Title: Simulation of Anyons Using Tensor Network Algorithms

Date: Jan 28, 2011 11:00 AM

URL: http://pirsa.org/11010110

Abstract: The simulation of systems of anyons offers a significant challenge to the condensed matter physicist. These systems are presently of substantial theoretical and experimental interest due to their potential for universal quantum computation, but due to their non-trivial exchange statistics, the tools available for their study have been limited. In this talk, I will present a formalism whereby any existing tensor network algorithm may be adapted for use with both Abelian and non-Abelian anyons, culminating in our recent simulations of infinite 1-D chains of interacting anyons using the Multi-scale Entanglement Renormalisation Ansatz, or MERA, demonstrating that tensor network algorithms may be effectively employed in the study of anyonic systems.

Anyonic Tensor Network Algorithms

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Anyons

- Non-bosonic, non-fermionic exchange statistics
- 2-D and quasi-1-D systems
- Currently a hot topic...

Tensor Network Algorithms

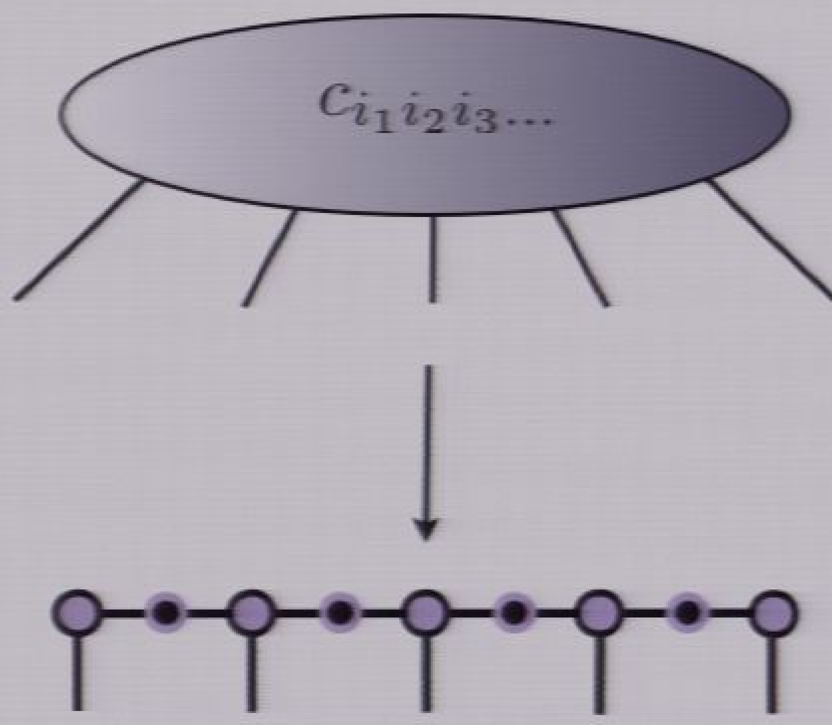
- Tensor network ansatz
- Update algorithm

$$|\psi\rangle = \sum c_{i_1 i_2 i_3 \ldots} |i_1, i_2, i_3, \ldots\rangle$$

$$= \sum \Gamma^{(1)j_1}_{i_1} \lambda^{(1)j_1} \Gamma^{(2)j_1 j_2}_{i_2} \lambda^{(2)j_2} \Gamma^{(3)j_2 j_3}_{i_3} \ldots |i_1, i_2, i_3, \ldots\rangle$$

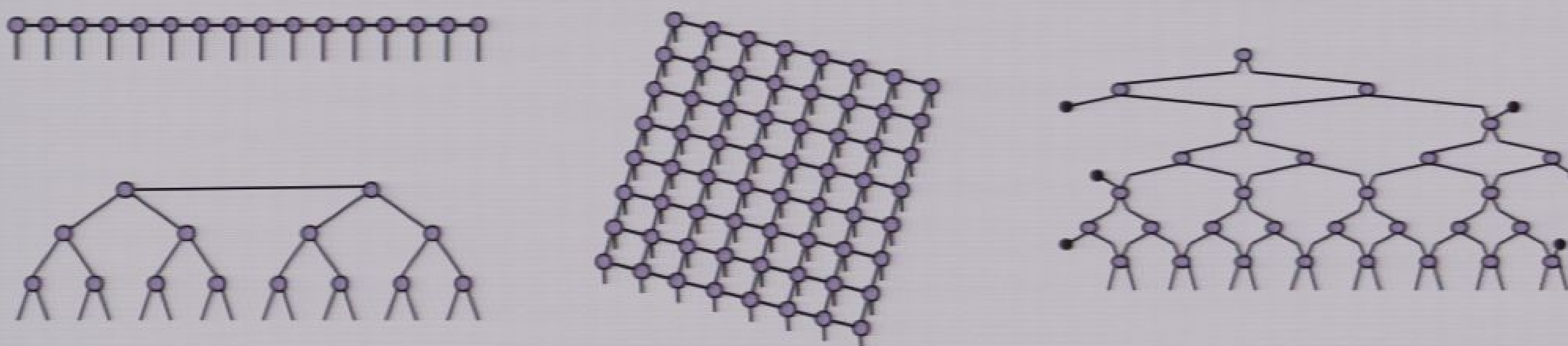
Tensor Network Algorithms

- Tensor network ansatz



Tensor Network Algorithms

- Tensor network ansatz

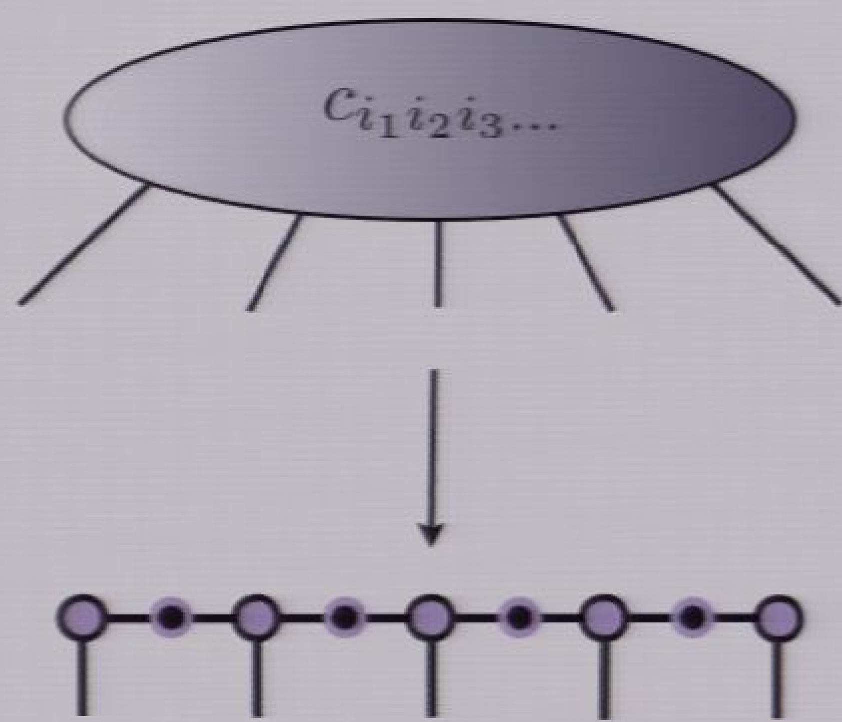


Tensor Network Algorithms

- Tensor network ansatz
- Update algorithm
  - To construct optimised representation of the ground state
  - Time evolution
  - ...

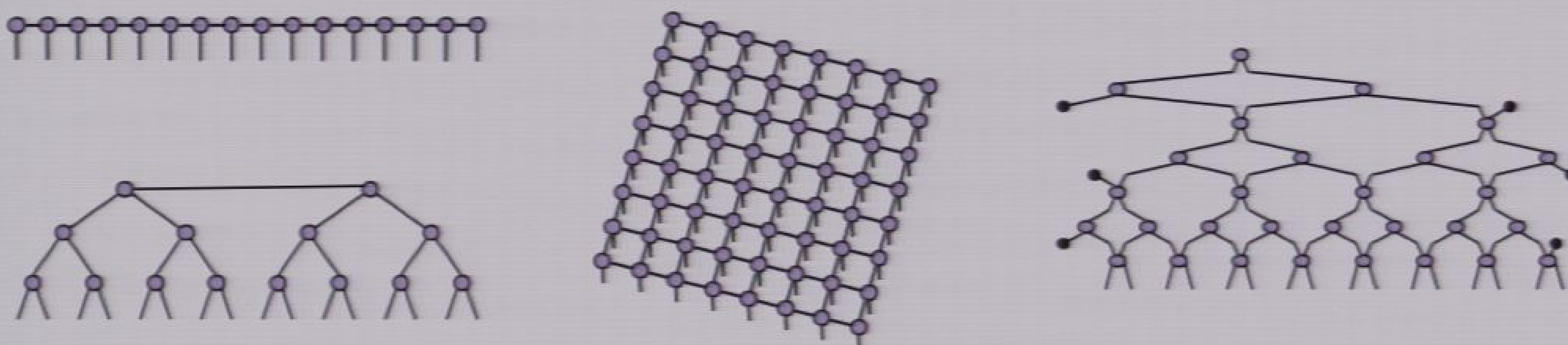
Tensor Network Algorithms

- Tensor network ansatz



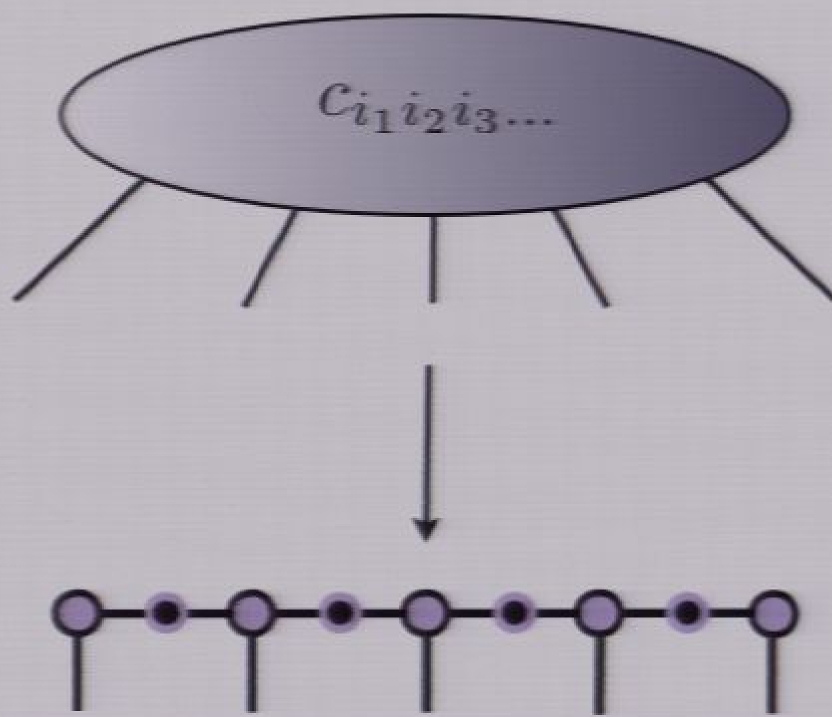
Tensor Network Algorithms

- Tensor network ansatz



Tensor Network Algorithms

- Tensor network ansatz



Tensor Network Algorithms

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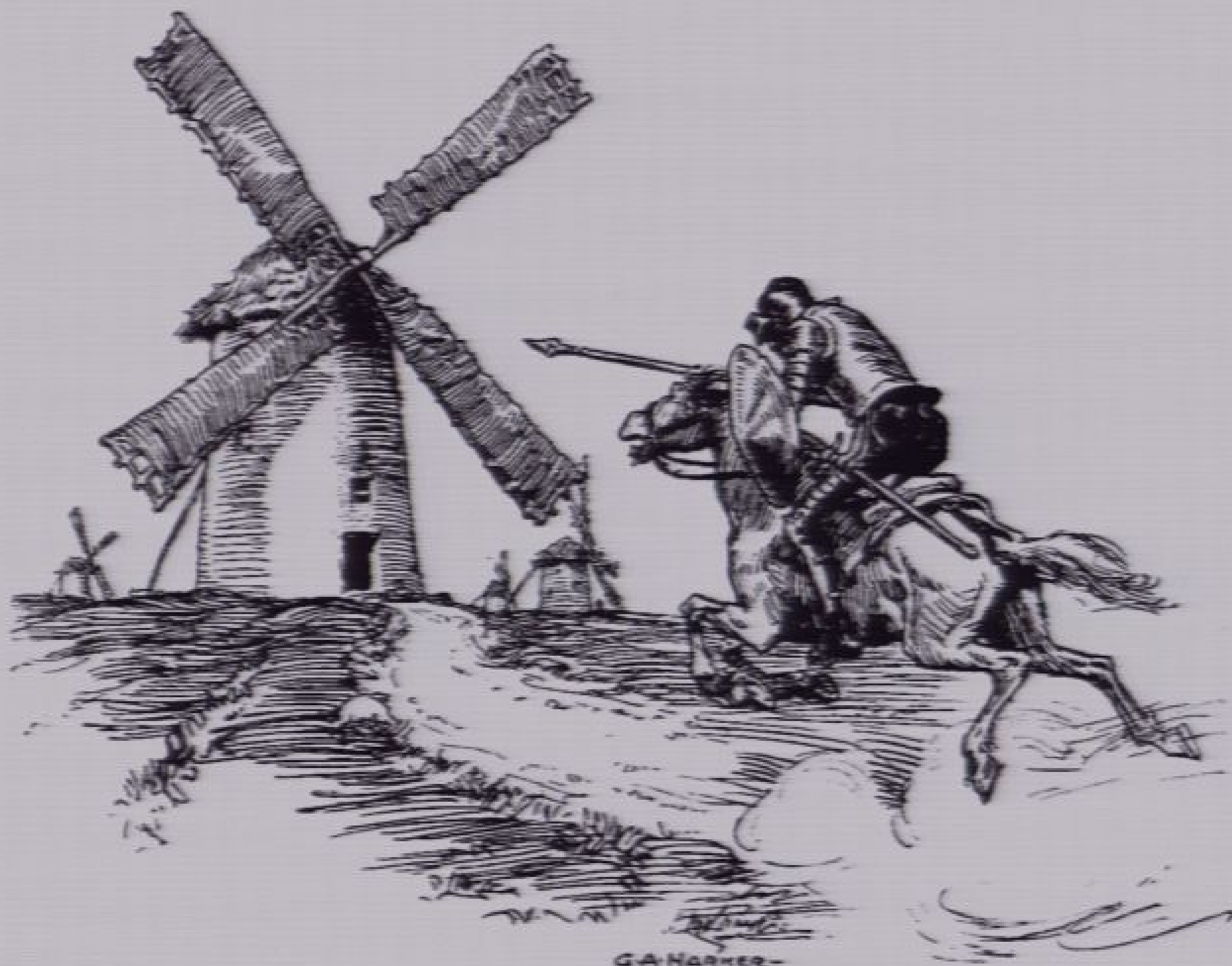
  - ...

Tensor Networks and Fermions

- Successful description of low-dimensional bosonic systems
- Recent extension to fermionic systems in 1-D and 2-D (e.g. Corboz et al., 2009)

Tensor Networks and Fermions

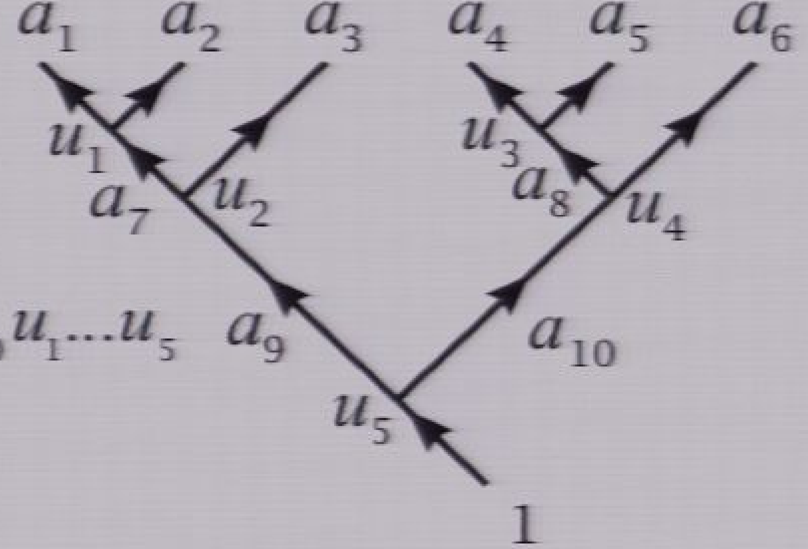


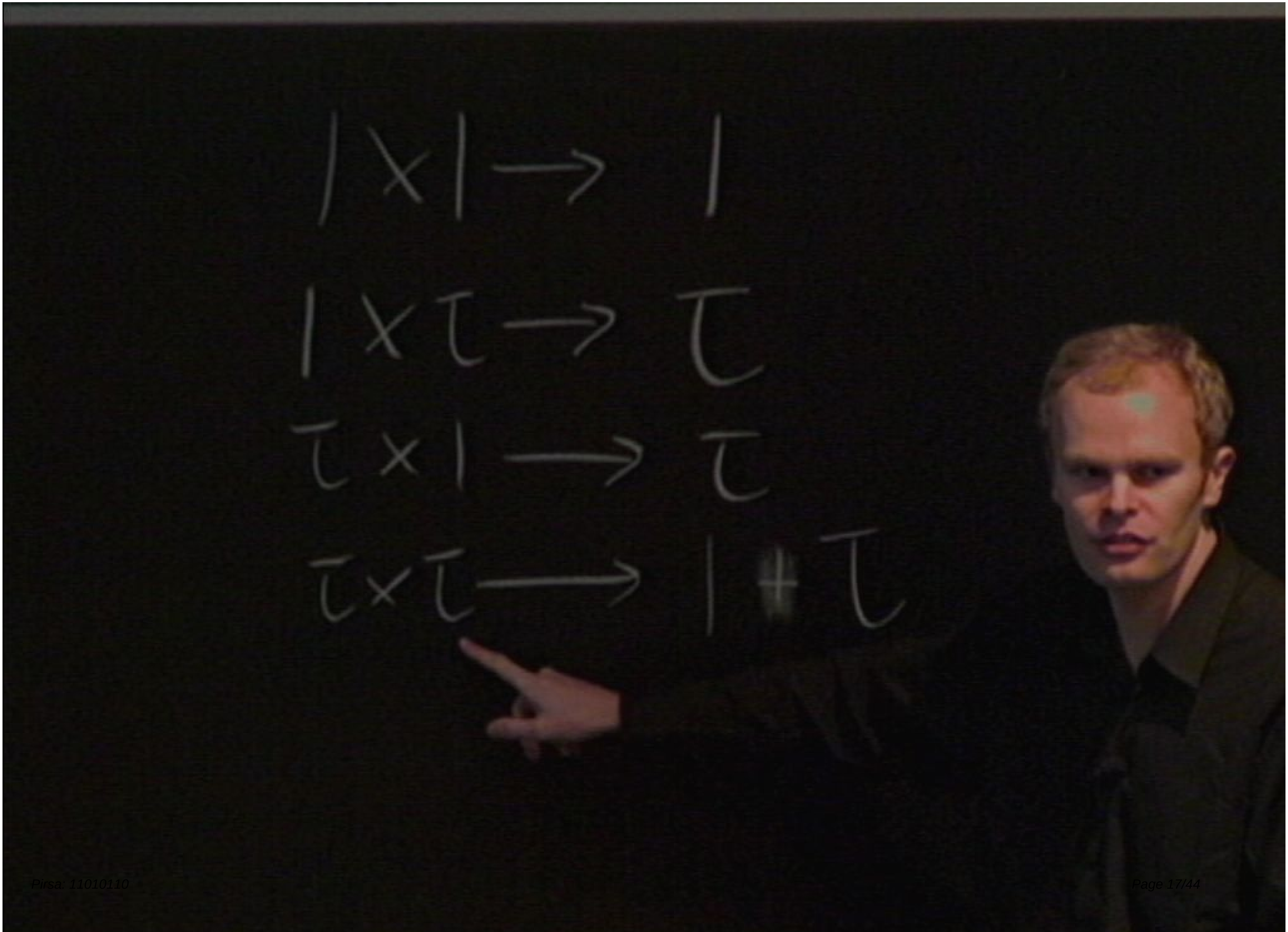
Tensor Networks and Fermions

- Successful description of low-dimensional bosonic systems
- Recent extension to fermionic systems in 1-D and 2-D (e.g. Corboz et al., 2009)
- No "sign problem"
- Can we generalise this success to anyons?

Tensor Networks and Anyons

$$|\psi\rangle = \sum_{\substack{a_1 \ldots a_{10} \\ u_1 \ldots u_5}} c_{a_1 \ldots a_{10} u_1 \ldots u_5}$$




1×1 → 1
1×τ → τ
τ×1 → τ
τ×τ → 1 + τ

$$1 \times 1 \to 1$$

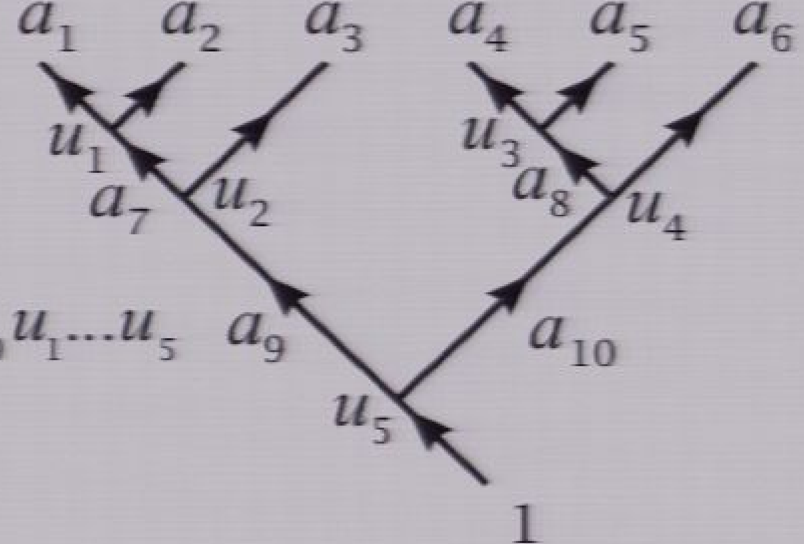
$$1 \times \tau \to \tau$$

$$\tau \times 1 \to \tau$$

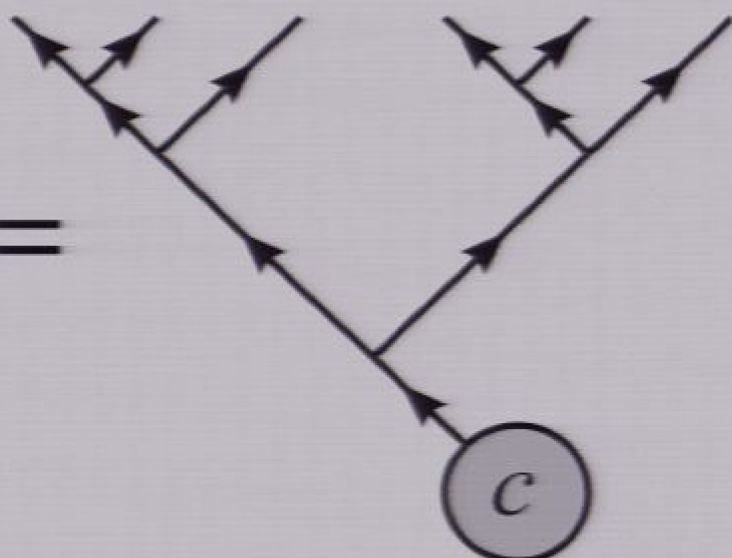
$$\tau \times \tau \to 1 + \tau$$

Tensor Networks and Anyons

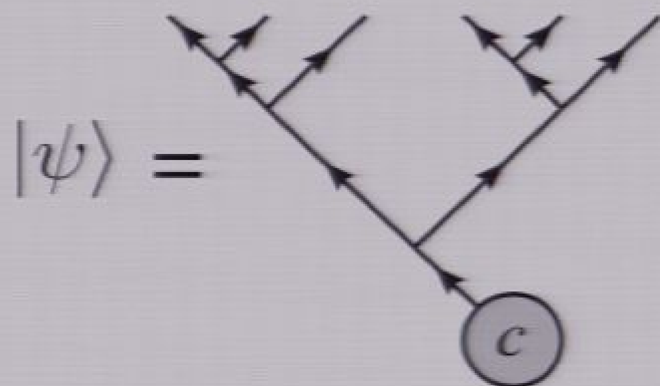
$$|\psi\rangle = \sum_{\substack{a_1 \ldots a_{10} \\ u_1 \ldots u_5}} c_{a_1 \ldots a_{10} u_1 \ldots u_5}$$

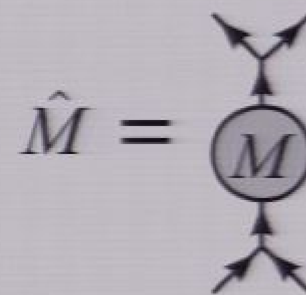


Tensor Networks and Anyons

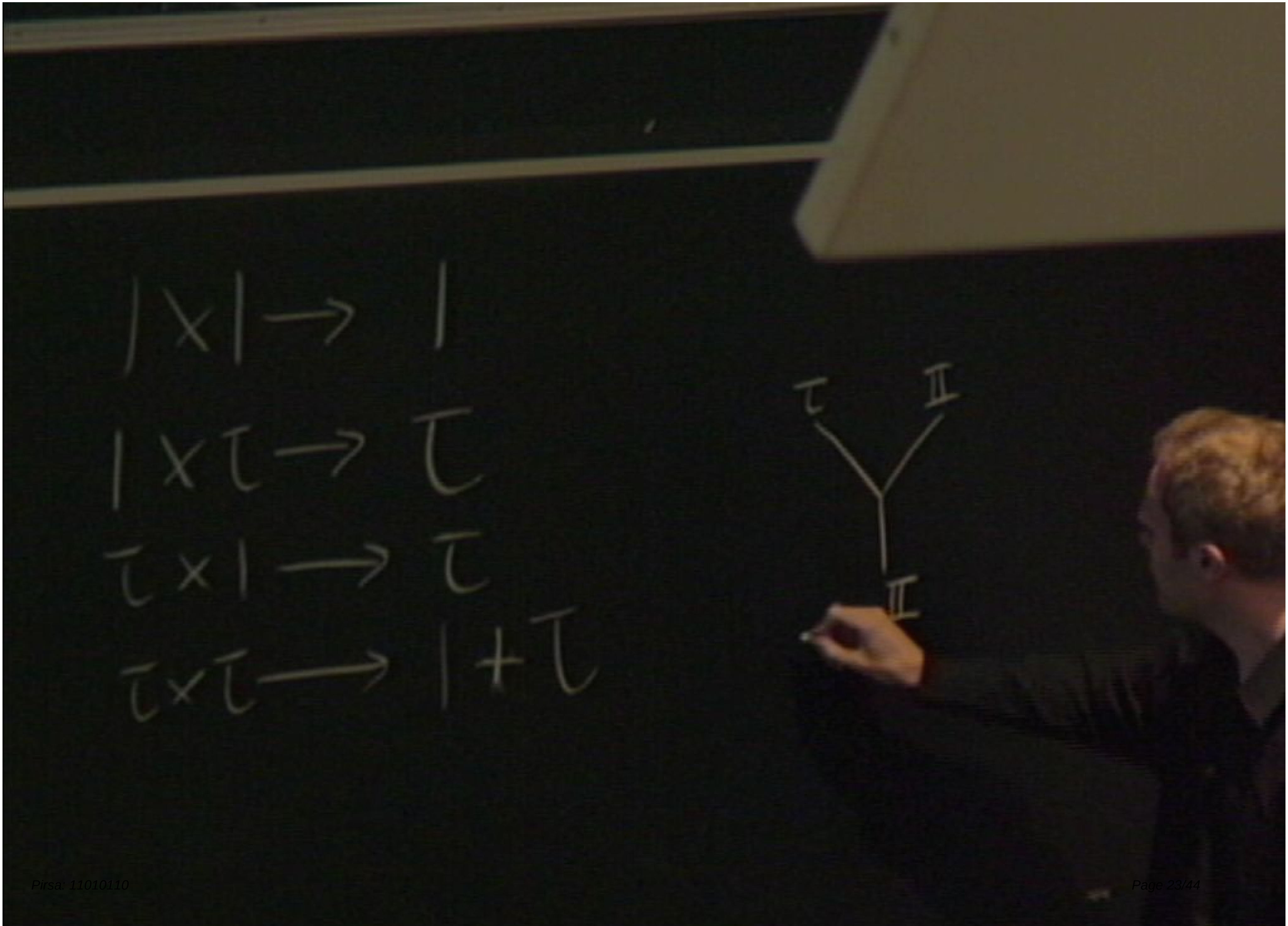
$|\psi\rangle =$ 

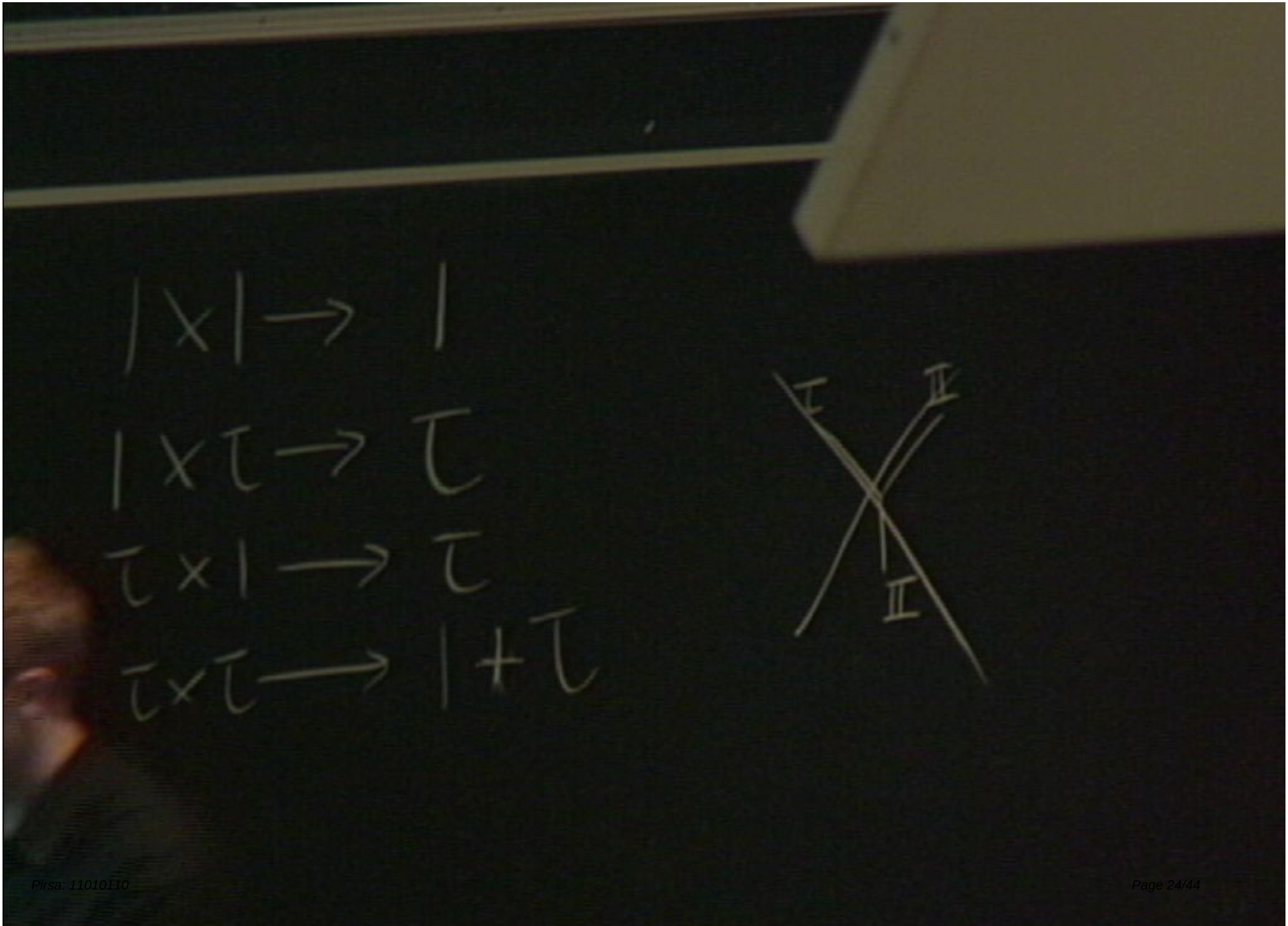
Tensor Networks and Anyons



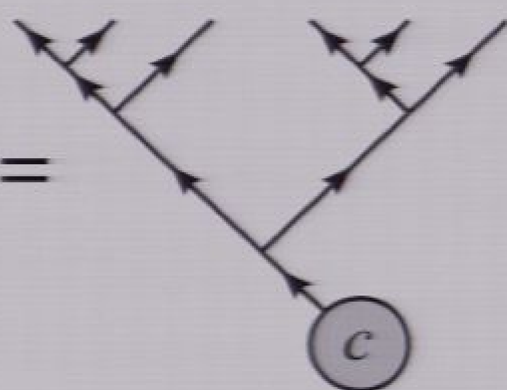


c^{α} M^{α}_{β}


1×1 → 1
1×τ → τ
τ×1 → τ
τ×τ → 1+τ
τ
II
II


1×1 → 1
1×τ → τ
τ×1 → τ
τ×τ → 1+τ
I
IV

Tensor Networks and Anyons

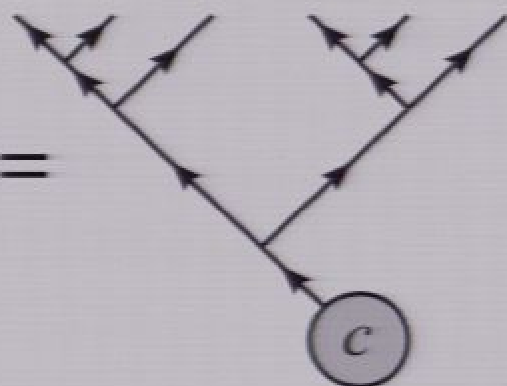
$|\psi\rangle =$ 

$\hat{M} =$ 

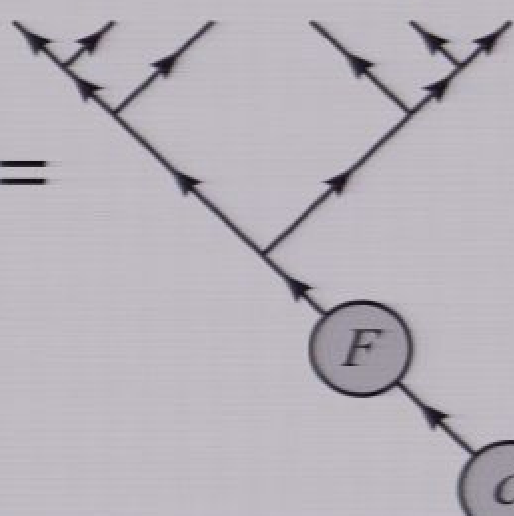
c^{α}

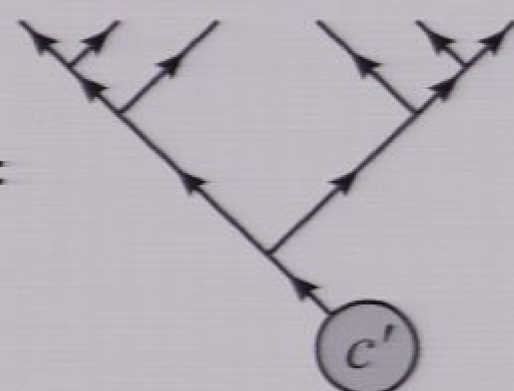
M^{α}_{β}

Tensor Networks and Anyons

$|\psi\rangle =$ 

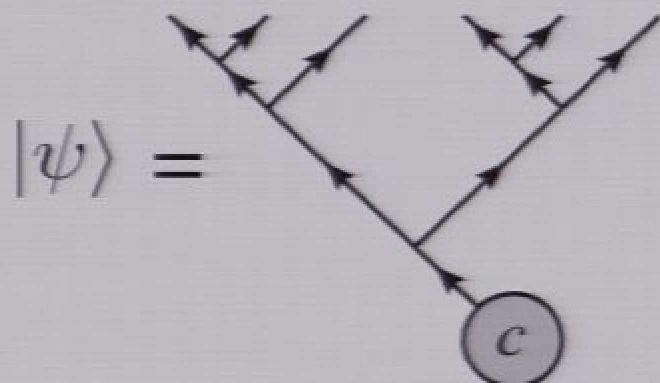
$\hat{M} =$ 

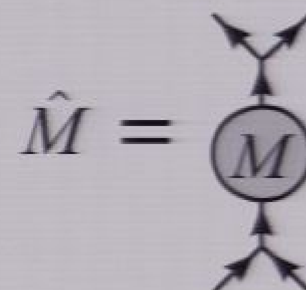
$|\psi\rangle =$ 

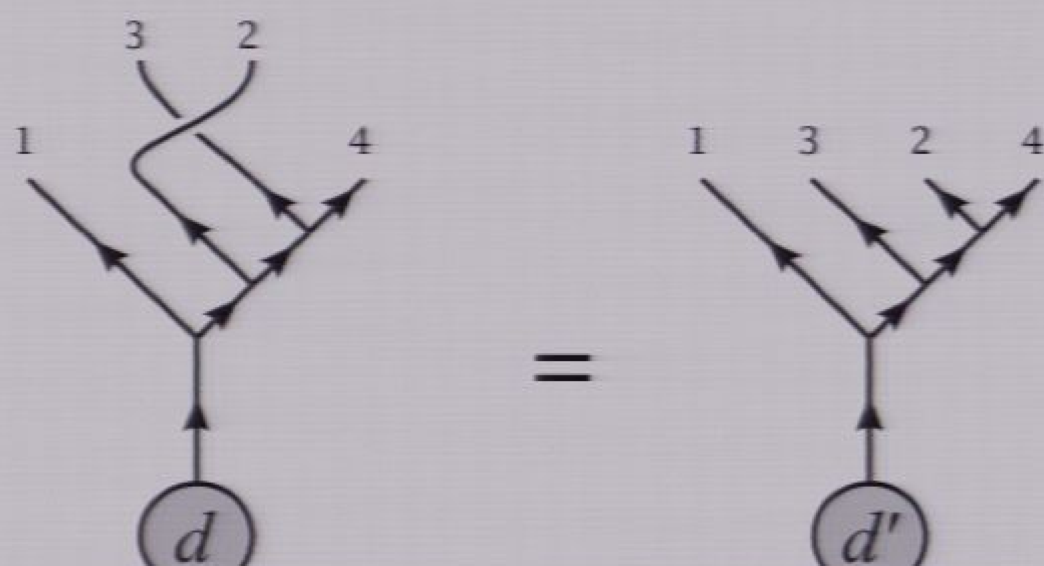
$=$ 

$$c'^{\alpha} = \sum_{\beta} c^{\beta} F^{\alpha}_{\beta}$$

Tensor Networks and Anyons

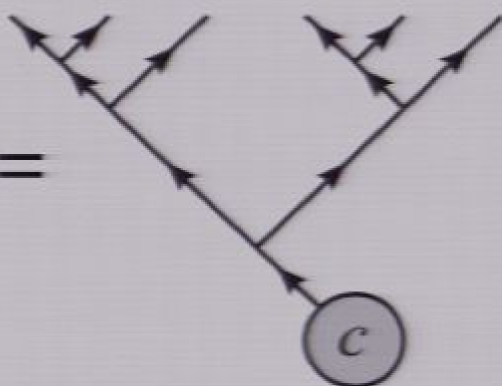






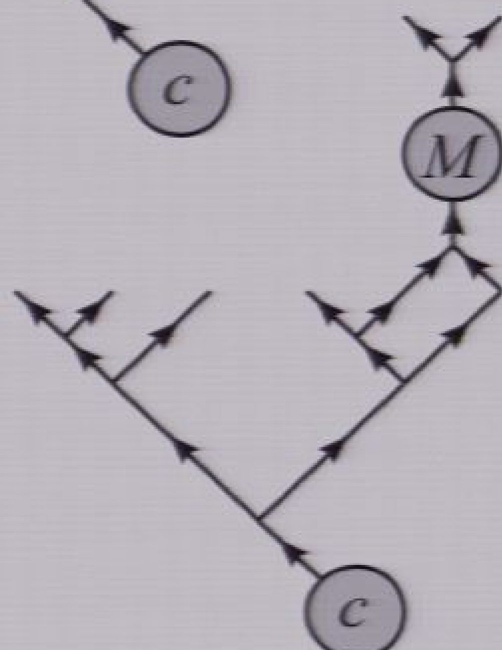
Tensor Networks and Anyons

$|\psi\rangle =$

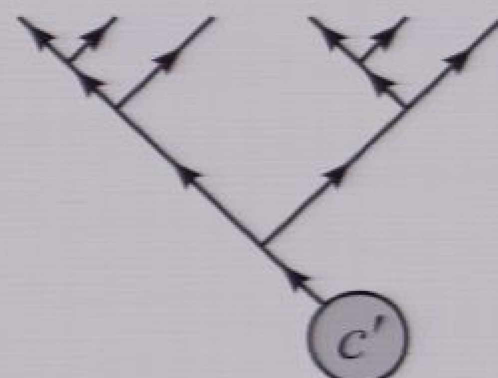


$\hat{M} =$

$\hat{M}|\psi\rangle =$



$=$

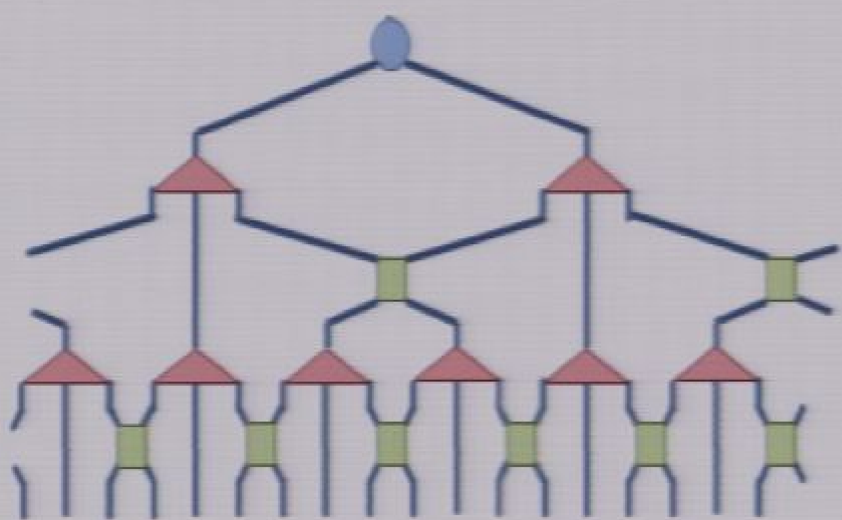


Anyonic Tensor Networks

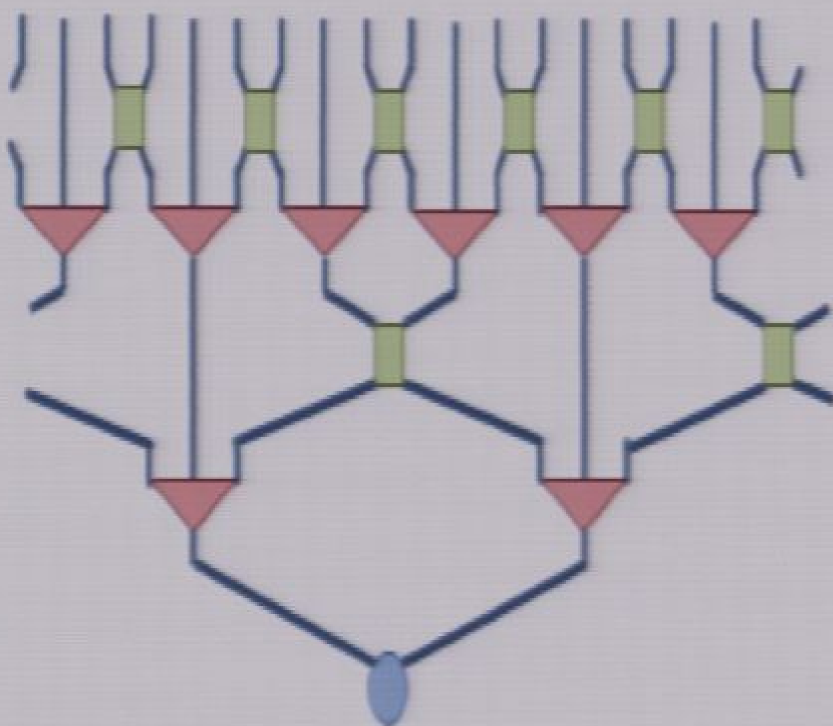
- Anyonic states and operators
  - Allows writing of an anyonic TN state
- Rules for manipulating anyonic trees
  - Allows translation of algorithms

Anyonic MERA!

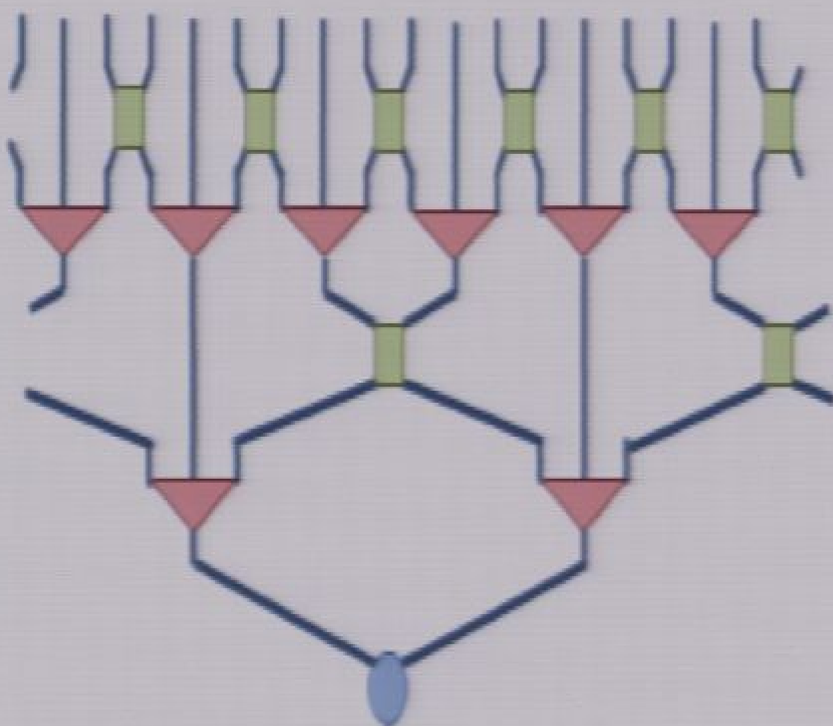
Anyonic MERA!

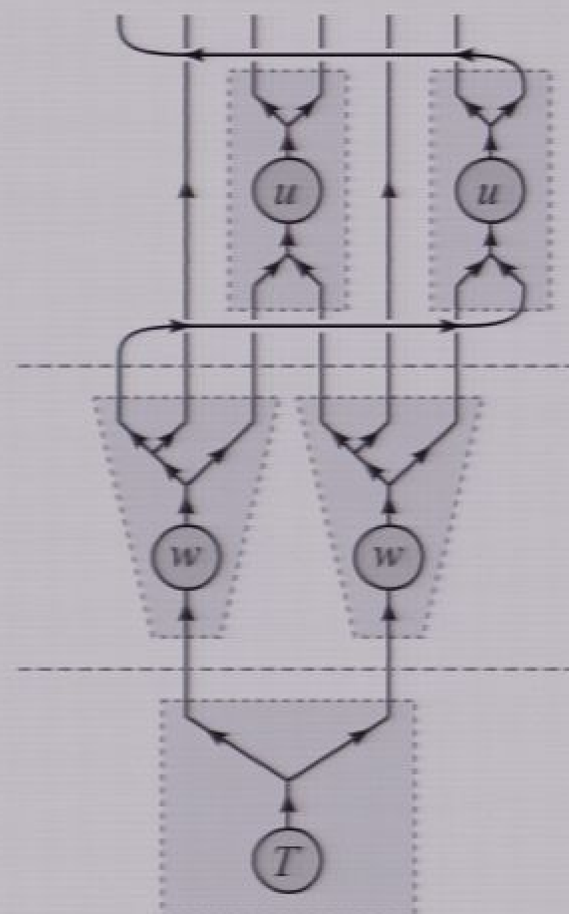


Anyonic MERA!



Anyonic MERA!

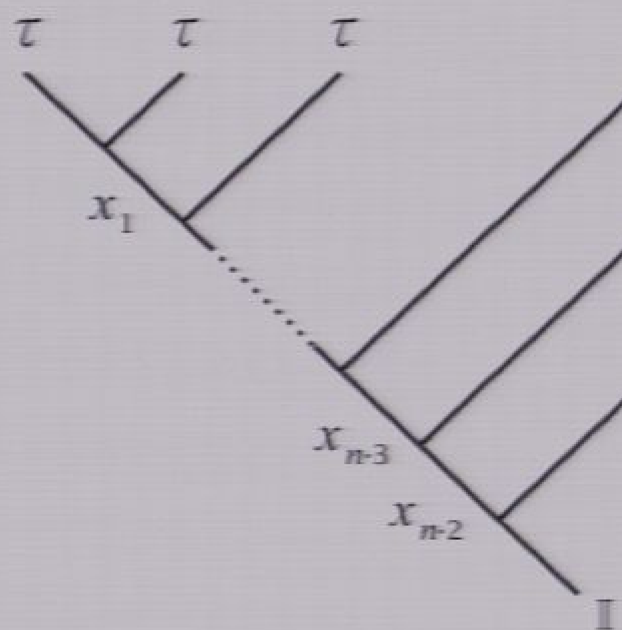


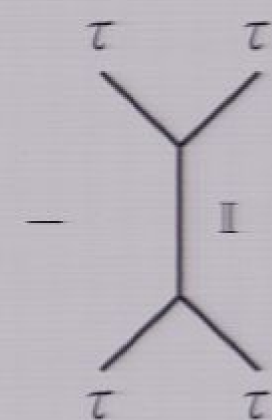


Golden Chain

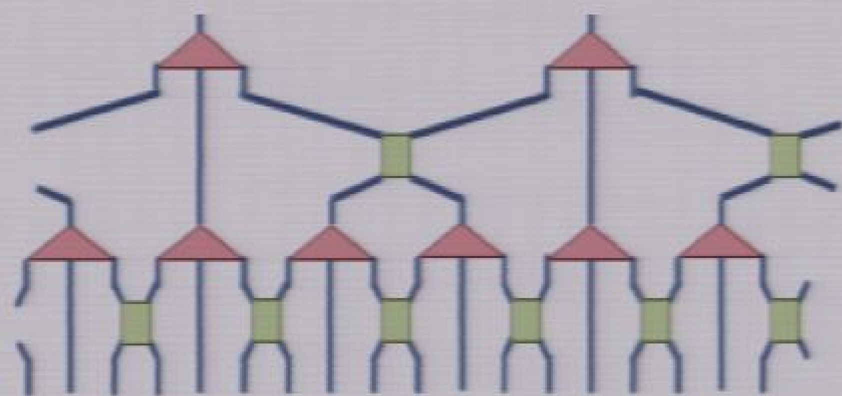
$$1 \times 1 \to 1 \qquad \tau \times 1 \to \tau$$

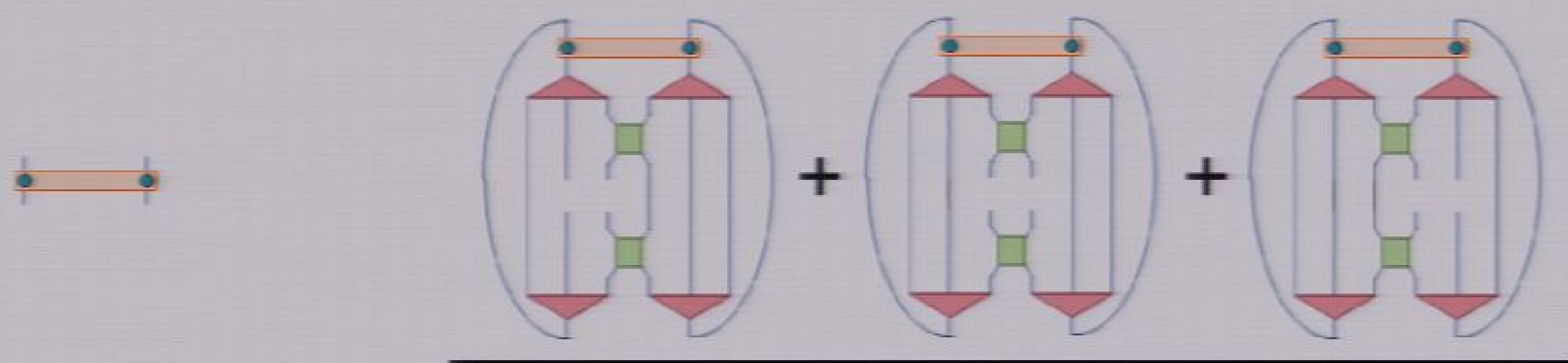
$$1 \times \tau \to \tau \qquad \tau \times \tau \to 1 + \tau$$





Scale-Invariant MERA





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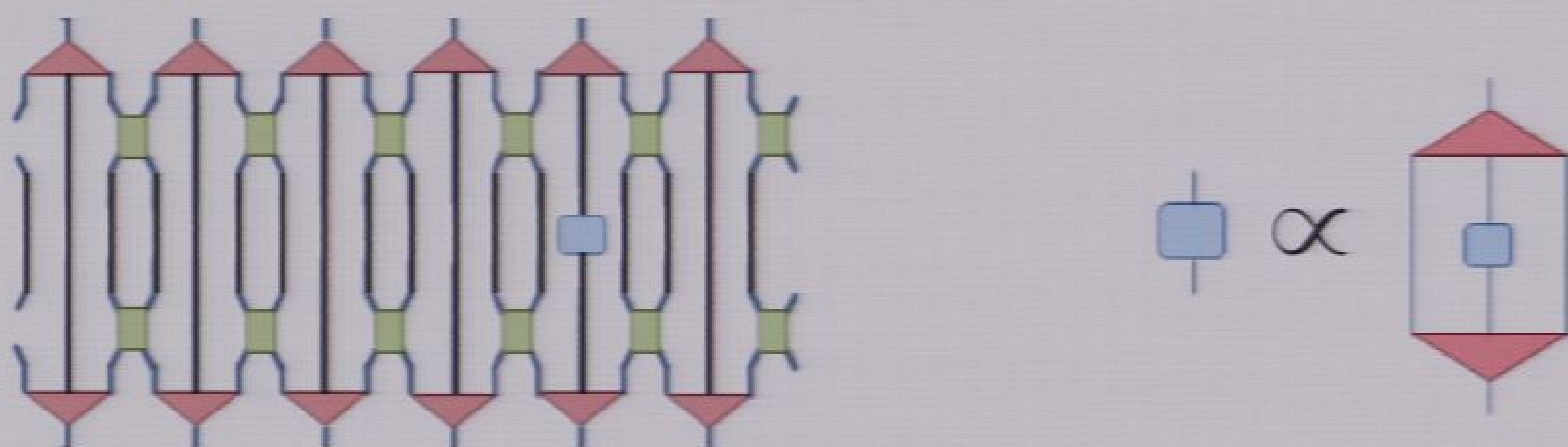
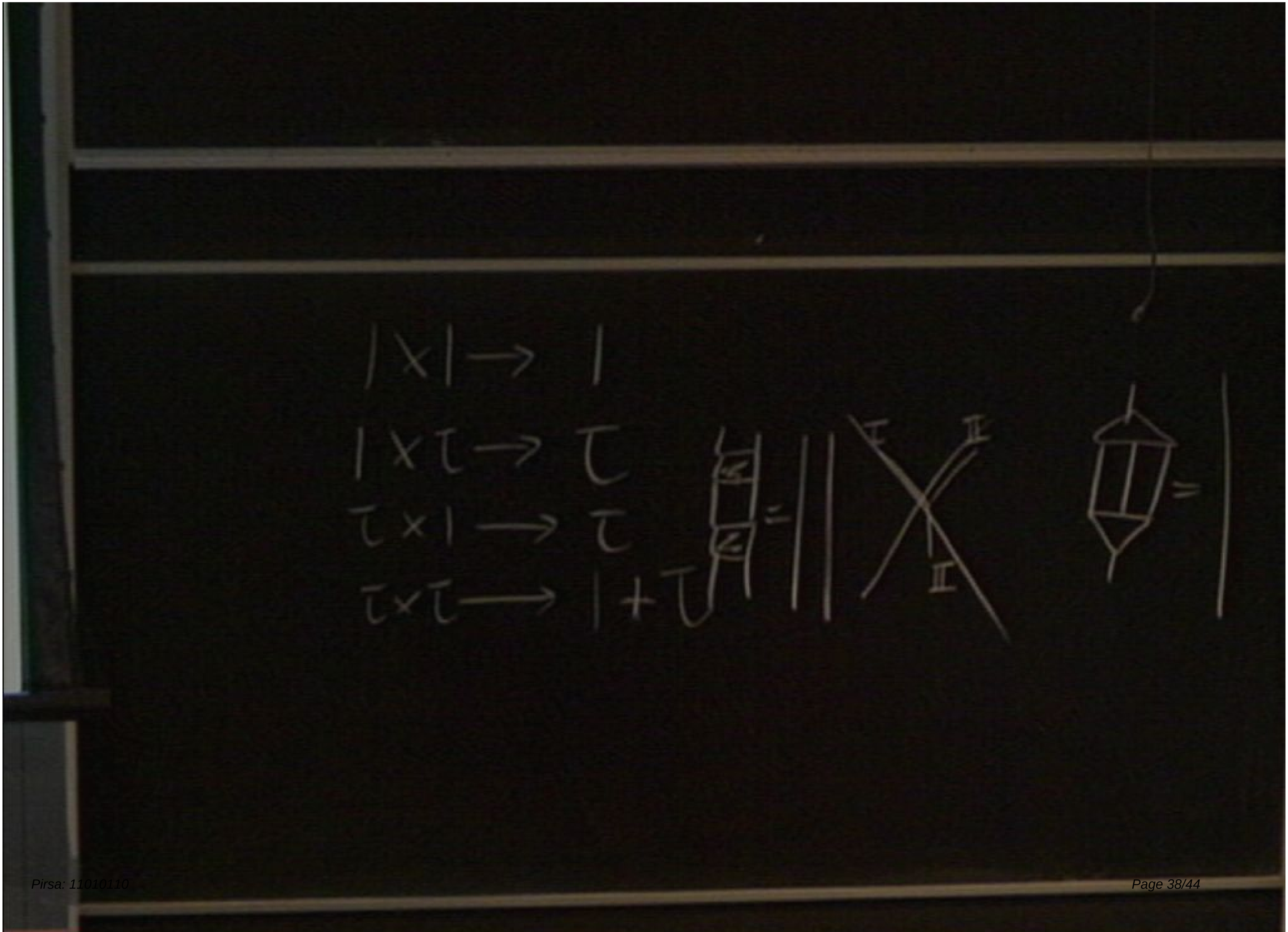
Scaling Operators




1×1 → 1
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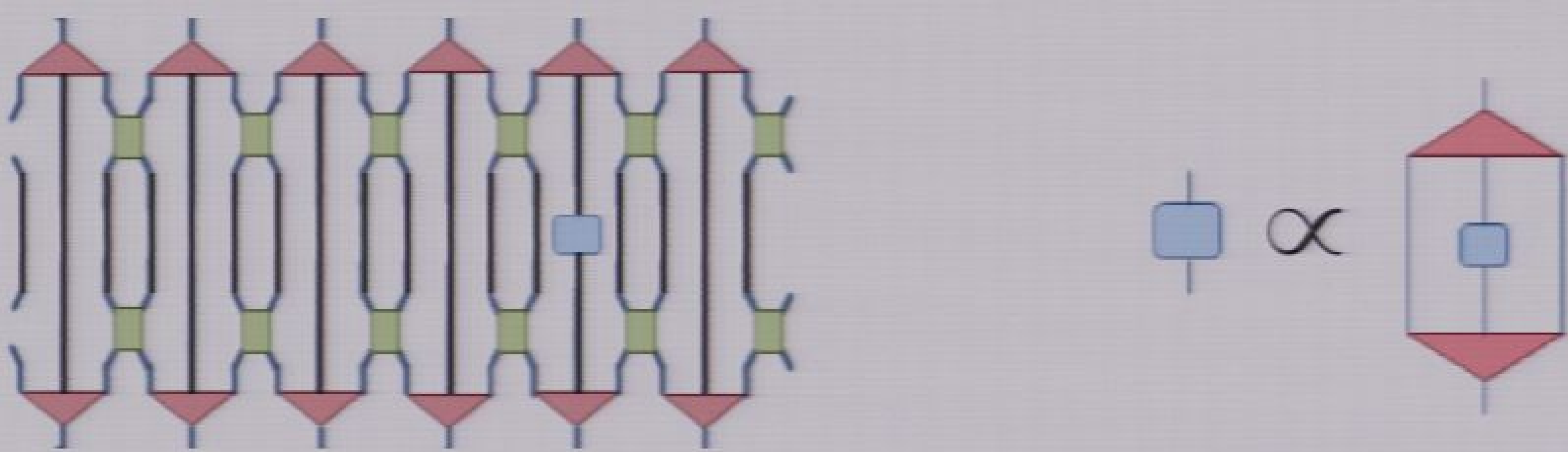
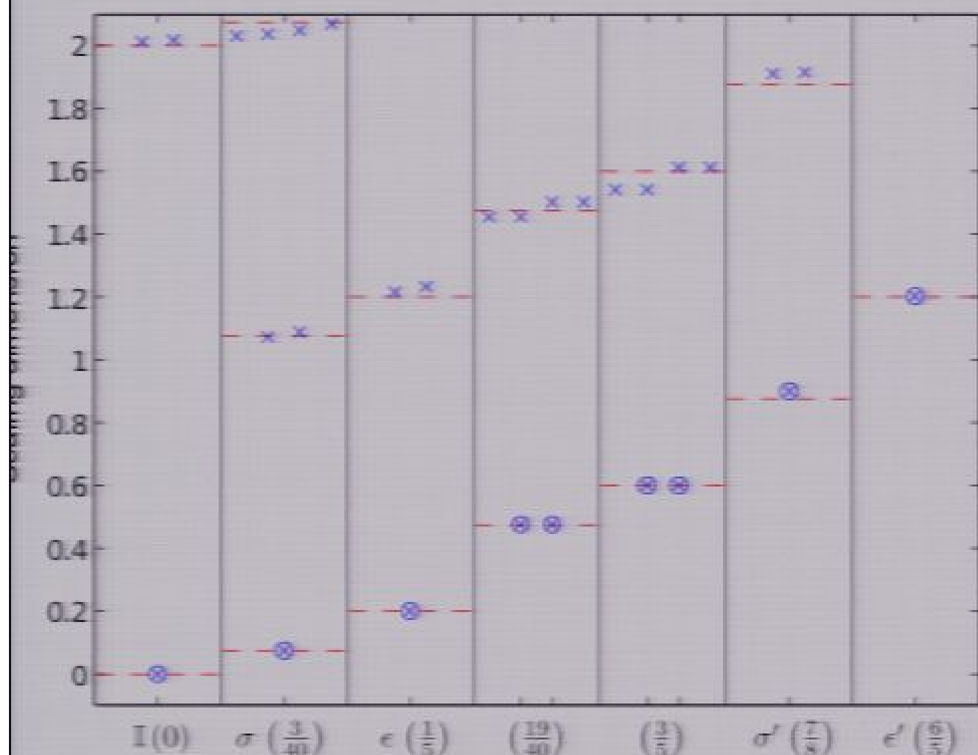
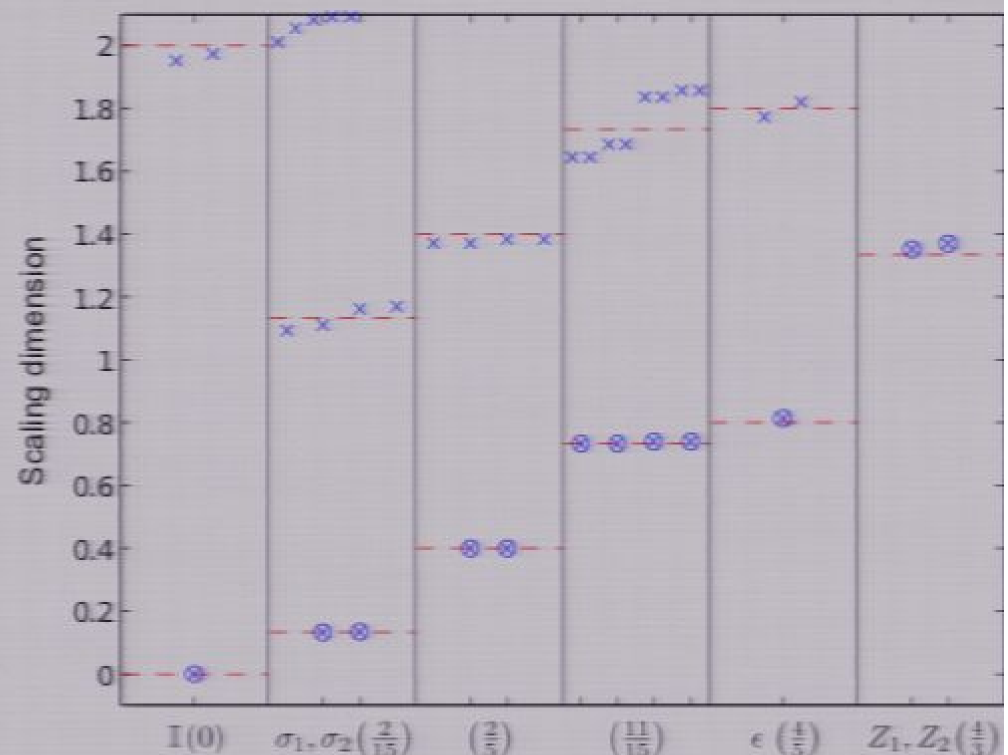
Scaling Operators



Results



AFM Golden Chain



FM Golden Chain

Conclusions

- Anyonic tensor networks:
  - possible
  - useful
  - efficient
  - shown to work!

Conclusions

- The microscopic behaviours of anyonic systems are still largely unstudied.

  Tensor networks for anyonic systems make their study possible.

  With these tools, there is a great deal of research to be done!