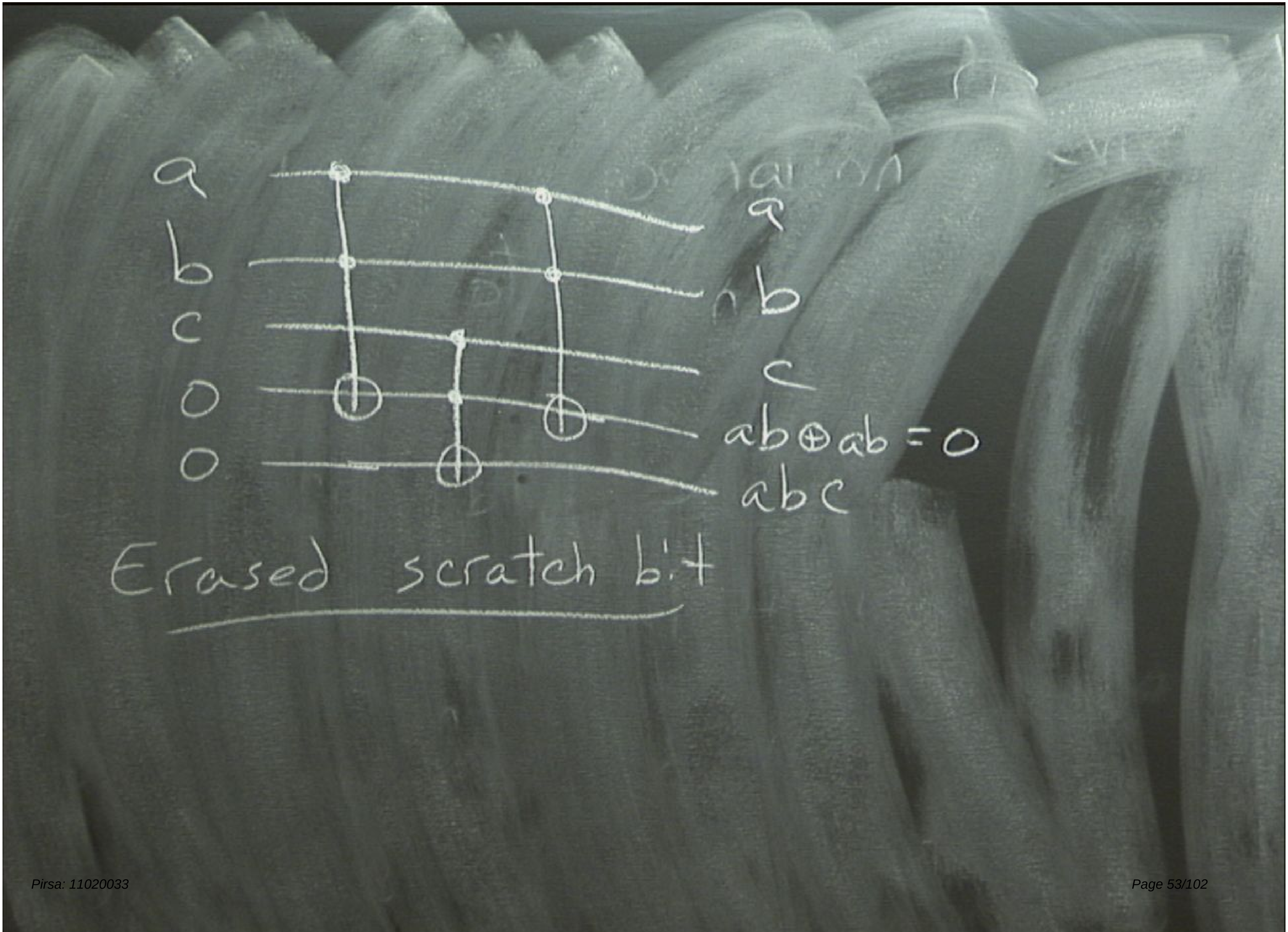

Quantum gates are unitary
⇒ they are reversible
Classical reversible computation
NOT
AND
XOR/CNOT
a XOR (a⊕b)
= a⊕(a⊕b) = b
Reversible
Toffoli

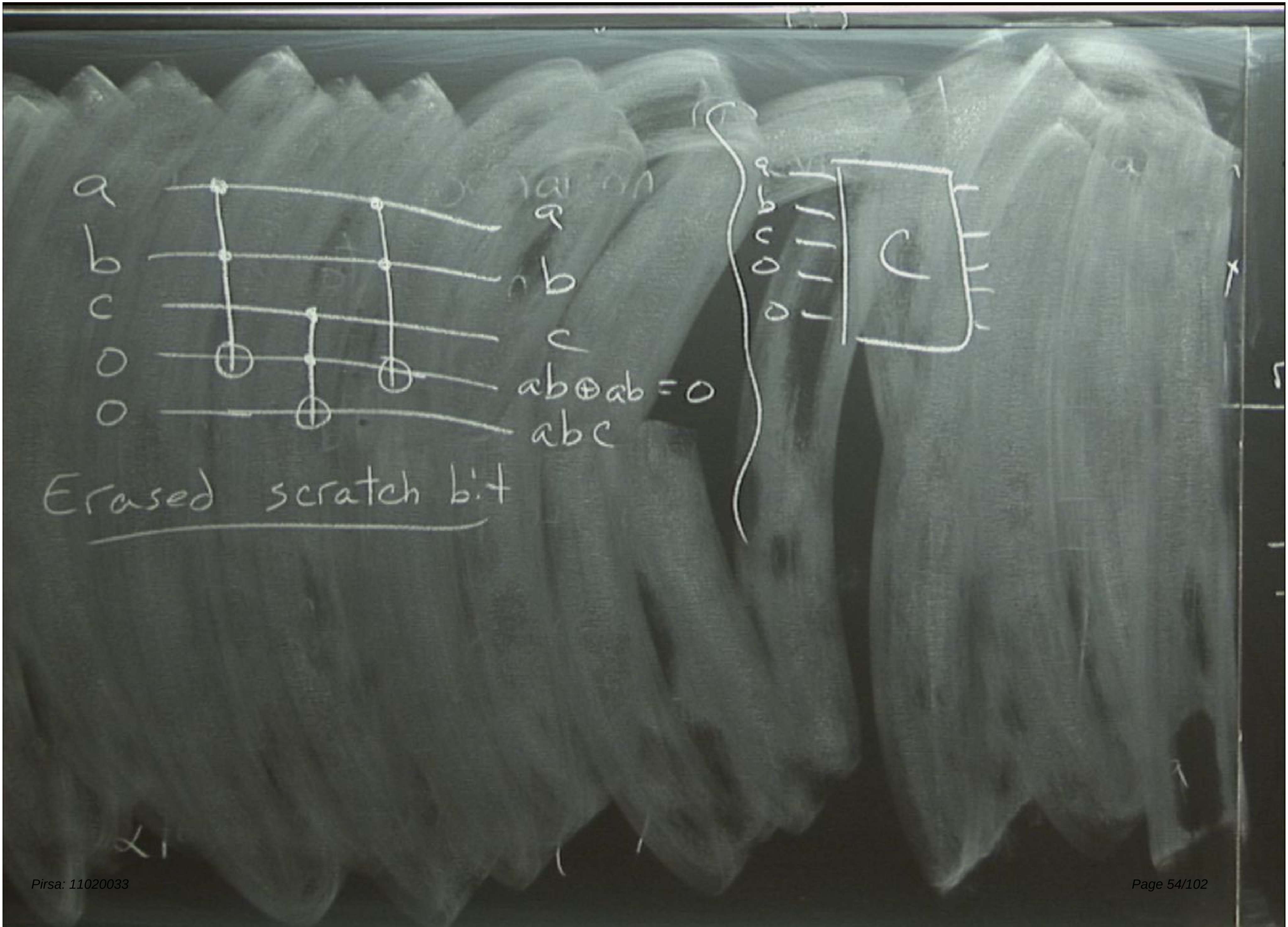

gates are unitary
are reversible
reversible computation
AND
in out
ab
XOR/CNOT
in out
a b a a⊕b
a XOR (a⊕b)
= a⊕(a⊕b) = b
Reversible
Toffoli gate
in out
a b c a b (c⊕ab)
(a,b,c⊕ab) → (a,b,(c⊕ab)⊕ab)
c

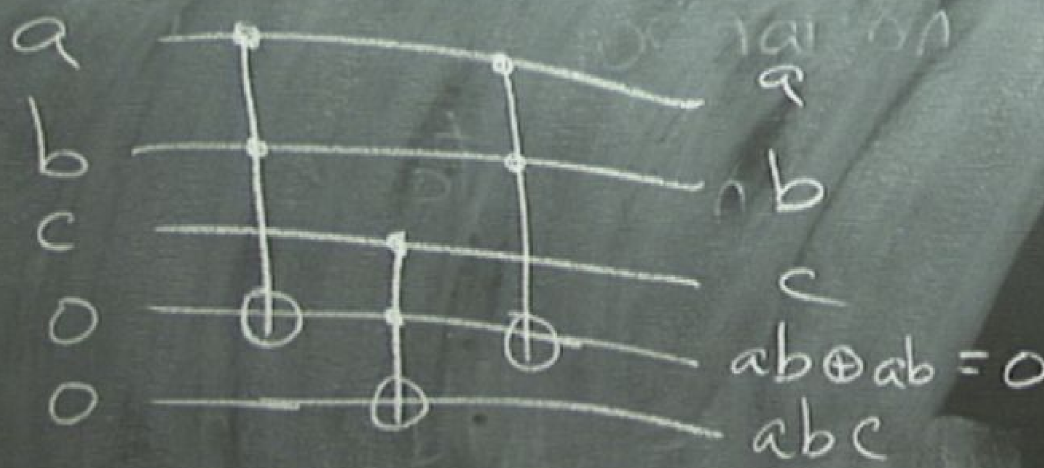

a
b
c
0
0

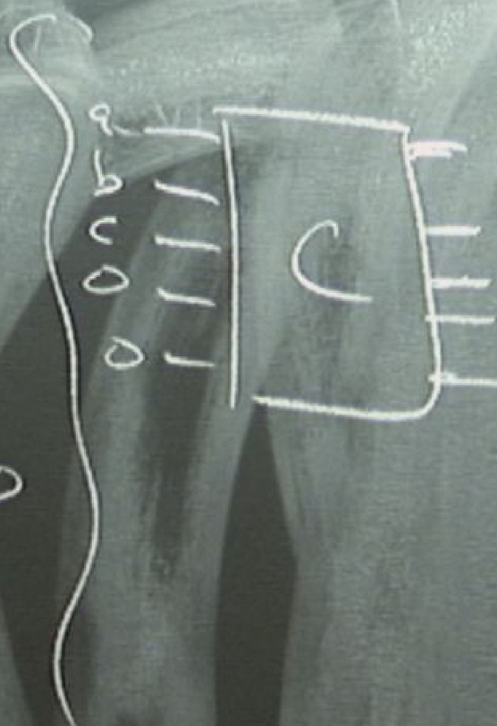

a
b
c
0
0
$\oplus ab = 0$
c


a
b
c
0
0
a
b
c
ab⊕ab=0
abc

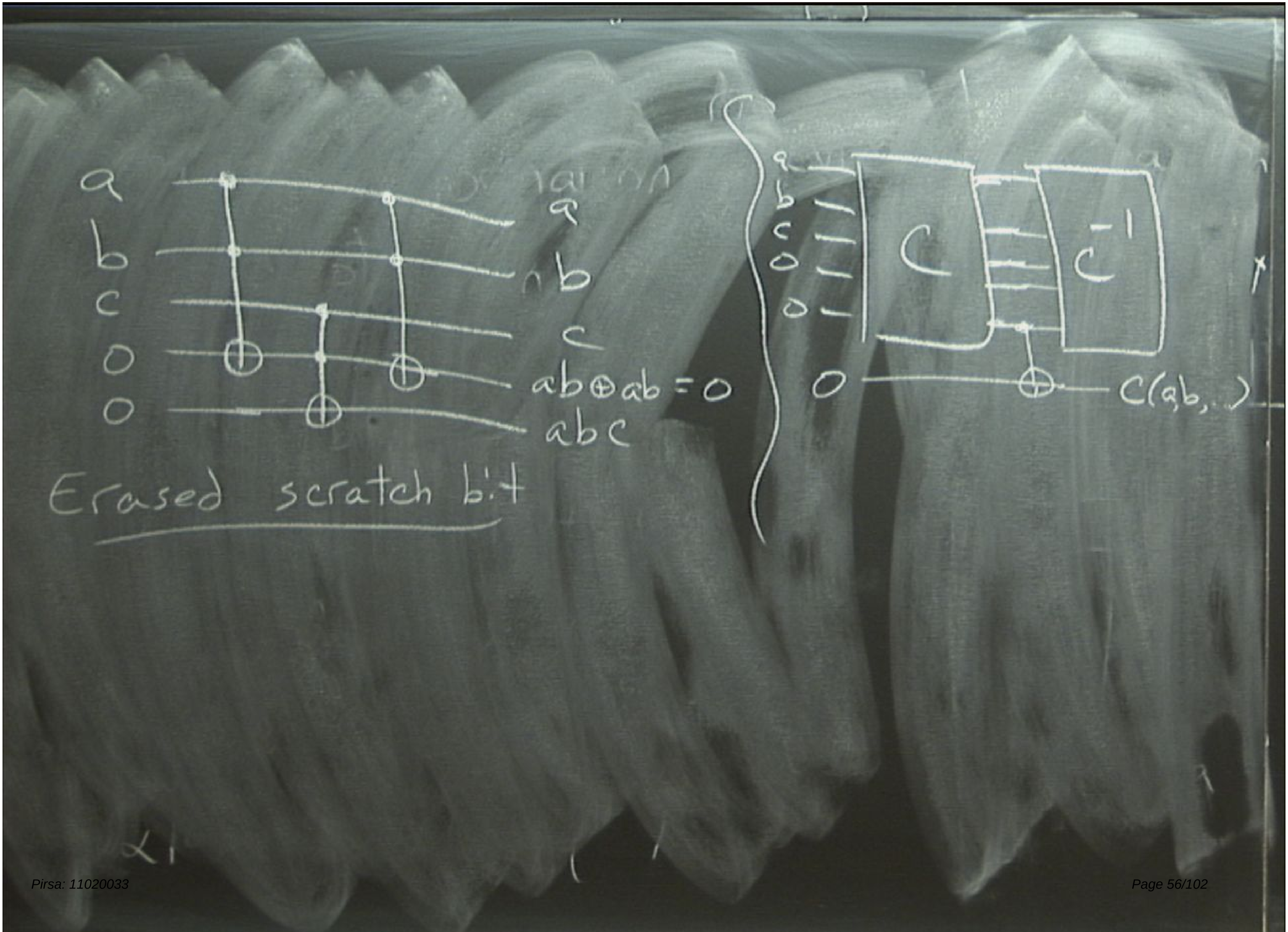

a
b
c
0
0
a
b
c
ab⊕ab=0
abc
Erased scratch bit

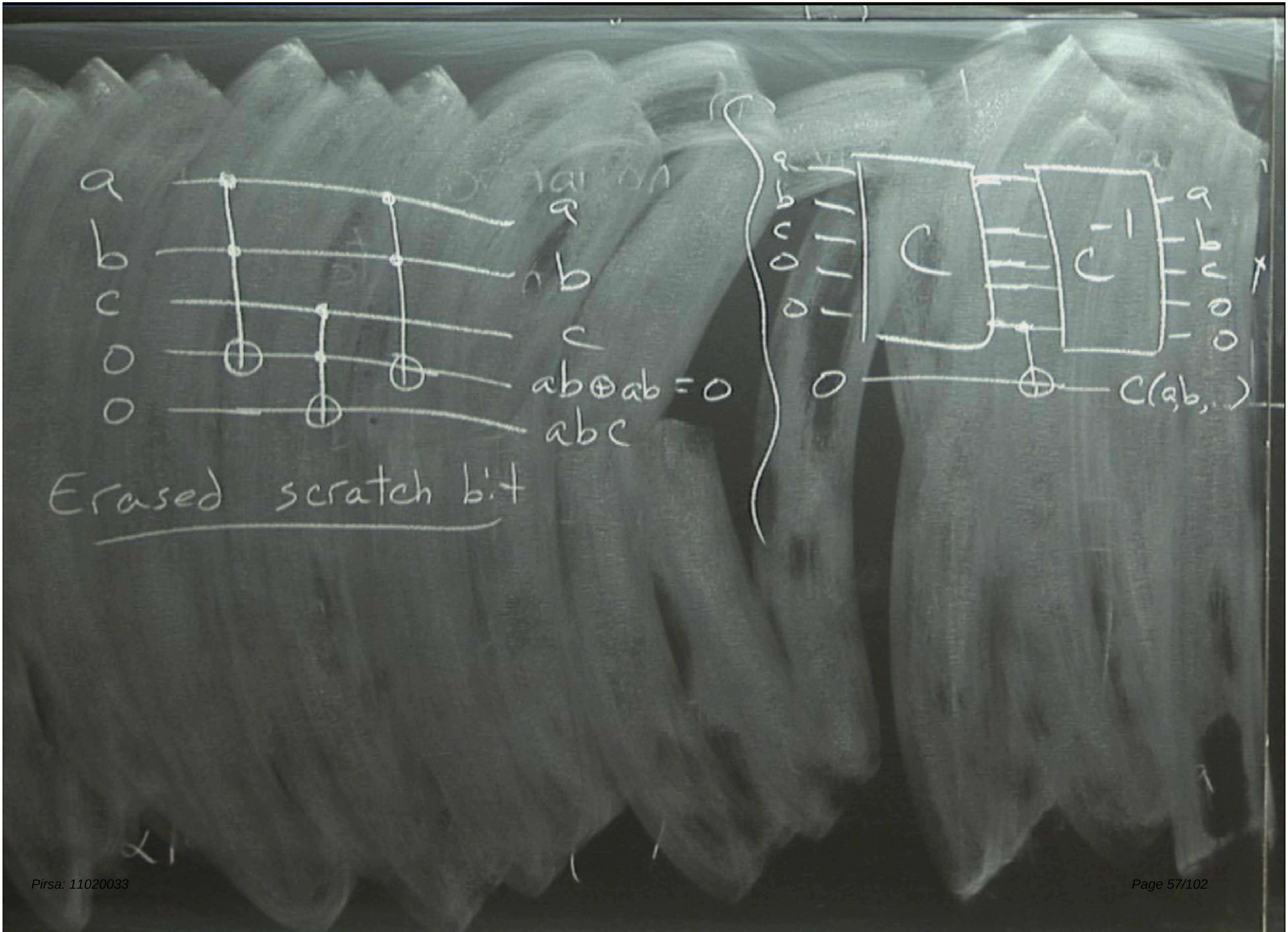

a
b
c
0
0
a
b
c
ab⊕ab=0
abc
Erased scratch bit
a
b
c
0
0
C

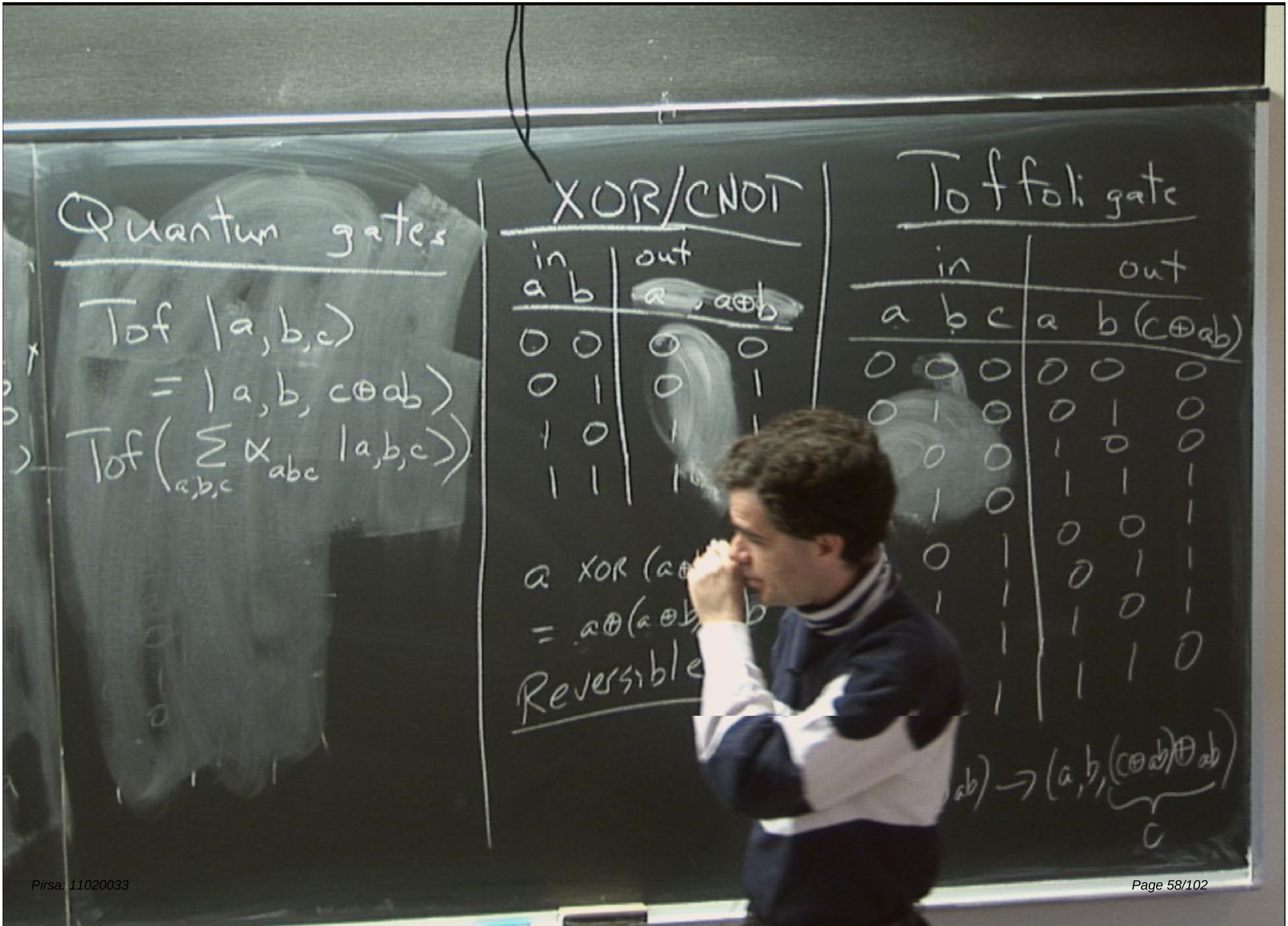


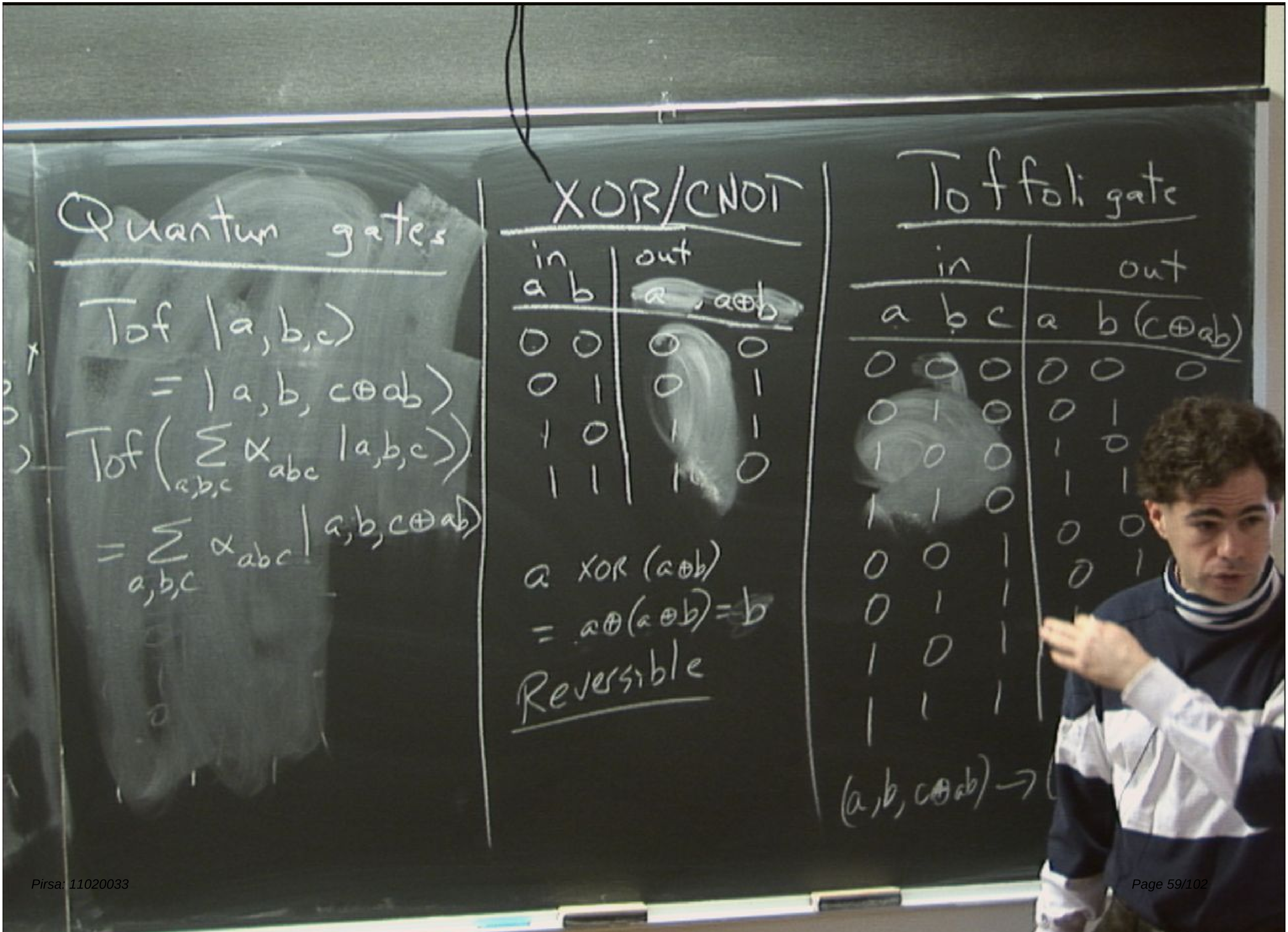


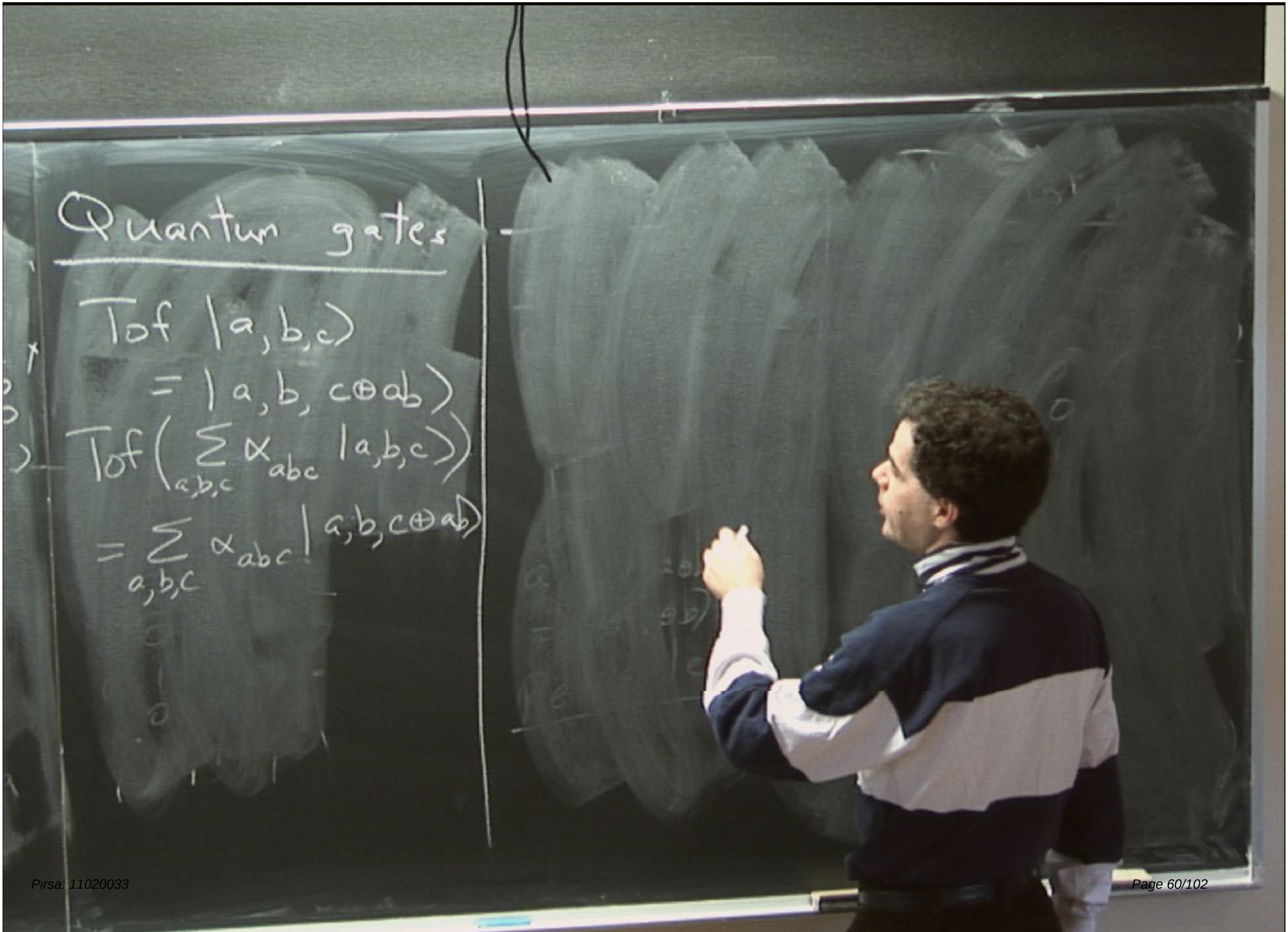
Erased scratch bit


a
b
c
0
0
a
b
c
ab⊕ab=0
abc
Erased scratch bit
a
b
c
0
0
C
C⁻¹
0
C(a,b,..)


a
b
c
0
0
a
b
c
ab⊕ab=0
abc
Erased scratch bit
a
b
c
0
0
C
C^-1
a
b
c
0
0
0
C(a,b,..)


Quantum gates
Tof |a, b, c⟩
= |a, b, c⊕ab⟩
Tof (Σ α_abc |a, b, c⟩)
a,b,c
XOR/CNOT
in
out
a b
a , a⊕b
0 0
0 0
0 1
0 1
1 0
1 1
1 1
1 0
Reversible
Toffoli gate
in
out
a b c
a b (c⊕ab)
c


Quantum gates
Tof |a,b,c)
= |a,b, c⊕ab)
Tof(Σ α_abc |a,b,c))
= Σ α_abc |a,b,c⊕ab)
XOR/CNOT
in out
a b a a⊕b
0 0 0 0
0 1 0 1
1 0 1 1
1 1 1 0
a XOR (a⊕b)
= a⊕(a⊕b) = b
Reversible
Toffoli gate
in out
a b c a b (c⊕ab)
(a,b,c⊕ab) →

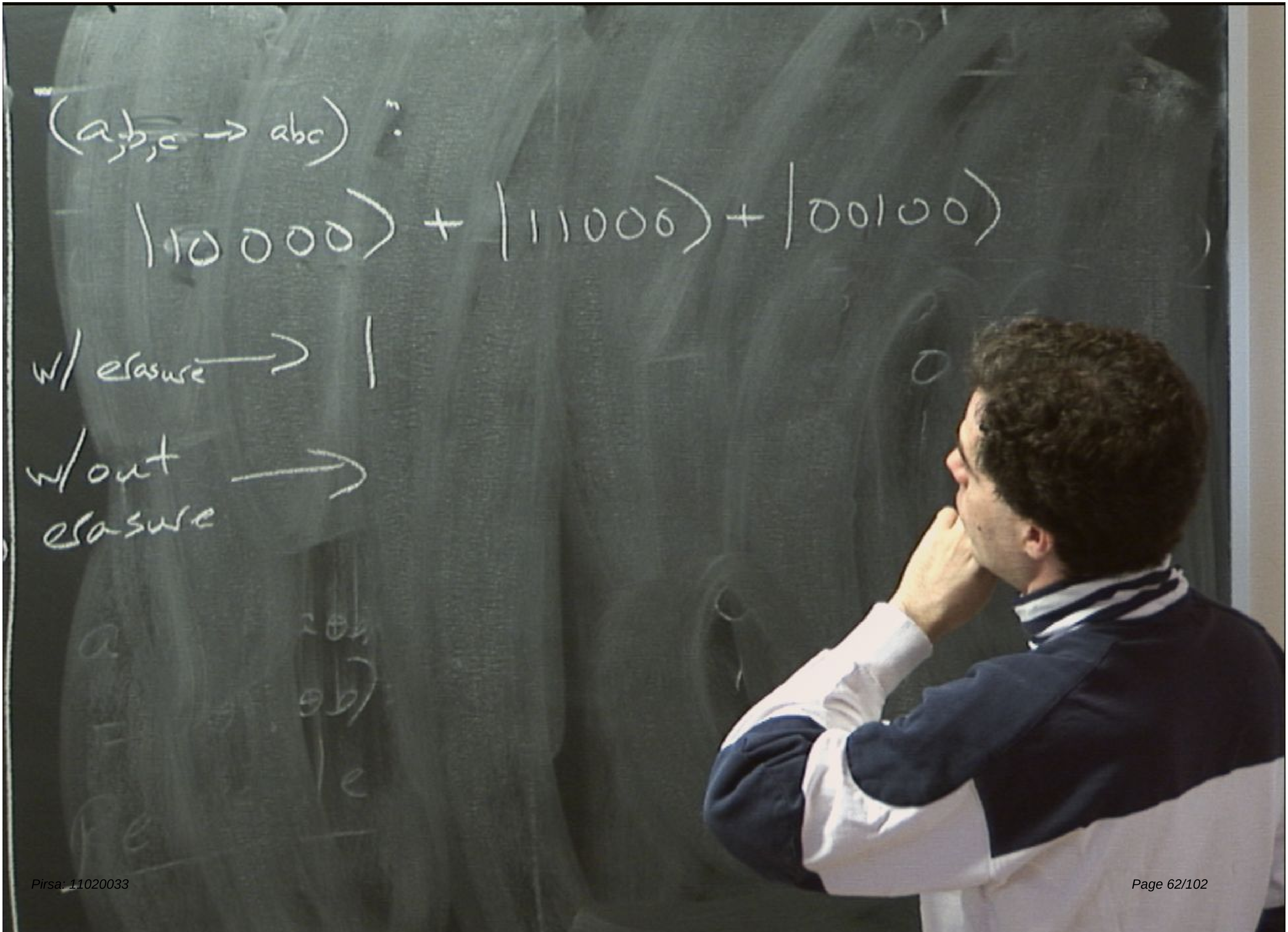

Quantum gates
$\mathrm{Tof}\,|a,b,c\rangle$
$= |a,b,c\oplus ab\rangle$
$\mathrm{Tof}\left(\sum_{a,b,c} \alpha_{abc} |a,b,c\rangle\right)$
$= \sum_{a,b,c} \alpha_{abc} |a,b,c\oplus ab\rangle$

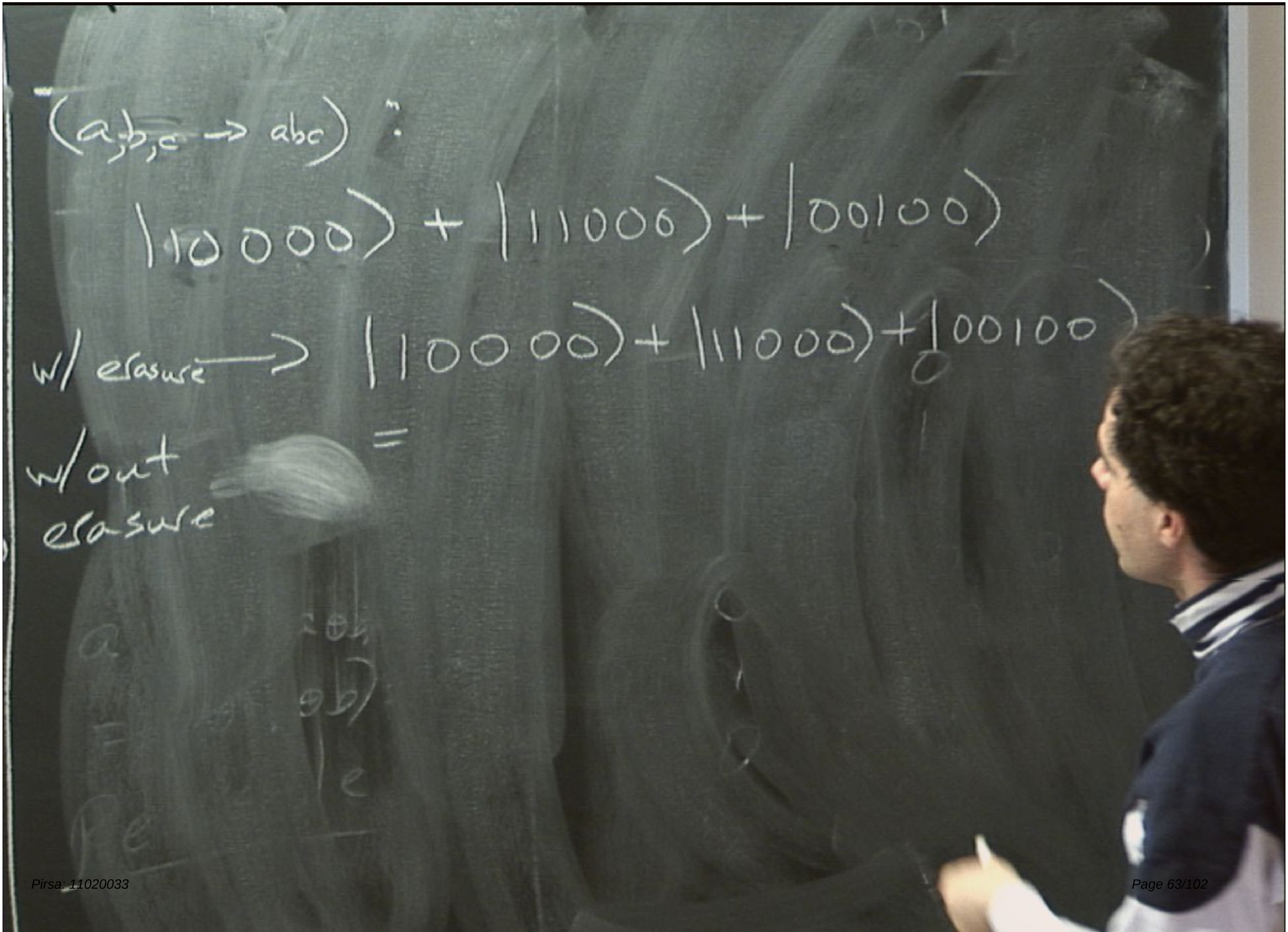
$(a,b,c \to abc)$:

$$|10000\rangle + |11000\rangle + |00100\rangle$$

w/ erasure $\longrightarrow$

w/out erasure $\longrightarrow$


(a,b,c → abc) :
|10000) + |11000) + |00100)
w/ erasure → |
w/out erasure →


(a,b,c → abc) :
|10000⟩ + |11000⟩ + |00100⟩
w/ erasure → |100000⟩ + |111000⟩ + |00100⟩
w/out erasure
=

$(a,b,c \to abc)$:

$$|10000\rangle + |11000\rangle + |00100\rangle$$

w/ erasure $\longrightarrow$ $|10000\rangle + |11000\rangle + |00100\rangle$

$$= (|100\rangle + |110\rangle + |001\rangle) \otimes |00$$

w/out erasure

$(a,b,c \to abc)$:

$$|10000\rangle + |11000\rangle + |00100\rangle$$

w/ erasure $\longrightarrow$ $|10000\rangle + |11000\rangle + |00100\rangle$

$= (|100\rangle + |110\rangle + |001\rangle) \otimes |00\rangle$

w/out erasure $\longrightarrow$ $|100\ \ 0\rangle$

$(a,b,c \to abc)$:

$$|10000\rangle + |11000\rangle + |00100\rangle$$

w/ erasure $\longrightarrow$ $|10000\rangle + |11000\rangle + |00100\rangle$

$$= \left(|100\rangle + |110\rangle + |001\rangle\right) \otimes |00\rangle$$

w/out erasure $\longrightarrow$ $|10000\rangle + |11010\rangle + |00100\rangle$

4th qubit, entangled w/ first 3.

$(a,b,c \rightarrow abc)$:

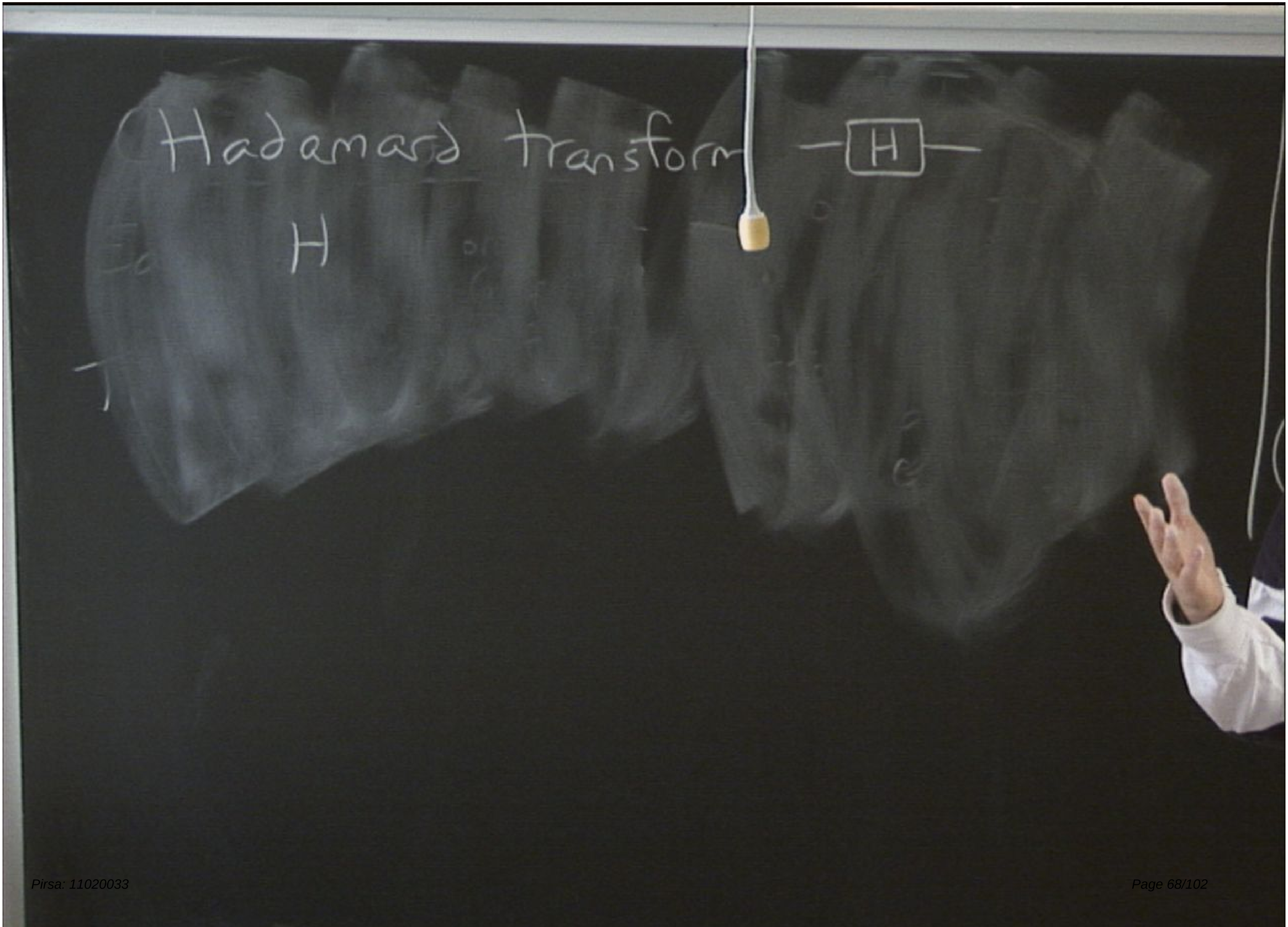
$$|10000\rangle + |11000\rangle + |00100\rangle$$

w/ erasure $\longrightarrow |10000\rangle + |11000\rangle + |00100\rangle$

$$= (|100\rangle + |110\rangle + |001\rangle) \otimes |00\rangle$$

w/out erasure $\longrightarrow |10000\rangle + |11010\rangle + |00100\rangle$

4th qubit, entangled w/ first 3.


Hadamard transform
H
H

Hadamard transform $-\boxed{H}-$

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Hadamard transform $-\boxed{H}-$

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$|0\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$|1\rangle \rightarrow$$

Hadamard transform $-\boxed{H}-$

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$|0\rangle \to \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$|1\rangle \to \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

Hadamard transform $-\boxed{H}-$

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$|0\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$|1\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

Hadamard transform $-\boxed{H}-$

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$|0\rangle \to \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$|1\rangle \to \frac{1}{\sqrt{2}}\left(|0\rangle -\right.$$

$$H^2 = I$$

Hadamard transform $\quad -\boxed{H}-$

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$|0\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$|1\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

$$H^2 = I$$

Hadamard transform $-\boxed{H}-$

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$|0\rangle \longrightarrow \overbrace{\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)}^{|+\rangle}$$

$$|1\rangle \longrightarrow \underbrace{\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)}_{|-\rangle}$$

$$H^2 = I$$

Hadamard transform

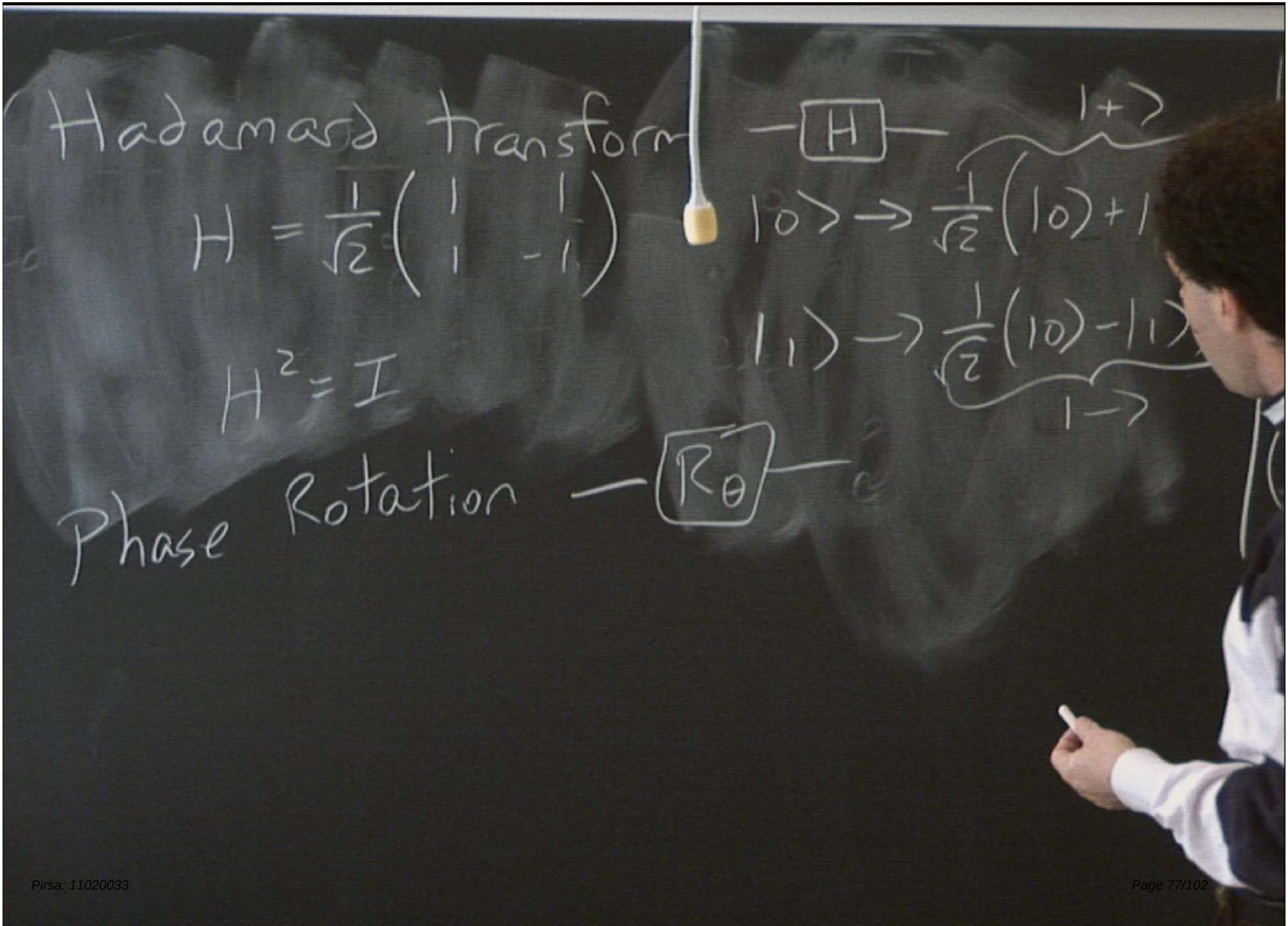
$$-\boxed{H}-$$

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$|0\rangle \to \overbrace{\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)}^{|+\rangle}$$

$$|1\rangle \to \underbrace{\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)}_{|-\rangle}$$

$$H^2 = I$$



Hadamard transform
$-\boxed{H}-$
$|+\rangle$
$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
$|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |$
$|1\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle$
$|-\rangle$
$H^2 = I$
Phase Rotation
$-\boxed{R_\theta}-$

Hadamard transform $-\boxed{H}-$

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$|0\rangle \to \overbrace{\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)}^{|+\rangle}$$

$$|1\rangle \to \underbrace{\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)}_{|-\rangle}$$

$$H^2 = I$$

Phase Rotation $-\boxed{R_\theta}-$

$$R_\theta = \begin{pmatrix} e^{-i\theta} & 0 \\ 0 & e^{i\theta} \end{pmatrix}$$

$$|0\rangle \to e^{-i\theta}|0\rangle, \quad |1\rangle \to e^{+i\theta}|1\rangle$$

Hadamard transform $-\boxed{H}-$

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$|0\rangle \to \overbrace{\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)}^{|+\rangle}$$

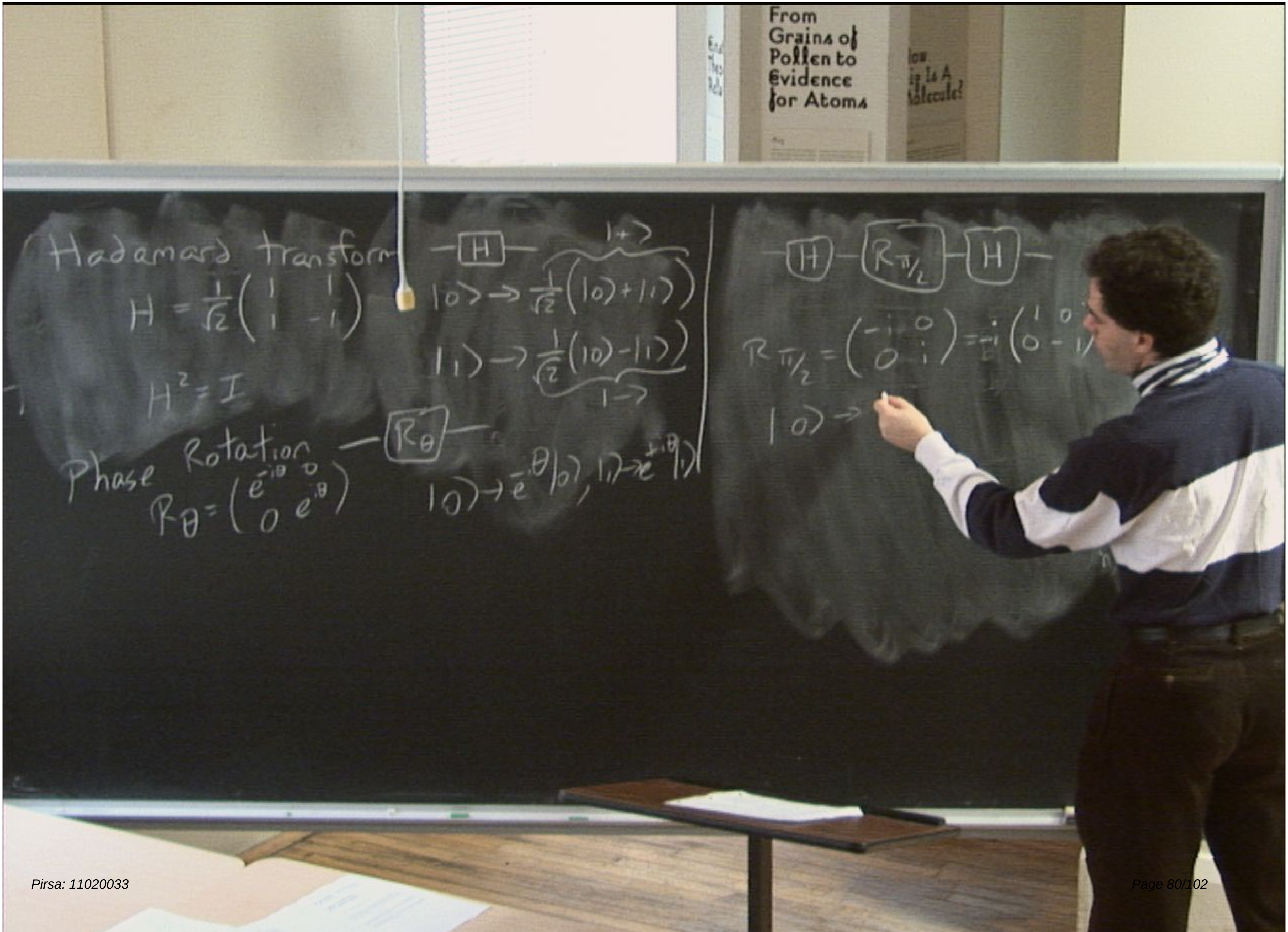
$$|1\rangle \to \underbrace{\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)}_{|-\rangle}$$

$$H^2 = I$$

Phase Rotation $-\boxed{R_\theta}-$

$$R_\theta = \begin{pmatrix} e^{-i\theta} & 0 \\ 0 & e^{i\theta} \end{pmatrix}$$

$$|0\rangle \to e^{-i\theta}|0\rangle, \quad |1\rangle \to e^{+i\theta}|1\rangle$$


From Grains of Pollen to Evidence for Atoms
Hadamard transform
H = 1/√2 (1 1; 1 -1)
|0> → 1/√2 (|0> + |1>)
|1> → 1/√2 (|0> - |1>)
H² = I
Phase Rotation
R_θ = (e^{-iθ} 0; 0 e^{iθ})
|0> → e^{-iθ}|0>, |1> → e^{+iθ}|1>
R_{π/2} = (-i 0; 0 i)
|0> →

$$-\boxed{H}-\boxed{R_{\pi/2}}-\boxed{H}-$$

$$R_{\pi/2} = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} = -i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$|0\rangle \xrightarrow{H} |+\rangle \xrightarrow{R_{\pi/2}} -i|-\rangle \rightarrow -i|1\rangle$$

$$-\boxed{H}-\boxed{R_{\pi/2}}-\boxed{H}-$$

$$R_{\pi/2} = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} = -i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

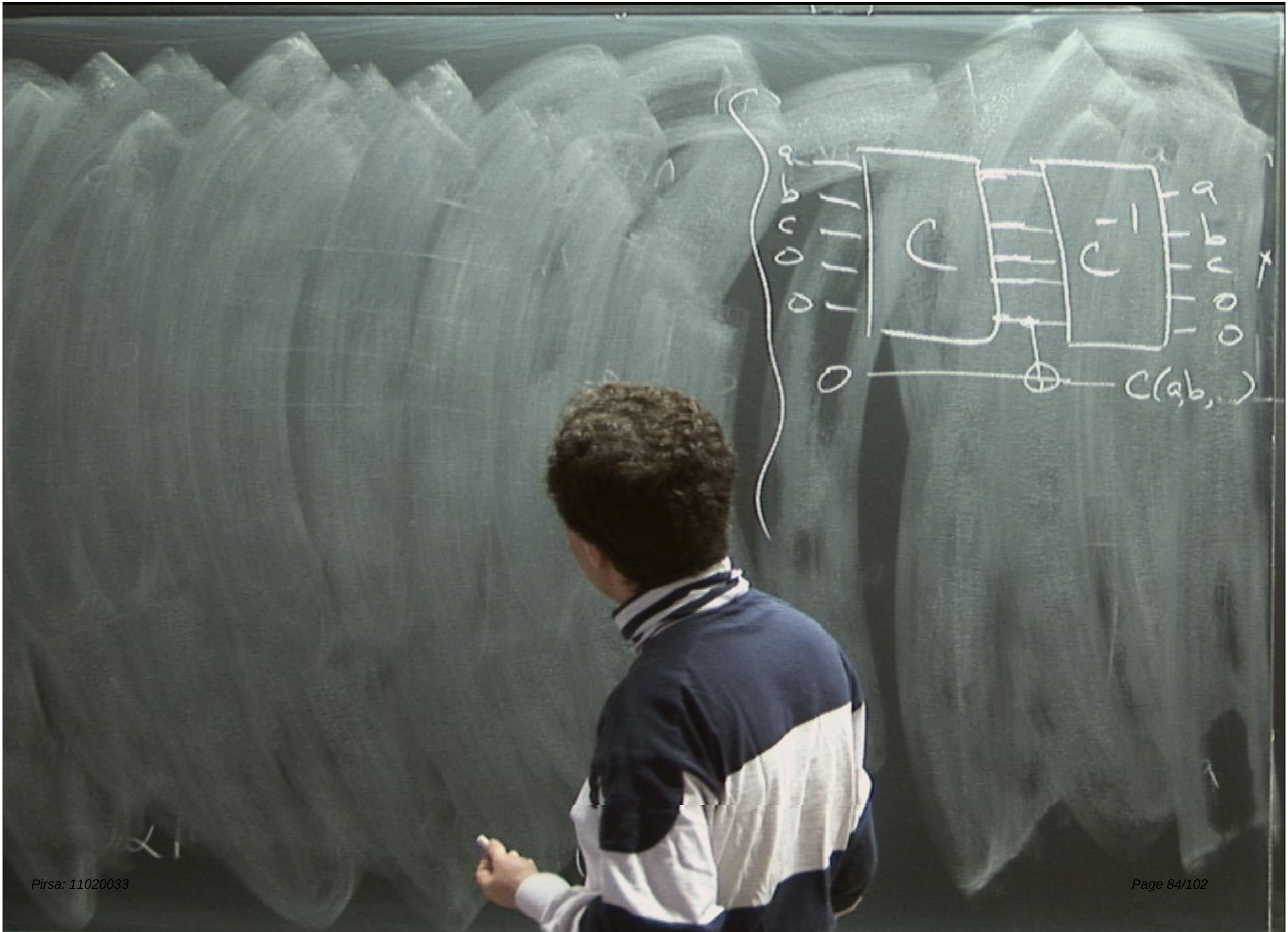
$$|0\rangle \xrightarrow{H} |+\rangle \xrightarrow{R_{\pi/2}} -i|-\rangle \rightarrow -i|1\rangle$$

$$-\boxed{H}-\boxed{R_{\pi/2}}-\boxed{H}- \;=\; -\oplus-$$

$$R_{\pi/2} = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} = -i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

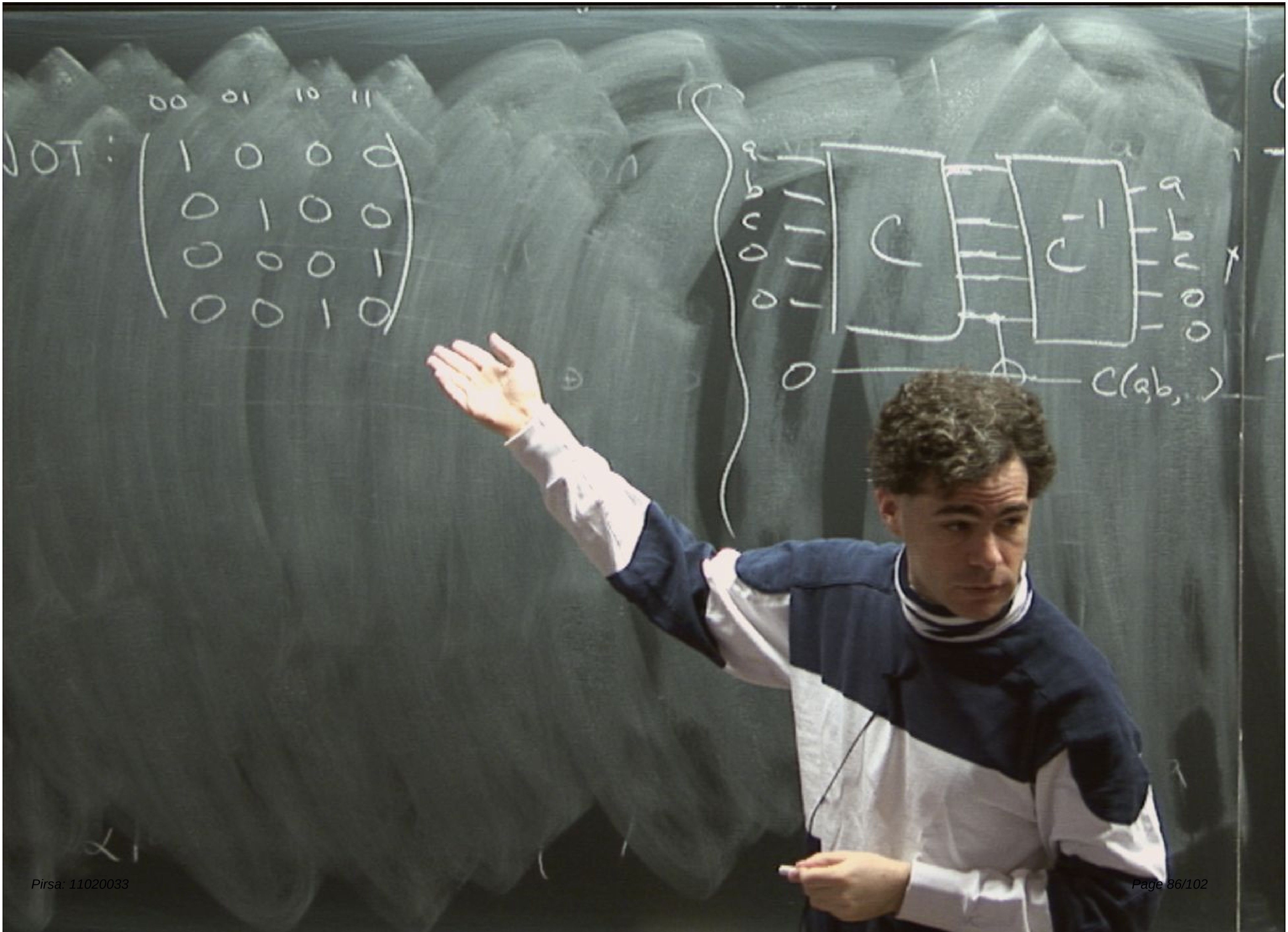
$$|0\rangle \xrightarrow{H} |+\rangle \xrightarrow{R_{\pi/2}} -i|-\rangle \xrightarrow{H} -i|1\rangle$$

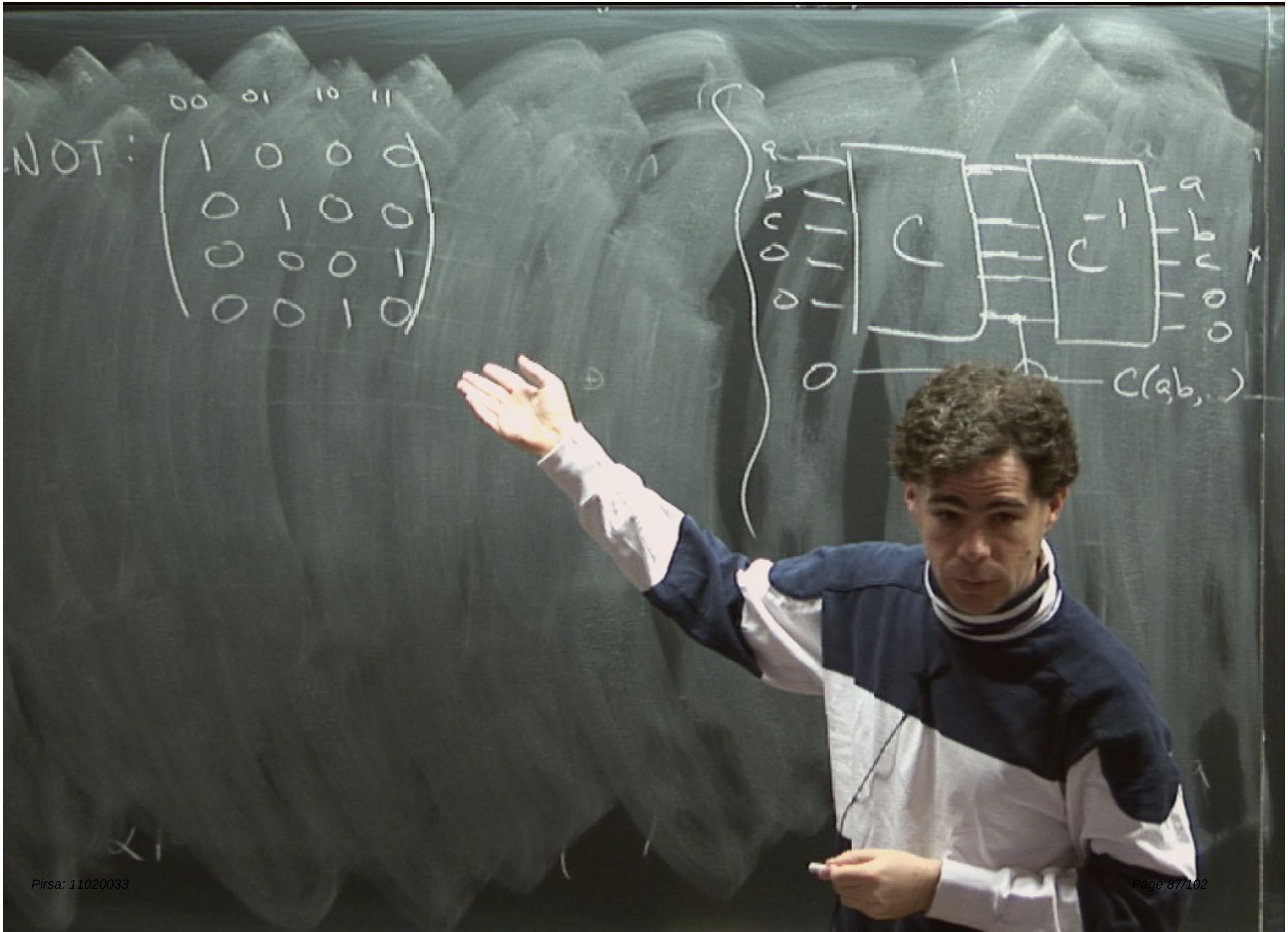
$$|1\rangle \longrightarrow |-\rangle \longrightarrow -i|+\rangle \longrightarrow -i|0\rangle$$

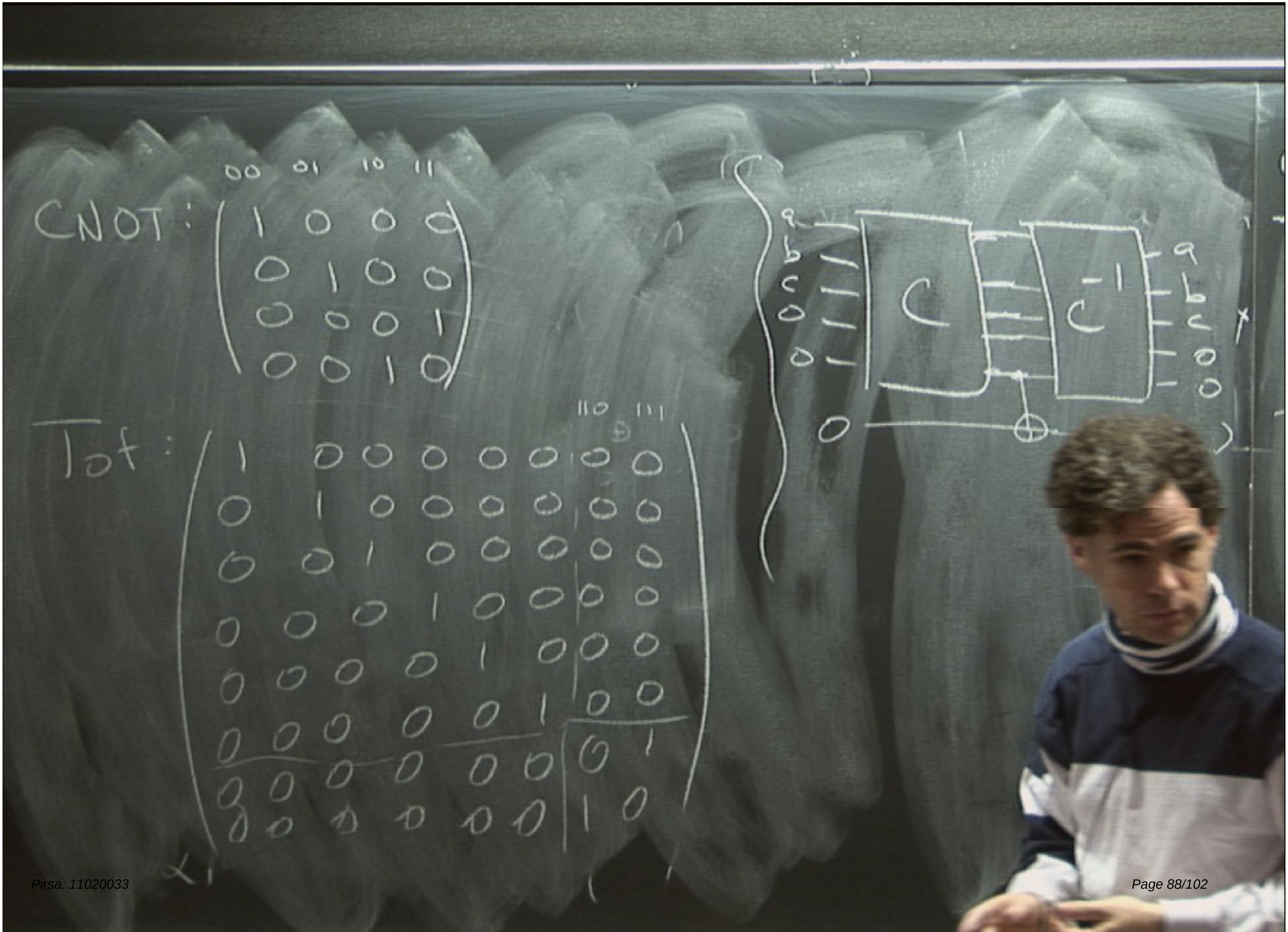

a
b
c
0
0
0
C
C⁻¹
a
b
c
0
0
C(a,b,..)

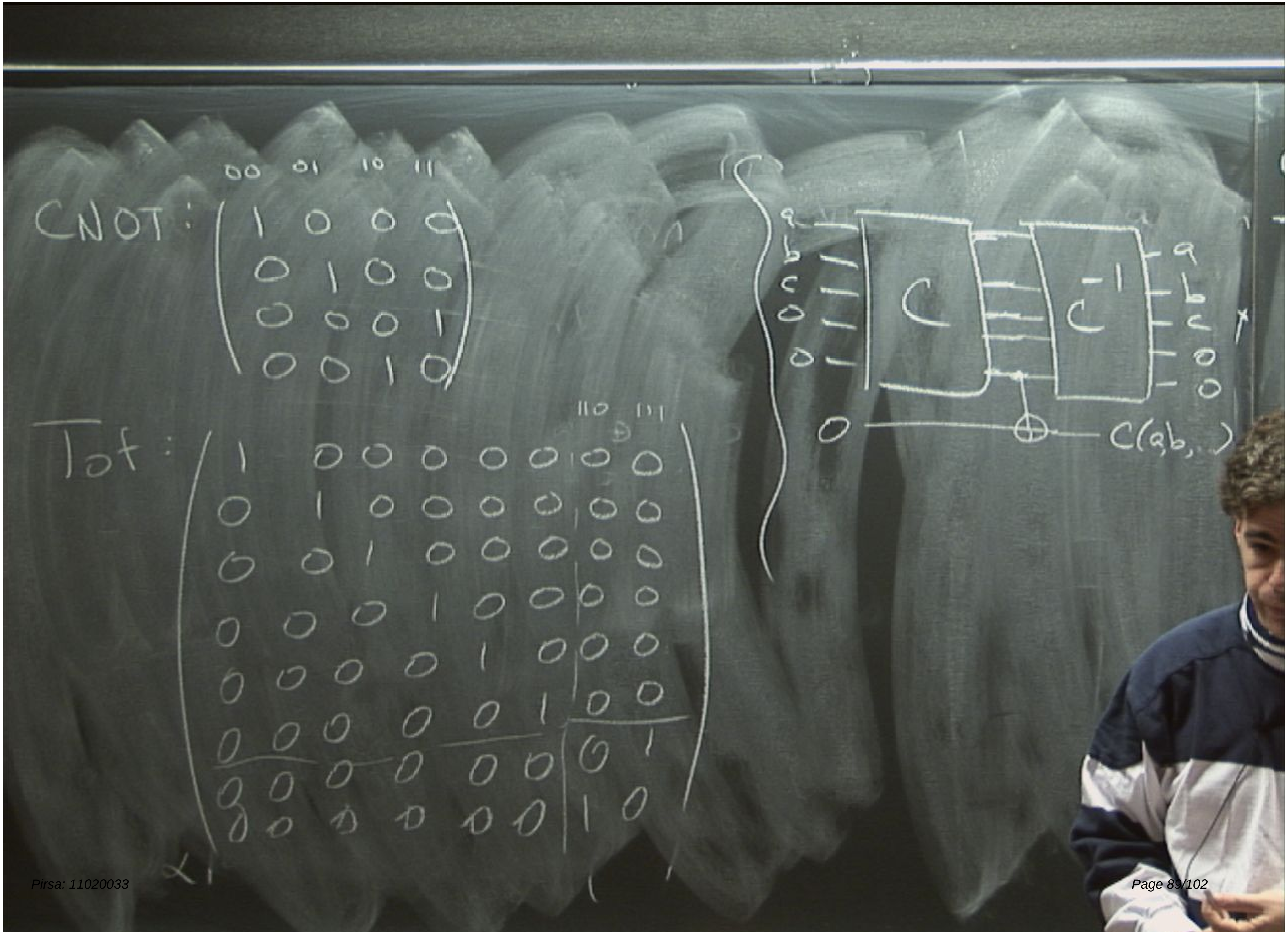
$$\text{CNOT}: \begin{array}{c} \begin{array}{cccc} 00 & 01 & 10 & 11 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & & \\ 0 & 0 & & \end{pmatrix} \end{array}$$

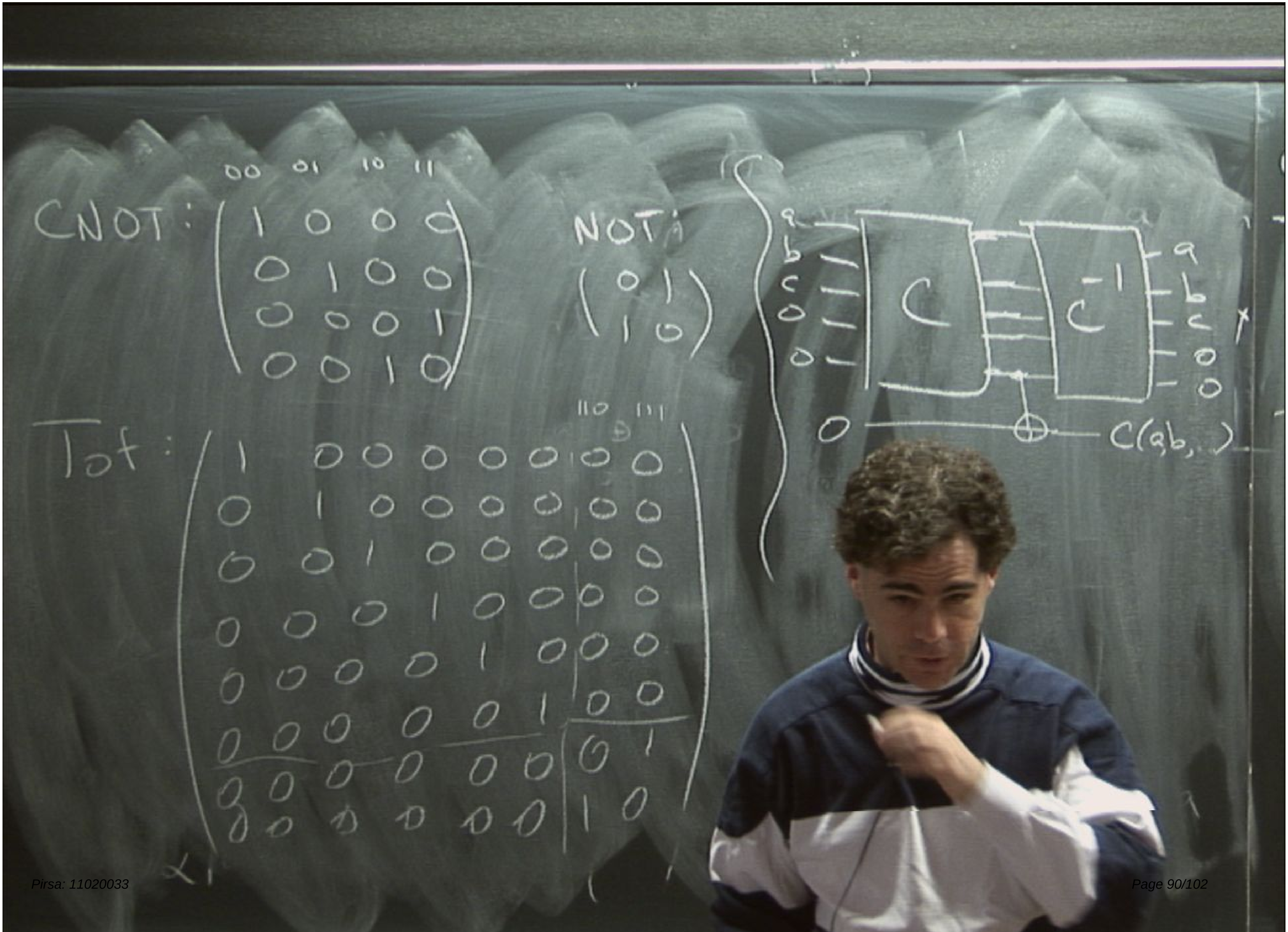
a
b
c
0
0


00 01 10 11
1 0 0 0
0 1 0 0
0 0 0 1
0 0 1 0
a
b
c
0
0
0
C
C⁻¹
C(a,b,..)


00 01 10 11
NOT:
1 0 0 0
0 1 0 0
0 0 0 1
0 0 1 0
a
b
c
0
0
0
C
C⁻¹
a
b
c
0
0
C(a,b,..)


00 01 10 11
CNOT:
Tof:
110 111
a
b
c
0
0
0
C
C^-1
a
b
c
0
0


00 01 10 11
CNOT:
1 0 0 0
0 1 0 0
0 0 0 1
0 0 1 0
110 111
Tof:
1 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0
0 0 1 0 0 0 0 0
0 0 0 1 0 0 0 0
0 0 0 0 1 0 0 0
0 0 0 0 0 1 0 0
0 0 0 0 0 0 0 1
0 0 0 0 0 0 1 0
a
b
c
0
0
C
C⁻¹
a
b
c
0
0
0
C(a,b,..)

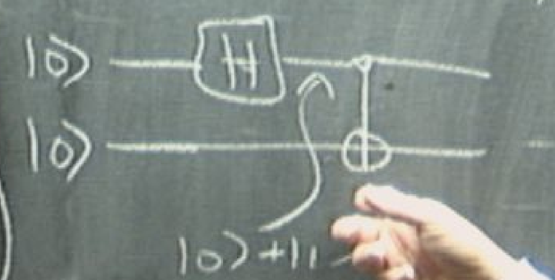

CNOT:
00 01 10 11
NOT:
Tof:
110 111
a
b
c
0
0
0
C
C^-1
C(a,b,..)

$$\text{CNOT}: \begin{array}{cccc} 00 & 01 & 10 & 11 \\ \end{array}$$
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\text{NOT}: \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

EPR pair

$$|00\rangle + |11\rangle$$



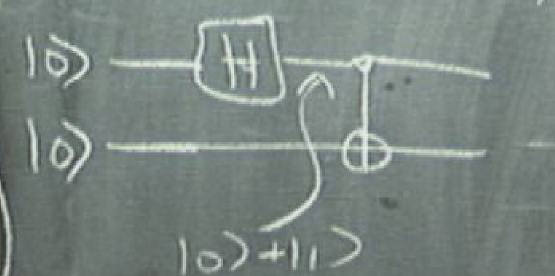
$$\text{Tof}: \begin{array}{cccccccc} & & & & & & 110 & 111 \end{array}$$
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\text{CNOT}: \begin{array}{c} \begin{matrix} 00 & 01 & 10 & 11 \end{matrix} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

$$\text{NOT}: \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

EPR pair

$$|00\rangle + |11\rangle$$



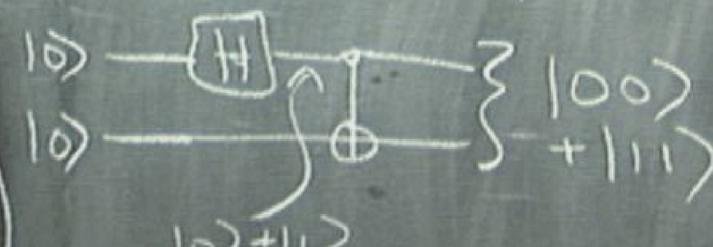
$$\text{Tof}: \begin{array}{c} \begin{matrix} & & & & & & 110 & 111 \end{matrix} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

$$\text{CNOT}: \begin{array}{c} \begin{matrix} 00 & 01 & 10 & 11 \end{matrix} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

$$\text{NOT}: \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

EPR pair

$|00\rangle + |11\rangle$



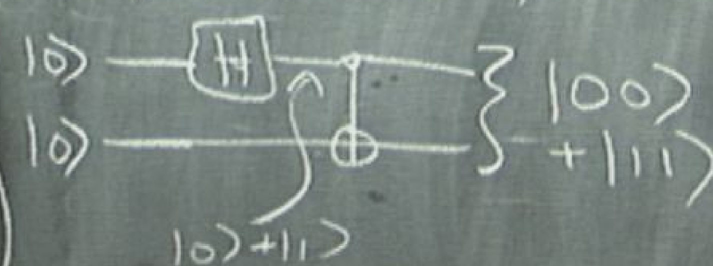
$$\text{Tof}: \begin{array}{c} \begin{matrix} & & & & & & 110 & 111 \end{matrix} \\ \left(\begin{array}{cccccc|cc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right) \end{array}$$

CNOT:
$$\begin{array}{cccc} 00 & 01 & 10 & 11 \end{array}$$
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

NOT
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

EPR pair

$|00\rangle + |11\rangle$



Tof:
$$\begin{array}{cccccccc} & & & & & & 110 & 111 \end{array}$$
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

UNIVERSAL NOT

$|\psi\rangle \to |\bar{\psi}\rangle$

NOT: $|0\rangle \to |1\rangle,\ |1\rangle \to |0\rangle$

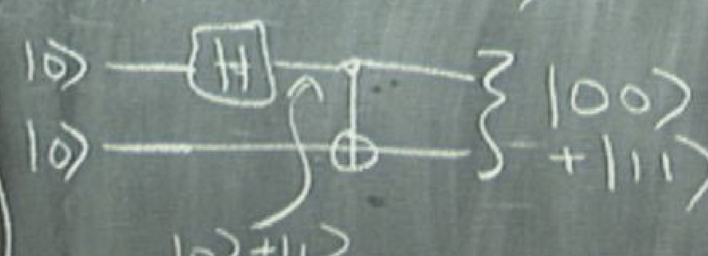
$|0\rangle + |1\rangle \to |1\rangle + |0\rangle$

$$\text{CNOT}: \begin{array}{cccc} 00 & 01 & 10 & 11 \\ \end{array}$$
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\text{NOT}: \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

EPR pair

$$|00\rangle + |11\rangle$$



$$\text{Tof}: \begin{array}{cccccccc} & & & & & & 110 & 111 \end{array}$$
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

UNIVERSAL NOT

$$|\psi\rangle \to |\bar{\psi}\rangle$$

NOT: $|0\rangle \to |1\rangle,\ |1\rangle \to |0\rangle$

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$$\text{CNOT}: \begin{array}{c} \begin{array}{cccc} 00 & 01 & 10 & 11 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

$$\text{NOT}: \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\text{Tof}: \begin{array}{c} \begin{array}{cccccccc} & & & & & & 110 & 111 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

EPR pair

$|00\rangle + |11\rangle$

$|0\rangle$ — $\boxed{H}$ — $\bullet$ —

$|0\rangle$ — — — $\oplus$ —

$|0\rangle + |1\rangle$

$\Big\}\ |00\rangle + |11\rangle$

UNIVERSAL NOT

$|\psi\rangle \to |\bar{\psi}\rangle$

NOT: $|0\rangle \to |1\rangle,\ |1\rangle \to |0\rangle$

NOT: $|0\rangle + |1\rangle \to |1\rangle + |0\rangle$

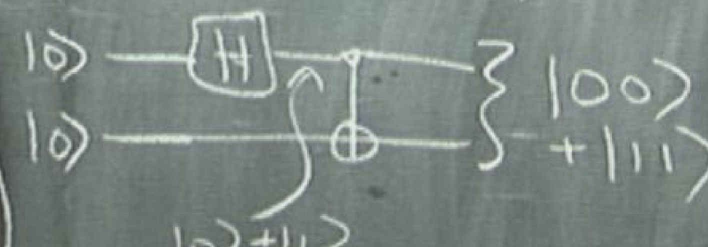
$$\text{CNOT}: \begin{array}{c} \begin{array}{cccc} 00 & 01 & 10 & 11 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

$$\text{NOT}: \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\text{Tof}: \begin{array}{c} \begin{array}{cccccccc} & & & & & & 110 & 111 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

EPR pair

$|00\rangle + |11\rangle$

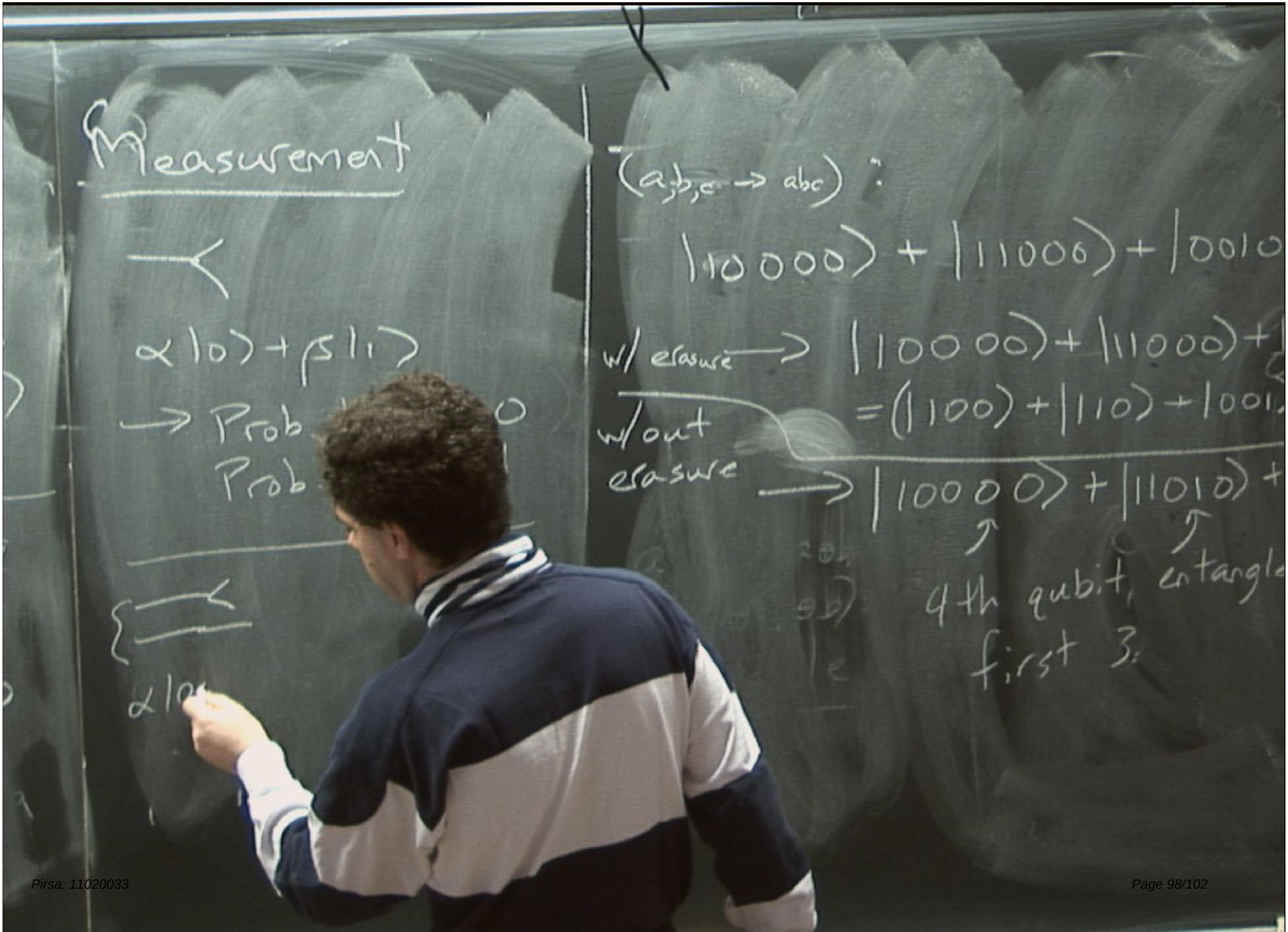


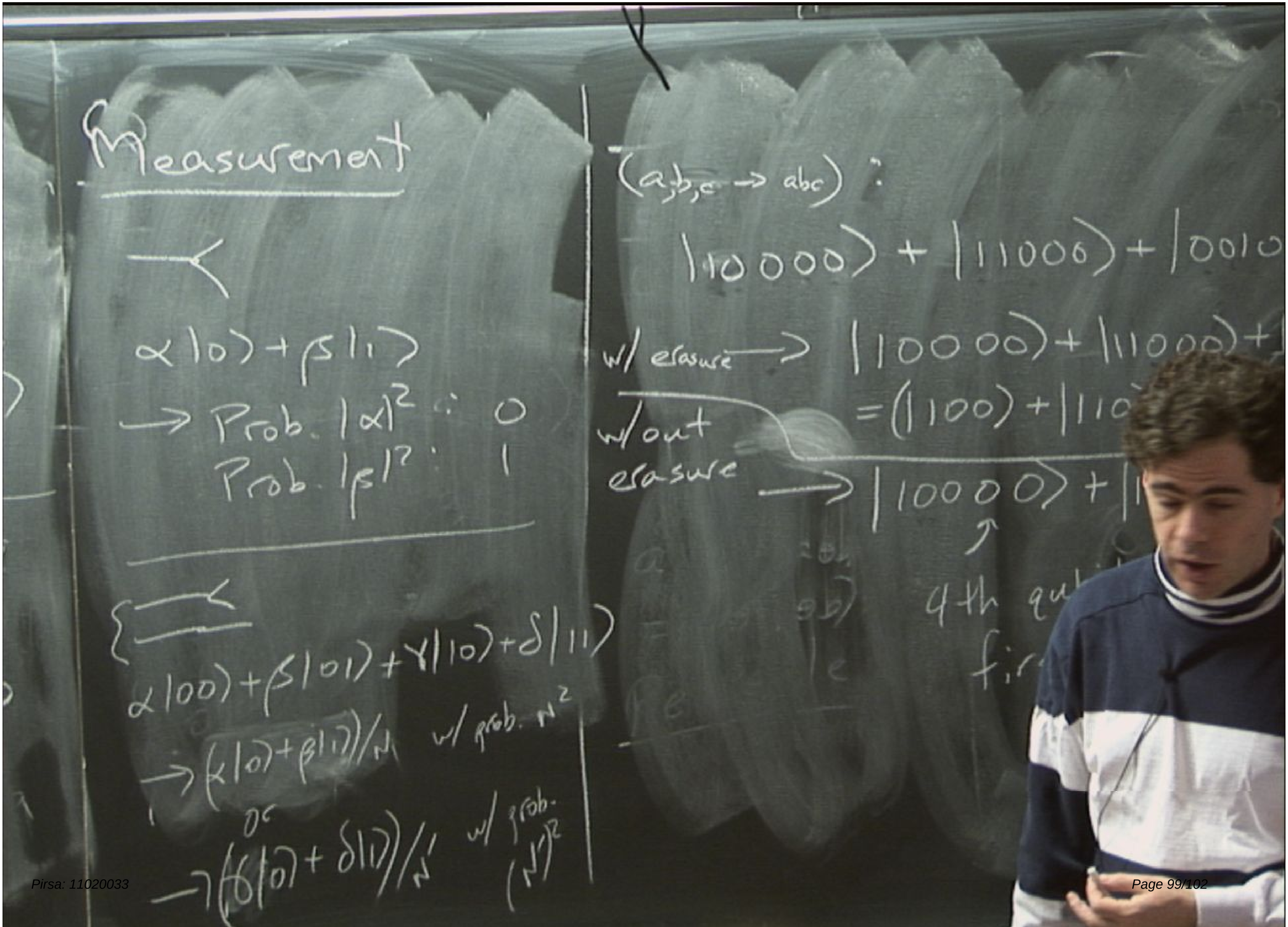
UNIVERSAL NOT

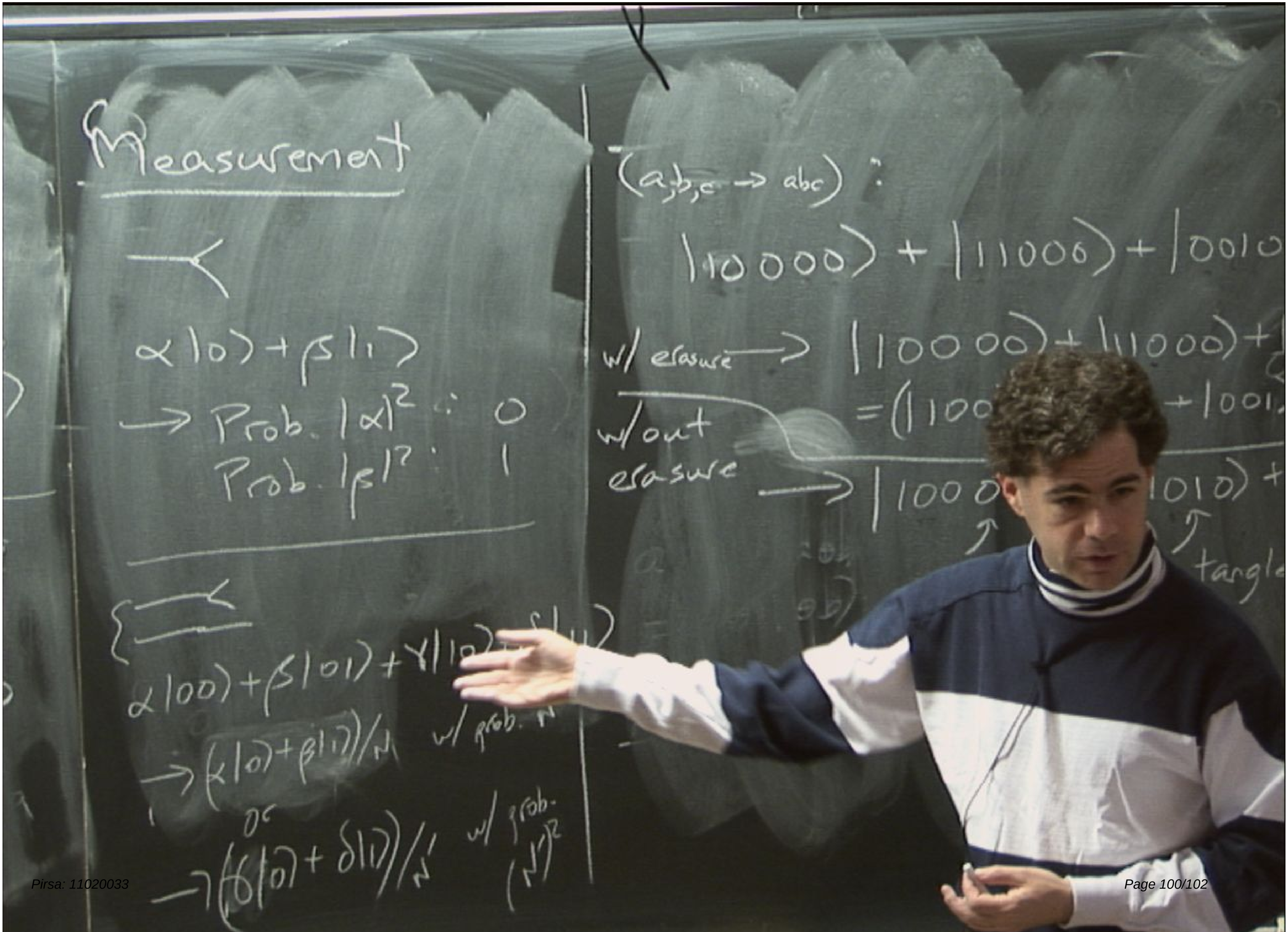
$|\psi\rangle \to |\bar{\psi}\rangle$

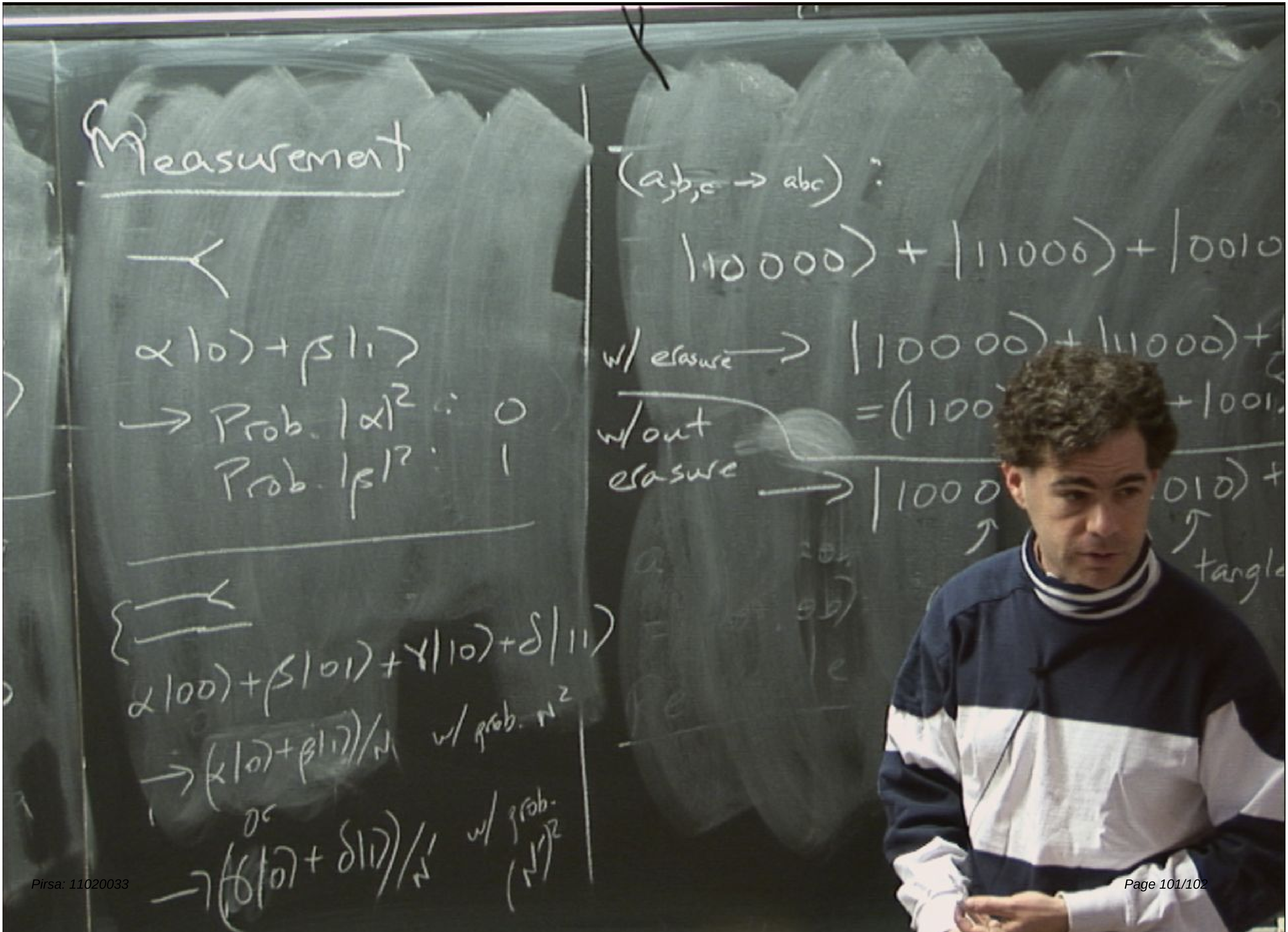
NOT: $|0\rangle \to |1\rangle$, $|1\rangle \to |0\rangle$

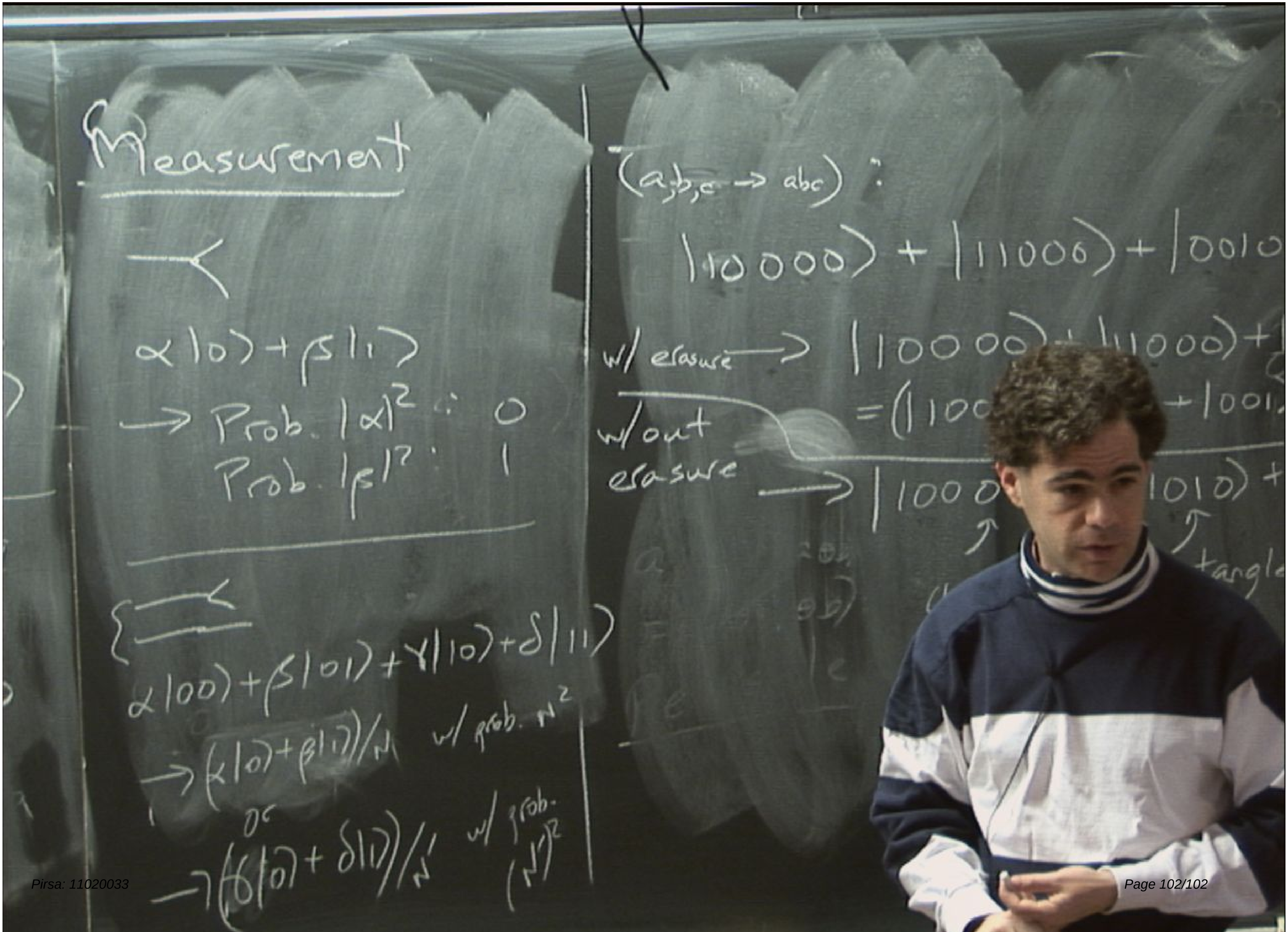
NOT: $|0\rangle + |1\rangle \to |1\rangle + |0\rangle$


Measurement
α|0⟩+β|1⟩
→ Prob
Prob
α|0
(a,b,c → abc) :
|10000⟩ + |11000⟩ + |0010
w/ erasure → |10000⟩ + |11000⟩ +
= (|100⟩ + |110⟩ + |001
w/out erasure → |10000⟩ + |11010⟩ +
4th qubit, entangle
first 3.


Measurement
α|0⟩ + β|1⟩
→ Prob. |α|² : 0
Prob. |β|² : 1
α|00⟩ + β|01⟩ + γ|10⟩ + δ|11⟩
→ (α|0⟩ + β|1⟩)/N w/ prob. N²
or
→ (γ|0⟩ + δ|1⟩)/N' w/ prob. (N')²
(a,b,c → abc) :
|10000⟩ + |11000⟩ + |0010
w/ erasure → |10000⟩ + |11000⟩ +
= (|100⟩ + |110
w/out erasure → |10000⟩ + |1
4th qu
fir


Measurement
α|0⟩ + β|1⟩
→ Prob. |α|² : 0
Prob. |β|² : 1
α|00⟩ + β|01⟩ + γ|10⟩ + δ|11⟩
→ (α|0⟩ + β|1⟩)/N w/ prob. N²
or
→ (γ|0⟩ + δ|1⟩)/N' w/ prob. (N')²
(a,b,c → abc) :
|10000⟩ + |11000⟩ + |0010
w/ erasure → |10000⟩ + |11000⟩ +
= (|1100
+ |001
w/out erasure → |1000
1010⟩ +
tangle


Measurement
α|0⟩ + β|1⟩
→ Prob. |α|²: 0
Prob. |β|²: 1
α|00⟩ + β|01⟩ + γ|10⟩ + δ|11⟩
→ (α|0⟩ + β|1⟩)/N w/ prob. N²
or
→ (γ|0⟩ + δ|1⟩)/N' w/ prob. (N')²
(a,b,c → abc) :
|10000⟩ + |11000⟩ + |0010
w/ erasure → |10000⟩ + |11000⟩ +
= (|1100
+ |001
w/out erasure → |1000
010⟩ +
tangle


Measurement
α|0⟩+β|1⟩
→ Prob. |α|² : 0
Prob. |β|² : 1
α|00⟩+β|01⟩+γ|10⟩+δ|11⟩
→ (α|0⟩+β|1⟩)/N w/ prob. N²
or
→ (γ|0⟩+δ|1⟩)/N' w/ prob. (N')²
(a,b,c → abc) :
|10000⟩ + |11000⟩ + |0010
w/ erasure → |10000⟩ + |11000⟩ +
= (|1100
+ |001
w/out erasure → |1000
|1010⟩ +
tangle