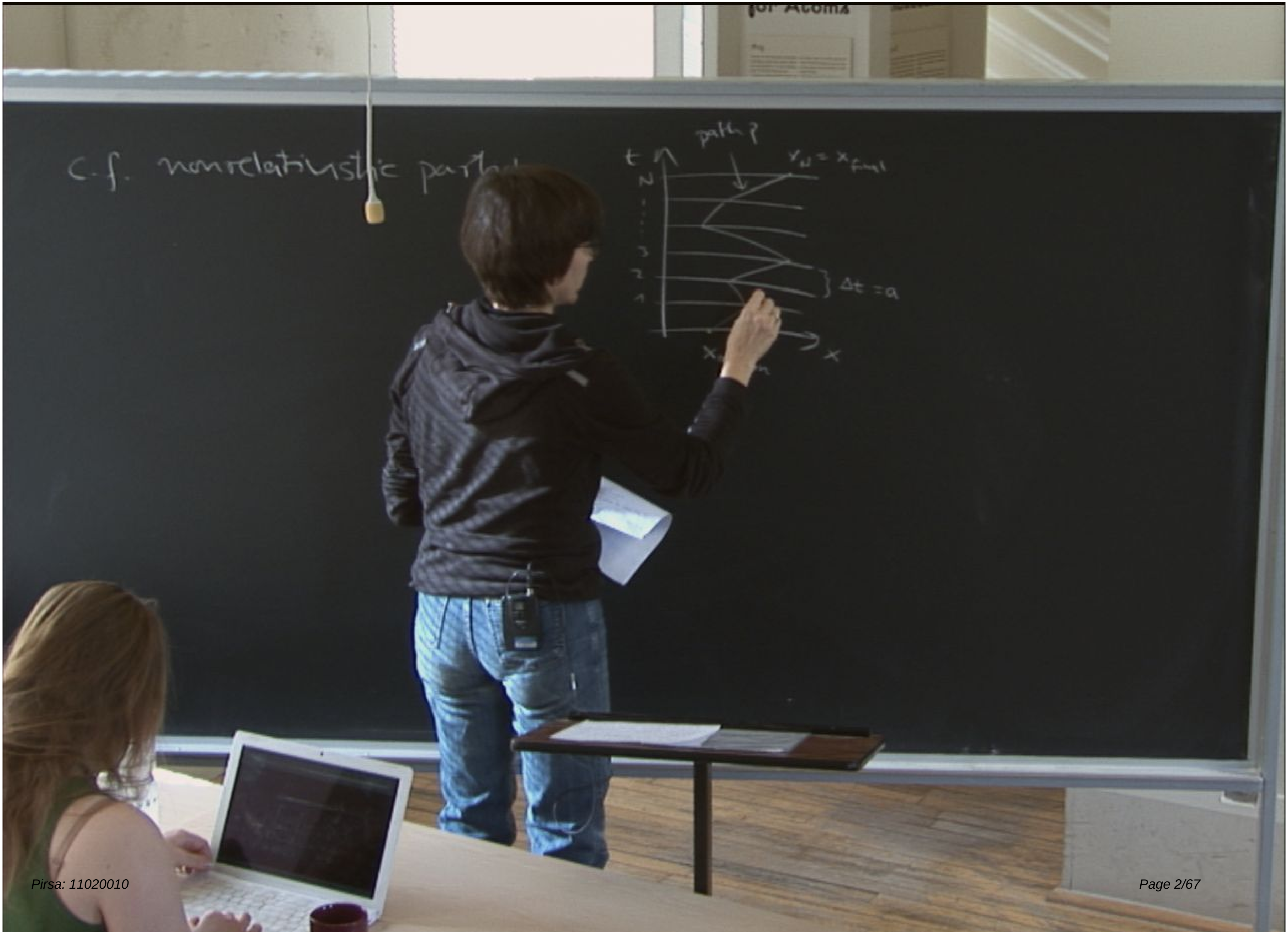
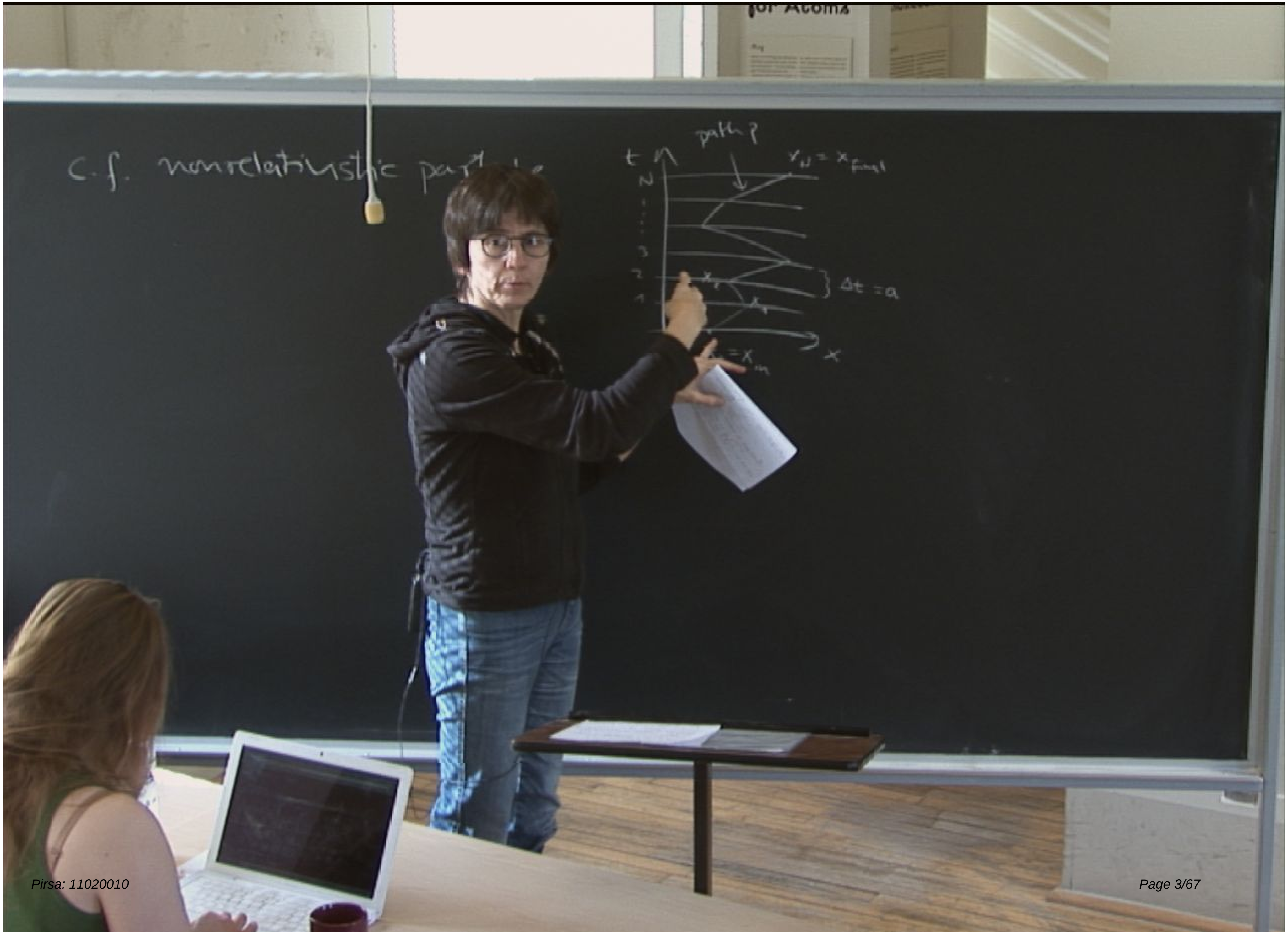
Title: Quantum Gravity Review - Lecture 13

Date: Feb 09, 2011 10:15 AM

URL: http://pirsa.org/11020010

Abstract:


c.f. nonrelativistic par
path ?
t
N
3
2
1
x_N = x_final
Δt = a
x


c.f. nonrelativistic
path?
t
N
3
2
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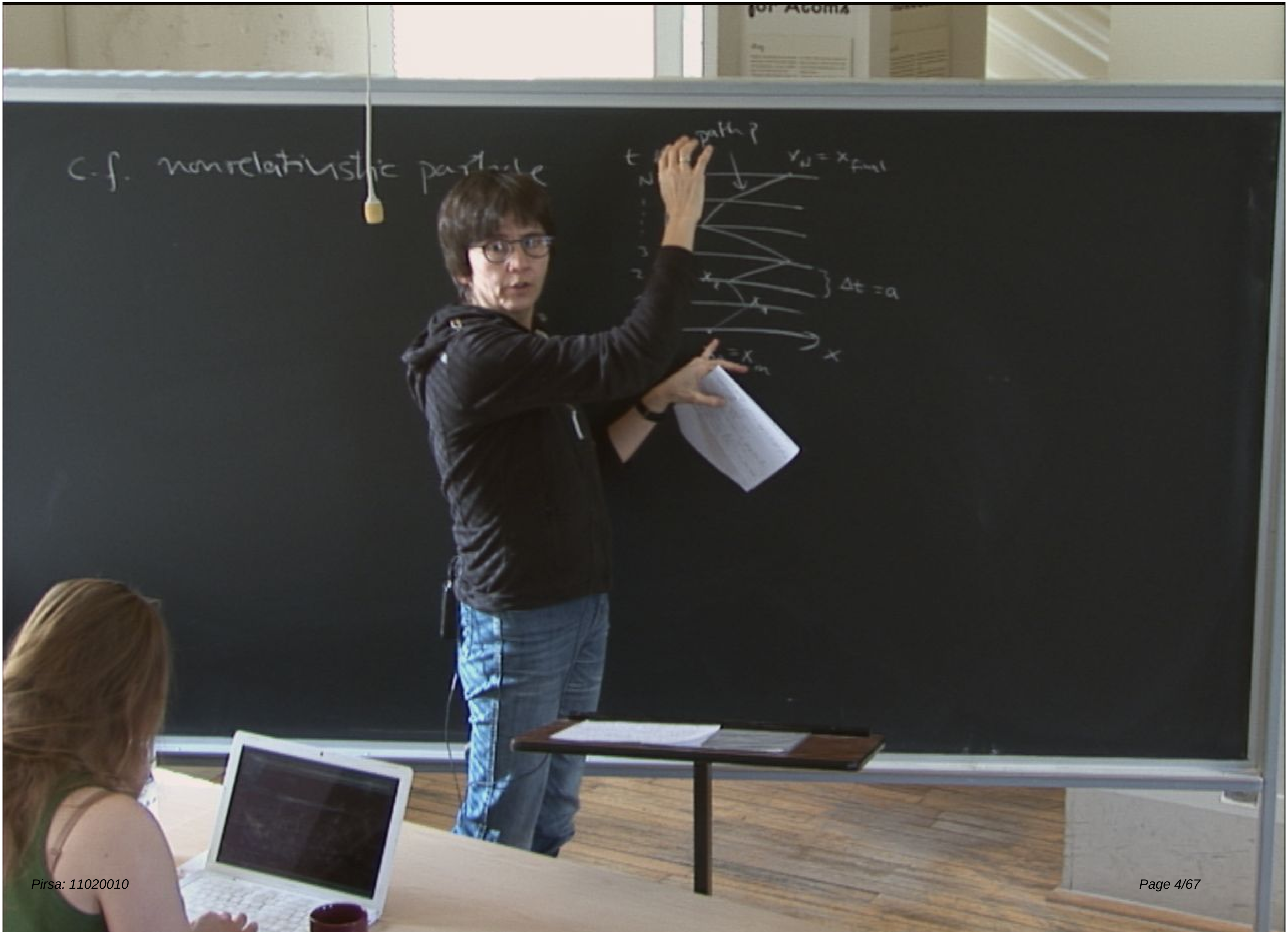
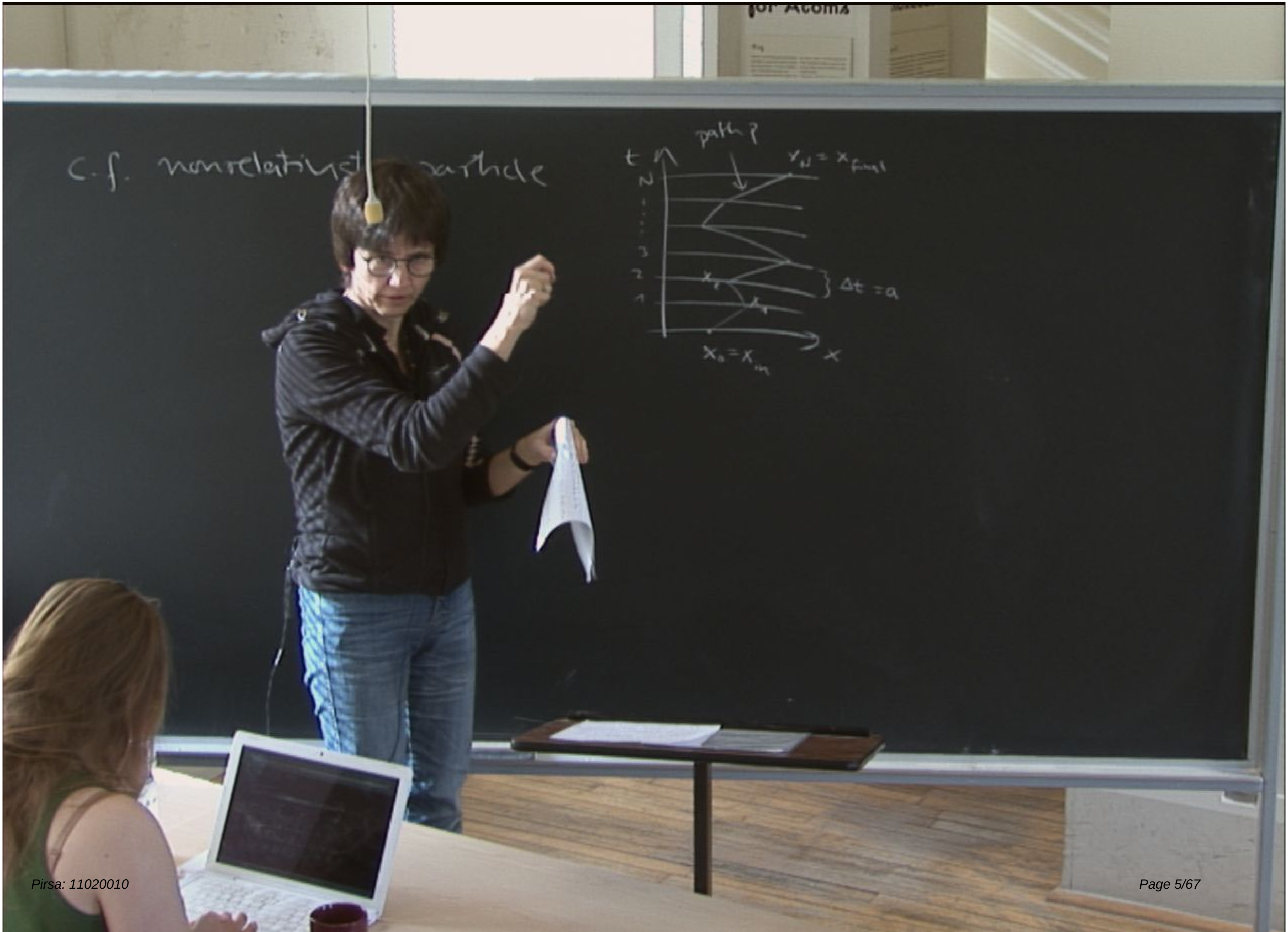
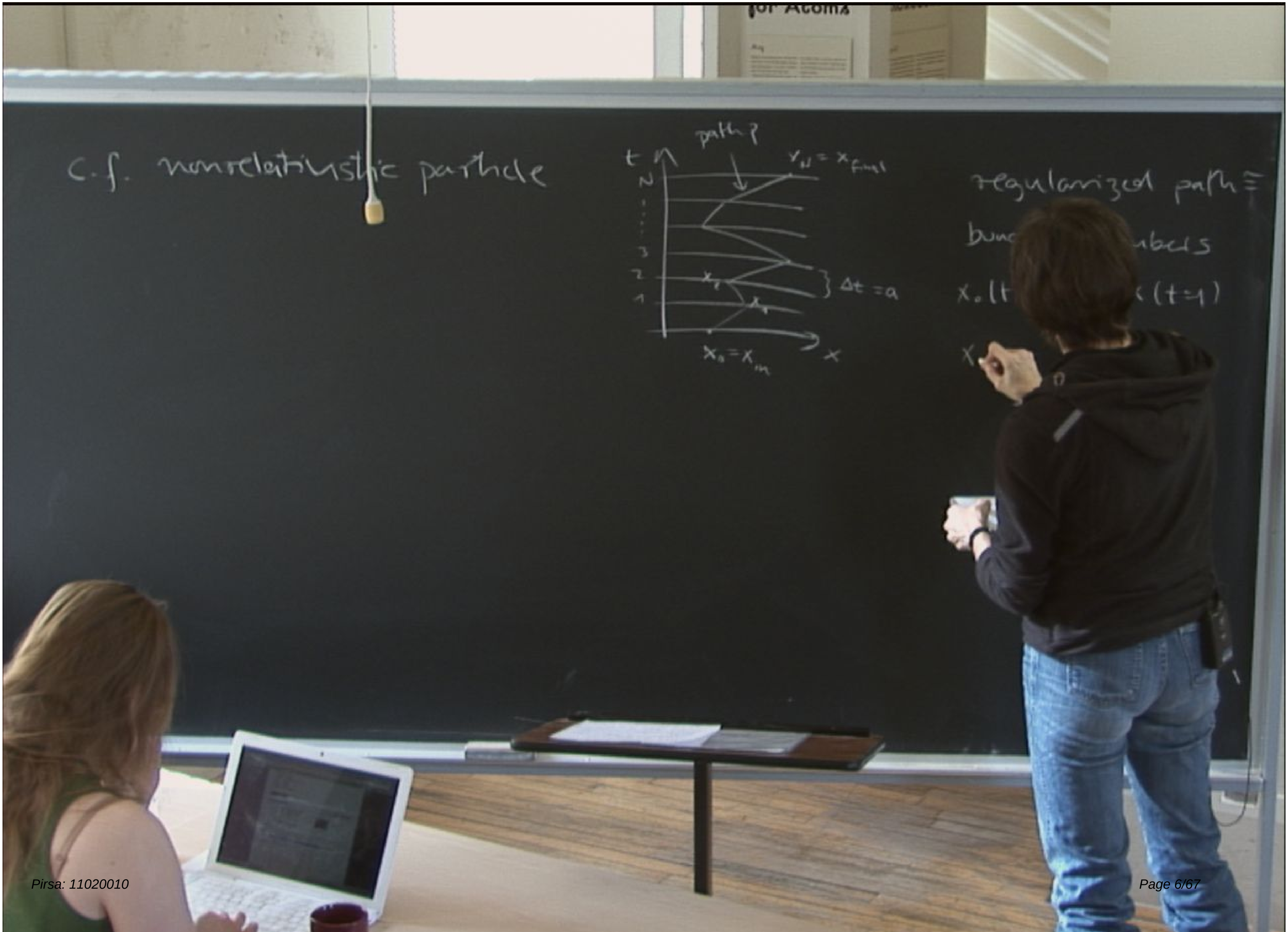
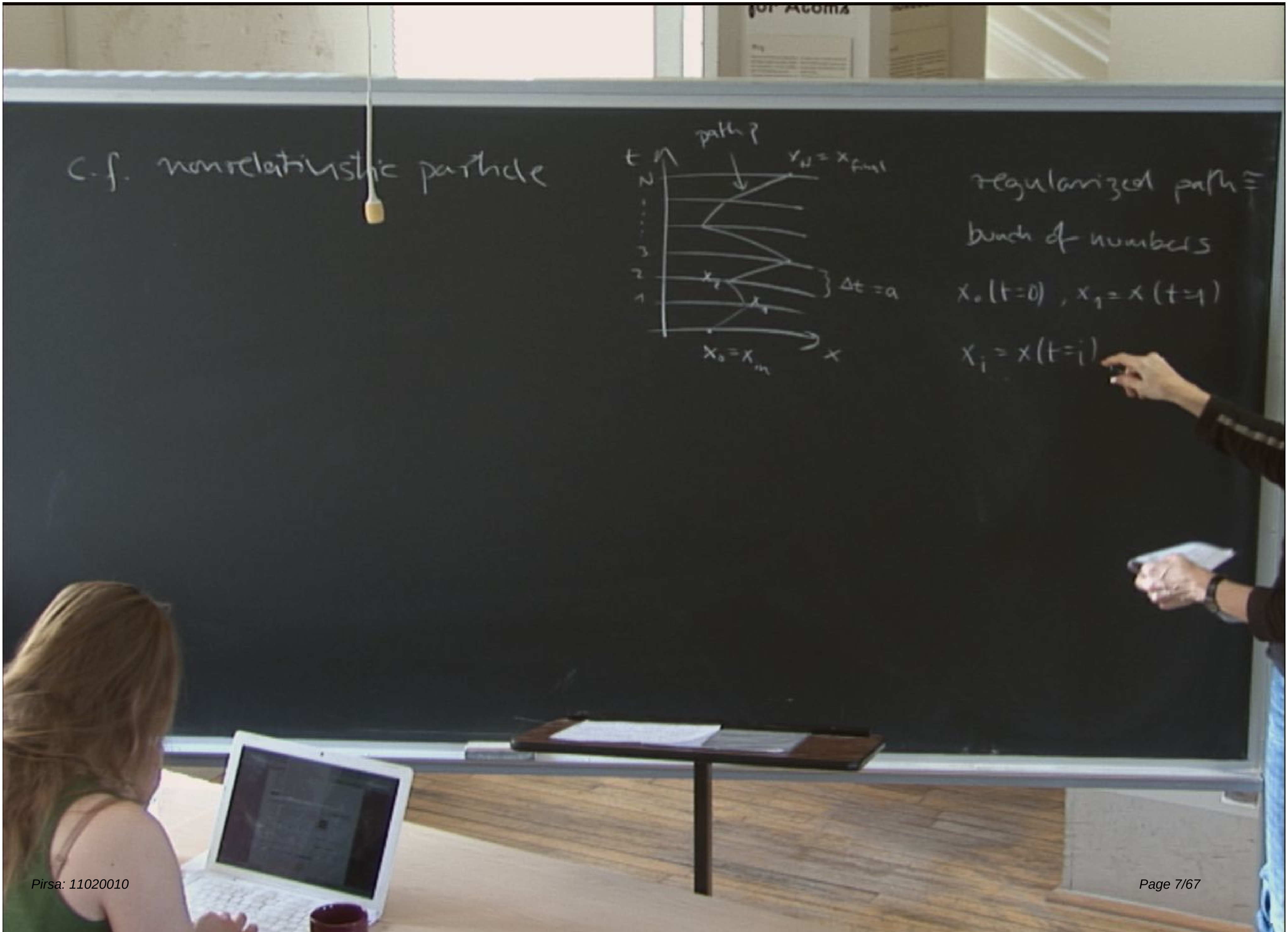
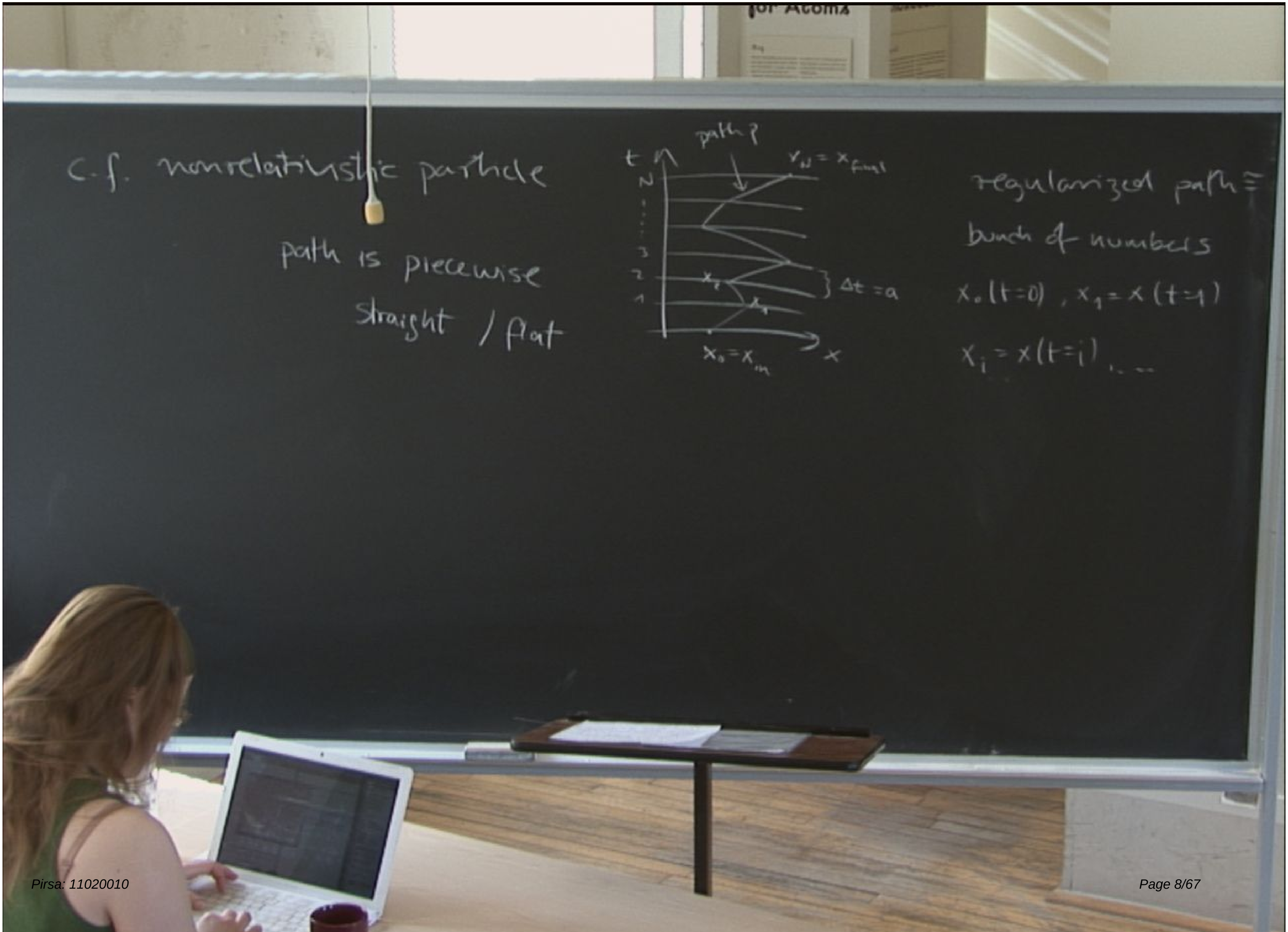
x


c.f. nonrelativistic particle
path?
$x_N = x_{final}$
$\Delta t = a$
x


c.f. nonrelativist particle
path?
t
N
3
2
1
x_N = x_final
Δt = a
x_0 = x_in
x

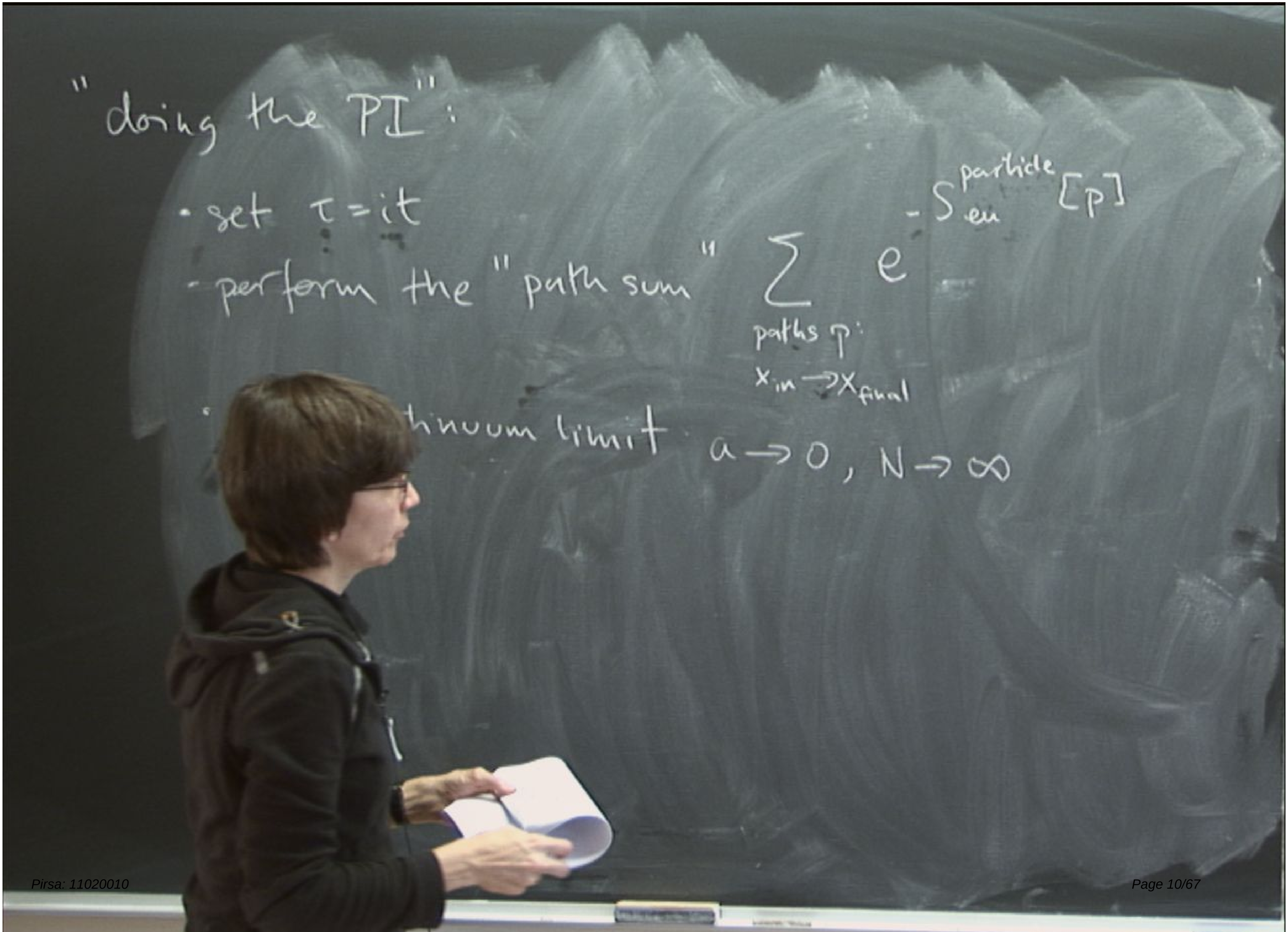

c.f. nonrelativistic particle
path ?
$x_N = x_{final}$
$\Delta t = a$
$x_0 = x_{in}$
regularized path ≡


c.f. nonrelativistic particle
path?
$x_N = x_{final}$
$\Delta t = a$
$x_0 = x_{in}$
regularized path ≡
bunch of numbers
$x_0 (t=0)$, $x_1 = x(t=1)$
$x_i = x(t=i)$


c.f. nonrelativistic particle
path is piecewise
straight / flat
path?
t
N
3
2
1
x_N = x_final
Δt = a
x_0 = x_in
x
regularized path ≡
bunch of numbers
x_0 (t=0) , x_1 = x (t=1)
x_i = x(t=i) , ...

"doing the PI":

- set $\tau = it$
- perform the "path sum" $\displaystyle\sum_{\substack{\text{paths } p: \\ x_{in} \to x_{final}}} e^{-S_{eu}^{particle}[p]}$


"doing the PI":
· set τ = it
· perform the "path sum" Σ e^(-S_eu^particle [p])
paths p: x_in → x_final
· tinuum limit a → 0, N → ∞

"doing the PI":

- set $\tau = it$
- perform the "path sum" $\sum\limits_{\substack{\text{paths } p: \\ x_{in} \to x_{final}}} e^{-S^{\text{particle}}_{eu}[p]}$
- take continuum limit $a \to 0,\ N \to \infty$
- Wick-rotate back

"doing the PI":

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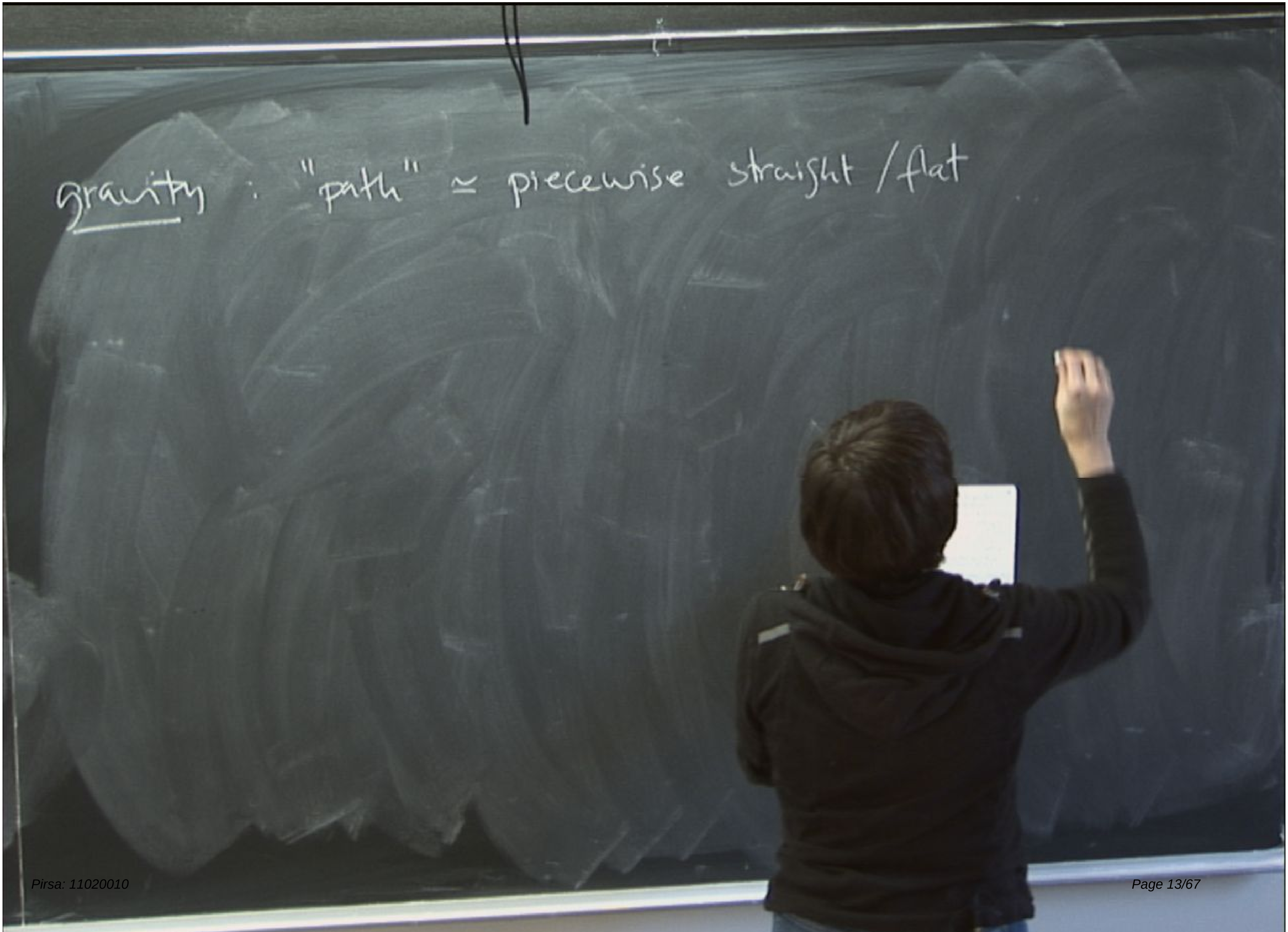
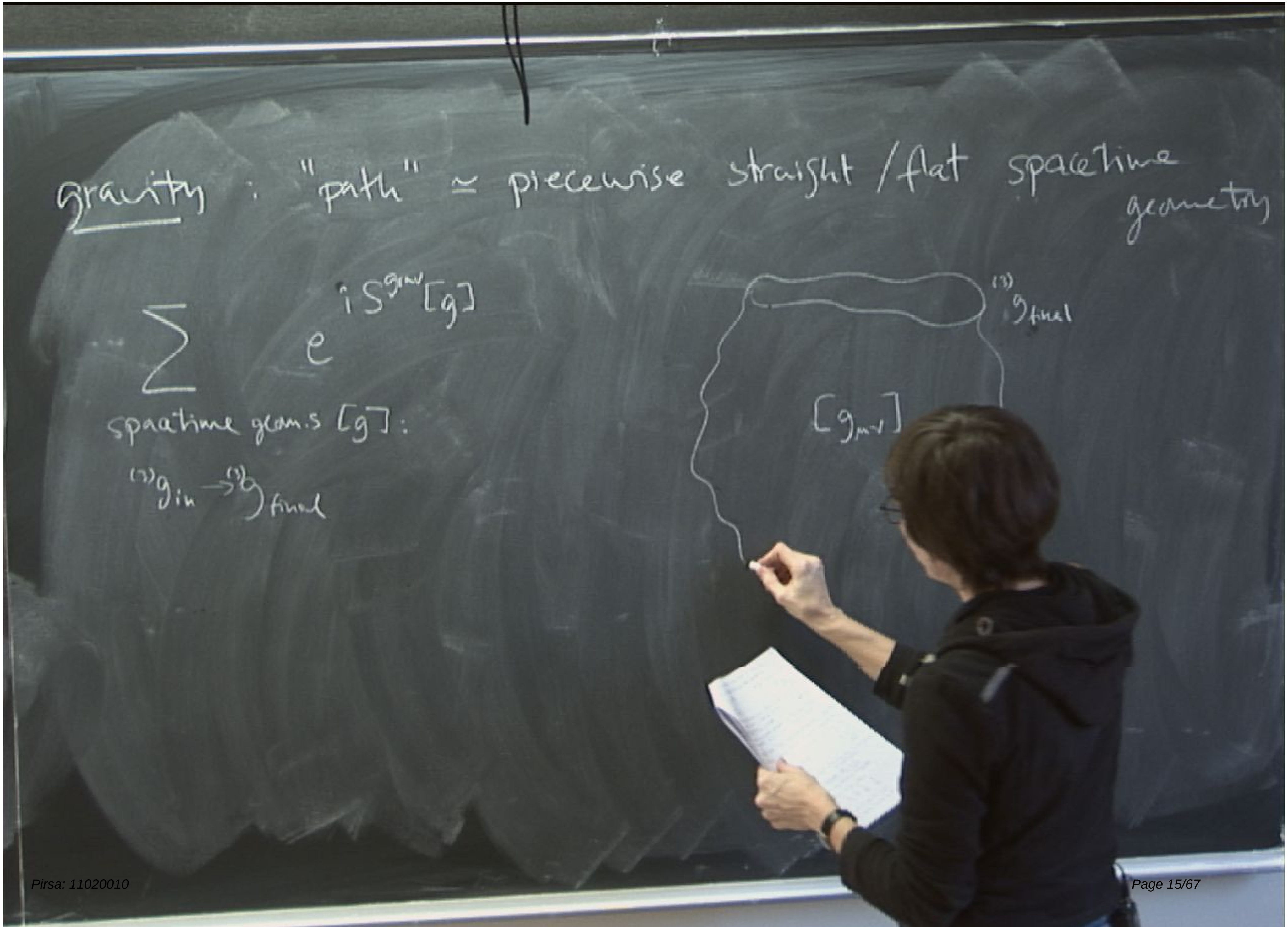

gravity : "path" ≃ piecewise straight / flat

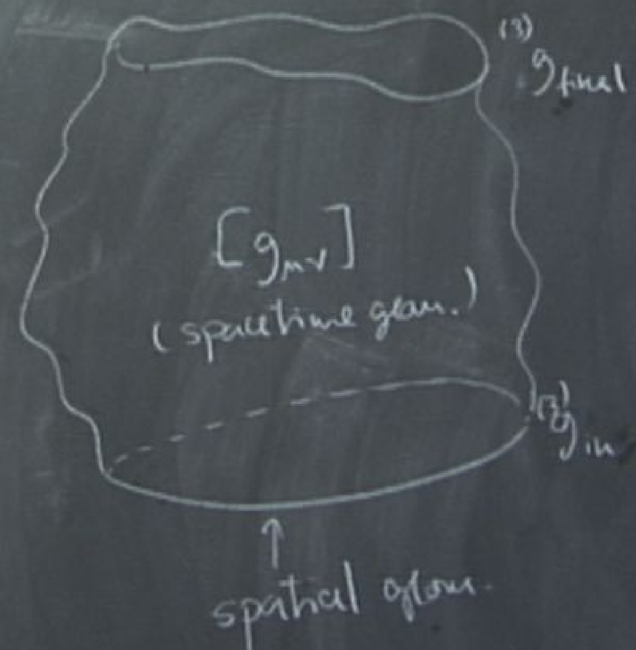
gravity : "path" $\simeq$ piecewise straight / flat spacetime geometry

$$\sum_{\text{spacetime geom.s } [g]:\ {}^{(3)}g_{in} \to {}^{(3)}g_{final}}$$


gravity : "path" $\simeq$ piecewise straight / flat spacetime geometry
$\sum e^{iS^{grav}[g]}$
spacetime geom.s [g]:
${}^{(3)}g_{in} \rightarrow {}^{(3)}g_{final}$
${}^{(3)}g_{final}$
$[g_{\mu\nu}]$

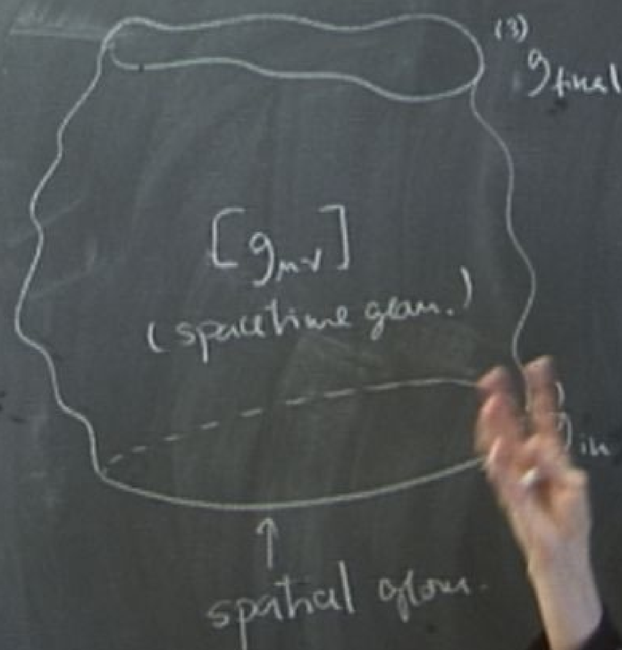
gravity : "path" $\simeq$ piecewise straight / flat spacetime geometry

$$\sum_{\substack{\text{spacetime geom.s } [g]: \\ {}^{(3)}g_{in} \to {}^{(3)}g_{final}}} e^{iS^{grav}[g]}$$



gravity : "path" $\simeq$ piecewise straight / flat spacetime geometry

$$\sum_{\text{spacetime geom.s } [g]:\; {}^{(3)}g_{in} \to {}^{(3)}g_{final}} e^{iS^{grav}[g]}$$



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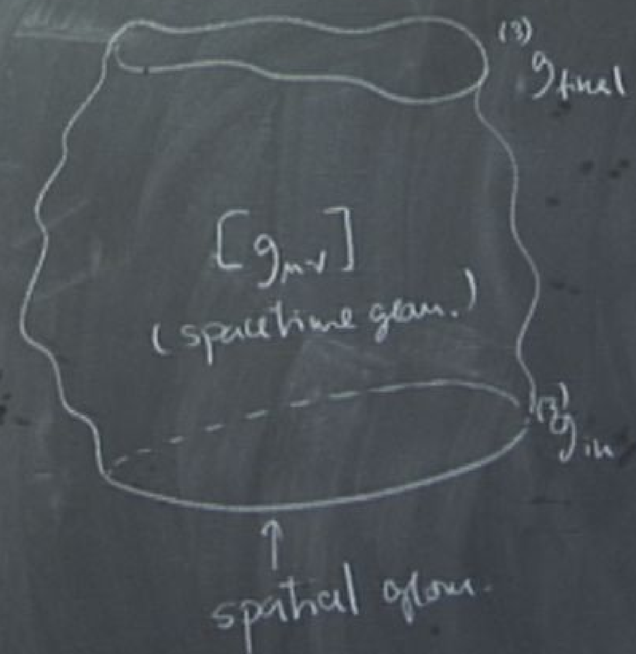
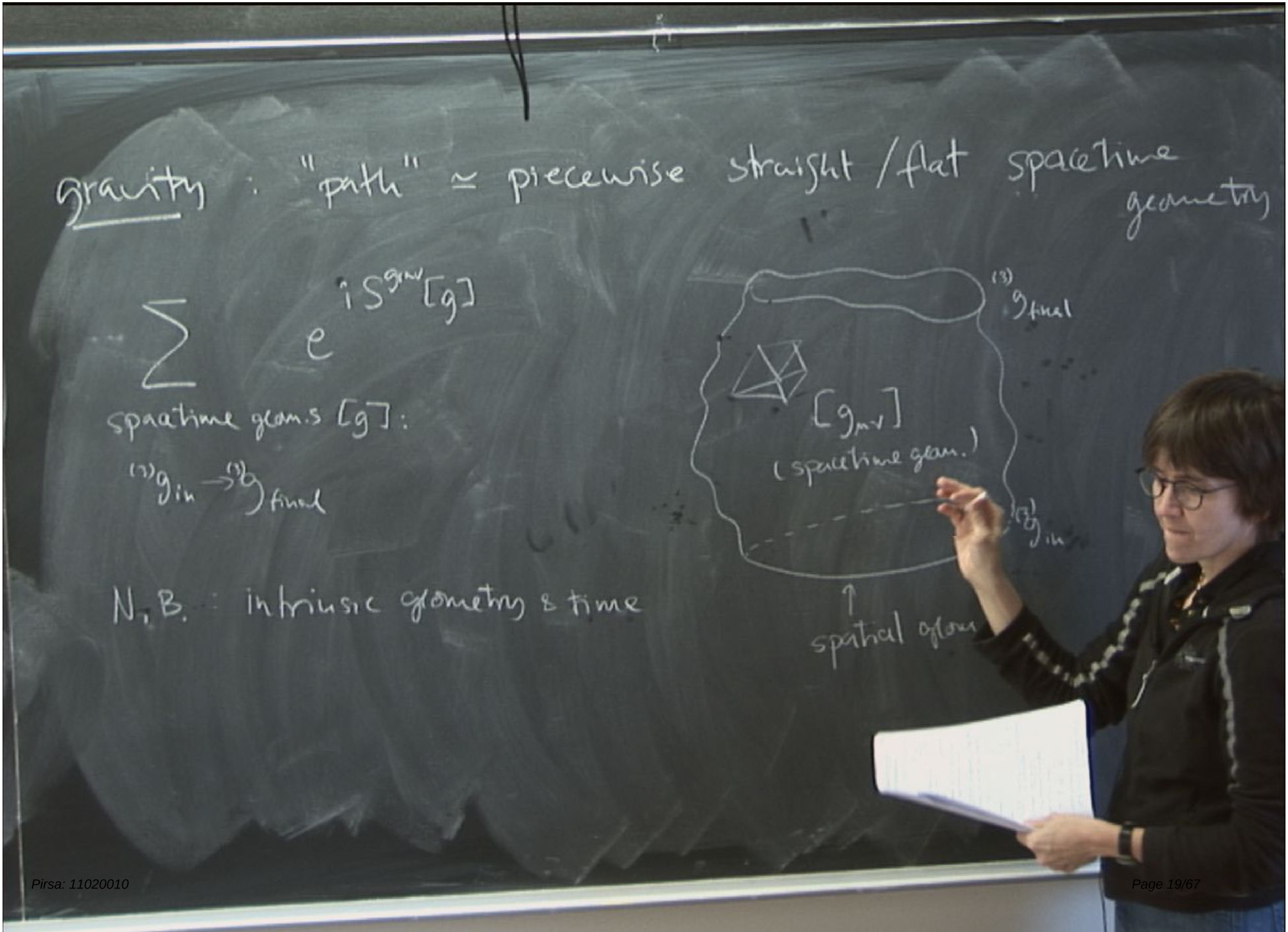
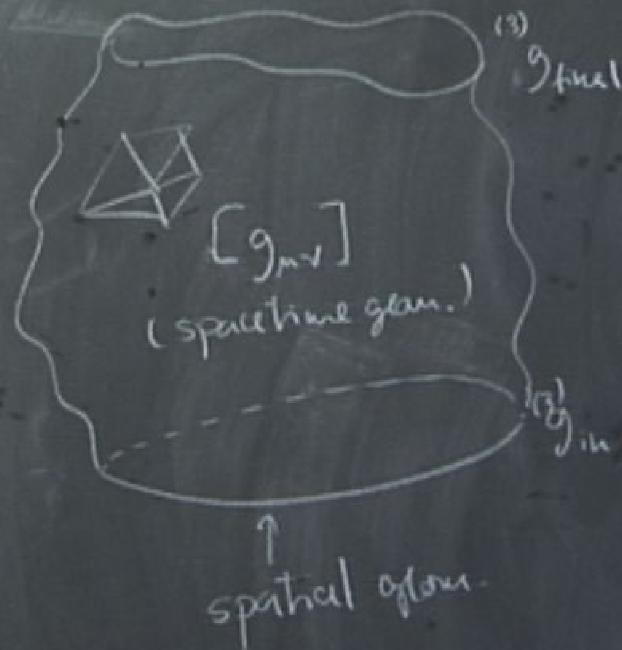


N.B.: intrinsic geometry & time


gravity : "path" ≃ piecewise straight / flat spacetime geometry
$\sum e^{iS^{grav}[g]}$
spacetime geom.s [g]:
$^{(3)}g_{in} \rightarrow {}^{(3)}g_{final}$
N.B. intrinsic geometry & time
$^{(3)}g_{final}$
$[g_{\mu\nu}]$
(spacetime geom.)
$^{(3)}g_{in}$
spatial geom

<u>gravity</u> : "path" $\simeq$ piecewise straight / flat spacetime geometry

$$\sum_{\substack{\text{spacetime geom.s } [g]: \\ {}^{(3)}g_{in} \to {}^{(3)}g_{final}}} e^{\,iS^{grav}[g]}$$



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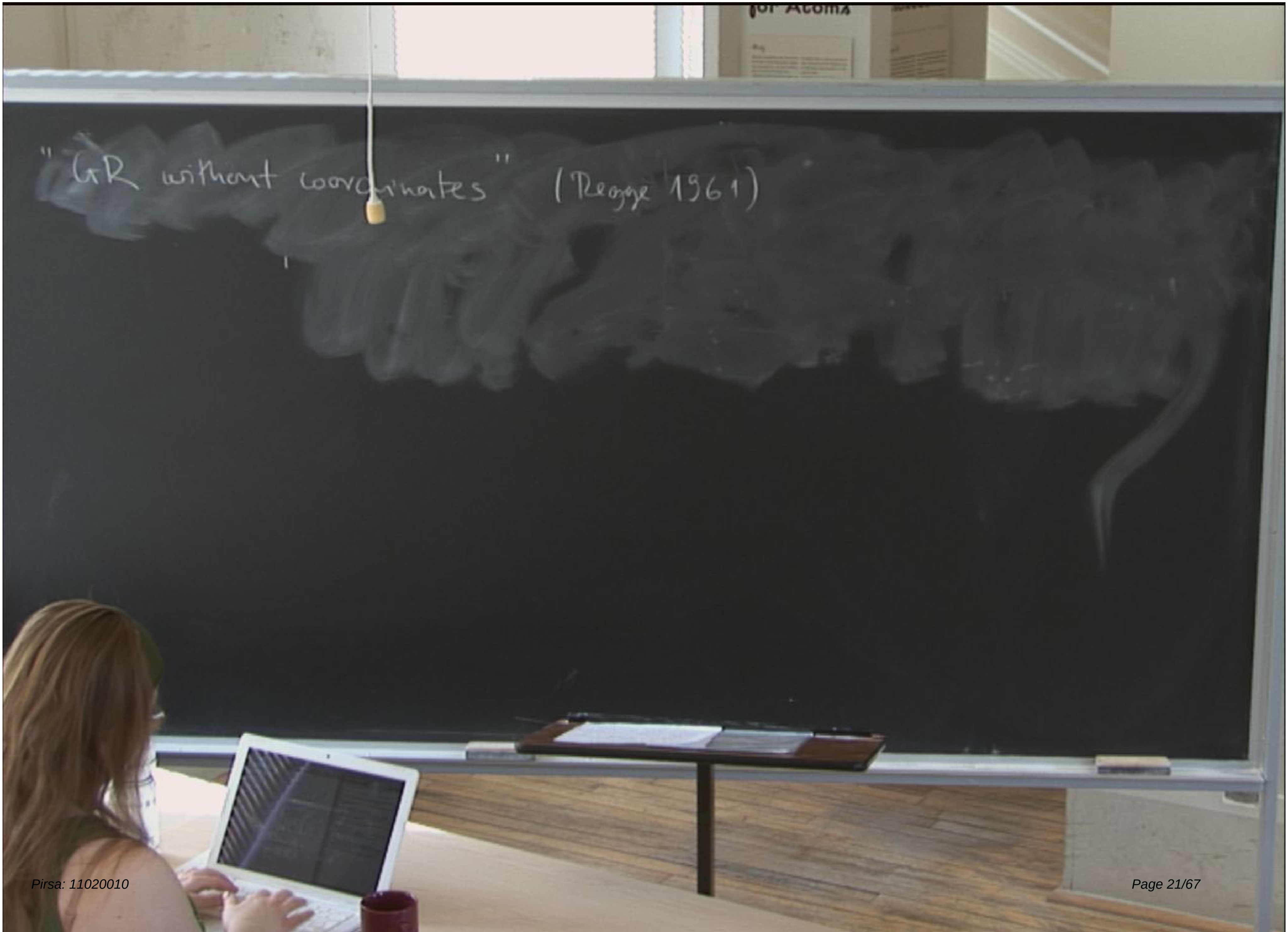
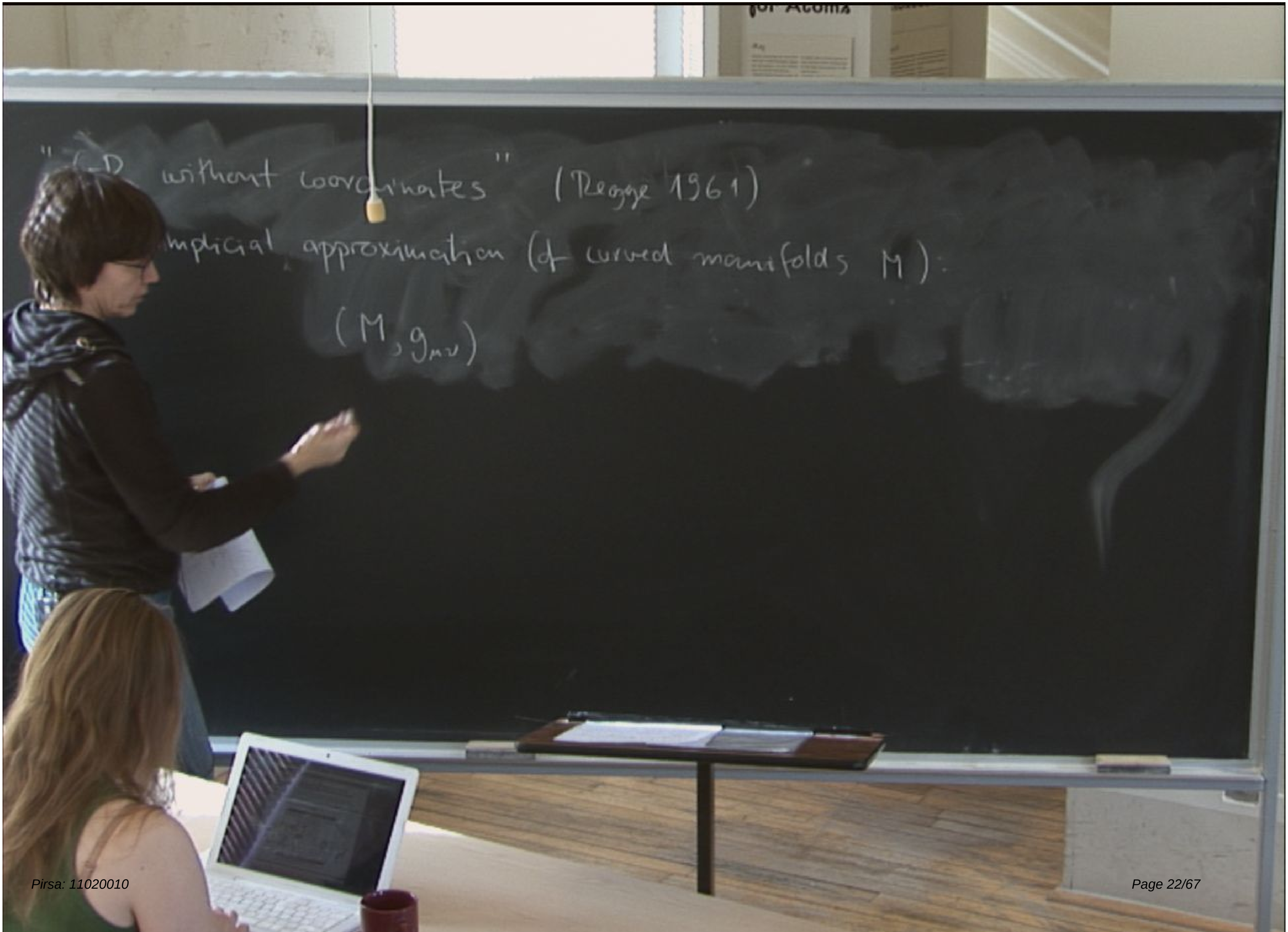
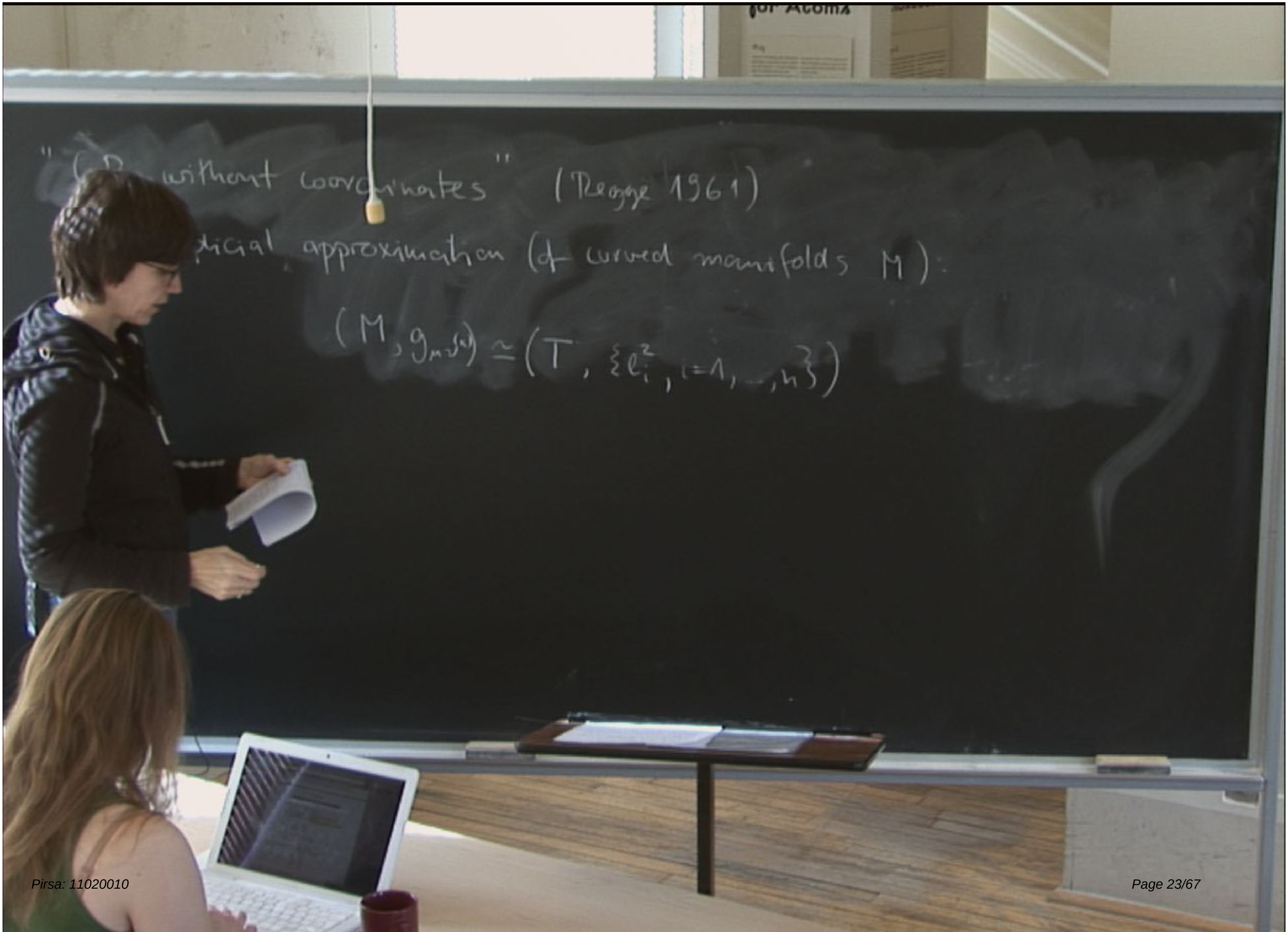
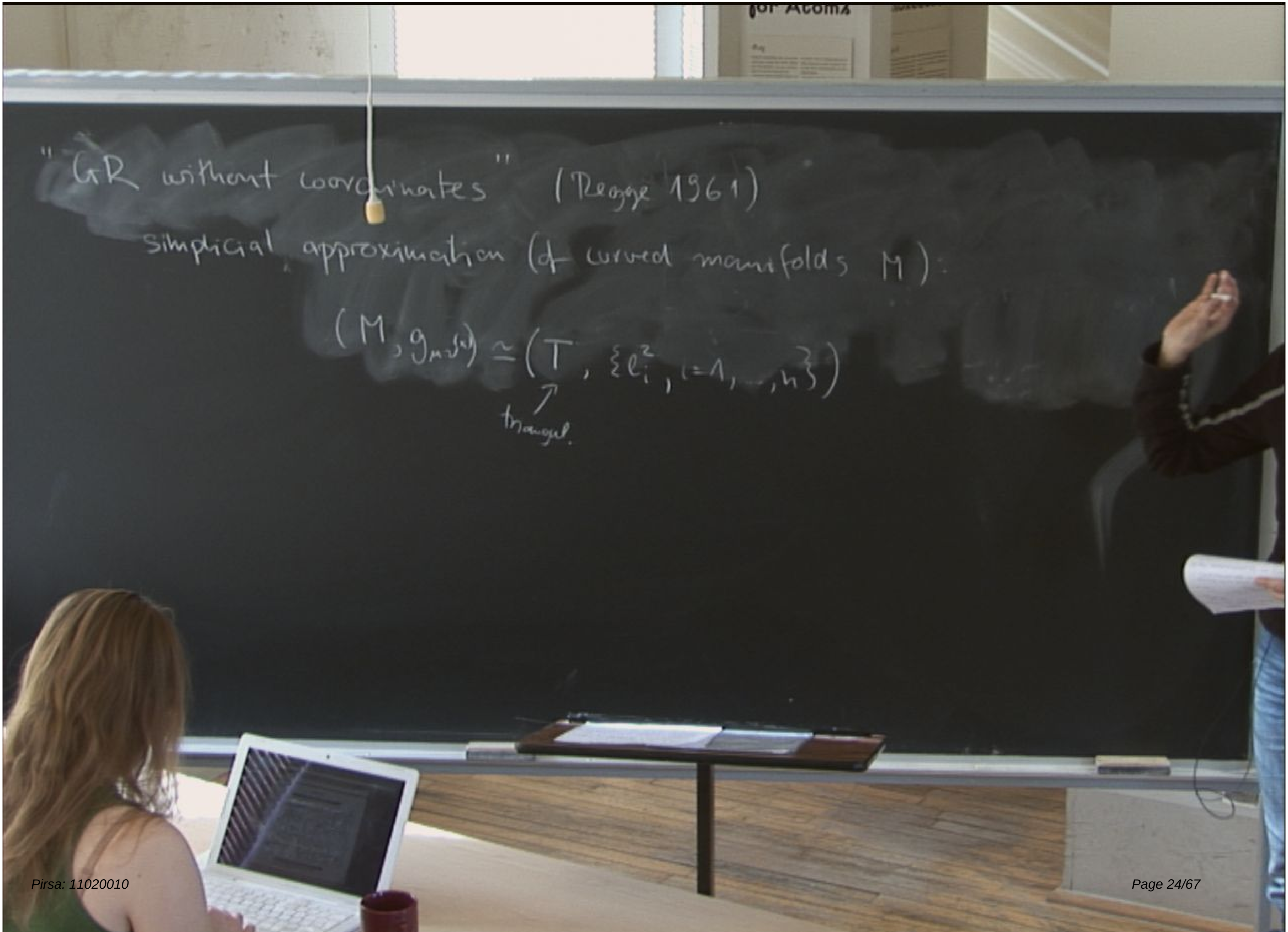
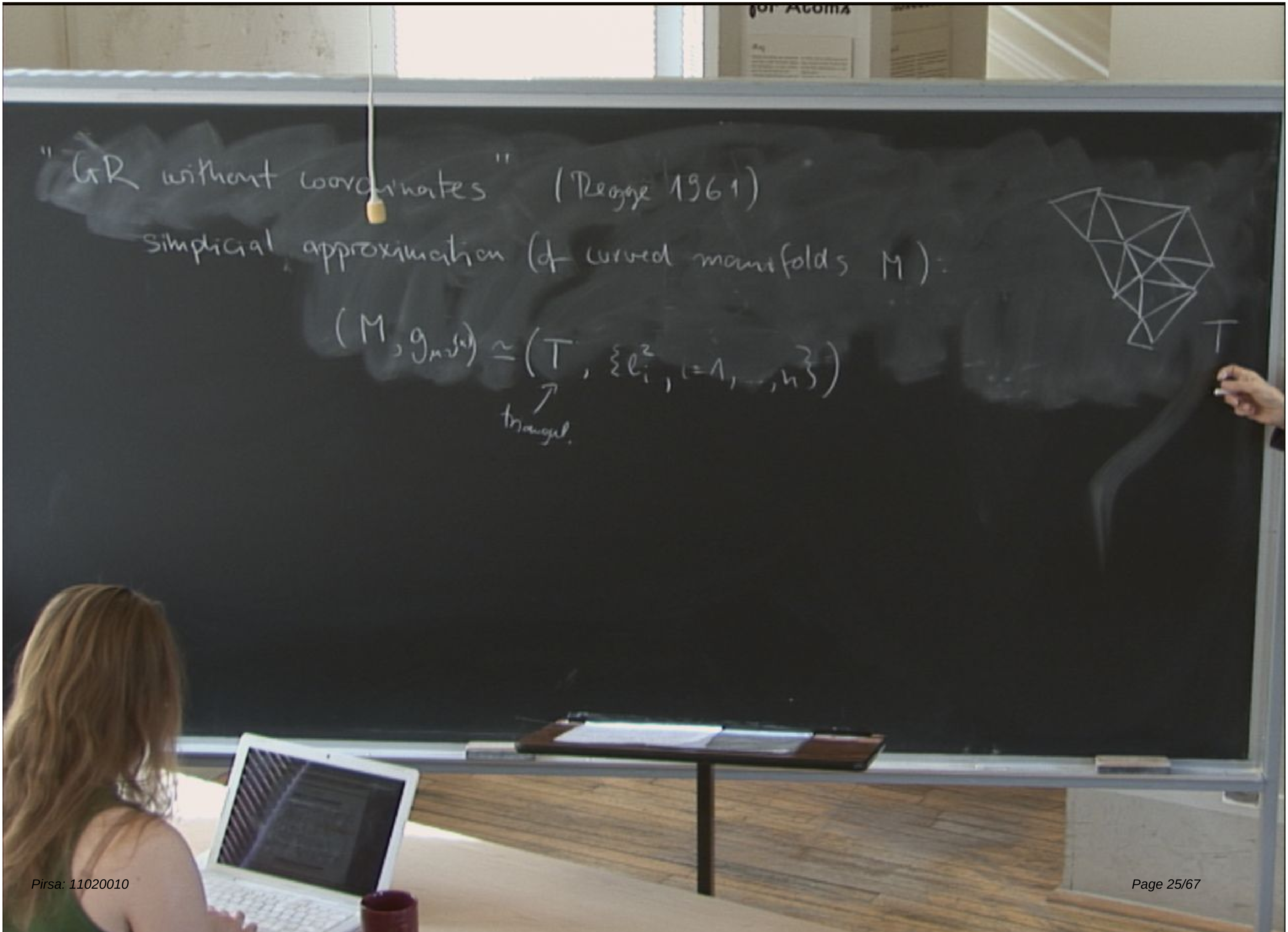
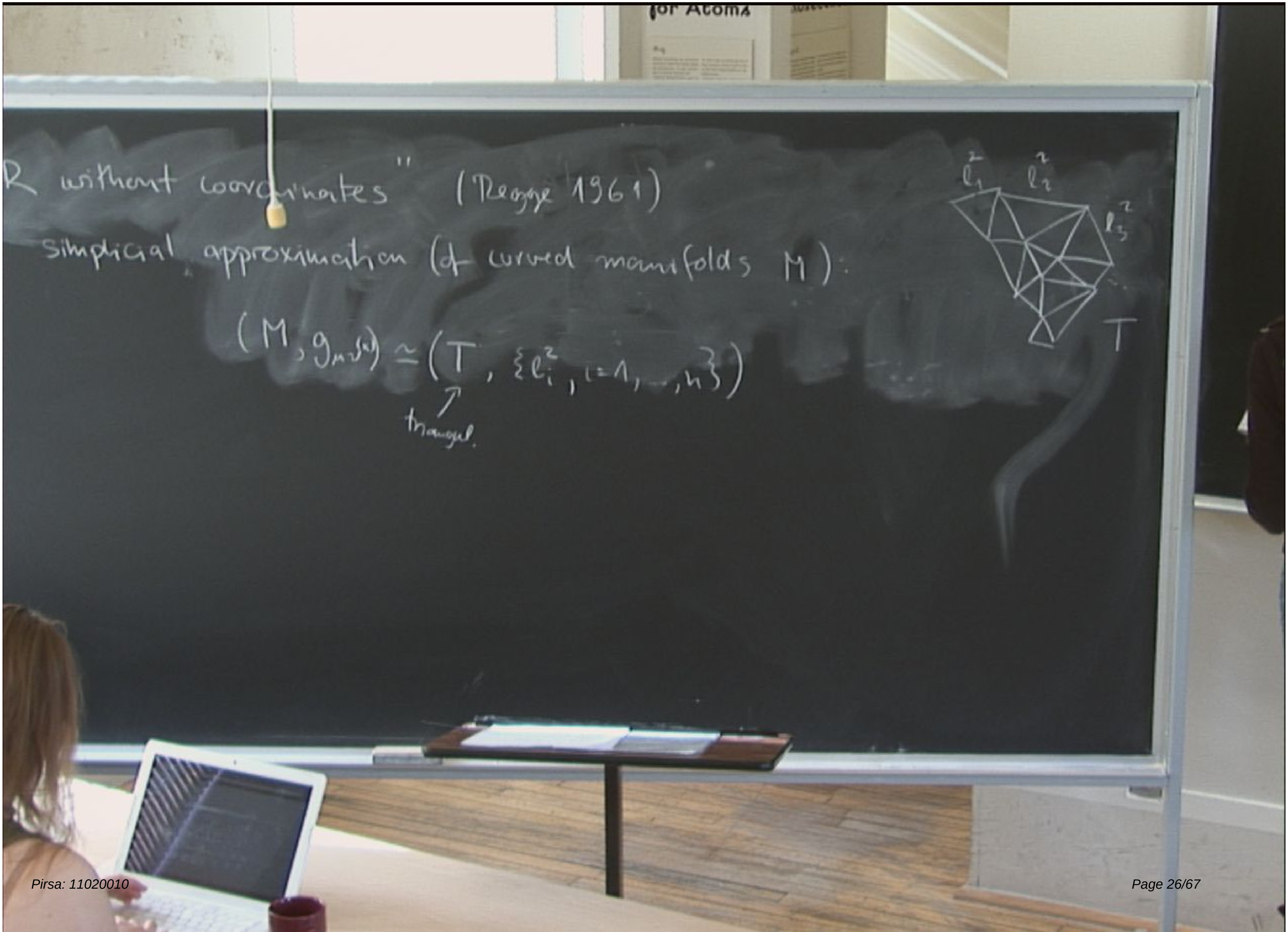

"GR without coordinates" (Regge 1961)

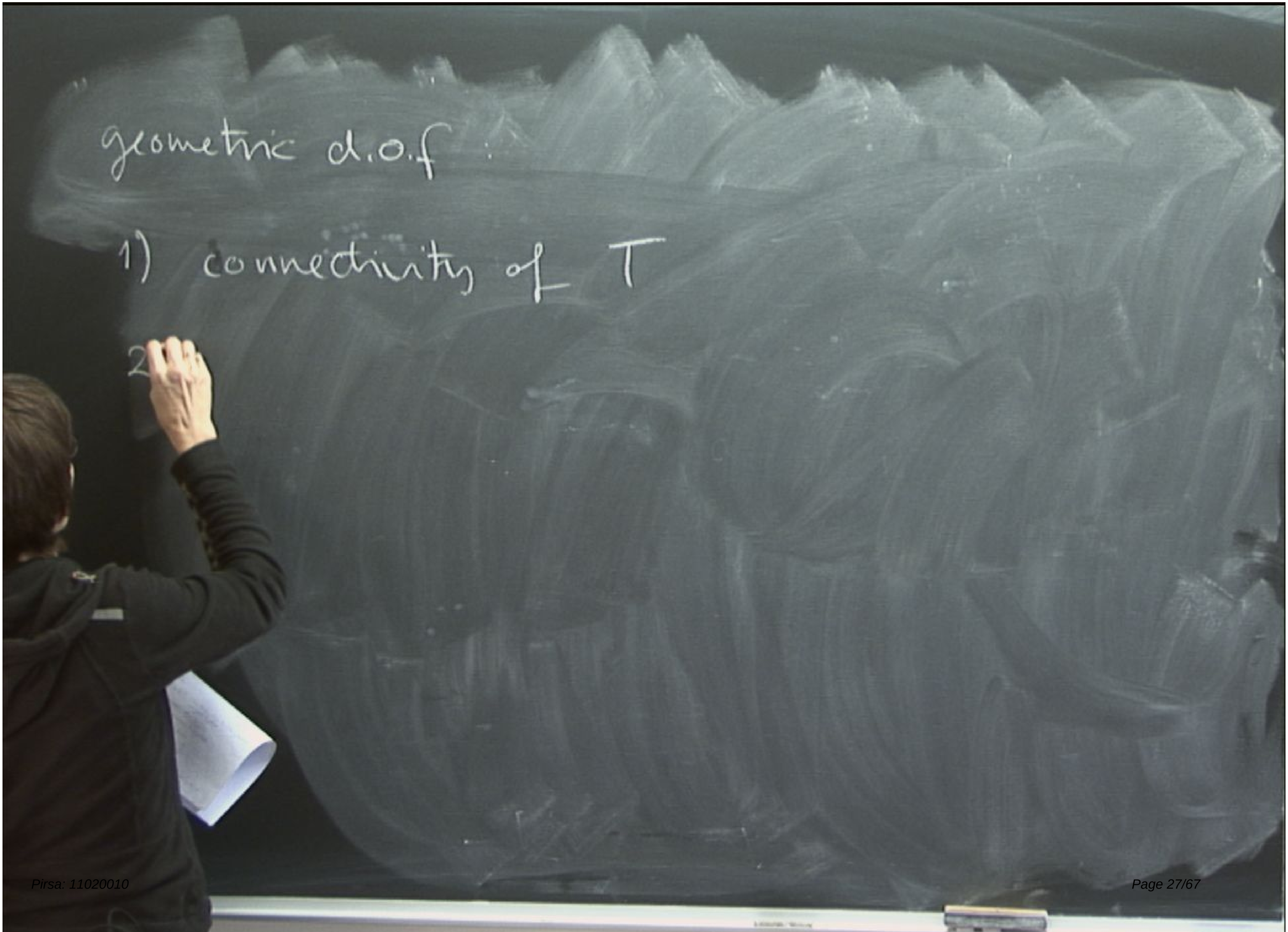

"without coordinates" (Regge 1961)
mplicial approximation (of curved manifolds M):
(M, g_{μν})


" without coordinates" (Regge 1961)
approximation (of curved manifolds M):
(M, g_{μν}(x)) ≃ (T, {l_i^2, i=1, …, n})


"GR without coordinates" (Regge 1961)
simplicial approximation (of curved manifolds M):
$(M, g_{\mu\nu}(x)) \simeq (T, \{l_i^2, i=1,\dots,n\})$
triangul.


"GR without coordinates" (Regge 1961)
simplicial approximation (of curved manifolds M):
(M, g_{μν}(x)) ≃ (T, {l_i^2, i=1, ..., n})
triangul.
T


for Atoms
R without coordinates" (Regge 1961)
simplicial approximation (of curved manifolds M):
$(M, g_{\mu\nu}(x)) \simeq (T, \{l_i^2, i=1,\dots,n\})$
triangul.
l_1^2
l_2^2
l_3^2
T


geometric d.o.f :
1) connectivity of T
2

geometric d.o.f

1) connectivity of T

2) (squared) edge lengths

geometric d.o.f :

1) connectivity of T

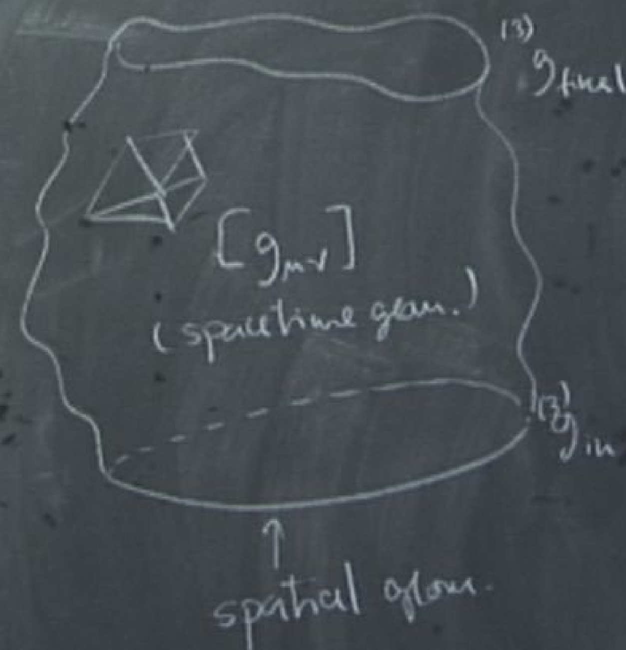
2) (squared) edge lengths

- simplicial building blocks
-

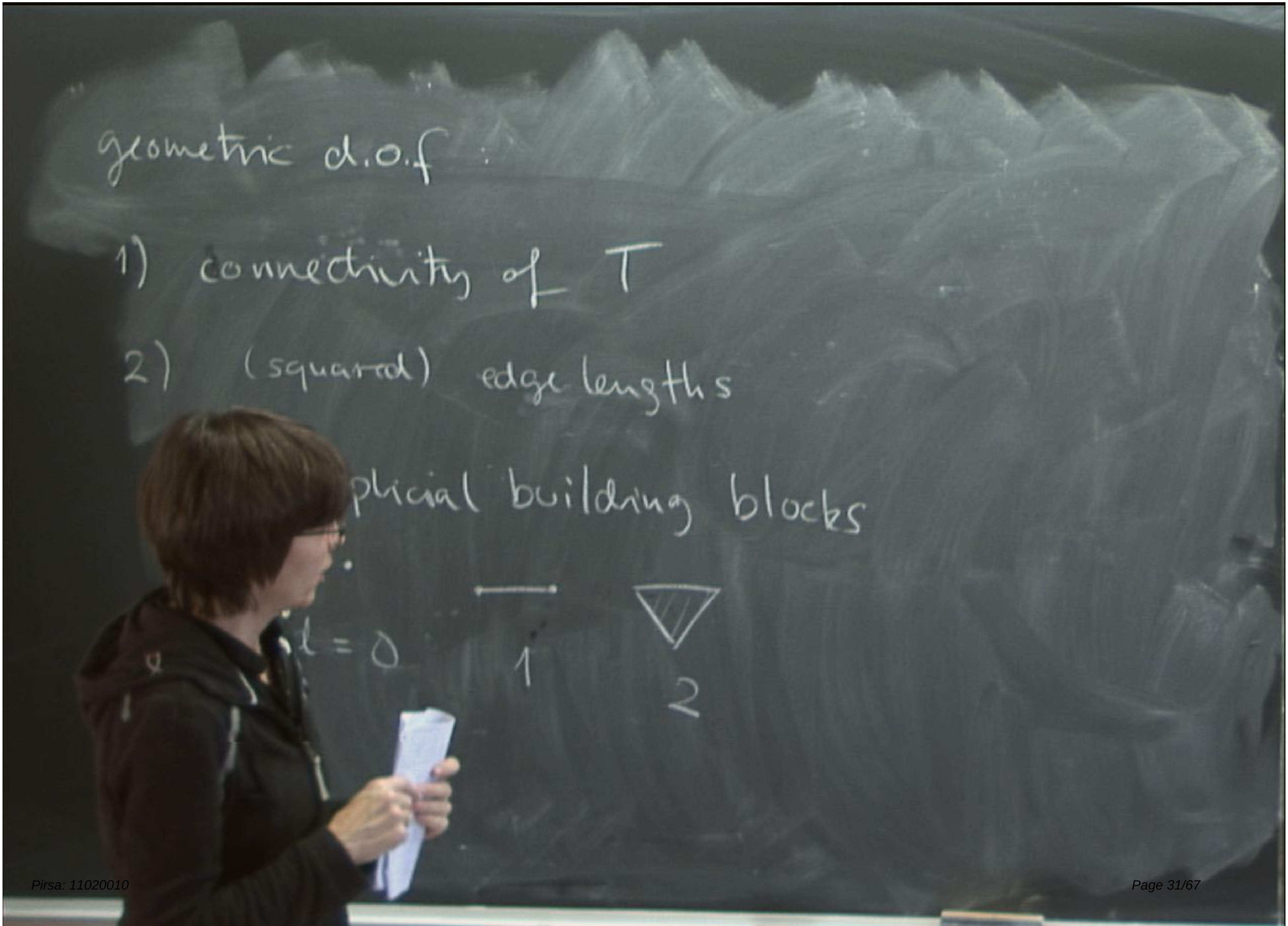
$d = 0$

gravity: "path" $\simeq$ piecewise straight / flat spacetime geometry

$$\sum_{\substack{\text{spacetime geom.s } [g]: \\ {}^{(3)}g_{in} \to {}^{(3)}g_{final}}} e^{iS^{grav}[g]}$$



N.B.: intrinsic geometry & time

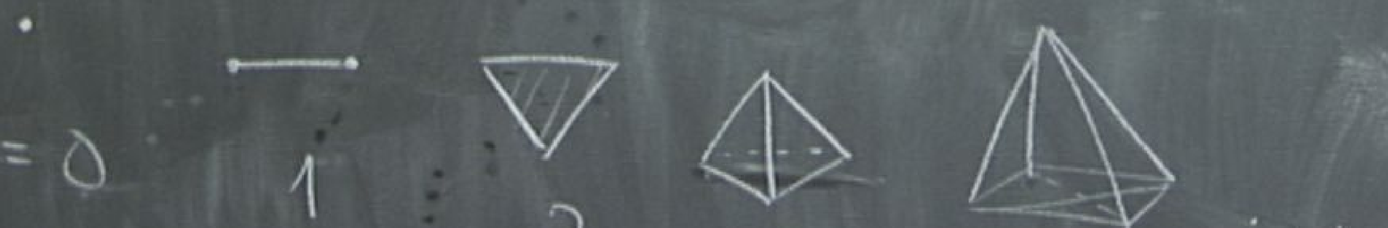

geometric d.o.f :
1) connectivity of T
2) (squared) edge lengths
plicial building blocks
d = 0
1
2

geometric d.o.f :

1) connectivity of T

2) (squared) edge lengths

• simplicial building blocks



$d = 0$ 1 2 3 4 ....

by def, interior geometry is flat

geometric d.o.f :

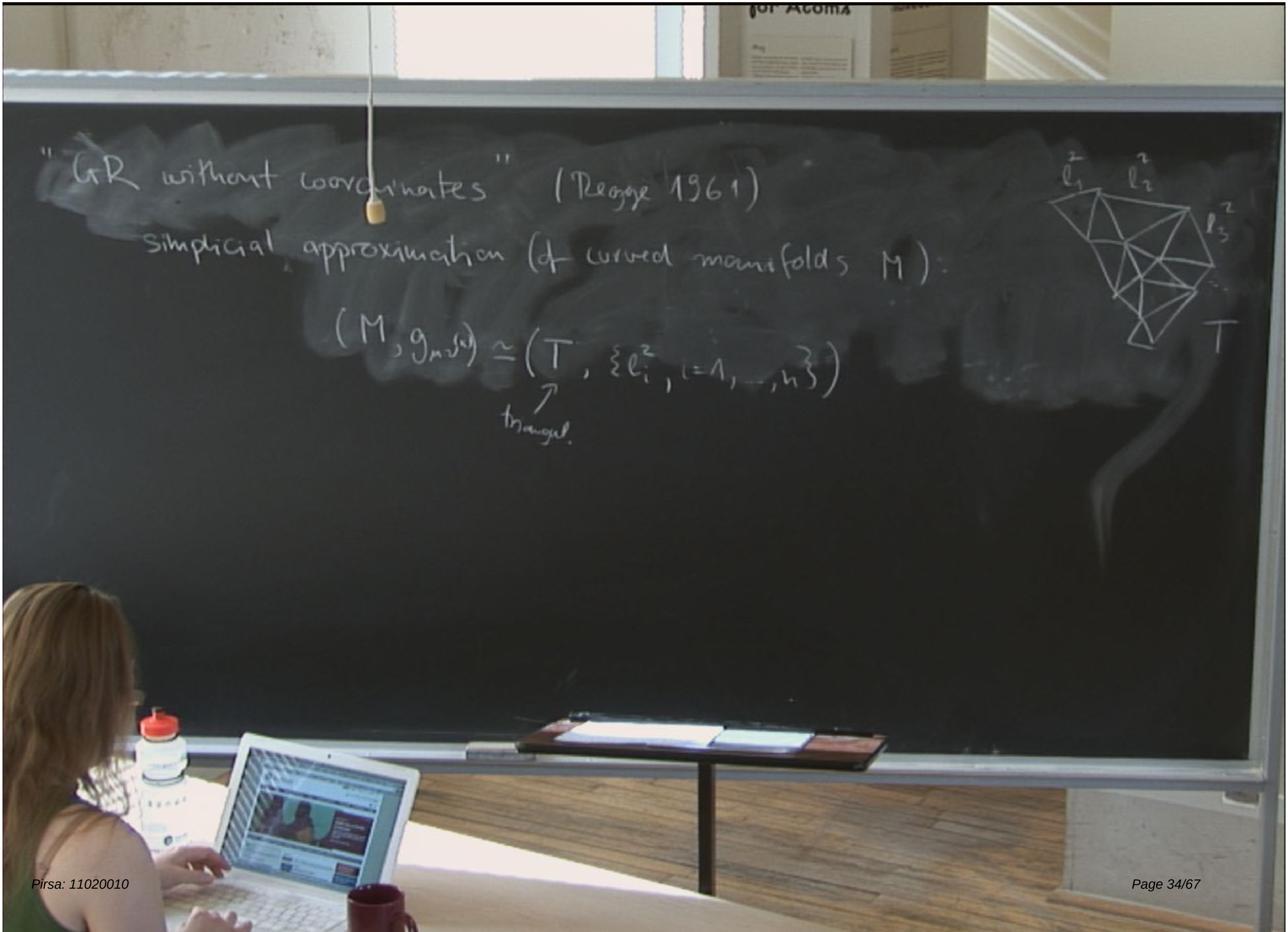
1) connectivity of T

2) (squared) edge lengths

- simplicial building blocks

$d = 0$ 1 2 3 4 ....

by def, interior geometry is flat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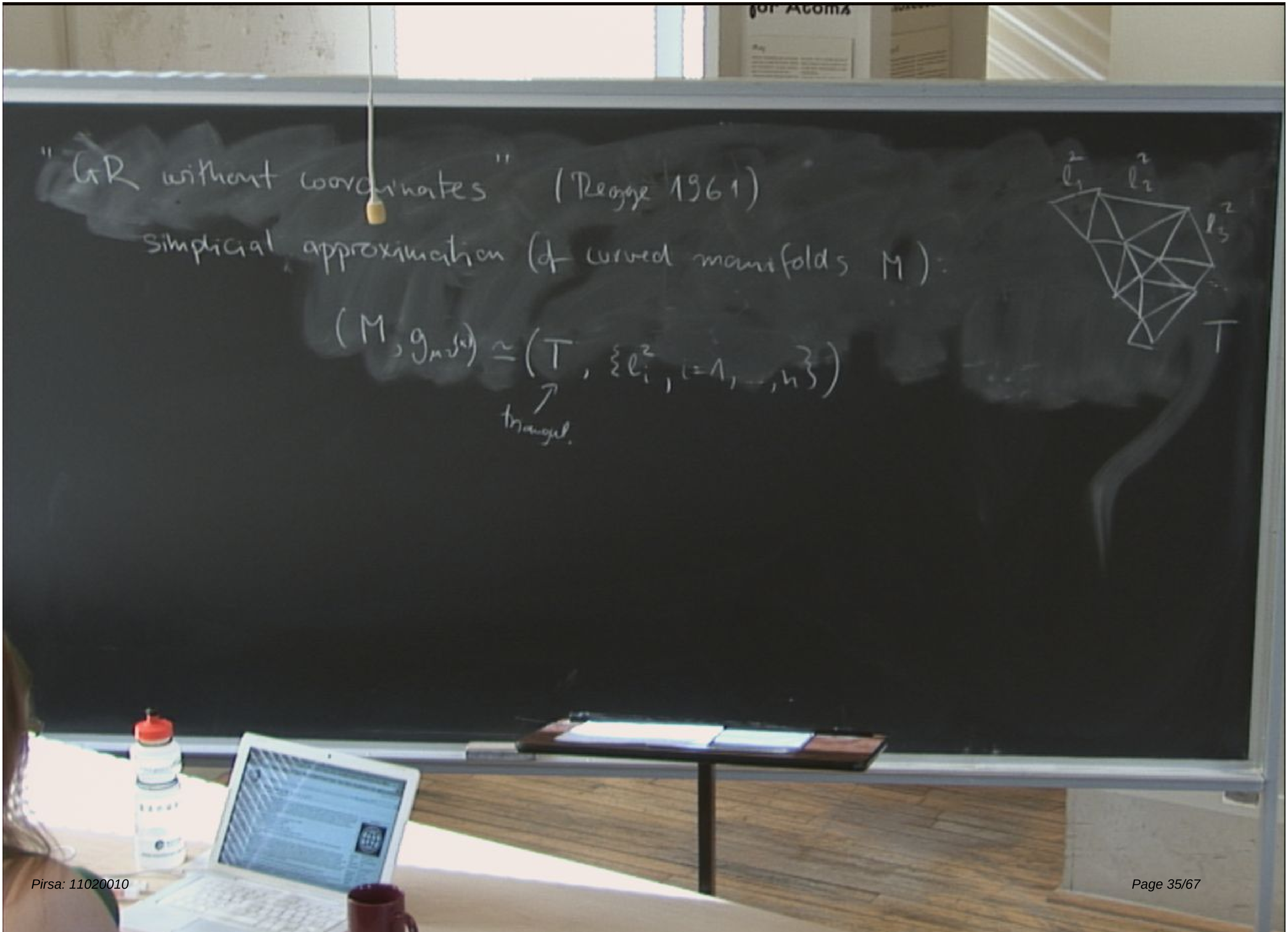
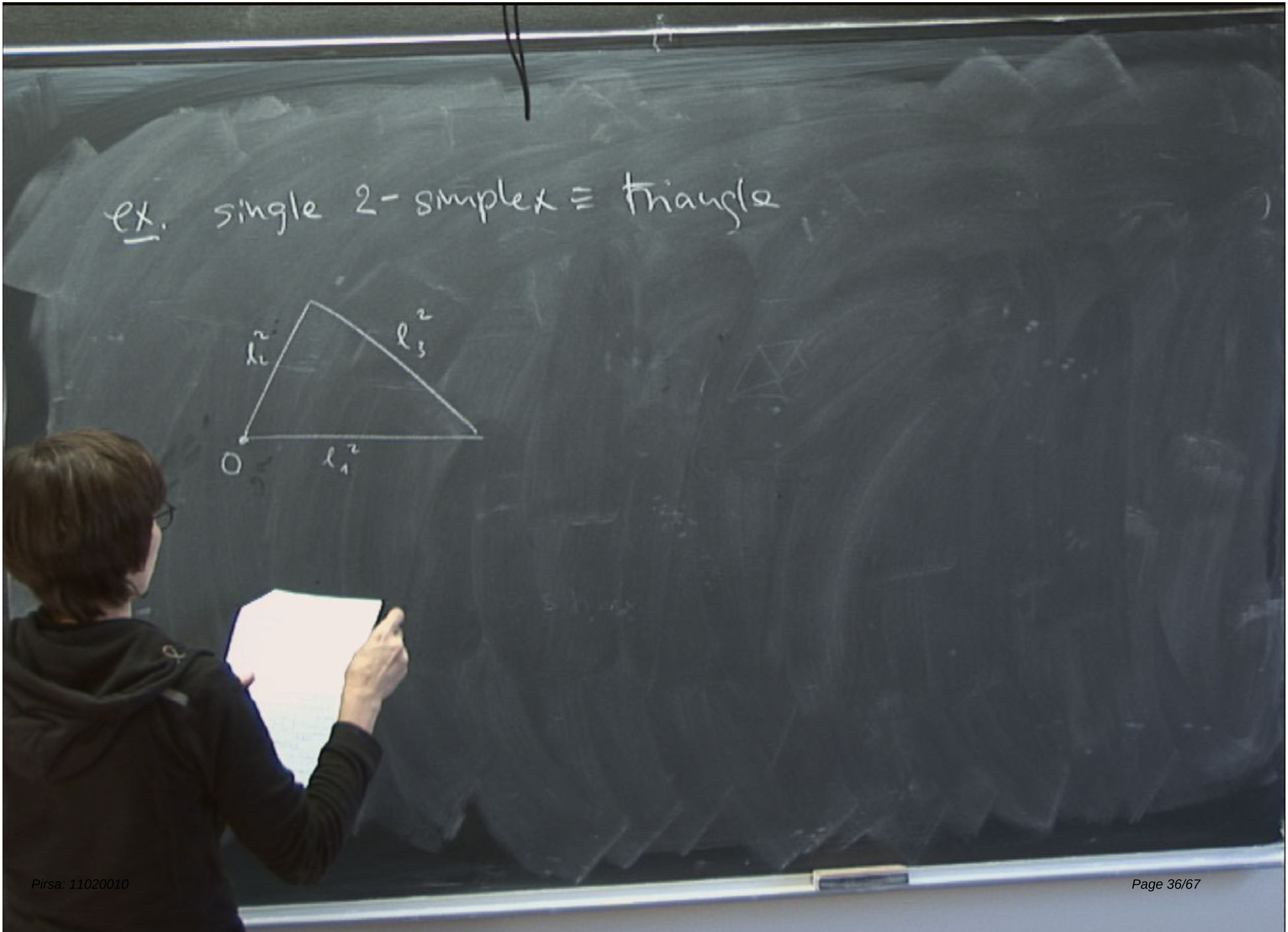
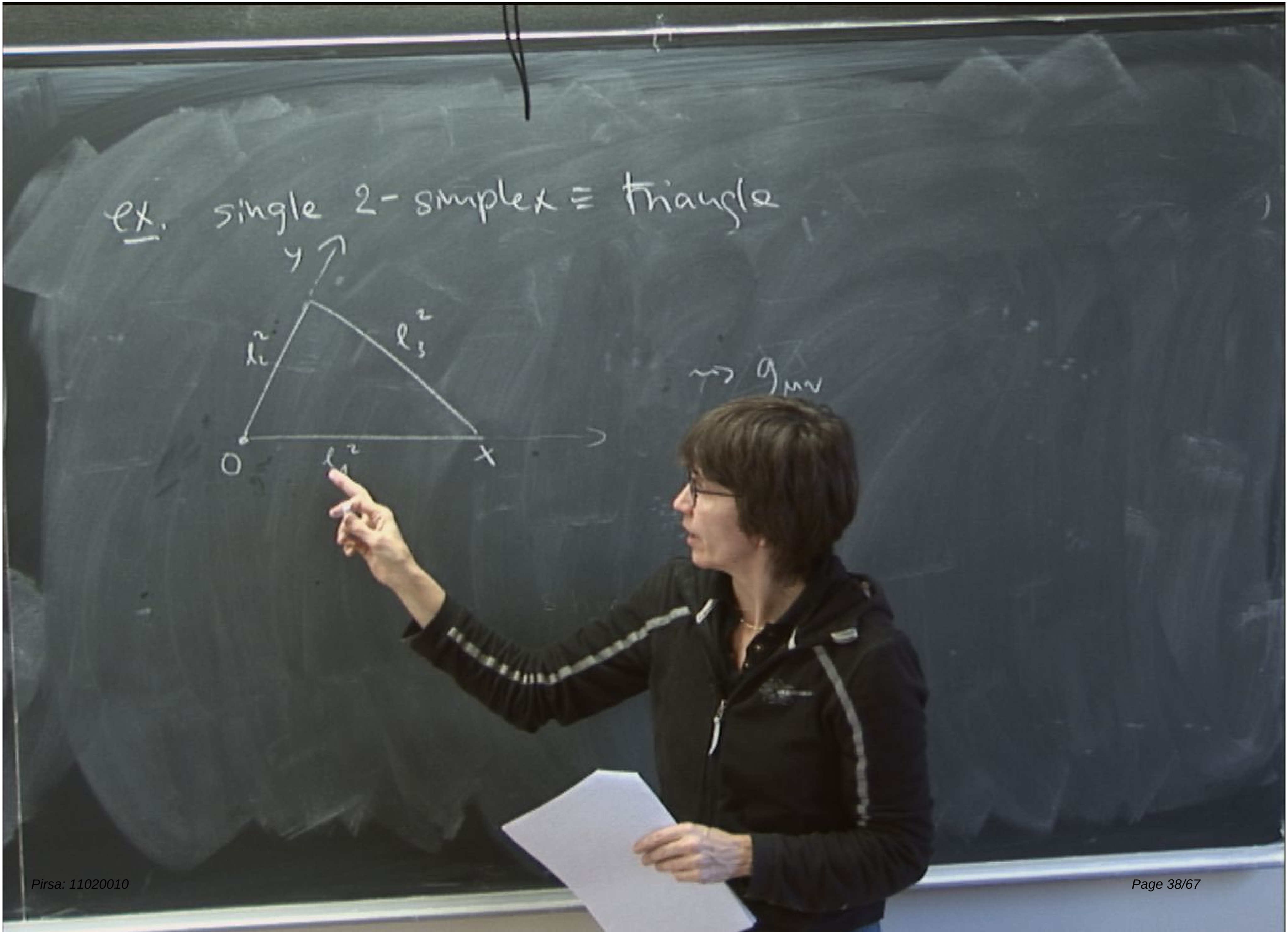
triangul.
l_1^2 l_2^2 l_3^2
T


"GR without coordinates" (Regge 1961)
simplicial approximation (of curved manifolds M):
(M, g_{μν}(x)) ≃ (T, {l_i^2, i=1, ..., n})
triangul.
l_1^2
l_2^2
l_3^2
T


ex. single 2-simplex ≡ triangle
ℓ_2^2
ℓ_3^2
O
ℓ_1^2

ex. single 2-simplex ≡ triangle

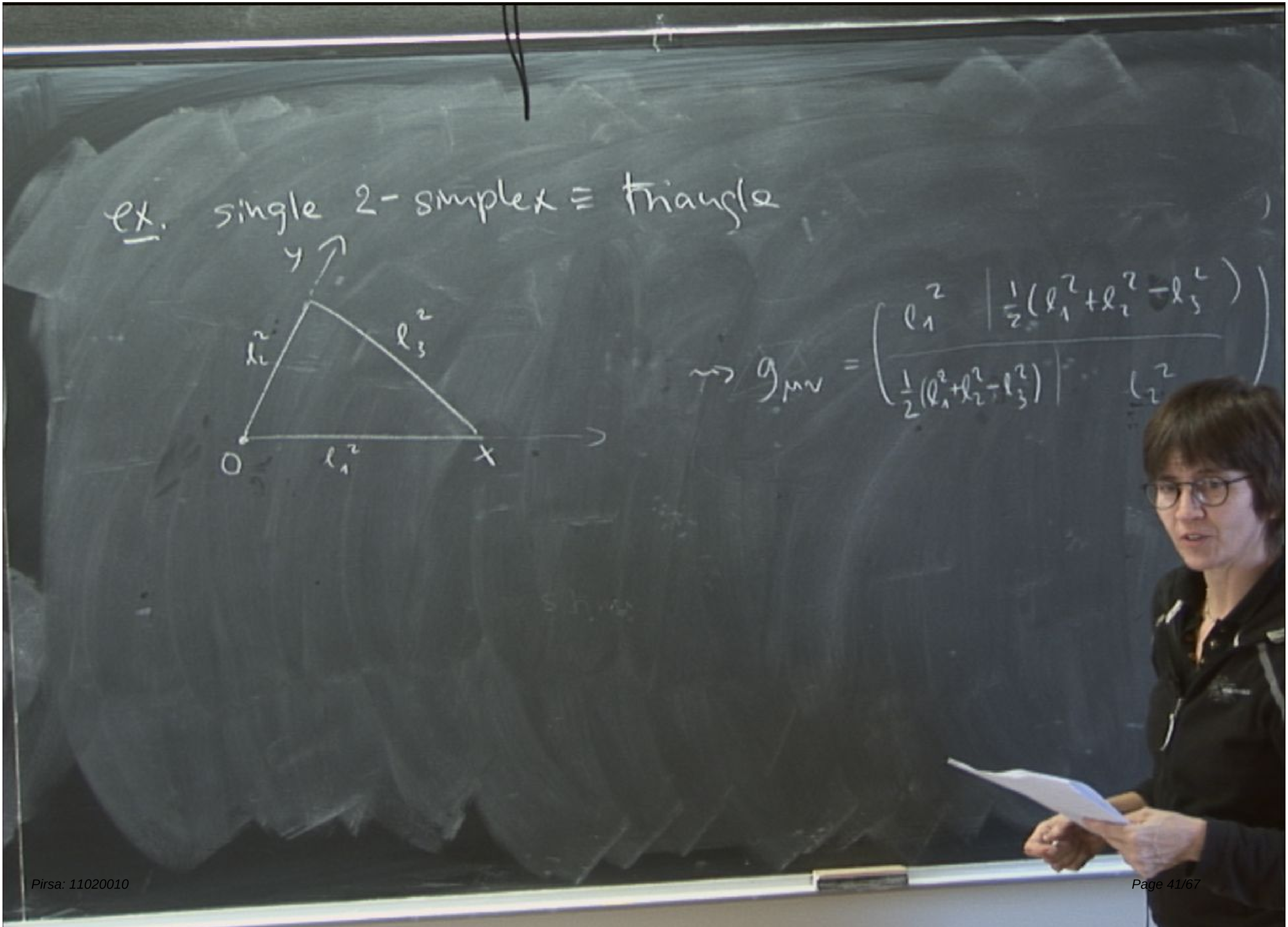

ex. single 2-simplex ≡ triangle
y
l_2^2
l_3^2
O
l_1^2
x
$\rightsquigarrow g_{\mu\nu}$

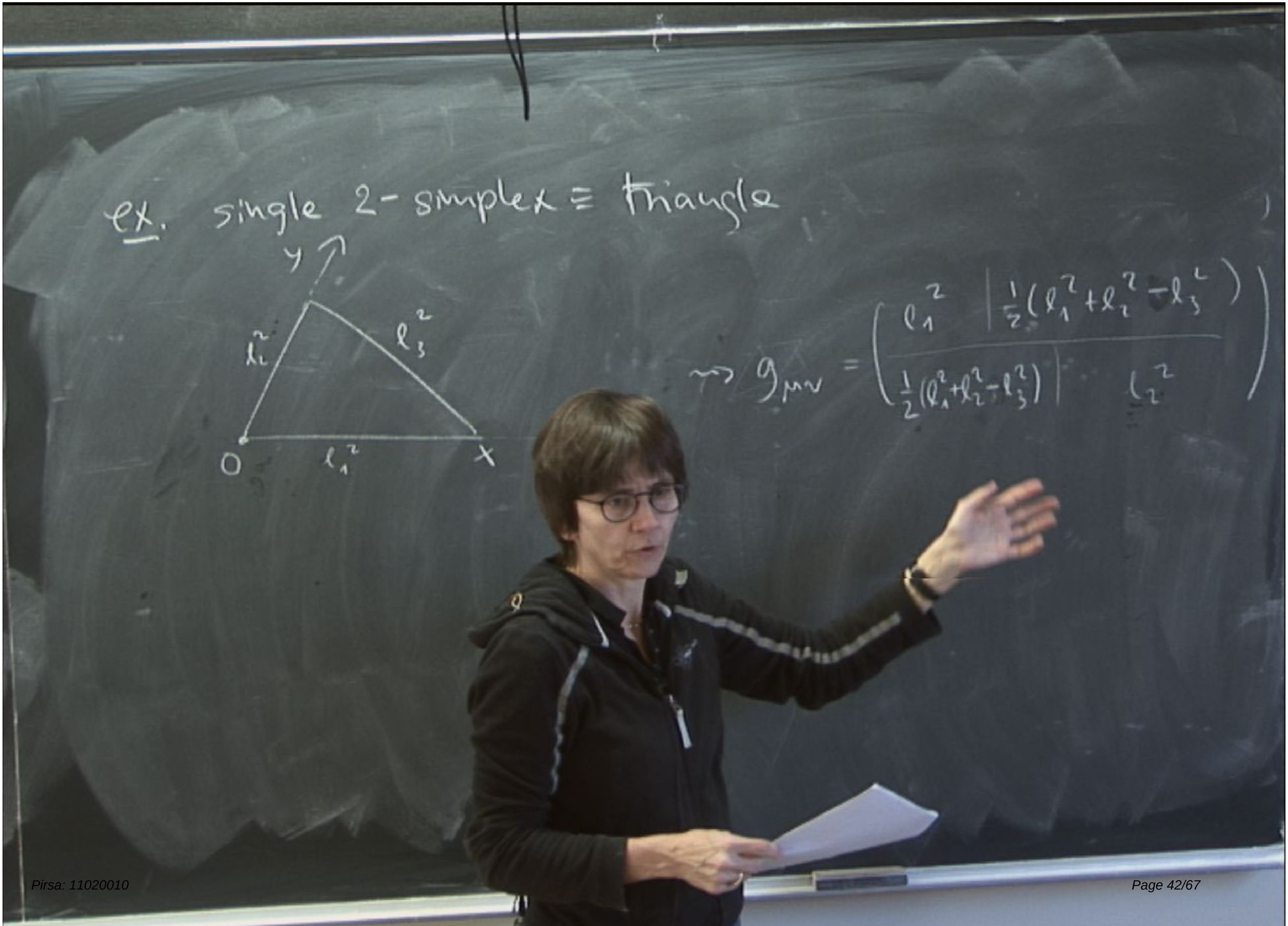
ex. single 2-simplex $\equiv$ triangle

$$\rightsquigarrow g_{\mu\nu} = \left(\begin{array}{c|c} l_1^2 & \frac{1}{2}(l_1^2+l_2^2+l_3^2) \\ \hline \frac{1}{2}(l_1^2+l_2^2+l_3^2) & \end{array} \right)$$

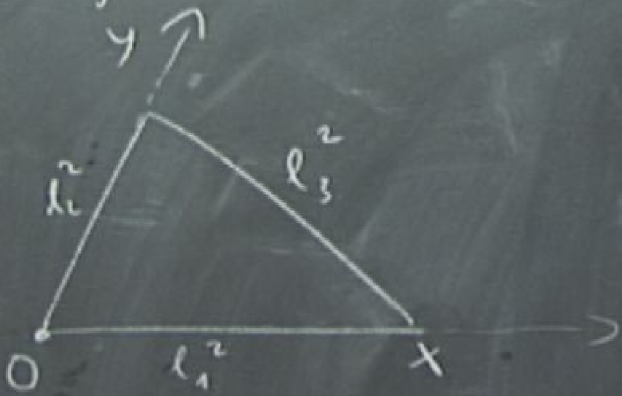
ex. single 2-simplex $\equiv$ triangle

$$\rightsquigarrow g_{\mu\nu} = \begin{pmatrix} l_1^2 & \frac{1}{2}(l_1^2+l_2^2+l_3^2) \\ \frac{1}{2}(l_1^2+l_2^2+l_3^2) & l_2^2 \end{pmatrix}$$


ex. single 2-simplex ≡ triangle
y
l_2^2
l_3^2
O
l_1^2
x
$\rightsquigarrow g_{\mu\nu} = \begin{pmatrix} l_1^2 & \frac{1}{2}(l_1^2+l_2^2-l_3^2) \\ \frac{1}{2}(l_1^2+l_2^2-l_3^2) & l_2^2 \end{pmatrix}$

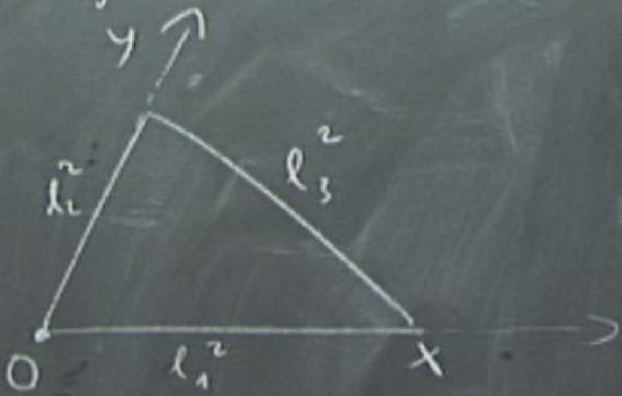

ex. single 2-simplex ≡ triangle
y
x
O
l_1^2
l_2^2
l_3^2
$\rightsquigarrow g_{\mu\nu} = \begin{pmatrix} l_1^2 & \frac{1}{2}(l_1^2+l_2^2-l_3^2) \\ \frac{1}{2}(l_1^2+l_2^2-l_3^2) & l_2^2 \end{pmatrix}$

ex. single 2-simplex $\equiv$ triangle



$$\rightsquigarrow g_{\mu\nu} = \begin{pmatrix} l_1^2 & \frac{1}{2}(l_1^2+l_2^2-l_3^2) \\ \frac{1}{2}(l_1^2+l_2^2-l_3^2) & l_2^2 \end{pmatrix}$$

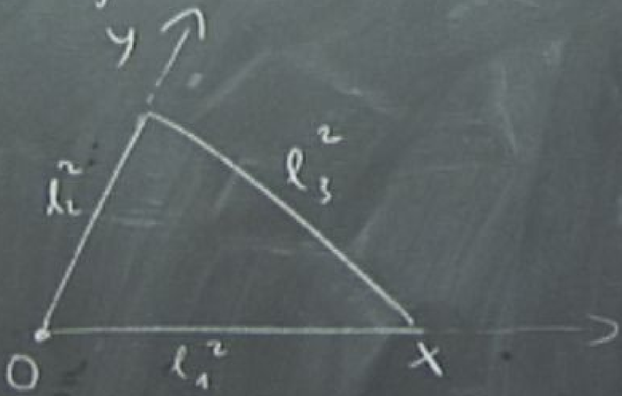
ex. single 2-simplex $\equiv$ triangle



$$\rightsquigarrow g_{\mu\nu} = \begin{pmatrix} l_1^2 & \frac{1}{2}(l_1^2+l_2^2-l_3^2) \\ \frac{1}{2}(l_1^2+l_2^2-l_3^2) & l_2^2 \end{pmatrix}$$

local (intrinsic) Gaussian curvature

ex. single 2-simplex ≡ triangle



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local (intrinsic) Gaussian curvature

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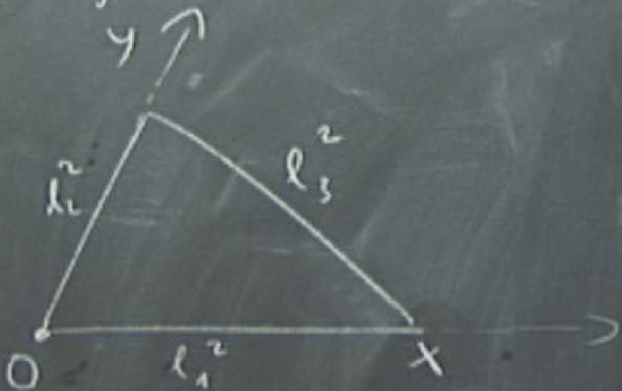
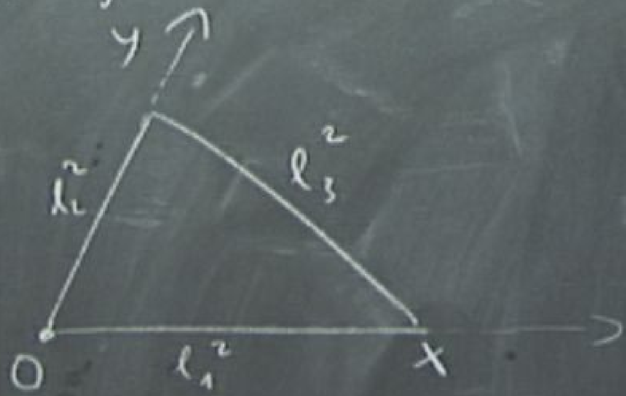


$$\rightsquigarrow g_{\mu\nu} = \begin{pmatrix} l_1^2 & \frac{1}{2}(l_1^2+l_2^2-l_3^2) \\ \frac{1}{2}(l_1^2+l_2^2-l_3^2) & l_2^2 \end{pmatrix}$$

local (intrinsic) Gaussian curvatures

$$= \text{deficit angle} \quad \delta = 2\pi - \sum_{i \supset h}$$

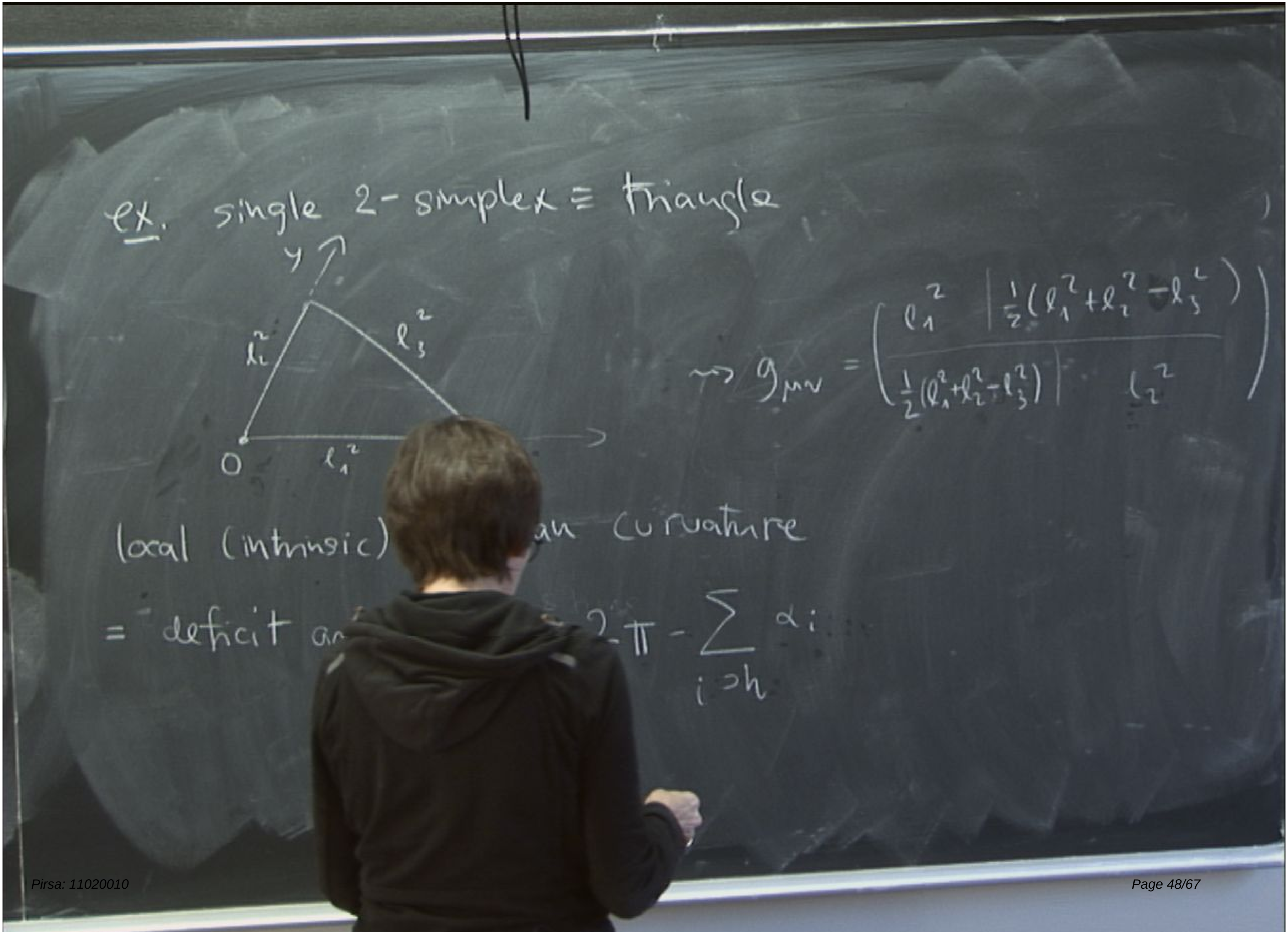
ex. single 2-simplex $\equiv$ triangle



$$\rightsquigarrow g_{\mu\nu} = \left(\begin{array}{c|c} l_1^2 & \frac{1}{2}(l_1^2 + l_2^2 - l_3^2) \\ \hline \frac{1}{2}(l_1^2 + l_2^2 - l_3^2) & l_2^2 \end{array} \right)$$

local (intrinsic) Gaussian curvature

$$= \text{deficit angle} \quad \delta = 2\pi - \sum_{i \supset h} \alpha_i$$


ex. single 2-simplex ≡ triangle
y
O
l_1^2
l_2^2
l_3^2
$\rightsquigarrow g_{\mu\nu} = \begin{pmatrix} l_1^2 & \frac{1}{2}(l_1^2+l_2^2-l_3^2) \\ \frac{1}{2}(l_1^2+l_2^2-l_3^2) & l_2^2 \end{pmatrix}$
local (intrinsic)
curvature
= deficit
$2\pi - \sum \alpha_i$

ex. single 2-simplex $\equiv$ triangle

$$\rightsquigarrow g_{\mu\nu} = \begin{pmatrix} l_1^2 & \frac{1}{2}(l_1^2+l_2^2-l_3^2) \\ \frac{1}{2}(l_1^2+l_2^2-l_3^2) & l_2^2 \end{pmatrix}$$

local (intrinsic) Gaussian curvature

$$= \text{deficit angle} \quad \delta = 2\pi - \sum_{i \supset h} \alpha_i$$

labels d-simp

ex. single 2-simplex ≡ triangle

$$\rightsquigarrow g_{\mu\nu} = \left(\begin{array}{c|c} l_1^2 & \frac{1}{2}(l_1^2+l_2^2-l_3^2) \\ \hline \frac{1}{2}(l_1^2+l_2^2-l_3^2) & l_2^2 \end{array}\right)$$

local (intrinsic) Gaussian curvature

= deficit angle $\delta = 2\pi - \sum_{i \supset h} \alpha_i$

labels d-simplices sharing a comm
(d-2)-simplex

ex. single 2-simplex ≡ triangle

$$\rightsquigarrow g_{\mu\nu} = \begin{pmatrix} \ell_1^2 & \frac{1}{2}(\ell_1^2+\ell_2^2-\ell_3^2) \\ \frac{1}{2}(\ell_1^2+\ell_2^2-\ell_3^2) & \ell_2^2 \end{pmatrix}$$

local (intrinsic) Gaussian curvature

$= \text{deficit angle} \quad \delta = 2\pi - \sum_{i \ni h} \alpha_i$

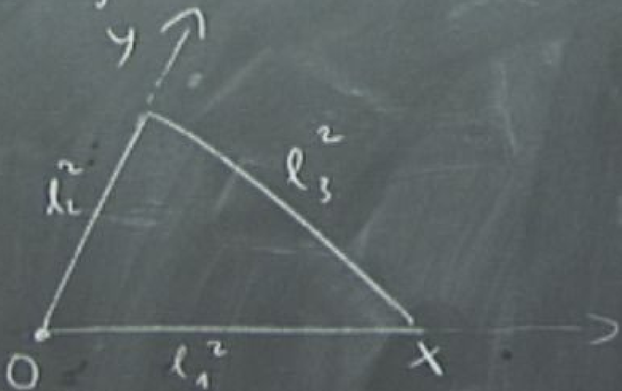
labels d-

sharing a

(d-2)-simplex

("h

ex. single 2-simplex ≡ triangle



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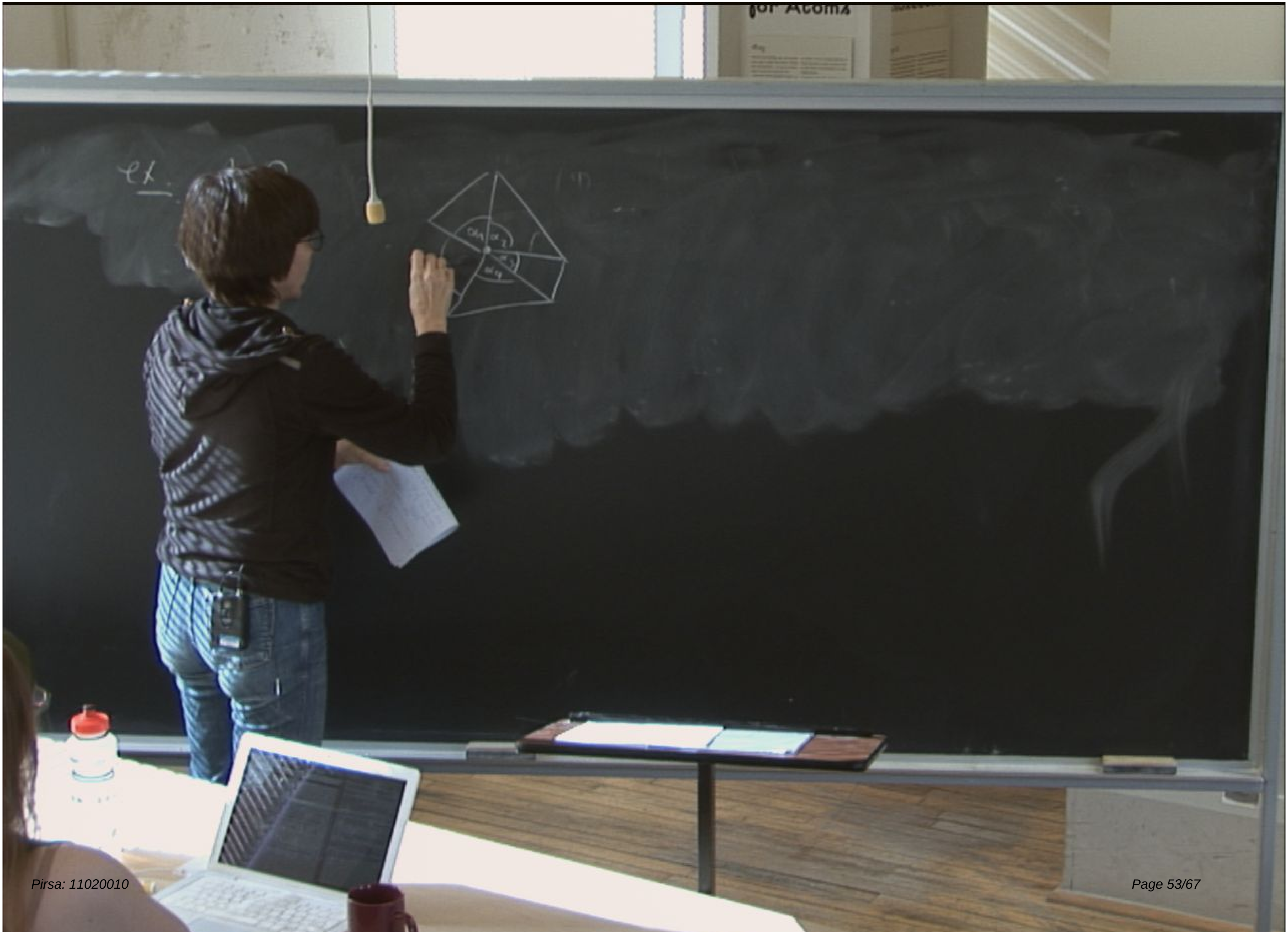
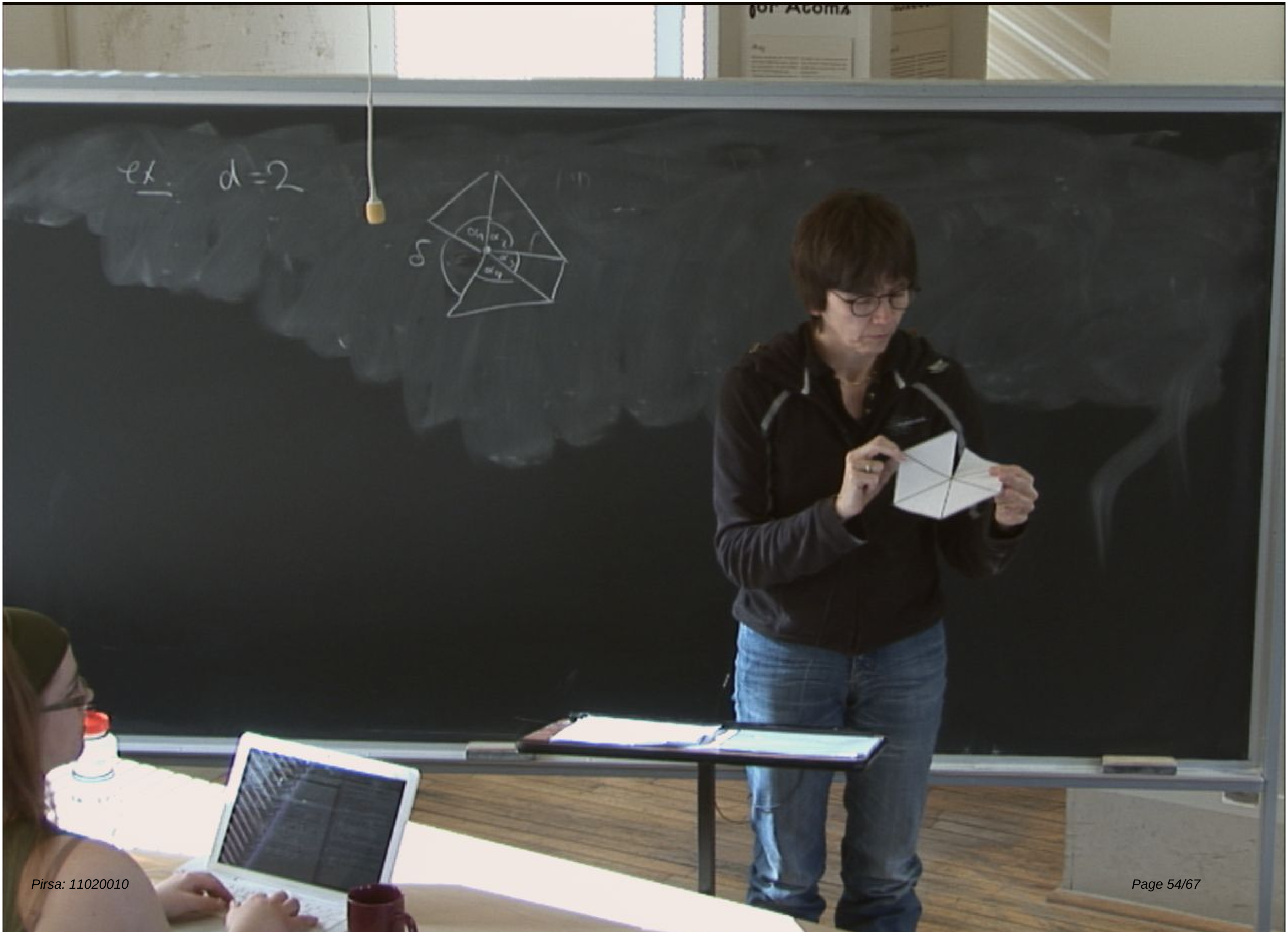
local (intrinsic) Gaussian curvature

$$= \text{deficit angle} \quad \delta = 2\pi - \sum_{i \ni h} \alpha_i$$

labels d-simplices sharing a common (d-2)-simplex ("hinge" h)




ex. d=2

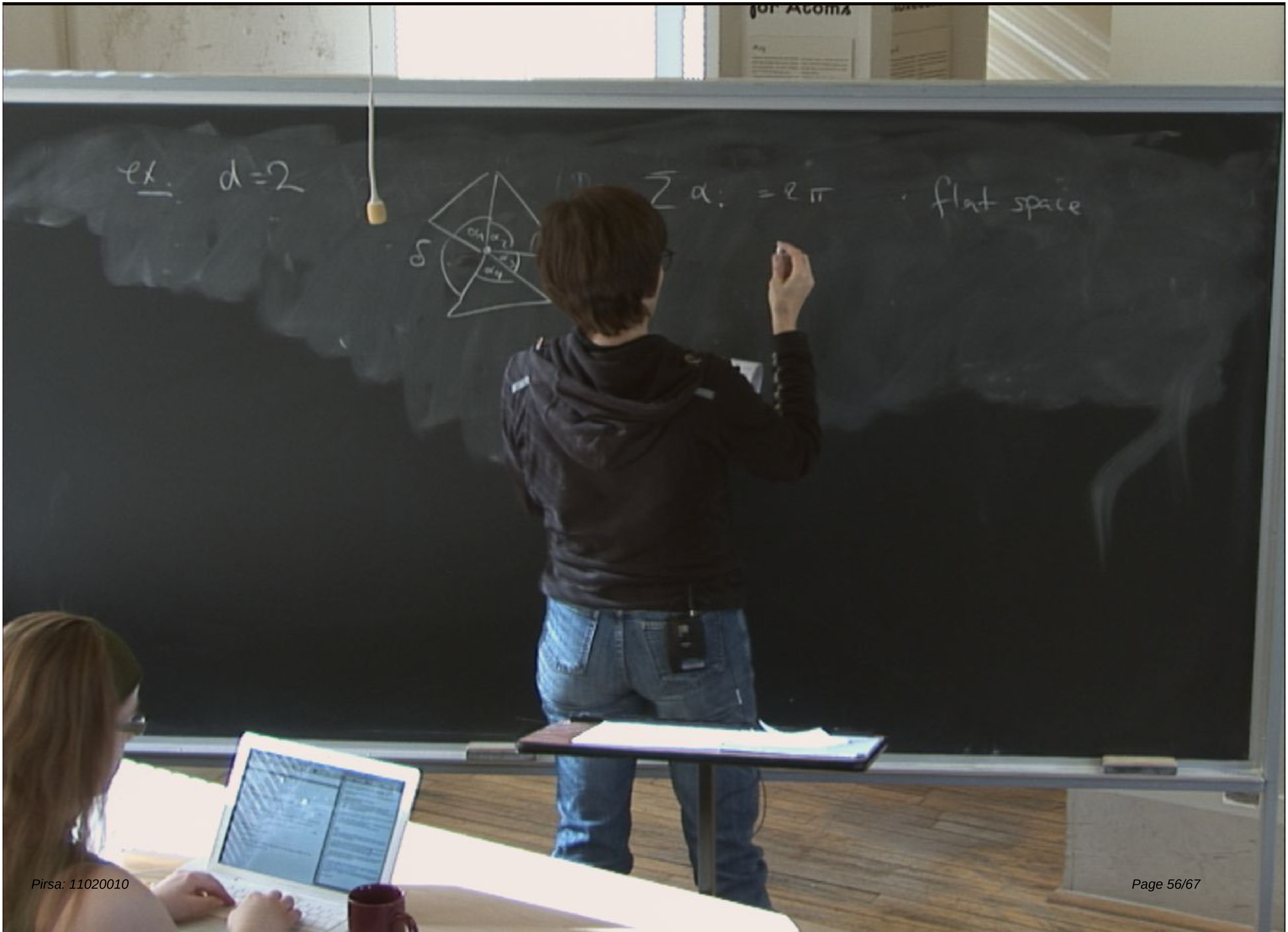
ex. single 2-simplex $\equiv$ triangle

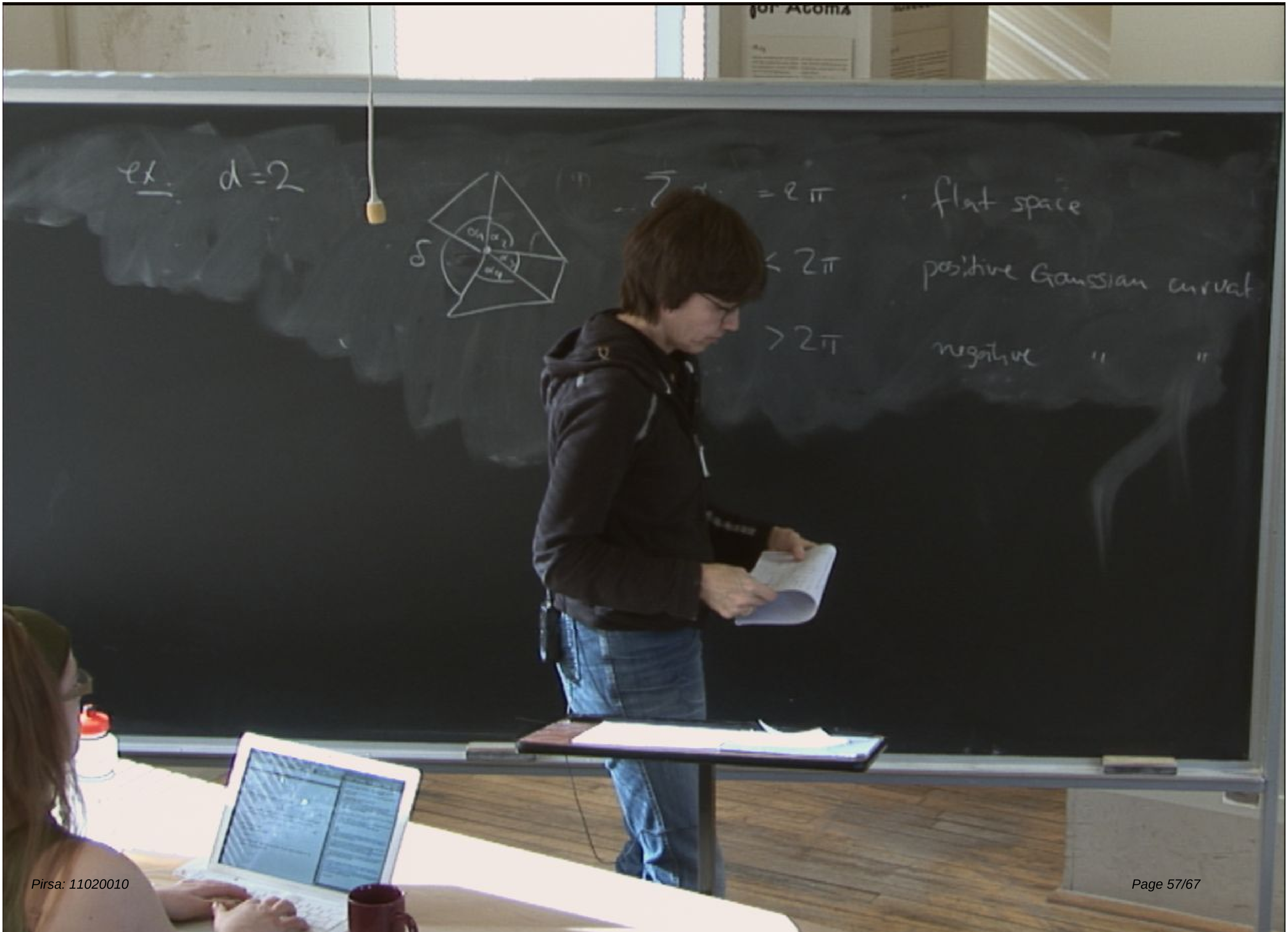
$$\rightsquigarrow g_{\mu\nu} = \begin{pmatrix} l_1^2 & \frac{1}{2}(l_1^2+l_2^2-l_3^2) \\ \frac{1}{2}(l_1^2+l_2^2-l_3^2) & l_2^2 \end{pmatrix}$$

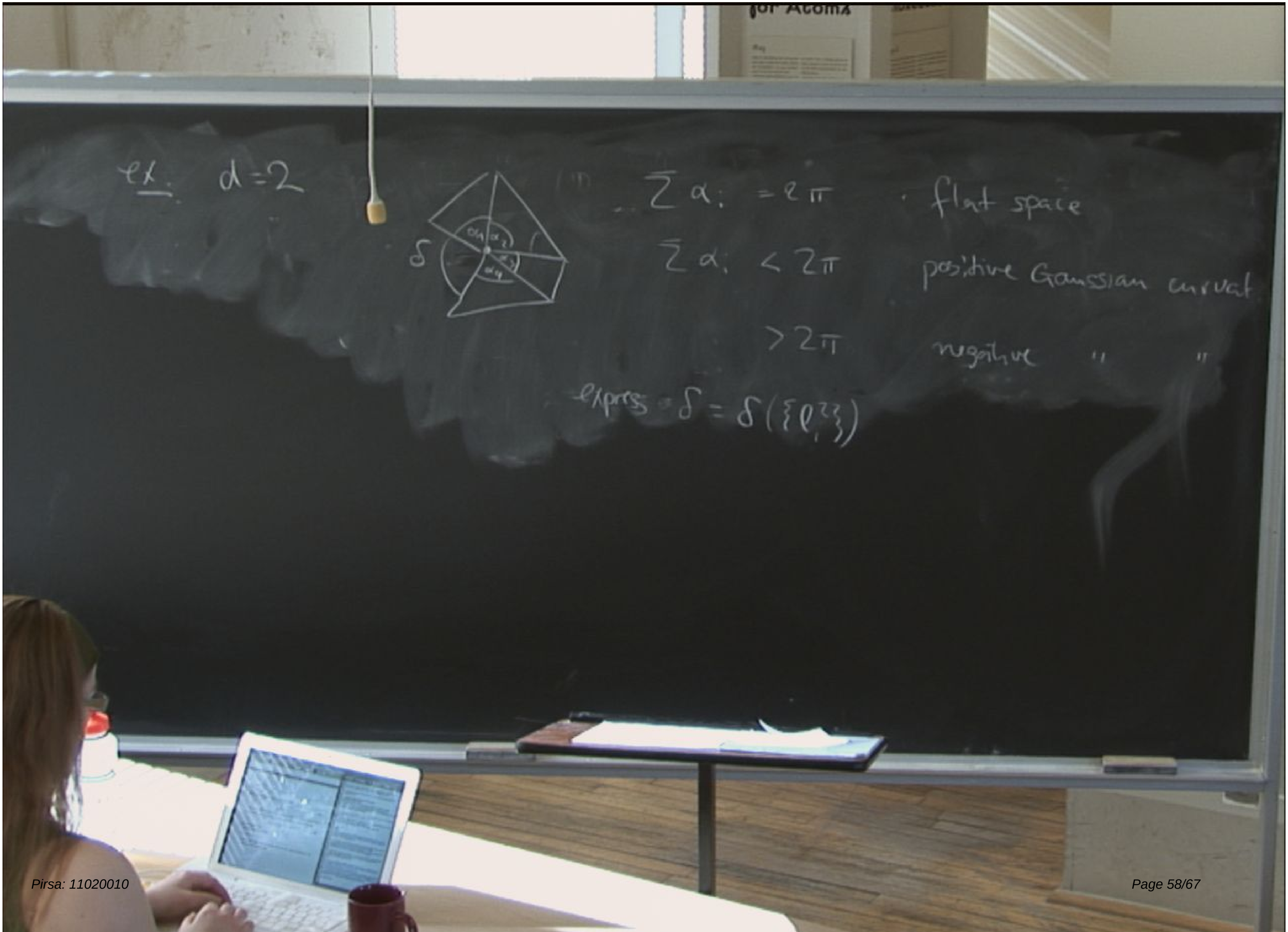
local (intrinsic) Gaussian curvature

$$= \text{deficit angle} \quad \delta = 2\pi - \sum_{i \supset h} \alpha_i$$

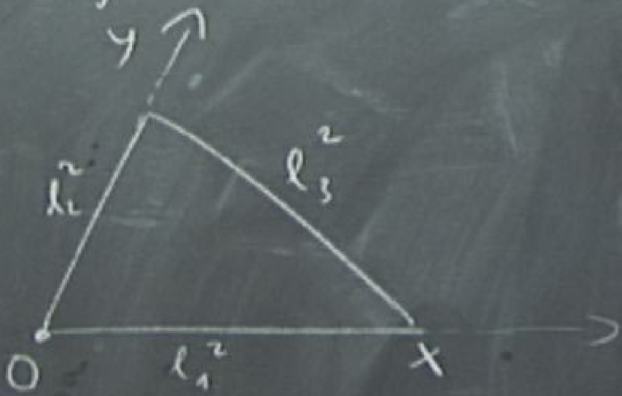
labels d-simplices sharing a common (d-2)-simplex ("hinge" h)


ex. d=2
flat space


ex. d=2
= 2π
flat space
< 2π
positive Gaussian curvat
> 2π
negative " "


ex. d=2
δ
α1 α2 α3 α4
Σαi = 2π flat space
Σαi < 2π positive Gaussian curvat
> 2π negative " "
express δ = δ({li²})

ex. single 2-simplex ≡ triangle



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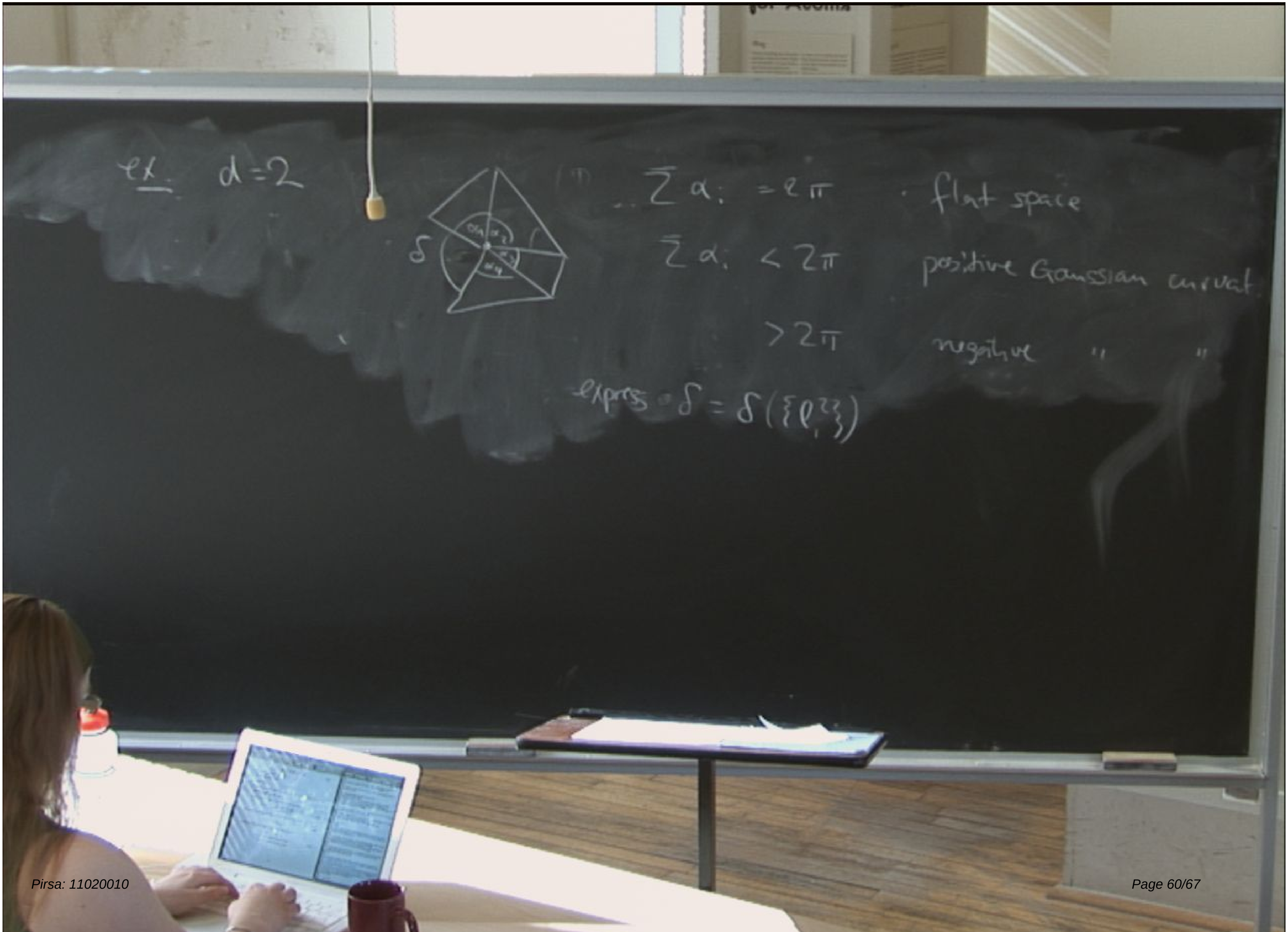
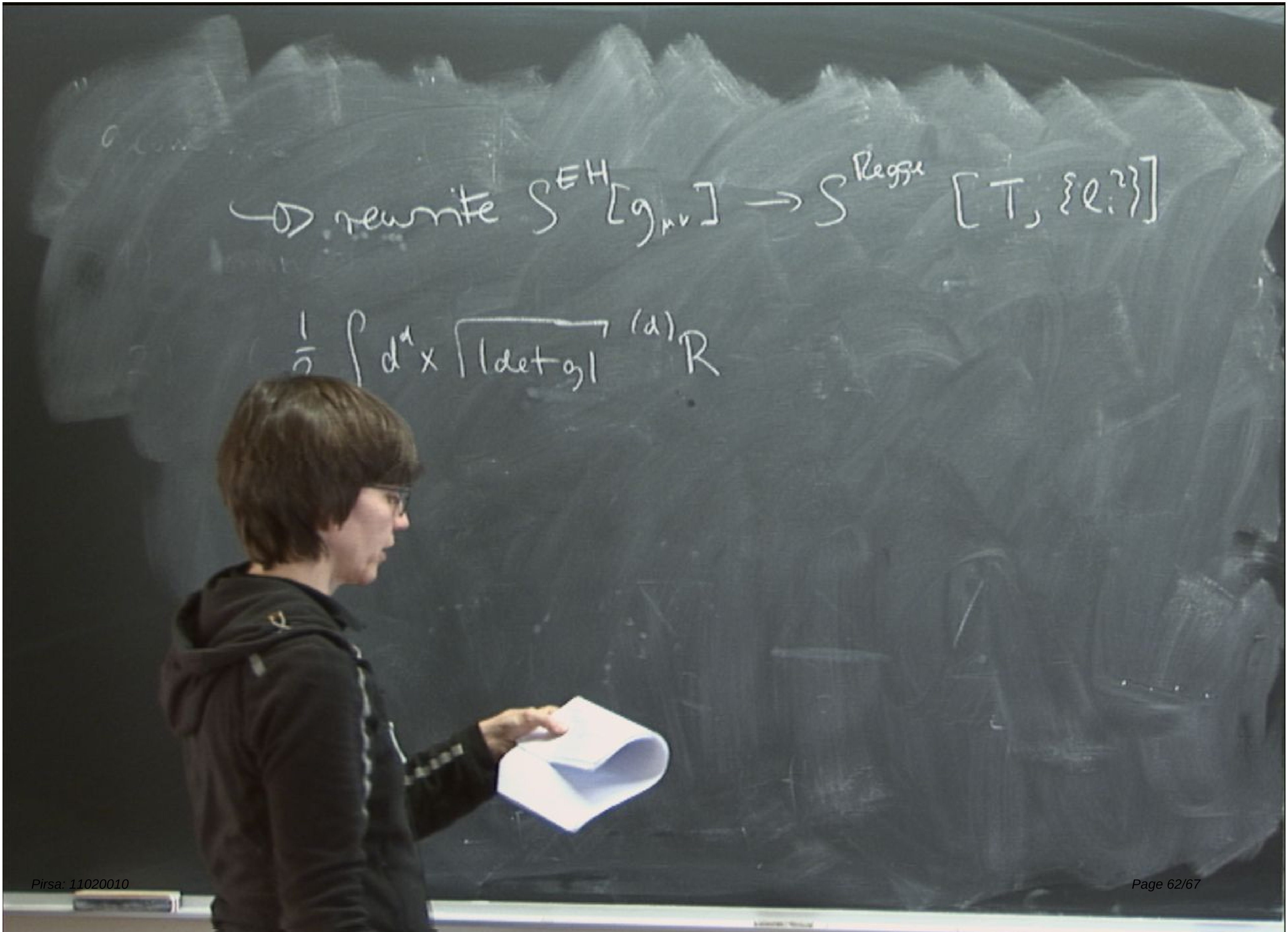
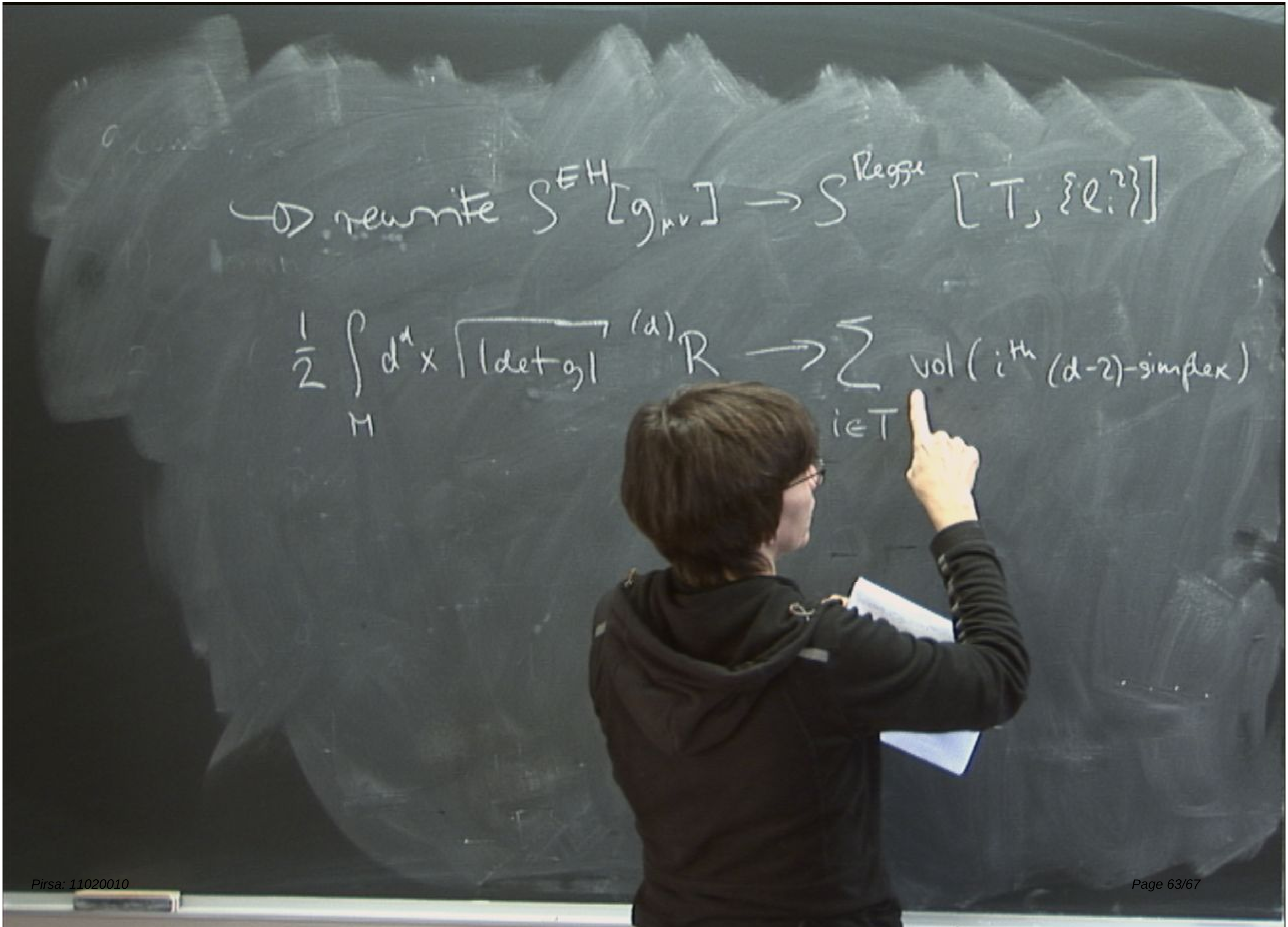

ex. d=2
Σ αᵢ = 2π flat space
Σ αᵢ < 2π positive Gaussian curvat
> 2π negative " "
express δ = δ({lᵢ²})

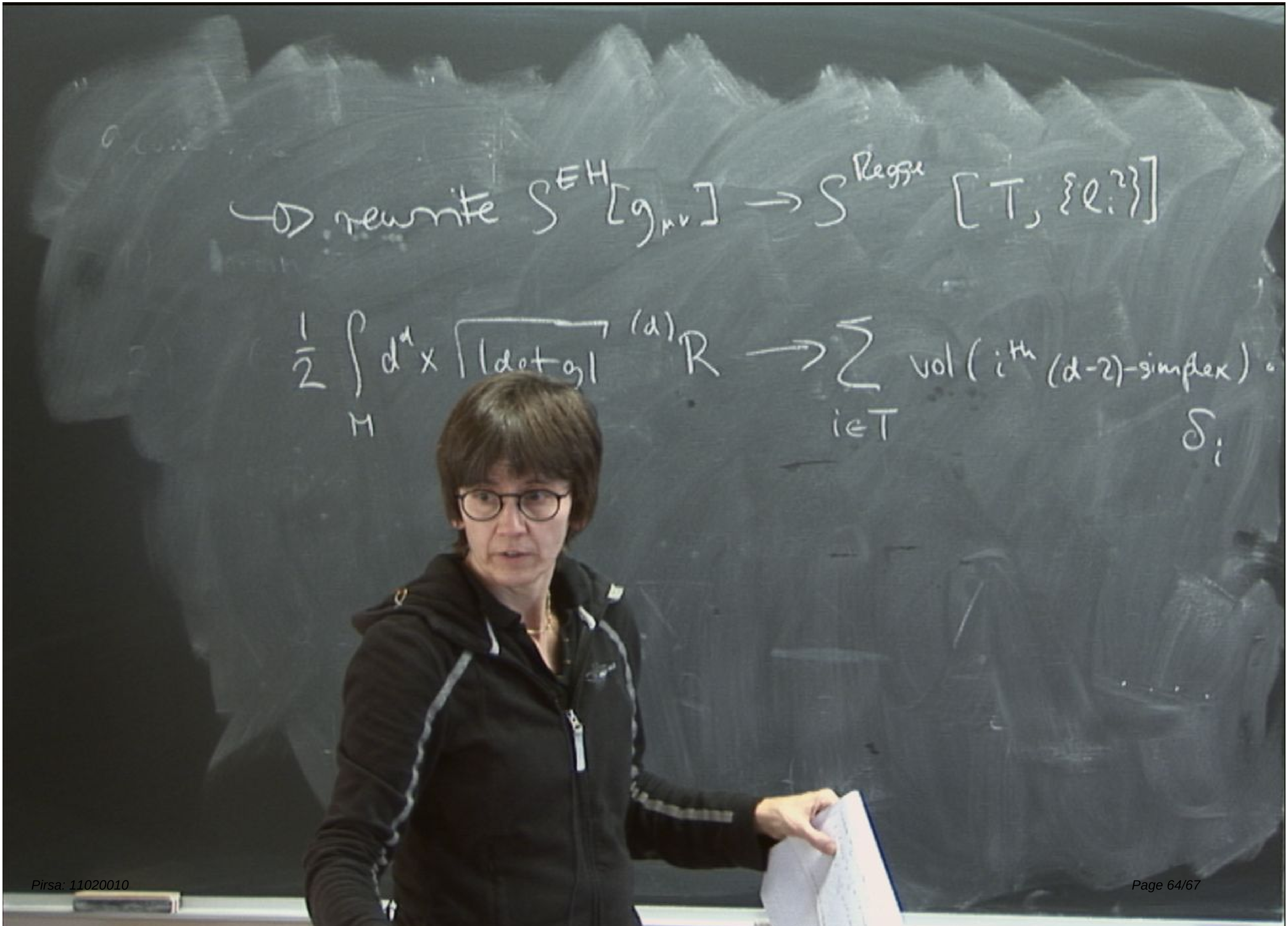
$\rightarrow$ rewrite $S^{EH}[g_{\mu\nu}] \rightarrow S^{Regge}[T, \{l_i^2\}]$

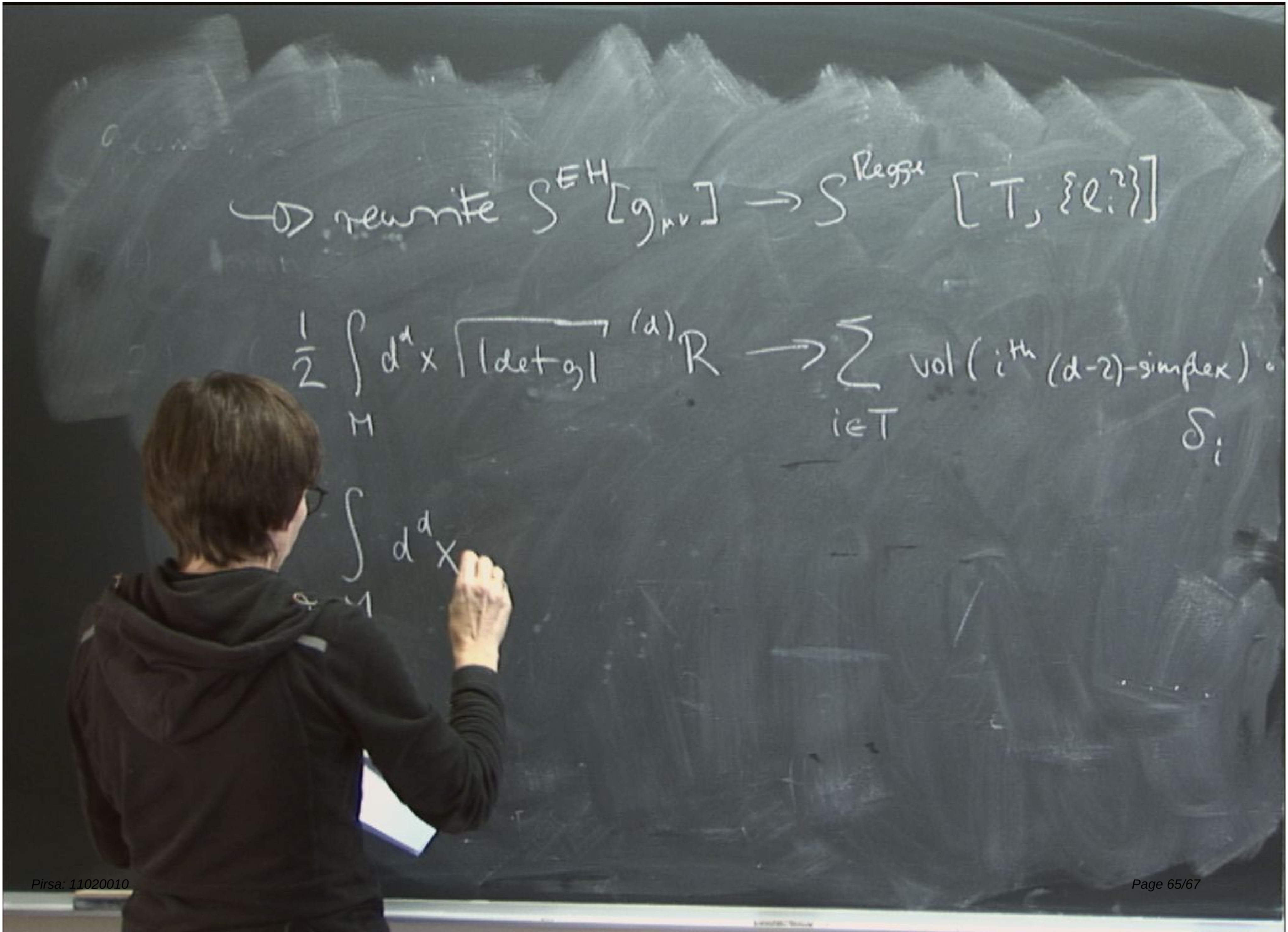
$$\frac{1}{2}\int_M d^d x$$



→ rewrite $S^{EH}[g_{\mu\nu}] \to S^{Regge}[T, \{l_i^2\}]$
$\frac{1}{2} \int d^d x \sqrt{|\det g|}\, {}^{(d)}R$


→ rewrite S^EH [g_μν] → S^Regge [T, {l_i^2}]
1/2 ∫_M d^d x √|det g| ^(d)R → Σ_{i∈T} vol(i^th (d-2)-simplex)


→ rewrite $S^{EH}[g_{\mu\nu}] \to S^{Regge}[T, \{l_i^2\}]$
$\frac{1}{2}\int_M d^dx \sqrt{|\det g|}\, {}^{(d)}R \to \sum_{i\in T}$ vol (i^{th} (d-2)-simplex) · δ_i



$\to$ rewrite $S^{EH}[g_{\mu\nu}] \to S^{Regge}[T, \{l_i^2\}]$
$\frac{1}{2}\int_M d^dx \sqrt{|\det g|}\; {}^{(d)}R \longrightarrow \sum_{i\in T} \mathrm{vol}(i^{th}\ (d-2)\text{-simplex}) \cdot \delta_i$
$\int_M d^dx$

$$\rightarrow \text{rewrite } S^{EH}[g_{\mu\nu}] \longrightarrow S^{\text{Regge}}\left[T, \{\ell_i^2\}\right]$$

$$\frac{1}{2}\int_M d^d x \sqrt{|\det g|}\; {}^{(d)}R \longrightarrow \sum_{i \in T} \text{vol}(i^{th}\ (d-2)\text{-simplex}) \cdot \delta_i$$

$$\int_M d^d x \sqrt{|\det g|} \longrightarrow \sum_{i \in T} \text{vol}(i^{th}\ d\text{-simplex})$$

$\rightarrow$ rewrite $S^{EH}[g_{\mu\nu}] \rightarrow S^{Regge}[T, \{l_i^2\}]$

$$\frac{1}{2}\int_M d^d x \sqrt{|\det g|}\, {}^{(d)}R \longrightarrow \sum_{i \in T} \mathrm{vol}(i^{th}\ (d-2)\text{-simplex}) \cdot \delta_i$$

$$\int_M d^d x \sqrt{|\det g|} \longrightarrow \sum_{i \in T} \mathrm{vol}(i^{th}\ d\text{-simplex})$$