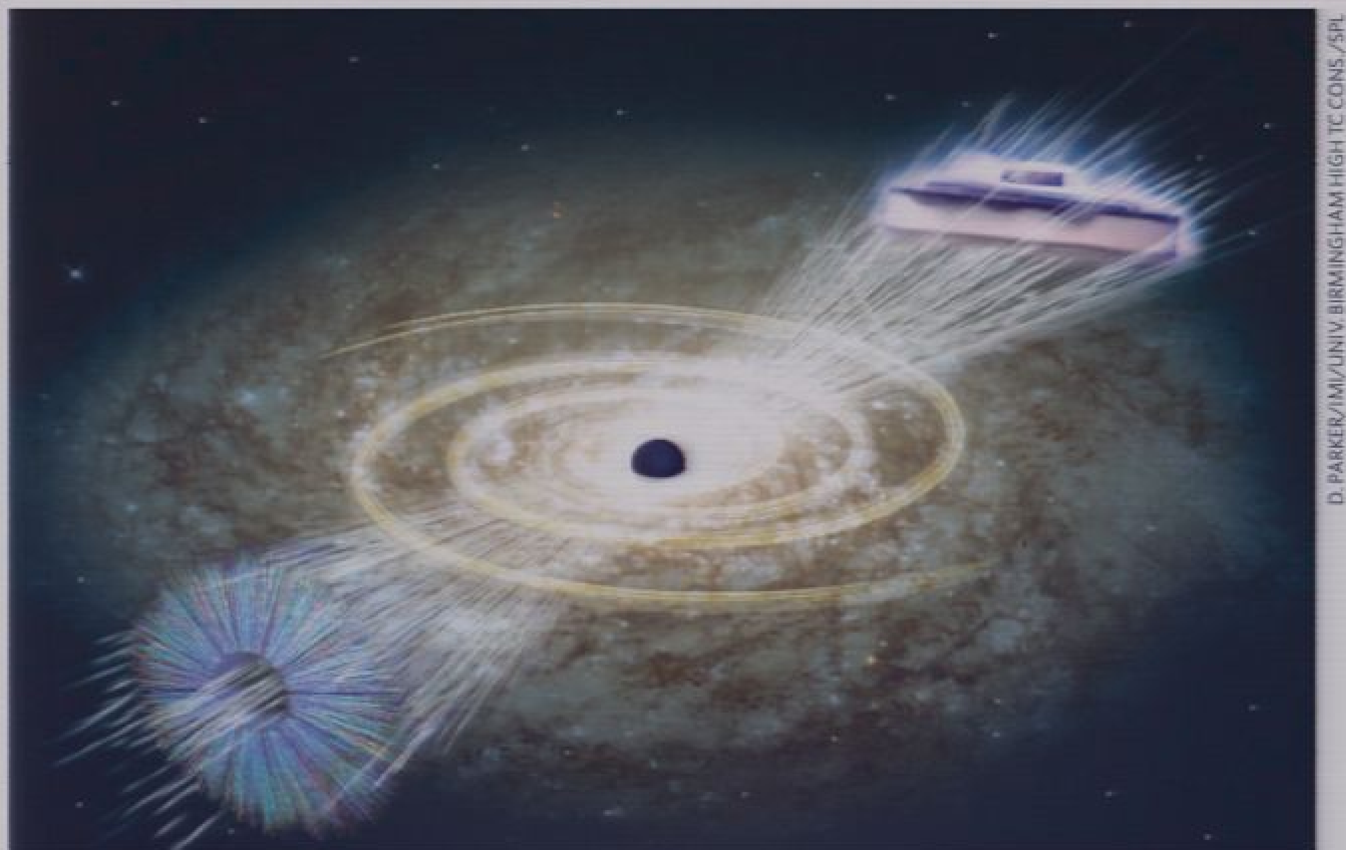
Title: How to get a superconductor from a black hole

Date: Jan 26, 2011 09:30 AM

URL: http://pirsa.org/11010112

Abstract: Gauge/gravity duality, a concept which emerged from string theory, holds promise for revealing the secrets of certain strongly interacting real world condensed matter systems. Historically, string theorists presented their subject as a promising framework for a quantum theory of gravity. More recently, the AdS/CFT correspondence and gauge/gravity dualities have emerged as powerful tools for using what we already know about gravity to investigate the properties of strongly interacting field theories. I will cherry pick and discuss a few recent developments where black holes are used to calculate the thermodynamic and transport properties of quantum critical systems, superconductors, and superfluids.



How to get a superconductor out of a black hole

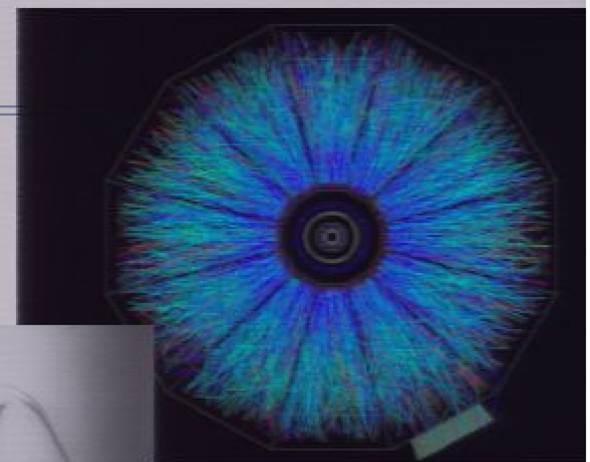
Christopher P. Herzog (Princeton)

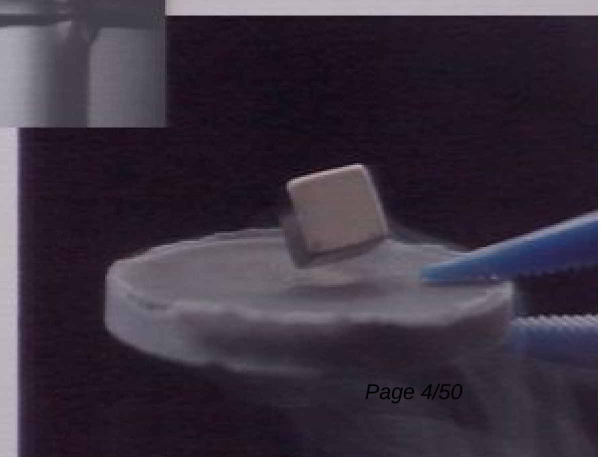
January 2011

Fundamental Theory or Tool?

- Historically, string theory was held out as a way of unifying quantum mechanics and gravity, providing a reductionist description of the universe in terms of tiny propagating one dimensional degrees of freedom called strings.
- An outgrowth of string theory - gauge/gravity duality (AdS/CFT correspondence) - can be used as a tool for studying strongly interacting systems.

Interesting Strongly Interacting Systems





- The quark-gluon plasma formed at the Relativistic Heavy Ion Collider.
- Superfluids such as helium-3 and helium-4.
- High Tc superconductivity: It is not at all clear if high Tc superconductors are the right system for an AdS/CFT approach, but they are, perhaps, the most interesting.

Traditional Methods

- Perturbation Theory: Start with a non-interacting system of free particles. Perturb by adding a small interaction.
- Numerical Simulation: Break the system up into lots of little boxes -- the lattice. Provide rules that govern the interaction between the boxes and evolution in time. Let the computer do the rest.
- Toy Models: Find a system that shares some properties with the one you are interested in but is simple enough to solve exactly.

The Issues

- Perturbation Theory: We're interested in strongly interacting systems!
- Numerical Simulation: Because of something called the "sign problem", it is not yet feasible to encode time evolution in a quantum mechanical setting.
- Toy Models: Right answer for the wrong system. These AdS/CFT constructions are sophisticated toy models. There is hope some of them may be more...

Superconductors: What and Why

- Perfect conductors: Conduct electricity without resistance.
- Meissner effect: They expel magnetic fields.
- Applications: electrical power distribution, MRI machines, particle accelerators, supercomputers, rf filters for cell phones

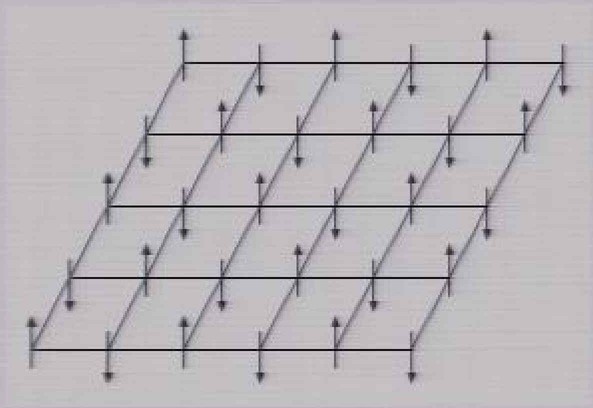


High Tc Superconductors

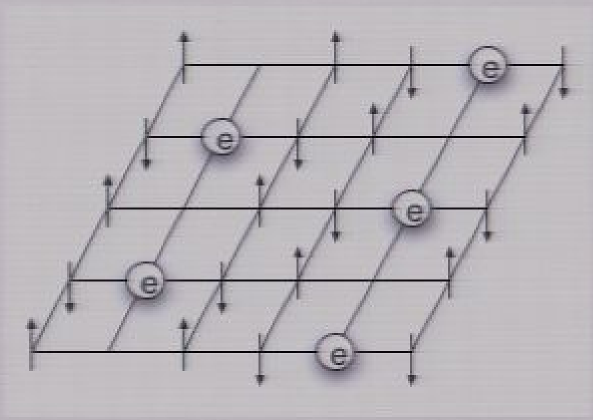
- Tc is the temperature below which the material superconducts.
- Traditional superconductors superconduct only at very low temperatures (< 20 K). Bardeen, Cooper, and Schrieffer (1957) provided a very good theoretical framework for these materials. Weakly interacting.
- Bednorz and Müller (1986) discovered the first high Tc superconductor (Tc = 30 K). The current record is 138 K. These ceramic materials are poorly understood. While it is difficult to make a wire out of ceramic, high Tc is clearly advantageous for applications

A High Tc Superconductor

La_2CuO_4



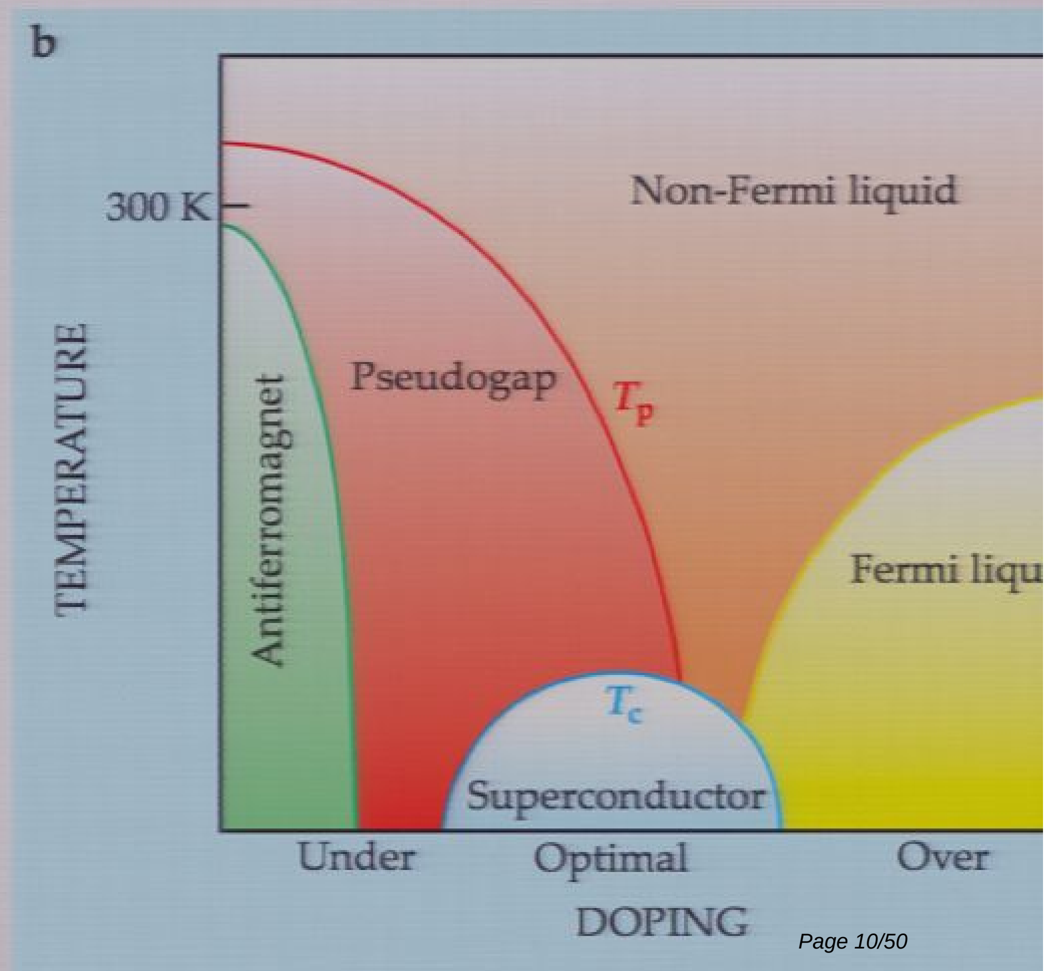
- 2d physics: The Cu atoms arrange themselves into a square lattice on separated sheets. For each unit cell, we believe there is only one electron we need to worry about.



- The material is an example of an antiferromagnetic insulator.
- Hole doping: substitute some of the La with Sr, $La_{2-x}Sr_xCuO_4$

A Phase Diagram for LSCO

- With enough added Sr, La_2CuO_4 becomes superconducting.
- Tc gets as high as 40 K.
- The over doped region is weakly interacting.
- The under doped region is strongly interacting.

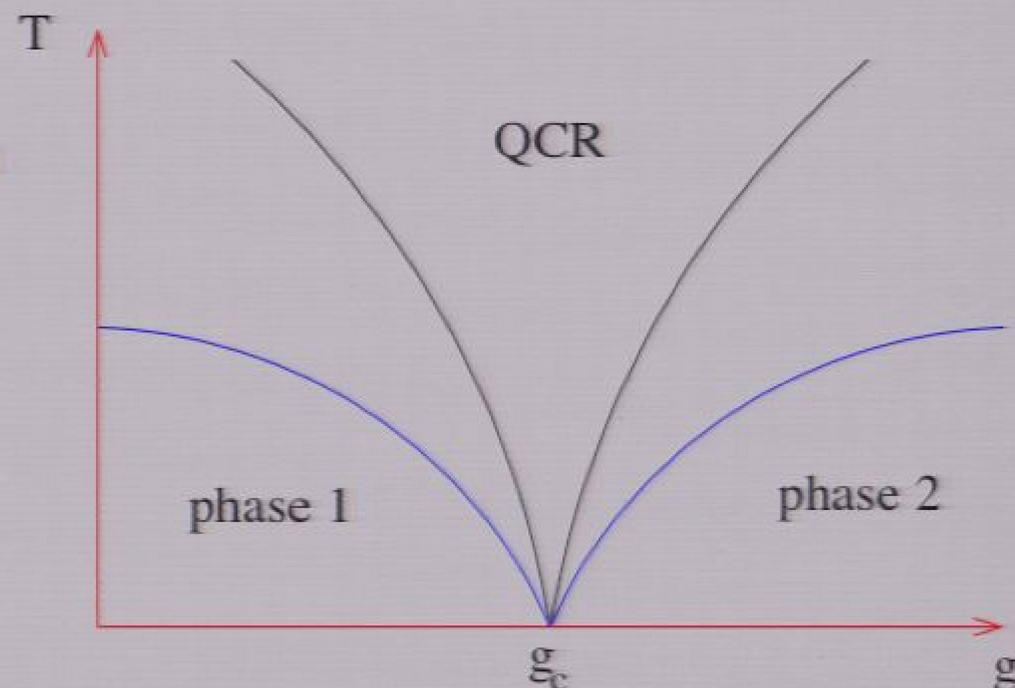


Quantum Phase Transition

- Phase transition at T=0.
- Driven by quantum rather than thermal fluctuations
- Emergent scale invariance:

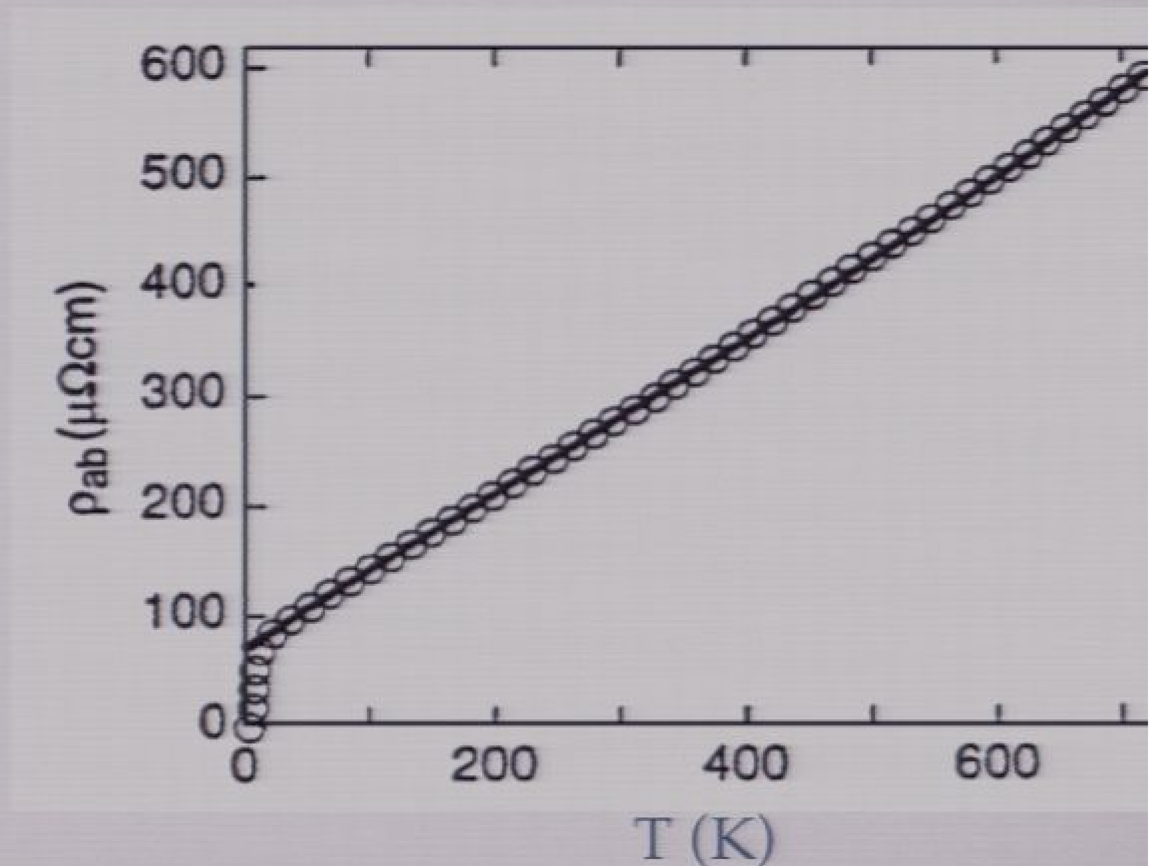
$$x \to \lambda x \text{ and } t \to \lambda^z t$$

- Continuum field theory can replace the lattice.



Linear Resistivity

- High Tc superconductors exhibit a linear rise in the resistivity in strange metal regime for T>Tc that has eluded explanation.
- A Fermi liquid, i.e. a weakly interacting system of electrons, would not yield this behavior
- On the right is a plot for the compound Bi-2201.



Martin, Fiory, Fleming, Schneemeyer, Waczczak, Phys. Rev. B 41 (1990) 846

The Nernst Effect

Apply ∇T

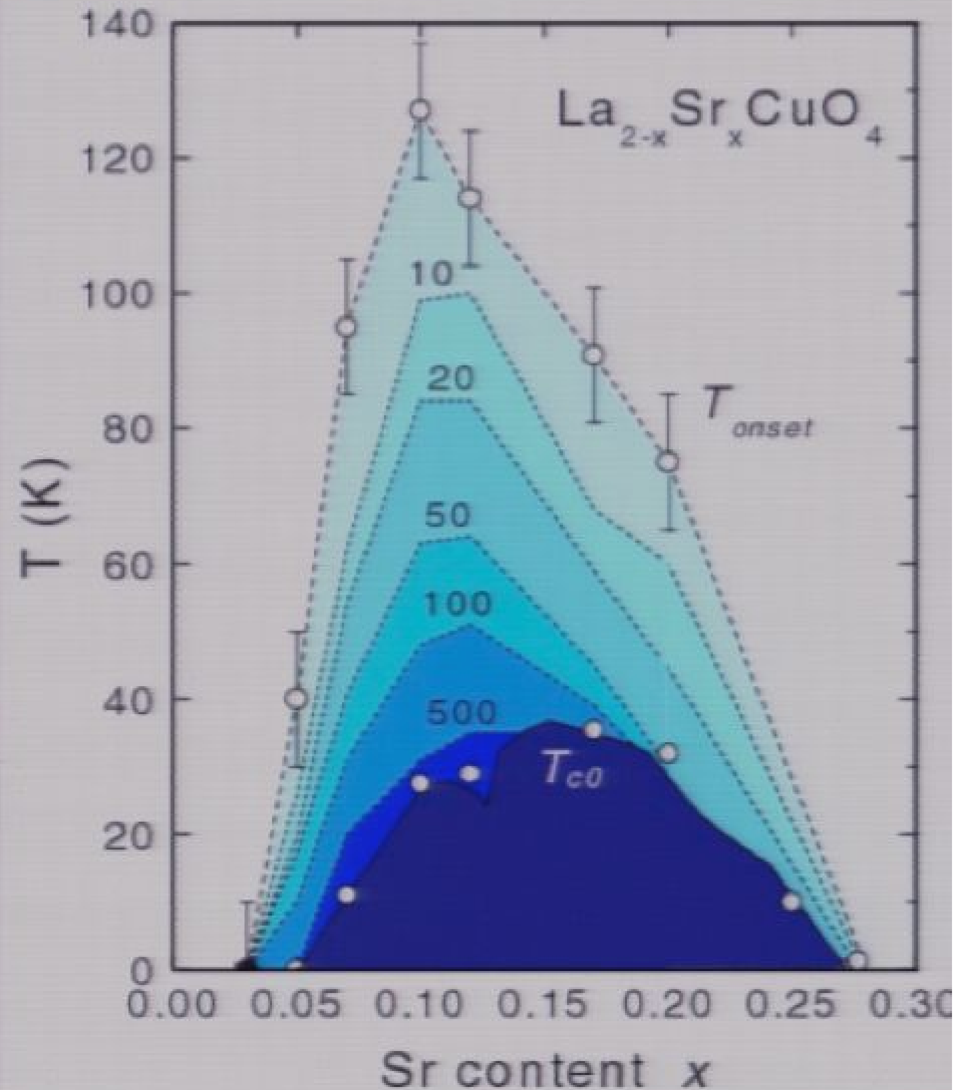
Apply $B \perp \nabla T$

Measure $E \parallel B \times \nabla T$

The Nernst Coefficient is $\nu = \dfrac{E}{B|\nabla T|}$

Nernst effect from e′s typically small (Sondheimer cancellation).

Scrambling degrees of freedom.



Wang, Li, Ong, Phys. Rev. B 73 (2006) 024510

AdS/CFT: An Answer in Search of a Question

The Nernst Effect

Apply ∇T

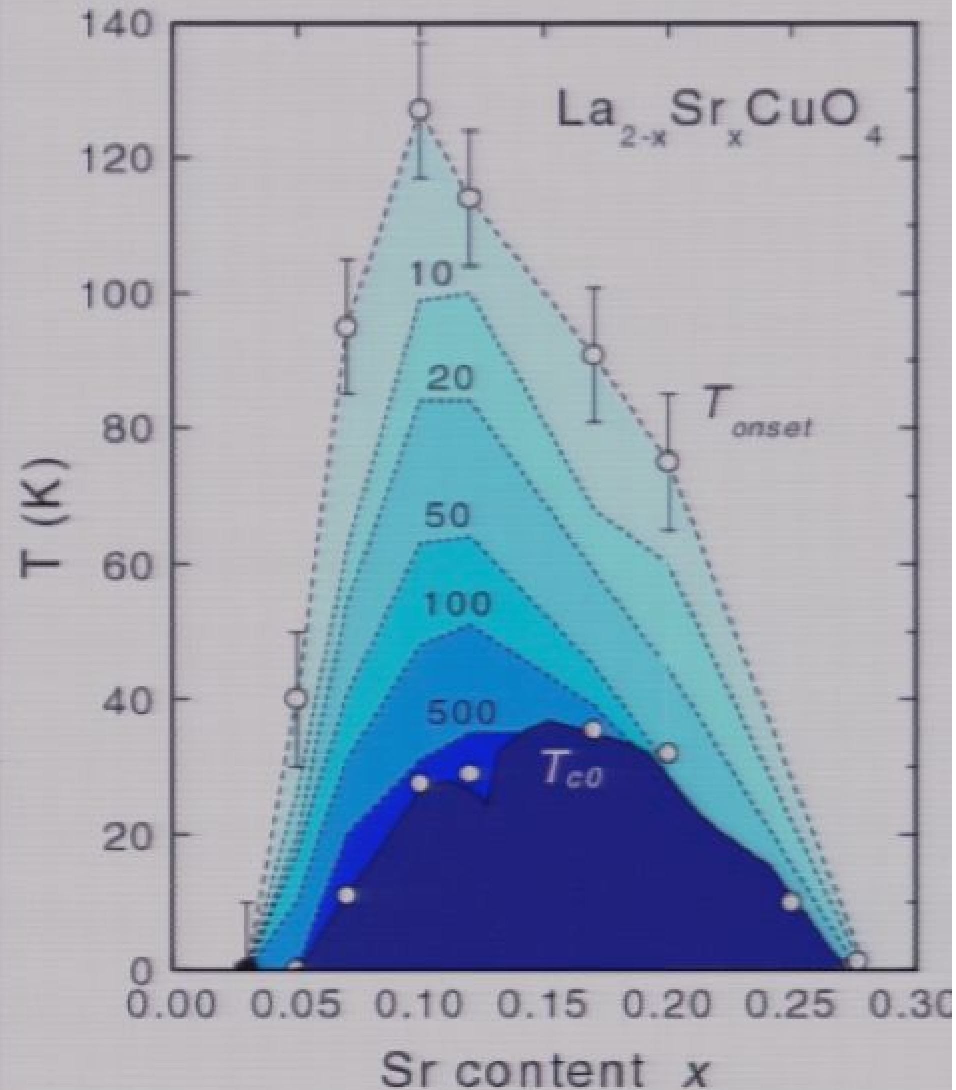
Apply $B \perp \nabla T$

Measure $E \parallel B \times \nabla T$

The Nernst Coefficient is $\nu = \dfrac{E}{B|\nabla T|}$

Nernst effect from e′s typically small (Sondheimer cancellation).

Scrambling degrees of freedom.



Wang, Li, Ong, Phys. Rev. B 73 (2006) 024510

AdS/CFT: An Answer in Search of a Question

AdS/CFT and the QCR

- AdS/CFT stands for anti-de Sitter space/conformal field theory correspondence. CFT's are scale invariant.
- The strongly interacting field theory is mapped to a classical gravity model in AdS in one dimension more -- holography.
- AdS/CFT optimized for the quantum critical point (QCP) and the quantum critical region (QCR).
- Maybe these AdS/CFT models can say something about the strange metal regime of a high Tc superconductor.

A Black Hole Full of Answers

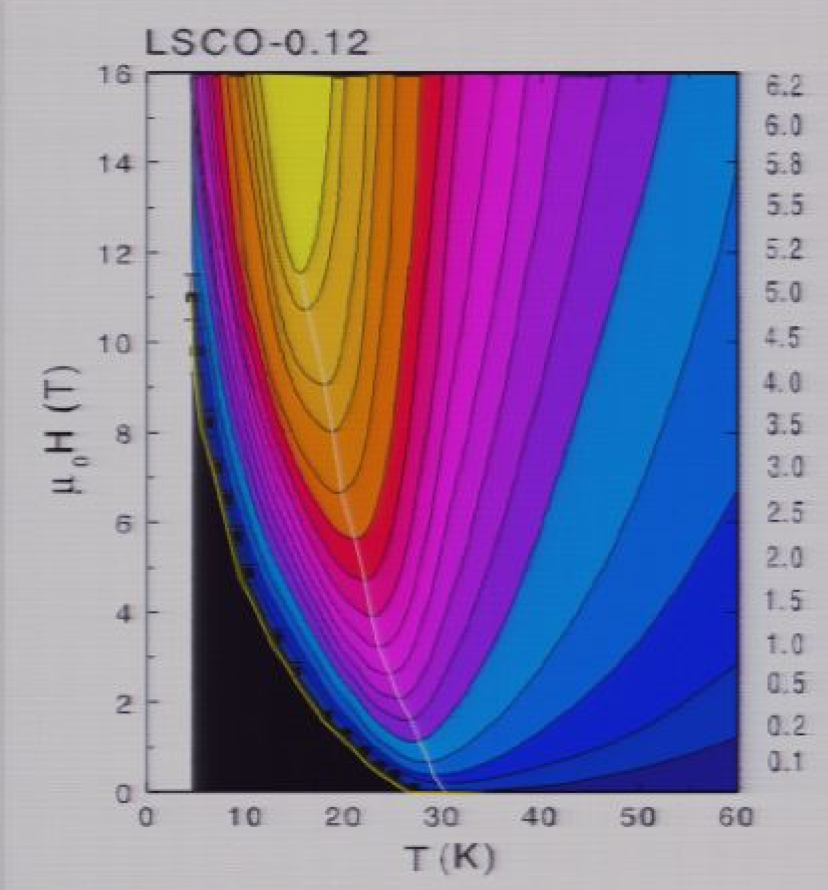
$$S = \frac{1}{2\kappa^2}\int d^{d+1}x\,\sqrt{-g}(R-2\Lambda) - \frac{1}{4g^2}\int d^{d+1}x\,\sqrt{-g}F_{\mu\nu}F^{\mu\nu}$$

Through AdS/CFT, a classical gravity action of this type allows us to calculate transport properties in the dual field theory (i.e. strongly interacting system).

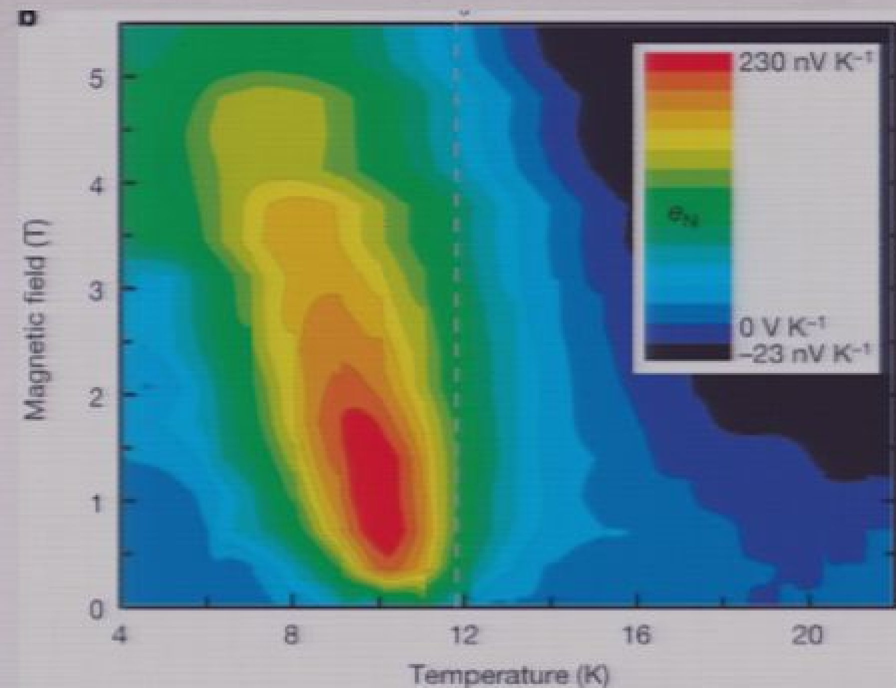
A electrically and magnetically charged black hole is a solution to the equations of motion of this action.

- The temperature of the black hole is the temperature of the field theory.
- The magnetic field of the black hole is the magnetic field of the field theory.
- The electric charge of the black hole is the charge density of the field theory.

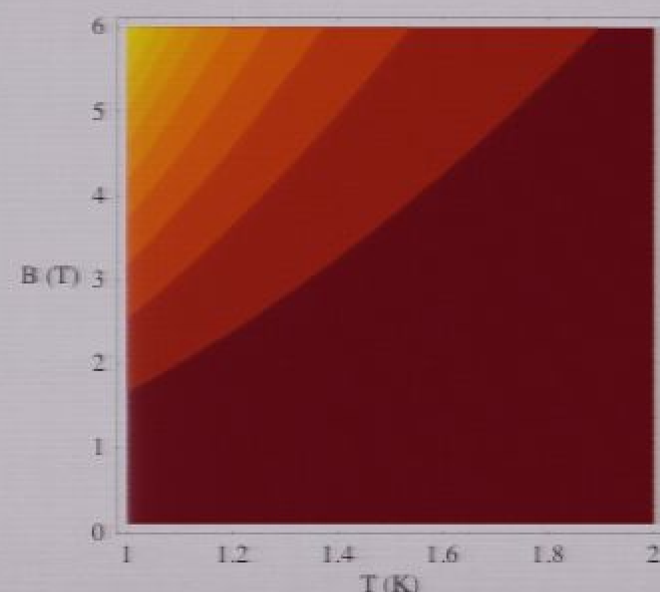
Answers in Search of Questions: Nernst Effect



Ong et al.



Ardavan et al.



Hartnoll and H (2008)

A Black Hole Full of Answers

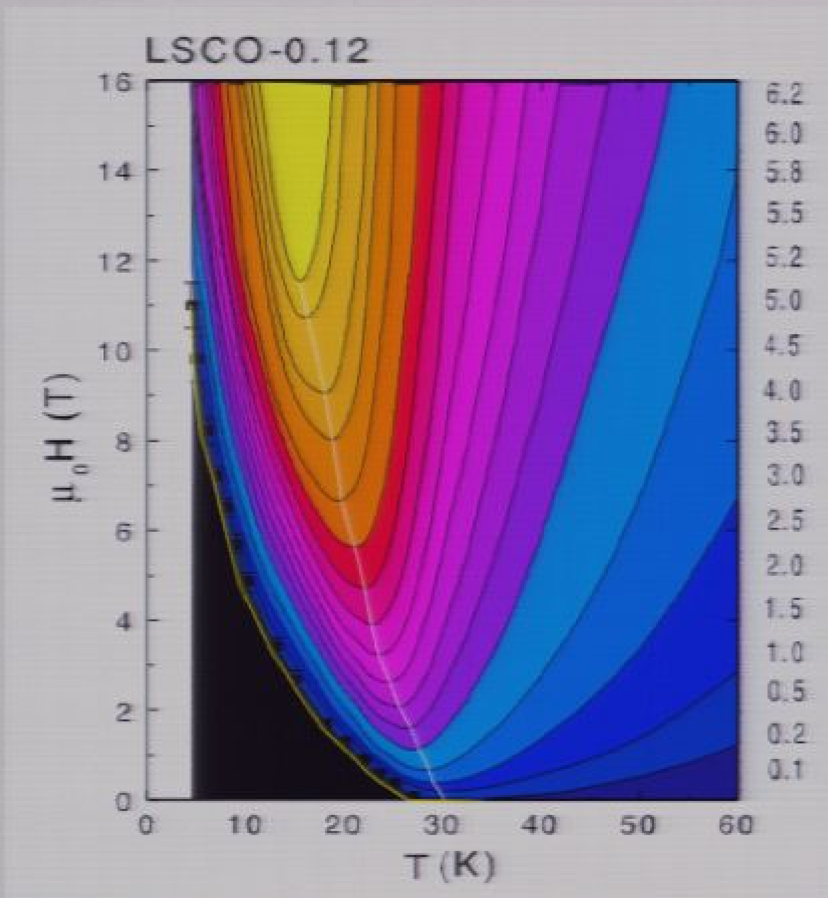
$$S = \frac{1}{2\kappa^2}\int d^{d+1}x\,\sqrt{-g}(R-2\Lambda) - \frac{1}{4g^2}\int d^{d+1}x\,\sqrt{-g}F_{\mu\nu}F^{\mu\nu}$$

Through AdS/CFT, a classical gravity action of this type allows us to calculate transport properties in the dual field theory (i.e. strongly interacting system).

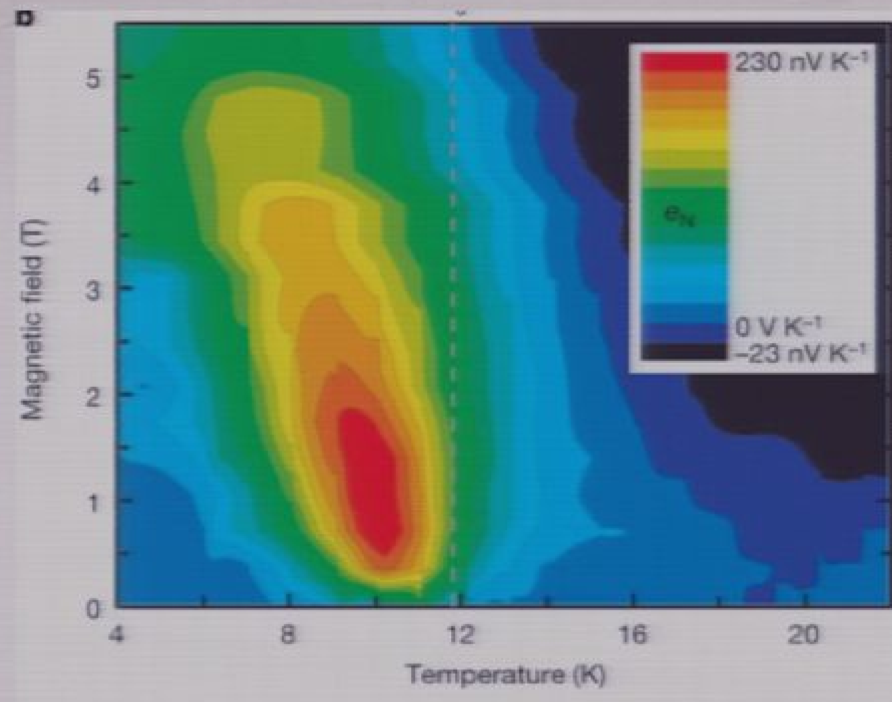
A electrically and magnetically charged black hole is a solution to the equations of motion of this action.

- The temperature of the black hole is the temperature of the field theory.
- The magnetic field of the black hole is the magnetic field of the field theory.
- The electric charge of the black hole is the charge density of the field theory.

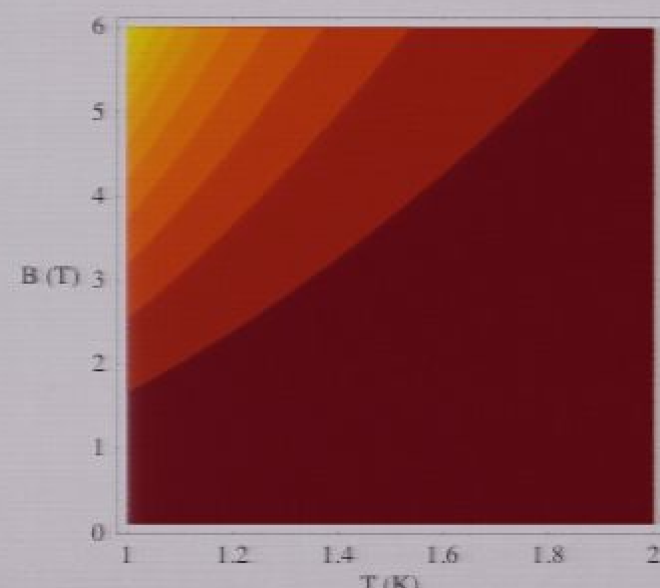
Answers in Search of Questions: Nernst Effect



Ong et al.



Ardavan et al.



Hartnoll and H (2008)

How to get a superconductor out of a black hole...

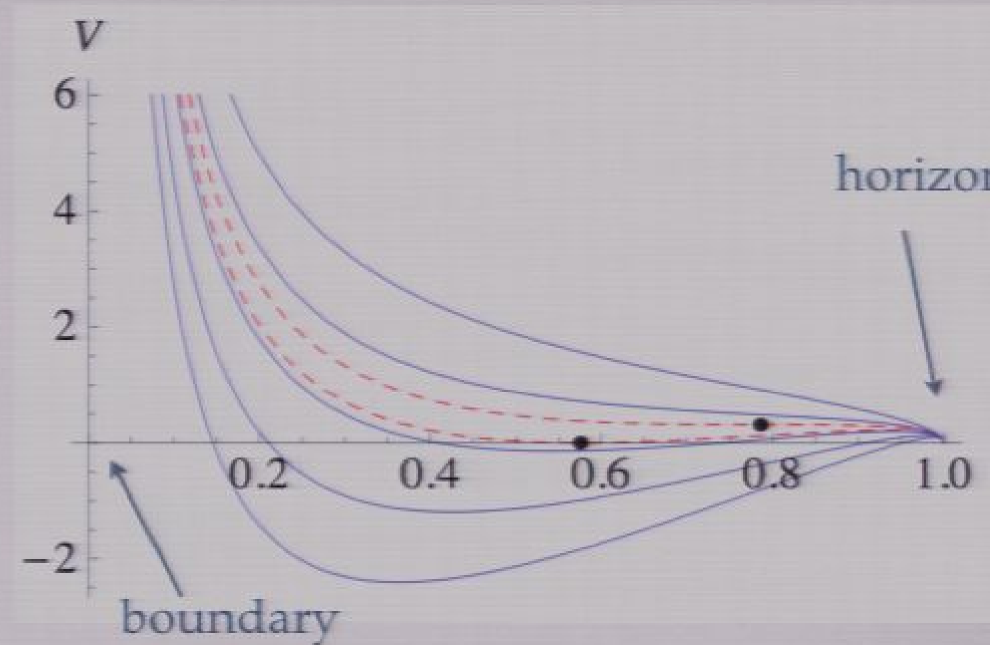
Does the Nernst effect story fit into a phase diagram that also contains a superconducting region?

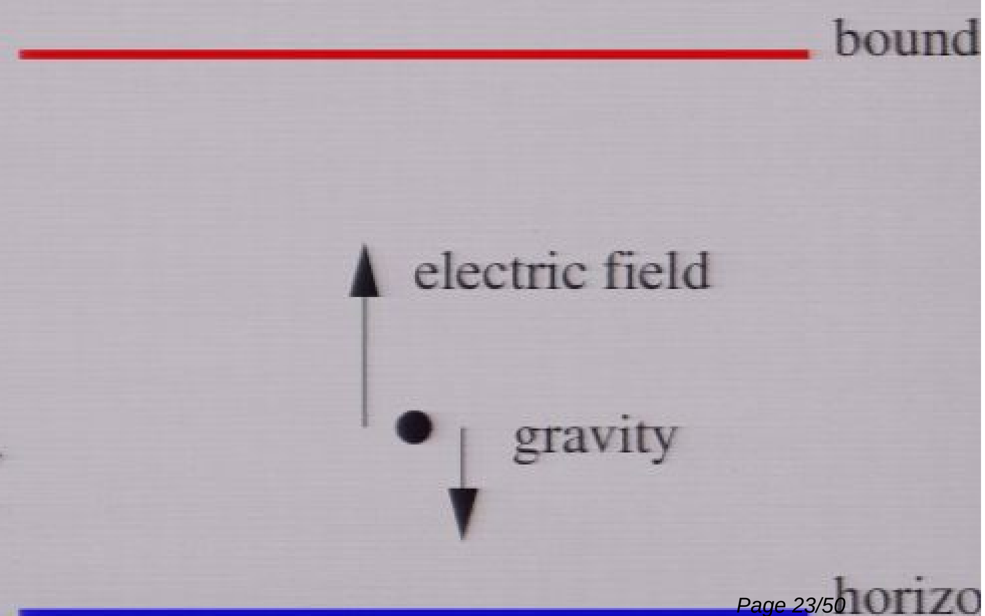
Holographic superconductors

- Add a charged scalar: gravitational attraction toward and electrostatic repulsion from the black hole.

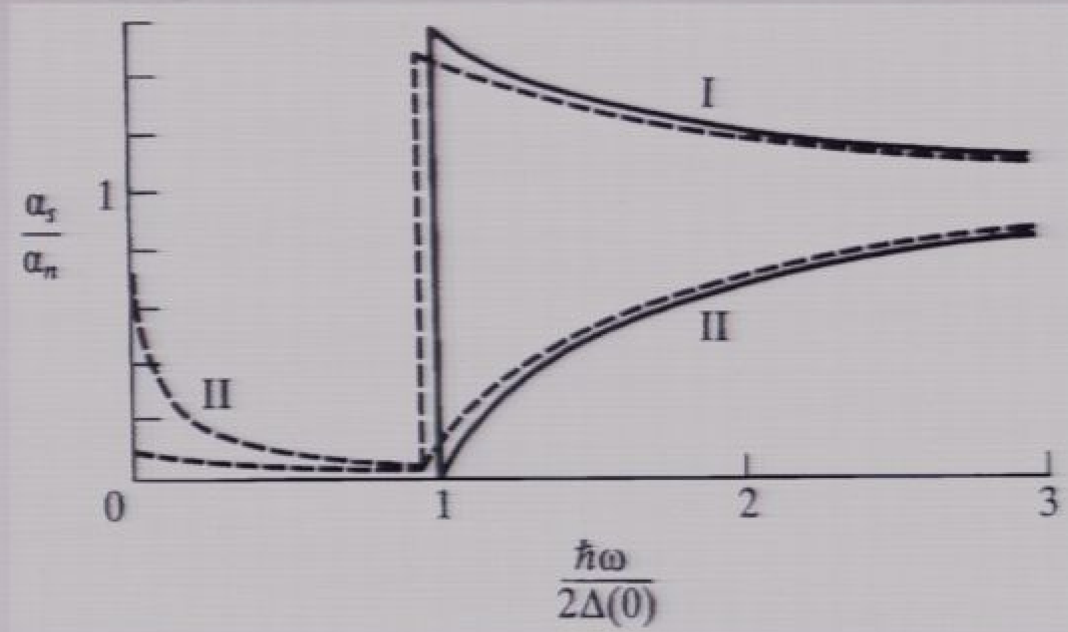
- As temperature decreases, black hole becomes less massive.

- Below some critical temperature Tc, the electrostatic repulsion wins, and the black hole develops scalar hair.

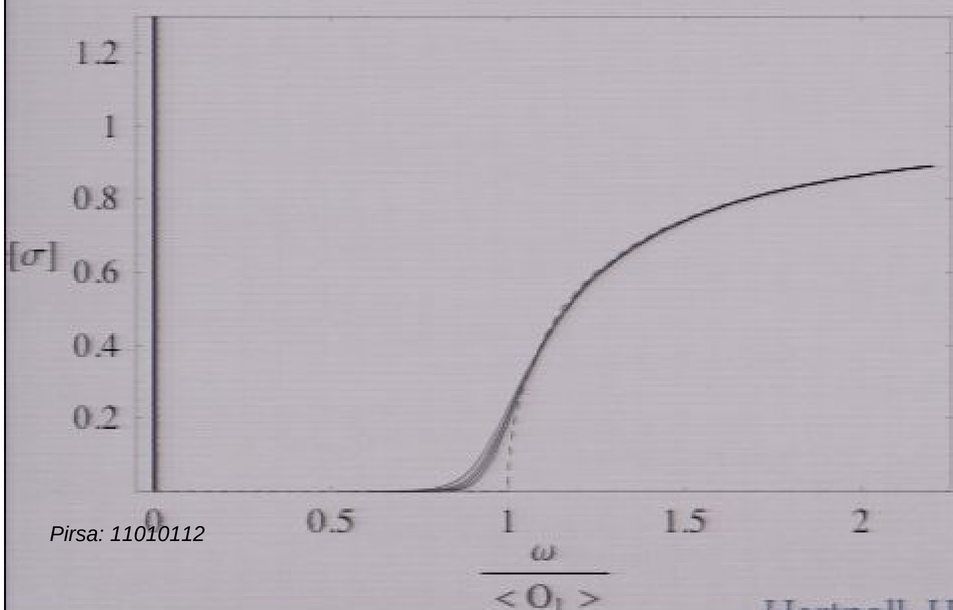


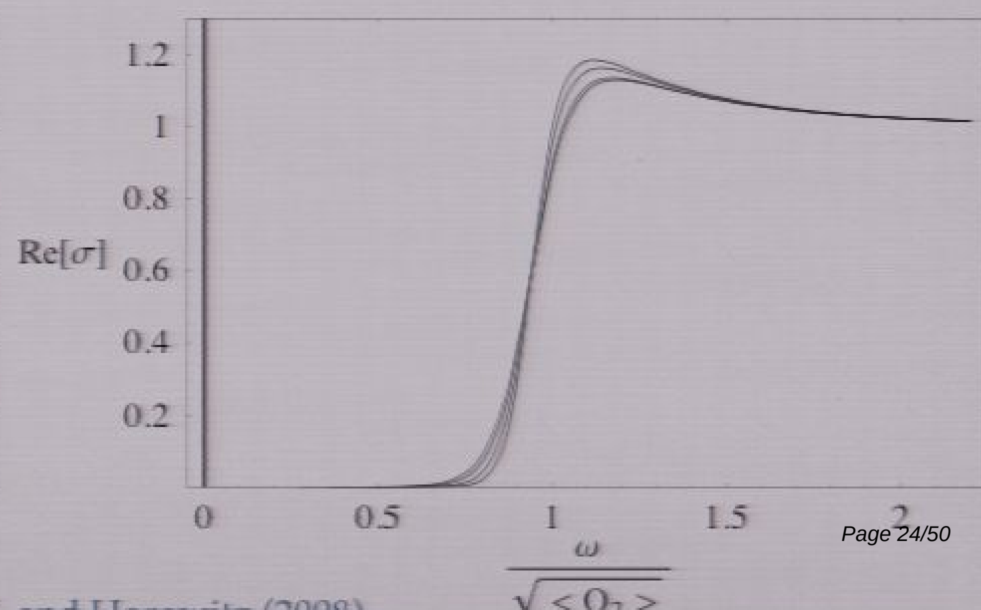


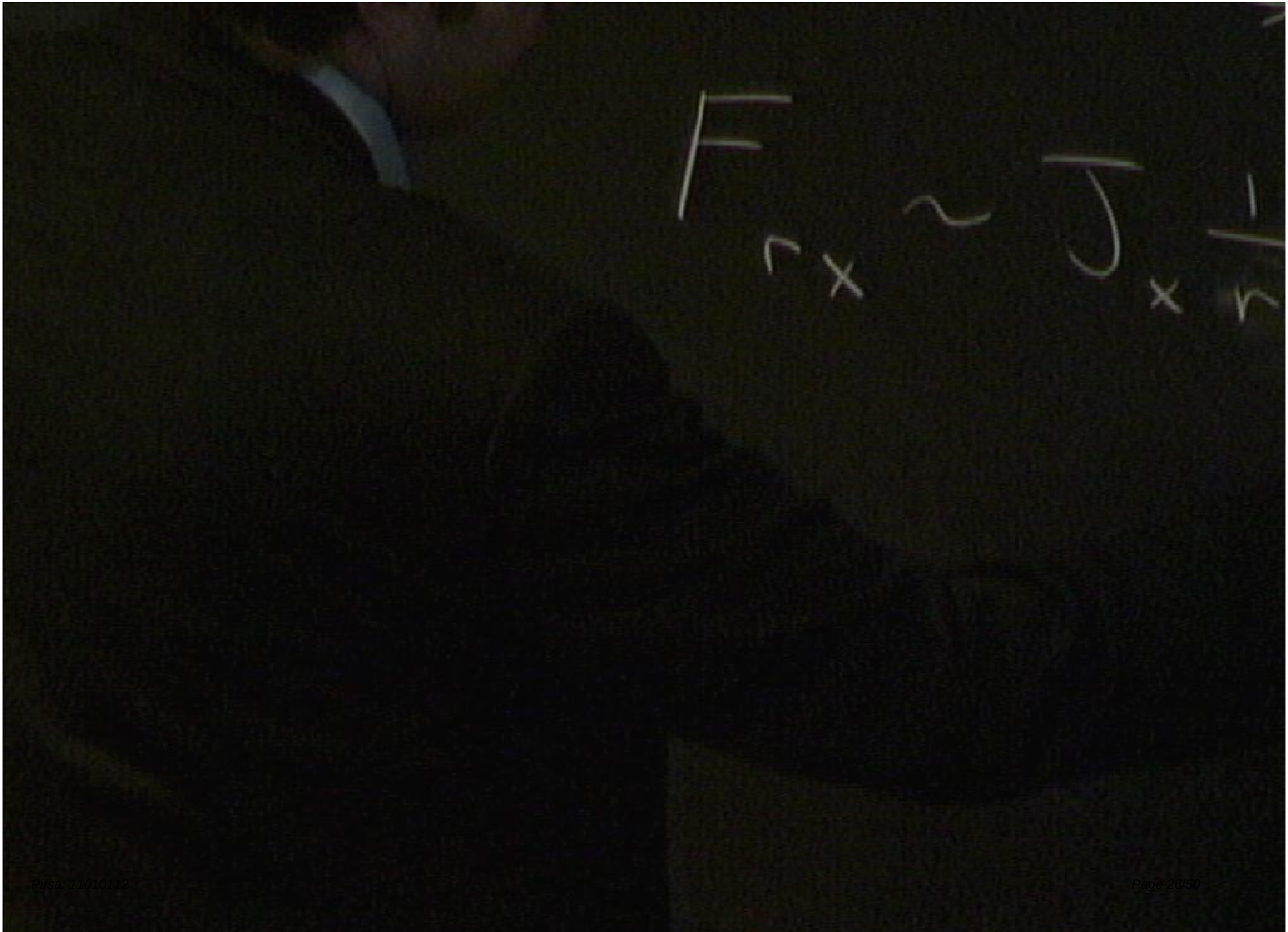
Frequency dependence of absorption processes obeying case I and II coherence factors at T = 0 (solid curves) and T ≈ 1/2 Tc (dashed curves). [Tinkham, Superconductivity, 2nd edition]

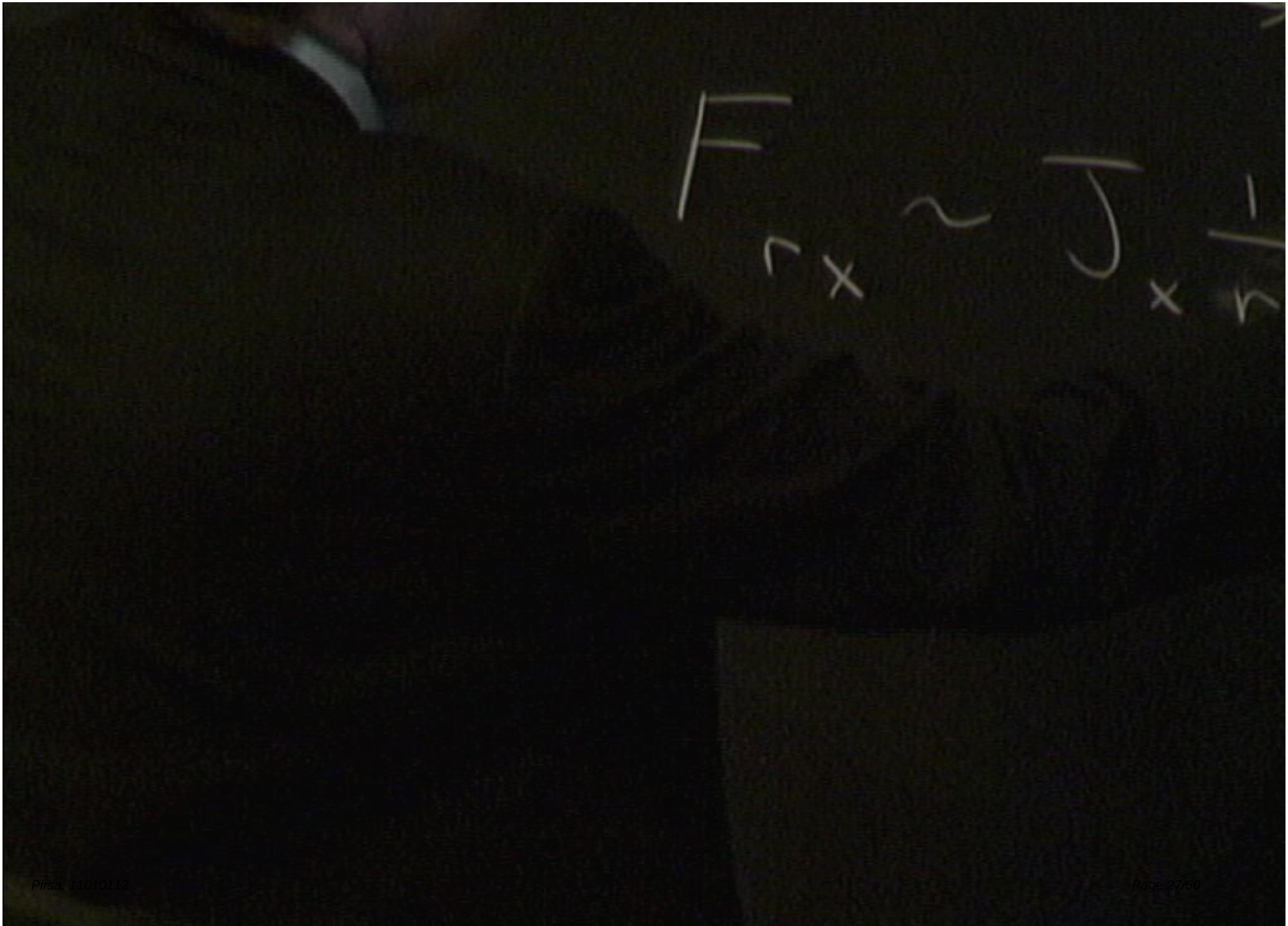


Two phenomenological models of a AdS/CFT superconductor ($\sigma = 1/\varrho$).





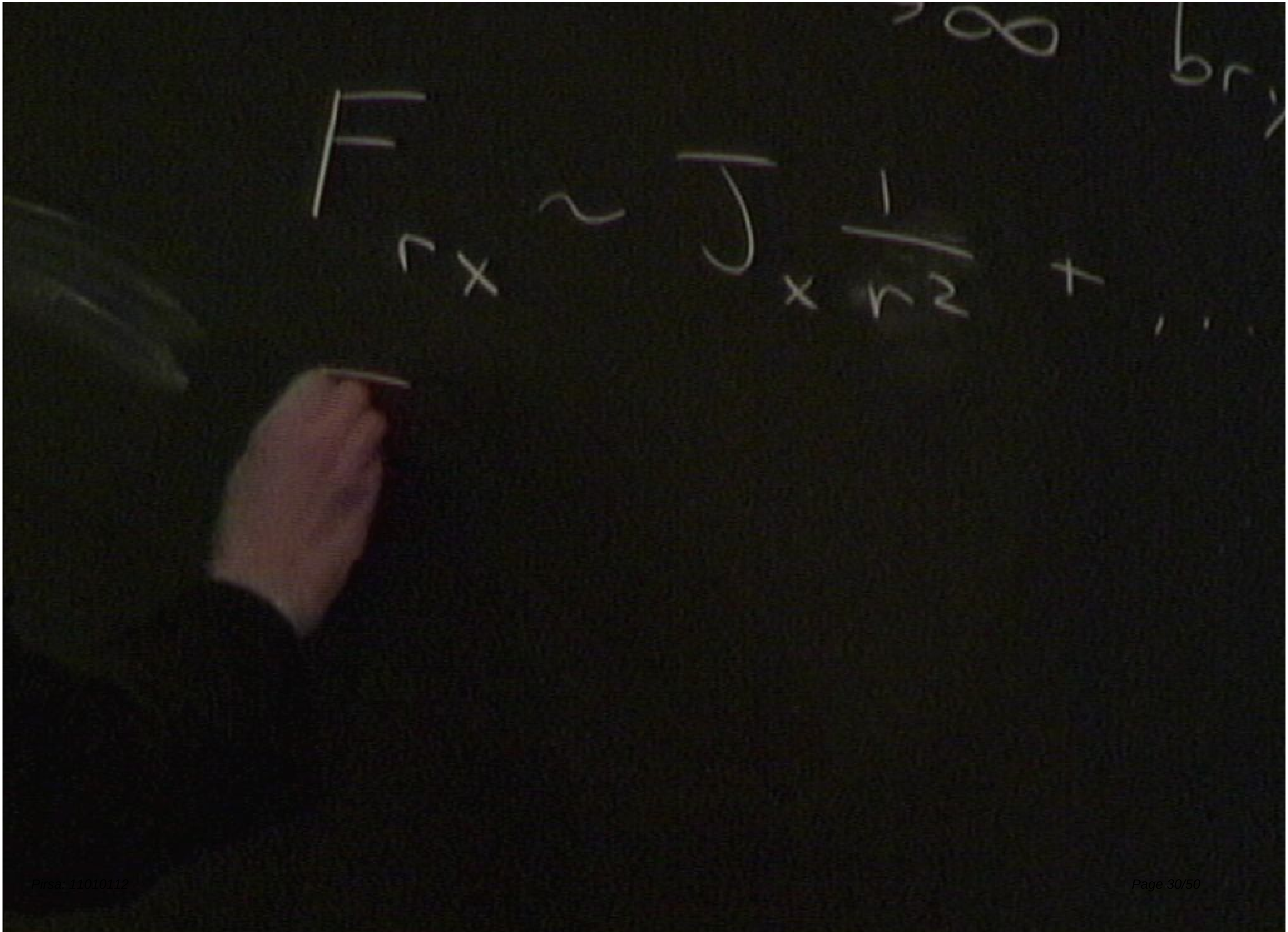

$F_{rx} \sim J_x \frac{1}{r}$

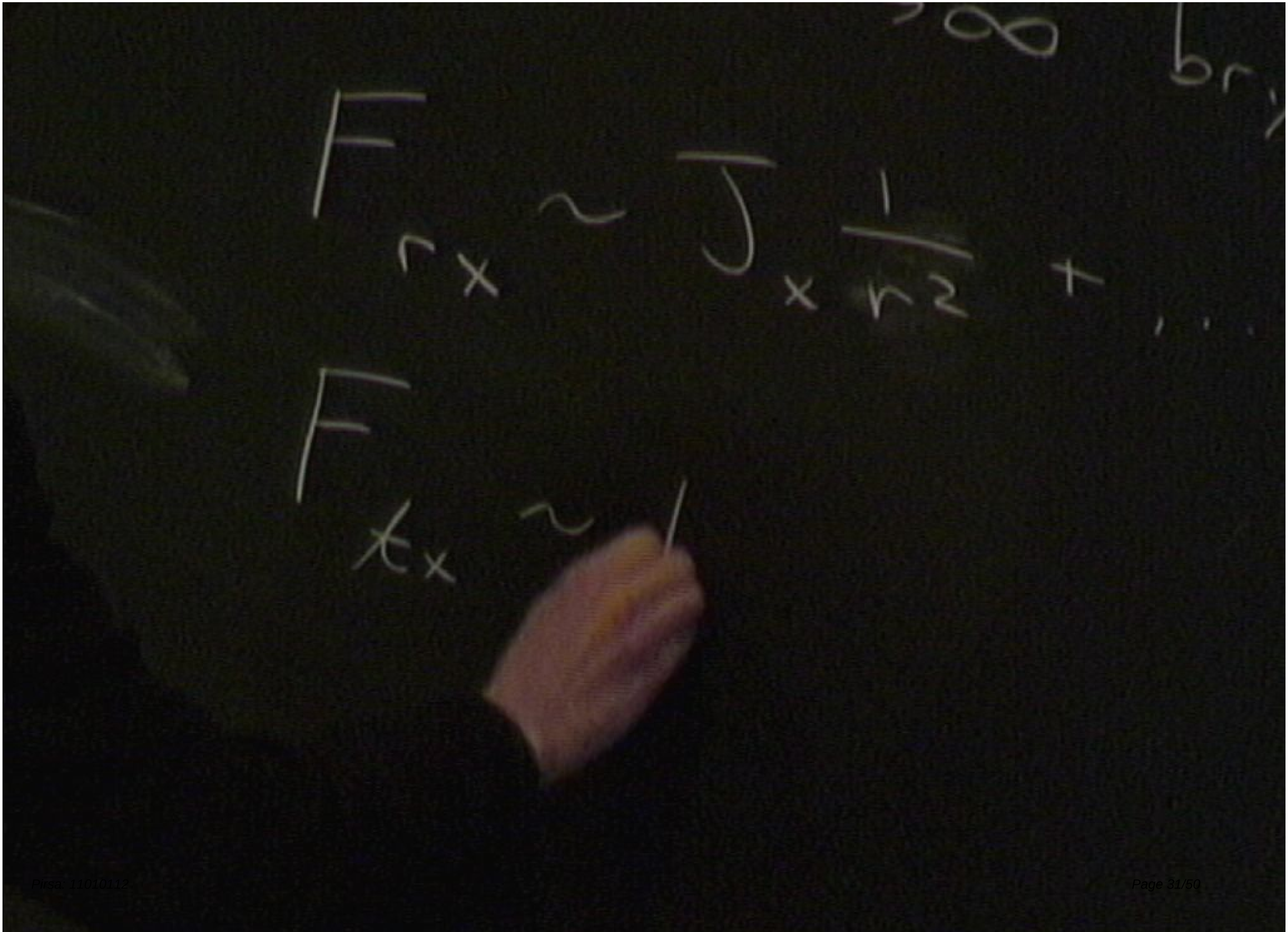


$$F_{rx} \sim J_x \frac{1}{r}$$

$\to \infty$ bry

$$F_{rx} \sim J_x \frac{1}{r^2} + \ldots$$


∞ bry
$F_{rx} \sim J_x \frac{1}{r^2} + \ldots$


∞
bry
F_rx ~ J_x 1/r^2 + ...
F_tx ~

$$F_{rx} \sim J_x \frac{1}{r^2} + \ldots$$

$$F_{tx} \sim E$$

$\to \infty$ bry

$$F_{rx} \sim J_x \frac{1}{r^2} + \ldots$$

$$F_{tx} \sim E_x + \ldots$$

$\to \infty$ bry

$$F_{rx} \sim J_x \frac{1}{r^2} + \dots$$

$$F_{tx} \sim E_x + \dots$$

∞ bry

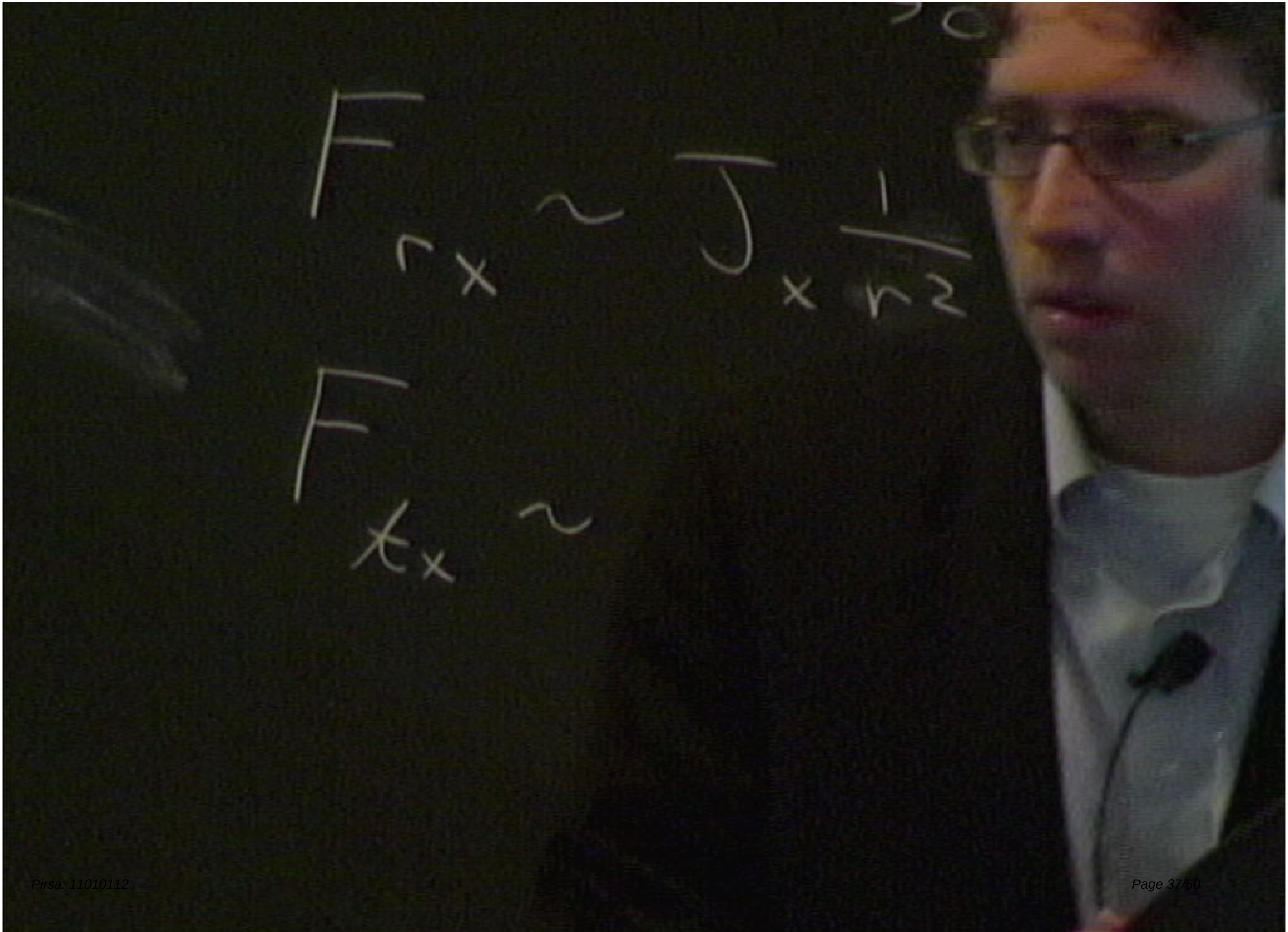
$$F_{rx} \sim J_x \frac{1}{r^2} + \ldots$$

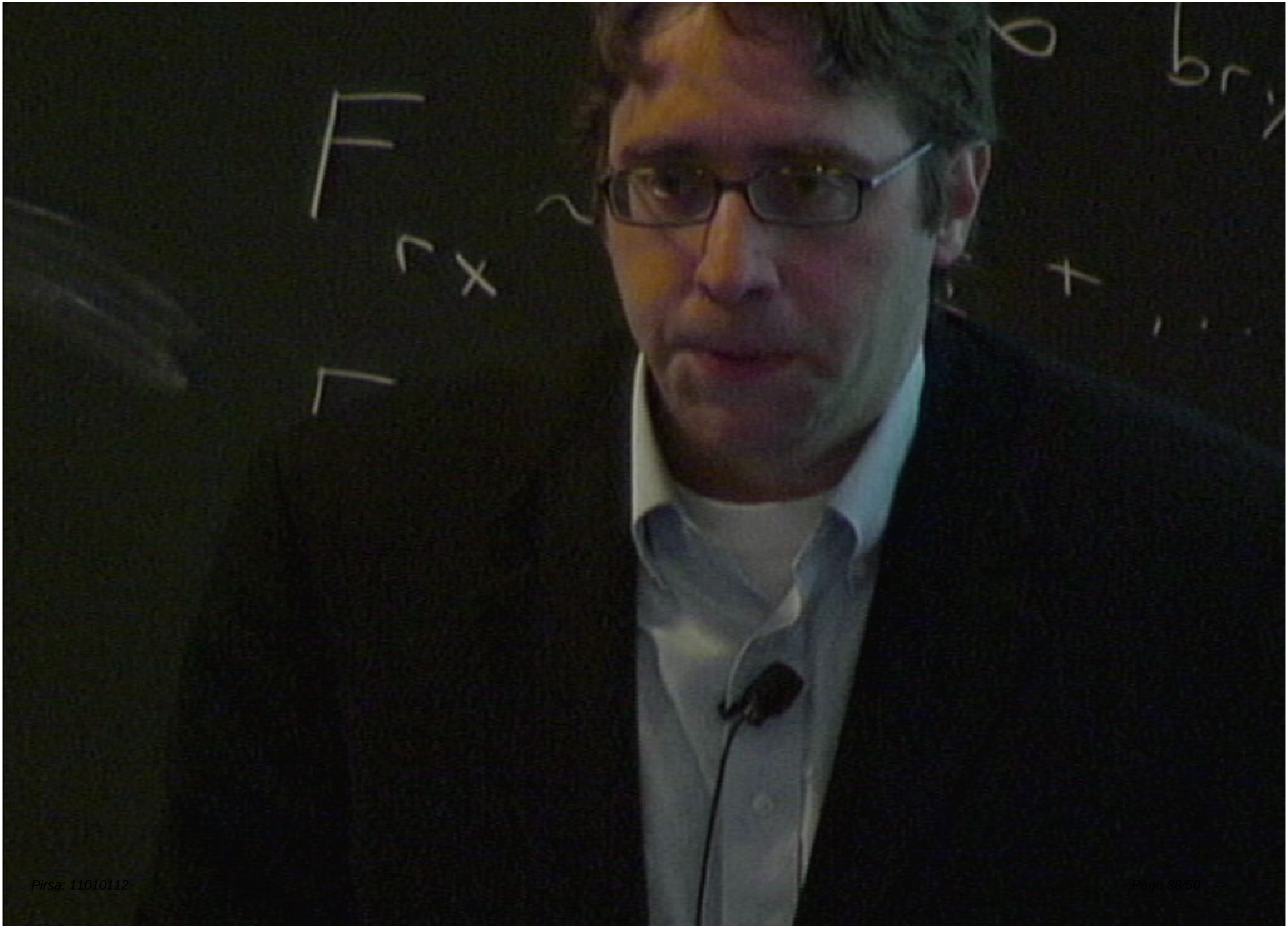
$$F_{tx} \sim E_x + \ldots$$

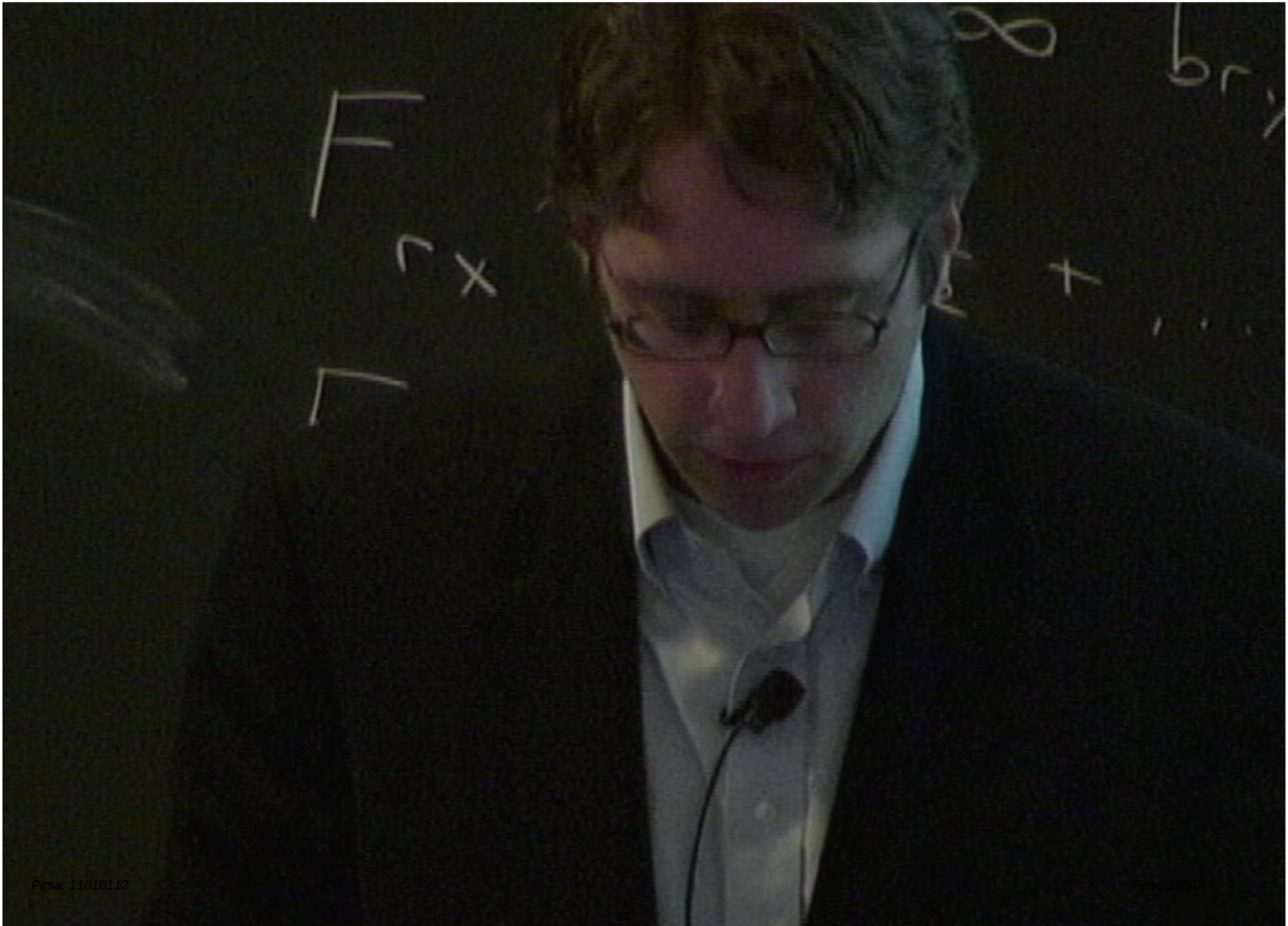
$\to \infty$ bry

$$F_{rx} \sim J_x \frac{1}{r^2} + \ldots$$

$$F_{tx} \sim E_x + \ldots$$


$F_{rx} \sim J_x \frac{1}{r^2}$
$F_{tx} \sim$





An Analytic Holographic Superconductor

- The original holographic superconductors required solving a system of nonlinear, coupled ordinary differential equations
- For a holographic superconductor in 3+1 dimensions with a specific choice of scalar field (saturates the BF bound), the equations can be solved exactly near the phase transition.
- $Tc = \mu/2\pi$ where μ is the chemical potential.
- Critical exponents are mean field, e.g. order parameter ~ $(Tc - T)^{1/2}$.

d-wave superconductor (with Benini and Yarom)

- The cuprates have a d-wave order parameter.
- One can construct a d-wave holographic superconductor by replacing the charged scalar field with a charged spin-2 field.
- Various technical difficulties associated with charged spin-2 fields which we did not overcome.
- Nevertheless, we find some qualitative similarities with the real cuprates, for example the existence of Fermi arcs (feature in the fermionic spectral density).

d-wave superconductor (with Benini and Yarom)

- The cuprates have a d-wave order parameter.
- One can construct a d-wave holographic superconductor by replacing the charged scalar field with a charged spin-2 field.
- Various technical difficulties associated with charged spin-2 fields which we did not overcome.
- Nevertheless, we find some qualitative similarities with the real cuprates, for example the existence of Fermi arcs (feature in the fermionic spectral density).

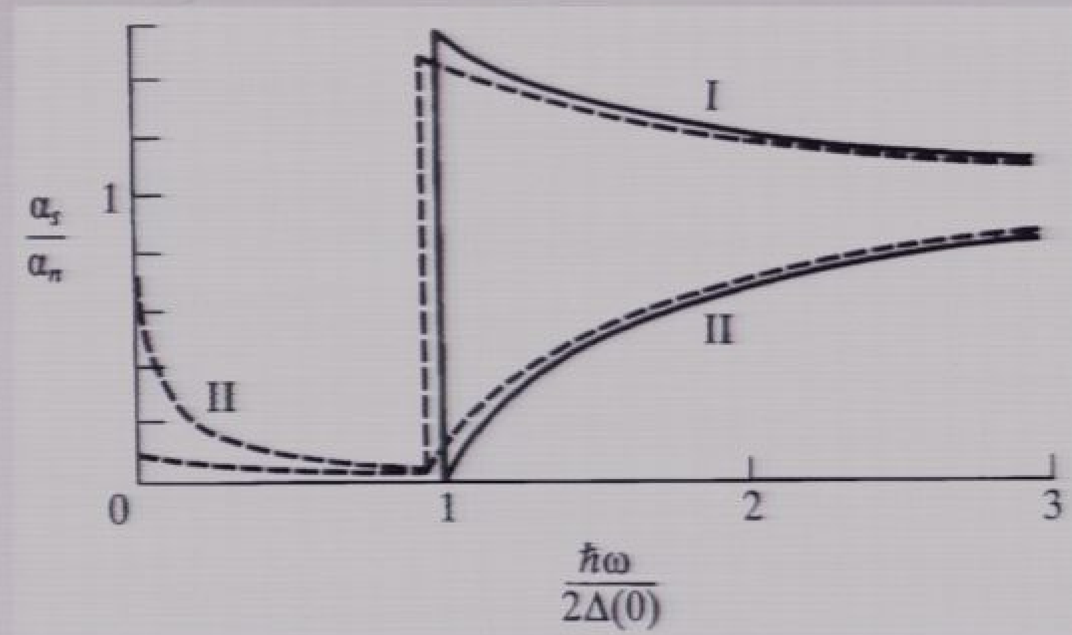
Sum Rules (with Gulotta and Kaminski)

- The conductivity satisfies an analog of the Ferrell-Glover-Tinkham sum rule.

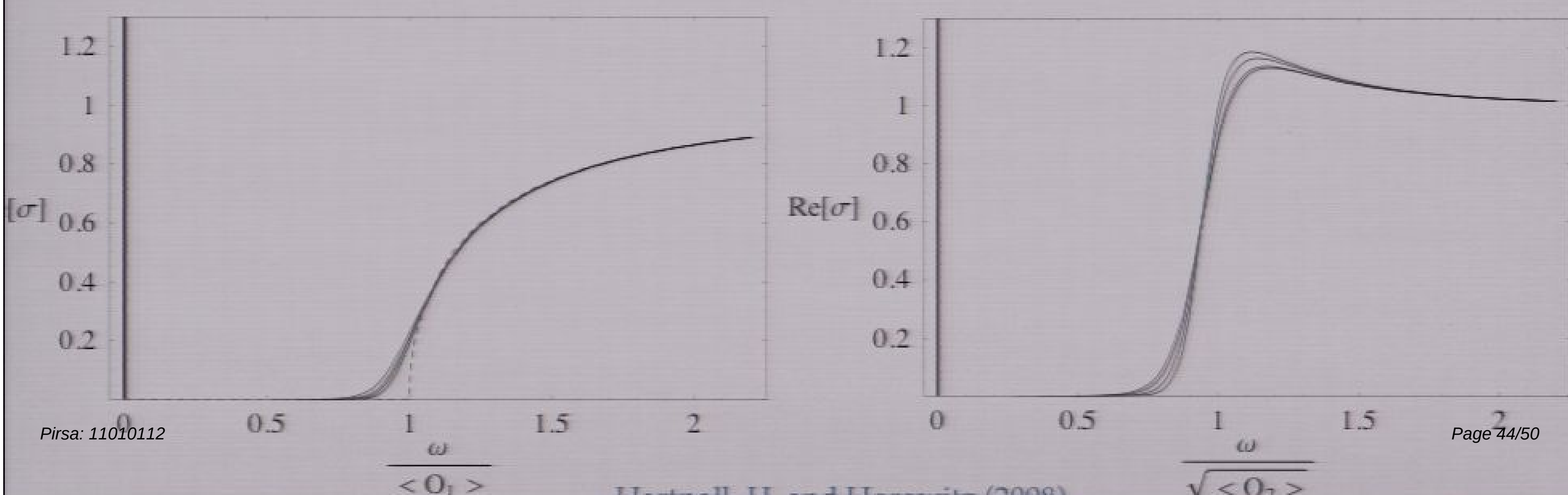
$$\lim_{\omega\to 0} \mathrm{Im}\, \omega\sigma(\omega) = \frac{2}{\pi}\int_0^\infty d\omega(\mathrm{Re}\,\sigma(\omega) - 1)$$

- Gravitational model ensures this sum rule is satisfied:
  - No poles in the upper half plane.
  - Proper large ω fall off
- The residue $\lim_{\omega\to 0} \mathrm{Im}\, \omega\sigma(\omega)$ is proportional to the superfluid density.

Frequency dependence of absorption processes obeying case I and II coherence factors at T = 0 (solid curves) and T ≈ 1/2 Tc (dashed curves). [Tinkham, Superconductivity, 2nd edition]



Two phenomenological models of a AdS/CFT superconductor (σ = 1/ϱ).



Sum Rules (with Gulotta and Kaminski)

- The conductivity satisfies an analog of the Ferrell-Glover-Tinkham sum rule.

$$\lim_{\omega\to 0} \mathrm{Im}\, \omega\sigma(\omega) = \frac{2}{\pi}\int_0^\infty d\omega(\mathrm{Re}\,\sigma(\omega) - 1)$$

- Gravitational model ensures this sum rule is satisfied:
  - No poles in the upper half plane.
  - Proper large ω fall off
- The residue $\lim_{\omega\to 0} \mathrm{Im}\, \omega\sigma(\omega)$ is proportional to the superfluid density.

Closing Remarks

- AdS/CFT is a useful tool for studying a class of strongly interacting field theories --- phase diagram and transport properties.
- Making the connection to the real world: universality or new physics.
- Open problems: fermions; moving beyond bottom up phenomenological constructions; large N; infinite coupling; supersymmetry...

Sum Rules (with Gulotta and Kaminski)

- The conductivity satisfies an analog of the Ferrell-Glover-Tinkham sum rule.

$$\lim_{\omega\to 0} \mathrm{Im}\, \omega\sigma(\omega) = \frac{2}{\pi}\int_0^\infty d\omega(\mathrm{Re}\,\sigma(\omega) - 1)$$

- Gravitational model ensures this sum rule is satisfied:
    - No poles in the upper half plane.
    - Proper large ω fall off
- The residue $\lim_{\omega\to 0} \mathrm{Im}\, \omega\sigma(\omega)$ is proportional to the superfluid density.

d-wave superconductor (with Benini and Yarom)

- The cuprates have a d-wave order parameter.
- One can construct a d-wave holographic superconductor by replacing the charged scalar field with a charged spin-2 field.
- Various technical difficulties associated with charged spin-2 fields which we did not overcome.
- Nevertheless, we find some qualitative similarities with the real cuprates, for example the existence of Fermi arcs (feature in the fermionic spectral density).

An Analytic Holographic Superconductor

- The original holographic superconductors required solving a system of nonlinear, coupled ordinary differential equations
- For a holographic superconductor in 3+1 dimensions with a specific choice of scalar field (saturates the BF bound), the equations can be solved exactly near the phase transition.
- $T_c = \mu/2\pi$ where μ is the chemical potential.
- Critical exponents are mean field, e.g. order parameter ~ $(T_c - T)^{1/2}$.

Holographic superconductors

- Add a charged scalar: gravitational attraction toward and electrostatic repulsion from the black hole.
- As temperature decreases, black hole becomes less massive.
- Below some critical temperature Tc, the electrostatic repulsion wins, and the black hole develops scalar hair.