Title: Space-Time, Quantum Mechanics and Scattering Amplitudes

Date: Jan 26, 2011 02:00 PM

URL: http://pirsa.org/11010111

Abstract: Scattering amplitudes in gauge theories and gravity have extraordinary properties that are completely invisible in the textbook formulation of quantum field theory using Feynman diagrams. In the standard approach--going back to the birth of quantum field theory--space-time locality and quantum-mechanical unitarity are made manifest at the cost of introducing huge gauge redundancies in our description of physics. As a consequence, apart from the very simplest processes, Feynman diagram calculations are enormously complicated, while the final results turn out to be amazingly simple, exhibiting hidden infinite-dimensional symmetries. This strongly suggests the existence of a new formulation of quantum field theory where locality and unitarity are derived concepts, while other physical principles are made more manifest. Rapid advances have been made towards uncovering this new picture, especially for the maximally supersymmetric gauge theory in four dimensions. These developments have interwoven and exposed connections between a remarkable collection of ideas from string theory, twistor theory and integrable systems, as well as a number of new mathematical structures in algebraic geometry. In this talk I will review the current state of this subject and and describe a number of ongoing directions of research.

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Space-Time, Quantum Mechanics

+

Scattering Amplitudes

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with

F. Cachazo
C. Cheung
J. Kaplan
J. Bourjaily
J. Trnka
S. Caron-Huot

also

E. Witten
L. Dolan
P. Goddard
M. Spradlin
A. Volovich
S. Goncharov

J. Maldacena
F. Alday
D. Gaiotto
P. Vieira
A. Sever
N. Beisert
M. Staudacher

Z. Bern
L. Dixon
D. Kosower
G. Korchemsky
E. Sokatchev
J. Henn
J. Drummond

R. Penrose
A. Hodges
L. Mason
D. Skinner
M. Bullimore

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Motivations

192 / Ne

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Gravity + QM

$\Downarrow$

"Space-time is Doomed"

$\Delta E \sim \frac{1}{\Delta t} \longrightarrow$ eventually make Black Hole!

$\leftarrow 10^{-33}$ cm $\rightarrow$

No Operational meaning to distance $< 10^{-33}$ cm, times $< 10^{-43}$ s, ...

End of Space-Time

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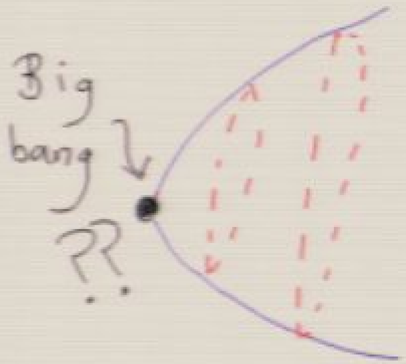

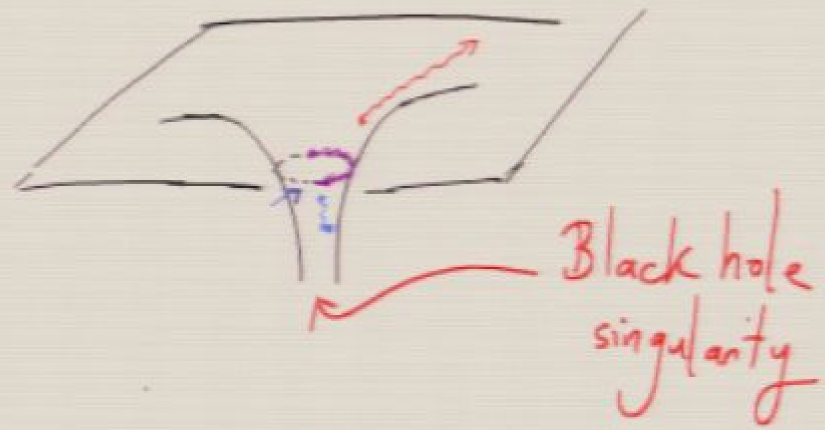

Our theories just break down when gravity is strong and quantum gravity effects are dominant.

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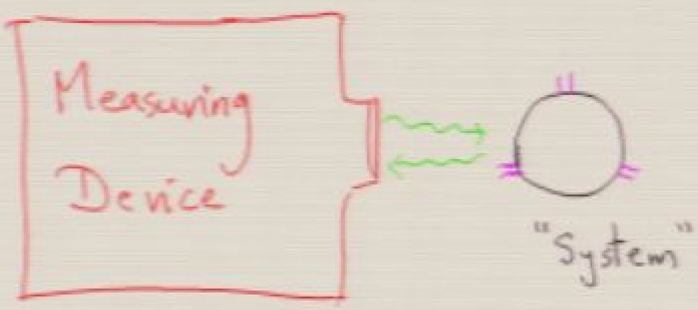
Exact Quantum Predictions



Infinitely many measurements with an Infinitely large measuring apparatus!

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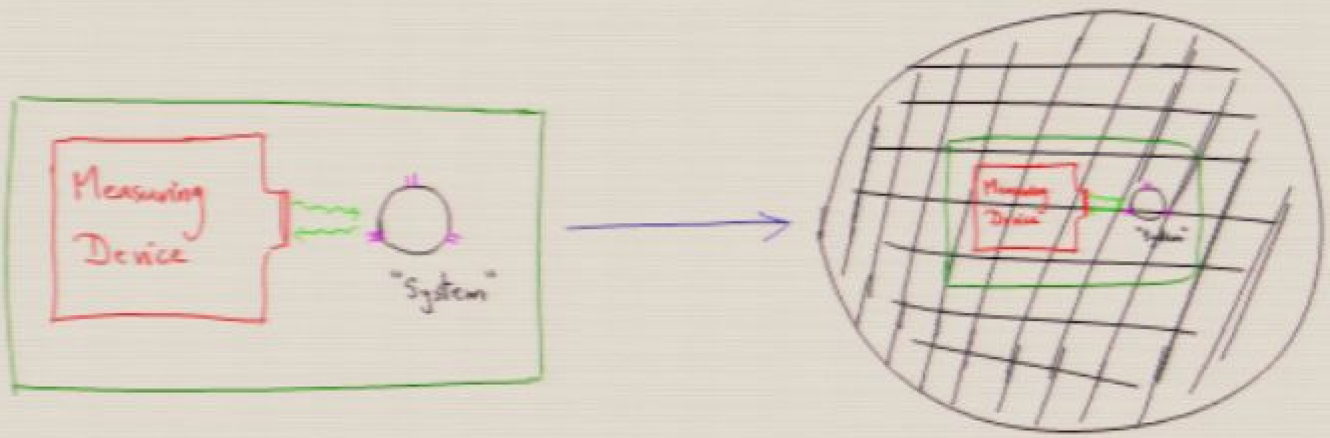
No Local Observables!



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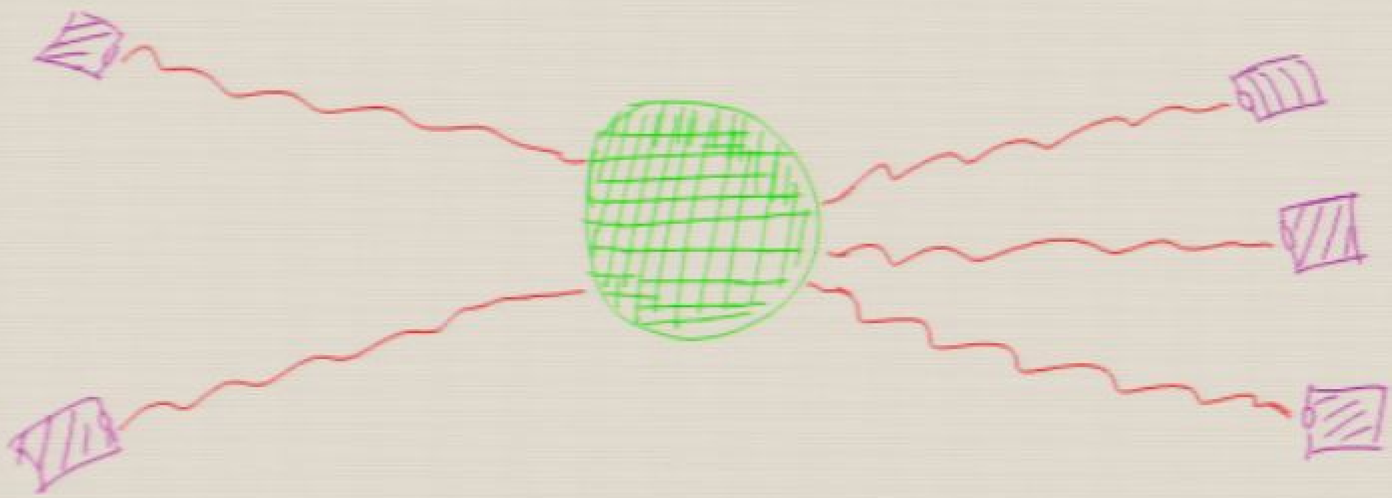
Observables on "Boundary at Infinity"



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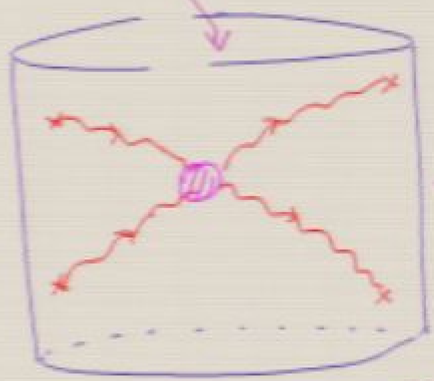
$(\text{Quantum Gravity})_{D+1} = (\text{Quantum Field Theory})_D$ (!)

Emergent Space, Gravity, Strings ...

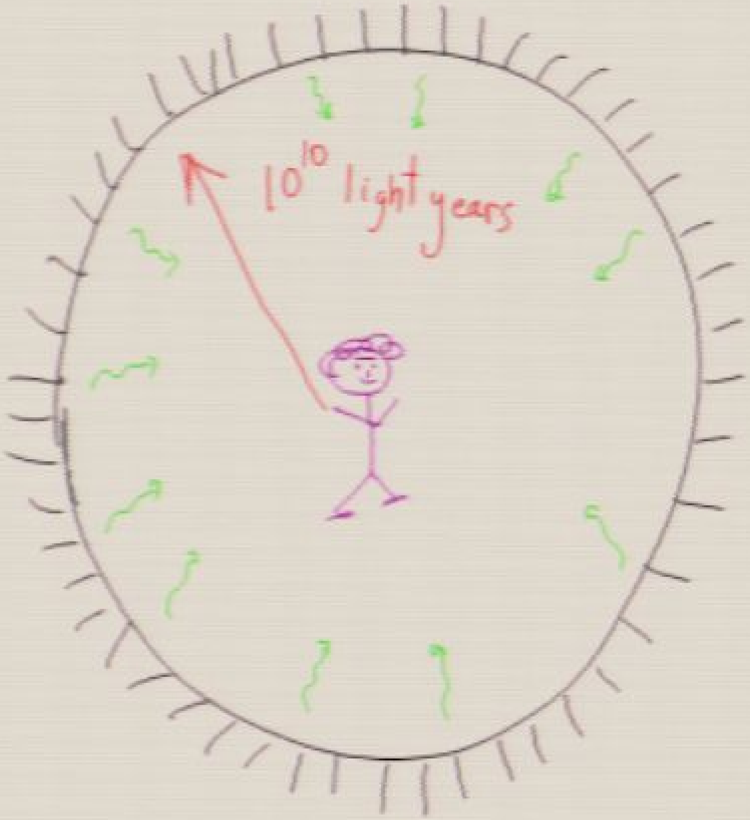
time



"Anti-de Sitter Space"

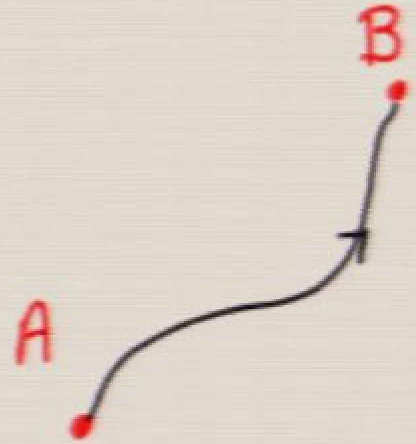
String Theory = Particle Physics

[Weakly interacting] [strongly interacting]



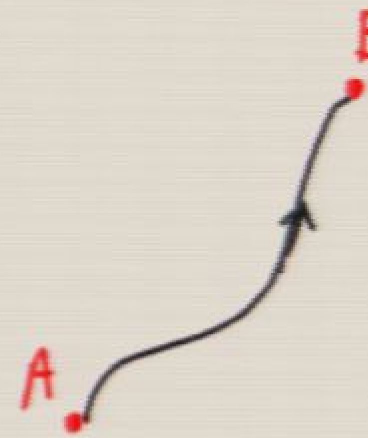
What are the correct observables ?!?!

Emergent Space-time ?



$$m\ddot{x} = -\frac{\partial V}{\partial x}$$

Manifestly Determinstic



$x(t)$ minimizes action

$$S = \int dt \left[\tfrac{1}{2} m \dot{x}^2 - V(x)\right]$$

Not manifestly determinstic

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Quantum Mechanics

All paths are taken.

$$\mathrm{Amp} = \sum_{\text{paths}} e^{iS/\hbar}$$

$\hbar \to 0$ limit of QM = Least action principle,

not $F = ma$!

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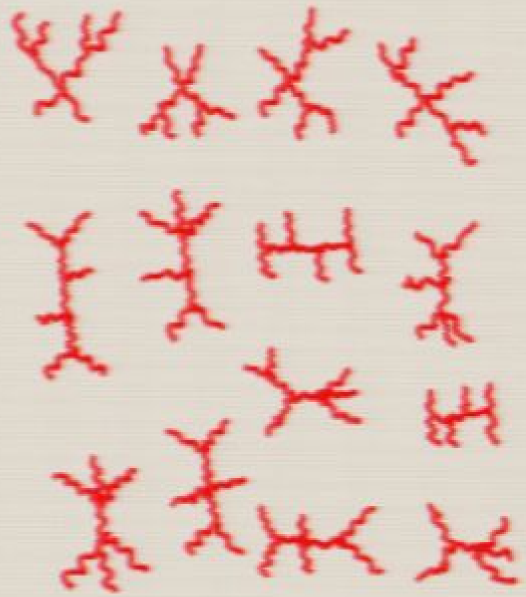
Feynman Follies



+ ···

220 Diagrams

10's of thousands of terms ···

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$$\mathrm{Amp}(1^+2^-3^+4^-5^+6^+) = \frac{\langle 24\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 56\rangle}\ !$$

"MHV Amplitudes" : i^-, j^-, rest plus

$$\frac{\langle i j\rangle^4}{\langle 12\rangle\langle 23\rangle \cdots \langle n1\rangle}$$

204 / Ne

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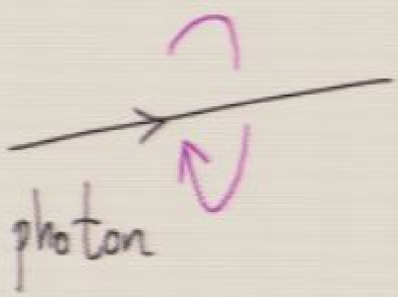
Q. What makes Feynman Diagrams so complicated, obscuring simplicity of answer?

A. Insistence on Manifest Locality + Unitarity!



$\underline{2}$ helicities ± 1

Locality $\Longrightarrow$ Field $\underbrace{A_\mu(x)}_{\text{4 components!}} = \epsilon_\mu e^{ipx}$

$\epsilon \cdot p = 0 \,, \quad \epsilon_\mu \sim \epsilon_\mu + \alpha p_\mu$

$A_\mu \sim A_\mu + \partial_\mu \Lambda$

Gauge Redundancy $\longrightarrow$ All the trouble!

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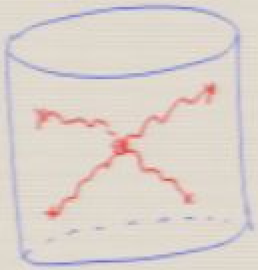
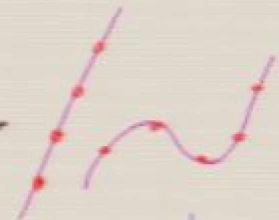
Sitting Under our Noses for 60 yrs



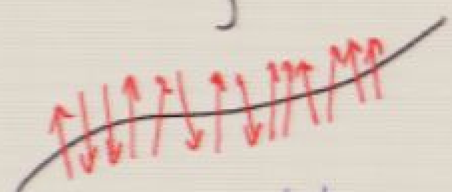
String Theory



Twistor Theory

Scattering without Spacetime – emergent locality + unitarity

Algebraic Geometry



Integrable systems

207 /

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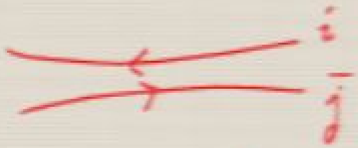
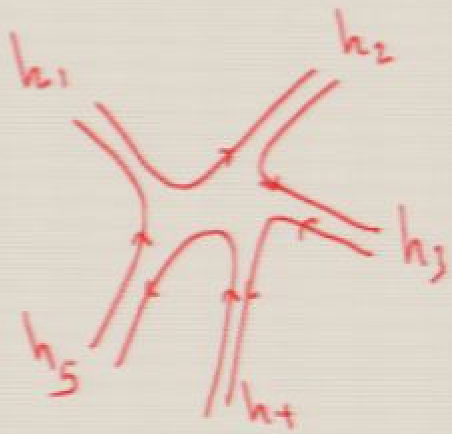
Cast of Characters

208 / Ne

Gluon Scattering Amplitudes

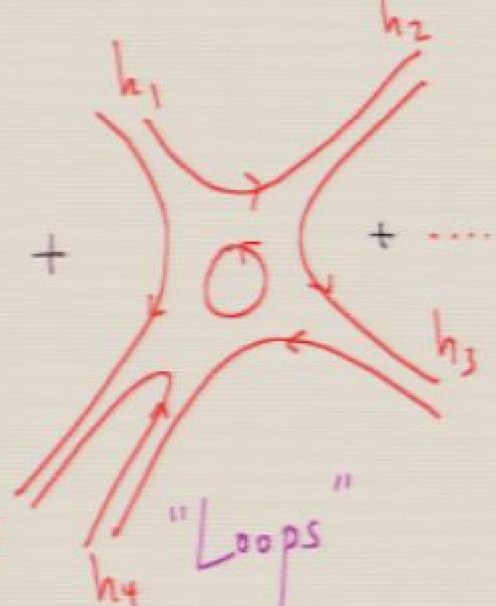


N_c "colors".



"trees"

+



+ ....

"Loops"

Restrict to <u>planar</u> diagrams

[dominate as $N_c \to \infty$]

<u>Important</u>: Planar Loop <u>Integrand</u> is well-defined

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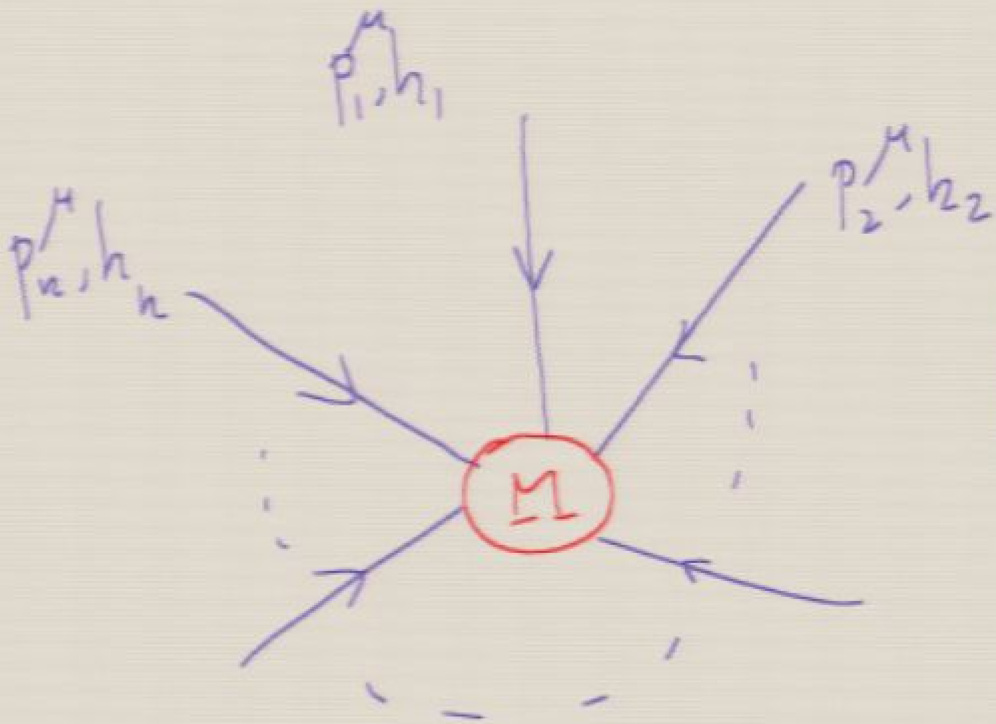

$$\left[p_1^\mu + \cdots + p_n^\mu = 0 \right]$$

$$M_n[p_a, h_a; l_i] \quad l_i : \text{loop momenta}$$

210 / Ne

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More Kinematics

$$p^\mu = (p^0, \vec{p}) \longleftrightarrow P_{A\dot{A}} = \begin{pmatrix} p^0+p^3 & p^1+ip^2 \\ p^1-ip^2 & p^0-p^3 \end{pmatrix}$$

$$\det P = p^2 = 0$$

$$\Longrightarrow \quad P_{A\dot{A}} = \lambda_A \tilde{\lambda}_{\dot{A}} \,. \qquad \text{Lorentz: } SL(2) \times SL(2)$$

Invariants $\langle \lambda_1 \lambda_2 \rangle = \epsilon^{AB} \lambda_{1A} \lambda_{2B}$

$[\tilde{\lambda}_1 \tilde{\lambda}_2] = \epsilon^{\dot{A}\dot{B}} \tilde{\lambda}_{1\dot{A}} \tilde{\lambda}_{2\dot{B}}$

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Manifest Little Group Transf.

$$M_n(\lambda_a, \tilde{\lambda}_a, h_a), \quad M_n(t_a \lambda_a, t_a^{-1} \tilde{\lambda}_a, h_a)$$

$$= t_a^{-2h_a} M_n(\lambda_a, \tilde{\lambda}_a, h_a)$$

e.g.

$$M_6(1^+ 2^- 3^+ 4^- 5^+ 6^+) = \frac{\langle 24 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 56 \rangle}$$

$$M_6(t_2 \lambda_2) - t_2^{2} M_6(\lambda_2) \Leftrightarrow$$ helicity of 2 is -1.

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More Kinematics

$$p^\mu = (p^0, \vec{p}) \longleftrightarrow p_{A\dot{A}} = \begin{pmatrix} p^0 + p^3 & p^1 + ip^2 \\ p^1 - ip^2 & p^0 - p^3 \end{pmatrix}$$

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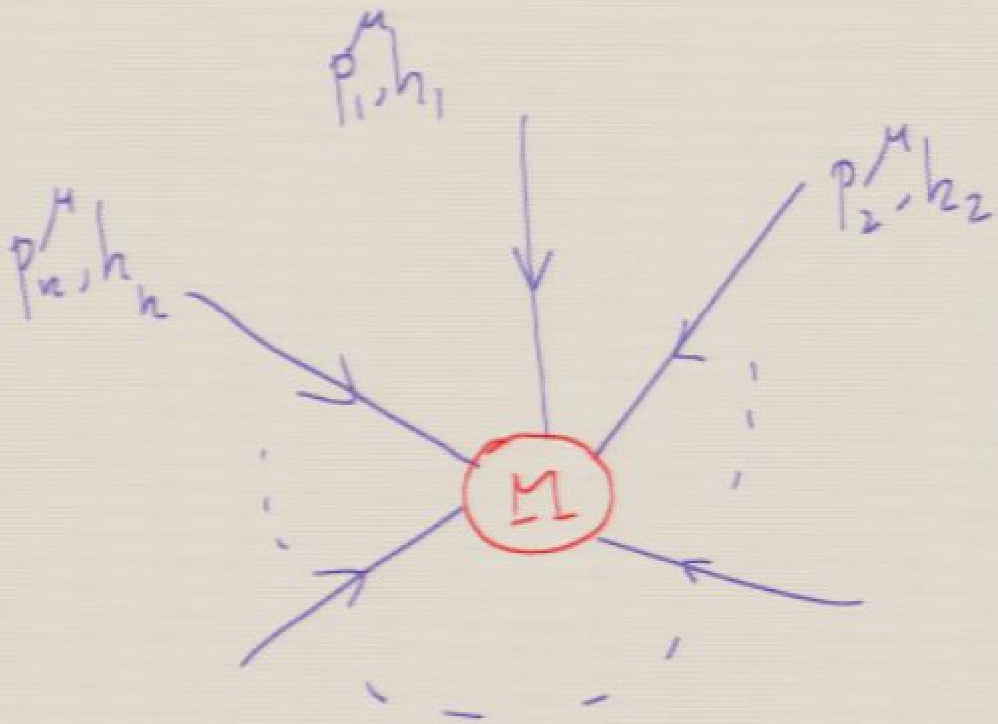

$$\left[p_1^\mu + \cdots + p_n^\mu = 0 \right]$$

$$M_n\left[p_a, h_a \,;\, l_i \right]$$ l_i : loop momenta

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$$M_6(t_2 \lambda_2) - t_2^2 M_6(\lambda_2) \Leftrightarrow$$ helicity of 2 is -1.

212 / 1 Ne

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Finally – simplest gauge theory of all is "maximally supersymmetric, $\mathcal{N}=4$ Super Yang Mills":

"Harmonic Oscillator of the 21st Century"

Unifies heliotles.

$$
\begin{array}{cc}
 & +1 \\
Q_{1,2,3,4} \downarrow & +\frac{1}{2} \\
 & 0 \\
\tilde{Q}_{1,2,3,4} \uparrow & -\frac{1}{2} \\
 & -1
\end{array}
\Bigg\}
$$

"Supermultiplet"

$$|\tilde{\eta}\rangle = e^{Q\tilde{\eta}}|+1\rangle$$

$$= |+1\rangle + \eta|+\tfrac{1}{2}\rangle + \cdots + \eta^4|-\rangle$$

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$$\mathcal{M}_n(\lambda_a, \tilde{\lambda}_a, \tilde{\eta}_a) = \sum_{k=0}^{n} \mathcal{M}_{n,k}(\lambda_a, \tilde{\lambda}_a, \tilde{\eta}_a)$$

contains amps with k - helicity gluons.

Turns out $\mathcal{M}_{n,0} = \mathcal{M}_{n,1} = 0$

$\mathcal{M}_{n,2}$ = "MHV" amplitude

$\mathcal{M}_{n,3}$ = "NMHV" amplitude

⋮

214 / 1

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Summary: We are after a theory for

$$M_{n,k}[\lambda_a, \tilde{\lambda}_a, \tilde{\eta}_a; l_i]$$

Without Unitary evolution through Spacetime

$$\left\{ \text{Emergent Space-time}, \text{Emergent QM} \right\}$$

215 / 1 Ne

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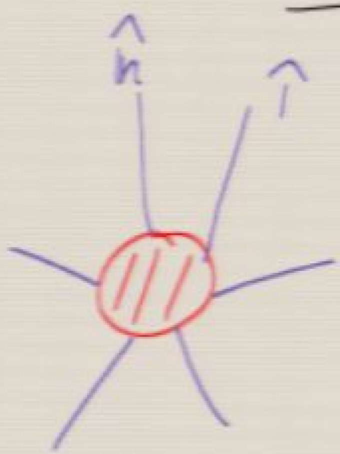
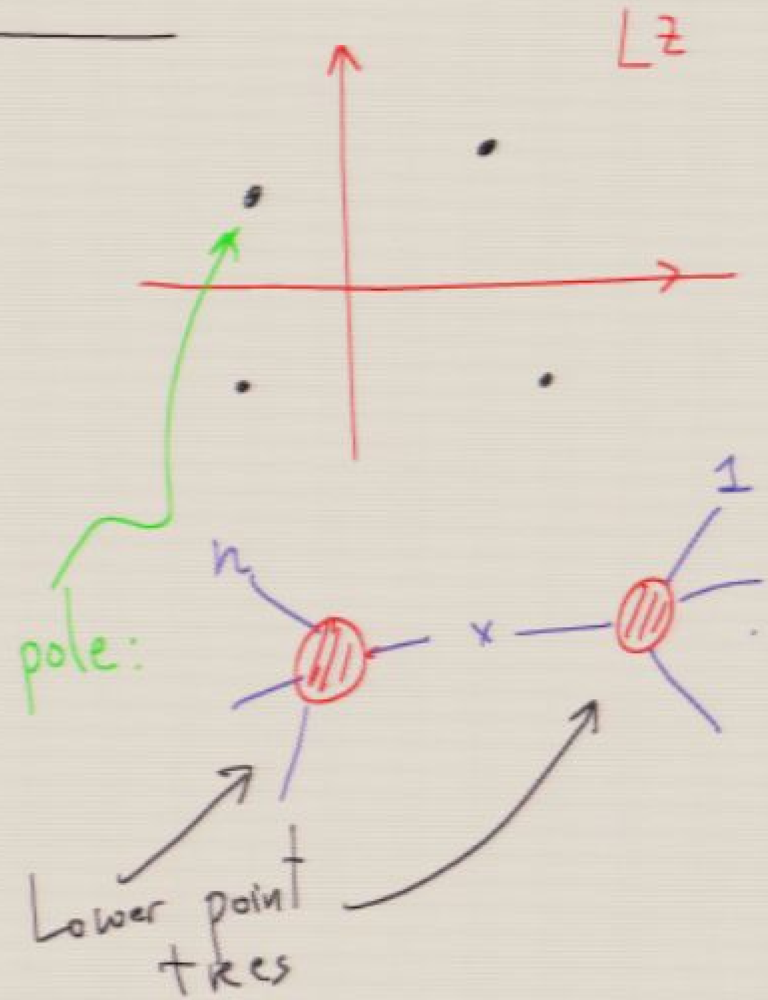
Tree Amplitudes :

Gathering "Data"

"BCFW Recursion"





Deform $\lambda_n \to \lambda_n + z\lambda_1$

$\tilde{\lambda}_1 \to \tilde{\lambda}_1 - z\tilde{\lambda}_n$

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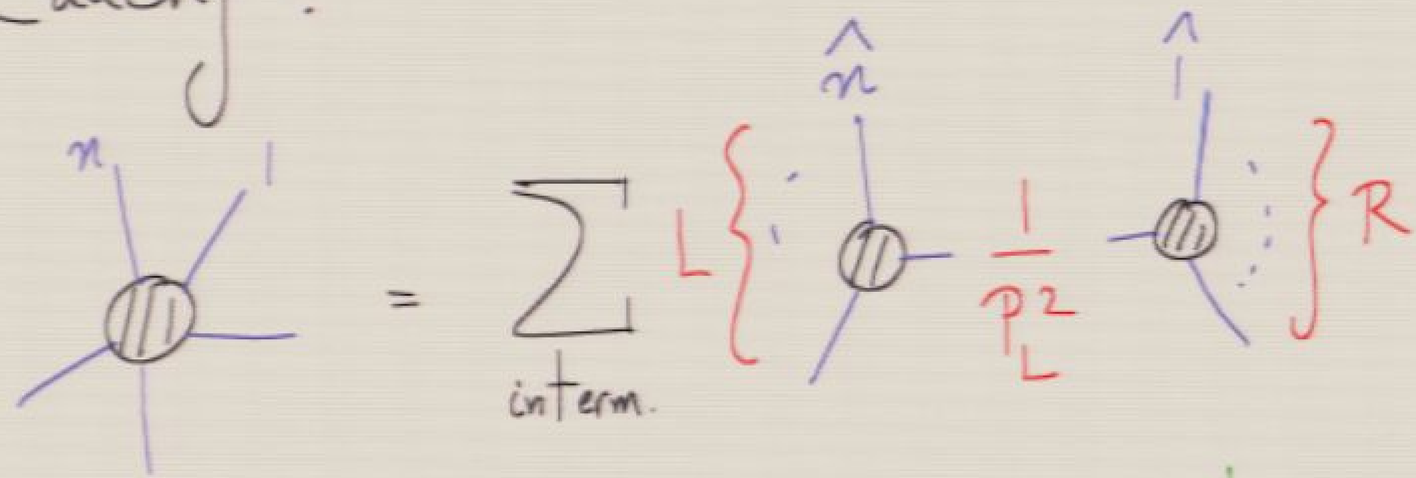
Cauchy :



On- Shell recursion relation !

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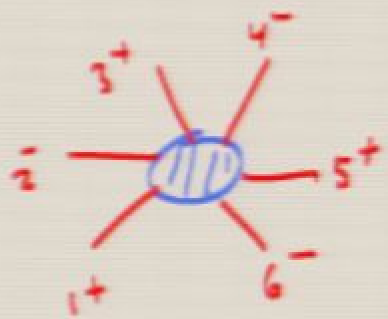
BCFW 6 pt



$$= \frac{\langle 46\rangle^4 [13]^4}{[12][23]\langle 45\rangle\langle 56\rangle} \frac{1}{(p_1+p_2+p_3)^2}$$

$$\times \frac{1}{\langle 6|5+4|3]} \frac{1}{\langle 4|5+6|1]}$$

"Spurious" Poles: Don't occur in local theories!

$$+ \quad \{i \to i+2\} + \{i \to i+4\}$$

Remarkable 6-term Id

$$\frac{\langle 46\rangle^4 [13]^4}{[12][23]\langle 45\rangle\langle 56\rangle} \frac{1}{(p_1+p_2+p_3)^2} \times \frac{1}{\langle 6|5+4|3]} \frac{1}{\langle 4|5+6|1]} + \{i \to i+2\} + \{i \to i+4\}$$

$$= \frac{\langle 3|(2+4)|6]^4}{[23][34]\langle 56\rangle\langle 61\rangle} \frac{1}{(p_5+p_6+p_1)^2} \times \frac{1}{\langle 1|6+5|4]} \frac{1}{\langle 5|6+1|2]} + \{i \to i+2\} + \{i \to i+4\}$$

Guarantees { Parity, Cyclicity, No Spurious Poles

7-pt 12 terms
8-pt 20 terms
40 terms
⋮ ⋮

SOME POWERFUL MATHEMATICAL STRUCTURE IS AT WORK!

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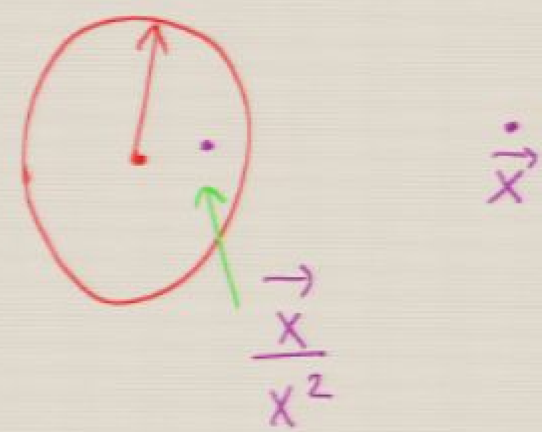
Infinitely Many Hidden Symmetries

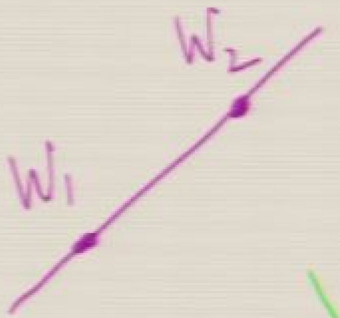
Theories of massless particles enjoy conformal invariance — the remarkable symmetry under inversions

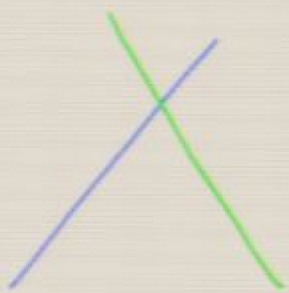
$$x^\mu \longrightarrow \frac{x^\mu}{x^2}$$



Twistor Space

- $W = \begin{pmatrix}\tilde{\mu}_{\dot A} \\ \tilde{\lambda}^{\dot A}\end{pmatrix}$, $W \to LW$, $\det L = 1$ are conf. transf.





Spacetime

$\tilde{\mu}_{A} = X_{A\dot A}\tilde{\lambda}^{\dot A}$

null ray

- $X = \frac{\mu_1 \lambda_2 - \mu_2 \lambda_1}{\langle 12 \rangle}$

$X \overset{\text{null}}{\longleftrightarrow} Y \qquad (X-Y)^2 = 0$

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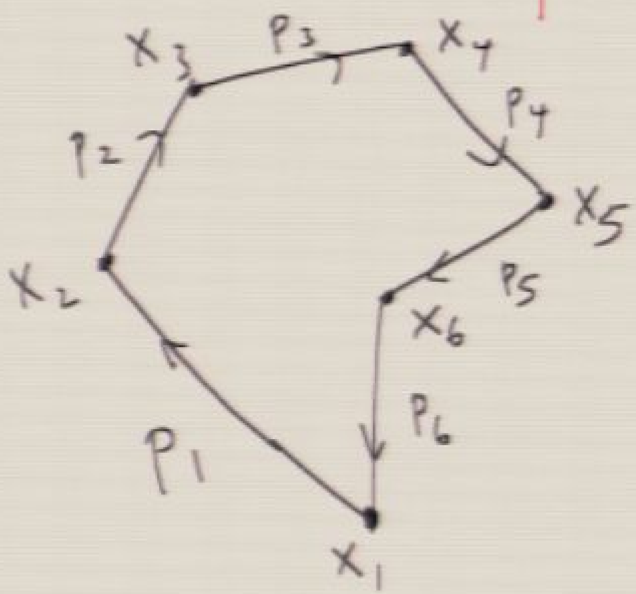
Dual (Super) Conformal Symmetry



$$p_a = x_{a+1} - x_a$$

"Experimental" observation

– amplitudes invariant under

Conf. transf. on

<u>this</u> x space!

[Term by term for BCFW form of trees]

224 / 1 Ne

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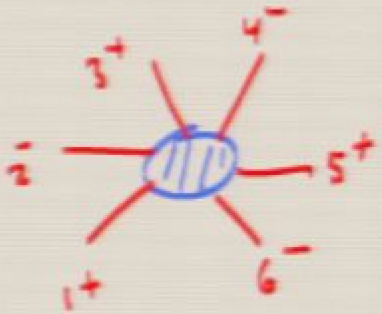


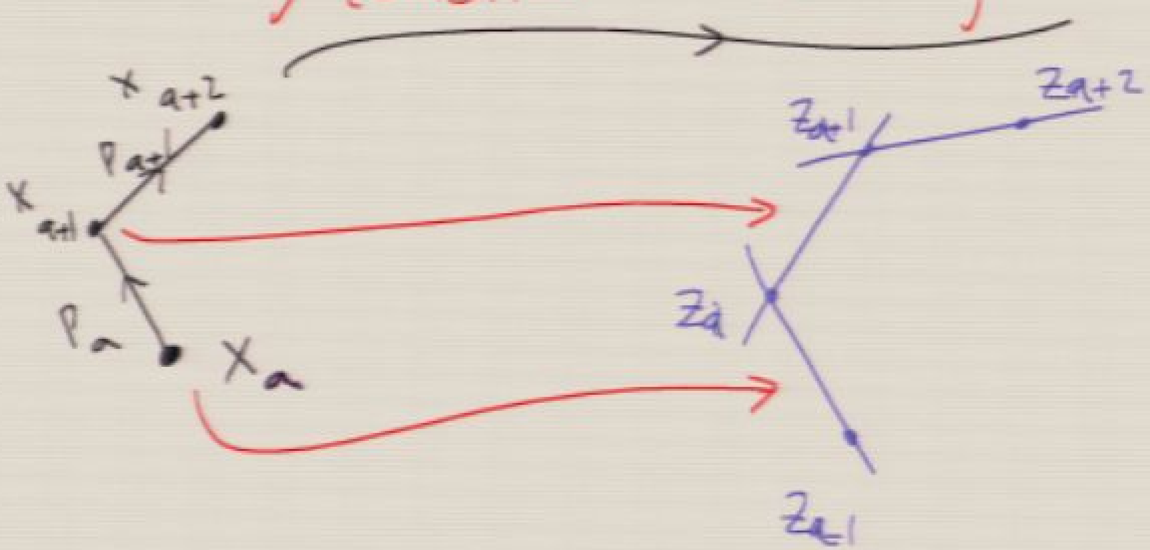
$$= \frac{\langle 46\rangle^4 [13]^4}{[12][23]\langle 45\rangle\langle 56\rangle} \, \frac{1}{(p_1+p_2+p_3)^2}$$

$$\times \; \frac{1}{\langle 6|5+4|3]} \, \frac{1}{\langle 4|5+6|1]}$$

"Spurious" Poles

Are there because these BCFW terms know about both spacetimes!

"Momentum" Twistor Space



$$Z_a = \begin{pmatrix} \mu_a \\ \lambda_a \\ \eta_a \end{pmatrix}, \quad \tilde{\lambda}_a = \frac{\langle a-1\, a\rangle \mu_{a+1} + \text{cyclic}}{\langle a-1\, a\rangle\langle a\, a+1\rangle}$$

$$\tilde{\eta}_a = \frac{\langle a{-}1\, a\rangle\, \eta_{a+1} + \text{cyc.}}{\langle a-1\, a\rangle\langle a\, a+1\rangle}$$

[Invariants $\langle Z_1 Z_2 Z_3 Z_4\rangle$, Schouten–Skinner identity $\langle \underbrace{a\,b\,c\,d\rangle\langle e}_{\text{cyclic}}\, 123\rangle = 0$]

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(Super) Conformal + Dual (Super) Conformal

$\Downarrow$ generate

"Yangian Algebra"

Infinite Dimensional Symmetry

Completely Invisible In $\mathcal{L}$

227 / 2 Ne

Very striking connection with

integrability ....

$$H = \sum_i S_i \cdot S_{i+1}$$

$$Q = \sum_{i<j} [S_i, S_j]$$

$\vdots$

+ AdS/CFT, spectrum of anom. dimensions in $\mathcal{N}=4$ SYM, + amplitudes!

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[In particular major breakthroughs in last ~ 5 yrs have solved the problem of determining anom. dimensions in $\mathcal{N}=4$ SYM — again no Feynman diagrams! — extension to amplitudes more physical + expect richer structure ...]

229 / 1 Ne

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(Super) Conformal + Dual (Super) Conformal

$\Downarrow$ generate

"Yangian Algebra"

Infinite Dimensional Symmetry

Completely Invisible In $\mathcal{L}$

Previou

227 / 262

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Very striking connection with

~~integrability~~ ....

$$\cancel{\uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\uparrow\downarrow}$$

$$H = \sum_i S_i \cdot S_{i+1}$$

$$Q = \sum_{i<j} [S_i, S_j]$$

$$\vdots$$

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[In particular major breakthroughs in last ~ 5 yrs have solved the problem of determining anom. dimensions in $\mathcal{N}=4$ SYM — again no Feynman diagrams! — extension to amplitudes more physical + expect richer structure ...]

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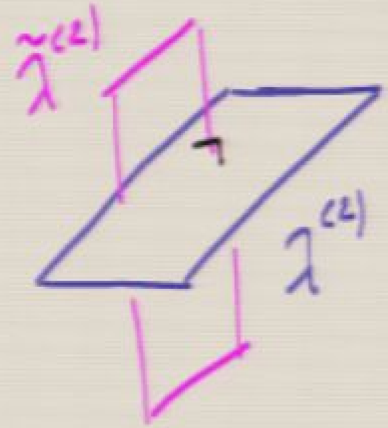
A New Formulation

230 / Ne

Start by thinking about momentum conservation afresh!

$$\lambda_A^{a}\,, \quad \tilde{\lambda}_{\dot{A}}^{\dot{a}}$$



mom. conservation:

$$\lambda \cdot \tilde{\lambda} = 0.$$

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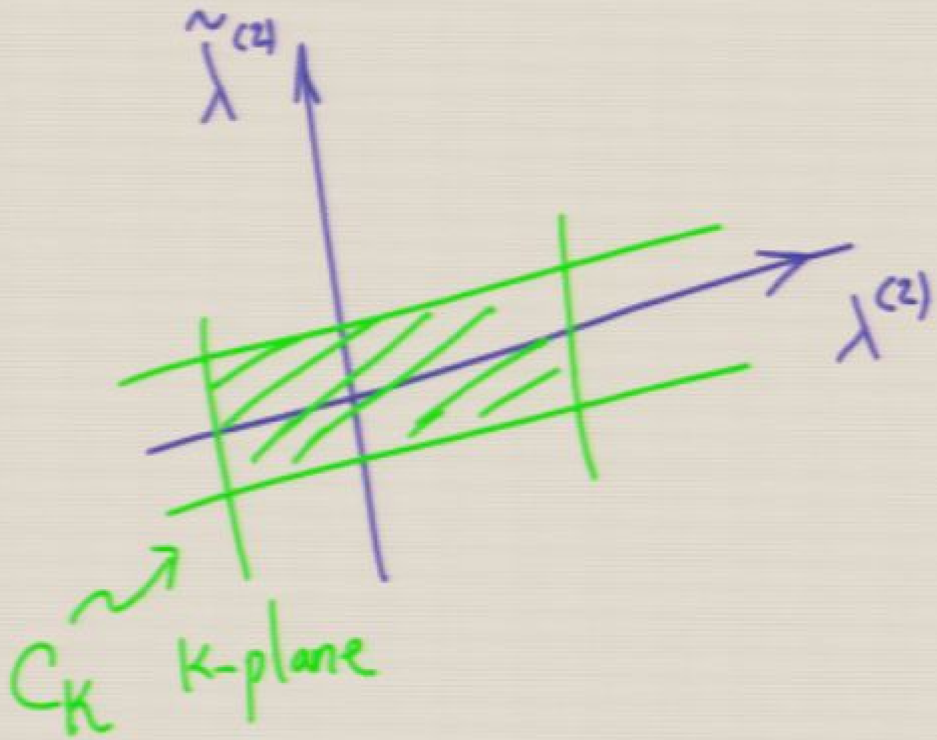

Note: parity invariant since

$\lambda \leftrightarrow \tilde{\lambda}$

k plane $\leftrightarrow$ n-k plane

Note: impossible for $k = 0, 1, n-1, n$.

Good!

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$E_{q^{ns}}$:

$$C = \begin{bmatrix} \vec{c}_1 \\ \vdots \\ \vec{z}_k \end{bmatrix} = C_{\alpha a}$$

Invariance under $GL(k)$ $C_{\alpha a} \to L_\alpha^{\ \beta} C_{\beta a}$.

Space of k-planes in n-dim : Grassmannian $G(k,n)$

$$\dim G(k,n) = kn - k^2 = k(n-k)$$

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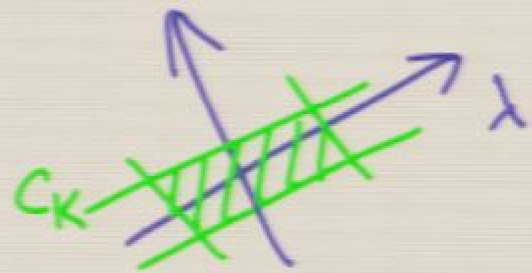

$$\int d^{2\times k}\rho_\alpha \; \underbrace{\delta^2[\,C_{\alpha a}\,\rho_\alpha - \lambda_a]}_{C \text{ contains } \lambda} \; \underbrace{\delta^2[C_{\alpha a}\tilde{\lambda}_a]}_{C \text{ orthogonal to } \tilde{\lambda}} \; \underbrace{\delta^4[C_{\alpha a}\tilde{\eta}_a]}_{\text{SUSY partner}}$$

Motivation : preserve GL(k)

234 / Ne

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This object is very simple in Twistor Space :

$$\prod_{\alpha=1}^{k} \delta^{4|4}\left[C_{\alpha a} W_a \right]$$

Manifests (Super) conformal symmetry

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$k=0,1,n-1,n$: no possible planes.

$k=2$ unique: $C = \lambda$ plane.

General k : integrate over all k-planes!

$$\int \frac{d^{k\times n} C_{\alpha a}}{(12..k)(23..k+1)..(n1..k-1)}$$

simplest + most natural $GL(k)$ invariant measure!

$(m_1 .. m_k)$: $k\times k$ minor of C made of columns $m_1,..,m_k$.

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$$\boxed{\mathcal{L}_{n,k} = \underbrace{\int \frac{d^{k(n-k)} C_{\alpha a}}{(12 \cdots k) \cdots (n1 \cdots k-1)}}_{\text{simplest measure}} \times \underbrace{\prod_\alpha \delta^{4|4}\left[C_{\alpha a} \mathcal{W}_a\right]}_{\text{simplest dependence on kinematics}}}$$

$$\Updownarrow$$

All-Loop Scattering in $\mathcal{N}=4$ SYM!

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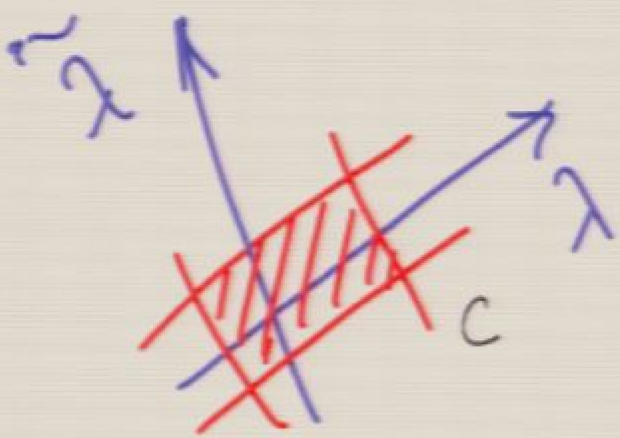
Manifest Dual Superconformal Invariance



C contains λ plane:

So really an integral over

$(k-2)$ planes in n dimensions!

Natural linear transformation mapping $k\times k$ minors to $(k-2)\times(k-2)$

minors ...

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$$\mathcal{L}_{n,k} \to \int \frac{d^{p\times(n-p)} D_{\alpha a}}{(12\cdots p)\cdots(n1\cdots(p-1))} \times \prod_{\alpha=1}^{p} \delta^{4|4}[D_{\alpha a} Z_a]$$

momentum-twistor variables

Identical Structure!

Dual superconformal symmetry manifest

239 / Nex

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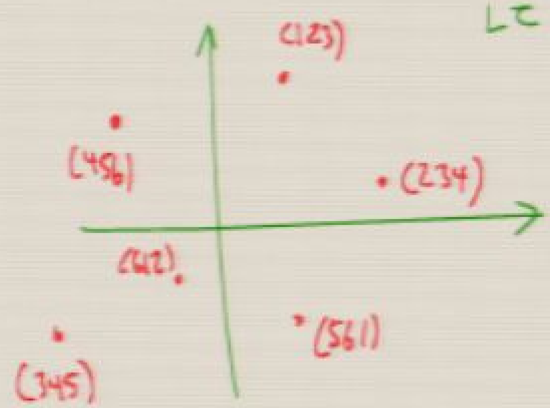
The Grassmannian Formulation makes no mention of locality or Unitarity — but makes all symmetries — The Yangian — manifest.

Quick Example

First non-trivial $k=3,\ n=6$, NMHV, $(k-2)(n-k-2) = 1$ variable!

$$\mathcal{Z}_{6,3} = \int \frac{d\tau}{(123)(\tau) \cdots (612)(\tau)}$$

each minor linear in τ



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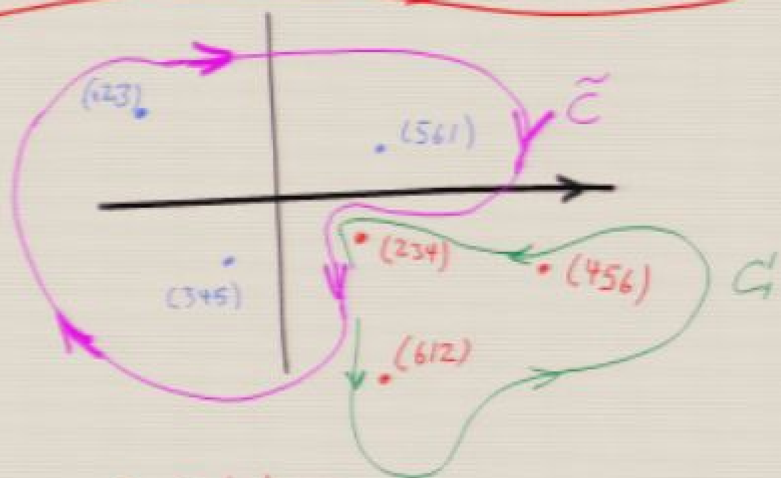
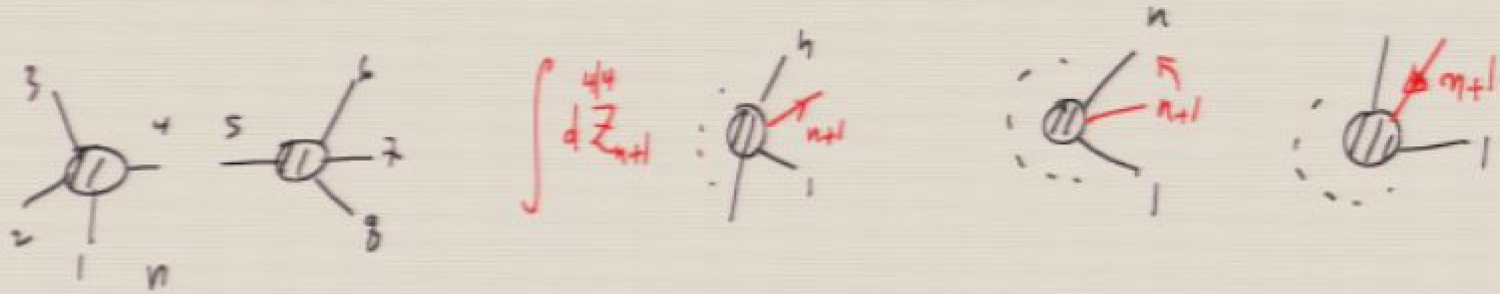
Tree Amplitude



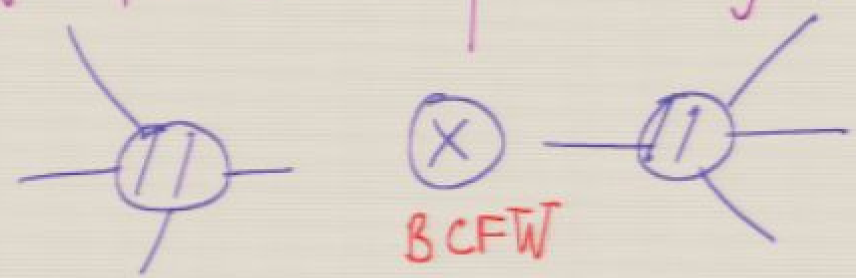
[Unique choices respecting cyclic symmetry]

- residues : BCFW terms
- residues : P[BCFW] terms
- Cauchy : BCFW = P[BCFW] = Remarkable 6-term identity!

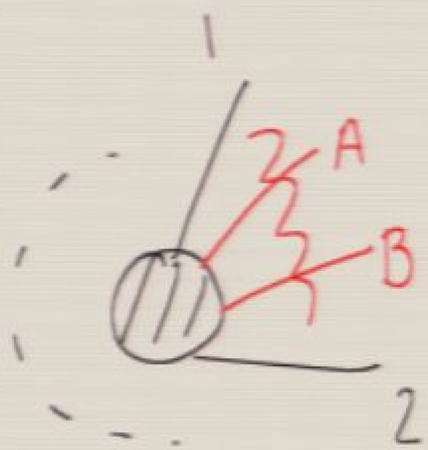
Basic Operations on Yangian Invariants



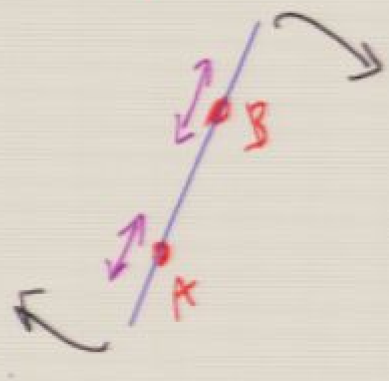
BCFW terms composed from these



Origin of Loops



$$\int d^{4|4} Z_A \, d^{4|4} Z_B$$



"Entangled removal" of a pair of particles

$\hookrightarrow$ Quantum Loop Corrections

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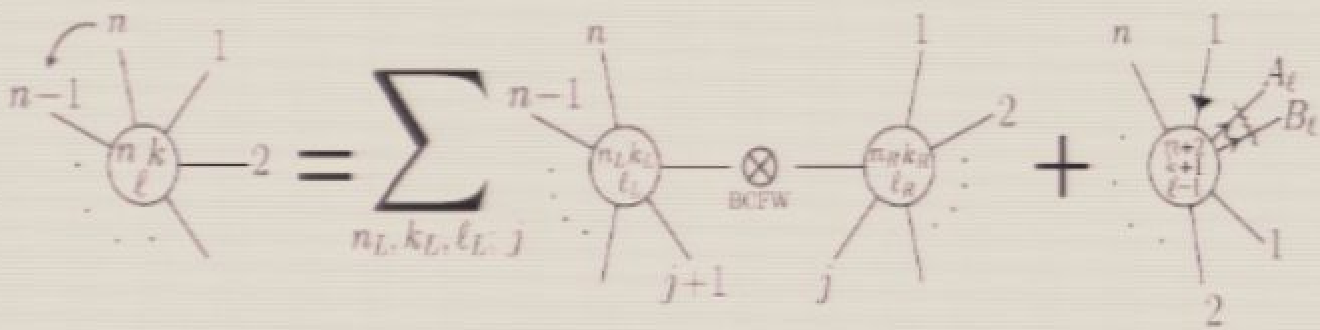
All-Loop Recursion



"Classical"

"Quantum"

246 / Ne

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Complete definition of theory,

Yangian symmetry manifest.

The words "spacetime", "Lagrangian",

"Path Integral", "Gauge Symmetry" ....

make no appearance.

247 / Ne

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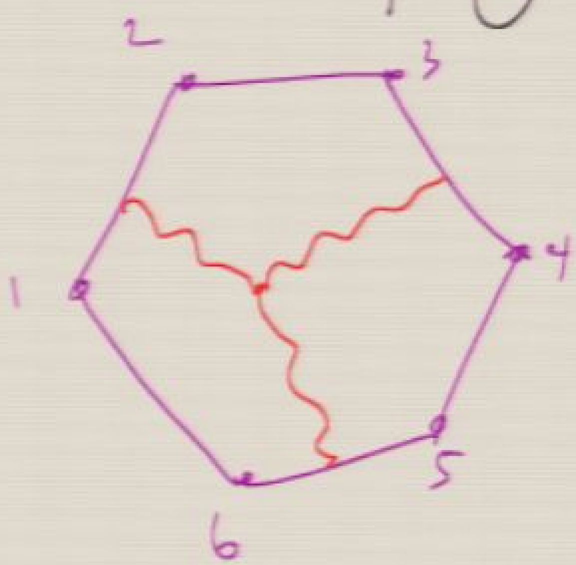
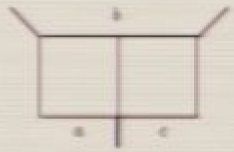
In the dual space-time, this object is interpreted as a certain supersymmetric Wilson loop:



Perfect symmetry has been established between both descriptions.

248 / Ne

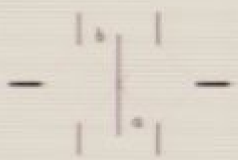


$$-\frac{1}{4}\begin{bmatrix} a & a+1 & b-1 \\ b+1 & c-1 & c \end{bmatrix} \tag{53}$$

B. Kissing double-box topologies

$$\begin{aligned}
&-\frac{1}{4}\begin{bmatrix} a & a+1 & b-1 & b \\ b & b+1 & a-1 & a \end{bmatrix}+\frac{1}{4}\begin{bmatrix} a & a+1 \\ b-1 & b \end{bmatrix}\begin{bmatrix} b & b+1 \\ a-1 & a \end{bmatrix}= \\
&\frac{1}{4}\Big(x^2_{a-1,b+1}x^2_{a+1,b-1}\left(x^2_{ab}\right)^2-x^2_{a-1,b-1}x^2_{a+1,b+1}\left(x^2_{ab}\right)^2+ \\
&+x^2_{a-1,a+1}x^2_{b-1,b+1}\left(x^2_{ab}\right)^2-x^2_{a-1,b}x^2_{a,b+1}x^2_{a+1,b-1}x^2_{ab}- \\
&-x^2_{a-1,b+1}x^2_{a,b-1}x^2_{a+1,b}x^2_{ab}+x^2_{a-1,b-1}x^2_{a,b+1}x^2_{a+1,b}x^2_{ab}+ \\
&+x^2_{a-1,b}x^2_{a,b-1}x^2_{a+1,b+1}x^2_{ab}\Big)
\end{aligned} \tag{54}$$

$$-\frac{1}{4}\begin{bmatrix} a+1 & a+2 & b-1 & b \\ b & b+1 & a-1 & a \end{bmatrix}+\frac{1}{4}\begin{bmatrix} a-1 & a \\ b & b+1 \end{bmatrix}\begin{bmatrix} a+1 & a+2 \\ b-1 & b \end{bmatrix} \tag{55}$$



$$-\frac{1}{4}\begin{bmatrix} a+1 & a+2 & b-1 & b \\ b+1 & b+2 & a-1 & a \end{bmatrix}+\frac{1}{4}\begin{bmatrix} a+1 & a+2 \\ b-1 & b \end{bmatrix}\begin{bmatrix} b+1 & b+2 \\ a-1 & a \end{bmatrix} \tag{56}$$

$$-\frac{1}{4}\begin{bmatrix} a & a+1 & b-1 & b \\ b & b+1 & c-1 & c \end{bmatrix}+\frac{1}{4}\begin{bmatrix} a & a+1 \\ b-1 & b \end{bmatrix}\begin{bmatrix} b & b+1 \\ c-1 & c \end{bmatrix} \tag{57}$$

$$-\frac{1}{4}\begin{bmatrix} a & a+1 & b-1 & b \\ b+1 & b+2 & c-1 & c \end{bmatrix}+\frac{1}{4}\begin{bmatrix} a & a+1 \\ b-1 & b \end{bmatrix}\begin{bmatrix} b+1 & b+2 \\ c-1 & c \end{bmatrix}$$

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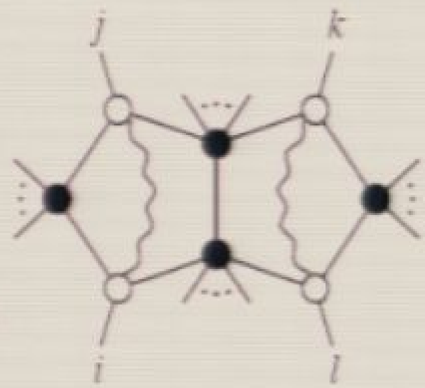
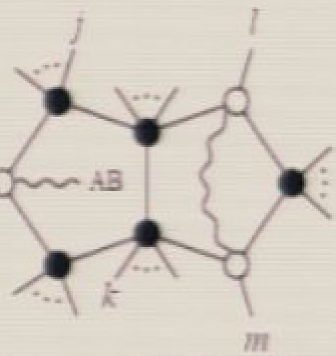
$$A^{2-\text{loop}}_{\text{MHV}} = \frac{1}{2}\sum_{i<j<k<l<i}$$

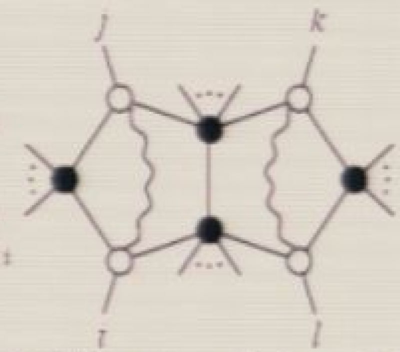


$$A^{2-\text{loop}}_{\text{NMHV}} = \sum_{\substack{i<j<l<m\leq k<i\\ i<j<k<l<m\leq i\\ i\leq l<m\leq j<k<i}}$$



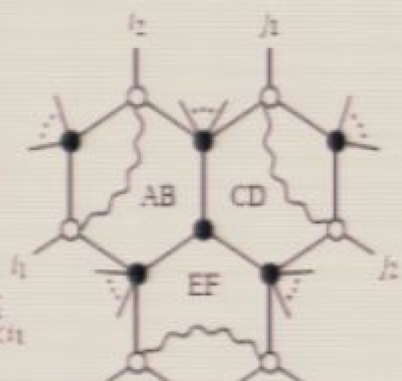
$$\times\,[i, j, j+1, k, k+1]$$

$$+\frac{1}{2}\sum_{i<j<k<l<i}$$

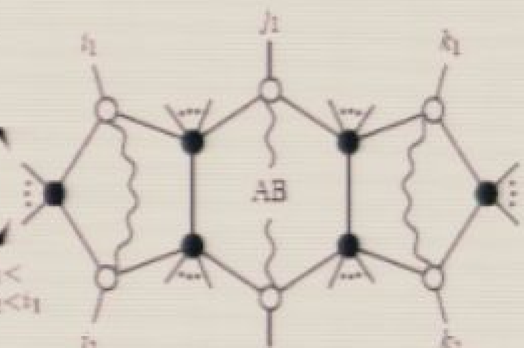


$$\times \left\{ \begin{array}{l} A^{\text{tree}}_{\text{NMHV}}(j,\ldots,k;\,l,\ldots,i) \\ \quad +A^{\text{tree}}_{\text{NMHV}}(i,\ldots,j) \\ \quad +A^{\text{tree}}_{\text{NMHV}}(k,\ldots,l) \end{array} \right\}$$

$$A^{3-\text{loop}}_{\text{MHV}} = \frac{1}{3}\sum_{\substack{i_1\leq i_2<j_1\leq \\ \leq j_2<k_1\leq k_2<i_1}}$$



$$+\,\frac{1}{2}\sum_{\substack{i_1\leq j_1<k_1< \\ <k_2\leq j_2<i_2<i_1}}$$



250 / Ne

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$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3}, v_{321}, \frac{1}{1-u_3}, 1; 1\right) + \frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3}, v_{321}, \frac{1}{1-u_3}, \frac{1}{1-u_3}; 1\right) +$$
$$\frac{1}{2}\mathcal{G}\left(v_{123}, 0, 1, \frac{1}{1-u_1}; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{123}, 0, \frac{1}{1-u_1}, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{123}, 1, 0, \frac{1}{1-u_1}; 1\right) -$$
$$\frac{5}{4}\mathcal{G}\left(v_{123}, 1, 1, \frac{1}{1-u_1}; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{123}, 1, \frac{1}{1-u_1}, 0; 1\right) - \frac{5}{4}\mathcal{G}\left(v_{123}, 1, \frac{1}{1-u_1}, 1; 1\right) +$$
$$\frac{1}{2}\mathcal{G}\left(v_{123}, 1, \frac{1}{1-u_1}, \frac{1}{1-u_1}; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{123}, \frac{1}{1-u_1}, 0, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{123}, \frac{1}{1-u_1}, 1, 0; 1\right) -$$
$$\frac{5}{4}\mathcal{G}\left(v_{123}, \frac{1}{1-u_1}, 1, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{123}, \frac{1}{1-u_1}, 1, \frac{1}{1-u_1}; 1\right) +$$
$$\frac{1}{2}\mathcal{G}\left(v_{123}, \frac{1}{1-u_1}, \frac{1}{1-u_1}, 1; 1\right) - \frac{1}{4}\mathcal{G}\left(v_{132}, 1, 1, \frac{1}{1-u_1}; 1\right) - \frac{1}{4}\mathcal{G}\left(v_{132}, 1, \frac{1}{1-u_1}, 1; 1\right) -$$
$$\frac{1}{4}\mathcal{G}\left(v_{132}, \frac{1}{1-u_1}, 1, 1; 1\right) - \frac{1}{4}\mathcal{G}\left(v_{213}, 1, 1, \frac{1}{1-u_2}; 1\right) - \frac{1}{4}\mathcal{G}\left(v_{213}, 1, \frac{1}{1-u_2}, 1; 1\right) -$$
$$\frac{1}{4}\mathcal{G}\left(v_{213}, \frac{1}{1-u_2}, 1, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{231}, 0, 1, \frac{1}{1-u_2}; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{231}, 0, \frac{1}{1-u_2}, 1; 1\right) +$$
$$\frac{1}{2}\mathcal{G}\left(v_{231}, 1, 0, \frac{1}{1-u_2}; 1\right) - \frac{5}{4}\mathcal{G}\left(v_{231}, 1, 1, \frac{1}{1-u_2}; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{231}, 1, \frac{1}{1-u_2}, 0; 1\right) -$$
$$\frac{5}{4}\mathcal{G}\left(v_{231}, 1, \frac{1}{1-u_2}, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{231}, 1, \frac{1}{1-u_2}, \frac{1}{1-u_2}; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{231}, \frac{1}{1-u_2}, 0, 1; 1\right) +$$
$$\frac{1}{2}\mathcal{G}\left(v_{231}, \frac{1}{1-u_2}, 1, 0; 1\right) - \frac{5}{4}\mathcal{G}\left(v_{231}, \frac{1}{1-u_2}, 1, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{231}, \frac{1}{1-u_2}, 1, \frac{1}{1-u_2}; 1\right) +$$
$$\frac{1}{2}\mathcal{G}\left(v_{231}, \frac{1}{1-u_2}, \frac{1}{1-u_2}, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{312}, 0, 1, \frac{1}{1-u_3}; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{312}, 0, \frac{1}{1-u_3}, 1; 1\right) +$$
$$\frac{1}{2}\mathcal{G}\left(v_{312}, 1, 0, \frac{1}{1-u_3}; 1\right) - \frac{5}{4}\mathcal{G}\left(v_{312}, 1, 1, \frac{1}{1-u_3}; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{312}, 1, \frac{1}{1-u_3}, 0; 1\right) -$$
$$\frac{5}{4}\mathcal{G}\left(v_{312}, 1, \frac{1}{1-u_3}, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{312}, 1, \frac{1}{1-u_3}, \frac{1}{1-u_3}; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{312}, \frac{1}{1-u_3}, 0, 1; 1\right) +$$
$$\frac{1}{2}\mathcal{G}\left(v_{312}, \frac{1}{1-u_3}, 1, 0; 1\right) - \frac{5}{4}\mathcal{G}\left(v_{312}, \frac{1}{1-u_3}, 1, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{312}, \frac{1}{1-u_3}, 1, \frac{1}{1-u_3}; 1\right) +$$
$$\frac{1}{2}\mathcal{G}\left(v_{312}, \frac{1}{1-u_3}, \frac{1}{1-u_3}, 1; 1\right) - \frac{1}{4}\mathcal{G}\left(v_{321}, 1, 1, \frac{1}{1-u_3}; 1\right) - \frac{1}{4}\mathcal{G}\left(v_{321}, 1, \frac{1}{1-u_3}, 1; 1\right) -$$
$$\frac{1}{4}\mathcal{G}\left(v_{321}, \frac{1}{1-u_3}, 1, 1; 1\right) - \frac{3}{4}G\left(0, \frac{1}{u_1}, \frac{1}{u_1+u_2}; 1\right) H(0; u_1) -$$

251 / Ne

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$$\frac{1}{2}\mathcal{G}\left(c_{123}, \frac{1}{1-u_1}, \frac{1}{1-u_1}, 1; 1\right) - \frac{1}{4}\mathcal{G}\left(c_{132}, 1, 1, \frac{1}{1-u_1}; 1\right) - \frac{1}{4}\mathcal{G}\left(c_{132}, 1, \frac{1}{1-u_1}, 1; 1\right) -$$

$$\frac{1}{4}\mathcal{G}\left(c_{132}, \frac{1}{1-u_1}, 1, 1; 1\right) - \frac{1}{4}\mathcal{G}\left(c_{213}, 1, 1, \frac{1}{1-u_2}; 1\right) - \frac{1}{4}\mathcal{G}\left(c_{213}, 1, \frac{1}{1-u_2}, 1; 1\right) -$$

$$\frac{1}{4}\mathcal{G}\left(c_{213}, \frac{1}{1-u_2}, 1, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(c_{231}, 0, 1, \frac{1}{1-u_2}; 1\right) + \frac{1}{2}\mathcal{G}\left(c_{231}, 0, \frac{1}{1-u_2}, 1; 1\right) +$$

$$\frac{1}{2}\mathcal{G}\left(c_{231}, 1, 0, \frac{1}{1-u_2}; 1\right) - \frac{5}{4}\mathcal{G}\left(c_{231}, 1, 1, \frac{1}{1-u_2}; 1\right) + \frac{1}{2}\mathcal{G}\left(c_{231}, 1, \frac{1}{1-u_2}, 0; 1\right) -$$

$$\frac{5}{4}\mathcal{G}\left(c_{231}, 1, \frac{1}{1-u_2}, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(c_{231}, 1, \frac{1}{1-u_2}, \frac{1}{1-u_2}; 1\right) + \frac{1}{2}\mathcal{G}\left(c_{231}, \frac{1}{1-u_2}, 0, 1; 1\right) +$$

$$\frac{1}{2}\mathcal{G}\left(c_{231}, \frac{1}{1-u_2}, 1, 0; 1\right) - \frac{5}{4}\mathcal{G}\left(c_{231}, \frac{1}{1-u_2}, 1, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(c_{231}, \frac{1}{1-u_2}, 1, \frac{1}{1-u_2}; 1\right) +$$

$$\frac{1}{2}\mathcal{G}\left(c_{231}, \frac{1}{1-u_2}, \frac{1}{1-u_2}, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(c_{312}, 0, 1, \frac{1}{1-u_3}; 1\right) + \frac{1}{2}\mathcal{G}\left(c_{312}, 0, \frac{1}{1-u_3}, 1; 1\right) +$$

$$\frac{1}{2}\mathcal{G}\left(c_{312}, 1, 0, \frac{1}{1-u_3}; 1\right) - \frac{5}{4}\mathcal{G}\left(c_{312}, 1, 1, \frac{1}{1-u_3}; 1\right) + \frac{1}{2}\mathcal{G}\left(c_{312}, 1, \frac{1}{1-u_3}, 0; 1\right) -$$

$$\frac{5}{4}\mathcal{G}\left(c_{312}, 1, \frac{1}{1-u_3}, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(c_{312}, 1, \frac{1}{1-u_3}, \frac{1}{1-u_3}; 1\right) + \frac{1}{2}\mathcal{G}\left(c_{312}, \frac{1}{1-u_3}, 0, 1; 1\right) +$$

$$\frac{1}{2}\mathcal{G}\left(c_{312}, \frac{1}{1-u_3}, 1, 0; 1\right) - \frac{5}{4}\mathcal{G}\left(c_{312}, \frac{1}{1-u_3}, 1, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(c_{312}, \frac{1}{1-u_3}, 1, \frac{1}{1-u_3}; 1\right) +$$

$$\frac{1}{2}\mathcal{G}\left(c_{312}, \frac{1}{1-u_3}, \frac{1}{1-u_3}, 1; 1\right) - \frac{1}{4}\mathcal{G}\left(c_{321}, 1, 1, \frac{1}{1-u_3}; 1\right) - \frac{1}{4}\mathcal{G}\left(c_{321}, 1, \frac{1}{1-u_3}, 1; 1\right) -$$

$$\frac{1}{4}\mathcal{G}\left(c_{321}, \frac{1}{1-u_3}, 1, 1; 1\right) - \frac{3}{4}G\left(0, \frac{1}{u_1}, \frac{1}{u_1+u_2}; 1\right) H(0; u_1) -$$

$$\frac{3}{4}G\left(0, \frac{1}{u_1}, \frac{1}{u_1+u_3}; 1\right) H(0; u_1) - \frac{1}{4}G\left(0, \frac{1}{u_2}, \frac{1}{u_1+u_2}; 1\right) H(0; u_1) -$$

$$\frac{1}{4}G\left(0, \frac{1}{u_3}, \frac{1}{u_1+u_3}; 1\right) H(0; u_1) - \frac{1}{4}G\left(0, \frac{u_1-1}{u_1+u_3-1}, \frac{1}{1-u_3}; 1\right) H(0; u_1) +$$

$$\frac{1}{4}G\left(0, \frac{u_3-1}{u_2+u_3-1}, \frac{1}{1-u_2}; 1\right) H(0; u_1) - \frac{3}{4}G\left(\frac{1}{u_1}, 0, \frac{1}{u_1+u_2}; 1\right) H(0; u_1) -$$

$$\frac{3}{4}G\left(\frac{1}{u_1}, 0, \frac{1}{u_1+u_3}; 1\right) H(0; u_1) + \frac{1}{2}G\left(\frac{1}{u_1}, \frac{1}{u_1}, \frac{1}{u_1+u_2}; 1\right) H(0; u_1) +$$

$$\frac{1}{2}G\left(\frac{1}{u_1}, \frac{1}{u_1}, \frac{1}{u_1+u_3}; 1\right) H(0; u_1) + \frac{1}{4}G\left(\frac{1}{u_1}, \frac{1}{u_2}, \frac{1}{u_1+u_2}; 1\right) H(0; u_1) +$$

$$\frac{1}{4}G\left(\frac{1}{u_1}, \frac{1}{u_3}, \frac{1}{u_1+u_3}; 1\right) H(0; u_1) - \frac{1}{4}G\left(\frac{1}{1-u_2}, 1, \frac{1}{u_1}; 1\right) H(0; u_1) +$$

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$$
\begin{aligned}
&\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3}, v_{321}, \frac{1}{1-u_3}, 1; 1\right) + \frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3}, v_{321}, \frac{1}{1-u_3}, \frac{1}{1-u_3}; 1\right) + \\
&\frac{1}{2}\mathcal{G}\left(v_{123}, 0, 1, \frac{1}{1-u_1}; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{123}, 0, \frac{1}{1-u_1}, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{123}, 1, 0, \frac{1}{1-u_1}; 1\right) - \\
&\frac{5}{4}\mathcal{G}\left(v_{123}, 1, 1, \frac{1}{1-u_1}; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{123}, 1, \frac{1}{1-u_1}, 0; 1\right) - \frac{5}{4}\mathcal{G}\left(v_{123}, 1, \frac{1}{1-u_1}, 1; 1\right) + \\
&\frac{1}{2}\mathcal{G}\left(v_{123}, 1, \frac{1}{1-u_1}, \frac{1}{1-u_1}; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{123}, \frac{1}{1-u_1}, 0, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{123}, \frac{1}{1-u_1}, 1, 0; 1\right) - \\
&\frac{5}{4}\mathcal{G}\left(v_{123}, \frac{1}{1-u_1}, 1, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{123}, \frac{1}{1-u_1}, 1, \frac{1}{1-u_1}; 1\right) + \\
&\frac{1}{2}\mathcal{G}\left(v_{123}, \frac{1}{1-u_1}, \frac{1}{1-u_1}, 1; 1\right) - \frac{1}{4}\mathcal{G}\left(v_{132}, 1, 1, \frac{1}{1-u_1}; 1\right) - \frac{1}{4}\mathcal{G}\left(v_{132}, 1, \frac{1}{1-u_1}, 1; 1\right) - \\
&\frac{1}{4}\mathcal{G}\left(v_{132}, \frac{1}{1-u_1}, 1, 1; 1\right) - \frac{1}{4}\mathcal{G}\left(v_{213}, 1, 1, \frac{1}{1-u_2}; 1\right) - \frac{1}{4}\mathcal{G}\left(v_{213}, 1, \frac{1}{1-u_2}, 1; 1\right) - \\
&\frac{1}{4}\mathcal{G}\left(v_{213}, \frac{1}{1-u_2}, 1, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{231}, 0, 1, \frac{1}{1-u_2}; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{231}, 0, \frac{1}{1-u_2}, 1; 1\right) + \\
&\frac{1}{2}\mathcal{G}\left(v_{231}, 1, 0, \frac{1}{1-u_2}; 1\right) - \frac{5}{4}\mathcal{G}\left(v_{231}, 1, 1, \frac{1}{1-u_2}; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{231}, 1, \frac{1}{1-u_2}, 0; 1\right) - \\
&\frac{5}{4}\mathcal{G}\left(v_{231}, 1, \frac{1}{1-u_2}, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{231}, 1, \frac{1}{1-u_2}, \frac{1}{1-u_2}; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{231}, \frac{1}{1-u_2}, 0, 1; 1\right) + \\
&\frac{1}{2}\mathcal{G}\left(v_{231}, \frac{1}{1-u_2}, 1, 0; 1\right) - \frac{5}{4}\mathcal{G}\left(v_{231}, \frac{1}{1-u_2}, 1, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{231}, \frac{1}{1-u_2}, 1, \frac{1}{1-u_2}; 1\right) + \\
&\frac{1}{2}\mathcal{G}\left(v_{231}, \frac{1}{1-u_2}, \frac{1}{1-u_2}, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{312}, 0, 1, \frac{1}{1-u_3}; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{312}, 0, \frac{1}{1-u_3}, 1; 1\right) + \\
&\frac{1}{2}\mathcal{G}\left(v_{312}, 1, 0, \frac{1}{1-u_3}; 1\right) - \frac{5}{4}\mathcal{G}\left(v_{312}, 1, 1, \frac{1}{1-u_3}; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{312}, 1, \frac{1}{1-u_3}, 0; 1\right) - \\
&\frac{5}{4}\mathcal{G}\left(v_{312}, 1, \frac{1}{1-u_3}, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{312}, 1, \frac{1}{1-u_3}, \frac{1}{1-u_3}; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{312}, \frac{1}{1-u_3}, 0, 1; 1\right) + \\
&\frac{1}{2}\mathcal{G}\left(v_{312}, \frac{1}{1-u_3}, 1, 0; 1\right) - \frac{5}{4}\mathcal{G}\left(v_{312}, \frac{1}{1-u_3}, 1, 1; 1\right) + \frac{1}{2}\mathcal{G}\left(v_{312}, \frac{1}{1-u_3}, 1, \frac{1}{1-u_3}; 1\right) + \\
&\frac{1}{2}\mathcal{G}\left(v_{312}, \frac{1}{1-u_3}, \frac{1}{1-u_3}, 1; 1\right) - \frac{1}{4}\mathcal{G}\left(v_{321}, 1, 1, \frac{1}{1-u_3}; 1\right) - \frac{1}{4}\mathcal{G}\left(v_{321}, 1, \frac{1}{1-u_3}, 1; 1\right) -
\end{aligned}
$$

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Stunning Simplification

$$R_6^{(2)}(u_1,u_2,u_3) = \sum_{i=1}^{3}\left(L_4(x_i^+,x_i^-) - \frac{1}{2}\mathrm{Li}_4(1-1/u_i)\right)$$
$$-\frac{1}{8}\left(\sum_{i=1}^{3}\mathrm{Li}_2(1-1/u_i)\right)^2 + \frac{1}{24}J^4 + \frac{\pi^2}{12}J^2 + \frac{\pi^4}{72}. \quad (3)$$

[Makes use of "theory of motives"]

252 / Next

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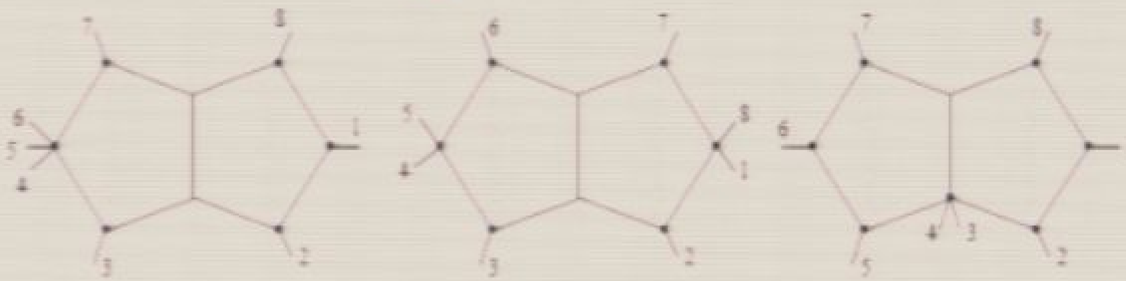
New Integrals are Simple!



(a) (b) (c)

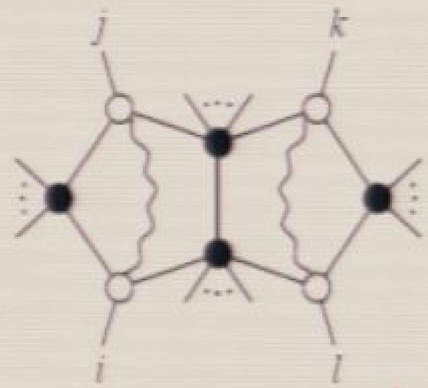
$$\partial_{\chi^+}\partial_{\chi^-} I^{total} = \tag{16}$$

$$\frac{1}{\chi^+\chi^-}\log\chi^+\log\chi^- + \frac{2(\chi^+-1)}{\chi^+\chi^-(1+\chi^+)}\log\chi^-\log(1+\chi^-) + \frac{2(\chi^--1)}{\chi^+\chi^-(1+\chi^-)}\log\chi^+\log(1+\chi^+) +$$

$$+\frac{4(\chi^+-1)}{\chi^+\chi^-(1+\chi^+)}\mathrm{Li}_2(-\chi^-) + \frac{4(\chi^--1)}{\chi^+\chi^-(1+\chi^-)}\mathrm{Li}_2(-\chi^+)$$

This Picture

$$A_{\mathrm{MHV}}^{2-\mathrm{loop}} = \frac{1}{2}\sum_{i<j<k<l<i}$$



is telling a geometric story —
once we understand it we'll just
write down the answer...

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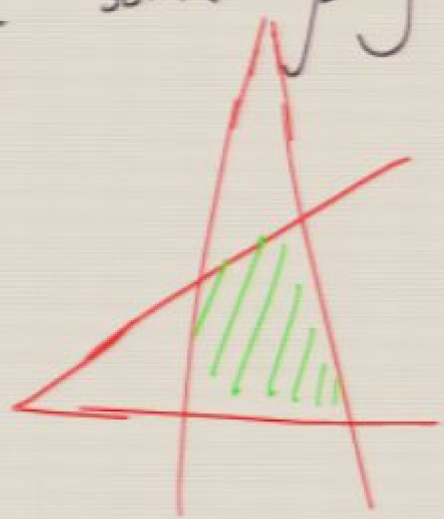
· In a specific sense, amplitudes are to be thought of as "the volume" of some polytope:



"Different triangulations" make different properties (Yangian, locality, Unitarity ...) manifest.

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Our solution should be thought of as providing one class of triangulations

– but we need to more deeply understanding what the object is that is being triangulated!

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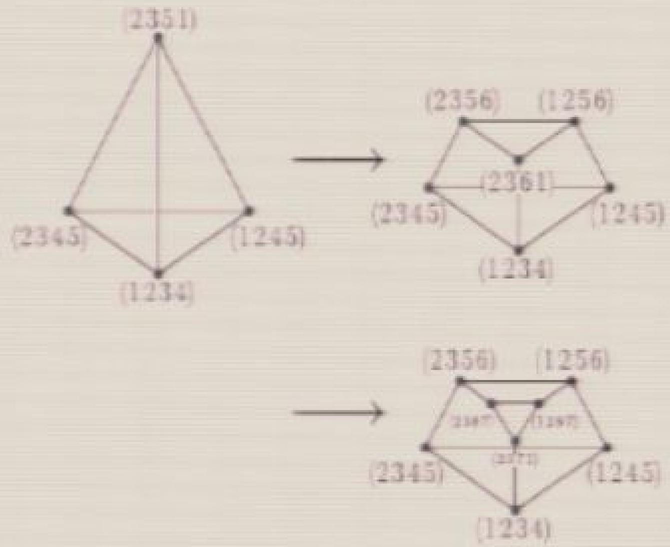
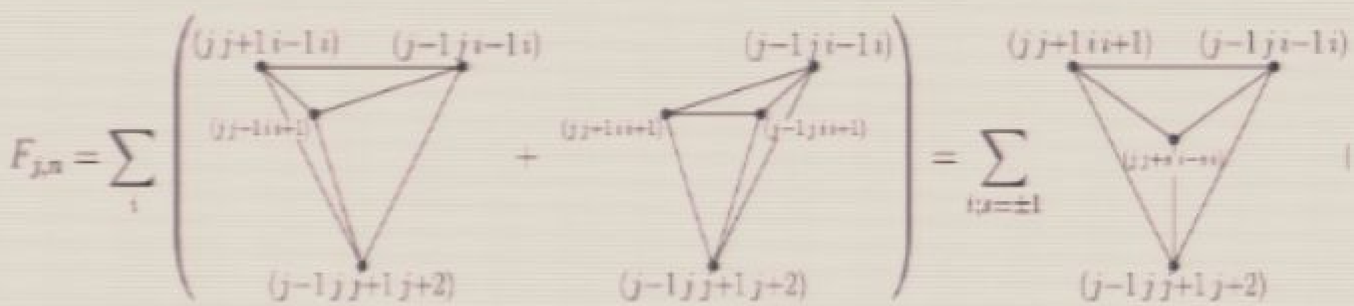


Understood
in Simple
Cases
⋮



17

$$M_n^{\mathrm{NMHV}} = \sum_{i,j;s=\pm1} \frac{\langle \eta_j . \{j-1\, j\, j+1\, j+2\, i\}.\{j-1\, j\, j+1\, i-s\, i\}.\{j\, j+s\, i-1\, i\, i+1\}\rangle}{\langle j-1\, j\, j+1\, j+2\rangle\langle j-1\, j\, i-1\, i\rangle.\langle j\, j+1\, i\, i+1\rangle\langle j\, j+s\, i-s\, i\rangle}$$

NEW LOCAL FORM!

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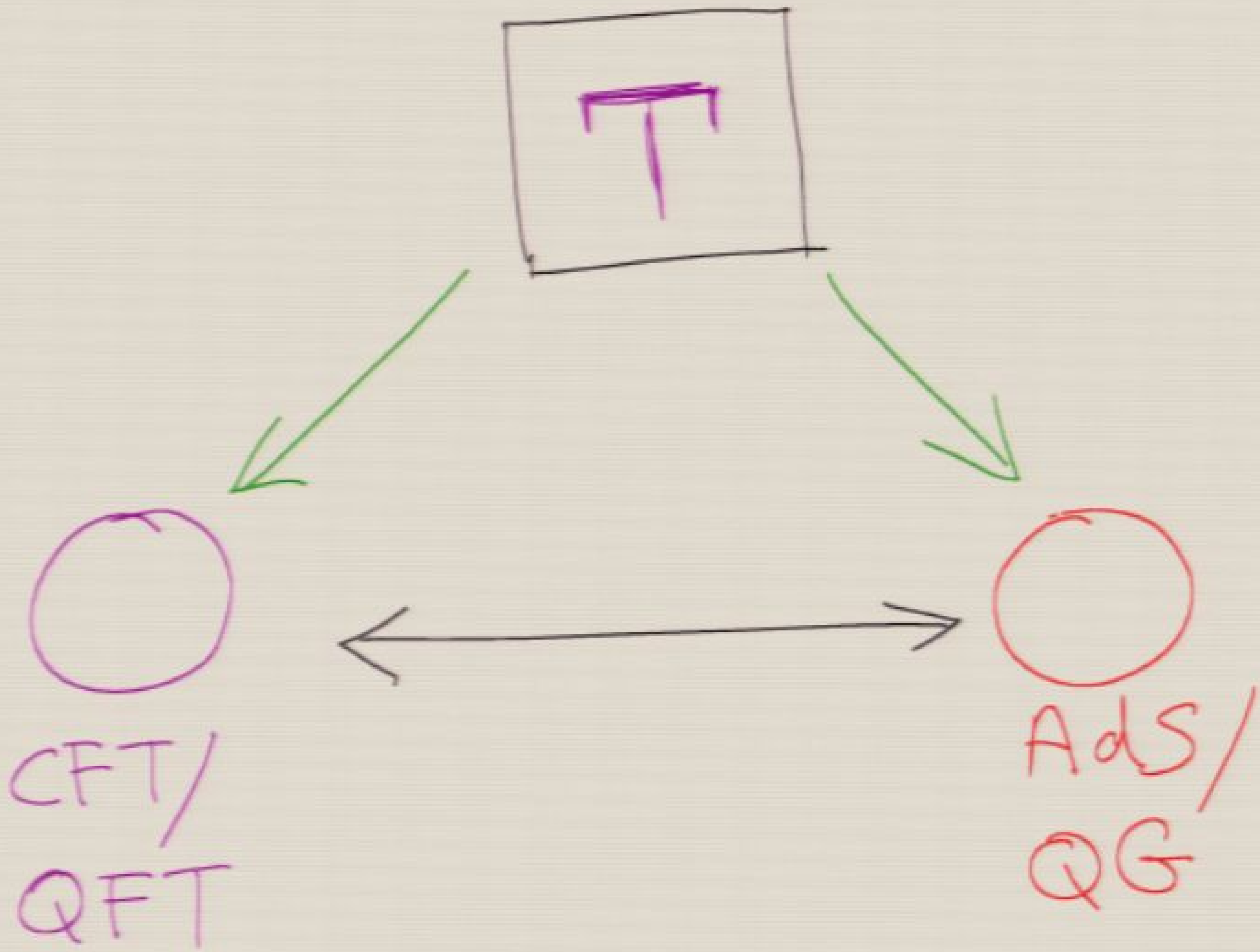

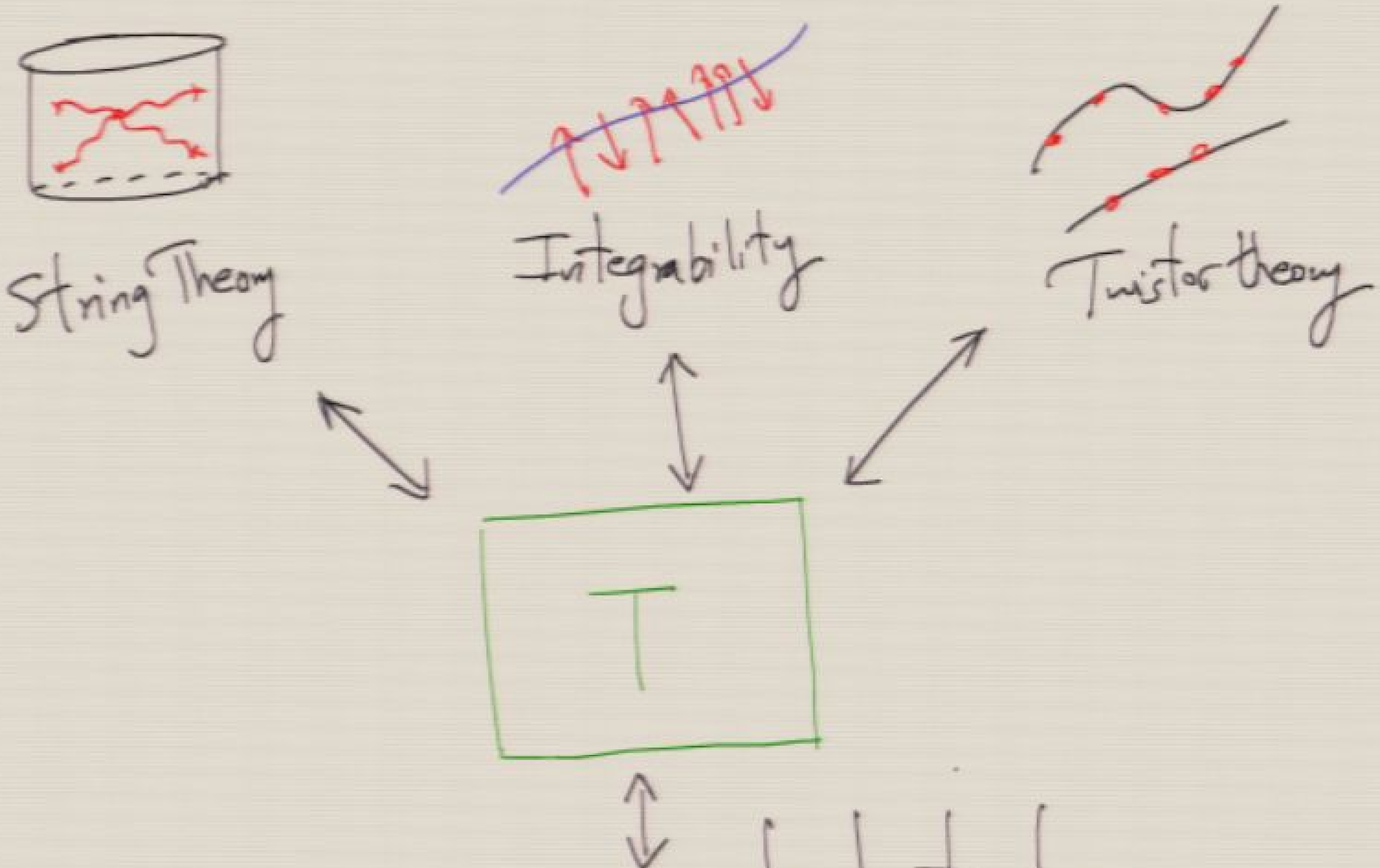

Amazing new mathematical structures:

(Grassmannians, Polylogs, "Motivic Galois Theory", ...)

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Still an enormous amount left to understand: what is the one-sentence "physical picture" behind all of this magic?

What moral lesson should we extract from $\mathcal{N}=4$ example?

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But there is strong encouragement to try + fully eviscerate Locality + Unitarity from our language for describing all of standard physics.

261 / Nex

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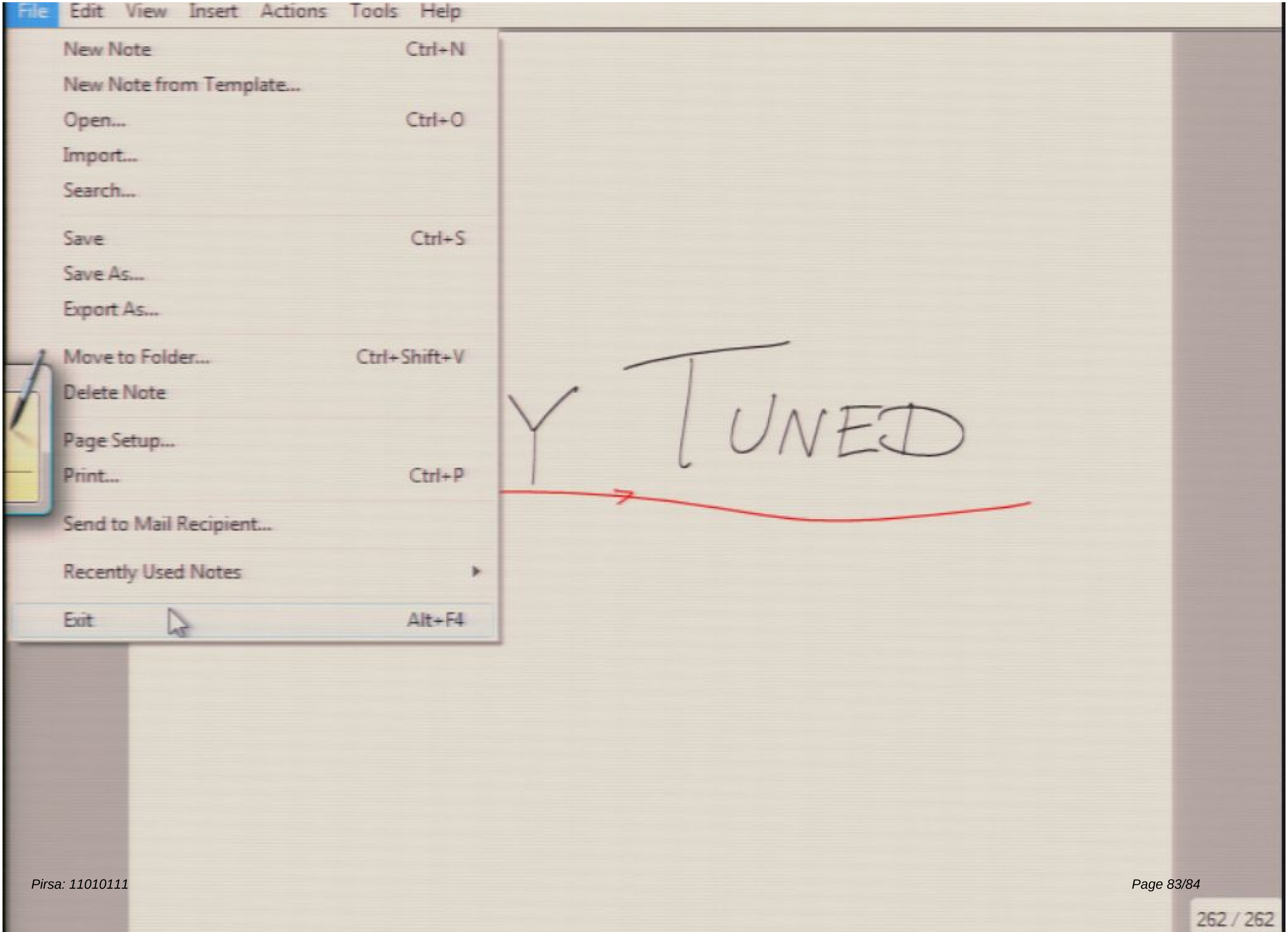
STAY TUNED

New Pag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Import...
Search...
Save Ctrl+S
Save As...
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Move to Folder... Ctrl+Shift+V
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Page Setup...
Print... Ctrl+P
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Recently Used Notes
Exit Alt+F4
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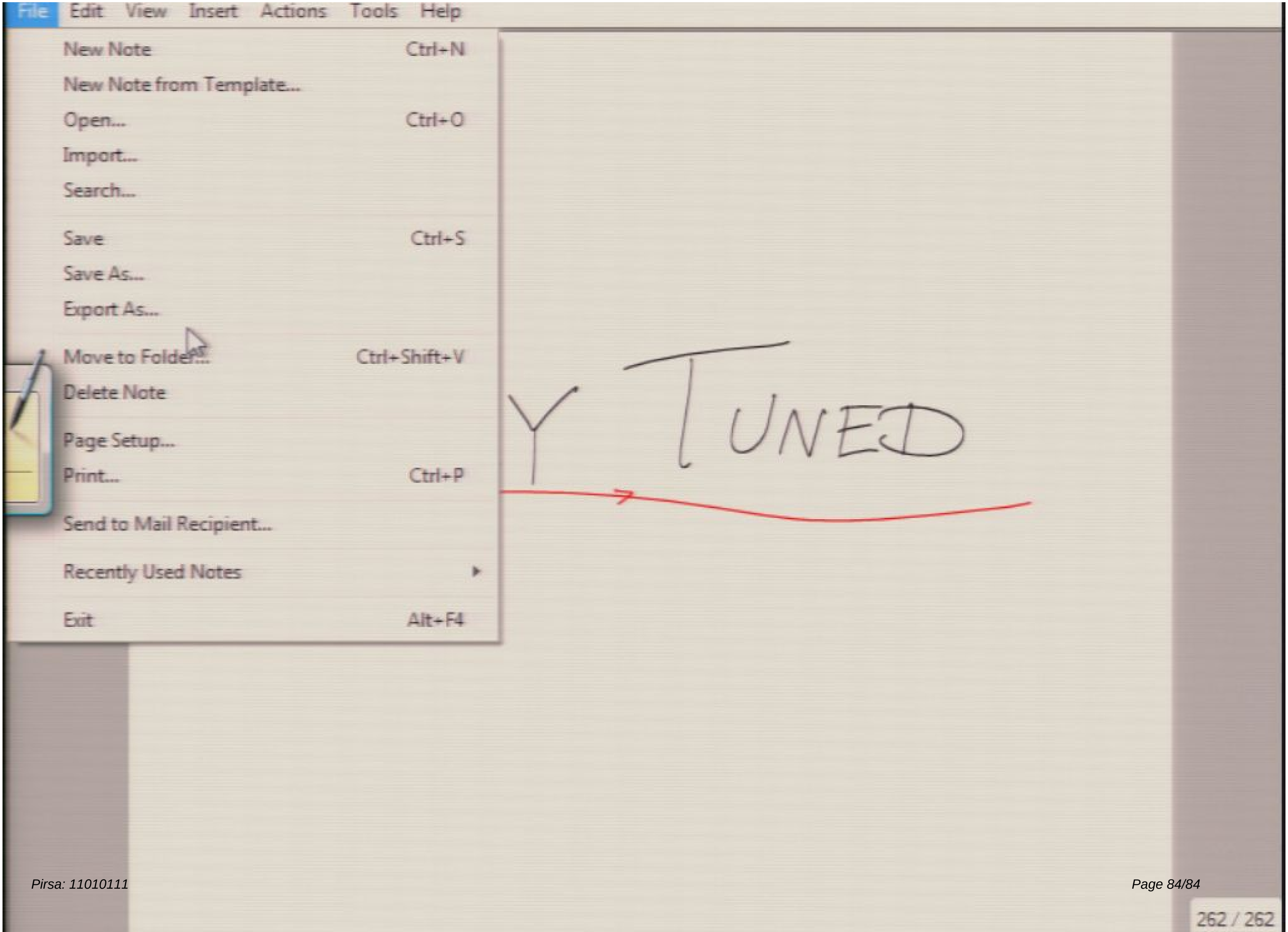
262 / 262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New Note from Template...
Open... Ctrl+O
Import...
Search...
Save Ctrl+S
Save As...
Export As...
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262 / 262