Title: The Shape of Inner Space

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Abstract: Mathematics and physics can come together to the benefit of both fields, particularly in the case of Calabi-Yau spaces and string theory---our leading attempt to explain the universe to date. The audience will gain a sense of how mathematicians think and approach the world and realize that mathematics does not have to be a wholly abstract discipline, disconnected from everyday phenomena, but it is instead crucial to our understanding of the physical world.

Introduction

I'd like to talk about how mathematics and physics can come together to the benefit of both fields, particularly in the case of Calabi-Yau spaces and string theory. This, not coincidentally, is the subject of a new book, THE SHAPE OF INNER SPACE, which I have written with Steve Nadis, a science writer.

Steve Nadis

Book Cover

This book tells the story of those spaces. It also tells some of my own story and a bit of the history of geometry as well. In that spirit, I'm going to back up and talk about my personal introduction to geometry and the evolution of the ideas that are discussed in this book.

I wanted to write this book to give people a sense of how mathematicians think and approach the world. I also want people to realize that mathematics does not have to be a wholly abstract discipline, disconnected from everyday phenomena, but is instead crucial to our understanding of the physical world.

So we're now going to step back in time a bit. Or perhaps I should say step back in spacetime...

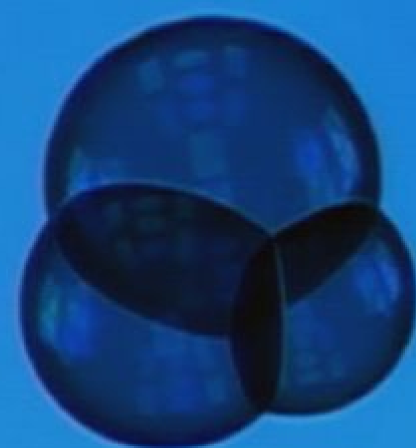
II. Geometry of Curved Space

Curved objects can be approximated by geometric figures that were studied in Euclidean geometry. In a sense, calculus gives us an estimation technique for extending Euclidean geometry through infinite processes.

When I was in college, I had a lot of fun studying the concept of curvature for arbitrary curves and surfaces in three-dimensional spaces. There are many interesting surfaces. One example is a surface defined by a soap film.

In the nineteenth century, Plateau posed the following question: Dip a wire into a pool of soap, take the wire out, and we see a soap film bounded by the wire. This film consists of the smallest possible area bounded by that wire. For any closed curve, is there a surface of minimal area bounded by that curve?

J. Plateau



Soap bubble

III. Riemannian Geometry

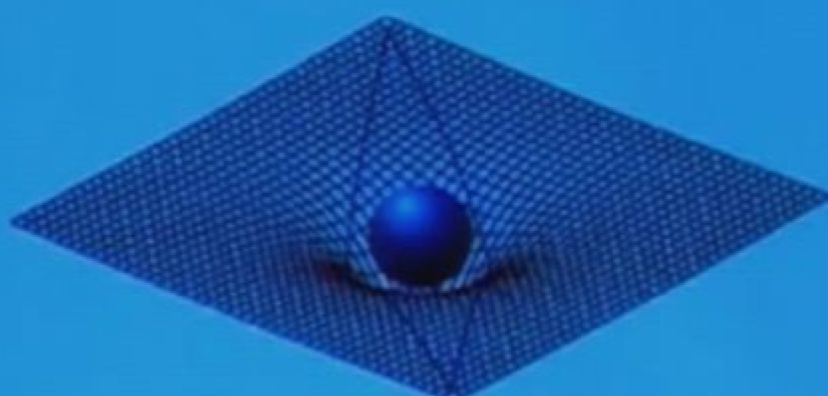
When I arrived in Berkeley in 1969 for graduate study, I learned that the concept of geometry had gone through a radical change in the 19th century, thanks to the contributions of Gauss and Riemann. Riemann revolutionized our notions of space. Objects no longer had to be confined to the flat, linear space of Euclidean geometry.

Gauss

Riemann

Riemann instead proposed a much more abstract conception of space — of any possible dimension — in which we could describe distance and curvature. In fact, one can develop a form of calculus that is especially suited to such an abstract space. It took about fifty years until Einstein realized that this kind of geometry, which involved curved spaces, was exactly what he needed to unify Newtonian gravity with special relativity. This insight culminated in his famous theory of general relativity.

Einstein

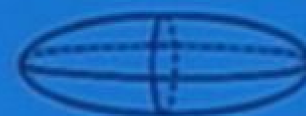


Curved Space-time

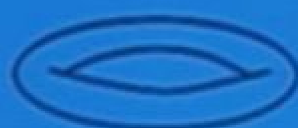
I learned about Riemannian geometry during my first year at Berkeley. After a couple of months, I started to toy around with some statements that related the curvature of a space—its exact shape or geometry—to a much cruder, more general way of characterizing shape, which we call topology.

At Berkeley 1969

For example:

1. The sphere  and the ellipsoid 

   have the same topology, but they have a different shape (geometry).

2. The thin donut  has the same topology as

    , but they have a different shape (geometry).

Einstein struggled in his attempt to obtain a fundamental description of gravity. But he got some help from his friend Grossman, a mathematician, who told him of the work of other mathematicians, Riemann and Ricci.

Riemann provided the framework of abstract space, as well as the means for defining distance and curvature in such a space. Riemann thus provided the background space or setting in which gravity, as Einstein formulated it, plays out.

But Einstein also drew on the work of Ricci, who defined a special kind of curvature that could be used to describe the distribution of matter in spacetime. Through general relativity, Einstein offered a geometric picture of gravity. Rather than considering gravity as an attractive force between massive objects, it could instead be thought of as the consequence of the curvature of spacetime due to the presence of massive objects. The precise way in which spacetime is curved tells us how matter is distributed.

When I looked at the equations of Einstein, I was intrigued by the fact that matter only controls part of the curvature of spacetime. I wondered whether we could construct a spacetime that is a vacuum, and thus has no matter, yet its curvature is still pronounced. Well, the famous Schwarzschild solution to Einstein's equations is such an example. This solution applies to a non-spinning black hole—a vacuum that, curiously, has mass owing to its extreme gravity. But that solution admits a singular point, or singularity—a place where the laws of physics break down.

Black hole

I became interested in a different situation — a smooth space without a singularity that was compact and closed, unlike the open, extended space of the Schwarzschild solution. The question was: Could there be a compact space that contained no matter — a closed vacuum universe, in other words — whose force of gravity was nontrivial? I was obsessed with this question and believed that such a space could not exist. If I could prove that, I was sure that it would be an elegant theorem in geometry.

For about three years, my friends and I tried to prove that the class of spaces proposed by Calabi could not exist. We, along with many others, considered them to be "too good to be true." But try as we might, we could not prove that such spaces do not exist.

With Prof. Calabi, 2004

Until one day I thought I found a way to demonstrate that Calabi was wrong. I made this discovery at a big conference at Stanford, and I was asked to give a talk about it. However, a few months later, while trying to write up my proof in a rigorous fashion, I found that I could not complete my argument.

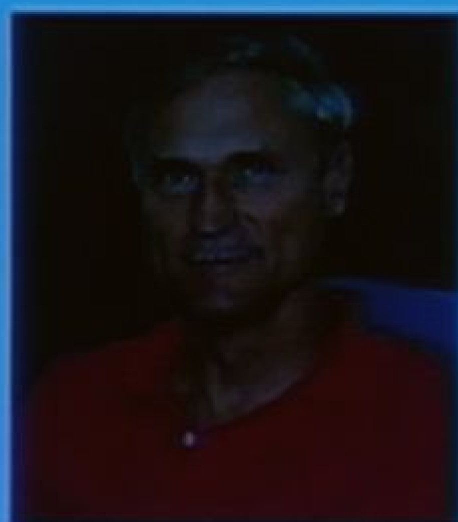
I finally decided that the Calabi conjecture must be right after all. It took me three more years to complete this proof, which provided confirmation of the Calabi conjecture. And it was finished right after I got married. I found a general mechanism to construct such spaces, which are now called Calabi-Yau spaces.

Calabi-Yau space

I had a strong sense that I had hit upon a great and beautiful piece of mathematics. And as such, I felt it must be relevant to physics and to our deepest understanding of nature. However, I did not know exactly where these ideas might fit in, as I didn't know much physics at the time.

Which isn't to say that I knew nothing about physics. For example, I had been interested in general relativity for a while. In 1973, I was exposed to a problem in general relativity called the positive mass conjecture, which the physicist Robert Geroch discussed at the same conference in Stanford.

I started working on this problem with my friend (and former student) Richard Schoen. Expressed in simple terms, the conjecture says that the mass or energy of our universe—or any other isolated physical system—must be positive. Our proof made use of the Plateau problem that I mentioned earlier in this talk. This work, moreover, brought me closer to my colleagues in physics.



Schoen

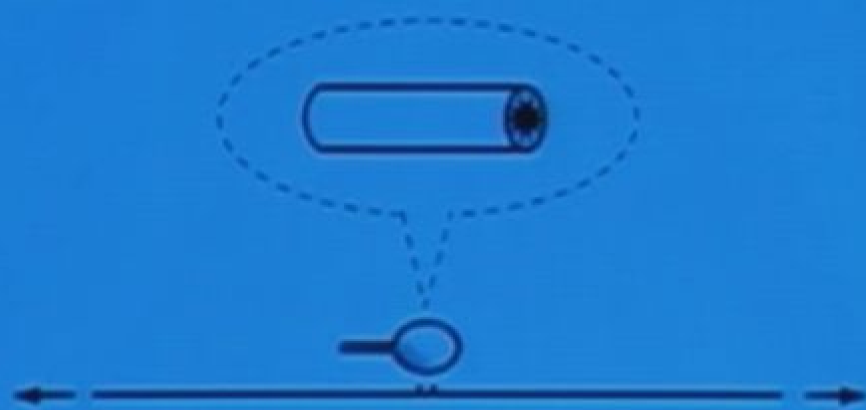
I ran a special year of geometry seminars at the Institute for Advanced Study in Princeton in 1979, where quite a few physicists participated. Subsequently I moved to the Institute for Advanced Study in Princeton as a faculty member. There were many young postdoctoral fellows at the institute. In 1981, I decided to offer Gary Horowitz a postdoctoral fellowship with the intention of studying classical relativity with him.

Gary Horowitz

Andy Strominger

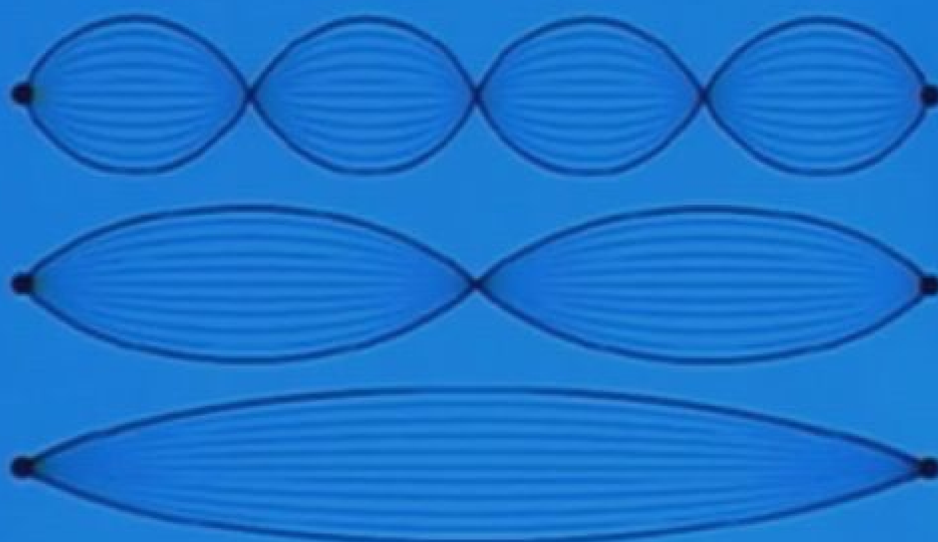
VI. String Theory

A couple of years later, when I visited my wife in San Diego, I got several phone calls. Horowitz and his colleague Andy Strominger said that they were very excited about a model for describing the vacuum state of the universe, based on a new theory called string theory.



Kaluza-Klein garden hose

String theory is built on the assumption that particles, at their most basic level, are made of vibrating bits of tiny strings. In order for the theory to be consistent with quantum theory, spacetime has a certain symmetry built into it called supersymmetry. Spacetime is also assumed to be ten dimensional.



Vibrating strings

Horowitz and Strominger were interested in the multidimensional spaces whose existence I proved, mathematically, in my confirmation of the Calabi conjecture. They believed that these spaces could play an important role in string theory, as they seemed to be endowed with the right kind of supersymmetry — a property deemed essential to their theory. They asked me if their assessment of the situation was correct and, to their delight, I told them that it was.

Then I got a phone call from Ed Witten whom I'd met in Princeton the year before. Witten told me that this was the one of the most exciting eras in theoretical physics. It was just like the time when quantum mechanics was being developed.

Witten

He told me that everyone who made contributions to quantum mechanics in early days left their name in the history of physics. He said that the important discoveries of early string theorists, such as Michael Green and John Schwarz, could lead to the grand unification of all forces—the goal that Einstein had spent the last 30 years of his life working toward.

Witten was now collaborating with Candelas, Horowitz, and Strominger, trying to figure out the shape, or geometry, of the six "extra" dimensions of string theory. The physicists believed these six dimensions were curled up in a tiny space, which they called Calabi-Yau space—the same family of spaces originally proposed by Calabi and later proved by me.

With Candelas, 2001

The existence of this extra-dimensional space is fantastic on its own, but string theory goes much farther. It says that the exact shape, or geometry, of Calabi-Yau space dictates the properties of our universe and the kind of physics we see. The shape of Calabi-Yau space—or the "shape of inner space," as we put it in our book—determines the kinds of particles that exist, their masses, the ways in which they interact, and maybe even the constants of nature.

While Einstein had said the phenomenon of gravity is really a manifestation of geometry, string theorists boldly proclaimed that the physics of our universe is a consequence of the geometry of Calabi-Yau space. That's why string theorists were so anxious to figure out the precise shape of this six—dimensional space—a problem we're still working on today.

Witten was eager to learn more about Calabi-Yau spaces. He flew from Princeton to San Diego to talk with me about how to construct them. He also wanted to know how many Calabi-Yau spaces there were for physicists to choose among. Initially, physicists thought there might only be a few examples—a few basic topologies—which made the goal of determining the shape that corresponds to our universe seem a lot more manageable. But we soon realized there were many more examples of Calabi-Yau spaces-many more possible topologies—than were originally anticipated.

The task of figuring out the shape of inner space suddenly seemed more daunting, and perhaps even hopeless if the number of possibilities turned out to be infinite. The latter question has yet to be settled, although I have always thought that the number of Calabi-Yau's is finite. That number is certain to be big, but I believe it is bounded.

The excitement over Calabi-Yau space started in 1984, when physicists first found out about them. That enthusiasm kept up for a couple years, before waning. But the excitement picked up again in the late 1980s, when Brian Greene, Ronen Plesser, Philip Candelas, and others began exploring the notion of "mirror symmetry."

Greene

Plesser

The basic idea here was that two different Calabi-Yau spaces, which had different topologies and seemed to have nothing in common, nevertheless gave rise to the same physics. This established a previously unknown kinship between so-called mirror pairs of Calabi-Yau's.

The connection uncovered through physics proved to be extremely powerful in the hands of mathematicians. When they were stumped trying to solve a problem involving one Calabi-Yau space, they could try solving the same problem on its mirror pair. On many occasions, this approach was successful. As a result, problems that had defied resolution, sometimes for as long as a century, were now being solved. And a branch of mathematics called enumerative geometry was suddenly rejuvenated. These advances gave mathematicians greater respect for physicists, as well as greater respect for string theory itself.

VII. Conclusion

Before we get too carried away, we should bear in mind that string theory, as the name suggests, is just a theory. It has not been confirmed by physical experiments, nor have any experiments yet been designed that could put that theory to a definitive test. So the jury is still out on the question of whether string theory actually describes nature, which was the original intent.

On the positive side of the ledger, some extremely intriguing, as well as powerful, mathematics has been inspired by string theory. Mathematical formulae developed through this connection have proved to be correct independent of the scientific validity of string theory. So far it stands as the only consistent theory that unifies the different forces. And it is beautiful. Moreover, the effort to unify the different forces of nature has unexpectedly led to the unification of different areas mathematics that at one time seemed unrelated.

We still don't know what the final word will be. In the past two thousand years, the concept of geometry has evolved over several important stages to the current state of modern geometry. Each time geometry has been transformed in a major way, the new version has incorporated our improved understanding of nature arrived at through advances in theoretical physics. It seems likely that we shall witness another major development in the 21st century, the advent of quantum geometry — a geometry that can incorporate quantum physics in the small and general relativity in the large.

The fact that abstract mathematics can reveal so much about nature is something I find both mysterious and fascinating. This is one of the ideas that my coauthor and I have tried to get across in our book, The Shape of Inner Space. We also hope that the book gives you a description of how mathematicians work. They are not necessarily weird people, such as a janitor who solves centuries-old math problems on the side while mopping and dusting floors, as described in the movie "Good Will Hunting". Nor does a brilliant mathematician have to be mentally ill, or exhibit otherwise bizarre behavior, as depicted in another popular movie and book.

Mathematicians are just scientists who look at nature from a different, more abstract point of view than the empiricists. But the work mathematicians do is still based on the truth and beauty of nature, the same as it is in physics. Our book tries to convey the thrill of working at the interface between mathematics and physics, showing how important ideas flow through different disciplines, with the result being the birth of new and important subjects.

In the case of string theory, geometry and physics have come together to produce some beautiful mathematics, as well as some very intriguing physics. The mathematics is so beautiful, in fact, and it has branched out into so many different areas, that it makes you wonder whether the physicists might be onto something after all.

The story is still unfolding, to be sure, and I consider myself lucky to have been part of it.